Supply chain planning for hurricane response with information updates

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Dissertation Abstract

Supply chain planning for hurricane response with information updates

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Planning inventories of supplies for the hurricane season can be challenging. For instance, in 2004, manufacturing and retail firms experienced stock outs because they were not prepared for responding to the demand caused by several hurricanes that swept through the state of Florida in the southeastern United States. In 2005, these firms again experienced shortages due to the extreme demand surge caused by Hurricane Katrina. These experiences motivated firms to be pro-active and more aggressive in their approach to stocking hurricane supplies in 2006, resulting in large amounts of excess inventory because of an inactive hurricane season.

While there are many issues, such as evacuation decisions and cooperation among government agencies that need to be addressed in terms of developing effective plans for responding to disastrous hurricanes, this research investigates stochastic production/inventory control problems that are relevant to planning for potential disaster relief activities associated with hurricane events. In particular, this study considers supply chain organizations

who experience demand surge for items such as flashlights, batteries, and gas-powered generators, where the magnitude of the demand surge is influenced by various characteristics of a hurricane season and/or a specific hurricane. These organizations are faced with challenging procurement and production decisions since the hurricane logistics planning process is complicated by the uncertainties associated with the number of hurricanes that will occur during a hurricane season, hurricane intensities, and locations affected during the season.

This study aims to assist major corporations to quickly and cost effectively respond to demand surges caused by hurricanes. In this dissertation, two different types of stochastic inventory models are introduced to determine the appropriate hurricane stocking levels for these organizations. The first two models address a hurricane stocking problem that is relevant to disaster recovery planning. In this context, the disaster recovery plan requires to determine optimal ordering/production policies for supply chain organizations for whom, the magnitude of the demand surge is influenced by various characteristics of an observed storm during the hurricane season. The third model introduces a multi period hurricane inventory control problem that allows the managers to adjust inventory decisions during the pre-season periods as demand realizes to reserve for the hurricane season demand. The model enables decision makers (DMs) to determine the optimal level of reserved hurricane stock while satisfying the demand associated with the pre-season periods. Finally, the work accomplished for each chapter of this dissertation is summarized with their relevance and usefulness, and possible extensions of this research and future study are proposed.

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Chapter 1

Introduction

This dissertation is motivated by the impact of increased hurricane activity in the United States, particularly in the Gulf Coast region. Many government agencies, not-for-profit organizations, and private corporations assume leading roles in positioning supplies, equipment, and personnel both during and after a major hurricane. These organizations are faced with challenging supply chain and logistics decisions to ensure that supplies, equipment, and personnel are readily available at the right places, at the right times, and in the right quantities. In addition to the complexities associated with supply chain and logistics planning in general, inventory planning decisions made before the hurricane season are complicated by forecasts related to the number of hurricanes that are expected to develop during the ensuing season. Similarly, supply chain decisions made during the hurricane season (after a tropical disturbance or depression is initially observed) are especially complicated by the dynamics and uncertainties associated with various hurricane characteristics such as its diameter, its projected path, and its intensity along the path. Therefore, the objective of this research can be stated more generally as determining the optimal level of supply chain readiness with respect to hurricane preparedness.

Chapter 2 investigates a disaster recovery planning problem encountered by manufacturing and retail organizations whose demand for products such as batteries, flashlights, and gas-powered generators is significantly influenced by the characteristics of an observed storm during the hurricane season. The planning horizon begins during the initial stages of storm development, when a particular tropical depression or disturbance is first observed, and ends when the storm dissipates. If an observed tropical disturbance or depression materializes into a major storm, then manufacturing and retail firms will often experience demand surge for hurricane supplies caused by increased consumer activity and additional requests from service organizations who use these and other supplies to carry out initial response operations. Without an effective disaster recovery plan in place, these manufacturing and retail firms are not likely to satify a hurricane induced demand surge. Consequently, the results of an ineffective or nonexistent disaster recovery plan include (i) stock outs of hurricane supplies such as those mentioned above, (ii) low service levels with respect to fulfilling hurricane related demand, and (iii) an extended recovery period associated with rebuilding inventory levels capable of supporting regular demand and order fulfillment not directly related to the storm. From this perspective, the disaster recovery plan for a manufacturing or retail firm with respect to preparing for a potential demand surge caused by an observed storm entails determining appropriate inventory levels for hurricane supplies that will enable the firm to (i) fulfill hurricane related demands during and immediately following the storm and (ii) minimize the time and resources invested in rebuilding target inventory levels to support normal operations after the storm.

In order to address these issues, this chapter proposes a framework that explicitly and dynamically incorporates hurricane predictions associated with an observed storm into the decision process, which also accounts for the inherent trade-off between hurricane forecast accuracy and logistics cost efficiency as a function of time. More specifically, predictions associated with a storm during its initial stages of development are often uninformative with respect to disaster recovery planning, at least relative to predictions during the later stages. From this perspective, it is more beneficial to postpone decisions until an accurate forecast is observed some time during the later stages of the planning horizon. On the other hand, it is also beneficial to implement supply chain decisions during the earlier stages of storm development to avoid potential inefficiencies and complications that may arise during the later stages. For instance, manufacturing firms constrained by production capacities may be forced into expensive overtime labour or outsourcing in order to respond to demand surge caused by a major hurricane. Manufacturing and retail firms may also incur additional expenses for faster modes of transportation in order to ensure that hurricane supplies are available when and where they are needed. Under more extreme circumstances, it might be impossible to deliver hurricane supplies during the later stages of the planning horizon because primary components of the transportation network could be inaccessible due to damages caused by the storm. Given the above-mentioned problems encountered in practice and trade-off between forecast accuracy and logistics cost, the objective of this research is to determine the optimal inventory level for hurricane supplies and how long after a tropical depression or disturbance is observed this inventory decision should be postponed such that the trade-off between logistics cost efficiency and hurricane forecast accuracy is optimized.

The forecast updating approach to hurricane supply chain planning described in this chapter emphasizes preparing for potential *extreme* hurricanes such as Hurricane Katrina. For the purposes of this chapter, an extreme hurricane is characterized by two specific features: (i) abnormal surge in demand for hurricane supplies and (ii) statistically low probability of occurrence. In order to minimize the expected costs associated with supply chain planning for a potential extreme event, a statistical decision model based on the traditional expected value approach that incorporates the cost and probability associated with the extreme event is not likely to lead to an appropriate and practical decision, just as the mean is often not an accurate indicator of central tendency for skewed data that contain outliers. However, the proposed Bayesian updating framework allows the decision-maker to postpone his decision until there is enough information available to accurately predict whether or not a hurricane will become extreme, which is more likely to lead to an appropriate decision than the traditional expected value approach.

Chapter 3 addresses a disaster recovery planning (DRP) problem for a one supplier multi retailer supply chain as a result of a hurricane event. This chapter investigates the suppliers' hurricane related inventory decisions to ensure that the right amount of hurricane supplies are readily available at the right retailers, at the right times, and in the right quantities. More specifically, the supplier is considered as the decision-maker (DM), for whom the DRP horizon begins when a particular tropical depression is first observed, and ends with the average storm life-cycle time (120h). The order/production decisions implemented earlier on the planning horizon will be less expensive for the supplier relative to those implemented in the later stages. On the other hand, the tropical cyclone information acquired during its initial development stage is not accurate enough to make stocking decisions. From this perspective, it is necessary for the supplier to balance the trade-off between cost efficiency and hurricane forecast accuracy. More specifically, the inventory decision should be postponed until this kind of optimization is achieved to minimize the financial risks associated with over/under preparation.

This chapter is an extension of previous work [58], which only focuses on the wind speed data to characterize the hurricane event. More specifically, this chapter accounts for the tropical cyclone path information in addition to the wind speed data to enhance the accuracy of predictions. In order to achieve this, the National Hurricane Center's (NHC) new wind speed probability model, which is based on the official forecasts of the cyclone's center position and intensity (maximum 1-min surface wind speed) is used. The official forecasts are issued every 6 hours, and each contains projections at cumulative 12 hour forecast periods up to 5 days. These forecasts are compared with the best track data, which are obtained after the NHC's post storm analysis of all available storm data. The NHC predicts hurricane's tracks and intensity by implementing various statistical, dynamical or combined models. Since 2005, the NHC has been making storm forecasts for locations based on wind speed probability maps. The wind speed probability model allows DMs to predict the current chance that a location will be hit by damaging tropical cyclone winds. Therefore, businesses and industry can better evaluate the risks associated with a tropical cyclone at their locations. For instance, insurers may obtain real-time information on the likelihood of a potential loss for their portfolios ([70]). This quantitative information is also important for many organizations that provide hurricane supplies to the retailers whose demands are affected by hurricanes. The NHC's wind speed probability predictions assist these organizations in their inventory related decisions, since they provide information as to how the demand will fluctuate at their locations. Therefore, they can have more accurate estimates of the characteristics associated with the hurricane season.

Chapter 4 presents a stochastic programming inventory model, which is based on general predictions regarding the ensuing hurricane season such as those issued by the NHC. This research considers allocating hurricane stock to meet the hurricane season demand while meeting the period's demand. In order to achieve this objective in a cost efficient way, this chapter introduces a stochastic programming model that is converted to a deterministic linear program where the potential realizations of the discrete demand distribution are introduced as the scenarios of the demand process. Additionally, the hurricane season demand distribution is described by implementing a Markov chain approach. The states of the Markov chain are defined by a finite number of hurricane landfall count rates. The hurricane count rate predictive probabilities are introduced as the stationary transition probabilities of the Markov chain. The underlying demand distribution is then described with respect to the pre-season and hurricane season demand distributions. The stochastic programming model is used to determine the optimal quantity and timing of the inventory decisions. The scenario reduction approach introduced by [38] is also implemented to determine the optimal ordering policy. The optimal values obtained from the solution of the stochastic programming model are compared with the results of the reduced models. The stochastic model presented in this chapter enables DMs to determine an appropriate stock level that should be available at the beginning of the hurricane season and allows multiple production/procurement decisions.

Chapter 2

Emergency Inventory Planning During the Hurricane Season

This chapter introduces a stochastic inventory control problem that is relevant to proactive disaster recovery planning as it relates to preparing for potential hurricane activity. The disaster recovery planning (DRP) problem encountered by manufacturing and retail organizations is characterized by (i) the financial risks associated with over-preparing if inventory levels exceed the demand caused by a hurricane, (ii) the financial risk of expensive emergency procurement / production if inventory levels are not sufficient to meet demand, (iii) the social risks characterized by long customer waiting times and prolonged human suffering also due to insufficient inventory levels, and (iv) the financial risks associated with restoring business continuity after a hurricane. As an example, the author has interacted with production and logistics managers for large scale manufacturing and retail organizations (who wish to remain anonymous) that experienced demand surges due to hurricane events. These firms felt that they could have been better prepared for responding to demand surge for hurricane related supplies caused by several hurricanes that affected the southeastern United States during the 2004 hurricane season, and again in 2005 after Hurricane Katrina. On the other hand, these organizations were stuck with large amounts of excess inventory at the end of the inactive 2006 hurricane season. In order to assist these organizations in their inventory related decisions, an optimal stopping model with Bayesian hurricane information updates is introduced. Additionally, a dynamic programming algorithm is implemented to solve the optimal stopping problem. The planning horizon for the Bayesian inventory model initiates when a tropical depression or disturbance is observed, and ends with the hurricane's life-cycle time. The objective is to determine the optimal ordering policy by leveraging the hurricane related information updates.

The remainder of this chapter is organized as follows: Section 2.1 reviews relevant literature, which includes disaster recovery planning, disaster relief planning, and inventory control with Bayesian updates. In section 2.2, the fundamental concepts related to optimal stopping problems are introduced, along with the notational conventions used in this chapter. In section 2.3, the mathematical formulation of the problem is presented, followed by the solution approach. Then, a numerical example is given to illustrate how the methodology can be implemented in practice. In section 2.6, an extension of the base model that accounts for the inability to carry out an ordering decision as a result of damages caused by the hurricane is presented. Finally in section 2.7, the conclusions and future research directions are presented.

2.1 Literature Review

Disruptions in business continuity caused by natural and man-made disasters demonstrate the need for organizations to develop effective Disaster Recovery Plans (DRPs). For example, immediately following the World Trade Center attacks of September 11, 2001, the United States government's protective measures inhibited the daily operations of many corporations. One such corporation was Ford Motor Company who eventually closed five U.S. plants and reported a 13% decline in vehicle production ([60]). Another example of business continuity disruption is Telefon AB L.M. Ericsson, a mobile phone manufacturer who estimated $400 million in lost sales during the year 2000 when a lightening induced fire shut down its sole supplier (Royal Phillips Electronics) of one of the chips needed for production ([23]).

The disaster recovery planning research literature with respect to developing an organization's business continuity plan seems to have originated with computer network security applications, although there is some early evidence of DRP in manufacturing environments (e.g., [43]). A noticeable increase of DRP applications in supply chain environments, which is often referred to as *disruption management* or *disruption planning*, occurred shortly after the World trade Center attacks mentioned above (e.g., [23], [49], and [73]). Quantitative approaches to DRP in supply chain management based on Operations Research and Management Science (ORMS) methods are discussed in [57] and [89]. ORMS approaches to DRP in airline operations, project management, machine scheduling, and other environments are presented in [89]. Finally, general frameworks for integrating DRP and ORMS are presented in [6], [16] and [79].

This research is also applicable to *disaster relief planning* problems encountered by military, government, and service organizations. While the focus of DRP is business continuity planning, disaster relief planning falls under the more general topic of emergency management. Therefore, the author also reviews relevant research from the emergency management / disaster relief logistics literature.

In terms of hurricane logistics planning, the majority of the literature entails the logistical aspects of the emergency evacuation process. Examples include a simulation based optimization approach to hurricane evacuation planning for Ocean City, Maryland ([92]), a methodology for establishing evacuation zones in the New Orleans, Louisiana area ([85]), and a Markov decision model that uses hurricane predictions to determine when and if an evacuation should be ordered ([67]). For a comprehensive review of hurricane evacuation planning and management protocols, the reader is referred to [86] and [87].

Hurricane logistics issues outside the realm of evacuation has received considerably less attention. Sheppard [74] discusses disaster relief logistics associated with the storage and distribution of public water utilities in the U.S. Virgin Islands, and [4] reports the experiences of firms such as Home Depot, Wal-mart, and CVS Pharmacy who were able to successfully support relief operations after Hurricane Katrina. To the best of the author's knowledge, there are no other published materials that address disaster relief logistics specific to hurricanes (except for other hurricane evacuation research). Additionally, note that neither [4] nor [74] involves quantitative approaches to hurricane logistics planning.

General frameworks for disaster relief logistics planning have also been presented in the research literature. For example, [81] discusses the role of private sector supply chains and their interactions with humanitarian organizations in providing logistics support to victims of disaster. Thomas [78] proposed a reliability and decision analysis framework for assessing the readiness of a contingency logistics network, and [39] proposed interaction network optimization as a general framework for disaster management. Disaster relief logistics planning associated with hazards other than hurricanes has also been addressed in the literature. Examples include helicopter logistics operations ([9]), inventory control for long-term humanitarian response ([13]), responding to pandemic outbreaks ([39]), and assigning electric power repair crews and depots to various areas in need after a natural disaster ([69]).

Finally, relevant research from the inventory control literature is reviewed. More specifically, the author discusses inventory models characterized by Bayesian forecast updates

(Sethi et al. [72] summarize non-Bayesian approaches to forecast updating that have appeared in the inventory literature). Bayesian updating has consistently been an active area of research in the inventory control literature since the pioneering work of Dvoretzky et al. [29]. A comprehensive review of this literature would then prove to be quite extensive. Hence, the related presentation is limited only to the more recent advances in Bayesian inventory research (some earlier papers that are often cited include [7], [8], [59], [62], [33], [42], and [71]). The motivation for applying Bayesian methodologies to problems of inventory control with stochastic demand is to make informed stocking decisions based on accurate demand forecasts. The general approach to doing so is to represent one or more of the demand distribution parameters as a random variable, but these parameter distributions are estimates. Bayesian inventory modeling then involves using early sales or demand information as it becomes available to improve the estimated parameter(s) distribution(s), thereby resulting in more information rich stocking decisions. This approach has recently been applied to spare parts inventory management [5], quick response inventory control [22], partially observed demand [27], supply chain contract design [88], quantity and pricing decisions [90], and products with short life cycles ([91]). The approach adopted in this chapter, which involves updating hurricane intensity predictions based on wind speed information updates, can be classified as a Bayesian inventory model with multiple delivery modes ([20], [21], [72]). Of these, this chapter is most closely related to [21], who model an inventory problem as an optimal stopping problem with Bayesian updates and normal demand distributions. This research is more general than [21] in the sense that neither the demand distribution nor its parameters are limited to normal distributions. Also, this research contributes to the inventory control literature in that hurricane predictions are explicitly incorporated into the Bayesian updating framework and inventory decision.

2.2 Background and Notations

The framework of *sequential statistical decision problems* is ideally suited to model the hurricane supply chain planning problem described in Section 1. Therefore, relevant concepts and terminology related to sequential decision problems based on [14] and [24] are first introduced and a model related to hurricane planning is presented later as a special case.

Consider a decision problem in which a decision-maker (DM) must specify a one-time decision δ that minimizes some loss function L. The loss function depends on a random variable E that has a known probability density function (pdf) $g(e, \Theta)$, where Θ is a random variable from an unknown distribution characterized by pdf $h(\theta)$. Before deciding upon a δ that minimizes L, DM has the opportunity to obtain more information about the unknown distribution parameter Θ by observing a *sequential random sample* $\mathbf{W}^t = (W_1, W_2, \ldots, W_t)$ from Θ, where the cost of observing W_j is C_j. Note that $\mathbf{W}^t = (W_1, W_2, \ldots, W_t)$ is called a sequential random sample if the W_j's are independently and identically distributed.

The definition of the sequential statistical decision problem is based on two fundamental ideas: (i) the *stopping time*, T, and (ii) the *decision rule*, $\boldsymbol{\delta}$. The stopping time T is derived from the concept of a stopping rule, $\boldsymbol{\tau}$, which is a series of functions $\tau_0, \tau_1(\mathbf{w}^1), \tau_2(\mathbf{w}^2), \ldots$ such that $\tau_t(\mathbf{w}^t)$ is the probability of terminating the sampling processes after t observations. Note that τ_0 is interpreted as the probability of giving the decision without sampling. In

[14] (page 442), the stopping time T is defined as

$$T\left(\mathbf{W}^t\right) = \min_{t\geq 0}\left\{\tau_t\left(\mathbf{W}^t\right) = 1\right\} \tag{2.1}$$

In words, $T \in \{0, 1, \ldots, t\}$ is the number of observations such that sampling is stopped and a decision δ_t is given as opposed to observing W_{t+1}.

Now the decision rule $\boldsymbol{\delta}$ is a set of functions $\delta_0, \delta_1\left(\mathbf{w}^1\right), \delta_2\left(\mathbf{w}^2\right), \ldots$ that specify which action is to be taken once sampling is stopped and the observed values from the sequential random sample is given by $\mathbf{w}^t = (w_1, w_2, \ldots, w_t)$. The sequential statistical decision problem then involves determining a stopping rule $\boldsymbol{\tau}$ and decision rule $\boldsymbol{\delta}$ that minimizes the loss function $L\left(\theta, \delta_t\left(\mathbf{w}^t\right), t\right)$ based on a sequential random sample $W_1, \ldots, W_t$.

More formally, define the *risk function* as the expected value of the loss function, denoted $R(\Theta, \mathbf{d})$, where $\boldsymbol{d = (\tau, \delta)}$. Also define $\Lambda = \left\{t \geq 1 : \tau_t\left(\mathbf{w}^t\right) = 1 \text{ and } \tau\left(\mathbf{w}^j\right) = 0\right\}$ for all $j < t$, which is the set of observations such that sampling terminates after t observations. Then from [14] (page 442),

$$\begin{aligned} R(\Theta, \mathbf{d}) &= \mathbb{E}\left[L\left(\Theta, \delta_T\left(\mathbf{W}^t\right), T\right)\right] \\ &= P(T=0)\cdot L(\Theta, \delta_0, 0) + \sum_{t=1}^{\infty}\int_{\Lambda} L(\Theta, \delta_t(\mathbf{w}^t), t)\mathrm{d}H_t(\mathbf{w}^t|\Theta) \\ &+ \sum_{t=1}^{\infty}\sum_{i=1}^{t} C_i P(T=t) \end{aligned} \tag{2.2}$$

Since Θ is also a random variable and has density $h(\theta)$, define the expected value with respect to Θ as follows.

$$r(h^t, \mathbf{d}, t) = \mathbb{E}\left[R(\Theta, \mathbf{d})\right] \tag{2.3}$$

Eq. (2.3) is known as the *Bayes risk function*. The density $h(\theta)$ is considered to be the prior probability distribution of Θ. Define $h^t\left(\mathbf{w}^t|\theta\right)$ as the likelihood function and $h^t(\theta|\mathbf{w}^t)$ as the posterior density of Θ after observing the sequential random sequence $\mathbf{W}^t = (W_1, \ldots, W_t)$. Then, the following posterior function will be obtained using the Bayes' theorem.

$$h^t(\theta|\mathbf{w}^t) = \frac{h^t\left(\mathbf{w}^t|\theta\right) \cdot h(\theta)}{\displaystyle\int_{\mathbb{R}} h^t\left(\mathbf{w}^t|\theta\right) \mathrm{d}H(\theta)} \tag{2.4}$$

The sequential statistical decision problem can now be formally stated as finding a decision and stopping rule $\mathbf{d}$ such that

$$r(h^t, t) = \inf_{\mathbf{d}} r(h^t, \mathbf{d}, t) \tag{2.5}$$

2.3 Model Formulation

In this section, the hurricane supply stocking problem is formulated as an optimal stopping problem within the general framework presented in the Section 2.2. Then, several important assumptions used in developing an appropriate single period loss function, which is a variation of the single product newsboy problem, are elaborated. Finally, a risk function based on the loss function is derived and incorporated into an optimal stopping problem framework in the form of Eq. (2.2) and Eq. (2.5).

2.3.1 Assumptions

The derivations of the loss and risk functions are based on the following assumptions.

Assumption 1 *Hurricane demand is a function of various hurricane characteristics such as its intensity and path.*

Let $\boldsymbol{\alpha} \in \mathbb{R}^m$ be a random vector, where each random component represents a distinct hurricane characteristic. Then Assumption 1 implies that hurricane demand (for a single product) is a random variable $X(\boldsymbol{\alpha})$. As an example, suppose $m = 2$ with $\boldsymbol{\alpha} = (L, W)$, where L is a random variable that represents a location affected by the hurricane and W is the hurricane's maximum sustained wind speed at location L. Then the demand corresponding to location L with maximum wind speed W is the random variable $X(\boldsymbol{\alpha})$ $= X(L, W)$.

A sequential random sample $\mathbf{W}^t$ can be observed during the disaster recovery planning horizon in order to obtain more information about $\boldsymbol{\alpha}$ as a new tropical depression evolves over time. Consequently, the sample $\mathbf{W}^t$ also reduces the uncertainty associated with $X(\boldsymbol{\alpha})$, which will enable DM to make better inventory decisions related to hurricane preparation.

Assumption 2 *Demand is a function of the storm's maximum sustained wind speed, W.*

According to Assumption 2, a hurricane is described by exactly one attribute (i.e., $\boldsymbol{\alpha} \in \mathbb{R}$), which is its maximum sustained wind speed W. This assumption implies that $X(\boldsymbol{\alpha})$ $= X(W)$, where W is also a random variable. Assumption 1 acknowledges the fact that realistically, hurricane demand X is a function of several hurricane characteristics including the locations it affects. However, Assumption 2 allows to develop the framework without being too distracted by the advanced statistical concepts required to implement multivariate Bayesian updating. Assumption 2 also suggests that storm intensity is a significant factor that influences demand for hurricane supplies.

Assumption 3 *Two classes of hurricane demand are considered:* $X_0(W)$ *= demand associated with a regular hurricane and* $X_1(W)$ *= demand associated with an extreme hurricane.*

The two classes of hurricanes are distinguished as: (i) "regular" and "extreme." An extreme hurricane is defined as one that (i) causes significant demand surge for hurricane supplies and (ii) has extremely low probability of occurrence. Based on Assumption 1 and Assumption 2, demand surge is correlated to W. In fact the most common classification scheme for hurricanes, known as the Saffir-Simpson scale, uses W to classify all hurricanes into one of five categories. Category 1 represents the least intense hurricane classification with maximum sustained wind speeds between 74 and 95 mph, while Category 5 represents the most intense hurricane classification with maximum sustained wind speeds greater than 155 mph. For the purposes of this study, hurricanes of Category 4 (i.e., maximum sustained wind speeds between 131 mph and 154 mph) and Category 5 will be classified as extreme, and hurricanes of Category 1, 2, and 3 will be classified as regular (or non-extreme). Although varying degrees of demand surge are likely to be caused by hurricanes of all Saffir-Simpson categories, the designations for extreme and regular hurricanes is based only on the second criteria (low probability of occurrence). Based on the analysis of HURDAT, which is a database maintained by the National Hurricane Center that records various attributes of each hurricane that has developed in the Atlantic Basin since the year 1851, it is found that Category 4 and 5 hurricanes collectively are less likely to occur than Category 3 hurricanes and significantly less likely than Category 1, 2, and 3 hurricanes collectively. Note that a "major" hurricane as defined by the National Hurricane Center is different than this definition of an extreme hurricane. In particular, a Category 3 hurricane is consider major, but not extreme. Assumption 3 can now be restated as follows.

Remark 1 *(Restatement of Assumption 3): The demand associated with a Category 1, 2, or 3 hurricane is a random variable $X_0(W)$, and the demand associated with a Category 4 or 5 hurricane is a random variable $X_1(W)$.*

Assumption 4 *Let E be a Bernoulli random variable with parameter P that specifies whether or not a hurricane is extreme. Also define $P = \Pr(E = 1)$ as the probability that a given hurricane is extreme and $P_t = \Pr(E = 1|W_t = w_t)$ as the probability that a given hurricane will ultimately be extreme after observing the t^{th} wind speed update. Then*

1. *P is a random variable with prior density $h(p)$, likelihood density $h^t\left(\mathbf{w}^t|p\right)$, and posterior density $h^t\left(p|\mathbf{w}^t\right)$, where $t = 1, \ldots, T$ and T is the number of hurricane prediction updates (T is also interpreted as the number of periods during the planning horizon).*

2. *The likelihood densities $h^t\left(\mathbf{w}^t|p\right)$ are Normally distributed for each $t = 1, \ldots, T$.*

3. *The likelihood and posterior densities, $h^t\left(\mathbf{w}^t|p\right)$ and $h^t\left(p|\mathbf{w}^t\right)$ respectively, are based on hurricane prediction updates that are given every 6 hours after a tropical depression or disturbance is initially observed.*

HURDAT (see narrative following Assumption 3) is used to determine a prior estimate of P after a tropical depression or disturbance is first observed. Then each time an updated hurricane forecast becomes available, the likelihood function $h^t(\mathbf{w}^t|p)$ associated with period t is used along with Bayes' formula to obtain the updated posterior distribution of P given by $h^t\left(p|\mathbf{w}^t\right)$. Since $W \geq 131$ constitutes an extreme hurricane based on Assumption 2 and

Remark 1, the following relationships among W, $\mathbf{W}^t$, and P for $t = 1, \ldots, T$ are derived.

$$P_0 = h(p) = \Pr(W \geq 131) \tag{2.6}$$

$$h^t\left(\mathbf{w}^t | p\right) = \Pr\left(\mathbf{W}^t = \mathbf{w}^t | W \geq 131\right) \tag{2.7}$$

$$P_t = h^t\left(p | \mathbf{w}^t\right) = \Pr\left(W \geq 131 | \mathbf{W}^t = \mathbf{w}^t\right) \tag{2.8}$$

After analyzing 10 years of hurricane data using HURDAT as described later in Section 2.5, it is determined that the likelihood densities $h^t(\mathbf{w}^t|p)$ can be represented by a Normal distribution for most of the periods $t \in \{1, \ldots, T\}$. Also note that HURDAT records various hurricane attributes for each storm since the year 1851 in 6-hour intervals. Consequently, the empirically derived distribution parameters discussed in Section 2.5.1 are based on updates that occur every six hours. However, it is important to note that the Bayesian methodology is not limited to 6-hour intervals and can in fact be used to derive an updated posterior distribution at any time when an information update becomes available in practice. This is discussed in more detail in Section 2.5.2.

Assumption 5 *Let c_t be the order (or production) cost associated with giving a decision after t forecast updates. Then $c_t \leq c_{t+1}$ for all $t = 0, 1, \ldots$.*

The hurricane supply stocking problem is characterized by a trade-off between forecast accuracy and logistics costs as a function of time. More specifically, forecast accuracy is an increasing function of t, but logistics cost is also an increasing function of t. This is reflected in Section 2.2 by the cost C_j associated with observing W_j in the sequential sampling process. Assumption 5 implies that $C_t = c_{t+1} - c_t$, where $t = 0, 1, 2, \ldots$ and $c_0 = 0$.

2.3.2 Single Period Loss Function

Assumptions 1, 2, and 3 are incorporated into the model by introducing a Bernoulli random variable E, with pdf $g(e, P) = P^e(1-P)^{1-e}$ and $e \in \{0, 1\}$, that specifies whether a hurricane is extreme ($E = 1$) or not ($E = 0$). If $E = 0$, then the optimal stocking policy that minimizes expected costs due to ordering (or producing), overstocking, and understocking is the critical fractile solution that corresponds to the newsboy problem with random demand X_0. Similarly, the random variable X_1 is applicable if $E = 1$. However, this approach to determining the optimal stock level Q^* requires the uncertainty associated with E to be resolved. Since E is a Bernoulli random variable with parameter P, the question then becomes "which newsboy problem should be solved?" One possible approach is to consider the newsvendor whose demand is X_1 with probability P and the newsvendor whose demand is X_0 with probability $1 - P$, which leads to the following loss function $L(Q)$:

$$L(Q) = (1-P) \cdot NB_0(Q) + P \cdot NB_1(Q) \tag{2.9}$$

where $NB_k(Q)$ is the expected cost function for the newsboy problem with demand distribution X_k for $k = 0, 1$. If c is the unit order/production cost, h the unit holding (or overstocking) cost, s the unit shortage cost, and $f_k(x_k)$ the density of X_k with $k \in \{0, 1\}$, then $NB_k(Q)$ is often expressed as

$$NB_k(Q) = Q \cdot c + h \cdot \int_0^Q (Q - x_k) f_k(x_k) \mathrm{d}x_k + s \cdot \int_Q^\infty (x_k - Q) f_k(x_k) \mathrm{d}x_k \tag{2.10}$$

2.3.3 Bayes Risk Function

The decision Q that minimizes the expected loss function given by Eq. (2.9) assumes that the decision is given without any sequential sampling process $\mathbf{W}^t$ related to the uncertain parameter P, and consequently does not consider the costs associated with such sampling. Additionally, Eq. (2.9) does not consider the fact that P is a random variable. In order to obtain a risk function similar to Eq. (2.2) and Eq. (2.3), and consistent with Assumption 4 and Assumption 5, the expected value operator is applied to Eq. (2.9) with respect to P, incorporating the posterior distribution of P given by Eq. (2.8), and adding the cost of sampling. The resulting Bayes risk function is

$$r\left(h^t, \mathbf{d}, t\right) = NB_0^t + \int_0^{\infty} w_t \cdot \left(NB_1^t - NB_0^t\right) \mathrm{d}H^t(p|\mathbf{w}^t) + \sum_{j=1}^{t} C_j \tag{2.11}$$

Note that NB_k^t is the newsboy expected cost function given by Eq. (2.10) with order/production cost c_t. Also note that from Assumption 5, C_j in Eq. (2.11) is given by

$$C_j = c_{j+1} - c_j, \qquad\qquad j = 0, 1, 2, \ldots \tag{2.12}$$

For a given prior $h(p)$ and likelihood $h^t(\mathbf{w}^t|p)$, the posterior distribution $h^t(p|\mathbf{w}^t)$ in Eq. (2.11) can be derived using Eq. (2.4). Recall from Assumption 5 that the likelihood density is a Normal distribution (with mean μ and variance σ^2). The likelihood is then

$$h^t\left(\mathbf{w}^t|p\right) = N\left(\mu, \sigma^2\right) = \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot \exp\left[-\frac{\left(w_t - \mu\right)^2}{2\sigma^2}\right] \tag{2.13}$$

By substituting Eq. (2.13) into Eq. (2.4), the posterior density can be expressed as

$$h^t(p|\mathbf{w}^t) = \frac{h(p) \cdot N\left(\mu, \sigma^2\right)}{\displaystyle\int_0^\infty N\left(\mu, \sigma^2\right) \mathrm{d}H(p)} \tag{2.14}$$

By substituting Eq. (2.12) and Eq. (2.14) into Eq. (2.11), the Bayes risk function becomes

$$\begin{aligned} r\left(h^t, \mathbf{d}, t\right) &= NB_0^t + \sum_{j=0}^{t} \left(c_{j+1} - c_j\right) \\ &+ \int_0^\infty \left\{ \frac{w_t \cdot h(p) \cdot \left(NB_1^t - NB_0^t\right) \cdot N\left(\mu, \sigma^2\right)}{\displaystyle\int_{\mathbb{R}} N\left(\mu, \sigma^2\right) \mathrm{d}H(p)} \right\} \mathrm{d}w_t \end{aligned} \tag{2.15}$$

Therefore by Eq. (2.5), the problem is to determine $\mathbf{d} = (t^*, Q)$ such that:

$$r(h^t, t) = \inf_{\mathbf{d}} r(h^t, \mathbf{d}, t) \tag{2.16}$$

where $r(h^t, \mathbf{d}, t)$ is Eq. (2.15).

2.3.4 Recursive Formulation

This chapter investigates a sequential statistical decision problem in which the maximum number of observations that can be taken is T. In other words, the sequential random sample $\mathbf{W}^T$ is bounded such that $t = 1, 2, \ldots, T$. Additionally, define:

$$r_T\left(h^t, t\right) = \inf_{\mathbf{d}} r\left(h^t, \mathbf{d}, t\right) \tag{2.17}$$

where $r\left(h^t, \mathbf{d}, t\right)$ is Eq. (2.15), but with the exception that $\mathbf{d}$ is now truncated at T. Also, $r_0(h^t, t)$ is the minimum Bayes risk associated with giving an immediate decision in period t, and $r_{T-t}(h^t, t)$ is the minimum Bayes risk associated with $T-t$ more observations. The following result then applies.

Theorem 2.1 *(Degroot [24]): Among all sequential decision procedures in which not more than T observations can be taken, the following procedure is optimal: If $r_0(h, 0) \leq r_T(h, 0)$, a decision δ_0 is chosen immediately without any observations. Otherwise, W_1 is observed. Furthermore, for $t = 1, \ldots, T-1$, suppose the sequential random sample $\mathbf{W}^t$ has been observed. If $r_0(h^t, t) \leq r_{T-t}(h^t, t)$, a decision δ_t is chosen immediately without further observations. Otherwise, W_{t+1} is observed. If sampling has not been terminated earlier, it must be terminated after W_T is observed.*

Based on Theorem 2.1, the sequential decision problem can be stated recursively as follows for $j = 2, \ldots, T$ (e.g., [14], page 449).

$$r_j\left(h^t, t\right) = \min_{\mathbf{d}} \left\{ r_0\left(h^t, t\right), \mathbb{E}\left[r_{j-1}\left(h^t\left(p|\mathbf{W}^{t+1}\right), t+1\right)\right] \right\} \tag{2.18}$$

The following section describes an algorithmic approach to solve Eq. (2.18).

2.4 Algorithm Development

The hurricane supply stocking problem represented by Eq. (2.18) involves determining an order/production quantity Q_t and single order/production period t^* that minimizes expected costs due to ordering/producing, overstocking, and understocking. Initially, the methodology used to determine Q_t is presented. Then, a procedure for obtaining t^* is described.

2.4.1 Optimal Order/Production Quantity

According to Theorem 2.1, the optimal decision rule $\boldsymbol{\delta}^t = (Q_1, \ldots, Q_t)$ is needed in order to determine the optimal stopping time t^*. Once an update w_t (which will be expressed as p_t by using the posterior density given by Eq. (2.8)) is observed, the optimal decision is Q_t that minimizes the loss function $L_t(Q)$ given by Eq. (2.9) with order/production cost c_t. The resulting decision rule is as follows.

Theorem 2.2 *(Lodree and Taskin [57]): Let $F_k(x_k), k \in \{0,1\}$ be the distribution function of random demand X_k. Then the optimal decision $\delta_t = Q_t$ that minimizes the loss function $L_t(Q_t)$ given by Eq. (2.9) with order/production cost c_t satisfies*

$$(1-p_t) \cdot F_0(Q_t) + p_t \cdot F_1(Q_t) = \frac{s-c}{s+h} \tag{2.19}$$

Another approach that could be used to determine an appropriate stock level is to choose the critical fractile Q_0 that minimizes NB_0^t if $p_t \leq 0.5$ and Q_1 that minimizes NB_1^t if $p_t > 0.5$. The DMs risk behavior can also be taken into account using this approach by choosing threshold values other than 0.5. However, an alternative loss function $L(Q)$ would then apply.

2.4.2 Optimal Stopping Time

In order to determine the optimal stopping time, the approach presented in [21] is adopted by using the concept of *cutting points* p_t^*. These cutting points are based on the fact that according to Theorem 2.1, it is optimal to stop only if $r_0(h^t, t) \leq r_{T-t}(h^t, t)$ for $t = 0, 1, \ldots, T$, where $t = 0$ is interpreted as giving an immediate decision without any sampling. A cutting point is then the point p_t^* that satisfies $r_0(h^t, t) = r_{T-t}(h^t, t)$, which can be considered a decision threshold that specifies whether or not sampling should continue after p_t is observed. Note that the parameter p_t in $r_j(h^t, t)$ is expressed as an expected value in terms of W using the posterior distribution $h(p_t|\mathbf{w}^t)$. Thus, to determine cutting point values, the loss function, which is expressed explicitly in terms of p_t, is used. Once the sample point p_t is observed, $r_0(h^t, t) \leq r_{T-t}(h^t, t)$ becomes $L_t(Q_t) \leq L_{t+1}(Q_{t+1})$ and the cutting point p_t^* satisfies $L_t(Q_t) = L_{t+1}(Q_{t+1})$. That is, Theorem 2.1 suggests that it is optimal to stop and place an order if $L_t(Q_t) \leq L_{t+1}(Q_{t+1})$ and the cutting point is obtained by solving

$$NB_0^t + p_t\left(NB_1^t - NB_0^t\right) = NB_0^{t+1} + p_t\left(NB_1^{t+1} - NB_0^{t+1}\right) \tag{2.20}$$

Note that $NB_k^t = NB_k^t\,(Q_t)$. Solving Eq. (2.20) for p_t yields the following cutting point p_t^*.

$$p_t^* = \frac{NB_0^{t+1} - NB_0^t}{\left(NB_0^{t+1} - NB_0^t\right) - \left(NB_1^{t+1} - NB_1^t\right)} \tag{2.21}$$

Solving the inequality $L_t(Q_t) \leq L_{t+1}(Q_{t+1})$ yields $p_t \leq p_t^*$. Thus the optimal procedure in Theorem 2.1 as applied to our hurricane stocking problem can be restated as follows.

Theorem 2.3 *If a sample update p_t is observed in period t and satisfies $p_t \leq p_t^*$, where p_t is given by Eq. (2.8) and p_t^* is given by Eq. (2.21), it is optimal to terminate sampling and give an immediate decision by stocking Q_t. Otherwise if $t < T$, it is optimal to observe the next update p_{t+1}.*

Theorem 2.3 suggests the following algorithm adapted from [21] for solving the hurricane stocking problem given by Eq. (2.18).

Step 1: Initialization: Set $t = 0$ and $p_T^* = 1$.

Step 2: Compute the cutting points p_t^* for all $t = 0, 1, \ldots, T - 1$.

Step 3: Convert the observed sample w_t to p_t using Eq. (2.8).

Step 4: If $p_t \leq p_t^*$, order/produce Q_t and terminate the sampling process. Otherwise if $p_t > p_t^*$, do not order/produce and continue sampling. Increment t and repeat steps 3 and 4.

Note that Theorem 2.3 requires the sequential random sample $\mathbf{W}^t = (W_1, \ldots, W_t)$ to be expressed as $\mathbf{P}^t = (P_1, \ldots, P_t)$, which is the reason for Step 3 in the above procedure. Also note the condition $p_T^* = 1$ ensures that an order is always placed. That is, if no order has been placed by period T, the condition $p_T \leq 1$ will always hold and force an order Q_T. Although the cutting point formula (2.21) is structurally the same as in [21], the details are more complicated because this research considers a more general case in which Bayesian updates are not conjugate. Furthermore, additional complexities arise because the optimal stocking policy in Theorem 2.2 cannot be expressed in closed form.

2.5 Empirical Study

In this section, the solution methodology presented in Section 2.4 is demonstrated using real hurricane data from the HURDAT database. The objective is to use historical wind speed data to simulate the evolution of the wind speeds associated with an observed tropical depression, and then apply the solution approach to determine a one-time stocking decision, as well as which period this stocking decision should be given. A sample of $N = 143$ hurricanes comprises our data set spanning the 10-year period 1995 - 2004. The sample is represented as a matrix $W_{T\times N}$ with T rows and N columns, where each row corresponds to a period t and each column corresponds to a particular hurricane n in the sample. Thus an entry w_{tn} represents the wind speed of hurricane n during period t. This data is used for two purposes: (i) to determine appropriate parameters for the Normal likelihood densities (Assumption 4) and (ii) to simulate wind speeds at 6-hour intervals for creating problem instances.

2.5.1 Empirical Likelihood Densities

Calculation of posterior densities requires that the distribution parameters associated with the likelihood densities $h^t\left(\mathbf{w}^t|p\right)$ be specified. To ensure the condition p in $h^t\left(\mathbf{w}^t|p\right)$ is satisfied, initially all hurricanes in the sample W_{TN} with $W \geq 131$ (recall that W is the maximum recorded wind speed associated with a specific hurricane over all periods t) are identified. The resulting sample is denoted $W_{T\times N_E}$, where $N_E = 23$ is the number of extreme hurricanes from the sample W_{TN}. To empirically determine parameters (using the maximum likelihood method) for the likelihood density $h^t\left(\mathbf{w}^t|p\right)$ that corresponds to period t, the wind speeds w_{tn} for $n = 1, \ldots, n_E$ are used resulting in T likelihood density functions. These likelihoods can then be used to transform an observation w_t into its corresponding p_t, which is needed for Step 3 of the algorithm presented in Section 2.4.2.

2.5.2 Simulating Wind Speeds

For illustrative purposes, the following elementary approach is applied to simulate hurricane wind speeds at 6-hour intervals for creating problem instances as follows (note that there is no need to simulate wind speeds in practice; wind speeds are simulated here only to create example problem instances):

Step 1: Initialize: Set $t = 1$.

Step 2: Simulate a random number r_t between 1 and N.

Step 3: Let w_t be the simulated wind speed period t and w_{t,r_t} be the wind speed associated with period t for the r_t^{th} hurricane in W_{TN}. Then $w_t = w_{t,r_t}$.

Step 4: Repeat steps 2 and 3 until $t = T$.

It is observed that the first several wind speed observations for many of the sample hurricanes (both extreme and otherwise) are either 30 mph or 35 mph. Since there is little to no distinction between storms in the initial stages, the analysis is limited to periods in which there are observable differences in wind speeds by defining

$$\underline{t} = \min\{t : w_t \geq 131\} \tag{2.22}$$

In words, $\underline{t}$ is the earliest period in which enough information is available to classify a hurricane as extreme. From a practitioners perspective, $\underline{t}$ represents the period in which managers should begin to take notice of wind speed updates and predictions.

Recall from Assumption 3 that hurricane demand is X_1 if $W \geq 131$ and X_0 otherwise. Therefore, if some observation w_t is at least 131, there is no need to continue observing wind speed updates. As a result, the number of wind speed simulations required to implement the analysis is limited by defining

$$\bar{t} = \max\{t : w_t \geq 131\} \tag{2.23}$$

By replacing $t = 1$ with $t = \underline{t}$ in Step 1 of the above wind speed simulation procedure, and also replacing $t = T$ with $t = \bar{t}$ in Step 5, the number of unnecessary simulated wind speed updates are reduced.

Now recall that the parameters associated with the likelihood densities are empirically derived based on 6-hour intervals between hurricane forecast updates because HURDAT data is recorded every six hours (Assumption 4). Although the available hurricane data is limited to 6-hour intervals, the Bayesian method for updating distribution parameters in general has no restrictions related to the times between successive information updates nor is there any restrictions on the number of updates. Furthermore, the amount of time between observed hurricane forecast updates in practice may be more than six hours during the earlier stages of the planning horizon, but the frequency of updates during the later stages is likely to increase resulting in times between observed updates that are less than six hours. The above procedure for simulating wind speed data can be modified as follows in order to account for hurricane predictions that are observed every k hours (or decision periods of length k), where k can also vary during the planning horizon.

1. If $k > 6$ is an integer, simulate wind speed data using the above procedure (which is based on forecast updates given every six hours) and solve the problem as usual. Then the modified decision period $\hat{t}$, which is based on updates given every $k > 6$ hours, is given by $\hat{t} = \max\{q, 1\}$, where q is the integer quotient resulting from the division $6t/k$. For example, suppose wind speed updates are observed every $k = 10$ hours, and the optimal decision period for the 6-hour interval problem is $t = 7$. First of all, the period $t = 7$ corresponds to 42 hours after the hurricane's initial observation. For the

case $k = 10$, this corresponds to period $\hat{t} = \max\{4, 1\} = 4$ since the integer quotient when 42 is divided by 10 is 4.

2. If $k < 6$, then more wind speed data points must be simulated during period t as follows.

   (a) If $k = 1$ between the 6-hour periods t and $t + 1$, then simulate $s = 6$ random numbers $r_{t1}, \ldots, r_{ts}$ in Step 2 of the above wind speed simulation procedure, where each $r_{tj} \in \{1, \ldots, N\}$ and $j = 1, \ldots, s$.

   (b) Then in Step 3, there will be $s = 6$ simulated wind speeds $w_{t1}, \ldots, w_{ts}$, where $w_{tj} = w_{t,r_{tj}}$, $w_{t,r_{tj}}$ is the wind speed in period t associated with the r_{tj}^{th} hurricane in W_{TN}, and $j = 1, \ldots, s$.

   (c) For $k = 2$ and $k = 3$, steps (a) and (b) can be repeated with $s = 3$ and $s = 2$ respectively. Also, it can be argued that $s = 2$ applies to both $k = 4$ and $k = 5$.

   (d) The optimal decision period is then determined by solving the problem with these additional wind speed updates, and will be based on the j^{th} hour of period t.

2.5.3 Numerical Example

The above wind speed generating procedure was used to produce 14 wind speed samples, W_{TN}. As shown in Table 2.1, seven of the examples were extreme and seven were not. The following data was used to solve each of the example problems, and the results are shown in Table 2.1: $c_1 = 20, c_{t+1} = c_t + 1, s = 100, h = 15, X_0 \sim N(98, 15^2)$, and $X_1 \sim$ Gumbel$(\alpha, \beta) = (153, 16)$. Note that $p_0 = \Pr\{W \geq 131\} = 0.16, \underline{t} = 11$, and $\overline{t} = 51$ for our sample of $N = 143$ hurricanes. Note that the Gumbel distribution, which is an extreme value distribution used for low probability events, was assumed for the demand associated with an extreme hurricane. Also note that Matlab™was used to perform numerical integration calculations associated with the posterior densities given by Eq. (2.14) for the example problems. Matlab™was also used to solve Eq. (2.19) to obtain decision rules for the example problems.

Table 2.1: Example results

Example	Hurricane Type	Update Interval	Decision Period t^*	Stocking Quantity Q_t
1	Not Extreme	6-hour	4	104.51
2	Not Extreme	6-hour	17	39.13
3	Not Extreme	6-hour	9	103.15
4	Not Extreme	6-hour	18	105.05
5	Not Extreme	6-hour	4	104.88
6	Not Extreme	3-hour	3	135.82
7	Not Extreme	10-hour	8	111.31
8	Extreme	6-hour	3	111.95
9	Extreme	6-hour	2	109.83
10	Extreme	6-hour	7	180.53
11	Extreme	6-hour	4	135.37
12	Extreme	6-hour	3	135.82
13	Extreme	3-hour	3	153.57
14	Extreme	10-hour	2	185.86

Table 2.1 suggests that the DMs are inclined to order/produce early for the extreme cases relative to the non-extreme cases (note that the decision period t^* in Table 2.1 actually corresponds to period $t^* + 11$ since $\underline{t} = 11$). More specifically, the HURDAT data shows that wind speeds tend to increase more quickly for extreme hurricanes than for non-extreme hurricanes during the hurricane's evolution. The results are also consistent with intuition in that the optimal order quantities associated with extreme hurricanes are larger, on average, than the optimal order quantities associated with non-extreme hurricanes.

2.6 Extension to Ordering Disruption

In this section, the base model presented in Section 2.4 is extended such that damages from an observed storm could prevent an ordering / producing decision from being carried out. That is, if the solution to the base model suggests ordering / producing a quantity Q_t in period t, then the extended model accounts for possible disruptions, such as damages to the transportation network or inaccessible overtime labour, that would prevent the decision from being implemented. In order to extend the base model, it is assumed that ordering disruptions are caused by the characteristics of an observed storm. That is, the disruptions caused by events other than the observed storm are not considered, which is consistent with Assumption 1. More specifically, it is assumed that an ordering disruption is a function of an observed storm's maximum sustained wind speed during period t, which is consistent with Assumption 2. Additionally, it is assumed that if an ordering disruption occurs during period t, then no order / production can occur during periods $t, t+1, \ldots, T$, which means that the ordering disruption cannot be resolved until after the storm dissipates.

Now let Z_t be a Bernoulli random variable, with parameter z_t, that assumes the value 1 if an ordering disruption occurs in period t and 0 otherwise. Then $z_t = \Pr(Z_t = 1)$ and $1 - z_t = \Pr(Z_t = 0)$. Furthermore, the parameter z_t is also assumed to be a random variable with prior probability z_0^t, and the following likelihood and posterior densities

$$g^t\left(\mathbf{w}^t | z\right) = \Pr\left(\mathbf{W}^t = \mathbf{w}^t | Z_t = 1\right) \tag{2.24}$$

$$g^t\left(z | \mathbf{w}^t\right) = \Pr\left(Z_t = 1 | \mathbf{W}^t = \mathbf{w}^t\right) \tag{2.25}$$

Note that unlike the Bernoulli random variable E introduced in Assumption 4, there are T random variables Z_t, each of which is unknown at the beginning of period t and known with certainty at the end of period t. Also note that the random sample w_t is observed at the beginning of period t, which is then used to calculate the posterior probability associated with z_t. Additionally, note that if an order / production decision is given in period t, it is given at the beginning of the period before the realization of Z_t. If $Z_t = 0$, then the order / production quantity satisfies Theorem 2.2. However, if $Z_t = 1$ for some period $t = s$, then $Q_t = 0$ for each $t = s, s+1, \ldots, T$ since $Z_t = 1$ implies that no ordering or producing can take place in period t or after period t. Under these conditions, the loss function $L_t(Q)$ is:

$$L_t(Q) = (1 - z_t) \cdot M_t(Q) + z_t \cdot M_t(0) \tag{2.26}$$

where

$$\begin{aligned} M_t(Q) &= (1 - p_t) \cdot NB_0^t(Q) + p_t \cdot NB_1^t(Q) \\ M_t(0) &= (1 - p_t) \cdot NB_0^t(0) + p_t \cdot NB_1^t(0) \end{aligned}$$

Similar to the base model, the cutting point for period t (see Section 2.4.2) can be derived by solving the equation $L_t(Q_t) = L_{t+1}(Q_{t+1})$, where $L_t(Q_t)$ is given by Eq. (2.26). The resulting cutting point is:

$$p_t^* = \frac{N_t(Q)}{N_t(Q) - R_t(Q)} \tag{2.27}$$

where

$$
\begin{aligned}
N_t(Q) &= NB_0^{t+1}(Q) - NB_0^t(Q) + z_t\left[NB_0^t(Q) - NB_0^t(0)\right] - z_{t+1}\left[NB_0^{t+1}(Q) - NB_0^{t+1}(0)\right] \\
R_t(Q) &= NB_1^{t+1}(Q) - NB_1^t(Q) + z_t\left[NB_1^t(Q) - NB_1^t(0)\right] - z_{t+1}\left[NB_1^{t+1}(Q) - NB_1^{t+1}(0)\right]
\end{aligned}
$$

The cutting point given by Eq. (2.27) can be used in a variation of the algorithm following Theorem 2.3 to compute the optimal stopping time. The modified algorithm requires an additional step that converts sample wind speeds w_t into their corresponding posterior probabilities z_t for each t. Consequently, the resulting algorithm would involve a more complex Bayesian updating environment that accounts for p and each z_t. Furthermore, additional data would need to be collected and compared to HURDAT in order to construct meaningful prior and likelihood distributions for each z_t.

2.7 Conclusion

In response to increased hurricane activity in the United States, particularly the devastating impact of Hurricane Katrina during the year 2005, this chapter addresses a disaster recovery planning problem encountered by manufacturing and retail organizations who experience demand surge for various products if an observed storm evolves into a catastrophic hurricane. The proposed model and solution method are also applicable to a closely related disaster relief planning problem relevant to the military, electric power companies, and other service organizations. Instead of formulating a general model that is applicable to preparing for any kind of disaster, the proposed approach leverages hurricane predictions to develop a disaster recovery plan that is most appropriate for managing the risks that are specific to hurricane events. This approach is consistent with fundamental concepts from emergency management in that risk identification is an integral part of the planning process (see [36]).

Relative to hazards such as earthquakes, terrorist attacks, and tornadoes, it is reasonable to expect more reliable disaster recovery plans for hurricanes because of (i) an abundance of historical data, (ii) the availability of sophisticated prediction models, (iii) the increasing accuracy of hurricane predictions as a storm evolves after its initial development, and (iv) the length of the planning horizon after a potential threat is first identified. This research leverages these characteristics by formulating the inventory problem as an optimal stopping problem with dynamic hurricane prediction updates. In particular, historical hurricane data is used to develop a statistical model for predicting whether or not an observed tropical depression or disturbance will evolve into an catastrophic hurricane. The prediction model entails Bayesian updates of hurricane wind speeds and is integrated with a decision model that specifies the optimal quantity and timing of the inventory decision such that the trade-off between forecast accuracy (better with time) and cost efficiency (worse with time) is optimized.

The framework presented in this chapter represents the initial stages of research needed to develop disaster recovery plans that would be effective in practice with respect to preparing for hurricane events. One possible extension is to integrate a more sophisticated hurricane prediction model into our decision framework, particularly one that includes both track and intensity predictions. This extension is more realistic because the magnitude of demand surge obviously depends on more than one hurricane characteristic. However, advanced statistical techniques would be required to facilitate a more complex Bayesian updating process. This chapter also introduces several other opportunities for further research. For example, the current model emphasizes planning for extreme hurricane events by considering two demand classes. A natural extension is to consider five demand classes (one for each hurricane category) to plan for any type of hurricane. Another possibility is to explore a multiple product version of the model. Finally, this chapter can also be extended based on other decision rules or loss functions, such as those described in Section 2.6 and in [57].

Chapter 3

Multi Location Inventory Model with Wind Speed Forecast Updates

This chapter investigates an inventory stocking problem encountered by suppliers whose demand for hurricane supplies is influenced by the hurricane season. More specifically, these suppliers' inventory related decisions made after a tropical depression is initially observed are affected by the uncertainties associated with the tropical cyclone's projected path, and its intensity along the path. Therefore, it is necessary for them to develop effective and efficient disaster recovery plans before the hurricane season initiates. In this chapter, the National Hurricane Center's (NHC) wind-speed probability model is used along with the Climatology and Persistence (CLIPER) tropical cyclone track prediction model to forecast various hurricane characteristics. These hurricane related forecasts and forecast updates are introduced as a Bayesian model. Then, this information is incorporated into an optimal stopping model to assist these suppliers in their inventory decisions for hurricane supplies.

This chapter discusses the DRP problem introduced in the previous chapter for a one supplier multi retailer supply chain. In this study, more accurate hurricane predictions are able to be obtained compared to the ones from ([58]). Lodree and Taskin [58] use historical wind-speed HURDAT data to simulate the evolution of the wind-speeds. In other words, an empirical methodology is implemented to obtain hurricane wind-speed forecast updates. However, in this chapter a widely accepted statistical prediction model known as the *wind speed probability* model is used. Starting in 2006, the wind-speed probabilities are issued by the Tropical Prediction Center/National Hurricane Center (TPC/NHC) for each hurricane season in the Atlantic and Eastern North Pacific basins. It is assumed that the supplier's DRP horizon corresponds to the average life-cycle time of hurricanes (120h). In the absence of having an effective disaster recovery plan in place, the supplier under consideration is not likely to withstand a hurricane induced demand surge. Lodree and Taskin [58] discuss the results of an ineffective or nonexistent disaster recovery plan. They mention that it might be impossible to deliver hurricane supplies to some locations within the supply chain network because of the inaccessibility of the transportation system. Additionally, the supplier may incur expenses for overtime, outsourcing or faster modes of transportation to accommodate the demand surges caused by hurricane events. Therefore, the supplier should determine the optimum inventory levels for the hurricane supplies such that the financial risks associated with over/under preparation are minimized as a result of improved hurricane forecast accuracy.

The remainder of this chapter is organized as follows: Section 3.1 reviews the hurricane prediction literature. In section 3.2, the hurricane prediction model is introduced. In section 3.3, the sequential statistical decision model associated with the optimal stopping problem is presented. In section 3.4, the mathematical formulation of the problem is presented, followed by the solution approach. Then, numerical example problems are given to illustrate how the methodology can be implemented in practice. Finally, conclusions are presented in section 3.6.

3.1 Hurricane Prediction Literature

There exists a substantial amount of research on hurricane prediction. Vickery and Twisdale [82] develop a simulation methodology using wind-field and filling models to obtain hurricane wind-speeds associated with various return periods. Simulation results reveal that the subregion identification is a critical factor for wind-speed prediction. Lehmiller, Kimberlain, and Elsner [56] use a multivariate discriminant analysis for making forecasts of hurricane activity both in the Gulf of Mexico and Caribbean Sea. The results of their statistical model identify different subsets of predictors within different prediction locations. Elsner, Niu, and Tsonis [44] develop an empirical Bayesian prediction algorithm to assess the potential usage of multi-season forecasts for the North Atlantic hurricane activity. Their analysis of the correlation values of fitted univariate time series reveals that the hurricane attributes can be well fitted by an univariate autoregressive moving average. Jagger, Niu, and Elsner [45] apply a space-time count process model to annual North Atlantic hurricane activity. They use the best-track data set of historical hurricane positions and intensities together with climate variables to determine the local space-time coefficients of a right-truncated Poisson process. The results show that on average, model forecast probabilities are larger in regions, in which hurricanes occur. Additionally, it is determined that there exists a hurricane path persistence among seasons. The results also indicate that this modeling procedure can be useful as a climate prediction tool since forecast skill above climatology is observed.

Klotzbach and Gray [51] develop an updated statistical scheme for forecasting tropical cyclone activity in the Atlantic basin. Their statistical findings reveal that the hurricane landfall probability shows considerable forecast skill from $1951-2000$ based on the net tropical cyclone activity prediction and the weighted North Atlantic See Surface Temperatures (SST). Elsner and Jagger [30] develop a hierarchical Bayesian strategy for modeling annual U.S. hurricane counts from $1851-2000$. The Bayesian analysis reveals that hurricane counts only from the twentieth century together with noninformative priors compares favourably to a traditional approach. Through the implementation of the Bayesian approach, climate relationships to U.S. hurricanes are also examined. The results of the Bayesian model also confirms a statistical relationship between climate patterns and coastal hurricane activity.

Weber [83] develops a method known as The Probabilistic Ensemble System for the Prediction of Tropical Cyclones (PEST) to develop geographical strike probability maps. The results of the model indicate that the mean annual errors of the deterministic position forecasts are comparable in quality to that of the current consensus approaches. In another paper, [84] presents a method for the maximum wind-speed prediction of tropical cyclones using PEST. In this study, he makes deterministic intensity predictions for all global tropical cyclone events during subsequent forecast periods of years 2001 and 2002, respectively. Post-analysis results reveal that the sizes of all intensity probability intervals give reliable estimates of future storm intensities. The model's deterministic forecasts are determined to have the same quality of the majority of all dynamical models. Nevertheless, the intensity predictions with PEST is observed to have lower overall quality than the position predictions with PEST discussed in [83].

Regnier [66] survey existing research on weather forecasts to assist DMs in their weather-sensitive decisions using Operations Research and Management Science (OR&MS) tools. DeMaria et al. [26] develop an experimental version of Statistical Hurricane Intensity Prediction Scheme (SHIPS). This new version includes the satellite observations for the 2002 and 2003 hurricane season. Predictors selected based on this version include the brightness temperature information from Geostationary Operational Environmental Science (GOES). The storm decay information is also incorporated to SHIPS to increase the accuracy of the hurricane forecast. The results of the analysis demonstrate that the inclusion of the effects of the decay over land beginning in year 2000 reduce the intensity errors up to 15%. Additionally, the combination of GOES and satellite altimetry improve the Atlantic forecasts by up to 3.5%. Regnier and Harr [68] develop a decision model to prepare for an oncoming hurricane where the DM monitors an evolving hurricane. The results indicate that the DM who has the flexibility to wait for an updated hurricane forecast can gain substantial value by adopting a dynamic approach to anticipate the improving forecast accuracy. Elsner and Jagger [31] develop a modeling strategy that uses May-June averaged values representing the North Oscillation Index (NOI), Southern Oscillation Index (SOI), and the Atlantic Multidecadal Oscillation (AMO) to predict the probabilities of observing U.S. hurricanes in the months ahead (July-November). A Bayesian approach is used to examine three different models that take the advantage of historical records extending back to 1851. These models are (i) a full model that includes all three predictors (NOI, SOI and AMO) (ii) a reduced model that includes NOI and SOI and (iii) a single-predictor model that includes only NOI. The statistical findings show that the NOI and SOI combination model and the NOI single predictor model performs best. The results of the model also show forecast skill above climatology for the years in which there are no hurricanes or more than two hurricanes. Additionally, it is determined that all three models capture annual variation in hurricane counts better than climatology does. Other recent research includes ([80, 64, 50, 52, 55, 51, 65, 25, 17]).

3.2 Hurricane prediction model

In this chapter, the National Hurricane Center (NHC)'s well-recognized hurricane prediction model, which is based on the wind-speed probability predictions, is used. The hurricane wind speed probability graphs show the probabilities of sustained surface wind-speeds of 74 mph (hurricane force) at different locations. More specifically, each graphic provides cumulative probabilities that wind-speeds of 74 mph will occur at any location officially during cumulative 12 hour intervals (i.e., $0-12$h, $0-24$h, $0-36$h,…,$0-120$h) and extends through a 5 day forecast. Figure 3.1 demonstrates an example wind-speed probability map made by the NHC.

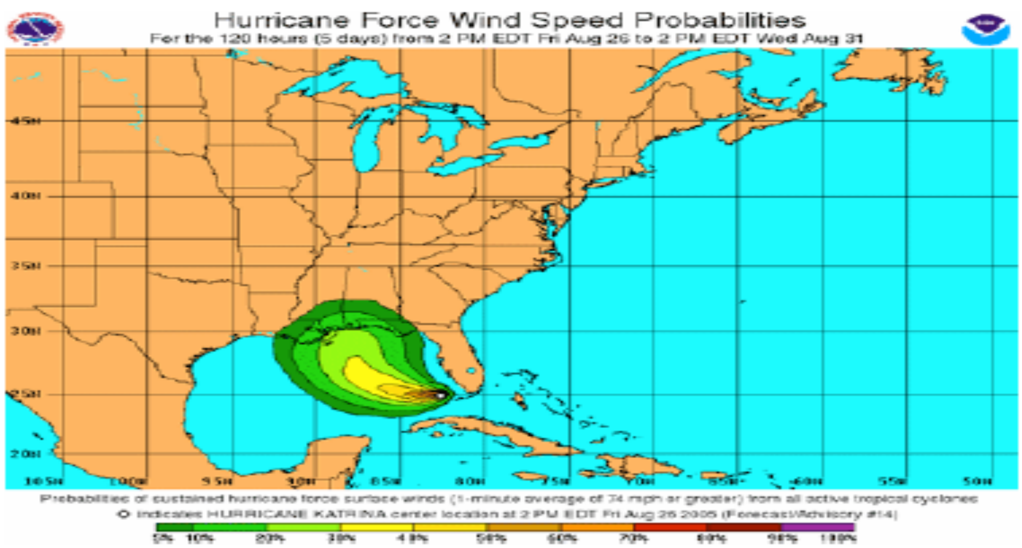

Figure 3.1: NHC wind-speed probability map.

As can be seen in Figure 3.1, the wind-speed probability graphs are organized such that the cumulative probabilities are given in percent from 1% to 100% in color-coded 10% bands, which indicate the probabilities of sustained surface winds that corresponds to 74 mph. Although, the focus of this research is on hurricane force winds only, wind-speed probability predictions also exist for tropical storm force wind-speeds.

The wind-speed probabilities are based on the official track, intensity, and wind radii forecasts, and on their corresponding forecast errors issued by the NHC. These forecast errors are determined by comparing the official forecasts with the best track database [1]. The track forecast error is determined as the circular distance between a cyclone's forecast position and the best track position. These errors are obtained by the distribution of along track (AT) and cross track (CT) forecast errors ([40]). While AT errors give an indication of the forecast of the tropical cyclone movement, CT errors are used to determine whether the model changes the path of the hurricane so frequently or not ([40]). On the other hand, the intensity error is forecasted as the absolute difference between the forecast and the best track intensity at each forecast verifying time. These forecasts and forecast errors can be found from the NHC's public resources, the Tropical Cyclone Forecast/Advisory and NHC Official Forecast Error Database. In addition to forecasting the center position and intensity of a tropical cyclone, the variability in tropical cyclone size (wind radii) is incorporated into the track forecasts via a *climatological wind radii forecast* model developed by ([35]). Based on their model, Monte Carlo (MC) simulation of the wind-speed probabilities are accomplished by creating a large sample of storm tracks and intensities relative to a given forecast track along with the climatological variations of tropical cyclone size. The corresponding wind radii forecast model formulated by [35] is:

$$V(r,\lambda) = (V_m - \alpha) \cdot (r_m/r)^x + \alpha \cdot [\cos(\lambda - \lambda_o)] \tag{3.1}$$

where $V(r,\lambda)$ is the wind-speed threshold, r is the radius from the storm center "wind radius" (nmi), λ is the angle measured counterclockwise starting from a direction 90^o to the

[1] obtained by the NHC's post-storm analysis of all observed storm data

right of the storm motion, V_m is the maximum wind (mph), r_m is the radius of maximum wind (nmi), x is a size parameter (non-dimensional), and α is an asymmetry parameter (kt) that is a function of the storm speed of motion.

Gross, DeMaria and Knaff [35] analyze the climatological model by fitting $x, r_m, \alpha, \lambda_o$ to the NHC wind radii forecasts for the $1988 - 2002$ Atlantic storms. They determine that r_m and x can be estimated in terms of V_m and latitude θ, and also α can be expressed as a function of the storm speed of motion c as given below.

$$r_m = 35.37 - 0.111 \cdot V_m + 0.570 \cdot (\theta - 25) \quad (3.2)$$

$$x = 0.285 + 0.0028 \cdot V_m \quad (3.3)$$

$$\alpha = 0.337 \cdot c - 0.003 \cdot c^2 \quad (3.4)$$

The wind radii for each MC track sample is simulated based on the *error distribution* of the x parameter. The initial value of the error in X is chosen as the difference between the value from the above climatological model and the best fit values to the observed radii at each storm quadrant. Various error values at forecasting times are then determined as a linear combination of the initial value and a random component to develop the error distribution. Then, the wind-speed probabilities are determined by counting the fraction of grid points that fall within the radius of a given wind-speed threshold $(39, 74)$ mph.

Initially, the Climatology and Persistence (CLIPER) is implemented to predict the storm center coordinates at forecasting periods. This model uses the current path of a tropical cyclone and an average of historical paths of similar cyclones to come up with a track. Additionally, CLIPER takes into account the size of the cyclone at the start of the forecast period as well as changes in size as the cyclone evolves in strength, motion and other factors. Two sets of regression equations are implemented to predict the storm track, where each predictand is either the zonal or meridional displacement observed at time t. Aberson [1] determine that the initial latitude, initial longitude, initial intensity, initial day number, initial zonal motion and the initial meridional motion impact the topical cyclone track. Among these predictors, [1] chose the significant predictors for the meridional displacement M, and zonal displacement Z in the Atlantic basin. Based on these selected predictors, regression analysis is implemented for various hurricane scenarios to forecast the storm centers. Eq. (3.5) gives the regression equations:

$$\begin{aligned} Z &= \beta_0 + \beta_1 \cdot U + \beta_2 \cdot LAT + \beta_3 \cdot (LON \cdot V) + \beta_4 \cdot (LAT \cdot V) \\ &\quad + \beta_5 \cdot (LON \cdot DAY) \beta_6 \cdot (LAT \cdot U) + \beta_7 \cdot (LAT \cdot LAT) + \beta_8 \cdot (DAY \cdot U) \\ &\quad + \beta_9 \cdot (U \cdot V) + \beta_{10} \cdot (INT \cdot V) \\ M &= \beta_0 + \beta_1 \cdot V + \beta_2 \cdot U + \beta_3 \cdot (INT \cdot U) + \beta_4 \cdot (LAT \cdot INT) + \beta_5 \cdot (INT \cdot V) \\ &\quad + \beta_6 \cdot (U \cdot V) + \beta_7 \cdot (DAY \cdot U) + \beta_8 \cdot (LON \cdot DAY) + \beta_9 \cdot (LAT \cdot V) \end{aligned} \tag{3.5}$$

where LAT is the initial latitude, LON is the initial longitude, INT is the initial intensity, DAY is the initial day number, U is the initial zonal motion, and finally V corresponds to the initial meridional motion.

In order to develop hurricane scenarios, mean values of these predictors and predictands are used for the chosen dependent data ($1931 - 1995$). Then, the wind-speed probability maps are simulated at each storm center to predict the probability of having hurricanes at different locations within the selected region.

3.3 Sequential Statistical Decision Model

In this chapter, a sequential statistical decision problem is considered to determine a stopping rule $\boldsymbol{\tau}$ and a decision rule $\boldsymbol{\delta}$ that minimizes the loss function $L\left(p, \delta_t\left(\mathbf{w}^t\right), t\right)$ based on a sequential random sample $\mathbf{w}^t$. The decision loss depends on random variable vector $W_{\vec{H}}$ that has a known probability density function with a parameter vector $\vec{P}$. Before deciding upon a δ that minimizes the loss function, the DM has the opportunity to obtain more information about the unknown distribution parameter vector $\vec{P}$ by observing a *sequential random sample* $\mathbf{W}^t$:

$$\mathbf{W}^t = \begin{pmatrix} W_{11} & W_{12} & \dots & W_{1t} \\ W_{21} & W_{22} & \dots & W_{2t} \\ \dots & \dots & \dots & \dots \\ W_{n1} & W_{n2} & \dots & W_{nt} \end{pmatrix} \tag{3.6}$$

where the cost of observing $\vec{W}_j = (W_{j1}, \dots, W_{jn})$ is C_j. Note that $\vec{W}_j$'s are independently and identically distributed.

In this study, the stopping time T is defined as the cumulative 30h interval such that sampling is stopped and a decision δ_t is given whether to observe $\vec{W}_{t+1}$. Then, the sampling cost of the statistical sequential model is determined as $\sum_{j=30}^{T} C_j$. Eq. (3.7) expresses the stopping time T as

$$T\left(\mathbf{W}^t\right) = \min_{t \in \{0,30,60,90,120\}} \left\{\tau_t\left(\mathbf{W}^t\right) = 1\right\} \tag{3.7}$$

with τ_t being the probability of stopping the sampling process after $\mathbf{W}^t$ is observed. Then the risk function associated with the sequential decision procedure $\boldsymbol{d} = (\boldsymbol{t}^*, \boldsymbol{Q})$ is the expected loss:

$$\begin{aligned} R(\vec{P}, \mathbf{d}) &= E\left[L\left(\vec{P}, \delta_T\left(\mathbf{W}^t\right), T\right)\right] \qquad (3.8) \\ &= P(T=0) \cdot L(\vec{P}, \delta_0, 0) + \sum_{t=30}^{120} \int_\Lambda L(\vec{P}, \delta_t(\mathbf{w}^t), t) \mathrm{d}H_t(\mathbf{w}^t | \vec{P}) \\ &+ \sum_{t=1}^{\infty} \sum_{j=1}^{t} C_j P(T=t) \end{aligned}$$

where the sequential sample $\mathbf{w}^t$ is

$$\mathbf{w}^t = \begin{pmatrix} w_{11} & w_{12} & \dots & w_{1t} \\ w_{21} & w_{22} & \dots & w_{2t} \\ \dots & \dots & \dots & \dots \\ w_{n1} & w_{n2} & \dots & w_{nt} \end{pmatrix} \qquad (3.9)$$

The Bayesian updating of the prior estimates $\tilde{\vec{p}}$ of the random variable $\vec{P}$ is achieved at each forecasting period t to obtain the corresponding posterior estimates. The Bayes risk function of the problem is developed using these predictions as given in Eq. (3.10).

$$r(\tilde{\vec{p}}_t, \mathbf{d}, t) = \mathbb{E}[R(\vec{P}, \mathbf{d})] \qquad (3.10)$$

Then the sequential decision problem can be formulated as follows.

$$r(\tilde{\vec{p}}_t, t) = \inf_{\mathbf{d}} r(\tilde{\vec{p}}_t, \mathbf{d}, t) \qquad (3.11)$$

3.4 Model Formulation

In this section, an optimal stopping framework is introduced to solve the hurricane supply stocking problem such that both the track and the intensity of the tropical cyclone are taken into account to determine an optimal inventory decision. Below, the assumptions and the details of the model are presented.

3.4.1 Assumptions

The derivations of the loss and risk functions are based on the following assumptions.

Assumption 6 *The supplier's demand is a random variable X, where X is a function of the observed storm's intensity along its path.*

Let $X_i, i = 1, \ldots, n$, be a random variable that represents the demand of retailer i, where n is the number of retailer locations whose demand can potentially be affected by the observed storm. Then X is a convolution of the random variables X_i. Each X_i depends on the intensity of the storm surrounding location i, which includes the possibility that the storm does not threaten the location at all. Let W_i be a random variable that represents the maximum sustained wind-speed at location i. Then Assumption 6 implies that $X_i = X(W_i)$ and $X = \sum X_i(W_i)$.

Assumption 7 *Two classes of demand are considered at each retailer location: demand associated with hurricane force winds and demand that corresponds to no hurricane force winds.*

Assumption 7 implies that the demand distribution at location i is one of two categories. Let Y_i denote demand at location i if no hurricane force winds are experienced and Z_i

be demand at location i if hurricane force winds are experienced, where both Y_i and Z_i are random variables for each $i = 1, \ldots, n$. Then $X_i \in \{Y_i, Z_i\}$. This represents the initial approach to modeling the impact of an observed storm on each retailer's demand, and hence the supplier's demand.

Assumption 8 *Let $\vec{H} = (H_1, \ldots, H_n)$ be a multivariate Bernoulli random variable such that $H_i = 1$ indicates hurricane force winds at location i, and $H_i = 0$ otherwise, where $i = 1, \ldots, n$. Let $\vec{P} = (P_1, \ldots, P_n)$ be the parameter vector associated with $\vec{H}$, where $P_i = Pr\{H_i = 1\}$, and $P_{it} = Pr\{H_{it} = 1 | \vec{A}_{it} = \vec{\alpha}_{it}\}$ is the probability of having hurricane at location i after observing the t^{th} wind-speed probability update, $\vec{\alpha}_{it}$. Then*

1. $\vec{P}$ is a random vector with prior densities $h_i(p_i)$, likelihood densities $h_i^t(\vec{\alpha}_i^t | p_i)$, and posterior densities $h_i^t(p_i | \vec{\alpha}_i^t)$. Here $\vec{\alpha}_{it}$ is a vector that represents an observation of storm attributes (namely location, intensity, and radius) that are used in wind-speed probability calculations, $t = 1, \ldots, T$ and T is the number of hurricane prediction updates (T is also interpreted as the number of periods during the planning horizon. . . see Assumption 10).

2. The likelihood and posterior densities, $h_i^t(\vec{\alpha}_i^t | p_i)$ and $h_i^t(p_i | \vec{\alpha}_i^t)$ respectively, are based on hurricane prediction updates that are given every 30 hours after a tropical depression or disturbance is initially observed. Thus t progresses in 30h intervals.

NHC publishes wind-speed probability maps (see Figure 3.1) typically in 12 hour intervals as an observed storm evolves. In this study, the wind-speed probability maps are updated in 30 hour intervals. Each map is generated based on an observation of $\vec{\alpha}_i^t$, given that $t - 1$

maps have been published before. Thus each wind-speed map is a posterior probability calculation, which is represented mathematically based on the notation as $h_i^t(p_i|\vec{\alpha}_i^t)$.

Assumption 9 *Let c_t be the production cost associated with giving a decision after t hours. Then $c_t \leq c_{t+30}$ for all $t = 0, 30, \ldots, 120$.*

Assumption 9 relates to the difficulty of implementing a production decision during the latter stages of the planning horizon. In reality, the problem with waiting for very accurate storm information is that there may not be enough time to meet demands because of limited production capacity. If the decision about the target inventory level is determined earlier, then the supplier can schedule production over a few days without paying a premium for additional capacity. Thus $C_j = c_{j+30} - c_j$ for $j = 0, 30, \ldots, 90$ and $c_0 = 0$ can be interpreted as the premium that the supplier has to pay for additional capacity in order to reach target inventory levels such that finished goods can be shipped to the disaster area immediately after the disaster strikes.

Assumption 10 *Demand realization happens at each location i exactly* 5 *days* (120 *hours) after the storm is initially observed.*

Assumption 10 allows us to specify that the length of the supplier's decision horizon is known with certainty. That is, $T = 120$. In actuality, the length of the supplier's decision horizon is also uncertain. However, useful results from sequential Bayesian decision theory are leveraged to model and solve the inventory control problem by assuming that T is known with certainty. Additionally, this assumption is used to introduce convoluted demand distributions.

3.4.2 Single Period Loss Function

Assumption 8 allows to introduce a multivariate Bernoulli random variable $\vec{H} = (H_1, \ldots, H_n)$ with density of $g(\vec{h}, \vec{P}) = \prod_{i=1}^{n} P_i^{h_i} \cdot (1 - P_i)^{(1-h_i)}$ and $h_i \in \{0, 1\}$, that specifies whether a hurricane is observed ($H_i = 1$) or not ($H_i = 0$) at location i. Then, a hurricane demand Z_i with probability P_i is considered at location i. Similarly, a regular demand Y_i with probability $1 - P_i$ is applicable at location i. Since the demand at different locations are independent and identically distributed, the loss function $\mathcal{L}(Q)$ is formed using a convolution of the random variables that represent demand at each respective location. The probability of observing a specific convoluted demand k at time t is introduced as the *scenario probability* q_{kt}. Once the hurricane demand probabilities $\vec{p_t} = \{p_{1t}, p_{2t}, \ldots, p_{nt}\}$ at each location $i = 1, 2, \ldots, n$ are observed, the corresponding scenario probabilities q_{kt} are calculated using the multiplication rule. Then, the expected loss at time t is expressed as:

$$\mathcal{L}_t(Q_t) = \sum_{k=1}^{k=2^n} q_{kt} \cdot NB_k^t \tag{3.12}$$

where n is the number of locations, and NB_k^t is the expected cost function for the newsboy problem with convoluted demand random variable X_k for $k = 1, \ldots, 2^n$. Note that exactly one scenario can occur out of $k = 2^n$ number of scenarios at time t. If c_t is the unit order/production cost, h_t the unit holding (or overstocking) cost, s_t the unit shortage cost at time t, and $f_k(x_k)$ the density of X_k, then NB_k^t is

$$NB_k^t = Q_t \cdot c_t + h_t \cdot \int_0^{Q_t} (Q_t - x_k) f_k(x_k) \mathrm{d}x_k + s_t \cdot \int_{Q_t}^{\infty} (x_k - Q_t) f_k(x_k) \mathrm{d}x_k \tag{3.13}$$

Table 3.1 demonstrates the scenario probabilities for a two location problem at time t. Note that for a n location problem, there will be $n+1$ different convoluted demand distributions given that the retailers have the same demand distribution parameters for the two classes of demand.

Table 3.1: Scenario probabilities

Scenario k	Scenario probability q_{kt}	Retailer1	Retailer2	Convoluted Demand X_k
1	$q_{1t} = p_{1t} \cdot p_{2t}$	Z_1	Z_2	$Z_1 + Z_2$
2	$q_{2t} = (1 - p_{1t}) \cdot p_{2t}$	Y_1	Z_2	$Y_1 + Z_2$
3	$q_{3t} = (1 - p_{1t}) \cdot (1 - p_{2t})$	Y_1	Y_2	$Y_1 + Y_2$
4	$q_{4t} = p_{1t} \cdot (1 - p_{2t})$	Z_1	Y_2	$Z_1 + Y_2$

3.4.3 Bayes Risk Function

In order to determine the ordering quantity Q, the Bayes risk function of the problem $r(\tilde{\vec{p_t}}, \mathbf{d}, t)$ is formalized by revising the loss function given in Eq. (3.12) such that the posterior predictions of $\vec{P} = \tilde{\vec{p}}$, and the cost of observing a sequential sample $\mathbf{W}^t$ associated with $\vec{P}$ are incorporated into the function. As a result, $r(\tilde{\vec{p_t}}, \mathbf{d}, t)$ is expressed as

$$r(\tilde{\vec{p_t}}, \mathbf{d}, t) = \sum_{k=1}^{k=2^n} q_{kt} \cdot NB_k^t + \sum_{j=30}^{t} C_j \qquad t = 30, \ldots, T \tag{3.14}$$

Then, the sequential problem is bounded such that

$$r_T(\tilde{\vec{p_t}}, t) = \inf_{\mathbf{d}} r(\tilde{\vec{p_t}}, \mathbf{d}, t) \tag{3.15}$$

with T being the maximum number of observations taken at locations. Additionally, define $r_0(\tilde{\vec{p_t}}, t)$ as the minimum Bayes risk associated with giving an immediate decision in period

t, and $r_{T-t}(\tilde{\vec{p_t}}, t)$ as the minimum Bayes risk associated with $(T-t)$ more observations at locations. Theorem 3.1 is introduced to state the sequential decision problem recursively.

Theorem 3.1 *(Degroot [24]): Among all sequential decision procedures in which not more than T observations can be taken, the following procedure is optimal: If $r_0(\tilde{\vec{p_t}}, 0) \leq r_T(\tilde{\vec{p_t}}, 0)$, a decision δ_0 is chosen immediately without any observations. Otherwise, $\vec{W}_1$ is observed. Furthermore, for $t = 30, 60, \ldots, T-30$, suppose the sequential random sample $\mathbf{W}^t$ has been observed. If $r_0(\tilde{\vec{p_t}}, t) \leq r_{T-t}(\tilde{\vec{p_t}}, t)$, a decision δ_t is chosen immediately without further observations. Otherwise, $\vec{W}_{t+1}$ is observed. If sampling has not been terminated earlier, it must be terminated after $\vec{W}_T$ is observed.*

Then, the recursive formulation of the sequential decision problem is

$$r_j(\tilde{\vec{p_t}}, t) = \min_{\mathbf{d}} \left\{ r_0(\tilde{\vec{p_t}}, t), \mathbb{E}\left[r_{j-30}(\vec{p_t} | \mathbf{W}^{t+1}, t+1) \right] \right\} \qquad j = 60, \ldots, T \tag{3.16}$$

3.5 Solution Methodology

In this section, the methodology used to determine the order/production quantity Q_t and single order/production period t^* that minimizes the expected loss associated with ordering/producing, overstocking, and understocking is described. Initially, posterior wind-speed probabilities $\tilde{\vec{p_t}}$ at locations are predicted based on the CLIPER's updated storm center forecast and the generated wind-speed probability map at this forecasted storm center. Then, the optimal order/production quantity is determined. More specifically, once $\vec{p_t}$ is observed, the optimal decision is Q_t that minimizes the loss function $L_t(Q_t)$ with order/production cost c_t. Theorem 3.2 describes the resulting decision rule.

Theorem 3.2 *(Lodree and Taskin [57]): Let $F_k(x_k)$, where $k = 1, \ldots, 2^n$ be the cumulative distribution function of random demand $X_k \in \{Y, Z\}$. Then the optimal decision $\delta_t = Q_t$ that minimizes the loss function $L_t(Q_t)$ with order/production cost c_t satisfies*

$$\sum_{k=1}^{k=2^n} q_{kt} \cdot F_k(Q_t^*) = \frac{s-c}{s+h} \tag{3.17}$$

Theorem 3.1 suggests that the optimal decision rule $\boldsymbol{\delta}^t = (Q_1, \ldots, Q_t)$ should be determined first to search for the optimal stopping time t^*. Then, it is optimal to stop and place an order if $L_t(Q_t) \leq L_{t+1}(Q_{t+1})$ is satisfied.

3.5.1 Numerical Example

In this section, the previously mentioned tropical cyclone forecasting methods are used to determine the wind-speed probabilities. More specifically, various wind-speed probability scenarios are considered to demonstrate the solution methodology. The CLIPER track prediction model is used to forecast the fictitious storm's track. The hypothetical tropical cyclone is assumed to start at a predetermined location for simplicity. The initial zonal (longitude) coordinate is $x = 75W^o$ and the initial meridional (latitude) coordinate is $y = 25N^o$. Then, the CLIPER regression equations are run for a large sample size at each forecasting time to come up with a track scenario. Table 3.2 demonstrates the zonal and meridional displacements over 30h forecasting intervals.

Table 3.2: CLIPER Output

Forecast Time	Zonal Displacement	Meridional Displacement	Storm Center Coordinate (W^o, N^o)
30	3.76	0.33	(78.76,25.33)
60	6.30	-1.82	(85.06,23.51)
90	3.27	-0.69	(88.33,22.82)
120	4.50	-1.73	(92.83,21.09)

Based on the CLIPER storm track forecast, the *wind-speed probability model* is implemented to predict $\vec{P} = (P_1, \ldots, P_n)$ at each forecasting period. Then, the predicted wind-speed probabilities, which are determined from the simulated maps, are considered for the following example problems. These values are presented as a matrix $P_{T \times N}$ with T rows and N columns, where each row corresponds to a period $t = 0, 30, 60, 90, 120$h and each column corresponds to a particular location. Thus an entry p_{tn} represents the wind-speed probability at location n during period t. The predicted wind speed probabilities at each forecasting period are presented in Table 3.3.

Table 3.3: Predicted wind-speed probabilities

Example	P_{TxN}	Example	P_{TxN}
1	$\begin{pmatrix} 0.6 & 0.62 & 0.64 & 0.66 & 0.68 \\ 0.5 & 0.52 & 0.54 & 0.56 & 0.58 \\ 0.4 & 0.42 & 0.44 & 0.46 & 0.48 \\ 0.3 & 0.32 & 0.34 & 0.36 & 0.38 \\ 0.2 & 0.22 & 0.24 & 0.26 & 0.28 \end{pmatrix}$	2	$\begin{pmatrix} 0.2 & 0.22 & 0.24 & 0.26 & 0.28 \\ 0.3 & 0.32 & 0.34 & 0.36 & 0.38 \\ 0.4 & 0.42 & 0.44 & 0.46 & 0.48 \\ 0.5 & 0.52 & 0.54 & 0.56 & 0.58 \\ 0.6 & 0.62 & 0.64 & 0.66 & 0.68 \end{pmatrix}$
3	$\begin{pmatrix} 0.5 & 0.45 & 0.6 & 0.55 & 0.7 \\ 0.5 & 0.45 & 0.6 & 0.55 & 0.7 \\ 0.5 & 0.45 & 0.6 & 0.55 & 0.7 \\ 0.2 & 0.25 & 0.35 & 0.4 & 0.55 \\ 0.3 & 0.35 & 0.4 & 0.45 & 0.5 \end{pmatrix}$	4	$\begin{pmatrix} 0.9 & 0.6 & 0.45 & 0.7 & 0.85 \\ 0.95 & 0.7 & 0.65 & 0.6 & 0.7 \\ 0.9 & 0.75 & 0.6 & 0.65 & 0.75 \\ 0.85 & 0.8 & 0.65 & 0.75 & 0.8 \\ 0.9 & 0.85 & 0.7 & 0.7 & 0.75 \end{pmatrix}$
5	$\begin{pmatrix} 0.7 & 0.65 & 0.55 & 0.48 & 0.1 \\ 0.6 & 0.65 & 0.15 & 0.2 & 0.9 \\ 0.2 & 0.25 & 0.25 & 0.15 & 0.55 \\ 0.2 & 0.25 & 0.25 & 0.15 & 0.55 \\ 0.3 & 0.35 & 0.4 & 0.35 & 0.5 \end{pmatrix}$	6	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0.9 \\ 0.3 & 0 & 0 & 0 & 0.8 \\ 0 & 0.25 & 0 & 0 & 0.55 \\ 0.2 & 0.25 & 0 & 0 & 0.55 \\ 0.3 & 0.35 & 0.4 & 0 & 0.5 \end{pmatrix}$
7	$\begin{pmatrix} 0.45 & 0 & 0.25 & 0.3 & 0.4 \\ 0.3 & 0 & 0.25 & 0.15 & 0.2 \\ 0.1 & 0 & 0 & 0.17 & 0.3 \\ 0.06 & 0 & 0 & 0.1 & 0.9 \\ 0.01 & 0 & 0 & 0 & 0.8 \end{pmatrix}$	8	$\begin{pmatrix} 0.2 & 0.22 & 0.24 & 0.26 & 0.28 \\ 0 & 0 & 0.24 & 0.26 & 0.28 \\ 0.2 & 0.1 & 0 & 0 & 0.3 \\ 0.1 & 0 & 0 & 0.46 & 0.48 \\ 0.2 & 0 & 0 & 0.56 & 0.58 \end{pmatrix}$
9	$\begin{pmatrix} 0.2 & 0.5 & 0.6 & 0.4 & 0.3 \\ 0.04 & 0.4 & 0.9 & 0 & 0 \\ 0.45 & 0.6 & 0.25 & 0 & 0 \\ 0.03 & 0.2 & 0.8 & 0 & 0 \\ 0.1 & 0.4 & 0.9 & 0 & 0 \end{pmatrix}$	10	$\begin{pmatrix} 0.1 & 0.4 & 0.6 & 0.5 & 0.3 \\ 0 & 0 & 0.6 & 0.55 & 0.45 \\ 0 & 0 & 0.34 & 0.36 & 0.38 \\ 0 & 0 & 0.2 & 0.36 & 0.38 \\ 0 & 0 & 0.54 & 0.56 & 0.58 \end{pmatrix}$

As can be seen in Table 3.3, the first five examples consider having hurricane force wind-speeds at all locations during forecasting periods. The other five also include locations that are expected to have no hurricane force winds speeds at a specific forecasting period.

Figures 3.2, 3.3, 3.4, 3.5 and 3.6 display the Matlab™wind-speed probability maps for Example 1 created at 0h, 30h, 60h, 90h, and 120h, respectively. Note that the green line corresponds to the forecasted storm track, and the red line represents the realized storm path up to that forecasting period. Figure 3.2 shows only the forecasted track since it is formed at the start of the forecasting period $t = 0$h. More specifically, it gives the prior wind-speed probability map, which is updated in 30h intervals to predict the posterior wind-speed probability maps as more wind-speed information becomes available. In fact, each wind speed probability map corresponds to a prior of its subsequent one. Similar maps can be created for the other example problems.

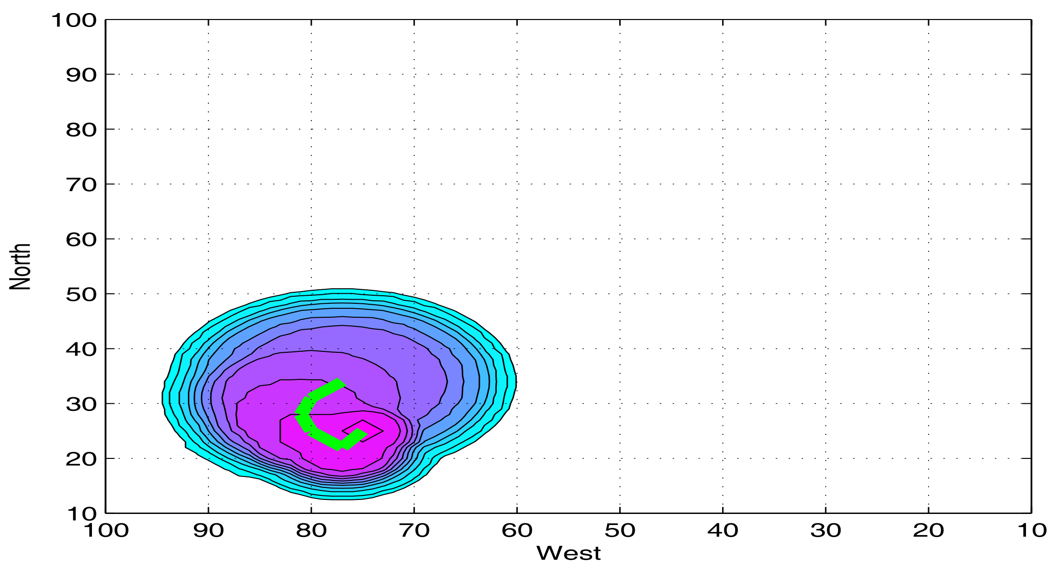

Figure 3.2: Wind-speed probability map at $t = 0$h.

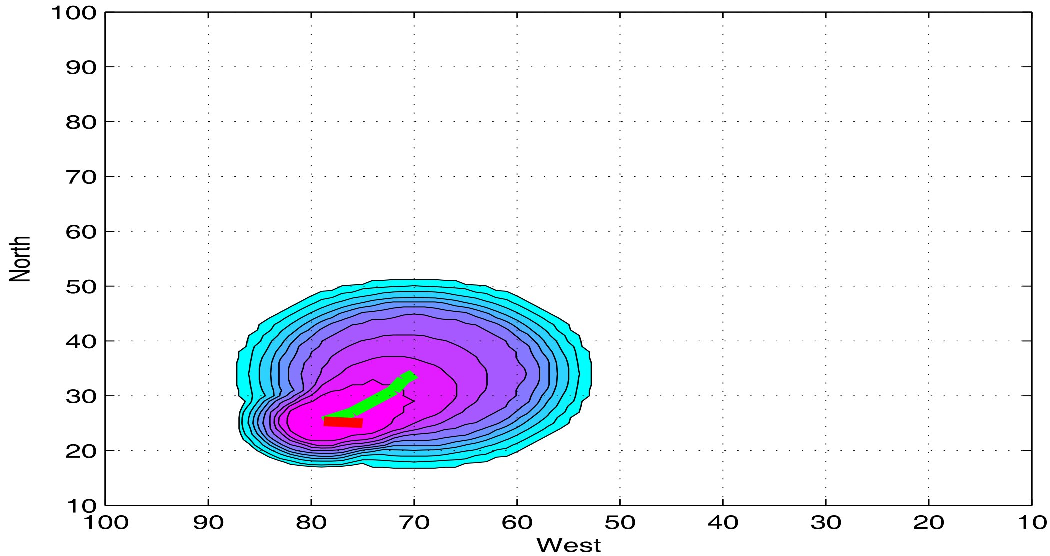

Figure 3.3: Wind-speed probability map at $t = 30$h.

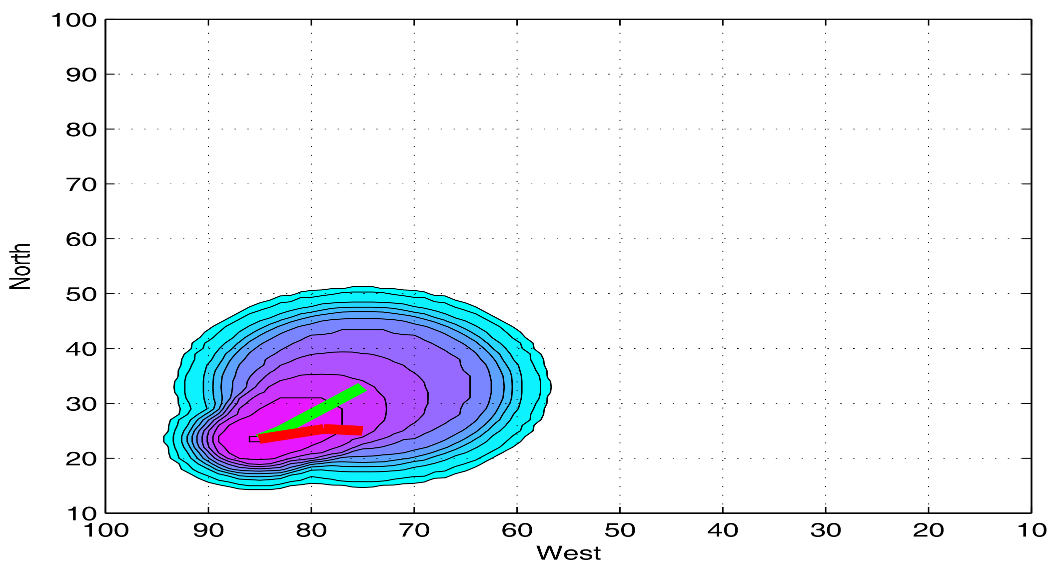

Figure 3.4: Wind-speed probability map at $t = 60$h.

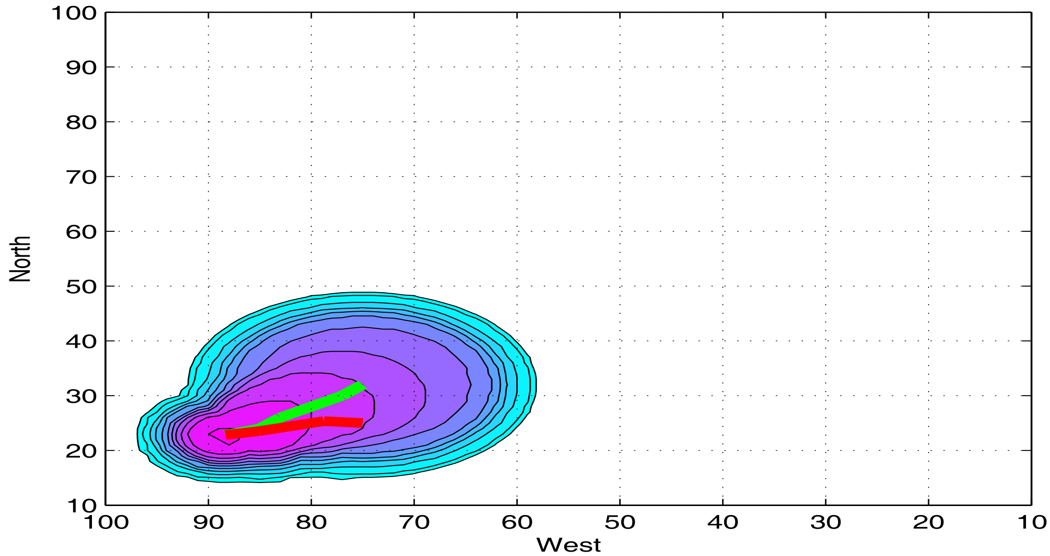

Figure 3.5: Wind-speed probability map at $t = 90$h.

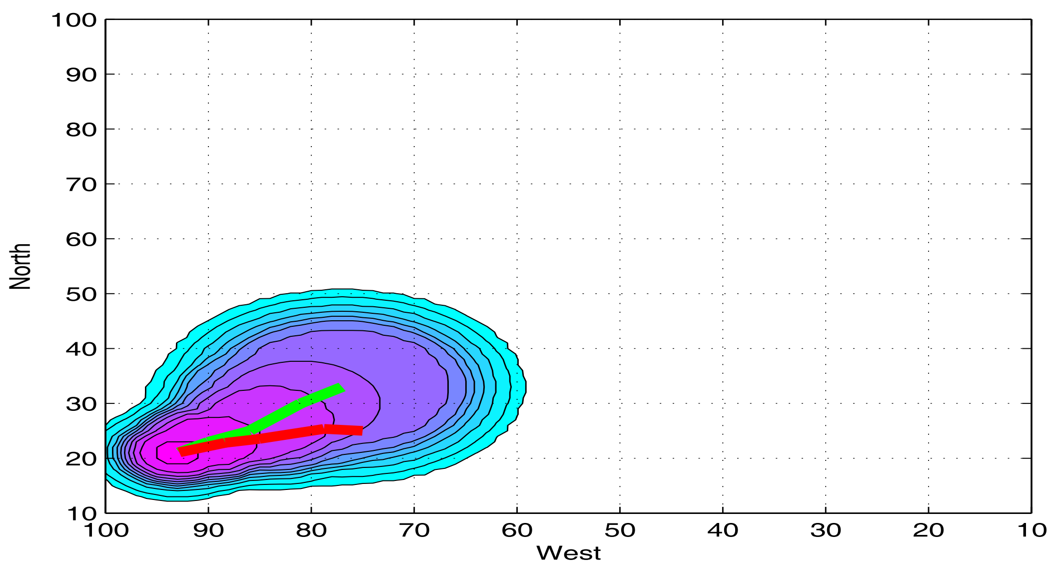

Figure 3.6: Wind-speed probability map at $t = 120$h.

The following data is used to solve each of the example problems: $c_1 = 20, c_{t+1} = c_t + 1, s = 100, h = 15, n = 5$ location. The demand information at locations are given in Table 3.4.

Table 3.4: Demand information

Location i	Regular Demand Y_i	Hurricane Demand Z_i
1	$N(500, 25)$	$N(1000, 100)$
2	$N(550, 30)$	$N(1500, 125)$
3	$N(600, 35)$	$N(2000, 150)$
4	$N(650, 40)$	$N(2500, 175)$
5	$N(700, 45)$	$N(3000, 200)$

Note that the DMs have the option of giving an order each time an updated wind-speed probability map is obtained. Therefore, only the cumulative 30 hour interval wind-speed probability updates are considered for the example problems. Mathematica 5.2™is also used to obtain decision rules for these problems. The results are shown in Table 3.5.

Table 3.5: Example results

Example	Decision Period t^*	Stocking Quantity Q_t	Expected Cost EC
1	1	8567.79	\$198,995
2	1	6299.83	\$140,188
3	1	7685.85	\$203,198
4	1	8619.32	\$174,569
5	2	6751.26	\$196,656
6	3	5295.66	\$155,180
7	3	7156.1	\$132,302
8	3	5829.81	\$122,755
9	4	4884.36	\$112,849
10	4	6250.24	\$145,707

The results suggest that the DMs are inclined to wait to give their order/production decisions related to the hurricane supplies as no hurricane force wind-speeds are predicted at some locations during a forecasting period. This finding is consistent with intuition in that the DMs would like to reduce the risk associated with demand uncertainty by obtaining more accurate wind-speed probability predictions as the storm evolves over time.

Table 3.5 also suggests that the optimal order quantities associated with the examples, in which all locations are expected to have hurricane force wind-speeds are larger, on average, than the other examples where cyclone wind-speeds that are less than hurricane force wind-speeds are predicted for some locations. Finally, the results are encouraging in the sense that they are consistent with the previous findings ([58]).

3.6 Summary

This chapter discusses a disaster recovery planning problem encountered by suppliers who experience a demand surge for various products such as flashlights, batteries, and gas-powered generators in case of a hurricane event. The decision framework is also applicable to a disaster relief problem faced by various service organizations. This chapter presents a more sophisticated hurricane prediction model than [58] by including both tropical cyclone track and intensity predictions into the decision framework. More specifically, the hurricane stocking problem is formulated as an optimal stopping problem with Bayesian prediction updates based on these hurricane characteristics. The proposed model and its solution methodology is based on wind-speed probabilities as opposed to maximum sustained wind-speeds used in [58]. The proposed approach utilizes wind-speed probability predictions to form a disaster recovery plan to manage the hurricane related risks. The Bayesian wind-speed probability updates are integrated into an optimal stopping framework to determine the optimal ordering policy such that a balance is maintained between forecast accuracy and cost efficiency. This approach provides suppliers a better forecast of the hurricane season so that they can accommodate demand fluctuations on time at their retailer locations at a minimum cost.

The solution methodology presented in this chapter will lead suppliers to establish disaster recovery plans that are most appropriate for managing the risks associated with hurricane events. An area for future research is to search for new methods, other than the wind-speed probabilities predicted within certain regions, to enhance the efficiency of the hurricane prediction model. Another possibility is to explore a multiple product version of the model with more sophisticated decision rules or loss functions.

Chapter 4

Advanced Inventory Planning for the Hurricane Season

This chapter addresses a stochastic inventory control problem for manufacturing and retail firms who expose to challenging procurement and production decisions caused by the hurricane events. The hurricane stocking decisions made in advance of the season are affected by the general predictions regarding the ensuing hurricane season, such as the expected number of hurricane landfalls. These kinds of predictions are issued by the NHC up to six months in advance of the season. The inventory planning problem is characterized by multiple periods before the hurricane season in which the inventory manager has the option of adjusting the inventory level during each of these planning periods. More specifically, it is assumed that the production / inventory planning horizon spans several months before the beginning of the hurricane season. During these pre-hurricane season months, manufacturing and retail organizations plan emergency supply inventory levels for the ensuing season in addition to satisfying demands for these products that occur before the season.

In this chapter, it is assumed that hurricane season demand predictions are revised at the beginning of each pre-season planning period, and that these demand predictions are correlated to landfall hurricane count rate predictions. A stochastic programming model is introduced to determine production / inventory decisions that account for pre-hurricane season demands as well as anticipated demands during the hurricane season. Hurricane season demand predictions, which are updated and observed at the beginning of each pre-season period, are represented as a Markov chain based on a hurricane landfall count prediction model. The historical hurricane landfall counts along with hurricane prediction related index values are used as for the historical data matrix. The states of the Markov chain are defined by a finite number of hurricane count rate predictions. The predicted hurricane count rate probabilities are empirically analyzed to calculate the stationary transition probabilities. The long-run properties of Markov chains are used to come up with the steady-state probabilities, over which the hurricane season demand distribution is described. Then, the underlying demand distribution is described over the weighted probabilities that are determined based on the hurricane season demand and pre-season demand distributions. The model and solution methodology described in this chapter optimize the trade-off between hurricane forecast accuracy and cost efficiency as a function of time. More specifically, the DMs can make more accurate decisions as the season draws near. On the other hand, procurement, production and logistics costs are more efficient during the earlier stages of planning.

In section 4.1 a review of related literature is presented. In section 4.2, the stochastic programming inventory model is explained. In section 4.2.1, the hurricane count prediction model and the selected predictors are introduced. In addition to this, the general idea behind the Markov Chain approach in generating demand scenarios is discussed. A numerical example is also implemented in section 4.2.2. In section 4.3, the scenario reduction approach in stochastic programming is introduced followed by a hurricane stocking example problem. In section 4.3.1, the numerical example is solved with a heuristic algorithm. Comparisons with optimal and heuristic reduction methodologies are also made. Finally, conclusions and managerial implications are presented in section 4.4.

4.1 Literature Review

In this section, relevant research from the supply chain inventory literature that is most applicable to this problem and to our solution approach is summarized. Relevant research includes inventory control for humanitarian relief and supply chain management, stochastic inventory models with Markovian demand predictions, and inventory models with more than one period to prepare for the selling season.

Beamon [10] compares and contrasts the commercial supply chain and the humanitarian relief chain to identify the challenges of relief logistics planning. Beamon [13] addresses this issue by developing a multiple supplier inventory model that determines optimal order quantities and reorder points for long-term emergency relief response. The expected number of units held per cycle and corresponding expected cycle lengths are determined both with and without emergency orders. Initially, the optimal order quantity is determined using a pre-specified stock out risk. Then the reorder quantity and level are optimized based on reordering, holding and back-order costs. Beamon and Balcik [12] evaluate inventory management strategies applied to emergency cases in South Sudan. They develop quantitative inventory management strategies for humanitarian relief. Kovács and Spens [54] describe the unique characteristics of humanitarian logistics, and emphasize that the humanitarian logistics should benefit from business logistics. Oloruntoba and Gray [63] identify the characteristics of business supply chains, and apply them to the humanitarian aid supply chain. They develop an agile supply chain model for humanitarian aid. Kapucu [46] examines the role of non-profit organizations with respect to responding to a catastrophic disaster via a case study. Lodree and Taskin [58] introduce newsvendor variants to assess the risks and benefits associated with inventory decisions with respect to preparing for supply chain disruptions or disaster relief efforts. Alexander Smirnov et al. [76] develop a decision-making approach for disaster response operations, and present the similarities of industrial environment and disaster relief operations in decision-making. Beamon and Balcik [11] develop an effective performance measurement system for the relief sector. They compare performance measurement in the humanitarian relief chain with in the commercial supply chain to develop new performance metrics for the humanitarian relief chain.

The inventory models where the demand distribution is defined via a Markovian process is also relevant to this research. Karlin and Fabens [47] introduce a Markovian demand model. They claim that if each demand state is defined by different numbers, a base-stock type inventory policy can be obtained. Iglehart and Karlin [41] prove that a base-stock policy is optimal for a demand process modeled by a discrete-time Markov chain. Song and Zipkin [77] examine an inventory model, in which the fluctuations in the demand rate is represented by a continuous-time Markov chain. They determine the optimal ordering policy for a linear cost model through a modified value-iteration algorithm. They also show that their algorithm yield slightly better solutions than a standard value iteration approach. A fixed ordering cost model is also explored and it is shown that a state-dependent base-stock policy is optimal. Beyer et al. [15] show the existence of an optimal Markov policy for the discounted and average-cost problems where the demand is introduced as unbounded, and costs have polynomial growth. They also prove that the base-stock policy is optimal even when the ordering cost consists of additional components to that of the fixed cost given a convex surplus cost function. Cheng and Sethi [19] examine an inventory-promotion decision problem, in which the demand state is represented both by the environmental factors and the promotion decisions. In this study, they determine a threshold inventory level such that if this level is exceeded, it is favorable to give a promotion decision. They use dynamic programming to come up with the optimal inventory and promotion decisions for the finite horizon problem. Abhyankar and Graves [2] consider a Markov-modulated Poisson demand process, and determine closed-form approximations both for the inventory and the service level. In order to hedge against cyclic demand variability, they suggest using an intermediate-decoupling inventory. Additionally, they develop an optimization model to determine the tradeoffs between inventory investments and customer service. Finally, [18] examine a serial multistage inventory problem with Markov-modulated demand. They prove that state-dependent base-stock policy is optimal. Additionally, they develop an algorithm to determine the optimal base-stock levels.

This study can also be described as an inventory model with more than one period to prepare for the selling season. The reader is referred to [75] for an extensive list of references related to this problem. In this section, representative papers for this type of inventory model are presented. For instance, [62] examine the sales potential of a product, which is treated as a subjective random variable whose distribution is updated adaptively using Bayes's rule as the sales data becomes available. They formulate this problem as a dynamic program and introduce computationally efficient procedures for special cases. Hausman and Peterson [37] extend the work in [62] to the case of multiple products with limited production capacity in each period. They develop and compare three heuristics to solve the multi-product production planning problem. Bitran et al. [34] investigate a system that produces several families of style goods. The problem is formulated as a deterministic mixed integer programming problem that provides an approximate solution. Matuso [61] uses a continuous treatment of time and formulated the problem examined by [34] as a two-stage stochastic sequencing model. A heuristic procedure is developed to solve the problem. Kodama [53] derives the optimality condition for a single-period problem in which partial returns and purchases are allowed in case of a surplus and shortage, respectively. The author shows that this problem is a special case of the single-period problem when surplus inventory cannot be carried over between periods.

4.2 Stochastic Programming Model

The objective of this study is to determine an optimal ordering policy such that (i) demand at each pre-hurricane season period is met and (ii) reserved supplies are stored for the ensuing hurricane season in a cost effective way. The pre-season planning problem is introduced as a stochastic programming model, in which the procurement/production decisions are given to minimize the expected total cost. The assumptions of the stochastic inventory model are given as follows.

Assumption 11 *The annual hurricane landfall count $\tilde{n}_h$ is assumed to follow a Poisson distribution with rate λ.*

The Poisson distribution is used to express the probability of hurricane counts occurring in a fixed period of time. From a statistical standpoint, it is appropriate to describe the distribution of the hurricane counts as a Poisson distribution because they occur independently of the time since the last event, and with a known average rate.

Assumption 12 *Hurricane season demand is a linear function of predicted hurricane landfall count rates λ_t during month t.*

Assumption 12 enables us to define the hurricane season demand distribution in terms of the hurricane count rate probabilities. In real applications, the demand is influenced by various attributes such as hurricane wind-speeds, radius of the storm, and the population of the locations hit by hurricanes. For illustrative purposes, the hurricane season demand is introduced as a linear function of λ_t.

Assumption 13 *Pre-season demand and hurricane season demand are introduced as independent random variables.*

As consistent with intuition, demand tends to be higher during the hurricane season compared to the demand observed during the pre-season months. More specifically, they are not correlated, and should be described as independent variables.

The nature of the stochastic problem requires making multi-period ordering decisions by considering the uncertainty associated with demand realizations. The inventory control theory introduces dynamic programming, optimal control and stochastic programming as the main approaches to solve multi-period inventory problems. In this study, the inventory control problem is formulated as a multi-stage stochastic programming with recourse that can be reduced to a discrete-equivalent linear program. Dupačová et al. [28] formulate the two-stage and multi-stage stochastic programs with recourse as follows:

$$\min_{d \in \mathbb{D}} \mathbb{E}_P f(x, \mathbf{d}) = \int_X f(x, \mathbf{d}) P(\mathrm{d}x) \tag{4.1}$$

where D is the set of feasible first-stage decisions, and $X \subset D$. $f(\cdot, \mathbf{d}), \mathbf{d} \in \mathbb{D}$ is the objective function of the stochastic model. P is the probability measure on the Borel σ-field and the subset X.

The stochastic programs determine the optimal decisions $\mathbf{d}$ given a realization of the stochastic process x. The basic idea behind this approach is the concept of *recourse*. Recourse gives DMs the flexibility to make further decisions after the realization of the stochastic elements of the problem. Recourse decisions preserve the feasibility of the constraints of the problem. For instance, a two-stage recourse problem requires to (i) choose one decision variable for each decision that must be made in stage-one (ii) determine the possible states of the world that might be realized next period (iii) take some recourse action after the realizations of the stochastic elements. This stochastic programming problem can be extended to a multi-stage case, in which the realizations of the stochastic elements are represented as scenarios. In this case, the stochastic elements are often considered to have discrete distributions.

Now the stochastic programming inventory problem is described as follows: A demand realization occurs at the end of each period t. Let Q_{kt} denote the order quantity at the beginning of period t under scenario k, and let c_t be the associated unit cost. Let X_{kt} be a discrete random variable representing the total demand for the item during period t under scenario k, and q_{kt} the corresponding scenario probability. Let v_{kt} denote the excess inventory observed at the end of period t under scenario k, and let h_t be the associated unit holding cost. Similarly, u_{kt} is the observed number of shortages at the end of period t under scenario k with corresponding unit cost s_t. Then the multi-stage stochastic programming problem can be expressed as the following linear program. The details about the model can be found in [3].

$$\min \sum_{t=1}^{T} \sum_{k=1}^{K} q_{kt} \cdot (c_t \cdot Q_{kt} + h_t \cdot v_{kt} + s_t \cdot u_{kt}) \tag{4.2}$$

$$\begin{aligned} Q_{kt} + v_{k(t-1)} + u_{kt} - v_{kt} &= x_{kt} && \\ v_{k0} &= 0, && k = 1, \ldots, K \\ Q_{kt}, v_{kt}, u_{kt} &\geq 0, && t = 1, \ldots, T,\ k = 1, \ldots, K \end{aligned}$$

This problem can be shortened by adding the *nonanticipativity* constraints:

$Q_{kt} = Q_{k't}$, $u_{kt} = u_{k't}$, and $v_{kt} = v_{k't}$ for all k, k' for which $x_{k,[1,t]} = x_{k',[1,t]}, t = 1, \ldots, T$

These constraints ensure that the decisions taken at period t do not depend on the future observations of the stochastic process, but on the information available up to period t, $x_{[1,t]}$.

4.2.1 Demand Scenario Probabilities

In order to generate scenarios for the demand process, the Markov chain associated with hurricane count rates is used. The stationary transition probability p_{ij} of the Markov chain corresponds to the probability of predicting hurricane landfall count rate of $j = 0, \ldots, 5$ during the current pre-season month given that the previous month's prediction is $i = 0, \ldots, 5$. These probabilities are predicted based on the hurricane count prediction model developed by [30]. Elsner et al. [32] further extend [30] to provide a six-month forecast horizon for annual hurricane counts along the U.S. coastline. Elsner et al. [32] determine that the North Atlantic Oscillation (NAO) and the Atlantic Sea-Surface Temperature (SST) variations are the most significant predictors of the annual hurricane landfall. The NAO is measured based on the fluctuations in the difference of sea-level pressure between the subtropical high and the polar low. The corresponding NAO index data is used to predict the general path hurricanes will take when they form. On the other hand, SST variations referred to as the Atlantic Multidecadal Oscillation (AMO) indicate how much temperatures depart from what is normal for that time of year. Therefore, AMO index is used to predict how active the basin will be in terms of the number of hurricanes. These AMO data are determined as the difference between the current observation and the corresponding climatological value. A Bayesian approach to regression analysis is conducted to generate samples of posterior parameters β_{t+1} given the prior estimates of these parameters β_t at period t. Eq. (4.3) shows the ensuing Poisson regression equation:

$$\log(\lambda) = \beta_{0_t} + \beta_{1_t} \cdot AMO + \beta_{2_t} \cdot NAO + \beta_{3_t} \cdot (AMO \cdot NAO) \tag{4.3}$$

where the response is the observed annual hurricane landfall count $n_h = 0, \ldots, 5$[1], and the predictors are the NAO and AMO index. Note that the regression coefficients $\beta_t = (\beta_{0_t}, \beta_{1_t}, \beta_{2_t}, \beta_{3_t})$ are introduced as random variables as opposed to constants.

The prior distribution for β_t is specified by a multivariate normal distribution, $MVN(\mu_t, \psi_t^{-1})$. In order to determine bootstrap prior estimates for μ_t and ψ_t, the regression equation is fitted to the set of hurricane counts using only the April index for NAO and AMO, and a large number of bootstrap samples are generated with replacement. Then, landfall count rate λ_t is predicted during pre-season month t for the forecasted hurricane season using these bootstrap samples and the observed NAO_t and AMO_t data during month t as given

[1]Note that response is n_h when observed data is used

in Eq. (4.4).

$$\log(\lambda_t) = \beta_{0_t} + \beta_{1_t} \cdot AMO_t + \beta_{2_t} \cdot NAO_t + \beta_{3_t} \cdot (AMO_t \cdot NAO_t) \tag{4.4}$$

The regression equation given in Eq. (4.3) is run with the corresponding May index data to form the likelihood function $g_t(\lambda_t|\beta_t)$. The posterior density of β_t is then determined conditioned on these observed hurricane landfall counts $f(\beta_{t+1}|\lambda_t)$. In order to analyze the posterior distribution $f(\beta_{t+1}|\lambda_t)$, the *Gibbs Sampler* algorithm is implemented. The Gibbs Sampler is a MCMC method used to generate samples from the joint distribution of two or more variables for high-dimensional situations. The sampler works by iteratively generating values from each distribution in turn, using parameter values from the previous iteration to generate successive values. The Gibbs Sampler is run to generate samples of regression coefficients. The posterior coefficient values $\beta_{t+1} = [\beta_{0_{t+1}}, \beta_{1_{t+1}}, \beta_{2_{t+1}}, \beta_{3_{t+1}}]$ are obtained after convergence of the Markov chain is achieved. These stationary coefficient values are used to predict landfall count rate λ_{t+1} for the ensuing hurricane season. Eq. (4.5) is used to generate predictive samples given the observed NAO and AMO index values associated with month $t + 1$.

$$\log(\lambda_{t+1}) = \beta_{0_{t+1}} + \beta_{1_{t+1}} \cdot AMO_{t+1} + \beta_{2_{t+1}} \cdot NAO_{t+1} + \beta_{3_{t+1}} \cdot (AMO_{t+1} \cdot NAO_{t+1}) \tag{4.5}$$

Eq. (4.6) gives the predictive density of hurricane landfall count rate $h_{t+1}(\lambda_{t+1}|\lambda_t)$:

$$h_{t+1}(\lambda_{t+1}|\lambda_t) = \int g_{t+1}(\lambda_{t+1}|\beta_{t+1}) f_{t+1}(\beta_{t+1}|\lambda_t) \mathrm{d}\beta \tag{4.6}$$

where λ_t corresponds to an observed prediction, and λ_{t+1} is the predicted prediction. Additionally, $g_{t+1}(\lambda_{t+1}|\beta_{t+1})$ is the density function of λ_{t+1} given the posterior parameter estimates β_{t+1}, and $f_{t+1}(\beta_{t+1}|\lambda_t)$ is the posterior distribution of β_{t+1}.

Predictive samples of λ_{t+1} are then generated using Gibbs sampler. The predictive probabilities associated with landfall count rate are used to determine the stationary transition probabilities. More specifically, the stationary transition probabilities, p_{ij}, $i, j = 0, \ldots, 5$, are obtained by evaluating $P\{\lambda_{t+1} = j|\lambda_t = i\}$. Then, the steady-state equations are formed based on these stationary transition probabilities. Eq. (4.7) gives the steady-state equations:

$$\begin{aligned} \pi_j &= \sum_{i=0}^{5} \pi_i p_{ij} \quad \text{for } j = 0, 1, \ldots, 5 \\ \sum_{j=0}^{5} \pi_j &= 1 \end{aligned} \tag{4.7}$$

where π_j is referred to as the steady state probability of the Markov chain.

The steady-state probabilities give the predictive probabilities of observing hurricane count states. For instance, π_1 gives the predictive probability of observing exactly one hurricane during the hurricane season. These probabilities are used to calculate the scenario probabilities for the demand process. The inventory control problem is then solved using the stochastic programming model described in the previous section.

4.2.2 Numerical Example

For the numerical example, the pre-season months of April and May are considered as the inventory planning periods. The hurricane season (June 1-November 30) as a whole is considered to be one period. Table 4.1 gives the historical NAO and AMO index values associated with the pre-season months of April and May along with the observed landfall counts for each hurricane season.

Table 4.1: Hurricane landfall count and April-May NAO and AMO index derived by the $1950 - 1979$ data

		April		May	
Year	n_h	NAO	AMO	NAO	AMO
1950	3	1.61	-0.16	-1.73	-0.34
1951	3	-0.45	0.43	-2.11	0.29
1952	0	2.79	0.06	-0.94	0.28
1953	1	-1.6	0.33	-0.75	0.14
1954	3	-0.26	-0.15	-0.91	0.14
1955	3	2.4	-0.18	0.33	-0.11
1956	3	-1.9	0.11	4.54	-0.14
1957	1	-0.62	-0.46	-0.84	-0.23
1958	1	1.79	1.02	1.1	0.78
1959	0	1.51	-0.08	-2.22	-0.36
1960	3	1.93	0.13	0.07	0.1
1961	2	0.71	-0.08	-0.94	-0.11
1962	1	0.74	0.33	-0.1	0.43
1963	0	-0.46	0.44	1.91	0.23
1964	1	0.95	-0.05	2.51	0
1965	4	2.14	-0.16	-0.08	-0.27
1966	1	1.18	0.47	1.51	0.2
1967	2	-0.76	0.18	-0.46	-0.12
1968	1	-0.71	0.18	-1.5	-0.14
1969	1	1.11	0.91	-0.23	0.78
1970	2	2.52	0.39	1.87	0.47
1971	1	-3.15	-0.23	-0.62	-0.36
1972	3	0.22	-0.08	1.24	-0.34
1973	1	-2.61	0.02	0.37	-0.11
1974	0	-2.3	-0.9	-0.01	-0.99
1975	1	-0.84	-0.47	-2.42	-0.72
1976	1	-1.53	-0.44	1.2	-0.68
1977	1	1.07	-0.19	-1.62	-0.14
1978	1	-3.12	0.53	0.37	0.14
1979	0	-0.79	0.4	1	0.46

This data is used to predict the hurricane landfall count rate for the forecasted hurricane season. The NAO values are obtained from the Climatic Research Unit and the AMO values are obtained from the Climatic Diagnostics Center. These data are used as inputs for the WinBUGS (Windows version of Bayesian inference using Gibbs Sampling) to analyze the predictive distribution of hurricane count rates via a Bayesian analysis. The hurricane landfall count rate is bounded such that $\lambda = 0, 1, \ldots, 5$. Initially, Bootstrap priors are specified for the model coefficients. These bootstrap priors and the NAO and AMO index data associated with period t are used to estimate λ_t, which corresponds to the predicted hurricane landfall count rate at period t. Then, the Gibbs sampler is run to update the priors until the convergence of the posterior coefficients is achieved. Table 4.2 is continuation of Table 4.1, and demonstrates the $1980 - 2007$ hurricane related data.

Table 4.2: Hurricane landfall count and April-May NAO and AMO index derived by the $1980 - 2007$ data

		April		May	
Year	n_h	NAO	AMO	NAO	AMO
1980	3	0.03	0.42	-2.26	0.73
1981	1	-3.04	0.48	0.05	0.56
1982	0	-0.99	-0.12	1.1	0.1
1983	0	-1.01	0.74	-0.57	0.57
1984	1	0.33	-0.33	-2.34	-0.33
1985	5	0.34	-0.55	-2.13	-0.57
1986	2	-0.93	-0.45	2.16	-0.45
1987	1	2.59	0.39	-0.81	0.36
1988	1	-2.39	0.33	-1.24	0.15
1989	3	-0.48	-0.7	1.16	-0.69
1990	0	1.77	0.2	-1.19	0.34
1991	1	1.48	-0.29	-0.04	-0.26
1992	1	1.32	-0.16	0.8	0
1993	1	0.83	-0.08	-2.59	0.05
1994	0	1.38	-0.45	-1.43	-0.53
1995	2	-1.81	0.41	-0.36	0.55
1996	2	-0.31	0.46	-1.5	0.38
1997	1	-0.97	0.2	-1.35	0.23
1998	3	-0.39	0.69	-1.26	0.92
1999	3	0.43	0.01	1.03	0.05
2000	0	-3.34	-0.19	0.31	-0.17
2001	0	1.24	-0.14	-0.09	-0.09
2002	1	0.91	0.17	-0.05	-0.08
2003	2	-1.74	-0.03	1.17	-0.09
2004	5	1.08	0.53	-0.67	0.27
2005	5	0.71	1	-0.13	1.18
2006	0	0.57	0.44	-0.22	0.5
2007	2	-0.1	0.47	0.62	0.19

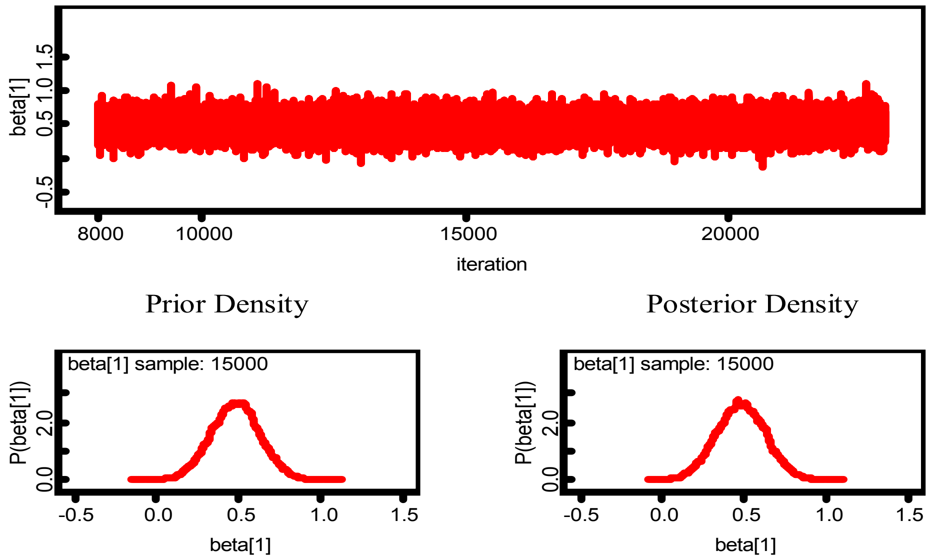

Figure 4.1: WinBugs posterior regression coefficients.

Figure 4.1 shows the obtained prior and posterior density function of the regression coefficient β_1. Similar density functions are obtained for the other coefficients.

Visual inspections reveal that the convergence of the chain is observed after $15,000$ simulations. The predictive inference for λ_{t+1} is made by setting the hurricane landfall count rate to NA (not available) for the forecasted hurricane season. More specifically, the Gibbs sampler is run once again to generate λ_{t+1} conditional on the posterior coefficients β_{t+1} and the observed NAO and AMO index data at period $t+1$. Figure 4.2 demonstrates the WinBugs output associated with the landfall count rate predictions, and Figure 4.3 shows the corresponding hurricane landfall count predictions.

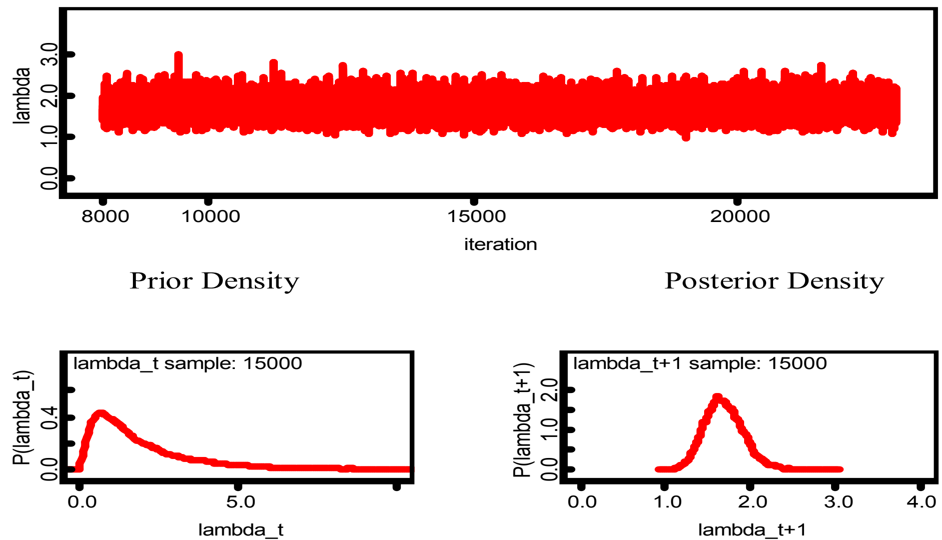

Figure 4.2: WinBugs predictive hurricane count rates.

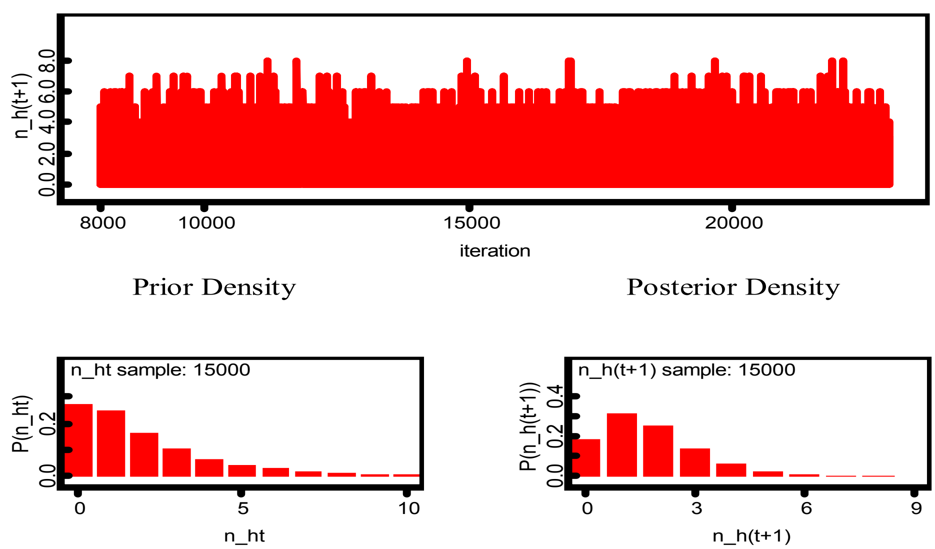

Figure 4.3: WinBugs predictive hurricane counts.

The predicted hurricane count rate probabilities are used to determine the stationary transition probabilities of the six-state Markov chain. For instance, p_{11} is obtained by evaluating $P\{\lambda_{t+1} = 1|\lambda_t = 1\} = 0.29$ empirically. Remaining entries are obtained in a similar manner. Eq. (4.8) shows the resulting transition matrix. Recall that the states of the Markov chain correspond to predicting exactly $\lambda = 0, \ldots, 5$, respectively.

$$\mathbf{P} = \begin{pmatrix} 0.23 & 0.29 & 0.22 & 0.15 & 0.09 & 0.02 \\ 0.22 & 0.29 & 0.22 & 0.15 & 0.09 & 0.03 \\ 0.23 & 0.29 & 0.22 & 0.13 & 0.1 & 0.03 \\ 0.21 & 0.28 & 0.22 & 0.16 & 0.1 & 0.03 \\ 0.2 & 0.3 & 0.25 & 0.18 & 0.05 & 0.02 \\ 0.21 & 0.31 & 0.18 & 0.18 & 0.1 & 0.02 \end{pmatrix} \tag{4.8}$$

By substituting p_{ij} values into the steady-sate equations, the following set of equations are obtained.

$$\begin{aligned} \pi_0 &= 0.23\pi_0 + 0.22\pi_1 + 0.23\pi_2 + 0.21\pi_3 + 0.2\pi_4 + 0.21\pi_5 \qquad (4.9) \\ \pi_1 &= 0.29\pi_0 + 0.29\pi_1 + 0.29\pi_2 + 0.28\pi_3 + 0.3\pi_4 + 0.31\pi_5 \\ \pi_2 &= 0.22\pi_0 + 0.22\pi_1 + 0.22\pi_2 + 0.22\pi_3 + 0.25\pi_4 + 0.18\pi_5 \\ \pi_3 &= 0.15\pi_0 + 0.15\pi_1 + 0.13\pi_2 + 0.16\pi_3 + 0.18\pi_4 + 0.18\pi_5 \\ \pi_4 &= 0.09\pi_0 + 0.09\pi_1 + 0.1\pi_2 + 0.1\pi_3 + 0.05\pi_4 + 0.1\pi_5 \\ \pi_5 &= 0.02\pi_0 + 0.03\pi_1 + 0.03\pi_2 + 0.03\pi_3 + 0.02\pi_4 + 0.02\pi_5 \\ 1 &= \pi_0 + \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 \end{aligned}$$

The simultaneous solutions to the last six equations provide a unique solution as

$$\begin{aligned}\pi_0 &= 0.22,\ \pi_1 = 0.29,\ \pi_2 = 0.22 \\ \pi_3 &= 0.15,\ \pi_4 = 0.09,\ \pi_5 = 0.03\end{aligned} \tag{4.10}$$

The steady-state probabilities given in Eq. (4.10) are used to define the hurricane season demand distribution.

Suppose the likely outcomes of the hurricane season demand are $200, 250, 300, 350, 400$, and 450 corresponding to $\lambda = 0, 1, \ldots, 5$, respectively. Assuming that each period's pre-season demand is equally likely to be 100 or 150, the underlying stochastic demand distribution can be described as shown in Table 4.3.

Table 4.3: Demand distribution

Pre-seasonal demand	Probability	Hurricane season demand	Probability	Demand	Weighted probability
100	0.5	200	0.22	650	0.17
150	0.5	250	0.29	750	0.27
		300	0.22	850	0.23
		350	0.15	950	0.17
		400	0.09	1050	0.12
		450	0.03	1150	0.04

Using the data in Table 4.3, the stochastic programming model (4.2) becomes

$$\min \sum_{t=1}^{2} \sum_{k=1}^{36} q_{kt} \cdot (c_t \cdot Q_{kt} + h_t \cdot v_{kt} + s_t \cdot u_{kt}) \tag{4.11}$$

$$\begin{aligned} Q_{kt} + v_{k(t-1)} + u_{kt} - v_{kt} &= x_{kt}, \qquad t = 1, \ldots, 2,\ k = 1, \ldots, 36 \\ v_{k0} &= 0, \qquad k = 1, \ldots, 36 \\ Q_{kt}, v_{kt}, u_{kt} &\geq 0, \qquad t = 1, \ldots, 2,\ \ k = 1, \ldots, 36 \end{aligned}$$

plus the nonanticipativity constraints:

$$
\begin{aligned}
Q_{k1} &= Q_1, \quad k = 1, \ldots, 36 \\
Q_{k2} &= Q_{21}, \; u_{k2} = u_{21}, \; v_{k2} = v_{21}, \quad k = 1, \ldots, 6 \\
Q_{k2} &= Q_{22}, \; u_{k2} = u_{22}, \; v_{k2} = v_{22}, \quad k = 7, \ldots, 12 \\
Q_{k2} &= Q_{23}, \; u_{k2} = u_{23}, \; v_{k2} = v_{23}, \quad k = 13, \ldots, 18 \\
Q_{k2} &= Q_{24}, \; u_{k2} = u_{24}, \; v_{k2} = v_{24}, \quad k = 19, \ldots, 24 \\
Q_{k2} &= Q_{25}, \; u_{k2} = u_{25}, \; v_{k2} = v_{25}, \quad k = 25, \ldots, 30 \\
Q_{k2} &= Q_{26}, \; u_{k2} = u_{26}, \; v_{k2} = v_{26}, \quad k = 31, \ldots, 36
\end{aligned}
\tag{4.12}
$$

where $t = 1, 2$ corresponds to the pre-season months of April and May, respectively. The weighted probabilities are determined using the pre-season and hurricane season demand probabilities. For instance, 0.17 is calculated as follows. Similar calculations are made to develop the demand distribution.

$$
0.17 = \frac{300 \cdot (0.22 \cdot 0.5) + 350 \cdot (0.22 \cdot 0.5)}{300 \cdot 0.11 + 350 \cdot 0.255 + 400 \cdot 0.255 + 450 \cdot 0.185 + 500 \cdot 0.12 + 550 \cdot 0.06 + 600 \cdot 0.015}
\tag{4.13}
$$

Excel Solver™is used to obtain the optimal ordering policy for the numerical example. The following data are used to solve the linear program, and the results are shown in Table 4.4: $c_1 = 20, c_{t+1} = c_t \cdot 4, s = 300, h_t = c_t/2$.

Table 4.4: Results of the original model

Q_1^*	Scenarios	Q_2^*	Expected Cost
1700	1	0	$87,243.6
	2	0	
	3	0	
	4	100	
	5	200	
	6	300	

Note that the nonanticipativity constraints are added to the problem. In other words, there is only one first period decision, namely Q_1, and there are 6 second period ordering and recourse decisions, one for each scenario. There are 91 variables and 42 constraints in the linear program. The solution yields a total expected cost of $87,243.6. The optimal solution values in Table 4.4 can be interpreted as follows: Order/produce 1700 units at the beginning of the current period (April). If the observed demand associated with the month of April is 950, then order/produce 100 units at the beginning of May. Similarly, order/produce 200 and 300 units at the beginning of May if April's demand is 1050 and 1150, respectively.

4.3 Scenario Reduction

In real-life problems the true probability distribution can have many realizations. In order to numerically solve such problems, it is necessary to find an approximation of the stochastic process that is defined by a finite number of realizations. This discretization process is referred to as a *scenario tree.* The scenario tree serves to model the uncertainty associated with the stochastic process. A scenario is then defined as a possible realization of the underlying stochastic process. In the literature, there exists a wide range of scenario generation methods such as moment matching, conditional sampling, bootstrap, Monte

Carlo sampling, and Markov chain. Note that the performance of the scenario generation method can also be improved by increasing the initial number of scenarios, or simply by improving the sampling method as discussed in ([48]).

In most practical cases, the original tree has a large scale branching structure. Therefore, the size of the tree should be decreased to eliminate the computational burden. For these situations, [38] and [28] introduce an optimal scenario reduction methodology based on the probability metric minimization. They define the optimal scenario reduction of a given discrete approximation as the determination of a scenario subset of prescribed cardinality that is closest to the original distribution. Heitsch and Römisch [38] show that the stochastic programs are stable in terms of a Fortet-Mourier probability metric. Let $P = \sum_{i=1}^{N} p_i \cdot \delta_{x_i}$ be the original discrete probability distribution, and $Q = \sum_{j \notin J} q_j \cdot \delta_{x_j}$ be its optimal reduced distribution where δ_x denotes the Dirac measure assigning unit mass to x. Then the probability metric with the Fortet-Mourier structure is defined in [38] as

$$\zeta_c(P, Q) := \sup_{f \in \mathcal{F}_c} \left| \int_X f(x) P(\mathrm{d}x) - \int_X f(x) Q(\mathrm{d}x) \right| \tag{4.14}$$

with $\mathcal{F}_c$ being the class of continuous functions having the form

$$\mathcal{F}_c := \{ f : X \to \mathbb{R} : f(x) - f(\tilde{x}) \leq c(x, \tilde{x}) \text{ for all } x, \tilde{x} \in X \} \tag{4.15}$$

Here, c is a continuous symmetric function that is selected such that the following Lipschitz condition described in [38] is satisfied given a nondecreasing function $g : \mathbb{R}_+ \to \mathbb{R}_+ \backslash \{0\}$

$$|f(x, d) - f(\tilde{x}, d)| \leq g(||d||) \cdot c(x, \tilde{x}) \tag{4.16}$$

In this model, c is defined as a metric such that $c(x, \tilde{x}) = ||x - \tilde{x}||$, and $||\cdot||$ is the Euclidean norm.

Dupačová et al. [28], and Heitsch and Römisch [38] prove that the Kantorovich function $\hat{\mu}_c(P, Q)$ is an estimate of the upper bound value of $\zeta_c(P, Q)$, i.e. $\zeta_c(P, Q) \leq \hat{\mu}_c(P, Q)$. They use the Kantorovich function $\hat{\mu}_c$, which represents the optimal value of a finite-dimensional linear program, to develop an optimal scenario reduction approach for a given discrete approximation Q of P. The optimal reduction approach described in their papers suggests considering the following Kantorovich probability distance:

$$\begin{aligned}\hat{\mu}_c(P, Q) &= \min \left\{ \sum_{\substack{i,j=1 \\ j \notin J}}^{N} c(x_i, x_j) \cdot \eta_{ij} : \eta_{ij} \geq 0, \sum_{i=1}^{N} \eta_{ij} = q_j, \sum_{\substack{j=1 \\ j \notin J}}^{N} \eta_{ij} = p_i \right\} \qquad (4.17) \\ D(J; q) &:= \hat{\mu}_c \left(\sum_{i=1}^{N} p_i \cdot \delta_{x_i}, \sum_{j \notin J} q_j \cdot \delta_{x_j} \right)\end{aligned}$$

where $J \subset \{1, \ldots, N\}$ is the index of withdrawn scenarios with fixed cardinality. Based on the optimal reduction concept, the optimal index set J^*, and the optimal weights q^* are determined such that $D(J; q)$ is minimized. Then, the new probabilistic weights $q_j, j \in \{1, \ldots, N\} \backslash J$ are assigned to each remaining scenario $x_j, j \notin J$ using the following optimal redistribution rule.

Theorem 4.1 *(Heitsch and Römisch [38]): Given $J \subset \{1, \ldots, N\}$, the probability distance is defined as follows.*

$$D_J = \min \left(D(J; q) : q_j \geq 0, \sum_{j \notin J} q_j = 1 \right) = \sum_{i \in J} p_i \cdot \min_{j \notin J} c(x_i, x_j) \qquad (4.18)$$

and the minimum value is attained at:

$$q_j^* = p_j + \sum_{i \in J_{j(i)}} p_i \quad \text{for each} \quad j \notin J \tag{4.19}$$

where $j(i) \in \arg\min_{j \notin J} c(x_i, x_j)$ *for each* $i \in J$*.*

Theorem 4.1 implies that the new probability of a kept scenario is equal to the sum of its original probability and of all probabilities of the closest withdrawn scenarios determined based on the c metric. Then, the optimal index set J^* for scenario reduction with given cardinality is determined by solving the following problem formulated by ([38]).

$$\min \left\{ D_J := \sum_{i \in J} p_i \cdot \min_{j \notin J} c(x_i, x_j) : J \subset \{1, \ldots, N\}, n(J) = N - n \right\} \tag{4.20}$$

As discussed by [38] when $n(J) = N - 1$, Eq. (4.20) reduces to

$$\min_{j \in \{1,\ldots,N\}} \sum_{i=1}^{N} p_i \cdot c(x_i, x_j) \tag{4.21}$$

Eq. (4.21) yields the best possible deterministic approximation of the initial distribution such that the redistribution rule assigns $q_j^* = 1$ to the preserved scenario.

Now, the numerical example presented in the previous section is resolved using the optimal scenario reduction concept. The number of deleted scenarios is fixed as $n(J) = 4$. Table 4.5 gives the selected index sets and their corresponding probability distances, and it reveals that $J^* = \{1, 3, 5, 6\}$ gives the minimum distance with $D_J^* = 60$. Therefore, these scenarios should be removed from the original set of scenarios.

Table 4.5: Probability distances

J	D_J	J	D_J	J	D_J
{1,2,3,4}	212	{1,2,5,6}	81	{2,3,4,5}	119
{1,2,3,5}	140	{1,3,4,5}	86	{2,3,4,6}	94
{1,2,3,6}	132	{1,3,4,6}	61	{2,3,5,6}	70
{1,2,4,5}	90	{1,3,5,6}	60	{2,4,5,6}	80
{1,2,4,6}	82	{1,4,5,6}	70	{3,4,5,6}	109

The D_J values shown in Table 4.5 are calculated using the Euclidean distances. For instance, D_J^* is determined as follows.

$$D_J^* = 100 \cdot 0.17 + 100 \cdot 0.23 + 100 \cdot 0.12 + 200 \cdot 0.04 = 60 \tag{4.22}$$

Table 4.6 gives the Euclidean distances $c(x_i, x_j)$ used to determine the optimal weights for the remaining scenarios.

Table 4.6: Euclidean distances (c metric)

(i,j)	2	4
1	100	300
3	**100**	**100**
5	300	100
6	400	200

The optimal weights for scenarios 2 and 4 are calculated using Eq. (4.19).

$$\begin{aligned} q_2^* &= \sum_{i=1}^{3} = 0.17 + 0.27 + 0.23 = 0.67 \\ q_4^* &= \sum_{i=4}^{6} = 0.17 + 0.12 + 0.04 = 0.33 \end{aligned} \tag{4.23}$$

Stochastic programs can have more than one candidate scenario that has the same probability distance to another scenario. For instance, in this example different values can be assigned to optimal weights by incorporating the probability of deleted scenario $i = 3$ to the scenario $j = 4$ since $i = 3$ has the same proximity both to $j = 2, 4$. Eq. (4.24) shows the resulting optimal weight values.

$$
\begin{aligned}
q_2^* &= \sum_{i=1}^{2} = 0.17 + 0.27 = 0.44 \\
q_4^* &= \sum_{i=3}^{6} = 0.23 + 0.17 + 0.12 + 0.04 = 0.56
\end{aligned}
\tag{4.24}
$$

Then, the reduced versions of the stochastic programming models are solved using the previously defined unit costs. The reduced models defined by $n = 2$ consist of 15 variables, and 6 constraints obtained by considering the nonanticipativity property of stochastic programs. Table 4.7 demonstrates the results of these reduced models.

Table 4.7: Results of reduced models

Reduced model	Q_1^*	Scenario i	q_i^*	Q_2^*	Expected Cost
1	1700	2	0.67	0	$76,180.8
		4	0.33	0	
2	1700	2	0.44	0	$82,290
		4	0.56	200	

The reduced model 1 recommends ordering 1700 units at the beginning of April, and no order should be given in May. Similar to this model, the reduced model 2 indicates that initially 1700 units should be ordered. However, if the demand realization at the end of April is 950, then this model suggests ordering an additional 200 units. Otherwise no order should be given at the beginning of May. Table 4.8 presents the solutions for all the potential reduced models given that $n(J) = N - n$.

Table 4.8: Optimal values (solutions) of reduced models

n	J^*	D_J^*	Q_1^*	Expected Cost	Relative Error
1	$J = \{1, 2, 4, 5, 6\}$	114	1700	\$63,750	$114/114 = 100\%$
2	$J = \{1, 3, 5, 6\}$	60	1700	\$76,180.8	$60/114 = 52.63\%$
3	$J = \{1, 5, 6\}$	37	750	\$127,695	$37/114 = 32.46\%$
4	$J = \{5, 6\}$	20	750	\$153,660	$20/114 = 17.54\%$
5	$J = \{6\}$	4	750	\$163,260	$4/114 = 3.50\%$

Table 4.8 also indicates that with increasing J, i.e., with increasing number of deleted scenarios, the accuracy of the approximation decreases. The *relative error* is defined based on the probability distances. More specifically, the error from the approximation of the initial distribution is defined relative to the minimum distance associated with the deterministic approximation given by the reduced model with $n = 1$. For detailed information about the relative error concept, the reader is referred to ([38]).

Another important issue that is worth exploring in stochastic programming is the evaluation of the quality of the reduced scenario trees. In this context, one does not search for the best approximation of the initial distribution but for the quality of the optimal solutions (values). In this study, the accuracy is defined as the ratio of the optimal first-stage decisions obtained from the solutions of the reduced model and the original model. Recall that only the (deterministic) first stage is the appropriate outcome of the stochastic program. The tree serves to model the demand. uncertainty. Table 4.8 reveals that the models carried by the number of scenarios $n = 1, 2$ are reduced in an optimal way since the value of first-stage optimal decisions obtained from both of the reduced models are exactly the same as that of the original model. On the other hand, reduced models having $n = 3, 4, 5$ scenarios have an accuracy of $\frac{750}{1700} \cdot 100 \approx 44\%$. These results suggest that the accuracy of the reduced trees tends to increase as they are supported with a small number of scenarios. In other words, while the stochastic process is approximated with less number of scenarios by implementing the scenario reduction approach, the accuracy of the values (solutions) obtained from reduced models increases. This finding is consistent with intuition such that one would expect to obtain more accurate results as the uncertainty associated with the stochastic process reduces. Similar interpretations can be made for the expected costs associated with the reduced models.

In order to evaluate the performance of the reduced models, the stochastic inventory problem is initially solved on the reduced tree. Then, the values of all the first-stage(root) variables are fixed, and resolved on the original tree. As a result, the out-of-sample performance of the reduced-tree solution is obtained assuming that the original tree is a good-enough approximation of the true distribution. The reduced models carried by $n = 1, 2$ give the same optimal expected cost value as the initial optimum value ($\$87, 243.6$). The reduced models having 44% solution accuracy result in an expected cost of ($\$174, 975$). These findings indicate that as the accuracy of the optimal first stage solutions of the reduced model decreases so too does the cost efficiency.

4.3.1 Heuristic Algorithm

In most of the stochastic problems where the stochastic process is represented by many scenarios, Eq. (4.20) can not be solved optimally. Therefore, [28] and [38] develop heuristic algorithms to approximate solutions of Eq. (4.20). In this study, the *simultaneous backward reduction* algorithm, which includes all the previously deleted scenarios in each backward step, is used. The algorithm determines an index set J to be removed from the original set of scenarios based on the solution of the following equation given in ([38]):

$$l_k \in \arg\min_{l \notin J^{[k-1]}} \sum_{i \in J^{[k-1]} \cup \{l\}} p_i \cdot \min_{j \notin J^{[k-1]} \cup \{l\}} c(x_i, x_j) \tag{4.25}$$

where $J^{[k-1]} = \{l_1, \dots, l_{k-1}\}$ is defined as the index set of deleted scenarios up to and including step $k-1$.

The stochastic programming inventory model is solved by implementing the simultaneous backward reduction algorithm to illustrate the application of the heuristic algorithm. The first step requires the deletion of only one scenario. Through the following steps, the index l_k is determined given that the previous index set $\{l_1, \dots, l_{k-1}\}$ is optimal. Finally, the optimal distribution rule given by Theorem 4.1 is implemented. Then, the number of withdrawn scenarios is set as $n(J) = 4$. In step 1 of the algorithm, initially the scenarios are defined as $l = 1, 2, 3, 4, 5, 6$. Then, the optimal scenario to be removed is selected based on the Euclidean probability distances. Table 4.9 shows the resulting distance matrix. Table 4.9 indicates that the minimum distance is achieved at $l_1 = 6$ with $D^*_{J^1} = 0.04 \cdot 100 = 4$.

Table 4.9: Euclidean distance matrix 1

(i,j)	1	2	3	4	5	6	$D_{J^1} = p_i \cdot c(x_i, x_j)$
1	-	100	200	300	400	500	17
2	100	-	100	200	300	400	27
3	200	100	-	100	200	300	23
4	300	200	100	-	100	200	17
5	400	300	200	100	-	100	12
6	500	400	300	200	100	-	4

In step 2, the Euclidean probability distances are calculated for the kept scenarios $l = 1, 2, 3, 4, 5$ as $D_{J^2} = 21, 31, 27, 21, 20$, respectively. The minimum distance is obtained as $D^*_{J^2} = 20$ at $l_2 = 5$. Table 4.10 shows the corresponding Euclidean distance matrix.

Table 4.10: Euclidean distance matrix 2

(i,j)	1	2	3	4	$D^*_{J^2} = \sum_{i\in\{6,5\}} p_i \cdot \min_{j\in\{1,2,3,4\}} c(x_i,x_j)$
6	500	400	300	200	$D^*_{J^2} = 0.04 \cdot 200 + 0.12 \cdot 100 = 20$
5	400	300	200	100	

In step 3, the remaining scenarios are revised as $l = 1, 2, 3, 4$. Then, the corresponding probability distances are calculated as $D_{J^3} = 37, 47, 43, 53$. The minimum distance is $D^*_{J^3} = 37$ that corresponds to $l_3 = 1$. Table 4.11 gives the Euclidean distances associated with $D^*_{J^3}$.

Table 4.11: Euclidean distance matrix 3

(i,j)	2	3	4	$\sum_{i\in\{6,5,4\}} p_i \cdot \min_{j\in\{2,3,4\}} c(x_i,x_j)$
6	400	300	200	$D^*_{J^3} = 0.04 \cdot 200 + 0.12 \cdot 100 + 0.17 \cdot 100 = 37$
5	300	200	100	
1	100	200	300	

In step 4, the scenarios $l = 2, 3, 4$ are considered for reduction. The probability distances are calculated as $D_{J^4} = 81, 60, 70$, respectively. It can be inferred that the minimum distance is $D^*_{J^4} = 60$ with $l_4 = 3$. Table 4.12 gives the Euclidean distances used to determine this minimum distance.

Table 4.12: Euclidean distance matrix 4

(i,j)	2	4	$\sum_{i\in\{6,5,1,3\}} p_i \cdot \min_{j\in\{2,4\}} c(x_i,x_j)$
6	400	200	$D^*_{J^4} = 0.04 \cdot 200 + 0.12 \cdot 100 + 0.17 \cdot 100 + 0.23 \cdot 100 = 60$
5	300	100	
1	100	300	
3	100	100	

Since $n(J) = 4$ is achieved, the algorithm is terminated. The reduced stochastic model is solved for the remaining scenarios $j = 2, 4$. As can be seen in Table 4.12, the optimal redistribution rule will result in the same optimal weights. Therefore, the obtained stochastic programming model will have the same arguments as that of the optimally reduced one. Additionally, the simultaneous backward reduction algorithm yields optimal solution (value) except for the reduced model supported by $n = 1$. This arises from the fact that the best possible scenario $i = 3$ has already been deleted in the previous backward step. In other words, while the optimal reduction method directly solves Eq. (4.20), the heuristic algorithm implements the scenario reduction process recursively in a stepwise fashion. For this deterministic problem, the algorithm yields an optimal order quantity of $Q_1^* = 1900$ with an expected cost of $\$71,250$. The index set $i = 6, 5, 1, 3, 2$ is deleted and so the reduced model is defined only by the scenario $j = 4$. These results indicate that the simultaneous backward reduction algorithm works reasonably well, and can be used in lieu of optimal reduction approach to reduce the number of scenarios where the initial distribution is represented by many scenarios.

4.4 Summary and Future Work

This chapter presents a stochastic inventory model that will assist organizations in determining their optimal ordering policies as related to an upcoming hurricane season. In this study, the pre-season demand distribution is assumed to be known to the inventory manager at the beginning of the inventory planning horizon. However, the hurricane season demand distribution is based on monthly information updating. The hurricane landfall count rate predictive probabilities, which are used to define the hurricane season demand distribution as a Markov chain, are estimated via a widely-accepted hurricane prediction model developed by ([30]).

This chapter considers making preparations for the hurricane season demand two months ahead of the season. The objective is to allocate some reserved stock to meet the hurricane season demand while satisfying each period's demand in a cost efficient way. The uncertainty in the stochastic model is represented by a finite number of discrete demand realizations. The cost minimization function, together with the constraints, constitutes the structure of the model. Depending on the context of the inventory decision model, a different type of solution approach can be implemented. For instance, dynamic programming has been widely used to solve inventory problems with the application of the principle of optimality due to its computational efficiency. However, a dynamic programming representation of this problem could not be found since the randomness required to be defined in the model increases with the addition of reserved stock and the Markovian hurricane season demand random variables. Therefore, a stochastic programming model that is written as a deterministic linear program is developed to determine the optimal ordering policy. Different scenarios of the stochastic process are defined based on the underlying demand distribution.

In real-life applications, the stochastic process is represented by many scenarios and/or stages. For these situations, the stochastic programming approach becomes less efficient, and requires the implementation of other algorithms. In this study, the scenario reduction approach introduced by [38] is implemented to find the optimal set of scenarios to represent the underlying distribution. It is determined that the optimum scenario reduction method yield approximately 44% accuracy when at least half of the scenarios are removed. Additionally, It is shown that both the accuracy and the performance of the reduced models increase as more scenarios are withdrawn from the original distribution. This finding suggests that as the uncertainty associated with the demand process decreases, the accuracy of the solution tends to increase.

In this study, it is also determined that the simultaneous backward reduction algorithm yields optimum results for the considered reduced models except for the reduced model corresponding to the deterministic approximation of the initial distribution $n = 1$. Therefore, it can be inferred that the heuristic algorithm developed by [38] can be used to find the reduced approximations of the stochastic processes introduced by many scenarios. However, as the problem gets larger, the running time of the algorithm substantially increases.

This chapter introduces a stochastic inventory model that will enable quick-response logistics decisions as related to hurricane disaster relief. For future study, one might want to develop approximations of a demand process described by many scenarios through the implementation of the scenario generators. It is also worth exploring the quality of the reduced scenario model where the discrete demand process is represented by many scenarios. Additionally, a case can be developed in which a different demand distribution is introduced for each state. Finally, the existence of an optimal state-dependent base-stock policy can be investigated.

Chapter 5

Conclusions and Proposed Future Study

This dissertation discusses supply chain organizations disaster recovery problem whose demand for hurricane supplies is influenced by various attributes of hurricane events. In this context, the objective of disaster recovery planning is to minimize interruption to business continuity during and after a hurricane. The proposed approach is applicable to predictable disasters and leverages hurricane predictions to develop disaster recovery plans. In a hurricane, it is reasonable to expect various logistics issues that affect the supplier's ability to adequately address the order at a given time. For instance, transportation may not be possible and overtime may be unavailable or available but inaccessible due to transportation issues. Therefore, the objective of this dissertation can be stated more generally as determining the optimal level of supply chain readiness with respect to hurricane preparedness(i.e. levels of supplies, equipment or personnel) and how long this preparation decision should be postponed such that the trade-off between logistics cost efficiency and hurricane forecast accuracy is optimized.

Although this dissertation is presented as a stochastic production / inventory control problem from the perspective of disaster recovery plan associated with a manufacturing facility or retail organization, the framework is also relevant to disaster relief planning problems encountered by service organizations. For example, military organizations and electric power companies often pre-position manpower and equipment in anticipation of a potential disaster-relief operation. This pre-positioning decision also inherits the risk of over-preparation if pre-positioning efforts exceed the demands of the disaster-relief operation, and the risk of under-preparation if demands exceed pre-positioning efforts. Not-for-profit service organizations such as the American Red Cross face similar risks and decisions related to stocking and staffing evacuation shelters. Therefore, the primary intention in this study is to develop an information updating framework as is relates to disaster recovery planning for managing the hurricane related risks faced by different types of organizations.

In this dissertation, three models are introduced to assist these organizations in their hurricane related inventory decisions. The first two model focus on an emergency inventory planing problem in which the ordering decision is given sometime during the season when a storm is first observed until it dissipates. The first model considers a one location problem. The second model is introduced as an extension of the first model where the multi location problem is taken into account. For both of these models, the unit ordering/production cost is introduced as an increasing function of time, which is influenced by the expectation of a demand surge during the evolution of an observed storm. Since hurricane characteristics can be predicted with more accuracy during the later stages of the planning horizon relative to the earlier stages, the inventory control problem is formulated as optimal stopping problem with Bayesian updates, where the updates are based on hurricane predictions. The information updating framework is introduced by applying a sequential statistical decision approach with fixed sample size. The samples consist of observed maximum wind-speeds at a specific location and sustained wind-speeds at different locations, respectively. Two different classes of demand are defined over the observed sequential samples. In the first model, empirical methodologies are implemented to illustrate the proposed approach. On the other hand, the second model is developed based on a widely-recognized statistical prediction model to investigate the managerial applications inferred from the decision process.

Additionally, Matlab™and Mathematica™software programs are used to solve the hurricane stocking problems and the corresponding optimal ordering policies are determined. The results of the models are encouraging since they are consistent. The obtained ordering policies reveal that the DMs tend to wait to give their inventory related decisions as no extreme hurricanes or hurricane force wind-speeds are observed at a specific location or at different locations, respectively. For these situations, the ordering quantities are also relatively smaller as consistent with intuition.

The extended model that accounts for the ordering disruptions is planned to be investigated based on other decision rules or loss functions. In this study, the stocking quantity represents either supplies, equipment or personnel. For future research, the author might explore an aggregated model that account for all the resources. Additionally, the demand rate or multiple demand classes will be defined over multiple hurricane attributes to make the model more realistic. The author also plans to examine the case with random demand realizations at different locations. In real-life applications the magnitude of the demand surge is influenced by the fluctuations in the market as a result of hurricane events. Therefore, the author plans to examine the relationship between the hurricane demand rate and the market value. In this context, the value of information may be investigated under a "Real Options" framework. In the first two models, Bayesian sequential decision models are developed based on a fixed sample size. As an extension of this base model, the author might also randomize the duration of an observed storm's evolution to account for various sample sizes.

The third model investigates pre-season inventory planning problem with respect to preparing for a potential hurricane activity. The objective is to determine an optimal ordering policy such that an appropriate amount of hurricane supplies are reserved for the ensuing season while meeting the period's demand in a cost efficient way. In order to determine the optimal ordering policies for the pre-season months, a stochastic programming model is introduced. The proposed model and solution approach gives DMs the flexibility to adjust their ordering decisions considering the hurricane season demand predictions as demand realizations occur.

In this model, the underlying demand distribution is approximated with a small number of scenarios corresponding to the demand realizations. As an extension, the author plans to investigate the case where the stochastic demand process is continuous or approximated with many scenarios. Therefore, the author will implement the scenario reduction approaches presented in this study to determine appropriate ordering policies for those organizations who make stocking decisions in advance of the hurricane season to prepare themselves for a potential demand surge. Additionally, the author plans to make sensitivity analysis to compare the accuracy (efficiency) of the optimal solutions (values) among the reduced models. She would also like to search for optimal state-dependent base-stock policies defined over the Markovian hurricane count distribution. The author might also explore optimal ordering policies using a Value at Risk (VaR) approach.

The hurricane inventory planning models and the information updating framework presented in this dissertation are also applicable for other predictable disasters such as floods and droughts. The disaster recovery plans associated with these kinds of predictable disasters are more reliable compared to the ones proposed for the hazards such as earthquakes, terrorist attacks, and tornadoes. For future research, the author plans to develop new decision models to account for earthquake events. Earthquakes are a common problem for all mankind and it is of vital importance to support research activities concerning them. In recent years, academic and scientific institutions develop various forecasting models associated with earthquakes. For instance, Istanbul Technical University conducts a project to develop an earthquake prediction system that is based on the electrical stress measurements of rocks. The objective of this project is to forecast the approximate place and time of earthquakes based on the analysis of the collected data. Afterwards, early warning studies will be initiated. The author plans to look for potential research collaborations with this project team. The author' s aim is to examine the application of this earthquake prediction model to develop decision models for supply chain organizations that are under the threat of earthquakes. The author may also explore an earthquake risk model to assess the economical impacts of earthquakes on profit-driven organizations.

Finally, the author plans to do research on "Humanitarian Relief Logistics". This is a new emerging topic in the disaster management and relief planning area. Therefore, there exist a scant amount of relevant research. In the long term, the author aims to develop statistical decision models such that the human factor is incorporated into the decision process.

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