A SUPPLY CHAIN APPROACH TO SHELF SPACE ALLOCATION

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# A SUPPLY CHAIN APPROACH TO SHELF SPACE ALLOCATION

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# THESIS ABSTRACT

## A SUPPLY CHAIN APPROACH TO SHELF SPACE ALLOCATION

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The growing intensity of retail competition is forcing stores to strive for excellence in operations. In this environment, retailers have to balance the interconnected operations, such as transportation from warehouse, shelf space and backroom space allocations in a way that the overall profit is maximized. This study introduces an analytical model for optimally allocating shelf and backroom space among items with stochastic demands, and defining cycle time for each while considering transportation utilization between the warehouse and store. A constructive heuristic and Genetic Algorithm method are developed to solve the non-linear model. 72 different scenarios with 720 different problem instances are generated to compare heuristics and also to

analyze the significance of several factors (shelf space, backroom space, truck cost, and problems size) on the results.

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# CHAPTER 1

# INTRODUCTION

The growing intensity of retail competition due to the emergence of new technology and shifts in customer needs is forcing stores to strive for excellence in operations. Extremely narrow profit margins leave little room for inefficiency and waste, thus the retailer has to balance all of the interrelated operations, such as shelf space allocation (SSA), in-store replenishment (ISR) and transportation in a way that the overall profit is maximized.

A store can be considered as real estate that is leased by a number of different items. The store does not have a sufficient display area for all of the items available in the market; therefore making the best use of the available space and allocating it among an optimum pool of items are very important. For example, there is no need to allocate excessive shelf space for a slow moving item; accordingly the item can be exchanged with a more profitable one or the allocated shelf space might be reduced.

In addition, space-planning, and location of items within departments are very important to create the maximum sales from every square foot of the store. Field experiments (Corstjens and Doyle (1981), Dreze et al. (1994), and Desmet and Renaudin (1998)) have shown that changes in number of facings of a product can affect customer attention. A facing can be defined as the front surface of an item that is visible to the customer. Altering the visibility of a product through changes in display area or location influences the probability of sales. Therefore, increased exposure and price reduction are two commonly used promotional activities.

In most stores, shelves are not the only stock keeping locations (SKLs) within the store. Some stores have backrooms where any items may be stored, thus allowing the retailer to achieve the availability of the items with timely and frequent replenishments. Furthermore, this also allows the store to have a wide range of items displayed on the shelves. By the same token, a warehouse acts like a high-capacity backroom where a set of items stored and distributed to retailers.

Consequently, there are three SKLs and two linkages in a typical retail supply chain. The objective of whole supply chain, maintaining logistics efficiency together with low inventory levels, should allow product availability and customer satisfaction. Therefore, the retailer has to balance many interrelated operations (when to order, how often to replenish, when to remove an item, when to introduce a new item, etc.) that often conflict. Consequently, trade-offs between the sales generated by the items and the other management decisions have to be examined well in order to survive in this competitive industry.

The retailers can increase profit either by increasing sales or decreasing cost, both marketing strategies and promotional activities ensure increased sales so operational strategies are applied to respond to the emerging requirements. As a result, these two objectives should be combined to optimize the whole system.

The store shelf is the final inventory location where any item meets the customer. No matter what the space allocation decisions are, retailers usually draw unsteady traffic and sell more or less than what they expect; however, customer demand must be met. Eventually, the main concern of store manager is the availability of items on the shelf at the time customers wish to purchase.

ISR is one of the major operational issues in supply chain systems. After all, supply chains are ultimately responsible for ensuring on-shelf availability of items. One of the reasons why customer will stop patronizing a particular store is because the store no longer has what the customer wants. The greatest customer service in the world won't save a retailer that does not stock what the customer wants and has constant stock-out positions on key items. Stock-outs mean lost sales opportunities for retailers and suppliers alike. They also create customer and brand-loyalty defections. On the other hand, it is also essential to recognize weak performers to get them out of the store as quickly as possible because they are occupying the space that could be allocated to more profitable items.

One of the challenges faced by retail stores is in controlling the total investment in inventories. The retailer wants to protect itself from facing stock-outs by having safety stock. Safety stock is additional inventory that is carried to buffer against uncertainties in supply and demand. For instance, a supplier may have a problem that causes a delivery from the warehouse to be delayed by one or two days. Demand variation may occur because customers may buy more than expected because the item gets more popular for a short time.

Controlling the flow of inventory has become more important than ever with the increase of local and global competition. Inventory is assumed to be the single and largest asset on the balance sheet for almost all retailers. Inventory control is an important factor both to get rid of stock-outs and to lower inventory holding cost.

Re-order cycle time (RCT), called lead time, is an important factor that impacts inventory as well. It is the amount of time from the point at which the store determines the need to order to the point at which the inventory is on hand and available for use. New items are ordered when the inventory on hand is depleted to a predetermined level. Intuitively, the longer the lead time the greater the amount of inventory that the store have to carry.

Turnover, which corresponds to the number of times a particular stock of items is sold and restocked during a given period of time, is used to determine the productivity of inventory. For example, if the retail store has an item that turns only twice a year, the store has to make a much higher profit on that item as it is only realizing the profit two times and yet paying holding costs to store the item the entire year. Contrast this with an item that store will sell three of per week or 156 per year and only need to pay to keep six on hand. The investment in the slow turning item is longer and therefore more costly.

When items are delivered from the warehouse to the store, transportation cost is incurred. There has to be a balance between transportation and the inventory cost. As the lead time is increased the warehouse can take advantage of the slower deliveries by waiting to fill its trucks with additional units. On the contrary, frequent shipments from the warehouse to the store will lower average inventory levels and raise transportation costs. Ultimately, the goal is to ensure that store has the right item, in the right amount, at the right place, at the right time, while decreasing inventory levels, and increasing inventory turns.

To determine where items should be located on the shelves, retailers of all types generate maps known as planograms. A planogram is a diagram created mostly by commercial software packages (Spaceman, Apollo, and etc.) that illustrates exactly where every stock keeping unit (SKU) should be placed. Software packages require the user to input UPC codes, profit margins, turnover, size of the item packaging or actual pictures of the packaging, and other applicable information into the program.

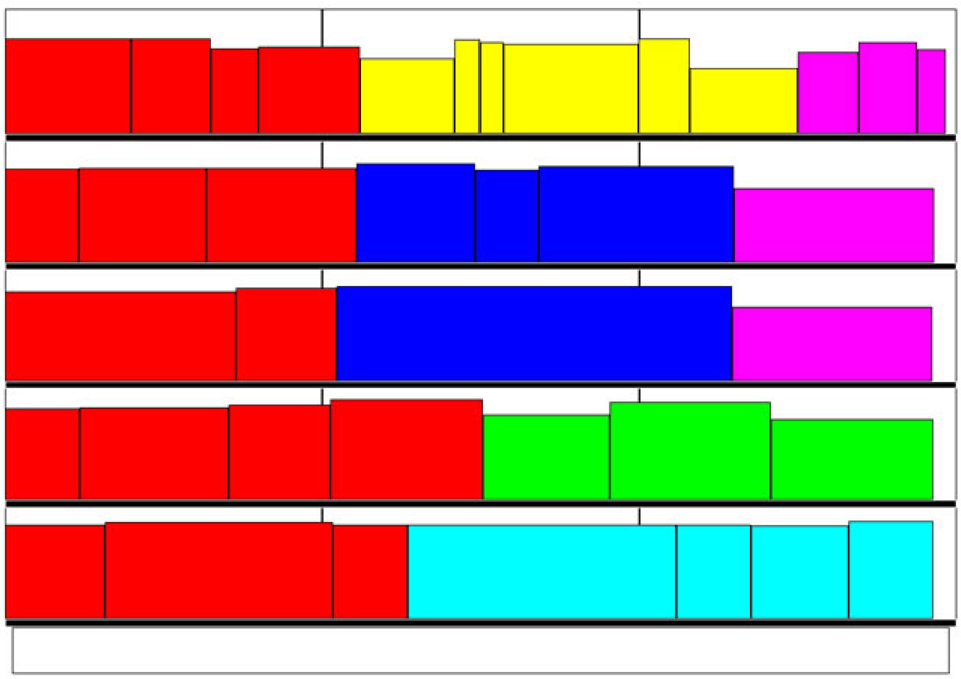


**Figure 1.1 A Planogram Prepared by Spaceman Application Builder for the Alcoholic Beverages Section of a Grocery Store**

There are two major drawbacks of Spaceman. First one is that it allows user to allocate the space based on a single criteria, such as movement, sales, profit and so forth. Single criteria allocation might not give any optimum solution in terms of maximizing profit. In order to determine the limitation of single-criteria solution, a problem with 7 items was created (see appendix 1). Table 1.1 demonstrates the number of facings allocated to the items, and corresponding profit value when certain allocation criterion is applied by Spaceman. Also, a simple heuristic was developed. First, the items were sorted descending by gross profit (price-cost) per cubic foot. Then, the shelf was stocked with items respectively (starting from the most profitable item) until the daily demand of each item was reached. The allocation process ended when the shelf was completely full, or daily demand for all items was met. As Table 1.1 indicates, item-3 has the smallest gross profit per cubic foot value. Because of the limited capacity of shelf space, the number of facings for item-3 was found 0. The solution found by the heuristic is better than the solutions generated by Spaceman in terms of GP.

**Table 1.1 Projected GP Values for Single-Criteria Solutions**

| **Criteria\Product Facings** | **P1** | **P2** | **P3** | **P4** | **P5** | **P6** | **P7** | **Projected GP** |
|---|---|---|---|---|---|---|---|---|
| Divide Equally | 19 | 22 | 17 | 19 | 29 | 17 | 21 | $20,489.00 |
| Movement | 18 | 21 | 18 | 18 | 30 | 17 | 22 | $20,307.00 |
| Sales | 19 | 21 | 17 | 19 | 32 | 16 | 21 | $20,202.00 |
| Profit | 19 | 19 | 16 | 19 | 35 | 19 | 18 | $19,943.00 |
| Market Share | 19 | 22 | 17 | 19 | 29 | 17 | 21 | $20,489.00 |
| Required Inventory | 18 | 21 | 18 | 18 | 30 | 17 | 22 | $20,307.00 |
| Force Minimum-As Is | 21 | 22 | 17 | 19 | 26 | 17 | 21 | $20,797.00 |
| Force Minimum-Minimize | 21 | 22 | 17 | 19 | 26 | 17 | 21 | $20,797.00 |
| **Heuristic** | 23 | 24 | 0 | 23 | 26 | 24 | 24 | **$21,196.00** |

The second problem with commercial package such as Spaceman is that transportation cost incurred because of the changes in frequency of replenishment from supplier is not considered. Still, Spaceman incorporates stock-out and holding costs while providing a user friendly interface, which make the software guide popular in industry.

This study has two major objectives. First, building a model for optimally allocating shelf and backroom space among the items, and defining cycle time for each while considering transportation utilization between the warehouse and store, the replenishment process between backroom inventory and shelf inventory, holding costs and stock-out costs in the store. Second, developing a solution method to solve the proposed non-linear model, and evaluating the results of the method against a constructive heuristic solution produced using shelf space allocation rule based on profitability of items. In Chapter 2, major drawbacks of field experiments to estimate the parameters used in SSA models are discussed. An overview of four comprehensive SSA models and relevant literature are also provided. In Chapter 3, problem definition, assumptions and mathematical formulation are presented. In Chapter 4, two heuristics are developed to solve the model. In Chapter 5, 72 (2x3x2x6) different scenarios with 720 different problem instances are generated to compare heuristics and also to analyze the significance of several factors (shelf space, backroom space, truck cost and problem size) on the results. Computational results are also shown.

# CHAPTER 2

# LITERATURE REVIEW

Space allocated to an item or a group of items has positive influence on the probability of the purchase. For instance, if an item is given a large space, it is more likely to have the customer's attention; therefore, altering the visibility of an item through changes in space and location should affect sales. In order to provide evidence to support this argument, field experiments have been conducted in space management concentrated on determining whether a relationship exists between the number of facings allocated to an item and that item's sales. The term, *elasticity*, is widely used in SSA literature and can be defined as the measure of sensitivity of one variable to another. For example, space elasticity is defined as increase in sales when the number of facings is doubled or the decrease in sales when the number of facings is halved.

Corstjens and Doyle (1981), Dreze et al. (1994), and Desmet and Renaudin (1998) conducted a series of experiments to measure the elasticities. Needed data was collected from retailers with high number of stores. Only the category (item group) elasticities have been estimated through these field experiments. However, space allocation decisions across and within categories can not be considered to be independent; in addition, elasticity parameters for each and every item within the store should be calculated in the evaluation process.

Experimental findings indicate that impulse buying categories have higher space elasticities; thus, increasing the dedicated space for them will also tend to increase the sales. On the other hand, Desmet and Renaudin (1998) discovered a number of item groups having almost no change in demand with respect to changes in number of facings. Moreover, Dreze et al. (1994) demonstrated that the number of facings allocated to an item was one of the less significant factors. Vertical location of the item was determined to be more significant than the number of facings. Motivated from these findings, we can expect that a store may have items with stochastic demand which is independent from allocated shelf space.

The major drawbacks of the experimentations are time, cost and inadequacy. First off, if the cross elasticities are to be considered in the evaluation process, data collection will take so much time. Moreover, with multiple item introductions and changes in demand for individual brands, the optimal SSA would be outdated before it could ever be implemented. On the other hand, ignoring cross elasticities and considering only main effects in the allocation process can lead to a major sub-optimization. For instance, a significant promotion in a substitute brand can totally change the demand pattern for two substitute items. The cheaper item will receive more demand and the other item will not grab as much attention as it used to do no matter the number of facings is. Likewise, allocating shelf space based on sales, while ignoring or simplifying cost side, has the same consequence. Second, elasticity parameters for each item do not remain same and should be evaluated continuously. Third, it’s assumed that shelves are always kept full; however, the number of facings may change between two consecutive in-store replenishments, thus the uncontrolled change in number of facings does not allow evaluating the space elasticity.

On the contrary, commercial SSA programs for retail industry allocate shelf space according to traditional criteria such as profit, sales and so forth. They are driven by operational concerns at item level. These commercial systems use sales data, item and shelf dimensions, and some relatively simple heuristics for developing operational guidelines, which is easy to implement in practice. Moreover, these programs suggest plans based on guidelines set by user.

The marketing community has formulated number of models by incorporating the demand rate as a function of the shelf space allocated to the item, after they recognized the relationship between number of facings and sales. Borin and et al. (1994) developed the most comprehensive model to date. The SSA problem was formulated as constrained optimization problem with two decision variables: assortment and allocation of space to the items in the assortment. Switching to substitute items in the event of stock-outs was considered in the demand function. Because of the non-linearities in the objective function a heuristic solution method based on simulated annealing was tested on a small problem with a known optimum (complete enumeration) as well as on a larger problem without known optimum. They compared the results of simulated annealing against a solution produced using shelf space allocation rule based on share of sales.

Urban (1998) presented a model where the demand rate is a function of instantaneous inventory level on the shelf. Backroom and shelved inventory were distinguished; thus, there was a limited amount of displayed inventory that had an effect on sales and was subject to shelf-space cost. A fixed procurement cost and a holding cost based on average inventory level were incorporated to the model as well. A greedy heuristic and a genetic algorithm (GA) were proposed for the solution to the problem. Exhaustive search was conducted for small problems. They compared the results of a greedy heuristic against the GA and a solution produced based on share of sales. Both the greedy heuristic and the GA performed well.

Yang (1999) made simplifications to the model of Corstjens and Doyle (1981). First, the profit of any item was assumed to be linear with respect to the allocated number of facings for that item. In fact, this is against the fact that there is a diminishing increase in sales due to the increase in shelf space. Second, the availability constraint was removed. A heuristic, similar to the algorithm for solving the knapsack problem, was proposed after these simplifications. Shelf space was allocated according to a descending order of sales profit for each item per display area or length. Small size problems were created to get the optimum solutions by applying complete enumeration. For the purpose of comparing performance, the heuristic compared against enumeration. An improved heuristic was also developed and found to be very efficient.

Hwang and et al. (2004) developed an integrated mathematical model for the shelf space allocation problem and inventory control problem with the aim of maximizing retailer's overall profit.  The demand rate was shaped by space, cross-space and location elasticity parameters. The items were restocked from the backroom to the shelves instantaneously and the restocking cost was ignored. A gradient search heuristic and a genetic algorithm are proposed to solve the model. Comparison of the proposed solution procedures with a total enumeration was demonstrated. Compared to the gradient search, the genetic algorithm performed better and generated near-optimum solutions.

The major drawbacks of the above four comprehensive models are as follows:

- Demand is assumed to be deterministic. Using deterministic demand in the model neglects the holding and stock-out costs caused by the variation in demand. The demand distribution for each item can be obtained by analyzing the point of sales (POS) data and forecasting decisions can also be integrated into the decision process. Finally, stochastic models would more accurately portray realistic inventory settings
- The lost sales due to stock-outs were not incorporated into the cost function. Current shelf space models focus on space responsiveness and neglect issues of assortment and stock-outs. The existing models attempt to allocate space to shelves using only space elasticities which have been shown to be weak. Murphy (2000) stated that availability of the items in the backroom or warehouse is at very high level but yet there are still stock-outs. The reason can be attributed to poor in-store management such as shelf space allocation and in-store replenishment. (Gruen and et al., 2002)
- The items were restocked from the backroom into the shelves continuously; as a consequence, none of the models explicitly differentiates between the backroom inventory and the displayed inventory.
- Transportation cost was not considered. Moreover, an item might trigger an order and this may cause less than truck load shipment; thus, the frequency of deliveries from warehouse influences transportation cost.

A comprehensive literature review on SSA models has recently been done by Urban (2004). Inventory control models incorporating inventory level dependent demand rate were split into two distinct streams: models in which the demand rate of an item is a function of the initial inventory level and those in which it is dependent on the instantaneous inventory level. Urban (2004) noted the lack of literature on models studying multi item case with stochastic demand.

The model developed in this thesis does not fit any of the streams above because the demand of an item is not a function of either shelf space or location; instead, it's a function of an appropriate probability distribution. It should be noted that, historical demand data can be used to model the probability distribution. Furthermore, it can be assumed that the change in demand of an item because of the change in shelf space and location is handled by variance of this distribution.

Cachon (2001) studied the management of transportation, shelf space and inventory costs for a retailer that sells multiple items with stochastic demand. Contrary to the marketing literature, the demand rate for each item was assumed to be independent of shelf-space allocation. In order to maintain analytical tractability, the Poisson distribution was used for demand function and the stochastic variables in the cost function were replaced with their mean. He assumed that all demand during stock-outs was backordered, which is doubtful for most retailers. The objective was to choose a truck dispatching policy and a SSA and an inventory policy to minimize total expected cost per unit time. Three different truck dispatching policies were compared. In this experimentation dispatching trucks whenever the cumulative orders across the products equals a constant threshold performed better than the two periodic review policies.

Speranza and Ukovich (1994) analyzed the problem of finding the frequencies that minimize the sum of transportation an inventory costs for several items on the same link. Items were assigned different/same frequencies and joint-transportation was taken into consideration. Items partially shipped at different frequencies found to be cost effective when they shared the same truck with those whose shipment happened to be simultaneous.

Lim and et al. (2004) indicated the lack of literature on heuristic solution methods to SSA problems. They developed two Metaheuristics, a Tabu Search and a hybrid of Squeaky-Wheel Optimization (SWO), and evaluated the performance of a number of heuristics including these two on the simplified problem proposed by Yang (1999). A new neighborhood move technique, "many-to-many move" was introduced and found to be well suited to SSA problem. Combining a local search technique with SWO has given better results in terms of obtained profit value.

In this study, the objective is to present a SSA model which incorporates stochastic demand (the impact of space elasticities due to impulse buying is not incorporated into the model) and joint transportation with different frequencies and to distinguish between backroom and shelf space introducing a new term called "shelf stock-out". Therefore, the articles of Cachon (2001), and Speranza and Ukovich (1994), establish a base to this study. A genetic algorithm with an efficient moving operator will be introduced and tested against a constructive heuristic method on randomly created test problems. A discussion on maintaining feasibility on SSA problems will be provided as well.

# CHAPTER 3

# PROBLEM DEFINITION

Our interest is in retail outlets (stores) where customers locate one or more desired items on shelf space within the store, purchase those items, and leave the store. The retailer brings the items from a warehouse and displays the items for sale on the store's shelves. Due to the number of different items available, the limited total shelf space, and the item delivery and stock-out costs, the retailer maintains a small inventory of items in the back of the store (backroom) and replenishes the shelves from this inventory on a periodic basis. Our concern is the allocation of shelf space and backroom space to specific items and the two replenishment operations (replenishment of the shelf from the backroom and replenishment to the store from the warehouse).

A *category* is an assortment of items with independent demand sharing limited shelf and backroom space. Suppose that there are shelves in a store and $N$ brands of items within a category are displayed on the shelves with limited capacity. Each of the items has inventory space in the backroom and the total backroom space for this certain category is limited as well. Using as little as possible inventory space for expensive items is one of the goals of the store to reduce inventory holding cost. Items on the shelf and in the backroom are subject to inventory holding cost and stock-out cost.

A category has multiple items with stochastic demand. Daily customer demand for item *i* is described by a random variable $X_i$ , with probability density function $f(x_i)$ for $i=1,\ldots,N$.

The store is replenished from a warehouse via trucks. $L$ is the time period between when an order is placed and when the order is received at the store. This time is assumed to be deterministic. Each truck has capacity $C$ in cubic feet, and costs \$$K$ per trip between the warehouse and the store. The number of units of each product available at the warehouse as well as the number of trucks available is not limited.

The shelves are periodically restocked from the backroom up to the capacity that has been allocated to the items. The time between two consecutive in-store replenishments is fixed. Daily in-store replenishment (ISR) is assumed to be realistic, because there are many items, which have very little space in a category resulting in frequent replenishments to maintain the availability of the items on the shelves. In addition to this, items need to be checked and re-arranged after the rush hour everyday to keep them accessible and visible.

Stock-outs occur in two forms. Shelf stock-out, which may occur anytime between two consecutive ISR due to excessive demand, occurs when the on-shelf quantity is less than the demand quantity. This type of shelf stock-out may be a result of items available in the back of the retail store that have not been transferred to the shelf where consumers can purchase them. Backroom stock-outs occur because the inventory on the shelf and in the back of the retail store has been completely depleted before the next order arrival in a cycle. We assume that empty shelves in the store result in lost sales opportunities and no backlogging is considered.

A cycle, $c_i$, is defined for each item as the time period between two successive arrivals of orders from the warehouse. It is assumed that only one order can be placed for each item during a cycle. Because of the availability of the products and the trucks at the warehouse, as soon as an order is placed, the required number of trucks is immediately dispatched; thus, the cycle time for each item must be bigger than the lead time. Multiple items having the same cycle times can be shipped together. This will allow reducing the inventory levels in the backroom as well as increasing transportation utilization. When the order arrives, a required amount of items go to shelves (another ISR), and the rest of them are accommodated in the store's backroom as usual.

The objective of this study is to build a model for optimally allocating shelf and backroom space among the items, and defining cycle time for each while considering transportation utilization between the warehouse and the store, the replenishment process between backroom inventory and shelf inventory, holding costs and stock-out costs in the store. The objective is to maximize the expected profit associated with the $N$ items in the category.

The assumptions of the model are as follows:

- The system involves $N$ brands of items within a category and each brand has a dedicated space on the shelf and in the backroom.
- ISR of items is joint replenishment and takes place instantaneously and periodically.
- Lead time is known and constant.
- It is not necessary to display all $N$ brands of items on the shelves (i.e., allocation of 0 shelf space for products is allowed).

- Demand for each item is based on a known continuous demand distribution and is not dependent on shelf space allocation or location. Moreover, these distributions do not change during the cycle.
- Any demand which exceeds the on-shelf quantity induces a stock-out cost and no backorders are allowed.
- All relevant costs of each product (stock-out cost, inventory holding cost, procurement cost and etc.) are known and constant.
- No more than one outstanding order from the warehouse is allowed for each item.

3.1 Notation and Model Formulation

Each item has:

| | |
|---|---|
| $X_i$ | Random variable for daily demand with density function $f(x_i)$ |
| $H_i$ | Random variable for average inventory level per cycle |
| $Y_i$ | Random variable for shelf stock-out per day |
| $W_i$ | Random variable for stock-out per cycle |
| $Z_i$ | Random variable for items sold per day |
| $R_i$ | Random variable for items sold per cycle |
| $Q_i$ | Random variable for days of inventory in a cycle |
| $v_i$ | Volume (in cubic feet) of item *i* |
| $e_i$ | Unit stock-out cost for item *i* |
| $h_i$ | Daily unit holding cost for item *i* |
| $g_i$ | Unit sales price for item *i* |
| $p_i$ | Unit purchase price for item *i* |
| $s_i$ | Shelf space allocated to item *i* |
| $r_i$ | Maximum number of items that can fit into $s_i$. Note that $r_i = \left\lfloor \frac{s_i}{v_i} \right\rfloor$ |
| $b_i$ | Backroom space allocated to item *i* |
| $d_i$ | Maximum number of items that can fit into $b_i$. Note that $d_i = \left\lfloor \frac{b_i}{v_i} \right\rfloor$ |
| $c_i$ | Number of days between two consecutive order arrivals for item *i* |

Other notations used in the model are as follows:

| | |
|---|---|
| $TP(s_i,b_i,c_i)$ | Objective function |
| $T$ | Random variable for number of trucks used per cycle |
| $K$ | Cost per delivery per truck |
| $C$ | Truck capacity (cubic feet) |
| $SC$ | Total shelf capacity (cubic feet) |
| $BC$ | Total backroom capacity (cubic feet) |
| $L$ | Lead time (in days) |
| $N$ | Number of items in the category |

Objective:

$$Max\ TP(s_i, b_i, c_i) = \sum_{i=1}^{N} [(g_i - p_i)E[R_i] - h_i E[H_i] - e_i E[W_i]] - KE[T] \qquad (1)$$

Subject to:

$$\sum_{i=1}^{N} s_i \le SC \qquad (2)$$

$$\sum_{i=1}^{N} b_i \le BC \qquad (3)$$

$$c_i \quad \ge \ L \quad i = 1,2,3,...N \qquad (4)$$

$$s_i, b_i, and\ c_i \ge 0 \qquad (5)$$

Consider the fact that demand is probabilistic and it is assumed to be independent from the availability of the product, so the expected demand per item per day is:

$$E[X_i] = \int_{x_i=0}^{\infty} x_i f(x_i) dx_i \qquad (6)$$

Thus;

Total expected demand per item per cycle= $E[X_i]c_i$ (7)

Assuming that the shelf is fully stocked, there are two factors which impact daily sales, namely, number of products on the shelf and the demand for the product. If $X_i$ is more than $r_i$, we can sell only $r_i$. Finally, the following relation is obtained:

$$\begin{aligned} E[Z_i] &= \int_{x_i=0}^{r_i} x_i f(x_i) dx_i + \int_{x_i=r_i}^{\infty} r_i f(x_i) dx_i \\ &= \int_{x_i=0}^{r_i} x_i f(x_i) dx_i + r_i \int_{x_i=r_i}^{\infty} f(x_i) dx_i \end{aligned} \qquad (8)$$

$E[Z_i] < r_i$ is always true. (9)

(9) implies that stock-out (hereafter shelf stock-out) may occur anytime between two consecutive ISR due to excessive demand regardless of the availability of the products in the backroom. Then, expected shelf stock-out per item per day is as follows:

From equations (6) and (8):

$$
\begin{aligned}
E[Y_i] &= E[X_i] - E[Z_i] \\
&= \int_{x_i=0}^{\infty} x_i f(x_i) dx_i - \int_{x_i=0}^{r_i} x_i f(x_i) dx_i - \int_{x_i=r_i}^{\infty} r_i f(x_i) dx_i \\
&= \int_{x_i=r_i}^{\infty} x_i f(x_i) dx_i - \int_{x_i=r_i}^{\infty} r_i f(x_i) dx_i \\
&= \int_{x_i=r_i}^{\infty} (x_i - r_i) f(x_i) dx_i
\end{aligned}
\tag{10}
$$

$E[Y_i]$ is not used in objective function, however it is explicitly shown to explain the occurrence of shelf stock-out even though we have backroom inventory.

The store may not have enough products to meet the total demand emerging in a cycle. Any demand, after all of the available products, $r_i + d_i$, are depleted, will be not be satisfied. Thus if backroom stock-out occurs before the end of a cycle, the expected daily stock-out turns out to be $E[X_i]$ for each day that remains in that cycle. The expected number of days of inventory in a cycle, expected time in which a given replenish-up-to level $(r_i + d_i)$ will deplete to zero, is shown below in order to calculate the total expected stock-out per cycle:

$$E[Q_i] = \frac{(r_i + d_i)}{E[Z_i]} \tag{11}$$

One important thing is that, increasing backroom space will not change total expected stock-out, if $E[Q_i] \geq c_i$. By the same token, the total expected stock-out in a cycle will always include $E[Y_i] \cdot c_i$.

Finally, the expected number of items sold per cycle, $E[R_i]$, can be written as:

$$E[R_i] = \begin{cases} E[Z_i]c_i & \text{if } E[Q_i] \geq c_i \\ (r_i + d_i) & \text{otherwise} \end{cases} \tag{12}$$

Figure 3.1 and 3.2 demonstrate two possible situations that may occur for each item during a cycle. In Figure 3.1, number of days of inventory in a cycle is less than cycle time, whereas in Figure 3.2, it is greater than or equal to cycle time.

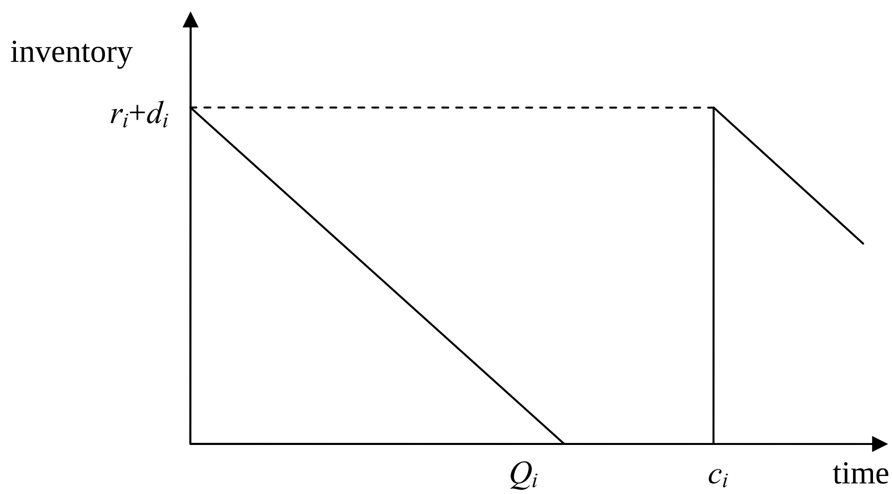



**Figure 3.1 Inventory Level When $Q_i$<$c_i$**

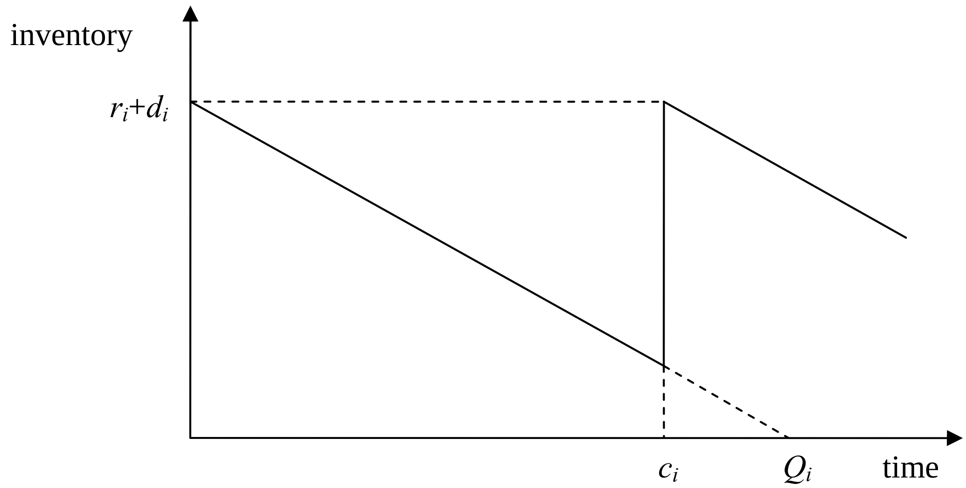



**Figure 3.2: Inventory Level When** $Q_i \geq c_i$

It should be noted that an item might trigger an order and this may cause less than truck load shipment. Figure 3.1 indicates that stock-out cost might be preferred to enable less transportation costs.

Let the inventory level be $r_i + d_i$ at the beginning of a cycle and $E[R_i]$ be the expected number of products sold during the cycle. Then, the expected inventory level at the end of the cycle is given by:

$$r_i + d_i - E[R_i] \tag{13}$$

Inventory level at the beginning of the cycle $= r_i + d_i$ (14)

From equation (12), (13) and (14):

$$E[H_i] = \begin{cases} \dfrac{(2(r_i + d_i) - E[Z_i]c_i)c_i}{2} & if\ E[Q_i] \geq c_i \\ \dfrac{(r_i + d_i)E[Q_i]}{2} & otherwise \end{cases} \tag{15}$$

The expected procurement cost per item per cycle is calculated as follows:

$$E[R_i]p_i \tag{16}$$

The expected sales revenue per item per cycle is calculated as follows:

$$E[R_i]g_i \tag{17}$$

If $E[Q_i] \geq c_i$, the total expected stock-out in a cycle, $E[W_i]$, will be $E[Y_i]c_i$. On the contrary if $E[Q_i] < c_i$, the total expected stock-out in a cycle will be as follows:

$$\begin{aligned} E[W_i] &= E[Y_i]E[Q_i] + E[X_i](c_i - E[Q_i]) \\ &= E[X_i]c_i - E[Q_i](E[X_i] - E[Y_i]) \\ &= E[X_i]c_i - E[Q_i]E[Z_i] \\ &= E[X_i]c_i - (r_i + d_i) \end{aligned}$$

Therefore;

$$E[W_i] = \begin{cases} E[Y_i]c_i & if \;\; E[Q_i] \geq c_i \\ E[X_i]c_i - (r_i + d_i) & otherwise \end{cases} \tag{18}$$

The total expected stock-out can also be shown as follows:

$$E[W_i] = E[X_i]c_i - E[R_i] \tag{19}$$

Trucks are used to transport inventory from warehouse to store at the end of each cycle, $c_i$. Each item can be shipped with those which have the same cycle time. As explained in more detail by Speranza and Ukovich (1994), the joint-transportation allows transportation utilization, therefore cheaper transportation costs. Total expected transportation cost per cycle is calculated as follows:

Steps for the pseudo code are as follows:

- Calculate the required number of trucks for each cycle
- Calculate the total required number of trucks
- Calculate the total expected transportation cost

$E[T] = 0$

For (j=L; j<=$max(c_i)$; j++) {

Capacity =0;

For each item {

If ( $c_i == j$ ) {

Capacity += $E[R_i]v_i$

}

}

$E[T] += \left\lceil \frac{Capacity}{C} \right\rceil$

}

Total expected transportation cost per cycle = $KE[T]$ (20)

Finally, the objective function can be shown as follows:

$$TP(s_i, b_i, c_i) = \sum_{i=1}^{N} [(g_i - p_i)E[R_i] - h_i E[H_i] - e_i(E[X_i]c_i - E[R_i])] - KE[T]$$

Where:

$$E[X_i] = \int_{x_i=0}^{\infty} x_i f(x_i) dx_i \tag{21}$$

$$E[Z_i] = \int_{x_i=0}^{r_i} x_i f(x_i) dx_i + \int_{x_i=r_i}^{\infty} r_i f(x_i) dx_i \tag{22}$$

$$E[Q_i] = \frac{(r_i + d_i)}{E[Z_i]} \tag{23}$$

$$E[R_i] = \begin{cases} E[Z_i]c_i & \text{if } E[Q_i] \geq c_i \\ (r_i + d_i) & \text{otherwise} \end{cases} \quad (24)$$

$$E[H_i] = \begin{cases} \dfrac{(2(r_i + d_i) - E[Z_i]c_i)c_i}{2} & \text{if } E[Q_i] \geq c_i \\ \dfrac{(r_i + d_i)E[Q_i]}{2} & \text{otherwise} \end{cases} \quad (25)$$

and $KE[T]$ (see pseudo code) (26)

The non-linearities and integrals in the objective function, and running a pseudo code program to get $E[T]$ disallow a closed form solution. Therefore, two heuristic solution are developed to find near-optimum and/or optimum solutions. In order to calculate the expected values, such as $E[Z_i]$ and $E[T]$, a computer program is developed. In Chapter 4, an adaptive optimization method called Genetic Algorithm (GA) is presented. A constructive heuristic solution is also provided to evaluate the performance of proposed GA.

# CHAPTER 4

# SOLUTION PROCEDURES

The shelf space allocation problem (SSAP) can be regarded as allocating limited capacity of shelf space among the demanding brands in the assortment list in a way that the total profit of the category is maximized. The model in this paper also includes backroom space and cycle time as decision variables.

Heuristics less often have been used in shelf space allocation studies and there is limited number of heuristic solutions in this area. Borin et al. (1994), Urban (1998), Yang (2001), and Tim et al. (2004) developed heuristic solutions to their models. Borin et al. used a heuristic solution based on simulated annealing (SA). They tested the heuristic on a small size problem, 6-item case, with a known optimum. Moreover, for a larger problem, 18-item case, the solution found by SA was compared against a common principle for shelf space allocation, in which the space allocated to a particular item is proportional to its sales. Urban developed a greedy heuristic and a genetic algorithm to solve his integrated model. He conducted an exhaustive search to obtain optimal solutions for 6-item case. Additional problems with 18 and 54 items were also generated, and both heuristic results compared with a solution produced using a shelf allocation rule based on share of sales. Yang simplified the non-linear model of Corstjens and Dolye (1981) and proposed a method which is similar to the algorithm used for solving a knapsack problem. He tested the performance of the proposed algorithm against optimum solutions on small size problems. Tim et al. (2004) extended the model of Yang by integrating product groupings and nonlinear profit function, and improved Yang's heuristic by introducing three new neighborhood moves. He developed two meta-heuristics, Tabu Search and Squeaky-Wheel Optimization. Simulated problems in different sizes, between 10 and 100 items, were generated to test the performance of the adjustment neighborhood moves and the proposed algorithms.

Since there is not any exact algorithm to solve SSAP proposed in this research, a hybrid heuristic method is applied with the aim of producing high-quality solutions in a reasonable time. In order to evaluate the performance of this method, a constructive heuristic is also applied to the same set of problems. There are not benchmarks available; therefore, both heuristics are experimented on randomly created problems with different sizes and the comparison is demonstrated in Chapter 5.

## 4.1 Heuristic Solution Methodologies

### 4.1.1 Genetic Algorithm

The first approach to solve the problem is an adaptive optimization technique known as a genetic algorithm (GA). Each individual represents a potential solution to the problem and each solution is evaluated to give some measure of its "fitness". In a typical GA the following steps are performed:

1. $\mu$ individuals (parents) are randomly generated forming the initial population solutions.

2. $\lambda$ individuals (offspring) are created, through the use of recombination and mutation, using the parents that are selected uniformly randomly (i.e., not based upon fitness) from the population.

3. The best $\mu$ survivors are chosen, based on their fitness values to form the current population, either from the offspring or the whole population,

4. Repeat steps 2 and 3 until the stopping criteria is met.

Therefore, over successive iterations, the best individual in the population is expected to approach to the global optimum. Problem specific modifications are as follows:

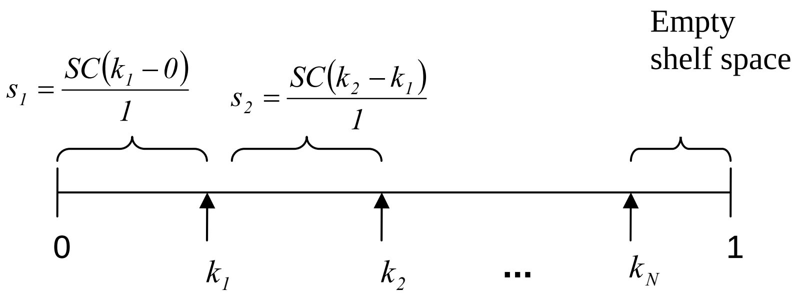
4.1.1.1 Representation

Phenotype is the actual representation of the decision variables ($s_i$ , $b_i$ , and $c_i$) as shown in previous chapter. On the other hand, genotype, the encoded representation of decision variables, is used for better implementation of the proposed heuristic. Genotypes facilitate simple crossover and mutation. Genotypes and phenotypes can be converted to each other as needed in evaluation step.

4.1.1.2 Encoding and Decoding Decision Variables

Shelf space:

$$s_1, s_2, \ldots, s_N \Rightarrow \textit{phenotype}$$
$$k_1, k_2, \ldots, k_N \Rightarrow \textit{genotype}$$




$$0 \le k_1 \le k_2 \le \ldots \le k_N \le 1$$

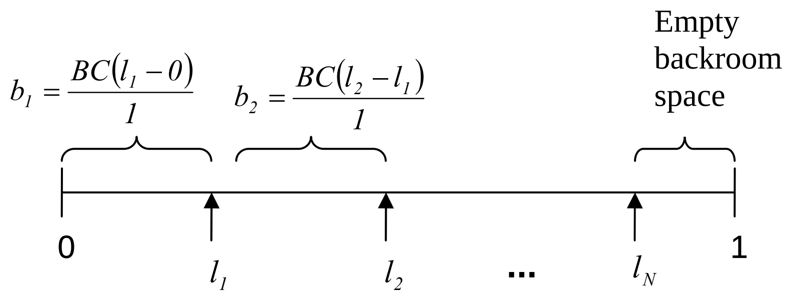
Equation (2) in Chapter 3 indicates that total shelf space allocated to the items is less than or equal to maximum shelf space capacity. This constraint also enables free shelf space. In genotype representation, “1” corresponds to maximum shelf space capacity, *SC*. [0,1] interval is divided into *N*+1 pieces. The first *N* pieces, $(k_1 - 0)$, $(k_2 - k_1)$,…, $(k_N - k_{N-1})$, are the percentages of the shelf space allocated to the items. $(1 - k_N)$ is the percentage of empty shelf space.

Calculating shelf space values for each item (genotype to phenotype):

```
For (i=1; i<=N; i++){
    If(i==1){
        s_i=SC(k_i-0)
    }else{
        s_i= SC(k_i -k_i-1)
    }
}
```

Backroom space:

$$b_1, b_2, \ldots, b_N \Rightarrow phenotype$$
$$l_1, l_2, \ldots, l_N \Rightarrow genotype$$




$$0 \le l_1 \le l_2 \le \ldots \le l_N \le 1$$

Equation (3) in Chapter 3 indicates that total backroom space allocated to the items are less than or equal to maximum backroom space capacity. This constraint also enables free backroom space. In genotype representation, “1” corresponds to maximum backroom space capacity, *BC*. [0,1] interval is divided into *N*+1 pieces. The first *N* pieces, $(l_1 - 0)$, $(l_2 - l_1)$,…, $(l_N - l_{N-1})$, are the percentages of the backroom space allocated to the items. $(1 - l_N)$ is the percentage of empty backroom space.

Calculating backroom space values for each item (genotype to phenotype):

```
For (i=1; i<=N; i++){
    If(i==1){
        b_i=BC(l_i-0)
    }else{
        b_i= BC(l_i -l_i-1)
    }
}
```

Cycle Times:

Phenotypes and genotypes are same.

$c_1, c_2, \ldots, c_N \Rightarrow phenotype$

$c_1, c_2, \ldots, c_N \Rightarrow genotype$

4.1.1.3 Initial Population

Each of the $\mu$ individuals in the initial population is randomly generated as follows:

1. Generate $N$ random numbers from the range [0,1], arrange these numbers in ascending order, and assign them as $k_1, k_2, \ldots, k_N$, such that $0 \leq k_1 \leq k_2 \leq \ldots \leq k_N \leq 1$

2. Generate $N$ random numbers from the range [0,1], arrange these numbers in ascending order, and assign them as $l_1, l_2, \ldots, l_N$, such that $0 \leq l_1 \leq l_2 \leq \ldots \leq l_N \leq 1$

3. Randomly generate cycle time values (in days) from the range $[0,m]$ where $m \geq 1$

4.1.1.4 Evaluation

The process of evaluating the fitness values of an individual consists of the following steps:

1. Convert to individual's genotype to its phenotype as explained in representation

2. Evaluate the objective function

In order to evaluate the objective function, first $E[Z_i]$ is calculated by using probability density function values for a given demand distribution. The procedure can be described as the summation of the probability of each possible outcome (number of items sold per day) multiplied by its value. Then $E[Q_i]$, $E[X_i]$, $E[H_i]$, and $E[T]$ are calculated respectively to get expected profit.

4.1.1.5 Selection

A 2-way tournament selection is adopted as the selection procedure. In this method, since the tournament size is 2, weak individuals have more chance to be selected. This approach consists of the following steps:

1. Calculate the fitness values of each individual in the population
2. Chose 2 individuals from the population at random
3. Select the better of these 2 individuals based on their fitness values (parent 1)
4. Repeat step 2 and 3 to select parent 2

The procedure will give more chance to individuals with less fitness to be selected more frequently, hence it also implies that genetic diversity is increased and further exploration of the solution space is provided. This helps keep the diversity of population large, preventing premature convergence on poor solutions.

4.1.1.6 Recombination

Two parents are chosen as described above in order to generate an offspring. The variables from each parent define the lower and upper bounds of the offspring variable, which is randomly selected within this interval. Each offspring variable is calculated as follows:

$$k_{inew} = k_{i1} + (k_{i2} - k_{i1}) \cdot U(0,1)$$

$$l_{inew} = l_{i1} + (l_{i2} - l_{i1}) \cdot U(0,1)$$

$$c_{inew} = \lfloor c_{i1} + (c_{i2} - c_{i1}) \cdot U(0,1) \rfloor$$

This implementation is called convex crossover and ensures that the offspring inherits traits from both parents.

4.1.1.7 Mutation

Mutation is an important part of the search as it helps to prevent the population from stagnating at local optima. Furthermore, it should also guarantee obtaining any solution in the feasible search space. Each variable in an individual is mutated with a probability of “f”. Each offspring variable is mutated as follows:

if( $k_i + N(0,a) \geq 0 \;\&\&\; k_i + N(0,a) \leq 1$ ) {

$$k_{inew} = k_i + N(0,a)$$

else{

$$k_{inew} = k_i - N(0,a)$$

}

if( $l_i + N(0,a) \geq 0 \ \&\& \ l_i + N(0,a) \leq 1$ ) {

$l_{inew} = l_i + N(0,a)$

else{

$l_{inew} = l_i - N(0,a)$

}

if(U(0,1)<n){

$c_{inew} = c_i + 1$

}else{

$c_{inew} = c_i - 1$

if( $c_{inew} \leq 1$ ){

$c_{inew} = 1$

}

}

where 0<*j*<1

In mutation, maintaining feasibility for a variable is to go back to its previous value when it is outside the feasible region. The feasible region is between 0 and 1 for $k_i$ and $l_i$; on the other hand, $c_i$ has only a lower bound which is 1. Then, $k_{inew}$ and $l_{inew}$ variables are calculated by adding or subtracting $|N(0,a)|$ according to which boundary is violated. $c_{inew}$ is set to 1 if the mutated variable is less than 1. On the other hand, in recombination, offspring always reside in feasible region no matter which parents are used for crossover operation.

**Table 4.1 Heuristic Parameters**

| | |
|---|---|
| $\mu$ | Number of parents |
| $\Lambda$ | Number of offspring |
| $t$ | Stopping criteria, number of unsuccessful iterations |
| $z$ | Number of maximum iterations |
| $f$ | Mutation rate for each variable |
| $a$ | Mutation parameter for shelf and backroom space |
| $n$ | Mutation parameter for cycle time |
| $m$ | Maximum value for initial cycle time |

4.1.1.8 Pseudo Code for GA

```
Set μ, λ, t, z, f, a, n, and m
Randomly generate initial population
Calculate fitness values
For each iteration until z {
    For each offspring {
        Select 2 parents
        Create an offspring by recombination
        Mutate the offspring
    }
    Calculate fitness values of offspring
    Sort the total population descending based on fitness values
    Assign first μ individuals as new parents
    Terminate if no-improvement is made after t iterations
}
```

In the proposed GA implementation, each individual (parent or offspring) represents how much space is allocated and what the cycle times are for each item. If the allocated space on the shelf for a particular item, $s_i$, is less than its unit volume, $v_i$, the item will not be placed on the shelf and $s_i$ will be added to the total free space. The same rule applies to the backroom space. Therefore, the heuristic enables product assortment and allocation decisions at the same time.

4.1.2 Constructive Heuristic

In order to evaluate the performance of above method, a constructive heuristic has also been developed.

**Table 4.2 Notation for Constructive Heuristic**

| | |
|---|---|
| Y | Total required shelf space to fulfill daily demand within $3\sigma_i$ <br> $Y = \sum_{i=1}^{N}((E[X_i] + 3\sigma_i)v_i)$ |
| $j_i$ | Profitability of an item based on expected daily demand, gross margin, and space requirement <br> $j_i = E[X_i]\frac{g_i - p_i}{v_i}$ |
| $J$ | Total profitability of items <br> $J = \sum_{i=1}^{N} j_i$ |

Since the daily in-store replenishment is assumed, there is no need to place more than maximum daily demand on the shelf to avoid any stock-outs. It is assumed that maximum daily demand for any item remains in the 3 sigma range. If the total required space, *Y*, is more than *SC* the allocation is done based on profitability of an item per its cubic volume, $j_i$. On the contrary, if it's less than SC, total required space is used to allocate all of the items. Cycle times are assumed to be “1 day”, resulting in daily shipments from warehouse to the backroom. Because of the daily transportation, there is no need to have any inventory in the backroom; thus, available backroom space is not used to allocate any item to minimize holding cost. The procedure for the constructive heuristic can be expressed as follows:

Calculate $Y$, $j_i$, and $J$

For each item $i${

If $(Y \leq SC)${

$$s_i = \left(E[X_i] + 3\sigma_I\right)v_i$$

}else{

$$s_i = \frac{SC \cdot j_i}{J}$$

}

$b_i = 0$
$c_i = 1$

}

$TP(s_i, b_i, c_i)$

It can be shown that the optimality of single criterion (such as sales, demand and profit) solution methods to allocate shelf space is problem dependent. For example, an item with a high demand is allocated more space if the allocation is done based on demand. However, this allocation might not give any optimum solution in terms of maximizing profit, if the gross margin of that item is relatively small. Consequently, selecting an appropriate solution method is vital on performance of the results.

The constructive heuristic developed in this study aims to avoid shortcomings of the single-criterion solutions. If the shelf space is scarce it allocates space among items based on gross margin, demand, and space requirements. On the contrary, if the shelf space is abundant, it does not allow the full space occupied by items resulting in unnecessary holding cost.

Since the constructive heuristic does not guarantee an optimal solution, another heuristic methodology called genetic algorithm was developed. In the following chapter, in order to analyze the significance of several factors (shelf space, backroom space, truck cost and problem size) on the performance of both heuristics, an experiment with 720 different problem instances is conducted. The problems are also solved with mixed seeded GA and constructive heuristic seeded to get improved results.

# CHAPTER 5

# EXPERIMENTATION AND NUMERICAL RESULTS

This chapter presents experimental results for the two algorithms that have been developed to solve the SSAP problem, which is described in detail in Chapter 4.

Since the proposed model is unique, there are no known benchmarks available. In order to evaluate the performance of both heuristics in different retailer settings, an experiment is conducted. The factors are defined as shelf space (2 levels), backroom space (3 levels), truck cost (2 levels), and problem size (6 levels) (see Table 5.1). Since demand is stochastic, 10 instance of each problem are generated. A total 720 (2x3x2x6x10) problems are randomly generated. Each row in Table 5.1 corresponds to 6 different scenarios with different problem sizes, 8, 16, 32, 64, 128, and 256 respectively.

**Table 5.1 Factor Levels of Scenarios**

| Scenario | Truck Cost | Shelf Space | Backroom Space | # of problems |
|---|---|---|---|---|
| 1-6 | 250 | Small | Small | 60 |
| 7-12 | 250 | Small | Medium | 60 |
| 13-18 | 250 | Small | Large | 60 |
| 19-24 | 250 | Large | Small | 60 |
| 25-30 | 250 | Large | Medium | 60 |
| 31-36 | 250 | Large | Large | 60 |
| 37-42 | 750 | Small | Small | 60 |
| 43-48 | 750 | Small | Medium | 60 |
| 49-54 | 750 | Small | Large | 60 |
| 55-60 | 750 | Large | Small | 60 |
| 61-66 | 750 | Large | Medium | 60 |
| 67-72 | 750 | Large | Large | 60 |

In order to be realistic, the levels of shelf space and backroom space are created as a function of expected demand, unit volume, and problem size. Small and large shelf space sizes for each problem are generated as follows:

Small size: $$SC = \left( \sum_{i=1}^{N} E[X_i] \right) \times 0.5$$

Large size: $$SC = \left( \sum_{i=1}^{N} E[X_i] \right) \times 2$$

Small, medium and large backroom space sizes for each problem are generated as follows:

Small size: $$BC = \left( \sum_{i=1}^{N} E[X_i] \right) \times 0.5$$

Medium size: $$BC = \left( \sum_{i=1}^{N} E[X_i] \right) \times 5$$

Large Size: $$BC = \left( \sum_{i=1}^{N} E[X_i] \right) \times 10$$

Demand was assumed to be following Poisson distribution, which is discrete and positive, for all items. The Poisson distribution has one parameter, mean, which was generated as discrete U(10,30) for each item.

The other parameter values used in problems are randomly generated as shown in Table 5.2.

**Table 5.2 Parameter Values Used in Problems**

| Parameters | Value |
|---|---|
| $v_i$ | Random(2,4) |
| $f_i$ | Random(30,50) |
| $p_i$ | Random(10,30) |
| $e_i$ | $g_i - p_i$ |
| $h_i$ | $\frac{6\,p_i}{36500}$ |
| $C$ | 2700 |

5.1 Numerical Results

When testing heuristics, the right choice of values for the search parameters has a considerable effect on the performance of the procedure. As the problem type and size change, the parameters should be fine tuned in order to get better results. However, it is desirable to have those parameters independent from the problem size. Therefore, a considerable amount of time was spent to fine tune the parameters (see Table 4.1 and Table 5.3) for the GA. They were set once while ensuring good results and used throughout the experiments. “a” is defined as a function of N, number of items, to avoid big jumps during the mutation process.

It is most desirable to test and evaluate the proposed GA by comparisons with optimal solutions in respect to the solution quality and computational effort. However, because of the problem size (and search space size) of numerical examples in this study, complete enumeration was not applicable. Therefore, the best known solutions obtained for the problems were those determined using a constructive heuristic.

In addition to the CH solution, each problem was solved 10 times with random initial seeds. This procedure is called random seeded GA (RSGA). Furthermore, CH solutions were used as initial population to study the significance of initial population on performance. If half of the initial population is CH seeded, the method is called mixed seeded GA (MSGA). If the whole initial population is CH seeded, the method is called CH seeded GA (CHSGA). Each problem was also solved 10 times with MSGA and CHSGA.

**Table 5.3 GA Parameters**

| Parameters | Values |
|---|---|
| $\mu$ | 30 |
| $\lambda$ | 40 |
| $t$ | 400 |
| z | 20,000 |
| a | 0.128/N |
| n | 0.1 |
| m | 10 |

Normalization is not applied to the results, because other than main factors, there are also problem dependent factors, such as SC and BC. Thus, instead of performing an Anova analysis, we preferred to demonstrate whether one method is better than the other in terms of fitness values. Moreover, the average difference between the fitness values of each method as well as standard deviation of these differences was also provided. Computational effort was demonstrated in terms of number of function evaluations until the stopping criteria is met.

### 5.1.1 Fitness Value Comparison

In each table (Table 5.4-5.6) below, 2 (hereafter m1 and m2) of the 4 different methods are compared. Only the maximum solution of ten seeds is taken into consideration for RSGA, CHSGA and MSGA. 720 problems are broken into 12 main groups in which are 6 scenarios. 60 problems are analyzed in each row while demonstrating comparison of fitness values. The following notation will be used in each table:

"A" refers to the percentage of better m1 (stands for the first method and will be defined for each table) than m2 (stands for the second method and will be defined for each table) solutions for 60 problems.

"B" refers to the average difference of the fitness values of m1 and m2 for 60 problems. The difference is calculated as m1-m2.

**Table 5.4 Comparison of RSGA and CH**

m1 : RSGA, and m2 : CH

| Scenario | A | B | FACTORS | | |
|---|---|---|---|---|---|
| | | | TC | SC | BC |
| 1-6 | 95% | 1,216 | 250 | small | small |
| 7-12 | 100% | 1,258 | 250 | small | medium |
| 13-18 | 100% | 1,303 | 250 | small | large |
| 19-24 | 67% | -351 | 250 | large | small |
| 25-30 | 43% | -341 | 250 | large | medium |
| 31-36 | 25% | -335 | 250 | large | large |
| 37-42 | 90% | 1,048 | 750 | small | small |
| 43-48 | 90% | 1,059 | 750 | small | medium |
| 49-54 | 97% | 1,154 | 750 | small | large |
| 55-60 | 68% | -325 | 750 | large | small |
| 61-66 | 57% | -271 | 750 | large | medium |
| 67-72 | 53% | -289 | 750 | large | large |
| 1-72 | 74% | 427 | | | |

Compared to CH, RSGA performs better for 534 problems (74% of the problems). Average difference for these problems is 427. On the other hand, CH solutions are better for 186 problem instances (26% of the problems).

It can be shown that larger shelf space will reduce the daily stock-outs, and less truck cost will maintain more frequent shipments from warehouse resulting in smaller cycle times. Since CH tries to allocate maximum daily demand on the shelf and assigns cycle time as “1” for each item, larger shelf space and less truck cost will enable CH to perform better. As the result of Table 5.4 indicates, the success rate and average difference (negative) in problems with large shelf space (scenarios 19-36 and 55-72). Besides, the average success rate for CH is found to be better in scenarios where truck cost is assigned 250, and shelf space is defined as large. Consequently, it can be noted that shelf space has significant effect on the performance of CH.

**Table 5.5 Comparison of CHSGA and RSGA**

m1 : CHSGA, and m2 : RSGA

| Scenario | A | B | FACTORS | | |
|---|---|---|---|---|---|
| | | | TC | SC | BC |
| 1-6 | 83% | 1,405 | 250 | small | small |
| 7-12 | 77% | 1,362 | 250 | small | medium |
| 13-18 | 73% | 1,405 | 250 | small | large |
| 19-24 | 42% | 353 | 250 | large | small |
| 25-30 | 80% | 342 | 250 | large | medium |
| 31-36 | 75% | 339 | 250 | large | large |
| 37-42 | 77% | 1,632 | 750 | small | small |
| 43-48 | 68% | 1,591 | 750 | small | medium |
| 49-54 | 68% | 1,537 | 750 | small | large |
| 55-60 | 32% | 334 | 750 | large | small |
| 61-66 | 50% | 272 | 750 | large | medium |
| 67-72 | 47% | 291 | 750 | large | large |
| 1-72 | 64% | 905 | | | |

**Table 5.6 Comparison of MSGA and RSGA**

m1 : MSGA, and m2 : RSGA

| Scenario | A | B | FACTORS | | |
|---|---|---|---|---|---|
| | | | TC | SC | BC |
| 1-6 | 85% | 1,393 | 250 | small | small |
| 7-12 | 75% | 1,369 | 250 | small | medium |
| 13-18 | 75% | 1,383 | 250 | small | large |
| 19-24 | 40% | 353 | 250 | large | small |
| 25-30 | 78% | 342 | 250 | large | medium |
| 31-36 | 75% | 338 | 250 | large | large |
| 37-42 | 75% | 1,657 | 750 | small | small |
| 43-48 | 68% | 1,585 | 750 | small | medium |
| 49-54 | 68% | 1,562 | 750 | small | large |
| 55-60 | 32% | 334 | 750 | large | small |
| 61-66 | 52% | 278 | 750 | large | medium |
| 67-72 | 47% | 297 | 750 | large | large |
| 1-72 | 64% | 908 | | | |

Table 5.5 and Table 5.6 demonstrate the comparison of modified heuristics and RSGA for 720 problems. Both CHSGA and MSGA are expected to perform better than RSGA, because the initial populations contain moderately good solutions. The quality of solutions is improved for 461 (64%) problems. However, there are still 259 (36%) problems where the RSGA performs better than CHSGA and MSGA. Recombination and mutation operators play important role in the performance of any genetic algorithm method. Convergence to an optimum solution will be achieved only by mutation, if the diversity of the initial population is not ensured. Therefore, premature convergence will be inevitable. Consequently, the solution quality will be dependent on the initial population when CHSGA and MSGA are used.

### 5.1.1.1 Effect of Problem Size on RSGA Performance

Another significant factor on RSGA's performance is problem size. As shown in Table 5.7, success rate tends to diminish as the problem size increases. For a particular scenario (TC, SC and BC are at a level), it can be shown that order quantity from warehouse increases as the number of items increases. Therefore, CH is likely to utilize truck capacity resulting in less transportation cost when large orders are placed. Consequently, decrease in success rate should also be attributed to transportation utilization and increase in number of decision variables.

**Table 5.7 Effect of Problem Size on RSGA Performance**

| Problem size | Success Rate |
|---|---|
| 8 | 100% |
| 16 | 95% |
| 32 | 78% |
| 64 | 83% |
| 128 | 51% |
| 256 | 36% |

### 5.1.2 Computational Effort Comparison

The choice of the stopping criteria is one of the key factors which decides the time complexity as well as the quality of the solutions. Thus, the stopping criteria should enforce fast convergence while retaining the quality of the solution to an acceptable value. The proposed stopping criteria in this study allows 20,000 iterations unless there is no improvement in any 400 consecutive iterations.

Computational effort is determined in terms of number of iterations executed until the stopping criteria is met. Since, the initial population of RSGA is randomly generated; it consists of relatively poor solutions. Hence, the computational results provided in this section demonstrate the average number of iterations to reach the CH solutions as well as the best solutions found so far (if RSGA is better than CH). On the other hand, only the average number of iterations, until the stopping criteria is met, is provided for CHSGA and MSGA, since, the initial population for those includes CH solutions.

In Table 5.8, only the problems, where RSGA outperforms CH, are taken into account. Column "A" refers to the average number of iterations until the stopping criteria is met. Column "B" refers to the average number of iterations to reach the CH solutions. It takes 991 iterations (for RSGA) on the average to reach the corresponding CH solutions. Furthermore, the average number of iterations is 6,559 at convergence.

**Table 5.8 Average Number of Iterations - RSGA**

| Scenario | A | B | FACTORS | | |
|---|---|---|---|---|---|
| | | | TC | SC | BC |
| 1-6 | 5,405 | 563 | 250 | small | small |
| 7-12 | 5,609 | 622 | 250 | small | medium |
| 13-18 | 5,870 | 566 | 250 | small | large |
| 19-24 | 7,662 | 1,516 | 250 | large | small |
| 25-30 | 7,184 | 1,848 | 250 | large | medium |
| 31-36 | 6,476 | 279 | 250 | large | large |
| 37-42 | 5,256 | 752 | 750 | small | small |
| 43-48 | 6,167 | 899 | 750 | small | medium |
| 49-54 | 5,762 | 1,141 | 750 | small | large |
| 55-60 | 7,807 | 1,153 | 750 | large | small |
| 61-66 | 9,343 | 1,464 | 750 | large | medium |
| 67-72 | 9,591 | 1,801 | 750 | large | large |
| 1-72 | 6,559 | 991 | | | |

Table 5.9 and Table 5.10 show the average number of iterations required for modified heuristics for 720 problems. Column “A” refers to the average number of iterations until the stopping criteria is met. Both CHSGA and MSGA converge faster than RSGA, since they start from moderately good solutions. The average number of iterations for CHSGA and MSGA to attain convergence are 3,427 and 3,396 respectively.

It should be noted that increasing the first stopping criteria, which is “maximum number of unsuccessful iterations”, could also improve the solution performance of GA.

**Table 5.9 Average Number of Iterations - CHSGA**

| Scenario | A | FACTORS | | |
|---|---|---|---|---|
| | | TC | SC | BC |
| 1-6 | 3,197 | 250 | small | small |
| 7-12 | 4,287 | 250 | small | medium |
| 13-18 | 4,549 | 250 | small | large |
| 19-24 | 2,574 | 250 | large | small |
| 25-30 | 2,874 | 250 | large | medium |
| 31-36 | 3,242 | 250 | large | large |
| 37-42 | 3,390 | 750 | small | small |
| 43-48 | 4,218 | 750 | small | medium |
| 49-54 | 4,314 | 750 | small | large |
| 55-60 | 2,557 | 750 | large | small |
| 61-66 | 2,722 | 750 | large | medium |
| 67-72 | 3,203 | 750 | large | large |
| 1-72 | 3,427 | | | |

**Table 5.10 Average Number of Iterations - MSGA**

| Scenario | A | FACTORS | | |
|---|---|---|---|---|
| | | TC | SC | BC |
| 1-6 | 3,240 | 250 | small | small |
| 7-12 | 4,131 | 250 | small | medium |
| 13-18 | 4,432 | 250 | small | large |
| 19-24 | 2,286 | 250 | large | small |
| 25-30 | 2,995 | 250 | large | medium |
| 31-36 | 2,975 | 250 | large | large |
| 37-42 | 3,436 | 750 | small | small |
| 43-48 | 4,417 | 750 | small | medium |
| 49-54 | 4,430 | 750 | small | large |
| 55-60 | 2,324 | 750 | large | small |
| 61-66 | 2,940 | 750 | large | medium |
| 67-72 | 3,144 | 750 | large | large |
| 1-72 | 3,396 | | | |

As a result, after solving 720 randomly generated problems, shelf space and problem size found to be more significant factors on the performance of proposed GA. CH seeded initial population has contributed to the GA, but this also gave rise to premature convergence because of similar individuals in current population. Performance of the CH was found to be dependent on problem type. It should be noted that the randomness among problems within each scenario is not enough to compare solution methods. Therefore, a wide variety of problem types should be taken into consideration in order to get a true comparison.

# CHAPTER 6

# CONCLUSIONS AND FUTURE WORK

This thesis studied the problem of shelf and backroom space allocations while explicitly considering stochastic demand and transportation costs. This is an important problem that is encountered in a wide variety of practical situations where space is scarce. A new model was proposed including holding cost and occurrence of stock-out (especially while items are available in the backroom).

Since there was not any exact algorithm to solve SSAP proposed in this research, a hybrid heuristic method was applied with the aim of producing high-quality solutions. In order to evaluate the performance of this method, a constructive heuristic was also applied the same problem. There were not benchmarks available; thus, both heuristics were experimented on randomly generated 720 problems. The heuristic parameters were set once and used throughout the experiments. GA performed better solutions for instances where the shelf space was small. It can be noted that in real life situations, the shelf space is usually scarce and each item has a lead time more than a day because of the lot-size and geographical limitations. The performance of CH solution is found to be problem dependent. It is observed that true comparison of different solution methods can only be valid, if wide variety of problems is taken into consideration. Not only problem size but also the other constraints should be evaluated.

The followings can be suggested for further studies:

1) Proposed model has practical relevance in a variety of situations; thus efforts must be made to apply the model on related problems. In this way, the performance of heuristic can be evaluated with existing benchmark problems.
2) Different selection, recombination and mutation operators can be applied to increase the performance of GA.
3) Growing manufacturing companies tend to use the available space for machinery and try to facilitate the warehouse space until the new business starts paying off. This model can also be implemented to a manufacturing plant where there are Kanban system and limited backroom capacity.

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# APPENDIX

## EXAMPLE PROBLEM

Relevant parameter definitions (from Spaceman manual):

| Term | Definition |
|---|---|
| Reg movement | The unit sales or case sales of a product for a specific period |
| Width | The distance from one side of a unit to the other side |
| Price | The retail price for a merchandized item |
| Cost | The cost per item of a product |
| Projected GP | Projected gross profit is the gross profit you can expect to earn if you maintain the current schedules and stock on shelves presently depicted on your planogram |

Problem input parameters:

| Product | Width (inches) | Price | Cost | Reg movement (daily demand) |
|---|---|---|---|---|
| 1 | 9 | $38 | $16 | 23 |
| 2 | 8 | $36 | $18 | 24 |
| 3 | 10 | $36 | $17 | 25 |
| 4 | 9 | $40 | $18 | 23 |
| 5 | 6 | $39 | $15 | 26 |
| 6 | 10 | $36 | $13 | 24 |
| 7 | 8 | $36 | $19 | 25 |

Period: 49 days

Shelf width: 1,200 inches