Mextram Bipolar Transistor Model Parameter Extraction Using Python

by

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Abstract

This work implements Mextram 504 parameter extraction using python. A 50 GHz SiGe HBT is used to demonstrate effectiveness of this developed extraction program. Comparisons with results extracted using built-in mextram 504 parameter extraction package of IC-CAP, excellent agreements are achieved between our python program and IC-CAP.
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Chapter 1
Introduction

1.1 SiGe HBT fundamentals

The Silicon-Germanium heterojunction bipolar transistor (SiGe HBT) has gained worldwide attention, since it could get higher current gain, higher cut-off frequency and lower base resistance compared with Si BJT. The heart of SiGe technology is a SiGe heterojunction bipolar transistor (HBT), which is the first practical bandgap engineering device realized in silicon, is easy to integrated with modern CMOS technology. Adding Ge to Si BJT introduced a number of exciting performance improvements. The base region of SiGe HBTs is typically the region where SiGe alloy is used instead of Si. The basic operational principle of SiGe HBT can be best understood by considering the band diagram shown in Figure 1.1. It illustrates the difference between SiGe HBT and Si BJT by showing the energy-band

![Energy band diagrams of a graded-base SiGe HBT and an Si BJT](image)

Figure 1.1: Energy band diagrams of a graded-base SiGe HBT and an Si BJT [3]
diagrams for both SiGe HBT and Si BJT biased identically in forward-active mode. The Ge profile linearly increases from zero near emitter-base (EB) junction to some maximum value near collector-base (CB) junction, and then rapidly ramps down to zero [4]. Since the energy bandgap of Ge is smaller than that of Si, adding germanium into the base region of the transistor leads to an additional bandgap shrinkage, which is approximately 75 meV for each 10 percent of Ge introduced[2]. The reduction of bandgap decreases band transit time, which gives a higher $f_T$. Smaller base bandgap also increases electron injection, which leads to a higher $\beta$. At same collector current density, compared with normal BJT, SiGe HBT allows us to have a higher base region doping concentration, which reduces base resistance.

### 1.2 Mextram basics

Mextram[1][11] model is a widely used vertical bipolar transistor model. It contains many features that the widely-used Ebers-Moll[5] and Gummel-Poon model[6] lacks. Mextram is the acronym of the ”most exquisite transistor model”. The first Mextram release was introduced as Level 501 in 1985[7] . Later Level 502[8], 503[9] and 504[10] were respectively released in 1986 , 1994 and 2000. And development was never stopped following the requirement of updated technology. The latest accessible version is Level 504.12[1] . Mextram contains descriptions for the following effects:

- Bias-dependent Early effect
- Low-level non-ideal base currents
- High-injection effects
- Ohmic resistance of the epilayer
- Velocity saturation effects on the resistance of the epilayer
- Hard and quasi-saturation (including Kirk effect)
- Weak avalanche in the collector-base junction (optionally including snap-back behaviour)
• Zener-tunneling current in the emitter-base junction
• Charge storage effects
• Split base-collector and base-emitter depletion capacitance
• Substrate effects and parasitic PNP
• Explicit modelling of inactive regions
• Current crowding and conductivity modulation of the base resistance
• First order approximation of distributed high frequency effects in
  the intrinsic base (high-frequency current crowding and excess phase-shift)
• Recombination in the base (meant for SiGe transistors)
• Early effect in the case of a graded bandgap (meant for SiGe transistors)
• Temperature scaling
• Self-heating

• Thermal noise, shot noise and 1/f-noise

   In Mextram model, there are five internal nodes and 79 parameters, including parameters of model flag, parameters of noise and the reference temperature, parameters of temperature scaling, parameters of individual transistor design and parameters to be determined by the fitting the model to the transistor characteristics of a specific device and at a specific temperature.

   Some parts of the model are optional and can be switched on or off by setting flags. These are the extended modeling of reverse behaviour, the distributed high-frequency effects, and the increase of the avalanche current when the current density in the epilayer exceeds the doping level.
Fig. 1.2 shows the equivalent circuit of Mextram model as it is specified in its latest release Level 504.12 [1]. The branches representing model currents and charges are schematically associated with different physical regions of a bipolar transistor separated by the base-emitter, base-collector, and substrate-collector junctions.

Figure 1.2: The full Mextram equivalent circuit for the vertical NPN transistor[1]
Chapter 2
Parameter extraction with Python

Python is a widely used general-purpose, high-level programming language[13]. Python supports multiple programming paradigms, including object-oriented, imperative and functional programming or procedural styles. It features a dynamic type system and automatic memory management and has a large and comprehensive standard libraries.[14] In our work, we use the lmfit package[17] which is a non-linear optimization interface based on Levenberg-Marquardt algorithm to realize the optimization routine.

2.1 Extraction using Python

Generally, there are two ways to extract Mextram transistor parameters. The first one is using build-in functions of extraction software. These functions has the simplified Mextram formulas implemented in and parameters can be extracted on different conditions of the transistor. However, the disadvantage of this method is that if we change the model of Mextram, these functions will be useless. In our python programming extraction routine, we also have simplified formulas of Mextram model implemented in the program I wrote. Unlike the extraction software, the program is still useful after we make change to the Mextram model as long as we update the formulas.

Another alternative way is using a circuit simulator[15] which has the full Mextram model behaviour. However, using the simulator take a very long time. Also some of the parameters, like the Early voltages and the avalanche parameters can be extracted without correct modelling of the absolute value of the current. For these extractions dummy parameters are optimised together with the Mextram parameters. For instance the extraction of
the avalanche parameters needs the correct base current at zero $V_{CB}$. This is more difficult when using a simulator[12].

2.2 Data Input

Generally, the measurement procedure is done throw device modeling software IC-CAP[16]. The output measurement data is saved in MDM file. In the MDM file, data within a file is organized into multiple groups of tabular data. Each group of tabular data is arranged in columns representing the innermost sweep data and its associated dependent data. In our program we writing a parser program to read the measurement MDM file and capture data. The code is:

```python
def mdm_reader(fname):
    f = open(fname, 'r')
    text = f.read()
    ldata = []
    for begin_match, end_match in zip(re.finditer('BEGIN_DB', text),
                                       re.finditer('END_DB', text)):
        s = begin_match.end()
        e = end_match.start()
        body = text[s:e]
        data = read_block(body)
        ldata.append(data)
    # convert to numpy array
    adata = np.array(ldata)
    f.close()
    return adata

def read_block(body):
```

6
2.3 Optimization with python

In parameter extraction routine, as the simulation data is calculated from the parameters we want to extract, changing the parameters and match the simulation data with measurement data will give us the value of the parameters. In our work we use lmfit package to realize the optimization function. Take the base-collector depletion capacitance extraction as an example, we first calculate $C_{bc}$ as function of $x_P$ and $p_C$ then use the minimize function which will change $x_P$ and $p_C$ to make calculated $C_{bc}$ match the measurement $C_{bc}$. And this is realized by code:

```python
import mdmreader
from lmfit import minimize, Parameters, Parameter, report_fit, fit_report
from myplot import *
```
import os

def cbc_reader(fname):
    data = mdmreader.mdm_reader(fname)
    vcb_m = data[:,:,0][0]
    cbc_m = data[:,:,1][0]
    x, y = vcb_m, cbc_m
    return x, y

# calculate cbc as function of xp and pc

def cbc_calculation(pars, x, y):
    x = -x
    vals = pars.valuesdict()
    CJC = vals['cjc']
    XP = vals['xp']
    VDC = vals['vdc']
    PC = vals['pc']
    CBCO = vals['cbco']
    AJC = vals['ajc']
    TEMP = vals['temp']
    KBdivQQ = vals['kq']

    Tk = TEMP + 273.15
    vt = KBdivQQ * Tk
    bjc = (AJC - XP) / (1 - XP);
    vfc = VDC * (1 - pow(bjc, -1.0 / PC))
    e = np.exp((x - vfc) / (0.1 * VDC))
vjc = x - 0.1*VDC*np.log(1+e)

dvjc_dvbc = 1.0/(1.0+e)

dvtc_dvbc = ( pow(1-vjc/VDC,-PC)-bjc ) * dvjc_dvbc + bjc

model = CJC*( (1-XP)*dvtc_dvbc + XP) + CBCO

return model - y

# find the value of cjc

def findcj(voltage, cap):
    for i in range(len(voltage)):
        if voltage[i] == 0:
            return cap[i]

    z = np.polyfit(voltage, cap,3)
    p = np.poly1d(z)
    return p(0)

def cbc_optimization(data, mdmpath):
    ofname = 'cap\POR_Vbc_Cbc_300K.mdm'
    fname = os.path.join(mdmpath, ofname)
    x,y = cbc_reader(fname)
    cbc_params = Parameters()
    cbc_params.add('cjc', value=findcj(x, y), min = 0, max = 1e-9)
    cbc_params.add('xp', value=data.xp, min = 0, max = 1)
    cbc_params.add('vdc', value = data.vdc, vary = False)
    cbc_params.add('pc', value = data.pc, min = 0.01, max = 1)
    cbc_params.add('cbco', value = data.cbco,vary = False)
    cbc_params.add('ajc', value = data.ajc,vary = False)
cbc_params.add('temp', value = data.temp, vary = False)
cbc_params.add('kq', value = data.kq, vary = False)

result = minimize(cbc_caculation, cbc_params, args=(x,y))

final = y + result.residual
report_fit(cbc_params)

2.4 Parameter extraction strategy

A reliable, robust and unambiguous parameter extraction method is very important. The use of a very accurate compact model with poorly extracted parameters will produce bad prediction of device and circuit performance. [18] The general strategy of parameter extraction is to put parameters in small groups and extract these parameters simultaneously out of measured data sensitive to these parameters. [12]

In Mextram, most of the parameters can be extracted directly from measured data, including depletion capacitance $C-V$, $dc$ Gummel plots, $dc$ output characteristics, $dc$ Early voltage measurement, and $ac$ S-parameter measurement. Some special measurements are taken to extract terminal resistance, such as $R_E$-flyback and $R_{Cc}$-active methods.

The electrical parameter extraction includes low-current parameters extraction and high-current parameters extraction. Low-current parameters extraction is straightforward. However, high-current parameters extraction is much more difficult because in that regime many physics effects play a role. In general, we first extract low-current parameters then high-current parameters, a typical sequence of the Mextram parameters extraction is given in Table 2.1. The frame of the extraction program is:

q = BJT()
Base-emitter depl. cap. \( C_{ji}, p_E, (V_{Dr}) \)
Base-collector depl. cap. \( C_{jc}, p_C, X_p \)
Substrate-emitter depl. cap. \( C_{jg}, p_S, (V_{ds}) \)

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<td>( V_{er} )</td>
</tr>
<tr>
<td>Forward-Early</td>
<td>( V_{ef} )</td>
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<td>Forward-Gummel</td>
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</tr>
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<tr>
<td>Reverse-Gummel</td>
<td>( I_{Ss}, (I_{ks}) )</td>
</tr>
<tr>
<td>Reverse-Gummel</td>
<td>( \beta_{ri}, I_{Br}, V_{Lr} )</td>
</tr>
<tr>
<td>Output-characteristic</td>
<td>( R_{th}, I_k )</td>
</tr>
<tr>
<td>Forward-Gummel</td>
<td>( R_{C_v}, (V_{dc}) )</td>
</tr>
<tr>
<td>Cut-off frequency</td>
<td>( SC R_{C_v}, I_{hc}, \tau_E, \tau_{epi}, (\tau_B, \alpha_X) )</td>
</tr>
</tbody>
</table>

Table 2.1: A typical grouping of parameters in Mextram that can be used in the extraction procedure.[12]
iks_optimization(q, mdmpath)
rth_optimization(q, mdmpath)
j = 0
for index in range(2):
    ik_optimization(q, mdmpath)
    icib_optimization(q, mdmpath)
    ik_optimization(q, mdmpath)
    ft_optimization(q, mdmpath)
reverse_optimization(q, mdmpath)
taue_optimization(q, mdmpath)
f = open('par.txt', 'w')
f.write(str(q.ParamDefs))
f.close()
Chapter 3

Measurement and Parameter Initialization

The first step for extraction of parameters of a single transistor is get the measurement data. In order to extract reliable parameters it is important that the measurements are done over a large range of collector, base and emitter biasing conditions.

3.1 Measurements

3.1.1 Forward-Gummel measurement

![Forward-Gummel measurement setup.](image)

In the forward-Gummel measurement setup, as shown in Figure 3.1 base and collector are grounded. Emitter voltage is swept. The reverse junction bias is 0 V to avoid the generation of avalanche current and self-heating effects. The measured $I_C$, $I_B$, $I_E$ versus $V_{BE}$ is shown is Figure 3.2
### Table 3.1: Forward-Gummel measurement

<table>
<thead>
<tr>
<th>T (K)</th>
<th>Start (V)</th>
<th>Stop (V)</th>
<th>Step (mV)</th>
<th>V_B (V)</th>
<th>V_C</th>
<th>V_SUB (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>393</td>
<td>0</td>
<td>-1.2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>300</td>
<td>-0.3</td>
<td>-1.2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>223</td>
<td>-0.5</td>
<td>-1.3</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

Figure 3.2: Forward-Gummel measurement $I_C, I_B, I_E$ versus $V_{BE}$.

### 3.1.2 Reverse-Gummel measurement

In the reverse-Gummel measurement, as shown in Figure 3.3, base and emitter are grounded. Collector voltage is swept. Figure 3.4 shows the measured $I_C, I_B, I_E$ and $I_{SUB}$ versus $V_{BC}$.

### Table 3.2: Reverse-Gummel measurement

<table>
<thead>
<tr>
<th>T (K)</th>
<th>$V_C$(V)</th>
<th>Start (V)</th>
<th>Stop (V)</th>
<th>Step (mV)</th>
<th>$V_B$ (V)</th>
<th>$V_E$(V)</th>
<th>$V_{SUB}$ (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>393</td>
<td>-1.2</td>
<td>0</td>
<td>-1.2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>-1.2</td>
</tr>
<tr>
<td>300</td>
<td>-0.3</td>
<td>-1.2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1.2</td>
</tr>
<tr>
<td>223</td>
<td>-0.5</td>
<td>-1.3</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1.2</td>
</tr>
</tbody>
</table>
Figure 3.3: Reverse-Gummel measurement setup.

Figure 3.4: Reverse-Gummel measurement $I_C, I_B, I_E$ and $I_{SUB}$ versus $V_{BC}$. 
<table>
<thead>
<tr>
<th>T (K)</th>
<th>$I_B$</th>
<th>$V_C$</th>
<th>$V_E$ (V)</th>
<th>$V_{SUB}$ (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>393</td>
<td>65.4n, 211.46n, 671.45n, 2.05u, 15.18u, 33.46u, 91.24u, 174.04u, 275.97u</td>
<td>0</td>
<td>4.4</td>
<td>50</td>
</tr>
<tr>
<td>300</td>
<td>39.54n, 85.74n, 185.22n, 480.67n, 2.44u, 13.84u, 23.82u, 37.69u, 57.72u</td>
<td>0</td>
<td>4.4</td>
<td>50</td>
</tr>
<tr>
<td>223</td>
<td>50.38n, 140.17n, 383.65n, 1.01u, 5.38u, 17.60u, 30.36u, 47.75u, 74.86u</td>
<td>0</td>
<td>4.4</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 3.3: Force-$I_B$ output characteristic measurement

### 3.1.3 Output characteristic measurement

In the output characteristic measurement shown in Figure 3.5, Force-$I_B$ method is used. Nine $I_B$ currents are used in Force-$I_B$ method. Figure 3.6 and 3.7 shows measured $I_C-V_{CE}$ and $V_{BE} - V_{CE}$ respectively.

Figure 3.5: Force-$I_B$ output characteristic measurement setup

### 3.1.4 Forward-Early measurement

In the forward-early measurement shown if Figure 3.8, the collector voltage $V_C$ is increased while keeping the base-emitter voltage constant. The maximum collector voltage
Figure 3.6: Force-$I_B$ output measurement $I_C$-$V_{CE}$.

<table>
<thead>
<tr>
<th>T (K)</th>
<th>Start (V)</th>
<th>Stop (V)</th>
<th>Step (mV)</th>
<th>$V_E$ (V)</th>
<th>$V_B$ (V)</th>
<th>$V_{SUB}$ (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>393</td>
<td>0</td>
<td>$V_{CB,max1}$</td>
<td>50</td>
<td>-0.60 V</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>300</td>
<td>0</td>
<td>$V_{CB,max2}$</td>
<td>50</td>
<td>-0.68 V</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>223</td>
<td>0</td>
<td>$V_{CB,max3}$</td>
<td>50</td>
<td>-0.8 V</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

Table 3.4: Forward-Early measurement

$V_{CB,max}$ is obtained from this measurement as the voltage where the base current becomes negative. The measured $I_B - V_{CB}$ and $I_C - V_{CB}$ are shown in Figure 3.9 and 3.10 respectively.
Figure 3.7: Force-$I_B$ output measurement $V_{BE} - V_{CE}$.

Figure 3.8: Forward-Early measurement setup
Figure 3.9: Forward-Early measurement $I_B - V_{CB}$ curve.

Figure 3.10: Forward-Early measurement $I_C - V_{CB}$ curve.
### Table 3.5: Reverse-Early measurement

<table>
<thead>
<tr>
<th>T (K)</th>
<th>Start (V)</th>
<th>Stop (V)</th>
<th>Step (mV)</th>
<th>$V_E$ (V)</th>
<th>$V_C$ (V)</th>
<th>$V_B$ (V)</th>
<th>$V_{SUB}$ (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>393</td>
<td>0</td>
<td>$V_{EB,max1}$</td>
<td>50</td>
<td>-0.50 V</td>
<td>0</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>0</td>
<td>$V_{EB,max2}$</td>
<td>50</td>
<td>-0.65 V</td>
<td>0</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>223</td>
<td>0</td>
<td>$V_{EB,max3}$</td>
<td>50</td>
<td>-0.76 V</td>
<td>0</td>
<td>-1</td>
<td></td>
</tr>
</tbody>
</table>

#### 3.1.5 Reverse-Early measurement

In the reverse-early measurement shown in Figure 3.11, the emitter voltage $V_E$ is increased while keeping the base-collector voltage constant. In the reverse-early measurement, the measured base current should be constant at first, then it will eventually decrease due to the generation of avalanche current in the reversed biased base-emitter junction. The maximum reverse emitter voltage $V_{EB,max}$ is obtained from the measurement by looking to the point where this effect occur. The measured $I_E$ and $I_B$ versus $V_{EB}$ are shown in Figure 3.12 and 3.13.

![Figure 3.11: Reverse-Early measurement setup](image-url)
In the $R_{CC}$ measurement shown in Figure 3.14, the base-collector is forward biased and the base-collector is 0.6V. The base-emitter voltage is swept from 0.6V to 1.4V. In this case the collector current remains positive and reasonably large. The voltage drop over the collector resistance makes that internally the base-collector bias that drives the substrate current is increased even further than the externally applied 0.6V. The substrate current can be measured and we can use a simple model for this current to extract $R_{CC}$. Figure 3.15 shows the measured $I_C$, $I_B$, $I_S$ versus $V_{BE}$.

<table>
<thead>
<tr>
<th>$T$ (K)</th>
<th>$V_{SUB}$ (V)</th>
<th>$V_E$ (V)</th>
<th>$V_B$ (V)</th>
<th>$V_C$ (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>393</td>
<td>-1</td>
<td>-0.6</td>
<td>-1.4</td>
<td>5</td>
</tr>
<tr>
<td>300</td>
<td>-1</td>
<td>-0.6</td>
<td>-1.4</td>
<td>5</td>
</tr>
<tr>
<td>223</td>
<td>-1</td>
<td>-0.6</td>
<td>-1.4</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 3.6: $R_{CC}$-active measurement setup
Figure 3.13: Reverse-Early $I_B - V_{EB}$.

Figure 3.14: $R_{CC}$-active measurement setup
Figure 3.15: \( R_{CC}\)-active measurement \( I_C, I_B, I_S \) versus \( V_{BE} \).
3.2 Parameter Initialization

The first step in the determination of parameter is to generate an initial parameter set. In general, the parameter extraction means to minimize the difference between the measurement data and simulation data. This is done by using some algorithm that optimises the value of the parameter. For the optimization, the initial value of the parameter is very important. If the initial value is good, the optimising process will be precise and fast. Furthermore, the chance of getting stuck in a local minimum with very unphysical values for the parameters is much smaller than stared with a random initial value.

3.2.1 Parameters that can be given an initial value

Table 3.7 shows the parameters and their initial values. Some of them are process dependent.

3.2.2 Parameters from measurements

Table 3.8 shows a list of parameters whose initial value can be obtained from measurement data and it also shows the way to get the values. Note that the last three quantities are help variables and not parameters of Mextram.

3.2.3 Parameter to be calculated

At last, we have same parameters whose initial value need to be calculated from laylout and process data (shows in Table ). We also need following constant to get the initial values.

Boltzmann constant is:

$$k = 1.3806226 \cdot 10^{-23} \text{ JK}^{-1},$$ (3.1)
<table>
<thead>
<tr>
<th>Parameter</th>
<th>value</th>
<th>remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEVEL</td>
<td>504</td>
<td></td>
</tr>
<tr>
<td>$T_{ref}$</td>
<td>TEMP</td>
<td>The actual measurement temperature</td>
</tr>
<tr>
<td>DTA</td>
<td>0</td>
<td>$T_{ref}$ already describes the actual temperature</td>
</tr>
<tr>
<td>EXMOD</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>EXPHI</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>EXAVL</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$m_{LF}$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>$X_{ib1}$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$V_{Lr}$</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>$X_{ext}$</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>$a_{xi}$</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>$V_{de}$</td>
<td>0.9</td>
<td>Somewhat depending on process</td>
</tr>
<tr>
<td>$p_{E}$</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>$C_{BEO}$</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>$C_{BCO}$</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>$m_{\tau}$</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>$\tau_{B}$</td>
<td>-1.0</td>
<td></td>
</tr>
<tr>
<td>$\tau_{R}$</td>
<td>-1.0</td>
<td></td>
</tr>
<tr>
<td>$d_{Eg}$</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>$X_{rec}$</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>$A_{QB0}$</td>
<td>0.3</td>
<td>Somewhat depending on process</td>
</tr>
<tr>
<td>$A_{E}$</td>
<td>0.0</td>
<td>Somewhat depending on process</td>
</tr>
<tr>
<td>$A_{B}$</td>
<td>1.0</td>
<td>Somewhat depending on process</td>
</tr>
<tr>
<td>$A_{ex}$</td>
<td>0.5</td>
<td>Somewhat depending on process</td>
</tr>
<tr>
<td>$A_{epi}$</td>
<td>2.0</td>
<td>Somewhat depending on process</td>
</tr>
<tr>
<td>$A_{C}$</td>
<td>0.5</td>
<td>Somewhat depending on process</td>
</tr>
<tr>
<td>$dV_{g3f}$</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>$dV_{g3r}$</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>$V_{gB}$</td>
<td>1.18</td>
<td></td>
</tr>
<tr>
<td>$V_{gC}$</td>
<td>1.18</td>
<td></td>
</tr>
<tr>
<td>$V_{gj}$</td>
<td>1.18</td>
<td></td>
</tr>
<tr>
<td>$dV_{gre}$</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>$V_{ds}$</td>
<td>0.6</td>
<td>Somewhat depending on process</td>
</tr>
<tr>
<td>$p_{S}$</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>$V_{qs}$</td>
<td>1.18</td>
<td></td>
</tr>
<tr>
<td>$A_{S}$</td>
<td>2.0</td>
<td>Somewhat depending on process</td>
</tr>
<tr>
<td>$R_{th}$</td>
<td>0.0</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.7: Parameter initial value [12]
<table>
<thead>
<tr>
<th>Param</th>
<th>Way of extraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{jE}$</td>
<td>Zero bias values of base-emitter capacitance</td>
</tr>
<tr>
<td>$C_{jC}$</td>
<td>Zero bias values of base-collector capacitance</td>
</tr>
<tr>
<td>$C_{jS}$</td>
<td>Zero bias values of substrate-collector capacitance</td>
</tr>
<tr>
<td>$R_E$</td>
<td>Numerical derivative in $R_E$-flyback measurement</td>
</tr>
<tr>
<td>$V_{er}$</td>
<td>Numerical derivative in reverse-Early measurement</td>
</tr>
<tr>
<td>$V_{ef}$</td>
<td>Numerical derivative in forward-Early measurement</td>
</tr>
<tr>
<td>$\beta_f$</td>
<td>Maximum of forward current gain</td>
</tr>
<tr>
<td>$\beta_{ri}$</td>
<td>Maximum of internal reverse current gain</td>
</tr>
<tr>
<td>$I_S$</td>
<td>From forward-gummel collector current without Early effect</td>
</tr>
<tr>
<td>$I_{Ss}$</td>
<td>From reverse-gummel substrate current without Early effect</td>
</tr>
<tr>
<td>$\tau_E, \tau_B$</td>
<td>$1/\left[10 \pi \max(f_T)\right]$</td>
</tr>
<tr>
<td>$I_{B0}$</td>
<td>Zero bias value of the base current in forward-Early measurement</td>
</tr>
<tr>
<td>$I_{C0}$</td>
<td>Zero bias value of the collector current in forward-Early measurement</td>
</tr>
<tr>
<td>$I_{E0}$</td>
<td>Zero bias value of the emitter current in reverse-Early measurement</td>
</tr>
</tbody>
</table>

Table 3.8: Parameters that can be extracted without optimising[12]

<table>
<thead>
<tr>
<th>MULT</th>
<th>Numbers of transistors in parallel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{em}$</td>
<td>Emitter width(Dimension on silicon)</td>
</tr>
<tr>
<td>$L_{em}$</td>
<td>Emitter strip length(dimension on silicon)</td>
</tr>
<tr>
<td>$\rho_P$</td>
<td>Pinched sheet resistance of the base</td>
</tr>
<tr>
<td>$N_{base}$</td>
<td>Number of base stripes</td>
</tr>
<tr>
<td>$N_{epi}$</td>
<td>Collector epilayer doping level</td>
</tr>
<tr>
<td>$W_{epi}$</td>
<td>Collector epilayer thickness</td>
</tr>
</tbody>
</table>

Table 3.9: The layout and process quantities[12]
The elementary charge can be described as:

\[ q = 1.6021918 \times 10^{-19} \text{C}, \quad (3.2) \]

Dielectric constant is shown as:

\[ \varepsilon = 1.036 \times 10^{-10} \text{C/Vm}, \quad (3.3) \]

The saturated drift velocity is:

\[ v_{\text{sat}} = 8.0 \times 10^4 \text{m/s}. \quad (3.4) \]

From layout data we can calculate the emitter surface and periphery

\[ A_{\text{em}} = H_{\text{em}} \cdot L_{\text{em}}, \quad (3.5) \]
\[ P_{\text{em}} = 2(H_{\text{em}} + L_{\text{em}}). \quad (3.6) \]

From the layout data and direct extraction estimates we can calculate the fraction of the BE depletion capacitance \( X_{\text{jE}} \) and the fraction of the BC depletion capacitance \( X_{\text{jC}} \)

\[ X_{\text{jE}} = \frac{P_{\text{em}}}{P_{\text{em}} + 6A_{\text{em}}/\mu\text{m}}, \quad (3.7) \]
\[ X_{\text{jC}} = X_{\text{jE}} \frac{V_{\text{er}} \cdot C_{\text{jE}}}{V_{\text{ef}} \cdot C_{\text{jC}}}. \quad (3.8) \]

The zero bias value of the variable base resistance \( R_{\text{Bv}} \) and the constant part of the base resistance \( R_{\text{Be}} \) can be estimated as

\[ R_{\text{Bv}} = \frac{H_{\text{em}} \rho_p}{3N_{\text{base}}^2 L_{\text{em}}}, \quad (3.9) \]
\[ R_{\text{Be}} = R_{\text{Cc}} = \frac{300\Omega \mu\text{m}}{L_{\text{em}}} \quad (3.10) \]
The initial values of the collector-emitter high injection knee current $I_K$ and base-substrate high injection knee current $I_{kS}$ are

$$I_K = \frac{V_{er}(1 - XC_{je})C_{je}}{\tau_B}, \quad (3.11)$$

$$I_{kS} = (500 \mu A/\mu m^2) * A_{em}. \quad (3.12)$$

The initial values of saturation current of the non-ideal reverse base current $I_{Br}$ and saturation current of the non-ideal forward base current $I_{Bf}$

$$I_{Br} = 100I_S, \quad (3.14)$$

$$I_{Bf} = 100I_S. \quad (3.15)$$

The epilayer thickness used in weak-avalanche model $W_{avl}$ and voltage determining curvature of avalanche current $V_{avl}$ can be estimated as

$$W_{avl} = W_{epi}, \quad (3.16)$$

$$V_{avl} = \frac{qN_{epi}W_{epi}^2}{2\varepsilon}. \quad (3.17)$$

We also need some spreading parameters for the epilayer: $\alpha_l$ is the spreading angle at low current levels ($I_C < I_{hc}$) while $\alpha_h$ is the spreading angle at high current levels. These quantities are process and geometry dependent. We can use the following values if we are
only interested in generating an initial parameters set:

\[
\tan \alpha_l = 0.5, \quad (3.18)
\]

\[
\tan \alpha_h = 1.0, \quad (3.19)
\]

\[
S_{Fl} = \tan \alpha_l W_{epi} \left( \frac{1}{H_{em}} + \frac{1}{L_{em}} \right), \quad (3.20)
\]

\[
S_{Fh} = \frac{2}{3} \tan \alpha_h W_{epi} \left( \frac{1}{H_{em}} + \frac{1}{L_{em}} \right). \quad (3.21)
\]

The latter quantity is the current spreading factor for high injection, and is a parameter used in the high-current avalanche model. We can now calculate the epilayer parameters.

The collector-base diffusion voltage:

\[
V_{dc} = V_T \ln \left( \frac{N_{epi}^2}{n_i^2} \right), \quad (3.22)
\]

Resistance of the un-modulated epilayer is:

\[
R_{Cv} = \frac{W_{epi}}{q\mu_0 N_{epi} A_{em}} \frac{1}{(1 + S_{Fl})^2}, \quad (3.23)
\]

Critical current for velocity saturation in the epilayer can be estimated as:

\[
I_{hc} = q\mu_0 N_{epi} A_{em} v_{sat} (1 + S_{Fl})^2, \quad (3.24)
\]

The initial value of space charge resistance of the epilayer is:

\[
SCR_{Cv} = \frac{W_{epi}^2}{2\varepsilon v_{sat} A_{em}} \frac{1}{(1 + S_{Fl})^2}, \quad (3.25)
\]
The constant part of $C_{jC}$, collector-base grading coefficient, coefficient for the current modulation of the collector-base depletion capacitance are:

$$X_p = x d_0 / W_{epi},$$  \hspace{1cm} (3.26)

$$p_C = 0.3 / (1 - X_p),$$  \hspace{1cm} (3.27)

$$m_C = (1 - X_p) / 2,$$  \hspace{1cm} (3.28)

The transit time of stored epilayer charge and transit time of reverse extrinsic stored base charge are:

$$\tau_{epi} = \frac{W_{epi}^2}{4 \mu_0 V_T},$$  \hspace{1cm} (3.29)

$$\tau_R = (\tau_B + \tau_{epi} \frac{1 - X C_{jc}}{X C_{jc}}),$$  \hspace{1cm} (3.30)
Chapter 4
Extraction of low current parameters

4.1 Base-emitter depletion capacitance

The bias dependence of the depletion capacitances is in Mextram generally considered as:

\[ C_j = \frac{C_0}{(1 - \frac{V_j}{V_{bi}})^p}, \]  \hspace{1cm} (4.1)

Here \( C_0 \) is the zero bias depletion capacitance, \( V_j \) is the internal P-N junction bias, \( V_{bi} \) is the junction built-in voltage and \( p \) is the grading coefficient. This method will cause singularity when applied voltage is equal to the built-in voltage. An effective junction bias \( V_{eff} \) is employed in Mextram model and the capacitance can be expressed as:

\[ C_j = \frac{C_0}{(1 - \frac{V_{eff}}{V_{bi}})^p}, \]  \hspace{1cm} (4.2)

The \( V_{eff} \) is given by:

\[ V_{eff} = V - V_{ch} \ln(1 + e^{\frac{V - V_F}{V_{ch}}}), \]  \hspace{1cm} (4.3)

The advantage of using \( V_{eff} \) is that when the applied voltage is larger than built-in voltage the \( V_{eff} \) is equal to applied voltage and when the applied voltage is larger than built-in voltage the \( V_{eff} \) is equal to a constant \( V_F \) and the value of the constant \( V_F \) is given:

\[ V_F = V_{bi}(1 - a_j^{-\frac{1}{p}}). \]  \hspace{1cm} (4.4)

Figure 4.1 and 4.2 shows the behavior of \( V_{eff} \) and \( V_j \):
The measurement $C_{BE}$ is consisted by three parts: depletion capacitance, overlap capacitance and diffusion capacitance. The depletion capacitance and overlap capacitance dominate as long as $V_{BE}$ bias is not too high. The base-emitter depletion capacitance can be expressed by the formula:

$$C_{BE,dep} = \frac{C_{jE}}{1 - \frac{V_{jE}}{V_{DE}}} p_E + C_{BEO},$$  \hspace{1cm} (4.5)

The constant capacitance $C_{BEO}$ describes any overlap capacitance between base and emitter. $C_{jE}$ is the zero-bias emitter base depletion capacitance, $V_{dE}$ is diffusion voltage and $p_E$ describes emitter-base grading coefficient. $V_{FE}$ is defined using the model constant $a_{jE}$.

$$V_{FE} = V_{dE} \left(1 - a_{jE}^{-1/p_E}\right),$$  \hspace{1cm} (4.6)

The effective junction bias is:

$$V_{jE} = V_{BE} - 0.1 V_{dE} \ln\left(1 + e^{-\frac{V_{BE} - V_{FE}}{0.1 V_{dE}}} \right).$$  \hspace{1cm} (4.7)

Actually, the sum of $C_{jE}$ and $C_{BEO}$ give the overall zero bias capacitance and cannot be separated clearly.
Figure 4.3: Measured (markers) and simulated (line) base emitter depletion capacitance by Python

Figure 4.4: Measured (markers) and build-in function simulated (line) base emitter depletion capacitance by IC-CAP

\[ C_{je}, p_E \text{ and } V_{de} \text{ can be extracted directly by applying Equation 4.5 to fit } C_{BE} - V_{BE}. \]

The extraction result is shown in Figure 4.3 – figure 4.4.
4.2 Base-collector depletion capacitance

The extraction of the base-collector depletion capacitance is almost the same with the base-emitter depletion capacitance. Parameter $X_p$ is introduced to describe the finite thickness of collector epilayer. The base-collector depletion capacitance can be written as:

$$C_{BC,dep} = \frac{(1 - X_p)C_{jc}}{1 - V_{jc}/V_{dc}} + X_p C_{jc} + C_{BCO},$$ \hspace{1cm} (4.8)

The constant capacitance $C_{BCO}$ describes base-collector overlap capacitance. $C_{jc}$ describes zero bias collector-base depletion capacitance, $p_C$ is collector-base grading coefficient, $V_{dc}$ is collector-base built-in voltage.

$V_{FC}$ is defined using the model constant $a_{jc}$.

$$V_{FC} = V_{dc} \left(1 - \frac{a_{jc} - X_p^{-1/p_c}}{1 - X_p}\right),$$ \hspace{1cm} (4.9)

The effective junction bias is:

$$V_{jc} = V_{BC} + 0.1V_{dc} \ln(1 + e^{(V_{BC} - V_{FC})/0.1V_{dc}}).$$ \hspace{1cm} (4.10)

The parameter can be extracted here is $C_{jc}$, $p_C$ and $X_P$ by fitting the $C_{BC} - V_{BC}$ curve as shown in Figure 4.5 - 4.6. Help parameter $V_{dc}$ will be re-extracted with other high current parameter later, because it has strong impact on the current gain roll-off, cut-off frequency roll-off and output characteristics quasi-saturation region.
Figure 4.5: Measured(markers) and simulated(line) base collector depletion capacitance by Python

Figure 4.6: Measured(markers) and build-in function simulated(line) base collector depletion capacitance by IC-CAP
4.3 Collector-substrate depletion capacitance

The extraction of collector-substrate capacitance is also similar to the base-emitter capacitance. The collector-substrate depletion capacitance can be written as:

\[ C_{SC} = \frac{C_{js}}{(1 - V_{js}/V_{ds})^{p_S}} + C_{p,CS}, \]  

(4.11)

\( C_{js} \) describes zero bias collector-substrate depletion capacitance and \( P_S \) is collector-substrate grading coefficient, \( V_{ds} \) is collector-substrate built-in voltage. \( C_{p,CS} \) is not a part of Mextram model, it is included here for convenience in the case of de-embedding problems.

\( V_{FS} \) is defined using the model constant \( a_{jc} \).

\[ V_{FS} = V_{ds} \left( 1 - a_{jc}^{-1/p_S} \right), \]

(4.12)

The effective junction bias is:

\[ V_{js} = V_{SC} + 0.1V_{ds} \ln(1 + e^{(V_{SC}-V_{FS})/0.1V_{ds}}). \]

(4.13)

\( C_{js} \) and \( p_S \) can be extracted directly through Equation 4.11. Figure 4.7 - 4.8 show the extracted \( C_{SC} - V_{SC} \) result.
Figure 4.7: Measured (markers) and simulated (line) collector substrate depletion capacitance by Python

Figure 4.8: Measured (markers) and build-in function simulated (line) collector substrate depletion capacitance by IC-CAP
4.4 Avalanche

In modern transistor because we have low breakdown voltage we can not neglect avalanche effect. The avalanche current is a result of impact ionisation in the epilayer due to the high electric fields. This generation of avalanche currents strongly depends on the maximum electric field. The low current avalanche parameters are extracted from base current under forward-early measurement shown in Figure 4.9. Because of avalanche current the base volt-

![Figure 4.9: Forward-Early measurement simplified circuit](image)

age will drop down as the increase of $V_{CB}$. So that the avalanche current can be written as:

$$I_{avl} = I_{B0} - I_B. \quad (4.14)$$

Here $I_{B0}$ is a help variable that shows the base current at $V_{CB} = 0$. The avalanche current can also be describe as:

$$I_{avl} = I_C G_{EM}, \quad (4.15)$$
$G_{EM}$ is the generation coefficient. From the measurement data we can calculate the measurement $G_{EM}$ as:

$$G_{EM} \cdot m = \frac{I_{B0} - I_B}{I_C} \quad (4.16)$$

The simulation $G_{EM}$ is:

$$G_{EM} = \frac{A_n}{B_n} E_M \lambda_D \{\exp\left[-\frac{B_n}{E_M}\right] - \exp\left[-\frac{B_n}{E_M} \left(1 + \frac{W_{avl}}{\lambda_D}\right)\right] \} \quad , \quad (4.17)$$

$A_n$ and $B_n$ are material constants, which are avalanche coefficient and critical electric field respectively. $\lambda_D$ is the extrapolated depletion thickness where the electric field is zero, effective width of the epilayer for avalanche current $W_{AVL}$ and voltage describing the curvature of the avalanche current $V_{AVL}$ are parameters we need to extract here.

$$\lambda_D = \frac{W_{avl}^2}{2V_{avl}} E_M \quad , \quad (4.18)$$

$E_M$ is the maximum electric field in the depletion region and it can be obtained from:

$$E_M = \frac{V_{dc} + V_{CB} + 2V_{avl}}{W_{avl}} \sqrt{\frac{V_{dc} + V_{CB}}{V_{dc} + V_{CB} + 2V_{avl}}} \quad . \quad (4.19)$$

$W_{AVL}$ and $V_{AVL}$ can be extracted from the comparison of the simulated and measured $G_{EM}$. The extraction result is shown in Figure 4.12- 4.13. As the collector-base bias gets larger, the base current drops below zero because of the avalanche effect.

### 4.5 Reverse Early effect

In Mextram model the forward and reverse Early effects are bias-dependent. The parameters $V_{er}$ and $V_{ef}$ are the values of these bias-dependent Early voltage when both the base-emitter and base-collector bias are zero. The reverse-early voltage $V_{er}$ is extracted from the reverse-Early measurement as shown in Figure 4.14. The emitter current is:
Figure 4.10: Measured (markers) and simulated (line) $G_{EM}$ in forward-Early measurement by Python.

Figure 4.11: Measured (markers) and simulated (line) $G_{EM}$ by IC-CAP.
Figure 4.12: Measured(markers) and simulated(line) base current in forward-Early measurement by Python

Figure 4.13: Measured(markers) and simulated(line) base current in forward-Early measurement by IC-CAP
Here $I_{E0}$ is a help variable which is the emitter current at $V_{BE} = 0$, $V_{tE}$ and $V_{tC}$ are variables related to base-emitter depletion charges and base-collector depletions charges. The forward early voltage parameter $V_{ef}$ also has a key effect on emitter current which means if we want to extract $V_{er}$, we must have a good estimate value of $V_{ef}$. Therefore, we will do the extraction twice, first we extract $V_{er}$ and the first time extraction result is shown in Figure 4.15 and 4.16, then $V_{ef}$, then redo this procedure again. For the calculation of the normalised charges $V_{tE}$ and $V_{tC}$ we neglect the voltage drop over the resistance.
\[ V_{tE} = \frac{V_{dE}}{1-P_E} \left[ 1 - \left( 1 - \frac{V_{jE}}{V_{dE}} \right)^{1-P_E} \right] + a_{jE} (V_{BE} - V_{jE}), \]  
\[ V_{tC} = (1 - X_p) \left( \frac{V_{dC}}{1-P_C} \left[ 1 - \left( 1 - \frac{V_{jC}}{V_{dC}} \right)^{1-P_C} \right] + b_{jC} (V_{BC} - V_{jC}) \right) + X_p V_{BC}. \]  
(4.21a) 
(4.21b)

The early voltage is given by:

\[ V_{\text{reverse-early}} = I_E \left( \frac{\partial I_E}{\partial V_{EB}} \right)^{-1} \]  
\[ = I_{E0} \frac{1 + \frac{V_{tC}}{V_{er}} + \frac{V_{tC}}{V_{ef}} \left( 1 + \frac{V_{tE}}{V_{er}} + \frac{V_{tC}}{V_{ef}} \right)^2}{1 + \frac{V_{tE}}{V_{er}} + \frac{V_{tC}}{V_{ef}}} I_{E0} \left( 1 + \frac{V_{tC}}{V_{er}} \right) \left( 1 + \frac{V_{tC}}{V_{ef}} \right) \]  
\[ = C_{JE} V_{er} \left( 1 + \frac{V_{tE}}{V_{er}} + \frac{V_{tC}}{V_{er}} \right). \]  
(4.22a) 
(4.22b) 
(4.22c)

Figure 4.15: Measured(markers) and simulated(line) emitter current of first time extraction in reverse-Early measurement by Python

Figure 4.16: First time extraction of actual reverse Early voltage by numerical differentiation of the measured(dash line) and simulated(solid line) collector current of in the reverse-Early measurement by Python
Figure 4.17: Measured(markers) and simulated(line) emitter current in reverse-Early measurement by Python

Figure 4.18: Measured(markers) and build-in function simulated(line) emitter current in reverse-Early measurement by IC-CAP

Figure 4.19: Actual reverse Early voltage by numerical differentiation of the measured(dash line) and simulated(solid line) collector current in the reverse-Early measurement by Python

Figure 4.20: Actual reverse Early voltage by numerical differentiation of the measured(dash line) and build-in function simulated(solid line) collector current in the reverse-Early measurement by IC-CAP

$C_{je}$ is the zero-bias emitter base depletion capacitance, $C_{BE}$ is emitter base depletion capacitance. Note that the measurement point of reverse-early voltage is not actual measurement data, it is calculated from measurement emitter current and measurement emitter voltage. As the result shown in Figure 4.17 to 4.20, we match the measurement and simulated emitter current and reverse early voltage together to extract $V_{er}$. 
4.6 Forward Early effect

The forward-early voltage parameter \( v_{ef} \) is extracted in forward-early measurement shown in Figure 4.9. The approximation for the collector current in forward-early measurement can be written as:

\[
I_C = I_{C0} \frac{1 + \frac{V_{te}}{V_{er}}}{1 + \frac{V_{ie}}{V_{er}} + \frac{V_{ic}}{V_{ef}}}.
\]  

(4.23)

\( I_{C0} \) is a help variable which stand for collector current at zero base collector bias.

Then we can calculate the Early voltage:

\[
V_{\text{forward-early}} = I_C \left( \frac{\partial I_C}{\partial V_{CB}} \right)^{-1}
\]

(4.24)

\[
= I_{C0} \frac{1 + \frac{V_{te}}{V_{er}}}{1 + \frac{V_{ie}}{V_{er}} + \frac{V_{ic}}{V_{ef}}} \frac{V_{ef} \left( 1 + \frac{V_{te}}{V_{er}} + \frac{V_{ic}}{V_{ef}} \right)^2}{I_{C0} \left( 1 + \frac{V_{ie}}{V_{er}} \right) \frac{dV_C}{dV_{CB}}}.
\]

(4.25)

\[
= \frac{C_{jc} V_{ef}}{C_{BC}} \left( 1 + \frac{V_{te}}{V_{er}} + \frac{V_{ic}}{V_{ef}} \right).
\]

(4.26)

\( C_{jc} \) describes zero bias collector-base depletion capacitance and \( C_{BC} \) is collector-base depletion capacitance. Changing \( V_{ef} \) can match the simulation collector current and forward early voltage to measurement data, so that \( V_{ef} \) can be extracted here. Result is shown in Figure 4.23 - 4.26.
Figure 4.21: Measured (markers) and simulated (line) collector current of first time extraction in forward-Early measurement by Python

Figure 4.22: First time extraction of actual forward Early voltage by numerical differentiation of the measured (dash line) and simulated (solid line) collector current of in the forward-Early measurement by Python
Figure 4.23: Measured (markers) and simulated (line) collector current in forward-Early measurement by Python

Figure 4.24: Measured (markers) and built-in function simulated (line) collector current in forward-Early measurement by IC-CAP

Figure 4.25: Actual forward Early voltage by numerical differentiation of the measured (dash line) and simulated (solid line) collector current in the forward-Early measurement by Python

Figure 4.26: Actual forward Early voltage by numerical differentiation of the measured (dash line) and simulated (solid line) collector current in the forward-Early measurement by IC-CAP
4.7 Collector saturation current

Since we have a good description of Early effect, we can extract the collector saturation current using forward-Gummel measurement (shown in Figure 4.27) at low base-emitter bias.

![Forward-Gummel measurement simplified circuit](image)

**Figure 4.27: Forward-Gummel measurement simplified circuit**

The collector current $I_S$ is proportional to the increase of base-emitter bias at low bias. At high bias the resistance effect cannot be neglected and the actual $V_{BE}$ will be much less than $V_{BE}$ we used, which is the reason that measurement data and simulation data is split in high bias. So we will only focus on the low bias behavior and optimize data at low bias area. The collector current can be describe as:

$$I_C = \frac{I_S e^{V_{BE}/V_T}}{1 + \frac{V_{BE}}{V_{CE}} + \frac{V_{CG}}{V_{CF}}}.$$  \hspace{1cm} (4.27)
$V_{t_E}$ and $V_{t_C}$ have been defined in (4.21). Here $V_T$ is the thermal voltage, which needs to be determined accurately from the actual absolute temperature. Changing collector saturation current $I_S$ can make the simulation collector current match the measurement collector current thus $I_S$ can be extracted and the result is shown in Figure 4.28 - 4.29.

4.8 Forward base current

The parameters of forward base current are extracted from base current in forward-Gummel measurements (shown in Figure 4.27) by comparing the measurement forward base
current and simulated forward base current. The forward current only depends on the internal base-emitter voltage $V_{B_2E_1}$ and neglect the voltage drops on base resistance.

We use the Equation (4.27) to calculate internal base-emitter bias from measured $I_C$:

$$V_{B_2E_1} = V_T \ln \left[ \frac{I_C}{I_S} \left( 1 + \frac{V_{E_1}}{V_{er}} + \frac{V_{I_C}}{V_{ef}} \right) \right], \quad (4.28)$$

As we take the internal bias equal to the external bias $V_{BE}$, we must make sure that resistance effects are not important so we need to fit the curve at low base-emitter bias.

$V_{E_1}$ and $V_{I_C}$ are given in (4.21). The ideal and non-ideal base current can be calculated:

1. $I_{B1} = \frac{I_S}{\beta_f} e^{V_{B_2E_1}/V_T}$, \quad (4.29a)
2. $I_{B2} = I_{Bf} \left( e^{V_{B_2E_1}/m_{Lf}V_T} - 1 \right), \quad (4.29b)$

Here ideal forward current gain $\beta_f$, saturation current of the non ideal forward base current $I_{Bf}$ and none-ideality factor of the non-ideal forward base current $m_{Lf}$ are used.

$\beta_f$, $I_{Bf}$ and $m_{Lf}$ are extracted here from the comparison of simulated forward base current and the measurement forward base current shown in Figure 4.30. Again we only
extract data from the low bias area at high bias the voltage drop at $R_{Cc}$ can not be ignored which is the reasons that measured and simulated base current can not match together at high bias.

### 4.9 Substrate saturation current

![Figure 4.31: Reverse-Gummel measurement simplified circuit](image)

The substrate saturation current is extracted in reverse-Gummel measurement as shown in Figure 4.31 by fitting the substrate current. We will neglect the voltage drop and the substrate current can be calculate by:

$$I_{sub} = I_{Ss} \exp\left(\frac{V_{BC}}{V_T}\right),$$

(4.30)
$I_{S}$ can be extracted directly from the substrate current. The extraction result is given in Figure 4.32. At high bias, because of the voltage drop over $R_{C_C}$ measured and simulated substrate current can not be fit at high base-collector bias.

![Figure 4.32: Measured (mark) and simulated (line) substrate current in reverse-Gummel measurement](image)

**4.10 Reverse current**

The reverse current parameters are extracted from reverse-Gummel measurement. In the reverse-Gummel measurement (shown in Figure 4.27) the base-collector is forward biased which means the external base current contains internal base current and substrate current.

The internal base-collector bias $V_{B_1C_1}$ is calculated:

$$V_{B_1C_1} = V_T \ln \left[ \frac{I_E}{I_S} \left( 1 + \frac{V_{I_E}}{V_{er}} + \frac{V_{I_C}}{V_{ef}} \right) \right], \quad (4.31)$$
Note here we use measurement emitter current. From $V_{B1C1}$ we can calculate extrinsic reverse base current $I_{ex}$ and non-ideal reverse base current $I_{B3}$. They can be described as:

$$I_{ex} = \frac{I_S}{\beta_{ri}} e^{\frac{V_{B1C1}}{V_T}},$$

$$I_{B3} = I_{Br} e^{\frac{V_{B1C1}}{2V_T}} + e^{\frac{V_{Lr}}{2V_T}}.$$  \hspace{1cm} (4.32)

$$I_{B3} = I_{Br} e^{\frac{V_{B1C1}}{2V_T}} + e^{\frac{V_{Lr}}{2V_T}}.$$  \hspace{1cm} (4.33)

The reverse current gain $\beta_{ri}$, Saturation current of the non-ideal reverse base current $I_{Br}$ and Cross-over voltage of the non-ideal reverse base current $V_{Lr}$ are variable of $I_{ex}$ and $I_{B3}$, so that they can be extracted by fitting the curve. The measured reverse base current and simulated reverse base current plot is given in Figure 4.33. Again we need only concentrate on the low base-collector bias. At high base-collector bias the voltage drop at $R_E$ can not be neglect and the internal base-emitter will be forward biased, and current will flow from base to emitter. This is the reason measured reverse base current is bigger than simulated reverse base current.

![Figure 4.33: The measured $I_B - I_{sub}$ and simulated $I_{ex} + I_{B3}$ in reverse-Gummel measurement](image-url)
4.11 Emitter series resistances

One of the simplest way to extract the emitter resistance is from the Giacoletto method [19] [20]. The collector current is kept zero and the $V_{BE}$ is increased. The collector-emitter saturation voltage can be estimated as $V_{CES} \approx I_E R_E$. Then the emitter resistance can be obtained by taking the derivative of $V_{CES}$ with regard to $I_E$:

$$R_E = \frac{\partial V_{CES}}{\partial I_E}. \quad (4.34)$$

However, here we use another method[21], here the emitter series resistances is extracted in forward-Gummel measurement (shown in Figure 4.27). The external base-emitter bias can be described as a sum of then internal junction bias and voltage drop over series resistance:

$$V_{BE} = V_{B_2E_1} + V_{B_1B_2} + I_B R_{Bc} + (I_B + I_C) R_E + V_{off,Rb}, \quad (4.35)$$

The internal junction bias $V_{B_2E_1}$ can be solved iteratively using the measurement base current and Equation 4.29.

The voltage drop over the pinched resistance gives only small contribution but we do want to include the variation of the resistance due to charge modulation and current crowding. The charge modulation can is:

$$R_{mod} = \frac{R_{By}}{q_B}, \quad (4.36)$$

$$I_C = \frac{I_f - I_r}{q_B} \approx \frac{I_s \exp(V_{B_2E_1}/V_T)}{q_B}, \quad (4.37)$$

This is only true under the assumption that quasi-saturation is not important which means that $I_r$ can be neglect. We also assume that non-ideal base current is negligible, the voltage drop is:

$$I_B R_{mod} \approx I_C R_{By}/\beta_f, \quad (4.38)$$
Figure 4.34: Measured (markers) and simulated (line) base-emitter bias as function of the base current in the forward-Gummel measurement by Python

Figure 4.35: Measured (markers) and build-in function simulated (line) base-emitter bias as function of the base current in the forward-Gummel measurement by IC-CAP

To include some current crowding effect and the fact that a part of the base current might go through the side-wall we can get:

\[ V_{B_1,B_2} = V_T \ln \left( 1 + \frac{R_{BV} I_C (1 - Xl_{B_1})}{\beta_f V_T} \right). \]  (4.39)
The emitter series resistance $R_E$ is the only variable in Equation 4.35 thus it can be extracted by fitting the curve. There are also some other methods to extract emitter series mentioned in [22] [23].

4.12 Collector series resistances

![Figure 4.36: $R_{Cc}$-active measurement simplified circuit](image)

The collector series resistance $R_{Cc}$ can be extracted from the substrate current in the dedicated $R_{Cc}$ – active measurement. The intrinsic base-collector bias can be shown as:

$$V_{B_1C_1} = V_{BC} + R_{Cc}I_C - R_{Bc}I_B,$$  \hspace{1cm} (4.40)
We will neglect the voltage drop so the substrate current can be described as:

\[
I_{sub} = \frac{2I_S e^{V_{B1C}/V_T}}{1 + \sqrt{1 + \frac{I_S e^{V_{B1C}/V_T}}{I_k}}}
\]  

(4.41)

Again we will only extract parameters using the low bias data. At high bias, because of the voltage drop over \(R_{bc1}\), \(V_{B1C_4}\) will be smaller than \(V_{B1C_1}\), so the measurement substrate current is smaller than simulated one at high bias. The collector series resistance and constant part of the base resistance \(R_{Be}\) can be extracted from the comparison of the simulated and measured substrate current.
Figure 4.37: Measured (markers) and simulated (line) substrate current in the Rcc-active measurement by Python

Figure 4.38: Measured (markers) and build-in function simulated (line) substrate current in the Rcc-active measurement by IC-CAP
Chapter 5

Extraction of high current parameters

5.1 Self-heating

For the high current parameters we will need include temperature scaling rules, see in next chapter. Thermal resistance $R_{th}$ is extracted from the base emitter voltage at high $V_{CE}$ using the output-characteristic measurements shown in Figure 5.1. Since the junction temperature is determined by the thermal resistance for a given power dissipation, accurate modeling of the thermal resistance $R_{th}$ is critical for the modeling of junction temperature,
and therefore the temperature characteristics of device. Due to the self-heating the temperature of the transistor will rise with an amount of:

$$\Delta T = R_{th} (I_B V_{BE} + I_C V_{CE}).$$ \hspace{1cm} (5.1)$$

The collector part is the most important to self-heating and the values of $R_{th}$ are about 100-500 °C/W. The temperature will increase with the increase of $V_{CE}$, and $V_{BE}$ can be express by:

$$V_{BE} \simeq V_T \ln \left( \frac{I_B \beta_f T}{I_s T} \right)$$ \hspace{1cm} (5.2)$$

Note that from here on we will using parameters that after temperature scaling rule. As the base current is constant and $\beta_f$ will decrease with the temperature growing and the $I_s$ increases with the increases of temperature. The base-emitter voltage will decrease. The thermal resistance can be extracted from drop of the base-emitter voltage in the output-characteristic measurement for which the base current is constant.

The base-emitter is:

$$V_{BE} = V_{B_2 E_1} + V_{B_1 B_2} + I_B R_{BcT} + (I_C + I_B) R_{ET},$$ \hspace{1cm} (5.3)$$

The voltage drop $V_{B_1 B_2}$ can be estimated as:

$$V_{B_1 B_2} = V_T \log \left( 1 + \frac{R_{BcT} I_B (1 - X_{iB_i})}{V_T q_B} \right).$$ \hspace{1cm} (5.4)$$

The only unknown part of this equation is the normalized base charge $q_B$, and it can be calculated:

$$q_1 = 1 + \frac{V_{I_E}}{V_{ot T}} + \frac{V_C}{V_{ef T}},$$ \hspace{1cm} (5.5)$$

$$q_B = q_1 (1 + 0.5 n_0 + 0.5 n_B).$$ \hspace{1cm} (5.6)$$
where \( n_0 \) and \( n_B \) are the electron densities in the base at the emitter edge and at the collector edge, the voltages \( V_{tE} \) and \( V_{tC} \) describe the curvature of the depletion charges as function of junction biases. In order to get \( q_B \) we need to know \( n_0 \), \( n_B \), \( V_{tE} \) and \( V_{tC} \).

The electron densities in the base at the emitter edge \( n_0 \) can be calculated from \( V_{B2E1} \) as:

\[
f_1 = \frac{4I_s T}{I_T} e^{V_{B2E1}/V_T},
\]

\[
n_0 = \frac{f_1}{1 + \sqrt{1 + f_1}}.
\]

The voltage \( V_{tE} \) also can be calculated from the voltage:

\[
V_{FE} = V_{dE} T \left( 1 - a_j^{-1/p_E} \right),
\]

\[
V_{jE} = V_{B2E1} - 0.1V_{dE} T \ln \{1 + \exp[(V_{B2E1} - V_{FE})/0.1V_{dE} T]\},
\]

\[
V_{tE} = \frac{V_{dE} T}{1 - p_E} \left[ 1 - (1 - V_{jE}/V_{dE} T)^{-1/p_E} \right] + a_j (V_{B2E1} - V_{jE}).
\]

The electron densities in the base at the collector edge \( n_B \) and the voltage \( V_{tC} \) is much more complicated, for the \( n_B \):

\[
f_2 = \frac{4I_s T}{I_T} e^{V_{B2C2}/V_T},
\]

\[
n_B = \frac{f_2}{1 + \sqrt{1 + f_2}}.
\]

We need to calculate \( V^*_{B2C2} \) for calculation of \( n_B \), The voltage and current at which quasi-saturation or Kirk effect start are given by:

\[
V^{th}_{qs} = V_{dC} T + 2V_T \ln \left( \frac{I_{C1C2} R_{CNT}}{2V_T} + 1 \right) - V_{B2C1},
\]
\[ V_{qs} = \frac{1}{2} \left( V_{th}^{qs} + \sqrt{(V_{th}^{qs})^2 + 4 \left( 0.1 V_{dcT} \right)^2} \right), \]  
\[ (5.13) \]

\[ I_{qs} = \frac{V_{qs}}{SCR_{Cv}} \frac{V_{qs} + I_{hc} SCR_{Cv}}{V_{qs} + I_{hc} R_{CVT}}, \]  
\[ (5.14) \]

Here \( R_{Cv} \) is ohmic resistance of the total epilayer, \( SCR_{Cv} \) is space-charge resistance of the epilayer, \( I_{hc} \) is critical current for hot carrier behaviour. \( V_{B2C1} \) can be obtained by:

\[ V_{B2C1} = V_{B2E1} + I_C R_{CVT} + (I_B + I_C) R_{ET} - V_{CE}, \]  
\[ (5.15) \]

From this we calculate:

\[ \alpha = \frac{1 + a_{xi} \ln \{1 + \exp[(I_{C1C2}/I_{qs} - 1)/a_{xi}]\}}{1 + a_{xi} \ln \{1 + \exp[-1/a_{xi}]\}}. \]  
\[ (5.16) \]

Here \( a_{xi} \) is smoothing parameter for the onset of quasi-saturation. We need to solve:

\[ \alpha I_{qs} = \frac{V_{qs}}{SCR_{Cv} y_i^2} \frac{V_{qs} + SCR_{Cv} I_{hc} y_i}{V_{qs} + R_{CVT} I_{hc}}, \]  
\[ (5.17) \]

which leads to:

\[ v = \frac{V_{qs}}{I_{hc} SCR_{Cv}}, \]  
\[ (5.18) \]

\[ y_i = \frac{1 + \sqrt{1 + 4 \alpha v (1 + v)}}{2 \alpha (1 + v)}. \]  
\[ (5.19) \]

The injection thickness is given by:

\[ \frac{x_i}{W_{epi}} = 1 - \frac{y_i}{1 + p_W y_i}, \]  
\[ (5.20) \]

Here \( p_W \) is normalized hole density in the collector epilayer at the buried layer edge.

\[ g = \frac{I_{C1C2} R_{CVT} x_i}{2V_T W_{epi}}, \]  
\[ (5.21) \]
The hole density $p_0^*$ at the base-collector junction is given by

$$p_0^* = \frac{g - 1}{2} + \sqrt{\left(\frac{g - 1}{2}\right)^2 + 2g + p_W(p_W + g + 1)},$$  \hspace{1cm} (5.22)$$

$$e^{V_{B_2C_2}/V_T} = p_0^*(p_0^* + 1) e^{V_{dC}/V_T}. \hspace{1cm} (5.23)$$

$V_{B_2C_2}^*$ is proportional to $V_{B_2E_1}$ and $I_{C_1C_2}$, so that $n_B$ can be calculated from $V_{B_2E_1}$ and $I_{C_1C_2}$.

Now we need to calculate the voltage $V_{C}$ is shown as:

$$B_1 = \frac{1}{2} \text{SCR}_{Cv}(I_{C_1C_2} - I_{hc}), \hspace{1cm} (5.24)$$

$$B_2 = \text{SCR}_{Cv} R_{CvT} I_{hec} I_{C_1C_2}, \hspace{1cm} (5.25)$$

$$V_{x_i=0} = B_1 + \sqrt{B_1^2 + B_2}, \hspace{1cm} (5.26)$$

The junction voltage is now the external voltage plus the voltage drop over the epilayer:

$$V_{junc} = V_{B_2C_1} + V_{x_i=0}, \hspace{1cm} (5.27)$$

$$V_{ch} = V_{dC} \left(0.1 + 2 \frac{I_{C_1C_2}}{I_{C_1C_2} + I_{qs}}\right), \hspace{1cm} (5.28)$$

$$b_{jc} = \frac{a_{jc} - X_{pT}}{1 - X_{pT}}, \hspace{1cm} (5.29)$$

$$V_{FC} = V_{dC} \left(1 - b_{jc}^{1/pC}\right), \hspace{1cm} (5.30)$$

$$V_{jC} = V_{junc} - V_{ch} \ln\{1 + \exp[(V_{junc} - V_{FC})/V_{ch}]\}, \hspace{1cm} (5.31)$$

$$I_{cap} = \frac{I_{hec} I_{C_1C_2}}{I_{hec} + I_{C_1C_2}}, \hspace{1cm} (5.32)$$

$$f_I = \left(1 - \frac{I_{cap}}{I_{hec}}\right)^{mc}, \hspace{1cm} (5.33)$$

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Here $f_I$ is collector current modulation coefficient. $m_C$ is collector current modulation coefficient.

$$V_{C_V} = \frac{V_{dC}}{1 - p_C} \left[ 1 - f_I \left( 1 - V_j / V_{dC} \right)^{1-p_C} \right] + f_I b_j \left( V_{\text{junc}} - V_j \right),$$

(5.34)

$$V_{t_c} = (1 - X_{pT}) V_{C_V} + X_{pT} V_{B_2 C_1}.$$  

(5.35)

$V_{t_c}$ is also proportional to $V_{B_2 E_1}$ and $I_{C_1 C_2}$. This means that in principle all quantities can be written as function of $V_{B_2 E_1}$ and $I_{C_1 C_2}$. The following two equation can be solved iteratively:

$$I_{B,m} = I_{B,s}(V_{B_2 E_1}, I_{C_1 C_2}),$$

(5.36a)

$$I_N((V_{B_2 E_1}, I_{C_1 C_2}) = I_{C_1 C_2}.$$

(5.36b)

Here $I_{B,m}$ is the measured base current, $I_{B,s}$ is the sum of all forward base currents. The ideal base current then is:

$$I_{B_1} = (1 - X_{iB_1}) \frac{I_S}{\beta_f} \left( e^{V_{B_2 E_1}/V_T} - 1 \right),$$

(5.37)

The Ideal side-wall base current is:

$$I_{B_1}^{S} = X_{iB_1} \frac{I_S}{\beta_f} \left( e^{V_{B_2 E_1}/V_T} - 1 \right),$$

(5.38)

The Non-ideal forward base current is:

$$I_{B_2} = I_{B f T} \left( e^{V_{B_2 E_2}/m_{LF} V_T} \right) + G_{\text{min}} (V_{B_2 E_1} + V_{B_2 C_1}),$$

(5.39)

The main current can be shown as:

$$I_N = I_{sT} * \frac{e^{V_{B_2 E_1}/V_T} - e^{V_{B_2 C_2}/V_T}}{qB},$$

(5.40)
The sum of all forward base current is:

\[ I_B = I_{B_1} + I_{B_1}^S + I_{B_2}. \] (5.41)

\( V_{B_2E_1} \) and \( I_{C_1C_2} \) can be solved iteratively together. Then simulated \( V_{BE} \) can be solved through Equation 5.3. As temperature scaling is included, changing thermal resistance \( R_{th} \) will change \( \Delta_T \) and thus all temperature scaling parameters will be influenced. Thus, \( R_{th} \) can be extracted from the comparison of the simulated and measured \( V_{BE} \) as shown in Figure 5.2 - 5.3. The base-emitter voltage increase sharply at beginning and drop down to keep the base-current constant. At high base-emitter bias the avalanche effect will happen which lead to the increase of base-emitter bias.
Figure 5.2: Measured (markers) and simulated (line) base-emitter voltage in the output characteristic measurement by Python.

Figure 5.3: Measured (markers) and simulated (line) base-emitter voltage in the output characteristic measurement by IC-CAP.
5.2 Knee current

The knee current is extracted from the collector current at high $V_{CE}$ using the output-characteristic measurements shown in Figure 5.1. The extraction of knee current use the same way that extract $R_{th}$ that we solve $I_{C1C2}$ and $V_{B2E_1}$ iteratively together. The collector current is:

$$I_{C1C2} = I_{sT} * \frac{e^{V_{B2E1}/V_T} - e^{V_{B2C2}/V_T}}{qB}.$$  \hspace{1cm} (5.42)

The knee current $I_K$ can change $I_{C1C2}$ by changing electron density in the base at emitter edge $n_0$ in Equation 5.7. The internal base-collector bias $V_{B2C2}^*$ can be calculate from current $I_{C1C2}$ and the internal base collector bias $V_{B2C1}$ as shown in Equation 5.15 – Equation 5.23. The base charge $q_B$ can also be calculated from $I_{C1C2}$, and $V_{B2E1}$. The knee current can be extracted by matching the simulation collector current to the measurement collector current as shown in Figure 5.4 - 5.5.

The collector current increase sharply as the collector-emitter bias increase from 0V. Then collector current curve become flat but increase slowly as the saturation current increase with increasing power dissipation. At high collector-emitter bias, the collector current increase because the avalanche effect.

5.3 Ohmic resistance

The ohmic resistance $R_{CV}$ is extracted from the decrease of the current gain in the forward-Gummel measurement shown in Figure 4.27. Here we also use the same way as before to solve $I_{C1C2}$ and $V_{B2E1}$ iteratively together. The current gain is:

$$h_{fe} = \frac{I_{C1C2}}{I_{B,m}}.$$ \hspace{1cm} (5.43)

Note here we use measurement base current. The ohmic resistance $R_{CV}$ can change $I_{C1C2}$ by changing $V_{qs}$ in Equation 5.12. Then $R_{CV}$ can be extracted from comparison of measurement current gain and simulation current gain and the result is shown in Figure 5.6 - 5.7. The
Figure 5.4: Measured (markers) and simulated (line) collector current in the output-characteristic measurement by Python

Figure 5.5: Measured (markers) and simulated (line) collector current in the output-characteristic measurement by IC-CAP
Figure 5.6: Measured (markers) and simulated (line) forward current gain as function of the measurement base current in forward-Gummel measurement by Python

Figure 5.7: Measured (markers) and simulated (line) forward current gain as function of the measurement base current in forward-Gummel measurement by IC-CAP

decrease of current gain is due to the quasi-saturation. The voltage drop over $R_{Cc}$ will result in the internal base-collector bias $V_{B2C2}$ forward biased and cause collector current decrease. However, the diffusion voltage $V_{dc}$ of the collector also cause current gain to decrease. In practice we fix the diffusion voltage $V_{dc}$ based on the doping level of the epilayer.

5.4 Cut-off frequency

The cut-off frequency $f_T$ is defined as the frequency where the current gain $h_{fe}$ becomes unity. The cut-off frequency is determined by:
\[ \frac{1}{2\pi f_T} = \tau_T = \frac{dQ}{dI_C} \bigg|_{v_{CE}}, \quad (5.44) \]

The \( \tau_T \) is the total emitter-collector transit time. It is related to the differential charge \( dQ \) and current \( dI_C \) under a constant collector-emitter bias.

The first step is to solve \( V_{B_2E_1} \) iteratively using Equation 5.3 - 5.41 as we do before. And here we use the measurement collector current as \( I_{C_1C_2} \).

Then we will determine the small signal variations

\[
\begin{align*}
    dV_{CE} &= \frac{\partial V_{CE}}{\partial V_{B_2E_1}} dV_{B_2E_1} + \frac{\partial V_{CE}}{\partial V_{B_2C_1}} dV_{B_2C_1} + \frac{\partial V_{CE}}{\partial I_{C_1C_2}} dI_{C_1C_2} = 0, \quad (5.45a) \\
    dI_N &= \frac{\partial I_N}{\partial V_{B_2E_1}} dV_{B_2E_1} + \frac{\partial I_N}{\partial V_{B_2C_1}} dV_{B_2C_1} + \frac{\partial I_N}{\partial I_{C_1C_2}} dI_{C_1C_2} = dI_{C_1C_2}, \quad (5.45b)
\end{align*}
\]

We can calculate \( dV_{B_2E_1}/dI_{C_1C_2} \) and \( dV_{B_2C_1}/dI_{C_1C_2} \) under a constant \( V_{CE} \) by solving these equations. The differential charge can be calculated similar to differential voltage and current above:

\[
    dQ = \frac{\partial Q}{\partial V_{B_2E_1}} dV_{B_2E_1} + \frac{\partial Q}{\partial V_{B_2C_1}} dV_{B_2C_1} + \frac{\partial Q}{\partial I_{C_1C_2}} dI_{C_1C_2}, \quad (5.46)
\]

Then the transit time can be calculated as:

\[
    \tau_T = \frac{\partial Q}{\partial V_{B_2E_1}} \frac{dV_{B_2E_1}}{dI_{C_1C_2}} + \frac{\partial Q}{\partial V_{B_2C_1}} \frac{dV_{B_2C_1}}{dI_{C_1C_2}} + \frac{\partial Q}{\partial I_{C_1C_2}}. \quad (5.47)
\]

In this way the cut-off frequency can be calculated as function of collector current and collector-emitter bias.

The parameters that can be extracted here are critical current for velocity saturation in the epilayer \( I_{he} \), space charge resistance of the epilayer \( SCR_{cv} \), transit time of stored epilayer charge \( \tau_{epi} \) and minimum transit time of stored emitter charge \( \tau_E \). The extraction result is given in Figure 5.8 and 5.9.
Figure 5.8: Measured(markers) and simulated(line) cut-off frequency in the S-parameter measurement by Python

Figure 5.9: Measured(markers) and simulated(line) cut-off frequency in the S-parameter measurement by IC-CAP
5.5 Reverse current at high injection

The partitioning factor of the extrinsic regions $X_{ext}$ is extracted at reverse-Gummel measurement from the absolute values of the reverse currents $I_B$, $I_E$ and $I_{sub}$. The main current is:

$$I_N = I_{ST} e^{V_{B_2}C_2/V_T} \frac{e^{V_{B_2}C_2/V_T} - 1}{qB}$$  \hspace{1cm} (5.48)

Mextram uses the Kull model in reverse and following the Kull model[24] we introduce:

$$k_0 = \sqrt{1 + 4 \exp \left( \frac{(V_{B_2}C_2 - V_{dc})}{V_T} \right)}$$ \hspace{1cm} (5.49a)

$$k_W = \sqrt{1 + 4 \exp \left( \frac{(V_{B_2}C_1 - V_{dc})}{V_T} \right)}$$ \hspace{1cm} (5.49b)

$$E_c = V_T \left[ 2k_0 - 2k_W - \ln \left( \frac{k_0 + 1}{k_W + 1} \right) \right]$$ \hspace{1cm} (5.49c)

$$I_{epi} = - \frac{(E_c + V_{B_2}C_2 - V_{B_2}C_1)}{R_{CV}}$$ \hspace{1cm} (5.49d)

The junction voltage $V_{B_2}C_1 = V_{B_1}C_1$ can be solve iteratively using $I_N = I_{epi}$. The non-ideal reverse base current is:

$$I_{B_3} = I_{BR_T} \frac{e^{V_{B_1}C_1/V_T} - 1}{e^{V_{B_1}C_1/2V_T} + e^{V_L/2V_T}}$$ \hspace{1cm} (5.50)

The substrate current includes high injection:

$$I_{sub} = \frac{2I_{ST}}{1 + \sqrt{1 + 4 \frac{I_{ST}}{I_{KT}} e^{V_{B_1}C_1/V_T}}} \frac{e^{V_{B_1}C_1/V_T} - 1}{1 + 4 \frac{I_{ST}}{I_{KT}} e^{V_{B_1}C_1/V_T}}$$ \hspace{1cm} (5.51)

The extrinsic base current is given by:

$$g_1 = \frac{4 I_{ST}}{I_{KT}} e^{V_{B_1}C_1/V_T}$$ \hspace{1cm} (5.52a)

$$n_{Bex} = \frac{4 I_{ST}}{I_{KT}} e^{V_{B_1}C_1/V_T} - 1 \frac{e^{V_{B_1}C_1/V_T} - 1}{1 + \sqrt{1 + g_1}}$$ \hspace{1cm} (5.52b)
\[ I_{ex} = (1 - X_{ext}) \frac{I_{kT}}{2 \beta_{HT}} n_{Bex} \]  

(5.52c)

The ideal base current then is:

\[ I_{B_1} = (1 - X_{I_{B_1}}) \frac{I_{kT}}{\beta_{R_T}} \left( e^{V_{B_2C_2}/V_T} \right) \]  

(5.53)

The junction voltage which we need for the calculation of \( XI_{ex} \) and \( XI_{sub} \) is:

\[ V_{BC_1} = V_{B_1C_1} + (I_{ex} + I_{sub} + I_{B_3}) R_{BCT} \]  

(5.54)

Extrinsic reverse base current is given:

\[ X_{g_1} = \frac{4 I_{ST}}{I_{kT}} e^{V_{BC_1}/V_T} \]  

(5.55a)

\[ X_{n_{Bex}} = \frac{4 I_{kT}}{I_{kT}} \frac{e^{V_{BC_1}/V_T} - 1}{1 + \sqrt{1 + X_{g_1}}} \]  

(5.55b)

\[ XI_{ex} = X_{ext} \frac{I_{kT}}{2 \beta_{HT}} X_{n_{Bex}} \]  

(5.55c)

Substrate current is described as:

\[ XI_{sub} = X_{ext} \frac{2I_{ST}}{1 + \sqrt{1 + 4 \frac{I_{eST}}{I_{kT}} e^{V_{BC_1}/V_T}}} \]  

(5.56)

Then we can calculate the external base-collector bias:

\[ V_{BC} = V_{BC_1} + (I_{ex} + XI_{ex} + I_N) R_{BCT} \]  

(5.57)

Note that \( V_{B_2C_2} \) need to be solve iteratively using equations above. Then we can get the following current:

\[ I_E = I_N \]  

(5.58a)
\[ I_B = I_{ex} + X I_{ex} + I_{sub} + X I_{sub} + I_{B3} \]  
\[ I_{sub,ext} = -I_{sub} - X I_{sub} \] 

Figure 5.10: Measured(markers) and simulated(line) emitter currents in the reverse-Gummel measurement by Python

Figure 5.11: Measured(red line) and simulated(blue line for build in function purple line for simulator) emitter currents in the reverse-Gummel measurement by IC-CAP

Figure 5.12: Measured(markers) and simulated(line) substrate currents in the reverse-Gummel measurement by Python

Figure 5.13: Measured(red line) and simulated(blue line for build in function purple line for simulator) substrate currents in the reverse-Gummel measurement by IC-CAP
Figure 5.14: Measured (markers) and simulated (line) base currents in the reverse-Gummel measurement by Python

Figure 5.15: Measured (red line) and simulated (blue line for build in function purple line for simulator) base currents in the reverse-Gummel measurement by IC-CAP
6.1 T-scaling rules

To be able to extract the high-current parameters correctly, it is important to take temperature scaling into account. In Table 6.1 the cross reference between the temperature parameters and the electrical parameters is listed.

The actual simulation temperature is denoted by TEMP (in °C). The temperature at which the parameters are determined is $T_{\text{ref}}$ (also in °C).

Conversion to Kelvin

$$T_K = \text{TEMP} + \text{DTA} + 273.15 + V_{dT}, \quad (6.1a)$$

$$T_{\text{amb}} = \text{TEMP} + \text{DTA} + 273.15, \quad (6.1b)$$

$$T_{RK} = T_{\text{ref}} + 273.15, \quad (6.2)$$

$$t_N = \frac{T_K}{T_{RK}}, \quad (6.3)$$

Depletion capacitances The junction diffusion voltages $V_{dE}$, $V_{dC}$, and $V_{dS}$ with respect to temperature are

$$U_{dE_T} = -3 V_T \ln t_N + V_{dE} t_N + (1 - t_N) V_{\text{fB}}, \quad (6.4)$$

$$V_{dE_T} = U_{dE_T} + V_T \ln \{1 + \exp[(V_{d,\text{low}} - U_{dE_T})/V_T]\}, \quad (6.5)$$
Table 6.1: Summery of the occurrence of the temperature parameters in the temperature scaling rules of the electrical parameters.

<table>
<thead>
<tr>
<th>No.</th>
<th>Parameter</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$A_{QB0}$</td>
<td>$V_{er}, V_{ef}, I_s, \beta_f, R_{Bv}, \tau_B, \tau_R, I_{ks}, dE_g$</td>
</tr>
<tr>
<td>2</td>
<td>$A_E$</td>
<td>$R_E, \beta_f$</td>
</tr>
<tr>
<td>3</td>
<td>$A_B$</td>
<td>$R_{Bv}, \beta_f, I_s, I_k, \tau_B, \tau_R, \tau_E, I_{ks}$</td>
</tr>
<tr>
<td>4</td>
<td>$A_{epi}$</td>
<td>$R_{c_e}, \tau_{epi}, \tau_R$</td>
</tr>
<tr>
<td>5</td>
<td>$A_{ex}$</td>
<td>$R_{Bc}$</td>
</tr>
<tr>
<td>6</td>
<td>$A_c$</td>
<td>$R_{Cc}$</td>
</tr>
<tr>
<td>7</td>
<td>$A_S$</td>
<td>$I_{SS}, I_{ks}$</td>
</tr>
<tr>
<td>8</td>
<td>$dV_{gBr}$</td>
<td>$\beta_{ri}$</td>
</tr>
<tr>
<td>9</td>
<td>$V_{gb}$</td>
<td>$I_s, C_{JE}, V_{de}, V_{er}, I_{ks}$</td>
</tr>
<tr>
<td>10</td>
<td>$V_{gc}$</td>
<td>$C_{JC}, V_{dc}, X_p, I_{Br}, V_{ef}$</td>
</tr>
<tr>
<td>11</td>
<td>$V_{gj}$</td>
<td>$I_{Br}$</td>
</tr>
<tr>
<td>12</td>
<td>$dV_{grE}$</td>
<td>$\tau_E$</td>
</tr>
<tr>
<td>13</td>
<td>$V_{gs}$</td>
<td>$I_{SS}, I_{ks}, C_{JS}, V_{ds}$</td>
</tr>
<tr>
<td>14</td>
<td>$dV_{gBf}$</td>
<td>$\beta_f$</td>
</tr>
</tbody>
</table>

\[ U_{dC_T} = -3V_T \ln t_N + V_{dC_T} t_N + (1 - t_N) V_{gc}, \]  
\[ V_{dC_T} = U_{dC_T} + V_T \ln \{ 1 + \exp \left[ \left( V_{d,low} - U_{dC_T} \right) / V_T \right] \}, \]  
\[ U_{dS_T} = -3V_T \ln t_N + V_{dS_T} t_N + (1 - t_N) V_{gs}, \]  
\[ V_{dS_T} = U_{dS_T} + V_T \ln \{ 1 + \exp \left[ \left( V_{d,low} - U_{dS_T} \right) / V_T \right] \}. \]

The zero-bias capacitances scale with temperature as

\[ C_{JE_T} = C_{JE} \left( \frac{V_{de}}{V_{de_T}} \right)^{pg}, \]  
\[ C_{JS_T} = C_{JS} \left( \frac{V_{ds}}{V_{ds_T}} \right)^{ps}, \]
The collector depletion capacitance is divided in a variable and a constant part. The constant part is temperature independent.

\[ C_{jcT} = C_{jc} \left[ (1 - X_p) \left( \frac{V_{dc}}{V_{dcT}} \right)^{PC} + X_p \right], \quad (6.12) \]

\[ X_{pT} = X_p \left[ (1 - X_p) \left( \frac{V_{dc}}{V_{dcT}} \right)^{PC} + X_p \right]^{-1}. \quad (6.13) \]

Resistances  The various parameters \( A \) describe the mobility of the corresponding regions:

\[ \mu \propto t_N^{-A}. \]  The temperature dependence of the zero-bias base charge goes as \( Q_{BO}/Q_{B0} = t_N^{A_{QB0}}. \]

\[ R_{ET} = R_E t_N^{A_E}, \quad (6.14) \]

\[ R_{BvT} = R_{Bv} t_N^{A_B-A_{QB0}}, \quad (6.15) \]

\[ R_{BcT} = R_{Bc} t_N^{A_{ex}}, \quad (6.16) \]

\[ R_{CvT} = R_{Cv} t_N^{A_{epi}}, \quad (6.17) \]

\[ R_{CcT} = R_{Cc} t_N^{A_C}. \quad (6.18) \]

Current gains

\[ \beta_{RT} = \beta_r t_N^{A_E-A_B-A_{QB0}} \exp[-dV_{gRT}/V_{\Delta T}], \quad (6.19) \]

\[ \beta_{RI} = \beta_{ri} \exp[-dV_{gRT}/V_{\Delta T}], \quad (6.20) \]

Currents and voltages

\[ I_{sT} = I_s t_N^{4-A_B-A_{QB0}+dA_s} \exp[-V_{gs}/V_{\Delta T}], \quad (6.21) \]

\[ I_{kT} = I_k t_N^{1-A_B}, \quad (6.22) \]
\[ I_{BfT} = I_{Bf} t_{N}^{(6-2m_{Lfi})} \exp[-V_{gL}/m_{Lf} V_{\Delta T}], \]  
\[ (6.23) \]

\[ I_{BrT} = I_{Br} t_{N}^{2} \exp[-V_{gC}/2V_{\Delta T}], \]  
\[ (6.24) \]

\[ V_{efT} = V_{ef} t_{N}^{A_{QBo}} \left[ (1 - X_{p}) \left( \frac{V_{dC}}{V_{dC} T} \right)^{PC} + X_{p} \right]^{-1}, \]  
\[ (6.25) \]

\[ V_{erT} = V_{er} t_{N}^{A_{QBo}} \left( \frac{V_{dE}}{V_{dE} T} \right)^{-pe}, \]  
\[ (6.26) \]

The temperature dependence of \( I_{Ss} \) and \( I_{ks} \) is given by \( A_{S} \) and \( V_{gs} \).

\( A_{S} \) equals \( A_{C} \) for a closed buried layer (BN) and \( A_{S} \) equals \( A_{epi} \) for an open buried layer.

\[ I_{SsT} = I_{Ss} t_{N}^{4-A_{S}} \exp[-V_{gs}/V_{\Delta T}], \]  
\[ (6.27) \]

\[ I_{ksT} = I_{ks} t_{N}^{1-A_{S}} \frac{I_{sT}}{I_{s}} \frac{I_{Ss}}{I_{SsT}}, \]  
\[ (6.28) \]

When either \( I_{s} = 0 \) or \( I_{SsT} = 0 \) we take \( I_{ksT} = I_{ks} t_{N}^{1-A_{S}} \).

**Transit times**

\[ \tau_{ET} = \tau_{E} t_{N}^{(A_{B} - 2)} \exp[-dV_{gE}]/V_{\Delta T}], \]  
\[ (6.29) \]

\[ \tau_{BT} = \tau_{B} t_{N}^{A_{QBo} + A_{B} - 1} \]  
\[ (6.30) \]

\[ \tau_{epiT} = \tau_{epi} t_{N}^{A_{epi} - 1}, \]  
\[ (6.31) \]

\[ \tau_{RT} = \tau_{R} \frac{\tau_{BT} + \tau_{epiT}}{\tau_{B} + \tau_{epi}}. \]  
\[ (6.32) \]

### 6.2 Temperature parameters

There are many different methods to extract temperature parameters. One method is optimizing the temperature parameters of all data over temperatures. The disadvantage of
this method is that we do not extract the individual parameters at each parameter. If there are unexpected differences between the model simulation and hardware data, it is difficult to know whether it is the weakness of the electrical model or temperature scaling model.

The second method is to extract the electrical parameters at all temperatures isothermally. The main advantage is that one can check the correctness of existing temperature scaling equations by comparing extracted and simulated electrical parameters.[18]

For each temperature parameter, we choose one electrical parameter from which we will extract the temperature parameter. This is easier than doing a fit over all the electrical parameters and it gives more control.

Figure 6.1: Extracted(markers) and simulated(line) values of the reverse Early voltage
Figure 6.2: Extracted (markers) and simulated (line) values of the forward Early voltage

Figure 6.3: Extracted (markers) and simulated (line) values of collector saturation current
Figure 6.4: Extracted (markers) and simulated (line) values of substrate saturation current

Figure 6.5: Extracted (markers) and simulated (line) values of forward current gain
Figure 6.6: Extracted (markers) and simulated (line) values of reverse current gain

Figure 6.7: Extracted (markers) and simulated (line) values of non-ideal forward base current
Figure 6.8: Extracted (markers) and simulated (line) values of non-ideal reverse base current
6.3 Extraction result

Since we have extracted all the parameters in Mextram 504 model. It is necessary for us to use a circuit simulator to compare measurement and simulation result. Here we use ADS [31] simulator in IC-CAP. Figure 6.9 - 6.10 show the $I_B$ and $I_C$ versus $V_{BE}$ in forward gummel measurement from 223-393K respectively. Figure 6.11 - 6.12 show the $I_B$ and $I_C$ versus $V_{BE}$ in reverse gummel measurement from 223-393K respectively. Figure 6.13 to 6.16 give $I_C$-$V_{CE}$ and $V_{BE}$-$V_{CE}$ in force- $I_B$ measurement from 223-393K. $f_T$-$I_C$ at $V_{CB}$ = -0.5, 0, 1, and 2V is shown in Figure 6.17.
Figure 6.9: Measured (symbol) and simulated (solid line) $I_B-V_{BE}$ from 223-393 K. (a) 223K. (b) 300K. (c) 393K.
Figure 6.10: Measured (symbol) and simulated (solid line) $I_C-V_{BE}$ from 223-393 K. (a) 223K. (b) 300K. (c) 393K.
Figure 6.11: Measured (symbol) and simulated (solid line) $I_{E-V_{BC}}$ from 2233-393 K. (a) 223K. (b) 300K. (c) 393K.
Figure 6.12: Measured (symbol) and simulated (solid line) $I_{Sub-V_{BC}}$ from 223-393 K. (a) 223K. (b) 300K. (c) 393K.
Figure 6.13: Measured (symbol) and simulated (solid line) $I_C-V_{CE}$ from 223-393K at high IB. (a) 223K. (b) 300K. (c) 393K.
Figure 6.14: Measured (symbol) and simulated (solid line) $I_C-V_{CE}$ from 223-393 K at low IB. (a) 223K. (b) 300K. (c) 393K.
Figure 6.15: Measured (symbol) and simulated (solid line) $V_{BE}-V_{CE}$ from 223-393 K at high IB. (a) 223K. (b) 300K. (c) 393K.
Figure 6.16: Measured (symbol) and simulated (solid line) $V_{BE}$-$V_{CE}$ from 223-393 K at low IB. (a) 223K. (b) 300K. (c) 393K.
Figure 6.17: Measured (symbol) and simulated (solid line) $f_T-I_C$ from 223-393 K. (a) 223K. (b) 300K. (c) 393K.
Bibliography


Appendices
Appendix A

Appendix A Full List of Mextram model Parameters

A.1 Compact model parameter list

Table A.1: Mextram 504.12 model parameters overview

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 LEVEL</td>
<td>Model level, must be set to 504</td>
</tr>
<tr>
<td>2 $T_{\text{ref}}$</td>
<td>Reference temperature. Default is 25°C</td>
</tr>
<tr>
<td>3 DTA</td>
<td>Difference between the local ambient and global ambient temperatures</td>
</tr>
<tr>
<td>4 EXMOD</td>
<td>Flag for extended modelling of the reverse current gain</td>
</tr>
<tr>
<td>5 EXPHI</td>
<td>* Flag for the distributed high-frequency effects in transient</td>
</tr>
<tr>
<td>6 EXAVL</td>
<td>Flag for extended modelling of avalanche currents</td>
</tr>
<tr>
<td>7 EXSUB</td>
<td>Flag for extended modelling of substrate currents</td>
</tr>
<tr>
<td>8 $I_s$</td>
<td>Collector-emitter saturation current</td>
</tr>
<tr>
<td>9 $I_k$</td>
<td>Collector-emitter high injection knee current</td>
</tr>
<tr>
<td>10 $V_{er}$</td>
<td>Reverse Early voltage</td>
</tr>
<tr>
<td>11 $V_{ef}$</td>
<td>Forward Early voltage</td>
</tr>
<tr>
<td>12 $\beta_f$</td>
<td>Ideal forward current gain</td>
</tr>
<tr>
<td>13 $I_{Bf}$</td>
<td>Saturation current of the non-ideal forward base current</td>
</tr>
<tr>
<td>14 $m_{Lf}$</td>
<td>Non-ideality factor of the non-ideal forward base current</td>
</tr>
<tr>
<td>15 $XI_{B_i}$</td>
<td>Part of ideal base current that belongs to the sidewall</td>
</tr>
<tr>
<td>16 $I_{zeb}$</td>
<td>Pre-factor of emitter-base Zener tunneling current</td>
</tr>
<tr>
<td>17 $N_{zeb}$</td>
<td>Coefficient of emitter-base Zener tunneling current</td>
</tr>
</tbody>
</table>
Table A.1 – continued from previous page

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\beta_{ri}$</td>
<td>Ideal reverse current gain</td>
</tr>
<tr>
<td>$I_{Br}$</td>
<td>Saturation current of the non-ideal reverse base current</td>
</tr>
<tr>
<td>$V_{Lr}$</td>
<td>Cross-over voltage of the non-ideal reverse base current</td>
</tr>
<tr>
<td>$X_{ext}$</td>
<td>Part of $I_{ex}$, $Q_{tex}$, $Q_{ex}$ and $I_{sub}$ that depends on $V_{BC_3}$ instead of $V_{B_1C_4}$</td>
</tr>
<tr>
<td>$W_{avl}$</td>
<td>Epilayer thickness used in weak-avalanche model</td>
</tr>
<tr>
<td>$V_{avl}$</td>
<td>Voltage determining curvature of avalanche current</td>
</tr>
<tr>
<td>$S_{IH}$</td>
<td>Current spreading factor of avalanche model (when EXAVL = 1)</td>
</tr>
<tr>
<td>$R_E$</td>
<td>Emitter resistance</td>
</tr>
<tr>
<td>$R_{Bc}$</td>
<td>Constant part of the base resistance</td>
</tr>
<tr>
<td>$R_{Bv}$</td>
<td>Zero-bias value of the variable part of the base resistance</td>
</tr>
<tr>
<td>$R_{Cc}$</td>
<td>Collector Contact resistance</td>
</tr>
<tr>
<td>$R_{cblx}$</td>
<td>Resistance of the Collector Buried Layer: eXtrinsic part</td>
</tr>
<tr>
<td>$R_{cbli}$</td>
<td>Resistance of the Collector Buried Layer: Intrinsic part</td>
</tr>
<tr>
<td>$R_{Cv}$</td>
<td>Resistance of the un-modulated epilayer</td>
</tr>
<tr>
<td>$SCR_{Cv}$</td>
<td>Space charge resistance of the epilayer</td>
</tr>
<tr>
<td>$I_{hc}$</td>
<td>Critical current for velocity saturation in the epilayer</td>
</tr>
<tr>
<td>$a_{x_i}$</td>
<td>Smoothness parameter for the onset of quasi-saturation</td>
</tr>
<tr>
<td>$C_{je}$</td>
<td>Zero-bias emitter-base depletion capacitance</td>
</tr>
<tr>
<td>$V_{dE}$</td>
<td>Emitter-base diffusion voltage</td>
</tr>
<tr>
<td>$p_E$</td>
<td>Emitter-base grading coefficient</td>
</tr>
<tr>
<td>$XC_{je}$</td>
<td>Fraction of the emitter-base depletion capacitance that belongs to the sidewall</td>
</tr>
<tr>
<td>$C_{BEO}$</td>
<td>Emitter-base overlap capacitance</td>
</tr>
<tr>
<td>$C_{jc}$</td>
<td>Zero-bias collector-base depletion capacitance</td>
</tr>
<tr>
<td>$V_{dC}$</td>
<td>Collector-base diffusion voltage</td>
</tr>
<tr>
<td>$p_C$</td>
<td>Collector-base grading coefficient</td>
</tr>
</tbody>
</table>
Table A.1 – continued from previous page

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_p$</td>
<td>Constant part of $C_{jc}$</td>
</tr>
<tr>
<td>$m_C$</td>
<td>Coefficient for the current modulation of the CB depletion capacitance</td>
</tr>
<tr>
<td>$XC_{jc}$</td>
<td>* Fraction of the collector-base depletion capacitance under the emitter</td>
</tr>
<tr>
<td>$C_{BCO}$</td>
<td>* Collector-base overlap capacitance</td>
</tr>
<tr>
<td>$m_r$</td>
<td>* Non-ideality factor of the emitter stored charge</td>
</tr>
<tr>
<td>$\tau_E$</td>
<td>* Minimum transit time of stored emitter charge</td>
</tr>
<tr>
<td>$\tau_B$</td>
<td>* Transit time of stored base charge</td>
</tr>
<tr>
<td>$\tau_{epi}$</td>
<td>* Transit time of stored epilayer charge</td>
</tr>
<tr>
<td>$\tau_R$</td>
<td>* Transit time of reverse extrinsic stored base charge</td>
</tr>
<tr>
<td>$dE_g$</td>
<td>Bandgap difference over the base</td>
</tr>
<tr>
<td>$X_{rec}$</td>
<td>Pre-factor of the recombination part of $I_{B1}$</td>
</tr>
<tr>
<td>$X_{Q_B}$</td>
<td>Fraction of the total base charge supplied by the collector</td>
</tr>
<tr>
<td>$A_{Q_{B0}}$</td>
<td>Temperature coefficient of the zero-bias base charge</td>
</tr>
<tr>
<td>$A_E$</td>
<td>Temperature coefficient of the resistivity of the emitter</td>
</tr>
<tr>
<td>$A_B$</td>
<td>Temperature coefficient of the resistivity of the base</td>
</tr>
<tr>
<td>$A_{epi}$</td>
<td>Temperature coefficient of the resistivity of the epilayer</td>
</tr>
<tr>
<td>$A_{ex}$</td>
<td>Temperature coefficient of the resistivity of the extrinsic base</td>
</tr>
<tr>
<td>$A_C$</td>
<td>Temperature coefficient of the resistivity of the collector contact</td>
</tr>
<tr>
<td>$A_{cbl}$</td>
<td>Temperature coefficient of the resistivity of the collector buried layer</td>
</tr>
<tr>
<td>$dA_{f_s}$</td>
<td>Parameter for fine tuning of temperature dependence of CE saturation current</td>
</tr>
<tr>
<td>$dV_{g\beta_f}$</td>
<td>Band-gap voltage difference of forward current gain</td>
</tr>
<tr>
<td>$dV_{g\beta_r}$</td>
<td>Band-gap voltage difference of reverse current gain</td>
</tr>
<tr>
<td>$V_{g_B}$</td>
<td>Band-gap voltage of the base</td>
</tr>
<tr>
<td>$V_{g_C}$</td>
<td>Band-gap voltage of the collector</td>
</tr>
<tr>
<td>$V_{g_{ij}}$</td>
<td>Band-gap voltage recombination emitter-base junction</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$V_{gzEB}$</td>
<td>Band-gap voltage at reference temperature relevant to the Zener effect in EB junction</td>
</tr>
<tr>
<td>$A_{Vgz}$</td>
<td>Temperature scaling coefficient of emitter-base Zener tunneling current</td>
</tr>
<tr>
<td>$T_{BVgz}$</td>
<td>Temperature scaling coefficient of emitter-base Zener tunneling current</td>
</tr>
<tr>
<td>$dV_{gtE}$</td>
<td>* Band-gap voltage difference of emitter stored charge</td>
</tr>
<tr>
<td>$A_f$</td>
<td>* Exponent of the Flicker-noise</td>
</tr>
<tr>
<td>$K_f$</td>
<td>* Flicker-noise coefficient of the ideal base current</td>
</tr>
<tr>
<td>$K_{fN}$</td>
<td>* Flicker-noise coefficient of the non-ideal base current</td>
</tr>
<tr>
<td>$K_{avd}$</td>
<td>* Switch for white noise contribution due to avalanche</td>
</tr>
<tr>
<td>$K_C$</td>
<td>* Switch for RF correlation noise model selection</td>
</tr>
<tr>
<td>$I_{SS}$</td>
<td>Base-substrate saturation current</td>
</tr>
<tr>
<td>$I_{SSF}$</td>
<td>Collector-substrate ideal saturation current</td>
</tr>
<tr>
<td>$I_{ks}$</td>
<td>Base-substrate high injection knee current</td>
</tr>
<tr>
<td>$C_{JS}$</td>
<td>* Zero-bias collector-substrate depletion capacitance</td>
</tr>
<tr>
<td>$V_{ds}$</td>
<td>* Collector-substrate diffusion voltage</td>
</tr>
<tr>
<td>$p_S$</td>
<td>* Collector-substrate grading coefficient</td>
</tr>
<tr>
<td>$V_{gs}$</td>
<td>Band-gap voltage of the substrate</td>
</tr>
<tr>
<td>$A_S$</td>
<td>For a closed buried layer: $A_S = A_C$, and for an open buried layer: $A_S = A_{\text{epi}}$</td>
</tr>
<tr>
<td>$A_{sub}$</td>
<td>Temperature coefficient for mobility of minorities in the substrate</td>
</tr>
<tr>
<td>$R_{th}$</td>
<td>Thermal resistance</td>
</tr>
<tr>
<td>$C_{th}$</td>
<td>* Thermal capacitance</td>
</tr>
<tr>
<td>$A_{th}$</td>
<td>Temperature coefficient of the thermal resistance</td>
</tr>
<tr>
<td>MULT</td>
<td>Multiplication factor</td>
</tr>
</tbody>
</table>