Reliability Analysis of Prestressed Concrete Box Girder Bridges in California

by

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A thesis submitted to the Graduate Faculty of
Auburn University
in partial fulfillment of the
requirement for the Degree of
Master of Science

Auburn, Alabama
August 7, 2021

Keywords: bridge, WIM, safety, reliability

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Abstract

Truck traffic increases day by day according to the economy. Road network infrastructure has to meet this growing demand. This work focuses on prestressed concrete box girder bridges in California, because many signs of deterioration, such as cracks, have been observed lately. The objective is to calculate the reliability index for a wide spectrum of those bridges. There are over 6,000 prestressed concrete box girder bridges in California, and fourteen of them were selected for the reliability analysis. Moments due to dead load, live load and resistance were calculated, and the Monte Carlo method was used to calculate the reliability indices for the ultimate limit state. The average and standard deviation for the reliability indices are 4.81 and 1.38, respectively. Results show that the reliability indices for simple spans decrease and then increase as the span length and the load effect increase, increasing and then decreasing for continuous spans.

Acknowledgments

I would like to express my gratitude to my advisor, Dr. Andrzej S. Nowak, for his guidance and support through my Master's thesis. I would like to thank Dr. Anton K. Schindler and Dr. James S. Davidson for serving on my committee.

Special thanks to my colleagues Sylwia Stawska, Jacek Chmielewski, Pablo Esteban Hurtado, and Karina Popok for their help, encouragement, and advice regarding my Master's thesis.

I would also like to thank and acknowledge my parents and siblings for their love, continuous support, and prayers.

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List of Abbreviations

AASHTO	American Association of State Highway and Transportation Officials
ACI	American Concrete Institute
ADT	Average Daily Traffic
ADTT	Average Daily Truck Traffic
ANPR	Automatic Number Plate Recognition
FHWA	Federal Highway Administration
GVW	Gross Vehicle Weight
HS-WIM	High-Speed Weigh-in-Motion
LRFD	Load and Resistance Factor Design
LRFR	Load and Resistance Factor Rating
MBE	Manual for Bridge Evaluation
NBI	National Bridge Inventory
NCHRP	National Cooperative Highway Research program
SWIM	Slow-Speed Weigh-in-Motion
WIM	Weigh-In-Motion

List of Symbols

β	Reliability index
Φ	Cumulative distribution function of the normal variable
Φ^{-1}	Inverse cumulative distribution function of the normal variable
L	Span length
N_c	Number of cells or boxes
P_f	Probability of failure
S	Spacing of one box girder
λ	Bias factor
V	Coefficient of variation
GDF	Girder distribution factor
CS	Continuous span
SS	Simple span

Chapter 1. Introduction

1.1 Problem Statement

Bridges are used every day for a variety of reasons including travelling to work or to a leisure place (lake, coast, etc.) and receiving packages from online retailers. The use of bridges has become more critical due to the increase in the population and the economy. As a result, the average daily traffic (ADT) and the average daily truck traffic (ADTT) also increase.

Many bridges in the state of California are functionally obsolete and structurally deficient. Functionally obsolete bridges are bridges that do not meet current traffic demands or current standards due to too few lanes or too narrow lanes and/or shoulders. Structurally deficient bridges are those that require significant maintenance, rehabilitation or replacement because critical load-carrying elements were found to be in poor condition due to deterioration or damage (ASCE, 2019).

According to the 2019 California infrastructure report card, California has the second largest percentage of functionally obsolete bridges. Figure 1-1 presents the evolution of ADT on obsolete bridges, and Figure 1-2 presents the evolution of the structurally deficient bridges and shows that the number of structurally deficient bridges decreases with age.

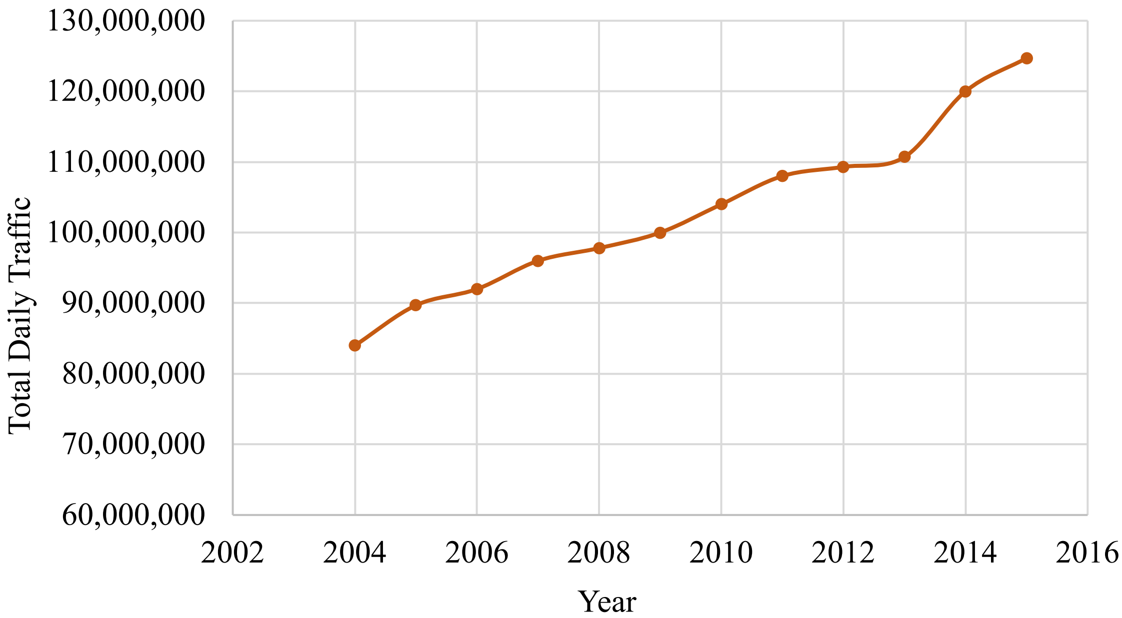


Figure 1-1. Total daily traffic on functionally obsolete bridges in the state of California (ASCE, 2019).

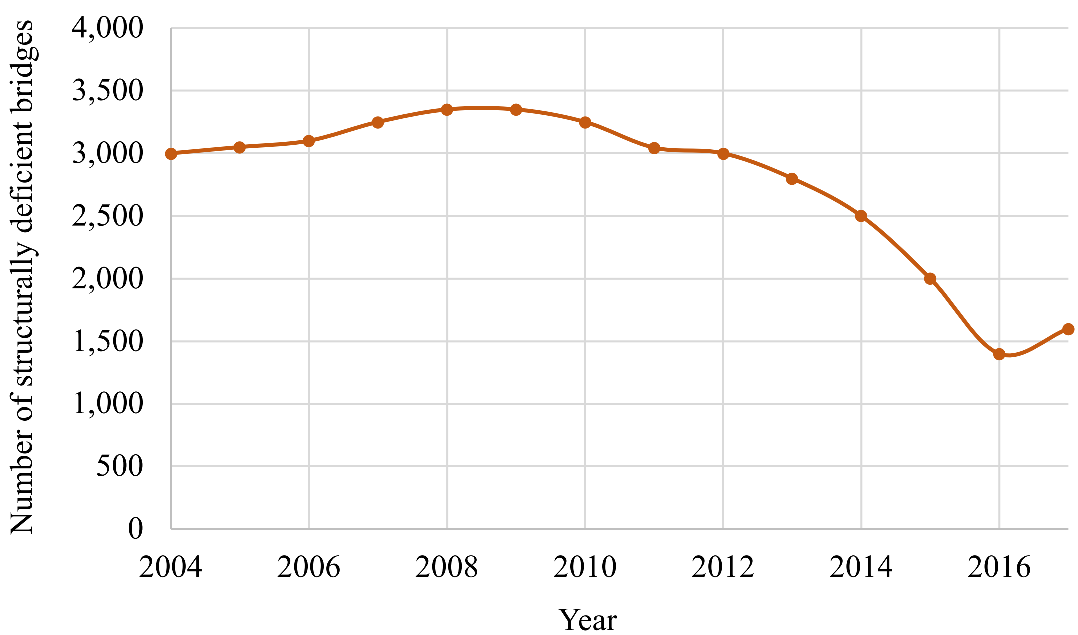


Figure 1-2. Structurally deficient bridges in the state of California (ASCE, 2019).

Bridge deterioration such as longitudinal, transverse, and pattern cracks, spalling, delamination, etc. are observed in decks of California bridges. Some examples of these cracks are displayed in Figure 1-3 and Figure 1-4.

Figure 1-3. Longitudinal deck cracks (Caltrans)

According to the 2019 California report card, 65% of the total bridge stock are in good condition, 30% are in fair condition and 5% are in poor condition. Maintenance is required so that bridges in the “fair” category do not fall into the “poor” category. In the fiscal year 2017-2018 State Budget, $131 million was allocated to bridge major maintenance; however, the ongoing annual need for bridge maintenance alone is estimated to require about $200 million (ASCE, 2019). Therefore, because of the scarcity of the funding, bridges with a higher structurally deficiency must be a priority in the process of repair.

Figure 1-4. Map pattern cracks (Caltrans).

1.2 Objective

The objective of the thesis is to calculate the reliability indices for a wide spectrum of prestressed concrete box girder bridges in California to investigate their safety under applied live load.

1.3 Outline

The thesis is outlined as follows:

- Chapter 1. Introduction, it provides some background and general information related to the bridges in California, and the objective and outline of the thesis report.
- Chapter 2. Literature Review provides a state of art review of literature on various subjects that include, bridge deterioration, weigh-in-motion, live loads, and reliability analysis.
- Chapter 3. Live Load Model in California, where the methodology used to determine statistical parameters using California weigh-in-motion data, is described.

- Chapter 4. Representative Bridges, it presents the bridge selection procedure for the reliability analysis .
- Chapter 5. Load and Resistance Models, it provides information about the dead load, live load and resistance models used in the reliability analysis.
- Chapter 6. Reliability analysis provides background about the reliability analysis topic, and gives the results of the reliability indices calculated for the selected bridges.
- Chapter 7. Summary and Conclusions, it focuses on the thesis report summary and conclusions.

1.4 Overview

In the National Bridge Inventory (NBI) database, issued in 2021, the bridge records in 2020 count 25,755 bridges of various types open for traffic in California. It makes California the fourth state in the U.S. in terms of bridge number after Texas, Ohio, and Illinois. Table 1-1 and Figure 1-5 show statistics of the bridge material types in California. The NBI database contains valuable information for each bridge, such as the name and bridge number, the location, the deck condition, the maximum span length, the year built and year reconstructed, etc. Details of each item are described in the *Recording and Coding Guide for the Structure Inventory and Appraisal of the Nation's Bridges*, written by the Federal Highway Administration (FHWA).

Table 1-1. Bridge material types in California

Bridge material type	Count
Aluminum, Wrought Iron, or Cast Iron	19
Concrete	5,176
Concrete continuous	10,720
Masonry	33
Other material	16
Prestressed concrete	3,200
Prestressed concrete continuous	3,101
Steel	2,373
Steel continuous	446
Wood or Timber	671
Total	**25,755**

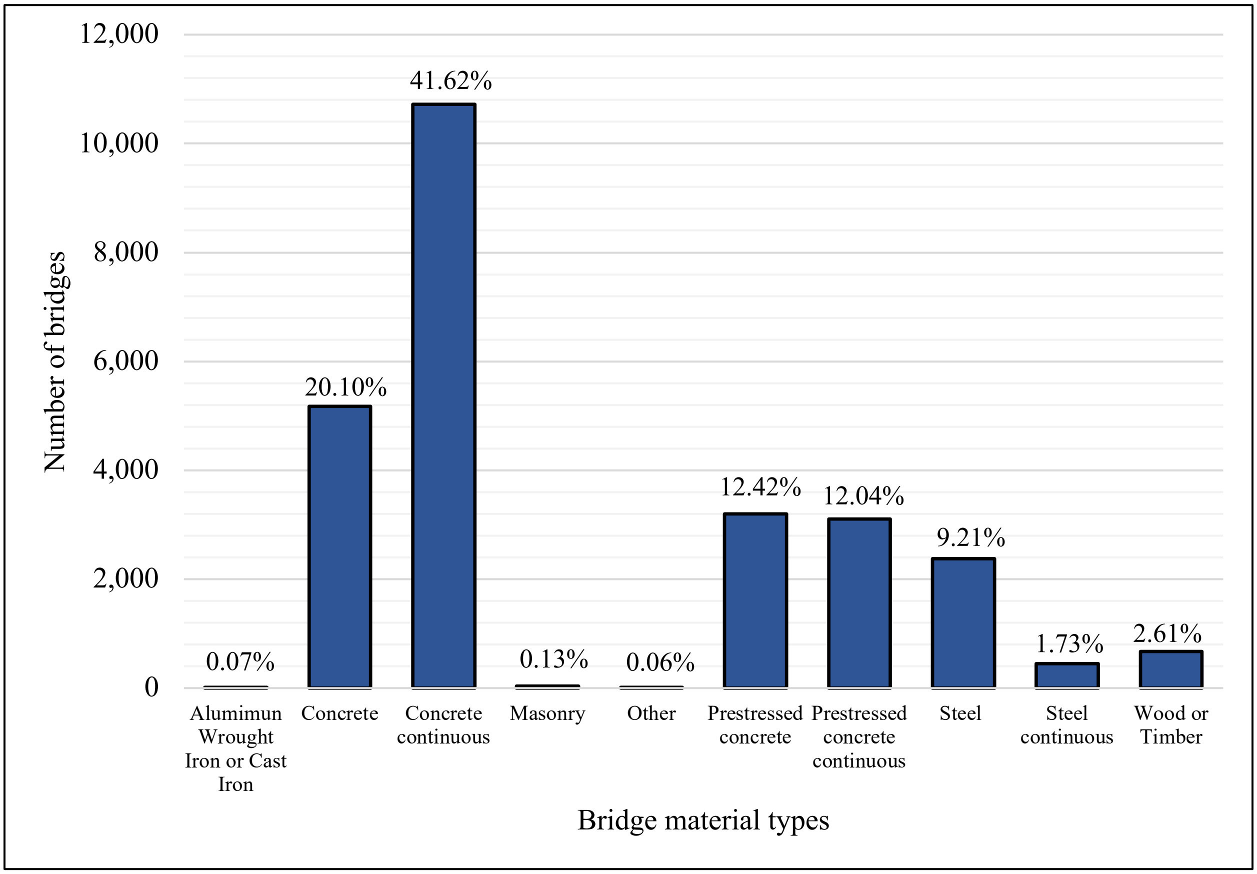


Figure 1-5. Bridge material type statistics in California

The percentage of bridge age, found using the NBI database, shows that approximately 50% of bridges in California have exceeded their design life (50-year lifespan); this includes 13% of the bridges that are over 75 years old, which is the design year according to AASHTO LRFD. Figure 1-6 classifies the bridge percentage in California by age. Various types of prestressed concrete bridges are found in California: girder, box, segmental and other types of bridges. Figure 1-7 presents the percentage of these bridges and shows that approximately 70% of prestressed concrete bridges in California are box girders, single and multi-boxes. Prestressed concrete box girder bridges are then selected for the analysis.

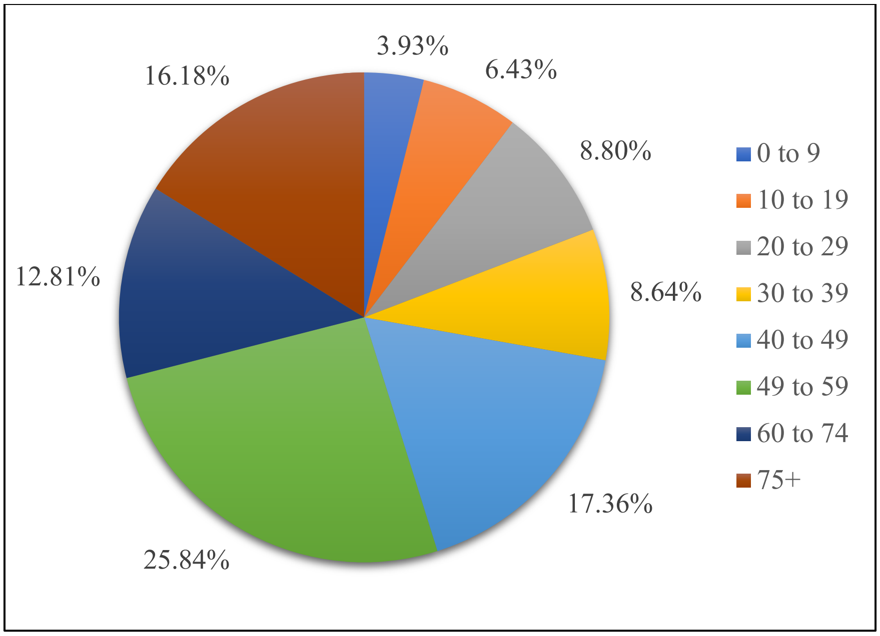


Figure 1-6. California bridges represented by age

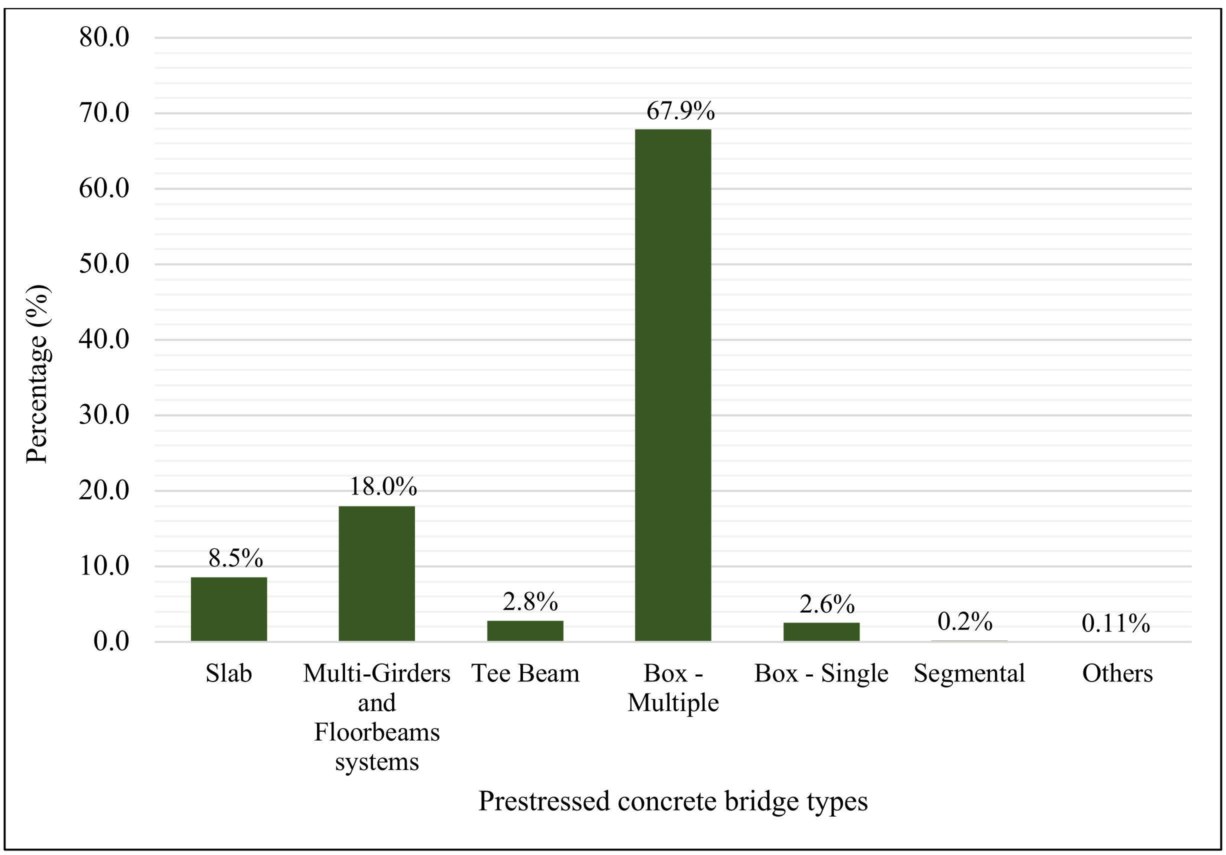


Figure 1-7. Percentage of prestressed concrete bridge type

Chapter 2. Literature Review

2.1 Bridge Deterioration

Concrete bridges behave and deteriorate in many ways. These include corrosion, exposure to the environment, drying shrinkage, thermal effects and traffic.

Corrosion is the destructive attack of a metal or an electrochemical reaction with the environment (Ahmad, 2006) in which water and oxygen must be present for the reaction to take place (Emmons, 1993). The process of corrosion is accelerated in reinforced or prestressed concrete structures if the aggressive chemicals are introduced into the concrete. This is the case of concrete structures located in marine environments or in contact with deicing salts because of the penetration of high concentration of chlorides present in sea waters and deicing salts. Corrosion causes tensile forces into the concrete forcing it to crack and spall. According to Emmons (1993), cracking and spalling of concrete reduce the croncrete effective cross section, thereby reduce the ultimate compressive load capacity.

Another form of deterioration is an exposure to the environment such as freeze and thaw, drying shrinkage and thermal effects. Freeze-thaw deterioration is induced when water penetrates the concrete. After many cycles at low temperatures, the water in the concrete expands causing tension, cracks and spalling. According to the American Concrete institute (ACI) website, drying shrinkage is the contraction in the concrete caused by moisture loss from drying concrete. Drying shrinkage is affected by the cement type, a short curing period, the placement temperature, aggregates size and type (Emmons, 1993). Thermal effects may also cause cracks. Concrete bridge structures are subjected to thermal effects due to the interaction with the environmental conditions or special effects, like the placement of hot asphalt on the deck or the development of heat of hydration during construction (Branco and Mendes, 1993).

Many bridges also suffer from serious deterioration due to repeated vehicle loads. The repeated vehicle loads cause accumulated damage due to fatigue and this damage accumulation has become more serious as the number of overloaded vehicles has increased recently (Oh et al., 2007). It was reported that the amount of repair of superstructures that indicates serious deck deteriorations are due to the direct contact of vehicular traffic, inducing tensile stress in the concrete decks. The combined longitudinal tensile stress on the decks due to shrinkage and wheels loads will induce parallel transverse cracks (Oh et al., 2007). For some bridge configuration (like short span and large width) and load position (heavy trucks in one lane), field measurements have shown that tensile stress can occur in the longitudinal direction at the top of the slab in the unloaded area (Nowak and Szerszen, 2003).

Bridge deterioration may be evaluated in terms of condition ratings. The condition ratings are important information that describes the condition of existing bridges (e.g. section, cracks, spalling, etc.). Table 2-1 contains the code and description of each condition rating as prescribed by the Federeal Highway Administration (FHWA).

Table 2-1. Condition rating codes and description (FHWA, 1995).

Codes	**Description**
N	Not applicable
9	Excellent condition
8	Very good condition - no problems noted
7	Good condition - some minor problems
6	Satisfactory condition - structural elements show minor deterioration

Codes	Description
5	Fair condition - all primary structural elements are sound but may have minor section loss, cracking, spalling or scour.
4	Poor condition - advanced section loss, deterioration, spalling or scour
3	Serious condition - loss of section, deterioration, spalling or scour have seriously affected primary structural components. Local failures are possible. Fatigue cracks in steel or shear cracks in concrete may be present.
2	Critical condition - advanced deterioration of primary structural elements. Fatigue cracks in steel or shear cracks in concrete may be present or scour may have removed substructure support. Unless closely monitored it may be necessary to close the bridge until corrective action is taken.
1	Imminent failure condition - major deterioration or section loss present in critical structural components or obvious vertical or horizontal movement affecting structure stability. Bridge is closed to traffic, but corrective action may put back in light service.
0	Failed condition - out of service - beyond corrective action

2.2 Weigh-in-Motion

In the U.S., bridge live loads are obtained using weigh-in-motion (WIM) technology. WIM systems are used for enforcement primarily to identify individual vehicles suspected of being in violation of weight or size laws and locate sites where relatively large numbers of probable weight, speed, or size violations occur (Miao and Chan, 2002). Several factors cause truck-induced damage. One of these factors is the weight of the vehicle and how that weight is distributed over the axles of the vehicle.

According to Haugen et al. (2016), WIM systems were first developed to preserve the road pavement and infrastructure, and to reduce the cost for weight enforcement resources. Weigh-in-motion technologies allow the control of vehicle weights without disrupting the traffic and freight operations (Haugen et al., 2016), and can be useful in bridge structural analysis, geometric design, safety analysis, facility planning and programming, and standard and policy development (FHWA, 2007). WIM systems capture information such as the vehicle classification, axle weight, gross vehicle weight (GVW), and vehicle speed. The Federal Highway Administration (FHWA) lists thirteen vehicle classes presented in Figure 2-1. The types of WIM systems available for use in application include the high-speed weigh-in-motion (HS-WIM) and the slow-speed weigh-in-motion (SWIM). The HS-WIM idenfies vehicles at highway speeds without hindering traffic flow and the SWIM accurately captures axle weights on-site to ensure over-the-road compliance. Figure 2-2 shows a diagram for high-speed weigh-in-motion. As per Ryguła et al. (2020), the basic components of a WIM system include weight sensors built into the road surface, automatic number plate recognition (ANPR) cameras, sensors to measure the height and length of the vehicle, inducive loops to measure the speed and generate a trigger signal, and telecommunications facilities with electronic controls.

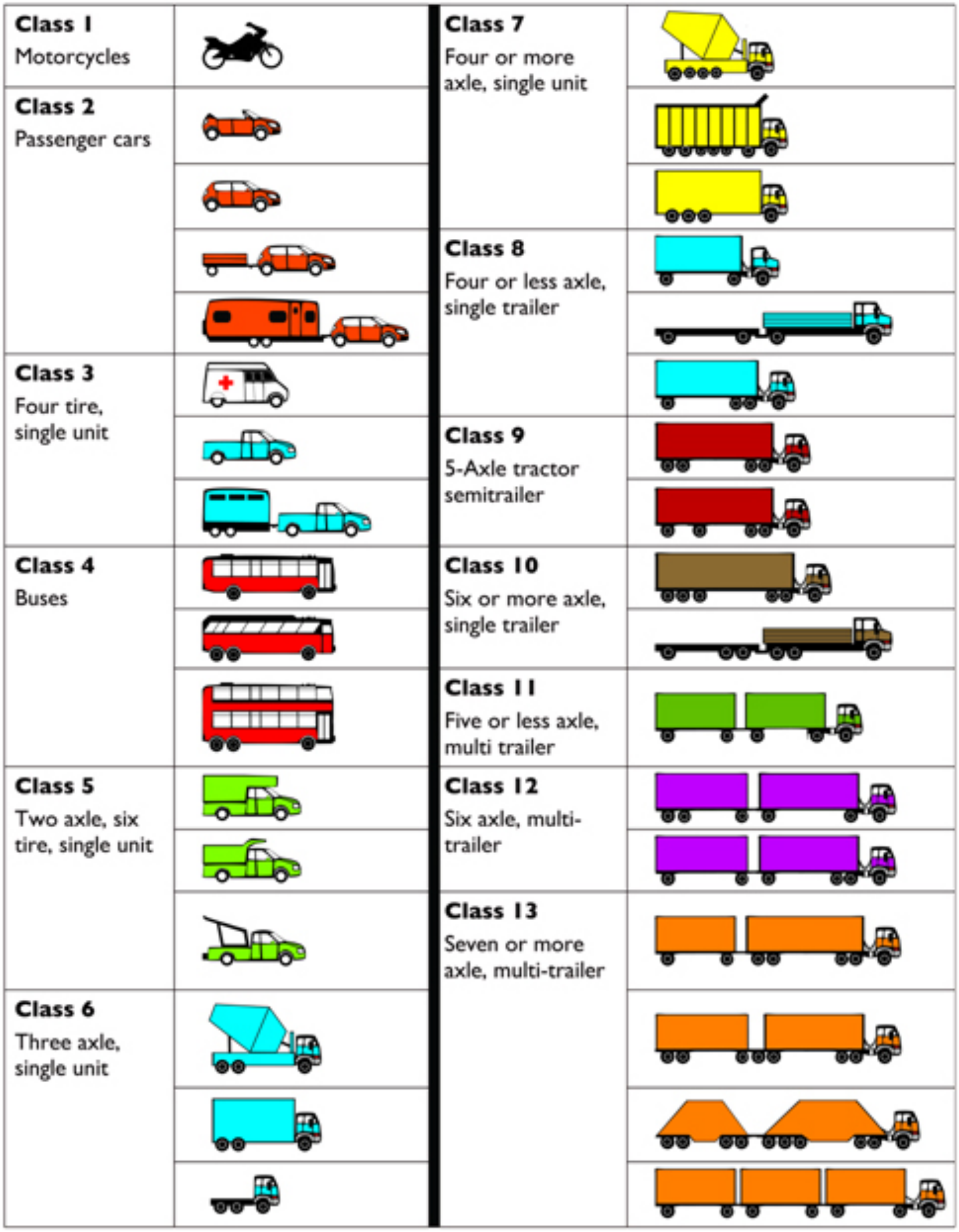


Figure 2-1. FHWA 13 Vehicle category classification

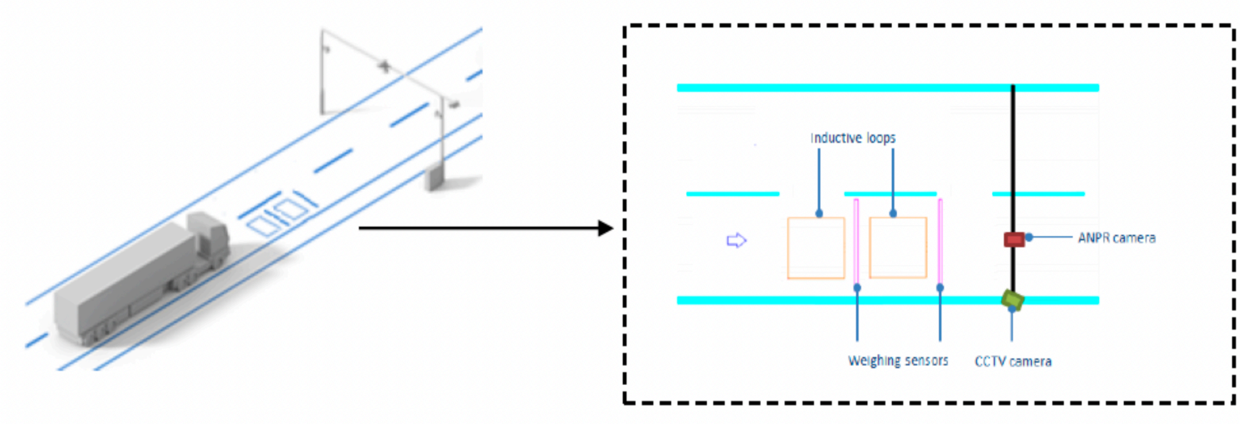


Figure 2-2. Diagram of High-Speed Weigh-in-Motion (HS-WIM) (Ryguła et al., 2020).

Despite the numerous benefits of weigh-in-motion system, some challenges and limitations are still identified. These include the functionality features of the WIM systems (i.e., the measurement uncertainty), and the difficulty of properly categorize the vehicles (Ryguła et al., 2020). According to Haugen et al., other challenges involve the calibration procedure, which has to be repeated often, the need for temperature correction due to sensor drift, as well as the short lifetime and decrease in accuracy of the sensors.

2.3 Live Loads

There is a growing interest in developing probability-based rational criteria for the design and evaluation of bridges; the live load model for load and resistance factor design (LRFD) is one of the most important parts of these new developments (Nowak and Hong, 1991). The effect of live load depends on many parameters including truck weight, axle loads, axle configurations, span length, position of the vehicle on the bridge (transverse and longitudinal), number of vehicles on the bridge (multiple presence) stiffnesses of structural members (slab and girders) and future growth (Moses and Ghosn, 1985; Nowak and Hong, 1991). According to AASHTO (2017), the live load effect is multiplied by the multiple presence factor, as shown in Table 2-2, to account for the probability of simultaneous lane occupation by the full HL93 design live load. Figure 2-3 represents the design truck, and Figure 2-4 represents the different cases for the design load according to AASHTO LRFD.

Table 2-2. Multiple Presence Factors, *m* (AASHTO, 2017).

Number of lanes loaded	Multiple presence factor, *m*
1	1.2
2	1
3	0.85
> 3	0.65

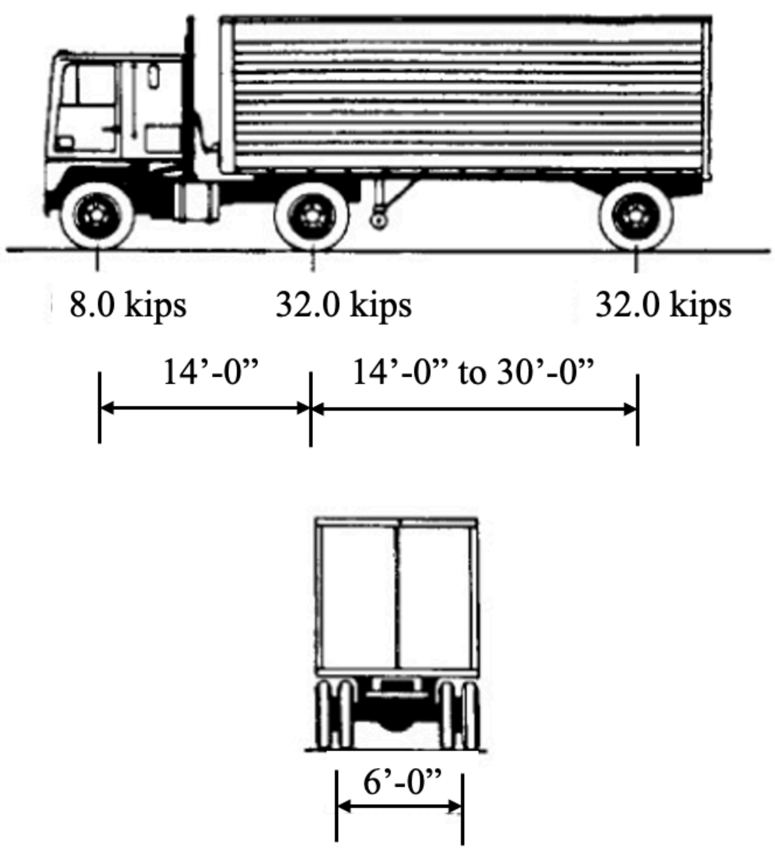


Figure 2-3. Characteristics of the design truck (AASHTO, 2017).

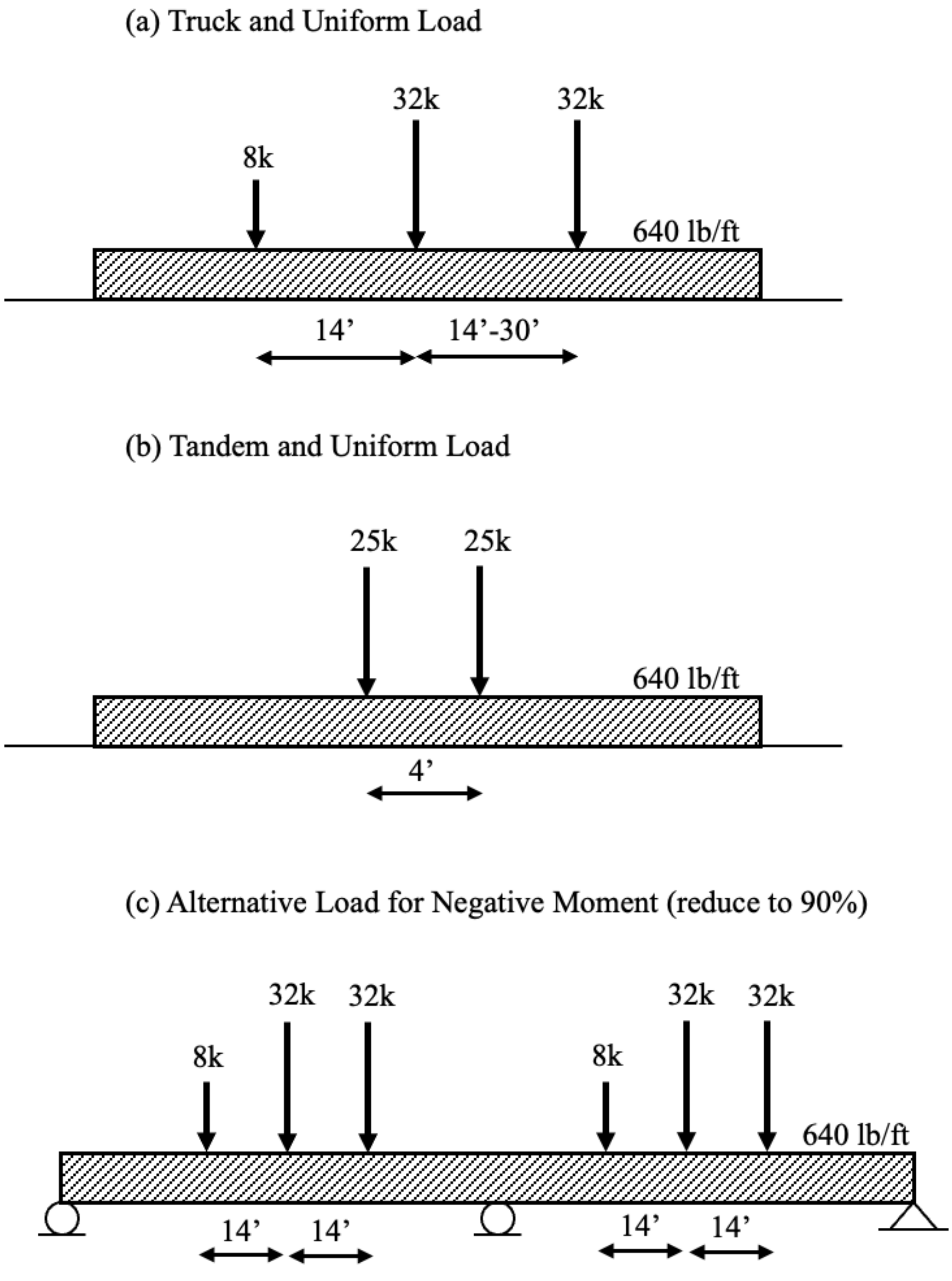


Figure 2-4. Design Live load according to AASHTO LRFD (Nowak, 1999).

The dynamic load allowance is defined as the maximum dynamic deflection, D_{dyn}, divided by the maximum static deflection, D_{sta} (Nowak, 1994), as shown in Figure 2-5. According to Nowak (1999), the dynamic load factor is a function of three major parameters: road surface roughness, bridge dynamics (frequency of vibration) and vehicle dynamics (suspension system). Dynamic load effect, I, is considered as an equivalent static load added to the live load, L (Nowak, 1999).

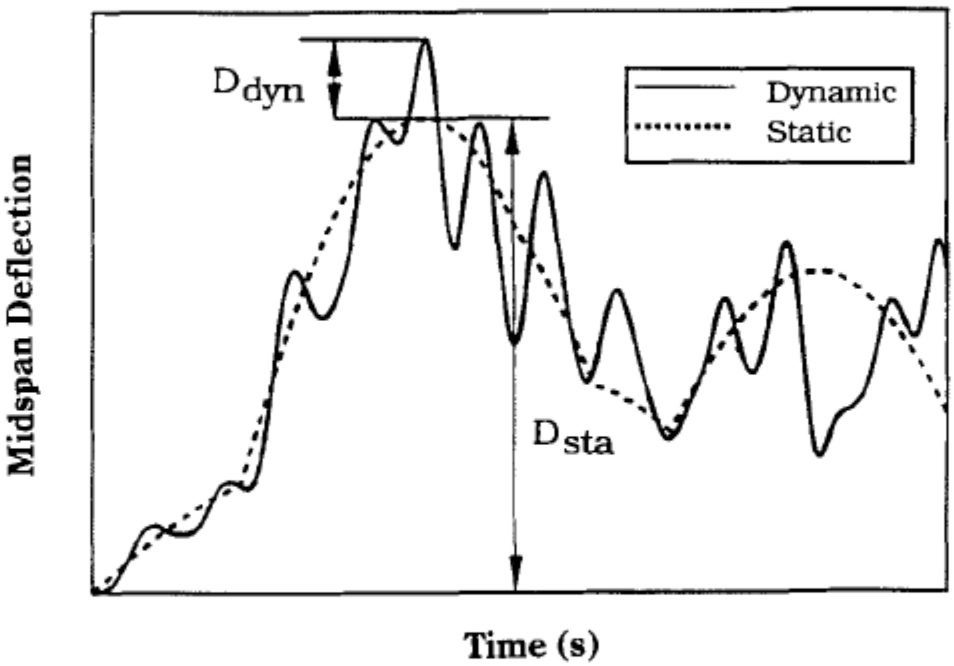


Figure 2-5. Time history for midspan deflection due to a single truck on a bridge (Nowak, 1994).

Live Load Models

The development of live load models is available in the literature. According to Shahid et al., (2017), live load models are usually developed from the existing truck data and depend on factors such as, gross vehicle weight (GVW), axle weight, axle spacing, truck configuration, annual average daily truck traffic (ADTT) (Babu et al., 2019; Shahid et al., 2017). Existing truck data are found in the weigh-in-motion data; according to Shahid et al., (2017), it is nearly impossible to estimate the traffic data for a long period of time (i.e., 75 year period). However, the method of extrapolation could be used to project the existing data to any period of time. This can also be used in the determination of the statistical parameters such as the bias factor, which is the mean-to-nominal ratio and the coefficient of variation which is the ratio of the standard deviation to the mean. In their research, Shahid et al. (2017) used three different methods to compare test results used in the extrapolation of live load effects to 75 years return period. These methods are the non-parametric fit method, the convolution method, and the cumulative distribution function (CDF) projection method. The non-parametric fit method does not involve any parameters like skew, mean, or variance, etc. One of its advantages is that it adjusts itself to the probability density function to any distribution function. The convolution method uses the linear fit to extrapolate the maximum expected value for any required period. It depends on the coefficient of determination, R^2, which shows how best the data was fitted. Lastly, the CDF projection method consists in using the ADTT to project the live load effects. Shahid et al. found that results obtained using the non-parametric fit were closer to the realistic data than the other method.

Similar research was carried out by Babu et al. (2019) to develop and compare statistical parameters of live loads in the United States and in Europe. For the analysis, WIM measurements from 44 U.S. locations were used as well as measurements from 5 European WIM stations, namely in Poland, Netherland, Czech Republic, Slovenia, and Slovakia. The extrapolation method used implies a maximum expected live load effect depending on a certain period of time T (in days), the ADTT and the cumulative distribution function of the traffic load, which are related by equations (1) and (2).

$$Z_{max} = -\Phi^{-1}\left(\frac{1}{N}\right) \tag{1}$$

$$N = T * ADTT \tag{2}$$

Where N is the number of expected vehicles for a return period T (in days), Z_{max} is the axis of standard deviation and Φ^{-1} is the inverse standard normal distribution function.

A linear extrapolation was then used at the upper tail of the CDFs as shown in Figure 2-6. The values located on the intersection of extrapolated CDFs and corresponding ordinates are the bias factors at the specified return period. Using different ADTTs (250, 1000, 2500, 5000, and 10000), bias factors for a 75-year live load effect were generated for the U.S. and Europe as presented in Figure 2-7.

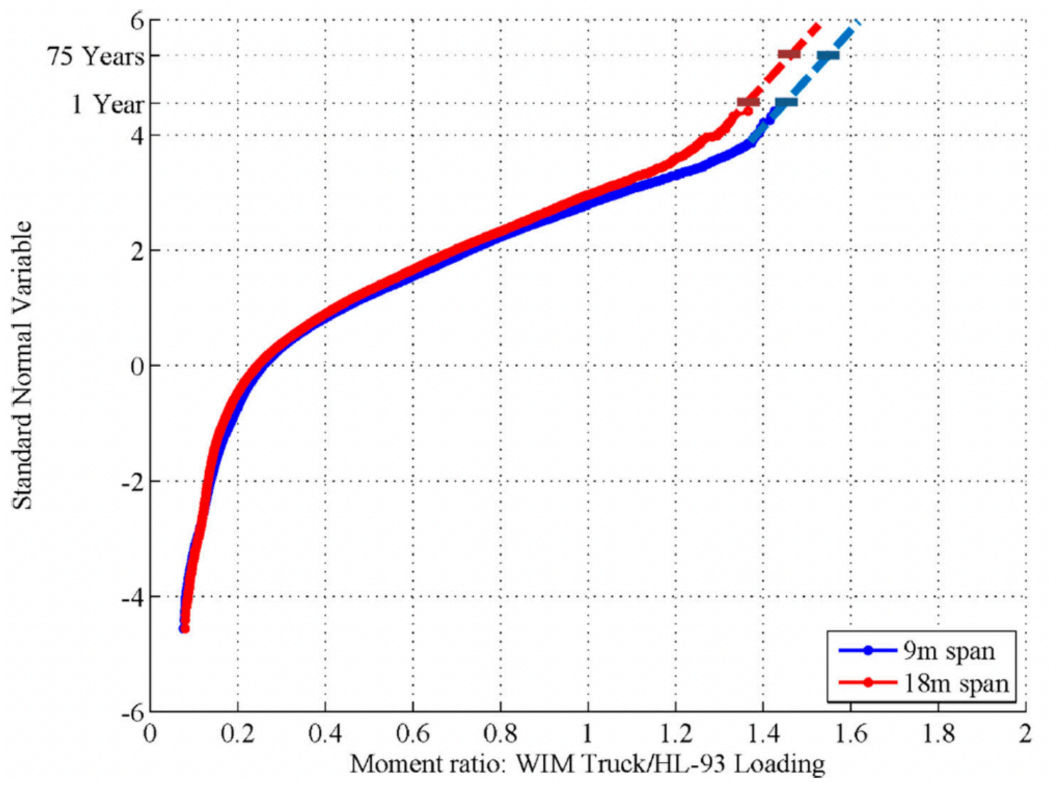


Figure 2-6. Linear extrapolation of the upper tail of the CDF of the moment ratio (Babu et al., 2019).

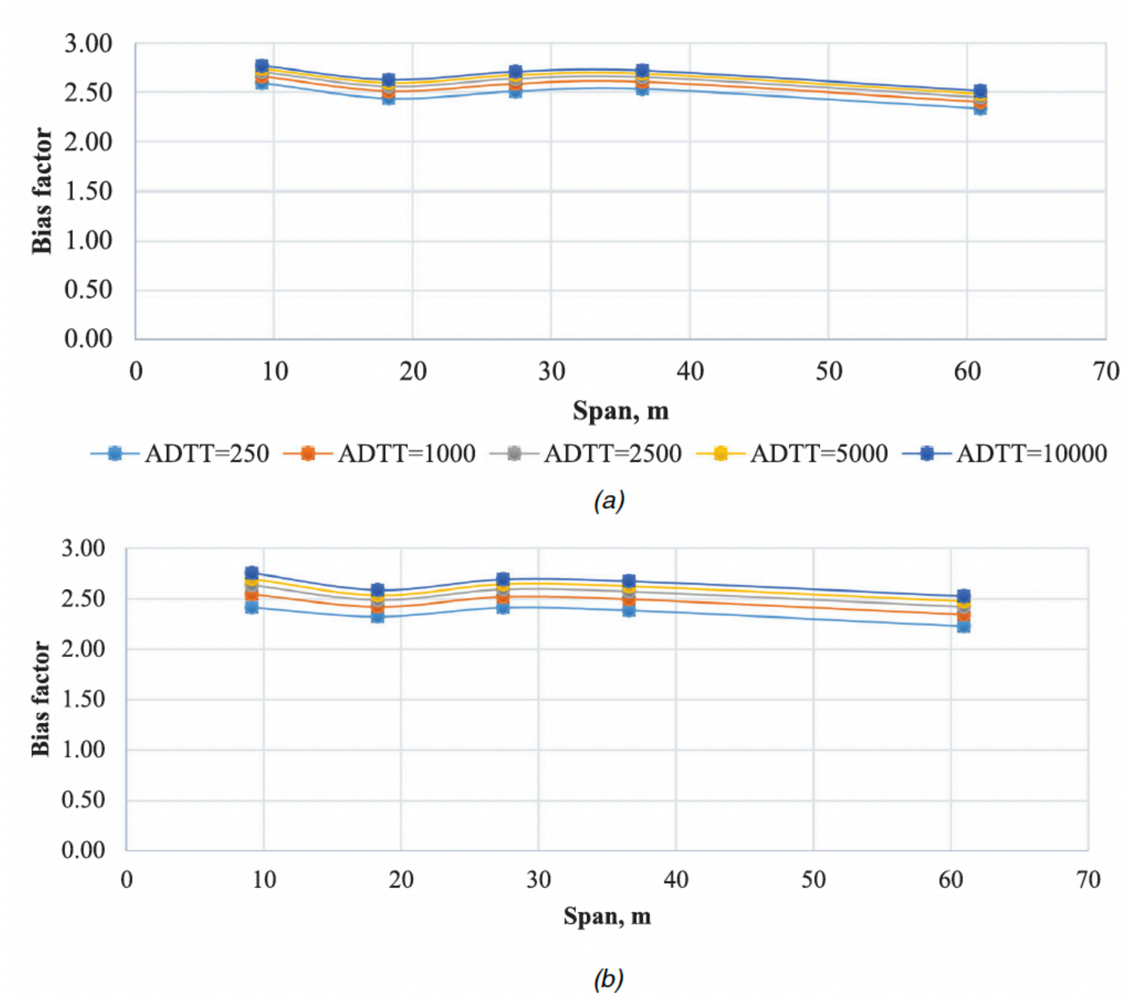


Figure 2-7. Bias factor of 75-year maximum moment ratios for different ADTT of the U.S. (a) and European data (b) (Babu et al., 2019)

2.4 Reliability Analysis

The reliability of a structure is its ability to fulfill its design purpose for some specified design lifetime (Nowak and Collins, 2013). According to Nowak (1999), the reliability index is an efficient measure of structural performance; reliability indices served as a basis for selecting the target reliability index, β_T. Nowak (1999) calculated the reliability indices for girder bridges (non-composite steel, composite steel, reinforced concrete and prestressed concrete) in his research. Figure 2-8 shows the results obtained for moment in prestressed concrete girders. The results show that the most important parameters determining the reliability index are girder spacing and span length. In general, the reliability indices are higher for larger girder spacing. This is due to more conservative girder distribution factor (GDF) compared to shorter spacings (Nowak, 1999).

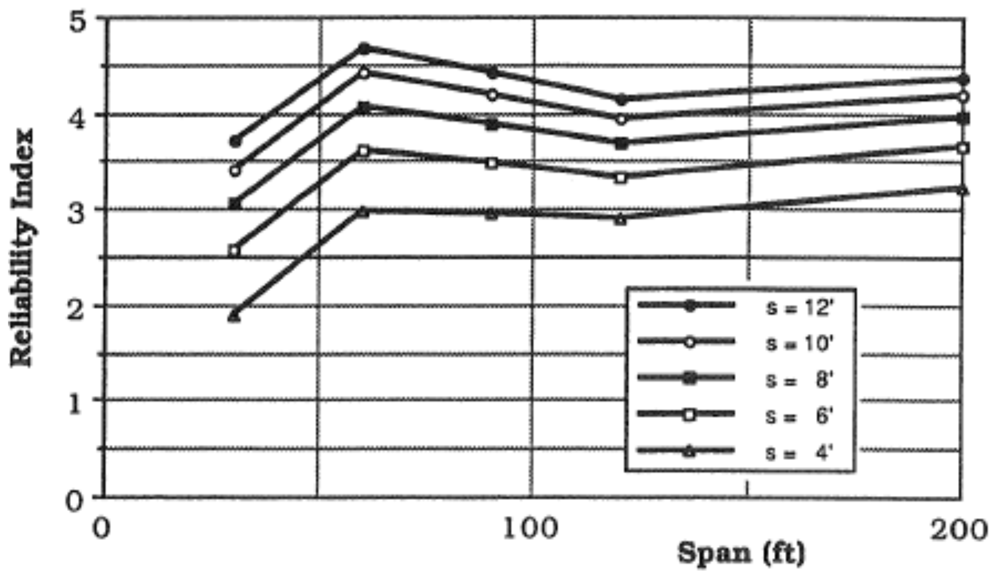


Figure 2-8. Reliability indices for simple span moment in prestressed concrete girders (Nowak, 1999)

Nowak and Tabsh (1991) also calculated the reliability index for noncomposite and composite steel girder, reinforced concrete T-beams and prestressed concrete girders, using available live load data from Nowak and Hong (1991), and weigh-in-motion data from Winsconsin and Florida. Failure at a single girder and failure of the whole bridge as a system was considered and a sensitivity analysis was also carried out to identify the parameters that mostly affect the bridge. The analysis was performed with thousands of Monte Carlo simulations using the statistical parameters for materials as shown in Table 2-3.

Table 2-3. Statistical parameters of material properties (Nowak and Tabsh, 1991).

Variable	Nominal (ksi)	Mean (ksi)	Mean-to-nominal ratio	V
F_y	36	38	1.005	0.10
f_y	40	45	1.125	0.12
f_y	60	67	1.115	0.10
f_{ps}	270	281	1.04	0.025
f_c	3	2.76	0.92	0.18
f'_c	5	4.03	0.805	0.15
E_s	29,000	29,000	1.0	0.06

The analysis shows that the reliability indices for noncomposite steel girders are 3.0-3.5, for composite steel 2.5-3.5, for reinforced and prestressed concrete 3.5-4.0. The system reliability indices range between 4.95 and 6.20. Figure 2-9 shows the sensitivity functions for prestressed concrete girders and indicates that the most important parameters are the effective depth *d* and the prestressing force $A_{ps}f_{ps}$.

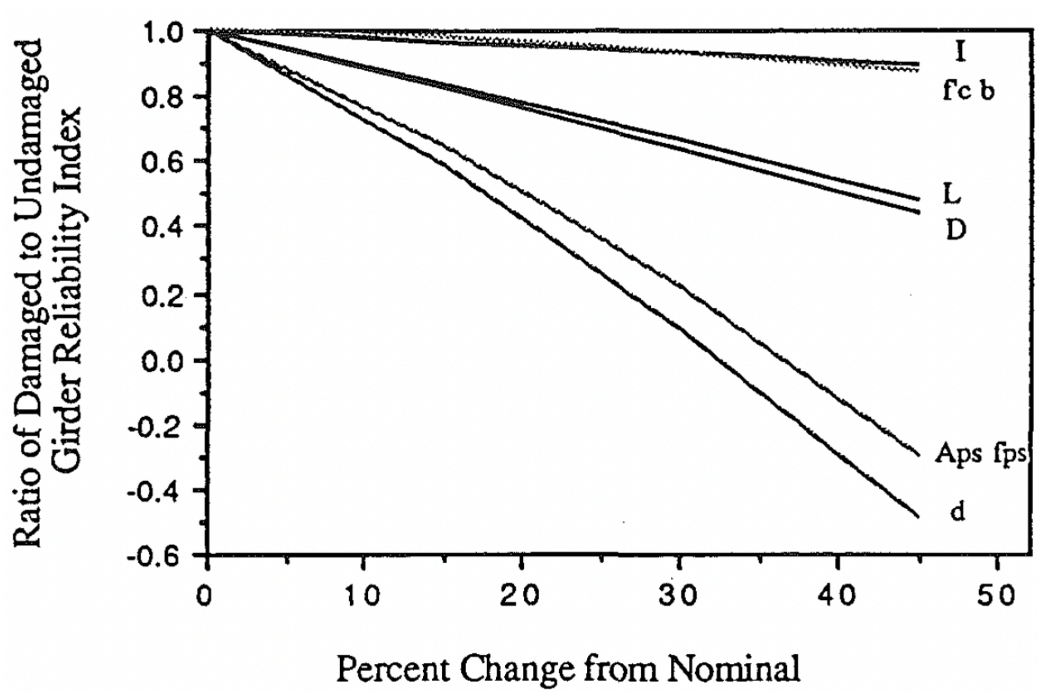


Figure 2-9. Sensitivity functions for prestressed concrete girders (Nowak and Tasbh, 1991).

Law and Li (2010) investigated if the measured information of a structural system could be integrated with reliability analysis to yield a safety estimate on the component or system. For their study, the authors used a finite element modeling of a three-span continuous prestressed concrete box bridge with a total length of 73 m. Figure 2-10 shows the cross section of the bridge; dimensions are in millimeters.

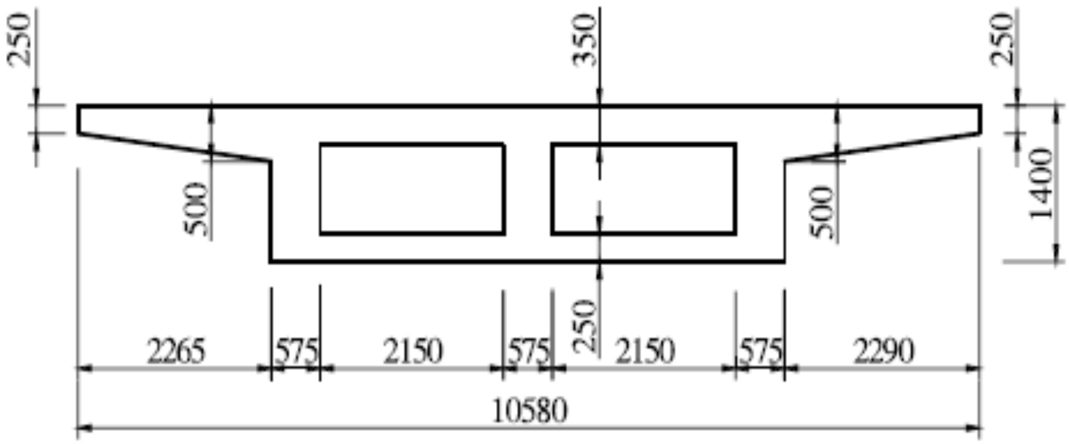


Figure 2-10. Cross section of the bridge superstructure (Law and Li, 2010).

Results obtained using the serviceability limit state are shown in Table 2-4 for the components and the overall system. Type I reliability index is calculated from the intact structure with initial coefficient of variation (COV) values of the modulus of elasticity for all elements; type II reliability index is calculated from the structure with true damage in the damage elements and initial COV values of the modulus of elasticity of all the elements; and type III is calculated from the structure with identified mean values of damage and identified COV values of the modulus of elasticity in all elements. The 18th and 19th elements are in the left web, the 54th and 55th elements are in the middle web and the 90th and 91st elements are in the right web as shown in Figure 2-11.

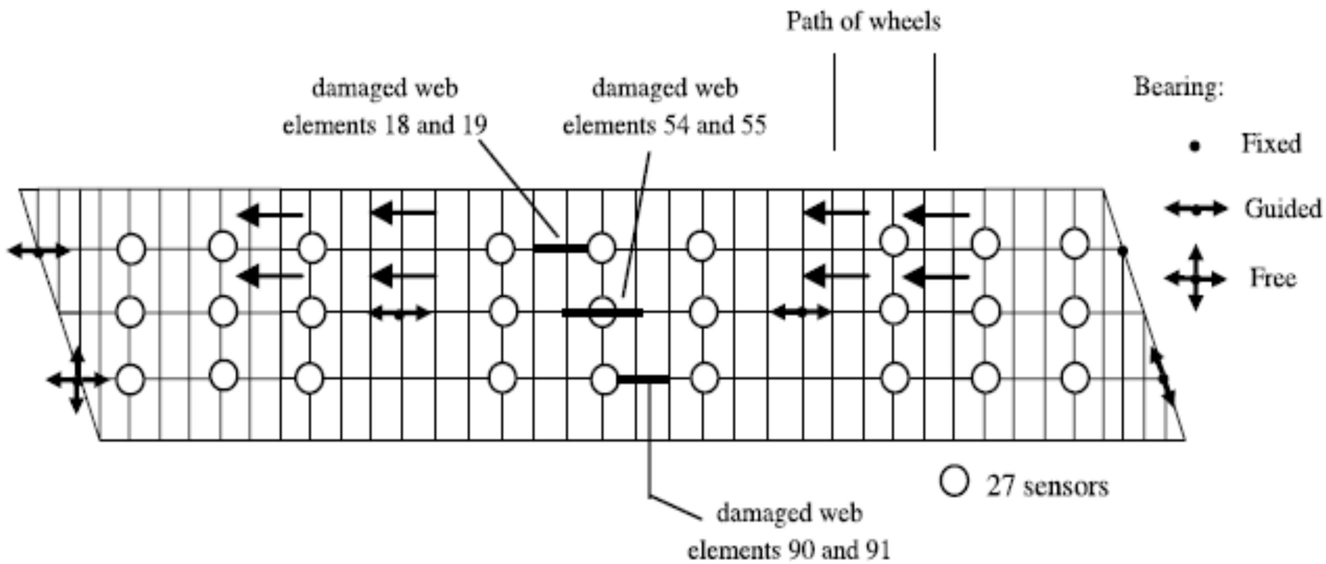


Figure 2-11. Plan of bridge and sensor configuration (Law and Li, 2010).

Table 2-4. Reliability indices (Law and Li, 2010).

Category	Element number	Intact with initial COV (Type I)	True damage with initial COV (Type II)	Identified damage with identified COV (Type III)
Component reliability index	18th	2.537	2.287	2.400
	19th	3.282	3.098	3.097
	54th	2.889	2.707	2.781
	55th	3.137	2.929	2.997
	90th	3.271	2.960	2.976
	91st	3.833	3.667	3.789
System reliability index		5.940	5.640	5.642

Chapter 3. Live Load Model in California

3.1 General Procedure

Developing the live load model for California requires finding statistical parameters such as the bias factor, defined as the ratio of mean-to-nominal and the coefficient of variation, which is the ratio of the standard deviation and the mean. The maximum expected mean is obtained from California WIM data and the nominal is taken as the AASHTO HL93 design value.

The first step in developing live load model in California is to collect and process California WIM data. WIM data from twenty-four (24) received by the California Department of Transportation, *Caltrans* was used. Figure 3-1 presents the location of the 24 California WIM stations. The most critical WIM station was chosen as a representative to determine the bias factor for one lane in California. The second step is to compute the live load effects. The influence line analysis was performed using a MATLAB algorithm. MATLAB is a programming and numeric platform used to analyze data, develop algorithms, and create models. The moment due to California vehicles and the AASHTO HL93 (design) moment were computed for span lengths ranging from 30 to 190 ft, for both simple spans and continuous spans (two identical spans). The ratio of moment due to California vehicles to moment due to the AASHTO design vehicle was then plotted for simple spans and continuous spans, separately for positive and negative moments as shown in Figure 3-2, Figure 3-3 and Figure 3-4. These graphs represent the cumulative distribution function (CDF) of the load effects. The plots show that approximately 97 % of California heavy vehicles cause a load effect less than the AASHTO design vehicle and approximately 3 % of California heavy vehicles exceed the AASHTO design vehicle.

Figure 3-1. Location of 24 California WIM stations

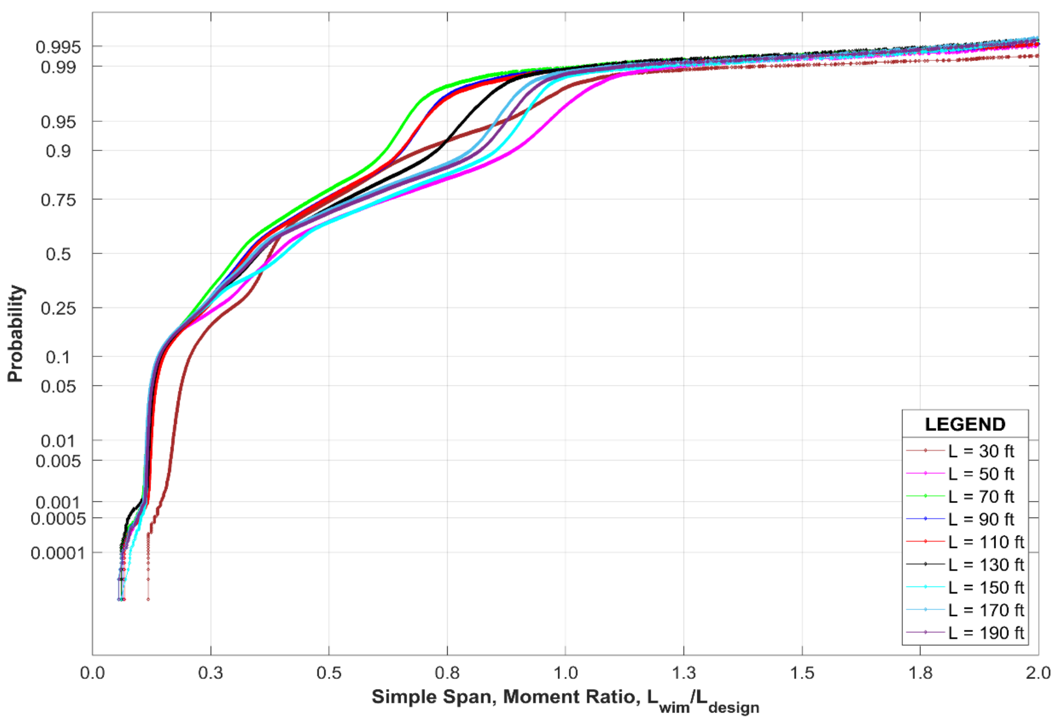


Figure 3-2. Simple span, positive moment ratio, L_{wim}/L_{design}

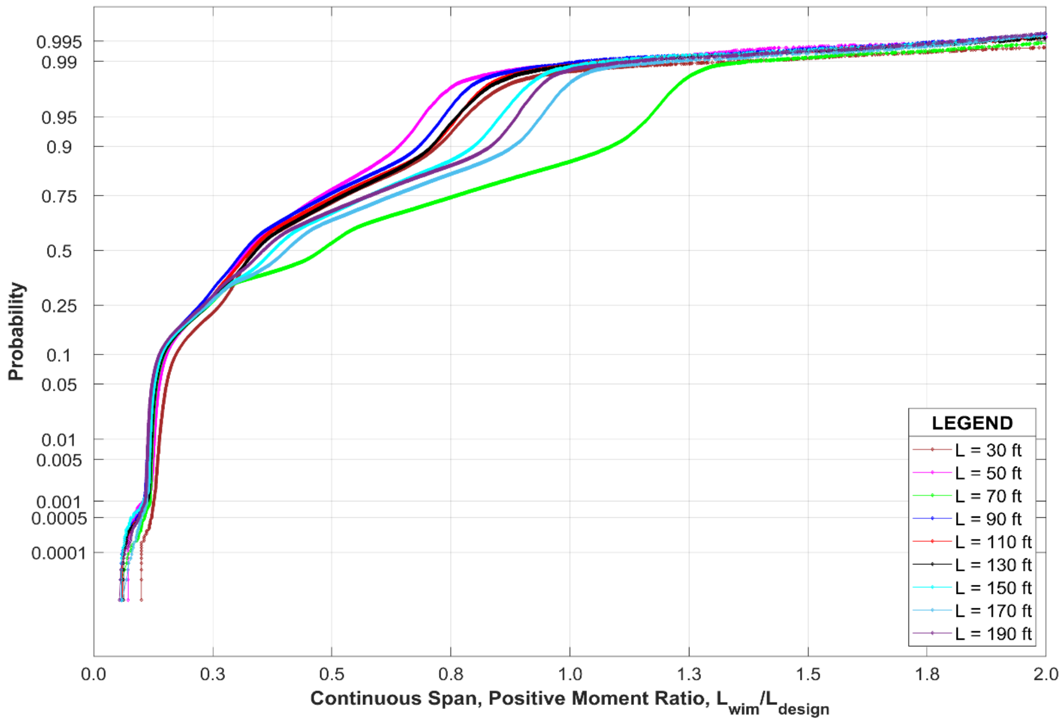


Figure 3-3. Continuous span, positive moment ratio, L_{wim}/L_{design}

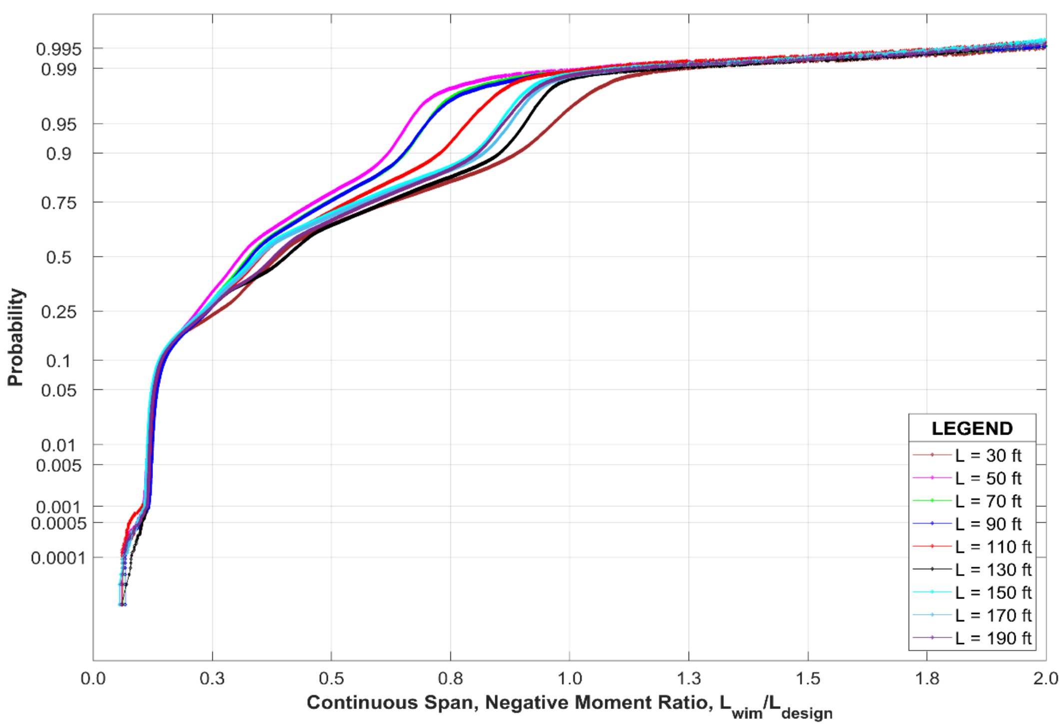


Figure 3-4. Continuous span, negative moment ratio, L_{wim}/L_{design}

The third step in developing California live load model includes the determination of the live load distribution. It is required, since there is no enough data to determine a maximum expected loading. Hence, there is a need for data extrapolation to find load ratio for a certain time period. The next steps of live load development are discussed in the next sections.

3.2 Live Load Distribution

Before extrapolating traffic data to any considered time period, knowing the distribution of the data is important. A MATLAB algorithm was used to find the best fit distribution that represents California traffic data. Four types of distribution were fitted to California data: the Extreme Value, Generalized Extreme Value, Gamma and Normal distributions. The probability density function and distribution types are shown in Figure 3-5. The cumulative probability and error distributions are also plotted in Figure 3-6. The Extreme Value and Generalized Extreme Value distributions best fitted California traffic data as shown in Figure 3-5. The cumulative probability and error distributions were plotted to verify and select only one distribution that represents best California data. The graphs in Figure 3-6 show that the Extreme Value distribution is closer to the empirical data and has an error distribution closer to zero. Therefore, California traffic data is represented by the Extreme Value distribution, also called Extreme type I distribution.

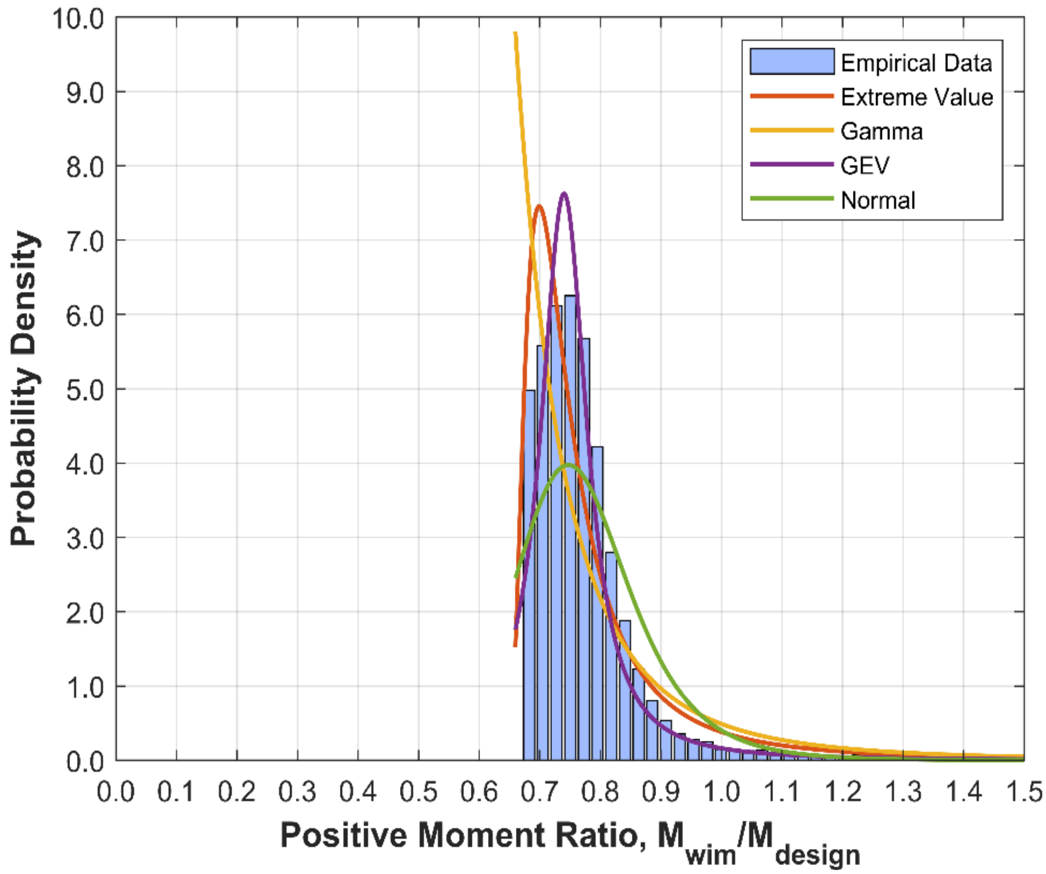
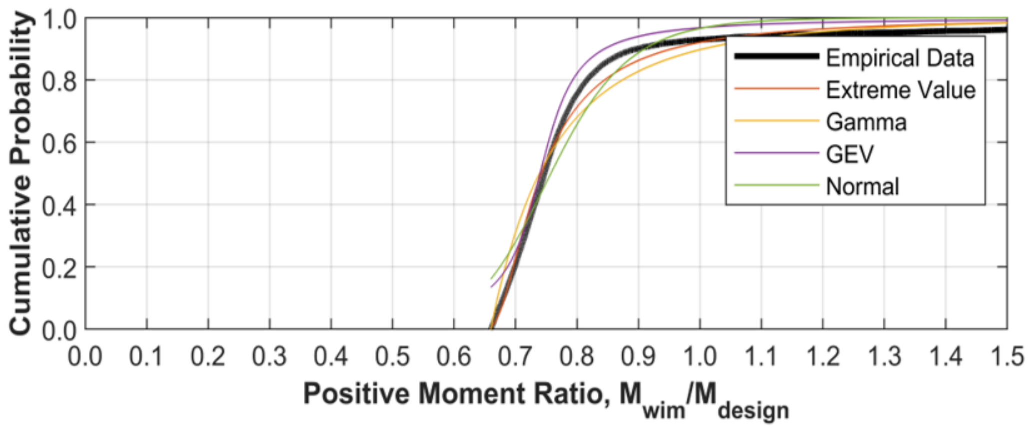
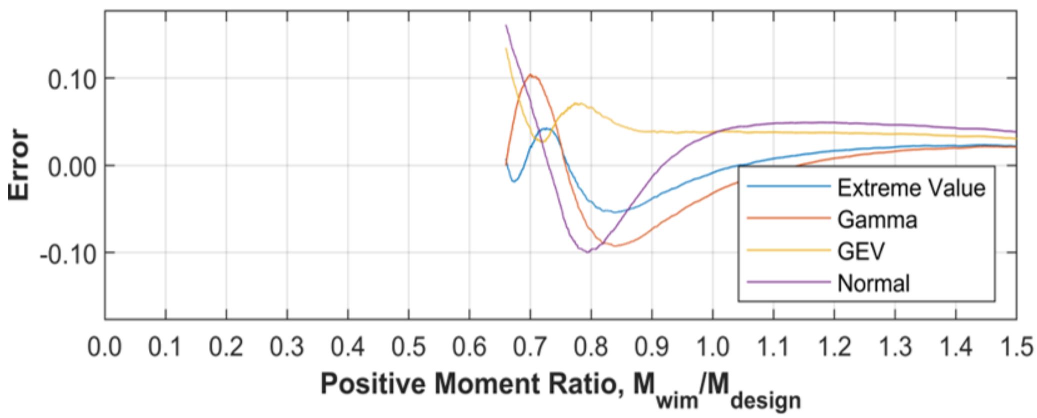


Figure 3-5. Probability density function for moment ratio, M_{WIM}/M_{Design}



(a) Cumulative probability vs. positive moment ratio



(b) Error distribution vs. positive moment ratio

Figure 3-6. Cumulative probability and error distribution for positive moment ratio

3.3 Extrapolation technique

The extrapolation of traffic data consists in predicting the maximum expected live load that the bridge structures can experience during a certain time period (i.e., the number of expected vehicles). The extrapolation of traffic data is performed to determine the load ratio for that period of time. According to the Manual for Bridge Evaluation (MBE), bridges are inspected every 2 or 4 years. The maximum expected live load for the analsyis is taken for a 5-year return period, based on the NCHRP Report 454 (Moses, 2001).

The number of expected vehicles is expressed in terms of the return period as the average daily truck traffic, as shown in equation (3).

$$N = T * ADTT \tag{3}$$

Where N is the number of expected vehicles for a return period T (in days), and a certain $ADTT$.

The upper tail of the cumulative distribution function represents the maximum values of the live load that can be obtained using a certain probability of occurrence $P*$ as shown in equation (4) or using the maximum moment at the the normal standard variable Z_{max} in equation (5)

$$P^* = 1 - \frac{1}{N} \tag{4}$$

$$Z_{max} = -\Phi^{-1}\left(\frac{1}{N}\right) \tag{5}$$

Where Φ^{-1} is the inverse standard normal distribution function.

The ADTT for each considered WIM station was determined as traffic count and number of days for which the data was recorded. Figure 3-7 presents a histogram of California ADTT per lane. According to Pande and Wolshon (2016), the 85th percentile speed is the speed at which 85% of the vehicles are travelling at or slower. Conversely, 15% of the vehicles travel faster than the posted speed. This method was applied in order to determine a representative ADTT for California. In Figure 3-7, 85% on the cumulative distribution function axis (right axis) corresponds to an ADTT of 5,000 vehicles. Therefore, an ADTT of 5,000 is used to determine statistical parameters of live load, and hence, the number of the maximum expected vehicles for a 5-year return period.

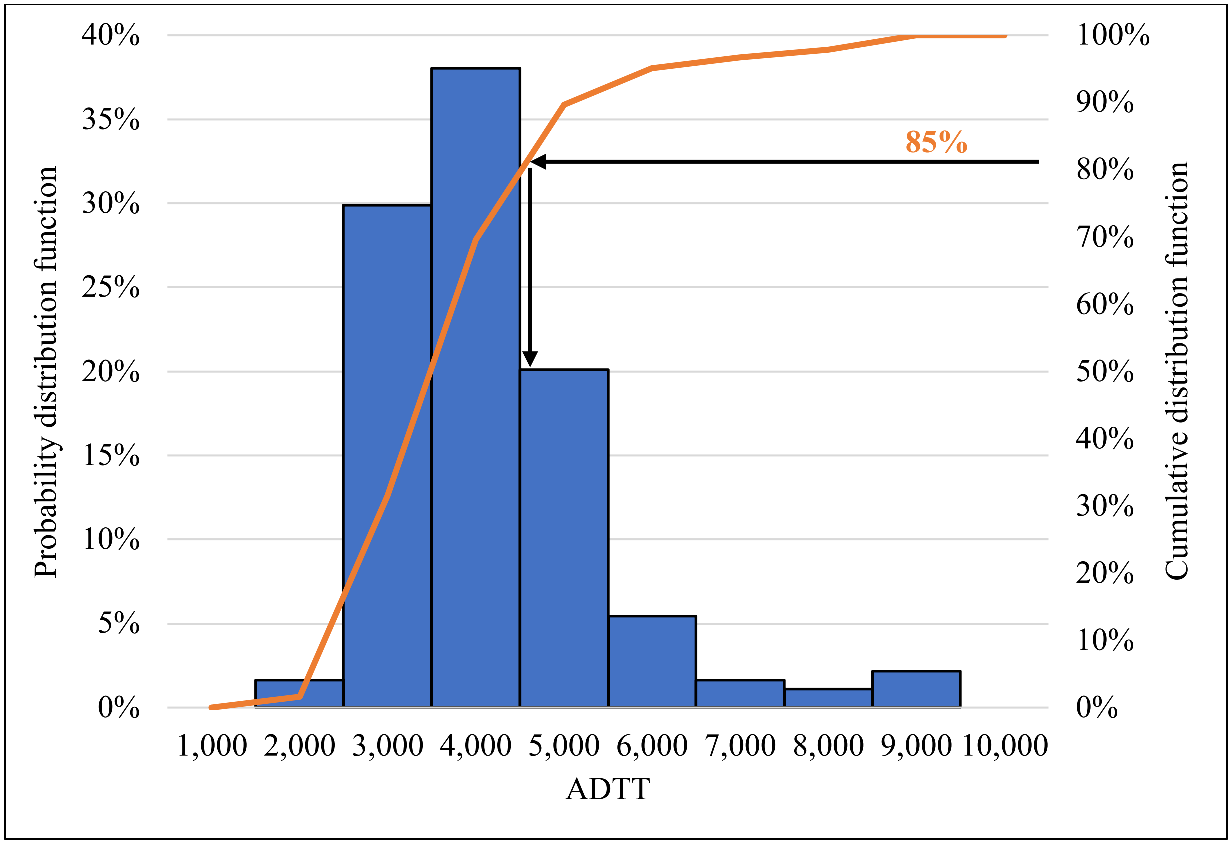


Figure 3-7. Determination of the expected average daily truck traffic.

The number of expected vehicles in a 5-year period, and the associated Z_{max} in equation (3) and (5) are therefore,

$$N = 365 * 5\ years * 5{,}000 = 9{,}125{,}000\ vehicles \tag{6}$$

$$Z_{max} = 5.18 \tag{7}$$

3.4 Statistical Parameters for live Load

Statistical parameters define the level of uncertainty for a certain variable. The CDF of the mean-to-nominal ratio was fitted at 15% of the top data (upper tail) with a normal distribution. The intersection between the probability $P*$ (or the associated Z_{max}) as shown in equations (4) and

(5) and the mean-to-nominal ratio was then plotted for each span length and each configuration (simple span and continuous span) as shown in Figure 3-8. To remain consistent throughout the analysis, the 85% rule by Pande and Wolshon (2016) was applied in order to determine the bias factor for California considering that 15% of the vehicles have a higher load ratio. Therefore the bias factor for California is 1.32 as shown in Figure 3-8, and the coefficient of variation is approximately 0.18.

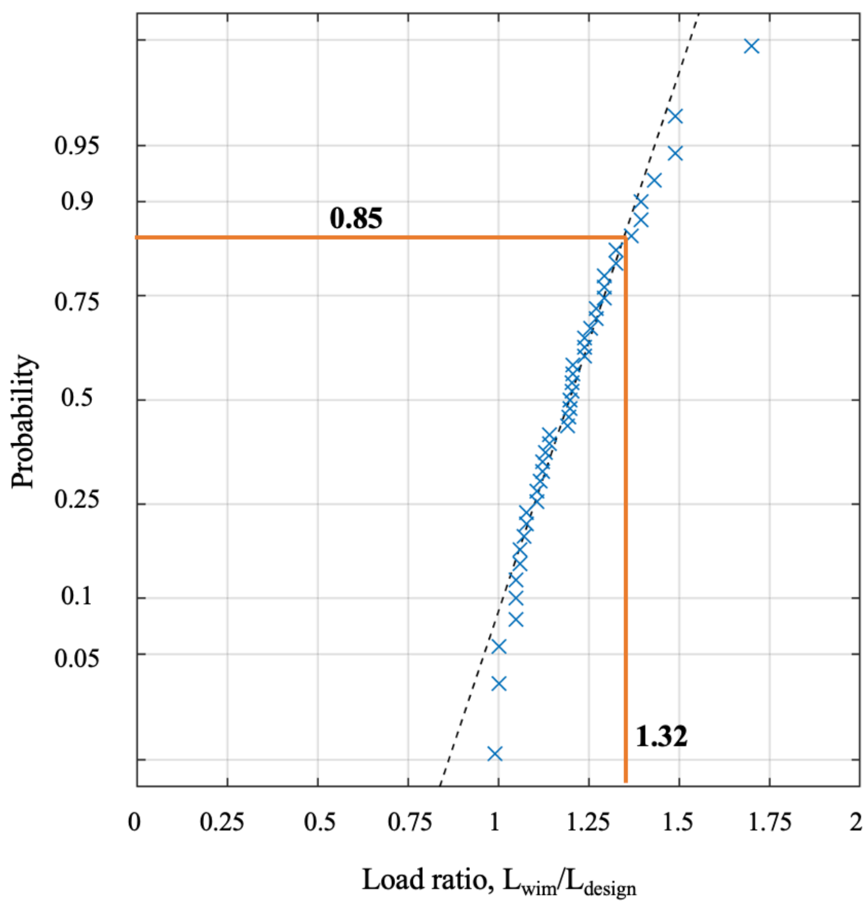


Figure 3-8. Determination of the bias factor

Chapter 4. Representative Bridges

Box girder bridge types represent the majority for prestressed concrete bridges in California. The reliability analysis of prestressed concrete box girder bridges first involves selecting a few bridges that will be considered as representative bridges for California.

Ninety-eight (98) bridge inspection reports were received from the California Department of Transportation, *Caltrans*. These reports contain several information about prestressed concrete box girder bridges built after 1990 such as the geometry of the bridges, defects, work recommendations, etc. Fourteen (14) prestressed concrete box girder bridges, including four simple span and ten continuous span bridges, were selected due to the availability of all necessary information required for the reliability analysis. These bridges are presented in the map in Figure 4-1.

Figure 4-1. Location of the fourteen selected prestressed box bridges

The selected prestressed box girder bridges have a range of span lengths between 62 and 231 ft, a box width between 7 and 10 ft, and a box height between 3 and 8 ft. Information about the selected bridges is provided in Table 4-1. Simple span bridges are denoted by *SS* and continuous span bridges, by *CS*.

Table 4-1. General information of selected prestressed concrete box girder bridges

Bridge Number	**Year Built**	**Span Type**	**Number of Spans**	**Span Length Considered (ft)**	**Deck Thickness (in.)**	**Bottom flange thickness (in.)**	**Number of Cells**	**Box width (ft)**	**Box Depth (ft)**
17 0030	1991	SS	1	176	8.75	6.63	8	9.58	8.00
29 0306L	1992	SS	1	158	7.38	5.50	8	7.25	7.00
37 0368L	1990	SS	1	124	8.13	6.38	5	9.54	6.00
55 0655	1992	SS	1	95	8.38	6.75	26	10.00	4.50
23 0020	1993	CS	2	112	7.38	5.50	5	7.25	4.42
23 0205L	1992	CS	3	126	7.25	5.50	5	7.00	5.00
37 0366L	1991	CS	3	85	7.25	6.00	7	7.00	3.50
37 0420L	1990	CS	3	62	7.63	5.50	6	7.96	3.00
37 0421L	1990	CS	3	77	7.38	5.50	7	7.17	3.50
37 0467L	1991	CS	10	231	8.25	6.63	7	9.75	6.50
49 0060L	1991	CS	5	160	7.63	5.50	4	8.00	6.50
49 0165L	1992	CS	3	98	8.00	6.00	4	8.88	4.00
54C0617	1991	CS	2	73	7.75	6.00	13	8.33	3.50
55 0670	1990	CS	7	156	8.25	6.63	7	9.67	6.25

Chapter 5. Load and Resistance models

5.1 Load Model

5.1.1 Dead Load

According to Nowak (1999), dead load D, is the gravity load due to the self-weight of the structural and non-structural elements permanently connected to the bridge. These include the weight of factory-made elements, such as steel and precast concrete, the weight of cast-in-place concrete, the weight of wearing surface (asphalt) and other types of weights such as railings and luminaries. Dead load due to self-weight of one box girder and dead load due to barrier and railings were considered for the reliability analysis; moment due to dead load was computed using SAP2000 software. SAP2000 is a civil-engineering software used for design and analysis of any type of structure. Dead load was assumed to follow a normal distribution. Statistical parameters used in reliability analysis include the bias factor and the coefficient of variation. Table 5-1 shows the statistical parameters of dead load.

Table 5-1. Statistical parameters of dead load (Nowak, 1999)

Component	Bias factor, λ	Coefficient of Variation, V
Factory-made members	1.03	0.08
Cast-in-place members	1.05	0.10
Asphalt	3.5 inch*	0.25
Miscellaneous	1.03-1.05	0.08-0.10

*mean thickness

In California, prestressed concrete box girder bridges are cast-in-place; thus, the bias factor and coefficient of variation of dead load used in the analysis are 1.05 and 0.10, respectively.

5.1.2 Live Load

The analysis involves determining the load in each lane and load distribution to girders (Nowak and Hong, 1991). According to AASHTO LRFD, the girder distribution factor (GDF) for a cast-in-place concrete multicell box is expressed as shown in equations (8) and (9).

- One lane loaded

$$\left(1.75+\frac{S}{3.6}\right)\left(\frac{1}{L}\right)^{0.35}\left(\frac{1}{N_c}\right)^{0.45} \tag{8}$$

- Two or more lanes loaded

$$\left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25} \tag{9}$$

Where S is the spacing of the box girder (ft), L is the span length (ft) and N_c is the number of cells for the prestressed concrete box girder bridge, in which $N_c = 8$ if the number of cells is greater than 8.

The AASHTO HL93, as shown in Figure 2-4, was used to determine the live load moment due to truck and lane load using SAP2000 software; this is the nominal live load. According to Nowak (1999), the mean dynamic load represents 10% of the mean live load. Calculations for live load are performed as follows.

The nominal live load (HL93) and nominal live load per box girder are

$$LL_{nominal} = truck + lane\ load \tag{10}$$

$$LL_{nominal/box} = (truck + lane\ load) * GDF \tag{11}$$

For one lane loaded, the bias factor is 1.32 as previously mentioned in Chapter 3, and for two lanes loaded, the bias factor for the live load can be taken as 1.10, according to Kulicki et al. (2007).

The mean live load, with girder distribution factor included and no dynamic load is expressed as,

$$LL_{mean} = \lambda_L * LL_{\frac{nominal}{box}} \tag{12}$$

where λ_L is the bias factor for live load.

The mean dynamic load represents 10% of the mean live load. The mean and standard deviation of live load and dynamic load are

$$LL + IM_{mean} = LL_{mean} * (1 + 0.10) \tag{13}$$

$$\sigma_{LL+IM} = COV_L * LL + IM_{mean} \tag{14}$$

Where COV_L is the coefficient of variation of live load.

5.2 Resistance Model (Strength Limit State)

The resistance model for the thesis follows the strength (or ultimate) limit state. The resistance of prestressed concrete box girder bridges used for the analysis was assumed to follow a lognormal distribution (Nowak, 1999) with a bias factor for resistance λ_R of 1.05 and a coefficient of variation, V_R of 0.075. The moment carrying capacity of one box girder was found by first determining the neutral axis location when the compression C, and tension forces T, are equal. The calculations are performed as follows.

$$C = T \tag{15}$$

$$\alpha * f'_c * b * a + A'_s * f'_s = A_{ps} * f_{ps} + A_s * f_s \tag{16}$$

Where α is 0.85 for a compressive strength f'_c less than 10 ksi, according to AASHTO LRFD 5.6.2.2. b is the width of one box, a is the depth of the compression block, A_s and A'_s are the area of reinforcing tension and compression steel, f_s and f'_s are the stresses in reinforcing steel at tension and compression fibers, respectively, A_{ps} is the prestressing area and f_{ps} is the average stress in prestressing steel.

The depth of compression block and the average stress in the prestressing steel are

$$a = \beta_1 * c \tag{17}$$

$$f_{ps} = f_{pu}\left(1 - k * \frac{c}{d_p}\right) \tag{18}$$

Therefore, the distance from the extreme compression fiber to the neutral axis is:

$$c = \frac{A_{ps}f_{pu} + A_s f_s - A'_s f'_s}{0.85 f'_c \beta_1 b + k A_{ps}\dfrac{f_{pu}}{d_p}} \tag{19}$$

f_s and f'_s are assumed to equal 60 ksi.

The value of *k* depends on the type of tendons that are used. 270-ksi low relaxation strands are used for California prestressed concrete bridges. According to AASHTO LRFD, *k* is calculated as,

$$k = 2 * \left(1.04 - \frac{f_{py}}{f_{pu}}\right) = 0.28 \tag{20}$$

as shown in Table 5-2.

Table 5-2. Value of *k* (AASHTO LRFD)

Type of tendons	F_{py}/f_{pu}	**Value of *k***
Low relaxation strand	0.90	0.28
Type 1 high-strength bar	0.85	0.38
Type 2 high-strength bar	0.80	0.48

The nominal capacity of one box is therefore,

$$M_n = \left[A_{ps} * f_{ps} * \left(d_p - \frac{a}{2}\right) + A_s * f_s * \left(d_s - \frac{a}{2}\right) - A'_s * f'_s * \left(d'_s - \frac{a}{2}\right)\right] \quad (21)$$

Where d_p, d_s, and d'_s are the distance from the extreme compression fiber to the centroid of the prestressing force, tensile steel, and compression steel, respectively.

The resistance determined for the fourteen prestressed concrete box girder bridges is calculated only for positive moment capacity.

Chapter 6. Reliability Analysis

6.1 Background

The reliability of a structure is its ability to fulfill its design purpose for some specified design lifetime (Nowak and Collins, 2013), in which three main questions are considered.

1) *How to measure the safety of structures?*

Safety is measured in terms of the reliability index β which is associated with the probability of failure. The probability of failure is defined as the probability that a structure will fail to perform as desired.

2) *How safe is safe enough?*

The proability of failure is a nonzero value. Reducing the probability of failure means increasing the reliability index. However, increasing the safety of a structure may be beyond a certain optimum level, which may not always be economical.

3) *How does a designer implement the optimum safety level?*

The optimum safety level is determined by the implementation of target reliability that can be accomplished through the development of probability-based design codes such as the American Association of State Highway and Transportation Officials (AASHTO) manual.

The reliability analysis of a structure also involves the use of random variables and limit states, which are discussed in the following sections.

6.1.1 Random Variables

According to Nowak and Collins (2013), a random variable is defined as a function that maps events onto intervals on the axis of real numbers as shown in Figure 6-1. This function, called the probability function, is categorized into three basic functions, which are the probability mass function (PMF), the cumulative distribution function (CDF), and the probability density function (PDF).

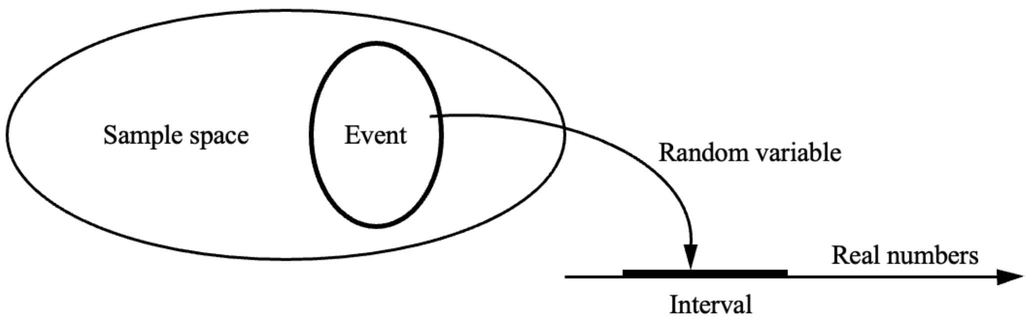


Figure 6-1. Representation of a random variable as a function (Nowak and Collins, 2013).

Probability Mass Function (PMF)

For a discrete random variable, the probability mass function is defined as follows.

$$p_x(x) = P(X = x) \tag{22}$$

Where $p_x(x)$ is the probability that a discrete random variable X is equal to a specific value x being a real number. For instance, a discrete random variable and probability mass function are shown in equation (23) and Figure 6-2.

$$X(f'_c) = \begin{Bmatrix} 1 & if & 0 & \leq & f'_c & < & 1{,}000\ psi \\ 2 & if & 1{,}000 & \leq & f'_c & < & 2{,}000\ psi \\ 3 & if & 2{,}000 & \leq & f'_c & < & 3{,}000\ psi \\ 4 & if & & & f'_c & & 3{,}000\ psi \end{Bmatrix} \tag{23}$$

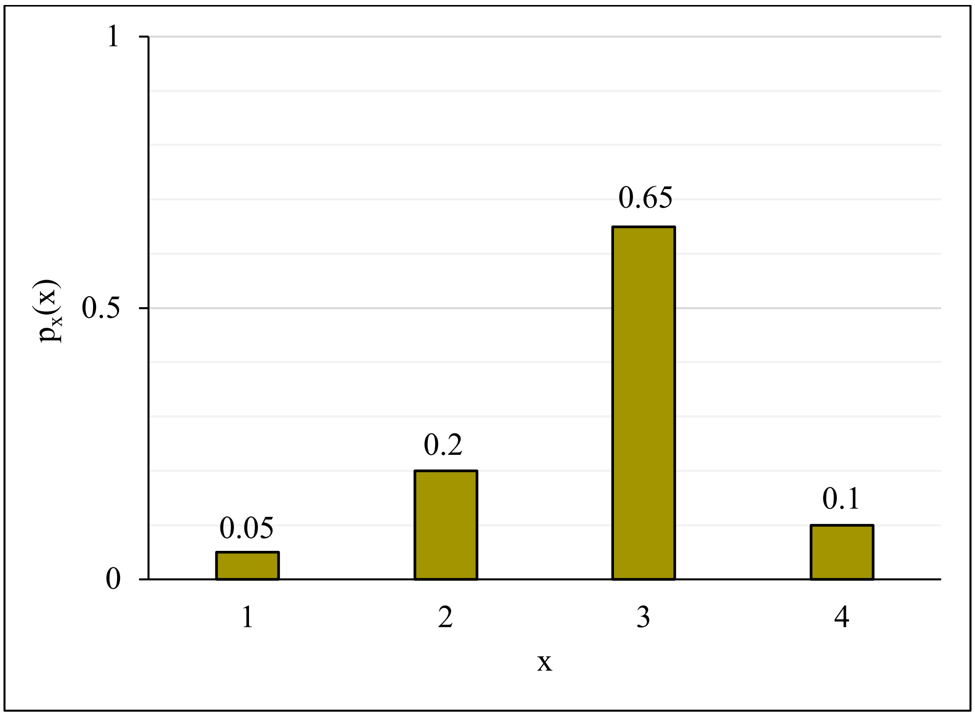


Figure 6-2. Probability mass function (Nowak and Collins, 2013).

<u>*Cumulative Distribution Function (CDF)*</u>

The cumulative distribution function is defined for both discrete and continuous variables as

$$F_x(x) = P(X \leq x) \tag{24}$$

Where $F_x(x)$ is the total sum or integral of all probability functions. An example of CDF is shown in Figure 6-3.

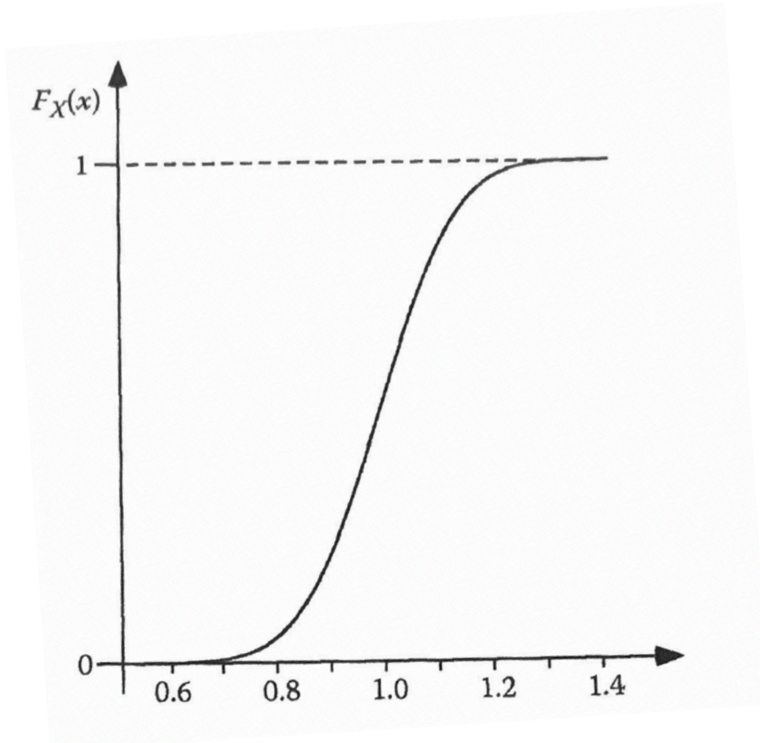


Figure 6-3. Example of a CDF (Nowak and Collins, 2013).

Probability Density Function (PDF)

The probability density function is defined as the first derivative of the CDF for continuous random variables. An example of PDF is shown in Figure 6-4.

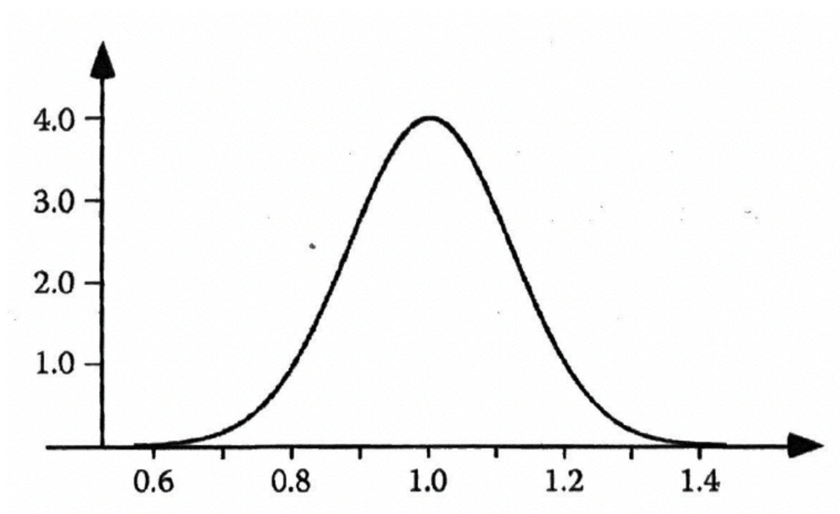


Figure 6-4. Example of a PDF (Nowak and Collins, 2013).

6.1.2 Limit States

The limit state is defined as the boundary between desired and undesired performance of a structure. According to Nowak and Collins (2013), this boundary is often represented mathematically by a limit state function called performance function, and can be formulated as follows.

$$g(R, Q) = R - Q \tag{25}$$

Where R is the resistance and Q is the load (or the demand).

$g \geq 0$ indicates that the structure is safe (desired performance) and $g < 0$, indicates that the structure is not safe (undesired performance).

The limit state function, g is associated with the probability that the undesired performance will occur; this is the probability of failure, P_f which is defined as

$$P_f = P(R - Q < 0) = P(g < 0) \tag{26}$$

For continuous random variables, namely variables that could take any positive values, the probability density function for the load effect, the resistance, and the safety margin is shown in Figure 6-5.

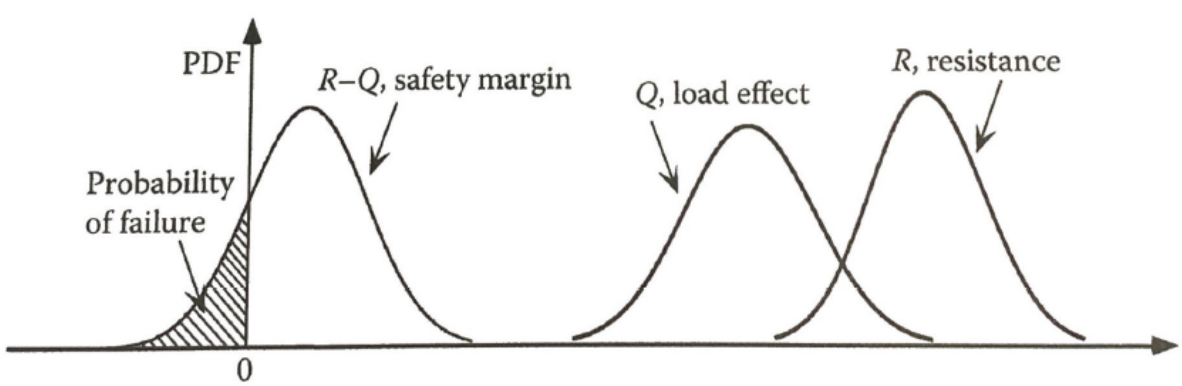


Figure 6-5. PDFs of load, resistance, and safety margin (Nowak and Collins, 2013)

Three types of limit states are generally considered in structural reliability analyses, which are:

Ultimate limit state (ULS)

The ultimate limit state is related to the loss of load-carrying capacity (i.e., buckling of the web, shear failure of web in steel beams, crushing of concrete, etc.)

Service limit state (SLS)

The service limit state is related to the gradual deterioration, user's comfort, or maintenance costs (i.e. excess deflections, excess vibrations, permanent deformations, and cracking)

Fatigue limit state (FLS)

The fatigue limit state is related to the loss of strength and accumulation of damage under repeated loads.

The considered limit state for the thesis is the ultimate limit state, also called the strength limit state.

6.1.3 Reliability index

According to Nowak and Collins (2013), the reliability index is defined as the shortest distance from the origin of reduced variables Z_R and Z_Q to the line $g\ (Z_R, Z_Q) = 0$. Figure 6-6 is a representation of the definition of the reliability index β. From the figure, the reliability index β for a normal distribution can be calculated as

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma^2{}_R + \sigma^2{}_Q}} \tag{27}$$

Where μ_R and σ_R are the mean and standard deviation of the resistance; μ_Q and σ_Q are the mean and standard deviation of the load.

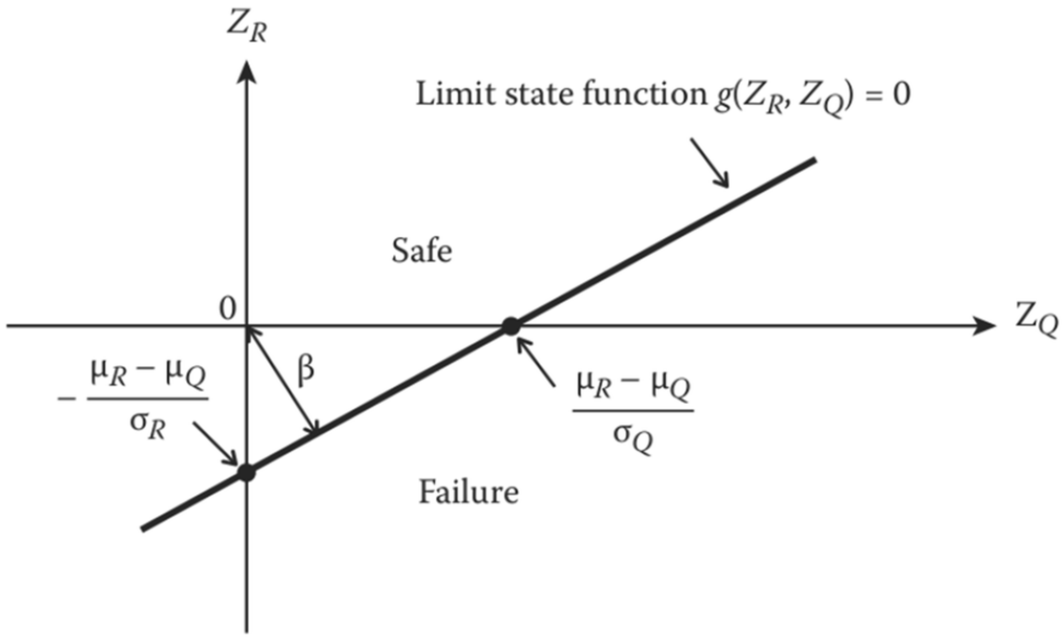


Figure 6-6. Reliability index defined as the shortest distance in the space of reduced variables (Nowak and Collins, 2013).

The reliability index is also related to the probability of failure by

$$\beta = -\Phi^{-1}(P_f) \text{ or } P_f = \Phi(-\beta) \tag{28}$$

Φ(z) is the cumulative distribution function (CDF) of the normal variable and z equals $-\beta$.

Other methods such as Monte Carlo, Rackwitz-Fiessler, or Hasofer-Lind methods can also be used to calculate the reliability index of a structure.

6.1.4 Probability Paper

According to Nowak and Collins (2013), probability paper is used to determine if certain data follow a particular distribution (normal, lognormal, etc.). The most common is the normal probability paper, in which a set of data are said to be normal when the CDF of the considered data follows a straight line. Figure 6-7 represents the normal probability paper. Note the difference in the dual vertical axes; the probability axis is unevenly spaced whereas the standard normal variable axis is evenly spaced. A probability of 0.5 represents a standard normal variable of zero (0). The mean and standard deviation of the distribution can also be determined using the slope and y-intercept of the graph as shown in Figure 6-8.

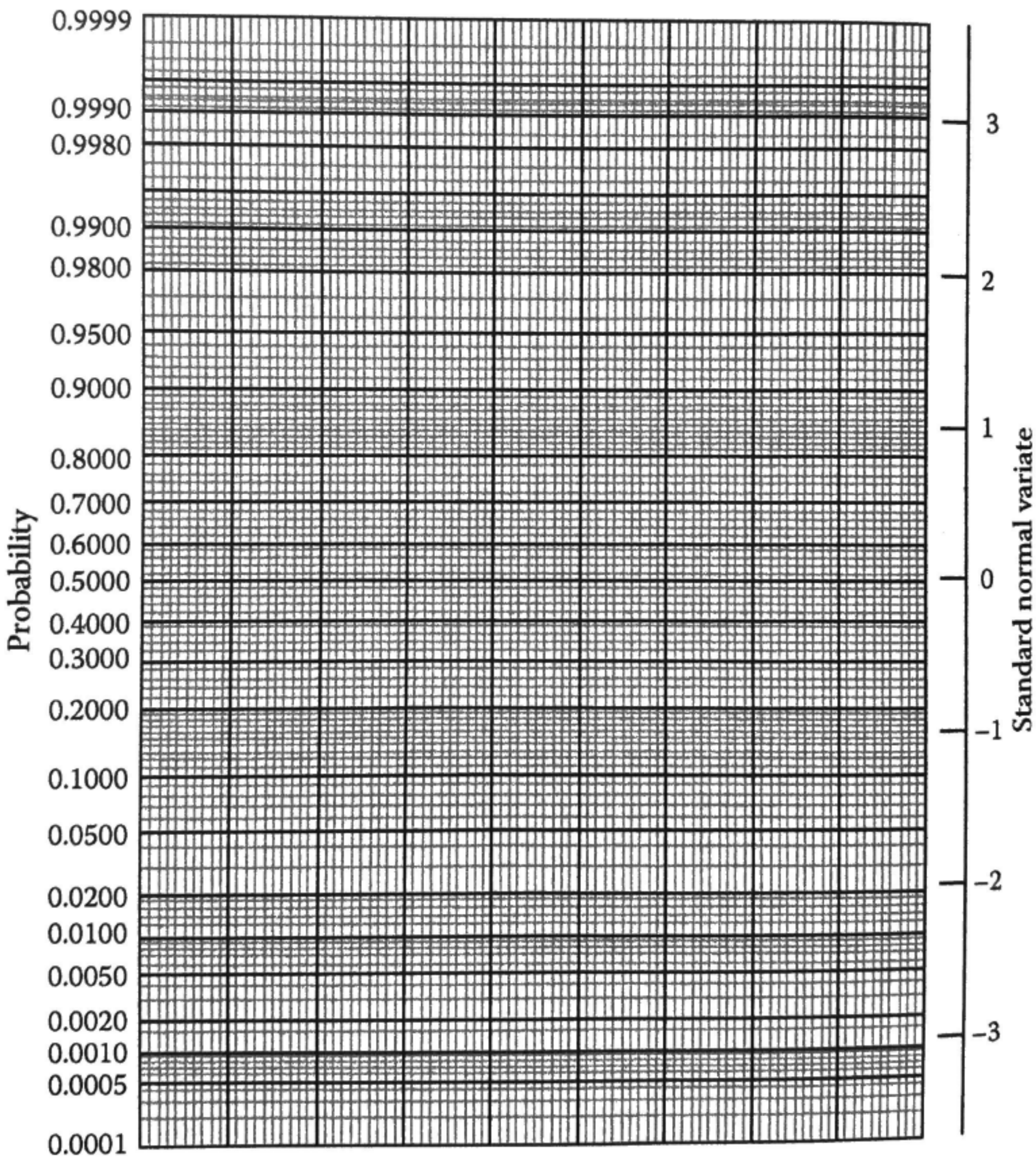


Figure 6-7. Example of normal probability paper (Nowak and Collins, 2013).

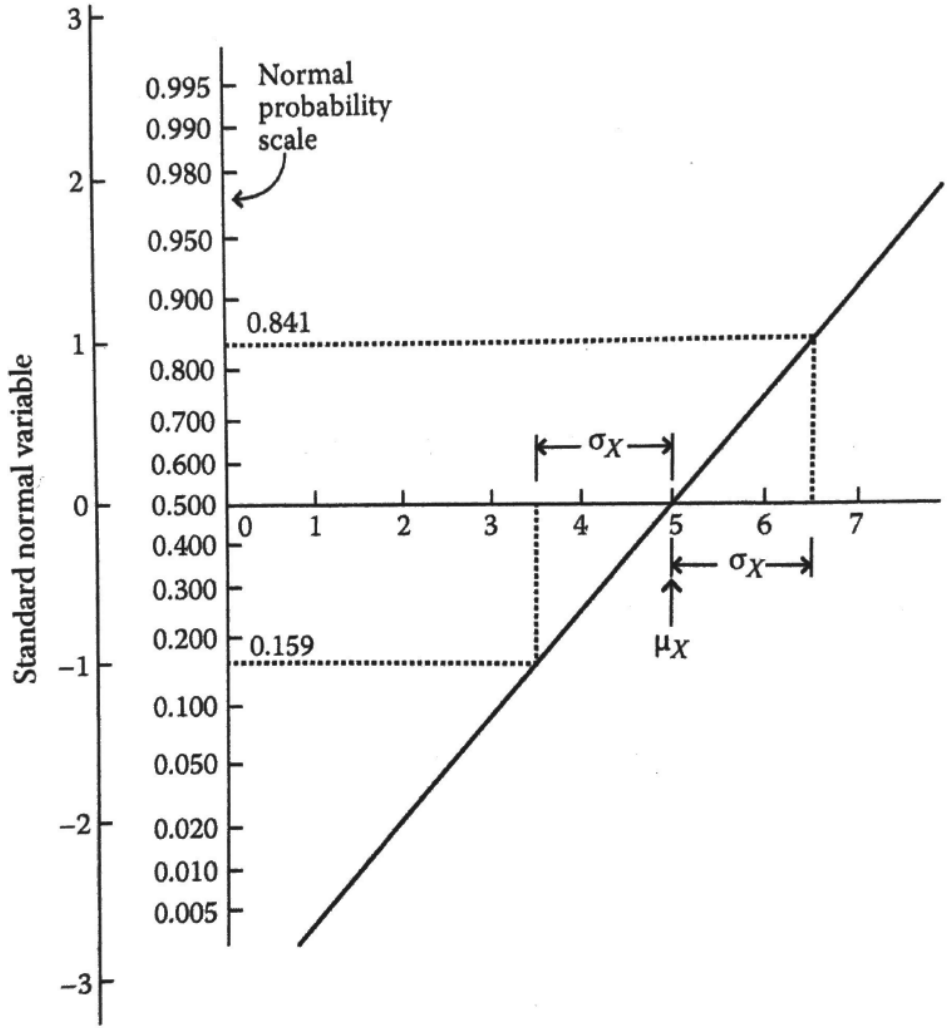


Figure 6-8. Interpretation of a straight-line plot on probability paper in terms of the mean and standard deviation of the normal random variable (Nowak and Collins, 2013).

The probability p based on the normal CDF and its inverse can be expressed as follows.

$$p = \Phi\left(\frac{x - \mu_X}{\sigma_X}\right) \tag{29}$$

$$\Phi^{-1}(p) = z = \frac{x - \mu_X}{\sigma_X} = \left(\frac{1}{\sigma_X}\right)x + \left(\frac{-\mu_X}{\sigma_X}\right) \tag{30}$$

Where p is the probability, $\Phi(z)$ and $\Phi^{-1}(z)$ are the cumulative distribution function (CDF) of the normal variable and its inverse, x is a certain value in a set of data of variable X, μ_X and σ_X are the mean and standard deviation of the normal random variable X.

6.2 Reliability Analysis Procedure

The reliability analysis used in this thesis was developed using the following procedure.

1. Select representative bridges.

   Fourteen prestressed box girder bridges were selected for California based on the availability of the information necessary to perform the reliability analysis.

2. Establish the statistical parameters and distribution types for the dead load, live load and resistance.

   The statistical parameters for dead load and resistance were found in the NCHRP 368 by Nowak (1999). Dead load and resistance were assumed to follow a normal and lognormal distributions. Live load distribution was found as Extreme Type I for California traffic. The bias factor for live load (mean-to-nominal factor) for one lane is determined using the weigh-in-motion data for California and the AASHTO HL93 truck and the bias factor for two lanes is taken as 1.10 as per Kulicki et al. (2007).

3. Calculate the nominal dead load moment for representative bridges using the self-weight of one interior box girder and the effect of barriers and/or railings per box girder.

4. Determine the nominal live load moment using the design live load as shown in Figure 2-4, for representative bridges.

   The positive moment is considered using the maximum moment due to one HL93 truck or tandem, and the negative moment is considered using two HL93 trucks with a headway distance of 50 ft as stated in AASHTO LRFD.

5. Calculate the positive nominal capacity of one prestressed concrete box girder.
6. Calculate the reliability index β.

   The reliability index for each selected prestressed concrete box girder is determined using the Monte Carlo method, described in section 6.3.

6.3 Monte Carlo Analysis

Monte Carlo is a simulation technique commonly used to resolve structural reliability problems. The basic idea behind simulations is, as the name implies, to numerically simulate some phenomenon and then observe the number of times some event of interest occurs (Nowak and Collins, 2013). The Monte Carlo method is often applied to solve complex problems for which solutions are either not possible or complex, problems with variables of different types of distribution, and to check the results of other solution techniques.

In this thesis is the ultimate limit state for flexure is checked. The basic limit state function can be formulated as,

$$g = R - D - L \tag{31}$$

Where R is the resistance (i.e., moment carrying capacity), D is the moment due to dead load, and L is the moment due to live load.

The Monte Carlo procedure is performed as follows.

1. Generate random numbers using the RAND function.

   The RAND function generates numbers between 0 and 1.
2. For each random number generated, calculate the normal standard variable z_i.
3. For each normal standard variable z_i, calculate the resistance R, the dead load D, and live load L according to their respective distributions, as described in sections 6.3.1 through 6.3.4.

4. Compute the limit state $g = R - D - L$.
5. Repeat step 1 to 4 until a sufficient number of g values have been generated.

   For the analysis 10,000 numbers have been simulated using MATLAB software.
6. Determine the reliability index β.

   The reliability index β can be determined using either the Monte Carlo analysis numerical method and/or graphical method.

Numerical method

The numerical method determines the reliability index using the limit state values that are less than zero. First, the number of simulations, where the load exceeds the resistance is determined. Then the probability of failure P_f is calculated as the ratio between the number of simulations where the limit state values are less than zero and the total number of simulations.

$$P_f = \frac{N_{g<0}}{N_{total}} \tag{32}$$

The reliability index β is therefore,

$$\beta = -\Phi^{-1}(P_f) \tag{33}$$

Graphical method

The graphical method for determining the reliability index uses the normal standard variable and the limit state values. First, values obtained through the limit state, $g = R - D - L$, were sorted from smallest to largest. Then for each limit state value, the probability, P_i and its associated normal standard variable, z_i are calculated, as shown as follows.

$$P_i = \frac{i}{N_{total} + 1} \tag{34}$$

$$z_i = \Phi^{-1}(P_i) \tag{35}$$

The normal standard variable is then plotted versus the limit state values. A linear trendline is fitted to the data to find the y-intercept. The y-intercept is the normal standard variable z value.

The reliability index and probability of failure are then,

$$\beta = -z \tag{36}$$

$$P_f = \Phi(z) \tag{37}$$

6.3.1 Generation of standard normal random numbers

The basis of the Monte Carlo method is the generation of random numbers u_i, which are values between 0 and 1. Here, the generated random numbers represent the probability and are used to determine the standard normal variables z_i, as shown in equation (38),

$$z_i = \Phi^{-1}(u_i) \tag{38}$$

Where Φ^{-1} is the inverse of the standard normal cumulative distribution function (CDF). A graphical representation is shown in Figure 6-9 and values for the cumulative distribution function for the standard normal variable are presented in Appendix B.

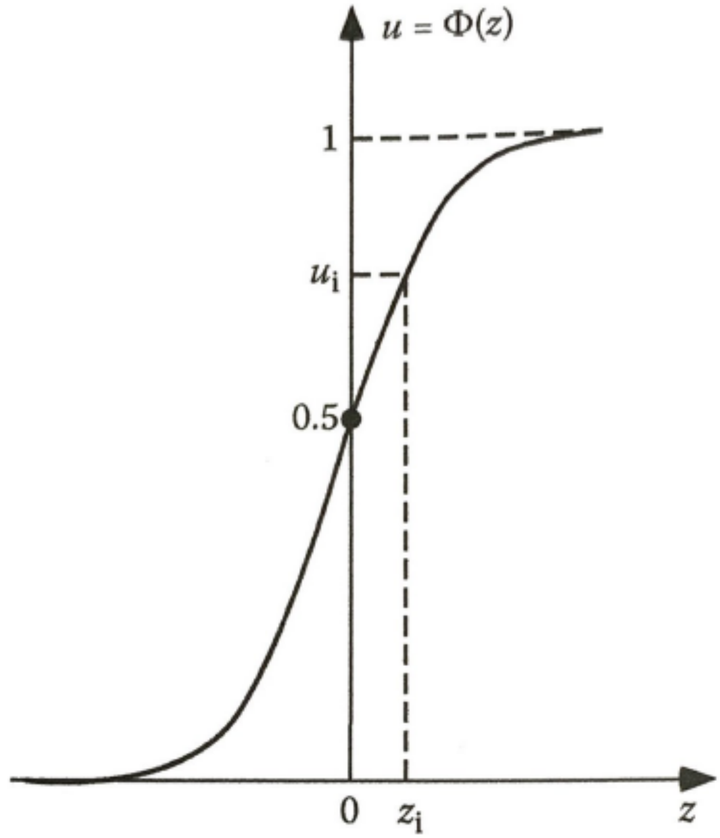


Figure 6-9. Generation of standard normal random variables (Nowak and Collins, 2013).

6.3.2 Generation of normal random numbers

The generation of normal random numbers implies the use of the standard normal variable z_i. For each random number generated, the basic relationship between a corresponding sample normal random variable x_i and its standard normal variable, z_i is,

$$x_i = \mu_X + z_i \sigma_X \tag{39}$$

Where μ_X and σ_X are the mean and standard deviation of a normally distributed random variable *X*.

6.3.3 Generation of lognormal random numbers

For a lognormal random variable with the mean and standard deviation, the relationship between a sample lognormal value x_i and its standard normal variable z_i is,

$$x_i = \exp\left[\mu_{lnX} + z_i \sigma_{lnX}\right] \tag{40}$$

where,

$$\sigma^2{}_{lnX} = \ln(V^2{}_X + 1) \tag{41}$$

$$\mu_{lnX} = \ln(\mu_X) - \frac{1}{2}\sigma^2{}_{lnX} \tag{42}$$

6.3.4 Generation of Extreme Type I random numbers

According to Nowak and Collins (2013), some of the most common distributions used in reliability analysis, such as the extreme type I distribution, do not consider the use of the standard normal variable z_i. However, a general procedure applicable to any type of distribution can be formulated as

$$x_i = F^{-1}{}_X(u_i) \tag{43}$$

Where $F^{-1}{}_x$ is the inverse of a random variable CDF, F_x.

A random variable q_i of extreme type I distribution is, therefore, evaluated as,

$$q_i = F^{-1}{}_X(u_i) = u - \frac{ln\,(ln(-p_i))}{\alpha} \tag{44}$$

Where α and u are the extreme type I parameters calculated as shown in equations (45) and (46).

$$\alpha = \frac{1.282}{\sigma_X} \tag{45}$$

$$u = \mu_X - 0.45 * \sigma_X \tag{46}$$

6.4 Results of Analysis

The reliability analysis procedure, previously described, was applied to the 14 selected prestressed concrete box girder bridges. Table 6-1 shows data about each bridge as well as the calculated reliability index. Simple span is denoted by *SS* and continuous span by *CS*. The resistance ratio is the ratio of the calculated load carrying capacity and the required load carrying capacity per box girder. The required load carrying capacity per box is

$$M_{n,required} = 1.25 * D_n + 1.75 * (LL + IM) * GDF \tag{47}$$

Where D_n is the moment due to dead load, *LL+IM* is the live and dynamic load, and *GDF* is the girder distribution factor.

Detailed calculations for the representative bridges can be found in Appendix A.

The results obtained through the reliability analysis of prestressed concrete box girders show that the reliability indices range between 5.64 and 6.54 for simple spans. The reliability indices lie between 2.32 and 5.79 for continuous spans. The average and standard deviation for the reliability indices for California are 4.81 and 1.38, respectively. These results are presented in Table 6-2. The reliability indices were plotted in terms of the span length, the girder spacing, and the total load effect (dead and live load) as shown in Figure 6-10, Figure 6-11, and Figure 6-12, respectively.

Table 6-1. Reliability analysis results

	Type	Span length (ft)	Mean Resistance R (kip·ft)	Mean Dead Load (kip·ft)	Mean Live and Dynamic per Box (kip·ft)	Resistance ratio	Reliability index β
17 0030	SS	175.5	34,079	11,857	3,395	1.63	6.54
29 0306L	SS	157.5	18,188	7,085	2,242	1.41	5.64
37 0368L	SS	124.3	15,334	5,208	2,527	1.38	5.69
55 0655	SS	94.6	10,100	2,846	1,665	1.52	6.52
23 0020	CS	111.8	7,091	2,348	1,336	1.32	5.36
23 0205L	CS	126.1	6,473	2,146	1,059	1.40	5.79
37 0366L	CS	85.0	2,894	1,174	777	1.00	3.24
37 0420L	CS	62.0	1,883	591	493	1.13	3.72
37 0421L	CS	77.0	2,561	773	588	1.24	5.20
37 0467L	CS	231.0	19,275	11,743	3,180	0.95	2.32
49 0060L	CS	160.0	12,998	5,481	2,095	1.23	4.62
49 0165L	CS	98.0	4,745	1,515	1,048	1.24	5.08
54C0617	CS	73.0	2,894	1,307	856	0.90	2.43
55 0670	CS	156.0	15,740	6,465	2,101	1.33	5.22

Table 6-2. General results of Monte Carlo analysis

Category	Simple Span	Continuous Span
Minimum β	5.64	2.32
Maximum β	6.54	5.79
Average	4.81	
Standard Deviation	1.38	

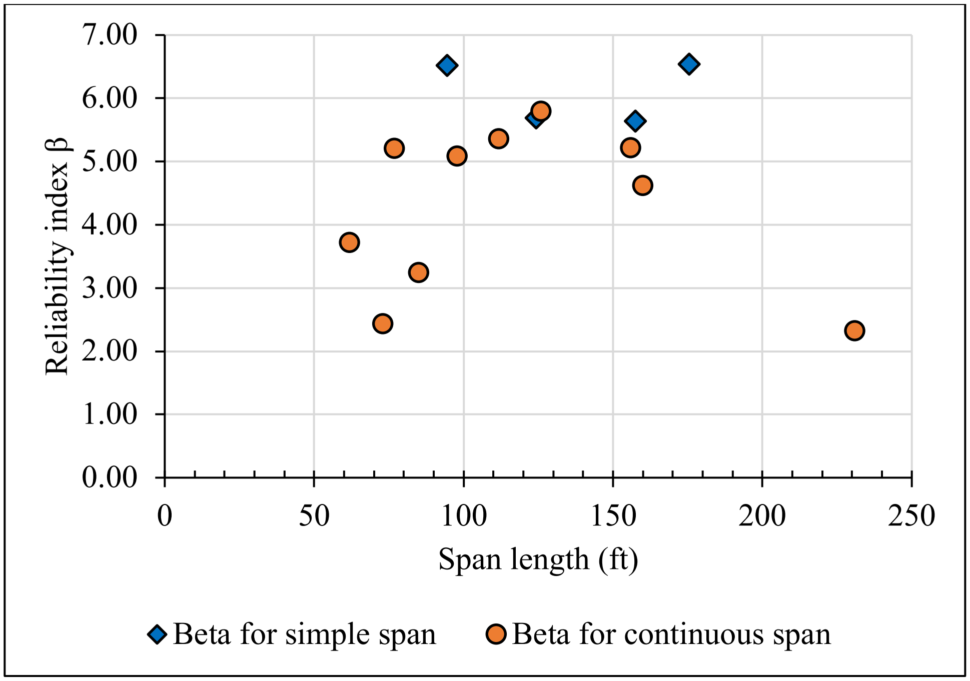


Figure 6-10. Scatter plot of reliability indices vs. span length

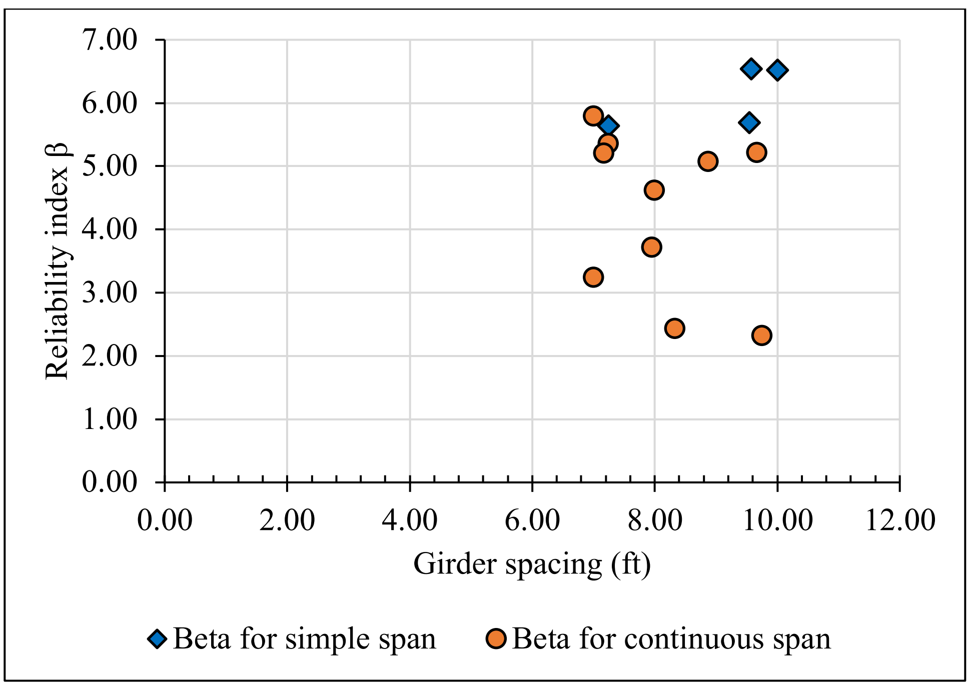


Figure 6-11. Scatter plot of reliability indices vs. girder spacing

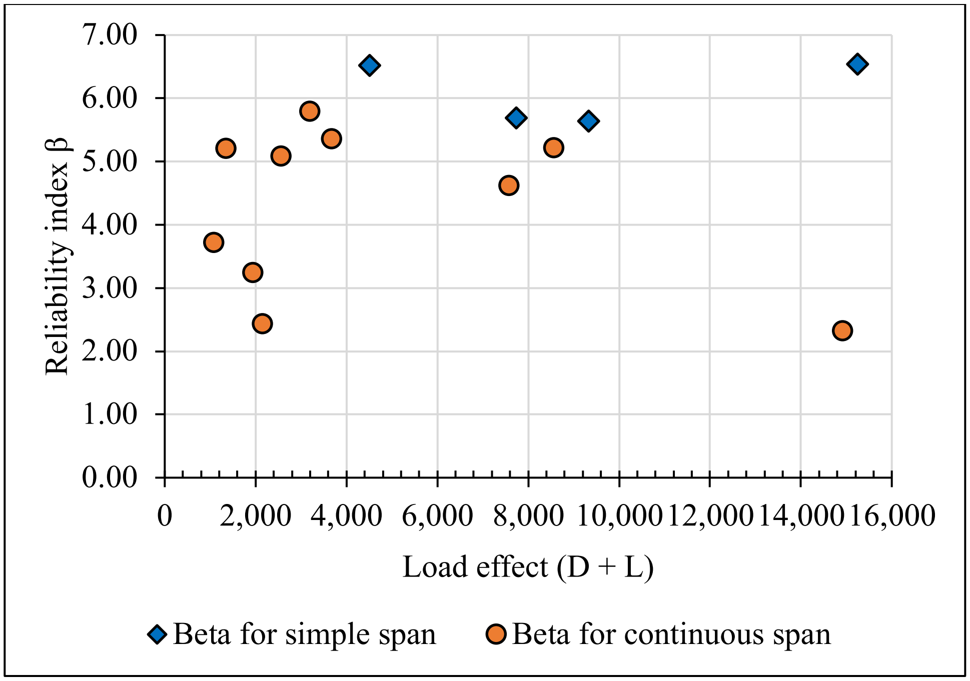


Figure 6-12. Scatter plot of reliability indices vs. load effect

Figure 6-10 and Figure 6-12 show that the reliability indices decrease for simple spans and then increase as the span length and the load effect increase. On the contrary, the reliability indices of continuous spans increase and then decrease as the span length and load effect increase.

Figure 6-11 shows that there is no particular relationship between the reliability indices and the girder spacing for both simple and continuous spans.

According to the AASHTO LRFD and AASTHO LRFR, the target reliability index for design is given as 3.5, while a target reliability index for evaluation of existing bridges is 2.5. Results show that the reliability indices for twelve of the considered prestressed box girder bridges in California are greater than the target reliability index for existing bridges, 2.5. Therefore, they are safe under the applied live loads, for the strength limit state. However, two of the bridges were found to have their reliability indices less than the target reliability indices 2.5; therefore, they will require further analysis.

Chapter 7. Summary and Conclusions

The prestressed concrete box girder bridges in the state of California have shown many signs of deck deterioration such as longitudinal, transverse and pattern cracks. The presence of heavy vehicles may cause these cracks. The objective of the thesis was to calculate the reliability indices for a wide spectrum of prestressed concrete box girder bridges in California, in order to investigate their safety under applied live loads.

Fourteen prestressed box girder bridges were selected for the reliability analysis. Moment due to dead load, live load, and resistance (only positive moment capacity) for one box, were calculated for each selected bridge. The Monte Carlo method was used to calculate the reliability indices for the ultimate limit state, for both simple and continuous spans.

The reliability indices range between 5.64 and 6.54 for simple spans and between 2.32 and 5.79 for continuous spans. The average and standard deviation for the reliability indices for California are 4.81 and 1.38, respectively. Results show that the reliability indices for simple spans decrease and then increase as the span length and the load effect increase. However, the reliability indices of continuous spans increase and then decrease as the span length and load effect increase.

Compared with the target reliability index for existing bridges of 2.5, the results obtained through the reliability analysis for twelve of the considered prestressed box girder bridges in California are greater than the target reliability index for existing bridges, 2.5. Therefore, they are safe under the applied live loads, for the Strength Limit State. However, two of the bridges were found to have their reliability indices less than the target reliability indices 2.5.

Additional research include performing a reliability analysis on the two prestressed box girder bridges that have their reliability indices less than the target reliability indices of 2.5, accounting for the distresses that are present on these bridges. Also, further research will be to calculate the reliability indices for the considered bridges for other limit states such as the Service Limit State and the Fatigue Limit State.

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Appendix A. Detailed Calculations

Bridge 29 0306L – Simple span

Bridge Properties

General information about bridge 29 0306L are presented below.

Span length:		157.5 ft
Concrete:		f'_c: 4.5 ksi
Prestressing steel:		270 ksi low relaxation strand
A_{ps}	=	82.57 in^2
Girder spacing	=	7.25 ft
Depth of beam	=	7 ft
Top flange	=	7.375 in
Bottom flange	=	5.5 in
Web thickness	=	12 in
Area of one box	=	1,973.63 in^2
Number of cells/box	=	8
Number of webs	=	9

The cross-section of bridge 29 0306L and prestressing notes are shown in Figure A-1 and Figure A-2.

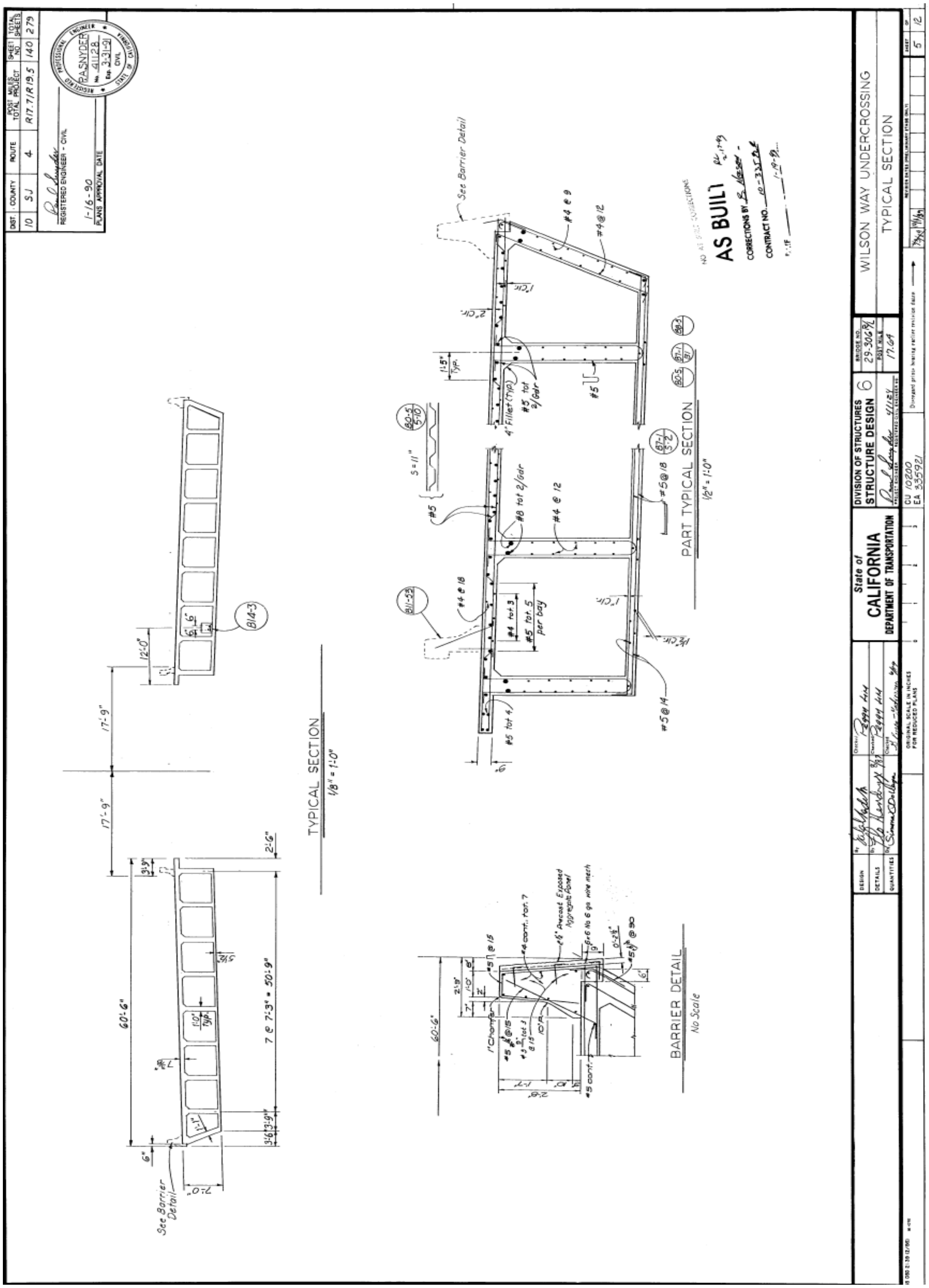

Figure A-1. Cross-section of bridge 29 0306L

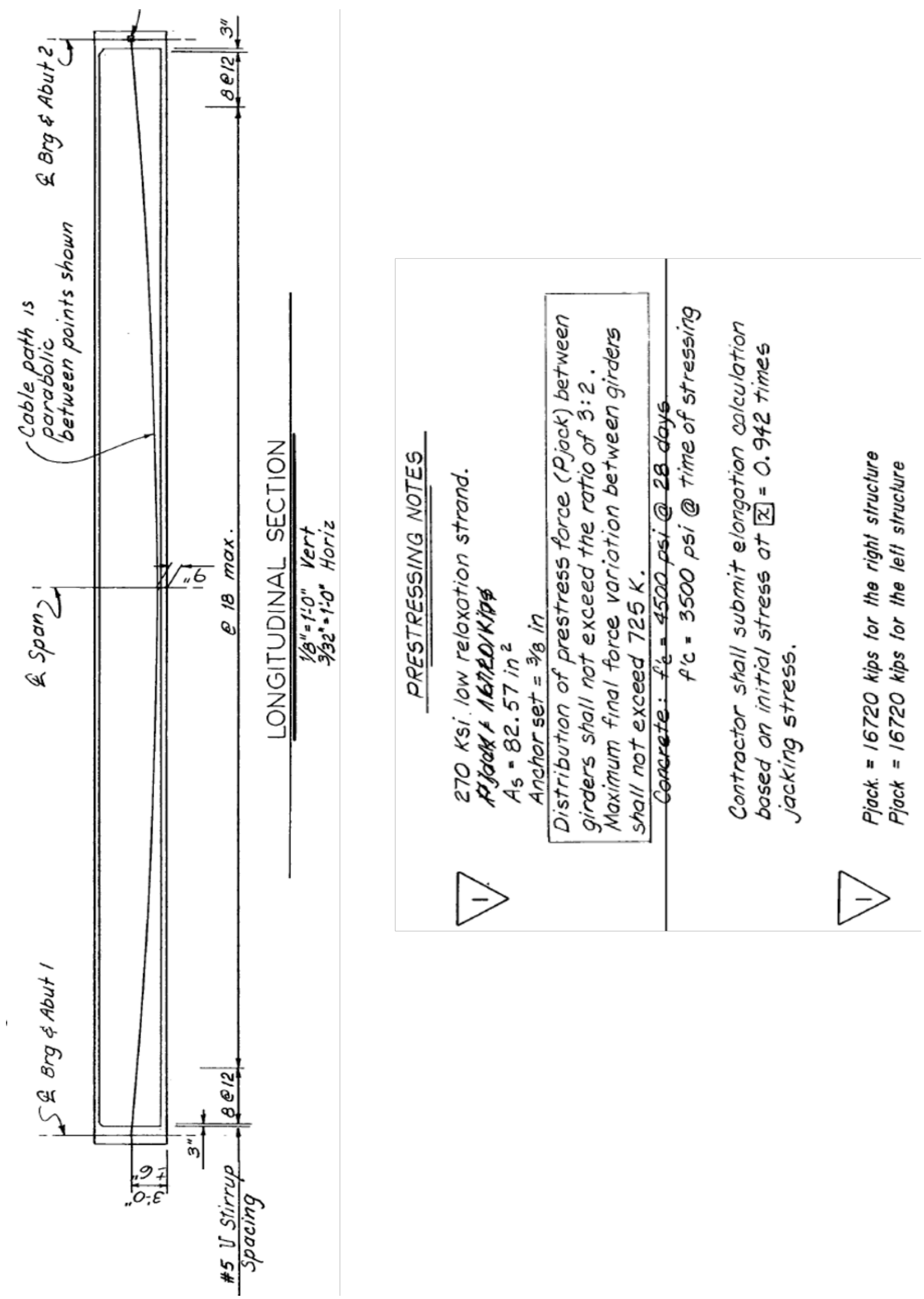

Figure A-2. Bridge 29 0306L details and prestressing notes

Dead Load Analysis – Interior Beam

Components and Attachments, *DC*

The unit weight of concrete is taken as 150 pcf.

$$Self - weight\ for\ one\ interior\ box\ =\ 0.150 * \left(\frac{1{,}973.63\ in^2}{144}\right)\ =\ 2.056\ kip/ft$$

$$Barrier\ and\ railing\ for\ one\ box\text{: } 0.150 * \left(\frac{319\ in^2}{144} + \frac{600\ in^2}{144}\right) * \frac{1}{8} =\ 0.120\ kip/ft$$

The self-weight and barrier loads were used in SAP2000 to find the moment due to dead load.

The moment due to self-weight is 6,375.21 kip.ft and the moment due to barrier and railing is 372.09 kip.ft

The total moment due to dead load for one interior box is

$$M_{DC}\ =\ 6{,}375.21\ +\ 372.09\ =\ \mathbf{6,747.3}\ \boldsymbol{kip{\cdot}ft}$$

Wearing surface, DW

There is no wearing surface wearing surface applied on the bridge. Therefore, the moment due to wearing surface is, M_{DW} = 0 kip·ft

Live Load Analysis – Interior Girder

Prestressed concrete box bridges in California are cast-in-place concrete multicell boxes as shown in Figure A-3, with a monolithic concrete deck.

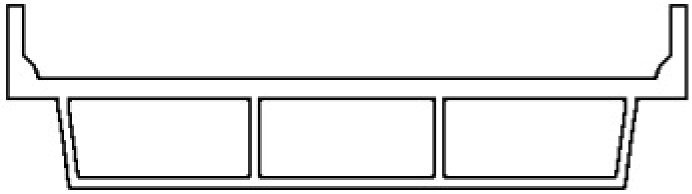

Figure A-3. Cast-in-place concrete multicell box (AASHTO, 2017).

Compute Live Load Distribution Factor for an Interior Box

The live load distribution factors were computed for one interior box. The girder distribution factors (GDFs) for one design lane and two design lanes loaded, are calculated according to AASHTO LRFD as follows.

- One design lane loaded:

$$\left(1.75+\frac{S}{3.6}\right)\left(\frac{1}{L}\right)^{0.35}\left(\frac{1}{N_c}\right)^{0.45}=\left(1.75+\frac{7.25}{3.6}\right)\left(\frac{1}{157.5}\right)^{0.35}\left(\frac{1}{8}\right)^{0.45}=0.2513$$

- Two or more design lanes loaded:

$$\left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25}=\left(\frac{13}{8}\right)^{0.3}\left(\frac{7.25}{5.8}\right)\left(\frac{1}{157.5}\right)^{0.25}=0.4082 \quad \leftarrow \mathit{Governs}$$

Further live load analysis is detailed in the reliability analysis section.

Nominal Flexural Resistance

The area of prestressing steel per box is calculated as follows.

$$A_{ps}=\frac{total\ prestressing\ area}{number\ of\ boxes}$$

$$A_{ps}=\frac{82.57\ in^2}{8}=10.32\ in^2$$

With a compressive strength of 4.5 ksi, the concrete stress block factor is computed as,

$$\beta_1 = 0.85 - 0.05 * (4.5\ ksi - 4)$$

$$\beta_1 = 0.825$$

The center of gravity of the prestressing steel at midspan is 9 inches. The distance from the extreme compression fiber to the centroid of the prestressing is,

$$d_p = 84 - 9 = 75\ in$$

Location of longitudinal reinforcement

The location of the longitudinal steel was estimated using the cross-section of the boxes. The table below shows the location of the longitudinal steel from the bottom to the top of one box.

Table A-1. Location of longitudinal reinforcement

Location	**Number of steel bars**	**Bar number**		**Location from bottom of the box girder (inches)**
Bottom flange	4	#5	==	1.42 in.
	3	#5	==	4.27 in.
Web	2	#4	==	14.24 in.
	2	#4	==	24.20 in.
	2	#4	==	35.59 in.
	2	#4	==	48.41 in.
	2	#4	==	58.37 in.
	1	#8	==	71.19 in.
	1	#8	==	72.61 in.
Top flange	3	#4	==	78.31 in.
	5	#5	==	79.73 in.
	2	#5	==	82.58 in.
	2	#5	==	82.58 in
	1	#4	==	82.58 in
Total	**32 bars**			

The location of the neutral axis was first estimated by computing the compression and tension forces, *C* and *T*.

$$C = \alpha * \beta_1 * f'_c * A_c + A'_s * f'_y = 0.85 * 0.825 * 4.5\ ksi * 87 * 7.375 + 3.59 * 60\ ksi$$

$$C = 2{,}240.13\ kips$$

$$T = A_{ps} * f_{pu} + A_s * f_y = 10.32\ in^2 * 270\ ksi + 5.75 * 60\ ksi$$

$$T = 3{,}131.74\ kips$$

$$C = 2{,}240.13\ kips < T = 3{,}131.74\ kips$$

Therefore, the neutral axis is located below the top flange of the box.

The location of the neutral axis is found when the compression and tension forces are equal. The formula is shown below.

$$C = T$$

$$\alpha * f'_c * b * a + A'_s * f'_s = A_{ps} * f_{ps} + A_s * f_s$$

Where α is 0.85 for f'_c less than 10 ksi, according to AASHTO LRFD 5.6.2.2.

$$a = \beta_1 * c$$

$$f_{ps} = f_{pu}\left(1 - k * \frac{c}{d_p}\right)$$

Therefore, for rectangular section behavior, the distance from the extreme compression fiber to the neutral axis is:

$$c = \frac{A_{ps}f_{pu} + A_s f_s - A'_s f'_s}{0.85 f'_c \beta_1 b + k A_{ps}\frac{f_{pu}}{d_p}}$$

f_s and f'_s are assumed to equal 60 ksi.

The value of k depends on the type of tendons that is used. According to AASHTO LRFD, k for low relaxation strand, is calculated as, $k = 2 * \left(1.04 - \frac{f_{py}}{f_{pu}}\right) = 2 * (1.04 - 0.90) = \mathbf{0.28}$

The neutral axis is located at,

$$c = \frac{A_{ps} f_{pu} + A_s f_s - A'_s f'_s}{\alpha f'_c \beta b + k A_{ps} \frac{f_{pu}}{d_p}} = \frac{10.32 * 270 + 5.75 * 60 - 3.59 * 60}{0.85 * 4.5 * 0.825 * 87 + 0.28 * 10.32 * \frac{270}{75}}$$

$$\boldsymbol{c = 10.23\ in} > t_s = 7.375\ in$$

The neutral axis is, therefore located below the top flange, as previously determined.

$$f_{ps} = f_{pu}\left(1 - k * \frac{c}{d_p}\right) = 270\left(1 - 0.28 * \frac{10.23}{75}\right) = 259.68\ ksi$$

$$a = \beta * c = 0.85 * 10.23 = 8.44\ in$$

The nominal capacity of one box is,

$$M_n = \left[A_{ps} * f_{ps} * \left(d_p - \frac{a}{2}\right) + A_s * f_s * \left(d_s - \frac{a}{2}\right) - A'_s * f'_s * \left(d'_s - \frac{a}{2}\right)\right]$$

$$M_n = \left[10.32 * 259.68 * \left(75 - \frac{8.44}{2}\right) + 5.75 * 60 * \left(56.42 - \frac{8.44}{2}\right) - 3.59 * 60 * \left(3.5 - \frac{8.44}{2}\right)\right]$$

$$\boldsymbol{M_n = 17{,}322\ kip{\cdot}ft}$$

Reliability Analysis

Table A-2 presents the statistical parameters for resistance and dead load.

Table A-2. Statistical parameters for resistance and dead load

Component	Bias factor, λ	Coefficient of variation, V	References
Resistance (flexure)	1.05	0.075	Nowak, 1999 (NCHRP 368)
Dead load (cast-in-place)	1.05	0.10	Nowak, 1999 (NCHRP 368)

The bias factor of live load for one lane is 1.32. The bias factor was found using weigh-in motion data for California. As per NCHRP 20-70/186, the bias factor for two lanes is taken as 1.10; the coefficient of variation for live load and dynamic load is 0.18.

Resistance

The distribution for resistance is assumed to be lognormal. The bias factor for flexural resistance is 1.05 and the coefficient of variation is 0.075.

The nominal capacity of one box M_n is 17,322 kip·ft.

The mean resistance, variance and mean of the logarithm resistance were expressed as follows

$$\mu_R = \lambda_R M_n = 1.05 * 17{,}322 = \mathbf{18{,}188}\ \ \boldsymbol{kip{\cdot}ft}$$

$$\sigma^2{}_{ln(R)} = ln(V^2{}_Y + 1) = ln(0.075^2 + 1) = \mathbf{0.00561}$$

$$\mu_{ln(R)} = ln(\mu_R) - \frac{1}{2}\sigma^2{}_{ln(R)} = ln(18{,}188) - \frac{1}{2} * 0.00561 = \mathbf{9.81}\ \ \boldsymbol{kip{\cdot}ft}$$

Dead Load

The distribution for dead load is assumed to be normal. It was calculated using the self-weight of one interior box and the effect of barrier and railing applied on the box.

The total dead load is 2.176 kip/ft and the moment due to the total dead load is 6,747 kip·ft, found using SAP2000. This is the nominal dead load.

The bias factor for the dead load λ_D is 1.05 for cast-in-place and the coefficient of variation COV_D is 0.10.

The mean dead load and the standard deviation of the dead load are

$$\mu_D = \lambda_D D_n = 1.05 * 6{,}747 = \mathbf{7{,}085} \quad \boldsymbol{kip \cdot ft}$$

$$\sigma_D = COV_D D_n = 0.10 * 6{,}747 = \mathbf{674.7} \quad \boldsymbol{kip \cdot ft}$$

Live Load

The distribution for the live load in California is the extreme type 1 distribution.

The nominal live load is HL93. The moment due to HL 93 truck and lane load was calculated using moving load in SAP2000 software. The moment due to truck load is 2,555 kip·ft and the moment due to lane load is 1,985 kip·ft.

The nominal live load (HL93) is

$$LL_{nominal} = truck \ + lane\ load = 2{,}555 + 1{,}985 = 4{,}540\ kip \cdot ft$$

The girder distribution factor for two lanes loaded governs. Therefore, the nominal live load per box with girder distribution factor (GDF) for two lanes included (no dynamic load) is,

$$LL_{nominal/box} = (truck + lane\ load) * GDF = (2{,}555 + 1{,}985) * 0.4082$$

$$LL_{nominal/box} = 1{,}853\ kip \cdot ft$$

The mean live load with girder distribution factor for two lanes (no dynamic load) is

$$LL_{mean} = \lambda_L * LL_{nominal/box} = 1.10 * 1{,}853$$

$$\boldsymbol{LL_{mean} = 2{,}038\ kip \cdot ft}$$

The mean dynamic load represents 10% of the mean live load. Therefore, the mean live load and dynamic load is

$$LL + IM_{mean} = LL_{mean} * (1 + 0.10) = 2{,}038 * (1 + 0.10)$$

$$\boldsymbol{LL + IM_{mean} = 2,242\ kip{\cdot}ft}$$

The standard deviation of the live and dynamic load is

$$\sigma_{LL+IM} = COV_L * LL + IM_{mean} = 0.18 * 2{,}242 = \mathbf{403.6}\ \boldsymbol{kip{\cdot}ft}$$

<u>*Extreme type 1 parameters*</u>

California live load best fits an extreme type 1 distribution. The parameters *α* and *u* are,

$$\alpha = \frac{1.282}{\sigma_{LL+IM}} = \frac{1.282}{403.6} = \mathbf{0.003176}$$

$$u = LL + IM_{mean} - 0.45 * \sigma_{LL+IM} = 2{,}242 - 0.45 * 403.6 = \mathbf{2,061}\ \ \boldsymbol{kip{\cdot}ft}$$

Monte Carlo

The limit state function is expressed as, $g = R - D - L$

Monte Carlo analysis was performed by running 10,000 simulations in Matlab software. Random numbers and their associated standard variable, *Z* were generated. The values for the dead load, live load and resistance were calculated using:

- For normal distribution (dead load)

$$D = \mu_D + Z_D * \sigma_D$$

- For lognormal distribution (resistance)

$$R = exp\ \left[\mu_{ln(R)} + Z_R \sigma_{ln(R)}\right]$$

- For extreme type 1 distribution (live load and dynamic load)

$$L = u - \frac{ln(ln(-p_i))}{\alpha}$$

Where p_i is the simulated random numbers.

The reliability index is determined using the numerical method or the graphical method as discussed in section 6.2.1. The graphical method is used for bridge 29 0306L.The normal standard variable is plotted versus the limit state values as shown in Figure A-4. A linear trendline is fitted to the data to find the y-intercept using Matlab. For bridge 29 0306L the y-intercept is, $z = -5.64$. Therefore, the reliability index and probability of failure are

$$\beta = -z = \mathbf{5.64}$$

$$P_f = \Phi^{-1}(-z) = \mathbf{8.5 * 10^{-9}}$$

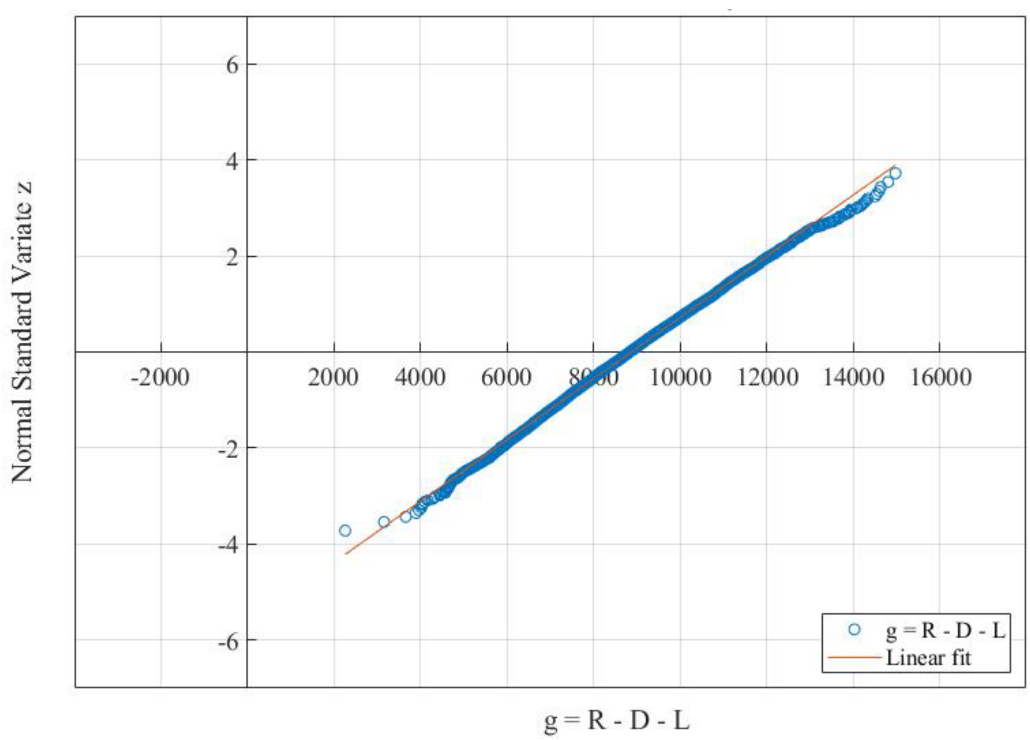

Figure A-4. Monte Carlo simulation for bridge 29 0306L

<u>**Bridge 17 0030 – Simple span**</u>

Bridge Properties

General information about bridge 17 0030 is presented below.

Span length:		175.5 ft
Concrete:		f'_c: 4.5 ksi
Prestressing steel:		270 ksi low relaxation strand
$A_{ps,total}$	=	54.32 in^2
Girder spacing	=	9.58 ft
Depth of beam	=	8 ft
Top flange	=	8.75 in
Bottom flange	=	6.625 in
Web thickness	=	12 in
Area of one box	=	2,735.63 in^2
Number of cells	=	8
Number of webs	=	11

The cross-section of bridge 17 0030 and prestressing notes are shown in Figure A-5 and Figure A-6.

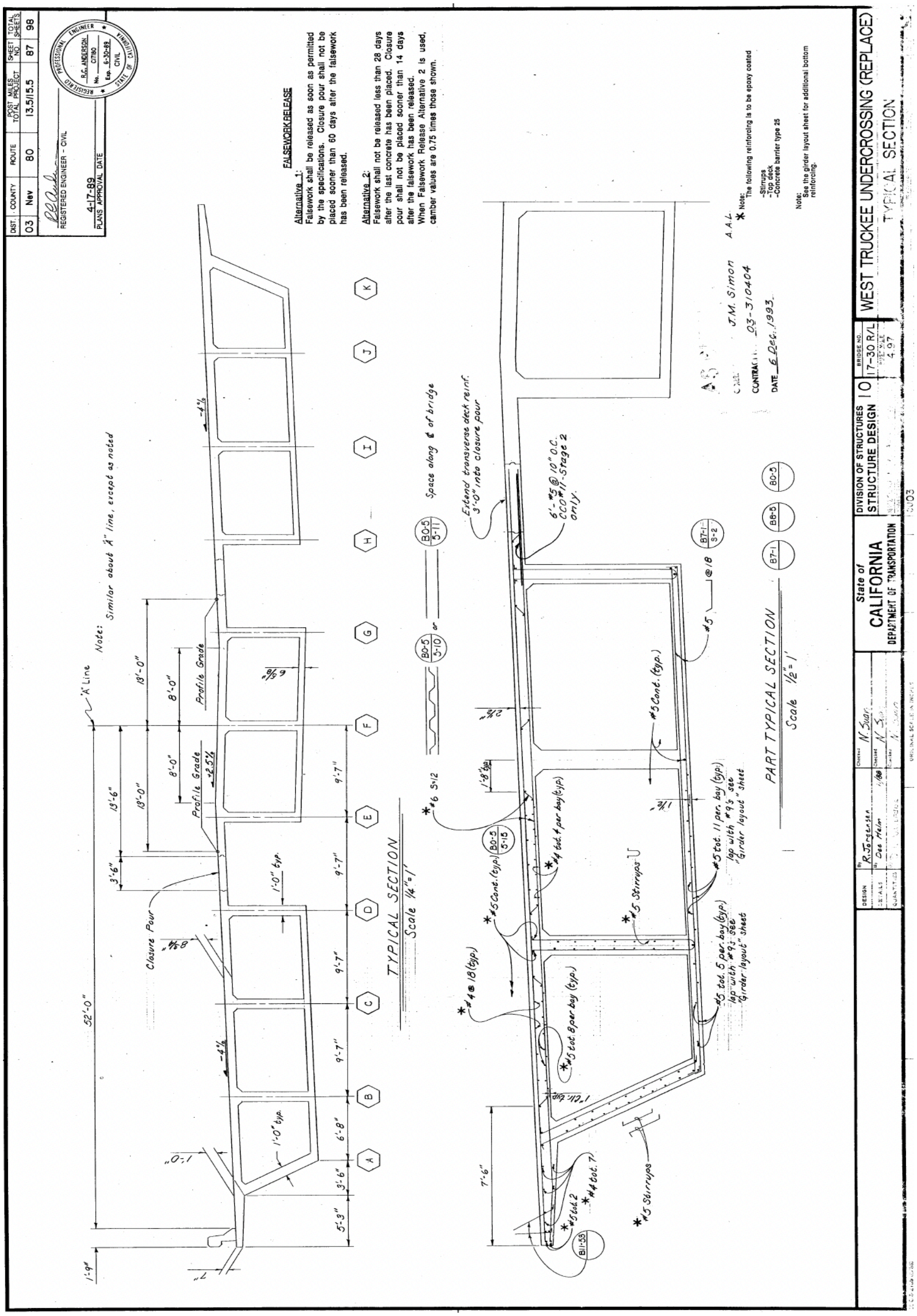

Figure A-5. Cross-section of bridge 17 0030.

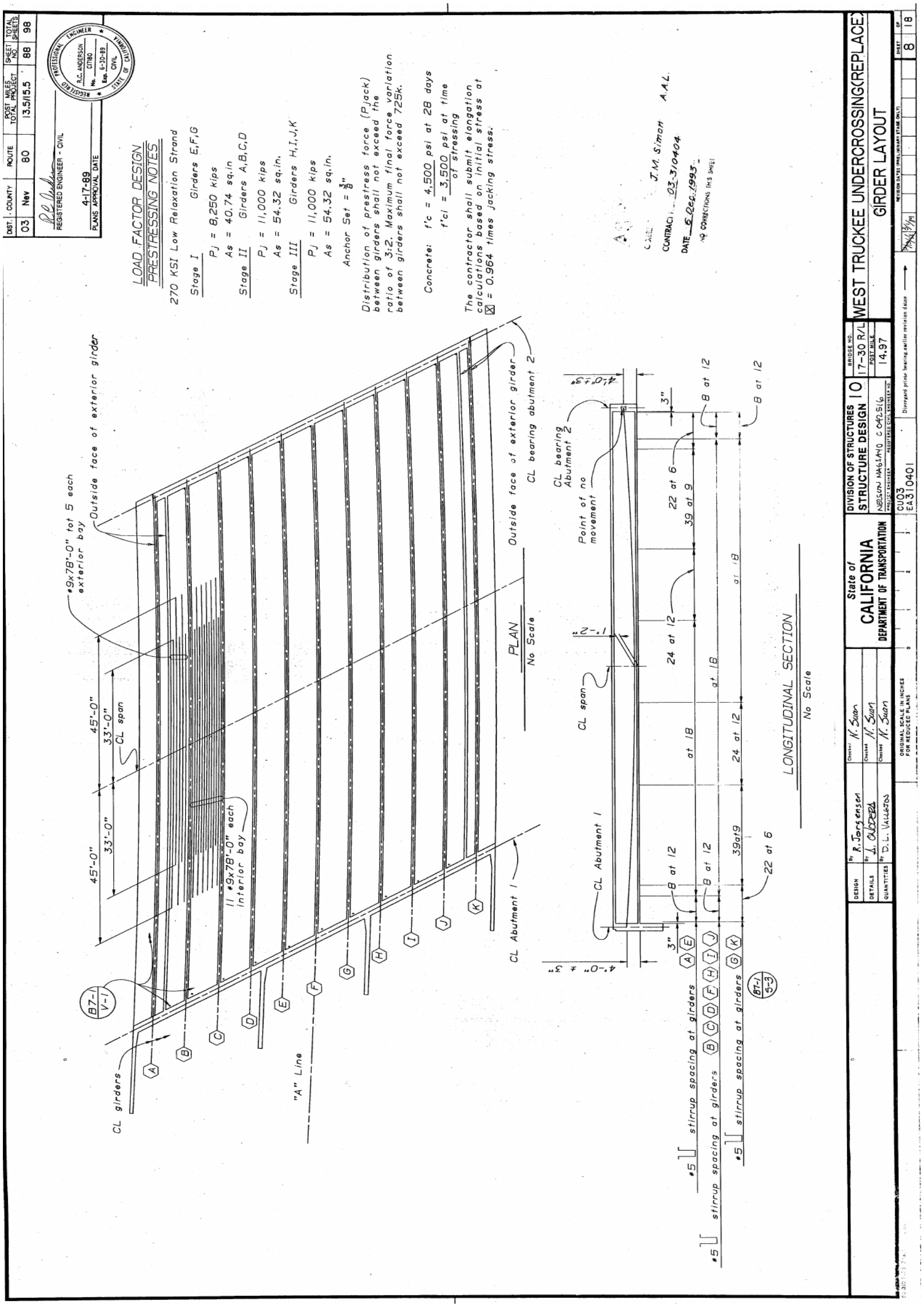

Figure A-6. Bridge 17 0030 details and prestressing notes

Dead Load Analysis – Interior Beam

Components and Attachments, *DC*

The total dead load applied on one box is

$$DL = 2.933\ kip/ft$$

The total moment due to dead load for one box is

$$M_{DC} = \mathbf{11,292.14}\ \boldsymbol{kip{\cdot}ft}$$

Wearing surface, DW

There is no wearing surface applied on the bridge. Therefore, the moment due to wearing surface is, M_{DW} = 0 kip·ft

Live Load Analysis – Interior Girder

Compute Live Load Distribution Factor for an Interior Girder

The girder distribution factors (GDF) for one design lane and two design lanes loaded are calculated according to AASHTO LRFD as follows.

- One design lane loaded:

$$\left(1.75+\frac{S}{3.6}\right)\left(\frac{1}{L}\right)^{0.35}\left(\frac{1}{N_c}\right)^{0.45} = \left(1.75+\frac{8}{3.6}\right)\left(\frac{1}{175.5}\right)^{0.35}\left(\frac{1}{8}\right)^{0.45} = 0.2836$$

- Two or more design lanes loaded:

$$\left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25} = \left(\frac{13}{8}\right)^{0.3}\left(\frac{8}{5.8}\right)\left(\frac{1}{175.5}\right)^{0.25} = 0.5251 \quad \leftarrow \ Governs$$

Nominal Flexural Resistance

The area of prestressing steel per box is $A_{ps} = 18.11\ in^2$

f_s and f'_s are assumed to equal 60 ksi and k is 0.28.

The neutral axis is located at,

$$c = \frac{A_{ps}f_{pu} + A_s f_s - A'_s f'_s}{\alpha f'_c \beta b + kA_{ps}\frac{f_{pu}}{d_p}} = \frac{18.11 * 270 + 8.37 * 60 - 5.1 * 60}{0.85 * 4.5 * 0.825 * 115 + 0.28 * 18.11 * \frac{270}{82}}$$

$$\boldsymbol{c = 13.396\ in} > t_s = 8.75\ in$$

$$f_{ps} = f_{pu}\left(1 - k * \frac{c}{d_p}\right) = 270\left(1 - 0.28 * \frac{18.11}{115}\right) = 257.65\ ksi$$

$$a = \beta * c = 0.825 * 13.396 = 11.05\ in$$

The nominal capacity of one box is,

$$M_n = \left[A_{ps} * f_{ps} * \left(d_p - \frac{a}{2}\right) + A_s * f_s * \left(d_s - \frac{a}{2}\right) - A'_s * f'_s * \left(d'_s - \frac{a}{2}\right)\right]$$

$$M_n = \left[18.11 * 257.65 * \left(75 - \frac{11.05}{2}\right) + 8.37 * 60 * \left(70.77 - \frac{11.05}{2}\right) - 5.1 * 60 * \left(5.71 - \frac{11.05}{2}\right)\right]$$

$$\boldsymbol{M_n = 32{,}456\ kip{\cdot}ft}$$

Reliability Analysis

Resistance

The mean resistance, variance, and mean of the logarithm resistance were expressed as follows

$$\mu_R = \lambda_R M_n = 1.05 * 32{,}456 = \boldsymbol{34{,}079\ \ kip.ft}$$

$$\sigma^2{}_{ln(R)} = ln(V^2{}_Y + 1) = ln(0.075^2 + 1) = \boldsymbol{0.00561}$$

$$\mu_{ln(R)} = ln(\mu_R) - \frac{1}{2}\sigma^2{}_{ln(R)} = ln(34{,}079) - \frac{1}{2} * 0.00561 = \boldsymbol{9.94\ \ kip{\cdot}ft}$$

Dead Load

The mean dead load and the standard deviation of the dead load are

$$\mu_D = \lambda_D D_n = 1.05 * 11{,}292.14 = \boldsymbol{11{,}857\ \ \ kip{\cdot}ft}$$

$$\sigma_D = COV_D D_n = 0.10 * 11{,}292.14 = \mathbf{1,129.2 \quad kip{\cdot}ft}$$

Live Load

The moment due to truck load is 2,879 kip·ft and the moment due to lane load is 2,464 kip·ft. The nominal live load (HL93) is

$$LL_{nominal} = truck + lane\ load = 2{,}879 + 2{,}464 = 5{,}343\ kip{\cdot}ft$$

The girder distribution factor for two lanes loaded governs. Therefore, the nominal live load per box with girder distribution factor for two lanes included (no dynamic load) is,

$$LL_{nominal/box} = (truck + lane\ load) * GDF = (2{,}879 + 2{,}464) * 0.5251$$

$$LL_{nominal/box} = 2{,}806\ kip{\cdot}ft$$

The mean live load with girder distribution factor for two lanes (no dynamic load) is

$$LL_{mean} = \lambda_L * LL_{nominal/box} = 1.10 * 2{,}806$$

$$\boldsymbol{LL_{mean} = 3,086\ kip{\cdot}ft}$$

The mean dynamic load represents 10% of the mean live load. The mean live load and dynamic load is

$$LL + IM_{mean} = LL_{mean} * (1 + 0.10) = 3{,}086 * (1 + 0.10)$$

$$\boldsymbol{LL + IM_{mean} = 3,395\ kip.ft}$$

The standard deviation of the live and dynamic load is

$$\sigma_{LL+IM} = COV_L * LL + IM_{mean} = 0.18 * 3{,}395 = \mathbf{611\ kip.ft}$$

Extreme type 1 parameters

California live load best fits an extreme type 1 distribution. The parameters α and u are,

$$\alpha = \frac{1.282}{\sigma_{LL+IM}} = \frac{1.282}{611} = \mathbf{0.002098}$$

$$u = LL + IM_{mean} - 0.45 * \sigma_{LL+IM} = 3{,}395 - 0.45 * 611 = \mathbf{3{,}120\ \ kip.ft}$$

Monte Carlo

Using the graphical method, the reliability index and probability of failure are

$$\beta = -z = \mathbf{6.54}$$

$$P_f = \Phi^{-1}(-z) = \mathbf{3.8 * 10^{-11}}$$

The normal standard variate is then plotted versus the limit state values as shown in Figure A-7.

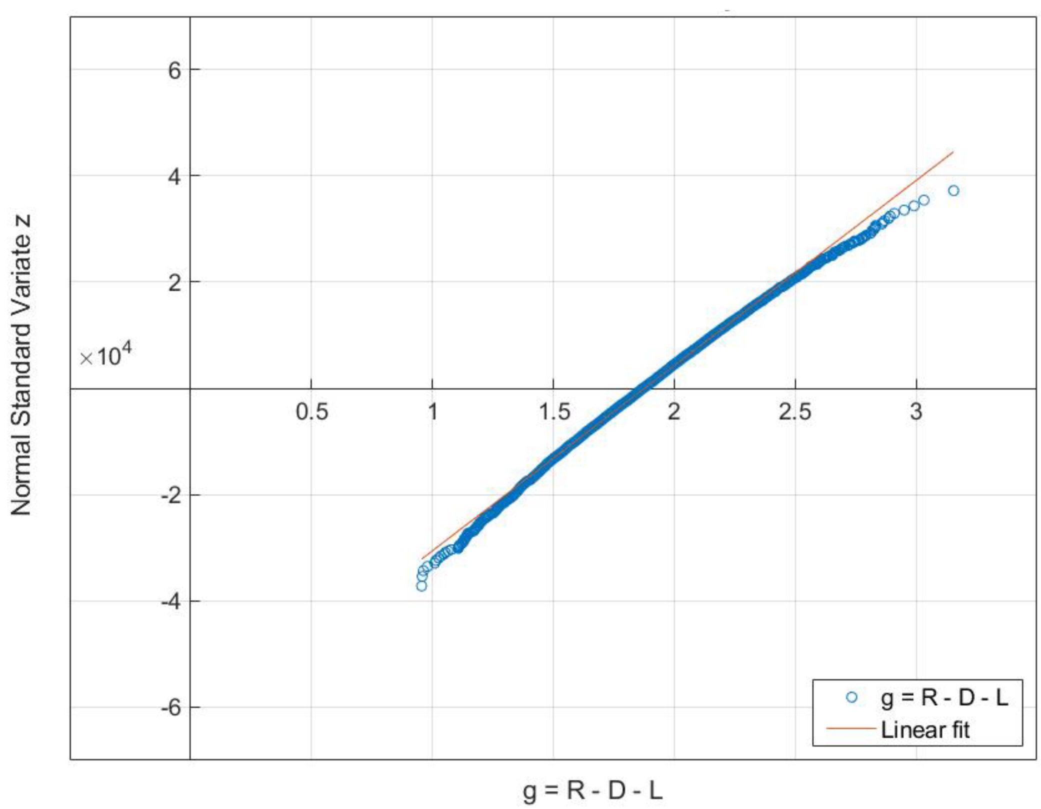

Figure A-7. Monte Carlo simulation for bridge 17 0030

Bridge 37 0368L – Simple span

Bridge Properties

General information about bridge 37 0368L is presented below.

Span length:		124.26 ft
Concrete:		f'_c: 4.0 ksi
Prestressing steel:		270 ksi low relaxation strand
$A_{ps,total}$	=	48.65 in^2
Girder spacing	=	9.54 ft
Depth of beam	=	6 ft
Top flange	=	8.125 in
Bottom flange	=	6.375 in
Web thickness	=	12 in
Area of one box	=	2,350.25 in^2
Number of cells	=	5
Number of webs	=	6

The cross-section of bridge 37 0368L and prestressing notes are shown in Figure A-8 and Figure A-9.

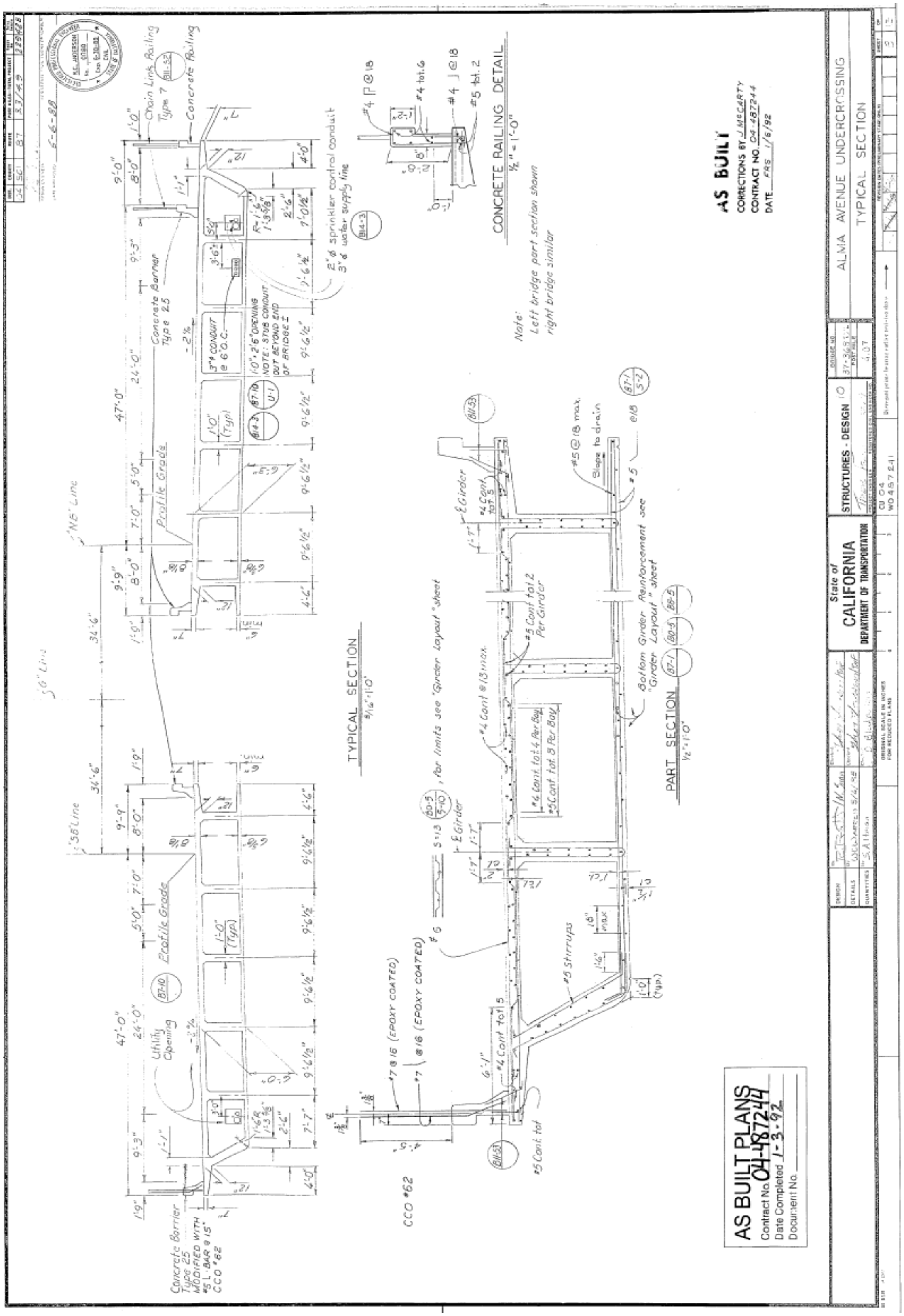

Figure A-8. Cross-section of bridge 37 0368L

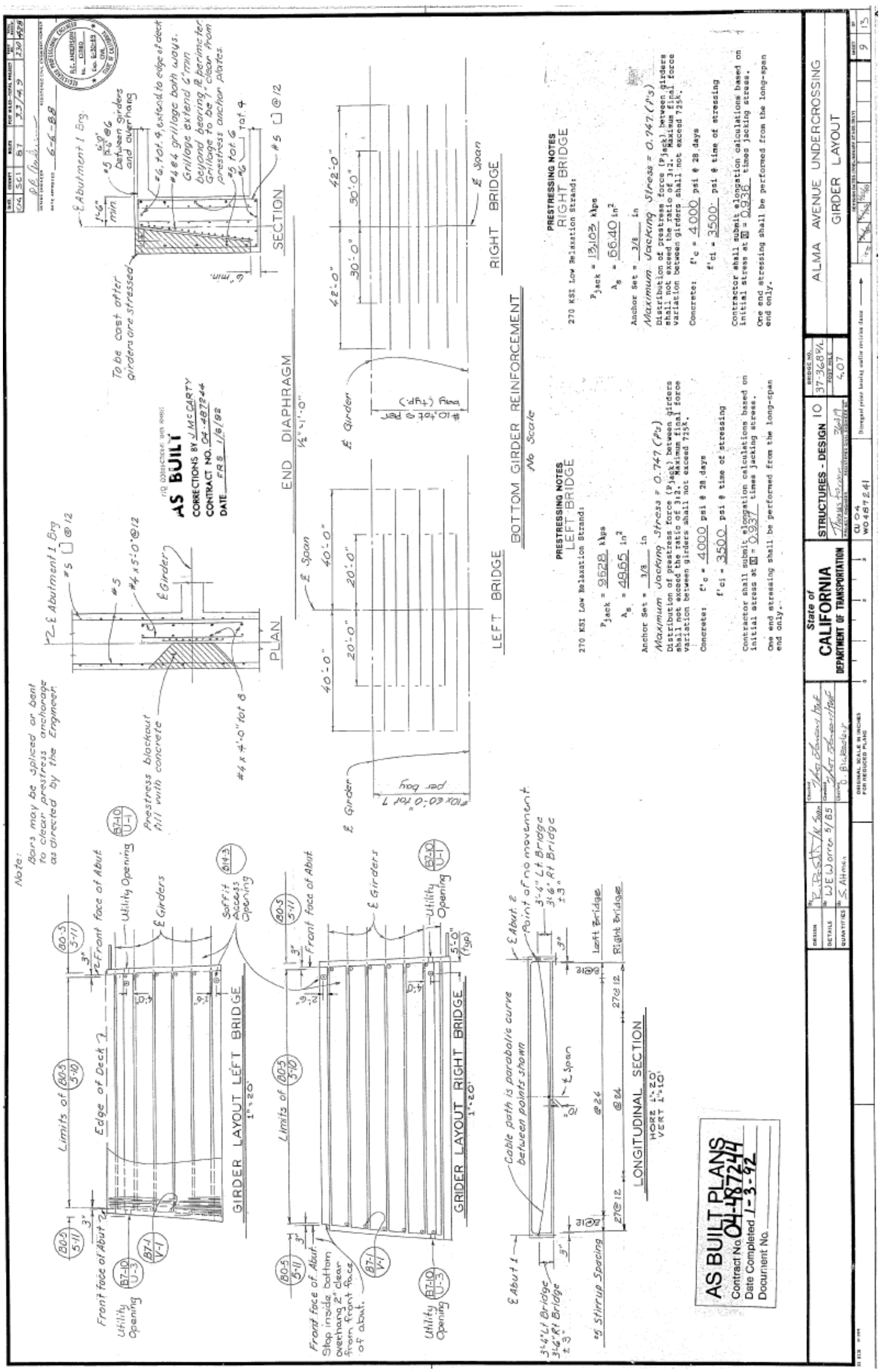

Figure A-9. Bridge 37 0368L details and prestressing notes

Dead Load Analysis – Interior Beam

Components and Attachments, *DC*

The total dead load applied on one box is

$$DL = 2.581\ kip/ft$$

The total moment due to dead load for one box is

$$M_{DC} = \mathbf{4,959.51}\ \boldsymbol{kip{\cdot}ft}$$

Wearing surface, DW

There is no wearing surface applied on the bridge. Therefore, the moment due to wearing surface is, M_{DW} = 0 kip-ft

Live Load Analysis – Interior Girder

Compute Live Load Distribution Factor for an Interior Girder

The girder distribution factors (GDF) for one design lane and two design lanes loaded are calculated according to AASHTO LRFD as follows.

- One design lane loaded:

$$\left(1.75+\frac{S}{3.6}\right)\left(\frac{1}{L}\right)^{0.35}\left(\frac{1}{N_c}\right)^{0.45}=\left(1.75+\frac{9.54}{3.6}\right)\left(\frac{1}{124.26}\right)^{0.35}\left(\frac{1}{5}\right)^{0.45} = 0.3944$$

- Two or more design lanes loaded:

$$\left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25}=\left(\frac{13}{5}\right)^{0.3}\left(\frac{9.54}{5.8}\right)\left(\frac{1}{124.26}\right)^{0.25} = 0.6563 \quad \leftarrow Governs$$

Nominal Flexural Resistance

The area of prestressing steel per box is $A_{ps} = 9.73\ in^2$

f_s and f'_s are assumed to equal 60 ksi and k is 0.28.

The neutral axis is located at,

$$c = \frac{A_{ps}f_{pu} + A_s f_s - A'_s f'_s}{\alpha f'_c \beta b + k A_{ps}\frac{f_{pu}}{d_p}} = \frac{9.73 * 270 + 9.08 * 60 - 5.1 * 60}{0.85 * 4.5 * 0.85 * 114.5 + 0.28 * 9.73 * \frac{270}{62}}$$

$$\boldsymbol{c = 8.361\ in} > t_s = 8.125\ in$$

$$f_{ps} = f_{pu}\left(1 - k * \frac{c}{d_p}\right) = 270\left(1 - 0.28 * \frac{9.73}{114.5}\right) = 259.8\ ksi$$

$$a = \beta * c = 0.85 * 8.361 = 7.11\ in$$

The nominal capacity of one box is,

$$M_n = \left[A_{ps} * f_{ps} * \left(d_p - \frac{a}{2}\right) + A_s * f_s * \left(d_s - \frac{a}{2}\right) - A'_s * f'_s * \left(d'_s - \frac{a}{2}\right)\right]$$

$$M_n = \left[9.73 * 259.8 * \left(62 - \frac{7.11}{2}\right) + 9.08 * 60 * \left(54.41 - \frac{7.11}{2}\right) - 5.1 * 60 * \left(4.21 - \frac{7.11}{2}\right)\right]$$

$$\boldsymbol{M_n = 14,604\ kip - ft}$$

Reliability Analysis

Resistance

The mean resistance, variance, and mean of the logarithm resistance were expressed as follows

$$\mu_R = \lambda_R M_n = 1.05 * 14{,}604 = \boldsymbol{15,334\ \ kip.ft}$$

$$\sigma^2{}_{ln(R)} = ln(V^2{}_Y + 1) = ln(0.075^2 + 1) = \boldsymbol{0.00561}$$

$$\mu_{ln(R)} = ln(\mu_R) - \frac{1}{2}\sigma^2{}_{ln(R)} = ln(15{,}334) - \frac{1}{2} * 0.00561 = \boldsymbol{9.63\ \ kip.ft}$$

Dead Load

The mean dead load and the standard deviation of the dead load are

$$\mu_D = \lambda_D D_n = 1.05 * 4{,}960 = \boldsymbol{5,208\ \ \ kip.ft}$$

$$\sigma_D = COV_D D_n = 0.10 * 4{,}960 = \mathbf{496} \quad \boldsymbol{kip.ft}$$

Live Load

The moment due to truck load is 1,952 kip-ft and the moment due to lane load is 1,230 kip-ft. The nominal live load (HL93) is

$$LL_{nominal} = truck + lane\ load = 1{,}952 + 1{,}230 = 3{,}182\ kip.ft$$

The girder distribution factor for two lanes loaded governs. Therefore, the nominal live load per box with girder distribution factor for two lanes included (no dynamic load) is,

$$LL_{nominal/box} = (truck + lane\ load) * GDF = (1{,}952 + 1{,}230) * 0.6563$$

$$LL_{nominal/box} = 2{,}088\ kip.ft$$

The mean live load with girder distribution factor for two lanes (no dynamic load) is expressed as,

$$LL_{mean} = \lambda_L * LL_{nominal/box} = 1.10 * 2{,}088$$

$$\boldsymbol{LL_{mean} = 2,297\ kip.ft}$$

The mean dynamic load represents 10% of the mean live load. The mean live load and dynamic load is

$$LL + IM_{mean} = LL_{mean} * (1 + 0.10) = 2{,}297 * (1 + 0.10)$$

$$\boldsymbol{LL + IM_{mean} = 2,527\ kip.ft}$$

The standard deviation of the live and dynamic load is

$$\sigma_{LL+IM} = COV_L * LL + IM_{mean} = 0.18 * 2{,}527 = \mathbf{252.7}\ \boldsymbol{kip.ft}$$

Extreme type 1 parameters

California live load best fits an extreme type 1 distribution. The parameters α and u are,

$$\alpha = \frac{1.282}{\sigma_{LL+IM}} = \frac{1.282}{252.7} = \mathbf{0.00507}$$

$$u = LL + IM_{mean} - 0.45 * \sigma_{LL+IM} = 2{,}527 - 0.45 * 252.7 = \mathbf{2{,}413\ \ \mathit{kip.ft}}$$

Monte Carlo

Using the graphical method, the reliability index and probability of failure are

$$\beta = -z = \mathbf{5.69}$$

$$P_f = \Phi^{-1}(-z) = \mathbf{6.35 * 10^{-9}}$$

The normal standard variate is then plotted versus the limit state values as shown in Figure A-10.

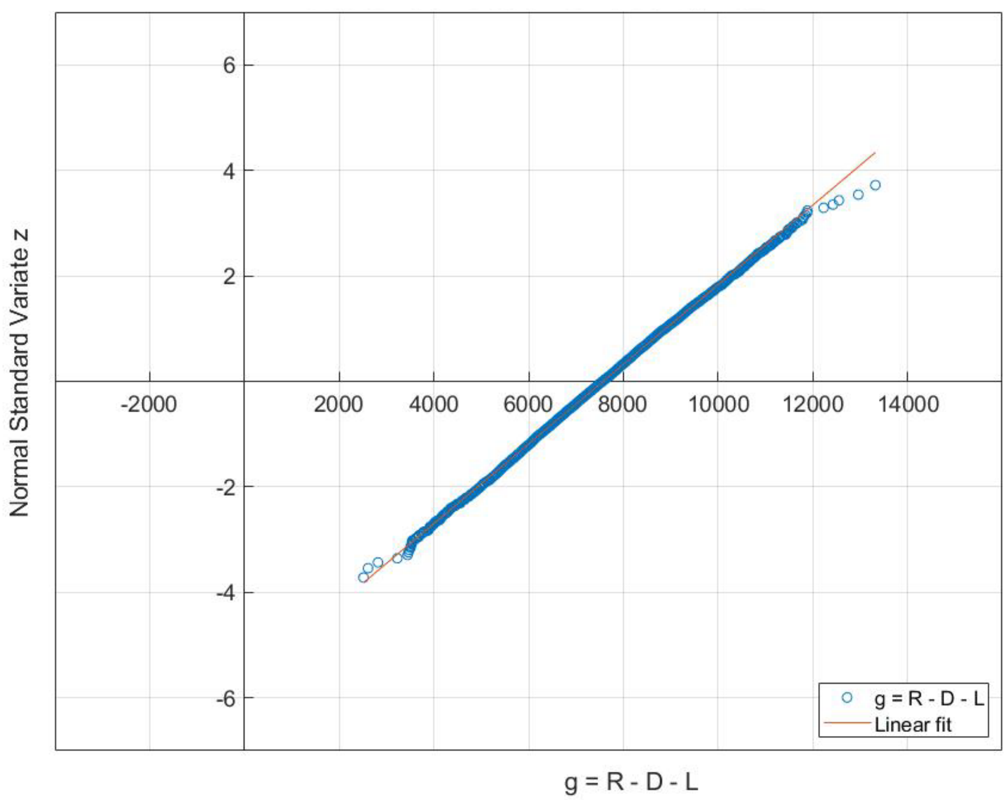

Figure A-10. Monte Carlo simulation for bridge 37 0368L

Bridge 55 0655 – Simple span

Bridge Properties

General information about bridge 55 0655 is presented below.

Span length:		94.58 ft
Concrete:		f'_c: 4.0 ksi
Prestressing steel:		270 ksi low relaxation strand
$A_{ps,total}$	=	41.82 in^2
Girder spacing	=	10 ft
Depth of beam	=	4.5 ft
Top flange	=	8.375 in
Bottom flange	=	6.75 in
Web thickness	=	12 in
Area of one box	=	2,281.5 in^2
Number of cells	=	26
Number of webs	=	27

The cross-section of bridge 55 0655 and prestressing notes are shown in Figure A-11 and Figure A-12.

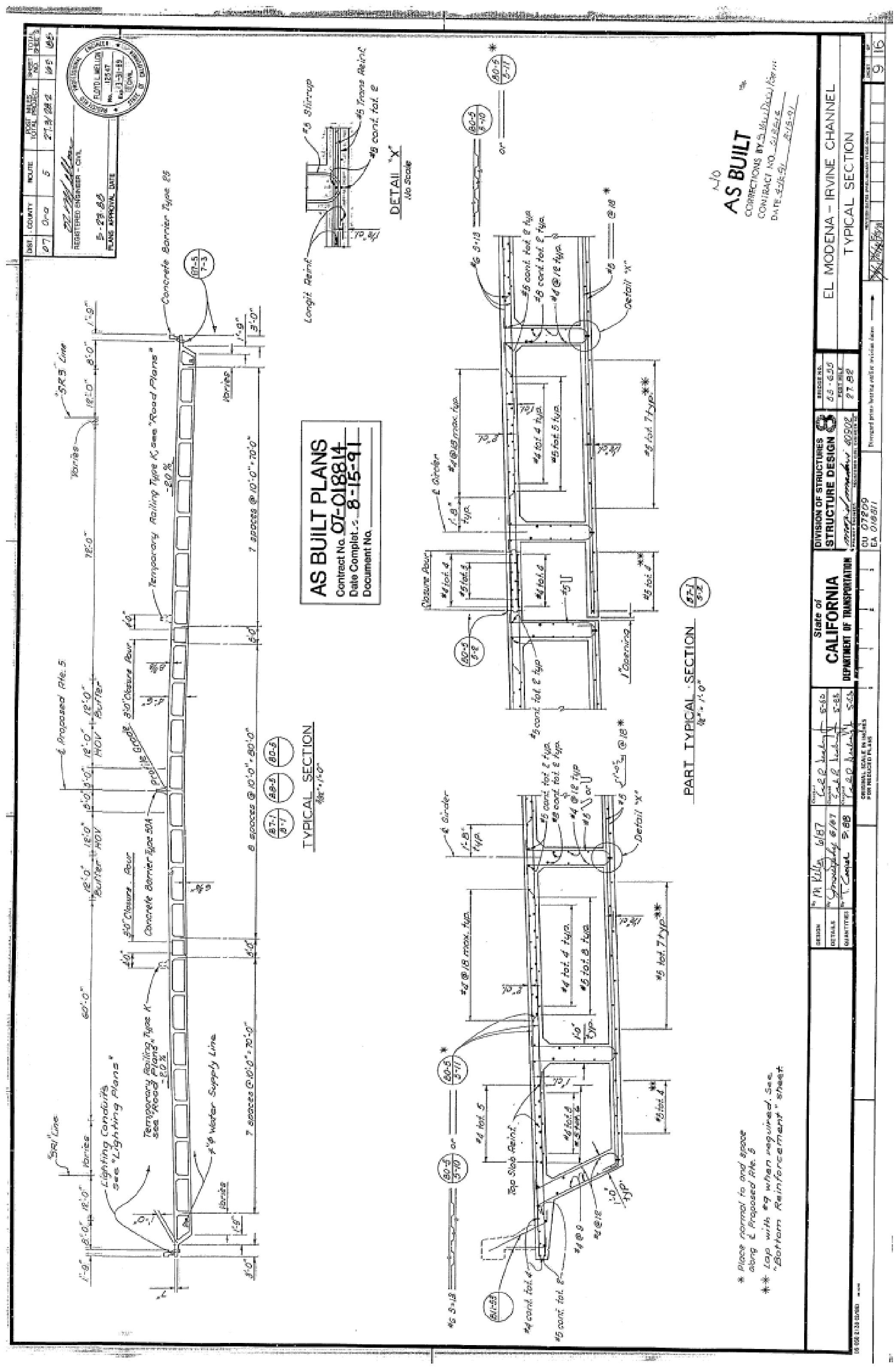

Figure A-11. Cross-section of bridge 55 0655

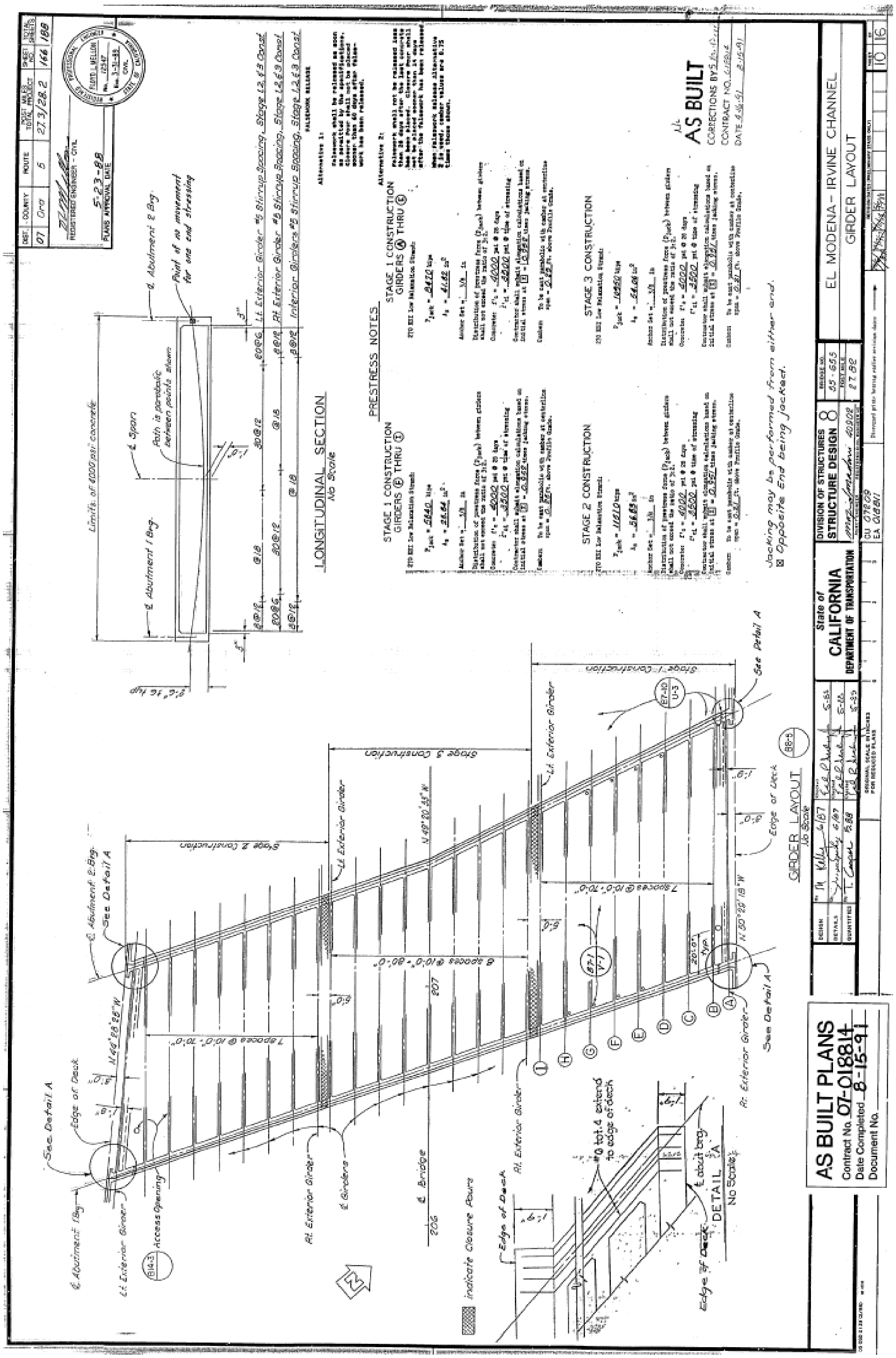

Figure A-12. Bridge 55 0655 details and prestressing notes

Dead Load Analysis – Interior Beam

Components and Attachments, *DC*

The total dead load applied on one box girder is

$$DL = 2.402\ kip/ft$$

The total moment due to dead load for one box girder is

$$M_{DC} = \mathbf{2,710.88}\ \boldsymbol{kip \cdot ft}$$

Wearing surface, DW

There is no wearing surface applied on the bridge. Therefore, the moment due to wearing surface is, M_{DW} = 0 kip-ft

Live Load Analysis – Interior Girder

Compute Live Load Distribution Factor for an Interior Girder

The girder distribution factors (GDF) for one design lane and two design lanes loaded are calculated according to AASHTO LRFD as follows.

- One design lane loaded:

$$\left(1.75 + \frac{S}{3.6}\right)\left(\frac{1}{L}\right)^{0.35}\left(\frac{1}{N_c}\right)^{0.45} = \left(1.75 + \frac{10}{3.6}\right)\left(\frac{1}{94.58}\right)^{0.35}\left(\frac{1}{8}\right)^{0.45} = 0.3614$$

- Two or more design lanes loaded:

$$\left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25} = \left(\frac{13}{8}\right)^{0.3}\left(\frac{10}{5.8}\right)\left(\frac{1}{94.58}\right)^{0.25} = 0.6396 \quad \leftarrow \ Governs$$

Nominal Flexural Resistance

The area of prestressing steel per box is $A_{ps} = 10.46\ in^2$

f_s and *f'_s* are assumed to equal 60 ksi and *k* is 0.28.

The neutral axis is located at,

$$c = \frac{A_{ps}f_{pu} + A_s f_s - A'_s f'_s}{\alpha f'_c \beta b + kA_{ps}\dfrac{f_{pu}}{d_p}} = \frac{10.46 * 270 + 5.17 * 60 - 5.1 * 60}{0.85 * 4.0 * 0.85 * 120 + 0.28 * 10.46 * \dfrac{270}{42}}$$

$$\boldsymbol{c = 7.732\ in} < t_s = 8.375\ in$$

$$f_{ps} = f_{pu}\left(1 - k * \frac{c}{d_p}\right) = 270\left(1 - 0.28 * \frac{10.46}{120}\right) = 256.08\ ksi$$

$$a = \beta * c = 0.825 * 7.732 = 6.57\ in$$

The nominal capacity of one box is,

$$M_n = \left[A_{ps} * f_{ps} * \left(d_p - \frac{a}{2}\right) + A_s * f_s * \left(d_s - \frac{a}{2}\right) - A'_s * f'_s * \left(d'_s - \frac{a}{2}\right)\right]$$

$$M_n = \left[10.46 * 256.08 * \left(42 - \frac{6.57}{2}\right) + 5.17 * 60 * \left(41.49 - \frac{6.57}{2}\right) - 5.1 * 60 * \left(3.54 - \frac{6.57}{2}\right)\right]$$

$$\boldsymbol{M_n = 9,619\ kip - ft}$$

Reliability Analysis

Resistance

The mean resistance, variance, and mean of the logarithm resistance were expressed as follows

$$\mu_R = \lambda_R M_n = 1.05 * 9{,}619 = \boldsymbol{10,100\ \ kip.ft}$$

$$\sigma^2{}_{ln(R)} = ln(V^2{}_Y + 1) = ln(0.075^2 + 1) = \boldsymbol{0.00561}$$

$$\mu_{ln(R)} = ln(\mu_R) - \frac{1}{2}\sigma^2{}_{ln(R)} = ln(10{,}100) - \frac{1}{2} * 0.00561 = \boldsymbol{9.22\ \ kip.ft}$$

Dead Load

The mean dead load and the standard deviation of the dead load are

$$\mu_D = \lambda_D D_n = 1.05 * 2{,}711 = \boldsymbol{2,846\quad kip.ft}$$

$$\sigma_D = COV_D D_n = 0.10 * 2{,}711 = \mathbf{271.1} \quad \boldsymbol{kip.ft}$$

Live Load

The moment due to truck load is 1,430 kip-ft and the moment due to lane load is 722 kip-ft. The nominal live load (HL93) is

$$LL_{nominal} = truck + lane\ load = 1{,}430 + 722 = 2{,}152\ kip.ft$$

The girder distribution factor for two lanes loaded governs. Therefore, the nominal live load per box with girder distribution factor for two lanes included (no dynamic load) is,

$$LL_{nominal/box} = (truck + lane\ load) * GDF = (1{,}430 + 722) * 0.6396$$

$$LL_{nominal/box} = 1{,}376\ kip.ft$$

The mean live load with girder distribution factor for two lanes (no dynamic load) is

$$LL_{mean} = \lambda_L * LL_{nominal/box} = 1.10 * 1{,}376$$

$$\boldsymbol{LL_{mean} = 1,514\ kip.ft}$$

The mean dynamic load represents 10% of the mean live load. The mean live load and dynamic load is

$$LL + IM_{mean} = LL_{mean} * (1 + 0.10) = 1{,}514 * (1 + 0.10)$$

$$\boldsymbol{LL + IM_{mean} = 1,665\ kip.ft}$$

The standard deviation of the live and dynamic load is

$$\sigma_{LL+IM} = COV_L * LL + IM_{mean} = 0.18 * 1{,}665 = \mathbf{299.7}\ \boldsymbol{kip.ft}$$

Extreme type 1 parameters

California live load best fits an extreme type 1 distribution. The parameters α and u are,

$$\alpha = \frac{1.282}{\sigma_{LL+IM}} = \frac{1.282}{299.7} = \mathbf{0.00428}$$

$$u = LL + IM_{mean} - 0.45 * \sigma_{LL+IM} = 1{,}665 - 0.45 * 299.7 = \mathbf{1{,}530\ \ kip.ft}$$

Monte Carlo

Using the graphical method, the reliability index and probability of failure are

$$\beta = -z = \mathbf{6.52}$$

$$P_f = \Phi^{-1}(-z) = \mathbf{3.52 * 10^{-11}}$$

The normal standard variate is then plotted versus the limit state values as shown in Figure A-13.

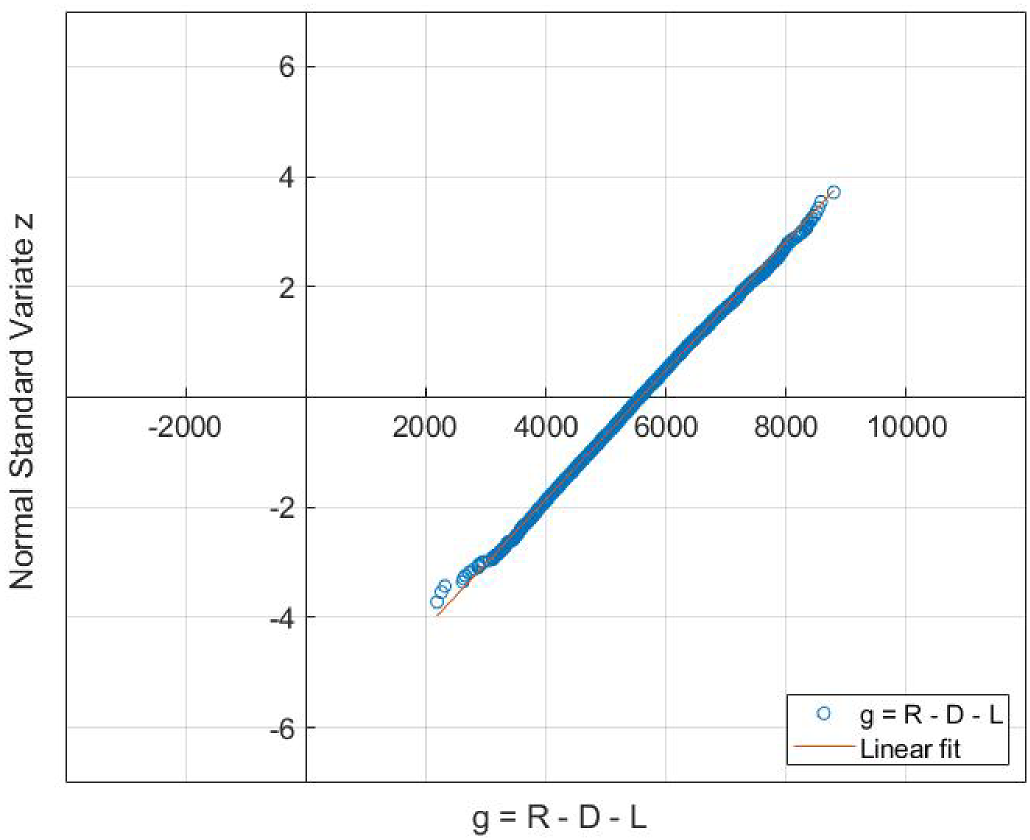

Figure A-13. Monte Carlo simulation for bridge 55 0655

Bridge 23 0020 – Continuous span

Bridge Properties

General information about bridge 23 0020 is presented below.

Span length:		111.8 ft
Concrete:		f'_c: 4.0 ksi
Prestressing steel:		270 ksi low relaxation strand
$A_{ps,total}$	=	34.07 in^2
Girder spacing	=	7.25 ft
Depth of beam	=	4.42 ft
Top flange	=	7.375 in
Bottom flange	=	5.5 in
Web thickness	=	12 in
Area of one box	=	1,601.63 in^2
Number of cells	=	5
Number of webs	=	6

The cross-section of bridge 23 0020 and prestressing notes are shown in Figure A-14.

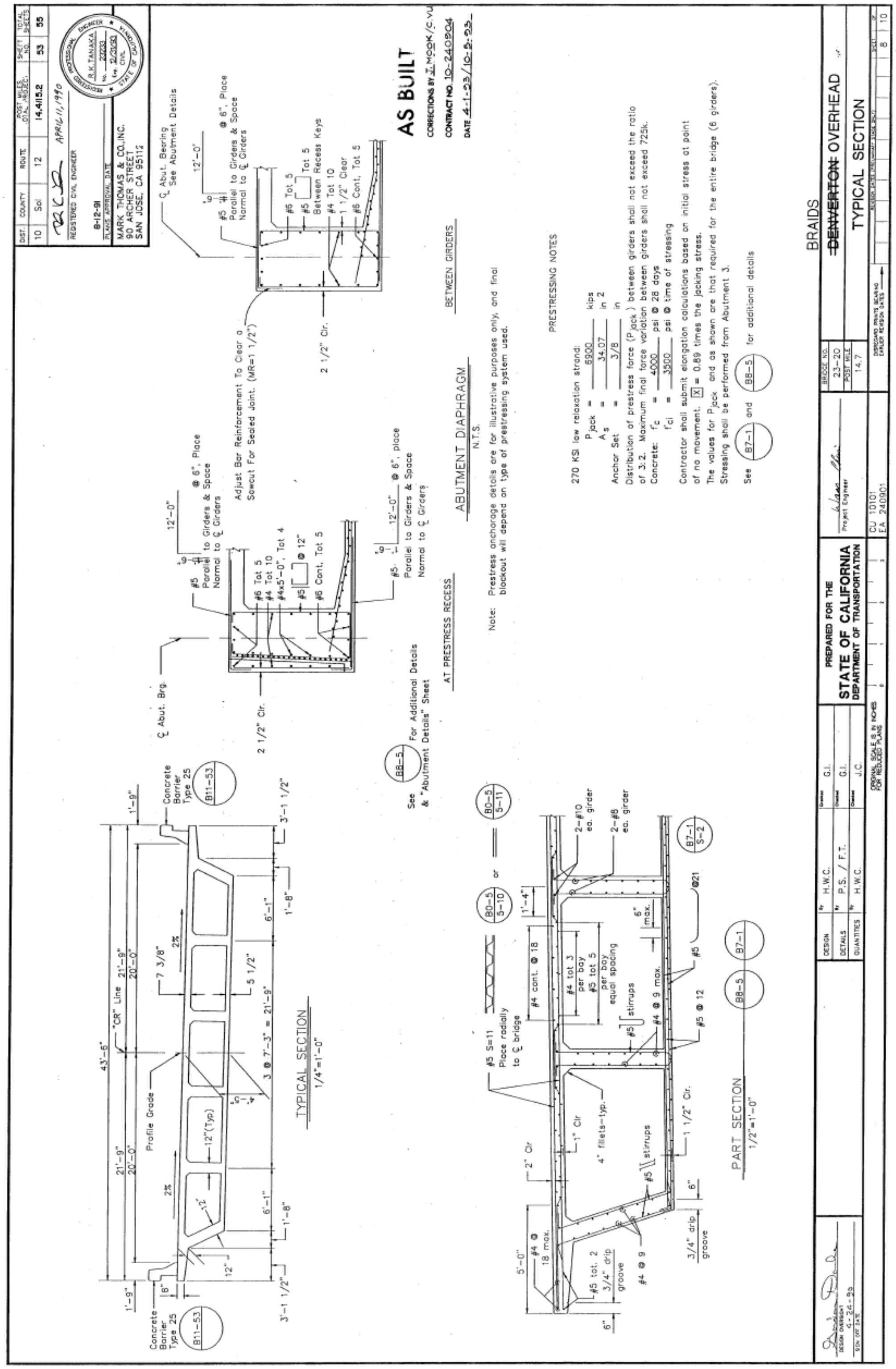

Figure A-14. Cross-section and details of bridge 23 0020

Dead Load Analysis – Interior Beam

Components and Attachments, *DC*

The total dead load applied on one box is

$$DL = 1.801\ kip/ft$$

The total moment due to dead load for one box is

$$M_{DC} = \mathbf{2,236.35\ kip - ft}$$

Wearing surface, DW

There is no wearing surface applied on the bridge. Therefore, the moment due to wearing surface is, M_{DW} = 0 kip-ft

Live Load Analysis – Interior Girder

Compute Live Load Distribution Factor for an Interior Girder

The girder distribution factors (GDF) for one design lane and two design lanes loaded are calculated according to AASHTO LRFD as follows.

- One design lane loaded:

$$\left(1.75 + \frac{S}{3.6}\right)\left(\frac{1}{L}\right)^{0.35}\left(\frac{1}{N_c}\right)^{0.45} = \left(1.75 + \frac{7.25}{3.6}\right)\left(\frac{1}{111.8}\right)^{0.35}\left(\frac{1}{5}\right)^{0.45} = 0.3501$$

- Two or more design lanes loaded:

$$\left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25} = \left(\frac{13}{5}\right)^{0.3}\left(\frac{7.25}{5.8}\right)\left(\frac{1}{111.8}\right)^{0.25} = 0.5120 \quad \leftarrow Governs$$

Nominal Flexural Resistance

The area of prestressing steel per box is $A_{ps} = 6.81\ in^2$

f_s and f'_s are assumed to equal 60 ksi and k is 0.28.

The neutral axis is located at,

$$c = \frac{A_{ps}f_{pu} + A_s f_s - A'_s f'_s}{\alpha f'_c \beta b + k A_{ps}\frac{f_{pu}}{d_p}} = \frac{6.81 * 270 + 8.85 * 60 - 5.89 * 60}{0.85 * 4.0 * 0.85 * 87 + 0.28 * 6.81 * \frac{270}{38}}$$

$$\boldsymbol{c = 7.613\ in} > t_s = 8.375\ in$$

$$f_{ps} = f_{pu}\left(1 - k * \frac{c}{d_p}\right) = 270\left(1 - 0.28 * \frac{6.81}{38}\right) = 254.85\ ksi$$

$$a = \beta * c = 0.85 * 7.613 = 6.47\ in$$

The nominal capacity of one box is,

$$M_n = \left[A_{ps} * f_{ps} * \left(d_p - \frac{a}{2}\right) + A_s * f_s * \left(d_s - \frac{a}{2}\right) - A'_s * f'_s * \left(d'_s - \frac{a}{2}\right)\right]$$

$$M_n = \left[6.81 * 254.85 * \left(38 - \frac{6.47}{2}\right) + 8.85 * 60 * \left(42.16 - \frac{6.47}{2}\right) - 5.89 * 60 * \left(2.48 - \frac{6.47}{2}\right)\right]$$

$$\boldsymbol{M_n = 6,753\ kip{\cdot}ft}$$

Reliability Analysis

Resistance

The mean resistance, variance, and mean of the logarithm resistance were expressed as follows

$$\mu_R = \lambda_R M_n = 1.05 * 6{,}753 = \boldsymbol{7,091\ \ kip{\cdot}ft}$$

$$\sigma^2{}_{ln(R)} = ln(V^2{}_Y + 1) = ln(0.075^2 + 1) = \boldsymbol{0.00561}$$

$$\mu_{ln(R)} = ln(\mu_R) - \frac{1}{2}\sigma^2{}_{ln(R)} = ln(7{,}091) - \frac{1}{2} * 0.00561 = \boldsymbol{8.86\ \ kip{\cdot}ft}$$

Dead Load

The mean dead load and the standard deviation of the dead load are

$$\mu_D = \lambda_D D_n = 1.05 * 2{,}236 = \boldsymbol{2,348\ \ \ kip{\cdot}ft}$$

$$\sigma_D = COV_D D_n = 0.10 * 2{,}236 = \mathbf{223.6 \quad kip{\cdot}ft}$$

Live Load

The moment due to truck load is 1,362 kip-ft and the moment due to lane load is 795 kip-ft. The nominal live load (HL93) is

$$LL_{nominal} = truck + lane\ load = 1{,}362 + 795 = 2{,}157\ kip{\cdot}ft$$

The girder distribution factor for two lanes loaded governs. Therefore, the nominal live load per box with girder distribution factor for two lanes included (no dynamic load) is,

$$LL_{nominal/box} = (truck + lane\ load) * GDF = (1{,}362 + 795) * 0.5120$$

$$LL_{nominal/box} = 1{,}104\ kip.ft$$

The mean live load with girder distribution factor for two lanes (no dynamic load) is

$$LL_{mean} = \lambda_L * LL_{\frac{nominal}{box}} == \ 1.10 * 1{,}104$$

$$\mathbf{LL_{mean} = 1{,}215\ kip.ft}$$

The mean dynamic load represents 10% of the mean live load. The mean live load and dynamic load is

$$LL + IM_{mean} = LL_{mean} * (1 + 0.10) = 1{,}215 * (1 + 0.10)$$

$$\mathbf{LL + IM_{mean} = 1{,}336\ kip.ft}$$

The standard deviation of the live and dynamic load is

$$\sigma_{LL+IM} = COV_L * LL + IM_{mean} = 0.18 * 1{,}336 = \mathbf{240.5\ kip.ft}$$

Extreme type 1 parameters

California live load best fits an extreme type 1 distribution. The parameters α and u are,

$$\alpha = \frac{1.282}{\sigma_{LL+IM}} = \frac{1.282}{240.5} = \mathbf{0.00533}$$

$$u = LL + IM_{mean} - 0.45 * \sigma_{LL+IM} = 1{,}336 - 0.45 * 240.5 = \mathbf{1{,}227.8\ \ kip.ft}$$

Monte Carlo

Using the graphical method, the reliability index and probability of failure are

$$\beta = -z = \mathbf{5.36}$$

$$P_f = \Phi^{-1}(-z) = \mathbf{4.16 * 10^{-8}}$$

The normal standard variate is then plotted versus the limit state values as shown in Figure A-15.

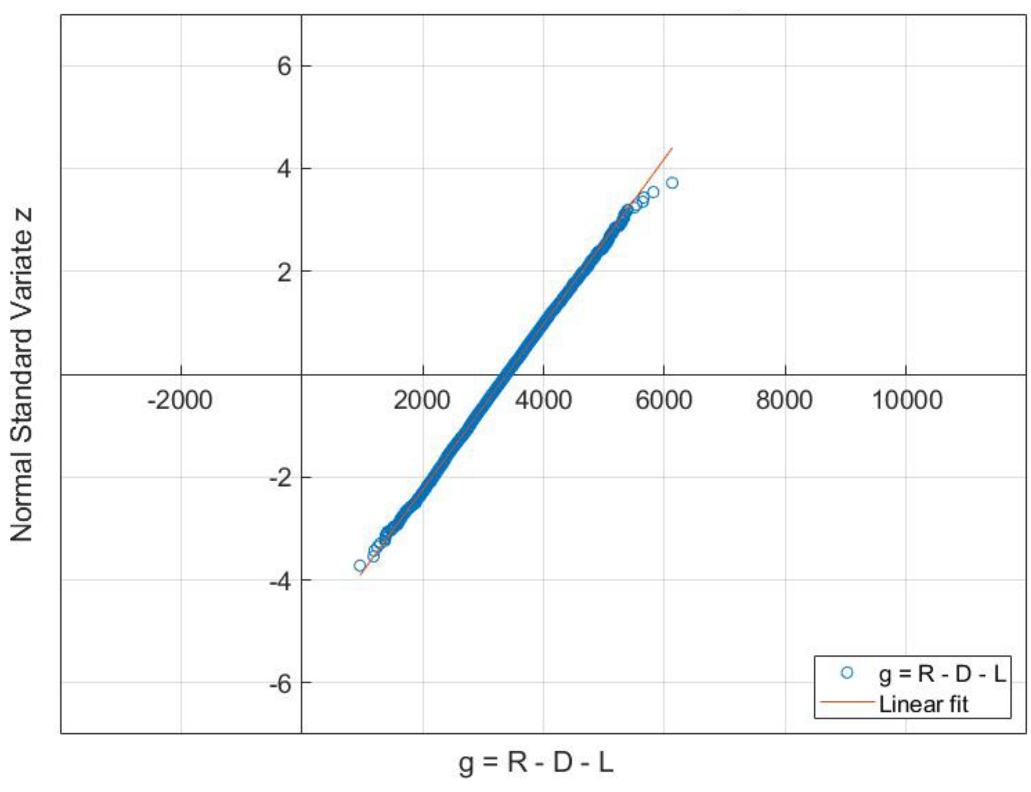

Figure A-15. Monte Carlo simulation for bridge 23 0020

Bridge 23 0205L – Continuous span

Bridge Properties

General information about bridge 23 0205L is presented below.

Span length:		126.1 ft
Concrete:		f'_c: 4.0 ksi
Prestressing steel:		270 ksi low relaxation strand
$A_{ps,total}$	=	25.43 in^2
Girder spacing	=	7 ft
Depth of beam	=	5 ft
Top flange	=	7.25 in
Bottom flange	=	5.5 in
Web thickness	=	12 in
Area of one box	=	1,638 in^2
Number of cells	=	5
Number of webs	=	6

The cross-section of bridge 23 0205L and prestressing notes are shown in Figure A-16 and Figure A-17.

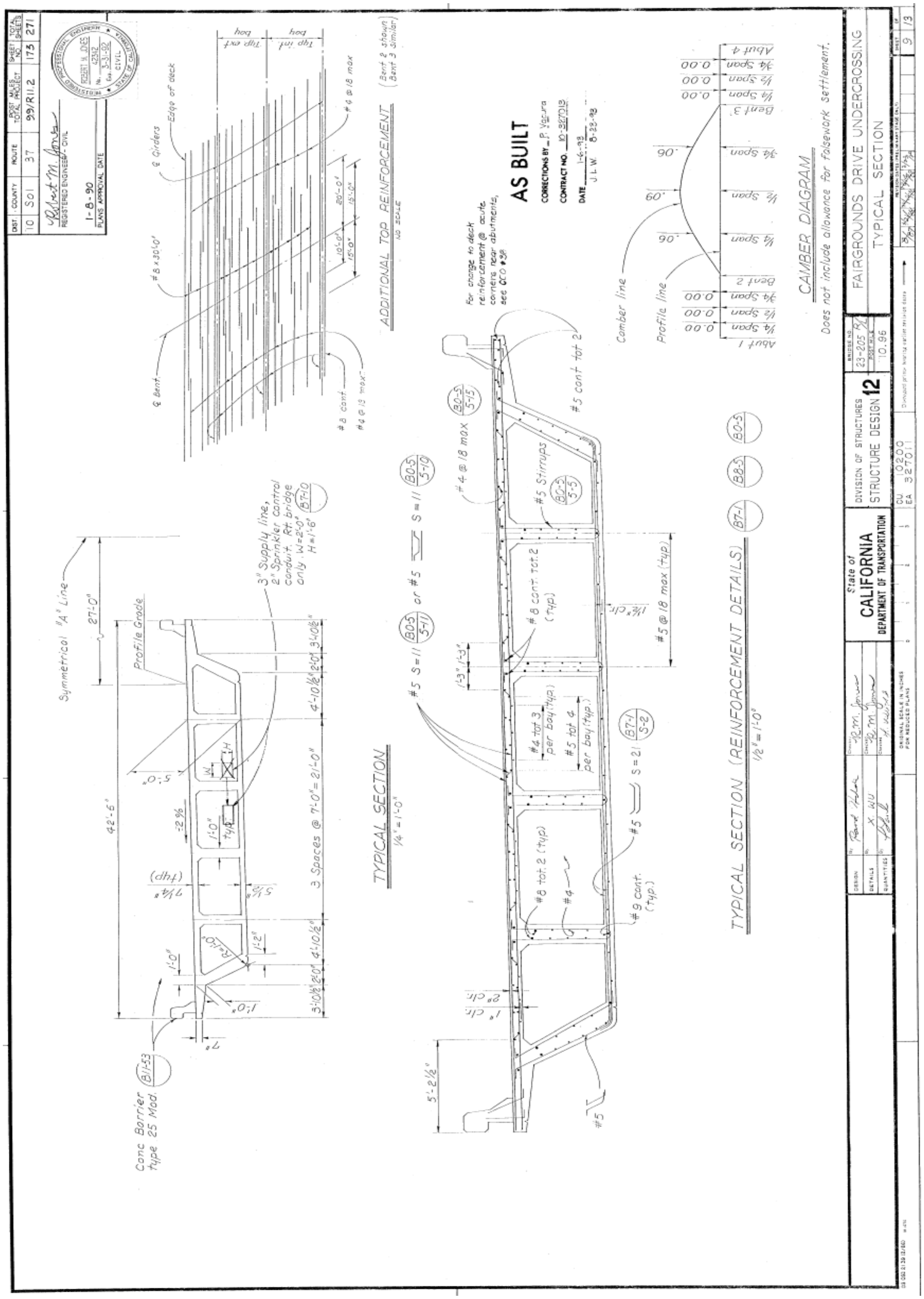

Figure A-16. Cross-section of bridge 23 0205L

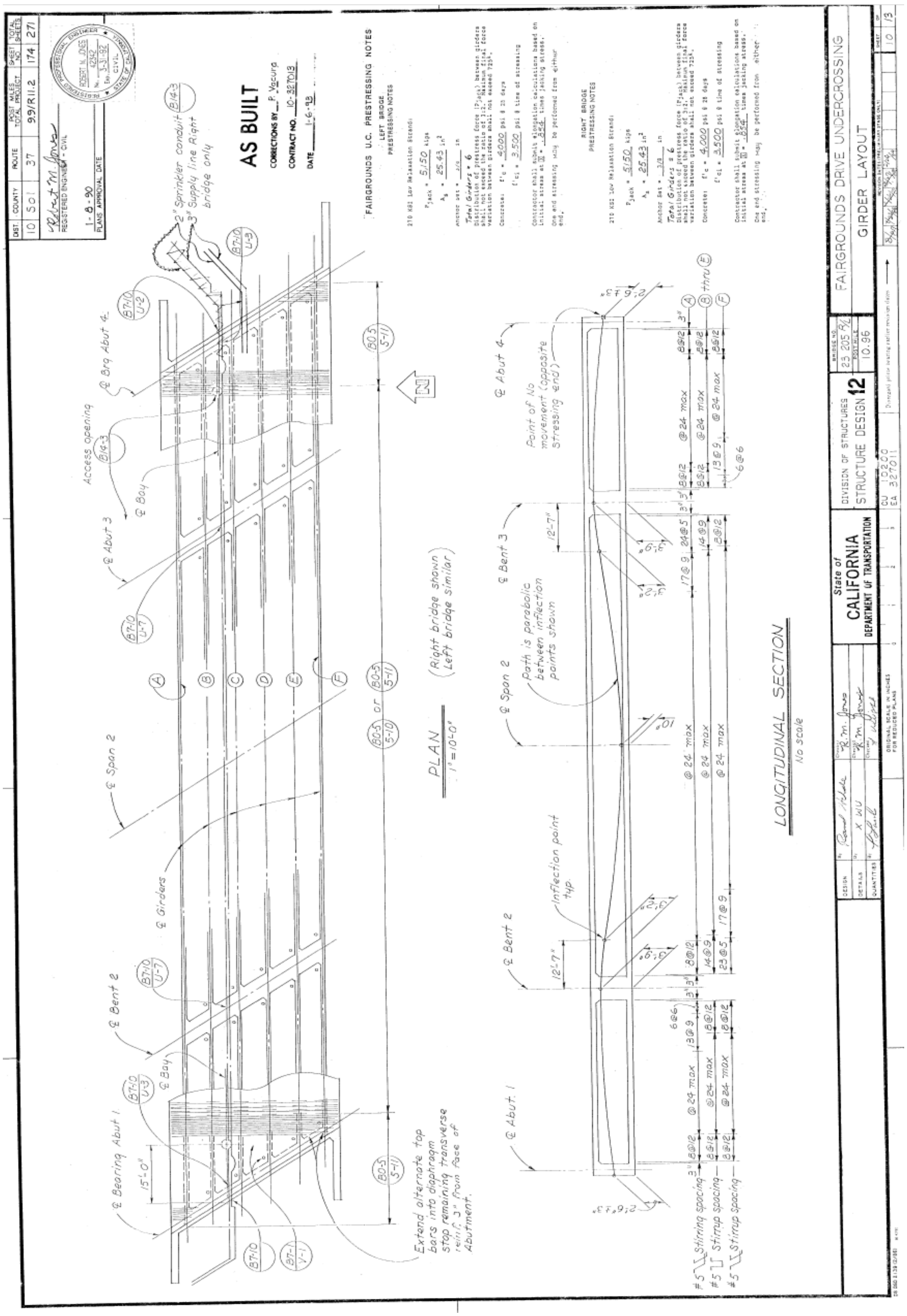

Figure A-17. Bridge 23 0205L details and prestressing notes

Dead Load Analysis – Interior Beam

Components and Attachments, *DC*

The total dead load applied on one box girder is

$$DL = 1.839\ kip/ft$$

The total moment due to dead load for one box girder is

$$M_{DC} = \mathbf{2,043.74\ kip - ft}$$

Wearing surface, DW

There is no wearing surface applied on the bridge. Therefore, the moment due to wearing surface is, M_{DW} = 0 kip-ft

Live Load Analysis – Interior Girder

Compute Live Load Distribution Factor for an Interior Girder

The girder distribution factors (GDF) for one design lane and two design lanes loaded are calculated according to AASHTO LRFD as follows.

- One design lane loaded:

$$\left(1.75+\frac{S}{3.6}\right)\left(\frac{1}{L}\right)^{0.35}\left(\frac{1}{N_c}\right)^{0.45} = \left(1.75+\frac{7}{3.6}\right)\left(\frac{1}{126.1}\right)^{0.35}\left(\frac{1}{5}\right)^{0.45} = 0.3294$$

- Two or more design lanes loaded:

$$\left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25} = \left(\frac{13}{5}\right)^{0.3}\left(\frac{7}{5.8}\right)\left(\frac{1}{126.1}\right)^{0.25} = 0.4797 \quad \leftarrow\ Governs$$

Nominal Flexural Resistance

The area of prestressing steel per box is $A_{ps} = 5.09\ in^2$

f_s and f'_s are assumed to equal 60 ksi and *k* is 0.28.

The neutral axis is located at,

$$c = \frac{A_{ps}f_{pu} + A_s f_s - A'_s f'_s}{\alpha f'_c \beta b + k A_{ps}\frac{f_{pu}}{d_p}} = \frac{5.09 * 270 + 5.620 * 60 - 3.62 * 60}{0.85 * 4.0 * 0.85 * 84 + 0.28 * 5.09 * \frac{270}{50}}$$

$$\boldsymbol{c = 5.962\ in} < t_s = 7.25\ in$$

$$f_{ps} = f_{pu}\left(1 - k * \frac{c}{d_p}\right) = 270\left(1 - 0.28 * \frac{5.09}{50}\right) = 262.34\ ksi$$

$$a = \beta * c = 0.85 * 5.962 = 5.07\ in$$

The nominal capacity of one box is,

$$M_n = \left[A_{ps} * f_{ps} * \left(d_p - \frac{a}{2}\right) + A_s * f_s * \left(d_s - \frac{a}{2}\right) - A'_s * f'_s * \left(d'_s - \frac{a}{2}\right)\right]$$

$$M_n = \left[5.09 * 262.34 * \left(50 - \frac{5.07}{2}\right) + 5.62 * 60 * \left(34.23 - \frac{5.07}{2}\right) - 3.62 * 60 * \left(2.71 - \frac{5.07}{2}\right)\right]$$

$$\boldsymbol{M_n = 6,165\ kip - ft}$$

Reliability Analysis

Resistance

The mean resistance, variance, and mean of the logarithm resistance were expressed as follows

$$\mu_R = \lambda_R M_n = 1.05 * 6{,}165 = \boldsymbol{6,473\ \ kip.ft}$$

$$\sigma^2{}_{ln(R)} = ln(V^2{}_Y + 1) = ln(0.075^2 + 1) = \boldsymbol{0.00561}$$

$$\mu_{ln(R)} = ln(\mu_R) - \frac{1}{2}\sigma^2{}_{ln(R)} = ln(6{,}473) - \frac{1}{2} * 0.00561 = \boldsymbol{8.77\ \ kip.ft}$$

Dead Load

The mean dead load and the standard deviation of the dead load are

$$\mu_D = \lambda_D D_n = 1.05 * 2{,}044 = \boldsymbol{2,146\quad kip.ft}$$

$$\sigma_D = COV_D D_n = 0.10 * 2{,}044 = \mathbf{204.4 \quad kip.ft}$$

Live Load

The moment due to truck load is 1,114 kip-ft and the moment due to lane load is 711 kip-ft. The nominal live load (HL93) is

$$LL_{nominal} = truck + lane\ load = 1{,}114 + 711 = 1{,}825\ kip.ft$$

The girder distribution factor for two lanes loaded governs. Therefore, the nominal live load per box with girder distribution factor for two lanes included (no dynamic load) is,

$$LL_{nominal/box} = (truck + lane\ load) * GDF = (1{,}114 + 711) * 0.4797$$

$$LL_{nominal/box} = 875\ kip.ft$$

The mean live load with girder distribution factor for two lanes (no dynamic load) is

$$LL_{mean} = \lambda_L * LL_{nominal/box} = 1.10 * 875$$

$$\boldsymbol{LL_{mean} = 963\ kip.ft}$$

The mean dynamic load represents 10% of the mean live load. The mean live load and dynamic load is

$$LL + IM_{mean} = LL_{mean} * (1 + 0.10) = 963 * (1 + 0.10)$$

$$\boldsymbol{LL + IM_{mean} = 1{,}059\ kip.ft}$$

The standard deviation of the live and dynamic load is

$$\sigma_{LL+IM} = COV_L * LL + IM_{mean} = 0.18 * 1{,}059 = \mathbf{190.6\ kip.ft}$$

Extreme type 1 parameters

California live load best fits an extreme type 1 distribution. The parameters α and u are,

$$\alpha = \frac{1.282}{\sigma_{LL+IM}} = \frac{1.282}{190.6} = \mathbf{0.00191}$$

$$u = LL + IM_{mean} - 0.45 * \sigma_{LL+IM} = 1{,}059 - 0.45 * 190.6 = \mathbf{973.23\ \ kip.ft}$$

Monte Carlo

Using the graphical method, the reliability index and probability of failure are

$$\beta = -z = \mathbf{5.79}$$

$$P_f = \Phi^{-1}(-z) = \mathbf{3.52 * 10^{-9}}$$

The normal standard variate is then plotted versus the limit state values as shown in Figure A-18.

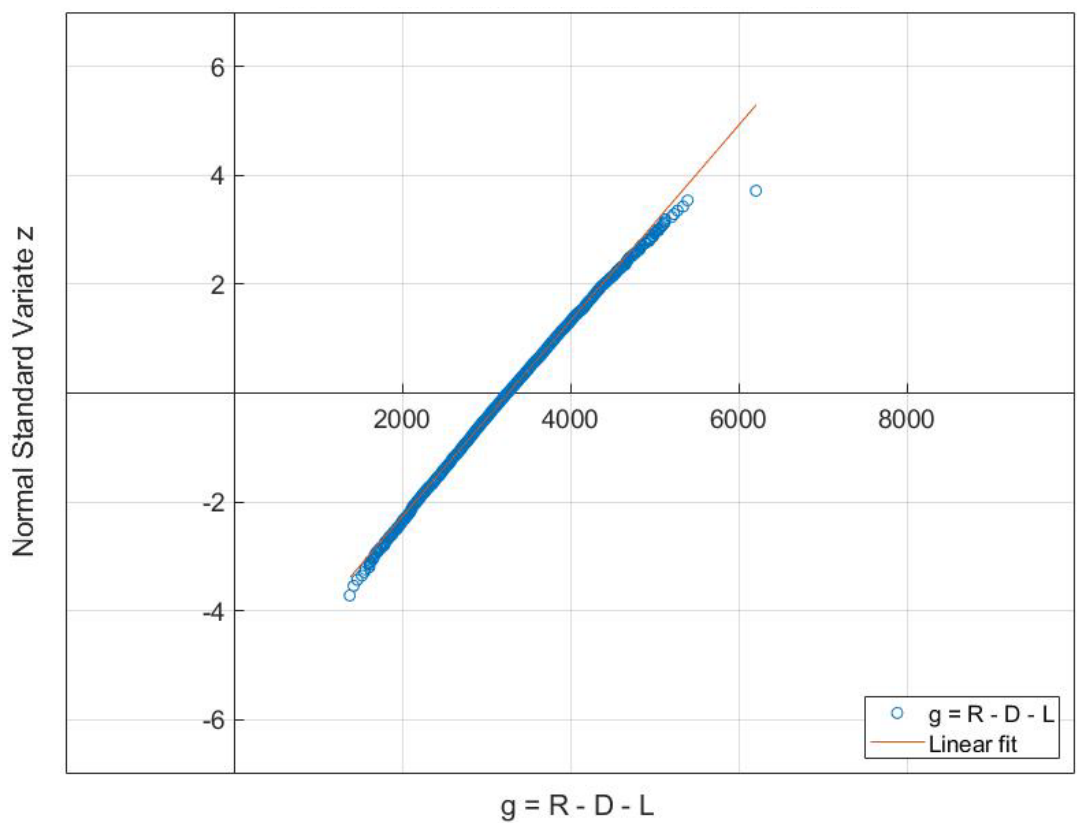

Figure A-18. Monte Carlo simulation for bridge 23 0205L

Bridge 37 0366L – Continuous span

Bridge Properties

General information about bridge 37 0366L is presented below.

Span length:		85 ft
Concrete:		f'_c: 4.0 ksi
Prestressing steel:		270 ksi low relaxation strand
$A_{ps,total}$	=	26.42 in^2
Girder spacing	=	7 ft
Depth of beam	=	3.5 ft
Top flange	=	7.25 in
Bottom flange	=	6 in
Web thickness	=	12 in
Area of one box	=	1,458 in^2
Number of cells	=	7
Number of webs	=	8

The cross-section of bridge 37 0366L and prestressing notes are shown in Figure A-19.

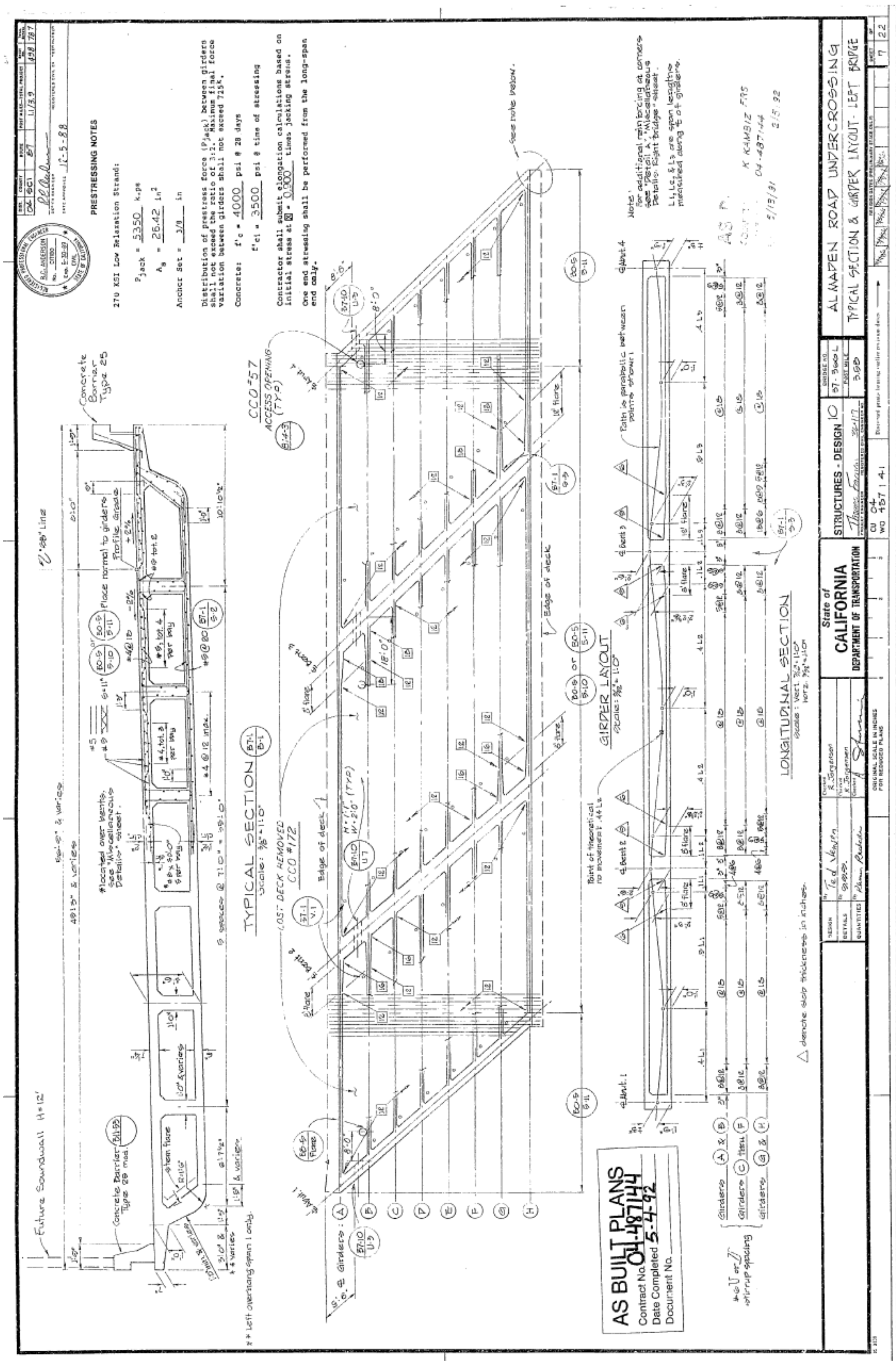

Figure A-19. Cross-section and prestressing details of bridge 37 0366L

Dead Load Analysis – Interior Beam

Components and Attachments, *DC*

The total dead load applied on one box girder is

$$DL = 1.88\ kip/ft$$

The total moment due to dead load for one box girder is

$$M_{DC} = \mathbf{1,118.17}\ \boldsymbol{kip{\cdot}ft}$$

Wearing surface, DW

There is no wearing surface applied on the bridge. Therefore, the moment due to wearing surface is, M_{DW} = 0 kip-ft

Live Load Analysis – Interior Girder

Compute Live Load Distribution Factor for an Interior Girder

The girder distribution factors (GDF) for one design lane and two design lanes loaded are calculated according to AASHTO LRFD as follows.

- One design lane loaded:

$$\left(1.75+\frac{S}{3.6}\right)\left(\frac{1}{L}\right)^{0.35}\left(\frac{1}{N_c}\right)^{0.45}=\left(1.75+\frac{7}{3.6}\right)\left(\frac{1}{85}\right)^{0.35}\left(\frac{1}{7}\right)^{0.45}=0.3251$$

- Two or more design lanes loaded:

$$\left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25}=\left(\frac{13}{7}\right)^{0.3}\left(\frac{7}{5.8}\right)\left(\frac{1}{85}\right)^{0.25}=0.4786\quad\leftarrow\ Governs$$

Nominal Flexural Resistance

The area of prestressing steel per box is $A_{ps} = 3.77\ in^2$

f_s and f'_s are assumed to equal 60 ksi and k is 0.28.

The neutral axis is located at,

$$c \;=\; \frac{A_{ps}f_{pu} + A_s f_s - A'_s f'_s}{\alpha f'_c \beta b + k A_{ps}\dfrac{f_{pu}}{d_p}} = \frac{3.77*270 + 4.24*60 - 5.370*60}{0.85*4.0*0.85*84 + 0.28*3.77*\dfrac{270}{30}}$$

$$\boldsymbol{c = 3.771\ in} > t_s = 7.25\ in$$

$$f_{ps} = f_{pu}\left(1 - k*\frac{c}{d_p}\right) = 270\left(1 - 0.28*\frac{3.77}{115}\right) = 260.5\ ksi$$

$$a = \beta * c = 0.85*3.771 = 3.21\ in$$

The nominal capacity of one box is,

$$M_n = \left[A_{ps}*f_{ps}*\left(d_p - \frac{a}{2}\right) + A_s*f_s*\left(d_s - \frac{a}{2}\right) - A'_s*f'_s*\left(d'_s - \frac{a}{2}\right)\right]$$

$$M_n = \left[3.77*260.5*\left(30 - \frac{3.21}{2}\right) + 4.24*60*\left(22.53 - \frac{3.21}{2}\right) - 5.37*60 *\left(2.15 - \frac{3.21}{2}\right)\right]$$

$$\boldsymbol{M_n = 2,756\ kip{\cdot}ft}$$

Reliability Analysis

Resistance

The mean resistance, variance, and mean of the logarithm resistance were expressed as follows

$$\mu_R = \lambda_R M_n = 1.05*2,756 = \boldsymbol{2,894\ \ kip{\cdot}ft}$$

$$\sigma^2{}_{ln(R)} = ln(V^2{}_Y + 1) = ln(0.075^2 + 1) = \boldsymbol{0.00561}$$

$$\mu_{ln(R)} = ln(\mu_R) - \frac{1}{2}\sigma^2{}_{ln(R)} = ln(2,894) - \frac{1}{2}*0.00561 = \boldsymbol{7.97\ \ kip{\cdot}ft}$$

Dead Load

The mean dead load and the standard deviation of the dead load are

$$\mu_D = \lambda_D D_n = 1.05*1,118 = \boldsymbol{1,174\ \ \ kip{\cdot}ft}$$

$$\sigma_D = COV_D D_n = 0.10 * 1{,}118 = \mathbf{111.8} \quad \boldsymbol{kip{\cdot}ft}$$

Live Load

The moment due to truck load is 898 kip·ft and the moment due to lane load is 443 kip·ft. The nominal live load (HL93) is

$$LL_{nominal} = truck + lane\ load = 898 + 443 = 1{,}341\ kip{\cdot}ft$$

The girder distribution factor for two lanes loaded governs. Therefore, the nominal live load per box with girder distribution factor for two lanes included (no dynamic load) is,

$$LL_{nominal/box} = (truck + lane\ load) * GDF = (898 + 443) * 0.4786$$

$$LL_{nominal/box} = 642\ kip{\cdot}ft$$

The mean live load with girder distribution factor for two lanes (no dynamic load) is

$$LL_{mean} = \lambda_L * LL_{nominal/box} = 1.10 * 642$$

$$\boldsymbol{LL_{mean} = 706\ kip{\cdot}ft}$$

The mean dynamic load represents 10% of the mean live load. The mean live load and dynamic load is

$$LL + IM_{mean} = LL_{mean} * (1 + 0.10) = 706 * (1 + 0.10)$$

$$\boldsymbol{LL + IM_{mean} = 777\ kip{\cdot}ft}$$

The standard deviation of the live and dynamic load is

$$\sigma_{LL+IM} = COV_L * LL + IM_{mean} = 0.18 * 777 = \mathbf{139.9}\ \boldsymbol{kip{\cdot}ft}$$

Extreme type 1 parameters

California live load best fits an extreme type 1 distribution. The parameters α and u are,

$$\alpha = \frac{1.282}{\sigma_{LL+IM}} = \frac{1.282}{139.9} = \mathbf{0.00916}$$

$$u = LL + IM_{mean} - 0.45 * \sigma_{LL+IM} = 777 - 0.45 * 139.9 = \mathbf{714\ \ kip{\cdot}ft}$$

Monte Carlo

Using the numerical method, the probability of failure P_f and the reliability index β are

$$P_f = \frac{6}{10{,}000} = \mathbf{6.00 * 10^{-4}}$$

$$\beta = \mathbf{3.24}$$

Bridge 37 0420L – Continuous span

Bridge Properties

General information about bridge 37 0420L is presented below.

Span length:		62 ft
Concrete:		f'_c: 4.0 ksi
Prestressing steel:		270 ksi low relaxation strand
$A_{ps,total}$	=	14.62 in^2
Girder spacing	=	7.96 ft
Depth of beam	=	3.0 ft
Top flange	=	7.625 in
Bottom flange	=	5.5 in
Web thickness	=	12 in
Area of one box	=	1,527.94 in^2
Number of cells	=	6
Number of webs	=	7

The cross-section of bridge 37 0420L and prestressing notes are shown in Figure A-20 and Figure A-21.

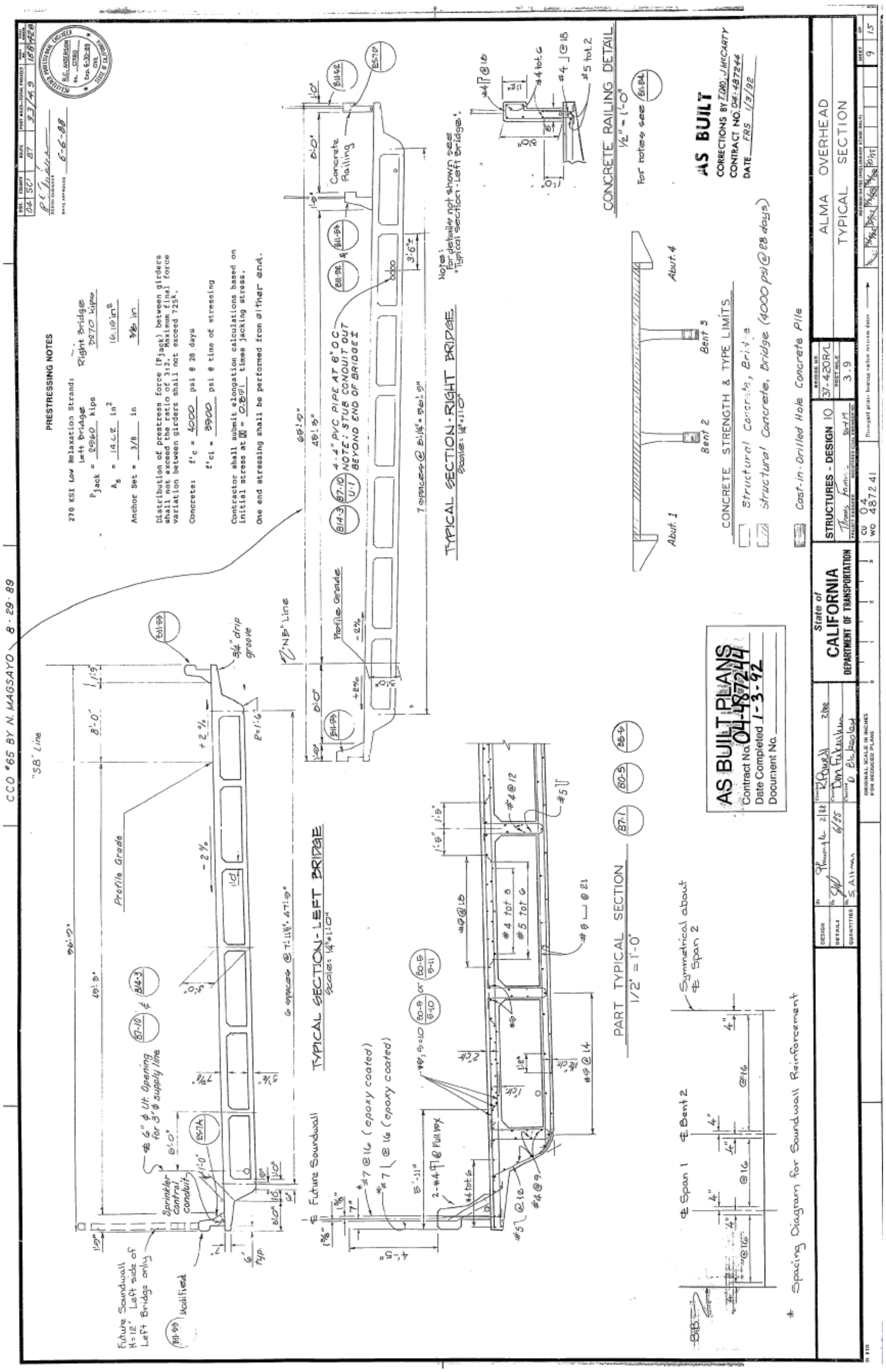

Figure A-20. Cross-section of bridge 37 0420L

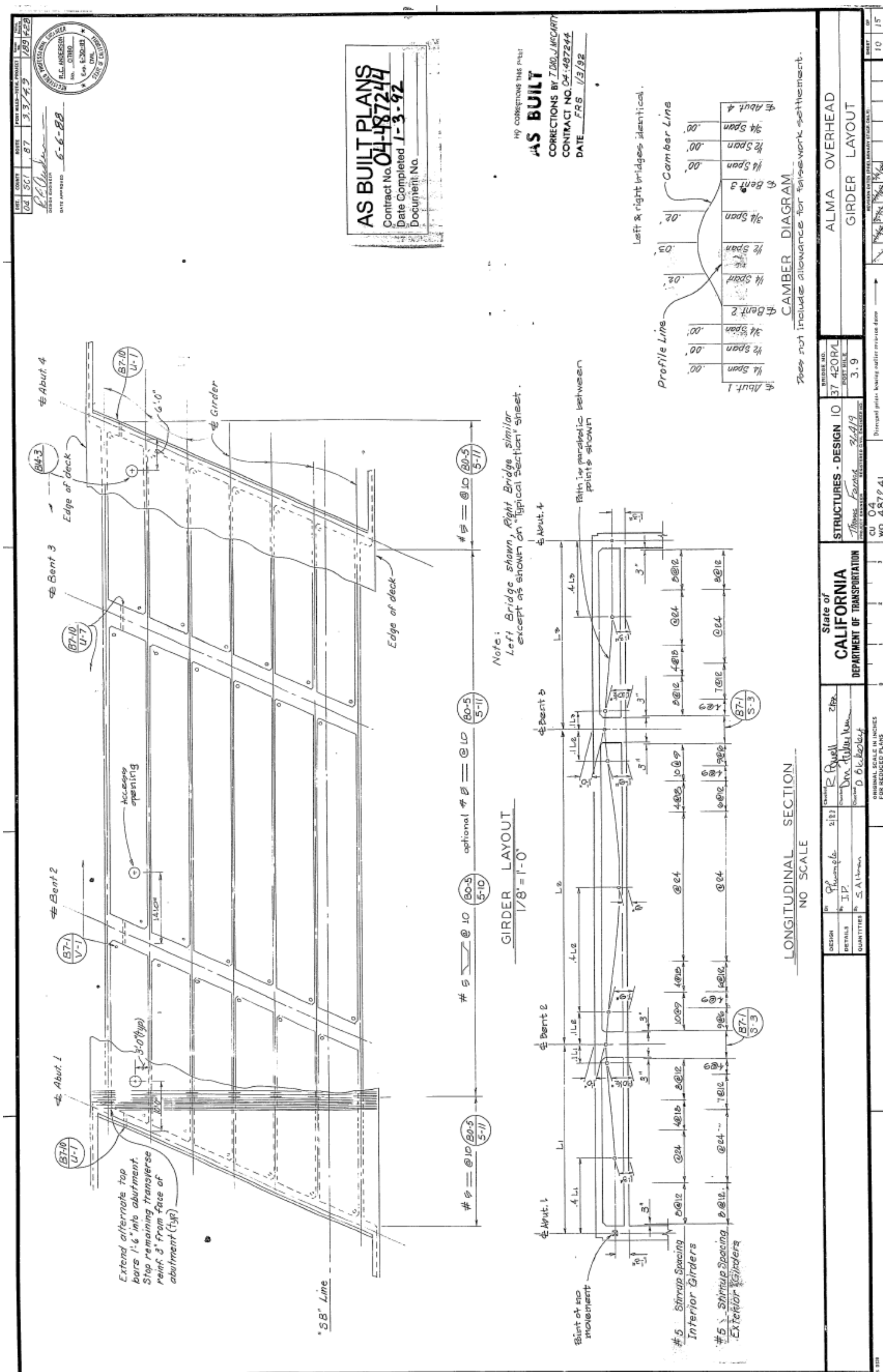

Figure A-21. Bridge 37 0420L details and prestressing notes

Dead Load Analysis – Interior Beam

Components and Attachments, *DC*

The total dead load applied on one box girder is

$$DL = 2.014\ kip/ft$$

The total moment due to dead load for one box girder is

$$M_{DC} = \mathbf{563.14}\ \boldsymbol{kip{\cdot}ft}$$

Wearing surface, DW

There is no wearing surface applied on the bridge. Therefore, the moment due to wearing surface is, M_{DW} = 0 kip-ft

Live Load Analysis – Interior Girder

Compute Live Load Distribution Factor for an Interior Girder

The girder distribution factors (GDF) for one design lane and two design lanes loaded are calculated according to AASHTO LRFD as follows.

- One design lane loaded:

$$\left(1.75+\frac{S}{3.6}\right)\left(\frac{1}{L}\right)^{0.35}\left(\frac{1}{N_c}\right)^{0.45}=\left(1.75+\frac{7.96}{3.6}\right)\left(\frac{1}{62}\right)^{0.35}\left(\frac{1}{6}\right)^{0.45}=0.4171$$

- Two or more design lanes loaded:

$$\left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25}=\left(\frac{13}{6}\right)^{0.3}\left(\frac{7.96}{5.8}\right)\left(\frac{1}{62}\right)^{0.25}=0.6166\quad\leftarrow Governs$$

Nominal Flexural Resistance

The area of prestressing steel per box is $A_{ps} = 2.44\ in^2$

f_s and f'_s are assumed to equal 60 ksi and k is 0.28.

The neutral axis is located at,

$$c = \frac{A_{ps}f_{pu} + A_s f_s - A'_s f'_s}{\alpha f'_c \beta b + k A_{ps}\frac{f_{pu}}{d_p}} = \frac{2.44 * 270 + 5.43 * 60 - 2.17 * 60}{0.85 * 4.0 * 0.85 * 95.5 + 0.28 * 2.44 * \frac{270}{27}}$$

$$\boldsymbol{c = 3.018\ in} > t_s = 7.625\ in$$

$$f_{ps} = f_{pu}\left(1 - k * \frac{c}{d_p}\right) = 270\left(1 - 0.28 * \frac{3.77}{27}\right) = 261.55\ ksi$$

$$a = \beta * c = 0.85 * 3.018 = 2.57\ in$$

The nominal capacity of one box is,

$$M_n = \left[A_{ps} * f_{ps} * \left(d_p - \frac{a}{2}\right) + A_s * f_s * \left(d_s - \frac{a}{2}\right) - A'_s * f'_s * \left(d'_s - \frac{a}{2}\right)\right]$$

$$M_n = \left[2.44 * 261.55 * \left(27 - \frac{2.57}{2}\right) + 5.43 * 60 * \left(17.46 - \frac{2.57}{2}\right) - 2.17 * 60 * \left(2.40 - \frac{2.57}{2}\right)\right]$$

$$\boldsymbol{M_n = 1{,}793\ kip{\cdot}ft}$$

Reliability Analysis

Resistance

The mean resistance, variance, and mean of the logarithm resistance were expressed as follows

$$\mu_R = \lambda_R M_n = 1.05 * 1{,}793 = \boldsymbol{1{,}883\ \ kip{\cdot}ft}$$

$$\sigma^2{}_{ln(R)} = ln(V^2{}_Y + 1) = ln(0.075^2 + 1) = \boldsymbol{0.00561}$$

$$\mu_{ln(R)} = ln(\mu_R) - \frac{1}{2}\sigma^2{}_{ln(R)} = ln(1{,}883) - \frac{1}{2} * 0.00561 = \boldsymbol{7.54\ \ kip{\cdot}ft}$$

Dead Load

The mean dead load and the standard deviation of the dead load are

$$\mu_D = \lambda_D D_n = 1.05 * 563 = \boldsymbol{591\quad kip{\cdot}ft}$$

$$\sigma_D = COV_D D_n = 0.10 * 563 = \mathbf{56.3 \quad kip{\cdot}ft}$$

Live Load

The moment due to truck load is 898 kip-ft and the moment due to lane load is 443 kip-ft. The nominal live load (HL93) is

$$LL_{nominal} = truck + lane\ load = 482 + 179 = 661\ kip{\cdot}ft$$

The girder distribution factor for two lanes loaded governs. Therefore, the nominal live load per box with girder distribution factor for two lanes included (no dynamic load) is,

$$LL_{nominal/box} = (truck + lane\ load) * GDF = (482 + 179) * 0.6166$$

$$LL_{nominal/box} = 408\ kip{\cdot}ft$$

The mean live load with girder distribution factor for two lanes (no dynamic load) is

$$LL_{mean} = \lambda_L * LL_{nominal/box} = 1.10 * 408$$

$$\boldsymbol{LL_{mean} = 448\ kip{\cdot}ft}$$

The mean dynamic load represents 10% of the mean live load. The mean live load and dynamic load is

$$LL + IM_{mean} = LL_{mean} * (1 + 0.10) = 448 * (1 + 0.10)$$

$$\boldsymbol{LL + IM_{mean} = 493\ kip{\cdot}ft}$$

The standard deviation of the live and dynamic load is

$$\sigma_{LL+IM} = COV_L * LL + IM_{mean} = 0.18 * 493 = \mathbf{88.74\ kip{\cdot}ft}$$

Extreme type 1 parameters

California live load best fits an extreme type 1 distribution. The parameters α and u are,

$$\alpha = \frac{1.282}{\sigma_{LL+IM}} = \frac{1.282}{88.94} = \mathbf{0.0144}$$

$$u = LL + IM_{mean} - 0.45 * \sigma_{LL+IM} = 493 - 0.45 * 88.74 = \mathbf{453.1\ \ kip{\cdot}ft}$$

Monte Carlo

Using the numerical method, the probability of failure P_f and the reliability index β are

$$P_f = \frac{1}{10{,}000} = \mathbf{1 * 10^{-4}}$$

$$\beta = \mathbf{3.72}$$

Bridge 37 0421L – Continuous span

Bridge Properties

General information about bridge 37 0421L is presented below.

Span length:		77 ft
Concrete:		f'_c: 4.0 ksi
Prestressing steel:		270 ksi low relaxation strand
$A_{ps,total}$	=	17.14 in^2
Girder spacing	=	7.17 ft
Depth of beam	=	3.5 ft
Top flange	=	7.375 in
Bottom flange	=	5.5 in
Web thickness	=	12 in
Area of one box	=	1,456.75 in^2
Number of cells	=	7
Number of webs	=	8

The cross-section of bridge 37 0421L and prestressing notes are shown in Figure A-22 and Figure A-23.

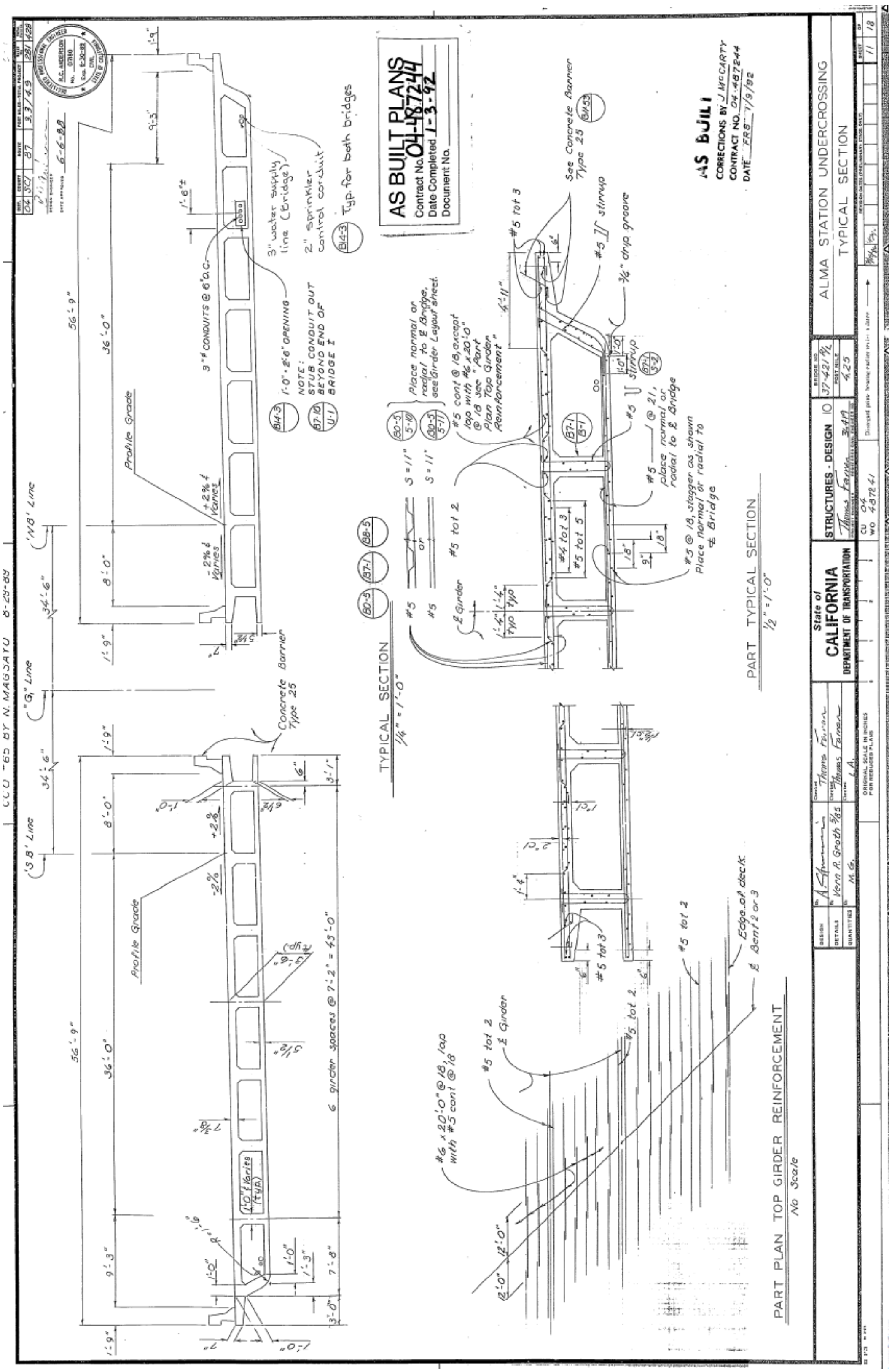

Figure A-22. Cross-section of bridge 37 0421L

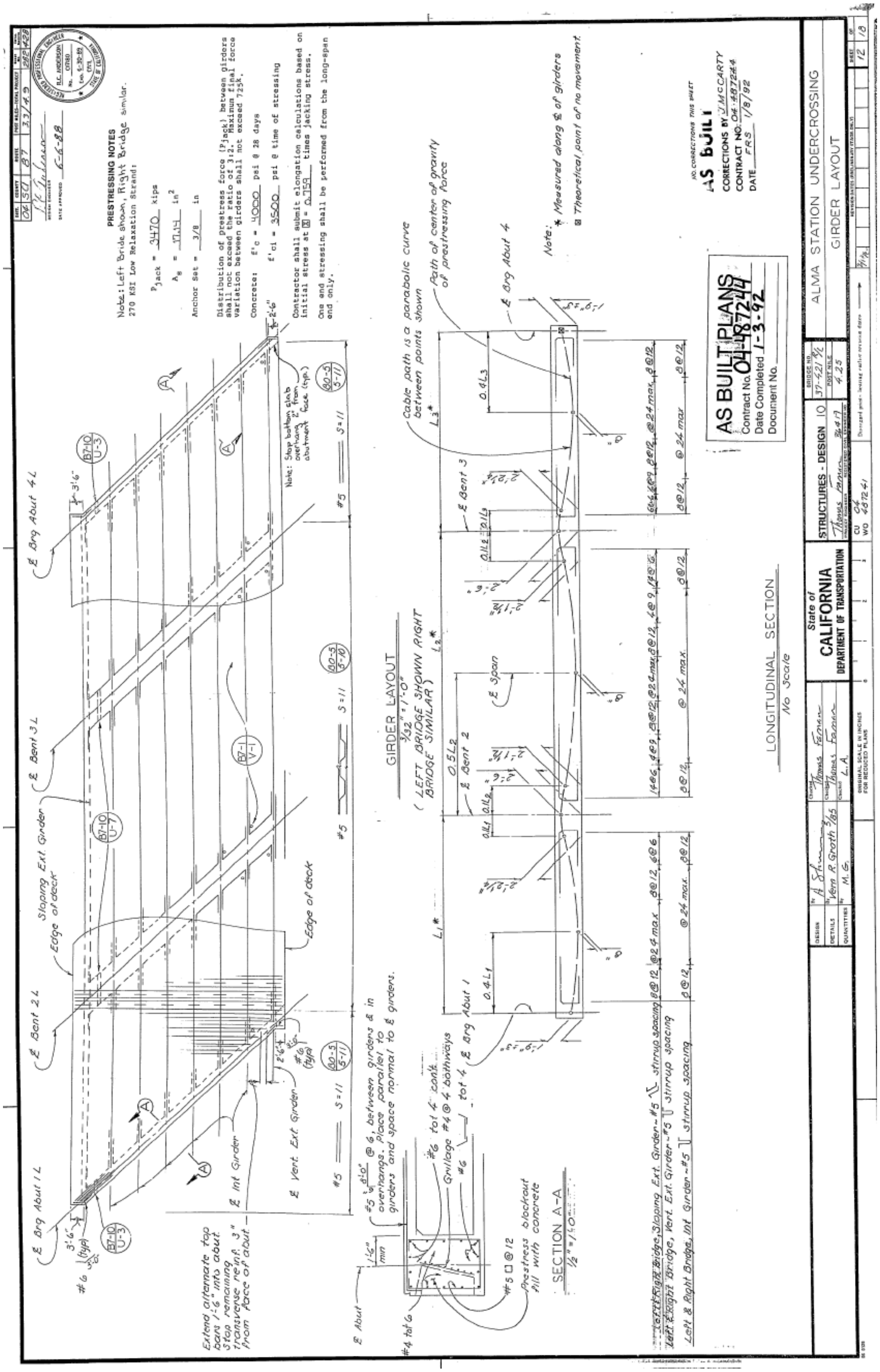

Figure A-23. Bridge 37 0421L details and prestressing notes

Dead Load Analysis – Interior Beam

Components and Attachments, *DC*

The total dead load applied on one box girder is

$$DL = 1.612\ kip/ft$$

The total moment due to dead load for one box girder is

$$M_{DC} = \mathbf{735.94\ kip{\cdot}ft}$$

Wearing surface, DW

There is no wearing surface applied on the bridge. Therefore, the moment due to wearing surface is, M_{DW} = 0 kip-ft

Live Load Analysis – Interior Girder

Compute Live Load Distribution Factor for an Interior Girder

The girder distribution factors (GDF) for one design lane and two design lanes loaded are calculated according to AASHTO LRFD as follows.

- One design lane loaded:

$$\left(1.75 + \frac{S}{3.6}\right)\left(\frac{1}{L}\right)^{0.35}\left(\frac{1}{N_c}\right)^{0.45} = \left(1.75 + \frac{7.17}{3.6}\right)\left(\frac{1}{77}\right)^{0.35}\left(\frac{1}{7}\right)^{0.45} = 0.3407$$

- Two or more design lanes loaded:

$$\left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25} = \left(\frac{13}{7}\right)^{0.3}\left(\frac{7.17}{5.8}\right)\left(\frac{1}{77}\right)^{0.25} = 0.5023 \quad \leftarrow Governs$$

Nominal Flexural Resistance

The area of prestressing steel per box is $A_{ps} = 2.45\ in^2$

f_s and f'_s are assumed to equal 60 ksi and k is 0.28.

The neutral axis is located at,

$$c \;=\; \frac{A_{ps}f_{pu} + \;A_s f_s \;-\; A'_s f'_s}{\alpha f'_c \beta b \;+\; kA_{ps}\dfrac{f_{pu}}{d_p}} = \frac{2.45 * 270 \;+\; 6.49 * 60 \;-\; 2.17 * 60}{0.85 * 4.0 * 0.85 * 86 \;+\; 0.28 * 2.45 * \dfrac{270}{34}}$$

$$\boldsymbol{c \;=\; 3.624\; in} \;>\; t_s = 7.25\; in$$

$$f_{ps} = f_{pu}\left(1 - k * \frac{c}{d_p}\right) = 270\left(1 - 0.28 * \frac{2.45}{34}\right) \;=\; 261.94\; ksi$$

$$a \;=\; \beta * c \;=\; 0.85 * 3.624 \;=\; 3.08\; in$$

The nominal capacity of one box is,

$$M_n = \left[A_{ps} * f_{ps} * \left(d_p - \frac{a}{2}\right) + \;A_s * f_s * \left(d_s \;-\; \frac{a}{2}\right) - A'_s * f'_s * \left(d'_s \;-\; \frac{a}{2}\right)\right]$$

$$M_n = \left[2.45 * 261.94 * \left(34 \;-\; \frac{3.08}{2}\right) + \;6.49 \;*\; 60 \;*\left(23.71 \;-\; \frac{3.08}{2}\right) - 2.17 * 60 \right.$$
$$\left. * \left(2.93 \;-\; \frac{3.08}{2}\right)\right]$$

$$\boldsymbol{M_n = \;2{,}439\; kip{\cdot}ft}$$

Reliability Analysis

Resistance

The mean resistance, variance, and mean of the logarithm resistance were expressed as follows

$$\mu_R = \lambda_R M_n = 1.05 * 2{,}439 = \boldsymbol{2{,}561\;\; kip{\cdot}ft}$$

$$\sigma^2{}_{ln(R)} = ln(V^2{}_Y \;+\; 1) = ln(0.075^2 + 1) = \boldsymbol{0.00561}$$

$$\mu_{ln(R)} = ln(\mu_R) - \frac{1}{2}\sigma^2{}_{ln(R)} = ln(2{,}561) - \frac{1}{2} * 0.00561 = \boldsymbol{7.35\;\; kip{\cdot}ft}$$

Dead Load

The mean dead load and the standard deviation of the dead load are

$$\mu_D = \lambda_D D_n = 1.05 * 736 = \boldsymbol{773 \quad kip{\cdot}ft}$$

$$\sigma_D = COV_D D_n = 0.10 * 736 = \mathbf{73.6 \quad kip \cdot ft}$$

Live Load

The moment due to truck load is 898 kip-ft and the moment due to lane load is 443 kip-ft. The nominal live load (HL93) is

$$LL_{nominal} = truck + lane\ load = 675 + 292 = 967\ kip \cdot ft$$

The girder distribution factor for two lanes loaded governs. Therefore, the nominal live load per box with girder distribution factor for two lanes included (no dynamic load) is,

$$LL_{nominal/box} = (truck + lane\ load) * GDF = (675 + 292) * 0.5023$$

$$LL_{nominal/box} = 486\ kip \cdot ft$$

The mean live load with girder distribution factor for two lanes (no dynamic load) is expressed as,

$$LL_{mean} = \lambda_L * LL_{nominal/box} == \ 1.10 * 486$$

$$\boldsymbol{LL_{mean} = 534\ kip \cdot ft}$$

The mean dynamic load represents 10% of the mean live load. The mean live load and dynamic load is

$$LL + IM_{mean} = LL_{mean} * (1 + 0.10) = 534 * (1 + 0.10)$$

$$\boldsymbol{LL + IM_{mean} = 588\ kip \cdot ft}$$

The standard deviation of the live and dynamic load is

$$\sigma_{LL+IM} = COV_L * LL + IM_{mean} = 0.18 * 588 = \mathbf{105.84\ kip \cdot ft}$$

Extreme type 1 parameters

California live load best fits an extreme type 1 distribution. The parameters α and u are,

$$\alpha = \frac{1.282}{\sigma_{LL+IM}} = \frac{1.282}{105.84} = \mathbf{0.0121}$$

$$u = LL + IM_{mean} - 0.45 * \sigma_{LL+IM} = 588 - 0.45 * 105.84 = \mathbf{540.37\ \ kip{\cdot}ft}$$

Monte Carlo

Using the graphical method, the reliability index and probability of failure are

$$\beta = -z = \mathbf{5.20}$$

$$P_f = \Phi^{-1}(-z) = \mathbf{9.08 * 10^{-8}}$$

The normal standard variate is then plotted versus the limit state values as shown in Figure A-24.

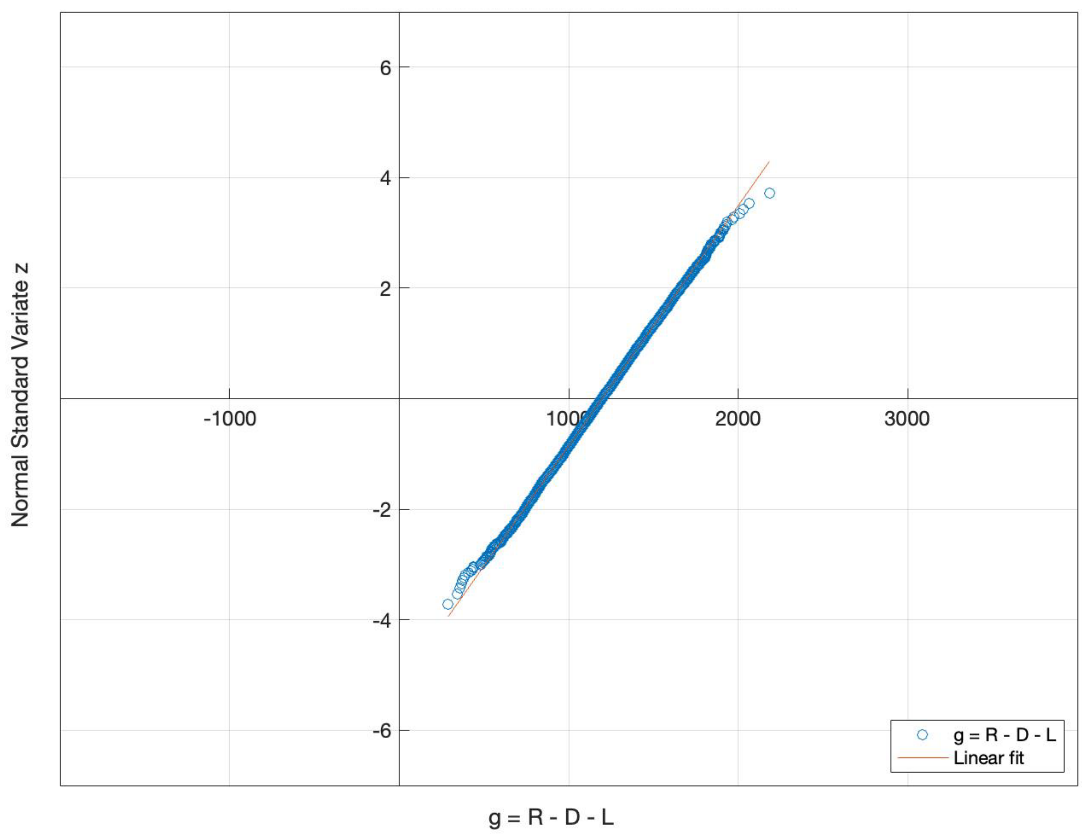

Figure A-24. Monte Carlo simulation for bridge 37 0421L

Bridge 37 0467L – Continuous span

Bridge Properties

General information about bridge 37 0467L is presented below.

Span length:		230.97 ft
Concrete:		f'_c: 4.0 ksi
Prestressing steel:		270 ksi low relaxation strand
$A_{ps,total}$	=	92.15 in^2
Girder spacing	=	9.75 ft
Depth of beam	=	6.5 ft
Top flange	=	8.25 in
Bottom flange	=	6.625 in
Web thickness	=	12 in
Area of one box	=	2,497.88 in^2
Number of cells	=	7
Number of webs	=	8

The cross-section of bridge 37 0467L and prestressing notes are shown in Figure A-25 and Figure A-26.

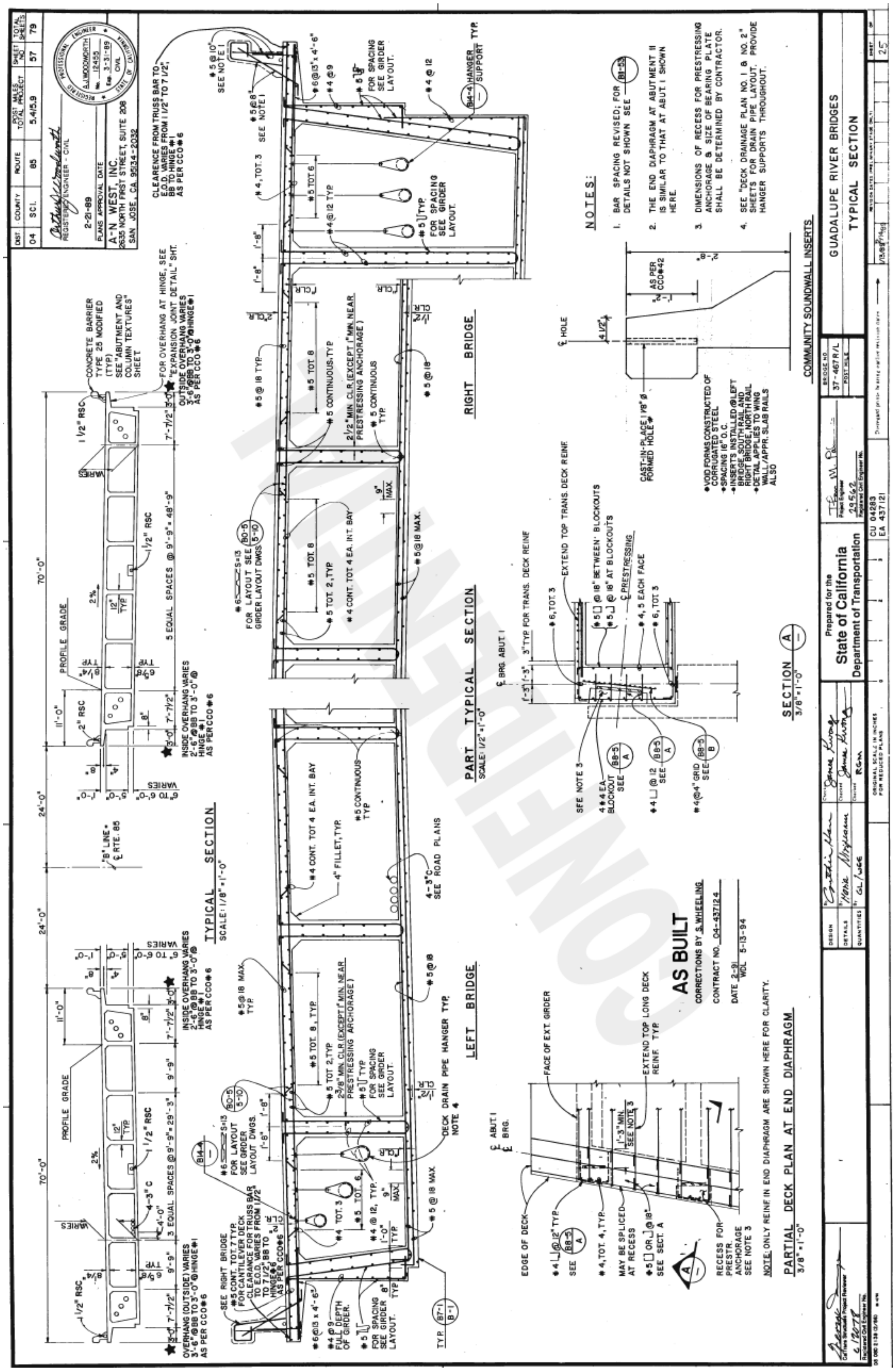

Figure A-25. Cross-section of bridge 37 0467L

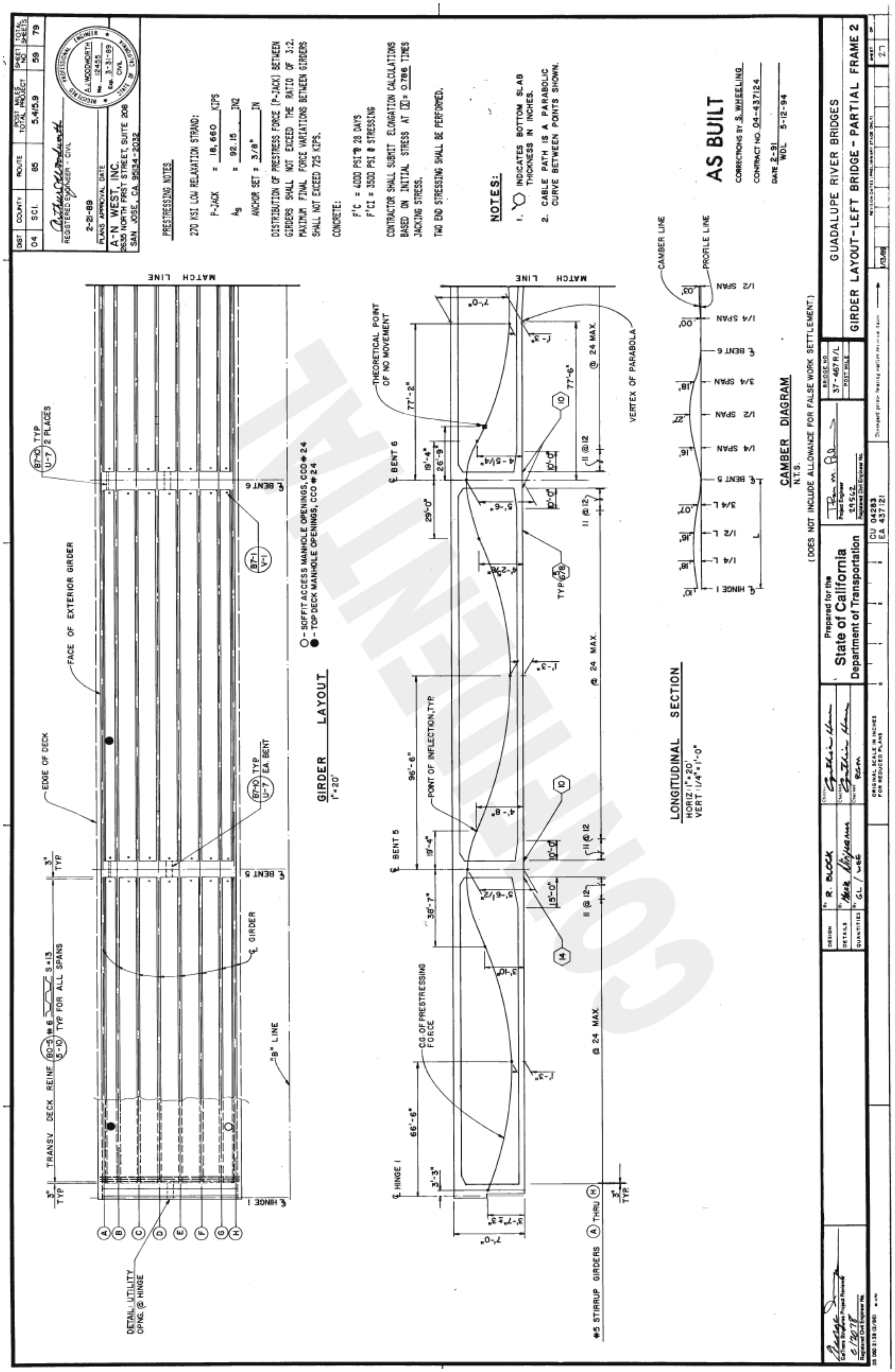

Figure A-26. Bridge 37 0467L details and prestressing notes

Dead Load Analysis – Interior Beam

Components and Attachments, *DC*

The total dead load applied on one box girder is

$$DL = 2.697\ kip/ft$$

The total moment due to dead load for one box girder is

$$M_{DC} = \mathbf{11,183.88}\ \boldsymbol{kip{\cdot}ft}$$

Wearing surface, DW

There is no wearing surface applied on the bridge. Therefore, the moment due to wearing surface is, M_{DW} = 0 kip-ft

Live Load Analysis – Interior Girder

Compute Live Load Distribution Factor for an Interior Girder

The girder distribution factors (GDF) for one design lane and two design lanes loaded are calculated according to AASHTO LRFD as follows.

- One design lane loaded:

$$\left(1.75+\frac{S}{3.6}\right)\left(\frac{1}{L}\right)^{0.35}\left(\frac{1}{N_c}\right)^{0.45}=\left(1.75+\frac{9.75}{3.6}\right)\left(\frac{1}{230.97}\right)^{0.35}\left(\frac{1}{7}\right)^{0.45} = 0.2765$$

- Two or more design lanes loaded:

$$\left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25}=\left(\frac{13}{7}\right)^{0.3}\left(\frac{9.75}{5.8}\right)\left(\frac{1}{230.97}\right)^{0.25} = 0.5192 \quad \leftarrow Governs$$

Nominal Flexural Resistance

The area of prestressing steel per box is $A_{ps} = 13.16\ in^2$

f_s and f'_s are assumed to equal 60 ksi and k is 0.28.

The neutral axis is located at,

$$c = \frac{A_{ps}f_{pu} + A_sf_s - A'_sf'_s}{\alpha f'_c\beta b + kA_{ps}\frac{f_{pu}}{d_p}} = \frac{13.16 * 270 + 6.34 * 60 - 7 * 60}{0.85 * 4.0 * 0.85 * 117 + 0.28 * 13.16 * \frac{270}{63}}$$

$$\boldsymbol{c = 9.931\ in} > t_s = 8.25\ in$$

$$f_{ps} = f_{pu}\left(1 - k * \frac{c}{d_p}\right) = 270\left(1 - 0.28 * \frac{9.931}{63}\right) = 258.08\ ksi$$

$$a = \beta * c = 0.85 * 9.931 = 8.44\ in$$

The nominal capacity of one box is,

$$M_n = \left[A_{ps} * f_{ps} * \left(d_p - \frac{a}{2}\right) + A_s * f_s * \left(d_s - \frac{a}{2}\right) - A'_s * f'_s * \left(d'_s - \frac{a}{2}\right)\right]$$

$$M_n = \left[13.16 * 258.08 * \left(63 - \frac{8.44}{2}\right) + 6.34 * 60 * \left(58.93 - \frac{8.44}{2}\right) - 7 * 60 * \left(4.77 - \frac{8.44}{2}\right)\right]$$

$$\boldsymbol{M_n = 18,375\ kip{\cdot}ft}$$

Reliability Analysis

Resistance

The mean resistance, variance, and mean of the logarithm resistance were expressed as follows

$$\mu_R = \lambda_R M_n = 1.05 * 18,357 = \boldsymbol{19,275\ \ kip{\cdot}ft}$$

$$\sigma^2{}_{ln(R)} = ln(V^2{}_Y + 1) = ln(0.075^2 + 1) = \boldsymbol{0.00561}$$

$$\mu_{ln(R)} = ln(\mu_R) - \frac{1}{2}\sigma^2{}_{ln(R)} = ln(19,275) - \frac{1}{2} * 0.00561 = \boldsymbol{9.86\ \ kip{\cdot}ft}$$

Dead Load

The mean dead load and the standard deviation of the dead load are

$$\mu_D = \lambda_D D_n = 1.05 * 11,184 = \boldsymbol{11,743\quad kip{\cdot}ft}$$

$$\sigma_D = COV_D D_n = 0.10 * 11{,}184 = \mathbf{1{,}118} \quad \boldsymbol{kip \cdot ft}$$

Live Load

The moment due to truck load is 2,673 kip·ft and the moment due to lane load is 2,389 kip·ft. The nominal live load (HL93) is

$$LL_{nominal} = truck + lane\ load = 2{,}673 + 2{,}389 = 5{,}062\ kip{\cdot}ft$$

The girder distribution factor for two lanes loaded governs. Therefore, the nominal live load per box with girder distribution factor for two lanes included (no dynamic load) is,

$$LL_{nominal/box} = (truck + lane\ load) * GDF = (2{,}673 + 2{,}389) * 0.5192$$

$$LL_{nominal/box} = 2{,}628\ kip.ft$$

The mean live load with girder distribution factor for two lanes (no dynamic load) is

$$LL_{mean} = \lambda_L * LL_{\frac{nominal}{box}} = \ 1.10 * 2{,}628$$

$$\boldsymbol{LL_{mean} = 2{,}891\ kip.ft}$$

The mean dynamic load represents 10% of the mean live load. The mean live load and dynamic load is

$$LL + IM_{mean} = LL_{mean} * (1 + 0.10) = 2{,}891 * (1 + 0.10)$$

$$\boldsymbol{LL + IM_{mean} = 3{,}180\ kip.ft}$$

The standard deviation of the live and dynamic load is

$$\sigma_{LL+IM} = COV_L * LL + IM_{mean} = 0.18 * 3{,}180 = \mathbf{572.4}\ \boldsymbol{kip.ft}$$

Extreme type 1 parameters

California live load best fits an extreme type 1 distribution. The parameters α and u are,

$$\alpha = \frac{1.282}{\sigma_{LL+IM}} = \frac{1.282}{572.4} = \mathbf{0.00224}$$

$$u = LL + IM_{mean} - 0.45 * \sigma_{LL+IM} = 3{,}180 - 0.45 * 572.4 = \mathbf{2,923\ \ kip.ft}$$

Monte Carlo

Using the numerical method, the probability of failure P_f and the reliability index β are

$$P_f = \frac{102}{10{,}000} = \mathbf{1.02 * 10^{-2}}$$

$$\beta = \mathbf{2.32}$$

Bridge 49 0060L – Continuous span

Bridge Properties

General information about bridge 49 0060L is presented below.

Span length:		160 ft
Concrete:		f'_c: 4.0 ksi
Prestressing steel:		270 ksi low relaxation strand
$A_{ps,total}$	=	32.31 in^2
Girder spacing	=	8 ft
Depth of beam	=	6.5 ft
Top flange	=	7.625 in
Bottom flange	=	5.5 in
Web thickness	=	12 in
Area of one box	=	2,038.5 in^2
Number of cells	=	4
Number of webs	=	5

The cross-section of bridge 49 0060L and prestressing notes are shown in Figure A-27 and Figure A-28.

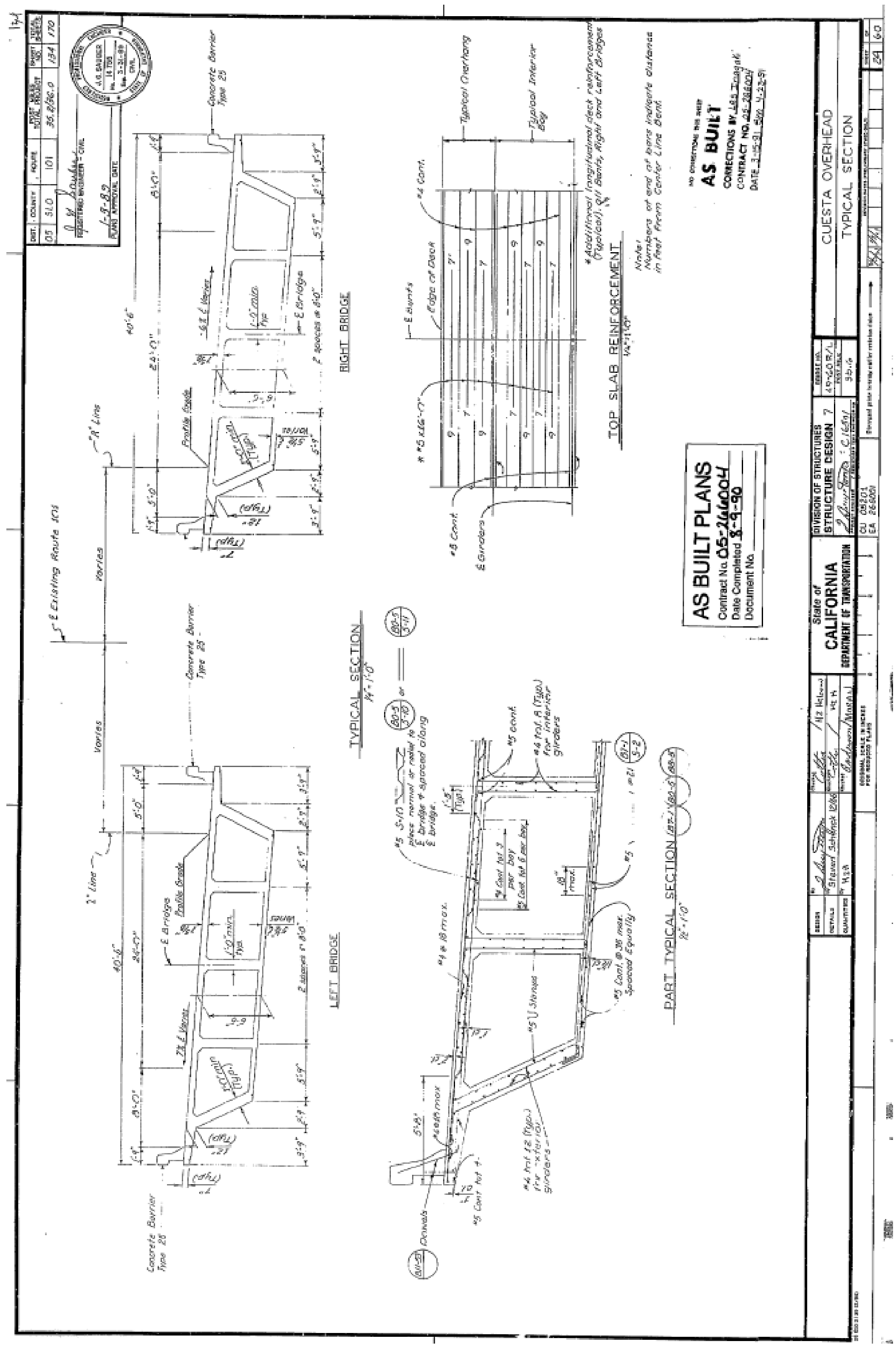

Figure A-27. Cross-section of bridge 49 0060L

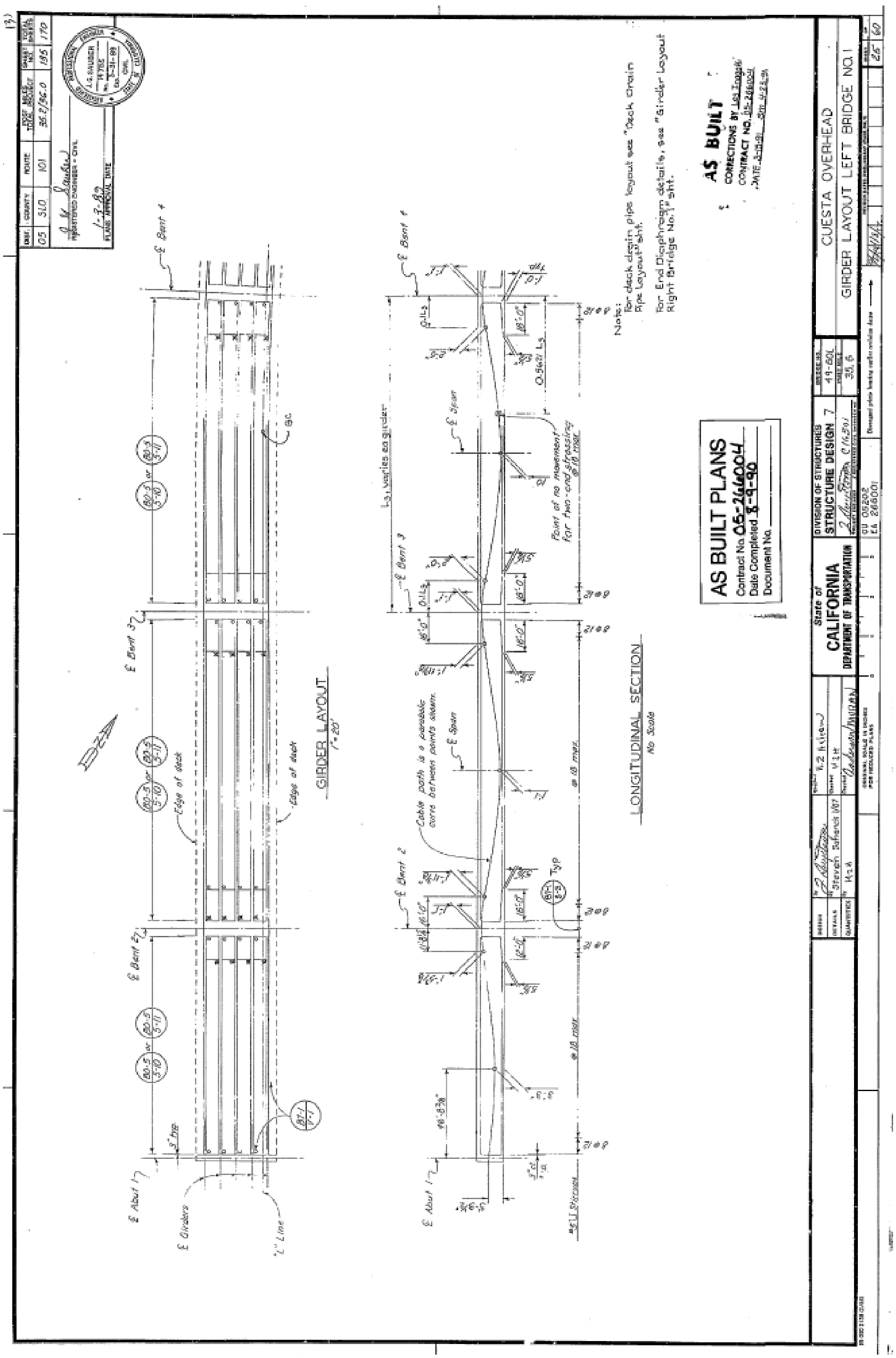

Figure A-28. Bridge 49 0060L details and prestressing notes

Dead Load Analysis – Interior Beam

Components and Attachments, *DC*

The total dead load applied on one box girder is

$$DL = 2.29\ kip/ft$$

The total moment due to dead load for one box girder is

$$M_{DC} = \mathbf{5,219.97}\ \boldsymbol{kip \cdot ft}$$

Wearing surface, DW

There is no wearing surface applied on the bridge. Therefore, the moment due to wearing surface is, M_{DW} = 0 kip·ft

Live Load Analysis – Interior Girder

Compute Live Load Distribution Factor for an Interior Girder

The girder distribution factors (GDF) for one design lane and two design lanes loaded are calculated according to AASHTO LRFD as follows.

- One design lane loaded:

$$\left(1.75+\frac{S}{3.6}\right)\left(\frac{1}{L}\right)^{0.35}\left(\frac{1}{N_c}\right)^{0.45}=\left(1.75+\frac{8}{3.6}\right)\left(\frac{1}{160}\right)^{0.35}\left(\frac{1}{4}\right)^{0.45} = 0.3603$$

- Two or more design lanes loaded:

$$\left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25}=\left(\frac{13}{4}\right)^{0.3}\left(\frac{8}{5.8}\right)\left(\frac{1}{160}\right)^{0.25} = 0.5523 \quad \leftarrow Governs$$

Nominal Flexural Resistance

The area of prestressing steel per box is $A_{ps} = 8.08\ in^2$

f_s and f'_s are assumed to equal 60 ksi and k is 0.28.

The neutral axis is located at,

$$c \;=\; \frac{A_{ps}f_{pu} + \; A_s f_s \; - \; A'_s f'_s}{\alpha f'_c \beta b \; + \; kA_{ps}\dfrac{f_{pu}}{d_p}} = \frac{8.08 * 270 \; + \; 4.08 * 60 \; - \; 4.08 * 60}{0.85 * 4.0 * 0.85 * 96 \; + \; 0.28 * 8.08 * \dfrac{270}{68}}$$

$$\boldsymbol{c \; = 7.614 \; in} < \; t_s = 7.625 \; in$$

$$f_{ps} = f_{pu}\left(1 - k * \frac{c}{d_p}\right) = 270\left(1 - 0.28 * \frac{7.614}{68}\right) \; = \; 261.53 \; ksi$$

$$a \; = \; \beta * c \; = \; 0.85 * 7.614 \; = \; 6.47 \; in$$

The nominal capacity of one box is,

$$M_n = \left[A_{ps} * f_{ps} * \left(d_p - \frac{a}{2}\right) + \; A_s * f_s * \left(d_s \; - \; \frac{a}{2}\right) - A'_s * f'_s * \left(d'_s \; - \; \frac{a}{2}\right)\right]$$

$$M_n = \left[8.08 * 261.53 * \left(68 \; - \; \frac{6.47}{2}\right) + \; 4.08 \; * \; 60 \; * \left(51.37 \; - \; \frac{6.47}{2}\right) - 4.08 * 60 \right.$$
$$\left. * \left(3.47 \; - \; \frac{6.47}{2}\right)\right]$$

$$\boldsymbol{M_n = \; 12,379 \; kip{\cdot}ft}$$

Reliability Analysis

Resistance

The mean resistance, variance, and mean of the logarithm resistance were expressed as follows

$$\mu_R = \lambda_R M_n = 1.05 * 12{,}379 = \boldsymbol{12,998 \;\; kip{\cdot}ft}$$

$$\sigma^2{}_{ln(R)} = ln(V^2{}_Y \; + \; 1) = ln(0.075^2 + 1) = \boldsymbol{0.00561}$$

$$\mu_{ln(R)} = ln(\mu_R) - \frac{1}{2}\sigma^2{}_{ln(R)} = ln(12{,}998) - \frac{1}{2} * 0.00561 = \boldsymbol{9.47 \;\; kip{\cdot}ft}$$

Dead Load

The mean dead load and the standard deviation of the dead load are

$$\mu_D = \lambda_D D_n = 1.05 * 5{,}220 = \boldsymbol{5,481 \quad kip{\cdot}ft}$$

$$\sigma_D = COV_D D_n = 0.10 * 5{,}220 = \mathbf{522} \quad \boldsymbol{kip{\cdot}ft}$$

Live Load

The moment due to truck load is 1,676 kip-ft and the moment due to lane load is 1,459 kip-ft. The nominal live load (HL93) is

$$LL_{nominal} = truck + lane\ load = 1{,}676 + 1{,}459 = 3{,}135\ kip{\cdot}ft$$

The girder distribution factor for two lanes loaded governs. Therefore, the nominal live load per box with girder distribution factor for two lanes included (no dynamic load) is,

$$LL_{nominal/box} = (truck + lane\ load) * GDF = (1{,}676 + 1{,}459) * 0.5523$$

$$LL_{nominal/box} = 1{,}732\ kip{\cdot}ft$$

The mean live load with girder distribution factor for two lanes (no dynamic load) is

$$LL_{mean} = \lambda_L * LL_{\frac{nominal}{box}} = 1.10 * 1{,}732$$

$$\boldsymbol{LL_{mean} = 1{,}905\ kip{\cdot}ft}$$

The mean dynamic load represents 10% of the mean live load. The mean live load and dynamic load is

$$LL + IM_{mean} = LL_{mean} * (1 + 0.10) = 1{,}905 * (1 + 0.10)$$

$$\boldsymbol{LL + IM_{mean} = 2{,}095\ kip{\cdot}ft}$$

The standard deviation of the live and dynamic load is

$$\sigma_{LL+IM} = COV_L * LL + IM_{mean} = 0.18 * 2{,}095 = \mathbf{377.1}\ \boldsymbol{kip{\cdot}ft}$$

Extreme type 1 parameters

California live load best fits an extreme type 1 distribution. The parameters α and u are,

$$\alpha = \frac{1.282}{\sigma_{LL+IM}} = \frac{1.282}{377.1} = \mathbf{0.0034}$$

$$u = LL + IM_{mean} - 0.45 * \sigma_{LL+IM} = 2{,}095 - 0.45 * 377.1 = \mathbf{1{,}925.3\ \ kip.ft}$$

Monte Carlo

Using the graphical method, the reliability index and probability of failure are

$$\beta = -z = \mathbf{4.62}$$

$$P_f = \Phi^{-1}(-z) = \mathbf{1.92 * 10^{-6}}$$

The normal standard variate is then plotted versus the limit state values as shown in Figure A-29.

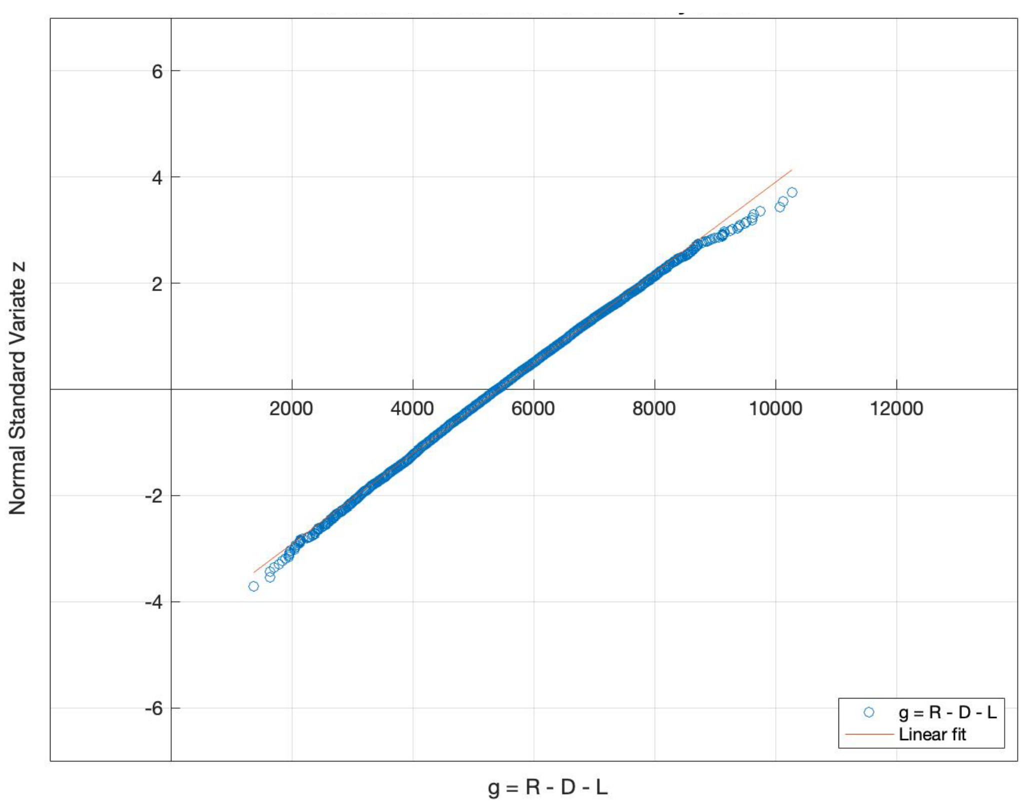

Figure A-29. Monte Carlo simulation for bridge 49 0060L

Bridge 49 0165L – Continuous span

Bridge Properties

General information about bridge 49 0165L is presented below.

Span length:		98 ft
Concrete:		f'_c: 4.0 ksi
Prestressing steel:		270 ksi low relaxation strand
$A_{ps,total}$	=	20.89 in^2
Girder spacing	=	8.88 ft
Depth of beam	=	4 ft
Top flange	=	8 in
Bottom flange	=	6 in
Web thickness	=	12 in
Area of one box	=	1,899 in^2
Number of cells	=	4
Number of webs	=	5

The cross-section of bridge 49 0165L and prestressing notes are shown in Figure A-30 and Figure A-31.

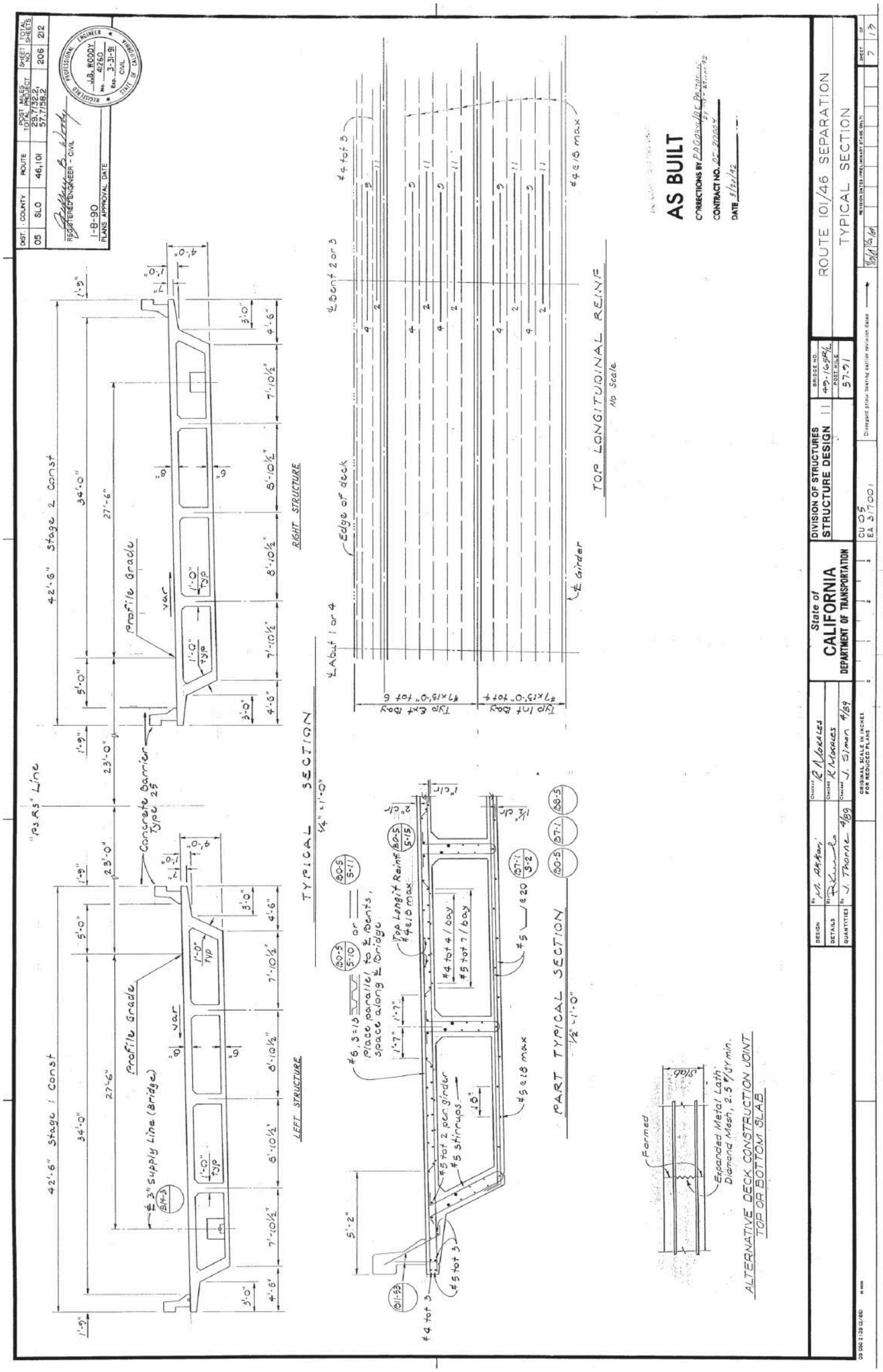

Figure A-30. Cross-section of bridge 49 0165L

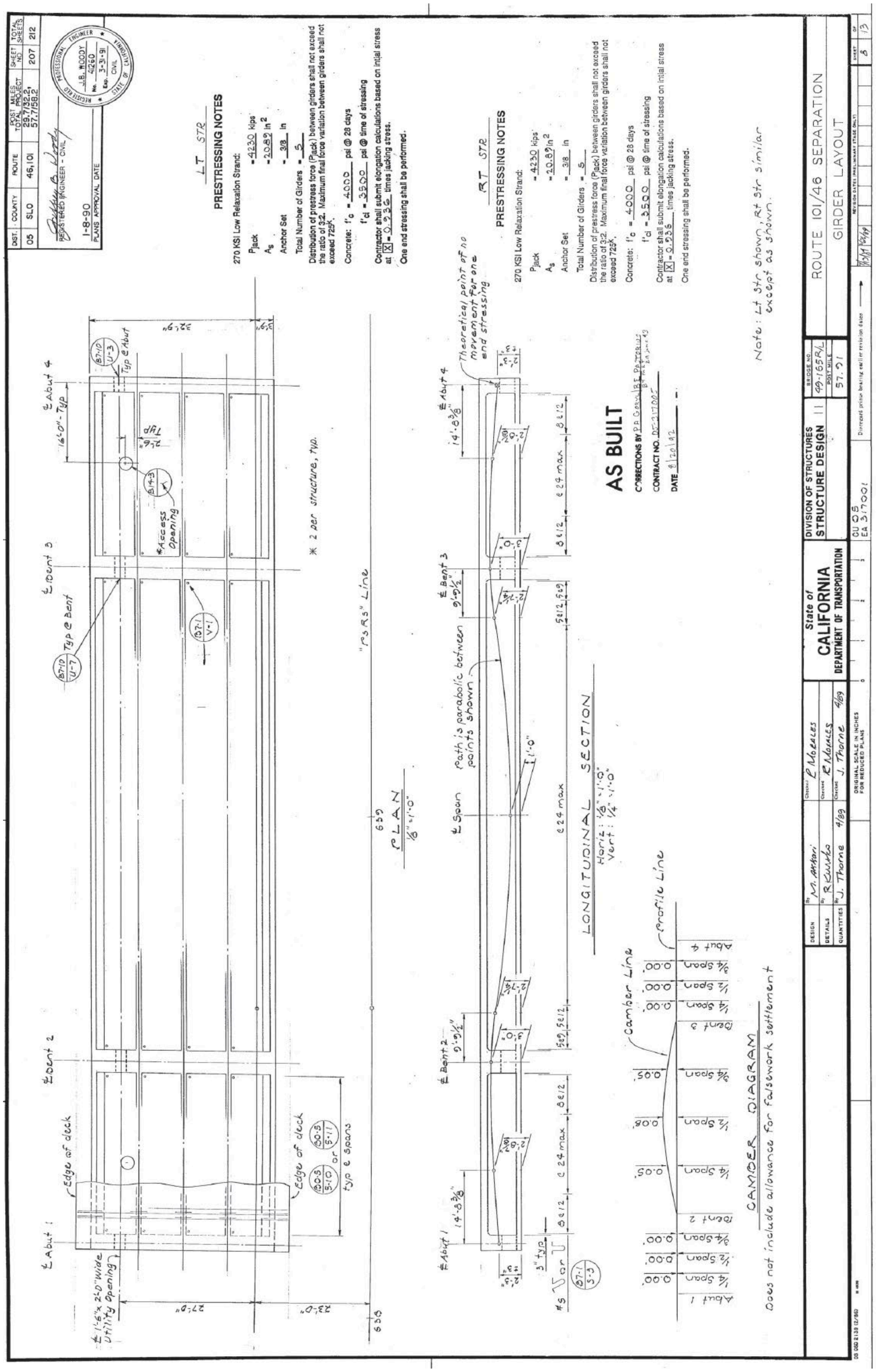

Figure A-31. Bridge 49 0165L details and prestressing notes

Dead Load Analysis – Interior Beam

Components and Attachments, *DC*

The total dead load applied on one box girder is

$$DL = 2.144\ kip/ft$$

The total moment due to dead load for one box girder is

$$M_{DC} = \mathbf{1,442.37\ kip \cdot ft}$$

Wearing surface, DW

There is no wearing surface applied on the bridge. Therefore, the moment due to wearing surface is, M_{DW} = 0 kip-ft

Live Load Analysis – Interior Girder

Compute Live Load Distribution Factor for an Interior Girder

The girder distribution factors (GDF) for one design lane and two design lanes loaded are calculated according to AASHTO LRFD as follows.

- One design lane loaded:

$$\left(1.75+\frac{S}{3.6}\right)\left(\frac{1}{L}\right)^{0.35}\left(\frac{1}{N_c}\right)^{0.45}=\left(1.75+\frac{8.88}{3.6}\right)\left(\frac{1}{98}\right)^{0.35}\left(\frac{1}{4}\right)^{0.45} = 0.4539$$

- Two or more design lanes loaded:

$$\left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25}=\left(\frac{13}{4}\right)^{0.3}\left(\frac{8.88}{5.8}\right)\left(\frac{1}{98}\right)^{0.25} = 0.6926 \quad \leftarrow \mathit{Governs}$$

Nominal Flexural Resistance

The area of prestressing steel per box is $A_{ps} = 5.22\ in^2$

f_s and f'_s are assumed to equal 60 ksi and k is 0.28.

The neutral axis is located at,

$$c = \frac{A_{ps}f_{pu} + A_s f_s - A'_s f'_s}{\alpha f'_c \beta b + kA_{ps}\frac{f_{pu}}{d_p}} = \frac{5.22 * 270 + 5.48 * 60 - 3.79 * 60}{0.85 * 4.0 * 0.85 * 106.5 + 0.28 * 5.22 * \frac{270}{36}}$$

$$\boldsymbol{c = 4.742\ in} < t_s = 8\ in$$

$$f_{ps} = f_{pu}\left(1 - k * \frac{c}{d_p}\right) = 270\left(1 - 0.28 * \frac{5.22}{36}\right) = 260.04\ ksi$$

$$a = \beta * c = 0.85 * 4.742 = 4.03\ in$$

The nominal capacity of one box is,

$$M_n = \left[A_{ps} * f_{ps} * \left(d_p - \frac{a}{2}\right) + A_s * f_s * \left(d_s - \frac{a}{2}\right) - A'_s * f'_s * \left(d'_s - \frac{a}{2}\right)\right]$$

$$M_n = \left[5.22 * 260.04 * \left(36 - \frac{4.03}{2}\right) + 5.48 * 60 * \left(27.87 - \frac{4.03}{2}\right) - 3.79 * 60 * \left(3.87 - \frac{4.03}{2}\right)\right]$$

$$\boldsymbol{M_n = 4,519\ kip{\cdot}ft}$$

Reliability Analysis

Resistance

The mean resistance, variance, and mean of the logarithm resistance were expressed as follows

$$\mu_R = \lambda_R M_n = 1.05 * 4{,}519 = \boldsymbol{4.745\ \ kip{\cdot}ft}$$

$$\sigma^2_{ln(R)} = ln(V^2_Y + 1) = ln(0.075^2 + 1) = \boldsymbol{0.00561}$$

$$\mu_{ln(R)} = ln(\mu_R) - \frac{1}{2}\sigma^2_{ln(R)} = ln(4{,}745) - \frac{1}{2} * 0.00561 = \boldsymbol{8.46\ \ kip{\cdot}ft}$$

Dead Load

The mean dead load and the standard deviation of the dead load are

$$\mu_D = \lambda_D D_n = 1.05 * 1{,}442 = \boldsymbol{1,515\ \ kip.ft}$$

$$\sigma_D = COV_D D_n = 0.10 * 1{,}442 = \mathbf{144.2 \quad kip{\cdot}ft}$$

Live Load

The moment due to truck load is 819 kip-ft and the moment due to lane load is 431 kip-ft. The nominal live load (HL93) is

$$LL_{nominal} = truck + lane\ load = 819 + 431 = 1{,}250\ kip.ft$$

The girder distribution factor for two lanes loaded governs. Therefore, the nominal live load per box with girder distribution factor for two lanes included (no dynamic load) is,

$$LL_{nominal/box} = (truck + lane\ load) * GDF = (819 + 431) * 0.6920$$

$$LL_{nominal/box} = 866\ kip{\cdot}ft$$

The mean live load with girder distribution factor for two lanes (no dynamic load) is expressed as,

$$LL_{mean} = \lambda_L * LL_{\frac{nominal}{box}} = 1.10 * 866$$

$$\boldsymbol{LL_{mean} = 952\ kip{\cdot}ft}$$

The mean dynamic load represents 10% of the mean live load. The mean live load and dynamic load is

$$LL + IM_{mean} = LL_{mean} * (1 + 0.10) = 952 * (1 + 0.10)$$

$$\boldsymbol{LL + IM_{mean} = 1{,}048\ kip{\cdot}ft}$$

The standard deviation of the live and dynamic load is

$$\sigma_{LL+IM} = COV_L * LL + IM_{mean} = 0.18 * 1{,}048 = \mathbf{188.6\ kip{\cdot}ft}$$

Extreme type 1 parameters

California live load best fits an extreme type 1 distribution. The parameters α and u are,

$$\alpha = \frac{1.282}{\sigma_{LL+IM}} = \frac{1.282}{188.6} = \mathbf{0.006797}$$

$$u = LL + IM_{mean} - 0.45 * \sigma_{LL+IM} = 1{,}048 - 0.45 * 188.6 = \mathbf{963.13\ \ \mathit{kip.ft}}$$

Monte Carlo

Using the graphical method, the reliability index and probability of failure are

$$\beta = -z = \mathbf{5.08}$$

$$P_f = \Phi^{-1}(-z) = \mathbf{1.91 * 10^{-7}}$$

The normal standard variate is then plotted versus the limit state values as shown in Figure A-32.

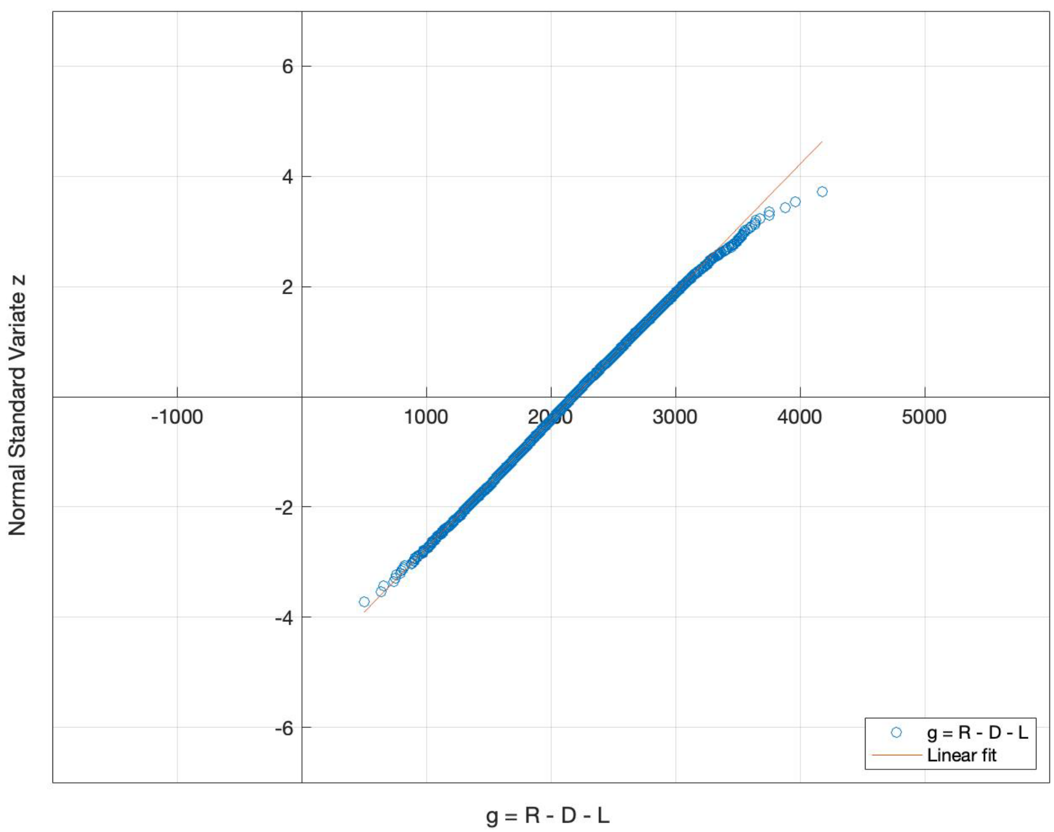

Figure A-32. Monte Carlo simulation for bridge 49 0165L

Bridge 54C0617 – Continuous span

Bridge Properties

General information about bridge 54C0617 is presented below.

Span length:		73 ft
Concrete:		f'_c: 4.0 ksi
Prestressing steel:		270 ksi low relaxation strand
$A_{ps,total}$	=	35.1 in^2
Girder spacing	=	8.33 ft
Depth of beam	=	3.5 ft
Top flange	=	7.75 in
Bottom flange	=	6 in
Web thickness	=	12 in
Area of one box	=	1,714 in^2
Number of cells	=	13
Number of webs	=	14

The cross-section of bridge 54C0617 and prestressing notes are shown in Figure A-33 and Figure A-34.

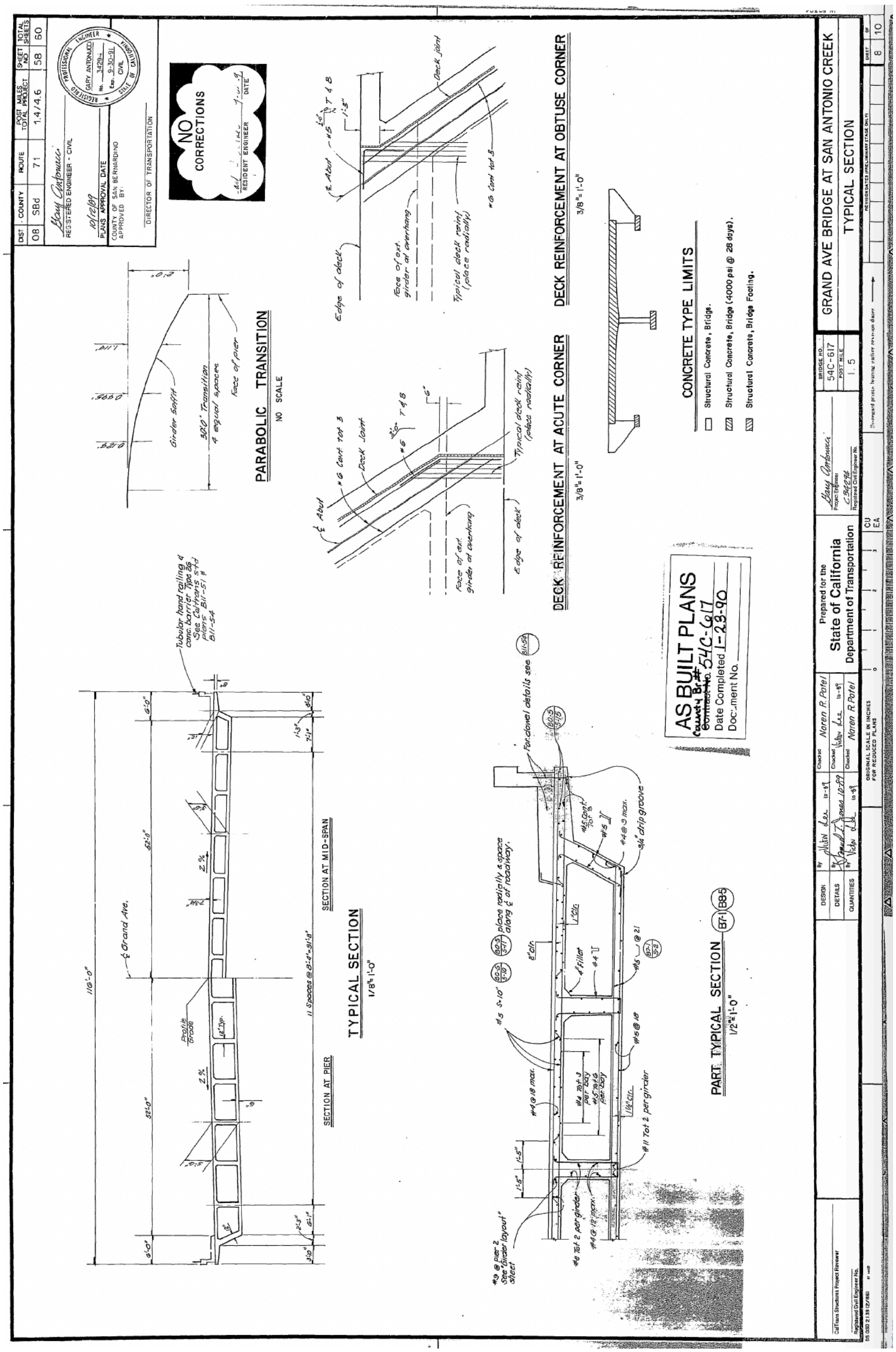

Figure A-33. Cross-section of bridge 54C0617

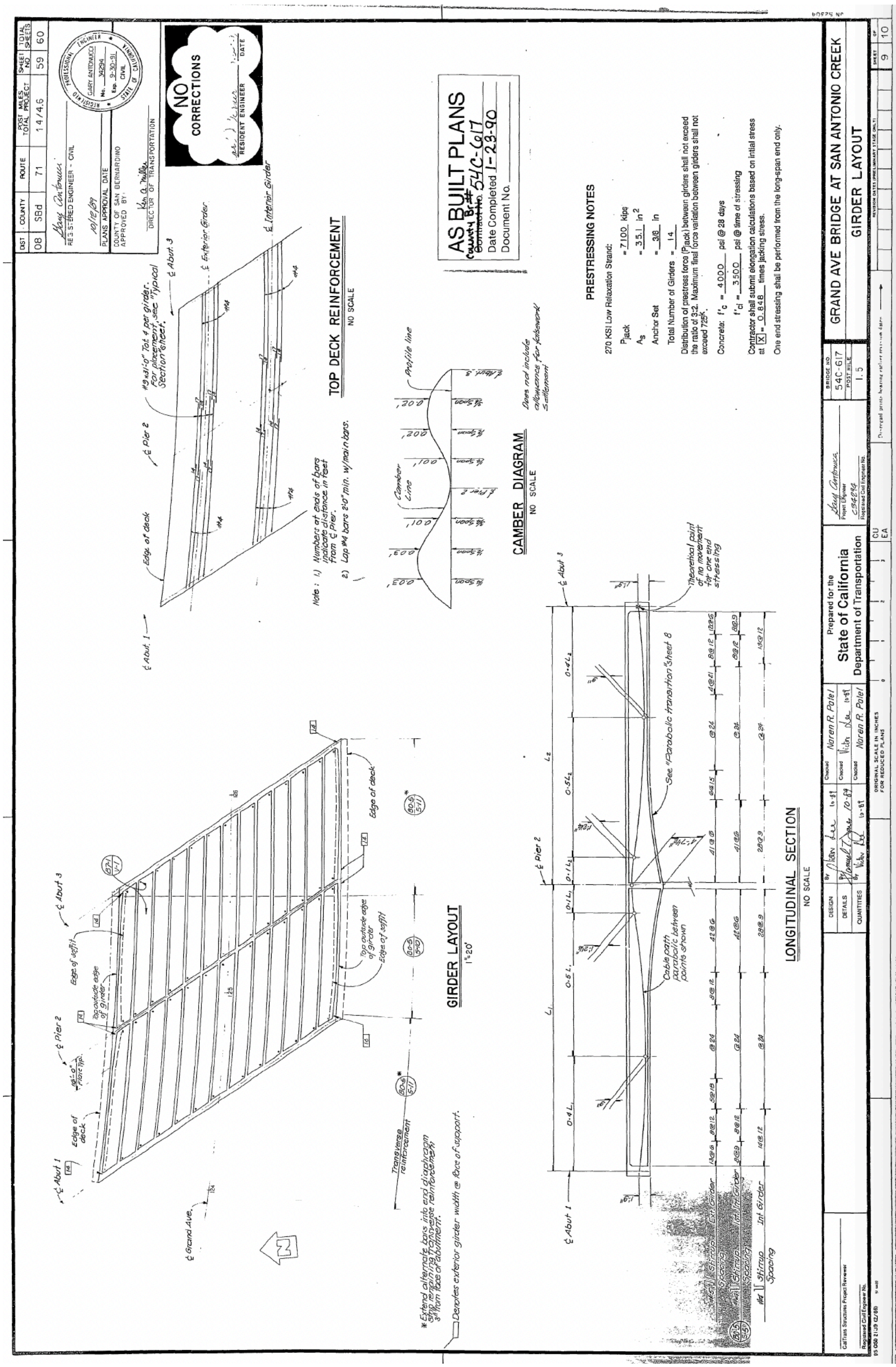

Figure A-34. Bridge 54C0617 details and prestressing notes

Dead Load Analysis – Interior Beam

Components and Attachments, *DC*

The total dead load applied on one box girder is

$$DL = 1.921\ kip/ft$$

The total moment due to dead load for one box girder is

$$M_{DC} = \mathbf{1,244.84}\ \boldsymbol{kip{\cdot}ft}$$

Wearing surface, DW

There is no wearing surface applied on the bridge. Therefore, the moment due to wearing surface is, M_{DW} = 0 kip-ft

Live Load Analysis – Interior Girder

Compute Live Load Distribution Factor for an Interior Girder

The girder distribution factors (GDF) for one design lane and two design lanes loaded are calculated according to AASHTO LRFD as follows.

- One design lane loaded:

$$\left(1.75+\frac{S}{3.6}\right)\left(\frac{1}{L}\right)^{0.35}\left(\frac{1}{N_c}\right)^{0.45}=\left(1.75+\frac{8.33}{3.6}\right)\left(\frac{1}{73}\right)^{0.35}\left(\frac{1}{8}\right)^{0.45} = 0.3552$$

- Two or more design lanes loaded:

$$\left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25}=\left(\frac{13}{8}\right)^{0.3}\left(\frac{8.33}{5.8}\right)\left(\frac{1}{73}\right)^{0.25} = 0.5686 \quad \leftarrow\ Governs$$

Nominal Flexural Resistance

The area of prestressing steel per box is $A_{ps} = 2.70in^2$

f_s and f'_s are assumed to equal 60 ksi and k is 0.28.

The neutral axis is located at,

$$c = \frac{A_{ps}f_{pu} + A_s f_s - A'_s f'_s}{\alpha f'_c \beta b + kA_{ps}\frac{f_{pu}}{d_p}} = \frac{2.70 * 270 + 9.31 * 60 - 3 * 60}{0.85 * 4.0 * 0.85 * 100 + 0.28 * 2.70 * \frac{270}{33}}$$

$$\boldsymbol{c = 3.752\ in} < t_s = 7.75\ in$$

$$f_{ps} = f_{pu}\left(1 - k * \frac{c}{d_p}\right) = 270\left(1 - 0.28 * \frac{3.752}{33}\right) = 261.4\ ksi$$

$$a = \beta * c = 0.85 * 3.752 = 3.19\ in$$

The nominal capacity of one box is,

$$M_n = \left[A_{ps} * f_{ps} * \left(d_p - \frac{a}{2}\right) + A_s * f_s * \left(d_s - \frac{a}{2}\right) - A'_s * f'_s * \left(d'_s - \frac{a}{2}\right)\right]$$

$$M_n = \left[2.70 * 261.4 * \left(33 - \frac{3.19}{2}\right) + 9.31 * 60 * \left(21.57 - \frac{3.19}{2}\right) - 3 * 60 * \left(3 - \frac{3.19}{2}\right)\right]$$

$$\boldsymbol{M_n = 2,756\ kip{\cdot}ft}$$

Reliability Analysis

Resistance

The mean resistance, variance, and mean of the logarithm resistance were expressed as follows

$$\mu_R = \lambda_R M_n = 1.05 * 2{,}756 = \boldsymbol{2,894\ \ kip{\cdot}ft}$$

$$\sigma^2{}_{ln(R)} = ln(V^2{}_Y + 1) = ln(0.075^2 + 1) = \boldsymbol{0.00561}$$

$$\mu_{ln(R)} = ln(\mu_R) - \frac{1}{2}\sigma^2{}_{ln(R)} = ln(2{,}894) - \frac{1}{2} * 0.00561 = \boldsymbol{7.97\ \ kip{\cdot}ft}$$

Dead Load

The mean dead load and the standard deviation of the dead load are

$$\mu_D = \lambda_D D_n = 1.05 * 1{,}245 = \boldsymbol{1,307\ \ kip{\cdot}ft}$$

$$\sigma_D = COV_D D_n = 0.10 * 1{,}245 = \mathbf{124.5} \quad \boldsymbol{kip.ft}$$

Live Load

The moment due to truck load is 829 kip-ft and the moment due to lane load is 415 kip-ft. The nominal live load (HL93) is

$$LL_{nominal} = truck + lane\ load = 829 + 415 = 1{,}244\ kip{\cdot}ft$$

The girder distribution factor for two lanes loaded governs. Therefore, the nominal live load per box with girder distribution factor for two lanes included (no dynamic load) is,

$$LL_{nominal/box} = (truck + lane\ load) * GDF = (898 + 443) * 0.5686$$

$$LL_{nominal/box} = 707\ kip{\cdot}ft$$

The mean live load with girder distribution factor for two lanes (no dynamic load) is expressed as,

$$LL_{mean} = \lambda_L * LL_{nominal/box} = 1.10 * 707$$

$$\boldsymbol{LL_{mean} = 778\ kip{\cdot}ft}$$

The mean dynamic load represents 10% of the mean live load. The mean live load and dynamic load is

$$LL + IM_{mean} = LL_{mean} * (1 + 0.10) = 778 * (1 + 0.10)$$

$$\boldsymbol{LL + IM_{mean} = 856\ kip{\cdot}ft}$$

The standard deviation of the live and dynamic load is

$$\sigma_{LL+IM} = COV_L * LL + IM_{mean} = 0.18 * 856 = \mathbf{154}\ \boldsymbol{kip{\cdot}ft}$$

Extreme type 1 parameters

California live load best fits an extreme type 1 distribution. The parameters α and u are,

$$\alpha = \frac{1.282}{\sigma_{LL+IM}} = \frac{1.282}{154} = \mathbf{0.00832}$$

$$u = LL + IM_{mean} - 0.45 * \sigma_{LL+IM} = 856 - 0.45 * 154 = \mathbf{786.7\ \ kip{\cdot}ft}$$

Monte Carlo

Using the numerical method, the probability of failure P_f and the reliability index β are

$$P_f = \frac{76}{10{,}000} = 7.\mathbf{60} * \mathbf{10^{-3}}$$

$$\beta = \mathbf{2.43}$$

Bridge 55 0670 – Continuous span

Bridge Properties

General information about bridge 55 0670 is presented below.

Span length:		156 ft
Concrete:		f'_c: 4.0 ksi
Prestressing steel:		270 ksi low relaxation strand
$A_{ps,total}$	=	78.37 in^2
Girder spacing	=	9.67 ft
Depth of beam	=	6.25 ft
Top flange	=	8.25 in
Bottom flange	=	6.625 in
Web thickness	=	12 in
Area of one box	=	2,447 in^2
Number of cells	=	7
Number of webs	=	8

The cross-section of bridge 55 0670 and prestressing notes are shown in Figure A-35 and Figure A-36.

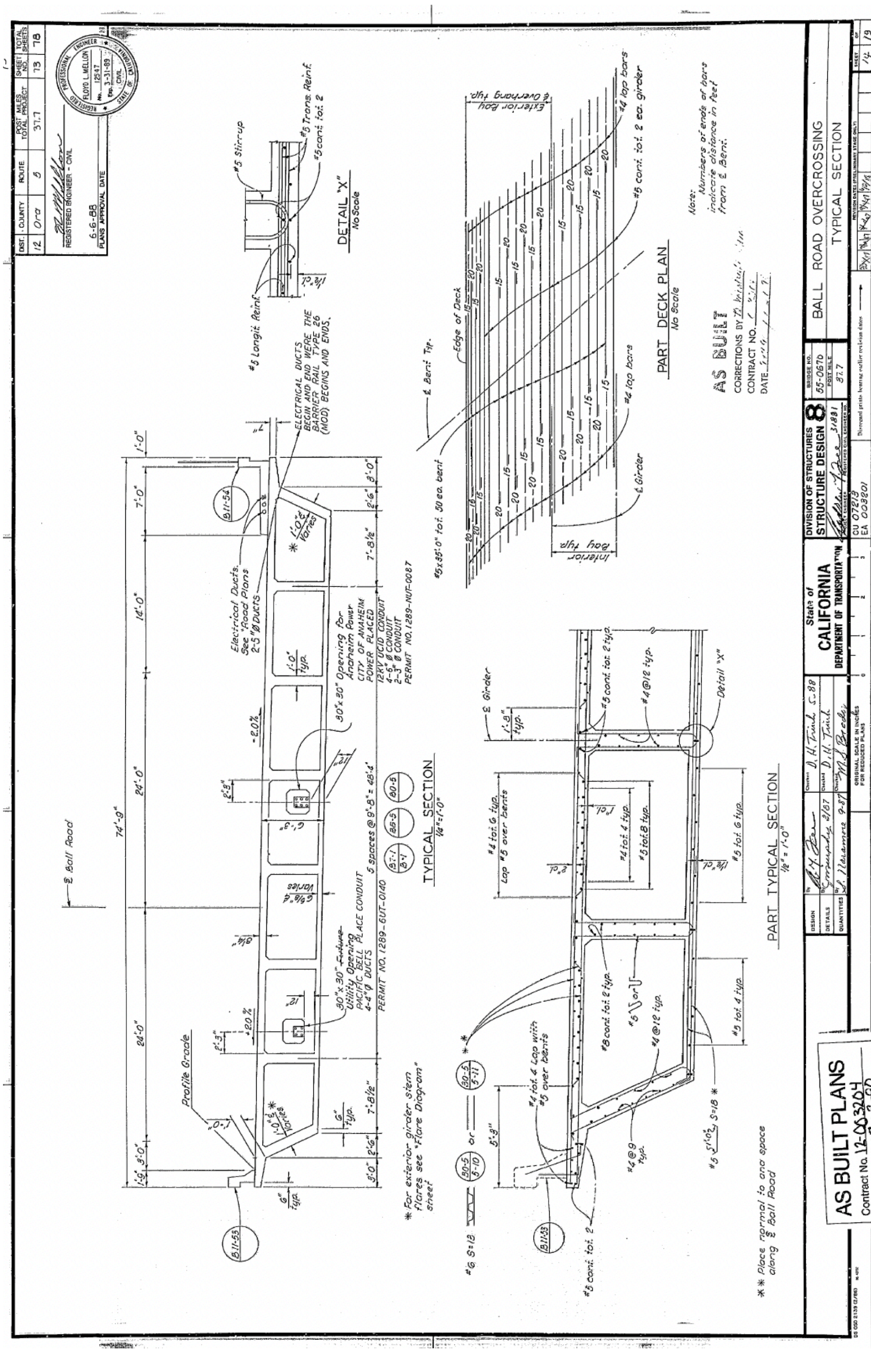

Figure A-35. Cross-section of bridge 55 0670

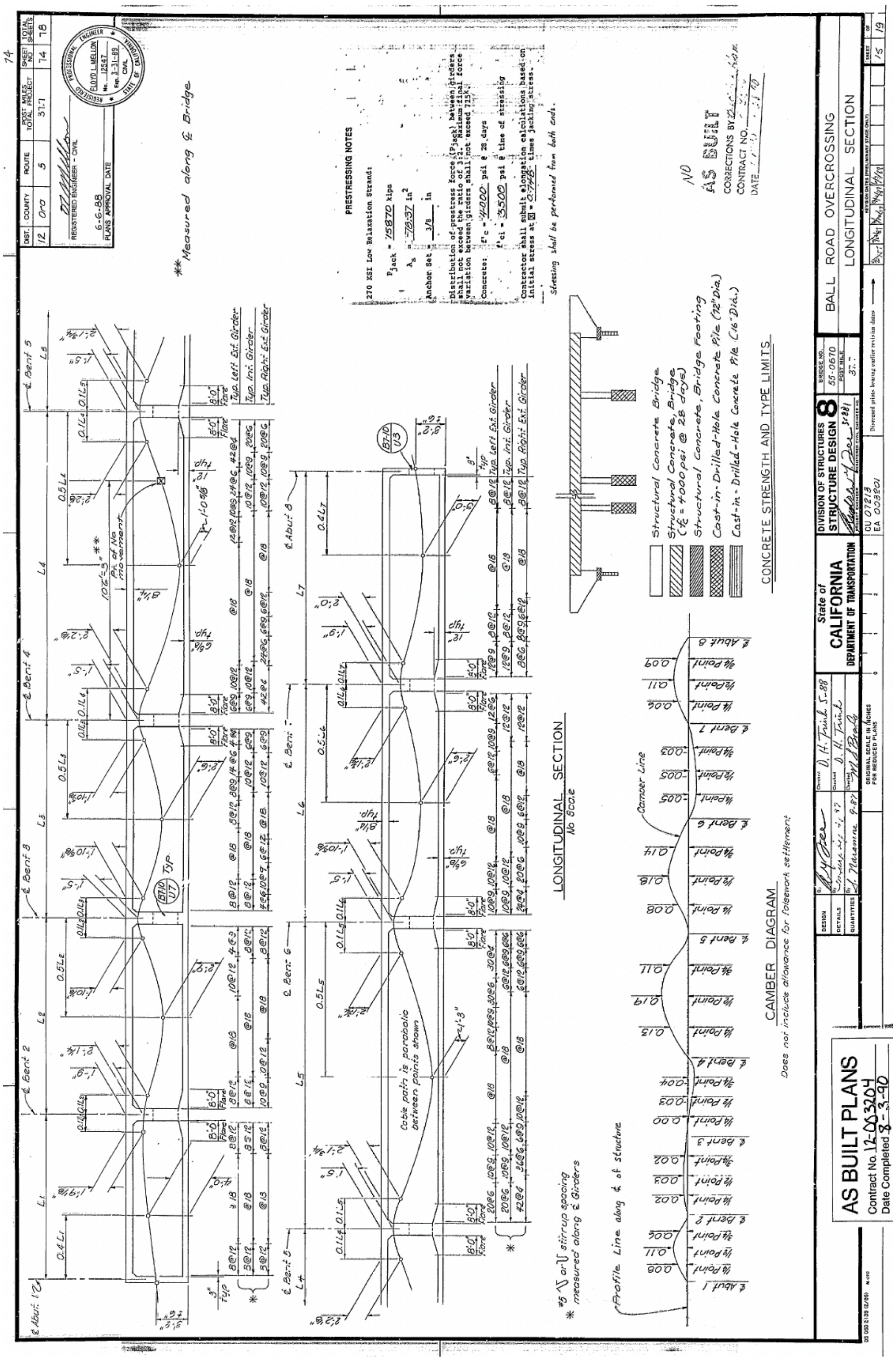

Figure A-36. Bridge 55 0670 details and prestressing notes

Dead Load Analysis – Interior Beam

Components and Attachments, *DC*

The total dead load applied on one box girder is

$$DL = 2.723\ kip/ft$$

The total moment due to dead load for one box girder is

$$M_{DC} = \boldsymbol{6,156.83\ kip{\cdot}ft}$$

Wearing surface, DW

There is no wearing surface applied on the bridge. Therefore, the moment due to wearing surface is, M_{DW} = 0 kip-ft

Live Load Analysis – Interior Girder

Compute Live Load Distribution Factor for an Interior Girder

The girder distribution factors (GDF) for one design lane and two design lanes loaded are calculated according to AASHTO LRFD as follows.

- One design lane loaded:

$$\left(1.75+\frac{S}{3.6}\right)\left(\frac{1}{L}\right)^{0.35}\left(\frac{1}{N_c}\right)^{0.45} = \left(1.75+\frac{9.67}{3.6}\right)\left(\frac{1}{156}\right)^{0.35}\left(\frac{1}{7}\right)^{0.45} = 0.3155$$

- Two or more design lanes loaded:

$$\left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25} = \left(\frac{13}{7}\right)^{0.3}\left(\frac{9.67}{5.8}\right)\left(\frac{1}{156}\right)^{0.25} = 0.5678 \quad \leftarrow\ Governs$$

Nominal Flexural Resistance

The area of prestressing steel per box is $A_{ps} = 11.20\ in^2$

f_s and f'_s are assumed to equal 60 ksi and k is 0.28.

The neutral axis is located at,

$$c = \frac{A_{ps}f_{pu} + A_s f_s - A'_s f'_s}{\alpha f'_c \beta b + k A_{ps}\frac{f_{pu}}{d_p}} = \frac{11.20 * 270 + 5.66 * 60 - 5.1 * 60}{0.85 * 4.0 * 0.85 * 116 + 0.28 * 11.20 * \frac{270}{60}}$$

$$\boldsymbol{c = 8.749\ in} > t_s = 8.25\ in$$

$$f_{ps} = f_{pu}\left(1 - k * \frac{c}{d_p}\right) = 270\left(1 - 0.28 * \frac{11.20}{60}\right) = 258.98\ ksi$$

$$a = \beta * c = 0.85 * 8.749 = 7.44\ in$$

The nominal capacity of one box is,

$$M_n = \left[A_{ps} * f_{ps} * \left(d_p - \frac{a}{2}\right) + A_s * f_s * \left(d_s - \frac{a}{2}\right) - A'_s * f'_s * \left(d'_s - \frac{a}{2}\right)\right]$$

$$M_n = \left[11.20 * 258.98 * \left(60 - \frac{7.44}{2}\right) + 5.66 * 60 * \left(53.47 - \frac{7.44}{2}\right) - 5.1 * 60 * \left(4.38 - \frac{7.44}{2}\right)\right]$$

$$\boldsymbol{M_n = 14,990\ kip{\cdot}ft}$$

Reliability Analysis

Resistance

The mean resistance, variance, and mean of the logarithm resistance were expressed as follows

$$\mu_R = \lambda_R M_n = 1.05 * 14{,}990 = \boldsymbol{15,740\ \ kip{\cdot}ft}$$

$$\sigma^2{}_{ln(R)} = ln(V^2{}_Y + 1) = ln(0.075^2 + 1) = \boldsymbol{0.00561}$$

$$\mu_{ln(R)} = ln(\mu_R) - \frac{1}{2}\sigma^2{}_{ln(R)} = ln(15{,}740) - \frac{1}{2} * 0.00561 = \boldsymbol{9.66\ \ kip{\cdot}ft}$$

Dead Load

The mean dead load and the standard deviation of the dead load are

$$\mu_D = \lambda_D D_n = 1.05 * 6{,}157 = \boldsymbol{6,465\quad kip{\cdot}ft}$$

$$\sigma_D = COV_D D_n = 0.10 * 6{,}157 = \mathbf{615.7\ \ kip{\cdot}ft}$$

Live Load

The moment due to truck load is 1,611 kip-ft and the moment due to lane load is 1,447 kip-ft. The nominal live load (HL93) is

$$LL_{nominal} = truck + lane\ load = 1{,}611 + 1{,}447 = 3{,}058\ kip{\cdot}ft$$

The girder distribution factor for two lanes loaded governs. Therefore, the nominal live load per box with girder distribution factor for two lanes included (no dynamic load) is,

$$LL_{nominal/box} = (truck + lane\ load) * GDF = (1{,}611 + 1{,}447) * 0.5678$$

$$LL_{nominal/box} = 1{,}736\ kip{\cdot}ft$$

The mean live load with girder distribution factor for two lanes (no dynamic load) is expressed as,

$$LL_{mean} = \lambda_L * LL_{\frac{nominal}{box}} == \ 1.10 * 1{,}736$$

$$\boldsymbol{LL_{mean} = 1{,}910\ kip{\cdot}ft}$$

The mean dynamic load represents 10% of the mean live load. The mean live load and dynamic load is

$$LL + IM_{mean} = LL_{mean} * (1 + 0.10) = 1{,}910 * (1 + 0.10)$$

$$\boldsymbol{LL + IM_{mean} = 2{,}101\ kip{\cdot}ft}$$

The standard deviation of the live and dynamic load is

$$\sigma_{LL+IM} = COV_L * LL + IM_{mean} = 0.18 * 2{,}101 = \mathbf{378.2\ kip{\cdot}ft}$$

Extreme type 1 parameters

California live load best fits an extreme type 1 distribution. The parameters α and u are,

$$\alpha = \frac{1.282}{\sigma_{LL+IM}} = \frac{1.282}{378.2} = \mathbf{0.00339}$$

$$u = LL + IM_{mean} - 0.45 * \sigma_{LL+IM} = 2{,}101 - 0.45 * 378.2 = \mathbf{1{,}930.8\ \mathit{kip{\cdot}ft}}$$

Monte Carlo

Using the graphical method, the reliability index and probability of failure are

$$\beta = -z = \mathbf{5.22}$$

$$P_f = \Phi^{-1}(-z) = \mathbf{9.17 * 10^{-8}}$$

The normal standard variate is then plotted versus the limit state values as shown in Figure A-37.

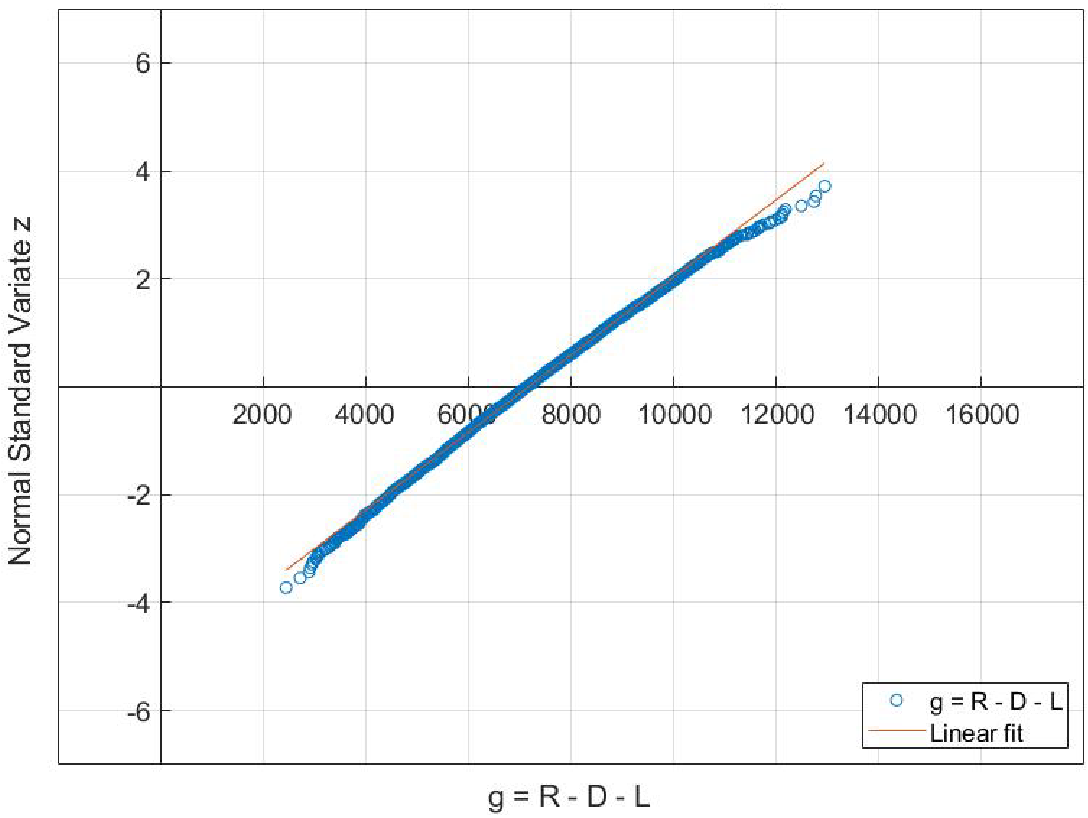

Figure A-37. Monte Carlo simulation for bridge 55 0670

Appendix B – Values of the CDF for the standard normal variable

Table B. Values of the cumulative distribution for the standard normal variable

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	5.00E-01	4.96E-01	4.92E-01	4.88E-01	4.84E-01	4.80E-01	4.76E-01	4.72E-01	4.68E-01	4.64E-01
-0.1	4.60E-01	4.56E-01	4.52E-01	4.48E-01	4.44E-01	4.40E-01	4.36E-01	4.33E-01	4.29E-01	4.25E-01
-0.2	4.21E-01	4.17E-01	4.13E-01	4.09E-01	4.05E-01	4.01E-01	3.97E-01	3.94E-01	3.90E-01	3.86E-01
-0.3	3.82E-01	3.78E-01	3.74E-01	3.71E-01	3.67E-01	3.63E-01	3.59E-01	3.56E-01	3.52E-01	3.48E-01
-0.4	3.45E-01	3.41E-01	3.37E-01	3.34E-01	3.30E-01	3.26E-01	3.23E-01	3.19E-01	3.16E-01	3.12E-01
-0.5	3.09E-01	3.05E-01	3.02E-01	2.98E-01	2.95E-01	2.91E-01	2.88E-01	2.84E-01	2.81E-01	2.78E-01
-0.6	2.74E-01	2.71E-01	2.68E-01	2.64E-01	2.61E-01	2.58E-01	2.55E-01	2.51E-01	2.48E-01	2.45E-01
-0.7	2.42E-01	2.39E-01	2.36E-01	2.33E-01	2.30E-01	2.27E-01	2.24E-01	2.21E-01	2.18E-01	2.15E-01
-0.8	2.12E-01	2.09E-01	2.06E-01	2.03E-01	2.00E-01	1.98E-01	1.95E-01	1.92E-01	1.89E-01	1.87E-01
-0.9	1.84E-01	1.81E-01	1.79E-01	1.76E-01	1.74E-01	1.71E-01	1.69E-01	1.66E-01	1.64E-01	1.61E-01
-1.0	1.59E-01	1.56E-01	1.54E-01	1.52E-01	1.49E-01	1.47E-01	1.45E-01	1.42E-01	1.40E-01	1.38E-01
-1.1	1.36E-01	1.33E-01	1.31E-01	1.29E-01	1.27E-01	1.25E-01	1.23E-01	1.21E-01	1.19E-01	1.17E-01
-1.2	1.15E-01	1.13E-01	1.11E-01	1.09E-01	1.07E-01	1.06E-01	1.04E-01	1.02E-01	1.00E-01	9.85E-02
-1.3	9.68E-02	9.51E-02	9.34E-02	9.18E-02	9.01E-02	8.85E-02	8.69E-02	8.53E-02	8.38E-02	8.23E-02
-1.4	8.08E-02	7.93E-02	7.78E-02	7.64E-02	7.49E-02	7.35E-02	7.21E-02	7.08E-02	6.94E-02	6.81E-02
-1.5	6.68E-02	6.55E-02	6.43E-02	6.30E-02	6.18E-02	6.06E-02	5.94E-02	5.82E-02	5.71E-02	5.59E-02
-1.6	5.48E-02	5.37E-02	5.26E-02	5.16E-02	5.05E-02	4.95E-02	4.85E-02	4.75E-02	4.65E-02	4.55E-02
-1.7	4.46E-02	4.36E-02	4.27E-02	4.18E-02	4.09E-02	4.01E-02	3.92E-02	3.84E-02	3.75E-02	3.67E-02
-1.8	3.59E-02	3.51E-02	3.44E-02	3.36E-02	3.29E-02	3.22E-02	3.14E-02	3.07E-02	3.01E-02	2.94E-02
-1.9	2.87E-02	2.81E-02	2.74E-02	2.68E-02	2.62E-02	2.56E-02	2.50E-02	2.44E-02	2.39E-02	2.33E-02
-2.0	2.28E-02	2.22E-02	2.17E-02	2.12E-02	2.07E-02	2.02E-02	1.97E-02	1.92E-02	1.88E-02	1.83E-02
-2.1	1.79E-02	1.74E-02	1.70E-02	1.66E-02	1.62E-02	1.58E-02	1.54E-02	1.50E-02	1.46E-02	1.43E-02

Table B. Values of the cumulative distribution for the standard normal variable (continued)

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-2.2	1.39E-02	1.36E-02	1.32E-02	1.29E-02	1.25E-02	1.22E-02	1.19E-02	1.16E-02	1.13E-02	1.10E-02
-2.3	1.07E-02	1.04E-02	1.02E-02	9.90E-03	9.64E-03	9.39E-03	9.14E-03	8.89E-03	8.66E-03	8.42E-03
-2.4	8.20E-03	7.98E-03	7.76E-03	7.55E-03	7.34E-03	7.14E-03	6.95E-03	6.76E-03	6.57E-03	6.39E-03
-2.5	6.21E-03	6.04E-03	5.87E-03	5.70E-03	5.54E-03	5.39E-03	5.23E-03	5.08E-03	4.94E-03	4.80E-03
-2.6	4.66E-03	4.53E-03	4.40E-03	4.27E-03	4.15E-03	4.02E-03	3.91E-03	3.79E-03	3.68E-03	3.57E-03
-2.7	3.47E-03	3.36E-03	3.26E-03	3.17E-03	3.07E-03	2.98E-03	2.89E-03	2.80E-03	2.72E-03	2.64E-03
-2.8	2.56E-03	2.48E-03	2.40E-03	2.33E-03	2.26E-03	2.19E-03	2.12E-03	2.05E-03	1.99E-03	1.93E-03
-2.9	1.87E-03	1.81E-03	1.75E-03	1.69E-03	1.64E-03	1.59E-03	1.54E-03	1.49E-03	1.44E-03	1.39E-03
-3.0	1.35E-03	1.31E-03	1.26E-03	1.22E-03	1.18E-03	1.14E-03	1.11E-03	1.07E-03	1.04E-03	1.00E-03
-3.1	9.68E-04	9.35E-04	9.04E-04	8.74E-04	8.45E-04	8.16E-04	7.89E-04	7.62E-04	7.36E-04	7.11E-04
-3.2	6.87E-04	6.64E-04	6.41E-04	6.19E-04	5.98E-04	5.77E-04	5.57E-04	5.38E-04	5.19E-04	5.01E-04
-3.3	4.83E-04	4.66E-04	4.50E-04	4.34E-04	4.19E-04	4.04E-04	3.90E-04	3.76E-04	3.62E-04	3.49E-04
-3.4	3.37E-04	3.25E-04	3.13E-04	3.02E-04	2.91E-04	2.80E-04	2.70E-04	2.60E-04	2.51E-04	2.42E-04
-3.5	2.33E-04	2.24E-04	2.16E-04	2.08E-04	2.00E-04	1.93E-04	1.85E-04	1.78E-04	1.72E-04	1.65E-04
-3.6	1.59E-04	1.53E-04	1.47E-04	1.42E-04	1.36E-04	1.31E-04	1.26E-04	1.21E-04	1.17E-04	1.12E-04
-3.7	1.08E-04	1.04E-04	9.96E-05	9.57E-05	9.20E-05	8.84E-05	8.50E-05	8.16E-05	7.84E-05	7.53E-05
-3.8	7.23E-05	6.95E-05	6.67E-05	6.41E-05	6.15E-05	5.91E-05	5.67E-05	5.44E-05	5.22E-05	5.01E-05
-3.9	4.81E-05	4.61E-05	4.43E-05	4.25E-05	4.07E-05	3.91E-05	3.75E-05	3.59E-05	3.45E-05	3.30E-05
-4.0	3.17E-05	3.04E-05	2.91E-05	2.79E-05	2.67E-05	2.56E-05	2.45E-05	2.35E-05	2.25E-05	2.16E-05
-4.1	2.07E-05	1.98E-05	1.89E-05	1.81E-05	1.74E-05	1.66E-05	1.59E-05	1.52E-05	1.46E-05	1.39E-05
-4.2	1.33E-05	1.28E-05	1.22E-05	1.17E-05	1.12E-05	1.07E-05	1.02E-05	9.77E-06	9.34E-06	8.93E-06
-4.3	8.54E-06	8.16E-06	7.80E-06	7.46E-06	7.12E-06	6.81E-06	6.50E-06	6.21E-06	5.93E-06	5.67E-06
-4.4	5.41E-06	5.17E-06	4.94E-06	4.71E-06	4.50E-06	4.29E-06	4.10E-06	3.91E-06	3.73E-06	3.56E-06

Table B. Values of the cumulative distribution for the standard normal variable (continued)

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-4.5	3.40E-06	3.24E-06	3.09E-06	2.95E-06	2.81E-06	2.68E-06	2.56E-06	2.44E-06	2.32E-06	2.22E-06
-4.6	2.11E-06	2.01E-06	1.92E-06	1.83E-06	1.74E-06	1.66E-06	1.58E-06	1.51E-06	1.43E-06	1.37E-06
-4.7	1.30E-06	1.24E-06	1.18E-06	1.12E-06	1.07E-06	1.02E-06	9.68E-07	9.21E-07	8.76E-07	8.34E-07
-4.8	7.93E-07	7.55E-07	7.18E-07	6.83E-07	6.49E-07	6.17E-07	5.87E-07	5.58E-07	5.30E-07	5.04E-07
-4.9	4.79E-07	4.55E-07	4.33E-07	4.11E-07	3.91E-07	3.71E-07	3.52E-07	3.35E-07	3.18E-07	3.02E-07
-5.0	2.87E-07	2.72E-07	2.58E-07	2.45E-07	2.33E-07	2.21E-07	2.10E-07	1.99E-07	1.89E-07	1.79E-07
-5.1	1.70E-07	1.61E-07	1.53E-07	1.45E-07	1.37E-07	1.30E-07	1.23E-07	1.17E-07	1.11E-07	1.05E-07
-5.2	9.96E-08	9.44E-08	8.95E-08	8.48E-08	8.03E-08	7.60E-08	7.20E-08	6.82E-08	6.46E-08	6.12E-08
-5.3	5.79E-08	5.48E-08	5.19E-08	4.91E-08	4.65E-08	4.40E-08	4.16E-08	3.94E-08	3.72E-08	3.52E-08
-5.4	3.33E-08	3.15E-08	2.98E-08	2.82E-08	2.66E-08	2.52E-08	2.38E-08	2.25E-08	2.13E-08	2.01E-08
-5.5	1.90E-08	1.79E-08	1.69E-08	1.60E-08	1.51E-08	1.43E-08	1.35E-08	1.27E-08	1.20E-08	1.14E-08
-5.6	1.07E-08	1.01E-08	9.55E-09	9.01E-09	8.50E-09	8.02E-09	7.57E-09	7.14E-09	6.73E-09	6.35E-09
-5.7	5.99E-09	5.65E-09	5.33E-09	5.02E-09	4.73E-09	4.46E-09	4.21E-09	3.96E-09	3.74E-09	3.52E-09
-5.8	3.32E-09	3.12E-09	2.94E-09	2.77E-09	2.61E-09	2.46E-09	2.31E-09	2.18E-09	2.05E-09	1.93E-09
-5.9	1.82E-09	1.71E-09	1.61E-09	1.51E-09	1.43E-09	1.34E-09	1.26E-09	1.19E-09	1.12E-09	1.05E-09
-6.0	9.87E-10	9.28E-10	8.72E-10	8.20E-10	7.71E-10	7.24E-10	6.81E-10	6.40E-10	6.01E-10	5.65E-10
-6.1	5.30E-10	4.98E-10	4.68E-10	4.39E-10	4.13E-10	3.87E-10	3.64E-10	3.41E-10	3.21E-10	3.01E-10
-6.2	2.82E-10	2.65E-10	2.49E-10	2.33E-10	2.19E-10	2.05E-10	1.92E-10	1.81E-10	1.69E-10	1.59E-10
-6.3	1.49E-10	1.40E-10	1.31E-10	1.23E-10	1.15E-10	1.08E-10	1.01E-10	9.45E-11	8.85E-11	8.29E-11
-6.4	7.77E-11	7.28E-11	6.81E-11	6.38E-11	5.97E-11	5.59E-11	5.24E-11	4.90E-11	4.59E-11	4.29E-11
-6.5	4.02E-11	3.76E-11	3.52E-11	3.29E-11	3.08E-11	2.88E-11	2.69E-11	2.52E-11	2.35E-11	2.20E-11
-6.6	2.06E-11	1.92E-11	1.80E-11	1.68E-11	1.57E-11	1.47E-11	1.37E-11	1.28E-11	1.19E-11	1.12E-11

Table B. Values of the cumulative distribution for the standard normal variable (continued)

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-6.7	1.04E-11	9.73E-12	9.09E-12	8.48E-12	7.92E-12	7.39E-12	6.90E-12	6.44E-12	6.01E-12	5.61E-12
-6.8	5.23E-12	4.88E-12	4.55E-12	4.25E-12	3.96E-12	3.69E-12	3.44E-12	3.21E-12	2.99E-12	2.79E-12
-6.9	2.60E-12	2.42E-12	2.26E-12	2.10E-12	1.96E-12	1.83E-12	1.70E-12	1.58E-12	1.48E-12	1.37E-12
-7.0	1.28E-12	1.19E-12	1.11E-12	1.03E-12	9.61E-13	8.95E-13	8.33E-13	7.75E-13	7.21E-13	6.71E-13
-7.1	6.24E-13	5.80E-13	5.40E-13	5.02E-13	4.67E-13	4.34E-13	4.03E-13	3.75E-13	3.49E-13	3.24E-13
-7.2	3.01E-13	2.80E-13	2.60E-13	2.41E-13	2.24E-13	2.08E-13	1.94E-13	1.80E-13	1.67E-13	1.55E-13
-7.3	1.44E-13	1.34E-13	1.24E-13	1.15E-13	1.07E-13	9.91E-14	9.20E-14	8.53E-14	7.91E-14	7.34E-14
-7.4	6.81E-14	6.31E-14	5.86E-14	5.43E-14	5.03E-14	4.67E-14	4.33E-14	4.01E-14	3.72E-14	3.44E-14
-7.5	3.19E-14	2.96E-14	2.74E-14	2.54E-14	2.35E-14	2.18E-14	2.02E-14	1.87E-14	1.73E-14	1.60E-14
-7.6	1.48E-14	1.37E-14	1.27E-14	1.17E-14	1.09E-14	1.00E-14	9.30E-15	8.60E-15	7.95E-15	7.36E-15
-7.7	6.80E-15	6.29E-15	5.82E-15	5.38E-15	4.97E-15	4.59E-15	4.25E-15	3.92E-15	3.63E-15	3.35E-15
-7.8	3.10E-15	2.86E-15	2.64E-15	2.44E-15	2.25E-15	2.08E-15	1.92E-15	1.77E-15	1.64E-15	1.51E-15
-7.9	1.39E-15	1.29E-15	1.19E-15	1.10E-15	1.01E-15	9.33E-16	8.60E-16	7.93E-16	7.32E-16	6.75E-16
-8.0	6.22E-16	5.74E-16	5.29E-16	4.87E-16	4.49E-16	4.14E-16	3.81E-16	3.51E-16	3.24E-16	2.98E-16
-8.1	2.75E-16	2.53E-16	2.33E-16	2.15E-16	1.98E-16	1.82E-16	1.68E-16	1.54E-16	1.42E-16	1.31E-16
-8.2	1.20E-16	1.11E-16	1.02E-16	9.36E-17	8.61E-17	7.92E-17	7.28E-17	6.70E-17	6.16E-17	5.66E-17
-8.3	5.21E-17	4.79E-17	4.40E-17	4.04E-17	3.71E-17	3.41E-17	3.14E-17	2.88E-17	2.65E-17	2.43E-17
-8.4	2.23E-17	2.05E-17	1.88E-17	1.73E-17	1.59E-17	1.46E-17	1.34E-17	1.23E-17	1.13E-17	1.03E-17
-8.5	9.48E-18	8.70E-18	7.98E-18	7.32E-18	6.71E-18	6.15E-18	5.64E-18	5.17E-18	4.74E-18	4.35E-18
-8.6	3.99E-18	3.65E-18	3.35E-18	3.07E-18	2.81E-18	2.57E-18	2.36E-18	2.16E-18	1.98E-18	1.81E-18
-8.7	1.66E-18	1.52E-18	1.39E-18	1.27E-18	1.17E-18	1.07E-18	9.76E-19	8.93E-19	8.17E-19	7.48E-19
-8.8	6.84E-19	6.26E-19	5.72E-19	5.23E-19	4.79E-19	4.38E-19	4.00E-19	3.66E-19	3.34E-19	3.06E-19
-8.9	2.79E-19	2.55E-19	2.33E-19	2.13E-19	1.95E-19	1.78E-19	1.62E-19	1.48E-19	1.35E-19	1.24E-19