Three Essays on Time Series Econometrics Analysis and Financial Market Applications

by

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Abstract

This dissertation first revisits the mean reversion properties of relative stock prices when the US and the UK sever as reference countries. We present strong evidence of nonlinear mean reversion of the stock price indices of OECD countries relative to that of the UK. However, the panel nonlinear unit root test failed to reject the null of nonstationarity even when the UK served as the reference country. The results appear inconsistent. Via principal component analysis and extensive Monte Carlo simulations, we demonstrate a potential pitfall in using panel unit root tests with cross-section dependence when a stationary common factor dominates nonstationary idiosyncratic components in small samples. We believe the empirical findings in this chapter provide useful implications for international asset market participants.

In the second chapter, we studied the impact of exchange rate shock on the commodity prices using VAR (Vector Autoregressive) Model, a forecasting technique in time series analysis. We report that first, the long-run adjustment of prices is very slow. Prices typically take 8 to 12 months to stabilize except for the oil prices which stabilize in about 4 months. Second, the responses of commodities are heterogeneous. Some commodities, like wheat, cocoa beans, beef, pork, chicken, bananas, oranges, and soft wood, under-correct, i.e. the price elasticities of these commodities are less than one. Others, like corn, lamb, sugar, hide, and crude oil adjust on par with the exchange rate movement. Finally, the prices of the commodities like barley, peanuts, rice, sunflower oil, olive oil, rubber, aluminum, nickel, and coal, over-correct. This might call for price stabilization policy implications especially for the developing countries.

The third chapter deals with the ordering of recursively identified VAR models and reports potentially useful facts that show under what circumstances these impulse response

functions are robust to this so-called Wold ordering. This adds important technical contributions to the existing multivariate time series model literature.

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Chapter 1

Introduction

In international finance it is important to know whether relative prices are mean reverting. If so, one can diversify international portfolios by short-selling well performing assets and purchasing poorly performing assets to obtain excess returns as shown by Balvers et al. (2000). Such a strategy is called the contrarian investment strategy, and it may imply that stocks become less risky in the long run and are attractive for long-term investors (Spierdijk et al. (2012)). On the contrary, if deviations are permanent, one should short worse performing assets while buying better performing ones, because winner-loser reversals are not likely to happen. This is called the momentum strategy.

In the first chapter we present strong evidence of nonlinear mean reversion of the stock price indices of OECD countries relative to that of the UK. However, the panel nonlinear unit root test failed to reject the null of nonstationarity even when the UK served as the reference country. The results appear inconsistent. Via principal component analysis and extensive Monte Carlo simulations, we demonstrate a potential pitfall in using panel unit root tests with cross-section dependence when a stationary common factor dominates nonstationary idiosyncratic components in small samples. We believe the empirical findings in this chapter provide useful implications for international asset market participants.

In the second chapter, We studied the impact of exchange rate shock on the commodity prices using VAR (Vector Autoregressive) Model, a forecasting technique in time series analysis. We report that first, the long-run adjustment of prices is very slow. Prices typically take 8 to 12 months to stabilize except for the oil prices which stabilize in about 4 months. Second, the responses of commodities are heterogeneous. Some commodities, like wheat, cocoa beans, beef, pork, chicken, bananas, oranges, and soft wood, under-correct, i.e. the price elasticities of these commodities are less than one. Others, like corn, lamb, sugar, hide, and crude oil adjust on par with the exchange rate movement. Finally, the prices of the commodities like barley, peanuts, rice, sunflower oil, olive oil, rubber, aluminum, nickel, and coal, over-correct. This might call for price stabilization policy implications especially for the developing countries.

The third chapter discusses potentially useful facts about the robustness of the IRF. Beard et al. (2012) investigated the effect of an increase in kidney donations from deceased donors on those from live donors. They show that the responses of live donantion and waiting times to the deceased donation are invariant to the ordering of live donation and waiting time, when the deceased donation is ordered first. We extend these results to a more general framework. We report potentially useful facts that show under what circumstances these impulse response functions are robust to this so-called Wold ordering. This adds important technical contributions to the existing multivariate time series model literature.

Chapter 2

Nonlinear Mean Reversion across National Stock Markets

2.1 Introduction

It is an interesting question in international finance whether asset arbitrage in international stock markets implies that deviations between stock indices are short-lived. If so, one can diversify international portfolios by short-selling well performing assets and purchasing poorly performing assets to obtain excess returns as shown by Balvers et al. (2000). Such a strategy is called the contrarian investment strategy, and it may imply that stocks become less risky in the long run and are attractive for long-term investors (Spierdijk et al. (2012)). On the contrary, if deviations are permanent, one should short worse performing assets while buying better performing ones, because winner-loser reversals are not likely to happen. This is called the momentum strategy.

Since the end of the 1980s, a lot of research work has examined mean reversion in international stock markets. Fama and French (1988) and Poterba and Summers (1988) were the first to provide the evidence in favor of mean reversion. Fama and French state that 25-40% of the variation in 3-5 year stock returns can be attributed to negative serial correlation. Poterba and Summers (1988) found that a substantial part of the variance of the US stock returns is due to a transitory component. However, Richardson and Smith (1991) showed that if the small-sample bias is controlled, there is be no evidence for long-term mean reversion. Kim et al. (1991) report very weak evidence of mean reversion in the post-war era. Jegadeesh (1991) shows that mean reversion in stock prices is entirely concentrated in January.

An array of researchers investigated possible cointegration properties of the stock indices and their fundamental variables. For example, Campbell and Shiller (2001) examine the mean-reverting behavior of the dividend yield and price-earnings ratio over time. If stock prices are high in comparison to company fundamentals, it is expected that adjustment toward an equilibrium will be made. They find that stock prices contribute most to adjusting the ratios towards an equilibrium level.

Balvers et al. (2000) considered relative stock price indices of eighteen OECD countries compared to a world index to get around the difficult task of specifying a fundamental or trend path. Under the assumption that the difference between the trend path of one country's stock price index and that of a reference index is stationary, and that the speeds of mean reversion in different countries are similar, they found substantial evidence of mean reversion of relative stock price indices with a half-life of approximately 3.5 years. Similar evidence has been reported by Chaudhuri and Wu (2004) for 17 emerging equity markets.

The assumption of a constant speed of mean reversion may be too restrictive, however, since the speed of mean reversion may depend on the economic and political environment, and also it may change over time. For example, Kim et al. (1991) conclude that mean reversion is a pre-World War II phenomenon only. Poterba and Summers (1988) find that the Great Depression had a significant influence on the speed of mean reversion. Additionally, their panel unit root test may have a serious size distortion problem in the presence of cross-section dependence (Phillips and Sul (2003)). Controlling for cross-section dependence, Kim (2009) reports much weaker evidence of mean reversion of relative stock prices across international stock markets.

In recent work, Spierdijk et al. (2012) employed a wild bootstrap method to get the median unbiased estimation and a rolling window approach to a long horizon data (1900-2009) for their analysis. They find that stock prices revert more rapidly to their fundamental value in periods of high economic uncertainty, caused by major economic and political events such as Great Depression and the start of World War II. They report a statistically significant mean reversion for most of their sub-sample periods, but their panel test results don't seem to match their univariate test results very well.[1]

Wälti (2011) studied the relationship between stock market co-movements and monetary integration. He reports that greater trade linkages and stronger financial integration contribute to greater stock market co-movements.[2]

In the present paper, we revisit the findings by Balvers et al. (2000). We re-examine the mean reversion of the relative stock price in international stock markets by using nonlinear unit root tests in addition to linear tests. Nonlinear models have been widely used in the study of financial data including exchange rates to account for state-dependent stochastic behavior due to market frictions such as transaction costs; examples include Obstfeld and Taylor (1997), Sarno et al. (2004), Lo and Zivot (2001), Sarno et al. (2004), Kim and Moh (2010) and Lee and Chou (2013) and to the study of commodity prices (for example, Balagtas and Holt (2009), Holt and Craig (2006), and Goodwin et al. (2011)) to address nonlinear adjustments towards the equilibrium due to costly transactions, government interventions, or different expectations by individuals (Arize (2011)).

Using a nonlinear unit root test (ESTAR), we find strong evidence of nonlinear mean reversion of relative stock prices when the UK serves as the reference country. We find very little evidence of linear mean reversion irrespective of the choice of the reference index. In addition, we employ a series of panel unit roots tests: the linear panel unit root test (Pesaran (2007)) and a newly developed nonlinear panel (PESTAR) unit root test (Cerrato et al. (2011)). These tests allow different mean reversion rates across countries and also allow for cross sectional dependence. Thus, our approach is less restrictive than Balvers et al. (2000) and should give more statistically reliable results.

[1]For example, with the US benchmark, only France shows mean reversion with the univariate test but there is a solid stationarity with the panel test.

[2]Also the author concludes that lower exchange rate volatility and joint EMU membership are associated with stronger stock market comovements. The joint significance of these two variables indicates that monetary integration raises return correlations by reducing transaction costs coming from exchange rate uncertainty, and through the common monetary policy and the convergence of inflation expectations leading to more homogeneous valuations.

We find no evidence of mean reversion from these panel unit root tests, a result which seems to be inconsistent with the univariate ESTAR test results with the UK as the reference index that provide strong evidence of mean reversion. To look into this seemingly conflicting statistical result, we conducted a principal component analysis via the PANIC method developed by Bai and Ng (2004). We note empirical evidence of stationarity of the estimated first common factor or cross-section means that served as proxy variables for the common factor in Pesaran (2007) and Cerrato et al. (2011) with the UK reference. When the stationary first common factor dominates idiosyncratic components that are quite persistent or even nonstationary, the panel unit root tests that filter out the stationary common factor may yield evidence against stationarity in the short-run, while the univariate test rejects the null of nonstationarity. Via Monte Carlo simulations, we confirm this conjecture.

In sum, our findings imply that the contrarian investment would be useful when national equity prices deviate sufficient from the UK stock index, while one may employ the momentum strategy with the US as a reference.

The rest of the paper is organized as follows. Section 2 constructs our baseline model of the relative stock indices. Sections 3 and 4 report univariate and panel unit root test results, respectively. Section 5 discusses our results using a dynamic factor analysis framework. Section 6 establishes and provides simulation results. Section 7 concludes.

2.2 The Baseline Model

We use a model of a stochastic process for national stock indices, employed in Kim (2009), that is a revised model of Balvers et al. (2000).

Let $p_{i,t}$ be the the national stock index and $f_{i,t}$ be its fundamental value in country i, all expressed in natural logarithms. We assume that $p_{i,t}$ and $f_{i,t}$ obey nonstationary stochastic processes. If $p_{i,t}$ and $f_{i,t}$ share a *unique* nonstationary component, deviations of $p_{i,t}$ from $f_{i,t}$ must die out eventually. That is, $p_{i,t}$ and $f_{i,t}$ are cointegrated with a known cointegrating vector $[1 \quad -1]$. Such a stochastic process can be modeled by the following error correction model.

$$\Delta(p_{i,t+1} - f_{i,t+1}) = a_i - \lambda_i(p_{i,t} - f_{i,t}) + \varepsilon_{i,t+1}, \tag{2.1}$$

where $0 < \lambda_i < 1$ represents the speed of convergence and $\varepsilon_{i,t}$ is a mean-zero stochastic process from an *unknown* distribution. The fundamental value $f_{i,t}$ is not directly observable, but is assumed to obey the following stochastic process:

$$f_{i,t} = c_i + p_{w,t} + \upsilon_{i,t}, \tag{2.2}$$

where c_i is a country-specific constant, $p_{w,t}$ denotes a reference stock index price, and $\upsilon_{i,t}$ is a zero-mean, possibly *serially correlated* stationary process from an *unknown* distribution.

Combining (4.1) and (4.12) and after controlling for serial correlation, we obtain the following augmented Dickey-Fuller equation for the relative stock price, $r_{i,t} = p_{i,t} - p_{w,t}$, for country i.

$$r_{i,t} = \alpha_i + \rho_i r_{i,t-1} + \sum_{j=1}^{k} \beta_{i,j} \Delta r_{i,t-j} + \eta_{i,t}, \tag{2.3}$$

That is, $r_{i,t}$ measures deviations of the stock index in country i from a reference index at time t. Note that $\rho_i \in (0,1)$ is the persistence parameter of the stock index deviation for country i.

It is easy to see that (4.5) is equivalent to Equation (4) in Balvers et al. (2000). It should be noted, however, that (4.5) does not require the homogeneity assumption for the convergence rate λ.[3] Furthermore, we do not need to impose any distributional assumptions on η_t.[4]

[3]In order to derive (4) in Balvers et al. (2000) from (1), one has to assume $\lambda^i = \lambda^w$ where w refers to the reference country. Otherwise, the unobserved term P^{*i}_{t+1} in their equation (1) cannot be cancelled out and remains in their estimation equation.

[4]Balvers et al. (2000) use Andrews (1993)'s methodology to calculate the median unbiased estimates and the corresponding confidence intervals, which requires Gaussian error terms.

2.3 Univariate Unit Root Tests

2.3.1 Data

Following Balvers et al. (2000), we use a panel of yearly observations of the Morgan Stanley Capital International (MSCI) stock price indices for 18 Developed Market group countries during the period 1969 to 2012 to test for mean reversion. The observations are end-of-period (December) value-weighted gross index prices in US dollar terms that include dividends. Table 1 provides summary statistics for the deviations of the logarithm of the relative stock indices of 17 countries to the two reference countries, US and UK respectively.

The mean values of the index deviations relative to the US index range from -0.981 for Italy to 1.583 for Hong Kong, and the standard deviations vary from 0.235 for UK to 0.709 for Japan. The mean values of the stock index deviations relative to the UK index range from -1.252 for Italy to 1.312 for Hong Kong, and the standard deviations vary from 0.235 for US to 0.639 for Japan. We also checked the normality of the data using the Jarque-Bera test. The test rejects the null hypothesis of normality at the 5% significance level for 3 and 6 countries with the US index and with the UK index, respectively.[5]

In the following two subsections for univariate tests we will drop country specific index i in the formulas for notational simplicity.

2.3.2 Linear Unit-Root Test Analysis

We first implement univariate linear unit root tests employing the following conventional augmented Dickey-Fuller (ADF) test.

$$r_t = \alpha + \psi t + \rho r_{t-1} + \sum_{j=1}^{k} \beta_j \Delta r_{t-j} + \eta_t, \tag{2.4}$$

[5] The Jarque-Bera test tends to reject the null hypothesis more often for higher frequency financial data. The test unanimously rejects the null of normality when we use the monthly frequency data. All results are available from authors upon requests.

where $\psi = 0$ for the ADF test with an intercept only. We implemented the test for the deviations of the logarithms of national stock price indices relative to that of the reference country (US or UK). Results are reported in Table 2.

When the US index serves as the reference, the test rejects the null of nonstationarity for 6 out of 17 countries at the 10% significance level when an intercept is included (Belgium, France, Germany, Hong Kong, Norway, and the UK). Allowing for trend stationarity, the test rejects for one additional country (Sweden) at the 5% level. When the UK index is used as the reference, the test rejects the null for 6 out of 17 countries when an intercept is included. Allowing the time trend, the test rejects for 3 additional countries (Italy, the Netherlands, and Sweden).

A rejection of the null hypothesis of nonstationarity implies that the national stock index tends to synchronize with that of the reference country, because deviations of the stock price from the reference index are not permanent. That is, short-selling a better-performing stock index and buying the other would generate financial gains on average. Put differently, stationarity of r_t suggests that a contrarian strategy would perform well for the pair of the national stock index and the reference index.

Confining our attention only to such linear piecewise convergence, our findings imply limited evidence in favor of the contrarian strategy, even though we observe a little stronger evidence using the contrarian strategy when the UK index serves as the reference.

2.3.3 Nonlinear Unit-Root Test Analysis

It is known that the linear ADF test has low power when the true data generating process (DGP) is nonlinear. One way to get around this difficulty is to use a nonlinear unit-root test. For this purpose, we revise the linear model (2.4) to a nonlinear model by allowing transitions of the stock price deviation r_t between the stationary and the nonstationary regime. Stock prices may adjust to long-run equilibrium only when the deviation is big enough in the presence of a fixed transaction cost. Then, r_t may follow a unit root process locally around the long-run equilibrium value. We employ a variation of such stochastic processes that allows gradual transitions between the regimes. Specifically, we assume the following exponential smooth transition autoregressive process for r_t.

$$r_t = r_{t-1} + \xi r_{t-1}\{1 - \exp(-\theta r_{t-d}^2)\} + \epsilon_t, \tag{2.5}$$

where θ is a strictly positive scale parameter so that $0 < \exp(-\theta r_{t-d}^2) < 1$, and d is a delay parameter. Note that when r_{t-d} is very big, that is, when national stock price indices substantially deviate from the reference index, $\exp(-\theta r_{t-d}^2)$ becomes smaller, converging to 0, which implies that the stochastic process (2.5) becomes a stationary AR(1) process $(1 + \xi = \rho < 1)$. On the other hand, if r_{t-d} is close to zero, then r_t becomes a unit root process. Alternatively, (2.5) can be rewritten as,

$$\Delta r_t = \xi r_{t-1}\{1 - \exp(-\theta r_{t-d}^2)\} + \epsilon_t, \tag{2.6}$$

Note that ξ is not identified under the unit root null hypothesis, which results in the so-called "Davies Problem." To deal with it, Kapetanios et al. (2003) transformed it as follows (assuming $d = 1$):

$$\Delta r_t = \delta r_{t-1}^3 + \epsilon_t \tag{2.7}$$

using the Taylor series expansion. They show that, under the unit root null, the least squares t-statistic for δ has the following asymptotic distribution

$$\frac{\frac{1}{4}W(1)^2 - \frac{3}{2}\int_0^1 W(1)^2\,ds}{\sqrt{\int_0^1 W(1)^6\,ds}} \tag{2.8}$$

where $W(z)$ is the standard Brownian motion defined on $s \in [0,1]$. When error terms (ε_t) are serially correlated, equation (8) can be augmented as follows

$$\Delta r_t = \delta r_{t-1}^3 + \sum_{j=1}^{k} \beta_j \Delta r_{t-j} + \epsilon_t. \tag{2.9}$$

We tested the data for both when an intercept is included and when an intercept and time trend are included. Results are shown in Table 3.

With the US index, the test rejects the null hypothesis of nonstationarity only for two countries, Hong Kong and the UK. With the UK as the reference country, however, the test rejects the null hypothesis for 10 countries at the 10% significance level. Allowing a time trend, the test rejects the null for an additional 2 countries, the Netherlands and Sweden. In combination with the results from the linear test results, our empirical findings yield a maximum of 14 rejections out of 17 countries at the 10% significance level, while we obtained a maximum 7 rejections out of 17 when the US serves as a reference country.[6] These findings imply that the UK stock index may be used as an anchor index in constructing international equity portfolios. When deviations of national equity indices from the UK index are large, one may short better performing assets while buying worse performing assets, since winner-loser reversals are likely to happen. When the US stock index serves as the reference, one should employ the momentum strategy because deviations of equity prices seem to be permanent.

2.4 Panel Unit Root Tests with Cross-Section Dependence Consideration

It is known that the univariate ADF test has low power in small samples. In this section we employ a series of panel unit root tests that are known to increase power over the univariate tests (Taylor and Sarno (1998)).

[6]Note that the linear test shows the relative prices of France and Norway vis-à-vis the UK are stationary, whereas the ESTAR does not. This may be due to the fact that the ESTAR test uses Taylor approximation and could miss some useful information. See Kim and Moh (2010) for some discussion on the issue.

As Phillips and Sul (2003) pointed out, however, the so-called first-generation panel unit root tests such as Maddala and Wu (1999), Levin et al. (2002), and Im et al. (2003) are known to be seriously over-sized (reject the null hypothesis too often) when the data is cross-sectionally dependent. We first test this issue by employing the statistic proposed by Pesaran (2004) described below in Equation (2.10).

$$CD = \sqrt{\frac{2T}{N(N-1)}} \left(\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \hat{\rho}_{i,j} \right) \overset{d}{\rightarrow} N(0,1) \tag{2.10}$$

where $\hat{\rho}_{i,j}$ is the pair-wise correlation coefficients from the residuals of the ADF regressions (2.4). We report results in Table 4. The results imply a very strong degree cross-section dependence. In what follows, therefore, we employ available second-generation panel unit root tests with cross-section dependence consideration.

2.4.1 Linear Panel Unit-Root Test Analysis

We first employ Pesaran (2007)'s cross-sectionally augmented panel ADF (PADF) test given by,

$$CIPS(N,T) = t_{N,T} = N^{-1} \sum_{i=1}^{N} t_i(N,T), \tag{2.11}$$

where $t_i(N,T)$ is the t-statistic for b_i from the following least squares regression,

$$\Delta r_{i,t} = a_i + b_i r_{i,t-1} + c_i \bar{r}_{t-1} + \sum_{j=0}^{p} d_{ij} \Delta \bar{r}_{t-j} + \sum_{j=1}^{p} \delta_{ij} \Delta r_{i,t-j} + e_{i,t}. \tag{2.12}$$

Here, $\bar{r}_t$ is the cross-section average at time t, which proxies the common factor component for $i = 1, ..., N$. Note that this is a cross-sectionally augmented version of the IPS (Im et al. (2003)) test.

We report test results in Table 5. In contrast to empirical evidence from Balvers et al. (2000), we obtain very weak panel evidence of stationarity even at the 10% significance level when we control for cross-section dependence, irrespective of the choice of the reference country. This implies that the strong evidence of stationarity in Balvers et al. (2000) could have been due to size distortion caused by a failure to account for the cross-section dependence.

2.4.2 Nonlinear Panel Unit-Root Test Analysis

We next explore the panel evidence of nonlinear stationarity by employing a test proposed by Cerrato et al. (2011). This test is an extension of the nonlinear ESTAR unit root test by Kapetanios et al. (2003) to a panel version test in combination with the methodology suggested by Pesaran (2007) to address the issue of cross-section dependence.

For this, we rewrite Equation (2.6) as the following set of equations.

$$\Delta r_{i,t} = \xi_i r_{i,t-1}\{1 - \exp(-\theta r_{i,t-d}^2)\} + \epsilon_{i,t}, \text{ and } \epsilon_{i,t} = \delta_i f_t + u_{i,t} \tag{2.13}$$

where δ_i is a country-specific factor loading, f_t is a common factor, and $u_{i,t}$ is a (possibly serially correlated) idiosyncratic shock. Cerrato et al. (2011) suggest the following nonlinear cross-section augmented IPS-type statistics:

$$t_{N,T} = N^{-1} \sum_{i=1}^{N} t_i(N,T) \tag{2.14}$$

where $t_i(N,T)$ is the t-statistic for $\beta_{i,0}$ from the following least squares regression,

$$\Delta r_{i,t} = \alpha_i + \beta_{i,0} r_{i,t-1}^3 + \gamma_{i,0} \overline{r}_{t-1}^3 + \sum_{j=1}^{p} (\beta_{i,j} \Delta r_{i,t-j} + \gamma_{i,j} \Delta \overline{r}_{t-j}^3) + e_{i,t}, \tag{2.15}$$

where $\bar{r}_t$ is the cross-section average at time t, which proxies the common factor component for $i = 1, ..., N$. In the absence of cross-section dependence, $\gamma_{i,j} = 0$ for all i and j, and the test statistic is reduced to nonlinear ESTAR test in Equation (2.9).

We report test results in Table 6. It is interesting to see that the test does not reject the null hypothesis for both reference cases at the 10% significance level. This is somewhat puzzling because we obtained strong evidence of nonlinear stationarity from the univariate ESTAR tests when the UK serves as the reference country. Since the panel test (2.14) has the alternative hypothesis that states that there are stationary $r_{i,t}$ for $i = 1, ..., N_1$ and $N_1 > 0$, and the univariate test rejects the null for 12 out of 17 countries, it would be natural to expect panel evidence of stationarity. Yet, we do not find it. To look into this apparent contradiction further, we turn to a dynamic factor analysis in what follows based on the following conjecture.

If the first common factor is stationary and has dominating effects on $r_{i,t}$ in the short-run, the stochastic properties of $r_{i,t}$ may resemble those of stationary variables even when the idiosyncratic component is nonstationary. Even though the nonstationary idiosyncratic component will dominate the stationary common factor in the long-run, unit root tests for finite horizon observations may reject the null of nonstationarity.

2.5 Dynamic Factor Analysis

In this section, we attempt to understand seemingly inconsistent statistical evidence from the univariate and the panel unit root test when the UK serves as the base country. We note that the panel unit root tests from the previous section control for the cross-section dependence by taking and including the first common factor in the regression. We employ the following factor structure motivated by the framework of the PANIC method by Bai and Ng (2004), described as follows. First we write

$$
\begin{aligned}
r_{i,t} &= a_i + \lambda_i' \mathbf{f}_t + e_{i,t} \\
(1 - \alpha L)\mathbf{f}_t &= \mathbf{A}(\mathbf{L})\mathbf{u}_t \qquad (2.16) \\
(1 - \rho_i L)e_{i,t} &= B_i(L)\varepsilon_{i,t}
\end{aligned}
$$

where a_i is a fixed effect intercept, $\mathbf{f}_t = [f_1 \dots f_r]'$ is a $r \times 1$ vector of (latent) common factors, $\lambda_i = [\lambda_{i,1} \dots \lambda_{i,r}]'$ denotes a $r \times 1$ vector of factor loadings for country i, and $e_{i,t}$ is the idiosyncratic error term. $\mathbf{A}(L)$ and $B_i(L)$ are lag polynomials. Finally, we assume that $\mathbf{u}_t, \varepsilon_{i,t}$, and λ_i are mutually independent.

Estimations are carried out by the method of principal components. When $e_{i,t}$ is stationary, $\mathbf{f}_t$ and λ_i can be consistently estimated irrespective of the order of $\mathbf{f}_t$. If $e_{i,t}$ is integrated, however, the estimator is inconsistent because a regression of $r_{i,t}$ on $\mathbf{f}_t$ is spurious. PANIC avoids such a problem by applying the method of principal components to the first-differenced data. That is,

$$\Delta r_{i,t} = \lambda_i' \Delta \mathbf{f}_t + \Delta e_{i,t} \tag{2.17}$$

for $t = 2, \cdots, T$. Let $\Delta \mathbf{r}_i = [\Delta r_{i,2} \cdots \Delta r_{i,T}]'$ and $\Delta \mathbf{r} = [\Delta \mathbf{r}_1 \cdots \Delta \mathbf{r}_N]$. After proper normalization,the method of principal components for $\Delta \mathbf{r} \Delta \mathbf{r}'$ yields estimated factors $\Delta \hat{\mathbf{f}}_t$, the associated factor loadings $\hat{\lambda}_i$, and the residuals $\Delta \hat{e}_{i,t} = \Delta r_{i,1} - \hat{\lambda}_i' \Delta \hat{\mathbf{f}}_t$. Re-integrating these, we obtain the following

$$\hat{\mathbf{f}}_t = \sum_{s=2}^{t} \Delta \hat{\mathbf{f}}_s, \quad \hat{e}_{i,t} = \sum_{s=2}^{t} \Delta \hat{e}_{i,s} \tag{2.18}$$

for $i = 1, \cdots, N$.

Bai and Ng (2004) show that when $k = 1$, the ADF test with an intercept can be used to test the null of a unit root for the single common component $\hat{\mathbf{f}}_t$. For each idiosyncratic component $\hat{e}_{i,t}$, the ADF test with no deterministic terms can first be applied. Then, a panel unit root test statistic for these idiosyncratic terms can be constructed as follows.

$$P_{\hat{e}} = \frac{-2\sum_{i=1}^{N} \ln p_{\hat{e}_i} - 2N}{2\sqrt{N}} \xrightarrow{d} N(0,1), \tag{2.19}$$

In Table 7, we report the linear and nonlinear unit root test for the estimated first common factor. The tests reject the null of nonstationarity only for the case with the UK, which implies that the first common factor is likely to be stationary.

In Figure 1, we plot the first five common factors and their relative portions of the stock price deviations with the UK as the reference. Starting with initial 50% observations, we use a recursive method to repeatedly estimate five common factors along with shares of variations explained by each common factor from each set of samples. The graph shows that the first common factor explains roughly about 45% of total variations, while other common factors play substantially smaller roles.[7] Put differently, the *stationary* first common factor seems to play a dominant role in determining the stochastic properties of $r_{i,t}$ in the short-run.[8] Also, we estimate idiosyncratic factor loading coefficients (λ_i) in Equation (2.16) that measure country-specific degrees of dependence of $r_{i,t}$ on the common factor. Estimates are reported in Figure 2. The results show that the first common factor represents each of $r_{i,t}$ fairly well with a few exceptions of Hong Kong and Singapore.[9]

Note on the other hand that this first common factor resembles the dynamics of the proxy common factor (cross-section means) in Equation (2.12) and (2.15) as we can see in Figure 2.

In addition to evidence of the linear and nonlinear stationarity of the common factor with the UK shown in Table 7, we compare the speeds of transitions from the ESTAR model specification for the common factors with the US and with the UK. For this purpose, we estimate the scale parameter θ in Equation (2.5) via the nonlinear least squares (NLLS) method to evaluate the speed of transitions across the stationarity and nonstationarity regimes. Note that we cannot estimate ξ and θ separately in Equations (2.6). Following Kapetanios et al. (2003), we assume $\xi = -1$.

We report a sample transition function estimate along with the 95% confidence bands in Figure 3. We note that the transition function for the common factor with the US reference may be consistent with nonstationarity, because the 95% confidence band of θ hits the zero lower bound, and we cannot reject the possibility of a single regime, which is the nonstationarity regime.[10] With the UK, the confidence band of the transition function remains compact ($\hat{\theta}$ was 1.308 and the standard error was 0.570).

[7] Similar patterns were observed when the US is the reference country.

[8] It will be eventually dominated by nonstationary idiosyncratic component in the long-run.

[9] Similar patterns were again observed when the US serves as the reference country.

[10] $\hat{\theta}$ was 1.746 and the standard error was 1.018, implying a negative value for the lower bound ($\hat{\theta} - 1.96 \cdot s.e.$). Since θ is bounded below zero, the estimate assumes 0 for the lower bound.

This evidence explains why the panel unit root tests fail to reject the null of nonstationarity, even when the univariate test rejects the null for many countries. To control for cross-section dependence, the test procedures incorporated in Equations (2.12) and (2.15) take out the dominant stationary common component, but leave the nonstationary idiosyncratic components. Hence, the panel tests might fail to reject the null of nonstationarity. However, the univariate unit root tests may reject the null because the dominant stationary component overpower the idiosyncratic component. We confirm this conjecture via Monte Carlo simulations in the next section.

2.6 Further investigation on Panel Results: Monte Carlo Simulation Analysis

We implement an array of Monte Carlo simulations in this section to see how plausible our conjecture from the previous section is. For this purpose, we construct 17 time series that have a factor structure with a nonlinear stationary common factor motivated by our panel ESTAR model. We assume that each of the 17 idiosyncratic components is nonstationary. That is, 17 time series variables $x_{i,t}$ share the following common component.

$$f_t = f_{t-1} + \xi f_{t-1}\{1 - \exp(-\theta f_{t-1}^2)\} + \mu_t, \tag{2.20}$$

where ξ is set at -1 following Kapetanios et al. (2003). The DGP assumes $\theta = 1.308$, which is the estimate from the previous section for the 17 relative stock price indices relative to the UK. In addition to Equation (2.20), we generate 17 independent nonstationary idiosyncratic components that are to be added to the common factor to construct each time series as follows:

$$x_{i,t} = \lambda_i \mathbf{f}_t + \varepsilon_{i,t}. \tag{2.21}$$

and

$$\varepsilon_{i,t} = \varepsilon_{i,t-1} + u_{i,t}, \tag{2.22}$$

where $u_{i,t} \sim N(0, 1)$. We used factor loading estimates (λ_i) from the PANIC estimations in the previous section. Then we employ a nonlinear univariate unit root test and the panel nonlinear unit root test. Repeating this process many times, we expect to see strong evidence of stationarity from the univariate tests and weak evidence from the panel tests in small samples, but weak evidence of stationarity from both types of tests in large samples where nonlinear idiosyncratic components must eventually dominate the stationary common factor.

We ran 3,000 Monte Carlo simulations for five different numbers of observations: 50, 100, 200, 300, and 500. In Table 8, we report the percentage of the mean and the median of the the frequency of the rejections of the null of unit roots out of 17 at the 5% significance level for the univariate ESTAR tests. For the panel test, we report the rejection rate at the 5% level for each exercise.

We confirm our conjecture by these simulations. When the number of observation is small, e.g., 50, the univariate ESTAR test rejects the null for many series about 50% frequency on average. This tendency disappears quickly as the number of observation increases. For example, when the number of observations is 500, only about 1 rejections out of 17 variables were observed. For all cases, the panel ESTAR that removes the effect of the stationary common factor rejected the null with near 0.5% frequency. Therefore, our empirical evidence suggests that stock indices with the UK as the reference country possess a dominating common factor that is nonlinear stationary, which makes it possible to profitably utilize a contrarian strategy when deviations are big.

2.7 Concluding Remarks

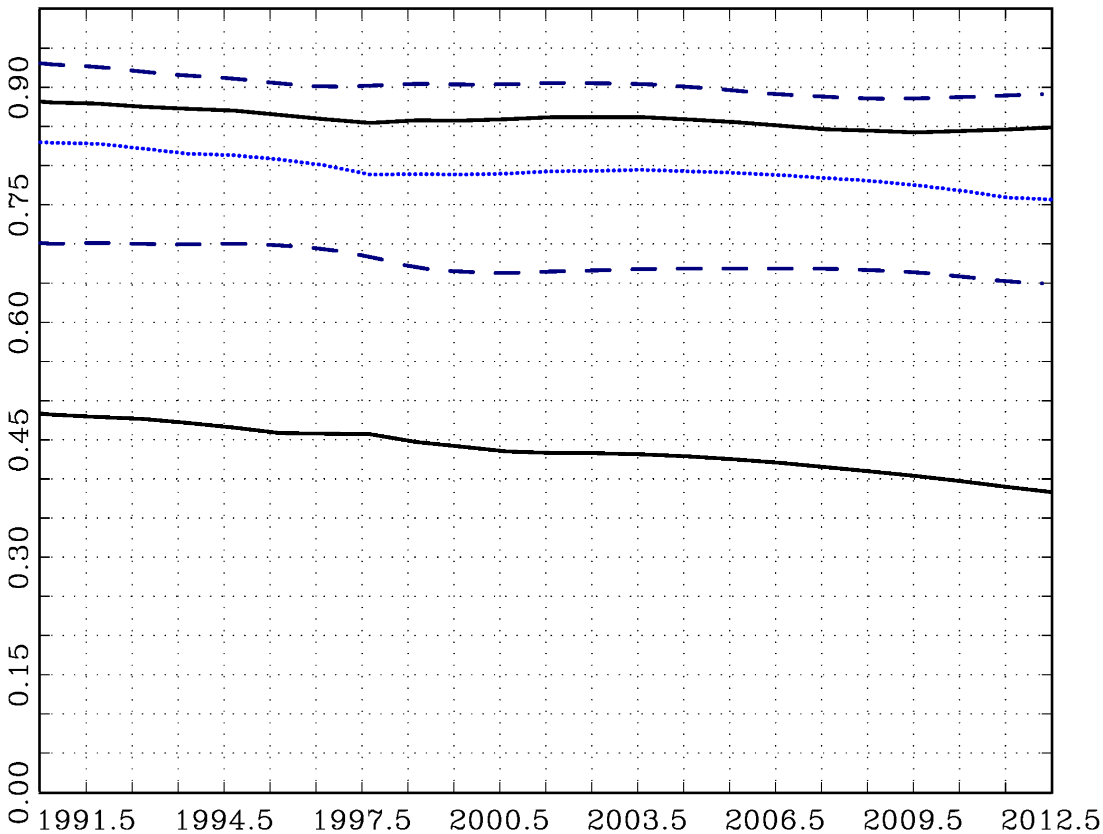
We revisited the topic of mean reversion in national stock prices across international stock markets relative to the US and the UK using the Morgan Stanley Capital International annual gross stock index data for 18 developed countries. We found strong evidence of mean reversion for a maximum 14 out of 17 countries in the case of the UK (but not the US) as the reference country, while very weak evidence of linear mean reversion was observed irrespective of the choice of the reference country.

Implementing panel version linear unit root tests while controlling for cross-section dependence provided weak evidence of stationarity in the univariate tests. The panel nonlinear unit root test also failed to reject the null of nonstationarity even when the UK served as the reference country. The results appear inconsistent.

To resolve this seeming puzzle, we estimated a common factor, then tested the null of nonstationarity with linear and nonlinear stationarity alternatives. Our tests strongly favor the stationarity for the first common factor from the panel when the UK serves as the reference country. These results imply that the first common factor with the UK is stationary and dominates nonstationary idiosyncratic components in small samples. That is, when the first common factor dominates the nonstationary idiosyncratic component, the panel unit root test that removes the influence of the stationary common factor may yield evidence against stationarity even though it behaves as a stationary variable in finite samples, even though it will become dominated by nonstationary variables in the long-run. Our Monte Carlo simulation analysis confirms our conjecture.

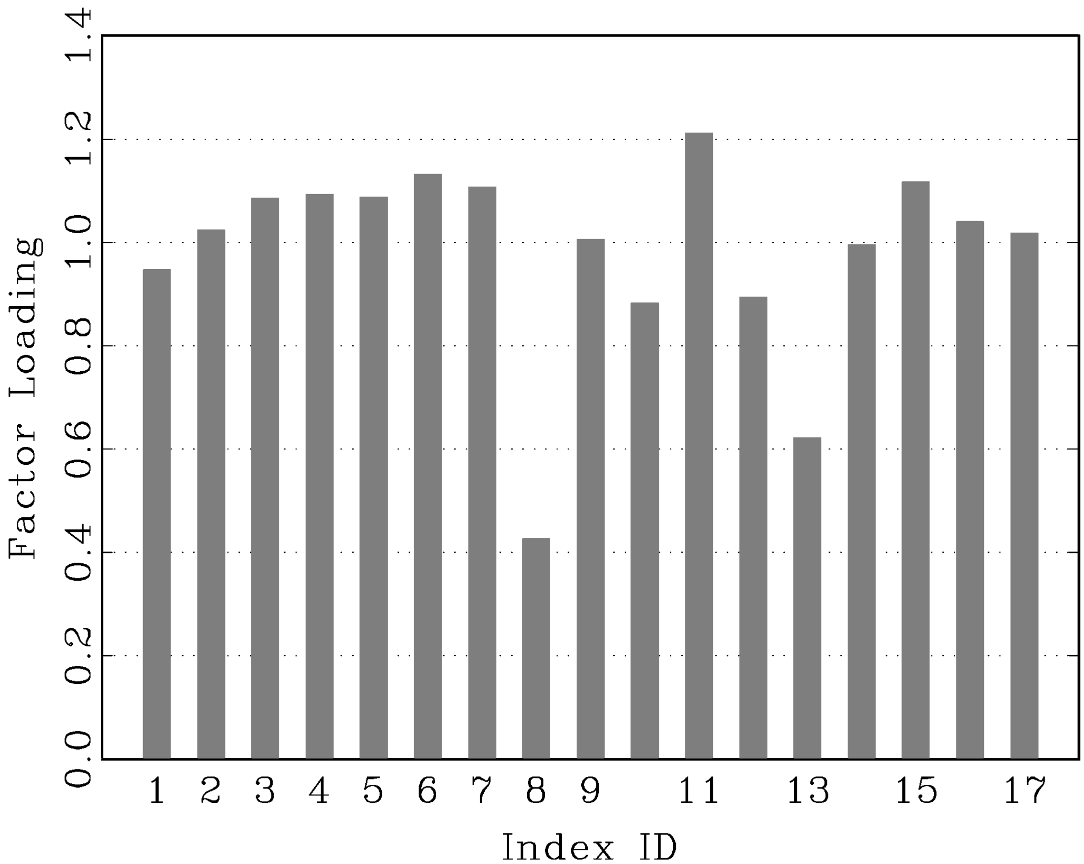
Our empirical findings suggest that the UK equity index may be used as an anchor in managing international equity portfolios. Big deviations of national equity prices from the UK index may be accompanied by winner-loser reversal soon. Therefore, one may consider short-selling better performing assets while buying worse performing ones. On the contrary, one should employ the momentum strategy with the US index, because deviations of equity prices are more likely to be permanent.

Figure 2.1: Cumulative Share of Variation by Five Common Factors: UK



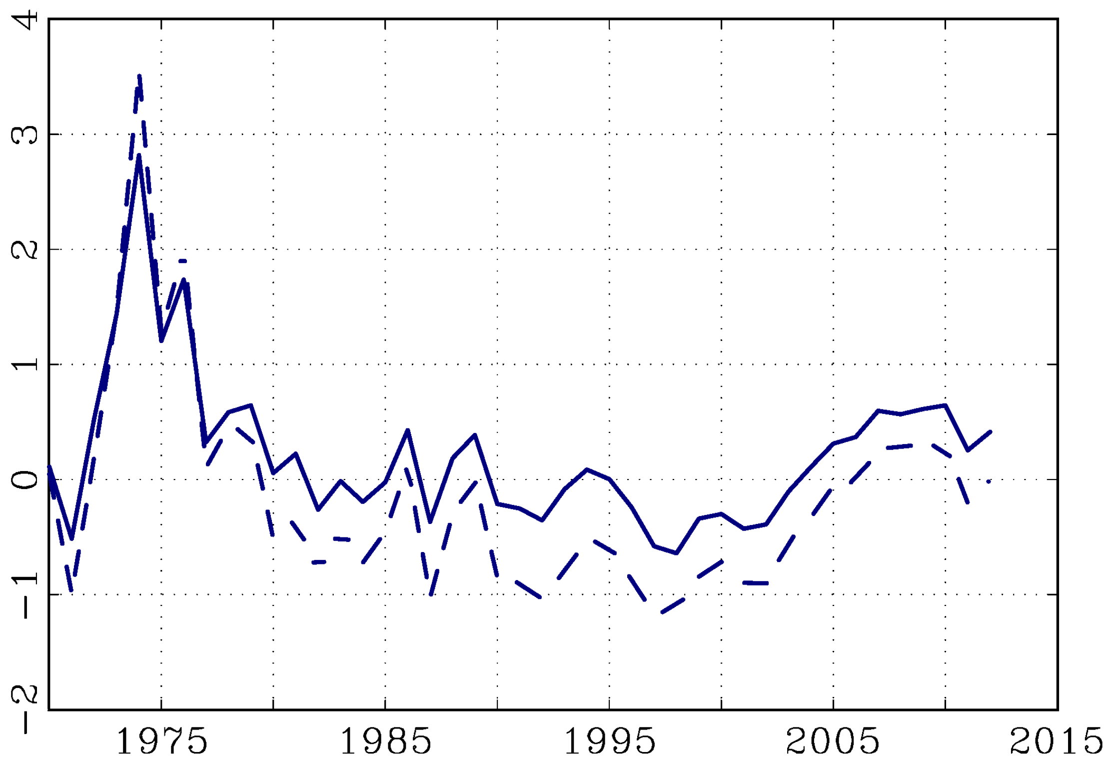
Note: A recursive method is used to repeatedly estimate the first five common factors using the initial 50% observations as the split point. We report shares of variations explained by the common factors from each set of samples.

Figure 2.2: Factor Loading Coefficients Estimation: UK



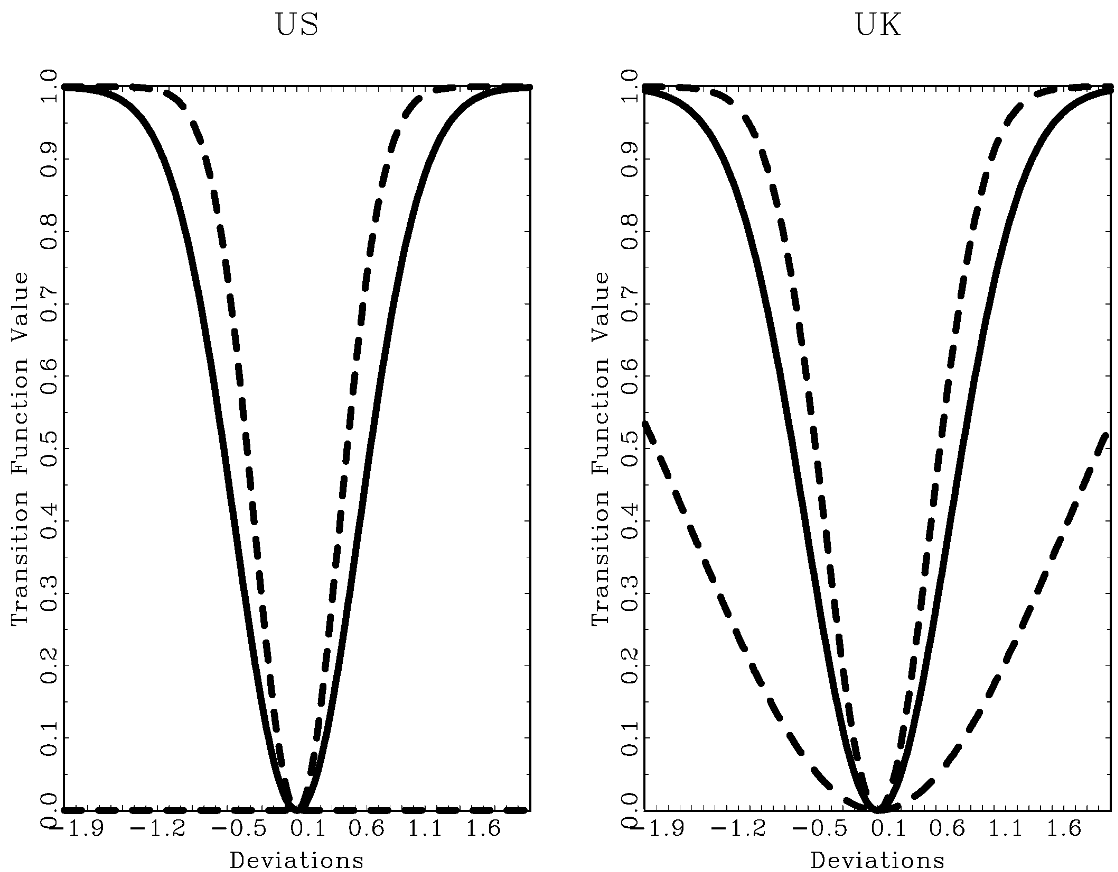
Note: We report factor loading coefficients (λ_i) in Equation (16). They represent the country-specific dependence on the common factor.

Figure 2.3: First Common Factor Estimates: UK



Note: We report two measures of the common factor: the first common factor (dashed line) via the PANIC (Bai and Ng, 2004) and the cross section mean (solid) as in Pesaran (2007).

Figure 2.4: Transition Function Estimates



Note: We report graphs of one minus the exponential transition function $1-\exp(-\theta x^2)$ for the common factor estimates with the US and the UK. We used $\theta = 1.746$ for the US reference and $\theta = 1.308$ for the UK reference, obtained from the data. Dashed lines are 95% confidence bands. The lower bound for the US is negative, so we used 0 because θ is bounded below zero.

Table 2.1: Summary Statistics

ID	Country	Mean	Std Dev	Skewness	Kurtosis	JB	Min	Max
Base Country: US								
1	Aus	−0.414	0.341	−0.363	2.443	1.500	−1.239	0.132
2	Aut	0.179	0.521	0.163	2.834	0.241	−0.927	1.047
3	Bel	0.683	0.352	−0.503	3.464	2.196	0.000	1.267
4	Can	0.013	0.357	−0.202	2.761	0.394	−0.915	0.597
5	Den	0.729	0.363	−0.265	2.784	0.585	−0.144	1.420
6	Fra	0.179	0.240	0.035	3.572	0.594	−0.295	0.565
7	Ger	0.150	0.255	0.748	3.570	4.596	−0.338	0.699
8	HK	1.583	0.531	−0.312	4.125	2.962	0.000	2.615
9	Ita	−0.981	0.436	0.687	3.712	4.286	−1.715	0.000
10	Jap	0.770	0.709	0.376	2.979	1.014	−0.191	2.345
11	Net	0.718	0.404	−0.088	2.054	1.659	−0.224	1.260
12	Nor	0.469	0.403	0.654	4.778	8.733	−0.465	1.237
13	Sing	0.767	0.514	0.287	4.215	3.235	−0.253	1.843
14	Spa	−0.282	0.457	−0.110	3.305	0.254	−1.361	0.851
15	Swe	0.819	0.518	0.117	1.906	2.243	−0.258	1.732
16	Swi	0.418	0.258	−0.061	2.869	0.057	−0.189	0.833
17	UK	0.271	0.235	−0.593	4.944	9.298	−0.461	0.694
Base Country: UK								
ID	Country	Mean	Std Dev	Skewness	Kurtosis	JB	Min	Max
1	Aus	−0.684	0.375	−0.225	2.247	1.378	−1.413	0.063
2	Aut	−0.092	0.541	0.910	4.223	8.615	−1.048	1.508
3	Bel	0.412	0.294	0.112	3.740	1.071	−0.169	1.165
4	Can	−0.258	0.440	−0.086	3.192	0.120	−1.121	0.859
5	Den	0.458	0.362	0.347	3.084	0.877	−0.304	1.358
6	Fra	−0.091	0.214	0.388	3.930	2.629	−0.549	0.602
7	Ger	−0.121	0.281	1.631	8.696	77.207	−0.539	0.867
8	HK	1.312	0.424	−0.024	3.317	0.185	0.000	2.077
9	Ita	−1.252	0.515	0.291	3.436	0.949	−1.991	0.141
10	Jap	0.499	0.639	0.491	3.038	1.728	−0.467	1.741
11	Net	0.447	0.302	−0.526	5.337	11.766	−0.384	0.832
12	Nor	0.198	0.426	0.988	5.015	14.271	−0.670	1.503
13	Sing	0.496	0.500	0.726	5.758	17.404	−0.458	1.479
14	Spa	−0.553	0.573	0.204	4.719	5.596	−1.576	1.312
15	Swe	0.548	0.457	−0.121	4.193	2.654	−0.293	1.404
16	Swi	0.147	0.278	0.355	4.238	3.653	−0.363	0.816
17	US	−0.271	0.235	0.593	4.944	9.298	−0.694	0.461

Note: JB refers the Jarque-Bera statistics, which has asymptotic χ^2 distribution with 2 degrees of freedom. For the US reference, most of the stock index deviation shows normality except for Norway and the UK whereas the stock index deviations for 6 countries (Austria, Germany, the Netherlands, Norway, Singapore and the US) show non-normality with the UK reference.

Table 2.2: Univariate Linear Unit Root Tests

	US		UK	
	ADF_c	ADF_t	ADF_c	ADF_t
Aus	-1.895	-1.722	-2.042	-1.499
Aut	-1.651	-2.268	-1.746	-2.659
Bel	-2.526*	-2.420	-3.099†	-3.098*
Can	-1.260	-1.247	-1.537	-1.466
Den	-2.060	-2.401	-2.285	-2.422
Fra	-2.663*	-2.804	-3.592‡	-3.553*
Ger	-2.667*	-2.671	-3.036†	-3.326*
HK	-3.275†	-3.692‡	-3.621‡	-4.204‡
Ita	-2.278	-2.735	-2.452	-3.067*
Jap	-1.030	-1.992	-1.172	-2.627
Net	-1.717	-1.416	-2.047	-3.394†
Nor	-3.013†	-3.005	-2.963†	-3.084
Sing	-2.268	-2.666	-2.139	-2.865
Spa	-1.840	-1.806	-1.794	-1.696
Swe	-1.303	-3.452†	-1.773	-3.866†
Swi	-2.161	-2.726	-2.270	-2.380
UK	-2.637*	-2.680	-	-
US	-	-	-2.637*	-2.680

Note: ADF_c and ADF_t denote the augmented Dickey-Fuller test statistic when an intercept and when both an intercept and time trend are present, respectively. *, †, and ‡ denote significance levels at the 10%, 5%, and 1% level, respectively.

Table 2.3: Univariate Nonlinear Unit Root Tests

	US		UK	
	$NLADF_c$	$NLADF_t$	$NLADF_c$	$NLADF_t$
Aus	-1.094	-1.078	-2.321	-1.440
Aut	-1.488	-2.113	-2.788*	-3.448†
Bel	-1.904	-1.763	-3.076†	-3.211*
Can	-1.883	-1.971	-3.469†	-2.978
Den	-1.930	-2.725	-3.923‡	-3.321*
Fra	-2.502	-2.503	-2.484	-2.473
Ger	-2.508	-2.541	-2.697*	-2.798
HK	-2.641*	-2.817	-2.639*	-3.495†
Ita	-2.180	-2.470	-3.095†	-4.288‡
Jap	-1.244	-1.864	-1.717	-2.044
Net	-1.401	-1.344	-1.899	-3.889†
Nor	-1.906	-1.962	-2.585	-2.724
Sing	-2.132	-2.500	-2.221	-3.007
Spa	-2.148	-2.166	-2.613*	-2.764
Swe	-1.416	-2.253	-1.299	-3.246*
Swi	-1.881	-2.628	-2.993†	-2.929
UK	-4.853‡	-4.858‡	-	-
US	-	-	-4.853‡	-4.858‡

Note: $NLADF_c$ and $NLADF_t$ denote the ESTAR test statistic (Kapetanios et al., 2003) when an intercept and when both an intercept and time trend are present, respectively. *, †, and ‡ denote significance levels at the 10%, 5%, and 1% level, respectively. Asymptotic critical values were obtained from Kapetanios et al. (2003).

Table 2.4: Cross-Section Dependence Test

	CSD	p-value
US	19.753	0.000
UK	24.586	0.000

Note: This test is proposed by Pesaran (2004).

Table 2.5: Panel Linear Unit Root Test Results

	$PADF_c$	$PADF_t$
US	-2.056	-2.573
UK	-2.012	-2.523

Note: Critical values were obtained from Pesaran (2007). The test fails to reject the null of nonstationarity for both reference countries.

Table 2.6: Panel Nonlinear Unit Root Test Results

	$NLPADF_c$	$NLPADF_t$
US	-1.345	-1.481
UK	-1.471	-1.588

Note: Critical values were obtained from Cerrato et al. (2011). The test fails to reject the null of nonstationarity for both reference countries.

Table 2.7: Test for the First Common Factor

	Linear		*Nonlinear*	
	ADF_c	ADF_t	$NLADF_c$	$NLADF_t$
US	-2.177	-2.499	-1.289	-1.804
UK	-2.845*	-2.967†	-3.728†	-3.919†

Note: *, †, and ‡ denote significance levels at the 10%, 5%, and 1% level, respectively.

Table 2.8: Simulation Results

		Univariate ESTAR		*Panel ESTAR*	
		$NLADF_c$	$NLADF_t$	$NLPADF_c$	$NLPADF_t$
nob=50	Median	41.2 %	47.1 %	0.7 %	0.4 %
	Mean	41.6 %	48.3 %		
nob=100	Median	29.4 %	35.3 %	0.3 %	0.5 %
	Mean	28.5 %	33.6 %		
nob=200	Median	17.6 %	17.6 %	0.1 %	0.0 %
	Mean	17.2 %	20.5 %		
nob=300	Median	11.8 %	11.8 %	0.2 %	0.0 %
	Mean	12.9 %	15.0 %		
nob=500	Median	5.9 %	11.8 %	0.2 %	0.0 %
	Mean	9.4 %	10.8 %		

Note: The table shows simulation results. Numbers in the Univariate ESTAR section represent percentage of the mean and median of the frequency of rejections of the null of unit roots when univariate ESTAR test is employed for the 3000 iterations. Numbers in the Panel ESTAR section represent percentage of rejections of the null of unit roots when the Panel ESTAR test is employed for the 3000 iterations.

Chapter 3

The Heterogeneous Responses of the World Commodity Prices to Exchange Rate Shocks

3.1 Introduction

During the recent great recession, we have observed big swings of the US exchange rate that were accompanied by similar and even more volatile movements of world commodity prices such as oil prices. See Figure 1. In his recent VOX article in December 2014, Jeffrey Frankel argued that commodity prices declined rapidly in 2014 mainly due to anticipation of a rise in the interest rate in the US, via the following four channels: the extraction channel (Hotelling (1931)), the inventory channel (Frankel (1986), Frankel (2014)), the financialization channel (Hamilton and Wu (2014)), and the exchange rate channel (Frankel (2006)).[1]

We are particularly interested in the exchange rate channel, noting that world commodity prices tend to exhibit a mirror image of the US dollar exchange rate as can be seen in Figure 1. Since world commodities are normally denominated in the US dollar, an appreciation of the US dollar results in an increase in the foreign price of the commodity in the rest of the world, which will induce adjustments in the commodity price. Since most world commodities are highly tradable, it is natural to assume that the law of one price (LOP) holds at least in the long-run. This paper investigates patterns of the price adjustment process in the world commodity market in response to unexpected changes in the US dollar exchange rate.

Since the seminal work of Obstfeld and Rogoff (1995), the profession has developed New Open Economy Macroeconomics (NOEM), which introduce sticky-price type economic frictions to open macronomic models. For example, prices of tradable goods are sticky in terms of exporter's currency under producer currency pricing (PCP; Obstfeld and Rogoff

[1]The article is available at http://www.voxeu.org/article/commodity-prices-down-dollars-euros.

1995), while prices are sticky in local consumers' currency under local currency pricing (LCP; Betts and Devereux 2000, Chari et al. 2002).

PCP implies 100% pass-through of the exchange rate to import prices, whereas the model results in 0% pass-through to export prices. The converse is true under LCP. Empirical literature finds mixed evidence for these predictions. For example, Campa and Goldberg (2002) report limited evidence on the degree of exchange rate pass-through into the import prices in 23 OECD countries, which is inconsistent with both PCP and LCP. Based on such empirical findings, some authors employ the assumption that there is a mix of firms using PCP and LCP in each economy (Choudhri and Hakura 2015). Overall, sticky prices seem to play an important role for the pass-through mechanism. Gopinath et al. (2014) offer a review of this literature.

What about the exchange rate pass-through to world commodity prices? There have been many studies on this issue, including Ridler and Yandle (1972), Dornbusch (1987), Fleisig and van Wijnbergen (1985), Giovannini (1988), Gilbert (1989), and Radetzki et al. (1990). But this issue has been somewhat overlooked in the current literature even though the profession started to pay an attention to the linkage between the exchange rate and commodity prices since the Great Recession, as one can note from Jeffrey Frankel's aforementioned VOX article.

Since world commodities are highly tradable, one may expect that the Law of One Price (LOP) should hold in the world commodity market at least in the long-run, because commodity arbitrages will occur otherwise (Goldberg and Verboven 2005, Eckard 2004, Pippenger and Phillips 2008).[2] Then an appreciation (depreciation) of the US dollar will result in a fall (rise) in dollar denominated commodity prices. In the presence of price stickiness, however, actual adjustments of the world commodity prices may not take place immediately in response to an exchange rate shock.

[2] There is a strand of studies that suggests evidence of the failure of the law of one price, such as Engel and Rogers (1999), Asplund and Friberg (2001), Goldberg and Verboven (2005). But Pippenger and Phillips (2008) maintain that all tests that fail to support the LOP are due to the result of ignoring important practical implications of arbitrage.

In what follows, we attempt to answer the following questions. First, how quickly do commodity prices adjust to the long-run equilibrium when there's an exchange rate shock? Is the speed of adjustment constant over time? Secondly, how homogeneous are the long-run responses of commodity prices to exchange rate shocks? Are long-run price elasticicities near one? Third, what are the policy implications of the volatility of the comoodity price?

We used monthly frequency world commodity prices from the IMF data base and estimated impulse response functions of commodity prices to exchange rate shocks using a recursively identified VAR framework. We further estimated dynamic exchange rate elasticities of the commodity prices. Our major findings are as follows. First, commodity prices tend to slowly adjust to their long-run equilibrium when the exchange rate shock occurs. Initial responses are typically much weaker than longer run responses, which implies a high degree price stickiness in the short-run. Most prices take 8 to 12 months to stabilize. One notable exception is oil prices which stabilize in about 4 months. Second, the responses of commodity prices exhibit high degree heterogeneity. Some commodities such as beef, pork, and logs under-correct to the exchange rate shock, that is, the price elasticities of these commodities are less than one. Some others, like corn, lamb, sugar, hide, and crude oil adjust on par with the exchange rate movement. Prices of the commodities like barley, peanuts, rubber, aluminum, and nickel over-correct.

For those commodities that over-react to the exchange rate shock, their prices in the rest of the world (outside the US) tend to rise permanently higher in the long-run when the US dollar depreciates unexpectedly. That is, US dollar exchange rate shocks would generate high volatility in these commodity prices. Put differently, not only fundamental demand/supply factors, but also financial factors may be responsible for the highly volatile movements in commodity prices we observed recently, which calls for attention from policymakers to financial markets dynamics in order to help stabilize commodity prices in the local markets.

The rest of the chpater is organized as follows. In Section 3.2, we present our baseline VAR model framework and analytical representations of the dynamic elasticity and our measure of price stickiness. Section 3.3 reports our major empirical findings. Section 4 concludes.

3.2 The Empirical Model

Let p_t^i be the log of the price of commodity i at time t, denominated in the US dollar and e_t be the log of the nominal effective exchange rate given as the price of US \$1 in terms of a basket of major foreign currencies. Most of commodity prices (p_t^i) we consider seem to follow nonstationary stochastic processes, as does the nominal exchange rate (e_t).[3] Since most series are integrated I(1) processes, we propose the following regression model with first differenced variables.

$$\Delta p_t^i = c_i + \lambda_i \Delta e_t + \varepsilon_t^i, \tag{3.1}$$

where c_i denotes the time invariant idiosyncratic intercept, λ_i is the commodity specific coefficient on the dollar appreciation rate and ε_t^i is the idiosyncratic error term that might capture disturbances in the demand-supply (fundamental) condition.

To measure dynamic effects of the exchange rate shock on each commodity price, we extend the model in (4.1) to the following bivariate vector autoregressive (VAR) model for log differences in the nominal exchange rate ($\triangle e_t$) and the commodity price ($\triangle p_t^i$),

$$\mathbf{x}_t = a + \mathbf{B}(L)\mathbf{x}_{t-1} + \mathbf{C}\mathbf{u}_t \tag{3.2}$$

where $\mathbf{x}_t = [\triangle e_t, \triangle p_t^i]$, $\mathbf{B}(L)$ denotes the lag polynomial matrix, $\mathbf{u}_t$ is a vector of normalized underlying shocks, and $\mathbf{C}$ is a matrix that describes the contemporaneous relationships

[3]Unit root test results are available upon request.

between $\triangle e_t$ and $\triangle p_t^i$. By putting $\triangle e_t$ first, we impose an assumption that US dollar appreciation rates are not contemporaneously influcenced by commodity price inflation within one month.[4]

We obtain the conventional orthogonalized impulse-response function (OIRF) for $\triangle e_t$ and $\triangle p_t^i$ as follows.

$$\begin{aligned} \theta_e^p(j) &= E\left(\Delta p_{t+j} | u_{e,t} = 1, \Omega_{t-1}\right) - E\left(\Delta p_{t+j} | \Omega_{t-1}\right), \\ \theta_e^e(j) &= E\left(\Delta e_{t+j} | u_{e,t} = 1, \Omega_{t-1}\right) - E\left(\Delta e_{t+j} | \Omega_{t-1}\right), \end{aligned} \tag{3.3}$$

where Ω_{t-1} is the adaptive information set at time $t-1$. Note that we normalize the size of the exchange rate shock to be one. Note also that the OIRFs in (4.3) are the same as the generalized impuluse-response function (GIRF) proposed by **?**, because $\triangle e_t$ is ordered first. We report the response function of level variables by cumulatively summing these response functions. That is,

$$\phi_e^p(j) = \sum_{s=0}^{j} \theta_e^p(j), \;\; \phi_e^e(j) = \sum_{s=0}^{j} \theta_e^e(j) \tag{3.4}$$

We also define the dynamic elasticity of a commodity price at time $t+j$ with respect to the exchange rate as follows.

$$\eta_e^p(j) = \frac{\phi_e^p(j)}{\phi_e^e(j)} \tag{3.5}$$

Note that $\eta_e^p(j)$ measures the elasticity of the commodity price with the time of impact $(j = 0)$ as a reference point, because $\phi(\cdot)$ measures cumulative responses of differenced variables from the initial steady state. All estimates are accompanied by the 95% confidence bands by taking 2.5% and 97.5% percentiles from residual-based bootstrap. In our empirical study below we consider a one percent positive shock in the exchange rate. Then $\eta_e^p(0)$ is the initial elasticity , the initial response of the commodity price, while $\eta_e^p(\infty)$, is the long-run elasticity when the responses are stabilized (24 months after the shock in our study.)

[4]This assumption seems to be reasonable, because it is hard to imagine that innovations in a single commodity market generate substantial fluctuations in the US dollar exchange rate.

In what follows, we also report a substantial degree price stickiness in the short-run from a majority of response functions even though these goods are highly tradable world commodities. We propose the following measures of price stickiness,

$$\zeta_e^p(1) = \eta_e^p(\infty) - \eta_e^p(0) \quad \text{or} \quad \zeta_e^p(2) = \frac{\eta_e^p(0)}{\eta_e^p(\infty)}. \tag{3.6}$$

The first is the difference of the long-run and the initial elasticity indicating how much more to be adjusted before the price is stabilized. And the second measure is the ratio of initial response to the long-run elasticity. It shows the percentage of the initial price adjustment to the long-run stabilized price.

3.3 Data Descriptions and the Empirical Findings

We used 49 primary commodity prices and the nominal US dollar exchange rate from January 1980 to November 2014. All commodity prices are denominated in the US dollar. We obtained the commodity price data from the International Monetary Fund (IMF) website. See Table 1 for data descriptions of all commodities: 23 items in the Food category (7 cereals, 5 vegetable oils, 4 meats, 3 seafoods, 4 other foods), 4 beverages, 9 agricultural raw materials, 8 metals, and 5 fuel prices. The foreign exchange rate is the trade-weighted average of the value of the US dollar against a subset of the major currencies (TWEXMMTH) obtained from the Federal Reserve Economic Data (FRED).[5]

3.3.1 Price Adjustments and Short-Run Price Stickiness

In Table 2, we report impulse-response function estimates of all 49 commodity prices when there is a one percent unexpected increase in the exchange rate. We report the initial response, $\phi_e^p(0)$ as well as the long-run response, $\phi_e^p(\infty)$, of the commodity price to the exchange rate shock. The long-run responses are measured by the response function after

[5]Major currency index includes the Euro Area, Canada, Japan, United Kingdom, Switzerland, Australia, and Sweden.

two years, which is long enough for the deviation to die out. We also report the long-run response of the exchange rate to its own shock, $\phi_e^e(\infty)$. All point estimates are accompanied by the 95% confidence bands that are obtained by 2000 nonparametric bootstrap replications.

There are couple of notable findings. First, exchange rate responses to the exchange rate shock are very similar in all 49 VAR models. After the initial 1% shock, the exchange rate increases for a while, then settles down to about 1.4%, exhibiting a mild hump-shape response function (see Figure 2, for example). All 95% confidence bands for $\phi_e^e(\infty)$ seem compact and again very similar in shape and size. Second, unlike the exchange rate responses, the response function estimates of the commodity prices are quite heterogeneous. For example, the initial responses $\phi_e^p(0)$ are insignificant at the 5% level and often negligible for 24 out of 49 prices. That is, we observe high degree price stickiness on impact even though these are highly tradable world commodities. We obtained statistically insignificant responses even in the long-run for 18 out of 49 prices, which is about 37% of all world commodity prices we consider.

In Figure 2, we report three set of impulse-response function estimates from the Food-Cereal category. As we mention previously, the responses of the exchange rate to the 1% exchange rate shock are very similar. They all show a mild degree hump-shape and stabilize around 1.4% in less than a year.

Responses of the cereal price are not uniform as can be seen in Figure 2. The barley price falls 2.5% in about 8 months, exhibiting an over-reaction as it responses more than the exchange rate changes in the long-run. The maize (corn) price falls 1.4 % in about 12 months which is on par with the exchange rate response, whereas the wheat price falls 0.8% in about 12 months a lot less than exchange rate response. Overall the commodities in the Cereal category show substantial and statistically significant responses with an exception of wheat (see Table 2). We also note substantial degree price stickiness in the short-run. Most cereals negatively responded, which is correct sign but far less than 1%. For example, $\phi_e^p(0)$ of the maize price was virtually 0%. Initial responses were often insignificant too.

The commodities in the meat subcategory show negligible and insignificant responses to the exchange rate shock with an exception of lamb (See Figure 3). For example, Poultry price show virtually no meaningful responses with the very narrow confidence band. Interestingly, the response of Lamb price exhibits a mirror image of the exchange rate response all the time. It's initial response was -1% that exactly offsets the 1% exchange rate shock. The long-run response point estimate was -1.46%, which is quite similar to that of the exchange rate in absolute value, which again offset the innovation in the exchange rate.

Agricultural raw materials show a wide range of heterogeneous responses. See Figure 4. Overall, forestry products such as Soft Logs and Soft Sawnwood show virtually no responses since the impact of the shock. Other products in this category show negligible initial responses (price-stickiness) but substantial price correction in about 8 months that are statistically significant. For instance, the rubber price drops only about 0.7% on impact but exhibits 3% correction within a year.

The prices of the items in the Metals category exhibit overall large and significant responses especially in the long-run with an exception of Zinc. See Figure 5. Most prices show substantial degree initial corrections as well. For example, the copper and the lead prices drop by more than 1% responding to the 1% exchange rate shock. The prices of nickel and aluminum show over-corrections in the long-run, implying a price fall in the rest of the world.

Among prices in the fuel category, all 4 oil prices decline initially by about 0.8%, then quickly reach to the long-run equilibrium of about -1.4% decreases in about 4 months, which offsets the increase in the exchange rate. See Figure 6. That is, oil prices show a midly sluggish adjustment in the short-run, but quickly restore the price before the exchange rate shock. The response of the Coal price show very sluggish adjustment in the short-run, but eventually over-correct the exchange rate shock in about 8 months.

3.3.2 Dynamic Elasticity Analysis

Estimates for the dynamic elasticity in the long-run, $\eta_e^p(\infty)$, are reported in Table 3 and its 95% confidence bands for the reported point estimates from 2000 nonparametric bootstraps are also included in the table. Note that $\eta_e^p(j) > 1$ implies an over-correction of the world commodity price in response to the exchange rate shock, while $\eta_e^p(j) < 1$ represents an under-correction.

We note that dynamic elasticity estimates greatly range from 0.05 (Soft Log) to -2.13 (Rubber). Elasticity estimates were highly significant for 31 out of 49 prices at the 5% level. We illustrated distribution of the dynamic elasticity in Figure 7 and calculated it's moments as mean = -0.98, standard error = 0.075, skewness = -0.08, and Kurtosis = 2.65. The median is -1.02. We also employed t-test and the the value is $t = 0.293$.

Dynamic elasticites for all cereal prices are significant with an exception of wheat. We observe an over-correction for the prices of barley, ground nut, and rice in the long-run, which implies higher volatility of these prices when the exchange rate shock occurs. That is, those countries that have high dependence on these grain products, probably developing countries, will face much higher domestic prices when exchange rate shocks occur. Maize, soybean meal, and soybean prices seem to (just) correct for the exchange rate shock just enough to maintain similar domestic prices. Dynamic elasticity estimate of wheat implies an under-correction, which is insignificant.

Most other food category prices and beverage prices show small and insignificant elasticity estimates with a couple of exceptions. Majority agricultural raw materials, metals, and fuel category prices exhibit highly significant dynamic elasticity estimates, which implies an active adjustment of the commodity price in response to the exchange rate shock. For example, oil prices show a just-correction from the short- to the long-run, which implies that exchange rate shock cause virtually no change in the domestic price in the rest of the world.

Lastly, we report our measures of price stickiness based on ζ_e^p in (4.8). Note that when $\zeta_e^p(1)$ is different from zero (negative in this exercise) or $\zeta_e^p(2)$ is smaller than one, the variable adjust more actively in the long-run, which may give some information about price-stickiness in the short-run. As we can see in Table 4 and Figure 9, the mean and the median of $\zeta_e^p(1)$ estimate are very different from zero. We also calculated its moments as mean $= -0.82$, median= -0.75, skewness = -0.25 Kurtosis = 2.56. That is, we observe very high degree price-stickiness in the short-run from the world commodity prices. From the alternative measure of stickiness, $\zeta_e^p(2)$, we find similar substantial price-stickiness behavior that the mean and the median are very different from one.

In a nutshell, irrespective of high degree tradability, we found substantial price rigidities in the world commodity markets in the short-run.

3.4 Interpretation of Price Responses

In Figure 8 the long run elasticity distribution is center around -1.00. We can interpret this that, overall, the commodity prices respond to the exchange rate shock in a way such that the the Law of One Price holds in the long-run.

However, in the short-run most of commodity prices respond less than the initial shock of the exchange rate (1% rise). From Figure 7 we observe that there are two modes. In particular some of them drop close to 1% but more of the commodity prices drop a lot less than the initial exchange rate shock, which shows substantial degree of price stickiness. What would cause these different responses? We believe this has to to with the market structure. From our observation, for the commodities whose market is local or domestic less adjustment is necessary. In the Cereal category, the prices of U.S. market based commodities drop a lot less than 1%. For example, corn price drops by only 0.08%, soybean by 0.54%, soybean meal by 0.52 % and wheat by 0.31% (see Figure 2.) The commodities in the Meat category show this tendency explicitly. Beef, pork and poultry markets are in the US showing very little initial responses whereas lamb market is in the UK and its price responses almost as much as the exchange rate with the initial response of 1% drop to accommodate the exchange rate rise. (see Figure 3.) Similar price adjustment is observed for the Seefood category, shrimp market is domestic and its price under-reacts but fishmeal and salmon market are foreign and their responses (fishmeal price drops by 0.82 % and salmon by 1.11%) are almost full. Most of the metal markets are in the UK and their prices respond fully or over to the exchange rate change (see Figure 5). The markets for oil is highly integrated and their prices adjust quickly and fully with the initial response -0.8% to the long-run response -1.47% in average (see Figure 6).

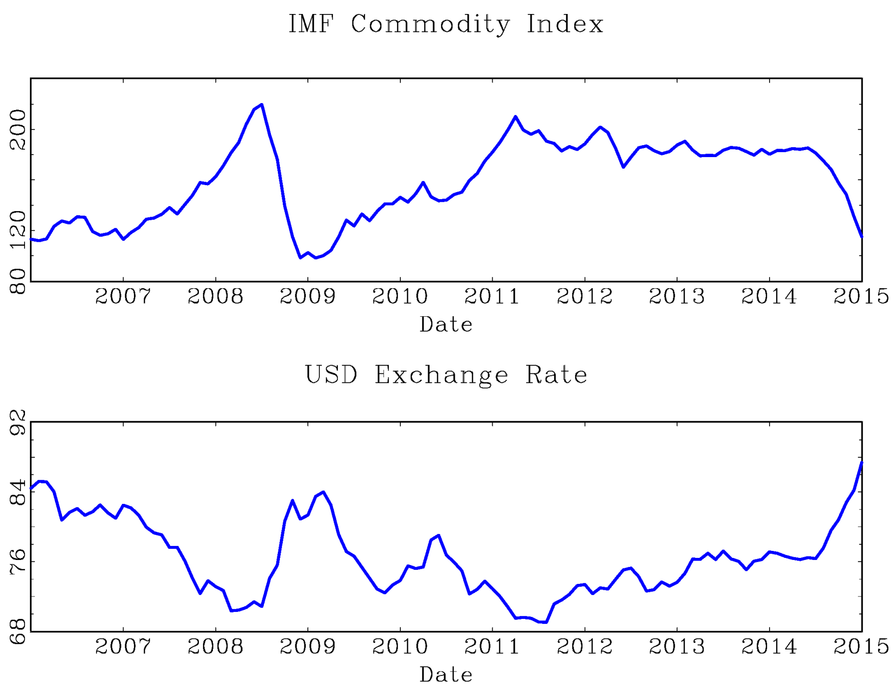
3.5 Concluding Remarks

This paper employs a VAR model to study how world commodity prices respond to exchange rate shocks. In the absence of economic friction, world commodity prices should adjust perfectly to changes in the exchange rate, because world commodities are denominated in the US dollar. Even though world commodities are highly tradable, we find a substantial degree of short-run price stickiness in a majority of cases. It takes 8 to 12 months for most prices to reach a long-run equilibrium, even though long-run responses are quite different across commodities. We also introduce a measure of price stickiness given by $\phi_e^p(\infty) - \phi_e^p(0)$, i.e., the difference between the long-run and the short-run elasticity of commodity prices with respect to the exchange rate. Our estimates range from -2.23 to 0.53. The mean measure is -0.85, which is quite different from 0, the case of instant adjustment.

We also find that the responses of commodity prices differ even within the same category. For example, in the Cereal category, the long-run response varies from -0.79% for wheat to -2.54% for peanuts. Among the 7 cereals, barley, peanuts and rice prices over-correct, soybeans, soybean meal and corn prices adjust to the exchange rate change, and wheat price under-corrects. Only oil prices show homogeneous responses within their category. We introduced the concept of dynamic elasticity and further characterized the heterogeneous responses. Long-run elasticities range from -2.13% for rubber to 0.05% for soft logs. About

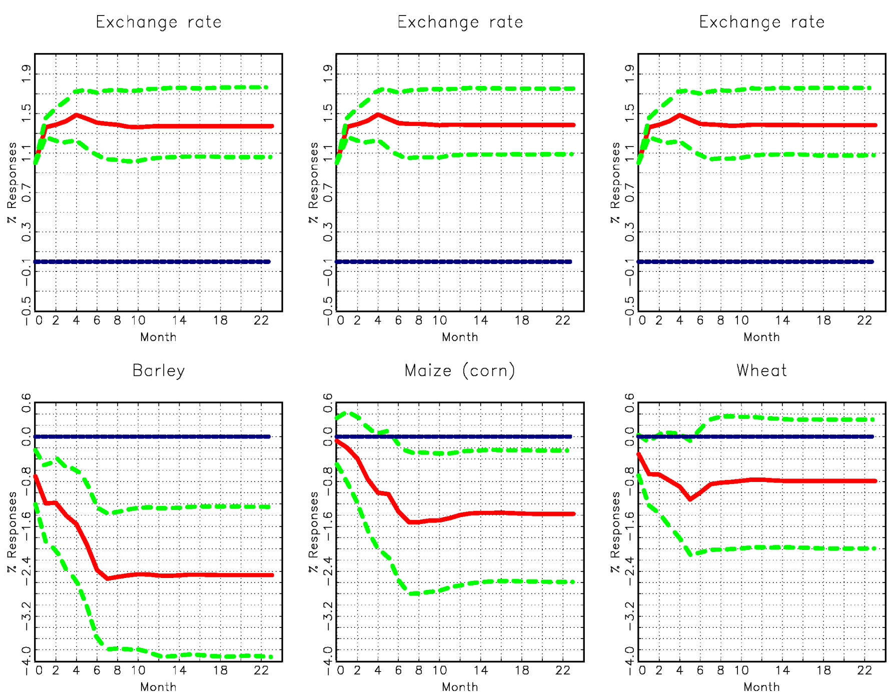
15 commodity prices including oil prices have long-run elasticity close to -1, i.e., they adjust to the exchange rate shock so that the local price remains the same. About 17 commodity prices including some food prices over-react, implying that these prices are more volatile than the exchange rate. Thus, local prices of these goods would rise if the US dollar depreciates unexpectedly, which may call for price stabilization policies.

Figure 3.1: Commodity Price and the USD Exchange Rate



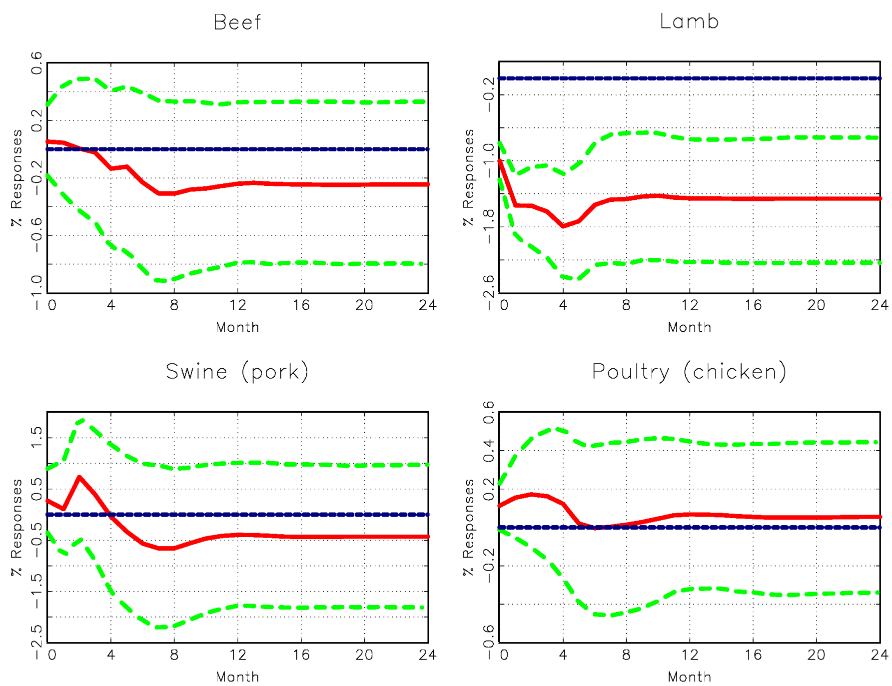
Note: The IMF commodity index was obtained from the IMF website. The USD exchange rate is the nominal effective exchange rate relative to major currencies obtained from the Federal Reserve Economic Data (FRED).

Figure 3.2: Impulse-Response Function Estimates: Food-Cereal



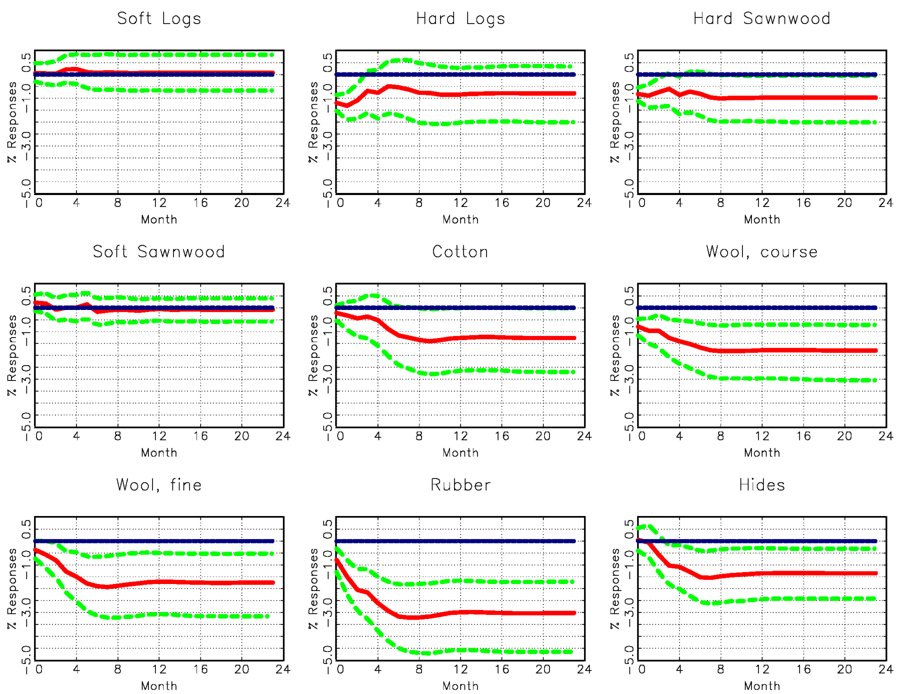
Note: All impulse-response function estimates are obtained from a bivariate VAR with the nominal exchange rate ordered first. 95% confidence bands were obtained from 2000 nonparametric bootstrap simulations.

Figure 3.3: Impulse-Response Function Estimates: Food-Meat



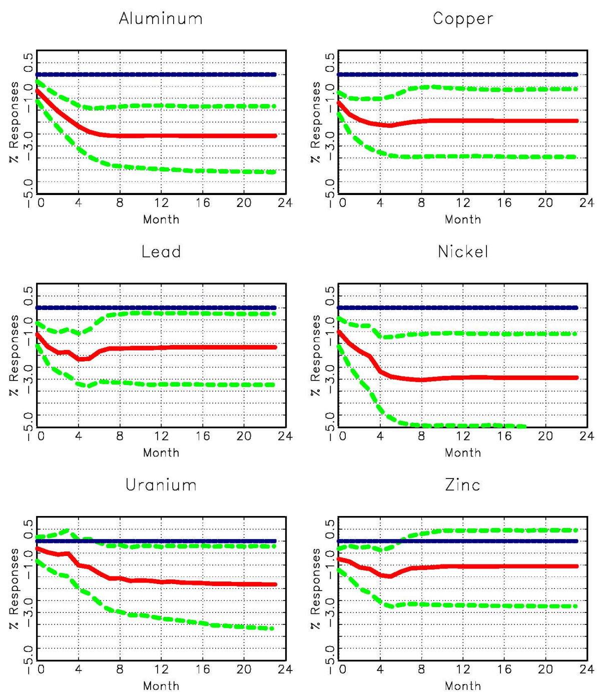
Note: All impulse-response function estimates are obtained from a bivariate VAR with the nominal exchange rate ordered first. 95% confidence bands were obtained from 2000 nonparametric bootstrap simulations.

Figure 3.4: Impulse-Response Function Estimates: Ag Raw Material



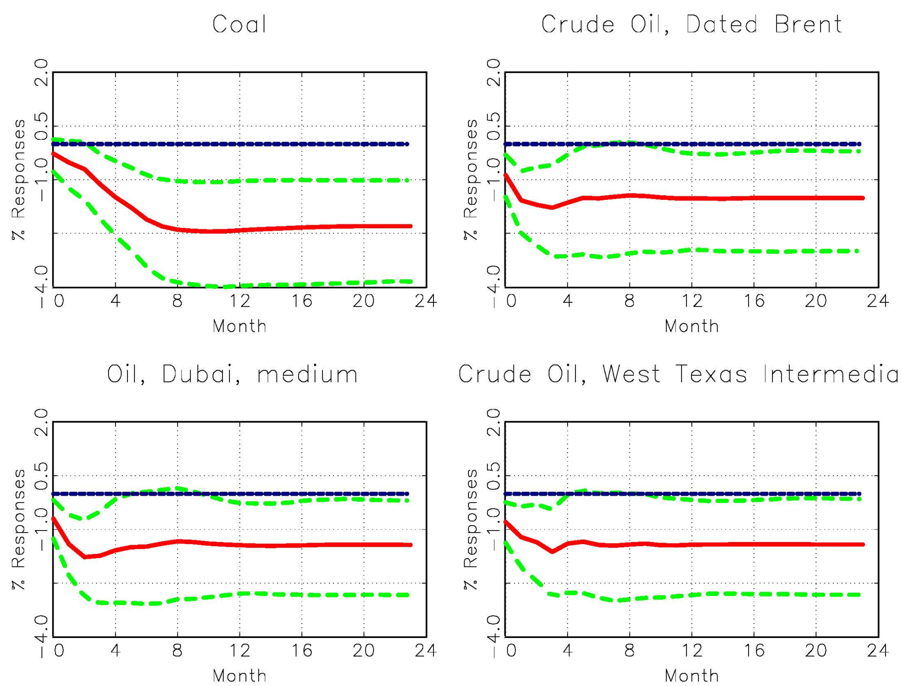
Note: All impulse-response function estimates are obtained from a bivariate VAR with the nominal exchange rate ordered first. 95% confidence bands were obtained from 2000 nonparametric bootstrap simulations.

Figure 3.5: Impulse-Response Function Estimates: Metals



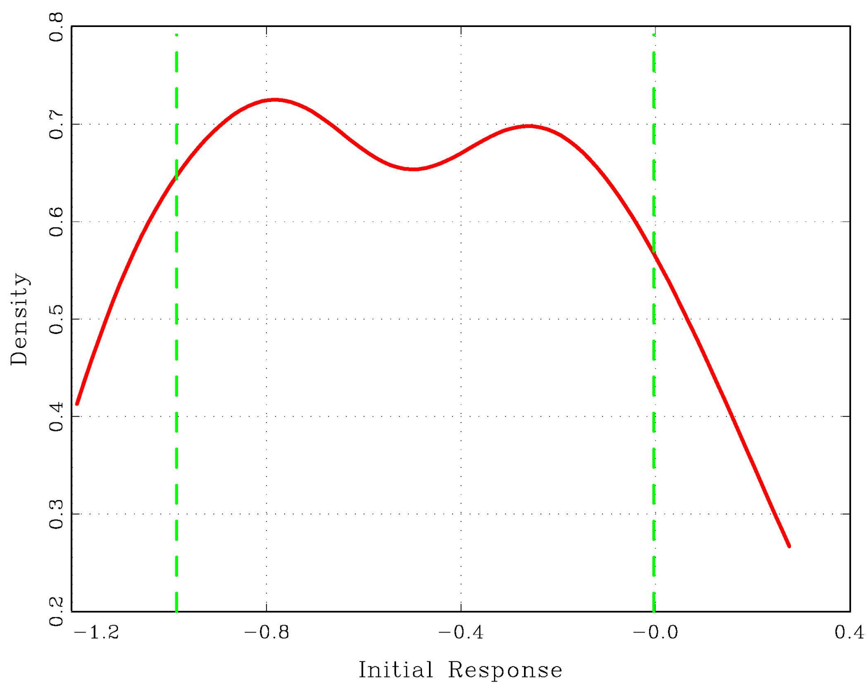
Note: All impulse-response function estimates are obtained from a bivariate VAR with the nominal exchange rate ordered first. 95% confidence bands were obtained from 2000 nonparametric bootstrap simulations.

Figure 3.6: Impulse-Response Function Estimates: Fuel



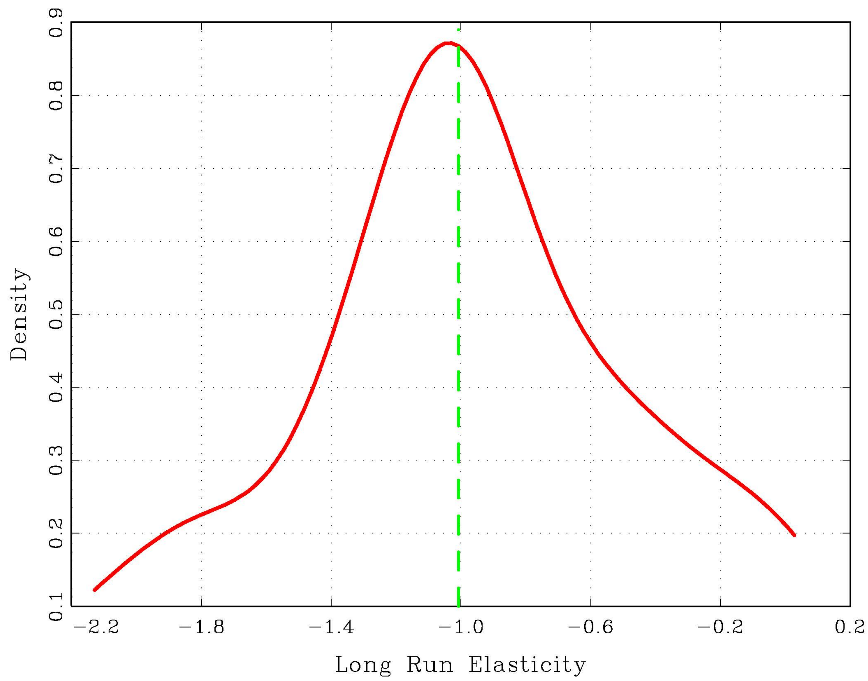
Note: All impulse-response function estimates are obtained from a bivariate VAR with the nominal exchange rate ordered first. 95% confidence bands were obtained from 2000 nonparametric bootstrap simulations.

Figure 3.7: Distribution of Initial Responses



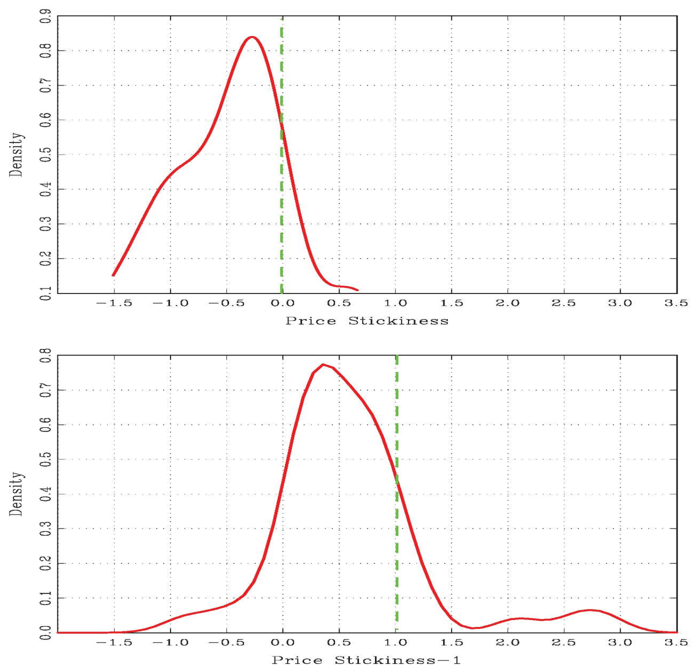
Note: We estimated non-parametric Kernel distribution using the Gaussian Kernel.

Figure 3.8: Distribution of Long Run Elasticity



Note: We estimated non-parametric Kernel distribution using the Gaussian Kernel.

Figure 3.9: Distribution of Price Stickiness



Note: We estimated non-parametric Kernel distribution using the Gaussian Kernel.

Table 3.1: Data Descriptions

Category	ID	IMF Code	Commodity
Cereal	1	PBARL	Barley, Canadian no.1 Western Barley
	2	PGNUTS	Groundnuts (peanuts), cif Argentina
	3	PMAIZMT	Maize (corn), U.S. No.2 Yellow, FOB Gulf of Mexico
	4	PRICENPQ	Rice, 5 percent broken milled white rice, Thailand price
	5	PSMEA	Soybean Meal, Chicago Soybean Meal Futures
	6	PSOYB	Soybeans, U.S. soybeans, Chicago Soybean futures contract
	7	PWHEAMT	Wheat, No.1 Hard Red Winter, FOB Gulf of Mexico
Vegetable Oil	8	PROIL	Rapeseed oil, crude, fob Rotterdam
	9	POLVOIL	Olive Oil, ex-tanker price U.K.
	10	PPOIL	Palm oil, Malaysia Palm Oil Futures
	11	PSOIL	Soybean Oil, Chicago Soybean Oil Futures
	12	PSUNO	Sunflower oil,US export price from Gulf of Mexico
Meat	13	PBEEF	Beef, Australian and New Zealand 85% lean fores
	14	PLAMB	Lamb, frozen carcass Smithfield London
	15	PPORK	Swine (pork), 51-52% lean Hogs, U.S. price
	16	PPOULT	Poultry (chicken), Whole bird spot price
Seafood	17	PFISH	Fishmeal, Peru Fish meal/pellets 65% protein, CIF
	18	PSALM	Fish (salmon), Farm Bred Norwegian Salmon, export price
	19	PSHRI	Shrimp, No.1 shell-on headless
Other Foods	20	PBANSOP	Bananas, Central American and Ecuador, FOB U.S. Ports
	21	PORANG	Oranges, miscellaneous oranges CIF French import price
	22	PSUGAISA	Sugar, Free Market, Coffee Sugar and Cocoa Exchange
	23	PSUGAUSA	Sugar, U.S. import price
Beverage	24	PCOCO	Cocoa beans, International Cocoa Organization cash price
	25	PCOFFOTM	Coffee, Arabica,New York cash price
	26	PCOFFROB	Coffee, Robusta, New York cash price
	27	PTEA	Tea, Mombasa, Kenya, US cents per kilogram
Ag Raw	28	PLOGORE	Soft Logs, Average Export price from the U.S. for Douglas Fir
	29	PLOGSK	Hard Logs, Best quality Malaysian meranti, import price Japan
	30	PSAWMAL	Hard Sawnwood, Dark Red Meranti, C & F U.K port
	31	PSAWORE	Soft Sawnwood, average export price of Douglas Fir, U.S. Price
	32	PCOTTIND	Cotton, Cotton Outlook 'A Index', CIF Liverpool
	33	PWOOLC	Wool, coarse, 23 micron, Australian Wool Exchange spot quote
	34	PWOOLF	Wool, fine, 19 micron, Australian Wool Exchange spot quote
	35	PRUBB	Rubber, Singapore Commodity Exchange, 1st contract
	36	PHIDE	Hides, Heavy native steers, over 53 pounds, US, Chicago

Category	ID	IMF Code	Commodity
Metals	37	PALUM	Aluminum, 99.5% minimum purity, LME spot price, CIF UK ports
	38	PCOPP	Copper, grade A cathode, LME spot price, CIF European ports
	39	PIORECR	Iron Ore Fines 62% FE spot (CFR Tianjin port), China import
	40	PLEAD	Lead, 99.97% pure, LME spot price, CIF European Ports
	41	PNICK	Nickel, melting grade, LME spot price, CIF European ports
	42	PTIN	Tin, standard grade, LME spot price
	43	PURAN	Uranium, NUEXCO, Restricted Price, Nuexco exchange spot
	44	PZINC	Zinc, high grade 98% pure
Fuel	45	PCOALAU	Coal, Australian thermal coal, 12,000- btu/pound
	46	POILAPSP	Crude Oil (petroleum), Price index, 2005 = 100
	47	POILBRE	Crude Oil (petroleum), Dated Brent, light blend 38 API, fob U.K.
	48	POILDUB	Oil; Dubai, medium, Fateh 32 API, fob Dubai Crude Oil
	49	POILWTI	Crude Oil (petroleum), West Texas Intermediate 40 API

Note: We obtained all commodity price data from the IMF website. The sample period is from January 1980 to November 2014.

Table 3.2: Impulse-Response Function Estimates

		Commodity Prices				*Exchange Rates*	
ID	IMF Code	$\phi_e^p(0)$	95% $C.I$	$\phi_e^p(\infty)$	95% $C.I$	$\phi_e^e(\infty)$	95% $C.I$
1	PBARL	-0.71	$[-1.20, -0.24]$	-2.47	$[-3.93, -1.25]$	1.37	$[1.06, 1.77]$
2	PGNUTS	-0.32	$[-0.75, 0.10]$	-2.55	$[-4.23, -1.02]$	1.39	$[1.08, 1.77]$
3	PMAIZMT	-0.08	$[-0.49, 0.33]$	-1.38	$[-2.59, -0.25]$	1.38	$[1.09, 1.75]$
4	PRICENPQ	-0.21	$[-0.55, 0.11]$	-1.80	$[-2.92, -0.73]$	1.34	$[1.05, 1.72]$
5	PSMEA	-0.54	$[-0.95, -0.15]$	-1.52	$[-2.86, -0.32]$	1.41	$[1.10, 1.80]$
6	PSOYB	-0.52	$[-0.94, -0.12]$	-1.59	$[-2.85, -0.45]$	1.41	$[1.10, 1.81]$
7	PWHEAMT	-0.31	$[-0.69, 0.03]$	-0.79	$[-1.99, 0.30]$	1.38	$[1.08, 1.76]$
8	PROIL	-1.06	$[-1.49, -0.63]$	-1.84	$[-3.20, -0.55]$	1.40	$[1.09, 1.77]$
9	POLVOIL	-1.10	$[-1.36, -0.86]$	-2.01	$[-2.96, -1.17]$	1.36	$[1.07, 1.72]$
10	PPOIL	-0.54	$[-1.10, 0.00]$	-1.16	$[-2.98, 0.59]$	1.39	$[1.08, 1.75]$
11	PSOIL	-0.37	$[-0.80, 0.05]$	-1.26	$[-2.57, 0.06]$	1.40	$[1.09, 1.79]$
12	PSUNO	-0.26	$[-0.85, 0.25]$	-1.95	$[-3.56, -0.54]$	1.30	$[1.01, 1.66]$
13	PBEEF	0.05	$[-0.18, 0.31]$	-0.25	$[-0.80, 0.33]$	1.39	$[1.09, 1.76]$
14	PLAMB	-1.00	$[-1.23, -0.78]$	-1.46	$[-2.24, -0.72]$	1.38	$[1.08, 1.76]$
15	PPORK	0.27	$[-0.35, 0.90]$	-0.43	$[-1.81, 0.98]$	1.38	$[1.08, 1.76]$
16	PPOULT	0.11	$[-0.01, 0.23]$	0.05	$[-0.34, 0.44]$	1.39	$[1.09, 1.78]$
17	PFISH	-0.82	$[-1.12, -0.53]$	-1.34	$[-2.55, -0.26]$	1.38	$[1.07, 1.74]$
18	PSALM	-1.11	$[-1.37, -0.85]$	-1.54	$[-2.41, -0.83]$	1.39	$[1.09, 1.78]$
19	PSHRI	-0.11	$[-0.36, 0.15]$	-0.61	$[-1.50, 0.31]$	1.37	$[1.07, 1.75]$
20	PBANSOP	0.29	$[-0.73, 1.31]$	-0.74	$[-1.93, 0.48]$	1.36	$[1.08, 1.73]$
21	PORANG	-1.11	$[-1.85, -0.45]$	-0.58	$[-1.93, 0.60]$	1.37	$[1.07, 1.75]$
22	PSUGAISA	-0.59	$[-1.20, 0.00]$	-1.44	$[-3.30, 0.32]$	1.36	$[1.07, 1.72]$
23	PSUGAUSA	-0.05	$[-0.24, 0.15]$	-0.48	$[-1.33, 0.43]$	1.38	$[1.07, 1.73]$
24	PCOCO	-0.63	$[-0.98, -0.28]$	-0.18	$[-1.23, 0.86]$	1.38	$[1.07, 1.75]$
25	PCOFFOTM	-0.06	$[-0.55, 0.41]$	-1.15	$[-2.77, 0.23]$	1.38	$[1.07, 1.75]$
26	PCOFFROB	-0.41	$[-0.81, -0.04]$	-1.47	$[-2.98, -0.19]$	1.39	$[1.07, 1.76]$
27	PTEA	-0.24	$[-0.68, 0.20]$	-0.86	$[-1.97, 0.38]$	1.39	$[1.09, 1.78]$
28	PLOGORE	0.06	$[-0.31, 0.48]$	0.07	$[-0.67, 0.83]$	1.39	$[1.09, 1.76]$
29	PLOGSK	-1.19	$[-1.53, -0.85]$	-0.79	$[-1.99, 0.34]$	1.39	$[1.08, 1.78]$
30	PSAWMAL	-0.82	$[-1.11, -0.55]$	-0.97	$[-2.01, -0.04]$	1.39	$[1.09, 1.76]$
31	PSAWORE	0.22	$[-0.13, 0.56]$	-0.08	$[-0.58, 0.39]$	1.40	$[1.09, 1.79]$
32	PCOTTIND	-0.22	$[-0.54, 0.11]$	-1.26	$[-2.70, -0.01]$	1.42	$[1.10, 1.83]$
33	PWOOLC	-0.80	$[-1.16, -0.46]$	-1.79	$[-3.05, -0.72]$	1.42	$[1.09, 1.81]$
34	PWOOLF	-0.37	$[-0.74, 0.00]$	-1.74	$[-3.16, -0.53]$	1.40	$[1.08, 1.80]$
35	PRUBB	-0.79	$[-1.30, -0.31]$	-3.02	$[-4.66, -1.71]$	1.42	$[1.10, 1.81]$
36	PHIDE	0.05	$[-0.41, 0.55]$	-1.35	$[-2.42, -0.32]$	1.35	$[1.06, 1.72]$
37	PALUM	-0.69	$[-1.11, -0.29]$	-2.57	$[-4.10, -1.33]$	1.42	$[1.09, 1.81]$
38	PCOPP	-1.19	$[-1.66, -0.76]$	-1.94	$[-3.46, -0.61]$	1.37	$[1.06, 1.72]$
39	PIORECR	-0.06	$[-0.36, 0.23]$	-1.27	$[-2.25, -0.36]$	1.42	$[1.09, 1.82]$
40	PLEAD	-1.12	$[-1.60, -0.64]$	-1.66	$[-3.23, -0.25]$	1.35	$[1.04, 1.71]$
41	PNICK	-1.01	$[-1.63, -0.44]$	-2.93	$[-5.01, -1.10]$	1.41	$[1.09, 1.80]$
42	PTIN	-0.99	$[-1.33, -0.64]$	-1.32	$[-2.73, -0.02]$	1.40	$[1.09, 1.78]$
43	PURAN	-0.31	$[-0.84, 0.17]$	-1.82	$[-3.69, -0.22]$	1.46	$[1.11, 1.89]$
44	PZINC	-0.75	$[-1.22, -0.32]$	-1.06	$[-2.74, 0.45]$	1.40	$[1.09, 1.77]$
45	PCOALAU	-0.27	$[-0.78, 0.13]$	-2.30	$[-3.84, -1.02]$	1.33	$[1.03, 1.69]$
46	POILAPSP	-0.81	$[-1.38, -0.27]$	-1.51	$[-2.88, -0.29]$	1.39	$[1.09, 1.78]$

Table 3.3: Dynamic Elasticity Estimates

ID	IMF Code	$\eta_e^p(\infty)$	95% C.I.	Id	IMF Code	$\eta_e^p(\infty)$	95% C.I.
1	PBARL	-1.80	$[-2.74, -1.00]$	26	PCOFFROB	-1.06	$[-2.02, -0.15]$
2	PGNUTS	-1.83	$[-2.96, -0.80]$	27	PTEA	-0.62	$[-1.46, 0.27]$
3	PMAIZMT	-0.99	$[-1.81, -0.18]$	28	PLOGORE	0.05	$[-0.48, 0.62]$
4	PRICENPQ	-1.34	$[-2.09, -0.56]$	29	PLOGSK	-0.57	$[-1.37, 0.25]$
5	PSMEA	-1.08	$[-1.90, -0.24]$	30	PSAWMAL	-0.70	$[-1.35, -0.03]$
6	PSOYB	-1.13	$[-1.93, -0.35]$	31	PSAWORE	-0.05	$[-0.40, 0.28]$
7	PWHEAMT	-0.57	$[-1.38, 0.22]$	32	PCOTTIND	-0.88	$[-1.74, -0.01]$
8	PROIL	-1.32	$[-2.22, -0.42]$	33	PWOOLC	-1.26	$[-1.95, -0.55]$
9	POLVOIL	-1.47	$[-2.13, -0.89]$	34	PWOOLF	-1.25	$[-2.02, -0.43]$
10	PPOIL	-0.84	$[-2.11, 0.42]$	35	PRUBB	-2.13	$[-2.90, -1.34]$
11	PSOIL	-0.90	$[-1.75, 0.05]$	36	PHIDE	-1.00	$[-1.71, -0.24]$
12	PSUNO	-1.50	$[-2.90, -0.39]$	37	PALUM	-1.82	$[-2.63, -1.03]$
13	PBEEF	-0.18	$[-0.58, 0.24]$	38	PCOPP	-1.42	$[-2.29, -0.48]$
14	PLAMB	-1.05	$[-1.59, -0.53]$	39	PIORECR	-0.90	$[-1.58, -0.27]$
15	PPORK	-0.31	$[-1.35, 0.70]$	40	PLEAD	-1.24	$[-2.18, -0.20]$
16	PPOULT	0.04	$[-0.25, 0.32]$	41	PNICK	-2.08	$[-3.33, -0.85]$
17	PFISH	-0.98	$[-1.80, -0.19]$	42	PTIN	-0.94	$[-1.90, -0.02]$
18	PSALM	-1.11	$[-1.62, -0.62]$	43	PURAN	-1.25	$[-2.29, -0.17]$
19	PSHRI	-0.44	$[-1.14, 0.22]$	44	PZINC	-0.76	$[-1.88, 0.34]$
20	PBANSOP	-0.55	$[-1.46, 0.33]$	45	PCOALAU	-1.73	$[-2.79, -0.77]$
21	PORANG	-0.42	$[-1.32, 0.46]$	46	POILAPSP	-1.08	$[-2.04, -0.21]$
22	PSUGAISA	-1.06	$[-2.29, 0.25]$	47	POILBRE	-1.08	$[-2.07, -0.15]$
23	PSUGAUSA	-0.35	$[-0.96, 0.30]$	48	POILDUB	-1.03	$[-1.99, -0.14]$
24	PCOCO	-0.13	$[-0.91, 0.63]$	49	POILWTI	-1.02	$[-1.95, -0.11]$
25	PCOFFOTM	-0.83	$[-1.90, 0.19]$				
	Mean: -0.98		Median: -1.02	skewness = -0.05		Kurtosis = 2.66	

Note: The long-run dynamic elasticity $\eta_e^p(\infty)$ is calculated by $\phi_e^p(\infty)/\phi_e^e(\infty)$. Long-run response functions are again measured by the 25-period ahead response function estimates. 95% confidence bands are obtained by taking 2.5% and 97.5% percentiles from 2000 nonparametric bootstrap iterations. We employed the T-test and t=0.293.

Table 3.4: Price Stickiness Estimates- measure 1

ID	IMF Code	$\eta_e^p(\infty) - \eta_e^p(0)$	Id	IMF Code	$\eta_e^p(\infty) - \eta_e^p(0)$
1	PBARL	-1.09	26	PCOFFROB	-0.64
2	PGNUTS	-1.51	27	PTEA	-0.38
3	PMAIZMT	-0.92	28	PLOGORE	-0.01
4	PRICENPQ	-1.13	29	PLOGSK	0.62
5	PSMEA	-0.54	30	PSAWMAL	0.12
6	PSOYB	-0.60	31	PSAWORE	-0.27
7	PWHEAMT	-0.26	32	PCOTTIND	-0.67
8	PROIL	-0.25	33	PWOOLC	-0.46
9	POLVOIL	-0.37	34	PWOOLF	-0.88
10	PPOIL	-0.29	35	PRUBB	-1.34
11	PSOIL	-0.53	36	PHIDE	-1.05
12	PSUNO	-1.24	37	PALUM	-1.12
13	PBEEF	-0.23	38	PCOPP	-0.23
14	PLAMB	-0.05	39	PIORECR	-0.84
15	PPORK	-0.59	40	PLEAD	-0.11
16	PPOULT	-0.07	41	PNICK	-1.07
17	PFISH	-0.16	42	PTIN	0.04
18	PSALM	0.01	43	PURAN	-0.93
19	PSHRI	-0.34	44	PZINC	-0.01
20	PBANSOP	-0.84	45	PCOALAU	-1.45
21	PORANG	0.68	46	POILAPSP	-0.27
22	PSUGAISA	-0.47	47	POILBRE	-0.21
23	PSUGAUSA	-0.30	48	POILDUB	-0.33
24	PCOCO	0.50	49	POILWTI	-0.22
25	PCOFFOTM	-0.78			
	Mean: -0.47	Median: -0.37	Skewness: -0.01	Kurtosis: 2.66	

Note: We employed the T-test and t= -6.543

Table 3.5: Price Stickiness Estimates - measure 2

ID	IMF Code	$\eta_e^p(0)/\eta_e^p(\infty)$	Id	IMF Code	$\eta_e^p(0)/\eta_e^p(\infty)$
1	PBARL	0.39	26	PCOFFROB	0.39
2	PGNUTS	0.18	27	PTEA	0.38
3	PMAIZMT	0.08	28	PLOGORE	1.24
4	PRICENPQ	0.16	29	PLOGSK	2.08
5	PSMEA	0.50	30	PSAWMAL	1.17
6	PSOYB	0.46	31	PSAWORE	-4.04
7	PWHEAMT	0.54	32	PCOTTIND	0.24
8	PROIL	0.81	33	PWOOLC	0.63
9	POLVOIL	0.75	34	PWOOLF	0.29
10	PPOIL	0.65	35	PRUBB	0.37
11	PSOIL	0.41	36	PHIDE	-0.05
12	PSUNO	0.17	37	PALUM	0.38
13	PBEEF	-0.30	38	PCOPP	0.84
14	PLAMB	0.95	39	PIORECR	0.06
15	PPORK	-0.88	40	PLEAD	0.91
16	PPOULT	2.87	41	PNICK	0.49
17	PFISH	0.84	42	PTIN	1.05
18	PSALM	1.00	43	PURAN	0.25
19	PSHRI	0.25	44	PZINC	0.99
20	PBANSOP	-0.53	45	PCOALAU	0.16
21	PORANG	2.61	46	POILAPSP	0.75
22	PSUGAISA	0.56	47	POILBRE	0.81
23	PSUGAUSA	0.15	48	POILDUB	0.68
24	PCOCO	4.84	49	POILWTI	0.78
25	PCOFFOTM	0.07			
	Mean: 0.58	Median: 0.49	Skewness: -0.04	Kurtosis: 11.03	

Note: We employed the T-test and t= -2.64

Chapter 4

On the Robustness of the Impulse-Response Function of Recursively Identified VAR Models

Abstract

As pointed out by Lütkepohl (1991), the impulse response function from recursively identified vector autoregressive models is not, in general, invariant to the ordering of the variables in the VAR. This paper reports potentially useful facts that show under what circumstances these impulse response functions are robust to this so-called Wold ordering. We demonstrate that all response functions to innovations in a group of the variables with known orderings are invariant to the order of remaining variables in the system among all possible alternative orderings as long as the former group is ordered first. We demonstrate that this fact applies to all recursively identified VAR models either by the short-run or by the long-run restrictions. Same principle applies to the vector error correction models too.

4.1 Introduction

Since the seminal work of Sims (1980), the impulse-response function (IRF) analysis for recursively identified Vector Autoregress (VAR) models has been frequently used in the empirical macroeconomic literature. As pointed out by Lütkepohl (1991), for instance, the IRFs are not invariant to the ordering of the variables in the VAR. That is, recursively identified VAR analysis may be subject to the so-called Wold-ordering problem. Pesaran and Shin (1998) introduced the generalized impulse-response function (GIRF) analysis for linear VAR models. However Kim (2013) demonstrated that the GIRF actually generates IRFs from a set of identifying assumptions that conflict each other.

This paper discusses potentially useful facts about the robustness of the IRF. Beard et al. (2012) investigated the effect of an increase in kidney donations from deceased donors on those from live donors. They show that the responses of live donantion and waiting times to the deceased donation are invariant to the ordering of live donation and waiting time, when the deceased donation is ordered first.

We extend these results to a more general framework. Suppose that there are two vectors of endogenous variables in a VAR, $\mathbf{x}_t$ and $\mathbf{z}_t$. $\mathbf{x}_t$ comes with a known Wold ordering while we don't have any a priori information on the ordering for $\mathbf{z}_t$. The present paper demonstrate that the IRFs to the variables in $\mathbf{x}_t$ shocks are independent of orderings for $\mathbf{z}_t$. That is, any arbitrary re-shuffling of $\mathbf{z}_t$ is irrelevant to IRFs to $\mathbf{x}_t$ shocks.

The organization of the present paper is as follows. Section 2 provides analytical demonstrations for our major findings. In Section 3, we discuss the potential benefit of our findings. Section 4 concludes.

4.2 Robustness

Consider the following VAR system with k endogenous variables.

$$\mathbf{y}_t = \mathbf{B_1}\mathbf{y}_{t-1} + \cdots \mathbf{B_p}\mathbf{y}_{t-p} + \mathbf{C}\mathbf{u}_t, \tag{4.1}$$

where $\mathbf{C}$ denotes the lower-triangular matrix that governs the contemporaneous relationships between the structural shocks, $\mathbf{u}_t$. We write $\mathbf{y}_t$ as

$$\mathbf{y}_t = [\mathbf{x}_t, \mathbf{z}_t]'.$$

and we assume that $\mathbf{x}_t$ is a $l \times 1$ vector of endogenous variables with a *known* Wold ordering, while $\mathbf{z}_t$ is a $(k-l) \times 1$ vector of endogenous variables with no *a priori* information as to the ordering.

Let $\mathbf{F}$ be the $kp \times kp$ state-space representation matrix of (4.1):

$$\mathbf{F} = \begin{bmatrix} \mathbf{B_1} \cdots \mathbf{B_p} & \\ \mathbf{I}_{k(p-1)} & \mathbf{0} \end{bmatrix},$$

where $\mathbf{I}_{k(p-1)}$ is a $k(p-1) \times k(p-1)$ identity matrix, while $\mathbf{0}$ is a $k(p-1) \times 1$ null vector.

$$\frac{\partial E_t \mathbf{y}_{t+h}}{\partial \mathbf{u}_t} = \mathbf{F}_{kk}(h)\mathbf{C}, \tag{4.2}$$

where $\mathbf{F}_{kk}(h)$ denotes the top-left $k \times k$ sub-matrix of $\mathbf{F}^h$. The $\mathbf{C}$ matrix can be identified by the following Choleski decomposition.

$$\mathbf{\Sigma} = \mathbf{C}\mathbf{C}', \tag{4.3}$$

where $\mathbf{\Sigma}$ is the least squares variance-covariance matrix estimate, which is symmetric and positive definite, thus $\mathbf{C}$, its associated Choeski factor, is the unique lower-triangular matrix.

4.2.1 Responses to a Scalar x_t

Assume that $\mathbf{x}_t$ is a scalar variable, that is, $p = 1$. (4.3) is then the following..

$$\begin{aligned} \begin{bmatrix} \sigma_{xx} & \sigma_{x\mathbf{z}} \\ \sigma_{x\mathbf{z}} & \mathbf{\Sigma_{zz}} \end{bmatrix} &= \begin{bmatrix} c_{xx} & \mathbf{0} \\ \mathbf{c}_{x\mathbf{z}} & \mathbf{C_{zz}} \end{bmatrix} \begin{bmatrix} c_{xx} & \mathbf{c}'_{x\mathbf{z}} \\ \mathbf{0} & \mathbf{C}'_{\mathbf{zz}} \end{bmatrix} \\ &= \begin{bmatrix} c_{xx}^2 & c_{xx}\mathbf{c}'_{x\mathbf{z}} \\ c_{xx}\mathbf{c}_{x\mathbf{z}} & \mathbf{c}_{x\mathbf{z}}\mathbf{c}'_{x\mathbf{z}} + \mathbf{C_{zz}}\mathbf{C}'_{\mathbf{zz}} \end{bmatrix} \end{aligned} \tag{4.4}$$

Also, assume that $q = 2$, that is, $\mathbf{z}_t = [y_t,\ z_t]'$. It turns out that the Choleski factor for this system is the following.

$$\mathbf{C} = \begin{bmatrix} \sqrt{\sigma_{xx}} & 0 & 0 \\ \frac{\sigma_{xy}}{\sqrt{\sigma_{xx}}} & \sqrt{\sigma_{yy} - \frac{\sigma_{xy}^2}{\sigma_{xx}}} & 0 \\ \frac{\sigma_{xz}}{\sqrt{\sigma_{xx}}} & \frac{\sigma_{xx}\sigma_{yz} - \sigma_{xy}\sigma_{xz}}{\sqrt{\sigma_{xx}^2\sigma_{yy} - \sigma_{xx}\sigma_{xy}^2}} & \sqrt{\sigma_{zz} - \frac{\sigma_{xz}^2}{\sigma_{xx}} - \frac{(\sigma_{xx}\sigma_{yz} - \sigma_{xy}\sigma_{xz})^2}{\sigma_{xx}^2\sigma_{yy} - \sigma_{xx}\sigma_{xy}^2}} \end{bmatrix} \tag{4.5}$$

where σ_{ij} denotes $(i,j)^{th}$ element of Σ. Note that the h-period ahead impulse-response functions of all variables to the shock to the first variable is the following.

$$\begin{aligned} \begin{bmatrix} f_{xx}(h) & f_{xy}(h) & f_{xz}(h) \\ f_{yx}(h) & f_{yy}(h) & f_{yz}(h) \\ f_{zx}(h) & f_{zy}(h) & f_{zz}(h) \end{bmatrix} * \begin{bmatrix} c_{xx} \\ c_{xy} \\ c_{xz} \end{bmatrix} &= \begin{bmatrix} c_{xx}f_{xx}(h) + c_{xy}f_{xy}(h) + c_{xz}f_{xz}(h) \\ c_{xx}f_{yx}(h) + c_{xy}f_{yy}(h) + c_{xz}f_{yz}(h) \\ c_{xx}f_{zx}(h) + c_{xy}f_{zy}(h) + c_{xz}f_{zz}(h) \end{bmatrix} \\ &= \begin{bmatrix} \frac{\partial E_t x_{t+h}}{\partial u_{x,t}} \\ \frac{\partial E_t y_{t+h}}{\partial u_{x,t}} \\ \frac{\partial E_t z_{t+h}}{\partial u_{x,t}} \end{bmatrix}, \end{aligned} \tag{4.6}$$

where $f_{ij}(h)$ is the $(i,j)^{th}$ element of $\mathbf{F}_{kk}(h)$.

Suppose now that we change the ordering for $\mathbf{z}_t = [y_t,\ z_t]'$ to $\tilde{\mathbf{z}}_t = [z_t,\ y_t]'$. That is, we re-shuffle $\mathbf{z}_t$ while keeping x_t in the first. Note that there is no change in $\mathbf{F}_{kk}(h)$

other than locations of its elements, because $\mathbf{F}$ is obtained from a reduced form equation by equation OLS estimations. Note also that the *first* column of $\mathbf{C}$ does not change other than the locations of the second and the third components. That is, the h-period ahead impulse-response functions of all variables to the shock to the first variable is,

$$\begin{bmatrix} f_{xx}(h) & f_{xz}(h) & f_{xy}(h) \\ f_{zx}(h) & f_{zz}(h) & f_{zy}(h) \\ f_{yx}(h) & f_{yz}(h) & f_{yy}(h) \end{bmatrix} * \begin{bmatrix} c_{xx} \\ c_{xz} \\ c_{xy} \end{bmatrix} = \begin{bmatrix} c_{xx}f_{xx}(h) + c_{xz}f_{xz}(h) + c_{xy}f_{xy}(h) \\ c_{xx}f_{zx}(h) + c_{xz}f_{zz}(h) + c_{xy}f_{zy}(h) \\ c_{xx}f_{yx}(h) + c_{xz}f_{yz}(h) + c_{xy}f_{yy}(h) \end{bmatrix} = \begin{bmatrix} \frac{\partial E_t x_{t+h}}{\partial u_{x,t}} \\ \frac{\partial E_t z_{t+h}}{\partial u_{x,t}} \\ \frac{\partial E_t y_{t+h}}{\partial u_{x,t}} \end{bmatrix}, \tag{4.7}$$

which confirmed our claim for this tri-variate VAR. It is straightforward to see that the same results apply to cases for any finite number q. In a nutshell, the impulse-response functions to the variable ordered first are the same for any Wold-ordering for $\mathbf{z}_t$. One the contrary, the response functions to the $\mathbf{z}_t$ variables change when we shuffle the ordering for $\mathbf{z}_t$.

4.2.2 Responses to Shocks to $\mathbf{x}_t$ Variables

Now consider the cases when $p \geq 2$. For example, let $\mathbf{x}_t = [x_t,\ y_t]'$ and $\mathbf{z}_t = [z_t,\ w_t]'$. Then, the Choleski decomposition for this quad-variate VAR is the following.

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} & \sigma_{xw} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} & \sigma_{yw} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} & \sigma_{zw} \\ \sigma_{xw} & \sigma_{yw} & \sigma_{zw} & \sigma_{ww} \end{bmatrix} = \begin{bmatrix} c_{xx} & 0 & 0 & 0 \\ c_{xy} & c_{yy} & 0 & 0 \\ c_{xz} & c_{yz} & c_{zz} & 0 \\ c_{xw} & c_{yw} & c_{zw} & c_{ww} \end{bmatrix} \begin{bmatrix} c_{xx} & c_{xy} & c_{xz} & c_{xw} \\ 0 & c_{yy} & c_{yz} & c_{yw} \\ 0 & 0 & c_{zz} & c_{zw} \\ 0 & 0 & 0 & c_{ww} \end{bmatrix} \tag{4.8}$$

It turns out the *first two* columns of $\mathbf{C}$ are,

$$c_{xx} = \sqrt{\sigma_{xx}}, c_{xy} = \frac{\sigma_{xy}}{\sqrt{\sigma_{xx}}}, c_{xz} = \frac{\sigma_{xz}}{\sqrt{\sigma_{xx}}}, c_{xw} = \frac{\sigma_{xw}}{\sqrt{\sigma_{xx}}}, \tag{4.9}$$

and

$$c_{yy} = \sqrt{\sigma_{yy} - \frac{\sigma_{xy}^2}{\sigma_{xx}}}, c_{yz} = \frac{\sigma_{xx}\sigma_{yz} - \sigma_{xy}\sigma_{xz}}{\sqrt{\sigma_{xx}^2\sigma_{yy} - \sigma_{xx}\sigma_{xy}^2}}, c_{yw} = \frac{\sigma_{xx}\sigma_{yw} - \sigma_{xy}\sigma_{xw}}{\sqrt{\sigma_{xx}^2\sigma_{yy} - \sigma_{xx}\sigma_{xy}^2}} \tag{4.10}$$

Note that re-shuffling $\mathbf{z}_t = [z_t,\ w_t]'$ to $\tilde{\mathbf{z}}_t = [w_t,\ z_t]'$does not change the first two columns of $\mathbf{C}$ other than the locations of its elements. Recall $\mathbf{F}(h)$ is ordering free so is $\mathbf{F}_{kk}(h)$. Because the h-period ahead impulse-response functions to the shock to the first two variables, $[x_t,\ y_t]'$, are determined by $\mathbf{F}_{kk}(h)$ and the first two columns of $\mathbf{C}$, they are the same irrespective to the Wold ordering for $\mathbf{z}_t$. Again, the response functions to $\mathbf{z}_t$ variables are not robust to different orderings for $\mathbf{z}_t$. Further generalization of $p > 2$ and $q > 2$ is cumbersome but straightforward. The following remark summarizes.

4.2.3 Long-Run Assumptions

Blanchard and Quah (1990) proposed an identifying scheme that relies on the long-run proposition in economics. Consider a reduced form VAR model

$$B(L)y_t = \varepsilon_t$$

where $B(L) = I - B_1L - \cdots - B_pL^p$. Let $\mathbf{C}$ be the Choleski factor of the symmetric and positive definite matrix

$$B(1)^{-1}\mathbf{\Sigma}B(1)^{-1'} \tag{4.11}$$

Then the h-period ahead impulse-response function can be calculated by

$$\mathbf{F}_{kk}(h)B(1)\mathbf{C}, \tag{4.12}$$

where $\mathbf{F}$ again is the $kp \times kp$ state-space representation matrix and $\mathbf{F}_{kk}(h)$ denotes the top-left $k \times k$ sub-matrix of $\mathbf{F}^h$.

Consider a k-variate finite order VAR with $[\mathbf{x}_t,\ \mathbf{z}_t]'$. Given a known Wold ordering for $\mathbf{x}_t$, it can be similarly demonstrated that the impulse-response functions to the $\mathbf{x}_t$ variable shocks are the same for any arbitrary orderings for $\mathbf{z}_t$. The same applies to VECM models and VAR models with differenced variables.

4.3 Concluding Remarks

This study claims that all response functions to innovations in a group of the variables with known orderings are invariant to the order of remaining variables in the system among all possible alternative orderings as long as the former group is ordered first. Similarly we can demonstrate this fact applies to all recursively identified VAR models either by the short-run or by the long-run restrictions. Same principle applies to the vector error correction models too.

Chapter 5

Conclusions

This dissertation focuses on quantitative methods and time series econometrics and their application to Applied Macroeconomics fields such as international finance and commodity prices. In the first chapter I studied time series properties of national stock prices using newly developed econometric techniques including nonlinear panel unit root tests. I believe my empirical findings in this chapter provide useful implications for international asset market participants. In the second chapter, I studied the impact of exchange rate shock on the commodity prices using VAR (Vector Autoregressive) Model, a forecasting technique in time series analysis. The findings in this chapter has important policy implications especially for the developing countries. The third chapter adds important technical contributions to the existing multivariate time series model literature that all response functions to innovations in a group of the variables with known orderings are invariant to the order of remaining variables in the system among all possible alternative orderings as long as the former group is ordered first.

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