Teaching Students to be Successful in Multiplication with Regrouping

by

Erin Noelle Blanton

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Approved by

Margaret M. Flores, Chair, Professor, Special Education, Rehabilitation and Counselling
Vanessa Hinton, Professor, Special Education, Rehabilitation and Counselling
Christine Drew, Professor, Special Education, Rehabilitation and Counselling
Suzanne Woods-Groves, Professor, Special Education, Rehabilitation and Counselling

Abstract

Concrete-Representational-Abstract (CRA) mathematical sequencing has been effective across individuals with varied disabilities and individuals who are at-risk and in teaching various skills. However, more research is needed to understand how CRA generalizes fully to elementary students with learning disabilities. Therefore, this study seeks to replicate research and expand the use of CRA by teaching students with learning disabilities fluency and accuracy in multiplication with the regrouping of two 2-digit numbers. The researcher used a multiple probe across participants to demonstrate a functional relation between the independent and dependent variables, as Horner and Baer (1978) described.

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Chapter 1: Introduction

Mathematics is a set of many skills individuals need to navigate their world, such as making purchases and maintaining a budget. Concrete-Representational-Abstract (CRA) and Strategic Instruction Model (SIM) mathematical sequencing is an intervention used to give students conceptual understanding, computational skills, and strategies to solve mathematical equations (Flores et al., 2014a). CRA-SIM has extensive research demonstrating functional relations and effective investigations across disabilities and at-risk populations (Flores & Franklin, 2014; Flores et al., 2014a; Flores et al., 2014b; Flores et al., 2016; Flores & Hinton, 2019; Flores et al., 2019a; Flores & Milton 2020; Flores et al., 2022). CRA had an impressive beginning with roots by Peterson et al. (1988) with place value for individuals with learning disabilities (LD). Mercer and Miller (1992) found CRA paired with SIM to be successful for individuals struggling in mathematics to learn place value and basic operations. Miller and Mercer (1993) replicated Mercer and Miller (1992) and used CRA with SIM to teach division to students with an LD. Due to the success of CRA-SIM, further research found CRA-SIM to be efficient in teaching single-digit multiplication skills (Harris et al., 1995; Morin & Miller, 1998) to individuals with an LD. With continued success, other researchers used CRA to teach subtraction with regrouping (Flores, 2009; Flores, 2010; Mancl et al., 2012). Compared to other instructional practices, Miller and Kaffar (2011) found CRA to be effective in teaching regrouping in addition to students with learning disabilities.

Statement of Research Problem

The line of research for teaching multiplication of multi-digit numbers began with the use of CRA-SIM to teach the standard algorithm (Flores et al., 2014a). This study included students with LD receiving services in a special education resource classroom. CRA-SIM was also effectively used to teach regrouping in subtraction and multiplication with the standard algorithm to students receiving tier-three instruction in a multitiered intervention framework (Flores et al., 2014b; Flores & Franklin, 2014). Flores et al. (2016) successfully used CRA to teach schema-based instruction to students at risk for failure and provided tiered intervention. Flores and Hinton (2019) replicated these results in regrouping with multiplication to students receiving multitiered interventions and adding a problem-solving component. Since Flores and Franklin (2014), Flores et al. (2014a), and Flores et al. (2014b) noted limitations in their studies related to lack of comparison, Flores et al. (2019) compared the effects of CRA-SIM versus Direct Instruction (DI) to teach the standard algorithm for multiplication with students in a remedial summer program. Expanding the research on CRA, Flores and Milton (2020) examined the effects of CRA using the partial products algorithm for individuals with other health impairments (OHI) and LD. In the most recent study, Flores et al. (2022) effectively investigated CRA to teach the standard multiplication algorithm but provided instruction using a remote format. Due to the gap in research pertaining to teaching individuals with LD using the standard algorithm, the need of research for evidence-based practices to improve instruction for students with LD to meet the rigor of mathematics the researcher sought further research through replication of studies with the inclusion of students with LD.

Justification of the Study

There is a lack of research in CRA in multiplication with regrouping that includes students with LD. Recently, Flores and Milton (2020) investigated CRA using partial products algorithm for individuals with OHI and LD; however, there is not a study investigating CRA teaching the standard algorithm to individuals with LD. Due to the gap in research pertaining to teaching individuals with LD using the standard algorithm the researcher sought further research through replication of studies with the inclusion of students with LD. According to Baer et al

(1968), single case research findings are generalized with replication overtime, across environments and individuals, and across related behaviors.

Purpose of Study

In the current line of CRA-SIM research for two 2-digit multiplication equations that require regrouping with students with LD, there has been one single case design study using partial products (Flores & Milton, 2020) and two single case design studies with the standard algorithm (Flores et al., 2014a; Flores et al., 2022). Due to a recent pandemic, Flores et al. (2022) used a remote-modified version of CRA to teach students with LD two 2-digit multiplication using the standard algorithm. Therefore, there needs to be more current literature on the effects of CRA-SIM using the standard algorithm with students with LD. Further research is warranted to replicate research findings for students with LD. The researcher replicated previous research and investigated the effects of CRA and SIM on individuals with LDs' fluency and accuracy in multiplication with the regrouping for equations with two 2-digit numbers (Flores et al., 2019b; Flores et al., 2022; Flores et al., 2014a).

Research Question

What are the effects of CRA-SIM intervention on individuals with learning disabilities' fluency and accuracy in multiplication with the standard algorithm for solving equations with two 2-digit numbers?

Definition of Terms

Concrete-Representational-Abstract (CRA) – A three stage mathematical sequence. The first stage is the concrete stage. During the concrete stage of instruction, the student uses three-dimensional objects to develop an understanding of the operation and mathematical concepts (Miller & Mercer, 1993). The next stage is the representational stage. In this stage the students use two-dimensional drawings to solve mathematical equations (Miller & Mercer, 1993). The final stage is the abstract stage. Here, the student uses numbers and only numbers in conjunction with the standard algorithm to solve mathematical equations.

Percentage Correct - the number of problems correct divided by the number of problems attempted.

Strategic Instruction Model (SIM)- a model that is applied across content areas using explicit instruction along with a mnemonic to aid in solving mathematics problems (Flores et al, 2014b).

Standard Algorithm for Multiplication – the traditional set of steps taught to solve mathematical equations involving 1-digit and 2-digit multipliers in multiplication equations.

Limitations of the Study

This study is limited due to the lessons being taught by the primary researcher. Furthermore, research implementation by a researcher rather than a teacher is problematic because a teacher trained by a researcher would be reflective of real-life conditions. A researcher has a level of expertise greater than that of a teacher; therefore, the instructional conditions are not realistic, and it is not clear whether the intervention would be feasible under normal conditions. Other limitations included lack of maintenance and generalization data for two groups. Due to groups needing to move more quickly than other studies, the study took longer to show a functional relation and the school year ended before some groups could move to maintenance and generalization.

Summary

This study sought to investigate the effects of CRA-SIM intervention on individuals with LDs' fluency and accuracy in multiplication with the standard algorithm for solving equations with 1-digit 2-digit multipliers. Currently, Flores and Milton (2020) investigated CRA using partial products algorithm for individuals with OHI and LD. There are two studies on the standard algorithm that include individuals with LD (Flores et al, 2014a; Flores et al, 2022).

Therefore, further research is warranted in this area according to Horner et al. (2005) for generalization. Due to the gap in research teaching individuals with LD using the standard algorithm the researcher sought further research through replication of studies with the inclusion of students with LD. Similar to previous studies, this study used materials from *The strategic math series: Multiplication with regrouping* manual (Flores et al., 2022; Flores & Kaffar, 2018). The researcher investigated what are the effects of CRA-SIM intervention on individuals with learning disabilities' fluency and accuracy in multiplication with the standard algorithm for solving equations with two 2-digit numbers?

Chapter 2: Review of Literature

Individuals who struggle academically often find mathematics challenging as they progress through their academic careers. Researchers have shown a functional relation between students who struggle in mathematics and the use of concrete-representational-abstract (CRA) mathematical sequence as an instructional sequence (Bouck et al., 2018a; Bouck et al. 2018b). CRA is an instructional sequence that assists students in mathematical thinking by incorporating objects, pictures, and numbers to develop an understanding of mathematical concepts and utilizes explicit instruction's practical components (Harris et al., 1995; Miller & Mercer, 1993). Explicit instruction designs and delivers instructional lessons that scaffold successful learning through clear, explicit language and expectations. Explicit instruction promotes active student engagement by requiring frequent and varied responses, followed by appropriate affirmative and corrective feedback, and helps long-term retention through purposeful practice strategies. The five components of explicit instruction are: segment the complex skills by breaking them into chunks. Next, the teacher gains attention to critical parts through modeling and thinking aloud; then, systematically fading supports and prompts through guided practice. Finally, provide opportunities for student responses and corrective feedback and provide purposeful practice (Miller & Mercer, 1993).

According to Miller and Mercer (1993), some individuals struggle academically, beginning with instruction in single-digit basic facts, and this struggle continues as students progress through school. The CRA sequence is based on the stages of representation (Bruner & Kenney, 1965). The three stages are the inactive, iconic, and symbolic stage. In the enactive stage, the learner learns with movement or action. In the iconic stage, the learner learns with images. Finally, in the symbolic stage, the learner learns with abstract symbols.

Similar to Bruner and Kenney (1965), CRA has three instructional stages that explicitly teach mathematics concepts using manipulative objects and that allow for conceptual understanding before procedures (Miller & Mercer, 1993). CRA introduces multisensory instructional techniques that build on previously taught lessons and scaffold conceptual understanding, procedural accuracy, and fluency through three stages (Bouck et al., 2018a; Bouck et al., 2018b). The first stage is the concrete stage. During the concrete stage of instruction, the teacher employs three-dimensional objects to develop an understanding of the operation and mathematical concepts (Miller & Mercer, 1993). The next stage is the representational stage. Students use two-dimensional drawings to solve problems (Miller & Mercer, 1993). The final stage is the abstract stage. Here, the abstract stage involves the use of numbers and only numbers. The teacher should no longer use manipulatives or pictures. Procedural knowledge and fluency become the focus in this stage. Morin and Miller (1998) used CRA paired with the Strategic Instruction Model (SIM) to teach students to solve basic multiplication facts. SIM teaches the student to use a mnemonic to aid in solving mathematics problems. The purpose of this paper is to provide an overview of the significant studies concerning CRA and its effectiveness.

According to recent research, CRA has been used to successfully teach individuals with and without disabilities, individuals who are at risk for failure, individuals with developmental disabilities, learning disabilities, autism spectrum disorders, specific learning disabilities, emotional disabilities, and low achieving students who struggle in mathematics. The skills addressed by CRA include single-digit addition, subtraction (Flores et al., 2014), and multiplication (Gibbs et al., 2017; Harris et al., 1995; Miller et al., 1998; Milton et al., 2019; Morin & Miller, 1998; Sealander et al., 2012), place value (Peterson et al., 1988) and number sense (Peterson et al., 1990), addition (Miller & Kaffar, 2011; Stroizer et al., 2015), subtraction (Flores 2009; Flores, 2010, & Mancl et al., 2012), and multiplication with regrouping (Flores & Hinton, 2019; Flores et al., 2014a; Flores et al., 2014b; Flores et al., 2019a; Flores & Milton 2020; and Flores et al., 2019b) and virtually (Bouck et al., 2014; Bouck et al., 2017a; Bouck et al., 2017b; Flores et al., 2022). CRA has been used with various learners in grades preschool through 12th grade. The author identified the studies in this paper by using the following research terms via the Auburn University Library: *concrete, representational, abstract, concrete-representational-abstract, CRA, mathematical sequence, semi-concrete, virtual, VRA, and intervention*. The author conducted an ancestral search using the reference lists of articles to find additional articles and research.

Single Digit Operations

The following section describes the investigation of CRA in single-digit operations in addition (Miller & Mercer, 1993), subtraction (Flores et al., 2014), multiplication (Gibbs et al., 2018; Harris et al., 1995; Miller et al., 1998; Milton et al., 2019; Morin & Miller, 1998; Sealander et al., 2012), word problems (Flores et al., 2016) and with video modeling (Root et al., 2017; Yakubova et al., 2016). These included students with learning disabilities, emotional disabilities, low achieving, intellectual disabilities, autism spectrum disorder, developmental disabilities, and neurotypical peers. The researchers taught using three-dimensional figures to solve equations at the concrete level, two-dimensional drawings at the representational level, and numbers only with a mnemonic at the abstract level. All the studies found that the CRA sequence successfully taught students to solve equations within single-digit operations.

The line of CRA using single-digit operations began with Miller and Mercer (1993). Miller and Mercer investigated the CRA sequence to teach students who struggle in mathematics to solve addition facts. Furthermore, Miller and Mercer investigated how students generalized and how students conceptually understand the fundamentals of mathematics using CRA. Additionally, Miller and Mercer explained the benefits of using data-based instruction to gather relevant information about students' current learning. In this study, nine students in second through third grade participated in a multiple probe across subjects' design to investigate how students conceptually understand the fundamentals of mathematics when using CRA sequence to teach addition problems. The researchers collected data via three abstract-level probe sheets containing sixty vertically aligned addition equations.

The teacher taught students in three phases. Phase one included baseline, in which the teacher gathered pretest data. Phase two included the intervention phase. The teacher used scripted lessons with four instructional steps: advanced organizer, modeling, guided practice, and independent practice. The intervention phase began with the concrete stage of instruction and progressed to the representational stage of instruction and the abstract stage of instruction. Phase three was the posttreatment phase, in which the researchers collected maintenance data one week after completing all intervention phases. Miller and Mercer (1993) found that, of the nine students who participated in the study, four experienced the crossover point during the concrete stage, and five experienced the crossover point during the representational stage of instruction.

Harris et al. (1995) and Miller et al. (1998) further extended the line of research conducted by Miller and Mercer (1993) to include teaching initial multiplication skills to neurotypical peers and to individuals who struggled with a learning disability or an emotional disability in general education classrooms. Harris et al. (1995) investigated the effectiveness of teaching CRA within the same classroom to twelve individuals with the exceptionality of learning disability (LD) and one individual with the exceptionality of emotional disability, and

99 neurotypical individuals. In this study, the researchers conducted two investigations simultaneously. Both investigations used a multiple baseline design.

The study took place in six general education classrooms in a public school located in north-central Florida. The researchers used two criteria for determining eligibility in the study. The first criteria were custodial consent. The second was to score 70% or higher on an experimenter-designed screening instrument named Prerequisite Skills Test. The Prerequisite Skills Test consisted of three parts. In the first part, the students had to write thirty numbers from memory for one minute. In the second part, the students had one minute to fill in the correct missing numbers up to eighty-one. Finally, the students had to calculate twenty single-digit problems with sums to 18 correctly in the third part. The criteria to be in the study required the students to score 70% or higher on the Prerequisite Skills Test.

The materials consisted of scripted manuals that were research-based and field-tested with twenty-two teachers and 109 elementary students and provided explicit instruction in multiplication computation and problem-solving using the CRA sequence. Harris et al. (1995) used four measurement instruments throughout the two investigations. The first instrument used was a 1-min timed probe sheet that contained six rows of nine multiplication problems with products up to eighty-one. The second and third instruments contained twenty single-digit multiplication facts, and the researchers used them as a pre-test and post-test. The fourth instrument used was a learning sheet that was different for each of the twenty-one scripted lessons. The learning sheets contained ten independent practice problems. The teacher used any additional problems for teaching purposes. For example, the learning sheet for lesson seven strictly listed the steps to the mnemonic device: a) Discover the sign, b) Read the problem, c) Answer or draw and check, and d) Write the answer (DRAW), and did not contain any computation problems. The researchers calculated interscorer reliability and procedural reliability. The result showed a 99.2% agreement score for baseline and lessons 1-10 and 96.9% agreement for lessons 11-21. To determine procedural reliability, the researcher observed each teacher in the study for three 10-min lessons. Procedural reliability was 92% for observation intervals.

Harris et al. (1995) divided the study into three phases. The first phase addressed training for teachers on explicit instruction and the CRA sequence. The second phase was the baseline procedures, in which they collected baseline data on 1-min probes until stability occurred. The final phase was intervention procedures, in which the teachers taught ten lessons. Each teacher taught three lessons at the concrete stage, three lessons at the representational stage, one lesson for the mnemonic strategy, and three lessons at the abstract stage. In the concrete stage, paper plates represented groups, and manipulatives represented the objects in the groups. The researchers used this to teach a conceptual understanding of multiplication. The first three lessons' instruction was very basic word problems and used the words paired with the manipulative to describe and teach computation. For example, during lesson two, the student learned the zero rule: anything multiplied by zero is zero. During lesson three, the students learned the one rule: anything multiplied by one equals the original number. Finally, in lessons four through six, the teacher taught the representational stage to the students to replace the concrete objects with drawings or pictures that consisted of boxes and circles containing dots.

During the mnemonic strategy lesson, teachers taught students to solve computation problems using a mnemonic, DRAW. Following lesson ten, the researchers gave a post-test that required students to score 90% or higher to advance to lesson 11. In lessons 11-21, the focus shifted to word problems and the rate of computation. During lesson eleven, the teachers taught the mnemonic FAST DRAW: a) Find what you are solving for, b) Ask yourself, "what are the parts of the problem," c) Set up the numbers, and d) Tie down the sign. The DRAW mnemonic remained the same as above. During lesson twelve, the teacher taught the students to solve word problems. During lesson thirteen, the students made up their word problems. The remaining lessons 14-21 consisted of independent practice following the advanced organizer. During these lessons, the researchers included at least three-word problems.

Regarding students with disabilities, all students improved from pretest to post-test in investigation one and investigation two. The difference between the investigations was that in investigation one, there were six participants, and in investigation two, there were seven participants. Additionally, all students improved in rate data. In investigation one, the students increased 8.4 correct digits and decreased 21.3 incorrect digits per minute. In investigation two, the students increased 11.6 correct digits and decreased 31.3 incorrect digits per minute.

Compared to neurotypical peers, individuals with disabilities were similar to their peers in concrete, representational, and abstract stages. When utilizing a strategy, individuals with disabilities were just as successful as their peers. However, during independent practice that required reading and writing, individuals with disabilities scored lower than their peers. Overall, as a whole, individuals with disabilities perform slightly better than their neurotypical peers. These findings suggest that all students can benefit from CRA instruction in a general education classroom.

Similar to Harris et al. (1995), Miller et al. (1998) investigated the effectiveness of teaching CRA within the same classroom to twenty-four individuals who were low achieving, had the exceptionality of LD or an emotional disability, as well as ninety-nine neurotypical individuals. In this study, the researchers conducted both investigations simultaneously. In addition, the researchers used a group design to show the effect of the dependent variable.

Miller et al. (1998) used CRA to teach single-digit multiplication in six general education classrooms in a public school located in a rural school district. The researchers determined eligibility in the study with the student's ability to score 70% or higher on an experimenter-designed screening instrument named Prerequisite Skills Test.

Miller et al. (1998) provided teacher training for all six second-grade teachers on basic multiplication concepts, skills, and principles using twenty-one scripted teachers' lessons. All six teachers implemented the twenty-one lessons from the manual to all six general education classes during the lesson implementation phases. There were no accommodations made to the lessons. During the instructional phases, the multiplication instruction progressed through seven phases. The instructional phases were similar to Harris et al. (1995).

In contrast to Harris et al. (1995), Miller et al. (1998) ended lesson one with the teachers and students discussing basic word problems; Harris et al. (1995) did not have a discussion. Miller et al. (1998) investigated whether there were significant differences between individuals with disabilities, lower achievers, and neurotypical individuals by analyzing the difference in the responses on the dependent variables between students' groups. There was a significant difference between individuals with disabilities and low achieving and their neurotypical peers. Individuals with disabilities and low achieving performed better than their neurotypical peers.

In contrast to Miller et al. (1998), who compared individuals with various disabilities and low achievers to neurotypical peers, Morin, and Miller (1998) used a single-subject multiple baseline across participants design. They taught multiplication to a specific group of students with disabilities, middle schoolers with intellectual disabilities. Morin and Miller (1998) selected the individuals in this study based on four criteria: a) teacher referral, b) a performance score of at least 90% on a Prerequisite Skills Test, c) a performance score of less than 80% on a pretest, and d) parent, guardian, and student consent/assent. This study took place in the Southeastern region of Alabama in an urban middle school. Materials consisted of 21 scripted manuals and procedures of SIM for teaching the CRA sequence. Morin and Miller (1998) assessed the dependent variable using assessments that included twenty single-digit multiplication facts.

Morin and Miller (1998) calculated interscorer reliability for all pretests, post-tests, probes, and learning sheets by the special education teacher and principal investigator. The results showed a 100% agreement. Procedural reliability was 98% for observation intervals.

Like Harris et al. (1995), Morin and Miller (1998) trained the special education teacher to use assessments and explicit CRA materials. The second phase was the baseline procedures in which the researcher collected baseline data using 1-min probes until stability occurred. The final phase was intervention procedures, in which the teachers taught ten lessons. Each teacher taught three lessons at the concrete stage, three lessons at the representational stage, one lesson for the mnemonic strategy DRAW, and three lessons at the abstract stage. Lessons 11-21 were the same as Harris et al. (1995) and Miller et al. (1998).

Regarding students with intellectual disabilities, all students improved from pretest to post-test. Morin and Miller (1998) used a single-subject multiple baseline across participates to demonstrate a functional relation by showing that once intervention began, the participants showed an immediate increase in the percentage of correct digits on the learning sheets. Morin and Miller noted that all students' scores dropped slightly during the advanced problem-solving practice. These findings are consistent with Harris et al. (1995) and Miller et al. (1998), who reported that during independent practice that required reading and writing, individuals with disabilities scored lower than their peers. These findings suggest that students with intellectual disabilities can benefit from CRA with SIM instruction.

Another extension of the CRA line of research involved changing instructional lessons' sequences (Milton et al., 2019). The researchers investigated alternating CRA multiplication and division instruction on students' mastery of unknown facts and conceptual understanding. This study consisted of five students in fourth and sixth grade using a multiple probe across participants design. The teacher followed the CRA instructional sequence, and all procedures for explicit instruction, use of materials, and DRAW strategy were all the same as previous research (Harris et al., 1995). The difference was alternating lessons that demonstrated the relation between multiplication and division and increased the student's fluency. The researchers measured student computation performance with researcher-created probes: one-minute multiplication probe, one-minute division probe, as well as an untimed division probe. The researchers recorded interviews with students that required students to describe and draw how they solved problems. They used qualitative procedures and deductive or theoretical thematic analysis to analyze data collected from videos. The researcher calculated interscorer reliability for 50% of the probes and agreement as 100%. The researchers assessed treatment fidelity. Interscorer agreement for fidelity was 100% for all sessions viewed, and treatment fidelity was 100%. Finally, the researchers collected social validity data. The social validity data showed that the teacher thought the intervention was easy to use and effective. The students liked the materials and thought it made multiplication and division easier.

The study results showed a functional relation between CRA instruction and accuracy in division facts and fluency in multiplication and division. Students wrote thirty correct digits in a minute on three consecutive probes with 100% accuracy. The researchers demonstrated these effects with three different students at three different points in time. The study results showed that, before the intervention, the students did not provide an accurate quotient or verbal, pictorial explanation of the intervention's division operation. In addition, the students did not use multiplication facts to solve division problems. After the intervention, the students used a verbal and pictorial explanation of division on the first request. All the students described the process accurately. Four students indicated knowledge of the inverse relationship between operations.

Additionally, all students gave responses that were lengthier than the responses given before intervention. This study differs from previous studies because the researcher alternated instruction between two different operations. Furthermore, it included a qualitative component and attempted to investigate students' understanding of operations.

Gibbs et al. (2018) extended the line of research with CRA by investigating the effects of explicit instruction using CRA to teach individuals to count in flexible ways and utilize skip counting as a strategy for solving multiplication computation problems. This case study consisted of fifteen third and fourth graders with disabilities located in an elementary school in a rural community in the southeastern part of the United States. To participate, the researcher obtained parental consent; students demonstrated a weakness in mathematics, specifically fluency in computation problems. The materials consisted of manipulatives for counting, paper, and flashcards with dots. The researchers administered a pretest/post-test assessment in multiplication and skip counting. The researchers timed both assessments, and students had two minutes to complete fifty one-digit facts. During the eleven intervention lessons, the teacher explicitly taught skip counting with numbers one through nine. Students played a game with flashcards to help them move from counting consecutively to skip counting. The teacher used flashcards as a review after mastery. The researchers used a checklist to assess procedural integrity data during the study. Procedural integrity was 100%.

Gibbs et al. (2018) used a paired-samples t-test to analyze the pretest and post-test growth. There was a significant difference in the multiplication assessment and the skip counting assessment. The findings support Harris et al. (1995), Miller et al. (1998), Morin and Miller (1998), and Milton et al. (2018) that CRA paired with explicit instruction is an effective way of closing the academic gaps in mathematics. Furthermore, this case study found that using CRA paired with explicit instruction helps students learn multiplication concepts and provide them with practical strategies for solving computation problems.

Flores et al. (2016) extended this line of research by investigating the effects of CRA and schema-based instruction to provide tiered intervention for students at risk for failure. The study took place in an intermediate elementary school located in the southern, eastern United States. The study consisted of three participants in the third grade. The assessment materials included twenty different probes that the researcher created that consisted of join and separating, part-part-whole, and compare problems. The researcher broke the study up into four lessons: (a) teaching the problem types, (b) concrete instruction with word problems, (c) representational instruction with word problems, and (d) abstract instruction with word problems. The materials for teaching word problems consisted of sheets of paper with a word problem that required one-step computation and included extraneous information. The sheet also included a problem-solving strategy. The materials used for representational instruction were the same except there were no prompting to act out, diagrams for problem type, and a prompt to draw the problem. All students completed written probes with no time limit. The instructional procedures included four phases that utilize explicit instruction. The first phase consisted of teaching problem types, the second phase consisted of the concrete stage, the third phase consisted of mnemonic, the final phase consisted of abstract level with the mnemonic and other visual aids.

Using a checklist of instructional behaviors, the researcher viewed live observations. A second researcher viewed 90% of the lessons. The researcher found treatment integrity to be 95%. Next, the researchers assessed probes for inter-observer agreement. The researchers found inter-observer agreement to be 100% for this study. Finally, the researchers assessed social validity using student and teacher surveys. Using work samples, the researcher found that the accuracy for the students was less than 25% before the intervention. Before the study, both teachers and students reported needing intervention. After the study, the teachers and students reported that their problem-solving skills improved. Using a multiple probe across students, all three participants demonstrated a functional relation, improved their problem-solving performance, and achieved mastery which the researcher defined as three probes at 100%. This study is significant because it demonstrated the effectiveness of schema-based instruction (SBI), CRA, and explicit instruction to improve performance in solving word problems.

Root et al. (2017) further extended the line of research with SBI by investigating the effects of modified schema-based instruction on solving word problems and compared the effectiveness of concrete and virtual manipulatives within the treatment package. Virtual manipulatives were consistent with previous concrete and representational materials; however, they differed in that they were displayed virtually on an iPad screen and were moveable. This study consisted of three participants with ASD and moderate intellectual disability (ID) and was in a public elementary school ages 7 to 11 years old in an urban school district in the southeast United States. The materials for this study consisted of (a) a problem-solving mat; (b) a graphic organizer either laminated or in on iPad (virtual); (c) a laminated student self-instruction sheet (task analysis); and (d) compare-type word problems.

Root et al. (2017) used a multiple probe across participants with an embedded alternating treatments design. All students began in baseline simultaneously until the researchers determined a stable level of performance which they defined as the student with the lowest and most stable level of performance. The researchers predetermined a sequence for each participant (no more than two consecutive sessions for each participant). Next, the researcher randomly picked one of the predetermined sequences for each participant. The researchers used the self-instruction sheet, a task analysis consisting of a chain analysis and dependent on the answer before. The instructor had to prompt or set up each step to determine a correct response. This gave a baseline for each participant, and the researcher used this to determine a functional relation. The instructor modeled each of the steps during a 3-day training period with the students. During these three days, the researchers collected no data, and the participants learned to follow the self-instruction sheet and check off each step they completed. After the training period, the instructor provided the least intrusive prompting to assist the participants in solving word problems. During baseline, the researchers provided daily math instruction for 30-45 mins that focused on their IEP goals. The special education teacher agreed not to provide instruction to the students on word problem solving during this study. During data collection in baseline, the first author served as the instructor and provided the participants with the following: (a) a student self-instruction sheet either concrete or virtual graphic organizers, (b) a problem-solving mat, either concrete or virtual manipulatives, and (c) the first word problem, which was read aloud to the participants if they asked. The instructor collected data on the number of steps completed in each self-instruction sheet. The instructor did not provide any additional prompting at baseline. The participants completed three-word problems using the self-instructed sheet for either concrete or virtual (on an iPad), and then they received time to play for three minutes on the iPad as a reinforcer. During the intervention, the instructor implemented all interventions with each participant in an individual session.

The lessons consisted of a model-lead-test format with an embedded system of least prompts (verbal, specific verbal, and a model prompt). Next, the instructor provided the self-instruction sheets for the appropriate lessons. The sessions lasted approximately 10 to 15 mins. The instructor decreased the time as the student became more fluent. Next, the instructor graphed correct independent responses on the self-instructed sheets for data collection. Then the instructor provided a single opportunity for the participants to complete an entire word problem independently. Next, the instructor moved the participants from intervention to the “choice phase” when they received at least eight out of ten points for two out of three problems for two consecutive days. The participant included correct responses on making sets, solving the problem, and writing the correct answer to receive points. The hierarchy consisted of three levels: verbal, specific verbal, and a model prompt. During the choice phase, the instructor allowed the participants to pick between the iPad or concrete self-instructed sheets. Finally, the instructor provided three novel word problems with no feedback and no prompting.

Root et al. (2017) collected IOA and procedural fidelity data on a range of 40% to 50% of baseline and 33% to 75% of intervention sessions. The researcher collected IOA by dividing the number of agreements and disagreements and then multiplying by 100%. The researchers calculated IOA at a range of 80% to 100%. Root et al. (2017) calculated procedural fidelity with a range of 98% to 100%. The researchers assessed social validity for this study. The researchers asked the students seven social validity questions, and they could answer yes or no. The researchers asked teachers to rate twelve items on a six-point Likert scale and one open-ended question and provide feedback on instructional methods and materials.

Root et al. (2017) demonstrated a functional relation on the effects of modified SBI on solving word problems in students with ASD and moderate ID with a focus on comparing the type of problems and the effects of concrete versus virtual manipulatives using a multiple probe across participants with an embedded alternating treatment design. The researchers found that all three participants improved in their ability to solve word problems after the intervention. Two of the three participants performed higher in the virtual condition and one participant the same in the concrete and virtual condition. All participants reported that they liked the mathematics lessons. All the teachers reported that the materials and lessons in the concrete and virtual conditions were helpful and would use them in the future. This study is beneficial in that it extends previous research where researchers showed that manipulatives are helpful for students to solve mathematics problems who had previously been unsuccessful.

There has been recent research related to single-digit multiplication, but Sealander et al. (2012) investigated single-digit multiplication and included the crossover point like Miller and Mercer's (1993) study to investigate the crossover point for individuals with learning and emotional disabilities. In this study, Sealander et al. (2012) examined the effect of discontinuing CRA instruction at the crossover point and what effects discontinuation of CRA had on the student's outcomes. This study recruited eight individuals who were either had an emotional disturbance or had a learning disability. Students were required to write numbers one through nine with 100% accuracy and have more incorrect digits than correct digits on a 24-item subtraction worksheet. Three special education teachers at different schools in the Southeastern part of the United States served as instructors in the study. The researchers provided training to the teachers in a 1-hr training session. The researchers assessed student progress through a pretest/post-test, daily abstract-level probes, and generalization tests. The pretest/post-test was a 24-item untimed test that consisted of subtraction problems. The abstract probes consisted of alternate forms of a 1-min timed worksheets that contained sixty subtraction problems with minuends one through nine. The researchers defined the crossover point as students writing more than thirty correct digits (Miller & Mercer, 1993). The national norm for single-digit fluency is thirty correct digits (Sealander et al., 2012). The researchers assessed generalization through three forms of probes with five listen/write word problems with minuends from 0 through 9 presented orally.

The materials consisted of teacher-created scripted lessons for teaching minuends 0 through 9. The instructional unit consisted of nine lessons: three at the concrete stage, three at the representational stage, and three at the abstract stage. Each lesson consisted of four sections using explicit instruction: a) provide an advance organizer, b) demonstrate and prompting, c) provide guided practice, and d) provide independent work. First, the teacher provided an advance organizer by reviewing past lessons and introducing new concepts. Then, during demonstration and prompting, the teacher modeled the individual skill. Next, the teacher provided guided practice in which they scaffolded the students through the skill. Finally, the teacher provided time for independent practice for the student to practice the skill independently. Sealander et al. (2012) assessed student learning using a multiple baseline across participant's design. The researchers collected daily data in the form of 1-min probes across conditions: pre baseline, baseline, intervention, and maintenance. The intervention was given individually to each student and consisted of the scripted lesson at the concrete, representational, and abstract stage. After each lesson, the teacher gave a ten-item mastery test. After the students showed mastery, which the researchers defined as receiving a 90% or higher score, the student progressed to the next level. If the student failed, the teacher gave additional instruction.

Sealander et al. (2012) assessed interobserver agreement on assessment probes and procedural integrity with instruction delivery. Inter-observer agreement was 98.7%. Results for teacher one's procedural integrity was 98%, teacher two was 99%, and teacher three was 97%.

When students demonstrated the crossover point, defined as days with more correct answers than incorrect answers (more than thirty correct digits) on probes, the researcher discontinued instruction (Miller & Mercer, 1993). Sealander et al. (2012) found that all students reached the crossover point before the teacher finished teaching the entire CRA sequence (Miller & Mercer, 1993). None of the students required teaching at the abstract level. The teacher taught an average of 4.38 lessons to reach the crossover point; however, three students had to repeat one or two lessons. During maintenance, seven students performed better than during instruction. The range of improvement from pretest and post-test was 60% to 100%. Generalization assessments showed that all students improved substantially in solving word problems. Overall, Sealander et al. (2012) found that discontinuing instruction at the crossover point did not adversely affect the student. Moreover, Sealander et al.'s (2012) finding suggests that when teachers use CRA paired with explicit instruction, teachers can use a data-based decision rule, a national norm of thirty correct digits, to determine whether to continue or discontinue instruction.

In comparison to Root et al. (2017), Flores et al. (2014) conducted a study that investigated the effects of CRA instructional sequence and SIM concerning mathematics computation performance of students ages five to twelve with autism spectrum disorders (ASD) and developmental disabilities (DD). Previous research had focused on high incidence disabilities and individuals who were at risk. Flores et al. (2014) taught single-digit addition and subtraction. Like Miller and Mercer (1993), Flores et al. (2014) used scripted lessons and teacher instructions. The researchers provided CRA-SIM instruction using the *Strategic Instruction Math Series: Addition Facts 0-9* (Miller & Mercer, 1998) and *Strategic Instruction Math Series: Subtraction Facts 0-9* (Miller & Mercer, 1998). Nine students received instruction in addition facts, while two students received instruction in subtraction facts.

To ensure the study demonstrated quality research, Flores et al. (2014) implemented several measures such as training all parties in the study and assessing treatment integrity and inter-observer agreement. Treatment integrity for this study was 92%. Inter-observer agreement for curriculum-based measures was 100%. Flores et al. (2014) used a paired sample t-test to compare the pretest and post-test differences. The results extended CRA research to individuals with ASD and DD since all previous research focused on students with high incidence disabilities and students at risk. The results indicated a significant improvement in the students' computation skills, progress, and performance from pretest to post-test and showed that individuals with ASD and DD could benefit from CRA-SIM using Strategic Math Series in basic computation.

To further extend the line of research regarding CRA and ASD, Yakubova et al. (2016) investigated the effects of video modeling intervention (VBI) in conjunction with CRA instructional sequence in teaching addition, subtraction, and number comparison to individuals with ASD. This study included four individuals enrolled in a private educational center that served individuals with ASD.

In contrast to previous studies discussed, Yakubova et al. (2016) paired CRA with VBI to deliver instruction. The researchers decided to use point-of-view VBI. In point-of-view VBI, the researchers filmed the video from a first-person perspective (Yakubova et al., 2016). This study's independent variables were the point-of-view VBI paired with CRA and a task-analysis derived from the videos. Using a multiple baseline across skills design, the researchers defined the dependent variable accuracy in solving single and double-digit addition, subtraction, and number comparison. The researchers evaluated each student before baseline. Then, the researchers made a set of problems that targeted the dependent variables. After input from the teacher and completion of the evaluation, the researchers developed one hundred problems. During the intervention, the researchers randomly selected four problems for baseline and intervention. The researchers did not repeat any problems in any of the sessions.

The researchers made five video clips one for single-digit addition, single-digit subtraction, 2-digit addition, 2-digit subtraction, and number comparison. The video consisted of an adult model working through each problem for each phase of CRA. Materials in the video consisted of colored manipulatives during the concrete stage and colored markers during the representational stage. During the abstract stage, the adult model solved the same problem using numbers and symbols. The researchers gave all students access to the same materials in the video. Also, the researchers gave each student a checklist derived directly from the videos. The checklist was in the form of task analysis in the steps required to solve problems.

Each student attended lessons for three days for a total of 23 days. The researchers taught three instructional lessons per week. The researcher taught five skills: one for single-digit addition, single-digit subtraction, 2-digit addition, 2-digit subtraction, and number comparison, and each day, the researchers taught a new session. Baseline lasted 10-15 mins for each session and had a minimum of five data points per skill. The researchers gave the students four problems per skill and did not provide intervention or assistance in solving the problems. The researcher gave the students a one-page paper with four problems set for each of the five skills and a pencil and paper. The researcher decided to move from baseline to intervention based on a visual analysis of the students' performance and a downward trend. During the intervention, the trainer gave the students the four problems per skill and an iPad with the training videos for each skill. Each intervention session lasted about 30 mins. The students watched the video clip in its entirety that consisted of a video model solving a subtraction problem, for example, at the concrete, representational, and abstract stage. The researcher allowed the students to watch any part of the video during any part of the intervention stage. The trainer guided the student through each session after the completion of the video. Students could re-watch the video as needed during the intervention. The trainer collected the response sheets and scored them. Students stayed in the intervention phase for a minimum of five sessions and moved to the next phase after mastering three out of four problems. The trainer gave verbal prompts to begin and positive reinforcement during each lesson. The maintenance phase was conducted three weeks after and lasted for ten minutes. The trainer gave the students the four problems and prompts. The trainer did not provide the videos to the student during the maintenance phase.

The researcher collected IOA data on 30% of the sessions by a second trained independent rater on the multiple baseline across skills design. The researcher collected IOA by dividing the number of agreements and disagreements and then multiplying by 100%. The researchers calculated IOA at 100%. The researchers assessed social validity for this study. The researchers asked the students and classroom teachers four social validity questions on Likert type scale.

This study demonstrated a functional relation between the point-of-view video clips paired with CRA and accuracy of solved problems in single and double-digit addition, subtraction, and number comparison. Furthermore, three students responded positively to the social validity questionnaire. One student expressed that he would not like to use video clips in the future. All teachers responded positively to the social validity questionnaire.

CRA Place Value and Number Sense

The following section describes the investigation of CRA in place value (Peterson et al., 1988) and number sense (Peterson et al., 1990). These included students with learning disabilities. The researchers taught using base ten blocks/popsicle sticks and place value strips at the concrete level, two-dimensional pictures (i.e., bundle sticks) at the representational level, and numbers only at the abstract level. All the studies found that students performed better using place value and number sense with CRA than their peers who used the abstract only method.

Peterson et al. (1988) compared CRA instruction to abstract-only instruction for teaching students with a learning disability initial place value skills. The researchers assigned twenty-four public elementary and middle school students to either CRA instruction or abstract-only instruction. The CRA group received nine lessons: three lessons for the concrete stage, three lessons for the representation stage, and three lessons for the abstract stage. The abstract-only group received nine lessons at the abstract stage. Of the twenty-four students receiving special education services, nineteen received them in a self-contained classroom, four students in a diagnostic classroom, and one student received services in the resource room. The criterion for participation was 70% or lowered on an assessment of acquisition and generalization.

Peterson et al. (1988) divided the study into three phases. The first phase addressed training for all teachers in the study, and two observers agreed (99% agreement) that teachers mastered the skills within the training. The second phase was instruction and data collection. The researchers created eighteen scripted lessons, nine for each group, which lasted about 10-15 minutes. The teacher gave scripted verbal praise if a student scored 80% or higher on individual lessons. Finally, the third phase collected posttreatment data for acquisition, maintenance, retention, and generalization. The researchers gathered data collection and posttreatment data via three-criterion-based teacher-made instruments consisting of an acquisition test given immediately, a maintenance test that was the same material as acquisition given one week later, and a different test generalization three weeks later.

The materials consisted of unfix cubes, place value sticks, and teacher-made place value strips in the concrete stage. The representational stage materials consisted of worksheets with pictorial representations from various published sources. The abstract stage materials consisted of worksheets without pictorial representations from various published sources. The abstract-only group did the same things as the CRA group during the abstract-level lessons. The researcher used a 2 x 3 mixed design with one between (treatment) and one within (performance over time). The researchers compared CRA's effects versus abstract only on students with LD's performance with a multivariate analysis of variance. Due to the instructional treatment's significant main effect, the researchers used a follow-up univariate analysis for acquisition and generalization. The researchers found that CRA was more effective than abstract only when teaching place value acquisition to students with LD. Students in both groups performed similarly on generalization. Students receiving the CRA instruction performed significantly higher than those who received the abstract-only instruction in place value acquisition. Peterson et al.'s (1988) findings suggest that the CRA sequence is more effective than abstract only when teaching initial place value skills to students with LD.

Peterson et al. (1990) continued this research line with three students with a LD in the first, second, and fourth grades. To be eligible to participate in the study, the students took a 10-

item test and were required to score a 70% or lower. Using a multiple baseline design, Peterson et al. (1990) implemented three phases: baseline, treatment, and posttreatment. During baseline, the students completed 1-min timings daily for three days. Similar to Peterson et al. (1988), the researcher taught the students place value skills using CRA during the treatment phase. The researchers used the same materials as Peterson et al. (1988) during the concrete and representational phases. The researchers taught nine lessons, three at each stage, using explicit instruction and 15-minute scripted lessons. The students worked through all nine lessons at their own pace. After completing the treatment phase, the researchers gave a post-test that was identical to the pretest in the baseline. Finally, the researchers administered a retention probe sheet, an alternate form of the baseline probe. Peterson et al.'s (1990) findings were similar to Peterson et al.'s (1988). All three students made gains, maintained, and retained the skills taught; this suggested that CRA was sufficient for all three students.

CRA Subtraction with Regrouping

The following section describes the investigation of CRA in subtraction with regrouping (Flores, 2009; Flores, 2010, & Mancl et al., 2012) and virtual (Bouck et al., 2014; Bouck et al., 2017a; Bouck et al., 2017b). These included students with SLD and who were at risk for failure. The researchers taught using base-ten blocks to solve equations at the concrete level, two-dimensional drawings at the representational level, and numbers only with a mnemonic at the abstract level. All the studies found that the CRA sequence successfully taught students to solve an equation in subtraction with regrouping.

The line of CRA instructional sequence using regrouping research began with Flores (2009). Flores sought to investigate the effects of the CRA instructional sequence on subtraction performance. Six students with a specific learning disability (SLD) and who were struggling in mathematics participated. The researcher used a multiple probe across groups design to investigate the use of CRA sequence to teach subtraction regrouping in the tens and hundreds (Flores, 2009). The researcher implemented the CRA instruction similar to Miller and Mercer (1993). The researcher taught three different phases in which the students learned to solve problems and regrouped numbers in the tens place. The concrete phase (lessons one to three) involved instruction with base-ten blocks and learning sheets. The researcher used base ten blocks as manipulatives to show the regrouping process and find the difference. The next phase was representational (lessons four to six), in which the student used drawings to represent ones, tens, and hundreds within the minuends. Abstract instruction was the final phase in which students memorized the DRAW strategy. During lessons eight through ten, students solved problems using numbers and symbols and DRAW.

Flores (2009) measured student progress with probes with regrouping in the tens. Once baselines were stable, CRA 10s instruction began. A stable baseline was three consecutive data points that did not vary more than 5% from an average responding rate. Once the researcher achieved stability, CRA instruction began with one student while the others stayed at the baseline. Once students achieved mastery, which was twenty correct digits for three consecutive data points, the students moved to maintenance. During this maintenance, students did not receive any instruction for four weeks. After four weeks, they completed a probe.

Flores (2009) demonstrated a functional relation between the CRA instruction and subtraction with regrouping performance. The researcher assessed interrater reliability for 100% of the probes, and the reliability score was 100%. The researcher used a treatment integrity checklist to assess integrity on 30% of the lessons in this study. The researchers addressed social validity with interviews with teachers and students before and after the study. The students reported before the study that subtraction was hard. After the study, the students reported that the CRA sequence made subtraction with regrouping easier. The teachers reported before the study that there was a need for this instructional sequence. After the study, the teachers reported that the intervention was sufficient and that students' scores increased.

Similar to Flores (2009), Flores (2010) sought to investigate the effects of the CRA instructional sequence on students' subtraction performance with problems that required regrouping in the tens and hundreds places. In this study, six students in the third grade, identified as at-risk for mathematics failure, participated in multiple probes across students with embedded changing behaviors design. (Flores, 2010). The criteria for phase changing and mastery was twenty correct digits on three consecutive two-minute probes. First, the researcher taught students to solve problems in which they regrouped numbers in the tens place. Then she taught students to solve problems that required regrouping in the tens and hundreds place. The phases for CRA 10s instruction and CRA 100s instruction consisted of a concrete phase (lessons one to three), in which the student learned with base-ten blocks. The next phase was representational (lessons four to six), in which the student used drawings. The students used the same materials and feedback that Flores (2009) used to represent the concrete manipulatives and solve problems.

The researcher measured student progress with probes with regrouping in the 10s and 10s, and 100s. Once baselines were stable, CRA 10s instruction began. Once students achieved mastery at CRA 10s, the student moved to baseline 10s and 100s. Once students achieved stability at baseline 10s and 100s, the student moved to instruction in regrouping in the 10s and 100s places. The students completed a maintenance phase at the end of six weeks.

Flores (2010) demonstrated a functional relation between the CRA instruction and students' subtraction with regrouping performance. The researcher assessed interrater reliability for 80% of the probes, and the reliability score was 97%. The researchers used a treatment integrity checklist to assess integrity on 30% of the lessons in this study. The researchers calculated treatment integrity at 100%.

To further extend, Flores (2009) and Flores (2010), Mancl et al. (2012) conducted a study that also used the CRA instructional sequence. This study investigated the effects of CRA instruction using explicit instruction with integrated cognitive strategies. This study consisted of six students in fourth and sixth grade using a multiple probe across participants design to determine if an implementation of eleven subtraction with regrouping lessons using the CRA sequence would improve students' performance (Mancl et al., 2012). This study is different from Flores (2009) in that it suggests that a smaller number of lessons would be sufficient; however, it is similar to Flores (2010) investigated subtraction with regrouping. This study also included word problems. The first five lessons taught concrete-level instruction with base ten blocks as well as a strategy for making decisions about regrouping. For example, the strategy said, "bigger number on Bottom means Break down and trade." Lessons six to eight taught representational-level instruction by using drawings. Lessons nine to eleven taught a specific strategy: (a) Read the problem, (b) Examine the one's column, (c) Note ones in the one's column, (d) Address the tens column, (e) Mark tens column, and (f) Examine and note hundreds; RENAME). Students completed probes at the end of instruction and completed the probe using the given lesson's CRA materials. This was different from the assessment procedures used by Flores (2010), in which probes were separate from instruction and assessed fluency at the abstract level from baseline through intervention and maintenance. Flores (2010) gave probes at the beginning of lessons, measuring what students learned from the previous lesson. Mancl et al.'s (2012) data came from independent practice portions of the lessons.

To ensure experimental control, the researchers assessed for interscorer reliability. Two independent observers scored 100% of the baseline and intervention probes. Interscorer reliability for the baseline phase was 99.52%, and the intervention phase was 98.95%. The researchers measured treatment fidelity with independent observers that used a checklist and were determined to be 99.15%. All students achieved immediate level gains after receiving intervention lessons. Baseline probe scores ranged from 0% to 40%, and intervention probe scores ranged from 40% to 100%.

Bouck et al. (2014) also taught subtraction with regrouping, but they used virtual manipulatives. They compared the effects of virtual and concrete manipulatives for solving mathematics problems for students with autism spectrum disorder. In this study, three male students with ASD participated in an alternating treatment design to examine the effectiveness of student's ability to solve mathematics problems using concrete and virtual manipulatives. Baseline, intervention, maintenance, and generalization sessions took place in an autism clinic. The materials consisted of paper and pencil, concrete manipulatives, and virtual manipulatives. The concrete manipulatives consisted of base-ten blocks. The virtual manipulatives consisted of base-ten blocks that were displayed on a computer screen and were movable. The researchers required the students to alternate between using concrete and virtual manipulatives. The researcher defined using concrete manipulatives as physically manipulating base-ten blocks. The researcher defined using virtual manipulatives as using a computer to manipulate virtual blocks provided by the National Library of Virtual Manipulatives.

The researcher used event recording to determine the effectiveness of both concrete and virtual manipulatives for solving subtraction problems. Each therapist recorded the percent of correctly completed subtraction problems, the percent of steps within each subtraction problem completed independently, and the number of prompts needed for each student. Before baseline data, the researchers trained all students on how to use concrete and virtual manipulatives. The training across thirty trials focused on addition problems. The researchers collected baseline data over six sessions: three for concrete and three for virtual manipulatives. The researchers provided the students with the required manipulatives to solve the subtraction problems. The researchers required the students to solve five problems per session based on their current level of performance (e.g., single-digit equations and equations with two 2-digit numbers). During the intervention, students alternated between concrete and virtual manipulatives. In the concrete manipulative's intervention conditions, the therapist gave the student a sheet of paper that contained five subtraction problems and base ten blocks. The therapist asked each student to select the correct number of manipulatives for the minuend. Next, the therapist asked the student to physically remove the number of base-ten blocks in the ones and tens one by one. When finished, the therapist asked the student to state the difference aloud and write it on paper. In the virtual manipulative's intervention conditions, the therapist gave the student five mathematics problems. The students interacted with the visual manipulatives and used these to solve subtraction problems with regrouping. The therapist set up the computer screen and did not provide any prompts.

To solve each problem in the virtual phase, the researchers required the students to drag/drop individual blocks in the one's column from the minuend to the subtrahend and place them on top of each other. Once accomplished, the blocks canceled each other out. During these interventions, the student used a mouse to drop and drag manipulatives in ones and then in the tens columns to solve the problem. The therapist did not provide any instruction. They only set up the problems and covered the differences with a sticky note. Once the student eliminated all blocks from the subtrahend, the answers prepopulated on the screen. Maintenance consisted of three sessions in which the student used the most effective method to solve subtraction problems. The final session consisted of application to the real world in a generalization phase. In the concrete condition, the researcher asked the students to simulate a token economy. The students earned tokens. The researcher gave the student a list of activities and asked the student to pick three activities. The researcher predetermined a value for the activities. Then, the students subtracted the value of the activity from their tokens to purchase the activity.

Bouck et al. (2014) assessed treatment fidelity in the alternating treatment design by using a checklist and recordings for 32.3% of the sessions in the study. The researchers found treatment integrity to be 100%. Before and after the study, the researchers interviewed the students and their therapists for social validity. Before the study, the researcher found that each student liked to solve mathematics problems using a computer. After the study, each student said they preferred using a computer to solve mathematics problems. The therapist expressed positive impressions when using virtual manipulatives. The researchers assessed for interobserver agreement for all of the sessions in the study using a recorder. The researchers found interobserver agreement to be 93%, with a range of 71% to 100%. Bouck et al. (2014) demonstrated a functional relation by showing that both concrete and virtual manipulatives increased students' ability to solve subtraction problems. When comparing both interventions across all three participants, the researchers found both conditions successful in assisting students in solving single- and double-digit subtraction problems; however, virtual manipulatives performed slightly better than concrete manipulatives. The researchers found that students could generalize the skills taught through concrete and virtual manipulatives by applying their skills to manage a token economy.

To further extend, Bouck et al. (2014), Bouck et al. (2017b) investigated the effects of using CRA to teach middle school students to solve change-making problems with coins. Bouck et al. (2017b) used a multiple probe across participants design to demonstrate a functional relation between CRA and abilities to solve change-making problems. The study consisted of four middle school students who had ID. This study took place in a public middle school in the Midwestern United States. The materials consisted of a learning sheet that contained two model problems, two guided practice problems, and five independent practice problems. First, each student took a preassessment in which they identified the name and the amount ($1.12) and solve adding money problems. Next, the student moved to baseline. The students solved five change-making problems with coins (e.g., $0.55 - $0.32) independently on the probes. Next, the researcher presented the numeral form problems that were not read aloud with no manipulatives. Once a stable baseline was determined, the students progressed to intervention.

The researcher defined a stable baseline as 80% of data falling within 20% of the median. During the intervention, the researchers followed the same procedures as traditional CRA. All students participated in nine sessions: (a) three for concrete, (b) three for representational, and (c) three for abstract. The researcher used explicit instruction to teach all lessons. During the concrete stage, the researchers used plastic coins and paper bills. The researcher modeled each problem and then provided think aloud to each student after two problems. The students used coin manipulatives during guided practice. After completing two guided practice problems, the students moved to the independent stage and completed five problems using the coin manipulatives. Once the student solved 80% of the problems correctly, they moved to the next stage in CRA. If they did not solve 80% correctly, they repeated the lesson. During the representational stage, the researchers explicitly taught each student how to use drawings to solve problems for each behavior. The researcher taught the students to draw a rectangle and write a one inside the rectangle to represent a one-dollar bill. The researcher taught the student to draw four different size circles and write 1, 5, 10, and 25 to represent the coins. The researcher modeled each problem and then provided think aloud to each student after two problems. The students used drawings during guided practice. After completing two guided practice problems, the students moved to the independent stage and completed five problems using the drawings. Once the student solved 80% of the problems correctly, they moved to the next stage in CRA. If they did not solve 80% correctly, they repeated the lesson. During the abstract stage, the researchers taught the student to solve the problems without manipulatives or drawings. The researcher modeled two problems, provided guided practice with two problems, and asked the individual to complete five or ten problems independently. Each student completed two maintenance probes two weeks after the completion of the study. The researcher did not provide any assistance during the maintenance. Additionally, the researchers kept each maintenance consistent with the independent portion of the study and baseline where the researcher did not provide and feedback.

Bouck et al. (2017b) calculated IOA with recordings for 33.3% of the probes. The researchers had two independent observers record one to two sessions per phase. Using a checklist, the researchers focused on whether each student received the materials, read aloud problems, and implemented explicit instruction. The researcher calculated treatment fidelity for 33.3% of the study. The researchers found treatment fidelity as 100% and inter-rater agreement as 100%. The students and teachers completed a social validity interview. The researcher asked the students question such as if they liked solving change-making problems with coins, drawings, or without anything. The researchers also asked if they like the CRA sequence. The students reported positive outcomes related to CRA. The researcher asked the teacher regarding student learning, the benefits, and the challenges of using CRA sequence. The teachers reported positive outcomes related to CRA. All students showed an increasing data path once the CRA intervention began. All four students maintained their skills after instruction ended. Bouck et al. (2017b) demonstrated a functional relation on the effects of CRA mathematical sequence on solving change-making problems for a student with ID.

To further extend, Bouck et al. (2014), Bouck et al. (2016), Bouck et al. (2017a) compared the effects of app-based manipulatives and concrete manipulatives for solving subtraction problems to support students with disabilities. In this study, three male students who received resource services participated in an alternating treatment design. Baseline, intervention, best treatment, and generalization sessions took place in a small Midwest town in a rural middle school. The materials consisted of paper and pencil, the researcher created probe sheets, concrete manipulatives, and virtual manipulatives. The concrete manipulatives consisted of base-ten blocks. The virtual manipulatives consisted of base-ten blocks that were displayed on an iPad screen and were movable. The students alternated between using concrete and app-based manipulatives. The researcher defined using concrete manipulatives as physically manipulating base-ten blocks. The researcher defined using application based manipulatives as using an iPad to manipulate virtual blocks provided by the Base 10 Blocks Manipulative iPad app.

The researcher used visual analysis to calculate level, trend, and effect size to determine the effectiveness of both concrete and app-based manipulatives for solving subtraction problems.

This study included four conditions: baseline, intervention, best treatment, and generalization. Before baseline data, the researchers trained all students on using concrete and app-based manipulatives to solve practice subtraction problems at their level, which consisted of double or triple-digit subtraction problems with regrouping. The researchers provided prompts if the student had a latency of 10 seconds. Once data were stable in baseline, the intervention began. The researcher defined stable data if 80% of the student's data fell within 20% of the median for the dependent variable. During the intervention, students alternated between concrete, app-based manipulatives, and no manipulatives. In the concrete manipulative's intervention conditions, the therapist gave the student two place value sheets, two different colored sets of base ten blocks, and a sheet of paper that contained five subtraction problems. The therapist asked each to begin. Students set up the correct number of blocks on their place value sheet, regrouped their blocks, and counted the remaining number to find the solution. Next, the therapist gave the student five mathematics problems, an iPad, Base 10 Blocks Manipulative iPad app, and a pencil in the app-based manipulative's intervention conditions. The students interacted with the virtual manipulatives and used these to solve regrouping problems by dropping and dragging manipulatives from a picture. In the no manipulative condition, the researchers extended the baseline phase to control and determine a functional relation. Best treatment consisted of three sessions in which the student used the best treatment for them determined from the intervention condition. Researchers used the percentage of nonoverlapping data to determine the best treatment by calculating which condition was more effective than dividing the sum by five and multiplying by one hundred. The final two sessions consisted of baseline and extended baseline in a generalization phase. The researcher gave the students five problems on a probe sheet and a pencil. The researchers did not provide any assistance to the students and no manipulatives in concrete or app-based.

Bouck et al. (2017a) assessed treatment fidelity by using a checklist and recordings for 40% of intervention sessions, 40% of baseline sessions, 33% for best treatment sessions, 50% of generalization sessions in the study. The researchers assessed for interobserver agreement for all of the sessions in the study. The researchers found interobserver agreement to be 100%. The researchers found treatment integrity to be 100%. After the study, the researchers interviewed the students and their therapist for social validity on their perceptions about each manipulative. After the study, two students said they preferred using an app-based manipulative to solve mathematics problems, and one student said they preferred using concrete manipulatives to solve mathematics problems. The therapist expressed positive impressions when using app-based manipulatives. Bouck et al. (2017a) extended Bouck et al. (2014) by demonstrating that both app-based and concrete manipulatives were effective for students with disabilities. The researchers found that two of the three students answered 100% of problems correctly while one student required additional assistance. When comparing both interventions across all three participates, Bouck et al. (2017a) found that app-based manipulatives can be just as effective as concrete manipulatives.

To further extend this line of research, Bouck et al. (2018a) investigated the effects of virtual-representational-abstract (VRA) mathematical sequence on the acquisition of four mathematical skills: place value, addition with regrouping, subtraction with regrouping, or single-digit multiplication using a multiple probe across behaviors design. Two middle school students with ID participated in this study. This study took place in a public middle school in the Midwestern United States. The materials included an app-based manipulative and a learning sheet with instructor-led problems, guided problems, and independent problems. The researchers used a multiple probe across participants to demonstrate a functional relation. The researchers created different probes to measure the dependent variable. Each probe consisted of five addition, subtraction, or multiplication problems, and the place value probe consisted of ten problems. The researchers gave a preassessment to measure the student's ability to identify place value. Each student scored fewer than 20% of the problems correctly. During baseline, the researchers gave probes for all skills measured: (a) place value; (b) subtraction with regrouping; (c) addition with regrouping; and (d) single-digit multiplication. The criteria to move from baseline to intervention the baseline needed to have a zero-acceleration or zero deceleration. During the intervention, the researchers followed the same procedures as traditional CRA.

Both students participated in nine sessions: (a) three for virtual, (b) three for representational, and (c) three for abstract. The researcher used explicit instruction to teach all lessons. During the virtual stage, the researchers explicitly taught each student how to use the app and manipulatives to solve problems for each behavior. During the representational stage, the researchers explicitly taught each student how to use the drawing to solve problems for each behavior—using the app-based manipulatives. Finally, during the abstract stage, the researchers asked the student to solve the problems using numbers and symbols. Each student completed two maintenance probes three weeks after the completion of the study. The researcher did not provide any assistance during the maintenance. Additionally, the researcher kept each maintenance consistent with the independent portion of the study and baseline where the researcher did not provide research.

Bouck et al. (2018a) calculated IOA for 33.3% of the probes for baseline and intervention and 50% of the probes for maintenance. Using a checklist, the researchers focused on the students using the correct tool for all three behaviors: app, drawings, or abstractly in treatment fidelity. The researchers found treatment fidelity as 100% and inter-rater agreement as 100%. The students completed a social validity interview that consisted of two open-ended questions. The students both reported positive outcomes related to VRA. Both students showed an increasing data path once the implementation of the independent variable began. All three students maintained their skills after instruction ended. Bouck et al. (2018a) demonstrated a functional relation to VRA mathematical sequence effects on the acquisition of four mathematical skills: place value, addition with regrouping, subtraction with regrouping, or single-digit multiplication.

CRA Addition with Regrouping

The following section describes the investigation of CRA to teach addition with regrouping (Miller & Kaffar, 2011; Stroizer et al., 2015). These included students who struggle with mathematics and students with autism spectrum disorder. The researchers taught using base-ten blocks to solve equations at the concrete level, two-dimensional drawings at the representational level, and numbers only with a mnemonic at the abstract level. All the studies found that the CRA sequence was successful in teaching students to solve equations in addition with regrouping.

Miller and Kaffar (2011) further extended the CRA regrouping line of research to investigate addition with regrouping when using a CRA sequence. Miller and Kaffar investigated instruction for solving multi-digit addition problems when using CRA. They compared the CRA sequence to textbook instruction when teaching addition with regrouping to students who struggled with mathematics.

In this study, a total of twenty-four second-grade students participated in six-week mathematics and reading summer camp. This study sought to measure the effectiveness of using the CRA teaching sequence with an integrated strategy for developing addition with regrouping. The mnemonic strategy included the following steps: (a) read the problem; (b) Examine the ones; (c) note the ones; (d) address the tens, and (e) examine the hundreds and exit with a check (RENAME). Similar to Flores' (2010) study, the researchers assessed the participants using two-timed measures, but they also used untimed measures.

In contrast, most research discussed above used single-case design; Miller and Kaffer (2011) used a pretest and post-test design and included different mnemonic strategies from previous research (Flores, 2010). The mnemonic strategies included RENAME, and they added FAST RENAME to solve word problems (Miller & Kaffer, 2011). Their study also differed because the authors did not implement instruction. Instead, teachers taught the treatment group using CRA instruction in sixteen lessons: five concrete-level lessons using base ten blocks, three representational-level lessons using drawings, and eight abstract-level lessons that involved solving problems without manipulatives. In addition, two abstract lessons used a learning strategy that gave the students steps for solving word problems during the abstract phase (Miller & Kaffer, 2011). Each lesson had a script for teachers to follow and a learning sheet. During the last six lessons, the treatment group also completed one-minute timings, whereas, in Flores' study, the students completed two-minute probes.

In contrast to the treatment group, the comparison group students received sixteen unscripted lessons from a second-grade mathematics textbook. One lesson involved concrete-level instruction using base ten blocks. None of the lessons included strategies. All lessons included the whole group and teacher-led instruction, in which the students received advance organizers and demonstrations, guided practice, and independent practice. During the last week, the comparison group completed one-minute timings. Similarly, both Flores (2010) and Miller and Kaffar (2011) used twenty problems on their assessments; however, Miller and Kaffar (2011) used two-word problems in their assessment and twenty problems on their pretest and post-test.

Miller and Kaffar (2011) determined inter-scorer reliability by randomly selecting samples and having two independent scorers grade 25% of the pretests and post-test. The reliability scores ranged from 98% to 100 %. To ensure treatment fidelity, the researchers provided each teacher with direct instruction for each lesson component (Miller & Kaffar, 2011). The findings were similar to Flores (2010) and found that students who received CRA intervention scored higher than the students who received traditional textbook instruction.

Stroizer et al. (2015) extended addition with regrouping research by examining the CRA sequence's effect in teaching elementary students with ASD. The researchers taught addition and subtraction with regrouping and multiplication facts from zero to five to students with ASD. Like Mancl et al. (2012), this study included three students in the third and fourth grades using a multiple probe across participants' design. Like Flores et al. (2014), this study took place in an extended school year program. The researchers used the same materials as Mancl et al. (2012) and Miller and Kaffar (2011), The teacher provided instruction in a small group setting that lasted for twenty minutes. Once students mastered six of the nine problems, they changed a phase. The first three lessons taught the concrete phase of instruction using manipulatives. Once students achieved mastery, they moved to the representational level of instruction for lessons five through six using drawings. Lesson seven introduced the RENAME strategy, similar to Mancl et al. (2012) and Miller and Kaffar (2011). Lesson eight and lessons that continued until the end of the program was the abstract level where the students used RENAME to remember problem-solving steps. The teacher used the same instructional sequence to teach the multiplication facts curriculum, but the mnemonic strategy was DRAW.

The researchers ensured that probes were reliable with Cronbach's Alpha Coefficients of .72 across probes. The researchers assessed treatment integrity for 27% of the sessions for baseline and addition with regrouping, 30% of the sessions for baseline and subtraction with regrouping, and 60% of the sessions for baseline and multiplication facts zero through five. The results were 88% for addition with regrouping condition, 97% for subtraction with regrouping condition and 81% for multiplication facts zero through five conditions. Overall inter-rater agreement across fidelity checklists was 98%. The researchers assessed the inter-observer agreement for all of the probes, and the agreement was 100%. All three students demonstrated steady progress across the three skill areas. The researchers demonstrated a functional relation between the CRA instruction and the behaviors of addition, subtraction with regrouping, and multiplication facts zero to five with three participants. Similar to Flores (2010), Flores et al. (2014); Mancl et al. (2012); and Miller and Kaffar (2011), Stroizer et al.'s (2015) CRA sequence resulted in increased computation performance of elementary students.

Multiplication with Regrouping

The following section describes the investigation of CRA in multiplication with regrouping (Flores & Hinton, 2019; Flores et al., 2014a; Flores et al., 2014b; Flores et al., 2019a; Flores & Milton 2020; and Flores et al., 2019b). These included students at risk for failure and students with SLD. The researchers taught using base-ten blocks to solve equations at the concrete level, two-dimensional drawings at the representational level, and numbers only with a mnemonic at the abstract level. All the studies found that the CRA sequence successfully taught students to solve an equation in multiplication with regrouping.

Flores et al. (2014b) used a multiple probe across behaviors design to pilot a study investigating the effects of CRA and SIM on teaching multiplication with regrouping to individuals. Three third-grade students located in a rural elementary school within the Southeastern United States participated in this study. The materials consisted of assessment probes and instructional items. The researchers developed four sets of assessments probes: (a) subtraction of three-digit numbers with regrouping in the ones' place, (b) subtraction of three-digit numbers with zeros in the tens' place that requiring regrouping in the ones' and tens' place, (c) multiplication of two-digit and one-digit numbers with regrouping, and (d) multiplication of 2 two-digit numbers with regrouping. Each probe consisted of twenty-four problems. The instructional materials varied from skill and the level of instruction. For subtraction, the researchers used the following in the concrete stage: (a) base ten blocks, (b) student learning sheets divided into three sections demonstration that included four problems, a guided practice that included four problems, and independent practice included five problems. For the representational stage or lessons four through six in subtraction, the materials included the following: (a) no manipulative objects because students made drawings, and (b) student learning sheets divided into three sections consistent with explicit instruction steps. Lesson seven for the subtraction included the mnemonic RENAME (Miller & Kaffer, 2011). For the abstract stage or lessons eight through ten with subtraction, the materials included the just student learning sheet with multiplication equations. The researchers used similar materials in the lessons used to teach subtraction in which there were zeros in the tens' place of minuend and requiring regrouping, multiplication with one-digit and two, and multiplication with two 2-digit multipliers with regrouping. The first author administrated probes until the baseline was stable. After the researcher established a stable baseline, the researcher administered probes before instruction to measure learning from the previous day. The researcher gave students two minutes to complete the probes. Instruction began with the subtraction with regrouping in the one's column. Once students mastered subtraction, they received instruction in the multiplication of two-digit and one digit with regrouping then multiplication with two two-digit regroupings.

Using a treatment observer checklist, two observers watched and scored 25% of the videos. Once they watched all videos, the observers compared their results. The observer checklist was 100%. Interobserver agreement was 100%. The researchers scored 50% probes in the study. Inter-scorer reliability was 96% for subtraction with regrouping in ones', 99% for subtraction with regrouping in the ones' and tens', 100% for two-digit by one-digit multiplication with regrouping, 100% for two-digit regrouping. Before and after the study, the student's general education teacher completed a social validity survey. After the study showed that students' performance and grades improved, students were engaged and less frustrated, and they would recommend the intervention to other teachers.

All three students showed an increasing data path once the implementation of the independent variable began across all behaviors. Two of the participants demonstrated a functional relation with systematic change in behaviors associated with subtraction with regrouping in the ones' place, subtraction with regrouping in the ones' and tens' place, and multiple regrouping one-digit multiplier. One participant demonstrated a function relation with systematic change in behaviors associated with subtraction with regrouping in the ones' place, subtraction with regrouping in the one's and ten's place, multiple regrouping one-digit multiplier, and multiple regrouping two-digit multiplier. All three students maintained their skills after instruction ended. The study was significant because it first investigated CRA and SIM in teaching multiplication with regrouping.

To further extend the CRA line of research, Flores et al. (2014a) investigated CRA's effects with SIM on multiplication with regrouping in students with LD using multiple probe across students' design. This study took place in the rural United States in an elementary school. The materials consisted of 25-probes made up of 2-digit multiplication with regrouping problems. The researchers developed four different probes used in assessment during baseline, intervention, and maintenance.

Additionally, the researchers used an instructional manual that provided scripted lessons, student learning sheets, place value mats, and based 10-blocks. Each learning sheet was divided into sections according to explicit instruction steps as described in previous research. The lessons for the concrete, representational, and abstract stages followed the same format as previous research (Flores, 2010; Miller & Kaffar, 2011; Mancl et al., 2012). Lesson seven consisted of the RENAME strategy (Miller & Kaffar, 2011). The students used base ten blocks and a place value mat to regroup and solve the problems.

Before instruction, the researchers administered 2-minute probes to establish baseline. Once instruction began, the researcher delivered all probes before the lessons until the students achieved mastery. The researcher administered probes two weeks after instruction ended to gather maintenance data. The researchers administered a generalization probe regrouping problem with a three-digit multiplicand and the two-digit multiplier at the end of the study. The instruction did not include these problems.

The second and third authors assessed treatment fidelity for two out of three sessions per week via live observations and a checklist. The researchers calculated treatment fidelity at 100% for this study. Additionally, the researcher assessed for inter-scorer agreement. A second observer scored 50% of all the probes in baseline and intervention. The agreement was 100% for three of the students and 97% for one. Finally, the researchers assessed social validity using an open-ended question format. The researchers found positive results from the social validity questionnaire.

Flores et al (2014a) demonstrated a functional relation on the effects of CRA-SIM on 2-digit multiplication with regrouping in individuals with LD. All four students showed an increasing data path once intervention began. In addition, all participants' maintenance and generalization probes demonstrated mastery. Flores et al. (2014) findings are significant because this study began a new line of research in CRA for students with SLD.

To further extend the CRA line of research in multiplication, Flores and Hinton (2019) replicated CRA's effects with SIM on multiplication with regrouping with elementary students using a multiple baseline across students. This study took place in the rural United States in an elementary school. Three elementary students qualified for the study due to receiving intervention under a failure prevention framework. None of the participants received special education services at the time of the study. The materials consisted of three types of assessments: (a) repeated computation probes, (b) a problem-solving assessment, and (c) an interview related to computation procedures.

Additionally, the researchers used an instructional manual that provided scripted lessons, student learning sheets, place value mats, and base 10-blocks. Rather than presenting just multiplication equations, the learning sheets included sentences about situations describing the combination of equal-sized groups. The research taught students to translate those sentences The students identified how many groups and how many items were in each group. Next, the students wrote a multiplication equation on the lesson sheet using numbers and symbols. Finally, the student and the teacher solved the problem using base ten blocks and a multiplication mat. The teachers used the same written sentences in the representational stage except that everything was verbal without the teachers' written prompts at the concrete stage.

Additionally, instead of using base ten blocks, the teacher drew on a place value mat: hundreds were squares, tens were horizontal lines, and the ones were vertical lines. The seventh lesson involved teaching the RENAME mnemonic (Miller & Kaffer, 2011). The teachers taught the abstract stage two ways: written numerals only in vertically written equations and one-step word problems that required regrouping in addition, subtraction, or multiplication.

The researcher administered the problem-solving probes that consisted of regrouping problems, subtraction, and multiplication after instruction ended. At the end of the study, the researchers interviewed the students to gather additional data on solving multiplication with regrouping problems. The questions consisted of computing the problem and identifying each numeral's place value in the problem. The researchers trained an observer on the implementation behaviors. The observers assessed treatment fidelity for a set of ten accurately scored probes in the study. The observers calculated treatment fidelity to be 100%.

Two researchers scored 100% of the probes in baseline and intervention. Additionally, the researcher assessed for inter-observer agreement. The agreement was 94%-100% for all of the students. Finally, the researchers assessed social validity using a student survey before and after the study. The researchers found positive results from the social validity survey.

Flores and Hinton (2019) demonstrated a functional relation between CRA-SIM and students' two-digit by one digit multiplication with regrouping performance. All three students showed an increasing data path once intervention began and maintained fluency in maintenance. This study is significant because it replicates previous research and adds to the research by extending current research that assessed the student's computation data by measuring correctly written digits and correct products. Before Flores and Hinton (2019), the research line only included the number of written digits or percentages of correct sums/differences/products/quotients. The researcher assessed the students' approach and understanding of solving multiplication problems with regrouping with interviews. Before the study, the researcher presented the participants with multiplication with regrouping problems. Then, the researchers asked the participants to explain how to solve the problems and wrote word for word what the participants said. The participants struggled to answer the interview questions; however, after the study, the participants were confident to answer all the questions. All three participants improved their computation performance. Additionally, all three participants achieved fluency in the computation of multiplication with regrouping. These findings are consistent with previous research findings (Flores et al., 2014a; Flores et al., 2014b).

Flores et al. (2019a) focused on closing a gap in the CRA research line. Until this study, the CRA line of research had not compared CRA to another method. The purpose of this study was to compare CRA-SIM to direct instruction (DI). Flores et al (2019a) used a between-group design with a pretest/posttest method in this study. This study took place in a remedial summer intervention program located at a combined elementary and middle school in Midwestern city in the United States. Instruction for CRA and DI occurred for 30 minutes simultaneously in separate classrooms. The materials for DI instruction included a manual that consisted of scripted lessons, a whiteboard, a student workbook, an activity page, a progress sheet, and a learning contract. The materials for the CRA instruction included an instructional manual, magnetic base ten blocks, a place value mat, a whiteboard or Smartboard, a learning contract, a progress chart, and learning sheets for each lesson.

DI and CRA-SIM are both explicit instruction methods. DI differs from CRA-SIM in that it requires choral responses and provides more opportunities to practice with repetitive problem-solving. CRA-SIM differs in that it focuses on conceptual understanding with concrete models and representational drawings before focus on procedures associated with the standard algorithm. The procedures for DI included a 50-minute lesson on lesson twenty-eight. Lesson 28 was the entry point in the program for students who have achieved mastery with basic multiplication. Lessons 28-44 involved multiplication with regrouping. Before and after each lesson, the students completed 2-minute probes for DI and CRA-SIM instruction. Instruction began with the teacher's verbal instruction on how to multiple 2-digit by 2-digit problems with regrouping. Next, the teacher gave the student a problem with grids and regrouping boxes printed on them. The students then completed the problem. A typical lesson consisted of rehearsing the basic multiplication facts, then completing addition with regrouping. Next, instruction was provided on place value and reading numbers and included word problems with addition, subtraction, and multiplication. For the CRA-SIM, the researchers conducted the study as previous research (Flores et al., 2014a; Flores et al., 2014b).

Using a checklist, the researchers watched 25% of live and videos lessons. Treatment integrity for DI instruction was 96%, with scores ranging from 80% to 100%. Treatment integrity for CRA-SIM was 93%. The researchers met with the teachers to ensure they addressed the areas of low scores. The researchers scored all of the probes in the study. Inter-scorer reliability was 97%. The teacher and students completed a social validity survey. Both the teacher and student reported positive outcomes related to CRA and partial products. Results showed that the CRA-SIM group performed slightly higher than the DI group. There was no difference in pretest before the study in the DI group or the CRA-SIM group; however, the posttest group showed a slight difference between DI and CRA-SIM. The results of this study are significant in that it was the first to compare CRA-SIM to another method (DI).

There have been different approaches in multiplying multi-digit numbers. Flores et al. (2019b) piloted a study to combine CRA with SIM and teach partial products. The researchers used a pre-experimental design to investigate the effects of CRA-SIM on students' fluency in solving two-digit multiplication problems using the partial product algorithm. This study took place in an urban elementary school located in the United States. Twelve students participated in this study. The materials consisted of untimed assessments of 2-digit multiplication problems, the written manual (Flores & Kaffar, 2018), a place value mat, a learning sheet, and base ten blocks. Each assessment consisted of ten multiplication problems with two 2-digit numbers and required regrouping. The researchers followed the scripted lesson from the instructor's manual and used explicit instruction to teach the CRA sequence. Using previous research as a paradigm, the teacher taught all lessons using the five steps of explicit instruction: advance organizer, modeling, guided practice, independent practice, and post organizer. The researchers set the learning sheets up like Flores et al. (2014a) and Flores et al. (2014b).

In the concrete stage, the teachers discussed the place value of each multiplier in the problem. The research discussed how one would multiply each of the four components to make partial products and that partial meant part. The researcher reminded the students about the commutative property. The researcher made equal groups on the place value table in the one's column. Next, the researcher modeled how to regroup by exchanging ones for tens. The student used the place value mat to write the product in the equation. The researcher modeled how to find the second partial product by making groups of tens and used the blocks shown on the place value mat to write the product in the equation. The researcher used the same process to show students how to compute the third and fourth partial products. Finally, the researcher modeled how to add all partial products together to find the total product. In the representational, the students used drawings instead of base ten blocks, and the teacher and students drew on a place value mat: hundreds were squares, tens were horizontal lines, and ones were vertical lines.

The researchers recorded one lesson once per week of lessons across the study. Using a treatment fidelity checklist, the teachers demonstrated a range of 84% to 92%, with an overall score of 88%. The researchers scored all of the assessments on the pre and post-tests in the study. Inter-scorer reliability was 100%. The researchers used a pre-experimental design to compare the students' performance before and after the study. The researchers investigated if there was a significant change over time in the participant's pre and post-test. The results showed that all students demonstrated a significant effect with substantial differences in scores and significantly improved solving 2-digit multiplication problems with CRA-SIM.

Flores and Milton (2020) replicated Flores et al. (2019b) and used a multiple probe across students' design. This study took place in an elementary school in the Southeastern United States. Three students with disabilities participated in the study. The criteria to participate in the study required a mastery of multiplication facts with digits 0-5, a deficit in multiplication with regrouping or less than 20 correct digits on one-minute probes, eligibility for special education services with a goal in multiplication, and parent consent. The researchers created four different versions of two-minute probes to measure the dependent variables. Each probe consisted of ten multiplication problems with 2-digit numbers between one and five, and the problem required regrouping. The teacher administered the probes before the instructional lesson to assess skills taught from previous lessons. The teacher used the same materials as (Flores et al., 2019). In addition, the teacher used explicit instruction using the same steps as previous research (Miller & Kaffar, 2011).

Using a fidelity checklist, the researchers trained the teacher on CRA mathematical sequence. The teacher demonstrated a 100% on the fidelity checklist. The researchers recorded 30% of all lessons across all three stages. Using a treatment fidelity checklist, two observers watched and scored the videos. Once they watched all videos, the observers compared their results. Treatment fidelity was 97.8%, and inter-rater agreement was 98.9%. The researchers scored all of the probes in the study. Inter-scorer reliability was 100%. The teacher and students completed a social validity survey that consisted of yes and no answers and two open-ended questions. Both the teacher and student reported positive outcomes related to CRA and partial products. All three students showed an increasing data path once the implementation of the independent variable began. Flores and Milton (2020) demonstrated a functional relation between CRA and student's performance using the partial products algorithm to solve 2-digit multiplication problems fluently. All three students maintained their skills after instruction ended.

The research discussed in this paper demonstrates that the CRA instructional approach has an extensive research base that spans multiple decades to the present across various mathematical skills and various populations. Therefore, CRA instructional sequence supports students with disabilities' performance across mathematics skills. The VRA instructional approach is a new extension to the CRA instructional approach (Bouck et al., 2018a). The research in the VRA instructional approach is not nearly as extensive as the CRA instructional approach, but it has been effective in place of CRA to support students. However, little research explores VRA and its effect on solving mathematical problems for individuals with various disabilities (Bouck et al., 2014).

Across all studies, researchers found that CRA was an effective way to teach students who struggle in solving equations in place value and number sense, single-digit operations, in addition, subtraction, and multiplication, as well as multi-digit operations in addition and subtraction, and multiplication. Additionally, the research discussed in this paper has found CRA to be successful with students who are at risk and receiving tiered supports, have developmental disabilities, learning disabilities, intellectual disabilities, specific learning disabilities, other health impairment, emotional disabilities, traumatic brain injury, speech impairments, autism spectrum disorder, and with and without disabilities. Across studies, generalization to other environments was a common limitation. Further research is needed to determine the effects of VRA. One area in which little VRA research exists is multiplication (2-digit by 2-digit with regrouping). Therefore, future research should focus on 2-digit by 2-digit with regrouping with VRA.

Chapter 3: Methodology

Participants

The researcher recruited participants according to a protocol approved by the Auburn University institutional review board. The criteria for participation were: (a) consent to participate in the study from a legal guardian as well as student assent, (b) eligibility for services under the exceptionality of LD or OHI, (c) currently receiving services in mathematics, (d) proficiency in addition with regrouping, and fluency in basic multiplication facts measured by 30 correct digits per minute, and (e) a deficit in multiplication with regrouping defined as less than 10 percent of problems correct on assessments (Flores et al., 2022). This study recruited seven participants: Patty, Edward, Kevin, Lacy, Alice, Tom, and Jeff. (See Table 1)

Patty was eligible for services under the exceptionality of LD. They received mathematical instruction daily in their general education classroom. Additionally, they received mathematics services in the resource room in a small group setting for 30 mins daily. They were proficient in addition with regrouping and fluent in basic multiplication facts. In the pretest timed assessment, Patty scored 0% and eight correct digits. In the pretest untimed assessment, Patty scored 0% and nine correct digits, showing a deficit in multiplication with regrouping.

Edward was eligible for services under the exceptionality of OHI. They received mathematical instruction daily in their general education classroom. Additionally, they received mathematics services in the resource room in a small group setting for 30 minutes daily. They were proficient in addition with regrouping and fluent in basic multiplication facts. In the pretest timed assessment, Edward scored 0% and 0 correct digits. In the pretest untimed assessment, Edward scored 0% and 0 correct digits, showing a deficit in multiplication with regrouping.

Kevin was eligible for services under the exceptionality of LD. They received mathematical instruction daily in their general education classroom. Additionally, they received mathematics services in the resource room in a small group setting for 30 minutes daily. They were proficient in addition with regrouping and fluent in basic multiplication facts. In the pretest timed assessment, Kevin scored 20% and 12 correct digits. In the pretest untimed assessment, Kevin scored 50% and 19 correct digits, showing a deficit in multiplication with regrouping.

Lacy was eligible for services under the exceptionality of OHI. They received mathematical instruction daily in their general education classroom. Additionally, they received mathematics services in the resource room in a small group setting for 30 minutes daily. They were proficient in addition with regrouping and fluent in basic multiplication facts. In the pretest timed assessment, Lacy scored 30% and 12 correct digits. In the pretest untimed assessment, Lacy scored 40% and 21 correct digits, showing a deficit in multiplication with regrouping.

Alice was eligible for services under the exceptionality of LD. They received mathematical instruction daily in their general education classroom. Additionally, they received mathematics services in the resource room in a small group setting for 30 minutes daily. They were proficient in addition with regrouping and fluent in basic multiplication facts. They were proficient in addition with regrouping and fluent in basic multiplication facts. In the pretest timed assessment, Alice scored 10% and 8 correct digits. In the pretest untimed assessment, Alice scored 50% and 32 correct digits, showing a deficit in multiplication with regrouping.

Tom was eligible for services under the exceptionality of LD. They received mathematical instruction daily in their general education classroom. Additionally, they received mathematics services in the resource room in a small group setting for 30 minutes daily. They were proficient in addition with regrouping and fluent in basic multiplication facts. In the pretest timed assessment, Tom scored 40% and 19 correct digits. In the pretest untimed assessment, Tom scored 40% and 16 correct digits, showing a deficit in multiplication with regrouping.

Jeff was eligible for services under the exceptionality of LD. They received mathematical instruction daily in their general education classroom. Additionally, they received mathematics services in the resource room in a small group setting for 30 minutes daily. They were proficient in addition with regrouping and fluent in basic multiplication facts. In the pretest timed assessment, Jeff scored 0% and 0 correct digits. In the pretest untimed assessment, Alice scored 0% and 0 correct digits, showing a deficit in multiplication with regrouping.

Table 1

Participant Characteristics

Student	Age	Grade	Disability	Cognitive Ability (IQ)	Mathematic Calculations/ Computations *	Mathematic Fact Fluency/Math Concepts and Applications *
Patty	10	5th	LD	82	SS 93	SS 96
Edward	11	6th	OHI	80	N/A	N/A
Kevin	12	6th	LD	84	SS 89	SS 87
Lacy	10	5th	OHI	91	SS 90	SS 76
Alice	11	6th	LD	88	SS 86	SS 94
Tom	13	6th	LD	76	SS 76 *	SS 72*
Jeff	11	6th	LD	80	SS 81	SS 83

Note. This table shows the age, grade level, disability, IQ, standard scores for mathematic calculations and computations*, and standard scores for mathematic fact fluency and math concepts and applications*.

Setting

The setting for the study was an elementary school in the southeastern United States. The study took place in a resource room at a kidney shape table during the participants mathematical instructional time for their Individuals Education Programs (IEPs) in the resource room. The primary researcher provided the intervention. She was a certified trained special education teacher with nine years of teaching experience. The researcher demonstrated 100 percent mastery to be deemed competent to implement the intervention. Mastery was determined by scoring 100 percent on treatment fidelity checklist. Instruction occurred in the students' resource room during regularly scheduled instruction. The student sat at a table with the researcher who provided one-on-one instruction. The intervention consisted of approximately sixteen sessions that lasted for 20 minutes.

Assessment Materials and Procedures

The study used multiplication probes to measure students' computation progress in fluency and accuracy. The probes were materials from *The strategic math series: Multiplication with regrouping* manual (Flores & Kaffar, 2018). There was one probe with three versions, it was printed on an 8-inch by 11-inch sheet with a total of ten multiplication equations written vertically, each requiring regrouping: six equations had two 2-digit multipliers and four equations had a two-digit and a one-digit multiplier. The researcher used a random generator to determine the order of administration and ensured students received different probes back-to-back to control for internal validity (Flores et al., 2022). The researcher used the same probes for maintenance. Generalization probes consisted of three-digit multiplicand and two-digit multiplier equations. These problems were not included in instruction so the researcher could assess for generalization of skills across participants.

Intervention Procedures

Before instruction began, the researcher collected baseline data. During baseline, the instructor placed probe material in direct view of the student, told the student to complete as many problems as possible, and started a timer. At the end of two minutes, the researcher said, "Times up, open your folder, put your probe in the front pocket, and pass me your folder," and collected the probe without providing feedback regarding the student's performance. During baseline, the students were individually administered the probe in their group.

During intervention, the researcher administered probes at the beginning of each lesson to measure learning from the previous day's lesson rather than immediately after receiving explicit instruction at the end of a lesson. The researcher placed the probe directly in front of the student and set a two-minute timer. Once the timer went off, the researcher collected the probe. Scores were obtained for fluency consistent with previous research as the number of correct digits (Flores et al., 2019; Flores et al., 2022; Flores & Hinton, 2019; Flores et al., 2014a; Flores et al., 2014b) written below the equation line. The researcher calculated accuracy as the percentage of completed problems that had an accurate product (Flores et al., 2022; Flores & Franklin, 2014; Flores & Hinton, 2019).

Instructional Materials

The researcher used materials from a published manual (i.e., Flores & Kaffar, 2018) that included instructional sample scripts, learning sheets with word problems and multiplication equations. The materials for concrete instruction included learning sheets, base ten blocks, and a place value mat printed on a L x W (11 X 17) poster with ones, tens, and hundreds column for manipulating base ten blocks (Flores & Kaffar, 2018). The materials for representational instruction included learning sheets that had place value tables printed next to each item that the students used to draw representations of ones, tens, and hundreds as they solved multiplication problems. The materials for abstract instruction were the learning sheets with word problems and multiplication problems and the FAST RENAME strategy written on the side of their learning sheet.

Training

The researcher created a treatment fidelity checklist with operational definitions prior to intervention to train for fidelity observations. Refer to Appendix A, Appendix B, and Appendix C for treatment fidelity checklist. The researcher used behavior skills training (BST) to train observers (Ward-Horner & Sturmey, 2012; Gianoumis & Sturrmey, 2012). BST consisted of (a) instruction, in which the researcher explained all the target skills; (b) modeling, in which the researcher showed the observers what to do; (c) rehearsal, in which the researcher allowed time for the trained observer to practice the skill with supervision, and (d) feedback in which the researcher provided feedback immediately to correct any errors (Ward-Horner & Sturrmey, 2012; Gianoumis & Sturrmey, 2012). Using the treatment fidelity checklist, the researcher trained the observers on target behaviors and procedures with examples and non-examples. The researcher required a 100% criterion for mastery across all sessions for concrete, representational, and abstract sessions for treatment fidelity. The researcher had two trained special education teachers rescore each probe for interrater agreement.

Research Design

This study employed a multiple probe across participant's design, as described by Horner and Baer (1978), to assess the effect of the independent variable of CRA and SIM using the standard algorithm on the dependent variable of fluency and accuracy for multiplication with regrouping for one and two 2-digit numbers with individuals with learning disabilities. According to O'Neill et al. (2011), once a person teaches a skill, it cannot be unlearned;

therefore, this design was appropriate for the study because removing the independent variable was not required to determine the effects on the dependent variable (O'Neill et al., 2011).

In single-case design studies, participants serve as their own control (Horner et al., 2005). Furthermore, this design was appropriate because the multiple probe design allowed the researcher to collect data intermittently instead of continuously collecting data; this may have reduced student frustration during baseline when given repeated difficult tasks (O'Neill et al., 2011). According to O'Neill et al. (2011), the advantages of a multiple probe design are that removing the independent variable is not required, demonstrates a functional relation across participants, and does not require extended experimental phases (O'Neill et al., 2011). According to O'Neill et al. (2011), the disadvantages of the multiple probe design for this study are delaying the implementation of the independent variable for some individuals and potential failure to show a functional relation. (O'Neill et al., 2011). The researcher controlled for significant threats to internal validity by using a within and between-subject comparison (Horner et al., 2005). The researcher compared baseline (control) and intervention data across three or more students. The researcher visually inspected the graph by noting changes in level (mean), the immediacy of effect, determining three data points above baseline to mastery, and the percentage of non-overlapping data points (PND) to determine a functional relation (O'Neill et al., 2011; Scruggs & Mastropieri, 2013). This study used systematic replication to demonstrate external validity or generalization (Horner et al., 2005). According to Scruggs and Mastropieri (2013), the researcher obtained PND by dividing the number of non-overlapping data points the total number of intervention data points and multiplying the quotient by one hundred. Consistent with Flores et al. (2022), Flores and Hinton (2019), and Flores et al. (2014a), the researcher measured magnitude of change with baseline-corrected Tau (Tarlow, 2017), and used an online calculator (Tarlow, 2016). Consistent with Flores et al. (2014a), the study administered maintenance probes two weeks after students mastered regrouping two 2-digit numbers and instruction ended, as well as generalization using probes consisting of three-digit multiplicand and two-digit multiplier equations. These problems were not included in the instruction so the researcher could assess for generalization of skills across participants.

Baseline data were collected using timed 2-minute probes until the student reached stability. Consistent with Flores et al. (2014a), the researcher defined stability as at least five data points that varied no more than 20% from the mean. Once baseline data were stable, the researcher randomly picked a student to begin the intervention. Once the first student had a stable baseline, they moved to intervention and continued until lesson sixteen. Once the first student demonstrated three consecutive intervention data points above baseline for correct digits, the second student moved to intervention with a stable baseline and continued until lesson sixteen or the end of the school year. This continued until all students moved to intervention.

Baseline Procedures and Intervention Procedures

During baseline, the researcher only administered probes. The researcher did not give any feedback regarding the students' performance. The baseline sessions ended when the student completed the probe.

During intervention, the instructor taught three different phases (concrete phase, representational phase, and abstract phase), in which students learned to solve multiplication equations that required regrouping using the standard algorithm (Flores & Hinton, 2019; Flores et al., 2020). The instructor taught one lesson per session. The instructor used the five steps of explicit instruction to teach all phases: (a) advance organizer, in which the instructor presented the lesson and stated behavioral expectations; (b) modeling, in which the instructor physically showed the process for solving mathematical equations while thinking aloud; (c) guided practice in which the instructor and student solved the equations together, and prompts as needed; (d) independent practice in which the student solved the equations independently; and (e) post organizer in which the researcher summarized the lesson and the instructor gave the student a preview of the next lesson (Flores et al., 2019; Flores & Hinton, 2019). Once students met mastery criteria, the instructor administered maintenance probes two weeks after instruction ended to collect maintenance data. The researcher administered generalization probes two weeks later.

Consistent with the instructional manual (Flores & Kaffar, 2018), the concrete phase (lessons one to four) involved instruction with base-ten blocks and learning sheets. The students used the learning sheets to read word problems and solve equations using the nine steps outlined in the manual. After determining that the given word problem required multiplication, the students used base-ten blocks to represent the top number by placing the correct number of blocks in the one's and ten’s column on the place value mat. Next, the students multiplied one's top number by the bottom number by making groups using the base ten blocks on the place value mat. For example, for 23 X 25, the student made five groups of three blocks. The students took note of the ones, and if there were more than ten, they went next door, meaning they removed groups from the one's column and added groups to the ten's column. For example, for 23 X 25, they physically exchanged tens ones for a ten block and put the tens block in the ten columns while writing five down on the paper under the ones column and writing a one above the tens column. The students looked at the tens column and multiplied twenty and five; they used the reversal rule to simplify their computation with the blocks (e.g., twenty groups of five or five groups of 20). The students made five groups of twenty on the place value mat and added the ten that was regrouped for eleven tens. Again, the students took note of the tens by using the rule that if there are more than ten, then go next door. The students physically exchanged ten tens blocks for a hundred block and wrote one ten on the paper under the tens column and a one above the hundreds place. The students notated the hundreds and wrote one hundred on the paper. Next, the students multiplied by the tens place of the bottom number. In a second row beneath the first partial product, they wrote a zero as a place holder in the ones place because the product can be no less than 20. They made three groups of 20 blocks. The students noted that regrouping is unnecessary and wrote sixty in the equation with a six in the tens place on the paper. The students examined the tens, making twenty groups of twenty, regrouping the base-ten blocks, and exchanging forty tens for four hundred blocks. Next, the students note the hundreds and write four hundred with a four written in the hundreds column of the equation on the paper. Finally, the students finished by adding the two partial products and checking the problem for errors (Flores et al., 2014a; Flores et al., 2018; Flores et al., 2019; Miller & Kaffar, 2011). During the representational phase (lessons five to eight), the students used drawings to represent ones, tens, and hundreds. Horizontal lines represented the ones, vertical lines represented the tens, and squares represented the hundreds.

The instructional procedures followed the same nine steps outlined above except drawings replaced base-ten blocks, and students drew circles around drawings to regroup. During lesson nine, students learned and memorized the following steps: (a) Read the problem; (b) Examine the ones; (c) Note the ones; (d) Address the tens; (e) Mark the tens; and (f) Examine the column, begin again, add or exit with a check (RENAME). During lesson nine, the researcher used sample equations to demonstrate how the strategy was used. Once the students memorized RENAME, they used the mnemonic and only numbers to help them solve equations in lesson ten through twelve; however, they did not use manipulatives or drawings in this phase They only use numbers, symbols, and mnemonics (Flores & Kaffar, 2018). In lesson eleven, the students learned additional steps to assist in differentiating the operations in word problems. They learned the following: (a) find what you are solving, (b) ask, “What are the parts of the problem?;” (c) set up the numbers; and (d) tie down the sign. These steps preceded RENAME for FAST RENAME. In lessons 12-16 the students distinguished between word problems and the operation and found solutions to word problems.

Treatment Fidelity and Interobserver Agreement

According to Gresham et al. (1993), it is the researcher's responsibility to report the fidelity of implementation for the independent variables. Consistent with the most recent research on CRA-SIM (Flores et al., 2022), the researcher measured treatment fidelity for 30% of the sessions across baseline, concrete, representational, and abstract lessons. O'Neill et al. (2011) recommended 25% of all intervention sessions with 90% or higher treatment fidelity during each observation. Following the guidelines set by Lane et al. (2004) and O'Neill et al. (2011) for direct observation, the researcher developed checklists for concrete (lessons one to four), representational (lessons five to eight), RENAME strategy (lesson 9), abstract (lesson 11), FAST RENAME strategy (lesson 11) and abstract lessons with word problem discrimination (lessons 12 to 15). The checklist had specific teacher behaviors which were noted as observed or not observed.

The researcher video-recorded each lesson. Using a random number generator, the researcher chose 30% percent of each student's baseline and intervention sessions for treatment fidelity assessment. Consistent with previous research (Flores et al., 2022) and exceeding current recommendations (Kratochwill et al., 2013), the researcher calculated inter-observer agreement IOA for 100% of all assessments by having two trained observers score each probe. For digits correct, each observer counted the number of digits written under equal line on a probe. The researcher calculated agreement by dividing the smaller sum by the larger sum. For accuracy, the observers scored each completed equation as correct or incorrect. The researcher calculated point by point agreement by dividing the sum of agreements by the total number of agreements and disagreements. (O'Neill et al., 2011).

Social Validity

Baer et al. (1968) expressed the importance of teaching socially significant behaviors; therefore, the researcher assessed the social validity of the intervention used in this study. Consistent with recent research (Flores et al., 2022), social validity data were collected prior to intervention and at the study's conclusion for students and their general education teachers. For teachers, prior to the study the researcher asked (Appendix G): (a) Do you believe there is a need for an intervention in mathematics (b) Are you familiar with CRA mathematical intervention? For the students, prior to the study the researcher asked (Appendix E): (a) do you feel confident in solving 2-digit by 2-digit multiplication problems (b) would you like to learn how to solve 2-digit by 2-digit multiplication problems? For the students, after the study the researcher asked (Appendix F): (a) did you like using the blocks (b) did you like using the drawings (d) did you like using FAST RENAME? (e) Do you feel confident in solving 2-digit by 2-digit multiplication problems? For the special education teachers (Appendix D), the survey after the intervention asked: (a) Do you believe the CRA method helped your students make academic gains in mathematics, (b) Do you find yourself using CRA in the future, (c) Would you recommend CRA to teachers, (d) Do you feel like there was a need for the interventions?

Chapter 4: Effects of CRA

Results

The following sections summarize the results for all participants. This section contains results for all the probes for the dependent variables (fluency and accuracy) for each participant across baseline and intervention phases. It also discusses PND and Tau with and without baseline correction.

Results for Patty

Results for Patty can be seen in Figure 1 for the number of correct digits below the line in solving 2-two-digit multiplication with regrouping using the standard algorithm. The baseline had a mean of 10.5 with a range of 9 to 12 digits correct. The intervention data had a mean of twenty-two with a range of 8 to 53 digits. Once intervention began, there was no immediate effect PND was 63%. Tau without baseline correction was Tau = 0.476, $p = 0.012$ (SE_{Tau} = 0.265)

Results for Patty can be seen in Figure 2 for accuracy in solving 2-two-digit multiplication with regrouping using the standard algorithm. The baseline for accuracy had a mean of 0% with a range of 0% correct. The intervention data had a mean of 38.75% with a range of 0% to 88% accuracy. Once intervention began, there was no immediate effect. PND was 50%. Tau without baseline correction was Tau = 0.476, $p = 0.012$ (SETau = 0.265).

Results for Edward

Results for Edward can be seen in Figure 1 for the number of correct digits below the line in solving 2-two-digit multiplication with regrouping using the standard algorithm. The baseline had a mean of 16.57 with a range of 10 to 19 digits correct. The intervention data had a mean of 17.87, ranging from 6 to 33 digits. Once intervention began, there was no immediate effect PND was 25%. Tau without baseline correction was Tau = -0.086, $p = 0.662$ (SE_{Tau} = 0.294).

Results for Edward can be seen in Figure 2 for accuracy in solving 2-two-digit multiplication with regrouping using the standard algorithm. The baseline for accuracy had a mean of 30.5% with a range of 14% to 40% correct. The intervention data had a mean of 43.37% with a 0% to 85% accuracy range. Once intervention began, there was no immediate effect. PND was 56%. Tau without baseline correction was Tau = -0.086, $p = 0.662$ ($SE_{Tau} = 0.294$).

Results for Kevin

Results for Kevin can be seen in Figure 1 for the number of correct digits below the line in solving 2-two-digit multiplication with regrouping using the standard algorithm. The baseline had a mean of 21.37 with a range of 18 to 20 digits correct. The intervention data had a mean of 21.37 with a range of 5 to 30 digits. Once intervention began, there was no immediate effect PND was 63%. Tau without baseline correction was Tau = 0.276, $p = 0.139$ ($SE_{Tau} = 0.283$).

Results for Kevin student 1.3 can be seen in Figure 2 for accuracy in solving 2-two-digit multiplication with regrouping using the standard algorithm. The baseline for accuracy had a mean of 40% with a range of 40% to 40% correct. The intervention data had a mean of 48.43%, with a range of 0% to 100% accuracy. Once intervention began, there was no immediate effect. PND was 38%. Tau without baseline correction was Tau = 0.276, $p = 0.139$ ($SE_{Tau} = 0.283$).

Results for Lacy

Results for Lacy can be seen in Figure 1 for the number of correct digits below the line in solving 2-two-digit multiplication with regrouping using the standard algorithm. The baseline had a mean of 19.5 with a range of 4 to 36 digits correct. The intervention data had a mean of 36.4 with a range of 23 to 45 digits. Once intervention began, there was no immediate effect PND was 69%. Tau with baseline correction was Tau = 0.215, $p = 0.121$ ($SE_{Tau} = 0.221$).

Results for Lacy can be seen in Figure 2 for accuracy in solving 2-two-digit multiplication with regrouping using the standard algorithm. The baseline for accuracy had a mean of 68.21% with a range of 0% to 85% correct. The intervention data had a mean of 94.33% with a range of 57% to 100% accuracy. Once intervention began, there was no immediate effect. PND was 87%. Tau with baseline correction was Tau = 0.215, p = 0.121 (SE_{Tau} = 0.221).

Results for Alice

Results for Alice can be seen in Figure 1 for the number of correct digits below the line in solving 2-two-digit multiplication with regrouping using the standard algorithm. The baseline had a mean of 21.17% with a range of 12 to 31 digits correct. The intervention data had a mean of 30.33 with a range of 14 to 55 digits. Once intervention began, there was no immediate effect PND was 40%. Tau without baseline correction was Tau = 0.348, p = 0.025 (SE_{Tau} = 0.238).

Results for Alice can be seen in Figure 2 for accuracy in solving 2-two-digit multiplication with regrouping using the standard algorithm. The baseline for accuracy had a mean of 40.18% with a range of 16% to 100% correct. The intervention data had a mean of 58.2% with a range of 20% to 100% accuracy. Once intervention began, there was no immediate effect. PND was 100%. Tau without baseline correction was Tau = 0.348, $p = 0.025$ (SE_{Tau} = 0.238).

Results for Tom

Results for Tom can be seen in Figure 1 for the number of correct digits below the line in solving 2-two-digit multiplication with regrouping using the standard algorithm. The baseline had a mean of 30.33 with a range of 17 to 20 digits correct. The intervention data had a mean of 24.4 with a range of 11 to 41 digits. Once intervention began, there was no immediate effect PND was 67%. Tau without baseline correction was Tau = 0.361, p = 0.019 (SE_{Tau} = 0.226).

Results for can Tom be seen in Figure 2 for accuracy in solving 2-two-digit multiplication with regrouping using the standard algorithm. The baseline for accuracy had a mean of 39.47% with a range of 30% to 40% correct. The intervention data had a mean of 58% with a range of

30% to 87% accuracy. Once intervention began, there was no immediate effect. PND was 67%. Tau without baseline correction was Tau = 0.361, p = 0.019 (SE_{Tau} = 0.226).

Results for Jeff

Results for Jeff can be seen in Figure 1 for the number of correct digits below the line in solving 2-two-digit multiplication with regrouping using the standard algorithm. The baseline had a mean of 6.55 with a range of 0 to 11 digits correct. The intervention data had a mean of 10.46 with a range of 0 to 24 digits. Once intervention began, there was no immediate effect PND was 20%. Tau without baseline correction was Tau = 0.300, p = 0.044 (SE_{Tau} = 0.228).

Results for Jeff can be seen in Figure 2 for accuracy in solving 2-two-digit multiplication with regrouping using the standard algorithm. The baseline for accuracy had a mean of 0% with a range of 0% correct. The intervention data had a mean of 28.46% with a range of 0% to 83% accuracy. Once intervention began, there was no immediate effect. PND was 53%. Tau without baseline correction was Tau = 0.300, p = 0.044 (SE_{Tau} = 0.228).

Figure 1

Fluency Results for Patty, Edward, Kevin, Alice, Lacy, Tom, and Jeff

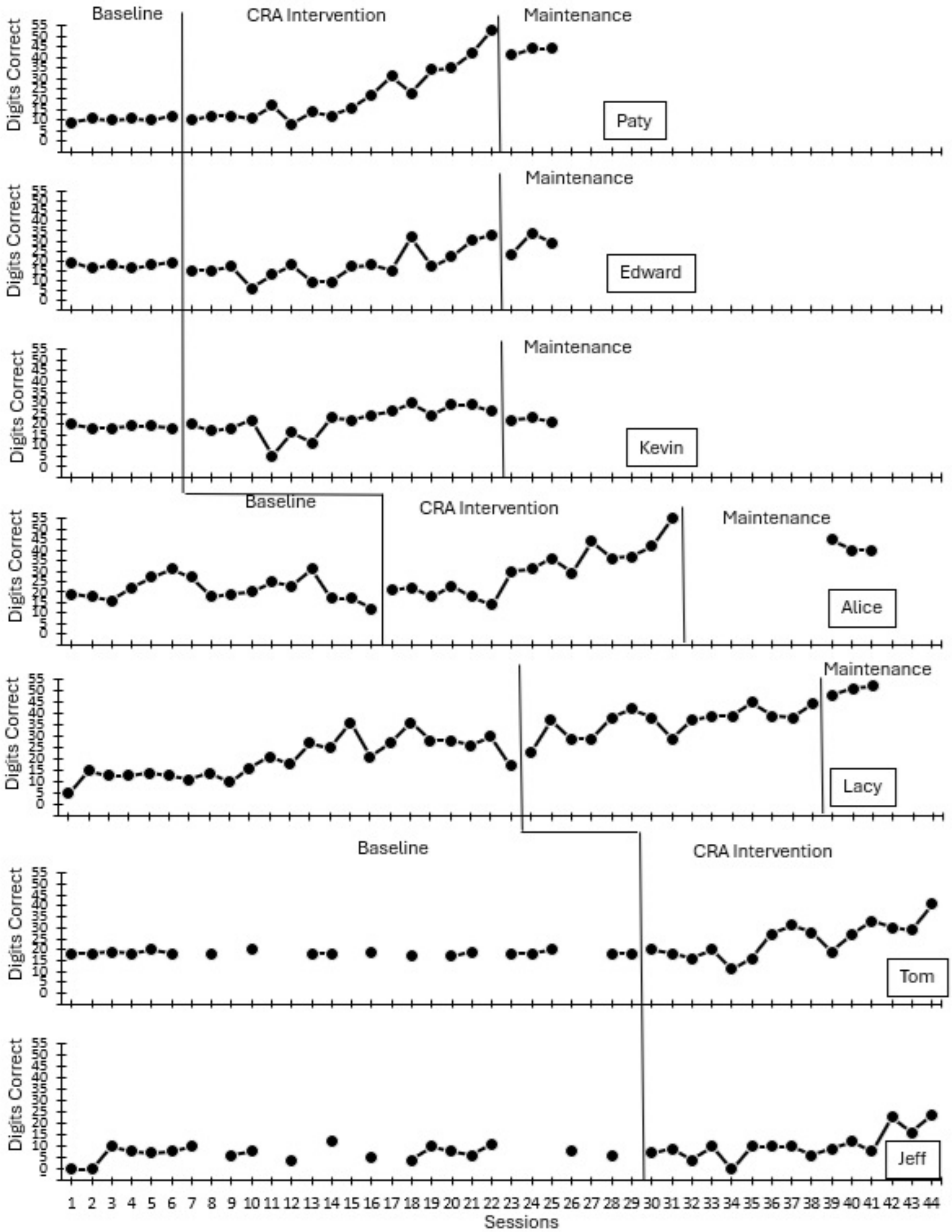
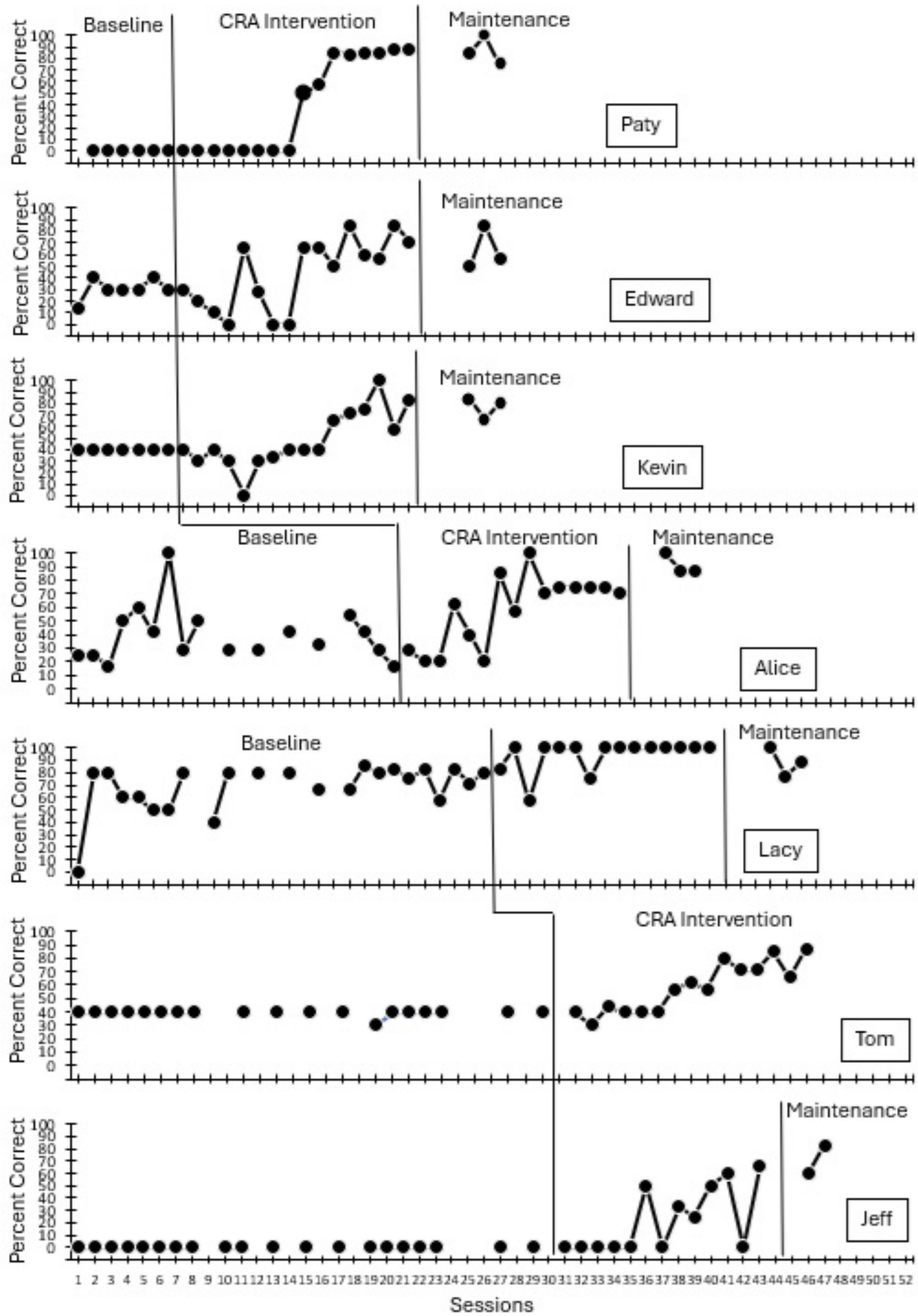


Figure 2

Percentage Results for Patty, Edward, Kevin, Alice, Lacy, Tom, and Jeff



Treatment Fidelity and Interobserver Agreement Results

All baseline, concrete, representational, and abstract lessons were recorded and using random generator the researcher selected 30% of all videos across baseline, concrete, representational, and abstract lessons. Two trained special education teachers watched 30% of videos across baseline, concrete, representational, and abstract lessons for treatment fidelity using a checklist that the researcher created prior to the study. After each trained observer scored each video, the checklists were compared, and treatment integrity was calculated by dividing the sum of agreements by the sum of agreements and disagreements and multiplying by one hundred. Patty's treatment fidelity was 100%, with both observers in agreement. Edward's treatment fidelity was 100%, with both observers in agreement. Kevin's treatment fidelity was 100%, with both observers in agreement. Lacy's treatment fidelity was 100%, with both observers in agreement. Alice's treatment fidelity was 100%, with both observers in agreement. Tom's treatment fidelity was 100%, with both observers in agreement. Jeff's treatment fidelity was 100%, with both observers in agreement.

Two trained special education teachers recalculated all scores for all the probes for the dependent variables (fluency and accuracy) for each participant across concrete, representational, and abstract probes. IOA was calculated by dividing the smaller sum by the larger sum and multiplying by one hundred. IOA was 100% for all fluency and accuracy probes across concrete, representational, and abstract probes.

Social validity data were taken for students. Patty, Edward, Kevin, Lacy, Alice, Tom, and Jeff stated pre-intervention that they were not confident in solving 2-digit by 2-digit multiplication problems. Patty, Edward, Kevin, Lacy, Alice, Tom, and Jeff also stated they would like to learn more about how to solve 2-digit by 2-digit multiplication problems. At the conclusion of the study Patty, Edward and Lacy enjoyed using the blocks, drawings, and FAST

RENAME. Alice enjoyed using the drawings and FAST RENAME. Tom and Kevin enjoyed using the blocks and FAST RENAME. Jeff enjoyed using FAST RENAME. Patty, Edward, Kevin, Lacy, Alice, Tom, and Jeff stated they now feel comfortable solving 2-digit by 2-digit multiplication problems.

Social Validity Results

Social validity data were collected from the students' teachers. At the beginning of the study, all of the teachers believed there was a need for an intervention in mathematics and all teachers were familiar with CRA mathematical intervention. At the conclusion of the study all the teachers believed that CRA method helped their students make academic gains in mathematics. They all see themselves using CRA in the future. They would all recommend CRA to other teachers. Finally, all the teachers felt like there was a need for the interventions.

Chapter 5: Conclusion and Recommendations

Discussion

The study's purpose was to investigate the effects of CRA-SIM intervention on individuals with disabilities' fluency and accuracy in multiplication with the standard algorithm for solving equations with 1-digit 2-digit multipliers. Visual analysis of the data shows a functional relation because there was a change in level, direction and range across three groups at three different points in time; however, there was not an immediate effect for most students and there were significant overlapping data. The students in the current study increased their fluency and accuracy as did the students in previous research (Flores et al., 2014a; Flores et al., 2014b; Flores & Franklin, 2014; Flores et al., 2022). Fluency was defined at the 30 correct digits below the answer line and percentage correct was the number of problems correct divided by the number of problems attempted. Patty, Edward, Lacy, Alice, and Tom had fluency of consecutive scores of 30-digits correct. Kevin scored thirty correct digits once during the intervention. Lacy reached consecutive scores of 100% accuracy during intervention and during maintenance and Patty scored 100% in maintenance. All students had significant overlap and there was a weak magnitude of change for all students, with Tau ranging from 0.012 to 0.370 when compared to Flores et al., (2014a) with a range of 0.1 to 0.7, Flores et al. (2014b) with a range of 0.83 to 1.0, and Flores et al. (2022) with a range of 0.50 to 0.75.

For fluency, Patty’s correct digits changed from twelve correct digits in baseline to fifty correct digits in intervention. Edward’s correct digits changed from nineteen correct digits in baseline to thirty-three correct digits in intervention. Kevin’s correct digits changed from twenty correct digits in baseline to thirty correct digits in intervention. Lacy’s correct digits changed from thirty-six correct digits in baseline to forty-five correct digits in intervention. Alice’s correct digits changed from thirty-one correct digits in baseline to fifty-five correct digits in intervention.

Tom's correct digits changed from twenty correct digits in baseline to forty-one correct digits in intervention. Jeff's correct digits changed from eleven correct digits in baseline to twenty-four correct digits in intervention.

Table 2

Fluency Change from Baseline to Intervention

Student	Digits in baseline	Digits in intervention
Patty	12	50
Edward	19	33
Kevin	20	30
Lacy	36	45
Alice	31	55
Tom	20	41
Jeff	11	24

Note. This table shows the correct digits in baseline and the correct digits in intervention*.

The researcher measured accuracy, the percentage of correct problems out of all problems attempted. For example, if eight problems were attempted, and seven were correct, the result was 87%. Patty's percentage changed from an average of 0%, with 0% being the highest in baseline, to an average of 28%, with 88% being the highest in intervention. Edward's percentage changed from an average of 30%, with 40% being the highest in baseline, to an average of 28%, with 85% being the highest in intervention. Kevin's percentage changed from an average of 40%, with

40% being the highest in baseline, to an average of 35%, with 100% being the highest in intervention. Lacy's percentage changed from an average of 38%, with 83% being the highest in baseline, to an average of 66%, with 100% being the highest in intervention. Alice's percentage changed from an average of 24%, with 100% being the highest in baseline, to an average of 43%, with 100% being the highest in intervention. Tom's percentage changed from an average of 39%, with 40% being the highest in baseline, to an average of 42%, with 87% being the highest in intervention. Jeff's percentage changed from an average of 0%, with 0% being the highest in baseline, to an average of 14%, with 83% being the highest in intervention. The next instructional step for these participants would be to continue instruction with three digits by digits multiplication with regrouping.

Alice and Jeff struggled with place value by transposing the ones with the tens when adding or multiplying. For example, when adding partial products, they would regroup a sum of nine plus four by writing 1 under the equal line and writing 4 as the crutch number. Furthermore, when multiplying, they would regroup the product, 2 x 7, by writing one under the equal line and writing four as the crutch number. Edward, Kevin, Tom, and Jeff would make mistakes with multiplication; they were not given access to multiplication sheets. For example, when multiplying 3x3 they would write the product as six. In the future, giving the students access to multiplication sheets would be beneficial. This would ensure they had the support needed to be able to solve problems without careless errors in multiplication. Patty, Edward, Kevin, Alice, and Jeff would not add their crutch numbers. For example, when multiplying 23 x 4, they would write eighty-two as the product and not 92. While Edward, Kevin, Jeff would rush to finish during the timed probes. All students completed independent practice items with 100%; however, the timed probes did present an added stressor with glancing up at the timer and losing their place in the problem, resulting in errors and loss of time. Patty was SLD in reading and Edward and Lacy were OHI, this would explain why they outperformed their peers who were SLD in mathematics.

All groups began baseline together: Patty, Edward, Kevin, Lacy, Alice, Tom, and Jeff. In group one, Patty, Edward, and Kevin all moved into intervention simultaneously. However, Alice moved into intervention before Lacy due to Lacy's unstable data. Once Lacy's data became stable, she moved on to intervention. Tom and Jeff moved into intervention at the same time in group three.

CRA is effective in teaching students with LD to multiply 2 2-digit multiplications problems with regrouping. These findings are consistent with Flores et al. (2014a) and Flores et al. (2022). While there was not an immediate effect when compared to other studies there is a functional relation showing that CRA is an effective intervention for teaching students the standard algorithm Flores et al. (2014a) and Flores et al (2022).

Limitations and Future Research

This study demonstrated that CRA is an effective intervention that teachers can use to meet the rigor of math instruction (Flores et al., 2014; Flores et al., 2022) such as solving 2 2-digit multiplication problems with regrouping. This study is limited due to the lessons being taught by the primary researcher. In the future, a non-bias trained individual should teach the lessons to eliminate any potential bias. Furthermore, research implementation by a researcher rather than a teacher is problematic because a teacher trained by a researcher would be reflective of real-life conditions. A researcher has a level of expertise greater than that of a teacher; therefore, the instructional conditions are not realistic and it is not clear whether the intervention would be feasible under normal conditions. Other limitations included lack of maintenance and generalization data. Due to the school year ending and groups needing to move more quickly than other studies, the study took longer to show a functional relation, and the school year ended before some groups could move to maintenance and generalization. Future research should include maintenance and generalization data for all participants. Future research should include a population of individuals with learning disabilities due to a gap in previous research with this population. Future research should consider the impact of the timed assessments. Participants struggled with timed assessments despite the fact that they made errors but scored 100% accuracy during independent practice which was not timed. Another limitations include not all individuals were SLD in mathematics. Future research should include a population of individuals SLD in mathematics. Finally, future research should consider giving participants multiplication sheets to accommodate individuals during assessments and could improve participants' scores and show this understanding of the algorithm rather than assessing their fact fluency.

Implications and Conclusions

These findings are promising for teachers and special education teachers, showing that CRA can be easily implemented in a special education classroom during resource time and not disrupt other educational endeavors. Due to the lack of mastery of the standard algorithm, more research is warranted to investigate the effect of CRA on students' performance. The students' performance on timed assessments has implications for teachers. Untimed assessments may allow students to focus on the task. Teachers might also consider the numbers of correct equations or 80% correct. Teacher might also consider the number of correct digits or 30 correct digits below the equation line. Finally, participants did not have access to multiplication sheets, resulting in errors. An implication for teachers is that students must have the pre-requisite skills to complete the assessment; the provision of multiplication sheets would provide the pre-requisite skills for students to complete the assessments.

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Appendix A

Treatment Fidelity Checklist for Concrete Phases: Lessons 1-3

Name: ______________________ Date:_______________________
Observer:____

Place an + when you observed the behavior and – when you did not observe the behavior.

Prior Lesson: Probe	**Observed**	
	Yes	No
Places the probe in direct view of the student and instruct the student: "Complete as many problems as you can until I tell you to stop."		
Sets timer for 2 minutes		
Tells the student to stop when the timer goes off and collects probes		
Explicit instruction (5-Steps)		
Advance Organizer	**Observed**	
	Yes	No
Provides an advanced organizer to the student and states learning objectives		
Modeling	**Observed**	
	Yes	No
Demonstrates two problems using instructional materials (base-ten blocks, learning sheets, place value mat), physical actions, and thinking aloud with an emphasis on the term: regrouping One-on-one setting Follows the scripts outlined in the instructional manual		
Guided Practice	**Observed**	
	Yes	No
Solves two equations with the student by taking turns and using base-ten blocks.		
Provides prompts as needed.		
The student solves problems correctly according to the CRA sequence. Follows the scripts outlined in the instructional manual		
Independent Practice	**Observed**	
	Yes	No

Provides problems and directions		
No prompting or assistance provided.		
Collects paper and scores problems		
Post Organizer	**Observed**	
	Yes	No
Summarizes lesson		
Provides corrective feedback and praise		
Score:		

Note sources used to complete the treatment fidelity checklist: Flores et al., 2022; Flores & Franklin, 2014; Flores & Hinton, 2019; Flores, Hinton, Schweck, 2014; Lane et al., 2004; O'Neill et al., 2011

Appendix B

Treatment Fidelity Checklist for Representational Phases: Lessons 4-6

Name: ______________________ Date:_______________________
Observer:____

Place an + when you observed the behavior and – when you did not observe the behavior.

Prior Lesson: Probe	**Observed**	
	Yes	No
Places the probe in direct view of the student and instruct the student: "Complete as many problems as you can until I tell you to stop."		
Sets timer for 2 minutes		
Tells the student to stop when the timer goes off and collects probes		
Explicit Instruction (5- Steps)		
Advance Organizer	**Observed**	
	Yes	No
Provides an advanced organizer to the student and states learning objectives		
Modeling	**Observed**	
	Yes	No
Demonstrates two problems using instructional materials (drawings, learning sheet, place value mat), physical actions, and thinking aloud with an emphasis on the term: regrouping One-on-one setting Follows the scripts outlined in the instructional manual		
Guided Practice	**Observed**	
	Yes	No
Solves two equations with the student by taking turns and using drawings		
Provides prompts as needed.		
The student solves the problem correctly according to the CRA sequence. Follows the scripts outlined in the instructional manual		
Independent Practice	**Observed**	
	Yes	No
Provides problems and directions		

No prompting or assistance provided.		
Collects paper and scores problems		
Post Organizer	**Observed**	
	Yes	No
Summarizes lesson		
Provides corrective feedback and praise		
Score:		

Note sources used to complete the treatment fidelity checklist: Flores et al., 2022; Flores & Franklin, 2014; Flores & Hinton, 2019; Flores, Hinton, Schweck, 2014; Lane et al., 2004; O’Neill et al., 2011

Appendix C

Treatment Fidelity Checklist for Abstract Phases: Lessons 7-12

Name: ______________________ Date:________________________
Observer:____

Place an + when you observed the behavior and – when you did not observe the behavior.

Prior Lesson: Probe	**Observed**	
	Yes	No
Places the probe in direct view of the student and instruct the student: "Complete as many problems as you can until I tell you to stop.”		
Sets timer for 2 minutes		
Tells the student to stop when the timer goes off and collects probes		
Explicit instruction (5-Steps)		
Advance Organizer	**Observed**	
	Yes	No
Provides an advanced organizer to the student and states learning objectives		
Modeling	**Observed**	
	Yes	No
Demonstrates RENAME strategy and practice problem using only numbers and symbols One-on-one setting Follows the scripts outlined in the instructional manual		
Guided Practice	**Observed**	
	Yes	No
Solves two equations with the student by taking turns and using RENAME strategy		
Provides prompts as needed.		
The student solves problems correctly according to the CRA sequence. Follows the scripts outlined in the instructional manual		
Independent Practice	**Observed**	
	Yes	No
Provides problems and directions		
No prompting or assistance provided.		

Collects paper and scores problems		
Post Organizer	**Observed**	
	Yes	No
Summarizes lesson		
Provides corrective feedback and praise		
Score:		

Note sources used to complete the treatment fidelity checklist: Flores et al., 2022; Flores & Franklin, 2014; Flores & Hinton, 2019; Flores, Hinton, Schweck, 2014; Lane et al., 2004; O'Neill et al., 2011

Appendix D

Examples of Social Validity Survey

Special Education Teachers

Instructions: Please circle the answer that best describes you.

1. Do you believe the CRA method helped your students make academic gains in mathematics?

   1. Yes   2. No

2. Do you find yourself using CRA in the future?

   1. Yes   2. No

3. Would you recommend CRA to teachers?

   1. Yes   2. No

4. Do you feel like there was a need for the interventions?

   1. Yes   2. No

Appendix E

Examples of Social Validity Survey

Students

Instructions: Please circle the answer that best describes you.

1. Do you feel confident in solving 2-digit by 2-digit multiplication problems?

   1. Yes   2. No

2. Would you like to learn how to solve 2-digit by 2-digit multiplication problems?

   1. Yes   2. No

Appendix F

Examples of Social Validity Survey

Students

Instructions: Please circle the answer that best describes you.

1. Did you like using the blocks?

   1. Yes   2. No

2. Did you like using the drawings?

   1. Yes   2. No

3. Did you like using FAST RENAME?

   1. Yes   2. No

4. Do you feel confident in solving 2-digit by 2-digit multiplication problems?

   1. Yes   2. No

Appendix G

Examples of Social Validity Survey

Special Education Teachers

Instructions: Please circle the answer that best describes you.

1. Do you believe there is a need for an intervention in mathematics?

   1. Yes 2. No

2. Are you familiar with CRA mathematical intervention?

   1. Yes 2. No

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NOTE: DO NOT AGREE TO PARTICIPATE UNLESS AN APPROVAL STAMP WITH CURRENT DATES HAS BEEN APPLIED TO THIS DOCUMENT

PARENTAL PERMISSION/CONSENT
For a Research Study entitled
"Teaching Students to be Successful in Multiplication with Regrouping."

Your child is invited to participate in a research study to examine the effects of how some children use tools they have learned to solve multiplication with regrouping problems. The study is being conducted by Erin Noelle Blanton a Doctoral Student in the Auburn University Department of Special Education, Rehabilitation, and Counseling for her dissertation. Your child was selected as a possible participant because he or she is enrolled in a special education program. Since your child is age 18 or younger, we must have your consent to include him/her in the research study.

If you decide to allow your child to be a part of this research study, your child will be assessed for a potential deficit in multiplication with regrouping, and if appropriate assigned to a group in his/her special education classroom program. The total in the class time will be fifteen weeks. Instruction will take place in your child's special education classroom. Instruction will be provided by a trained staff/teacher and by Erin Noelle Blanton. Instruction will involve solving multiplication with regrouping equations using objects, pictures, and numbers. During this time, your child will be asked to learn problem-solving tools and use the tools to solve equations.

The risks associated with participating in this study are minimal risk or discomfort. To minimize these risks, we will monitor for signs of increased anxiety or discomfort. If we notice any increased anxiety the student will be removed from the study immediately. Discomfort will be minimized by preparing students prior to daily instruction. In addition, we will provide modeling with manipulatives to demonstrate all expectations, guided practice where the student will be supported with verbal cues and prompts as well as additional time to practice the new skill.

If your child participates in this study, your child can expect to improve his/her math skills in solving multiplication with regrouping problems. We cannot promise you that your child will receive any or all the benefits described.

Your child will not receive any compensation for participation. To thank your child for participating, your child will receive knowledge in solving multiplication with regrouping problems.

There are not monetary fees or cost if you decide to allow your child to participate. The service is free.

2084 Haley Center, Auburn, AL 36849-5222; Telephone: 334-844-7676; Fax: 334-844-7677
www.auburn.edu/serc

NOTE: DO NOT AGREE TO PARTICIPATE UNLESS AN APPROVAL STAMP WITH CURRENT DATES HAS BEEN APPLIED TO THIS DOCUMENT

If you change your mind about your child's participation at any time in the study, your child can be withdrawn from the study immediately. Your child's participation is completely voluntary. If you choose to withdraw your child, your child's data can be withdrawn if it is identifiable. Your decision about whether to allow your child to participate or to stop participating will not jeopardize you or your child's future relations with Auburn University, the Department of Special Education, Rehabilitation, Counseling or your school system.

Any information obtained in connection with this study will remain confidential. The data collected will be protected by Erin Noelle Blanton. Findings from this study may be published in an educational journal or presented at a conference. Your child will not be identified personally.

If you have questions about your child's rights as a research participant, you may contact the Auburn University Office of Human Subjects Research or the Institutional Review Board by phone (334-844-5966 or email at hsubjec@auburn.edu or IRBChair@auburn.edu.

HAVING READ THE INFORMATION PROVIDED, YOU MUST DECIDE WHETHER OR NOT YOU WISH FOR YOUR CHILD TO PARTICIPATE IN THIS RESEARCH STUDY. YOUR SIGNATURE INDICATES YOUR WILLINGNESS TO ALLOW YOUR CHILD TO PARTICIPATE.

______________________________ Parent/Guardian Signature	______________________________ Investigator obtaining consent Date
______________________________ Printed Name	______________________________ Printed Name
____________________ Date ______________	______________________________ Date ____________________

Child's Name ____

____
Co-Investigator Date

____
Printed Name

Page 2 of 2

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DEPARTMENT OF
SPECIAL EDUCATION,
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MINOR ASSENT

For a research study entitled

"Teaching Students to be Successful in Multiplication with Regrouping."

We are asking you and your parents or guardian, for your participation in a research study to help us understand how some children use tools they have learned to solve multiplication with regrouping. This study will also help you solve multiplication with regrouping problems in the future on test and other assignments.

If you decide to be part of this research study, you will be assigned to a class. Your total in the class time will be fifteen weeks. During this time, you will learn a problem-solving tool and you will use that tool to solve multiplication with regrouping problems. You will start out using base ten blocks to help you solve multiplication with regrouping problems. After a while, you will be able to solve multiplication with regrouping problems without blocks or any other help. The lessons will take place during your special education class.

We will be taking a video of you while we are in class. We need the video to study later to make sure we did all the steps correctly. We can only make the video if you and your parent(s) or guardians give us permission to do that.

You can stop the class any time during the study. Just let your parents or your teacher know if don't want to be in the class anymore. No one will be angry with you if you stop the class. You will go back to your regular special education class. Your grade will not be affected if you stop participation in this study. No one will be mad, angry, or upset with you if you stop participation in this study. Participation in this study is only voluntary.

If you have any questions about what you will do in the study or what will happen during class time, please ask your parents or guardian, or ask Ms. Blanton now. If you have any questions while you are in class, we want you to ask us.

If you have decided to help us, please sign or print your name on the line below.

____________________________	______________________________	
Child's Signature	Print Name	Date
____________________________	______________________________	
Parent/Guardian Signature	Print Name	Date

(Parent/Guardian must also sign Parent/Guardian Permission form!)

____________________________	______________________________	
Investigator obtaining consent	Print Name	Date

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Video Release – Minor

During your child's participation in the research study titled, **"Teaching Students to be Successful in Multiplication with Regrouping"** your child will be videotaped. Your signature on the Informed Consent gives us permission to videotape your child for education purposes during the allotted study time.

Your signature on this document gives us permission to use the videotape(s) for publication in peer reviewed journals and training in the future. These videotapes will not be destroyed at the end of this research. The videotapes will be retained in a secure location until the final manuscript has been accepted for publication.

I give my permission for videotapes produced in the study, "Teaching Students to be Successful in Multiplication with Regrouping" which contain images of my child, to be used for the purposes listed above, and to also be retained until the final manuscript has been accepted for publication.

______________________	______________________
Parent/Guardian's Signature Date	Investigator's Signature Date
______________________	______________________
Parent/Guardian Printed Name	Investigator Printed Name
______________________	______________________
Minor's Signature Date	Co-investigator Signature Date
______________________	______________________
Minor's Printed Name	Printed Name

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