The Space Allocation Problem for Temporary Displays in Supermarkets

by

Ilknur Uludag

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Approved by

Alice E. Smith, Joe W. Forehand, Jr. Distinguished Professor Department of Industrial and Systems Engineering, Auburn University
Daniel Silva, Associate Professor, Industrial and Systems Engineering, Auburn University
Richard Sesek, Tim Cook Professor, Auburn University
Mark Clark, Lecturer, Department of Supply Chain Management

Abstract

In this research, a novel approach to the shelf-space allocation problem of product categories to temporary displays in a supermarket is proposed. The main store aisles display similar products, which are categorized and named according to their functional characteristics such as cleaning, bakery, dairy, etc. These aisles are considered permanent locations for the product categories. Endcaps and island displays have products that are intended to stimulate additional purchases for a short time. These displays are considered temporary locations for promotional products, seasonal products, and high impulse purchase products. A store changes the products and their display area in temporary displays frequently, usually weekly. We aim to maximize sales by allocating product categories to temporary displays while considering the location and area of these displays. While we assume that the permanent locations of the product categories are fixed. This shelf-space allocation problem for temporary displays is solved both for a single promotional period and two promotional periods as this best captures the store policy. We consider two versions of the MINLP – one where each display contains only a single product category and the second where product categories may share a temporary display. Working a major supermarket chain in Turkey, we were able to compare the resulting revenues with that from the designs ordinarily implemented by the store management. Our analytical approach provides significant increase in revenue by optimally siting and sizing the product displays.

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List of Abbreviations

NLP	Nonlinear Programming Model
MINLP	Mixed Integer Nonlinear Programming Model
IP	Integer Programming Model
LP	Linear Programming Model

Chapter 1 Introduction

Supermarkets are the preferred locations to purchase food, groceries, dairy products household, personal care, cosmetics, gift, and textile products. The U.S. Census Bureau stated that the retail and food services in U.S. was $540.9 billion in December 2020 (U.S. Census Bureau, 2021). In this competitive market, the allocation of products in a store has a significant impact on the revenue and customer purchasing behavior (Aloysius and Binu, 2013). The allocation of products specifies where to place the products and how much shelf-space to allocate to these products. Due to the high number of products, product categories are defined based on similarities of products. A product category such as cleaning, organic, or baby contains similar goods and individual products.

In the retail industry, category management and aisle management are the most commonly preferred approaches for product-assortment and shelf arrangement decisions. Category management combines sales data, user profiles, and store information to improve the performance of product categories (Pepe, 2012). Aisle management seeks higher traffic, sales, and profits of product categories in pre-determined zones in stores (Larson, 2006). In both approaches, retailers want to increase the potential of planned and unplanned purchases. Unplanned purchases are approximately 70% of purchasing decisions in a supermarket (Chen et al., 2006), so retailers seek the most profitable strategies to increase unplanned purchases, or impulse purchases. For this reason, it is common to find the products of different product categories next to each other.

Each store may have slightly different product assortments and display strategies to attract customers. There are three common store layouts in the retail industry: grid form, free form, and racetrack (Levy and Weitz, 2001). The grid store layout is the preferred layout for a supermarket, and we build our models based on the common grid store layout. The area at the entrance of the store, main aisles, end of aisle displays, off-shelf displays, and the checkout area all display a wide variety of products. An example of a grid store layout and its displays are shown in Figure 1.1. Island and checkout displays are called off-shelf displays and commonly used as temporary displays for the products, which may vary according to the promotional periods and seasons (Buttle, 1984). End of aisles displays are also known as endcaps (Dreze et al., 1994) and gondola-ends (Buttle, 1984) in the retail management literature.

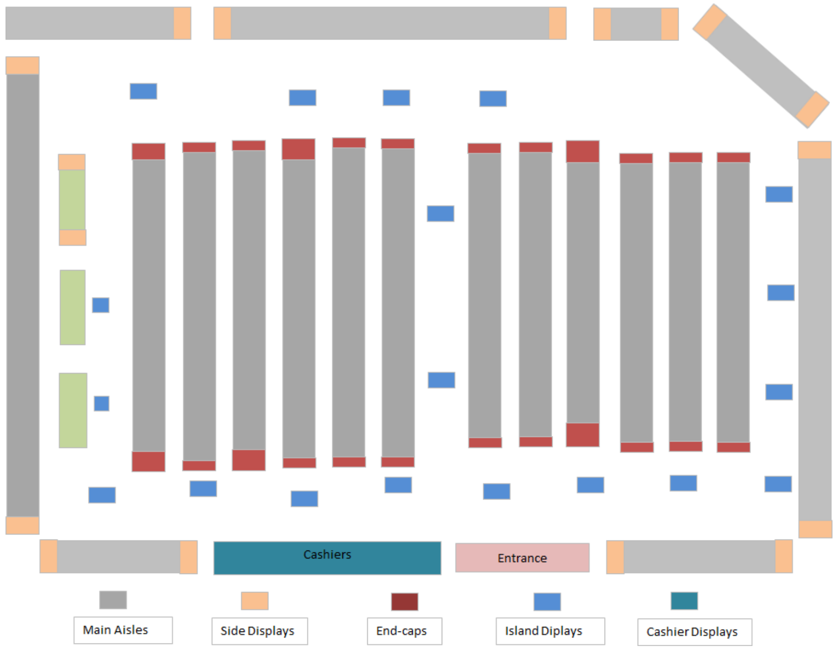


Figure 1.1. An Example of a Grid Store Layout

Each area in the store has its purchasing characteristics (Zentes and et al., 2007). The entrance is designed to welcome the customers to a pleasant atmosphere. The main aisles are the permanent locations of general food and household products. The locations of these products are not changed until a significant event occurs such as an extension of a store or new management.

Figure 1.2a. Typical End of Aisle Display[1] Figure 1.2b. Seasonal End of Aisle Display[2]

Endcaps have a significant impact on increasing sales while customers are on their path to buy their planned purchases (Valenzuela and Raghubir, 2009). Stern (1962) defined impulse

[1] www.kellytheculinarian.com/2012_01_01_archieve.htm

[2] www.creativedisplaysnow.com/six-inspired-examples-of-sales-boosting-grocery-store-endcaps

purchase items as those purchased by customers without planning prior to the store visit. In most cases, promotional products and seasonal products are not planned purchases. These products are commonly allocated to endcaps or island displays. Figure 1.2a shows an endcap with a single product. Figure 1.2b shows a design to hold various products for a Thanksgiving meal. Island display areas are designed to highlight a product, brand, or product group. These areas increase the awareness of the product, so it is common to allocate new products at island displays (Ozcan and Esnaf, 2013). An example of an island display with multiple products is given in Figure 1.3. The checkout area is the place where each customer completes their purchases by cashiers or self-checkout machines and these areas typically have small displays of high- impulse purchase products such as candy or magazines.

Figure 1.3. Typical Island Display[3]

The supply of a wide variety of products in the store requires the categorization of these products according to their functional or consumption similarities (Borges, 2003). Desmet and Renaudin (1998) presented a hierarchical classification for products in a retail chain. There are six levels of hierarchical classification. Starting from the most individual level, these are the stock keeping unit (SKU), the sub-product category, product categories, groups of product categories, departments, and finally, store (Desmet and Renaudin, 1998). The levels of product classification can be seen in Figure 1.4. In this research, we focus on the product category level.

Each product category has sale characteristics and relationships with other product categories. A sale in a product category can affect sales of other product categories. Corstjens and Doyle (1981) defined the term of cross-space elasticities to explain this sales effect. Cross-

[3] www.drugstoreshelving.com/4-way-island_display/index.htm

space elasticities have a significant impact on sales (Corstjens and Doyle, 1981). Therefore, understanding cross-category sales may provide improved product allocation and promotional activities (Larson, 2006).

Short-term price discounts, the replacement of products in the store, and advertising are components of promotional activities of retailers to achieve higher revenues. Promotional activities are so crucial for the supermarket industry that the U.S. and European companies have increased their promotional expenditures dramatically in recent decades (Foekens et al., 1999). The length and frequency of the promotional period and season may vary from store to store. There are four primary goals to apply promotional strategies in supermarkets: increase sales, reduce stocks, increase brand switching, and increase cross-category sales. First, we expect to increase sales and decrease the stock of promotional products or brands during a promotional period. Furthermore, there will be more customers in the areas of promotional products and these customers may purchase products from other brands (brand switching) and other product categories (cross-category) (Kamakura and Kang, 2007).

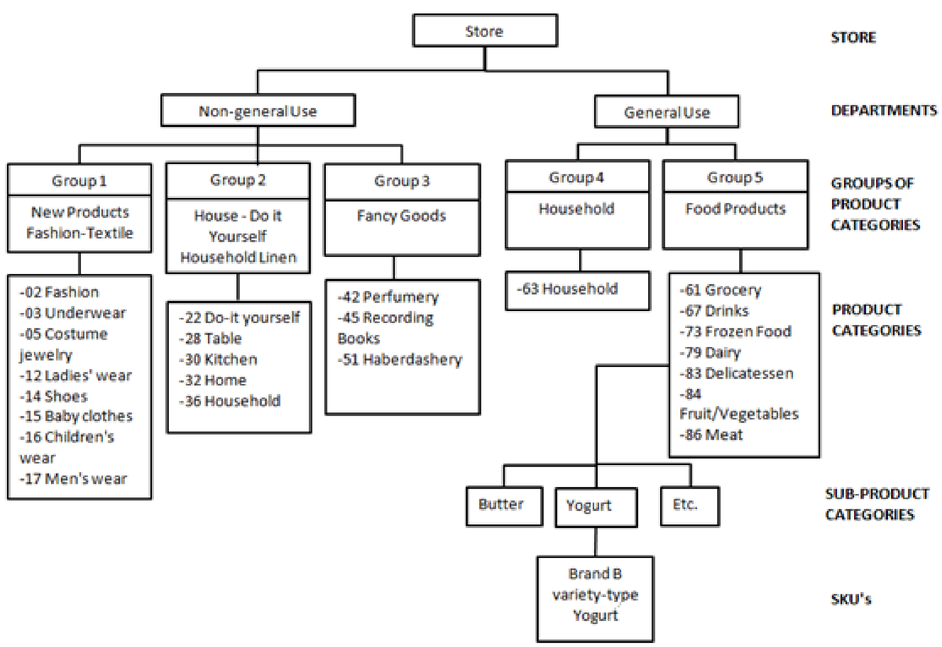


Figure 1.4. A Hierarchical Classification of Products in a Retail Chain (Desmet and Renaudin, 1998).

While allocating promotional product categories to their locations in the store, it is necessary to consider the visibility of the locations. Desmet and Renaudin (1998) indicated that spaces with more visibility had positive impacts on sales for a specific item or product category.

Campo et al. (2000) stated that places close to the entrance are more noticeable to customers than those at the back corner of the store. Furthermore, customers perceived that products on the end-of-aisles were discounted even if they were not discounted (Inman et al., 1990).

1.1. Problem Statement

A common problem in supermarkets is to allocate product categories to their permanent and temporary locations. Each product belongs to at least one product category according to the store's assortment strategy. Main aisles and wall displays are considered permanent locations to display product categories. However, store managers have to choose on the products and the areas of products in temporary locations every promotional period. Temporary displays have positive impressions on sales (Desmet and Renaudin, 1998, Campo et al., 2000, and Inman et al., 1990). In this research, we focus on product categories and their display areas in temporary displays for one or more promotional periods. The sales of a product category are associated with its location, total area of a display, and its relationship between other nearby categories. It will be beneficial to allocate product categories with high cross-space elasticities next to each other. The shelves at temporary displays are commonly occupied with promotional products, seasonal products, and high impulse purchase products. The selection of the product categories for these displays is very important for store sales and is done frequently. In this research, our goal is to answer the following research questions by developing and solving our allocation models for product categories in temporary displays.

Research Question 1. Which product categories should be allocated to which temporary displays if it is assumed that a product category occupies the entire temporary displays?

Each product category is assigned to at least one permanent location in the grid store layout. Temporary locations, which are endcaps and island displays, are used during a promotional or seasonal period. Endcaps are located at the end of the main aisles. Therefore, their spaces within the store are limited. Island displays also have limited area because they are located in the remaining areas leftover from the main aisles. When the customers are in the store to purchase their planned products, they visit the permanent locations of product categories, and they see other products in temporary displays. Even though the products at the temporary displays may not be included among their planned purchases, they may see a product from these displays and decide to buy it (i.e., make an impulse purchase). Therefore, supermarkets want to increase their sales by allocating promotional and seasonal products to endcaps and island displays because of the temporary display visual advantages. The limited area of the endcaps and island displays should be filled with the most profitable products with high relationships between the product categories in nearby main aisles for each promotional period.

Research Question 2. How will be the solution change, if multiple product categories can occupy a single display?

The choice of a single product category or multiple product category allocations to a temporary display during promotional periods is affected by the inventory levels of products, impulse purchase likelihoods, prices, cross-category sales with nearby product categories, and customer traffic rates at the allocated area. Multiple products can be combined for a special theme during a promotional period. If a temporary display has only one product category, it is termed a non-split allocation. If the display includes more than one product category, it is termed a split allocation. Supermarket managers seek single product category allocations and multiple product category allocations at temporary displays. Cross-category sales are considered for the closest permanent and temporary displays for single product category allocations. In the case of multiple product categories at a temporary display, cross-category sales within the temporary display are added to cross-category sales. The school opening season is an example of how the multiple product categories can be combined on a single display. The possible products that the customers can see are hand sanitizers, notebooks, clothes, and lunch bags. These products belong to the cleaning supplies, school supplies, clothes, and glassware product categories.

Research Question 3. How do the allocations change if a second period of promotion follows the first?

In a real store environment, there are multiple promotional activities. It is conceivable to have a period with overlapping promotional activities with different promotional products. For example, assume that the first promotional period starts with 60 items for 16 days, the second promotional period starts on the 10th day of the first promotional period with 10 items, and ends on the 16th day. It is unknown which products will be in the second promotional period at the beginning of the first promotional period. To solve this problem, it is necessary to develop another model for the second promotional period by considering the current layout of temporary displays during the first promotional period.

In this research, the shelf-space allocation models for temporary displays are designed for a supermarket. In addition to the supermarkets, the new allocation models could be applied to hardware stores, bookstores, and similar stores, which have permanent and temporary displays.

1.2. Research Objectives and Major Contributions

In this research, we aim to develop effective and realistic allocation models for temporary displays in supermarkets during promotional periods. The models are designed to achieve the following objectives:

- To determine the locations and sizes of the product category displays at temporary displays using mathematical models for a single promotional period and two consecutive promotional periods
    - A non-split mathematical model with the assumption of a single product category at a temporary display
    - A split mathematical model with the assumption of multiple product categories at a temporary display
- To evaluate the current store situation and proposed solutions by implementation of the solution in real store environment.

We provide two major contributions to retail operations and the retail literature with our research.

Contribution 1. We give mathematical models for the temporary display allocation problem with assumptions of a single product category and multiple product categories at a temporary display during a promotional period.

Our study is the first that considers the allocation problem of temporary displays at the product category level while considering impulse purchases and cross-category sales. We assume that the permanent locations of the product categories are pre-determined. The objective function maximizes sales of product categories at the temporary displays. The revenue of temporary displays includes impulse purchases of product categories and cross-category sales at permanent and temporary displays. Constraints are defined to satisfy the area limitations of product categories.

In our research, we solve these models by using the GAMS optimization package, which includes several MINLP solvers. In this dissertation we selected three MINLP solvers that we obtained optimal solutions in most of test problems. These MINLP solvers are BONMIN (Bonami and Lee, 2009), DICOPT (Grossmann et al., 2002), and LINDOGLOBAL (Bussieck and Vigerske, 2010). In some of the test problems, the MINLP solvers conclude with the local or global optimum solution within the allotted solution time. If the obtained solutions during the solution time are not listed as local or global optimal, the solution is listed as the best solution found for the test case.

Contribution 2. We give mathematical models for the temporary display allocation problem for two overlapping promotional periods.

This research will be the first that creates two mixed integer nonlinear mathematical models for the overlapping consecutive promotional periods where the second depends on the first. The last days of a promotional period may be the first days of another promotional period with different products from the previous promotional period. In order to benefit from both promotional periods, it is important to provide a layout for promotional products of both promotional periods. The first model is constructed with the assumption of a single product category at a temporary display. The second model is designed to evaluate multiple product categories at a temporary display. In both models, it is necessary to state a current layout of temporary displays for the first period. A current layout could be designed by the store manager or obtained from the single promotional period models. In our test problems, we used the solution from the single promotional period models.

Importantly, the results of the mathematical models are evaluated in the real store environment in one of the stores of the Yunus Supermarket Chain. This study provides a solution methodology with mathematical models to solve the temporary display allocation problem, which had previously been solved by trial-and-error and/or the store managers' experience.

1.3. The Solution Methodology

This research presents a solution methodology to the product allocation problem of temporary displays for promotional periods in a supermarket environment. We use the real time shopping basket data from the Yunus Supermarket Chain in Turkey. Non-split mathematical models and split mathematical models are proposed for both a single promotional period and two promotional periods. Figure 1.5 illustrates the classification of the proposed mathematical models in this dissertation.

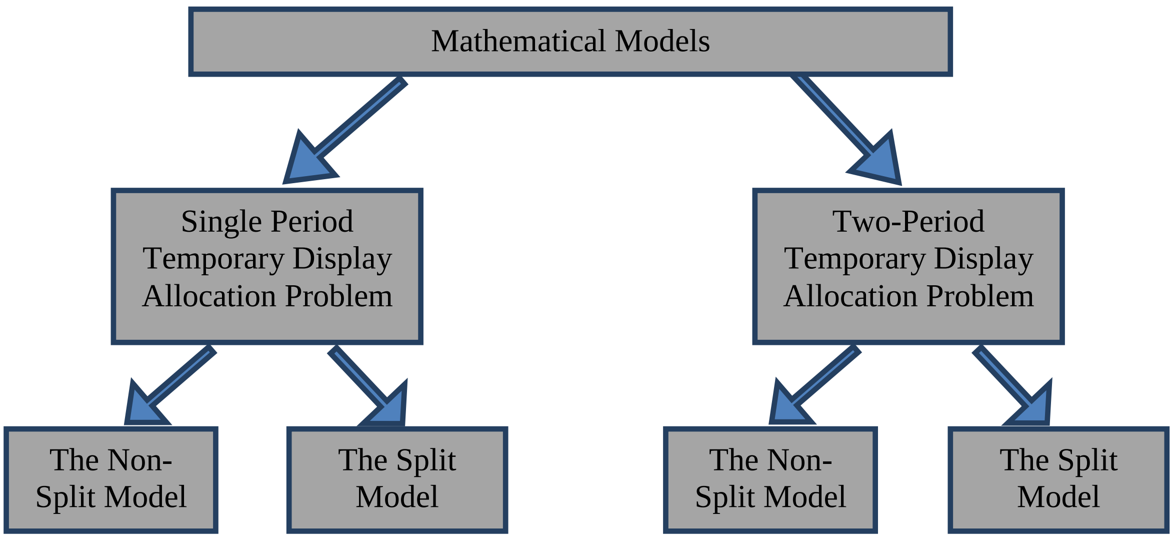


Figure 1.5. The Outline of the Proposed Mathematical Models

The following step is the evaluation of the problem as the single period problem and the two-period problem. Both problems are solved by considering the assumptions of non-split and split product category allocations. Due to the complexity of the nonlinear and integer programming models, GAMS program finalizes the solution search with the best solutions and optimal solutions for the real size problems. The model results and store sales are compared for validation of the proposed mathematical models.

The proposed mathematical models for a promotional period are mixed integer nonlinear programming models to solve the space allocation problem in temporary displays with the assumption of a single product category and multiple product categories at a display. Our goal is to find the best allocation of product categories at temporary displays by considering impulse purchases, space elasticities, and traffic rates. We aim to maximize sales with deterministic assumptions of the space elasticities, impulse purchases, and traffic rates. The space elasticity parameters of product categories aim to calculate the expected sales of these categories. We assume that impulse purchases are those sales that exceed the expected sales of a product category. The objective function maximizes impulse sales of the product categories in temporary displays and cross-category sales between the product categories in temporary displays and permanent displays. Binary and continuous decision variables are defined for the location and allocated area of the product categories. Two main constraints are stated in the problem. First, the allocated area for each product category should satisfy the lower and upper area limits of the product categories. Second, the total allocated area and the number of product categories at each temporary display cannot exceed the capacity of a temporary display. Test problems are solved by the selected MINLP solvers in the GAMS program.

As a next step, the temporary display allocation problem is solved for the consecutive two periods. The solution to the single-period problem is implemented in the store and the store managers will not want to remove and replace all temporary displays for the second period. The solution of the single-period problem is adapted to the two-period problem and the new problem is solved for non-split and split product category allocations. The objective functions and the constraints are similar to the single-period model. The only difference is that the total number of replaced temporary displays is restricted by the store manager's preferences. Test problems are solved by the selected MINLP solvers in the GAMS program.

Then, the implementation results of the proposed models are explained at the ends of Chapters 4 and 5. The output of our solution methodology is to identify the most profitable temporary locations for product categories and the allocated area of each product category in these temporary displays. The store managers allocated the promotional products to the temporary displays accordingly. Actual sales are compared to previous promotional period sales to show the benefit of our models. For validation of the proposed models in Chapters 4 and 5, we compare store sales and the model results at the product category level.

1.4. Validation of Models

In this research, we propose single and two period mathematical models for the temporary display allocation problem. It is essential to determine that our proposed models are valid for our problem. A model's representation of the problem entity and the model's structure, logic, and mathematical and causal relationships should be "reasonable" for the intended purpose of the model (Sargent, 2005). Historical data validation includes using some part of the current data to build the model and the other part of the data to test the proposed models (Sargent, 2005).

The validation process has two parts: validation of assumptions and input-output transformations (Table 1.1). The assumptions of models include structural and data assumptions. The structural assumptions and input-output transformations are discussed with the store managers to create reasonable mathematical models and realistic layout.

Table 1.1. Proposed Models' Structural and Input-Output Transformations

Models	Structural Assumptions	Input-Output Transformations
The Proposed Mathematical Models	Decision variables, objective function, and constraints	Comparison of the proposed models with actual store data using the current store layouts

In the validation process of mathematical models, we compared the models' objective function values with the current sales. In order to see the improvement rate of our solution approach, the sales data from the current store allocation, which is done by store managers' experiences, is compared with the sales data that we have obtained from proposed layouts.

1.5. Organization of the Dissertation

We summarize the related work for the shelf-space allocation problem in the retail industry, customer' shopping paths, and retail operations in Chapter 2. Retail operations include shelf-space elasticity, impulse purchases, location factors, promotional activities, and cross-category sales. The mutual parameters that apply to all models are presented in Chapter 3. We explain the additional parameters, decision variables, objective functions, constraints, assumptions, and results of the mathematical models for the temporary display allocation problem in a single promotional period in Chapter 4. The details of the temporary display allocation problem and results for two consecutive promotional periods are stated in Chapter 5. Finally, in Chapter 6, we summarize the main contributions of this thesis and provide some promising research areas that we plan to explore in future studies.

Chapter 2 Literature Review

The retail allocation problem has been studied by many researchers in the literature. Many of the studies provided decision models, optimization algorithms, and statistical models. The models from the literature and retail industry practices need to be integrated for successful business applications. The following research and review articles summarized and pointed out the key parameters for the current and future retail allocation models.

This chapter contains the relevant literature review of our research. The literature review includes four sections: 1) retail operations, 2) the retail shelf-space allocation problem, and 3) customer shopping paths. In the first section, we discuss five main topics in retail operations: shelf space elasticity, impulse purchases, location factors, promotional activities, and cross-category sales. In the second section, we explain the retail shelf-space allocation problem and provide an extensive literature review about it. In the third section, approaches to understand customer shopping paths are reviewed.

2.1. Retail Operations

Today's retail companies commonly have a grid, free-form, racetrack, or serpentine store layout to display their products (Levy and Weitz, 2001, Li, 2010). In most cases, they pick a single type, but it is also possible to see a combination of these designs. The grid store layout is the preferred layout in supermarkets and drugstores.

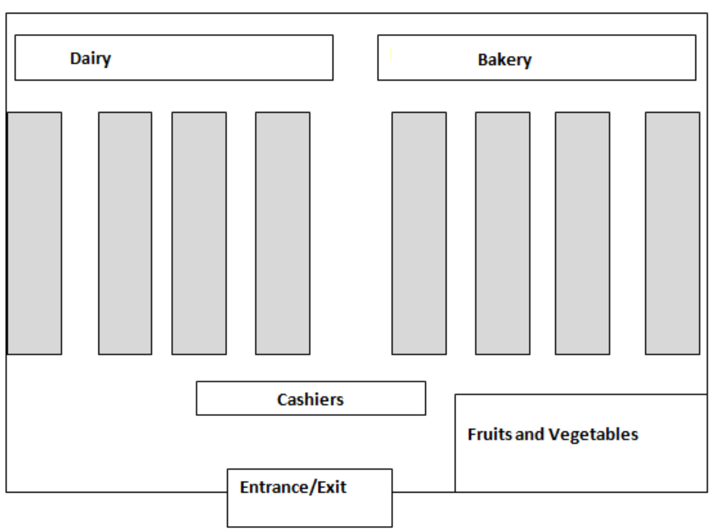


Figure 2.1 An Example of a Grid Store Layout

An example of a grid layout is illustrated in Figure 2.1. The advantage of the grid layout is having more display areas than other layouts. On the other hand, the browsing opportunities of the grid layout are limited (Garip and Unlu, 2012), and the store atmosphere is not as enjoyable as other layouts (Levy and Weitz, 2001). Products are categorized based on their functional or consumption similarities (Borges, 2003). The products within the product categories are determined according to the assortment policy of a store (Desmet and Renaudin, 1998).

In a store environment, the store managers consider many issues to increase sales. Space elasticity, impulse purchases, location impacts, promotional activities, and cross-category sales are important components of store sales.

Each product and product category is assigned to a designated space in the store. The allocated space and sales are associated, and the association is explained as shelf space elasticity (Corstjens and Doyle, 1981). Product, shelf, and store characteristics potentially influence shelf space elasticity (Eisend, 2014). Desmet and Renaudin (1998) focus on shelf space elasticities at the product category level. They provide an empirical estimation of the relationship between sales and the space of product categories. The monthly sales, space and margin data of product categories were collected from the French store chain, which had 200 stores. They state that space elasticities at the product level are between -0.44 and 0.80. Their results show that product categories with higher impulse purchase rates have higher space elasticities such as jewelry and fruit/vegetables. Cross space elasticities are not considered in the estimation model. Also, their model is not a generalized model to estimate shelf space elasticities.

Impulse purchases are one of the components of supermarket sales. Most customers purchase products which are not on their shopping list. The relationship between impulse purchase likelihood of products and customer purchases has a significant impact on sales (Kollat and Willet, 1967). Impulse purchase effects have been discussed by many researchers in the retail literature (Stern, 1962, Desmet and Renaudin, 1998, Bell et al., 2013, and Hulten and Vanyushyn, 2011). The most detailed empirical work about impulse purchases was done by Bell et al. (2013). They propose a multi-level empirical model to estimate the impulse purchase likelihood of product categories. They collected store data from 21 stores in Europe. The data includes 18,000 purchases and 58 product categories.

The decisions of a retail manager include not only the assortment strategy for products but also choosing the best display area for each product (Hariga et al., 2007). Retailers are aware of the importance of prominent and attractive display arrangements to capture the attention of customers (Abbott and Palekar, 2008). Desmet and Renaudin (1998) indicate that better visual space has positive impacts on sales for a specific product or product category.

Places close to the entrance are more noticeable by customers than other places, especially near the back corner of the store (Campo et al., 2000). End-of-aisle and island displays have advantages and greater impacts on sales compared with regular aisles. Inman et al. (1990) point out that customers perceive the products at the end-of-aisles as discounted even if they are not. Inman et al. (1990) compare displays at the end of the aisle, checkout counter, and within the regular aisle. Unplanned purchases of displays at the end of the aisle are higher than displays at the checkout counter and within the regular aisle. Burton et al. (1998)'s findings show that national brands may have better sales when they are located at the end-of-aisles rather than in regular aisles. Garrido-Morgado and Gonzalez-Benito (2015) investigate sales of milk and liquid soap/gel categories when they are allocated at end-of-aisle and island displays. Their results show that there is the significant increase in sales of both product categories when they are allocated to these temporary displays.

Bezawada et al. (2009) propose a spatial model to evaluate the cross-category effects of aisles and displays. Cross-category effects are considered as asymmetric sales affinities between product categories in their studies. Their results confirm that aisle and display areas have significant and substantial effects on cross-category sales (Bezawada et al., 2009).

Short-term price discounts, rotation of products in a store, and advertising are considered promotional strategies to achieve higher revenues in the retail industry. Promotional strategies are so crucial for the supermarket industry that U.S. and European companies have increased their promotional expenditures dramatically in recent decades (Foekens et al., 1999). Promotional activities occur at the SKU level, sub-category level, and product category level. There are four main goals of promotional strategies in supermarkets. They expect to increase sales of the promoted product or brand, decrease the stock of the promoted product and brand, increase cross-category sales, and/or increase sales of other brands (Kamakura and Kang, 2007).

The location effects of promoted products have been well studied in the literature. Promoted products affect the sales of other products (Leeflang and Parreno-Selva, 2012). The affected products may be complements or substitutes of the promoted product (Campo et al., 2000). Leeflang and Parreno-Selva (2012)'s findings show that there is a strong relationship between cross-category promotion elasticities and the physical distance between products.

Market basket analysis is a technique in the retail industry to understand the relationship between purchased products. The associations between items are explained as a set of rules in the analysis. A rule may show the relationship between flour, eggs, and sugar, for example. This rule may appear in multiple customer shopping transactions. Association rules are helpful for store-based decisions such as pricing and promotions. Russell and Petersen (2000) present an extension of market basket analysis, which focuses on the cross-category choice for each customer and his/her previous purchases. They investigate the demand relationship across complementary, independent, or substitution categories.

Recent retail literature is reviewed by Mou et al. (2018). They discuss demand forecasting, in-store logistics, backroom storage, shelf replenishment, inventory management, assortment and display, product promotion, employee management, and checkout operations. They describe the important operations and decisions of a retail store, summarize the current situation of operations, and identify the research gaps for future studies.

Cetin et al. (2019) proposed a nested multinomial logit framework to address the problem of where to allocate products in a promotional display. The optimal solution to this problem depends on the aisle attractiveness, which is defined as how an aisle attracts customers by considering the assortment size, customer heterogeneity, and walking distance to aisles (transit cost). Their findings show that the effect of a promotional display on a product category is to increase demand of the product category or estimate the product category sales.

The future of retail operations is discussed in the review article of Caro et al. (2020). The potential dominator of the retail industry is seen as e-commerce. New business models in the retail industry have been applied to improve retail practice. Brick-and-mortar retailers have services for home delivery and in-store pick-up. Also, originally online retailers now have physical stores. The main research topics of the future are listed as distribution approach (e-commerce and omnichannel), analytics, and social aspects. The most promising research topics for the distribution approach are listed as role of stores, last-mile logistics, omnichannel fulfillment, and inventory positioning. The areas of analytics include demand modeling, assortment optimization, online display optimization, personalization, and pricing. The authors addressed opportunities for assortment optimization and online display optimization. The new optimization models could provide the customization of the solution for each customer. The social aspects are classified as consumerism, jobs and labor relations, market concentration, waste, and supplier visibility/compliance.

2.2. The Retail Shelf-Space Allocation Problem

Allocation of products in a supermarket is one of the most critical factors to affect store revenue according to the retail literature. Two main solution approaches have been proposed for the problem: association rules-based data mining algorithms (Altuntas, 2017, Cil, 2012, and Aloysius and Binu, 2013) and shelf-space optimization models (Corstjens and Doyle, 1981, Yang and Chen, 1999, Yang, 2001, and Hwang et al., 2005).

Bianchi-Aguiar et al. (2020) provide a comprehensive review and classification framework for retail shelf space planning problems. This study asserts that future research studies about the retail shelf space planning problem should address four main areas. First, cross-elasticity effects, and traffic paths of the problem need to be further investigated. Second, merchandising rules, product category groups, and complementary products could be added to the constraints. Third, current solution approaches require many complicated procedures, and they are not easily applied in practice. Simple solution methodologies will provide more benefit to practice especially with multiple stores. Finally, current research papers have pointed out that macro-shelf space planning, assortment, pricing, promotion, and replenishment planning should be considered with comprehensive and integrated decision models.

2.2.1. Data Mining to Establish Buying Behavior

Association rules-based data mining algorithms aim to evaluate the relationship among products or product categories based on sales data. Strong and weak associations among products are detected and products which have strong associations should be allocated next to each other in the store. The physical space and space elasticity of products are not directly considered in these algorithms.

Cil (2012) explores product category correlation by using association rule mining and rearrangements of product categories based on strong associations. An association rule has two components: antecedent (product A) and consequent (product B). Each rule has a percentage value, which shows the percentage of customers that have purchased both product A and product

B during their shopping trip. They apply the well-known algorithm "Apriori", which was proposed by Agrawal and Srikant (1994) to discover the frequently purchased products.

Aloysius and Binu (2013) apply a data mining approach to investigate buying patterns in supermarkets and allocate products based on the discovered buying patterns. They propose a PrexSpan algorithm. The proposed algorithm has two steps. The first step is designed to discover the sequence of the customer product category purchases. The second step explores the most profitable products considering buying pattern. They show the effectiveness of their algorithm for small size datasets, which includes seven products in three product categories and five transactions. These small size datasets show that the PrexSpan algorithm promises good solutions to the product allocation problem in supermarkets.

Altuntas (2017) proposes a utility-based data mining algorithm to re-allocate product categories. The algorithm discovers strong and weak associations among product categories by evaluating the quantities and prices of purchased items. High utility product families define a large portion of the total profit. These families are selected and evaluated in this study. The algorithm does not provide allocation of products within the product categories. The author included a real case study from Turkey. The closeness ratings for product categories are calculated for the existing layout and the proposed layout. The possible revenue increase is calculated by multiplication of average prices of product categories and the average number purchased for each product category.

2.2.2. Optimization Models

Optimization models of the shelf-space allocation problem aim to maximize revenue from the limited shelf-space in the store. Space and cross space elasticity of each product category are the main aspects of these linear and nonlinear mathematical models. Space elasticity is a parameter to show the relationship between space and sales. Cross space elasticity refers to the sales reaction of two products when they are allocated next to each other. Along with nonlinear and integer programming models, efficient heuristic approaches have been proposed to exploit this relationship.

In the literature, researchers have proposed mixed integer nonlinear and linear mathematical models for the problem. Due to the demand function, most proposed optimization models are nonlinear. Mixed integer nonlinear programming models combine continuous and integer decision variables and nonlinear constraints or objective functions. The mixed integer programming models are combinatorial models, so there is no exact algorithm for large size problems. Finding optimum solutions is not easy for nonlinear models because the feasible region is not always convex. Heuristic approaches have been developed to solve the MINLP and MILP formulations. The proposed heuristics can be classified as greedy (Urban, 1998), branch-and-bound (Murray et al., 2010), genetic algorithm (Hwang, et al., 2005), tabu search (Yapicioglu, 2008), and other heuristic approaches (Yang, 2001, Hübner and Schaal, 2017).

The most well-known optimization model for the shelf-space allocation problem was proposed by Corstjens and Doyle (1981). Their nonlinear mathematical model for the problem considers operational costs and profits of products. Operational costs include carrying costs and out-of-stock costs. Profits are related to space elasticities of products, and cross-space elasticities among products. The parameters are explained in Table 2.1.

Table 2.1. Parameters of Corstjens and Doyle (1981)'s Optimization Model

K: Set of products $\boldsymbol{w_i}$: Sales price of product *i* $\boldsymbol{q_i}$: Demand for product *i* $\boldsymbol{\beta_i}$: Space elasticity for product *i* $\boldsymbol{\delta_{ij}}$: Cross space elasticity between product *i* and product *j* $\boldsymbol{\alpha_i}$: Demand multiplier for product *i*	$\boldsymbol{\gamma_i}$: Aggregate cost of product *i* $\boldsymbol{\tau_i}$: Operating cost elasticity of product *i* $\boldsymbol{S^*}$: Available shelf space $\boldsymbol{Q_i^*}$: Production availability limit for product *i* $\boldsymbol{s_i^L}$: Lower bounds on shelf space for product *i*$\boldsymbol{s_i^U}$: Upper bounds on shelf space for product *i*

The objective function of the model maximizes the total profit in the store considering the limited shelf space where the decision variables (s_i) represent the shelf space allocated to product *i*. The function $\alpha_i s_i^{\beta_i} \prod_{\substack{j=1 \\ j\neq i}}^{K} s_j^{\delta_{ij}}$ calculates the demand for each product. Their proposed nonlinear mathematical model is presented below.

$$Maximize \sum_{i=1}^{K} w_i \left(\alpha_i s_i^{\beta_i} \prod_{\substack{j=1 \\ j\neq i}}^{K} s_j^{\delta_{ij}} \right) - \sum_{i=1}^{K} \gamma_i \left(\alpha_i^{\tau_i} s_i^{\beta_i \tau_i} \prod_{\substack{j=1 \\ j\neq i}}^{K} s_j^{\delta_{ij}\tau_i} \right) \quad (2.1)$$

$$Subject\ to \qquad \sum_{i=1}^{K} s_i \leq S^* \quad (2.2)$$

$$\alpha_i s_i^{\beta_i} \prod_{\substack{j=1 \\ j \neq i}}^{K} s_j^{\delta_{ij}} \leq Q_i^* \qquad i = 1, \ldots, K \qquad (2.3)$$

$$s_i^L \leq s_i \leq s_i^U \qquad i = 1, \ldots, K \qquad (2.4)$$

$$s_i \geq 0 \qquad i = 1, \ldots, K \qquad (2.5)$$

Urban (1998) considers inventory decisions while solving the product allocation problem. The demand function of a product includes the shelf display inventory and the backroom inventory. The shelf display inventory is replenished until there is no backroom inventory. Prices of the products are pre-determined. The author provides a mixed integer nonlinear model for the multi-product shelf-space allocation problem. Greedy heuristic and genetic algorithms are proposed as solution methods to the problem. There are limitations of this study. The model would have been enriched with stochastic demand, promotion effects of products, and different inventory models.

Yang and Chen (1999) propose a simplified version of Corstjens and Doyle (1981)'s model and a multi-stage integer programming model for the shelf space allocation model. They assume that demand is linear if the number of product facings is between the lower and upper bounds. The continuous decision variable for the allocated space of product is altered with the integer decision variable of the number of product facings. A solution methodology in three steps provides good solutions in reasonable time. The problem is solved for departments, product categories, and products at each solution step.

Irion et al. (2012) propose mixed integer and mixed integer nonlinear models for the single category shelf space allocation problem with many products. The model includes in-store costs, and space and cross space elasticities. Their first model is nonlinear models because of the cross space elasticities among products. They apply a piecewise linearization technique to obtain a linear model. The mixed integer model allows solving realistic problem sizes close to optimally. The mixed integer nonlinear model allows solving the problems optimally, but it can only solve the small size problems.

Hariga et al. (2007) combine shelf space allocation, product assortment, and inventory control in one optimization model. The mixed integer nonlinear optimization model has decision variables for display locations (y_{ik}), display space allocation (s_{ik}), display quantity (q_{ik}) and inventory costs. They solve the model optimally in LINGO for four products at four locations. A heuristic model is necessary for realistically size problems.

Hwang et al. (2005) combine the multi-level shelf space allocation problem and inventory-control problem into one model for a single product category. The model seeks the optimum product allocation at multi-level shelves for products within a product category. The demand function considers the effects of location and inventory levels. Their proposed model is a nonlinear programming model, so they provide two heuristic approaches as the solution methodologies: a gradient search and a genetic algorithm.

Yapicioglu (2008) proposes mathematical models and a solution methodology to the allocation of departments in a racetrack layout. There are two main areas in the racetrack layout to allocate the departments: an inner region and an outer rim. Customers have access to departments when they follow the central pathway. The objective function of the model maximizes the revenue, which considers the allocated area and adjacency relationships among the departments. The author's model includes the length and width of a department where binary decision variables represent the locations. A constructive heuristic and tabu search are developed for the solution of the mixed integer nonlinear mathematical model.

Murray et al. (2010) aim to solve the shelf-space allocation problem for a single product category. They consider product prices, display facing areas, display orientations, and shelf-space locations for products within a product category in their mixed integer nonlinear mathematical model. Display facing areas are defined as the width and height of a product. Display orientations are added to the model to meet the three-dimensional area requirements. A branch-and-bound based heuristic algorithm is proposed for the solution of their model.

Geismar et al. (2014) decompose the two-dimensional shelf space allocation problem for multi-level shelves into two phases. In the first phase, they aim to choose the cabinets for multiple products. In the second phase, the goal is to determine the position of products within a display. They propose a mixed integer programming model for the problem of the first phase. A network representation is applied to the model of the second phase. Their solution approaches provide good solutions for actual and simulated data.

Hübner and Schaal (2017) propose a stochastic shelf-space allocation problem while considering the space and cross-space elasticities of products. They assume that demand is stochastic, and the vertical shelf position of a product affects the demand and sales. The objective function of the mixed integer linear model maximizes the shelf revenue. The decision variables (γ_{ikhp}) are defined as binary variables, which show the combination of facings (*k*), occupied shelf levels (*h*), and occupied uppermost shelf level (*p*) for each product (*i*). Heuristic algorithms are proposed for their mathematical model.

Düsterhöft et al. (2020) proposed optimization models for the shelf-space and replenishment problem. Their models provide solutions of optimal shelf quantities, an item's optimal shelf segment, and optimal frequencies of replenishments. The results were applied to a grocery retailer in Germany. The improvement in profits across the different categories and stores is stated as 9.1%.

Table 2.2. Optimization Models for the Shelf-Space Allocation Problem

Authors	Contribution	Formulation	Solution Methodology
Corstjens and Doyle (1981)	Shelf-space elasticity and cross-elasticities	Nonlinear programming model (NLP)	Dynamic programming
Urban (1998)	Inventory control and shelf-space allocation model	Mixed integer nonlinear programming model (MINLP)	Greedy heuristic and genetic algorithm
Yang and Chen (1999)	Linear profit function	Integer programming model (IP)	IP and multi-stage decision support system
Yang (2001)	Linear objective function	Linear programming model (LP)	IP and heuristic algorithm
Irion et al. (2012)	A linearized model	Linearized mixed integer and nonlinear programming model	Mixed integer programming
Hwang et al. (2005)	Effects of vertical locations and inventory control assumptions	Nonlinear objective function	Gradient search heuristic and genetic algorithm
Hariga et al. (2007)	Shelf space allocation, product assortment, and inventory control	MINLP	LINGO Optimization Package
Yapicioglu (2008)	Allocate departments to a racetrack layout	MINLP	Constructive heuristic and tabu search
Murray et al. (2010)	Product prices, display facing areas, display orientations, and	MINLP	Branch-and-bound based

	shelf-space locations		heuristic
Geismar et al. (2014)	2-dimensional displays: effects of vertical positions considering multiple shelves	MINLP and network formulation	IP and heuristic algorithm
Hübner and Schaal (2017)	Stochastic demand	MINLP model	Heuristic algorithms
Düsterhöft et al. (2020)	Shelf-space and replenishment problems	Nonlinear integer programming model	Optimal solutions
Our research	Temporary displays: space elasticity, cross space elasticity, impulse purchase, and traffic rate	MINLP for a single and two consecutive promotional periods	Optimal solutions by GAMS program

Table 2.2 summarizes the optimization models of the shelf-space allocation problem in the literature. Deterministic demand, space elasticity, cross-category elasticity, multi-level shelves, and inventory decisions are considered in various models. Most of the studies solve the problem as a single period shelf allocation problem. Analytic (Corstjens and Doyle, 1981, Yang and Chen, 1999, and Irion et al., 2012), heuristic (Yang, 2001, and Murray, 2010), and meta-heuristics approaches (Hwang et al., 2005, and Yapicioglu, 2008) exist in the literature. CPLEX Optimization software is often used for analytic approaches. Analytic approaches provide optimum solutions for small size problems while heuristic and meta-heuristic approaches generate good solutions for large size problems. Heuristic approaches are designed to find an optimum quickly, but the solution may be a local optimum. Meta-heuristic approaches are more complicated than greedy heuristics and have procedures to avoid local optima. Hansen et al. (2010) provide an extensive review of the solution approaches to the shelf space allocation problem.

Recently, Jil et al. (2023) proposed a mathematical programming approach for the shelf layout problem. They aimed to address the attention and relevancy of commodities, adjacency values and replacements, and provide analysis of the impact of different locations of commodities. The comprehensive related value-based commodity layout optimization model is proposed and the solutions are obtained from a genetic algorithm.

The retail store operations, customer purchase behaviors, the retail shelf-space allocation problem, and optimization models are discussed in this section. This dissertation aims to state practical model to the grocery retail environments. We contribute to the literature the shelf-space allocation literature by developing optimization models for the shelf space allocation problem at temporary displays at the product category level for a single promotional period and two consecutive promotional periods. Both mathematical models consider parameters of space elasticity, cross space elasticity, impulse purchases, and traffic rates with two assumptions. The first version assumes that each display contains only a single product category. The second version extends the previous assumption that multiple product categories may share a temporary display.

2.3. Customer Shopping Paths

In today's retail industry, understanding customer preferences and offering more purchasing opportunities to them are important factors to serve them better. Supermarkets commonly track customers' purchases using loyalty cards and sales data. The obtained data can be investigated by data mining techniques to identify their customer preferences. Since the obtained data explains where the customer has chosen the products, we need to examine customer shopping paths to design a better sales environment.

Tracking customers and simulation models are common approaches to identify customer paths. Customers can be tracked by staff or devices. Tracking customers by staff is very costly and time-consuming. Also, customers may not feel comfortable shopping with someone and may not show their real shopping behavior. RFID technology is newer technology to monitor customers in the store. Each customer is tracked by an RFID tag, which is attached to the customers shopping baskets. The installation of RFID tags on each shopping cart requires additional cost and labor.

The customer paths and clustering of customers in supermarkets have been studied by many researchers (see for example, Larson et al., 2005, Hui et al., 2009, Ballester et al., 2014). Larson et al. (2005) collected customers' purchasing and tracking data from 27,000 shoppers in a store by RFID tags in the U.S. The store is divided into zones of a racetrack, aisles, produce, convenience store (c-store), checkout and extremity. The products in the convenience store are high impulse purchase products, so customers can easily purchase them. The shopping durations at each zone were tracked and evaluated for clustering of shopping paths. They applied a multivariate clustering algorithm (K-medoids), which aims to minimize the distance between members in a cluster. They discovered fourteen most common customer shopping paths and three duration periods in the supermarket. The shopping duration periods are defined as low, medium, and high. They provide several observations about customer shopping paths. Many shoppers select aisles and travel within these aisles instead of traveling all aisles in the store. Customers tend to travel only along a portion of the aisle instead of passing along the entire aisle. They observe that the cluster of short trips includes more travel around the perimeter and in the convenience store area. Their analyses showed that the length of the shopping path and the duration of the shopping trip are positively correlated. The number of visited aisles in the low shopping duration is less than the number of visited aisles for the high shopping duration. These results are not surprising, of course.

Hui et al. (2009) investigated similarities between actual shopping trips and shortest paths of the purchased product categories. They collected the shopping trip data using RFID tags from a supermarket in the U.S. Each shopping trip data includes the purchased product categories and the observed shopping path. The observed shopping path shows the customer purchasing sequence and any additional travels. A customer purchasing sequence is called an observed order. The distances of the observed order are calculated based on the minimum distances between product categories. An observed shopping path is investigated as being comprised of three parts: the shortest path, order deviation, and travel deviation. The shortest path of the purchased product categories is defined based on the minimum distances between the purchased product categories. An order deviation of a shopping trip shows the difference between the observed order and the shortest path of the purchased product categories. In Figure 2.2, the shortest path is B-A-C and the total distance of the shortest path is 6. However, the customer follows the path of B-C-A and the total distance of the observed path is 8. The difference between the observed path and shortest path is calculated the order deviation and calculated as 2. Customers do not always follow the minimum paths and they have additional walking. A travel deviation of a shopping trip illustrates the additional walking without purchasing. In other words, the difference between the observed path and the minimum total distance of the observed order is called the travel deviation. In Figure 2.2, the shortest travel distance of the observed order is 8, and the observed travel is 11. The travel deviation is calculated as 3. Their results showed that the average order deviation was 3% greater than the shortest path and it is strongly associated with the basket size. The average travel deviation was 69% and had no strong correlation to the basket size.

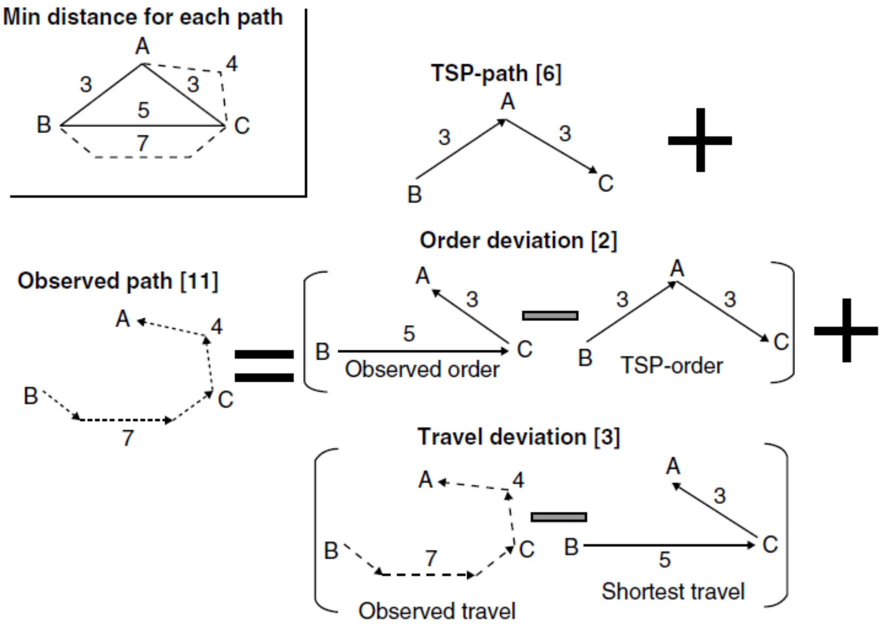
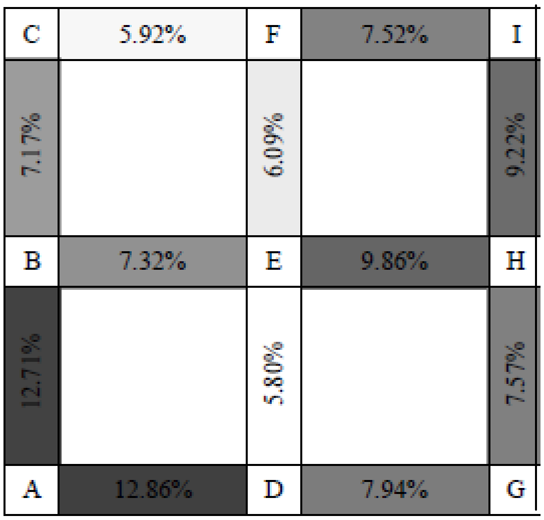
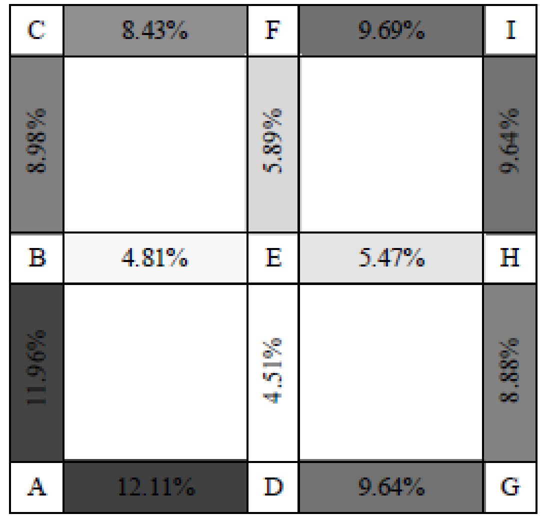


Figure 2.2. Examples of an Order Deviation and a Travel Deviation (Hui et al., 2009)

Ballester et al. (2014) investigated customer travel behavior and traffic density zones in a supermarket using a simulation model. They used real shopping data for the simulation model. They used a network representation for the store. Areas of the stores are shown by nodes while customer paths are shown by edges in the network representation. The number of purchased products in one trip, the locations of these products, the layout of the store, and all possible shortest paths for a customer are defined to calculate all possible travel paths. According to their initial solutions, they propose a balanced traffic density so that the most purchased products may not be located at the back of the store. Figure 2.3 illustrates the traffic densities for the current layout of the store and the layout of near-equally distributed products. Their idea is different than what is applied in supermarkets and generally proposed in the literature.



Node	% of Items Purchased
B	16.20%
C	6.10%
D	14.10%
E	17.94%
F	6.10%
G	7.32%
H	21.58%
I	10.66%



Node	% of Items Purchased
B	11.51%
C	13.10%
D	10.35%
E	12.20%
F	11.53%
G	14.10%
H	10.69%
I	16.51%

a) Approximation of the Real-World Layout b) Layout of Near-Equally Distributed Products

Figure 2.3. Traffic Densities of Different Layouts (Ballester et al., 2014)

Tsai et al. (2017) proposed a shopping behavior prediction system for supermarkets. The system has four modules: a database for customer movement and purchasing behavior, a behavior module for frequent shopping patterns, a similarity module for similar stores and products, and a behavior prediction module to create a list of shopping suggestions for customers. This is the first study in the literature that considers the quantities of purchased products to identify store and product similarities of items and stores.

Pfeiffer et al. (2020) investigated the customers' shopping motives in physical shops and virtual shopping environments by using eye-tracking applications. Machine learning techniques were applied to the classification of two customer shopping motives, which are the goal-oriented and explanatory search. Eye movements are analyzed to understand the customer search behavior, choices, and shopping motives.

In this section, we discussed the literature about customer shopping paths in-store environments. Identifying customer paths in stores help to analyze customers and design the store based on the customer traffic. Customer traffic rates will be used as parameters to calculate revenue in our mathematical models. Customer shopping paths in stores are still promising research area to study in the future.

Chapter 3 Parameters of the Mathematical Models

This research is the first study that evaluates the impulse purchase likelihoods of product categories and develops mathematical models for the temporary display allocation problem. In the retail literature, impulse purchase likelihoods of product categories have been studied by empirical methods (Kollat and Willet, 1967, Bell et al., 2013, Akyuz, 2018). However, it is difficult to know for a given store environment the impulse purchase likelihoods for the various product categories. This may also be influenced by time of year, by culture, by economic situation, and so on. The results from these studies indicated that impulse purchase likelihoods of product categories are low, medium, or high. Prices of products, the necessity of products, locations of products, promotions, and visibility of products all affect impulse purchases. Examining and utilizing these effects may result in an enhanced store environment to attract more purchases. On grocery shopping trips, customers have shopping lists before entering the store but, these customers usually purchase additional products. Impulse purchases in stores offer various research opportunities to understand customer purchase behavior. As mentioned in the literature, three levels of impulse purchase likelihood levels can be enhanced for better understanding the impulse sales in a store environment. We will use a value for the impulse purchase likelihood of each product category and this value will fall into one of three levels: low, medium, or high. Impulse purchase likelihoods from Bell et al. (2013), Hulten and Vanyushyn (2011), and store manager opinions are integrated for this research. In view of the inherent challenge in accurately forecasting the probability of impulse purchases within a given product category, our approach involves conducting a sensitivity analysis of the models resulting several superior temporary display allocations using the proposed mathematical models under different impulse purchase likelihood scenarios. In the base case, the store manager opinions lead the impulse purchase likelihoods by considering literature findings and the manager's professional experiences. In addition to the base case, a worst case, a median case, and a best case of impulse purchase likelihoods will be discussed. The mathematical results will demonstrate how the solution changes for different levels of impulse purchase likelihoods of product categories.

In this research, we developed mathematical models for the allocation of temporary displays at the product category level during a promotional period. The objective function of the proposed mathematical model is to maximize the sales at temporary displays by allocating promotional product categories. These sales are calculated by considering impulse purchase likelihoods, space elasticities, cross-category sales, and customer traffic rates. There are various studies about the impulse purchase likelihoods of product categories (Hodge, 2004, Bell et al., 2011, Hulten and Vanyushyn, 2011) and we used the results from the studies of Bell et al. (2013), Hulten and Vanyushyn (2011), and store managers' opinions. As a next step, we provided sensitivity analysis for impulse purchase likelihoods of product categories. We used the space elasticities of product categories in our mathematical models from Corstjens and Doyle (1981). Customer traffic rates and cross-category sales are calculated from actual data of the Yunus Store used as our case study. The revenue functions of the proposed mathematical models consider both direct sales of the total allocated areas of product categories and cross-category sales between the adjacent product categories.

In this chapter, we explain the parameters that are required for the proposed mathematical models contained in later chapters. These parameters are impulse purchase likelihoods at the product category level, product category allocation areas, the revenue function, space elasticities at the product category level, and customer traffic rates. Possible research directions concerning the parameters are discussed at the end of Chapter 3.

3.1. Impulse Purchase Likelihoods

Impulse sales are vital components of sales in supermarkets (Memon et al., 2019). Kollat and Willet (1967) stated the importance of the relationship between the impulse purchase likelihood of products and customer purchases. Stern (1962) and Hulten and Vanyushyn (2011) investigated the discounted price effect on impulse purchases. In the literature, impulse purchase effects on store sales have been discussed by many researchers (e.g., Stern, 1962; Desmet and Renaudin, 1998; Bell et al., 2013; and Hulten and Vanyushyn, 2001, Akyuz, 2018). Bell et al. (2013) provided an extensive study about impulse purchases at the product category level. They collected data from 21 stores in Europe including 18,000 purchases and 58 product categories. Akyuz (2018) explored the factors of impulse purchase behavior in Turkish supermarkets and concluded that impulse purchases and sales promotion, credit card usage, and shopping moods are positively correlated.

Impulse purchases are crucial to estimate the sales at temporary displays in grocery stores. Figure 3.1 illustrates factors for unplanned purchases as taken from the study of Bell et al. (2011). These factors are overall shopping trip goal, store-specific goal, the interaction between out-of-store exposure and use of in-store marketing materials, out-of-store control variables, in-store variables, and exposure variables. In the literature, there is no model to estimate impulse purchase likelihoods of product categories based on customer profile and product features.

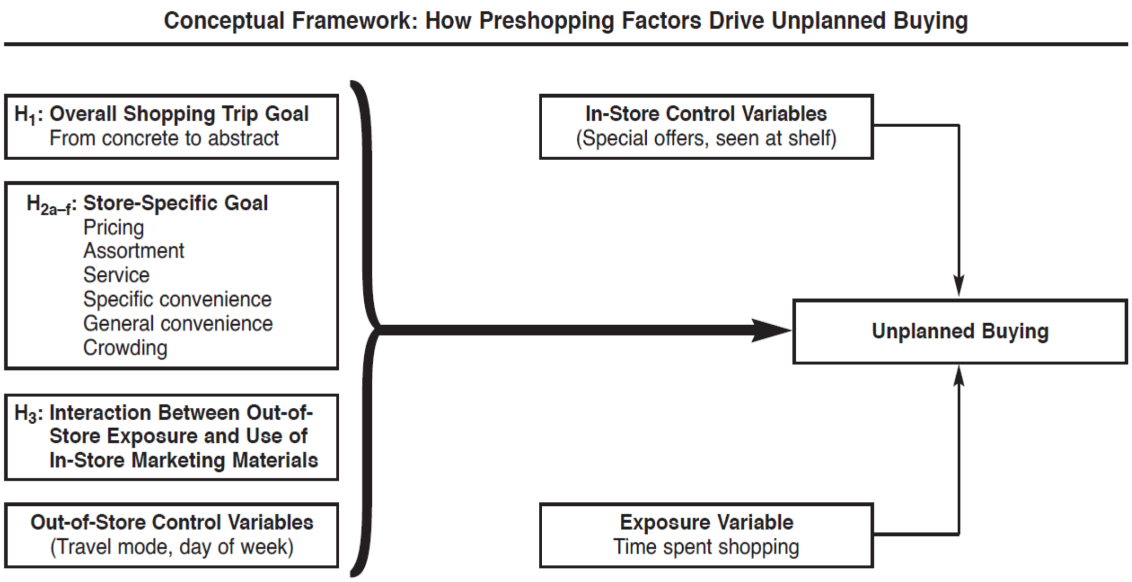


Figure 3.1. Conceptual Framework: How Pre-shopping Factors Drive Unplanned Buying (Bell et al., 2011)

In this dissertation, the literature findings and store managers' opinions are used to estimate the impulse purchase likelihood of product categories of the Yunus Store. Table 3.1 shows the impulse purchase likelihoods of product categories (Ratio A) based on the literature findings and store managers' opinions. The most detailed studies about impulse purchases at the product category level in grocery stores were done by Bell et al. (2013) and Hulten and Vanyushyn (2011). If the impulse purchase likelihood of a product category cited from the literature was not appropriate for this store, the store manager replaced it with a suitable one. If the store manager agreed that the impulse purchase likelihood from the literature fits the store, then it is used. The store manager has been working in this store in Aksaray for more than 10 years. According to monthly sales in August 2021, the store in Aksaray is ranked within the top 10 stores among 98 stores spread over 11 cities for the Yunus store chain.

Table 3.1 Impulse Purchase Likelihood of Product Categories

The Source of Impulse Purchase Likelihoods	Product Category	Impulse Purchase Likelihood of a Product Category (Ratio A)
Bell et al. (2013)	Dairy	0.14
	Bakery	0.16
	Cleaning Supplies	0.17
	Coffees	0.16
	Fish	0.17
	Frozen Food	0.27
	Meat2	0.17
	Dry Beans	0.21
	Fruits and Vegetables	0.17
	Cosmetics	0.23
	Perfumery	0.23
Hulten and Vanyushyn (2011)	Meat1	0.12
	Bread	0.06
	Clothes	0.16
	Textile	0.16
Store Manager	Cigarettes	0.05
	General	0.10
	Middle Section	0.10
	Glassware	0.20
	School Supplies	0.20
	Dried Fruits and Nuts	0.20
	Toys	0.25
	Shoes	0.25
	Spices	0.25
	Electronics	0.30

The impulse purchase likelihoods of the product categories of bakery, cleaning supplies, cleaning supplies, coffees, dairy, dried beans, fish, frozen food, meat1, and meat 2 are adopted from Bell et al. (2013). The impulse purchase likelihoods of product categories of clothes, cosmetics, fruits and vegetables, perfumery, and textile are adopted from Hulten and Vanyushyn (2011). The impulse purchase likelihoods of cigarettes, dried fruits and nuts, electronics, general, glassware, middle section, school supplies, shoes, spices, and toys are determined by the store manager. The studies of Bell et al. (2013) and Hulten and Vanyushyn (2011) investigated the impulse purchase likelihood of electronics and they stated the impulse purchase likelihood of electronics is 0.30. The lowest impulse purchase likelihood from the literature is 0.14 for dairy by Bell et al. (2013). However, the store manager states that the lowest impulse purchase likelihood is 0.05 for cigarettes. Table 3.1 shows the impulse purchase likelihoods of product categories that are adopted from Bell et al. (2013), Hulten and Vanyushyn (2011), and store manager.

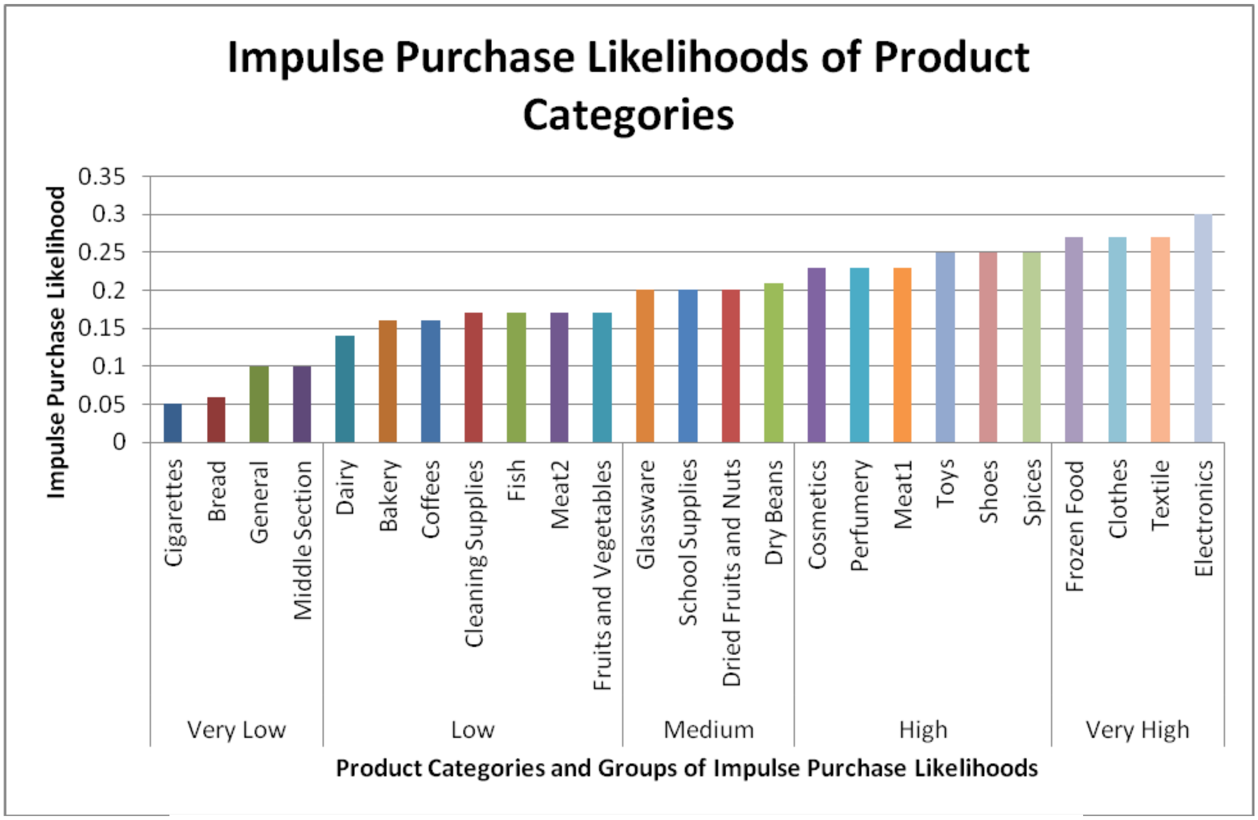


Figure 3.2. Sorted Impulse Purchase Likelihoods of Product Categories

Figure 3.2 illustrates the groups of product categories with the ranking of impulse purchase likelihoods that we obtained from the literature and store manager's opinion along with five groups. The highest impulse purchase likelihood is cited as electronics by the store manager.

Because these impulse purchase likelihoods are imprecise, we decided to investigate the sensitivity of changes to them using three scenarios. The literature findings and store managers' opinions about impulse purchase likelihoods possible differences are listed in Table 3.2. The product categories in the store are determined by the store chain management so not all impulse purchase likelihoods of product categories are cited from the literature. Hulten and Vanyushyn (2011) cited seven product categories from two stores data in France and Sweden. There are thirteen product categories listed in the study of Bell et al. (2013). The store manager assigned impulse purchase likelihoods to all product categories. The minimum, median, and maximum values of impulse purchase likelihoods of a product category from the literature and also assigned by the store manager are used for the worst, median, and best cases. The minimum value of impulse purchase likelihoods in each product category is the Worst Case Scenario. Similarly, the median value of impulse purchase likelihoods in each product category is the Median Case Scenario. Lastly, the maximum value of impulse purchase likelihoods in each product category is the Best Case. The highest impulse purchase likelihood of all product categories is stated as 0.30 with the product category of electronics by the store manager. According to the findings, the worst, median, and best scenario values of the impulse purchase likelihood of electronics are 0.04, 0.08, and 0.30. If the impulse purchase likelihood of a product category is only listed by the store manager, we use the same value for the product category in all cases. The mathematical models in Chapters 4 and 5 are solved by using these scenarios to see the changes in the temporary display allocations with different impulse purchase likelihood levels.

Table 3.2. Scenarios of Impulse Purchase Likelihoods

Product Categories	Store Managers' Opinions	Bell et al. (2013)	Hulten and Vanyushyn (2011) France	Hulten and Vanyushyn (2011) Sweden	Worst Case Scenario	Median Case Scenario	Best Case Scenario
Bakery	0.15	0.16	N/A	N/A	0.15	0.16	0.16
Bread	0.15	0.14	0.06	0.01	0.01	0.10	0.15
Cigarettes	0.05	N/A	N/A	N/A	0.05	0.05	0.05
Cleaning Supplies	0.20	0.17	N/A	N/A	0.17	0.19	0.20
Clothes	0.15	0.17	0.07	0.16	0.07	0.16	0.17
Coffees	0.15	0.16	N/A	N/A	0.15	0.16	0.16
Cosmetics	0.20	0.23	N/A	N/A	0.20	0.22	0.23
Dairy	0.15	0.14	0.08	0.09	0.08	0.12	0.15
Dried Fruits and Nuts	0.20	N/A	N/A	N/A	0.20	0.20	0.20

Dry Beans	0.20	0.21	N/A	N/A	0.20	0.21	0.21
Electronics	0.30	N/A	0.04	0.08	0.04	0.08	0.30
Fish	0.15	0.17	N/A	N/A	0.15	0.16	0.17
Frozen Food	0.25	0.27	N/A	N/A	0.25	0.26	0.27
Fruits and Vegetables	0.20	0.17	N/A	N/A	0.17	0.19	0.20
General	0.10	N/A	N/A	N/A	0.10	0.10	0.10
Glassware	0.20	N/A	N/A	N/A	0.20	0.20	0.20
Meat1	0.10	N/A	0.06	0.12	0.06	0.07	0.12
Meat2	0.15	0.17	N/A	N/A	0.15	0.16	0.17
Middle Section	0.10	N/A	N/A	N/A	0.10	0.10	0.10
Perfumery	0.25	0.23	0.06	0.07	0.06	0.15	0.25
School Supplies	0.20	N/A	N/A	N/A	0.20	0.20	0.20
Shoes	0.25	N/A	N/A	N/A	0.25	0.25	0.25
Spices	0.25	N/A	N/A	N/A	0.25	0.25	0.25
Textile	0.15	0.17	0.07	0.16	0.07	0.16	0.17
Toys	0.25	N/A	N/A	N/A	0.25	0.25	0.25

The impulse purchase rates of product categories in Table 3.2 are multiplied by total sales of product categories to calculate product categories' total impulse sales in Table 3.3. Total impulse sales are 13%-16% of total sales in the four scenarios (Table 3.3) the baseline, the worst, the median, and the best. In our proposed models, we allocate promotional product categories to temporary displays, so the total sales at temporary displays are related to the promotional products during the promotional period and their impulse purchase likelihoods.

Table 3.3. Impulse Sales (TL) for Alternative Values of Impulse Purchase Likelihoods – Sales per month

Product Category	Total Sales (TL)	Impulse Sales for Ratio A (Total Sales x Ratio A) (TL)	The Worst Case Scenario (Total Sales x Ratio B) (TL)	The Median Case Scenario (Total Sales x Ratio C) (TL)	The Best Case Scenario (Total Sales x Ratio D) (TL)
Bakery	3293.64	526.98	494.05	526.98	526.98
Bread	30676.14	1840.57	306.76	3067.61	4601.42
Cigarettes	93850.69	4692.53	4692.53	4692.53	4692.53
Cleaning Supplies	171159.20	29097.06	29097.06	32520.25	34231.84
Clothes	70.45	11.27	4.93	11.27	11.98
Coffees	10860.39	1737.66	1629.06	1737.66	1737.66
Cosmetics	18621.63	4282.97	3724.33	4096.76	4282.97
Dairy	126557.10	17717.99	10124.57	15186.85	18983.57
Dried Fruits& Nuts	59798.95	11959.79	11959.79	11959.79	11959.79
Dry Beans	14712.59	3089.64	2942.52	3089.64	3089.64
Electronics	1910.27	573.08	76.41	152.82	573.08
Fish	24443.52	4155.40	3666.53	3910.96	4155.40
Frozen Food	32127.60	8674.45	8031.90	8353.18	8674.45
Fruits and Vegetables	175189.60	29782.23	29782.23	33286.02	35037.92
General	145.06	14.51	14.51	14.51	14.51
Glassware	39909.57	7981.91	7981.91	7981.91	7981.91
Meat1	160065.80	19207.90	9603.95	11204.61	19207.90
Meat2	175332.80	29806.58	26299.92	28053.25	29806.58
Middle Section	287000.10	28700.01	28700.01	28700.01	28700.01
Perfumery	23866.41	5489.27	1431.98	3579.96	5966.60
School Supplies	1350.94	270.19	270.19	270.19	270.19
Shoes	1570.30	392.58	392.58	392.58	392.58

Spices	4841.23	1210.31	1210.31	1210.31	1210.31
Textile	14195.55	2271.29	993.69	2271.29	2413.24
Toys	10061.52	2515.38	2515.38	2515.38	2515.38
Total Sales	1481611.00	216001.56	185947.09	208786.33	231038.44
Ratio of Impulse Sales/Total Sales	-	**0.15**	**0.13**	**0.14**	**0.16**

There are research opportunities concerning impulse purchases in stores. To construct customized models for impulse purchases in each store of a grocery chain, research questionnaires for impulse purchases could be created and customers queried. Regression models and artificial neural network models were applied to actual sales data to forecast grocery sales in the study of Yeasmin et al. (2022). Similarly, these models would be another approach to estimate the impulse purchases of product categories. Historical grocery sales could be used as the input and output values in these models. The patterns of impulse purchases for the data set could potentially be discovered by these models.

3.2. Product Category Display Areas

For the mathematical models, the sizes (area) of displays for each product category need to be assigned. In the literature, Corstjens and Doyle (1981) proposed a model for shelf space allocation at the product level. The objective function of their model maximizes the total revenue of product categories (Equation 3.1). The parameters of the objective function are the unit revenue of a product (r_i), the allocated shelf length (s_i), the space elasticity (β_i), and the cross space elasticities (δ_{ij}). In our model we consider total revenue much like their definition.

$$Maximize \ r_i s_i^{\beta_i} \prod_{\substack{j=1 \\ i \neq j}}^{K} s_j^{\delta_{ij}} \qquad (3.1)$$

Yang (2001) enriched Corstjens and Doyle (1981)'s model by adding upper (s_i^U) and lower (s_i^L) bounds on the shelf space (Equation 3.2). The total revenues of product categories are calculated by Equation 3.3.

$$s_i^L \leq s_i \leq s_i^U \tag{3.2}$$

$$R_i = r_i s_i^L + r_i (s_i - s_i^L)^{\beta_i} \tag{3.3}$$

Yapicioglu (2012) addressed the retail store layout problem for a racetrack aisle with a mathematical model. The objective function is defined to maximize the revenue of a retail store. Total revenue has two parts: the adjacency coefficient of product categories and the revenues of product categories. A REL chart (Muther, 1973) is used to explain the desired adjacency relationship between the categories this is used to estimate the affinity of facilities in terms of proximity in the work of (Yapicioglu, 2008). The traditional REL ratings and scores are shown in Table 3.4 (Muther, 1973).

Table 3.4. REL Ratings and Scores (Muther, 1973)

Rating	Definition	C_{ij}
A	Absolutely Necessary	125
E	Especially Important	25
I	Important	5
O, U	Ordinary Closeness	1
X	Undesirable	-25
XX	Prohibited	125

The adjacency coefficient is calculated by using Equation 3.4, where c_{ij}^{+} represents a positive REL score and c_{ij}^{-} represents a negative REL score (Yapicioglu, 2012).

$$\varepsilon = \frac{REL^p}{REL^*} = \frac{\sum_{i=1}^{n} \sum_{j>i}^{n} c_{ij}^{+} x_{ij} - \sum_{i=1}^{n} \sum_{j>i}^{n} c_{ij}^{-} (1 - x_{ij})}{\sum_{i=1}^{n} \sum_{j>i}^{n} c_{ij}^{+} x_{ij} - \sum_{i=1}^{n} \sum_{j>i}^{n} c_{ij}^{-} x_{ij}} \tag{3.4}$$

$$R_i = \frac{\bar{R}_i}{1 + \sum_{k=1}^{K} y_{ik} [\max\ (0, z_k - d_i)]} \tag{3.5}$$

The revenue of a product category is presented in Equation 3.5, where $\bar{R}_i$ is the upper revenue bound of product category *i*, z_k is the traffic density of display *k*, and d_i is the impulse purchase likelihood of product category *i*. x_{ij} is a binary decision variable and it is equal to one when product categories *i* and *j* are adjacent. y_{ik} is a binary decision variable, which shows that product category *i* is in zone *k* when it is equal to 1. The traffic density rates and impulse purchase likelihoods are evaluated at three levels: low, medium, and high as adapted from Yapicioglu (2008). In our revenue function, the upper revenue bound for each product category is estimated by the store managers considering the stock levels and promotional agreements.

These upper revenue bounds are used to calculate the revenue of product categories per square meter and the total revenue of the allocated area of product categories, which is different than the revenue function of Yapicioglu (2008). The revenue of the product categories cannot be greater than the products in the store's stock levels. Furthermore, the store has agreements for promotional products, and they cannot order more once the promotional period starts. If there are no upper revenue bounds, the objective function will maximize the total revenue without any boundaries and can create unrealistic results. The revenue function of our proposed models is described in Section 3.3.

In the proposed mathematical models, the goal is to solve the allocation problem of temporary displays at the product category level for a promotional period under the following assumptions:

- We assume that product categories are permanently assigned to at least one location in the store.
- We assume that the available areas of temporary displays are different from each other and determined by the store manager.
- We assume that the candidate product categories for temporary displays are chosen by the store manager for each promotional period. The product categories to allocate to temporary displays are usually discounted products, seasonal products, and/or new products.

There are two types of decision variables in the proposed mathematical models. Binary decision variables are defined to represent the product categories in temporary displays (Equation 3.6). Continuous decision variables between 0 and 1 are defined to decide the proportion of the temporary display (Equation 3.7).

For example, if a temporary display is allocated to only promotional products of cosmetics, its continuous decision variable (Y_{ik}) will be 1. If there are promotional products from toys and cosmetics together in a temporary display, the continuous decision variable will show the ratio of these product categories. For example, $Y_{ik} = 0.6$ for toys and $Y_{ik} = 0.4$ for cosmetics. Some proposed models have additional decision variables, and these are explained in Chapters 4 and 5.

$$X_{ik} = \begin{cases} 1, & if\ product\ category\ i\ is\ assigned\ to\ temporary\ display\ k \\ 0, & Otherwise \end{cases} \quad (3.6)$$

$$Y_{ik}: Proportion\ of\ product\ category\ i\ at\ temporary\ display\ k \quad (3.7)$$

We construct our objective function based on the objective functions of Corstjens and Doyle (1981) and Yapicioglu (2008). We maximize revenue by choosing product categories for temporary displays by considering space elasticity, cross-category sales, traffic density, impulse purchase likelihoods, and adjacencies between product categories. The objective functions and constraints of each proposed model will are in Chapters 4 and 5.

The common parameters of the proposed mathematical models are listed in Table 3.5. There are sets for product categories (C), temporary displays (D), and adjacent product categories of displays (V_k). The set of V_k is defined to represent adjacent product categories for a temporary display *k*. In the traditional facility layout problem, two facilities are adjacent if they have a shared portion of a boundary. In our problem, we use the same adjacency definition for temporary displays to calculate cross-category sales. A detailed explanation of what constitutes adjacent in our work is given below.

Table 3.5. The Mutual Sets and Parameters of the Proposed Mathematical Models

Sets	**C:** Set of product categories **D:** Set of temporary display areas V_k**:** Set of adjacent product categories of display *k*
Parameters	d_i**:** Impulse purchase likelihood of product category *i* e_{ij}**:** Cross-category sale effect between product category *i* and product category *j* (ratio) R_{ik}**:** Revenue of allocating product category *i* to temporary display *k* s_i^L**:** Lower area limit for product category *i* s_i^U**:** Upper area limit for product category *i* t_k**:** Total area of temporary display *k* z_k**:** Traffic density rate of display *k* β_i**:** Space elasticity of product category *i*

Adjacent displays of temporary displays can be permanent displays or a combination of permanent and temporary displays. The product categories at adjacent displays are considered adjacent to a temporary display.

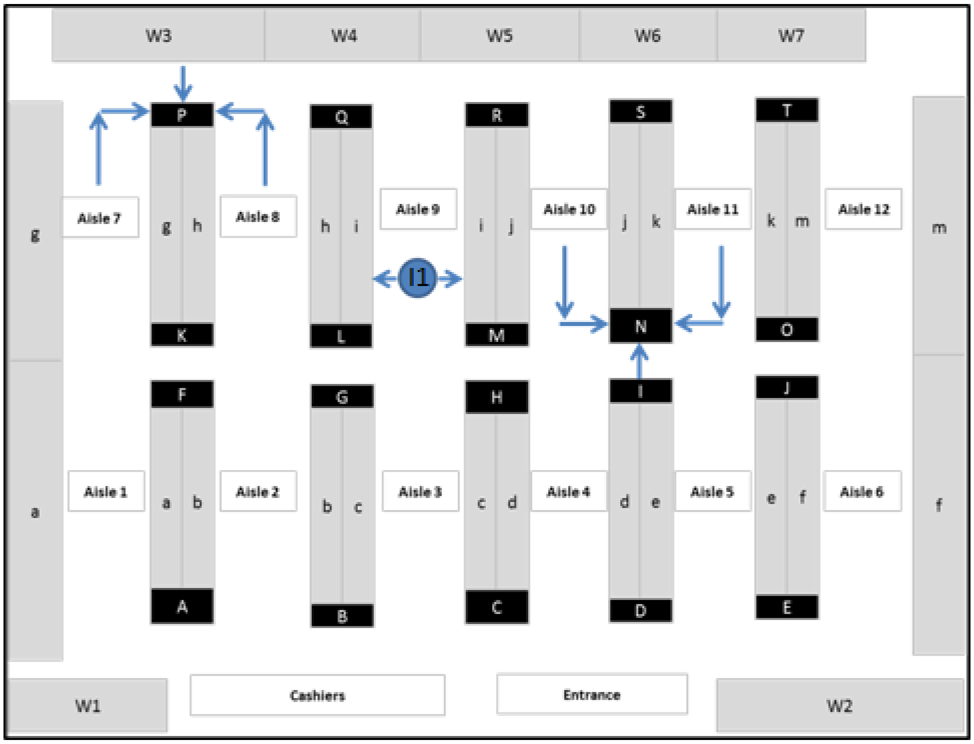


Figure 3.3. Examples of Adjacent Displays for Three Temporary Displays

Figure 3.3 illustrates adjacent displays for three temporary displays: an island display in aisle 9 (I1), an endcap display (*P*) at the end of aisles 7 and 8, and an endcap display (*N*) at the beginning of aisles 10 and 11. The adjacent product categories of the island display are the permanently allocated product categories within aisle 9. Endcap display *N* has common boundaries with aisles 10 and 11, and endcap display *I*. Endcap display *P* shares boundaries with aisles 7 and 8, and the outside of the racetrack. To define the adjacent displays for the endcaps at the back, the edge of the store is considered in multiple segments. The boundaries of these segments are determined based on the closest endcaps. For instance, there are five segments at the back of the store (W3, W4, W5, W6, and W7). The boundaries of the segment W3 include endcap display *P* and the boundaries of the segment of W7 include endcap display *T*.

Each temporary display has a limited area to display products and t_k stands for the total allocated area of temporary displays. We use area (height x length (or width)) for parameter t_k since temporary displays may have different height and length. We assume depth for each display is identical. Each product category must be allocated within the product category's total area limits. The lower area and upper area limits of product categories are represented by s_i^L and s_i^U in the proposed mathematical models.

3.3. Revenue Function

The revenue of a product category at a temporary display has two main components: revenue of the product category based on the allocated area and the adjacency scores of the nearby product categories. The area of a temporary display is limited, so the product category revenue from the display must be limited. Additionally, the stock availability of each product category is determined by store managers and the maximum revenue cannot be higher than the available stock levels (that is the ordered levels).

To calculate the revenue of a product category based on the total allocated area, the upper revenue bound ($\bar{R}_i$) of a product category is defined as the maximum expected revenue for a product category based on the store manager's order of the product. The store manager orders promotional products before the promotional period starts by evaluating the maximum allocation area, promotional activities, stock availabilities, and seasonality effects to determine the amount to order. In the Yunus store, there are no "rain checks". In other words, the store can only sell what has been ordered.

The promotional periods usually spans 14 days. However, if the promotional period starts mid-week, then it may extend to 18 to 19 days. In the proposed mathematical models, the parameter of r_i is defined as the average revenue of product category *i* per square meter for the promotional period, as shown in Equation 3.8 (without considering cross-category effects). The upper revenue bound $\bar{R}_i$ is divided by the upper area limits of product category *i*, s_i^u.

$$r_i = \frac{\bar{R}_i}{s_i^u} \qquad \text{Equation 3.8}$$

3.3.1. Space Elasticities

The space elasticities of product categories are designed to explain the relationship between the assigned area and sales (Corstjens and Doyle, 1981). Space elasticities are shown as the exponent (β_i) of the assigned area for a product category ($Total\ allocated\ area\ of\ product\ category\ i^{\beta_i}$) in the objective functions of the proposed mathematical models. In the literature, the average shelf-space elasticity at the product level was determined as 0.21 by Curhan (1972) and later empirical estimation of space elasticity at the product category level was done by Desmet and Renaudin (1998). The space elasticities for product categories of different store types were estimated by their studies (Table 3.6) as shown below. Store types were categorized based on their respective store areas and the average turnover per square meters. The standard store area and average turnover per square meters were established as reference values of 100. In essential stores, the average surface index is 64, while the average turnover per square meter is 77. Conversely, in plus stores, these values rise to 135 and 164 respectively. Based on their study, the categories of costume jewelry and fruits/vegetables have the highest space elasticity values. Note occasional negative space elasticities this is unusual but can happen due to the shelf arrangement and dilution of the focus of customers.

Table 3.6. Space Elasticities for Each Product Category and Store Type (Desmet and Renaudin, 1998)

	Plus Stores	**Standard Stores**	**Essential Stores**
03 Underwear	0.67	**0.55**	0.8
05 Costume Jewelry	0.68	**0.57**	0.44
12 Ladies' wear	0.13	**0.01**	-0.44
14 Shoes	0.46	**0.5**	0.12
15 Baby clothes	0.12	**0.03**	0.14
16 Children' wear	0.04	**-0.1**	0.24
17 Men’s wear	0.06	**0.05**	0.3
22 Do-it-yourself	0.08	**0.02**	0.02
28 Table	0.11	**0.16**	0.06
30 Kitchen	0.03	**0.06**	-0.1
32 Home	0.1	**0.11**	0.26
36 Household linen	0.15	**0.16**	0.41
42 Perfumery	0.5	**0.39**	0.57
25 Recording/Books	0.13	**0.18**	0.53
63 Household	0.15	**0.17**	-0.1
61 Grocery	0.29	**0.22**	-0.11
67 Drinks	0.39	**0.39**	0.21
73 Frozen food	0.1	**0.17**	0.1
79 Dairy	0.23	**0.23**	0.11
83 Delicatessen	0.25	**0.22**	0.16
84 Fruit/Vegetables	0.58	**0.57**	0.56
86 Meat	0.39	**0.33**	0.46

A discussion with the Yunus store manager revealed that space elasticities of the study of Desmet and Renaudin (1994) are appropriate for the Yunus store. If a product category of the Yunus store is not included in the study of Desmet and Renaudin (1998), the store manager’s opinion is taken as the space elasticity parameter. Table 3.7 shows space elasticity values that used in the proposed mathematical models.

Table 3.7. Space Elasticities of Product Categories

Number	Product Category	Space Elasticity	Data Source
1	Bakery	0.22	Desmet and Renaudin (1994)
2	Bread	0.10	Store Manager
3	Cigarettes	0.01	Store Manager
4	Cleaning Supplies	0.17	Store Manager
5	Clothes	0.03	Desmet and Renaudin (1994)
6	Coffees	0.22	Store Manager
7	Cosmetics	0.39	Desmet and Renaudin (1994)
8	Dairy	0.23	Desmet and Renaudin (1994)
9	Dried Fruits and Nuts	0.22	Store Manager
10	Dry Beans	0.22	Store Manager
11	Electronics	0.11	Store Manager
12	Fish	0.18	Store Manager
13	Frozen Food	0.17	Desmet and Renaudin (1994)
14	Fruits and Vegetables	0.57	Desmet and Renaudin (1994)
15	General	0.16	Store Manager
16	Glassware	0.11	Store Manager
17	Meat1	0.33	Desmet and Renaudin (1994)
18	Meat2	0.22	Store Manager
19	Middle Section	0.22	Store Manager
20	Perfumery	0.39	Desmet and Renaudin (1994)
21	School Supplies	0.18	Desmet and Renaudin (1994)
22	Shoes	0.50	Desmet and Renaudin (1994)
23	Spices	0.22	Store Manager
24	Textile	0.10	Store Manager
25	Toys	0.20	Store Manager

3.3.2. Cross-category Sales

A customer purchases several products from multiple product categories during a single shopping trip. Even if a customer plans to purchase only one product, it is possible to make an impulse purchase from another product category. The sale effect of a product category on other product categories is defined as cross-category sales by Corstjens and Doyle (1981). Irion et al. (2012) proposed a linear Mixed Integer Programming (MIP) model for allocating shelf space for products within a product category. Cross space elasticities are considered at the product level in their model.

Effects of cross-category sales among product categories are shown by e_{ij} in the proposed mathematical models. This parameter is generally considered in the decision of the locations of product categories in the store. High cross-category rates should be allocated next to each other. For instance, the product categories of beverages and chips can be seen at adjacent aisles in stores. In some cases, product categories with high cross-category sales are allocated far apart from each other, so the store layout encourages the customers to explore the store. For instance, the product categories of fruits/vegetables and bakery have high cross-category sales. Many stores allocate one of them at the entrance while the other at the back of the store. In the temporary display allocation problem, we aim to stimulate high cross-category sales between the temporary and permanent displays during the promotional period and use a data mining approach to achieve this.

One month of data consists of 63,661 customers with 1,622,709 individual product purchases. In a shopping trip, if a customer purchases at least one product from product category *i* and product category *j*, the cross-category sale occurrence between product categories *i* and *j* is increased by one. Then, cross-category occurrences are divided by the total number of product category purchases. Table 3.8 shows the cross-category sale effects between product categories. The product category names can be seen in Table 3.7.

Table 3.8. Effects of Cross-Category Sales between Product Category *i* and Product Category *j*

		Product Category No																								
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Product Category No	1	0	0.1692	0.0314	0.0634	0.0001	0.0079	0.0075	0.1111	0.036	0.013	0.0007	0.0079	0.0216	0.1469	0.0001	0.021	0.0398	0.0977	0.171	0.0124	0.0009	0.0013	0.0051	0.0082	0.0026
	2	0.0203	0	0.0477	0.0738	0.0001	0.0129	0.0096	0.1312	0.0395	0.0143	0.0006	0.0112	0.0244	0.1722	0.0001	0.0226	0.056	0.1056	0.1944	0.0131	0.0012	0.0009	0.0083	0.0068	0.0027
	3	0.0083	0.1056	0	0.0748	0.0001	0.0188	0.0127	0.0937	0.0335	0.0128	0.0005	0.0056	0.0189	0.1341	0.0001	0.0226	0.0426	0.075	0.1878	0.0165	0.0012	0.0011	0.0065	0.0078	0.0038
	4	0.0089	0.0868	0.0397	0	0.0001	0.0173	0.0194	0.1115	0.0386	0.0192	0.0007	0.0098	0.0248	0.1589	0.0001	0.036	0.053	0.0911	0.1878	0.0277	0.0017	0.0015	0.0101	0.0124	0.0039
	5	0.0001	0.0244	0.0001	0.0732	0	0.0001	0.0001	0.0732	0.0488	0.0001	0.0001	0.0244	0.0244	0.1707	0.0001	0.122	0.0001	0.0976	0.0976	0.0001	0.0001	0.0001	0.0001	0.1707	0.0244
	6	0.0058	0.0791	0.0518	0.0897	0.0001	0	0.0146	0.1088	0.0426	0.022	0.0008	0.0066	0.026	0.1225	0.0001	0.0277	0.0387	0.0842	0.2035	0.0213	0.0021	0.0013	0.0084	0.0079	0.0036
	7	0.0066	0.0706	0.0422	0.1217	0.0001	0.0176	0	0.093	0.0355	0.0148	0.0011	0.0085	0.0221	0.132	0.0002	0.0358	0.0387	0.0732	0.1631	0.0553	0.0032	0.0023	0.0069	0.0155	0.0037
	8	0.0119	0.1169	0.0377	0.0845	0.0001	0.0159	0.0113	0	0.0462	0.0186	0.0006	0.0072	0.0283	0.1625	0.0001	0.024	0.0544	0.1093	0.2064	0.0157	0.0015	0.001	0.0095	0.0075	0.0035
	9	0.01	0.0921	0.0352	0.0764	0.0001	0.0162	0.0112	0.1206	0	0.0183	0.0007	0.0085	0.0272	0.1554	0.0001	0.0226	0.0464	0.0929	0.1941	0.0161	0.0012	0.001	0.0096	0.0086	0.0027
	10	0.0086	0.0797	0.0321	0.0909	0.0001	0.02	0.0112	0.1158	0.0437	0	0.001	0.0078	0.0225	0.1473	0.0001	0.0251	0.0675	0.0926	0.1725	0.0183	0.0021	0.0011	0.0151	0.0092	0.003
	11	0.0109	0.0734	0.0272	0.0761	0.0001	0.0163	0.019	0.0788	0.0353	0.0217	0	0.0136	0.0163	0.1196	0.0001	0.1005	0.0299	0.0707	0.1793	0.0163	0.0054	0.0027	0.0054	0.0299	0.0082
	12	0.0095	0.1132	0.0256	0.0844	0.0002	0.0108	0.0117	0.0816	0.0367	0.0141	0.0011	0	0.0203	0.2275	0.0001	0.0217	0.0371	0.0836	0.1603	0.0108	0.002	0.0018	0.008	0.0108	0.0044
	13	0.0098	0.0924	0.0323	0.0799	0.0001	0.0161	0.0114	0.1201	0.0442	0.0153	0.0005	0.0076	0	0.14	0.0001	0.0283	0.0507	0.0865	0.1929	0.0159	0.0019	0.0012	0.0073	0.009	0.0035
	14	0.0117	0.1144	0.0402	0.0897	0.0001	0.0133	0.0119	0.1211	0.0443	0.0176	0.0006	0.015	0.0246	0	0.0001	0.0259	0.065	0.1038	0.1923	0.0151	0.0014	0.0011	0.009	0.0081	0.0035
	15	0.0001	0.0714	0.0001	0.1429	0.0001	0.0001	0.0357	0.1071	0.0357	0.0357	0.0001	0.0001	0.0001	0.1429	0	0.0714	0.0001	0.1071	0.1429	0.0714	0.0001	0.0001	0.0001	0.0001	0.0001
	16	0.0085	0.0761	0.0344	0.1033	0.0004	0.0153	0.0164	0.0909	0.0328	0.0152	0.0027	0.0072	0.0252	0.1316	0.0001	0	0.0462	0.078	0.1869	0.0206	0.005	0.0034	0.0109	0.0224	0.0123
	17	0.0088	0.1033	0.0354	0.0831	0.0001	0.0117	0.0097	0.1124	0.0367	0.0224	0.0004	0.0068	0.0246	0.1802	0.0001	0.0253	0	0.1117	0.1683	0.015	0.0011	0.0009	0.0127	0.0077	0.0026
	18	0.0127	0.1143	0.0366	0.0838	0.0001	0.0149	0.0108	0.1327	0.0431	0.018	0.0006	0.009	0.0247	0.169	0.0001	0.025	0.0656	0	0.1818	0.0148	0.0013	0.001	0.0101	0.0071	0.0028
	19	0.0115	0.1093	0.0476	0.0898	0.0001	0.0187	0.0125	0.1302	0.0469	0.0175	0.0008	0.0089	0.0287	0.1628	0.0001	0.0312	0.0514	0.0945	0	0.0166	0.002	0.0013	0.009	0.0095	0.0044
	20	0.0082	0.0718	0.0409	0.1292	0.0001	0.0191	0.0412	0.0967	0.0379	0.018	0.0007	0.0059	0.023	0.1247	0.0002	0.0335	0.0445	0.075	0.162	0	0.0026	0.0016	0.0103	0.013	0.004
	21	0.0058	0.0655	0.0292	0.0784	0.0001	0.0187	0.0234	0.0901	0.0281	0.0199	0.0023	0.0105	0.0269	0.1135	0.0001	0.0795	0.0316	0.0655	0.1883	0.0257	0	0.0047	0.0105	0.0211	0.0211
	22	0.0116	0.0697	0.0365	0.0978	0.0001	0.0166	0.0232	0.0813	0.0315	0.0149	0.0017	0.0133	0.0249	0.1277	0.0001	0.0763	0.0365	0.0713	0.1708	0.0216	0.0066	0	0.0083	0.0431	0.0149
	23	0.0064	0.0864	0.0302	0.0892	0.0001	0.0143	0.0098	0.111	0.0428	0.0282	0.0005	0.0082	0.02	0.1415	0.0001	0.0334	0.0719	0.0973	0.1669	0.0196	0.002	0.0011	0	0.008	0.0045
	24	0.0098	0.0682	0.0352	0.1051	0.0015	0.0129	0.021	0.0839	0.0369	0.0166	0.0024	0.0107	0.0236	0.1213	0.0001	0.0664	0.0415	0.0658	0.1682	0.0236	0.0039	0.0057	0.0076	0	0.0103
	25	0.0071	0.0632	0.0398	0.078	0.0005	0.0138	0.0117	0.0923	0.027	0.0127	0.0015	0.0102	0.0214	0.1213	0.0001	0.0851	0.0331	0.0607	0.184	0.0168	0.0092	0.0046	0.0102	0.024	0

3.3.3. Traffic Density Rates

The customer shopping paths in a store are crucial for the retail industry to understand customer purchasing behavior. The locations of the product categories in the store affect customer paths. Yapicioglu (2008) defined the traffic density as the relative number of customer visits to a zone.

There are very few studies in the literature to evaluate customer shopping paths and store traffic. Farley and Ring (1966) proposed a stochastic model to understand customer traffic flow in a supermarket, which was divided into multiple zones. The transition probabilities of zones are included in their stochastic model. Larson et al. (2005) provided an extensive study of customer shopping paths, which were collected with Radio Frequency Identification (RFID) Technology.

Purchase likelihoods of product categories were investigated using our Yunus data to estimate the traffic density rates in the store (Table 3.9). To obtain traffic density rates, we labeled each customer purchase according to its product category and counted the total number of customers who purchased from each product category. Then, the total number of customers buying at each product category (N_i) was divided by the total number of shopping trips (N^t) in a month to calculate the purchase likelihood of product category *i* (n_i). The customer traffic density rate of a display (z_k) represents the purchases of the product categories in display *k*. If a customer purchases from a product category, we assume that the customer visits the permanent display of the product category. Note that the traffic density calculations do not include customers who may walk by because of longer than necessary routes for their ultimate purchases. We only include the customer traffic rates based on the shortest paths for the purchased items. Table 3.9 summarizes the purchase likelihoods of product categories for the dataset with 63,661 customers during a month. The most often purchased product categories are fruits/vegetables and the middle section.

Table 3.9. Purchase Likelihoods of Product Categories in the Yunus Store

Product Categories	Total Number of Shopping Trips with Product Category *i* ($\boldsymbol{N_i}$)	Number of Trips in the Data Set ($\boldsymbol{N^t}$)	Purchase Likelihood of Product Categories ($\boldsymbol{n_i = N_i / N^t}$)
Bakery	1685	63661	0.026
Bread	15364	63661	0.241

Cigarettes	8446	63661	0.133
Cleaning Supplies	12331	63661	0.194
Clothes	45	63661	0.001
Coffees	2110	63661	0.033
Cosmetics	1860	63661	0.029
Dairy	16275	63661	0.256
Dried Fruits and Nuts	6218	63661	0.098
Dry Beans	1979	63661	0.031
Electronics	144	63661	0.002
Fish	1432	63661	0.022
Frozen Food	3599	63661	0.057
Fruits and Vegetables	26584	63661	0.418
General	35	63661	0.001
Glassware	4507	63661	0.071
Meat 1	7114	63661	0.112
Meat 2	12577	63661	0.198
Middle Section	33765	63661	0.530
Perfumery	2407	63661	0.038
Purses	30	63661	0.000
School Supplies	296	63661	0.005
Shoes	214	63661	0.003
Spices	1010	63661	0.016
Textile	1526	63661	0.024
Toys	726	63661	0.011

3.4. Summary and Possible Enhancements

To summarize this chapter, we defined all mutual parameters for the proposed mathematical models. Based on these definitions, the basic revenue functions in proposed models have two components as shown in Equation 3.9. The revenue from the allocated area of product categories and cross-category sales between the adjacent displays are the two terms. The first component includes the space elasticity (β_i), the total area of temporary display k (t_k), the impulse purchase likelihood (d_i), and the revenue of product category per square meter (r_i). It is similar to the revenue function of Corstjens and Doyle (1981). The second component includes the revenue of the product category per square meter (r_i), the total area of the temporary display k (t_k), *the* customer traffic rate of display k (z_k), and the cross-category sale effect between product category i and product category j (e_{ij}). The cross-category coefficient in adjacent aisles

(e_{ij}) is multiplied by the total allocated areas of category *i* ($r_i t_i$) and category *j* ($t_k Y_{jk}$) where product category *i* is a member of the set of adjacent product categories of display *k*. We use the customer traffic rates at the adjacent permanent displays to estimate the traffic rate of a temporary display as shown in Table 3.9. Based on Han et al. (2021) study, the front end cap displays are the most successful at increasing category purchase rates, whereas shelf displays have the highest impact on influencing brand selection. Also, the store manager experienced that customers tended to pay more attention to the front-of-store end cap displays compared to those at the back due to the concluding phase of their shopping experience which always occurs in front. In the test cases, the store has one entrance and the cashier area has fiver cashiers, so we assume that 95% of the customers walk by the temporary displays at the front side of the store. (These front temporary displays are very close to the cashier in our test cases.) If the store is quite large or has more than one entrance, the traffic rates at the front temporary displays may differ in the extra attraction they receive. The customer traffic rate of display *k* is added to the second component to account for the customer traffic at different displays. These traffic densities are likely to be estimated too low because we do not include walk bys who do not purchase nearby products.

$$\sum_{i \in C} d_i r_i \left\{ \sum_{k \in D} t_k Y_{ik} \right\}^{\beta_i} + \sum_{j \in C} \sum_{k \in D} \sum_{i \in V_k} z_k e_{ij} (r_i t_i)(t_k Y_{jk}) \qquad 3.9.$$

There are not many papers published, especially recently, concerning the important parameters needed to estimate sales in a supermarket. These parameters govern vital aspects such as space elasticity, cross-category sales, and traffic patterns in the store. Models could be constructed to exploit the effects of customer profile, the total area of the store, the size of the parking lot, and the location of the store on the parameter of space elasticity. Studies of cross-category sales could be extended to sub- product categories and products. Customer movement through the store could potentially be tracked by smartphone apps or video cameras to improve understanding of paths.

Although the models in subsequent chapters have different decision variables, constraints, and assumptions, they have mutual parameters which were explained in this chapter. In Chapter 4, we propose mathematical models for the temporary display allocation problem during a single promotional period. In Chapter 5, we propose mathematical models that solve the temporary display allocation problem for a two-period promotional period. In all models, the objective functions aim to maximize the revenue by considering the total allocated space of product categories, space elasticities, cross-category sales, and customer traffic rates under the assumptions of both split and non-split product categories within a single temporary display.

Chapter 4 The Allocation Problem of Temporary Displays for a Single Promotional Period

In this chapter, we propose two mixed integer nonlinear mathematical models to determine locations and allocated areas of product categories to temporary displays in a single promotional period. Both models have continuous and discrete variables with nonlinear objective functions because of the revenue functions and constraints. The main difference between these mathematical models is the number of product categories at a temporary display. The first mathematical model assumes that there should be only one product category at a temporary display, and it is termed the *non-split model*. The second mathematical model extends this assumption and allows a maximum of three product categories at a temporary display, and it is called the *split model*. It is possible for a temporary display to have both one product category and multiple product categories in a store environment. The store manager's decision of allocating one product category to a temporary display is related to the number of promotional products on a shelf (product size), brand, and available products in store stocks. Multiple product categories at an end-of-aisle display or an island display are commonly designed as a theme such as fall/winter/spring/summer time, religious holidays, New Year's Eve, school opening, etc. These temporary displays usually have multiple items from diffrerent product categories that are commonly sold in the same promotional period. In this chapter, we solve the temporary display allocation problem for both the split and non-split allocation assumptions during a single promotional period.

In a single promotional period, there are more than 40 products on the promotional list and the management plans to pick at least one product from the main product categories in the Yunus Store chain. A promotional period is typically defined as seven days, two weeks, or sixteen days. In some cases, a promotional period ends on Friday then they extend the promotional period for the weekend. It is necessary to solve the problem weekly or twice a month. In practice, the store manager chooses the location of the promotional products based on his experience and product characteristics, which are product dimensions, brand agreements, and stock availabilities of products. Some of the promotional products stay in their permanent locations while others will be allocated to the temporary displays. In this research, we focus on allocating promotional products to temporary displays, which are end-of-isles, cashier displays, and island displays between the permanent displays. The number of temporary displays represents the summation of all types of temporary displays, which is larger than the number of promotional product categories. In some cases, the number of temporary displays may be higher than the number of permanent displays, which are regular racks in the main aisles and can carry more item categories. The total usable area of a temporary display defines the limits of the temporary display and determines how many products can be allocated to the temporary display. The total usable area of all temporary displays is less than the total usable area of all permanent displays. After the promotional items list is determined by the store chain management, the store manager makes the decision of which the product categories can be allocated to temporary displays for this promotional period. The selected promotional product categories are called candidate promotional product categories. A candidate promotional product category can be allocated to more than one temporary display. In a store environment, the lower area bound of a promotional category is related to the stock level, price, and promotional discount amount.

In the following sections, the proposed non-split and split mathematical models are explained with their assumptions, parameters, decision variables, objective functions, and constraints. In the verification processes of the models, the real store data and model results are compared. The proposed models are solved by the GAMS optimization program for real sized problems. Real size problems might have more than six product categories to allocate to 25 to 75 temporary displays in a single promotional period.

4.1. The Mathematical Model for the Temporary Display Allocation Problem with Non-Split Product Categories during a Single Promotional Period- Model-1A

In the temporary display allocation problem, our goal is to find the highest revenue locations for the preselected product categories during a promotional period. Suppose there are n temporary displays and m preselected product categories to be allocated for a promotional period. We aim to allocate m product categories to n temporary displays by considering the highest revenue and the area restrictions during a promotional period. In our test cases, the number of temporary displays (n) is greater than the number of product categories (m). However, there might be some special cases where $m > n$. In a store environment, some of the temporary displays are allocated by promotional categories while others are allocated by non-promotional categories, which are identified in the preselected product categories. The store managers and the brand representatives determine the non-promotional product categories displays. Since these locations are fixed, the decision variables in our model are defined only for temporary displays, that is, those that carry promotional categories during a promotional period.

Permanent displays are intended for customer planned purchases while temporary displays are designed to attract customers to have impulse purchases. In this study, we assume that customer purchasing decisions at temporary displays are all impulse purchases. There might be a small number of customers who shop for discounted and promotional products at temporary displays intentionally. Nevertheless, we ignore these customer purchases in this research because they would be a small minority of purchases from the total and define the purchases at temporary displays as impulse purchases.

Table 4.1. Definitions of Sets

	Notation	Definition	Source of Data	Range of Data
Sets	$\boldsymbol{C}$	Set of candidate product categories	The chosen promotional product categories	The number of product categories
	$\boldsymbol{D}$	Set of temporary displays	Layout of the store	The number of temporary displays
	$\boldsymbol{P_k}$	Set of product categories that are allocated to permanent displays adjacent to temporary display *k*	Layout of the store	Subset of product categories
	$\boldsymbol{F_k}$	Set of product categories that should not be allocated at the temporary display *k*	Store policies, such as health conditions, brand agreements, etc.	0 - (the number of product categories-1)
	$\boldsymbol{T_k}$	Set of temporary displays that are adjacent to temporary display *k*	Layout of the store	Subset of temporary displays

In the mathematical formulation of the temporary display allocation problem with non-split product categories during a single period (TDAP-NSS), the decision variables are represented by Y_{ik} which is whether to allocate *product category i* to *temporary display k*. Each decision variable Y_{ik} is defined as a binary variable that is equal to 1 when *product category i* is allocated to *temporary display k*, and 0 otherwise. In this version of the problem, there will be only one product category per temporary display. All definitions of sets are listed in Table 4.1. The paremeters are explained in Table 4.2.

Table 4.2. Definitions of Parameters

	Notation	Definition	Source of Data	Range of Data
Parameters	$\boldsymbol{a_{ki}}$	Total area of product category *i* at permanent displays that are adjacent to temporary display *k*	Layout of the store	0 - 20 m^2
	$\boldsymbol{b_i^L}$	Lower bound of the allocated area of product category *i* in all temporary displays (square meters)	Store policies and stock levels	0 - the largest s_k
	$\boldsymbol{b_i^U}$	Upper bound of the allocated area of product category *i* to all temporary displays (square meters)	Store policies and stock levels	The smallest s_k - all temporary displays
	$\boldsymbol{e_{ij}}$	Cross-category sales effect between product category *i* and product category *j* (ratio)	Analysis of the historical data	0 - 1
	$\boldsymbol{m_{ki}}$	Expected traffic density of product category *i* at adjacent permanent displays of temporary display *k* (ratio)	Analysis of the historical data	0.1 - 1
	$\boldsymbol{n_k}$	Expected traffic density at temporary display *k* (ratio)	Analysis of the historical data	0.01 - 1
	$\boldsymbol{r_i}$	Average revenue of product category *i* per square meter for a promotional period	Analysis of the historical data	0 - 30,000 TRY
	$\boldsymbol{s_k}$	Total area of temporary display *k* (square meters)	Layout of the store	0 m^2- 6 m^2
	$\boldsymbol{w_i}$	Impulse purchase rate of product category *i* (ratio)	The analysis of the historical data	0 - 0.35
	$\boldsymbol{\beta_i}$	Space elasticity of product category *i* (ratio)	The findings by Corstjens and Doyle (1981)	0 - 1

Total Promotional Revenue=

$$\sum_{i\in C} w_i r_i \left\{\sum_{k\in D} s_k Y_{ik}\right\}^{\beta_i} + \sum_{j\in C}\sum_{k\in D}\sum_{i\in P_k} m_{ki}(r_i a_{ki}) e_{ij}(s_k Y_{jk}) + \sum_{i\in C}\sum_{\substack{j\in C\\ i\neq j}}\sum_{k\in D}\sum_{v\in T_k} n_k r_i (s_v Y_{iv}) e_{ij}(s_k Y_{jk})$$

$Equation$ 4.1

In the objective function (Equation 4.1) of the mathematical formulation of the TDAP-NSS, there are three revenue components. The first revenue component of Equation 4.1 represents the area related revenue which considers the impulse purchases of product category *i* (w_i), the average revenue of product category *i* per square meter (r_i), the product categories allocated areas at temporary displays ($s_k Y_{ik}$), and their space elasticities (β_i). The second and third revenue components of Equation 4.1 are regarded as cross-category sales (e_{ij}), which are also impulsive purchases where decisions are made at neighboring permanent and temporary displays.

Each component of the objective function is explained in detail as follows:

$$\sum_{i\in C} w_i r_i \left\{\sum_{k\in D} s_k Y_{ik}\right\}^{\beta_i} \qquad Equation\ 4.2$$

Equation 4.2 handles the impulse purchase revenue related to the total allocated area at temporary displays. The space elasticity of a product category explains the sales increase according to the allocated space of the product categories. The total allocated areas of a product category in all temporary displays is calculated by $\sum_{k\in D} s_k Y_{ik}$. The average revenue (r_i) of a product category is multiplied by the total allocated space of that product category to the power of its space elasticity factor ($\{\sum_{k\in K} s_k Y_{ik}\}^{\beta_i}$). To summarize, in this term we calculate the expected sales at temporary displays, which are related to the total allocated areas of the product categories and the space elasticities of product categories.

$$\sum_{j\in C}\sum_{k\in D}\sum_{i\in P_k} m_{ki}(r_i a_{ki}) e_{ij}(s_k Y_{jk}) + \sum_{i\in C}\sum_{\substack{j\in C\\ i\neq j}}\sum_{k\in D}\sum_{v\in T_k} n_k r_i (s_v Y_{iv}) e_{ij}(s_k Y_{jk}) \qquad Equation\ 4.3$$

Equation 4.3 defines the cross-category sales at temporary displays. Cross-category sales of a temporary display are related to adjacent permanent and temporary displays. The first component of Equation 4.3 shows the summation of cross-category sales between a product category at a temporary display and the product categories at its adjacent permanent displays. The second component of Equation 4.3 shows the summation of cross-category sales between a product category at a temporary display and the product categories at its adjacent temporary displays. In a store layout, one permanent display is usually allocated with one or two product categories. Due to the adjacency definition, a temporary display can be adjacent to the one or multiple permanent displays.

Figure 4.1 shows the illustration of the adjacencies of temporary displays that are located at three different locations in the store. The adjacency relationships of a temporary display are demonstrated with blue arrows. Endcap *P* is adjacent to the product categories at Aisle 7, Aisle 8, and permanent display W3. The island display at Aisle 9 is adjacent to the product categories at Aisle 9. Endcap *N* is adjacent to the product categories at Aisles 10, Aisle 11, Endcap M, Endcap I, and Endcap O.

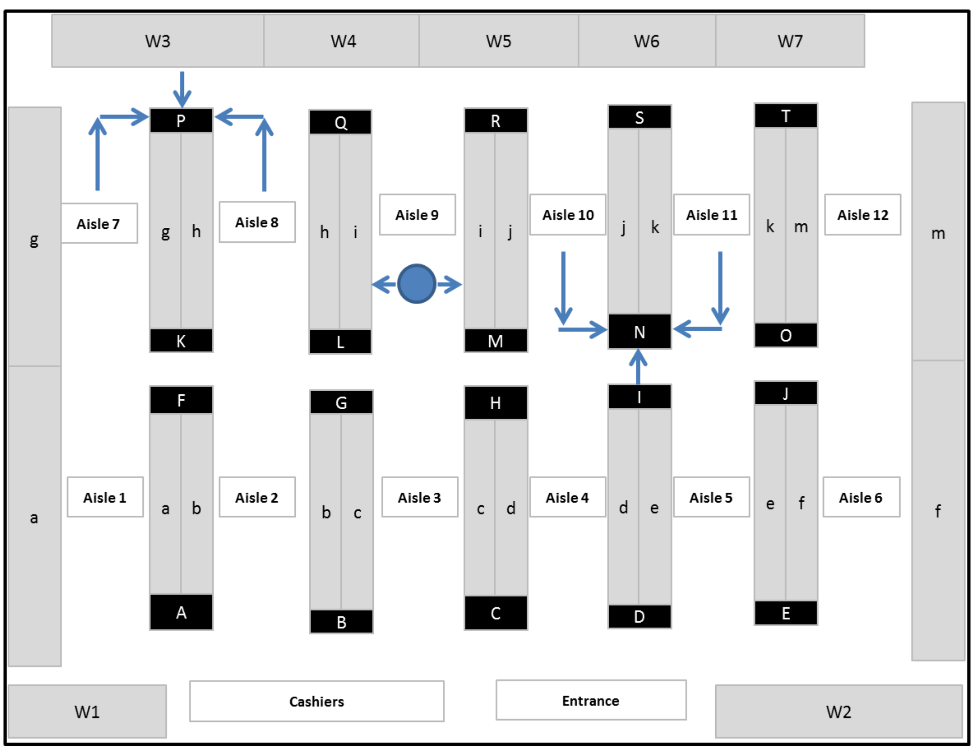


Figure 4.1. Illustration of the Adjacencies of Temporary Displays at Three Different Locations

In the first component of Equation 4.3, we calculate the cross-category sales between the product categories at temporary displays by evaluating the expected revenue of product categories at adjacent permanent displays ($(r_i a_{ki})e_{ij}$). m_{ki} is the expected customer traffic rate of product category *i* at adjacent permanent displays of temporary display *k*. P_k represents the adjacent product categories of temporary display *k*. In the term of $r_i a_{ki}$ generates the sales of the permanent displays where a_{ki} shows the total allocated area of product category *i* that is adjacent to the temporary display *k*. Then we can calculate the cross-category sales (e_{ij}) between the product category at permanent displays and the product category at temporary displays ($s_k Y_{jk}$). In the second component of Equation 4.3, the cross-category sales between the product categories at adjacent temporary displays are calculated.

n_k is the expected traffic density at temporary display *k*. The customer traffic rate at a temporary display is approximated by considering the summation of the purchase likelihood of product categories along the shortest path between the store entrance and the temporary display. C_i^s is the set of product categories along the shortest path between the store entrance and temporary display *i*. The formula of the traffic rate at the temporary display is given in Equation 4.4. Note that this is a conservative, that is low, estimate because customers who take longer paths through the store are not included.

$$n_k = \sum_{i \in C_i^s} \frac{Number\ of\ customers\ who\ purchase\ from\ product\ category\ i}{Total\ Number\ of\ Shopping\ Trips} \qquad Equation 4.4$$

The temporary displays at the front of the store have higher customer traffic rates because most customers can see them when they enter the store. T_k represents the set of adjacent temporary displays of temporary display *k*. $r_i(s_v Y_{iv})e_{ij}$ defines the expected revenue of cross-category sales between product category *i* at temporary display *v* which is adjacent to product category *j* at temporary display *k*. The cross-category sale effects are defined as the sales interaction between product categories *i* and *j* in the historical sales data. For instance, the cross-category sale between fruits / vegetables and bread is 0.17. The ratio represents 17% of the customers who purchased fruits vegetables purchase bread.

The revenues of product categories are not only related to their allocated space and impulse purchase likelihood but also related to the customer traffic in the store. All of the temporary displays are designed to be seen by customers. However, the store traffic at each temporary display is different from each other. The highest customer traffic can be seen at the temporary displays at the entrance of the store. These temporary displays are seen by all the customers and the traffic rate is equal to one. If the allocation of a selected product category is in the high traffic areas, the impulse purchase likelihood of the product category will increase. Hence, it is necessary to include customer traffic rates at temporary displays (n_k) as multipliers of Equation 4.3 in the model. Because we do not have actual path data of customers through the store, we mine the historic sales date to estimate the customer traffic patterns. The customer traffic rate of a temporary display is calculated by the number of customers who purchase from adjacent permanent displays of that temporary display divided by the total number of customers in the store. The customer traffic rates at temporary displays and permanent displays can be different and are defined as two parameters. m_{ki} represents the customer traffic rates of permanent displays which are adjacent to the temporary display *k*. m_{ki} is calculated by the following formula (Equation 4.5). Again, this is conservative as it does not include customers who are inefficient in their travels through the store.

$$m_{ki} = \frac{\begin{array}{c}\text{Number of customers who purchase from product category } i \\ \text{that is adjacent to temporary display } k\end{array}}{\text{Total number of shopping trips}} \qquad \text{Equation 4.5}$$

The temporary display allocation problem has four main constraints. The first constraint is that a temporary display should be fulfilled by one product category (Equation 4.6) only.

$$\sum_{i \in C} Y_{ik} = 1 \quad \forall k \in D \qquad \text{Equation 4.6}$$

The allocated total space of each product category must be between the area limitations of the product category (Equations 4.7 and 4.8). Each product category has lower and upper area boundaries that are predefined by considering stock levels and brand agreements.

$$\sum_{k \in D} Y_{ik} s_k \geq b_i^L \quad \forall i \in C \qquad \text{Equation 4.7}$$

$$\sum_{k \in D} Y_{ik} s_k \leq b_i^U \quad \forall i \in C \qquad \text{Equation 4.8}$$

Equation 4.9 restricts the product categories that should not be allocated to specified temporary displays due to safety and health reasons. For instance, baby products are not allocated next to pet foods.

$$\sum_{k \in D} \sum_{i \in F_k} Y_{ik} = 0 \qquad Equation\ 4.9$$

Equation 4.10 is added to the formulation for definitions of decision variables. If, product category *i* is allocated to temporary display k, Y_{ik} will be 1. Otherwise, Y_{ik} will be 0.

$$Y_{ik} \in \{0,1\} \quad \forall i \in C, k \in D \qquad Equation\ 4.10$$

The goal of the temporary display allocation problem is to allocate *m* product categories to *n* temporary displays. It is a different version of the bin packing problem and quadratic assignment problem, because *m* may be larger than *n* and not all m will be allocated every period and there are interactions between the product categories. In the quadratic assignment problem, the problem size depends on the number of binary decision variables as does our problem. The number of binary variables is critical to evaluate the solution time of the problem. In the literature, Marzetta and Brungger (1999) cited that it is difficult to provide an optimum solution for the quadratic assignment problem with 20 departments to 20 locations. The number of binary variables for our problem has two parameters, which are the number of product categories and the number of temporary displays.

Each grocery chain may have its assorting strategy and have a different number of product categories. In our case study, the Yunus store has 26 product categories while a typical Walmart has 24 departments and 310 product categories (https://www.walmart.com/all-departments). Each product category is allocated to at least one permanent display which has at least two end-of-aisle displays. It is also possible to allocate multiple product categories to a permanent display in a small size grocery store which has only 10 permanent displays and 20 end-of-aisle displays. In this case, the small size of the problem has 10 product categories to 20 temporary displays and the number of decision variables is $2^{20} = 1048576$. There is no algorithm to solve this problem in reasonable-time due to the number of binary variables and nonlinearity in the objective function. We can solve small and medium size problems by using GAMS, a nonlinear solver. In the next section, the objective function calculations and test results are discussed.

4.2. Actual vs. Projected Promotional Sales with the Non-split Model during a Single Promotional Period

In this section, we will explain the objective function calculations for the TDAP-NSS illustrated by a test case. This test case has been selected by the Yunus store managers to illustrate the problem in the typical store environment and the store is located in Aksaray, Turkiye. In Test Case 1, there are 15 product categories, 39 permanent displays, and 26 temporary displays. Our goal is to determine the best location of each product category at each temporary display. The product categories and their permanent displays for Test Case 1 are listed in Table 4.3. The layout of the store, which shows the permanent and temporary displays in different colors, is illustrated in Figure 4.2. The product categories in permanent displays are pre-determined so the abbreviations of product categories are written in the permanent displays in the figure while temporary displays have only their numbers. The store has 7 permanent displays with refrigeration (light orange), 32 permanent displays without refrigeration (blue), 20 endcap type temporary displays (pink), 4 pallet type temporary displays (light blue), and 2 temporary displays with refrigeration (green).

Table 4.3. Abbreviations of Product Categories

Abbreviation	Product Category
C1	Fruits and Vegetables
C2	Dairy
C3	Frozen Food
C4	Meat
C5	Breakfast Items
C6	Glassware
C7	Household Items (Paper Products & Plastics)
C8	Bread
C9	Bakery
C10	Textile
C11	Toys
C12	Glassware
C13	Cleaning Supplies
C14	Cosmetics
C15	Middle Section

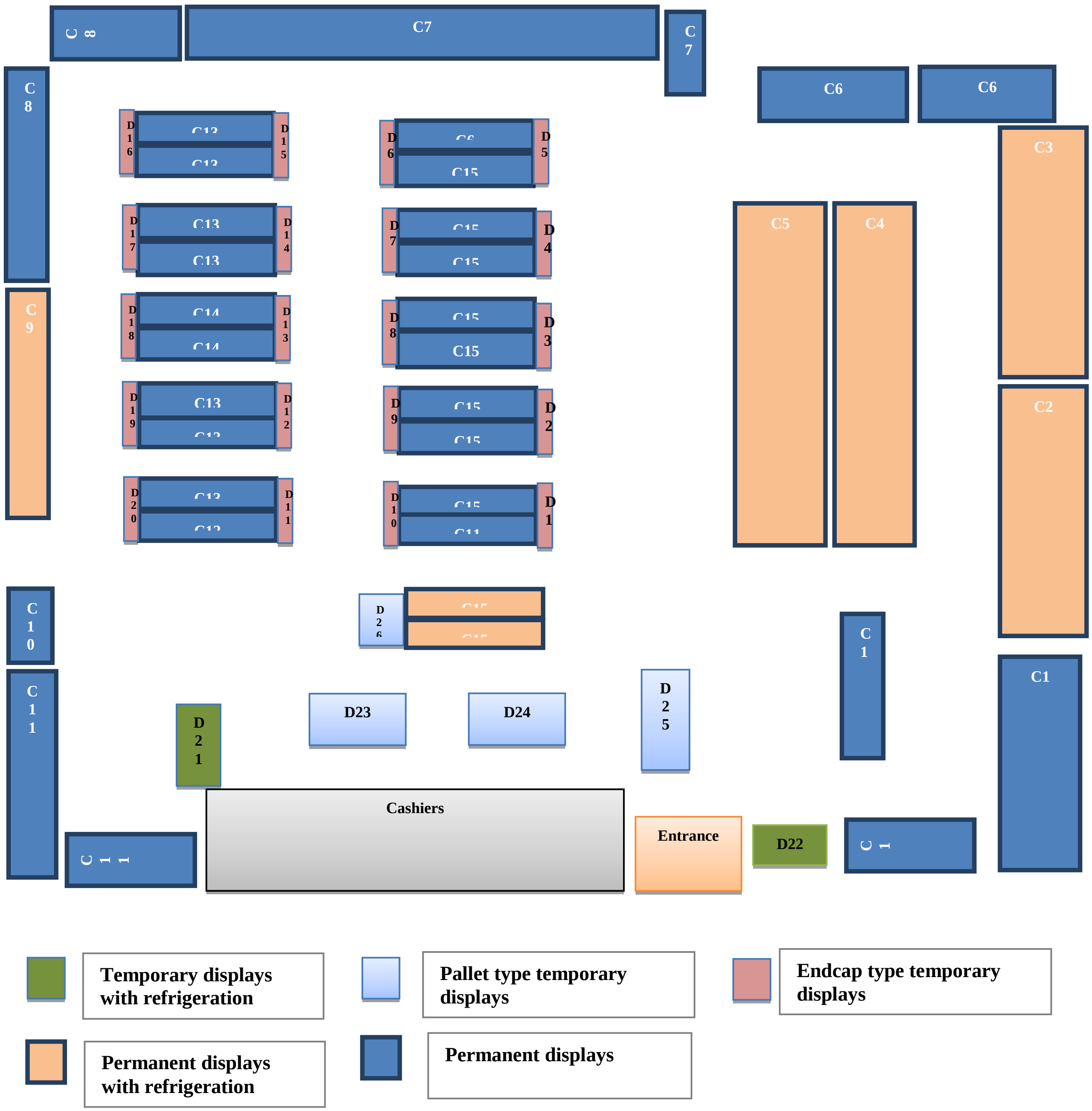


Figure 4.2. The Store Layout of the Test Case for 26 Temporary Displays

We use the data sets that we explained in Chapter 3 for the mathematical models. The sets and the related data are shown in Table 4.4. The size of the matrix of the cross-category sales is 15x15. The data of the effects of cross-category sales for product categories is given in Table 4.7. The rest of the parameters and their values are shown in Table 4.8.

Table 4.4. Sets of Test Case 1

Sets	Definition	Data
$\boldsymbol{C}$	Set of candidate product categories	{C1,C2,C3,C4,C5,C6,C7,C8,C9,C10, C11,C12,C13,C14,C15}
$\boldsymbol{D}$	Set of temporary displays	{D1,D2,D3,D4,D5,D6,D7,D8,D9,D10,D11, D12,D13,D14,D15,D16,D17,D18,D19,D20, D21,D22,D23,D24,D25,D26}
$\boldsymbol{P_k}$	Set of product categories that are allocated to permanent displays adjacent to temporary display *k*	See Table 4.5
$\boldsymbol{F_k}$	Set of product categories that should not be allocated at temporary display *k*	$F_{D1} = \{C13\}$ $F_{D2} = \{C13\}$ $F_{D20} = \{C4, C5\}$
$\boldsymbol{T_k}$	Set of temporary displays that are adjacent to temporary display *k*	See Table 4.6

Table 4.5. $\boldsymbol{P_k}$ (Set of product categories that are allocated to permanent displays adjacent to temporary display *k)*

k	**Adjacent Product Categories**		
D1	C11	C15	
D2	C15		
D3	C15		
D4	C15		
D5	C6	C7	
D6	C6	C7	C15
D7	C15		
D8	C15		
D9	C15		
D10	C11	C15	
D11	C12	C13	
D12	C13		
D13	C13	C14	
D14	C13	C14	
D15	C7	C13	
D16	C8	C13	
D17	C8	C13	
D18	C9	C13	C14
D19	C9	C13	C14
D20	C9	C12	C13
D21	C10	C11	C12
D22	C1		

D23	C15		
D24	C15		
D25	C5	C15	
D26	C15		

Table 4.6. $\boldsymbol{T_k}$ Set of temporary displays that are adjacent to temporary display *k*

k	**Adjacent Temporary Displays**		
D1	D2		
D2	D1	D3	
D3	D2	D4	
D4	D3	D5	
D5	D4		
D6	D7	D15	
D7	D6	D8	D14
D8	D7	D9	D13
D9	D8	D10	D12
D10	D9	D11	D26
D11	D10	D12	
D12	D9	D11	D13
D13	D8	D12	D14
D14	D7	D13	D15
D15	D6	D14	
D16	D17		
D17	D16	D18	
D18	D17	D19	
D19	D18	D20	
D20	D19		
D21	N/A		
D22	N/A		
D23	D24		
D24	D25	D23	
D25	D24		
D26	D10		

Table 4.7. Cross-Category Sale Effects of Product Categories *i* and *j* (e_{ij}- ratios)(see Chapter 3.3.2)

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15
C1		0.121	0.025	0.104	0.065	0.026	0.090	0.114	0.012	0.008	0.003	0.026	0.090	0.012	0.192
C2	0.163		0.028	0.109	0.054	0.024	0.085	0.117	0.012	0.007	0.004	0.024	0.085	0.011	0.206
C3	0.140	0.120		0.086	0.051	0.028	0.080	0.092	0.010	0.009	0.003	0.028	0.080	0.011	0.193
C4	0.169	0.133	0.025		0.066	0.025	0.084	0.114	0.013	0.007	0.003	0.025	0.084	0.011	0.182
C5	0.180	0.112	0.025	0.112		0.025	0.083	0.103	0.009	0.008	0.003	0.025	0.083	0.010	0.168
C6	0.132	0.091	0.025	0.078	0.046		0.103	0.076	0.008	0.022	0.012	0.000	0.103	0.016	0.187
C7	0.159	0.112	0.025	0.091	0.053	0.036		0.087	0.009	0.012	0.004	0.036	0.000	0.019	0.188
C8	0.172	0.131	0.024	0.106	0.056	0.023	0.074		0.020	0.007	0.003	0.023	0.074	0.010	0.194
C9	0.147	0.111	0.022	0.098	0.040	0.021	0.063	0.169		0.008	0.003	0.021	0.063	0.007	0.171
C10	0.121	0.084	0.024	0.066	0.042	0.066	0.105	0.068	0.010		0.010	0.066	0.105	0.021	0.168
C11	0.121	0.092	0.021	0.061	0.033	0.085	0.078	0.063	0.007	0.024		0.085	0.078	**0.012**	0.184
C12	0.132	0.091	0.025	0.078	0.046	0.000	0.103	0.076	0.008	0.022	0.012		0.103	0.016	0.187
C13	0.159	0.112	0.025	0.091	0.053	0.036	0.000	0.087	0.009	0.012	0.004	0.036		0.019	0.188
C14	0.132	0.093	0.022	0.073	0.039	0.036	0.122	0.071	0.007	0.015	0.004	0.036	0.122		0.163
C15	0.163	0.130	0.029	0.095	0.051	0.031	0.090	0.109	0.012	0.009	0.004	0.031	0.090	**0.012**	

Table 4.8. Parameters and Case Study Data

Notation	Definition	Data
a_{ki}	Total area of product category $i \; where \; i \in P_k$ at permanent displays that are adjacent to temporary display *k*	See Table 4.8.a
b_i^L	Lower bound of the allocated area of product category *i* in all temporary displays (square meters)	See Table 4.8.b
b_i^U	Upper bound of the allocated area of product category *i* in all temporary displays (square meters)	See Table 4.8.c
m_{ki}	Traffic density of product category *i* $where \; i \in P_k$ at adjacent permanent displays of temporary display *k* (ratio)	See Table 4.8.d
n_k	Traffic density at temporary display *k* (ratio)	See Table 4.8.e
r_i	Average revenue of product category *i* per square meter	See Table 4.8.f
s_k	Total area of temporary display *k* (square meters)	See Table 4.8.g
w_i	Impulse purchase rate of the product	See Table

	category *i* (ratio)	4.8.h
β_i	Space elasticity of product category *i* (ratio)	See Table 4.8.i

Table 4.8.a: a_{ki}: Total area (square meters) of each product category *I* at permanent displays that are adjacent to temporary display *k*. Each temporary display has one to three adjacent permanent display categories.

k	a_{ki}
D1	20,60,0
D2	20,0,0
D3	20,0,0
D4	20,0,0
D5	20,20,20
D6	20,20,20
D7	20,0,0
D8	20,0,0
D9	20,0,0
D10	20,60,0
D11	20,20,0
D12	20,0,0
D13	20,0,0

k	a_{ki}
D14	20,0,0
D15	20,20,0
D16	20,20,20
D17	20,40,0
D18	20,40,0
D19	20,40,0
D20	20,40,0
D21	60,20,0
D22	40,0,0
D23	30,0,0
D24	30,0,0
D25	30,20,0
D26	20,60,0

Table 4.8.b: b_i^L **:** Lower bound of the allocated area of product category *i* in all temporary displays (square meters)

i	b_i^L
C1	0
C2	0
C3	0
C4	0
C5	0
C6	0
C7	0
C8	0

i	b_i^L
C9	0
C10	0
C11	0
C12	0
C13	0
C14	0
C15	0

Table 4.8.c: b_i^U: Upper bound of the allocated area of product category *i* in all temporary displays (square meters)

i	b_i^U
C1	6
C2	6
C3	6
C4	6
C5	6
C6	6
C7	6
C8	6

i	b_i^U
C9	6
C10	6
C11	6
C12	6
C13	6
C14	6
C15	6

Table 4.8.d: m_{ki}: Traffic density of product category *i* at adjacent permanent displays of temporary display *k* (ratio)

k	m_{ki}		
D1	0.60	0.65	
D2	0.65		
D3	0.60		
D4	0.60		
D5	0.70	0.60	0.70
D6	0.70	0.60	0.70
D7	0.60		
D8	0.60		
D9	0.65		
D10	0.65	0.60	
D11	0.13	0.45	
D12	0.45		
D13	0.45		
D14	0.45		
D15	0.70	0.5	
D16	0.80	0.25	0.45
D17	0.80	0.45	
D18	0.80	0.45	
D19	0.80	0.50	
D20	0.80	0.5	
D21	0.30	0.45	
D22	0.95		
D23	0.95		

D24	0.95		
D25	0.95	0.80	
D26	0.80		

Table 4.8.e: n_k Traffic density at temporary display *k* (ratio)

k	n_k
D1	0.80
D2	0.70
D3	0.70
D4	0.75
D5	0.80
D6	0.85
D7	0.60
D8	0.60
D9	0.60
D10	0.90
D11	0.90
D12	0.60
D13	0.60

k	n_k
D14	0.60
D15	0.85
D16	0.85
D17	0.60
D18	0.60
D19	0.60
D20	0.90
D21	0.80
D22	0.99
D23	0.90
D24	0.90
D25	0.90
D26	0.90

Table 4.8. f : r_i **:** Average revenue per month in TL of product category *i* per square meter.

i	r_i
C1	3300
C2	2500
C3	3600
C4	6920
C5	2500
C6	3050
C7	3900
C8*	70

i	r_i
C9	880
C10	3050
C11	4900
C12	3050
C13	4300
C14	2690
C15	360

* Note that C8, Category of bread, is very low. This is because bread prices in Turkiye are kept very low.

Table 4.8.g: s_k: Total area of temporary display *k* (square meters)

k	s_k
D1	3
D2	3
D3	3
D4	3

k	s_k
D14	3
D15	3
D16	3
D17	3

D5	3		**D18**	3
D6	3		**D19**	3
D7	3		**D20**	3
D8	3		**D21**	4
D9	3		**D22**	4
D10	3		**D23**	6
D11	3		**D24**	6
D12	3		**D25**	6
D13	3		**D26**	6

Table 4.8.h: w_i**:** Impulse purchase rate of product category *i* (ratio) (see Chapter 3.1)

i	w_i		*i*	w_i
C1	0.090		**C9**	0.290
C2	0.150		**C10**	0.185
C3	0.100		**C11**	0.190
C4	0.165		**C12**	0.185
C5	0.290		**C13**	0.140
C6	0.185		**C14**	0.180
C7	0.140		**C15**	0.015
C8	0.050			

Table 4.8.i: β_i: Space elasticity of product category *i* (ratio) (see Chapter 3.3.1)

i	β_i		*i*	β_i
C1	0.570		**C9**	0.220
C2	0.230		**C10**	0.145
C3	0.240		**C11**	0.360
C4	0.330		**C12**	0.145
C5	0.230		**C13**	0.060
C6	0.145		**C14**	0.390
C7	0.060		**C15**	0.220
C8	0.030			

The mathematical formulation of the problem is as follows.

Maximize

$$\sum_{i \in C} w_i r_i \left\{ \sum_{k \in D} s_k Y_{ik} \right\}^{\beta_i} + \sum_{j \in C} \sum_{k \in D} \sum_{i \in P_k} m_{ki} (r_i a_{ki}) e_{ij} (s_k Y_{jk})$$
$$+ \sum_{i \in C} \sum_{\substack{j \in C \\ i \neq j}} \sum_{k \in D} \sum_{v \in T_k} n_k r_i (s_v Y_{iv}) e_{ij} (s_k Y_{jk})$$

$Equation\ 4.11$

Subject to:

$$\sum_{i\in C} Y_{ik} = 1 \quad \forall k \in D \qquad Equation\ 4.12$$

$$\sum_{k\in D} Y_{ik}s_k \geq b_i^L \quad \forall i \in C \qquad Equation\ 4.13$$

$$\sum_{k\in D} Y_{ik}s_k \leq b_i^U \quad \forall i \in C \qquad Equation\ 4.14$$

$$\sum_{k\in D}\sum_{i\in F_k} Y_{ik} = 0 \qquad Equation\ 4.15$$

$$Y_{ik} \in \{0,1\} \quad \forall i \in C, k \in D \qquad Equation\ 4.16$$

We provide explanations of the calculation of the objective function for one category as an example. In the first term, we consider impulse purchase sales. Let us assume product category *C14* is assigned to only one temporary display, which is temporary display *D10*, then the expected sales will be 0.18*2,690*(3)$^{0.39}$ = 743.19 based on the equation of $w_{C14}r_{C14}\{\sum_{k\in D} s_{D10}Y_{C14D10}\}^{\beta_{C14}}$.

In the second term, we consider the cross–category sales between the temporary displays and the permanent displays. Again, let us assume product category *C14* is assigned to temporary display *D10*. According to the adjacency definition, we calculate cross-category sales between the product categories *C15* and *C11*. The calculation of the second term for this assignment will be the summation of $m_{D10C15}(r_{C15}a_{D10C15})e_{c15C14}s_{D10}Y_{C14D10}$ and $m_{D10C11}(r_{C11}a_{D10C11})e_{C11C14}s_kY_{C14D10}$. The calculation of the term will be as follows 0.65*360*20*0.012*3+0.6*4,900*20*0.012*3= 2,285.28.

In the last term, we consider the cross-category sales between the temporary displays. Temporary display *D10* is adjacent to temporary displays *D9*, *D11*, and *D26*. According to the results, product category *C15* is assigned to temporary display location *D26* *(*$Y_{C15D26} = 1$*)*. Let us assume that product category *C11* is assigned *to* locations *D9* and *D11*. The third term calculation is the summation of following equations:

- $n_{D9}r_{C11}(s_{D9}Y_{C11D9})e_{C11C14}s_{D10}Y_{C14D10}$
- $n_{D11}r_{C11}(s_{D11}Y_{C11D11})e_{C11C14}s_{D10}Y_{C14D10}$
- $n_{D26}r_{C15}(s_{D26}Y_{C15D26})e_{C15C14}s_{D10}Y_{C14D10}$

The result of the third term is as follows:

0.6*4,900*3*0.012*3 + 0.s9*4,900*3*0.012*3 + 0.9*360*6*0.012*3= 863.78 TRY.

Finally, the expected total sales of assigning product category *C14* to temporary display location *D10* is 743.19 + 2,285.28 + 863.78 = 3,892.25 TRY during the promotional period.

4.3. Test Problems

We evaluate three test cases to compare the actual sales of promotional products experienced by the store and the predicted sales for this same layout. We then compare these to the projected sales of the optimized layout of the promotional products. The main difference between these three cases is the promotional products and relative categories. These are all actual test cases from the Yunus store.

In Test Case 1, 91 products have discounted prices during the 16-day promotional period. 91 products are to be assigned to 75 temporary displays. These promotional products belong to 8 product categories, which are shoes, electronics, cosmetics, middle section, meat-2, dairy, cleaning supplies, and glassware. During the promotional period, the actual data shows **117,041.70 TRY of** promotional sales. The total promotional sales during the promotional period has three components, sales from direct sales (that is, not affected by cross-category sales), cross-category sales of promotional product categories (cross-category sales at temporary displays), and cross-category sales of non-promotional product categories (cross-category sales at permanent displays). From the total actual sales, we calculate the cross-category sales of promotional categories at temporary displays as **72,870.70 TRY** and the cross-category sales of non-promotional product categories at permanent displays as **38,838.99 TRY**. We calculated the space elasticity sales as **5,332.01 TRY** and the total cross-category sales as **111,709.69 TRY** using the actual data and the revenue function of the proposed mathematical model (Table 4.9). For the same test case, the calculated sales amount from the proposed mathematical model for Test Case 1 is **115,120.89 TRY**, a difference of only 1.6%. The mathematical model generates the space elasticity sales as **4,760.25TRY** which is 10.7% less than the actual corresponding value.

Since we have used the space elasticity parameters from the Desmet and Renaudin (1998)'s study, we expect a tolerable difference between the actual sales and the predicted sales. Borin et al. (1994) states that retail store specific evaluations provide the most reliable models to predict space elasticity and cross-category elasticity. According to the Yunus store manager's experience, the expected difference between the expected and actual sales can be 0% to 20%.

At permanent displays, the cross-category sales on non-promotional product categories is **34,628.77 TRY** and this is 10.8% lower than the actual sales. The difference shows that the positive impact of temporary display purchases on permanent display purchases is considerably less than the impact of permanent display purchases on temporary display purchases, at least in this Test Case. From the mathematical model, the cross–category sales amount of promotional product categories is **75,731.87 TRY** which is 3.9% higher than the actual sales. The reasons behind the difference are related to impulse purchases and customer reactions to specific promotional items. It is difficult to predict these kinds of customer perceptions and purchases by any model. Overall, our model did a very good job of estimating sales from the promotional displays. For the current layout, Table 4.9 shows the comparison of the actual sales amount at the store vs. the estimated sales using the proposed mathematical model.

Table 4.9. Actual Promotional Sales vs. Mathematical Model Results for the Existing Layout (Test Case 1)

Revenue Components of the Objective Function	**Actual Promotional Sales (TRY)**	**Calculated Sales by the proposed Mathematical Model for the Current Layout (TRY)**	**Absolute Proportion Difference**
Space Elasticity Sales (Direct sales)	5,332.01	4,760.25	0.107
Cross-Category Sales of Non-Promotional Product Categories (Cross-category Sales of Permanent Displays)	38,838.99	34,628.77	0.108
Cross-Category Sales of Promotional Product Categories (Cross-category Sales of Temporary Displays)	72,870.70	75,731.87	0.039
Total Sales	117,041.70	115,120.89	0.016

The revenue components of the best solution for Test Case 1 are shown in Table 4.10 with the total revenue of **140,708.59 TRY,** which is **22.2%** higher than the estimated sales using the mathematical model. Of course, this is the goal of the research, that is to improve sales by optimal location of the items on promotion. The sales components of the revenue function are the space elasticity sales, cross-category sales of permanent displays, and cross-category sales of temporary displays. The space-elasticity sales is calculated as **8,045.91 TRY**, which is **69%** higher than the calculated sales for the actual layout. This result shows that the total allocated areas of product categories changed and resulted in much increased space-elasticity sales. The cross-category sales of permanent displays is calculated as **37,630.59 TRY**, which is **8.6%** higher than the estimated sales of the actual layout. The cross-category sales of temporary displays is **95,042.09TRY**, which is **25.5%** higher than the estimated sales of the actual layout. Due to the complexity of the problem and the usage of various solvers (DICOPT, BONMIN) within GAMS, the program ends up with the best integer solution for this test problem. The proposed solution for the non-split mathematical model for Test Case 1 is listed in Table 4.11. If the product category in the display changed, the product category in the proposed solution will be highlighted in bold.

Table 4.10. Comparison of the Optimized Solution and the Current Promotional Sales for Test Case 1

Revenue Components of the Objective Function	**Actual Promotional Sales (TRY)**	**Calculated Sales by the proposed Mathematical Model for the Current Layout (TRY)**	**Absolute Proportion Difference**
Space Elasticity Sales (Direct sales)	4,760.25	8,045.91	0.690
Cross-Category Sales of Non-Promotional Product Categories (Cross-category Sales of Permanent Displays)	34,628.77	37,620.59	0.086
Cross-Category Sales of Promotional Product Categories (Cross-category Sales of Temporary Displays)	75,731.87	95,042.09	0.255
Total Sales	115,120.89	140,708.59	0.222

Table 4.11. Product Category Allocations of the Optimized Solution for Test Case 1

Temporary Display No	Current Allocation (Product Category No)	Proposed Layout (Product Category No)
D1	C23	**C18**
D2	C20	C20
D3	C14	**C18**
D4	C20	**C14**
D5	C14	**C20**
D6	C23	**C18**
D7	C20	**C14**
D8	C14	**C20**
D9	C20	**C18**
D10	C14	C14
D11	C20	C20
D12	C14	C14
D13	C20	C20
D14	C14	**C18**
D15	C20	**C14**
D16	C25	**C18**
D17	C24	**C20**
D18	C14	C14
D19	C20	**C18**
D20	C14	C14
D21	C25	**C11**
D22	C6	**C1**
D23	C23	**C20**
D24	C20	**C18**
D25	C14	C14
D26	C23	**C20**
D27	C20	**C18**
D28	C14	**C20**
D29	C20	**C18**
D30	C14	C14
D31	C20	**C18**
D32	C14	**C20**
D33	C23	**C14**
D34	C20	**C18**
D35	C14	C14
D36	C20	C20
D37	C14	C14
D38	C24	**C20**
D39	C20	**C18**
D40	C14	C14

D41	C23	**C18**
D42	C24	**C20**
D43	C14	C14
D44	C20	**C18**
D45	C14	C14
D46	C1	**C11**
D47	C1	**C23**
D48	C20	**C18**
D49	C23	**C14**
D50	C14	**C20**
D51	C23	**C18**
D52	C20	C20
D53	C14	C14
D54	C20	**C18**
D55	C24	**C14**
D56	C20	**C18**
D57	C14	**C20**
D58	C24	**C14**
D59	C20	**C18**
D60	C14	C14
D61	C20	C20
D62	C14	C14
D63	C23	**C18**
D64	C20	**C14**
D65	C14	**C20**
D66	C1	**C18**
D67	C25	**C14**
D68	C23	**C20**
D69	C20	**C18**
D70	C14	C14
D71	C6	C6
D72	C6	**C25**
D73	C20	**C18**
D74	C23	**C14**
D75	C14	**C20**

In Test Case 2, 60 items from 6 product categories are chosen to be allocated at 75 temporary displays during the promotional period of 17 days. Promotional categories are cosmetics, middle section, meat2, dairy, cleaning supplies, and glassware. The total actual sales during the promotional period was **98,457.47 TRY** for Test Case 2. The cross-category sales of non-promotional product categories is estimated based on the actual data for the cross-category coefficients as **20,240.51 TRY**. The estimated space elasticity sales is **4,206.85 TRY** and the estimated cross-category sales of promotional products at temporary displays is **74,010.12 TRY**. Table 4.12 summarizes the revenue function for the actual layout.

Table 4.12. Actual Sales vs. the Mathematical Results (Test Case 2)

Revenue Components of the Objective Function	Actual Promotional Sales (TRY)	Calculated Sales by the proposed Mathematical Model for the Current Layout (TRY)	Absolute Proportion Difference
Space Elasticity Sales (Direct sales)	4,206.85	8,040.22	0.9112
Cross-Category Sales of Non-Promotional Product Categories (Cross-category Sales of Permanent Displays)	20,240.51	20,306.2	0.0032
Cross-Category Sales of Promotional Product Categories (Cross-category Sales of Temporary Displays)	74,010.12	65,212.31	0.1189
Total Sales	98,457.47	93,558.73	0.0498

For Test Case 2, the mathematical model generates 4.9% lower revenue than the actual sales during the promotional period. The mathematical model generates the space elasticity sales as **8,040.22 TRY** which is 91.2% higher than the estimated such sales. The cross-category sales of non-promotional product categories at permanent displays are **20,306.2 TRY**. This is 0.3% lower than the actual sales, so nearly in agreement. The cross–category sales of promotional product categories from our model is **65,212.31 TRY**. This is 11.9% lower than the estimated sales. A reason behind the difference is related to having some seasonal items within cleaning supplies, which are at the end of their season. Additionally, the impulse purchases are lower than the model expectations. Overall, the total sales of the model are 4.9 % lower than the actual sales. As the predicted total promotional sales are lower than actual sales, this result is similar to Test Case 1, where our model is conservative relative to the actual sales of Yunus. Table 4.12 shows the actual sales at the store. It also shows the calculated sales using the model vs. the actual sales for the existing layout of Test Case 2.

Since we do not know the promotional sales proportion within the actual sales data, we have calculated and derived promotional sales considering our model assumptions. Next, we have solved and reached the best solution to maximize promotional sales. For the best solution of

Test Case 2, the total revenue is **140,348.93 TRY**, which is 50% higher than the calculated sales by the proposed mathematical model. The best solution's revenue components for Test Case 2 are shown in Table 4.13. The space-elasticity sales are calculated as **7,932.95 TRY**, which is 1.3% lower than the calculated sales for the existing layout. This result shows that the total allocated areas of the product categories are similar. Cross-category sales of permanent displays are calculated as **37,855.06 TRY**, which is 8.6**%** higher than the calculated sales of the existing layout. Cross-category sales of temporary displays are **94,560.93 TRY**, which is 45**%** higher than the calculated sales of the current layout. The proposed layout expects more cross-category sales with the new allocation of promotional product categories at temporary displays as they better complement each other. Table 4.14 shows the best solution of the proposed mathematical model for Test Case 2.

Table 4.13. Current Promotional Sales vs. the Optimized Solution of Model Results for Test Case 2

Revenue Components of the Objective Function	**Actual Promotional Sales (TRY)**	**Calculated Sales by the proposed Mathematical Model for the Current Layout (TRY)**	**Absolute Proportion Difference**
Space Elasticity Sales (Direct sales)	8,040.22	7,932.95	-0.013
Cross-Category Sales of Non-Promotional Product Categories (Cross-category Sales of Permanent Displays)	20,306.2	37,855.06	0.864
Cross-Category Sales of Promotional Product Categories (Cross-category Sales of Temporary Displays)	65,212.31	94,560.93	0.450
Total Sales	93,558.73	140,348.93	0.500

Table 4.14. Product Category Allocations of the Optimized Solution for Test Case 2

Temporary Display No	Current Allocation (Product Category No)	Proposed Layout (Product Category No)
D1	C20	C20
D2	C18	C18
D3	C11	**C14**
D4	C23	**C18**
D5	C11	**C14**
D6	C23	**C18**
D7	C20	C20
D8	C25	**C14**
D9	C23	**C18**
D10	C23	**C14**
D11	C20	C20
D12	C23	**C14**
D13	C20	C20
D14	C23	**C18**
D15	C23	**C14**
D16	C18	C18
D17	C20	C20
D18	C11	**C14**
D19	C18	C18
D20	C14	C14
D21	C11	C11
D22	C25	C25
D23	C20	C20
D24	C18	C18
D25	C14	C14
D26	C20	C20
D27	C18	C18
D28	C20	C20
D29	C18	C18
D30	C14	C14
D31	C18	C18
D32	C20	C20
D33	C14	C14
D34	C18	C18
D35	C14	C14
D36	C20	C20
D37	C14	C14
D38	C20	C20
D39	C18	C18
D40	C14	C14
D41	C25	**C18**

D42	C20	C20
D43	C14	C14
D44	C25	**C18**
D45	C14	C14
D46	C11	C11
D47	C25	C25
D48	C25	**C18**
D49	C14	C14
D50	C20	C20
D51	C25	**C18**
D52	C14	C14
D53	C20	C20
D54	C23	**C18**
D55	C14	C14
D56	C23	**C18**
D57	C20	C20
D58	C18	C18
D59	C14	C14
D60	C18	C18
D61	C14	C14
D62	C20	C20
D63	C18	C18
D64	C14	C14
D65	C20	C20
D66	C18	C18
D67	C14	C14
D68	C20	C20
D69	C18	C18
D70	C14	C14
D71	C23	C23
D72	C23	C23
D73	C18	C18
D74	C14	C14
D75	C20	C20

The differences between the actual sales and the total sales of the objective function of the model from the mathematical model for Test Case 1 and Test Case 2 are 1.6% and 4.98%, respectively. These results show that the mathematical model mimics actual sales with less than the 5% difference. The differences between the actual direct sales in the store and the proposed mathematical model are 10.7% and 91.1% for Test Case 1 and Test Case 2, respectively. Since overall store space-elasticity sales may not be well predicted, we recommend customizing the product category specific space-elasticity parameters to a particular store or store type. For the actual derived cross-category sales at permanent displays vs. those solved from the model, differences are 10.8% and 0.32% for Test Case 1 and Test Case 2, respectively. These differences are very modest. For the actual derived cross-category sales at temporary displays vs. those solved from the model, differences are 3.9% and 11.8% for Test Case 1 and Test Case 2, respectively. Again, very modest differences. Although promotional sales depend on item prices, seasonality, brand, and customer purchase behavior, the proposed mathematical model does not include these parameters. Therefore, these variations of cross-category sales in Test Case 1 and Test Case 2 are acceptable.

Table 4.15. The Model's Solution vs. the Actual Derived Sales for the Existing Layout (Test Case 3)

Revenue Components of the Objective Function	**Actual Promotional Sales (TRY)**	**Calculated Sales by the proposed Mathematical Model for the Current Layout (TRY)**	**Absolute Proportion Difference**
Space Elasticity Sales (Direct sales)	8,352.15	8,352.15	-
Cross-Category Sales of Non-Promotional Product Categories (Cross-category Sales of Permanent Displays)	35,895.79	49,427.20	0.38
Cross-Category Sales of Promotional Product Categories (Cross-category Sales of Temporary Displays)	30,025.00	42,374.36	0.41
Total Sales	74,272.86	100,153.70	0.35

In Test Case 3, there are 60 promotional items from 8 product categories during the promotional period of 17 days. The promotional product categories are shoes, electronics, cosmetics, middle section, meat2, dairy, cleaning supplies, and glassware. In the actual data, the total promotional sales is **77,936.98 TRY** during the promotional period. Once we applied the current store allocation in our objective function, the total promotional sales is **74,272.86 TRY**. Our model conversely predicts the total promotional sales. Table 4.15 summarizes the space elasticity sales (**8,352.15 TRY**), the non-promotional product category sales (**35,895.79 TRY**), and promotional cross-category sales (**30,025 TRY**) of Test Case 3. Use of our model improves these sales by 35%.

For the Test Case 3's optimized layout, the total sales are **100,153.70 TRY**, which is 35 % higher than the actual derived sales from the existing layout. We have shown the best result's revenue components for Test Case 3 in Table 4.15. We have calculated the space-elasticity sales as **8,352.15 TRY**. Since the size of the allocated product categories are the same as in the current layout, the space elasticity sales are not changed in the optimized solution. We have calculated the cross-category sales at permanent displays as **49,427.20 TRY**, which is improved 38% more than the current layout's actual derived sales. The cross-category sales at temporary displays are **42,374.36 TRY**, which is improved 41% more than the current layout's actual derived sales. The actual store allocation and the proposed mathematical model's optimized solution for Test Case 3 is shown in Table 4.16. If a temporary display has a different product category than the current layout, it is highlighted in bold.

Table 4.16. Product Category Allocations of the Optimized Solution for Test Case 3

Temporary Display No	Current Allocation (Product Category No)	Proposed Layout (Product Category No)
D1	C1	**C11**
D2	C1	C1
D3	C11	C11
D4	C11	**C1**
D5	C6	C6
D6	C14	**C20**
D7	C14	C14
D8	C18	C18
D9	C18	**C20**
D10	C20	**C23**
D11	C20	**C11**
D12	C11	**C18**
D13	C18	**C20**
D14	C21	C21
D15	C13	**C14**
D16	C23	**C13**
D17	C13	C13
D18	C14	C14
D19	C25	C25
D20	C1	C1
D21	C25	C25
D22	C23	C25

D23	C20	C23
D24	C23	**C18**
D25	C25	**C23**

In conclusion, three test cases show similar revenue components results for the proposed model's objective function. The three cases' space elasticity sales are higher than the actual sales for the space elasticity sales. This result points out that it is necessary to have a detailed study for the product category specific space elasticity coefficients of the selected store. The difference between the cross-category sales on permanent displays and temporary displays by proposed model vs. the actual sales are in acceptable ranges. The best solutions in all cases provide 22%, 50%, and 35% improvements for the current layouts.

4.4. The Mathematical Model Split Product Categories during a Single Promotional Period

In this section, we present the split product category model for the allocation problem at temporary displays. This model is a mixed integer nonlinear mathematical model where multiple product categories are allowed in a temporary display. Again, we investigate using the Yunus store's sales data. At a temporary display, the number of product categories is not generally more than three product categories in our retail partner. From the store manager's experience, if a customer sees more than three product categories in a display, the customer may get distracted. Thus, he/she might be less likely to purchase an item from the display. The product categories are horizontally separated on a temporary display to attract customers. In order to represent this constraint of the problem, we propose a split mathematical model for the temporary display allocation problem which allows multiple product categories within one display.

The formulation has continuous and binary decision variables to choose the allocation areas and product category locations. We explain the proposed mathematical model assumptions, parameters, decision variables, objective functions, and constraints in the following sections. Realistically sized problems have more than 6 product categories to allocate at 75 temporary displays for a promotional period. The split model is solved by the GAMS program, a leading commercial nonlinear solver. We discuss the real store data vs. model results comparison in subsequent sections.

First, we list the constraints.

Constraint 1: At least one product category must be allocated to a temporary display.

Constraint 2: A product category's total allocated area should be greater than or equal to the product category's lower area bound.

Constraint 3: A product category's total allocated area should be less than or equal to the product category's upper area bound.

Constraint 4: The number of product categories at a temporary display can not exceed three.

Constraint 5: If a product category is assigned to a temporary display, it cannot be less than 25% of the area.

The main differences of the second model from the first model are the number of product categories in a display (Constraint 4) and the product category area at temporary displays (Constraint 5). For multiple product categories at a temporary display, it is necessary to create a new model with two types of decision variables. The first decision variable type is similar to the previous mathematical model. In the first model, we have a Y_{ik} binary variable that represents where to allocate product category *i* to temporary display *k*. If product category *i* is allocated at temporary display *k*, Y_{ik} will be 1, otherwise 0. In the previous model, is only one product category per temporary display. However, the second model assumes that the maximum number of product categories per temporary display is three. The second decision variables type is a continuous variable between 0 and 1. It shows the product category proportion at a temporary display (X_{ik}). Based on the store experience, it is common to allocate at least 25% of the temporary display for a product category, hence constraint 5. That is almost equal to one shelf of a typical temporary display.

All parameter definitions are the same as the previous mathematical formulation. They are listed in Table 4.17 and Table 4.18.

Table 4.17. Set Definitions for the Split Model

	Notation	Definition	Data	Range of Data
Sets	$\boldsymbol{C}$	Set of candidate product categories	The chosen promotional product categories in the promotional period	1 - the number of product categories
	$\boldsymbol{D}$	Set of temporary displays	Layout of the store	The number of temporary displays (the number of endcaps and the number of island displays)
	$\boldsymbol{P_k}$	Set of product categories that are allocated to permanent displays adjacent to temporary display *k*	Store layout	1 - subset of all product categories in the store
	$\boldsymbol{F_k}$	Set of product categories that should not be allocated at temporary display *k*	Store policies, such as health, brand agreements, etc.	0 - (the number of product categories-1)
	$\boldsymbol{T_k}$	Set of temporary displays that are adjacent to temporary display *k*	Store layout	1- all temporary displays

Table 4.18. Parameter Definitions for the Split Model

	Notation	Definition	Data	Range of Data
Parameters	$\boldsymbol{a_{ki}}$	Total area of product category *i* at permanent displays that are adjacent to temporary display *k*	Store layout	0 - 20 m^2
	$\boldsymbol{b_i^L}$	Lower bound of product category *i*'s allocated area in all temporary displays (square meters)	Store policies	0 - the largest s_k
	$\boldsymbol{b_i^U}$	Upper bound of product category *i*'s allocated area in all temporary displays (square meters)	Store policies	The smallest s_k - all temporary displays
	$\boldsymbol{e_{ij}}$	Cross-category sale	Historical data	0 - 1

		effect between product category *i* and product category *j* (ratio)	analysis	
	m_{ki}	Traffic density of product category *i* at adjacent permanent display's temporary display *k* (ratio)	Historical data analysis	0.1 - 1
	n_k	Traffic density at temporary display *k* (ratio)	Historical data analysis	0.01 - 1
	r_i	Average revenue of product category *i* per square meter for a promotional period	Historical data analysis	0 - 30,000 TRY
	s_k	Total area of temporary display *k* (square meters)	Store layout	0 - 6 m^2
	w_i	Impulse purchase rate of product category *i* (ratio)	Historical data analysis and Bell et al. (2009)	0 - 0.35
	β_i	Space elasticity of product category *i* (ratio)	Corstjens and Doyle (1981)	0 - 1

Maximize

Total Promotional Revenue=

$$\sum_{i\in C} w_i r_i \left\{\sum_{k\in D} s_k X_{ik}\right\}^{\beta_i}$$
$$+\sum_{j\in C}\sum_{k\in D}\sum_{i\in P_k} m_{ki}(r_i a_{ki}) e_{ij}(s_k X_{jk}) + \sum_{i\in C}\sum_{\substack{j\in C\\ i\neq j}}\sum_{k\in D}\sum_{v\in T_k} n_k r_i (s_v X_{iv}) e_{ij}(s_k X_{jk})$$
$$+\sum_{i\in C}\sum_{\substack{j\in C\\ i\neq j}}\sum_{k\in D} n_k r_i (s_k X_{ik})\, e_{ij}(s_k X_{jk}) \qquad \text{Equation 4.16}$$

Equation 4.16 represents the allocation model's objective function for temporary displays with split product categories to maximize the revenue. In the split model, the objective function components are area related revenue, cross-category sales between temporary and adjacent permanent displays, cross-category sales between neighboring temporary displays, and cross-category sales within a temporary display (this last component is new from the previous models).

Although the split model components are similar to the non-split model, it is necessary to alter the appropriate decision variables for the revenue components. In the first component, it is necessary to calculate the product category's total allocated area by multiplication of the temporary display total area and the product category's proportion at the temporary display *k* ($s_k X_{ik}$). For the rest of the objective function, we replace the term $s_k Y_{jk}$ with the term $s_k X_{jk}$ to calculate the cross-category sales at permanent displays.

Subject to:

$$\sum_{i \in C} Y_{ik} \geq 1 \quad \forall k \in D \qquad \textit{Equation 4.17}$$

$$\sum_{k \in D} X_{ik} s_k \geq b_i^L \quad \forall i \in C \qquad \textit{Equation 4.18}$$

$$\sum_{k \in D} X_{ik} s_k \leq b_i^U \quad \forall i \in C \qquad \textit{Equation 4.19}$$

$$\sum_{k \in D} \sum_{i \in F_k} Y_{ik} = 0 \qquad \textit{Equation 4.20}$$

$$\sum_{i \in C} Y_{ik} \leq 3 \quad \forall k \in D \qquad \textit{Equation 4.21}$$

$$X_{ik} \leq Y_{ik} \quad \forall\, i \in C, k \in D \qquad \textit{Equation 4.22}$$

$$X_{ik} \geq 0.25 Y_{ik} \quad \forall\, i \in C, k \in D \qquad \textit{Equation 4.23}$$

$$Y_{ik} \in \{0,1\} \quad \forall i \in C, k \in D \qquad \textit{Equation 4.24}$$

$$0 \leq X_{ik} \leq 1 \; \forall i \; \in C, k \in D \qquad \textit{Equation 4.25}$$

The second mathematical model's first four constraints are similar to the first mathematical model.

Constraint 1 (Equation 4.17) represents that at least one product category should be assigned to each temporary display.

Constraints 2 and 3 (Equations 4.18 and 4.19) state that a product category's lower and upper area bounds must be satisfied.

Constraint 4 (Equation 4.20) restricts any product categories that will not be assigned to temporary displays during the selected promotional period.

Constraint 5 (Equation 4.21) assures that the maximum number of product categories at temporary displays is three.

Constraint 6 (Equation 4.22) is a connection constraint between the binary and continuous decision variables. If product category *i* is not selected to be assigned at temporary display *k* ($Y_{ik} = 0$), then the corresponding continuous variable will be zero (X_{ik}=0). If product category *i* is assigned at temporary display *k* ($Y_{ik} = 1$), then the corresponding continuous variable will be greater than zero ($X_{ik} \leq 1$). Since this constraint does not restrict the assigned product category total area, it is necessary to have another constraint to set the minimum area.

Constraint 7 (Equation 4.23) ensures that at least 25% of the temporary display is assigned to a given product category.

Constraints 8 and 9 (Equations 4.24 and 4.25) represent the binary and continuous decision variable definitions.

4.5. Test Results with Split Product Categories during a Single Promotional Period

The academic version of GAMS can only solve 25 product categories and 25 temporary displays for the mathematical model with split product categories we use Test Case 3 with 25 temporary displays. The promotional product categories are shoes, cosmetics, middle section, meat2, dairy, cleaning supplies and glassware. There are 75 temporary displays during the promotional period, but the academic version of GAMS program is only able to solve up to 25 temporary displays. After discussions with the store manager, we decided to solve the test problem with only endcap displays and the five temporary displays which are close to the cashiers.

We have solved the split model for Test Problem 3 with 25 temporary displays. The best promotional sales are found as **117,659.28 TRY**. This is 51% higher than the actual promotional sales for the existing layout for Test Case 3 (**77,936.98 TRY**). Since the split model allows multiple product categories, we expect to have more cross-category sales opportunities. Table 4.19 shows the displays allocations for the split model.

Table 4.19. The Optimal Layout Product Category Allocations for Test Case 1 Using 25 Temporary Displays

Temporary Display No	Product Category No in Split-Model	Allocation Ratio
D1	C18	0.50
D1	C20	0.50
D2	C14	1
D3	C20	1

D4	C14	1
D5	C18	0.50
D5	C23	0.25
D5	C25	0.25
D6	C18	1
D7	C14	1
D8	C20	1
D9	C14	1
D10	C18	0.55
D10	C20	0.45
D11	C14	1
D12	C18	1
D13	C1	0.25
D13	C6	0.25
D13	C14	0.50
D14	C20	1
D15	C14	0.75
D15	C20	0.25
D16	C6	0.25
D16	C11	0.25
D16	C20	0.50
D17	C1	0.25
D17	C11	0.25
D17	C18	0.50
D18	C14	1
D19	C11	0.25
D19	C20	0.50
D19	C25	0.25
D20	C14	0.70
D20	C23	0.30
D21	C1	0.25
D21	C6	0.25
D21	C18	0.50
D22	C18	0.43
D22	C23	0.32
D22	C25	0.25
D23	C20	1
D24	C14	1
D25	C20	1

The objective function values of current allocation, non-split model, and split model are compared in Table 4.20. The non-split model promises 29% promotional sales improvement while split model promises 50% promotional sales improvement.

Table 4.20. Promotional Sales Comparison for Test Case 3

	Current Allocation	Non-Split Model	Split Model
Total Promotional Sales	77,936.98 TRY	100,153.70 TRY	117,659.28 TRY
Improvement		29%	51%

4.6. Conclusion

In this chapter, we introduced mathematical models for the temporary display allocation problem during a single promotional period. The temporary displays aim to attract purchases while customers are collecting their planned purchases throughout the store. In these models, our goal is to identify the temporary display locations with the highest revenue for a set of preselected product categories during a promotional period. We assume that there will be only one product category in a temporary display in the first model. In the second model, we assume that there may be up to three product categories. With the same goal in both mathematical models, there are some different variables and constraints to accommodate splitting and non-splitting the temporary display. The comparison between actual sales and the objective function values shows that the mathematical models mimic the sales data very closely. Three real sized test problems were solved using the non-split mathematical model. There were quite significant improvements in revenue when the layouts were optimized.

GAMS Optimization Package offers multiple solvers for MINLPs. We solved the test problems by using BONMIN (Bonami and Lee, 2009), DICOPT (Grossmann et al., 2002), and LINDOGLOBAL (Bussieck and Vigerske, 2010). We ran the test problems with these solvers individually for both 1000 seconds and 1500 seconds. We obtained the same objective function values for all test problems. The results for the single period test problems are listed in Table

4.21. Extending beyond 1000 seconds had no effect on the solutions identified for any of the four runs, so we maintained 1000 seconds as the computational limit.

Table 4.21. Results for the Single-Period Models

Mathematical Model and Test Problem	**DICOPT/ BONMIN/ LINDOGLOBAL (Run time=1000sec)**	**Run time (1500sec)**
Single Period Non-Split Model Test Problem I (25 Product Categories X 75 Temporary Displays)	140,708.59 – Best Integer Solution found	There is no improvement in the objective function.
Single Period Non-Split Model Test Problem II (25 Product Categories X 75 Temporary Displays)	140,348.93 – Best Integer Solution found	There is no improvement in the objective function.
Single Period Non-Split Model Test Problem III (25 Product Categories X 75 Temporary Displays)	100,153.70 – Best Integer Solution found	There is no improvement in the objective function.
Single Period Split Model Test Problem I (25 Product Categories X 25Temporary Displays)	**117,659.28 TRY – Optimal Solution found**	Since it is optimal, there is no need to run more.

GAMS can solve the MINLP with up to 2500 variables and 2500 constraints. The test case with 25 product categories and 25 temporary displays has 1351 constraints and 1250 decision variables. The test case with 25 product categories and 50 temporary displays has 2651 constraints and 2500 decision variables. Due to GAMS problem size constraints, only one real size test problem was solved using the split mathematical model and this one has 25 temporary displays which only includes endcaps and island displays closer to the entrance. The store manager felt this problem would be the best reflection of the real life split problem. The next chapter will consider allocation models for temporary displays for two promotional periods.

Chapter 5 The Allocation Problem for Temporary Displays for Two Consecutive Promotional Periods

The mathematical models in the previous chapter allocate promotional product categories at temporary displays during a single promotional period. In addition to displaying the promotional items, the store chain management advertises new promotional items on billboards. The research question is where to allocate these billboard items at temporary displays while considering also retaining some of the promotional items from the initial period. In this chapter, we propose non-split and split nonlinear mathematical models for the temporary display allocation problem during two consecutive promotional periods (the first period being the conventional one while the second period includes the extra products being promoted on the new billboard advertisements). The new formulations are proposed as a two-period temporary display allocation problem at the product category level. The objective functions maximize revenue by allocating the product categories to temporary displays for two promotional periods. The constraints of these models are similar to the single promotional period models in Chapter 4 and include area restrictions that should be satisfied for temporary displays and product categories. Furthermore, there are product categories that must not be allocated to the specified temporary displays due to safety and health reasons. Additionally, some products from the first period must remain in their assigned locations. This may happen because of contractual arrangements with the product vendors or because rearranging some items is too time consuming. Each temporary display should be allocated to only one product category in the non-split model and to multiple product categories in the split model.

5.1. The Mathematical Formulation of the Two-Period Temporary Display Allocation Problem for Non-Split Product Categories (Model-2A)

In this section, we propose a two-step solution methodology for the two-period temporary display allocation problem. As a first step, we solve the first promotional period problem with the mathematical model of the single period temporary display allocation problem from Chapter 4. Then, the solution of the single period allocation problem will be added as constraints and additional constraints to portray the interactions between periods are added to the new formulation. Items retained from the previous period must be retained at or above their minimum allocated area. The objective function maximizes the revenue of allocating the billboard items to temporary displays and considers the loss revenue when product categories are removed from temporary displays. There is no explicit cost with moving a product category from one display to another.

Decision variables that represent where to allocate the product categories during periods are defined as below.

$$Y1_{ik} = \begin{cases} 1 \text{ if the product category } i \text{ is allocated to tempoary display } k \text{ during the first period} \\ 0 \text{ otherwise} \end{cases}$$

$$Y2_{ik} = \begin{cases} 1 \text{ if the product category } i \text{ is allocated to temporary display } k \text{ during the second period} \\ 0 \text{ otherwise} \end{cases}$$

In addition to these decision variables, it is necessary to address the decisions of which temporary displays are changed in the two-periods. The following decision variables are added to the formulation.

$$U_{ik} = \begin{cases} 1 \text{ if the product category } i \text{ is removed from tempoary display } k \text{ during the second period} \\ 0 \text{ otherwise} \end{cases}$$

$$Q_{ik} = \begin{cases} 1 \text{ if the product category } i \text{ is added to temporary display } k \text{ during the second period} \\ 0 \text{ otherwise} \end{cases}$$

The first three revenue components in the objective function are similar to the revenue components from Chapter 4. The first component represents direct sales. The second component is for the total cross-category sales between a product category at a temporary display and the product categories at its adjacent permanent displays. The third component shows the total of cross-category sales between a product category at a temporary display and the product categories at its adjacent temporary displays.

Maximize

$$\begin{aligned} &\sum_{i \in C} w_i r_i \left\{ \sum_{k \in D} s_k Y2_{ik} \right\}^{\beta_i} + \sum_{j \in C} \sum_{k \in D} \sum_{i \in P_k} m_{ki} (r_i a_{ki}) e_{ij} (s_k Y2_{jk}) \\ &+ \sum_{i \in C} \sum_{\substack{j \in C \\ i \neq j}} \sum_{k \in D} \sum_{v \in T_k} n_k r_i (s_v Y2_{iv}) e_{ij} (s_k Y2_{jk}) - \sum_{i \in C} w_i r_i \left\{ \sum_{k \in D} s_k U_{ik} \right\}^{\beta_i} \\ &- \sum_{j \in C} \sum_{k \in D} \sum_{i \in P_k} m_{ki} (r_i a_{ki}) e_{ij} (s_k U_{jk}) \\ &- \sum_{i \in C} \sum_{\substack{j \in C \\ i \neq j}} \sum_{k \in D} \sum_{v \in T_k} n_k r_i (s_v Y2_{iv}) e_{ij} (s_k U_{jk}) \qquad \textit{Equation } 5.1 \end{aligned}$$

In the objective function (Equation 5.1), the revenue function is modified by considering that some product categories are removed from the temporary display in the second period and thus that revenue would be lost. Thus, we term these costs of removing a product category. If we remove the product category from its assigned display of the first promotional period, we assume that we will lose its promotional revenue. Thus, the last three components reflect the cost of removing the product categories after the first promotional period. The objective function of the new formulation is represented as follows:

The constraints of the two-period temporary display allocation problem for non-split product categories are mainly the same as in Chapter 4. In addition to those, the solution of the single period problem adds one constraint. The set of constraints for the interaction between the periods are addressed in the last constraint.

The constraint that each temporary display should be allocated to one product category should be satisfied for both periods (Equation 5.2 and Equation 5.3).

$$\sum_{i \in C} Y1_{ik} = 1 \quad \forall k \in D \qquad \textit{Equation } 5.2$$

$$\sum_{i \in C} Y2_{ik} = 1 \quad \forall k \in D \qquad \textit{Equation } 5.3$$

The lower and upper area bounds of each product category should be satisfied for both periods (Equations 5.4, 5.5, 5.6, and 5.7) however the first period may have different area bounds than the second period.

$$\sum_{k \in D} Y1_{ik} s_k \geq b1_i^L \quad \forall i \in C \qquad \textit{Equation } 5.4$$

$$\sum_{k \in D} Y1_{ik} s_k \leq b1_i^U \quad \forall i \in C \qquad \textit{Equation } 5.5$$

$$\sum_{k \in D} Y2_{ik} s_k \geq b2_i^L \quad \forall i \in C \qquad \textit{Equation } 5.6$$

$$\sum_{k \in D} Y2_{ik} s_k \leq b2_i^U \quad \forall i \in C \qquad \textit{Equation } 5.7$$

Product categories that should not be allocated to the specified temporary displays are restricted by the following constraints (Equations 5.8 and 5.9).

$$\sum_{k \in D} \sum_{i \in F_k} Y1_{ik} = 0 \qquad \textit{Equation } 5.8$$

$$\sum_{k \in D} \sum_{i \in F_k} Y2_{ik} = 0 \qquad Equation\ 5.9$$

The solution of the single period allocation problem is added as a constraint for the two-period allocation problem (Equation 5.10). The sets of the single period solution where $Y1_{ik}$s equal to 1 are labeled as C_1 *and* D_1. The summation of the allocated product categories in all temporary displays in the first promotional period should be the number of temporary displays that is shown as *d*.

$$\sum_{i \in C_1} \sum_{k \in D_1} Y1_{ik} = d \qquad Equation\ 5.10$$

The new constraint to consider temporary displays that change product categories between promotional periods is listed below (Equation 5.11). Assume that product category *i* is allocated to temporary display *k* for the first period ($Y1_{ik} = 1$) and product category *i* is removed from temporary display *k* before the second period ($Y2_{ik} = 0$). Then the removed product category *i* at temporary display *k* becomes 1 (U_{ik}=1). If product category *i* is not allocated to temporary display *k* for the first period ($Y1_{ik} = 0$) and product category *i* is allocated to the temporary display *k* for the second period ($Y2_{ik} = 1$), then the replaced product category *i* at temporary display *k* becomes 1 (Q_{ik}=1). Lastly, product categories at temporary displays during the first period that are remaining in place for the second period have Q_{ik} and U_{ik} equal to 0.

$$Q_{ik} - U_{ik} = Y2_{ik} - Y1_{ik} \qquad \forall i \in C, k \in D \qquad Equation\ 5.11$$

This model will satisfy the area constraints and seek higher revenue by replacing the product categories from the first promotional period during the second period. It is possible to see a solution with numerous changes in the solution for the second period. The store manager seeks locations for the product categories during the second period, but some product categories must remain in their same locations and not be moved or removed or resized. The parameter *g* is defined as the number of promotional products during the second period. The following constraint restricts that the total changes to temporary displays during the second period should be equal to parameter *g* (Equation 5.12).

$$\sum_{i \in C} \sum_{k \in D} Q_{ik} = g \qquad Equation\ 5.12$$

The last constraints are the definitions of the decision variables, which are binary decision variables (Equations 5.13, 5.14, 5.15, and 5.16).

$$Y1_{ik} \in \{0,1\} \qquad \forall i \in C, k \in D \qquad Equation\ 5.13$$

$$Y2_{ik} \in \{0,1\} \qquad \forall i \in C, k \in D \qquad Equation\ 5.14$$

$$Q_{ik} \in \{0,1\} \qquad \forall i \in C, k \in D \qquad Equation\ 5.15$$

$$U_{ik} \in \{0,1\} \qquad \forall i \in C, k \in D \qquad Equation\ 5.16$$

5.2. Test Case for Model-2A

In this test case, we choose the best temporary displays for product categories during the second promotional period while considering removing or replacing the existing product categories at these temporary displays that were placed there for the first promotional period. In the second promotional period, some of these temporary displays retain the product categories that were allocated during the first promotional period, and some will be replaced with different product categories. Our goal is to find the most valuable locations for product categories during the second promotional period while considering removing the product categories at temporary displays. Table 5.1 shows the promotional product categories and their area bounds for the first promotional period. The additional promotional product categories and the updated area bounds for period 2 are shown in Table 5.2.

Table 5.1. Area Bounds of Promotional Product Categories for the First Promotional Period

Product Categories	Lower Area Bounds of Product Categories	Upper Area Bounds of Product Categories
C9	0.00	12.00
C10	11.25	22.50
C11	22.50	45.00
C14	33.75	67.50
C16	0.00	12.00
C20	22.50	45.00
C23	67.50	90.00
C25	22.50	45.00

Table 5.2. Area Bounds of Promotional Product Categories for the Second Promotional Period

Product Categories	Lower Area Bounds of Product Categories	Upper Area Bounds of Product Categories
C5	0.00	**15.0**
C9	0.00	**15.0**
C10	11.25	**25.5**
C11	22.50	**48.0**
C13	0.00	**9.0**
C14	33.75	**71.5**
C16	0.00	**15.0**
C18	0.00	**15.0**
C20	22.50	**48.0**
C22	0.00	**12.0**
C23	67.50	**93.0**
C25	22.50	**48.0**

Since there are 10 additional products in the new promotional period, the store manager wants to replace the product categories of 10 temporary displays. This constraint is added to the multi-period layout model and the model aims to increase the revenue by replacing the product categories of 10 temporary displays for the second period.

Table 5.3. Replaced Product Categories of Temporary Displays from the First Period to the Second Period

The First Promotional Period		The Second Promotional Period	
Product Category No	Temporary Display No	**Product Category No**	Temporary Display No
C14	D3	**C18**	D3
C14	D32	**C20**	D32
C20	D39	**C21**	D39
C20	D40	**C14**	D40
C20	D42	**C13**	D42
C9	D49	**C20**	D49
C14	D50	**C9**	D50
C14	D58	**C5**	D58
C9	D6	**C21**	D6
C14	D9	**C20**	D9

In the first promotional period, the best-found objective function is **147,555.88 TRY**. In the second promotional period, the best-found objective function is **169,633.12 TRY**. The improvement rate of the proposed re-layout is 14.96%. Remember this is for the non-split model-that is, there is only one product category per temporary display. The changes to the product categories at temporary displays are summarized in Table 5.3. The product categories at the rest of the temporary displays during the second promotional period remain the same as the first promotional period. Table 5.4 shows the product categories at temporary displays during the first and second promotional periods.

Table 5.4. The Allocations of Product Categories for the First and Second Promotional Periods

The First Promotional Period Allocations		The Second Promotional Period Allocations		Changes in the Second Period	
Product Category No	**Temporary Display No**	**Product Category No**	**Temporary Display No**	**Removed Categories**	**Added Categories**
C16	D1	**C16**	D1		
C23	D10	**C23**	D10		
C25	D11	**C25**	D11		
C23	D12	**C23**	D12		
C25	D13	**C25**	D13		
C23	D14	**C23**	D14		
C14	D15	**C14**	D15		
C13	D16	**C13**	D16		
C13	D17	**C13**	D17		
C14	D18	**C14**	D18		
C23	D19	**C23**	D19		
C23	D2	**C23**	D2		
C25	D20	**C25**	D20		
C11	D21	**C11**	D21		
C10	D22	**C10**	D22		
C23	D23	**C23**	D23		
C14	D24	**C14**	D24		
C23	D25	**C23**	D25		
C16	D26	**C16**	D26		
C11	D27	**C11**	D27		
C23	D28	**C23**	D28		
C14	D29	**C14**	D29		
C14	D3	**C18**	D3	**C14 Removed**	**C18 Added**
C23	D30	**C23**	D30		
C9	D31	**C9**	D31		
C14	D32	**C20**	D32	**C14 Removed**	**C20 Added**
C23	D33	**C23**	D33		
C14	D34	**C14**	D34		
C23	D35	**C23**	D35		
C20	D36	**C20**	D36		
C20	D37	**C20**	D37		
C20	D38	**C20**	D38		
C20	D39	**C21**	D39	**C20 Removed**	**C21 Added**

C23	D4	**C23**	D4		
C20	D40	**C14**	D40	**C20 Removed**	**C14 Added**
C20	D41	**C20**	D41		
C20	D42	**C13**	D42	**C20 Removed**	**C13 Added**
C14	D43	**C14**	D43		
C20	D44	**C20**	D44		
C14	D45	**C14**	D45		
C11	D46	**C11**	D46		
C10	D47	**C10**	D47		
C23	D48	**C23**	D48		
C9	D49	**C20**	D49	**C9 Removed**	**C20 Added**
C25	D5	**C25**	D5		
C14	D50	**C9**	D50	**C14 Removed**	**C9 Added**
C11	D51	**C11**	D51		
C11	D52	**C11**	D52		
C23	D53	**C23**	D53		
C14	D54	**C14**	D54		
C23	D55	**C23**	D55		
C9	D56	**C9**	D56		
C23	D57	**C23**	D57		
C14	D58	**C5**	D58	**C14 Removed**	**C5 Added**
C23	D59	**C23**	D59		
C9	D6	**C21**	D6	**C9 Removed**	**C21 Added**
C25	D60	**C25**	D60		
C23	D61	**C23**	D61		
C25	D62	**C25**	D62		
C23	D63	**C23**	D63		
C14	D64	**C14**	D64		
C23	D65	**C23**	D65		
C10	D66	**C10**	D66		
C11	D67	**C11**	D67		
C14	D68	**C14**	D68		
C23	D69	**C23**	D69		
C14	D7	**C14**	D7		
C25	D70	**C25**	D70		

C11	D71	**C11**	D71		
C10	D72	**C10**	D72		
C11	D73	**C11**	D73		
C23	D74	**C23**	D74		
C25	D75	**C25**	D75		
C23	D8	**C23**	D8		
C14	D9	**C20**	D9	**C14 Removed**	**C20 Added**

5.3. The Mathematical Formulation of the Two-Period Temporary Display Allocation Problem for Split Product Categories (Model-2B)

In this section, we introduce a new formulation to the solution of the temporary display allocation problem with split product categories during two consecutive promotional periods. In both promotional periods, multiple product categories can be allocated to a single temporary display. The new formulation is proposed as a two-period temporary display allocation problem at the product category level. In the formulation of the two-period allocation problem, the solution of the single period allocation problem is added as constraints along with additional constraints to portray the interactions between periods. The objective function of the two-period allocation problem is to maximize the revenue of allocating the billboard items to temporary displays and consider the lost revenue of removing product categories already at temporary displays. Decision variables that represent where to allocate the product categories for each period are defined as below. Similar to the first split mathematical model of Chapter 4, there are continuous decision variables for the total allocated area of product category *i* at temporary display k ($Xnew_{ik}$) and discrete decision variables for the location of the product categories ($Ynew_{ik}$).

$$Ynew_{ik} = \begin{cases} 1 \text{ if the product category } i \text{ is allocated to temporary display } k \text{ for the second period} \\ 0 \text{ otherwise} \end{cases}$$

The two-period split allocation model considers the product category allocations in the previous promotional period and these allocations are added as the parameters in the model. Continuous decision variables, which explain the total area allocations of product categories at temporary displays, are labeled as parameter of X_{ik} and the binary decision variables, which state where to allocate the product categories at temporary displays, are shown by parameter of

Y_{ik}. The cost of loss revenue from removing product categories at temporary displays is evaluated in the objective function.

Two additional binary decision variables are added to the new model to specify the replacing and the replaced product categories at temporary displays. The replaced product categories are represented by Q_{ik} and the removed product categories are represented by U_{ik}. The replacing product categories are defined as the product categories that were not allocated at the selected temporary display for the first promotional period but are in the second. As an example, product category *i* is allocated at the temporary display *k* and the allocation ratio is 0.4 ($X_{ik} = 0.4\ and\ Y_{ik}$ =1) for the first promotional period. It is possible assign product category *i* at temporary display *k* with a higher ratio such as 0.6 ($XNew_{ik} = 0.6\ and\ YNew_{ik} = 1$). In this case, decision variables $Q_{ik}\ and\ U_{ik}$ will be zero in the solution because product category *i* is not removed from the temporary display nor newly added to the temporary display *k*.

$$Q_{ik} = \begin{Bmatrix} 1\ if\ product\ category\ i\ is\ added\ to\ temporary\ display\ k\ for\ the\ second\ period \\ 0\ otherwise \end{Bmatrix}$$

$$U_{ik} = \begin{Bmatrix} 1\ if\ product\ category\ i\ is\ removed\ from\ temporary\ display\ k\ for\ the\ second\ period \\ 0\ otherwise \end{Bmatrix}$$

The objective function maximizes the revenue by allocating new product categories and removing existing product categories at temporary displays. The value of the current solution is calculated by Equation 5.17. The objective function of the new model subtracts the current solution value from the new product category allocation.

$$Current\ Allocation = \sum_{i\in C} w_i r_i \left\{\sum_{k\in D} s_k X_{ik}\right\}^{\beta_i} + \sum_{j\in C}\sum_{k\in D}\sum_{i\in P_k} m_{ki}(r_i a_{ki}) e_{ij}(s_k X_{jk}) + \sum_{i\in C}\sum_{\substack{j\in C \\ i\neq j}}\sum_{k\in D}\sum_{v\in T_k} n_k r_i (s_v X_{iv}) e_{ij}(s_k X_{jk}) + \sum_{i\in C}\sum_{\substack{j\in C \\ i\neq j}}\sum_{k\in D} n_k r_i (s_k X_{ik})\, e_{ij}(s_k X_{jk}) \qquad Equation\ 5.17$$

Maximize

$$\sum_{i\in C} w_i r_i \left\{\sum_{k\in D} s_k Xnew_{ik}\right\}^{\beta_i} + \sum_{j\in C}\sum_{k\in D}\sum_{i\in P_k} m_{ki}(r_i a_{ki}) e_{ij}(s_k Xnew_{jk})$$
$$+ \sum_{i\in C}\sum_{\substack{j\in C\\ i\neq j}}\sum_{k\in D}\sum_{v\in T_k} n_k r_i (s_v Xnew_{iv}) e_{ij}(s_k Xnew_{jk}) + \sum_{i\in C}\sum_{\substack{j\in C\\ i\neq j}}\sum_{k\in D} n_k r_i (s_k Xnew_{ik})\, e_{ij}(s_k Xnew_{jk})$$
$$- Current\ Allocation \qquad Equation\ 5.18$$

Constraints

$$\sum_{i\in C} YNew_{ik} s_k \geq 1 \qquad \forall i \in D \qquad Equation\ 5.19$$

$$\sum_{i\in C} YNew_{ik} s_k \leq 3 \qquad \forall i \in D \qquad Equation\ 5.20$$

$$\sum_{k\in D} XNew_{ik} s_k \geq b_i^L \qquad \forall i \in C \qquad Equation\ 5.21$$

$$\sum_{k\in D} XNew_{ik} s_k \leq b_i^U \qquad \forall i \in C \qquad Equation\ 5.22$$

$$XNew_{ik} \leq Ynew_{ik} \qquad \forall i \in C, k \in D \qquad Equation\ 5.23$$

$$XNew_{ik} \geq 0.25 * Ynew_{ik} \qquad \forall i \in C, k \in D \qquad Equation\ 5.24$$

$$\sum_{i\in C} XNew_{ik} \leq 1 \qquad \forall k \in D \qquad Equation\ 5.25$$

$$\sum_{k\in D}\sum_{i\in F_k} Ynew_{ik} = 0 \qquad Equation\ 5.26$$

$$Q_{ik} - U_{ik} = Ynew_{ik} - Y_{ik} \qquad \forall i \in C, k \in D \qquad Equation\ 5.27$$

$$\sum_{i\in C}\sum_{k\in D} Q_{ik} = 10 \qquad Equation\ 5.28$$

$$0 \leq Xnew_{ik} \leq 1 \qquad \forall i \in C, k \in D \qquad Equation\ 5.29$$

$$Ynew_{ik} \in \{0,1\} \qquad \forall i \in C, k \in D \qquad Equation\ 5.30$$

$$Q_{ik} \in \{0,1\} \qquad \forall i \in C, k \in D \qquad Equation\ 5.31$$

$$U_{ik} \in \{0,1\} \qquad \forall i \in C, k \in D \qquad Equation\ 5.32$$

The constraints of the two-period temporary display allocation problem for non-split product categories are mainly the same as the single period temporary display allocation problem for non-split product categories.

Equations 5.19 and 5.20 state that each temporary display should be allocated at least one product category or at most three product categories. The maximum number of product categories of three based on the store manager’s preference.

Equations 5.21 and 5.22 state that the lower and upper area bounds of each product category must be satisfied for the second period.

Equations 5.23 and 5.24 connect the binary and continuous decision variables for the model. Equation 5.24 states if a product category is allocated to the temporary display; the allocated area should be greater than the 25% of the temporary display area.

Equation 5.25 restricts that the total allocation of the temporary display should be equal to the total area of the temporary display.

Equation 5.26 represents that the product categories that should not be allocated to the specified temporary displays are restricted.

Equation 5.27 states the relationship between the replacing product categories and the replaced product categories. Equation 5.28 restricts the total number of replacing product categories to a predetermined number by the store manager. The areas of the replaced product categories are not restricted in the model; it is possible to assign the product categories to larger areas than the first promotional period, however the total allocated areas are limited by the upper area limits.

The last constraints are the definitions of the decision variables. Equation 5.29 represents the continuous variables for the total area allocations of product categories and Equations 5.30, 5.31, and 5.32 are the binary decision variables for the location of product categories, replacing product categories, and replaced product categories for the second promotional period.

5.4. Test Case for Model-2B

Test Case 2 is created to evaluate the two-period temporary display allocation model for split product categories. To evaluate the new split model, the model is run with no restriction on the number of replacing product categories. The first split model results are shown in Table 5.5 and used as data in the new model. The objective function value of the first model is **64,780.19 TRY**. The best found solution of the two-period split model is presented in Table 5.6. The objective function value is reported as **-2,225.18 TRY** (the amount of revenue lost compared to leaving the first period allocation in place for the second period), which shows that new allocation is not better than the previous promotion allocation for this test case. However, this is the best solution that we can find because some new promotional products needed to be included in the second period.

The changes in the allocated areas of product categories between the first and second periods are shown in Table 5.7. If the product category's allocated area in the second period is larger than in the first period, it is labeled "Increase". If the product category's allocated area in the second period is less than in the first period, it is labeled "Decrease". If the product category is assigned to the display during the first period but is not assigned at that display for the second period, it is labeled "Removed". If there is no change in the assignment, it is shown as blank.

Table 5.5. Product Category Allocation Ratios for the First Promotional Period

	Product Categories						
Display No	**C6**	**C11**	**C14**	**C18**	**C20**	**C23**	**C25**
D1	0	0	0.38	0	0.62	0	0
D2	0	0	0.58	0.42	0	0	0
D3	0	0	0	0	0.55	0.45	0
D4	0	0	0.35	0.40	0.25	0	0
D5	0	0	0.52	0	0	0.48	0
D6	0	0	0	0.66	0.34	0	0
D7	0	0	0.45	0	0.55	0	0
D8	0	0	1	0	0	0	0
D9	0	0	0	1	0	0	0
D10	0	0	0.75	0	0.25	0	0
D11	0	0	0	0.43	0.57	0	0
D12	0	0	0.75	0	0	0.25	0
D13	0	0	0	0	0	0	1
D14	0	0	0	0.25	0.25	0.5	0
D15	0	0	1	0	0	0	0
D16	0.25	0.75	0	0	0	0	0
D17	0	0.25	0	0	0	0	0.75
D18	0	0	1	0	0	0	0
D19	0	0	0	0.41	0	0.59	0
D20	0	0	0.5	0	0.25	0.25	0
D21	0	1	0	0	0	0	0
D22	0	0	0	0	0	0	1
D23	0	0	0.40	0	0.60	0	0
D24	0	0	0.42	0.58	0	0	0
D25	0	0	0.52	0	0.48	0	0
Objective Function Value	64,780.1931						

Table 5.6. The Best Solution Found for the Two-Period Temporary Display Allocation Problem with Split Product Categories

	Product Categories						
Display No	**C6**	**C11**	**C14**	**C18**	**C20**	**C23**	**C25**
D1	0	0	0	0.26	0.34	0.40	0
D2	0	0	0.48	0.52	0	0	0
D3	0	0	0.25	0	0.5	0.25	0
D4	0	0	0.33	0.67	0	0	0
D5	0	0	0.40	0	0.35	0.25	0
D6	0	0	0	0.75	0.25	0	0
D7	0	0	0.56	0	0.44	0	0
D8	0	0	0.25	0.25	0	0.5	0
D9	0	0	0	0	0	0	1
D10	0	1	0	0	0	0	0
D11	0	0.27	0	0	0	0	0.73
D12	1	0	0	0	0	0	0
D13	0	0.25	0	0	0	0	0.75
D14	0	0	0	0.61	0	0.39	0
D15	0	0	0.75	0	0.25	0	0
D16	1	0	0	0	0	0	0
D17	0	0.47	0	0	0	0	0.52
D18	0	0	1	0	0	0	0
D19	0	0	0	0.71	0.29	0	0
D20	0	0	0.5	0	0.25	0.25	0
D21	0	1	0	0	0	0	0
D22	1	0	0	0	0	0	0
D23	0	0	0.25	0.43	0.32	0	0
D24	0	0	0.37	0.37	0.26	0	0
D25	0	0	0.38	0	0.25	0.37	0
Objective Function Value	-2,225.18 TRY						

Table 5.7. Area and Allocation Changes for the Second Promotional Period

Display No	C6	C11	C14	C18	C20	C23	C25
	Product Categories						
D1			Remove	Increase	Decrease	Increase	
D2			Decrease	Increase			
D3			Increase		Decrease	Decrease	
D4			Decrease	Increase	Remove		
D5			Decrease		Increase	Decrease	
D6				Increase	Decrease		
D7			Increase		Decrease		
D8			Decrease	Increase		Increase	
D9				Remove			Increase
D10		Increase	Remove		Remove		
D11		Increase		Remove	Remove		Increase
D12	Increase		Remove			Remove	
D13		Increase					Decrease
D14				Increase	Remove	Decrease	
D15			Decrease		Increase		
D16	Increase	Remove					
D17		Increase					Decrease
D18							
D19				Increase	Increase	Remove	
D20							
D21							
D22	Increase						Remove
D23			Decrease	Increase	Decrease		
D24			Decrease	Decrease	Increase		
D25			Decrease		Decrease	Increase	

5.5. Conclusion

In this chapter, we introduced mathematical models for the temporary display allocation problem for two consecutive promotional periods. In the Yunus supermarket chain, it is a common practice to have different promotional activities in consecutive promotional periods. It is necessary to have a solution that considers the intersection of two promotional periods. The problems in this chapter have more constraints than the problems in Chapter 4. Additional constraints restrict the solution area and the objective function values are lower what we have in the previous chapter because new products are forced into the second period. Due to the capacity of GAMS, one real size test problem for each mathematical model was solved. The next chapter will further discuss the mathematical models and their implications.

Chapter 6 Conclusions

In this dissertation, we developed a new methodology to allocate product categories at temporary display areas at supermarkets. While there are fixed locations of product categories within the main store aisles, temporary display allocation changes periodically. The general expectation is to have fixed locations for product categories at main aisles for a long period of time (until a major rearrangement). Supermarkets aim to encourage further buying by displaying products at endcaps and island shelf areas. Supermarket management can have various reasons to employ promotional applications such as increasing revenue, reducing stocks on a particular product, and changing customer brand preferences. Seasonal products, on sale merchandise, and small size items are typically located at these temporary display areas. These items are switched with different products as the season changes or other items go on sale. Supermarkets constantly evaluate product category allocation and sizing at temporary displays to maximize sales. Therefore, the layout and sizing problem at temporary displays occurs frequently but has been largely neglected in the analytical literature.

We solved the shelf area assignment problem at temporary displays using a mixed integer nonlinear programming model. As the initial problem, we focused on choosing a product category that covers a whole temporary display area. This is generally a seasonal or promotional product category that has a strong relationship with the main aisle fixed product categories. As a second phase, we looked at allocating multiple product categories at a single temporary display area. In many cases, a more comprehensive season theme may include multiple related product categories that affect each other on sales as well as main aisle permanent product category sales. These are the non-split split models.

Furthermore, as we discussed with store managers, we realized that it is common to have overlapping running promotional product categories. To address the problem of having more than one promotional period, we have extended the models to two promotional periods. Therefore, we were able to choose product category allocation and sizing for the second promotional period, while the first promotional period is still running. Although the supermarkets are the focus of our work, any store with a similar nature such as hardware, hobby, and clothing stores can apply these models.

We were able to utilize actual shopping basket data from the Yunus stores, a large Turkish supermarket chain. Furthermore, to actually see the effects on sales, we were able to implement our solutions at one (typical) Yunus store. Our product category placement solutions resulted in major gains in revenue by optimally allocating and sizing product categories at temporary display areas.

In contributions to the literature, our objective was to build practical and plausible models, which maximize sales by providing product category allotting and assignment. To achieve this goal, we estimated impulse buying likelihoods of product groups. We established the specific sizes and locations of the product categories at temporary displays. We were able to review the current condition of a store with the data including market basket and the expertise of store managers.

Our model is the first to provide an optimized solution on sizing and allotment of product categories at temporary display shelf space. Since a store can run more than one promotional period simultaneously, we proposed an approach to solve the product category allocation and sizing problem with two overlapping promotional periods. Therefore, we can assign product categories optimally to the temporary displays with these two different promotional activities. Ours is also the first model to have a two-promotional period methodology for a practical application at a store. By using data mining, analytics, and optimization the resulting layouts improved sales significantly as verified in actual physical implementations at a Yunus store. We compared the model results with the promotional sales for the current layout to determine the improvement rates.

Table 6.1 summarizes average improvements for Test Case (25 Product Categories x 25 Temporary Displays) in the split, non-split, single period, and two-period models. In the single-period models, the average improvement rate is 50% when comparing them with the sales from the actual store layout sales. In the two-period models when comparing with the single period model results, the improvement averages 11%. Moving from a non-split to a split model improves results fairly dramatically for the single period. In the two period results, we are comparing only model results (not with the actual store layout) so in the split case we worsen the results. This can be because in the second period, some new promotional items are added which may be less profitable than the ones that must be removed to make room for them.

The total promotional sales for the existing layout are **77,936.98 TRY**. The objective function value from the single period non-split model is **116,356.90 TRY** which is 49% higher than the promotional sales achieved using the existing store layout. The objective function for the single period split model is **117,659.28 TRY,** 51% higher. Therefore, splitting product categories within a display can result in improved sales.

The objective function from the two-period non-split model is **82,722.81 TRY**. The objective function from the two-period split model is **89,820.64 TRY.** The objective function value of two-period model is worse than the single period model, but it is still promising 15% more promotional sales than the existing layout. Note that these sales are for the same period of time, but this time is either one promotional period or two promotional periods depending on if the product mix changes halfway through. Evidently the change in promotional products is detrimental to the sales achieved in the second half of the time for the two-period models. The model objective functions are compared with the existing layout sales in Table 6.1. The allocations in these models are shown in Tables 6.2, 6.3, 6.4, and 6.5.

Table 6.1. Improvement Rates in All Models
(Test Case - 25 Product Categories X 25 Temporary Displays)

Model	Objective Function Value	Improvement Rate (Compared with the existing layout sales-**77,936.98 TRY**)
Single Period Non-Split Model	116,356.90 TRY	49%
Single Period Split Model	117,659.28 TRY	51%
Two Period Non-Split Model	82,722.81 TRY	6%
Two-Period Split Model	89,820.64 TRY	15%

Table 6.2. Single Period Non-Split Model Allocation for Test Case (25 Product Categories X 25 Temporary Displays)

Temporary Display	Product Category
D1	C1
D10	C6
D11	C11
D12	C21
D13	C14
D14	C21
D15	C20
D16	C13
D17	C13
D18	C14
D19	C23
D2	C25
D20	C25
D21	C20
D22	C23
D23	C18
D24	C20
D25	C18
D3	C11
D4	C25
D5	C11
D6	C18
D7	C23
D8	C21
D9	C14

Table 6.3. Single Period Split Model Allocation for Test Case (25 Product Categories X 25 Temporary Displays)

Temporary Display No	Product Category No	Allocation Ratio
D1	C18	0.50
D1	C20	0.50
D2	C14	1.00
D3	C20	1.00
D4	C14	1.00
D5	C18	0.50
D5	C23	0.25
D5	C25	0.25

D6	C18	1.00
D7	C14	1.00
D8	C20	1.00
D9	C14	1.00
D10	C18	0.55
D10	C20	0.45
D11	C14	1.00
D12	C18	1.00
D13	C1	0.25
D13	C6	0.25
D13	C14	0.50
D14	C20	1.00
D15	C14	0.75
D15	C20	0.25
D16	C6	0.25
D16	C11	0.25
D16	C20	0.50
D17	C1	0.25
D17	C11	0.25
D17	C18	0.50
D18	C14	1.00
D19	C11	0.25
D19	C20	0.50
D19	C25	0.25
D20	C14	0.70
D20	C23	0.30
D21	C1	0.25
D21	C6	0.25
D21	C18	0.50
D22	C18	0.43
D22	C23	0.32
D22	C25	0.25
D23	C20	1.00
D24	C14	1.00
D25	C20	1.00

Table 6.4. Two-period Non-Split Model Allocation for Test Case (25 Product Categories X 25 Temporary Displays)

Temporary Display	Product Category
D1	C18
D10	C14
D11	C20
D12	C14
D13	C25
D14	C18
D15	C14
D16	C18
D17	C20
D18	C14
D19	C18
D2	C14
D20	C14
D21	C11
D22	C23
D23	C20
D24	C18
D25	C14
D3	C20
D4	C18
D5	C14
D6	C18
D7	C20
D8	C14
D9	C20

Table 6.5. Two-period Split Model Allocation for
Test Case (25 Product Categories X 25 Temporary Displays)

Temporary Display	Product Category	Allocation Ratio
D1	C14	0.27
D1	C18	0.39
D1	C20	0.34
D10	C14	0.70
D10	C18	0.30
D11	C18	0.43
D11	C20	0.57
D12	C14	1.00
D13	C20	1.00
D14	C18	1.00
D15	C14	0.67
D15	C20	0.33
D16	C14	0.29
D16	C18	0.43
D16	C20	0.28
D17	C14	0.35
D17	C18	0.25
D17	C20	0.40
D18	C1	0.25
D18	C6	0.25
D18	C23	0.50
D19	C1	0.25
D19	C6	0.25
D19	C25	0.50
D2	C11	0.25
D2	C23	0.50
D2	C25	0.25
D20	C1	0.25
D20	C6	0.25
D20	C11	0.50
D21	C11	0.25
D21	C23	0.50
D21	C25	0.25
D22	C14	0.36
D22	C18	0.33

D22	C20	0.31
D23	C14	0.25
D23	C18	0.45
D23	C20	0.30
D24	C14	0.36
D24	C18	0.33
D24	C20	0.31
D25	C14	0.56
D25	C20	0.44
D3	C18	0.40
D3	C20	0.60
D4	C14	0.50
D4	C18	0.50
D5	C14	0.49
D5	C20	0.51
D6	C18	1.00
D7	C14	0.41
D7	C20	0.59
D8	C14	0.51
D8	C18	0.49
D9	C18	0.37
D9	C20	0.63

The summary of test problem solutions is listed in Table 6.6. We were able to find the optimal solution of the Single Period Split Model only for the test problem with 25 product categories and 25 temporary displays. Due to the complexity (MINLP) of the Temporary Display Allocation Problem, GAMS ends with the best integer solutions in most cases, rather than the true optimal. To improve the solutions, we tried different variable bounds for each test case. Also, the partial solutions of test cases were used to try to find better solutions. There are some strategies to improve the found integer solutions in GAMS. These strategies include a warm start, variable bound, and variable/constraint aggregation. The warm start provides a better initial feasible solution, so the MINLP solvers could converge to a better integer solution. The tighter bounds on the integer variables can help to guide the solvers towards better and optimum solutions. Combining multiple variables or constraints so there are fewer can simplify the problem structure and reduce its size. The objective function of the two-period model split model seeks a better solution than the single period split model solution. The store or chain managers have the option to expand the number of promotional items in the second promotional period, but this does not necessarily lead to higher sales for the second period.

Table 6.6. Summary of the Test Problem Solutions for All Mathematical Models

Mathematical Model and Test Problem	**Solutions**
Single Period Non-Split Model Test Problem I (25 Product Categories X 75 Temporary Displays)	140,708.59 – Best Integer Solution found
Single Period Non-Split Model Test Problem II (25 Product Categories X 75 Temporary Displays)	140,348.93 – Best Integer Solution found
Single Period Non-Split Model Test Problem III (25 Product Categories X 75 Temporary Displays)	100,153.70 – Best Integer Solution found
Two-Period Non-Split Model Test Problem I (25 Product Categories X 75 Temporary Displays)	169,633.12 – Best Integer Solution found
Single Period Non-Split Model Test Problem (25 Product X 25 Temporary Displays)	116,356.90 – Best Integer Solution found
Single Period Split Model Test Problem I (25 Product Categories X 25 Temporary Displays)	**117,659.28 – Optimal Solution found**
Two-Period Non-Split Model Test Problem (25 Product Categories X 25 Temporary Displays)	82,722.81 – Best Integer Solution found
Two-Period Split Model Test Problem I (25 Product Categories X 25 Temporary Displays)	89,820.64– Best Integer Solution found

As future research, one aspect would be to model the shopper traffic more precisely in the store. This is problematic since trackers or films could invade personal privacy. But it would be useful to know better the typical pathways shoppers take on the way to their planned purchases. Also, since the e-commerce shopping experience is moving towards virtual reality, a more realistic store arrangement could be aimed for internet shopping. Electronics companies continue to develop virtual reality headsets for an improved user experience. That opens up the opportunity to have a more sensory experience for online shopping. We can work on and optimize the virtual 3D store layout provided by these e-commerce websites to maximize revenues by impulse purchases at temporary displays. This kind of setup still can benefit from cross category sales and impulse purchase impact. It will be closer to a walking setup of an actual store environment. It will relax some of the constraints such as the physical size of the goods. It will also provide more options for placing the products since it will not have physical constraints such as placing items above the shopper. This product display environment will require a whole new set of constraints and variables to develop models and provide solutions for this virtual reality shopping experience.

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