Aisle Designs for Unit-Load Warehouses with

Multiple Pickup and Deposit Points

Except where reference is made to the work of others, the work described in this thesis is my own or was done in collaboration with my advisory committee. This thesis does not include proprietary or classified information.

Goran Ivanović

Certificate of Approval:

Russell D. Meller
Professor
Industrial Engineering
University of Arkansas

Kevin R. Gue, Chair
Associate Professor
Industrial and Systems Engineering

Emmett J. Lodree, Jr.
Assistant Professor
Industrial and Systems Engineering

George T. Flowers
Interim Dean, Graduate School

Aisle Designs for Unit-Load Warehouses with Multiple Pickup and Deposit Points

Goran Ivanović

A Thesis

Submitted to

the Graduate Faculty of

Auburn University

in Partial Fulfillment of the

Requirements for the

Degree of

Master of Science

Auburn, Alabama
December 17, 2007

Aisle Designs for Unit-Load Warehouses with

Multiple Pickup and Deposit Points

Goran Ivanović

Signature of Author

Date of Graduation

Vita

Goran Ivanović, son of Stoiša and Milanka Ivanović, was born on April 9th, 1979, in Ćuprija, Serbia. Soon after, his family relocated to a nearby town of Jagodina, where he spent his childhood and teenage years. He studied at University of Belgrade, School of Traffic and Transportation Engineering, and graduated from Department of Logistics in June 2004. Prior to entering Auburn University in May 2006, he worked as a logistics consultant.

Thesis Abstract

Aisle Designs for Unit-Load Warehouses with Multiple Pickup and Deposit Points

Goran Ivanović

Master of Science, December 17, 2007
(B.S., University of Belgrade, 2004)

77 Typed Pages

Directed by Kevin Gue

This thesis focuses on unit load warehouses having multiple pickup and deposit (P&D) locations on the warehouse dock(s) — i.e., a P&D region. Our aim is to design a cross aisle structure that will reduce the cost of operating a unit load warehouse. We concentrate on the labor cost of material handling, or more specifically, on the travel cost associated with unit load putaway and retrieval. We define two "candidate shapes." Namely, the Flying-V shape and Λ shape. We state a nonlinear optimization problem to find an improved cross aisle configuration by minimizing the expected cost of a single pickup and retrieval task. We separately carry out optimization for each candidate shape and choose the more economical one. Our results are a sort of worst case analysis for the designs we consider and the benefits from our solutions range typically from 3% to 6.5%.

The second part of this thesis is concerned with applications of the same design principles to floor storage. A less rigorous, but effective approach is developed. We devise a number of aisle and lane designs and apply them to a realistic setting.

Acknowledgments

Serbian saying goes: "Man without a woman is like a rotten tree stump by the side of the road." I thank my wife Anica, for all her love, trust and patience.

If I try to write a "real acknowledgement" to my adviser, Dr. Gue, it might end up either being too formal, or too saccharine. I am most grateful for his help and support.

Style manual or journal used Latex: A Document Preparation System : User's Guide and Reference Manual, by Leslie Lamport (together with the style known as "aums").

Computer software used The document preparation package TEX (specifically LATEX) together with the departmental style-file `aums.sty`.

Table of Contents

List of Figures

Chapter 1

Aisle Designs for Unit Load Warehouses with Selective Pallet Racks

Very rarely is it possible to transport, sell, consume or utilize goods immediately after they have been produced. Instead, often times it is necessary to store the goods temporarily and wait for the right moment to introduce them for consumption or utilization. This interruption of the natural product flow through the supply chain usually appears to be a result of interaction of numerous factors. Production, logistics and distribution systems are becoming more and more complex, and at an unprecedented rate. Even the operations taking place in basic building blocks of logistics systems, such as warehouses, have been raised to a new level by numerous technological advances over the course of the last two decades. Notwithstanding, RFID, sophisticated order-picking systems, warehouse management systems, on-board AGV computers and other technological novelties, have not changed the need to put a pallet on the floor (or in a rack) and let it dwell there for some time. Apart from the technological changes warehouses have undergone, there is a constantly increasing pressure from both upstream and downstream supply chain structures to adopt numerous value-added activities: final product assembly, thermal processing, labeling and relabeling, etc. However, value-added activities do not change the fundamentally unproductive nature of the warehouse rationale — they only make opportune use of the unfavorable environment, that is a warehouse. That being the case, we can freely say that reasons to have a warehouse have remained unchanged for decades, if not centuries.

Due to physical, chemical or nutritional properties of the product it is often desirable or inevitable to store and therefore preserve the product for future use. Examples may include various perishable goods, such as agricultural and pharmaceutical products. Secondly, timing and mismatch between production and consumption processes dictate that commodities be stored. This is almost mandatory for goods with highly pronounced seasonal production or consumption characteristics. The former are impossible to sell right after being finalized and have to be stored since they are usually consumed at a slower rate. As for the latter, it is impossible to meet seasonal peaks of demand without keeping significant quantity of a product in stock. At last, uncertainty in demand or supply usually leads to buildup of a so-called safety stock. This is "just-in-case" inventory, held in order to be able to respond to unexpected changes in demand or sudden interruptions in supply. It is usually caused by the inability to forecast market caprices and uncertainty inherent in the outcome of even the most carefully planned human activities.[1]

These are all but not the only reasons to store goods in a warehouse. Nevertheless, they are sufficient to sketch the most important characteristics of warehousing: Usually one does not garner and store because he wants, but because he must. Hence, warehousing is in essence an unproductive endeavor. Modern times might have changed the technology of, but not the rationale for warehousing. In addition, the last two decades have witnessed several strategies in industry that attempt to reduce the amount of the unproductive inventory. In the realm of warehousing, crossdocking is the most prominent one. By means of coordination

[1] Note that the order in which we present motives to store the goods is very likely to correspond chronologically to the evolution of warehousing. In the ancient times people stored goods to protect them from decay. With development of markets and trading it became desirable to store, say in the Fall, in order to trade in the Spring. At last, with emergence of the first supply chains (e.g., silk road) it became prudent for the traders to protect themselves from the highly risky "supply chain failures" of those times.

and aggregation, crossdocking aims to cut the transportation and inventory holding cost of a traditional supply chain.

Prior to the last two decades of the Twentieth Century, warehousing costs have not raised significant concern with the industry. Since then however, decreasing profit margins of western economies have imposed a strong incentive to deal with many of the costs that were previously considered peripheral. With time, this became the case with a large class of warehousing and material handling problems. Consequently, we believe attention is being given to warehouse design issues that would have until recently been regarded as unimportant.

1.1 Unit load warehouse and aisle design in practice and literature

The main purpose of this work is to explore the opportunities for reduction of operational costs in a widely encountered class of warehouses. More specifically, we propose and evaluate a number of alternative cross aisle designs for unit load warehouses. Our analysis focuses on warehouses having multiple pickup and deposit (P&D) locations on the warehouse dock(s) — i.e., a P&D region. The motivation for this thesis stems from Gue and Meller (2006), in which they explore a similar problem with only one P&D point.

Unit load storage is characterized by homogenous dimensions of the stored skus. A typical unit of handling and storage is a pallet. Storage equipment commonly includes one-deep and two-deep selective pallet racks, and the most common material handling device is the counterbalance forklift truck. This type of warehouse is usually considered to be the simplest, because the cost of operation is straightforwardly described by the overall cost of stowing, keeping and retrieving the pallets. The unit load warehouse takes on a number of forms depending on the logistics system it is supporting. Physically, it can be a stand-alone facility or a part of a larger warehouse that houses other forms of storage in addition to the unit load module. Functionally, the unit load warehouse serves as a finished goods warehouse, a 3$^{\text{rd}}$ party transshipment warehouse, or a distribution center (DC).

Theoretical work on unit load storage design and operations can be traced back to the 1960's. In fact, Roberts and Reed (1972) claim that Francis (1967) was the first to address warehouse design analytically. Traditional unit load design with orthogonal aisle and rack structure has been discussed by Francis (1967). This work gives sufficient conditions for optimal facility layout, under certain assumptions. Roberts and Reed (1972), develop a comprehensive cost scheme for warehouse optimization. Their approach includes both operational and construction cost of employing a certain bay structure. Yet, they only describe one P&D point configurations with orthogonal rack and aisle structure. It is suggested that their model can be used for warehouses with a P&D region, provided that individual P&D points serve a dedicated set of racks and aisles.

First discussions on non-orthogonal aisle and rack layout come from Berry (1968) and White (1972). Berry considered floor storage in particular, rather than unit load warehouses in general. He devised a design with a diagonal aisle structure and showed that layouts which minimize travel distance and maximize space utilization are different ones. White examined radial aisle designs with a single P&D point dock, but neglected the issue of radial aisle width. Also, picking aisles needed to access items deeper in storage space, i.e., ones not directly accessible from the radial aisles, are not modeled.

Finally, Gue and Meller (2006) address the aisle design problem from a different perspective. They utilize a prescriptive modeling approach and formulate a non-linear optimization function to find an improved non-orthogonal aisle structure for a single P&D point problem. In doing so, they relax two "unspoken design rules:"

1. Picking aisles must be straight, and they must be parallel.

2. If present, cross aisles must be straight, and they must meet picking aisles at right angles.

The authors provide solutions for a range of warehouse sizes and compare them with traditional, orthogonal designs.

The challenge we are facing is more demanding, since Gue and Meller only deal with a single P&D point. Geometry and operational procedures of P&D regions suggest different shapes and more complicated travel patterns than those of single P&D point. We also explore the potential for redesign of unit-load warehouses and DC's by introducing a non-standard cross aisle and thus relaxing the second "unspoken rule." We essentially generalize the initial single P&D point model to provide solution for a more complicated case. In addition, we address some practical aspects of new aisle designs.

1.2 Model assumptions

The P&D region appears in variety of forms across a number of warehouses and DC's we have visited. In our model (Figure 1.1), n P&D points are equally spaced along the "bottom" of the warehouse. We assume n to be an odd number. It is important to note that n equals both the number of picking aisles and the number of doors along the bottom of the warehouse rectangle. Picking aisles are orthogonal to the bottom of the warehouse. For

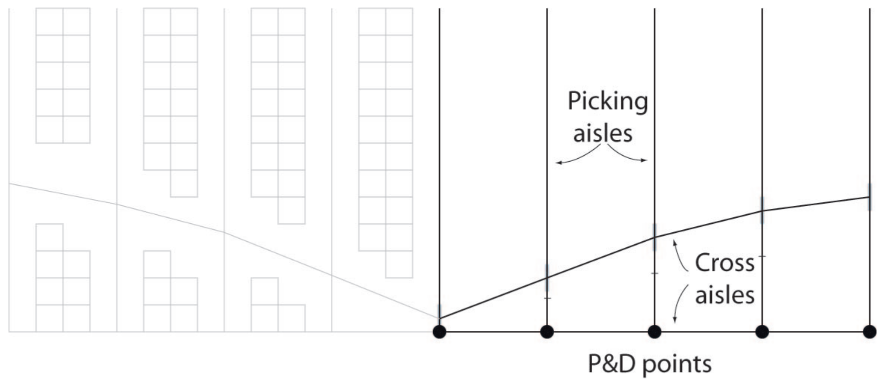

Figure 1.1: Model assumptions — warehouse layout with a cross aisle

the purpose of this work we define “bottom” as the side of the rectangle with the shipping dock. Strictly speaking, this is adequate only for those facilities where the entire volume of shipping and receiving is concentrated along a single side of the warehouse rectangle. In all other cases, for ease of representation, we would rotate the rectangle so that shipping assumes the “southern,” bottom part.

Besides equal spacing of dock doors, which is common in practice, we assume that incoming and outgoing trailers choose among dock doors with equal probabilities. This is not likely to be the case in real life since centrally located doors are closer to more good storage locations than doors farther from the center (see Bartholdi and Hackman, 2007, chap. 6). The reason for taking this approach is that trailer assignment can be considered a form of optimization, and we wish to investigate only the impact of aisle structure on travel cost. As we will show, our results are a sort of worst case analysis for the designs we consider.

The storage policy is random, which means that the probability of making a pick (or a stow) from (to) a randomly chosen storage location is equal to the probability of making a pick from any other storage location. In other words, picking is uniformly distributed inside the storage space. This assumption greatly simplifies the mathematical model. Similar to the case with trailer assignment, more elaborate analysis of storage policies would lead to loss of precision in judging the effect of new aisle structure on travel costs. It should be recognized that few warehouses strictly employ random or dedicated storage. It is more common to assign one storage policy to portions of a warehouse. Also, changes in storage policy can be a matter of adaptation and evolution. According to some managers we spoke with, random storage policy is common in the incipient stages of warehouse life. After a "warm up" period, material flow reaches "steady state" and it becomes much easier to decide upon the right storage policy.

Forklift drivers utilize P&D points as bases for their travels into the storage space and back. All of our models assume a single command cycle. For incoming trailers this means that a forklift unloads the pallet from the trailer, travels to a random part of storage area, unloads the pallet into an empty rack cell and travels back to the same P&D point. For outgoing trailers the process is reversed: the forklift travels to the randomly chosen storage location, loads the pallet, travels back to the P&D point and unloads the pallet into a trailer. An alternative is the dual command cycle which reduces deadheading but brings in complications in forklift fleet supervision and guidance.

Our aim is to design an aisle structure that will reduce the cost of operating a unit load warehouse. We concentrate on the labor cost of material handling, or more specifically, on the travel cost associated with unit load putaway and retrieval. This cost is defined as the average distance traveled to make a pick or a stow. Because of the single command cycle, the same distances are traveled into the storage space and out of it, which enables us to consider only the half of the actual distance crossed. In our case, that is going to be the travel into the storage space, for both activities.

Unit load length is the adopted measure of distance. For easier representation, our units have a square-shaped footprint. All activities in a warehouse revolve around the unit load and we found that adopting it as a basic measure provides easier description.

1.3 Upper and lower bounds for a single pickup and retrieval travel task

Traditional orthogonal aisle and rack structures impose rectilinear travel patterns. Travel distances resulting from these patterns are one way to describe the cost of making a stow or a pick in such a warehouse. Gue and Meller (2006) claim that it is often inefficient to use rectilinear travel patterns and that such travels can be made more economical.

Consider the following example: Two picks need to be made from a P&D point located in the central bottom part of the warehouse to two storage locations in the left warehouse half-space. Figure 1.2 depicts two possible travel patterns available to a picker. Dashed lines represent the first — rectilinear, whereas continuous lines connect the P&D point and picking locations in a straight fashion — with a Travel-by-flight path. In the Euclidean geometry, a line connecting two points represents the shortest distance between them. Therefore, minimum travel cost for any single pick up and retrieval task is achieved if the travel path to the pick location is a straight line. Such is the case of a hypothetical warehouse in which an order picker could fly directly to the desired pick location. While travel-by-flight design is implausible, the expected distance to make a pick remains an ideal lower bound for the cost of a single pickup and retrieval task. Traditional rectilinear travel is the upper bound for the cost of a single pickup and retrieval travel task.

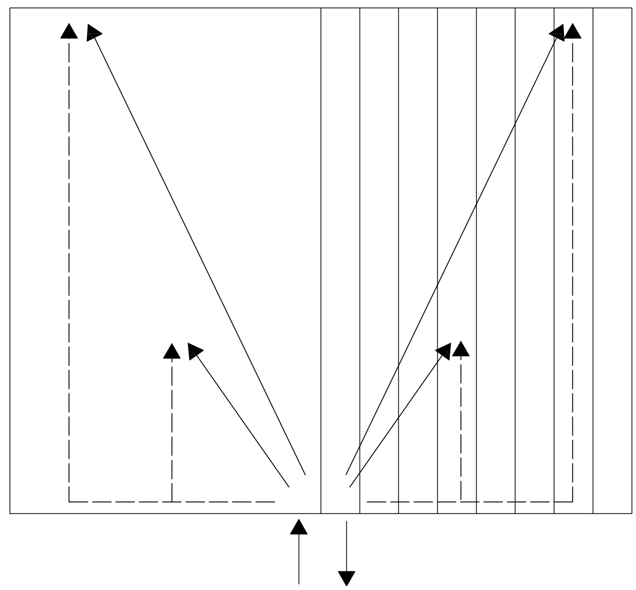

Figure 1.2: Continuous vs. discrete upper and lower travel bound

Both upper and lower bounds can be modeled as discrete or continuous. In the previous example we concentrated on the left warehouse half-space which does not display any aisle, rack or lane structure at all, while right half-space features a discrete, aisle-like layout. Discrete modeling assumes picking activity in a clearly defined number of racks only, and is modeled through sums of costs. Because there is no rack or aisle division, a continuous model theoretically allows for picking of pallets with infinitesimally small dimensions, where travel cost is modeled as an integral. Despite the apparent formal difference, our experiments show that bounds perform almost identically.

1.3.1 Continuous and discrete upper bounds — Rectilinear travel

Upper bounds result from travel paths where a picker is first moving along the bottom of the rectangle and then up along the picking aisle until he reaches the desired pick location. This pattern forms a rectilinear travel and holds for both continuous and discrete bound.

For a continuous bound, let A denote the width and let h be the height of a given warehouse. If x_1 is the x coordinate of a randomly chosen P&D point, and x_2 and y_2 are the coordinates of a randomly chosen point in the storage space, we express the average cost of a rectilinear single pickup or deposit travel as:

$$\begin{aligned} U_c &= \frac{1}{A^2 h} \int_0^h \int_0^A \int_0^A (|x_1 - x_2| + y_2) dx_1 dx_2 dy_2 \\ &= \frac{A}{3} + \frac{h}{2}. \end{aligned}$$

For a discrete bound, let n and i be the total number of picking aisles in a warehouse, and the index of a randomly chosen P&D point, respectively. Let a be the horizontal distance between two consecutive picking aisles. We can express the average cost of a single pickup or deposit travel as:

$$\begin{aligned} U_d = \frac{1}{n} \Bigg[&\sum_{i=2}^{n} \sum_{j=1}^{i-1} \frac{1}{nh} \int_0^h \Big[(i-j)a + x \Big] dx + \frac{1}{nh} \sum_{i=1}^{n} \int_0^h x dx + \\ + &\sum_{i=1}^{n-1} \sum_{k=i+1}^{n} \frac{1}{nh} \int_0^h \Big[(k-i)a + x \Big] dx \Bigg] \\ = &\frac{a(n^2-1)}{3n} + \frac{h}{2} \ . \end{aligned}$$

The two expressions have similar structure. Knowing that for the length of the warehouse rectangle we have that $A = a/2 + (n-1)a + a/2 = na$, let us hypothesize a discrete warehouse with an infinite number of picking aisles. In that case we obtain:

$$\lim_{n \to \infty} \frac{a(n^2-1)}{3n} = \frac{A}{3}.$$

So, for an infinitely wide discrete warehouse, discretization of picking space is irrelevant. Our tests show that for realistic warehouse sizes this inaccuracy is less than one tenth of a percent.

1.3.2 Continuous and discrete lower bounds — Travel-by-flight

Lower travel bounds arise from a straight travel-by-flight pick. The average cost of such a pick is the expected distance from a randomly chosen point on the bottom side of the rectangle to a randomly chosen point somewhere in the storage space. Using the same notation we obtain both bounds:

$$
\begin{aligned}
L_c &= \frac{1}{A^2 h} \int_0^h \int_0^A \int_0^A \sqrt{(x_1 - x_2)^2 + {y_2}^2}\; dx_1 dx_2 dy_2 \\
&= \frac{1}{12 A^2 h} \Bigg[2h^4 + 3A^2 h \sqrt{A^2 + h^2} - 2h^3 \sqrt{A^2 + h^2} - \\
&\quad - 2Ah^3 \ln(-A + \sqrt{A^2 + h^2}) + 2Ah^3 \ln(A + \sqrt{A^2 + h^2}) - \\
&\quad - A^4 \ln \frac{A}{h + \sqrt{A^2 + h^2}} \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
L_d = \frac{1}{n} \Bigg[& \sum_{i=2}^{n} \sum_{j=1}^{i-1} \frac{1}{nh} \int_0^h \sqrt{a^2 (i-j)^2 + x^2} dx + \frac{1}{nh} \sum_{i=1}^{n} \int_0^h x dx + \\
& + \sum_{i=1}^{n-1} \sum_{k=i+1}^{n} \frac{1}{nh} \int_0^h \sqrt{a^2 (k-i)^2 + x^2} dx \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
= \frac{1}{2n^2h}\Bigg[& \sum_{i=2}^{n}\sum_{j=1}^{i-1}(h\sqrt{h^2+a^2(i-j)^2}+a^2(i-j)^2\ln(h+\sqrt{h^2+a^2(i-j)^2})- \\
& - a^2(i-j)^2\ln(a(i-j))) + \sum_{i=1}^{n} h^2 + \\
& + \sum_{i=1}^{n-1}\sum_{k=i+1}^{n}(h\sqrt{h^2+a^2(k-i)^2}+a^2(k-i)^2\ln(h+\sqrt{h^2+a^2(k-i)^2})- \\
& - a^2(k-i)^2\ln(a(k-i)))\Bigg] .
\end{aligned}
$$

A similar but more complicated problem is faced by electrical engineers and power network designers. The expected wire distance between two randomly chosen terminals in a rectangle is equal to the problem of travel-by-flight distance between two randomly chosen storage locations inside a warehouse. This problem is more complicated than ours in that the y-coordinate of the first terminal point is randomized, as opposed to the y-coordinate of a P&D point always being zero. This seemingly trivial added complexity produces a very difficult quadruple integral whose closed form solution was only obtained in 1994 by Lazoff and Sherman.

1.4 The models

We have approximated the cost of operating a warehouse in terms of expected distance to make a pick or a stow. Hence, our goal is to design a rack and aisle structure that minimizes this cost. This can be achieved in two ways:

1. Choose a number of cross aisle shapes. For each shape, route pickers in the most efficient way, and then choose the most economical shape.

2. Define a number of sets of routing rules. For each set of rules, let the model choose the aisle structure, and then choose the most economical shape.

For the designs we consider, there is a one-one correspondence between a set of routing rules and a cross aisle shape. At this point in our discussion, we opt for the first approach. However, after laying the ground and providing some insight into more intuitive issues of the aisle designs, we explain the routing rules paradigm in a separate section.

To begin, we define two "candidate shapes." Namely, the Flying-V shape and Λ shape (Figures 1.3 and 1.4). Since we choose warehouses with an odd number of picking aisles, we also assume symmetry about the central picking aisle. Based on geometrical features of the shapes, we define the most efficient routing of pickers. Subsequently, for a given warehouse, defined by its design parameters — number of aisles (n), height of a warehouse (h), width of picking ($a-2$) and cross ($2w$) aisles — we state a nonlinear optimization problem to find an improved cross aisle configuration by minimizing the expected cost of a single pickup and retrieval task. We separately carry out optimization for each candidate shape and choose the more economical one. We solve the optimization problems with MATHEMATICA.

1.4.1 Flying-V cross aisle

We examine the Flying-V shape first, depicted in Figure 1.3. Because we assume the cross aisle is symmetric about the center aisle, we consider only the right half of the warehouse. We number picking aisles in the right half-space starting from the center and ending at the rightmost aisle. Thus, $i = 0, \ldots, m$, where $m = (n-1)/2$. We describe a V-shaped cross aisle as a vector $\overrightarrow{b} = \{b_0, b_1, \ldots, b_m\}$. Every point b_i is a y coordinate of the intersection of the cross aisle and the picking aisle i. In order to provide pickers with sufficient maneuvering space we require the cross aisle to have a width of $2w$ at the intersection points. That is, below and above the b_i point, the cross aisle consumes w units of storage space. A more realistic alternative, where one would define a constant cross-aisle width independent from the intersection points, would lead to a more complicated mathematical model. We believe that possible increase in precision would not be sufficient to justify the added complication. For the same reason, Gue and Meller (2006) refer to the $2w$ convention as a "judicious modeling choice."

Introducing the cross aisle penalty width decreases available storage space. Therefore, any possible travel savings from the insertion of a cross aisle comes at the cost of this wasted space. To illustrate (Figure 1.3), assume that a pick is being made from the P&D point i into the picking aisle i. In other words, the travel path is a simple straight line. If a cross aisle is present, every time the pick is above it, there will be an increase of $2w$ in the travel distance.[2]

We assume that P&D points are located exactly at the bottoms of the picking aisles (Figure 1.3). This is an idealization, because in reality, they would be w units below, in keeping with the bottom cross aisle. The effect of our assumption is a slight overstatement of the estimated improvements.

Next, we define the cost structure of a V-shaped cross aisle. We divide the picking space according to appropriate travel patterns (Figure 1.3). We define the following notation

- $E[R_i]$ is the expected cost of making a pick to the ***right*** of aisle i;
- $E[S_i]$ is the expected cost of making a pick in aisle i;

[2]If, however, there is no cross aisle, all the picks that were previously above it will be $2w$ units closer, which will result in a more economic travel. For some warehouses with smaller h this is the reason why rectilinear travel, and therefore the upper travel limit too, might perform better than any cross-aisle-based improvement.

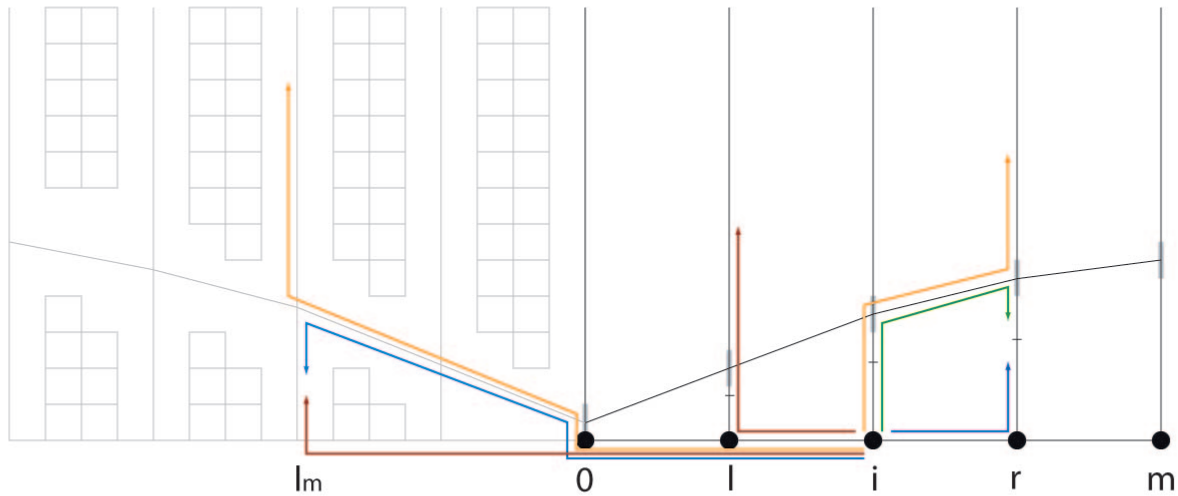

Figure 1.3: Flying-V shape - scheme of travel patterns and related picking regions

- $E[L_i]$ is the expected cost of making a pick in an aisle located ***left*** of aisle i, but no farther than the center picking aisle; and

- $E[L_i^{most}]$ is the expected cost of making a pick in an aisle to the left of aisle i, from the first aisle after the center aisle to the ***leftmost*** aisle.

Now, we express the expected cost of picking from a randomly chosen P&D point i located right from the central picking aisle, and denote it by $E[C_i]$:

$$E[C_i] = p_{R_i} E[R_i] + p_{S_i} E[S_i] + p_{L_i} E[L_i] + p_{M_i} E[L_i^{most}],$$

where $p_{R_i} = (m-i)/n, p_{S_i} = 1/n, p_{L_i} = i/n$, and $p_{M_i} = m/n$ are the probabilities of making a pick in the respective regions.

The total expected picking cost for all P&D points $i = 0, \ldots, n$ is:

$$E[C] = \sum_{i=0}^{n} p_i E[C_i],$$

where $p_i = 1/n$ is the probability of choosing a random P&D point. Because of the symmetry we assume, the total picking cost becomes:

$$E[C] = p_0 E[C_0] + 2\sum_{i=1}^{m} p_i E[C_i].$$

The cost structure above conveys basic information about the regions of a warehouse and their relationship to the picking cost. To be able to derive detailed mathematical expressions for all of the above cost components, we need to learn more about the routing of workers and its relationship with mathematical formulation of the picking cost. In that sense, especially important is the case of $E[R_i]$. Consider aisles i and r, such that $i < r$, and assume that a pick is being made from P&D point i into a random picking point in the aisle r. Depending on the location of the random pick in the aisle r, the picker should take different routes to access the pick location. If the desired pick is above the cross aisle, the picker will travel straight ahead in aisle i, along the cross aisle, and then up aisle r — the upper path. If the pick is somewhere between the bottom of the aisle r and the intersection of the aisle r with the cross aisle $(b_r - w)$, there are two possible routes from which to chose: In the part closer to the $b_r - w$, straight ahead the aisle i, along the cross aisle and down the aisle r — the middle path. In the part closer to the bottom of the aisle r, along the bottom of the warehouse and up the aisle r — the lower path. There exists a delimiter point — q_{ir} (q-delimiter henceforth), where middle and lower paths are equidistant. For a given set of parameters i, r, a, h, w and appropriate vector $\overrightarrow{b}$, we have that cross aisle length between two consecutive picking aisles and any two picking aisles respectively, is:

$$
\begin{aligned}
d_j &= \sqrt{a^2 + (b_j - b_{j-1})^2} \\
d_{ir} &= \sum_{j=i+1}^{r} d_j,
\end{aligned}
$$

also q-delimiter between any two picking aisles is derived as:

$$
\begin{aligned}
q_{ir} + (r-i)a &= b_i + d_{ir} + b_r - q_{ir} \\
q_{ir} &= \frac{1}{2}\Big[b_i + d_{ir} + b_r - (r-i)a\Big].
\end{aligned}
$$

For the picks above the q-delimiter, the middle path is preferred and for those below, the lower path is best. Yet, there are cases of an anomaly that renders the middle travel path unusable. Knowing that every b_i, and hence every b_r point, has an associated $2w$ penalty, we impose the constraint $w \leq b_r \leq h - w$. On the other hand, constraints that govern behavior of the q-delimiter have to be chosen very carefully. Since we defined q_{ir} as a point of indifference for the middle and lower paths, we expect $w \leq q_{ir} \leq b_r - w$ to hold. So, for every i and r such that $i < r$ we have that:

$$
\begin{aligned}
q_{ir} &\leq b_r - w \\
\frac{1}{2}(b_i + d_{ir} + b_r - (r-i)a) &\leq b_r - w \\
d_{ir} - (b_r - b_i) &\leq (r-i)a - 2w \\
\sum_{j=i+1}^{r} \sqrt{a^2 + (b_j - b_{j-1})^2} - (b_r - b_i) &\leq (r-i)a - 2w.
\end{aligned}
$$

Adjacent aisles are especially sensitive to this constraint. In this case,

$$\sqrt{a^2 + (b_j - b_{j-1})^2} - (b_j - b_{j-1}) \leq a - 2w$$
$$b_j - b_{j-1} \geq \frac{2w(a-w)}{a-2w},$$

which effectively imposes a lower bound on the slope of the consecutive cross aisles segments. The constraint tends to be tight whenever height h is relatively small compared to the width $A = na$. In extreme cases, the result is a straight V-shaped cross aisle with the rightmost b_i's either being very close to, or reaching $h - w$. If the latter occurs, i.e. $b_r = h - w$ and $r < m$, all aisles with indices greater than r do not have a cross aisle.

The previously discussed constraint is in no way imposed by the needs of a warehouse; rather, it is an artifact of the model. If the constraint is simply removed, a cross aisle with a very low slope might cause $E[R_i]$ to become negative. If the constraint is held as it is, whenever the height to width ratio is small, the cross aisle will be limited by the lower bound slope. To address this, we force the model to choose a cross aisle slope that excludes the middle travel pattern whenever it is prudent to do so. That is, we define $\mathbf{q_{ir}} = \min(q_{ir}, b_r - w)$. In the case of $q_{ir} > b_r - w$, the middle travel path vanishes and $\mathbf{q_{ir}} = b_r - w$.

Finally, we derive expressions for each component of the total picking cost:

$$E[R_i] = \sum_{r=i+1}^{m} \frac{1}{(m-i)(h-2w)} \left[\int_0^{\mathbf{q_{ir}}} [(r-i)a + x]dx \; + \int_{\mathbf{q_{ir}}}^{b_r - w} [b_i + d_{ir} + b_r - x]dx + \right.$$
$$\left. + \int_{b_{r+w}}^{h} [b_i + d_{ir} + x - b_r]dx \right]$$

$$
\begin{aligned}
&= \sum_{r=i+1}^{m} \frac{1}{(m-i)(h-2w)} \Bigg[(r-i)a\mathbf{q_{ir}} + \frac{\mathbf{q_{ir}}^2}{2} + (b_r - w - \mathbf{q_{ir}}) \Bigg[b_i + d_{ir} + b_r - \\
&\quad - \frac{1}{2}(b_r - w + \mathbf{q_{ir}}) \Bigg] + (h - b_r - w) \Bigg[b_i + d_{ir} - b_r + \frac{1}{2}(h + b_r + w) \Bigg] \Bigg] \\
&= \sum_{r=i+1}^{m} \frac{1}{(m-i)(h-2w)} \Bigg[\mathbf{q_{ir}} \Bigg[(r-i)a + \frac{\mathbf{q_{ir}}}{2} \Bigg] + (b_r - w - \mathbf{q_{ir}}) \Bigg[b_i + d_{ir} + \\
&\quad + \frac{1}{2}(b_r + w - \mathbf{q_{ir}}) \Bigg] + (h - b_r - w) \Bigg[b_i + d_{ir} + \frac{1}{2}(h - b_r + w) \Bigg] \Bigg],
\end{aligned}
$$

where

$$
\begin{aligned}
d_j &= \sqrt{a^2 + (b_j - b_{j-1})^2} \\
d_{ir} &= \sum_{j=i+1}^{r} d_j \\
q_{ir} &= \frac{1}{2} \Bigg[b_i + d_{ir} + b_r - (r-i)a \Bigg] \\
\mathbf{q_{ir}} &= \min(q_{ir}, b_r - w).
\end{aligned}
$$

$$
\begin{aligned}
E[S_i] &= \frac{1}{(h-2w)} \left[\int_0^h x dx \; - \int_{b_i - w}^{b_i + w} x dx \right] \\
&= \frac{1}{(h-2w)} \left[\frac{h^2}{2} - 2b_i w \right], \\
E[L_i] &= \sum_{l=0}^{i-1} \frac{1}{i(h-2w)} \left[\int_0^h [(i-l)a + x] dx \; - \int_{b_l - w}^{b_l + w} [(i-l)a + x] dx \right] \\
&= \sum_{l=0}^{i-1} \frac{1}{i(h-2w)} \left[a(i-l)(h-2w) + \frac{h^2}{2} - 2b_l w \right], \text{ and}
\end{aligned}
$$

$$
\begin{aligned}
E[L_i^{most}] = &\sum_{l_m=1}^{m} \frac{1}{m(h-2w)} \Bigg[\int_0^{\mathbf{q_{0l_m}}} \Big[ia + l_m a + x \Big] dx + \\
&+ \int_{\mathbf{q_{0l_m}}}^{b_{l_m}-w} \Big[ia + b_0 + d_{0l_m} + b_{l_m} - x \Big] dx \; + \int_{b_{l_m}+w}^{h} \Big[ia + b_0 + d_{0l_m} + x - b_{l_m} \Big] dx \Bigg] \\
&= \sum_{l_m=1}^{m} \frac{1}{m(h-2w)} \Bigg[(i+l_m)a\mathbf{q_{0l_m}} + \frac{\mathbf{q_{0l_m}}^2}{2} + (b_{l_m} - w - \mathbf{q_{0l_m}}) \Big[ia + b_0 + d_{0l_m} + \\
&+ b_{l_m} - \frac{1}{2}(b_{l_m} - w + \mathbf{q_{0l_m}}) \Big] + (h - b_{l_m} - w) \Big[ia + b_0 + d_{0l_m} - b_{l_m} + \frac{1}{2}(h + b_{l_m} + w) \Big] \Bigg] \\
&= \sum_{l_m=1}^{m} \frac{1}{m(h-2w)} \Bigg[\mathbf{q_{0l_m}} \Big[(i+l_m)a + \frac{\mathbf{q_{0l_m}}}{2} \Big] + (b_{l_m} - w - \mathbf{q_{0l_m}}) \Big[ia + b_0 + d_{0l_m} + \\
&+ \frac{1}{2}(b_{l_m} + w - \mathbf{q_{0l_m}}) \Big] + (h - b_{l_m} - w) \Big[ia + b_0 + d_{0l_m} + \frac{1}{2}(h - b_{l_m} + w) \Big] \Bigg].
\end{aligned}
$$

Finally let us consider the special case of the central picking aisle. Since costs of traveling to the right and left from the central aisle are identical, we can simplify the regular cost expression to:

$$
E[C_0] = 2p_{R_0} E[R_0] + p_{S_0} E[S_0],
$$

where $p_{R_0} = m/n$ and $p_0 = 1/n$ are the probabilities of picking to the right of aisle zero and the probability of picking in aisle 0.

$$
\begin{aligned}
E[R_0] = &\sum_{r=1}^{m} \frac{1}{m(h-2w)} \Bigg[\int_0^{\mathbf{q_{0r}}} [ra + x] dx \; + \int_{\mathbf{q_{0r}}}^{b_r - w} [b_0 + d_{0r} + b_r - x] dx + \\
&+ \int_{b_r+w}^{h} [b_0 + d_{0r} + x - b_r] dx \Bigg] \\
&= \sum_{r=1}^{m} \frac{1}{m(h-2w)} \Bigg[ra\mathbf{q_{0r}} + \frac{\mathbf{q_{0r}^2}}{2} + (b_r - w - \mathbf{q_{0r}}) \Big[b_0 + d_{0r} + b_r - \frac{1}{2}(b_r - w + \mathbf{q_{0r}}) \Big] + \\
&+ (h - b_r - w) \Big[b_0 + d_{0r} - b_r + \frac{1}{2}(h + b_r + w) \Big] \Bigg]
\end{aligned}
$$

$$= \sum_{r=1}^{m} \frac{1}{m(h-2w)} \left[\mathbf{q_{0r}}(ra + \frac{\mathbf{q_{0r}}}{2}) + (b_r - w - \mathbf{q_{0r}}) \left[b_0 + d_{0r} + \frac{1}{2}(b_r + w - \mathbf{q_{0r}}) \right] + \right.$$
$$\left. + (h - b_r - w) \left[b_0 + d_{0r} + \frac{1}{2}(h - b_r + w) \right] \right],$$

and the last component — the cost of picking straight ahead, in the central aisle:

$$E[S_0] = \frac{1}{(h-2w)} \left[\int_0^h x dx \ - \int_{b_0-w}^{b_0+w} x dx \ \right]$$
$$= \frac{1}{(h-2w)} \left[\frac{h^2}{2} - 2b_0 w \right].$$

1.4.2 Λ-shaped cross aisle

We refer to our second candidate shape as a Λ shape, because it is a concave structure (Figure 1.4) with a single saddle point at b_0. This shape imposes different travel patterns and results in a more complicated cost function. Comparison with the Flying-V shape reveals similarity in some travel paths. If we choose a random P&D point in the right half of the dock, picking to the right in a Λ structure is equivalent to picking to the left in the Flying-V structure. The same relationship holds for picking to the left (no further than the central aisle) in the Λ structure and picking to the right in a Flying-V structure. Both shapes share similar q-delimiters and constraints. For the Λ shape, we also assume that P&D points are located exactly at the bottoms of picking aisles.

We follow the same steps applied to the Flying-V cross aisle, and establish cost expression:

$$E[C_i] = p_{R_i} E[R_i] + p_{S_i} E[S_i] + p_{L_i} E[L_i] + p_{M_i} E[L_i^{most}].$$

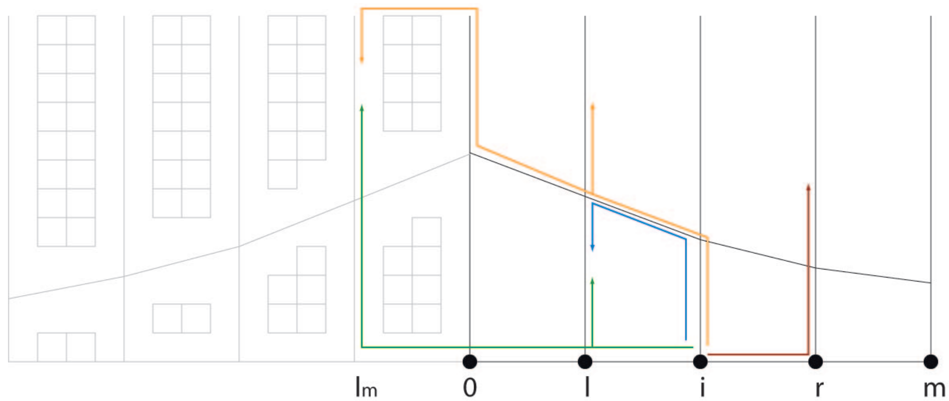

Figure 1.4: Λ shape - scheme of travel patterns and related picking regions

Development of the first three components is as before; the fourth is slightly more complicated and we discuss it in detail.

$$
\begin{aligned}
E[R_i] &= \sum_{r=i+1}^{m} \frac{1}{(m-i)(h-2w)} \left[\int_0^h [(r-i)a + x]dx \; - \int_{b_r-w}^{b_r+w} [(r-i)a+x]dx \right. \\
&= \sum_{r=i+1}^{m} \frac{1}{(m-i)(h-2w)} \left[a(r-i)(h-2w) + \frac{h^2}{2} - 2b_r w \right] \\
E[S_i] &= \frac{1}{(h-2w)} \left[\int_0^h x dx \; - \int_{b_i-w}^{b_i+w} x dx \right] \\
&= \frac{1}{(h-2w)} \left[\frac{h^2}{2} - 2b_i w \right] \\
E[L_i] &= \sum_{l=0}^{i-1} \frac{1}{i(h-2w)} \left[\int_0^{\mathbf{q_{il}}} [(i-l)a + x]dx \; + \int_{\mathbf{q_{il}}}^{b_l-w} [b_i + d_{li} + b_l - x]dx + \right. \\
&\quad \left. + \int_{b_l+w}^{h} [b_i + d_{li} + x - b_l]dx \right] \\
&= \sum_{l=0}^{i-1} \frac{1}{i(h-2w)} \left[(i-l)a\mathbf{q_{il}} + \frac{\mathbf{q_{il}}^2}{2} + (b_l - w - \mathbf{q_{il}}) \left[b_i + d_{il} + b_l - \frac{1}{2}(b_l - w + \mathbf{q_{il}}) \right] + \right. \\
&\quad \left. + (h - b_l - w) \left[b_i + d_{il} - b_l + \frac{1}{2}(h + b_l + w) \right] \right]
\end{aligned}
$$

$$= \sum_{l=0}^{i-1} \frac{1}{i(h-2w)} \left[\mathbf{q_{il}}((i-l)a + \frac{\mathbf{q_{il}}}{2}) + (b_l - w - \mathbf{q_{il}}) \left[b_i + d_{il} + \frac{1}{2}(b_l + w - \mathbf{q_{il}}) \right] + \right.$$
$$\left. + (h - b_l - w) \left[b_i + d_{il} + \frac{1}{2}(h - b_l + w) \right] \right]$$

When picking from some P&D points in the right half-space into the left storage half-space, one can chose between two travel patterns: the simpler one, depicted in Figure 1.4 and the more complicated, given in Figure 1.5. The illustrations reveal that picks at the top and bottom are always picked in the same way. For picks in the middle region, Figure 1.5 shows that it is possible to use the cross aisle. In the next paragraph we explain when and how the cross aisle is used.

For a P&D point i in the right and picking aisle l_m in the left warehouse half-space, we introduce $bel(i, l_m) = (i + l_m)a + b_{l_m}$ and $ab(i, l_m) = b_i + d_{i0} + d_{l_m 0}$, which are the lower and upper shortest paths from the i^{th} P&D point to the b_{l_m} point. If $ab(i, l_m) < bel(i, l_m)$, it is cheaper to access b_{l_m} from above, via the Λ cross aisle. In this situation, it is better to use the cross aisle, which would produce the pattern given in Figure 1.5. Note that this pattern creates two q-delimiters in aisle l_m, one above and one below b_{l_m}. This holds for every aisle l_m such that $ab(i, l_m) < bel(i, l_m)$ and $1 \leq l_m < i$.

For the q-delimiter below the cross aisle:

$$(l_m + i)a + q^L_{il_m} = b_i + d_{i0} + d_{l_m 0} + b_{l_m} - q^L_{il_m}$$
$$q^L_{il_m} = \frac{1}{2} \left[b_i + d_{i0} + d_{l_m 0} + b_{l_m} - (l_m + i)a \right].$$

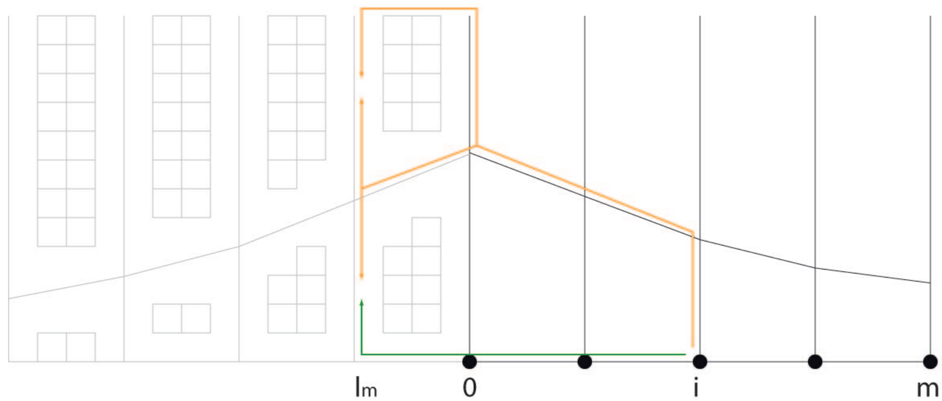

Figure 1.5: Λ shape — a rarely used and complicated travel pattern

and for the q-delimiter above the cross aisle:

$$\begin{aligned} d_{l_m0} + q^U_{il_m} - b_{l_m} &= h - b_0 + l_m a + h - q^U_{il_m} \\ q^U_{il_m} &= \frac{1}{2}\Big[b_{l_m} + 2h - b_0 + l_m a - d_{l_m0}\Big]. \end{aligned}$$

In order to avoid imposing a bound for the slope of the Λ cross aisle and the resulting cost anomalies, we define, as before:

$$\begin{aligned} \mathbf{q^L_{il_m}} &= \max(w, \min{(q^L_{il_m}, b_{l_m} - w)}) \\ \mathbf{q^U_{il_m}} &= \max(b_{l_m} + w, \min(q^U_{il_m}, h - w)). \end{aligned}$$

The other possibility is that $bel(i,x) < ab(i,x)$. Now it becomes better not to use the cross aisle and the outcome is the pattern depicted in Figure 1.4. The travel is less complicated and features only one q-delimiter above the cross aisle:

$$\begin{aligned} (i+l_m)a + q_{il_m}^{l_m} &= b_i + d_{i0} + h - b_0 + l_m a + h - q_{il_m}^{l_m} \\ q_{il_m}^{l_m} &= \frac{1}{2}\Big[b_i + d_{i0} + 2h - b_0 - ia\Big]. \end{aligned}$$

As before, we need to let the model decide freely on the slope of the aisle, so we define:

$$\mathbf{q_{il_m}^{l_m}} = \max(b_{l_m} + w, \min{(q_{il_m}^{l_m}, h - w)}).$$

For a fixed P&D point i, we choose the cheaper of the two arising travel patterns for every aisle l_m in the left half-space. Travel cost for the travel pattern with two q-delimiters is given by:

$$\begin{aligned} L_{i,l_m}^{most}(ab) = & \frac{1}{m(h-2w)}\Bigg[\int_0^{\mathbf{q_{il_m}^L}} [(i+l_m)a + x]dx \ + \int_{\mathbf{q_{il_m}^L}}^{b_{l_m}-w} [b_i + d_{i0} + d_{l_m0} + b_{l_m} - x]dx + \\ & + \int_{b_{l_m}+w}^{\mathbf{q_{il_m}^U}} [b_i + d_{i0} + d_{l_m0} + x - b_{l_m}]dx + \int_{\mathbf{q_{il_m}^U}}^{h} [b_i + d_{i0} + h - b_0 + l_m a + h - x]dx\Bigg] \\ & = \frac{1}{m(h-2w)}\Bigg[(i+l_m)a\mathbf{q_{il_m}^L} + \frac{\mathbf{q_{il_m}^L}^2}{2} + (b_{l_m} - w - \mathbf{q_{il_m}^L})\Big[b_i + d_{i0} + d_{l_m0} + b_{l_m} - \\ & - \frac{1}{2}(b_{l_m} - w + \mathbf{q_{il_m}^L})\Big] + (\mathbf{q_{il_m}^U} - b_{l_m} - w)\Big[b_i + d_{i0} + d_{l_m0} - b_{l_m} + \frac{1}{2}(\mathbf{q_{il_m}^U} + b_{l_m} + w)\Big] + \\ & + (h - \mathbf{q_{il_m}^U})\Big[b_i + d_{i0} + 2h - b_0 + l_m a - \frac{1}{2}(h + \mathbf{q_{il_m}^U})\Big]\Bigg] \\ & = \frac{1}{m(h-2w)}\Bigg[\mathbf{q_{il_m}^L}((i+l_m)a + \frac{\mathbf{q_{il_m}^L}}{2}) + (b_{l_m} - w - \mathbf{q_{il_m}^L})\Big[b_i + d_{i0} + d_{l_m0} + \\ & + \frac{1}{2}(b_{l_m} + w - \mathbf{q_{il_m}^L})\Big] + (\mathbf{q_{il_m}^U} - b_{l_m} - w)\Big[b_i + d_{i0} + d_{l_m0} + \frac{1}{2}(\mathbf{q_{il_m}^U} - b_{l_m} + w)\Big] + \\ & + (h - \mathbf{q_{il_m}^U})\Big[b_i + d_{i0} + 2h - b_0 + l_m a - \frac{1}{2}(h + \mathbf{q_{il_m}^U})\Big]\Bigg]. \end{aligned}$$

For the travel pattern with a single q-delimiter above the cross aisle we proceed in a similar manner:

$$\begin{aligned}
L_{i,l_m}^{most}(bel) &= \frac{1}{m(h-2w)}\Bigg[\int_0^{b_{l_m}-w}\Big[(i+l_m)a+x\Big]dx \;+\int_{b_{l_m}+w}^{\mathbf{q_{il_m}^{l_m}}}\Big[(i+l_m)a+x\Big]dx\;+ \\
&+\int_{\mathbf{q_{il_m}^{l_m}}}^{h}\Big[b_i+d_{i0}+h-b_0+l_ma+h-x\Big]dx\Bigg] \\
&= \frac{1}{m(h-2w)}\Bigg[(i+l_m)a(b_{l_m}-w)+\frac{(b_{l_m}-w)^2}{2}+(\mathbf{q_{il_m}^{l_m}}-b_{l_m}-w)\Big[(i+l_m)a+ \\
&+\frac{1}{2}(\mathbf{q_{il_m}^{l_m}}+b_{l_m}+w)\Big]+(h-\mathbf{q_{il_m}^{l_m}})\Big[b_i+d_{i0}+2h-b_0+l_ma-\frac{1}{2}(h+\mathbf{q_{il_m}^{l_m}})\Big]\Bigg].
\end{aligned}$$

Finally, we arrive at the expression that favors cheaper travel pattern:

$$E[L_i^{most}] = \sum_{l_m=1}^{m}\min\left(L_{i,l_m}^{most}(bel), L_{i,l_m}^{most}(ab)\right).$$

Cost for picking from a central P&D point is calculated similar to the V-shaped cross aisle cost.

$$E[C_0] = 2p_{R_0}E[R_0]+p_{S_0}E[S_0]$$

$$\begin{aligned}
E[R_0] &= \sum_{r=1}^{m}\frac{1}{m(h-2w)}\Bigg[\int_0^h(ra+x)dx \;-\int_{b_r-w}^{b_r+w}(ra+x)dx \\
&= \sum_{r=1}^{m}\frac{1}{m(h-2w)}\Bigg[ar(h-2w)+\frac{h^2}{2}+2b_rw\Bigg] \\
E[S_0] &= \frac{1}{n}\int S \\
&= \frac{1}{h-2w}\Bigg[\int_0^h xdx\;-\int_{b_0-w}^{b_0+w}xdx\;\Bigg] \\
&= \frac{1}{h-2w}\Bigg[\frac{h^2}{2}-2b_0w\Bigg].
\end{aligned}$$

1.4.3 The most economical cross aisle

The most economical cross aisle enables a single pickup and retrieval travel into the storage space at a minimum cost. We optimize candidate shapes one at a time, compare them and choose a more cost-effective solution. Due to the complexity and nonlinearity of the cost function, it is not possible to apply any calculus based methods for finding minima. Being unable to generate closed form solutions, we resort to nonlinear optimization. Both optimization problems are of the following form:

$$\text{Min } E[C]$$

$$\text{S.T. } w \leq b_i \leq h - w,\ i \in [0, m].$$

We make use of Mathematica's `NMinimize` function, which employs a number of standard algorithms for solving nonlinear problems. In the series of experiments we carried out, it never took more than fifteen minutes to solve problems for warehouses with a large number of P&D points. Smaller instances of the problem are solved in a matter of seconds.

1.5 Results

We carry out the experiment on a variety of warehouse configurations. We perform the following changes in basic design parameters:

- $n = 11 - 39$ in increments of 4,
- $w = 1 - 3$ in increments of 0.5, and
- $h = 50 - 125$ in increments of 25.

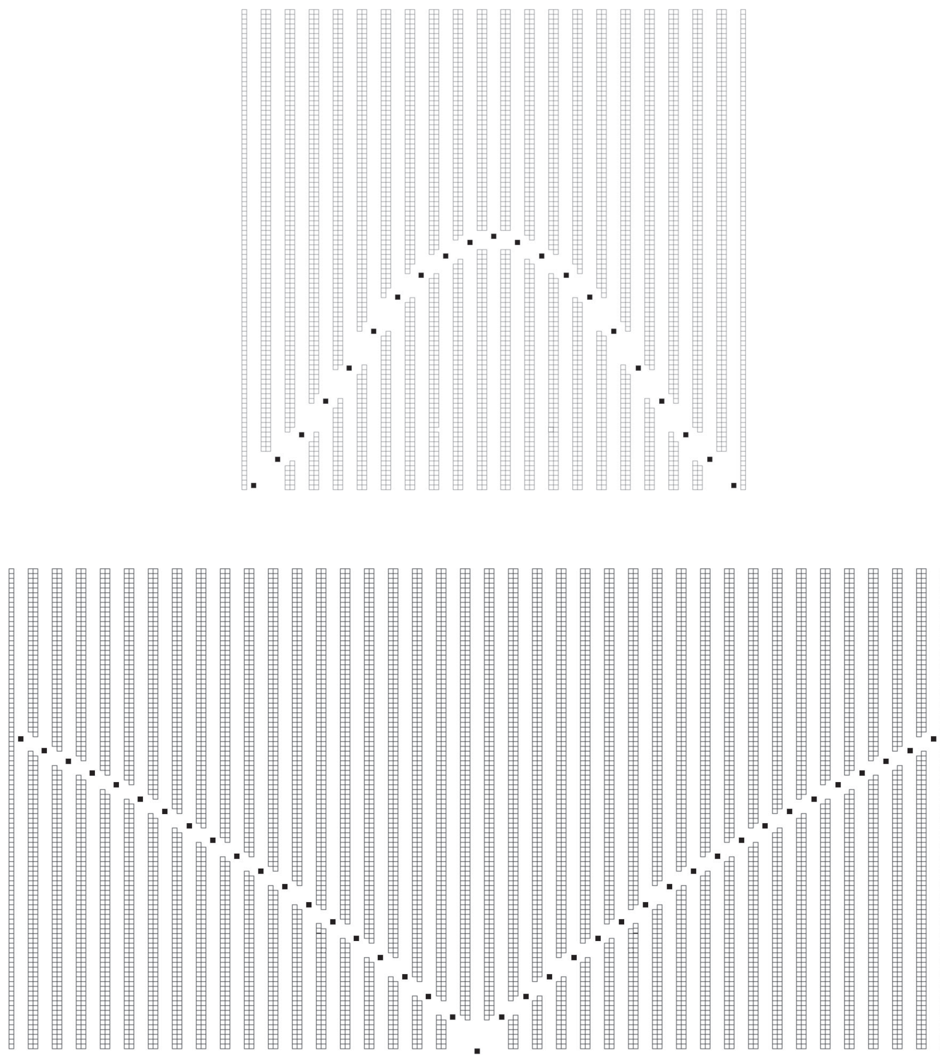

Figure 1.6: Typical Flying-V ($n = 39$, $w = 2$, $a = 5$, $h = 100$) and Λ ($n = 21$, $w = 2$, $a = 5$, $h = 100$) shape cross aisles

We keep a at value of 5 because our designs assume use of counterbalance forklifts. For this type of fork truck picking aisles are usually 12 feet wide, which corresponds to $a-2$ pallet units in our notation. Among the obtained population of 160 instances, not all layouts are realistic. In fact, one could say that those with $n = 15-30, w = 1.5-2, h = 50-100$ are perhaps closest to existing facilities, while others diverge mostly because of unusual width to height ratio or cross aisle width. Nevertheless, a larger experiment, encompassing the applicable and less pragmatic alike, helps build some intuition about laws governing the quality of a layout. We stipulate that configurations being compared have the same overall length of picking aisles. For example, Λ and Flying-V shape with $w = 1.5, h = 100$ together with traditional no-cross-aisle layout of $h = 97$ represent a valid comparison triplet.

Before discussing the results we note a few physical and visual properties of typical Flying-V and Λ shapes given in Figure 1.6. Our experiments confirm the expectation that whenever it is advantageous to have a V-shaped cross aisle, b_0 is equal to w. Gue and Meller (2006) explain this observation in detail. Suffice it to say that having a b_0 point as low as possible shortens all right and left travel traversing P&D point zero or originating at it. Similarly, for all justifiable Λ layouts, travel originating at the leftmost and rightmost P&D points are shortened with $b_m = w$. For 39 P&D points (aisles), 100 units of height, and 4 units wide cross aisle, the Flying-V shape yields a 5.64% travel savings over the traditional design. For the same height and cross aisle width, a Λ warehouse with 23 P&D points (aisles) does not confer any benefit. Step-by-step increase in number of P&D points induces small benefits. Yet, a Λ warehouse with 39 P&D points, 100 units of height, and 4 units wide cross aisle results in only 1.09% savings.

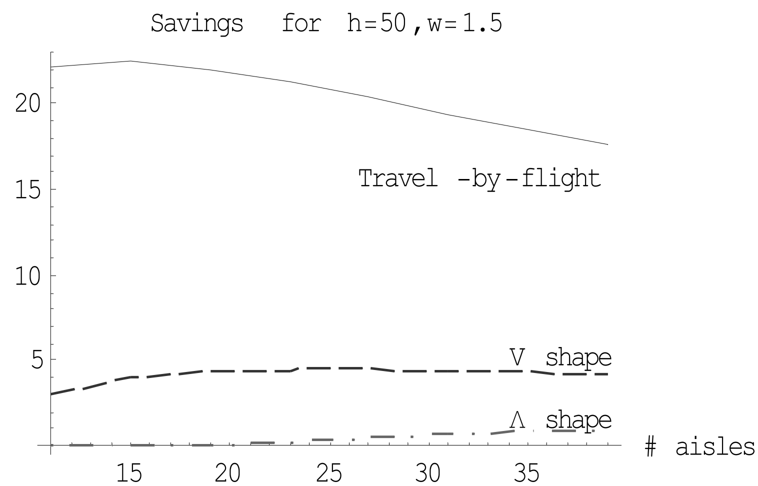

Figure 1.7: Example comparison of savings over orthogonal design for Flying-V and Λ-shape layouts, and Travel-by-flight

Neither of the shapes comes close to the Travel-by-flight savings (Figure 1.7), which are about 20% for all configurations we consider. In Figure 1.8 we present the summary of savings for applicable layouts for Λ shape and Flying-V shape. A comprehensive comparison is given in Table 1 of the Appendix. A glimpse at Figure 1.8 or Table 1 makes it clear that the V shape always performs better than Λ shape.

For Λ shapes with up to $n = 31$ aisles, heights of $h = 50, 75$ or 100, benefits do not exceed 2.16%. For cross aisle width of $w = 2$ (or more), a Λ aisle provides no benefit and the optimization model returns a vector $\overrightarrow{b} = \{h - w, h - w, \ldots, h - w\}$, which corresponds to a traditional orthogonal design.

With respect to the Flying-V shape, narrow aisles (of $w = 1$) offer savings as high as 7.24%. Regrettably, small increases in cross aisle width cause considerable drop in savings. A large percentage of travel crosses the $2w$ space and increases the average distance to make a pick. This disadvantage accumulates rather quickly, thereby making the overall average

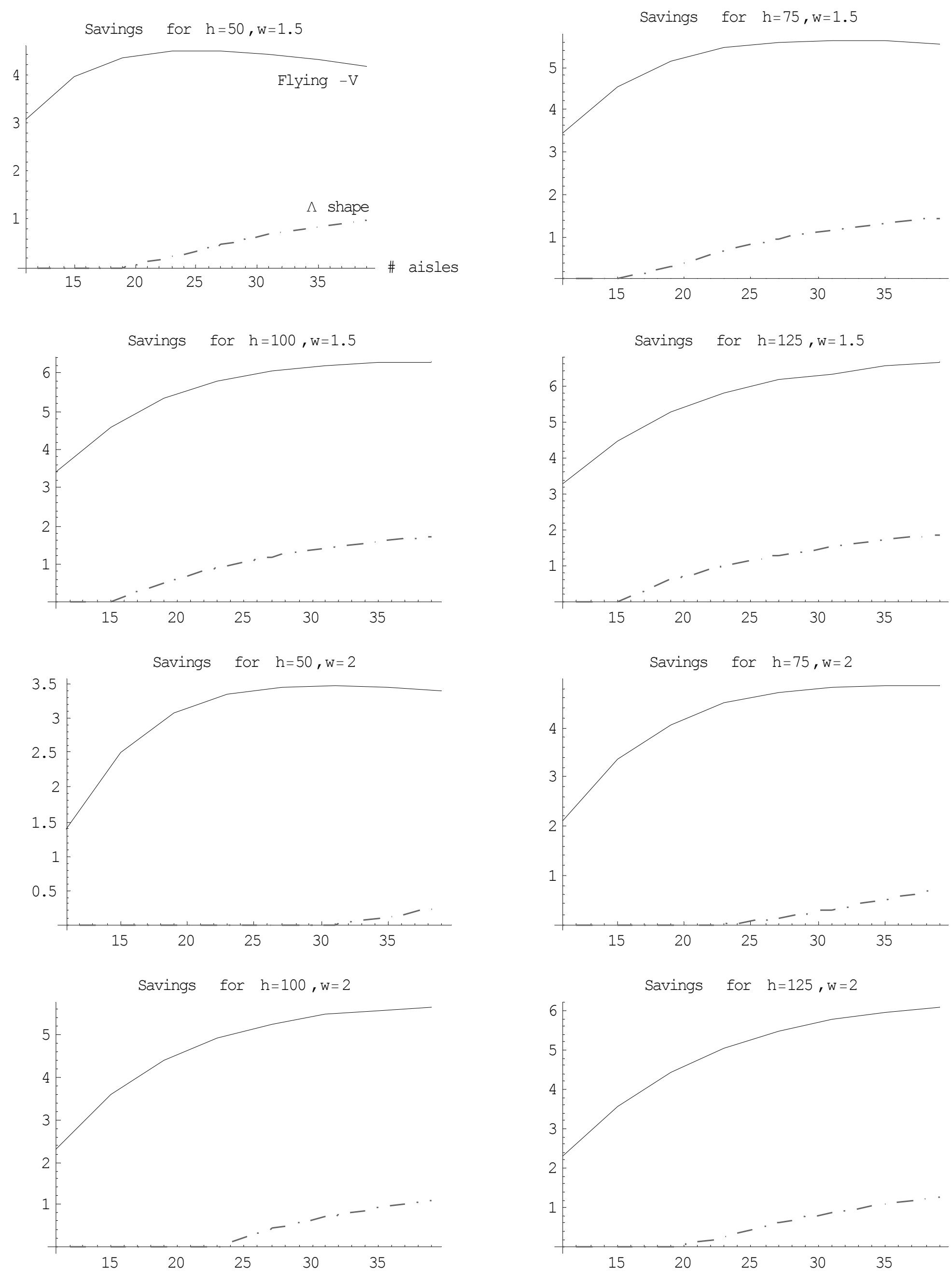

Figure 1.8: Savings over orthogonal design for Flying-V and Λ-shape layouts

cost very sensitive to it. Consequently, Flying-V shape layouts with realistic cross aisle widths carry benefits not higher than 6.07% for $w = 2$, or 6.65% for $w = 1.5$.

In the worst case, for some warehouses it is less costly to keep the traditional design. The break-even point for a warehouse with a fixed number of P&D points can be reached in two ways: by reducing h or by increasing w. Both of those have the same meaning — the picking space is shrinking. When decreasing h we diminish the ability of a cross aisle to provide savings. When h becomes small enough, economies of scale that were enabling V-shape savings vanish. The number of farther locations that were more prudent to access via the cross aisle is reduced to insignificance. There is a similar effect with an increase in cross aisle width. Very often a travel into the storage space includes the cross aisle width, which is not the case with the traditional design. An increasing width adds to the travel distance more than before, while a smaller traditional warehouse with height of $h - 2w$ does not have this handicap. In Figure 1.9 we give an example of break-even configurations for a fixed value of $w = 1.5$. As the number of picking aisles in a warehouse increases, break-even heights decrease.

1.6 A flow-through crossdock

A flow-through crossdock receives pallets on one side and ships from the opposite. In keeping with our terminology, receiving occurs on the top and shipping at the bottom. If a V-shaped cross aisle is inserted, a picker on the shipping side will perceive it as a V shape, whereas his coworker on the receiving side will see a Λ shape. Therefore, a flow-through crossdock is a natural composition of Flying-V and Λ shape travel patterns and hence, cost

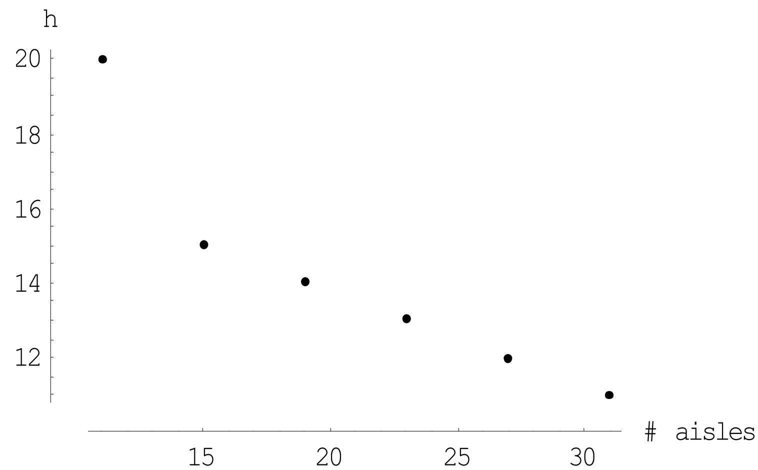

Figure 1.9: Example of warehouse configurations ($w = 1.5$, $n = 11 - 35$) for which it is better not to have a cross aisle

functions. The optimal cross aisle is found with respect to optimizing the sum of Flying-V and Λ shape travel costs divided by two.

To determine the potential advantage of having a single cross aisle in a crossdock, we conducted an experiment over the same combinations of n, w, and h as in the Section 1.5. Detailed results of the experiment are given in the Table 1 of the Appendix. In Figure 1.10 we present the crossdock savings for applicable combinations of h and w and graphically compare them with savings of Λ and Flying-V shapes. Benefits, if any, are relatively close to the arithmetic mean of the Λ and Flying-V shape savings. Additionally, for instances of Λ shape that incur loss (compared to the upper travel bound) part of the Flying-V shape gains is wasted on making up for the deficit.

Knowing that Flying-V shape savings dominate those of Λ, we would also expect to see $b_0 = w$, which indeed occurs in all instances of our experiment. As regards the Λ shape, not even once does the b_m equal $h - w$.

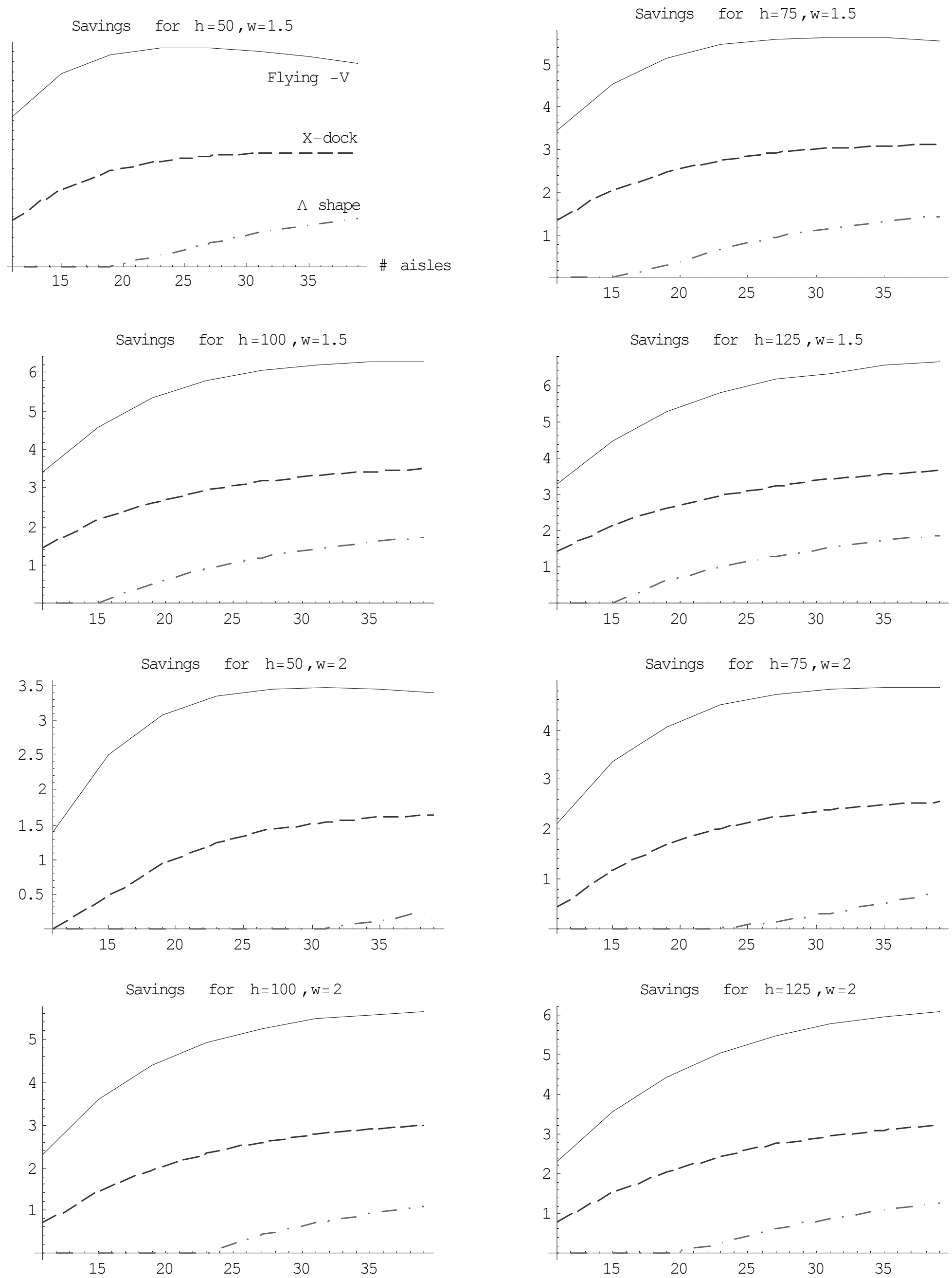

Figure 1.10: Comparisson of savings over orthogonal design for crossdock configuration, Flying-V and Λ-shape layouts

1.7 Duality of representation

In this section we return to the alternative modeling paradigm we defined in Section 1.4. The methodology we have used so far can be summarized as choosing a number of cross aisle shapes, routing workers to determine shortest travels into the storage space, and then choosing the best aisle shape. Alternatively, we could first specify a number of sets of rules governing the travel, let the model choose the shape resulting from each set of rules, and then choose the best aisle shape. Let us demonstrate by fully defining one set of rules:

1. If the P&D point is centrally located
   (a) for a "right" pick above the cross aisle, travel to the cross aisle along picking aisle zero, along the cross aisle and then up the destination aisle.
   (b) for a "right" pick below the cross aisle and above the q-delimiter (point of indifference), travel to the cross aisle along picking aisle zero, along the cross aisle, and down the destination aisle.
   (c) for a "right" pick below the point of indifference, travel right along the bottom and up the destination aisle.
   (d) for a "straight" pick, travel up the picking aisle zero.
   (e) for a "left" pick "mirror" the "right" pick travel rules 1(a)-1(d).
2. P&D point is in the right half-space.
   (a) for a "right" pick above the cross aisle, travel up along the current picking aisle, along the cross aisle and up the destination aisle.

(b) for a “right” pick below the cross aisle and above the point of indifference, travel up along the current picking aisle, along the cross aisle, and down the destination aisle.

(c) for a “right” pick below the point of indifference, travel right along the bottom and up the destination aisle.

(d) for a “straight” pick, travel up the current aisle.

(e) for a left pick in the right half-space including the central aisle, travel left along the bottom, and up the destination aisle.

(f) for a left pick in the left half-space not-including the central aisle, travel left along the bottom to reach central P&D point and then follow the rule 1(e).

3. P&D point is in the left half-space.

   (a) “mirror” the rules 2(a)-2(f).

Flying-V cross aisle depiction (Figure 1.3) reveals analogy between the optimum routing of workers illustrated, and the routing rules we define here. The difference is that in the first case cross aisle shape is known *a priori*, and routing rules are inferred from shortest path travel within the V shape. Conversely, in the second approach routing rules are established first and cross aisle shape is therefore a *consequence* of routing rules.

To demonstrate the validity of an alternative representation we conduct the following test. Let us define a set of routing rules:

1. any P&D point

   (a) for a “right” pick above the cross aisle, travel up, along the cross aisle and up the destination aisle.

(b) for a “right” pick below the cross aisle and above the point of indifference, travel up, along the cross aisle, and down the destination aisle.

(c) for a “right” pick below the point of indifference, travel right along the bottom and up the destination aisle.

(d) for a “straight” pick, travel up the current aisle.

(e) for a left pick “mirror” the rules 1(a)-1(d).

The rules state that for each aisle a pick can be made to the right, straight and left from the current aisle, which in terms of cost structure can be stated as:

$$E[C_i] = p_{R_i}E[R_i] + p_{S_i}E[S_i] + p_{L_i}E[L_i].$$

The rules further translate into cost expressions below. Because derivation of final formulae is straightforward and has been presented before, we include only the initial expressions:

$$\begin{aligned}
E[R_i] &= \sum_{r=i+1}^{n} \frac{1}{(n-i)(h-2w)} \left[\int_0^{\mathbf{q_{ir}}} [(r-i)a + x]dx \; + \int_{\mathbf{q_{ir}}}^{b_r - w} [b_i + d_{ir} + b_r - x]dx \; + \right. \\
&\quad \left. + \int_{b_{r+w}}^{h} [b_i + d_{ir} + x - b_r]dx \right] \\
E[S_i] &= \frac{1}{(h-2w)} \left[\int_0^h xdx \; - \int_{b_i - w}^{b_i + w} xdx \; \right] \\
E[L_i] &= \sum_{l=1}^{i-1} \frac{1}{(i-1)(h-2w)} \left[\int_0^{\mathbf{q_{il}}} [(i-l)a + x]dx \; + \int_{\mathbf{q_{il}}}^{b_l - w} [b_i + d_{il} + b_l - x]dx \; + \right. \\
&\quad \left. + \int_{b_l + w}^{h} [b_i + d_{il} + x - b_l]dx \right].
\end{aligned}$$

The total cost of picking is expressed with:

$$\begin{aligned} E[C] &= \sum_{i=1}^{n} p_i E[C_i] \\ &= \frac{1}{n} \sum_{i=1}^{n} (p_{R_i} E[R_i] + p_{S_i} E[S_i] + p_{L_i} E[L_i]). \end{aligned}$$

To find the most economic cross aisle shape for this set of routing rules, we state an optimization problem:

$$\text{Min } E[C]$$

$$\text{S.T. } w \le b_i \le h - w,\ i \in [1, n].$$

Observe that this set of rules has no mention of central aisle or central P&D point. For any pick above the point of indifference (which also includes the points above the cross aisle) the picker travels directly to the cross aisle and then along the cross aisle. An "educated guess" that neither Flying-V nor Λ shape would perform well is actually correct. We ran the experiment on the same sample of 160 warehouses and the result was always a traditional, orthogonal layout with the "cross aisle" at $\overrightarrow{b} = \{h - w, h - w, \dots, h - w\}$, which defines no cross aisle at all.

We have demonstrated two examples where there is a complete analogy between prescribing a cross aisle shape, and choosing a set of routing rules to serve as a basis for optimization. However, we believe that the latter approach is more general and more appropriate from the methodological point of view. The former is easier to grasp and especially easier to present in a written form, but we stress that the shapes we dealt with result in a relatively simple travel and cost structure. We allow the possibility that future work in this area might benefit from the travel rules paradigm, since there may exist sets of travel rules where it would not be possible to predict the aisle shape ahead of optimization.

1.8 Some practical issues

Through several contacts with engineers, logistics analysts and managers we received valuable feedback on practical issues of cross aisle implementation. In addition to the cross aisle width problem, worker orientation and adaptation to new designs were among most cited concerns. To quote a supervisor who has been suggested to implement our designs: "Whatever we do, we must keep it as simple as possible." We set out to examine the implications of "simple." When making a right pick in the right half-space of a Flying-V-shaped warehouse, a travel pattern of the form "up, along the cross aisle and down the destination aisle" is the most confusing. It is much easier to instruct workers to use the cross aisle only for the picks above it and eliminate the problematic travel pattern instead. In connection with this, several managers have broached the subject of traffic safety and forklift stability when making a sharp right turn down into the picking aisle. We proposed the same solution for both problems. Since we have simplified the accompanying travel pattern, we call this new shape V-simple. Results show that the new shape offers less benefit, but only slightly so — maximum reduction in savings measured across all configurations is only 0.85%. For convenience, we give complete results of this experiment in the Table 2 of the Appendix.

Another potential problem in practice is the curvature of cross aisles. Most of our shapes are curved slightly, and this was perceived as an obstacle to visibility and traffic safety. There were concerns about perception, suspecting that workers would not have a "feel" for the warehouse space and that non-orthogonal curved aisles might cause them to lose orientation. We evaluated straight instead of the curved, Flying-V-shaped cross aisle. This straight V shape changes the field of view and improves visibility at a maximum increase in travel distance of 0.33% across all configurations, and 0.21% in realistic ones. Again, we give complete results in the Table 2 of the Appendix.

1.9 Conclusions

In this chapter we focused on optimization of picking cost in unit load warehouses with rack storage. We presented two cross aisle designs, Λ and Flying-V, discussed their combination in a flow-through crossdock and quantified the potential for cost reduction.

To summarize, Λ shape savings generally are nonexistent or too small to justify implementation. The same applies to crossdock, where associated benefits can be roughly thought of as the averaged savings of the two aisle shapes. Regarding the Flying-V shape, benefits are small and considerably less than those of Flying-V shape with a single P&D point (Gue and Meller, 2006), but may be worth implementing. A simple explanation exists: With a single P&D point a greater portion of travel uses the cross aisle, whereas the region does not have this advantage. As P&D points get farther away from the center, travel gains less and less advantage from the cross aisle. It becomes necessary to travel greater distances up the aisle i just to reach the cross aisle.

Flying-V shape is superior to Λ , which is a consequence of their respective geometries and characteristics of the material flow we hypothesize. Assuming uniform pick density, material flows along a linear region will be greatest in the center. The Flying-V shape facilitates such flows by providing a "launching point" from which to enter the picking space after travel along the bottom aisle. On the other hand, Λ shape provides no such advantage. The lowest b_i points of a Λ shape, the ones with highest "launching potential," are located in the corners of the warehouse rectangle where only a small fraction of material flow can benefit from them.

As we have stated, our results are a sort of worst case analysis for the designs we consider. This particularly holds for the assumption about uniform flows across the entire shipping region. Wise implementation would concentrate more flows into the central doors and results should improve. Moreover, decreasing the number of P&D points should provide more benefit and is expected to come closer to the results of Gue and Meller (2006). Also, one should be aware that all benefits discussed in this work represent a relative measure and are tied to the expected cost of making a single pick. When deciding on implementation of the cross aisle, the designer should use the performance metrics such as daily or monthly flow through the warehouse, total number of picks per shift and similar to realize the absolute financial benefit.

Chapter 2

Application of aisle design principles to floor storage — a case study

In the process of developing the cross aisle designs (Chapter 1), we made a number of assumptions that help alleviate the complexity associated with the model. They do not significantly affect the accuracy of our results, but do not fully represent the factual state either. For example, we model individual rack locations as continuous, as opposed to them being discrete in reality. Strictly speaking, insertion of a w units wide cross aisle into a continuous rack space results in unrealistic storage space allocation. For example, a b_i at 12.35, with the $w = 1.5$, means that the storage space below the aisle numbers 10.85 locations. Theoretically, it is not possible to utilize 0.85 units of storage space in a unit load warehouse. Similar problem occurs with the space above the aisle. Let us recall that we integrate across this nonexisting storage space and therefore increase the actual travel cost. Even though we account for the increase by calculating the average cost, we indeed alter the real cost, which although slight, is an inaccurate representation of reality *per se*.

When one tries to implement a theoretical result or principle in practice, it is necessary to adopt an even more more flexible approach. In this chapter, we demonstrate a case of a large home appliance manufacturer in the Southeast. We present a less rigorous modeling approach and apply the main ideas of our designs to a new problem — unit load storage using floor stacking. We discuss several possible amendments to the general floor storage layouts with different positions of P&D points. We develop rather simple estimates of the savings associated with our designs. Finally, we propose a new layout for the manufacturer's warehouse and calculate the approximate savings in the putaway and picking costs.

2.1 Problem background

The company operates a large unit-load warehouse, with floor storage area that is used to keep the finished goods being delivered from the adjacent manufacturing facility. Figure 2.1 depicts the current layout. As can be inferred from the figure, finished goods are delivered from the left side of the warehouse rectangle and the loading dock is located on the bottom side. Upon order receipt, pickers travel into the storage space from two P&D points located in close proximity of the loading dock. In reality, picking activity is not uniformly distributed since every effort is made to reserve the most convenient picking locations A_1, A_2, A_3, A_4 and G, for the fastest moving skus. Less frequent items are stored in blocks B_1, B_2, C, D, H, I, L and K, whereas slow movers are in blocks E, F, J and M. This storage policy is best described in terms of dedicated storage. After being picked, the item is transported to the same P&D point from which the picker started the trip (single command cycle) for consolidation. In the end, it is loaded onto one of the outgoing trailers. The loading dock houses twelve doors where each P&D point is serving only the closer half of the doors. An alternative is a centrally located P&D and consolidation point serving all the doors. The former solution provides significant savings in the average distance traveled from P&D points to the doors and is preferred by the warehouse management.

Before proceeding to aisle design implementation, let us acknowledge several properties of alternative aisle designs as they are seen from the practical point of view. In our model, we express expected cost of a single pick or a stow as average distance traveled to complete these tasks. It is only advisable to do so if we assume constant or close to constant traveling speed. Several factors might impair the soundness of this assumption. First of all, pickers will have to turn at least once in order to reach the destination aisle. In our opinion,

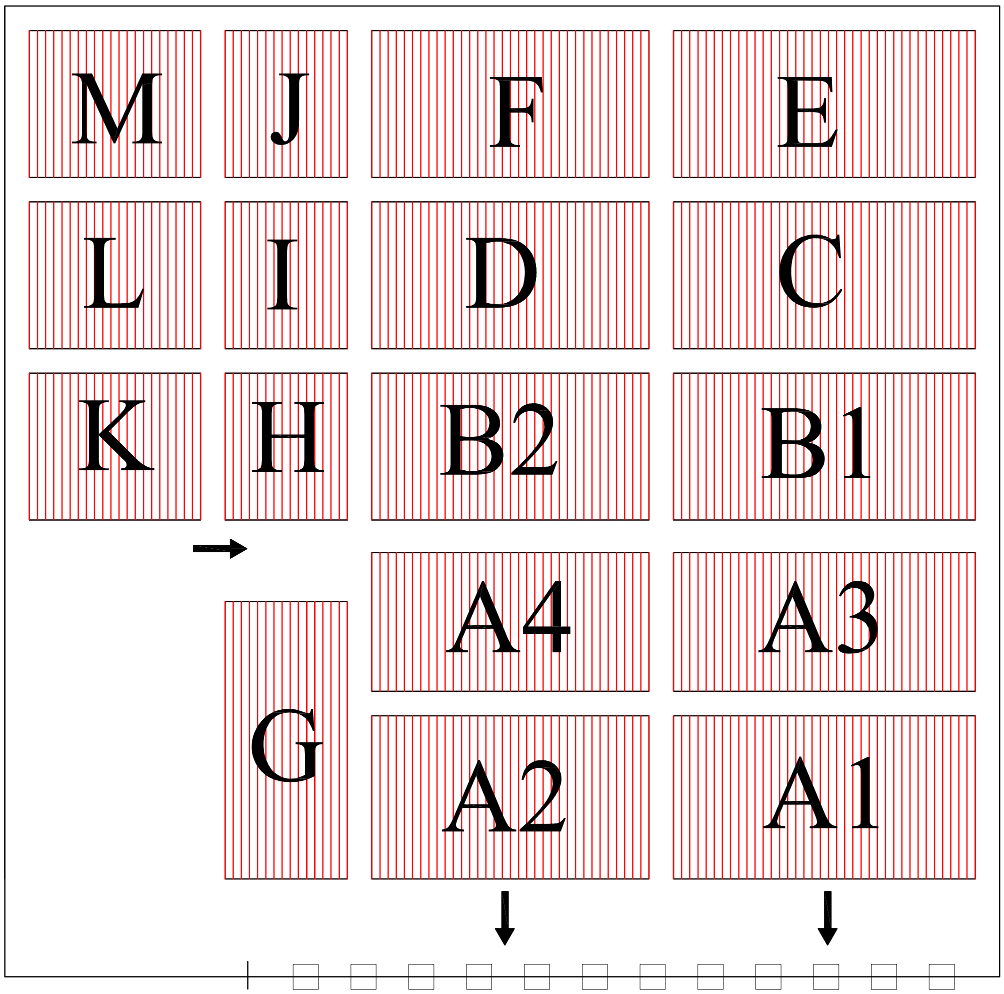

Figure 2.1: Current layout. Arrows pointing outward from the storage space represent P&D points with retrieval activity. Arrow pointing into the storage space denotes inflow of goods and putaway activity. Letters, or combinations of letters and numbers represent “labels” of the storage blocks.

this will not affect significantly the average speed. More importantly, we assume ideal traffic conditions, i.e. congestion does not occur in the cross and picking aisles. Therefore, our model does not predict any negative effects caused by possible traffic impediment. A separate model is needed to represent the reduction in savings due to congestion.

Cost estimate by distance only includes reaching the point in the center of the picking aisle and opposite to the unit to be picked from the rack, not the actual travel needed to reach and load the item. If one needs to include the cost of reaching and loading the pallet onto a material handling device (most often a forklift), distance ceases to be an accurate estimator, because the half-width of the picking aisle is very likely to be crossed with speed significantly different from the average traveling speed. After a forklift reaches the pallet rack, only the pallet handling device is moving and is doing so in a vertical plane with a speed that is again different from the forklift traveling speed. This means that reaching distance and elevation cannot be simply added to the original distance traveled. In situations where exact assessment is needed, it is more suitable to use time as the cost estimator — distance to reach the center of the picking aisle is divided by the average speed and the time interval needed to reach and load the pallet is added. Usually, this interval is broken up into time to rotate the truck for 90°, elevate the forks, position them, load the pallet, lower the forks and again rotate for -90°. Unfortunately, all the loading components may vary significantly due to different forklift model performance, driver experience, pallet, rack and floor properties. Hence, quantifying the loading cost may be very tedious and prescribing a generic formula not appropriate. In the end, if distance is the chosen cost estimator and one needs to compare the cost of two alternative layout solutions, he can only do so accurately for layouts with the same rack depths (single deep with single deep and double deep with double deep) and same heights (number of levels). However, we believe that for most applications of rack layout, distance is the appropriate estimator and that loading/extraction issues become more important in floor storage.

One generally opts for floor storage when it is necessary to achieve greater density and better floor space utilization, or when access to each individual unit is not required at all times. The classical problem pertaining to this warehousing technology is determining the optimal lane depth, one that would minimize the loss from honeycombing. Although Bartholdi and Hackman (2007) provide a straightforward and elegant solution, in reality it is often not possible to calculate optimal lane depths, because it is difficult to maintain an appropriate database of sku inventory information.

The essential advantage of floor storage is the improved footprint utilization, but an apparent disadvantage is the lack of access to individual items. Double deep selective pallet racks suffer from this shortcoming too, but in a less severe form. Since floor storage is arranged in blocks and lanes within them, a picker might travel significant distances through the empty part of the lane in order to reach the intended item (extraction distance). As Bartholdi and Hackman note, deeper lanes increase the space utilization but also increase insertion/extraction times. If there is a single lane depth or a small number of lane depths throughout the warehouse, cost can be safely expressed in terms of reaching the center of the picking aisle. Nonetheless, when floor storage features a variety of lane depths, extraction distance needs to be included. If there is a substantial difference in traveling vs. extraction speed, slower extraction should be accounted for. In our case, on-site observations suggested approximately equal travel and extraction speeds, and we made this assumption throughout.

Depending on the layout, replenishment and picking in a lane can be performed from different directions, and consequently lead to different storage queueing disciplines. When replenishment is done from the back and picking from the front, FIFO is preserved; both replenishment and picking at the front lead to LIFO discipline.

2.2 Simple floor storage

The practical problem we discuss brings in a number of special considerations. One is putaway from the side (Figure 2.1), a question we did not deal with in Chapter 1. To get acquainted with and better build our understanding of the floor storage and potential improvements, we first discuss a layout analogous to one of the rack storage, with a single P&D point and both putaway and picking at the bottom of the warehouse rectangle.

In Figure 2.2(a) we present a classical floor storage while in Figures 2.2(b), 2.2(c) and 2.2(d), we give a schematic depiction of some alternative improvements. Note that the numbering in storage locations in all of Figures 2.2 represents the order in which individual pallet positions are being restocked and subsequently picked from — that is to say, a storage discipline. Upon emptying the lane of a rectilinear (classical) floor layout the restocker first takes care of the positions in the upper part of the warehouse rectangle. Once the whole lane has been restocked, the picker will start from the lowermost positions, therefore establishing the LIFO storage discipline.

Figure 2.2(b) shows a layout similar to Flying-V cross aisle of rack storage. The left half-space features all three travel patterns present in a Flying-V. It can be inferred that q-delimiters (depicted with curvy irregular line) govern the storage discipline. If one wishes to utilize all three travel patterns, the storage discipline in the positions below the aisle will

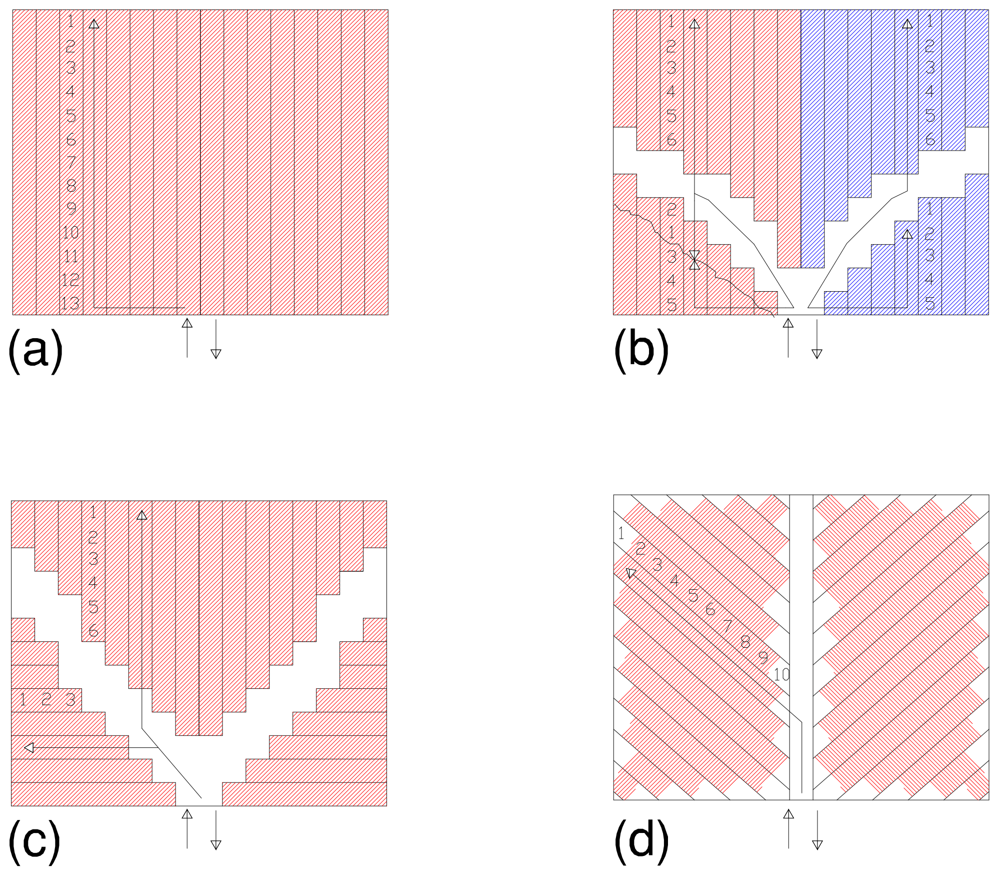

Figure 2.2: One P&D point with putaway and picking from the same side

be neither FIFO nor LIFO. In rack storage, a so called "middle travel pattern" complicates picking. We showed in Chapter 1 that using only rectilinear travel below the cross aisle results in lesser savings, but simpler travel patterns. Same is the case with floor storage, though with an additional benefit — simpler, LIFO storage discipline. In such a layout (Figure 2.2(b), blue half-space), all positions, below and above the aisle, have LIFO storage.

Translating fishbone principles to the floor storage results in the layout of Figure 2.2(c).[1] It confers more travel savings but changes the lane orientation in the space below the aisle. For a fishbone and chevron layout (Figure 2.2(d)) where the fishbone aisle is parallel to the chevron storage lanes, chevron lanes are deeper since they represent the hypotenuse and fishbone lanes resemble the cathetus. Therefore, the choice between the two might depend on the lane depth preferences. Note that other practical issues might complicate the designs and the choice to be made among them. For example, various safety concerns might impose an additional central aisle in the fishbone structure. Such an aisle would cut through the existing storage space and facilitate evacuation from the storage area.

2.2.1 Side putaway

A more complicated task is designing a layout where the inflow of skus occurs at an entry point located on the left side of the warehouse rectangle. Unlike the previous example, we now encounter *conflicting flows* in some parts of the warehouse, i.e., in the same portions of the storage space, travel patterns resulting from a new design might offer advantage for picking and induce cost for replenishment, and vice versa. Figure 2.3 represents some of the choices available to the designer faced with this problem.

[1] For more details on fishbone design see Gue and Meller (2006)

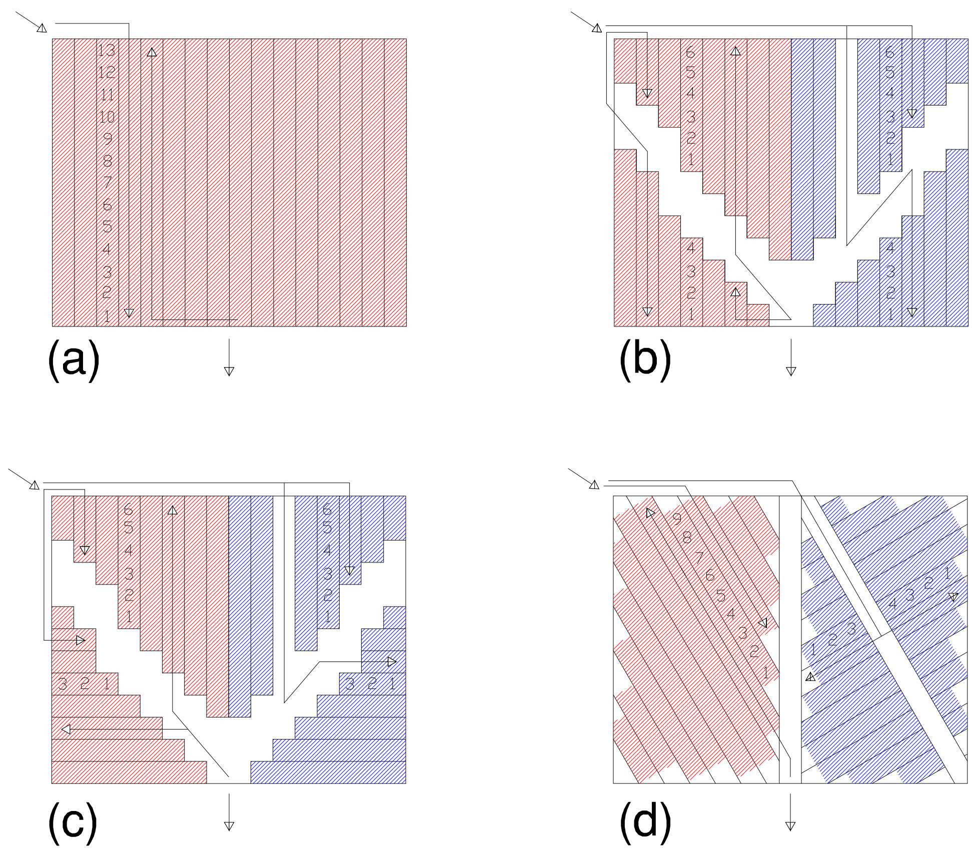

Figure 2.3: One P&D point with putaway and picking at different sides

In Figures 2.3(b) and 2.3(c) the right storage half-space is the region with conflicting flows. In the left half-space in both designs, the picker and restocker will simultaneously enjoy the benefit of the diagonal aisle. Unfortunately, in the right half-space restocker will be forced to take on the route that is inferior to his original rectilinear travel, in order for picker to benefit from the diagonal aisle. To resolve this situation one must take a closer look at the relationship between picking and replenishing and possibly investigate for daily peaks in both of the activities. If the number of picks per day is considerably greater than number of replenishments per day, the layout in Figures 2.3(b) and 2.3(c) should remain intact. In that case, loss on the putaway side will be offset by picking savings. If there exists a period during the day where surge in picking activity occurs, new aisle designs should be given serious thought. Since an average pick would take less time, the system would be more responsive and the peak period would not last as long. If one cannot judge about the picking to putaway ratio or daily peaks, or simply finds that there is no difference between picking and putaway, it is safest to employ traditional rectilinear lane and aisle structure in the right half-space.

Another way to support a disproportional increase in picking activity is proposed in Figure 2.3(d), where a cut-through diagonal aisle is inserted in the right half-space. On the picking side, the aisle is simply treated as a "no-storage space". In other words, once a picker has emptied the locations in a specific lane that are below the aisle, he proceeds to those in the same lane above the aisle. For a picker, a cut-through aisle does not bear any meaning except that he needs to traverse its empty space on his way to the locations above it. On the putaway side, however, things are different. The cut-through aisle is used to avoid the losses from other alternative designs. If a cut-through aisle and a storage lane intersect at approximately a right angle, the putaway travel pattern becomes rectilinear and resembles that of traditional storage. Provided that lane depths are acceptable, this is a safe way to keep the picking savings and as we will show later, gain small savings on the putaway side.

2.2.2 Two P&D points

Incentive to increase the number of P&D points might stem from a number of opportunities to realize savings, or from legitimate constraints imposed on the warehouse operations. So far, we have treated P&D points as simple origins of travel into the storage space. They might involve much more in reality: stretch wrappers or other consolidation devices, inspection stations, labeling and repackaging points and so on. What is usually tied to these small hubs within the warehouse are space requirements. For example, in a busy unit load warehouse items may be subject to a detailed and lengthy inspection taking place after the picking. In the case of a single P&D point at the loading dock, congestion is very likely to occur. A logical solution is to increase the number of P&D points. Simplified communication on the dock, less traffic, improved safety; all those might be motives for having more P&D points. It is worth remarking that many of the benefits obtained are not possible to quantify. While it provides unquestionable operational improvements, separating the points makes the problems with conflicting flows more severe. In Figures 2.4 and 2.5, we depict some of the design alternatives available to the decision maker.

Figure 2.4 involves putaway and picking from the bottom of the warehouse, whereas Figure 2.5 moves the putaway to the left side. In Figure 2.4 we see that conflicting flows can occur without inflow from the side. We assume that picking is evenly distributed inside

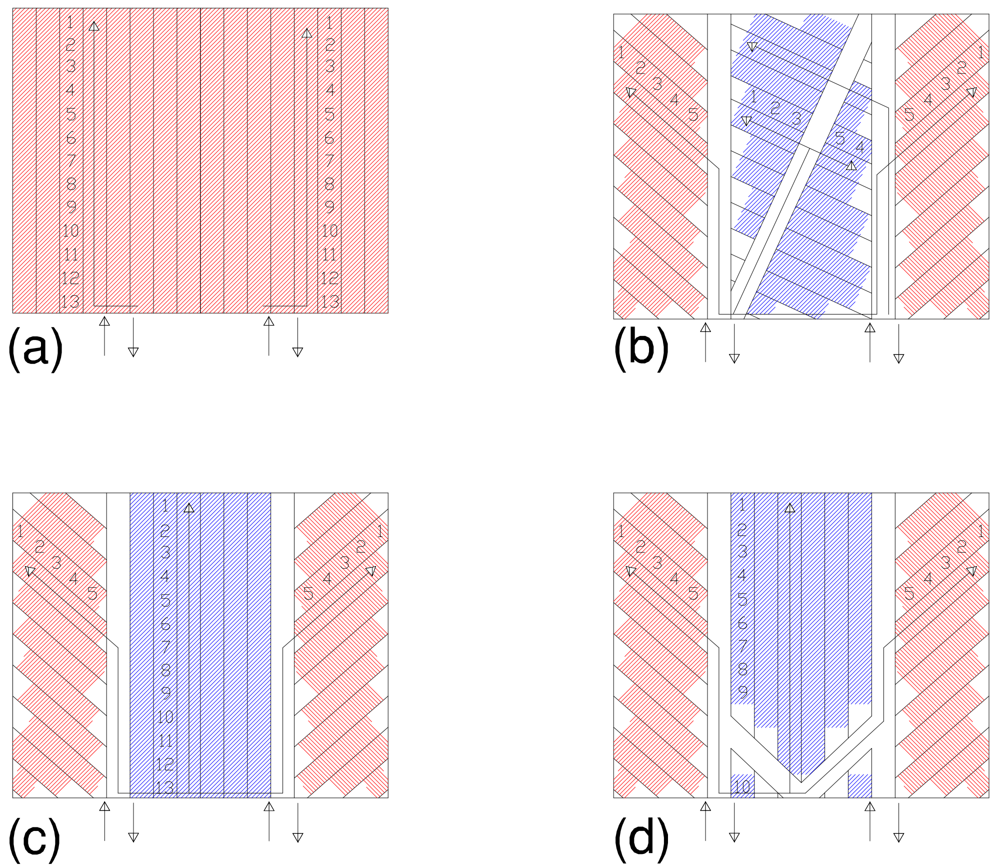

Figure 2.4: Two P&D point with putaway and picking from the same side

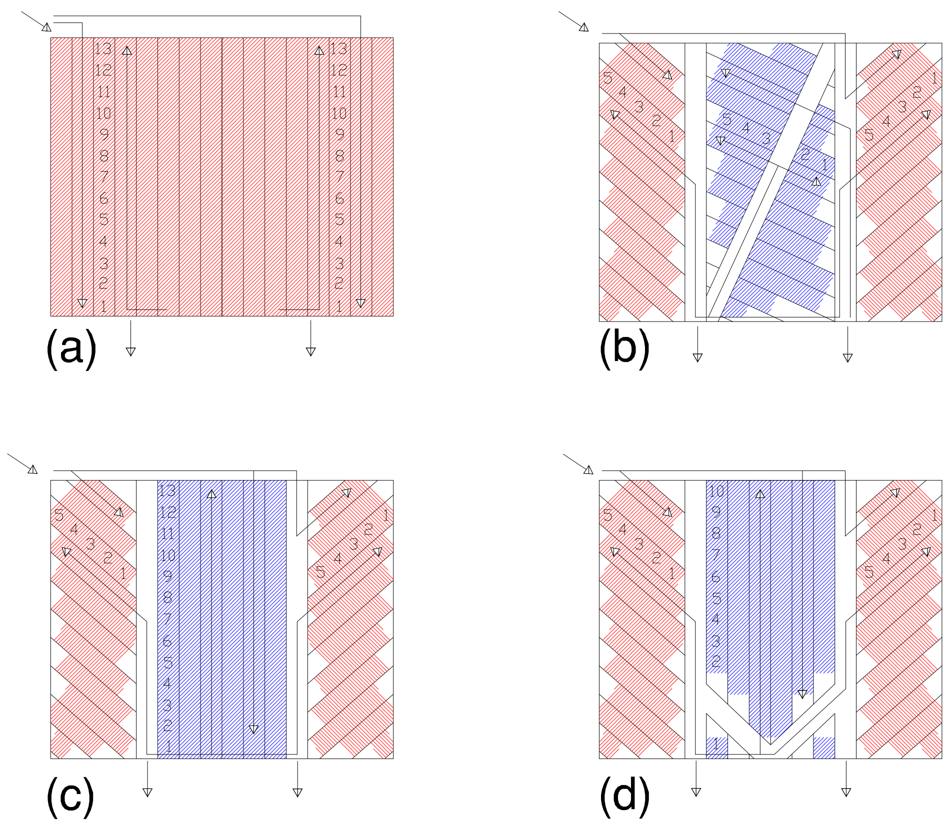

Figure 2.5: Two P&D points with putaway and picking at different sides

the whole warehouse space and that P&D points are chosen with equal probability. We designate the middle (blue) segment of the Figures 2.4(c) and 2.4(d) is designated to keep the traditional lane structure. A small improvement is achieved in 2.4(d), where we insert a diagonal aisle in the middle region to facilitate travels from the left to the right region and *vice versa*. Figure 2.4(b) features a cut-through aisle that defines the "neutral," middle region in a different way.

The same principles apply to the configurations with two P&D points and side putaway. The only difference is that all of the alternative designs suffer from conflicting flows in two of the three regions. Note that now there are three conflicting flows: two coming from picking and one from the putaway. This leads us to mirror the same middle region designs as in Figure 2.4. As for the rightmost region, we depict the alternative, diagonal lanes, and note that they should be given preference only if picking is more labor intensive than putaway.

2.3 Alternative layout of the facility

Having presented the basic modeling choices and underlying principles, we now tackle the manufacturer's layout. The management identified three major groups of skus — fast movers, less frequent and slow movers, and assigned them to appropriate regions in the warehouse. The topmost regions contain slow movers and we do not optimize its lane and aisle structure. For the two faster regions we devise a new aisle and lane structure and propose the layout in Figure 2.6.

In our proposition, we mostly use diagonal lanes and cut-through aisles, but we retain classical floor storage where necessary. In the regions A_1 and B_1 we are free to use diagonal lanes, but in the areas A_2, B_2 and B_3 we resort to cut-through aisles due to the conflicting

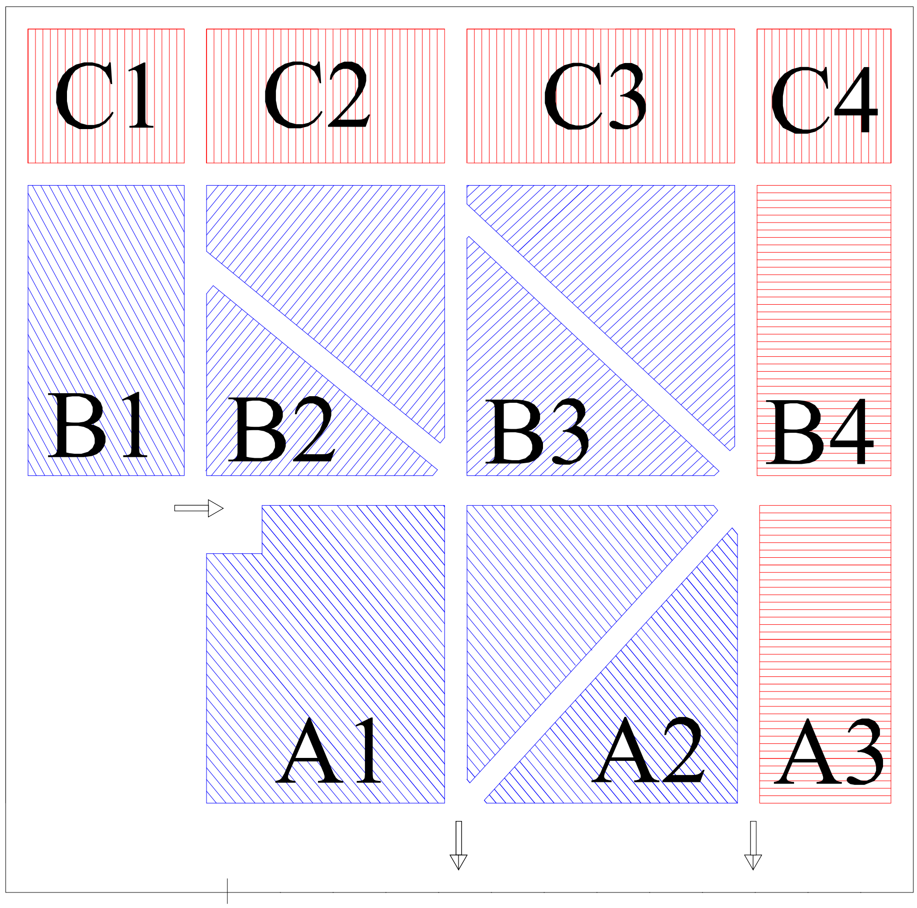

Figure 2.6: Improved Layout

flows. For the region A_3 which is the intersection of conflicting flows, and B_4 which is not, we keep the traditional storage.

Next, we estimate potential savings from the new layout. For this purpose, we develop a small computational aid. Improved storage blocks feature diagonal lanes and cut-through aisles, so our assessment of benefits needs to address these novelties. In doing so, we make one simplifying assumption — we disregard the width of the cut-through aisle. Although this leads to an overstatement of the anticipated savings, lost storage locations account for less than 1% of the total storage capacity of this warehouse. Therefore, we do not expect our assumption to significantly reduce the precision of the estimate.

We begin by expressing the cost of picking in a rectangular half-space with diagonal lanes and one P&D point in the lower left corner (Figure 2.7). The diagonal divides the storage space into two equal regions and we denote the average cost of picking in the lower and upper region as $E[L]$ and $E[U]$, respectively. In the region L, the picker is moving along the bottom of the rectangle and then parallel to the diagonal. In the region U, travel is along the left edge of the rectangle and then parallel to the diagonal. Because we assume that picking is uniformly distributed throughout the space, probabilities of choosing a pick in upper or lower regions will be equal, i.e., $p_l = p_u = 1/2$. From the mean value theorem for multiple integrals, expected cost of a single pick is:

$$E[C] = p_l E[L] + p_u E[U] = p_l \frac{1}{m_l} \int \int L + p_u \frac{1}{m_u} \int \int U \, .$$

Specifying the measures of the areas of integration,

$$m_l = \int_0^A \int_0^{A-x} 1 \, dydx = \frac{A^2}{2}$$
$$m_u = \int_0^h \int_0^{h-x} 1 \, dydx = \frac{h^2}{2} \,,$$

and from similarity of triangles, we obtain the following expressions:

$$\int\int L = \int_0^A \int_0^{A-x} (x + \frac{y\sqrt{A^2+h^2}}{A})dydx = \frac{1}{12}A^2(A + \sqrt{A^2+h^2})$$
$$\int\int U = \int_0^A \int_0^{h-x} (x + \frac{y\sqrt{A^2+h^2}}{h})dydx = \frac{1}{12}h^2(h + \sqrt{A^2+h^2})\,.$$

By means of elementary substitution we arrive at the average cost of a single pick:

$$E[C] = \frac{1}{6}(A + h + 2\sqrt{A^2+h^2})$$
$$= \frac{A+h}{6} + \frac{D}{3}\,.$$

This formula can be interpreted as traveling $(A+h)/6$ units along the bottom of the longer side of the warehouse rectangle and then $D/3$ units parallel to the diagonal.

In the case of the cut-through aisle, it is possible to develop the cost expression in a similar manner. A diagonal divides the warehouse rectangle into two equal triangle-spaces. Triangle altitudes (h_t) further divide the lower and upper triangles into four cost regions: L_1, L_2, U_1 and U_2. For simplicity, in Figure 2.8 we show only the structure of regions L_1 and L_2. A picker travels along the diagonal of the rectangle, and then parallel to the altitude into the space of the lower (or upper) triangle. Expected cost of making a pick is

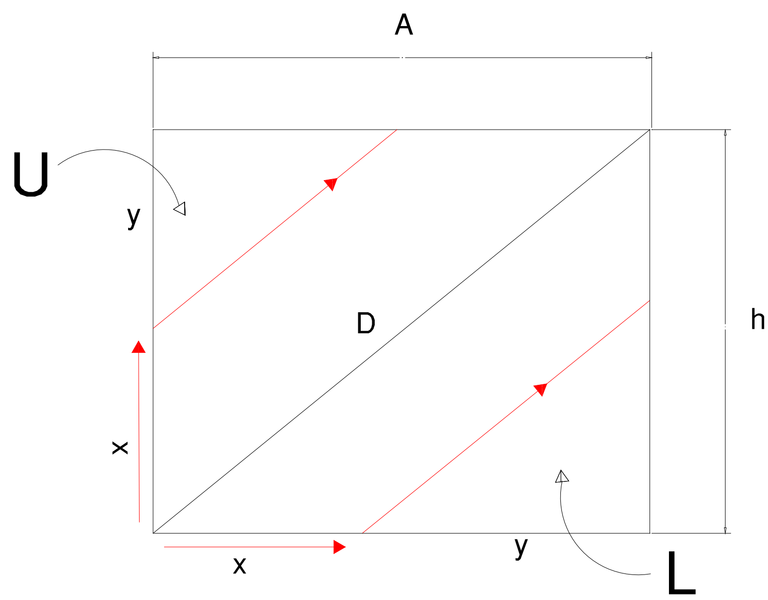

Figure 2.7: Travel patterns in a floor storage with diagonal lanes

given with:

$$
\begin{aligned}
E[C] &= p_{l_1}E[L_1] + p_{l_2}E[L_2] + p_{u_1}E[U_1] + p_{u_2}E[U_2] \\
&= p_{l_1}\frac{1}{m_{l_1}}\int\int L_1 + p_{l_2}\frac{1}{m_{l_2}}\int\int L_2 + p_{u_1}\frac{1}{m_{u_1}}\int\int U_1 + p_{u_2}\frac{1}{m_{u_2}}\int\int U_2 \,.
\end{aligned}
$$

Geometric properties of the warehouse layout lead to basic geometric relations below:

$$
\begin{aligned}
D &= \sqrt{A^2 + h^2} \\
h_t &= \frac{Ah}{D} = \frac{Ah}{\sqrt{A^2 + h^2}} \\
d_B &= \frac{A^2}{D} = \frac{A^2}{\sqrt{A^2 + h^2}} \\
A_{l_1} = A_{u_2} &= \frac{A^3 h}{2(A^2 + h^2)} \\
A_{l_2} = A_{u_1} &= \frac{A h^3}{2(A^2 + h^2)} \,,
\end{aligned}
$$

where d_B simply stands for the length of the diagonal from the vertex to the foot of the altitude, and other variables are self-explanatory.

Probabilities of choosing one among four regions are ratios of storage space in the picking region and total storage space available:

$$\begin{aligned}
p_{l_1} = p_{u_2} &= \frac{A_{l_1}}{Ah} = \frac{A_{u_2}}{Ah} = \frac{A^2}{2(A^2+h^2)} \\
p_{l_2} = p_{u_1} &= \frac{A_{l_2}}{Ah} = \frac{A_{u_1}}{Ah} = \frac{h^2}{2(A^2+h^2)}.
\end{aligned}$$

The first part of the pick travel is done along the diagonal, but the second part, along the line parallel to the altitude, depends on the length of the first part. We denote these functions as $d_1(x)$, $d_2(x)$, $d_3(x)$ and $d_4(x)$. They pertain to the regions L_1, L_2, U_1 and U_2 respectively:

$$\begin{aligned}
d_1(x) &= \frac{xh}{A} \\
d_2(x) &= \frac{A(D-x)}{h} \\
d_3(x) &= \frac{xA}{h} \\
d_4(x) &= \frac{h(D-x)}{A}.
\end{aligned}$$

Cost integrals for four picking regions are given with:

$$\begin{aligned}
\int\int L_1 &= \int_0^{d_B}\int_0^{d_1(x)} (x+y)dydx = \frac{A^4h(2A+h)}{6(A^2+h^2)^{3/2}} \\
\int\int L_2 &= \int_{d_B}^{D}\int_0^{d_2(x)} (x+y)dydx = \frac{Ah^3(3A^2+Ah+h^2)}{6(A^2+h^2)^{3/2}} \\
\int\int U_1 &= \int_0^{D-d_B}\int_0^{d_3(x)} (x+y)dydx = \frac{Ah^4(A+2h)}{6(A^2+h^2)^{3/2}} \\
\int\int U_2 &= \int_{D-d_B}^{D}\int_0^{d_4(x)} (x+y)dydx = \frac{A^3h(A^2+Ah+3h^2)}{6(A^2+h^2)^{3/2}}.
\end{aligned}$$

Measures of areas of integration are:

$$
\begin{aligned}
m_{l_1} &= \int_0^{d_B} \int_0^{d_1(x)} 1 \; dydx = \frac{A^3 h}{2(A^2+h^2)} \\
m_{l_2} &= \int_{d_B}^{D} \int_0^{d_2(x)} 1 \; dydx = \frac{Ah^3}{2(A^2+h^2)} \\
m_{u_1} &= \int_0^{D-d_B} \int_0^{d_3(x)} 1 \; dydx = \frac{Ah^3}{2(A^2+h^2)} \\
m_{u_2} &= \int_{D-d_B}^{D} \int_0^{d_4(x)} 1 \; dydx = \frac{Ah^3}{2(A^2+h^2)} \,.
\end{aligned}
$$

Since we derived all the necessary expressions, we proceed to formulate the final picking cost:

$$
\begin{aligned}
E[C] &= \frac{3A^2 + 2ah + 3h^2}{6\sqrt{A^2+h^2}} \\
&= \frac{D}{2} + \frac{Ah}{3D} \\
&= \frac{D}{2} + \frac{h_t}{3} \,,
\end{aligned}
$$

which is conveniently interpreted as first traveling one half of the diagonal's length, and then one third of the altitude length along a line parallel to the altitude.

The natural use of these formulae is to calculate the expected cost of a pick from the P&D point that is located at or near a vertex of a rectangular region. In the layout we propose, not all travels have the convenience of direct access to the storage space. Most of the time, a picker will travel a certain straight or rectilinear distance and then gain access to the picking locations through one of the points (not necessarily the P&D point) at the vertices of a rectangular storage space. Therefore, the approximate average cost of picking

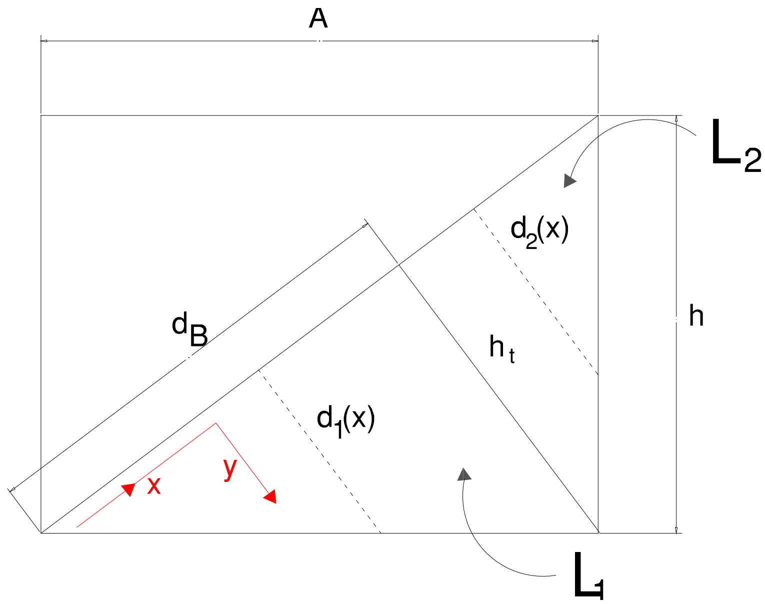

Figure 2.8: Travel patterns in a floor storage with diagonal lanes and cut-through aisles

in the block B_i from the P&D point j is expressed as:

$$E[B_{ji}] = \frac{S_i}{S}(R_{ji} + E[C_i]),$$

where S_i is the storage space in block i, S is the total storage space, R_{ji} is the distance from P&D point j to the vertex of a block i, and $E[C_i]$ is the expected cost of picking in a block from its vertex. Multiplying every expected cost B_{ji} by probability of choosing the P&D point j and summing across all P&D points and blocks, gives the total expected cost of picking in a number of blocks.

2.4 Results and conclusions

In CHAPTER 1 we assumed uniformly distributed activity throughout the entire storage space. Yet, we have stated that the manufacturer's operations indicate the contrary — dedicated storage policy, with separate regions reserved for skus with different activity. Therefore, we believe that the costs should be separated region by region, with random picking assumed inside each region, and the cost of operation measured with the formulae we developed. More than that, we believe that for practical purposes, picking and putaway costs should be separated in a warehouse with conflicting flows. Dividing the costs by region and type (picking or putaway) helps to assess accurately the importance of the regions from the cost standpoint and possible prevalence of a single cost type.

We calculated the expected cost and savings for picking and putaway in the regions holding fast movers (lower region) and less frequent skus (middle region). Picking savings for the lower and upper region are 10.54% and 9.18%, respectively. When it comes to the putaway cost, the new aisle and lane structure saves 10.57% in the lower region and 9.84% in the upper region. The numbers do look promising but should be put in the right context. Because of a number of assumptions we made, we think of this result as an approximation or even as a proof that significant savings are possible, rather than the guarantee that the savings are going to amount exactly to what we have calculated. However, the principles we presented, if applied correctly, promise improvement.

In addition to the quantitative aspect of the layout we propose, there exist other, more practical issues that need to be addressed. Similar to our cross aisle designs for rack storage, worker orientation and routing might become an obstacle. A very deep diagonal lane might cause a worker to lose orientation. Frequent visits to lanes with different orientations might have a similar effect. Additional clearance on the exit of a cut-through aisle may be required to prevent interference with a worker's field of vision.

Although models to determine optimal lane depth for each sku exist (Bartholdi and Hackman (2007)), it is our impression that most warehouse managers prefer having only a small number of lane depths. Such a solution is robust with respect to day-to-day operations, but it utilizes the floor space optimally only if skus have similar optimal lane depths. On the other hand, a design with diagonal lanes features a wide variety of lane depths. If the lane depths are uniform across the sku spectrum, there is not much sense in insisting on a diagonal design. However, if that is not the case, and the warehouse keeps track of its sku data in a way that enables easy calculation of the optimal lane depths, diagonal designs are a promising choice. Another potential problem is that the deepest diagonal lanes might be much deeper than any lane in the traditional design. If the new lane depths are not suitable for the skus currently present in the warehouse, one possible solution is to design smaller blocks with shorter diagonals.

Bibliography

Barthold, J. J. and Hackman, S. T. (2007). *Warehouse and Distribution Science*. Version 0.85.

Berry, J. R. (1968). Elements of warehouse layout. *International Journal of Production Research*, 7(2):105–121.

Francis, R. L. (1967). Sufficient conditions for some optimum property facility designs. *Operations Research*, 15(3):448–466.

Gue, K. R. and Meller, R. D. (2006). Aisle configurations for unit load warehouses. Technical report, Auburn University, Department of Industrial and Systems Engineering, Auburn, AL 36849.

Lazoff, D. M. and Sherman, A. T. (1994). An exact formula for the expected wire length between two randomly chosen intervals. Technical Report TR CS-94-08, University of Maryland Baltimore County, Computer Science Department, Baltimore, MD 21228-5398.

Roberts, S. D. and Reed, R. (1972). Optimum warehouse bay configurations. *AIIE Transactions*, 4(3):178–185.

White, J. A. (1972). Optimum design of warehouses having radial aisles. *AIIE Transactions*, 4(4):333–336.

Appendix — Tables of savings over orthogonal aisle design

	h=50				h=75				h=100				h=125			
n/w	Λ	X	V	LB	Λ	X	V	LB	Λ	X	V	LB	Λ	X	V	LB
11/1	0	2.21	4.78	22.10	0	2.32	4.85	20.16	0	2.23	4.60	18.05	0	2.09	4.28	16.20
15/1	0.66	2.72	5.41	22.50	0.94	2.92	5.76	21.81	1.01	2.86	5.64	20.32	1.02	2.72	5.38	18.75
19/1	1.06	2.97	5.62	22.10	1.36	3.28	6.23	22.46	1.45	3.28	6.27	21.61	1.45	3.17	6.10	20.41
23/1	1.31	3.08	5.62	21.36	1.65	3.49	6.44	22.55	1.76	3.56	6.65	22.28	1.77	3.49	6.58	21.46
27/1	1.48	3.12	5.51	20.50	1.85	3.62	6.50	22.31	1.99	3.75	6.85	22.55	2.02	3.73	6.90	22.10
31/1	1.59	3.11	5.35	19.60	1.99	3.68	6.47	21.90	2.16	3.87	6.94	22.56	2.21	3.89	7.09	22.45
35/1	1.67	3.07	5.17	18.73	2.09	3.70	6.38	21.38	2.28	3.95	6.96	22.40	2.36	4.00	7.20	22.58
39/1	1.72	3.02	4.99	17.89	2.16	3.69	6.26	20.82	2.37	3.99	6.93	22.13	2.47	4.09	7.24	22.57
11/1.5	0	0.93	3.06	22.15	0	1.36	3.46	20.25	0	1.46	3.43	18.13	0	1.44	3.28	16.27
15/1.5	0	1.59	3.95	22.49	0	2.05	4.55	21.87	0	2.15	4.60	20.38	0	2.12	4.47	18.81
19/1.5	0	1.96	4.33	22.03	0.30	2.48	5.14	22.48	0.51	2.62	5.33	21.65	0.61	2.61	5.27	20.46
23/1.5	0.23	2.16	4.47	21.26	0.68	2.76	5.46	22.54	0.90	2.95	5.79	22.30	1.00	2.97	5.82	21.50
27/1.5	0.47	2.27	4.48	20.37	0.96	2.93	5.60	22.28	1.19	3.17	6.06	22.56	1.30	3.23	6.19	22.13
31/1.5	0.69	2.32	4.41	19.45	1.17	3.04	5.64	21.85	1.42	3.33	6.21	22.55	1.53	3.42	6.32	22.46
35/1.5	0.85	2.34	4.31	18.57	1.33	3.09	5.61	21.31	1.59	3.42	6.27	22.38	1.72	3.56	6.57	22.58
39/1.5	0.97	2.33	4.18	17.72	1.45	3.12	5.55	20.74	1.72	3.49	6.28	22.10	1.87	3.66	6.65	22.56
11/2	0	0	1.40	22.20	0	0.39	2.10	20.33	0	0.68	2.29	18.21	0	0.80	2.29	16.33
15/2	0	0.45	2.52	22.47	0	1.18	3.35	21.92	0	1.44	3.58	20.45	0	1.52	3.57	18.87
19/2	0	0.93	3.08	21.96	0	1.68	4.07	22.50	0	1.96	4.40	21.70	0	2.04	4.45	20.51
23/2	0	1.23	3.35	21.15	0	2.02	4.49	22.52	0	2.33	4.93	22.33	0.25	2.43	5.06	21.54
27/2	0	1.42	3.47	20.23	0.13	2.24	4.72	22.24	0.43	2.59	5.27	22.56	0.60	2.72	5.48	22.15
31/2	0	1.54	3.49	19.30	0.28	2.39	4.82	21.79	0.70	2.78	5.48	22.54	0.88	2.94	5.76	22.47
35/2	0.10	1.61	3.46	18.40	0.51	2.49	4.86	21.25	0.92	2.91	5.59	22.36	1.10	3.1	5.95	22.59
39/2	0.25	1.65	3.40	17.55	0.71	2.54	4.84	20.65	1.09	3.00	5.64	22.06	1.28	3.23	6.07	22.55
11/2.5	0	0	0	22.25	0	0	0.78	20.42	0	0	1.16	18.29	0	0.14	1.31	16.40
15/2.5	0	0	1.14	22.44	0	0.29	2.17	21.96	0	0.72	2.56	20.51	0	0.91	2.68	18.93
19/2.5	0	0	1.87	21.88	0	0.87	3.02	22.51	0	1.29	3.48	21.74	0	1.48	3.64	20.56
23/2.5	0	0.30	2.27	21.03	0	1.27	3.53	22.51	0	1.71	4.09	22.35	0	1.9	4.31	21.57
27/2.5	0	0.56	2.49	20.09	0	1.55	3.84	22.21	0	2.01	4.49	22.57	0	2.22	4.78	22.18
31/2.5	0	0.74	2.60	19.14	0	1.74	4.02	21.73	0.02	2.23	4.75	22.53	0.24	2.46	5.11	22.48
35/2.5	0	0.87	2.65	18.23	0	1.87	4.11	21.17	0.27	2.39	4.91	22.33	0.50	2.65	5.34	22.59
39/2.5	0	0.95	2.65	17.37	0.12	1.97	4.14	20.57	0.48	2.50	5.00	22.03	0.71	2.79	5.49	22.55
11/3	0	0	0	22.29	0	0	0	20.50	0	0	0.07	18.37	0	0	0.36	16.47
15/3	0	0	0	22.41	0	0	1.03	22.01	0	0	1.57	20.57	0	0.30	1.81	18.99
19/3	0	0	0.71	21.80	0	0.05	1.99	22.52	0	0.62	2.58	21.78	0	0.90	2.83	20.61
23/3	0	0	1.23	20.91	0	0.52	2.60	22.49	0	1.08	3.26	22.37	0	1.36	3.56	21.61
27/3	0	0	1.55	19.94	0	0.85	2.99	22.16	0	1.42	3.72	22.57	0	1.71	4.08	22.20
31/3	0	0	1.75	18.98	0	1.09	3.23	21.67	0	1.67	4.04	22.52	0	1.98	4.46	22.50
35/3	0	0.13	1.86	18.05	0	1.26	3.37	21.10	0	1.85	4.24	22.31	0	2.19	4.72	22.59
39/3	0	0.26	1.92	17.19	0	1.39	3.45	20.48	0	2.00	4.37	21.99	0.16	2.35	4.91	22.54

Table 1: Comparison of savings over orthogonal aisle design — Λ shape, crossdock, Flying-V shape and Travel-by-flight (Lower bound for cost)

	h=50			h=75			h=100			h=125		
n/w	Smp	Str	V	Smp	Str	V	Smp	Str	V	Smp	Str	V
11/1	4.33	4.73	4.78	4.55	4.83	4.85	4.38	4.58	4.60	4.12	4.27	4.28
15/1	4.81	5.35	5.41	5.32	5.73	5.76	5.31	5.62	5.64	5.12	5.36	5.38
19/1	4.91	5.53	5.62	5.66	6.19	6.23	5.83	6.24	6.27	5.75	6.08	6.10
23/1	4.84	5.52	5.62	5.78	6.39	6.44	6.11	6.61	6.65	6.14	6.56	6.58
27/1	4.69	5.40	5.51	5.77	6.44	6.50	6.23	6.81	6.85	6.38	6.87	6.90
31/1	4.51	5.23	5.35	5.68	6.40	6.47	6.26	6.90	6.94	6.50	7.06	7.09
35/1	4.32	5.05	5.17	5.56	6.30	6.38	6.22	6.91	6.96	6.55	7.16	7.20
39/1	4.14	4.85	4.99	5.41	6.18	6.26	6.14	6.87	6.93	6.54	7.20	7.24
11/1.5	2.67	2.98	3.06	3.20	3.42	3.46	3.24	3.41	3.43	3.14	3.26	3.28
15/1.5	3.40	3.85	3.95	4.14	4.50	4.55	4.30	4.57	4.60	4.24	4.45	4.47
19/1.5	3.68	4.22	4.33	4.62	5.08	5.14	4.92	5.30	5.33	4.95	5.25	5.27
23/1.5	3.75	4.34	4.47	4.84	5.39	5.46	5.28	5.74	5.79	5.41	5.79	5.82
27/1.5	3.71	4.33	4.48	4.91	5.52	5.60	5.47	6.01	6.06	5.69	6.15	6.19
31/1.5	3.63	4.26	4.41	4.90	5.55	5.64	5.55	6.15	6.21	5.79	6.28	6.32
35/1.5	3.51	4.14	4.31	4.83	5.52	5.61	5.56	6.21	6.27	5.95	6.53	6.57
39/1.5	3.39	4.01	4.18	4.73	5.44	5.55	5.53	6.21	6.28	5.98	6.60	6.65
11/2	1.05	1.27	1.4	1.87	2.04	2.10	2.12	2.25	2.29	2.16	2.26	2.29
15/2	2.02	2.37	2.52	2.98	3.28	3.35	3.30	3.53	3.58	3.36	3.54	3.57
19/2	2.49	2.92	3.08	3.59	3.99	4.07	4.02	4.35	4.40	4.15	4.42	4.45
23/2	2.69	3.18	3.35	3.91	4.40	4.49	4.46	4.88	4.93	4.67	5.02	5.06
27/2	2.76	3.28	3.47	4.07	4.62	4.72	4.72	5.21	5.27	5.01	5.44	5.48
31/2	2.76	3.30	3.49	4.12	4.72	4.82	4.85	5.41	5.48	5.23	5.72	5.76
35/2	2.72	3.26	3.46	4.11	4.74	4.86	4.91	5.51	5.59	5.35	5.90	5.95
39/2	2.66	3.19	3.40	4.07	4.71	4.84	4.92	5.55	5.64	5.42	6.01	6.07
11/2.5	0	0	0	0.57	0.68	0.78	1.01	1.10	1.16	1.20	1.27	1.31
15/2.5	0.70	0.93	1.14	1.84	2.08	2.17	2.31	2.51	2.56	2.49	2.64	2.68
19/2.5	1.33	1.65	1.87	2.57	2.92	3.02	3.13	3.42	3.48	3.35	3.59	3.64
23/2.5	1.66	2.05	2.27	3.00	3.42	3.53	3.65	4.02	4.09	3.94	4.26	4.31
27/2.5	1.84	2.25	2.49	3.23	3.72	3.84	3.97	4.42	4.49	4.34	4.73	4.78
31/2.5	1.93	2.35	2.60	3.36	3.89	4.02	4.16	4.67	4.75	4.60	5.05	5.11
35/2.5	1.96	2.39	2.65	3.40	3.97	4.11	4.27	4.82	4.91	4.76	5.27	5.34
39/2.5	1.95	2.38	2.65	3.41	3.99	4.14	4.31	4.90	5.00	4.86	5.41	5.49
11/3	0	0	0	0	0	0	0	0	0.07	0.27	0.30	0.36
15/3	0	0	0	0.73	0.90	1.03	1.34	1.49	1.57	1.63	1.75	1.81
19/3	0.22	0.42	0.71	1.58	1.85	1.99	2.25	2.50	2.58	2.57	2.78	2.83
23/3	0.68	0.94	1.23	2.10	2.46	2.60	2.84	3.17	3.26	3.22	3.50	3.56
27/3	0.96	1.25	1.55	2.42	2.83	2.99	3.23	3.63	3.72	3.66	4.02	4.08
31/3	1.12	1.43	1.75	2.60	3.07	3.23	3.48	3.94	4.04	3.97	4.39	4.46
35/3	1.22	1.54	1.86	2.71	3.20	3.37	3.63	4.13	4.24	4.18	4.65	4.72
39/3	1.28	1.60	1.92	2.76	3.27	3.45	3.71	4.26	4.37	4.31	4.83	4.91

Table 2: Comparison of savings over orthogonal aisle design — Simple-V, Straight-V and Flying-V shapes