

FACTORS AFFECTING VARIATION IN  
GROWTH OF CHANNEL CATFISH FINGERLINGS  
AND MATHEMATICAL MODELLING OF  
THEIR POPULATION STRUCTURES

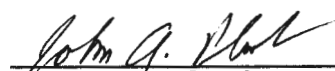
Valentin Abe


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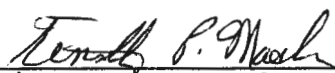
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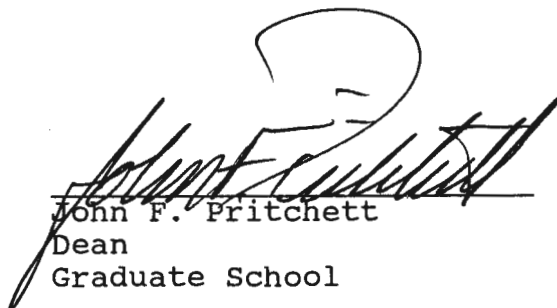
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FACTORS AFFECTING VARIATION IN GROWTH OF CHANNEL CATFISH  
FINGERLINGS AND MATHEMATICAL MODELLING OF THEIR  
POPULATION STRUCTURES

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DISSERTATION ABSTRACT  
FACTORS AFFECTING VARIATION IN GROWTH OF CHANNEL CATFISH  
FINGERLINGS AND MATHEMATICAL MODELLING OF  
THEIR POPULATION STRUCTURES

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A partial differential equation was used to predict population structures in channel catfish, Ictalurus punctatus, fingerlings stocked at low (150,000/ha), medium (300,000/ha), or high (600,000/ha) densities in earthen ponds. An exponential growth rate function incorporated into the model gave better prediction than a constant mean growth rate at low and medium densities. However, both expressions of the growth rate could be used at high fish density without affecting the solutions of the model. Mortality did not have substantial effect on the solutions of the model at either of the three densities.

The components of the total variance in size in channel catfish fingerlings were also determined using populations

cultured in tanks. After a 90 day culture period, 86% of the total variance for length and 65% of the total variance in weight could be explained by conditions of the environment. A variance model was used to describe the rate of increase in the coefficient of variation (CV). Initial stocking density and time had substantial impact on the rate of increase in the CV. However, fish size did not account for the increase in CV over time. The variance model suggested an asymptotic CV for each population, a value of which was a function of the environment in which the fingerlings were cultured.

Mortality was predicted using variations of the length-based method. The traditional method underestimated mortality rates. However, using the descending limb above the modal length improved the prediction of mortality.

Growth rate of channel catfish fingerlings was related to feeding levels and stocking density. In 1992, growth rate was similar at 150,000/ha and 300,000/ha, but greatly reduced at 600,000/ha. Increasing the stocking density from 150,000 to 300,000/ha resulted in 50% more fish harvested and a yield of 2,745 kg/ha rather than 1,809 kg/ha. However, increasing the density from 300,000 to 600,000/ha resulted in only 9% higher fish harvest and lower yield (1,213 kg/ha). Partial harvest increased yield and mean size but not survival.

## ACKNOWLEDGEMENTS

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## I. INTRODUCTION

In the United States, catfish farming has become the largest aquacultural industry. Commercial production increased from 12 million kilograms of farm-raised channel catfish Ictalurus punctatus in the 1970s to about 276 million kilograms in 1989 (Engle et al. 1989; Masser et al. 1991).

The ability to forecast production and population structures in fingerling culture is important in commercial aquaculture. As the catfish market requires a more standardized size fish, knowledge of the factors that contribute to size variation and to the prediction of size structure can aid in the development of feeding and harvesting policies, the assessment of operating conditions, and the evaluation of economic aspects of food-size fish and fingerling production.

Stochastic models coupled with growth models, can be used to predict size structures of fish populations (Ricker 1979). Several types of stochastic models, such as the Sinko-Streifer partial differential equation and population balance models, have been used to describe and predict the structure of distinct populations and the associated growth rate of individuals within them (Sinko and Streifer 1967;



DeAngelis and Coutant 1979; Thompson and Cauley 1979; DeAngelis and Huston 1987).

Variance models describe the increase in variation in size with time and provide additional information, which otherwise, could not be perceived by simple observation of size-frequency distribution charts (Arnasson et al. 1992). It is also well documented that environmental, behavioral, and genetic factors play an important role in growth variation. Variance models help test and quantify the effects of those factors and their interactions.

Studies leading to the development of population balance and variance models are needed to better understand the effects of environmental, behavioral, and genetic factors on growth and production of channel catfish fingerlings with the ultimate goal of optimizing production while minimizing operating cost.

The major objectives of this series of studies were:

1. Develop a mathematical model to predict the population structure of channel catfish fingerlings in ponds
2. Determine the relative importance of genetic and environmental factors on growth variability and the modelling of the coefficient of variation.
3. Evaluation of the effects of density, partial harvest and survival on catfish fingerling growth, yield, and size distribution.

## II. LITERATURE REVIEW

Mathematical models have been used to describe the effects of factors such as temperature, food, density, and size on fish growth in capture fisheries (Ricker 1979; DeAngelis and Coutant 1979) and more recently in aquaculture (Cuenco et al. 1985; Cacho 1990). In both areas, the models were either deterministic or stochastic.

Deterministic models are models in which the transition processes involved always give the same response for the same initial set of conditions (Kitchling 1983). In aquaculture, deterministic models are used to predict mean weight or length (Arnason et al. 1992).

Stochastic (or probabilistic) models define transition relationships in terms of probability where several responses may be obtained for the same process if different computations of the transition are possible, even with the same initial set of conditions (Kitchling 1983). Added to deterministic growth models, stochastic equations may be used to predict fish size structure (Arnason et al. 1992).

Generally three parameters are used to describe size structure in a population: the coefficient of variation (CV), the skewness, and the kurtosis. The CV is a dimensionless quantity which describes variability in

measurements about their mean within and between populations. CVs have proven to be a convenient measure of uniformity of size in cohorts of animals (DeAngelis and Coutant 1979; Thompson and Cauley 1979), and plants (Hara 1984; DeAngelis and Huston 1987).

Recent studies have focused on the use of CV in models of genetic and fish culture experiments as well as in commercial production with the purpose of providing a more comprehensive comparison among strains or culture conditions (Wismer et al. 1985; Arnason et al. 1992). It is often difficult to separate between the confounding effects of genetic differences and other factors in promoting heterogeneity in growth within and between fish populations. However, some models including CV can help elucidate these confounding effects (Arnason et al. 1992).

In general the magnitude of the CV for linear, surface and volume measurements of the same population are often different. Yablokov (1974) reported that CV for measurements of body length are smaller than those of body weights in 218 of 221 cases studied. Lande (1977) suggested a ratio of approximately 1:3 between CV measurements for length and weight for the same population. It is, therefore, important when comparing CV across growth studies that the same units of measurement be used.

What magnitude of CV should a set of measurements have for a population to be regarded as uniform? Several authors

considered CV below 10% to be small and values above 20% large (Lande 1977; Arnason et al. 1992). This infers that a population with a CV below 10% is considered uniform while a population with CV values above 20% is heterogeneous. A population may be uniform in body length and heterogeneous in body weight.

Consideration of Skewness shows the symmetry of size distribution around the mean. For a symmetric distribution, the median and the mean coincide and the skewness value is zero. When the number of large individuals is less than the number of individuals having size equal to the mean of the population, the distribution is skewed to the left or negatively skewed. In this case, the mean is less than the median. Inversely, when the number of large individuals is more than the number of individuals having the mean size, the distribution is skewed to the right, or positively skewed. McNew and Summerfelt (1978) reported a wide range of coefficient of skewness from -1.77 to 1.88 in 10 populations of largemouth bass Micropterus salmoides which length-frequency distributions at age were determined by scale reading. In aquaculture, McGinty (1980) found that the skewness of channel catfish fingerling populations were affected by the form of the diet they receive. Fish fed pelleted feed were positively skewed while their counterparts fed with crumbled pellet were negatively

skewed. Wohlfarth (1977) working extensively with carps came to the following conclusions:

1. The quantity and quality of food are primary factors in determining skewness. Limited food environment produces highly skewed populations.
2. If larger individuals are removed from a population and the remainder are allowed to grow, then frequency distribution becomes skewed again. This secondary skewness is a negative function of size and age.
3. Differences in stocking densities have no direct influence on skewness. Populations of different stocking densities with food levels proportional to the number of fish stocked may produce similar skewness coefficient.

Kurtosis measures the spread (peakness or flatness) of the distribution. When most of the data are squeezed into the middle of the distribution, the kurtosis has a positive value. A negative kurtosis suggests many extreme values that spread over a wide range. McNew and Summerfelt (1978) recorded kurtosis values ranging from 1.82 to 8.42 in largemouth bass populations.

Convenient statistics for comparing frequency distributions use normal distributions distributed approximately as  $\chi^2$ . The Pearson chi-square, usually known as chi-square, is the most widely used, especially when the

sample size is large (Steel and Torrie 1980). The Pearson chi-square is defined by:

$$\chi^2 = \sum_i \sum_j (O_{ij} - E_{ij})^2 / E_{ij}$$

where O and E are the observed and the expected frequencies of the considered parameters.

The continuity-adjusted chi-square and the likelihood-ratio chi-square are very similar to the Pearson chi-square and are more useful for smaller sample sizes. The continuity-adjusted chi-square, as the name suggests, is adjusted for the continuity of the  $\chi^2$  distribution and 0.5 is added to include end most observations. The continuity-adjusted chi-square is used only when the degree of freedom is equal to one. Likelihood-ratio chi-square is simply derived from a general test procedure and involves the ratios between the observed and the expected frequencies. Other statistics such as the Mantel-Haenszel chi-square and the Fisher's exact test are also used to test differences between frequency distributions, although rarely used in biology.

Population balance models are stochastic equations that describe the changes in population structures as a specific variable (or variables) changes. The population balance equation for a fish population has been expressed by the following formula (Thompson and Cauley 1979):

$$\delta N / \delta t + \delta (NG\phi) / \delta \phi + \delta (NG_L) / \delta L = B - D + Dg\delta^2 N / \delta L^2$$

Where  $N$  is the density function of fish number,  $t$  is time elapse,  $\phi$  is age,  $L$  is the length,  $G_L$  is the growth rate,  $B$  is the birth rate,  $D$  is the death rate, and  $D_g$  is the growth diffusivity. In fingerling populations, the birth rate could be ignored. Since most of the individuals are of the same age,  $\delta(NG\phi)d\phi$  could also be ignored. However, the solutions of the equation are complex because of the presence of the growth diffusivity function. Growth diffusivity is a function of random individual fish growth rates relative to the mean growth rate of the population from which they are drawn. The population balance equation developed by Thompson and Cauley (1979) is a variation of an earlier model developed and known as the Sinko-Streifer differential equation (Sinko and Streifer 1967):

$$\delta\eta(t, a, m)/\delta t + \delta\eta(t, a, m)/\delta a + \delta g(t, a, m)\eta(t, a, m)/\delta m = -D(t, a, m)\eta(t, a, m)$$

where  $\eta(t, a, m)$  is the density function, i.e. number of individuals at time  $t$ , between ages  $a$  and  $a + \Delta a$ , and of masses  $m$ ,  $g$  is the growth rate,  $D$  is the death rate.

Sinko and Streifer (1967) used the partial differential equation to describe dynamic size distribution in a population of Cladocerans, Daphnia pulex, through time. The main difference between the population balance equation and the Sinko-Streifer model is the presence of the growth diffusivity expression. The Sinko-Streifer equation is greatly simplified in cases where there is no mortality.

DeAngelis and Coutant (1979) and Wismer et al. (1985) used a partial differential equation similar to that of Sinko and Streifer to describe population growth rate and size distribution of different age groups of smallmouth bass Micropterus dolomieu. In their analysis, DeAngelis and Coutant (1979) were able to show that the average length of fish increased linearly with time during the first few weeks of their life. The variance associated to the length increased in a quadratic form with time when fingerlings had different growth rates. Wismer et al. (1985) used the same model to predict the length-frequency distribution of smallmouth bass in Lake Huron. They assumed a size-dependent growth rate within the population but no size-dependent mortality and concluded that size-dependent growth was very important in explaining the spread in the length-frequency distributions. On the other hand, they suggested that size-dependent mortality had absolutely no effect on the change in the distribution over time.

Thompson and Cauley (1979) applied their population balance model to a population of brown trout Salmo trutta. All individuals in the fish population described by Thompson and Cauley (1979) were of the same age which greatly simplified the solutions of the equations. Thompson and Cauley (1979) also addressed two major problems usually encountered when trying to predict the size distribution



within a population: the growth rate and the mortality rate.

They came to the following conclusions:

1. When all individuals in a size distribution grow at randomly different rates relative to the mean growth rate, the CV of the predicted population increases with time. However, if the growth rate is assumed identical for all individuals within the population, then the CV remains constant.
2. When mortality is assumed to be zero, the CV remains constant over time, and when mortality is different from zero, the CV changes over time.

Brett (1969) defined two groups of environmental factors, abiotic and biotic, that influence growth and variation in growth among individuals within the same population. The first group, the abiotic factors (temperature, light, seasons, oxygen, etc.) are usually less influential in themselves but strongly interact with some biotic factors such as food quantity and quality, density, size, and other social behaviours and genetic factors to foster growth. Among the abiotic factors, temperature is the most important factor that can influence growth of fish. Fry (1971) depicted temperature as the controlling factor which operates at all levels of the hierarchical arrangement of growth processes. Temperature generally governs the rate of metabolism (catabolic reactions) involving food requirements and food processing. Although, Stauffer (1973)

stressed the fact that any growth model should at least include temperature as an independent variable though less is known on the role or importance of this factor in explaining size variation within a population

While attempting to explain size distribution and variation in growth within a population, it is often difficult to separate the effect of food availability, density, space, and eventually size because of the multiple intercorrelation among the effects. Brett (1979) stressed the importance of biotic factors such as food quantity and quality on growth and growth variation in studies conducted in communal systems. He suggested that the quality of food was directly involved with the increase in weight and feed conversion, but only the quantity of the ration, coupled with some social interactions such as competition, can influence growth variability and subsequently, size differences within a population.

The competitive interaction for food caused by high population density and the resulting differential growth rate, or growth depensation, has been the subject of extensive speculation. It has been suggested (Weatherley 1972, 1976) that in high density populations, larger individuals which are dominant in the system may prevent the smaller ones access to the food source and therefore, grow faster while the smaller fish grow more slowly. As a

result, differential mortality occurs in the system thereby lowering the density of fish and reducing competition.

Hanes and Ciborowski (1991) studied the effect of density and food limitation on size variation and mortality in larval mayfly Hexagenia rigida (Ephemeroptera: Ephemeridae, Class: Insecta). They found that stocking densities and food levels have no effect on growth, survival, and structure at 30 and 60 days. However, densities and food levels significantly influenced growth, survival, and size structure at 90 and 120 days, with the lowest density and high food level combination giving the highest growth rate and lesser variation in size.

In channel catfish, few studies have demonstrated the importance of the genetic make-up on the variation in growth and the correlation between the genetic factor and the environmental factors. Can the growth patterns in fish be inherited? The answer has to be yes, though there may be no genetic variation in growth pattern which can be selected. It is known that fish can display intraspecific growth differences but how this relates to the intensity of competition for food has not received appropriate attention by biologists. Moav and Wohlfarth (1974) observed that initial size differences compounded with genetic differences were magnified over time as the fish grew and the final size differences were greater when the initial size differences were greater.

Weatherley and Rogers (1978) argued that only the upper limits of growth, which are species-specific in fish, could be inherited. Because these upper limits to growth are achieved during the early development stages, initial uniformity in size during the nursery phases cannot be improved through selection. Brown (1957) observed that growth in brown trout was related to egg size. Similarly, Smitherman and Duhnam (1985) suggested that initial variation in size in channel catfish was associated with egg quality and size. Individuals hatched from large and good quality eggs were larger. However, this competitive advantage is lost after the first month due to the influence of environmental factors (Reagan and Conley 1977). Fowler (1972) also came to a similar conclusion when studying the effect of egg size on growth and mortality of fingerling chinook salmon Oncorhynchus tshawytscha. He concluded that larger fry hatched from larger eggs but their size superiority lasted only 11 to 12 weeks. This trend also held true for brown trout (Brown 1957).

El-Ibiary et al. (1976) reported that male channel catfish grew faster than female. In an experiment where 50 fry were stocked in 3-m<sup>3</sup> fiberglass tanks and hand-fed a 40% commercial ration four times daily, they found that male catfish were 33% heavier and 9% longer after 14 months than females. Simco et al. (1989) showed that this differential

growth between sexes did not occur in ponds until the fish were 10 months old or weighed more than 50g.

Two types of analysis have been used to determine the proportion of the genetic and environmental components in the expression of the variance in size: the analysis of variance (ANOVA) and the analysis of covariance (ANCOVA). The analysis of variance is a statistical method which identify and measure sources of variation within a collection of data. Its main purpose is to partition the total variation of the data into its component parts. The analysis of covariance, on the other hand, takes into account the correlation between the different sources of variation (Kachigan 1986).

Using ANOVA, Wetherington et al. (1989) found that about 10% to 13% of the total variation in growth in fish hemiclones (Poeciliopsis: Poeciliidae) could be explained by genetic differences among strains within 60 days of culture. Hutchings (1991) used ANOVA to describe juvenile survival probabilities as a function of egg size. He found that the influence of egg size on juvenile survival in brook trout, Salvelinus fontinalis, ceased after 50 days of exogenous feeding and that maternal fitness had no effect on the individual growth of the juvenile after 50 days.

Arnason et al. (1992) argued that in growth variability analysis with multiple interactions among factors, an ANOVA would provide information only on the proportion of the

different components in the expression of the CV while an ANCOVA, not only would provide the same information, but also would help eliminate the confounding effects by using the factors that interact as covariates. As a result, group effects can be adjusted to account for differences among groups with regards to these factors. However, for the ANCOVA to be effective, the dependence of the CV on the covariates has to be correctly modelled.

A variance model is a general expression for a model that explains the change in variation in size within a population or among populations. Sparre (1976) first developed a variance model for fish with the same initial weight and grown under the same environmental conditions (temperature and feeding levels). He found that the growth conditions, temperature and feeding levels, or the initial weight have no effect on the prediction of the final variance-in-weight. Only the final weight was the contributor in the expression of the variance. Arnason et al. (1992) underlined the importance of a time factor in the expression of the variance. The inclusion of a time factor in a variance model explains the increase in variance when the environmental conditions and the mean size are constant.

Variance models have been used to predict and test for genetic and environmental effects on growth variation and to quantify such effects on variance over time. Using three feeding levels (0, 75, and 150% of table values), two

temperatures (10.5 and 14°C), and different genetic strains of rainbow trout Oncorhynchus mykiss and Arctic char Salvelinus alpinus, Arnason et al. (1992) were able to demonstrate that genetics had little influence on the rate of increase in variance-in-weight, but did have a strong influence on the increase in weight itself. Feeding rates were found to be a poor predictor of the variance-in-weight, probably because feeding rate is difficult to quantify in a meaningful way. On the other hand, temperature was found to be a stronger predictor of the CV.

Mathematical models have been used to predict or calculate growth and mortality rates in biological populations. In fisheries, those models often used size-frequency distributions or empirical relationships between length and age estimated from otoliths (MacDonald and Pitcher 1979). Ricker (1975) described a model that predicts mortality known as instantaneous mortality based upon evolution of a cohort over a period of time. Johnston and Mathias (1993) used Ricker's formula to calculate mortality rates in postlarval walleye, Stizostedion vitreum, populations in ponds with variable success. Another method of predicting mortality is the length-based method which uses the slope of the descending limb of the cumulative length-frequency distribution of the biological population (Smith and Richardson 1977; Pepin 1993). This method, unlike Ricker's formula which expresses mortality per unit

of time, calculates mortality as number per unit of length. The length-based method uses size-specific mortality and growth rates as vital assumptions and its accuracy depends upon the shape of the size distribution. Van Sickle (1977) discussed the importance of the shape of the size distribution in estimating mortalities. Using data from length-frequency distributions from Black abalone Haliotis cracherodii and Pismo clams Tivela stultorum, he showed that mortalities were greatly underestimated when the slope of the descending limb of the size distribution was not steep. On the other hand, dome-shaped size distributions with steeper slopes were most likely to give high mortality estimates.



### III. POSSIBLE APPLICATION OF PARTIAL DIFFERENTIAL EQUATIONS IN FORECASTING POPULATION STRUCTURES OF CHANNEL CATFISH FINGERLINGS IN PONDS

#### Introduction

Biological processes that regulate population structures in fish have been the subject of much speculation. Among these processes, growth and mortality are thought to be selective so that some individuals are more likely to survive or grow faster than others, based on their genetic, physical, behavioral and physiological characteristics. The selective forces are effective in shaping size structures within cohorts (Hutchinson 1978; Weatherley and Rogers 1978; McGinty 1980; and Rice et al. 1987).

The ability to forecast growth and population structure of fish is important in commercial aquaculture. Knowing the number and range of sizes of fish that may be expected at any time during the growing period would allow refinement of feeding, pond management, and harvest practices.

Mathematical models have been developed in fisheries biology to describe growth rate and size distribution at age in natural environments (DeAngelis and Coutant 1979; Wismer

et al. 1985) and have been applied in aquaculture. Cuenco et al. (1985) discussed the importance of mathematical models to improve fish production. Fox and Flower (1990) used growth models to describe the effects of fish density on growth and survival of walleye, Stizostedion vitreum, in ponds. Cacho (1990) also developed a bioenergetical model that described the effect of fish weight, feeding levels and environmental temperature on fish growth and body composition and speculated upon the benefits of the models in aquacultural production. However, few reports on the use of mathematical models to predict size distribution and growth rate in fishpond have been published. Thompson and Cauley (1979) used a population balance equation to predict size structures in a brown trout, Salmo trutta, fingerling population. Ricker (1979) suggested a strong correlation between growth and size that could have some applications in the field of aquaculture.

In the present study, the possibility of using mathematical models to forecast population structures in channel catfish, Ictalurus punctatus, cultured in earthen ponds at different densities is evaluated. Our major goal is not to predict populations that are exactly identical to the actual fingerling populations but to construct size structures that are very close to those observed in ponds. The models used are derived from population structure based models previously used in modeling microbial growth,

particle size distributions in crystals, and fish (Von Foerster 1959; Sinko and Streifer 1969; Randolph and Larsen 1971; and Thompson and Cauley 1979). With adequate assumptions and accurate estimation of the different parameters, valuable information on the growth rate and size distribution can be developed and used in production plans.

The objectives of this study are to evaluate the use of partial differential equations in predicting population structure of channel catfish fingerlings in ponds. The sensitivity of the prediction to growth and mortality rates are also investigated.

### Materials and Methods

#### Experimental methods

The field experiment was conducted at the Fisheries Research Unit, Alabama Agricultural Experimental Station, Auburn University, Alabama. In 1992, 15-day old channel catfish fry were stocked in 12 ponds (0.04 ha) for 120 days. Fry were stocked at low density of 150,000/ha, medium density of 300,000/ha, or high density of 600,000/ha with four replicates per treatment.

In 1993, channel catfish fry were stocked at 300,000, and 600,000/ha with six replicates per density. In three of the medium density ponds, an estimated 10% and 3% (by weight) of the population were partially harvested at 60 and 90 days post stocking, respectively. An estimated 25% and

6% (by weight) of the population were also removed from three high density ponds at the same time intervals to simulate mortality.

In all ponds, fry were fed a 36% protein crumble catfish feed twice a day, increasing the rate to a maximum of 40 kg/ha/day during the experiment. The feeding level was identical in every pond regardless of the initial stocking density.

At the end of each 30-day culture period, a sample of 100 fingerlings was removed from each pond and individually weighed to the nearest gram and measured to the nearest millimeter total length. Ponds were harvest after 120 days. Total number of fish harvested was estimated by dividing the total harvest weight by the average weight.

#### Approach to modeling

A series of simulations were designed to examine the sensitivity of model to growth rates and mortality. In these simulations, each group of fish was assigned either a constant growth rate equivalent to the absolute mean growth during the 30-day time interval or a variable growth rate as an exponential function. Exponential growth has been observed in fingerlings of several species of fish (Thompson and Cauley 1979; Drew and Hecht 1994).

The model developed in this study is a variation of the partial differential equation known as the Von Foerster (1959) or the Sinko-Streifer's (1969) equation:

$$\delta N(l,t)/\delta t + \delta N(l,t)G(l,t)/\delta l = -Z(l,t)N(l,t) \quad (1)$$

where  $\delta N(l,t)/\delta t$  is the rate of change overtime in the number at length  $l$ ,  $\delta N(l,t)G(l,t)/\delta l$  is the net rate of change in the number at size  $l$  resulting from growth ( $G$ ) in size of individuals, and  $-Z(l,t)N(l,t)$  is the rate of decrease in number at size  $l$  due to mortality ( $Z$ ).  $N(l,t)$  is the density function of fish number at length  $l$  and time  $t$ ,  $G(l,t)$  is the growth function and  $Z(l,t)$  is the mortality function.

Although many experiments have been carried out to estimate survival and growth rates, very little data exist on the mortality distribution within a given population and the specific growth rate of each individual in ponds. Usually, what are known are a gross estimate of mortality, monthly average length and standard deviations, and monthly length-frequency distributions, as it is the case in the present study. Thus there are not sufficient data to specify all the terms in equation (1). To solve equation (1) the following assumptions are made and the sensitivity of the model to these assumptions are tested:

1. There is no mortality in the population. Equation (1) becomes:

$$\delta N(l,t)/\delta t + \delta N(l,t)G(l,t)/\delta l = 0 \quad (2)$$

2. There is either a constant growth rate for all individuals during time  $t + \Delta t$  ( $dG(l,t)/dt=c$ ) or a variable growth rate that is size and time dependent.
3. Finally, there is no fish having zero length ( $N(l,t)=0, l=0$ ).

The absolute growth rate equation, as defined by Ricker (1979), can be expressed by:

$$G=l_2-l_1/t_2-t_1 \quad (3)$$

where  $l_2$  and  $l_1$  are characteristic lengths at time  $t_2$  and  $t_1$ , respectively. A growth rate function that characterizes the dependence of growth to size and time can be represented by the general expression:

$$dl/dt=Kf(t) \quad (4)$$

with  $Kf(t)$  being always a positive expression.  $K$  is a constant that can be determined experimentally. In the present study,  $f(t)$  may be expressed in the form:

$$f(t)=e^{-\alpha t} \quad (5)$$

where  $\alpha$  is a positive constant. After equation (5) is incorporated into (4), the following result is obtained after integration:

$$l = l_0 \exp\{(-K/a)(e^{-\alpha t} - e^{-\alpha t_0})\} \quad (6)$$

where  $l_0$  is the initial length at time  $t_0$  that is the time at which the initial size distribution is known. The growth rate function  $G(l,t)$  is found by combining equations (5) and (4):

$$G(l,t)=dl/dt=Kle^{-\alpha t} \quad (7)$$

Equation (7) indicates that growth rate is both size- and time-dependent. The different coefficients in the growth function were estimated by first determining the average length for each month. These average lengths were then plotted through the growing season and an exponential function fitting the plot was generated.

Either equation (3) or (7) can be incorporated into equation (1) to describe the length-frequency distribution of the channel catfish fingerlings at any time. The solutions of equation (1) are formulated by the initial and boundary conditions. The initial length  $l_0$  chosen to solve equation (1) is derived from the length-frequency distribution at 30 d. The length distribution at 30 d was chosen over the length distribution at stocking because fry were near uniform size at stocking and did not constitute a good baseline for the prediction.

A Pearson chi-square statistic ( $\chi^2$ ) was used to determine the goodness-of-fit of the modeled frequencies to the observed frequencies. Individual lengths were grouped into 10-millimeter size classes. Classes with less than five individuals were included with adjoining classes to meet the minimum required for a proper use of  $\chi^2$  (Steel and Torrie 1980).

The standard deviation,  $\sigma$ , is given by the expression:

$$\sigma^2 = \sigma_0^2 \exp\{(-2K/\alpha)(e^{-\alpha t} - e^{-\alpha t_0})\} \quad (8)$$

where  $\sigma$  is the standard deviation of the initial length-frequency distribution.

The coefficient of variation, CV, is expressed by:

$$CV = \sigma / \text{mean } l \quad (9)$$

The skewness is defined in terms of its third moment about the mean:

$$Sk = E(l - \text{mean } l)^3 / \sigma^3 = E(l_0 - \text{mean } l_0)^3 / \sigma_0^3 \quad (10)$$

and the kurtosis (or coefficient of kurtosis) is measured as the fourth moment about the mean:

$$K = [E(l - \text{mean } l)^4 / \sigma^4] - 3 = [E(l_0 - \text{mean } l_0)^4 / \sigma_0^4] - 3 \quad (11)$$

The effect of mortality on the prediction was also investigated by assuming a 20% mortality in the population at the end of the 30 d period. The choice of a 20% mortality was dictated by previous mortality rates observed in pond at the Alabama experiment Station, Auburn University (McGinty 1980; Phelps and DeGomez 1990). Mortality was assumed to be random with respect to length. The impact of this mortality was then assessed at 60, 90, and 120 d.

The model was solved numerically for each density using finite difference techniques. Most simulations were run with 400 individuals per stocking density. Simulations were run using 30-days intervals to increase the quality of the predictions. It has been shown that prediction using large time intervals usually poor goodness of fit (Randolph and Larsen 1971).



## Results

First, the 1992 fingerling data was used to build and test the model. To determine the effects of growth rate on the prediction, 18 simulations were run assuming no mortality. This assumption seems most appropriate based upon observation of the monthly plots of the length-frequency distribution since size-dependent mortality that impacts a length-frequency distribution can easily be detected on frequency plots as shown by Thompson and Cauley (1979). A change in the growth rate from an exponential function (for an individual fish) to a constant mean growth rate (entire population) resulted in higher  $\chi^2$  values at 150,000/ha and 300,000/ha but had no effect at 600,000/ha (Df = 395). Moreover, a constant mean growth shifted the predicted frequencies to the left (Fig. 1, 2 & 3).

The effects of mortality on the prediction of the size distribution were evaluated. Nine simulations were run over the 120 d period for each initial stocking density assuming a 20% randomly distributed with respect to length and assessed at the end of the first 30 d. The simulations were run using a variable growth rate. The inclusion of mortality values in the models had no substantial effects on the prediction at all densities. Values of  $\chi^2$  increased from 78 to 86 (Df = 394) at 60 d and from 83 to 91 (Df = 394) at 120 d at low densities. Similar magnitudes of increase were observed at the medium density. On the other

Figure 1. Predicted (Pred.) and observed (Obs.) length-frequency distributions of channel catfish fingerlings stocked at 150,000/ha and fed up to 40 kg/ha/day for 120 d. Predicted frequencies were generated assuming no mortality. (a): 60 d; (b): 90 d; and (c): 120 d.

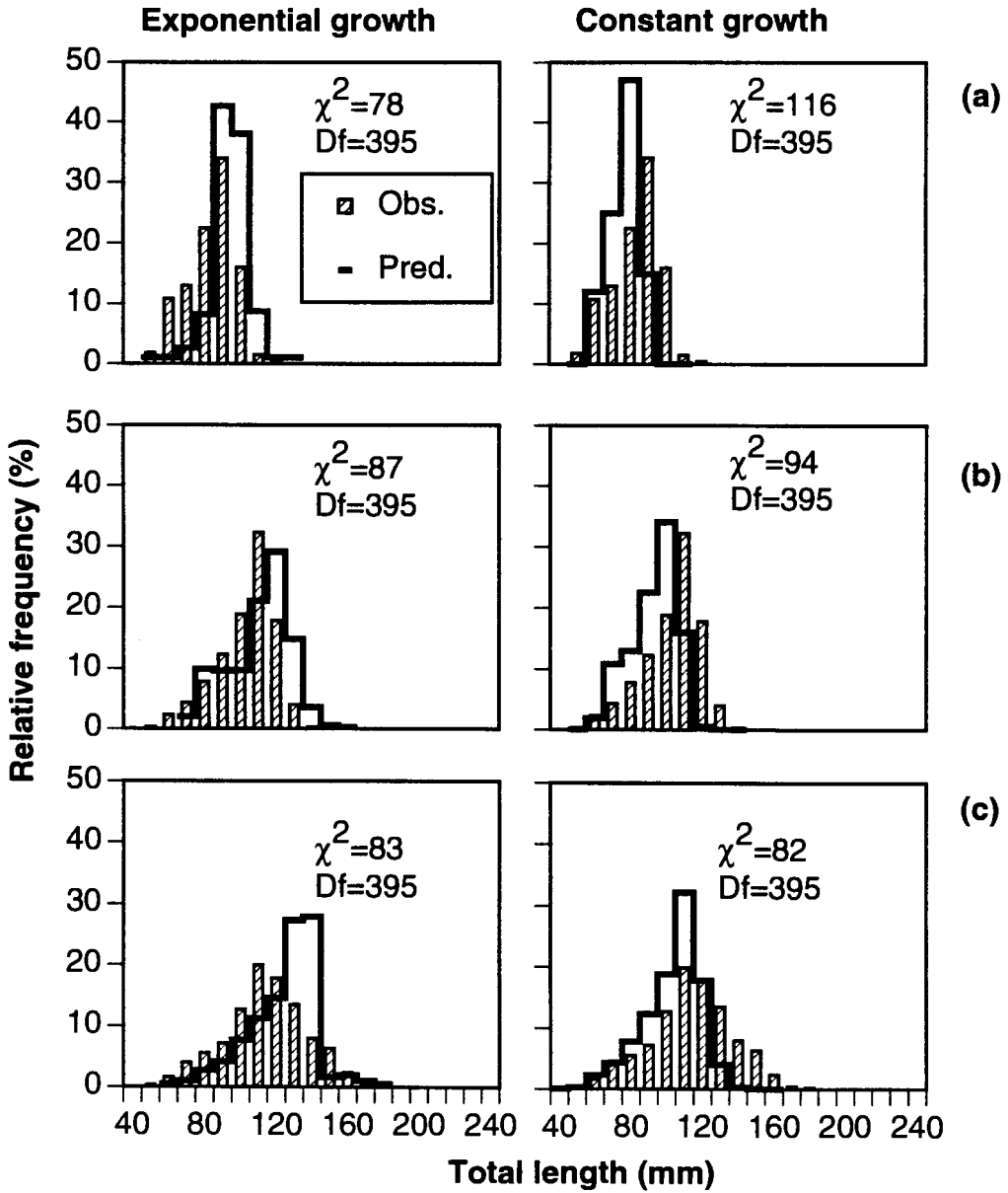


Figure 2. Predicted (Pred.) and observed (Obs.) length-frequency distributions of channel catfish fingerlings stocked at 300,000/ha and fed up 40 kg/ha/day for 120 d. Predicted frequencies were generated assuming no mortality. (a): 60 d; (b): 90 d; and (c): 120 d.

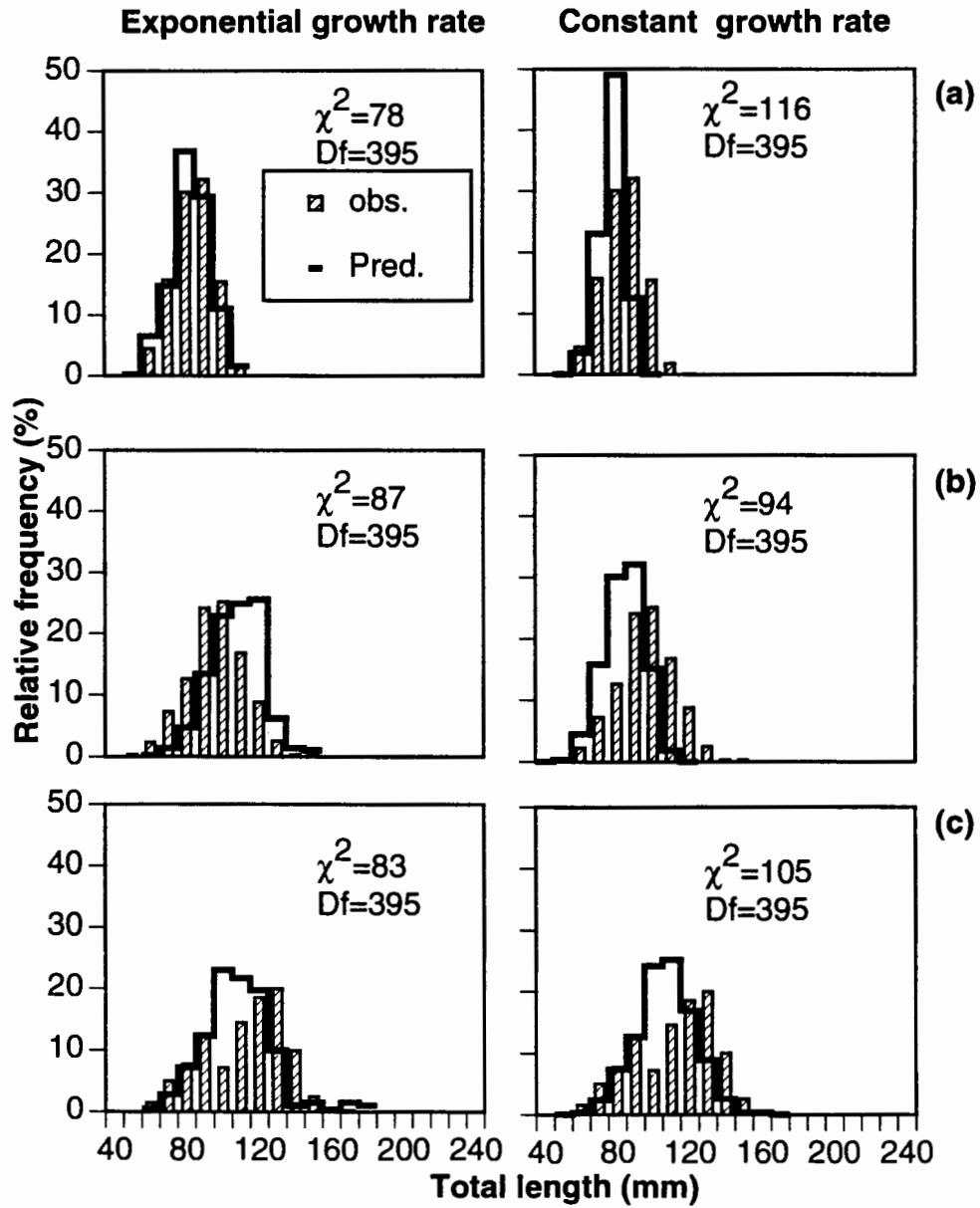
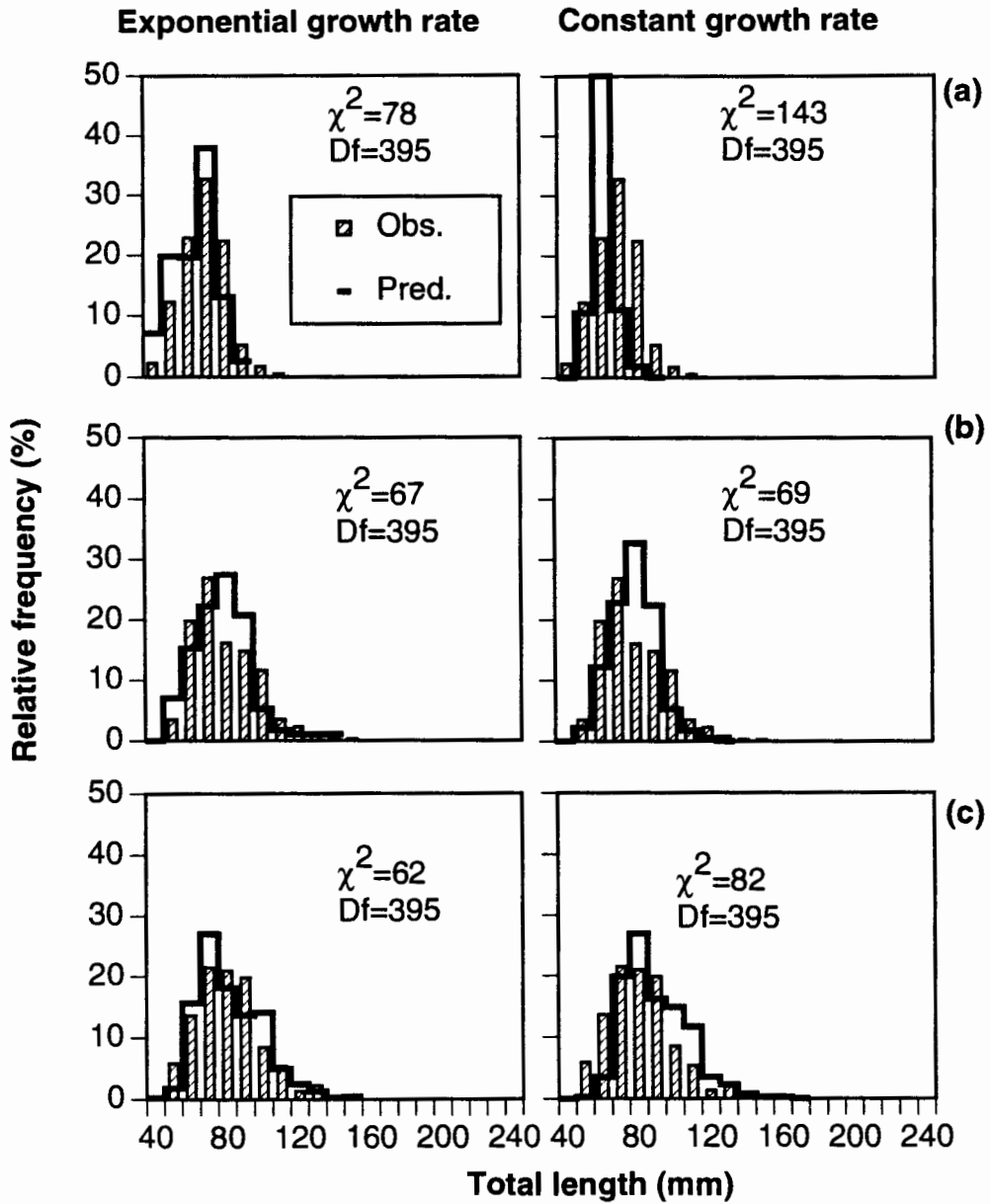


Figure 3. Predicted (Pred.) and observed (Obs.) length-frequency distributions of channel catfish fingerlings stocked at 600,000/ha and fed up to 40/ha/day for 120 d. Predicted frequencies were generated assuming no mortality. (a): 60 d; (b): 90 d; and (c): 120 d.



hand, higher magnitudes of increase in  $\chi^2$  were observed at high density with values increasing from 62 to 73 (Df = 394) at 120 d (Fig.4).

Second, a sensitivity analysis was done on the model using different levels (or expressions) of growth and mortality. To determine the sensitivity of the model to growth rates, simulations were run assuming no mortality. Increasing both coefficients (K and  $\alpha$ ) in the growth function ( $G(l,t) = Kle^{-\alpha t}$ ) spread the predicted distributions over a wider range and increased the  $\chi^2$  statistics by two folds. Similarly, increasing K and decreasing  $\alpha$  also spread the predicted distributions. However, decreasing K and increasing  $\alpha$  not only narrowed the size range but also shifted the predicted distributions to the left. When different values of constant growth rate were introduced into the model, smaller values shifted the distributions to the right while larger values shifted those distributions to the left without affecting the size range.

Simulations were run using either a constant growth or an exponential growth with different levels of mortality. Increasing the mortality up to 60% had no effect on the predicted distributions. However, increasing mortality above 60% truncated the predicted distributions by removing individuals at both ends. The model was also sensitive to the initial size distribution. When applied to the initial size distribution at stocking where only two size classes



were present, it failed to provide size structures commonly observed in fingerling populations after 30 d.

The model was then applied to channel catfish fingerling data collected in 1993. Series of simulations were run with the partial harvest data assuming either an exponential or a constant mean growth rate. The removal of fingerlings during the partial harvest is technically equivalent to mortality. When the same growth rates used in 1992 were included in the model to predict the population structures in 1993,  $\chi^2$  values increased from 78 to 86 (Df = 394) at 60 d but increase drastically from 83 to 158 (Df = 394) at 120 d for the medium density of 300,000/ha for the exponential growth function. The  $\chi^2$  values were even higher with the constant growth rate increasing from 105 to 254 (Df = 394) at 120 d. Replacing the 1992 growth rate expressions by the actual growth (either exponential or constant) observed in 1993 generated predicted size distributions closer to the actual distributions observed in the ponds (Fig. 5 & 6).

The coefficient of variation (CV), skewness, and kurtosis remained constant between time intervals when no mortality was assumed. A growth function (exponential) tends to give higher mean values (Table 1). The introduction of the mortality term in the model also gave higher mean lengths when compared to the observed values.

Figure 4. Predicted (Pred.) and observed (Obs.) length-frequency distributions of channel catfish fingerlings stocked at three densities. Predicted frequencies were generated assuming a variable growth rate and a 20% mortality assessed at 30 d and assuming no additional mortality between 30 and 60 d (a), between 60 and 90 d (b) and between 90 and 120 d (c).

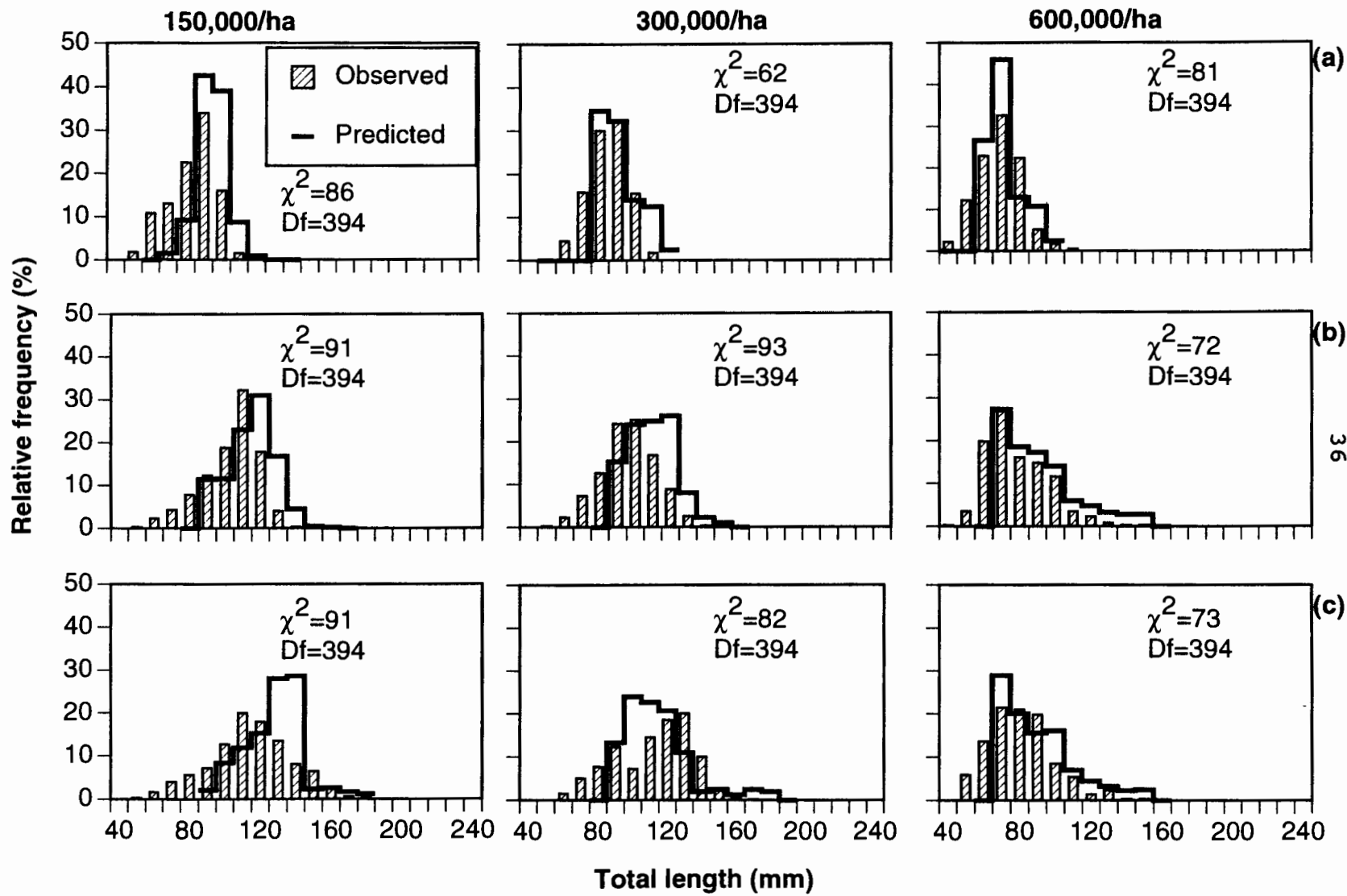


Figure 5. Predicted (Pred.) and observed (Obs.) length-frequency distributions of channel catfish fingerlings stocked at 300,000/ha and partially harvested at 60 and 90 d. (a): 60 d; (b): 90d; and (c): 120 d.

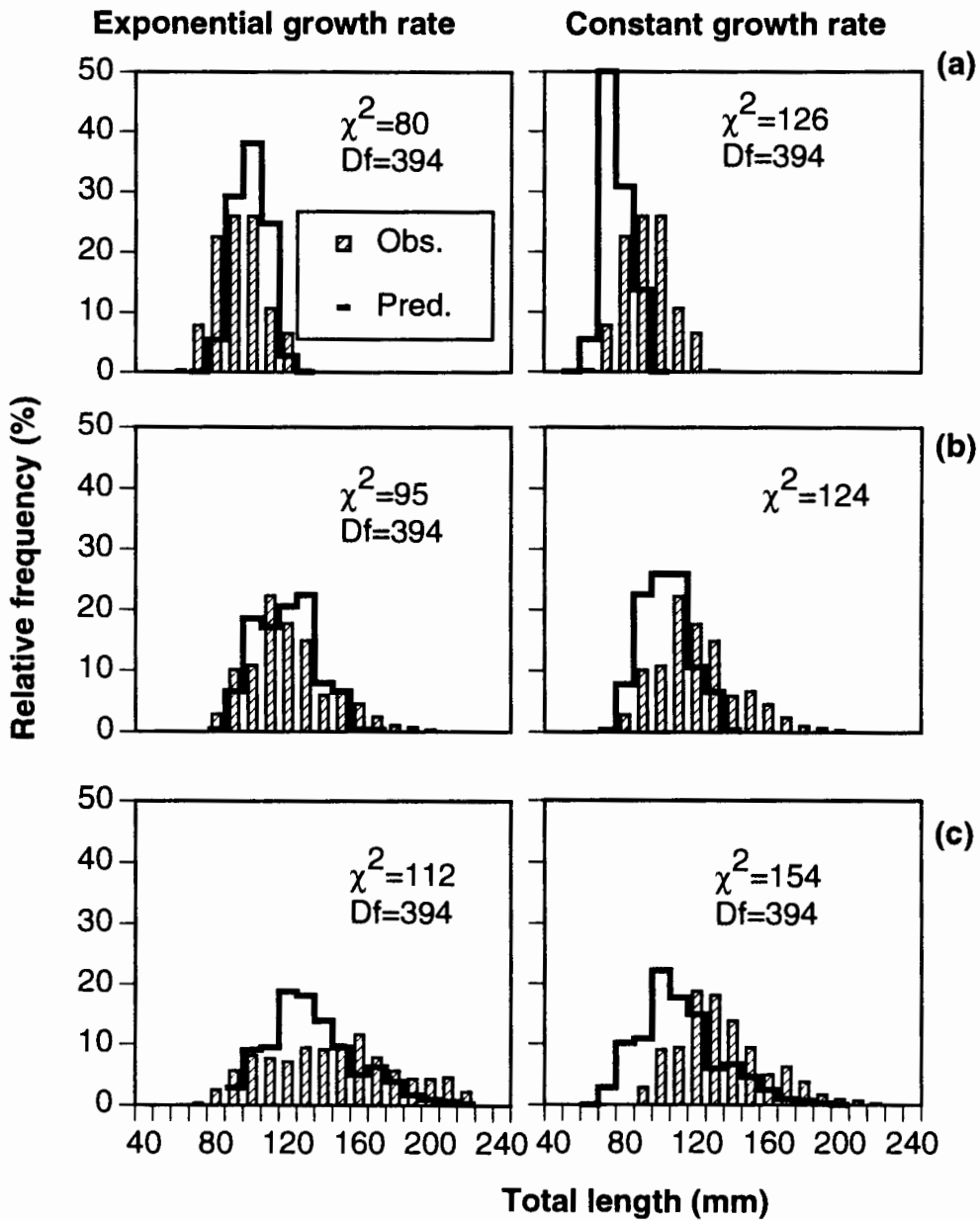
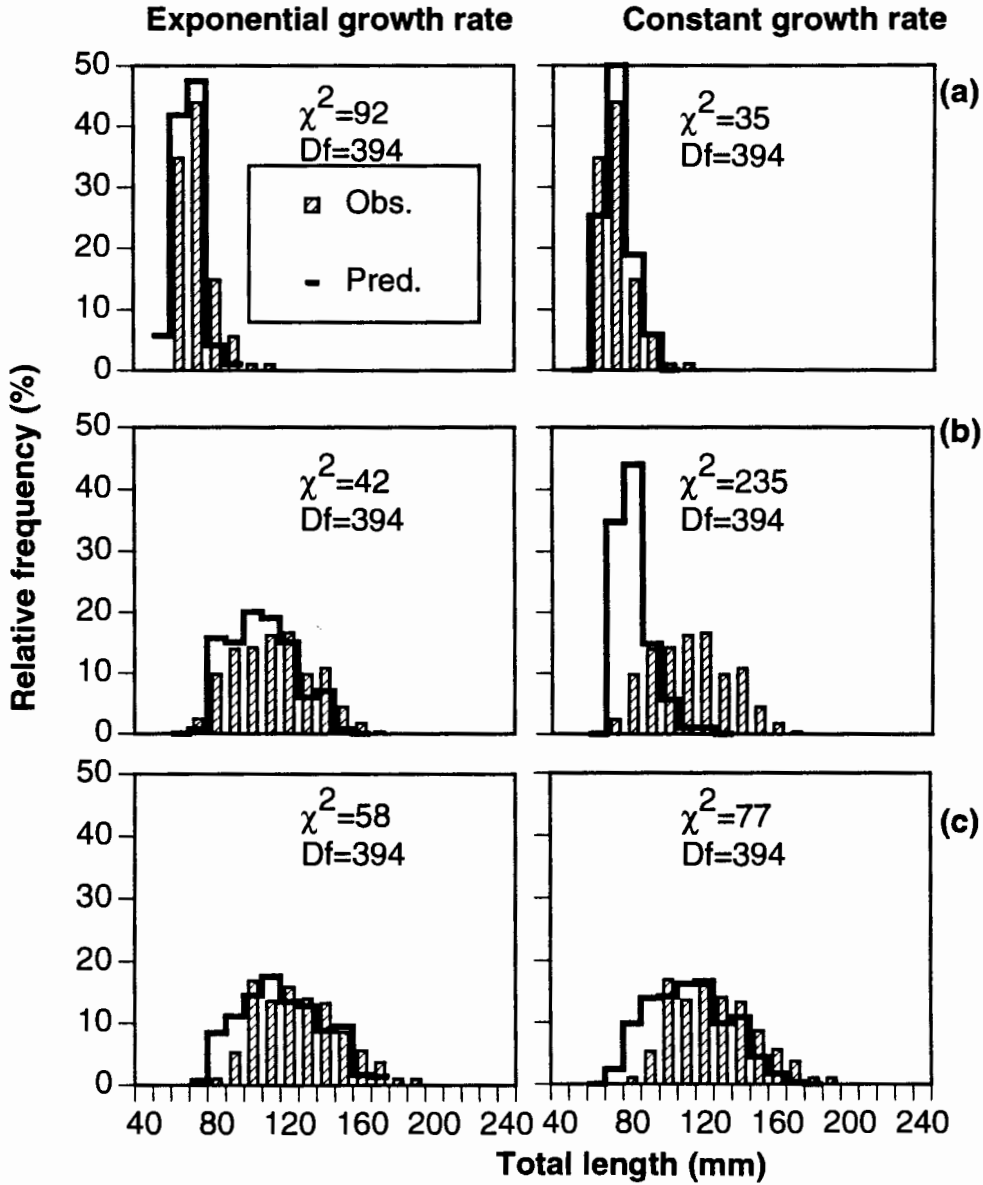


Figure 6. Predicted (Pred.) and observed (Obs.) length-frequency distributions of channel catfish fingerlings stocked at 600,000/ha and partially harvested at 60 and 90 d. (a): 60 d; (b): 90 d; and (c): 120 d.



However, CV, skewness, and kurtosis were lower than their respective observed values but increased with time (Table 2).

### Discussion

The solutions of the models clearly show that partial differential equations can be used to predict size distributions with results close to actual size structures in aquaculture ponds. Those results also suggest that growth rate, more than mortality has a greater impact on the accuracy of the prediction of population structure in fingerlings in ponds. Similar conclusions were drawn by Wismer et al. (1985) on the impact of size dependent growth and size-dependent mortality on the prediction of length-frequency distribution of smallmouth bass. Indeed, discernable mortality that appears in the form of marked depressions on the length-frequency distribution or the absence of size classes of smaller fish has to be observed for a size-dependent mortality to substantially impact the prediction of the size structures. Such mortality is not observed in the present study where all size classes were basically represented in each length-frequency plot. This evidence that size-dependent mortality does not naturally occur in channel catfish population, at least after the first month of the growing season. McGinty (1980) observed



Table 1. Mean length, standard deviation (SD), coefficient of variation (CV), skewness (Sk), and kurtosis (Ku) of channel catfish fingerlings at 60, 90, and 120 d after stocking in ponds at three densities.

Parameters	Densities (fish/ha)					
	150,000		300,000		600,000	
	Obs.	Pred.*	Obs.	Pred.	Obs.	Pred.
<u>60 days</u>						
Mean length (mm)	88	103	88	97	73	68
SD	13.1	5.3	11.2	12.3	12.4	8.6
CV (%)	14.9	8.1	12.7	12.3	17.0	17.1
Sk	- 0.4	0.3	- 0.4	- 0.1	0.2	- 0.1
Ku	- 0.4	1.7	0.8	- 0.3	- 0.0	- 0.5
<u>90 days</u>						
Mean Length (mm)	107	116	101	109	83	75
SD	16.1	13.1	15.4	11.2	18.1	12.4
CV(%)	15.0	15.0	15.3	12.7	21.7	17.0
Sk	- 0.7	- 0.4	0.0	- 0.4	- 0.1	0.8
Ku	0.3	- 0.4	- 0.1	0.8	0.8	0.0
<u>120 day</u>						
<u>S</u> Mean length (mm)	118	119	116	114	85	87
SD	22.5	16.1	15.4	18.6	20.8	21.7
Sk	- 0.1	- 0.7	- 0.3	- 0.1	0.7	0.8
Ku	- 0.1	- 0.3	- 0.7	- 0.1	0.7	0.8

Obs. = Observed

Pred.= Predicted

\* Predicted values were generated assuming a variable growth rate and no mortality.

Table 2. Mean length, standard deviation (SD), coefficient of variation (CV), skewness (Sk), Kurtosis (Ku) of channel catfish fingerlings at 60, 90, and 120 and assuming a 20% mortality.

Parameters	Densities (fish/ha)					
	150,000		300,000		600,000	
	Obs.	Pred.*	Obs.	Pred.	Obs.	Pred.
	<u>60 days</u>					
Mean length (mm)	88	107	88	101	73	71
SD	13.1	4.9	11.3	4.2	12.3	8.1
CV(%)	14.9	10.9	12.7	8.4	13.1	8.2
Sk	- 0.4	0.0	- 0.4	0.4	0.7	0.0
Ku	- 0.4	0.5	0.8	- 0.8	0.4	- 0.4
	<u>90 days</u>					
Mean length (mm)	107	114	101	110	83	87
SD	16.1	9.1	15.4	9.3	18.0	12.0
Sk	- 0.7	0.0	0.0	0.5	0.9	1.0
Ku	0.3	- 0.1	- 0.1	- 0.1	0.8	1.1
	<u>120 days</u>					
Mean length (mm)	118	126	116	118	85	86
SD	22.5	15.5	21.6	14.4	17.6	17.7
CV(%)	19.3	15.0	18.6	19.6	20.8	19.7
Sk	- 0.1	0.4	- 0.3	0.5	0.7	0.9
Ku	- 0.1	0.2	- 0.7	0.1	0.7	1.1

Obs. = Observed

Pred.= Predicted

\* Predicted values were generated assuming a variable growth rate and a 20% mortality assessed at the end of first 30-day period.

a size-dependent mortality due to predation in 3-day old channel catfish fry stocked in earthen ponds treated with diesel fuel.

The solutions of the models also suggest that channel catfish fingerlings could have different expressions of the growth rate dependent upon the culture conditions. Clearly, it is not realistic for all individuals within a population to have the same growth rate throughout a culture period. Individual fish would have growth rates that vary from day to day according to their size, density, and other environmental conditions such as feed quantity and quality and water quality (Rice et al. 1993; Ricker 1979). The prediction of channel catfish fingerling populations at different time intervals while assuming a constant growth rate would be a rather conservative approach relative to the more realistic assumption that juvenile fish grow at randomly different rates. This is seen at low and medium densities where the predicted frequencies are shifted to the left comparative to the observed frequencies and a better fit is observed with the exponential growth rate (Fig. 1 & 2). However, either a constant growth or a variable growth could be used at high density without affecting the solutions of the models. The use of a constant growth at high density does not by any means downgrade the assertion that growth is size-dependent. At high density, larger fish may still grow at different rate (or faster) than the

smaller fish but their proportion in the population is so small that they have very little impact on the overall growth rate of the entire population.

The results of the simulations ran for the 1993 data validate the use of partial differential equations to predict size structure in population with high mortality. However, growth expressions have to be adjusted to accommodate the mortality rates. This confirms the assertion that growth expressions, more than mortality, have greater impact on the solutions of the model.

Although, the predicted frequencies are close to the observed frequencies which make this model a good tool in projecting length structures of the fingerling populations, it failed to provide an overall accurate prediction of the population structure parameters. To support this quantitatively, we compared the mean length and standard deviations of the model and the field data. The model showed an increase in the mean length but the standard deviations did not increase proportionally. Moreover, the CV, skewness, and kurtosis remained constant between two sample dates although they were expected to change over time. It is possible that an uneven mortality had occurred within the population. However, this mortality was not severe enough to alter the length-frequency distribution and, therefore, had little impact on the prediction per se. Thompson and Cauley (1979) came to similar conclusions when

applying their population balance equation to actual brown trout data. It is likewise possible that mortality was evenly distributed over all size classes and the changes in CV, skewness and kurtosis are the results of real effects of factors such as feeding efficiency and density which the model does not account for.

### Conclusion

The present study showed that mathematical models previously used in fisheries biology can also be applied in aquaculture with much accuracy. The solutions of the partial differential equation allowed us to meet our goal that is to predict size structures close to those of the actual populations in pond. Poor agreement of the model to the increase in CV, and the change in skewness and kurtosis should not undermine the usefulness of the model in predicting size structure for management purposes. The solutions of the model also underline the correctness of size-dependent growth rate which help explain the spread of the length-frequency distribution with time.

IV. FACTORS AFFECTING SIZE VARIATION AND MODELING OF THE  
COEFFICIENT OF VARIATION AND MORTALITY IN CHANNEL  
CATFISH FINGERLING POPULATIONS

Introduction

Variability in fish growth is impacted by behavioral, environmental and genetic factors and their interactions (Weatherley and Rogers 1978; Brett 1979; Arnason et al. 1992). Understanding how those factors contribute to fish growth variability is one of the most important questions in commercial fish culture as variation in size often results in poor harvest and increased operating costs.

Several models have been introduced to predict variation in size in fish populations (Thompson and Cauley 1979; Arnason et al. 1992). However, very few addressed the important question relative to the effective interactions of factors that influence variation in size in controlled and uncontrolled environmental conditions and their relative importance. How much variation is due to the genetic make-up of the fish population, and what is the contribution of the environmental factors to variation are very important questions, answers to which should provide valuable

information for assessing stock performance, marketing, and production planning.

Moav and Wohlfarth (1974) observed that initial size differences confounded with genetic differences were magnified when initial size differences were greater in carps. Smitherman and Dunham (1985) found that initial size variation in channel catfish was associated with egg size and quality. Individuals hatched from large and good quality eggs were larger. However, this competitive advantage was lost after the first month because of environmental effects (Reagan and Conley 1977). Fowler (1972) came to a similar conclusion when studying the growth and mortality of fingerling chinook salmon as affected by egg size. He also concluded that larger fry hatched from larger eggs but the size superiority lasted only 11 to 12 weeks.

Coefficient of variation in channel catfish fingerling growth ( $CV_t$ ) can be expressed as:

$$CV_t = CV_g + CV_e$$

where  $CV_g$  is the coefficient of variation due to genetic differences and  $CV_e$  is the coefficient of variation due to the response to environmental factors. Understanding the contribution of  $CV_g$  and  $CV_e$  assuming no genetic-environment interaction and the ability to predict  $CV_t$  at any time during the culture period will facilitate the production of a more uniform crop of fish.

The main objectives of this study were:

1. To estimate the variance components of juvenile channel catfish due to genetic differences among families and that due to differences in environmental conditions.
2. To Predict the coefficient of variation and discuss the importance of the different factors affecting the CV and its implications in channel catfish fingerling productions.
3. To assess predicted CV as a criterion for estimating mortality among channel fingerlings.

### Materials and Methods

#### Experiment 1

This experiment was conducted at the Fisheries Research Unit, Alabama Agricultural Experimental Station, Auburn University, Alabama. Sixteen families of Marion channel catfish were used. Female broodfish were either three or four years old. Each female was mated to a single male (Table 3). Individual pairs were allowed to spawn naturally in pen and eggs were hatched in separate aquaria. Full-sib fry from each mating, 1 to 3-day post swim-up, were stocked separately in 20-m<sup>2</sup> outdoor concrete tanks at a density of



Table 3. Allocation and characteristics of each family of channel catfish at the start of experiment 1.

Family	Weight of female (kg)	Weight eggs (g)	Age female (year)
1	1.45	214.0	3
2	2.30	787.0	4
3	2.25	1,176.0	4
4	2.20	389.0	4
5	3.60	532.0	4
6	3.60	487.0	4
7	2.90	961.0	4
8	2.75	954.0	4
9	1.25	420.0	3
10	3.30	790.0	4
11	0.80	267.0	3
12	1.55	454.0	3

600 fry per tank. Before stocking, an additional 100 fry from each full-sib family was randomly taken to estimate average weight, and each fry was then measured to the nearest millimeter to determine the initial size distribution and the CV.

Fry were fed a 36% protein crumble catfish feed twice a day, increasing the rate to a maximum of 40 kg/ha/day during the experiment. At the end of the 90-day culture period, tanks were drained and all fingerlings were collected and counted to determine survival. A sample of 100 fingerlings was taken from each tank and fingerlings were individually weighed and measured to the nearest millimeter total length.

## Experiment 2

Three-day post swim-up fry from the same family were stocked in indoor circular tanks (1.55 diameter, 0.6 m deep) containing 1.1 m<sup>3</sup> of water at the Fisheries Research Unit, Alabama Agricultural Experimental Station, Auburn University. Water flow rate was set at 4.5 L/min and reset once a week. Photoperiod was a diurnal light:dark cycle of 14:10 using incandescent lighting set on a timer.

Fry were stocked in each tank at the density of 4,500 fry/1.1 m<sup>3</sup>. Fry averaged 0.036 g and 15 mm at the time of stocking. Fry were fed a 36% protein trout chow. Diets were fed twice daily to triplicate groups at the rates of 5, 10, and 20% wet body weight per day for a 30-day period.

One hundred fish from each tank were randomly selected, weighed each week and amount of feed adjusted accordingly.

At the end of the experiment, tanks were drained and all fish weighed. A sample of 100 fish was once again randomly selected from each tank and total length of each fish measured to the nearest millimeter.

### Experiment 3

Experiment 3 was also conducted at the Fisheries Research Unit, Alabama Agricultural Experiment Station, Auburn University. In 1992, 15 day-old channel catfish fry from different families were stocked in 12 earthen ponds (0.04 ha) for 120 d. Fry were stocked at the low density of 150,000/ha, medium density of 300,000/ha, or high density of 600,000/ha with four replicates per treatment.

In 1993, fry were stocked at 300,000/ha and 600,000/ha with six replicates per density. In three of the medium density ponds, an estimated 10% and 3% (by weight) of the population were partially harvested at 60 and 90 d post stocking, respectively. An estimated 25% and 6% (by weight) of the population were also removed in three other high density ponds for the same periods.

In both experiments, fry were fed a 36% protein crumble catfish feed twice a day, increasing the rate to a maximum of 40 kg/ha/day in all treatments. Descriptions of the three experiments are summarized in Table 4.

Table 4. Summary of design factors for the three experiments on channel catfish fingerling populations in different environmental conditions.

Experiment	Stock	Controlled factors	Environment	Treatment
1	full-sib families	same density (30/m <sup>2</sup> ) same feeding levels	outdoor tanks	None
2	mixed families	same density (4,500/1.1m <sup>3</sup> ) flow rate: 4.5 L/min Temperature: 24-25°C Culture period: 30 days	indoor tank	3 feeding levels: 5, 10, and 120%
3	mixed families	same feeding culture period: 120 days	earthen ponds	3 densities: - 150,000/ha - 300,000/ha - 600,000/ha

## Statistical analysis

Experiment 1

Coefficients of correlations were estimated between the female weight, the weight of egg mass, the percent survival, the initial CV, and the final CV. The correlations between the initial CV and the weight of egg mass, and the percent survival led to the division of the 12 families into two groups. Within each group, families were divided into two sub-groups (high and low survival).

The coefficient of variation (CV) was expressed using a model similar to that used by El-Ibiary (1976):

$$CV_{ijk} = \mu + g_i + f_{ij} + e_{ik} + (fe)_{ijk} + \epsilon_{ijk}$$

where  $CV_{ijk}$  = the coefficient of variation in the  $k$ th environment in the  $j$ th family in the  $i$ th group.

$\mu$  = the actual observed value,

$g_i$  = the effect of the  $i$ th survival group,

$f_{ij}$  = the effect of the  $j$ th full-sib family in the  $i$ th group,

$e_{ik}$  = the effect of the environment in the  $i$ th group,

$(fe)_{ijk}$  = the interaction between the  $k$ th environment and the  $j$ th family in the  $i$ th group,

$\epsilon_{ijk}$  = normally distributed uncontrolled environmental and genetic deviation between families and between tanks.

Variance components due to differences between groups ( $\sigma^2_g$ ), families in groups ( $\sigma^2_{fg}$ ), environment in group ( $\sigma^2_{eg}$ ), families and environment interaction in group ( $\sigma^2_{feg}$ ) were estimated in a one-way analysis of variance with a nested classification (Steel and Torrie 1980). Negative variances are considered zero in the analysis.

### Experiment 2

The contribution of the feeding level in the expression of the CV was estimated using a simple linear regression.

### Experiment 3

An ANCOVA was used to determine the effects of some factors on the change of the CV over time. For the analysis, the increase in variance was analyzed and the CV was derived from the expression of the variance model. The increase in variance-in-length with time is a non-linear function which was expressed by the general equation (Sparre 1976; Arnason et al. 1992):

$$\sigma^2_t = \alpha t^\beta \mu_t^\gamma \quad (1)$$

where  $\sigma^2_t$  is the increase in the CV for the mean size,  $\mu_t$  (length or weight), and is a function of time (t). Alfa ( $\alpha$ ) specifies the rate of increase in the CV per unit of time.

Beta ( $\beta$ ) and gamma ( $\gamma$ ) are coefficients assigned to  $t$  and  $\mu$ . Using the logarithmic transformation, we obtain:

$$\log(\sigma^2_t) = \log(\alpha) + \beta \log(t) + \gamma \log(\mu_t) \quad (2)$$

We can now fit the model with a weighted multiple linear regression in the form of :

$$y_i = y_0 + bt'_i + g\mu'_i + \epsilon_i \quad (3)$$

where  $y_i = \log(\sigma^2_t)$ ,  $t'_i = \log(t_i)$ , and  $\mu'_i = \log(\mu_i)$ .  $\epsilon_i$  is the residual error which is assumed to be normally distributed with mean 0 and variance proportional to  $1/n_i$  (sample size).  $y_0$ ,  $b$ , and  $g$  are the estimates of  $\log(\alpha)$ ,  $\beta$ , and  $\gamma$

## Results

### Experiment 1

From the 16 families initially stocked, only 12 were used for the analysis because of high mortality in the other four families. Final CVs for length ranged from 9.6 to 21.1%. Final CVs for weight ranged from 21 to 102%. Survival in tanks was highly variable and ranged from 55 to 97% (Table 5). Correlation coefficients were relatively high and positive between survival and final CVs for length ( $r = 0.74$ ) and between survival and weight ( $r = 0.694$ ). There was high correlation ( $r = 0.968$ ) between average weight and final average length. Similarly, final CV for weight and final CV for length were highly correlated ( $r = 0.968$ ). However, there were no correlations between the

Table 5. Initial and final length, weight, CV for length and weight of full-sib families of channel catfish cultured in outdoor tank and fed up to 40 kg/ha/day for 90 d (init. wt for all families = 0.036g).

Variables	Families											
	1	2	3	4	5	6	7	8	9	10	11	12
Initial length (mm)	14	15	13	14	14	15	13	13	13	12	12	14
Final length (mm)	117	101	115	113	117	142	121	113	95	114	96	112
Initial CV TL (%)	3.9	6.6	5.0	4.6	3.0	4.9	3.7	3.2	3.9	4.0	5.0	4.6
Final CV TL (%)	18.2	17.2	16.3	12.9	15.4	9.6	19.3	12.3	17.3	18.2	21.1	14.6
Final weight (g)	13	9	10	13	11	22	15	11	8	14	9	12
Final CV weight (%)	55	53	50	38	21	28	58	40	67	57	102	54
Survival (%)	95	92	78	57	67	55	96	59	88	92	97	78



final and initial CV for length ( $r = 0.049$ ). The initial CV for length were negatively correlated to the weight of the egg mass ( $r = -0.528$ ) and to the female weight ( $r = -0.436$ ) (Table 6). The variances for components of the CV ranged from 0 to 65% for total length and from 0 to 86% for weight (Tables 7 & 8).

### Experiment 2

Food availability can be a significant factor in size variability of small catfish. Fish fed at 20% body weight had a CV of 7.8% while fish fed at 5% body weight had a CV of 11.7% (Table 9). The coefficient of variation (CV) was quadratic, decreasing with an increase feeding rate but at a declining rate (Fig. 7).

### Experiment 3

The analysis of covariance was carried out in a number of stages. Covariates were  $t'$  (log elapse time) and  $\mu'$  (log final length). Only the analysis for length was done and the principle applied also to weight. The class variable was initial stocking density.

The results of the ANCOVA showed the importance of the class variable initial stocking density (ISD) ( $P < 0.01$ ) in explaining the increase in variance among treatment. This high level of significance scored by the class variable ISD is evidence that  $\alpha$  is not the same at all densities.

Table 6. Estimates of coefficient of correlation between pairs of variables for full-sib channel catfish fingerlings cultured in outdoor tanks and fed up to 40 kg/ha for 30 d.

Variables	Variables*							
	Weight				Length			
	Female	Egg	Average	Final CV	Initial CV	Final CV	Average	Surv.
Female	***	0.32	0.71	0.71	-0.44	-0.44	0.77	-0.44
Egg	0.32	-	0.09	-0.26	-0.53	-0.06	0.13	-0.04
Average	0.71	0.09	-	-0.49	-0.10	-0.52	0.97	-0.42
Final CV	0.71	-0.26	-0.49	-	0.26	0.94	-0.65	0.69
Initial CV	-0.44	-0.53	-0.10	0.26	-	0.05	-0.18	0.19
Final CV	-0.44	-0.06	-0.52	0.94	0.05	-	-0.61	0.74
Average	0.77	-0.06	-0.49	-0.65	-0.18	-0.61	-	0.69
Surv.	-0.40	-0.04	-0.42	0.69	0.19	0.74	-0.52	-

59

\* Surv.= survival.

\*\* All coefficients are significant at  $P < 0.01$  except the coefficients between initial CV and final length and initial CV and survival ( $P > 0.1$ ).

Table 7. Analysis of variance and estimation of the variance components of the coefficient of variation (CV) for weight of full-sib channel catfish fingerlings. Fingerlings were fed up to 40 kg/ha/day for 90 d.

Sources	Df	Mean square ( $10^{-3}$ )	Variance model	Estimates	Proportion of variance (%)
Between groups	1	0.234	$\sigma^2_{feg} + 2\sigma^2_{eg} + 6\sigma^2_{fg} + 12\sigma^2_g$	$\sigma^2_g = -0.01$	0.0
Within group					
-Families	5	0.094	$\sigma^2_{feg} + 2\sigma^2_{eg} + 6\sigma^2_{fg}$	$\sigma^2_{fg} = -0.60$	0.0
-Environment	1	3.723	$\sigma^2_{feg} + 2\sigma^2_{eg}$	$\sigma^2_{eg} = 1.72$	86.2
Fam.x Envir. within group	4	0.268	$\sigma^2_{feg}$	$\sigma^2_{feg} = 0.27$	6.2

Table 8. Analysis of variance and estimation of the variance components of the coefficient of variation (CV) for length of full-sib channel catfish fingerlings. Fingerlings were fed up to 40 kg/ha/day for 90 d.

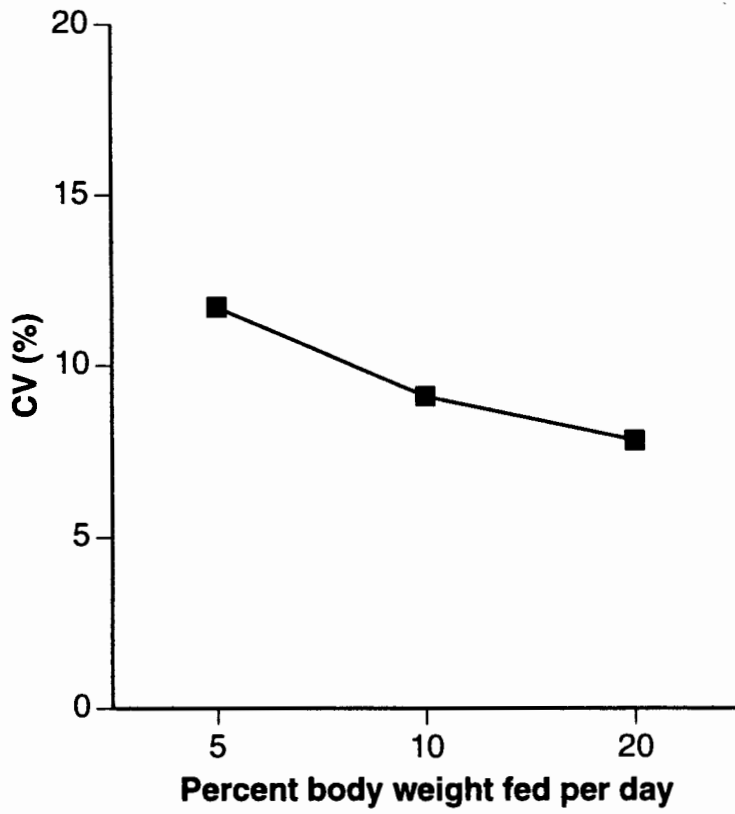
Sources	Df	Mean square	Variance model	Estimate	Proport. of var. (%)
Between groups	1	0.0080	$\sigma^2_{feg} + 2\sigma^2_{eg} + 6\sigma^2_{fg} + 12\sigma^2_g$	$\sigma^2_g = -0.001$	0.0
Within group					
-Families	5	0.0216	$\sigma^2_{feg} + 2\sigma^2_{eg} + 6\sigma^2_{fg}$	$\sigma^2_{fg} = -0.034$	0.0
-Environment	1	0.2287	$\sigma^2_{feg} + 2\sigma^2_{eg}$	$\sigma^2_{eg} = 0.090$	65.0
Fam. x Envir.	4	0.0486	$\sigma^2_{feg}$	$\sigma^2_{feg} = 0.048$	35.0

Table 9. Average length and standard deviation (SD) for length, final weight, initial and final CV for length of channel catfish fingerlings cultured in indoor tanks for 30 d (Initial TL = 15 mm; Initial Wt = 0.036 g)

Parameters						
Feeding levels (% body weight)	Ave. TL (mm)	SD	Ave. Wt (g)	Init. CV (%)	Final CV (%)	Surv (%)
5%	33	5.36	0.24	3.1	11.7a*	85
10%	40	3.56	0.49	3.0	9.1ab	81
20%	43	3.67	0.69	3.0	7.8b	84

\* Treatments with the same letter are not statistically different at the 5% level

Figure 7. Coefficient of variation for length of channel catfish fingerlings cultured in indoor tanks and fed at 5%, 10%, and 20% body weight per day for 30 d.



The covariate  $t'$  was also important ( $P < 0.001$ ) in explaining the increase in variance which made the coefficient  $\beta$  also important. However, the interaction term  $t' \times \text{ISD}$  was not significant enough ( $P = 0.0670$ ) to suggest different values of  $\beta$  for each stocking density. The term ( $\mu' = \log \{\text{final length}\}$ ) was less important and contributed very little in the expression of the variance-in-length. The interaction term  $\mu' \times \text{ISD}$  contributed very little into the model ( $P = 0.0682$ ) (Table 10). Therefore, the interaction terms  $t' \times \text{ISD}$  and  $\mu' \times \text{ISD}$  could be dropped without affecting the overall goodness of fit of the complete model (a) (Table 11). Dropping the interaction terms decreased the  $r^2$  from 94 to 74% in model (b). However, the drop in  $r^2$  did not necessarily mean that model (b) is significantly poorer than model (a) and the reduction in  $r^2$  may largely be attributed to fewer parameters being used in model (b). Finally, the simplest model (c) gave the lowest  $r^2$  ( $r^2 = 0.65$ ) but better estimates of the coefficients (Table 12).

The analysis of the estimate of the coefficients in model (c) ( $Y = Y_0 + bt' + g\mu'$ ) or in its original form ( $\log(\sigma^2_t) = \log(\alpha) + \beta \log(t) + \gamma \log(\mu_t)$ ) showed that the increase in the initial stocking density from 150,000 to 300,000/ha had no substantial impact on the rate of increase in the variance.  $\log(\alpha)$  increases from 0.26 to 0.33 between the low and medium density ponds (Table 12). Similarly,  $\log(\alpha)$  increased by two-fold between the low



Table 10. Analysis of Covariance ANCOVA on  $y = \log$  variance increase-in-length of channel catfish fingerlings. The class variable is the initial stocking density (ISD).  $t'$  is log time elapse and  $\mu'$  is log final length.

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Sources	Df*	F**	P***	$r^2$ ****
ISD	2	3.21	0.01	
$t'$	3	19.85	0.0001	
$\mu'$	3	1.77	0.0852	
$\mu' \times \text{ISD}$	6	2.96	0.0682	
$t' \times \text{ISD}$	6	2.96	0.0670	
Overall goodness of fit	20,30	7.18	0.001	0.9432

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\* Df is the degree of freedom

\*\* F gives the stepwise significance

\*\*\* P is the probability level for significance

\*\*\*\*  $r^2$  is the stepwise percent of total variance in y explained by the term

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Table 11. Summary of model fits of the ANCOVA on  $y = \log$  variance increase in length. The covariates were  $t'$  (log elapse time) and  $\mu'$  (log final length). The class variable was the initial stocking density (ISD)

Model designation	Model	Df*	$r^{2**}$
a	$y = y_0 + bt' + gL\mu' + x_i \text{ISD}$ + interaction terms	20,30	0.94
b	$y = y_0 + bt' + g\mu' + x_i \text{ISD}$	8,44	0.74
c	$y = y_0 + bt' + g\mu'$	6,48	0.65

\*Df is the degree of freedom

\*\* $r^2$  gives the goodness of fit of the models as explained by the parameters.

Table 12. Parameter estimates for the model (d)  $y=y_0 + bLt$ .  $\log(\alpha)$  is the rate of increase in the variance-in-length,  $\beta$  is the temporal effect on  $y$ , and  $\gamma$  is the size effect.

Parameters	Estimates	SE*	$P> T $ **
$\log(\alpha)$			
-150,000/ha	0.2620	0.13	0.055
-300,000/ha	0.3312	0.12	0.043
-600,000/ha	0.5789	0.15	0.001
$\beta$	1.5737	0.27	0.000
$\gamma$	-0.8782	0.48	0.084

\* SE is the standard error of the estimate

\*\* $P>|T|$  is the probability for which the estimate of the parameter is different from zero.

and high density. The estimate of  $\beta$  is different from 1 suggesting a non-linear response of  $\log(\alpha)$  to time. The estimate of  $\gamma$  also showed that fish length played very small role in explaining the increase in variance and could be dropped without affecting the prediction. It is important to signal that values of  $\log(\alpha)$ ,  $\beta$ , and  $\gamma$  vary change as different models are used. Since the estimate of the parameter  $\gamma$  is not different from 0 ( $P = 0.084$ ), it could further be dropped in the expression of the variance in length. The final expression of the variance in length becomes:

$$y = y_0 + bt' \quad (4)$$

or:

$$\log(\sigma^2_t) = \log(\alpha) + \beta \log(t) \quad (5)$$

If the reduced model (5) is back-transformed, the following expression of variance-in-length is obtained:

$$\alpha^{2t} = \alpha(d)t^\beta \quad (6)$$

with  $\alpha(d)$  being a function of density. The expected CV is given by the following:

$$E(CV_t) = [\alpha(d)t^\beta]^{1/2} \quad (7)$$

where  $\alpha(d)$  is the relative rate of increase of the variance due to the initial stocking density, and  $\beta$  describes the temporal effect on the rate of increase. Equation (7) presents two important characteristics:

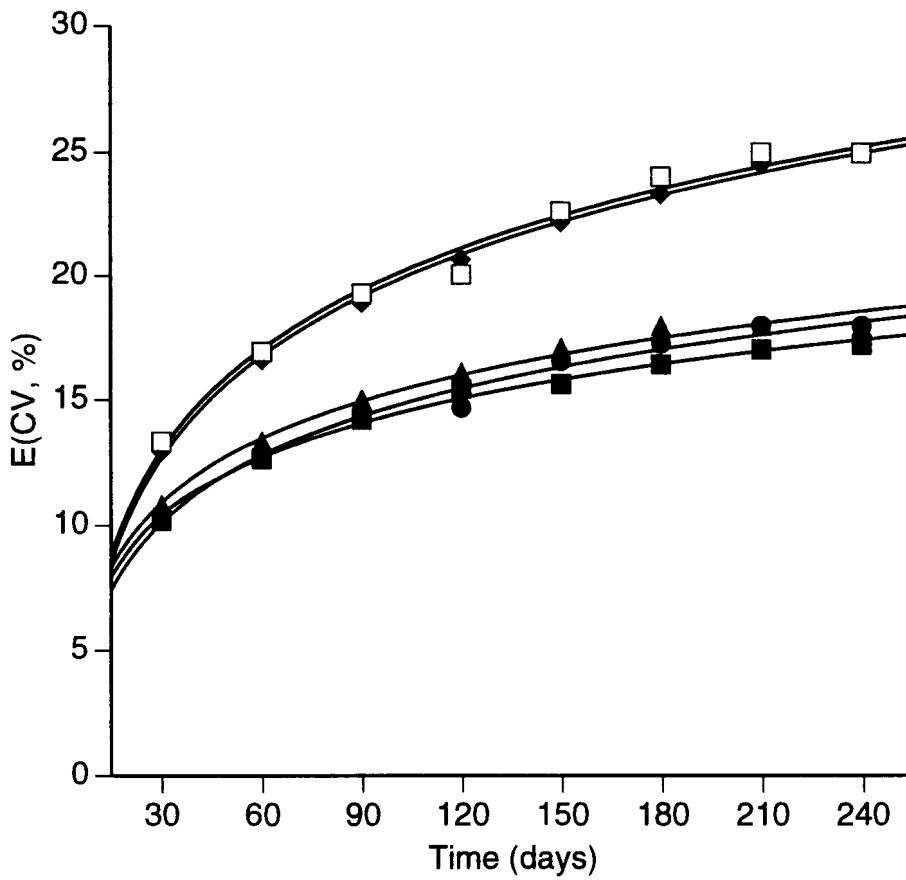
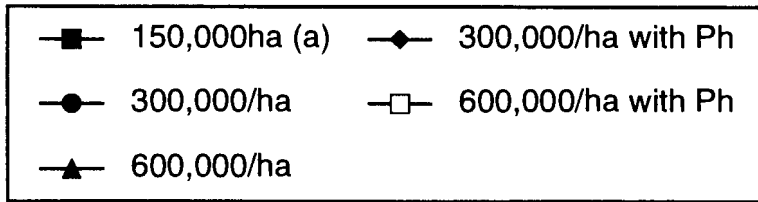
1. The rate of increase of the CV is independent of the fingerling size as suggested by the present study.
2. Each treatment density tends to have an asymptotic CV over time when other environmental factors are held constant (Fig. 8).

To validate the model, it was applied to channel catfish fingerlings stocked in earthen ponds at 300,000 and 600,000/ha and partially harvested at 60 d and 90 d in 1993. Because of high mortality incidence in the non-harvested ponds, the number of fish at harvest was not different from that of the partially harvested ponds (Appendix B).

The ANCOVA for this study gave  $\log(\alpha)$  and  $\beta$  values higher than those observed with in 1992. The plots of the expected CV ( $E(CV)$ ) versus time showed an increase in the CV which ultimately reached asymptotic values with time confirming the results previously obtained (Fig. 8).

The expected CV generated by equation (7) also gave some interesting insights into the dynamics of fingerling population structures. It appears from the plot of  $E(CV)$  over time that the CV limit is dependent of the environmental conditions. At high density, the expected CV increased rapidly and reached CV limit earlier. This is seen by extending the plot over the 120-day period.

Figure 8. Predicted coefficient of variation (CV) of channel catfish fingerlings cultured in ponds at different densities. Expected CV  $\{E(CV)\}$  were generated using the model  $E(CV_t) = [\alpha t^\beta]^{1/2}$ . (a): 150,000/ha; (b): 300,000/ha; (c): 600,000/ha; (d): 300,000/ha with partial harvests; (e): 600,000/ha with partial harvest.



The medium and low densities followed the same pattern and reached their asymptotic values later. The partial harvest not only postponed the time at which the population reached the asymptotic value but also increased that value.

### Discussion

The original assumption in the general formulation of the coefficient of variation was:

$$CV_t = CV_g + CV_e$$

where  $CV_g$  and  $CV_e$  represent the genetic and environmental components of the coefficient of variation.

Experiment 1 showed that the initial genetic contribution in regards to broodfish and egg characteristics had no effect on the expression of the CV at harvest. The proportion of 86 and 65% of the total variance in weight and in length scored by variance component due to the environment showed the overwhelming importance of the environmental factors in the expression of the CV.

factors in the expression of the CV.

The low family-environment interaction for length (6%) and the higher interaction for weight (35%) may be justified by the theory of Weatherley and Rogers (1978) in which they suggested that length is species-specific and does not change easily with the conditions of the environment as opposed to weight as fish grow older. This is illustrated by the lower CV for length when compared to the values for



weight. As genetic influences fade away with time, it is legitimate to express the CV in the form  $CV = CV_e$  based on the results of experiment 1. A more elaborate expression of the CV is given by the following:

$$CV_{jk} = \mu + e_{jk} + (fe)_{jk} + \epsilon_{jk}$$

with:

$$CV_e = CV_{jk}$$

Where  $CV_{jk}$  is the coefficient of variation in the  $k$ th environment in the  $j$ th family,  $\mu$  is the observed value,  $(fe)$  is the family-environment interaction, and  $\epsilon$  is the error term.

Brown (1957) first concluded that the effects of parental factors on the growth disappeared within the first month when studying the growth pattern in brown trout. Fowler (1972) later found in chinook salmon fingerlings that initial variation in size was related to egg quality and size. He observed that larger fry hatched from larger eggs but their size superiority lasted only 11 to 12 weeks. Silverstein and Hersberger (1992) found in coho salmon Onchorhynchus kisutch the maternal factors lasted up to 3 months.

In channel catfish, Smitherman and Dunham (1985) suggested that initial size variation was also associated with egg size and quality. Larger individuals hatched from larger eggs would have competitive advantage over smaller ones. However, as Reagan and Conley (1977) suggested

earlier, this competitive advantage is lost within the first month due to environmental effects. Those conclusions support the lack of correlation between the parental characteristics and size variation at harvest and also between the initial variation and the size variation at harvest. It is evident that the environmental conditions are the driving factors that govern the growth variability in a catfish fingerling population.

Experiment 2 provides the simplest formulation of the CV for length. From this experiment, it appears that CV can be expressed as a quadratic function of the feeding levels. This experiment showed that feeding levels have to be increased in a large order of magnitude to effect size variability. As an example, there was no statistical difference was observed between 5 and 10% and between 10 and 20% but statistical difference was observed between 5 and 20% ( $P < 0.05$ ).

Experiment 3 provides a good demonstration of a non-linear function of CV for a more extended period of time. The inclusion of the power term  $t^\beta$  for time is necessary because it helps explain the increase in CV even when mean length remains constant or changes very little. This is seen in high stocking density ponds where growth rate is reduced with time (Fig. 8)

In the present study, the overwhelming effect of the initial stocking densities on the rate of increase  $\log(\alpha)$  on

the variance-in-length (thus on the increase in the CV) justifies the fact that fingerlings stocked at different densities would have different rates of increase in their CVs. However, the CVs ultimately reach asymptotic values which are specified by the time coefficient  $\beta$ . The theory of asymptotic CVs also find support in the fact that the number of fish harvested (or survival) had very little effect on the change in CV over time.

The model,  $E(CV_t)=[\alpha(d)t^\beta]^{1/2}$ , presents some elegant characteristics with tremendous implications in channel catfish fingerling production. Both  $\alpha$  and  $\beta$  vary according to the culture conditions. The coefficient  $\alpha$  is lower in low density condition and increases with increasing stocking density as shown in the 1992 pond experiment with  $\alpha$  being the lowest (0.26) at the low density of 150,000/ha and the highest (0.58) at the high of 600,000/ha. However, the ANCOVA gave much lower values of  $\log(\alpha)$  (0.21) at 300,000/ha and 0.41 at 600,000/ha for the 1993 pond experiment than those observed in 1992. On the other hand,  $\beta$  was higher in 1993 (1.88) than in 1992 (1.41). Consequently, it appears that  $\log(\alpha)$  and  $\beta$  vary not only with the initial density but more importantly with the survival rate within a given population. Therefore, those two parameters could be used to assess mortality rate in fish populations.

The model also showed that the CV ultimately reaches an asymptotic value regardless of the initial stocking density.

At high density, the rate of increase is higher and the CV reaches the asymptotic value earlier than in the case of the medium and the low densities.

### Conclusion

The multiple regression model for CV provides meaningful information on the different components of CV, especially in situations where performance of fish from different origins are evaluated. This study showed that the parental conditions have no effect on the expression of the CV and only environmental factors coupled with some interactions contribute to the variation-in-size.

The different expressions of CV can find several applications in aquaculture. Selection for uniform growth may not be feasible because environmental factors result in competition which translates into size variation.

V. INTERACTIONS OF DENSITY AND SURVIVAL AND THEIR IMPACT  
ON GROWTH AND YIELD OF CHANNEL CATFISH, ICTALURUS  
PUNCTATUS, FINGERLINGS

Introduction

In the United States, catfish farming is the largest aquacultural industry with more than 3.75 million hectares of water producing 276 million kilograms of farm-raised catfish in 1989 (Masser et al. 1991). Catfish farming has evolved from the traditional two-season production cycle to a multiple stocking-multiple harvest strategy. In the traditional cycle, broodfish are spawned in May-June and fry are stocked in the nursery ponds at densities varying from 25,000 to 500,000 fry per hectare. Fingerlings are harvested in the fall-winter when they average 10 to 15 cm, and restocked in grow-out ponds at densities from 5,000 to 7,500/ha to final weights of 300 to 450 g the following fall. Most of the fish were sold either live or dressed through local outlets.

The majority of catfish now produced is sold to processing plants which require year round supplies of fish. More catfish are being sold as filets, requiring at least a 675 g fish whole weight for processing. The new orientation

of the market resulted in a change in the production technology. This included higher stocking rates of 10,000 to 12,500 fish per hectare in grow-out ponds, multiple harvests and multiple restocking over several years before a complete harvest.

In the attempt to explain growth within a population, it is often difficult to separate the direct effects and interactions of food availability, density, space and size. Brett (1979) stressed the importance of biotic factors such as food quantity and quality on growth and growth variability in communal systems. He suggested that the quality of food was directly involved in the increase in weight and feed conversion, but only the quantity of food coupled with some social interactions affect growth variability, and subsequently, size differences within fish populations. These interactions are not well understood as they affect channel catfish fingerling production.

Partial harvest techniques have been used in young Atlantic salmon Salmo salar and Arctic charr Salvelinus alpinus production to improve growth rate and produce uniform-size fish (Gunnes 1976; Wallace and Kolbeinhavn 1988). The theory behind the partial harvest technique is that removal of some individuals in the population increases food availability and reduces competition.

The objective of this study was to determine the effects of density and survival on growth rate and yield of

fingerling channel catfish in fingerling ponds. The effects of partial harvests on the size distribution and change in the population structure was also examined.

### Materials and Methods

This study was conducted at the Fisheries Research Unit, Alabama Agricultural Experimental Station, Auburn University, Alabama. In 1992, 15 day-old channel catfish fry were stocked in 12 earthen ponds (0.04 ha) and cultured for 120 days. Fry were stocked at the low density of 150,000/ha, medium density of 300,000/ha, and high density of 600,000/ha with four replicates per treatment.

In 1993, channel catfish fry were stocked at 300,000/ha and 600,000/ha with six replicates per density. In three of the medium density ponds, an estimated 10% and 3% (by weight) of the population were partially harvested at 60 and 90 day post stocking, respectively. An estimated 25% and 6% (by weight) of the population were also removed in three other high density ponds for the same periods to reduce the estimated standing crop to 1,000 Kg/ha.

In all treatments, fry were fed 36% protein crumble catfish feed twice a day, increasing the rate to a maximum of 40 kg/ha/day during the experiment. The feeding level was identical in every pond regardless of the initial stocking density.

At the end of each 30-day culture period, a sample of 100 fingerlings was removed from each pond, individually weighed to the nearest gram and measured to the nearest millimeter total length. Ponds were harvested after 120 days. Total number of fish harvested was estimated by dividing the total harvest weight by the average weight of fingerlings.

Growth variability was described as the coefficient of variation (Brett 1979):

$$CV (\%) = (SD/\text{mean } L) \times 100$$

where SD is the standard deviation, and L is the length.

The growth rate function is given by the expression (Thompson and Cauley 1979):

$$G_L = KLe^{-\alpha t}$$

where L is the length, t is the age (or time) and K and  $\alpha$  are constants. The growth coefficients K and  $\alpha$  were determined graphically by fitting growth curves via Ricker's growth models (Ricker 1975).

## Results

### Growth and production

In 1992 the average weight of fish between the low and medium densities was not significantly different but average weight at high density was significantly lower from the medium density ponds ( $P < 0.05$ ). Survival was a function of density, 90% at low density, 70% at medium density, and 44%



at high density. Consequently, a 100% difference in the stocking density between the low and the medium densities resulted in 50% more fish being harvested in the medium treatment; and a 100% difference in the stocking rate between the medium and the high density ponds resulted only in 9% more fish harvested at the high density. Yield was the highest at the medium density (2,745 kg/ha) and the lowest at the high density (1,213kg/ha) (Table 13).

In 1993, the average weight of fish was not significantly different between partial and batch-harvested ponds within and across all treatments. Yield and survival were higher in partially harvested ponds than in batch-harvested ones but not significantly different in all treatments (Table 14).

Values for  $K$  increased and  $\alpha$  decreased as density decreased. The growth curves associated with the coefficients  $K$  and  $\alpha$  showed that maximum growth occurred during the first month of the culture period with the low density ponds having the greatest maximum growth. The growth curves of batch-harvested and partially harvested medium density ponds are identical. In high density ponds, the growth rate of the population was reduced at 30 and 60 days, and was near zero at 90 days. Partial harvest at 60 days increased the growth rate (Fig. 9).

Table 13. Growth, survival, yield, and feed conversion ratio (FCR) of channel catfish fingerlings stocked at three densities and fed up to 40kg/ha/day. FCR is the ratio of weight of feed to weight gained of fish.

Variables	Stocking densities (fish/ha)		
	150,000	300,000	600,000
Initial length (mm)	17	17	17
Final length (mm)	118	117	85
Initial weight (g)	0.04	0.04	0.04
Final weight (g)	13.4	13.0	4.9
Survival (%)	90.0	70.0	44.5
Yield (kg/ha)	1,809	2,745	1,213
FCR	2.3	1.8	2.3

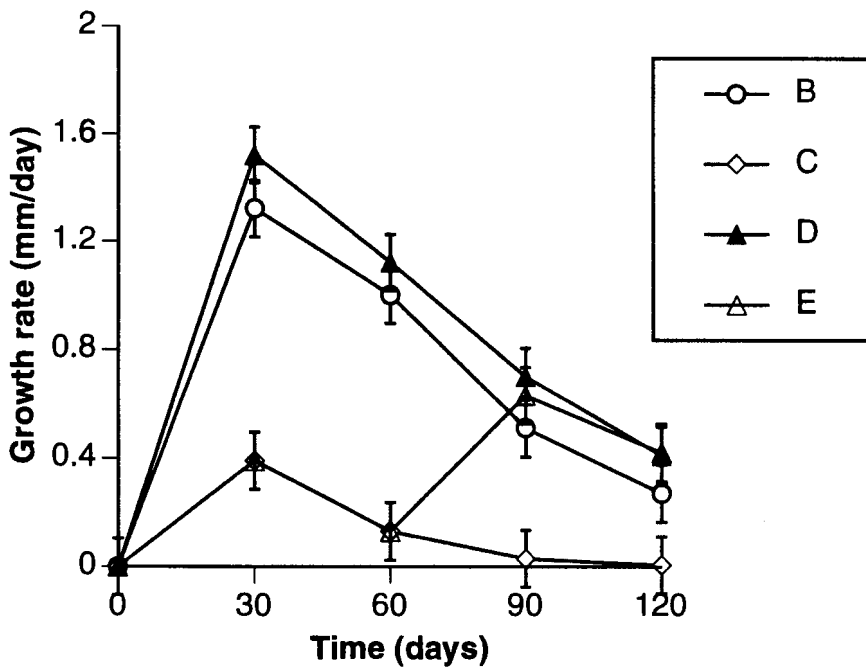
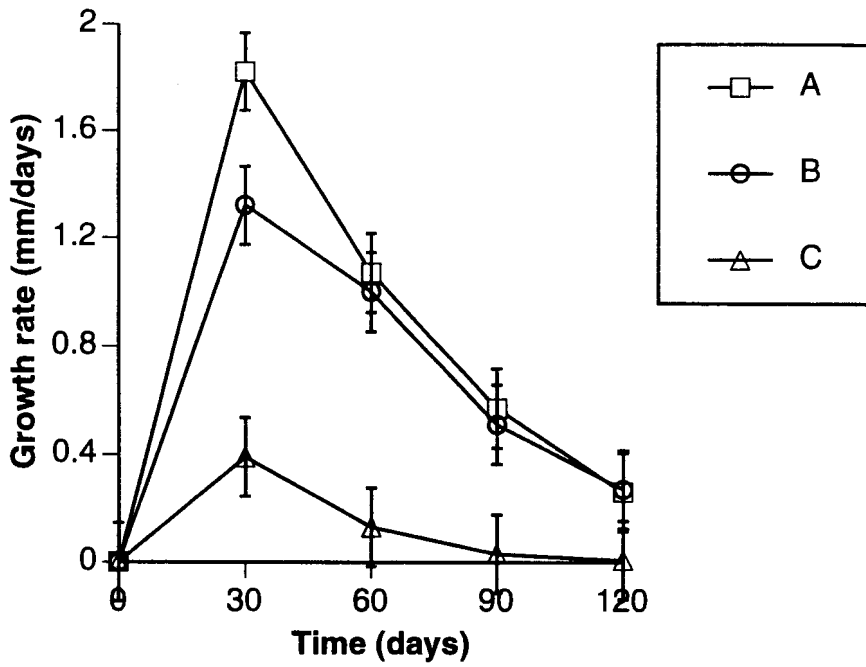
Table 14. Production data for channel catfish fingerlings cultured in ponds at the densities of 300,000 and 600,000/ha with or without partial harvest at 60 and 90 d post stocking and fed up to 40 kg/ha/day.

Variables	Stocking density (fish/ha)			
	300,000	300,000 <sup>Ph</sup>	600,000	600,000 <sup>Ph</sup>
Initial length (mm)	15	15	15	15
Final length (mm)	144	148	126	128
Initial weight (g)	0.04	0.04	0.04	0.04
Final weight (g)	26	31	24	20
Number harvested (per ha)	89,838	73,650	96,225	67,283
Number removed (per ha)	-	39,980	-	184,283
Total number	89,838	103,630	96,225	251,533
Survival (%)*	30	38	16	42
Yield	2,309	2,305	2,285	1,332
Total yield	2,309	3,557	2,285	4,980

Ph = Partial harvest

\* Survival includes number harvested and number removed.

Figure 9. Growth curves of channel catfish fingerlings cultured in ponds and partially harvested at 60 and 90 days. The treatments were A: 150,000/ha, B: 300,000/ha, C: 600,000/ha, D: 300,000 with partial harvest and E: 600,000/ha with partial harvest.



### Population structure

At low and medium densities, the populations were similar at 30 and 60 days and the CVs increased with age. About 10 and 20% of the population were at or above 100 mm (acceptable size) at 60 d, 75 and 80% were of acceptable size at 120 days in the low and medium densities, respectively. However, about 10% of the population failed to increase their length after 60 days.

At high density, population structures at 30 and 60 days were similar. At 60 days, 3% of the population was above 100 mm. At 120 days, only 10% of the population was above 100 mm (Fig. 10).

In the partially harvested medium density treatment, over 90% of the population was above 10 cm after 120 days. In partially harvested high density ponds, about 50% was above 10 cm for the same period (Fig. 11).

The coefficient of variation (CV) for total length increased with time and is related to the number of fish harvested ( $r = 0.69$ ). At low and medium densities, the CVs increased from 3 to 19%, and similarly, increased from 3 to 21% in the high density ponds after 120 days. The increase in CVs were not statistically different across densities. The increase in CV over time was inversely related to the estimated percent body weight fed ( $r = 0.77$ ). Partial harvest did not improve the uniformity in size within each

Figure 10. Structure of channel catfish fingerling populations over 120-day culture period in earthen ponds and at three densities (A: population at 30 days; B: populations at 60 days; C: populations at 120 days).

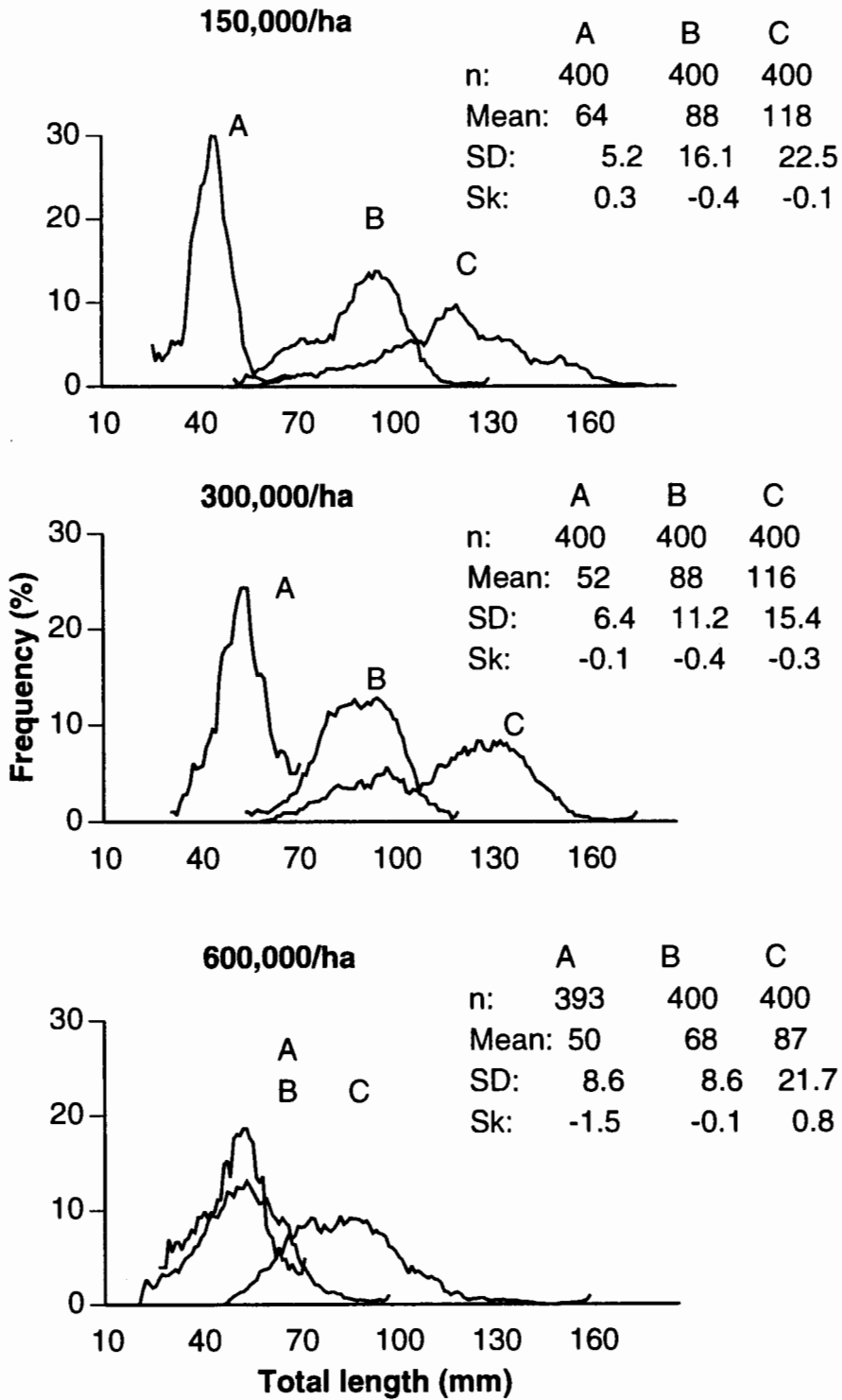
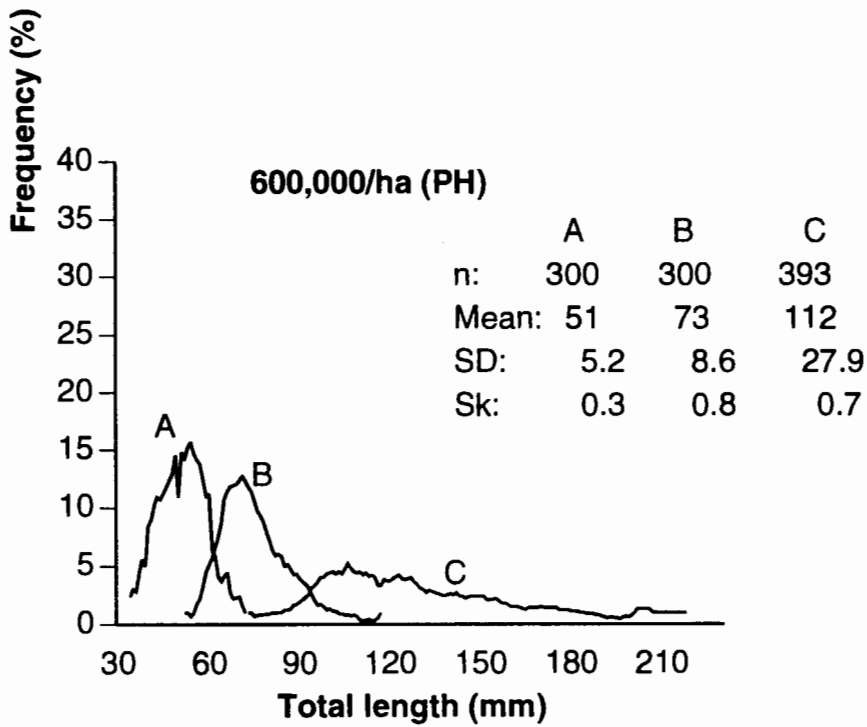
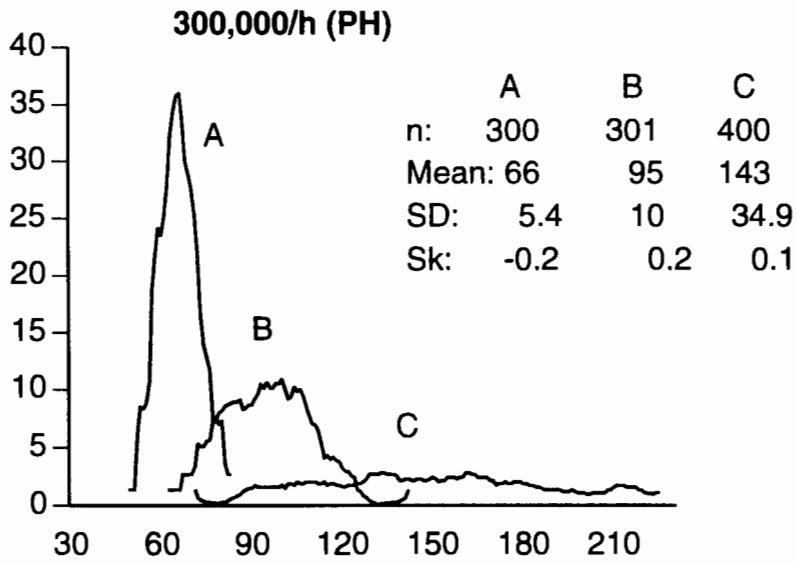




Figure 11. Structure of channel catfish fingerling populations over a 120-day culture period at two densities and with partial harvest at 60 and 90 d (A: populations at 30 days; B: populations at 60 days; and C: populations at 120 days).



population. Harvest CV increased to 21% at medium density and 22% at high density.

Skewness changed throughout the culture period. The populations were negatively skewed in the low and medium density ponds and positively skewed in the high density population at 60 and 120 days. After the partial harvest, the populations were all negatively skewed in all treatments.

### Discussion

Goals in fish farming include maximizing the use of pond space and maintaining biomass near the critical standing crop while minimizing inputs. In the present study, those goals were best reached at a stocking density of 300,000/ha. Growth rates were similar in both low and medium density ponds. The populations were negatively skewed in the low and medium density reaching harvestable size by 120 days but with the absolute number of harvestable-size fish lower at low density. The additional 150,000 fry stocked to obtain a density of 300,000/ha gave 75,650 more fish (50%) than that obtained at 150,000/ha. Therefore, pond space and resources were not fully used at 150,000/ha. A stocking rate of 600,000/ha gave the greatest number of fish at harvest, but growth rate and the percentage of harvestable-size fish were low. The additional 300,000 catfish fry stocked to obtain the density

of 600,000/ha resulted in the production of only 27,925 more fish (9%) than that obtained when fry were stocked at 300,000/ha.

Low and medium stocking densities were fed at approximately 15 and 11% body weight on day 30 and 9 and 6% on day 60. These percentages were well above the maximum values of 4.5 to 6.5% body weight needed for sustained growth for channel catfish fingerlings grown in aquaria (Mangalik 1986). For fingerlings grown in ponds at 30°C, Lovell (1989) also advocated maximum feeding rate from 4.5 to 10% from swim-up fry to fish of 16 cm in size and grown in ponds at 30°C. There was both individual and population growth in low and medium density ponds at 30 and 60 days. At 90 days, fish at low and medium densities received approximately 3.5 and 2.5% body weight, the standing crop increased above 1,000 kg/ha and a depression in growth was observed.

High stocking density ponds were fed an estimated 7 and 4% body weight at 30 and 60 days. Despite these percentages being above the assumed minimum required, growth rate was lower than that in the low and medium densities. Fingerlings were fed 1.4 and 1.3% at 90 and 120 days resulting in a reduction in the growth rate by 25% between day 90 and day 120.

At the high stocking density, crowding appeared to be the controlling factor. Brown (1946) came to similar

conclusion when culturing brown trout, Salmo trutta fingerlings where highly crowded conditions (0.33 fish/liter) resulted in a reduced feed conversion efficiency despite an increase in the feeding level. In the present study, the feed conversion ratio increased at the high density to 2.41 which one would have expected to be similar to that of the medium density. Magnuson (1962) who examined the effect of density on growth of the medaka Oryzias latipes found that increasing the population four-fold reduced growth rate; he attributed growth reduction to both feeding and social behavior. In the case of the channel catfish fingerlings, the feeding and schooling behaviors may help explain the effect of crowding on growth. Weatherley (1972) associated size variability to feeding mode and schooling behavior and concluded that species which take relatively long period of time to complete their meal were likely to exhibit high size variation. Nibbling is the feeding mode in Ictalurids and the crowded condition is antagonistic for this feeding behavior. At high density, nibbling becomes less efficient because of the physical interference between fish resulting in feed loss and in turn less efficient feed conversion (Brett 1985).

Coefficient of variation (CV) increased with time. This is in harmony with the expression of the growth rate function which suggests that larger individuals grow faster while the smaller grow slowly. At all densities, the

smallest size ranges continued to grow at a slower rate than the larger fish resulting in greater variations after 60 days. This trend was also evident in the nature of the skewness of the populations as affected by density. At 150,000 and 300,000/ha, the populations were negatively skewed implying that a high proportion of the fish were in the larger size ranges. At 600,000/ha the population was positively skewed with a small portion able to continue a high growth rate while the majority grew very slowly resulting in the modal length being less than the mean length of the population. The amplitude of the CV and the degree of skewness increased noticeably once the estimated standing crop was in excess of 1,000 kg/ha and the feed input fell below 4% body weight, independent of stocking density.

The primary objective of partial harvest was to reduce the standing crop below 1,000 kg/ha to permit a feed input of 5% body weight without exceeding 40 kg/ha and to allow fish in the smallest size ranges to increase in size. Partial harvest did reduce the standing crop, increased the yield at harvest, and allowed both individual and population to increase in length and weight. However, partial harvest, did not prevent increasing size variation in the population. Smaller individuals increased by only 10% while larger fish increased by 50 to 100% between 60 and 90 days. It is suspected that factors other than feed input and standing

crop controlled the individual growth rate in the ponds after the 60th day.

### Conclusion

The growth rate of fish population and individual growth are affected by density. The 1992 production data showed that optimum standing crop is a combination of survival and the final mean weight. Increasing stocking density rates decreased survival and at high density of 600,000/ha reduced average weight yield at harvest.

In all densities, the portion of the population in the smallest size ranges grew at a slower rate than the remaining portion of the population resulting in greater size variation. Partial harvest did reduce the standing crop and allowed the fish population to grow but did not control the increasing size variation of the population. Finally, partial harvest did increase the total yield.

VI. USE OF SIZE DISTRIBUTION DATA TO ESTIMATE MORTALITY  
RATES IN CHANNEL CATFISH FINGERLING POPULATIONS  
IN PONDS

Introduction

Proper management of fish populations requires accurate and precise knowledge of both growth and mortality rates (Ebert 1973; Pepin 1991). In fisheries, mathematical models using length-frequency distributions or empirical relationships between length and age estimated from otoliths have been used with success to calculate or predict growth rate (MacDonald and Pitcher 1979). However, fisheries biologists have been less successful in estimating mortality rates from size distributions mainly because of the difficulty in sampling an entire population for a long period of time (Pepin 1993).

In aquaculture, prediction of mortality rates using mathematical models have received limited attention. Traditional methods of calculating mortality rates involve harvesting the entire population and dividing the number of fish harvested by the total number stocked. Methods such as Ricker's formula (Ricker 1975) and the length-based method (Smith and Richardson 1977; Pepin 1993) can also be applied



with appropriate adjustments to estimate mortality. Ricker's formula estimates mortality as a function of time. The length-based method uses the slope of the descending limb of the cumulative length-frequency distribution of the fish population and estimates mortality as a function of length. This method is based upon the assumption that natural mortality is size dependent and is effective in natural fisheries (Houde 1989; Pepin 1993). However, in aquaculture, the size dependency of natural mortality has not yet been established and the use of this method becomes limited, especially for individuals of nearly the same age.

In our quest for alternative methods of estimating mortality rates in populations, we analyzed the possibility of using a variation of the length-based method. Using the same principle as the length-based method, we considered the descending limb of the length-frequency distribution above the modal length rather than the entire population to solve the population number density equation. Our method is based upon the assumption that fish mortality is a function of the shape of the size structure and that the right hand limb of a size-frequency curve is more convincingly representative of mortality and growth pattern (Van Sickle 1977).

The objective of this study is to evaluate an alternative method of determining mortality rates in fish populations using length-frequency distributions and compare this method to the more traditional length-based method.

## Materials and Methods

### Fish culture and sampling

Fifteen-day old channel catfish fry were stocked in 12 earthen ponds (0.04 ha) for 120 days at the Fisheries Research Unit, Alabama Agricultural Experimental Station, Auburn University. Fry were stocked at the low density of 150,000/ha, medium density of 300,000/ha, or high density of 600,000/ha with four replicates per treatment.

In 1993, fry were stocked at 300,000/ha and 600,000/ha with six replicates per density. In three of the medium density ponds, an estimated 10 and 3% (by weight) of the population were partially harvested at 60 and 90 d post stocking, respectively. An estimated 25 and 6% (by weight) of the population were also removed in three other high density ponds for the same periods. In both experiments, fry were fed a 36% protein crumble catfish feed twice a day, increasing the rate to a maximum of 40 kg/ha/day in all treatments.

In 1994, fry were stocked at 600,000/ha in nine ponds. In three ponds, fry were fed up to 40 kg/ha/day and in the other six, fry were fed up to 80 kg/ha/day. In three of the six ponds fed at 80 kg/ha/day, fry were partially harvested at 60 and 90 d. At the end of each 30-day culture period, a sample of 100 fingerlings was removed from each pond and individually weighed to the nearest gram and measured to the

nearest millimeter total length. Ponds were harvested after 90 d. Total number of fish harvested was estimated by dividing total harvest weight by the average weight.

#### Estimates of mortality rates

The length-based method of estimating mortality rates was described by Smith and Richardson (1977) and Pepin (1993). The length-frequency distribution for the entire population is divided into size classes, and it is assumed that mortality is the same for all individuals in each size class. The mortality rate is estimated by using the linear regression of the log-transformation of the number density equation (Smith and Richardson 1977; Pepin 1993):

$$N = ae^{-zL}$$

where  $N$  is the number of fish,  $L$  is the length,  $z$  is the mortality rate and  $a$  is a constant which is estimated graphically. This method requires a sample truly representative of the population. In addition, it assumes no migration and that growth rate is constant for individuals of the same size.

#### Method 1

The number density equation was applied to the entire population of channel catfish fingerlings cultured in ponds within different environmental conditions in 1992 and 1993. The equation was applied to the entire population using a

size class width of 10 mm. The slope of the descending limb was mathematically determined from the plot of the log-transformation of the number of fish against fish total length.

### Method 2

The number density equation was applied to the population above the modal length for each length-frequency distribution using the same size class width of 10 mm. The slopes of the descending limb were again used to estimate mortality rates.

Both methods were applied to channel catfish fingerling population cultured in ponds in 1992 and 1993. The data collected in 1994 was used to validate their applications.

### Results

Mortality estimates from each method were compared to the actual mortality observed at harvest. The length-based method applied to the entire population (Method 1) gave lower estimates of mortality ranging from 14 to 47% (Table 15). Relatively flat slopes and an overall poor general relationship describing the length-based survival curve with  $r^2$  values ranging from 0.18 to 0.91 were observed (Fig. 12). Moreover, some estimates were not significantly different from 0 ( $P>0.05$ ). Method 2 gave steeper slopes (Fig. 13) and higher mortality rates which ranged from 25 to 80%

Table 15. Initial stocking density, actual and estimated mortality rates using the entire population (method 1), and estimated mortality using the portion of the population above the modal length (method 2) in channel catfish fingerling populations.

Yr/Trt (fish/ha)	Length interval (mm)	Actual mort. (%)	Method 1		Method 2	
			Fish mort. $r^2$ (%)	Fish mort. $r^{2*}$ (%)		
1992						
150,000	56-186	10	18	0.17	25	0.90
300,000	64-170	30	18	0.16	38	0.84
600,000	52-155	56	41	0.35	64	0.92
1993						
300,000Ph (a)	76-226	84	14	0.09	44	0.51
300,000	82-226	70	17	0.16	47	0.76
600,000Ph (b)	86-191	89	42	0.41	42	0.95
600,000	69-198	84	20	0.35	44	0.91
1994						
600,000/80NG (b)	60-120	76	43	0.91	80	0.94
600,000/80G (c)	72-135	87	47	0.82	80	0.91
600,000/40NG (d)	45-140	53	35	0.91	60	0.94

\*  $r^2$  represents the goodness-of-fit of the log-linear transformation of log cumulative number.

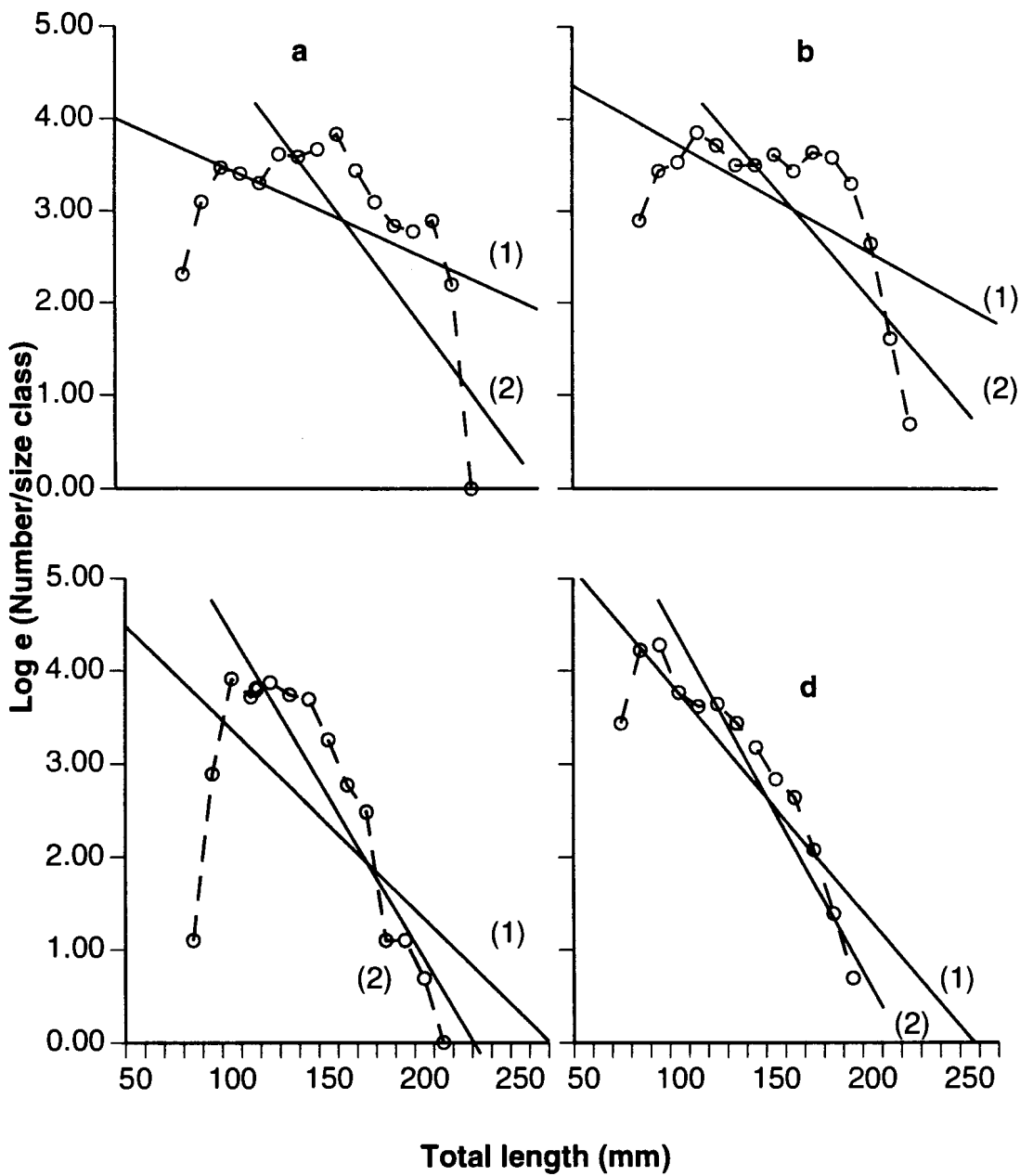
(a) Ph: partial harvest

(b) ponds fed at 80 kg/ha/day with no partial harvest

(c) ponds fed at 80 kg/ha/day with partial harvest

(d) ponds fed at 40 kg/ha/day with no partial harvest

Figure 12. Cumulative length-frequency distributions of channel catfish fingerlings. The broken lines show the average cumulative abundance (a: 300,000/ha with partial harvest; b: 300,000/ha without partial harvest; c: 600,000/ha with partial harvest; and d: 600,000/ha without partial harvest). The slopes of the solid lines give the mortality estimates using equation 1. Width of size class: 10 mm. (1) represents slope from method 1 and (2) slope from method 2.



(Table 15). However, the estimates of mortality for the 1993 data deviated from the cumulative mortality at harvest by about 40%.

To further our investigation, we remove extreme points within the distributions. By doing so, we were able to increase the slopes and consequently increase mortality estimates.

The analysis of the length-frequency distributions showed that the combined effect modal frequency, the size range, and the skewness dictate the mortality estimation. The 1994 data showed size ranges smaller than those observed in 1992 and 1993. When the method was applied to the 1994 channel catfish fingerling data, we observed mortality estimates from method 1 higher than those of the previous years using the same method. Smaller size ranges gave steeper slopes and higher mortality estimates (i.e at 600,000/ha, size range: 103 mm; modal frequency: 42%; estimated mortality: 64%). Similarly, larger size range coupled with smaller modal frequency gave lower mortality estimates (i.e at 600,000/ha, size range: 129 mm; modal frequency: 25%; estimated mortality: 44%) (Table 16). Similarly, we observed that size structures with positive skewness gave lower mortality estimates than those with a negative skewness (Fig. 14). However, we were able to increase the mortality estimates by removing the extreme values within the frequency distributions.

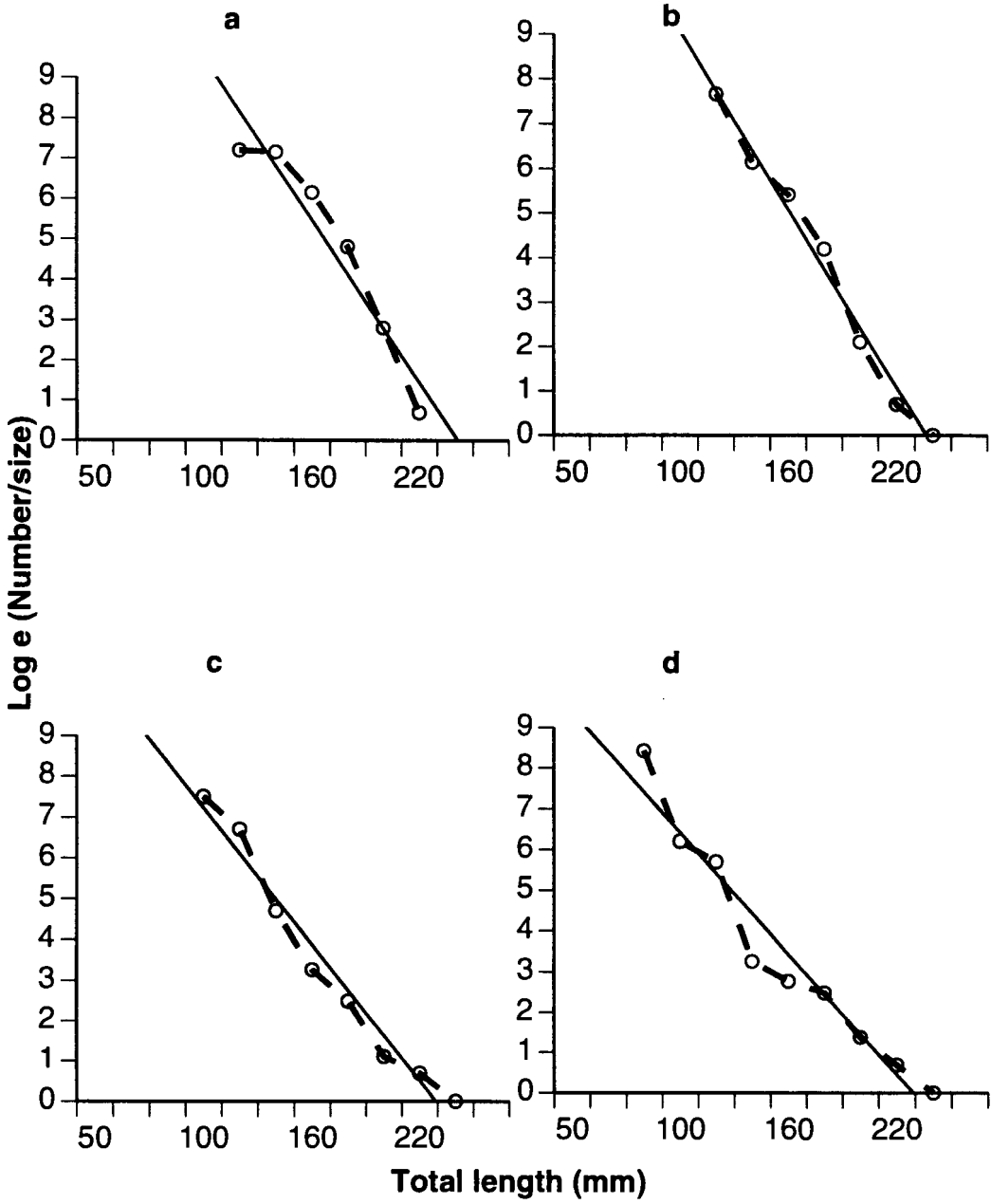


Table 16. Initial stocking density, size range, modal frequency, estimate and actual mortality rates in channel catfish fingerling populations cultured in earthen ponds.

Treatment (fish/ha) (%)	Size range (mm)	Freq. of mode		Estimated mort. (%)		Actual mort.
		Width size classes		Width size classes		
		10 mm	20 mm	10 mm	20 mm	
1992						
150,000	56-186	51	-	25		10
300,000	64-170	27	-	38	-	30
600,000	52-155	42	-	61	-	56
1993						
300,000PH	76-226	16	52	44	78	84
300,000	82-226	31	40	47	74	70
600,000PH	86-198	23	53	42	80	89
600,000	69-198	25	52	44	78	84

PH: Partial harvest.

Figure 13. Cumulative length-frequency distributions and estimates of mortality rates in channel catfish fingerlings. The broken lines show the average cumulative abundance (a: 300,000/ha with partial harvest; b: 300,000/ha without partial harvest; c: 600,000/ha with partial harvest; and d: 600,000/ha without partial harvest). The slope of the solid lines gives mortality estimates from number density equation.



### Discussion

When size-frequency data are available, it is possible to estimate mortality on a size-specific basis. Ricker (1975) and Van Sickle (1977) regarded mortality and growth rate as vital parameters in shaping size-frequency distributions. They established a strong relationship between the quality of the prediction of mortality and the shape of the size-frequency distribution. As Table 15 shows, the estimation of mortality rates from all size classes using method 1 gives values far less than the actual mortality estimates. The poor estimation of mortality is explained mathematically by less steep slopes which are due to the poorly defined size classes and the generally dome shape of channel catfish fingerling size-frequency distributions.

Method 2 helps circumvent this problem by using only the distribution above the modal length. This method generates higher slope values and subsequently higher mortality estimates. The strategy of using one portion of a length-frequency distribution was described by Van Sickle (1977) when comparing different methods of estimating mortality rates from distributions from populations of Pismo clams (Tivelia stultorum). He showed that considering only a segment of the distribution did not affect the prediction of mortality when he was confronted with similar problem.

In the present study, Method 2 gave higher mortality estimation for the channel catfish fingerling data in 1992.

In the 1993 data, both methods failed to give values close to the actual mortality, probably because individuals within each size class were sufficiently dispersed in the size distribution resulting in a wide size range. The 1993 data differed from that of 1992 by the size range of the different populations and the modal frequency. The channel catfish fingerlings in 1992 had smaller size ranges and higher modal frequencies than the populations in 1993. It appears from this study that smaller size ranges and higher modal frequencies give higher mortality estimates. The size ranges of the channel catfish fingerling populations in 1994 varied from 60 to 95 mm and were on the average half of the size range of the populations in 1993. Mortality estimates in 1994 from Method 2 were about twice the estimates in 1993.

To compensate for the wide size range and the low modal frequency, an alternative strategy is to increase the width of the size classes. Based on this study, increasing the width from 10 to 20 mm decreases the dispersion of the size-frequency distributions and increases the slope estimates and eventually increases the mortality estimates.

The methods discussed in the present study can be useful to hatchery managers and fish culturists when size distribution of fingerling populations are available.

Unlike Ricker's formula (Ricker 1975) which estimates mortality over time, the methods described here estimate size-specific mortality rates when the stationary size distribution is known. The choice of either of the two methods should be made after analysis of the length-distribution.

The relationship between the mortality estimation and the skewness using both method can be better understood after observations of length-frequency distributions. Positively skewed populations have long right tail as opposed to negatively skewed distributions. Populations highly skewed to the left (positive skewness) have flat slopes as compared to populations skewed to the right (negative skewness). As a result, mortality estimates are lower in positively skewed populations.

### Conclusion

The present study shows the possibility of using length-frequency distributions to predict mortality rates of fish populations in aquaculture ponds. Both methods discussed are practical in estimating mortality would require a precise estimate of growth rate.

Both methods 1 and 2 provide mortality estimates of fingerlings in ponds. The proposal to use lengths above mode may not be good in some instances where it failed to give mortality estimates close to the actual mortality in

the ponds. It is also questionable to alter the data by removing extreme values within the length-frequency distributions to increase the mortality estimates. However, those questions should not undermine the usefulness of method 2 in management fish populations in aquacultural ponds.

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## APPENDICES

APPENDIX A  
1992 CHANNEL CATFISH SAMPLING DATA

Table 17. Final mean length, standard deviation (SD), CV, skewness of channel catfish fingerlings stocked at three densities and fed up to 40kg/ha/day (initial length = 16.5 mm; initial CV = 3 to 4%).

Period (days)	Trt/Pond (fish/ha)	Mean length (mm)	SD	CV (%)	SK
30	150,000				
	E-39	64.6	5.48	8.48	0.43
	E-42	62.4	4.70	7.54	-0.63
	E-47	64.7	5.28	8.17	0.61
	E-48	65.5	5.70	7.61	0.70
	300,000				
	E-41	51.7	6.39	12.36	-0.002
	E-43	51.0	6.43	12.61	-0.075
	E-44	51.5	6.42	12.45	0.089
	E-46	53.7	6.15	11.45	-0.700
	600,000				
	E-37	54.3	7.52	13.86	-0.061
	E-38	46.9	8.74	18.65	0.220
	E-45	50.1	8.84	17.65	-0.270
	E-55	50.8	7.87	15.49	-0.240
	60	150,000			
E-39		92.6	7.09	7.65	-0.090
E-42		69.8	7.24	10.37	0.270
E-47		91.1	7.42	8.10	0.110
E-48		97.3	9.18	9.43	-0.001
300,000					
E-41		82.2	10.58	12.87	-0.069
E-43		96.6	9.27	9.60	-0.290
E-44		85.9	10.85	12.64	0.320
E-46		89.3	8.91	9.9	0.320
600,000					
E-37		73.2	9.02	12.33	0.089
E-38		61.2	10.20	16.65	0.820
E-45		78.4	11.53	14.71	0.280
E-55		78.3	10.12	12.92	0.640

Table 17 (continued): Final mean length, standard deviation (SD), CV, skewness (Sk) of channel catfish fingerlings stocked at three densities and fed up to 40 kg/ha/day (initial length = 16.5 mm; initial CV = 3 to 4%).

Period (Days)	Trt/Pond (fish/ha)	Mean length (mm)	SD	CV (%)	SK
90	150,000				
	E-39	113.7	10.29	9.05	-0.160
	E-42	86.9	12.93	14.88	0.026
	E-47	112.7	10.98	9.75	0.670
	E-48	116.1	8.97	7.73	-0.270
	300,000				
	E-41	86.0	13.30	15.47	0.440
	E-43	102.6	9.73	9.48	0.340
	E-44	111.9	13.99	12.51	-0.390
	E-46	102.2	11.98	11.72	0.400
	600,000				
	E-37	83.5	12.89	15.44	0.079
	E-38	79.1	16.19	20.46	0.530
	E-45	71.9	10.45	14.53	0.810
	E-55	98.5	19.85	20.14	0.530
120	150,000				
	E-39	132.7	20.64	15.55	0.210
	E-42	94.8	18.37	19.39	0.150
	E-47	121.7	14.99	12.32	0.250
	E-48	124.4	15.59	12.53	0.210
	300,000				
	E-41	114.0	24.35	21.35	-0.410
	E-43	126.6	15.20	12.00	0.050
	E-44	114.4	19.56	17.10	-0.050
	E-46	111.2	23.20	20.89	0.090
	600,000				
	E-37	82.7	13.68	21.35	0.960
	E-38	75.0	18.27	24.38	1.200
	E-45	83.5	13.84	16.57	0.070
	E-55	97.6	16.40	16.80	-1.260



APPENDIX B  
1993 CHANNEL CATFISH SAMPLING DATA

Table 18. Final mean length, standard deviation (SD), CV, skewness of channel catfish fingerlings stocked at two densities with partial harvest and fed up to 40 kg/ha/day (initial length = 15.7 mm; initial CV =3 to 6.1%).

Period (days)	Trt/Pond (fish/ha)	Mean length (mm)	SD	CV (%)	SK
30	300,000				
	E-38	67	4.81	7.19	-0.02
	E-40	69	4.85	7.07	-0.46
	E-44	71	4.07	5.76	-0.21
	E-47	64	3.67	5.71	0.21
	E-48	64	4.98	7.81	-0.14
	600,000				
	E-39	57	5.12	9.20	-0.13
	E-41	51	6.24	12.29	0.48
	E-42	56	6.64	11.91	-0.10
	E-43	49	7.07	14.48	0.48
	E-45	52	3.80	7.36	0.03
60	300,000				
	E-38	104	8.99	8.69	0.41
	E-40	99	8.12	8.22	0.42
	E-44	105	10.49	10.00	0.53
	E-47	84	7.37	8.78	1.28
	E-48	91	10.22	11.24	0.74
	600,000				
	E-39	73	6.67	9.09	0.49
	E-41	64	6.66	7.14	0.36
	E-42	88	6.89	7.82	0.19
	E-43	86	10.07	11.78	0.51
	E-45	86	10.12	10.57	0.22

Table 18 (continued): Final length, standard deviation (SD), CV, skewness (SK) of channel catfish fingerlings stocked at two densities with partial harvest and fed up to 40 kg/ha/day (initial length = 15.7 mm; initial CV = 3 to 6.1%).

Period (day)	Trt/Pond (fish/ha)	Mean length (mm)	SD (%)	CV (%)	Sk
90	300,000 with partial harvest				
	E-38	130	19.51	15.02	0.54
	E-44	133	23.45	17.65	0.99
	E-47	112	18.35	16.43	0.50
	300,000 no partial harvest				
	E-40	132	19.65	14.85	0.42
	E-48	121	22.78	18.84	0.28
	600,000 with partial harvest				
	E-39	103	18.04	17.59	0.45
	E-42	124	18.71	15.12	0.23
	E-43	121	20.57	16.94	0.11
	600,000 no partial harvest				
	E-41	93	17.19	18.50	1.00
	E-45	100	20.64	20.66	0.38
120	300,000 with partial harvest				
	E-38	165	31.66	19.19	0.14
	E-44	134	33.32	24.90	0.39
	E-47	139	32.14	23.10	0.24
	300,000 no partial harvest				
	E-40	155	31.17	20.07	0.20
	E-48	133	34.79	26.11	0.26
	600,000 with partial harvest				
	E-39	134	23.31	17.39	0.23
	E-42	121	18.10	14.93	0.58
	E-43	122	19.10	15.04	0.44
	600,000 no partial harvest				
	E-41	113	23.18	20.44	0.86
	E-45	110	31.86	28.94	0.71

APPENDIX C  
PARTIAL DIFFERENTIAL EQUATION

The model used to predict the population structures is a variation of the partial differential equation described by Sinko and Streifer (1969):

$$\delta N(l,t)/\delta t + \delta N(l,t)G(l,t)/\delta l = -Z(l,t)N(l,t) \quad (1)$$

where  $\delta N(l,t)/\delta t$  is the rate of change in the number at size  $l$ ,  $\delta N(l,t)G(l,t)/\delta l$  is the rate of change in the net number at size  $l$  resulting from growth  $G$ .  $-Z(l,t)N(l,t)$  is the mortality rate.

If no mortality, then equation (1) becomes:

$$\delta N(l,t)/\delta t + \delta N(l,t)G(l,t)/\delta l = 0 \quad (2)$$

and using the differential form:

$$dN(l,t)dt + dN(l,t)G(l,t)/dl = 0 \quad (3)$$

Equation (3) is then solved numerically using SAS (SAS 1985).

When a growth rate function is used such as:

$$G(l) = Kle^{-\alpha t} \quad (4)$$

$K$  and  $\alpha$  are obtained by plotting successive values of  $G$  against time and fitting an exponential curve to the data. Once the values of  $K$  and  $\alpha$  are obtained, calculate the different values of length ( $l$ ) through the equation:

$$l = l_0 \exp\{(-K/\alpha)(e^{-\alpha t} - e^{-\alpha t_0})\} \quad (5)$$

where  $l_0$  and  $t_0$  are length and time at a selected initial distribution. The values of  $l$  give the new length-frequency distribution. The univariate procedure in SAS (SAS 1985) gives the standard deviation, the coefficient of variation (CV), the skewness (Sk), and the kurtosis (Ku).

Table 19. Growth rate functions and coefficient for channel catfish fingerlings growth functions after 120 days when stocked at densities of 150,000, 300,000, and 600,000/ha (t = time (days); L = length (mm); PH = partial harvest).

Stocking density (fish/ha)	K	$\alpha$	Growth function
150,000/ha	2.0	0.8	$2.0Le^{-0.8t}$
300,000/ha	1.7	0.8	$1.7Le^{-0.8t}$
600,000/ha	1.1	1.3	$1.1Le^{-1.3t}$
300,000/ha <sup>PH</sup>	1.4	0.7	$1.4Le^{-0.7t}$
600,000/ha <sup>PH</sup>	1.1	0.6	$1.1Le^{-0.6t}$

APPENDIX D  
LENGTH-BASED METHOD

The mortality rate is estimated by using the linear regression of the log-transformation of the following equation:

$$N = ae^{-z l} \quad (1)$$

where  $N$  is the density number of fish,  $l$  is the length, and  $z$  is the mortality. Using the logarithmic transformation, we obtain:

$$\log (N) = \log (a) - z l \quad (2)$$

First, generate the cumulative frequency distribution and plot against the length ( $l$ ). The slope of the linear regression of equation (2) gives the estimated value of  $z$  which also estimates the mortality rate in fish per unit of length. The value of  $z$  is further converted into a percentage of the population.

### Example

The channel catfish fingerling population stocked at 150,000/ha in 1992 gave a slope of 0.8 which corresponded to an average mortality estimate of 0.8 fish per millimeter.

The size range for this population was 130 mm.

The number of fish dying:  $0.8 \times 130 = 74$  fish.

Since 397 fish were used to build the length-frequency distribution, the mortality as a percent is:

$$74/397 = 0.18 \text{ (or 18\%)}$$



Table 20. Slopes (z) and values of the log transformed of the number density equation  $N=ae^{-Zl}$ . N is the number density function; l is the length (mm). Method 1 uses the entire population and Method 2 uses the population above the modal length.

Stocking density (fish/ha)	Method 1		Method 2	
	z	a	z	a
150,000	0.80	5.32	1.4	1.19
300,000	0.72	4.23	1.2	1.51
600,000	0.98	4.02	1.7	8.03
300,000PH	0.4	4.05	0.4	6.39
300,000	0.58	4.42	0.67	9.04
600,000PH	0.20	4.58	0.41	8.02
600,000	0.25	5.22	3.52	6.32

PH: Partial harvest

