Narrative Case Study of Three Secondary Mathematics Teachers that Use Technology Effectively in Classroom Instruction

by

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Abstract

Despite calls for the implementation of technology in today's classrooms to improve the teaching and learning of mathematics, very few teachers implement technology in effective ways, even when they are adequately prepared and supported to use technology in the mathematics classroom. This study sought to answer the question, what inspires some teachers but not others to implement technology despite having similar opportunities? A narrative case study of three teachers who effectively used technology in the mathematics classroom for teaching and learning was conducted, where effective technology use referred to the use of mathematical action technology that requires students to engage in reflection, sense making, and reasoning about the mathematics. Using narrative interviews and classroom observations, the events experienced by these exemplary teachers related to learning to effectively use technology in the secondary mathematics classroom were first identified, then a cross-case analysis was undertaken to identify how the events and experiences in the personal narratives of those teachers influenced their implementation practices of technology in the context of reform mathematics. Several important factors associated with their technology implementation practices in the secondary mathematics classroom were identified across the cases, including leadership roles related to technology use, access to resources and support, and a growth mindset with respect to teaching and technology. Many of these factors were related to their participation in a comprehensive professional development program to improve mathematics teaching and learning, as well as their preservice preparation.

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1. Introduction

Policy documents and recommendations from educational bodies frequently advocate for the use of technology in the mathematics classroom as a tool to foster more robust mathematical understandings in students and to expose students to the power of emerging technology all around us (International Society for Technology in Education [ISTE], 2016; Association of Mathematics Teacher Educator [AMTE], 2015; National Council of Teachers of Mathematics [NCTM], 2014). In much of the research that is available on the impact of technology use in the mathematics classroom on student learning, students show the most gains in learning when technology is used in meaningful and effective ways (Li & Ma, 2010). Effective technology use in the mathematics classroom does not mean using a calculator to complete a worksheet more efficiently, but rather using a graphing calculator or statistical software to investigate, reason about, and make sense of challenging and authentic problems to build robust mathematical understandings (Dick & Hollebrands, 2011).

Despite these calls, very little progress has been made in implementing technology in mathematics classroom in effective ways (Handal, Campbell, Cavanagh, Petocz, & Kelly, 2013; ISTE, 2008). Unfortunately, teachers often implement technology in ineffective ways because they are unaware of how technology should be used effectively in the classroom (Handal, Campbell, Cavanagh, Petocz, & Kelly, 2013). The effective use of technology and related pedagogical methods that have been shown to increase student learning in mathematics are especially important due to the recent decline in mathematics achievement of students across the United States (National Center for Educational Statistics, 2015; Organisation for Economic Cooperation and Development [OECD], 2015). In order for technology use in the classroom to result in an increase in student achievement, students must be given opportunities to use technology tools to model mathematics, make connections among mathematical ideas, and explore concepts behind procedures (Laborde, 2007; Hollebrands, 2007; NCTM, 2014; Peressini & Knuth, 2005).

Researchers have identified the knowledge that is essential for teachers to use technology effectively in the classroom and have also studied the ways in which that knowledge is acquired (Koehler, Mishra, Hershey, & Peruski, 2004; Koehler & Mishra, 2005; Guerrero, 2010; Niess, 2011; Niess, Sadri, & Lee, 2007; Niess et al., 2009; Richardson, 2009). Studies have also shown that teachers who value technology as a teaching and learning tool, receive on-going support, have adequate technology resources, have access to exemplary models of practice, and are given opportunities to engage with technology as a learning tool are more likely to use technology in the classroom (Keengwe, Onchwari, & Wachira, 2008; Olive & Lobato, 2008; Bennison & Goos, 2010).

Statement of the Problem

When given adequate resources, experiences to develop necessary knowledge, and continuing support, very few teachers implement technology in ways consistent with effective use in the classroom (Bennison & Goos, 2010; Ertmer, 2005 Dick & Hollebrands, 2011; Dunham & Hennessy, 2008; Guerrero, Walker, & Dugdale, 2004). Some researchers have suggested that factors such as beliefs about the nature of learning and mathematics may be the overriding reasons for teachers' implementation practices (Ertmer, 2005; Wachira & Keengwe, 2011), but others have found that even teachers who have beliefs aligned with these practices do not necessarily use technology as an effective teaching and learning tool (Wachira & Keengwe, 2011; Smith, Kim, & McIntyre, 2015). Thus, the question remains: what inspires some teachers but not others to implement technology despite having similar opportunities to learn, adequate resources, and sufficient supports? Research has shown that teachers' processes for making pedagogical decisions are highly complex (Okumus, Lewis, Wiebe, & Hollebrands, 2016; Guerrero, 2010) and there is a need to study the issues of knowledge development and technology implementation practices of exemplary teachers in a manner that will allow a deeper analysis of the associated complex elements such as knowledge, experiences, and beliefs.

Purpose of the study

The purpose of this study is to provide a broad and contextual view of how teachers that effectively use technology developed their knowledge related to using technology and also provide a more robust understanding of the set of factors that may have contributed to their technology implementation practices. In her study on teachers' responses to implementing new pedagogical methods, Drake (2006) found that using a narrative research approach allowed for a more robust examination of the complex set of factors involved. Narrative research refers to research based on life story interviews and often observations. Examining these teachers to explore not just their knowledge but also their experiences and beliefs in the contexts of their mathematics, teaching, technology, and teaching mathematics with technology life stories may lead to insights into how and why they came to be effective users of technology in the classroom. Thus, the research questions addressed in this study are as follows:

1. What are the events experienced by teachers related to learning to use technology as a reasoning and sense-making tool in the secondary mathematics classroom?

2. How did the events and experiences in the personal narratives of secondary mathematics teachers influence their implementation practices of technology in the context of reform mathematics?

2. Review of Related Literature

This chapter provides a review of the research and literature that was used to inform this study. It begins with a discussion of reports on current student performance in mathematics and calls for improving mathematics education, followed by evidence of the impact technology can have in the mathematics classroom and ways that technology can be implemented effectively. Following is a discussion of what is known about preparing teachers to use technology in the classroom. Finally, literature that informed the choice of the methodology is described.

Current Student Performance in Mathematics

The 2015 National Assessment of Educational Progress (NAEP) results showed an overall decrease in performances at the fourth, eighth, and twelfth grade levels in mathematics (National Center for Education Statistics [NCES], 2015). In 2015, only 33% of eighth grade students were achieving at or above proficient levels in mathematics (NCES, 2015), and even more alarming, in 2015, only 25% of twelfth graders were considered proficient in mathematics (NCES, 2015). Students in the United States have been performing at mediocre levels in mathematics as compared to their international counterparts and certainly well below the level that is necessary to maintain a leadership position in the world (National Mathematics Advisory Panel, 2008). For example, in the 2015 TIMSS report, eighth graders in the U.S. scored well below their counterparts from countries such as Singapore, Chinese Taipei, Japan, and Russia (Mullis, Martin, Foy, & Hooper, 2016). Results from the 2015 PISA, an international survey of 15 year olds, showed that U.S. students scored below the mean in mathematics (OECD, 2015). Additionally, according to the National Mathematics Advisory Panel's *Final Report* (NMAP, 2008), community colleges and four-year institutions have reported an increased demand for remedial mathematics courses due to insufficient preparation of students for doing college-level mathematics. More recently, a survey by the Community College Resource Center found that 68% of community college students had to take at least one developmental course and most often in mathematics (Jaggars & Stacey, 2014). Finally, achievement gaps among different ethnicities and also significant gaps in mathematical achievement between students living in poverty and students in the middle class have been reported (NCES, 2015; Ball, 2003). For example, NAEP results at the eighth grade level showed that 43% of white students were at or above the proficient level while only 13% of black students and 19% of Hispanic students were at or above the proficient level (NCES, 2015). Table 2.0 displays average mathematics scores for recent years from the NAEP assessment. Although some progress in student performance has been reported in recent years (NCES, 2015), it has not been enough to reduce the achievement gaps between groups, to prepare students for college, or to raise the level of achievement to be consistent with that of the top performing countries. Clearly, mathematics performance across all groups remains at unsatisfactory levels.

Calls for Reform of Mathematics Teaching

In this section, I will discuss educational practices consistent with the reform mathematics movement that have been associated with increasing student performance and lessening achievement gaps. In 2014, *Principles to actions: Ensuring mathematical success for all* (National Council of Teachers of Mathematics [NCTM], 2014), highlighted the importance of teaching mathematics with a focus on developing deeper understandings, mathematical practices, and mathematical habits of mind among students. *Principles to actions* (NCTM, 2014) outlined the necessary actions needed to "realize the potential of educating all students- under any standards or in any educational setting" (p. vii). However, such calls for change are hardly new.

Table 2.0

Performance on National Assessment of Educational Progress (National Center for Education Statistics, 2015)

Grade level	Year	Average scale score	Percent at or above proficient
4	2013	242	42
	2015	240	40
8	2013	285	35
	2015	282	33
12	2013	153	26
	2015	152	25

Early calls for reform. In 1980, the National Council of Teachers of Mathematics stated that "problem solving must be the focus of school mathematics" (NCTM, 1980, "Agenda for Action: Problem Solving," Para. 1) and fully outlined recommended actions for teachers to take toward this goal in their report, *An Agenda for Action*. NCTM (1980) recommended a decade of reforms in mathematics education including increased teacher competency, a challenging curriculum for all students, and improved teacher education programs. They called upon teachers to develop a more professional attitude by seeking continuing education and actively participating in programs that would improve teaching and learning. Recommendations were set concerning basic skills, calculators and computers, content standards, assessment, and increasing mathematics study in schools, and in each area the emphasis was on including problem solving in the instruction and learning. Additionally, problem solving was the underlying theme in the

Basic Skills recommendations as they suggested a decreased emphasis on skill drilling and complicated computations by hand and an increased emphasis on an "expanded concept of basic skills" (NCTM, 1980, "Agenda for Action: Basic Skills," Para. 15) to include skills such as applying what is learned, estimation, and making predictions (NCTM, 1980). Although some viewed this stance as a de-emphasis of basic skills, NCTM (1980) acknowledged both the importance of basic skills fluency and application with understanding by stating that

> there must be an acceptance of the full spectrum of basic skills and recognition that there is a wide variety of such skills beyond the mere computational if we are to design a basic skills component of the curriculum that enhances rather than undermines education (NCTM, 1980, "Agenda for Action: Basic Skills," Para. 1).

The impact of this document was great as many organizations and government commissions began to follow NCTM's lead in recommending major shifts in how and what mathematics should be taught.

In 1983, the report *A Nation at Risk* (National Commission on Excellence in Education [NCEE]) also called for reform in education. Recommendations included requiring more mathematics at the secondary level and providing challenging mathematics curriculum for all students not just for those considered college bound (NCEE, 1983). The recommendations for mathematics included terms such as understand, estimate, apply, and test conjectures. The use of these terms reflected the shift toward the goals of reasoning and understanding in mathematics.

NCTM aggressively continued their efforts to bring about such change with a series of standards publications. The first, *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) articulated the shift more clearly by focusing on communication, connections, and reasoning in addition to problem solving. Throughout the book, there was a clear theme that “the study of mathematics should emphasize reasoning so that students can believe that mathematics makes sense” (NCTM, 1989, p. 29). Teachers and other education professionals were provided with standards for content, but it was the Evaluation Standards that provided the most support for changing practices in the classroom. The Evaluation Standards outlined what teachers should emphasize in order to determine what students could do with mathematics (NCTM, 1989). For example, teachers were urged to move away from using the number of correct answers on a test as the only evidence of student learning and from assigning only procedural exercises to students. Instead, teachers should incorporate multiple assessment methods, including the use of manipulatives and technology in assessments, and develop assessments that require students to apply a broad range of skills (NCTM, 1989).

The next document, *Professional Standards for Teaching Mathematics* (NCTM, 1991), supported the previous set of standards by making clear for teachers, teacher educators, and teacher evaluators the knowledge and skills needed to implement the new goals and how the teachers should be evaluated in light of this different form of instruction. Clear descriptions and example vignettes showed how teachers and students should participate in a classroom that is engaged in worthwhile tasks, productive discourse, and establishing a successful learning environment (NCTM, 1991). In order to help evaluators of teachers to support this new vision of teaching in the mathematics classroom, the Professional Standards also included specific statements about what should be observed. For example, in order to evaluate the teaching of mathematical content including concepts, procedures, and connections, an evaluator should see evidence of the teacher engaging students in tasks and discourse that develop understanding in students (NCTM, 1991). NCTM then released a third document in the standards series, *Assessment Standards for School Mathematics* (1995), to help teachers and education professionals begin to assess students in ways that reflected the new goals of understanding and reasoning.

Reform efforts since 2000. As the reform efforts gained momentum, the evaluation and revision of the standards became necessary to remain current in the ever-changing educational landscape, so NCTM once again produced a valuable resource, this time addressing curriculum, teaching and assessment together in *Principles and standards for school mathematics* (NCTM, 2000). This updated version of the standards provided mathematics educators with an evolved set of Content Standards (Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability) and Process Standards (Problem Solving, Reasoning and Proof, Communication, Connections, and Representation) (NCTM, 2000). For example, the Representation Standard, added from the 1989 Standards, described how mathematical representations throughout the grade levels are essential for developing student understanding, and students must be able to use a variety of representations to reason about and communicate mathematical ideas (NCTM, 2000). Also outlined were the underlying Principles, the important issues and considerations for school mathematics not inherently specific to mathematics that guided the development of the Standards (See Table 2.1). All Content Standards were included at each grade level to promote teaching and learning of an integrated mathematics curriculum while recognizing different levels of emphasis at appropriate grade bands. Figure 2.0 provides a visual representation of this integrated focus. As with the previous standards documents, when developing programs for teacher education or professional development for teachers already in the field to improve the mathematics education of students, *Principles and standards for school mathematics* (NCTM, 2000) highlighted that how the mathematics is taught is as important as what mathematics is taught.

Table 2.1

Principles from Principles and standards for school mathematics (NCTM, 2000, p. 22)

Principle	Description
Equity	Excellence in mathematics education requires high expectations and strong support for all students
Curriculum	It must be coherent, focused on important mathematics, and well articulated across the grades
Teaching	Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well
Learning	Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge
Assessment	Assessment should support the learning of important mathematics and furnish useful information to both teachers and student
Technology	Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning

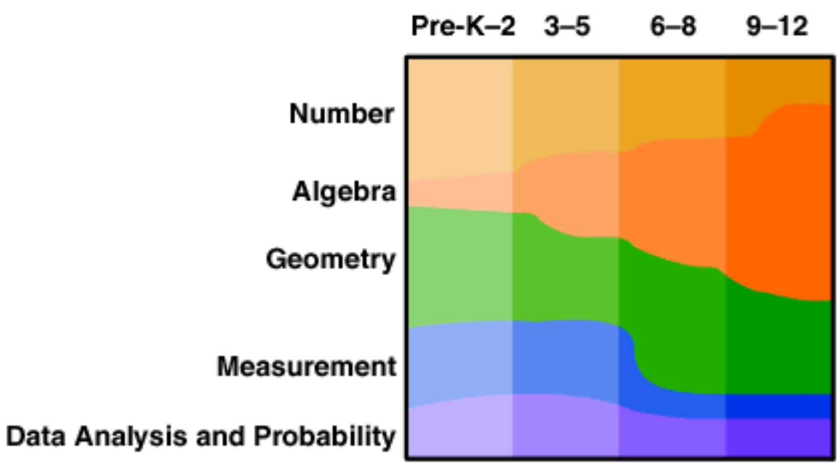


Figure 2.0. Content emphases separated by grade bands. Reprinted from *Principles and standards for school mathematics* by NCTM, 2000, Reston, VA: NCTM. Copyright 2000 by The National Council of Teachers of Mathematics, Inc. Reprinted with permission.

The National Mathematics Advisory Panel, in their *Final Report* (2008), recommended that students need a variety of instructional strategies and the body of research they examined that met their standards for high quality research supported neither a completely teacher-centered nor a completely student-centered approach. Throughout the long list of recommendations that they proposed for improving mathematics proficiency in schools, there was a general tone that a mixed-methods approach to teaching is needed in the classroom today. For example, the recommendation given to prepare students for algebra stated,

> The curriculum must simultaneously develop conceptual understanding, computational fluency, and problem-solving skills. Debates regarding the relative importance of these aspects of mathematical knowledge are misguided. These capabilities are mutually supportive, each facilitating learning of the others. Teachers should emphasize these interrelations; taken together, conceptual understanding of mathematical operations, fluent execution of procedures, and fast access to number combinations jointly support effective and efficient problem solving (p. xix, National Mathematics Advisory Panel, 2008).

Clearly, there has been an abundance of recommendations for what teachers can do in the classroom to improve the learning of mathematics and recommendations for improving mathematics achievement for all students.

In order to engage in the mathematical processes described by the NCTM (2000) Process Standards, reasoning and sense making must be emphasized in the mathematics classroom (NCTM, 2009). In *Focus in high school mathematics: Reasoning and sense making* (NCTM, 2009), the authors argued for significant change in the mathematics classroom through a shift in focus from the traditional classroom most students experience today that emphasizes learning and practicing algorithms to a classroom that emphasizes investigating, justifying, explaining, reasoning, and sense making. The emphasis on reasoning and sense making described included developing reasoning habits by students. Teachers should provide students with opportunities and experiences that can help them to develop mathematical reasoning habits including analyzing a problem, implementing a strategy, seeking and using connections, and reflecting on a solution (NCTM, 2009). For example, when analyzing a problem, students should be able to identify important concepts or connections, make predictions or conjectures, clearly identifying the necessary variables or constraints involved in a problem, and identify patterns or related simplified cases (NCTM, 2009). NCTM (2009) explained that the reasoning habits should be an integrated part of the daily mathematics classroom activities in order to truly become habit, and some strategies are recommended for use in the classroom in order to develop these habits. For example, teachers should encourage discourse that promotes reasoning through effective questioning, providing "think time" or explaining reasoning to others. Reasoning and sense making are described as the purpose and means for mathematics learning (NCTM, 2009). Thus, reasoning and sense making "are the foundation for true mathematical competence" (NCTM, 2009, p. 14), without which students view mathematics as an unrelated and complex set of rules to be memorized. *Focus in high school mathematics* (NCTM, 2009) also provided direction to improve curriculum and instruction for developing these reasoning habits with a vision to create a new norm for mathematics classrooms.

One could consider the *Common Core State Standards in Mathematics* (*CCSSM*) (National Governors Association Center for Best Practices [NGA Center] & Council of State School Officers [CCSSO], 2010) to be the culmination of the efforts over the past several decades to reform the teaching and learning mathematics in that it required teachers to help students to develop mathematical practices of reasoning and sense making in order to be in compliance with teaching all standards. The majority of the states have adopted the Common Core State Standards in Mathematics (NGA Center & CCSSO, n.d.), and common assessments that test students in the same way that they are expected to be taught have been developed. The Mathematical Practice Standards, skills that teachers are expected to guide students in developing, are

- make sense of problems and persevere in solving them;
- reason abstractly and quantitatively;
- construct viable arguments and critique the reasoning of others;
- model the mathematics;
- use appropriate tools strategically;
- attend to precision;
- look for and make use of structure;
- look for and express regularity in repeated reasoning (NGA Center & CCSSO, 2010, p. 6-8)

Teachers are expected to provide opportunities for students to develop proficiency with mathematical practices in tandem with developing understanding of content. Thus, teachers have little choice but to strive to improve their quality of teaching, incorporate new strategies, and provide students with opportunities to engage in the mathematical practices if they are to be in compliance with the standards required in the CCSSM (NGA Center & CCSSO, 2010).

The Mathematical Practice Standards are a culmination of several documents that identified important mathematics skills that students need. The Mathematical Practices were developed as a synthesis of the NCTM's Process Standards (2000) and the Mathematical

Proficiency Strands presented in *Adding It Up* (Kilpatrick, Swafford, & Findell, 2001), and they mirror the goal of developing reasoning and sense making habits of mind in students described in *Focus in high school mathematics* (NCTM, 2009) In *Adding it up: Helping children learn mathematics*, a report produced by the National Research Council (Kilpatrick, Swafford, & Findell, 2001), the educational professionals that formed the Committee on Mathematics Learning described mathematical proficiency as a function of five interconnected strands; conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. These strands required that students understand mathematical concepts and connections, competently carry out computations and procedures, create and solve problems, reflect and justify, and perceive mathematics as worthwhile and possible in order to be considered mathematically proficient (Kilpatrick, Swafford, & Findell, 2001). However, the most important characteristic of the five strands is that they are "interwoven and interdependent" (Kilpatrick, Swafford, & Findell, 2001, p. 5). This report was aimed at improving the mathematics proficiency of K-8 students and stressed the importance of creating a solid foundation of mathematical understanding. The RAND Mathematics Study Panel's report *Mathematical proficiency for all students* (Ball, 2003) also used the term Mathematical Practices to describe the actions that successful mathematicians take.

Most recently*, Principles to actions: Ensuring mathematical success for all* (NCTM, 2014) provided teachers, teacher educators, and other stakeholders with an updated version of the Principles first reported in *Principles and standards for school mathematics* (NCTM, 2000) as well as specific practices at grades K-12 aimed at helping move all students toward success in mathematics. In the discussion of the Teaching and Learning Principle, NCTM identified eight "high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics" (NCTM, 2014, p. 9). The eight Mathematics Teaching Practices are displayed in table 2.2. For example, teachers should help students to develop a strong conceptual understanding from which to build procedures skills (NCTM, 2014). In the classroom that facilitates this conceptual to procedural development, teachers should be encouraging students to create and share their own reasoning when solving problems, elicit explanations for procedures used, and help students to connect their invented strategies to established procedures when useful or appropriate.

Table 2.2

Mathematics Teaching Practices from Principles to Actions, (NCTM, 2014), p. 10.

Practice
Establish mathematics goals to focus learning.
Implement tasks that promote reasoning and problem solving.
Use and connect mathematical representations.
Facilitate meaningful mathematical discourse.
Pose purposeful questions.
Build procedural fluency from conceptual understanding.
Support productive struggle in learning mathematics.
Elicit and use evidence of student thinking.

Additionally, *Principles to actions* (NCTM, 2014) included other principles that form a set of essential features of mathematics programs that support and facilitate effective teaching and learning. Schools and systems should focus on providing equity in and access to quality mathematics instruction, a strong and meaningful mathematics curriculum, appropriate tools including technology, varied assessments that are used to inform instructional decisions, and promoting professionalism that makes each teacher accountable for the success of all students. Particularly helpful in illustrating the shift in mindset needed to accomplish the goals set forth in *Principles to actions* (NCTM, 2014) are the examples of unproductive versus productive beliefs that influence instructional practices related to each of the essential features. For example, the belief that all students can achieve when provided identical learning opportunities is an unproductive belief in that it does not acknowledge the specialized instruction or supports that some students need in order to be successful. Rather, teachers are encouraged to move from an equality stance toward an equity stance so that students receive the differentiated supports needed. This focus is on what teachers can and should do in the mathematics classroom to improve teaching and learning.

Table 2.3

Summary of Influential Documents in the Reform Mathematics Movement

Author, Document	Year	Contribution Summary
NCTM, *An agenda for action*	1980	Advocated for a focus on problem solving and an expanded definition of basic skills to include reasoning skills such as estimation
NCEE, *A nation at risk*	1983	Recommended mathematics education reform including more mathematics at the secondary level and more challenging curriculum
NCTM, *Curriculum and evaluation standards for school mathematics*	1989	Established standards by gradeband; communication, connections, and reasoning in addition to problem solving should be incorporated in classrooms
NCTM, *Professional standards for teaching mathematics*	1991	Recommended support for teachers for improving mathematics education (engaging students in worthwhile tasks, facilitating discourse, and a successful learning environment)
NCTM, *Assessment standards for school mathematics*	1995	Described how assessments should be aligned to support and enhance mathematics learning.

NCTM, *Principles and standards for school mathematics*	2000	Described principles, process standards, and content standards to improve mathematics teaching and learning.
Kilpatrick, Swafford, & Findell; *Adding it up: Helping children learn mathematics*	2001	Described mathematical proficiency: five interconnected strands; conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition
Ball, *Mathematical proficiency for all s*tudents	2003	Emphasized the importance of students engaging in mathematical practices
NCTM, *Focus in high school mathematics: Reasoning and sense making*	2009	Defined categories of reasoning habits, emphasized reasoning and sense making as essential to high school mathematics
NGAC & CCSSO, *Common core state standards for mathe*matics	2010	Established a common curriculum to be used by most states and mathematical practices to engage all students
NCTM, *Principles to actions: Ensuring mathematical success for all*	2014	Described actions to be taken by schools and teachers to ensure mathematical learning for all students

Technology in the Mathematics Classroom

Technology implementation has been included as a recommendation in many of the standards and policy documents discussed in the previous section as an essential component of helping students to develop mathematical fluency and understanding. The inclusion of technology is emphasized by the CCSSM’s (2010) Mathematical Practice Standard that requires students to “use appropriate tools strategically” (NGA Center & CSSO, 2010, p. 7). Technologies such as calculators, spreadsheets, and geometry software are listed alongside tools found in more traditional classrooms such as protractors or pencil and paper. In other words, digital technology is no longer separated from non-technology tools in the mathematics classroom, but rather technology is referred to as a resource one should expect to find in today’s classroom just as you would expect to see a student using a ruler. The focus on technology can be credited to the various influential documents and recommendations by several governing bodies and organizations in an effort to improve student learning outcomes (NCTM 1980, 1989, 1991, 1995, 2000; NGA Center & CCSSO, 2010; Public Law 107-110, 2001) as well as the infusion of technology into our everyday world.

The emergence of technology in mathematics education. One of the earliest reports to highlight the importance of computers and calculators, *An agenda for action* (NCTM, 1980), included technology as one of eight recommendations aimed at improving mathematics education. In this report, NCTM (1980) explained the role that schools should play to promote computer literacy and instruction of mathematics that included technology-enhanced problem-solving opportunities for all students as being essential in producing technologically capable citizens in the 1980's. The report also called for professional development for teachers, access to technology for all students, development of curriculum materials to support and extend technology use, computer science courses at the secondary level, technology focus in teacher preparation programs, and the addition of technology literacy in teacher certification requirements (NCTM, 1980). Following the recommendations of *An agenda for action* (NCTM, 1980), the report, *A nation at risk*, (NCEE, 1983) indicated that the lack of technology education in American schools put the country at even greater risk of continuing to fall behind other countries. The report detailed how technology advancements were rapidly increasing the need for highly skilled workers. NCEE (1983) indicated that computers, laser technology, robotics, and other forms of developing technology "are penetrating every aspect of our lives" (p. 12).

NCTM continued their efforts to influence the reform of mathematics education within the sets of standards that clearly identified technology as an essential part of this reform. In *Curriculum and evaluation standards for school mathematics* (NCTM, 1989) the authors explained that the use of graphing calculators and computers for “all students at all times” (p. 124) is considered an underlying assumption for the standards pertaining to the 9-12 curriculum. Technology is acknowledged not only for its role as a tool but also as an influence on the type of mathematics that is important to be learned. Additionally, technology was touted as a tool to enhance the mathematics learned but not to replace mathematical literacy. The NCTM (1989) document made clear that no evidence has been found to support the concerns of many teachers that students will regress in basic skills or become dependent on them, and instead they stressed the importance of helping students to develop the skills to decide when to use the technology to enhance learning and what is the most appropriate tool. This judicious use of technology is reiterated in the *Common core state standards for mathematics* (NGA Center & CSSO, 2010) as one of the Mathematical Practices in which students are expected to develop knowledge and proficiency with various forms of technology so that they can judge the usefulness and reasonableness of answers given by a tool when solving a problem.

In *Professional standards for teaching mathematics* (NCTM, 1991), knowledge of technology tools and helping students to use the tools for investigative purposes is considered an essential characteristic of a teacher that can foster the development of mathematical power in students. Mathematical power refers to “the ability to explore, conjecture, and reason logically; to solve non-routine problems; to communicate about and through mathematics; and to connect ideas within mathematics and between mathematics and other intellectual activity” (NCTM, 1991, p. 1). Technology is described as a tool to promote mathematical discourse as it allows students to create and analyze multiple representations. Thus, the incorporation of technology in the mathematics classroom has been presented as an essential resource for teachers in order to help students engage in the mathematical practices.

In 2000, technology was recognized as an essential characteristic of high quality mathematics education in *Principles and standards for school mathematics* (NCTM, 2000). The Technology Principle included three major claims about technology in mathematics. First, the level of mathematical learning is greatly increased by technology. For example, advanced computations previously inaccessible can be simplified by the use of technology. Particularly in situations where the computation is not the main objective of the lesson, this is important as it allows students to focus on the concepts, connections, and conceptualizing of the problem at hand. Also, multiple representations provided by technology tools allow students to analyze and focus on concepts in addition to procedural understanding. Next, technology can be used to support teachers to teach effectively. Technology allows teachers to create the situations and problem-solving opportunities for students that can make the mathematics they learn meaningful and the learning itself more effective, and multiple representations allow more of a variety of learners to access the mathematical concepts. Thirdly, the authors expanded on the previous statement in the 1989 NCTM Standards that the mathematics taught in the classroom today is directly influenced by the technology available and the technology skills needed to become effective problem solvers. They explained how previously inaccessible content was now possible due to tools that aid in difficult computations, organize large amounts of data for analysis, and provide opportunities for frequent experimentation. In a summary of research findings on teaching and learning with technology, Laborde (2007) also indicated that technology must be highly incorporated into schools in order to prepare students for a world so immersed in technology. However, she indicated that technology should not be incorporated solely for the sake of incorporating technology but, as is consistent with the outside world, should be used frequently in meaningful ways.

These documents, among others, contributed to state and federal requirements and recommendations of technology education and the incorporation of technology in mathematics education. Public Law 107-110, also known as No Child Left Behind Act of 2002 (No Child Left Behind [NCLB], 2002), specifically outlined several technology goals that public schools throughout the United States were required to work toward. The goals of this legislation required schools to use technology to enhance student achievement, provide access for all students to essential technology in order to ensure technology literacy, and focus on research-based teacher training and resource procurement that will contribute to the effective use of technology in classrooms. NCTM's standards documents and NCLB (2002) helped bring about the current focus on technology in mathematics education and also detailed the importance of improving teacher training in order for technology to be used effectively.

Recent calls for technology in mathematics teaching and learning. *Principles to actions* (NCTM, 2014) identified tools such as technology as essential to effective teaching and learning of mathematics. The guiding principle states,

> An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking. Available tools and technology help teachers and students visualize and concretize mathematics abstractions, and when these resources are used appropriately, they support effective teaching and meaningful learning (NCTM, 2014, p. 4).

Both NCTM and the Association of Mathematics Teacher Educators (AMTE) have put out position statements advocating for meaningful and authentic implementation of technology and the sufficient preparation of teachers to use technology in effective ways (NCTM, 2011; AMTE, 2015).

The ISTE (2017) Technology Standards for Teachers also provided a framework for educators in general to use technology in the classroom. For example, recommendations included collaborating with colleagues to improve classroom practice, develop activities to engage students in learning, and to mentor and model responsible digital citizenship for students. ISTE (2016) also published a set of technology standards for students that identified seven roles that students should engage in when using and learning with technology. The roles included empowered learner, digital citizen, knowledge constructor, innovative designer, computational thinker, creative communicator, and global collaborator.

Technology tools for the mathematics classroom. This section addresses the various ways technologies are being used in the mathematics classroom including the effective use of technology that is the focus of this study. In *Focus in high school mathematics: Technology to support reasoning and sense making*, Dick and Hollebrands (2011) introduced two broad categories of technology that can be found in the mathematics classroom. One categorization was "conveyance technologies" (Dick & Hollebrands, 2011, p. xii), aptly named for their ability to convey information. This would include presentation technologies such as interactive whiteboards, presentation software, and projection devices such as projectors and document cameras. Other technologies that are included in the conveyance category are communication technologies such as the Internet, collaboration technologies, and monitoring technologies such as computer monitoring software and clicker devices. Another common use for technology in the classroom is for evaluating students. Computer-based assessments, clicker tools to record student responses, and interactive websites are just a few examples of how teachers can assess student progress and then use that information to improve their teaching or provide individual feedback to students. Intervention and skill practice software programs also are a type of assessment method. These software programs use the computer to convey the mathematics concepts to students, mostly serving as an alternate medium to books or teacher instruction and do not necessary act in mathematical ways in response to student input. While these types of technology can facilitate classroom tasks that provide reasoning and sense-making opportunities in the mathematics classroom, they “do not intrinsically provide any mathematical substance for sense making and reasoning” (Dick & Hollebrands, 2011, p. xii). Instead, it is the mathematical tasks and the facilitation of the tasks by the teacher that provides the rich mathematical opportunities for learning and the technologies are secondary, or often unnecessary, to the process.

The second type of technology described by Dick and Hollebrands (2011) is referred to as “mathematical action technology” (Dick & Hollebrands, 2011, p. xii). This type of technology refers to those that are able to function and respond to the user mathematically. Common mathematical action technologies (MAT) include graphing calculators and software, computer algebra systems, spreadsheet software, and dynamic geometry software. Laborde (2007) also distinguished technologies in general from mathematically-specific technologies. She indicated that these technologies offered mathematical models and further allowed users "the possibility of 'seeing' mathematical objects behaving mathematically" (Laborde, 2007, p. 5). Such MAT allows students and teachers to focus on experimentation and understanding mathematical properties because of the mathematical knowledge inherent in the technology, and the vast affordances (features that allow experimenting such as dragging a point in a dynamic geometry environment) available when computing, graphing, or modeling. *Principles to actions*

(NCTM, 2014) also identified MAT as valuable tools for the mathematics classroom because they "provide opportunities for students to interact with mathematical ideas" (p. 84). Further, the affordances of MAT expand the content that may be taught and the pedagogical skills that can be used (NCTM, 2014).

There are various types of mathematical action technologies; included in this category are computational tools, dynamic geometry software, microworlds such as virtual manipulatives, and computer simulations (Dick & Hollebrands, 2011). Computational tools include spreadsheets, various calculators, and computer algebra systems (CAS). Dynamic geometry software or dynamic geometry environment (DGE) refers to technology that incorporates the traditional construction tools in an environment where objects can be manipulated. The various mathematical action technologies have unique and shared characteristics that can be utilized to expand mathematical understanding and encourage students to interact with mathematical concepts (Dick & Hollebrands, 2011). The following sections discuss several MATs.

Dynamic geometry environments. Dynamic geometry environments (DGE) are effective tools for encouraging conjecturing, exploring, and reasoning by students (Hadas, Hershkowitz, & Schwarz, 2000; Ubus, Ustun, & Erbas, 2009). DGEs (such as Geometer's Sketchpad, Cabri, TI-Nspire, and Geogebra) include tools to construct, draw, measure, and manipulate geometric objects such as points, lines, rays, and polygons. The DGE allow students to control and reason about the behavior of geometric objects, utilize the constraints or properties of objects to make constructions, find patterns and properties and test them to determine their generalizability, and test and refine conjectures (Hollebrands & Dick, 2011). They have many advantages to more traditional methods of exploring geometric objects such as manipulations not possible with a ruler and compass (i.e. objects retain properties related to the construction when manipulated), efficient and precise revisions and measures, and highlighting relationships not noticeable in a static environment (Hollebrands & Dick, 2011). For example, points on geometric objects constructed in Geogebra can be dragged or moved to different positions to allow students to observe and conjecture about the general case.

Graphing calculators. Due to the affordability and inclusion in state standards and standardized exams, graphing calculators are often reported to be the most frequently used form of technology in the secondary classroom (Dunham & Hennessy, 2008). Graphing calculators build on the computational assistance offered by basic calculators and include features that allow numeric, graphic, and symbolic representations. Students can use the different representations to identify and understand how changes in each representation impact the other two. For example, students can explore how changing the coefficients in a symbolically entered function will change the graphical representation of the function. Graphing calculators can be used to collect data and perform statistical analyses. They also include programming capabilities and a variety of applications that can be downloaded for use. Graphing calculator applications such as Desmos can be downloaded on smartphones and tablets.

Computer algebra systems. Arguably the most rapidly developing technology tool currently being used in some secondary mathematics classrooms is the computer algebra system (CAS). Computer algebra systems (CAS) include traditional computer-based systems such as Maple and Mathematica, as well as handheld versions such as the TI-Nspire and CAS smartphone applications. WolframAlpha is a publicly available CAS that can access through its website or downloaded as a smartphone application. The handheld availability and relatively inexpensive applications available for smartphones could explain the recent increase in popularity of computer algebra systems despite having been available in less mobile forms since the 1980s. Computer algebra systems (CAS) are different from other computational tools because they can perform symbolic manipulations as opposed to numerical manipulations only. As early as 1988, researchers have identified CAS as powerful tools for allowing an increased focus on conceptual understanding, problem solving and investigations due to the increased computational efficiency and the ability to do more complex computations with CAS (Heid, 1988). Although many have argued that graphing calculators in addition to CAS have the potential for diminishing the computational skills of students, Ball and Stacey (2005) found that good teaching and judicious use of CAS could contribute to developing the mathematical power of students.

Effective Technology Use in the Context of Reform Mathematics

In order to for the technology described in the previous section to have a positive impact on student achievement, it must be used in effective ways. MAT allows teachers and students to use technology as the means to model, investigate and create mathematical situations that could not be accomplished equivalently without the technology. Technology is no longer just an organized way to display and disseminate mathematical concepts, but many teachers are unaware of the most effective ways to use MATs (Handal, Campbell, Cavanagh, Petocz, & Kelly, 2013). Technology is often incorrectly labeled as a hindrance to the development of mathematical skills and understandings because it is frequently used in ineffective ways or at inappropriate times (e.g. when pencil and paper strategies are more efficient). According to NCTM (2014), "the value of the technology depends on whether students actually engage with specific technologies or tools in ways that promote mathematical reasoning and sense making" (p. 80). NCTM (2014) stated that technology can provide students opportunities to develop deeper mathematical understandings due to the multiple representations of concepts available through technology. Teachers must recognize ways to utilize technology tools effectively, such as:

> help[ing] students connect their observations from exploration with understanding of the mathematics behind the situation. Students and teachers need to understand both the power and limitations of tools and technology, acknowledging the need to ensure that answers are considered both for their reasonableness and for their applicability to the context in which the manipulation or computation took place (NCTM, 2014, p. 84).

Tools such as technology are necessary for helping students to increase their mathematical understandings by engaging in reasoning and sense making (NCTM, 2014).

Peressini and Knuth (2005) claimed that improving student understanding of mathematical concepts is the most important role for technology "from the perspective of school mathematics reform" (p. 280). According to Peressini and Knuth (2005), effective technology use promoted student understanding by providing representations of the mathematics that specifically support conceptual understanding. Laborde (2007) stated that technological environments could facilitate student learning but would not inherently do so. There are three conditions that she found across the studies that she reviewed that determined the effectiveness of technology on learning including the choice of technology tool, the mathematical task presented, and the ways in which the teacher facilitated the task (Laborde, 2007). The perception that technology should simplify or reduce what is learned is not consistent to the ways in which technology has been found to "contribute to enriching or totally renewing meanings and understandings students can construct in carefully designed situations" (Laborde, 2007, p. 2).

Student use of technology to support mathematics learning. Students should be taught ways to use technology that promote effective use. Hollebrands (2007) conducted interviews with students about their mathematical knowledge and also observed tasks being completed. In order to gain a better understanding of the mathematical interpretations the students had, she also gathered sample student work and analyzed transcripts from recorded lessons. From Hollebrands' (2007) analysis, two major strategies that students employed while working with the dynamic geometry software emerged that could also be used to distinguish effective from non-effective use of technology. The most commonly employed strategy was referred to as reactive. The students' decisions were reactive in the sense that the investigative decisions were based on reacting to what the technology told them about the mathematics in order to investigate a problem. The second type of strategy was proactive, meaning the students considered how the technology would react and made a plan of action according to those assumptions. According to Hollebrands (2007), students who used reactive strategies did so because they were relying on the visual cues given to them from the computer and lacked a general understanding of the mathematical concepts. On the other hand, “students who are using the tool proactively might have certain expectations of what they want to do with the technology, determine what actions will achieve their desired result, and then perform the actions and reflect on the results that appear on the screen" (Hollebrands, 2007, p. 21). Hollebrands (2007) noticed that how and when each strategy was employed was a reflection of the student's mathematical and technological understanding. According to Hollebrands’ (2007) findings, effective technology use requires students to use their mathematical understandings and knowledge of the technology affordances to explore mathematics in purposeful ways that include reflecting on results, predicting outcomes of technological actions, and modeling mathematical concepts.

In an analysis of what is known about technology-based learning, Galbraith (2006) identified four levels of interaction with technology that represent increasingly effective uses among mathematics students. They are technology as master, technology as servant, technology as partner, and technology as an extension of self, where master is the least effective level of interaction and extension of self is the greatest level of interaction (Galbraith, 2006). When a student is unable to judge the reasonableness of the output or unable to utilize the complex affordances when using technology, then they are working at the technology as master level. If students simply use calculators to replace computations done mentally or on pencil and paper, then they are using the tool as a servant. Many who do not support the use of technology as a tool that enhances student understanding often refer only to this level of interaction (Dick & Hollebrands, 2011) and label technology as a crutch for students that have not developed proficient basic skills. The task servant role of technology is similar to Hollebrands (2007) description of using reactive strategies in the dynamic geometry environment because in both cases there is a lack of reflection and reasoning about the mathematics and technology relationship by the students. However, the effective use of technology calls for students to interact with technology on levels above that of the task master or task servant.

When students better understand how to use technology to solve problems they "appear to interact directly with the technology (e.g. graphical calculator), treating it almost as a human partner that responds to their commands" (Galbraith, 2006, p. 10). They use the calculator or other technology as a useful tool for testing ideas, facilitating discussions, and validating their mathematical discoveries. Essentially, the technology is treated as a partner with whom they coordinate knowledge to solve a problem. At the most effective level of interaction, students see

> technology as an extension of self. This is the highest level of functioning, in which users incorporate technological expertise as an integral part of their mathematical repertoire. The partnership between student and technology merges to a single identity,

> so that rather than existing as a third party, technology is used to support mathematical argumentation as naturally as intellectual resources (Galbraith, 2006, p. 10).

However, students should be encouraged to consider the most appropriate tool for a task rather than relying on a single tool at all times (NCTM, 2014). The concept that technology should be interwoven within the mathematics curriculum means that it is a seamless part of the classroom activities. Keengwe, Onchwari, and Wachira (2008) also reported on the concept of forming a collaborative relationship between students and computers, and they stated that this level of use of technology is essential for meaningful learning to occur.

Dick and Hollebrands (2011) identified four important skills that students must develop to manage and make appropriate decisions about using technology in order to interact with the technology at the highest levels. First, students should seek to understand a problem as a whole before starting to solve, and then identify the most appropriate technology and how it is suited to help them reach their goals. Next, students should purposefully employ a strategy, including assessing and revising the strategy when necessary. Students also need to be taught to use connections found among different representations and concepts. Finally, students should become reflective users of technology. They need to learn strategies for determining reasonableness, limitations, errors, comparing approaches, and to connect and apply mathematical results into the problem's context (Dick & Hollebrands, 2011).

Pedagogical strategies for effective technology use. Hollebrands (2007) identified several ways to help students shift toward using a more proactive approach when using technology. First, teachers should select mathematical tasks that help students to focus on the mathematical relationships beyond the specific visual characteristics of a single representation. Next, teachers should provide students with opportunities to collaborate, explore, and discuss activities that require students to start using technology in a proactive manner, such as activities in which students determine what technology input is needed to result in a particular outcome. Zengin and Tatar (2017) also found that students benefitted from technology when they were able to collaborate. In their study of impact on student learning with lessons utilizing dynamic geometry environment software, Zengin and Tatar (2017) found that the software used in the context of a cooperative learning environment had a positive effect on student achievement. Finally, students should be engaged in tasks that help them make connections among their new and learned mathematical understandings (Hollebrands, 2007).

In the introduction to *Focus in high school mathematics*, Dick and Hollebrands (2011) provided a definition for teachers and teacher educators of the type of task or learning opportunity that employs the effective use of technology as "scenarios [that] should allow students to take deliberate, purposeful, and mathematically meaningful actions and provide immediate, perceptible (usually visual), and mathematically meaningful consequences to those actions" (p. xiv). In addition to tool and content aspects, implementation issues must also be considered when identifying effective uses of technology in mathematics.

In Mistretta's (2005) report that described a teacher education course that focused on teaching pre-service teachers how to use technology in the mathematics classroom, a checklist was presented that was used to develop effective technology-based lessons. This checklist (Appendix A) resulted from discussions with the pre-service teachers and was adapted from Roblyer (2003) as cited by Mistretta (2005). Mistretta (2005) integrated technology training units into a required pre-service course and reported on the confidence levels and reactions of 70 pre-service teachers that participated. The pre-service teachers explored and interacted with multiple K-12 mathematics instructional software and websites. They collaborated on lesson planning, exploring activities, and evaluating the effectiveness of the lessons. The evaluation and lesson plan criteria provided a clear list of focal points for identifying effective technology-infused lessons.

The checklist found in Appendix A includes many of the common characteristics that researchers have found as essential to using technology effectively such as a focus on problem solving, the seamless integration of technology and curriculum, and the natural integration of technology tools into the learning process. Several of the characteristics reported here in individual studies are commonly found in other reports that seek to define the most effective uses of technology. For example, many researchers stated that incorporating technology in meaningful ways should allow a teacher to gain insight into the mathematical thinking of students, provide models of mathematical concepts, provide opportunities to solve problems otherwise inaccessible, and provide opportunities for students at multiple levels to make sense of the mathematics (Dick & Hollebrands, 2011). Ultimately, these characteristics center around the goal of helping students learn to reason and make sense of the mathematics through the use of technology. In addition to the characteristics discussed above, incorporating technology in meaningful ways should allow a teacher to gain insight into the mathematical thinking of students, provide models of mathematical concepts, provide opportunities to present problems otherwise inaccessible, and provide opportunities for students at multiple levels to make sense of the mathematics (Dick & Hollebrands, 2011).

A definition of effective technology use. Effective technology use is defined for the purpose of this study as the use of mathematical action technology that requires students to engage in "reflection, sense making, and reasoning" (Dick & Hollebrands, 2011, p. xvii) about the mathematics. This definition has two important components. First, it incorporates only mathematical action technologies and deliberately excludes the use of conveyance technologies. The exclusion of conveyance technologies is not intended to suggest that conveyance technologies cannot promote learning in the mathematics classroom; rather it is a reflection of the claim that it is primarily the facilitation of and the structure of the task itself that makes the conveyance technology valuable in promoting reasoning and sense making (Dick & Hollebrands, 2011). Second, this definition requires that students engage in reasoning and sense making as a result of the technology being used. Certainly, there are less cognitively demanding uses for mathematical action technology such as computational assistance (i.e. using a graphing calculator to complete a procedural worksheet on graphing linear equations); however, these uses do not necessarily promote student understanding and some have reported that they could actually hinder mathematics learning in some cases (NCTM, 2014). NCTM (2014) explained that "these unproductive uses of tools and technology limit students' opportunities to reason with and about mathematics" (p. 81).

Summary. Table 2.4 shows a summary of the many characteristics that contributed to this definition for effective technology use. It is not necessary for every characteristic to be present and the list in table 2.4 does not include all possible characteristics of effective technology use in the mathematics classroom. In sum, this definition is a result of a synthesis of the research available on the best practices and essential characteristics for using technology effectively in the mathematics classroom as discussed in this section and will be used for the remainder of this document. Next, I will report on the benefits of using technology effectively in the secondary mathematics classroom.

Table 2.4

Summary of characteristics of effective technology use

Author(s)	Year	Characteristic(s)
NCTM	2014	Technology promotes reasoning and sense making
Dick and Hollebrands	2011	Tool is a mathematical action technology, most appropriate for the task; Users are reflective about the tool, strategies and findings; Tool allows for previously inaccessible mathematics to be made accessible; Users are engaged in reasoning and sense making; Tool allows insight into student's reasoning process and provides mathematical models.
Peressini and Knuth	2005	Technology is a cognitive tool that supports conceptual understanding; Tool used as part of complex tasks that require exploration
Laborde	2007	Tool used with carefully designed tasks
Keengwe et al.	2008	Technology used as a part of constructivist approach
Hollebrands	2007	Tool used while employing proactive strategies and making connections among mathematical ideas
Galbraith	2006	Technology tool is an extension of self
	2005	Tool is seamless part of lesson so that focus is on the mathematical content; Tool is necessary part of lesson

Benefits of Effective Technology Use in the Secondary Mathematics Classroom

Since the mid 1990s, researchers have documented the many advantages of technology use in the mathematics classroom (Bokosmaty, Mavilidi, & Paas, 2017; Hegedus, Dalton, & Tapper, 2015; Guerrero, Walker, & Dugdale, 2004; Li & Ma, 2010; Weglinsky, 1998). Weglinsky (1998) analyzed the data from the 1996 National Assessment of Educational Progress in Mathematics to determine the level of access, frequency of use and types of uses of technology among a national sample of fourth and eighth graders. As he uncovered patterns of usage among different ethnicities and socioeconomic groups in different regions, he found that higher frequency of use did not benefit students. He suggested that a higher frequency of computer use may be associated with ineffective uses of technology such as drill and practice software or games unrelated to academic objectives (Weglinsky, 1998). However, he reported that, "professional development and using computers for higher-order thinking skills were each associated with more than a one-third of a grade level increase" (Weglinsky, 1998, p. 5) at the eighth grade level. Weglinsky (1998) also asserted that in order for the benefits of technology to be actualized, teachers must be adequately prepared and supported for using technology and, most importantly, the technology must be used in meaningful or effective ways. Others have found that students benefit the most by using technology in ways consistent with the definition in this study for effective use of technology such as for complex problem solving, engaging in discourse triggered by findings in the technological environment, and developing and using research skills (Keengwe et al., 2008).

The benefits of technology for learning particular mathematical content have also been widely studied (Huang & Zbiek, 2017; Heid & Blume, 2008; Olive & Lobato, 2008; Hadas et al., 2000; Laborde, 2000; Mariotti, 2000) In their review of research on preparing pre-service teachers to use technology, Huang and Zbiek (2017) reported that pre-service teachers' use of technology contributed to greater content knowledge in areas such as statistics and limits of sequences. Hadas et al. (2000) found that activities that were designed to lead students to a surprise or contradiction and then to provide explanations moved students toward engaging in traditionally challenging formal proof activity. In another study, students that used a mathematical simulation software to explore algebra developed a deeper understanding of functions than those without technology (Hegedus, Dalton, & Tapper, 2015).

Olive and Lobato (2008) summarized the findings of nine published studies on the use of technology to teach rational number concepts and the effectiveness of this approach into five conclusions. Some actions performed in the technology environments that facilitated the understanding of rational numbers were "difficult, if not impossible, to perform without technology" (Olive & Lobato, 2008, p. 33). The technology allowed students to create representations that are considered to be richer than those that would normally be available. Also, Olive and Lobato (2008) concluded that the studies as a whole contributed greatly to the efforts to understand how students conceptualized rational numbers and how they related whole numbers and rational numbers as they were building their knowledge. The final conclusion noted that technology can also allow for the application of incorrect and misleading strategies that were not necessarily easy to redirect. Their findings indicated that technology can positively impact the development of rational number concepts in students if the tools are designed to interact with how children think about fractional concepts and careful consideration is given to what tasks are appropriate. They also indicated that teachers must be properly supported and prepared for using the tool to facilitate meaningful learning opportunities (Olive & Lobato, 2008).

Heid and Blume (2008) examined the research related to algebra and functions and found that technology has been shown to enhance the development of these concepts. Their synthesis of the available research suggested that some technology advantages included students engaged in the act of creating multiple representations versus analyzing predetermined representations, increased spatial-visual skills, and more focus on and better development opportunities of the conceptual versus procedural skills (Heid & Blume, 2008). For example, although spreadsheets do not allow students to manipulate variables, they do allow students to develop and analyze a series of values that varies and thus contributes to the conceptualization of a variable. Another important finding was that students had "no substantial deficit in by-hand skills" (Heid & Blume, 2008, p. 98) because of the use of technology such as computer algebra systems (CAS), and further they found that students could develop such skills by first engaging in technological experiences.

Benefits of technology for traditionally underachieving groups. Often, traditionally underachieving groups of students receive instruction with technology in the least effective ways (Strutchens & Silver, 2000). Teachers that use technology in the least effective ways often lack the specialized knowledge (Mishra & Koehler, 2006) required. Also, there is little argument that the inequitable funding of technology, deficient resources, unfair structures and school practices, and inadequate teacher preparation for using technology in the classroom results in inequitable access to technology for students (Dick & Hollebrands, 2011; Dunham & Hennessy, 2008; Guerrero, Walker, & Dugdale, 2004). Previous reports on the National Assessment of Education Progress (Strutchens & Silver, 2000) have shown inequities related to how technology is used differently with different groups. For example, among eighth grade students, white students were more likely to have experiences with technology involving higher-order thinking skills, problem-solving and simulation activities than black students. Also, black students were more likely to experience remedial or drill usage of technology than white students (Strutchens & Silver, 2000). More recently, the 2015 NAEP report showed that 59% of 8th grade students rarely or never use computers in mathematics and students from higher poverty schools are more likely to use computers for drill and practice (NCES, 2015).

However, in their report on technology use, Darling-Hammond, Zielezinski, and Goldman (2014) identified characteristics of technology use that have been shown to benefit underachieving and at-risk students. They reported that using technology as a tool to support interactive learning so that students can explore concepts with a variety of representations has been show to benefit all students in general and at-risk students in particular. Also, technology should be used by students to create content as part of the learning process. Finally, at-risk students can benefit from technology use when used with an appropriate combination of technology engagement, teacher support, and collaboration among students (Darling-Hammond et al., 2014).

Effective technology implementation in the mathematics classroom is important for reducing inequities in mathematics education because it has also been found to benefit other traditionally underachieving groups (Guerrero, Walker, & Dugdale, 2004; Kim & Chang, 2010; Lyublinskaya & Tournaki, 2011). Kim and Chang (2010) examined the effects of home computer use, computer use in mathematics, and computer use for various purposes among ELL and native English-speaking students in grades kindergarten through fifth grade using data from the Early Childhood Longitudinal Survey. Although Kim and Chang (2010) showed mixed results of the effects of computer use at home and at school for a variety of purposes across all racial and language ability groups, they did find a reduction in the mathematics achievement gap between ELL students and non-ELL students. Particularly, Hispanic and Asian ELL students had significantly higher rates of growth in mathematics performance compared to English-speaking students when computers were used frequently in mathematics classes. The researchers noted previously identified benefits for ELL students when using technology that can support their developing language skills and contribute additional support for learning mathematics such as translation features, multiple modes of content delivery including pictures, text, and video, and dictation features (text-to-speech or speech-to-text) (Kim & Chang, 2010). Guerrero, Walker, and Dugdale (2004) conducted a review of research related to middle grades students' achievement in terms of mathematics skills and conceptual understanding associated with technology use. The reported benefits for middle grades students using technology included increased confidence in mathematical abilities, increased motivation to engage in mathematics, and a positive attitude about the use of a technology tool to increase mathematics learning and understanding in environments where technology is used for exploration and problem-solving (Guerrero, Walker, & Dugdale, 2004).

Students with special needs can also benefit from mathematics instruction that incorporates technology effectively and appropriately (Murray, Silver-Pacuilla, & Helsel, 2007; Little, 2009). Instruction that integrates technology can improve mathematics learning by students with disabilities by providing interactive representations that can be related to more abstract representations, compensating for lack of fine motor skills needed to explore with manipulatives and measuring tools in a non-digital environment, and creating environments that increase the level of interaction with mathematical concepts (Little, 2009). For example, a student that struggles with the physical act of measuring angles using a protractor due to poor development of fine motor skills or behavioral issues triggered by frustration can measure angles in a dynamic geometry environment with more ease and accuracy and be able to fully engage in an investigation. This does not suggest that the development of physical measuring skills is not valuable, but rather when it is not the content goal, the lack of such skills does not have to impede student learning of mathematical concepts. Murray, Silver-Pacuilla, & Helsel (2007) also reported on various ways technology has been found to support learning of students with special needs. Students with disabilities that would discourage their engagement with mathematics can be supported by "the flexibility and interactivity that are inherent in technology" (Murray, et al., 2007, p. 2). They also reported that technology could support the development of conceptual understanding by using manipulatives that allow students to make connections between concrete representations, representational images, and formal abstract or symbolic representations.

In a study comparing the performance of algebra students taught using graphing calculators to the performance of algebra students using TI-Nspire interactive handheld devices, the researchers found that girls significantly outperformed boys in the group using the TI-Nspire for instruction (Lyublinskaya & Tournaki, 2011). This finding suggested that further research is needed to explore the effects of an interactive and investigative technology tool on the motivation and mathematics achievement of females as it was not explained by the data collected or in associated research findings (Lyublinskaya & Tournaki, 2011). Lyublinskaya and Tournaki (2011) also compared low and high achieving secondary students using handheld graphing devices (control group) versus handheld CAS devices (experimental group) to determine the effectiveness of the more advanced CAS technology tool. They found that lower achieving students in the experimental groups performed at the same level as the higher achieving students in the control groups, suggesting that the more "dynamic and interactive explorations" (Lyublinskaya & Tournaki, 2011, p. 27) used with the CAS devices could positively impact previously unengaged students. They also reported higher passing rates on the standardized state exam in each racial group for students taught using CAS devices compared to those taught using graphing calculators although their scores were not significantly different. However, Lyublinskaya and Tournaki (2011) claimed that the increased passing rates indicated that interactive and dynamic technological tools could contribute to reducing the mathematics achievement gap.

The lack of access to technology in schools is often cited as a barrier to equity (Dunham & Hennessy, 2008; Van Roekel, 2008). The lack of access or differentiated access to technology impacts equity in a variety of ways. For example, states have differing policies concerning calculator use on high stakes exams that can affect outcomes (Dunham & Hennessy, 2008). Access issues related to home availability, school internet connectivity, course assignment and associated technology, and varying models and capabilities of technology have also been associated with reinforcing differences among populations of students (Dunham & Hennessy, 2008). Also, Dunham and Hennessy (2008) reported that minority students are disproportionately assigned to mathematics classes designed for lower-achieving students and subsequently have less experience with technology tools that promote higher order thinking skills because they are often utilized in higher achieving classes only. School structures such as these, along with inequitable funding for schools serving poor students (NCTM, 2014), contribute to inequities associated with technology access in mathematics classrooms. Clearly there is potential for technology to further benefit groups of students that have traditionally underperformed in mathematics, and additional research is needed in this area that can be enabled by increasing the frequency and quality of teacher use of technology in the classroom.

Benefits of dynamic geometry environments (DGEs). Dynamic geometry environments such as Geometer's Sketchpad and Geogebra have been studied in order to identify the benefits particular to their unique features (Bokosmaty, Mavilidi, & Paas, 2017; Hollebrands, Laborde, & Straber, 2008; Ubuz, Ustun, & Erbas, 2009). For example, Bokosmaty, Mavilidi, and Paas (2017) compared the impact of DGE manipulations of triangles versus presentation of concepts using static models on student learning outcomes of triangle properties. Students in a control group examined static pictures of shapes demonstrating triangle properties while students in experimental groups manipulated a triangle in a DGE and were instructed to examine the same properties as the control group. They found that students that had engaged in manipulation of triangles performed better on assessment tasks when asked to explain or justify properties of triangles. Dynamic geometry software can also provide students with an exploratory environment to construct representations to develop more traditionally difficult skills such as formal proof and reasoning skills (Hadas et al., 2000; Laborde, 2000; Mariotti, 2000).

Four common themes were found in the analysis of over 200 published studies on the use of technology (primarily Dynamic Geometry Environments [DGE]) for the teaching and learning of secondary geometry concepts (Hollebrands, Laborde, & Straber, 2008). The majority of the studies examined one or more of the following elements of DGE implementation; DGE constructions and representations and the connections among the representations, the tasks used in DGE, student actions and constructions in the DGE, and the relationship between the construction of knowledge of the DGE and geometric concepts. One commonly noted advantage to using a dynamic geometry environment was that teachers gained evidence of student thinking and understanding. For example, the choices of actions a student took in a dynamic geometry environment often reflected the conceptions a student held about the mathematics and the behavior of geometric objects in a constrained environment. According to Hollebrands et al. (2008) the cognitive processes of a student "are externalized" (p. 190) as the student interacted with the computer during processes of exploration and reflection.

Also, Hollebrands et al. (2008) found that explorations conducted in dynamic geometry environment often required students to apply geometric knowledge strategically. For example, some students began tasks using visually motivated strategies in constructions. When those strategies failed to produce the expected results, students often developed more geometrically sound strategies that reflected their development of both geometric knowledge and knowledge of the tool itself (Hollebrands et al., 2008). This strategic application of geometric knowledge versus empirical-based approaches is similar to the proactive strategies discussed previously and was also found to support students' development of deductive proof skills. Many studies found that in carefully constructed activities, students used deductive justifications to explain a new geometric finding as opposed to simply a means to verify an already established fact (Hollebrands et al., 2008). Thus, students were able to engage in meaningful tasks that allowed them to experience deductive proof as a purposeful mathematical activity through the aid of technology.

Ubuz, Ustun, and Erbas (2009) also looked at the effects of instruction incorporating dynamic geometry environment compared to lecture based instruction with seventh grade Turkish students on learning concepts of lines, angles, and polygons. They hypothesized that students would develop deeper understandings after engaging in problems that allowed students to become "conceptualizing participants" (Ubuz, Ustun, & Erbas, 2009, p. 12) and also be able to make connections among properties of related shapes. After comparing pretest and posttest definitions provided by students, they found that students more frequently identified critical attributes of the definitions of shapes in the group taught using a dynamic geometry environment (DGE) than in the control group. They attributed this shift toward recognizing the critical properties by the students using DGE to having experienced distinguishing between a drawing and a construction as required by the DGE activities (Ubuz, et al., 2009). Constructions required more purposeful actions and critical thinking prior to creating a shape than a drawing required.

Thus, DGE in the secondary geometry classroom provided teachers a means to better understand student thinking, a tool that required students to employ geometric strategies by constructing their knowledge, and offered alternative perspectives on the meaning and application of deductive proof skills. Also, the results suggested that the frequent use of a DGE can encourage a classroom culture that engages in conjecturing, exploring, and reasoning as part of a daily routine.

Benefits of graphing calculators. According to NCTM's research brief on graphing calculators, various studies have shown benefits such as increased mathematical understandings and improved problem-solving skills. In addition, after an exhaustive examination of almost 200 studies, no evidence has been found to show that graphing calculators do not benefit students (Ronau et al., 2011). The benefits of graphing calculators on student mathematics achievement has also been examined in various studies that Ellington (2006) used in her meta-analysis on the effects of non-CAS graphing calculators. After comparing 42 published studies that each included an experimental group using graphing calculators and a control group that did not use graphing calculators in instruction or assessment and also provided the data from which the effect sizes were calculated, her findings identified calculators as another beneficial tool for students but only when used in appropriate contexts. For example, when students were assessed on conceptual skills there was a significant difference in favor of the group using the graphing calculator for instruction regardless if the calculator was permitted to be used on assessments or not. When instruction included graphing calculators but testing did not and the assessment was of procedural skills, no significant difference was found; however, when the calculators were permitted for testing, the experimental group significantly outperformed the control group (Ellington, 2006). Ellington (2006) also reported that students who had access to graphing calculators during instruction had a better attitude toward mathematics than those who did not have access. These findings are consistent with others in that graphing calculator technology effectively used during instruction and assessment positively impacts the development of procedural and conceptual mathematics skills (Ellington, 2003; Laumakis & Herman, 2008).

Benefits of computer technology. In their study examining the impact of implementing a mathematics simulation software in a high school algebra course, Hegedus, Dalton, and Tapper (2015) found that positive gains on tests after engaging in lessons with the software. Students explored different algebraic functions by creating and running simulations with the software. In addition to improved test scores, students showed an increase in engagement and they developed higher-order problem solving skills (Hegedus, Dalton, & Tapper, 2015). Keengwe, Onchwari, and Wachira (2008) reviewed and summarized studies about the uses of computers in education and found that meaningful or authentic use of computers in the classroom required a shift in beliefs about teaching and learning that must include constructivist approaches. Technology use often increased student achievement when it was used for problem solving, research, critical thinking, and experimenting (Keengwe et al., 2008). When implemented and supported properly, technology "can positively impact important dimensions of learning such as active learning, critical thinking, cooperative learning, communication skills, instructional effectiveness, multisensory delivery, motivation, and multicultural education " (Keengwe et al., 2008, p. 10).

Li and Ma (2010) conducted a meta-analysis of 46 studies that explored the impact of technology on mathematics achievement. They focused on computer technology, indicating that the benefits of graphing calculators have been previously established in individual studies and several meta-analyses (Li & Ma, 2010). The studies included met rigorous criteria such as the use of the technology used with experimental groups had to be instructional in nature, as opposed to drill and practice; the studies had to have been published between 1990 and 2006; and an effect size statistic was available (or able to be calculated with the data presented) comparing control groups and experimental groups of students in K-12 classrooms. Their meta-analysis showed that technology used in a meaningful way, particularly in a constructivist environment, resulted in greater mathematics achievement for students (Li & Ma, 2010). They also reported larger effects on mathematics achievement for the following categories; elementary students, special needs students, assessment occurring using non-standardized tests, and when the technology intervention was applied for shorter periods of time. They offered several possible explanations for their findings. For example, they suggested that shorter interventions were more effective because in longer interventions, the novelty effect of the technology tends to wane. Also, they indicated that the higher achievement of elementary students over secondary students was not surprising because the "visual and hands-on approaches [with technology] suit more to the learning styles of elementary school students" (Li & Ma, 2010, p. 234). Li and Ma (2010) concluded that technology should be included as a necessary tool for promoting mathematics student achievement, but that contextual issues such as teaching approaches and student characteristics must be considered when planning to use technology effectively.

Several characteristics of classroom instruction that incorporated computer tools have been found to increase student mathematics achievement in the studies reviewed by Keengwe, Onchwari, and Wachira (2008). In their analysis, they concluded that students engaged in computer activities using constructivist strategies instead of procedural drilling activities achieved higher scores. It is important to note that all the studies examined by Li and Ma (2010) and others (Keengwe, Onchwari, & Wachira, 2008) indicated that although there were at times no significant differences among experimental groups using technology and control groups, there often were significant differences in favor of the experimental groups, and perhaps most importantly no evidence that indicated that technology use was detrimental to mathematics learning.

Summary. The effective and meaningful implementation of technology "can positively impact important dimensions of learning such as active learning, critical thinking, cooperative learning, communication skills, instructional effectiveness, multisensory delivery, motivation, and multicultural education " (Keengwe et al., 2008, p. 10). In their summary of the literature on the use of computers in education, Keengwe, et.al (2008) found that meaningful or authentic use of computers in the classroom that resulted in increased student achievement required a shift in teacher knowledge and beliefs about the roles of students and teachers, the value of technology as an essential tool in the mathematics classroom, and what it means for students to learn and do mathematics. Using technology effectively should support mathematical reasoning and sense making and also help students to develop procedural skills, but technology should not be used to replace procedural skills. Creating opportunities for students to engage at such effective levels requires that teachers themselves experience this level of interaction and model it in the classroom (Galbraith, 2006). Thus, it is important to help students and teachers to develop their proficiency and inquiry skills in both mathematics and the use of technology in order to help them to become effective users of technology.

Factors That Impact Effective Technology Implementation

The advantages described in the previous section of incorporating technology in the classroom can only be exploited if the technology is implemented effectively; therefore, it is important to consider what we already know and what we need to know about preparing and supporting teachers to effectively use technology in the mathematics classroom. The Association of Mathematics Teacher Educators (AMTE) (2017) recommended that beginning teachers utilize tools including technology as a standard for all teacher education programs. Preparation of teachers should include experiences for pre-service teachers to use technology to learn mathematics and learn to use technology to teach mathematics. The recommendations also stated that by the end of their teacher preparation program, "well-prepared beginning high school mathematics teachers are comfortable using technology to engage in mathematics and to effectively support meaningful mathematics learning" (AMTE, 2017, p. 125). It is also recommended that beginning teachers develop mindsets about technology as essential to learning and as consistently available learning tools for students in their future classrooms (AMTE, 2017).

Teacher knowledge required for effective technology use. NCTM (2014) stated that teachers must be made aware of the ways that technology can support reasoning and sense making in the classroom. They stated that teachers

> need to develop deep understandings of how technology and tools can be used to investigate mathematical ideas, generate multiple representations of a mathematical construct, and solve mathematics problems. They need to reflect on how their students might use these tools and how the tools might be incorporated into the curriculum in a meaningful way (NCTM, 2014, p.85).

In *Professional standards for teaching mathematics*, NCTM (1991) acknowledged the importance of preparing teachers to use technology effectively in the classroom and that "such instruction requires substantial changes in the philosophy and strategies of mathematics" (p. 128). In order to understand how to implement technology effectively in the classroom, a teacher must first understand how to apply the concepts of social constructivism into her instruction (Ertmer, Ottenbreit-Leftwich, & York, 2007; Koehler & Mishra, 2005). Implementing constructivist practices in the classroom requires a shift towards beliefs such as those expressed by Piaget (1970). He expressed that "each time one prematurely teaches a child something he could have discovered himself, the child is kept from inventing it and consequently from understanding it completely" (Piaget, 1970, p. 115). Constructivism is a collection of learning theories that describe learning as an active process where learners must build knowledge through the social interaction with the physical world and negotiate the new knowledge within their current body of knowledge (Jaworski, 1994). Ertmer et al. (2007) defined expert teachers using technology as "those who employ technology in learner-centered, constructivist environments" (p. 1). Thus, the effective use of technology in the mathematics classroom requires that students develop understanding in this way through engagement with the mathematics and technology.

Vygotsky (1978) added to the theories of knowledge development and claimed "learning awakens a variety of internal development processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers" (p. 35). He claimed that students in the Actual Development Level (this includes what a child already knows and can do independently) require carefully implemented tasks including interaction in order to progress to their Zone of Proximal Development (the potential knowledge of a child) (Vygotsky, 1978). Similarly, when students are using technology effectively to learn mathematics, they must incorporate previous knowledge and skills and utilize the technology to develop new understandings. Thus, any professional development or teacher preparation program that aims to provide teachers with the skills to use technology in meaningful ways must be grounded in these learning theories.

Knowledge for teaching. Before examining the specialized knowledge needed by teachers in order to effectively use technology in the classroom, it is important to understand what is known about the knowledge needed for effective teaching in general. Shulman (1986) recognized that most research and teacher education programs were focused on knowledge of content and general pedagogical strategies such as organizing classrooms and planning lessons. Beyond this type of knowledge, there must be focus on content knowledge specific to teaching such as the

> most useful forms of representation…, the most powerful analogies, illustrations, examples, explanations, and demonstrations- in a word, the ways of representing and formulating the subject that make it comprehensible to others (Shulman, 1986, p. 7).

Shulman (1986) referred to this specialized knowledge as Pedagogical Content Knowledge. Although he also acknowledged previously identified forms of knowledge needed for teaching such as general pedagogical knowledge, knowledge of how students learn, and knowledge of specific curriculum and content, he proposed that pedagogical content knowledge was the fusion of the many types of knowledge and thus a new domain in itself (Shulman, 1987).

In order to identify and examine pedagogical content knowledge, Shulman (1987) focused on the development of knowledge of content and pedagogy of 12 beginning teachers over a period of three years. They observed and conducted interviews of the teachers to understand the knowledge that teachers developed that is unique and essential to teaching. They sought to understand how teachers are

> able to comprehend subject matter for themselves…to elucidate subject matter in new ways, reorganize and partition it, clothe it in activities and emotions, metaphors and exercises, and in examples and demonstrations, so that it can be grasped by students" (Shulman, 1987, p. 13).

They found that teachers must have the knowledge of both content and pedagogy and to be able to critically reason about what choices to make in the classroom (Shulman, 1987). In other words, the knowledge of how and what to teach was not sufficient without the ability to reason about the pedagogical choices that are made during instruction and applied. Thus, Shulman (1986) suggested that a true assessment of teacher knowledge and teacher preparedness must include pedagogical content knowledge and its application. He noted that much of the research on teacher preparation and teaching resulted in simplified lists of basic knowledge and skills that have been mistakenly accepted as complete sets of standards by policy makers. Since 1986, many other researchers have also attempted to answer questions about the unique knowledge needed for teaching mathematics such as what knowledge is needed, how can it be measured, and how is it developed (Hill, Ball, & Schilling, 2008; Ronau & Rakes, 2012; Ball, Thames, & Phelps, 2008).

Mathematical knowledge for teaching (MKT) (Hill, Ball, & Schilling, 2008) provided a model for the knowledge needed by mathematics teachers specifically and was developed from the broader concept of pedagogical content knowledge. The model of mathematical knowledge for teaching proposed by Hill, Ball, and Schilling (2008) described various types of knowledge of content and teaching including pedagogical content knowledge. The strands (see figure 2.1) that were included as part of the mathematical knowledge for teaching were common content knowledge (CCK), knowledge at the mathematical horizon, specialized content knowledge

(SCK), knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of curriculum (Hill et al., 2008). Common content knowledge is the knowledge that would be common among others and not exclusive to teachers. Specialized content knowledge includes the skills and knowledge required for what educational professionals often refer to as unpacking mathematics for students.

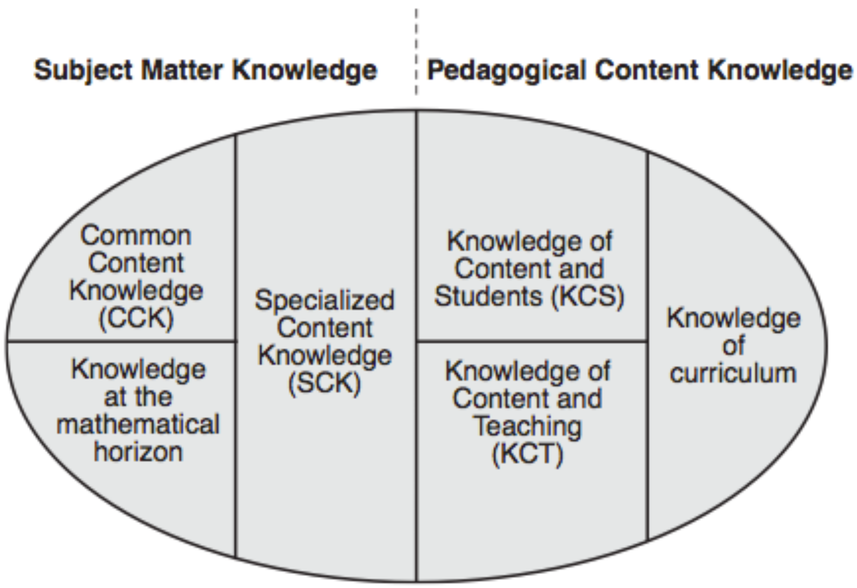


Figure 2.1. Domain map for mathematical knowledge. Reprinted from "Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students." by H.C. Hill, D.L. Ball, and S.G. Schilling, 2008, *Journal for Research in Mathematics Education, 39*(4), p. 377. Copyright 2008 by *Journal for Research in Mathematics Education*. Reprinted with permission.

In their model, knowledge of content and students and knowledge of content and teaching are included under the umbrella of pedagogical content knowledge. Teaching tasks such as anticipating common misconceptions in students often require teachers to consider how content and student factors interact or to use their knowledge of content and students. The interaction of mathematical and pedagogical understandings, or knowledge of content and teaching, is evident in tasks such as sequencing a lesson or identifying the best instructional method. Their stated purpose for identifying the many different types of knowledge and for further research aimed at measuring such knowledge is to continue to identify the most significant factors associated with student achievement. It has been found that the mathematics courses taken by elementary teachers are not a predictor for elementary student achievement (National Mathematics Advisory Panel, 2008); however, Ball, Thames, and Phelps (2008) proposed that perhaps other areas of knowledge for teaching such as specialized content knowledge may have a significant impact on student achievement.

In order to better understand mathematical knowledge for teaching, Ball, Thames, and Phelps (2008) identified many tasks that were common in the teaching of mathematics and sought to pinpoint the mathematical knowledge that was required to perform such tasks. These tasks included assessing student explanations and conjectures, adjusting mathematical tasks to meet the needs of students, choosing representations and examples for particular purposes, and asking purposeful questions to further understanding (Ball et al., 2008). They concurred with Shulman's (1986) findings that pedagogical content knowledge must be included as part of teacher preparation programs and also investigated to determine how this knowledge is related to effective teaching (Ball et al., 2008). Further, the Conference Board of Mathematical Sciences (CBMS) (2012) stated that the development of the mathematical knowledge for teaching requires more collaboration between mathematicians and mathematics educators in order to develop courses for pre-service teachers that emphasize doing mathematics in the ways they will be expected to teach. Thus, their recommendations include providing pre-service teachers with opportunities to "acquire mathematical practices from carefully designed experiences of doing mathematics" (CBMS, 2012, p. 11).

Technological, pedagogical and content knowledge (TPACK). Research on the knowledge needed for teaching with technology has emerged from the PCK concept proposed by Shulman (1986). Many researchers have examined the types of knowledge that a teacher must have in order to integrate technology effectively into their teaching (Koehler, Mishra, Hershey, & Peruski, 2004; Koehler & Mishra, 2005; Guerrero, 2010; Niess, 2011; Niess, Sadri, & Lee, 2007; Niess et al., 2009; Richardson, 2009). Most notably, the Technological Pedagogical and Content Knowledge (TPACK) model was introduced by Koehler and Mishra (2005) and combined their own research with research from others on teacher knowledge. They incorporated these ideas into their study of how TPACK developed among four faculty members and 14 graduate students that participated collaboratively in a project creating online courses. Koehler, Mishra, Hershey, and Peruski (2004) began to explore how technology, content, and pedagogy interact as university faculty members and graduate students worked together to build online courses using a method called learning by design. Their previous experiences with traditional online course design approaches that included utilizing a technology team to build the online courses based on the pedagogical and content design from faculty members, resulted in what they considered to be poorly designed online courses due to the lack of collaboration among parties and lack of pedagogical and content knowledge of the technology team. They implemented the learning by design method that included all members of the team- faculty and graduate students focused on content and pedagogy and technology specialists working together to make decisions to create an online course so that content, pedagogy, and technology were all addressed.

Building from Shulman's (1987) Pedagogical Content Knowledge framework that emphasized that teachers must understand the interactions between content and pedagogy in order to be an effective teacher, Koehler et al. (2004) used the observations from the learning by design seminar to support and revise their TPACK model in its infancy. They argued that the three components have a transactional relationship that must be understood in order for good teaching to result and that the “formulation of representations to be communicated to students is an iterative process” (Koehler et al., 2004, p. 49). As their work progressed, they added technology to Shulman’s Pedagogical Content Knowledge framework to form the TPACK model that highlighted the importance of understanding not just the three individual components but also the interactions among the components. They hypothesized that in order to develop TPACK their participants must engage in collaborative activities that required them to sort through this "complex web of relationships between content, pedagogy, and technology" (Mishra & Koehler, 2006, p. 1019). They compared the results of pre- and post- treatment surveys assessing how the participants perceived their TPACK to develop and the levels of usefulness of their tasks. Participants reported positive attitudes toward their learning experiences while collaboratively producing online courses through a process of inquiry and research that resulted in increased understandings of the complex relationships in the TPACK framework (Mishra & Koehler, 2006).

Figure 2.2 shows how the components interact and Mishra and Koehler (2008) claim that each area is as important as the others- including the individual areas and the areas representing the relationships between the components. Table 2.5 describes each of the three components and also details the elements involved in the different areas of interaction (Mishra & Koehler, 2008). According to Mishra and Koehler (2006) teaching with technology requires a constructivist approach so that teachers engage students in interactions with the technology, thus merely providing technology tools in the classroom will not result in effective implementation of technology. They described good teaching as going beyond simply adding technology to what is already being done and instead recognizing that "the introduction of technology causes the representation of new concepts and requires developing a sensitivity to the dynamic, transactional relationship between all three components" (Mishra & Koehler, 2006, p. 4).

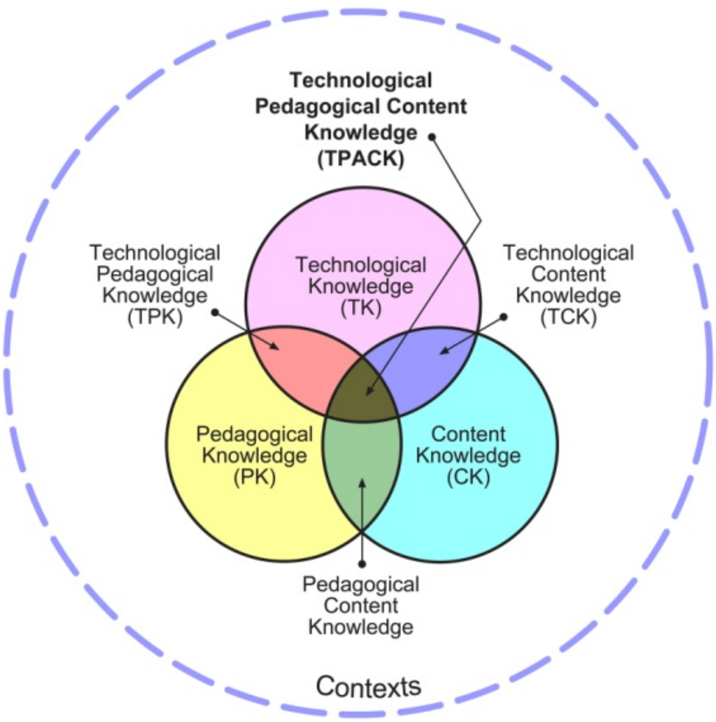


Figure 2.2. TPACK Framework. Reprinted from TPACK.org, by P. Mishra & M. J. Koehler, 2008, Retrieved from http://TPACK.org/. Copyright 2008 by TPACK.org. Reprinted with permission.

Table 2.5

Technological Pedagogical and Content Knowledge Components and Descriptions from Mishra and Koehler (2008, pp. 3-10)

Component	Description
Technology Knowledge	Skills required to use technologies
Content Knowledge	The subject content being taught
Pedagogical Knowledge	Methods of teaching and student learning
Technological Content Knowledge	How content can best be represented through the different technologies and what technology is best suited for different content
Pedagogical Content Knowledge	Interpret subject matter, find multiple ways to represent it, and adapt instructional materials to alternative conceptions and students' prior knowledge
Technological Pedagogical Knowledge	An understanding of how teaching and learning changes when particular technologies are used including understanding the pedagogical affordances and constraints of a range of technological tools
Technological Pedagogical and Content Knowledge	The intersection of all three bodies of knowledge including an understanding of how to represent concepts with technologies, pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn and how technology can help students learn; knowledge of students' prior knowledge and theories of epistemology; and knowledge of how technologies can be used to build on existing knowledge and to develop new epistemologies or strengthen old ones

Principles to actions (NCTM, 2014) states that teachers must have developed the knowledge necessary in order to implement technology in effective ways. In particular, the TPACK (and all components of TPACK including content knowledge [CK], pedagogical knowledge [PK], technology knowledge [TK], pedagogical content knowledge [PCK], pedagogical technology knowledge [PTK], technological content knowledge [TCK] and technological pedagogical content knowledge [TPACK]) of a teacher is essential in understanding and implementing effective technology learning opportunities for students (Mishra & Koehler, 2006). For example, in planning a task for constructing an isosceles triangle using Geometer's Sketchpad (GSP), there are many elements to consider. First, the teacher must be aware of the content knowledge (CK) that students are bringing into the activity. If students are not familiar with constructing circles or properties associated with circles, then it may be difficult for the students to conceptualize that particular construction method. Next, a teacher must consider pedagogical issues (PK) such as how the task can be constructed to fit the needs of students and what questions to ask in order to properly facilitate the activity. Also, the teacher and students must be familiar with at least some of the functions, constraints and affordances of GSP (TK).

Knowledge at the intersections of technology, pedagogy, and content must also be considered. The choice of GSP as a proper and effective technology tool to construct an isosceles triangle reflects the TCK of the teacher. Teachers must also be aware of the variety of ways an isosceles triangle can be constructed and the geometrical understandings behind each construction in case students use unexpected strategies (PCK). Next, teachers must be aware of the common mistakes and misconceptions that arise in GSP (TPK) such as drawing as opposed to constructing. Finally, teachers must be able to combine all types of knowledge in order to what and how prior knowledge can be used in the GSP environment to build upon and develop new geometric understandings related to constructing an isosceles triangle (TPACK). This description also demonstrates the interconnectedness of each element of TPACK since it could be argued that some of the examples did not fit completely into the category that I identified and could be combined with others. Although this example was given in the context of a dynamic geometry environment, the knowledge described is needed for the effective implementation of any mathematical action technology.

Others have reported on the TPACK framework in order to inform teacher educators and program developers on how teachers become effective users of technology in their classroom Manfra and Spires (2013) suggested that pedagogical knowledge be represented with a larger circle than content and technology in the Venn diagram due to the priority placed on that type of knowledge by the teachers in their study on TPACK development. (Guerrero, 2010; Niess, 2011). For example, Richardson (2009) found that teachers participating in professional development to integrate technology into algebra instruction needed to specifically engage in activities that moved them from TCK to TPACK. In other words, teachers were able to successfully progress from technology as a way of presenting mathematical concepts to a tool that allowed students to explore and develop their mathematical understandings when specifically supported to do so.

Guerrero (2010) further unpacked the components of TPACK by identifying four key elements within the Technological Pedagogical and Content Knowledge intersection. These include a) conception and use of technology, b) technology-based mathematics instruction, c) management issues, and d) depth and breadth of mathematics content (Guerrero, 2010). According to Guerrero (2010), the beliefs that teachers hold about the usefulness and appropriate applications of technology, the mathematics that can be learned via technology, and how technology can support learning all impact teachers' decisions about how technology is used. Next, Guerrero (2010) stated that technology should be used when it is the tool that most appropriately assists in the development of the intended content. Technology-based mathematics instruction also requires that teachers allow for a shift toward a student-centered classroom and be able to take on new roles in the instructional process. Additionally, teachers must be able to manage the nuances that occur in a technology-infused classroom such as collaborative learning environments, physical placement issues, technical problems, and varying levels of student engagement (Guerrero, 2010). Finally, the advantages of technology-based instruction include creating opportunities for in-depth explorations and inquiries that may go beyond the mathematical understandings and comfort level of some teachers. Thus, Guerrero (2010) stated that it is important for teachers to be confident in their own abilities to inquire and be comfortable learning with the students at times.

Niess (2011) identified insights and challenges from the TPACK framework and their implications for teacher education programs. She recognized the impact that TPACK has had on shifting the focus in teacher education from learning about technology tools isolated from the contexts of content and pedagogy toward the relationships among the components of TPACK. Guerrero (2010) stated that many components have been proposed to be added to the TPACK framework in order to further the understanding of how teachers come to effectively use technology. For example, Niess (2011) suggested that "Teaching and Learning" better described the robust set of variables that make up the "Pedagogical" component. Finally, teachers for the most part are not exposed to learning mathematics in a technology-based environment, and Niess (2011) reported that teacher educators are challenged to create opportunities for future teachers to have these experiences.

TPACK development. In the 2010 survey conducted by the National Center for Education Statistics on teachers' use of technology in the classroom, 61% of teachers reported that they had adequate training to learn to use technology effectively in the classroom through professional development or on-site training (Gray, Thomas, & Lewis, 2010). Additionally, 78% felt that they were prepared to use technology effectively through independent learning. However, studies have shown that teachers lack TPACK and knowledge of effective uses of technology (Handal, Campbell, Cavahagh, Petocz, & Kelly, 2013). Handal et al. (2013) noted that teachers report awareness of the potential of technology tools to improve teaching and learning but lack the skills needed for successful implementation. Thus, it is important to consider the ways in which TPACK is developed.

While Koehler and Mishra (2005) used the learning by design model to study the development of TPACK, Niess et al. (2009) identified five developmental stages that teachers moved through as they were observed learning to use spreadsheets in mathematics instruction. Figure 2.3 shows the visual description of the stages teachers may move through as they develop their TPACK (Niess et al., 2009). At the recognizing stage, teachers have not yet integrated what they know about using technology with their knowledge of teaching and learning mathematics but there is some recognition of alignment between technology and mathematical content. This stage of TPACK development also does not require that one has had any experience teaching. Teachers at the accepting or persuasion stage of TPACK development engage in activities that allow them to form attitudes (favorable or unfavorable) about teaching mathematics with technology. The next level of TPACK development is the adapting or decision-making stage. Here teachers engage in activities that result in the adoption or rejection of technology as a teaching and learning tool in the mathematics classroom. Teachers at the exploring or implementation stage engage in teaching mathematics using technology. When teachers reach the advancing stage, they are able to adapt content, pedagogy, and technology to meet their curriculum goals and aid students in using the technology as well (Niess et al., 2009).

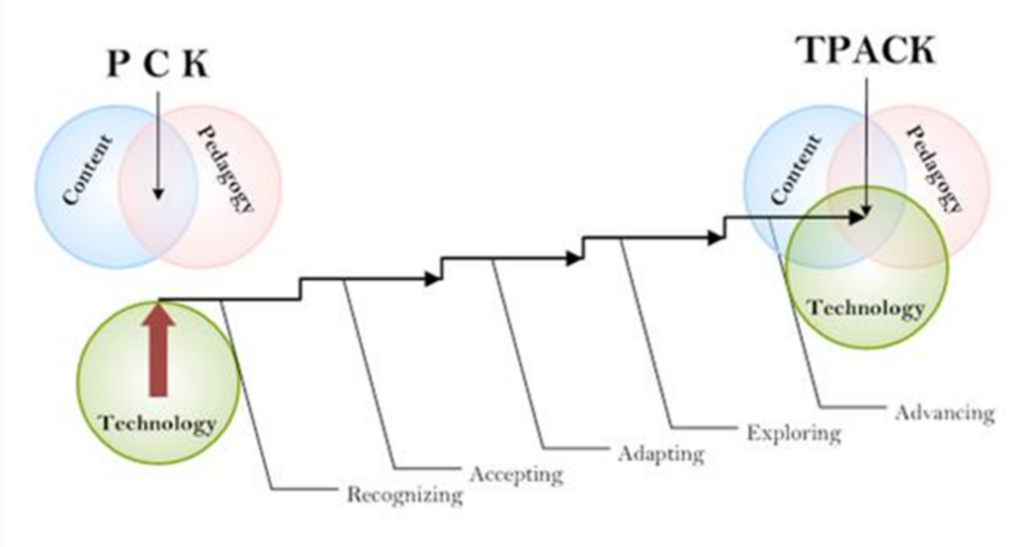


Figure 2.3. Developmental stages for TPACK. Reprinted from "Mathematics teacher TPACK standards and development model," by M. L. Niess et al., 2009, *Contemporary Issues in Technology and Teacher Education*, *9*(1), 4-24. Copyright by M. L. Niess et. al. Reprinted with permission.

Niess et al. (2009) identified four areas related to teacher knowledge for which evidence of TPACK could be observed. These areas included curriculum and assessment, learning with technology, teaching with technology, and access to technology. Various descriptors of teacher actions were identified at each level of TPACK development. Appendix B shows the TPACK Development Model level descriptors identified by Niess et al. (2009). Research in the area of TPACK development remains incomplete as teacher educators are still developing best practices that will aid teachers in moving through the stages and increase the likelihood of future implementation of technology in all mathematics classrooms.

In their meta-analysis of research reports on TPACK, Chai, Koh, and Tsai (2013) reported that several studies found that engaging teachers in a learn by design model was a successful model for developing TPACK. Teachers that collaborated to develop technology-based projects or lessons during a course or workshop were able to enhance their TPACK in 28 out of 32 studies examined. Findings on how TPACK is developed in teachers were also summarized by Niess (2011). First, TPACK development should be incorporated across the teacher preparation courses and not isolated to a single technology-intensive course in order to increase their exposure to technology-based mathematics learning. These should include opportunities to explore and discuss the usefulness and appropriateness of various technologies; however, the mathematical content should remain at the forefront of the explorations and the technology should be presented as one possible tool for teaching and learning the content. Next, teacher educators should guide future teachers in developing more complex TPACK (Niess, 2011). For example, a student may start out with a general understanding of how a certain technology tool aligns with the mathematical content but they need to be provided opportunities to develop positive attitudes toward learning with technology, acquire skills to judge the usefulness and appropriateness of technology, and learn to integrate technology into their teaching plans (Niess, 2011). Niess, Sadri, and Lee (2007) recognized that although TPACK is a complete model for understanding what knowledge is needed for teachers to effectively integrate technology into instruction, teacher educators still needed to understand how such knowledge is developed.

In a review of research available on developing TPACK in pre-service teachers, Yigit (2014) found several common themes. First, several studies reported that pre-service teachers develop their TPACK in courses that were designed using the integrated approach of TPACK to engage the pre-service teachers. Next, pre-service teachers developed TPACK when they engaged in lessons that required them to consider how technology would be implemented in their own classrooms in the future. Finally, he found several studies that indicated that technology should be fully integrated and modeled in teacher education programs including mathematics courses in order to help pre-service teachers develop their TPACK.

Barriers and enablers to technology implementation. Why teachers implement technology in the mathematics classroom is important to understand because of the significant role technology now plays in the learning of mathematics. Factors impacting technology implementation can be both barriers and enablers. According to Ertmer et al. (2007), barriers and enablers can be either extrinsic such as lack of technology resources or intrinsic such as teacher beliefs. Ertmer et al. (2007) indicated that enablers and barriers often have an inverse association meaning a factor such as administrative support can be a barrier if the support is lacking or can be an enabler if there is sufficient support for teachers. In this section, I will discuss the factors that affect technology integration as enablers, barriers, or both as most factors are not strictly enablers or strictly barriers.

Despite the clear benefits and ever-increasing availability of technology, it is often reported that very few teachers use technology in the mathematics classroom as an integrated teaching and learning tool even if prepared to do so (Bennison & Goos, 2010; Ertmer, 2005). For example, Bennison and Goos (2010) conducted a survey of 485 secondary mathematics teachers in Australia on their professional development experiences and implementation of technology in the classroom. They found that the same common barriers to technology integration that are discussed by other researchers (Stoilescu, 2014; Norton, McRobbie, & Cooper, 2000; Sturdivant, Dunham, & Jardine, 2009) were given by their participants as reasons for not implementing technology including lack of teacher knowledge, undervalued roles of technology, lack of resources, insufficient learning experiences, and lack of adequate professional development.

Okumus, Lewis, Wiebe, and Hollebrands (2016) studied teachers' decisions to use Geometer's Sketchpad and examined how their perceived ease of use and perceived usefulness impacted those decisions. In particular, they wanted to find out what made teachers consider the technology useful and easy to use. They used interviews and observations of 34 teachers that used technology in a 1-1 classroom environment. The interviews were used to attempt to understand the teacher's perceived usefulness and ease of use of the technology and what factors led to those beliefs. They found that the most influential factor in considering the technology useful was its alignment with the curricular objectives of the teacher. However, other factors included the teachers' levels of expertise with the technology, support from colleagues, and alignment with the pedagogical approaches of the teacher. Stoilescu (2015) conducted a multiple case study of three secondary mathematics teacher to examine their TPACK and roles for technology. Through multiple observations and interviews, he found that teachers implemented technology because they found it useful for five primary purposes. First, teachers used technology to make the mathematics more accessible and engaging for students. Next, teachers used the technology to support students to take more ownership of the learning process. Students required less direct instruction from the teacher as they learned to develop their own knowledge. Teachers also implemented technology to aid in their mathematical explanations through visualization. Next, Stoilescu (2015) reported that teachers found technology useful for assessment purposes. Finally, the technology served as a tool to improve communication between teacher and student, as well as among students.

Bennison and Goos (2010) summarized many of the essential factors related to preparing teachers to use technology and classified them into three categories; factors pertaining to the school environment, opportunities for teachers to learn, and teachers' personal beliefs and knowledge. The researchers focused on these elements to analyze the relationships between the mathematics teachers' use of technology and their subject knowledge and beliefs, their availability of resources and support, and professional development opportunities. Some school environment factors included the required curriculum, student variables, school culture and norms, resource availability, and teacher support structures. Opportunities to learn included learning experiences from pre-service through in-service professional development. Also, factors associated with beliefs and knowledge included beliefs about teaching, learning, and students and many different types of knowledge needed for teaching. Each of the factors can impact technology use by teachers in the classroom either negatively if not addressed or positively if the development program and continuing teacher support provide resources and experiences related to these (Bennison & Goos, 2010).

Throughout the research on technology in mathematics classrooms the ineffective use of technology due to lack of teacher knowledge of how, when, and what technology to implement was a barrier to implementation of effective technology use (Bennison & Goos, 2010). Teachers often enter educational programs, leave educational programs, and teach in schools while lacking a true understanding of how technology can and should be used effectively in the classroom (Wachira, Keengwe, & Onchwari, 2008). In their study of the beliefs of 20 pre-service middle school mathematics teachers about effective technology use prior to taking a technology methods course, Wachira et al. (2008) found that teachers come into pre-service programs without having experienced using technology to explore and investigate mathematical concepts during their mathematical education. They claim that the lack of meaningful experiences led to teachers' beliefs about technology not being an investigative tool that is reflected in their teaching practices. For instance, teachers that have experienced graphing calculators only as computational aids are more likely to view graphing calculators as a tool to be used after basic computational skills have been developed and not as a means to develop these skills. According to Sturdivant et al. (2009), teachers blame their lack of knowledge with all aspects of incorporating technology as a barrier for doing so. They lack experience in facilitating groups engaged in different tasks, specific features of a technological tool, engaging with the tool mathematical, and how technology enhances a lesson.

Bennison and Goos (2010) also found that teachers who did not participate in a professional development program on the use of technology were less aware of how computers and graphing calculators could be used to enhance learning of advanced concepts than those who did participate. Also, participation in professional development increased teacher confidence and developed more positive attitudes about student benefits of technology. Further, their research suggested that professional development could have a positive effect on beliefs about the benefits of technology and on teacher confidence in using technology in the classroom. Bennison and Goos (2010) concluded that effective professional development is needed to help teachers to develop technological, pedagogical and content knowledge (TPACK) through mathematical learning, and should not be limited to an overview of a particular technological tool. However, Okumus, Lewis, Wiebe, and Hollebrands (2016) reported that in their study of teachers that engaged in a professional development program for using a dynamic computer software program that the ability to implement within the workshop environment did not translate into teachers being ready to incorporate the technology into their own classrooms.

They lacked the self-efficacy to employ the technology outside of the supportive professional development environment. Thus, it is important to consider what long term supports may be needed to help teachers successfully implement technology.

The impact of school environment on technology implementation. According to Norton, McRobbie, and Cooper (2000) the institutional culture can limit the integration of pedagogical changes such as incorporating technology in the mathematics classroom. In addition to the findings of Bennison and Goos (2010) that school culture could reverse positive change, Norton et al. (2000) claimed that certain environments could prevent receptivity of new pedagogical methods such as implementing technology. Norton et al. (2000) conducted five case studies of secondary mathematics teachers within one Australian private school that had sufficient technology resources. The researchers were able to eliminate lack of resources as a barrier for technology implementation due to the site where the study was conducted. Using surveys, interviews, observations, and the provision of professional development, the researchers gathered data about the teachers' beliefs, practices, and responses to the professional development. Both before and after the professional development, it was reported that all five teachers rarely or never used the technology resources available to them. Only one of the five teachers, Mary, saw computers as a tool to develop conceptual understanding among students, and Mary was also the only one to express a desire to integrate more technology in the classroom.

In Norton et al.'s (2000) analysis of pedagogical styles and beliefs about teaching and learning among the five teachers, they found that Mary viewed teaching and learning through a social constructivist lens, while the other four teachers considered teaching and learning a transmission-absorption process. Additionally, Mary's pedagogical style was learner-centered whereas her colleagues used content-focused pedagogical methods. Finally, even though all five teachers reported seldom using technology, Mary's perceived barrier to use technology was different from the rest of the group. Mary reported that her barrier for use was her lack of models for integrating technology into her teaching; however, the other participants all reported other barriers including lack of knowledge or views of technology as inefficient, ineffective, or a hindrance to learning (Norton, et al., 2000). Due to Mary's conflicting beliefs about the teaching and learning of mathematics and technology, she was "ideologically isolated" (Norton et al., 2000. p. 103) from her peers, and she reported being unable to influence change or collaborate with her colleagues. The ideas and beliefs of those in leadership positions in the department influenced the staff in such a way that it prevented computers from being viewed as an effective teaching and learning tool.

Norton et al. (2000) concluded that the beliefs and attitudes most predominate or most aggressively asserted among faculty could prevent teachers from implementing change even when other reported barriers for use are removed. For example, they noted that although the teachers were offered training for mathematical computer software and the researchers volunteered to co-teach the instruction in the teachers' classrooms, the participants did not request such assistance. Even after rejecting this support, participants still noted that their lack of knowledge of software was a barrier to its use. Thus, Norton et al. (2000) hypothesized that the teachers' refusal of support and opportunities to learn allowed them to continue to not implement technology, and that the lack of knowledge may not have been the true barrier. The researchers also concluded that the beliefs about teaching and learning mathematics and about technology in the mathematics classroom held by professional development participants must be carefully examined and addressed by those creating the learning opportunities. Finally, they identified the dynamics among the staff members as a contributing factor to the success or failure of a professional development program; therefore, it was recommended that leadership development be incorporated in any program aimed at increasing teacher use of technology (Norton et al., 2000). Bennison and Goos (2010) also found that school environment and constraints could undo the positive impact of effective professional development. Thus, successful professional development programs must not only help teachers to develop the knowledge and skills needed to implement technology but must also prepare teachers for the school constraints unique to their environment.

Perceived value of technology as learning tool. Teachers must also actively engage in the use of technology to learn mathematics in order to understand the effectiveness of technology in their own classrooms (DeVilliers, 2004; NCTM 2000; Kurz & Middleton, 2006; Hardy, 2008; Wachira et al., 2008). When pre-service teachers were required to explore different technology tools and then compare their features to determine appropriate uses for each tool, Kurz and Middleton (2006) noted that participants acquired knowledge of the different technologies including their assorted features and an understanding of how the learning process could be enhanced by these features. Using carefully designed tasks that provided opportunities for teachers to engage in using dynamic geometry software to explain, discover, organize concepts or generalize findings, the teachers gained a better understanding of the various roles of proof (DeVilliers, 2004). DeVilliers (2004) reported that teachers who did not have positive experiences learning proof concepts in high school benefitted the most from the activities using dynamic geometry software, and suggested that teachers need to experience these types of activities as students to understand why and what is needed to teach in this way. Mistretta (2005) also stressed the importance of pre-service teachers engaging in mathematical activities that require them to actually use the technology tools and develop their skills.

Secondary and middle school mathematics teachers who participated in the Technology in Mathematics Education (TIME) Project (Hardy, 2008) reported a significant increase in their attitudes toward technology and confidence in their ability to incorporate technology into their teaching after engaging in a course that included activities that infused technology throughout as they explored various mathematics problems. Hardy (2008) reported employing other effective methods including modeling the effective use of technology, supporting participants in creating their own technology-based lessons, and providing on-going support. Similar to Kurz and Middleton (2006), Hardy prepared teachers to make decisions about what tools were most appropriate by having them critique the different technologies. Teachers in this study recognized the benefits of engaging in learning by using the technology since they reported a desire to add more practice time to the course (Hardy, 2008).

Chamblee et al. (2008) analyzed the concerns that mathematics teachers had for the value of integrating graphing calculator technology in their classroom. The concerns were measured using The Concerns Based Adoption Model (CBAM) (Hall & Hord, 2006 as cited by Chamblee et al. 2008) before and after a two-week professional development workshop in which mathematics and science teachers paired up to explore learning with technology and integrated instructional planning (see table 2.6). Results indicated that most teachers entered into the workshop beyond concern levels of “awareness” and “informational” (Chamblee et al., 2008). The researchers suggested that because of the familiarity with the graphing calculator, professional development programs for this tool should not focus on demonstrating and explaining features, but should include components that address more advanced concerns such as exploring and experimenting.

However, the study revealed that the “personal,” “management,” and “consequence” stages were the most common stages of concern for participants. This finding suggests that professional development on graphing calculators should allow participants to engage in activities that include topics most relevant to their classrooms and to require participants to consider aspects of implementation in their personal situations (Chamblee et al., 2008). In their study on the impact of graduate studies on TPACK development, Manfra and Spires (2013) found the greatest impact on TPACK development occurred when teachers made connections to the needs in their own classrooms. They stated that teachers “were much more likely to integrate teaching strategies learned in the program if they could identify a direct connection to their practice” (Manfra & Spires, 2013, p. 406).

Table 2.6

Concerns Based Adoption Model Stages of Concern from Chamblee et al., 2008, p. 186

Stage	Description
Awareness	Individual has little concern and involvement with the innovation
Informational	Individual has general awareness of the innovation and interest in learning more about the innovation
Personal	Individual is uncertain about the demands of the innovation and role in the innovation
Management	Individual’s attention is focused on the processes and tasks of using the innovation and the best use of information and resources
Consequence	Individual focuses on impact of the innovation on their students
Collaboration	Individuals focus is on coordination and cooperation with others regarding use of the innovation
Refocusing	Individuals focus on the exploration of more universal benefits from the innovation

Long term teacher support for technology implementation. Chamblee et al. (2008) found no significant differences in pre- and post-concern level scores among teachers that participated in the two-week graphing calculator professional development, but indicated that the results were not surprising because longer periods of sustained professional development are often needed to move through the more advanced stages of concern. The need for on-going support over a long period of time is essential for adopting major pedagogical changes such as the incorporation of technology (Tyminski, Haltiwanger, Zambak, Horton, & Hedetniemi, 2013; Chamblee, Slough, and Wunsch, 2008.)

Using an intensive and long-term professional development treatment, Tyminski et al. (2013) found that the middle grades mathematics teachers were able to develop their knowledge of inquiry-based instruction with technology. Participants in the first year of professional development focused on developing inquiry-based pedagogical skills and then the participants in the second year (a few were added to those from year one) integrated technology into their new teaching approached. Results indicated that the process of preparing teachers to use technology within an inquiry-based teaching model required substantial support and time, and it may be necessary to support teachers in developing their inquiry skills before introducing technology.

Beliefs impacting teacher practice. Links between beliefs and practice are often described in the research on how teachers' pedagogical beliefs impact their classroom practice (Kagan, 1992; Ertmer, 2005). Wachira and Keengwe (2011) found that beliefs about teaching and learning motivate the actions of teachers in the classroom. In her review of empirical studies during the 1980's, Kagan (1992) found two common claims that have since been studied further and used to inform teacher education and professional development. First, it is very difficult to change teachers' beliefs. Second, the beliefs that teachers hold are generally consistent with their pedagogical styles and often influence classroom decisions more than teacher knowledge. The findings summarized in Kagan's (1992) report have led to further research in this area. For example, Ertmer (2005) reviewed and summarized more recent reports on teacher pedagogical beliefs that have consistently shown that teachers' beliefs do influence teacher practice. However, in some cases teacher practice was inconsistent with teacher beliefs due to "contextual constraints, such as curricular requirements or social pressure exerted by parents, peers, or administrators" (Ertmer, 2005, p. 29) or as a result of competing beliefs.

The shift toward a social constructivist approach is often presented in mathematics research as significant for developing the beliefs, skills, and knowledge to effectively implement reform methods in the classroom (Ball, 1994; Ertmer, 2005; Ertmer, Ottenbreit-Leftwich, Sadik, Sendurur, & Sendurur, 2012). According to Ball (1994) the prior experiences, beliefs, and knowledge of teachers entering professional development have a significant impact on the effectiveness of the program and must be considered by those providing the learning experiences. She explained that teachers face challenges when engaged in professional development on reform methods because it requires

> revising deeply-held notions about learning and knowledge, reconsidering one's assumptions about students and images of oneself both as mathematical thinker, a cultural and political being, and teacher- all this, and developing new ways of teaching, reflecting and assessing one's work (Ball, 1994, p. 20).

Often preparing teachers to use general reform methods requires that teachers interpret and understand mathematics and the learning of mathematics in ways they have not before and at times they must change their beliefs about teaching and learning mathematics (Drake, 2006).

The value of an instructional tool or the teacher's perceived importance of that tool is associated with the concept of teacher beliefs (Ertmer & Ottenbreit-Leftwich, 2010). Teachers often place higher value on a particular approach or tool if they believe that the approach or tool will help to accomplish the goals they deem important (Tonduer, Hermans, van Braak, & Valcke, 2008). For example, if a new pedagogical approach is learned by a teacher within content specific to their grade or subject area, then the teacher is more likely to hold a higher value belief about the tool. Development of value beliefs is important particularly with respect to technology in the classroom because teachers with higher value beliefs about a tool are more likely to implement that tool in the classroom (Zhao, Pugh, Sheldon, & Byers, 2002)

Teachers' elementary mathematics teachers' life stories were used to gain insight into the beliefs and knowledge behind their implementation of an elementary reform curriculum (Drake, 2006). In Drake's (2006) study, she described how she applied a sense-making theoretical framework, citing Weick (1995), to interpret the mathematical life stories of the participants. She stated that the teacher must not only understand the policy or program to be implemented but also, as part of the process of sense-making, make these new ideas fit among his or her own beliefs and understandings in order for implementation to occur. Six of her twenty participants told stories that included turning-point situations and Drake claimed that these turning points opened up the teachers to making sense of the reform curriculum through the processes of noticing, interpreting, and implementing. Drake (2006) found that out of twenty participants, only two teachers showed true reform-oriented practices. She reported that the mathematical life stories of the two reform-oriented teachers were similar in that they described turning points that involved new and deeper understandings of mathematical concepts, as opposed to others who described turning points that involved understanding the benefits of particular tools such as manipulatives. The teachers who experienced content-based turning points held beliefs about the reform curriculum as a way to further develop their new mathematical understandings; however, the tool-based teachers attempted to adapt the curriculum to incorporate the new tools in ways that did not always coincide with the curriculum developer's goals. Drake (2006) suggested that teachers who experience meaningful mathematics learning through reform methods are more likely to integrate the methods in their own teaching. Teachers whose stories included developing a deeper understanding of mathematics at some turning point were more likely to look for a tool or curriculum that will result in the same level of understanding for students when used in the classroom (Drake, 2006). However, further research is needed to determine how teachers' mathematical learning experiences and beliefs about learning mathematics can significantly influence the implementation of other new teaching strategies such as technology as an exploration tool.

Belief specifically impacting the use of technology in the classroom. Okumus et al. (2016) reported on their study of thirty-four teachers that engaged in in-person and online professional development for using Geometer's Sketchpad [GSP] then implemented the technology in their classrooms. The researchers used observations and teacher interviews to explore the decisions that teachers made when integrating GSP. After participating in the professional development, the interviews with teachers revealed that their implementation practices were related to their perceived ease of use of the technology. The beliefs that teachers held about GSP's alignment with the curricular goals, prior experiences with GSP, and comfort levels all impacted the decisions of teachers to use the technology.

Smith, Kim, and McIntyre (2015) also explored the connection between mathematics teachers' beliefs about the nature of mathematics, role of teacher, and role of technology and their TPACK. They found that the belief that technology should be used to explore mathematics was associated with a high level of TPACK. In addition, teachers with student-centered views about the role of teaching were more likely to integrate technology in effective ways but did not guarantee that the use of technology would be effective. Finally, the teachers that did not hold a view of technology use in the classroom consistent with effective use were unable to design appropriate technology activities for students.

Ertmer (2005) summarized the findings of others to argue for the need for research on how beliefs impact teacher practices in regards to using technology and even suggested that pedagogical beliefs may be “the final frontier in our quest for technology integration” (p. 1). The findings common among the studies also provided three major implications for professional development and teacher education. First, teachers must be provided with experiences through which they can develop beliefs or change beliefs that impact technology integration. Next, vicarious experiences such as observing an effective cooperating teacher may provide an “informational and motivational” (Ertmer, 2005, p. 33) model, increasing the likelihood of successful implementation. Finally, professional learning communities can be used to support teacher implementation of technology by influencing the social-cultural factors such as the values and norms of the school environment.

Ertmer and her colleagues have produced a substantial amount of research on the association between teacher beliefs and technology use in the classroom (Ertmer, 2005; Ertmer & Ottenbreit-Leftwich, 2010; Ertmer et al., 2012; Ertmer et al., 2007; Park & Ertmer, 2008; Sadaf, Newby, & Ertmer, 2012). Across their studies, they indicated that teacher beliefs are a critical component to technology integration. In a study that examined the impact of a problem-based learning approach on pre-service teachers’ beliefs about technology use, Park and Ertmer

(2008) found that the pre-service teachers' indicated significant changes in intended teaching practices toward a more student-centered approaches after engaging in various problem-based learning experiences. Changes in teacher beliefs often follow successful experiences with new teaching practices (Ertmer, 2005); therefore, although the problem-based learning approach did not result in significant changes in teacher beliefs, Park and Ertmer (2008) indicated that this approach could be used to increase the likelihood of technology use by teachers and in turn impact their beliefs. Although these studies support the importance of teacher beliefs, Ertmer (2005) recommended that research is needed to further examine the relationship between beliefs about technology and pedagogical beliefs and also to examine how pedagogical beliefs are formed.

The focus of several other studies by Ertmer and her colleagues was on expert or exemplary technology-using teachers and the factors that were associated with their success (Ertmer et al., 2012; Ertmer et al., 2007). The rationale for the focus on exemplary technology-using teachers was to identify specific factors that have contributed to the success of the teachers. Barriers are often discussed and identified in the literature, but Ertmer et al. (2007) indicated that the enablers or specific factors that contribute to overcoming barriers need to be identified and understood. Twenty-five teachers identified as expert users of technology due to their statuses as technology award winners were surveyed about their perceptions of what factors contributed to their successful technology integration. Enablers and barriers alike that are intrinsic such as teacher beliefs and self-efficacy were rated by the teachers as having had more impact on their technology use than extrinsic enablers and barriers (Ertmer et al., 2007). The intrinsic factors inner drive and personal beliefs were indicated as most influential by the teachers (Ertmer et al., 2007). They concluded that "intrinsic belief systems appear to be strong, if not the primary, contributing factor in teachers' efforts" (Ertmer et al., 2007, p. 57).

Other researchers also noted the importance of transforming beliefs and attitudes about technology in preparing teachers to use technology in the mathematics classroom (Williams, Foulger, and Wetzel, 2009; Kurz and Middleton, 2006.). In their study, Williams et al. (2009) used the context of a pre-service education course to explore how collaboration in small groups, scaffolded exploration of technology tools, and subsequent "mini-teach" lessons that engaged classmates in an activity supported by the newly discovered technology tool affected participants' attitudes and beliefs about technology and possibly influenced their use in the future. Participating in the discovery of a technology tool and integrating it in the context of learning mathematics resulted in a reported increase in comfort and confidence with using technology, expressed intentions to conduct reform-oriented classrooms, and overall significant shifts toward viewing technology positively as a valuable teaching and learning tool by the participants (Williams et al., 2009).

Summary. Studies have identified the knowledge needed for effective teaching with technology (Mishra & Koehler, 2008) and ways that TPACK can be developed (Niess et al., 2009). Many teachers come into the field prepared to begin using technology but by in large do not use the technology available and if so, very rarely is it in effective ways. Barriers and enablers to technology implementation have also been widely studied. Although several researchers (Mistretta, 2005; Williams et al., 2009) have indicated that many methods for preparing teachers to use technology resulted in a change of attitude toward and more confidence with technology, there is little research indicating that teachers subsequently use technology in the classroom. Research on the technology implementation practices of teachers is still incomplete likely due to the complexity of the factors that impact teachers' pedagogical decisions (Drake, 2000). Ultimately, "teachers' responses to reform appear individual and idiosyncratic" (Drake, 2000, p. 53).

Table 2.7

Summary of Research on Teacher Implementation of Technology

Author (Year)	Major finding(s)
Keengwe et al. (2008)	Teacher must hold social constructivist views.
Guerrero (2010)	TPACK needs to be developed.
Williams, Foulger, and Wetzel (2009)	Knowledge was developed as a learner of mathematics with technology; profession development models good teaching.
Norton et al. (2000)	School norms and culture can support or impede implementation.
Bennison and Goos (2010)	School environment, beliefs and knowledge, and opportunities to learn can support or impede implementation.
Zhao et al. (2002); Ottenbreit-Leftwich, Glazewski, Newby, and Ertmer (2010)	Implementation may be determined by the value judgment by the teacher of technology for student learning.
Ertmer (2005)	Beliefs about teaching and learning can support or impede implementation.
Chamblee, Slough, and Wunsch (2008)	On-going supports are neccessary.

Narrative Inquiry in Education

Prior knowledge, experiences, and beliefs and their complex interplay influence how teachers respond to reform (Ball, 1994), suggesting that a narrative approach to inquiry might be useful for this study as it would allow for the development of deeper understanding of the elements influencing implementation of technology. Narrative inquiry practices in education can be traced back to John Dewey who considered both experience and continuity of experiences as essential impacts on education (Clandinin & Connelly, 2004). Clandinin and Connelly (2004), suggest that when considering issues in education, one must balance both personal and social factors, in the context of the past, present, and future to gain a clear understanding of what issues are at play. Their narrative approach to educational research reflects their views of teaching and teacher knowledge as narrative in nature, embedded in personal and social landscapes and changing over time. Also, Clandinin and Connelly (2004) claim that narrative describes both the "phenomena under study and the method of study" (p. 4) that best address the various factors and their relationships that impact educational experiences.

Clandinin and Connelly (2004) reported that narrative inquiry has been used in multiple fields, and a commonality that they found was the fields were highly transitional. Experiences and events are described in the context of a life story rather than as stand-alone elements. This allows the researcher to gain a deep understanding of the experiences as a whole, as opposed to unconnected events. In this way, Clandinin and Connelly (2004) argued that narrative researchers are interested in "day-by-day experiences that are contextualized within a longer-term historical narrative" (p. 19). Comparing narrative inquiry to some other forms of inquiry is like comparing open-ended questions on a mathematics test to closed-ended questions. A single numerical answer can tell you if a student can do a particular problem in a particular way at a particular time, but it does not tell you the extent of the knowledge the student had about an objective, their ability to make connections among objectives, or their ability to apply their learning in different ways.

Teachers develop mathematical identities as teachers and as learners of mathematics in complex ways. According to Drake (2000), these identities are closely linked to the ways teachers respond to reform. Narrative provides opportunities to explore the "knowledge, beliefs, and experiences" (Drake, 2000, p. 58) that shape teacher identities and influence the implementation practices of teachers. Ertmer et al. (2007) expressed the need for a deeper understanding after identifying several important factors that contributed to the implementation of technology by teachers. They stated that

> It is important to more fully understand these factors (such as beliefs, practice, and developmental processes) in order to encourage other teachers to achieve similar levels of technology integration. Thus, future researchers should investigate the critical experiences, beliefs, and practices that have contributed to teachers' ability to integrate technology successfully in order to help us understand how to achieve similar results with other teachers (Ertmer et al., 2007, p. 58).

The plan for this study was modeled after Drake's (2006) study in which she explored elementary teachers' mathematical life stories in order to understand their choices pertaining to the implementation of a reform curriculum. This study is different in that I used life stories to understand the experiences, beliefs and knowledge related to teaching with technology and the impact on secondary teachers' decisions related to implementing technology.

Drake (2006) described significant turning points in elementary teachers' mathematical life stories that she claimed directly affected the implementation of reform curriculum, and she explained how the narrative methodology provided "a more contextualized and integrated view of teachers' beliefs, knowledge, and prior experience" (p. 580) allowing her to identify the turning points and their impact. Although it was indicated that this methodology only allows for an understanding of the subjects at a particular point in time, Drake (2006) argued that this provided for an opportunity to better compare the stories across subjects and to identify broad patterns. Ultimately, the understanding of the experiences and beliefs that make up teachers' mathematical life stories can aid researchers in understanding the implementation behaviors of teachers.

Conclusions and Questions

In conclusion, the studies on preparing teachers to use technology effectively do not explain the idiosyncratic practices of teachers (Chai, Koh, & Tsai, 2013; Yigit, 2014) since we do not understand why professional development and teacher preparation programs designed to help teachers infuse the technology into the mathematics instruction have very little effect on most, nor do we understand what events occurred that made it successful for others. Studies often examine factors as isolated influences and fail to consider the complex environment in which teachers must navigate as they attempt to drastically shift their teaching methods (Smith, Kim, & McIntyre, 2015). Along with the evolution of the technology tools, mathematics learned with technology, approaches to teaching with technology, and technological abilities of students, also evolve and expand. It is important to continue to examine cases of successful technology implementation and understand the factors that contributed to this success. It is also important to consider a wide variety of contextual factors that contribute to the varied implementation practices of teachers that are prepared and supported to use technology. A narrative approach to inquiry was chosen to provide a more contextual view of how the teachers in my study came to effectively use technology and the contributions of their TPACK, beliefs, and experiences.

The specific research questions for this study follow:

1. What are the events experienced by teachers related to learning to use technology as a reasoning and sense-making tool in the secondary mathematics classroom?

2. How did the events and experiences in the personal narratives of secondary mathematics teachers influence their implementation practices of technology in the context of reform mathematics?

3: Methodology

The purpose of this qualitative study was to understand the events that have influenced the implementation practices of three secondary mathematics teachers identified as effective in using technology as a tool to support their students' mathematical reasoning and sense-making. Throughout this study, the implementation of technology as a reasoning and sense-making tool during instruction in the secondary mathematics classroom is defined as teachers engaging students in inquiry-based learning involving mathematical technology tools that foster robust mathematical understandings (Dick & Hollebrands, 2011). Qualitative research in general and narrative inquiry in particular allowed for an in-depth examination of the subjects. This chapter's discussion begins with the theoretical framework, followed by the general research design, the procedures used in this study for data collection and analysis, and finally issues related to the trustworthiness of the findings.

Theoretical Framework

This study is grounded in social constructivism. First, I held a social constructivist worldview as I sought to understand each participant's "subjective meanings of their experiences" (Creswell, 2007, p. 20) related to learning to use technology in their classrooms. The goal of my research was to "inductively develop a … pattern of meaning" (Creswell, 2007, p. 21) about the events related to teachers becoming effective technology users rather than test a predetermined conjecture. Social constructivism states meanings are developed through social interactions and contexts, and I thus need to acknowledge that any interpretations I made are "shaped by [my] own experiences and background" (Creswell, 2007, p. 21). This guided my selection of a qualitative research design described in the next section.

Second, this study is grounded in the stance that effective technology use requires that a teacher knows how to apply practices consistent with social constructivism into his or her instruction (Ertmer, Ottenbreit-Leftwich, & York, 2007; Koehler & Mishra, 2005). Social constructivism in the classroom is described as learning in an active manner where learners must build knowledge through the social interaction with the physical world and negotiate the new knowledge within their current body of knowledge (Jaworski, 1994). Thus, since this study explored teachers' experiences as they became effective users of technology with practices consistent with social constructivism, this stance informed my choice of procedures including data collection, code development, and case analysis.

Research Design

Qualitative researchers seek to study experiences, interpret these particular experiences, and find meaning in experiences (Eisner, 1998). A major tenet of qualitative research is that it is conducted in the natural setting of the phenomenon under study (Creswell, 2009). The term "naturalistic" is often associated with what qualitative researchers study, meaning situations are studied as they are (Lincoln & Guba, 1985; Creswell, 2007). In this way, researchers are able to observe behaviors and interactions within the contexts that they would normally occur. A second major characteristic of qualitative research is that the instrument through which one studies experience is the researcher (Creswell, 2009; Eisner, 1998). Data are collected from observations, interviews, and artifacts by the researcher. Many critics of qualitative inquiry point to the subjective nature of how experiences are observed, interpreted, and represented. How events are perceived "is influenced by skill, point of view, focus, language, and framework"

(Eisner, 1998, p. 46). Eisner (1998) explained that educational connoisseurship, or the refined skills of perception, is a required trait of a qualitative researcher in an educational setting. However, qualitative inquiry requires a researcher to go beyond perceiving to making meaning out of what has been observed. It was important for me as the researcher to recognize how my history and understandings influence the interpretation of data.

Third, qualitative researchers use multiple sources of data to find themes or patterns and also to support their findings (Creswell, 2009). Qualitative researchers create a representation of the experience that is "supported by evidence that is never incontestable; there will always be alternative interpretations" (Eisner, 1998, p. 86). Eisner (1998) used the term "structural corroboration" for using multiple sources of evidence to support conclusions. He compared this strategy to a puzzle where the more pieces of information provided, the clearer the picture becomes. This is the strategy that was employed to corroborate the claims made in this research.

A reasonable interpretation of events in qualitative research is built using inductive data analysis (Creswell, 2009), another major characteristic of qualitative research. Eisner (1998) pointed out that qualitative researchers must be able to identify significant events or behaviors and be able to interpret what is observed with "unique insight" (p. 35) but in a way that can be judged as reasonable by others. The inductive process, highlighted by building patterns and themes through an iterative process of data collection and analysis that is comprehensive and often collaborative between researcher and participant, supports the reasonableness of the researcher's findings. Finally, qualitative methods are employed to gain a broad picture of the experiences of the teachers related to their technology use because qualitative studies can provide a holistic understanding of a complex situation (Creswell, 2009).

Case study research. Case study research is commonly used in the social sciences for the examination of one or more cases of a particular phenomenon. Purposeful sampling is often used in case study research in order to examine a specific issue across one or more cases (Creswell, 2007). The purpose of this multiple case study was to examine an issue across several cases of teachers who use technology effectively using multiple sources of data. Within-case analysis can provide a holistic and in-depth description of each case and any themes within each case (Creswell, 2007). A secondary analysis to identify common themes among the different cases is called cross-case analysis (Creswell, 2007). During this research, three cases of teachers who effectively use technology were examined using both within-case analysis and cross-case analysis. Although themes were identified within and across the cases, the findings are not meant to be generalizable but rather useful for further understanding of the complex factors that impact teachers' technology implementation decisions.

Narrative inquiry. Narrative research includes "activities involved in generating and analyzing stories of life experiences and reporting that kind of research" (Schwandt, 2007, p. 203). Narratives are a natural way to reason about and understand life experiences (Kvale and Brinkman, 2009). The narrative methodology was used to allow teachers to tell about their experiences in order to help to understand their journeys to becoming effective technology users. A case study design built on narrative inquiry was used because there was a need to study multiple subjects in a way that elicited contextualized stories about their experiences with learning to use and implemented technology.

Clandinin and Connelly (2004) described a three-dimensional narrative inquiry space to provide a visualization of how the elements of narrative inquiry fit together and are considered individually (see Figure 3.0). The first dimension, interaction, places experiences on a scale of personal to social. The second dimension, continuity, describes where in time an experience can be placed: past, present, or future. The third dimension is labeled place or situation to describe the specifics of the environment present in the experiences.

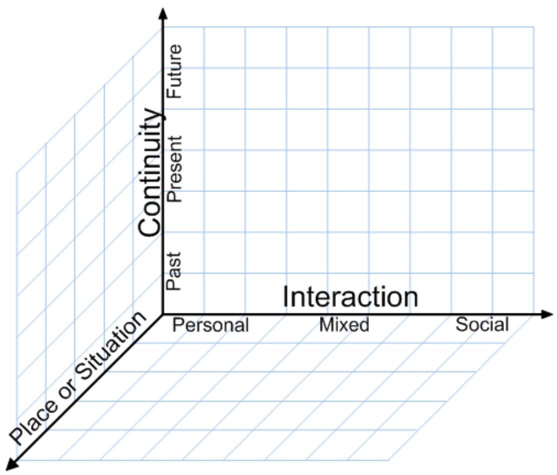


Figure 3.0. The three dimensional life space. Reprinted from *Narrative inquiry: Experience and story in qualitative research* (p. 20), by D. J. Clandinin and F. M. Connelly, F. M., 2004, San Francisco: Jossey-Bass. Copyright 2004 by John Wiley and Sons.

The three-dimensional life space was considered when planning the narrative interview in order to elicit stories that are highly contextual and provide the most holistic understanding of the experience. The three-dimensional life space also helped to organize the analysis of each subject's life narrative. Most types of narrative interviews are designed to have broad life history and oral history as their culminating product; however, narrative interviews can also be designed to focus on a specific time period and course of action that will result in a snapshot of that time period (Kvale and Brinkman, 2009). The purpose of this study was to understand how teachers learned to use technology effectively and what events in their lives motivated or enabled this use. Using a narrative interview allowed me, as the researcher, to directly ask about the events and stories that helped shape the beliefs and knowledge related to technology of each teacher and to collaborate with the interviewee to create an accurate and understandable story.

Narrative self-study. The researcher participated in this study as both a subject and researcher. Characteristics of autoethnography were used "to describe and systematically analyze (*graphy*) personal experience (*auto*) in order to understand cultural experience (*ethno*)" (Ellis, Adams, & Bochner, 2011, para.1). This methodology embraces a researcher's individual subjectivity and stance that influences the research, but many argue it must be combined with acceptable qualitative analysis conventions (Ellis et al., 2011, Para. 3). Researchers using a self-narrative or autoethnography methodology need to be selective in the events and stories that are considered data; thus, they need to focus on finding connections between self and others. Significant stories such as "epiphanies" (Ellis et al., 2011, para.8) relevant to the purpose and/or audience of the study should be analyzed using theoretical frameworks, methodological tools, and/or research literature. Additional issues related to autoethnography will be discussed at relevant points later in the chapter.

Researcher Stance and Bias

In this study, I served as the researcher, as a subject of a case study, and as a participant in some of the events described by the other subjects. Thus, it is important to clearly identify my role as both the researcher and as a participant and to critically examine my background that not only led me to this study, but also shaped the values and biases that have impacted my interpretations throughout the study (Creswell, 2009). Ultimately, I proposed this study due to my own experiences as a teacher and teacher leader heavily involved in providing professional development opportunities to colleagues since my first year as a teacher. These experiences have served to strengthen my belief in this teaching approach of technology to support student learning in the classroom. I conducted this study to understand why some teachers had made the implementation of technology in the classroom a priority and others did not, despite having similar educational and/or professional development experiences.

In addition to understanding the events that led me to my research questions, it is important for readers to know how my own experiences may have affected the collection and reporting of data. I had collaborated professionally with the participants from this study in the past. Thus, I went into the study with assumptions about each participant that needed to be corroborated or disproved with evidence collected. I took care to use evidence in reporting my results and frequently checked my claims to be sure they did not rely on my assumptions about the subjects. In addition, I had developed a friendly rapport with each teacher. Due to this rapport, I feel that each participant was very open and honest about his or her experiences and there was a mutual trust. However, one participant repeatedly expressed concern about one of his observed lessons not being as inquiry-based as he "would have liked it to be." As with the participant in the study other than myself, I took care to assure them that I was not inquiring about their experiences to judge them but rather to understand how effective teachers develop. Finally, my role as a mentor teacher equipped me with skills that were used to critique the use of technology by the teachers in this study.

Subjects and Context for Study

This section provides a description of a related pilot study and how it informed the current study, a summary of the participants and how they were selected for this study, and the context for the study. Next, the methods used for data collection including interviews, observations, and artifacts for each teacher are discussed. Then, the coding and data analysis procedures including cross case analysis are explained.

Pilot study. A pilot study was conducted prior to proposing this study that informed some of the methodological choices. The teacher in the pilot study agreed to record two lessons and participated in a face-to-face life story interview and one debriefing by email. Given that there was only one participant in the pilot study, comparing cases or finding themes across participants was not possible; however, the events in the teacher's narrative were compared to the current research literature to determine how the robust understanding of the participant's mathematical and teacher story could support better understanding of her teaching practices with technology. Close attention was paid to how my methodological decisions either contributed to that understanding or how they needed to be modified for future research. The impact of the pilot study is further discussed throughout the details of the methodology for the main study.

Participant selection. The participants, including the researcher, were selected as "highly unusual manifestations of the phenomenon" (Miles & Huberman, 1994, p. 28) to be studied. A reputational case selection method (Miles & Huberman, 1994) was used to identify the participants as extreme cases of effective technology users by the researcher. This was informed by first hand classroom observations of participants or interactions with the participants in a multi-faceted professional development program including conversations in which they discussed their commitment to using technology to support student learning. This group of participants was a convenience sample in that they were easily accessible to the researcher and already known to be using technology effectively in the classroom.

Table 3.0 shows a summary of details about each participant including a list of professional development participation that is described further in the next section. Prior to the study, the participants demonstrated a commitment to enhancing their professional knowledge in order to improve their teaching by continuing their graduate studies; participating in nonmandatory professional development for increasing their technological, mathematical and pedagogical understandings; promoting innovative technology-enhanced instructional practices through mentoring pre-service teachers; and serving as teacher leaders at their schools to encourage and act as resources for other practicing teachers wanting to learn to implement new instructional approaches such as effective technology integration.

Table 3.0

Summary of Participants

	Mrs. Alpha	Mr. Beta	Mrs. Chi
Age	36	27	63
Years of teaching experience	10	5	35
Education	Bachelor’s and Master’s degrees in mathematics education	Bachelor’s and Master’s degrees in mathematics education	Bachelor’s degree in mathematics education
Subjects taught	Geometry Discrete Math	Algebra I	Algebra I Pre-Calculus AP Calculus
Professional development involvement	Summer, quarterly, and technology-based professional development program; teacher leader fellowship program	Teacher leader fellowship program	Summer and quarterly professional development program

The participants were employed as secondary mathematics teachers at the time of data collection, although each taught in a different district and was at a different stage of his or her teaching career. The Participant Informed Consent Form (see Appendix C) was used to ensure that each participant was fully aware of the purpose of the study, their expected participation, any risks, and how the data would be used. Each superintendent and principal was provided with a Study Proposal Form (see Appendix D) once the participant's consent was gained in order to inform the institutional gatekeepers of the purpose of the study and the participant's responsibilities, and also to gain permission to conduct this research in the school setting (Creswell, 2009). Relationships between participant and observer change over the course of the inquiry, and the levels of trust and openness in the relationship also play a part in the story being told and retold (Clandinin & Connelly, 2004). It is important for researchers to clearly establish their purpose and at the same time help the participant to understand that multiple purposes are often shaping and developing throughout the process of inquiry. Thus, the participants were informed as part of the consent that the recorded lessons did not serve as evidence to judge their teaching but only provided more evidence of their uses and implementation practices with technology.

Common professional development experiences. As noted earlier, all of the participants had significant common experiences in which they collaborated with each other in a long-term, multi-faceted professional development program prior to the collection of data. The Mathematics and Science Partnership (MSP) program was supported by funding from the National Science Foundation. The MSP's mission was to improve the mathematics achievement of K-12 students by addressing curriculum alignment, improving the preparation of new and practicing teachers to employ best practices in the classroom, and providing on-going support and resources for teachers, including the development of school- and district-based teacher leaders (Martin, Strutchens, Stuckwisch, & Qazi, 2011).

Several principles guided the development and implementation of the MSP (Martin et al., 2011). First, leadership teams included mathematicians and mathematics educators, as well as

K-12 teachers, evaluators, and administrators in order to engage a variety of stakeholders in the planning process. Next, the goal of professional development was to

> produce an articulated school effort to enhance students' motivation and achievement by improving teachers' attitudes toward, and use of, reform practices. The practices are student-centered, and they provide instructional scaffolding for students that allows them to move from what they know to what they do not know (Martin et al., 2011, p.108).

Additionally, the MSP program worked to cultivate parental and community involvement. Finally, the MSP's planning team remained committed to using research-based practices and relied on participant feedback to develop, adjust and implement each component.

Components of the professional development program. The activities or components of the MSP program are outlined in table 3.1. During the initial meetings facilitated by the program's planning team, the program's mission was collaboratively developed in partnership with more than a dozen school districts and two local universities (Martin, et al., 2011). K-12 teachers, mathematicians, and mathematics educators collaborated to develop a curriculum guide as an initial step to guide the next steps of the program. Additionally, the teams worked to examine textbooks and gave recommendations for participating school systems. Teacher leaders were identified from participating schools and districs; they attending quarterly workshops to develop the knowledge and skills needed to serve as leaders to support change in the schools and as liaisons to the MSP program. Teacher leaders were also provided workshops on increasing community engagement.

Table 3.1

Components of the Mathematics and Science Partnership and their Descriptions

Component	Description
Curriculum guide development	Analysis of local, state, and national standards; development of a common curriculum guide to guide instruction; annual updates to curriculum guide.
Textbook adoption	Analysis of textbooks for alignment with curriculum guide; recommendations for participating districts for adoption.
School and district-level teacher leader development	Quarterly, half-day workshops focused on developing leadership skills; leaders served as a liaison to the program and to provide knowledge and assistance to teachers in building to supporting implementation of program goals.
Summer professional development	Activities designed and modeled to deepen mathematical knowledge and develop new pedagogical skills to be implemented in the classroom; to develop a learning community of teachers; to learn to use the curriculum guide and recommended textbook resources. Sessions were also held for administrators and guidance counselors
Quarterly professional development	Half-day Saturday meetings building on and extending summer professional development
Pre-service preparation	Mathematics content courses for teachers; alignment of preparation for pre-service teachers with program recommentations; use of MSP teacher leaders and participants as mentors for field experiences.
Parent and community involvement	Workshops held to introduce teacher leaders to programs that promote community involvement, such as family mathematics night

Secondary mathematics capstone course	Secondary mathematics education course designed to model the best practices consistent with the MSP; content to show connections between the 6-12 curriculum and higher mathematics.
Technology-focused professional development (add-on program funded by a private foundation)	Workshops to engage participants in technology-based explorations to develop deeper mathematical knowledge and pedagogical skills needed to implement technology; technology resources provided.
Teacher leader fellowship program (three year program funded as a supplement to the initial MSP grant)	Support for secondary mathematics teacher leaders to pursue an advanced degree in mathematics education and develop advanced leadership skills. (A second program was offered to elementary teacher leaders through the NSP Noyce Scholarship Program)

Two-week summer professional development institutes were held for teachers K-12 aligned with the goals and principles of the MSP program, with a one-week follow-up institute the next summer. Presenter teams included mathematics educations, mathematicians, and K-12 personnel. The recommended textbooks were used to develop modules of instruction to prepare teachers to use the materials in their classrooms. Teachers attended a two week summer professional development session followed by a one week follow up session the following summer. In addition, teachers attended quarterly meetings on Saturday's during the school year following each summer institute. The professional development consisted of sessions for both pedagogy and mathematics content. Teachers discussed issues such as equity, discourse, and conceptual versus procedural understanding in the pedagogical courses. The content courses included modules from the selected textbooks that teachers worked through as students in order to better understand the content they were teaching, along with related pedagogical issues. A secondary mathematics capstone course was developed as part of a preservice component of the MSP program, designed to show the connections between secondary and high-level mathematics.

Several components emerged as the program progressed and additional funding was procured. A technology-focused series of professional development workshops funded by a private foundation were held to improve teaching and learning with technology. Funds from this component also provided technology resources for participants' schools. A fellowship program for secondary mathematics teacher leaders also was established as a supplement to the original MSP grant. Teachers who met the criteria, such as working at a high needs school, and were accepted as fellows in the program, participated in leadership workshops, courses to pursue an advanced degree, teacher leader roles, and served as mentors for pre-service teachers. Many also took a graduate version of the capstone course.

Subjects' participation in the professional development program. Both Mrs. Chi and Mrs. Alpha participated in the curriculum guide development, textbook adoption, and teacher leader development components during their 25th and first year of teaching, respectively. Since their beliefs about teaching and learning mathematics were aligned with those of the MSP program and they had participated as teacher leaders, they were asked to participate in training and subsequently lead summer and quarterly professional development sessions for other teachers over five years. They engaged in week-long training sessions each summer prior to the two-week summer institutes provided for the teachers. As part of the training and preparation, Mrs. Chi and Mrs. Alpha participated in pedagogy-focused sessions before presenting them, with leadership team members modeling the student-centered methods they were expected to implement as presenters, and collaborated with other presenters to design the mathematical content modules to be used during professional development that were aligned with the goals of the MSP program. They presented at the two-week summer institutes and the follow up quarterly meetings during the school year.

Mr. Beta was in the fourth year of his undergraduate program in mathematics education, which was also aligned with the goals and practices of the MSP program, during the final year of the summer professional development during which he worked as a student assistant for the MSP program. Also, Mr. Beta and Mrs. Alpha both completed the mathematics education capstone course that was a mathematics content course designed to be in alignment with the goals of the MSP program (Martin et al., 2011). In this course, the instructors modeled student-centered pedagogical methods, and they were engaged in activities that allowed them to build new understandings of higher mathematics from our knowledge of 6-12 level mathematics.

Mrs. Alpha also participated in the technology-focused professional development component of the MSP program, again as a presenter. She collaborated with another teacher leader to develop and implement professional development modules for using Geometer's Sketchpad to teach algebra and geometry. She was tasked to use the same student-centered methods to guide the participants to develop and/or deepen mathematical knowledge while engaged in technology-based explorations. Presenters and participants were also rewarded for their participation with classroom sets of the technology tools such as graphing calculators or software site licenses.

Throughout these MSP components, a cohort of teacher leaders, including the participants in this study, from a wide variety of school systems emerged from the program. The MSP led to the natural development of a professional learning community (PLC) for many teacher leaders. They collaborated with each other, with mathematics teacher educators, and with mathematicians to enhance their professional knowledge and act as a support system for each other as they continued to advocate for improved mathematics teaching in each of their school districts. The members of this professional learning community seemed to share a common vision of teaching and learning mathematics to support the development of mathematical power for all students that was often in conflict with the norms and pervasive beliefs in their individual schools. This common vision was evident by their commitment to enhancing their professional knowledge, their reported school and classroom experiences, and other actions designed to support and elicit change in their individual schools.

Mr. Beta and Mrs. Alpha were involved with a final component that emerged from the MSP program, the teacher leader fellowship program funded by the National Science Foundation. As a part of this program, Mrs. Alpha continued to serve ,and Mr. Beta began to serve, as a teacher leaders in their respective schools. The program also provided tuition scholarships and salary supplements to support their pursuit of advanced degrees in mathematics. The graduate courses included mathematics courses, some of which were designed specifically to develop their mathematical knowledge for teaching while others were designed to increase their general content knowledge, and mathematics education courses with themes such as equity, teaching methods, and research (Strutchens & Martin, 2017). They continued to attend quarterly professional development to develop their leadership skills and served as mentors to preservice teachers in one of the universities' secondary mathematics teacher preparation programs, which was involved in the MSP. It is also important to note that Mr. Beta and Mrs. Alpha both went through the same mathematics education pre-service program, which was aligned with the MSP's vision and goals. These experiences with the MSP and related programs provide a common backdrop for the events described in this study.

Data collection.

For each teacher, the researcher conducted a narrative interview, observed classroom lessons, held observation debriefings, asked follow up questions after several phases of analysis, and gathered additional artifacts that in total told a narrative story of how each teacher came to use technology in the mathematics classroom. Table 3.2 shows a summary of data collected for each case and Table 3.3 shows the timeline for data collection.

Table 3.2

Summary of Data Collected for Each Case

Teacher	Data collected			
	Interviews	Lessons observed	Lesson artifacts	Debriefing questionnaire
Mrs. Alpha	Audio recorded interview and follow up	2 video and audio recorded observations- Discrete mathematics and geometry	Student work samples for geometry lesson only; Lesson outlines	Both observed lessons
Mr. Beta	Audio recorded interview and follow up	1 video and audio recorded observations: algebra I	Student work samples for lesson observed	Via email for 1 lesson observed
Mrs. Chi	Audio recorded interview and follow up	2 video recorded observations: pre-calculus and algebra I	Student work samples for both observations	Via email for both observed lessons

Table 3.3

Timeline of Data Collection Procedures

Collection method	Week(s) of the study
Life story interviews	1-4
Member checks of life story outlines	5-6
Lessons incorporating technology recorded by subjects	5-9
Recorded lessons and related artifacts sent by subjects to researcher	10
Interviews probing recorded lessons (via email following transcription of each recorded lesson)	11-15
Follow up interviews (questions that arose during data analysis)	15 and after

Interviews. Narrative interview was chosen for this study because it allowed the researcher to directly ask for significant stories, and it allowed the teachers to describe the events in a way that encouraged details of characters, settings, time periods, emotions, and other salient pieces of evidence that were essential for understanding the many factors involved. In particular, each participant was asked to recall and retell the events that significantly impacted his or her teaching life trajectory. The interview protocol was based on McAdams' Life Story Interview protocol (McAdams, 1993) and Drake's (2000) adaptation of the protocol. The purpose of the interview was to allow participants to tell stories of significant events in their mathematical, technological, and teaching stories as they understood them. The interview questions encouraged the participants to provide detailed recollections of positive and negative events related to mathematics, technology, and teaching and other events that shaped their beliefs and knowledge about teaching, mathematics, and technology. Interviewees were also asked to identify the major influences on the development of their knowledge of technology.

During the pilot study, it appeared that it might be useful to include a question asking participants to describe their lowest point related to learning mathematics and it was added to the final interview protocol. Also, it seemed that it might be helpful to provide the participants with a copy of the protocol questions and sub-questions in advance of the interview in order to elicit more details about each event due to the length of each question. Also, to enhance the reliability of the interviews, the researcher engaged in interpretation during the interview, asked appropriate follow up questions to clarify or verify my interpretations during the interview, and maintained a "good listener" stance to encourage rich, detailed stories (Kvale & Brinkmann, 2009). The final protocol is given in Appendix E.

Each interview lasted about 90 minutes. Each interview was subsequently transcribed. Conventions were established and recorded during the transcription process to ensure consistency throughout the project (Kvale & Brinkmann, 2009). For example, abbreviations "EC," "IS," and "SS" were used in an additional column to indicate when the subject was speaking to the entire class, an individual student, or several students respectively during periods of dialogue.

As the researcher, I conducted the interview of myself, and I attended to several issues to ensure reliability. First, one particular event in my life story was very detailed and seemed significant at the time of the interview; however, after further examination it did not contribute to answering either research question. This mirrored the observations related to self-interview of Delamont (2009), who realized that although some incidents were important to her, they were not significant to the project's purpose or to perhaps a reader. Thus, during the interview process

I often considered what should be said about myself and what was unimportant as far as contributing useful insights relevant to the questions, while at the same time being careful to not eliminate potentially important details. Second, there may be issues of reliability related to self-study. However, it is possible that autoethnography can minimize some of the issues normally found in other qualitative research related to misinterpreting the stories of others. For example, in reporting about her autoethnographic writing, Wall (2008) stated, "It seems that unless data about personal experience are collected and somehow transformed by another researcher, they fail to qualify as legitimate" (p. 45). Autobiographical writing is as if there is one less child to distort the information passed along in the childhood game of "telephone". However, I do not make the claim that my story is more significant or contributes more to understanding the phenomenon than that of the other participants; thus, I chose to report my case in third person.

Observations and debriefings. Clandinin and Connelly (2004) explained that narrative inquiry requires more than just listening to participants' stories; the researcher must try to experience the events in the on-going narrative (Clandinin and Connelly, 2004). Thus, recorded classroom lessons and follow-up interviews were utilized to experience how the participants made decisions about and implemented technology in the classroom. After the initial interview, each participant provided one or two video-recorded lessons over a 4-5 week time period that reflected what they considered good examples of their integration of technology into the mathematics learning in their classroom. Two of the teachers wore audio-recording devices around their neck to ensure that the conversations with individual students or small groups were captured for transcription and analysis; the third teacher's recorder device followed her around the room, capturing conversations with students. The pilot study verified that the combination of video recording the full classroom with a focus on the teacher and audio recording for more individual discourse was adequate for collecting the data sought in the observations.

Following the researchers' viewing of the recorded lessons, participants were then asked to reflect and elaborate on their recorded lessons in a follow up interview in response to emailed questions. This method allowed for modification of the questions if needed to ensure understanding of the participant's intentions (i.e., content objectives and purpose of technology use) during the lesson and the history (i.e., source of resources used or the source of inspiration for the lesson) and development (i.e., whether this was the first time the lesson was being used or how it was modified from the first implementation and why modifications were made) of the lesson. Appendix F shows the debriefing template that was used for each participant. For my own debriefings, I created written reflections using the same set of questions to document the purpose of each lesson and the developmental history of the lesson.

Finally, the participants provided copies of work related to the recorded lessons from at least five different students with names and any other identifiable data blacked out or removed to ensure the anonymity of the students. Teachers were instructed to choose papers from students with a variety of levels of success in the class. These artifacts aided in the interpretation of the recorded lessons when the audio or video of the lesson left some doubt about the content of the lesson or conversation. Second, they provided more evidence of participant's use of technology to promote reasoning and sense making among students. I recognized that the effective use of technology could be different in each participant's classroom from my own, so the artifacts were necessary in exploring the extent to which the participants were in effectively using technology.

Additional interviews. After analysis of the data collected, I recognized that there were some additional questions about particular events in the teachers' narratives, so a set of follow up questions was submitted to each teacher. This also allowed me to ask questions to corroborate some of the conclusions that were made during the data analysis.

Maintaining confidentiality. Several steps were taken to ensure the confidentiality of the participants. Pseudonyms were used in all transcriptions and identifying details such school names were not included on transcripts. Date were kept confidential in a locked file box when not in use. All recordings were destroyed and audio-recordings were deleted from the recording device after the data had been transcribed. Also, names on students' work were redacted before being collected from participants.

Data Analysis

Table 3.4 summaries the procedures for data analysis used for each data set, including interviews, observations and related artifacts, debriefings, and follow up interviews. The table is organized chronologically in the order in which the entire process unfolded and mirrors the discussion that follows. The qualitative data analysis software Nvivo was used to code the transcriptions at each phase. Query reports, used to gather all units throughout the data with the same code(s), were created in Nvivo and used during subsequent analysis phases to ensure that all pertinent coded sections were included in each phase of analysis. A dictionary encompassing all the codes used in the study is given in Appendix G.

Table 3.4

Data Analysis Procedures by Data Set

Stage of analysis Procedures	Data source
Interviews, coding phase 1 Use of phase 1 a priori codes Coding check 1	Interview transcriptions

Interviews, coding phase 2 Use of phase 2 a priori codes Coding check 2	Interview transcriptions
Initial outline of events for each case Construction of outline of events Member check of outline of events Analysis of events in outline to identify events that explain implementation decisions	Events identified in interview transcripts
Coding and analysis of observations Use of codes for indicators of effective use of technology codes Analysis for evidence of effective use	Transcriptions of recorded lessons; student work artifacts
Cross case analysis	
Coding and analysis of TPACK development and barriers to use of technology Use of codes for barriers to technology, and TPACK stages and indicators codes Analysis of barriers to technology to identify evidence of implementation decisions and TPACK trajectories Addition of TPACK stages and barriers to technology to outline of events Analysis across cases for common events, stages of TPACK development indicators, and barriers	Transcriptions of interviews and lesson follow-up interviews
Identification and analysis of codes for factors related to technology implementation Identification of factors related to technology implementation codes Analysis across cases for factors for emergent themes	Transcriptions of interviews and lesson follow-up interviews
Additional interviews coding and analysis Member check of factors Additional follow up questions Use of codes for factors related to technology implementation codes Additional analysis of coding of factors for emergent themes	Transcriptions of additional interviews

Interviews coding, phase 1. In the first phase, the interview transcripts were coded using an a priori framework based on Clandinin and Connelly's (2004) three-dimensional life space as listed in Appendix G, Elements of Events, Phase 1. Each unit or event was first coded by the time period, place/situation, and interaction to identify the placement in the three-dimensional life space (see figure 3.0). Since the life story interview was designed to elicit a variety of stories from each participant, a "unit" for coding and analysis was defined as one event or story described by the participant. Several stories may have been given to answer a question, and some stories were told in the context of larger stories and were considered both a part of the larger story and a sub-story. For example, Mr. Beta described his internship and several stories were embedded within the story of his internship. The entire internship story was considered one unit for coding and analysis, the embedded story of learning to use conveyance technologies was coded and analyzed as one unit, and the embedded story of using mathematical action technology to get students to explore during his internship was coded and analyzed as one unit. During this phase of coding, a colleague in graduate school participated in a coding check exercise. We each coded 12 pages of transcripts and had an inter-coder reliability rating of 94%.

Interviews coding, phase 2. In the next phase, events were coded using codes adapted from Drake's (2000) study, including type (high point, low point, turning point), content (mathematical, pedagogical, technological, pedagogical including technology), tone (positive, negative, neutral), and specificity (not, slightly, fairly, very). During the coding process in the pilot study, an established definition was needed for coding positive, negative, and neutral events, as well as for the different levels of specificity. Drake (2000) used a comparison of the number of positive affect words to the number of negative affect words to determine the tone of an event when coding the events in the mathematical life stories of elementary teachers. Also, she used a four-point scale to determine specificity based on the description of the event, the time period that it occurred and the mathematical content involved (Drake, 2000).

During this process of identifying elements of the events, a second round of coding checks was conducted. Drake (2000) also employed the interrater reliability tests for tone and specificity that were used in this study. The same graduate student and I coded ten pages of transcripts and the inter-rater reliability for the check was 82%. The main reason for the decrease in percentage was due to an unclear definition of "turning point" since all but two of the non-matching coded units were in this area. After discussion, the definition of a turning point event was changed to the definition in Appendix G, and all previously coded sections were recoded to reflect the new definition. The final list of codes used is given in Appendix G, Elements of Events, Phase 2.

Initial outline of events for each case. For each participant, the events whose content was coded as "technological" or "pedagogical including technological" were arranged chronologically into an outline using the time period codes. Using this sorting method resulted in events that were clearly part of each teacher's development as an effective user or technology in the classroom such as professional development opportunities. Other events that may have contributed to the development were also included in the initial chronological outline of events but marked "may have contributed."

At this point, each teacher was asked to participate in a member checking exercise (Lincoln & Guba, 1985) to corroborate the outline of events in two ways. First, participants were asked to verify that the outline was an accurate representation of the events discussed in the interview. Secondly, each participant was asked if any of the events marked as "may have contributed" actually did contribute to his or her development into an effective technology user in the classroom or to his or her decisions to use technology. The feedback resulted in some timeline adjustments of events but did not result in any changes in the content or the representation of the events themselves. The feedback from each participant's member checking was used to develop the final list of events that are summarized in the findings in Chapter 4; thus, the a priori coding was used to recreate the "plot" of each story (Kvale & Brinkmann, 2009).

Coding and analysis of observations. Next, for each participant, the video- and audio-recorded lessons were transcribed. A two-column organization was used for this process to transcribe the dialogue that occurred in one column and the corresponding classroom actions in the other column. The observations' transcripts were coded using a checklist of observed behaviors that are indicators of effective technology use in the classroom (Mistretta, 2005) and other factors identified in the literature on effective technology use. The Observations section of Appendix G shows the codes used at this phase.

The unit of analysis that was used to code the observations was one complete action description, one interaction, or one conversation; a unit was coded if it contained evidence of an indicator of effective technology use. These codes were used to identify and explain specific examples of effective use of technology by each teacher in order to better understand their current use of technology. It was not the claim or goal to show that each lesson contained only indicators of effective technology use but rather that each lesson contained some aspects of effective technology use. Also, note that this list of codes is not exhaustive, as there are certainly other types of necessary and effective technology lessons, such as instructing students how to use technology. However, for the purpose of this study, the focus was on effective use of technology in lessons with mathematical objectives. Student work was examined and coded for effective use of technology indicators. From the observations, evidence was found of effective use of technology by each of the three teachers that is discussed in chapter four. These claims were corroborated using additional evidence from the observations and student artifacts as outlined in the next chapter.

Coding and analysis of TPACK development and barriers to use. During the analysis of the outline of events experienced by each participant, it became clear that the significant types of stories that were associated with teachers' decisions to change in Drake's study (2000) were not present in my study. Findings did not include significant high point, low point, or turning point stories that explained each teacher's technology use development or implementation practices. As the foremost theory on what teachers need to know in order to implement technology effectively, it became clear that ideas related to the development of TPACK were present throughout the events in each participant's development and needed to be explored. Thus, an additional phase of coding was undertaken, given that TPACK is the knowledge necessary to implement technology in effective ways. Indicators of the stages of TPACK development were used to code the stories in order to specify the stage of development described in each event in the outline. Figure 2.3 shows the TPACK stages of development, and Appendix G, TPACK Stages, shows the indicators that were used to code each experience. The interviews and transcripts from the lesson follow-up interviews were then also coded for evidence of the indicators of TPACK development stages. A TPACK stage was identified for an event if at least two specific indicators were found in an event that were consistent with that stage.

During the initial two rounds of coding of the events, descriptions of potential barriers to technology use and barriers to the development of productive attitudes toward technology use were also noted. Teachers often use barriers to technology use as reasons for not implementing technology, yet the teachers in this study did implement technology (Bennison & Goos, 2010).

Thus, it seemed likely that factors related to implementation practices may be associated with the stories that contained barriers. As a result, the transcriptions of the interviews and lesson follow-up interviews were coded for potential barriers to use, barriers to attitude, and barriers to use related to technology use and developing attitudes about technology as a teaching and learning tool. A barrier to use or attitude development was identified if the story included description of a factor did negatively impact the teacher's technology use (i.e. feelings of isolation in department due to beliefs about technology use), and a potential barrier was identified if the story included description of a factor that could possibly impact technology use. The codes used are given in Appendix G, Barriers to technology.

After the coding for TPACK and barriers was completed, the stories of the participants were further organized to show how their development into an effective user of technology included their development of TPACK aligned with the model in Figure 2.3. In addition, barriers were embedded into that organization. Thus, the outline of each participant's story showed their development of their use of technology in effective ways in alignment with the TPACK stages, with barriers embedded throughout those events. This organization became the basis for each case reported in Chapter 5.

Cross case analysis. In order to examine across all three cases, a mix of a case-oriented and variable-oriented approach was used (Miles, Huberman, & Saldana, 2014). First, developmental experiences and factors within each case were explored. Next, themes common across the cases were examined. According to Miles, Huberman, & Saldana (2014), the former method is useful for finding patterns among this sample set of cases; however, this method is particularistic and does not lead to generalizing. The latter method is helpful for identifying key themes that may not seem as significant in each individual case but may be a key variable in explaining a phenomenon when examined across cases. The purpose of this analysis was to deepen the understanding of the cases through comparison and identify any factors that may help explain teacher implementation practices with technology.

At this stage of analysis, the stories of each teacher were examined for any reasons teachers gave for implementing technology or for factors that were related to technology implementation practices. The codes in this phase were created by examining each event for direct statements or less direct evidence suggesting that the event contributed to the decisions that teachers made about the implementation of technology. For example, if a teacher stated a reason that technology was used, then it was coded for that reason. These new codes are referred to as factors due to the likelihood that they contributed to the implementation decisions of teachers. In order to increase the validity of the findings, once a code was developed, then each event across cases was examined for both evidence confirming or disconfirming it (Miles & Huberman, 1994). Queries were then run for each new code to gather and analyze all passages determined to contained evidence for each factor.

Several factors were identified in one case but not identified in other cases. In order to determine if all of the factors were themes across all cases, it was determined that additional interview questions were required. The additional interviews were developed to determine if the participant's story did in fact contain a given factor but was just not included in the original interview. Next, this second round of interviews were coded for the factors related to technology implementation. This disconfirmed two possible factors as themes (strong personal interest in technology and learning secondary mathematics through technology) and helped to corroborate the others. Seven themes emerged across the cases from the factors that influenced the technology implementation decisions of the teachers in the classroom and are the themes presented in the next chapter. Appendix G, Emergent Themes, shows the definitions for the final list of emergent themes that were coded.

Establishing Trustworthiness

An overview of the steps that were taken to ensure that the findings are valid and reliable are detailed in this section. Proper permissions and approval were obtained that were needed to conduct the study, to interview each participant, and to enter the schools of the participants. Multiple recording methods were used to ensure accurate and detailed transcriptions of the data as well as creating and maintaining consistent transcription conventions. Multiple sources of data were collected to aid in corroboration of the claims (Eisner, 1998). During the coding process, interrater reliability tests were conducted (Miles & Huberman, 1994); the code book was updated to address discrepancies that were identified. After identifying and outlining the significant events in each teacher's story, a member check with the participants was conducted (Creswell, 2007). Findings in the next chapter are presented with the thick descriptions (Denzin, 1989) to increase the credibility of the claims. Finally, these methods and findings have been reviewed and critiqued by the five professors on my committee from fields related to the study: mathematics education, mathematics, and educational research; their critiques through multiple phases of feedback have both supported and challenged the methods and findings.

4: Research Findings

In this chapter, I will first present a description of the case of each of the three teachers. Each case includes background information, findings related to each teachers' current use of technology to show how each teacher used technology in effective ways in his/her classroom, and a description of the events and corresponding stages of TPACK development that make up the narrative of how each teacher came to use technology in effective ways. Within each case, I will describe the barriers for technology use based on the identification and coding of barriers. In the second section of this chapter, I will present the themes that emerged during the analysis across cases that may be factors associated with each teacher's technology implementation decisions in the classroom.

The Case of Mrs. Alpha

At the time of data collection, Mrs. Alpha was in her tenth year of teaching high school mathematics at her second school of employment. The school she was employed at was in a rural college town in the Southeastern U.S. Mrs. Alpha had earned a bachelor's and master's degree in mathematics education and taught geometry and discrete mathematics. During the last ten years, she had also taught pre-algebra, algebra I, algebra II, and an International Baccalaureate high-level mathematics course. Mrs. Alpha reported that she frequently used Geometer's Sketchpad during parts of her instruction for geometry, and that she also used Excel and Geometer's Sketchpad occasionally during discrete mathematics instruction. She also indicated that she encouraged the use of graphing calculators for all of her students. In addition to teaching high school mathematics, Mrs. Alpha was very active in the MSP program participating in the curriculum guide development, textbook adoption, school-level teacher leader development, summer professional development, quarterly professional development, secondary mathematics capstone course, technology-focused professional development and the teacher leader fellowship program.

Current technology use. Two video recorded lessons showed Mrs. Alpha using technology in effective ways as a teaching and learning tool in the mathematics classroom. In this section, the lessons that were observed through recordings and the information gathered from the debriefing sessions are presented. The lesson observation transcripts were coded for any indicators that were consistent with established criteria in the literature about effective technology use in the mathematics classroom.

Observation 1. Mrs. Alpha's twelfth grade discrete mathematics class of 14 students used Geometer's Sketchpad to construct fractals including Lipinski's triangle and the Koch snowflake. Figure 4.0 shows a Sierpinski's triangle and figure 4.1 shows a Koch snowflake both with several levels of iteration, constructed using Geometer's Sketchpad. The observed class lasted 96 minutes. The students were given written step-by-step instructions to build the fractals each at their own computer. The students asked questions of each other and the teacher when they were unsure of a step in the written instructions. Mrs. Alpha clarified several steps to the entire class when multiple students were having trouble understanding how to execute certain instructions. By the end of the observation, each student had completed and printed at least one fractal and had begun to create formulas to determine the area inside the fractal at each level of iteration. A handful of students had completed the assignment by finding the correct formula for area and were moving around the room talking with other students. The conversations that were observed were mostly focused on offering assistance to classmates who were not finished with the activity. Mrs. Alpha ended the class with instructions for all students to have formulas ready for the next class period for discussion.

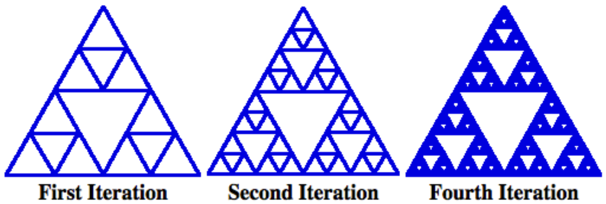


Figure 4.0. Several iterations of Sierpinski's triangle constructed using Geometer's Sketchpad.

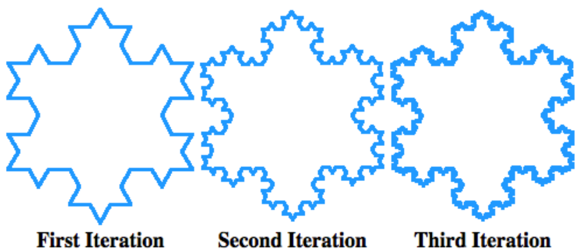


Figure 4.1. Several iterations of Koch Snowflake constructed using Geometer's Sketchpad.

Aspects of the observation were coded as indicating effective use of technology, based on the criteria set forth by Mistretta (2005). First, the lesson was coded as "standards-based" because there was a connection to particular mathematics standards. During the debriefing, Mrs. Alpha indicated that the purpose of the lesson was for students to engage in the recursive process of constructing a Koch snowflake and to create a formula for the area inside the fractal that could be applied at any level of iteration.

The lesson also included elements that were coded as "worthwhile task" because it promoted student reasoning and sense-making. For example, multiple times the dialogue during the lesson indicated that students were engaged in reasoning and sense making. Following is an example of that dialogue.

Mrs. Alpha: You are going to come up with the area formula for the fractal based on the previous area. How is that related to what we have been doing?

Student 1: It's a recursive formula.

Student 2: So it's just the area of the triangle?

Mrs. Alpha: No, you will change the triangle into a snowflake. So how will we use the area of triangle?

Student 1: Start with the area of a triangle and then see how it changes. And then we can change our formula.

Mrs. Alpha: How do you think the shape and area will change?

Student 2: It's going to get bigger.

Mrs. Alpha: Why do you think that?

Student 1: Because you are bending it outward so there is like more area in there.

In another passage of the debriefing that was coded as "worthwhile task," Mrs. Alpha described how she had found a version of the lesson on the internet and used it to create her own lesson, but the instructions in the version on the internet were too detailed and did not require her students to think about how they would complete subsequent steps. So, she rewrote the lessons to include less detail in the steps provided.

This lesson was also coded as "necessary" because the use of Geometer's Sketchpad was necessary to the success of the lesson. Mrs. Alpha stated that she had used the lesson one time before and changed it to require the completion of just one fractal. She noted that the construction of the fractals on the computer was time consuming but they would require multiple class periods if they had done them by hand. She further explained in the debriefing about the advantages of using technology for this lesson as follows.

> The technology created a more efficient and precise way to construct as opposed to the tediousness of a by-hand construction. There are some kids that would not be able to complete even the first level of iteration or would have given up entirely. So these kids were encouraged to keep going because they could be successful in the construction.

Thus, the technology was essential given the complicated nature of the constructions, the time constraint, and the importance of the precision for making correct conclusions.

Another aspect of effective technology use was coded as a "proactive strategy" (Hollebrands, 2007). The students in Mrs. Alpha's class used proactive strategies when predicting the next stage of fractal construction. One student stated, "If I use the same steps [to complete the next iteration] then it will just [iterate] this part [of the snowflake]. How do I get it to iterate the whole thing?" He was predicting what the outcome would be and trying to adjust how he was completing the next step in order to get the desired outcome. Next, this lesson was coded as "appropriate for task" (Dick and Hollebrands, 2011) since the tool enabled students to construct in an efficient and precise manner allowing them to focus on analyzing the process. Finally, the task was coded as "carefully designed" (Laborde, 2007), since it had been modified by Mrs. Alpha to connect to previous work. Mrs. Alpha stated in the debriefing that she used this lesson because it allowed students to connect their previous work with recursive and explicit formulas to the processes of constructing fractals and determining area.

Observation 2. Mrs. Alpha's tenth grade geometry class used graphing calculators to explore lines and their equations during the second observation. There were 28 tenth grade students present who were doing a graphing calculator-based exploration, with a goal of finding lines that formed isosceles triangles. Although many students started out strictly using a "guess and check" method by typing in a random equation and then looking at its graph, most began to predict how parts of the equation would impact the graph and were able to strategically choose equations to meet their goals. The students worked in pairs and copied solutions on to their own paper. After 30 minutes of partner exploration, Mrs. Alpha called students to the front of the room to present their answers to the group using the graphing calculators and their sheets under the document camera. Figure 4.2 shows a student's solution to finding two equations whose lines would form an isosceles triangle with the y-axis. There were multiple solutions shown for each problem and students discussed multiple strategies for finding the solutions.

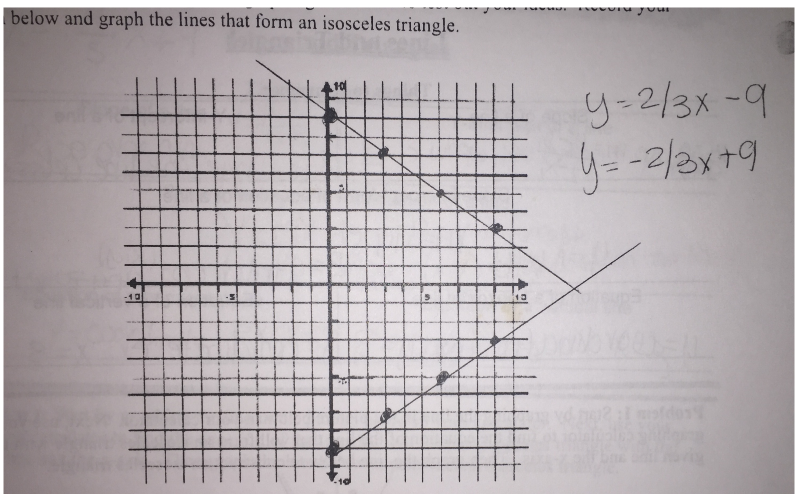


Figure 4.2. A student's solution to finding two equations whose lines would form an isosceles triangle with the y-axis.

Mrs. Alpha stated during the debriefing that the activity had come from a textbook that she had been encouraged to use during her internship that focused on using an investigative approach to learning geometry. She had modified it over the years to include different requirements to suit the needs of her students. For example, the original activity asked students to find three equations that form an isosceles triangle. Mrs. Alpha modified the lesson to include finding two equations that form an isosceles triangle with the x-axis and with the y-axis before finding three equations.

Several of Mistretta's (2005) indicators for an effective technology-based lesson were present during the observation. First, this lesson was coded as "standards-based" since several mathematics standards were incorporated in the lesson including using linear equations, properties of triangles, and using coordinate geometry to solve problems. Next, this lesson included elements that were coded as "worthwhile task" since students engaged in reasoning and sense-making. For example, students were engaged in connecting concepts of linear equations, geometric properties of triangles, and proof. Also, students used problem-solving skills to solve the problem and it was not a scripted step-by-step activity. In another example, students were engaged in reasoning and sense making (Dick and Hollebrands, 2011) as they experimented with different equations and discussed how they needed to be changed to fit the constraints of the problem. In the following excerpt, students made a connection to previous content.

Mrs. Alpha: Ok so we think it is isosceles? I agree. Somebody said something about symmetry or reflection. Who said that?

Student 1: Me.

Mrs. Alpha: Can you repeat and explain please?

Student 1: I just said that you have to take the original line and reflect it to the other quadrant.

Mrs. Alpha: Reflect over what?

Student 1: Y.

Mrs. Alpha: The y-axis?

Student 2: Yes! It's just like when we did the reflections on the graph and used the line to flip it over.

Student 2 was able to use what they had learned about transformations and apply it to make sense of the new problem. The lesson was also coded as “mathematics focused” since the technology was used to explore the mathematics and at no time was the focus of the lesson on technology in a way that took away from learning mathematics. Next, the students remained engaged most of the lesson and seemed to be benefitting from the use of technology based on the accuracy of their solutions so this lesson was coded as “engaged and benefitted.”

In other parts of the lesson, students employed “proactive strategies” (Hollebrands, 2007) as they predicted what changes were needed to make a line behave in a certain way. Below is an example of dialogue that was coded for having “proactive strategies” by students as they began to find two equations whose lines would make an isosceles triangle with the x-axis.

> Mrs. Alpha: So put some sort of equation in there but you don’t want to just throw anything in there. Tell me one thing we should know about the line?
>
> Student 1: You shouldn’t have a negative two there again (the equation previously entered had a slope of -2).
>
> Mrs. Alpha: It shouldn’t be a negative two? Why not?
>
> Student 2: They would be going in the same direction. We want them to go opposite like this (motions with hands one going up and the other going down).
>
> Mrs. Alpha: Ok so again what should we know about the line?
>
> Student 1: Use a positive two [slope]?
>
> Mrs. Alpha: Interesting, ok try that.
>
> Student 2: What about the [y-intercept]? Should we use the same?
>
> Mrs. Alpha: What would happen if it was the same?
>
> Student 2: Oh yeah, that would be the top of the triangle because they could meet there.

Finally, the lesson was coded as "seamless" since the technology was used in a way that allowed the students to focus on the mathematical content of finding equations that would form lines arranged in an isosceles triangle and not become distracted by how to use the graphing calculators. For example, some students were observed graphing a dozen or more different equations before arriving at an answer but doing so in a short period of time. Overall, Mrs. Alpha demonstrated a number of effective uses of technology in her mathematics classroom.

Events and corresponding stages of TPACK development. This section will present the events that Mrs. Alpha experienced related to using technology as a teaching and learning tool and their corresponding TPACK development stages (Niess et al., 2009) to show her progression towards becoming an effective technology user (see figure 2.3). Descriptions of the indicators established by Niess et al. (2009) that were used to identify the developmental stages are also included. Appendix G shows the complete list of indicators at each stage. Figure 4.3, adapted from Niess et al. (2009), shows a summary of Mrs. Alpha's events organized by phase (pre-college, college, and career) and by TPACK development stages. Also included in this section are the events that were coded as "barriers" to developing a positive attitude toward technology implementation or to developing the knowledge and skills needed to use technology effectively. The purpose for identifying the potential barriers to technology use was to determine if factors related to technology implementation decisions were present in an event. They are presented in this section in the context of their story in order to provide a better understanding of why they were coded as barriers and how they relate to the development of the knowledge for teaching with technology. Barriers will be discussed further in the cross case analysis section.

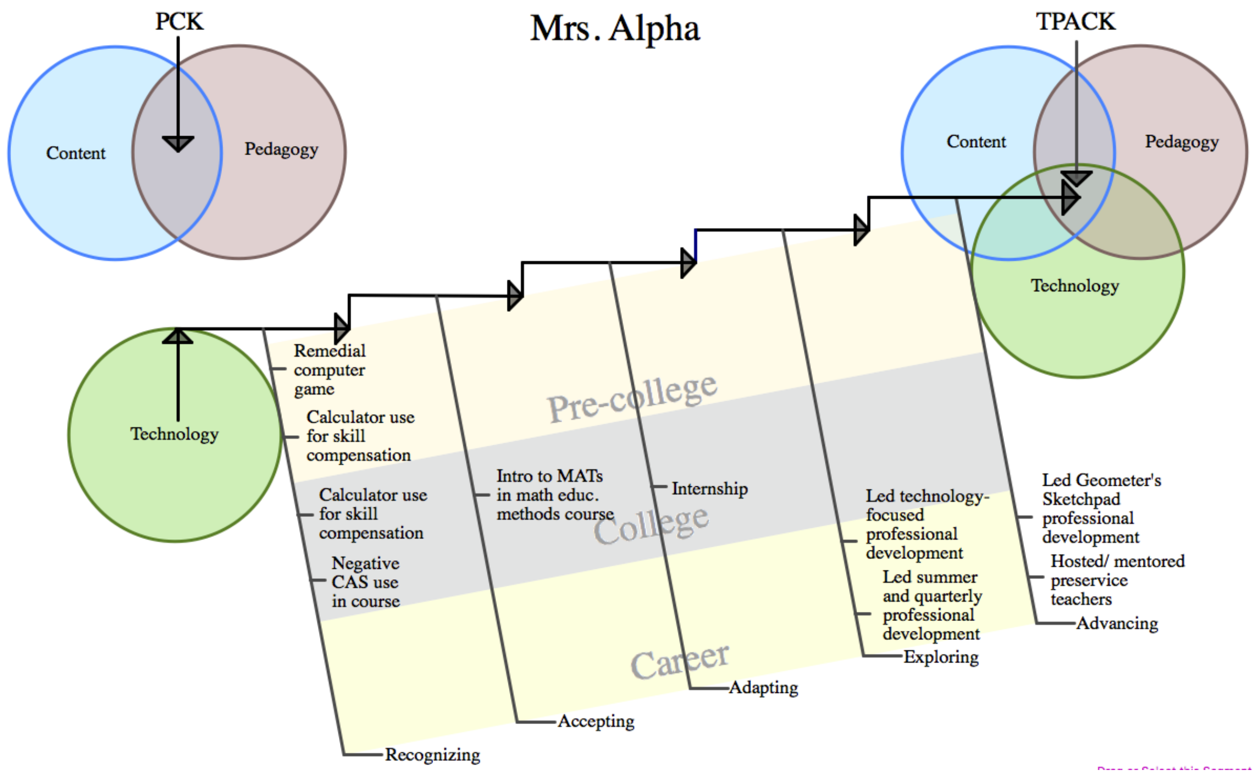


Figure 4.3. Mrs. Alpha's events organized by phase and TPACK development stage. Adapted from "Mathematics teacher TPACK standards and development model," by M. L. Niess et al., 2009, *Contemporary Issues in Technology and Teacher Education*, *9*(1), 4-24. Copyright 2009 by M. L. Niess et. al.

Pre-college technology experiences. Mrs. Alpha had very limited experiences with technology before entering college. Her experiences are consistent with the "recognizing" stage of TPACK development, defined by a budding knowledge of how technology and mathematics are integrated. Note that this stage of TPACK development does not require understanding of teaching integrated with mathematics and technology. The indicators at this stage included "able to use technology for personal use", "use is without teacher instructions," and "technology used for rote activities, not learning." For example, in elementary school, she used a computer program that included a multiplication game. She explained,

> I had to stay in during recess and play this game on a computer that reinforced fast recall of multiplication facts. I remember it being like a centipede's game where I had to type in the answer to a problem as fast as possible. I started to prefer it to recess because I was catching up with the other kids. But I didn't really learn. I still mixed up sevens and eights but I was faster.

The program promoted rote procedures and was used for remedial purposes when she fell behind other students in mathematics.

During high school, Mrs. Alpha's stated that her use of technology was limited to a graphing calculator. Mrs. Alpha stated that she used the calculator to compensate for poor mathematical skills or lack of understanding of concepts. The following was coded as "barrier" because she did not experience technology as a learning tool.

> [I] never used [the graphing calculator] during my education for anything more than a crutch and a place to type information to use during a test. It helped me get through lots of mathematics that I didn't have the complete background for. I don't remember any teacher showing me how to use it.

Also coded as "barrier," Mrs. Alpha explained that she did not have many opportunities before college to develop a positive attitude about using technology because her limited experiences with technology made it seem intimidating. Despite this early ineffective use of technology, Mrs. Alpha found value in the technology for helping her to "keep pace" with other students.

College technology experiences. Early college events were identified as being at the "recognizing" stage of TPACK development since she was still developing an understanding of the mathematics/technology alignment. The indicator codes that were identified in these events included "able to use technology for personal use," "technology used for rote activities, not learning," and "use is without teacher instructions." For example, Mrs. Alpha again used the graphing calculator to compensate for poor mathematical skills. Mrs. Alpha described her experience using a computer algebra system (CAS) during a calculus course as unsuccessful and indicated she felt "completely lost and intimidated" by the technology. The course included lectures for introduction of material and computer lab time to practice and apply what was learning during lecture. So, this event was also coded "technology used after mastering concepts," a "recognizing" stage indicator. Coded as "attitude barrier" she stated that the technology had made the content even more difficult to understand.

However, later in her college education Mrs. Alpha was introduced to various mathematics action technologies (MATs) during her mathematics education methods courses. Coding of this experience was consistent with the "accepting" stage of TPACK development. This experience was coded as "form favorable attitude toward technology learning and teaching mathematics" because during these courses, she used the technology to explore mathematical concepts. Also, she stated that she was re-learning mathematics while at the same time learning to use technology such as Geometer's Sketchpad, graphing calculators, and Excel spreadsheets as a teacher. She stated,

> I started to understand geometry and some algebra in ways that I didn't even realize that I did not know before. That is when I started to realize how weak I had been in my classes. So I also was using a teacher's lens at the time and I could see how amazing this tool [Geometer's Sketchpad] would be in the classroom as far as having students investigate. I was able to start seeing mathematics myself in more complete ways, making connections, and in turn making me a better teacher because I have multiple ways of understanding concepts.

Mrs. Alpha stated that during these courses she would complete mathematics activities with groups then discuss results with other mathematics education students. In addition, she began to develop lessons using technology to explore mathematics in ways that allow students to build their knowledge from explorations, and she was able to practice a few lessons in a laboratory experience at a local high school. The coded indicators of the "accepting" stage also included "may mimic exact PD lesson" because she was only using lessons presented by her instructor during her laboratory experience and methods courses.

Mrs. Alpha's college education experiences concluded with her internship, which she identified as the most significant influence on her implementation practices during her career. She stated that several characteristics of her internship influenced her current implementation practices: she was required to use technology as an inquiry tool with her students, she was supported in the process of learning to use technology, she had sufficient access to technology resources, and she was praised for her use of MATs in her instruction. Multiple indicators showed that this experience was consistent with the "adapting" stage in which the teacher engages in activities that lead to full adoption. This event was coded as "students explore for part of the lesson" and "teacher maintains control of technology activity progress." Mrs. Alpha indicated that she predominately used Geometer's Sketchpad as a tool to have students gather data and make conjectures from their observations but then returned to the regular classroom for the remainder of the lesson. Also, Mrs. Alpha stated that students explored using Geometer's Sketchpad for part of the lesson, and she maintained control of the progress of the activity in order to lead them to a particular discovery. Also, this "adapting" stage event was coded as "students develop mathematics thinking skills using technology." She explained,

> I loved being able to lead them to an "aha moment" in class so that they could start to investigate for themselves. I liked how easy it seemed to get kids to state the mathematical ideas themselves instead of reading a theorem out of the book to them and then giving examples.

Mrs. Alpha stated that the positive experiences during her internship prepared her to continue using technology during her career.

Career technology experiences. Several experiences early in Mrs. Alpha's teaching career were identified as significant in her development as she continued to grow in her knowledge of technology use in the classroom. First, learning of content through technology use continued into Mrs. Alpha's teaching career. Mrs. Alpha reported that because she was using technology to see and understand mathematics better, that in turn made it easier to teach the mathematics. Other events, associated with her experiences in the MSP program, were identified as consistent with the "exploring" stage of TPACK development because she implemented technology integrated with teaching and learning. For example, as part of her participation in the MSP program, she led technology-focused professional development that also helped her to develop her skills using multiple MATs including graphing calculators, dynamic geometry software, and computer algebra systems; so this was coded as "shares with and seeks out others that are using technology similarly." Mrs. Alpha explained that she created modules of instruction that participating teachers engaged in as part of the technology-focused professional development "that required teachers to explore mathematical concepts and build knowledge through their explorations with the technology." Thus, this event was also coded "plans with concern for student understanding/thinking" and "technology plays more integral role for developing/exploring new mathematical ideas," both indicators of the "exploring" stage.

Mrs. Alpha progressed in her TPACK development through her heavy involved in the MSP's summer and quarterly professional development program as a presenter. Throughout the interview, Mrs. Alpha repeated that her experiences in the MSP program's summer and quarterly professional development had resulted in better content knowledge and reinforced her beliefs about the effectiveness of technology in learning mathematics as she experienced it for herself. For example, during the summer professional development program, she

> provided or assisted in sessions for teachers on fostering productive discourse among students, using and creating lessons that develop conceptual understanding with procedural skills, and using inquiry-based instruction with and without technology in the mathematics classroom which in turn helped [her] to understand how to incorporate all of these things better in [her] own classroom.

She also described how her experiences helped her to develop her TPACK as follows.

> We [her professional development group of teachers] often met on Saturday's to work new and interesting mathematics problems and often with the aid of some technology tool such as graphing calculators or Geometer's Sketchpad. I was able to expand the number of topics we could explore with technology and I started to feel very confident in using the technology. We also worked together to create new lessons that focused on learning mathematics through investigations to present to other teachers. The teachers would work through the lessons as students to experience learning in this new way and then we would discuss the pedagogical issues using our "teacher hats." I was able to start doing this my first year of teaching and this continued for more than 5 years.

These events were also coded as "adapts own lessons to incorporate technology" and "seeks solutions to barriers for technology use," which are "exploring" stage indicators.

Mrs. Alpha stated that she lacked access to technology early in her teaching career because her classroom the was not equipped with anything other than marker boards; this was coded as "use barrier." However, the summer, quarterly and technology-focused professional development opportunities provided Mrs. Alpha with many technology resources for her classroom and helped her to develop a network of teachers with whom to collaborate. Mrs. Alpha explained that she also purchased her own resources for use in the classroom with funds she had earned during the MSP program. In addition to the MSP's technology-focused professional development component, Mrs. Alpha provided professional development sessions for other teachers specifically interested in learning to use Geometer's Sketchpad in the classroom. This event was coded with several indicators consistent with the "advancing" stage of TPACK development, where teachers are confirmed in their decisions to use and begin to evaluate the effectiveness of use. First, this event was coded as "teacher is a model for novel ideas of using technology to help students" because Mrs. Alpha stated that she was considered a leader in her community of teachers for teaching with technology. This event was also coded as "create-plan-implement-reflect cycle" since she explained that she often engaged in a cycle of planning, implementing, reflecting and revising her technology lessons as a part of her role as a professional development leader. Finally, this event was coded as "resolves perceived barriers for use" because she worked to remove barriers to technology use when she acquired technology resources through professional development opportunities and collaborated with participants on ways to address classroom management issues that arise when incorporating technology.

Mrs. Alpha stated several times during the interview that she had feelings of isolation from her department due to differing philosophies about technology's purpose in the mathematics classroom. She also felt rejection from her department when trying to share technology uses. The following was coded as “use barrier:”

> Several veteran teachers in my department have developed a social climate that rejects and belittles using constructivist approaches and innovative teaching methods. One teacher listened to a very innovative lesson that my [mathematics education undergraduate student intern from one of the universities participating in the MSP] had been implementing using Geometer’s Sketchpad and commented that she was glad that she went to a college where she was given a “practical education and didn’t have to worry about all this stuff.”

Although the teachers in her own school did not show interest in these sessions, she was able to work with and create a network with teachers in her region through her participation in the MSP program. Thus, this was also coded “potential barrier” since Mrs. Alpha’s professional learning community helped her to overcome this barrier to technology use.

Lastly, Mrs. Alpha reported that she often hosted mathematics education students for laboratory experiences and internships as a requirement of the MSP’s teacher leader fellowship program, and a major responsibility included helping them learn to use technology in their lessons that were implemented in Mrs. Alpha’s classroom. This event was consistent with the “advancing” stage. Elements of this event were coded as “engages others to incorporate technology,” “create-plan-implement-reflect cycle,” and “teacher is model for novel ideas of using technology to help students.” Mrs. Alpha explained,

> Working with interns has made me evaluate how I am using technology and think about the best ways to use it in the classroom to help students learn. They generally have an understanding of how the technology works and an understanding of the mathematics but

> its challenging to help them to implement a good investigation because they often lack the underlying knowledge that comes with experience, with just doing the lesson a few times. I have learned a lot from trying to help these students close the gap between planning and implementation.

This was also coded as "personal conviction to enhance student understanding" because of her described focus on student learning. Mrs. Alpha indicated that her leadership roles often impacted her technology implementation decisions.

Summary. Mrs. Alpha's trajectory of events that indicated that her level of TPACK development moved from the "recognizing" stage to the "advancing" stage. This progression occurred over a period of 12 years. She identified her internship as a supportive environment that included adequate access to technology resources and her experiences within the MSP programs the events when she experienced the most significant growth in her ability to use technology in the mathematics classroom. Her continued education experiences served to provide her with technology resources, support through a network of like-minded teachers, multiple leadership roles, and opportunities to build her knowledge of effective uses of technology in the mathematics classroom.

The Case of Mr. Beta.

At the time of data collection, Mr. Beta was in the fifth year of his career at a small rural high school in the southeastern U.S. He had completed his internship at the same school and was immediately hired upon completion. He had earned his bachelor's and master's degrees in mathematics education and was currently teaching algebra I courses. During his final year of his undergraduate program, Mr. Beta worked as a student assistant in the MSP program. Some of his responsibilities included assisting presenters with sessions, working alongside presenters to assist teachers as they worked through mathematics problems, participating as a teacher in sessions, and clerical tasks such as making copies and gathering supplies. Mr. Beta participated in the training sessions for presenters as well as being present for summer and quarterly professional development. During his undergraduate education, he completed the mathematics education capstone course developed through the MSP program. Finally, he completed his master's degree in mathematics education while participating in the MSP's teacher leader fellowship program.

Current technology use. Due to conflicts with required testing in Mr. Beta's school, he was only able to record and submit one lesson for observation. During this lesson, he demonstrated an effective use of technology in mathematics instruction based evidence coded using the observed indicators consistent with established literature on effective technology use. Presented in this section first is a discussion of the lesson observed, then the observed established criteria for effective technology use.

Mr. Beta's recorded a lesson of his algebra I class of 15 ninth-grade students. The lesson included creating box and whisker plots on the graphing calculator and interpreting the meaning of the data. Figure 4.4 shows an example of a box and whisker plot. The students were asked to make observations about two sets of class test scores, create the box and whisker plots for the data in the calculator, and then talk about the validity of their observations based on their findings. Mr. Beta asked students a variety of follow up questions that required that they interpret the data representation they had created and apply their knowledge of statistics. For example,

> Mr. Beta: Ok so this calculator can quickly do it but the important part is not so much that, but how you read something like this. What was this about?

Student 1: We were figuring out the calcium in the food.

Mr. Beta: Ok what about this value right here? What was that 450 about?

Student 2: Yogurt

Mr. Beta: What about yogurt?

Student 1: Yogurt has the most calcium.

Mr. Beta: Hmm.. does that make sense? More than milk?

Student 2: Well in our data, yes its more.

Students remained engaged in the lesson during the majority of the observation and actively engaged in the discussion led by Mr. Beta. In the debriefing, Mr. Beta explained that the purpose of the lesson was to have students practice applying the concepts of box-and-whisker plots and interpreting the information in the context of real data. He also indicated that this was one in a series of lessons would continue to add more statistics concepts as they went along but felt that this lesson showed how technology could be used to "help [his] students learn about interpreting data representations."

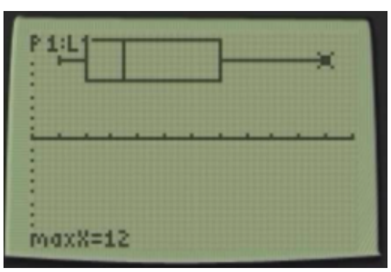


Figure 4.4. A box and whisker plot constructed using the TI-84 graphing calculator. The minimum value, lower quartile, median value, upper quartile and maximum value for a set of data are shown.

A number of aspects of the observation of Mr. Beta's lesson, examination of student work, and the lesson debriefing were coded as effective technology use. First, the lesson was coded as “standards-based” because a mathematics standard appropriate to the course and students was the focus of the lesson. Next, it was coded as “worthwhile task” because students gained an understanding of statistics concepts as evidenced by the discourse that occurred between teacher and students. Students built conceptual understanding and skills as well as engaged in problem solving to interpret statistics. For example, a student had difficulty entering data in correctly to the calculator and she was not getting the same results in her calculator as the rest of the students around her. Mr. Beta asked the class what type of data entry mistake could have caused the error she was experiencing. He continued pressing students to think about what other mistakes could have caused her troubles and students began discussing the possible results of making particular entry mistakes. He also asked a variety questions such as “Does [student’s] answer make sense?” and “What is represented?” to engage students in the discussions.

Several parts of the lesson were also coded as “natural tool.” For example, students were clearly familiar with the calculators as they learned to create the box and whisker plots, and the technology was not a distraction as they were simultaneously answering these conceptual questions. They appeared to use technology without frequent prompting. Finally, the lesson was coded “mathematics focused” because the representations provided by the technology allowed the focus of the lesson to remain on the content being taught. Mr. Beta stated that in the debriefing that this approach made the concepts of interpreting data and their discussions the focus rather than the construction of the representations (Dick & Hollebrands, 2011). Finally, the lesson was coded as “necessary” because during the debriefing, Mr. Beta stated that the technology was a necessary part of the lesson, allowing its completion in the allotted time frame.

Events and corresponding stages of TPACK development. Mr. Beta described events that contributed to his development into an effective user of technology as a teaching and learning tool that included barriers to use and indicators of stages of TPACK development during his pre-college years, college, and his teaching career. The events are summarized in figure 4.5 and are organized by phase and TPACK development stage adapted from Niess et al. (2009).

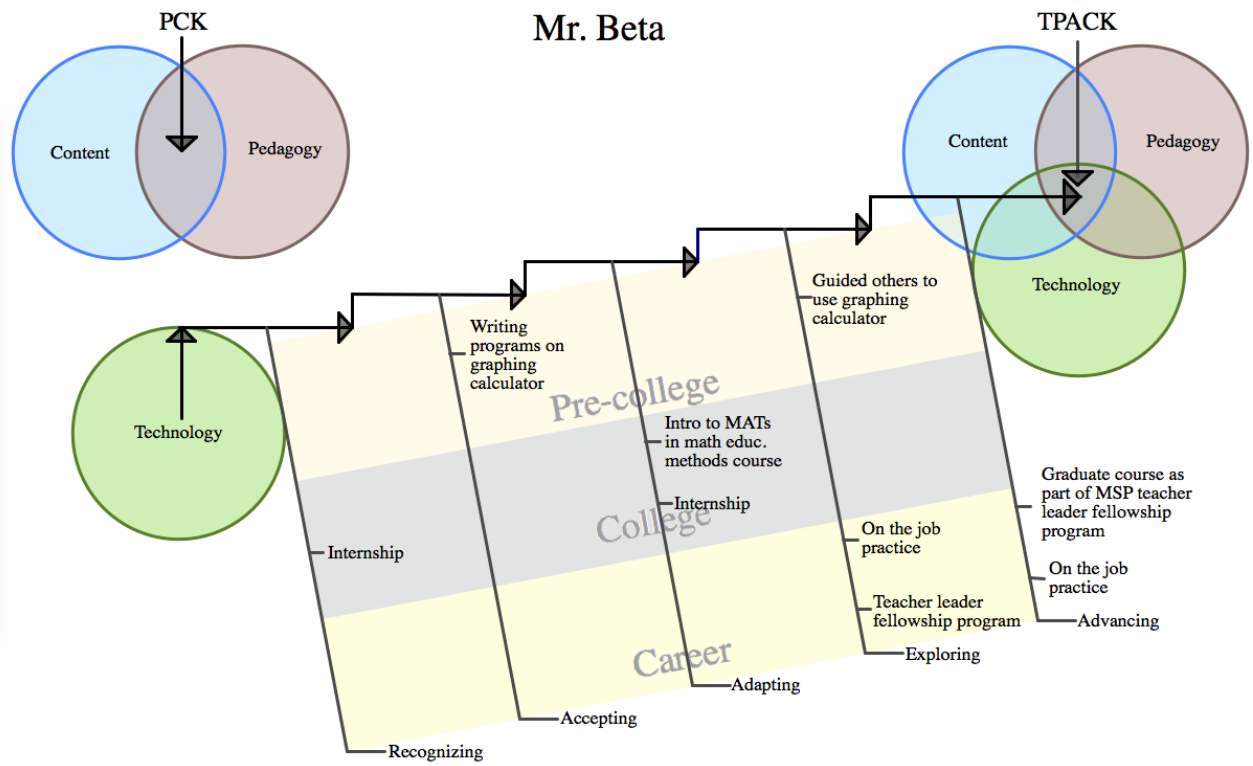


Figure 4.5. Mr. Beta's events organized by phase and TPACK development stage. Adapted from "Mathematics teacher TPACK standards and development model," by M. L. Niess et al., 2009, *Contemporary Issues in Technology and Teacher Education*, *9*(1), 4-24. Copyright 2009 by M. L. Niess et. al.

Pre- college technology experiences. Several events were coded for "technology" prior to college related Mr. Beta learning to use technology. Mr. Beta described his earliest use of technology as a problem-solving opportunity as his family challenged him to think about the different parts of the computer or video game system. They encouraged him to engage in problem solving and figure out how the different parts of the systems communicated with each other. He indicated that this problem-solving approach sparked his personal interest in learning to use technology.

Mr. Beta described his own high school experiences with using a graphing calculator as a learning tool as exclusively an add-on activity that followed traditional instruction. However, Mr. Beta then began developing his TPACK during his high school education when he engaged in writing calculator programs about concepts taught by the teacher without technology and then teaching them to others. This event is consistent with the "accepting" level of TPACK development. The following passage was coded as "form favorable attitude toward technology learning and mathematics." He stated,

> I have never had an issue with my affect. I never had an issue with wanting to use it. Never had an issue about learning the importance of it. I mean it was just like yep I believe that, that makes sense because I had such uses of my own [for the technology] in my calculus class. And if you used it in high school, I felt like why not use it.

Mr. Beta explained that he was using technology with step-by-step instructions for "add on" activities and this was coded as "technology used for days off," an "accepting" level indicator. He indicated that he had to teach himself how to use the graphing calculator as a high school student in a calculus class. He used the manual to learn to program the graphing calculator.

However, Mr. Beta's description of how he promoted student engagement by using questioning techniques to help other students to use the technology was coded with indicators of the "exploring" level of TPACK development. Coded as "engages/guides students in high-level thinking activities" and "develops curriculum enhancements due to technology," Mr. Beta explained that during lessons he would ask fellow students questions such as, "Why don't you use [the calculator] to do this?" or "Why don't you calculate the intersection this way?" in an effort to teach them something they did not learn during the regular lesson and also tried to help others learn the programs he had created on the calculator. Finally, Mr. Beta stated that he lacked access to technology due to his economic status growing up with the exception of his high school's graphing calculator and this was coded as "attitude barrier." Also identified as an "attitude barrier," prior to his college education Mr. Beta did not have many opportunities to see technology being used in effective ways as a learning tool.

College technology experiences. During his undergraduate courses, Mr. Beta continued to have experiences consistent with the "adapting" stage of TPACK development as he was prepared to use various technology tools to teach mathematics. Coded as "some technology benefits understood" and "explore some mathematics with technology," Mr. Beta reported that he learned to use tools in the context of exploring mathematics during his undergraduate methods courses. He stated that he learned to implement certain tools for particular content without considering other factors such as individual needs of students. For example, he recognized that the calculator was a great tool for helping students to graph lines but had not yet considered how to help his students to use the calculator to promote deeper understandings such as "how to identify key features of the line by looking at the graph."

Mr. Beta's earliest focus on technology in the classroom as a pre-teacher was for presenting material only and not for use as a reasoning and learning tool. Mr. Beta reported that he focused on learning to use conveyance technologies during his internship because his cooperating teacher was unfamiliar with the new tools she had just received. He explained,

> Of course in the classroom I feel like starting from day one I wanted to use graphing calculators in my internship. I had to grow in my use and understanding of the presentation technology. Like a document camera. During my internship I think that was

> an important use of that kind of technology to show representations when I was teaching or showing student work. But I couldn't get it to not be blurry. Later on I realized that all I had to do was plug it into the projector. And so I used it more effectively after I learned how to connect it correctly.

He stated that he used the document camera primarily for teacher presentations during lecture-based lessons to help his students visualize the mathematics as he was explaining.

Mr. Beta also reported that he felt that he needed to focus more on management and lesson-building during his internship because he felt that these were weaker areas for him than technology use. He described his experience,

> I think at that point though [technology] was more of a way for me to kind of [let] the kids visualize what was being said. And I could just write and look at the students because as an intern I was not where I needed to be yet. It was a lot more about teacher presentations than it should have been.

Although this event was coded as "useful for visual representations" and "no curriculum changes to reflect technology" which is consistent with the "recognizing" stage of TPACK development, Mr. Beta stated that he was aware of more advanced uses for the technology which was coded as "some technology benefits understood" and is consistent with the "adapting" stage. Also, other evidence of his internship that were consistent with the "adapting" stage included background information reported by Mr. Beta. For example, Mr. Beta's description of using inquiry-based technology lessons as a requirement in his internship was coded as "explore some mathematics with technology."

Career technology experiences. Most of the events that Mr. Beta described about using technology during his teaching career are consistent with the "exploring" stage of TPACK development. For example, Mr. Beta reported that he used technology in the classroom at the time of the interview with expectations that students explore and learn to use the technology through exploration which was coded as “manages activities but promotes student engagement and self direction.” He stated that he learned to help students figure out how to use the technology themselves through his line of questioning which was coded “plans with concern for student thinking.”

Mr. Beta reported that he often struggled with consistently implementing constructivist-based instruction including technology-based lessons and this was coded as “barrier.” Instead of using technology, he often found himself focused on what he referred to as “coverage,” teaching all required objectives even if only at a surface level that results in a lack of depth of student learning. He experienced frustration using technology due to a perceived lack of basic skills in his students and not “getting them as far” as he hoped. He lamented that he lacked understanding at that time in his career of how to help his students’ progress and sometimes felt that technology was getting in the way of their progress. However, during a lesson, Mr. Beta’s students questioned his exclusive use of technology, which caused him to rethink his practices. Mr. Beta explained that he experienced time management issues as a teacher that caused him to debate the effectiveness of technology due to the class time needed to implement the technology.

> And so to present things quicker, these is just a time crunch within your content and getting used to that, I have gotten into a bad habit of doing presentations myself. Because if I know there is going to be an issue with the technology, I can fix it quicker and go over it… but just the fact that they called me out on best practices. They’re not trained in that. But they called me out and really it was true.

This resulted in Mr. Beta expanding his use of conveyance technologies to regularly promote student discourse through student-led presentations. He stated, "I told them, 'You are presenting problems now, you are presenting this.' So, they go up there now and they are pretty good at talking about the mathematics." This event was coded as "more integral role for developing mathematics ideas" and "manages activities but promotes student engagement and self direction" which are indicators of the "exploring" stage.

Mr. Beta stated that the most influential experience for developing his TPACK and impacting how he implemented technology in the classroom was learning and engaging in discussions about TPACK and other technology tools during a graduate course in mathematics education as part of the MSP's teacher leader fellowship program. During this course, he studied the concepts of TPACK and explored a variety of technology tools beyond what he had reported using previously such as iPad mathematics applications. This was coded as "seeks PD for emergent technology." Coded as "create-plan-implement-reflect cycle" and "assesses student understanding of mathematics embedded in technology," he indicated that learning about TPACK enabled him to "take [his] technology use to a higher level" through the awareness of the various components that made up TPACK. He explained,

> It gave me words for things that I was doing but didn't know. That shaped my beliefs the most since then because now when I think about using the calculator I think about this. It just changed the way I think about how I am going to approach teaching in a situation. And so I think getting words for things, like oh I need to go to the pedagogical side to address this misconception here or content-wise they are missing this idea here. Or technology- they just don't realize how to use the features on the calculator. So it lets me completely think it out and figure out why a student doesn't understand.

These indicators are consistent with the "advancing" stage of TPACK development as Mr. Beta was evaluating its effectiveness.

During the interview, Mr. Beta indicated that learning on the job by using the graphing calculators during his classes also had a great impact on his developing his skills as an effective technology user. His early teaching experiences included the availability of some technology resources, but he lacked an in-house mentor to assist in learning how to effectively use the technology. Coded as "create-plan-implement-reflect cycle," he stated that he increased his uses and knowledge of technology every year and continued to try to improve. Mr. Beta also reported that his technology skills grew because he was often asked to assist other teachers with troubleshooting their technology although he had no formal training in this area. He used web searches and online tutorials to teach himself how to fix common technology problems which was coded as "resolves perceived barriers for use." Thus, this event contained indicators consistent with the "advancing" stage of TPACK development.

Finally, Mr. Beta participated in the MSP's teacher leader fellowship program. The purpose of the program was to prepare teachers to promote, model, and support others to transform the way they teach mathematics to include more reform methods with a goal of improving student learning. Mr. Beta indicated that he had greater access to technology resources through the teacher leader fellowship program which was coded as the "exploring" stage indicator "resolves perceived barriers for use." He also indicated that the program provided him with opportunities to collaborate with teachers "more like [him]" in terms of using technology in the classroom. Mr. Beta reported feeling isolation in his department because his colleagues used technology for "low level tasks" and even suggested that Mr. Beta was allowing students to use technology to replace basic skills. During the teacher leader fellowship program, Mr. Beta shared lesson ideas and collaborated with others outside of his school to reflect on and improve their current approaches in the classroom including technology and this was coded as "teacher is a model for novel ideas of using technology to help students" and "create-plan-implement-reflect cycle". Thus, Mr. Beta's described experiences in the MSP's teacher leader fellowship program were consistent with the "exploring" stage.

Summary. Mr. Beta's journey to becoming an effective technology user in the mathematics classroom included high school experiences with a graphing calculator and an undergraduate education and internship with opportunities to learn effective uses of technology. Throughout his development he overcame barriers to use and developed a positive attitude toward technology through his continued education and the events ranged from "recognizing" to "advancing" stages of TPACK development. Finally, he participated in the MSP's teacher leadership fellowship program, engaged in leadership roles related to technology, and had multiple opportunities outside the classroom over time to collaborate and enhance understanding about technology while applying learned knowledge in the classroom.

The Case of Mrs. Chi

Mrs. Chi was in her 35^{th} and final year of teaching at the time of data collection. She had taught the majority of the years at her current school of employment and was teaching both algebra I and pre-calculus classes. She had taught a variety of mathematics and computer programming courses during her teaching career. She was teaching in the small rural southeastern U.S. town where she had grown up. Her recorded lessons featured graphing calculators but she also reported that she used Geometer's Sketchpad at times in her instruction. Mrs. Chi also participated in multiple components of the MSP program. She participated in the curriculum guide development, textbook adoption, district and school-level teacher leader development, summer professional development, and quarterly professional development.

Current technology use. In this section, the results from the analysis of both observations and debriefings will be presented. Mrs. Chi demonstrated effective uses of technology in both observations, and the criteria found will be discussed.

Observation 1. Mrs. Chi's students engaged in a lesson on examining data and lines of best fit during the first observed lesson of her classes. The recording device failed to record for approximately 15 minutes of the lesson but the audio recordings and student artifacts provided evidence of the classroom activity during that time. This occurred in a 9th grade algebra I course with 24 students present. Students used the graphing calculator to make conclusions about the rate of cricket chirps compared to temperatures. The objectives of the lesson were to use real life data to create a mathematical model of the relationship between the rate of cricket chirps and the temperature, determine the linear regression equation to represent the data, determine the measure of "fit" based on the r^2 value, make predictions based on the line of best fit equation, accurately calculate the linear regression equation based on the given data, and to provide an efficient model of the relationship between the cricket chirps and temperature. Figures 4.6 and 4.7 show examples of student work from the lesson. Students were observed working in groups to enter data and answer questions about their observations. Later in the class, Mrs. Chi facilitated a full group discussion to discuss the results of the students' explorations. In the debriefing, Mrs. Chi stated that the lesson was chosen to allow her students to investigate using a real data set and the graphing calculator features allowed for "efficient and accurate calculations of the line of best fit." Mrs. Chi stated that before she had the graphing calculators, this same lesson was very tedious and students often gave up if they made "one tiny mistake" because they would have to start over.

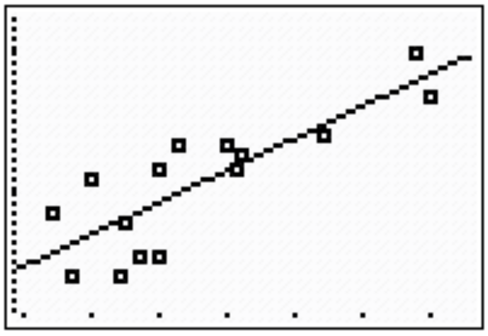
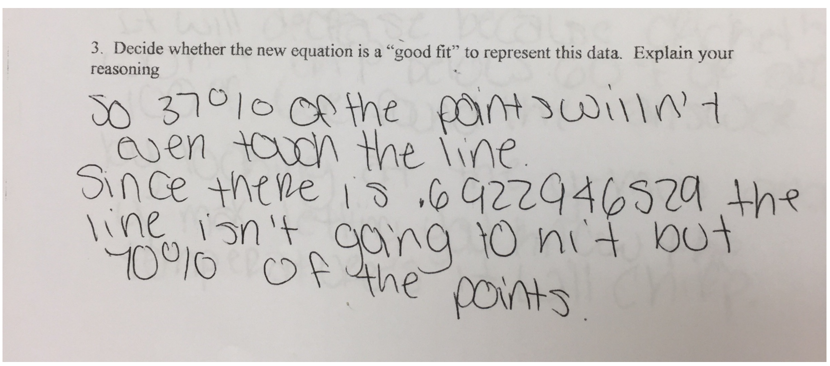


Figure 4.6. Screenshot of data entered into the TI-84 graphing calculator and the line of best fit



3. Decide whether the new equation is a "good fit" to represent this data. Explain your reasoning

So 37% of the points willn't even touch the line. Since there is .6922946529 the line isn't going to hit but 70% of the points.

Figure 4.7. A student's answer analyzing the accuracy of the line of best fit. These answers were used to facilitate a discussion about acceptable r^2 values.

The lesson was coded as "standard-based" and "appropriate level" since the focus of the lesson was a mathematical standard appropriate for the student level. The lesson was also coded as "seamless" because the technology was seamless in that the discourse and learning focus was on the mathematics and context of the investigation. Thus, this was also coded as "mathematics focused." The following discourse is an example of how the focus remained on the mathematics when two students had different answers given by their calculators.

Student 1: Wait, I didn't get the same r squared as him!

Mrs. Chi: Ok let me help you out a minute. What did you get?

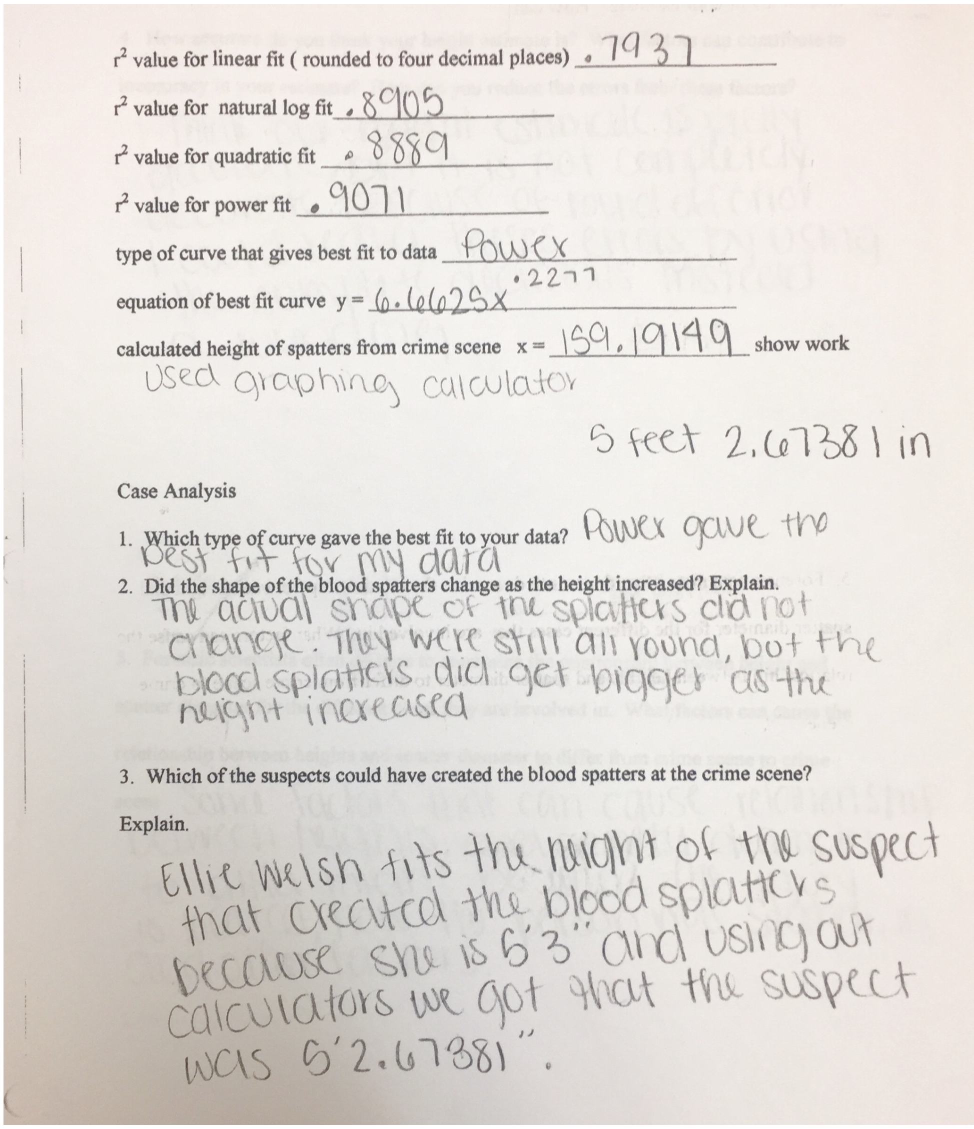
Student 2: Wait, you are trying to change something. Look and see if you got what I got or you put something in wrong.

Mrs. Chi: Ok well how do you know? How do you know that you put something in wrong?

Student 2: It's the same [data] so we should be getting the same value.

Mrs. Chi urged the students to consider the reasonableness of getting two different solutions when using the same data set instead of allowing students to just blindly type data into the calculator. Lastly, the lesson was coded as "skill variety" because it included skill building (plotting data and examining it), conceptual understanding (deciding if the line of fit was a good fit for the data), and problem solving (using the line of best fit to make conclusions about the cricket chirps and temperature). Next, the lesson was coded as "carefully designed" since students employed a constructivist approach (Keengwe et al., 2008) as they built understanding from previous knowledge through investigation of real data. Finally, the lesson was coded "worthwhile task" because the students were observed engaged in reasoning and sense-making discourse (Dick and Hollebrands, 2011).

Observation 2. Mrs. Chi's pre-calculus class of 19 students completed a lesson that consisted of students measuring drops of fake blood and comparing the data to the heights of possible suspects in order to determine who could have committed the crime. The students used the graphing calculators to enter data and created a curve of best fit. Then they made conclusions based on their data. Figure 4.8 shows an example of a student's work from class. After students completed the handout, Mrs. Chi held a full class discussion and students shared their work under the document camera. Mrs. Chi indicated that she used this lesson and the graphing calculator tool in order for students to explore a "real life example" of analyzing data and making decisions. She stated that the graphing calculator was a familiar tool for her students and this point in the semester and they would be able to use the statistics functions to find their solutions.



r^2 value for linear fit (rounded to four decimal places) .7937

r^2 value for natural log fit .8905

r^2 value for quadratic fit .8889

r^2 value for power fit .9071

type of curve that gives best fit to data Power

equation of best fit curve y = $6.6625x^{.2277}$

calculated height of spatters from crime scene x = 159.19149 show work

Used graphing calculator

5 feet 2.67381 in

Case Analysis

1. Which type of curve gave the best fit to your data? Power gave the best fit for my data
2. Did the shape of the blood spatters change as the height increased? Explain.
The actual shape of the splatters did not change. They were still all round, but the blood splatters did get bigger as the height increased
3. Which of the suspects could have created the blood spatters at the crime scene?

Explain.

Ellie Welsh fits the height of the suspect that created the blood splatters because she is 5'3" and using out calculators we got that the suspect was 5'2.67381".

Figure 4.8. Student work from the blood drop activity in Mrs. Chi's class

Criteria for an effective technology-based lesson from Mistretta (2005) and others were observed during this lesson also. First, since students were focused on the context and mathematics and not distracted by the technology the lesson was coded as "mathematics focused." Also, the lesson was coded as "engaged and benefitting" because students were engaged throughout the lesson and participated in the discourse. The task was also "carefully designed" (Laborde, 2007) to accomplish the goal of finding the curve of best fit with the gathered data. Next, the use of the graphing calculator was coded as "conceptual with procedural understanding" because the students used the calculator to supported conceptual understanding (Peressini and Knuth, 2005) as representations were accurately and efficiently constructed in order to draw conclusions about curves of best fit before they were engaged in constructing them without the use of the calculator. Finally, the technology was part of the exploration and coded as a "natural tool" (Galbraith, 2006) because students were not told to enter data into the calculators but did so instinctively. During the observations, it was clear that Mrs. Chi's students were very familiar with the uses of technology and were engaged in effective ways.

Events and corresponding stages of TPACK development. Mrs. Chi's development of TPACK mostly occurred very late in her teaching career although there a few events earlier in her career that contributed to her TPACK development. In this section, the events and corresponding stages of TPACK development are outlined. Unlike the previous cases that were organized by pre-college, college, and career time periods, Mrs. Chi's case is organized by early career and late career experiences since no events related to technology occurred before her entry into the teaching field. However, she still experienced barriers to implementing technology in the mathematics classroom. In high school, her mathematics teacher told her that mathematics was not an appropriate area of study for females. Nevertheless, she persisted. No technology was available at the time when Mrs. Chi went through high school and college, so she did not have an opportunity to develop her attitude about the use of technology in the classroom before starting her career. Both of these events were coded as "attitude barrier." Despite these early barriers to technology use, Mrs. Chi's experiences during her career led her to become an effective technology user in mathematics instruction as described previously. Figure 4.9 shows Mrs. Chi's events arranged by phase and TPACK stage development adapted from Niess et al. (2009).

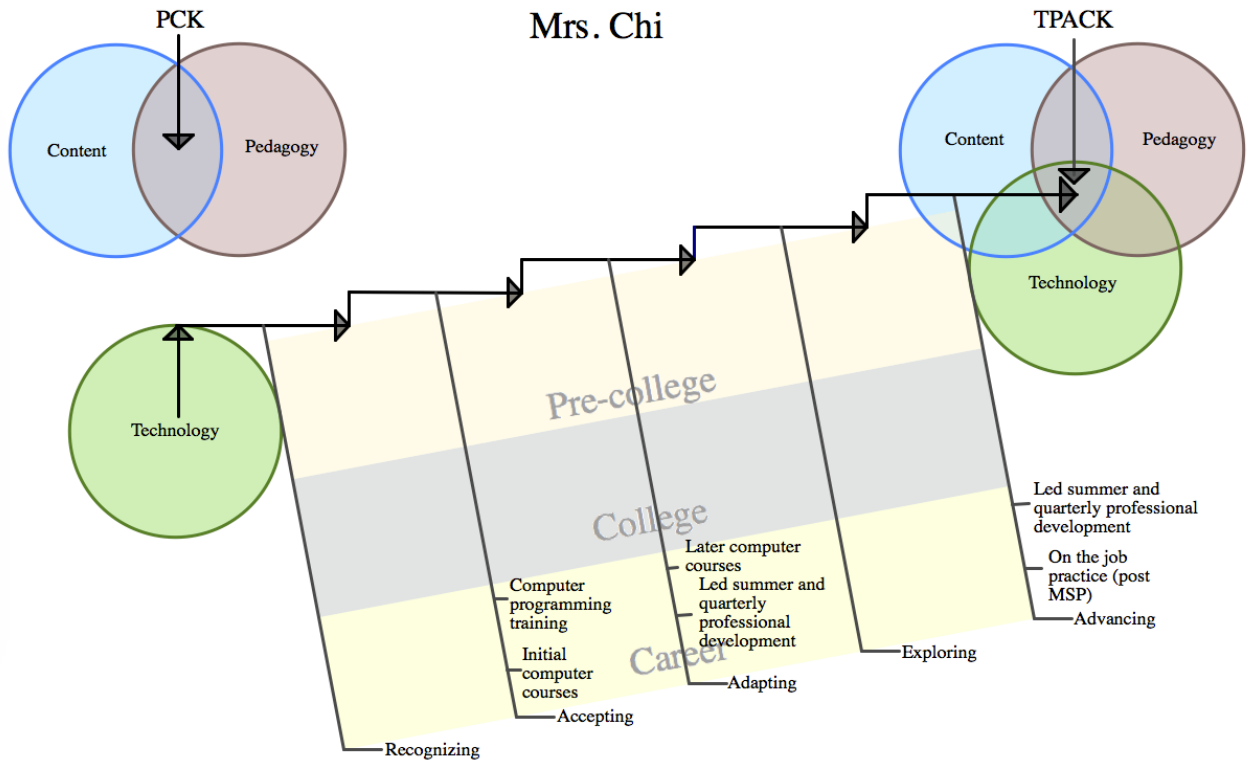


Figure 4.9. Mrs. Chi's events organized by phase and TPACK development stage. Adapted from "Mathematics teacher TPACK standards and development model," by M. L. Niess et al., 2009, *Contemporary Issues in Technology and Teacher Education*, *9*(1), 4-24. Copyright 2009 by M. L. Niess et. al.

Early teaching career technology experiences. Technology resources were very scarce, especially during the beginning and middle of Mrs. Chi's career. However, she did have some early career experiences that were centered around the use of computers outside of the mathematics classroom that was very progressive for the time. She began learning to use computers by going to a three-day course in 1979 on basics of programming in order to prepare for the school's first computer course that she had volunteered to teach despite having no experience. This event was coded as "seeks out PD" and is an indicator of the "accepting" stage. This training focused on computer programming only and did not incorporate any type of traditional mathematics concepts. While teaching the computer class, Mrs. Chi recognized the connections between computer programming and concepts in mathematics, and she started to consider the usefulness in the mathematics classroom. She stated,

> Using the data and the programming, I could immediately see how I could use it. I would give them the data and have them write a program. Then I would ask like in 1955 such and such happened. What would happen in 1972? What would happen in 1965? So I immediately saw how it could be used in the classroom.

This passage was coded as "form favorable attitude toward technology learning" and is an indicator of the "accepting" stage of TPACK development.

Mrs. Chi began teaching her computer students to program mathematical formulas and investigated along with the students to further understand how this could be accomplished. This event was coded as "some technology benefits understood" and "students develop mathematics thinking skills using technology." This is consistent with the "adapting" stage of TPACK development because Mrs. Chi was engaging in activities that lead to full integration of technology in mathematics instruction. It was not until late in her career that Mrs. Chi began to use technology in a way that was fully integrated into her mathematics instruction.

Later career technology experiences. Mrs. Chi's most significant late career experiences related to learning to implement technology in the mathematics classroom were due to the MSP program. The MSP program's aim was to help teachers transform the way they taught mathematics in order to increase students' learning of mathematics. At the start of the program, Mrs. Chi stated that she observed and worked with other teachers in the MSP's summer and quarterly professional development program using graphing calculators to teach mathematics. This was coded as "continues PD in one area of technology" and "shares knowledge with other teachers," both indicators of the "adapting" stage. As part of her training as a presenter for the summer and quarterly professional development, she participated in informational sessions about promoting student discourse, using inquiry-based lessons, and equity issues as well as content-based sessions where the participants engaged in inquiry-based learning of mathematics similar to what they were being prepared to implement in their own classrooms. She explained how the experiences impacted her teaching as follows:

> Now in the classroom, I think [the MSP summer and quarterly professional development program] did more for me in the classroom than anything. Because watching everyone else do it. And the graphing calculators, giving me a set of graphing calculators. So that more than anything else, [the MSP summer and quarterly professional development program] was very influential.

Using the TPACK indicators, this event was consistent with the "adapting" stage of TPACK development as she was engaging in activities that led to adoption of technology.

As the MSP's summer and quarterly professional development programs progressed, so did Mrs. Chi's ability to implement technology in the classroom in effective ways and her TPACK grew. Mrs. Chi's description of her participation in the later years of the MSP program included indicators of the "advancing" stage. For example, Mrs. Chi expressed a desire to implement a consistent presence of technology in the classroom but her school system could not afford the resources. This event was coded as "resolves perceived barriers to use" because Mrs.

Chi was able to overcome this barrier by receiving technology resources through her participation in the summer and quarterly professional development programs. Also during the later years of the MSP program, Mrs. Chi participated as a presenter for the summer and quarterly professional development on using graphing calculators. She planned and implemented lessons with graphing calculators for this professional development and stated that these experiences helped her to become more competent and aware of the uses of the calculator. She stated that she provided assistance and lesson ideas to other teachers in the summer and quarterly professional development program that were attempting to implement graphing calculators in new ways. This event was coded as "teacher is a model for novel ideas of using technology to help students" which is an indicator of the "advancing" stage.

Mrs. Chi reported that when comparing her career and teaching methods before and after participating in the MSP, it seemed like "two different careers." She elaborated, "I truly don't remember what it feels like to not teach with [technology] anymore. Graphing calculators changed my life in teaching. It changed my life." Mrs. Chi reported that she made graphing calculators available to students every day after receiving a class set from her participation in the summer and quarterly professional development program and incorporated them into her lessons. Mrs. Chi had begun using graphing calculators in her room with just a few students passing them around to share. She reported that this use was ineffective and time consuming because students were not able to consistently use them. However, after receiving a class set of calculators, she described her classroom differently: "You have these calculators instead of passing one or two calculators around to let them play with, they had one in their hands. Everyday. Every single day and everything that we did. Very seldom did I say you cannot use calculators." This passage was coded as "consistent acceptance of technology as tool for learning." These indicators are consistent with the "advancing" stage of TPACK development as Mrs. Chi described "confirmation" of the benefits of technology. The professional development events were reported by Mrs. Chi as being the most significant in learning to use technology in the classroom and being able to implement it.

Mrs. Chi explained how she changed the way that she had previously taught after learning to effectively implement technology throughout her participation in the MSP program. First, she reported that she changed the way that mathematics was taught using graphing calculators by moving from rote skill practice to using real life data, graphing data, using investigative approaches, group work, increased dialogue among teacher and students, and comparing representation in an efficient manner to increase learning. This was coded as "essential to modify curriculum to effectively incorporate technology," an "advancing" stage indicator. She also claimed that her instruction and student understanding had improved due to changes in instruction. She explained,

> Yeah, I don't teach the way I used to. The discovery is more. They teach their, hmm, how do I say this? Instead of me telling them what to do, they tell me what they did. They tell each other what they did. And what result they got. They seem to understand it more and can talk about it more. So, it just, teaching is so different. I mean it's not really teaching, its leading.

This passage was coded as "high level thinking and high levels of mathematics." This event is consistent with the "advancing" level.

Finally, Mrs. Chi reported that she started to incorporate Geometer's Sketchpad for teaching certain units such as conic sections after learning to use it at the summer and quarterly professional development program. Coded as "personal conviction to enhance student understanding," Mrs. Chi explained that she felt she was not doing enough in this unit to help her students to understand. She changed how she taught this unit so that students could better understand through multiple dynamic representations rather than completing the unit using very procedural methods as before. There are several indicators that showed these experiences are consistent with the "advancing" stage of TPACK development. This event was coded as "essential to modify curriculum to effectively incorporate technology" and "create-plan-implement- reflect cycle."

Summary. Mrs. Chi's TPACK development and journey to becoming an effective user of technology in her instruction occurred at a much later phase of her career than the other two participants but she still was able to progress to the advancing stage of TPACK development through sustained professional development and leadership roles related to technology. Also, she used technology in ways in her classroom to change how she had previously taught and to address learning needs of her students.

Summary of Cases

The observations, debriefings, and student artifacts for each observation were used to identify ways that each teacher used technology in effective ways, corroborating their selection as effective users of technology. In order to use technology in effective ways, each teacher had developed the TPACK necessary to plan and execute such a lesson. Thus, the most significant events in each teacher's development of TPACK and the barriers that were present in the events were discussed. Each teacher's TPACK trajectories progressed with experiences and education (see figures 4.3, 4.5, and 4.9). The teachers in each of the cases showed their greatest progress toward the "exploring" or "advancing" stages at differing stages of their careers: Mrs. Alpha during her internship, Mr. Beta during his early career, and Mrs. Chi during her late career.

However, this might be attributed to differences in the availability of technology, as each progressed the most during the time period when they had access to the most technology.

Emergent Themes Across Cases

The objectives of this section are two-fold: to deepen the understanding of the cases through comparison and identify key factors that may help explain the teachers' implementation practices with technology. After identifying the significant events and barriers in each participant's story that contributed to the development of TPACK and technology use, the events were coded to identify why the teachers implemented technology in the ways that they did. Each event was examined for factors that could describe an event's contribution to the teacher's implementation practices or that could describe teacher variables that impacted technology implementation such as a description of overcoming a barrier to technology use. The process of coding for these factors and then comparing across cases resulted in seven themes developed directly from the factor codes that appear to be contributing to each teacher's implementation practices. It is not the claim that the themes definitively explain or predict each teacher's implementation practices but rather they could contribute to the decisions that the teachers made to implement technology effectively.

Development of TPACK and technology implementation during preservice internship. The first theme, which emerged from the events related to TPACK development, was that Mr. Beta and Mrs. Alpha both had internships that helped them to develop their TPACK. Mrs. Alpha's internship experience was very significant in developing her TPACK and implementation practices. First, technology integration was a required part of her undergraduate work and internship so she began implementing technology in effective ways. Next, resources such as computer labs and graphing calculators were available for her use during her internship;

therefore, equipment access barriers did not impede her efforts to implement technology. Also, Mrs. Alpha stated that she showed an interest in using technology and was paired with a "tech savvy" cooperating teacher during her internship. She indicated that her internship prepared her and made her "comfortable enough with technology" to continue to use and explore the use of other technology in her teaching.

Mrs. Alpha described several experiences during her internship that were particularly positive and reinforced her use of technology. She stated that her students appeared very engaged in learning during technology-infused lessons. She also explained that the cooperating teacher and university supervisor for her internship provided her with support and praise, as well as resources and lessons to support her technology use. She described the impact of her internship on her continued use of technology.

> I think that the initial success [during the internship] and feeling that my students were really able to understand the mathematics has been something that I have carried with me since and used to continue to value a constructivist or investigative approach in the classroom even in the face of opposition or extreme frustration.

Overall, Mrs. Alpha's internship included multiple technology resources to implement lesson with students, various and consistent support, and successful student outcomes.

Mr. Beta did not indicate that his internship had the same level of impact on learning to use technology in effective ways, as he explained he already felt prepared to implement technology in the classroom and needed to focus on other pedagogical skills such as classroom management. However, he also was required to use technology during inquiry-based lessons during his internship; therefore, he gained experience with using technology in a learning environment with support before having to implement technology on his own in his own classroom. Thus, while it is possible that he also had significant growth in his TPACK during his internship, he considered it as secondary to the other pedagogical skills he felt that he needed to develop during his internship.

Both Mr. Beta and Mrs. Alpha experienced learning to integrate technology as a teaching and learning tool during their respective internships, but their internships differed in their focus. Mrs. Alpha felt that she had a strong focus on using activities with Geometer's Sketchpad to learn to help her students to explore and make conjectures in geometry. Mr. Beta felt that the focus of his internship was more on learning to use conveyance technologies and managing a classroom since he already felt confident in his abilities to implement technology in effective ways. In sum, both internship experiences required the use of technology as a teaching and learning tool, occurred in supportive and technology-rich environments, and likely contributed to their implementation decisions.

Development of TPACK through experiences in the MSP. Another TPACK theme that was identified in several events in the stories of the participants described professional development and educational experiences of the teachers as part of the MSP program as instrumental in their development of TPACK. Table 4.0 shows the timeline of MSP program participation for each teacher. Mr. Beta and Mrs. Alpha specified that they explicitly learned about the concepts of TPACK, while Mrs. Chi did not. Although Mrs. Chi did not indicate that she specifically learned about the concept of TPACK, the indicators of TPACK development were present throughout the events in Mrs. Chi's story. Mrs. Chi's experiences in her summer and quarterly professional development program both as a participant and a presenter helped her to develop her TPACK. For example, Mrs. Chi worked with others and watched other participants model how to use the graphing calculators to teach mathematics and that helped her skills to grow. She explained,

> It put a graphing calculator in my hand. I may have had one, I didn't play with it like I should. But then when we were working together coming up with activities and all that kind of stuff, it was just like the whole world of mathematics is opened up now. I mean it's like it is a different subject almost. It's not solving equations. It's using the equations to solve real life problems.

Mr. Beta stated that learning about the concept of TPACK during a graduate course that was part of the teacher leader fellowship program was the most influential on his classroom practices related to technology use.

Table 4.0

Timeline of MSP program participation

Teacher	Years in Career	MSP program component
Mrs. Alpha	1-2	Summer and quarterly professional development- participated and provided in PD related to implementing reform mathematics methods including technology
	3-5	Graduate school- master's degree in mathematics education (not part of MSP but aligned with goals and principles of MSP), Summer, quarterly, and technology-focused professional development- participated and provided in PD related to implementing reform mathematics methods including technology
	6-10	Graduate school as part of teacher leader fellowship program- began PhD program in mathematics education, Teacher leader fellowship program- participated in PLC aimed at developing teacher leader skills to influence change among peers related to reform mathematics
Mr. Beta	1-5	Graduate school as part of teacher leader fellowship program- master's degree in mathematics education, Teacher leader fellowship program- participated in PLC aimed at developing teacher leader skills to influence change among peers

Mrs. Chi	3-5	Professional development- to learn basics of computer programming
	6-10	n/a
	11+	Summer and quarterly professional development- participated and provided in PD related to implementing reform mathematics methods including technology

Each teacher reported that they had seen the effective use of technology modeled during their graduate education and/or the MSP professional development and it enhanced their knowledge of how to use technology in the classroom. Through Mrs. Alpha's experiences providing professional development, participating in a professional learning community, and taking graduate courses in mathematics and mathematics education supported her development of TPACK. She was exposed to a variety of technology uses in the classroom and interacted with teachers that used technology effectively. Mr. Beta also stated that he shared lessons and found new lesson ideas, including his recorded lesson, during his professional development sessions with other teacher leaders. He also reported that many of his lessons that use the graphing calculator for exploration were from participating the professional development program. Mr. Beta stated that he continued to share ideas and lessons with the teachers that participated in the teacher leader fellowship program and/or that were in his graduate classes in mathematics and mathematics education.

Also, each teacher reported that they engaged in improvement cycles due to their experiences in the MSP. For example, Mrs. Chi reported that she had to reflect on her own practices and continued to improve as she prepared to provide professional development to others in using Geometer's Sketchpad. Mrs. Chi and Mr. Beta indicated that since they were participating in professional development while they were teaching in the classroom, they were able to plan with others, implement their plans in the classroom, and then report back to their groups for reflection and improvement. Mr. Beta implemented what he learned in graduate school and continued the plan-implement-reflect cycle after graduating to further develop his TPACK. He explained,

> You have to be under the mindset that technology is not what makes them successful. What kinds of questions would I ask and how would I teach it? Then when I bring in the technology, how will that enhance it? How could I take it to the next level with technology? And that's not how I would have done it before. Before I just would have figured how could I teach this concept with technology. And that's part of it but not all of it. But it is not a replacement by any means of content or teacher.

Also, Mrs. Alpha stated that providing professional development for others had improved her TPACK because she engaged in a plan, implement, reflect and improve cycle as part of her preparation for the professional development process. She explained,

> I was forced to reflect on my own practices and I saw the impact on other teachers and their students as well. It also allowed me to interact with others and reflect on how I was using technology and to improve upon it… It was a powerful exercise for me to have to write these things down and then reflect on their effectiveness before presenting them to the group.

Mrs. Chi reported that the other teachers in the MSP's summer and quarterly professional development program were the source of her lessons that she chose to record for this study, and she had participated in sessions where she discussed the implementation and then adjusted the lessons. Thus, Mrs. Chi's professional development experiences also served as support for engaging in a plan-implement-reflect cycle that enhanced her TPACK.

In summary, the teachers each participated in continuing education in the MSP program that included an increased availability of resources, support from a community of teachers, long-term sustained professional development, and opportunities to learn to use technology effectively in the context of mathematics. These characteristics of continuing education contributed to the development of the TPACK needed by each teacher to implement technology in effective ways.

Continued education to overcome barriers. Although each teacher experienced multiple barriers to technology use, they each found ways to overcome or remove barriers in order to implement technology. After analysis of each event, examples of continuing education were identified such as the MSP's professional development programs, graduate education courses, other professional development, or professional learning communities that contributed to removing, overcoming or preventing common barriers to technology implementation in the classroom. Each teachers story contained several events that were coded for continuing education to overcome barriers.

Mr. Beta, Mrs. Alpha, and Mrs. Chi experienced some similar barriers to implementing technology in the mathematics classroom and developing positive attitudes toward implementation. For example, they each considered themselves to be outsiders in their department with respect to their beliefs about technology and their implementation practices. Their experiences ranged from colleagues that chose not to implement technology in effective ways to outright rejection and isolation from their mathematics departments.

However, Mrs. Alpha reported that the community in her professional development circle of teachers and her experiences in graduate school provided support and resources when she felt isolated in her ideology. She explained, "The [MSP professional development] workshops sort of let me know that I wasn't the only one who saw [technology] as a powerful tool." Mr. Beta stated that graduate school and professional development provided him with opportunities to collaborate with other teachers since none of his colleagues were using technology in the same ways; thus, the lack of in-house support did not prevent his implementation of technology. Also, Mrs. Chi reported that her collaboration with teachers from the summer and quarterly professional development programs led her to conclude that she was different from the other teachers in her building. She explained,

> The older I have gotten, the more I see that I am different. There are some teachers that just come and teach, do their job and go home. When I'm up here at 4 or 5 o'clock and I am working on the lesson for tomorrow, trying to figure out a great activity to start my class with or end with. And I am looking at teachers going home, beating the students out of the parking lot and I am thinking how can they do that? So I do see that I am different. Then at [professional development] I see teachers doing what they are supposed to do.

Thus, each teacher was able to overcome barriers associated with isolation due to their participation in continuing education.

Also, at some point in their careers, each teacher experienced a lack of technology resources in their classrooms, although the amount lacking varied greatly among the teachers. Mrs. Alpha, Mr. Beta and Mrs. Chi each indicated that at least some of the technology resources were provided by the MSP program. Mrs. Alpha also described how many of her lesson ideas or changes she made to her lessons that incorporated technology came from other teachers in her professional development community that developed from the MSP program. Mr. Beta explained that graduate school courses offered as part of the teacher leader fellowship program provided support for learning to use a wider variety of technology for exploration in the classroom and it helped him move away from ineffective technology for uses such as displaying notes or remedial work. Mrs. Chi stated that, “We have been blessed with on-going professional development for years. Anything that I ever needed and asked for, I have gotten from them. I mean, it’s just amazing.” She noted that collaboration with peers, multiple opportunities to explore technology, various lesson resources, and technology resources for her school were key factors in learning to use and implementing technology.

Not only did Mrs. Chi not have any technology experiences prior to entering the teaching field, but she was also discouraged from entering the field of mathematics entirely. Also, Mrs. Chi received very little training and instruction before implementing technology for the first time in the classroom. Mrs. Chi’s professional development journey began in 1979 when she attended a workshop provided by Apple to learn some basics about computer programming in preparation for teaching her computer course. Although she did not feel completely prepared, she stated that it was enough to allow her to continue teaching herself about programming.

Additionally, although Mr. Beta and Mrs. Alpha reported being adequately prepared by their undergraduate education to implement technology, they experienced classroom factors that are often barriers to technology use. Mr. Beta reported frustration with time constraints and content coverage with respect to his implementation of technology but stated that he had discussed in a graduate course how to revisit topics throughout the semester to help with time constraints. Mrs. Alpha reported experiencing classroom management issues related to resistance to inquiry methods and frustration from students as they used technology in the classroom. However, Mrs. Alpha stated that she found motivation to continue despite these struggles due to the expectations of teachers involved with the professional development that she had provided.

She stated that it was important to be "living out [her] message," and she used these experiences to reflect on and improve upon her use of technology.

Interestingly, none of the teachers reported that they stopped using technology due to the barriers described. Rather, they described the barriers in most cases as situations that needed to be fixed or worked through (i.e. provided their own technology, adjusted instruction to fit the needs of students). In the case of the department isolation barrier, no teacher reported that it led him/her to stop using technology or even to question his/her implementation practices, and all three teachers stated that their peers were in the wrong. Many of the barriers discussed have been established as common reasons for teachers to not use technology in the classroom. However, the experiences of each teacher related to continuing education through providing professional development, participating in a professional learning community, and/or graduate school courses in mathematics and mathematics education all contributed to removing or preventing barriers to technology implementation.

Access to resources and support. Clearly each teacher had access to resources and support at certain times during their careers as they each discussed the technology that they used and the ways they learned to use the technology. For example, Mrs. Chi stated that once she began her involvement with the MSP program, she was given technology resources to use in her classroom including class sets of graphing calculators. Both Mr. Beta and Mrs. Alpha repeatedly mentioned throughout their many experiences examples of on-going support for learning to use technology throughout their careers and the multiple sources of technology tools that made it possible for them to implement technology in the classroom. At the time of the interviews, each teacher had adequate access to technology and support resources; however, the sources of the technology and support mostly came from outside their own schools of employment.

It was clear throughout Mrs. Alpha's story that she had adequate access to resources and support for developing her technology use and lessons. Technology such as computer labs and graphing calculator technology were available for her use from her internship through her current position. She reported receiving encouragement and praise from mentors related to her technology implementation. Also, school technology resources were mostly provided as needed. For example, she described the support she received in her first teaching position.

> It became such an important part of my initial repertoire of teaching skills that when I was offered my first teaching job, I conditionally accepted on the terms that the system would purchase Geometer's Sketchpad for the computer lab. Surprisingly, the superintendent that hired me easily agreed and came through.

When additional technology was not available through the school system, Mrs. Alpha purchased her own technology early in her career using funds earned through the MSP program. Conducting professional development Geometer's Sketchpad workshops also resulted in technology resources for her school as well as new lessons and ideas.

Later in her career, Mrs. Alpha explained that she was often treated as a priority for getting new technology due to her reputation as a frequent user such as getting new tools to pilot or contributing to technology budgeting decisions. Finally, Mrs. Alpha reported that she retained a connection with her university advisor pertaining to technology as a personal hobby as well as a classroom tool. Although Mrs. Alpha indicated that she would have liked to have had more technology to use in her classroom and did at times during her early career in particular lack access to technology, she did not express a lack of access during the majority of her career.

Throughout Mr. Beta's TPACK development experiences, he had many resources and supports available to him. For example, the observed lesson was modified from its original version in order to meet the needs of his students after discussing the lesson with peers during a teacher leader fellowship program professional development experience. Also, Mr. Beta felt supported by his principal during his career because he featured Mr. Beta in a news report about technology use in the school. He often had access to technology resources from his school of employment, contributed to the budgetary decisions concerning technology, and was able to earn a grant to purchase graphing calculators for himself and other teachers in his building. Although Mr. Beta stated that he lacked formal assistance with technology early in his career, he reported that he became proficient with using online resources to troubleshoot technology. Finally, support for Mr. Beta's technology use can be traced back to his high school calculus teacher who allowed him to teach other students what he discovered about programming on the graphing calculator related to the class content. Each teacher received adequate support and resources in order to become effective technology users in the classroom.

Leadership roles related to technology. The next theme that was identified relates to the leadership roles related to technology use in the classroom that all three teachers held within the MSP program. Within these events, the ways in which these roles may have impacted their implementation practices were identified. To do this, statements that indicated that a teacher was acting in a leadership or mentorship role were coded as leadership. Then each piece of data that was coded as pertaining to technology if the role included teaching someone to use technology, having a reputation as a leader with technology, or other event that included both technology and leadership aspects. Each teacher's story contained at least two events that were coded for leadership roles related to technology.

Some of the leadership roles of the teachers were similar. First, each teacher reported having a reputation as a technology leader either within their school, within the MSP program's community, or both and were considered a source for innovative lesson ideas incorporating technology. Mrs. Chi and Mrs. Alpha both held leadership roles by providing professional development sessions for other teachers regionally as part of their participation in the MSP program. They reported that as a result of the professional development, they were considered leaders regionally and their own understandings related to using technology in the mathematics classroom were enhanced. For example, Mrs. Alpha reported that she led professional development sessions for regional teachers on Geometer's Sketchpad. She described her experiences,

> I used lessons from my classroom and others that I found in textbooks for using Geometer's Sketchpad and shared them with the teachers. I also shared little tips that I had learned through my experiences that just wouldn't be obvious otherwise. For example, I had begun using a small laser pointer to point at things on students' screens instead of getting in their space or giving in to the bad habit of just taking over their computers. Just little tips like that, the teachers said were very helpful. I still have teachers that attended the PD contact me asking for lesson ideas or tips for issues they are experiencing with GSP.

Also, Mrs. Chi served as a summer and quarterly professional development presenter and helped other teachers in her school and region to learn to implement graphing calculator and computer technology in their mathematics instruction. Mr. Beta was asked to provide professional development opportunities to the other teachers in his mathematics department related to using the graphing calculator to teach mathematics. In addition, early in Mr. Beta's career, he procured class sets of graphing calculators through a grant that led to him being considered the teacher consultant to the administration on how the technology budget should be used.

Mr. Beta and Mrs. Alpha both reported a sense of pride for their technology use and felt recognized by others as effective users. For example, due to his reputation as an innovative technology user in the classroom, Mr. Beta's principal chose him to be featured in a local news report highlighting technology use in the schools. Mr. Beta stated that his reputation as an advanced user of technology in the classroom made him a leader among his peers. He reported that other teachers as well as administrators regularly sought his help with technology including mathematical action technology, conveyance technology, and clerical technology uses despite not having formal training in troubleshooting technology.

Both Mrs. Chi and Mrs. Alpha reported that these roles made them accountable for using technology in their classrooms. Mrs. Alpha stated that she felt others viewed her as an innovative and proficient technology user and was asked to be a guest speaker on technology use in the mathematics classroom at another school system due to her reputation. Although she did not feel that she was valued in her own school, she stated that her leadership roles allowed her to receive recognition outside of her school for her use of technology. This recognition, along with the responsibility she felt to continue to reflect and improve on her technology practices as a leader in her region, were cited as factors during her interview and observation debriefings that influenced her implementation practices.

Mrs. Chi felt that she was viewed in her school as a leader in graphing calculator implementation and as a source of novel ideas for technology-based lessons. She described herself as the "head honcho" at her school when it came to innovative lessons with the graphing calculator. She stated, "If anybody wants to know anything about graphing calculators, then they come see me. Or if they need an idea for how to use the graphing calculator." In response to the follow up questions about her leadership roles, Mrs. Chi stated that she felt the pressure to be a good role model at all times in order to influence change among the teachers in her school. She explained,

> I needed to held accountable for the teachers because they needed to see how to be professional. The complaining teachers won't try anything new if they think the complaints are going to fly with me so if I am not walking the walk then I can't expect them to walk the walk. I am going to walk the walk either way. With or without them. I was not going to change. They would go with me or not. This was even for others that were not mathematics teachers, like the older teachers that saw me doing new things that I hadn't done before and I kept telling everyone that it is whole lot more fun. But yes it had to be do as I do, not just do as I say.

Both Mrs. Alpha and Mrs. Chi both reported being mentors to students was important to their development as technology users. Mrs. Chi had been a mentor to several of her former students that became mathematics teachers or computer programmers, and both Mrs. Chi and Mrs. Alpha served as a mentor for university students working on degrees in mathematics education. Mrs. Alpha stated that she was often asked to work with undergraduate students in her classroom as a supervising teacher for technology-focused lab experiences and internships with the university program which she was prepared and as a requirement for her participation in the teacher leader fellowship program. She indicated that both of these roles held her accountable for using technology in her own classroom. Mrs. Chi reported that she also worked with university students in her classroom to use technology but felt that they more often than not were well prepared to use the technology in their lessons.

The teachers described other leadership roles related to technology in the classroom. At the beginning of her career, Mrs. Chi was recruited by her current school of employment to teach a computer course because she was known to be able to teach computer programming at a time when there were very few computer programming teachers; thus, she became a leader in introducing technology into her school. Also, both Mr. Beta and Mrs. Alpha participated in the teacher leader fellowship program to become a school teacher leader for reforming mathematics education in their schools that included the incorporation of technology in the classroom. They attended sessions presented by mathematics educators that focused on implementing workshops in their own schools to engage colleagues in activities and discussions about using inquiry-based techniques in the classroom such as technology-based explorations.

Clearly, each of the teachers engaged in activities as leaders that promoted the use of technology in the classroom and allowed them to continue to develop their skills. In Mrs. Alpha's case, she indicated that these roles directly impacted her decisions related to technology implementation in the classroom. Mrs. Chi also indicated that she felt accountable for implementing technology in their classroom in effective ways.

Technology fulfills a need. Throughout the interviews and observation debriefings, the teachers often made statements such as "I use technology because of ______" or "Technology helps with ________ in my classroom." These types of statements were coded as "fulfilling a need" because they directly indicated the expected results of the uses for technology in each teacher's classroom, thus addressing some classroom need identified by the teacher. A common thread throughout each case was that the uses showed that teachers valued the use of technology in the classroom to improve student learning in particular. Table 4.1 summarizes the learning needs addressed by technology according to each teacher and the needs that were common among all three teachers.

Table 4.1

Summary of Student Learning Needs Addressed by Technology Including the Needs that Were Common Among the Teachers and the Needs Unique to Each Teacher

Teacher	Learning need
Common	Provides multiple representations to enhance understanding Students understand mathematics better because of visualization and explanations Needed exposure to technology and complex tasks Students investigate and discover Students that cannot learn through straight lecture, are struggling, or are weak in mathematics need other methods of learning
Mrs. Alpha	Remediation Compensate for disabilities Learning phase supported by technology is more effective
Mr. Beta	Kept students engaged to help develop procedural skills Improved focus Efficient and more can be accomplished Show variety of methods used by students
Mrs. Chi	Focus can be on understanding of bigger concepts Can see real world applications of mathematics

There were several common needs addressed by the use of technology discussed by each teacher. First, they each felt that technology provided students with multiple representations of the mathematics they were learning to help them better understand through enhanced discussions and visualizations. Mrs. Alpha stated that technology was used to help students that rely on visual representations to understand mathematics. In her interview, Mrs. Alpha stated that she felt that all of her students learned mathematics better when the learning phase was supported by representations with technology. Mrs. Chi stated that she had felt she was not doing enough in her instruction to help students to understand the mathematics, but after introducing graphing calculators to her students, they were able to see and understand mathematics in new ways. She explained how she changed a particular unit using technology.

> I do conic sections on Geometer's Sketchpad (GSP). I teach conic sections and that was the one thing I hated more than anything else was teaching conic sections. All that completing the square and all that kind of stuff, it was so boring. The kids were bored and struggled to understand. Not anymore. There's GSP for conic sections. Oh my goodness yes. You can see it. You can move it; you can change it. Before you would do the same thing over and over and over again. You can say step one this, step two do this and you might could graph it by hand and its ok. And I still teach that but now it's like what does that mean? What does that bunch of numbers mean? Well let's look at GSP and see what happened.

Mr. Beta also used technology to help students visualize the mathematics being taught. For example, he used his technology to show different ways to approach problems by displaying student work and also showed multiple representations during instruction.

Each teacher also used technology to help struggling students, students with weak mathematical skills, and students that could not learn well with a strictly lecture-based approach to instruction. Mrs. Alpha valued technology in her classroom as a tool to help differentiate instruction and to help students with disabilities. She talked about the different abilities in her classroom and the need to address a variety of different skill levels. For example, technology was used by Mrs. Alpha to make sure that all students had opportunities to learn. She explained,

> I believe that technology should be used to help students with disabilities to achieve the same level of understanding as non-disabled students. For example, in geometry I often choose to use GSP to do constructions. Many of my students have had disabilities that

make them physically unable to measure accurately with traditional tools. Also at times when the measuring and by-hand constructions are not the focus or objectives of the lesson, I can use technology to help them bypass some human errors that might make discoveries more difficult. I guess I look at technology as the great equalizer in mathematics. This says about me as a person that I think it is my job as a teacher to find ways to make sure every student has an opportunity to learn and understand mathematics.

Mr. Beta reported that for struggling students, he used technology to help explain ideas and they responded better to those lessons. He indicated that technology often helped him to focus better and that he found it did the same for his students. He stated, "I am finding that with technology I can get it across what I am asking. Whereas if we had to do it all by hand, a lot of my kids are not going to want to sit there." Next, Mr. Beta described a particular lesson in which he used a graphing calculator activity to teach struggling students to graph a line by having them graph lots of lines on paper that were copied from their work on the calculator. They were able to repeat the process over and over until they started to recognize the patterns themselves and not have to rely on the calculator. Mr. Beta stated that using the calculator gave them a place to start to understand.

Mrs. Chi recognized that not all students were able to learn mathematics the way that she had, through lecture-based instruction and by-hand calculations. She stated that the technology helped these students to understand the mathematics. She described the advantages as follows.

> I can't tell you what the graphing calculator has done for my students. Groups, being able to talk, being able to look at that calculator and talk about what you are seeing on the calculator. They can look at the graphs, understand the graphs. I could do fine in a

> classroom where they just lectured to me and I did it because I could understand it. I could understand the procedure. Not everybody is that way.

The teachers also valued technology for the opportunities it afforded students to explore and investigate deeper mathematics on their own. Mrs. Alpha described multiple ways that she used technology in the classroom to help students gain a more complete understanding of mathematics. During the observation debriefing, Mrs. Alpha explained that the use of graphing calculators with her geometry students allowed them to focus on how changing the equation would change the graph of the line without "getting lost in the process of graphing." She noted that while graphing was an important skill, it was not the purpose of the lesson. She felt students gained a better understanding of the connection between the algebra and geometry concepts presented in this lesson because the calculator allowed students to focus on broader ideas. She found technology essential for helping students to learn how to discover mathematics through investigations. In Mrs. Alpha's classroom, technology was used to make complex tasks more efficient and precise to allow students to focus on the overall concepts and not get caught up in the steps.

Finally, they all indicated that exposure to technology and complex problems that could be solved with technology was part of their responsibility for preparing students for beyond the high school classroom. Mrs. Alpha stated that it was essential for students to be exposed to a variety of technology tools in order to prepare them for their lives after high school. During the interview, Mr. Beta stated several ways that he used technology specifically to help students to understand mathematics in ways they would not be able to without the technology. Mrs. Chi also explained that she used technology in her classroom to give students opportunities to see applications of mathematics that might be relevant outside of the classroom. She stated, "You can actually see the real world right there on the graphing calculator. It is real. It is not just pretend problems or just solving equations. It's real life, however you want to use it." Next, Mrs. Chi used technology to help her students to focus on the larger concepts being taught in a lesson and not get caught up in tedious by-hand calculations like she had experienced during her education. As she recalled her own education, she stated, "We spent our time on arithmetic, instead of mathematics." Also, Mrs. Chi felt that her students needed to know about technology so their exposure to various technologies in mathematics class helped to prepare them for the outside world. She explained,

> I mean, the world is different. I can't stay the same. The world is different. I am teaching my kids to be in the world, not in my little fantasy world. In the real world, which is technology. So they can't survive unless they know technology. They can't.

Mr. Beta also stated that students needed to be exposed to using technology to solve complex problems. He explained,

> I've got unfortunately a lot of student that will not go to college but that may go into a job where they may have to have a heavy use of technology. So why not give them a chance right now to learn complicated tasks. Why not have a task where you have to use the calculator for [a lesson] that involves 20 or 30 steps. You have to remember. And you go through, do this, do that, you have to explain to other people how to do it. Because on the job, I'm sure that everyone doesn't go to work and just hit one button and a key or something. Or they have to know how to do complicated things with technology.

Mr. Beta also stated that although many mathematics professors at a nearby college did not allow mathematics students to use a graphing calculator, that his student still needed to be exposed to its use.

Mr. Beta further stated that he had used technology as a student to make unchallenging classes more interesting for him and wanted to provide that type of environment for his students. He explained,

> I wasn't sitting there bored in class and I got to figure out things that maybe I wouldn't have gotten the chance to figure out [without the technology]. I think it changes maybe the way I will try to present a task. I will want to present it how it would have been for me, so that people can go ahead and figure things out before we go ahead and discuss it. And so, I want people to be able to figure things out so I try to pick tasks where they can actually get a change to figure things out.

Finally, Mr. Beta often expressed concern about running out of time in the classroom and he felt that he had progressed in his use of technology to a point where he could get more accomplished during a class period by using technology.

The teachers reported that they used technology in the classroom to address a variety of student learning needs and that technology was an effective and appropriate tool. Thus, their decisions to use technology in the classroom were likely associated with the value they placed on technology as a tool to fulfill classroom needs pertaining to student learning.

Growth mindset related to teaching and technology. Results suggest that a growth mindset with respect to teaching and technology may have developed in the teachers in the study as they engaged in long-term learning experiences. Each teacher was an effective user of technology in part because they were open to learning about new technology tools and demonstrated a desire to expand their TPACK, as well as their pedagogical knowledge in general. Events that showed a desire to improve his or her teaching in an effort to become a more effective teacher with or without technology were coded as "growth mindset." Each teacher described events where they learned to use technology in an effort to improve their teaching, but Mrs. Chi and Mr. Beta described many events that showed they had a growth mindset throughout their careers related to teaching and using technology. Additionally, both Mrs. Chi and Mr. Beta showed a willingness to learn in front of their own students and even were willing to allow students to teach them. They both reported a need to improve but differed in what they felt needed to improve. For example, Mrs. Chi was satisfied with her teaching methods but felt that learning to use technology had helped her take her teaching to "another level," whereas Mr. Beta was satisfied with his knowledge of technology but often felt that his teaching methods in general needed to improve in order to better implement technology. Overall, Mr. Beta and Mrs. Chi recognized that they could improve their teaching in ways that would help student learning with technology and expressed a responsibility to do so.

Throughout Mr. Beta's experiences related to development of TPACK he often showed a growth mindset toward teaching skills and technology use. Mr. Beta stated that he often looked for ways to improve his teaching and expressed that he was very critical of himself as a teacher. When asked about a high point in his teaching career, Mr. Beta struggled to find an example. He explained,

> I am always seeing myself as wanting to improve so I don't tend to think "Oh that was some hot stuff right there" so this is a hard question. I'm always like I wish I could do this better or that better so everything I think needs improvement. I'm always trying to improve.

After he described a situation in which he felt successful in teaching with technology, he continued to be critical of his teaching. He stated, “So all I was thinking at that particular moment was ways in which I could improve or this didn’t go well, that didn’t go well, and I wanted this to get better.” Throughout the interview, Mr. Beta talked about ways that he reflected on student learning during a lesson and looked for ways to improve. For example, he indicated that using technology in the classroom and reflecting on the use served as one of the greatest opportunities to learn and improve on his implementation of graphing calculators.

Mr. Beta reported other examples of having a growth mindset. He reported that he learned the importance of the influences of the community on his students and how it impacts his students’ performance. Also, he described experiences implementing ideas learned in graduate school including concepts of student discourse, student reasoning, and sense-making strategies. Mr. Beta explained that learning about TPACK made him reflect on how he was using the graphing calculators, and he reported that he now engaged in a cycle of plan-implement-reflect-revise with his lessons using graphing calculators. Next, Mr. Beta stated that a student had “called him out on best practices” and he took the critical feedback from students with regards to technology use when he was monopolizing the technology and not allowing students to use it. Finally, Mr. Beta demonstrated a growth mindset when he stated that technology use in the classroom for him was a progression and he had been willing to try things and possibly have a lesson fail in an attempt to become proficient with using technology.

Mrs. Chi also often showed a growth mindset related to teaching and learning with technology throughout her described experiences. During her early career, she volunteered to teach the school’s first computer classes with no experience at all because she thought that was the direction that education was going and expressed a desire to change and grow with the times.

She commented, “I thought it would be fun. Yeah but you know you could see that [technology] is where it was going. I mean you had to know something about computers.” While she was teaching the computer courses, Mrs. Chi stated that she was mostly self-taught and described a willingness to learn with her students. She allowed students to teach her and the other students what they knew and were learning about the technology.

Mrs. Chi also stated that students were very immersed in technology outside the classroom so it was her responsibility to learn to use more technology in the classroom in order to prepare them for their lives after school. She explained,

> You can’t get away from it. These kids, that’s all they know is technology. That’s all they know and you can’t change it. You are not going to change the way they look at life. I mean it is technology everywhere. Either you join them or you are left behind. You have got to do it; you don’t have an option.

She expressed frustration with colleagues who often left school “with the kids” and did not try to improve while she stayed late working to revise lessons to improve student understanding. She stated that she felt that the way that she taught before using technology was not enough to help her students to truly understand the mathematics so she had looked for something to increase student understanding and immediately saw technology as the key. She stated,

> I was having a little yearning to do something different because I knew that what I was doing was not [good enough]. Well I was getting good results, but it was not, I knew it was wrong. It wasn’t enough, that was it. But when I went to my first meeting [of the professional development program]. I knew immediately, there was my light switch right there. I knew we had to be a part of it.

Mrs. Chi showed a growth mindset and willingness to learn about technology throughout her teaching career.

It is interesting to note that Mrs. Alpha did not make statements during her interview or debriefings about needing or wanting to improve her teaching or use of technology in the classroom. However, there was evidence in her narrative that indicated that she had a growth mindset. For example, she participated in multiple professional development opportunities and continued her education by pursuing a graduate degree. None of these activities were required for her teaching position; thus, the voluntary participation may be an indication of having a growth mindset related to teaching mathematics. She also stated that she engaged in plan-implement-reflect-revise cycles when preparing for professional development.

Each teacher showed a willingness to learn how to use technology in their stories even if a need for improvement was not explicitly stated. Clearly, not all teachers that have a growth mindset with respect to teaching value technology as a tool for teaching and learning. However, having a growth mindset with respect to teaching and technology appears to be related to developing the knowledge needed to become an effective user of technology and teachers' implementation practices. The long term professional development experiences in the MSP program provided multiple models of exemplary practice with technology and likely contributed to the participants' desire to improve their own teaching skills.

Summary of Findings

The findings of this study are displayed in figure 4.10. Evidence suggested that the teachers used technology in their classrooms in effective ways, validating their section as cases in this study. Each teacher's story included evidence of effective technology use (in light blue) and themes (in purple) throughout the events they reported, including developing TPACK through internship and MSP experiences, removing barriers through continuing education, access to resources and support, leadership roles, technology fulfilling a classroom need, and growth mindsets. These themes were identified as factors that appeared to have influenced the implementation practices of the teachers. Various events made up each teacher's stories; the most impactful events (in dark blue) appeared to be their undergraduate education, internship experiences, and continuing education opportunities. Teachers' descriptions of the significant events in the development of the knowledge and skills needed to implement technology effectively contained common characteristics (in yellow, green, and pink).

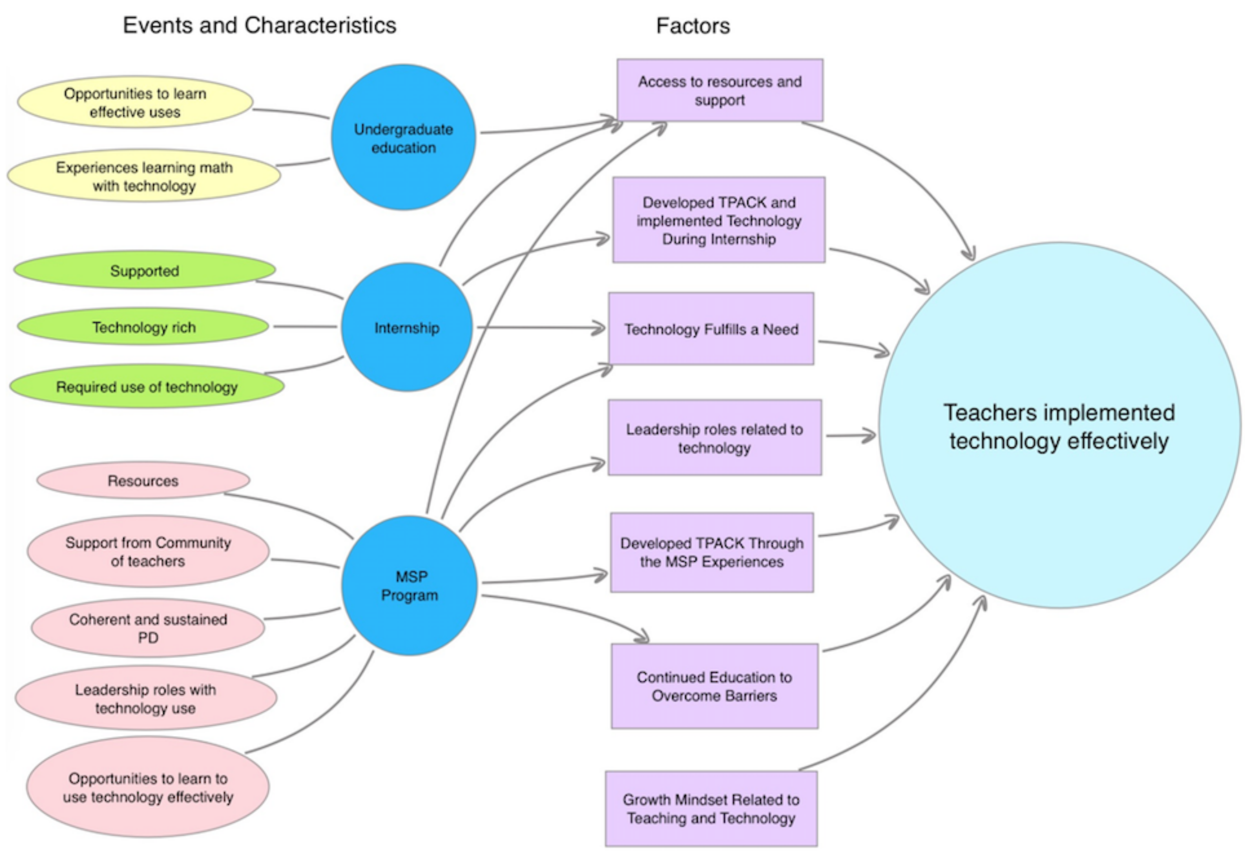


Figure 4.10. Summary of research findings

5: Limitations, Conclusions, and Implications

Recent assessments of student progress in the U.S. have shown that far too few students are performing at or above proficient levels in mathematics (NCES, 2015; OECD, 2015). Calls for implementing effective pedagogical methods such as using technology to reason and make sense of mathematics are based on research that has shown that such methods can result in student learning gains (AMTE, 2015; NCTM, 2014). The focus of this study was to examine the types of experiences teachers had as they developed the knowledge and skills needed to effectively use technology and to identify factors that may have influenced their technology implementation practices.

Three secondary mathematics teachers identified as effective technology users in the classroom participated in interviews that elicited stories about the events that contributed to their use of technology in the classroom. The teachers had common experiences in a comprehensive professional development program whose aim was to prepare teachers to use research-based teaching methods in the mathematics classroom in order to improve student learning outcomes. Classroom observations and student work were also collected and analyzed. A priori and emergent coding were used to identify themes across cases that were factors related to technology implementation. This chapter begins with a discussion of the limitations of this study, then addresses how the findings answer the research questions with reference to relevant literature, and concludes with a discussion of the implications of the study for various audiences and for future research.

Limitations

There were several limitations related to the methodology employed. First, the results rely on the accounts of the participants and video-recorded lessons; thus, there is not an extensive use of sources to corroborate their accounts. Second, my own biases and my personal knowledge of the participants may have prejudiced my interpretations of recorded lessons and the events described by the participants despite the precautions discussed in chapter 3 taken to avoid such influence on interpretation.

Next, the small sample size of this study limits the generalizability of the findings. However, the findings are presented to better understand the implementation practices of three exemplary teachers with respect to technology use and provide insight into factors that could impact their decisions to use technology. The identification of these factors is important for possible future research using a larger sample of teachers. Lastly, the collection of some field data was incomplete in that Mr. Beta was only able to provide one video-recorded lesson and the video camera fell flat for approximately 15 minutes of one of the lessons. However, evidence of effective use was identified in the recordings that were provided.

Conclusions

In this section, key results are used to answer the research questions along with associated literature.

Research question 1. *What are the events experienced by teachers related to learning to use technology as a reasoning and sense-making tool in the secondary mathematics classroom?*

A number of events were identified that were critical in teachers' development of the knowledge need to implement technology effectively. In two cases, the undergraduate education programs included instruction that modeled effective uses of technology and experiences learning mathematics in the context of inquiry with technology. Researchers have reported that teachers develop positive attitudes toward teaching with technology most often when they can engage in learning mathematics with technology to understand the effectiveness of the tool (DeVilliers, 2004; NCTM, 2000; Kurz & Middleton, 2006; Hardy, 2008; Wachira et al., 2008). In particular, in those two cases, their internship experiences were characterized by a supportive environment, sufficient technology resources, and a requirement to use technology in lessons to promote reasoning and sense-making, also supported the development of technological, pedagogical, and content knowledge (TPACK) (Mishra & Koehler, 2008) and positive attitudes toward teaching with technology.

Each teacher had events coded for indicators at the “advancing” level of TPACK development during their experiences in or associated with a Mathematics and Science Partnership (MSP) program funded by the National. Mishra and Koehler (2006) found that teachers who collaborated on activities that focused on the relationships among content, pedagogy, and technology developed their TPACK. In two cases, the participants engaged in a planning process as presenters for the MSP’s summer and quarterly professional development program, in which they designed modules to help teachers learn to use inquiry-based instructional methods, often with technology. As presenters, they had opportunities to collaborate on the interplay of content, pedagogy, and technology as they planned and implemented the modules with participants. Sturdivant et al. (2009) found that teachers unable to effectively implement technology lacked experiences with integrating the various components in a technology-based lesson such as facilitating groups and enhancing lessons with appropriate technology.

Although some of the participants' experiences related to learning to use technology effectively in the mathematics classroom were different, their events were coded for several common TPACK stage indicators. In the events consistent with the "adapting" stage of TPACK development, all three teachers began using lessons in their classroom that required their students to develop mathematical thinking skills while using technology. This is consistent with Drake's (2006) claim that teachers much change their beliefs about what teaching and learning mathematics means in the context of reform mathematics methods. At the "exploring" stage of TPACK development, experiences of all three teachers were coded "teacher manages technology activities while at the same time promoting student engagement and self-direction." Park and Ertmer (2008) found that effective technology use required a shift by teachers toward a more student-centered approach to teaching. The teachers each engaged in collaborative cycles of improvement with technology-based lessons during events coded as "advancing."

Research question 2. *How did the events and experiences in the personal narratives of secondary mathematics teachers influence their implementation practices of technology in the context of reform mathematics*?

The events of the teachers' stories did not necessarily explain their decisions about implementation practices but factors related to the facilitation of technology use were identified across each of the cases. Their continuing education experiences in general and MSP program experiences in particular were associated with overcoming common barriers to technology use as a teaching and learning tool. Continuing education experiences in the MSP program that may have removed potential barriers for technology use for each teacher included plentiful technology resources, support from a community of like-minded teachers, long term sustained supports, leadership roles related to technology use, and opportunities to use technology to explore mathematics. Norton et al. (2000) and Bennison and Goos (2010) found that the school environment can negatively impact efforts for pedagogical change. The professional community of teachers that emerged from the MSP program and included each of the participants provided necessary support for overcoming the negative social climate each teacher experienced. The lack of knowledge in all aspects of technology use is a commonly cited barrier for use of technology in the mathematics classroom (Stoilescu, 2014; Sturdivant et al., 2009; Norton et al., 2000). However, each teacher had opportunities to learn about various types of technology tools and how they can be used in the mathematics classroom including the MSP program professional development and preservice methods courses also aligned with the goals of the MSP program. Okumus et al. (2016) noted that support for teachers for implementing technology needed to extend beyond the professional development environment to ensure their development of skills and self-efficacy. Chamblee et al. (2008) also reported the need for prolonged support for teachers to adopt major pedagogical changes such as technology implementation. All three teachers in this study participated in long term professional development through the MSP program that provided support for their growth as teachers of mathematics and development as effective users of technology.

Technology was valued by each teacher in this study as a tool to address their students' learning needs and identified as a motivating factor for implementation. Chamblee et al. (2008) emphasized that teachers need professional development opportunities that address their personal situations and include topics most relevant to their own classrooms. Several other researchers have concluded that when teachers find value in a tool to accomplish the curricular goals that they deem important, they are more likely to implement the tool in their classroom (Manfra & Spires, 2013; Okumus et al., 2005; Tonduer et al., 2008; Zhao et al., 2002).

Each teacher held leadership roles outside of the classroom with respect to technology implementation and indicated that the roles influenced their practices in their own classrooms due to accountability associated with the roles. Okumus, Lewis, Wiebe, and Hollebrands (2016) found that teachers' perceived levels of expertise with the technology influenced how and if they implemented technology in the classroom. The teachers in this study indicated that they were considered by colleagues to have a high level of expertise with a particular technology tool due to their leadership roles.

The teachers reported multiple successful learning experiences as they gained the knowledge and skills needed to implement technology effectively, and that likely contributed to their development of a growth mindset over time. Bennison and Goos (2010) found that successful professional development experiences increased teacher confidence and developed more positive attitudes about technology. It is likely that increased confidence and positive attitude toward technology supported each teacher's willingness and desire to learn and implement new pedagogical methods including effective technology. The teachers in this study also described being comfortable with learning with the students which is consistent with findings by Guerrero (2010) who found that teachers that used technology effectively were confident in their own abilities to inquire with technology.

Implications

In this section, the implications of this study for several audiences are given, including for teacher educators that are tasked with preparing pre-service mathematics teachers to use technology in the classroom, for professional development providers related to supporting mathematics teachers to use technology, and for practicing teachers of mathematics.

Teacher educators. This study suggests that an important factor in helping mathematics teachers develop practices that support effective technology use is engaging them as preservice teachers with technology in the meaningful ways that they are expected to use the technology in their own classrooms prior to teaching experiences. Mathematical action technologies in particular should be a primary focus of methods courses and mathematics courses aimed at prospective teachers. These courses need not focus solely on the basics of technology or activities consistent with the "recognizing" and "accepting" stages of TPACK development. Instead, they should give preservice teachers opportunities to engage in learning mathematics with technology using activities consistent with the "adapting" or even "exploring" stage of TPACK development. Moreover, pre-service teachers should be made aware of resources that are available to eliminate barriers to technology implementation so that they are prepared to take a more proactive approach in their own classrooms. For example, pre-service teachers should explore grant writing techniques and be encouraged to seek out online or local professional learning communities for support.

Professional development providers. None of the teachers had indicators in the "advancing" stage during their initial teacher education program. This suggests that further training beyond the teacher education program and during their careers must be made available for teachers to have the types of experiences needed to reach the advancing level of TPACK development. This research also suggests that a professional development program that is sustained and coherent can result in effective technology implementation. Such professional development should include support for learning and for continuing to implement what was learned. An increased emphasis on student learning needs addressed by technology during professional development for teaching with technology may provide increased motivation for teachers to subsequently incorporate technology into their classrooms. Long-term support well beyond an initial professional development experience or teacher preparation program is necessary to ensure that teachers continue to develop their TPACK and receive the necessary support, guidance, and resources as needed. Although schools often lack resources to properly support teachers using technology, such support does not need to be provided in house and can be designed to serve a regionally connected set of teachers (i.e. university programs that serve multiple school systems). Professional development providers should develop partnerships with school districts and administrators to procure resources and engage teachers in a coherent program.

There were several experiences that appeared to move the teachers forward in their TPACK and were common among all three teachers. Teachers should be provided with continuing education opportunities that include engaging in improvement cycles with a professional learning community, using technology to have students develop mathematical thinking skills in the classroom, and implementing lessons that promote student engagement and self-direction instead of teacher-focused lessons. Professional development should engage teachers in activities to further their mathematical understandings by exploring with technology and not on the "ins and outs" of technology tools. Additionally, engaging teachers in leadership roles related to learning to use technology in the classroom may increase the likelihood of implementation due to a sense of accountability associated with the leadership roles.

Teachers. The importance of a professional learning community is highlighted throughout this study. Teachers should seek out professional learning communities that can provide support, access to exemplary models, lesson ideas, and opportunities to continue enhancing professional knowledge. NCTM (2014) identified the importance of this professional collaboration as essential to creating a collective sense of responsibility for the success of all students in the professionalism principle from *Principles to actions: Ensuring mathematical success for all*. Also, the teachers in this study reached the advancing level of TPACK development only after participating in continuing education experiences beyond their initial teacher education, suggesting that teachers should seek continued education to increase their skills and knowledge related to technology use. Finally, teachers should strive to resolve their own issues related to barriers to technology use when possible.

Future Research

While this study provided a deeper understanding of the events that impacted each teachers' knowledge and skill development and their technology implementation practices, some findings of this study might warrant further investigation. This study suggests that teachers valued technology that addresses the learning needs of student in their personal classrooms. This finding might be explored more in depth by examining how technology-focused professional development designed to address the most relevant learning needs of each participants' students impacts levels of technology implementation. While this appeared to be a motivating factor for technology use by the teachers in this study, more research is needed to determine the impact of this approach on a larger set of teachers.

Clearly the participation in the MSP program impacted the participants' implementation of technology; however, additional research could further examine the impact of this or similar professional development programs. For example, what are the views on the value and roles of technology for teaching and learning mathematics held by other participants that were not presenters or did not hold leadership roles? Also, how did the professional learning community emerge from the MSP? Have teachers in this PLC continued their collaborations? What purpose does the PLC currently serve with respect to technology implementation in the classrooms of the participants? An extended study of a larger set of teachers that participated in the MSP program could be helpful in answering these questions.

Further research on the impact and development of growth mindsets among teachers may also be helpful. While the growth mindsets of the teachers in this study contributed to their development of TPACK, further research exploring the connection between growth mindsets and technology implementation practices is needed. Also, further studies could examine how teacher educators or professional development providers can help teachers to develop such a growth mindset about the use of technology and effective practice to promote mathematics learning.

A final extension would be to explore the impact of the implementation of technology on student learning in the classrooms of the participants and others that participated in the MSP's technology-focused professional development and professional learning communities. Student learning outcomes were beyond the scope of this study but would be helpful for informing what uses of technology are most effective in terms of student learning.

Conclusion

This study has explored why some mathematics teachers become effective users of technology in their instructional practice while others do not by examining the learning experiences of three exemplary teachers who increased their ability to effectively implement technology over a span of several years. While this study may not have found interventions that guarantee that effective technology uses will be implemented, several factors that may be associated with TPACK development and technology implementation practices of teachers have been identified and should be explored further so that more students have opportunities to develop deeper mathematical understandings through explorations with technology.

References

Association of Mathematics Teacher Educators [AMTE]. (2015). *Position of the Association of Mathematics Teacher Educators on technology*. Retrieved from https://amte.net/sites/default/files/technologypositionstatement-nov2015.pdf

AMTE. (2017). *Standards for Preparing Teachers of Mathematics*. Retrieved from amte.net/standards.

Ball, D. L. (1994). *Developing mathematics reform: What don't we know about teacher learning- but would make good working hypotheses?* Paper presented at the Teacher Enhancement in Mathematics K-6, Arlington, Virginia. Retrieved from ncrtl.msu.edu/http/craftp/html/pdf/cp954.pdg

Ball, D. L. (2003). Mathematical proficiency for all students: Toward a strategic research and development program in mathematics education. Santa Monica, Calif.: RAND Corporation.

Ball, L. and Stacey, K. (2005). Teaching strategies for developing judicious technology use. In W. J. Masalski & P. C. Elliott (Eds.), *Technology- Supported mathematics learning environments: 2005 National Council of Teachers of Mathematics yearbook* (pp. 3-15). Reston, VA: National Council of Teachers of Mathematics.

Ball, D. L., Thames, M. H., & Phelps, G.. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407.

Bennison, A. & Goos, M. (2010). Learning to teach mathematics with technology: A survey of professional development needs, experiences, and impacts. *Mathematics Education Research Journal*, *22*(1), 31-56.

Bokosmaty, S., Mavilidi, M., & Paas, F. (2017). Making versus observing manipulations of geometric properties of triangles to learn geometry using dynamic geometry software. *Computers & Education*, *113*(1), 313-326.

Chai, C.S., Koh, J. H.L., & Tsai, C.C. (2013). A review of technological pedagogical content knowledge. *Educational Technology & Society*, *16* (2), 31–51.

Chamblee, G., Slough, S., & Wunsch, G. (2008). Measuring yearlong study. *Journal of Computers in Mathematics and Science Teaching, 27*(2), 183-194.

Clandinin, D. J., & Connelly, F. M. (2004). *Narrative inquiry: Experience and story in qualitative research*. San Francisco: Jossey-Bass.

Conference Board of the Mathematical Sciences (2012). *The mathematical education of teachers II*. Providence RI and Washington DC: American Mathematical Society and Mathematical Association of America.

Creswell, J. W. (2007). *Qualitative inquiry and research design: Choosing among five approaches*. Thousand Oaks, California: Sage Publications, Inc.

Creswell, J.W. (2009) *Research design: Qualitative, quantitative, and mixed methods approaches*. 3rd Edition. Los Angeles: Sage Publications, Inc.

Darling-Hammond, L., Zielezinski, M. B., & Goldman, S. (2014). Using technology to support at-risk students' learning. *Stanford Center for Opportunity Policy in Education.* Retrieved from https://edpolicy.stanford.edu/publications/pubs/1241.

Delamont, S. (2009). The only honest thing: Autoethnography, reflexivity and small crises in fieldwork. *Ethnography and Education*, *4*(1), 51-63.

Denzin, N. K. (1989). *Interpretive biography* (Vol. 17). Thousand Oaks, California: Sage Publications, Inc.

De Villiers, M. (2004). Using dynamic geometry to expand mathematics teachers' understanding of proof. *International Journal of Mathematical Education in Science and Technology*, *35*(5), 703-724.

Dick, T., & Hollebrands, K. (2011). *Focus in high school mathematics: Technology to support reasoning and sense making.* Reston, VA: National Council of Teachers of Mathematics.

Drake, C. (2000). *Stories and stages: Teacher development and mathematics education reform.* (Unpublished doctoral dissertation). Northwestern University, Evanston, IL.

Drake, C. (2006). Turning points: Using teachers' mathematics life stories to understand the implementation of mathematics education reform. *Journal of Mathematics Teacher Education*, *9*(6), 579-608.

Dunham, P., & Hennessy, S. (2008). Equity and use of educational technology in mathematics. In M.K. Heid & G. G. W. Blume (Eds.), *Research on technology and the teaching and learning of mathematics: Syntheses, cases, and perspectives* (345-418). Charlotte, North Carolina: Information Age Publishing, Inc.

Eisner, E. (1998). The enlightened eye: Qualitative inquiry and the enhancement of educational practice. Upper Saddle River, NJ: Prentice-Hall.

Ellington, A. J. (2003). A meta-analysis of the effects of calculators on students' achievement and attitude levels in precollege mathematics classes. *Journal for Research in Mathematics Education*, *34* (5), 433-463.

Ellington, A. J. (2006). The effects of non-CAS graphing calculators on student achievement and attitude levels in mathematics: A meta-analysis. *School Science and Mathematics*, *106*(1), 16-26.

Ellis, C., Adams, T. E., & Bochner, A. P. (2011). Autoethnography: An overview. *Historical Social Research/Historische Sozialforschung*, *12*(1), 273-290.

Ertmer, P. A. (2005). Teacher pedagogical beliefs: The final frontier in our quest for technology integration? *Educational Technology Research and Development, 53*(4), 25-39.

Ertmer, P. A., & Ottenbreit-Leftwich, A. (2010). Teacher technology change: How knowledge, beliefs, and culture intersect. *Journal of Research on Technology in Education, 42*(3), 255-284.

Ertmer, P. A., Ottenbreit-Leftwich, A. T., Sadik, O., Sendurur, E., & Sendurur, P. (2012). Teacher beliefs and technology integration practices: A critical relationship. *Computers & Education, 59*(2), 423-435.

Ertmer, P. A., Ottenbreit-Leftwich, A., & York, C. S. (2007). Exemplary technology-using teachers: Perceptions of factors influencing success. *Journal of Computing in Teacher Education, 23*(2), 55-61.

Galbraith, P. (2006). Students, mathematics, and technology: assessing the present–challenging the future. *International Journal of Mathematical Education in Science and Technology*, *37*(3), 277-290.

Gray, L., Thomas, N., and Lewis, L. (2010). *Teachers' use of educational technology in U.S. public schools: 2009 (NCES 2010-040)*. National Center for Education Statistics, Institute of Education Sciences, U.S. Department of Education. Washington, DC. Retrieved from https://nces.ed.gov/pubs2010/2010040.pdf

Guerrero, S. (2010). Technological pedagogical content knowledge in the mathematics classroom. *Journal of Digital Learning in Teacher Education*, *26*(4), 132-139.

Guerrero, S., Walker, N., & Dugdale, S. (2004). Technology in support of middle grade mathematics: What have we learned? *Journal of Computers in Mathematics and Science Teaching*, *23*(1), 5-20.

Hadas, N., Hershkowitz, R., & Schwarz, B. (2000). The role of contradiction and uncertainty in promoting the need to prove in dynamic geometry environments. *Educational Studies in Mathematics*, *44*(1-3), 127-150.

Handal, B., Campbell, C., Cavanagh, M., Petocz, P., & Kelly, N. (2013). Technological pedagogical content knowledge of secondary mathematics teachers. *Contemporary Issues in Technology and Teacher Education, 13*(1), 22-40.

Hardy, M. (2008). It's TIME for technology: The technology in mathematics education project. *The Journal of Computers in Mathematics and Science Teaching*, *27*(2), 221.

Heid, M. K. (1988). Resequencing skills and concepts in applied calculus using the computer as a tool. *Journal for research in mathematics education*, *(19)*1, 3-25.

Heid, M. K., & Blume, G. W. (2008). Algebra and function development. In M.K. Heid & G. G. W. Blume (Eds.), *Research on technology and the teaching and learning of mathematics: Syntheses, cases, and perspectives*. Charlotte, North Carolina: Information Age Publishing, Inc.

Hegedus, S.J., Dalton, S., & Tapper, J.R. (2015). The impact of technology-enhanced curriculum on learning advanced algebra in US high school classrooms. *Educational Technology Research and Development, 63 (2)*, 203-228.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372-400.

Hollebrands, K. F. (2007). The role of a dynamic software program for geometry in the strategies high school mathematics students employ. *Journal for research in mathematics education*, *38*(2), 164-192.

Hollebrands, K., Laborde, C., & Straber, R. (2008). Technology and the learning of geometry at the secondary level. In M. K. Heid & G. W. Blume (Eds.), *Research on technology in the teaching and learning of mathematics: Research syntheses* (pp. 155–205). Charlotte, NC: Information Age Publishing.

Huang, R., & Zbiek, R. M. (2017). Prospective secondary mathematics teacher preparation and technology. In M.E. Strutchens, R. Huant, L. Losano, D. Potari, J. P. da Ponte, M. C. d. C. T. Cyrino, *The mathematics education of prospective secondary teachers around the world* (pp. 17-23). Switzerland: Springer International Publishing.

International Society for Technology in Education [ISTE]. (2016). *ISTE standards for students.* Retrieved from https://www.iste.org/my-profile/standards-download.

ISTE. (2017). *ISTE standards for educators*. Retrieved from https://www.iste.org/my-profile/standards-download.

Jaggars, S. S., & Stacey, G. W. (2014). *What we know about developmental education outcomes. Research overview*. Community College Research Center, Teachers College: Columbia University.

Jaworski, B. (1994). Investigating mathematics teaching: A constructivist enquiry. London: The Falmer Press.

Kagan, D. M. (1992). Implication of research on teacher belief. *Educational Psychologist*, *27*(1), 65-90.

Keengwe, J., Onchwari, G., & Wachira, P. (2008). The use of computer tools to support meaningful learning. *Association for the Advancement of Computing in Education Journal, 16*(1), 77-92.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.

Kim, S., & Chang, M. (2010). Computer games for the mathematics achievement of diverse students. *Journal of Educational Technology & Society*, *13*(3), 224-233.

Koehler, M. J., & Mishra, P. (2005). What happens when teachers design educational technology? The development of technological pedagogical content knowledge. *Journal of Educational Computing Research, 32*(2), 131–152.

Koehler, M. J., Mishra, P., Hershey, K., & Peruski, L. (2004). With a little help from your students: A new model for faculty development and online course design. *Journal of Technology and Teacher Education, 12*(1), 25–55.

Kurz, T. L., & Middleton, J. A. (2006). Using a functional approach to change preservice teachers' understanding of mathematics software. *Journal of Research on Technology in Education*, *39*(1), 45-65.

Kvale, S., & Brinkmann, S. (2009). *Learning the craft of qualitative research interviewing*. Thousand Oaks: Sage Publications.

Laborde, C. (2000). Dynamic geometry environments as a source of rich learning contexts for the complex activity of proving. *Educational Studies in Mathematics*, *44*(1), 151-161.

Laborde, C. (2007). The role and uses of technologies in mathematics classrooms: Between challenge and modus vivendi. *Canadian Journal of Math, Science & Technology Education*, *7*(1), 68-92.

Laumakis, P., & Herman, M. (2008). The effect of a calculator training workshop for high school teachers on their students' performance on Florida state-wide assessments. *International Journal for Technology in Mathematics Education*, *15*(3), 8-21.

Li, Q., & Ma, X. (2010). A meta-analysis of the effects of computer technology on school students' mathematics learning. *Educational Psychology Review*, *22*(3), 215-243.

Little, C. (2009). Interactive geometry in the classroom: Old barriers & new opportunities. *Mathematics in School*, *38*(2), 9-11.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry* (Vol. 75). Thousand Oaks, California: Sage Publications.

Lyublinskaya, I. & Tournaki, N. (2011, March). The effects of teacher content authoring on TPACK and on student achievement in algebra: Research on instruction with the TI-Nspire handheld. In M. Koehler, & P. Mishra (Eds.), *Proceedings of SITE 2011--Society for Information Technology & Teacher Education International Conference* (pp. 4396-4401). Nashville, Tennessee, USA: Association for the Advancement of Computing in Education (AACE).

Manfra, M. M., & Spires, H. A. (2013). Creative synthesis and TPACK: Supporting teachers through a technology and inquiry-rich graduate degree program. *Contemporary Issues in Technology and Teacher Education*, *13*(4), 386-418.

Mariotti, M. A. (2000). Introduction to proof: The mediation of a dynamic software environment. *Educational studies in mathematics*, *44*(1), 25-53.

Martin, W.G, Strutchens, M.E., Stuckwisch, S., & Qazi, M. (2011). Transforming east Alabama mathematics (TEAM-Math): Promoting systemic change in schools and universities. In W.F. Tate, C. Rousseau, & K. King (eds.), *Disrupting tradition: Research and practice pathways in mathematics education* (pp. 105-118). Reston, VA: National Council of Teachers of Mathematics.

McAdams, D. P. (1993). The stories we live by: Personal myths and the making of the self. New York: Guilford Press.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook*. Thousand Oaks, California: Sage Publications.

Miles, M. B., Huberman, A. M., & Saldana, J. (2014). *Qualitative data analysis: A method sourcebook.* Thousand Oaks, California: Sage Publications.

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record*, *108*(6), 1017.

Mishra, P., & Koehler, M. J. (2008). *Introducing technological pedagogical content knowledge.* Paper presented at the Annual Meeting of the American Educational Research Association, New York City, New York: American Educational Research Association.

Mistretta, R. M. (2005). Integrating technology into the mathematics classroom: The role of teacher preparation programs. *Mathematics Educator*, *15*(1), 18-24.

Mullis, I. V. S., Martin, M. O., Foy, P., & Hooper, M. (2016). *TIMSS 2015 international results in mathematics.* Retrieved http://timssandpirls.bc.edu/timss2015/international-results/

Murray, B., Silver-Pacuilla, H., & Helsel, F. (2007). Improving basic mathematics instruction: Promising technology resources for students with special needs. *Technology in Action, 2(5), 1-8.*

National Center for Education Statistics (NCES). (2015). NAEP data explorer. Retrieved from https://nces.ed.gov/nationsreportcard/naepdata/

National Commission on Excellence in Education [NCEE]. (1983). *A nation at risk: The imperative for educational reform.* Washington, D.C.: U.S. Government printing office.

National Council of Teachers of Mathematics (NCTM). (1980). *Agenda for action*. Retrieved from nctm.org.

NCTM. (1989). Curriculum and evaluations standards for school mathematics. Reston, VA: NCTM.

NCTM. (1991). Professional standards for teaching mathematics. Reston, VA: NCTM.

NCTM. (1995). Assessment standards for school mathematics. Reston, VA: NCTM.

NCTM. (2000). Principles and standards for school mathematics. Reston, VA: NCTM.

NCTM. (2009). Focus in high school mathematics: Reasoning and sense making. Reston, VA: NCTM.

NCTM. (2014). Principles to actions: Ensuring mathematical success for all. Reston, VA: NCTM.

National Governors Association Center for Best Practices [NGA Center] & Council of State School Officers [CCSSO]. (2010). *Common core state standards for mathematics.*

Washington, D.C.: NGA Center and CCSSO. Retrieved from http://www.corestandards.org.

NGA Center & CCSSO (n.d.) *Standards in your state*. Retrieved from http://www.corestandards.org/standards-in-your-state/

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the national mathematics advisory panel*. Washington, D.C.: U.S. Department of Education.

Niess, M. L. (2011). Investigating TPACK: Knowledge growth in teaching with technology. *Journal of educational computing research*, *44*(3), 299-317.

Niess, M. L., Sadri, P., & Lee, K. (2007, April). *Dynamic spreadsheets as learning technology tools: Developing teachers' technology pedagogical content knowledge (TPCK).* Paper presented at the annual meeting of the American Educational Research Association Annual Conference, Chicago, IL.

Niess, M. L., Ronau, R. N., Shafer, K. G., Driskell, S. O., Harper, S. R., Johnston, C., ... & Kersaint, G. (2009). Mathematics teacher TPACK standards and development model. *Contemporary Issues in Technology and Teacher Education*, *9*(1), 4-24.

Norton, S., McRobbie, C., & Cooper, T. (2000). Exploring secondary mathematics teachers' reasons for not using computers in their teacher: Five case studies. *Journal of Research on Computing in Education, 33*(1), 87-109.

Okumus, S., Lewis, L., Wiebe, E., and Hollebrands, K. (2016). Utility and usability as factors influencing teacher decisions about software integration. *Education Technology Research Development*, *(64)*6, 1227-1249.

Olive, J., & Lobato, J. (2008). The learning of rational number concepts using technology. In M. K. Heid & G. W. Blume (Eds.), *Research on technology in the teaching and learning of mathematics: Research syntheses* (pp. 1–53). Charlotte, NC: Information Age Publishing.

Organisation for Economic Co-operation and Development. (2015). *PISA 2015 results in focus.* Retrieved from http://www.oecd.org/pisa/pisa-2015-results-in-focus.pdf

Park, S. H., & Ertmer, P. A. (2008). Impact of problem-based learning (PBL) on teachers' beliefs regarding technology use. *Journal of Research on Technology in Education, 40*(2), 247-267.

Peressini, D. D., & Knuth, E. J. (2005). The role of technology in representing mathematical problem situations and concepts. In W.J. Masalski & P. C. Elliott (eds.), *Technology-supported mathematics learning environments, 67th Yearbook of the National Council of Teachers of Mathematics* (pp.277-290). Reston, VA: National Council of Teachers of Mathematics.

Piaget, J. (1970). Science of education and the psychology of the child. New York: Orion Press.

Public Law 107-110. (2002). 2001 No Child Left Behind (NCLB) Act of 2001, Pub. L. No. 107-110, § 115, Stat.

Richardson, S. (2009). Mathematics teachers' development, exploration, and advancement of technological pedagogical content knowledge in the teaching and learning of algebra. *Contemporary Issues in Technology and Teacher Education*, *9*(2), 117-130.

Ronau, R. N., & Rakes, C. R. (2012). Making the grade: Reporting educational technology and teacher knowledge research. In R. N. Ronau (Ed), *Educational technology, teacher knowledge, and classroom impact: A research handbook on frameworks and approaches* (pp. 323-332). IGI Global.

Ronau, R.N., Rakes, C.R., Bush, S.B., Driskell, S., Niess, M.L., & Pugalee, D. (2011, September). *Using calculators for teaching and learning mathematics.* (NCTM research brief). Retrieved from http://www.nctm.org/uploadedFiles/Research_and_Advocacy/research_brief_and_clips/2011-Research_brief_18-calculator.pdf

Sadaf, A., Newby, T. J., & Ertmer, P. A. (2012). Exploring factors that predict preservice teachers' intentions to use Web 2.0 technologies using decomposed theory of planned behavior. *Journal of Research on Technology in Education, 45*(2), 171-196.

Schwandt, T. (2007). *The SAGE dictionary of qualitative inquiry. 3rd Ed.* Thousand Oaks: SAGE Publications.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4-14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1-22.

Smith, R., Kim, S., & McIntyre, L. (2015). Relationships between prospective mathematics teachers' beliefs and TPACK. In T.B. Bartell, K.N. Bieda, R. T. Putnam, K. Bradfield, & H. Dominguez, (Eds.). (2015). *Proceedings of the 37th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. East Lansing, MI: Michigan State University.

Stoilescu, D. (2014). *Studying challenges in integrating techology in secondary mathematics with technological pedagogical and content knowledge.* Paper presented at International Conferences on Educational Technologies 2014 and Sustainability, Technlogy, and

Education 2014, New Taipai City, Taiwan. Retrieved from https://eric.ed.gov/?id=ED557320

Strutchens, M. E. & Martin, W.G. (2017). The TEAM-Math teacher leader academies. In N. Rigelman and M. McGatha (eds.), Elementary mathematics specialists: Developing, refining, and examining programs that support mathematics teaching and learning. (pp.) Charlotte, NC: Information Age Publishing, Inc.

Strutchens, M. E., & Silver, E. A. (2000). NAEP findings regarding race/ethnicity: Students' performance, school experiences, and attitudes and beliefs. *Results from the seventh mathematics assessment of the National Assessment of Educational Progress*, 45-72.

Sturdivant, R. X., Dunham, P., & Jardine, R. (2009). Preparing mathematics teachers for technology-rich environments. *Primus: Problems, Resources & Issues in Mathematics Undergraduate Studies, 19*(2), 161-173. doi: 10.1080/10511970802409180

Tonduer, J., Hermans, R., van Braak, J., & Valcke, M. (2008). Exploring the link between teachers: Educational belief profiles and different types of computer use in the classroom. *Computers in Human Behavior, 24*, 2541-2553.

Tyminski, A. M., Haltiwanger, L., Zambak, V. S., Horton, R., & Hedetniemi, T. (2013). Developing inquiry practices in middle grades mathematics teachers: Examining the introduction of technology. *Contemporary Issues in Technology and Teacher Education*, *13*(4), 325-359.

Ubuz, B., Üstün, I., & Erbaş, A. K. (2009). Effect of dynamic geometry environment on immediate and retention level achievements of seventh grade students. *Eurasian Journal of Educational Research*, *(35)*, 147-164.

Vygotsky, L. (1978). Interaction between learning and development. *Readings on the Development of Children*, *23*(3), 34-41.

Wachira, P., & Keengwe, J. (2011). Technology integration barriers: Urban school mathematics teachers' perspectives. *Journal of Science Education and Technology*, *20*(1), 17-25.

Wachira, P., Keengwe, J., & Onchwari, G.. (2008). Mathematics preservice teachers' beliefs and conceptions of appropriate technology use. *Association for the Advancement in Computing Education Journal, 16*(3), 293-306.

Wall, S. (2008). Easier said than done: Writing an autoethnography. *International Journal of Qualitative Methods*, *7*(1), 38-53.

Weglinsky, H. (1998). Does it compute. The relationship between educational technology and student achievement in mathematics. Princeton, NJ: Educational Testing Service.

Weick, K.E. (1995). *Sensemaking in Organizations*. Thousand Oaks, CA: Sage.

Williams, M. K., Foulger, T. S., & Wetzel, K. (2009). Preparing preservice teachers for 21st century classrooms: Transforming attitudes and behaviors about innovative technology. *Journal of Technology and Teacher Education, 17*(3), 393-418

Yigit, M. (2014). A review of the literature: How pre-service mathematics teachers develop their technological, pedagogical, and content knowledge. *International Journal of Education in Mathematics, Science and Technology*, *2*(1), 26-35.

Zengin, Y., & Tatar, E. (2017). Integrating dynamic mathematics software into cooperative learning environments in mathematics. *Educational Technology & Society*, *20*(2), 74–88.

Zhao, Y., Pugh, K., Sheldon, S., & Byers, J. L. (2002). Conditions for classroom technology innovations. *Teachers College Record, 104*(3), 482-515.

Appendices

Appendix A: Criteria for an effective technology-based lesson from Mistretta (2005)

A. A connection to mathematics standards.
B. Appropriate approach to mathematics topics with respect to grade, ability, and reading level(s).
C. Worthwhile mathematical task.
D. Presence of conceptual development, skill building, and problem solving/ higher order thinking skills.
E. Use of practical applications and interdisciplinary connections.
F. The technology activity is a seamless part of the lesson.
G. The students are focusing on learning with the technology, not on the technology itself.
H. Lesson objectives could not be accomplished or accomplished as well if the technology weren't there.
I. The contributions of the instructional technology are evident.
J. All students are engaged with the technology and benefiting from it.

Appendix B: TPACK Development Model level descriptors summary

<u>Recognizing</u>- knowledge of the tech/mathematics alignment but not integrating it
1. able to use the tech for personal use
2. recognize a mathematics/tech alignment but not used in instruction
3. useful for visual representations
4. no tech on assessments
5. closed view of learning mathematics w/o tech
6. tech used in activities outside regular learning activities
7. learning tech takes too much time
8. student use is w/o teacher instructions
9. tech used for rote activities, not learning
10. tech used to reinforce concepts after by hand competence
11. considers attending PD
12. tech used after mastering concepts
13. no curriculum changes to reflect tech
14. authentic problems that require tech use may be extra credit
1. form (un)/favorable attitude toward tech learning & teaching mathematics
2. may not know what concepts are best taught with tech
3. may mimic exact PD lessons
4. Acknowledge a limited use of tech during assessment
5. retest via pencil/paper if tested via tech
6. concern that too much tech focus = lessened mathematics focus
7. limits tech when developing key ideas
8. thinking skills will be hindered if tech used to verify conjectures
9. redo/relearn concepts w/o tech to be sure learned
10. tech used for "days off"
11. activities requiring advanced tech skills are avoided
12. any tech activity is tightly managed/ orchestrated using step by step instructions
13. Feels more PD is needed and seeks out PD
14. tech can expand number of examples explored

<u>Adapting</u>- decision, engage in activities that lead to adoption
1. some tech benefits understood
2. may use tech to teach/demo then students use to verify
3. different questions are needed for tech use in assessment- more concepts
4. Explore SOME mathematics with tech
5. students develop mathematics thinking skills using tech
6. most assessment without tech
7. may enhance/reinforce previous knowledge
8. Adapts PD lessons
9.teacher maintains control of tech activity progress
10. students explore for part of a lesson
11. continues PD in one area of tech
12. Shares with teachers
13. some curriculum. Changes

Exploring- Implementation, actively integrate teaching/learning/tech
1. more integral role for developing mathematics ideas, explore NEW ideas
2. adapts own lessons to incorporate tech
3. develops curriculum enhancements due to tech
4. uses varied tech assessments that extend beyond pen/paper capable.
5. plans with concern for student understanding, student thinking
6. manages activities but promotes student engagement and self direction
7. engages/ guides students in high-level thinking activities
8. shares with and seeks out others that are using tech similarly
9. Tech available and encouraged most of class
10. seeks solutions to barriers for tech use
11. multiple representations and connections are made

Advancing- confirmation, evaluate its effectiveness
1. essential to modify curriculum. to effectively/ efficiently incorporate tech
2. uses advantages of tech to expand beyond traditional curriculum
3. assesses student understanding of mathematics embedded in technology
4. creates, plans, implements, reflects, cycle!
5. personal conviction to enhance student understanding
6. tech is integral to learning of mathematics
7. high level thinking and high levels of mathematics
8. consistent acceptance of tech as tool for learning
9. teacher is model for novel ideas of using tech to help students
10. activities managed to MAINTAIN self direction and engagement
11. seeks PD for emergent techs
12. engages others to incorporate and adjust and expand curriculum
13. Tech is always permitted
14. Resolves perceived barriers for use

Appendix C: Informed Consent Form

COLLEGE OF EDUCATION

DEPARTMENT OF CURRICULUM AND TEACHING

(NOTE: DO NOT SIGN THIS DOCUMENT UNLESS AN IRB APPROVAL STAMP WITH CURRENT DATES HAS BEEN APPLIED TO THIS DOCUMENT.)

INFORMED CONSENT
for a Research Study entitled
"Exploring Narratives of Secondary Teachers That Use Technology Effectively in the Mathematics Classroom"

You are invited to participate in a narrative design research study to explore how the experiences, knowledge, and beliefs of teachers related to their implementation of technology in the context of reform mathematics are shaped by the events in their personal narrative. The study is being conducted by Nancee Garcia, graduate student, under the direction of Dr. W. Gary Martin, Professor in the Auburn University Department of Curriculum and Teaching. You were selected as a possible participant because you are known to the researcher to be a teacher who is an effective and frequent user of technology in the mathematics classroom and are age 19 or older.

What will be involved if you participate? If you decide to participate in this research study, you will be asked to participate in an initial narrative interview, to video record three technology-infused lessons of your choice and equipment will be provided, to provide researcher with copies of student work, to participate in a debriefing interview, to provide written reflections of recorded lessons, and to participate in a follow-up interview. Your total time commitment will be approximately 45 minutes to 1 hour per interview and approximately 15 minutes per reflection over a period of 6 to 8 weeks.

Are there any risks or discomforts? The risks associated with participating in this study are you may recall unpleasant prior experiences and others may find out your personal information. To minimize these risks, you will have the right to withdraw from the study at any time. Additionally, your personal information will be kept in a locked file box and it will eventually be destroyed. No person other than the researcher and her advisor will view the video recordings.

Are there any benefits to yourself or others? If you participate in this study, you can expect to engage in reflection of your implementation practices and share stories related to your expertise in using technology. Findings from this study may be used to inform mathematics teacher educators or professional development providers of the needs of teachers in order to increase the likelihood of implementing technology in the classroom. We/I cannot promise you that you will receive any or all of the benefits described.

College of Education

Department of Curriculum and Teaching

Will you receive compensation for participating? There is no compensation being offered for participation.

Are there any costs? If you decide to participate, you will need to provide your time over the period mentioned above and attend interviews.

If you change your mind about participating, you can withdraw at any time during the study. Your participation is completely voluntary. If you choose to withdraw, your data can be withdrawn as long as it is identifiable. Your decision about whether or not to participate or to stop participating will not jeopardize your future relations with Auburn University, the Department of Curriculum and Teaching, or your school system.

Your privacy will be protected. Any information obtained in connection with this study will remain confidential and/or anonymous. Information obtained through your participation may be used to fulfill the requirements for an advanced degree, published in a professional journal, presented at a professional meeting, used for educational purposes, etc.

If you have questions about this study, *please ask them now or* contact Nancee Garcia at 334-329-0341 or Dr. W. Gary Martin at wgarymartin@me.com. A copy of this document will be given to you to keep.

If you have questions about your rights as a research participant, you may contact the Auburn University Office of Human Subjects Research or the Institutional Review Board by phone (334)-844-5966 or e-mail at hsubjec@auburn.edu or IRBChair@auburn.edu.

HAVING READ THE INFORMATION PROVIDED, YOU MUST DEC
IDE WHETHER OR NOT YOU WISH TO PARTICIPATE IN THIS RESEARCH STUDY. YOUR SIGNATURE INDICATES YOUR WILLINGNESS TO PARTICIPATE.

_______________________________ _______________________________

Participant's signature Date Investigator obtaining consent Date

_______________________________ _______________________________

Printed Name Printed Name

Appendix D: Study Proposal Form

Dear [superintendent/principal],

My name is Nancee Garcia and I am a doctoral student under Dr. W. Gary Martin in the department of Curriculum and Teaching, mathematics education, at Auburn University. I am conducting a research study exploring the narratives of exemplary technology-using mathematics teachers in order to understand the factors that have contributed to their success. For this narrative case study, I need to interview and observe exemplary teachers that use technology in the classroom to engage students in meaningful mathematical learning opportunities. [teacher] at [school] has been identified as an exemplary teacher that would be an asset to this study. There will be 1 interview with the teacher and 3 classroom observations and debriefings over a period of 4 to 8 weeks of technology infused lessons of [his or her] choice. Classroom observations will be video and audio taped for data purposes only. All transcriptions from this study will maintain the anonymity of the participating teachers and students. Classroom disruption for the students will be minimal, as the purpose of the study is to understand the process of becoming an exemplary technology-using teacher. The semester for collecting data for this study is Spring 2013. I would greatly appreciate your permission for [teacher] to take part in this study. This study is an opportunity for exceptional teachers like [teacher] to contribute to the literature on knowledge of how teachers can be supported to become exemplary teachers.

Your signature on the line at the bottom right corner of this page will be considered your consent for participation. With your consent [principal, teacher] will receive similar letters explaining the study. Please e-mail this back to nrgarcia@auburnschools.org or fax to (334) 844-0124.

If you have any questions concerning the research project, please contact:

Nancee Garcia: (334) 329-0341 or nrgarcia@auburnschools.org

Dr. W. Gary Martin: wgarymartin@me.com

Thank you for your time and attention to this matter.

Sincerely,

Nancee Garcia

Appendix E: Life Story Interview Protocol

Adapted from Mathematics and Technology Life Story Protocol (McAdams, 1994; Drake, 2006)

I. Introduction to Interview
The purpose of this interview is to hear the story of your life experiences with technology and mathematics teaching and learning. I will ask you to focus on a few key scenes in your life. There are no right or wrong answers. Instead, your task is simply to tell me about some of the most important things that have happened in your life related to technology and mathematics teaching and learning. A key event should be a specific happening, a critical incident, a significant episode in your past set in a particular time and place. It is helpful to think of such an event as constituting a specific moment that stand out for some reason in your experiences with mathematics or technology. Events that occurred over a long period of time would not qualify.

Please rest assured that my purpose in doing this interview is not to judge your teaching or attempt to figure out how you might improve. My only purpose is to hear your story and those of others in order to try to identify significant commonalities, differences, or events in the stories of your experiences with mathematics and technology. Everything you say is voluntary, anonymous, and confidential. Do you have any questions?

II. Critical Events Related to Math and Teaching

Event #1: High Point Related to Math
A high point would be a peak experience in your story about mathematics in your life- perhaps the high point. It would be a moment or episode in the story in which you experienced extremely positive emotions; like joy, excitement, great happiness, uplifting, or even deep inner peace after some mathematics experience. Tell me exactly what happened, where it happened, who was involved, what you did, what you were thinking and feeling, what impact this experience may have had upon you, and what this experience says about who you were or who you are now as a teacher.

Event #2 High Point Related to Teaching
Describe a moment or episode in your story about teaching which you experienced extremely positive emotions or a wonderful moment in your story about teaching. Give me as much detail about the event as possible and also tell me why you think this particular moment was so good and what it says about you as a person or as a teacher.

Event #3 Low Point Related to Learning Math
This is the opposite of a peak experience. Thinking back over your entire life, please identify a scene that stands out as a low point, or the low point, in your mathematics story. Try to remember a specific experience in which you felt extremely negative emotions about mathematics. What happened? Who was involved? When? What did you do? What were you thinking and feeling? What impact has the event had on you? What does the event say about you as a person or a teacher?

Event #4 Low Point Related to Teaching Mathematics

Try to remember a specific experience in which you felt extremely negative emotions about teaching mathematics. What happened? Who was involved? When? What did you do? What were you thinking and feeling? What impact has the event had on you? What does the event say about you as a person or a teacher?

Event #5 Introduction to Technology
Describe the earliest experience you can recall using technology as a teaching and learning tool in the classroom. What was the purpose of the technology? How did you learn to use it effectively? What key influences led you to use technology? Why did you decide to use technology in the classroom? What impact did it have on you as a teacher? How have your skills grown since starting to use technology? How have you faced, handled, or dealt with the challenges of learning to use and implementing technology in the classroom? Have other people impacted or assisted you in dealing with the challenges?

Event #6 The purpose of technology
Can you identify a significant event that shaped your beliefs about using technology in the classroom? How did that event shape your beliefs? Why do you think it is important to use technology in the classroom? What/who were the key influences in developing these beliefs? What impact do your beliefs have on your teaching and implementation of technology? What do you think these ideas say about you as a person?

Event #7 High Point related to teaching with Technology
Describe a moment or episode in which you experienced extremely positive emotions or a wonderful moment in your story about teaching with technology. Why do you think that this moment was so successful? Give me as much detail about the event as possible and also tell me why you think this particular moment was so good and what it says about you as a person or as a teacher.

III. Positive Influence
Positive: Looking back over your life story, please identify the single person, group of persons, or organization/institution that has or have had the greatest positive influence on your perspective of mathematics and technology. Please describe this person or group and the way in which he, she, it, or they have had a positive impact on your story.

IV. Additional Questions
Why do you think you decided to be a teacher of mathematics?

Do you see yourself as different from the average mathematics teacher?

How do you feel you are perceived by your colleagues and administration at your school with regards to your technology use in the mathematics classroom and why do you feel they perceive you in that way?

What does it mean to you for a student to be successful in mathematics?

What role do you think technology plays in helping a student be successful in mathematics?

V. Wrap up

Is there anything else that you would like to tell me that you feel is important for helping me to understand what impacts or influences your use of technology in the classroom?

Appendix F: Debriefing Protocol

1. Intentions: I understood the objectives of the lesson to be [insert objectives] and the purpose of the technology to be [insert technology purpose]. Were these the intended objectives and purposes? Are there others that I did not identify?

2. History: Tell me about the history of the lesson. How long have you used that lesson? What is the source of that lesson?

3. Development: How has this lesson changed from your first implementation of it? Why were the changes needed?

4. Advantages: Why do you think that [specific technology] was particularly useful for this lesson's objectives?

5. Other questions specific to this lesson.

6. Is there anything else you would like to tell me about this lesson or the use of technology in this lesson?

Appendix G: Code Book and Definitions

UNIT= one complete description of an event. A unit may contain other sub-units when events are discussed embedded in another event. The sub-units are part of the original unit and a separate unit for coding purposes.

Interviews: A Priori codes

Category	Sub-category	Code	Definition
Elements of Events, phase 1	Time period (continuity)	K-12	During K-12 education
		Undergrad	During undergraduate education not including internship
		Internship	During undergraduate internship
		Early career	During years 0-5 of teaching career
		Mid-career	During years 6-15 of teaching career
		Late career	During years 16+ of teaching career
	Interaction	Personal	Was action done completely independently?
		Social	No part was done independently
		Mixed	Was any part completely independent?
	Situation/ Place	Family home	Event occurred at personal home
		School	Event occurred at k-12 school as student
		Post-secondary school	Event occurred at university- Undergrad or graduate
		School of employment	Event occurred at school of employment as teacher
		Other educational institution	Event occurred at other educational institution like site of professional development
		Other	Event occurred at place not mentioned above
Elements of events, phase 2	Type & Content (Might not be one of	High point	Must state high point or answer the question
		Low point	Must state low point or answer the question
		Turning point	Participant states that event is a turning point or describes change as a result, change may be small or significant

	first three!)	Pedagogical	Event contents is pedagogical and may include mathematics but not technology
		Mathematical	Event includes mathematics but no pedagogical aspects
		Technological	No pedagogical but can include learning mathematics
		Pedagogical including technology	Events includes elements of pedagogy and technology and may also include mathematics
	Tone	Positive	(If $P \geq N + 3$) Nice=1 but very nice =2
		Negative	(If $N \geq P + 3$)
		Neutral	(If total is ≥ 5 or N, P differ by ≤ 2) Notes: Count positive and negative words, positive or negative only if the mix count is greater than 5 or if P or N = 0).
	Specificity	1 Not specific	Some of who, what, when why where how is missing and/or NO details about each given
		2 Slightly specific	Most there and some details given
		3 Fairly specific	All there, some details given
		4 Very specific	All there, most include details

Observations

Evidence of effective use	Mistretta checklist	Standards-based	The lesson includes a connection to mathematics standards.
		Appropriate level	Appropriate approach to mathematics topics with respect to grade, ability, and reading level(s).
		Worthwhile task	Worthwhile mathematical task that promotes student reasoning and sense making
		Skill variety	Presence of conceptual development, skill building, and problem solving/ higher order thinking skills.
		Applications	Use of practical applications and interdisciplinary connections.
		Standards-based	The lesson includes a connection to mathematics standards.
		Appropriate level	Appropriate approach to mathematics topics with respect to grade, ability, and reading level(s).
		Seamless	The technology activity is a seamless part of the lesson.
	Other	Math focused	The students are focusing on learning with the technology, not on the

			technology itself.
		Necessary	Lesson objectives could not be accomplished or accomplished as well if the technology weren't there.
		Contributions evident	The contributions of the instructional technology are evident.
		Engaged and benefitting	All students are engaged with the technology and benefiting from it.

TPACK development

	Stage code	Indicator code and definition
Technology, Pedagogy, And Content Knowledge (TPACK) Stages	Recognizing	Shows knowledge of the technology/ mathematics alignment but is not integrating the use of technology into teaching mathematics. Descriptors follow: 1. Able to use the technology for personal use 2. Recognize a mathematics/technology alignment but not used in instruction 3. Useful for visual representations 4. No technology on assessments 5. Closed view of learning mathematics without technology 6. Technology used in activities outside regular learning activities 7. View that learning technology takes too much time 8. Student use is without teacher instructions 9. Technology used for rote activities, not learning 10. Technology used to reinforce concepts after by hand competence 11. Considers attending Professional development (PD) 12. Technology used after mastering concepts 13. No curriculum changes to reflect technology 14. Authentic problems that require technology use may be extra credit
	Accepting	Engages in activities to form attitude about technology use in teaching and learning mathematics, persuasion stage. Descriptors follow: 1. Form (un)/favorable attitude toward technology learning & teaching mathematics 2. May not know what concepts are best taught with technology

		3. May mimic exact PD lessons 4. Acknowledge a limited use of technology during assessment 5. Retest via pencil/paper if tested via technology 6. Concern that too much technology focus leads to lessened mathematics focus 7. Limits technology when developing key ideas 8. Thinking skills will be hindered if technology used to verify conjectures 9. Redo/relearn concepts without technology to be sure learned 10. Technology used for “days off” 11. Activities requiring advanced technology skills are avoided 12. Any technology activity is tightly managed/ orchestrated using step by step instructions 13. Feels more PD is needed and seeks out PD 14. Technology can expand number of examples explored
	Adapting	Decision stage, engage in activities that lead to adoption. Descriptors follow: 1. Some technology benefits understood 2. May use technology to teach/demonstrate, then students use to verify 3. Different questions are needed for technology use in assessment- more concepts 4. Explore SOME mathematics with technology 5. Students develop mathematical thinking skills using technology 6. Most assessment without technology 7. May enhance/reinforce previous knowledge 8. Adapts PD lessons 9. Teacher maintains control of technology activity progress 10. Students explore for part of a lesson 11. Continues PD in one area of technology 12. Shares with teachers 13. Some curriculum changes
	Exploring	Implementation stage, actively integrate technology into teaching and

		learning. Descriptors follow: 1. More integral role for developing mathematical ideas, explore NEW ideas 2. Adapts own lessons to incorporate technology 3. Develops curriculum enhancements due to technology 4. Uses varied technology assessments that extend beyond pen/paper capabilities 5. Plans with concern for student understanding, student thinking 6. Manages activities but promotes student engagement and self direction 7. Engages/ guides students in high-level thinking activities 8. Shares with and seeks out others that are using technology similarly 9. Technology available and encouraged most of class 10. Seeks solutions to barriers for technology use 11. Multiple representations and connections are made
	Advancing	Confirmation stage, evaluate technology's effectiveness. Descriptors follow: 1. Essential to modify curriculum to effectively/ efficiently incorporate technology 2. Uses advantages of technology to expand beyond traditional curriculum 3. Assesses student understanding of mathematics embedded in technology 4. Create-plan-implement-reflect cycle 5. Personal conviction to enhance student understanding 6. Technology is integral to learning of mathematics 7. High level thinking and high levels of mathematics 8. Consistent acceptance of technology as tool for learning 9. Teacher is model for novel ideas of using technology to help students 10. Activities managed to MAINTAIN self direction and engagement 11. Seeks PD for emergent technology 12. Engages others to incorporate and adjust and expand curriculum 13. Technology is always permitted 14. Resolves perceived barriers for use
Barriers to technology		

	Code	Definition
Barriers	Use barrier	Event or element of an event that is a barrier to implementing technology in the classroom as a teaching and learning tool
	Potential barrier	Perceived barrier, or possible barrier to implementing technology in the classroom as a teaching and learning tool
	Attitude barrier	Event or element of an event that is a barrier to forming a positive attitude to using technology as a teaching and learning tool

Factors related to technology implementation (emergent themes)

	Code	Definition
Emergent themes	Development of TPACK and technology implementation during preservice internship	Indication that learning to use technology while implementing during internship as a teaching and learning tool was very significant in increasing understanding of TPACK and/or impacting implementation decisions (i.e. teacher describes how positive feedback during internship led to increased use of technology in the classroom)
	Continued education to overcome barriers	Overcoming or preventing common barriers to technology use in events or elements of events that are attributed to continuing education which may include professional development, graduate education or professional learning communities (PLCs) and may impact implementation decisions (i.e. teacher participation in professional development leads to greater access to technology resources)
	Development of TPACK through experiences in the MSP	Description of the MSP's professional development, graduate education or PLCs that contributed to the TPACK development of the teacher and/or implementation decisions (i.e. teacher describes how participation in MSP session introduced her to new technology tool)
	Leadership roles related to technology	Teacher participated in a leadership role related to the implementation of technology in the mathematics classroom and may have impacted implementation decisions (i.e. teacher is asked to lead professional development in technology for other teachers)
	Access to resources and support	Access to technology resources and lessons and support for implementing technology-based lessons and may have impacted implementation decisions (i.e. teacher is supported by administration in using technology by frequent praise)

	Growth mindset related to teaching and technology	Teacher shows growth mindset related to teaching and learning with technology and a willingness to learn with students that may have impacted implementation decisions (i.e. teacher expresses or shows evidence of learning to use a new technology tool to increase student learning)
	Technology fulfills a need	Technology is implemented in the classroom because it fulfills a specific student or teacher need related to learning mathematics (i.e. teacher uses technology in classroom to help students with physical disabilities to do geometric constructions)