KINEMATIC AND KINETIC COMPARISON OF OVERHAND AND UNDERHAND

PITCHING: IMPLICATIONS TO PROXIMAL-TO-DISTAL SEQUENCING

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KINEMATIC AND KINETIC COMPARISON OF OVERHAND AND UNDERHAND PITCHING: IMPLICATIONS TO PROXIMAL-TO-DISTAL SEQUENCING

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PITCHING: IMPLICATIONS TO PROXIMAL-TO-DISTAL SEQUENCING

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The author was born February 17, 1979, in McComb, MS. He is the only child of Dr. and Mrs. John C. Garner, Jr. (Diane). In 1997, he graduated as valedictorian from Brookhaven High School in Brookhaven, MS. That same year, he accepted a scholarship to play baseball and study biology at Delta State University in Cleveland, MS. He completed an All-American baseball career and graduated with honors with a Bachelor of Science Degree in 2002. In the fall of 2002, the author entered graduate school at Auburn University. In May of 2005, he completed a Masters of Science in Exercise Science, and has since been pursuing a Doctorate in Biomechanics.

DISSERTATION ABSTRACT

KINEMATIC AND KINETIC COMPARISON OF OVERHAND AND UNDERHAND PITCHING: IMPLICATIONS TO PROXIMAL-TO-DISTAL SEQUENCING

John C. Garner, III

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Because the segments of the body are linked, the movement of one component affects the action of all the other components of that segment, suggesting that there is an interaction between segments in an open kinetic chain movement. Typically, these segments interact in a sequence from the segment that is most proximal to a segment that is most distal. This interaction is known as proximal-to-distal sequencing. This sequence results in a summation of speed at the most distal segment producing a maximal end segment velocity. Although there is no question that this principle occurs, the mechanism of this interaction is still under scrutiny. Currently there are two explanations for the proximal-to-distal sequence, both based on the principle of conservation of angular momentum. Theory One states that once the motion of the system begins, an angular momentum is developed in the system and the distal segment lags behind. As the proximal segment approaches maximum velocity, an external force opposes this motion, which negatively accelerates the proximal segment, allowing inertia to propel the distal segment forward. Theory Two contends that no external torque is applied to the system after the initial acceleration of the system takes place. The system, with some mass, is said to move with a given angular velocity, thus having an angular momentum, which is conserved throughout the action. In this theory, as the proximal segment reaches its maximum angular velocity, an internal muscle moment is applied between the proximal and distal segments to accelerate the distal segment. Currently there is a wealth of information on the kinetics and kinematics of the overhand baseball throw, but surprisingly little on the underhand softball pitch, which includes a flexion action. Even more surprising is that these two pitching motions have received no direct study to analyze the overhand motion in terms of proximal-to-distal sequencing compared to the somewhat analogous underhand pitch in softball. Furthermore, the agonist/antagonist activation of the primary musculature that would cause the desired joint action at the elbow (extensors in baseball, flexors in softball) has also been neglected in the literature. Using motion capture 3-D analysis, the results of this study confirmed the existence of proximal-to-distal sequencing in both throwing types and illustrated that both types can be theoretically categorized as Theory One, with inertial acceleration of the distal segment. Furthermore, the musculature acting at the elbow can be considered analogous in both types of pitches.

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Chapter I

Introduction

The segments of the human body are linked together to make up one system. When the extremities of the system are moving in space (e.g. throwing a ball, swinging the leg through while running), they are considered to be an open kinetic chain. Because the segments of the body are linked, the movement of one component affects the action of all the other components of that segment, suggesting that there is an interaction between the segments in an open kinetic chain movement.

Typically, these segments interact in a sequence from the segment that is most proximal to a segment that is most distal. This interaction is known as proximal-to-distal sequencing. This sequence results in a summation of speed at the most distal segment producing a maximal end segment velocity (Bunn, 1972). This interaction principle was first proposed as a computer model by Plagenhoef (Plagenhoef, 1971), and suggested in humans one year later in a coaching text by Bunn (Bunn, 1972). Plagenhoef proposed that segments acted in a proximal-to-distal sequence which summated the speed of each previous segment to create a maximal end velocity of the most distal segment (Plagenhoef, 1971). Understanding of this process was further advanced when it was found that as the motion progresses towards the distal segment, each segment starts its motion at the instant of greatest velocity of the preceding segment. This allows the more distal, adjacent segment to reach a maximum velocity greater than that of the predecessor (Kreighbaum & Barthels, 1985) by building on the velocity of the preceding segment. In addition, it has been well documented that in open kinetic chain motions that demonstrate a proximal-to-distal sequence, the proximal segment will be at its minimum value just as the distal segment is at its maximum velocity, which will then be greater than that of the more proximal segment (Bunn, 1972; Atwater, 1979; Putnam, 1983; Feltner & Depena, 1986; Putnam & Dunn, 1987; Feltner, 1989; Putnam, 1991, 1993; Fleisig et al., 1995; Sorenson et al., 1996).

Although there is no question that this principle occurs, the mechanism of this interaction is still in question. Currently there are two explanations for the proximal-to-distal sequence, both based on the principle of conservation of angular momentum. These two theories were previously explained by Ford in an unpublished dissertation (Ford, 1998), which was based on contradictory statements in an early study by Kreighbaum and Barthels (Kreighbaum & Barthels, 1985).

Theory One (Ford, 1998) states that once the motion of the system begins, an angular momentum is developed in the system and the distal segment lags behind. As the proximal segment approaches maximum velocity, an external force opposes this motion, which negatively accelerates the proximal segment, allowing inertia to propel the distal segment forward. Kreighbaum and Barthels explained that as a result of the negative acceleration of the proximal segment, the axis of rotation for the more proximal segment shifts from the proximal end to the distal portion, decreasing the radius of gyration, thus decreasing the rotational inertial for the more distal segments (Kreighbaum & Barthels, 1985).

This theory is the more widely accepted of the two theories and has been studied in various movements (e.g. throwing, kicking, striking). The most common analysis of Theory One is the overhand baseball throw. In a study simulating the overhand throw, Herring concluded that a series of events including: (1) proximal-to-distal sequence of the onset of joint moments, (2) simultaneous peaking of segment velocities at the instant of the preceding segments minimal velocity and (3) reversing moments, created an inertial influence on the acceleration of the distal segment (Herring & Chapman, 1992).

Another study that supported Theory One utilized a model to fractionate the three-dimensional angular acceleration vector of the segments during an overhand throw into the two-dimensional kinetic and kinematic parameters (Feltner, 1989). The results of this computer model revealed that the elbow extensor muscles created no extension moment as the distal segment began its rapid extension, suggesting that the inertial component of this joint action was the responsible agent for elbow extension. While Feltner was the first to propose that the elbow extensors did not contribute substantially to elbow extension during the overhand throw, it was not until Dobbins (Roberts, 1971) who, through the use of a differential radial nerve block to eliminate triceps activity, was able to provide substantive proof. Dobbins reported that after six practice trials, the participants were able to throw at greater than 85% of the velocity attained before the nerve block. These findings give credence to Feltner's suggestion of the relative lack of contribution of the elbow extensor muscles during the rapid elbow extension during the overhand throw. The notion that the elbow extensors sparsely contribute to elbow extension in the overhand throw was further supported by two other groups, who reported very little elbow extensor activity during the rapid elbow extension using EMG analysis (Toyoshima et al., 1976; Atwater, 1979; Fleisig et al., 1995), each supporting the inertial based Theory One (Ford, 1998)

Theory Two (Ford, 1998) contends that no external torque is applied to the system after the initial acceleration of the system takes place. The system, with some mass, is said to move with a given angular velocity, thus having an angular momentum, which is conserved throughout the action. In this theory, as the proximal segment reaches its maximum angular velocity, an internal muscle moment is applied between the proximal and distal segments to accelerate the distal segment. Because this in an internal moment, the angular momentum is conserved, allowing the more distal segments to rotate faster as they gain the momentum lost by the more proximal adjacent segment, thereby changing the angular velocities but not the angular momentum (Kreighbaum & Barthels, 1985). Very little research was done that supported Theory Two (Ford, 1998) until Putnam's 1983 soccer kicking analysis (Putnam, 1983). Putnam's continued research on proximal-to-distal sequencing included an analysis of punting versus place kicking (Putnam & Dunn, 1987) and comparing kicking, walking, and running (Putnam, 1991), all of which favor the internal moment producing the increase in angular velocity of the distal segment, or Theory Two. Conclusions from the 1991 comparison study revealed that a decrease in thigh angular velocity can occur in the presence of a large hip flexing moment, due to the "resultant joint moment acting at the hip which counteracts the effect of the leg's motion on the thigh, limiting the loss of thigh angular velocity" (Putnam, 1991). Putnam continued by stating that there was no support that the negative angular acceleration of a proximal segment "aids the positive angular acceleration of an adjacent more distal segment" (Putnam, 1991), which is a direct contradiction of Newton's Third Law of Motion (Newton, 1972).

There have been only two overhand studies that have found support for this theory. One study, analyzed female handball players and concluded that an internal moment created the increase in angular velocity of the distal segments (Joris et al., 1985). It must be noted that the increase in the size and mass of the ball was not mentioned as a factor in the overhand throwing pattern of his subjects. Present literature suggests that the increased size and mass of the ball may have altered the throwing pattern, thus altering the muscle activation of the throwing arm (Alexander, 1991) and thereby altering the throwing mechanics. In an electromyographic analysis of the musculature of the throwing arm, a medical group, found significant tricep (major elbow extensor) activity only during the arm acceleration phase of overhand throwing, which includes a rapid elbow extension action (Jobe et al., 1984). The authors of the study did not directly attribute the triceps activity to elbow extension, but this concept was highly inferred, supporting Theory Two.

A comparison of studies supporting Theory One and Theory Two revealed that all analyses used an extension action, whether it was at the knee or at the elbow. To date, there are no studies that have efficiently analyzed a flexion moment with regard to proximal-to-distal sequencing. The flexion moments create a segment position as well as movement pattern that is quite different than an extension moment in either the leg or the arm. The angle between the segments goes from great to small, with a decreased angular position difference at the end of the motion. It would also include a decreased need for joint protection through the contraction of the antagonistic muscles as noted in several throwing papers (Atwater, 1979; Jobe et al., 1983; Wilson et al., 1983; Jobe et al., 1984; Feltner & Depena, 1986, 1988; Werner et al., 1993; Zehr & Sale, 1994; Fleisig et al., 1995; Barrentine et al., 1998; Werner et al., 2006).

Currently there is a wealth of information on the kinetics and kinematics of the overhand baseball throw, which includes a joint extension, but surprisingly little on the underhand softball pitch, which includes a flexion action. Even more surprising is that these two pitching motions have received no direct study to analyze the overhand motion in terms of proximal-to-distal sequencing compared to the somewhat analogous underhand pitch in softball. Furthermore, the agonist/antagonist activation of the primary musculature that would cause the desired joint action at the elbow (extensors in baseball, flexors in softball) has also been neglected in the literature.

Purpose

Therefore, the purpose of this study was to investigate the interaction between segments in relation of Theory One and Theory Two (Ford, 1998) as they function in an overhand baseball throw and an underhand softball throw. In addition, this study endeavored to determine if the musculature that crosses the elbow are analogous in an overhand and underhand throw via electromyographic analysis synchronized with the motion capture system. A tertiary purpose of this study was to expand the current literature by creating an equation that quantifies the moments produced at the shoulder in both the overhand and underhand throwing motions. To date, a moment equation has only been constructed for the elbow (Feltner & Depena, 1989).

<u>Hypotheses</u>

The hypotheses for this study are as follows:

H_{o1}: Overhand and underhand throwing will not alter the way in which maximum velocity of the most distal segment is achieved via proximal-to-distal sequencing.

H_{a1}: Overhand and underhand throwing will utilize different proximal-to-distal sequencing patterns to maximize end-ball velocity.

Research (Roberts, 1971; Atwater, 1979; Feltner & Depena, 1986; Feltner, 1987, 1989; Fleisig et al., 1995) has supported that the overhand throw will elicit a sequential pattern that fits within the structure of Theory One (Ford, 1998). The underhand softball pitch has not been researched in this manner. It is hypothesized that the flexion moment at the elbow within the windmill pitch will fit the model for Theory Two (Ford, 1998), due to the joint position and for the need to protect the joint during the rapid flexion moment (Atwater, 1979; Barrentine et al., 1998; Ford, 1998; Werner et al., 2006).

H_{o2}: There will be no difference in elbow extensor muscle activation within the overhand baseball throw and elbow flexor muscle activation within the underhand softball throw.

H_{a2}: To date, no conclusive evidence supports the hypothesis that the elbow flexors and elbow extensors are analogous with respect to the particular type of pitch. There is a plethora of evidence to suggest the role of the elbow extensors in the overhand throw, but none to suggest the role of the elbow flexors in the underhand throw. However, it is implied that the flexion moment of the underhand throw would necessitate the activity of a flexor muscle activation via Theory Two of proximal-to-distal sequencing (Ford, 1998; Werner et al., 2006).

Limitations

The limitations to this study are listed below:

1. Subjects were recruited from Auburn University varsity baseball and softball teams
2. Five apparently healthy, females (underhand pitch) and males (overhand pitch) above the age of 19 with similar experience and ability levels in their respective sport were used in this study.
3. The participants threw inside a laboratory space while wearing pertinent data collection tools into a net placed 15 feet in front of them.

Delimitations

The delimitations to this study are listed below:

1. Only varsity athletes of the particular sport were included in this study in order to reduce the confounding of gender and ability on the outcome measures.
2. Self reported health and experience levels were used.
3. Athletes were allowed to warm up "as needed" and were allowed to throw their pitches when they felt they were ready to throw at game speed.
4. Athletes were instructed to throw at game speed with that intensity not directly measured.

Definitions

Proximal-to-Distal Sequencing:

A motion is initiated with the larger, heavier, slower, central body segments; then, as the energy increases, the motion proceeds outward to the smaller, lighter, and faster segments, each summating the speed upon the other to create maximal end segment velocity (Plagenhoef, 1971; Bunn, 1972; Zernicke & Roberts, 1976; Putnam, 1983; Kreighbaum & Barthels, 1985; Robertson & Mosher, 1985; Marshall & Elliot, 2000).

Kinetics:

A branch of classical mechanics that focuses on the effects of forces on the motions of an object.

Kinematics:

A branch of classical mechanics that focuses on describing the motions of an object without considering the factors that cause or affect the motion.

Electromyography (EMG):

Electromyography (EMG) is a medical technique for evaluating and recording physiologic properties of muscles at rest and while contracting. EMG is performed using an instrument called an electromyograph, to produce a record called an electromyogram. An electromyograph represents the spatial and temporal summation of all motor unit action potentials in the proximity of the recording electrode. It is indicative of the level of muscle activity via the motor unit recruitment and rate coding.

Moment of Inertia:

Also known as Rotational Inertia, it is the measure of an objects resistance to a change in its rotational motion.

Moment:

The angular equivalent to Newton's Second Law of Motion, $\Sigma F = ma$. The unbalanced force within the sum of all forces produces an acceleration of the said mass.

Joint Force:

The net force of the distal segment acting upon the proximal segment, as analysis proceeds up the segmental chain, Newton's Third Law states that the directionality of the force must be reversed as one moves the analysis from one segment to the next.

Inverse Dynamic Model:

A method in which the equations of motion are employed to determine the unknown components of the forces that produce the motion being analyzed (Allard et al., 1985).

Newton's Third Law of Motion:

> *"Lex III: Actioni contrariam semper et æqualem esse reactionem: sive corporum duorum actiones in se mutuo semper esse æquales et in partes contrarias dirigi.*

All forces occur in pairs, and these two forces are equal in magnitude and opposite in direction.

Chapter II

Review of Literature

The purpose of this investigation is to examine the theories of how adjacent segments, within an open kinetic chain, interact during the acceleration phase of two different pitching patterns. This chapter will be divided into four major sections. The first section will include a discussion of underhand pitching, which will be followed by a similar review of the literature pertaining to overhand pitching. The third section will contain a description and comparison of the two theories of proximal-to-distal sequencing. The final section will be a brief review of the analysis of electromyography of ballistic movements, such as pitching.

Underhand Pitching

Softball first arose as a version of indoor baseball in 1887 and has matured into America's number one team sport, played by over 40 million men and women (Monteleone & Crisfield, 1999; van Wyk et al., 1999). Although there has been a significant amount of research on the biomechanics and injury occurrences in overhand pitching, there is a scarcity of published literature on fast-pitch softball pitching. This limited amount of information may be attributed to the perception that underhand, also known as "windmill" pitching creates less stress on the arm than overhand pitching, and therefore, does not pose as great an injury risk as overhand pitching (Barrentine et al., 1998). This is certainly not the case. In a study of pitchers participating in the 1989 College Softball World Series, researchers (Loosli et al., 1992) determined that approximately 80% of the pitchers reported some type of pitching related arm injury. Furthermore, 82% of those reported time-loss injuries of the shoulder or elbow (Loosli et al., 1992).

Research on the kinematics and kinetics of underhand pitching has shown that the windmill pitching motion can elicit motion variables comparable to overhand pitching and thus put the shoulder/elbow at just as much risk as overhand pitching. For example, research by Barrentine, et al., has shown that the forces and moments at the shoulder and elbow in underhand pitching were between 70-95% (Barrentine et al., 1998) and in some instances (Werner et al., 2006) exceeded those encountered by overhand baseball pitchers. In addition, internal rotational velocity has been measured as high as 5000^{o}/sec in underhand pitching (Barrentine et al., 1998; Hill et al., 2004) which is only slightly below that of overhand pitching, which reaches angular velocities as high as 7510°/sec (Pappas et al., 1985; Feltner & Depena, 1986; Dillman et al., 1993; Fleisig et al., 1995). Furthermore, a study by Maffet and colleagues has shown that the same musculature is the muscles most involved for both pitching motions. Specifically, Maffet and colleagues showed that the major power producer of shoulder motion in both sports was the pectoralis major, while the serratus anterior works to synchronize the motions at the shoulder (Maffet et al., 1997). It was also noted that there are higher levels of muscle activity for the pectoralis minor and subscapularis muscles, which contribute to internal rotation of the humerus during windmill pitching, than in overhand pitching (Maffet et al., 1997). Although the similarities of risk are evident, unfortunately there remains little literature on the biomechanics of the underhand pitch.

To better understand the underhand pitching motion it is advantageous to break the motion down into components. Specifically, Barrentine, 1998, has broken the underhand, windmill pitching motion into 4 distinct phases. (Figure 1) (Barrentine et al., 1998). These four phases are the wind-up, stride, delivery, and follow-through. The wind up phase is defined as the time from the initial movement of the pitcher until lead foot toe-off. During this phase and the stride phase, the majority of the kinetic and kinematic variables of the upper extremity remain minimal (Barrentine et al., 1998; Werner et al., 2006). In the wind-up phase, the shoulder is hyperextended at the shoulder as the pitcher pushes off the pitching rubber with the pivot foot, to initiate movement of the body towards home plate. The next phase is the stride phase, and is defined as the time from lead foot toe-off to lead foot contact with the ground. This phase emphasizes the forward translation and is noted by the maximal linear velocity of the system of the center of mass of the system (Barrentine et al., 1998; Werner et al., 2006). As the pitcher reaches foot contact, the trunk rotates toward the appropriate foul line; for example a right handed pitcher rotates toward the third base line, with the shoulder flexing past 180° to a slightly extended position (Barrentine et al., 1998). The delivery phase is considered the most important phase biomechanically, and contains most of the integral kinetic and kinematic data.

The delivery phase, defined as the time from foot contact to the release of the ball, is the most ballistic portion of the motion and where most of the kinetic and kinematic analyses are conducted. During the delivery phase, the pitcher uses a combination of trunk rotation (pelvis and upper torso), internal rotation at the shoulder, and flexion at the elbow to apply maximum acceleration to the ball. This maximum acceleration is achieved through the arm developing the highest angular velocity and force values that occur during the windmill pitching

Due to the importance of this phase it is valuable to break this phase down further. During the first half of the delivery phase, a flexion moment at the shoulder is used to generate a shoulder flexion velocity that reaches over 5,000° per second (Barrentine et al., 1998; Hill et al., 2004; Werner et al., 2006). Furthermore, at this time, an internal rotation moment is applied to the shoulder to generate internal rotation in preparation for release of the ball. Amazingly, the magnitude of this moment relative to body weight appears to be greater for underhand pitching than overhand pitching (Barrentine et al., 1998) and is similar, in magnitude, to that found in overhand pitching (Barrentine et al., 1998; Werner et al., 2006).

During the middle part of the delivery phase, maximum pelvis and upper torso velocities are reached. As the humerus is flexed, the forearm extends at the elbow, creating a maximum extension velocity of 570°/s, which is dramatically lower than in overhand motions (Barrentine et al., 1998). A flexion moment is then initiated at the elbow and reaches its maximum at the end of the delivery phase (Barrentine et al., 1998). During this time, a maximum compressive force (70 percent of body weight) is experienced at the elbow, followed by a maximum superior force at the shoulder (Barrentine et al., 1998). Similar magnitudes for forces resisting distraction at the shoulder have been calculated for overhand pitching (Feltner & Depena, 1986; Escamilla et al., 1994; Fleisig et al., 1995). Yet, there is a definite difference between the two pitching motions with regard to the timing of peak kinematic and kinetic variables. For example, during underhand pitching, maximum values occur during the delivery or acceleration phase, while maximum values during overhand pitching occur during the negative, follow through phase suggesting the two different pitching patterns will have different sequences of motion (Barrentine et al., 1998). These differences are most likely, due to the more circular nature of the underhand throw, where there does not seem to be as dramatic a moment acting at the elbow as seen in overhand pitching.

Just prior to ball release in the underhand pitch, maximum internal rotation velocity is reached. At this time, a maximum abduction moment and a maximum extension moment are generated at the shoulder to help transfer momentum to the more distal segment and initiate negative acceleration of the upper arm. In the analysis of the kinematics of the upper extremity, Alexander and Haddow (Alexander & Haddow, 1982) observed a sequence of motions that includes the more proximal segments attaining peak velocities before the more distal segments. After reaching peak velocity, the proximal segment is negatively accelerated in order to transfer momentum to the distal segment (Barrentine et al., 1998). During underhand pitching a peak shoulder extension moment is reached as elbow flexion is initiated, enabling the momentum from the upper arm to be transferred to the lower arm (Barrentine et al., 1998). Werner also observed this "windmilling moment" just prior to ball release and believed the purpose was to control the windmill motion (Werner et al., 2006). This action/reaction of the segments indicates that there is a summation of velocites moving in a proximal to distal sequence down the kinetic chain in the underhand, windmill pitching motion.

During the acceleration or delivery phase of the underhand pitch, maximum lateral force and a valgus moment are generated at the elbow (Barrentine et al., 1998). Conversely, in baseball pitching, there is a production of a varus moment to resist the valgus motion, often leading to ulnar collateral ligament injuries (Fleisig et al., 1995), which is considered an uncommon injury in underhand pitching (Loosli et al., 1992). These differences in moments at the elbow during analogous phases of the throw may suggest a different mechanism for reaching maximal ball velocity.

The next phase of the underhand pitch is the follow through. The time period of this phase occurs from the instant of ball release until the forward motion of the pitching arm has stopped and the upper and lower arm are negatively accelerated to a stop. Barrentine reported that a second peak shoulder extension moment occurred during this phase in order to aid in the negative acceleration (Barrentine et al., 1998).

Although it is difficult to determine how these forces and moments are directly tied to the increasing amount of injuries seen by the athletes who play softball, the types of injuries reported by Loosli et al. appear to be related to overuse and the accumulative stress at the shoulder and elbow (Loosli et al., 1992). Tendonitis, rotator cuff strain, tendon strain, and ulnar nerve damage comprised the majority of injuries reported, for all grades of injuries, incurred by underhand throwers (Loosli et al., 1992; Hill et al., 2004). Similar to overhand pitching, the causes of these injuries may be related to maintaining joint stability (Loosli et al., 1992; Fleisig et al., 1995; Hill et al., 2004). However, in the overhand motion, it is maintaining joint stability in the face of high velocities, and high forces, while in underhand motion, it is most likely in the overuse and the relatively high forces encountered during the motion.

Although the position of the shoulder during underhand pitching does not create the same joint instability of the shoulder that occurs in overhand pitching (Fleisig et al., 1995; Barrentine et al., 1998), the motion does require resistance to distraction while also controlling internal rotation and elbow extension. As Werner noted, the forces at the shoulder had the effect of maintaining joint stability by resisting the distraction of the humerus from the shoulder joint caused by the windmill motion (Werner et al., 2006). These resistance forces are considered the cause of many of the underhand pitching injuries that occur in the shoulder, especially in the labrum (Loosli et al., 1992; Barrentine et al., 1998).

A common complaint of softball pitchers is anterior shoulder discomfort near the insertion of the long head of the biceps brachii tendon (Hill et al., 2004). The long head of the biceps brachii tendon, by reason of its insertion into the superior glenoid labrum, normally functions as a humeral head depressor. As discussed by Fleisig et al., the biceps brachii functions to provide elbow flexion moments and aids in resisting humeral distraction during overhand pitching (Fleisig et al., 1995). The same mechanism can be applied to underhand pitching during the delivery phase. Forces required to resist distraction reach a peak during delivery when an elbow flexion moment is exerted to control elbow extension and to initiate elbow flexion (Barrentine et al., 1998). The demand on the bicep/labrum complex to both resist glenohumeral distraction and produce elbow flexion moment makes this structure susceptible to overuse injury (Barrentine et al., 1998; Hill et al., 2004). Internal rotation at the shoulder and pronation of the radio-ulnar joint further complicates the mechanism (Tanabe et al., 1991; Barrentine et al., 1998; Hill et al., 2004). The torsional stress that occurs as the forearm is pronated, through the ball release phase, has been related to stress fracture injuries in underhand pitchers (Tanabe et al., 1991).

Maffet et al. has shown that the teres minor is very active in negatively accelerating internal rotation of the humerus during the delivery phase of the underhand pitch (Maffet et al., 1997), which has also been noted in baseball pitching (Fleisig et al., 1995; Maffet et al., 1997). Eccentric loading and stretching of the posterior muscles of the shoulder girdle through overuse, which does not cause the eccentric loading and stretching, could significantly contribute to dynamic anterior instability of the humeral head. Ultimately, failure or the inability of the posterior support structures to keep the humeral head properly within the glenoid fossa, could advance the symptoms of posterior shoulder pathology (Atwater, 1979; Fleisig et al., 1995; Kvitne et al., 1995). This can result in the rapid deterioration of the shoulder capsule as a result of this cyclical process of fatigued posterior muscular allowing anterior joint movement, causing increased demands on the posterior musculature.

The potential for overuse injuries in underhand pitching is directly related to the volume of pitching. It is not uncommon for softball pitchers to pitch multiple games in one day and/or pitch on consecutive days throughout the season (Hill et al., 2004). It is reasonable to speculate that even with sound mechanics, overuse-type injuries may occur. Loosli et al., as well as Hill et al, found that pitchers reporting grade I or grade II type injuries, which did not result in missed games or practices, pitched more innings per season, on average, than uninjured pitchers (Loosli et al., 1992; Hill et al., 2004) suggesting that volume may play a significant role in injury development. While underhand and overhand pitching motions are different in the overall movement, it is apparent that underhand pitching does subject the underhand pitcher to similar forces and therefore similar injuries. While the similarities are apparent, it is surprising that more research is not done on the injuries associated with fast-pitch underhand pitching.

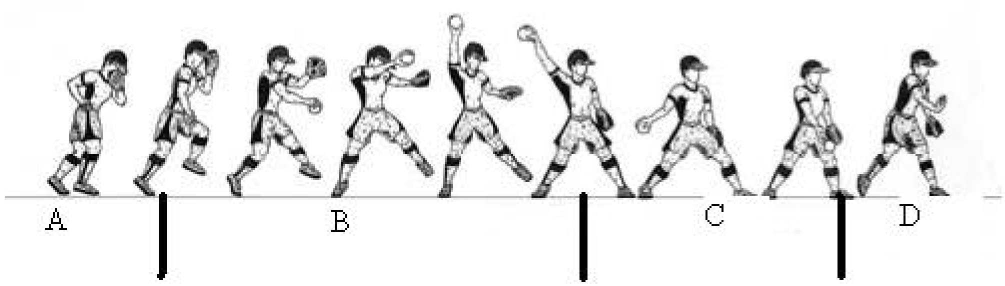

Figure 1. Phases of Underhand Pitch. The four phases of the underhand softball pitch, adapted from Werner et al (Werner et al., 2006). The phases are broken down into A) Wind-up, B) Stride, C) Delivery, and D) Follow-Through.

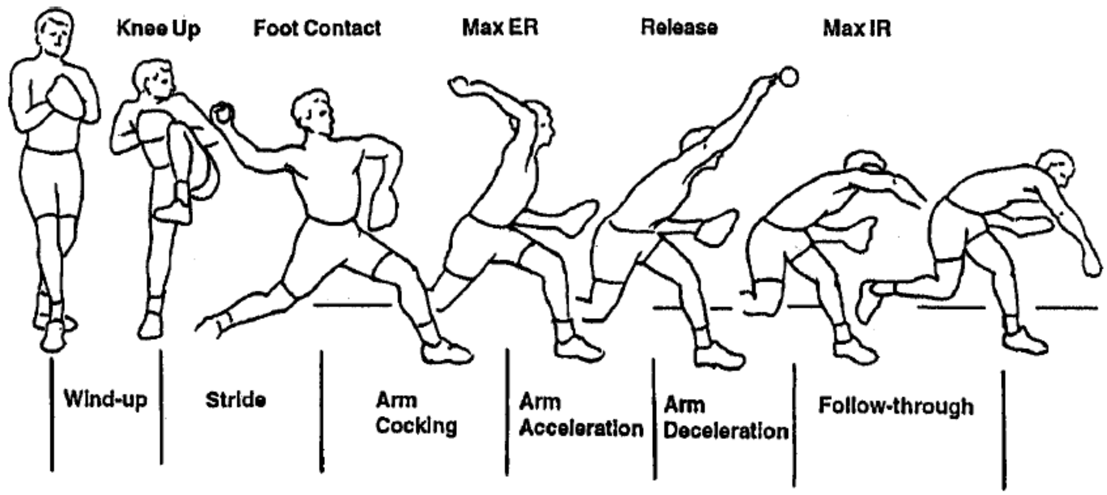

Figure 2. Six Phases of Overhand Pitching. . The six phases of pitching. Images represent the instances separating the phases: initial motion, balance point, stride foot contact, maximum external rotation, release, and maximum internal rotation. Adapted from Fleisig et al (Fleisig et al., 1996a).

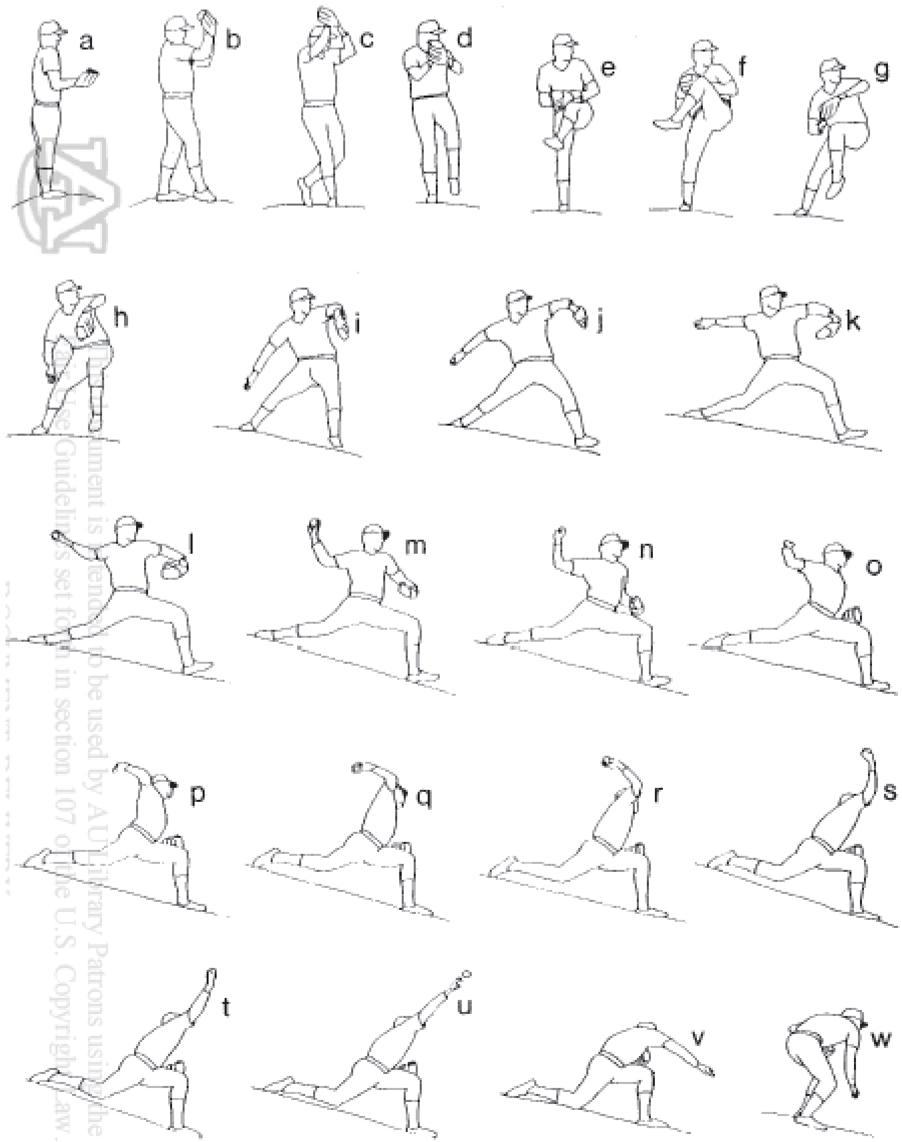

Figure 3. Phases of Pitching. Phases of pitching, adapted from Feltner and Depena (Feltner & Depena, 1986)

<u>Overhand Pitching</u>

To review the topic of overhand pitching is a massive undertaking. However, this review does not attempt to cover all components of the over hand throw rather only those studies that considered variables of interest to the present investigation will be addressed. Specifically, this review will be confined to the kinematics and kinetics of the overhand throw and the implication of those variables to proximal to distal sequencing as well as to mechanisms of injury.

The first analysis of overhand pitching and its implications to injury was completed in 1979 (Atwater, 1979), however, in the years following, there have been relatively few research projects that have analyzed the kinetics of the overhand throw (Gainor et al., 1980; Feltner & Depena, 1986; Werner et al., 1993; Fleisig et al., 1995). Nevertheless, a plethora of information exists on the kinematic variables of the overhand throw (Pappas et al., 1985; Neal et al., 1991; Dillman et al., 1993; Wang et al., 1995; Escamilla et al., 1998; Newsham et al., 1998; Escamilla et al., 2001; Matsuo et al., 2001; Stodden et al., 2001; Hirashima et al., 2002; Reagan et al., 2002).

The first kinematic analysis began with the division of the overhand pitching motion into three distinct phases (Pappas et al., 1985). This three phase analysis was soon expanded as knowledge of the overhand pitch increased. Presently, the literature has accepted the six step stage (Figure 4) presented by Fleisig, et al., and Dillman, et al. (Fleisig et al., 1989; Dillman et al., 1993), which was adapted from the multistage presentation of Feltner and Depena, Figure 3 (Feltner & Depena, 1986). The first stage is considered the wind-up and begins when the player has initiated movement. This phase ends when the pitcher has rotated his trunk 90° from the target and is in a completely balanced position with the stride leg flexed at the hip and knee. From this position, the pitcher strides toward the target and continues until foot contact with the mound is made. Arm cocking begins with foot contact and ends with maximal external glenohumeral rotation, which may reach angles greater than 90° from the typical 90-90 measurement position (Feltner & Depena, 1986; Dillman et al., 1993; Fleisig et al., 1995).

Acceleration, which is the most dynamic stage of the throw, begins with the initiation of internal rotation and horizontal adduction of the glenohumeral joint and continues until maximum internal rotation angular velocity at the glenohumeral joint is reached and the ball is released. At ball release, negative acceleration of the arm is initiated and continues until maximal internal rotation, where the follow-through begins. Within the throw, it is well established that the only phases in which kinetic and kinematic variables play a critical role, in terms of understanding the motion are the arm cocking, arm acceleration, and arm negative acceleration (Feltner & Depena, 1986; Feltner, 1987; Fleisig et al., 1989; Dillman et al., 1993; Werner et al., 1993; Fleisig et al., 1995).

Kinematic results from most studies have been very similar (Feltner & Depena, 1986; Dillman et al., 1993; Fleisig et al., 1995; Wang et al., 1995; Matsuo et al., 2001). All of the cited kinematic analyses have started at the point of maximum external rotation, after the completion of the wind-up, stride phases, at the end of the cocking phase. At this point, the arm is positioned at approximately 15° of adduction as taken from 90°, 10° of horizontal adduction, and 0° of internal/external rotation, with the elbow flexed to roughly 90°(Feltner & Depena, 1986; Dillman et al., 1993; Werner et al., 1993; Fleisig et al., 1995). The upper arm is then internally rotated, adducted, and horizontally adducted until ball release. The kinematic analyses have indicated that during this phase, maximum angular acceleration of glenohumeral internal rotation is reached and has been considered one of the fastest known human movements (Atwater, 1979; Feltner & Depena, 1986; Dillman et al., 1993; Fleisig et al., 1995). In fact, the peak angular velocity, which coincides with ball release, has been shown to vary from 6100 to 7510°/sec in collegiate and professional pitchers (Pappas et al., 1985; Feltner & Depena, 1986; Dillman et al., 1993; Fleisig et al., 1995). During this phase the elbow joint is also rapidly extended and reaches its peak angular velocity (2200-4500°/s) just before ball release (Pappas et al., 1985; Feltner & Depena, 1986; Werner et al., 1993), with a maximum extension value of ~20° (Pappas et al., 1985; Feltner & Depena, 1986; Werner et al., 1993). At the instant of ball release, the angles of abduction and horizontal adduction are both extremely small (~2°) (Pappas et al., 1985; Feltner & Depena, 1986), while the arm was still in an externally rotated position (15-25°) (Atwater, 1979; Pappas et al., 1985; Feltner & Depena, 1986).

After ball release, the negative acceleration of the arm phase begins and the upper arm continues to undergo internal rotation, and also begins to abduct and horizontally adduct (Pappas et al., 1985; Feltner & Depena, 1986; Werner et al., 1993). Neutral glenohumeral internal/external rotation (0°) is typically achieved 1-4 ms after ball release (Feltner & Depena, 1986; Feltner, 1989), as the arm negative accelerates after release. This process continues to negatively accelerate the arm to rest near the contralateral knee.

During the initial stages of the overhand pitching motion, the wind-up and stride stages, critical kinetic and kinematic data is minimal. It is in this phase that the forces, moments, and velocities are small, due to the fact that the arm is simply being brought to a position to create maximal velocity. In these phases the arm positioned at approximately 15° of adduction, 10° of horizontal adduction, and 0° of internal/external rotation, elbow flexion of approximately 90°(Feltner & Depena, 1986; Dillman et al., 1993; Werner et al., 1993; Fleisig et al., 1995). From this position, the elbow is moved forward by a horizontal adduction moment at the glenohumeral joint. This muscle moment has been verified by several studies that have analyzed the EMG activity of the pectoralis major and anterior deltoid (Jobe et al., 1983; Jobe et al., 1984; Feltner & Depena, 1986; Feltner, 1989).

The next phase of the motion, arm cocking, is marked by the pitcher exerting an abduction moment, just after foot contact and is the first critical stage for understanding the kinetic contribution to mechanisms of injury during this motion. This abduction moment lifts the elbow, while the pitcher rotates the trunk toward the target and laterally flexes the lumbar toward the non-pitching side. (Feltner & Depena, 1986). At this point, the shoulder is not only maximally externally rotated to approximately 150-180° (Feltner & Depena, 1986; Dillman et al., 1993; Werner et al., 1993; Fleisig et al., 1995), but is also abducted to an average of 20°, and horizontally abducted to an average of 26° (Feltner & Depena, 1986; Dillman et al., 1993; Werner et al., 1993; Fleisig et al., 1995). Interestingly, just prior to the maximal external rotation, a peak abduction and internal rotation moment were found in all of the kinetic analyses (Feltner & Depena, 1986; Dillman et al., 1993; Werner et al., 1993; Fleisig et al., 1995) suggesting that an opposite moment is applied just prior to maximum external rotation. This finding has tremendous implications for understanding proximal to distal sequencing and the beginning mechanisms of injury. Although there is a large range of maximal external rotation occurring during this stage, it is occurring against the aforementioned internal rotation moment (Feltner & Depena, 1986; Feltner, 1989; Fleisig et al., 1995). This motion was explained by Feltner and Depena in the following description (Feltner & Depena, 1986). The anterior muscles of the shoulder create a horizontal adduction moment, which creates a external rotation, angular acceleration and a linear adduction acceleration of the humerus at the shoulder. This forward acceleration creates a negative moment on the lower arm causing it to externally rotate the glenohumeral joint. This indirect mechanism allows the muscles that create horizontal adduction to also create external rotation against an internal rotation moment (Feltner & Depena, 1986; Fleisig et al., 1995). This mechanism agrees with a similar mechanism proposed by Kreighbaum and Barthels (Kreighbaum & Barthels, 1985) in that the external rotation is produced by the inertial lag of the forearm and ball as the proximal segments accelerate forward. While the external rotation is occurring due to the combination of abduction and horizontal adduction (Feltner & Depena, 1986; Fleisig et al., 1995), a portion of the internal rotation moment is transmitted down the humerus to the elbow and, as said earlier, causes the forearm to exert an external rotation moment at the shoulder. In effect, this combination causes the upper arm and lower arm to move in opposite directions at the elbow (the upper arm internally rotating about its long axis, and the forearm rotating counter-clockwise about the long axis of the upper arm, at the elbow). This arrangement places tremendous stress at the elbow, especially to the ulnar collateral ligament and the medial portion of the elbow (Atwater, 1979; Feltner & Depena, 1986; Werner et al., 1993).

It is this stress to the ulnar collateral ligament, reported as high as 120 Nm (Fleisig et al., 1995), that is thought to cause most of the valgus stress injuries at the elbow, which has been shown to withstand only 60-68 Nm of moment (Atwater, 1979; Wilson et al., 1983; Feltner & Depena, 1986; Feltner, 1989; Werner et al., 1993; Fleisig et al., 1995). This situation has been termed valgus extension overload (Wilson et al., 1983) due to the relative commonality of the injury to overhand pitchers. It is important to note that at this point, the elbow is also flexed to an angle of 88-105° (Feltner & Depena, 1986; Dillman et al., 1993; Werner et al., 1993; Fleisig et al., 1995), with the ulna collateral ligament (UCL) generating 54% of the varus moment needed to resist the valgus stress placed on the elbow (Morrey & An, 1983). If the mean valgus moment of 64 Nm at the elbow is assumed, with the ulna collateral ligament providing 34.6 Nm (54% of 64 Nm) and cadaveric evidence showing the UCL can withstand only ~33 Nm of moment before failing (Dillman et al., 1991), it is easy to see why there is such an abundance of UCL injuries occurring in pitchers at higher levels of play. Furthermore, it is hypothesized that in adolescents whose ligaments are still developing, the failure point may be much lower. This hypothesis, along with the increased training load of younger athletes, helps to explain the incidence of a growing number of UCL reconstruction surgeries being done on adolescent throwers.

Toward the end of this arm cocking phase, just before maximal external rotation, rapid elbow extension begins (~2250°/s)(Feltner & Depena, 1986; Werner et al., 1993; Fleisig et al., 1995), however there is a very small elbow extension muscular moment (~30 Nm) found at this point of the motion (Atwater, 1979; Feltner & Depena, 1986; Feltner, 1989; Werner et al., 1993). This minimal elbow extension moment has been verified through EMG analysis of the triceps during the overhand throw, which has shown extremely little elbow extensor activation (the triceps are relatively quiet) while the elbow is extending during the acceleration phase of the pitch (Roberts, 1971; DiGiovine et al., 1992). These data suggest that the extension of the elbow is not due exclusively to the action of the triceps, but to the resultant joint forces exerted on to the lower arm by the upper arm at the elbow (Roberts, 1971; Feltner & Depena, 1986). The activation of the triceps that has been reported has been considered to be associated with the resistance of a centripetal flexion moment (Feltner & Depena, 1986),maintaining elbow position and providing the most effective moment-arm for the more forceful rotations that are utilized to propel the ball at large velocities (Feltner & Depena, 1986; Feltner, 1989; DiGiovine et al., 1992).

The transition into the next phase of the motion, the arm acceleration phase, occurs at the instant of maximal glenohumeral external rotation, where (Atwater, 1979; Feltner & Depena, 1986; Fleisig et al., 1989), the arm undergoes an extremely rapid motion of glenohumeral internal rotation with the rapid elbow extension (Atwater, 1979; Feltner & Depena, 1986; Fleisig et al., 1989; Fleisig et al., 1995). It is thought that this internal rotation is due to the release of stored elastic potential energy as a result of the stretch of the internal rotation musculature during excessive external glenohumeral rotation, and the inability of the abduction and horizontal adduction moments to cause any external rotation when the elbow is extended (Feltner & Depena, 1986; Feltner, 1989). Just before full elbow extension and at maximum, internal rotation angular velocity, ball release occurs.

Ball release marks the transition between the arm cocking and the arm negative acceleration phases of the motion. At this point, a flexion moment occurs at the elbow along with the internal rotation and flexion moment at the shoulder. It is thought that these moments contribute to reducing the stress placed on the elbow during maximal external rotation by contributing a varus resistant moment enabling the UCL to better withstand the valgus loading (Feltner & Depena, 1986; Werner et al., 1993; Fleisig et al., 1995) rather than helping to resist full elbow extension just after ball release or negatively accelerating the arm.

Following ball release, a second critical kinetic phase occurs in relation to injury, as the body attempts to negatively accelerate the arm (Fleisig et al., 1995). At this point, the arm is outstretched towards the target with the elbow flexed only ~20°, the glenohumeral joint externally rotated ~65°, abducted to ~90°, and horizontally adducted to ~5° (Feltner & Depena, 1986; Fleisig et al., 1995). This anatomical arrangement imparts a maximum compressive force on the shoulder of ~900 N (Feltner & Depena, 1986; Fleisig et al., 1995), potentially creating a degenerative force in the labrum due to the "shoulder grinding factor" (McLeod & Andrews, 1986; Fleisig et al., 1995). This continual degenerative force along with the elbow flexor moment, which originates at the glenoid labrum through the long head of the biceps and is active during the negative acceleration phase to inhibit elbow flexion (DiGiovine et al., 1992), is the mechanism that is considered to be responsible for most SLAP Lesion injuries (tear to the superior labrum anterior and posterior) (Andrews et al., 1985; McLeod & Andrews, 1986; Snyder et al., 1990).

Proximal-to-distal Sequencing

Several biomechanical principles have been devised in order to explain the interaction of adjacent segments within the kinetic chain during various activities. The most prominent include the summation of speed principle, summation of force principle, and the transfer of angular momentum principle. The most overriding idea of these concepts is the summation of speed principle (Bunn, 1972), which describes proximal-to-distal sequencing, but gives no mechanical explanation of the interaction between adjacent segments (Putnam, 1993). Proximal-to-distal sequencing states that a motion is initiated by the larger, slower central body segments, then transfers the energy on to the small, faster distal segments which build on the speed of the larger proximal segments (Bunn, 1972; Zernicke & Roberts, 1976; Kreighbaum & Barthels, 1985; Robertson & Mosher, 1985). The summation of speed principle was first introduced, although not in its current form, in 1950 by Moorehouse and Cooper (Morehouse & Cooper, 1950) when they introduced the "early" and "late" sub-optimal patterns of activity. The early pattern states that the distal segments initiate forward movement before the proximal segments reach peak angular velocity, while the "late" pattern states that the distal segments initiate forward movement after the proximal segments reach peak angular velocity (Morehouse & Cooper, 1950). Although it is possible to extract the basic tenants of proximal-to-distal sequencing from the patterns states approached posited by Morehouse and colleagues, this group was not given credit for presenting the basic premise of proximal to distal sequencing. Roberts and Metcalfe (Roberts & Mechalfe, 1967) also provided a basis for the idea of proximal to distal sequencing when they provided the first detailed description of a soccer kick, which stated that the thigh came to rest prior to contact with the ball and that this must somehow influence the speed of the distal segment. They were still unclear, however, on how the body achieved the speed of the distal segment (Roberts & Mechalfe, 1967). John Bunn was given credit for formally introducing the concept of proximal-to-distal sequencing, when he noted the transfer of speed down the segments and stated that the "jerky movements" were not as effective as the "smooth rhythmic passing of speed" in the pitching motion (Bunn, 1972). Bunn further explained that in order to achieve maximal velocity in pitching, the movement should start with the more proximal segments and progress to the more distal segments. In addition, Bunn suggested that the motion of the distal segment would begin at the instant that the more proximal, adjacent segment reaches maximum speed. This progression would therefore, allow the more distal segment to achieve a greater speed than the more proximal, adjacent segment (Bunn, 1972). Although Bunn (Bunn, 1972) was given credit for the development of this theory, a computer simulated model, by Plagenhoef (Plagenhoef, 1971) using Dempster's anthropometric data (Dempster, 1955), predicted a similar segmental interaction one year before Bunn's textbook was published (Bunn, 1972). The Plagenhoef model proposed that the segments acted in a proximal-to-distal sequence which summated the speed of each segment to create a maximal end velocity of the most distal segment (Plagenhoef, 1971). Kreighbaum and Barthels further expanded the theory when they stated that "each distal segment comes forward as the movement of its proximal segment reaches its greatest angular velocity" (Kreighbaum & Barthels, 1985). This same group also concluded that this sequential motion is used by experienced throwers or strikers when attempting to maximize velocity (Kreighbaum & Barthels, 1985). Next, Alexander (Alexander, 1991) created three distinct models of pitching: overhand sequential, underhand sequential, and overhand simultaneous, each modeled with two segments. Each of these motions was shown to produce the most end ball velocity when there was some sort of sequential acceleration of the segments of the kinetic chain. It is important to note that the sequential motion in the more simultaneous, push throw was seen in the lower body (Zatsiorsky et al., 1981) then to the upper body; in no way indicating the upper body motion was sequential. Alexander also stated and was supported in textbooks (Nordin & Frankel, 2001; Enoka, 2002) that this sequential manner of pitching was an important marker for skilled performers of pitching tasks and plays a role in preventing injury, as well.

It must be noted that the original work of Kreighbaum and Barthels greatly enhanced the knowledge in this area, but also unintentionally created an area of great debate. In one statement of their book, they state that the angular momentum of the more distal segments is being conserved, but the angular momentum of the original system is not conserved because of the application of an external force (Kreighbaum & Barthels, 1985). This external force acts to negatively accelerate the proximal segment as it reaches its maximal angular velocity, allowing the distal segment to accelerate rapidly with regard to the adjacent segment. Later in the text, the authors state that an internal moment changes the angular velocity of the individual segments in the system, but not the angular momentum of the entire system (Kreighbaum & Barthels, 1985). The authors proposed that this internal moment is created by a muscle action that crosses from the proximal to distal segment (Kreighbaum & Barthels, 1985). The former statement by Kreighbaum and Barthels suggests that an external force causes a negative acceleration of the proximal segment, which would cause a the distal segment to be propelled forward, yet the latter statement concludes that the distal segment is thrust forward due to the internal muscle moment (Kreighbaum & Barthels, 1985). These two conflicting statements have led to a division in the field of biomechanics regarding the mechanism of proximal-to-distal sequencing. Although both theories are based on the summation of speed principle in conjunction with the principle of conservation of angular momentum (Atwater, 1979; Alexander & Haddow, 1982), they are not analogous. It is this discrepancy that has given rise to the following two theories.

Theory One: External Moment

Theory one (Ford, 1998) states that once the motion of the system begins, angular momentum is developed within the system with the distal segment lagging behind the proximal one. As the proximal segment attains maximal angular velocity, an external force opposes the motion and negatively accelerates the proximal segment, allowing the inertia of the distal segment to drive the distal segment forward. These “external” forces are often defined as muscular forces acting on the proximal end of the proximal segment. For example in the overhand throw, the negative accelerators or external rotators (teres minor, posterior deltoid, and infraspinatus) act on the proximal portion of the humerus to slow down the internal rotation moment during the end of the acceleration phase. Because the mechanical system used to describe proximal-to-distal sequencing typically only includes the distal end of the proximal segment to the proximal end of the distal segment and the muscles that cross that articulation, these muscular forces can be considered external (Plagenhoef, 1971).

There is support for Theory One in both the kicking and pitching literature. The most prominent support comes from the seminal paper of overhand pitching authored by Michael Feltner and Jesus Depena (Feltner & Depena, 1986), and was followed by their application papers 3 years later (Feltner, 1989; Feltner & Depena, 1989). In a series of studies dedicated to the kinetic analysis and interpretation of the cause-effect relationship between the musculature and motions of the arm in overhand pitching, Feltner and Depena conducted a three-dimensional analysis of the shoulder during pitches made in practice situations (Feltner & Depena, 1986). The study included 8 collegiate pitchers, with the pitching arm being modeled as a four-link kinetic chain composed of the upper arm, forearm, hand, and ball. The study concluded that the extension moment at the elbow was extremely small compared to the other moments during the acceleration phases of the throw. This finding suggests that the extension of the elbow was not due to muscular force from the triceps, but rather due to the inertia of the lower arm obtained from the resultant force from the upper arm, which would point from the elbow to the shoulder. The authors proposed that this force could be associated with the resistance to a centripetal acceleration force at the elbow joint as the upper arm completed the abduction and horizontal adduction about the shoulder joint and (Kreighbaum & Barthels, 1985) the linear acceleration of the trunk towards the non-pitching side during the acceleration phase of the overhand throw. This supports Theory One by providing evidence that an internal moment was not needed to propel the distal segment during a maximal velocity throw.

In the later application paper, Feltner (Feltner, 1989) attempted to clarify the mechanical relationships that produce the motions of the pitching arm segments during baseball pitching. He used a similar subject pool, but proposed a different model. Specifically, the researchers used a 2 segment, kinetic chain model that was composed of the upper and lower arms. The conclusions of this paper also supported Theory One in that it found that the musculature at the shoulder (external force) produced an external moment at the elbow causing the lower arm to increase its velocity and whip forward. The authors also proposed that the flexion moment at the elbow is, in fact, used to negate an external extension moment, which is thought to be an inertial force, potentially passing the acceleration down the kinetic chain, allowing for an increased angular velocity at the wrist (Feltner, 1989).

In addition to the work of previous researchers, one group (Wang et al., 1995) studied maximum external rotation of the shoulder and subsequent time to maximal acceleration to pitching performance. The study reported a decrease in wrist velocity when an increased negative acceleration at the wrist occurs just prior to an increased velocity and acceleration of the hand just prior to ball release. This revelation lends support for a negative acceleration causing a positive inertial based acceleration of the distal segment.

Furthermore, in an attempt to quantify total body sequencing, Elliot and colleagues (Elliot et al., 1988) studied the timing of lower limb stabilization to that of the pitching limb. The group considered the lead leg the proximal segment with the upper body and arms the distal segment, and concluded that when the front leg is stabilized (negatively accelerated to a stable position), it caused the upper body to sequentially angularly accelerate as suggested by Theory One (Ford, 1998).

Herring and Chapman also found support for Theory One (Ford, 1998) (Herring & Chapman, 1992) using simulated throws. An overhand throw in the sagittal plane was simulated using a three-segment model representing the upper arm, forearm, and hand plus ball. Moment inputs at each joint were turned on at systematically varied times and maintained once initiated. The aim was to determine the sequence of onset of joint moments that gave maximal velocity of the ball irrespective of direction of ball release. Best throws were noted during those throws in which the onset of moments was sequential, in a proximal to distal temporal sequence. The direction of the shoulder and elbow moments was reversed instantaneously at peak velocity to represent the use of antagonistic muscles. This led to increased end ball velocity if performed late in the throw and in conjunction with a proximal-to-distal sequence. It was concluded that the use of antagonistic muscles, to negatively accelerate the segment, leads to beneficial transfer of angular velocity from more proximal segments to more distal segments.

Research from kicking studies lend more support to Theory One (Ford, 1998). Similar studies completed by Robertson and Mosher (Robertson & Mosher, 1985) and Zernicke and Roberts (Zernicke & Roberts, 1976) as well as a kicking simulation conducted by Marshall (Marshall & Wood, 1986) indicated that an activation of the hip extensors negatively accelerates the upper leg just prior to ball contact, allowing the lower leg to "whip" (Robertson & Mosher, 1985) forward to kick the ball. In addition, support for this theory was found with similar agonist and antagonist muscle activation pattern noted when kicking a soccer ball suggesting that there is no internal agonist muscle activation causing the lower leg to move forward.

Further work was conducted on the lower extremity during the running motion. In a computer modeled simulated running pattern, the thigh motion was found to have significant effects on the motion of the lower leg. Specifically, the hip flexion moments were decreased as knee extension increased, without the aid of muscular activation (Phillips et al., 1983).

An interesting study (Roberts, 1971), used a differential radial nerve block technique to eliminate the any contribution of the triceps to overhand pitching. The study revealed that following the nerve block, which eliminated the contribution of the triceps, pitchers were able to throw at over 80% of their maximal velocity before the nerve block, after only 6 trials. This shows the relative unimportance of the triceps, the primary elbow extensor, for elbow extension during the overhand pitching. The results of this study give substantial support to Theory One, by showing that an internal muscle moment was not needed to achieve maximal velocity at the distal segment. Further support that the elbow extensors were inactive during the overhand throw was provided in an early study that analyzed the muscle activity of the body during the overhand throw. This study, using collegiate pitchers, revealed that the triceps were virtually silent during the acceleration phase of the throw (Toyoshima et al., 1976).

It must be noted that with each article that supports Theory One, the motion analyzed was an extension movement, where the joint angle increased as the time period moved towards ball release or ball contact. To date, there is no published research to support the idea that Theory One can occur with a flexion moment.

Theory Two: Internal Moment

Theory Two (Ford, 1998) of proximal-to-distal sequencing states that the angular velocity of the system implies an angular momentum. As the proximal segment achieves maximal angular velocity, an internal muscle moment is applied to the distal segment causing it to rotate about the articulation. Due to the internal moment applied to the distal segment, it will have an increased velocity via an increased acceleration. For this to happen however, the principle of conservation of momentum maintains that as a force is applied to the more distal segment to cause it to positively accelerate, increasing its velocity, an equal but opposite force must be applied to the proximal segment causing it to be negatively accelerated, causing a decrease in velocity of the proximal segment

(Ford, 1998). This relationship thus satisfies Newton's Third Law of Motion, which states that for every action, there is a reactive force that is equal in magnitude, but opposite in direction (Newton, 1972).

Research based conclusions supporting this theory were first put forth in the early 1980's (Putnam, 1983) when researchers began to study the interactions of kinetic segments within the kicking motion. This first work indicated a decrease in the angular velocity of the upper leg as the lower leg accelerated, leading Putnam to the conclusion that the increased angular velocity of the lower leg was responsible for the decreased angular velocity of the upper leg. This conclusion supports the application of the action/reaction principle (Newton, 1972) as it pertains to Theory Two (Ford, 1998). Putnam's research has been a strong proponent of Theory Two (Ford, 1998), and has stated in several of her research articles that there was no evidence to suggest the negative angular acceleration of the proximal segment aids in the positive angular acceleration of the more distal segment (Putnam, 1983, 1991, 1993). The same author's later kicking (Putnam & Dunn, 1987) research came to the same conclusions when a kicking motion was analyzed at three different speeds. In 1991 (Putnam, 1991) compared the actions of the kicking motions to that of the leg swings of walking and running in order to determine the interaction between segments in the leg during movements of different speeds (fast, medium, slow). Viewed as a two segment model (upper leg and lower leg), Putnam stated that as the shank achieved maximum angular velocity, the resultant moments caused the upper leg to negatively accelerate, meeting the internal moment criteria set forth for Theory Two (Ford, 1998). Still within the kicking literature, but shifted to martial arts, Sorenson, et al. stated that there was no active negative acceleration of the thigh even with the noticeable decrease in velocity during the time period that the shank experienced a dramatic positive angular acceleration (Sorenson et al., 1996). Luhtanen and colleagues also drew these same conclusions about the upper body in a study of the interaction of the segments during a volleyball strike (Luhtanen, 1988). The authors analyzed the kinetics and kinematics of volleyball players of different levels, revealing no reactive accelerations in the kinetic chain (Luhtanen, 1988).

In a 1993 review of literature on the sequence of segmental interaction, (Putnam, 1993), Putnam indicated that the "forward acceleration of the distal segment is largely a result of the way the proximal segment interacts with the distal segment as a function of the segment's angular velocity" (Putnam, 1993). The negative acceleration of the proximal segment was thought to be the consequence of the interactive moments at the proximal end of the distal segment due to the change in angular acceleration of the proximal segment (Putnam, 1993). Putnam also supported the internal moment theory early in her career using a computer simulated model of sprinting that elicited data suggesting that when a positive increment of the knee's reactive joint moment was introduced to the system, there was a concurrent positive acceleration of the lower leg and a negative acceleration of the thigh (Putnam et al., 1987).

Further evidence of Theory Two in overhand throwing was reported by Joris and colleagues (Joris et al., 1985). Using 52 handball players to calculate the segment interactions of the upper arm to the lower arm and the flow of energy in the overhand throw, the research group (Joris et al., 1985) concluded that it seemed more likely that the negative acceleration of the proximal segments is simply explained by Newton's Third Law, which says that for every action on a more distal segment, there must be an equal in magnitude but opposite in direction action on the more proximal segment (Joris et al., 1985), which is the basis of Theory Two (Ford, 1998). Joris continued by stating that the increase in velocity of the distal segment is mainly the result of the preceding actions of the more proximal segments (Joris et al., 1985).

In an electromyographic analysis of the muscular activity of 29 muscles of the shoulder girdle and upper extremity during overhand pitching using collegiate or professional level athletes, it was found that the triceps were extremely active during the acceleration phase of the throw (DiGiovine et al., 1992). This was hesitantly attributed to the resistance of the centripetal flexion moments at the elbow. The researchers later contradicted themselves when they stated that the “triceps-activated” elbow extension was used in the acceleration phase or to resist the negative flexion moment (DiGiovine et al., 1992). It must be noted, that for a muscle to be in a non-isometric contractile state, an extension moment is needed to resist a flexion moment is the same as creating an internal force to accelerate an extension of the joint, lending credence to Theory Two. Members of the same research group led by Dr. Frank Jobe also performed two EMG studies of the biceps, triceps, pectoralis major, latissimus dorsi, seratus anterior, and brachialis during the pitching motion of four professional baseball pitchers (Jobe et al., 1983; Jobe et al., 1984). In these reports, it was noted that the triceps started to contract at the end of the cocking phase and maintained a high level of activity throughout the acceleration phase. The research group also concluded that due to the continuation of the triceps activity into the follow through phase, it was an active motion, not just a passive phenomenon (Jobe et al., 1984). It must be noted that this is the only literature to suggest this notion.

Summary:

To date, there has been very little research to actively compare the two theories of proximal-to-distal sequencing within the same study. Most of the literature supporting the external moment theory (Theory One) has been conducted using overhand pitching, but still remains inconclusive. Similarly, much of the research conducted supporting Theory Two has been completed by one research group (Ford, 1998), with little support from other research. There has been one study, an unpublished doctoral dissertation (Ford, 1998), that attempted to compare the two competing theories. Within that study, the author compared overhand softball throws and volleyball serves as well as underhand softball throws and volleyball serves. For purposes related to this paper, the review of the research of Ford (Ford, 1998) will focus mainly on the pitching motions, excluding the volleyball serves. In both cases, the linear acceleration of the shoulder as well as the shoulder moments caused the acceleration of the proximal segment. In the overhand throw, elbow joint moments caused an acceleration, which caused the distal forearm segment to lag behind in the flexed position. In the underhand throw, the linear acceleration of the shoulder and the angular acceleration of the upper arm caused the distal segment to lag in an extended position. In both instances, the linear acceleration of the shoulder and the moments at the elbow caused the distal segment to accelerate forward (extension for overhand, flexion for underhand), with the major contribution of slowing down the proximal segment being attributed to the linear acceleration of the shoulder. Each of these conclusions made by the researcher (Ford, 1998) suggests that the kinetic segments follow the pattern of Theory One, regardless of the overhand or underhand action (Ford, 1998). The results of this study must be taken with some caution. The subjects of this study were volleyball players with very little, if any softball experience. This would play a significant role in the sequential, segmental interactions during the throws (Atwater, 1979; Alexander, 1991; Fleisig et al., 1995; Enoka, 2002). This introduction of a novice task was implied by the author (Ford, 1998), but not stated, yet has been thoroughly discussed in many research articles (Atwater, 1979; Alexander, 1991; Fleisig et al., 1995; Enoka, 2002). The study was also done using a constructed three-dimensional model, allowing for greater error within the data collection.

Given the absence of published material on the comparison of the two theories, this research project intends to combine the methods of several studies in order to analyze the two theories of proximal-to-distal sequencing within the context of overhand and underhand throwers.

Electromyography

Propelling a ball for maximal velocity can certainly be considered a ballistic movement. While the term ballistic is often referred to as rapidly moving segments, the true definition includes not only the muscle actions used to initiate the movement, but also includes the momentum that stops the motion (Luttgens & Hamilton, 2002). This same group (Luttgens & Hamilton, 2002) indicated that ballistic movements are brought to an end in one of three distinct patterns. These patterns include the activation of the antagonist muscle(s), the segment reaching its maximum range of motion, or interaction of with an outside object. In pitching studies, these three possibilities can be reduced as pitching motions will only utilize patterns one or two.

In his text, Roger Enoka (Enoka, 2002) states that the various throwing activation patterns can be seen in electromyographic analysis of the desired movement. The traditional response to a ballistic movement is a triphasic EMG response (Enoka, 2002).

This three bust pattern of EMG is seen in the accelerating muscles when an individual performs goal directed movements, such as propelling a ball at maximum velocity with maximum accuracy. This pattern includes an initial burst of agonist activity followed by a burst of the antagonist, immediately followed by the second burst of the agonist muscle (Wachholder & Altenburger, 1926; Enoka, 2002). As the amplitude and specificity of the movement increases, so will the amplitude and duration of the first burst, followed by a longer delay to the onset of the antagonist muscle activity, and a following larger burst of the agonist activity (Corcos et al., 1989). This finding led to the hypothesis of a speed dependent strategy proposed by the same research group (Almeida et al., 1995) that involves an increase in EMG activity as the moment increases along with a decrease in the latency to the onset of the antagonist muscle activity.

While this theory was considered the rule for several years, it has recently fallen out of favor with researchers attempting to analyze ballistic movements. Currently, research is geared towards the synchronization of EMG equipment to motion analysis equipment, allowing for direct analysis of muscle activity during specified motions. Based on the specifications of the system used in this research, the current method of direct comparison will be used to analyze specific muscle activity in order to compare segment interactions between overhand and underhand pitching. The present study will use similar analyses used by Jobe and DiGiovine (Jobe et al., 1983; Jobe et al., 1984; DiGiovine et al., 1992) to analyze major muscle groups of the throwing arm during the pitching motion, such as the pectoralis major, triceps, anterior deltoid, and biceps brachii.

Chapter III

Methods

The purpose of this study was to compare the kinetics and kinematics of the overhand baseball pitch and the underhand baseball pitch. Specifically, an analysis was be conducted to categorize the method of proximal-to-distal sequencing used by each of the throwing types. Furthermore, this study compared the muscular contribution of the appropriate musculature that crosses the elbow for the two throwing types. The purpose of this chapter is to provide the reader with an overview of how the study was conducted, in order to allow for the systematic replication of the research project.

Sample

Ten total collegiate level athletes (5 male baseball pitchers and 5 female softball pitchers), were recruited from the Division I-A Varsity Baseball and Softball Teams from a university in the Southeast United States, respectively. Only those participants who are of varsity level, free of current injury, and active on the team were included in the sample. Each participant was then asked to sign a university approved informed consent form to indicate his or her voluntary willingness to participate in this project (Appendix F). Prior to the initiation of the throwing portion of the research, the following anthropometric measurements were taken: body height, body mass, and limb segment lengths. These lengths were taken to aid in data analysis.

Throwing Motions

The throwing motions were conducted at game speed by the study participants. In order to ensure maximal results, the athlete was allowed to warm up as much as needed.

Underhand Softball Pitch:

The pitchers were asked to throw 5 rise ball pitches at game speed into net placed 15 feet in front of the athlete. The rise ball pitch is considered the most common pitch thrown by this group of athletes (Werner et al., 2006). To throw a rise ball, the pitcher positions the throwing hand under the ball, with the palm up and the wrist radially deviated at ball release. The forearm then supinates during the delivery phase to impart backspin to the ball to make it “rise”. The pitch viewed to be the highest quality motion capture at the greatest end velocity will be chosen for kinetic and kinematic analysis. The ball had a circumference of 12 in (30.5 cm), and the mass of the ball was 6.5 oz (0.18 kg).

Overhand Baseball Pitch:

The baseball pitchers were asked to throw 5 “four seam” fastball pitches at game speed into a net placed 15 feet front of the athlete. The “four seam” fastball is the most common pitch thrown by an overhand pitcher. To throw a “four seam” fastball, the pitcher positions the hand behind the ball, with his fingers across the seams. The wrist will then flex, while the forearm slightly pronates upon release. This type of pitch is used to impart maximal velocity upon the ball, with as little motion as possible. The pitch viewed to be the highest quality motion capture and greatest velocity will be chosen for kinetic and kinematic analysis. The ball had a circumference of 9 in (22.9 cm), and the mass of the ball was 5 oz (0.14 kg). In order to use a game like situation, the overhand athletes will throw off of a custom manufactured pitching mound that meets NCAA requirements of slope (1 inch of drop per 1 foot of distance). It has been shown in previous research that the throwing kinetics are significantly different when throwing from the flat ground and from a mound (Fleisig et al., 1996b; Badura et al., 2003).

Instrumentation

The model configuration at a sequence of instants throughout the duration of the pitching motion will be determined by video. Highly reflective markers were attached to the subject. Six video cameras, backlit with spotlights, were used to record the motion. The six cameras were positioned to surround the subject. The cameras acquired fields at 180 Hz using a 1/1000 second shutter speed and operate in a synchronous fashion. Any markers visible to a camera appeared as a bright spot. A digital image processing system (Motion Reality Inc., Marietta, GA) was used to locate and record the positions of all markers visible in each of the cameras. Upon identification of the markers in the first captured field, subsequent tracking and identification was performed automatically by the system. Three-dimensional locations of any markers visible in two or more cameras were generated. A model configuration for each field was then generated that is as consistent as possible with the marker locations.

The determination of the system variable values from the marker locations were achieved through an optimization process. It is enabled by a number of preliminary measures that will establish the model characteristics. There were a total of thirty-five permanent markers and six temporary markers, however only a portion of these were used for the study. The permanent markers used for a analysis in this study were placed on the second and fifth carpo-phalangeal joints, the radial stylus, the lateral epicondyle of the elbow, at the center of mass on the lateral portion of the upper arm and the acromion process of the scapula. Additional markers were placed on the center of the spine of the scapula, the apex of the lower angle of the scapula, and near the center of the sternum (Figure 4 & 5). Additionally, the two temporary markers were used to establish the relationship between all of the markers and the underlying anatomical landmarks. One temporary marker was placed on the anterior deltoid, while the other was placed on the medial portion of the elbow, near the antecubital space. The performer then stood in the camera volume with the shoulders abducted to approximately 70°, so that the positions of the markers can be captured to define the coordinates of each marker. Following the removal of the temporary markers, the participant assumed the preparatory position for the appropriate type of pitch. Upon a signal from the experimenter, the participant executed a pitch 5 consecutive times. The pitch producing the maximal end velocity was used for analysis.

The optimal model configuration was that configuration that was most consistent with the measured global marker coordinates. Consistency was defined as a minimized error function. The error function is considered the sum of squared error terms over each marker. The error for any marker was the difference between its measured global position (by the camera) and its predicted position. The predicted position is the global position that one would predict the marker to be based upon its local coordinates and the current model configurations. Therefore, the system variables were determined as those values that minimize the sum, over all the markers, of the squared distances between the global position of the marker and its predicted position. Furthermore, since these variables depend upon all of the marker coordinates; their sensitivity to errors in any one marker coordinate should be reduced (Lu & O'Connor, 1999).

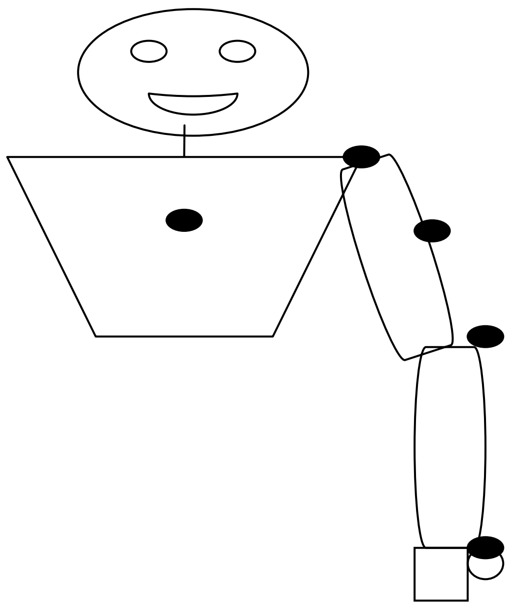

Figure 4. Anterior View of Marker Locations

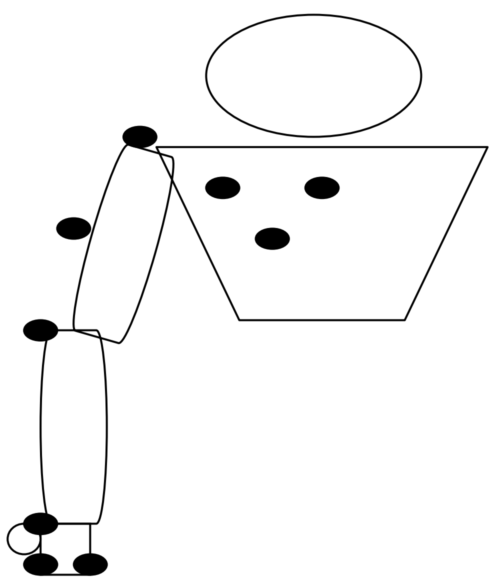

Figure 5. Posterior View of Marker Locations

<u>System/Model</u>

To date, any exploration of the interaction between two adjacent segments (the upper and lower arm) during the overhand and underhand throwing motion, has been based on the model constructed by the work of Feltner and Depena (Feltner & Depena, 1989). An important aspect of this design is the two-segment three dimensional mathematical model of the arm that was used in determining the interactions between the two segments during the pitch. This model consists of the upper arm as the proximal segment and the forearm and hand as the distal segment, including the ball as part of the distal segment (Feltner & Depena, 1989). This model used equations of motion which allow for the

> "fractionation of the three-dimensional components of the angular acceleration vector of each segment into angular acceleration terms associated with the joint moments made on the segment into motion dependent angular acceleration terms associated with the kinematic variables of the arm segments" (Feltner & Depena, 1989).

The variables of interest for the proximal segment included:

α_{UPJT} Angular acceleration of the proximal joint moment exerted on the proximal segment

α_{UDJT} Angular acceleration of the proximal joint moment exerted on the distal segment

α_{USH} Motion dependent angular acceleration associated with the linear acceleration of the shoulder

α_{UDA} Motion dependent angular acceleration associated with the angular acceleration of the forearm

α_{UDV} Motion dependent angular acceleration associated with the angular velocity of the forearm

α_{UGR} Motion dependent angular acceleration associated with the force necessary to counter gravity

The terms for the distal segment are as follows:

α_{DJT} Angular acceleration of the proximal joint moment exerted distal segment

α_{DSH} Motion dependent angular acceleration of the shoulder

α_{DUA} Motion dependent angular acceleration associated with the angular acceleration of forearm

α_{DUV} Motion dependent angular acceleration associated with the angular velocity of the upper arm

α_{DGR} Motion dependent angular acceleration associated with the force necessary to counter gravity

These terms are used to determine the relative contributions to the motion of each segment. Figure 6 a, b, and c, present the free body diagrams that correspond to the equations developed by Feltner and Depena (Feltner & Depena, 1989)

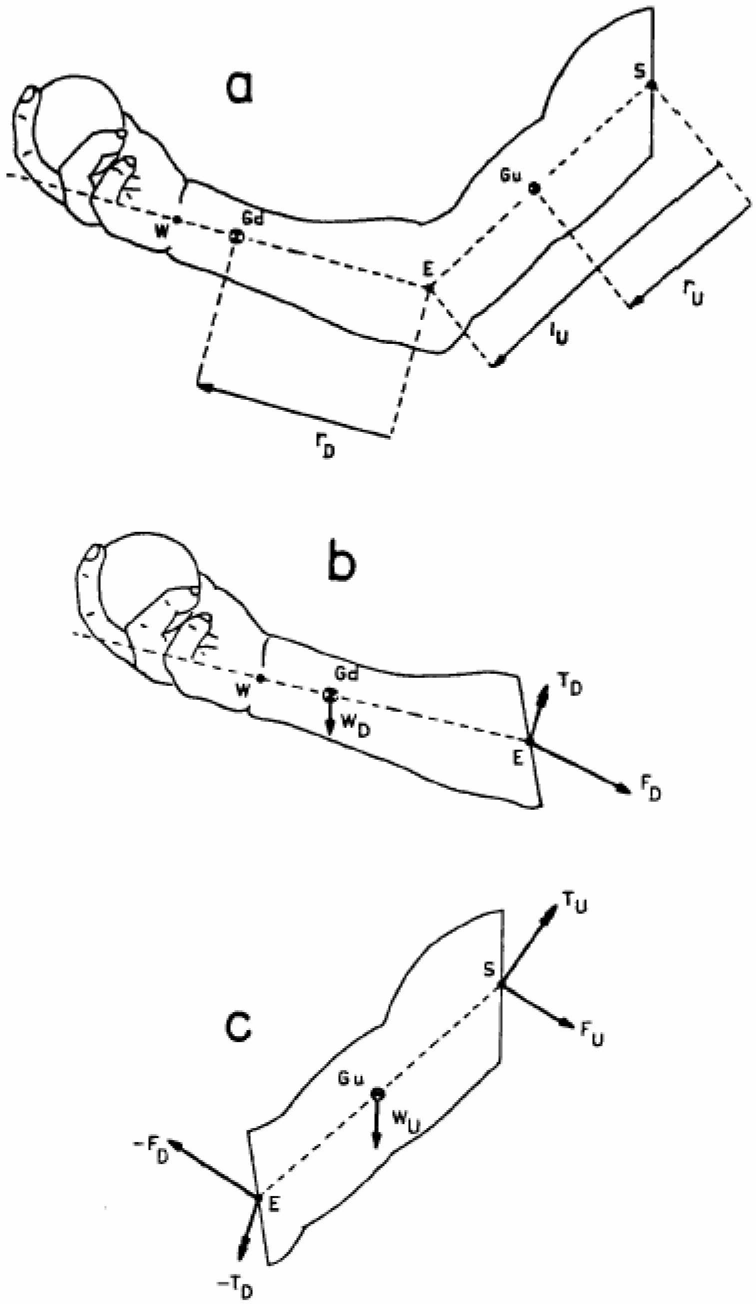

Figure 6. Free Body Diagram. The free body diagram pertaining to the equations developed by Feltner & Depena (Feltner & Depena, 1989).

Feltner and Depena's three dimensional analysis of a two segment chain in the upper extremity (Feltner & Depena, 1989) is as follows:

The linear acceleration of the distal segment:

$$\overline{a}_{Gd} = \overline{a}_s + \left(\overline{\alpha}_U \times \overline{r}_U\right) + \left(\overline{\omega}_U \times \left(\overline{\omega}_U \times \overline{r}_U\right)\right) + \left(\alpha_D \times \overline{r}_D\right) + \left(\overline{\omega}_D \times \left(\overline{\omega}_D \times \overline{r}_D\right)\right)$$

Gd = Mass Center of Distal Segment

S = Shoulder

U = Upper Arm

D = Lower Arm

Where:

First Term: Linear Acceleration of the Shoulder (a_s)

Second Term: Tangential Angular Acceleration of Upper Arm (α_U x r_U)

Third Term: Centripetal Velocity of Upper Arm (ω_U x (ω_U x r_U))

Fourth Term: Tangential Angular Acceleration of Lower Arm (α_D x r_D)

Fifth Term: Centripetal Angular Velocity of Lower Arm (ω_D x (ω_D x r_D))

The linear acceleration for the proximal segment:

$$\overline{a}_{G1} = \overline{a}_S + \left(\overline{\omega}_U \times \left(\overline{\omega}_U \times \overline{r}_{G1/S}\right)\right) + \left(\overline{\alpha}_U \times \overline{r}_{G1/S}\right)$$

G1 = Center of Mass of Upper Arm (proximal Segment)

Where:

First Term: Linear Acceleration of the Shoulder (a_s)

Second Term: Centripetal Velocity of Upper Arm (ω_U x (ω_U x $r_{G1/s}$))

Third Term: Tangential Acceleration of Upper Arm (a_U x $r_{G1/s}$)

The following equation will be used to determine the sum of forces exerted on the distal segment:

$$m_D a_{Gd} = F_D + m_D g$$

m_D mass of distal segment

a_{Gd} linear acceleration of the mass center of the distal segment

F_D Force exerted on the distal segment at the elbow

g acceleration due to gravity

The following equation will be used to determine the sum of the forces exerted on the upper arm:

$$m_U a_{Gu} = F_U - F_D + m_D g$$

m_U mass of proximal segment

a_{Gu} linear acceleration of the mass center of the proximal segment

F_U force exerted on the upper ram at the shoulder

Sum of Moments:

The following equation represents the moment about the center of mass of the distal segment.

$$\Sigma M_{Gd} = [I_{MD}\alpha_{DX}]i + [I_{MD}\alpha_{DY}]j + [I_{MD}\alpha_{DZ}]k$$

Where:

M_{Gd} Sum of moments acting at the lower arm center of mass

I_{MD} Mass moment of inertia of the mass center of the distal segment

α_D Angular acceleration of the distal segment

The following equation represents the moment about the center of mass of the proximal segment.

$$\Sigma M_{Gu} = [I_{MU}\alpha_{UX}]i + [I_{MU}\alpha_{UY}]j + [I_{MU}\alpha_{UZ}]k$$

Where:

M_{Gu} Sum of moments acting at the upper arm center of mass

I_{Mu} Mass moment of inertia of the mass center of the proximal segment

α_U Angular acceleration of the proximal segment

Although the moment equations presented by Feltner and Depena (Feltner & Depena, 1989) are correct, there are implied variables that can be expanded upon to gain an accurate understanding of the output variables in the three-dimensional overhand throw. It was one of the goals of this project to further partition the individual accelerations in order to further explain the contribution of each motion at the particular joints, rather than express the movement in a summative statement for the entire joint. In particular, the researcher investigated the effect of including the angular velocity and angular acceleration components neglected in the aforementioned analysis (Feltner & Depena, 1989). In the early stages of the development of this project the researcher attempted to employ these equations in a manner that would determine whether the angular acceleration (change in angular velocity) of a segment was due to: a) the joint moments exerted at its proximal end; b) the linear acceleration of the proximal endpoint in the kinetic chain, shoulder; c) gravity; or d) the angular velocity of angular acceleration of the adjacent segment in the kinetic chain. The determination of these variables would dictate the labeling of the action as either Theory One or Theory Two (Ford, 1998).

The current study will use a similar setup in analyzing the two segment interaction during the pitch. The first segment will be the upper arm. The second segment will be the lower arm, hand, ball combination. This model will also neglect any relative motion between the hand and the lower arm, and between the hand and the ball. This model will have four degrees of freedom. The first two degrees of freedom will be the absolute position of the head of the humerus (X_H, Y_H). The next degree of freedom will be the absolute angular position of the upper arm (θ_U). The final degree of freedom will be the angular position of the composite lower arm segment relative to the upper arm (θ_L).

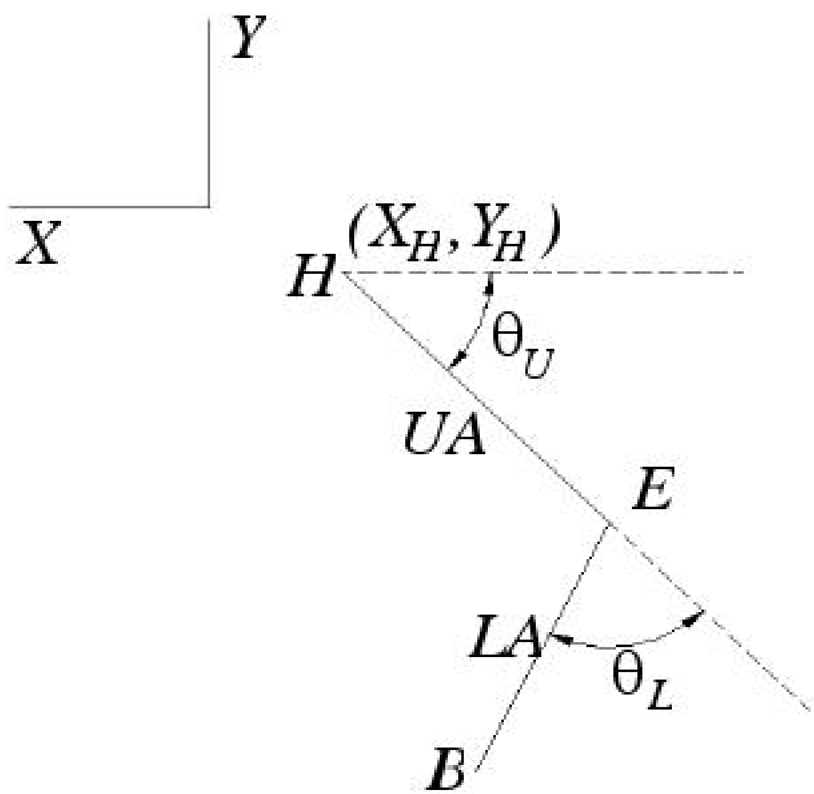

Figure 7. Coordinate Axes.

The figure depicts the coordinate axes (X,Y), the location of the humeral head, H, the location of the elbow, E, and the location of the ball, B. The X-axis is horizontal, positive in the target direction, and the Y-axis is vertical, positive in the upward direction. The two segments are shown and labeled, UA for the upper arm segment, and LA for the composite lower arm, hand, ball segment. The four degrees of freedom are depicted including the absolute location of the humeral head, X_H, Y_H, the angle made by the upper arm with the horizontal, θ_U, and the relative angle between a ray that extends the upper arm and the lower arm, θ_L.

While these mathematical models are tremendously valuable and have served as the basis for many other research projects, choices were made by Feltner and Depena (Feltner & Depena, 1986; Feltner, 1987; Feltner & Depena, 1988; Feltner, 1989; Feltner & Depena, 1989) presumably to aid in the development of moment calculations at the elbow and shoulder that limit the value of these equations and the conclusions that can be drawn from the employment of these equations. As such, this current project has selected a different partitioning scheme that will more readily provide definitive parameters that will elucidate the variables that will contribute to the understanding of Theory One or Theory Two as well as the role of the musculature in the joint actions of the elbow. For example, the partitioning scheme developed for this project will allow the researcher to be able to identify the direction of the moments at the shoulder and elbow. Based upon the direction of the moments at specific instants during the motion, it will be clear: (1) whether the throw is a Theory One or Theory Two motion and (2) whether the EMG activity when considered with the directions of the moments will demonstrate an analogous relationship between the bicep for the underhand throw and the tricep for the overhand throw.

While still following the same mechanical principles, this study chose to use the following equations in order to extract the data needed for the classification of the throw as either Theory One or Theory Two. Full derivation of the equations is given in Appendix A (underhand kinematics), Appendix B (underhand kinetics), Appendix C (overhand kinematics), and Appendix D (overhand kinetics).

Electromyographic Analysis

In order to accomplish the secondary purpose of this study, comparing muscular activity during the throw, electrodes will be placed on the area of greatest bulk for both the elbow flexors and elbow extensors. This will measure the muscle activity of this musculature throughout the throwing pattern (long head of the triceps brachii and short head of the biceps brachii). Using a custom synchronization process, the muscle activity will be synchronized to the throwing motion, allowing the researchers to analyze when the muscles were active or inactive, helping to determine the pattern of sequential action used to maximize the velocity of the distal segment

Chapter IV

Results

The purpose of this investigation was to examine the theories of how adjacent segments, within an open kinetic chain, interact during the acceleration phase of two different pitching patterns. The results of this study will be presented in the following order: subject characteristics, underhand pitch kinematics, underhand pitch kinetics, underhand pitch electromyography, overhand pitch kinematics, overhand pitch kinetics, and overhand pitch electromyography.

Subject Characteristics

Underhand

Five females who volunteered met the initial qualifications. Upon preliminary screening, all volunteers met the criteria for participation in the study, and signed an institutionally approved Informed Consent document (Appendix F). Participants consisted of 4 Caucasians and 1 African-American. The participants' anthropometric characteristics are summarized in Table 1. It is important to note that the participants had significant training in the underhand softball pitch.

Participant	Height (m)	Mass (kg)	Segment Length Upper Arm (m)	 Lower Arm (m)
1	1.78	75	0.299	0.255
2	1.73	77.27	0.292	0.246
3	1.65	59.09	0.268	0.232
4	1.68	52.27	0.284	0.243
5	1.78	86.36	0.295	0.253

Table 1. Underhand Subject Characteristics

Overhand

Five males who volunteered met the initial qualifications. Upon preliminary screening, all volunteers met the criteria for participation in the study, and signed an institutionally approved Informed Consent document (Appendix F). Participants consisted of 5 Caucasians. The participants' anthropometric characteristics are summarized in Table 2. It is important to note that the participants had significant training in the overhand baseball pitch.

			Segment Length	
Participant	Height (m)	Mass (kg)	Upper Arm (m)	Lower Arm (m)
1	1.93	90.91	0.359	0.281
2	1.88	85.00	0.350	0.274
3	1.75	79.55	0.326	0.256
4	1.83	85.45	0.340	0.267
5	1.78	95.45	0.331	0.260

Table 2. Overhand Subject Characteristics

<u>Underhand Pitch Kinematics</u>

Figures 8-10 present a graphical representation of the paths followed by the shoulder, elbow, and wrist, respectively, during the motion of one pitch (where y is the vertical direction and x is the horizontal direction). The data for these figures were taken from a single, representative participant, as there was very little variability in the patterns of these graphs across participants. This path length measure was computed using the following equation:

$$path-length = \sum \sqrt{\left[(x_2 - x_1)^2 + (y_2 - y_1)^2\right]}$$

The path encompasses the duration of the one pitch that was used for analysis. The beginning of the pitch was defined as the set position where there is no movement with the hand and ball together near the middle of the stomach. The end of the pitch was defined as the point where the shoulder began to extend after the release of the ball. The reader should take note that the scales of the path graphs are not identical; therefore the graph lines are not an accurate representation of the slope of the position in space, thus, angles at the shoulder are not identically represented at the elbow and so forth. The (0,0) point at each graph is indicative of the shoulder position at the time of ball release.

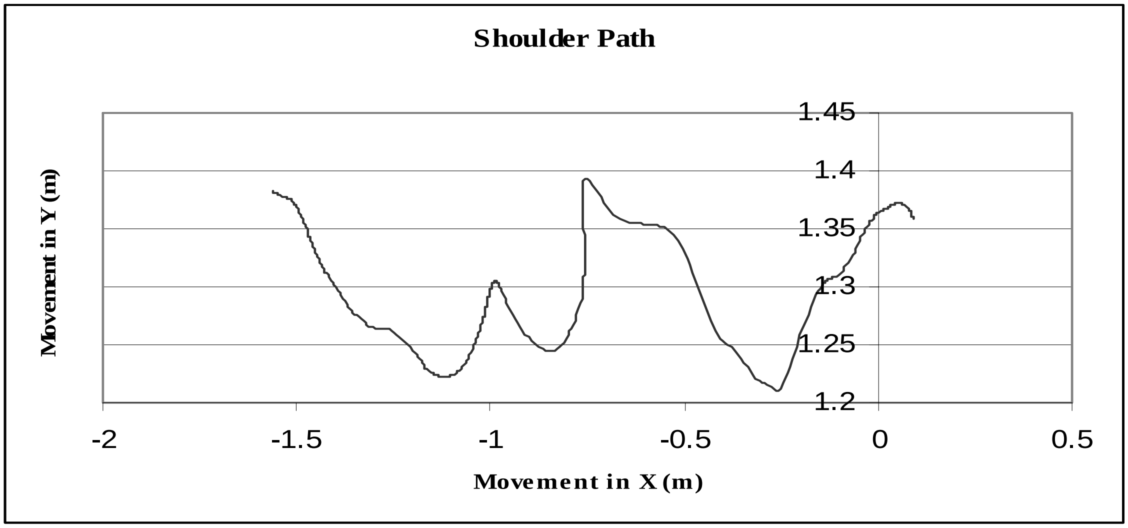

Figure 8. Path of Shoulder during one pitch. Release is marked by the y-axis.

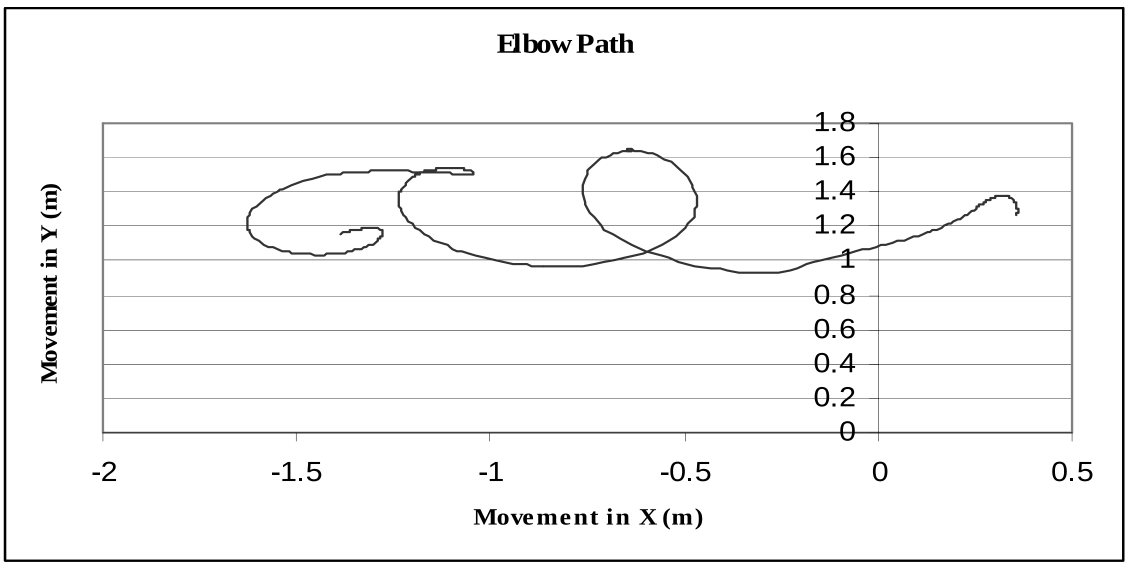

Figure 9. Path of Elbow during one pitch. Release is marked by the y-axis.

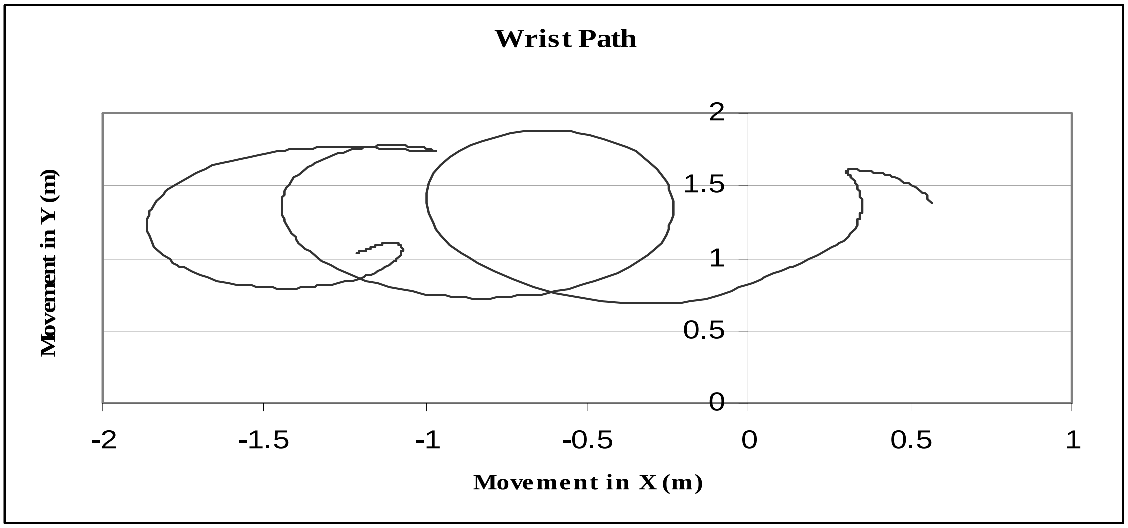

Figure 10. Path of Wrist during one pitch. Release is marked by the y-axis**.**

The linear velocity pattern of the underhand pitch from the position of arm up during the late stride phase to just after the release of the ball is illustrated in Figure 11. This figure demonstrates a proximal-to-distal sequencing between the two segments with a summation of speed noted in the distal segment as the peak wrist speed is achieved at a point near a minimum of the elbow speed. Although this graph represents only one participant, the graphs of each participant followed the same general pattern.

Figure 12 presents the rates of change of the angles constructed at the proximal end of each segments. Theta U (θ_U) represents the angle between the x-axis and the humerus, while Theta L/U ($\theta_{L/U}$) is an angle that represents an inside angle between the upper arm and lower arm (often termed the flexion angle). This is defined in the diagram depicted in Figure 7. Figure 12 also illustrates the proximal-to-distal sequencing between the two adjacent segments as the angular rate of changes of the preceding segment is at a minimum when the angular rate of change is at a maximum for the more distal segment. It also demonstrates the transfer of angular momentum from the proximal to distal segment as indicated by the change in angular rate of speed. This graph is representative of the pattern demonstrated by each of the participants.

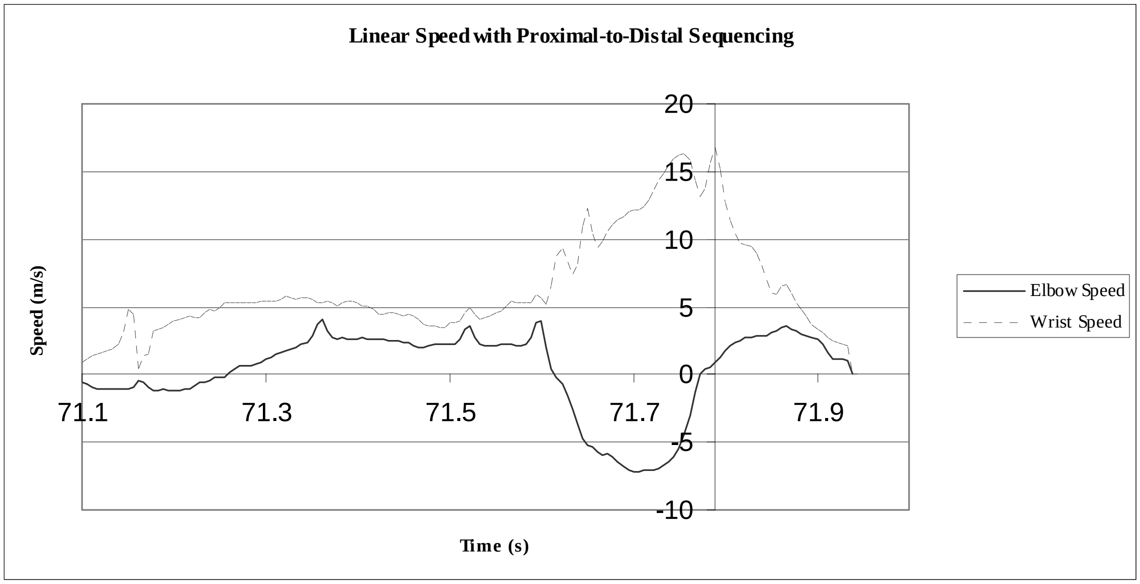

Figure 11. Linear Speed.   Linear speed of the distal portions of the adjacent segments demonstrating proximal-to-distal sequencing. Release is marked by the y-axis.

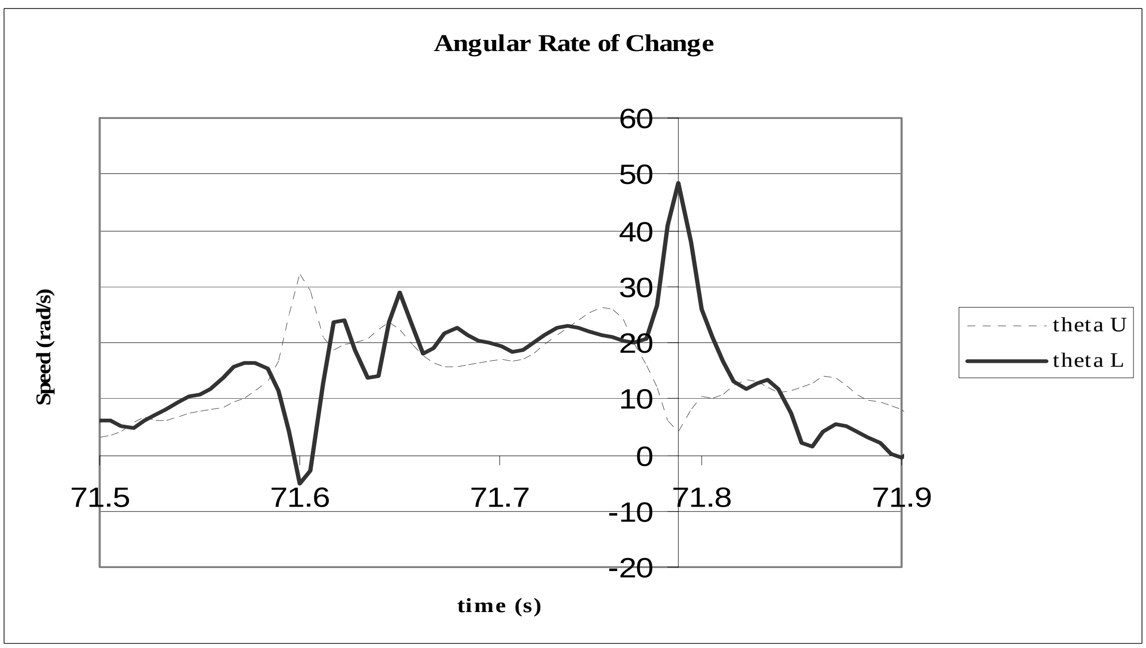

Figure 12. Angular Rate of Change. Angular rate of change of the angle at the shoulder and at the elbow demonstrating proximal-to-distal sequencing. Release is marked by the y-axis.

The angular accelerations of θ_U and θ_L are presented in Figure 12. The graph demonstrates the maximal negative acceleration of the proximal segment occurring concurrently with the maximal positive acceleration of the distal segment. This trend maintains the proximal-to-distal sequencing between the two segments as the distal segment increases its speed relative to the decrement in speed of the proximal segment.

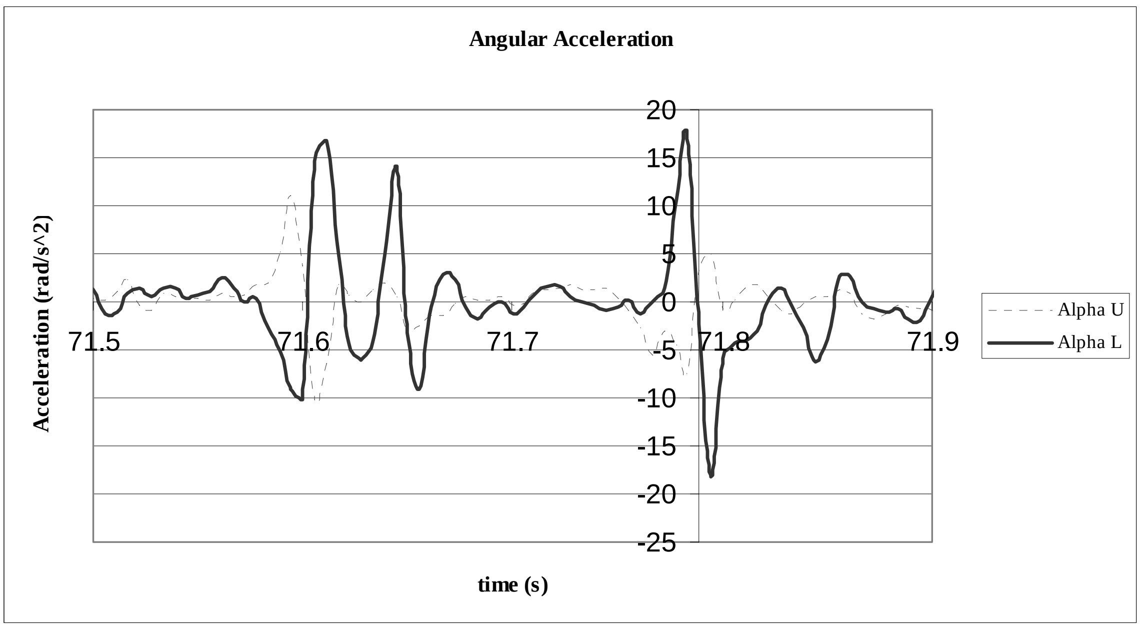

Figure 13. Angular Acceleration of θ_U and θ_L. Release is marked by the y-axis.

Underhand Pitch Kinetics

The kinetics associated with the underhand softball pitch include the moment about the shoulder joint (z-axis) and the moment about the elbow joint (z-axis). The direction of the moment at the shoulder in conjunction with the moment at the elbow allowed the researcher to classify the proximal-to-distal sequencing seen in the underhand pitch as Theory One (inertial acceleration of the distal segment) or Theory 2 (muscular acceleration of the distal segment). Figure 14 is a graphical representation of the moment about the shoulder during the underhand pitch. Figure 15 is a graphical representation of the moment about the elbow during the underhand pitch. The instant of release is marked by the location of the y-axis on the graph. The negative moment at the shoulder with the small negative moment at the elbow, during the time of separation of velocity lines between the upper arm and lower arm, suggest that the pitch can be categorized as an inertial acceleration of the lower arm, Theory One (Ford, 1998), with the negative moment indicative of the reactive moments produced by the hard end point of the bone on bone contact at the joint during the transfer of momentum from the proximal-to-distal sequencing between the two segments.

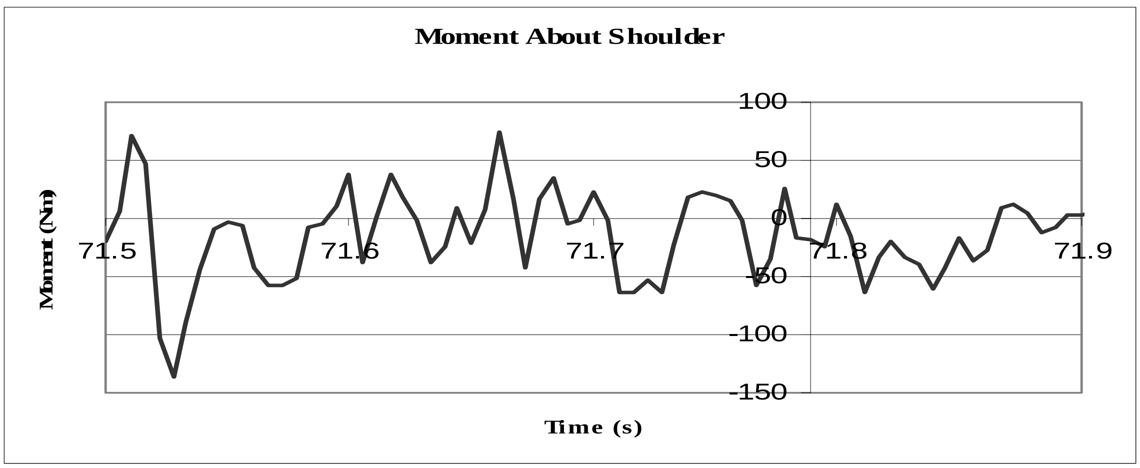

Figure 14. Moment about the Shoulder. Moment about the shoulder during the underhand pitch. The negative moment about the shoulder prior to release is characteristic of an inertial type acceleration of the lower arm (Theory One). Release is marked by the y-axis.

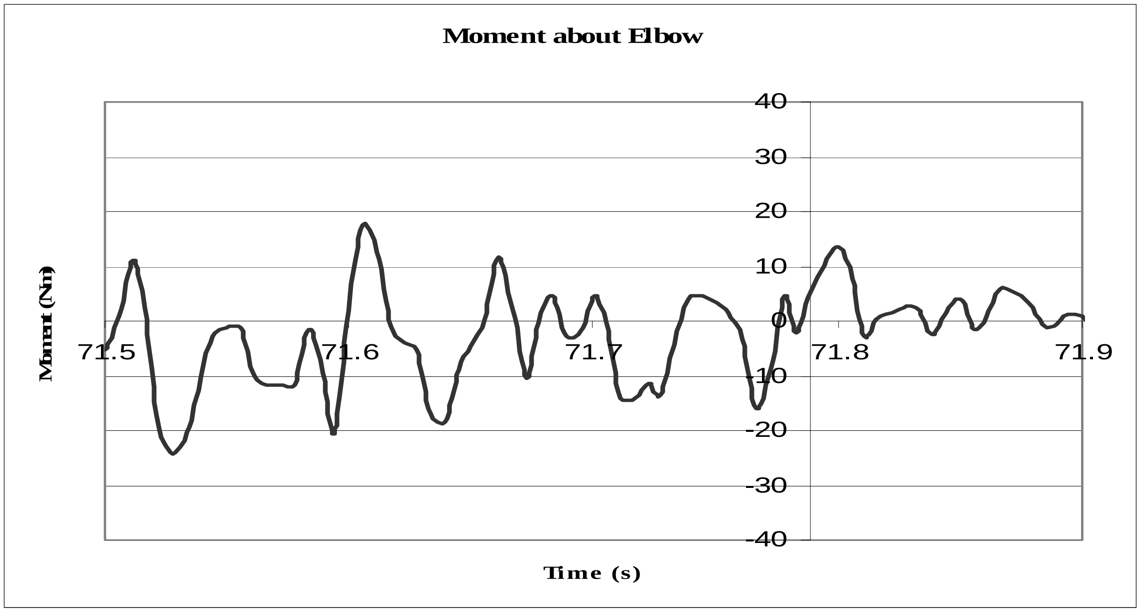

Figure 15. Moment about the Elbow. Moment about the elbow during the underhand pitch. Release is marked by the y-axis.

Underhand Pitch Electromyography

The electromyogram of the underhand throw for each of the participants illustrated increased tricep activation with an attenuated bicep contribution just before release. This pattern of muscle activity suggests that the bicep does not contribute substantially to the elbow flexion during the underhand throw. Figure 15 presents the typical electromyogram of the underhand throw. The pattern was similar among each of the participants.

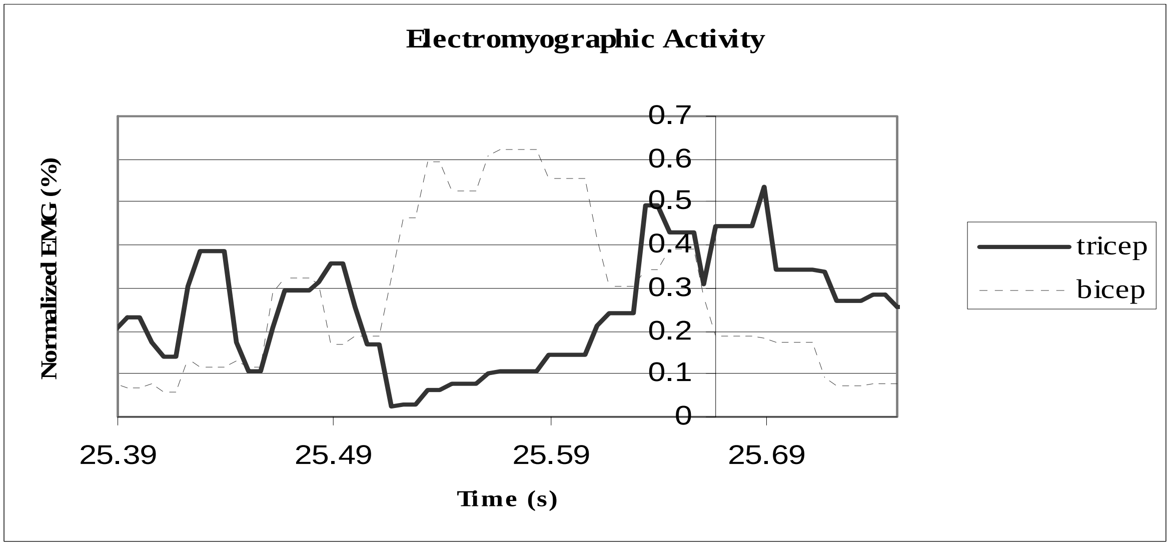

Figure 16. Underhand Pitch Electromyograph. Electromyograph of muscle activity during the underhand pitch of the long head of the triceps brachii and the short head of the biceps brachii. Release is marked by the y-axis.

Overhand Pitch Kinematics

Figures 17-19 present a graphical representation of the paths followed by the shoulder, elbow, and wrist, respectively, during the motion of one overhand pitch (where y is the vertical direction and x is the horizontal direction). The data for these figures were taken from a single, representative participant, as there was very little variability in the patterns of these graphs across participants. This path length measure was computed using the following equation:

$$path-length = \sum \sqrt{\left[(x_2 - x_1)^2 + (y_2 - y_1)^2\right]}$$

The path encompasses the duration of the overhand pitch that was used for analysis. The beginning of the pitch was defined as the position where the toe of the lead leg of the pitcher made contact with the mound where the shoulder is near 90° abduction with the ball facing away from the target. The end of the pitch was defined as 60 frames after the release of the ball. This was typically a position where the wrist and elbow of the throwing arm were near the opposite knee. The reader should take note that the scales of the path graphs are not identical; therefore the graph lines are not an accurate representation of the slope of the position in space, thus, angles at the shoulder are not identically represented at the elbow and so forth.   The (0,0) point at each graph is indicative of the shoulder position at the time of ball release.

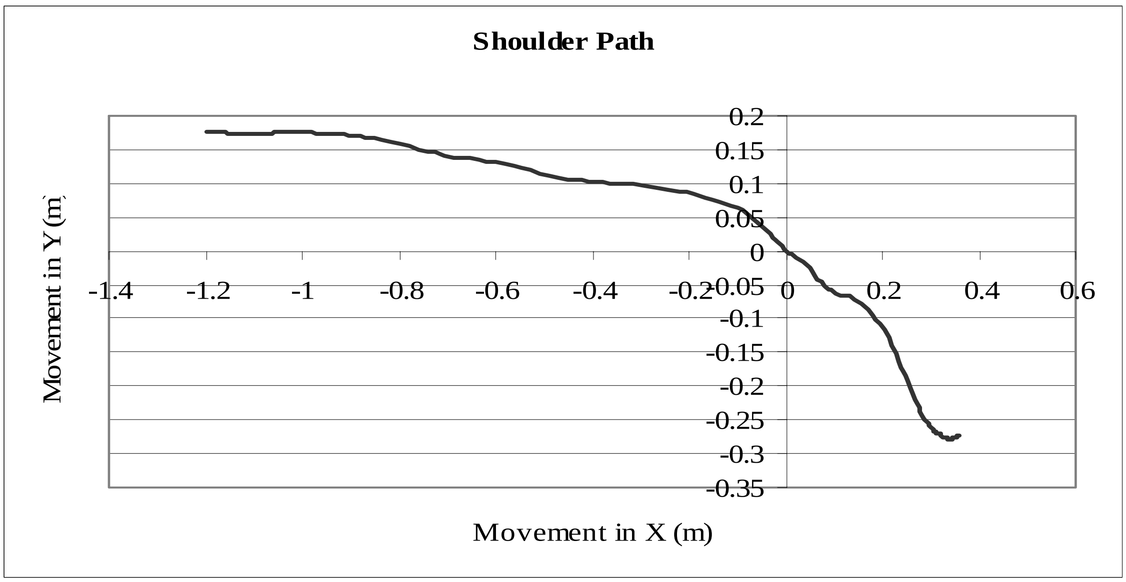

Figure 17. Path of Shoulder during one pitch. Release is marked by the y-axis.

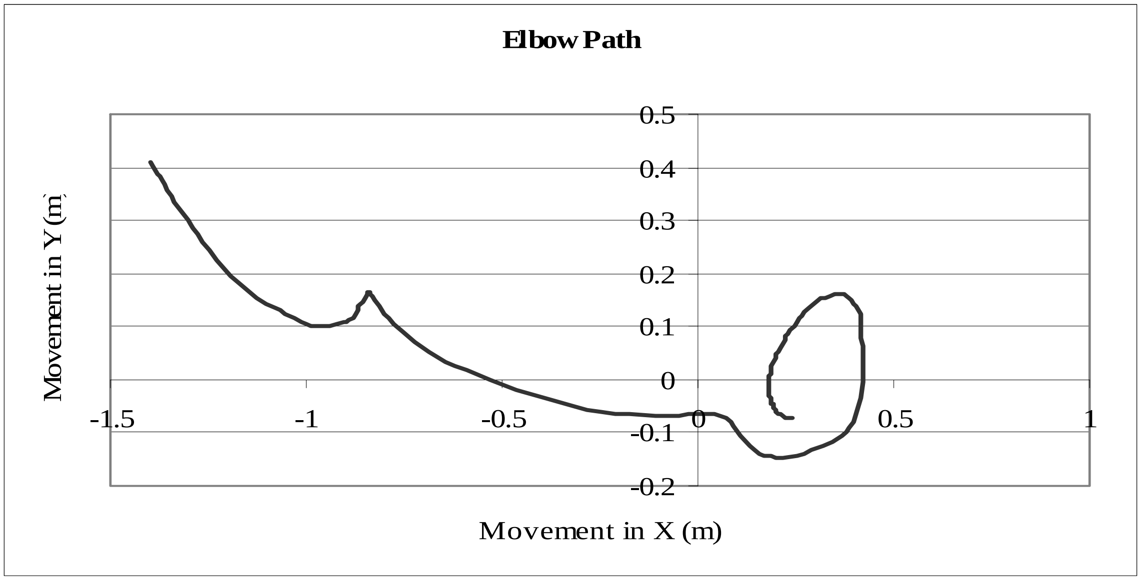

Figure 18. Path of Elbow during one pitch. Release is marked by the y-axis.

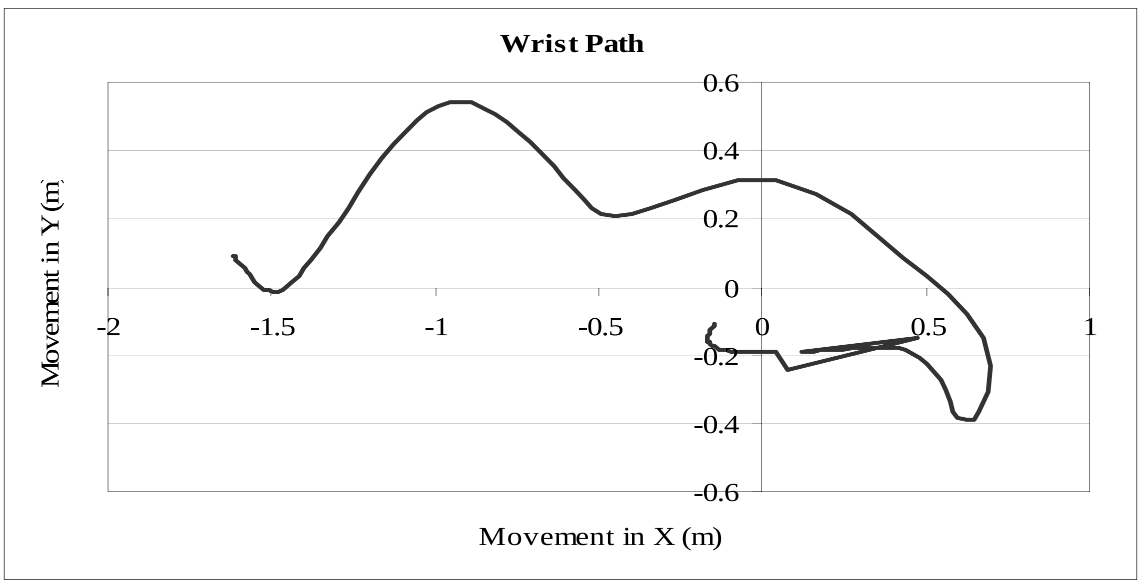

Figure 19. Path of Wrist during one pitch. Release is marked by the y-axis.

The linear velocity pattern of the overhand pitch from the cocked position at toe down to just after release (the acceleration phase), is illustrated in Figure 20. The y-axis denotes ball release. The figure illustrates the sequential increase in velocity between the segments, as a summation of speed is noted in the lower arm as the peak wrist velocity is maximal at a time where the upper arm is at a minimum velocity. Although this graph represents only one participant, the graphs of each participant followed a similar pattern.

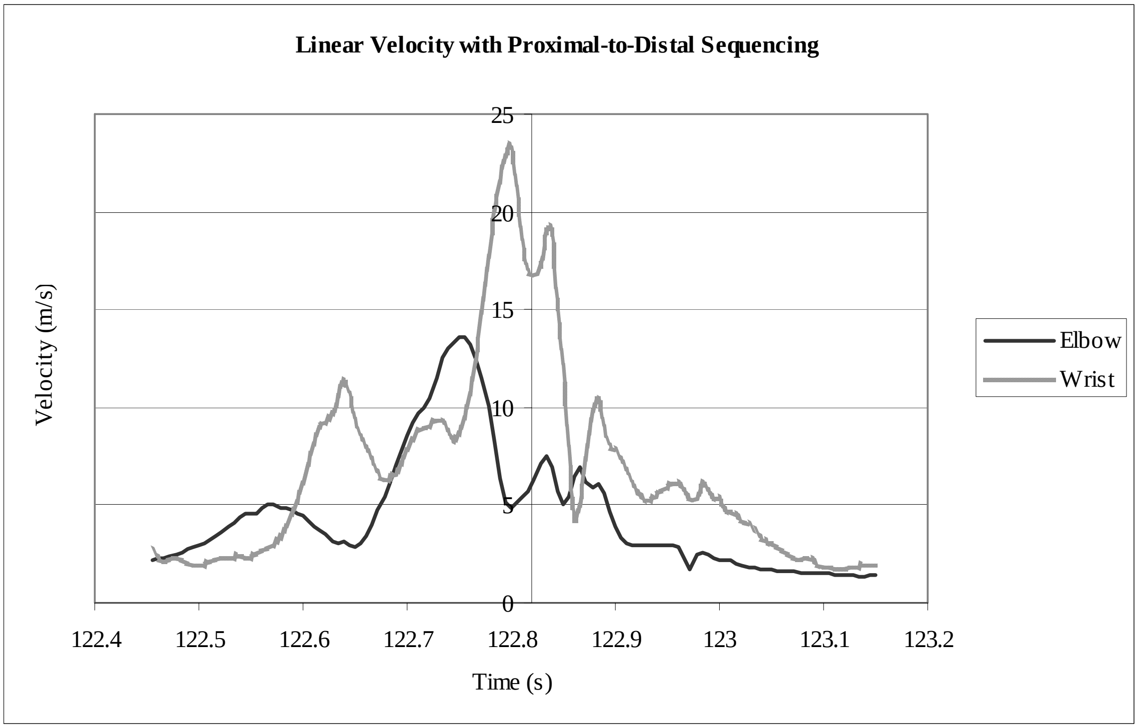

Figure 20. Linear Velocity. Linear velocity of the distal portions of the adjacent segments demonstrating proximal-to-distal sequencing. Release is marked by the y-axis.

Figure 21 presents the angular rate of change of the shoulder and elbow. This graph also demonstrates a proximal-to-distal sequence between the segments in the decrease in the shoulder angular rate of change with an increase in the angular rate of change of the elbow leading up to release. It must be noted that the elbow angle in the overhand throw is described as the external angle of the elbow, thus the downward slope of the elbow velocity illustrates and that the angle is extending at an increasing rate. The key patterns to notice are the shoulder internal rotation angle and the elbow angle, as these are the major factors in ball velocity. This pattern is representative of each participant.

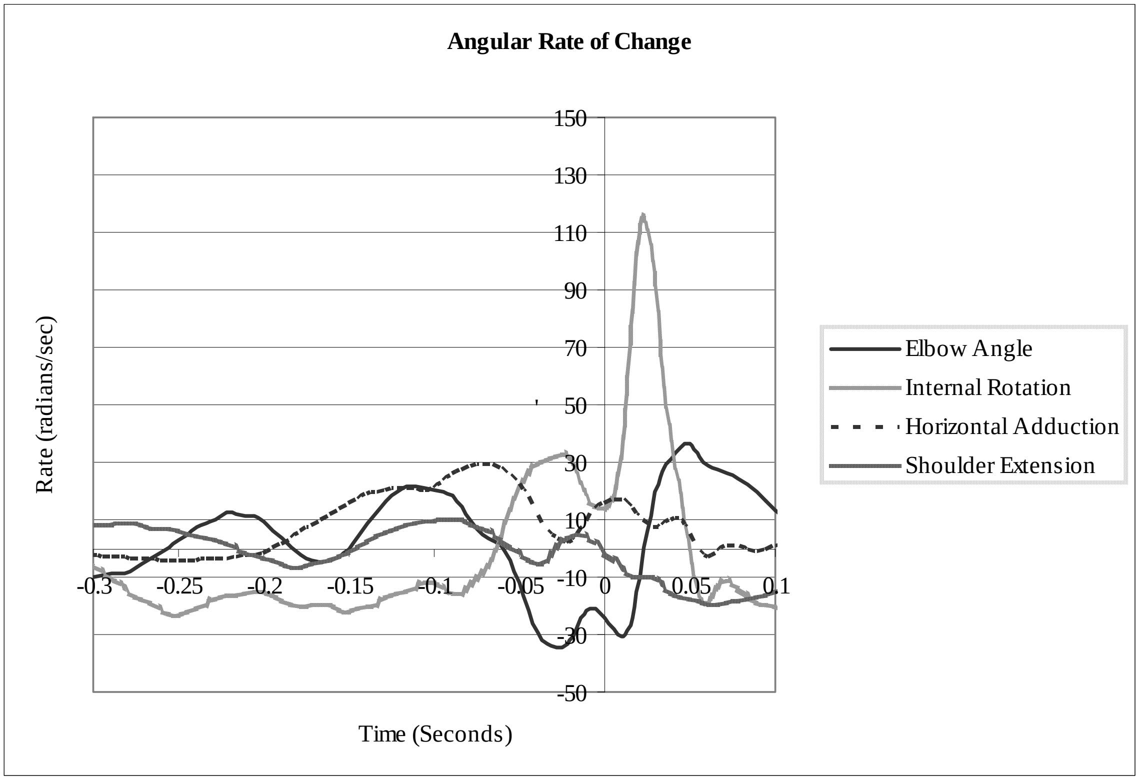

Figure 21. Angular rates of change. Angular rate of change of the angles at the shoulder and at the elbow. Release is marked by the y-axis.

The second angular derivatives of all three angular movements at the shoulder and at the elbow are presented in Figure 22. Again, the major shoulder motion to examine is the internal rotation, or rotation about the z-axis. The negative acceleration of the internal rotation along with the acceleration of elbow extension demonstrates the typical proximal-to-distal sequencing.

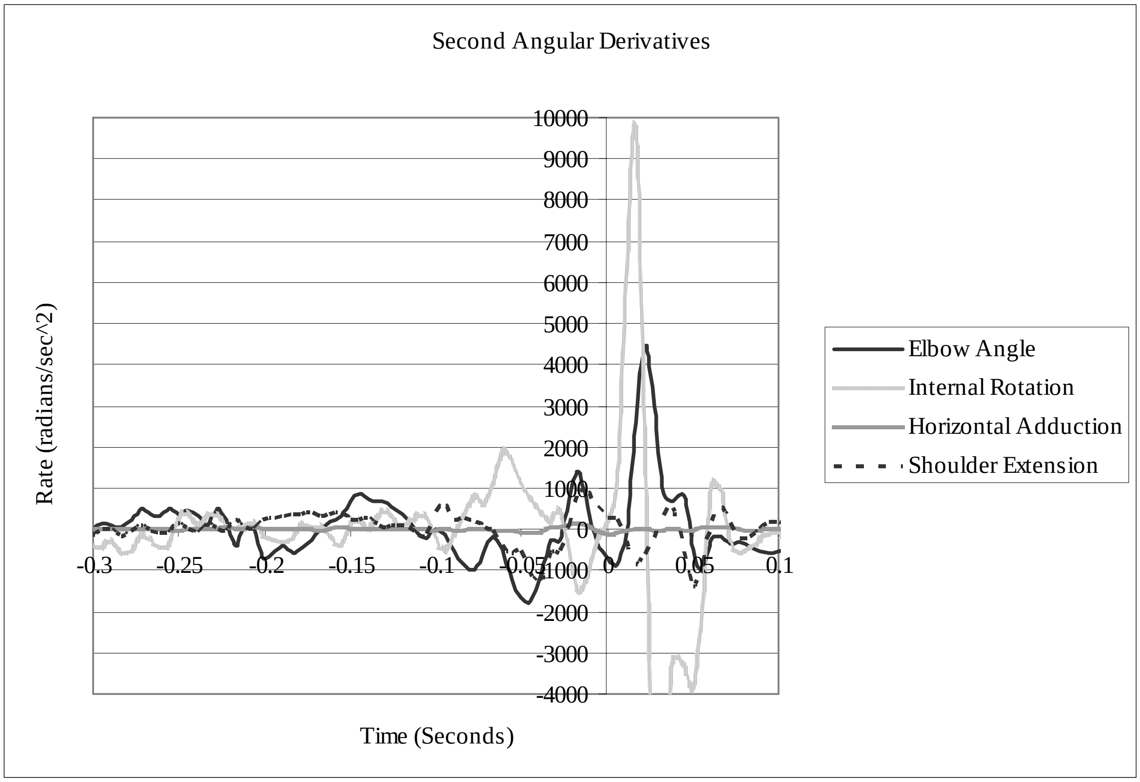

Figure 22. Second Angular Derivatives. Second angular derivatives of shoulder and elbow. Release is marked by the y-axis.

Overhand Pitch Kinetics

The kinetics associated to the overhand baseball pitch that are relevant to this study are the internal rotation moments of the shoulder about the z-axis and the moment about the elbow. The direction of the moment at the shoulder in conjunction with the moment at the elbow allow the researcher to theoretically classify the proximal-to-distal sequencing seen in the overhand baseball pitch as an inertial acceleration of the distal segment or a muscular acceleration of the distal segment. Figure 23 illustrates the moment about the z-axis, or the horizontal adducti moment, of the shoulder during the analyzed portion of the pitch. Figure 24 is a graphical representation of the moment about the elbow during the overhand baseball pitch. The negative moment at the shoulder, followed immediately by a negative moment at the elbow suggests a negative acceleration of the respective segments allowing for the distal segment to inertially accelerate.

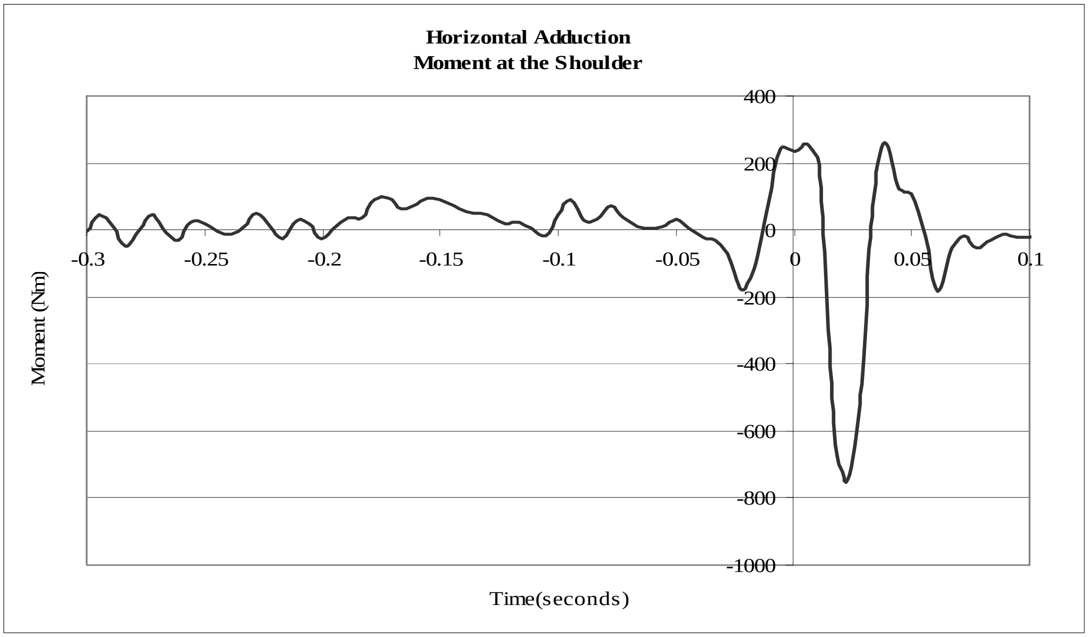

Figure 23. Moment at the shoulder. Horizontal adduction moment at the Shoulder. Release is marked as the y axis.

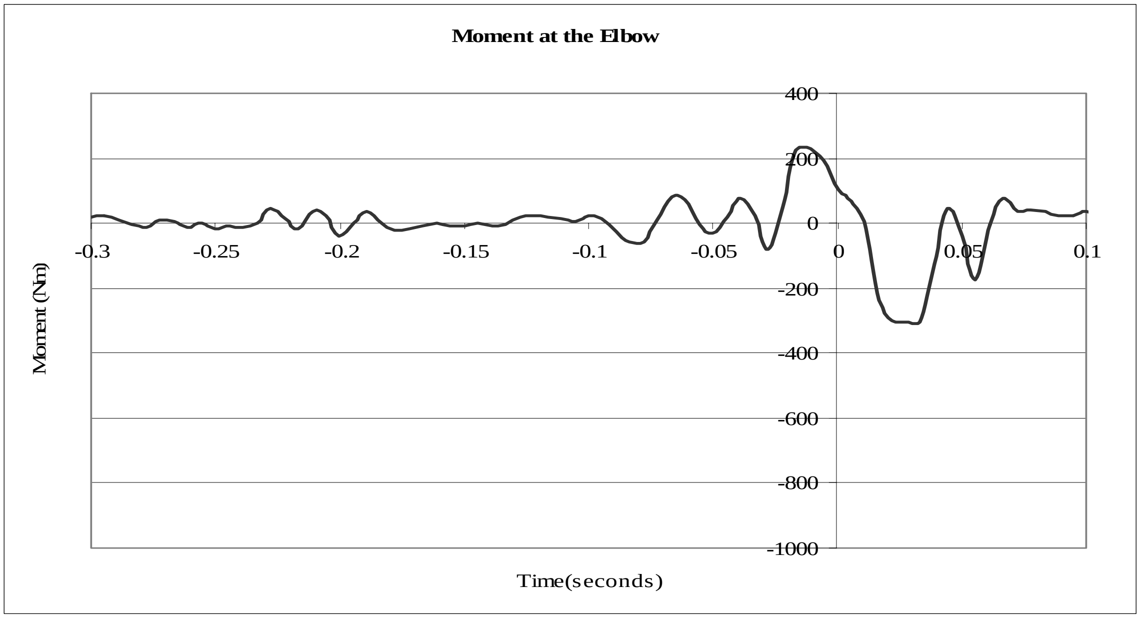

Figure 24. Moment at the Elbow. Release is marked by y-axis.

Overhand Pitch Electromyography

The electromyogram of the overhand throw for each of the participants illustrated an elevated level of muscle activity prior to release, but showed very little activity at or around release. Following release, there was a strong co-contraction in an attempt to negatively accelerate the entire arm at release.

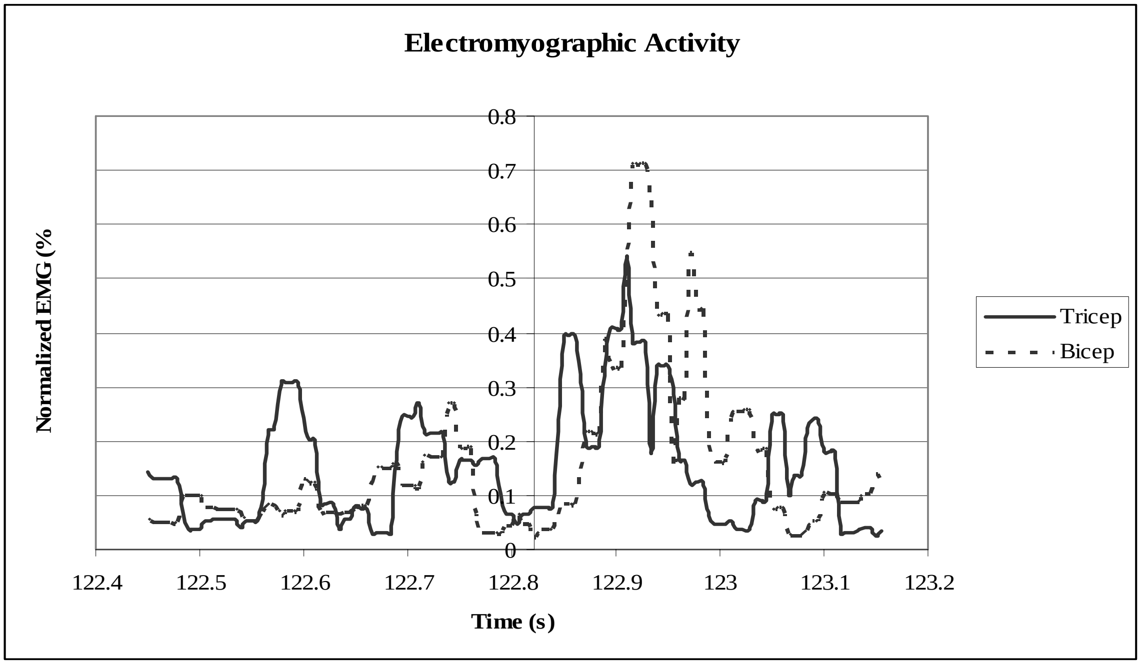

Figure 25. Electromyograph of the Overhand Throw. Electromyograph of muscle activity during the overhand pitch of the long head of the triceps brachii and the biceps brachii. Release is marked by the y-axis.

Chapter V

Discussion

The purpose of this study was to accurately investigate the interaction between adjacent segments in relation to Theory One and Theory Two of proximal to distal sequencing (Ford, 1998) as they function in an overhand baseball pitch and an underhand softball pitch. Second, this study endeavored to determine if the musculature that crosses the elbow is analogous in an overhand and underhand throw via electromyographic analysis synchronized with the motion capture system. A tertiary purpose of this study was to expand the current literature by creating an equation that expands the mathematical models at both the shoulder and elbow for the overhand and underhand throwing motions.

Underhand Pitch

There is limited information on the kinematic and kinetic variables associated with the underhand softball pitch (Alexander & Haddow, 1982; Hill et al., 2004; Werner et al., 2006), however, previous researchers have all suggested a sequential action in the throw. A purpose of this study was to validate the existence of proximal-to-distal sequencing in the arm during the underhand softball fast pitch, and if found, classify that sequence as either a Theory One or Theory Two sequence of motions (Ford, 1998). The results of this study supported the notion that the underhand softball pitch follows the tenets of proximal-to-distal sequencing. The basic premise of any proximal-to-distal sequence is an increase in the velocity of the distal segment, building upon the speed of the proximal segment and utilizing the summation of speed principle to achieve a greater speed than could have been achieved as a single segment and allowing for the transfer of angular moment along the kinetic chain. As shown in the results (Figures 7-8), the pattern of proximal-to-distal sequencing is seen both in the linear speed and angular rate of change in the pitches. In Figure 7, it is apparent that as the linear speed of the proximal segment (denoted by the elbow) reached its maximum velocity and began to slow down, the speed of the distal segment (denoted by the wrist) increased and achieved a greater maximal speed. Furthermore, in Figure 8, which depicts the angular rate of change, a similar pattern emerged. As the angular rate of change of the upper segment (denoted by θ_U) reached its maximum and began to slow down, the lower segment (denoted by θ_L) began to accelerate and achieve its maximum velocity as the lower segment was able to build upon the velocity of the upper segment (combining or summing the velocities of the adjacent segments). The existence of a proximal to distal sequence in the underhand pitching motion is further supported by the reversal of angular accelerations seen in these segments (Figure 9). These combinations of angular and linear speed graphs give substantial evidence to the notion that the underhand softball pitch exhibits a proximal-to-distal sequence.

The next goal of the study was to classify this proximal-to-distal sequencing into one of two theoretical models (Ford, 1998). In order to do this, one must look at the moments produced about the proximal and distal attachments. To be classified as a

Theory One (Ford, 1998) motion, a negative moment at the shoulder must be seen at the instant of negative acceleration of the upper segment, as well as zero or negative moment about the elbow. This would suggest that a muscular action that is external to the segment articulation of interest acts to slow the proximal segment, allowing for an inertial acceleration of the distal segment. The negative or zero moment at the elbow would indicate that there is no active muscular contribution to the flexion exhibited at the elbow during the phase of interest. Furthermore, electromyographic data of secondary shoulder extensors and shoulder flexors can clarify the classification by providing support of muscular contribution to the actions. As seen in the data (Figure 10), there is a negative (extension) moment at the shoulder at the instant of separation between the linear and angular speeds of the proximal and distal segments. Therefore, the negative moment at the shoulder, the small negative (extension) moment at the elbow, the increased tricep activity (shoulder extensor) and attenuated bicep activity (both elbow flexor and shoulder flexor) all suggest that the underhand softball pitch can be classified as a Theory One (Ford, 1998) motion. Data supports the idea that the bicep is active as an elbow flexor, but only after release as it is trying to stop the arm during the follow through. The relative small bicep activity also supports the Theory One notion, as a primary elbow flexor was not on to cause the requisite moment at the elbow that is required to slow the proximal segment which would be in line with Theory Two.

The third purpose of this study was to expand the mathematical model of the interaction of two adjacent segments during the throwing motion (Feltner, 1987; Feltner & Depena, 1988; Feltner & Depena, 1989). While the basic premise of the model and calculations of moments were the same as previously designed equations (Feltner &

Depena, 1986; Feltner, 1987; Feltner & Depena, 1988; Feltner & Depena, 1989), there were two main expansions of the equations for the underhand pitch. First, the input of derived kinematic variables which were expressed in terms of their system variable derivatives. The linear and angular velocities and accelerations used in Feltner and Depena's work (Feltner, 1987; Feltner & Depena, 1988; Feltner & Depena, 1989) were based on the inertial positions; however in this study, the values used were those relative to the body segments. The benefit of this choice is that it gives the practitioner the ability to look at movements in terms of the time rates of physically observable and physically controllable attributes of the performer. The following is the expansion of the moment equations compared to that of Feltner and Depena (Feltner, 1987; Feltner & Depena, 1988; Feltner & Depena, 1989):

Feltner and Depena (Feltner, 1987; Feltner & Depena, 1988; Feltner & Depena, 1989) ΣM:

$$\Sigma M_{Gd} = [I_{MD}\alpha_{DX}]i + [I_{MD}\alpha_{DY}]j + [I_{MD}\alpha_{DZ}]k$$

$$\Sigma M_{Gu} = [I_{MU}\alpha_{UX}]i + [I_{MU}\alpha_{UY}]j + [I_{MU}\alpha_{UZ}]k$$

Partitioning of same equation that allows for manipulation of physically controllable characteristics:

$$M_E = \begin{Bmatrix} m_L \cdot L_{LG} \cdot \left[\sin\left(\theta_U + \theta_L\right) \quad -\cos\left(\theta_U + \theta_L\right) \right] \cdot \begin{bmatrix} \ddot{X}_H \\ \ddot{Y}_H \end{bmatrix} \\ + \left[I_{LE} + m_L \cdot L_{LG} \cdot L_U \cdot \cos\left(\theta_L\right) \quad I_{LE} \right] \cdot \begin{bmatrix} \ddot{\theta}_U \\ \ddot{\theta}_L \end{bmatrix} \\ + \left[\dot{\theta}_U \quad \dot{\theta}_L \right] \cdot \begin{bmatrix} m_L \cdot L_{LG} \cdot L_U \cdot \sin\left(\theta_L\right) & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_U \\ \dot{\theta}_L \end{bmatrix} \end{Bmatrix}$$

This study also developed a similarly partitioned equation for the moments acting about the shoulder during the underhand pitch:

$$M_H = \begin{Bmatrix} \left[M_{HX} \quad M_{HY} \right] \cdot \begin{bmatrix} \ddot{X}_H \\ \ddot{Y}_H \end{bmatrix} + \left[M_{HU} \quad M_{HL} \right] \cdot \begin{bmatrix} \ddot{\theta}_U \\ \ddot{\theta}_L \end{bmatrix} \\ + \left[\dot{\theta}_U \quad \dot{\theta}_L \right] \cdot \begin{bmatrix} M_{HUU} & M_{HUL} \\ M_{HLU} & M_{HLL} \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_U \\ \dot{\theta}_L \end{bmatrix} \end{Bmatrix}$$

Where:

$$M_{HU} = I_{UH} + I_{LE} + m_L \cdot \left(L_U^2 + 2 \cdot L_U \cdot L_{LG} \cdot \cos\left(\theta_L\right) \right)$$
$$M_{HL} = I_{LE} + m_L \cdot L_{LG} \cdot L_U \cdot \cos\left(\theta_L\right)$$

$$M_{HX} = \left(m_U \cdot L_{UG} + m_L \cdot L_U \right) \cdot \sin\left(\theta_U\right) + m_L \cdot L_{LG} \cdot \sin\left(\theta_U + \theta_L\right)$$

$$M_{HY} = -\left(m_U \cdot L_{UG} + m_L \cdot L_U \right) \cdot \cos\left(\theta_U\right) - m_L \cdot L_{LG} \cdot \cos\left(\theta_U + \theta_L\right)$$

$$M_{HUU} = \vec{0}$$

Second, the equations of Feltner and Depena (Feltner & Depena, 1986; Feltner, 1987; Feltner & Depena, 1988; Feltner & Depena, 1989) were further altered for this study by inverting the equations to express physical outputs in terms of their physical inputs. For example, output variables of the system would be the acceleration of the ball (a_b), as well as the second time derivative of the angular positions of the upper arm and lower arm (θ_U and θ_L). The input values, which are the values that drive the motion, are values that the thrower would control; including the inertial position of the shoulder (x, y) and the moments about the shoulder and elbow, which can be controlled with the activation or dampening of a particular muscle group. The benefits of inverting these equations in this manner is that it allows the athlete or coach to directly relate the physical inputs to the physical outputs by allowing for mechanical manipulation of physically active motions.

Implications

The data that emerged from this study supports the taxonomy of the underhand softball pitch into Theory One (Ford, 1998) of proximal-to-distal sequencing, due to the negative shoulder moments and negligible elbow moment seen during the time leading up to release. The classification of this type of sequence suggests that there is eccentric loading on the shoulder extensor muscles, which decreases the velocity of the proximal segment, allowing for an inertial acceleration of the distal segment as the arm moves toward release. There are two major shoulder muscular implications that can be taken from this data. Firstly, due to the eccentric loading of the musculature, which is the type of loading that most injuries occur, the shoulder extensors, mainly the latissimus dorsi, rear deltoids, and long head of the triceps, should be trained in this manner in order to reduce the risk of injury. The incidence of injury of these muscles is typically rather low due to the relative size and strength of the latissimus dorsi, as well as the twisting nature of the tendon as it moves toward the insertion point on the medial humerus. The twisting of the tendon actually allows for reduction in tension on the muscle during internal rotation of the humerus, even though an extension moment is still being applied.

This extension moment is concurrently supported by the long head of the triceps brachii and the posterior deltoid. Furthermore, the long head of the biceps is additionally active as a shoulder flexor due to the decrement of elbow flexion during the acceleration phase of the pitch. Therefore, it would be beneficial for the underhand pitching athletes to train the external rotation musculature as it is used, in an eccentric manner. By training the shoulder extensor musculature in an eccentric manner, the benefits would be two-fold. The first would be as a preventative measure for injury during the ballistic motion of the pitch. Second, a strengthening of the musculature that indirectly accelerates the distal segment during a motion that attempts to maximize velocity could lead to an increase in the inertial acceleration of that distal segment due to the conservation of angular momentum theorem. This increase in inertial acceleration could benefit the athlete by allowing for more velocity of that segment, thus more momentum being passed to the further distal segments at release and potentially resulting in greater velocities of the pitched ball.

The most common occurrence of reported injury to the underhand pitching athlete has typically been at the anterior shoulder near the origin of the long head of the biceps brachii (Loosli et al., 1992; Maffet et al., 1997; Barrentine et al., 1998; Werner et al., 2006), which acts as a shoulder flexor, elbow flexor, and a radio-ulnar suppinator. The long head of the biceps tendon inserts into the superior glenoid labrum and also assists in the resistance to distraction of the humeral head. This function has lead to the theory that this tendon acts as the fifth rotator cuff. Each of these authors (Loosli et al., 1992; Maffet et al., 1997; Barrentine et al., 1998; Werner et al., 2006) suggested that anterior shoulder injuries occur due to the dramatic eccentric loading placed on the biceps brachii muscle as it is activated in both shoulder flexion and elbow flexion. It most be noted that during both shoulder and elbow flexion the muscle would actually be shortening. However, it was noticed that there was a large amount of radio-ulnar pronation occurring during the acceleration phase of the throwing motion. Furthermore, the biceps brachii functions mainly as a suppinator at the radio-ulnar joint, where it will be eccentrically loaded during pronation, concentrically loaded at the shoulder during flexion and concentrically loaded at the elbow as a flexor. It is the idea of this researcher that it is not eccentric loading of the musculature, but the overloading of active insufficiency of the musculature as it attempts to function at all three joints that causes the degradation/injury at the anterior shoulder where it resists the anterior distraction of the humerus. Furthermore, the small amount of elbow motion relative to the shoulder during the acceleration and release phases of the pitch gives support to the notion that the action of the bicep brachii at the shoulder and radio-ulnar joint is much more important than that at the elbow.

Future Research

Research (Werner et al., 2006) supports the notion that internal rotation of the shoulder is the driving factor in the velocity of the softball at release. The next step in underhand pitching research should be the development of a three-dimensional analysis of the pitching motion in an attempt to partition the accelerations of not only the typical sagital plane motions of flexion and extension but to also include both the frontal plane motions of abduction and adduction at the shoulder as well as the transverse plane motions of internal and external rotation at the shoulder and pronation and suppination at the radio-ulnar joint. This will allow for a complete analysis and full explanation of the sequence of segmental interactions that occur to maximize velocity in the underhand pitch. Furthermore, this three-dimensional analysis will allow for more preventative and rehabilitation suggestions for conditioning exercises. In addition, the partitioning of the three-dimensional accelerations, as done in this study, will allow for coaching and teaching items that will allow the athlete to put the science of mathematical modeling into practice on the field.

Overhand Pitch

The overhand baseball pitch is one of the most biomechanically analyzed three dimensional motions. However, there remains some controversy as to which of the proximal-to-distal sequencing theoretical models the overhand throw fits. A purpose of this study was to confirm the existence of proximal-to-distal sequencing in the overhand baseball pitch in skilled participants, and if found, classify that sequence as either a Theory One or Theory Two sequence of motions (Ford, 1998). The results of this study confirmed the proximal-to-distal sequential action of the overhand baseball pitch using both kinematic and kinetic data (See Figures 20-25).

Proximal-to-distal sequencing is based upon the summation of speed principle which states that the speed of the distal segment summates upon the maximal speed of the proximal segment, allowing for a greater end-segment speed (Plagenhoef, 1971; Bunn, 1972; Kreighbaum & Barthels, 1985), while conserving and passing the angular momentum down the kinetic chain. In the baseball pitch, both the linear and angular rates of change (Figures 20 and 21) follow textbook examples (Plagenhoef, 1971) of proximal-to-distal sequencing, shown as a decrease in speed of the proximal segment and an increase of speed of the distal segment as the action moves toward release of the ball. This is further illustrated in the angular acceleration graph (Figure 22) where a reversal of the angular accelerations occurs, again following the premise of proximal-to-distal sequencing. These combinations of angular and linear speeds give substantial evidence that there is in fact a proximal-to-distal sequence in the overhand baseball pitch in skilled performers.

The secondary goal of this project was to classify the proximal-to-distal sequence into one of two theoretical models (Ford, 1998). The overhand pitch can be classified by comparing the temporal position derivatives to the moments produced about the proximal attachments of both segments. In order to classify the overhand baseball pitch as a Theory One (Ford, 1998) inertial acceleration, as some have suggested (Roberts, 1971; Toyoshima et al., 1976; Atwater, 1979; Fleisig et al., 1995) there must be a negative, external rotation, moment seen at the shoulder. The negative, external rotation, moment would suggest a negative acceleration of the internal rotation of the upper segment, causing a whipping or inertial acceleration of the lower arm segment. Furthermore, there should be an attenuated tricep activity (elbow extension) that coincides with this external rotation moment. To classify the pitch as a Theory Two (Ford, 1998) muscular acceleration of the distal segment, the moment at the shoulder should remain positive, furthering the internal rotation, with a spike in tricep activity (elbow extension) showing a muscular acceleration of the lower arm segment. The representative graphs of shoulder internal rotation moment (Figure 23) as well as the electromyograph (Figure 25) of the representative pitch indicate that the overhand baseball pitch is indeed a motion that follows the tenets of Theory One (Ford, 1998) of proximal-to-distal sequencing. The negative, external rotation, of the shoulder preceding release, along with a relatively quiet EMG reading suggest that the elbow is extended through inertial acceleration rather than a muscular acceleration. Unfortunately, the moment about the elbow is not as definitive an indicator in this study as it was for the underhand pitch. This may be explained mechanically through interaction of two rigid bodies as the proximal segment rotates about an axis. In the underhand pitch the segments rotated about the bi-lateral axis, however in the overhand throw the segments are rotating about multiple axes and as such the system will endeavor to reduce the load placed upon its structures. In this case, that means that since the upper arm and lower arm segments are connected with a hinge joint, the distal segment will attempt to move to a position of 90° flexion. This position of 90° of elbow flexion is desirable for the system and as such the musculature will attempt to maintain this elbow position in order to maximize the radius of the circle. Second, this 90° angle at the elbow is a position that is naturally achieved as a result of the rigid configuration of the upper arm and lower arm during internal rotation at the shoulder. It is this tendency of the segments to move to this conformation and the desire of the body to maintain this position accounts for the oscillations of the elbow about that point and is responsible for the muscular activation of the triceps or biceps to maintain this relative position, and not to accelerate the distal segment.

A tertiary purpose of this study was to expand the mathematical model created for the interaction of adjacent segment in the overhand pitch (Feltner, 1987; Feltner & Depena, 1988; Feltner & Depena, 1989). This expansion was similar to that done in the underhand pitch in that the kinematic terms are expressed in terms of the system variable derivatives, allowing the researcher to observe the time rates of change of the physically observable or controllable variables. Also, the equations were inverted to express the physical outputs in terms of their physical inputs, which will allow for better input from practitioners and better mechanical manipulation of the physically controllable movements. The following is the partitioning of the aforementioned moment equation (Feltner, 1987; Feltner & Depena, 1988; Feltner & Depena, 1989) which would allow for manipulation of physically controllable characteristics:

Moment about the elbow:

$$\vec{M}_E = m_L \cdot L_{LG} \cdot g \cdot \vec{k}_L \times \vec{j} + \left\{ \begin{array}{l} \left[\vec{M}_{EX} \quad \vec{M}_{EY} \quad \vec{M}_{EZ} \quad \vec{M}_{E\theta Y} \quad \vec{M}_{E\theta X} \quad \vec{M}_{E\theta Z} \quad \vec{M}_{E\theta E} \right] \cdot \\ \left[\ddot{X}_H \quad \ddot{Y}_H \quad \ddot{Z}_H \quad \ddot{\theta}_Y \quad \ddot{\theta}_X \quad \ddot{\theta}_Z \quad \ddot{\theta}_E \right]^T \end{array} \right\}$$

$$+ \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \\ \dot{\theta}_E \end{bmatrix}^T \cdot \begin{bmatrix} \vec{M}_{E\theta Y\theta Y} & \vec{M}_{E\theta Y\theta X} & \vec{M}_{E\theta Y\theta Z} & \vec{M}_{E\theta Y\theta E} \\ \vec{M}_{E\theta X\theta Y} & \vec{M}_{E\theta X\theta X} & \vec{M}_{E\theta X\theta Z} & \vec{M}_{E\theta X\theta E} \\ \vec{M}_{E\theta Z\theta Y} & \vec{M}_{E\theta Z\theta X} & \vec{M}_{E\theta Z\theta Z} & \vec{M}_{E\theta Z\theta E} \\ \vec{M}_{E\theta E\theta Y} & \vec{M}_{E\theta E\theta X} & \vec{M}_{E\theta E\theta Z} & \vec{M}_{E\theta E\theta E} \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \\ \dot{\theta}_E \end{bmatrix}$$

Where:

$$\vec{M}_{EX} = m_L \cdot L_{LG} \cdot \vec{k}_L \times \vec{i}$$
$$\vec{M}_{EY} = m_L \cdot L_{LG} \cdot \vec{k}_L \times \vec{j}$$
$$\vec{M}_{EZ} = m_L \cdot L_{LG} \cdot \vec{k}_L \times \vec{k}$$
$$\vec{M}_{E\theta Y} = m_L \cdot \left\{ L_U \cdot L_{LG} \cdot \vec{k}_L \times \left(\vec{j}_1 \times \vec{k}_2 \right) + \left(L_{LG}^2 + k_{LB}^2 \right) \cdot \vec{j}_1 - \left(L_{LG}^2 + k_{LB}^2 \cdot (1 - r_L) \right) \cdot \vec{k}_L \cdot \vec{k}_L \square \vec{j}_1 \right\}$$
$$\vec{M}_{E\theta X} = m_L \cdot \left\{ L_U \cdot L_{LG} \cdot \vec{j}_2 \times \vec{k}_L + \left(L_{LG}^2 + k_{LB}^2 \right) \cdot \vec{i}_2 - \left(L_{LG}^2 + k_{LB}^2 \cdot (1 - r_L) \right) \cdot \vec{k}_L \cdot \vec{k}_L \square \vec{i}_2 \right\}$$
$$\vec{M}_{E\theta Z} = m_L \cdot \left\{ \vec{k}_U \cdot \left(L_{LG}^2 + k_{LB}^2 \right) - \vec{k}_L \cdot \cos\left(\theta_E\right) \cdot \left(L_{LG}^2 + k_{LB}^2 \cdot (1 - r_L) \right) \right\}$$
$$\vec{M}_{E\theta E} = m_L \cdot \left(L_{LG}^2 + k_{LB}^2 \right) \cdot \vec{j}_L$$

Moment about the shoulder:

$$\vec{M}_H = \left\{ m_U \cdot L_{UG} \cdot \vec{k}_U + m_L \cdot \left(L_U \cdot \vec{k}_U + L_{LG} \cdot \vec{k}_L \right) \right\} \times g \cdot \vec{j}$$
$$+ \left\{ \begin{array}{l} \left[\vec{M}_{HX} \quad \vec{M}_{HY} \quad \vec{M}_{HZ} \quad \vec{M}_{H\theta Y} \quad \vec{M}_{H\theta X} \quad \vec{M}_{H\theta Z} \quad \vec{M}_{H\theta E} \right] \cdot \\ \left[\ddot{X}_H \quad \ddot{Y}_H \quad \ddot{Z}_H \quad \ddot{\theta}_Y \quad \ddot{\theta}_X \quad \ddot{\theta}_Z \quad \ddot{\theta}_E \right]^T \end{array} \right\}$$
$$+ \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \\ \dot{\theta}_E \end{bmatrix}^T \cdot \begin{bmatrix} \vec{M}_{H\theta Y\theta Y} & \vec{M}_{H\theta Y\theta X} & \vec{M}_{H\theta Y\theta Z} & \vec{M}_{H\theta Y\theta E} \\ \vec{M}_{H\theta X\theta Y} & \vec{M}_{H\theta X\theta X} & \vec{M}_{H\theta X\theta Z} & \vec{M}_{H\theta X\theta E} \\ \vec{M}_{H\theta Z\theta Y} & \vec{M}_{H\theta Z\theta X} & \vec{M}_{H\theta Z\theta Z} & \vec{M}_{H\theta Z\theta E} \\ \vec{M}_{H\theta E\theta Y} & \vec{M}_{H\theta E\theta X} & \vec{M}_{H\theta E\theta Z} & \vec{M}_{H\theta E\theta E} \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \\ \dot{\theta}_E \end{bmatrix}$$

Where:

$$\vec{M}_{HX} = \left(\left(m_U \cdot L_{UG} + m_L \cdot L_U \right) \cdot \vec{k}_U + m_L \cdot L_{LG} \cdot \vec{k}_L \right) \times \vec{i}$$
$$\vec{M}_{HY} = \left(\left(m_U \cdot L_{UG} + m_L \cdot L_U \right) \cdot \vec{k}_U + m_L \cdot L_{LG} \cdot \vec{k}_L \right) \times \vec{j}$$
$$\vec{M}_{HZ} = \left(\left(m_U \cdot L_{UG} + m_L \cdot L_U \right) \cdot \vec{k}_U + m_L \cdot L_{LG} \cdot \vec{k}_L \right) \times \vec{k}$$

$$\vec{M}_{H\theta Y} = \left\{ \begin{array}{l} \left(m_U \cdot \left(L_{UG}^2 + k_{UB}^2 \right) + m_L \cdot \left(L_U^2 + L_{LG}^2 + 2 \cdot L_U \cdot L_{LG} \cdot \cos\left(\theta_E\right) \right) + k_{LB}^2 \right) \cdot \vec{j}_1 \\ -\left(\left(m_L \cdot L_U^2 + m_U \cdot \left(L_{UG}^2 + k_{UB}^2 \cdot \left(1 - r_U\right) \right) \right) \cdot \vec{j}_1 \bullet \vec{k}_2 + m_L \cdot L_U \cdot L_{LG} \cdot \vec{j}_1 \bullet \vec{k}_L \right) \cdot \vec{k}_U \\ -m_L \cdot \left(\left(L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right) \right) \cdot \vec{j}_1 \bullet \vec{k}_L + L_U \cdot L_{LG} \cdot \vec{j}_1 \bullet \vec{k}_2 \right) \cdot \vec{k}_L \end{array} \right\}$$

$$\vec{M}_{H\theta X} = \left\{ \begin{array}{l} \left(m_U \cdot \left(L_{UG}^2 + k_{UB}^2 \right) + m_L \cdot \left(L_U^2 + L_{LG}^2 + L_{LG} \cdot L_U \cdot \cos\left(\theta_E\right) + k_{LB}^2 \right) \right) \cdot \vec{i}_2 \\ -m_L \cdot \left(\left(L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right) \right) \cdot \vec{k}_L \bullet \vec{i}_2 \cdot \vec{k}_L + L_{LG} \cdot L_U \cdot \vec{k}_L \times \vec{j}_2 \right) \end{array} \right\}$$

$$\vec{M}_{H\theta Z} = \left\{ \begin{array}{l} \left(m_U \cdot k_{UB}^2 \cdot r_U + m_L \cdot \left(L_{LG}^2 + L_U \cdot L_{LG} \cdot \cos\left(\theta_E\right) + k_{LB}^2 \right) \right) \cdot \vec{k}_U \\ -m_L \cdot \left(\left(L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right) \right) \cdot \cos\left(\theta_E\right) + L_U \cdot L_{LG} \right) \cdot \vec{k}_L \end{array} \right\}$$

$$\vec{M}_{H\theta E} = m_L \cdot \left(L_{LG} \cdot L_U \cdot \cos\left(\theta_E\right) + L_{LG}^2 + k_{LB}^2 \right) \cdot \vec{j}_L$$

$$\vec{M}_{H\theta Y\theta Y} = -\left\{ \begin{array}{l} \vec{j}_1 \times \vec{k}_2 \cdot \left(\left(m_U \cdot \left(L_{UG}^2 + k_{UB}^2 \cdot \left(1 - r_U\right) \right) + m_L \cdot L_U^2 \right) \cdot \vec{j}_1 \bullet \vec{k}_2 + m_L \cdot L_{LG} \cdot L_U \cdot \vec{j}_1 \bullet \vec{k}_L \right) \\ + \vec{j}_1 \times \vec{k}_L \cdot m_L \cdot \left(\left(L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right) \right) \cdot \vec{j}_1 \bullet \vec{k}_L + L_{LG} \cdot L_U \cdot \vec{j}_1 \bullet \vec{k}_2 \right) \end{array} \right\}$$

$$\vec{M}_{H\theta X\theta X} = m_L \cdot \left\{ L_U \cdot L_{LG} \cdot \sin\left(\theta_E\right) \cdot \left(\vec{j}_U - \vec{i}_2 \cdot \vec{i}_2 \cdot \vec{j}_U \right) - \left(L_U \cdot L_{LG} + L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right) \right) \vec{i}_2 \times \vec{k}_L \cdot \vec{k}_L \cdot \vec{i}_2 \right\}$$

$$\vec{M}_{H\theta X\theta Y} = \left\{ \begin{array}{l} -\vec{k}_1 \cdot \left(m_U \cdot k_{UB}^2 + m_L \cdot k_{LB}^2 \right) + \vec{k}_U \cdot m_U \cdot k_{UB}^2 \cdot \left(1 - r_U\right) \cdot \cos\left(\theta_X\right) - \vec{j}_1 \times \vec{k}_L \cdot m_L \cdot k_{LB}^2 \cdot \left(1 - r_L\right) \cdot \vec{k}_L \cdot \vec{i}_2 \\ + \vec{j}_2 \cdot \left(m_U \cdot \left(2 \cdot L_{UG}^2 + k_{UB}^2 \cdot \left(1 - r_U\right) \right) \cdot \vec{j}_1 \cdot \vec{k}_2 + m_L \cdot 2 \cdot L_U \cdot \left(L_{LG} \cdot \vec{j}_1 \cdot \vec{k}_L - L_U \cdot \sin\left(\theta_X\right) \right) \right) \\ + \vec{k}_L \times \vec{i}_2 \cdot \left(m_L \cdot \left(\left(k_{LB}^2 \cdot \left(1 - r_L\right) + 2 \cdot L_{LG}^2 \right) \cdot \vec{j}_1 \cdot \vec{k}_L - 2 \cdot L_{LG} \cdot L_U \cdot \sin\left(\theta_X\right) \right) \right) + \vec{k}_L \cdot m_L \cdot k_{LB}^2 \cdot \left(1 - r_L\right) \cdot \vec{k}_1 \cdot \vec{k}_L \end{array} \right\}$$

$$\vec{M}_{H\theta Z\theta Z} = -\vec{j}_U \cdot m_L \cdot \sin\left(\theta_E\right) \cdot \left(\left(k_{LB}^2 \cdot \left(1 - r_L\right) + L_{LG}^2 \right) \cdot \cos\left(\theta_E\right) + L_U \cdot L_{LG} \right)$$

$$\vec{M}_{H\theta Z\theta Y} = \left\{ \begin{array}{l} \vec{i}_2 \cdot \cos\left(\theta_X\right) \cdot \left(m_U \cdot k_{UB}^2 \cdot r_U + m_L \cdot k_{LB}^2 \right) - m_L \cdot k_{LB}^2 \cdot \left(1 - r_L\right) \cdot \left(\sin\left(\theta_E\right) \cdot \vec{k}_L \cdot \vec{j}_L \cdot \vec{j}_1 + \vec{j}_1 \times \vec{k}_L \cdot \cos\left(\theta_E\right) \right) \\ -m_L \cdot \vec{j}_L \cdot \sin\left(\theta_E\right) \cdot \left(2 \cdot L_U \cdot L_{LG} \cdot \vec{k}_U \cdot \vec{j}_1 + \left(2 \cdot L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right) \right) \cdot \vec{k}_L \cdot \vec{j}_1 \right) \end{array} \right\}$$

$$\vec{M}_{H\theta Z\theta X} = \left\{ \begin{array}{l} -\vec{j}_2 \cdot \left(m_U \cdot k_{UB}^2 \cdot r_U + m_L \cdot k_{LB}^2 \right) - \vec{j}_U \cdot m_L \cdot \sin\left(\theta_E\right) \cdot \vec{k}_L \cdot \vec{i}_2 \cdot \left(2 \cdot L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right) \right) \\ -m_L \cdot k_{LB}^2 \cdot \left(1 - r_L\right) \cdot \left(\sin\left(\theta_E\right) \cdot \left(\vec{k}_L \cdot \vec{j}_U \cdot \vec{i}_2 \right) + \vec{i}_2 \times \vec{k}_L \cdot \cos\left(\theta_E\right) \right) \end{array} \right\}$$

$$\vec{M}_{H\theta E\theta E} = -m_L \cdot L_U \cdot L_{LG} \cdot \sin\left(\theta_E\right) \cdot \vec{j}_U$$

$$\vec{M}_{H\theta E\theta Y} = -m_L \cdot \left\{ \begin{array}{l} \vec{i}_L \cdot \left(\left(2 \cdot L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right) \right) \cdot \vec{k}_L \cdot \vec{j}_1 + 2 \cdot L_{LG} \cdot L_U \cdot \vec{k}_U \cdot \vec{j}_1 \right) + \vec{j}_1 \cdot 2 \cdot L_{LG} \cdot L_U \cdot \sin\left(\theta_E\right) \\ + k_{LB}^2 \cdot \left(\left(1 - r_L\right) \cdot \vec{k}_L \cdot \vec{i}_L \cdot \vec{j}_1 - \vec{j}_1 \times \vec{j}_U \right) \end{array} \right\}$$

$$\vec{M}_{H\theta E\theta X} = -m_L \cdot \left\{ \begin{array}{l} \vec{i}_L \cdot \left(2 \cdot L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right) \right) \cdot \vec{k}_L \cdot \vec{i}_2 + \vec{i}_2 \cdot 2 \cdot L_U \cdot L_{LG} \cdot \sin\left(\theta_E\right) \\ + k_{LB}^2 \cdot \left(\left(1 - r_L\right) \cdot \vec{k}_L \cdot \vec{i}_L \cdot \vec{i}_2 - \left(\vec{i}_2 \times \vec{j}_U \right) \right) \end{array} \right\}$$

$$\vec{M}_{H\theta E\theta Z} = -m_L \cdot \left\{ \begin{array}{l} \left(2 \cdot L_U \cdot L_{LG} \cdot \cos\left(\theta_E\right) + k_{LB}^2 \right) \cdot \vec{i}_U + \left(2 \cdot L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right) \right) \cdot \cos\left(\theta_E\right) \cdot \vec{i}_L \\ -k_{LB}^2 \cdot \left(1 - r_L\right) \cdot \vec{k}_L \cdot \sin\left(\theta_E\right) \end{array} \right\}$$

Implications

The data from this study supported previous research which classified the overhand baseball pitch into Theory One (Ford, 1998) which suggests there is a negative acceleration at the proximal portion of the proximal segment, causing an inertial acceleration of the distal segment. This implies there is an eccentric load placed on the external rotators of the shoulder (teres minor, infraspinatus, and posterior deltoid) as they produce a negative moment in an attempt to negatively accelerate the arm moving forward. These muscles are relatively weak compared to the larger internal rotators of the shoulder (latissimus dorsi, pectoralis major, teres major, and anterior deltoid). This inequality in strength may be what causes these small rotator cuff muscles to be the ones that are injured more often than any other shoulder musculature (Fleisig et al., 1995). This musculature may also be an overlooked area of strength training. If the negative accelerators are indirectly responsible for the inertial acceleration of elbow extension, it would benefit the athlete to train this musculature in that manner. By training the musculature in the way it is used, as a negative accelerator of internal rotation, injury prevalence may be reduced and performance enhanced.

Future Research

Future research on this motion should continue up the kinetic chain in an attempt to fully explain the mathematical and kinesiological factors that lead to maximal velocity during the overhand pitch. Furthermore, a three-dimensional analysis of both elbow and shoulder joints with the current partitioning scheme of accelerations should be the next endeavor on this topic. To date, there is a mechanical model of the throw, but not a model that is worthwhile to the practitioner. By partitioning the accelerations of this motion, it allows the confluence of the mechanical world and the applied world, allowing for a true scientific base of training and teaching.

Hypotheses Results*:*

H_{a1}: Overhand and underhand throwing will utilize different proximal-to-distal sequencing patterns to maximize end-ball velocity.

Research (Roberts, 1971; Atwater, 1979; Feltner & Depena, 1986; Feltner, 1987, 1989; Fleisig et al., 1995) has supported that the overhand throw will elicit a sequential pattern that fits within the structure of Theory One (Ford, 1998). The underhand softball pitch has not been researched in this manner. It is hypothesized that the flexion moment at the elbow within the windmill pitch will fit the model for Theory Two (Ford, 1998), due to the joint position and for the need to protect the joint during the rapid flexion moment (Atwater, 1979; Barrentine et al., 1998; Ford, 1998; Werner et al., 2006).

Result: The results of this study suggest that the overhand and underhand pitches both exhibit the markers that allow for classification into (Ford, 1998) Theory One of proximal-to-distal sequencing.

H_{a2}: To date, no conclusive evidence supports the hypothesis that the elbow flexors and elbow extensors are analogous with respect to the particular type of pitch. There is a plethora of evidence to suggest the role of the elbow extensors in the overhand throw, but none to suggest the role of the elbow flexors in the underhand throw. However, it is implied that the flexion moment of the underhand throw would necessitate the activity of a flexor muscle activation via Theory Two of proximal-to-distal sequencing (Ford, 1998; Werner et al., 2006).

Result: The results of this study suggest that the biceps brachii, acting at the elbow, is analogous to the triceps brachii in their respective function during the overhand and underhand pitch. This conclusion must be taken with some care due to the multiarticular nature of both of the long heads of the muscles. The muscles are in fact analogous in their function at the elbow, but not at the shoulder.

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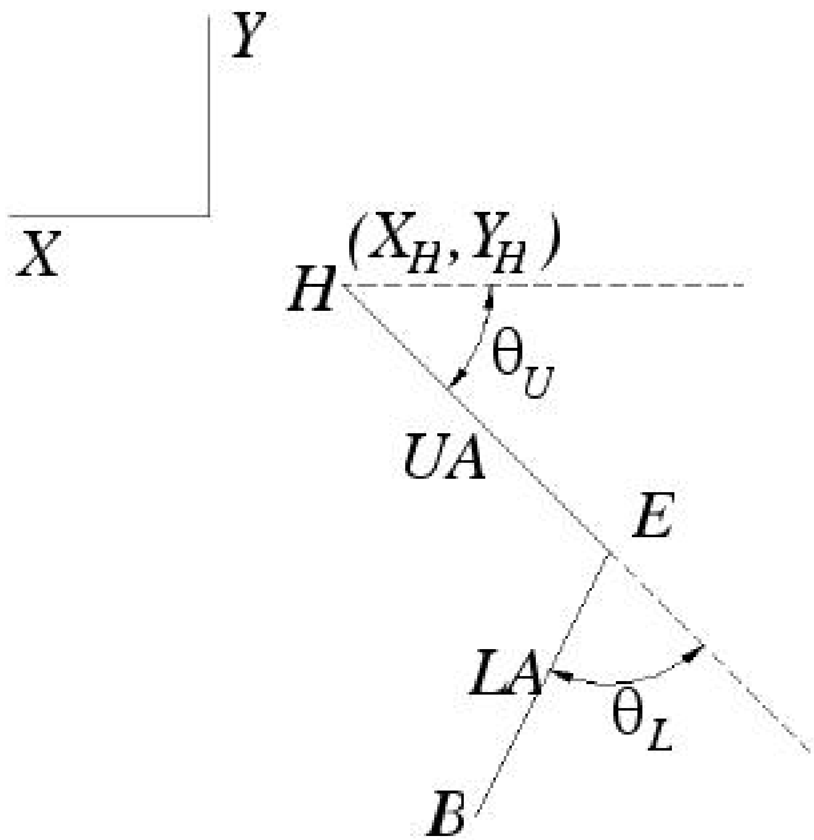
Appendix A

The Expanded Underhand Throwing Kinematic Equations

All equations were based on the following drawing:



All velocities will be determined based upon the unit vectors listed below

$$\begin{bmatrix} \vec{i}_U \\ \vec{j}_U \end{bmatrix} = \begin{bmatrix} \cos(\theta_U) & \sin(\theta_U) \\ -\sin(\theta_U) & \cos(\theta_U) \end{bmatrix} \cdot \begin{bmatrix} \vec{i} \\ \vec{j} \end{bmatrix}$$

$$\begin{bmatrix} \vec{i}_L \\ \vec{j}_L \end{bmatrix} = \begin{bmatrix} \cos(\theta_L) & \sin(\theta_L) \\ -\sin(\theta_L) & \cos(\theta_L) \end{bmatrix} \cdot \begin{bmatrix} \vec{i}_U \\ \vec{j}_U \end{bmatrix}$$

I. Angular Velocities

Angular Velocity of θ_U:

$$\vec{\omega}_U = \vec{k} \cdot \frac{d}{dt}\theta_U = \dot{\theta}_U \cdot \vec{k} = \vec{k}_U \cdot \frac{d}{dt}\theta_U = \dot{\theta}_U \cdot \vec{k}_U$$

Angular Velocity of $\theta_{U/L}$

$$\vec{\omega}_{L/U} = \vec{k}_U \cdot \frac{d}{dt}\theta_L = \dot{\theta}_L \cdot \vec{k}_U = \vec{k}_L \cdot \frac{d}{dt}\theta_L = \dot{\theta}_L \cdot \vec{k}_L$$

Angular Velocity of θ_L

$$\vec{\omega}_L = \vec{\omega}_{L/U} + \vec{\omega}_U = \vec{k} \cdot \frac{d}{dt}\theta_U + \vec{k} \cdot \frac{d}{dt}\theta_L = \vec{k} \cdot \frac{d}{dt}(\theta_U + \theta_L) = \vec{k} \cdot \left(\dot{\theta}_U + \dot{\theta}_L\right)$$

II Linear Velocities

Velocity of Humeral Head:

$$\vec{v}_H = \vec{i} \cdot \frac{d}{dt} X_H + \vec{j} \cdot \frac{d}{dt} Y_H = \vec{i} \cdot \dot{X}_H + \vec{j} \cdot \dot{Y}_H$$

Velocity of the Elbow:

$$\vec{v}_E = \vec{v}_H - L_U \cdot \left(\frac{d}{dt}\theta_u\right) \cdot \vec{j}_U$$

Velocity of the Mass Center of the Upper Arm

$$\vec{v}_{UG} = \vec{v}_H - L_{UG} \cdot \left(\frac{d}{dt} \theta_u \right) \cdot \vec{j}_U$$

Velocity of the Wrist/Ball

$$\vec{v}_B = \vec{v}_E - L_L \cdot \left(\frac{d}{dt} \theta_u + \frac{d}{dt} \theta_L \right) \cdot \vec{j}_L$$

Velocity of the Mass Center of the Lower Arm

$$\vec{v}_{LG} = \vec{v}_E - L_{LG} \cdot \left(\frac{d}{dt} \theta_u + \frac{d}{dt} \theta_L \right) \cdot \vec{j}_L$$

III. Angular Accelerations

Angular Acceleration of θ_U

$$\vec{\alpha}_U = \frac{d}{dt} \vec{\omega}_U = \vec{k} \cdot \frac{d^2}{dt^2} \theta_U = \vec{k} \cdot \ddot{\theta}_U$$

Angular Acceleration of $\theta_{L/U}$

$$\vec{\alpha}_{L/U} = \vec{k}_u \frac{d}{dt} \omega_L = \vec{k} \cdot \frac{d^2}{dt^2} \Theta_L = \vec{k} \cdot \ddot{\Theta}_U$$

Angular Acceleration of θ_L

$$\vec{\alpha}_L = \frac{d}{dt} \vec{\omega}_L = \vec{k} \cdot \frac{d^2}{dt^2} \left(\theta_U + \theta_L \right) = \vec{k} \cdot \left(\ddot{\theta}_U + \ddot{\theta}_L \right)$$

.

IV. Linear Accelerations

Acceleration of the Shoulder

$$\vec{a}_H = \frac{d}{dt}\vec{v}_H = \vec{i} \cdot \frac{d^2}{dt^2} X_H + \vec{j} \cdot \frac{d^2}{dt^2} Y_H = \vec{i} \cdot \ddot{X}_H + \vec{j} \cdot \ddot{Y}_H$$

Acceleration of the Elbow:

$$\vec{a}_E = \vec{a}_H - L_U \cdot \vec{j}_U \cdot \frac{d^2}{dt^2}\theta_U + L_U \cdot \vec{i}_U \cdot \left(\frac{d}{dt}\theta_U \right)^2$$

Acceleration of the Mass Center of the Upper Arm

$$\vec{a}_{UG} = \vec{a}_H - L_{UG} \cdot \vec{j}_U \cdot \frac{d^2}{dt^2}\theta_U + L_{UG} \cdot \vec{i}_U \cdot \left(\frac{d}{dt}\theta_U \right)^2$$

Acceleration of the Wrist/Ball

$$\vec{a}_B = \vec{a}_E - L_L \cdot \vec{j}_L \cdot \left(\frac{d^2}{dt^2}\theta_U + \frac{d^2}{dt^2}\theta_L \right) + L_L \cdot \vec{i}_L \cdot \left(\frac{d}{dt}\theta_U + \frac{d}{dt}\theta_L \right)^2$$

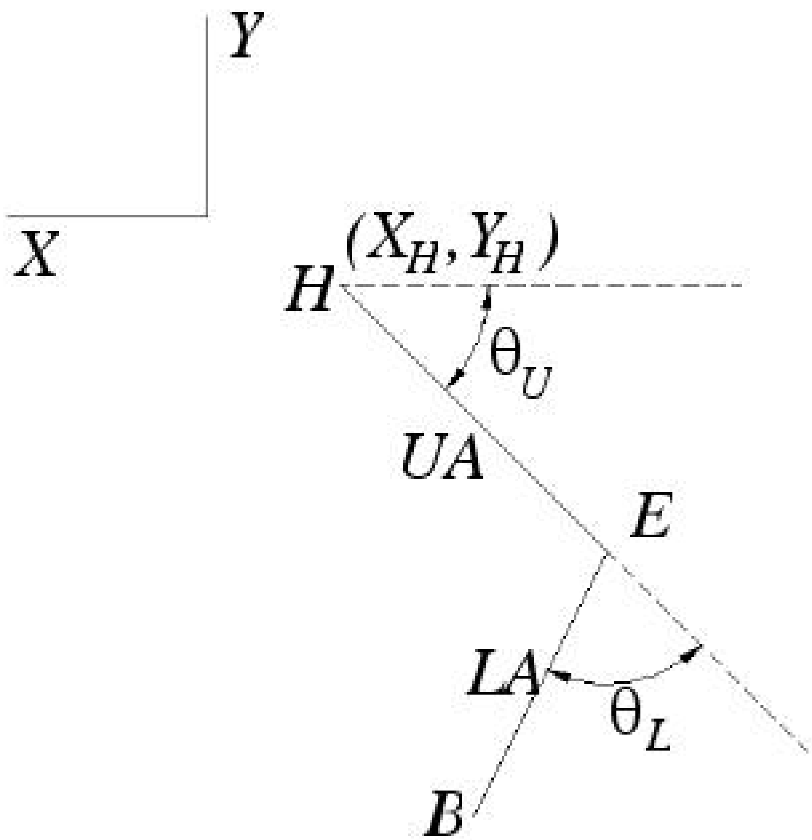
Acceleration of the Mass Center of the Lower Arm

$$\vec{a}_{LG} = \vec{a}_E - L_{LG} \cdot \vec{j}_L \cdot \left(\frac{d^2}{dt^2}\theta_U + \frac{d^2}{dt^2}\theta_L \right) + L_{LG} \cdot \vec{i}_L \cdot \left(\frac{d}{dt}\theta_U + \frac{d}{dt}\theta_L \right)^2$$

Appendix B

The Expanded Underhand Throwing Kinetic Equations

All equations were based on the following drawing:



All velocities will be determined based upon the unit vectors listed below

$$\begin{bmatrix} \vec{i}_U \\ \vec{j}_U \end{bmatrix} = \begin{bmatrix} \cos(\theta_U) & \sin(\theta_U) \\ -\sin(\theta_U) & \cos(\theta_U) \end{bmatrix} \cdot \begin{bmatrix} \vec{i} \\ \vec{j} \end{bmatrix}$$

$$\begin{bmatrix} \vec{i}_L \\ \vec{j}_L \end{bmatrix} = \begin{bmatrix} \cos(\theta_L) & \sin(\theta_L) \\ -\sin(\theta_L) & \cos(\theta_L) \end{bmatrix} \cdot \begin{bmatrix} \vec{i}_U \\ \vec{j}_U \end{bmatrix}$$

I. Moment about the Elbow (M_E)

$$M_E = \left\{ \begin{aligned} & \begin{bmatrix} M_{EX} & M_{EY} \end{bmatrix} \cdot \begin{bmatrix} \ddot{X}_H \\ \ddot{Y}_H \end{bmatrix} + \begin{bmatrix} M_{EU} & M_{EL} \end{bmatrix} \cdot \begin{bmatrix} \ddot{\theta}_U \\ \ddot{\theta}_L \end{bmatrix} \\ & + \begin{bmatrix} \dot{\theta}_U & \dot{\theta}_L \end{bmatrix} \cdot \begin{bmatrix} M_{EUU} & M_{EUL} \\ M_{ELU} & M_{ELL} \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_U \\ \dot{\theta}_L \end{bmatrix} \end{aligned} \right\}$$

where:

$$M_{EU} = I_{LE} + m_L \cdot L_{LG} \cdot L_U \cdot \cos(\theta_L)$$

$$M_{EL} = I_{LE}$$

$$M_{EX} = m_L \cdot L_{LG} \cdot \sin(\theta_U + \theta_L)$$

$$M_{EY} = -m_L \cdot L_{LG} \cdot \cos(\theta_U + \theta_L)$$

$$M_{EUU} = m_L \cdot L_{LG} \cdot L_U \cdot \sin(\theta_L)$$

$$M_{EUL} = M_{ELU} = 0$$

$$M_{ELL} = 0$$

II. Moment about the Shoulder (M_H):

$$M_H = \left\{ \begin{aligned} & \begin{bmatrix} M_{HX} & M_{HY} \end{bmatrix} \cdot \begin{bmatrix} \ddot{X}_H \\ \ddot{Y}_H \end{bmatrix} + \begin{bmatrix} M_{HU} & M_{HL} \end{bmatrix} \cdot \begin{bmatrix} \ddot{\theta}_U \\ \ddot{\theta}_L \end{bmatrix} \\ & + \begin{bmatrix} \dot{\theta}_U & \dot{\theta}_L \end{bmatrix} \cdot \begin{bmatrix} M_{HUU} & M_{HUL} \\ M_{HLU} & M_{HLL} \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_U \\ \dot{\theta}_L \end{bmatrix} \end{aligned} \right\}$$

where:

$$\vec{M}_{HX} = \left(\left(m_U \cdot L_{UG} + m_L \cdot L_U \right) \cdot \vec{k}_U + m_L \cdot L_{LG} \cdot \vec{k}_L \right) \times \vec{i}$$

$$\vec{M}_{HY} = \left(\left(m_U \cdot L_{UG} + m_L \cdot L_U \right) \cdot \vec{k}_U + m_L \cdot L_{LG} \cdot \vec{k}_L \right) \times \vec{j}$$

$$\vec{M}_{HZ} = \left(\left(m_U \cdot L_{UG} + m_L \cdot L_U \right) \cdot \vec{k}_U + m_L \cdot L_{LG} \cdot \vec{k}_L \right) \times \vec{k}$$

$$\vec{M}_{H\theta Y} = \left\{ \begin{array}{l} \left(m_U \cdot \left(L_{UG}^2 + k_{UB}^2 \right) + m_L \cdot \left(L_U^2 + L_{LG}^2 + 2 \cdot L_U \cdot L_{LG} \cdot \cos\left(\theta_E\right) \right) + k_{LB}^2 \right) \cdot \vec{j}_1 \\ - \left(\left(m_L \cdot L_U^2 + m_U \cdot \left(L_{UG}^2 + k_{UB}^2 \cdot \left(1 - r_U\right) \right) \right) \cdot \vec{j}_1 \bullet \vec{k}_2 + m_L \cdot L_U \cdot L_{LG} \cdot \vec{j}_1 \bullet \vec{k}_L \right) \cdot \vec{k}_U \\ - m_L \cdot \left(\left(L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right) \right) \cdot \vec{j}_1 \bullet \vec{k}_L + L_U \cdot L_{LG} \cdot \vec{j}_1 \bullet \vec{k}_2 \right) \cdot \vec{k}_L \end{array} \right\}$$

$$\vec{M}_{H\theta X} = \left\{ \begin{array}{l} \left(m_U \cdot \left(L_{UG}^2 + k_{UB}^2 \right) + m_L \cdot \left(L_U^2 + L_{LG}^2 + L_{LG} \cdot L_U \cdot \cos\left(\theta_E\right) + k_{LB}^2 \right) \right) \cdot \vec{i}_2 \\ - m_L \cdot \left(\left(L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right) \right) \cdot \vec{k}_L \bullet \vec{i}_2 \cdot \vec{k}_L + L_{LG} \cdot L_U \cdot \vec{k}_L \times \vec{j}_2 \right) \end{array} \right\}$$

$$\vec{M}_{H\theta Z} = \left\{ \begin{array}{l} \left(m_U \cdot k_{UB}^2 \cdot r_U + m_L \cdot \left(L_{LG}^2 + L_U \cdot L_{LG} \cdot \cos\left(\theta_E\right) + k_{LB}^2 \right) \right) \cdot \vec{k}_U \\ - m_L \cdot \left(\left(L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right) \right) \cdot \cos\left(\theta_E\right) + L_U \cdot L_{LG} \right) \cdot \vec{k}_L \end{array} \right\}$$

$$\vec{M}_{H\theta E} = m_L \cdot \left(L_{LG} \cdot L_U \cdot \cos\left(\theta_E\right) + L_{LG}^2 + k_{LB}^2 \right) \cdot \vec{j}_L$$

$$\vec{M}_{H\theta Y\theta Y} = - \left\{ \begin{array}{l} \vec{j}_1 \times \vec{k}_2 \cdot \left(\left(m_U \cdot \left(L_{UG}^2 + k_{UB}^2 \cdot \left(1 - r_U\right) \right) + m_L \cdot L_U^2 \right) \cdot \vec{j}_1 \bullet \vec{k}_2 + m_L \cdot L_{LG} \cdot L_U \cdot \vec{j}_1 \bullet \vec{k}_L \right) \\ + \vec{j}_1 \times \vec{k}_L \cdot m_L \cdot \left(\left(L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right) \right) \cdot \vec{j}_1 \bullet \vec{k}_L + L_{LG} \cdot L_U \cdot \vec{j}_1 \bullet \vec{k}_2 \right) \end{array} \right\}$$

$$\vec{M}_{H\theta X\theta X} = m_L \cdot \left\{ L_U \cdot L_{LG} \cdot \sin\left(\theta_E\right) \cdot \left(\vec{j}_U - \vec{i}_2 \cdot \vec{i}_2 \square \vec{j}_U \right) - \left(L_U \cdot L_{LG} + L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right) \right) \vec{i}_2 \times \vec{k}_L \cdot \vec{k}_L \square \vec{i}_2 \right\}$$

$$\vec{M}_{H\theta X\theta Y} = \left\{ \begin{array}{l} -\vec{k}_1 \cdot \left(m_U \cdot k_{UB}^2 + m_L \cdot k_{LB}^2 \right) + \vec{k}_U \cdot m_U \cdot k_{UB}^2 \cdot \left(1 - r_U\right) \cdot \cos\left(\theta_X\right) - \vec{j}_1 \times \vec{k}_L \cdot m_L \cdot k_{LB}^2 \cdot \left(1 - r_L\right) \cdot \vec{k}_L \square \vec{i}_2 \\ + \vec{j}_2 \cdot \left(m_U \cdot \left(2 \cdot L_{UG}^2 + k_{UB}^2 \cdot \left(1 - r_U\right) \right) \cdot \vec{j}_1 \square \vec{k}_2 + m_L \cdot 2 \cdot L_U \cdot \left(L_{LG} \cdot \vec{j}_1 \square \vec{k}_L - L_U \cdot \sin\left(\theta_X\right) \right) \right) \\ + \vec{k}_L \times \vec{i}_2 \cdot \left(m_L \cdot \left(\left(k_{LB}^2 \cdot \left(1 - r_L\right) + 2 \cdot L_{LG}^2 \right) \cdot \vec{j}_1 \square \vec{k}_L - 2 \cdot L_{LG} \cdot L_U \cdot \sin\left(\theta_X\right) \right) \right) + \vec{k}_L \cdot m_L \cdot k_{LB}^2 \cdot \left(1 - r_L\right) \cdot \vec{k}_1 \square \vec{k}_L \end{array} \right\}$$

$$\vec{M}_{H\theta Z\theta Z} = - \vec{j}_U \cdot m_L \cdot \sin\left(\theta_E\right) \cdot \left(\left(k_{LB}^2 \cdot \left(1 - r_L\right) + L_{LG}^2 \right) \cdot \cos\left(\theta_E\right) + L_U \cdot L_{LG} \right)$$

$$\vec{M}_{H\theta Z\theta Y}=\begin{Bmatrix}\vec{i}_2\cdot\cos\left(\theta_X\right)\cdot\left(m_U\cdot k_{UB}^2\cdot r_U+m_L\cdot k_{LB}^2\right)-m_L\cdot k_{LB}^2\cdot\left(1-r_L\right)\cdot\left(\sin\left(\theta_E\right)\cdot\vec{k}_L\cdot\vec{j}_L\square\vec{j}_1+\vec{j}_1\times\vec{k}_L\cdot\cos\left(\theta_E\right)\right)\\-m_L\cdot\vec{j}_L\cdot\sin\left(\theta_E\right)\cdot\left(2\cdot L_U\cdot L_{LG}\cdot\vec{k}_U\square\vec{j}_1+\left(2\cdot L_{LG}^2+k_{LB}^2\cdot\left(1-r_L\right)\right)\cdot\vec{k}_L\square\vec{j}_1\right)\end{Bmatrix}$$

$$\vec{M}_{H\theta Z\theta X}=\begin{Bmatrix}-\vec{j}_2\cdot\left(m_U\cdot k_{UB}^2\cdot r_U+m_L\cdot k_{LB}^2\right)-\vec{j}_U\cdot m_L\cdot\sin\left(\theta_E\right)\cdot\vec{k}_L\square\vec{i}_2\cdot\left(2\cdot L_{LG}^2+k_{LB}^2\cdot\left(1-r_L\right)\right)\\-m_L\cdot k_{LB}^2\cdot\left(1-r_L\right)\cdot\left(\sin\left(\theta_E\right)\cdot\left(\vec{k}_L\cdot\vec{j}_U\square\vec{i}_2\right)+\vec{i}_2\times\vec{k}_L\cdot\cos\left(\theta_E\right)\right)\end{Bmatrix}$$

$$\vec{M}_{H\theta E\theta E}=-m_L\cdot L_U\cdot L_{LG}\cdot\sin\left(\theta_E\right)\cdot\vec{j}_U$$

$$\vec{M}_{H\theta E\theta Y}=-m_L\cdot\begin{Bmatrix}\vec{i}_L\cdot\left(\left(2\cdot L_{LG}^2+k_{LB}^2\cdot\left(1-r_L\right)\right)\cdot\vec{k}_L\square\vec{j}_1+2\cdot L_{LG}\cdot L_U\cdot\vec{k}_U\square\vec{j}_1\right)+\vec{j}_1\cdot 2\cdot L_{LG}\cdot L_U\cdot\sin\left(\theta_E\right)\\+k_{LB}^2\cdot\left(\left(1-r_L\right)\cdot\vec{k}_L\cdot\vec{i}_L\square\vec{j}_1-\vec{j}_1\times\vec{j}_U\right)\end{Bmatrix}$$

$$\vec{M}_{H\theta E\theta X}=-m_L\cdot\begin{Bmatrix}\vec{i}_L\cdot\left(2\cdot L_{LG}^2+k_{LB}^2\cdot\left(1-r_L\right)\right)\cdot\vec{k}_L\square\vec{i}_2+\vec{i}_2\cdot 2\cdot L_U\cdot L_{LG}\cdot\sin\left(\theta_E\right)\\+k_{LB}^2\cdot\left(\left(1-r_L\right)\cdot\vec{k}_L\cdot\vec{i}_L\square\vec{i}_2-\left(\vec{i}_2\times\vec{j}_U\right)\right)\end{Bmatrix}$$

$$\vec{M}_{H\theta E\theta Z}=-m_L\cdot\begin{Bmatrix}\left(2\cdot L_U\cdot L_{LG}\cdot\cos\left(\theta_E\right)+k_{LB}^2\right)\cdot\vec{i}_U+\left(2\cdot L_{LG}^2+k_{LB}^2\cdot\left(1-r_L\right)\right)\cdot\cos\left(\theta_E\right)\cdot\vec{i}_L\\-k_{LB}^2\cdot\left(1-r_L\right)\cdot\vec{k}_L\cdot\sin\left(\theta_E\right)\end{Bmatrix}$$

$$M_{HU}=I_{UH}+I_{LE}+m_L\cdot\left(L_U^2+2\cdot L_U\cdot L_{LG}\cdot\cos\left(\theta_L\right)\right)$$

$$M_{HL}=I_{LE}+m_L\cdot L_{LG}\cdot L_U\cdot\cos\left(\theta_L\right)$$

$$M_{HX}=\left(m_U\cdot L_{UG}+m_L\cdot L_U\right)\cdot\sin\left(\theta_U\right)+m_L\cdot L_{LG}\cdot\sin\left(\theta_U+\theta_L\right)$$

$$M_{HY}=-\left(m_U\cdot L_{UG}+m_L\cdot L_U\right)\cdot\cos\left(\theta_U\right)-m_L\cdot L_{LG}\cdot\cos\left(\theta_U+\theta_L\right)$$

$$M_{HUU}=\vec{0}$$

$$M_{HLL}=-m_L\cdot L_U\cdot L_{LG}\cdot\sin\left(\theta_L\right)$$

$$M_{HUL}=M_{HLU}=-m_L\cdot L_U\cdot L_{LG}\cdot\sin\left(\theta_L\right)$$

Appendix C

The Expanded Overhand Throwing Kinematic Equations

Global coordinate directions

Y – Vertical, positive upward.

X – Horizontal, positive in the direction of the target

Z – Horizontal, positive to the pitcher's right when they are facing the target.

Body fixed axes:

ZU = Along upper arm, positive from humeral head to elbow.

YU = Parallel to elbow axis of rotation, positive directed to the "outside" of the elbow.

XU = Completes right handed triad.

ZL = Along lower arm, positive from elbow to ball/wrist

YL = Parallel to elbow axis of rotation (same as YU)

XL = Completes right handed triad

Velocity of the humeral head:

$$\vec{V}_H = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \end{bmatrix} \cdot \begin{bmatrix} \dot{X}_H \\ \dot{Y}_H \\ \dot{Z}_H \end{bmatrix}$$

Acceleration of the humeral head:

$$\vec{a}_H = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \end{bmatrix} \cdot \begin{bmatrix} \ddot{X}_H \\ \ddot{Y}_H \\ \ddot{Z}_H \end{bmatrix}$$

Angular velocity of the upper arm:

$$\vec{\omega}_U = \begin{bmatrix} \vec{j} & \vec{i}_1 & \vec{k}_2 \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \end{bmatrix} = \begin{bmatrix} \vec{j}_1 & \vec{i}_2 & \vec{k}_U \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \end{bmatrix}$$

Angular velocity of the lower arm relative to the upper arm:

$$\vec{\omega}_{L/U} = \vec{j}_U \cdot \dot{\theta}_E = \vec{j}_L \cdot \dot{\theta}_E$$

Angular velocity of the lower arm:

$$\vec{\omega}_L = \vec{\omega}_U + \vec{\omega}_{L/U} = \begin{bmatrix} \vec{j} & \vec{i}_1 & \vec{k}_2 & \vec{j}_U \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \\ \dot{\theta}_E \end{bmatrix} = \begin{bmatrix} \vec{j}_1 & \vec{i}_2 & \vec{k}_U & \vec{j}_L \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \\ \dot{\theta}_E \end{bmatrix}$$

If the distance between the humeral head and the elbow is LU, then the velocity of the elbow is given by:

$$\vec{V}_E = \vec{V}_H + \vec{\omega}_U \times L_U \cdot \vec{k}_U = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} & L_U \cdot \vec{j}_1 \times \vec{k}_2 & -L_U \cdot \vec{j}_2 & \vec{0} \end{bmatrix} \cdot \begin{bmatrix} \dot{X}_H & \dot{Y}_H & \dot{Z}_H & \dot{\theta}_Y & \dot{\theta}_X & \dot{\theta}_Z \end{bmatrix}^T$$

If the mass center of the upper arm, U_G, is on the Z_U axis running from the humeral head to the elbow, and the distance from the humeral head to the upper arm mass center is L_{UG}, then the velocity of the mass center of the upper arm is given by:

$$\vec{V}_{UG} = \vec{V}_H + \vec{\omega}_U \times L_{UG} \cdot \vec{k}_U = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} & L_{UG} \cdot \vec{j}_1 \times \vec{k}_2 & -L_{UG} \cdot \vec{j}_2 & \vec{0} \end{bmatrix} \cdot \begin{bmatrix} \dot{X}_H & \dot{Y}_H & \dot{Z}_H & \dot{\theta}_Y & \dot{\theta}_X & \dot{\theta}_Z \end{bmatrix}^T$$

If the distance between the elbow and the ball/wrist is LL, then the velocity of the ball/wrist is given by:

$$\vec{V}_B = \vec{V}_E + \vec{\omega}_L \times L_L \cdot \vec{k}_L = \left\{ \begin{array}{l} \left[\vec{i} \quad \vec{j} \quad \vec{k} \quad \vec{j}_1 \times \left(L_U \cdot \vec{k}_2 + L_L \cdot \vec{k}_L \right) \quad -L_U \cdot \vec{j}_2 + L_L \cdot \vec{i}_2 \times \vec{k}_L \quad L_L \cdot \vec{k}_U \times \vec{k}_L \quad L_L \cdot \vec{i}_L \right] \\ \cdot \left[\dot{X}_H \quad \dot{Y}_H \quad \dot{Z}_H \quad \dot{\theta}_Y \quad \dot{\theta}_X \quad \dot{\theta}_Z \quad \dot{\theta}_E \right]^T \end{array} \right\}$$

If the mass center of the lower arm, LG, is on the line between the elbow and the ball/wrist, and the distance from the elbow to the mass center of the lower arm is LLG, then the velocity of the mass center of the lower arm is given by:

$$\vec{V}_{LG} = \vec{V}_E + \vec{\omega}_L \times L_{LG} \cdot \vec{k}_L$$
$$= \left\{ \begin{array}{l} \left[\vec{i} \quad \vec{j} \quad \vec{k} \quad \vec{j}_1 \times \left(L_U \cdot \vec{k}_2 + L_{LG} \cdot \vec{k}_L \right) \quad -L_U \cdot \vec{j}_2 + L_{LG} \cdot \vec{i}_2 \times \vec{k}_L \quad L_{LG} \cdot \vec{k}_U \times \vec{k}_L \quad L_{LG} \cdot \vec{i}_L \right] \\ \cdot \left[\dot{X}_H \quad \dot{Y}_H \quad \dot{Z}_H \quad \dot{\theta}_Y \quad \dot{\theta}_X \quad \dot{\theta}_Z \quad \dot{\theta}_E \right]^T \end{array} \right\}$$

Angular acceleration of the upper arm is given by:

$$\vec{\alpha}_U = \left[\vec{j} \quad \vec{i}_1 \quad \vec{k}_2 \right] \cdot \begin{bmatrix} \ddot{\theta}_Y \\ \ddot{\theta}_X \\ \ddot{\theta}_Z \end{bmatrix} + \left[\dot{\theta}_Y \quad \dot{\theta}_X \quad \dot{\theta}_Z \right] \cdot \begin{bmatrix} \vec{0} & -\vec{k}_1 & \vec{j}_1 \times \vec{k}_2 \\ \vec{0} & \vec{0} & -\vec{j}_2 \\ \vec{0} & \vec{0} & \vec{0} \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \end{bmatrix}$$

Angular acceleration of the lower arm is given by:

$$\vec{\alpha}_L = \left[\vec{j} \quad \vec{i}_1 \quad \vec{k}_2 \quad \vec{j}_U \right] \cdot \begin{bmatrix} \ddot{\theta}_Y \\ \ddot{\theta}_X \\ \ddot{\theta}_Z \\ \ddot{\theta}_E \end{bmatrix} + \left[\dot{\theta}_Y \quad \dot{\theta}_X \quad \dot{\theta}_Z \quad \dot{\theta}_E \right] \cdot \begin{bmatrix} \vec{0} & -\vec{k}_1 & \vec{j}_1 \times \vec{k}_2 & \vec{j}_1 \times \vec{j}_U \\ \vec{0} & \vec{0} & -\vec{j}_2 & \vec{i}_2 \times \vec{j}_U \\ \vec{0} & \vec{0} & \vec{0} & -\vec{i}_U \\ \vec{0} & \vec{0} & \vec{0} & \vec{0} \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \\ \dot{\theta}_E \end{bmatrix}$$

Acceleration of the elbow is given by:

$$\vec{a}_E = \vec{a}_H + \vec{\omega}_U \times \left(\vec{\omega}_U \times L_U \cdot \vec{k}_U\right) + \vec{\alpha}_U \times L_U \cdot \vec{k}_U$$

$$= \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} & L_U \cdot \vec{j}_1 \times \vec{k}_2 & -L_U \cdot \vec{j}_2 & \vec{0} \end{bmatrix} \cdot \begin{bmatrix} \ddot{X}_H & \ddot{Y}_H & \ddot{Z}_H & \ddot{\theta}_Y & \ddot{\theta}_X & \ddot{\theta}_Z \end{bmatrix}^T$$

$$+ \begin{bmatrix} \dot{\theta}_Y & \dot{\theta}_X & \dot{\theta}_Z \end{bmatrix} \cdot \begin{bmatrix} L_U \cdot \vec{j}_1 \times \left(\vec{j}_1 \times \vec{k}_2\right) & -L_U \cdot \vec{j}_1 \times \vec{j}_2 & \vec{0} \\ -L_U \cdot \vec{j}_1 \times \vec{j}_2 & -L_U \cdot \vec{k}_2 & \vec{0} \\ \vec{0} & \vec{0} & \vec{0} \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \end{bmatrix}$$

Acceleration of the mass center of the upper arm is given by:

$$\vec{a}_{UG} = \vec{a}_H + \vec{\omega}_U \times \left(\vec{\omega}_U \times L_{UG} \cdot \vec{k}_U\right) + \vec{\alpha}_U \times L_{UG} \cdot \vec{k}_U$$

$$= \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} & L_{UG} \cdot \vec{j}_1 \times \vec{k}_2 & -L_{UG} \cdot \vec{j}_2 & \vec{0} \end{bmatrix} \cdot \begin{bmatrix} \ddot{X}_H & \ddot{Y}_H & \ddot{Z}_H & \ddot{\theta}_Y & \ddot{\theta}_X & \ddot{\theta}_Z \end{bmatrix}^T$$

$$+ \begin{bmatrix} \dot{\theta}_Y & \dot{\theta}_X & \dot{\theta}_Z \end{bmatrix} \cdot \begin{bmatrix} L_{UG} \cdot \vec{j}_1 \times \left(\vec{j}_1 \times \vec{k}_2\right) & -L_{UG} \cdot \vec{j}_1 \times \vec{j}_2 & \vec{0} \\ -L_{UG} \cdot \vec{j}_1 \times \vec{j}_2 & -L_{UG} \cdot \vec{k}_2 & \vec{0} \\ \vec{0} & \vec{0} & \vec{0} \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \end{bmatrix}$$

The acceleration of the ball/wrist is given by:

$$\vec{a}_B = \vec{a}_E + \vec{\omega}_L \times \left(\vec{\omega}_L \times L_L \cdot \vec{k}_L\right) + \vec{\alpha}_L \times L_L \cdot \vec{k}_L$$

$$= \left\{ \begin{array}{l} \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} & \vec{j}_1 \times \left(L_U \cdot \vec{k}_2 + L_L \cdot \vec{k}_L\right) & -L_U \cdot \vec{j}_2 + L_L \cdot \vec{i}_2 \times \vec{k}_L & L_L \cdot \vec{k}_U \times \vec{k}_L & L_L \cdot \vec{i}_L \end{bmatrix} \\ \cdot \begin{bmatrix} \ddot{X}_H & \ddot{Y}_H & \ddot{Z}_H & \ddot{\theta}_Y & \ddot{\theta}_X & \ddot{\theta}_Z & \ddot{\theta}_E \end{bmatrix}^T \end{array} \right\}$$

$$+ \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \\ \dot{\theta}_E \end{bmatrix}^T \cdot \begin{bmatrix} \vec{a}_{BYY} & \vec{a}_{BYX} & \vec{a}_{BYZ} & L_L \cdot \vec{j}_1 \times \vec{i}_L \\ \vec{a}_{BXY} & \vec{a}_{BXX} & \vec{a}_{BXZ} & L_L \cdot \vec{i}_2 \times \vec{i}_L \\ \vec{a}_{BZY} & \vec{a}_{BZX} & \vec{a}_{BZZ} & L_L \cdot \cos\left(\theta_E\right) \cdot \vec{j}_L \\ \vec{a}_{BEY} & \vec{a}_{BEX} & \vec{a}_{BEZ} & -L_L \cdot \vec{k}_L \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \\ \dot{\theta}_E \end{bmatrix}$$

Where:

$$\vec{a}_{BYY} = \vec{j}_1 \times \left(\vec{j}_1 \times \left(L_U \cdot \vec{k}_2 + L_L \cdot \vec{k}_L \right) \right)$$

$$\vec{a}_{BXY} = \vec{j}_1 \times \left(-L_U \cdot \vec{j}_2 + L_L \cdot \vec{i}_2 \times \vec{k}_L \right) = \left(L_L \cdot \vec{j}_1 \square \vec{k}_L - L_U \cdot \sin\left(\theta_X\right) \right) \cdot \vec{i}_2$$

$$\vec{a}_{BZY} = L_L \cdot \sin\left(\theta_E\right) \cdot \vec{j}_1 \times \vec{j}_L$$

$$\vec{a}_{BEY} = L_L \cdot \vec{j}_1 \times \vec{i}_L$$

$$\vec{a}_{BYX} = \vec{j}_1 \times \left(-L_U \cdot \vec{j}_2 + L_L \cdot \vec{i}_2 \times \vec{k}_L \right) = \left(L_L \cdot \vec{j}_1 \square \vec{k}_L - L_U \cdot \sin\left(\theta_X\right) \right) \cdot \vec{i}_2$$

$$\vec{a}_{BXX} = -L_U \cdot \vec{k}_2 + L_L \cdot \vec{i}_2 \times \left(\vec{i}_2 \times \vec{k}_L \right)$$

$$\vec{a}_{BZX} = L_L \cdot \sin\left(\theta_E\right) \cdot \vec{i}_2 \times \vec{j}_L$$

$$\vec{a}_{BEX} = L_L \cdot \vec{i}_2 \times \vec{i}_L$$

$$\vec{a}_{BYZ} = L_L \cdot \sin\left(\theta_E\right) \cdot \vec{j}_1 \times \vec{j}_L$$

$$\vec{a}_{BXZ} = L_L \cdot \sin\left(\theta_E\right) \cdot \vec{i}_2 \times \vec{j}_L$$

$$\vec{a}_{BZZ} = -L_L \cdot \sin\left(\theta_E\right) \cdot \vec{i}_U$$

$$\vec{a}_{BEZ} = L_L \cdot \cos\left(\theta_E\right) \cdot \vec{j}_L$$

The acceleration of the mass center of the lower arm is given by:

$$\vec{a}_{LG} = \vec{a}_E + \vec{\omega}_L \times \left(\vec{\omega}_L \times L_{LG} \cdot \vec{k}_L \right) + \vec{\alpha}_L \times L_{LG} \cdot \vec{k}_L$$

$$= \left\{ \begin{array}{l} \left[\begin{array}{ccccccc} \vec{i} & \vec{j} & \vec{k} & \vec{j}_1 \times \left(L_U \cdot \vec{k}_2 + L_{LG} \cdot \vec{k}_L \right) & -L_U \cdot \vec{j}_2 + L_{LG} \cdot \vec{i}_2 \times \vec{k}_L & L_{LG} \cdot \vec{k}_U \times \vec{k}_L & L_{LG} \cdot \vec{i}_L \end{array} \right] \\ \cdot \left[\begin{array}{ccccccc} \ddot{X}_H & \ddot{Y}_H & \ddot{Z}_H & \ddot{\theta}_Y & \ddot{\theta}_X & \ddot{\theta}_Z & \ddot{\theta}_E \end{array} \right]^T \end{array} \right\}$$

$$+ \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \\ \dot{\theta}_E \end{bmatrix}^T \cdot \begin{bmatrix} \vec{a}_{LGYY} & \vec{a}_{LGYX} & \vec{a}_{LGYZ} & L_{LG} \cdot \vec{j}_1 \times \vec{i}_L \\ \vec{a}_{LGXY} & \vec{a}_{LGXX} & \vec{a}_{LGXZ} & L_{LG} \cdot \vec{i}_2 \times \vec{i}_L \\ \vec{a}_{LGZY} & \vec{a}_{LGZX} & \vec{a}_{LGZZ} & L_{LG} \cdot \cos\left(\theta_E\right) \cdot \vec{j}_L \\ \vec{a}_{LGEY} & \vec{a}_{LGEX} & \vec{a}_{LGEZ} & -L_{LG} \cdot \vec{k}_L \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \\ \dot{\theta}_E \end{bmatrix}$$

Where:

$$\vec{a}_{LGYY} = \vec{j}_1 \times \left(\vec{j}_1 \times \left(L_U \cdot \vec{k}_2 + L_{LG} \cdot \vec{k}_L \right) \right)$$

$$\vec{a}_{LGXY} = \vec{j}_1 \times \left(-L_U \cdot \vec{j}_2 + L_{LG} \cdot \vec{i}_2 \times \vec{k}_L \right) = \left(L_{LG} \cdot \vec{j}_1 \square \vec{k}_L - L_U \cdot \sin\left(\theta_X\right) \right) \cdot \vec{i}_2$$

$$\vec{a}_{LGZY} = L_{LG} \cdot \sin\left(\theta_E\right) \cdot \vec{j}_1 \times \vec{j}_L$$

$$\vec{a}_{LGEY} = L_{LG} \cdot \vec{j}_1 \times \vec{i}_L$$

$$\vec{a}_{LGYX} = \vec{j}_1 \times \left(-L_U \cdot \vec{j}_2 + L_{LG} \cdot \vec{i}_2 \times \vec{k}_L \right) = \left(L_{LG} \cdot \vec{j}_1 \square \vec{k}_L - L_U \cdot \sin\left(\theta_X\right) \right) \cdot \vec{i}_2$$

$$\vec{a}_{LGXX} = -L_U \cdot \vec{k}_2 + L_{LG} \cdot \vec{i}_2 \times \left(\vec{i}_2 \times \vec{k}_L \right)$$

$$\vec{a}_{LGZX} = L_{LG} \cdot \sin\left(\theta_E\right) \cdot \vec{i}_2 \times \vec{j}_L$$

$$\vec{a}_{LGEX} = L_{LG} \cdot \vec{i}_2 \times \vec{i}_L$$

$$\vec{a}_{LGYZ} = L_{LG} \cdot \sin\left(\theta_E\right) \cdot \vec{j}_1 \times \vec{j}_L$$

$$\vec{a}_{LGXZ} = L_{LG} \cdot \sin\left(\theta_E\right) \cdot \vec{i}_2 \times \vec{j}_L$$

$$\vec{a}_{LGZZ} = -L_{LG} \cdot \sin\left(\theta_E\right) \cdot \vec{i}_U$$

$$\vec{a}_{LGEZ} = L_{LG} \cdot \cos\left(\theta_E\right) \cdot \vec{j}_L$$

Appendix D

The Expanded Overhand Throwing Kinetic Equations

The angular momentum of the upper arm about its mass center can be expressed as:

$$\vec{H}_{UG} = m_U \cdot \left(k_{US}^2 \cdot \vec{k}_U \cdot \vec{\omega}_U \square \vec{k}_U + k_{UB}^2 \cdot \left(\vec{\omega}_U - \vec{k}_U \cdot \vec{\omega}_U \square \vec{k}_U \right) \right)$$

$$\vec{H}_{UG} = m_U \cdot k_{UB}^2 \cdot \begin{bmatrix} \vec{j}_1 - \left(1 - r_U\right) \cdot \vec{k}_U \cdot \vec{j}_1 \square \vec{k}_2 & \vec{i}_2 & r_U \cdot \vec{k}_U \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \end{bmatrix}$$

$$r_U = \frac{k_{US}^2}{k_{UB}^2}$$

The corresponding radii of gyration will be denoted kLB and kLS. Using this notation, the angular momentum of the lower arm about its mass center can be expressed as:

$$\vec{H}_{LG} = m_L \cdot \left(k_{LS}^2 \cdot \vec{k}_L \cdot \vec{\omega}_L \square \vec{k}_L + k_{LB}^2 \cdot \left(\vec{\omega}_L - \vec{k}_L \cdot \vec{\omega}_L \square \vec{k}_L \right) \right)$$

$$\vec{H}_{LG} = m_L \cdot k_{LB}^2 \cdot \begin{bmatrix} \vec{j}_1 - \left(1 - r_L\right) \cdot \vec{k}_L \cdot \vec{k}_L \square \vec{j}_1 & \vec{i}_2 - \left(1 - r_L\right) \cdot \vec{k}_L \cdot \vec{k}_L \square \vec{i}_2 & \vec{k}_U - \left(1 - r_L\right) \cdot \vec{k}_L \cdot \cos\left(\theta_E\right) & \vec{j}_L \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \\ \dot{\theta}_E \end{bmatrix}$$

$$r_L = \frac{k_{LS}^2}{k_{LB}^2}$$

The time derivative of the angular momentum of the upper arm is:

$$\dot{\vec{H}}_{UG} = m_U \cdot k_{UB}^2 \cdot \begin{bmatrix} \vec{j}_1 - \left(1-r_U\right)\cdot \vec{k}_U \cdot \vec{j}_1 \square \vec{k}_2 & \vec{i}_2 & r_U \cdot \vec{k}_U \end{bmatrix} \cdot \begin{bmatrix} \ddot{\theta}_Y \\ \ddot{\theta}_X \\ \ddot{\theta}_Z \end{bmatrix}$$

$$+m_U \cdot k_{UB}^2 \cdot \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \end{bmatrix}^T \cdot \begin{bmatrix} -\left(1-r_U\right)\cdot \vec{j}_1 \times \vec{k}_U \cdot \vec{j}_1 \square \vec{k}_2 & -\vec{k}_1 & r_U \cdot \vec{j}_1 \times \vec{k}_2 \\ \left(1-r_U\right)\cdot\left(\vec{j}_2 \cdot \vec{j}_1 \square \vec{k}_2 + \vec{k}_U \cdot \vec{j}_1 \square \vec{j}_2\right) & \vec{0} & -r_U \cdot \vec{j}_2 \\ \vec{0} & \vec{0} & \vec{0} \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \end{bmatrix}$$

The time derivative of the angular momentum of the lower arm is:

$$\dot{\vec{H}}_{LG} = m_L \cdot k_{LB}^2 \cdot \begin{bmatrix} \vec{j}_1 - \left(1-r_L\right)\cdot \vec{k}_L \cdot \vec{k}_L \square \vec{j}_1 & \vec{i}_2 - \left(1-r_L\right)\cdot \vec{k}_L \cdot \vec{k}_L \square \vec{i}_2 & \vec{k}_U - \left(1-r_L\right)\cdot \vec{k}_L \cdot \cos\left(\theta_E\right) & \vec{j}_L \end{bmatrix} \cdot \begin{bmatrix} \ddot{\theta}_Y \\ \ddot{\theta}_X \\ \ddot{\theta}_Z \\ \ddot{\theta}_E \end{bmatrix}$$

$$+m_L \cdot k_{LB}^2 \cdot \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \\ \dot{\theta}_E \end{bmatrix}^T \cdot \begin{bmatrix} \vec{T}_{YY} & \vec{T}_{YX} & \vec{T}_{YZ} & \vec{j}_1 \times \vec{j}_U \\ \vec{T}_{XY} & \vec{T}_{XX} & \vec{T}_{XZ} & \vec{i}_2 \times \vec{j}_U \\ \vec{T}_{ZY} & \vec{T}_{ZX} & \vec{T}_{ZZ} & -\vec{i}_U \\ \vec{T}_{EY} & \vec{T}_{EX} & \vec{T}_{EZ} & \vec{0} \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \\ \dot{\theta}_E \end{bmatrix}$$

Where:

$$\vec{T}_{YY} = -\left(1 - r_L\right)\cdot \vec{j}_1 \times \vec{k}_L \cdot \vec{k}_L \square \vec{j}_1$$
$$\vec{T}_{XY} = -\left(1 - r_L\right)\cdot\left(\vec{i}_2 \times \vec{k}_L \cdot \vec{k}_L \square \vec{j}_1 - \vec{k}_L \cdot \vec{k}_1 \square \vec{k}_L\right)$$
$$\vec{T}_{ZY} = -\left(1 - r_L\right)\cdot \sin\left(\theta_E\right)\cdot\left(\vec{j}_L \cdot \vec{k}_L \square \vec{j}_1 + \vec{k}_L \cdot \vec{j}_L \square \vec{j}_1\right)$$
$$\vec{T}_{EY} = -\left(1 - r_L\right)\cdot\left(\vec{i}_L \cdot \vec{k}_L \square \vec{j}_1 + \vec{k}_L \cdot \vec{i}_L \square \vec{j}_1\right)$$
$$\vec{T}_{YX} = -\vec{k}_1 - \left(1 - r_L\right)\cdot \vec{j}_1 \times \vec{k}_L \cdot \vec{k}_L \square \vec{i}_2$$
$$\vec{T}_{XX} = -\left(1 - r_L\right)\cdot \vec{i}_2 \times \vec{k}_L \cdot \vec{k}_L \square \vec{i}_2$$
$$\vec{T}_{ZX} = -\left(1 - r_L\right)\cdot \sin\left(\theta_E\right)\cdot\left(\vec{j}_U \cdot \vec{k}_L \square \vec{i}_2 + \vec{k}_L \cdot \vec{j}_U \square \vec{i}_2\right)$$
$$\vec{T}_{EX} = -\left(1 - r_L\right)\cdot\left(\vec{i}_L \cdot \vec{k}_L \square \vec{i}_2 + \vec{k}_L \cdot \vec{i}_L \square \vec{i}_2\right)$$
$$\vec{T}_{YZ} = \vec{j}_1 \times\left(\vec{k}_U - \left(1 - r_L\right)\cdot \vec{k}_L \cdot \cos\left(\theta_E\right)\right)$$
$$\vec{T}_{XZ} = -\vec{j}_2 - \left(1 - r_L\right)\cdot \vec{i}_2 \times \vec{k}_L \cdot \cos\left(\theta_E\right)$$
$$\vec{T}_{ZZ} = -\left(1 - r_L\right)\cdot \sin\left(\theta_E\right)\cdot \cos\left(\theta_E\right)\cdot \vec{j}_L$$
$$\vec{T}_{EZ} = \left(1 - r_L\right)\cdot\left(\vec{k}_L \cdot \sin\left(\theta_E\right) - \vec{i}_L \cdot \cos\left(\theta_E\right)\right)$$

The net force at the elbow must be equal to:

$$\vec{F}_E = m_L \cdot\left(\vec{a}_{LG} + g \cdot \vec{j}\right)$$

The sum of the moments about the mass center has two contributors. The first is the net force at the elbow and the second is the net moment at the elbow. Thus:

$$\vec{M}_E - L_{LG} \cdot \vec{k}_L \times \vec{F}_E = \dot{\vec{H}}_{LG}$$

$$\vec{M}_E = m_L \cdot L_{LG} \cdot g \cdot \vec{k}_L \times \vec{j} + \left\{ \begin{array}{l} \left[\vec{M}_{EX} \quad \vec{M}_{EY} \quad \vec{M}_{EZ} \quad \vec{M}_{E\theta Y} \quad \vec{M}_{E\theta X} \quad \vec{M}_{E\theta Z} \quad \vec{M}_{E\theta E}\right]\cdot \\ \left[\ddot{X}_H \quad \ddot{Y}_H \quad \ddot{Z}_H \quad \ddot{\theta}_Y \quad \ddot{\theta}_X \quad \ddot{\theta}_Z \quad \ddot{\theta}_E\right]^T \end{array} \right\}$$
$$+ \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \\ \dot{\theta}_E \end{bmatrix}^T \cdot \begin{bmatrix} \vec{M}_{E\theta Y\theta Y} & \vec{M}_{E\theta Y\theta X} & \vec{M}_{E\theta Y\theta Z} & \vec{M}_{E\theta Y\theta E} \\ \vec{M}_{E\theta X\theta Y} & \vec{M}_{E\theta X\theta X} & \vec{M}_{E\theta X\theta Z} & \vec{M}_{E\theta X\theta E} \\ \vec{M}_{E\theta Z\theta Y} & \vec{M}_{E\theta Z\theta X} & \vec{M}_{E\theta Z\theta Z} & \vec{M}_{E\theta Z\theta E} \\ \vec{M}_{E\theta E\theta Y} & \vec{M}_{E\theta E\theta X} & \vec{M}_{E\theta E\theta Z} & \vec{M}_{E\theta E\theta E} \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \\ \dot{\theta}_E \end{bmatrix}$$

The vector coefficients of the terms linear in the second time derivatives of the variables can be seen to be:

$$\vec{M}_{EX} = m_L \cdot L_{LG} \cdot \vec{k}_L \times \vec{i}$$
$$\vec{M}_{EY} = m_L \cdot L_{LG} \cdot \vec{k}_L \times \vec{j}$$
$$\vec{M}_{EZ} = m_L \cdot L_{LG} \cdot \vec{k}_L \times \vec{k}$$
$$\vec{M}_{E\theta Y} = m_L \cdot \left\{ L_U \cdot L_{LG} \cdot \vec{k}_L \times \left(\vec{j}_1 \times \vec{k}_2 \right) + \left(L_{LG}^2 + k_{LB}^2 \right) \cdot \vec{j}_1 - \left(L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right) \right) \cdot \vec{k}_L \cdot \vec{k}_L \square \vec{j}_1 \right\}$$
$$\vec{M}_{E\theta X} = m_L \cdot \left\{ L_U \cdot L_{LG} \cdot \vec{j}_2 \times \vec{k}_L + \left(L_{LG}^2 + k_{LB}^2 \right) \cdot \vec{i}_2 - \left(L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right) \right) \cdot \vec{k}_L \cdot \vec{k}_L \square \vec{i}_2 \right\}$$
$$\vec{M}_{E\theta Z} = m_L \cdot \left\{ \vec{k}_U \cdot \left(L_{LG}^2 + k_{LB}^2 \right) - \vec{k}_L \cdot \cos\left(\theta_E\right) \cdot \left(L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right) \right) \right\}$$
$$\vec{M}_{E\theta E} = m_L \cdot \left(L_{LG}^2 + k_{LB}^2 \right) \cdot \vec{j}_L$$

The vector coefficients of the terms quadratic in the first time derivatives of the angles can be seen to be:

$$\vec{M}_{E\theta Y\theta Y} = m_L \left\{ L_U \cdot L_{LG} \cdot \sin\left(\theta_E\right) \cdot \vec{j}_1 \cdot \vec{j}_1 \cdot \vec{j}_L - \vec{j}_1 \times \left(L_U \cdot L_{LG} \cdot \vec{k}_U + \left(L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right)\right) \cdot \vec{k}_L \right) \cdot \vec{k}_L \cdot \vec{j}_1 \right\}$$

$$\vec{M}_{E\theta X\theta Y} = m_L \cdot k_{LB}^2 \cdot \left\{ \left(1 - r_L\right) \cdot \vec{k}_L \cdot \vec{k}_1 \cdot \vec{k}_L - \vec{k}_1 - \left(1 - r_L\right) \cdot \vec{j}_1 \times \vec{k}_L \cdot \vec{k}_L \cdot \vec{i}_2 \right\}$$

$$\vec{M}_{E\theta Z\theta Y} = -m_L \cdot \left(2 \cdot L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right)\right) \cdot \sin\left(\theta_E\right) \cdot \vec{j}_L \cdot \vec{k}_L \cdot \vec{j}_1 - m_L \cdot k_{LB}^2 \cdot \left(1 - r_L\right) \cdot \sin\left(\theta_E\right) \cdot \vec{k}_L \cdot \vec{j}_L \cdot \vec{j}_1$$

$$\vec{M}_{E\theta E\theta Y} = -m_L \cdot \left(2 \cdot L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right)\right) \cdot \vec{i}_L \cdot \vec{k}_L \cdot \vec{j}_1 - m_L \cdot k_{LB}^2 \cdot \left(1 - r_L\right) \cdot \vec{k}_L \cdot \vec{i}_L \cdot \vec{j}_1$$

$$\vec{M}_{E\theta Y\theta X} = m_L \cdot \left(\left(2 \cdot L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right)\right) \cdot \vec{j}_1 \cdot \vec{k}_L - 2 \cdot L_U \cdot L_{LG} \cdot \sin\left(\theta_X\right) \right) \cdot \vec{k}_L \times \vec{i}_2$$

$$\vec{M}_{E\theta X\theta X} = m_L \cdot L_U \cdot L_{LG} \cdot \sin\left(\theta_E\right) \cdot \vec{j}_L - m_L \cdot \left(L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right)\right) \cdot \vec{i}_2 \times \vec{k}_L \cdot \vec{k}_L \cdot \vec{i}_2$$

$$\vec{M}_{E\theta Z\theta X} = -m_L \cdot \left(2 \cdot L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right)\right) \cdot \sin\left(\theta_E\right) \cdot \vec{j}_U \cdot \vec{k}_L \cdot \vec{i}_2 - m_L \cdot k_{LB}^2 \cdot \left(1 - r_L\right) \cdot \sin\left(\theta_E\right) \cdot \vec{k}_L \cdot \vec{j}_U \cdot \vec{i}_2$$

$$\vec{M}_{E\theta E\theta X} = -m_L \cdot \left(2 \cdot L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right)\right) \cdot \vec{i}_L \cdot \vec{k}_L \cdot \vec{i}_2 - m_L \cdot k_{LB}^2 \cdot \left(1 - r_L\right) \cdot \vec{k}_L \cdot \vec{i}_L \cdot \vec{i}_2$$

$$\vec{M}_{E\theta Y\theta Z} = m_L \cdot k_{LB}^2 \cdot \vec{j}_1 \times \left(\vec{k}_U - \left(1 - r_L\right) \cdot \vec{k}_L \cdot \cos\left(\theta_E\right) \right)$$

$$\vec{M}_{E\theta X\theta Z} = -m_L \cdot k_{LB}^2 \cdot \left\{ \vec{j}_2 + \left(1 - r_L\right) \cdot \vec{i}_2 \times \vec{k}_L \cdot \cos\left(\theta_E\right) \right\}$$

$$\vec{M}_{E\theta Z\theta Z} = -m_L \cdot \left(L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right)\right) \cdot \sin\left(\theta_E\right) \cdot \cos\left(\theta_E\right) \cdot \vec{j}_L$$

$$\vec{M}_{E\theta E\theta Z} = -m_L \cdot \left(2 \cdot L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right)\right) \cdot \vec{i}_L \cdot \cos\left(\theta_E\right) + m_L \cdot k_{LB}^2 \cdot \left(1 - r_L\right) \cdot \vec{k}_L \cdot \sin\left(\theta_E\right)$$

$$\vec{M}_{E\theta Y\theta E} = m_L \cdot k_{LB}^2 \cdot \vec{j}_1 \times \vec{j}_U$$

$$\vec{M}_{E\theta X\theta E} = m_L \cdot k_{LB}^2 \cdot \vec{i}_2 \times \vec{j}_U$$

$$\vec{M}_{E\theta Z\theta E} = -m_L \cdot k_{LB}^2 \cdot \vec{i}_U$$

$$\vec{M}_{E\theta E\theta E} = \vec{0}$$

The force equation for the upper arm gives us:

$$\vec{F}_H = \vec{F}_E + m_U \cdot g \cdot \vec{j} + m_U \cdot \vec{a}_{UG}$$

Recalling the result for the force at the elbow:

$$\vec{F}_H = \left(m_U + m_L\right) \cdot g \cdot \vec{j} + m_U \cdot \vec{a}_{UG} + m_L \cdot \vec{a}_{LG}$$

The moment equation for the upper arm reveals:

$$\vec{M}_H - L_{UG} \cdot \vec{k}_U \times \vec{F}_H - \vec{M}_E - \left(L_U - L_{UG}\right) \cdot \vec{k}_U \times \vec{F}_E = \dot{\vec{H}}_{UG}$$

Using the expressions for the joint forces and the elbow joint moment in terms of the primary kinematic variables:

$$\vec{M}_H = \dot{\vec{H}}_{UG} + \dot{\vec{H}}_{LG} + m_U \cdot L_{UG} \cdot \vec{k}_U \times \vec{a}_{UG} + m_L \cdot \left(L_U \cdot \vec{k}_U + L_{LG} \cdot \vec{k}_L \right) \times \vec{a}_{LG}$$
$$+ \left\{ m_U \cdot L_{UG} \cdot \vec{k}_U + m_L \cdot \left(L_U \cdot \vec{k}_U + L_{LG} \cdot \vec{k}_L \right) \right\} \times g \cdot \vec{j}$$

We can express the moment crossing the shoulder as:

$$\vec{M}_H = \left\{ m_U \cdot L_{UG} \cdot \vec{k}_U + m_L \cdot \left(L_U \cdot \vec{k}_U + L_{LG} \cdot \vec{k}_L \right) \right\} \times g \cdot \vec{j}$$
$$+ \left\{ \begin{array}{l} \left[\vec{M}_{HX} \quad \vec{M}_{HY} \quad \vec{M}_{HZ} \quad \vec{M}_{H\theta Y} \quad \vec{M}_{H\theta X} \quad \vec{M}_{H\theta Z} \quad \vec{M}_{H\theta E} \right] \cdot \\ \left[\ddot{X}_H \quad \ddot{Y}_H \quad \ddot{Z}_H \quad \ddot{\theta}_Y \quad \ddot{\theta}_X \quad \ddot{\theta}_Z \quad \ddot{\theta}_E \right]^T \end{array} \right\}$$
$$+ \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \\ \dot{\theta}_E \end{bmatrix}^T \cdot \begin{bmatrix} \vec{M}_{H\theta Y\theta Y} & \vec{M}_{H\theta Y\theta X} & \vec{M}_{H\theta Y\theta Z} & \vec{M}_{H\theta Y\theta E} \\ \vec{M}_{H\theta X\theta Y} & \vec{M}_{H\theta X\theta X} & \vec{M}_{H\theta X\theta Z} & \vec{M}_{H\theta X\theta E} \\ \vec{M}_{H\theta Z\theta Y} & \vec{M}_{H\theta Z\theta X} & \vec{M}_{H\theta Z\theta Z} & \vec{M}_{H\theta Z\theta E} \\ \vec{M}_{H\theta E\theta Y} & \vec{M}_{H\theta E\theta X} & \vec{M}_{H\theta E\theta Z} & \vec{M}_{H\theta E\theta E} \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \\ \dot{\theta}_E \end{bmatrix}$$

Where:

$$\vec{M}_{HX} = \left(\left(m_U \cdot L_{UG} + m_L \cdot L_U \right) \cdot \vec{k}_U + m_L \cdot L_{LG} \cdot \vec{k}_L \right) \times \vec{i}$$
$$\vec{M}_{HY} = \left(\left(m_U \cdot L_{UG} + m_L \cdot L_U \right) \cdot \vec{k}_U + m_L \cdot L_{LG} \cdot \vec{k}_L \right) \times \vec{j}$$
$$\vec{M}_{HZ} = \left(\left(m_U \cdot L_{UG} + m_L \cdot L_U \right) \cdot \vec{k}_U + m_L \cdot L_{LG} \cdot \vec{k}_L \right) \times \vec{k}$$

$$\vec{M}_{H\theta Y} = \left\{ \begin{array}{l} \left(m_U \cdot \left(L_{UG}^2 + k_{UB}^2 \right) + m_L \cdot \left(L_U^2 + L_{LG}^2 + 2 \cdot L_U \cdot L_{LG} \cdot \cos\left(\theta_E\right) \right) + k_{LB}^2 \right) \cdot \vec{j}_1 \\ -\left(\left(m_L \cdot L_U^2 + m_U \cdot \left(L_{UG}^2 + k_{UB}^2 \cdot \left(1 - r_U\right) \right) \right) \cdot \vec{j}_1 \bullet \vec{k}_2 + m_L \cdot L_U \cdot L_{LG} \cdot \vec{j}_1 \bullet \vec{k}_L \right) \cdot \vec{k}_U \\ -m_L \cdot \left(\left(L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right) \right) \cdot \vec{j}_1 \bullet \vec{k}_L + L_U \cdot L_{LG} \cdot \vec{j}_1 \bullet \vec{k}_2 \right) \cdot \vec{k}_L \end{array} \right\}$$

$$\vec{M}_{H\theta X} = \left\{ \begin{array}{l} \left(m_U \cdot \left(L_{UG}^2 + k_{UB}^2 \right) + m_L \cdot \left(L_U^2 + L_{LG}^2 + L_{LG} \cdot L_U \cdot \cos\left(\theta_E\right) + k_{LB}^2 \right) \right) \cdot \vec{i}_2 \\ -m_L \cdot \left(\left(L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right) \right) \cdot \vec{k}_L \bullet \vec{i}_2 \cdot \vec{k}_L + L_{LG} \cdot L_U \cdot \vec{k}_L \times \vec{j}_2 \right) \end{array} \right\}$$

$$\vec{M}_{H\theta Z} = \left\{ \begin{array}{l} \left(m_U \cdot k_{UB}^2 \cdot r_U + m_L \cdot \left(L_{LG}^2 + L_U \cdot L_{LG} \cdot \cos\left(\theta_E\right) + k_{LB}^2 \right) \right) \cdot \vec{k}_U \\ -m_L \cdot \left(\left(L_{LG}^2 + k_{LB}^2 \cdot \left(1 - r_L\right) \right) \cdot \cos\left(\theta_E\right) + L_U \cdot L_{LG} \right) \cdot \vec{k}_L \end{array} \right\}$$

$$\vec{M}_{H\theta E} = m_L \cdot \left(L_{LG} \cdot L_U \cdot \cos\left(\theta_E\right) + L_{LG}^2 + k_{LB}^2 \right) \cdot \vec{j}_L$$

$$\vec{M}_{H\theta Y\theta Y} = -\left\{ \begin{array}{l} \vec{j}_1 \times \vec{k}_2 \cdot \left(\left(m_U \cdot \left(L_{UG}^2 + k_{UB}^2 \cdot (1-r_U) \right) + m_L \cdot L_U^2 \right) \cdot \vec{j}_1 \bullet \vec{k}_2 + m_L \cdot L_{LG} \cdot L_U \cdot \vec{j}_1 \bullet \vec{k}_L \right) \\ + \vec{j}_1 \times \vec{k}_L \cdot m_L \cdot \left(\left(L_{LG}^2 + k_{LB}^2 \cdot (1-r_L) \right) \cdot \vec{j}_1 \bullet \vec{k}_L + L_{LG} \cdot L_U \cdot \vec{j}_1 \bullet \vec{k}_2 \right) \end{array} \right\}$$

$$\vec{M}_{H\theta X\theta X} = m_L \cdot \left\{ L_U \cdot L_{LG} \cdot \sin(\theta_E) \cdot \left(\vec{j}_U - \vec{i}_2 \cdot \vec{i}_2 \bullet \vec{j}_U \right) - \left(L_U \cdot L_{LG} + L_{LG}^2 + k_{LB}^2 \cdot (1-r_L) \right) \vec{i}_2 \times \vec{k}_L \cdot \vec{k}_L \bullet \vec{i}_2 \right\}$$

$$\vec{M}_{H\theta X\theta Y} = \left\{ \begin{array}{l} -\vec{k}_1 \cdot \left(m_U \cdot k_{UB}^2 + m_L \cdot k_{LB}^2 \right) + \vec{k}_U \cdot m_U \cdot k_{UB}^2 \cdot (1-r_U) \cdot \cos(\theta_X) - \vec{j}_1 \times \vec{k}_L \cdot m_L \cdot k_{LB}^2 \cdot (1-r_L) \cdot \vec{k}_L \bullet \vec{i}_2 \\ + \vec{j}_2 \cdot \left(m_U \cdot \left(2 \cdot L_{UG}^2 + k_{UB}^2 \cdot (1-r_U) \right) \cdot \vec{j}_1 \bullet \vec{k}_2 + m_L \cdot 2 \cdot L_U \cdot \left(L_{LG} \cdot \vec{j}_1 \bullet \vec{k}_L - L_U \cdot \sin(\theta_X) \right) \right) \\ + \vec{k}_L \times \vec{i}_2 \cdot \left(m_L \cdot \left(\left(k_{LB}^2 \cdot (1-r_L) + 2 \cdot L_{LG}^2 \right) \cdot \vec{j}_1 \bullet \vec{k}_L - 2 \cdot L_{LG} \cdot L_U \cdot \sin(\theta_X) \right) \right) + \vec{k}_L \cdot m_L \cdot k_{LB}^2 \cdot (1-r_L) \cdot \vec{k}_1 \bullet \vec{k}_L \end{array} \right\}$$

$$\vec{M}_{H\theta Z\theta Z} = -\vec{j}_U \cdot m_L \cdot \sin(\theta_E) \cdot \left(\left(k_{LB}^2 \cdot (1-r_L) + L_{LG}^2 \right) \cdot \cos(\theta_E) + L_U \cdot L_{LG} \right)$$

$$\vec{M}_{H\theta Z\theta Y} = \left\{ \begin{array}{l} \vec{i}_2 \cdot \cos(\theta_X) \cdot \left(m_U \cdot k_{UB}^2 \cdot r_U + m_L \cdot k_{LB}^2 \right) - m_L \cdot k_{LB}^2 \cdot (1-r_L) \cdot \left(\sin(\theta_E) \cdot \vec{k}_L \cdot \vec{j}_L \bullet \vec{j}_1 + \vec{j}_1 \times \vec{k}_L \cdot \cos(\theta_E) \right) \\ -m_L \cdot \vec{j}_L \cdot \sin(\theta_E) \cdot \left(2 \cdot L_U \cdot L_{LG} \cdot \vec{k}_U \bullet \vec{j}_1 + \left(2 \cdot L_{LG}^2 + k_{LB}^2 \cdot (1-r_L) \right) \cdot \vec{k}_L \bullet \vec{j}_1 \right) \end{array} \right\}$$

$$\vec{M}_{H\theta Z\theta X} = \left\{ \begin{array}{l} -\vec{j}_2 \cdot \left(m_U \cdot k_{UB}^2 \cdot r_U + m_L \cdot k_{LB}^2 \right) - \vec{j}_U \cdot m_L \cdot \sin(\theta_E) \cdot \vec{k}_L \bullet \vec{i}_2 \cdot \left(2 \cdot L_{LG}^2 + k_{LB}^2 \cdot (1-r_L) \right) \\ -m_L \cdot k_{LB}^2 \cdot (1-r_L) \cdot \left(\sin(\theta_E) \cdot \left(\vec{k}_L \cdot \vec{j}_U \bullet \vec{i}_2 \right) + \vec{i}_2 \times \vec{k}_L \cdot \cos(\theta_E) \right) \end{array} \right\}$$

$$\vec{M}_{H\theta E\theta E} = -m_L \cdot L_U \cdot L_{LG} \cdot \sin(\theta_E) \cdot \vec{j}_U$$

$$\vec{M}_{H\theta E\theta Y} = -m_L \cdot \left\{ \begin{array}{l} \vec{i}_L \cdot \left(\left(2 \cdot L_{LG}^2 + k_{LB}^2 \cdot (1-r_L) \right) \cdot \vec{k}_L \bullet \vec{j}_1 + 2 \cdot L_{LG} \cdot L_U \cdot \vec{k}_U \bullet \vec{j}_1 \right) + \vec{j}_1 \cdot 2 \cdot L_{LG} \cdot L_U \cdot \sin(\theta_E) \\ + k_{LB}^2 \cdot \left((1-r_L) \cdot \vec{k}_L \cdot \vec{i}_L \bullet \vec{j}_1 - \vec{j}_1 \times \vec{j}_U \right) \end{array} \right\}$$

$$\vec{M}_{H\theta E\theta X} = -m_L \cdot \left\{ \begin{array}{l} \vec{i}_L \cdot \left(2 \cdot L_{LG}^2 + k_{LB}^2 \cdot (1-r_L) \right) \cdot \vec{k}_L \bullet \vec{i}_2 + \vec{i}_2 \cdot 2 \cdot L_U \cdot L_{LG} \cdot \sin(\theta_E) \\ + k_{LB}^2 \cdot \left((1-r_L) \cdot \vec{k}_L \cdot \vec{i}_L \bullet \vec{i}_2 - \left(\vec{i}_2 \times \vec{j}_U \right) \right) \end{array} \right\}$$

$$\vec{M}_{H\theta E\theta Z} = -m_L \cdot \left\{ \begin{array}{l} \left(2 \cdot L_U \cdot L_{LG} \cdot \cos(\theta_E) + k_{LB}^2 \right) \cdot \vec{i}_U + \left(2 \cdot L_{LG}^2 + k_{LB}^2 \cdot (1-r_L) \right) \cdot \cos(\theta_E) \cdot \vec{i}_L \\ -k_{LB}^2 \cdot (1-r_L) \cdot \vec{k}_L \cdot \sin(\theta_E) \end{array} \right\}$$

The forces at the shoulder and the elbow can also be expressed in this form:

$$\vec{F}_H = \left(m_U + m_L\right)\cdot g\cdot\vec{j} + \left\{\begin{matrix}\left[\vec{F}_{HX} \quad \vec{F}_{HY} \quad \vec{F}_{HZ} \quad \vec{F}_{H\theta Y} \quad \vec{F}_{H\theta X} \quad \vec{F}_{H\theta Z} \quad \vec{F}_{H\theta E}\right]\cdot \\ \left[\ddot{X}_H \quad \ddot{Y}_H \quad \ddot{Z}_H \quad \ddot{\theta}_Y \quad \ddot{\theta}_X \quad \ddot{\theta}_Z \quad \ddot{\theta}_E\right]^T\end{matrix}\right\}$$

$$+\begin{bmatrix}\dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \\ \dot{\theta}_E\end{bmatrix}^T \cdot \begin{bmatrix}\vec{F}_{H\theta Y\theta Y} & \vec{F}_{H\theta Y\theta X} & \vec{F}_{H\theta Y\theta Z} & \vec{F}_{H\theta Y\theta E} \\ \vec{F}_{H\theta X\theta Y} & \vec{F}_{H\theta X\theta X} & \vec{F}_{H\theta X\theta Z} & \vec{F}_{H\theta X\theta E} \\ \vec{F}_{H\theta Z\theta Y} & \vec{F}_{H\theta Z\theta X} & \vec{F}_{H\theta Z\theta Z} & \vec{F}_{H\theta Z\theta E} \\ \vec{F}_{H\theta E\theta Y} & \vec{F}_{H\theta E\theta X} & \vec{F}_{H\theta E\theta Z} & \vec{F}_{H\theta E\theta E}\end{bmatrix}\cdot\begin{bmatrix}\dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \\ \dot{\theta}_E\end{bmatrix}$$

Where:

$$\vec{F}_{HX} = \left(m_U + m_L\right)\cdot\vec{i}$$
$$\vec{F}_{HY} = \left(m_U + m_L\right)\cdot\vec{j}$$
$$\vec{F}_{HZ} = \left(m_U + m_L\right)\cdot\vec{k}$$
$$\vec{F}_{H\theta Y} = \vec{j}_1 \times \left(\left(m_U\cdot L_{UG} + m_L\cdot L_U\right)\cdot\vec{k}_2 + m_L\cdot L_{LG}\cdot\vec{k}_L\right)$$
$$\vec{F}_{H\theta X} = -\left(m_U\cdot L_{UG} + m_L\cdot L_U\right)\cdot\vec{j}_2 + m_L\cdot L_{LG}\cdot\vec{i}_2\times\vec{k}_L$$
$$\vec{F}_{H\theta Z} = m_L\cdot L_{LG}\cdot\sin\left(\theta_E\right)\cdot\vec{j}_U$$
$$\vec{F}_{H\theta E} = m_L\cdot L_{LG}\cdot\vec{i}_L$$

$$\vec{F}_{H\theta Y\theta Y} = \vec{j}_1 \times \left(\vec{j}_1 \times \left(\left(m_U \cdot L_{UG} + m_L \cdot L_U \right) \cdot \vec{k}_2 + m_L \cdot L_{LG} \cdot \vec{k}_L \right) \right)$$

$$\vec{F}_{H\theta X\theta Y} = \vec{j}_1 \times \left(-\left(m_U \cdot L_{UG} + m_L \cdot L_U \right) \cdot \vec{j}_2 + m_L \cdot L_{LG} \cdot \vec{i}_2 \times \vec{k}_L \right) = \left(m_L \cdot L_{LG} \cdot \vec{j}_1 \times \vec{k}_L - \left(m_U \cdot L_{UG} + m_L \cdot L_U \right) \cdot \sin\left(\theta_X \right) \right) \cdot \vec{i}_2$$

$$\vec{F}_{H\theta Z\theta Y} = m_L \cdot L_{LG} \cdot \sin\left(\theta_E \right) \cdot \vec{j}_1 \times \vec{j}_L$$

$$\vec{F}_{H\theta E\theta Y} = m_L \cdot L_{LG} \cdot \vec{j}_1 \times \vec{i}_L$$

$$\vec{F}_{H\theta Y\theta X} = \vec{j}_1 \times \left(-\left(m_U \cdot L_{UG} + m_L \cdot L_U \right) \cdot \vec{j}_2 + m_L \cdot L_{LG} \cdot \vec{i}_2 \times \vec{k}_L \right) = \left(m_L \cdot L_{LG} \cdot \vec{j}_1 \times \vec{k}_L - \left(m_U \cdot L_{UG} + m_L \cdot L_U \right) \cdot \sin\left(\theta_X \right) \right) \cdot \vec{i}_2$$

$$\vec{F}_{H\theta X\theta X} = -\left(m_U \cdot L_{UG} + m_L \cdot L_U \right) \cdot \vec{k}_2 + m_L \cdot L_{LG} \cdot \vec{i}_2 \times \left(\vec{i}_2 \times \vec{k}_L \right)$$

$$\vec{F}_{H\theta Z\theta X} = m_L \cdot L_{LG} \cdot \sin\left(\theta_E \right) \cdot \vec{i}_2 \times \vec{j}_L$$

$$\vec{F}_{H\theta E\theta X} = m_L \cdot L_{LG} \cdot \vec{i}_2 \times \vec{i}_L$$

$$\vec{F}_{H\theta Y\theta Z} = m_L \cdot L_{LG} \cdot \sin\left(\theta_E \right) \cdot \vec{j}_1 \times \vec{j}_L$$

$$\vec{F}_{H\theta X\theta Z} = m_L \cdot L_{LG} \cdot \sin\left(\theta_E \right) \cdot \vec{i}_2 \times \vec{j}_L$$

$$\vec{F}_{H\theta Z\theta Z} = -m_L \cdot L_{LG} \cdot \sin\left(\theta_E \right) \cdot \vec{i}_U$$

$$\vec{F}_{H\theta E\theta Z} = m_L \cdot L_{LG} \cdot \cos\left(\theta_E \right) \cdot \vec{j}_L$$

$$\vec{F}_{H\theta Y\theta E} = m_L \cdot L_{LG} \cdot \vec{j}_1 \times \vec{i}_L$$

$$\vec{F}_{H\theta X\theta E} = m_L \cdot L_{LG} \cdot \vec{i}_2 \times \vec{i}_L$$

$$\vec{F}_{H\theta Z\theta E} = m_L \cdot L_{LG} \cdot \cos\left(\theta_E \right) \cdot \vec{j}_L$$

$$\vec{F}_{H\theta E\theta E} = -m_L \cdot L_{LG} \cdot \vec{k}_L$$

And at the elbow:

$$\vec{F}_E = m_L \cdot g \cdot \vec{j} + \left\{ \begin{array}{l} \left[\begin{array}{ccccccc} \vec{F}_{EX} & \vec{F}_{EY} & \vec{F}_{EZ} & \vec{F}_{E\theta Y} & \vec{F}_{E\theta X} & \vec{F}_{E\theta Z} & \vec{F}_{E\theta E} \end{array} \right] \cdot \\ \left[\begin{array}{ccccccc} \ddot{X}_H & \ddot{Y}_H & \ddot{Z}_H & \ddot{\theta}_Y & \ddot{\theta}_X & \ddot{\theta}_Z & \ddot{\theta}_E \end{array} \right]^T \end{array} \right\}$$

$$+ \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \\ \dot{\theta}_E \end{bmatrix}^T \cdot \begin{bmatrix} \vec{F}_{E\theta Y\theta Y} & \vec{F}_{E\theta Y\theta X} & \vec{F}_{E\theta Y\theta Z} & \vec{F}_{E\theta Y\theta E} \\ \vec{F}_{E\theta X\theta Y} & \vec{F}_{E\theta X\theta X} & \vec{F}_{E\theta X\theta Z} & \vec{F}_{E\theta X\theta E} \\ \vec{F}_{E\theta Z\theta Y} & \vec{F}_{E\theta Z\theta X} & \vec{F}_{E\theta Z\theta Z} & \vec{F}_{E\theta Z\theta E} \\ \vec{F}_{E\theta E\theta Y} & \vec{F}_{E\theta E\theta X} & \vec{F}_{E\theta E\theta Z} & \vec{F}_{E\theta E\theta E} \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_Y \\ \dot{\theta}_X \\ \dot{\theta}_Z \\ \dot{\theta}_E \end{bmatrix}$$

Where

$$\vec{F}_{EX} = m_L \cdot \vec{i}$$
$$\vec{F}_{EY} = m_L \cdot \vec{j}$$
$$\vec{F}_{EZ} = m_L \cdot \vec{k}$$
$$\vec{F}_{E\theta Y} = m_L \cdot \vec{j}_1 \times \left(L_U \cdot \vec{k}_2 + L_{LG} \cdot \vec{k}_L \right)$$
$$\vec{F}_{E\theta X} = m_L \cdot \left(-L_U \cdot \vec{j}_2 + L_{LG} \cdot \vec{i}_2 \times \vec{k}_L \right)$$
$$\vec{F}_{E\theta Z} = m_L \cdot L_{LG} \cdot \sin\left(\theta_E\right) \cdot \vec{j}_U$$
$$\vec{F}_{E\theta E} = m_L \cdot L_{LG} \cdot \vec{i}_L$$

$$\vec{F}_{E\theta Y\theta Y} = m_L \cdot \vec{j}_1 \times \left(\vec{j}_1 \times \left(L_U \cdot \vec{k}_2 + L_{LG} \cdot \vec{k}_L \right) \right)$$
$$\vec{F}_{E\theta X\theta Y} = m_L \cdot \vec{j}_1 \times \left(-L_U \cdot \vec{j}_2 + L_{LG} \cdot \vec{i}_2 \times \vec{k}_L \right) = m_L \cdot \left(L_{LG} \cdot \vec{j}_1 \square \vec{k}_L - L_U \cdot \sin\left(\theta_X\right) \right) \cdot \vec{i}_2$$
$$\vec{F}_{E\theta Z\theta Y} = m_L \cdot L_{LG} \cdot \sin\left(\theta_E\right) \cdot \vec{j}_1 \times \vec{j}_L$$
$$\vec{F}_{E\theta E\theta Y} = m_L \cdot L_{LG} \cdot \vec{j}_1 \times \vec{i}_L$$
$$\vec{F}_{E\theta Y\theta X} = m_L \cdot \vec{j}_1 \times \left(-L_U \cdot \vec{j}_2 + L_{LG} \cdot \vec{i}_2 \times \vec{k}_L \right) = m_L \cdot \left(L_{LG} \cdot \vec{j}_1 \square \vec{k}_L - L_U \cdot \sin\left(\theta_X\right) \right) \cdot \vec{i}_2$$
$$\vec{F}_{E\theta X\theta X} = m_L \cdot \left(-L_U \cdot \vec{k}_2 + L_{LG} \cdot \vec{i}_2 \times \left(\vec{i}_2 \times \vec{k}_L \right) \right)$$
$$\vec{F}_{E\theta Z\theta X} = m_L \cdot L_{LG} \cdot \sin\left(\theta_E\right) \cdot \vec{i}_2 \times \vec{j}_L$$
$$\vec{F}_{E\theta E\theta X} = m_L \cdot L_{LG} \cdot \vec{i}_2 \times \vec{i}_L$$
$$\vec{F}_{E\theta Y\theta Z} = m_L \cdot L_{LG} \cdot \sin\left(\theta_E\right) \cdot \vec{j}_1 \times \vec{j}_L$$
$$\vec{F}_{E\theta X\theta Z} = m_L \cdot L_{LG} \cdot \sin\left(\theta_E\right) \cdot \vec{i}_2 \times \vec{j}_L$$
$$\vec{F}_{E\theta Z\theta Z} = -m_L \cdot L_{LG} \cdot \sin\left(\theta_E\right) \cdot \vec{i}_U$$
$$\vec{F}_{E\theta E\theta Z} = m_L \cdot L_{LG} \cdot \cos\left(\theta_E\right) \cdot \vec{j}_L$$
$$\vec{F}_{E\theta Y\theta E} = m_L \cdot L_{LG} \cdot \vec{j}_1 \times \vec{i}_L$$
$$\vec{F}_{E\theta X\theta E} = m_L \cdot L_{LG} \cdot \vec{i}_2 \times \vec{i}_L$$
$$\vec{F}_{E\theta Z\theta E} = m_L \cdot L_{LG} \cdot \cos\left(\theta_E\right) \cdot \vec{j}_L$$
$$\vec{F}_{E\theta E\theta E} = -m_L \cdot L_{LG} \cdot \vec{k}_L$$

Appendix E

List of System Parameters

A complete list of system parameters follows:

$\vec{i}, \vec{j}, \vec{k} =$ Globally fixed unit vectors, horizontally in the target direction, vertical,horizontal to the right of the target direction.

L_U, L_{UG}, L_L, L_{LG} = System lengths including the length of the upper arm, the distance from the humeral head to the upper arm mass center, the length of the lower arm, and the distance from the elbow to the mass center of the lower arm.

m_U, m_L = Masses of the upper arm and lower arm

k_{UB}, k_{LB}, k_{US}, k_{LS}, r_U, r_S = Larger principal radii of gyration of the upper and lower arms, smaller principal radii of gyration of the upper and lower arms, or the ratio of the smaller to the larger for the upper and lower arms.

g = magnitude of local accerleration of gravity (no implied direction or sign)

Appendix F

Institutionally Approved Informed Consent Document

Auburn University
Sport Biomechanics Laboratory

Department of Heath and Human Performance
2050 Beard Eaves Memorial Coliseum
Auburn University, 36849

Phone: 334.844.1468
Fax: 334.844.1467

email: sportbiomechanicslab@auburn.edu

"Kinematic and Kinetic Comparison of Overhand and Underhand Throws: Implications to Proximal to Distal Sequencing"

Informed Consent

You are invited to participate in a study that compares the motions of the overhand baseball pitch and the underhand windmill softball pitch. This study is being conducted by John Garner, Dr. Wendi Weimar, Dr. Nels Madsen, and Adam Knight. We hope to compare the in muscle activity and forces acting at the elbow and shoulder in overhand and underhand pitching mechanics. You were selected as a possible participant because you met the following criteria:

1. Age 19-30
2. Varsity Athlete (pitcher)
3. No history of surgery in the throwing arm in the last year
4. No history of injury to the dominant arm (throwing arm) within the previous year.
5. Physically active, participating in at least 20 minutes of physical activity for a minimum of 3 times per week.

If you decide to participate, we will ask you to report to the Industrial and Systems Engineering Shop Building 3 Motion Capture Laboratory for one testing session that will last approximately 45 minutes, including paper work. I will then weigh you and measure your height, limb lengths, and clavicle lengths. You will then be allowed to warm up and prepare to throw 5 pitches at game speed. I will then fit you with small sensors on the front and back of your upper throwing arm. These sensors will allow me to measure the activity of your muscles during the throwing motion. You should not feel any discomfort whatsoever from them. You will then be asked to make 5 pitches into the net placed 15 feet in front of you. After you have thrown all five pitches, you will have completed your responsibilities as a participant in this study.

There are minimal risks associated with participation in this study. The risks that may be present are similar to that if you were pitching in practice or a game. Though the potential for injury is minimal, in the event of injury resulting from participation in this study, you will be financially responsible for any medical costs incurred through participation in this study.

The Auburn University Medical Clinic and/or East Alabama Medical Center will be available for minor risk injuries.

A phone will be available at all times for 911 emergencies. You will be allowed to discontinue participation at any time for any reason without penalty. If you have any questions or problems after you leave the laboratory as a result of your participation in this study, please inform John Garner (telephone: 844-1468; email: garnejc@auburn.edu).

This study may benefit society in general in that the results will contribute to the body of knowledge regarding how the musculature is activated and is stressed during both the overhand baseball pitch and the underhand windmill softball pitch.

Any information obtained in connection with this study with which you can be identified will remain confidential. Information collected through your participation may be published in a professional journal, and/or presented at a professional meeting, and if so, none of your identifiable information will be included.

Your decision whether or not to participate in this study will not jeopardize your future relations with Auburn University or the Department of Health and Human Performance. If you have any questions we invite you to ask them now. If you have questions later, we will be happy to answer them (telephone: 844-1648; email: garnejc@auburn.edu). You will be provided a copy of this form to keep for your records.

For more information regarding your rights as a research participant you may contact the Auburn University Office of Human Subjects Research or the Institutional Review Board by phone (334)-844-5966 or e-mail at hsubjec@auburn.edu or IRBChair@auburn.edu.

YOU ARE MAKING A DECISION WHETHER OR NOT TO PARTICPATE. YOUR SIGNATURE INDICATES THAT YOU HAVE DECIDED TO PARTICIPATE HAVING READ THE INFORMATION PROVIDED ABOVE.

______________________________ __________________

Participant's Name (Printed) Date

____

Participant's Signature

______________________________ __________________

Investigator's Signature Date