GROUP VERSUS STAGGERED REPLACEMENT POLICY-

STRATEGIC REPLACEMENT DECISIONS

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GROUP VERSUS STAGGERED REPLACEMENT POLICY-

STRATEGIC REPLACEMENT DECISIONS

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STRATEGIC REPLACEMENT DECISIONS

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THESIS ABSTRACT

GROUP VERSUS STAGGERED REPLACEMENT POLICY-

STRATEGIC REPLACEMENT DECISIONS

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Replacement decisions are critical in most businesses, because assets are subject to deterioration or obsolescence with usage and time. In addition, technological improvement affects the replacement cycle of assets. In our paper, we focus on a fleet replacement problem with a single-unit. The main problems of fleet replacement decisions are first, when we should replace existing assets with new assets, and second, how many assets to replace at once. To solve these problems, we introduce two policies for fleet replacement: group replacement and staggered replacement. To address these issues, we develop mathematical models and analyze results to find the preferable policy under certain conditions.

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CHAPTER 1. INTRODUCTION

Replacement decisions are critical in most businesses, because assets are subject to deterioration or obsolescence with usage and time. In addition, technological improvement affects the replacement cycle of assets. In this research, we focus on a fleet replacement problem concerning a single-unit. The main problems of fleet replacement decisions are: 1) when we should replace existing assets with new assets, and 2) how many assets should be replaced at one time. To solve these problems, we introduce two types of fleet replacement policies: Group replacement and Staggered replacement. To address these issues, we develop mathematical models and analyze the results to find the preferable policy under certain conditions.

1.1. BACKGROUND

Replacement is inevitable in business. Replacement costs consist of three main components: 1) the initial costs, 2) operating and maintenance costs, and 3) resale values. As equipment ages, operating and maintenance costs gradually increase, and resale values gradually decrease. The initial costs are also affected by technological improvement. Therefore, at some point in time, the retention costs for old assets may exceed the costs of purchasing and operating new assets.

We focus on a fleet replacement problem in this research. In practice, there are two replacement strategies for this problem. One is Group replacement, which replaces all assets at once during each service life cycle of assets. The second design is Staggered replacement, which replaces an equal portion of the fleet every year. We chose replacement of the same numbers of assets every year instead of replacing a different number of assets every two or three years. According to Jones and Zydizk (1993), their main result suggests that fleet operators would want an equal number of assets in each replacement group.

Industry under ongoing technological improvement makes new products which may be cheaper and more efficient. When we are faced with replacing equipment which has technological improvement over time, the important questions are “should we change all equipment at once or follow a Staggered policy?” The purpose of our research is to suggest a decision tool for replacement decisions in specific cases.

1.2. PROBLEM STATEMENT

This research presents an analysis of the replacement decisions of a company that has single assets to replace periodically in an infinite horizon period. Also, it considers the technological progress of assets, which changes the costs of new investment and operating and maintenance. Productivity is also affected by technological progress. According to a recent survey that the research firm Gartner conducted with 177 large businesses, the average life span of a PC is 36-43 months. Traditionally, many businesses replaced their PCs in staggered, one-third-per-year increments over a three-year cycle. More recently, large companies are replacing all their PCs at once rather than in staggered cycles (Dunn, 2005). The main inspiration for CIOs to make this change is the benefit, which includes reduced maintenance costs. Considering the limited budgets of companies, we need to know exactly how much benefit is possible.

1.3. RESEARCH METHODOLOGY

Our research presents the procedure for finding the optimal replacement policy in fleet replacement. We use the net present value decision as a cost comparison approach, because modern replacement theory is based on discounted cash-flow. Besides, the problem is solved in a cost-minimizing framework: we find the minimization of total present worth costs.

To find the optimal replacement policy in a fleet replacement problem, we construct closed form mathematical models: the Group replacement policy and the Staggered replacement policy in both the basic model and the model under technological improvement. These models apply an exponential form of technological progress to show ongoing technological progress. Furthermore, we simulate our models under ongoing technological progress to illustrate and analyze the uncertain situation using @Risk, a risk analysis plug-in for Microsoft Excel.

1.4. RESEARCH PLAN

This research proceeds as follows. Section 2 describes the literature review of general, group and staggered replacement. Section 3 constructs the basic mathematical models of two policies: group and staggered replacement without considering technological changes. We also examine numerical examples to demonstrate our models.

Section 4 develops mathematical models for each replacement policy under ongoing technological progress. Here we analyze the models with the same numerical example as the example in Section 3. Section 5 illustrates the uncertain situation of these models, which we intend to account for using @Risk software. Section 6 summarizes and presents conclusions, including contributions of the proposed research.

CHAPTER 2. LITERATURE REVIEW

When we place an asset in service, we need to replace the asset at some point in the future. Obsolescence and deterioration are the two major reasons for considering the replacement of an existing asset. The issue of when to replace an existing asset is one of the critical operating decisions in business. Consequently, many researchers have investigated a variety issues related to asset replacement. However, our literature review will focus only on two types of replacement policies—Group replacement and Staggered replacement, as our ultimate goal is to examine which replacement policy is more cost effective.

2.1. GENERAL REPLACEMENT

James S. Taylor (1923) and Harold Hotelling (1925) developed a mathematical theory of depreciation for an asset which loses value over time. Roos (1928) is one of the early researchers who studied replacement decision problems in a systematic way for a single machine by considering the cost of production and the changing market value of the machine. Preinreich (1940) recognized the importance of depreciation in finding the optimum economic life of a machine. "All rules of economic life are also rules of depreciation." Terborgh (1949), considered the father of modern replacement theory, developed a simple and complete rule prescribing the time at which existing production equipment should be replaced. His essential contribution is the integration of obsolescence into the applied theory of replacement as detailed in *Dynamic Equipment Policy*.

2.2. GROUP REPLACEMENT

When replacing identical assets placed in service, one policy to follow is to replace all assets together at the end of their economic service life. This is known as a group replacement policy. As outlined in Figure 2.1, group replacement policies are classified further into three major classes according to when units are replaced. The first class, the T-age policy, says that when the age of a unit reaches a prescribed point, units are replaced periodically. In the second class, M-Failure, units are replaced when the number of failure reaches a prescribed number, m. The third class, (m, T), considers both T-age and m-failure.

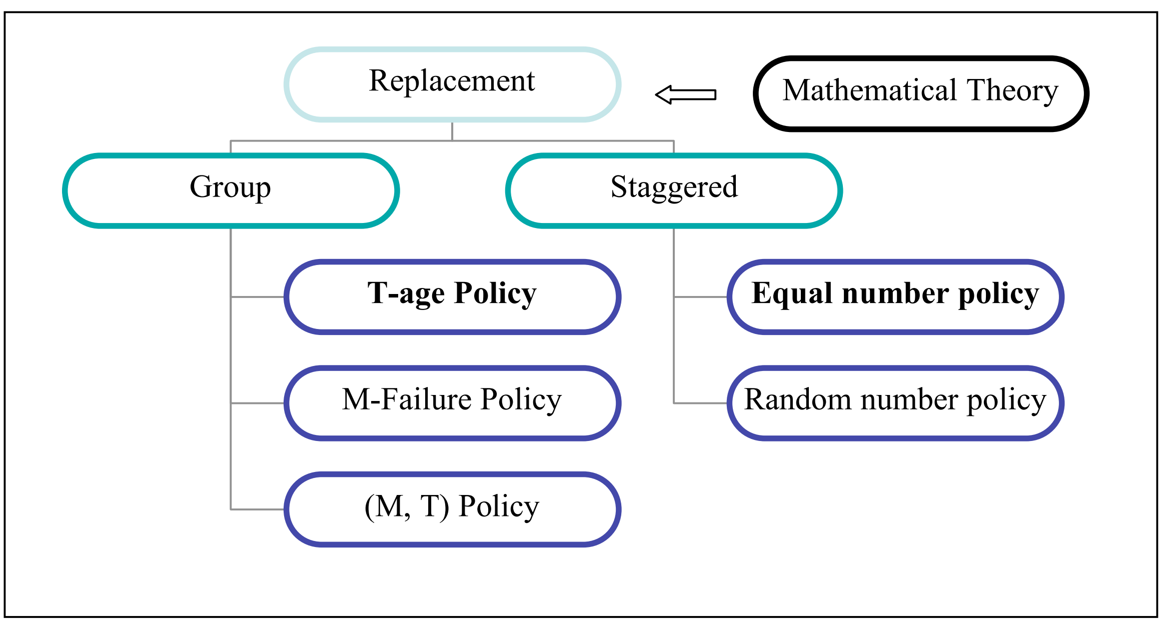


Figure 2.1 Summary of replacement research

2.2.1. T-AGE REPLACEMENT POLICY

Barlow and Hunter (1960) first introduced the periodic replacement policy with minimal repair at failure, which takes a negligible amount of time. They further considered two preventive maintenance policies: one for simple equipment which operates continuously without failure, and one for complex systems which operate with minimal repairs. In this model it is assumed that the failure rate of a unit or system is not changed after repair.

Tahara and Nishida (1973) introduced a preventive maintenance policy that considers repairable systems. The failure rate of systems in their models increases because the system is not able to recover completely after repair, and the service life of the system decreases after repair. Okumoto and Elsayed (1983) extended Barlow and Hunter’s model, which is basically the optimal scheduled time for preventive maintenance. They further provided an optimal group replacement policy of single units with an exponential failure distribution during a given interval. Recently, Park and Yoo (2004) considered the same replacement problem under minimal repair. Then they compared three types of replacement policies. First, all units are replaced periodically. Second, the group replacement interval considers both repair and waiting times. Third, the minimal repair for each unit is conducted during the repair interval. They recommended the third policy to be most economical among the three policies.

2.2.2. M-FAILURE REPLACEMENT POLICY

Gertsbakh (1984) provided an optimal repair policy: repair is conducted when the number of failed machines reaches some prescribed number. Gertsbakh assumed that a group of machines has *n* independent but identical machines, and each machine has an exponential lifetime.

Assaf and Shanthikumar (1987) proposed the group maintenance policy under continuous and periodic inspections with stochastic failures. The idea is that they decided to repair the failed machine after inspection. Assaf and Shanthikumar examined the optimal repair policy: if the number of failed machines reaches a prescribed number n, m machines are repaired ($m < n$).

Wilson and Benmerzouga (1990) extended Assaf and Shanthikumar's optimal m-failure replacement policy. They assumed the failure times of *n* machines are independent but identically distributed exponential random variables. They developed a cost function to use in accordance with the behavior of the optimal policy.

More recently, Liu (2004) developed an m-failure group replacement policy for M/M/N queuing systems which are unreliable with identically exponential failure times. They formulated a matrix-geometric model to consider the steady-state situation.

2.2.3. (M, T) REPLACEMENT POLICY

Morimura (1970) introduced an (m, T) policy which combined two policies: m-failure replacement and T-age replacement. They considered the number of failed machines and the operating time of the machines to find a minimum replacement cost. If the number of failed machines reaches a prescribed number m before the T-age of a machine, or the T-age comes before the m-failure for the machines, they are repaired.

Nakagawa (1983) considered counting the number of failed machines and recording the age of machines over a fixed replacement time and then repairing the failed machines if either the replacment age T or the number N of a predetermined number of machines fail first.

More recently, Ritchken and Wilson (1990) considered the same problem with two decision variables: a fixed time interval and a fixed number of failed units. If one of the two variables occurs, all failed machines are replaced with new ones that perform perfectly. They provided an algorithm to obtain the optimal (m, T) policy and demonstrated it with a numerical example.

2.3. STAGGERED REPLACEMENT

The term "Staggered replacement" was first mentioned by Cook and Cohen (1958). Although they did not outline any specific replacement policy, the purpose of the Staggered replacement policy is to smooth out the required lump sum capital outlay over time. The Staggered replacement policy is commonly practiced in many industrial settings.

Jones and Zydiak (1993) formally considered Staggered replacement by comparing two prevalent replacement designs; one replaces an equal portion of a fleet every year, and the other replaces larger bunches less frequently in order to account for the fleet management problem. In their paper, they defined that the second case is a Staggered replacement policy. They concluded that the first policy is better than the other.

2.4. SUMMARY

Many researchers have been studied to find optimal replacement policies of the group replacement models for single-unit systems. There are three main types of group replacement. The first is the age replacement policy, the second is the m-failure (failure number) policy, and the third combines (m, T) policies. While extensive research has been done for Group replacement policy, not much work has been done to determine the effectiveness of a Staggered replacement strategy. Therefore, our research focus is to compare the Group replacement policy with a Staggered replacement policy to find which policy is more strategically cost-effective. We will focus mainly on the T-age Group replacement policy and the equal number Staggered replacement policy.

CHAPTER 3. BASIC MODEL

We will first examine replacement problems without technological changes in assets being considered for replacement. This basic model focuses on fleet replacement decisions about identical assets such as PCs, delivery trucks, buses and airplanes. Two types of replacement policy will be examined—Group replacement and Staggered replacement. The Group replacement policy calls for replacing all assets once at the end of the economic service life of the each of assets. On the other hand, the Staggered replacement policy recommends that businesses replace a predetermined number of assets during a specified time interval (possibly every year). In this chapter we will present mathematical models for each replacement policy and give numerical examples to demonstrate how the models work in a specific replacement environment. We will also interpret the results to determine which policy is more economically preferable.

3.1. ASSUMPTION

In order to decide whether to replace existing assets, we assumed the following factors: First, we chose the infinite planning horizon for a corporation whose business requires the same type of assets for an indefinite period. Second, we used the net present value of the total cost for the entire planning horizon as a decision criterion to compare the results between the Group and Staggered replacement models. Third, we considered a replacement policy under a stable economy, meaning that the asset prices and operating and maintenance cost would remain constant in the absence of inflation. The concept of an infinite sequence of replacements can be generalized to the situation in which the life of an asset is a decision variable. A common example of this type of problem is deciding on the replacement interval for an automobile.

3.2. REPLACEMENT MODELS

Three types of cash flows are considered in developing a basic replacement model: First is the sequence of asset purchases over the planning horizon ($\mathrm{PW}(i')_1$). Second is the sequence of salvage values for the assets purchased at the time of each replacement cycle ($\mathrm{PW}(i')_2$). Third is the sequence of cash flows related to the operating and maintenance costs of the assets over the entire planning horizon ($\mathrm{PW}(i')_3$). Since we are dealing with cash flows over an indefinite period, these cash flows must be discounted at an inflation-free interest rate. The total present cost of a typical replacement policy is then simply the sum of these three present values ($\mathrm{PW}(i')_1 - \mathrm{PW}(i')_2 + \mathrm{PW}(i')_3$). We will use the following set of notations in describing the replacement models:

P = purchase price of a new asset without volume discount at time 0, the cost per unit multiplied by number of assets

P_n = purchase price of the asset at time n

d = volume discount multiplier for purchase cost, where $d < 1$

i' = inflation-free (real) interest rate

b = multiplier for end-of-year-1 salvage value, where $b < 1$

c = annual multiplier for subsequent-year salvage values, $c < 1$

A = first-year O&M costs for assets purchased at time 0

p = annual multiplier for O&M costs for given assets, where $p > 1$

3.2.1. GROUP REPLACEMENT MODEL

In this section, we will develop the group replacement model where all assets are replaced in a group when they reach the end of their economic service life, *N*. One of the advantages of the group replacement policy is to obtain some form of volume discount when purchasing the new assets. The degree of volume discount depends on the nature of assets, but these savings must be considered in the model.

The Purchase Cost

Suppose we begin in year 0. Assets cost P_0 at time 0; the cost includes discounts for volume of purchased assets as follows:

$$P_0 = (1-d)P \tag{1}$$

Under the inflation-free environment with no technological improvement in future assets, we can assume the purchase cost at times *N*, 2*N*, …, *kN* will be the same as the initial purchase cost at time 0.

$$P_{kN} = P_{2N} = P_N = P_0 \tag{2}$$

Here we also further assume that the volume discount for future replacements would remain the same. Then, we determine $\text{PW}(i')_1$ by discounting the initial cost cash flow streams when assets are purchased every *N*-year as follows:

$$
\begin{aligned}
\mathrm{PW}(i')_1 &= P_0 + \frac{P_N}{(1+i')^N} + \frac{P_{2N}}{(1+i')^{2N}} + \cdots \\
&= P_0 + \frac{P_0}{(1+i')^N} + \frac{P_0}{(1+i')^{2N}} + \cdots \\
&= P_0 \left[\frac{1}{1 - \dfrac{1}{(1+i')^N}} \right] \\
&= P_0 \left[\frac{(1+i')^N}{(1+i')^N - 1} \right] \qquad \left(\frac{1}{1+i'} < 1 \right)
\end{aligned} \tag{3}
$$

Salvage Values

While we retain the asset for N years, the value of the asset will continue to decrease over the holding period. Let's consider a sequence of salvage values. If we sell the asset purchased at time 0 after one year, we would receive

$$S_1 = bP_0 \tag{4}$$

If we sell the asset after two years of use, we would receive

$$S_2 = bcP_0 \tag{5}$$

Here, the parameter c, which less than 1, represents the scaling factor related to calculating subsequent-year salvage values. The main logic for introducing a new multiplier (c) is that most assets lose a greater portion of their values during the first year of ownership, implying $b \le c$. This assumption is also considered by Park and Gunter (1990). If we consider a sequence of asset retirements for subsequent replacement cycles, the salvage value of the kth replacement cycle can be expressed by

$$S_{kN} = bc^{N-1} P_{kN} = bc^{N-1} P_0 \tag{6}$$

The contribution to the total PW of costs from a sequence of asset replacements every N year is

$$\begin{aligned}
\mathrm{PW}(i)_2 &= \frac{bc^{N-1}P_N}{(1+i')^N} + \frac{bc^{N-1}P_{2N}}{(1+i')^{2N}} + \cdots \\
&= \frac{bc^{N-1}P_0}{(1+i')^N} + \frac{bc^{N-1}P_0}{(1+i')^{2N}} + \cdots = \frac{bc^{N-1}P_0}{(1+i')^N}\left[\frac{1}{1-\frac{1}{(1+i')^N}}\right] \\
&= \frac{bc^{N-1}P_0}{(1+i')^N}\left[\frac{(1+i')^N}{(1+i')^N-1}\right] \\
&= \frac{bc^{N-1}P_0}{(1+i')^N-1}
\end{aligned} \tag{7}$$

O&M Cost

First-year operating and maintenance costs (O&M costs) often are considered to follow a negative exponential curve based on the O&M costs of the current year's model. Then, the O&M costs of future models are usually assumed to follow the same pattern as that of the O&M costs of the current year's model. With this assumption, the expression for the O&M costs is more tedious, but it follows along similar lines as the previous two cash flow sequences. Recall that each replacement cycle contributes N-year O&M cost terms, with each successive year showing a higher cost than that of the previous year. Once again, with no technological improvement in future assets, the O&M cost series in the first replacement cycle will repeat in future replacement cycles. The first O&M cost term in each cycle is

$$A_1 = A \tag{8}$$

The second O&M cost term will be higher than the first cost by

$$A_2 = pA \tag{9}$$

The O&M cost term in N^{th} period will be

$$A_N = p^{N-1}A \tag{10}$$

As mentioned previously, the O&M cost terms in the second replacement cycle will be the same as those during the first cycle.

$$\begin{aligned} A_{N+1} &= A_1 = A \\ A_{N+2} &= A_2 = pA \\ &\vdots \\ A_{2N} &= A_N = p^{N-1}A \end{aligned} \tag{11}$$

As this O&M cost series repeats for subsequent replacement cycles, the closed form expression for the PW of the O&M costs is

$$\begin{aligned} \text{PW}(i)_3 &= \frac{A}{(1+i')} + \frac{pA}{(1+i')^2} + \cdots + \frac{p^{N-1}A}{(1+i')^N} + \frac{A}{(1+i')^{N+1}} + \cdots + \frac{p^{N-1}A}{(1+i')^N} + \cdots \\ &= \frac{\left[\dfrac{A}{p}\displaystyle\sum_{k=1}^{N}\left(\frac{p}{1+i'}\right)^k\right]}{1-\left(\dfrac{1}{1+i'}\right)^N} = A\sum_{k=1}^{N}\left(\frac{p^{k-1}}{(1+i')^k}\right)\left(\frac{(1+i')^N}{(1+i')^N-1}\right) \end{aligned} \tag{12}$$

The Total PW Cost Function for the Group Replacement Policy

The total PW cost function for the group replacement policy can be summarized as follows:

$$\begin{aligned} \text{PW}(i')_{\text{Group}} &= \text{PW}(i')_1 - \text{PW}(i')_2 + \text{PW}(i')_3 \\ &= P_0\left[\frac{(1+i')^N}{(1+i')^N-1}\right] - \frac{bc^{N-1}P_0}{(1+i')^N-1} + A\sum_{k=1}^{N}\left(\frac{p^{k-1}}{(1+i')^k}\right)\left(\frac{(1+i')^N}{(1+i')^N-1}\right) \end{aligned} \tag{13}$$

Note that the second PW term needs to be subtracted from the total cost function, as these are salvage values, which reduce the total cost of the replacement cycles.

3.2.2. STAGGERED REPLACEMENT MODEL

While a Group replacement policy replaces all assets once, a Staggered replacement policy replaces an equal number of assets during the economic service life of assets. As with the Group replacement policy model, we have three types of cash flow streams, and the modeling scheme is quite similar to the Group replacement model.

The Initial Cost Stream

With the Staggered replacement policy, we have many different ways of staggering the replacement assets. Staggering options themselves lead to another optimization problem, that is, what the best way to stagger the replacement assets is. However, as shown in Jones and Zydiak (1993), we will assume that one- N^{th} of assets is purchased every year over the economic service life. We also can consider some form of volume discount for the smaller scale of purchases, even though the discount may not be as high as with group replacement; if we assume a uniform rate of volume discount, the periodic purchase cost can be expressed as follows:

$$P_0 = (1-d)(P) \tag{14}$$

With one- N^{th} replacement each period, the initial cost stream will be

$$P_k = P_2 = P_1 = \left(1-\frac{d}{N}\right)\left(\frac{P}{N}\right) \tag{15}$$

The closed form expression for the PW cost of purchase streams is

$$\text{PW}(i')_1 = P_0 + \frac{P_1}{(1+i')^1} + \frac{P_2}{(1+i')^2} + \cdots = P_0 + \frac{P_1}{(1+i')^1} + \frac{P_1}{(1+i')^2} + \cdots$$

$$= P_0 + \frac{P_1}{(1+i')}\left[\frac{1}{1-\frac{1}{(1+i')}}\right] = P_0 + \frac{P_1}{(1+i')}\left[\frac{1+i'}{i'}\right] = P_0 + P_1\left(\frac{1}{i'}\right) \tag{16}$$

Certainly, if the volume discount itself is a function of volume, then we need to adjust the scaling factor $\left(1-\frac{d}{N}\right)$ as $\left(1-\frac{d_N}{N}\right)$. However, for the sake of simplicity, we will consider a uniform discount.

Salvage Values

Unlike the group replacement model, we need to estimate the salvage value of the asset as a function of the asset age until it reaches the end of its economics service life. The difference between the resale price of the assets under Group and Staggered models will be the same after the N^{th} year.

If we sell the one- N^{th} asset purchased at time 0 after one-year, we could receive

$$S_1 = b\left(\frac{P_0}{N}\right) \tag{16}$$

Then in the second year, we also sell the one- N^{th} asset purchased at time 0.

$$S_2 = bc\left(\frac{P_0}{N}\right) \tag{17}$$

In the third year,

$$S_3 = bc^2\left(\frac{P_0}{N}\right)$$

When we get to the N^{th} year, the asset group purchased at time 0 has been completely replaced.

$$S_N = bc^{N-1}\left(\frac{P_0}{N}\right) \tag{18}$$

After the N^{th} year, the salvage value of the Staggered model will be the same as the salvage value of the N+1th year.

$$S_{N+\infty} = S_{N+1} = bc^{N-1}P_1 \tag{19}$$

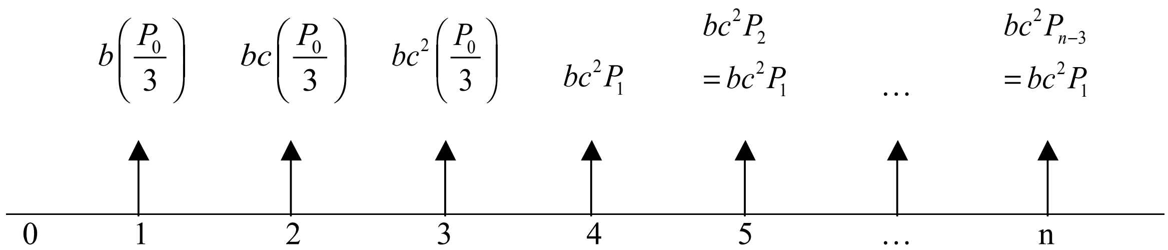
Time	0	1	2	3	4 - ∞
Rate of Quantity	1 (New)	2/3 (1 year)	1/3 (2 year)	(2 year)	(2 year)
			1/3 (1 year)	(1 year)	(1 year)
		1/3 (New)	1/3 (New)	(New)	(New)

First asset Second asset Third asset

Figure 3.1 A graphical representation of one- N^{th} Staggered replacement policy

For example, suppose that the service life is 3 years. Figure 3.1 illustrates process of staggering the replacement assets. New assets will be placed in service in year 0. Then, at the end of year 1, one third of the assets placed in service in year 0 will be replaced. At year 2, another one third of the assets placed in service at period 0 will be replaced. Then, in period 2, we will have three types of assets: one third of the old assets placed in service at period 0, one-third of new assets placed in service at period 1, and one-third of new assets purchased in period 2. Finally at period 3, the old assets placed in service in period 0 will be completely gone. The composition of the assets includes (1) one-third of the asset group purchased at period 1, (2) one-third of the asset group purchased at period 2, and (3) one-third of new asset group purchased at period 3. After year 3, the asset composition will be the same as that of year 3—that is 1/3 of assets are two years old, 1/3 of the assets are one year old, and 1/3 of the assets are brand new.

In terms of the sequence of salvage value, Figure 3.1 can be translated as follows:



The PW of the salvage value stream is

$$\begin{aligned}\mathrm{PW}(i')_2 &= \frac{bP_0}{N(1+i')^1} + \frac{bcP_0}{N(1+i')^2} + \frac{bc^2P_0}{N(1+i')^3} + \cdots + \frac{bc^{N-1}P_0}{(1+i')^N} + \frac{bc^{N-1}P_1}{(1+i')^{N+1}} + \cdots \\ &= \frac{bP_0}{N}\sum_{k=1}^{N}\frac{c^{k-1}}{(1+i')^k} + \frac{bc^{N-1}P_1}{(1+i')^{N+1}}\left[\frac{1+i'}{i'}\right].\end{aligned} \qquad (20)$$

O&M Cost Streams

The O&M cost stream is a bit more involved. Referring to Figure 3.1, the O&M costs in each year must account for the composition of assets placed in service at different points in time. For example, with N = 3 years, the O&M costs at period 1 consist of only assets purchased in year 0:

$$A_1 = A$$

The O&M costs at period 2 consist of two different assets: two-thirds is from the first asset group, and one-third is from the second asset group.

$$A_2 = \left(\frac{2}{3}\right)pA + \left(\frac{1}{3}\right)A$$

Then, the O&M costs at period 3 will be

$$A_3 = \left(\frac{1}{3}\right)p^2 A + \left(\frac{1}{3}\right)pA + \left(\frac{1}{3}\right)A$$

After the N^{th} period, say year 4 in our example, the O&M costs would take the following expression:

$$A_4 = \left(\frac{1}{3}\right)p^2 A + \left(\frac{1}{3}\right)pA + \left(\frac{1}{3}\right)A$$

Then, the O&M costs beyond the N period would be exactly the same as those of N-th period, because the composition of the asset groups is exactly same—the one year old asset group, the two year old asset group, and brand new asset group. Therefore, we can generalize the O&M cost at time N as

$$\begin{aligned} A_N &= \left(\frac{1}{N}\right)p^{N-1}A + \left(\frac{1}{N}\right)p^{N-2}A + \left(\frac{1}{N}\right)p^{N-3}A + \cdots + \left(\frac{1}{N}\right)pA + \left(\frac{1}{N}\right)A \\ &= \left(\frac{A}{N}\right)\left(p^{N-1} + p^{N-2} + \ldots + p^{N-N}\right) = \left(\frac{A}{N}\right)\sum_{k=1}^{N} p^{N-k} \end{aligned} \tag{21}$$

And

$$A_{N+\infty} = \ldots = A_{N+1} = A_N \tag{22}$$

The equivalent total PW cost of the O&M cash flows is as follows:

$$\mathrm{PW}(i')_3 = \frac{A}{(1+i')^1} + \frac{\left(1-\frac{1}{N}\right)pA + \left(\frac{A}{N}\right)}{(1+i')^2} + \frac{\left(1-\frac{2}{N}\right)p^2 A + \left(\frac{A}{N}\right)p + \left(\frac{A}{N}\right)}{(1+i')^2}$$

$$+ \cdots + \frac{\left(\frac{A}{N}\right)\sum_{k=1}^{N} p^{N-k}}{(1+i')^N} + \frac{\left(\frac{A}{N}\right)\sum_{k=1}^{N} p^{N-k}}{(1+i')^{N+1}} + \cdots \quad (23)$$

$$= \frac{A}{(1+i')^1} + \frac{\left(1-\frac{1}{N}\right)pA + \left(\frac{A}{N}\right)}{(1+i')^2} + \cdots + \frac{\left(\frac{A}{N}\right)\sum_{k=1}^{N} p^{N-k}}{(1+i')^N}\left[\frac{1+i'}{i'}\right]$$

The Total PW Cost for the Staggered Replacement Policy

We obtain the total PW cost expression for the Staggered replacement policy without technology improvement as follows:

$$\mathrm{PW}(i')_{\text{Staggered}} = P_0 + P_1\left(\frac{1}{i'}\right) - \frac{bP_0}{N}\sum_{k=1}^{N}\frac{c^{k-1}}{(1+i')^k} + \frac{bc^{N-1}P_1}{(1+i')^{N+1}}\left[\frac{1+i'}{i'}\right]$$

$$+ \frac{A}{(1+i')} + \frac{\left(1-\frac{1}{N}\right)pA + \left(\frac{A}{N}\right)}{(1+i')^2} + \cdots + \frac{\left(\frac{A}{N}\right)\sum_{k=1}^{N} p^{N-k}}{(1+i')^N}\left[\frac{1+i'}{i'}\right] \quad (24)$$

We will use Equations (13) and (24) to compare the effectiveness of a given replacement policy by minimizing the equivalent total PW cost of entire replacement cycles.

3.3. ECONOMIC ANALYSIS

To compare the effectiveness of the two different replacement policies, we will consider a case example and give an economic interpretation of the results. We will further conduct a series of sensitivity analyses for the key input parameters.

3.3.1. An Illustrative Case Example

The K-Company is considering replacing their old copy machines with new ones. The unit price of a new copy machine is $500, and they have 100 machines. If they buy 100 machines once, they can get 10% discount, and for each 10 machines the volume discount is 1%. The value of each machine decreases to 60% of the original purchase cost after using it for 1 year. Then, the value will decease 20% each subsequent year. O&M costs in the first year are $50 per unit, and O&M costs will increase 25% each year. Each copy machine has an economic service life of five years. The interest rate is 10%. A summary of key input parameters follows:

Parameter	Value
P	$50,000
d	10%
i'	10%
b	60%
c	80%
A	$5,000
p	125%

Table 3.1 Summary of example data

It is assumed that the K-Company has enough money to replace all the assets at once if the group replacement policy is considered to be more economical. Here, all analysis is done on a before-tax basis.

3.3.2. Economic Interpretation of the Numerical Results

With the set of parameters assumed in the previous section, the Group replacement policy appeared more cost effective when compared with the Staggered replacement policy. Figure 3.2 illustrates the incremental cost of the Staggered replacement policy over the Group replacement policy. Note that the Group replacement policy takes a stair-shaped curve because of the chunk of costs occurs at the replacement period. In terms of the PW cost of the entire cash flow stream, Group replacement results in $176,318, while Staggered replacement costs $189,030 with a planning horizon of 41 years and an economic service life of the assets at $N = 5$ years. The incremental cost of choosing the Staggered replacement policy over Group replacement is $12,712 in present value. The cost differentials will vary as a function of planning horizon, but the Group replacement policy will be more cost effective for a wide-range of planning horizons.

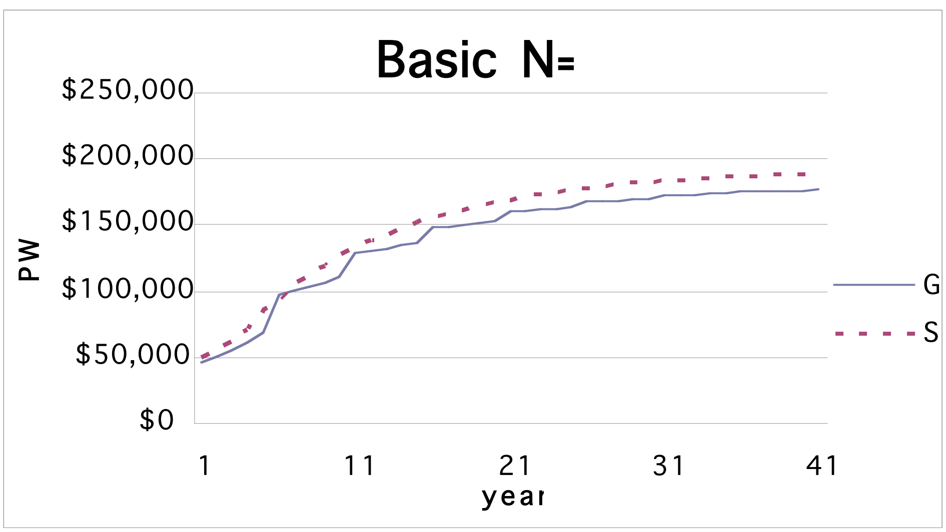


Figure 3.2 under N=5 (base case)

Certainly, we should not conclude that the Staggered replacement policy is always inferior to the Group replacement policy. To answer this question, we need to conduct a series of "what if" analyses.

3.3.3. Sensitivity analysis

To determine what conditions make Group replacement cost effective, we will perform a series of sensitivity analyses on the key input variables. The three key input parameters considered are (1) economic service life, which dictates the replacement intervals for the Group replacement policy, (2) the amount of volume discount with Group replacement as well as staggered replacement, and (3) the discount rate used in comparing the two replacement policies.

Replacement Interval *N*

Ideally the best replacement interval is the economic service life of the asset. However, as we vary the replacement interval from N = 3 years to N = 10 years, we obtain the present worth cost of each replacement policy as follows:

Type	N=3	N=5	N=7	N=10
Difference (G-S)	-$11,504	**-$12,712**	$4,721	$8,889

Table 3.2 The Difference between the NPW of Group and Staggered models under varying *N*.

As expected, with the replacement interval set at the economic service life of the asset (N = 5 years, which is our base case), the Group replacement policy is more cost effective.

As we deviate from this base further out, the cost differential gap between the two policies narrows.

If we further examine the PW cost differential with N = 7 (that is, we keep the assets two more years beyond their economic service life), we observe that the Staggered policy turned out to be more cost effective, as depicted in Figure 3.3.

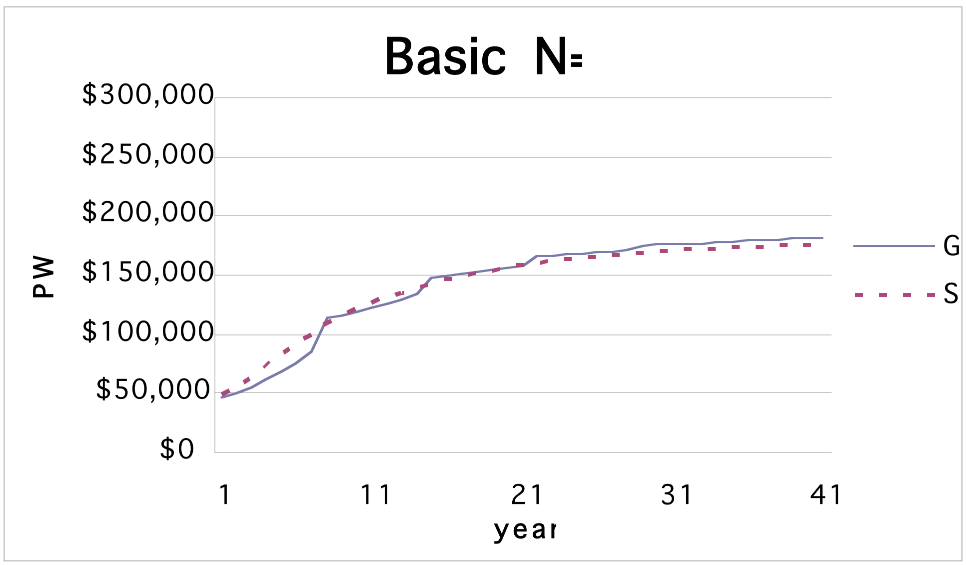


Figure 3.3 under N=7

This is simply because the replacement cost for the Group model increases if the assets are replaced at an interval other than the economic service life, which minimizes the total equivalent cost. This also clearly illustrates that if we go with the Group replacement policy, the assets must be replaced at their economic service life.

Volume Discount (d)

The amount of volume discount available will be an important parameter, as the volume discount reduces the capital cost for the replacement chains for both Group and Staggered policies. The Group policy will enjoy a higher volume discount as compared with the Staggered policy where the purchased amount is spread over the N-period. As we vary the volume discount from 5% to 20%, the preference for the group policy is furthered evidenced.

N=5(d)	5%	10%	15%	20%
Difference(G-S)	-$8,363	-$12,712	-$17,061	-$21,410

Table 3.3 The difference between the NPW of Group and Staggered models under N=5.

Figures 3.4 - 3.7 illustrate how the total present worth cost functions according to the Group and Staggered policies over a wide planning horizon. As expected, the gap between the two policies widens as we increase the volume discount, which favors the Group policy.

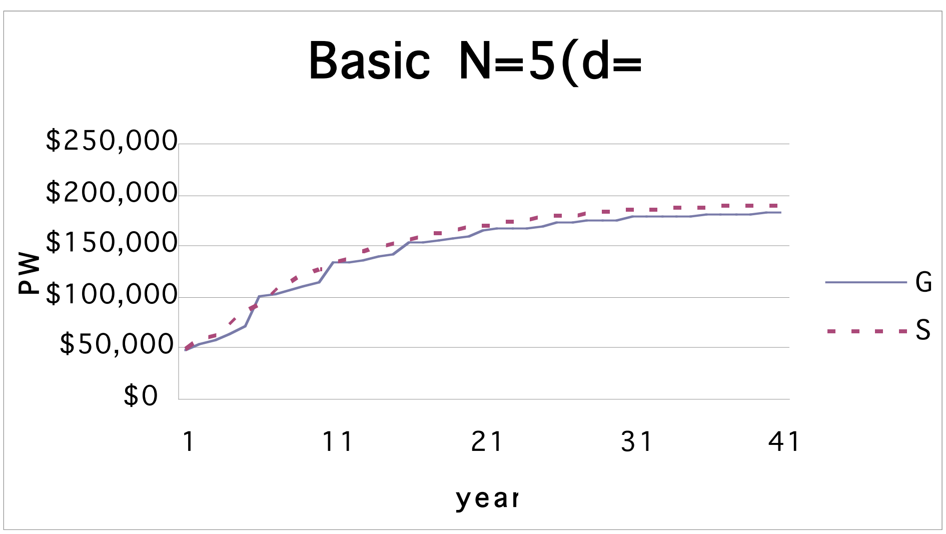


Figure 3.4 under d=5%

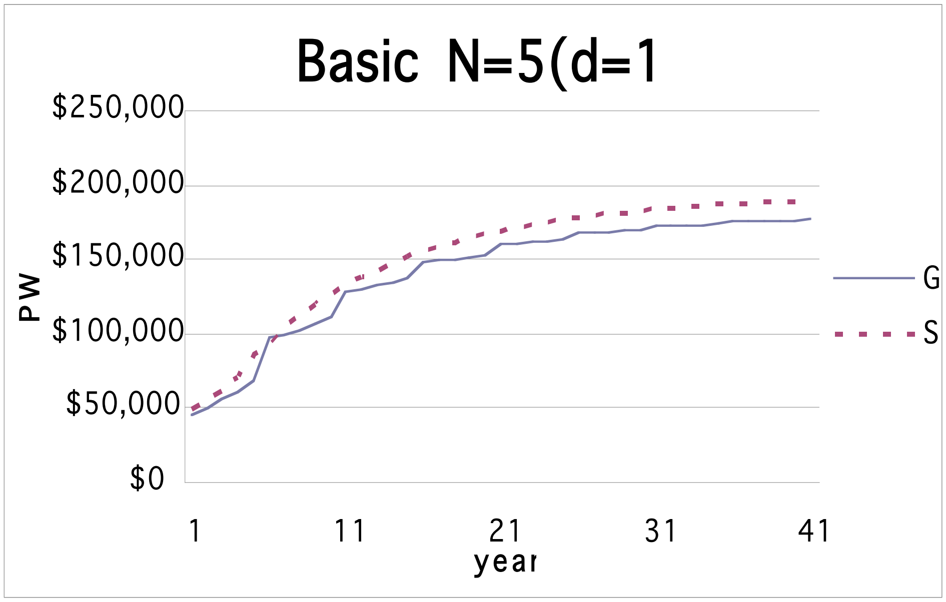


Figure 3.5 under d=10%

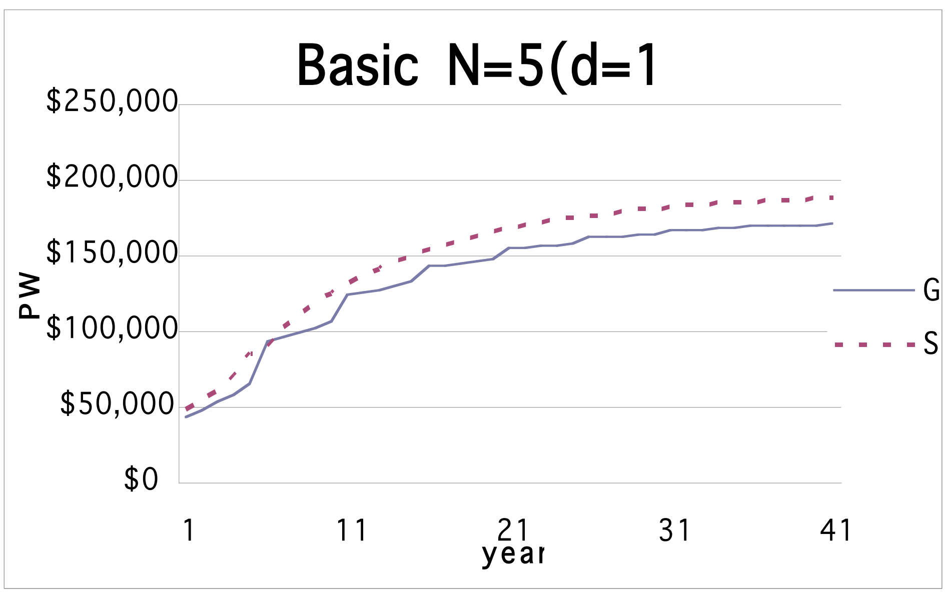


Figure 3.6 under 15%

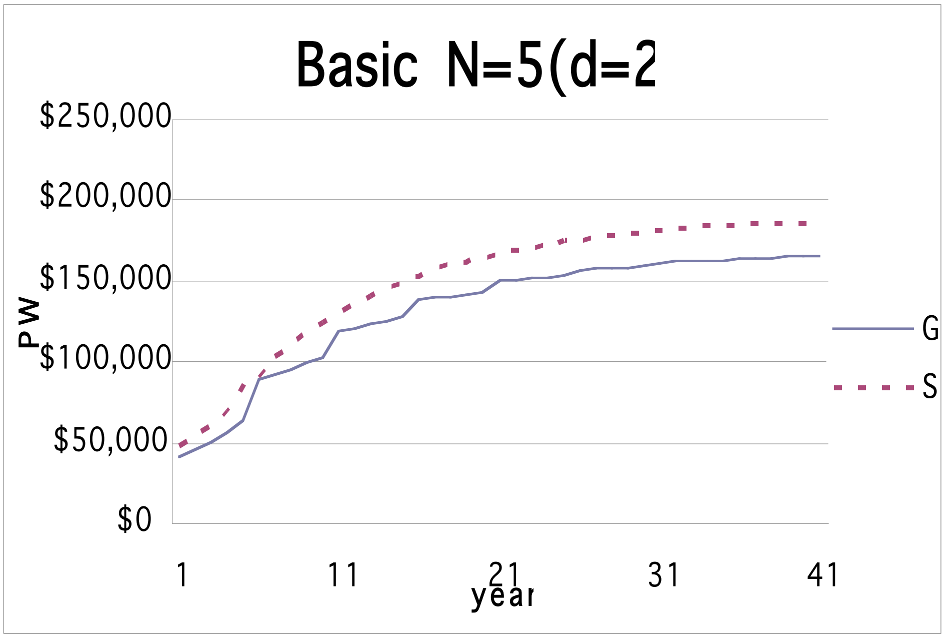
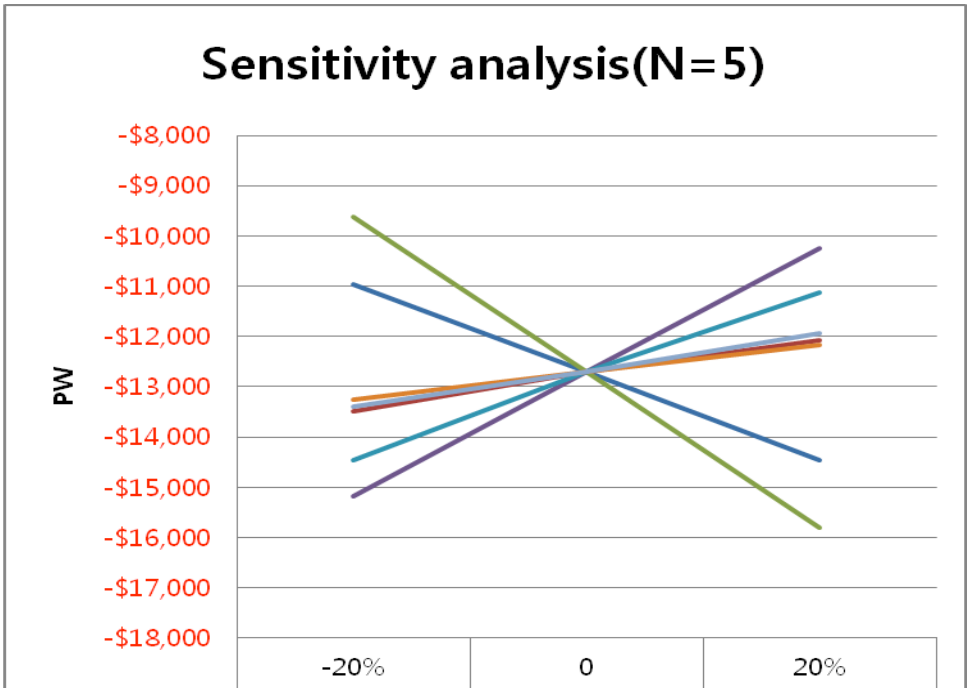


Figure 3.7 under d=20%

Obtaining an Overall Sensitivity Graphs

Figure 3.8 shows the sensitivity graphs for seven of the key input variables. The base-case PW cost differential (Group – Staggered) is plotted on the ordinate of the graph at the value of 0 (0% deviation) on the abscissa. Next, the value of volume discount is reduced to 80% of its base-case value, and the PW cost differential is recomputed, with all other variables held at their base-case value. We repeat the process by either decreasing or increasing the relative deviation from the base-case. The lines for the variable interest rate (i'), purchase price (*P*), and other parameters such as *b*, *c*, *A*, and *q* are obtained in a similar manner.



	-20%	0	20%
d	-$10,973	-$12,712	-$14,452
i	-$13,487	-$12,712	-$12,080
P	-$9,625	-$12,712	-$15,799
b	-$15,176	-$12,712	-$10,248
c	-$14,450	-$12,712	-$11,123
A	-$13,257	-$12,712	-$12,168
p	-$13,398	-$12,712	-$11,944

Figure 3.8 Sensitivity graph for the PW cost differential between Group and Staggered replacement policies

In Figure 3.8, we see that the group replacement policy is quite cost effective for the range of values examined. In particular, the cost differential is (*1*) most sensitive to change in purchase price (*P*) and the first-year's loss of market value of the asset (*b*), (2) fairly sensitive to changes in the volume discount (*d*) and the scaling factor of the market value of the asset (*c*) , and (3) relatively insensitive to changes in the interest rate (*i*'), initial O&M cost (*A*) and the scaling factor of the future O&M cost (*p*).

CHAPTER 4. MODEL UNDER TECHNOLOGICAL PROGRESS

In Chapter 3, we presented two types of replacement models (Group and Staggered) without considering any technological changes in future replacement assets. However, technology improvement is one of the critical factors that can change the purchase prices and operating and maintenance costs of future assets in years to come. In this chapter, we will develop mathematical models for each replacement policy and examine which replacement policy is more cost effective when we experience technological progress in future replacement assets. To compare the results with the basic models, we will use the same numerical example as used in Chapter 3.

4.1. CONSIDERING TECHNOLOGY IMPROVEMENT IN REPLACEMENT DECISIONS

Technological improvement in future assets is one of the main reasons for replacing existing assets, since the future assets should be more efficient in many aspects: improved efficiency (productivity), reduced operating and maintenance costs, and lower purchase costs. However, it is rather difficult to predict the trend of efficiency and the price of assets over several years in any precise fashion. The problem of replacement under technological progress has been studied by many researchers. Grinyer (1973) and

Bethuye (1998) examined the influence of technological progress and concluded that technology may lead to an increase of the economic service lives of assets in some cases. In contrast, Howe and McCabe (1983) and Rogers and Hartman (2005) explained that technological change makes the replacement cycle of assets shorter than in a stationary situation. In practical models of replacement under technological progress, Terborgh (1949), a previous researcher, applied a linear form for technological change, but Grinyer (1973) recommended a geometric form after comparing the linear form with the geometric form.

4.2. ASSUMPTION

In our basic case, we assume three factors: (1) an infinite planning horizon, (2) the PW of the total cost as a decision criterion to compare both models, and (3) the asset prices and operating and maintenance costs remain constant in the absence of inflation. In order to compare with the basic models, we will further assume three additional factors. First, the asset price keeps decreasing (or remains relatively stable) due to technological progress. Second, the operating and maintenance costs for the future replacement assets will continue to decrease compared with those assets purchased in the previous replacement cycles, but they increase each year during the holding period. Third, the productivity of assets decreases every year during the holding period.

4.3. REPLACEMENT MODELS

As with the basic model, we will consider three types of cash flows: first is the sequence of asset purchases ($\text{PW}(i')_1$). Second is the sequence of salvage values

($\mathrm{PW}(i')_2$). Third is the sequence of the operating and maintenance costs ($\mathrm{PW}(i')_3$). The total present worth cost, which is obtained by discounting the combined cash flows at an inflation-free interest rate, is then simply the sum of these three present values ($\mathrm{PW}(i')_1 - \mathrm{PW}(i')_2 + \mathrm{PW}(i')_3$). We will use the same notations as in the basic models, while introducing three additional variables (*a, q, s*), in developing the replacement models:

P = purchase price of a new asset without volume discount at time 0, the cost per unit multiplied by number of assets

P_n = purchase price of the asset at time n

d = volume discount multiplier for purchase cost, where $d < 1$

i' = inflation-free (real) interest rate

b = multiplier for end-of-year-1 salvage value, where $b < 1$

c = annual multiplier for subsequent-year salvage values, where $c < 1$

A = first-year O&M costs for assets purchased at time 0

p = annual multiplier for O&M costs for given assets, where $p > 1$

a = annual multiplier to calculate purchase price, where $a < 1$

q = annual multiplier to calculate first-year O&M costs for an asset purchased after time 0

s = productivity loss multiplier for O&M costs, where $s < 1$

4.3.1. GROUP REPLACEMENT MODEL

In this section, we will first develop a group replacement model when assets are replaced in a group when they reach the end of their economic service life, *N*. In general, technological progress leads to a reduction in the purchase costs and operating and maintenance costs of future assets, even though the operating and maintenance costs increase as assets age during the replacement cycle. Further, as new assets tend to have a higher productivity rate, keeping existing assets longer implies a productivity loss. We will consider all these factors in developing the group replacement model.

The Purchase Cost

Suppose that the firm purchases brand new assets at period 0, meaning that there are no existing assets to consider at time 0. Let's assume that assets cost P_0 at time 0:

$$P_0 = (1-d)P \tag{1}$$

Since the purchase cost of subsequent assets decreases over time, the purchase cost in year one is

$$P_1 = aP_0 \tag{2}$$

Then, if we make a new purchase when the asset placed in service in year 0 reaches its economic service life of *N* years, the asset costs would be

$$P_N = a^N P_0 \tag{3}$$

In group replacement, we purchase assets every *N*-year, therefore

$$P_{kN} = a^{kN} P_0 \tag{4}$$

The contribution to the total PW of costs from a sequence of asset replacements every N-year is

$$\begin{aligned}
\mathrm{PW}(i')_1 &= P_0 + \frac{P_N}{(1+i')^N} + \frac{P_{2N}}{(1+i')^{2N}} + \cdots \\
&= P_0 + \frac{a^N P_0}{(1+i')^N} + \frac{a^{2N} P_0}{(1+i')^{2N}} + \cdots \\
&= P_0 \left[\frac{1}{1-\left(\frac{a}{1+i'}\right)^N} \right] = P_0 \left[\frac{(1+i')^N}{(1+i')^N - a^N} \right]
\end{aligned} \tag{5}$$

Salvage Values

A sequence of salvage values over an asset's economic service life is the same as in the basic Group replacement model.

$$\begin{aligned}
S_1 &= bP_0 \\
S_2 &= bcP_0 \\
&\vdots \\
S_N &= bc^{N-1}P_0
\end{aligned} \tag{6}$$

However, as the purchase cost decreases in the second replacement cycle, the sequence of salvage values during the second replacement cycle also decreases in the following fashion:

$$\begin{aligned}
S_{N+1} &= ba^N P_0 \\
&\vdots \\
S_{2N} &= bc^{N-1}P_N = bc^{N-1}a^N P_0
\end{aligned} \tag{7}$$

We can obtain the closed form expression for the PW of the salvage values over the infinite planning horizon as follows:

$$\begin{aligned}
\mathrm{PW}(i')_2 &= \frac{S_N}{(1+i')^N} + \frac{S_{2N}}{(1+i')^{2N}} + \cdots = \frac{bc^{N-1}P_0}{(1+i')^N} + \frac{bc^{N-1}a^N P_0}{(1+i')^{2N}} + \cdots \\
&= \frac{bc^{N-1}P_0}{(1+i')^N}\left[\frac{1}{1-\left(\frac{a}{1+i'}\right)^N}\right] \\
&= \frac{bc^{N-1}P_0}{(1+i')^N}\left[\frac{(1+i')^N}{(1+i')^N - a^N}\right] \\
&= \frac{bc^{N-1}P_0}{(1+i')^N - a^N}
\end{aligned} \qquad (8)$$

O&M Cost

The trend of operating and maintenance costs for the assets purchased in year 0 is quite similar to the basic Group replacement model. In the basic Group replacement model, the operating and maintenance cost follows a negative exponential curve over its service life of the assets. With technology improvement, we need to consider another factor-- productivity loss. Since brand new assets tend to have a higher productivity rate (they produce more with the same amount of operating hours), we will experience some sort of productivity loss as we delay replacing the old assets. This productivity loss needs to be captured in terms of operating cost as well. In other words, if we retain the assets longer, the O&M costs increase on two fronts: requiring more frequent maintenance, and increasing productivity loss due to aging assets. Recall that each replacement cycle contributes *N*-year O&M cost terms, with each year showing a higher cost than that of the previous year because of aging assets. To reflect the two different sources for accounting for O&M costs, we will introduce an additional factor, productivity loss (*s*), in our Group replacement model.

The first O&M cost term during the first replacement cycle is

$$A_1 = A \tag{9}$$

The second O&M cost term will be higher than the first cost by

$$A_2 = (p+s)A \tag{10}$$

The O&M cost term in N^{th} period will be

$$A_N = (p+s)^{N-1} A \tag{11}$$

Now we enter the second replacement cycle with brand new assets which will have less O&M costs compared with the assets placed in service during the first cycle. By introducing a new annual multiplier q, which is less than 1, the O&M cost term at time N+1 will be

$$A_{N+1} = q^N A \tag{12}$$

The sequence of O&M costs during the second replacement cycle is

$$\begin{aligned} A_{N+2} &= (p+s)^1 q^N A \\ &\vdots \\ A_{2N} &= (p+s)^{N-1} q^N A \end{aligned} \tag{13}$$

As this O&M cost series repeats for subsequent replacement cycles, then we determine $\text{PW}(i')_3$ as follows:

$$\begin{aligned} \text{PW}(i')_3 &= \frac{A_1}{(1+i')^1} + \frac{A_2}{(1+i')^2} + \cdots + \frac{A_N}{(1+i')^N} + \frac{A_{N+1}}{(1+i')^{N+1}} + \cdots + \frac{A_{2N}}{(1+i')^{2N}} + \cdots \\ &= \frac{A}{(1+i')^1} + \frac{(p+s)A}{(1+i')^2} + \cdots + \frac{(p+s)^{N-1} A}{(1+i')^N} + \frac{q^N A}{(1+i')^{N+1}} + \cdots + \frac{(p+s)^{N-1} q^N A}{(1+i')^{2N}} + \cdots \\ &= \frac{\left[\dfrac{A}{(p+s)} \displaystyle\sum_{k=1}^{N} \left(\frac{p+s}{1+i'} \right)^k \right]}{1 - \left(\dfrac{q}{1+i'} \right)^N} = A \sum_{k=1}^{N} \left(\frac{(p+s)^{k-1}}{(1+i')^k} \right) \left(\frac{(1+i')^N}{(1+i')^N - q^N} \right) \end{aligned} \tag{14}$$

The Total PW Cost Function for the Group Replacement Policy

The total PW cost function for the Group replacement policy with technology improvement can be summarized as follows:

$$\begin{aligned} \text{PW}(i')_{\text{Group}} &= \text{PW}(i')_1 - \text{PW}(i')_2 + \text{PW}(i')_3 \\ &= P_0\left[\frac{(1+i')^N}{(1+i')^N - a^N}\right] - \frac{bc^{N-1}P_0}{(1+i')^N - a^N} + A\sum_{k=1}^{N}\left(\frac{(p+s)^{k-1}}{(1+i')^k}\right)\left(\frac{(1+i')^N}{(1+i')^N - q^N}\right) \end{aligned} \tag{15}$$

Recall that the second PW term needs to be subtracted from the total cost function, as these are salvage values which reduce the total cost of the replacement cycles.

4.3.2. STAGGERED REPLACEMENT MODEL

As we mentioned in the basic model, the Staggered replacement policy calls for the replacement of an equal number of assets during the economic service life of the assets. This model also has three types of cash flow streams, and we will follow the same modeling scheme as in the Group replacement model.

The Initial Cost Streams

Although the Staggered replacement policy replaces an equal number of assets in each period, the amount of assets purchased in year 0 will be the same as with the Group replacement policy. That is, we start with the same number of assets. The first purchase cost can be expressed as follows:

$$P_0 = (1-d)P \tag{16}$$

With one- N^{th} replacement in each period, the initial purchase cost at time 1 will be

$$P_1 = \left(1 - \frac{d}{N}\right)\left(\frac{aP}{N}\right) \tag{17}$$

The sequence of the initial purchase cost stream will be

$$\begin{aligned} P_2 &= aP_1 = \left(1 - \frac{d}{N}\right)\left(\frac{a^2 P}{N}\right) \\ P_3 &= a^2 P_1 = \left(1 - \frac{d}{N}\right)\left(\frac{a^3 P}{N}\right) \\ &\vdots \\ P_k &= a^{k-1} P_1 \qquad (2 \le k) \end{aligned} \tag{18}$$

The closed form expression for the PW cost of the initial purchase cost stream is

$$\begin{aligned} \text{PW}(i')_1 &= P_0 + \frac{P_1}{(1+i')^1} + \frac{P_2}{(1+i')^2} + \cdots = P_0 + \frac{P_1}{(1+i')^1} + \frac{aP_1}{(1+i')^2} + \cdots \\ &= P_0 + \frac{P_1}{(1+i')}\left[\frac{1}{1 - \left(\frac{a}{1+i'}\right)}\right] = P_0 + \frac{P_1}{(1+i'-a)} \end{aligned} \tag{19}$$

Salvage Values

As we express the salvage value as a function of the initial purchase cost, the salvage value would be the same after the N^{th} year in the basic model. Although the scheme of salvage values under Staggered replacement is quite similar to the Group replacement model, unlike in the basic model, the salvage values considering technological change are smaller than they are under the basic model because the initial purchase cost continues to decrease under ongoing technological progress. As we will see, the sequence of salvage values during the first replacement cycle is the same as the basic Staggered replacement model in Chapter 3.

$$
\begin{aligned}
S_1 &= b\left(\frac{P_0}{N}\right) \\
S_2 &= bc\left(\frac{P_0}{N}\right) \\
S_3 &= bc^2\left(\frac{P_0}{N}\right) \\
&\vdots \\
S_N &= bc^{N-1}\left(\frac{P_0}{N}\right)
\end{aligned} \tag{20}
$$

After the N^{th} year, we start replacing the assets purchased during the first cycle. The salvage value stream, after the N^{th} year, will be

$$
\begin{aligned}
S_{N+1} &= bc^{N-1}P_1 \\
S_{N+2} &= bc^{N-1}P_2 = bc^{N-1}aP_1 \\
S_{N+3} &= bc^{N-1}P_3 = bc^{N-1}a^2P_1 \\
&\vdots \\
S_{N+k} &= aS_{N+k-1} \qquad (2 \le k)
\end{aligned} \tag{21}
$$

The PW of the salvage value stream is

$$
\begin{aligned}
\mathrm{PW}(i')_2 &= \frac{bP_0}{N(1+i')^1} + \frac{bcP_0}{N(1+i')^2} + \cdots + \frac{bc^{N-1}P_0}{N(1+i')^N} + \frac{bc^{N-1}P_1}{(1+i')^{N+1}} + \frac{bc^{N-1}P_2}{(1+i')^{N+2}} + \cdots \\
&= \frac{bP_0}{N(1+i')^1} + \frac{bcP_0}{N(1+i')^2} + \cdots + \frac{bc^{N-1}P_0}{N(1+i')^N} + \frac{bc^{N-1}P_1}{(1+i')^{N+1}} + \frac{bc^{N-1}aP_1}{(1+i')^{N+2}} + \cdots \\
&= \frac{bP_0}{N}\sum_{k=1}^{N}\frac{c^{k-1}}{(1+i')^k} + \frac{bc^{N-1}P_1}{(1+i')^{N+1}}\left[\frac{1}{1-\left(\frac{a}{1+i'}\right)}\right] \\
&= \frac{bP_0}{N}\sum_{k=1}^{N}\frac{c^{k-1}}{(1+i')^k} + \frac{bc^{N-1}P_1}{(1+i')^N(1+i'-a)}
\end{aligned} \tag{22}
$$

O&M Cost Streams

Referring to Figure 3.1, the composition of assets in each year is composed of several groups of assets which were purchased at different points in time. This implies that each group of assets within the same period has different O&M costs. Therefore, we need to consider all these variations of the O&M costs in each year. Although the O&M cost streams are similar to those of the basic model, the scale of the O&M costs can change due to the productivity loss factor that we have mentioned earlier. For example, with $N = 3$ years, the O&M costs at period 1 consist of only assets purchased in year 0:

$$A_1 = A \tag{23}$$

The O&M costs at period 2 consist of two different assets: two-thirds is from the first asset group, and one-third is from the second asset group.

$$A_2 = \left(\frac{2}{3}\right)(p+s)A + \left(\frac{1}{3}\right)qA \tag{24}$$

The O&M costs at period 3 will be

$$A_3 = \left(\frac{1}{3}\right)(p+s)^2 A + \left(\frac{1}{3}\right)(p+s)qA + \left(\frac{1}{3}\right)q^2 A \tag{25}$$

After the N^{th} period, year 4 in our example, the O&M costs would take the following expression:

$$\begin{aligned} A_4 &= \left(\frac{1}{3}\right)(p+s)^2 qA + \left(\frac{1}{3}\right)(p+s)q^1 A + \left(\frac{1}{3}\right)q^2 A \\ &= q\left(\left(\frac{1}{3}\right)(p+s)^2 A + \left(\frac{1}{3}\right)(p+s)qA + \left(\frac{1}{3}\right)q^2 A\right) \\ &= qA_3 \end{aligned} \tag{26}$$

The general form of the O&M cost stream is then as follows:

$$A_2 = \left(1-\frac{1}{N}\right)(p+s)A + \left(\frac{1}{N}\right)qA$$

$$\vdots$$

$$A_N = \left(\frac{A}{N}\right)\sum_{k=1}^{N}\left[(p+s)^{N-k}q^{k-1}\right] \tag{27}$$

$$A_{N+k} = qA_{N+k-1} \qquad (1 \le k)$$

The PW cost expression for the entire O&M cost stream over the planning horizon is obtained as follows:

$$\begin{aligned} \mathrm{PW}(i')_3 &= \frac{A_1}{(1+i')^1} + \frac{A_2}{(1+i')^2} + \cdots + \frac{A_N}{(1+i')^N} + \frac{qA_N}{(1+i')^{N+1}} + \cdots \\ &= \frac{A_1}{(1+i')^1} + \frac{A_2}{(1+i')^2} + \cdots + \frac{A_N}{(1+i')^N}\left[\frac{1}{1-\frac{q}{(1+i')}}\right] \\ &= \frac{A_1}{(1+i')^1} + \frac{A_2}{(1+i')^2} + \cdots + \frac{A_N}{(1+i')^{N-1}(1+i'-a)} \end{aligned} \tag{28}$$

The Total PW Cost of the Staggered Replacement Policy

We obtain the total PW cost expression for the Staggered replacement policy under technology improvement as follows:

$$\begin{aligned} \mathrm{PW}(i')_{\text{Staggered}} &= P_0 + \frac{P_1}{(1+i'-a)} + \frac{bP_0}{N}\sum_{k=1}^{N}\frac{c^{k-1}}{(1+i')^k} + \frac{bc^{N-1}P_1}{(1+i')^N(1+i'-a)} \\ &+ \frac{A_1}{(1+i')^1} + \frac{A_2}{(1+i')^2} + \cdots + \frac{A_N}{(1+i')^{N-1}(1+i'-a)} \end{aligned} \tag{29}$$

Since we have developed the total PW cost expressions for both Group and Staggered models considering technology improvement in future replacement assets, we will examine the effectiveness of each policy with the illustrating case example.

4.4. ECONOMIC ANALYSIS

In the basic model, we compared the results between the Group and Staggered replacement policies through a case example. In this section, using the same case example, we will follow a similar scheme. Further, we will compare the results for both replacement models under the basic replacement model and after considering technological progress.

4.4.1. AN ILLUSTRATIVE CASE EXAMPLE

Recall the case example in chapter 3. The owner of K-Company gets some information about technological improvement in the copy machine market. The price and operating and maintenance costs of the copy machine will decrease 10% each year. Further, the speed of the new machine will increase 5% each year. The parameter a represents the annual multiplier for the purchase cost, and parameters q and s represent the multipliers for operating and maintenance cost and productivity loss respectively. The summary of the case example is as follows:

Parameter	Value	
P	\$50,000	Previous information
d	10%	
i'	10%	
b	60%	
c	80%	

A	$5,000	
p	125%	
a	90%	Additional information
q	90%	
s	5%	

Table 4.1 Summary of example parameters and values.

4.4.2. ECONOMIC INTERPRETATION OF THE NUMERICAL RESULTS

Recall that we considered additional parameters to reflect technological progress. As we mentioned, technological improvement can affect the economic service life of the asset. Therefore, we should check whether the economic service life of assets is still N=5 under ongoing technological progress. Table 4.1, the PW cost of Group replacement under ongoing technological progress, indicates N=3 is the economic service life in this case.

Service life (N)	The PW cost of group replacement
2	$112,125
3	**$106,752**
4	$107,500
5	$111,736
7	$111,963

Table 4.2 The summary of service life

Figure 4.1 illustrates the PW cost of the Group and Staggered replacement policies when the economic service life is N=3. Figure 4.1 indicates that the incremental cost of the

Staggered replacement policy is greater than the Group replacement policy along the entire cash flow stream. The PW cost of Group replacement is $108,346, while Staggered replacement costs $114,102 with a planning horizon of 41 years. The incremental cost of choosing Staggered replacement is $5,756 in present value.

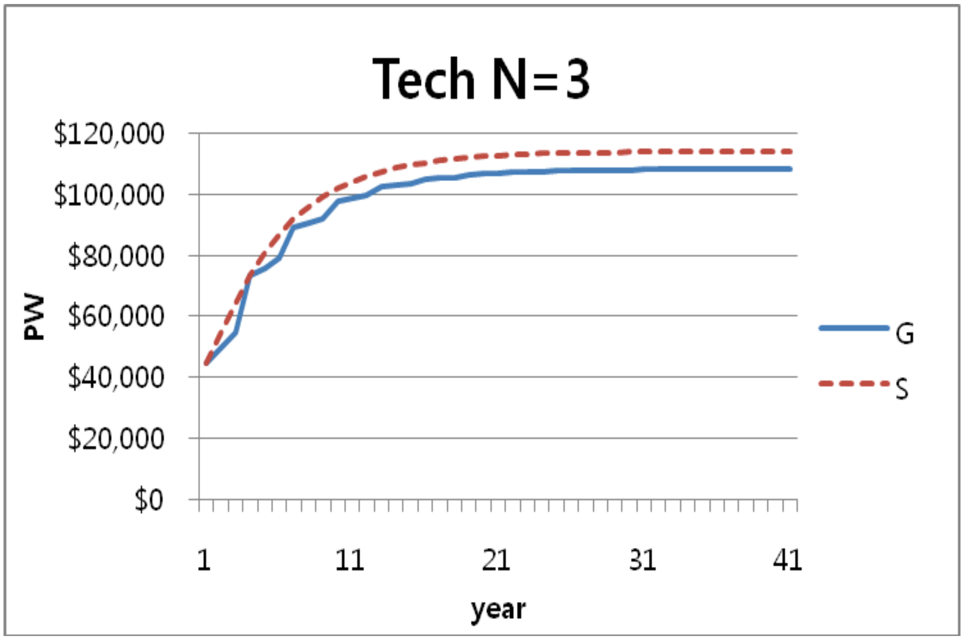


Figure 4.1 under N=3(under ongoing technological progress model)

When we adjust the service life to account for replacement under technological progress, Group replacement is more cost effective when compared with Staggered replacement. To verify, we will conduct sensitivity analyses.

4.4.3. SENSITIVITY ANALYSIS

We will perform a series of sensitivity analyses to see the results under different conditions, as we did in chapter 3. The procedure for this sensitivity analysis is the same as it is for the basic model. The key input parameters are as follows: (1) service life, (2) the annual multiplier for the purchase cost (*a*), and (3) overall sensitivity graphs of the difference between Group and Staggered replacement models.

Replacement Interval, N

For this replacement problem under ongoing technological change, we found that the economic service life is *N*=3 years. However, we observe the difference between two PW costs when the replacement intervals change from *N*=2 years to *N*=7 years in table 4.2.

Type	N=2	N=3	N=5	N=7
Difference (G-S)	-$14,116	-$5,756	-$2,131	$6,438

Table 4.3 The difference between the PW of Group and Staggered model under *N* change

In this case, the Group replacement policy still has slightly less total cost than the Staggered replacement policy. Here, let's consider *N*=5—that is, we keep the assets two more years beyond its economic service life. Figure 4.2 illustrates that the total cost of the Staggered replacement policy is higher than the Group replacement policy during most periods when *N*=5 years. With a planning horizon of 41 years and an economic service life for assets of 5 years, the Group replacement policy costs $116,528, while the Staggered replacement policy costs $118,658. The cost difference of both of the policies is $2,131 in present value.

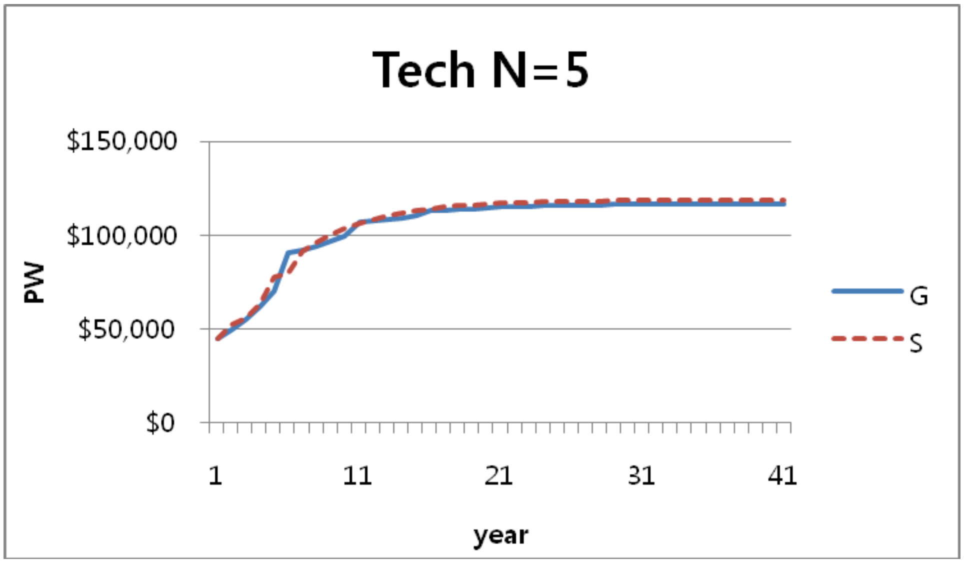


Figure 4.2 under N=5(under the ongoing technological model)

However, the Staggered replacement policy appears to be more cost effective when compared with the Group replacement policy between the fifth and seventh year. That means that if K-company keeps the assets from five to seven years, Staggered replacement is more cost effective than Group replacement. Figure 4.3 illustrates that Staggered replacement is more efficient when N=7.

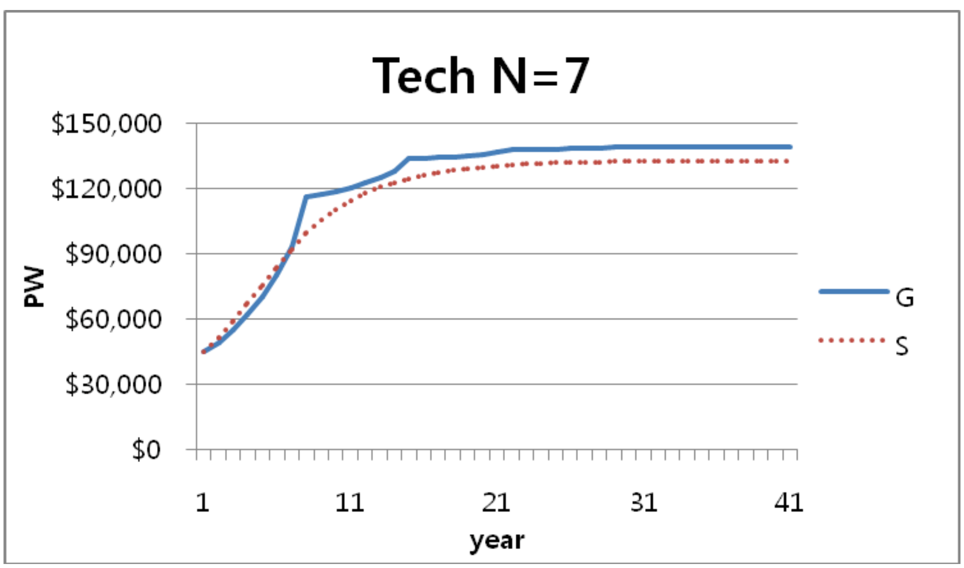


Figure 4.3 under N=7

This result explains that if we keep the assets beyond their economic service life, the total cost of Group replacement increases.

Annual Multiplier for the Purchase Cost (*a*)

We added three parameters (*a, q, s*) to reflect technological progress in our replacement problem. The annual multiplier *a* is the most significant factor among them. It affects the total PW cost of both replacement models; a relatively small *a* means that the new purchase cost will decrease because technological improvement of assets relatively much increases in market. The parameter *a* reduces the capital cost for both Group and Staggered replacement models. As we vary the volume discount from 85% to 95%, the trend is demonstrated in table 4.3.

Under N=3	*a*=85%	*a*=90%	*a*=95%
Difference(G-S)	-$4,724	-$5,956	-$7,569

Table 4.4 The difference between the PW of Group and Staggered models under various *a*

Figures 4.4 - 4.7 illustrate more detail for the results presented in table 4.3; the gap between the Group and Staggered replacement policies increases according to an increase in the annual multiplier *a*.

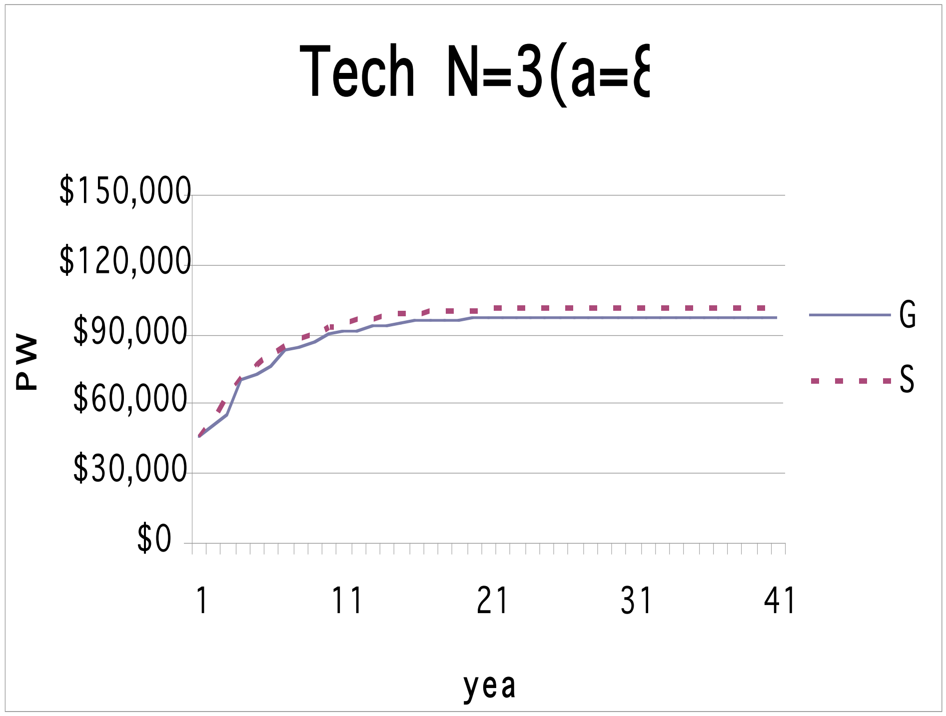


Figure 4.4 under a=85%

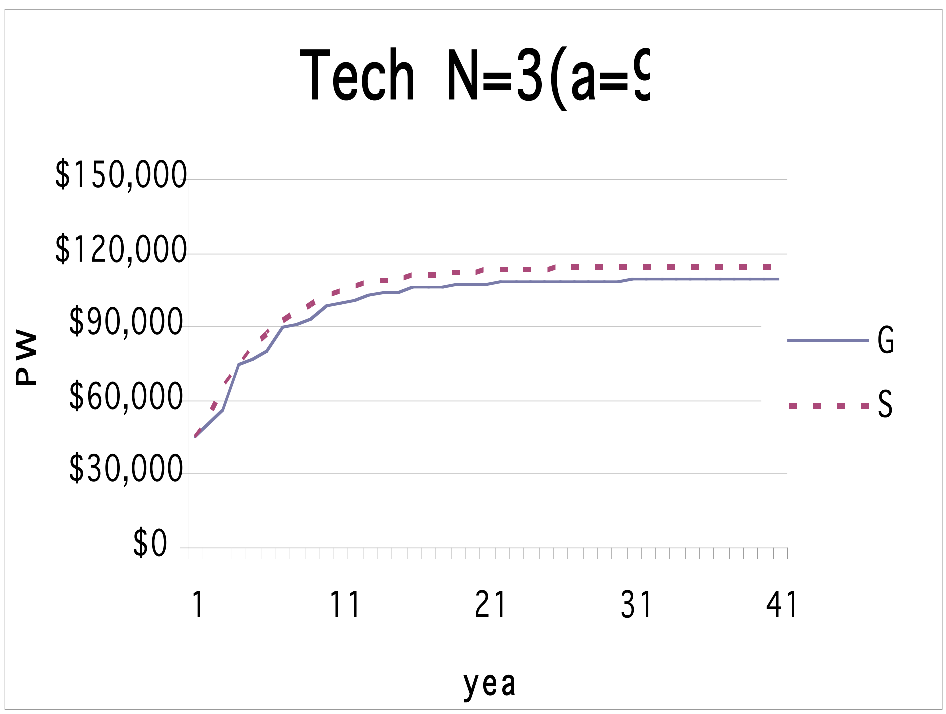


Figure 4.5 under a=90%

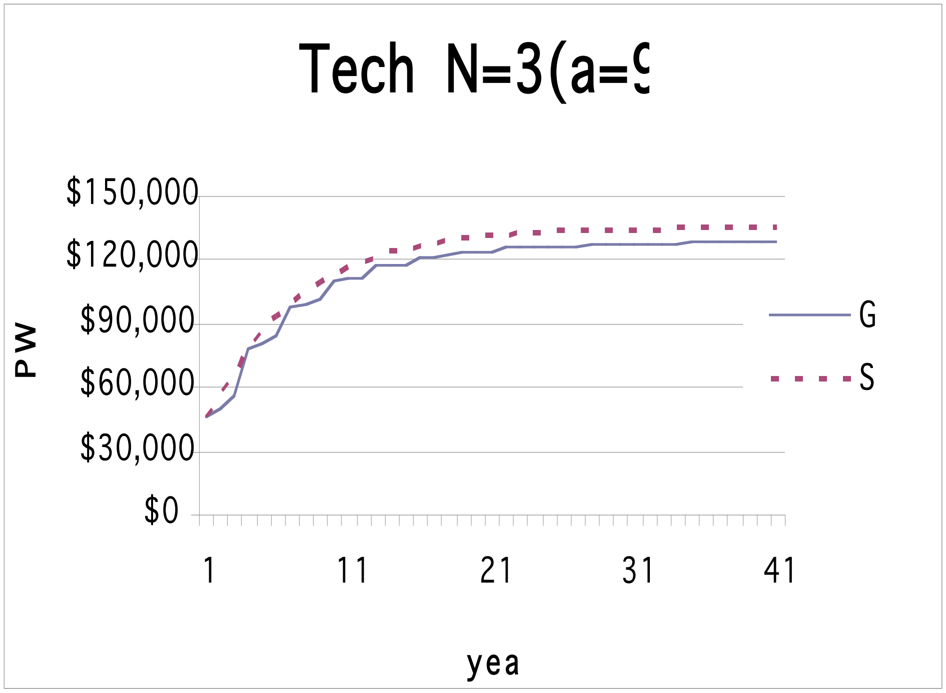
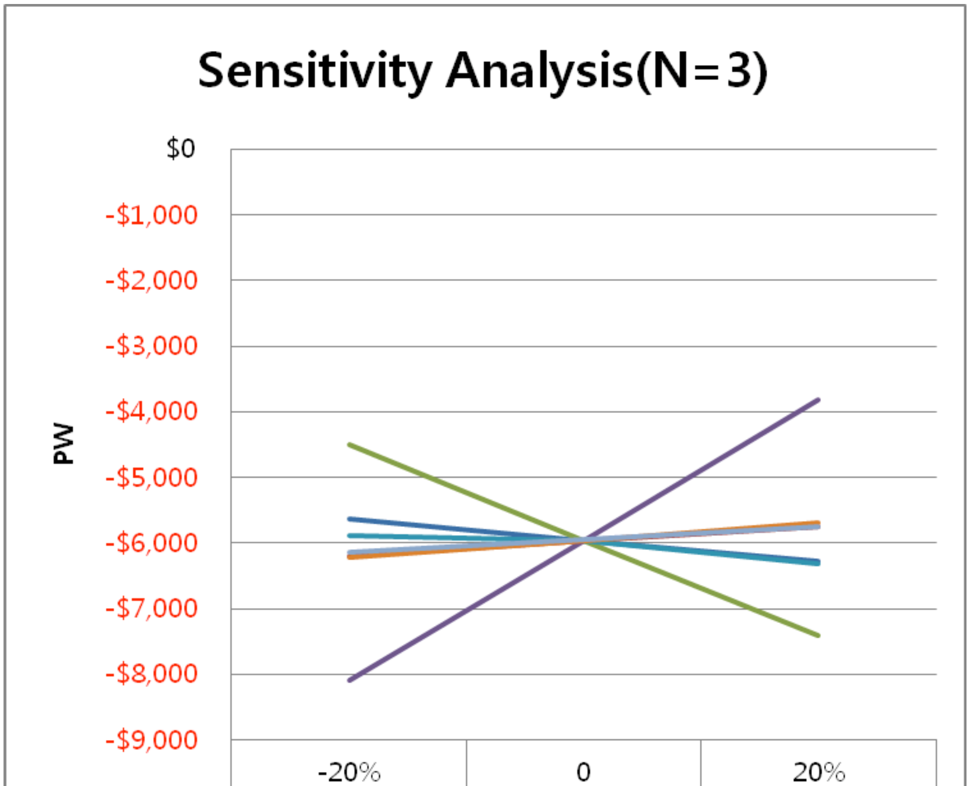


Figure 4.6 under a=95%

Obtaining an Overall Sensitivity Graphs

Figure 4.7 illustrates the sensitivity analysis for seven of the key input variables, which are the same variables as in the replacement problem under no technological progress. Figure 4.8 shows the sensitivity analysis for the three key input variables we added because of technological progress. In the first sensitivity analysis, the values of variables change plus or minus 20% from the base-case value. In the second sensitivity analysis, the range of variables is plus or minus 10% from the base-case because when we increase the value of *a* to 20%, it is possible for the value of *a* to go over 1, and this would violate the restriction that *a* is less than 1.



	-20%	0	20%
d	-$5,642	-$5,956	-$6,270
i	-$6,192	-$5,956	-$5,747
P	-$4,497	-$5,956	-$7,415
b	-$8,084	-$5,956	-$3,828
c	-$5,883	-$5,956	-$6,315
A	-$6,224	-$5,956	-$5,688
p	-$6,149	-$5,956	-$5,758

Figure 4.7 Sensitivity graph for the PW cost differential between Group and Staggered replacement policies.

In figure 4.7, we see that the total cost of the Staggered replacement policy is higher than the Group replacement policy for the range of values examined. In particular, the first-year's loss of market value of the asset (*b*) is the most sensitive variable, and the initial

O&M cost (A) and the scaling factor of the future O&M cost (p) are relatively insensitive variables.

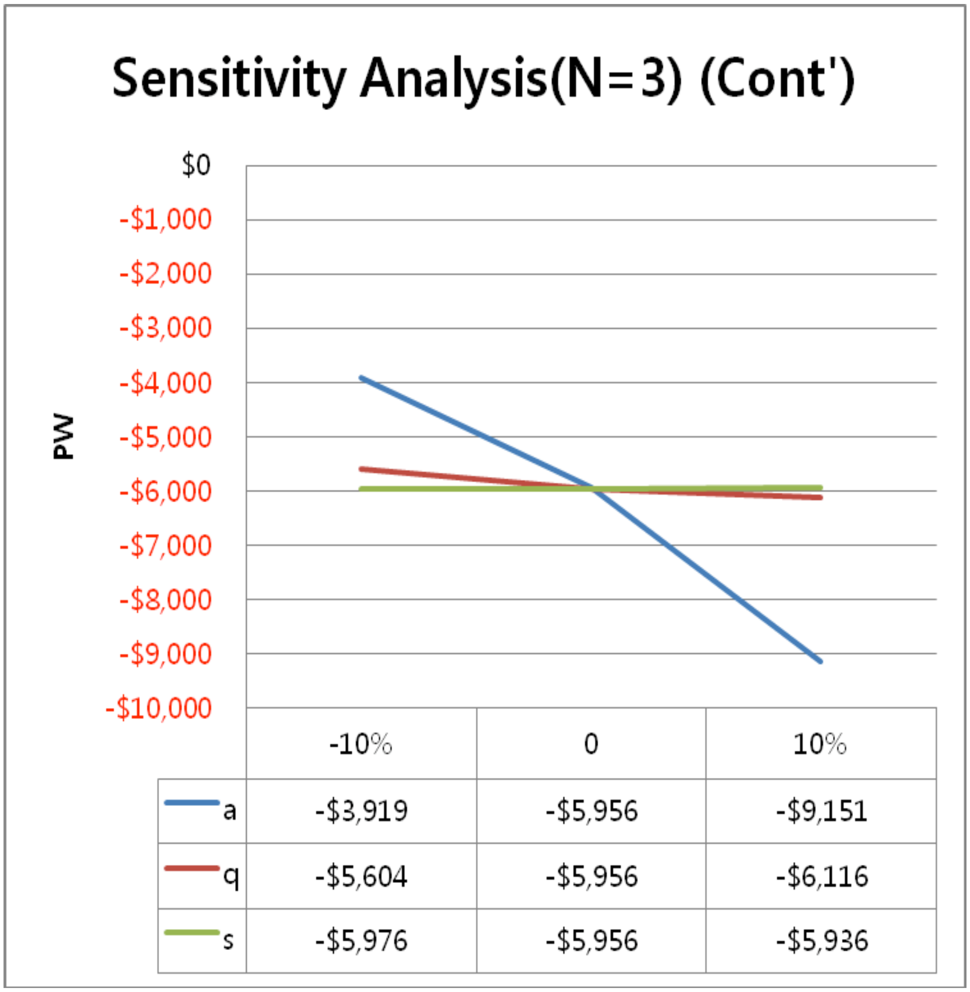


Figure 4.8 Sensitivity graph (Cont') for the PW cost differential between Group and Staggered replacement policies.

Figure 4.8 also shows that the Group replacement policy is quite cost effective for the range of values examined. The annual multiplier for the purchase cost (a) is the most sensitive variable among them.

CHAPTER 5. MODEL UNDER RISK

In Chapter 4, we developed two replacement models (Group and Staggered) under ongoing technological progress. In those models, we assumed all parameters to be known with reasonable certainty. However, this certainty assumption is rather naïve, as it is very difficult to predict the price or operating and maintenance costs of assets in any precise fashion. One practical way to estimate these parameters is to observe the trend of costs of similar assets during past periods. To introduce possible variations in our decision parameters, we will treat some key input parameters as random variables. Since we are not likely to attain an analytical solution, we will rely on computer simulation using @RISK

5.1. RISK SIMULATION PROCEDURES USING @ RISK

To conduct a risk simulation, we will use a Microsoft Excel plug-in known as @ Risk. All we have to do is to develop an Excel worksheet to calculate the net present cost of either the Group or Staggered replacement policy over the planning horizon. These worksheets were already developed as functions of key input parameters presented in Chapters 3 and 4. Then, we need to identify the random variables in the replacement models. With @RISK, we have a variety of probability distributions to choose from to describe our beliefs about the random variables of interest. Running an analysis with @RISK involves five steps:

Step 1: Create a cash flow statement within Excel in which the cash flow entries are a function of the input variables.

Step 2: Define Uncertainty. Here we start by replacing uncertain values in our spreadsheet model with @RISK probability distribution functions. As shown in Figure 5.1, @RISK provides a wide range of probability functions to choose from. In our demonstration, however, we will assume a Beta distribution for each random variable. The Beta distribution has been chosen primarily for convenience, as we can easily make three-point estimates: an optimistic estimate, a pessimistic estimate, and a most likely estimate. These three estimates are used as the upper bound, the lower bound, and the mode of the corresponding input parameter distribution. Then, the probability distribution itself is assumed to be a Beta distribution with a standard deviation of one-sixth for the spread between the upper and lower bounds (Park and Gunter, 1990).

Step 3: Pick Your Bottom Line. With @RISK, we need to designate our output cells, which are the bottom line cells whose values we are interested in. In our case, this is the total present worth cost of each replacement policy.

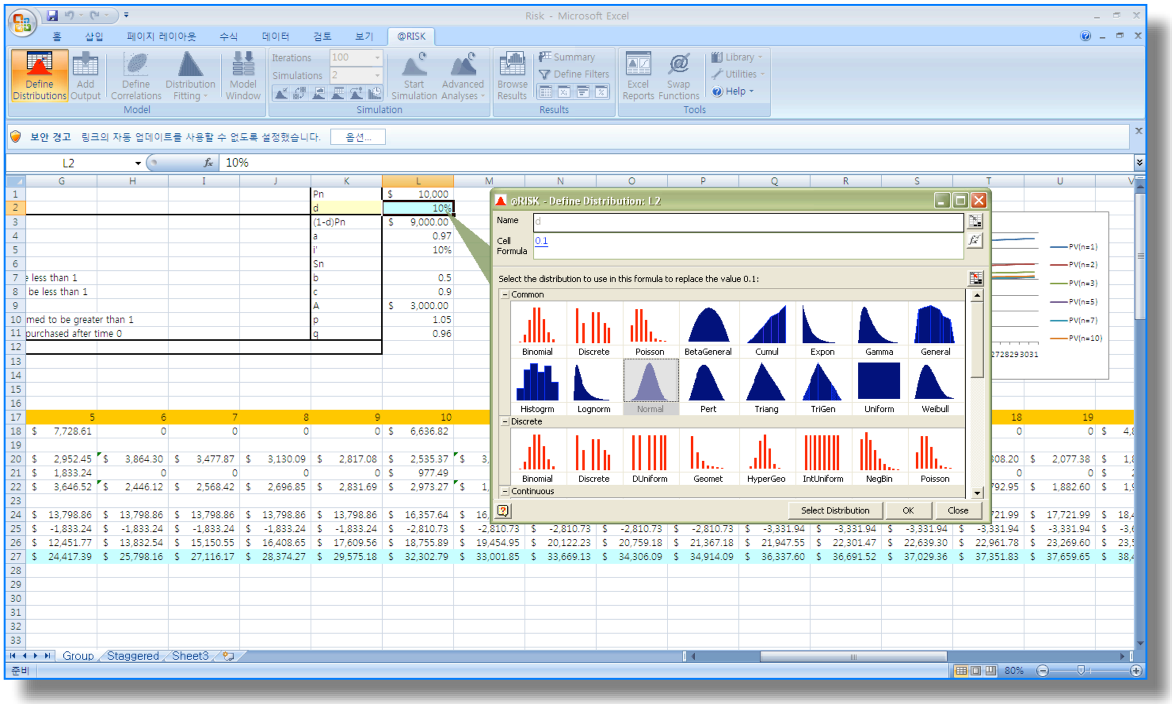


Figure 5.1 Selecting a distribution function in @Risk

Step 4: Simulate. Once we have completed Step 3, we are ready to simulate. There is no limit to the number of different scenarios we can look at in our simulations. Each time, @RISK samples random values from the @RISK functions we entered in Step 2 and records the resulting outcome (present worth cost of adopting a Group (or Staggered) replacement policy). With 100 iterations for each scenario (or any number of iterations), we obtain the probability distribution of the present worth cost function for each replacement strategy.

Step 5: Analyze the Simulation Results. Once we obtain the probability distribution for each replacement policy, we have a way to compare the effectiveness of one policy over the other. As shown in Figure 5.2, @RISK provides a full statistical report with a wide range of graphing options for interpreting and presenting the simulation results.

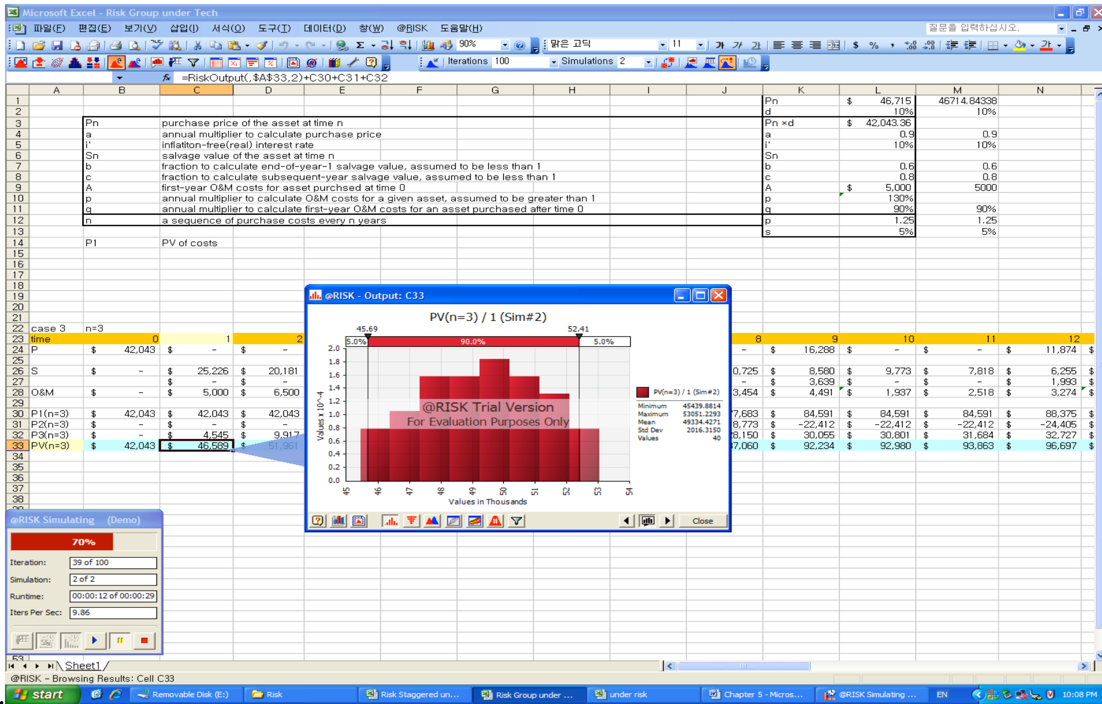


Figure 5.2 Displaying the simulation results

5.2. DEVELOPING A SIMULATION MODEL

To illustrate the process of developing a simulation model and the impact of uncertainty in choosing a replacement policy, we could consider one critical random variable (such as purchase cost, *P*) at a time, just like conducting a sensitivity analysis. Since we consider one single random variable at a time, we don't need to define any statistical relationship with other input parameters. In Section 5.3, we will extend our basic simulation model by considering all input parameters to be random variables.

5.2.1. PURCHASE COST (*P*) AS A SINGLE RANDOM VARIABLE

Let's assume that the purchase cost (*P*) is the only random variable among the input parameters. In that case, we select a Beta distribution to describe the nature of uncertainty associated with the purchase cost. Using the @RISK distribution function, the Beta probability distribution for *P* looks like the following:

Name	Function	Min	Mean	MAx
P_n	RiskBetaGeneral(2,2,45000,55000,RiskStatic(50000))	45000	50000	55000

Table 5.1 Beta Distribution Function for Purchase Cost, *P*

With 100 iterations, @RISK produces the simulation outputs as summarized in Figures 5.3 and 5.4. Recall that the economic service life for group replacement was 3 years (N = 3) in Chapter 4. In Figures 5.3 and 5.4, we see how the mean, standard deviation and percentiles of the PW cost for each year change over the planning horizon. Different colors were used to display the mean value in yellow, ±1 standard deviation in red and the range between the lower 5th percentile and upper 5th percentile in green. Clearly, as we further extend the planning horizon, the variability of the PW cost continues to increase, but eventually it reaches some form of steady state after 40 years. Recall that the PW cost function we have developed in Chapters 3 and 4 was based on the infinite planning horizon. Therefore, the steady-state results are more important for replacement decisions.

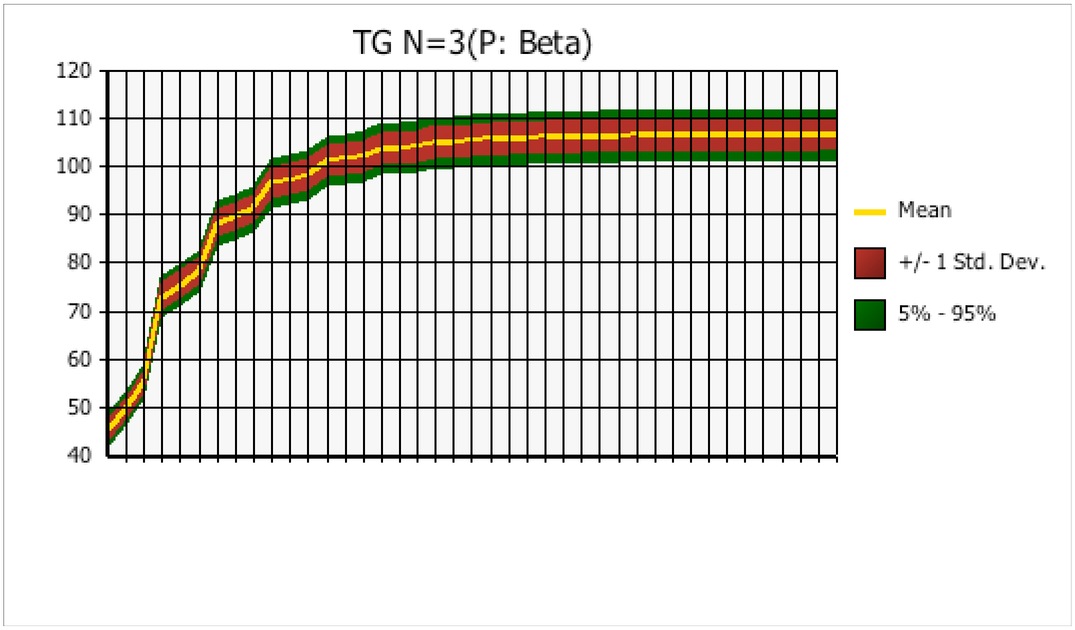


Figure 5.3 The PW cost for Group replacement as a function of the planning horizon

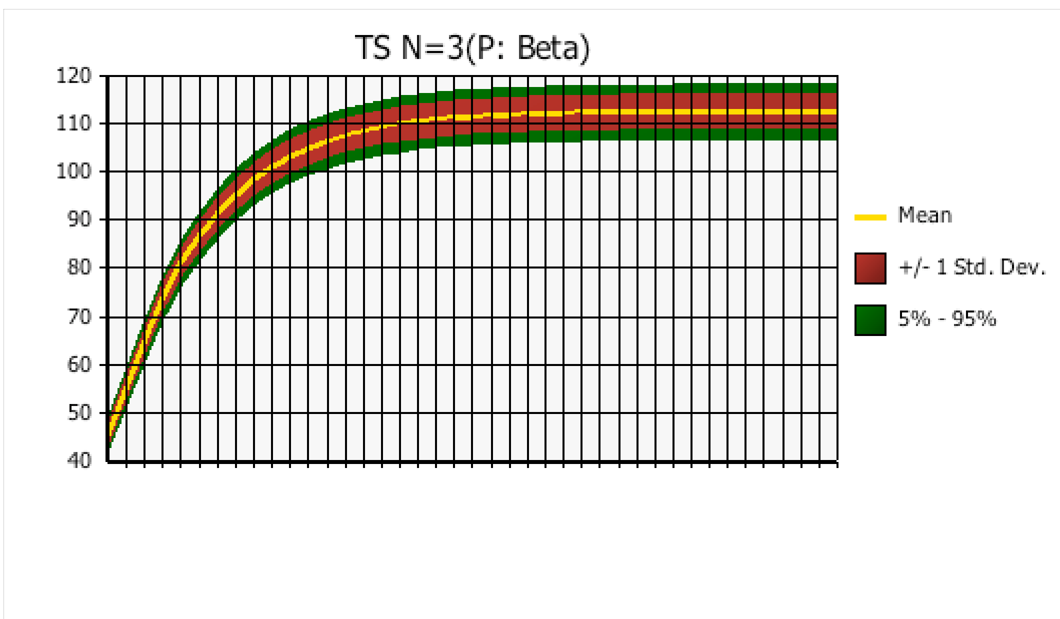


Figure 5.4 The PW cost for Staggered replacement as a function of the planning horizon

Table 5.2 summarizes the trend of the PW cost for both models. Although the mean PW of the group replacement policy is smaller than that of the Staggered replacement model, it has yet to be verified for any instance of clear stochastic dominance between the two policies, which will be shown in Section 5.2.1.2.

	95 Per.	+1 Std. Dev.	Mean	-1 Std. Dev.	5 Per.
Group	$ 111,854	$ 109,982	$ 106,789	$ 103,596	$ 101,528
Staggered	$ 118,278	$ 116,260	$ 112,749	$ 109,238	$ 106,844

Table 5.2 Summary of PW Cost for the Group and Staggered Replacement Policies

Figures 5.5 and 5.6 show the cumulative probability distribution charts for the two policies. With these cumulative probability distributions, we can assess the likelihood of incurring a certain level of replacement cost over the infinite planning horizon. For example, if a firm targets the total replacement cost for a certain asset group at $110,000, we see that the Group replacement policy will meet this target level with an 82% probability, whereas the Staggered replacement policy meets this target level with only a 25% probability. Even though we cannot say in an absolute sense that the Group policy dominates the Staggered policy, we can say clearly that the Group policy appears to be more cost effective in a general sense.

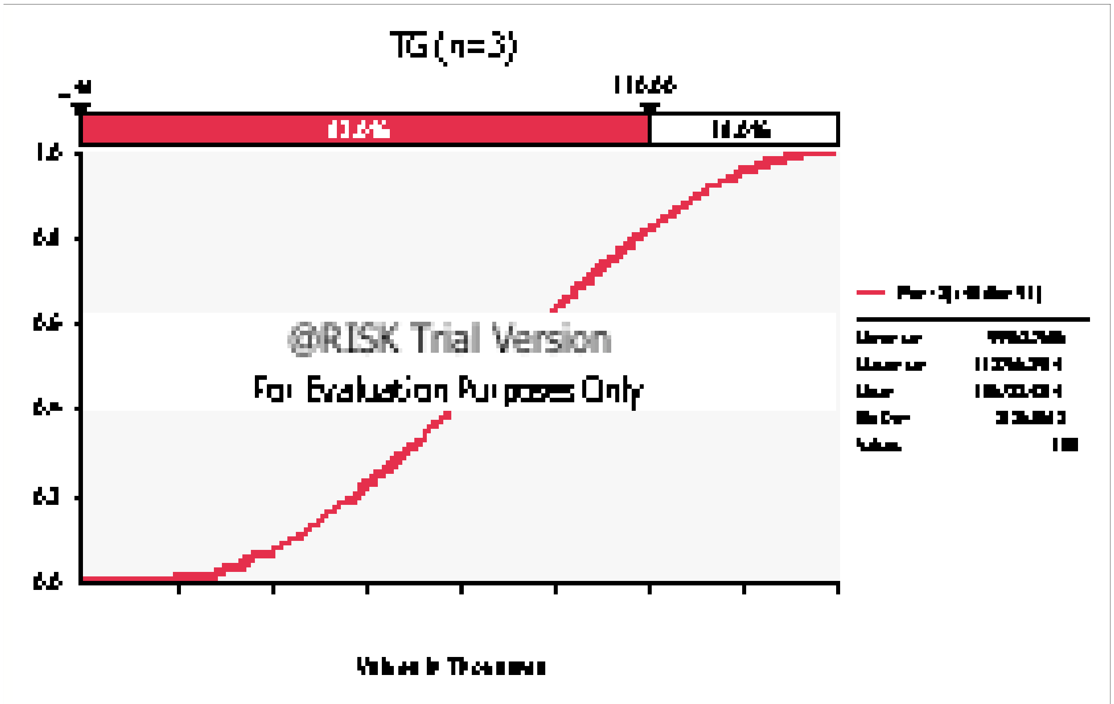


Figure 5.5 The cumulative ascending graph for Group replacement

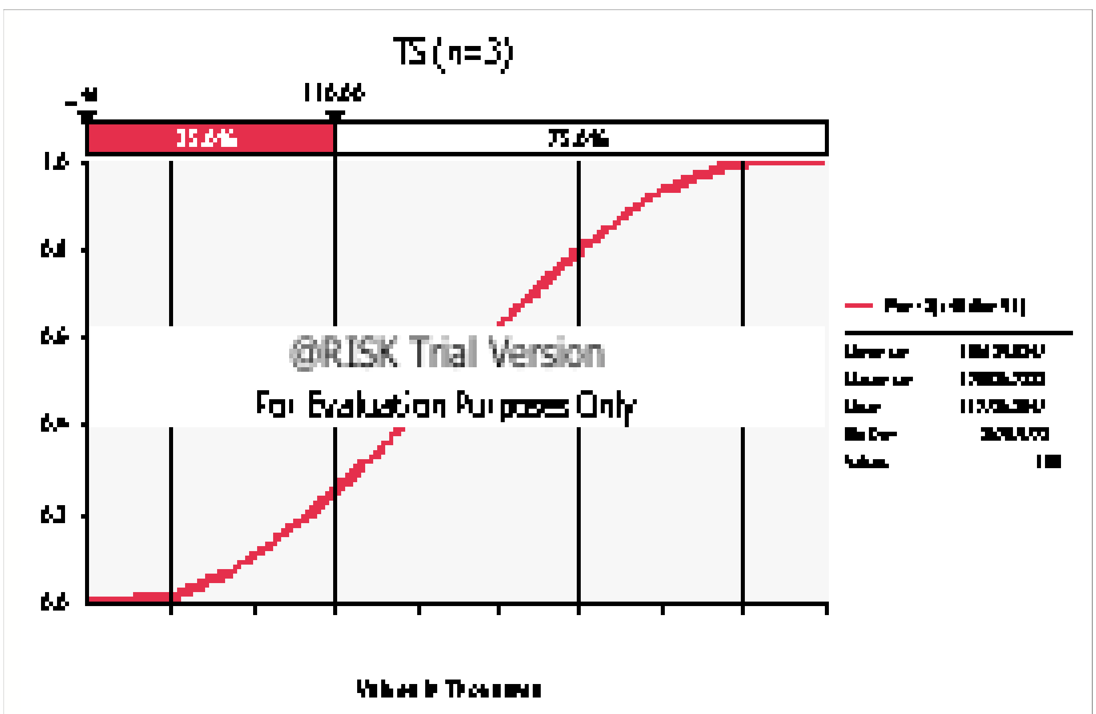


Figure 5.6 The cumulative ascending graph for Staggered replacement

Incremental Cost of Selecting the Group Policy

One practical way to compare these two policies is to develop the incremental cost between the two policies as shown in Figure 5.7. We can see that the differential cost (G-S) is in wide swing in either direction (positive or negative) until it reaches the steady-state. As mentioned earlier, however, we only consider the results in the steady-state condition, because we assumed the study period of an infinite planning horizon. A negative difference between the PW for the Group and Staggered replacement policies implies that the company can benefit from choosing the Group replacement policy. Table 5.3 shows with certainty that the Group replacement policy is more cost effective than the Staggered replacement policy under an infinite planning horizon. With the data set assumed in our model, the company would save $6,013 on average by choosing the Group replacement policy.

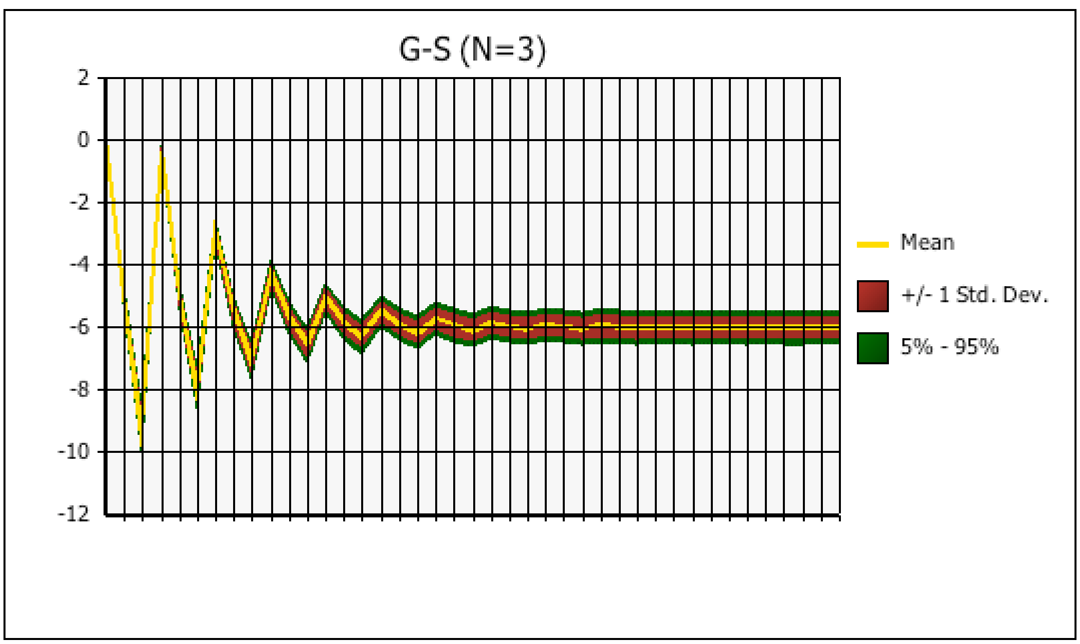


Figure 5.7 The PW trend for the difference between the two replacement models

	95 Per.	+1 Std. Dev.	Mean	-1 Std. Dev.	5 Per.
G-S	-$5,494	-$5,684	-$6,013	-$6,342	-$6,550

Table 5.3 Summary of PW Cost Distribution with Random Variable of P (G-S)

Validation of the Simulation Results

Since the risk simulation model contains random elements (such as the purchase cost, *P*), outputs from the simulation are limited to the number of observed samples of this random variable. As a consequence, any decisions made on the basis of simulation results should consider the variability of the simulation outputs. Our ultimate question is how close an estimator (e.g., mean value of the differential PW cost) is to the true measure. The common approach to assessing the accuracy of an estimator is to construct a confidence interval for the true measure—we determine an interval about the mean within which the true value may be expected to fall with a certain probability.

As we have seen in Figure 5.7, the results of the simulation show that the PW cost of Staggered replacement is higher than Group replacement in most periods. To analyze the output data, we will use the method of replication. Our goal is to obtain point and interval estimates of the difference in mean performance. Table 5.4 gives the summary of simulation output data for a random sample from a Beta distribution and the sample mean and variance from 100 iterations.

Model	Iteration					Sample Mean($\bar{Y}$)	Sample Variance
	1	2	3	…	100		
Group (θ_1)	\$106,130	\$109,836	\$107,126	…	\$108,930	\$106,734	$3{,}185^2$
Staggered (θ_2)	\$112,080	\$116,170	\$113,181	…	\$115,170	\$112,747	$3{,}515^2$

Table 5-4 Simulation Output Data and Summary Measures for Comparing Two Models.

To define a confidence interval, we denote $\triangle\theta = \theta_1 - \theta_2$ which is an interval estimate of the difference in mean performance. A two-sided 100(1 – α) confidence interval for $\triangle\theta$ will take the following form:

$$\triangle\bar{Y} - t_{\alpha/2,v}\triangle\hat{\sigma} \;\le\; \triangle\theta \;\le\; \triangle\bar{Y} + t_{\alpha/2,v}\triangle\hat{\sigma} \qquad (5.1)$$

where $\triangle\bar{Y} = \bar{Y}_1 - \bar{Y}_2$ and $\triangle\hat{\sigma} = \sqrt{\frac{S_1^{\,2}}{n_1} + \frac{S_2^{\,2}}{n_2}}$.

Then, the sample mean and variance based on 100 iterations are

$$\triangle\bar{Y} = \bar{Y}_1 - \bar{Y}_2 = \$106,734 - \$112,747 = -\$6,013$$

$$\triangle\hat{\sigma} = \sqrt{\frac{S_1^{\,2}}{n_1} + \frac{S_2^{\,2}}{n_2}} = \sqrt{\frac{3{,}185^2}{100} + \frac{3{,}515^2}{100}} = 351.95$$

The degree of freedom is

$$v_0 = \frac{\left(\frac{S_1^{\,2}}{n_1} + \frac{S_2^{\,2}}{n_2}\right)^2}{\frac{(S_1^{\,2}/n_1)^2}{n_1 - 1} + \frac{(S_2^{\,2}/n_2)^2}{n_2 - 1}} = \frac{\left(\frac{3{,}185^2}{100} + \frac{3{,}515^2}{100}\right)^2}{\frac{(3{,}185^2/100)^2}{100-1} + \frac{(3{,}515^2/100)^2}{100-1}} = 196.11$$

$$v = v_0 - 2 = 196.11 - 2 = 194.11$$

Finally, a 95% confidence interval is

$$-\$6{,}013 - 1.96 \times 352 \le \Delta\theta \le -\$6{,}013 + 1.96 \times 352$$

$$-\$6{,}592 \le \Delta\theta \le -\$5{,}434$$

which indicates that the confidence interval for $\Delta\theta$ is totally to the left of zero, so we may conclude $\theta_1 < \theta_2$, and Group replacement is preferable to Staggered replacement.

5.3. MULTIPLE RANDOM VARIABLES

In Section 5.2, we have demonstrated how we consider uncertainty in estimating a parameter through the @RISK simulation. Recall that we have a total of nine key input parameters (where the parameter *P* is just one of these nine). As we treat other input parameters as random variables, we need to explicitly consider the correlations among them. Basically, it requires constructing a matrix of correlation coefficients. To include a specific correlation between two random variables in @RISK, we just specify a correlation coefficient (ρ) between -1 and +1. We will examine how the PW cost for each replacement policy changes with different statistical relationships among random variables.

5.3.1. CASE 1-ALL RANDOM VARIABLES ARE MUTUALLY INDEPENDENT

We will first assume that all random variables are mutually independent from each other. Even though this assumption is not likely valid in the real world, we consider this extreme case for comparison purposes. First, we need to assess the degree of randomness for each random variable using the three-point estimates as shown in Table 5.5:

Parameter	Low	Most likely	High
P_n	$45,000	$50,000	$55,000
d	0.09	0.1	0.11
a	0.81	0.9	0.99
i'	0.09	0.1	0.11
b	0.54	0.6	0.66
c	0.72	0.8	0.88
A	$4,500	$5,000	$5,500
q	0.81	0.9	0.99
p	1.125	1.25	1.375
s	0.045	0.05	0.055

Table 5.5 Three-Point Estimates for Key Input Variables

These three-point estimates for each random variable are then converted into a corresponding Beta distribution as shown in Table 5.6

:

@RISK Model Inputs

Performed By: Kyongsun Kim

Date: Sunday, March 16, 2008 11:14:19

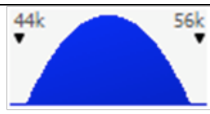
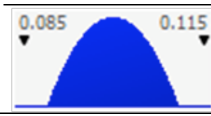
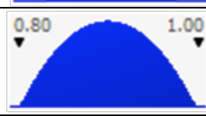
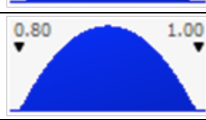
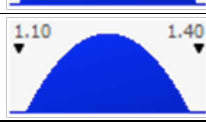
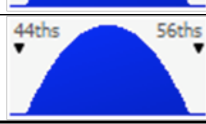
Name	Graph	Function	Min	Mean	Max
P_n		RiskBetaGeneral(2,2,45000,55000,RiskStatic(50000))	45000	50000	55000
d		RiskBetaGeneral(2,2,0.09,0.11,RiskStatic(0.1))	0.09	0.1	0.11
a		RiskBetaGeneral(2,2,0.81,0.99,RiskStatic(0.9))	0.81	0.9	0.99
i'		RiskBetaGeneral(2,2,0.09,0.11,RiskStatic(0.1))	0.09	0.1	0.11
b		RiskBetaGeneral(2,2,0.54,0.66,RiskStatic(0.6))	0.54	0.6	0.66
c		RiskBetaGeneral(2,2,0.72,0.88,RiskStatic(0.8))	0.72	0.8	0.88
A		RiskBetaGeneral(2,2,4500,5500,RiskStatic(5000))	4500	5000	5500
q		RiskBetaGeneral(2,2,0.81,0.99,RiskStatic(0.9))	0.81	0.9	0.99
p		RiskBetaGeneral(2,2,1.125,1.375,RiskStatic(1.25))	1.125	1.25	1.375
s		RiskBetaGeneral(2,2,0.045,0.055,RiskStatic(0.05))	0.045	0.05	0.055

Table 5.6 Beta Distribution Functions for Key Input Variables

PW Cost Distributions

Figures 5.8 and 5.9 depict the trend of PW cost for each replacement policy as a function of the planning horizon using these multiple random variables. Note that the economic service life for the Group replacement policy under the deterministic condition was 3 years. As we expected, the range of the PW cost distribution for each policy is wider than when we consider just one random variable, but the preference for the Group replacement policy still remains unchanged, as summarized in Table 5.7

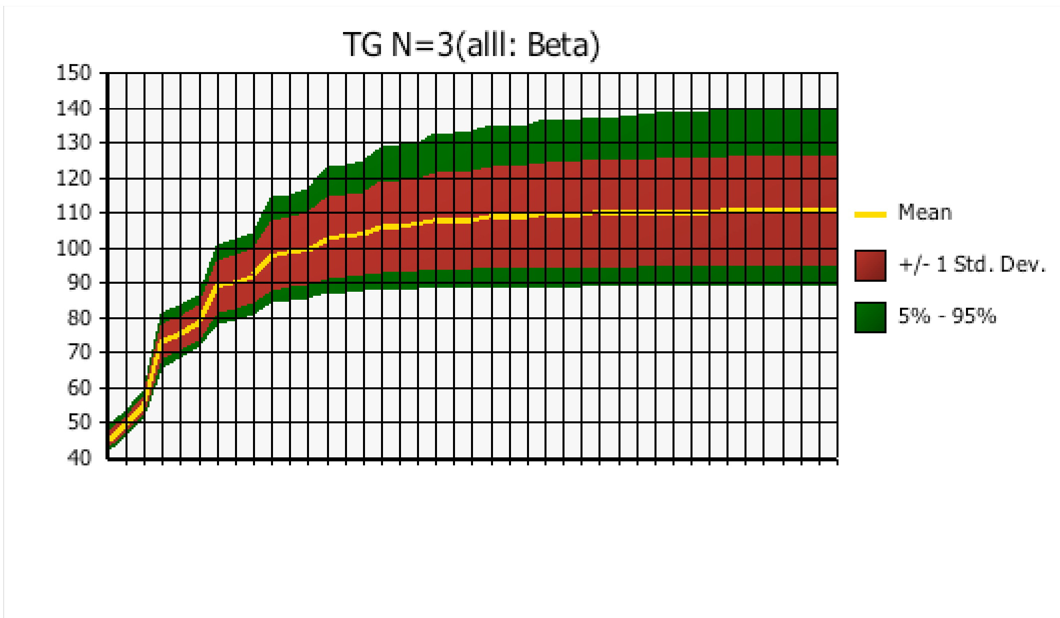


Figure 5.8 The PW trend for Group replacement under all Beta distributions

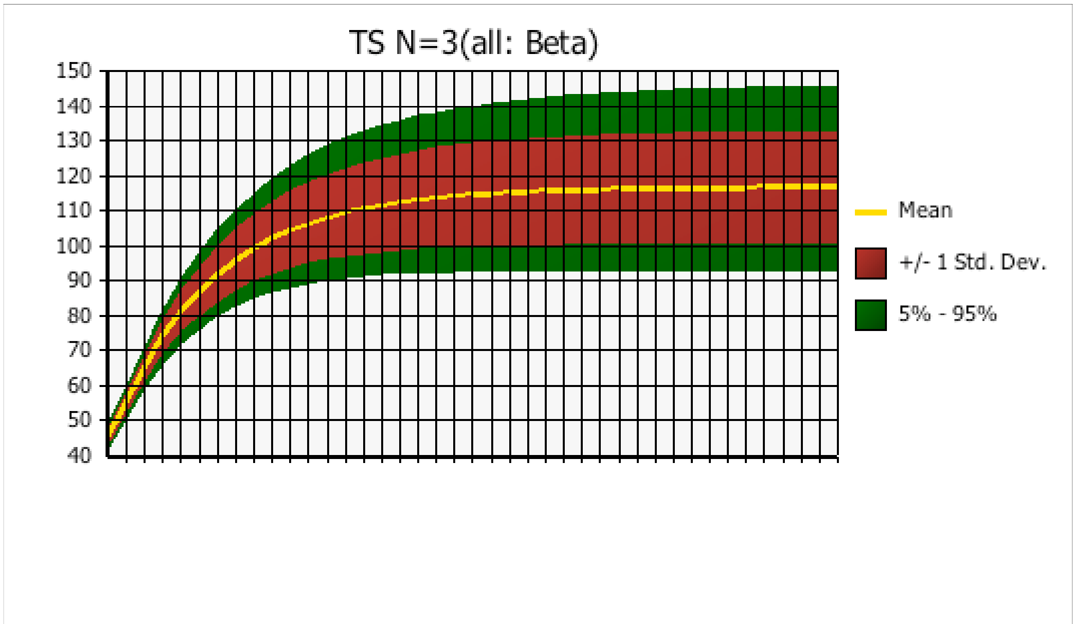


Figure 5.9 The PW trend for Group replacement under all Beta distributions

	95% Per.	+1 Std. Dev.	Mean	-1 Std. Dev.	5% Per.
Group	$143,702	$128,123	$110,809	$93,495	$87,489
Staggered	$150,778	$135,634	$117,020	$98,406	$92,819

Table 5.7 Summary of the PW Cost Distributions for Group and Staggered Replacement Policies

Figures 5.10 and 5.11 are the cumulative probability distribution charts for the two policies. As before, if a firm targets the total replacement cost for a certain asset group at $110,000, we see that the Group replacement policy will meet this target level with a 54% probability, whereas the Staggered replacement policy meets this target level with a 40% probability. Even though these probabilities are smaller than they are in the single random variable situation, the preference for the group replacement policy is still evident.

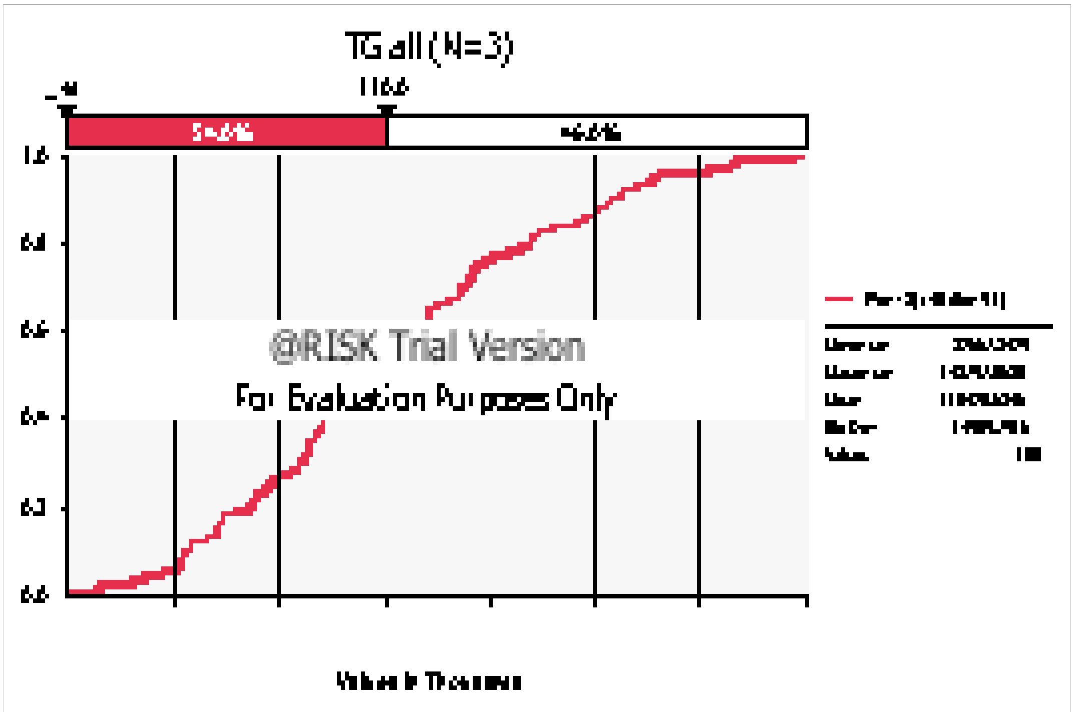


Figure 5-10 The cumulative ascending graph for the Group replacement model

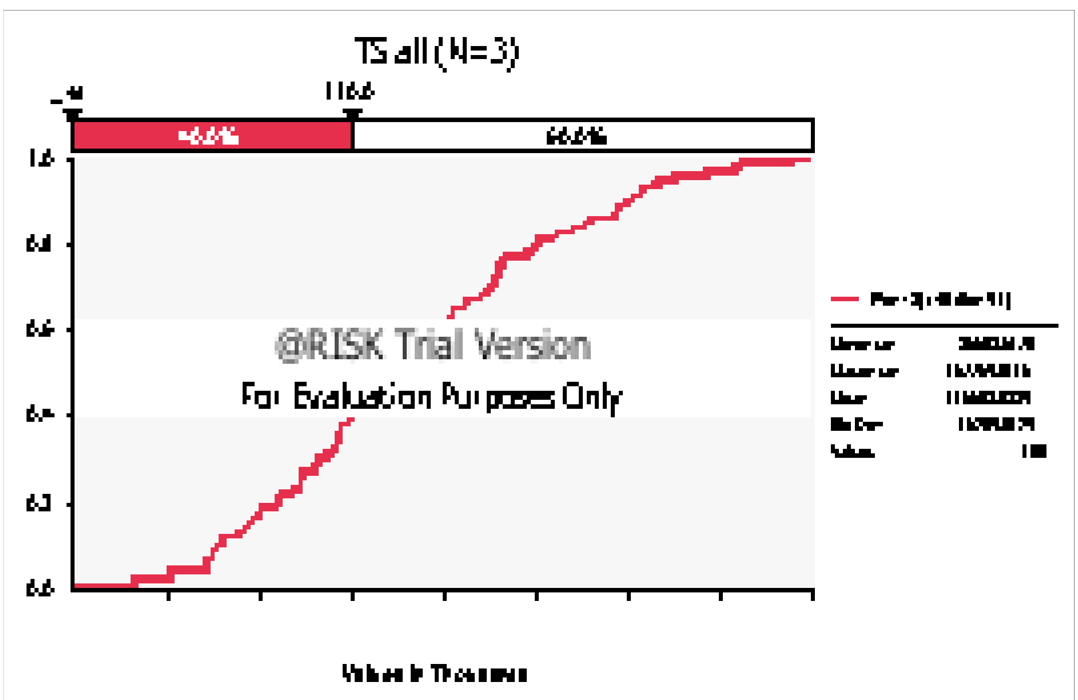


Figure 5-11 The cumulative ascending graph for the Staggered replacement model

Incremental Analysis

We developed a differential cost distribution (G – S) with the multiple random variables, and our results are shown in Figure 5.12. As summarized in Table 5.8, although the mean value of the differential cost does not change much (-$6,013 versus -$6,203), the variability in the differential PW cost increases significantly when compared with the results shown in Table 5. 3.

	95% Per.	+1 Std. Dev.	Mean	-1 Std. Dev.	5% Per.
G-S	-$4,105	-$4,699	-$6,203	-$7,707	-$9,027

Table 5-8 Summary of PW Differential Cost Distributions (G-S)

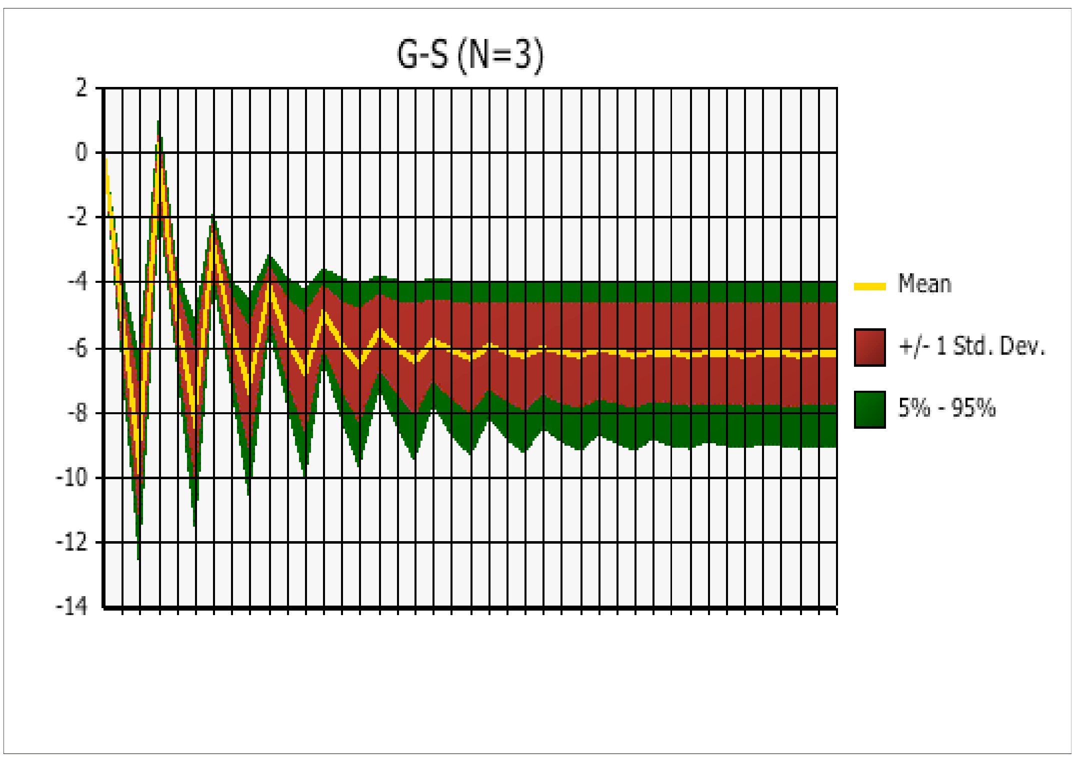


Figure 5-12 The trend of the difference between two models under completely uncertain conditions

Table 5.9 has been prepared to develop a 95% confidence interval for the differential cost ($\triangle\theta$).

Type	Iteration					Sample Mean	Sample Variance
	1	2	3	…	100		
Group	\$100,438	\$110,309	\$117,509		\$96,095	\$110,599	$15{,}604^2$
Staggered	\$105,697	\$114,320	\$124,688		\$101,541	\$116,800	$16{,}872^2$

Table 5.9 Summary of Simulation Output Data with Multiple Random Variables

The sample mean and variance are

$$\triangle\overline{Y} = \overline{Y}_1 - \overline{Y}_2 = \$110,599 - \$116,800 = -\$6,201$$

$$\triangle\hat{\sigma} = \sqrt{\frac{S_1^{\ 2}}{n_1} + \frac{S_2^{\ 2}}{n_2}} = \sqrt{\frac{15,064^2}{100} + \frac{16,872^2}{100}} = 2,262$$

The degree of freedom is

$$v_0 = \frac{\left(\frac{S_1^{\ 2}}{n_1} + \frac{S_2^{\ 2}}{n_2}\right)^2}{\frac{(S_1^{\ 2}/n_1)^2}{n_1 - 1} + \frac{(S_2^{\ 2}/n_2)^2}{n_2 - 1}} = \frac{\left(\frac{15,604^2}{100} + \frac{16,872^2}{100}\right)^2}{\frac{(15,604^2/100)^2}{100-1} + \frac{(16,872^2/100)^2}{100-1}} = 196.80$$

$$v = v_0 - 2 = 196.80 - 2 = 194.80$$

The 95% confidence interval is then

$$\text{-\$6,013-1.96}\times 352 \ \le\ \triangle\theta\ \le\ \text{-\$6,013+1.96}\times 352$$

$$\text{-\$6,592} \ \le\ \triangle\theta\ \le\ \text{-\$5,434}$$

Since the confidence interval for $\triangle\theta$ is totally to the left of zero, we may conclude $\theta_1 < \theta_2$; Group replacement is preferred to Staggered replacement when we assume a statistical independence among key input random variables.

5.3.2. CASE 2-CONSIDERING CORRELATION AMONG RANDOM VARIABLES

@RISK provides an easy way to specify dependent relationships among paired random variables with a matrix of correlation coefficients. In practice, the tasks of estimating these correlation coefficients can be very difficult, since we normally do not have a good database to go by. Nevertheless, if we can construct such a matrix of correlation coefficients as in Table 5.10, with just three random variables for our own demonstration purposes, @RISK will sample the cash flow streams for each period according to these dependent relationships. As shown in Figure 5.13 through Figure 5.15, the random variables *a* and *q* are assumed to be positively correlated, both pairs of the random variables *a* and *s* and *q* and *s* are negatively correlated.

@RISK Correlations	*a* in L5	*q* in L12	*s* in L14
a in L5	1	0.8	-0.8
q in L12	0.8	1	-0.8
s in L14	-0.8	-0.8	1

Table 5.10 Matrix of Correlation Coefficients with @RISK

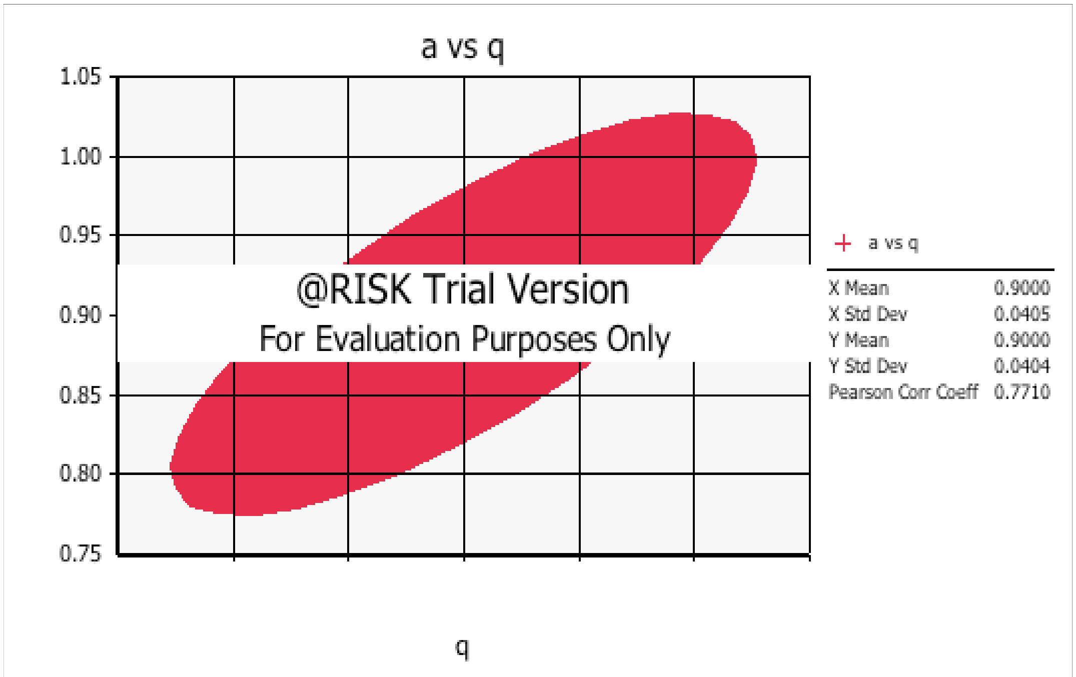


Figure 5.13 Cross Plots of Simulated Dependent Random Deviates (*a* vs *q*)

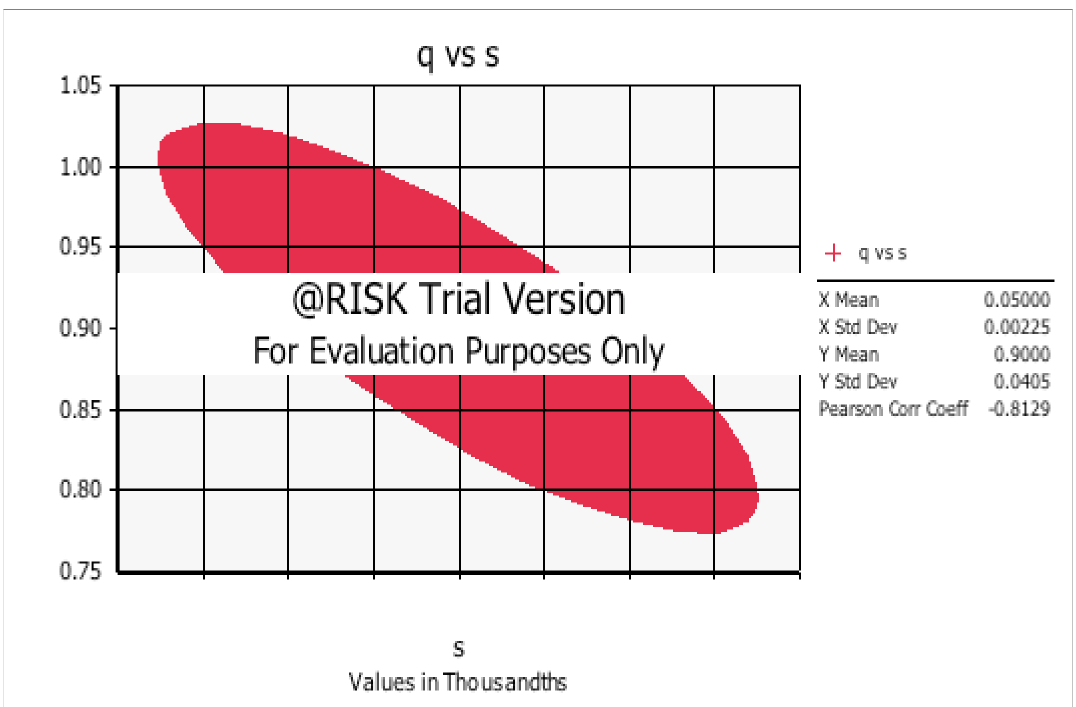


Figure 5.14 Cross Plots of Simulated Dependent Random Deviates (*q* vs. *s*)

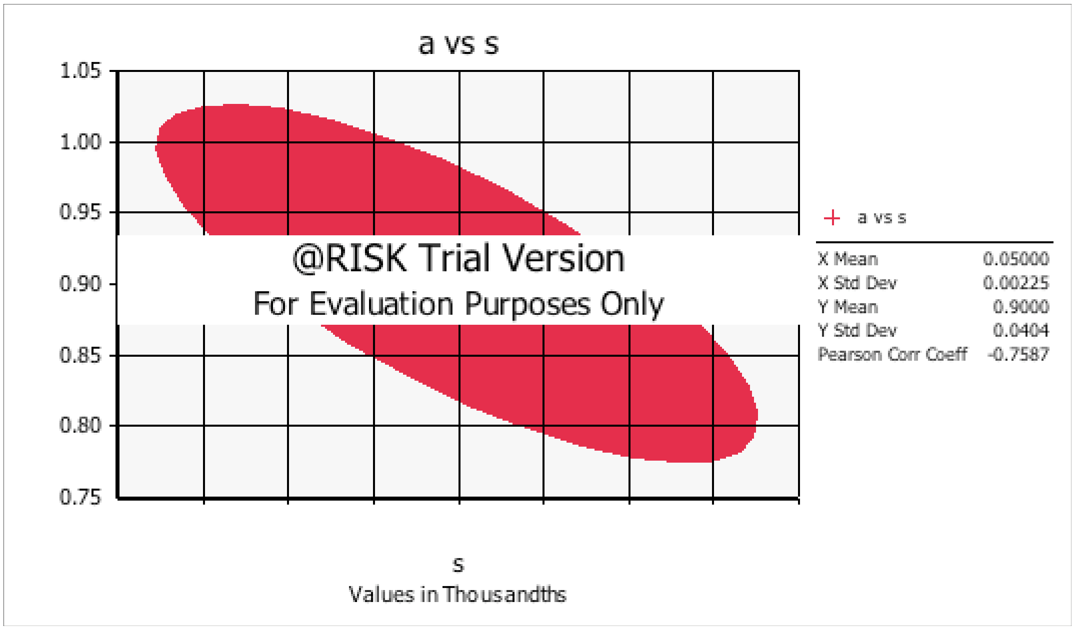


Figure 5.15 Cross Plots of Simulated Dependent Random Deviates (*a* vs. *s*)

With these dependent relations, we obtain the PW cost distributions as a function of planning horizon (Figure 5.16 and 5.17), the differential PW cost distribution (G – S) in Figure 5.18.

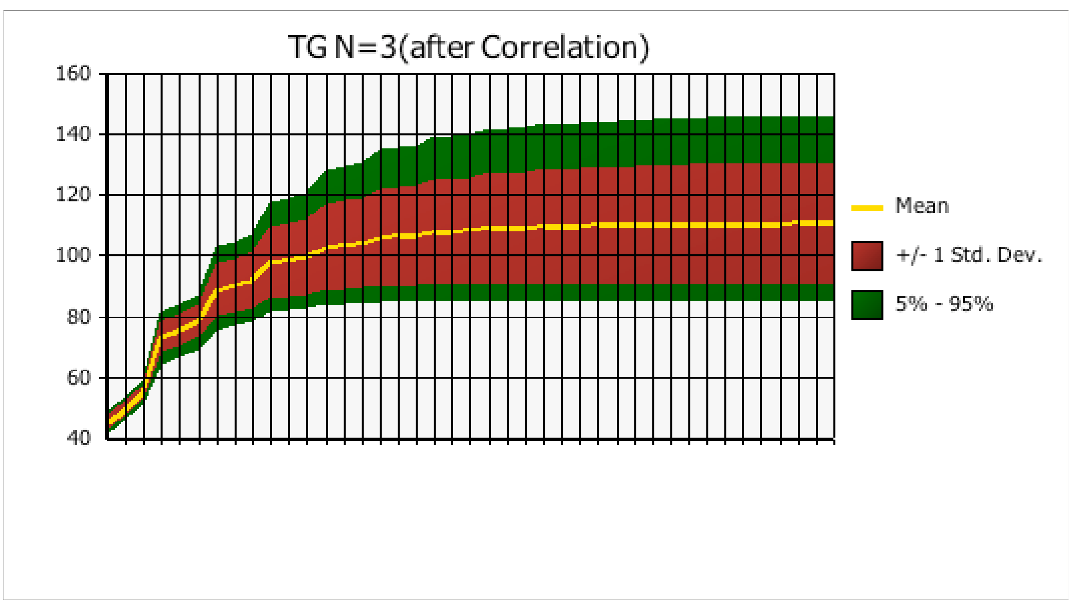


Figure 5.16 PW Cost Distributions as a Function of Planning Horizon (Group Policy)

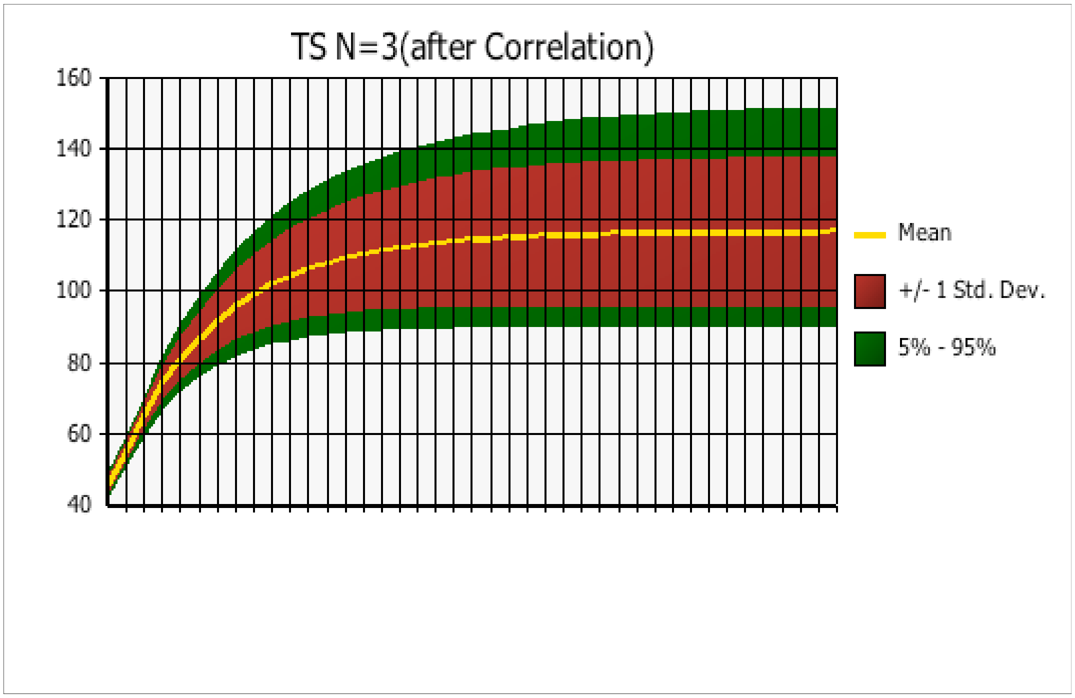


Figure 5.17 PW Cost Distributions as a Function of Planning Horizon (Staggered Policy)

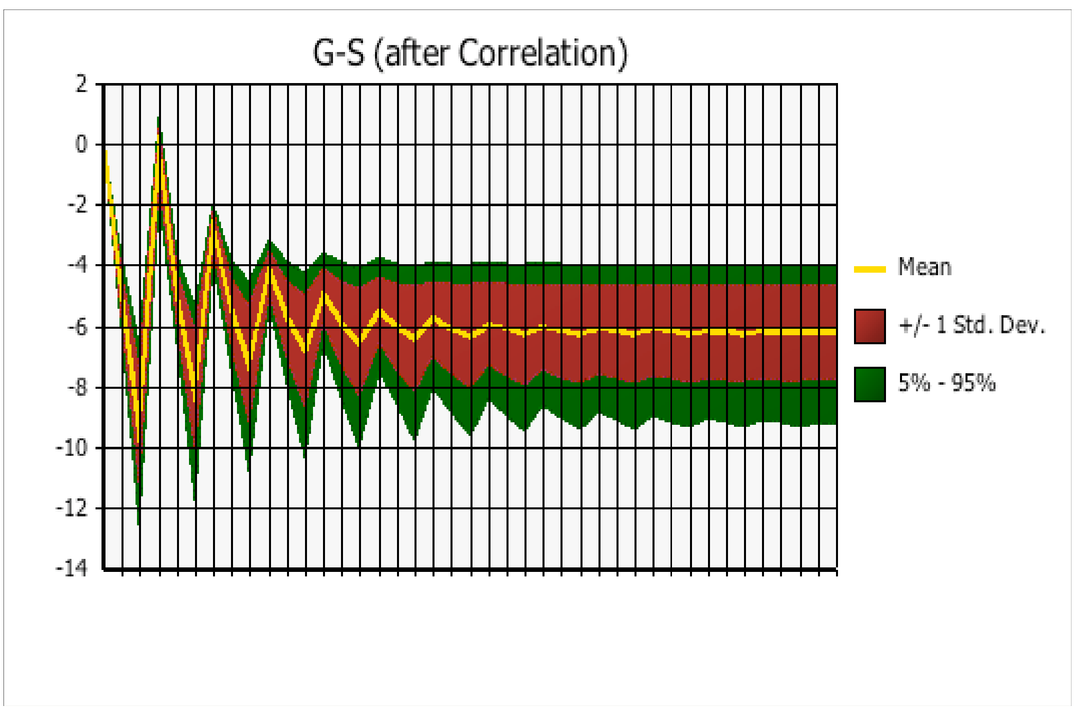


Figure 5.18 PW Differential Cost Distribution (G – S)

Table 5.11 also summarizes the detailed PW distribution statistics for both replacement policies. Even though we do not present a 95% confidence interval for this case, it is shown that the confidence interval for $\Delta\theta$ is again totally to the left of zero, or $\theta_1 < \theta_2$.

	95 Per.	+1 Std. Dev.	Mean	-1 Std. Dev.	5 Per.
Group	\$141,503	\$129,917	\$110,609	\$91,302	\$82,001
Staggered	\$151,472	\$137,647	\$116,795	\$95,943	\$90,309
G-S	-\$3,950	-\$4,614	-\$6,206	-\$7,798	-\$9,241

Table 5.11 Summary of the PW Cost Distribution Statistics

What we have shown in this chapter is the process of incorporating the uncertainty associated with key input parameters through @RISK. Depending on the input data assumed, we may not rule out the possibility of having a situation where the Staggered replacement policy could be more cost effective.

CHAPTER 6. SUMMARY AND CONCLUSION

In this research, we have studied replacement decision for a single-fleet replacement problem according to two different policies. We have shown that there are some situations in which either the Group or Staggered replacement policies could be considered under certain circumstances. However, since the results depend on many variables, such as the purchase cost, interest rate, and operating cost etc., these two policies can only be compared numerically for a particular situation.

Chapter 2 introduced the previous work on replacement problems. Many researchers have studied Group replacement; we classified three types as follows: 1) the T-age policy, 2) the m-failure policy, and 3) the (m, T) policy. However, Staggered replacement has not been clearly defined yet in existing literature.

Chapter 3 constructed the basic mathematical models for Group and Staggered replacement policies without technological progress in assets and gave numerical examples to demonstrate our basic models. We focused on Group replacement and Staggered replacement under similar conditions. The results showed that the Group replacement policy is more cost effective when compared with the Staggered replacement policy.

Chapter 4 developed the mathematical models for each replacement policy under ongoing technological progress and compared the result of two policies (Group vs.

Staggered) with the same numerical example as that in Chapter 3. The Group replacement policy is still preferable to the Staggered replacement policy under ongoing technological progress.

Chapter 5 simulated our model under technological progress using @Risk, which is risk analysis software, to consider uncertainty in future costs. To consider the uncertain conditions, we presented the results of simulations we did with @Risk. The results of the simulations verified our mathematical models. Furthermore, they showed the probability of different results under the uncertain conditions. The results of comparing the two policies using identical numerical examples were as follows. First, we can calculate the differential costs between Group and Staggered replacement using the net present value method. The differential costs implied the benefit or loss when we chose Group replacement policy instead of Staggered replacement policy. Second, the sensitivity analysis explained behaviors of each variable to show which variable is more important. Our result showed that Group replacement is more cost effective than Staggered replacement in our example. However, we should not conclude that Group replacement is always superior to the Staggered replacement policy. The effectiveness of each replacement policy changes according to the variables.

Our contribution is that we developed the replacement decision procedure or application to find the optimal replacement policy in a single-fleet replacement problem. We focused on comparing the Group replacement and Staggered replacement policies. First, we constructed mathematical models which apply to both Group and Staggered replacement. Second, our procedure determined the optimal model under technological improvement conditions. Third, we used @ Risk software to cover uncertain conditions in our model. The advantage of these models is that they are applicable to general situations.

Even though our work has been exhaustive, there is still further research to conduct. In our research, we assumed that a new asset would be the same quality as the asset it replaces, but due to technological improvement, the price of the asset would decrease. However, if we assume the same purchase cost for new assets, the outcome may change. When we consider replacement, we replace with more efficient assets instead of cheaper assets because we can purchase better assets for the same price as we paid for the older assets. Therefore, it is desirable to extend our mathematical models to consider these variations.

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APPENDIX 1. OUTPUT DATA OF UNCERTAIN PARAMETER *P*

	PW(G) / 40	PW(S) / 40		PW(G) / 40	PW(S) / 40
	Output (Sim#1)	Output (Sim#1)		Output (Sim#1)	Output (Sim#1)
1	$ 106,130.14	$ 112,080.29	51	$ 104,280.47	$ 110,038.97
2	$ 109,836.09	$ 116,170.23	52	$ 112,808.57	$ 119,450.70
3	$ 107,127.55	$ 113,181.05	53	$ 108,482.17	$ 114,676.02
4	$ 109,329.67	$ 115,611.34	54	$ 107,295.99	$ 113,366.94
5	$ 104,530.99	$ 110,315.45	55	$ 102,688.95	$ 108,282.55
6	$ 103,449.36	$ 109,121.75	56	$ 106,191.35	$ 112,147.85
7	$ 101,856.18	$ 107,363.49	57	$ 106,339.20	$ 112,311.02
8	$ 111,308.37	$ 117,795.06	58	$ 103,346.30	$ 109,008.01
9	$ 106,739.82	$ 112,753.14	59	$ 110,154.30	$ 116,521.41
10	$ 109,097.04	$ 115,354.61	60	$ 112,079.87	$ 118,646.50
11	$ 111,048.03	$ 117,507.75	61	$ 104,175.39	$ 109,923.00
12	$ 102,959.99	$ 108,581.67	62	$ 107,667.24	$ 113,776.66
13	$ 103,769.80	$ 109,475.39	63	$ 104,787.64	$ 110,598.70
14	$ 105,316.65	$ 111,182.51	64	$ 105,721.97	$ 111,629.83
15	$ 101,045.18	$ 106,468.46	65	$ 106,609.54	$ 112,609.37
16	$ 105,929.06	$ 111,858.38	66	$ 101,544.98	$ 107,020.04

17	$	105,996.25	$	111,932.53	67	$	109,901.62	$	116,242.55
18	$	111,437.38	$	117,937.44	68	$	112,293.14	$	118,881.87
19	$	108,320.14	$	114,497.20	69	$	110,943.34	$	117,392.21
20	$	110,483.95	$	116,885.22	70	$	102,048.58	$	107,575.82
21	$	103,887.31	$	109,605.08	71	$	113,501.75	$	120,215.71
22	$	104,950.64	$	110,778.58	72	$	100,382.90	$	105,737.56
23	$	111,814.20	$	118,353.30	73	$	104,608.70	$	110,401.21
24	$	108,662.23	$	114,874.74	74	$	104,379.44	$	110,148.20
25	$	105,477.95	$	111,360.53	75	$	106,481.13	$	112,467.65
26	$	105,570.57	$	111,462.74	76	$	105,170.91	$	111,021.68
27	$	106,941.96	$	112,976.23	77	$	103,492.40	$	109,169.25
28	$	106,376.20	$	112,351.84	78	$	105,229.27	$	111,086.08
29	$	111,575.16	$	118,089.49	79	$	110,727.98	$	117,154.54
30	$	103,050.34	$	108,681.38	80	$	108,147.85	$	114,307.06
31	$	105,599.37	$	111,494.53	81	$	108,013.05	$	114,158.30
32	$	102,509.45	$	108,084.45	82	$	105,017.60	$	110,852.48
33	$	107,757.85	$	113,876.65	83	$	110,222.42	$	116,596.59
34	$	100,576.20	$	105,950.89	84	$	102,116.32	$	107,650.59
35	$	109,451.49	$	115,745.78	85	$	108,765.01	$	114,988.17
36	$	107,106.70	$	113,158.04	86	$	107,391.27	$	113,472.09
37	$	107,810.89	$	113,935.19	87	$	101,285.34	$	106,733.50
38	$	104,147.36	$	109,892.07	88	$	107,920.40	$	114,056.05

39	$	102,774.15	$	108,376.58	89	$	107,419.87	$	113,503.66
40	$	106,671.13	$	112,677.34	90	$	110,443.61	$	116,840.69
41	$	107,555.69	$	113,653.55	91	$	109,581.57	$	115,889.33
42	$	101,592.98	$	107,073.02	92	$	103,957.18	$	109,682.19
43	$	108,231.28	$	114,399.14	93	$	110,777.66	$	117,209.37
44	$	103,664.96	$	109,359.69	94	$	105,785.52	$	111,699.97
45	$	109,692.37	$	116,011.62	95	$	102,297.42	$	107,850.45
46	$	103,209.63	$	108,857.18	96	$	109,271.44	$	115,547.08
47	$	104,858.53	$	110,676.93	97	$	112,509.32	$	119,120.45
48	$	108,807.56	$	115,035.13	98	$	106,874.83	$	112,902.15
49	$	108,364.95	$	114,546.66	99	$	108,990.14	$	115,236.63
50	$	109,983.43	$	116,332.84	100	$	108,930.12	$	115,170.39

APPENDIX 2. OUTPUT DATA OF ALL UNCERTAIN PARAMETER

	PV(G) / 40	PV(S) / 40		PV(G) / 41	PV(S) / 41
	Output (Sim#1)	Output (Sim#1)		Output (Sim#1)	Output (Sim#1)
1	$ 100,437.92	$ 105,697.27	51	$ 105,419.91	$ 111,473.01
2	$ 110,308.93	$ 114,319.82	52	$ 88,179.42	$ 91,995.84
3	$ 117,509.36	$ 124,687.72	53	$ 98,543.37	$ 103,230.45
4	$ 100,050.20	$ 105,441.65	54	$ 88,713.76	$ 93,169.02
5	$ 136,891.19	$ 144,267.52	55	$ 123,247.54	$ 130,431.55
6	$ 108,212.90	$ 115,019.70	56	$ 108,056.36	$ 114,117.52
7	$ 118,760.88	$ 126,310.09	57	$ 132,849.87	$ 140,897.45
8	$ 106,595.35	$ 112,320.85	58	$ 94,500.93	$ 99,660.81
9	$ 125,338.52	$ 133,677.42	59	$ 101,645.33	$ 106,429.35
10	$ 110,964.35	$ 116,810.79	60	$ 112,045.10	$ 118,965.23
11	$ 108,915.30	$ 114,826.60	61	$ 122,625.81	$ 129,779.46
12	$ 99,208.74	$ 104,153.13	62	$ 127,197.21	$ 133,968.56
13	$ 97,018.04	$ 102,903.97	63	$ 114,791.30	$ 121,783.85
14	$ 136,965.27	$ 145,521.00	64	$ 105,320.34	$ 111,695.07
15	$ 121,195.14	$ 128,140.50	65	$ 140,867.07	$ 148,658.57
16	$ 117,062.60	$ 123,333.52	66	$ 91,920.79	$ 96,130.13

17	$ 155,993.24	$ 165,958.98	67	$ 112,742.38	$ 118,831.61
18	$ 81,327.30	$ 86,051.07	68	$ 103,361.44	$ 109,240.00
19	$ 114,706.68	$ 120,093.72	69	$ 105,911.34	$ 111,768.75
20	$ 128,981.98	$ 138,334.97	70	$ 85,221.55	$ 88,801.78
21	$ 105,716.28	$ 111,035.73	71	$ 121,879.97	$ 128,267.89
22	$ 94,881.33	$ 100,094.94	72	$ 109,733.83	$ 116,484.86
23	$ 100,208.94	$ 104,634.78	73	$ 100,832.74	$ 106,289.44
24	$ 112,651.31	$ 118,278.75	74	$ 114,361.19	$ 122,287.89
25	$ 102,275.10	$ 108,711.41	75	$ 87,365.41	$ 92,155.34
26	$ 91,450.06	$ 96,830.37	76	$ 105,231.85	$ 111,008.52
27	$ 92,858.02	$ 98,077.04	77	$ 120,845.66	$ 127,759.29
28	$ 104,153.15	$ 110,210.33	78	$ 113,186.56	$ 120,678.82
29	$ 92,168.70	$ 96,021.83	79	$ 90,981.11	$ 96,396.92
30	$ 100,853.50	$ 106,588.40	80	$ 116,457.31	$ 122,263.32
31	$ 146,193.64	$ 156,938.91	81	$ 97,792.95	$ 104,040.67
32	$ 134,145.76	$ 141,050.24	82	$ 150,875.66	$ 160,308.65
33	$ 108,593.68	$ 113,649.07	83	$ 95,502.96	$ 99,950.42
34	$ 129,575.38	$ 137,630.60	84	$ 109,970.54	$ 115,995.34
35	$ 91,852.57	$ 96,156.64	85	$ 113,944.04	$ 120,137.67
36	$ 122,843.63	$ 129,734.76	86	$ 105,405.03	$ 112,043.30
37	$ 104,300.17	$ 110,277.76	87	$ 104,309.79	$ 110,298.34
38	$ 125,903.35	$ 132,166.46	88	$ 113,800.26	$ 120,721.69

39	$ 127,492.49	$ 137,053.52	89	$ 107,941.43	$ 114,354.09
40	$ 121,474.40	$ 128,319.30	90	$ 83,675.14	$ 87,356.31
41	$ 109,346.66	$ 116,381.64	91	$ 105,480.69	$ 110,696.62
42	$ 116,710.60	$ 123,339.08	92	$ 135,655.09	$ 143,963.75
43	$ 119,266.26	$ 125,804.31	93	$ 158,959.32	$ 168,965.74
44	$ 117,134.14	$ 122,779.34	94	$ 107,227.34	$ 113,990.13
45	$ 116,822.39	$ 123,478.05	95	$ 108,993.23	$ 112,798.02
46	$ 110,512.12	$ 116,783.32	96	$ 94,903.85	$ 100,156.10
47	$ 115,543.77	$ 122,730.18	97	$ 91,282.77	$ 95,484.36
48	$ 97,359.81	$ 102,425.42	98	$ 108,795.59	$ 114,424.30
49	$ 101,990.00	$ 106,987.14	99	$ 94,613.30	$ 99,818.73
50	$ 119,894.64	$ 126,263.96	100	$ 96,095.12	$ 101,541.25