VALUATION OF POWER GENERATION INVESTMENTS IN DEREGULATED CAPACITY MARKETS

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VALUATION OF POWER GENERATION INVESTMENTS IN DEREGULATED CAPACITY MARKETS

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VALUATION OF POWER GENERATION INVESTMENTS IN DEREGULATED

CAPACITY MARKETS

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Huseyin Hakan Balci, son of Mehmet Cetin Balci and Saadet Balci, was born on May 26, 1977, Isparta, Turkey. He graduated with the degrees of Bachelor of Science (Textile Engineering) in 1999 and Master of Science (Industrial Engineering) in 2001, both from Istanbul Technical University, Istanbul, Turkey. He entered Graduate School, Auburn University, in August 2001.

DISSERTATION ABSTRACT

VALUATION OF POWER GENERATION INVESTMENTS IN DEREGULATED CAPACITY MARKETS

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Electricity is a very unique product that has yet to become efficiently storable, and it is uniform in its nature independent of what technology is being used to produce it. This factor makes the valuation of a power generator asset very complex, particularly in a deregulated capacity market environment. While the value of a power generator asset depends heavily on the price of electricity and the cost of fuel, the price of electricity itself is defined by the price and volume bids submitted to the market.

This research is aimed at studying the valuation of a power generator asset in a deregulated capacity market environment. The analysis performed in this study follows a three-step procedure: (1) investigating the distribution of demand in the Pennsylvania-New Jersey-Maryland electricity market within specific time intervals, (2) modeling the

behavior of market participants with a set of generator technologies and with in-depth analysis of the various fuel cost scenarios that affect the actual price of electricity, and (3) implementing real options valuation to assess the value of a power generator operating within the capacity market environment. Cournot game theoretical model is assumed in all three studies. Transmission congestion and availability of the generators are not considered.

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1. INTRODUCTION AND REVIEW OF LITERATURE

In this research, a methodology is developed for the valuation of a power generator asset, which is operated in a deregulated capacity market. The value of a power generator depends greatly on the price of electricity and the cost of fuel. The methodology presented in this research involves a three-step procedure. First, it investigates the distribution of demand in the Pennsylvania – New Jersey – Maryland (PJM) electricity market within specific time intervals. Second, it models the market participants with a set of generator technologies and investigates the effects of different fuel cost scenarios on the price of electricity. Third, it uses real options valuation to assess the value of a power generator operating in this capacity market environment. The Cournot game theoretical model is assumed in this research. Transmission congestion and availability of the generators are not considered.

Market participants submit their sell bids to the market and the price of electricity is determined as a result of this bidding process. Energy companies, which possess a set of generators, would like to withhold their capacity in order to make more profits because they are price makers and the demand side participants are price takers. Electricity is a very unique product and has yet to become efficiently storable. It is a uniform product in spite of various technology used to produce it.

This dissertation is organized as follows. Chapter 1 outlines the deregulation of the electric power industry and elaborates on the Pennsylvania–New Jersey–Maryland

Interconnection (PJM). PJM is currently the largest wholesale energy market in the US and has been portrayed as a successful implementation of competitive spot, day-ahead, and bilateral energy markets. This chapter also explains the structure, volume, and market concentration (how perfect the competition is) of the capacity market in PJM. Game theoretical models as well as the previous implementations of real options valuation on energy markets are also described in this chapter.

In Chapter 2, an approach is presented to determine electricity prices considering stochastic demand. The effect of the probability distribution of the nominal demand on price is analyzed by using different probability distributions. First, the annual demand (load) data for PJM is divided into 12 periods such as off-peak/on-peak, weekdays/ weekends and winter/summer/fall months. Three probability distributions are found to statistically fit the demand data. They are normal, lognormal, and Weibull distributions. Second, the density function for the price of electricity is constructed based upon the probability distribution of the demand. Resulting price is shown for a market with five firms.

Chapter 3 provides an approach for determining the price of electricity considering that power companies possess capacity resources with different fuel technologies. It is assumed that a market participant owns and operates four types of technologies: nuclear, coal, natural gas, and petroleum. Power generation cost functions are constructed using the cost functions of each generation type. Low, medium, and high fuel cost scenarios are considered for different generation technologies. The effect of those fuel cost scenarios on the price of electricity is investigated.

In Chapter 4, a procedure is described to compute the value of a power generator that operates in a deregulated capacity market. The physical structure of the power generation system is described by utilities' operating constraints and costs. Instead of modeling the price as a time series model or a mean reverting process, a bottom-up approach is used. The price is calculated as an outcome of the competition among market participants. In an n-firm market, the competitors bid according to a Cournot competition model. The value of a generator is computed by the total of expected revenues and the corresponding costs of operating the power generator over its economic service life.

1.1. World-wide Deregulation

In 1996, the Federal Energy Regulatory Commission (FERC) of United States facilitated the deregulation of the electricity market by establishing a guideline that provides open access to transmission lines. This policy removed restrictions on ownership of power generation facilities and granted non-utility power producers to allow open access to transmission lines. Power generation facilities represent 75% of the generation while non-utility power producers represent the rest. Non-utility power producers are not actual power companies, however they own small generators with which they can serve independently (www.eia.doe.gov). By separating the generation from the transmission and distribution functions, electricity markets attempt to stimulate competition among suppliers to deliver power to consumers at a competitive price while providing sensible signals for investment and new entry (Ott, 2003). Then in late 1990s, federal and state regulators started forming independent system and transmission operators to ensure the electricity supply and delivery reliability by overseeing the transactions within the established market environments. These grid operators are called Independent System Operators (ISO) or Regional Transmission Organizations (RTO). While eight of these ISO/RTOs (PJM, NYISO, ISO-NE, ERCOT, CAL ISO, Midwest ISO, Northwest ISO and SPP) are currently operating in US, deregulation has also changed the structure of power generation and trade in the UK, France, Japan, New Zealand and the Nordic countries since the 1990s.

Competitive markets are thought to provide better economic incentives and opportunities to both buyers and sellers. It is well known that perfect competition can be achieved, if the good is homogenous. In this case, there is a large number of buyers and sellers, and there are market mechanisms that allow easy entry of new firms. A homogenous product allows buyers to get access to a perfect substitute of the same product (e.g. electricity). The existence of a large number of buyers and sellers reduces market power and consequently prevents a firm from dominating the market. Enabling a firm to enter or exit the market allows competitors to take advantage of any economic opportunity. Availability of distributed resources from various suppliers also ensures supply reliability throughout the operation in case of a peak demand or an unexpected outage.

Two models, the pool and the bilateral model, have been considered in the framework of market competition (Bower and Bunn, 1999; Stoft, 2002). The pool model involves an ISO that receives bids for load and generation and determines a market-clearing price at which energy is bought and sold. In the bilateral model, on the other hand, suppliers and wholesale consumers sign a contract for the delivery of power over a period of time. In the pool framework, the market-clearing price of electricity is determined by solving an optimization model that matches supply and demand while minimizing the total revealed cost of power delivery. The supply and load bids are aggregated so that the price at which the aggregated demand (load) curve equals the aggregated supply curve determines the market-clearing price. Market-clearing price can be uniformly applied over the market. Transmission costs and losses for a location on the grid can be added on this price.

Day-ahead markets and real-time markets are the most common energy market structures (Stoft, 2002). The day-ahead market is a forward market in which hourly clearing prices are calculated for each hour of the next operating day based on generation offers, demand bids, virtual supply offers, virtual demand bids, and bilateral transaction schedules, which are submitted into the day-ahead market. This market is cleared after PJM's calculations using least-cost security-constrained unit commitment and economic dispatch. The real-time energy market is a balancing market, in which the clearing prices are calculated every five minutes based on the actual system operations.

1.2. Pennsylvania–New Jersey–Maryland Interconnection

In this study, the analysis of demand distribution is inspired by the Pennsylvania–New Jersey–Maryland Interconnection (PJM) regulations and parameters. Also, use of different fuel types is related with the study of effect of fuel cost on the long-term electricity price estimations. In the first part of this section, the background of PJM is presented. Latter, current distribution of supply resources and power generation by fuel type in US are explained.

PJM began in 1927 with only three power utilities. Since April 1, 1997, PJM operates a neutral and independent bid-based energy market. In 1999, PJM started operating a Daily Capacity Market and Monthly, Multi-monthly, and Interval Capacity Markets. Day-ahead and Real-time Energy Markets were initiated in 2000. Later, a Spinning Reserves Market was introduced in 2002, and Annual and Monthly Auctions Market for Fixed Transmission Rights were introduced in 2003 (PJM MMU, 2006).

Since FERC encouraged the formation of an RTO to operate the transmission system in multi-state areas and to advance the development of competitive wholesale power markets, PJM became the USA's first fully functioning RTO in 2002, with the expansion of the Mid-Atlantic energy market to form PJM West. For the first time, a single market operated across North American Electricity Reliability Council (NERC). In 2005, Midwest ISO, PJM, and Tennessee Valley Authority (TVA) signed a Joint Reliability Coordination Agreement (JRCA) for comprehensive reliability management and congestion relief among these wholesale electricity markets. In the same year, PJM also signed a JRCA with Progress Energy Carolinas, Inc. in order to provide market-to-non market coordination (PJM MMU, 2006).

Nowadays, the PJM energy market is the world's largest wholesale electricity market and one of the major power grids of North America. PJM completely or partially covers fourteen states –PA, NJ, MD, DE, IL, IN, OH, MI, VA, WV, TN, KY, NC and DC– within its control area, and serves more than 51 million people. The company dispatches more than 164,600 megawatts of generation capacity over more than 50,000 miles of transmission lines. PJM has administered more than $20 billion in energy and energy-service trades since 1997 (Ott, 2003 and PJM MMU, 2006).

PJM now has 390 market participants –members and/or customers– with more than 900 diverse power generating facilities, including coal, oil and gas fired units, nuclear plants, and hydroelectric facilities. Primarily due to the integration with other markets and regions, the installed capacity grew from 50,000MW to 164,000MW between 1999 and 2005. Percentages of the total installed capacity of coal, natural gas, nuclear, oil, hydroelectric, and solid waste is respectively, 41.5, 27.5, 19.1, 7.2, 4.3, and 0.3. The nuclear generators and coal-fired units are operated to supply the base load, because of their low operating and fuel cost. So nuclear generators and coal-fired units run at a high efficiency and operate in most of the market hours. Gas-fired units are very popular for their low investment cost. Although they have higher operating costs, they hold the design parameters to respond sudden changes in the market demand. Therefore, gas-fired units run as the marginal units and they serve the marginal load most of the time. They operate fewer hours compared to base load units. As a result, shares of the total generation of coal, natural gas, nuclear, oil, hydroelectric, and solid waste in 2005 was, 56, 6, 34, 1, 2, and 0.6, respectively. Gas fired units supplied 6% of the electricity, while representing 27.5% of the installed capacity in 2005.

1.3. Capacity Markets

In the first part of this section, the structure and the operations of capacity markets are explained. Then, the market volume data of the capacity markets and the bilateral transactions in PJM are presented. Lastly, the market concentration results for PJM markets are outlined in order to understand how competitive these markets are.

The main purpose of introducing capacity markets into the deregulated environment is to achieve adequate capacity resources to commit expected loads while providing investment incentives for power suppliers.

Stoft and Cramton (2005) state that pricing only the primary commodity is sufficient in most of the markets. Normally, supply and demand functions submitted by the market participants define the price, which results in the short-term benefits. In this scenario, because the customer demand is not responsive to the price, the prices can reach extreme values. Market administrators placed price caps in order to limit the peak prices, which occur at peak demand periods or with unexpected outages. Price caps in various markets are between $1,000/MWh and $6,000/MWh.

With the price caps in effect, the investors do not see the opportunity for them to invest in new resources. As a result, the regulatory effect of price caps and the lack of demand side response caused the failure of the electricity markets (Creti and Fabra, 2004). These two failures cause two more problems: The concerns about long-term reliability of the network and lack of investment signals to entrepreneurs (Kiesling, 2005). Kiesling (2005) emphasizes that a market should be double-sided with Load Serving Entities (LSEs) on the demand side and the generation, transmission, and demand reduction owners on the supply side. Kiesling (2005) suggests the longest term possible for the transactions in order to have some investment incentives for the generation and demand side participants.

Creta and Fabra (2004) classify the capacity markets as price-based and quantity-based. In price-based capacity markets (e.g. Spain, England) the generators withhold capacity to increase the price, therefore they are not working as efficiently as they were aimed. Quantity-based capacity markets are common in US (e.g. PJM, CA ISO, NY ISO). In the US, LSEs are required by ISO/RTOs to procure more than their expected peak load plus a reserve margin either by bilateral transactions or capacity markets. The amount they have to buy is called their capacity obligation (PJM, 2006).

PJM's Capacity Credit Market enables suppliers to offer and buyers to purchase unforced capacity credits in PJM. All bids and offers are confidential. PJM validates and processes buy bids and sell offers to clear the market at a uniform market-clearing price. PJM Capacity Credit Market consists of the Daily, Monthly, and Multi-monthly Capacity Credit Markets. Capacity markets have three intervals for commitment: January to May, June to September, and October to December. PJM defines Monday to Friday as on-peak days and weekends as off-peak days. PJM also defines hours between 7 a.m. and 11 p.m. as on-peak hours and hours between 11 p.m. and 7 a.m. as off-peak hours (PJM, 2006).

Prospective buyers or sellers of capacity credits submit buy bids or sell offers indicating the maximum amounts (in megawatts) and prices (dollars/megawatt-day) they are willing to pay or accept for capacity credits for a given period of time. Buy bids and sell offers must be in the increments of 0.1MW. Sellers should submit the minimum price they accept and buyers should submit the maximum price they accept. The bids for the daily market have to be submitted at least 19 days prior to the actual date of the transaction. The bids for the monthly and multi-monthly markets are submitted on a specific bidding day determined before the interval.

Suppliers can offer the energy generated by their units or the energy that they will import from outside resources. Even though the participation in the market is voluntary, the buyers have to acquire at least their capacity obligation amount. Thus, the demand amount is fixed by the expectations made for the future peak load. The capacity obligation in the PJM-East is calculated annually, while it is calculated daily in the PJM-West. In 2006, PJM obligated all LSEs to acquire capacities for three years. If the LSE is short of capacity, a mandatory bid is submitted on its behalf and the remaining capacity is acquired by the capacity deficiency charge ($160/MWh). If the market is also in capacity shortage for that day, the buyer is charged twice the capacity deficiency charge. Capacity owners with excess capacity share the capacity deficiency revenues proportional to their excess capacity amounts (PJM, 2006).

1.3.1. Trade Volume and Prices in PJM Markets

In this section, the trade volume of the PJM Capacity Market and Bilateral Transactions are summarized. In 2005, the total average obligation calculated was 139,736 MW. The capacity markets had an average of 6,892 MW. The total bilateral transactions had an average of 150,597 MW. Bilateral transactions are twofold: Unit-specific transactions averaged 11,789 MW and capacity credit transactions averaged 133,057 MW.

The average activity of the day-ahead and real-time markets accounted for 28,531 MW and 31,536 MW, respectively. The volume of bilateral/capacity market transactions is 2.6 times larger than the volume of the spot market trade. 58.9% of the total demand obligation is served on the effective dates. PJM Market Monitoring Unit (MMU) also states that most of the spot market transactions involve long-term bilateral transactions cleared that day.

The total of daily, monthly, and multi-monthly capacity market volumes increased from 2.6% of the obligation of the market in 1999 to 4.9% of the obligation in 2005. Meanwhile, the weighted average price of electricity has decreased from \$52.24/MW-day to \$6.12/MW-day. The average real-time market prices increased from \$21.82/MWh to \$58.02/MWh and the day-ahead market prices increased from \$20.21/MWh to \$57.83/MWh between 2000 and 2005. The reader should consider the 46% increase in oil prices between 2004 and 2005. Because of the higher volume supplied to the capacity/bilateral transactions, the prices realized are more competitive than spot markets. However, in order to understand the market competitiveness, concentration ratios of markets should be analyzed. This would give more insight about market participants' behaviors.

1.3.2. Market Concentration

The Hirschmann-Herfindahl Index (HHI) is a very common measurement tool for market concentration and it is calculated by the addition of squares of market shares of all market participants. If there is only one market participant (monopoly), HHI is equal to 10,000 and if there are 10 market participants with equal share, HHI is equal to 1,000. If HHI is below 1,000, the market is considered as unconcentrated. If HHI is between 1,000 and 1,800, there is medium concentration and above 1,800, there is high concentration (FERC, 1997).

In this section, the market concentration and its relation to efficiency for PJM markets are explained using the values reported by PJM MMU (PJM, 2006). The HHI of combined PJM Spot Market was 1,275, so it was moderately concentrated in 2005. However, PJM distinguished the analysis between base, intermediate and peak load units, and corresponding average HHI reported for those units are 1,451, 3,078 and 4,612, respectively. HHI of peak loads reached 10,000 at congested time periods. PJM outlines that there were 861 concentrated hours, in which pivotal suppliers were able to manipulate the price. This makes almost 10% of total hours in 2006 (PJM 2006). A total of 5% or fewer companies owned 85% of marginal units.

HHI values reported by PJM MMU for the Capacity Markets are as follows: The average HHI for Daily Market was 1,036, while for Monthly and Multi-Monthly Market it was 1,865 (highly concentrated). HHI averages exceeded 1,800 in 0.5% of the auctions (365) of Daily Market and in 44.4% of the auctions (63) of Monthly and Multi-Monthly Market. There were no pivotal suppliers in Daily Market auctions. However in 92.5% of Multi-Monthly Market auctions, there were three or less pivotal suppliers. In PJM capacity markets, the share of the biggest market participant was 16.6% and the average HHI was 917 on December 31, 2005.

The findings outlined above show that there is medium to high market concentration and existence of pivotal suppliers. This challenges the perfect competition idea behind the introduction of deregulated markets and suggests the assumption of a

Cournot model in modeling the market participant behavior in current deregulated electricity markets.

1.4. Game Theoretical Models

In a competitive market, the decision problem for market participants is to determine the price and/or the quantity for the product. The theory on oligopoly models explains the behavior of players (market participants) in a game where they look for maximizing their profits. In the following subsections, three oligopoly models, Cournot, Bertrand, and Supply Function Equilibrium models are summarized.

Cournot Model

Under a Cournot game framework, strategic decisions of firms relate to quantity choices in order to supply uniform goods. In this setting, there is an aggregate demand function to represent customers, who are not active agents in the game. The sellers, who actively participate in the game, have non-cooperative actions. The sellers make simultaneous optimal choices of quantities to respond to their competitors' optimal choices (Thomas, 1984 and Daughety, 1988).

Bertrand Model

In the Bertrand game model, firms choose price as a strategic decision variable instead of quantity. The Bertrand model introduces consumers as active participants of the game with the assumption that customers search the market to find the lowest price. Thus, in the Bertrand model, firms simultaneously choose prices and consumers choose where to buy. This dynamic interaction of the game leads to a market-clearing price equal to the marginal costs of sellers.

Supply Function Equilibrium (SFE) Model

In this model, the competition is neither on price (as in Bertrand models) nor in quantity (as in Cournot models), but on a set of price-quantity pairs, which are supply functions. The Bertrand and Cournot models are limited cases of SFE models. The SFE model seems to fit very well with the market structure of many restructured markets. The SFE model appears to be more attractive compared to the other models, because it represents the electricity markets more realistically in which bidding rules require suppliers to submit a price-quantity curve at each hour (Klemperer and Meyer, 1988). SFE models explain the markups of electricity prices better, which empirical studies have shown are higher than Bertrand equilibrium, but lower than Cournot model. The problem with the use of SFE models is that in general there is not a unique equilibrium. The existence of many equilibrium points makes it difficult to predict the likely outcome of strategic interaction among players. There are some factors that reduce the range of feasible equilibriums: uncertainty of demand and capacity constraints.

1.4.1. Comparison of Competition Models

All of the three competition models summarized have advantages as well as disadvantages in modeling deregulated power markets. While the Bertrand type competition results in a perfectly competitive environment with an outcome of electricity prices equal to the short-run marginal costs of generating power, the Cournot type competition usually results in prices exceeding the short-run marginal costs because of some withholding of capacity from the market by the generation owners. However, neither of these assumptions appears realistic enough to capture essential elements of electricity markets.

In the Bertrand model the competitors quote a fixed price. It is assumed that generators use all of their capacity before competing to sell all of the production with a price that covers at least their costs. This is inapplicable in the real electricity markets, because electricity cannot be stored in a large quantity and generators have to submit price bids before the production of the commodity. Therefore, the resultant electricity prices are not as low as the marginal cost in real markets.

In the Cournot model where the competitors quote a fixed quantity, it is assumed that generators commit to production only after establishing price requirements, and it is also assumed that generators have perfectly flexible control over the level of production. This flexibility enables them to change their commitments instantaneously in order to affect the market clearing prices. However, in the real electricity markets generators do not have the full flexibility to switch on or off instantly and to increase or decrease the production quantity substantially because of their operational constraints.

More realistically, the electricity markets require generators to submit supply functions. Depending on the market type, these bids have to be submitted hourly or daily in advance. Even though SFE suits better the real power markets compared to Cournot and Bertrand models, it is not clear which equilibrium will be observed in practice or if the electricity market will converge to any equilibrium when SFE is applied.

1.5. Real Options Valuation and Applications on Energy Markets

Real Options Valuation assumes flexibility in decision-making at the management level of a company unlike the net present value approach. It may also assume different discount rates related with various expected revenues contradicting the decision tree approach (Copeland and Antikarov, 2001). An option is the right, but not the obligation, to undertake an investment decision in future. If the underlying asset's value on the expiration date is more than the exercise price of the option contract, the holder of the option contract exercises the option. Real options involve tangible assets where financial options involve stocks, foreign currencies, stock indices and future contracts (Hull, 2000). Real assets have carrying costs, physical constraints, and private risks associated with the project. Real assets are not tradable like the financial assets. Even though the real options valuation is inspired by the financial option valuation models such as binomial options models and Black-Scholes models, the differences between the assets resulted in different modeling approaches (Borison, 2001).

Borison (2001) summarizes the real option methodologies in four categories:

1. Financial option models with no arbitrage assumption and that uses financial market data.

2. Financial option models with no specific replicating portfolio where the model lets the decision-maker decide the appropriate volatility measures.

3. Market Asset Disclaimer Method assuming the underlying asset is not traded and the asset prices follow a geometric Brownian motion process. This model uses a binomial lattice to compute the value.

4. Revised classic approach, which considers two types of investment whether they sustain more public risks or private risks. If the project involves private risks, financial option models are suggested. If private risks are involved, a decision tree analysis is constructed to represent investment alternatives and risk measures are subjectively assigned.

Several researchers applied real options valuation to the energy asset valuation problem. Tseng and Barz (2002) present the methodology for the valuation of power plants with unit commitment constraints over a short-term period. In their study, the price follows an Ito process and the obtained model computes the value of the plant for one week.

Gitelman (2002) defines the low and high demand scenarios and values the exchange options of the assets. Volatility of the option is captured by the volatility of free cash flows to be consistent with the underlying asset. The model computes the moneyness of a generator investment for alternative fuel types.

Näsäkkälä and Fleten (2003) value gas fired plant investments for long term planning based on electricity and gas forward prices. They model the difference of the price of electricity and the gas prices in order to obtain the spark spread assuming Brownian motion process. Comparison of the values of base load and peak-load plants is presented. The authors emphasize that the operating flexibility of the generator has huge positive effect on the value of the plant.

Kaneva (2006) values the generator investment as a call option using the Black-Scholes equation. Volatility of the energy price and the energy cost are obtained from annual historical values. The valuation horizon is 3 years in this method and the author suggests using a shorter time horizon to maturity and more price data to be used in the volatility calculation for accuracy of the method.

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2. MODELING ELECTRICITY PRICE WITH DEMAND UNCERTAINTY IN DEREGULATED CAPACITY MARKETS

Abstract

With the deregulation of electricity markets in several countries, the forecast of future electricity prices has become more essential for managers of energy companies. Electricity prices are not predefined by tariffs anymore. Instead, the competition between power producers determines the price. In this study, a new approach to model electricity prices in deregulated capacity markets is presented. An n-firm Cournot market environment is assumed where firms compete by bidding quantities into the market. The effect of the uncertainty of the nominal demand on price is analyzed by using different probability distributions. First, the demand data of PJM for one year is divided into 12 periods such as off-peak/on-peak, weekdays/weekends and winter/summer/fall months. The hourly demand in years between 2000 and 2006 is analyzed. Three probability distributions are found to fit well demand data: Normal, lognormal, and Weibull distributions. Second, the density function for the price of electricity is constructed. Third, the first and second moments of price are computed for each interval using the best fitted statistical distribution of demand for a specific time interval. The obtained price model is generic and it is suitable for any statistical distribution of demand. As a case study, the first and second moments of price are calculated for a 5-firm market. Results

show that the consideration of time intervals leads to better price information as expected. However, contradicting to the idea that lognormal and Weibull distributions represent the price of electricity much better, the prices obtained using the normal distribution to fit the demand data are good enough. The differences between mean prices using the normal distribution and mean prices using other distributions are less than 0.3%.

2.1. Introduction

The importance of electric energy has tremendously increased because of its usage in most of the equipments that are needed in daily life. Meanwhile, the power system has changed to serve the increasing amount of consumption more efficiently (Denny and Dismukes, 2001).

In 1996, the Federal Energy Regulatory Commission of United States facilitated the deregulation of the electricity market by establishing a guideline that provided open access to transmission lines. This policy removed restrictions on ownership of power generation facilities and granted non-utility power producers open access to transmission lines. By separating the generation from the transmission and distribution functions, electricity markets attempt to stimulate competition among suppliers to deliver power to consumers at a competitive price, while providing sensible signals for investment and new entry.

Competitive markets are thought to provide better economic incentives and opportunities for both buyers and sellers. It is well known that perfect competition can be achieved if the good is homogenous. In this case, there is a large number of buyers and sellers, and there are market mechanisms that allows easy entry of new firms. A homogenous product allows buyers to get access to a perfect substitute of the same product (e.g. electricity). The existence of large numbers of buyers and sellers reduces market power and consequently prevents a firm from dominating the market. Having firms enter or exit the market allows competitors to take advantage of any economic opportunity. However, the prices in the power markets are not realized near to the cost of electricity generation. Although the price reduction in the long term was one of the main reasons of promoting deregulation, perfect competition could not be reached in these markets on the contrary to Bertrand game theoretical model.

The volatile structure of the price of electricity is illustrated in Figure 1. Notice that the price data shown in the graph belong to the average of market-clearing prices of 17 zones in the PJM wholesale market. Market players respond to the demand (load) requested by the customer by changing the price of electricity for a specific time period.

The electricity demand generally follows the daily and/or seasonal cycles depending on weather conditions and lifestyle of consumers. However, the price of electricity is not directly proportional to these changes. The demand and the price of electricity in PJM-East are illustrated in Figure 2 and Figure 3, respectively, for the week between May 29 and June 4 in 2006.

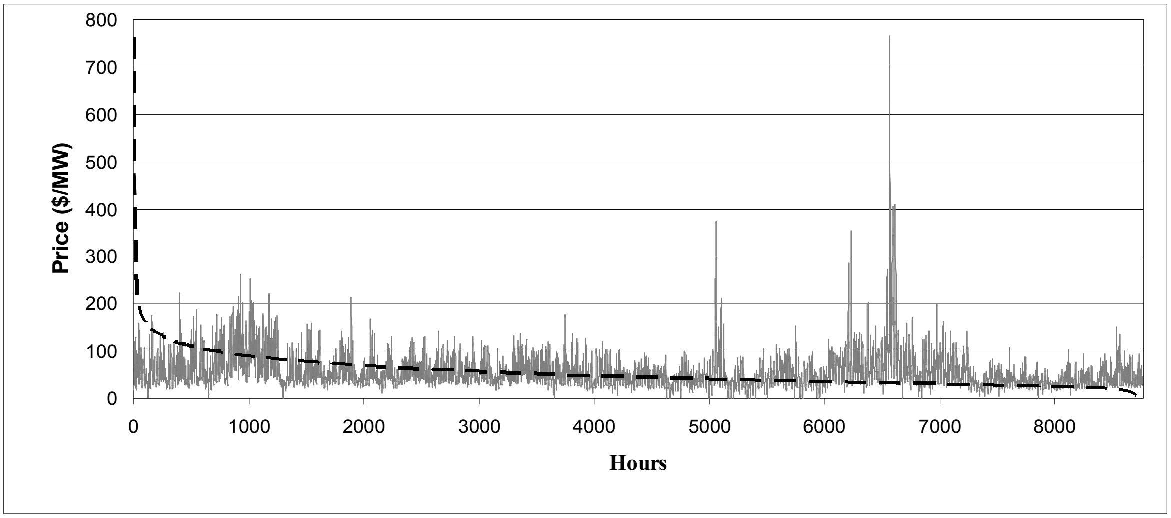

Figure 1. Hourly (continuous line) and sorted (dashed line) price of electricity for PJM wholesale market between November 2005 and October 2006.

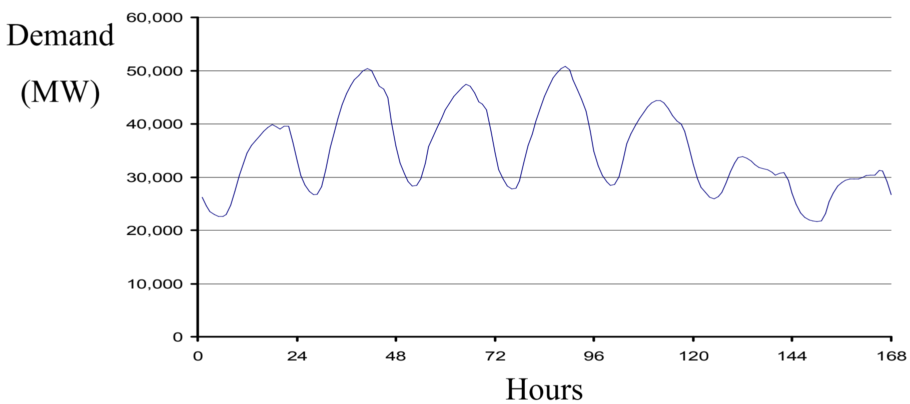

Figure 2. Hourly demand (MW) in PJM-East for the week of May 29-June 4 in 2006.

As it can be seen in Figure 2, in the 2nd and 4th days of that week the demand of electricity followed almost the same load scheme. While the price of electricity reached \$150/MWh on the 2nd day, the market competition resulted at \$270/MWh and \$760/MWh on the 3rd and 4th days, respectively. This effect of competition on the price diminishes as the demand of electricity decreases at the end of the week. This clearly shows that market participants take advantage of peak load occurrence. When they predict the peak load in advance they tend to submit higher price bids to the market. This is an example of Cournot behavior.

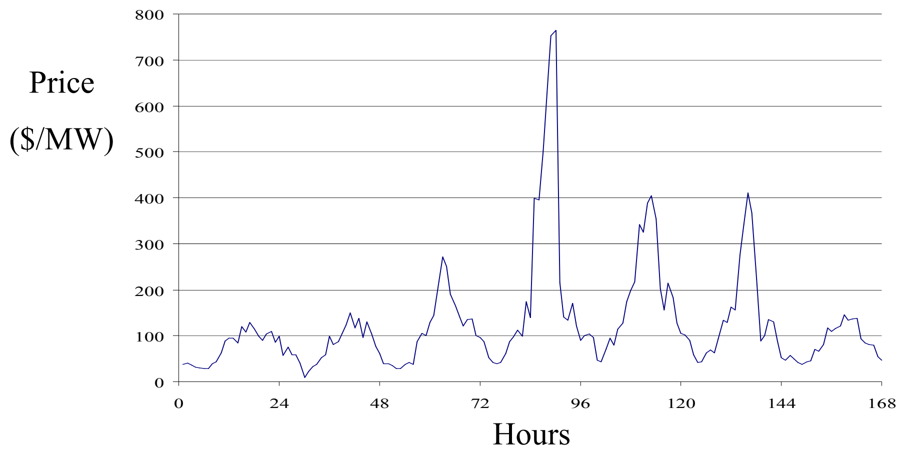

Figure 3. Hourly price ($/MW) in PJM-East for the week of May 29-June 4 in 2006.

The price of electricity depends on economic factors such as strategic bidding and load elasticity. Also, over a long-term it is a quantity that depends on physical factors such as production cost, load, generation availability, unit commitment, and transmission constraints.

2.2. Literature Review

Many researchers have attempted to forecast electricity prices. Deng (2000) shows a mean reverting time series model, which also accounts for price jumps and different price regimes. Szkuta et al. (1999) uses an artificial neural network model to forecast the next 48 half-hourly electricity prices. Contreras et al. (2003) implements auto regressive integrated moving average models to analyze time series in order to predict the next-day electricity prices. Davison et al. (2002) models the price of options on electrical power using time series models, and models for demand and capacity.

The previous models are mostly based on time series and require a large amount of historical price data. Skantze et al. (2000) asserts that the amount of data from current markets is not available for long-term periods. Most of the research aims to forecast short-term electricity prices such as next-day to next week. However, investment decisions of a power asset require price information for longer time periods such as five to ten years.

According to the Energy Information Administration (Energy Information Administration website), overnight capital investment cost of a power generator ranges from $600/kW to $1800/kW if a gas turbine, a coal-fired plant or a nuclear generator is considered. Thus, the cost of construction for a 700MW plant ranges from 400 million USD to 1.2 billion USD. The investment is in massive amounts and the payoffs are expected in decades. This shows the substantial value of accurate long-term price information. Knittel and Roberts (2001) conclude that the structural models of supply and demand would bring more insight into the price generating process.

In this study, a price model that can be used for long-term project valuation is developed. The price of electricity is modeled using a bottom-up approach. This study incorporates the strategic behavior of the market participants as well as the price elasticity of the demand. It considers the physical characteristics of the market participants in developing the model for price of electricity. The Cournot competition model is assumed for the bidding structure of firms in the market. Cournot competition better resembles the operation of today's markets. The intention behind introducing electricity capacity markets is to provide reliable energy while providing the suppliers the investment incentive and providing the demand side (load serving entities) more choice of supply. However, there is still high concentration in the markets. Although the PJM Capacity Market is thought to be an efficient electricity market, high concentration occurs in certain time intervals as outlined in Section 1.3.2.

The remainder of this chapter is organized as follows. In Section 2.3, a methodology to develop a long-term model for price of electricity is provided. In Section 2.4, the analysis to characterize the probability distribution of the demand at specific time intervals is presented. The effect of using different statistical distributions for the demand on the price of electricity and on the square of price of electricity are shown in Section 2.5. The conclusions are given in Section 2.6.

2.3. Model Description

In this section, a procedure is described to compute the price of electricity in a deregulated capacity market. Instead of modeling the price as a time series or a mean reverting process, a bottom-up approach is used. In contrast with the top-down approaches which focus on the economic feedbacks received form the market, the bottom-up approach considers the physical properties and the behavior of the market participants. The physical properties of the power system are described by utilities' operating constraints and costs. The price at each period is calculated as an outcome of the competitive behavior among market participants. An n-firm market is considered, in which the competitors bid according to a Cournot model. The model uses the following notation:

n	Number of firms
ξ	Slope parameter for demand
$\pi_{i,t}$	Profit of firm i in period t ($)
K_t	Demand intercept (MWh) (The demand, when the price of electricity is zero) in period t
P_t	Price at period t ($/MWh)
$Q_i^{\max}$	Maximum production of firm i (MW)
$Q_{i,t}$	Total production of firm i at period t (MW)
$D_t(P_t)$	Actual demand of the power system at period t as a function price P_t (MW)
$\phi_i(Q)$	Power generation costs of firm i of producing Q MWh ($)
η_j	Slope parameter of price for the price regime j

2.3.1. Model Assumptions

a) System demand (load)

The actual demand of the system, D_t, which is assumed to be price sensitive, is represented by the following linear relationship (Figure 4):

$$D_t(P_t) = K_t - \xi P_t$$

where ξ is the elasticity of demand and $\xi \geq 0$.

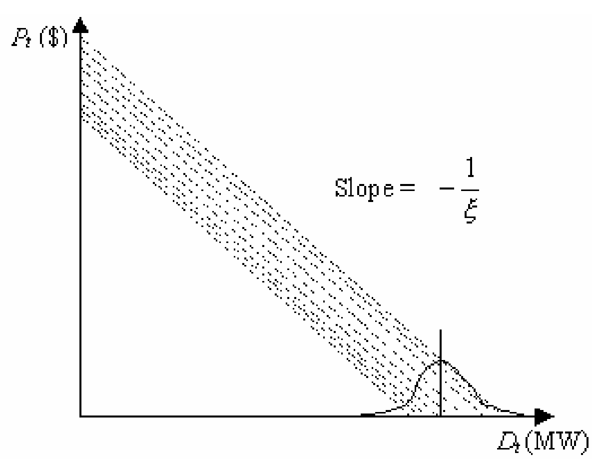

Figure 4. The relationship between price and the actual demand

The demand intercept, K_t, is assumed to be a random variable with an expected value of μ_t and a variance of σ_t^2.

b) Power producers

Producer firms that are bidding in the market have a set of power generation assets. It is assumed that all generators of those firms are always available. Consequently, the minimum up-time, down-time and ramping constraints are not considered. Power producers bid according to their aggregate cost function. The aggregate cost function of firm *i* is assumed in quadratic form and is as follows:

$$\phi_i(Q_{i,t}) = A_i + B_i Q_{i,t} + C_i Q_{i,t}^2 \text{ and } 0 \le Q_{i,t} \le Q_i^{\max} \qquad i \in \{1,2,\ldots,n\}$$

According to the Cournot model, firms submit their bids as generation quantities (MWh). Firm *k* knows the total quantity produced by the other firms ($Q_{-k,t}$), then calculates the amount of power ($Q_{k,t}$) that maximizes its profit for a given value of $Q_{-k,t}$. Thus, the profit of firm *k* is as follows:

$$\pi_{k,t} = P_t(Q_{k,t}; Q_{-k,t}) Q_{k,t} - \phi_k(Q_{k,t})$$

where P_t is price of electricity at time period *t*.

Notice that P_t depends on the quantities $Q_{k,t}$ and $Q_{-k,t}$. Each firm responds to the other firms' bid with a new quantity in order to maximize its profit, and then simultaneous responses result in the Nash equilibrium solution. The Nash equilibrium quantities that maximize each firm's profit are denoted by $Q_{k,t}^*$. The following procedure is applied for each period *t*:

For i=1,…n
Repeat
For i=1,…n
$Q^*_{-i,t} = \sum_{j=1, j\neq i}^{n} Q_{j,t}$
$Q^*_{i,t} = Q$ such as $\text{Max}\{\pi_{i,t} = P_t(Q)Q - \phi_i(Q)\}$
$\Delta Q^*_{i,t} = \text{Abs}(Q^*_{i,t} - Q_{i,t})$
$Q_{i,t} = Q^*_{i,t}$
End
Until And ($\Delta Q_{1,t} < \varepsilon, \Delta Q_{2,t} < \varepsilon, \ldots, \Delta Q_{n,t} < \varepsilon$)

c) Market clearing

The actual demand served by the system at time period t is equal to the total of the firms' generations. The actual demand is equal to the following:

$$D_t(P_t) = \sum_{i=1}^{n} Q_{i,t}$$

2.3.2. Modeling the Price

The price of electricity at the Nash equilibrium is as follows:

$$P_t = \frac{K_t - \sum_{i=1}^{n} Q^*_{i,t}}{\xi}$$

Empirically, it is observed that P_t is a piece-wise linear function of K_t. As the value of demand intercept increases, firms bid larger quantities of power to the market. When a firm reaches its maximum generation capacity, the firm cannot increase its quantity bid anymore.

The slope of the price function changes when firm i reaches its maximum generation capacity, $Q_i^{\max}$ (Figure 5). By decreasing number of firms in the competition, the competition weakens and the slope of the price function becomes larger. Each slope switching point is denoted by s_i. Therefore, the price, P_t, can be characterized as a piece-wise linear function of demand intercept, K_t, as shown in Figure 5.

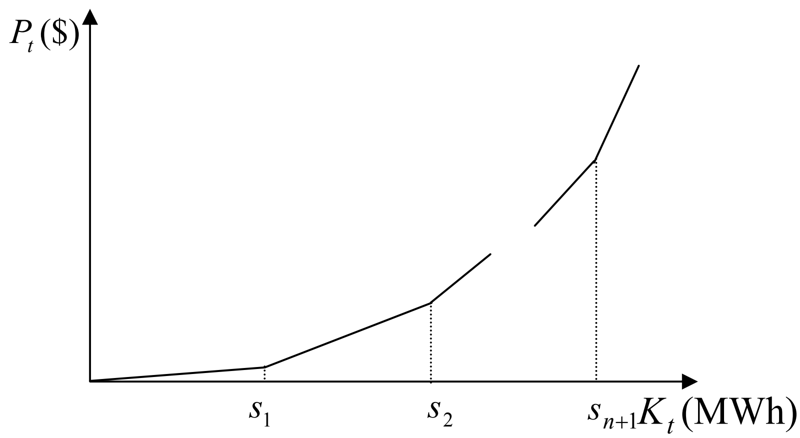

Figure 5. Price over demand intercept

The price regimes are obtained as follows:

$$P_t = \begin{cases} \eta_1 K_t & \text{if } K_t \le s_1 \\ s_1(\eta_1 - \eta_2) + \eta_2 K_t & \text{if } K_t > s_1 \text{ and } K_t \le s_2 \\ \quad \ldots & \quad \ldots \\ s_1\eta_1 + (s_2 - s_1)\eta_2 + \ldots + (s_n - s_{n-1})\eta_n - s_{n+1}\eta_{n+1} + \eta_{n+1}K_t & \text{if } K_t > s_{n+1} \end{cases} \tag{2}$$

where η_{n+1} is the slope of the $(n+1)^{th}$ price regime.

If the demand intercept, K_t, follows a general distribution function, the density function of price, $f_{K,t}(x)$, and cumulative distribution function $F_{K,t}(x)$ are obtained as follows:

$$f_{p,t}(x) = f_{k,t}\left(\frac{x}{\eta_1}\right)\mathrm{F}_{\mathrm{K,t}}(s_1) + f_{k,t}\left(\frac{x-(\eta_1-\eta_2)s_1}{\eta_2}\right)\left[\mathrm{F}_{\mathrm{K,t}}(s_2) - \mathrm{F}_{\mathrm{K,t}}(s_1)\right]$$

$$+ f_{K,t}\left(\frac{x-(\eta_1-\eta_2)s_1-(\eta_2-\eta_1)s_2}{n_3}\right)\left[\mathrm{F}_{K,t}(s_3) - \mathrm{F}_{K,t}(s_2)\right] + \ldots \qquad (3)$$

This density function is generic. It can be used with any distribution function of the nominal demand.

2.3.3. Computing $E[P_t\ ]$ and $E[P_t^2]$

The expected value of the price is calculated considering the *n+1* price regimes as follows:

$$E[P_t] = \sum_{i=1}^{n+1} E[P_t | K_t \in R_i]\Pr[K_t \in R_i] \qquad (4)$$

where R_i is the set of nominal demand values in price regime *i*.

The values of $\Pr[K_t \in R_i]$ are denoted by $Z_{i,t}$. By using (2), (4) can be written as;

$$E[P_t] = \eta_1\mu_t Z_{1,t} + (s_1(\eta_1-\eta_2) + \eta_2\mu_t)Z_{2,t} + \ldots$$

$$+ (s_1\eta_1 + (s_2-s_1)\eta_2 + \ldots + (s_n - s_{n-1})\eta_n - s_n\eta_{n+1} + \eta_{n+1}\mu_t)Z_{n+1,t} \qquad (5)$$

where $Z_{1,t} = F_{K,t}(s_1)$ and $Z_{i,t} = F_{K,t}(s_i) - F_{K,t}(s_{i-1})$.

(5) can be written for normal, Weibull, and lognormal distributions where

$Z_{i,t} = \Phi(s_i;\mu_t,\sigma_t^2) - \Phi(s_{i-1};\mu_t,\sigma_t^2)$ for normal distribution,

$Z_{i,t} = \Theta(s_i;x_0,\alpha_t,\beta_t) - \Theta(s_{i-1};x_0,\alpha_t,\beta_t)$ for Weibull distribution and

$Z_{i,t} = \Psi(s_i;x_0,\mu_t,\sigma_t) - \Psi(s_{i-1};x_0,\mu_t,\sigma_t)$ for lognormal distribution.

Similarly, the second moment of price is computed as follows:

$$E[P_t^2] = \sum_{i=1}^{n+1} E[P_t^2 | K_t \in R_i] \Pr[K_t \in R_i] \quad (6)$$

By using (2), (6) can be written as;

$$\begin{aligned} E[P_t^2] &= (\eta_1\mu_t)^2 Z_{1,t} + \left((s_1(\eta_1 - \eta_2))^2 + 2(s_1(\eta_1 - \eta_2))(\eta_2\mu_t) + (\eta_2\mu_t)^2\right) Z_{2,t} \\ &+ \left((s_1\eta_1 + (s_2 - s_1)\eta_2 + \ldots + (s_n - s_{n-1})\eta_n - s_n\eta_{n+1})^2 + \right. \\ &\left. 2(s_1\eta_1 + (s_2 - s_1)\eta_2 + \ldots + (s_n - s_{n-1})\eta_n - s_n\eta_{n+1})(\eta_{n+1}\mu_t) + (\eta_{n+1}\mu_t)^2\right) Z_{n+1,t} \end{aligned} \quad (7)$$

2.4. Numerical Results

In this section, the methodology explained in Section 2.3 is implemented in a market environment with 5 firms. Cost parameters and capacities for these firms are listed in Table 1.

Table 1. Characteristics of market participants.

Characteristic	Firm 1	Firm 2	Firm 3	Firm 4	Firm 5
Q^{max} (MW)	30,000	20,000	25,000	10,000	12,000
Q^{min} (MW)	0	0	0	0	0
A	400	175	250	300	120
B	7.654	7.054	6.265	6.605	8.31
C	0.0016	0.0032	0.0022	0.00012	0.0027

The firms' quantity bids are calculated according to the Cournot competition for 10MW increments of nominal demand between 0 and 160,000MW using the CPLEX 8.0 solver called by AMPL. The electricity price depending on the demand intercept is illustrated in Figure 6.

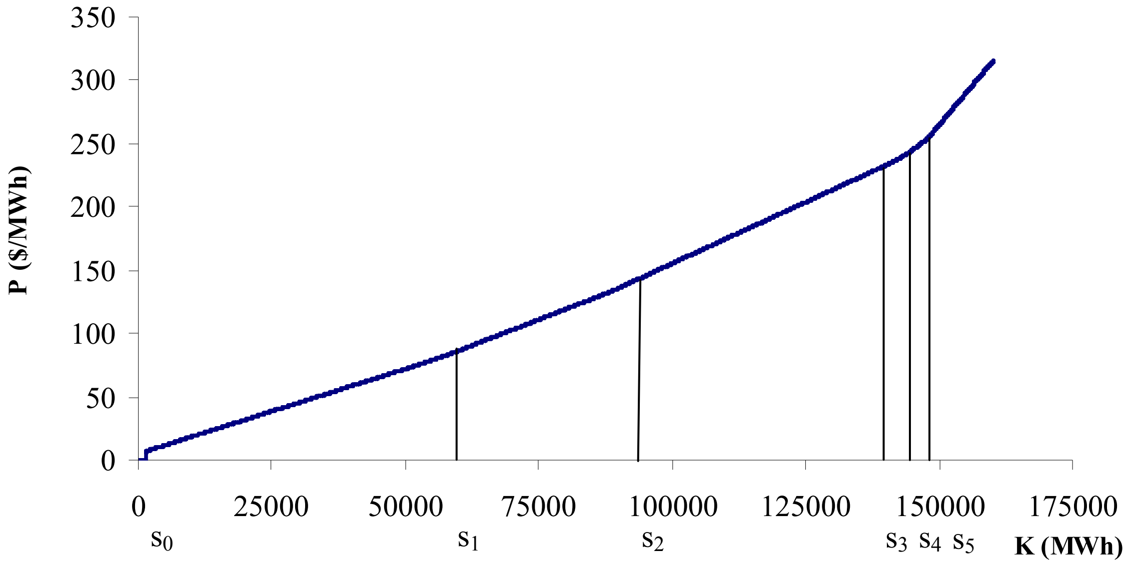

Figure 6. The function of price versus the demand intercept for 5-firm market.

Price switching points, s_0, s_1, s_2, s_3, s_4 and s_5 are calculated as 2,310 MWh, 56,330 MWh, 88,470 MWh, 141,090 MWh, 143,750 MWh, and 147,740 MWh, respectively.

2.4.1. Computing the Density Function of the Price

In this section, the statistical analysis performed to characterize the distribution of the demand for one complete year is described. It is very common in the literature to assume that the demand for electricity is normally distributed (Veall, 1983 and Valenzuela, 2005).

Instead of using arbitrary numbers, the normal distribution is fit to the hourly demand data of the PJM market for 2006 and demand distribution parameters are obtained. The hourly demand in the PJM-East market in 2006 is shown in Figure 7. The histogram and fitted normal distribution (dashed line) are shown in Figure 8.

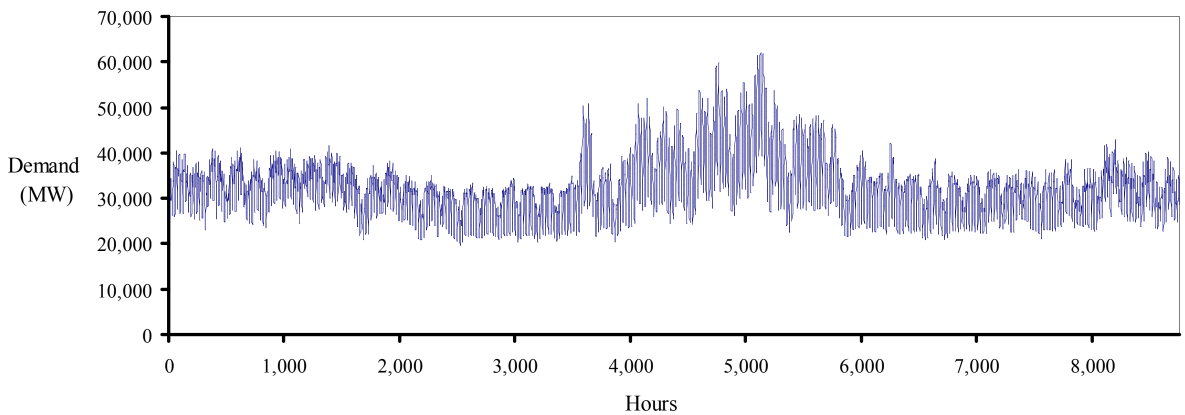

Figure 7. Hourly demand (MW) for PJM-East in 2006.

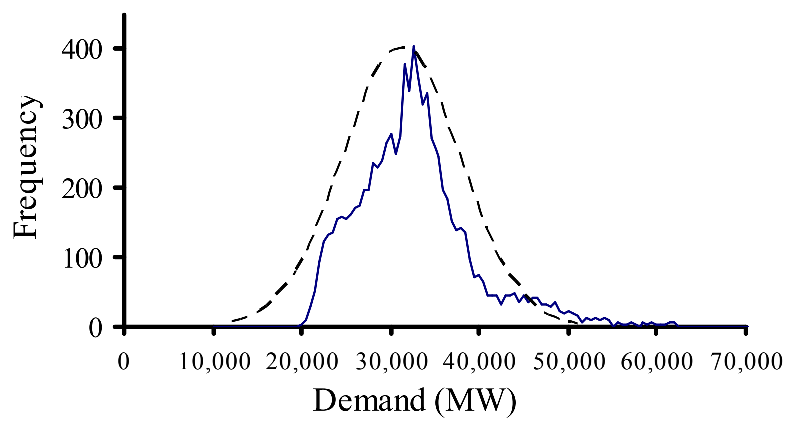

Figure 8. The histogram and normal distribution fit (dashed line) of hourly demand in PJM-East in 2006.

The mean and the standard deviation of the fitted normal distribution are found as 32,407MWh and 6,586MWh, respectively. The density function of the market price is constructed by using (3) in Section 2.3.2 with this set of normal distribution parameters as shown in Figure 9. The density function is calculated with \$1/MWh increments between

\$0/MWh and \$200/MWh. Then, the first moment ($E[P_t]$) and second moment ($E[P_t^2]$) of price are computed as \$50.7/MWh and \$2,641/MWh, respectively.

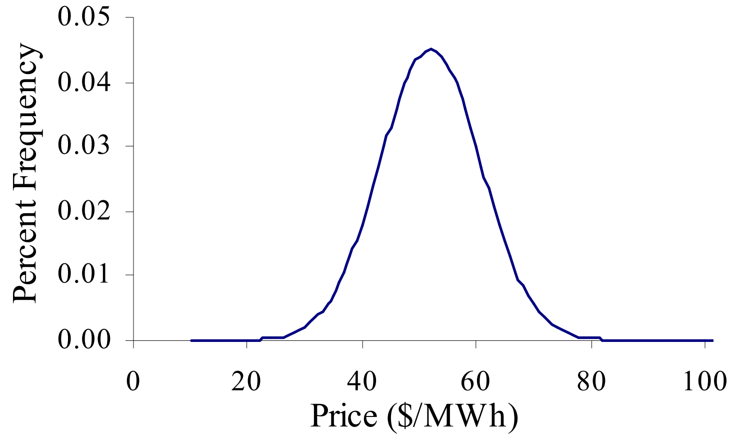

Figure 9. The density function of price after competition of five firms.

2.4.2. Characterizing the Distribution of Demand

In this section, the appropriate type of distribution for demand of electricity is investigated in a more detailed fashion. As it is stated earlier, the PJM capacity market operates considering several time intervals due to the cyclic behavior of the demand. Thus, the annual hourly demand data is divided into 12 periods such as off-peak/on-peak hours, weekdays/ weekends, and winter (January-May)/summer (June-September)/fall (October-December) periods. On-peak hours are between 7 a.m. and 11 p.m. and off-peak hours are between 11 p.m. and 7 a.m. Then, the statistical distribution of demand is analyzed in PJM-East for each time interval between 2000 and 2006 using the Palisade's Best Fit Software (ver.4.5.5).

The Chi Square values (with 95% confidence level) for the best fitted distributions and the normal distribution for comparison are listed in Table 2. The best fit distribution for an interval is highlighted in bold. Smaller Chi Square values indicate a better fit. Lognormal and Weibull distributions are found to be the distinct distributions that fit the demand data better for the time intervals compared to other major statistical distributions such as normal, triangular, uniform, exponential and gamma distributions.

Table 2. The Chi Square values for each interval averaged between 2000 and 2006.

Distribution	weekdays winter on peak	weekdays winter off peak	weekdays summer on peak	weekdays summer off peak	weekdays fall on peak	weekdays fall off peak
Weibull	424.50	69.53	224.80	32.84	474.10	38.90
Lognormal	165.60	62.01	156.00	51.49	258.90	44.86
Normal	616.50	116.90	288.30	164.50	340.30	171.10

Distribution	weekends winter on peak	weekends winter off peak	weekends summer on peak	weekends summer off peak	weekends fall on peak	weekends fall off peak
Weibull	84.46	21.77	39.92	4.44	67.74	15.50
Lognormal	57.71	18.44	81.04	8.89	68.56	11.51
Normal	89.54	38.72	72.10	36.89	72.42	60.63

The time intervals in winter season are better represented by the Lognormal distribution, while the time intervals in the summer season are better represented by the Weibull distribution (Table 2).

The distribution of the demand for Weekdays/ On-Peak Hours/ Winter interval in PJM market in 2006 is shown in Figure 10. According to the analysis (listed in Table 2), the best fit distribution of demand for this time interval is the Lognormal distribution. Appendix A includes distributions of the demand for all 12 intervals.

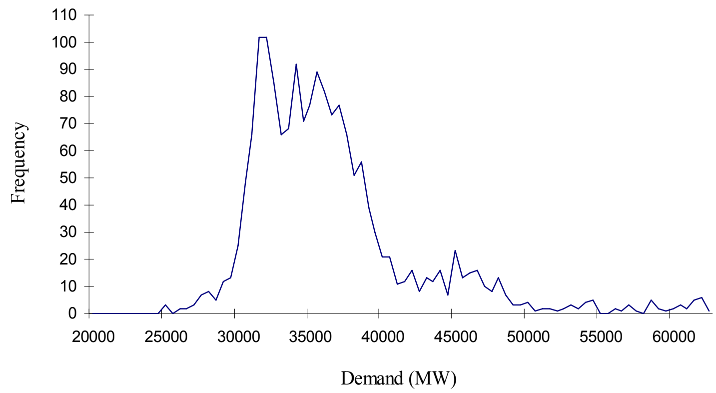

Figure 10. Frequency plot of the electricity demand for Weekdays/ On-Peak Hours/ Winter in PJM East in 2006.

The effects of using the best-fit distribution of demand on the price of electricity are investigated. The density functions of price for the nominal demand are constructed using a lognormal distribution (Figure 11) and a Weibull distribution (Figure 12) for the same interval.

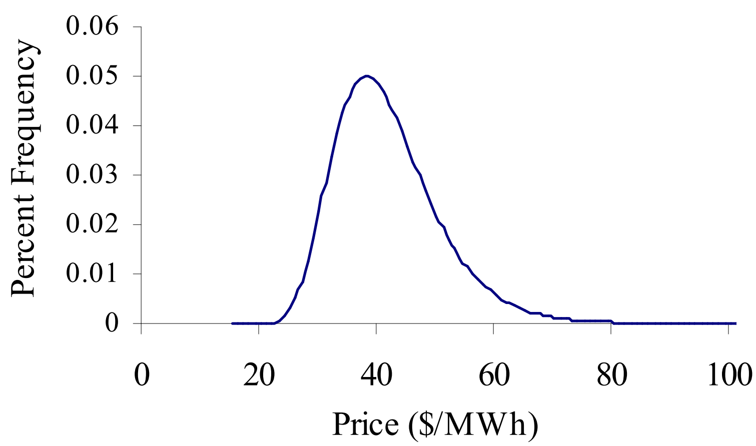

Figure 11. Density function of price computed using Lognormal distributed nominal demand (μ =21,631, σ =6,563 and shift=10,775)

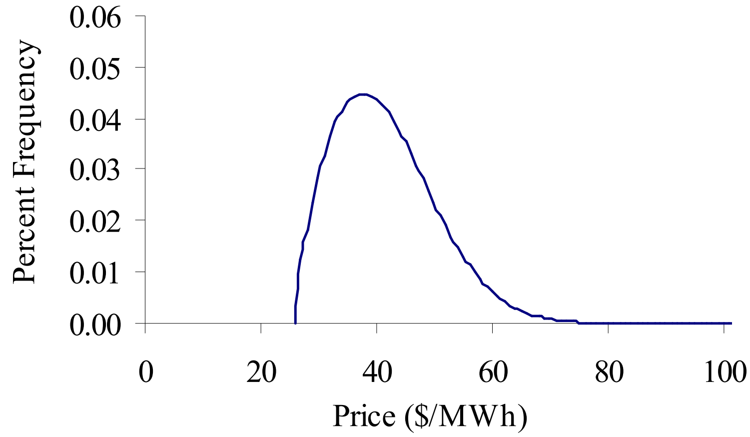

Figure 12. Density function of price computed using Weibull distributed nominal demand (α=2.03, β=14,376 and shift=19,673)

Then, the first and second moments of price ($E[P_t]$ and $E[P_t^2]$) are calculated for Lognormal distributed fit results are 50.6MWh and 2,638MWh^2, respectively and are 50.7MWh and 2,641MWh^2, respectively, for the Weibull distributed fit.

The performed analysis for Weekdays/On-Peak Hours/Winter is repeated for all 12 time intervals within the year of 2006 for the PJM-East capacity market. The parameters of demand with normal distribution (μ and σ), lognormal distribution (shift, μ and σ), and Weibull distribution (shift, α and β) for the 12 intervals are listed in Table 3, Table 4 and Table 5, respectively. Then, $E[P_t]$ and $E[P_t^2]$ are calculated for each interval (Table 6, Table 7 and Table 8) for the five-firm Cournot market.

Table 3. The parameters (μ, σ) of normal distribution for intervals.

Time Interval		Winter		Summer		Fall	
		μ	σ	μ	σ	μ	σ
Weekdays	On-peak	36,199	5,860	39,547	7,231	33,312	3,177
Weekdays	Off-peak	29,114	4,681	29,292	5,305	26,125	3,505
Weekends	On-peak	31,507	4,892	34,529	6,714	29,089	2,965
Weekends	Off-peak	27,437	4,129	27,772	4,891	25,246	3,151

Table 4. The parameters (shift, μ and σ) of lognormal distribution for intervals.

Time Interval	Winter			Summer			Fall		
	shift	μ	σ	Shift	μ	σ	shift	μ	σ
Wdays/On-Peak	22493	13666	5442	17288	22266	7394	14460	18848	3076
Wdays/Off-Peak	13838	15273	4649	19610	9765	6076	19590	6565	3807
Wend/On-Peak	10569	20936	4860	27240	7243	6101	12033	3807	2960
Wend/Off-Peak	3673	23765	4153	17870	9939	5280	17045	2960	3274

Table 5. The parameters (shift, α and β) of the Weibull distribution for intervals.

Time Interval	Winter			Summer			Fall		
	shift	α	β	shift	α	β	shift	α	β
Wdays/On-Peak	24474	2.1	13269	22356	2.55	19410	22832	3.22	11600
Wdays/Off-Peak	20292	1.96	9944	21749	1.41	8281	20948	1.51	5749
Wend/On-Peak	20116	2.44	12833	20349	2.24	16023	19386	3.52	10752
Wend/Off-Peak	19105	2.12	9400	20350	1.55	8256	20139	1.64	5699

Table 6. $E[P_t]$ and $E[P_t^2]$ with the normal distributed demand.

Time Interval	Winter	Summer	Fall
Weekdays/ On-Peak Hours	(55.6, 3150)	(59.9, 3677)	(51.9, 2706)
Weekdays/ Off-peak Hours	(46.4, 2190)	(46.6, 2222)	(42.5, 1828)
Weekends/ On-Peak Hours	(49.5, 2491)	(53.4, 2931)	(46.4, 2164)
Weekends/ Off-peak Hours	(44.2, 1984)	(44.6, 2034)	(41.4, 1728)

Table 7. $E[P_t]$ and $E[P_t^2]$ with the lognormal distributed demand.

Time Interval	Winter	Summer	Fall
Weekdays/ On-Peak Hours	(55.5, 3133)	(59.8, 3672)	(51.9, 2704)
Weekdays/ Off-peak Hours	(46.4, 2189)	(46.7, 2243)	(42.6, 1835)
Weekends/ On-Peak Hours	(49.5, 2490)	(53.3, 2904)	(46.5, 2172)
Weekends/ Off-peak Hours	(44.2, 1984)	(44.7, 2044)	(41.4, 1730)

Table 8. $E[P_t]$ and $E[P_t^2]$ with the Weibull distributed demand.

Time Interval	Winter	Summer	Fall
Weekdays/ On-Peak Hours	(55.6, 3153)	(59.8, 3681)	(51.7, 2698)
Weekdays/ Off-peak Hours	(46.4, 2189)	(46.6, 2223)	(42.5, 1829)
Weekends/ On-Peak Hours	(49.5, 2491)	(53.4, 2931)	(46.3, 2162)
Weekends/ Off-peak Hours	(44.2, 1983)	(44.7, 2034)	(41.4, 1728)

2.5. Analysis of the Effect of Using Different Demand Distributions

This section analyzes the effects of using different demand distributions such as normal, Weibull, and lognormal distributions on the first and the second moments of price. First, it compares $E[P_t]$ values obtained by the Weibull and lognormal distributions with the values obtained by the normal distribution. Then the same comparison is performed for $E[P_t^2]$.

The comparisons for $E[P_t]$ as percent differences are shown in Table 9. Bold values in Table 9 denotes the best fit distribution (Weibull or lognormal) for that time interval.

Table 9. Percent difference between of $E[P_t]$ obtained by Weibull and lognormal distributions and $E[P_t]$ obtained by fitting normal distribution.

Time Interval			Weibull vs. Normal (% diff.)	Lognormal vs. Normal (% diff.)
Winter	Weekdays	On-peak	0.05	**-0.15**
Winter	Weekdays	Off-peak	-0.02	**-0.01**
Winter	Weekends	On-peak	-0.03	**-0.01**
Winter	Weekends	Off-peak	-0.02	**0**
Summer	Weekdays	On-peak	0.04	**-0.16**
Summer	Weekdays	Off-peak	**-0.02**	0.14
Summer	Weekends	On-peak	**0.01**	-0.29
Summer	Weekends	Off-peak	**0.01**	0.07
Fall	Weekdays	On-peak	-0.22	**-0.01**
Fall	Weekdays	Off-peak	**0.03**	0.09
Fall	Weekends	On-peak	**-0.07**	0.19
Fall	Weekends	Off-peak	-0.03	**0.03**
Ave. of Absolute % Differences			0.04	0.10

$E[P_t]$ values obtained with the Weibull distribution and the lognormal distribution are generally slightly lower than those of normal distribution. However, the differences are negligibly small (less then 0.3%). The highest differences occur in Summer/Weekends/On-Peak Hours and Fall/Weekdays/On-Peak Hours intervals, where the difference between other distributions and normal distribution reach –0.29% and –0.22%, respectively.

When the prices calculated with normally distributed demand are compared with the prices with best-fit functions (bold values), the prices are 0.03% higher on average. Therefore, according to Table 9 the use of the normal distribution overvalues the price compared to both the Weibull and lognormal distributions on average. The percent difference for each time interval is shown in Table 10.

Table 10. Percent difference between $E[P_t]$ obtained by best-fitted distribution (Weibull and lognormal) and normal distribution fitted $E[P_t]$.

Time Interval	Winter	Summer	Fall
Weekdays/ On-Peak	-0.15	-0.16	-0.01
Weekdays/ Off-peak	-0.01	-0.02	0.03
Weekends/ On-Peak	-0.01	0.01	-0.07
Weekends/ Off-Peak	0	0.01	0.03

Using the normal distribution parameters results in higher prices during weekdays (0.06% on average). However, the prices are almost the same for weekends as the differences are relatively small (-0.005% on average).

$E[P_t^2]$ values obtained by the Weibull and lognormal distributions are also compared to the values obtained by the normal distribution. These comparisons as percent differences are shown Table 11. Bold values in Table 11 denote the best fit distribution (Weibull or lognormal) for that time interval.

Percent differences are all less then 1%. $E[P_t^2]$ values obtained with the Weibull distribution and the lognormal distribution differ 0.2% on average from those of the normal distribution. The percent differences between Weibull and normal distributions are higher than the percent differences between lognormal and normal distributions (only except Fall/ Weekdays/ On-Peak Hours).

Table 11. Percent difference between $E[P_t^2]$ obtained by Weibull and lognormal distributions and normal distribution $E[P_t^2]$.

Time Interval			Weibull vs. Normal (% diff.)	Lognormal vs. Normal (% diff.)
Winter	Weekdays	On-peak	-0.54	**0.10**
Winter	Weekdays	Off-peak	-0.05	**-0.02**
Winter	Weekends	On-peak	-0.04	**0.00**
Winter	Weekends	Off-peak	0.02	**-0.04**
Summer	Weekdays	On-peak	-0.16	**0.09**
Summer	Weekdays	Off-peak	**0.97**	0.06
Summer	Weekends	On-peak	**-0.91**	0.01
Summer	Weekends	Off-peak	**0.49**	0.02
Fall	Weekdays	On-peak	-0.06	**-0.28**
Fall	Weekdays	Off-peak	**0.38**	0.05
Fall	Weekends	On-peak	**0.37**	-0.11
Fall	Weekends	Off-peak	0.13	**-0.03**
Ave. of Absolute % Differences			0.34	0.07

2.5.1. Weighted Average Price of Electricity for the Intervals

The weighted average price of electricity ($/MWh) is calculated for winter, summer and fall seasons (Table 12). The expected value of price is presented. The number of hours in off-peak (8 hours) and on-peak (16 hours) periods and number of days in each season are considered.

Table 12. Weighted average price ($/MWh) for a complete time interval.

Distribution	Winter	Summer	Fall
Normal	51.16	54.06	47.58
Lognormal	51.17	54.07	47.52
Weibull	51.12	53.99	47.61

The interval specific electricity prices give the market participant more insight. The price fluctuates up to 14% between different seasons. However, the use of different probability distributions does not significantly affect the calculated weighted average price of electricity for the interval (within $\pm 0.2\%$) as shown in Tables 9, 10, and 11.

Notice that the expected price of electricity calculated in Section 2.4.1 for the year 2006, without considering any intervals, was \$50.7/MWh. This is misestimating the price from 1% to 6% compared to interval specific prices. Thus, it is better to consider the time intervals.

The weighted average values of $E[P_t^2]$ are shown in Table 13 for winter, summer, and fall intervals. Percent difference of $E[P_t^2]$ using different distributions are less than 0.4%. The lognormal distribution resulted in higher $E[P_t^2]$ in all of the three seasons.

Table 13. Weighted average value of $E[P_t^2]$ ($\$^2/MWh^2$) for a complete time interval.

Distribution	Winter	Summer	Fall
Normal	2684.7	3032.1	2300.5
Lognormal	2686.1	3034.1	2296.6
Weibull	2676.2	3030.4	2303.1

At the last step of the analysis, best-fit distributions (Table 2) are used to compute the price of electricity for each interval. The density functions of price for all of the 12 time intervals are shown in Appendix B for normal, lognormal, and Weibull distributions. The weighted average price of electricity for winter, summer, and fall are computed to be \$51.12/MWh, \$54/MWh and \$47.57/MWh, respectively.

In the weighted average price calculation, the variance of price has no effect. However, the variance of price is important since it provides more insight to the decision maker, especially for the long-term decisions. A model that considers the variance of price in the valuation of a generator is presented in Chapter 4.

2.6. Conclusions

In this study, the price of electricity was modeled with a bottom-up approach considering the characteristics and the behavior of the market participants. The Cournot competition model assumption enabled better resemblance of the operation of today's markets, especially much better than a perfect competition.

A methodology was developed to generate the density function of price based on the distribution of demand. First, using the relationship between the demand and the price, the density function of price was constructed. The equation is generic, so it suits different statistical distribution functions of demand. The obtained density function provides a better understanding of the market outcomes to the decision maker. Second, the demand was computed at different time intervals (e.g. winter/summer/fall, on-peak/off-peak, weekdays/ weekends) in PJM capacity market. Third, the expected value of price and the expected value of square of price were calculated for each interval using the selected statistical distributions of demand. The effect of different statistical distributions of demand on the price of electricity was analyzed.

The normal distribution is commonly used for the demand of electricity in the literature. The analysis in this chapter showed that lognormal and Weibull distributions are the dominant distributions that fit better the demand data in certain intervals. However, the use of these distributions did not affect the market price of electricity significantly. Thus, using normally distributed price is found to be an appropriate approach. The reason behind opposing the normal distribution is that it allows the demand and price values to be negative. With the same notion, since neither of the lognormal and Weibull distributions realize at a negative value, it is favorable to use lognormal and Weibull distributions to represent the load and the electricity of price better.

As the first result of this analysis, the prices of electricity obtained with these distributions are not significantly different compared to prices obtained by fitting a normal distribution to the demand (within $\pm 0.3\%$ for all time intervals). Second, a negative price is never realized, since the mean demand value of the fitted normal distribution is so high (e.g. 32,407) compared to the standard deviation (e.g. 6,586) (Section 2.4). There are more than 4σ to reach negative demand values on the left tail of the distribution. This probability is considerably small.

When the price for a long-term period is desired, each time interval has to be considered separately instead of considering a year as the time interval. The price of electricity is 1% to 6% different when 12 time intervals within a year are considered. Lastly, the aggregated price of electricity for each season (winter, summer and fall) becomes a decision tool for the investor during the bidding process to the market.

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3. MODELING ELECTRICITY PRICE WITH UNCERTAIN FUEL COST AND DEMAND IN DEREGULATED CAPACITY MARKETS

Abstract

In this study, a model is developed for the price of electricity considering that power companies operate a set of capacity resources with different fuel technologies. The price determination is achieved by the Cournot competition of the firms in the market. Major fuel types to generate electricity in US are coal (32%), petroleum (6%), natural gas (39%), nuclear (10%), and hydroelectric (8%). The companies operate a series of generators that use different type of fuels. The fuel costs are subject to uncertainty. Each generating technology possesses a different electricity cost function which affects the cost function of the firm. The aggregated cost function of a firm is constructed depending on the capacities and parameters of each generating technology. The demand for electricity is assumed stochastic. Low, medium and high cost scenarios are considered for the cost of fuel. The expected price of electricity is calculated based on the probabilities of these cost scenarios.

3.1. Introduction

In the last decade, the electric power industry has been facing the challenge of competition as well as the new dynamic structure of fuel costs. The Energy Information Administration (EIA) forecasts in the Energy Outlook 2007 that the price of natural gas, coal, and oil in US will increase 32%, 29% and 67%, respectively, in the next 25 years. Meanwhile the increase for the price of electricity is forecasted at just 5%. According to the historical data, the electricity demand has increased 2% annually since 1980 (EIA website). The historical figures and the forecasts show that more electricity generators have to be built. As a result, energy companies will face two challenges: Selling cheaper because of the presence of increasing number of competitors (price pressure) and generating electricity cheaper than others because of the increasing fuel costs (fuel cost pressure).

The companies generally find their way out of this competition by diversifying their power generation types. Strategically some of the companies aim for the base load generation by investing more in coal and nuclear plants, however some firms invest more on combined cycle gas turbines (CCGTs) to step in the market during peak load hours, which enables them to make higher profits. CCGTs are generally smaller in generation capacity (MWs) than the coal or nuclear plants. CCGTs have less startup costs and less on/off time requirements, which make them more demand-responsive.

In US, the percentage of available capacity in 2005 was mainly distributed to five main types: Coal (32%), petroleum (6%), natural gas (39%), nuclear (10%), and hydroelectric (8%). Since the generation costs and operating constraints are different depending on the generation type, the shares of these types in the generation are different. Power companies commit electricity to the power grid under the marginal price and the generator availability constraints. In 2005, the electricity generation by coal, petroleum, natural gas, nuclear, and hydroelectric was respectively 50%, 3%, 19%, 19%, and 7%.

These generation totals are reached by annual average utilization of the natural gas fired units at 21% and the nuclear plants at 90%. According to EIA data, the fuel cost constitutes 79.4% and 91.1% of the cost of electricity for coal fired plants and combined cycle gas turbines, respectively (EIA, 2006). The fuel cost is a very important factor to consider and it varies between \$4.7/MWh (nuclear) and \$53.7/MWh (natural gas). Since the fuel costs of nuclear plants are much lower than those of natural gas fired units, nuclear plants are operating at a much higher utilization and they serve the base load.

In this study, a model for the price of electricity is proposed considering that the power companies in the market operate capacity resources with different fuel technologies and the fuel costs are subject to uncertainty. The price determination is achieved by the Cournot competition of the firms in the market. The demand for electricity is also uncertain.

It is assumed that the generating technologies possessed by a firm have different fuel cost coefficients that constitute the aggregated cost function of the firm. The aggregated cost function of the firm is constructed depending on the capacities and cost parameters of each generating technology.

Nuclear, coal, natural gas, and petroleum are considered as the available generating technologies for the market participants. Hydroelectric power plants are not considered, since their availability and operation are contingent upon environmental conditions and they supply 7% of the total electricity generation in US (EIA website, 2006).

3.2. Literature Review

Many researchers have attempted to forecast electricity prices using a time series model with price jumps (Deng, 2000; Davison et al. 2002), an artificial neural network model (Szkuta et al., 1999), and a regression model (Contreras et al., 2003). Gonzalez et al. (2005) indicates that time series based models (regression, regime switching models e.g.) are appropriate for short-term price modeling.

Ventosa et al. (2005) summarizes the market modeling trends. A single firm's decision may be based on a demand-price function. However a multi-firm environment requires consideration of the behavior of the market participants. Agent based simulation models and the equilibrium-based models (Supply function equilibrium (SFE) and Cournot equilibrium) are suitable and commonly used for this oligopolistic competition. In the SFE model, firms compete by submitting price-quantity curves to the market at each hour. Although the SFE model seems to fit very well with the market structure of many restructured markets, there is no unique solution. Very rarely is there a closed form solution. In the Cournot model the sellers make simultaneous optimal choices of quantities to respond to their competitors' optimal choices (Daughety, 1988). The Cournot model is more flexible and tractable than SFE.

Yao et al. (2005) uses Cournot equilibrium in a two-settlement market considering transmission congestions. They focus on the effects of price cap on forward and spot prices. Valenzuela and Mazumdar (2007) consider the Cournot model to represent the price of electricity considering stochastic demand and generator availability.

The Energy Information Administration (EIA) has been performing extensive research on price estimation and has been publishing the results. Since 1996, EIA's Office of Integrated Analysis and Forecasting has produced the Evaluation of Projections for the years after 1982. These projections on the total consumption and production and imports of fuels have been done for the following 25 years. Average percent differences for those projections are summarized in Table 14.

Table 14. Average percent differences over the annual estimations between actual and projected values

Parameter	% Difference
Energy Consumption	2.2
Petroleum Consumption	2.9
Natural Gas Consumption	6.7
Coal Consumption	4.0
Petroleum Production	4.9
Natural Gas Production	5.5
Coal Production	4.2
Average Electricity Prices	19.8
World Oil Prices	52.9
Natural Gas Wellhead Prices	63.5
Coal Prices to Power Generators	46.7

Table 14 shows that EIA estimations for consumption and production parameters were within 2% to 7%, however the estimations for fuel prices were between 46.7% and 63.5% off. The analysts' comment on this is "As regulatory reforms that increased the role of competitive markets were implemented in the mid-1980s, the behavior of natural gas was especially difficult to predict." (EIA Website, 2006)

The focus of this study is to investigate the effects of the uncertainty of demand and fuel cost on the price of electricity. Each firm owns several units. These units' marginal costs vary due to the cost of fuels used. Firms may own one or more different generation technologies such as nuclear, natural gas, coal or petroleum. Fuel costs are considered as stochastic. Three levels of scenarios are possible for the cost of each fuel: Low, medium and high.

In this study, a game theoretical approach is implemented to model the price of electricity. The Cournot model allows firms to withhold their possible electricity generations to increase their profits. The price curve is constructed due to the nominal demand. If a specific demand or a distribution of demand is introduced, the price for that interval is determined. It is assumed that the demand is an inverse linear function of price. In this model, the generators are always available and there is no transmission congestion in the system.

This study is organized as follows: In Section 3.3, the models for demand, cost function, and price are explained. The numerical results with different scenarios of fuel costs are presented in Section 3.4, and the conclusions are outlined in Section 3.5.

3.3. Modeling Competition

In this section, a procedure is described in which the price of electricity is computed in an n-firm market. The characteristics of the firms are their aggregated operating costs and capacity constraints. The model uses the following notation:

n	Number of firms
M	Number of generating technologies available for firms
ξ	Slope parameter of demand
$\pi_{i,t}$	Profit of firm i in period t ($)
K_t	Demand intercept (MWh) (The load, when the price of electricity is zero)
P_t	Price at period t ($/MWh)
$Q_i^{\max}$	Maximum production of firm i (MWh)
$Q_{i,t}$	Total production of firm i at period t (MWh)
$q_{i,j}^{\max}$	Maximum capacity of generator type j of firm i (MW)
$q_{i,j,t}$	Production of generator type j of firm i at time period t (MWh)
$D_t(P_t)$	Actual demand of the power system at period t as a function price P_t (MWh)
$\phi_i(Q)$	Power generation costs of firm i of producing Q MWh ($)
η_j	Slope parameter of price for the price regime j

3.3.1. Modeling the System Load

The actual demand (load) of the system, $D_t(P_t)$, is assumed to be price sensitive and it is represented by the following linear relationship:

$$D_t(P_t) = K_t - \xi P_t$$

where ξ is the slope of demand and $\xi \geq 0$.

In this study, the demand intercept, K_t, is assumed to have a uniform distribution. The generation quantity for firm i at time period t, $Q_{i,t}$ (MW), is the sum of generation quantities of all generation technologies and it is calculated as follow:

$$Q_{i,t} = \sum_{j=1}^{m} q_{i,j,t} \quad \text{for } j \in \{1, 2, \ldots, m\}$$

The actual demand (load) of the system for time period t is equal to the total of all firms' generation amounts and is calculated as follows:

$$D_t(P_t) = \sum_{i=1}^{n} Q_{i,t} \quad \text{for } i \in \{1, 2, \ldots, n\}$$

where $Q_{i,t}$ is the generation quantity for firm *i* at period *t*.

3.3.2. Modeling the Price

The firms, which are bidding in the market, have distributed power generation assets. In Chapter 2, the cost function of a firm is related with one type of generator, thus one type of fuel cost. However, in this chapter, it is assumed that a firm possesses power generators with various technologies. Different types of power generators are considered to have different fuel cost processes in time. The focus is on how the price is affected by changes in fuel costs.

According to the Cournot model, firms compute and submit their bids as generation quantities (MW). The firms bid generation quantities starting from their cheapest available generator type. Therefore, the marginal cost of firm *i* is equal to the marginal generator type's fuel cost, $b_{i,j}$. Firm *k* calculates the amount of power ($Q_{k,t}$) that maximizes its profit knowing the total quantity produced by the other firms ($Q_{-k,t}$). Thus, the profit of firm *k* is as follows:

$$\pi_{k,t} = P_t Q_{k,t} - \phi_k(Q_{k,t})$$

where P_t is price of electricity at time *t*.

The cost of generating $Q_{k,t}$ (MWh) amount of electricity for firm *k* at time period *t* is;

$$\phi_k(Q_{k,t}) = \sum_{j=1}^{m} \phi_k(q_{k,j,t}) \quad j \in \{1,2,\ldots,m\}$$

The cost function of a firm is stepwise linear function and is shown Figure 13. Four types of generators are considered (m=4); nuclear, coal, natural gas, and petroleum and they are denoted by the index 1, 2, 3 and 4, respectively. Thus, four regions are obtained each of which belongs to a generation technology.

Firm k operates its generators according to the economic dispatch rule. At any time, the firm operates the generator which is available with the cheapest marginal cost. Hence, the decision maker sorts different generators according to their marginal cost (from the cheapest to most expensive) and commits electricity in this order. Figure 13 shows the cumulative generation cost of firm k.

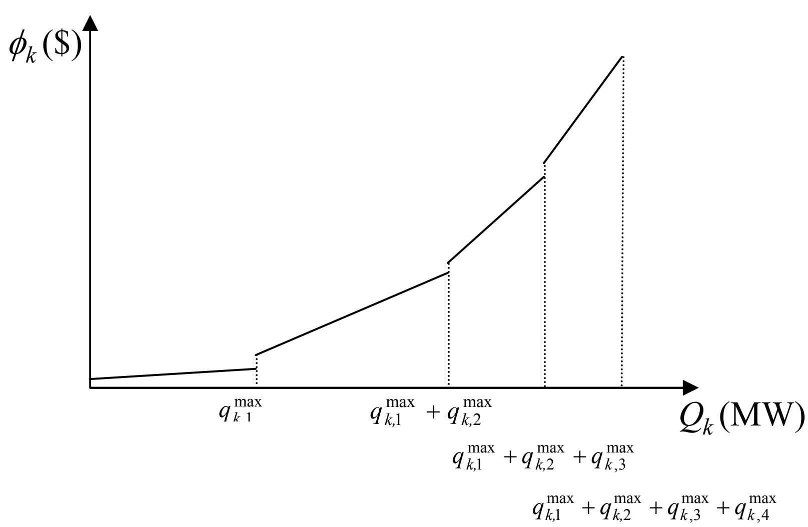

Figure 13. Cost function of firm k.

Each firm responds to the others' bid with a new quantity in order to maximize its profit. Simultaneous responses result in the Nash equilibrium solution. The following procedure is applied in order to find the optimum quantity for each period t:

For i=1,…n
Repeat
For i=1,…n

$Q^*_{-i,t} = \sum_{j=1, j\neq i}^{n} Q_{j,t}$

$Q^*_{i,t} = Q$ such as $\text{Max}\{\pi_{i,t} = P_t(Q)Q - \phi_i(Q)\}$

$\Delta Q^*_{i,t} = \text{Abs}(Q^*_{i,t} - Q_{i,t})$

$Q_{i,t} = Q^*_{i,t}$

End
Until And ($\Delta Q_{1,t} < \varepsilon, \Delta Q_{2,t} < \varepsilon, \ldots, \Delta Q_{n,t} < \varepsilon$)

The quantities that maximize each firm's profit are denoted by $Q^*_{k,t}$. The price of electricity at the equilibrium is as follows:

$$P_t = \frac{K_t - \sum_{i=1}^{n} Q^*_{i,t}}{\xi}$$

As the demand intercept increases, firms bid larger quantities of electricity to the market. In Chapter 2, a detailed analysis of price function depending on K_t was presented. In this study, since firms are composed of several technologies, the slope of the price function changes when a generator type j of firm i reaches its maximum generation capacity, $q^{\max}_{i,j}$.

Firms start to produce when the price level reaches the marginal cost of the cheapest type. Thus, the minimum nominal required load for generating electricity, s_0, is calculated as follows:

$$s_0 = b^{\min}_{i,j}\,\xi$$

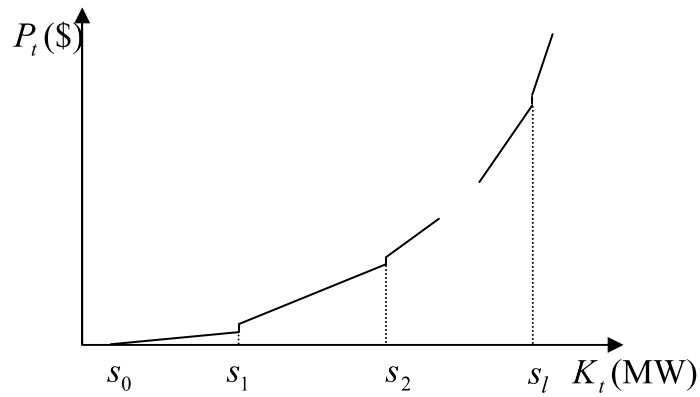

Figure 14. Price over demand intercept.

The demand intercept value, where the slope of price function changes, is called a price switching point, s_i. This means a firm gets into a condition where it can ask a higher price ($) for an additional 1 MWh demand.

By decreasing the number of generators and firms in the competition, the competition weakens and the slope of the price becomes larger. Therefore, the price can be characterized as a piece-wise linear function of K_t as shown in Figure 14.

3.3.3. Modeling the Fuel Cost

Each firm has four types of power generation technology: nuclear, coal, natural gas, and petroleum. Each type of generation is represented with one cost function. The costs of these fuels are assumed stochastic.

The fuel cost is not following a time series or regression model. Instead, the possibility of low ($b_{i,j}^{low}$), medium ($b_{i,j}^{med}$) and high ($b_{i,j}^{high}$) cost scenarios are considered. Three scenarios for four types of generator constitute 81 scenario combinations. Using probabilities for those scenarios one can compute an expected value of price of electricity. The probabilities of these scenarios are assumed to be uniform.

3.4. Numerical Results

In this section, the methodology explained in the previous sections is implemented for a five-firm market. Characteristics of these firms are listed in Table 15. Cost parameters indicating fuel cost, $b_{i,j}$, are represented with their current value in Table 15. Available generator types are nuclear, coal, natural gas, and petroleum and they are denoted with 1, 2, 3 and 4, respectively. For example, $q_{i,1}^{\max}$ represents the capacity of nuclear generation for firm *i*.

Table 15. Characteristics of market participants

Characteristic	Firm 1	Firm 2	Firm 3	Firm 4	Firm 5
$Q_i^{\max}$ (MW)	30,000	20,000	25,000	10,000	12,000
$Q_i^{\min}$ (MW)	0	0	0	0	0
$q_{i,1}^{\max}$ (MW)	6,000	9,500	4,000	3,000	2,500
$q_{i,2}^{\max}$ (MW)	21,000	7,000	7,000	4,000	2,500
$q_{i,3}^{\max}$ (MW)	2,500	3,000	13,000	2,000	4,500
$q_{i,4}^{\max}$ (MW)	500	500	1000	1,000	2,500
a_i	400	175	250	300	120
$b_{i,1}$	7.654	7.054	6.265	6.605	7.102
$b_{i,2}$	14.28	13.56	11.24	15.82	12.42
$b_{i,3}$	45.61	43.29	38.17	47.86	36.37
$b_{i,4}$	77.44	68.15	64.71	70.77	61.26

Total capacities of each generation type in the market are similar to the available capacities in US market. In Table 16, low, medium and high price scenarios for each market participant are presented. The values are the multipliers, when multiplied with the current fuel cost, give the future cost. Since the specific generators vary from one firm to another, marginal cost of one type of technology differs between firms.

Table 16. Price scenarios for major fuels for 2017.

	Low	Medium	High
$b_{i,1}$	0.7	1.1	1.2
$b_{i,2}$	0.8	1.2	1.3
$b_{i,3}$	0.8	1.3	1.8
$b_{i,4}$	0.6	1.2	1.4

The firms' quantity bids and resulting electricity prices are calculated using MATLAB according to Cournot competition. The function of price vs. the demand intercept (between 2,000 MWh and 160,000 MWh) is represented in Figure 15. As the value of demand intercept increases, the price also increases. When a generator of a firm reaches a capacity limit, the marginal cost of the system changes. One less generator remains in the competition. Thus, the slope of the price function changes.

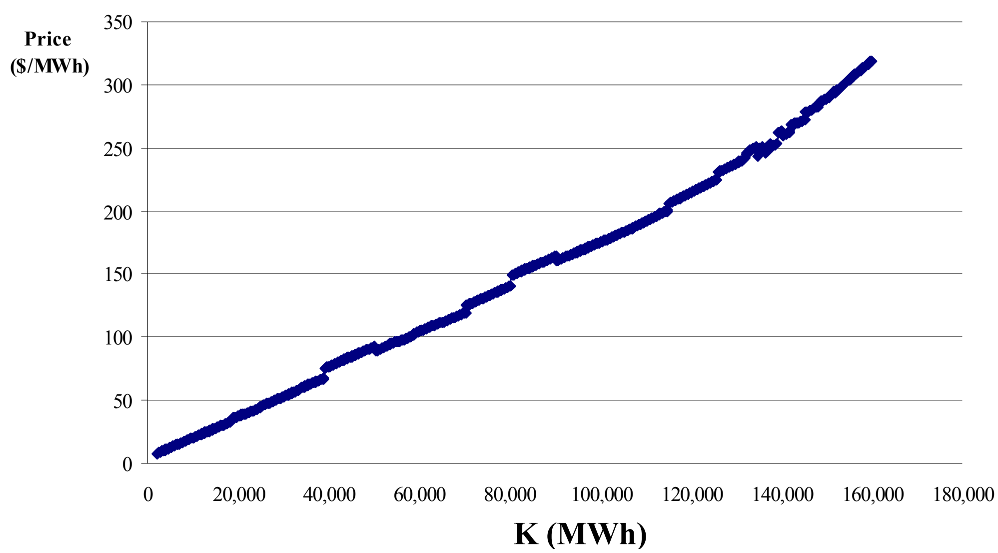

Figure 15. Price vs. demand intercept

The price of electricity for each fuel cost scenario is shown in Table 17, in which the demand intercept is fixed to 60,000MWh. As the fuel cost scenarios involve medium and high values for cost coefficients, the price of electricity increases. The electricity price varies between \$23.55/MWh and \$300/MWh. The same data is plotted in Figure 16.

Table 17. The price (\$/MWh) of electricity for 81 fuel cost scenarios

No	$b_{i,1}$	$b_{i,2}$	$b_{i,3}$	$b_{i,4}$	Price		No	$b_{i,1}$	$b_{i,2}$	$b_{i,3}$	$b_{i,4}$	Price
1	Low	Low	Low	Low	65.37		42	Med	Med.	Med.	High	78.49
2	Low	Low	Low	Med.	57.94		43	Med	Med.	High	Low	123.14
3	Low	Low	Low	High	56.63		44	Med	Med.	High	Med.	101.18
4	Low	Low	Med.	Low	60.19		45	Med	Med.	High	High	97.74
5	Low	Low	Med.	Med.	48.62		46	Med	High	Low	Low	166.19
6	Low	Low	Med.	High	64.98		47	Med	High	Low	Med.	191.44
7	Low	Low	High	Low	69.59		48	Med	High	Low	High	178.70
8	Low	Low	High	Med.	82.30		49	Med	High	Med.	Low	211.69
9	Low	Low	High	High	23.55		50	Med	High	Med.	Med.	230.85
10	Low	Med.	Low	Low	81.89		51	Med	High	Med.	High	237.80
11	Low	Med.	Low	Med.	82.81		52	Med	High	High	Low	300.00
12	Low	Med.	Low	High	24.25		53	Med	High	High	Med.	250.00
13	Low	Med.	Med.	Low	83.24		54	Med	High	High	High	240.00
14	Low	Med.	Med.	Med.	144.79		55	High	Low	Low	Low	300.00
15	Low	Med.	Med.	High	25.66		56	High	Low	Low	Med.	250.00
16	Low	Med.	High	Low	195.48		57	High	Low	Low	High	240.00
17	Low	Med.	High	Med.	148.12		58	High	Low	Med.	Low	299.97
18	Low	Med.	High	High	27.91		59	High	Low	Med.	Med.	231.83
19	Low	High	Low	Low	200.08		60	High	Low	Med.	High	83.02
20	Low	High	Low	Med.	92.64		61	High	Low	High	Low	125.81
21	Low	High	Low	High	56.70		62	High	Low	High	Med.	196.84
22	Low	High	Med.	Low	102.35		63	High	Low	High	High	121.72
23	Low	High	Med.	Med.	86.83		64	High	Med.	Low	Low	195.33
24	Low	High	Med.	High	83.14		65	High	Med.	Low	Med.	130.06
25	Low	High	High	Low	135.42		66	High	Med.	Low	High	154.70
26	Low	High	High	Med.	150.17		67	High	Med.	Med.	Low	215.76
27	Low	High	High	High	127.64		68	High	Med.	Med.	Med.	246.40
28	Med.	Low	Low	Low	156.73		69	High	Med.	Med.	High	220.39
29	Med.	Low	Low	Med.	108.05		70	High	Med.	High	Low	300.00
30	Med.	Low	Low	High	85.22		71	High	Med.	High	Med.	250.00
31	Med.	Low	Med.	Low	116.38		72	High	Med.	High	High	240.00
32	Med.	Low	Med.	Med.	85.31		73	High	High	Low	Low	300.00
33	Med.	Low	Med.	High	59.67		74	High	High	Low	Med.	250.00
34	Med.	Low	High	Low	207.10		75	High	High	Low	High	240.00
35	Med.	Low	High	Med.	55.40		76	High	High	Med.	Low	300.00
36	Med.	Low	High	High	30.14		77	High	High	Med.	Med.	250.00
37	Med.	Med.	Low	Low	203.19		78	High	High	Med.	High	240.00
38	Med.	Med.	Low	Med.	56.83		79	High	High	High	Low	300.00
39	Med.	Med.	Low	High	57.88		80	High	High	High	Med.	250.00
40	Med.	Med.	Med.	Low	102.42		81	High	High	High	High	240.00
41	Med.	Med.	Med.	Med.	85.08							

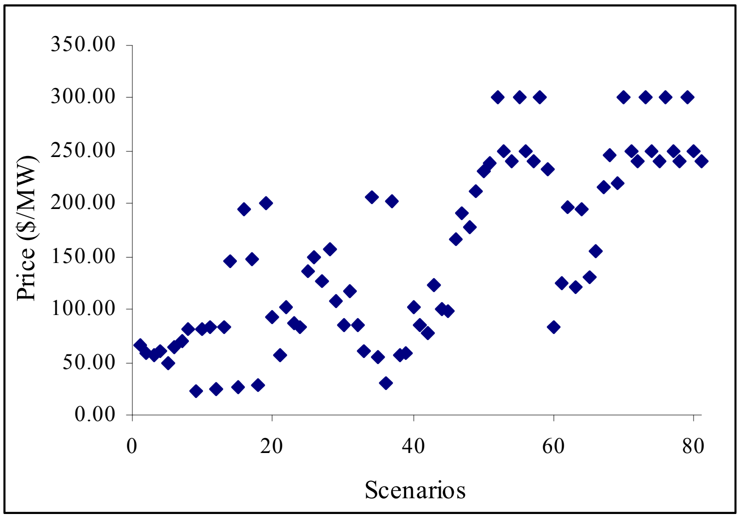

Figure 16. The price of electricity for 81 fuel cost scenarios

The price of electricity varies between $23/MWh and $300/MWh in 81 scenarios. The price of electricity in the first third of the plot is less then the rest because the scenarios 1 to 27 have the low price scenario for the nuclear fuel cost. Since nuclear plants are the base units for the system, the price of nuclear fuel affects the price of electricity directly. The high prices (around $200/MWh) occurring within the first 27 scenarios, when the cost scenario for the coal units is medium or high.

Between scenarios 46 and 81, the price reaches the highest value several times. There are two reasons: the price scenario for nuclear fuel is high in all of these scenarios and the price scenario for coal fuel is either medium or high.

There are low price sections in the mid section of both the first 40 and the second 40 scenarios. The reason is that the price scenario for coal is either low or medium.

Results shown in this section reflect the price of electricity for one specific demand value, 60,000 MWh. In Figure 17, the price of electricity for all scenarios with a changing value of demand intercept (Figure 17) is presented. The value of demand intercept is considered between 2,000 MWh and 160,000 MWh with 1,000MWh increments. For the first twenty scenarios the increase in the price is smooth; however for the medium and high price scenarios, the price shows a steep increase.

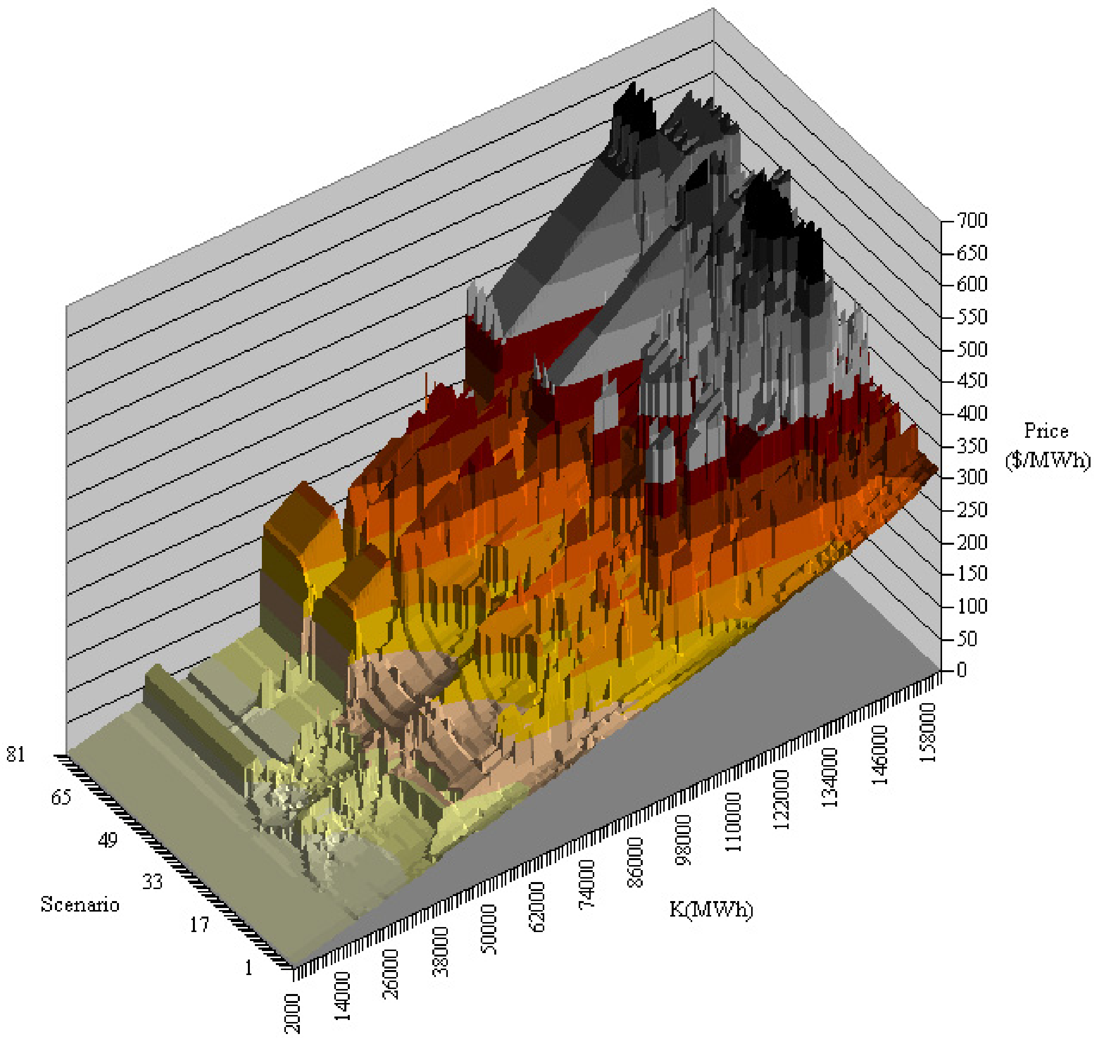

Figure 17. Price of electricity for varying demand with all fuel cost scenarios

3.5. Conclusions

In this study, availability of different types of generators was considered for all market participants. The cost function of a firm was constructed using four types of technologies: nuclear, coal, natural gas and petroleum. Low, medium and high price scenarios are considered for each fuel type and the market price was computed accordingly. The price of electricity showed a steep increase as (1) the value of demand intercept increases and (2) as the fuel cost scenarios with high prices occur (towards the 18^{th} scenario) as shown in Figure 16.

Results confirmed that when a capacity limit is reached for a generator, competition becomes weaker so the price of electricity increases with a higher rate. More investigation of demand intercept would explain which generation technology to be considered at certain time periods.

Availability of generators and transmission congestion are two factors to consider, when the Cournot prices are constructed. Exercise of market power would be more if these constraints were considered.

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4. REAL OPTIONS VALUATION OF A POWER GENERATOR IN DEREGULATED CAPACITY MARKETS WITH UNCERTAIN FUEL COST AND DEMAND

4.1. Introduction

The emerging competitive electricity wholesale markets will end the common practice of using long-term demand forecasts to decide on investments in electricity generation installations. In the future, investments by competitive producers will be highly dependent on market electricity prices. The price of electricity over a long-term period is a quantity that depends on physical factors such as production cost, load, generation availability, unit commitment, and transmission constraints (Denny and Dismukes, 2001). It also depends on economic factors, such as strategic bidding and load elasticity.

When the restructuring of the electricity market took place in many different regions it was done in the belief that it would result in a purely competitive market structure. The motive was that the electricity prices would be predictable using the Bertrand model (Clarker, 1986). There is, however, strong empirical evidence that firms have been able to raise prices well above competitive levels by exercising market power. Many analysts believe that the Cournot model is able to represent the electricity market better as it has evolved (Daughety, 1988). The Supply Function Equilibrium (Rudkevich, 2003; Klemperer and Meyer, 1989) framework is also a strong favorite of certain authors for modeling the electricity prices.

4.2. Valuation of a Generator Asset

In this section, a procedure is described to compute the value of a power generator in a deregulated market. The structure of the power generation is described by utilities' operating constraints and costs. An n-firm market is considered, in which the competitors bid according to a Cournot model. Instead of modeling the price as a time series model or a mean reverting process, a bottom-up approach is used. The price is calculated as an outcome of the competition among market participants. The value of a generator is defined as the expected total benefits and corresponding costs of operating the power generator in a deregulated market environment. The model uses the following notation:

n	Number of firms
ξ	Slope parameter for demand
$\pi_{i,t}$	Profit of firm *i* in period *t* ($)
K_t	Demand intercept (MWh) (The demand, when the price of electricity is zero)
P_t	Price at period *t* ($/MWh)
$Q_i^{\max}$	Maximum production of firm *i* (MWh)
$Q_{i,t}$	Total production of firm *I* at period *t* (MWh)
$D_t(P_t)$	Actual demand of the power system at period *t* as a function price P_t (MWh)
$\phi_i(Q)$	Power generation costs of firm *i* of producing *Q* MWh ($)
η_j	Slope parameter of price for the price regime *j*
q_t	Production of the new generator unit at time period *t* (MW)
$q^{\max}$	Maximum capacity of the new generator

The new generator/capacity, which is evaluated as an investment alternative, is assumed to have a negligible effect on the spot price. Therefore, the new generator/capacity is a price taker. During the operation, it will acquire income with the prices realized at the market. In computing the value of the new generator, the spot price is assumed exogenous to the valuation model. It is also assumed that the power quantity to be generated by the new generator is calculated after the realization of the spot price, which is considered as a random variable. It is also assumed that the new generator does not have minimum up-time/down-time and ramping constraints. Therefore, the generator is always available. The system load (demand) is assumed to be random.

4.2.1. Valuation Model Description

To calculate the value of a power generator, the real options approach is used (Copeland and Antikarov, 2001). In the real options framework, operating a power generator can be viewed as a call (deferral) option. In a call option the decision maker waits until the spot price exceeds the exercise price to exercise the option. A power generator is a real asset and the real option is whether to run the generator to commit a certain amount of power or to turn it off. In this real option, the decision of whether to turn on or off the generator depends on the price of electricity and the cost of generation. The Operating Profit of a Generator with Flexibility (OPF) over a period of T for a generator can be calculated as follows:

$$\text{OPF} = E\left[\sum_{t=1}^{T} \pi_t e^{-rt}\right]$$

where r is the risk-adjusted discount rate and continuous discounting is applied.

The power generator is assumed to have a quadratic cost function as follows:

$$C(q_t) = a + bq_t + cq_t^2$$

The value of "a" includes capital and maintenance costs of the generator, while the parameters "b" and "c" relate to the fuel costs. The profit of the generator at time t can be calculated as follows:

$$\pi_t = P_t q_t^*(P_t) - C(q_t^*(P_t))$$

After the spot price is realized, the generator produces the quantity that maximizes its profit. The optimal quantity is given by

$$q_t^*(P_t) = Min\left\{q^{\max}, Max\left\{0, \frac{P_t - b}{2c}\right\}\right\}$$

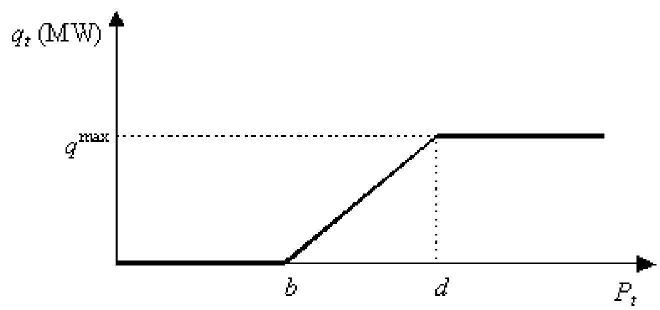

Figure 18. Produced quantity versus price

Notice that the generator produces power only if the price is larger than the value of "b." In this case, the power produced increases linearly with respect to price (Figure 18) until the generator reaches its capacity, $q^{\max}$. The price of electricity at this point is denoted with d and it can be calculated as follows:

$$d = 2cq^{\max} + b$$

The profit function of the generator is shown in Figure 19. Notice that the profit function is a piece-wise function depending on the price. When P_t is less or equal than b, the profit is constant and equal to -a. However, when P_t is between b and d, the profit is a quadratic function and a linear function when P_t is greater than d.

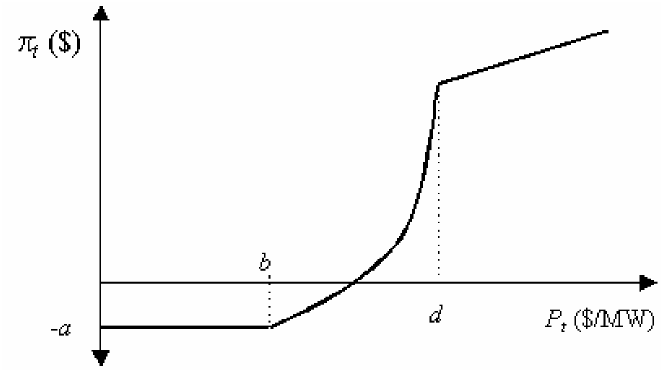

Figure 19. Profit vs. price.

To calculate the value of E[π_t], first the expected revenue is computed. This calculation is conditioned on the value of the price. The price is split in the intervals $(0,b)$,$[b,d)$, and $[d,\infty]$. The expected revenue can be written as follows:

$$E[P_t q_t^*] = E[P_t q_t^* | P_t < b]\Pr[P_t < b] + E[P_t q_t^* | P_t \in [b,d)]\Pr[P_t \in [b,d)] \\ + E[P_t q_t^* | P_t \geq d]\Pr[P_t \geq d]$$

The probabilities that the price lies in the intervals $(0,b)$,$[b,d)$, and $[d,\infty]$ are defined as γ_t, α_t, and β_t, respectively. Then the expected revenue is given by the following equations:

$$E[P_t q_t^*] = \frac{\alpha_t}{2c} E[P_t^2] + \left(\beta_t q^{\max} - \frac{\alpha_t b}{2c} \right) E[P_t]$$

Similarly, the expected cost equation is derived. The expression for the expected cost is as follows:

$$E[C(q_t^*)] = \frac{\alpha_t}{4c} E[P^2] - \frac{\alpha_t b^2}{4c} + a + (bq^{\max} + cq^{\max 2})\beta_t$$

In summary, the expected revenue and the expected cost are added together. The OPF over T periods is as follows:

$$\text{OPF} = \sum_{t=1}^{T} e^{-rt} E\left[\begin{array}{l} \dfrac{\alpha_t}{4c} E[P_t^2] + \left(\beta_t q^{\max} - \dfrac{\alpha_t b}{2c} \right) E[P_t] \\ + \dfrac{\alpha_t b^2}{4c} - a - (bq^{\max} + cq^{\max 2}) \beta_t \end{array} \right] \quad (1)$$

$E[P_t]$, $E[P_t^2]$, γ_t, α_t, and β_t are required to be calculated.

4.2.2. Modeling the System Load

The actual load of the system is assumed to be price sensitive. The actual load is represented by the following linear relationship (Figure 20):

$$D_t(P_t) = K_t - \xi P_t$$

where ξ is the elasticity of demand and $\xi \geq 0$.

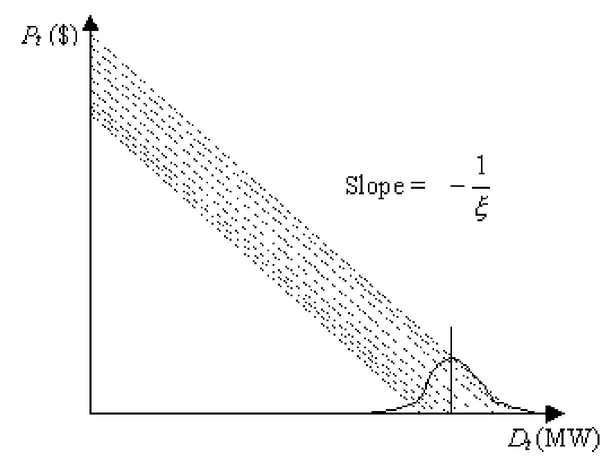

Figure 20. The relationship between price and the demand intercept.

The demand intercept, K_t, is assumed to be normally distributed with an expected value of μ_t and a variance of σ_t^2 as a result of Chapter 2. The actual load of the system for time *t* is equal to the total of the firms' generations and is calculated as follows:

$$D_t(P_t) = \sum_{i=1}^{n} Q_{i,t}$$

4.2.3. Modeling the Price

Each firm has several generators and the firms' aggregate production cost functions are quadratic. The cost functions for the firms are as follow:

$$\phi_i(Q_{i,t}) = A_i + B_i Q_{i,t} + C_i Q_{i,t}^2 \qquad i \in \{1,2,\ldots,n\} \text{ and } 0 \le Q_{i,t} \le Q_i^{\max}$$

According to the Cournot model, firms compute their bids as generation quantities (MWh). Firm *k* calculates the amount of power ($Q_{k,t}$) that maximizes its profit knowing the total quantity produced by the other firms ($Q_{-k,t}$). Thus, the profit of firm *k* is calculated as follows:

$$\pi_{k,t} = P_t Q_{k,t} - \phi_k(Q_{k,t})$$

Each firm responds to the others' bid with a new quantity in order to maximize their profit. Simultaneous responses result in the Nash equilibrium. The quantities that maximize each firm's profit are denoted by $Q_{k,t}^*(K_t)$. The price of electricity at the equilibrium is as follows:

$$P_t = \frac{K_t - \sum_{i=1}^{n} Q_{i,t}^*(K_t)}{\xi}$$

Empirically, it is seen that P_t is a piece-wise linear function of K_t. As the nominal demand increases, firms bid larger quantities of power to the market. When a firm reaches its maximum generation capacity, it cannot increase its quantity bid anymore.

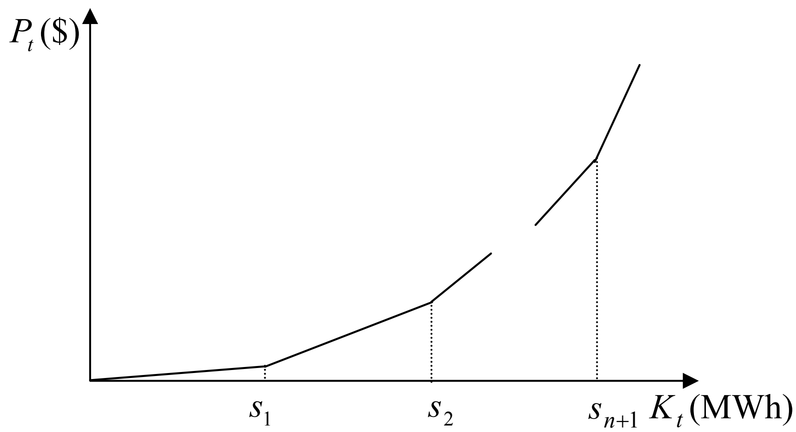

Figure 21. Price over demand intercept

The slope of the price function changes when firm *i* reaches its maximum generation capacity, $Q_i^{\max}$ (Figure 21). By decreasing the number of firms in the competition, the competition weakens and the slope of the price function becomes larger. Each price switching point is denoted by s_i. Therefore, the price, P_t, can be characterized as a piece-wise linear function of demand intercept, K_t, as shown in Figure 21.

The price regimes are obtained as follows:

$$P_t = \begin{cases} \eta_1 K_t & \text{if } K_t \leq s_1 \\ s_1(\eta_1 - \eta_2) + \eta_2 K_t & \text{if } K_t > s_1 \text{ and } K_t \leq s_2 \\ \quad \cdots & \quad \cdots \\ s_1\eta_1 + (s_2 - s_1)\eta_2 + \ldots + (s_n - s_{n-1})\eta_n - s_{n+1}\eta_{n+1} + \eta_{n+1}K_t & \text{if } K_t > s_{n+1} \end{cases} \tag{2}$$

where η_{n+1} is the slope of the $(n+1)^{th}$ price regime.

If the demand intercept, K_t, follows a general distribution function, the density function of price, $f_p(x)$, can be defined in terms of the density function, $f_K(x)$, and cumulative distribution function, $F_K(x)$, as follows:

$$f_p(x) = f_k\left(\frac{x}{\eta_1}\right)\mathrm{F_K}(s_1) + f_k\left(\frac{x-(\eta_1-\eta_2)s_1}{\eta_2}\right)[\mathrm{F_K}(s_2)-\mathrm{F_K}(s_1)]$$

$$+ f_K\left(\frac{x-(\eta_1-\eta_2)s_1-(\eta_2-\eta_1)s_2}{n_3}\right)[\mathrm{F}_K(s_3)-\mathrm{F}_K(s_2)]+\ldots \qquad (3)$$

This density function is generic in that it can be used with any distribution function of the nominal demand.

4.2.4. Computing $E[P_t\,]$ and $E[P_t^2]$

The expected value of the price is calculated considering the *n+1* price regime as follows:

$$E[P_t] = \sum_{i=1}^{n+1} E[P_t | K_t \in R_i]\Pr[K_t \in R_i] \qquad (4)$$

where R_i is the set of demand intercept values in price regime *i*.

The values of $\Pr[K_t \in R_i]$ are denoted by $Z_{i,t}$. By using (2), (4) can be written as

$$E[P_t] = \eta_1\mu_t Z_{1,t} + (s_1(\eta_1-\eta_2)+\eta_2\mu_t)Z_{2,t} + \ldots$$

$$+ (s_1\eta_1 + (s_2-s_1)\eta_2 + \ldots + (s_n - s_{n-1})\eta_n - s_n\eta_{n+1} + \eta_{n+1}\mu_t)Z_{n+1,t} \qquad (5)$$

where $Z_{i,t} = \Phi(s_i;\mu_t,\sigma_t^2) - \Phi(s_{i-1};\mu_t,\sigma_t^2)$ for a normal distribution and μ_t is the expected value of demand intercept for that distribution.

Similarly, the second moment of price is computed as follows:

$$E[P_t^2] = \sum_{i=1}^{n+1} E[P_t^2 | K_t \in R_i] \Pr[K_t \in R_i] \tag{6}$$

where R_i is the set of demand intercept values in price regime *i*.

By using (2), (6) can be written as;

$$\begin{aligned} E[P_t^2] = {} & (\eta_1 \mu_t)^2 Z_{1,t} + \left((s_1(\eta_1 - \eta_2))^2 + 2(s_1(\eta_1 - \eta_2))(\eta_2 \mu_t) + (\eta_2 \mu_t)^2 \right) Z_{2,t} \\ & + \left((s_1\eta_1 + (s_2 - s_1)\eta_2 + \ldots + (s_n - s_{n-1})\eta_n - s_n \eta_{n+1})^2 + \right. \\ & \left. 2(s_1\eta_1 + (s_2 - s_1)\eta_2 + \ldots + (s_n - s_{n-1})\eta_n - s_n\eta_{n+1})(\eta_{n+1}\mu_t) + (\eta_{n+1}\mu_t)^2 \right) Z_{n+1,t} \end{aligned} \tag{7}$$

where $Z_{i,t} = \Phi(s_i; \mu_t, \sigma_t^2) - \Phi(s_{i-1}; \mu_t, \sigma_t^2) = \Pr[K_t \in R_i]$ for a normal distribution and μ_t is the expected value of demand intercept K_t for that distribution.

4.3. Numerical Results

4.3.1. 2-Firm Market

The characteristics of market participants are shown in Table 18. The value of ξ is assumed to be 200. The risk adjusted discount rate, *r*, is assumed as 9% and continuous discounting is applied.

Table 18. Characteristics of market participants (firms).

Characteristic	Firm 1	Firm 2
Q^{max} (MWh)	25,000	10,000
Q^{min} (MWh)	0	0
A	250	300
B	6.265	6.605
C	0.0022	0.0012

The newly introduced generator has a maximum capacity of 650 MW and its cost function is as follows:

$$C(q) = 670 + 43q + 0.0056q^2 \quad (\$/\text{MWh})$$

The parameters of the demand intercept at each time interval hour are shown in Table 19.

Table 19. The parameters (μ, σ) of normal distributed demand intercept for 12 intervals.

Time Interval	Winter	Summer	Fall
Weekdays/ On-Peak Hours	(36199, 5860)	(39547, 7231)	(33312, 3177)
Weekdays/ Off-peak Hours	(29114, 4681)	(29292, 5305)	(26125, 3505)
Weekends/ On-Peak Hours	(31507, 4892)	(34529, 6714)	(29089, 2965)
Weekends/ Off-peak Hours	(27437, 4129)	(27772, 4891)	(25246, 3151)

The overnight cost for a combined cycle gas turbine with these operating costs and constraints is shown as $490/kW on average (between $360/kW and $620/kW), which results in 318.5 million USD initial investment (EIA website, 2006).

The values of s_0, s_1 and s_2 are found to be 1,254MWh, 31,184MWh and 61,251MWh, respectively. The first and the second moments of price ($E[P_t]$ and $E[P_t^2]$) are shown in Table 20 for the 12 time intervals.

Table 20. $E[P_t]$ and $E[P_t^2]$ for the normal distributed demand.

Time Interval	Winter	Summer	Fall
Weekdays/ On-Peak Hours	(67.9, 4802)	(76.1, 6111)	(61.0, 3780)
Weekdays/ Off-peak Hours	(52.1, 2806)	(52.4, 2864)	(47.4, 2287)
Weekends/ On-Peak Hours	(56.9, 3341)	(63.7, 4288)	(52.3, 2763)
Weekends/ Off-peak Hours	(49.4, 2495)	(49.8, 2567)	(46.1, 2156)

The Operating Profit of a Generator with Flexibility (OPF) for each interval is calculated (Table 21). As the expected values of demand are higher for the Summer/Weekdays/On-Peak, Winter/Weekdays/On-Peak and Summer/Weekends/On-

Peak intervals, the electricity prices and the OPF values are also higher when compared with other intervals. The price of electricity is always higher than the marginal cost of the generator, thus the generator is ON during all time intervals.

Table 21. OPF for 12 time intervals.

Time Interval	Price ($/MWh)	Generator State	OPF ($/Hour)
Winter/Weekdays/On-Peak	67.9	ON	14,113.6
Winter/Weekdays/Off-Peak	52.1	ON	3,361.7
Winter/Weekends/On-Peak	56.9	ON	19,736.3
Winter/Weekends/Off-peak	49.4	ON	3,617.2
Summer/Weekdays/On-Peak	76.1	ON	9,119.5
Summer/Weekdays/Off-Peak	52.4	ON	630.8
Summer/Weekends/On-Peak	63.7	ON	6,520.1
Summer/Weekends/Off-peak	49.8	ON	1,653.7
Fall/Weekdays/On-Peak	61.0	ON	11,305.9
Fall/Weekdays/Off-Peak	47.4	ON	2,023.1
Fall/Weekends/On-Peak	52.3	ON	3,294.1
Fall/Weekends/Off-peak	46.1	ON	120.4

The generator's OPF for each interval is calculated using (1) in Section 4.2.1. The present value of the total OPF for one year is 77.5 million USD. The present values of the cash flows for 20 years is shown in Table 22. The present value of the generator for an economic lifetime of 20 years are calculated as 432.6 million USD. Break-even time for positive net value or payoff period is reached in 3.2 years.

The internal rate of return is found 22%. When considering this high rate of return, one should bear in mind that this is a 2-firm market with an HHI value of 5,918. This high HHI value shows there is high concentration and there is almost no competition.

Table 22. Present values of the cash flows for 20 years.

Year	Cash flow (Million USD)	Year	Cash flow (Million USD)
0	-318.5	11	31.5
1	77.5	12	28.8
2	70.8	13	26.3
3	64.7	14	24.0
4	59.1	15	22.0
5	54.0	16	20.1
6	49.4	17	18.4
7	45.1	18	16.8
8	41.3	19	15.3
9	37.7	20	14.0
10	34.5	Total PV	432.6

4.3.2. 5-Firm Market

The characteristics of market participants are shown in Table 23. Parameters of the demand intercept for 12 time intervals and the properties of the evaluated power generator are the same as the 2-firm market example.

Table 23. Characteristics of market participants.

Characteristic	Firm 1	Firm 2	Firm 3	Firm 4	Firm 5
Q^{max} (MW)	30,000	20,000	25,000	10,000	12,000
Q^{min} (MW)	0	0	0	0	0
A	400	175	250	300	120
B	7.654	7.054	6.265	6.605	8.31
C	0.0016	0.0032	0.0022	0.0012	0.0027

The values of s_0, s_1, s_2, s_3, s_4 and s_5 are found to be 2,310 MWh, 56,330 MWh, 88,470 MWh, 141,090 MWh, 143750 MWh and 147,740 MWh, respectively. OPF for 12 intervals are calculated using (1) in Section 4.2.1 and are shown in Table 24.

In four of the intervals, all of the Fall and Summer off-peak hours, the OPF is zero, because the prices of electricity ($42.51/MWh, $44.22/MWh, $44.65/MWh and $41.37/MWh) during these intervals are not high enough to generate a positive value as revenues, variable operating costs ($43/MWh), and fixed costs ($670/hour) are considered.

Table 24. OPF for 12 time intervals.

Time Interval			Price ($/MWh)	Generator State	OPF ($/Hour)
Winter	Weekdays	On-Peak	55.60	ON	5,616.3
Winter	Weekdays	Off-Peak	46.40	ON	263.8
Winter	Weekends	On-Peak	59.90	ON	8,586.0
Winter	Weekends	Off-Peak	46.63	ON	418.0
Summer	Weekdays	On-Peak	51.85	ON	2,956.5
Summer	Weekdays	Off-Peak	42.51	OFF	0.0
Summer	Weekends	On-Peak	49.51	ON	1,631.9
Summer	Weekends	Off-Peak	44.22	OFF	0.0
Fall	Weekdays	On-Peak	53.43	ON	4,166.1
Fall	Weekdays	Off-Peak	44.65	OFF	0.0
Fall	Weekends	On-Peak	46.36	ON	65.8
Fall	Weekends	Off-Peak	41.37	OFF	0.0

The present value of the total OPF for one year is 27.7 million USD. The present values of the cash flows for 20 years is shown in Table 25.

Table 25. Present values of the cash flows for 20 years.

Year	Cash flow (million USD)	Year	Cash flow (million USD)
0	-318.5	11	11.3
1	27.7	12	10.3
2	25.3	13	9.4
3	23.2	14	8.6
4	21.2	15	7.9
5	19.3	16	7.2
6	17.7	17	6.6
7	16.2	18	6.0
8	14.8	19	5.5
9	13.5	20	5.0
10	12.3	Total PV	-49.6

As shown on Table 25, the present value of the generator for a lifetime of 20 years is calculated as -49.6 million USD. As a result, break-even time could not be reached in 20 years. Internal rate of return is found 6.73%. This low rate of return is also expected as a result of the smaller HHI value of 2,305 instead of 5,819 in the 2-firm market, which is a sign of more competitive environment. Even though the market became more competitive, HHI value of 2,305 still shows evidence of little concentration since it is higher than 1,800.

4.3.3. Comparison of the Price between 2-firm Market and 5-firm Market

The prices obtained in the 2-firm market and in the 5-firm market are different. The functions of price in these two different markets are shown in Figure 22. The corresponding slope values at each price regime are shown in Figure 23. In both figures, the continuous line represents the 2-firm market and the dashed line represents the 5-firm market.

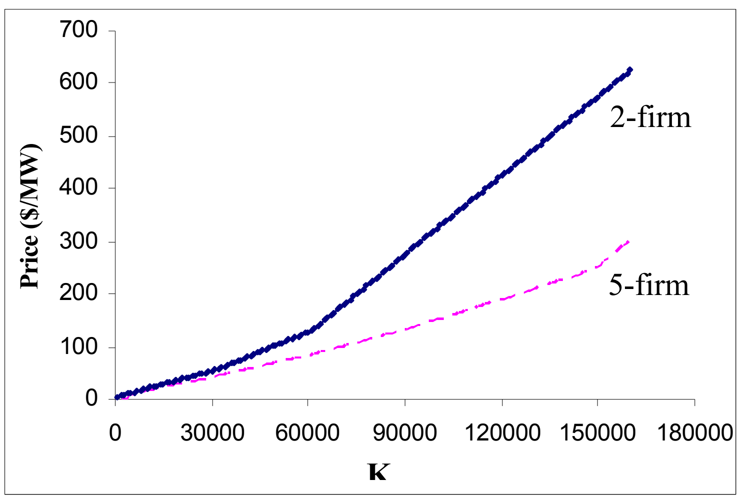

Figure 22. Price versus demand intercept

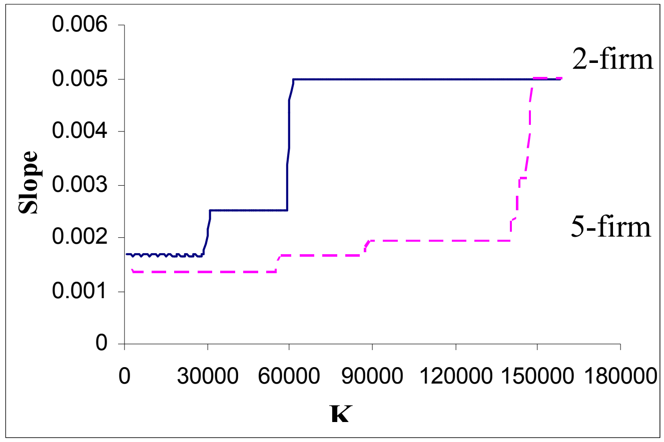

Figure 23. Slope of price over demand intercept

Involving less number of firms, the 2-firm market has higher electricity prices As the demand intercept, K, increases, firms reach their capacity limits earlier in the 2-firm market compared to the 5-firm market because of the availability of less resources. Thus, the price of electricity realized is much higher, especially after 61,251MWh (s_2 for the 2-firm market). If one firm is only bidding a certain extra capacity to the market, the slope of the price function reaches maximum and is equal to 0.005 (equal to $1/\xi$) (Figure 23).

4.3.4. Sensitivity Analysis

The sensitivity analysis is performed in order to understand how the present value of the generator changes with changing marginal cost. Different types of generator investments such as coal and nuclear are also considered.

The marginal cost parameter, *b*, for the generator being evaluated has a substantial effect on the value obtained. Also, there are several technologies available with varying generation costs that need to be considered. According to Energy Information Administration, the variable costs of generation for combined cycle gas fired electricity generators range between \$40/MWh and \$70/MWh (Annual Energy Outlook, 2007). The change in present value of the generator is illustrated in Figure 24 for both 2-firm and 5-firm markets by the change in marginal cost, *b*, between \$30/MWh and \$60/MWh. Risk-adjusted discount rate is 9%.

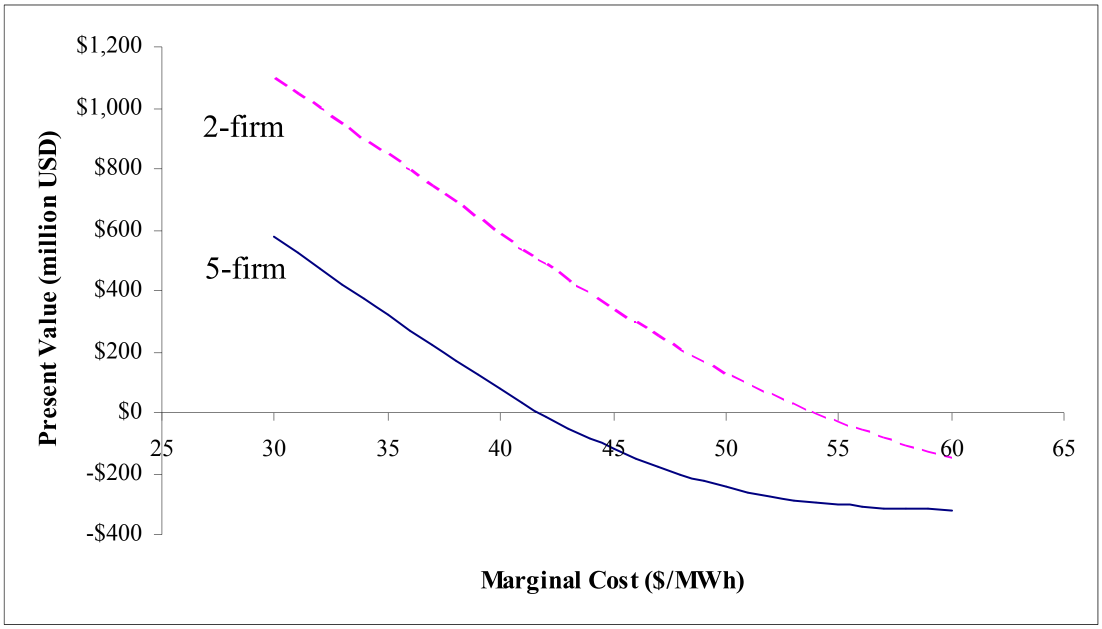

Figure 24. The present value of generator with flexibility vs. changing marginal cost.

According to Figure 24, the present value drops at a decreasing rate while the marginal cost increases. The present value of the generator is higher in the 2-firm case; however it decreases below zero in both the 2-firm and the 5-firm cases if marginal costs are more than \$42/MWh and \$54/MWh, respectively.

Investing in different types of generation technologies such as coal and nuclear can be considered as well. Table 26 shows the characteristics of a typical coal plant and a nuclear plant.

Table 26. Characteristics of the coal and nuclear plants

Characteristic	Coal	Nuclear
q^{max} (MW)	700	1,100
q^{min} (MW)	0	0
A	800	750
b (range)	20-40	15-35
C	0.0021	0.0019

Overnight investment cost of a coal plant and a nuclear plant are $1,400/kWh and $1,800/kWh, respectively. Thus the total investment cost for a coal plant and a nuclear plant are 980 million USD and 1,980 million USD, respectively. Ranges of costs are arbitrarily selected to reflect the effect of marginal cost on the present value as well. Present values for the 2-firm and the 5-firm market are illustrated in Figure 25.

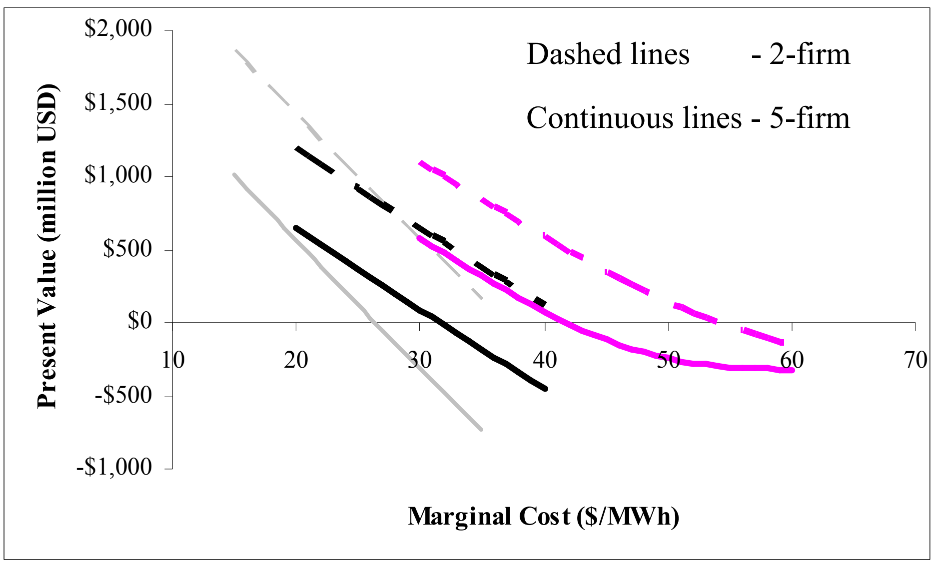

Figure 25. Present value for gas (dark grey), coal (black) and nuclear (light grey).

According to Figure 25, the present value for a nuclear plant is higher than that of coal plant for their average marginal costs. Average marginal costs for coal, nuclear and gas-fired plants are $30/MWh, $25/MWh, and $45/MWh, respectively. The present value of the nuclear plant decreases sharply compared to coal and gas-fired units by increasing marginal cost. The internal rate of return and the present value are shown in Table 27 for coal, nuclear and gas fired units in 2-firm and 5-firm markets.

Table 27. The present value and internal rate of return for different markets and generators.

	2-firm Market			5-firm Market		
	Coal	Nuclear	Gas	Coal	Nuclear	Gas
Present Value (million USD)	649	989	336	91	122	-119
Internal Rate of Return (%)	17.3	15.4	22	10.2	9.8	6.7

For the 2-firm market, despite the fact that the present values of coal and nuclear plants are much higher than the gas-fired generator, the internal rate of return of the gas-fired unit is higher. This is because of the low investment cost of gas-fired units (0.3 million USD) compared to coal plants (1 billion USD) and nuclear plants (2 billion USD). When a 2 billion USD investment is considered, investors would prefer to invest on multiple gas plants in different strategic locations in a network instead of investing in nuclear or coal plants when the competition is still weak. This advantage vanishes when the competition gets tougher. The value of a gas plant is negative for the 5-firm market and the internal rate of return is less than the risk-adjusted discount rate.

4.4. Conclusion

In this study, analytical models and procedures were developed based on power system data and load data for probabilistically representing a Cournot market competition of firms. The models were used to value a power plant using the real options approach.

The sensitivity analysis results showed that the value of a generator/new capacity depends on the level of competition. Understanding the competitiveness of the market is very crucial while making the generation investment decision. When the market consists of five firms, the present value of the new plant is resulted to be lower than the case when the market includes two firms. In this particular example, the decrease in internal rate of return was between 40% and 70%.

The OPF value calculated using real options for each interval permits the decision maker to consider different revenue alternatives for each interval. These revenue alternatives might be the contracts for direct sales or the hydroelectricity pump-backs or sales to other energy markets for specific time intervals in which the value of generator in the competitive market is zero.

Although the effects of unit commitment, transmission congestion, and transmission outages were not considered, it is expected that the methodology proposed in this study will provide a suitable tool for the decision maker in assessing the value of a power plant. The consideration of capacities, costs and market behavior of the competitors has provided the expected profit of operating a power generator with flexibility.

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5. CONCLUSIONS

This research investigated the valuation of a power generation asset in a deregulated capacity market environment where the price of electricity is determined after competitive bidding. A 3-step procedure was followed. First, the price of electricity was modeled considering the distribution of electricity demand in the Pennsylvania – New Jersey – Maryland (PJM) electricity market. Second, the price of electricity is modeled considering stochastic fuel cost and availability of a number of generating technologies in the energy firms. Third, real options valuation was performed to assess the value of a power generator that operates in this capacity market.

The developed price model considered competitive behavior of the market participants. It was found that the slope of the price function change as the companies reached their capacities according to increasing demand. The closed form equation for the first moment and the second moment of price were derived. The density function of price was calculated.

An extensive statistical analysis was performed to understand the demand distribution and the effect of this distribution on those moments. A year was divided into twelve time intervals similar to the intervals considered in PJM market. After fitting various statistical distributions to the demand data of the selected market, it was found that lognormal and Weibull distributions fit better to the demand data compared to other distributions. The first and second moments of price were calculated for lognormal, Weibull and normal distributions for comparison. The comparison of those results showed that the percent differences of prices are negligible. The percent difference between Weibull distribution and normal distribution was found 0.04% in average. The percent difference between lognormal distribution and normal distribution was found 0.1% on average. In general, lognormal and Weibull distributions were thought more suitable to fit the price data, because of the non-negativity as a consequence of curtailed left tail. The results represented in this research showed that normal distribution represents the demand of electricity as good as Weibull and lognormal distributions when the standard deviation of the demand is very small compared to the average demand which makes the fitted distribution practically impossible to be negative. It requires more than 6σ on the left tail to be negative.

The consideration of stochastic fuel cost provided a deeper insight about the investment decision because of the long time period required to obtain benefits out of the power generator investment. It was considered that four types of generators are available at each firm: nuclear, coal, natural gas, and petroleum. Low, medium, and high cost scenarios were considered for each fuel type. Chapter 3 included 81 cost scenarios for the firms with various generators. The price of electricity increased more than three times between the scenario with all-low costs and the scenario with all-high costs.

The present value of a generator wais defined as the total of expected benefits and corresponding costs of operating the power generator with flexibility for a period of time in a deregulated market environment. The value of a nuclear, coal and natural gas fired generators were calculated for both 2-firm and 5-firm markets. The present value of the generator drops as the number of firms increased in the market. Nuclear and coal fired units had higher rate of return than gas fired units when the competition is strong. However, they require 2 to 3 times more investment.

The difference between the marginal cost and the price of electricity is very important. As the marginal cost of a unit increases the net real option value could be negative.

5.1. Future work

In this research, transmission congestion is not considered. It may be interesting to analyze the effects of the network configuration on the price. It could diminish the competition depending on both occurrences of congestion. This would also increase the exercise of market power by the market participants.

The planned outages and breakdowns have an effect on the price of electricity and the value of a generator. In this study, the availability of the generators was not considered since a long-term valuation is performed. While serving the capacity markets (mostly base load), the availability would have a negligible effect on the price of electricity.

Another extension would be exploring supply function equilibrium (SFE) models. The implementation of SFE models to energy markets has been limited because of higher complexity of the equations and the availability of multiple Nash equilibrium solutions. Exploratory studies using SFE models would give more insight about the value of an investment alternative.

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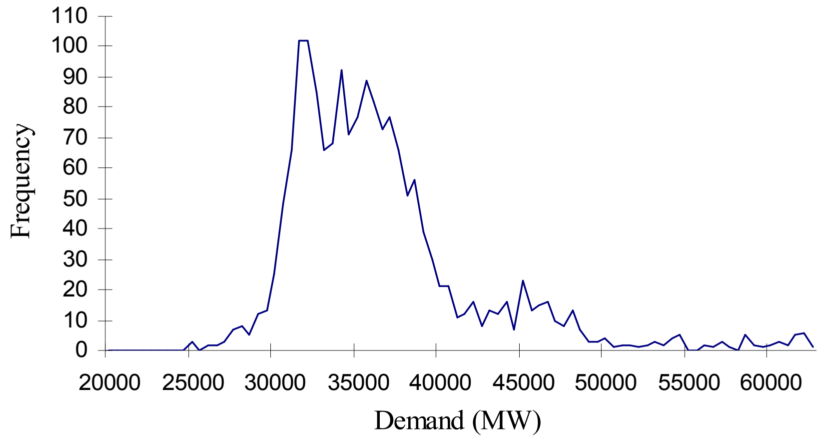
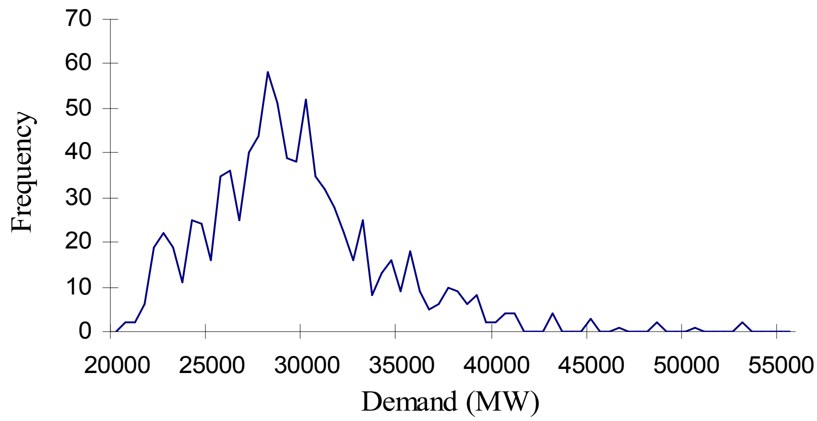
7. APPENDICES

7.1. Appendix A: The frequency of the electricity demand in 12 time intervals in PJM East during 2006.



Winter/ Weekdays/ On-Peak Hours



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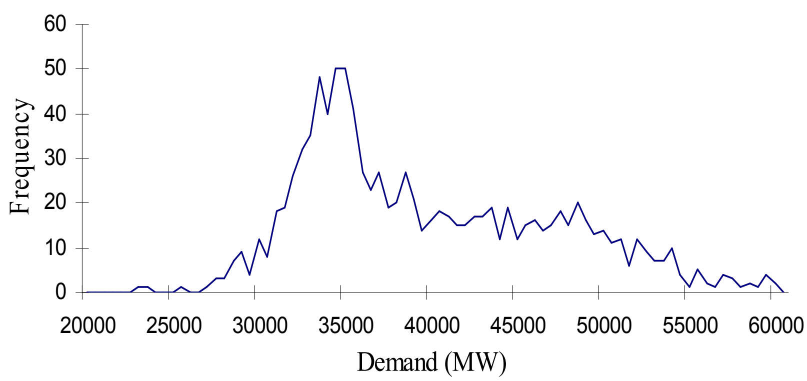

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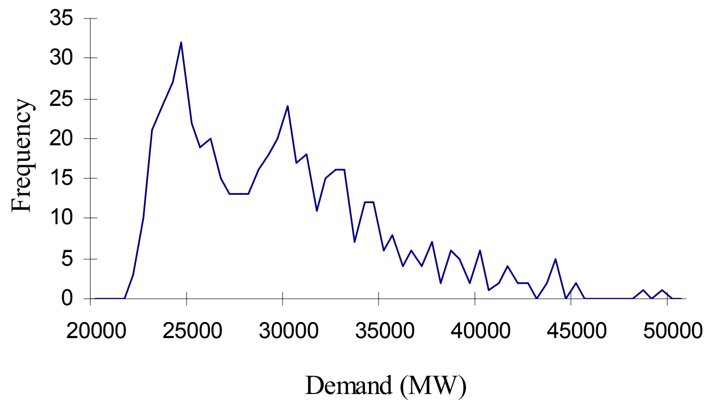

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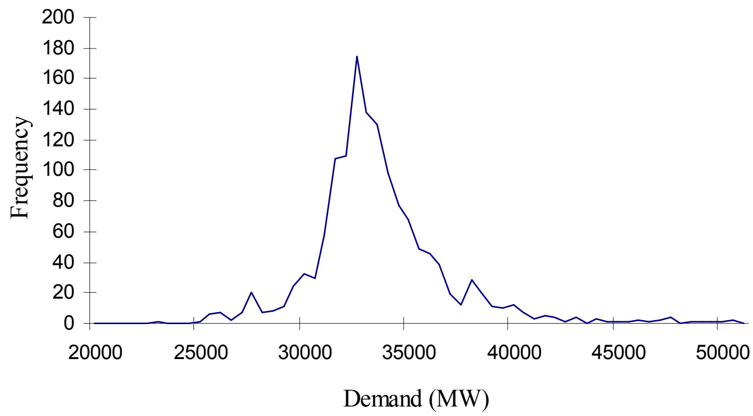
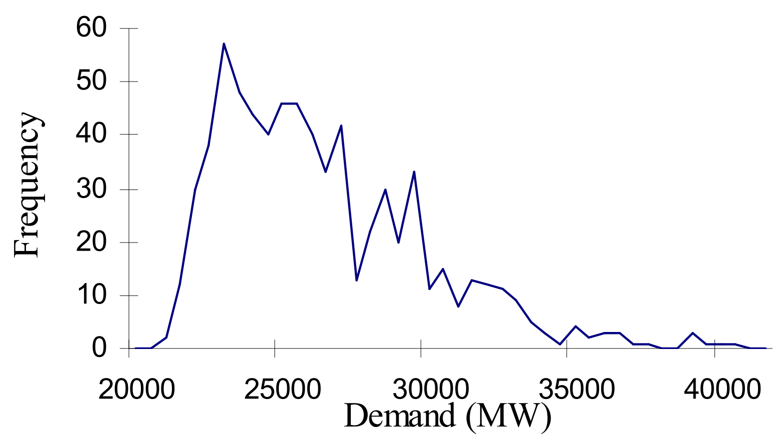


Fall/ Weekdays/ On-Peak Hours



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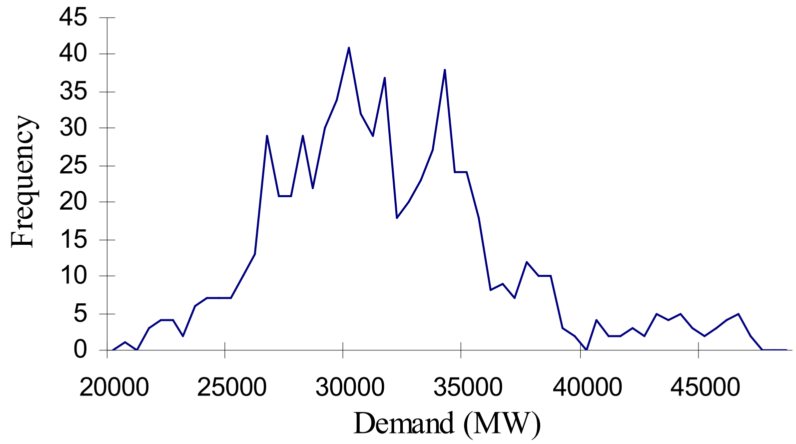

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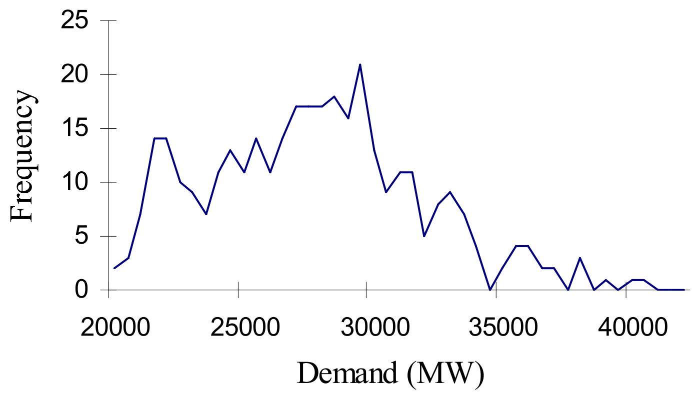

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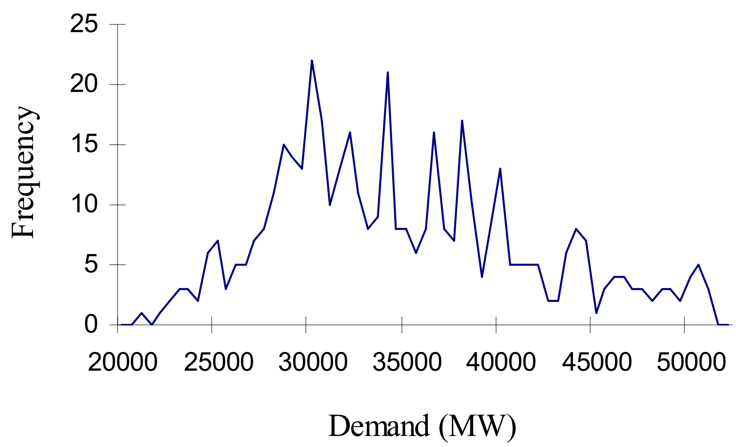

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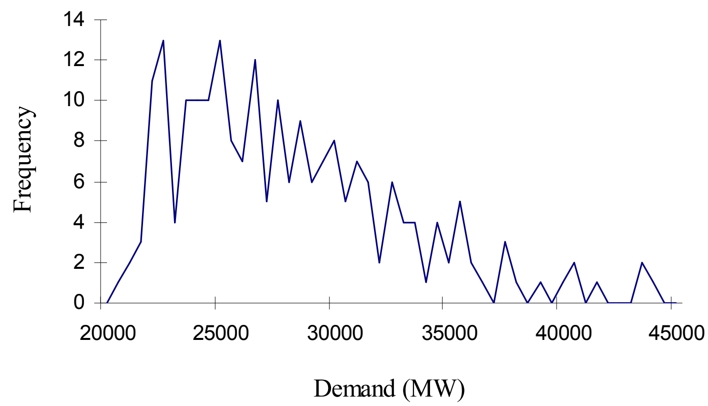

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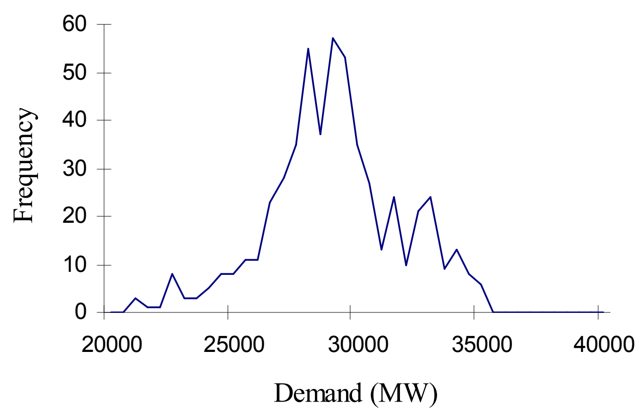

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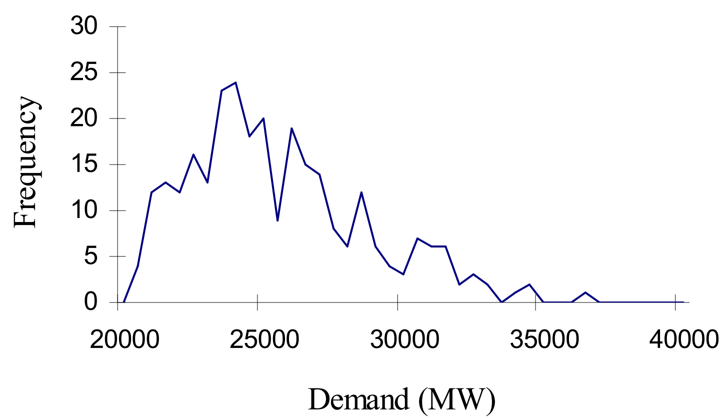
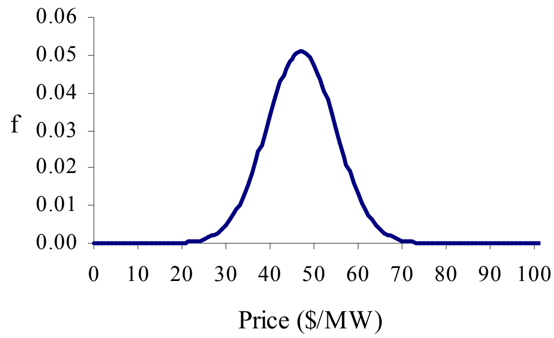


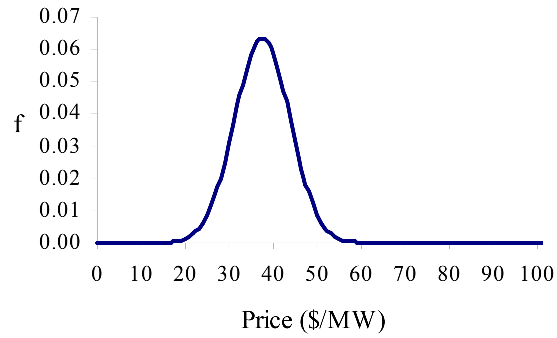
Fall/ Weekends/ Off-Peak Hours

7.2. Appendix B: Density Function of Price for the price of electricity in 12 time intervals in PJM East during 2006.

- Normal Distribution



Weekdays/ On-Peak Hours/ Winter



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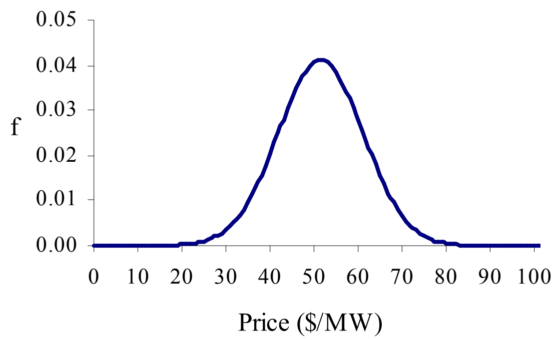
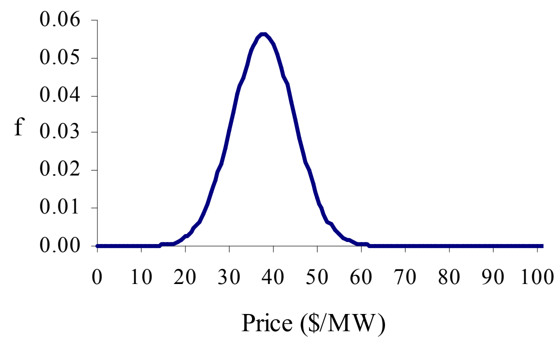


Weekdays/ On-peak Hours/ Summer



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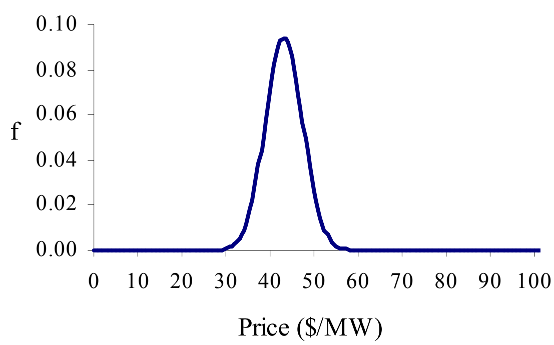
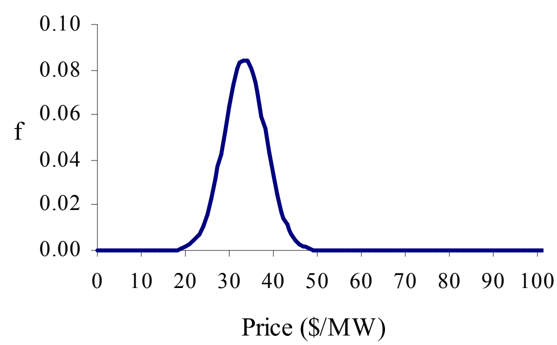
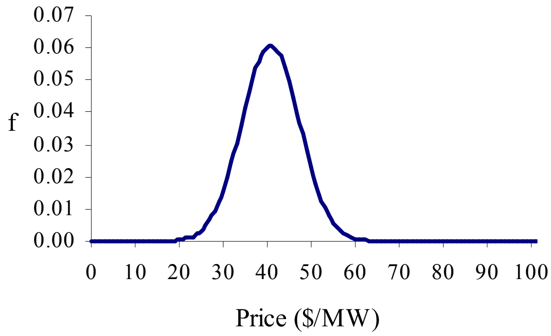


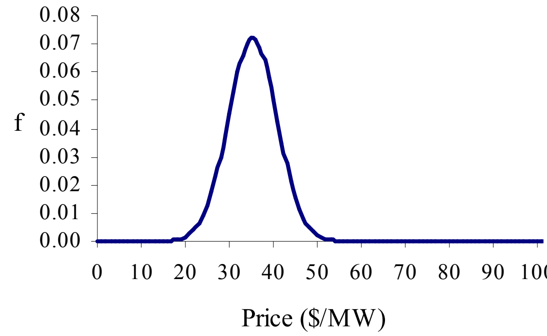
Weekdays/ On-peak Hours/ Fall



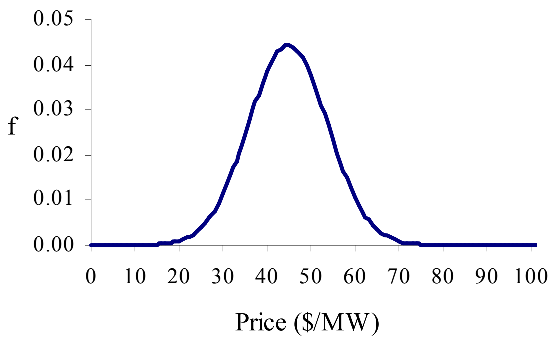
Weekdays/ Off-peak Hours/ Fall



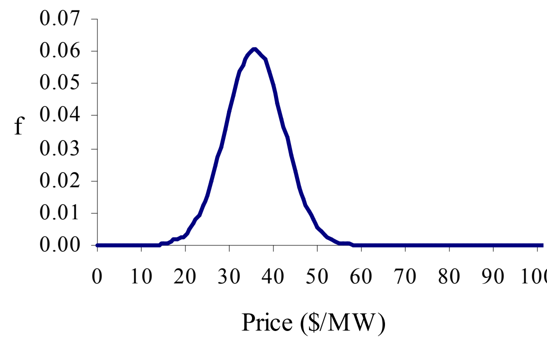
Weekends/ On-Peak Hours/ Winter



Weekends/ Off-Peak Hours/ Winter



Weekends/ On-peak Hours/ Summer



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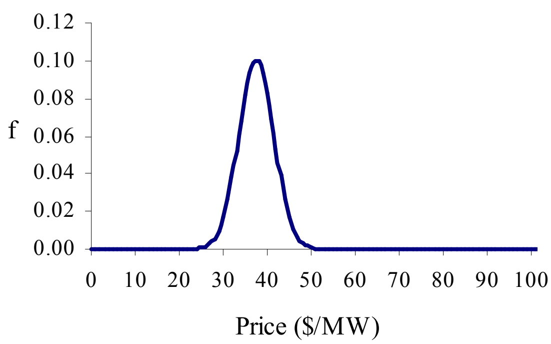
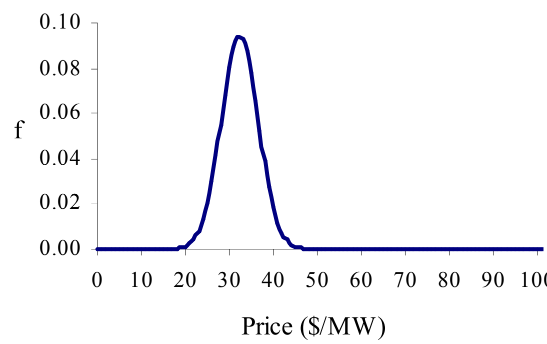
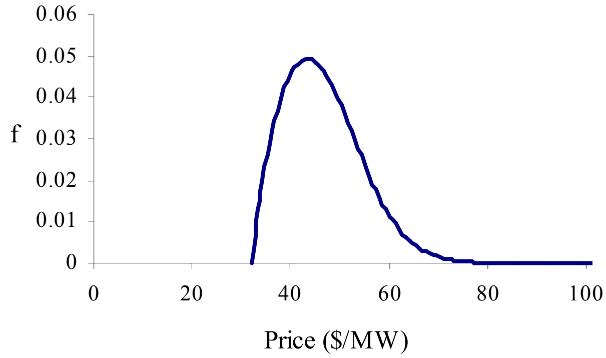


Weekends/ On-peak Hours/ Fall

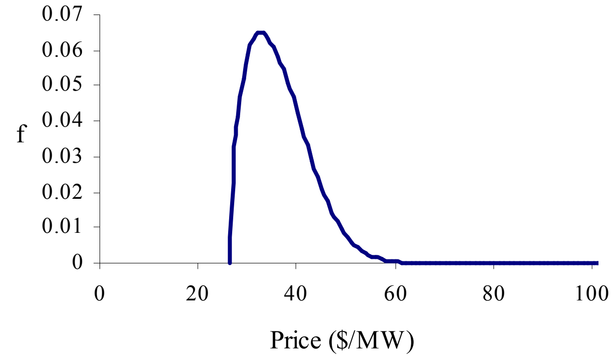


Weekends/ Off-peak Hours/ Fall

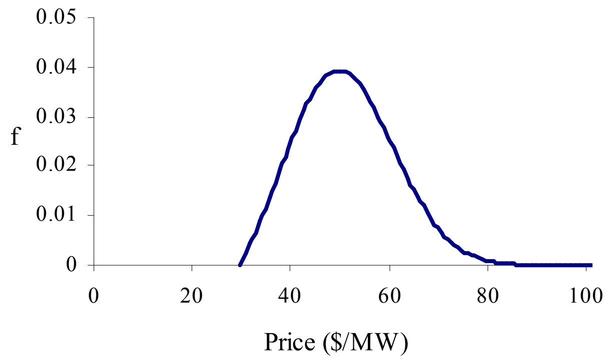
- Weibull Distribution



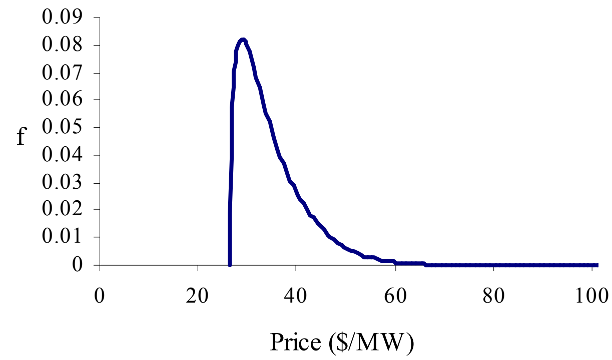
Weekdays/ On-Peak Hours/ Winter



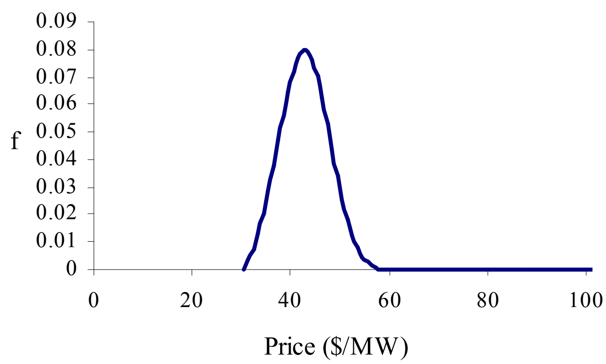
Weekdays/ Off-Peak Hours/ Winter



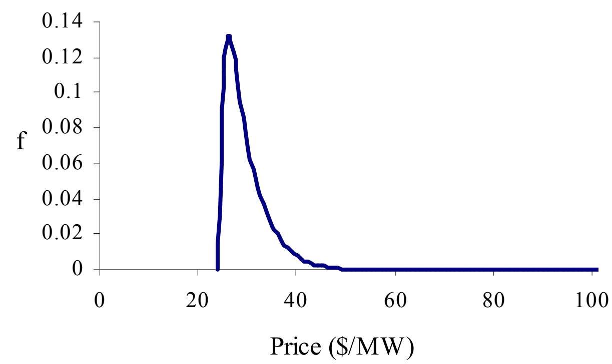
Weekdays/ On-peak Hours/ Summer



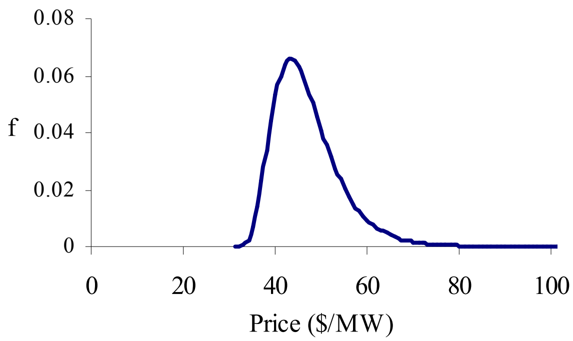
Weekdays/ Off-peak Hours/Summer



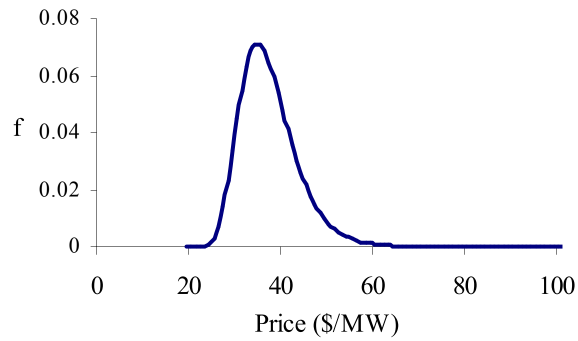
Weekdays/ On-peak Hours/ Fall



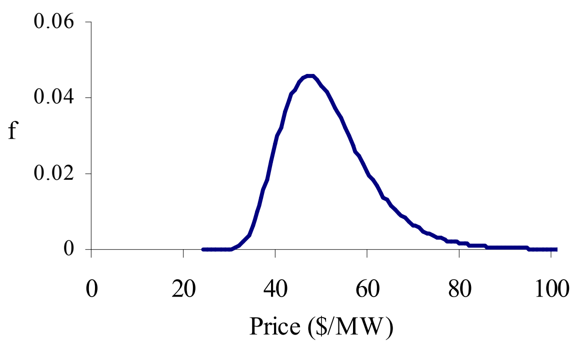
Weekdays/ Off-peak Hours/ Fall



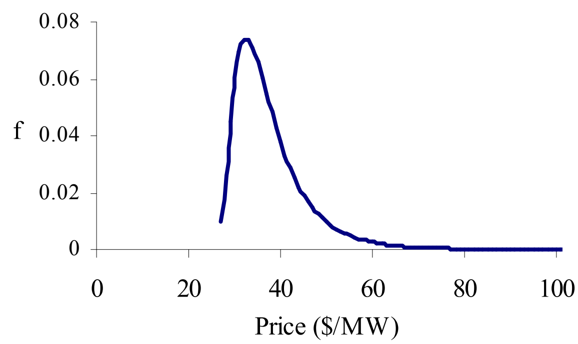
Weekends/ On-Peak Hours/ Winter



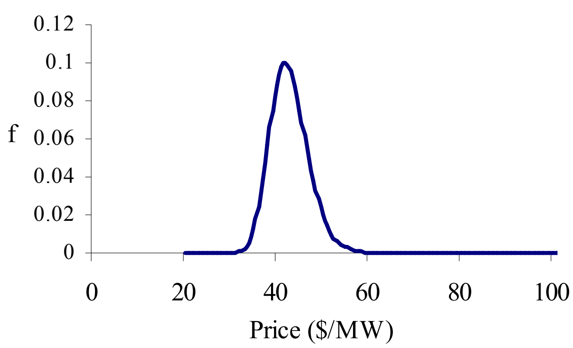
Weekends/ Off-Peak Hours/ Winter



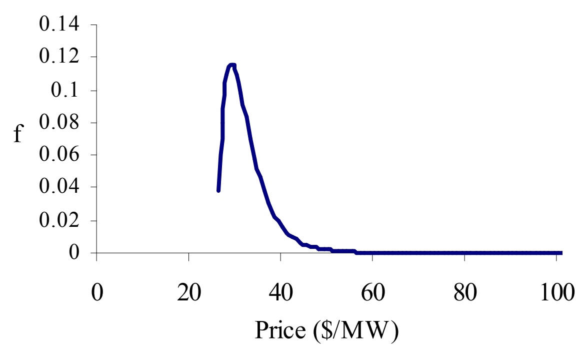
Weekends/ On-peak Hours/ Summer



Weekends/ Off-peak Hours/Summer

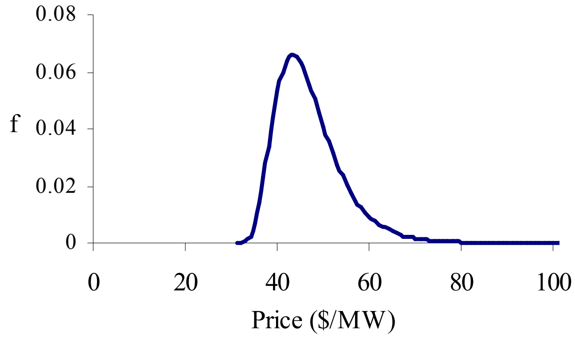


Weekends/ On-peak Hours/ Fall

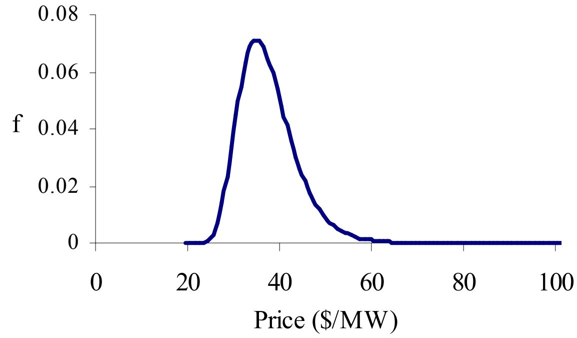


Weekends/ Off-peak Hours/ Fall

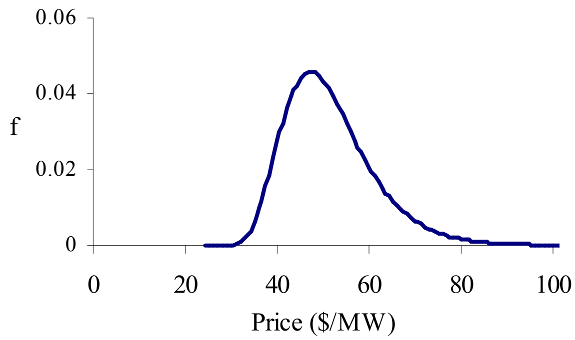
- Lognormal Distribution



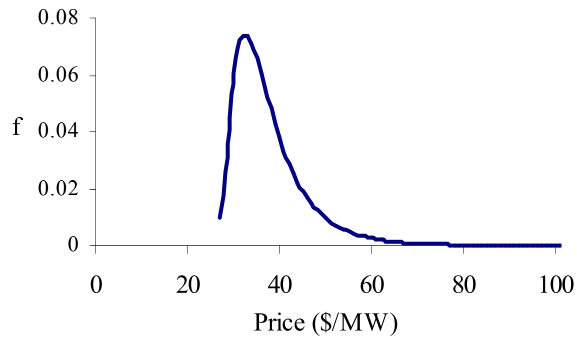
Weekdays/ On-Peak Hours/ Winter



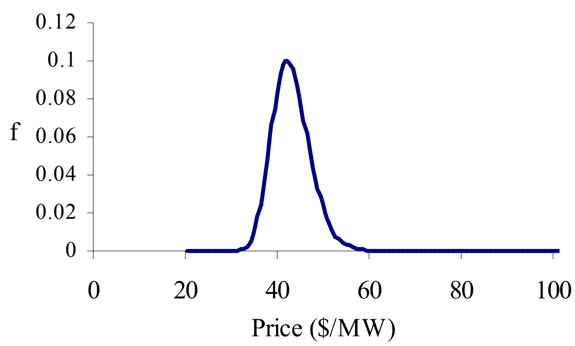
Weekdays/ Off-Peak Hours/ Winter



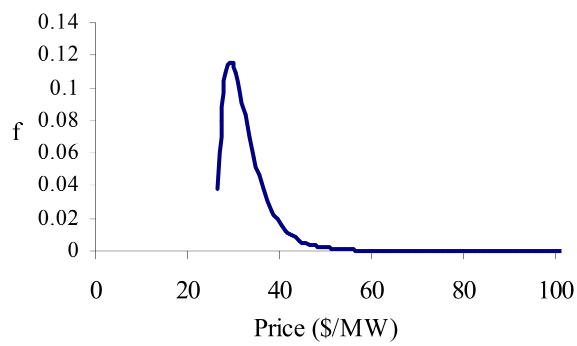
Weekdays/ On-peak Hours/ Summer



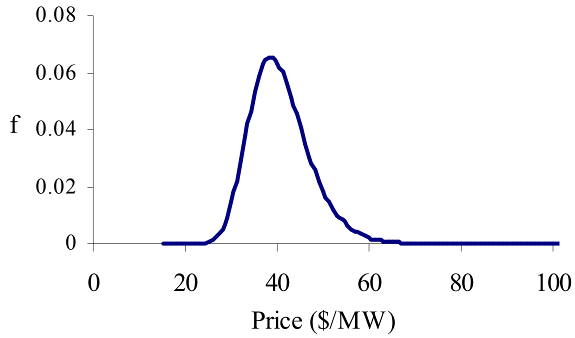
Weekdays/ Off-peak Hours/Summer



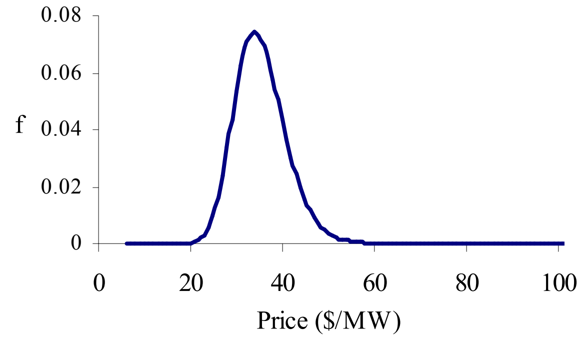
Weekdays/ On-peak Hours/ Fall



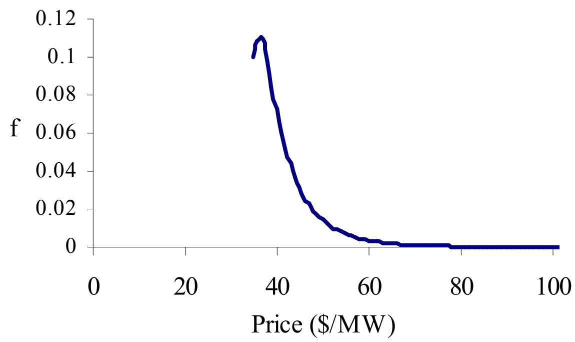
Weekdays/ Off-peak Hours/ Fall



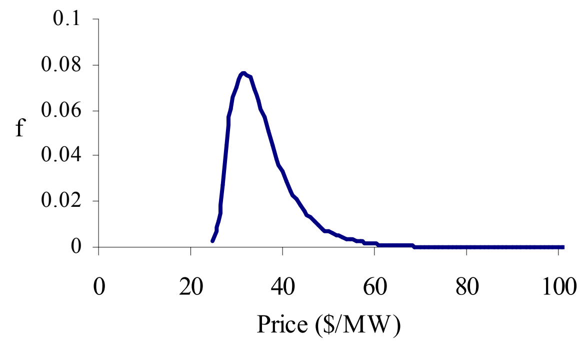
Weekends/ On-Peak Hours/ Winter



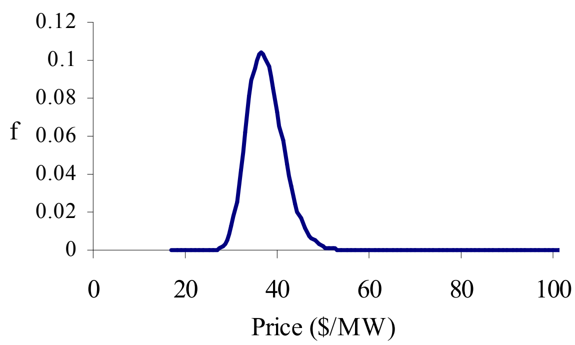
Weekends/ Off-Peak Hours/ Winter



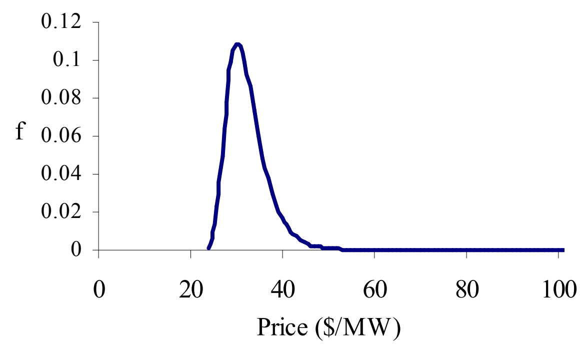
Weekends/ On-peak Hours/ Summer



Weekends/ Off-peak Hours/Summer



Weekends/ On-peak Hours/ Fall



Weekends/ Off-peak Hours/ Fall