Examining Nonlinear Business Cycle Effects on U.S. Unemployment and Capacity Utilization

by

Peter Joseph Santoro

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Approved by

John D. Jackson, Chair, Professor, Economics
Henry Thomson, Professor, Economics
Hyeongwoo Kim, Associate Professor, Economics

Abstract

This thesis develops nonlinear smooth transition autoregressive (STAR) models to capture the business cycle dynamics of the U.S. rates of unemployment and capacity utilization. This choice of model is motivated by a general inquiry as to whether the two series behave in a similar fashion during periods of recession, expansion, or somewhere in between. While STAR models for unemployment have been developed by several authors, capacity utilization has not (to our knowledge) been analyzed in the same context. Our results for unemployment are in accord with those of previous studies: namely, that unemployment follows essentially two business cycle regimes of expansion and contraction. By contrast, our results for capacity utilization indicate that it follows a three-regime process: those of recession, high growth, and normal growth. The model suggests that dynamic adjustments are roughly equal for the first and third regimes, but different for the second.

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CHAPTER I

INTRODUCTION

The rates of capacity utilization and unemployment are economic variables that measure the degree of resource utilization for overall production in the economy. The fluctuations of the two series have long been regarded by economists as reflections of the ebb and flow of short-run macroeconomic activity. As such, they provide relatively timely signals of the trajectory on which the economy is moving. More important than the trajectory, however, is the speed with which these variables might tend in one direction or another, as more rapid movements generally prefigure periods of boom and slowdown in the short-run. Such movements are referred to as business cycles. The prospect of business cycles engenders uncertainty on the part of the economy's producers, and hence may undermine the long-run growth trend toward which the economy is tending. Therefore, insofar as both unemployment and capacity utilization reflect producers' expectations, they abide in their importance as indicators of macroeconomic stability.

If, however, unemployment and capacity utilization offer a snapshot of an economy's short-run path, do they necessarily give the same picture? Do they follow similar dynamics during periods of expansion or contraction? How do they adjust to demand and supply shocks? Do their relative magnitudes portend the same situation, given changes in their exogenous components like population growth and the state of technology? Finally, given the peaks and troughs of business cycles, how smoothly does– and for how long before – each of the two measures revert to their long-run averages? It is these questions, particularly the last one, which this thesis will attempt to address.

This thesis will seek to test empirically whether the local dynamics of the two series are fundamentally similar over different phases of the business cycle. To that end, a univariate, nonlinear smooth transition autoregressive (STAR) model, formalized by Timo Terasvirta (1994), will be constructed for the two series. STAR models are designed to model smooth, continuous transitions from one regime to another. In contrast to linear, autoregressive (AR) models whose parameters are deterministic, STAR models append a nonlinear function to the autoregressive lag polynomial, rendering the AR parameters stochastic and regime-dependent. Understood in its usual economic context, the parameters are dependent on the underlying structure of the economy over time. By convention, the nonlinear function will take either a logistic or exponential form, and must be designed by the model builder to contain information about the regime at discrete moments in time. They have proved useful in modeling situations where there is reason to believe, whether from economic theory or by *a priori* beliefs of the data-generating process, that local, dynamic adjustment depends on the regime – an expanding or contracting economy, for instance – within which adjustment takes place. This phenomenon has come to be called 'asymmetry', and has been noted by economists for at least as far back as Keynes (1936).

Asymmetry is but one form of nonlinearity. Pesaran and Potter (1992) offer a good, succinct definition of nonlinearity when they describe linear models: "One can think of a dynamic system as being linear if its global properties can be completely characterized by its local behavior." If global and local behavior cannot be characterized by the same dynamic process, then linear models are bound to be inadequate. The reasoning behind this choice of model will become clearer as this thesis progresses.

STAR models have been applied to several economic variables. Terasvirta and Anderson (1992) used STAR models to verify the existence of business cycle asymmetries in industrial production series for several countries. Granger and Terasvirta (1993) reinforced Terasvirta's and Anderson's work by applying the models to the same variables for countries the latter did not consider. Ocal and Osborn (2000) applied STAR models to the UK series for consumption and production, again to compare possible nonlinearities over the business cycle. Awokuse and Christophoulos (2009) analyzed the nonlinear relation between exports and GDP growth using STAR models. STAR models have been applied to U.S. unemployment by Bianchi and Zoega (1998), Skalin and Terasvirta (2002), van Dijk, et.al (2002), and

Terasvirta, Tjostheim, and Granger (2010). To our knowledge, STAR models have yet to be applied to capacity utilization. This is surprising given the significance ascribed by much of the economics profession to capacity utilization as a business cycle indicator; see Corrado and Mattey (1997) for a survey. Following many of these authors, this thesis will test using STAR models whether capacity utilization and unemployment follow similar paths across the business cycle

. Our results for the unemployment series are on par with those of the studies cited above. The results suggest that the short-run dynamics of unemployment are governed according to a two-regime framework. Depending on whether the economy is growing or contracting, the short-run adjustment follows a stochastic process that changes monotonically according to the overall state of the economy. This scenario has ample theoretical backing in the literature. There is, for example, the famous search model of Diamond (1982) where opportunities for work surface stochastically according to a Poisson process. Cabellero and Hammour (1994) followed up on Diamond's work by testing it empirically. In a different area, Bentoila and Bertola (1990) exposited the theory of job creation and destruction in light of technological advancement. Hamermesh and Pfann (1996) studied the disproportion of hiring and firing costs as a source of friction in the labor market.

Data for capacity utilization, heretofore unanalyzed in this context of business cycle asymmetry, revealed quite a different scenario. Our results suggest that the stochastic component of the series is better characterized by a three-regime process, with dynamic adjustment being roughly equal for the first and third regimes, but different for the second. This result lends itself to some significant economic interpretations. It suggests that dynamic adjustment follows the same process when the economy has been rooted in its recessionary trough for some extended period and when it has been expanding persistently toward its business cycle peak, again for an extended period. The dynamics are different when the economy at the moment is characterized by some intermediate scenario, namely, when the economy is beginning to experience an uptick in activity, compared with a relative lull that preceded it. The work of Bansak, Morin, & Starr (2007) point to possible explanations for the kind of behavior exhibited by capacity utilization indicated by our STAR model. They discuss different decisions on the part of plant managers in light of new information about future demand. One explanation is that a firm will change output and capacity using existing technology in light of a demand shock. The shock triggers a dynamic adjustment, whereby a new level of utilization is reached when adjustment is complete, at which it then remains steady in much the same way as was the case before the shock. Perhaps the difference in dynamics of the middle regime can also be attributed to differences over time of the relative costs of 'excess capacity' (the degree to which firms operate their capital below their full potential). In addition, there are differences in the scale and magnitude of the expansionary periods, which are also likely to influence the dynamics of the series during such periods. We note that Smyth and Jackson (1984) found evidence to that effect in a ratchet model framework.

While there is, of course, no shortage of interpretations which can be adduced, this thesis will be largely empirical and data-driven in its intent. It will follow in the same vein as that begun by Terasvirta and Anderson (1992), and followed by, among others, Granger and Terasvirta (1993), Ocal and Osborn (2000) and Akram (2005) in characterizing the local dynamics of macroeconomic series over the business cycle. It is worth noting that, for a long time, research in macroeconomics operated under the assumption that local and global properties of time series could be captured by the same specification. This assumption motivated the classification of series as being either stationary or nonstationary. Models were then applied to a series depending on this classification. It is no longer taken for granted that local and global properties can be characterized in the same way, though it is very often the case that they can be. Unemployment and capacity utilization may prove to be two exceptions; that the former is in fact an exception is a hypothesis about which a lot of evidence has already been accumulated. One of the operative assumptions behind the building of nonlinear models is that the series being considered are globally stationary, but locally nonstationary; see Pesaran and Potter (1992) for an intuitive discussion. This study will seek to reinforce this hypothesis concerning the two series it will study.

This thesis will proceed as follows. Chapter II will outline the theoretical background that relates unemployment and capacity utilization. This background is grounded in a traditional Keynesian framework. Chapter II will also review some of the conflicting economic interpretations of the business cycle, which have become more salient in light of the most recent 2007-2009 recession. Chapter III will give a description of the data. Chapter IV provides the formal background of the STAR methodology. Preceding this, however, is an attempt to explain how and why traditional tests for common stochastic trends have proved inadequate for data that exhibit some form of nonlinearity. In Chapter V, unemployment and capacity utilization will be subjected to the STAR modeling framework crystallized by Terasvirta (1994). Results will then be reported. Chapter VI gives the conclusion, which includes some proposals for future research. All tables and figures referenced can be found at the end of the paper.

CHAPTER II

THEORY

The economic theory that relates capacity utilization and unemployment is grounded in traditional Keynesian models of aggregate demand and supply. Keynes (1936) defined an employment function that corresponds to aggregate levels of output, reflected in the aggregate demand function. Finn (1995) reviews the comparative static model that locates some full employment level of output, Y^*, at the kink of the discontinuous aggregate supply function. The supply function is perfectly horizontal up until Y^*, beyond which it slopes upward at an increasing rate to indicate that demand for higher real wages is higher, now that labor is working overtime. The second kink occurs at some output level, Y^c, which indicates full capacity, at which point the supply function becomes vertical, and therefore increases in aggregate demand through increases in consumption, investment, government spending, the money supply, or some combination thereof, can only result in a higher price level, but not higher output. Finn notes that the Federal Reserve's measure of capacity utilization can be understood to equal the ratio of actual output –"effective demand" in Keynes's terms – to the full capacity level of output, namely, Y/Y^c. However, Y^c is greater than Y^*, and the space between them refers to that volume of output resulting from overtime on the part of labor. Moreover, $Y^c - Y$ can be viewed as 'excess capacity', that is, the amount by which production is lower than that capable at full capacity. A more appropriate measure of slack in the labor market according to this model might be $(Y^* - Y)/Y^c$, where Y is the current level of output as dictated by the level of current effective demand.

This model presents the short-run comparative statics whereby cyclical fluctuations in capacity utilization and unemployment correspond to deviations of actual, effective demand-driven output, from the full-employment level of output. The long-run phenomenon of growth stems from increases in exogenous factors like the size of the workforce, the capital stock, and technology, all of which are assumed fixed in the short-run. Both capacity utilization and employment, therefore, are determined by where the aggregate demand curve intersects the aggregate supply curve, and by the region within which the point of intersection exists.

The economic factors that influence aggregate demand are consumption (and the marginal propensity to consume), government spending (net of taxes), the money supply, and investment. However, it is the fourth factor, namely the volume of investment, that is the more volatile among the others and that serves as the driver of the cyclical fluctuations of demand and thus of employment and capacity. For Keynes, the equilibrium solution occurs when the interest rate equals a measure he called the 'marginal efficiency of capital', a term, vague in definition, that runs throughout *The General Theory*. Because the marginal efficiency of capital is a function of many things, Keynes believed that cyclical fluctuations in output stayed within a normal range. In Keynes's words, "there is some recognizable degree of regularity in the time-sequence and duration of the upward and downward movements" (1936, pg. 314).

There is however, an exception to this regularity, namely, the phenomenon of "the crisis". The marginal efficiency of capital is determined, as Keynes (1936, pg. 316) put it, "by the uncontrollable and disobedient psychology of the business world," and that, "when disillusion falls upon an over-optimistic and overbought market, it should fall with sudden and even catastrophic force". The resulting uncertainty as to when the market will begin to recover induces a "liquidity-preference" which, in the Keynesian story, lays the 'liquidity trap' that renders monetary policy of money expansion impotent when interest rates are sufficiently low. This becomes a seemingly intractable problem as long as the post-crisis level of aggregate demand remains rooted in its trough. Thus, for Keynes, the business cycle moves according to the volume of investment which behaves in a cyclical fashion, except in times of crisis, when the uncertainty as to the future yield of investment becomes so grave as to precipitate a downturn in investment that cannot be offset by an increase in the marginal propensity to consume. The fall in the cycle is sudden, whereas, in the time when the economy starts to recover, there is “no such sharp turning point.”

Modern studies of asymmetry (e.g., Neftci, 1984, Terasvirta and Anderson, 1992) cite Keynes as one of the first to propose the idea. An earlier study by Mitchell (1927) relied less on theory and more on statistics to validate it, ultimately to mixed results. Keynes was silent as to whether the crisis would have permanent or temporary effects of the overall economy. The asymmetry phenomenon was also not presented in *The General Theory* as though it were a characteristic of all business cycles, a fact overlooked by those modern studies that cite Keynes as one of the progenitors of this idea. Nonetheless, it should be noted that the discontinuous property of the aggregate supply function suggests differences in regime according to the region of the aggregate supply function within which the economy is operating.

This chapter will close with a brief discussion of some of the conflicting economic interpretations of business cycles, particularly in light of the most recent 2007-2009 recession. One of the canonical explanations offered by economists for the onset of recessions has been declines in worker productivity. Traditional labor market models consist, first, of a demand function which is derived from the marginal product of labor (MPL), and second, a supply function which is based on the marginal rate of substitution (MRS) between labor and leisure that characterize the behavior of a representative household. Equilibrium in the labor market is defined by equality of the MPL with the MRS. Declines in the marginal product of labor not only indicate lower productivity, but also induce the demand shortages that explain unemployment. Declines in both productivity and employment are two of the most salient features of economic recessions, the former effecting the latter.

This explanation of unemployment as one of productivity decline is often associated with the seminal work of Kydland and Prescott (1982) on real business cycle theory. Their work analyzes deviations from the equilibrium conditions of different input markets. In the labor market, for instance, positive (negative) labor deviation is defined by the level of employment that is above (below) the level consistent with the marginal product of labor. If one considers total productivity as determined by a production function of labor and capital, positive (negative) productivity deviations are defined as the level of output that is above (below) the level generated by the capital and labor inputs. This deviation is referred to as the 'Solow residual' and is often used to measure changes in productivity. In short, real business cycle theory says that changes in productivity go hand in hand with disequilibrium in the markets for labor and capital.

Recently, questions have been brought to bear on this theory, particularly in light of the recent 2007-2009 recession. In the United States, postwar recessions – excluding the 2007 recession – have, on average, seen a -2.4 percent labor deviation and a -2.2 percent productivity deviation. The 2007 recession, by sharp contrast, has seen a -12.9 percent labor deviation, along with a mere -.1 percent productivity deviation. For a good overview of how the 2007 recession differs from other postwar recessions, see Ohanian (2010). In other words, productivity has barely dipped in this most recent and very deep recession, which, as noted by Ohanian, can be completely characterized by a labor market slump from which we are still recovering as of the fall of 2012.

Thus, the salience of the 2007-2009 recession lies in there being a relative lack of productivity decline that is conspicuously coupled with a labor market which has been persistently marred by high unemployment. As noted, this is an aberration from previous postwar recessions (Ohanian, 2010). It is this persistence that this thesis will seek to both account for and quantify over the postwar period. An assumption is that the peaks and troughs of the business cycle make for the appropriate backdrop against which one can account for how slowly or swiftly employment and capacity recover in the face of a recession. As noted, while the traditional Keynesian framework does not explain the nature of recovery, it does suggest through its discontinuous aggregate supply function that the macroeconomy is governed by several regimes based on the level of overall output. However, it is an open question as to whether all recessions are followed by swift, or slow, rises in employment and capacity. This thesis shall seek to contribute to filling in this gap.

CHAPTER III

DESCRIPTION OF THE DATA

The empirical analysis is based on quarterly data for the U.S. civilian rate of unemployment, and for the capacity utilization series, both for the period 1948:Q1 – 2009Q3. The unemployment series was obtained from the St. Louis Federal Reserve Databank (FRED). Data for capacity utilization is available from the Federal Reserve Board of Governor's official website.

Figure 1 plots the two quarterly series over this period (Panel A for capacity utilization, Panel B for unemployment). Figure 2, Panel B plots the two overlapping, showing that they are roughly the mirror image of each other. Figure 2, Panel A displays capacity utilization and employment overlapping; the vertical axes are drawn differently to make comparison more visible. A cursory evaluation of Figure 2 suggests that, from their trough, recessionary levels, adjustments of capacity utilization tend to be smoother and more secular compared to employment. Examples of this are the recessions of the early 1970s and 2000s. An exception is the relative smoothness with which both variables recovered from their respective troughs following the early 1980s recession. The period of the mid- to late-1990s is shaded orange on the graph in order to emphasize the anomalous situation seen during this period: despite strong economic growth, capacity utilization saw a persistent decline going right into the early 2001 recession. Discussion of this apparent anomaly will take place in the next chapter.

CHAPTER IV

ANALYSIS AND METHODOLOGY

This chapter is divided into three parts. Each dovetails into the other in that an ancillary purpose of this chapter is to chronicle how the STAR method was chosen in the effort to test the hypothesis stated in Chapter I. The first will report results of traditional linear tests for common stochastic trends between unemployment and capacity utilization. This first part will review some of the theories concerning the two series. The review is placed in this chapter because these theories have direct bearing on the assumptions the researcher must make in determining the kind of tests that will be implemented. Corrado and Mattey (1997) contend that, "much of the variance in capacity utilization is common to other 'business cycle clocks' such as unemployment." Tests for verifying this claim fall generally under the rubric of 'cointegration', developed by Engle and Granger (1987). Cointegration as originally defined states that the two or more variables are cointegrated if they follow the same stochastic trend; accordingly, their divergences from one another are merely temporary. The second part will review some of the models which have been developed to incorporate changes in the regime such that the parameters of the data-generating process change. It is important to remember that regime-change is different from what is referred to in the literature as 'structural change'. The latter is conventionally understood to result from an exogenous shock that leaves a permanent effect on the series' evolution. The third and final section presents the STAR methodology, specifically the well-specified procedure that leads up to estimation.

Part 1: Testing for Common Stochastic Trends under the Assumption of Linear Adjustment

In accordance with the framework laid out by Engle and Granger (1987), our analysis begins by determining the order of integration d, labeled $I(d)$, where d is the number of times the difference operator Δ must be applied to a time series before it can be characterized as a (covariance) stationary process. That most annual macroeconomic variables are $I(1)$ has been well known since the seminal work of Nelson and Plosser (1982), who applied Dickey-Fuller tests of stationarity to fourteen macroeconomic variables and found that all but one contained a unit root. (It is worth mentioning that the one exception was the U.S. unemployment rate.) This result ran counter to the then-prevailing view that most macroeconomic series were stationary around a deterministic trend, which is termed 'trend stationary'. Trend stationarity of a variable, say *y*, implies that the residuals of the regression $y = c + \delta t + \varepsilon_t$ are stationary. The Nelson and Plosser results indicate that most variables are stationary when differenced once, but random walks when analyzed in levels. This finding opened up a new issue that researchers face when applying regressions of individual time series, namely, the issue of 'spurious regression'.

The concept of spurious regression goes as far back as Yule's (1926) discussion on spurious correlation, when he showed that significant levels of correlation between two unrelated random walks could result even in large samples. The R^2 of a regression of two such variables should tend to zero as the sample size increases. However, because both variables contain a deterministic component, spurious (or false) correlation could be reflected in the form of significant *t*-statistics, along with an R^2 significantly different from zero. When a regression between $I(1)$ time series (expressed in levels) yields these two results, along with a low Durbin-Watson (D-W) statistic, the researcher should be cautious in drawing conclusions. The crux of the problem is that two variables may well follow a shared deterministic trend – and hence the signs of correlation – but not the same stochastic trend.

The spurious regression problem was revisited by Granger and Newbold (1974), when they showed how autocorrelated residuals, in addition to causing inefficient OLS estimates, can also render invalid the traditional *t*-tests for significance. They took two random walks,

$$y_t = y_{t-1} + u_t$$

$$x_t = x_{t-1} + v_t,$$

and generated artificial data for u_t and v_t according to different distribution parameters, thus allowing *x* and *y* to follow two different stochastic paths. They then regressed *y* on *x* plus a constant and found a high R^2 but a low D-W statistic. When regressing in first differences, the R^2 was close to zero and the D-W was close to 2, thus showing that *y* and *x* were in fact unrelated, despite the significant results of the regression that was run in levels. In short, one of the rules of thumb in applying OLS to time series is that, if $R^2 >$ D-W, this is a symptom of a spurious regression.

Understanding the concept of spurious regression is helpful in thinking about the way in which the theory of cointegration was originally developed. In light of Granger‘s and Newbold’s findings, an obvious question arises: what if a regression of one random walk on another yields a residual series that is stationary? The residuals would be, by definition, a mean-zero process bounded by a finite variance. This would indicate that the variables do not drift too far apart from one another, and converge in the long-run. This condition gives rise to the concept of cointegration. The economic interpretation of cointegration is that some long-run, equilibrium relationship exists between two or more variables. Moreover, these cointegrated variables would share a common trend, such as some leading economic indicator like money growth or interest-rate spreads. The formal notation of cointegration is summarized as follows: if $y_t \sim I(1)$ and $x_t \sim I(1)$ and there exists some β such that $y_t = \beta x_t + \varepsilon_t$, in which $\varepsilon_t \sim I(0)$, then this is evidence that y_t and x_t share a common trend and are cointegrated in the long-run. Engle and Granger (1987) define cointegration more generally, stating that if, for all the $I(d)$ components of some vector x_t, there exists a vector α so that $z_t = a'x_t \sim I(d-b), b > 0$, then the components of that vector are cointegrated. In other words, for cointegration to exist, the components must be integrated of the same order *d*, and a linear combination of those components must be integrated of an order less than d. An exception would be a case where one of the variables is $I(2)$ and the rest are $I(1)$. It is possible that a linear combination of these two is $I(1)$, in which case we would have what is called ‘multicointegration’. It is also possible that d is a fraction, in which case the series is ‘fractionally integrated’, characterized as being not quite in possession of a unit root, but, at the same time, highly persistent; this persistence is reflected in an autocorrelation function that declines at a slow, hyperbolic rate (Hjalmarsson & Osterholm, 2010). For now, we will confine our attention to processes whose orders of integration are discrete integers ($d = 0,1,2 \dots$). Engle's and Granger's definition will become useful as we try to ascertain the existence of a cointegrating relationship between the two variables of this study, capacity utilization and unemployment, a task to which we will soon turn.

First, we will review some of the modern theories of the two variables, beginning with unemployment. The two most prominent theories of unemployment are the 'natural rate' hypothesis advanced by Friedman (1968), and the hysteresis hypothesis propounded by Blanchard and Summers (1987). These two theories have important implications on time series models of unemployment and the assumptions on which they are based.

Friedman extended the concept of the 'natural rate of interest' of Wicksell, which says that there is some natural rate of interest that the Fed cannot influence without inducing inflation, by applying the same idea to unemployment, which the Fed might also try to influence. Friedman was clear on the point, often misunderstood, that the natural rate of unemployment does not stay the same, but can rise largely because, says Friedman, of government policies that create frictions in the labor market. Nevertheless, the natural rate theory suggests that, *ceteris paribus*, unemployment does revert back to some natural rate. This precludes unemployment from being guided by a unit root process that is defined as having a mean rate that varies with time.

In contrast to Friedman, Blanchard and Summers (1987) were writing in the wake of high unemployment in Europe that had been persisting for nearly twenty years. They offered a framework that is often called the 'insider-outsider' theory of employment, whereby, in the event of a shock that causes some (the outsiders) to lose jobs, firms retain those more valuable workers (the insiders) who would then wield bargaining power in order to ensure their own job security. Such bargaining activity pushes the equilibrium wage higher, thus obviating competition for jobs that outsiders present, in light of their recent joblessness. Blanchard and Summers point out that, from 1980 to 1986, unemployment rates in three major European countries – the U.K., France, and West Germany – had nearly doubled through steady increases over those six years, without much sign of any mean-reverting tendency. Such a persistent rise could not be explained by either monetarist or Keynesian theories. The contribution of Blanchard's and Summers' research is that it has shed light on the phenomenon of 'persistence' (long-memory) , which describes how unemployment rates are in part functions of historical shocks, the effects of which never fully die out. This is now regarded as one of the more important stylized facts of unemployment of which theorists and model-builders alike must take account.

Turning to capacity utilization, this measure has long been of interest to macroeconomists, particularly in its being viewed as a leading indicator of inflation (for a brief survey, see Finn (1995)). Garner (1994), for example, argued that both the unemployment rate and capacity utilization, as measures of 'resource tightness' for labor and capital respectively, give "consistent signals about U.S. inflationary pressure". Garner also argued for the existence of a "stable-inflation" rate of capacity utilization" – a 'natural rate of capacity utilization', as it were – of about 82%. Several other studies (e.g., Davies, 1994), many of which just barely predate the investment boom of the late 1990s, reinforce the view of capacity utilization as a harbinger of inflation.

Recently, however, this view has been questioned. Bansak, Morin, and Starr (2007) speculate that, during the investment boom of the late 1990s, new technology might have enhanced capacity in a way that made excess capacity cheaper, and hence afford firms the opportunity to operate at lower average capacity so as to be able to handle any potential upswings in demand. By the same token, expansion of capacity is also made cheaper, so that firms need not raise their levels of excess capacity. The effects of technology on capacity utilization are far from obvious. However, Bansak, et.al. found using panel data of 111 manufacturing industries that technological change tends to dampen the levels of utilization among manufacturers.

Should the so-called 'technology-boom' of the late 1990s be seen as a 'structural break' in terms of how firms utilize their capacity levels against the backdrop of the overall economy? Even in the absence of the results of Bansak, et al., few would deny the marked differences in how capital is utilized today, versus how they had been utilized in the 1940s and 1950s, with assembly lines of workers, complemented by fixed large-scale machinery. Further, theory must account for how capacity utilization had either remained flat or had been declining during the late ’1990s, when investment boomed and unemployment fell below 4% (see Figure 2, Panel A). On the whole, one would expect unemployment and capacity utilization to be negatively correlated; perhaps anomalously, they had been trending in the same direction during much of this boom period.

Because theory is either silent or conflicted as to the true statistical nature of these two variables, the applied researcher’s efforts are hampered by a kind of *a priori* uncertainty as to whether to consider unemployment and/or capacity utilization as stationary or not. Generally, one should not proceed to test for cointegration if one or more variables is $I(0)$, given how different the data-generating process would be from that of an $I(1)$ variable. It is clear that, in testing for stationarity, the results one arrives at depend crucially on the decisions made by the empirical researcher, such as the number of autoregressive lags to include, the sample size, the sample period, and the model specification. There are several ways to deal with this uncertainty. Osterholm (2010) and Emerson (2011), to give two examples, both test for cointegration – the latter for the U.S., the former for Sweden –between rates of unemployment and labor force participation by applying the DF-GLS test of Elliot, Rothenberg, and Stock (1996). This test is useful for near-integrated variables for which the ADF tests have low power; in other words, the latter set of tests tends to accept the null hypothesis too often. The DF-GLS is designed to take local dynamics into account. Mustafa and Rahman (1995) test for cointegration between U.S. capacity utilization and U.S. inflation. Their ADF results suggest that capacity utilization is nonstationary; however, they use monthly data, which in general are ‘noisier’ (more local variation), but at the same time augment the local trend effects due to the data’s low frequency. Their sample size of eleven years is also relatively small. Transformation of the data is another way to overcome uncertainty. Koop and Potter (1999) remind us that working with series like unemployment and capacity utilization is especially difficult because, on the one hand, they are “bounded variables” (i.e., they are percentages) that preclude global unit root behavior, but on the other hand, they are near-integrated variables that exhibit strong, local persistence that confounds traditional unit root tests – and thus traditional tests for cointegration. They suggest taking a (nonlinear) logistic transformation, $\ln(x_t * (1 - x_t)^{-1})$, in order to dampen the local trend-following behavior.

Presented in Table 1 are our results of the ADF tests on both capacity utilization and unemployment. We underscore how pronounced the uncertainty is by reporting the results of different sample periods. All equations include a constant, and different lag lengths, given by ρ, as recommended by the Akaike Info Criterion (AIC) and the Schwarz Info Criterion (SIC) are also reported. The ADF regression equation is the following:

$$\Delta y_t = \alpha_0 + \delta y_{t-1} + \sum_{i=1}^{\rho} \beta_i \Delta y_{t-i} + \varepsilon_t$$

It is well known that having too many lags will decreases the power of the test, and having too few will create enough autocorrelation to bias the test in favor of rejecting the null hypothesis of a unit root. However, for our data, it appears that adding more lags seems to produce higher *p*-vales and hence lower rejections rates of the null. What the results also reflect is that, as you increase the sample size but keep the number of lags low, there is a higher probability of rejecting the null.

The reader will be able to note the ambiguity of these results. For unemployment, a sample size of forty years and a specification that includes nine lags yields a conclusion according to which we cannot reject the null, that is, unemployment has a unit root and is therefore nonstationary. By contrast, a sample size encompassing ten years, with a lag length of two suggests rejection of the null, i.e., unemployment is stationary. The results on capacity utilization provide roughly the same picture.

The augmented Dickey-Fuller results reinforce the *a priori* uncertainty discussed earlier. Unreported here, stationarity tests using the DF-GLS procedure of Elliot, et.al (1996) give evidence that both series possess a unit root (nonstationary). It is the results of this procedure that are often reported in empirical work to support the hypothesis that the series in question possesses a unit root. We now, in the face of this uncertainty, take the step taken by many of the authors cited above and proceed to test for cointegration using the two-step Engle-Granger (1987) procedure.

The first step is to estimate the long-run equation, the second, to test the residuals for stationarity. One of the pitfalls of this two-step procedure is that, in the absence of theory, there is little guidance in deciding which variable to set on the left-hand side. We run the long-run equation twice, once with unemployment (denoted UE_t) as the dependent variable and once with capacity utilization ($CAPU_t$) as the dependent variable,

$$CAPU_t = 93.065 - 2.177UE_t + \hat{\varepsilon}_t$$

$$(.965) \quad (.165)$$

$$[96.422] \quad [-13.203]$$

$$N = 247 \qquad R^2 = .416 \qquad \sigma_\varepsilon = 3.986 \qquad D - W = .153$$

$$UE_t = 21.074 - 0.191CAPU_t + \hat{\varepsilon}_t$$

$$(1.171) \quad (.014)$$

$$[18.001] \quad [-13.203]$$

$$N = 247 \qquad R^2 = .416 \qquad \sigma_\varepsilon = 1.17 \qquad D - W = .105$$

where OLS standard errors are given in parentheses below the coefficient estimates, *t*-values are given in brackets, R^2 is the coefficient of determination, which tells us the portion of the variation in the dependent variable that is explained by the regression, σ_ε is the standard deviation of the residuals, and $D - W$ is the Durbin-Watson statistic of first-order autocorrelation in the residuals. As neither of the dependent variables follows a global trend, the addition of a linear time trend did little to contribute to the fit of the regressions. Nor did it assist in raising the Durbin-Watson statistic. Also, given that for both regressions, $R^2 > D - W$, and that our *t*-statistics are all highly significant, these two equations bear the traits of the aforementioned spurious regression.

Figure 3 plots the residuals of the two long-run equations (Panel A for the equation with capacity utilization as the dependent variable, Panel B for that with unemployment as the dependent variable). It is evident that the two series exhibit strong persistence, but globally behave in a cyclical fashion. It is clear *a priori* that feedback from unit root tests depends crucially on how pronounced the cyclicality is.

We proceed to the second step and test for the presence of a unit root in the residuals by applying the augmented Engle-Granger tests – the counterpart to the ADF tests – and using the critical values calculated by Mackinnon (1991). A different set of critical values is needed because the residuals we are testing are estimates and thus follow a distribution different from that followed by actual observations. We specify the E-G tests in a way to ensure that the stochastic disturbances (the epsilon term) are as close to being white noise as possible. Because of the highly persistent nature of the two series, it is likely that any autocorrelation in the residuals of the E-G test may go beyond the first order. Thus, we implement the Breusch-Godfrey LM (*k*) test, which tests the joint hypothesis that all the autoregressive coefficients of the residuals up to lag *k* are all zero. Once we achieve white noise in our specification, we can compare the test statistic of our estimate against the Engle-Granger critical values. Table 2 reports the results of the E-G tests. We also report some of the summary statistics of the epsilon term of the tests, because they are informative in helping us ascertain the existence of white noise.

Neither of the test-statistics are significant, from which we conclude that there is no evidence of cointegration between the two series. Both Durbin-Watson statistics indicate that there is no first-order autocorrelation. σ_ε (again) denotes the standard error of the regression; it is placed alongside the summary statistics because it can be thought of as the standard deviation of the epsilon term. As to the white noise of the epsilon term, the Breusch-Godfrey test recommends a lag of $k = 3$ for both equations. Both the mean and median are very close to 0. However, both the skewness and kurtosis cast considerable doubt on the normality assumption of epsilon. A skewness that is significantly different than zero, and a kurtosis significantly different from three, resulted in a high Jarque-Bera statistic in both cases (not reported, the J-B stat tests for the null of normality), and hence a very low *p*-value. The two high levels of kurtosis are conspicuous; they suggest there are many outliers on both tails. The equation in which unemployment is on the left-hand side generates error-terms that are closer to being normally distributed.

We will now, in light of our results, review the limitations of the two-step Engle-Granger procedure. First, it has been amply noted by many authors that the long-run equation is static, that is, there are no lagged dependent variables, and hence in most cases there is little information about short-run dynamics. In the Engle and Granger (1987) framework, the first step is to establish that there is a long-run relationship, which is then followed by estimation of an 'error-correction model'. It is in this latter model where the information on short-run dynamics can be seen more clearly. Engle and Granger (1987) proposed a famous theorem known as the 'Granger representation theorem', which says that if two variables are cointegrated in the long-run, then their short run dynamics can be well captured by an error-correction model of the following general form:

$$\Delta y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \Delta y_{t-i} - \gamma \widehat{z_{t-1}} + \varepsilon_t$$

where $\widehat{z_{t-1}}$ is the lagged term of the 'equilibrium error', that is, the error term of the long-run equation, $\widehat{\varepsilon_{t-1}}$. In the bivariate case, $\widehat{z_t} = Y_t - \hat{\alpha}_0 - \hat{\beta} x_t$, where $\hat{\beta}$ is the cointegrating parameter which was estimated in the first step. This error-correction specification is quite useful. First, note that γ is necessarily negative because the sign of the equilibrium error at time 't-1' should then be of the opposite sign at time 't'. γ can also tell us in percentage terms how much of the discrepancy between the long- and short-term relationship is corrected within one time period. If, for example, $\hat{\gamma} = -.1254$ and our data are quarterly, then about 12.54% of the discrepancy is corrected within one quarter, or three months. In short, the estimated coefficient on the lagged equilibrium error is a measure of the persistence of the disequilibrium, or, how long it takes for the two variables to converge to their long-run values, *ceteris paribus.*

Despite its usefulness, one cannot build error-correction models if one cannot establish the existence of a long-run relationship between the variables. To this end, an unbiased and efficient estimate of β is essential. One of the first criticisms of the Engle-Granger approach is the small sample biases in estimates of β. Banerjee, et al. (1986) showed this using Monte Carlo evidence and suggested that a dynamic equation serve as the cointegrating regression. Cointegration has been defined as a special kind of long-run relationship, and a sample consisting of monthly data spanning eight years (96 observations) might not produce as efficient an estimate as a sample of quarterly data spanning fifteen years (60 observations). Numerous other methods of estimating long-run relations between time series using dynamic specifications have been proposed (see Maddala and Kim, 1998, for a survey).

Another, more practical issue is the decision of which variable to classify as dependent. Of course, this is not an issue when economic theory already has an answer. Further, in the case of a bivariate cointegrating regression, β should be unique asymptotically, and the fit of the regression depending on which variable to place on the left-hand side should not be very different. In the equations we have estimated, the two R^2 values are identical but both standard errors and our β estimates are quite different. Perhaps the issue can be explained in part by the findings of Ng and Perron (1997), when they studied the normalization problem in cointegrating regressions containing two variables. They showed that, in a bivariate cointegrating equation, having the less integrated variable as the explanatory variable creates a bias that makes the superconsistency property of the OLS estimators discovered by Stock (1987) no longer valid. They advised selecting as the regressand the variable that is less integrated. To consider variables whose order of integration is a non-integer would take us into the realm of the ARFIMA (autoregressive fractionally integrated moving average) models introduced by Granger and Joyeux (1980). We will sidestep those models and consider instead what Ng and Perron recommended: they suggest taking each series in first differences and ranking them according to their spectral density at zero frequency. Eviews 6 includes a menu option on spectral estimation when using its built-in function for the Philips-Perron unit root test. In accordance with Ng and Perron, we apply the quadratic kernel to the least-squares residual, and used the Andrews procedure to select the bandwidth. The bandwidth on $CAPU$ is 1.96, showing a slight increase over that for UE of 1.72. This suggests that capacity utilization is integrated of a lower order (i.e., closer to zero) than unemployment, and that therefore, according to Ng and Perron, the equation with capacity utilization on the left-hand side contains less of a bias than does the equation with unemployment on the left-hand side.

We close this section by mentioning that some of these issues are circumvented by applying the VAR-based test of cointegration of Johansen (1995). Unreported here, the results from the Johanssen procedure found evidence of cointegration between the two variables. We then followed the methodology proposed in Osterholm (2010) of carrying out a Wald-type test whereby the following restrictions on the cointegrating vector are implemented: $\beta = (1\ 0)'$ and $\beta = (0\ 1)'$. If these restrictions are not rejected, then the result of finding cointegration is attributed to the fact that one of the series is stationary. This is particularly relevant when working with near-integrated series: namely series that are not exactly in possession of a unit root, but at the same time highly persistent. Both Wald tests were implemented and neither of them could be rejected.

Part 2:A Review of Models of Discrete Regime-Switches:

Time series models that incorporate regime-dependent autoregressive processes were popularized mainly by the threshold autoregressive (TAR) models of Tong (1990). A good description of the TAR model is given in Enders and Siklos (2001): "The basic TAR model…allows the degree of autoregressive decay to depend on the state [that is, the regime] of the variable of interest." If we take a simple, AR(1) model,

$$y_t = \rho y_{t-1} + \varepsilon_t$$

and modify it in such a way that, for every 't', y_t is generated by one of two linear models, with the first-order lag determining that model, , the AR(1) model becomes,

$$y_t = \rho_1 y_{t-1} I(y_{t-1} \leq c) + \rho_2 y_{t-1} I(y_{t-1} > c) + \varepsilon_t$$

where $I(\cdot)$ Is an indicator function, whereby if the condition of I holds, then $I = 1$ and equals zero otherwise. This is the simplest form of the TAR specification; more complicated ones include more regimes. This general concept of regime-dependence has been extended to unit root theory by Enders and Granger (1998), when they analyzed interest rate spreads. It has also been applied to cointegration theory by Balke and Fomby (1997), Enders and Siklos (2001) and Granger and Yoon (2002). Enders and Granger (1998) allowed for asymmetric equilibrium adjustment to short- and long-term interest rates by attaching a Heaviside function to the coefficient of the first-order lag to the variable denoting interest-rate spreads. Balke and Fomby (1997) theorized a scenario where equilibrium adjustment between two or more cointegrated variables might not be active within a given threshold and, moreover, the equilibrium adjustment process depends on the sign of the equilibrium error. They suggest using Wald tests to determine whether such a specification is correct when doing applied work (their paper lacked an empirical component). Enders and Siklos (2001) applied the TAR-like specification to Dickey-Fuller equations, when they, too, evaluate the relationship between short- and long-term interest rates. Granger and Yoon (2002) develop the idea of 'hidden cointegration', whereby different error-correction equations are specified for positive and negative equilibrium errors. They suggest that failure to detect cointegration using the Engle-Granger procedure might mask the relationship two or more variables might have in how they respond to positive or negative shocks in their common factor restriction. In other words, there might be a common attractor between the two variables in the event of a positive or negative shock.[1]

In each of these studies, however, it is important to note (at least) two common threads. First, it is assumed that switches in regime are sudden, discrete, and discontinuous events. What these models say is that, given the value of the indicator function, which tells us whether the adjustment process is within some specified range, the variable will be set on some given regime. The second is the persistent problem of how to estimate the cointegrating vector when the dynamic adjustment process is nonlinear. Of course, when studying interest rates or exchange rates, economic theory already has something to say about long-run relationships and one therefore need not estimate the long-run equation. Since Engle and Granger (1987), a lot of work has focused on how to circumvent the problem of testing for cointegration without having to estimate the cointegrating vector. The literature on this topic is too vast to summarize but perhaps its best reference work is Maddala and Kim (1998).

[1] Hidden cointegration is, conceptually, perhaps the most straightforward way to test for nonlinear cointegration. However, computationally it is somewhat cumbersome, in that you must difference each series you are working with, separate out the positive and negative values, and calculate the cumulative functions for each. The difference in these cumulative functions is what substitutes for the equilibrium error term in the traditional ECM model. We estimated the two ECM equations and found some evidence that capacity utilization and unemployment respond in a similar way to negative shocks.

Part 3: Smooth Continuous Regime-Switches and the STAR Modeling Procedure

What if the transition from one regime to another is not discrete and sudden, but continuous and smooth? There have been several models that include smooth transition functions , of which one of the more famous is the smooth transition autoregression (STAR) model that has been popularized by Terasvirta and Anderson (1992). A STAR model takes the following general form:

$$y_t = \alpha_0 + \alpha_1' x_t + (\theta_0 + \theta_1' x_t) F(y_{t-d}) + \mu_t$$

where $\mu_t \sim iid(0, \sigma^2)$, $x_t = (y_{t-1}, \dots, y_{t-p})$, $\alpha_1 = (\alpha_{11}, \dots, \alpha_{1p})$, and $\theta_1 = (\theta_{11}, \dots, \theta_{1p})$. $F(y_{t-d})$ is a continuous transition function of some lagged value of y_t, where *d* is referred to as the delay parameter. In the STAR framework, the delay parameter is determined empirically through a sequence of Lagrange multiplier tests (about which more will be said in due course). One of the better known transition functions is the logistic transition function. STAR models that incorporate the logistic function are denoted 'LSTAR'. We will consider first the logistic function, *G*, which takes the following form:

$$G(\gamma, c, s_t) = \{1 + \exp[-\gamma \prod_{k=1}^{K} (s_t - c_k)]\}^{-1}, \qquad \gamma > 0, \qquad c_1 \le c_2 \le \cdots \le c_k$$

where γ is often referred to as the slope parameter, which measures the speed with which the series moves from one regime to another. s_t is a stochastic transition variable that determines the regime of the series at time *t*. Finally, c_K is a constant term and is dependent on K, where K + 1 is the number of regimes. The most common model is one in which it is assumed that $K = 1$; such models have been applied by Terasvirta and Anderson (1992) to industrial production series for different countries, and by Skalin and Terasvirta (2002) to unemployment series, also for different countries. This model is used to characterize asymmetric behavior of a two-regime scenario, whereby the dynamic adjustment process is different depending on the regime. In the context of business cycles, the two-regime scenario often considered is that between expansion versus contraction of the overall economy. It should be noted that there is a lot of flexibility available to the researcher in choosing the transition variable s_t. It can be a variable other than the dependent variable or a linear combination of other variables. A deterministic time function could even serve; this latter choice makes for a model, abbreviated TSTAR, which has been applied by Dueker, Owyang, and Sola (2010) to the unemployment rate in order to allow for variation in the natural rate of unemployment. One will also note in $G(\cdot)$ that, when $\gamma = 0, G(\cdot) = \frac{1}{2}$, and the LSTAR equation thereby degenerates into a linear model. On the other hand, as $\gamma \rightarrow \infty$, the model becomes the TAR model which, as discussed previously, includes the Heaviside function as an indicator of regime.

If we restrict ourselves to the case where $K = 1$ and $s_t \equiv y_{t-d}$, the transition function now takes the following form:

$$G(\gamma, c, s_t) = \{1 + \exp[-\gamma(y_{t-d} - c)]\}^{-1}$$

It is observed that, as $y_{t-d} \rightarrow +\infty, G \rightarrow 1$ and as $y_{t-d} \rightarrow -\infty, G \rightarrow 0$; in both cases, the transition function will degenerate into a constant. G is thus a continuous function bounded between 0 and 1. Those autoregressive coefficients of the terms that are products of the transition variable will change smoothly as the transition variable moves within these two extremes.

An alternative, exponential specification takes the following form:

$$G(\gamma, s_t, c) = 1 - \exp[-\gamma(y_{t-d} - c)]^2$$

STAR models that have this function serve as the transition variable are denoted ESTAR. In contrast to the LSTAR model, the exponential function is symmetric around $y_{t-d} = c$, because $G(\cdot) = 0$ as $y_{t-d} \rightarrow c$, and $G \rightarrow 1$ as $y_{t-d} \rightarrow \pm\infty$. The ESTAR can be used in situations where large deviations from a long-run relationship initiate the mean-reversion process, but small deviations do not. A common application is exchange rates, where small deviation from purchasing power parity will not be corrected through arbitrage due to the presence of transaction costs. Large deviations, however, are far more likely to induce arbitrage behavior. It is the transaction costs, such as the premium on a futures contract, that are the frictions that preclude PPP from holding when discrepancies from equilibrium are small. In short, the speed of mean reversion is dependent on the size of the discrepancy. Small to mid-size discrepancies are eliminated much more slowly than large discrepancies.

Our task is to test the hypothesis of linearity of our two series against a STAR alternative. This amounts to testing for restrictions on coefficients. A conventional way of carrying this out involves estimating a restricted and unrestricted model, finding for each the residual sum of squares, and calculating an *F*-statistic to which we compare the critical values of an *F*-distribution. Another way to test a hypothesis would be to estimate only an unrestricted model, and perform a *t*-test on the restriction. This is known as the Wald procedure. The *F*-test requires the estimation of both models, and the Wald procedure only requires estimation of the unrestricted model. A third method, the Lagrange multiplier test, requires only that we estimate the restricted model. Our hypothesis tests place linearity as the null hypothesis, on which we impose the restriction that the coefficients on nonlinear terms are all zero. The alternative hypothesis is that the data generating process follows a nonlinear STAR specification, which is our unrestricted model. We thus elect to apply the LM test in that it affords us the convenience of only needing to estimate the restricted, linear model.

The LM-test essentially calls for regressing the OLS residuals from the linear portion of the model, on the partial derivatives of the dependent variable with respect to each of the model's parameters. Suppose we had the following[2]:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \alpha_3 y_{t-1} y_{t-2} + \varepsilon_t$$

We find $\partial y_t / \partial \alpha_i \; i = 0,1,2,3$, and then estimate an auxiliary regression,

$$\varepsilon_t = \delta_0 + \delta_1 y_{t-1} + \delta_2 y_{t-2} + \delta_3 y_{t-1} y_{t-2} + u_t$$

where δ_i , $i = 1,2,3$ is the coefficient for each of the partial derivatives of y_t with respect to $\alpha_i \;\; \forall i,$ and α_0 is a vector of ones since $\partial y_t / \partial \alpha_0 = 1$. After estimating the auxiliary regression, our test

[2] This example is taken from Enders (2004, pg. 410)

statistic is TR^2, which asymptotically follows a chi-square distribution with $p + 1$ degrees of freedom, p being the number of parameters of the equation. If our test statistic exceeds the chi-square critical value, we reject the null of linearity.

Now, let us implement the same procedure to an LSTAR model:

$$y_t = \alpha_0 + \alpha_1 x_t + (\beta_0 + \beta_1 x_t)\{1 + \exp[-\gamma(y_{t-d} - c)]\}^{-1} + \varepsilon_t$$

where $x_t = (y_{t-1}, \dots, y_{t-p})$, $\alpha_1 = (\alpha_{11}, \dots, \alpha_{1p})$, and $\beta_1 = (\beta_{11}, \dots, \beta_{1p})$. Our null hypothesis is that the true model is linear, enforcing the restriction that $\gamma = 0$. However, there is a problem. As noted, at $\gamma = 0, G = \frac{1}{2}$. But the partial derivatives of y_t with respect to each of the parameters, evaluated at $\gamma = 0$, are 0 for $\beta_0, \beta_1,$ and c. In other words, those latter three parameters are unidentified under the null, and as such are classified as 'nuisance parameters'.

Therefore, the LM test cannot be applied for hypothesis testing purposes. Normally, in this case, the applied researcher might be compelled to follow either the F-test or Wald procedures mentioned earlier. This would be cumbersome because, on the one hand, both procedures would involve estimation of a nonlinear equation, which is generally more difficult compared to linear ones. On the other hand, she would also have to run these procedures for a number of models allowing for different nonlinear alternatives, then running the diagnostic tests for each, and then, finally, using judgment as to which model provides the best fit. All of this, of course, would have to take place in the absence of any guidance economic theory might provide as to the nature of the nonlinear relationship.

Despite the bleakness of the situation, there is a framework which has been designed to deal effectively with the nuisance parameter problem. Testing for linearity using Lagrange multiplier tests was considered by Luukkonen, et al. (1988) and then more fully in Granger and Terasvirta (1993). The former suggested taking a Taylor expansion of the transition function around $\gamma = 0$, and reparameterizing the auxiliary equation by substituting the resulting Taylor expansion for the transition function. Let us reformulate our LSTAR model in this way:

$$y_t = \phi_1' x_t + (\phi_1 + \phi_2)' x_t \, G(\cdot) + \varepsilon_t$$

where $x_t = (1, y_{t-1}, \dots, y_{t-p}), \phi_i = (\phi_{i,0}, \phi_{i,1}, \dots, \phi_{i,p}),\ i = 1,2,$ and $G(\cdot)$ is the logistic transition function. If we apply a first-order Taylor expansion to $G(\cdot)$ and reparameterize, our auxiliary equation becomes,

$$y_t = \beta_0' x_t + \beta_1' x_t s_t + v_t$$

where $\beta_i = (\beta_{i,0}, \beta_{i,1}, \dots, \beta_{i,p})\ i = 0,1,$ s_t is our transition function (a lagged value of y_t) and $v_t = \varepsilon_t + (\phi_2 + \phi_1)' x_t \, R$, where R is the remainder term from the Taylor expansion. Under the null, however, $R = 0 \;\; => \;\; v_t = \varepsilon_t$. Our null can now be specified:

$$H_0\colon \gamma = 0 \quad => \ \beta_{0,j} \neq 0 \ \text{ and } \ \beta_{1,j} = 0 \quad j \in [0, p]$$

The test statistic, denoted LM_1, with the subscript reflecting the fact that the test statistic is based on a Taylor-expansion of the first order, is $\text{LM}_1 \sim \chi^2_{p+1}$. We reject the null of linearity if the test statistic is greater than the chi-square critical value.

As noted, we select as the transition variable a lagged value of y_t, y_{t-d}, where d is the delay parameter. In this case the regressors are lags, either in levels or differences. Further, as noted by Luukkonen, et al. (1988), the LM_1 specification does not have power in the case in which only the intercept differs across regimes. A test that does is the LM_3 test, which requires a third-order Taylor expansion of G. Let $\theta = \{1 + \exp[-\gamma(y_{t-d} - c)]\}^{-1} = \{1 + \exp(-m_{t-d})^{-1}$, where $m_{t-d} = -\gamma(y_{t-d} - c)$. In order to take a third-order Taylor-expansion, we take the first three partial derivatives of θ with respect to m_{t-d}, evaluated at $m_{t-d} = 0$

$$\left.\frac{\partial\theta}{\partial m_{t-d}}\right|_{m_{t-d}=0} = \frac{1}{4}, \qquad \left.\frac{\partial^2\theta}{\partial m_{t-d}^2}\right|_{m_{t-d}=0} = 0, \qquad \left.\frac{\partial^3\theta}{\partial m_{t-d}^3}\right|_{m_{t-d}=0} = -\frac{1}{8}$$

Now, we insert these into the form of the Taylor-expansion:

$$\theta = \frac{1}{4}m_{t-d} + \frac{1}{2!}(0)\left(m_{t-d}^2\right) + \frac{1}{3!}\left(-\frac{1}{8}\right)m_{t-d}^3 = \frac{m_{t-d}}{4} - \frac{m_{t-d}^3}{48} = \frac{\gamma(y_{t-d}-c)}{4} - \frac{\gamma^3(y_{t-d}-c)^3}{48}$$

We reparameterize our nonlinear transition function such that both the linear and cubed terms are included, leaving out the square term since the second derivative is zero. We are now ready to test for linearity.

However, an additional complication arises in deciding what conclusion to draw if we were to reject linearity. As noted, the alternative has nested within it several nonlinear specifications. Thus, rejecting the null does not necessarily mean that we can accept the alternative, which we have arbitrarily set up as being of the LSTAR form. This point is discussed in detail in Granger and Terasvirta (1993, pg. 101), and becomes clearer as we remind ourselves that, under the LM procedure, we are not estimating the nonlinear equation.

If linearity were to be rejected, how might the mystery of the exact nonlinear nature of the data-generating process be solved? Fortunately, there is a procedure, discussed in Granger and Terasvirta (1993, pg. 150) and Terasvirta (1994). We have already taken the first three partial derivatives of the logistic transition function, and found that the second one, evaluated at $m_{t-d} = 0$, is zero. If we were to do the same thing for the exponential transition function, defined above, we would find that the first and third derivatives, again evaluated at $m_{t-d} = 0$, are both zero but the second is not zero. These facts will have some bearing when we test the restricted models. Thus, in testing for linearity, we nest the squared term between the linear and cubic terms. As noted by Enders (2004, pg. 412), “the key insight in Terasvirta (1994) is that the auxiliary equation for the ESTAR model is nested within that for an LSTAR model. If the ESTAR is appropriate, it should be possible to exclude all of the terms in the cubic expression.” Our new, reparameterized test equation for linearity is the following:

$$y_t = \beta_0' z_t + \beta_1' z_t y_{t-d} + \beta_2' z_t y_{t-d}^2 + \beta_3' z_t y_{t-d}^3 + \varepsilon_t$$

where $z_t = (1, y_{t-1}, \ldots, y_{t-p})$. It is to this equation, newly freed of any nuisance parameters and based on the Taylor expansion, that we shall apply our tests for linearity. Our test for linearity tests the joint hypothesis that the coefficients of the terms that include as factors y_{t-d} are all zero: $H_0\colon \beta_1 = \beta_2 = \beta_3 = 0$. Due to the size distortions of the chi-square distribution for small samples, we will instead calculate test statistics following an F-test. Table 3 gives the template, which includes the regression specification along with the degrees of freedom for each of the hypotheses. The delay parameter, *d*, is determined from the data, and should be large enough to capture adequately the transition from expansion to contraction, or vice versa. The hypothesis is run for a number of different delay parameters. If linearity is rejected for more than one value of *d*, the decision rule is to select that value with the lowest corresponding *p*-value. We conduct the LM_3 test for values of *d* ranging from 2 to the largest lag, *p*.

The question of which kind of nonlinearity – ESTAR or LSTAR – to accept as our alternative model in the event of rejecting the null remains unanswered. Different sets of decision criteria have been adopted in different studies. We will use those criteria adopted by Terasvirta and Anderson (1992), Granger and Terasvirta (1993), and Akram (2005). The sequence of LM tests is based on the following hypotheses:

$$H_0\colon \beta_1 = \beta_2 = \beta_3 = 0$$

$$H_{03}\colon \beta_{3i} = 0$$

$$H_{02}\colon \beta_{2i} = 0 \mid \beta_{3i} = 0$$

$$H_{01}\colon \beta_{1i} = 0 \mid \beta_{2i} = \beta_{3i} = 0$$

If H_{02} (which is run for a number of different delay parameters) is rejected most strongly (yields the smallest *p*-value), an ESTAR or LSTAR(2) model is appropriate. Otherwise, the LSTAR method should be selected. These decision criteria were recommended by Terasvirta (1994).

To summarize the testing procedure, a three-step approach is followed:

1. Estimate the linear AR(*p*) model, both to determine the lag-order *p* and obtain the residuals
2. Test linearity using a sequence of LM-tests on the auxiliary equation for various delay parameters and for the various hypotheses, thus establishing both the proper delay parameter and the nonlinear functional form in the event of rejecting linearity.
3. If linearity is rejected, estimate the appropriate STAR model using nonlinear least squares (NLS).

It will be clear once we complete the second step whether a nonlinear STAR model is appropriate. If it is appropriate for both series, we will be able to compare the nonlinear behavior of the two series by graphing each of their transition variables, both across time and across different values of their respective transition variables. The purpose of the former is to verify that the estimated transition function corresponds to the timing of the business cycle (for which we use NBER data). The purpose of the latter is to attain an understanding of how smooth the transitions between one regime and another are, which is largely based on how large the estimated slope parameter, gamma, is, as larger values indicate very swift transitions. We subject unemployment and capacity utilization to the STAR analysis in the next chapter.

CHAPTER V

MODEL ESTIMATION

The first step of the STAR modeling procedure is to estimate the linear portion of the model, both to determine the lag order p and obtain the residuals. Following Terasvirta, Tjostheim, and Granger (2010), our AR models will be regressed in first differences, with a constant and first-order lag included on the right-hand-side. Also following these authors, we will also, in the second stage of the modeling procedure, nominate as possible transition variables lags of 'long differences', with the lags ranging from 2 to 9: $\Delta_d y_{t-1} = y_{t-1} - y_{t-d}, \quad d \in [2,9]$.

We begin by estimating the linear portion of the unemployment series, denoted $\mathrm{y_t}$, by fitting an AR model with a lag of 9 as recommended by the AIC:

$$\underset{}{\Delta y_t =} \underset{(.010)}{.274} - \underset{(.017)}{.047 y_{t-1}} + \underset{(.065)}{.478\Delta y_{t-1}} + \underset{(.068)}{.254\Delta y_{t-2}} - \underset{(.038)}{.057\Delta y_{t-3}} - \underset{(.038)}{.277\Delta y_{t-4}} + \underset{(.073)}{.076\Delta y_{t-5}} -$$

$$\underset{(.069)}{.186\Delta y_{t-6}} - \underset{(.069)}{.042\Delta y_{t-7}} - \underset{(.068)}{.311\Delta y_{t-8}} + \underset{(.066)}{.176\Delta y_{t-9}} + \hat{\varepsilon}$$

$N = 237,\ R^2 = .382,\ \sigma_\varepsilon = .334, SK = .651, EK = 1.56, LBQ(1) = .988, LBQ(12) = .94\ (ARCH(3) = 7.664(10^{-4}), LM_{BG}(1) = .943(.332), LM_{BG}(4) = .747(.561), LM_{BG}(8) = .654(.732)$

where OLS standard errors are given in parentheses below the parameters, $\hat{\sigma}_\varepsilon$ is the residual standard deviation, SK and EK are, respectively, skewness and excess kurtosis in the residuals, $LBQ(p)$ is the p-value for the Ljung-Box Q-statistic for nonstationarity of the residuals based on an autocorrelation function of an order up until and including p, (a very high p-value is indicative of 'white noise').

ARCH(q) is the Engle LM test for ARCH effects of order *q*, and $LM_{BG}(j)$ is the Breusch-Godfrey LM test for autocorrelation in the residuals up till and including lag *j*. The numbers inside the parentheses following the LM tests are the corresponding *p*-values. We note that the residuals are positively skewed, with a high kurtosis. We also note strong ARCH effects, as all ARCH tests for lags lower than 10 yield a *p*-value less than 10^{-5}. Nonetheless, the above model seems adequate in that it appears entirely free of autocorrelation.

Let us now construct the AR model for capacity utilization, denoted x_t:

$$\underset{}{\Delta x_t} = \underset{(2.303)}{4.84} - \underset{(.028)}{.060x_{t-1}} + \underset{(.068)}{.409\Delta x_{t-1}} + \underset{(.074)}{.065x_{t-2}} + \underset{(.073)}{.075\Delta x_{t-3}} - \underset{(.073)}{.205\Delta x_{t-4}} - \underset{(.072)}{.081\Delta x_{t-5}}$$

$$+\underset{(.071)}{.188\Delta x_{t-6}} - \underset{(.072)}{.053\Delta x_{t-7}} - \underset{(.071)}{.169\Delta x_{t-8}} - \underset{(.069)}{.055\Delta x_{t-9}} + \underset{(.069)}{.123\Delta x_{t-10}} + \underset{(.069)}{.003\Delta x_{t-11}} - \underset{(.066)}{.140\Delta x_{t-12}} + \hat{\varepsilon}_t$$

$N = 237,\ R^2 = .30,\ \sigma_\varepsilon = 1.52, SK = .010, EK = 3.85, LBQ(1) = .931, LBQ(12) = 1, ARCH(3) = 4.87(.0027), LM_{BG}(1) = .004(.949), LM_{BG}(4) = .210(.933), LM_{BG}(8) = .369(.936)$

The AIC recommends twelve lags for the capacity utilization series. We have arrived at results similar to those for the unemployment series, namely, that there is evidence of ARCH effects in the residuals, and the kurtosis for the residual series is very high. The latter effect may be attributed to a few high residuals in the beginning of the series. Like the unemployment series, the Breusch-Godfrey tests indicate that there is no autocorrelation, thus making the above, linear equation a suitable restricted model against which we will test for a STAR alternative. We note in passing that we attempted to include quarterly dummies in each of the AR equations in the hope of capturing the well-known seasonality properties of the two series. None of the quarterly dummies proved significant.

The second step is to conduct the sequence of LM tests for the coefficient restrictions of the reparameterized equation on page 30. As noted, the purpose of the LM tests is twofold: to determine the correct delay parameter *d*, and to infer the nonlinear form of the data. Table 3 reports the series of null hypotheses (the restriction to be tested), their corresponding specifications and degrees of freedom. For the regression equations, $\hat{u}_t$ are the estimated residuals from the linear, AR regressions. The idea is to test whether the residuals exhibit the hypothesized nonlinearity that was not captured in the AR reregressions. Table 4 presents the results from the LM tests; the values reported are *p*-values of the *F*-distribution. For the unemployment series, a delay parameter of two was rejected with the lowest *p*-value for all hypotheses, though values of eight and nine are close contenders. We suspect that a delay parameter of two is perhaps too small, in that it may not contain timely information about the regime of which the transition function is designed to take account. The results for capacity utilization are more interesting and equivocal: a delay parameter of two was rejected most strongly for H_{01}, but a delay parameter of nine was rejected about as strongly for H_{02}, in that both yielded roughly the same *p*-value. Strongest rejection of the latter hypothesis suggests that either an ESTAR or LSTAR (2) specification is needed. Therefore, we will attempt estimation of three models for capacity utilization: an LSTAR(1) model with a delay parameter of two, an LSTAR(2) model, and an ESTAR model, both including a delay parameter of nine.

Proceeding to the third step of model estimation, we have successfully estimated an LSTAR(1) model for unemployment. However, the model incorporating the delay parameter of eight provided a slightly better fit than the model in which the delay parameter of two was used:

$$\begin{aligned}\Delta y_t = {} & \underset{(.008)}{-.020 y_{t-1}} + \underset{(.086)}{.359\Delta y_{t-2}} - \underset{(.080)}{.043\Delta y_{t-3}} - \underset{(.088)}{.245\Delta y_{t-4}} - \underset{(.075)}{.046\Delta y_{t-5}} + \underset{(.079)}{.110\Delta y_{t-6}} + \underset{(.083)}{.073\Delta y_{t-7}} \\ & - \underset{(.099)}{.089\Delta y_{t-8}} + \underset{(.079)}{.052\Delta y_{t-9}} - \underset{(.084)}{.028\Delta y_{t-10}} + \underset{(.085)}{.014\Delta y_{t-11}} - \underset{(.077)}{.113\Delta y_{t-12}} + (\underset{(.385)}{1.360} - \underset{(.280)}{.711\Delta y_{t-2}} \\ & - \underset{(.429)}{.097\Delta y_{t-3}} - \underset{(.346)}{1.05\Delta y_{t-4}} + \underset{(.421)}{.487\Delta y_{t-5}} - \underset{(.336)}{1.08\Delta y_{t-6}} + \underset{(.402)}{.233\Delta y_{t-7}} - \underset{(.443)}{.333\Delta y_{t-8}} + \underset{(.406)}{.281\Delta y_{t-9}} \\ & + \underset{(.340)}{.675\Delta y_{t-10}} + \underset{(.364)}{.056\Delta y_{t-11}} - \underset{(.305)}{.291\Delta y_{t-12}} - \underset{(.241)}{.168\Delta y_{t-13}}) * \\ & \left\{1 + \exp\left[\left(\frac{-4.458}{\sigma_{\Delta_8 y_{t-1}}}\right)(\Delta_8 y_{t-1} - .603)\right]\right\}^{-1} \\ & \qquad (\mathrm{SE}_\gamma = 1.35) \qquad (\mathrm{SE}_c = .108)\end{aligned}$$

$$R^2 = .496, \quad \hat{\sigma} = .308, \quad \frac{\hat{\sigma}}{\hat{\sigma}_L} = .923, \quad p\text{JB} = 2.3 * 10^{-4} \quad p\text{LM}(1) = .30, \quad p\text{LM}(4) = .477$$

where *p*JB is the *p*-value for the Jarque-Bera test of normality of the residuals, with the null hypothesis being that the residuals are normally distributed. $\hat{\sigma}_L$ is the standard error of the linear model. $\text{LM}_{\text{AR}}(q)$ is a modified version of the Breusch-Godfrey Lagrange multiplier test of no autocorrelation of up till lag *q*. The *p*-values reported above are based on the *F*-version of the test. The null hypothesis is that there is no autocorrelation. The test is modified for nonlinear specifications, such that, instead of regressing the residuals on lagged dependent variables, they are regressed on the numerical derivatives of the left-hand-side expression with respect to each of the model's parameters (Terasvirta, et.al. (2010, pg. 382)). The left-hand expression in matrix form is:

$$\mathrm{m}(\mathrm{z}_t;\ \theta) = \psi' \mathrm{z}_t + \varphi' \mathrm{z}_t \mathrm{G}(\gamma, \mathrm{c}, \mathrm{s}_t)$$

And the derivatives on which the residuals are regressed are given by:

$$\frac{\partial \mathrm{m}(\mathrm{z}_t\ ;\ \theta)}{\partial \theta}, \forall \theta$$

where $\theta = (\psi', \varphi', \gamma, \mathrm{c})'$. The residuals are also regressed on their own lags up to order *q*.

The LSTAR(1) equation for unemployment shows significant improvement in its R^2 over its linear version, from .382 to .496 (about a 30% improvement). Further, the standard error of the LSTAR equation is about 8% less than that for the AR(9) linear specification. These results are on par with Terasvirta, et.al. (2010, pg. 406), who estimated an LSTAR(1) model for the monthly unemployment series. While the JB *p*-value remains low, it also shows significant improvement over the linear model, which yielded a value less than 10^{-7}. The JB is sensitive to violations of the well-known characteristics of the normal distribution, namely, that there are no outliers (and thus a low kurtosis) and no skewness in the distribution. It is evident that the LSTAR model has taken more account of some of the outliers in the residuals of the linear model. Finally, the modified LM tests of no autocorrelation yield satisfactory results in accepting the null.

Moving on to capacity utilization, we have found that the LSTAR(1) and ESTAR models have consistently failed, in that they invariably resulted in exploding standard errors. Estimation of STAR models becomes quite difficult when the true value of the smoothness parameter, gamma, is large. The STAR model reduces to a switching TAR model when gamma is sufficiently large. This is a common problem in STAR models, especially when the sample size is small, and is clearly indicated by an unreasonably high standard error in the estimate of gamma. There is also the issue of setting initial starting values for the coefficients, which is often required when estimating equations by nonlinear least squares. If starting values are far off the mark, the built-in algorithms will have difficulty minimizing the sum of squared residuals function. In such cases, standard errors and *t*-statistics will often either fail to converge, or find merely a local as opposed to global minimum in the SSR function.

The LSTAR(2) specification, with the delay parameter of nine as indicated by our LM tests, converged with much greater ease, and yielded the following:

$$\Delta x_t = \underset{(2.972)}{2.034} - \underset{(.036)}{.025}x_{t-1} + \underset{(.080)}{.445}\Delta x_{t-1} - \underset{(.104)}{.035}\Delta x_{t-2} - \underset{(.090)}{.004}\Delta x_{t-3} - \underset{(.098)}{.057}\Delta x_{t-4} - \underset{(.094)}{.154}\Delta x_{t-5} +$$

$$\underset{(.102)}{.116}\Delta x_{t-6} - \underset{(.104)}{.073}x_{t-7} + \underset{(.080)}{.171}\Delta x_{t-8} - \underset{(.111)}{.173}\Delta x_{t-9} - \underset{(.097)}{.011}\Delta x_{t-10} - \underset{(.085)}{.069}\Delta x_{t-11} - \underset{(.076)}{.187}\Delta x_{t-12} +$$

$$(\underset{(5.08)}{10.71} - \underset{(.063)}{.132}x_{t-1} + \underset{(.156)}{.405}\Delta x_{t-2} + \underset{(.168)}{.150}\Delta x_{t-3} - \underset{(.162)}{.416}\Delta x_{t-4} + \underset{(.152)}{.198}\Delta x_{t-5} + \underset{(.156)}{.285}\Delta x_{t-6} + \underset{(.149)}{.128}\Delta x_{t-7}$$

$$-\underset{(.102)}{.154}\Delta x_{t-8} - \underset{(.158)}{.425}\Delta x_{t-10} - \underset{(.160)}{.148}\Delta x_{t-11} + \underset{(.159)}{.144}\Delta x_{t-12}) *$$

$$\left\{1 + \exp\left[\left(\frac{-27.936}{\hat{\sigma}^2_{\Delta_9 x_{t-1}}}\right)(\Delta_9 x_{t-1} - 1.09)(\Delta_9 x_{t-1} - 3.86)\right]\right\}^{-1}$$

$$(SE_\gamma = 61.02) \quad (SE_{c_1} = .094) \quad (SE_{c_2} = .317)$$

$$R^2 = .374, \quad \hat{\sigma} = 1.49, \quad \frac{\hat{\sigma}}{\hat{\sigma}_L} = .98, \quad pJB < 10^{-7}, \quad pLM_{AR}(1) = .035, \quad pLM_{AR}(4) = .02$$

Compared to unemployment, the LSTAR(2) equation for capacity utilization showed less improvement over its AR(12) specification. The R^2 did increase from .299 to .374, about a 25% improvement. However, the standard deviation of the residuals for the nonlinear model only showed about a 2%

improvement over the linear model. The JB *p*-value remains low, and there is evidence that some autocorrelation remains in the residuals.

Figure 4 compares the residuals of the linear AR model with those of the STAR models for both the series. Panel A compares the residuals between the linear and nonlinear specifications for unemployment, Panel B for capacity utilization. It is seen that the residuals' divergence from zero peak at a lower spot than do the residuals for the linear models. This may indicate either that the strong ARCH effects or seasonality found in the linear models may have been attenuated by the transition function.

Figure 5 plots the transition function for unemployment , both over time and over different values of the transition variable. Panel A displays the latter, Panel B, the former. Panel B includes shaded areas representing NBER-dated recessions. The transition function shows some consistency in its adjustments during period of recessions. The dot plot, shown in Panel A, of the transition function is plotted against the transition variable on the horizontal axis, where each dot represents at least one observation, and reflects the monotonic increase characteristic of the stochastic process underlying the LSTAR(1) model.

Figure 6 does the same for the capacity utilization series. The graph to the left shows a situation quite different from that seen in Figure 5. It shows that there are three regimes: the white region toward the right, and the two orange regions on both sides of the white region. What this scenario indicates is that capacity utilization follows the same dynamic adjustment process when the economy is near its trough, and when it is quickly approaching its business cycle peak (the two orange regions). It follows a different process when the economy is in some intermediate state (the white region). That the white region is toward the right indicates that this different dynamic adjustment process occurs as the economy is closer to its peak than to its trough. This makes empirical sense, in that, at this region, uncertainty as to the economy's recovery process will have been alleviated as it nears the white region.

To recapitulate what the different models between the two series suggest, the LSTAR(1) model contains one constant term in the transition function, indicating that there are two regimes. This constant term is understood to be the threshold at which the series changes in its dynamic adjustment. The gamma parameter, often referred to as the 'smoothness parameter' or 'slope parameter', reflects how smoothly this change in regime takes place, with larger values of gamma indicating that the transition takes place more or less instantaneously, and smaller values suggest that transition is more gradual. The LSTAR(2) model contains two constant terms, indicating three regimes. As gamma approaches infinity, the model becomes a TAR model. The LSTAR(2) model also nests a restricted three-regime self-exciting TAR model (SE-TAR), where the restriction is that the linear models in the outer regimes are identical.

This chapter will close with a few observations on the STAR model process. While STAR models are, of course, not without their faults, their power lies in their being able to model how macroeconomic time series vary according to different states in the economy according to their transition variables. This power is augmented by the fact that regime-changes need not be discrete (that is, marked by a zero or one in an indicator function, as is the case with switching TAR models), and no *a priori* information as to the timing of the regime need be available. Also, there is flexibility in the number of regimes by which the data-generating process is governed. There is also considerable flexibility available to the researcher in choosing the transition variable, which can be any variable or a combination of variables. However, some common sense must be exercised in that the transition variable must contain timely and accurate information about the regimes. The LM tests for linearity are by no means flawless, in that linearity may well be rejected equally decisively with several transition variable candidates. The protocol in this case is to experiment with the different transition variables, and compare their relative fits. Nonetheless, the series of LM tests will often prove to yield valuable information – as they had in our case – as to the nonlinear nature of the variable.

Perhaps the chief disadvantage of STAR models is that they are extremely sensitive to specification error. This makes them generally quite difficult to estimate. Nonlinear least squares estimation will often fail to converge if the transition variable is misspecified, or, if it does converge, it will often do so at a local as opposed to global minimum of the sum of squared residuals function, in which case, neither the coefficient estimates, standard errors, nor *t*-statistics will be asymptotically valid. This is the most difficult feature of nonlinear least squares estimation. Often, it will be necessary to carry out a grid search for the two parameters of the transition function, and iterate estimation of the AR parameters using starting values for the γ and c until the sum of squares are minimized. For an intuitive discussion, see Griffiths, Hill, and Judge (1993). Clearly, there is the problem of how to choose the initial values, which are usually far from obvious. In addition, in the case where there are many parameters for lag variables, having redundant variables may confound results, and there is often no *a priori* knowledge as to which of the lag terms are redundant. Maximum likelihood is an alternative method. However, ML is sensitive to the assumption of white noise error terms, for which STAR models may not be adequate in the first place.

When estimating STAR models, one should first look at the standard errors to make sure they are reasonable. If they are too high, they are likely to explode as redundant variables are successively deleted, which would then render the entire model undefined. It is common practice (for instance, van Dijk, et.al. (2002)), to identify and then successively delete redundant parameters when their *t*-statistics are less than one in absolute value

CHAPTER VI

CONCLUSION

This thesis has compared the local dynamics of the U.S. rates of unemployment and capacity utilization over different phases of the business cycle. To this end, we have successfully modeled the two series via the STAR specification of Terasvirta (1994). These specifications allow for regime changes – that is, changes in the dynamic properties according to broader economic influences – as well as for a continuous transition process. It has been demonstrated that the asymmetry phenomenon is ever-present in unemployment in the context of business cycles, though its severity varies according to the depth of the trough and the height of the peak. It has also been demonstrated that there is more inter-regime symmetry in the adjustment of capacity utilization, with two identical processes within the two outer regimes of strong growth and recession, but different within the intermediate regime of slow to moderate growth.

Our results for the unemployment series are on par with previous work, including that of Skalin and Terasvirta (2002), van Dijk. et. al. (2002), and Terasvirta, Tjostheim, and Granger (2010). This thesis makes a contribution in modeling U.S. capacity utilization for the first time in a STAR context. It was found that capacity utilization is best characterized by a three-regime, LSTAR(2) process, whereby the local dynamics of the two outer regimes are more or less identical, yet different from the middle regime. This finding may prove to be the basis of future research in comparing how input markets react to broader economic influences.

Our STAR models for the two series indicate that capacity utilization reacts more quickly to shocks than unemployment does and that the former exhibits the same local behavior at both the peak and trough of the business cycle, but different behavior in the intermediate phase. This is empirical evidence that capacity utilization is governed by a three-regime process. By contrast, unemployment follows a two-regime process, with different local adjustment depending on whether the economy is expanding or contracting. In terms of the stationary versus nonstationary dichotomy, which has long been operative in macroeconomic research, these results cast the dichotomy in a different light. It was shown in Chapter 4 that, over the sample period of 1948-2009, unemployment and capacity utilization are stationary processes. The mean-reversion feature by which the stationary process is characterized actually reflects the two series' reversion to their local, regime-dependent equilibrium, from which it then diverges to a new equilibrium when the economy switches to a different regime, *ceteris paribus*. The stationarity of the two series as verified by Dickey-Fuller regressions is a reflection of the cyclical nature of this multiple equilibrium process, as opposed to a single equilibrium to which the series reverts with varying speed in the event of a shock. This, we speculate, may be the reason why the results of the stationarity tests yielded different conclusions for different time periods. This may also explain why linear tests for cointegration between two persistent (fractionally integrated) variables like unemployment and capacity utilization, tend to suggest either spurious OLS estimates (in the case of Engle and Granger (1987)), or spurious cointegration (in the case Johanssen (1995)).

In closing, it is worth praising STAR models for their flexibility. There is, however, a trade-off between their flexibility and their difficulty, the latter occurring primarily at the nonlinear estimation stage. They have been largely implemented in a univariate setting, though multivariate STAR models are as of now a burgeoning field. This thesis also demonstrates that univariate STAR models may be a useful alternative to methods of finding nonlinear cointegration, which remains a nascent field in macroeconometrics.

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TABLES

Table 1:
Augmented Dickey-Fuller Regression Results

Table 1: Results of ADF Regressions of Unemployment and Capacity Utilization

		Unemployment				Capacity Utilization			
		Information Criterion				Information Criterion			
		AIC		SIC		AIC		SIC	
Sample Period	# of Years in Sample	Lag Length	τ-Stat	Lag Length	τ-Stat	Lag Length	τ-Stat	Lag Length	τ-Stat
1948:Q1-1952:Q4	5	1	-1.56	1	-1.56	4	-0.68	1	-1.5
1948:Q1-1957:Q4	10	2	-3.38**	2	-3.38**	9	-1.99	1	-2.86*
1948:Q1-1967:Q4	20	9	-1.95	1	-3.38**	6	-2.20	1	-3.36**
1948:Q1-1987:Q12	40	9	-2.02	1	-3.17**	6	-3.25**	1	-4.18***
1948:Q1-2007:Q4	60	12	-2.69*	1	-3.73***	12	-2.73*	1	-4.63***

Notes: AIC is the Akaike Info Criterion

SIC is the Schwarz Info Criterion

Critical Values are adapted from Fuller (1976, pg. 373)

* refers to rejection of null of nonstationarity at the 10% level

** at 5%

*** at 1%

Table 2:
Engle-Granger Tests of Cointegration Results*

$$\Delta\hat{e_t} = \alpha + \beta\widehat{e_{t-1}} + \sum_{i=1}^{k}\delta_i\widehat{\Delta e_{t-\iota}} + \varepsilon_t$$

Dependent Variable	β	t-Stat	D-W	σ	Mean	Median	Skewness	Kurtosis
				ε				
Capacity Utilization	-0.067	-2.223	2	3.98	-8.63E-16	1.04	-0.64	3.03
Unemployment	-0.054	-2.38	2.02	1.18	1.99E-15	0.012	0.11	2.76

*Critical Values taken from MacKinnon (1991)

10% Critical Value:	-3.4959
5%	-3.7809
1%	-4.3266

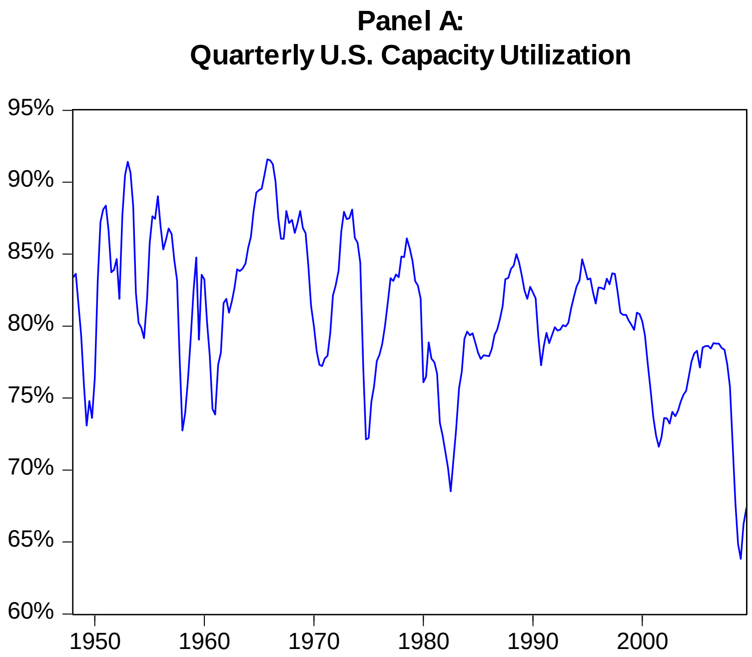
Table 3:
Template for Lagrange Multiplier Tests of Linearity

Hypothesis	Regression Equation	Chi-Square Degrees of Freedom	F-Distribution Degrees of Freedom
$H_0: \beta_1 = \beta_2 = \beta_3 = 0$	$\hat{u}_t = \beta_0' z_t + \sum_{j=1}^{3} \beta_j' z_t s_t^j + \varepsilon_t$	$3(p+1)$	$3p \;/\; T - 4p - 1$
$H_{03}: \beta_3 = 0$	$\hat{u}_t = \beta_0' z_t + \sum_{j=1}^{3} \beta_j' z_t s_t^j + \varepsilon_t$	$p+1$	$p \;/\; T - p - 1$
$H_{02}: \beta_2 = 0 \vert \beta_3 = 0$	$\hat{u}_t = \beta_0' z_t + \sum_{j=1}^{2} \beta_j' z_t s_t^j + \varepsilon_t$	$2(p+1)$	$2p \;/\; T - 2p - 1$
$H_{01}: \beta_1 = 0 \vert \beta_2 = \beta_3 = 0$	$\hat{u}_t = \beta_0' z_t + \beta_1' z_t s_t + \varepsilon_t$	$3(p+1)$	$3p \;/\; T - 4p - 1$

Table 4:
Results of LM Tests for Linearity

Table 4: LM Tests for STAR Nonlinearity for Quarterly Unemployment and Capacity Utilization: 1948Q1-2009Q3								
Transition Variable	Quarterly Unemployment Series				Quarterly Capacity Utilization Series			
$\Delta_d y_{t-d}$,	Hypothesis				Hypothesis			
$d =$...	H_0	H_{03}	H_{02}	H_{01}	H_0	H_{03}	H_{02}	H_{01}
2	$2.00*10^{-19}$	$5.5*10^{-10}$	$1.36*10^{-12}$	$8.55*10^{-10}$	$1.19*10^{-28}$	$4.61*10^{-16}$	$2.99*10^{-13}$	$2.3*10^{-5}$
3	$3.99*10^{-11}$	$8.62*10^{-6}$	$1.86*10^{-5}$	.436	$1.48*10^{-19}$	$1.49*10^{-10}$	$7.53*10^{-9}$	$2.8*10^{-5}$
4	$2.06*10^{-13}$	$6.67*10^{-7}$	$2.64*10^{-5}$	.0110	$2.55*10^{-20}$	$5.42*10^{-11}$	$3.26*10^{-11}$	$1.4*10^{-9}$
5	$8.96*10^{-7}$	$9.27*10^{-4}$	$6.65*10^{-9}$	.994	$2.61*10^{-21}$	$1.44*10^{-11}$	$3.44*10^{-8}$	$1.9*10^{-4}$
6	$5.07*10^{-12}$	$3.19*10^{-6}$	$4.67*10^{-6}$	.153	$4.06*10^{-24}$	$3.07*10^{-13}$	$4.54*10^{-10}$	$9.3*10^{-6}$
7	$1.67*10^{-13}$	$6.01*10^{-7}$	$7.95*10^{-8}$	.128	$1.41*10^{-16}$	$6.97*10^{-9}$	$5.11*10^{-8}$	.379
8	$4.31*10^{-16}$	$3.01*10^{-8}$	$3.29*10^{-7}$	.0231	$4.19*10^{-19}$	$2.7*10^{-10}$	$2.61*10^{-7}$	.023
9	$8.93*10^{-16}$	$4.37*10^{-8}$	$2.09*10^{-7}$	.201	$6.1*10^{-28}$	$1.31*10^{-15}$	$3.34*10^{-16}$	$4.3*10^{-7}$
10	-	-	-	-	$2*10^{-23}$	$8.04*10^{-13}$	$2.27*10^{-16}$	.432
11	-	-	-	-	$5.75*10^{-26}$	$2.26*10^{-14}$	$2.37*10^{-12}$	.691
12	-	-	-	-	$8.29*10^{-27}$	$6.76*10^{-15}$	$3.58*10^{-13}$	.099
d value with smallest *p*-value	**2**	**2**	**2**	**2**	**2**	**2**	**9**	**4**

Figure 1:
Quarterly U.S. Unemployment and Capacity Utilization



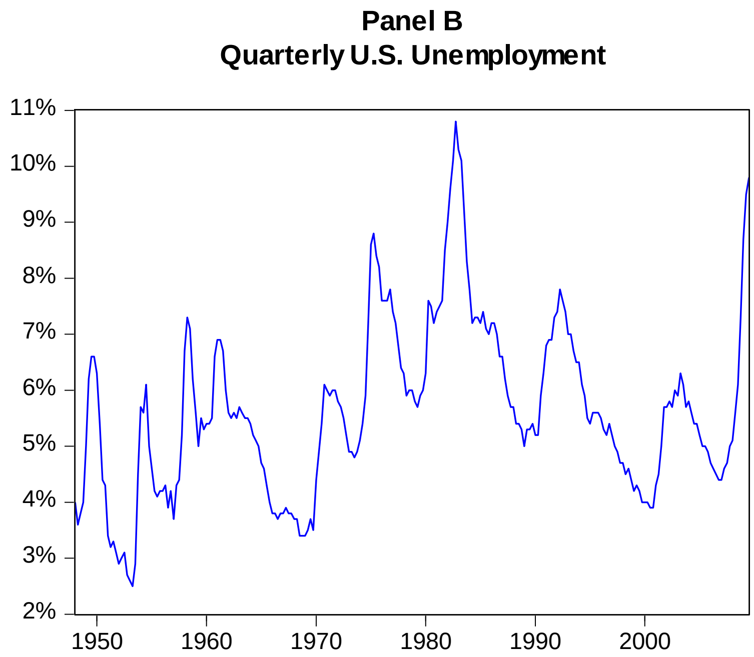
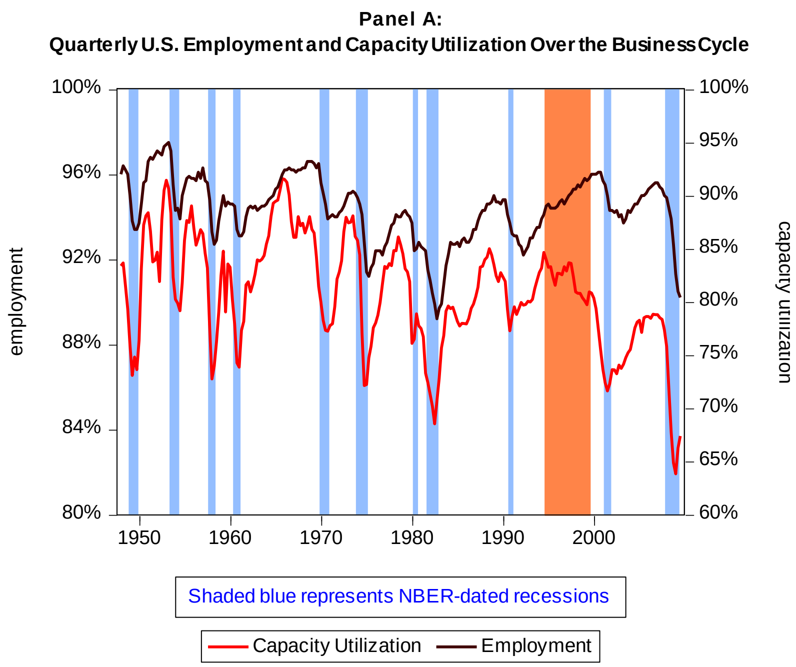


Figure 2:
(Un)employment and Capacity Utilization:
Over Time and Over the Business Cycle



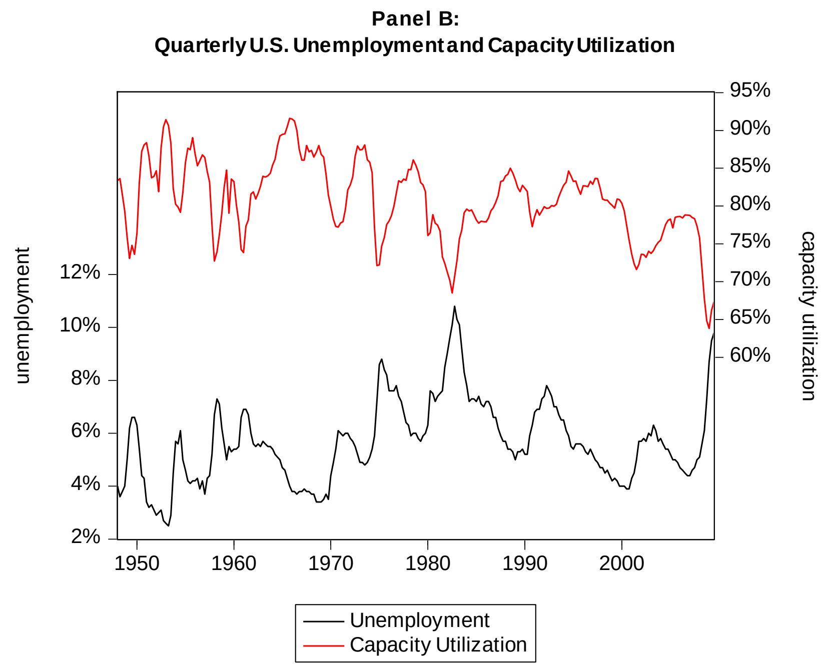
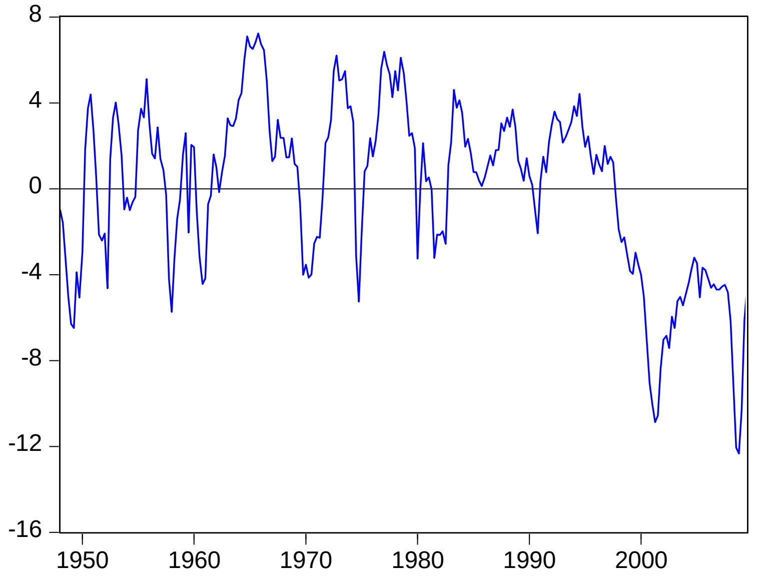


Figure 3:
Residuals of Engle-Granger Test

**Panel A:
Residuals of Engle-Granger Test:
Dependent Variable: Capacity Utilization**



**Panel B:
Residuals of Engle-Granger Test
Dependent Variable: Unemployment**

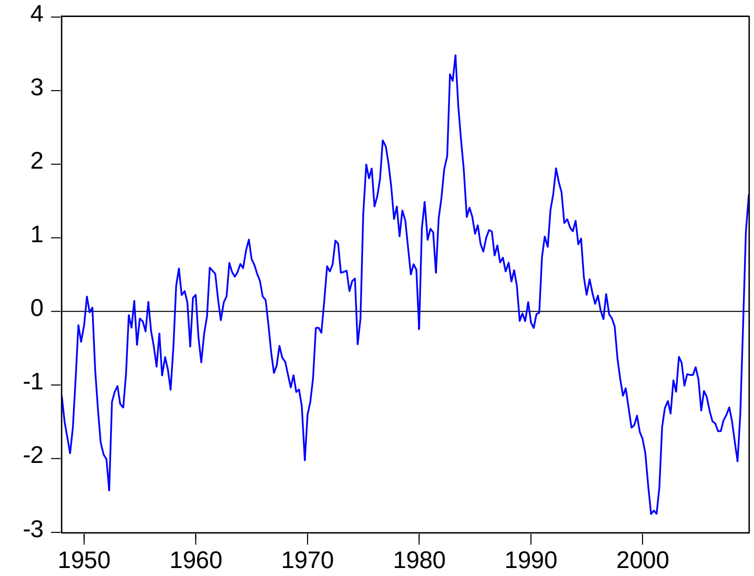
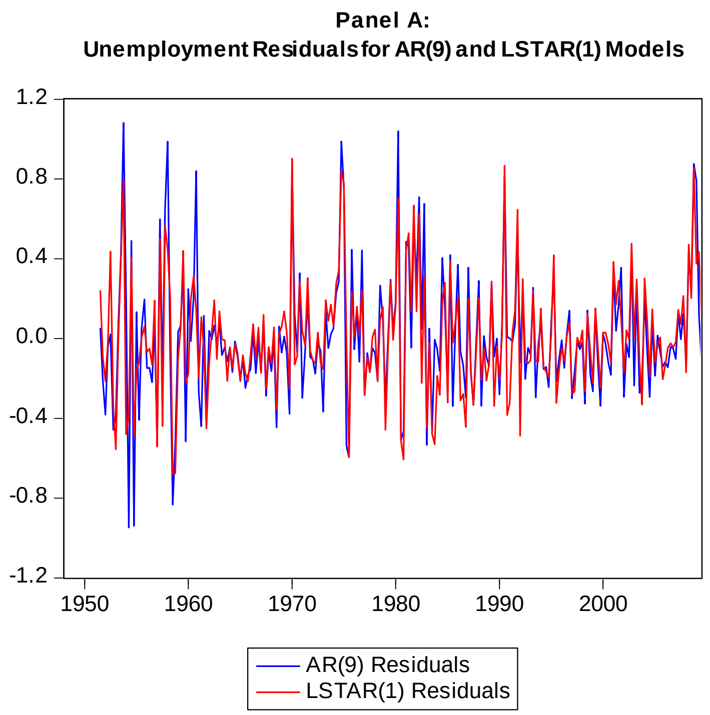


Figure 4:
Comparison of Residuals between AR and LSTAR(q) models



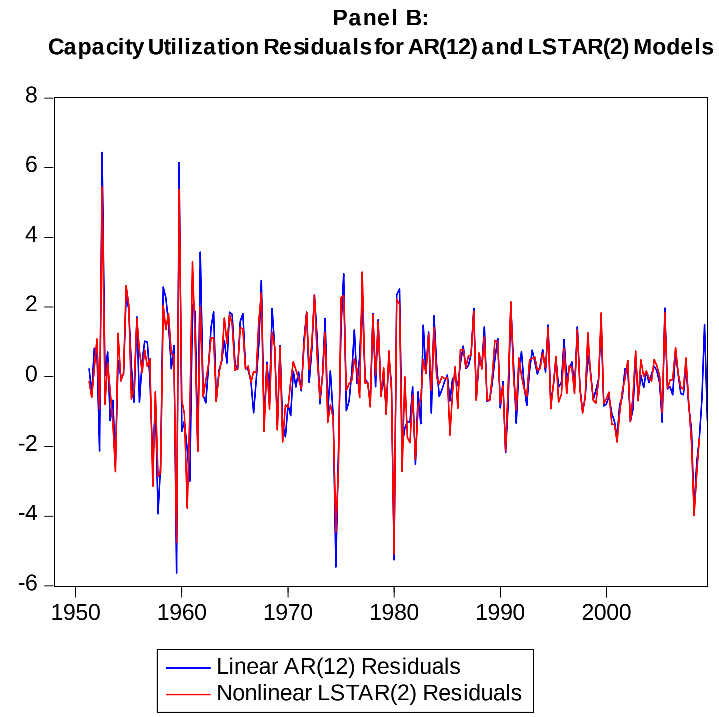
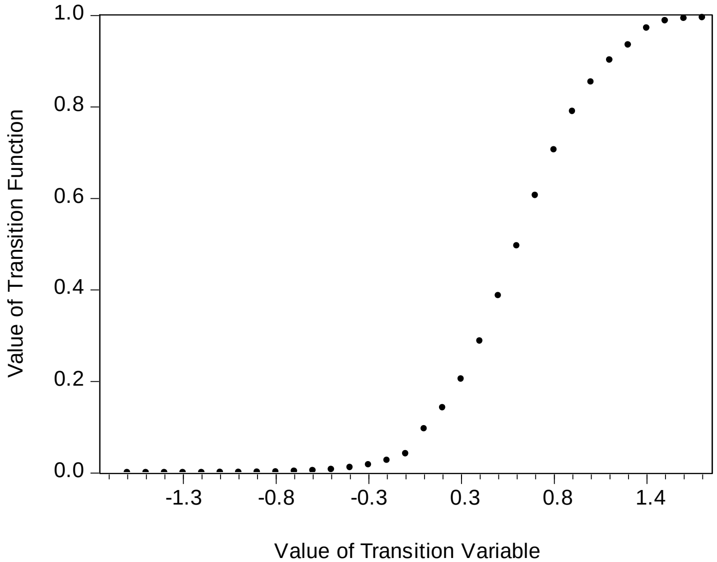


Figure 5:
Transition Function for the LSTAR model of Unemployment:

Panel A:
Transition Function of U.S. Quarterly Unemployment
Graphed Against Transition Variable



Panel B:
Transition Function of U.S. Quarterly Unemployment
Graphed Against Time

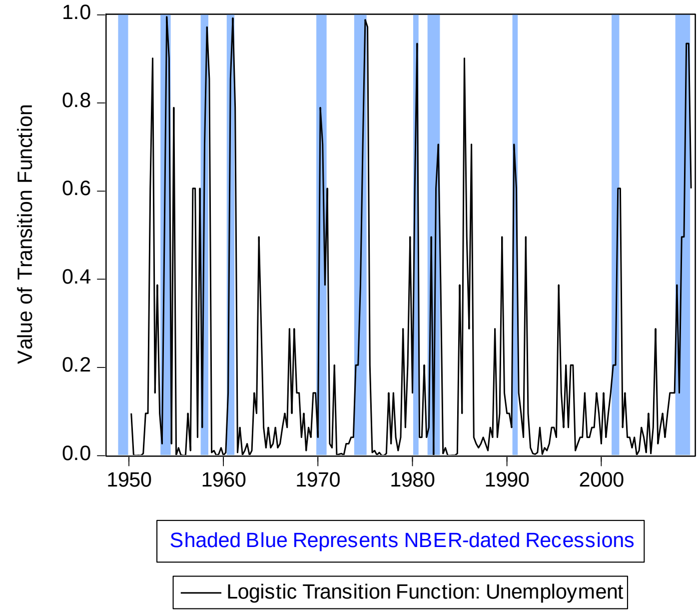
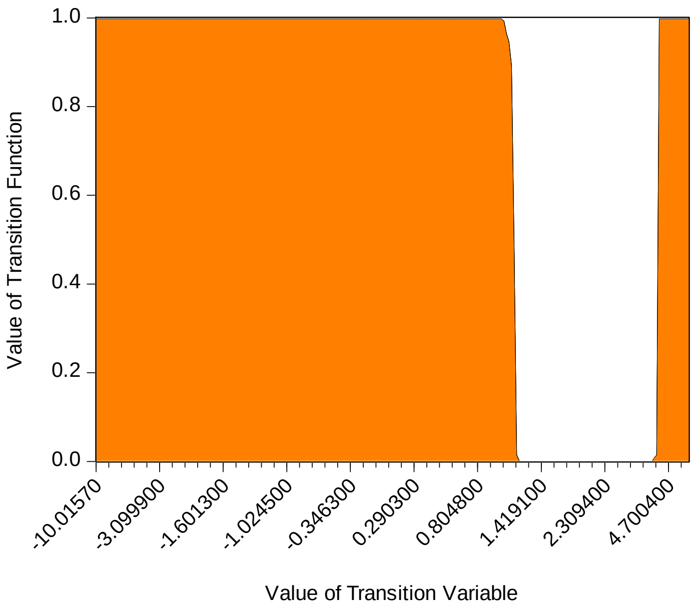


Figure 6:
Transition Function for the LSTAR(2) model of Capacity Utilization:

Panel A:
Transition Function of U.S. Quarterly Capacity Utilization
Graphed Against Transition Variable



Panel B:
Transition Function of U.S. Quarterly Capacity Utilization
Graphed Against Time