ELECTRIC POWER GENERATION EXPANSION

IN DEREGULATED MARKETS

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DISSERTATION ABSTRACT

ELECTRIC POWER GENERATION EXPANSION

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The generation expansion problem involves increasing electric power generation capacity in an existing power network. In competitive environment, power producers, distributors, and consumers all make their own decisions with the aim of maximizing their own utilities from power transactions. Therefore, models that incorporate a behavioral structure in decision making of power producers are needed to analyze market operation and to make better decisions.

In this dissertation, the main purpose is to develop generation expansion models for deregulated power markets using game theoretic approaches in order to make accurate analysis of generation capacity under transmission constraints in the power network and uncertainties in the power markets. Three new generation expansion models are

introduced. The first model is a generation expansion model that incorporates the features of the transmission grid to the investment model to study the interaction between competition and network transfer capabilities in capacity expansion. The other two models are generation capacity expansion models under competition that incorporate the uncertainty on electricity demand and fuel costs. Numerical examples and analysis on the use of the models are given using a 5-bus and a 24-bus power network.

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Computer software used	AMPL
	CPLEX
	MATLAB
	Microsoft Office Excel
	Microsoft Office Word

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NOMENCLATURE

Parameters

$\mathcal{L}$: Set of lines, $ij \in \mathcal{L}$

$\mathcal{B}$: Set of buses, $k \in \mathcal{B}$

$\mathcal{P}$: Set of producers, $p, q \in \mathcal{P}$

$\mathcal{T}$: Set of states of Nature

$\mathcal{F}$: Set of flowgates

$\mathcal{F}_f$: Set of transmission lines of flowgate f

$\mathcal{U}$: Set of fuel types, $u \in \mathcal{U}$

G_{pk}^{0}: Current capacity of producer p at node k

$G_{pk}^{\max}$: Maximum new capacity installation of producer p at node k

C_{pk}^{0}: Cost of power generation from existing capacity of producer p at node k

C_{pk}: Cost of power generation of producer p at node k

I_{pk}: Capital investment cost for capacity expansion of firm p at node k

T_{ij}: Thermal limit on line i-j

F_f: Maximum capacity of the flowgate f

$PTDF_{ij}^{k}$: Power transfer distribution factor of node k for link ij

G_{pk}^{u} Upper bound on generation at real-time for producer p at node k

G_{pk}^{l} Lower bound on generation at real-time for producer p at node k

α_k, β_k : Vertical intercept and the slope of the inverse demand function at node k

T : Number of states

C_{tuk} : Cost of power generation at state t from fuel type u at node k

C_{tuk}^{0} : Cost of power generation from existing capacity at state t of producer p at node k

I_{puk} : Capital investment cost for expansion of firm p on fuel type u at node k

α_{tk}, β_{tk} : Vertical intercept and the slope of the inverse demand function at state t at node k

G_{puk}^{0} Existing capacity limit of producer p from fuel type u at node k

$G_{puk}^{\max}$ Maximum capacity installation of producer p from fuel type u at node k

Decision Variables

g_{pk}^{0} : Amount of power generated at node k from existing capacity of producer p

g_{pk} : Amount of power generated at node k from new capacity of producer p

G_{pk} : Total amount generated at real-time by producer p at node k

s_{pk}^{0} : Amount of demand met at node k from existing capacity of producer p

s_{pk} : Amount of demand met at node k from new capacity of producer p

D_k : Demand at node *k* for

P_k : Price of power at node *k*

y_k : Amount of power transmitted by the grid operator from hub to node *k*

λ_k : Congestion based transmission price at node *k*

g^0_{tpuk} : Amount of power generated at state *t* from existing capacity of producer *p* from fuel type *u* at node *k*

g_{tpuk} : Amount of power generated at state *t* from new capacity of producer *p* from fuel type *u* at node *k*

g^n_{puk} : Amount of new capacity of producer *p* from fuel type *u* at node *k*

s_{tpk} : Amount of demand met by producer *p* at state *t* at node *k*

D_{tk} : Demand for at state *t* at node *k*

P_{tk}: Price of power at state *t* at node *k*

y_{tk} : Amount of power transmitted by the grid operator at state *t* from hub to node *k*

λ_{tk} : Congestion based transmission price at state *t* at node *k*

$E(y_k)$: Expected transmission quantity at node *k*

$ER(p)$: Expected revenue of producer *p*

$EC(p)$: Expected cost of producer *p*

ECR : Expected congestion revenue for transmission owner

CHAPTER 1

INTRODUCTION

The electric power industry throughout the world has been undergoing restructuring in many power markets, which includes major markets in the United States such as Pennsylvania-New Jersey-Maryland Power Market (PJM), California Independent System Operator (CAISO), and New York Independent System Operator (NYISO). Restructuring has also many examples in Europe, Africa, and Latin America.

The main aim of the introduction of deregulation to the electricity industry is to obtain more liberalized and therefore more efficient power markets. In competitive environment, power producers, distributors, and consumers all make their own decisions with the aim of optimizing their own objectives from power transactions. For that purpose, these market agents have to consider the actions of other market players in their operational and planning decisions. Thus, planning decisions such as addition of new generation capacity is expected to be highly influenced by competition.

The generation expansion problem involves increasing electric power generation capacity in an existing power network. In traditional (vertically integrated) power markets, the expansion decisions are made by the central operator of the power network, considering all existing generation and available transmission capacity in the network as well as the electricity demand. As the central operator decides which generator to operate to meet the demand, dispatches the power, and decides on the selling price of power, the regulated model of generation capacity expansion can be easily formulated with a least-cost structure of the operation and investment costs of capacity expansion over the whole system. The model then becomes a minimization problem subject to operational constraints. In deregulated markets, the generators are not owned by a central operator. The electric power companies decide on the power quantities for their generation as well as expanding their generation capacities. With the restructuring, the generation companies now can make their own decisions in order to maximize their profits. While these companies maximize their profits, they need to consider that the market operates under a competitive structure with the existence of other market players. That is, the decisions and behaviors of other generation companies operating in the market are equally important for a power producer in making decisions as the demand quantities or transmission availability. Therefore, models that incorporate a behavioral structure in decision making of power producers are needed to analyze market operation and to make better decisions. Game theoretic approaches are appropriate to study these kinds of behavioral decision structures in power markets as they provide tool for modeling competitive markets. Generation expansion under competition remains one important topic to be explored using game theoretic approaches as very few research projects are now emerging in this area.

In this research, the main purpose is to develop generation expansion models for deregulated power markets, which incorporate the new power market structure. The term "new power structure" refers to the changes in power trading, demand's price sensitivity, transmission system's functioning, and system operation. In order to build a generation expansion model for deregulated power markets, it is necessary to incorporate the above elements in the decision model. This research's purpose is to construct capacity investment models under competition using game theoretic approaches with the aim of making accurate analysis of generation capacity and its effect on power market and network operation. With this purpose, two main factors in power markets are considered while modeling capacity investments: transmission network and uncertainty in power markets.

The first important factor that is considered in this dissertation is the power transmission network in power generation expansion. Capacity expansion may have several effects on the market's situation. One of these is the effect on the power network's operation. Increased generation capacity in a power network may encounter transmission congestion in the transmission network as a result of excess regional capacity investments. Expansion, which actually aims to increase the investors bargaining power, may in fact have a negative impact on generator companies' profits as a result of the increased transmission congestion and decreased ability to transfer power to demand locations. The first model that is introduced in this dissertation is a generation expansion model that incorporates the features of the transmission grid to the investment model to study the interaction between competition and network transfer capabilities in capacity expansion.

The second important factor that is considered in this dissertation is mitigating the risk involved in the decision of capacity expansion in restructured power markets. Since generation capacity expansion is an investment decision that needs to be made with the current information available to the investor generation companies, there is an uncertainty involved in this decision, which brings risk to the investment decisions. Therefore, the generation expansion models, either aiming to analyze generation capacity or the network operation, need to incorporate the uncertainty involved in the power markets. Two models are introduced in this dissertation for generation capacity expansion under competition and uncertainty in electricity demand and fuel cost.

The remainder of this dissertation is organized as follows: In chapter 2, a review of related research currently available and power market concepts, preliminary information and assumptions used for formulating the restructured power markets in this dissertation are explained. Chapter 3 describes the generation expansion framework under transmission constraints. Chapter 4 describes the generation expansion models under uncertainty, competition, and network constraints. In chapter 5, conclusions are presented.

CHAPTER 2

REVIEW OF LITERATURE AND BACKGROUND

2.1. Literature Review

The generation expansion problem involves increasing electric power generation capacity in an existing power network. Generation expansion decisions involve the *location* and the *amount* of new capacity to be built. Generation expansion in the regulated power market structure aims to optimize the total generation capacity in the whole power network, considering the future electricity demand. As in the regulated markets the generators are owned by the system operator, one decision model that minimizes the total costs to the entire generator set is sufficient. This approach is known as the least-cost investment approach and has been extensively studied in centralized electricity markets. The generation expansion model in this case is called the traditional and static generation expansion planning model.

Among the major objectives of the traditional generation expansion planning are the minimization of investment and operating costs of capacity, and meeting the demand criteria [1]. This approach has been used with different optimization techniques such as stochastic programming [2], nonlinear programming [3], heuristic optimization methods as genetic algorithms [4], [5], and evolutionary programming [6]. In addition, multiple criteria decision making approaches were applied to the generation expansion problem [7], [8]. In the static model approach the decisions are based on inelastic fixed demand quantities. In restructured markets, one important change in production cost modeling is the introduction of the elasticity of demand [9], which refers to the decentralization of the decisions after deregulation.

With the deregulation, the generation companies are owned by independent power producers, transmission network is owned by independent entities, and the market involves competition of the suppliers. Models that incorporate the competition of generators have been introduced to the power market literature. These models mainly focus on the decision of bidding strategies of the generator companies under competition using game theoretic approaches. Demonstrating the usefulness of the game theoretic approaches is amongst the most important research needs in electricity resource planning [10]. Finding the equilibrium solution for market games is desirable for market participants and regulators [11]. According to [12], searching for the Nash-equilibrium is important as it provides participants' best strategies in different market conditions. Recent examples of research on bidding strategy modeling using game-theoretic approaches involve [13], [14], and [15]. In addition these kinds of approaches have been used for environmental problems in deregulated markets [16] and modeling the network effects on strategic bidding [17] and [18]. However, few models have been proposed for studying the capacity investment decisions in oligopolistic power markets. Generation capacity expansion remains one important topic to be explored using these approaches as very few research projects are now emerging in this area.

In power markets, restructuring has been initiated in order to introduce competition among power producers to obtain more liberalized and therefore more efficient power markets. In other words, the expectation is that the power markets will benefit from the competition of generation companies with decreasing market power and power prices. Under this market structure, the generators are competing in an oligopoly in order to maximize their profit objectives. Although the power market restructuring had the aim to obtain perfectly competitive markets in which generators bid under perfect competition for prices at competitive levels, in the power system literature, most of the time, perfect competition is referred to as unrealistic as it is a very strong assumption [19]. Perfect competition is used widely in the literature for comparison purposes [20].

Chuang et al. [21] have proposed a single-period Cournot competition model that incorporates plant capacity and energy balance constraints. Murphy and Smeers [19], [22] have introduced three new models concerning capacity expansion under competition in electricity markets. The open-loop Cournot model given in [19] is based on a single-stage decision process where the capacity expansion and production decisions are made at the same stage. The other two models refer to the two-stage (closed-loop) and three-stage decision processes in which the decisions are also based on Cournot competition. These two models are more realistic. However, they are also more difficult to solve and the equilibrium solution is not guaranteed. Neither of these models considers the transmission network in the decision framework to prevent additional complexity. Adding new generation capacity to a power network may have positive or negative impacts depending on where the new capacity will be installed and the amount of such capacity. Shahidehpour et al. [23] have pointed out that generation companies' investment decisions may challenge the transmission system security. Therefore, it is important for transmission owners and operators to make investments in transmission to overcome future conditions and demand growth in the power network. The first model that is presented aims to contribute to the lack of models in the literature that incorporate the transmission grid in a power system subject to competitive generation expansion decisions using game theoretic approaches. The competition is based on Cournot which is known as the "competition on quantities". The assumption of Cournot competition is based on three major facts:

a) although power companies may compete on price in the short term, they also need to make long-term commitments to capacity, which supports adopting competition on quantities, i.e. the Cournot model

b) power plant owners may reduce generation to deliberately induce higher profits, a feature of Cournot competition

c) the Cournot model can render simple analytical expressions that can be easy to manipulate.

Another important factor in power markets that may effect the investment decisions is the uncertainty in market parameters on which the investment decisions are based. As the investment decisions are made today in consideration of future market conditions, a level of risk is involved in these decisions that needs to be incorporated for more realistic models and investment decisions. Two important sources of risk in power markets are the demand uncertainty and fuel cost uncertainty. In regulated power markets, these kinds of risks were generally passed to the consumers by reflecting the uncertainties onto the power prices [19]. In deregulated power markets, the need for the power prices to stay in competitive levels gives rise to the need for considering these factors as part of the decision models in order to obtain a complete investment analysis. These kinds of investment models in deregulated markets can create investment plans for producers that can mitigate the above mentioned risk factors and also may help power prices to stay in competitive levels.

In [19] and [22], capacity expansion under Cournot competition is studied with several models. However, in these models instead of accounting for the uncertainty in power markets, the risks are considered with increased discount rates with respect to the risk-free rate. Although, in [19] it has been noted that representation of uncertainty, such as found in [24] is necessary for capacity investment decisions. The second model that is introduced in this dissertation aims to fill this research need with a generation expansion model under competition which incorporates the uncertainty in power demand and uncertainty in fuel costs with network constraints. Therefore, the second model adds up on the first model of generation expansion under transmission network constraints with the purpose of creating a risk mitigating model by considering the sources of uncertainty in the power markets.

2.2. Background

Electricity markets involve suppliers and consumers that bid for power generation and consumption. Many electricity markets throughout the world include some form of supplier bidding in which suppliers submit MW outputs, along with the associated prices [25]. All markets based on spot pricing consist of supplier bidding for electricity, in terms of power quantities or prices, which allows the consumers to react to power prices [25]. As represented in figure 2.1, several power producers in the power markets compete to meet the power demand under the constraints of the power network.

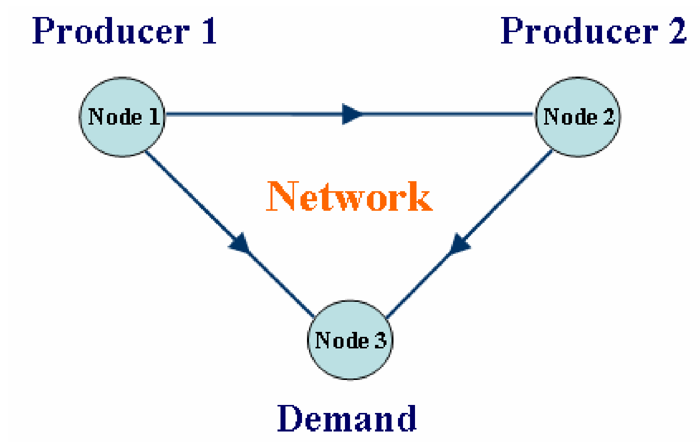

Fig. 2.1. Restructured market operation

2.2.1. Consumer Demand

In traditional production cost models, an inelastic demand is assumed and this demand has to be met with a penalty for unserved load [9]. To model the capacity expansion in deregulated markets, at first it is needed to identify the behavior of consumers. In this study, a linear inverse demand function is assumed as it is a good approximation of a nonlinear demand curve in the neighborhood of the equilibrium, and it simplifies the mathematics of the models [19]. In figure 2.2, the curve shows a general form of a linear inverse demand function that is used in this dissertation and indicates the price responsiveness of the consumers.

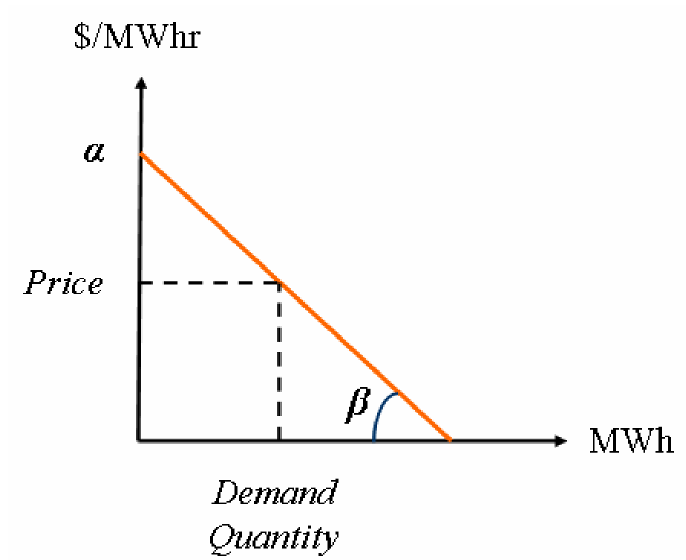

Fig. 2.2. Consumer demand curve

It can be observed that the power price and the demand quantity are inversely proportional. The reason for this linear function is to simplify the mathematics of the framework and to ensure that a unique equilibrium solution exists. According to the demand function illustrated, the price at node *k* is defined in (2.1).

$$P_k = \alpha_k - \beta_k D_k \tag{2.1}$$

The demand function is used in the producer decision model to calculate its revenue defined as the price multiplied by the quantity sold. With a linear demand curve, the benefit function of the consumers (the integral of the demand function) can be calculated as in (2.2) [20], [25].

$$\text{Consumer benefit} = \alpha_k D_k - \frac{1}{2}\beta_k D_k^2 \tag{2.2}$$

This benefit function is used in the equilibrium model to compute the total social welfare defined as the benefit to the consumers minus the cost to the generators [20]. The demand bids may vary for each customer. The variation may be depending on a single parameter which affects both the intercept and the slope as in [25], or it may be assumed that only the intercept of the curves is under manipulation as in [26]. In the numerical examples in this research, only the variation on the intercept is considered.

2.2.2. Cost of New Generation

Regarding the cost of new generation, two types of cost need to be considered: operating and capital costs. An operating cost is incurred every time the power plant is operated and it includes costs such as maintenance and fuel costs. The capital cost, on the other hand, is non-recurrent and it involves the initial investment and the salvage value at the time of disposal of the power plant. The operating cost is usually expressed in $/MWh and the capital cost is expressed in $/MW. To write these two quantities in similar measurement units, the capacity pricing method suggested by Stoft in [27] is adopted. In this method, the total capital cost is annualized over the life of the plant using an appropriate discount rate, and then divided by the number of operating hours per year.

The investment decision is based on long term power purchases, which are made bilaterally between consumers and producers, to meet the base load. Therefore, the new capacity is expected to be fully used in order to meet the resultant load. Under the assumption that the plant will operate at full-capacity (a capacity factor equal to one), the cost of new generation can be represented by investment and operation terms both expressed in $/MWh. As suggested in [27], in this research, the cost of new capacity is represented with the per unit annualized cost of investment, which is also followed in [19] and [22]. The cost of investment of the power generators are given per unit values with constant marginal cost of operation.

2.2.3. Transmission System Owner

A power transmission system model usually includes constraints such as thermal limits on transmission lines and power balances at each node of the network. Capacity expansion may encounter congestion in the transmission network by constrained single-line limits as well as flowgate transfer capabilities. In [28], flowgates are defined as bottlenecks in transmission lines where congestion takes place during certain operation conditions. In addition to single-line constraints, flowgates may be used for stability purposes to control the reliability of the network. A flowgate is a group of lines in the network that has limited total flow capability in its lines. Flowgates may be used for thermal or voltage based reasons in a power network. Flowgates may also be imposed to limit the flows within interregional coordinated markets, which help to control the regional operations.

Flowgates represent the available transfer capability through a congested region including more than one transmission line of the network. The basic idea is to assign a portion of the flowgate limit to each operating generator in the constrained area using a linear combination of the PTDF matrix elements. With these PTDFs, a single inequality constraint based on flowgate total can be added to the power dispatch model in a manner exactly analogous to that used when mitigating thermal constraints on single elements [29]. The inequality constraint is of the form shown in (2.3) where y_k is the amount of power transmission to node k, and ij is summed over all lines that make up the flowgate to get a flowgate total. This total must be kept less than some maximum limiting value, F_f.

$$\sum_{ij \in \mathcal{F}_f} \left(\sum_k PTDF_{ij}^k * y_k \right) \le F_f \quad \forall f \in \mathcal{F} \qquad (2.3)$$

Flowgate constraints, like other transmission related constraints such as thermal limits of transmission lines, are for securing the transmission network operation and therefore they should be included in the Transmission System Owner's decision model. Thus, transmission line and flowgate constraints can both be modeled mathematically by making use of Power Transfer Distribution Factors (PTDFs) and be included in the decision models that consider the transmission network.

Transmission System Owner (TSO) decides what amount of power needs to be transmitted to each node of the network using the transmission lines, while safeguarding the single line and flowgate limits. In order to model the TSO, the approach proposed in

[13] is used. In this approach the transmission owner is assumed to behave under the Bertrand competition assumption, where it cannot control the power prices. The revenue of the TSO is the amount of total congestion rent that is collected from the network for transmitting power through the congested regions. With this aim, the objective function and the constraints given in [13] are used to formulate the equilibrium model for generation expansion explained in the proceeding section.

The TSO is modeled based on [13] with the following profit maximizing objective:

$$Max \sum_{k} \lambda_k y_k \tag{2.4}$$

subject to

thermal capacity constraints:

$$\sum_{k} PTDF_{ij}^{k} y_k \leq T_{ij} \quad \forall ij \in \mathcal{L} \quad (\mu_{ij}^{+}) \tag{2.5}$$

$$-\sum_{k} PTDF_{ij}^{k} y_k \leq T_{ij} \quad \forall ij \in \mathcal{L} \quad (\mu_{ij}^{-}) \tag{2.6}$$

and the flowgate constraints:

$$\sum_{ij \in \mathcal{F}_f} \left(\sum_{k} PTDF_{ij}^{k} * y_k \right) \leq F_f \quad \forall f \in \mathcal{F} \quad (\mu_f) \tag{2.7}$$

In this model, the coefficient λ_k denotes the congestion based transmission prices that need to be paid for transmitting power from hub to node *k* which is the dual price of the market clearing constraint written for that node. TSO is paid λ_k to get power from node *k* to the hub and pays λ_k to convey power from the hub to node *k*. The TSO model includes the constraints associated with both thermal and network stability. The thermal constraints are represented using the PTDFs of each line following the framework in [29]. This framework coincides with the network stability modeling that is explained above and used in the TSO model, as both the thermal and flowgate stability consideration of the network are represented in terms of generation PTDF amounts.

2.2.4. Market Clearing

Given that the power transmission amounts are decided by the Transmission System Owner and the generation and sales quantities are decided by the power producers, then these quantites need to be balanced for every node at the power network with market clearing constraints given below.

$$\sum_{p\in\mathcal{P}}(s^0_{pk}+s_{pk})-\sum_{p\in\mathcal{P}}(g^0_{pk}+g_{pk})=y_k \quad \forall k\in\mathcal{B} \quad (\lambda_k) \tag{2.8}$$

The variables g^0_{pk} and g_{pk} denote the generation quantities from existing and new capacity installed of producer *p* at node *k*, whereas s^0_{pk} and s_{pk} denote the power sales of producer *p* to node *k* from existing and new capacity. The balancing of the power quantities is performed by an Independent System Operator in order to maintain feasible operation of the power system. In addition, this constraint provides the marginal cost of power transmission to each node *k* from the hub, λ_k. This marginal cost is the cost that the power producer pays for transferring power from the hub and receives for transferring power to the hub [13]. The cost of transmission is used in formulating the profit maximization objective of the power producer.

2.2.5. Spot Market Dispatch

In the spot market, an independent central system operator ensures network reliability and sets the prices for the energy and transmission rights using an Optimal Power Flow (OPF) model. With a flow-based structure, the OPF can be represented by the following model.

$$\underset{G_{pk},y_k}{Min} \sum_{p,k} C_{pk} G_{pk} \tag{2.9}$$

subject to

balance constraint:

$$\sum_{p,k} G_{pk} - \sum_{k} D_k = 0 \quad (\lambda) \tag{2.10}$$

thermal capacity constraints:

$$\sum_{k} PTDF_{ij}^{k} y_k \le T_{ij} \quad \forall ij \in \mathcal{L} \quad (\mu_{ij}^{+}) \tag{2.11}$$

$$-\sum_{k} PTDF_{ij}^{k} y_k \le T_{ij} \quad \forall ij \in \mathcal{L} \quad (\mu_{ij}^{-}) \tag{2.12}$$

flowgate constraints:

$$\sum_{ij \in \mathcal{F}_f} \left(\sum_{k} PTDF_{ij}^{k} y_k \right) \le F_f \quad \forall f \in \mathcal{F} \quad (\mu_f) \tag{2.13}$$

generation capacity limits:

$$G_{pk}^{l} \le G_{pk} \le G_{pk}^{u} \tag{2.14}$$

and nonnegativity constraints:

$$G_{pk} \ge 0 \tag{2.15}$$

In this formulation, the objective is minimizing the total generation cost (dispatch cost). The constraint (2.10) is the balance of total generation and total load in the system, and the constraints (2.11)-(2.13) are the power network feasibility constraints. Equation (2.14) is the upper and lower bounds on the generation quantity. The electricity prices in the real-time market are calculated using the Locational Marginal Pricing (LMP) concept. This model provides one dual price (λ) for the system balance equation, which represents the LMP of the hub. The LMPs for other nodes can be calculated by using the dual prices of the single circuit and flowgate limits, which correspond to the congestion prices associated with those limits.

2.2.6. Optimization Theory

2.2.6.1. Complementarity

A complementarity condition between a nonnegative variable x_i and a function $g_i(x)$ of a vector of variables $x = \{x_i\}$ can be defined as in [20]:

$$x_i \geq 0 \tag{2.16}$$

$$g_i(x) \leq 0 \tag{2.17}$$

$$x_i g_i(x) = 0 \tag{2.18}$$

This can also be expressed as $0 \leq x_i \perp g_i(x) \leq 0$. A complementarity problem is to find the x such that $0 \leq x \perp g(x) \leq 0$, where $g(x) = \{g_i(x)\}$. Specifically, given a set of functions $g(x)$, the complementarity problem is to find a vector x such that

$$x \geq 0 \tag{2.19}$$

$$g(x) \leq 0 \tag{2.20}$$

$$x^T g(x) = 0 \tag{2.21}$$

If all the $g_i(x)$ are linear functions, then complementarity problem is referred to as a linear complementarity problem [20]. Mixed linear complemetarity problem is a more general form of a linear complementarity problem with additional equality constraints. Let y be a second vector of variables, and $h(x, y)$ be a vector valued function. An example of a mixed linear complementarity problem (MLCP) can be stated as finding x and y such that [20]:

$$x \geq 0 \tag{2.22}$$

$$g(x) \leq 0 \tag{2.23}$$

$x^T g(x) = 0$ (2.24)

and $h(x, y) = 0$. (2.25)

2.2.6.2. Karush-Kuhn-Tucker Conditions

Now let us assume a constrained optimization problem in order to visualize how the market equilibrium conditions are obtained using the first order necessary conditions for an optimizer vector. Consider the following optimization problem:

Max $f(x, y)$ (2.26)

subject to

$g(x, y) \leq 0$ (2.27)

$h(x, y) = 0$ (2.28)

$x \geq 0$ (2.29)

The first order, Karush-Kuhn-Tucker (KKT) conditions for this problem are written by using complementarity and equality constraints. Assuming that λ and μ are the dual variables associated with the inequality and equality constraints respectively, the KKT conditions are [13]:

for *x*: $$\frac{\partial f}{\partial x} - \lambda \frac{\partial g}{\partial x} - \mu \frac{\partial h}{\partial x} \leq 0,\ x \geq 0,\ x\left(\frac{\partial f}{\partial x} - \lambda \frac{\partial g}{\partial x} - \mu \frac{\partial h}{\partial x} \leq 0\right) = 0 \quad (2.30)$$

for y: $$\frac{\partial f}{\partial y} - \lambda \frac{\partial g}{\partial y} - \mu \frac{\partial h}{\partial y} = 0 \tag{2.31}$$

for λ: $$g(x, y) \leq 0,\ \lambda \geq 0,\ \lambda g(x, y) = 0 \tag{2.32}$$

for μ: $$h(x, y) = 0 \tag{2.33}$$

This formulation of the KKT conditions will be used to obtain the market equilibrium conditions in the power generations expansion model in the following chapters.

2.2.6.3. Quadratic Programming

Quadratic programming deals with the problem of maximizing (or minimizing) a quadratic objective function over a polyhedral feasible region [30]. The mixed linear complementarity problems provide a natural setting for the Karsuh-Kuhn-Tucker conditions of a quadratic program with general equality and inequality constraints [30]. Consider the quadratic program:

$$\text{Max } \frac{1}{2} x^T Q x + c^T x \tag{2.34}$$

subject to

$$Ax \geq b \tag{2.35}$$

$$Cx = d \tag{2.36}$$

$$x \geq 0 \tag{2.37}$$

with Q a symmetric square matrix. The KKT conditions for a locally optimal solution of the above quadratic program are written as follows:

for x: $$Qx + c - A^T\lambda - C^T\mu = 0 \tag{2.38}$$

for λ: $$Ax - b \geq 0,\ \lambda \geq 0,\ \lambda(Ax - b) = 0 \tag{2.39}$$

for μ: $$Cx - d = 0 \tag{2.40}$$

These conditions are an example of a mixed linear complementarity problem where λ and μ are the dual variables associated with the inequality and equality constraints respectively. These are the necessary conditions for x to be a local optimizer of the problem. In addition, in the case where Q is a negative semi-definite matrix, then the objective function of the quadratic program is convex, and thus the KKT conditions are sufficient for x to be the global optimal solution [30].

CHAPTER 3

TRANSMISSION CONGESTION AND COMPETITION ON POWER GENERATION EXPANSION

In this section, a generation expansion model that incorporates the features of the transmission grid is developed to study the interaction between competition and network transfer capabilities in capacity expansion. In this framework, a DC-approximation model that includes network constraints is used to represent the transmission grid while a Cournot oligopolistic market model is used to represent competition. Hobbs [13] proposed a Nash-equilibrium modeling approach to strategic bidding in deregulated markets under network constraints. The same approach is undertaken in this research to find the equilibrium solution under network consideration for generation capacity expansion. The proposed model is a single-stage deterministic Cournot capacity expansion model where the decision to invest and operate are made at the same stage with bilateral contracts.

3.1. Power Producer Model

Generators have two basic decisions concerning the amount of power trading for maximizing their profit: Amount of power sales to each node and amount of capacity expansion. In order to model the generation expansion decision problem, an open-loop Cournot approach similar to [19] is undertaken. In this approach, the producers are assumed to make long-term power agreements while they make decisions on capacity expansion. That is, the decision on where and how much to expand is given together with how much to offer into the power market. Firms decide on their output and capacity expansion in Cournot behavior. This approach helps integrate the two decisions in one producer profit maximization model and also represents the imperfect competition. The generation expansion model for the producers is written by undertaking the strategic bidding approach in [13], which is a Cournot competition model that maximizes the profit of generation firms while deciding on the optimum power sales of each firm. The power producer model proposed in this paper is an investment model that uses this approach, but it assumes that each power producer may have existing generation capacity and is able to expand capacity at any node of the power network. Hobbs's model is extended by including capacity expansion decision variables and investment cost terms to the power producer profit maximization model. Unlike [19] and [22], this model distinguishes between existing and new capacity of the power producers so that new entrants are modeled considering the current conditions of the competitors. Adding new nodes to the system has not been included, as this issue needs to be studied together with the transmission expansion problem.

In deregulated power markets, the amount of demand supplied from each power producer in the network may vary. However, the demand at one node is equal to the amount that is sold to that node by different producers. The plant owners receive the price obtained from the inverse demand function (2.1) per MW sold and pay for the operation, transmission and capital investment costs. The resulting producer's objective is the revenue from sales minus transmission cost, operating cost and investment cost. The decision model of a producer p ($\forall p \in \mathcal{P}$) is as follows:

$$\begin{aligned}\text{Max} \sum_k \left(\alpha_k - \beta_k [s^0_{pk} + s_{pk} + s^0_{-pk} + s_{-pk}]\right)\left(s^0_{pk} + s_{pk}\right) - \sum_k \lambda_k \left(s^0_{pk} + s_{pk} - g^0_{pk} - g_{pk}\right) \\ - \sum_k C^0_{pk} g^0_{pk} - \sum_k C_{pk} g_{pk} - \sum_k I_{pk} g_{pk}\end{aligned} \tag{3.1}$$

subject to

generator capacity constraints:

$$g^0_{pk} \le G^0_{pk} \ \forall k \in \mathcal{B} \ (\rho^0_{pk}) \tag{3.2}$$

$$g_{pk} \le G^{\max}_{pk} \ \forall k \in \mathcal{B} \ (\rho_{pk}) \tag{3.3}$$

firm energy balance:

$$\sum_k s^0_{pk} = \sum_k g^0_{pk} \ \ (\theta^0_p) \tag{3.4}$$

$$\sum_k s_{pk} = \sum_k g_{pk} \ \ (\theta_p) \tag{3.5}$$

and non-negativity constraints:

$$\forall s^0_{pk}, s_{pk}, g^0_{pk}, g_{pk} \ge 0 \tag{3.6}$$

In this model, the variable s^0_{pk} and s_{pk} denote the sales of producer p at node k, whereas s^0_{-pk} and s_{-pk} represent the sales of the remaining producers, from their existing and new capacity respectively. The variables given in parenthesis correspond to the dual variables of the related constraints. For instance, the variable θ^0_p can be interpreted as the marginal cost of production of the existing capacity of producer *p*. The new capacity variables g_{pk} and s_{pk} are introduced to the problem in [13] with additional constraints, which are maximum capacity installation constraints (3.3) and sales from the new capacity constraints (3.5). Furthermore, the objective function (3.1) includes the cost terms $\sum_k C_{pk} g_{pk}$ and $\sum_k I_{pk} g_{pk}$, which represent the new capacity operating costs and investment costs, respectively. The model optimizes the sales and generation variables from existing and new capacity while deciding on the capacity expansion amount.

3.2. Equilibrium model

The power producers need to maximize their own profit maximizing objectives while deciding on the capacity expansion quantities. Therefore, each power producer needs to solve the Power Producer Model given in the previous section iteratively until an equilibrium solution is reached among power producers. Transmission System Owner solves its own profit maximizing problem and then the outcomes of these models are balanced in the market clearing constraint by the ISO as explained in chapter 2. The framework of the generation expansion equilibrium under transmission constraints is represented in figure 3.1 based on the bidding procedure in [13].

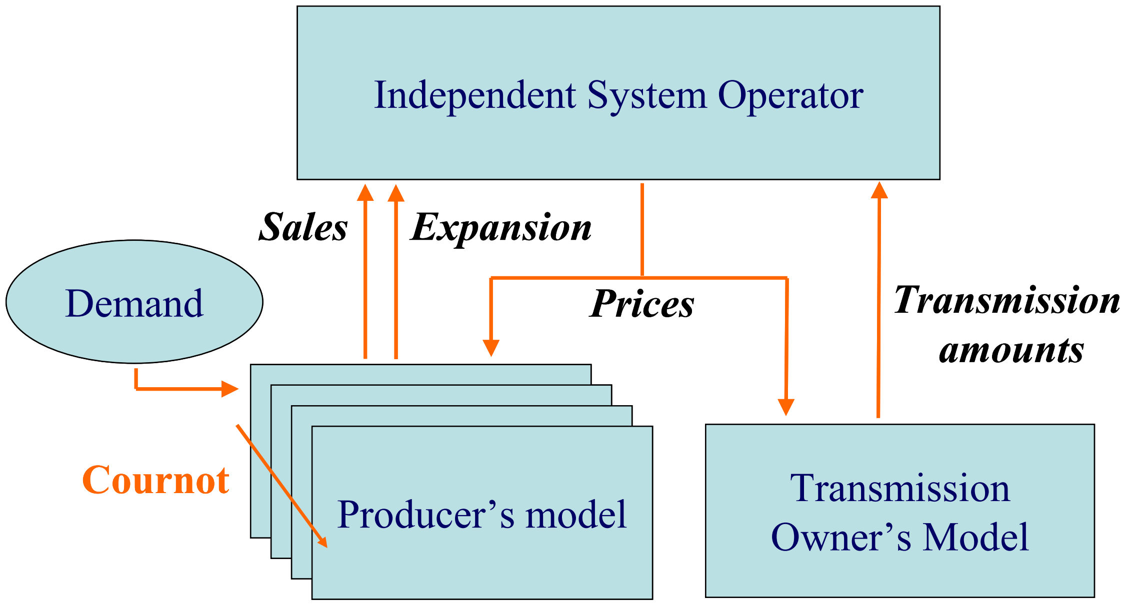

Fig. 3.1. Equilibrium Procedure

A solution that is optimal for each producer, transmission system owner, and the market clearing optimization models is considered an equilibrium solution. A Nash-equilibrium capacity solution is the quantities of expansion and power sales that optimize every power producer's profit model. Thus, the equilibrium solution needs to satisfy the optimality conditions of the producer models written for each power producer. The Karush-Kuhn-Tucker (KKT) optimality conditions of the producer model for each producer are written as follows.

KKTs related to the Power Producer Model:

for g^0_{pk}: $\lambda_k - C^0_{pk} - \rho^0_{pk} + \theta^0_p \le 0, g^0_{pk} \ge 0,\ g^0_{pk}(\lambda_k - C^0_{pk} - \rho^0_{pk} + \theta^0_p) = 0$ (3.7)

for g_{pk}: $\lambda_k - C_{pk} - I_{pk} - \rho_{pk} + \theta_p \le 0, g_{pk} \ge 0,$ (3.8)

$$g_{pk}(\lambda_k - C_{pk} - I_{pk} - \rho_{pk} + \theta_p) = 0$$

for θ_p^0: $$\sum_k s_{pk}^0 = \sum_k g_{pk}^0 \quad (3.9)$$

for θ_p: $$\sum_k s_{pk} = \sum_k g_{pk} \quad (3.10)$$

for ρ_{pk}^0: $$g_{pk}^0 \le G_{pk}^0, \rho_{pk}^0 \ge 0, \rho_{pk}^0(g_{pk}^0 - G_{pk}^0) = 0 \quad (3.11)$$

for ρ_{pk}: $$g_{pk} \le G_{pk}, \rho_{pk} \ge 0, \rho_{pk}(g_{pk} - G_{pk}) = 0 \quad (3.12)$$

for s_{pk}^0: $$\alpha_k - \beta_k\left[2\left(s_{pk}^0 + s_{pk}\right) + s_{-pk}^0 + s_{-pk}\right] - \lambda_k - \theta_p^0 \le 0, s_{pk}^0 \ge 0, \quad (3.13)$$

$$s_{pk}^0(\alpha_k - \beta_k\left[2\left(s_{pk}^0 + s_{pk}\right) + s_{-pk}^0 + s_{-pk}\right] - \lambda_k - \theta_p^0) = 0$$

for s_{pk}: $$\alpha_k - \beta_k\left[2\left(s_{pk}^0 + s_{pk}\right) + s_{-pk}^0 + s_{-pk}\right] - \lambda_k - \theta_p \le 0, s_{pk} \ge 0, \quad (3.14)$$

$$s_{pi}(\alpha_k - \beta_k\left[2\left(s_{pk}^0 + s_{pk}\right) + s_{-pk}^0 + s_{-pk}\right] - \lambda_k - \theta_p) = 0$$

The solution also needs to satisfy the optimality conditions of the Transmission System Owner and the ISO models simultaneously for market equilibrium. Applying the same framework the KKT optimality conditions of the TSO and ISO can be obtained as follows.

KKTs related to the transmission owner's model:

for y_k: $$\lambda_k + PTDF_{ij}^k \mu_{ij}^- - PTDF_{ij}^k \mu_{ij}^+ - \sum_{ij} PTDF_{ij}^k \mu_f = 0 \quad (3.15)$$

for μ_{ij}^-: $$-\sum_k PTDF_{ij}^k y_k \le T_{ij}, \mu_{ij}^- \ge 0, \ \mu_{ij}^-(\sum_k PTDF_{ij}^k y_k + T_{ij}) = 0 \quad (3.16)$$

for μ_{ij}^{+} : $$\sum_{k} PTDF_{ij}^{k} y_k \le T_{ij} , \mu_{ij}^{+} \ge 0 , \; \mu_{ij}^{+} (\sum_{k} PTDF_{ij}^{k} y_k - T_{ij}) = 0 \tag{3.17}$$

for μ_f : $$\sum_{ij} \sum_{k} PTDF_{ij}^{k} y_k \le F_f , \mu_f \ge 0 , \; \mu_f (\sum_{ij} \sum_{k} PTDF_{ij}^{k} y_k - F_f) = 0 \tag{3.18}$$

KKT related to the market clearing:

for λ_k : $$\sum_{p \in \mathcal{P}} (s_{pk}^{0} + s_{pk}) - \sum_{p \in \mathcal{P}} (g_{pk}^{0} + g_{pk}) = y_k \tag{3.19}$$

By implementing the procedure presented in [13], a single equivalent optimization model is built using the social welfare concept and the constraints of the optimization models of all market participants. This procedure consists of transforming the Linear Complementarity Constraints that represents the equilibrium solution into a Quadratic Programming model, called the equilibrium model. The resulting equilibrium model has the same Karush-Kuhn-Tucker (KKT) optimality conditions as the combination of the KKT conditions of the producer model, transmission system owner model, and a market clearing constraint. Thus, the equilibrium generation expansion solution can be obtained by solving the following equilibrium model.

$$\begin{aligned} \text{Max} \sum_{k} [\alpha_k \sum_{p \in \mathcal{P}} \left(s_{pk}^{0} + s_{pk}\right) - \frac{\beta_k}{2} [\sum_{p \in \mathcal{P}} \left(s_{pk}^{0} + s_{pk}\right)]^2 - \frac{\beta_k}{2} \sum_{p \in \mathcal{P}} \left(s_{pk}^{0} + s_{pk}\right)^2 \\ - \sum_{p} (C_{pk}^{0} g_{pk}^{0} + C_{pk} g_{pk} + I_{pk} g_{pk})] \end{aligned} \tag{3.20}$$

subject to

$$g_{pk}^{0} \leq G_{pk}^{0} \quad \forall p \in \mathcal{P}, k \in \mathcal{B} \quad (\rho_{pk}^{0}) \tag{3.21}$$

$$g_{pk} \leq G_{pk}^{\max} \quad \forall p \in \mathcal{P}, k \in \mathcal{B} \quad (\rho_{pk}) \tag{3.22}$$

$$\sum_{k} s_{pk}^{0} = \sum_{k} g_{pk}^{0} \quad \forall p \in \mathcal{P} \quad (\theta_{p}^{0}) \tag{3.23}$$

$$\sum_{k} s_{pk} = \sum_{k} g_{pk} \quad \forall p \in \mathcal{P} \quad (\theta_{p}) \tag{3.24}$$

$$\sum_{k} PTDF_{ij}^{k} y_{k} \leq T_{ij} \quad \forall ij \in \mathcal{L} \quad (\mu_{ij}^{+}) \tag{3.25}$$

$$-\sum_{k} PTDF_{ij}^{k} y_{k} \leq T_{ij} \quad \forall ij \in \mathcal{L} \quad (\mu_{ij}^{-}) \tag{3.26}$$

$$\sum_{ij \in \mathcal{F}_{f}} \left(\sum_{k} PTDF_{ij}^{k} y_{k} \right) \leq F_{f} \quad \forall f \in \mathcal{F} \quad (\mu_{f}) \tag{3.27}$$

$$\sum_{p \in \mathcal{P}} (s_{pk}^{0} + s_{pk}) - \sum_{p \in \mathcal{P}} (g_{pk}^{0} + g_{pk}) = y_{k} \quad \forall k \in \mathcal{B} \quad (\lambda_{k}) \tag{3.28}$$

$$\forall s_{pk}^{0}, s_{pk}, g_{pk}^{0}, g_{pk} \geq 0 \tag{3.29}$$

Equation (3.28) that includes the amount of power transmitted y_k is defined as the market clearing condition, and is explained in more detail in [13] and in chapter 2. It balances the difference between the power produced and demanded at one node to the amount of power transmitted to that node by the transmission system operator. The model optimizes the amount of power transmitted, sales and generation variables from existing and new capacity.

Similar to Hobbs' model, the concavity of the objective function assures a unique solution if a solution exists. The objective function of the equilibrium model represents the total welfare of the system, which is the benefit to the consumers minus the Cournot producers' effect on the benefit and the operation and investment cost to the producers. For more detail the reader should refer to [20]. The above Quadratic Programming model can be solved using a commercial solver engine.

In the case of perfect competition, the market players do not act as if they can alter the market prices. The objective for the perfect competition model is to maximize the social welfare which can be calculated as the benefit to the consumers minus the cost to the producer, and does not include the price alteration term used in the Cournot objective [20]. The perfect competition model is used for comparison purposes.

3.3. Numerical Example and Analysis

To study the interaction of the transmission constraints and competition on optimal generation expansion decisions, two different examples are used: the 5-bus, 4-generator power system [31] and the IEEE Reliability Test System [32]. The commercial package CPLEX is used to solve the models of both systems.

3.3.1. 5-Bus System

For illustrative purposes, the model is first applied to a small power network based on [31] and shown in figure 3.2. The generator and line data for the 5-bus system are given in table 3.1 and 3.2. As this system is easier to illustrate, detailed analysis including perfect competition among the generators and monopoly cases are given using this example.

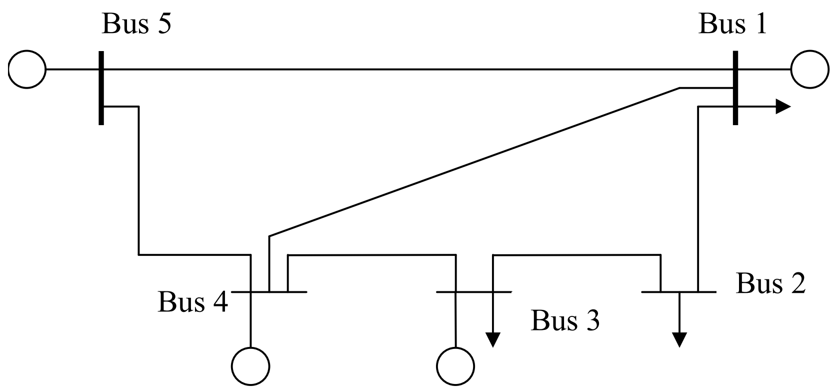

Fig. 3.2. 5-bus, 4-generator power system based on [31]

Table 3.1. Line Data for 5-Bus System

Line	Reactance (per unit)	$PTDF_{ij}^{k}$				
		1	2	3	4	5
1-2	0.0138	0.2941	-0.5294	-0.3529	-0.1765	0
2-3	0.0138	0.2941	0.4706	-0.3529	-0.1765	0
4-3	0.0138	-0.2941	-0.4706	-0.6471	0.1765	0
5-4	0.0414	-0.5147	-0.5735	-0.6324	-0.6912	0
4-1	0.0552	-0.2206	-0.1029	0.0147	0.1324	0
5-1	0.0690	-0.4853	-0.4265	-0.3676	-0.3088	0

Table 3.2. Generator Data

Generator	Capacity (MW)	Cost ($/MWh)
Bus 1	110	14
Bus 3	520	30
Bus 4	200	25
Bus 5	600	10

In this system, the flows on lines 5-1, 4-1, and 4-3 have been selected as the lines that constitute a transmission flowgate. The total flow over these three lines is limited to 670

MW. There are no flow constraints on single lines. The PTDF values for the transmission lines of this are calculated using the MATLAB software and are given in table 3.1.

The demand locations are at nodes 1, 2, and 3 of the network and the inverse demand functions of the three demand nodes are defined by $P_i = 80 - 0.04 D_i$. As the focus of this dissertation is on the effect of transmission constraints, the same function is used to represent the demand at each node. The existence of two producers (duopoly), namely producer *p* and *q*, is considered. Producer *p* owns the generators at buses 1 and 3, while producer *q* owns the generators at buses 4 and 5. These buses are selected such that the producers are separated by the transmission flowgate. This is done in order to see the effect on the producers at the different sides of the flowgate (demand side and non-demand side). It is assumed that the producers may expand their capacity at nodes where they currently own generators. The capital cost per hour of adding extra capacity is assumed to be $25 per MW at each node, while the operating costs are given in table 3.2.

The problem is solved using the Cournot generation expansion model explained in chapter 3.2. The expansion solution obtained is given in table 3.3. The notation WO/TC (without transmission constraints) and W/TC (with transmission constraints) indicates the presence or not of transmission constraints in the model. These results indicate that producer *p* (which is at the demand side of the flowgate limit) expands more when the flowgate limit is active, and the expansion is at the same location in both cases. On the other hand, producer *q* at the non-demand side of the flowgate limit does not expand any of its plant capacities when the flowgate constraint is present, independently of the type of market competition. The profits of the firms for one hour are also given in table 3.3. Without the transmission constraint, producer *q* has a much higher profit than producer *p*.

However, the transmission constraint changes the market conditions by decreasing the producer q's profit and increasing p's profit in all cases. These profit figures indicate the relative effect of a constrained power network on different power producers. Using these numbers, additional decisions such as transmission capacity investments can be made.

Table 3.3. Cournot Equilibrium Solution

Firm	Node	Expansion (MW)		Profit ($)	
		W/O TC	W/ TC	W/O TC	W/ TC
p	1	295	572	18,838	26,710
	3	0	0		
q	4	0	0	37,008	14,985
	5	425	0		

In addition to the Cournot solution, for comparison purposes, the 5-bus system is also solved with the perfect competition equilibrium model. The results are shown in table 3.4. By comparing tables 3.3 and 3.4, it can be seen that the expansion decisions depend also on the type of market competition. Producer q at the non-demand side of the flowgate limit does not expand any of its plant capacities when the flowgate constraint is present, independently of the type of market competition as it cannot sell more because of the transmission limits. Another observation is that under Cournot competition the optimal expansion quantities are lower than in perfect competition as a result of the tendency to withhold production capacity of the Cournot suppliers. As expected, the profits of the producers are higher with the Cournot competition than with the perfect competition. Under both competition structures, the transmission limits prevent q to sell more to the demand side, which decreases q's profit highly while increasing p's profit to some extent.

Table 3.4. Perfect Competition Equilibrium Solution

Firm	Node	Expansion (MW)		Profit ($)	
		W/O TC	W/ TC	W/O TC	W/ TC
p	1	0	1775	4,910	7,430
	3	0	0		
q	4	0	0	17,000	9,000
	5	1945	0		

Social welfare can be calculated as the sum of the producer profits, consumer surplus, and the TSO revenue. In figure 3.3, the components that build up the social welfare amounts in each case are given. The social welfare of the monopoly case, assuming that all power plants are owned by a single company, is shown also in this figure. Under monopoly, the producer has a better control on the market and therefore has the ability to make higher profits. The consideration of a flowgate decreases the total producer profit in all cases, while increasing the TSO revenue. The increase in total producer profit is more remarkable in this case while the model is solved under Cournot competition. In addition, the decrease in the consumer surplus is more significant for the case when the monopoly assumption is applied.

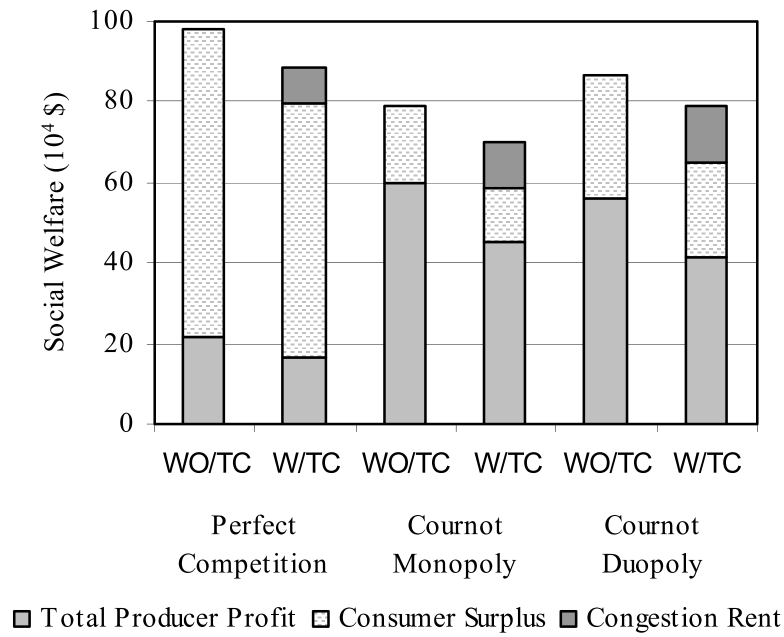

Fig. 3.3. Components of the social welfare

3.3.2. IEEE 24-Bus System

To illustrate the Cournot equilibrium results in a larger system, the model is applied to the IEEE Reliability Test System [32]. The 24-bus system has 38 transmission lines with 4 double lines. For simplicity, the double lines have been merged into single lines with adjusted reactance and thermal limits. As the purpose of this example is to study the effect of transmission congestion, one forth of the flow limits of the lines given in [32] is used. The network is illustrated in figure 3.4 and line reactances and limits for the 34 lines are given in table 3.5. The PTDF values for the lines are calculated using MATLAB software and are given for every node in Appendix A1.

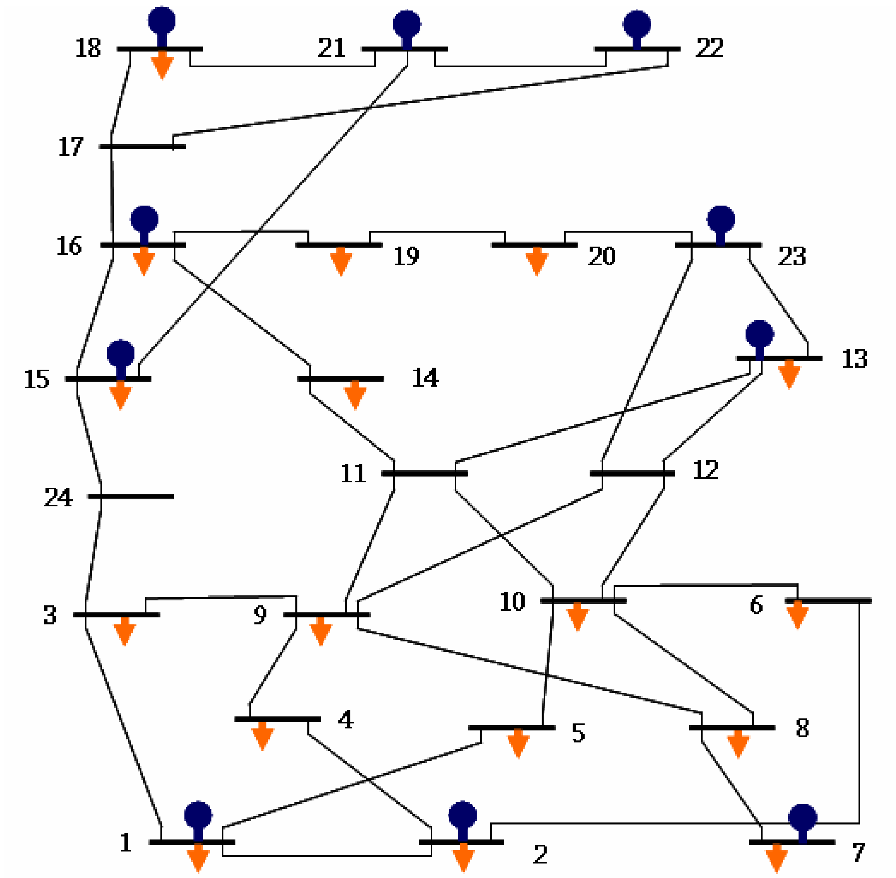

Fig. 3.4. 24-bus system network

The demand functions are defined by using the suggested peak demand values in [32]. The demand intercept is defined by multiplying the peak demand values at each node by 3, which is selected arbitrarily. The same slope for the inverse demand functions is used for all demand nodes and it is selected as 0.15. The generator and demand locations are the same as the original system. Two different firms, *p* and *q* are defined. The operating costs for the existing and new generators are assumed to be 20 $/MWh and $15 $/MWh respectively, whereas the capital cost per hour of adding extra capacity is $30 per MW at each node. The generator ownership, capacities, and the price intercepts of the inverse demand functions are given for each node in table 3.6.

Table 3.5. 24-Bus Network Data

Line	Reactance (per unit)	Limit (MW)	Line	Reactance (per unit)	Limit (MW)
1-2	0.014	43.75	11-13	0.048	125
1-3	0.211	43.75	11-14	0.042	125
1-5	0.085	43.75	12-13	0.048	125
2-4	0.127	43.75	12-23	0.097	125
2-6	0.192	43.75	13-23	0.086	125
3-9	0.119	43.75	14-16	0.049	125
3-24	0.084	100	15-16	0.017	125
4-9	0.104	43.75	15-21	0.024	250
5-10	0.088	43.75	15-24	0.052	125
6-10	0.061	43.75	16-17	0.026	125
7-8	0.061	43.75	16-19	0.023	125
8-9	0.165	43.75	17-18	0.014	125
8-10	0.165	43.75	17-22	0.105	125
9-11	0.084	100	18-21	0.013	250
9-12	0.084	100	19-20	0.020	250
10-11	0.084	100	20-23	0.011	250
10-12	0.084	100	21-22	0.068	125

Table 3.6. 24-Bus System Data

Node	Capacity		α	Node	Capacity		α
	p	q			p	q	
1	192	0	48.6	13	0	591	119.3
2	192	0	43.65	14	0	0	87.3
3	0	0	81	15	0	215	142.7
4	0	0	33.3	16	0	155	45
5	0	0	31.95	17	0	0	0
6	0	0	61.2	18	0	400	149.9
7	300	0	56.25	19	0	0	81.45
8	0	0	76.95	20	0	0	57.6
9	0	0	78.75	21	0	400	0
10	0	0	87.75	22	0	300	0
11	0	0	0	23	660	0	0
12	0	0	0	24	0	0	0

The expansion solutions obtained by solving the Cournot model are given in table 3.7. These results indicate that the existence of the transmission limits have a more important effect on the expansion decision of producer *p* than producer *q* under Cournot competition. When the results are observed, it can be seen that producer *p*'s decision is highly affected when the transmission limits are inserted to the model. On the other hand, producer *q* does not need to expand under Cournot competition without transmission limits, and it needs to expand a small quantity when limits are included.

Table 3.7. 24-Bus Cournot Expansion Solution (MW)

Producer	WO/ TC	W/ TC
p	244.8	664.5
q	0	55.7

The profits of the firms and other economic analysis information are given in table 3.8. When the profits are observed, it can be said that the existence of the transmission constraints in the analysis decreases the profits of both firms. In other words, the transmission limits diminish the producers' ability to manipulate the prices by the quantities they sell. As a result, the total consumer surplus value (see table 3.8) is higher, but the total welfare is lower.

Table 3.8. 24-Bus Cournot Equilibrium Solution ($)

Metric		WO/ TC	W/ TC
Profit	*p*	59,078	57,104
	q	83,706	72,324
Congestion rent		0	21,115
Consumer surplus		52,536	55,597
Total Welfare		179,403	170,786

3.4. Effects of Expansion Decisions on the Network Congestion

In this section, an analysis of how the generation expansion decisions could impact the congestion of the network is conveyed. The case where transmission constraints are considered in the expansion decisions and the case where these constraints are neglected are examined. Since the electricity market balances at real-time and long-term power purchases are financially binding economic contracts, the schedules will liquidate in the spot market.

In order to evaluate the performance of the expansion decisions on the network, the dispatch model (2.9)-(2.15) is solved using the expanded capacity solutions obtained from the Cournot equilibrium model of the 24-Bus system example. Notice that the dispatch model is always solved including the transmission constraints regardless of the expansion model used (WO/TC or W/TC). When the expansion decisions are obtained including the transmission constraint, the dispatch model gives feasible solutions. However, when the transmission constraints are excluded in the expansion planning, the dispatch model could not find a feasible solution. In order to obtain a feasible power dispatch, the equilibrium load is curtailed. The load curtailments necessary to make the model feasible are given in table 3.9.

Table 3.9. Load Curtailments (MWh)

Node	Unmet Demand	Node	Unmet Demand
1	0	13	0
2	0	14	152.81
3	81.72	15	0
4	0	16	79.02
5	12.78	17	0
6	88.90	18	0
7	0	19	35.37
8	77.57	20	0
9	77.65	21	0
10	91.19	22	0
11	0	23	0
12	0	24	0

After solving the dispatch model using the new capacities obtained by the expansion models WO/TC and W/TC, the congestion rents, consumer benefits, and consumer losses are calculated for each solution dispatch. Table 3.10 gives the results for both cases. The consumer benefits have been calculated using (2.2). The consumer loss is the difference of consumer benefits before and after the load curtailments.

Table 3.10. Network Economic performance ($)

Metric	WO/ TC	W/ TC
Congestion rent	26,686	0
Consumer benefit	239,403	276,807
Consumer loss	35,032	0

In order to observe the effects on market prices, averages of the LMPs for both expansion solutions are shown in table 3.11. The weighted averages LMPs have been calculated by multiplying the LMPs at each node by the ratio of the load at that node relative to the total system load.

Table 3.11. Prices ($/MWh)

LMP	WO/ TC	W/ TC
Average	32.80	20
Weighted Avg.	34.03	20

In table 3.10, when expansion decisions are made using the model W/TC the congestion rent resulted to be zero, i.e. there is no congestion since it has been relieved with new generation. Furthermore, the model W/TC provides a higher consumer's benefit than the model WO/TC. The reason is that in the latter case the load needed to be curtailed. From table 3.11, the decision model W/TC produces lower average spot prices than the model WO/TC.

This example demonstrates that the inclusion of transmission network constraints in Cournot decision models for expanding generation is crucial. Since when they are neglected from the decision process, the expansion decisions can lead to obligatory load curtailments and induce higher congestion charges and higher spot prices.

3.5. Conclusion

A methodology for analyzing the effect of system congestion and Cournot competition on generation expansion decisions in deregulated electricity markets is proposed. The presented approach considers the behavioral models of the power market players and the transmission network in the generation expansion modeling. By using the suggested model, effects of the network transmission constraints on the generation expansion decision under perfect and imperfect competition are evaluated. Transmission constraints represented in terms of PTDFs are included in modeling generation expansion decisions under Cournot competition. It has been shown that transmission constraints may affect the outcome of competition by limiting expansion decisions of producers according to where the generators are located relative to the transmission limits. It has also been shown that if transmission constraints are not considered in planning decisions, the consumers benefit may diminish due to higher prices and higher congestion charges.

CHAPTER 4

POWER GENERATION EXPANSION IN DEREGULATED MARKETS UNDER UNCERTAINTY

The generation expansion problem involves increasing electric power generation capacity in an existing power network. Capacity expansion decisions, like in other investment decision problems, are subject to risk due to future conditions of the system. To be able to cope with the uncertainties in the investment problem, the uncertainty in the information available to the decision makers needs to be integrated into the investment models. In this section, models that integrate the uncertainty in power markets to the decision of generation expansion are introduced. The aim of the models is to mitigate the risk involved in the decision of capacity expansion in restructured power markets. Based on the power market modeling principals and assumptions described in chapter 2 and the generation expansion framework described in chapter 3, two new generation expansion models have been developed.

The first model developed for generation expansion under uncertainty in restructured markets is an optimization model that considers the uncertainty of the information from the producer point of view. In this new generation expansion model, the producers have additional information on the sources of uncertainty that could have an effect on generation expansion decisions. Thus, the generation expansion equilibrium model is built considering this information.

The second model developed for generation expansion under uncertainty considers the uncertainty to all decision makers in the market. In this generation expansion model, all market participants have the same information on the data.

In both of the above models, the capacity expansion is represented as a Cournot competition game whereas the Transmission Owner is assumed to be a Bertrand decision maker. These assumptions comply with the generation expansion model in the previous section. Two sources of uncertainty are considered in this research: Uncertainty in power demand and uncertainty in fuel costs. These two factors have high volatility and directly affect the firm profits.

4.1. Uncertainty in Power Demand

According to [19] and [24], uncertain demand can be formulated using a set of demand states with associated probabilities. In order to model the uncertainty in the demand, possible load scenarios are considered in different demand states. Different linear demand functions as explained in chapter 2 are used for all demand states in this formulation. As shown in figure 4.1, these demand functions may have different intercepts and slopes for all demand states depending on the price sensitivity of the load at the corresponding network node. At every demand state, the price function is defined by a different price intercept α_t and demand slope $\beta_{t.}$ where *t* denotes a particular state of nature.

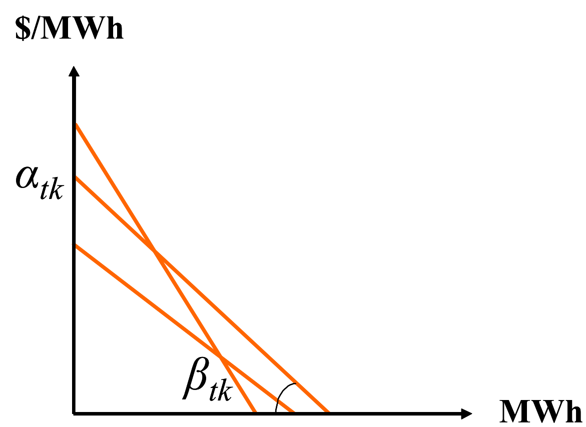

Fig. 4.1. Demand states

In this framework, the inverse demand function for node k at state t can be written as in (4.1). The inverse demand function is used in the producer decision model to calculate its expected revenue from the power sales over the different states.

$$P_{tk} = \alpha_{tk} - \beta_{tk} D_{tk} \tag{4.1}$$

4.2. Uncertainty in the Fuel Costs

The uncertainty in the fuel costs has much importance as the operational costs of the power generators are directly affected from changes in fuel costs. To formulate the uncertainty of fuel costs, T states of fuel costs are used. At each state, the operation cost of a power generator is affected from the change in fuel cost. Therefore, T states of the operational cost are included and written as:

$$\text{Operational Cost} = \sum_u \sum_k C^0_{tuk} g^0_{tpuk} + \sum_u \sum_k C_{tuk} g_{tpuk} \tag{4.2}$$

where the first term relates to the existing capacity available and the second term to the new generation units that will be available after the capacity investment of firm p is implemented. The summation is for each fuel type at each node. The subscript t denotes the index of the state.

4.3. Power Generation Expansion Under Uncertainty – Model 1

Considering the different states of nature, the objective of the power producer is to maximize its expected profit. The expected profit is calculated by the difference of the expected revenue and expected cost terms, minus the total investment cost of the producer.

In this model, equal probability of occurrence is assumed for all possible states of power demand and fuel costs. Therefore, by using the nodal power price definition given in chapter 3 and adapting it for the uncertainty model, the expected revenue for producer p under uncertainty can be written as follows:

$$\text{ER}\ (p) = \frac{1}{T} \sum_t \sum_k \left(\left[\alpha_{tk} - \beta_{tk} \left(s_{tpk} + s_{t-pk} \right) \right] \left(s_{tpk} \right) - \lambda_k \left(s_{tpk} - \sum_u g^0_{tpuk} - \sum_u g_{tpuk} \right) \right) \tag{4.3}$$

In the revenue calculation for the producers, the congestion rent that needs to be paid by producer p needs to be taken into account, thus the transmission prices that are defined by the ISO model are needed. In this model, the transmission price for each node λ_k is independent of the states and is defined by the market clearing constraint. In this formulation the variable s_{tpk} denotes the sales of producer *p* at node *k* for the realization of state *t*, s_{t-pk} refers to the sales of producers other than producer *p* at state *t* and node *k*. The variables g^0_{tpuk} and g_{tpuk} denote the generation at state *t*, of producer *p*, from fuel type *u*, at node *k*, from existing and new capacities respectively. Similarly, by using the operational cost formulation explained previously, the expected total generation cost for producer *p* from its existing capacity in the network and its new capacity that will be available after the investment takes place can be formulated as below:

$$\mathrm{EC}\,(p) = \frac{1}{T}\sum_t \sum_k \left(\sum_u C^0_{tuk}\, g^0_{tpuk} + \sum_u C_{tuk}\, g_{tpuk} \right) \tag{4.4}$$

The coefficients C^0_{tuk} and C_{tuk} correspond to the operational cost at state *t*, for generation unit of fuel type *u*, at node *k*, from existing and new capacities. The producer model's objective is to maximize the expected profit subject to the capacity constraints and balance constraints, which needs to be written for each state of demand and fuel cost. The decision model of a producer *p* ($\forall p \in \mathcal{P}$) is as follows:

$$\text{Max } \frac{1}{T}\sum_{t}\sum_{k}\left(\begin{array}{l}\left[\alpha_{tk}-\beta_{tk}\left(s_{tpk}+s_{t-pk}\right)\right]\left(s_{tpk}\right)-\lambda_{k}\left(s_{tpk}-\sum_{u}g^{0}_{tpuk}-\sum_{u}g_{tpuk}\right)\\ -\sum_{u}C^{0}_{tuk}g^{0}_{tpuk}-\sum_{u}C_{tuk}g_{tpuk}\end{array}\right) \\ -\sum_{u}\sum_{k}I_{puk}g^{n}_{puk} \tag{4.5}$$

$$g^{0}_{tpuk} \le G^{0}_{puk} \quad \forall t \in \mathcal{T} \quad \forall u \in \mathcal{U} \quad \forall k \in \mathcal{B} \ (\rho^{0}_{tpuk}) \tag{4.6}$$

$$g_{tpuk} \le g^{n}_{puk} \quad \forall t \in \mathcal{T} \quad \forall u \in \mathcal{U} \quad \forall k \in \mathcal{B} \ \ (\rho_{tpuk}) \tag{4.7}$$

$$g^{n}_{puk} \le G^{\max}_{puk} \quad \forall u \in \mathcal{U} \quad \forall k \in \mathcal{B} \ \ (\rho^{n}_{puk}) \tag{4.8}$$

$$\sum_{k} s_{tpk} = \sum_{k}\left(\sum_{u} g^{0}_{tpuk} + \sum_{u} g_{tpuk}\right) \ \forall t \in \mathcal{T} \ \ (\theta_{tp}) \tag{4.9}$$

$$\forall s_{tpk}, g^{0}_{tpuk}, g_{tpuk}, g^{n}_{puk} \ge 0 \tag{4.10}$$

The objective function of the producer model represents the difference of the total expected revenue for producer p and the total investment cost of all new fuel type capacity investments of producer p which is given by $\sum_{u}\sum_{k} I_{puk} g^{n}_{puk}$. As in chapter 3, the variables given in parenthesis correspond to the dual variables of the related constraints. Here, a new capacity investment variable g^{n}_{puk} is introduced with additional constraints, which are maximum capacity installation constraints given by equation (4.8). Notice that these constraints are independent of the realization of a specific state. In addition, in this model the generation quantity from the new capacity g_{puk} is limited with the variable g^{n}_{puk} denoting the investment amount, which is represented in (4.7). The model optimizes the sales and generation variables from existing and new capacity while deciding on the capacity expansion amounts.

In addition to the power producers, the TSO's model and the market clearing constraints need to be modified under the given state structure. The associated TSO model has an objective of maximizing the congestion rent by deciding on the expected transmission quantities, which are defined as $E(y_k)$. The existence of different states of demand and fuel cost are not taken into consideration from the TSO viewpoint. The TSO model is then formulated as:

$$\text{Max} \sum_k \lambda_k E(y_k) \tag{4.11}$$

$$\sum_k PTDF_{ij}^k E(y_k) \le T_{ij} \quad \forall ij \in \mathcal{L} \quad (\mu_{ij}^+) \tag{4.12}$$

$$-\sum_k PTDF_{ij}^k E(y_k) \le T_{ij} \quad \forall ij \in \mathcal{L} \quad (\mu_{ij}^-) \tag{4.13}$$

$$\sum_{ij \in \mathcal{F}_f} \left(\sum_k PTDF_{ij}^k E(y_k) \right) \le F_f \quad \forall f \in \mathcal{F} \quad (\mu_f) \tag{4.14}$$

The Independent System Operator, on the other hand, balances the expected value of total power sales to each node, the expected value of the generation quantities from existing and new capacities, and the transmission values for each node. This balance is written for every node in the power network. The market clearing constraint becomes:

$$\frac{1}{T} \sum_{p \in \mathcal{P}} s_{tpk} - \frac{1}{T} \sum_{p \in \mathcal{P}} \left(\sum_u g_{tpuk}^0 + \sum_u g_{tpuk} \right) = E(y_k) \quad \forall k \in \mathcal{B} \quad (\lambda_k) \tag{4.15}$$

The solution of this problem needs to satisfy the optimality conditions of all three models for all the market participants as explained previously. The same framework explained by figure 3.1 needs to be satisfied by an equilibrium solution to the given three models if a solution exists. The equilibrium solution of these models can be found with a quadratic programming approach as done in chapter 3. In the quadratic programming approach, a quadratic programming model that has the same KKT conditions as the KKT conditions of the models for the different market participants combined is defined. These models include the models written for all the power producers, transmission network owner, and independent system operator. The KKT conditions of the generation expansion problem under uncertainty for each of these entities are given below:

KKTs related to the Power Producer Model:

for s_{tpk}: $$\frac{1}{T}\left(\alpha_{tk} - \beta_{tk}\left[2s_{tpk} + s_{t-pk}\right] - \lambda_k\right) - \theta_{tp} \leq 0, s_{tpk} \geq 0, \tag{4.16}$$

$$s_{tpk}\left(\frac{1}{T}\left(\alpha_{tk} - \beta_{tk}\left[2s_{tpk} + s_{t-pk}\right] - \lambda_k\right) - \theta_{tp}\right) = 0$$

for g^0_{tpuk}: $$\frac{1}{T}\left(\lambda_k - C^0_{tuk}\right) + \theta_{tp} - \rho^0_{tpuk} \leq 0, g^0_{tpuk} \geq 0, \tag{4.17}$$

$$g^0_{tpk}\left(\frac{1}{T}\left(\lambda_k - C^0_{tuk}\right) + \theta_{tp} - \rho^0_{tpuk}\right) = 0$$

for g_{tpuk}: $$\frac{1}{T}\left(\lambda_k - C_{tuk}\right) + \theta_{tp} - \rho_{tpuk} \leq 0, g_{tpuk} \geq 0, \tag{4.18}$$

$$g^0_{tpk}\left(\frac{1}{T}\left(\lambda_k - C_{tuk}\right) + \theta_{tp} - \rho_{tpuk}\right) = 0$$

for g^n_{puk} : $$-I_{puk} + \sum_t \rho_{tpuk} - \rho^n_{puk} \le 0, g^n_{puk} \ge 0, \tag{4.19}$$

$$g^n_{pk}(-I_{puk} + \sum_t \rho_{tpuk} - \rho^n_{puk}) = 0$$

for θ_{tp} : $$\sum_k s_{tpk} = \sum_k \left(\sum_u g^0_{tpuk} + \sum_u g_{tpuk} \right) \tag{4.20}$$

for ρ^0_{tpuk} : $$g^0_{tpuk} \le G^0_{puk}, \rho^0_{tpuk} \ge 0, \rho^0_{tpuk}(g^0_{tpuk} - G^0_{puk}) = 0 \tag{4.21}$$

for ρ_{tpuk} : $$g_{tpuk} \le g^n_{puk}, \rho_{tpuk} \ge 0, \rho_{tpuk}(g_{tpuk} - g^n_{puk}) = 0 \tag{4.22}$$

for ρ^n_{puk} : $$g^n_{puk} \le G^{\max}_{puk}, \rho^n_{puk} \ge 0, \rho^n_{puk}(g^n_{puk} - G^{\max}_{puk}) = 0 \tag{4.23}$$

KKTs related to the transmission owner's model:

for $E(y_k)$: $$E(\lambda_k) + \sum_{ij} PTDF^k_{ij} \mu^-_{ij} - \sum_{ij} PTDF^k_{ij} \mu^+_{ij} - \sum_{ij} PTDF^k_{ij} \mu_f = 0 \tag{4.24}$$

for $E(\mu^-_{ij})$: $$-\sum_k PTDF^k_{ij} E(y_k) \le T_{ij}, \mu^-_{ij} \ge 0, \ \mu^-_{ij}(\sum_k PTDF^k_{ij} E(y_k) + T_{ij}) = 0 \tag{4.25}$$

for $E(\mu^+_{ij})$: $$\sum_k PTDF^k_{ij} E(y_k) \le T_{ij}, \mu^+_{ij} \ge 0, \ \mu^+_{ij}(\sum_k PTDF^k_{ij} E(y_k) - T_{ij}) = 0 \tag{4.26}$$

for $E(\mu_f)$: $$\sum_{ij} \sum_k PTDF^k_{ij} E(y_k) \le F_f, \mu_f \ge 0, \tag{4.27}$$

$$\mu_f (\sum_{ij} \sum_k PTDF^k_{ij} E(y_k) - F_f) = 0$$

KKT related to the market clearing:

for λ_k : $$\frac{1}{T}\sum_{p\in\mathcal{P}} s_{tpk} - \frac{1}{T}\sum_{p\in\mathcal{P}}(\sum_u g^0_{tpuk} + \sum_u g_{tpuk}) = E(y_k) \tag{4.28}$$

The KKT conditions of the generation expansion under uncertainty problem (4.16)-(4.28) form a mixed linear complementarity problem with both equality constraints and complementarity constraints. These KKT conditions are equivalent to that of the below generation expansion equilibrium model, which is obtained by adapting the equilibrium model in chapter 3 for the uncertainty problem. The objective function represents the total welfare of the system under uncertainty, which is the benefit to the consumers minus the Cournot producers' effect on the benefit and the operation and investment cost to the producers. The constraints are equal to the producer model constraints written for all producers (4.30)-(4.33) and (4.38), TSO model constraints (4.34)-(4.36), and the ISO constraint (4.37). The equilibrium model for the power generation expansion under uncertainty – Model 1 is defined by (4.29)-(4.38).

$$\text{Max } \frac{1}{T}\sum_t\sum_k\left(\alpha_{tk}\sum_p s_{tpk} - \frac{\beta_{tk}}{2}\left(\sum_p s_{tpk}\right)^2 - \frac{\beta_{tk}}{2}\sum_p\left(s_{tpk}\right)^2 - \sum_p\sum_u C^0_{tuk}g^0_{tpuk} - \sum_p\sum_u C_{tuk}g_{tpuk}\right) - \sum_p\sum_u\sum_k I_{puk}g^n_{puk} \tag{4.29}$$

$$g^0_{tpuk} \le G^0_{puk} \quad \forall t\in\mathcal{T} \quad \forall p\in\mathcal{P} \quad \forall u\in\mathcal{U} \quad \forall k\in\mathcal{B} \quad (\rho^0_{tpuk}) \tag{4.30}$$

$$g_{tpuk} \le g^n_{puk} \quad \forall t\in\mathcal{T} \quad \forall p\in\mathcal{P} \quad \forall u\in\mathcal{U} \quad \forall k\in\mathcal{B} \quad (\rho_{tpuk}) \tag{4.31}$$

$$g_{puk}^{n} \le G_{puk}^{\max} \quad \forall u \in \mathcal{U} \quad \forall k \in \mathcal{B} \quad (\rho_{puk}^{n}) \tag{4.32}$$

$$\sum_{k} s_{tpk} = \sum_{k} \left(\sum_{u} g_{tpuk}^{0} + \sum_{u} g_{tpuk} \right) \quad \forall t \in \mathcal{T} \quad \forall p \in \mathcal{P} \; (\theta_{tp}) \tag{4.33}$$

$$\sum_{k} PTDF_{ij}^{k} E(y_{k}) \le T_{ij} \quad \forall ij \in \mathcal{L} \quad (\mu_{ij}^{+}) \tag{4.34}$$

$$-\sum_{k} PTDF_{ij}^{k} E(y_{k}) \le T_{ij} \quad \forall ij \in \mathcal{L} \quad (\mu_{ij}^{-}) \tag{4.35}$$

$$\sum_{ij \in \mathcal{F}_f} \left(\sum_{k} PTDF_{ij}^{k} E(y_{k}) \right) \le F_{f} \quad \forall f \in \mathcal{F} \quad (\mu_{f}) \tag{4.36}$$

$$\frac{1}{T} \sum_{p \in \mathcal{P}} s_{tpk} - \frac{1}{T} \sum_{p \in \mathcal{P}} \left(\sum_{u} g_{tpuk}^{0} + \sum_{u} g_{tpuk} \right) = E(y_{k}) \quad \forall k \in \mathcal{B} \quad (\lambda_{k}) \tag{4.37}$$

$$\forall s_{tpk}, g_{tpuk}^{0}, g_{tpuk}, g_{puk}^{n} \ge 0 \tag{4.38}$$

Similar to the Cournot equilibrium model in the previous section, the concavity of the objective function assures a unique solution. The above Quadratic Programming model can be solved using a commercial quadratic programming solver.

4.4. Power Generation Expansion under Uncertainty – Model 2

In the previous model, the different states of demand and fuel costs were only considered in the Cournot capacity expansion game of the producers. In the power generation expansion under uncertainty – Model 2 the uncertainty in demand and fuel costs are brought to the transmission and market clearing models. With consideration of different states of nature in the TSO and ISO models, the power producer model is modified with the new objective calculated by the different transmission prices for each state t, which are the dual prices of the modified market clearing constraint. The expected profit, similar to Model 1, can be calculated by the difference of the expected revenue and expected cost terms, minus the total investment cost of producer p. Once again, equal probability of occurrence is assumed for all possible states of power demand and fuel costs. Therefore, with state dependent transmission prices λ_{tk}, the expected revenue for producer p for Model 2 can be written as follows:

$$\text{ER}\ (p) = \frac{1}{T}\sum_{t}\sum_{k}\left(\left[\alpha_{tk} - \beta_{tk}\left(s_{tpk} + s_{t-pk}\right)\right]\left(s_{tpk}\right) - \lambda_{tk}\left(s_{tpk} - \sum_{u} g^{0}_{tpuk} - \sum_{u} g_{tpuk}\right)\right) \tag{4.39}$$

The producer model's objective is to maximize the expected profit for producer p subject to the capacity constraints and balance constraints, which needs to be written for each state of demand and fuel cost. The decision model of a producer p ($\forall p \in \mathcal{P}$) is as follows:

$$\begin{aligned} \text{Max}\ \frac{1}{T}\sum_{t}\sum_{k} & \left(\begin{array}{l} \left[\alpha_{tk} - \beta_{tk}\left(s_{tpk} + s_{t-pk}\right)\right]\left(s_{tpk}\right) - \lambda_{tk}\left(s_{tpk} - \sum_{u} g^{0}_{tpuk} - \sum_{u} g_{tpuk}\right) \\ -\sum_{u} C^{0}_{tuk} g^{0}_{tpuk} - \sum_{u} C_{tuk} g_{tpuk} \end{array}\right) \\ & -\sum_{u}\sum_{k} I_{puk} g^{n}_{puk} \end{aligned} \tag{4.40}$$

$$g^{0}_{tpuk} \le G^{0}_{puk} \quad \forall t \in \mathcal{T} \quad \forall u \in \mathcal{U} \quad \forall k \in \mathcal{B} \ (\rho^{0}_{tpuk}) \tag{4.41}$$

$$g_{tpuk} \le g^{n}_{puk} \quad \forall t \in \mathcal{T} \quad \forall u \in \mathcal{U} \quad \forall k \in \mathcal{B} \ \ (\rho_{tpuk}) \tag{4.42}$$

$$g_{puk}^{n} \leq G_{puk}^{\max} \quad \forall u \in \mathcal{U} \quad \forall k \in \mathcal{B} \quad (\rho_{puk}^{n}) \tag{4.43}$$

$$\sum_{k} s_{tpk} = \sum_{k} \left(\sum_{u} g_{tpuk}^{0} + \sum_{u} g_{tpuk} \right) \quad \forall t \in \mathcal{T} \quad (\theta_{tp}) \tag{4.44}$$

$$\forall s_{tpk}, g_{tpuk}^{0}, g_{tpuk}, g_{puk}^{n} \geq 0 \tag{4.45}$$

In Model 2, the availability of uncertain information on states of demand quantities and fuel costs is considered in the TSO model and the ISO model in addition to the power producer game. Therefore, the TSO's objective function is modified under the given state structure. The associated TSO model has an objective of maximizing the expected congestion rent which is calculated, considering *T* equal probability states, as:

$$ECR = \frac{1}{T} \sum_{t} \sum_{k} \lambda_{tk} y_{tk} \tag{4.46}$$

Therefore, the new TSO model can be formulated as below:

$$\text{Max } \frac{1}{T} \sum_{t} \sum_{k} \lambda_{tk} y_{tk} \tag{4.47}$$

$$\sum_{k} PTDF_{ij}^{k} y_{tk} \leq T_{ij} \quad \forall ij \in \mathcal{L} \quad \forall t \in \mathcal{T} \quad (\mu_{tij}^{+}) \tag{4.48}$$

$$-\sum_{k} PTDF_{ij}^{k} y_{tk} \leq T_{ij} \quad \forall ij \in \mathcal{L} \quad \forall t \in \mathcal{T} \quad (\mu_{tij}^{-}) \tag{4.49}$$

$$\sum_{ij \in \mathcal{F}_f} \left(\sum_{k} PTDF_{ij}^{k} * y_{tk} \right) \leq F_f \quad \forall f \in \mathcal{F} \quad \forall t \in \mathcal{T} \quad (\mu_{tf}) \tag{4.50}$$

Since there are T different demand and fuel cost states, the balance equation needs to be written according to these states. The balance of sales, generation, and transmission values at each node should be satisfied at all different states of nature. Therefore, T different balance equations for the ISO model are written and the market clearing constraint becomes:

$$\sum_{p\in\mathcal{P}} s_{tpk} - \sum_{p\in\mathcal{P}} (\sum_{u} g^{0}_{tpuk} + \sum_{u} g_{tpuk}) = y_{tk} \quad \forall k \in \mathcal{B} \quad \forall t \in \mathcal{T} \quad (\lambda_{tk}) \tag{4.51}$$

To find the equilibrium solution of these three models, yet again the quadratic programming approach is applied. With this aim, the KKT conditions of the generation expansion problem under uncertainty for each of market participants are given below:

KKTs related to the Power Producer Model:

for s_{tpk}:
$$\frac{1}{T}\left(\alpha_{tk} - \beta_{tk}\left[2s_{tpk} + s_{t-pk}\right] - \lambda_{tk}\right) - \theta_{tp} \le 0, s_{tpk} \ge 0, \tag{4.52}$$

$$s_{tpk}\left(\frac{1}{T}\left(\alpha_{tk} - \beta_{tk}\left[2s_{tpk} + s_{t-pk}\right] - \lambda_{tk}\right) - \theta_{tp}\right) = 0$$

for g^{0}_{tpuk}:
$$\frac{1}{T}\left(\lambda_{tk} - C^{0}_{tuk}\right) + \theta_{tp} - \rho^{0}_{tpuk} \le 0, g^{0}_{tpuk} \ge 0, \tag{4.53}$$

$$g^{0}_{tpk}\left(\frac{1}{T}\left(\lambda_{tk} - C^{0}_{tuk}\right) + \theta_{tp} - \rho^{0}_{tpuk}\right) = 0$$

for g_{tpuk}:
$$\frac{1}{T}\left(\lambda_{tk} - C_{tuk}\right) + \theta_{tp} - \rho_{tpuk} \le 0, g_{tpuk} \ge 0, \tag{4.54}$$

$$g_{tpk}^{0}\left(\frac{1}{T}\left(\lambda_{tk} - C_{tuk}\right) + \theta_{tp} - \rho_{tpuk}\right) = 0$$

for g_{puk}^{n} : $$-I_{puk} + \sum_{t} \rho_{tpuk} - \rho_{puk}^{n} \le 0, g_{puk}^{n} \ge 0, \quad (4.55)$$

$$g_{pk}^{n}(-I_{puk} + \sum_{t} \rho_{tpuk} - \rho_{puk}^{n}) = 0$$

for θ_{tp} : $$\sum_{k} s_{tpk} = \sum_{k}\left(\sum_{u} g_{tpuk}^{0} + \sum_{u} g_{tpuk}\right) \quad (4.56)$$

for ρ_{tpuk}^{0} : $$g_{tpuk}^{0} \le G_{puk}^{0}, \rho_{tpuk}^{0} \ge 0, \rho_{tpuk}^{0}(g_{tpuk}^{0} - G_{puk}^{0}) = 0 \quad (4.57)$$

for ρ_{tpuk} : $$g_{tpuk} \le g_{puk}^{n}, \rho_{tpuk} \ge 0, \rho_{tpuk}(g_{tpuk} - g_{puk}^{n}) = 0 \quad (4.58)$$

for ρ_{puk}^{n} : $$g_{puk}^{n} \le G_{puk}^{\max}, \rho_{puk}^{n} \ge 0, \rho_{puk}^{n}(g_{puk}^{n} - G_{puk}^{\max}) = 0 \quad (4.59)$$

KKTs related to the transmission owner's model:

for y_{tk} : $$\frac{1}{T}\lambda_{tk} + \sum_{ij} PTDF_{ij}^{k} \mu_{tij}^{-} - \sum_{ij} PTDF_{ij}^{k} \mu_{tij}^{+} - \sum_{ij} PTDF_{ij}^{k} \mu_{tf} = 0 \quad (4.60)$$

for μ_{tij}^{-} : $$-\sum_{k} PTDF_{ij}^{k} y_{tk} \le T_{ij}, \mu_{tij}^{-} \ge 0, \ \mu_{tij}^{-}(\sum_{k} PTDF_{ij}^{k} y_{tk} + T_{ij}) = 0 \quad (4.61)$$

for μ_{tij}^{+} : $$\sum_{k} PTDF_{ij}^{k} y_{tk} \le T_{ij}, \mu_{tij}^{+} \ge 0, \ \mu_{tij}^{+}(\sum_{k} PTDF_{ij}^{k} y_{tk} - T_{ij}) = 0 \quad (4.62)$$

for μ_{tf} : $$\sum_{ij}\sum_{k} PTDF_{ij}^{k} y_{tk} \le F_{f}, \mu_{tf} \ge 0, \ \mu_{tf}(\sum_{ij}\sum_{k} PTDF_{ij}^{k} y_{tk} - F_{f}) = 0 \quad (4.63)$$

KKT related to the market clearing:

$$\text{for } \lambda_{tk}: \qquad \sum_{p\in\mathcal{P}} s_{tpk} - \sum_{p\in\mathcal{P}} (\sum_u g^0_{tpuk} + \sum_u g_{tpuk}) = y_{tk} \tag{4.64}$$

The KKT conditions of this problem also form a mixed linear complementarity problem with both equality constraints and complementarity constraints. Power generation expansion under uncertainty – Model 2 that is solved in this research is the equivalent quadratic programming problem, which is defined by (4.65)-(4.74).

$$\begin{aligned}\text{Max } & \frac{1}{T}\sum_t\sum_k\left(\alpha_{tk}\sum_p s_{tpk} - \frac{\beta_{tk}}{2}\left(\sum_p s_{tpk}\right)^2 - \frac{\beta_{tk}}{2}\sum_p\left(s_{tpk}\right)^2 - \sum_p\sum_u C^0_{tuk}g^0_{tpuk} - \sum_p\sum_u C_{tuk}g_{tpuk}\right) \\ & -\sum_p\sum_u\sum_k I_{puk}g^n_{puk}\end{aligned} \tag{4.65}$$

$$g^0_{tpuk} \le G^0_{puk} \quad \forall t\in\mathcal{T} \quad \forall p\in\mathcal{P} \quad \forall u\in\mathcal{U} \quad \forall k\in\mathcal{B} \quad (\rho^0_{tpuk}) \tag{4.66}$$

$$g_{tpuk} \le g^n_{puk} \quad \forall t\in\mathcal{T} \quad \forall p\in\mathcal{P} \quad \forall u\in\mathcal{U} \quad \forall k\in\mathcal{B} \quad (\rho_{tpuk}) \tag{4.67}$$

$$g^n_{puk} \le G^{\max}_{puk} \quad \forall u\in\mathcal{U} \quad \forall k\in\mathcal{B} \quad (\rho^n_{puk}) \tag{4.68}$$

$$\sum_k s_{tpk} = \sum_k\left(\sum_u g^0_{tpuk} + \sum_u g_{tpuk}\right) \quad \forall t\in\mathcal{T} \quad \forall p\in\mathcal{P} \quad (\theta_{tp}) \tag{4.69}$$

$$\sum_k PTDF^k_{ij}\, y_{tk} \le T_{ij} \quad \forall ij\in\mathcal{L} \quad \forall t\in\mathcal{T} \quad (\mu^+_{tij}) \tag{4.70}$$

$$-\sum_k PTDF^k_{ij}\, y_{tk} \le T_{ij} \quad \forall ij\in\mathcal{L} \quad \forall t\in\mathcal{T} \quad (\mu^-_{tij}) \tag{4.71}$$

$$\sum_{ij \in \mathcal{F}_f} \left(\sum_k PTDF_{ij}^k * y_{tk} \right) \leq F_f \quad \forall f \in \mathcal{F} \quad \forall t \in \mathcal{T} \quad (\mu_{tf}) \tag{4.72}$$

$$\frac{1}{T}\left(\sum_{p \in \mathcal{P}} s_{tpk} - \sum_{p \in \mathcal{P}} (\sum_u g_{tpuk}^0 + \sum_u g_{tpuk}) \right) = \frac{1}{T} y_{tk} \quad \forall k \in \mathcal{B} \quad \forall t \in \mathcal{T} \quad (\lambda_{tk}) \tag{4.73}$$

$$\forall s_{tpk}, g_{tpuk}^0, g_{tpuk}, g_{puk}^n \geq 0 \tag{4.74}$$

Notice that in this model, unlike Model 1, the transmission quantity variable y_{tk} has two indexes. It depends on both the state realized and the nodes of the network. Thus, constraints (4.70)-(4.73) are revised according to the new TSO and ISO models. Notice that the constraints are again written for all the power producers and that this quadratic program needs to be solved only once to find the equilibrium solution.

4.5. Numerical Example and Analysis

To study the effect of uncertainties on optimal generation expansion decisions under competition, the IEEE Reliability Test System [32] is used. The MOSEK solver available in NEOS server [33], [34], [35] is used to solve the Generation Expansion Equilibrium Model 1 and Model 2.

As detailed in chapter 3, the 24-bus system has 34 transmission lines with the merging of the double lines into a single line. The network line reactances and limits for the 34 lines are given in table 3.5. The PTDF values are given in Appendix A1 and the full network is described in figure 3.4.

The demand functions are defined by using the suggested peak demand values in [32]. The base price intercept which is assumed to be the current price intercept for each demand location is defined proportional to the peak demand values at each node. The yearly growth in power demand is assumed to be uniformly distributed between 1% and 3%, and thus the price intercept of the inverse demand functions are assumed to be uniformly distributed within this interval. The existing generator locations and demand locations are the same as the original system. Ten possible states of nature with equal probability of occurrence are assumed. The demand parameters are generated using the random numbers generated with help of the Microsoft Excel software. The current price intercepts, the price intercepts for ten states and the average of the states are shown in table 4.1.

Table 4.1. Price intercept of inverse demand functions ($/MWh)

Node	Base	State										Avg
		1	2	3	4	5	6	7	8	9	10	
1	72.9	84.1	83.5	89.5	91.1	83.6	85.4	93.4	83.1	87.2	89.2	87.0
2	65.5	75.6	75.0	80.4	81.8	75.1	76.7	83.9	74.6	78.3	80.1	78.2
3	121.5	140.2	139.2	149.1	151.8	139.3	142.4	155.7	138.5	145.3	148.7	145.0
4	50.0	57.7	57.2	61.3	62.4	57.3	58.5	64.0	56.9	59.7	61.1	59.6
5	47.9	55.3	54.9	58.8	59.9	55.0	56.2	61.4	54.6	57.3	58.7	57.2
6	91.8	106.0	105.1	112.7	114.7	105.3	107.6	117.6	104.6	109.8	112.4	109.6
7	84.4	97.4	96.6	103.6	105.4	96.7	98.9	108.1	96.2	100.9	103.3	100.7
8	115.4	133.2	132.2	141.7	144.2	132.4	135.3	147.9	131.6	138.0	141.3	137.8
9	118.1	136.3	135.3	145.0	147.6	135.4	138.4	151.3	134.6	141.3	144.6	141.0
10	131.6	151.9	150.7	161.6	164.5	150.9	154.3	168.6	150.0	157.4	161.1	157.1
13	178.9	206.5	204.9	219.6	223.5	205.1	209.6	229.2	203.9	213.9	219.0	213.5
14	131.0	151.1	150.0	160.7	163.6	150.2	153.5	167.8	149.3	156.6	160.3	156.3
15	214.0	247.0	245.1	262.6	267.3	245.4	250.8	274.1	243.9	255.9	261.9	255.4
16	67.5	77.9	77.3	82.9	84.3	77.4	79.1	86.5	76.9	80.7	82.6	80.6
18	224.8	259.4	257.4	275.9	280.8	257.7	263.4	288.0	256.2	268.8	275.1	268.3
19	122.2	141.0	139.9	150.0	152.6	140.1	143.2	156.5	139.3	146.1	149.6	145.8
20	86.4	99.7	99.0	106.0	107.9	99.1	101.3	110.7	98.5	103.3	105.8	103.1

As in the previous example in chapter 3, the same slope for the inverse demand functions is used for all demand nodes and it is selected as one of the three values 0.09, 0.15, or 0.18 ($/MWh^2) to have different levels of price sensitivity over the demand states. The slopes of the inverse demand functions for the ten states of nature and the average of the states are shown in table 4.2.

Table 4.2. Slope of inverse demand functions ($/MWh^2)

Node	State										
	1	2	3	4	5	6	7	8	9	10	Avg
1	0.15	0.15	0.09	0.09	0.15	0.15	0.18	0.18	0.15	0.15	0.14
2	0.15	0.15	0.09	0.09	0.15	0.15	0.18	0.18	0.15	0.15	0.14
3	0.15	0.15	0.09	0.09	0.15	0.15	0.18	0.18	0.15	0.15	0.14
4	0.15	0.15	0.09	0.09	0.15	0.15	0.18	0.18	0.15	0.15	0.14
5	0.15	0.15	0.09	0.09	0.15	0.15	0.18	0.18	0.15	0.15	0.14
6	0.15	0.15	0.09	0.09	0.15	0.15	0.18	0.18	0.15	0.15	0.14
7	0.15	0.15	0.09	0.09	0.15	0.15	0.18	0.18	0.15	0.15	0.14
8	0.15	0.15	0.09	0.09	0.15	0.15	0.18	0.18	0.15	0.15	0.14
9	0.15	0.15	0.09	0.09	0.15	0.15	0.18	0.18	0.15	0.15	0.14
10	0.15	0.15	0.09	0.09	0.15	0.15	0.18	0.18	0.15	0.15	0.14
13	0.15	0.15	0.09	0.09	0.15	0.15	0.18	0.18	0.15	0.15	0.14
14	0.15	0.15	0.09	0.09	0.15	0.15	0.18	0.18	0.15	0.15	0.14
15	0.15	0.15	0.09	0.09	0.15	0.15	0.18	0.18	0.15	0.15	0.14
16	0.15	0.15	0.09	0.09	0.15	0.15	0.18	0.18	0.15	0.15	0.14
18	0.15	0.15	0.09	0.09	0.15	0.15	0.18	0.18	0.15	0.15	0.14
19	0.15	0.15	0.09	0.09	0.15	0.15	0.18	0.18	0.15	0.15	0.14
20	0.15	0.15	0.09	0.09	0.15	0.15	0.18	0.18	0.15	0.15	0.14

As for the different fuel types, different fuel types existing in the system were not considered in the deterministic equilibrium model in chapter 3 in order to be able to focus on the transmission constraints and keep the network simple. However, in the uncertainty model, one of the uncertainty sources is the fuel cost, which can have a huge effect on the generator operational costs. For that reason, the existing units in the network are separated according to their fuel types based on [32]. The existing generators in the 24-bus network with their fuel types are given in table 4.3. In this problem, two different producers, *p* and *q* are studied. Producer *p* is assumed to own the existing generators at nodes 1 through 13, whereas the producer *q* is assumed to own the generators at nodes 14 through 24. However each producer is able to invest unlimited capacity at any node in the network.

Table 4.3. Different fuel types and capacities in the 24-Bus System (MW)

Node	Fuel Type			
	Oil	Hydro	Coal	Nuclear
1	40		152	
2	40		152	
7	300			
13	591			
15	60		155	
16			155	
18				400
21				400
22		300		
23			660	

To consider the fuel cost uncertainty, fuel price growth is assumed normally distributed with the means taken from the Annual Energy Outlook 2007 official statistics of the US Government [36] for the future of power markets. In addition, it is assumed that two types of existing units are affected from the fuel cost uncertainty, namely coal and oil units. In addition, the capacity investments are assumed to be made in one of the coal or gas fuel types. The price of coal and oil is expected to increase with the associate means, whereas the price of gas is expected to decrease. The parameters of the fuel cost price growth are given in table 4.4.

Table 4.4. Fuel price change parameters

Fuel type	Growth	
	Mean	Std
Coal	4.575	0.15
Oil	19.13	0.25
Gas	-32.78	0.23

As the operational costs of the power generators are directly affected by the changes in fuel costs, the operational cost data is generated based on the above fuel cost data. The Microsoft Excel software is used to construct tables 4.5 and 4.6. These tables show the operational cost of existing and new generators depending on the location in the power network.

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Table 4.5. Operational costs of existing generators in the 24-Bus System ($/MWh)

Node	Current	Segment										Avg
		1	2	3	4	5	6	7	8	9	10	
Coal												
1	13.24	13.84	13.86	13.84	13.81	13.84	13.84	13.82	13.85	13.85	13.83	13.84
2	13.24	13.84	13.86	13.84	13.81	13.84	13.84	13.82	13.85	13.85	13.83	13.84
15	10.42	10.89	10.91	10.89	10.87	10.89	10.89	10.88	10.90	10.90	10.88	10.89
16	10.42	10.89	10.91	10.89	10.87	10.89	10.89	10.88	10.90	10.90	10.88	10.89
23	10.52	11	11.01	10.99	10.97	11	10.99	10.98	11	11	10.99	10.99
Oil												
1	32.36	38.52	38.61	38.51	38.41	38.52	38.51	38.45	38.57	38.57	38.47	38.51
2	32.57	38.77	38.86	38.76	38.66	38.77	38.76	38.70	38.82	38.82	38.72	38.76
7	20.7	24.64	24.70	24.63	24.57	24.64	24.63	24.60	24.67	24.67	24.61	24.64
13	20.47	24.37	24.42	24.36	24.30	24.37	24.36	24.32	24.40	24.40	24.34	24.36
15	25.95	30.89	30.96	30.88	30.80	30.89	30.88	30.84	30.93	30.93	30.85	30.89
Hydro												
22	10	10	10	10	10	10	10	10	10	10	10	10
Oil												
18	5.46	5.46	5.46	5.46	5.46	5.46	5.46	5.46	5.46	5.46	5.46	5.46
21	5.46	5.46	5.46	5.46	5.46	5.46	5.46	5.46	5.46	5.46	5.46	5.46

Table 4.6. Operational costs of new generators in 24-Bus System ($/MWh)

Node	Current	Segment										Avg
		1	2	3	4	5	6	7	8	9	10	
Coal												
1	10	10.45	10.45	10.48	10.47	10.45	10.45	10.45	10.48	10.45	10.47	10.46
2	10	10.45	10.45	10.48	10.47	10.45	10.45	10.45	10.48	10.45	10.47	10.46
3	10	10.45	10.45	10.48	10.47	10.45	10.45	10.45	10.48	10.45	10.47	10.46
4	10	10.45	10.45	10.48	10.47	10.45	10.45	10.45	10.48	10.45	10.47	10.46
5	10	10.45	10.45	10.48	10.47	10.45	10.45	10.45	10.48	10.45	10.47	10.46
6	10	10.45	10.45	10.48	10.47	10.45	10.45	10.45	10.48	10.45	10.47	10.46
7	10	10.45	10.45	10.48	10.47	10.45	10.45	10.45	10.48	10.45	10.47	10.46
8	10	10.45	10.45	10.48	10.47	10.45	10.45	10.45	10.48	10.45	10.47	10.46
9	16.1	16.83	16.83	16.88	16.86	16.83	16.83	16.83	16.87	16.83	16.86	16.84
10	16.1	16.83	16.83	16.88	16.86	16.83	16.83	16.83	16.87	16.83	16.86	16.84
11	16.1	16.83	16.83	16.88	16.86	16.83	16.83	16.83	16.87	16.83	16.86	16.84
12	16.1	16.83	16.83	16.88	16.86	16.83	16.83	16.83	16.87	16.83	16.86	16.84
13	10	10.45	10.45	10.48	10.47	10.45	10.45	10.45	10.48	10.45	10.47	10.46
14	10	10.45	10.45	10.48	10.47	10.45	10.45	10.45	10.48	10.45	10.47	10.46
15	10	10.45	10.45	10.48	10.47	10.45	10.45	10.45	10.48	10.45	10.47	10.46
16	10	10.45	10.45	10.48	10.47	10.45	10.45	10.45	10.48	10.45	10.47	10.46
17	10	10.45	10.45	10.48	10.47	10.45	10.45	10.45	10.48	10.45	10.47	10.46
18	17.35	18.13	18.13	18.19	18.17	18.13	18.14	18.14	18.19	18.14	18.17	18.15
19	10	10.45	10.45	10.48	10.47	10.45	10.45	10.45	10.48	10.45	10.47	10.46
20	10	10.45	10.45	10.48	10.47	10.45	10.45	10.45	10.48	10.45	10.47	10.46
21	17.35	18.13	18.13	18.19	18.17	18.13	18.14	18.14	18.19	18.14	18.17	18.15
22	10	10.45	10.45	10.48	10.47	10.45	10.45	10.45	10.48	10.45	10.47	10.46
23	10	10.45	10.45	10.48	10.47	10.45	10.45	10.45	10.48	10.45	10.47	10.46
24	10	10.45	10.45	10.48	10.47	10.45	10.45	10.45	10.48	10.45	10.47	10.46
Gas												
1	35	23.54	23.48	23.52	23.44	23.64	23.46	23.38	23.41	23.45	23.57	23.49
2	35	23.54	23.48	23.52	23.44	23.64	23.46	23.38	23.41	23.45	23.57	23.49
3	35	23.54	23.48	23.52	23.44	23.64	23.46	23.38	23.41	23.45	23.57	23.49
4	30.1	20.25	20.19	20.23	20.16	20.33	20.18	20.10	20.13	20.17	20.27	20.20
5	30.1	20.25	20.19	20.23	20.16	20.33	20.18	20.10	20.13	20.17	20.27	20.20
6	35	23.54	23.48	23.52	23.44	23.64	23.46	23.38	23.41	23.45	23.57	23.49
7	35	23.54	23.48	23.52	23.44	23.64	23.46	23.38	23.41	23.45	23.57	23.49
8	35	23.54	23.48	23.52	23.44	23.64	23.46	23.38	23.41	23.45	23.57	23.49
9	35	23.54	23.48	23.52	23.44	23.64	23.46	23.38	23.41	23.45	23.57	23.49
10	35	23.54	23.48	23.52	23.44	23.64	23.46	23.38	23.41	23.45	23.57	23.49
11	35	23.54	23.48	23.52	23.44	23.64	23.46	23.38	23.41	23.45	23.57	23.49
12	35	23.54	23.48	23.52	23.44	23.64	23.46	23.38	23.41	23.45	23.57	23.49
13	45.8	30.81	30.72	30.78	30.67	30.93	30.70	30.59	30.63	30.69	30.84	30.74
14	35	23.54	23.48	23.52	23.44	23.64	23.46	23.38	23.41	23.45	23.57	23.49
15	35	23.54	23.48	23.52	23.44	23.64	23.46	23.38	23.41	23.45	23.57	23.49
16	35	23.54	23.48	23.52	23.44	23.64	23.46	23.38	23.41	23.45	23.57	23.49
17	35	23.54	23.48	23.52	23.44	23.64	23.46	23.38	23.41	23.45	23.57	23.49
18	35	23.54	23.48	23.52	23.44	23.64	23.46	23.38	23.41	23.45	23.57	23.49
19	35	23.54	23.48	23.52	23.44	23.64	23.46	23.38	23.41	23.45	23.57	23.49
20	45.8	30.81	30.72	30.78	30.67	30.93	30.70	30.59	30.63	30.69	30.84	30.74
21	35	23.54	23.48	23.52	23.44	23.64	23.46	23.38	23.41	23.45	23.57	23.49
22	35	23.54	23.48	23.52	23.44	23.64	23.46	23.38	23.41	23.45	23.57	23.49
23	45.8	30.81	30.72	30.78	30.67	30.93	30.70	30.59	30.63	30.69	30.84	30.74
24	35	23.54	23.48	23.52	23.44	23.64	23.46	23.38	23.41	23.45	23.57	23.49

As stated previously, the investments can be made on two types of units which are coal and gas. These fuel types are selected as they reflect two cases of generator unit investments. Coal units have larger investment costs and smaller operational costs. Gas units on the other hand have relatively smaller investment costs and larger operational costs. Therefore, coal units can be an example of base-load generation units, whereas gas units are similar to the peak-load generation units. In table 4.7, the investment costs of these unit types are shown.

Table 4.7. Investment costs of new generators in the 24-Bus System ($/MWh)

Node	Coal	Gas	Node	Coal	Gas
1	12.21	4.62	13	12.21	7.2
2	12.21	4.62	14	12.21	4.62
3	12.21	4.62	15	12.21	4.62
4	12.21	3.5	16	12.21	4.62
5	12.21	3.5	17	12.21	4.62
6	12.21	4.62	18	20.1	4.62
7	12.21	4.62	19	12.21	4.62
8	12.21	4.62	20	12.21	7.2
9	18.5	4.62	21	20.1	4.62
10	18.5	4.62	22	12.21	4.62
11	18.5	4.62	23	12.21	7.2
12	18.5	4.62	24	12.21	4.62

The capacity expansion solutions of both producers in both fuel types are obtained by solving the Generation Expansion under Uncertainty models with the ten states. The capacity expansion quantities resulting from Model 1 and Model 2 are given in table 4.8. According to these results, Model 1 and Model 2 give similar results in terms of total capacity expansion. The results indicate that the producer q chooses to invest in similar capacity quantities to both coal and gas units; however producer p invests mostly in coal type units. The reason for that is the lower need of producer q for base-load units. Producer q already owns less expensive units such as hydro and nuclear units, which have relatively smaller generation costs. Thus, peak-load generators may be more profitable for producer q after investing a certain amount of capacity on coal units.

Table 4.8. Capacity expansion comparison (MW)

Producer	Gen. Exp. Model 1			Gen. Exp. Model 2			Certainty Model		
	Coal	Gas	Total	Coal	Gas	Total	Coal	Gas	Total
p	3157	1874	5031	3375	1454	4829	3553	528	4081
q	2344	1812	4156	2105	2121	4226	2192	394	2587
Total	5501	3686	9187	5480	3574	9054	5746	922	6668

The results of the Generation Expansion problem are compared as in [24] with the Certainty game (known also as the Expected Value Solution) in order to see the effect of modeling the uncertainties in generation expansion. For the Certainty game solution, the averages of the demand quantity and operational costs (given in Avg column of the data tables) are used over all the states to solve the equilibrium problems described. It can be observed that in the Certainty game, the investment on gas units are less than the generation expansion under uncertainty solution of Model 1 and Model 2 for both producers, whereas the coal investments are not that affected. This shows that when the Certainty values are used in the model the results indicate less need in peak-load units as high and low demand states are combined in an average demand state. The capacity expansion solution results with their locations in the network for all three models are given in Appendix A2-A4.

The profits of the firms and other economic analysis information are given in table 4.9. With the generation expansion model under uncertainty Model 1 and Model 2, it can be observed that Model 1 produces higher producer profits. Since the information on the states is not considered in the TSO model, this is reflected on the producer profits. Model 1 and Model 2 producer profits are both larger than the profits obtained in the Certainty approach. This shows that the uncertainty game can produce more profitable results for both of the producer players. Similarly, the higher consumer benefit, and total welfare values given in table 4.9 indicate that the uncertainty model produces more beneficial results.

Table 4.9. Economic analysis comparison ($)

Metric		Gen. Exp. Model 1	Gen. Exp. Model 2	Certainty Model
Profit	*p*	226,105	226,005	215,875
	q	252,555	248,398	241,779
	Total	478,660	474,404	457,654
Consumer Benefit		1,107,236	1,102,707	1,050,929
Congestion Rent		16,278	21,487	21,067
Total Welfare		696,957	691,255	667,626

4.6. Evaluation of Expansion Decisions with Cournot Competition Model

In this section, a comparative analysis of the generation expansion decisions made using the different models is conveyed. Since Model 2 is constructed with the probability of occurrence of each state of nature, the decisions of all market players are provided by the model for each state of nature. That is, by solving Model 2 not only the expansion decisions are obtained, but also the sales of the producers, the transmission quantities, and the prices at each node for all the possible states. However, in Model 1 and the Certainty game, the expansion decisions need to be evaluated considering the possible states of nature in order to obtain the market operation decisions of all players. That is, all possible scenarios sales, transmission amounts, and power prices should be calculated again using the bidding under competition model. With this aim, the Cournot competition model presented in [13] is used. This equilibrium model is rewritten using the notation of this dissertation and given in (4.75)-(4.81). The subscript h indicates the generating facility index at a certain node.

$$\text{Max} \sum_k [\alpha_k \sum_{p\in\mathcal{P}} s_{pk} - \frac{\beta_k}{2}[\sum_{p\in\mathcal{P}} (s_{pk})]^2 - \frac{\beta_k}{2}\sum_{p\in\mathcal{P}} (s_{pk})^2 - \sum_p \sum_h C_{pkh} g_{pkh}] \tag{4.75}$$

subject to

$$g_{pkh} \le G_{pkh}^{\max} \quad \forall p \in \mathcal{P}, k \in \mathcal{B}, h \in \mathcal{H} \quad (\rho_{pkh}) \tag{4.76}$$

$$\sum_k s_{pk} = \sum_k \sum_h s_{pkh} \quad \forall p \in \mathcal{P} \quad (\theta_p) \tag{4.77}$$

$$\sum_k PTDF_{ij}^k y_k \le T_{ij} \quad \forall ij \in \mathcal{L} \quad (\mu_{ij}^+) \tag{4.78}$$

$$-\sum_k PTDF_{ij}^k y_k \le T_{ij} \quad \forall ij \in \mathcal{L} \quad (\mu_{ij}^-) \tag{4.79}$$

$$\sum_{p\in\mathcal{P}} s_{pk} - \sum_{p\in\mathcal{P}} \sum_{h\in\mathcal{H}} g_{pkh} = y_k \quad \forall k \in \mathcal{B} \quad (\lambda_k) \tag{4.80}$$

$$\forall s_{pk}, g_{pkh} \ge 0 \tag{4.81}$$

In order to evaluate the performance of the expansion decisions on the network, the Cournot equilibrium model is solved using the expanded capacity solutions obtained from the generation expansion equilibrium Model 1 and the Certainty game of the 24-Bus system example and compared to the results obtained from Model 2. The Cournot competition model is solved using the data of all 10 possible states taking the capacity expansion solutions of Model 1 and the Certainty game under consideration separately. Afterwards, the results obtained from the different states are averaged and compared to those obtained by solving Model 2. These results are represented in tables 4.10-4.15.

Table 4.10. Comparison of the producer profits ($)

Profit	Gen. Exp. Mod. 1	Gen. Exp. Model 2	Certainty Model
p	270,015	273,880	277,702
q	281,555	283,848	274,699
Total	551,571	557,728	552,401

The economic analysis results for the producers are shown in table 4.10. According to these results it is observed that when the expansion decisions are obtained considering the different states of nature, generation expansion Model 2 produces the highest total producer profit.

After solving the Cournot model using the new capacities obtained by the expansion models, the congestion rents, consumer benefits, and total welfare are also calculated for each Cournot solution. The results for all three cases are represented in table 4.11. It can be indicated that the proposed generation expansion model under uncertainty Model 2 is beneficial as higher consumer benefit and higher overall system welfare can be obtained by using this equilibrium capacity investment model. In addition, it is worth mentioning that Model 1 and the Certainty Model also produces much higher congestion rents.

Table 4.11. Cournot economic analysis ($)

Metric	Gen. Exp. Model 1	Gen. Exp. Model 2	Certainty Model
Consumer Benefit	1,066,620	1,102,707	1,053,276
Congestion Rent	33,287	21,487	36,608
Total Welfare	767,333	774,579	752,944

For a more through analysis of the results on the system and the power consumers, the power prices at every load location, the amounts that are paid by the consumers and the consumer benefits at the demand nodes, and the transmission revenues collected for each node of the network are given in tables 4.12-4.15. The Cournot prices given in table 4.12 give an idea of the different result provided by the three models. Although in certain demand nodes Model 2 results in higher power prices as a result of the network constraints, in general terms it can be concluded that lower prices can be obtained with the expansion solution found by Model 2. This also explains the reason why the total consumer benefit in table 4.11 is higher for Model 2.

Table 4.12. Cournot prices ($/MWh)

Node	Gen. Exp. Model 1	Gen. Exp. Model 2	Certainty Model
1	44.51	44.12	43.94
2	41.35	41.16	41.33
3	63.55	63.46	65.54
4	33.26	34.99	35.18
5	32.45	34.18	34.52
6	51.31	51.64	52.68
7	46.14	48.68	45.19
8	63.80	61.04	62.68
9	68.91	65.74	70.14
10	74.64	71.11	76.37
13	86.03	86.28	81.67
14	65.55	67.21	67.04
15	105.36	100.25	107.14
16	41.37	41.97	43.81
18	116.66	108.17	119.10
19	63.50	63.72	66.63
20	49.59	49.49	50.10

Table 4.13 represents the total amounts that need to be paid by each load location in the network. These amounts show the quantities paid to receive the optimal amount of power for the three models. However, these amounts are not significant for comparison as the power quantities purchased by the consumers are different for all three models. Therefore, it is reasonable that the total amounts paid by the consumers are also different.

Table 4.13. Total amount paid by the consumer at each node ($)

Node	Gen. Exp. Model 1	Gen. Exp. Model 2	Certainty Model
1	13,522	13,658	13,562
2	10,801	10,878	10,913
3	36,686	37,995	35,014
4	6,495	6,046	6,404
5	5,951	5,483	5,896
6	20,957	21,627	20,486
7	18,608	18,681	18,241
8	33,610	34,311	32,190
9	35,373	36,402	33,603
10	44,120	45,117	41,484
13	81,383	81,471	75,641
14	43,219	44,173	41,508
15	113,147	115,458	106,341
16	10,536	11,368	10,424
18	127,422	128,534	118,073
19	37,624	38,430	35,858
20	18,053	19,025	18,199
Total	657,508	668,659	623,837

In order to be able compare the consumer viewpoint to the three models, consumer benefit at each load location is calculated and given in table 4.14. Consumer benefit is defined as the willingness of the load to pay for a certain quantity of power. Therefore, the consumer benefit better represents the actual advantage of the load from the power transactions instead of the total amount paid. Depending on the nodal power prices, Model 2 results in lower consumer benefits in a few demand locations, however; as the prices obtained by Model 2 are lower in most nodes the consumer benefits are higher, which is reflected in the total consumer benefits. The consumer surplus values given in table 4.14 show the difference between what the consumer is willing to pay for the power and what they actually pay. It is again observed that the highest consumer surplus is obtained by the expansion solution of Model 2.

Table 4.14. Total consumer benefit at each node ($)

Node	Gen. Exp. Model 1	Gen. Exp. Model 2	Certainty Model
1	19,840	20,109	20,045
2	15,528	15,654	15,484
3	59,851	61,565	58,508
4	9,093	8,158	8,073
5	8,243	7,327	7,253
6	32,780	33,392	32,223
7	29,402	28,441	29,623
8	52,685	55,129	53,086
9	53,543	56,636	52,360
10	67,941	71,634	66,037
13	140,315	140,261	143,229
14	72,164	72,471	71,345
15	191,858	202,284	189,837
16	15,960	16,549	15,148
18	208,008	221,698	203,462
19	61,349	62,325	59,225
20	28,059	29,075	28,336
Total	1,066,620	1,102,707	1,053,276
Consumer Surplus	409,111	434,048	429,439

Finally, how the transmission revenue collected from each node of the network is analyzed. The transmission revenues collected from each node are given in table 4.15. It can be observed that these quantities show much difference depending on which model is used for finding the generation expansion solution. All three models result in a different dispatch of power through the transmission lines. Thus, the transmission revenue collected for each node is different for all three models with Model 2 providing the least congestion-based transmission revenue.

Table 4.15. Total transmission price collected at each node ($)

Node	Gen. Exp. Model 1	Gen. Exp. Model 2	Certainty Model
1	-66	-59	535
2	12	15	160
3	-43	11	-159
4	264	48	133
5	563	79	-11
6	67	4	99
7	533	278	574
8	-464	-2	119
9	3,069	1,790	1,868
10	2,981	1,792	2,273
11	0	0	0
12	0	-75	0
13	516	26	1,303
14	330	-158	755
15	3,106	8	1,860
16	125	-7	552
17	1,160	0	0
18	6,511	1,573	3,945
19	-492	-10	-916
20	112	25	0
21	7,179	7,687	10,602
22	1,847	2,745	4,324
23	5,976	5,715	8,591
24	0	0	0
Total	33,287	21,487	36,608

4.7. Conclusion

Two new models for finding the generation expansion decisions in deregulated electricity markets under Cournot competition and uncertainty are proposed. The presented models consider the behavioral models of the power market players and the uncertainty in the generation expansion modeling. By using the suggested generation expansion models for a given power system additional information can be added in the generation expansion modeling process with the utilization of states of nature of demand quantities and fuel costs. Comparing these models to the Certainty game solution, it has been shown that modeling the uncertainties in capacity expansion may provide higher benefits for the market participants and the overall system welfare. It has also been shown that considering the states in all market participants' models may results in much lower congestion rents in Cournot equilibrium.

CHAPTER 5

CONCLUSION AND FURTHER RESEARCH DIRECTIONS

In this research, models of generation capacity investments in deregulated power markets were introduced. The first model aimed to analyze the effect of transmission congestion and Cournot competition on generation expansion decisions in deregulated electricity markets. Additionaly, two new models were introduced for finding the generation expansion decisions in deregulated electricity markets under Cournot competition and uncertainty. The models presented considered the behavioral models of the power market players and the transmission network in the generation expansion modeling. In addition, uncertainties in power demand and fuel costs were taken into consideration in the second model.

By using the suggested models, the effects of the network transmission constraints on the generation expansion decision can be analyzed under perfect and imperfect competition. In addition, how decisions are affected by uncertainty can be evaluated with the help of the generation expansion under uncertainty models.

In the model presented in chapter 3, transmission constraints represented in terms of PTDFs were included in modeling generation expansion decisions under Cournot competition. It has been shown that transmission constraints may affect the outcome of competition by limiting expansion decisions of producers according to where the generators are located relative to the transmission limits. It has also been shown that if transmission constraints are not considered in planning decisions, the consumer's benefit may diminish due to higher prices and higher congestion charges.

In the models presented in chapter 4, the behavioral models of the power market players and the uncertainty in the generation expansion modeling were both taken into account. By using the suggested generation expansion models for a given power system additional information has been added in the generation expansion modeling process with the utilization of states of nature of demand quantities and fuel costs. Comparing the generation expansion models under uncertainty and the Certainty game solutions, it has been shown that modeling the uncertainties in capacity expansion provides higher total benefits for the market participants and the overall system welfare. It has also been shown that considering the states in all market participants' models may result in much lower congestion rents in Cournot equilibrium.

For future research, the sensitivity analysis concept in quadratic programming can be considered. The equilibrium models presented in this research are convex quadratic programs and sensitivity analysis on parameters of the models can be conveyed using optimization theory. In recent theory, it has been proved that in a quadratic program, sensitivity analysis on the coefficients of the right-hand-side vector and the objective function results in a partially quadratic objective value function [37]. This property will be used in further research to obtain a mapping of demand and fuel cost parameters on the objective value function.

Moreover, in this dissertation transmission expansion has not been taken into consideration to avoid additional complexity. The transmission network has been accounted for using the current network. In future research, models that relate generation expansion and transmission expansion decision in deregulated markets will be studied.

Furthermore, in this research the uncertainties in power demand and fuel costs are modeled using equal probability states. Using probability distributions in competitive models result in very complicated models, especially under generation capacity and network constraints. One additional area of research in the future could be to study modeling generation expansion under deregulation using probability distribution functions of the sources of uncertainties.

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APPENDIX

Table A1. 24-Bus System PTDF data

Link	1	2	3	4	5	6	7	8
1-2	0.380	-0.563	0.041	-0.304	0.171	-0.171	-0.070	-0.070
1-3	0.360	0.337	-0.085	0.211	0.273	0.220	0.145	0.145
1-5	0.260	0.226	0.044	0.093	-0.445	-0.049	-0.075	-0.075
2-4	0.224	0.252	0.013	-0.385	0.135	0.093	-0.022	-0.022
2-6	0.157	0.185	0.028	0.081	0.036	-0.264	-0.048	-0.048
3-9	-0.161	-0.177	0.181	-0.284	-0.203	-0.230	-0.309	-0.309
3-24	0.521	0.514	0.735	0.495	0.476	0.450	0.454	0.454
4-9	0.224	0.252	0.013	0.615	0.135	0.093	-0.022	-0.022
5-10	0.260	0.226	0.044	0.093	0.556	-0.049	-0.075	-0.075
6-10	0.157	0.185	0.028	0.081	0.036	0.736	-0.048	-0.048
7-8	0	0	0	0	0	0	1	0
8-9	0.036	0.034	-0.012	-0.016	0.067	0.084	0.521	0.521
8-10	-0.036	-0.034	0.012	0.016	-0.067	-0.084	0.479	0.479
9-11	0.070	0.076	0.102	0.180	0.022	-0.003	0.119	0.119
9-12	0.028	0.033	0.079	0.136	-0.024	-0.051	0.071	0.071
10-11	0.212	0.210	0.054	0.117	0.285	0.326	0.202	0.202
10-12	0.170	0.167	0.031	0.073	0.239	0.278	0.154	0.154
11-13	0.006	0.006	0.004	0.007	0.007	0.007	0.007	0.007
11-14	0.275	0.279	0.153	0.291	0.301	0.316	0.314	0.314
12-13	0.080	0.082	0.045	0.085	0.088	0.092	0.092	0.092
12-23	0.117	0.119	0.065	0.124	0.128	0.135	0.134	0.134
13-23	0.087	0.088	0.048	0.091	0.095	0.099	0.099	0.099
14-16	0.275	0.279	0.153	0.291	0.301	0.316	0.314	0.314
15-16	-0.413	-0.419	-0.229	-0.436	-0.451	-0.474	-0.470	-0.470
15-21	-0.066	-0.067	-0.037	-0.070	-0.073	-0.076	-0.076	-0.076
15-24	0.479	0.486	0.265	0.506	0.524	0.550	0.546	0.546
16-17	0.066	0.067	0.037	0.070	0.073	0.076	0.076	0.076
16-19	-0.204	-0.207	-0.113	-0.215	-0.223	-0.234	-0.232	-0.232
17-18	0.054	0.055	0.030	0.057	0.059	0.062	0.061	0.061
17-22	0.013	0.013	0.007	0.013	0.014	0.014	0.014	0.014
18-21	0.054	0.055	0.030	0.057	0.059	0.062	0.061	0.061
19-20	-0.204	-0.207	-0.113	-0.215	-0.223	-0.234	-0.232	-0.232
20-23	-0.204	-0.207	-0.113	-0.215	-0.223	-0.234	-0.232	-0.232
21-22	-0.013	-0.013	-0.007	-0.013	-0.014	-0.014	-0.014	-0.014

Table A1. 24-Bus System PTDF data – Continued 1

Link	9	10	11	12	13	14	15	16
1-2	-0.093	-0.047	-0.058	-0.062	-0.058	-0.046	-0.025	-0.033
1-3	0.108	0.183	0.120	0.128	0.120	0.096	0.053	0.068
1-5	-0.015	-0.136	-0.063	-0.066	-0.062	-0.050	-0.027	-0.035
2-4	-0.087	0.043	-0.018	-0.020	-0.018	-0.015	-0.008	-0.010
2-6	-0.005	-0.090	-0.040	-0.042	-0.039	-0.032	-0.017	-0.022
3-9	-0.371	-0.247	-0.256	-0.272	-0.255	-0.204	-0.112	-0.144
3-24	0.479	0.430	0.376	0.400	0.374	0.300	0.164	0.211
4-9	-0.087	0.043	-0.018	-0.020	-0.018	-0.015	-0.008	-0.010
5-10	-0.015	-0.136	-0.063	-0.066	-0.062	-0.050	-0.027	-0.035
6-10	-0.005	-0.090	-0.040	-0.042	-0.039	-0.032	-0.017	-0.022
7-8	0	0	0	0	0	0	0	0
8-9	-0.057	0.099	0.017	0.019	0.017	0.014	0.008	0.010
8-10	0.057	-0.099	-0.017	-0.019	-0.017	-0.014	-0.008	-0.010
9-11	0.265	-0.028	-0.215	0.004	-0.096	-0.153	-0.063	-0.081
9-12	0.220	-0.078	-0.042	-0.276	-0.160	-0.052	-0.049	-0.063
10-11	0.041	0.362	-0.146	0.077	-0.027	-0.098	-0.033	-0.043
10-12	-0.005	0.312	0.027	-0.203	-0.091	0.003	-0.019	-0.024
11-13	0.007	0.008	0.216	-0.200	-0.431	0.115	-0.002	-0.003
11-14	0.300	0.328	0.423	0.281	0.308	-0.367	-0.094	-0.122
12-13	0.088	0.096	-0.088	0.293	-0.318	-0.064	-0.028	-0.035
12-23	0.128	0.139	0.072	0.227	0.067	0.015	-0.040	-0.052
13-23	0.094	0.103	0.129	0.093	0.250	0.052	-0.030	-0.038
14-16	0.300	0.328	0.423	0.281	0.308	0.633	-0.094	-0.122
15-16	-0.449	-0.491	-0.537	-0.517	-0.539	-0.603	0.142	-0.679
15-21	-0.072	-0.079	-0.086	-0.083	-0.087	-0.097	0.023	-0.109
15-24	0.522	0.570	0.624	0.601	0.626	0.700	0.836	0.789
16-17	0.072	0.079	0.086	0.083	0.087	0.097	-0.023	0.109
16-19	-0.222	-0.242	-0.201	-0.320	-0.318	-0.067	0.070	0.090
17-18	0.059	0.064	0.070	0.068	0.070	0.079	-0.019	0.089
17-22	0.014	0.015	0.016	0.016	0.016	0.018	-0.004	0.021
18-21	0.059	0.064	0.070	0.068	0.070	0.079	-0.019	0.089
19-20	-0.222	-0.242	-0.201	-0.320	-0.318	-0.067	0.070	0.090
20-23	-0.222	-0.242	-0.201	-0.320	-0.318	-0.067	0.070	0.090
21-22	-0.014	-0.015	-0.016	-0.016	-0.016	-0.018	0.004	-0.021

Table A1. 24-Bus System PTDF data – Continued 2

Link	17	18	19	20	21	22	23	24
1-2	-0.031	-0.030	-0.037	-0.046	-0.029	-0.029	-0.050	0
1-3	0.064	0.062	0.078	0.095	0.059	0.061	0.104	0
1-5	-0.033	-0.032	-0.040	-0.049	-0.031	-0.032	-0.054	0
2-4	-0.010	-0.010	-0.012	-0.014	-0.009	-0.009	-0.016	0
2-6	-0.021	-0.021	-0.026	-0.031	-0.020	-0.020	-0.034	0
3-9	-0.136	-0.133	-0.165	-0.201	-0.126	-0.130	-0.221	0
3-24	0.200	0.195	0.242	0.296	0.186	0.191	0.325	0
4-9	-0.010	-0.010	-0.012	-0.014	-0.009	-0.009	-0.016	0
5-10	-0.033	-0.032	-0.040	-0.049	-0.031	-0.032	-0.054	0
6-10	-0.021	-0.021	-0.026	-0.031	-0.020	-0.020	-0.034	0
7-8	0	0	0	0	0	0	0	0
8-9	0.009	0.009	0.011	0.014	0.009	0.009	0.015	0
8-10	-0.009	-0.009	-0.011	-0.014	-0.009	-0.009	-0.015	0
9-11	-0.077	-0.075	-0.076	-0.066	-0.072	-0.074	-0.060	0
9-12	-0.060	-0.058	-0.090	-0.136	-0.055	-0.057	-0.161	0
10-11	-0.041	-0.040	-0.031	-0.012	-0.038	-0.039	-0.001	0
10-12	-0.023	-0.022	-0.046	-0.082	-0.021	-0.022	-0.102	0
11-13	-0.003	-0.003	-0.060	-0.157	-0.002	-0.003	-0.210	0
11-14	-0.115	-0.112	-0.047	0.080	-0.107	-0.110	0.149	0
12-13	-0.034	-0.033	-0.034	-0.033	-0.031	-0.032	-0.032	0
12-23	-0.049	-0.048	-0.101	-0.186	-0.045	-0.047	-0.232	0
13-23	-0.036	-0.035	-0.094	-0.190	-0.034	-0.035	-0.242	0
14-16	-0.115	-0.112	-0.047	0.080	-0.107	-0.110	0.149	0
15-16	-0.482	-0.393	-0.653	-0.607	-0.232	-0.330	-0.582	0
15-21	-0.318	-0.412	-0.105	-0.098	-0.582	-0.479	-0.094	0
15-24	0.800	0.805	0.758	0.704	0.814	0.809	0.675	0
16-17	-0.682	-0.588	0.105	0.098	-0.418	-0.521	0.094	0
16-19	0.085	0.083	-0.805	-0.625	0.079	0.081	-0.527	0
17-18	0.258	-0.598	0.085	0.079	-0.339	-0.105	0.076	0
17-22	0.060	0.010	0.020	0.018	-0.079	-0.416	0.018	0
18-21	0.258	0.402	0.085	0.079	-0.339	-0.105	0.076	0
19-20	0.085	0.083	0.195	-0.625	0.079	0.081	-0.527	0
20-23	0.085	0.083	0.195	0.375	0.079	0.081	-0.527	0
21-22	-0.060	-0.010	-0.020	-0.018	0.079	-0.584	-0.018	0

Table A2. Capacity expansion solution of Model 1 (MW)

Fuel Type	Coal		Gas	
Node	*p*	*q*	*p*	*q*
1	63	0	59	0
2	3	0	3	0
3	333	0	246	0
4	0	493	0	824
5	0	482	0	797
6	266	0	210	0
7	181	0	154	0
8	359	0	260	0
9	0	207	0	58
10	0	289	0	66
11	0	0	0	0
12	0	0	0	0
13	529	0	331	0
14	543	0	353	0
15	290	0	219	0
16	0	0	0	0
17	88	0	81	0
18	0	403	0	67
19	217	0	177	0
20	178	0	151	0
21	0	0	0	0
22	0	0	0	0
23	0	0	0	0
24	107	0	97	0
Total	3157	1874	2344	1812

Table A3. Capacity expansion solution of Model 2 (MW)

Fuel Type	Coal		Gas	
Node	*p*	*q*	*p*	*q*
1	97	82	0	0
2	89	76	0	0
3	344	207	17	18
4	103	86	19	20
5	87	75	24	25
6	246	167	3	3
7	166	126	0	0
8	417	232	50	58
9	0	0	237	301
10	0	0	291	370
11	0	0	0	0
12	0	0	16	15
13	561	265	0	0
14	498	253	0	0
15	335	200	194	405
16	117	96	0	0
17	0	0	0	0
18	0	0	441	626
19	155	119	161	279
20	160	122	0	0
21	0	0	0	0
22	0	0	0	0
23	0	0	0	0
24	0	0	0	0
Total	3375	2105	1454	2121

Table A4. Capacity expansion solution of the Certainty model (MW)

Fuel Type	Coal		Gas	
Node	*p*	*q*	*p*	*q*
1	134	110	0	0
2	53	49	0	0
3	316	204	0	0
4	105	90	0	0
5	85	75	0	0
6	248	175	0	0
7	208	155	0	0
8	388	230	0	0
9	0	0	103	88
10	0	0	148	118
11	0	0	0	0
12	0	0	0	0
13	645	290	0	0
14	580	288	0	0
15	254	175	0	0
16	46	43	0	0
17	0	0	0	0
18	0	0	277	188
19	121	98	0	0
20	226	147	0	0
21	0	0	0	0
22	0	0	0	0
23	0	0	0	0
24	142	62	0	0
Total	3553	2192	528	394