AN EXPLORATION OF THE ROLE OF THE TEACHER THROUGH THE LENSES OF FOUR COMPONENTS OF EFFECTIVE TEACHING IN THE ALGEBRA I CLASSROOM

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AN EXPLORATION OF THE ROLE OF THE TEACHER THROUGH THE LENSES OF FOUR COMPONENTS OF EFFECTIVE TEACHING IN THE ALGEBRA I CLASSROOM

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Lora Merchant Joseph, daughter of Howard and Nova Merchant, was born October 20, 1968, in Bloomfield, Iowa. She graduated from Springwood School in Lanett, Alabama in 1986. She graduated from Auburn University at Montgomery with a Bachelor of Science degree in Secondary Mathematics Education in December, 1993. She taught high school in Georgia and Alabama. While teaching at Southern Union State Community College, she graduated from Auburn University with a Master of Science degree in Discrete Mathematics in May, 2002, after which she taught in the mathematics department at Auburn University. She has three beautiful children, Jonathan, Jeremy, and Jamey.

DISSERTATION ABSTRACT

AN EXPLORATION OF THE ROLE OF THE TEACHER THROUGH THE LENSES OF FOUR COMPONENTS OF EFFECTIVE TEACHING IN THE ALGEBRA I CLASSROOM

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As the importance of mathematics continues to grow in our society, so does the question of how to effectively teach mathematics. Although the claim that teaching does make a difference seems obvious, the question of what makes mathematics teachers effective is not easily answered. Evidence from research shows that students' mathematics learning is influenced by the teaching they experience at school.

Using case studies, five teachers were followed in order to explore their role in effective teaching. Specifically, the study explored the role of these five teachers in implementing the four components of effective teaching: 1) content, 2) discourse, 3) equity, and 4) connections. All the teachers were teaching the second half of a year long Algebra I course at a school participating in a mathematics initiative. Various types of

data were collected to better understand the role of the teacher including: classroom observations, a teacher interview and questionnaire, and a student test given at the beginning and end of the course, along with a student survey.

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I. INTRODUCTION

A growing concern in the United States' education system is the equipping of students to function and participate in society, particularly mathematics (National Council of Teachers of Mathematics [NCTM], 2000). Mathematics is no longer an isolated topic for the gifted few, but has evolved into an essential component of everyday life (Alabama Department of Education, 2005; Anrig & Lapointe, 1989; Empson, 2002; Kehle et al., 2004; Knuth & Jones, 1991; NCTM, 2000; U. S. Department of Education, 2002, 2005). Understanding and using mathematics in everyday life and in the workplace has never been greater (NCTM, 2000).

However, research shows that students in the United States are particularly weak in mathematics. According to the National Assessment of Educational Progress (NAEP), while the average scores of fourth graders have improved considerably over the last fifteen years, scores of eighth graders have only improved slightly, and little or no improvement has been evident in twelfth grade math scores (Kehle et al., 2004; National Center for Educational Statistics [NCES], 2008; U. S Department of Education, 2005). NAEP results also show that 39% of our students are at or above the "proficient" level in Grade 8 (U.S. Department of Education, 2005), but only 23% are at that level by Grade 12 (U. S. Department of Education, 2005). "Proficient" is defined as meeting or exceeding academic content standards (NCES, 2008). NAEP shows that 27% of eighth-graders cannot correctly shade 1/3 of a rectangle and 45% are unable to solve a word problem that involves dividing fractions (NCES, 2008). Although students generally do well on computational skills, they are lacking in understanding of basic mathematical concepts and are noticeably deficient in application and problem solving (Kehle et al., 2004).

Thus, mathematical literacy is a serious problem in our nation (U. S. Department of Education, 2008a). Other international comparisons also show that American students do not achieve as well in the eighth grade as they do the fourth; and students do even less well by grade twelve. Studies from the Trends in International Mathematical and Science Study (TIMSS) suggest that students in the United States perform more poorly than students from other countries (U. S. Department of Education, 2008b). In another international study, the Programme for International Student Assessment (PISA), students in the U.S ranked 35th in mathematical literacy out of the 57 nations that participated in the study (Organization for Economic Co-Operation and Development [OECD], 2006). On average, U.S. mathematics teachers spend little time on engaging students in problem solving and reasoning that lead to the understanding of mathematics (U. S. Department of Education, 2008b).

Algebra

Because action must be taken to strengthen America in mathematical learning, the President created the National Mathematics Advisory Panel (U. S. Department of Education, 2008a). The charge given to the panel was to “foster greater knowledge of and among improved performance in mathematics among American students…with respect of the conduct, evaluation, and effective use of the results of the research relating to proven-effective and evidence-based mathematics instruction" (U. S. Department of Education, 2008a). Policy makers and educators have focused on how students can be best prepared for mathematics (U. S. Department of Education, 2008a). Algebra has emerged as a central concern in improving student achievement even though students encounter difficulties with many aspects of mathematics. The sharp falloff in mathematics achievement in the U. S. begins as students reach late middle school, where, for more and more students, algebra course work begins (U. S. Department of Education, 2008a). Naturally, the Panel turned their focus on how students can be best prepared for entry into algebra. This focus has consequences since the formal study of algebra is a demonstrable gateway to later achievement (U. S. Department of Education, 2008a). Students need formal study of algebra for any form of higher mathematics later in high school; moreover, research shows that completion of Algebra II correlates significantly with success in college and earnings from employment (U. S. Department of Education, 2008a). In fact, students who complete Algebra II are more than twice as likely to graduate from college compared to students with less mathematical preparation (U. S. Department of Education, 2008a).

In the Middle Ages, algebra meant calculating by rules or using algorithms to find solutions. Algebra has continued to evolve and by the Renaissance has come to mean calculation with signs and symbols—using *x*'s and *y*'s instead of numbers (Steen, 1999). In subsequent centuries following the Renaissance, algebra was primarily about solving equations and determining unknowns (Steen, 1999). By the twentieth century, algebra became the science of arithmetic, but today has evolved to include more than just the four operations found in arithmetic (Steen, 1999). Expectations for student understanding in algebra include: understanding patterns, relations, and functions; representing and analyzing mathematical situations and structures using algebraic symbols; using mathematical models to represent and understand quantitative relationships; and analyzing change in various contexts (NCTM, 2000).

As the topics in algebra evolved, so did the trends in algebra course taking. Algebra has a well-established reputation as one of the primary gatekeepers for access to college-required mathematics courses (American Institutes for Research [AIR], 2006; Steen, 1999; U. S. Department of Education, 2008). Before the 1980s, high school algebra served as a critical filter to separate college-bound students from their work-bound classmates (Steen, 1999). Then, the notion that only a few could learn algebra began to drastically change. The Mathematics Education Trust (MET) Committee recognized that algebra is important to all of our students (NCTM, 1990). At the 1988 NCTM's annual meeting, MET invited a group to address the need to teach the fundamentals of algebra to the entire population (NCTM, 1990). During the next decade, algebra became the key to unlock doors to productive careers and serve as an agent of change in equity, thus unleashing "algebra for all" (Chazan, 1996). Robert Moses (1995) referred to algebra as "the new civil right." By 2004, key findings from NAEP showed that a higher percentage of thirteen year olds were enrolled in algebra in 2004 than in any previous assessment year, and the percentage of seventeen year olds taking second year algebra has increased from 37% in 1978 to 53% in 2004 (NCES, 2008). Moses and Cobb (2001) suggest:

> Once solely in place as the gatekeeper for higher math and the priesthood who gained access to it, [algebra] now is the gatekeeper for citizenship, and people who don't have it are like the people who couldn't read and write in the industrial age … it has become not a barrier to college entrance, but a barrier to citizenship. That's the importance of algebra that has emerged with the new higher technology. (p. 14)

At the least, algebra is essential for any student to be well prepared for the future (U. S. Department of Education, 2008). However, despite the fact that higher percentages of students are taking algebra, statistics also show that all students are not passing these classes (American Institutes for Research [AIR], 2006). Just taking and completing an algebra course does not signify long term success (AIR, 2006).

Given the importance of mathematics education, we must also take a hard look at who will be teaching this subject in school. All the efforts to ensure that mathematics is given the attention it deserves in the nation's schools will be for nothing without an adequate supply of mathematically knowledgeable and properly trained mathematics teachers (U. S. Department of Education, 2008).

Teachers and Teaching

In the wake of efforts to raise academic standards at the state and federal level, improving teacher quality has become a widely acknowledged concern. Teaching is well documented to significantly affect the nature and level of students' learning (Hiebert & Grouws, 2007). Although the claim that teaching does make a difference seems obvious, the question of what makes mathematics teachers effective is not easily answered. Policy makers tend to focus on nonclassroom aspects of teacher quality such as teacher education levels and years of experience, rather than the nature of teaching and learning that occurs in the classroom.

In a review targeting teacher quality, Wayne and Youngs (2003) synthesized the results of such studies into four categories of teacher characteristics: college ratings, test scores, degrees and coursework, and certification status. The objective of the review was to create a clear interpretation of the research by looking at all the research in a more systematic way, by rendering joint interpretations of the studies, and by offering implications for future research. One of the categories, for example, is teachers' test scores. Of the seven studies involving teacher test scores, five findings supported the contention that students learn more from teachers with higher test scores, while two did not. Wayne and Youngs (2003) suggested several explanations for the divergent findings. That these seven determinate findings could have occurred randomly—and that there is in fact no relationship between student achievement and teacher test scores. Wayne and Youngs (2003) also offered that the relationship between student gains and teacher assessment scores might depend upon alignment of the underlying instruments.
In the end, the most plausible explanation for the divergent findings emerged through examination of what control subjects were used in each of the seven studies. The implications from the study that follow are quite apparent, a need for more research.

Evidence from multiple research studies show that students' mathematics learning and their dispositions toward mathematics are influenced by the teaching they experience at school (Mewborn, 2003). NCTM (2000) and the American Council of Education (1999) assert "students' understanding of mathematics, their ability to solve problems, and their confidence in, and disposition toward, mathematics are all shaped by the teaching they encounter in school" and "the success of the student depends most of all on the quality of the teacher", respectively. In the simplest sense, effective teachers are those who get students to learn. Studies done in the past deem teachers to be "effective" based on their students' standardized test scores. These studies did not however identify specific classroom practices that characterize "effectiveness" in teaching (Mewborn, 2003). In an attempt to link teacher characteristics and learning in mathematics, Begle (1979) concluded that there is no doubt that teachers play a key role in the students' learning of mathematics. However, the specific ways teacher characteristics affect student learning are not understood. This conclusion is still true today (Mewborn, 2003).

Although much research has been done on teachers and teaching, some questions remain unanswered (Mewborn, 2003). One such question is what do effective teachers do differently in their classrooms than less effective teachers? One of the most reliable predictor of effective teaching is "opportunity to learn." Students learn what they are given opportunities to learn. The National Research Council (2005) defines opportunity to learn as "circumstances that allow students to engage in and spend time on academic tasks…" (p. 333). Providing an opportunity to learn means providing students the conditions in which they are likely to engage in tasks that involve relevant mathematics (Hiebert, 2003). This engagement may include listening, talking, writing, and reasoning (Hiebert, 2003). Teaching is a complex activity and there is not a set recipe for what approach will always work, but teaching plays a major role in shaping students' learning opportunities (Hiebert & Grouws, 2007; Mewborn, 2003). Much of the current research does not provide the depth of understanding of the phenomena under investigation, or the detailed characteristics of what effective teaching looks like. It is therefore necessary to explore this issue by looking at the classroom in depth, moving beyond the research that has been done before.

The Purpose of the Study

Students are not achieving at the levels they should be in mathematics. As a result, policy makers and educators have focused on the best ways to get students to learn. In doing so, they have turned their attention to algebra as the key to students being prepared and being successful in mathematics. Also, teachers and the ways they teach are considered a key component of mathematical learning. Although effective teaching no doubt leads to learning, it is unclear what effective practices in the classroom actually are.

This study is designed to gain an understanding of effective teaching and learning by providing depth and detail of the role of the teacher inside an Algebra I classroom. This study will examine the roles of the teacher and how they impact student learning in the Algebra I classroom, focusing on four main components of the classroom: content, discourse, equity, and connections. The specific questions addressed in this study are

1. What is the role of the teacher in implementing content, discourse, equity, and connections in the Algebra I classroom?
2. How do these roles affect the role of the student and student learning?

An outline of the remaining chapters follows: chapter two will provide a review of literature pertinent to effective teaching, including four main components of effective teaching; chapter three will discuss the methodology used for this study, including the data analysis process; chapter four describes the findings from the study; and chapter five provides conclusions from the study.

II. REVIEW OF LITERATURE

The purpose of this study is to explore and gain a better understanding of the role of the teacher in effective mathematics teaching in the Algebra I classroom. In the following chapter, I will first frame teaching and learning from a constructivist view. Then, I will look at how effective teaching is defined. Finally, the role of the teacher will be examined in the context of effective teaching within the mathematics classroom, applying the components of content, discourse, equity, and connections.

Theoretical Framework

This exploration of effective teaching and learning of mathematics is based in the theory of constructivism. Constructivism is defined as the individual student forming knowledge of himself, and not relying on what someone says is true (Jonassen et al., 2005). In constructivism, the student acts as the creator of his own meaning (Jonassen et al., 2005). Constructivism, particularly in its "social" forms suggests that the learner is much more actively involved with the teacher in constructing new meanings (Atherton, 2005). John Dewey (1963) talked about the interaction between the learner and the environment. He believed that because students need to interact with their environment in order to think, every student should be engaged in activities (Dewey, 1963). Dewey (1963) said these activities needed to fit the student's interests, involve the student actively, have intrinsic worth, and present problems that would lead to new questions and inquiry.

Piaget (1973) was a forerunner in constructivist theory by arguing that a teacher "telling" students knowledge required that the teacher and the student have a mutual communication framework. According to Piaget, learners construct their own knowledge schemes in relation to previous and current experiences, and learning is a process of equilibrium in response to external stimuli (Piaget, 1973).

Other researchers, such as Vygotsky (1934/1978) and Bruner (1985), have questioned aspects of Piaget's theory, arguing that while the shift to a more child-centered learning style has been essential for education, there has remained some ambiguity regarding the role of the teacher. Vygotsky (1934/1978) and Bruner (1985) placed more emphasis on the part played by language and other people in enabling children to learn. Vygotsky (1934/1978) laid the base of social constructivist theory by introducing a concept to provide some measure of a learner's development related to instruction (Vygotsky, 1934/1978). The Zone of Proximal Development (ZPD) is an account of how the more competent assist the young and the less competent to reach the higher ground from which to reflect more abstractly about the nature of things (Vygotsky, 1934/1978). It is within this notion that Bruner (1985) termed *scaffolding* in describing the interaction between adult and child. The term scaffolding describes interactions through prompts, cues, answers, questions, modeled behaviors and suggestions leading eventually to successful task completion. The teacher structures tasks to allow the students to participate in tasks that would otherwise be beyond their grasp (Bruner, 1985; Vygotsky, 1934/1978). Therefore, active engagement by the student is necessary (Bruner, 1985, 1990; Vygotsky, 1934/1978). There is also the potential for the interaction between students to create collaboration in the ZPD. By working together, students have the opportunity to create their own mathematical insights. Learning is shifting away from acquisition and moving towards learning as participation. (Bruner, 1985; Vygotsky, 1934/1978).

More recently, social constructivism continues to be extended and enriched. In contrast to Vygotsky's view that key elements of teacher-student interactions are imitated, practiced, and internalized by the student, social constructivism defines learning as socially shared cognition that is "co-constructed" within a community of participants (Green & Gredler, 2002). In other words, people's social interaction constructs and reconstructs contexts, knowledge, and meanings (Green & Gredler, 2002). Social constructivist theory recognizes the need to give attention to the institutional context of social interactions, the importance of interpersonal relationships in teaching and learning, and that "thinking" is closely linked to forms of social practice (Forman, Minick, & Stone, 2003). Researchers have given their attention to the relationship between instructional practices and learning outcomes and looked at the concept of learning mathematics in a community of practice (Forman, Minick, & Stone, 2003). Explanations of students' mathematical activity and learning given in individualistic psychological terms has proven inadequate in evaluating the effectiveness of instructional activities and strategies. As a result, a primary theoretical objective became that of accounting for mathematical development of the student as it occurs in the social context of the classroom (Cobb & Yackel, 1996).

Based on constructivist theory, I believe that learning does not come from the transmission of facts, but from students actively constructing knowledge. Students should be empowered to make contributions in their mathematics learning, and students should connect new ideas to prior knowledge. In order for this to happen, teachers must know what practices are necessary to help the students become successful mathematical learners. Thus, effective teaching requires knowing and understanding the mathematics to be learned. It also means having the pedagogical tools to support students' mathematics learning. "Teaching mathematics well involves creating, enriching, maintaining, and adapting instruction to move toward mathematical goals, capture and sustain interest, and engage students in building mathematical understanding" (NCTM, 2000, p. 18).

Thus, in this study, the classroom will be viewed as a mini society, a community of learners. Some characteristics founded on constructivism include: all students actively participate in learning activities; the teacher and students work together to help one another learn; interaction is not exclusively between the teacher and student, students interact with each other; teachers and students respect the diversity of student interests, thinking, and progress; and the thinking involved in learning activities, such as problem solving, is as important as the outcome. These learners should be actively engaged in activity, discourse, and reflection on their mathematical experience. The teacher should provide concrete and contextually meaningful lessons where students ask questions and construct their own meaning and understanding in the mathematics. The teacher should establish an environment where the students are comfortable communicating and discussing their mathematical thinking. The teacher should provide an opportunity for all students to learn meaningful mathematics. The teacher should provide opportunities for the students to make connections within the mathematics discipline itself as well as providing real world contexts outside of mathematics. This study will look at the role of the teacher in effective teaching inside the classroom based on this vision of what student learning entails.

Definition of Effective Teaching

Before considering the role of the teacher in effective teaching, effective teaching itself must be defined from a review of literature pertinent to mathematics instruction. One approach to searching for what defines teacher effectiveness is known as the process-product paradigm (Shulman, 1986). This research examines how classroom teaching behaviors (process) effect student achievement (product). The research has moved beyond "process" factors such as personal characteristics to what teachers actually *do* in the classroom. Elaborations of this paradigm have added attention to classroom activity, both associated with teacher actions and with student learning (Shulman, 1986).

Studies Related to Effective Teaching

In reviewing the literature for reliable demonstrable recommendations for effective teaching practices, several significant studies were chosen pertaining to effective teaching. These studies were chosen because they took a comprehensive view of teaching rather than focusing on one particular aspect of teaching. The studies encompassed a broad scope of effective teaching practices and explored many relevant facets of successful mathematics instruction. After holistically examining the characteristics of effective teaching they described, the studies were synthesized to provide a framework for effective teaching to be used in this study. Each of theses studies will be discussed and then the commonalities found in effective instruction will be drawn from the studies to establish my framework for effective teaching.

Open and Closed Mathematics

The first comprehensive study in this review was designed to look at different approaches to teaching. Boaler (1998) conducted this study due to the growing concern of how to help students to learn mathematics. Boaler (1998) discussed the findings of an ethnographic, three year case studies of two schools. Case studies were chosen to monitor the relationships between the students' day to day experiences in classrooms and their developing understanding of mathematics. While the questions of the case studies were not explicitly given, the author stated that one of the aims of the research was to investigate student learning, particularly "to discover whether different forms of teaching would create different forms of knowledge, which might then cause students to interact differently with the demands of new and unusual situations" (Boaler, 1998, p. 42).

The student populations of the two schools reflected equivalent demographics. The schools were located close to a predominately white, working class community (Boaler, 1998). There was no significant difference in socioeconomic status of the two schools. A study conducted at the start of the research, based on the results of a national test, showed little differences in the averages of the scores of the students at the two schools. Prior to entering years 9–11, students from both of the schools used individualized booklets that introduced students to mathematical procedures and techniques, then provided practice questions. Teachers interacted individually with students. However, upon entering years 9–11, the teaching methods in the schools were very different. One school used a traditional, textbook approach, and the other, open-ended activities at all times (Boaler, 1998).

Amber Hill, the first school, was a large mixed comprehensive school run by an authoritarian trying to improve the school's academic record by enforcing traditional practices (Boaler, 1998). The school contained quiet, calm classrooms with students sitting in rows or small groups watching the chalkboard or working exercises. In years 9-11, these students moved from the individualized booklets to a more traditional textbook. These textbooks presented a particular method, along with a series of exercises for the students to practice. All the teachers presented techniques for 15 to 20 minutes at the start of the class, then gave students questions to work. This continued throughout the year, all but 3 weeks when the students were given open-ended tasks (Boaler, 1998).

The second school, Phoenix Park, encouraged students to take responsibility for their own actions and to be independent thinkers. This school had a relaxed atmosphere. The students there worked on open-ended projects in mixed ability groups at all times until January when they started to practice examination techniques. Students were encouraged to develop their own ideas, formulate and extend problems, and use their mathematics. Phoenix Park had the philosophy that students should encounter a need to use mathematics in situations that are realistic and meaningful to them. The teacher taught the students any math they did not know. Each project would last 2–3 weeks and at the end the students were required to turn in descriptions of their work and their mathematics activities (Boaler, 1998).

The results indicated Amber Hill, the "traditional" school had hard-working, disciplined students, who were bored and uninterested in mathematics and could not make a connection between the mathematics they were learning and the real-world (Boaler, 1998). There were no indications that traditional mathematics provided the content knowledge needed for standard assessment and seemed to be lacking in real-world situations (Boaler, 1998). Students in these classrooms were at a disadvantage. On the other hand, Phoenix Park had students that while not overly eager to work, could use mathematics as a tool they could adapt and apply. The non-traditional students did better on the tests and could solve everyday problems. They understood the mathematics and were able to relate this understanding to situations outside the classroom which equipped them for life.

There was also a significant difference in the achievement between boys and girls in the traditional approach, as opposed to no significant differences in attainment between boys and girls (Boaler, 1998). Boaler (1998) observed that the students who were taught using a traditional approach developed an inert, procedural knowledge that was of limited use to them in anything but textbook situations. They could not interpret any unfamiliar questions and could not see how to apply them. Students from the school which advocated problems that create meaning in mathematics had the belief that mathematics involved active and flexible thought and the students had the ability to adapt and change methods in new situations (Boaler, 1998).

From this study, I draw the following key observations. First, this study showed that effective teaching encompasses students needing to take an active role in their learning so that the mathematics will become meaningful and real to them. Students must be given the opportunity to engage in worthwhile mathematics and then be given the opportunity to make connections within mathematics as well as contexts outside of mathematics. Students need to be given opportunities to utilize mathematical knowledge in situations outside the classroom context or applied to new situations.

Second, teaching methods that focus on standard textbook questions encourage the development of procedural knowledge that is of limited use in nonschool situations. A collection of exercises, rules, and equations that need to be learned are often not recalled and students to not know what to do if the situation is slightly different or does not contain a rule to use.

Open-ended, practical and investigative work that requires students to make their own decisions, plan their own strategies through tasks, choose methods, and apply their mathematical knowledge, however, lead to students making mathematical connections and developing meaning. Mathematical situations that are meaningful and realistic to the students lead to active and flexible thinking, the ability to adapt and change methods to fit new situations, and students select appropriate procedures.

Middle Grade Teachers' Learning Through Students' Engagement With Modeling Tasks

My second comprehensive study reported on how teachers develop and use subject matter and pedagogical content knowledge in the act of teaching certain tasks. The key to this study was how the features of these tasks can promote teacher's knowledge development and their support of student learning (Doerr & English, 2006). In this study, the researchers used modeling tasks that were intended to maximize the learning of both the students and the teachers to provide a window to the development of teacher knowledge. The researchers chose a sequence of mathematical modeling tasks because of their known potential for engaging the students with realistic problems, for their potential in revealing multiple ways a student may think about the task, and for their capacity to engage the students in evaluating the usefulness of their solutions (Doerr & English, 2006). The questions addressed in the study were:

- How do middle grade teachers develop new subject matter knowledge and pedagogical content knowledge as they implement a novel model-eliciting task?
- How do middle grade teachers learn to support their students learning during the implementation of model-eliciting tasks?
- How do the characteristics of a model-eliciting task promote teacher learning, in terms of their knowledge development and their support of student learning? (Doerr & English, 2006).

Model-eliciting tasks are defined here as those in which student's thinking processes are explicitly revealed via their descriptions, explanations, justifications, and representations both as they engage with the task and as they present their end products (Lesh, Hoover, Hole, Kelly, & Post, 2000). Model eliciting tasks involve meaningful problem situations. They provide a basis for subsequent model exploration and application. Multiple interpretations and approaches are encouraged, mathematical communication is emphasized, they require the documentation of the end processes, and self-evaluation is an inherent component of model-eliciting tasks (Doerr & English, 2006).

Seven middle grade teachers and their students participated in this study, but two teachers were selected for an in depth case study. The study revealed that both teachers engaged in developing new subject matter knowledge and pedagogical content knowledge (Doerr & English, 2006). However, one of the teachers developed knowledge of the mathematics content as her students engaged in model-eliciting tasks, while the other teacher developed new pedagogical content knowledge. Both teachers were able to support the students in making sense of these tasks in realistic ways (Doerr & English, 2006). Another significant feature of model-eliciting tasks was that the students could evaluate the usefulness of their strategies used to find the solution to the task (Doerr & English, 2006). The students were engaged in evaluating of the usefulness of the models. These tasks also encouraged the students to develop and revise their own ideas. Model-eliciting tasks provided alternate ways of thinking about the problem, which led the teacher to attend to the ways the students were thinking about the problem and away from the students being judges according to the teacher's way of thinking about the problem. Students were encouraged to create and explain representations that were useful and meaningful to them (Doerr & English, 2006).

This study suggests that effective teaching involves worthwhile tasks that enable teachers to develop new understandings of the mathematics content. The teacher is able to examine the ways in which student ideas develop and are represented, and to adapt to new roles in their interactions with the students, including focusing on listening and asking questions for understanding.

These worthwhile tasks lead to students' engagement in the mathematics which allows the teacher to develop new understandings of the content and how student ideas are developed and represented. New roles of the teacher in interactions with the students include listening, observing, and asking questions for understanding and clarification. Teachers must understand how to implement the content and how to present the concepts and facts to promote learning. Teachers must understand student's understandings and misunderstandings.

Modeling tasks, as shown in this study, promote student engagement by providing a realistic problem, multiple ways to think about the problem, and the capacity to engage students in evaluating the tasks usefulness. Aspects of a meaningful task include the development of mathematical understanding, reasoning, and sense-making. These tasks also provide opportunities for multiple representations, multiple solution approaches, and mathematical communication.

Classrooms that utilize these types of tasks help students make sense through their own knowledge and experiences rather than conform to teachers' notion of a particular way to think about the problem. Students in these classrooms judge for themselves if responses are good enough rather than referring to an external authority. Students in these classrooms reveal explicitly how they are thinking about problem documentation and representation. These types of classrooms require listening, interpreting, and understanding various ways students try to make sense of the problem. There is a shift in the role of the teacher from evaluating correctness of work to devising ways for student evaluation. Teachers need to understand and interpret student thinking and representations. The role of the teacher is for teachers to learn lessons as the lessons are taught and as they interact with the student as the students engage with the mathematics.

Tracing the Evolution of Pedagogical Content Knowledge as the Development of Interanimated Discourses

In the third comprehensive study, Seymour and Lehrer (2006) tracked the development of one teacher's pedagogical content knowledge, as defined in the previous section. *Growth* in pedagogical content knowledge for the purposes of this study is characterized as the interanimation or the interweaving of two discourses as defined by Gee (1999). One discourse refers to the ways that students talk and act about mathematics; and the other discourse refers to the way the teacher talks and acts as she guides the development of students' mathematical thinking (Seymour & Lehrer, 2006).

The context of the study was a unit on geometric similarity, where the researchers tracked one sixth-grade teacher's efforts to support student understanding of linear functions (Seymour & Lehrer, 2006). How students might think about linear functions, especially in their relation to geometric similarity, were all new to this teacher (Seymour & Lehrer, 2006).

During the course of the study, the researchers observed several transitions in the teacher's pedagogical content knowledge. For example, in year one, the students were asked to translate between two representational forms, equations and graphs. The students were asked to relate what they were looking at, to their previous efforts to develop algebraic expressions (Seymour & Lehrer, 2006). Since students were having difficulty understanding, so the teacher expanded her instructions for the entire class:

> Okay so now can we come up with a rule here that tells us what a line will be? We want to come up with a rule. Okay, we've got a description. How can we write a rule? Okay, that's what I want you to do in your small groups right now. Write a rule for the steepness of a line. (Seymour & Lehrer, 2006, p. 559)

The teacher seemed to rely on general heuristics about the importance of student thinking. She asked students to explain their thinking, revoiced student's contributions, and emphasized the need for justification (Seymour & Lehrer, 2006). By the end of the second year, the teacher increasingly recognized patterns of student talk and reasoning and acted in productive ways to transform mathematical thinking (Seymour & Lehrer, 2006). For example, when students were unsure about the relations between graphs and equations, the teacher foregrounded meaning by asking questions she had learned from student activity the year before. The teacher could anticipate student difficulties and responded to them in more productive ways. The classroom conversation was more interanimated and student and teacher talk more coordinated (Seymour & Lehrer, 2006).

One implication of this study is that pedagogical content knowledge can be obtained as teachers attempt discourse. In order to develop mathematical understanding, conversations must tune to particular elements of students' mathematical talk and activity. Effective teachers understand mathematics central conceptual structures and forms of argument and develop activities that support student participation. The goal is not for the student to repeat "right" answers, but to participate in accurate mathematical discourses. This process is gradual as the teacher and students begin to recognize one another's meanings.

Cognitively Guided Instruction

The fourth comprehensive study is Cognitively Guided Instruction (CGI) is a research based professional development initiative for teachers in grades K-6, aimed at raising teacher content and pedagogical knowledge (Carpenter, Fennema, & Franke, 1996). CGI focuses specifically on: 1) the development of students' mathematical thinking, 2) instruction that influences that development, 3) teachers knowledge and beliefs that influence their instructional practices, and 4) the way that teachers' knowledge, beliefs, and practices are influenced by their understanding of students' mathematical thinking. CGI focuses on students' understanding of mathematical concepts to provide a basis for teachers to develop their knowledge more broadly (Carpenter, Fennema, & Franke, 1996). The primary goal of CGI is to provide teachers with a framework to assess students' understanding. Students' understanding is characterized in terms of how the students make connections from prior knowledge to new ideas. The nature of this knowledge is explicitly portrayed. Teachers can learn the processes of how students solve mathematical problems using concrete material and then move to more formal, abstract operations (Carpenter, Fennema, & Franke, 1996). The emphasis in CGI is on what students can do, rather than on what they cannot do. This leads to a very different approach regarding incorrect answers. Incorrect answers are viewed as an opportunity to assess student thinking, what they know and what they don't know. With the CGI approach, teachers focus on what students know and help them build mathematical understanding (Carpenter, Fennema, & Franke, 1996).

In the first CGI study, twenty CGI teachers were compared with twenty control teachers (Carpenter, Fennema, & Franke, 1996). The study found that the CGI teachers placed more emphasis on problem solving and less emphasis on computational skills. The CGI teachers also expected more multiple solution strategies, rather than a single method. They listened to their students more and knew more about their students thinking than the control teachers (Carpenter, Fennema, & Franke, 1996). This initial study also showed that the CGI classes had significantly higher levels of achievement in problem solving than the control class. Another part of the study indicated that teachers who had not participated in the CGI program had intuitive knowledge about children's mathematical thinking, but because this knowledge was fragmented, it did not play a key role in teacher decision making (Carpenter, Fennema, & Franke, 1996).

CGI followed the initial study with a three year longitudinal study of 21 teachers to examine the nature and pattern of change among teachers and the relations between beliefs and instruction. The levels in becoming a CGI teacher are identified as follows. Level 1 teachers believe that children have to be explicitly taught mathematics. Instruction in their classroom is usually guided by textbook and focuses on specific skills. Teachers usually demonstrate the procedure and the students are expected to practice (Carpenter, Fennema, & Franke, 1996). Level 2 teachers begin to question whether explicit instruction is necessary and may provide opportunities for students to use their own strategies to solve a problem (Carpenter, Fennema, & Franke, 1996). Level 3 teachers believe that students can solve problems without having a strategy provided for them (Carpenter, Fennema, & Franke, 1996). Classrooms are characterized by students solving and reporting their solutions. These classrooms are strongly influenced by the teacher's understanding of students' thinking and know the appropriate questions and problems to pose to elicit this thinking. Level 4 teachers conceptualize instruction in terms of student thinking in their classroom. These teachers continually reflect, modify, and adapt their models to learn based on their students (Carpenter, Fennema, & Franke, 1996).

This study proposed that an understanding of student thinking can provide a framework with which to construct coherence to teachers' pedagogical content knowledge and their knowledge of mathematics. The basis for student thinking is that students construct knowledge, rather than assimilate some part of what they are taught.

Students intuitively solve word problems by modeling the actions and relations described in the problems. Mathematical ideas develop in students and they construct concepts.

A key component in student development is meaningful problems. As students develop efficient ways to solve meaningful problems, their understanding on how to apply the knowledge needed to solve the problem increases. Students can acquire the skills and concepts required to solve the problems as they are solving the problems. In CGI, for example, student's invented algorithms were constructed through progressive abstraction of their modeling procedures (Carpenter, Fennema, & Franke, 1996).

Student understanding is characterized in terms of how students connect new ideas to existing knowledge. Teachers must learn how students use concrete materials to solve problems, then how the students evolve to formal, abstract operations. Teachers must know what questions to ask and what to listen for from the student.

Quantitative Understanding: Amplifying Student Achievement and Reasoning and The Algebra Project

Because of similar themes, the fifth and sixth comprehensive studies are reviewed together. First, the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) project is another effort designed to improve mathematical proficiency by developing desirable instruction that fosters student learning (Silver & Stein, 1996). The goal of the QUASAR project was to show that it is "both feasible and responsible to implement instructional programs that foster the acquisition of mathematical thinking and reasoning skills by students attending middle schools in economically disadvantaged communities" (Williams & Baxter, 1996, p. 29). QUASAR is based on the premise that in many cases low levels of participation and performance in mathematics for poor urban students in the middle grades are not due to the students' lack of ability. Rather, the educational practices may fail to provide the opportunity to learn high quality mathematics. QUASAR aims to provide instruction based on meaningful learning opportunities and high expectations for all students. QUASAR provides instruction that encourages conceptual understanding and thinking, reasoning, and problem solving in mathematics (Silver & Stein, 1996).

QUASAR is a complex research study designed to make certain conditions that appear to be conducive to student success; to derive instruction principles for effective mathematics instruction for middle school students; to describe effective instructional programs in ways that will allow their adaption to other schools, and to devise new assessment tools to measure growth in mathematical thinking, reasoning, and communication (Silver & Stein, 1996).

The QUASAR project first began in 4 schools, then increased to a total of six schools. Schools were selected based on application by collaborative teams at each school. The schools were all located in urban school districts, five of which are among the 75 largest in the U. S. (Silver & Stein, 1996). There was considerable diversity among the schools with respect to ethnicity and language, but all the QUASAR schools served students who live primarily in poverty. This was also a fairly high rate of student transience (Silver & Stein, 1996).

Teachers in the QUASAR project expect students to understand the mathematics they are asked to learn, the instructional practices support the development of students understanding, and the use of mathematical tasks provide challenging settings where students develop and apply this understanding. Students know how to execute and recall factual knowledge, but also when and why to apply the procedures (Silver & Stein, 1996).

Observations collected over the course of the project found that about 75% of the instructional tasks were intended to engage the students in conceptual understanding, reasoning, or problem solving. Only about 1/5 of the tasks were set up for computation or memorization without the development of understanding. Two-thirds of these tasks involved multiple solution strategies and representations, as well as student's explanations (Silver & Stein, 1996).

Another feature of the QUASAR classroom was the emphasis on curriculum topics typically not focused on in conventional middle school mathematics instruction. Topics such as statistics, algebraic reasoning, patterns and functions, and probability were included. In QUASAR classrooms, teachers emphasized discourse through student communication and collaboration. Most of these classrooms were set up so students could use each other as resources (Silver & Stein, 1996; Williams & Baxter, 1996).

Results show that QUASAR students outperformed NAEP's disadvantaged urban sample in all areas of the mathematical content assessed by NAEP and did especially well on problem solving and open ended or short answers (Williams & Baxter, 1996). Evidence that QUASAR instruction is providing increased numbers of students qualifying for placement in ninth-grade algebra. The number of students who qualified for algebra increased from about 8% at the end of the first year to more than 40% by the end of the fourth year. The study also showed that many of the students were able to sustain this level of performance after making the transition to high school classrooms (Silver & Stein, 1996).

The instruction offered in this study was instruction that is oriented toward helping students develop a meaningful understanding of mathematical ideas through engagement with challenging mathematical tasks. Teachers involved in the QUASAR project have engaged in developing and implementing the type of instruction that leads to desirable forms of mathematical proficiency (Silver & Stein, 1996).

The next study, similar in ideals to the QUASAR project, is the Algebra Project. The Algebra Project is a middle school mathematics program to increase student achievement in mathematics, and prepare students to succeed in college preparatory math and science courses at the high school level (Silva, Moses, Rivers, & Johnson, 1990). The project evolved after Robert Moses visited his daughter's classroom and recognized that ability grouping channeled most students into a non-algebra track (Silva et al., 1990). The three broad goals of the project are an attempt to point students, teachers and school communities toward a new vision. First, the Algebra Project seeks to develop students who are mathematically literate, self-competent, and motivated in order to succeed and master the mathematics necessary for higher learning and the mathematics needed for mathematics related careers (Silva et al., 1990). Second, the Project seeks to reform mathematics instruction in the middle school that is relevant to the students' lives and their socially constructed knowledge base. Third, the Algebra Project seeks to organize supportive communities which understand "mathematics education as a problem of mathematical literacy and who understand the question of students capability as learners as a matter of effective effort (Silva et al., 1990, p. 379). In order to achieve these goals, the Algebra Project focuses on 1) curriculum development, 2) teacher education, 3) expansion of the project and replication in other schools, and 4) developing community support (Silva et al., 1990).

While working on the project, Moses identified that students had trouble making the transition from arithmetic to algebra. For example, children at an early age learn about numbers as a quantity or "how many." As they make the transition to Algebra numbers become more qualitative or "which way." Through a five step curricular process known as the Transition Curriculum, Moses set out to eliminate the conceptual barrier (Silva et al., 1990).

- Step 1— Experiencing a physical event as a group
- Step 2— Representing the physical event through modeling or drawing
- Step 3 — Describing the event informally or intuitively, using their own language
- Step 4 — Translating their description into formal edited English
- Step 5 — Creating symbolic representation of the event using mathematical language (Silva et al., 1990).

One example of how students were initiated to this five step process by taking a field trip on a subway. The students had to reconstruct their journey using a map which served as a number line to illustrate "how many" and "which direction" (positive and negative numbers), and equivalence (Silva et al., 1990). Using this procedure, students transition from concrete physical events to abstract understanding and representation (Silva et al., 1990).

The results show that studying algebra in the seventh and eighth grade is reasonable. Since the Project began, students engage in serious ongoing work in mathematics. Before the Project, virtually no students took or passed the optional ninth grade mathematics placement exam. During the first five years of the Project, more than half of the students took the placement exam and 79% of the students passed it. All students who participated in the project were prepared to enter the high school algebra sequence in the ninth grade and several entered honors algebra or geometry classes (Silva et al., 1990).

QUASAR and The Algebra Project believe that every student has the right to a quality education, and if given the opportunity, can succeed (Silva et al., 1990; Williams & Baxter, 1996). These studies showed that by creating classroom environments that were rich in mathematical activities and tasks that support the development of understanding of mathematical ideas, performance improved. Students increased their mathematical understanding, competency in mathematical understanding and problem solving, and communicated mathematical explanations (Silva et al., 1990; Williams & Baxter, 1996).

Meaningful understanding of mathematical ideas can be developed through engagement with challenging mathematical tasks. These types of tasks lead to a deeper and more meaningful understanding of mathematical ideas, help students demonstrate a proficiency in reasoning, and enable students to solve complex tasks.

Most mathematical tasks used in the classroom are used to illustrate the mathematical technique at hand, rather than representing real problems that engage students' thinking. Most classrooms are teacher centered and offer whole class instruction with seatwork and recitations with little or no student interaction. In classrooms where students are expected to memorize facts and procedures and to imitate the use of formulas, algorithms, and other procedures without much attention to why or when it makes sense, students gradually come to expect there is only one way to solve every problem, that the methods should be supplied by the teacher, and the students should not take the time to check the reasonableness of the method.

On the other hand, the use of tasks that require students to construct meaning and or relate concepts to symbols, rules and procedures lead to students' conceptual understanding, thinking, reasoning, and problem solving. Mathematical tasks need to challenge the students to apply and extend their developing understanding. Students need not only know the algorithms and how to execute them, but when and why to apply the procedures and knowledge, and to reason and solve complex problems. Meaningful tasks are intended to provoke student engagement. Tasks need to involve multiple solution strategies and representations that include student explanation in addition to finding a solution.

Along with meaningful tasks, teachers must expect students to understand the mathematics they are asked to learn. Teachers must provide an environment where students are encouraged to discuss ideas. Communication and collaboration is expected and valued. Student thinking is nurtured.

Inside the Classroom

A final comprehensive study looked at mathematics instruction in the U. S. and the factors that shape the instruction, but also provided a framework for rating effective teaching. This study, funded by the National Science Foundation and conducted by Horizon Research Inc. (HRI) called *Inside the Classroom* was designed to get a snapshot view of what goes on in the nation's science and mathematics classrooms (Weiss, Pasley, Smith, Banilower, & Heck, 2003). The target population of the study was private and public schools in all 50 states including the District of Columbia. HRI selected a subset of 40 middle schools from the schools that participated in the 2000 National Survey of Science and Mathematics Education. To ensure sample sites were representative of the nation, HRI used systematic sampling with implicit stratification. When a middle school agreed to participate, the feeder elementary and high school(s) were randomly sampled. Thus, each site consisted of three schools. For classroom observations, a simple random sample was drawn from the math and science teachers in the selected school. One class each of 2 math teachers was observed at each school (Weiss et al., 2003). The questions of the study are:

- To what extent are mathematics portrayed as inert collections of facts and algorithms, as opposed to dynamic bodies of knowledge continually enriched by conjecture, investigation, analysis, and proof/justification?
- To what extent do mathematics lessons engage students intellectually with important mathematics disciplinary content?
- Is teacher-presented information accurate?
- Do teachers display an understanding of mathematics concepts in their dialogue with students?
- When teachers ask questions, are they posed in such a way that is likely to enhance the development of student conceptual understanding?
- Are adequate time and structure provided for student reflection and sense-making?

- To what extent is there a climate of respect for students' ideas, questions, and contributions?
- Are students encouraged to generate ideas, questions, and conjectures?
- Are students actively engaged in pursuing questions of interest to them, or simply "going through the motions," whether they are doing individual "seatwork" or working in groups? (Weiss et al., 2003)

The quality of the lessons that the teachers designed and enacted varied significantly. To determine which lessons were very effective and which were reported ineffective, the authors did an in-depth analysis of lesson descriptors from observations and interviews. Researchers took detailed field notes, including describing what teachers and students were doing throughout the lesson. The researchers were asked to pay attention to: the significance, accuracy, and developmental appropriateness of the content; the extent of intellectual engagement on the part of the students; the nature of the teachers questions and student responses; whether the lesson included appropriate sense-making and closure; and the extent to which the classroom culture encouraged all the students to participate (Weiss et al., 2003).

In the interview, teachers were asked the learning goals of the lesson, characteristics of students, and the instructional materials used. They were asked how comfortable they were in implementing the lesson. The teachers were also asked about the context in which they teach and how that context influences what and how they teach (Weiss et al., 2003).

The observation and analytic protocol used in the study consisted of three parts. Part I was used in describing and assessing the quality of the lesson. This included descriptive information about the purpose of the lesson as well as how class time was spent. The majority was comprised describing and assessing the lessons in four component areas: 1) lesson design, 2) implementation, 3) mathematics content, and 4) classroom culture. The researcher also provided a one or two page summary of the lesson and quality including “rich detail” for the reader to get a clear view of what was happening in that particular classroom (Weiss et al., 2003). Part II used information provided in the interview to document the extent to which each of a number of factors that influenced the lesson. This also included the teacher’s description of the students in the class, the physical description of the classroom, and how various influences interacted (Weiss et al., 2003). Part III was used to pull all of the information together, including any additional information the researcher wanted to include (Weiss et al., 2003).

The results showed that only 15% of the lessons were classified high quality, 27% were classified medium in quality, and 59% were considered low (Weiss et al., 2003). Factor analysis was used to identify qualities of mathematics lessons as a whole in the United States. Characteristics that emerged from the study that distinguished the effective components from the ineffective components in an attempt to answer these questions, were the ability to: 1) engage students with mathematics content; 2) create an environment conducive to learning; 3) ensure access for all students; 4) use questioning to ensure and promote understanding; and 5) help students make sense of the mathematics content (Weiss et al., 2003).

Synthesis and Framework for Effective Teaching

For the purposes of this study, several significant comprehensive studies were examined in order to compare research findings of effective mathematics instruction. In looking at the characteristics of what made the instruction effective, several common themes recurred. These themes have been synthesized into four main components of effective teaching.

1) *Content* is defined as how teachers use their content knowledge to teach mathematics lessons that are meaningful and worthwhile All of the studies showed that one of the most important, if not the most important component in effective instruction, is the mathematical content (Boaler, 1998; Carpenter, Fennema, & Franke, 1996; Silva et al., 1990; Silver & Stein, 1996; Weiss et al., 2003). However, important content does not stand alone (Weiss et al., 2003). Content must be significant and worthwhile (Boaler, 1998; Carpenter, Fennema, & Franke, 1996; NCTM, 2000; Silva et al., 1990; Silver & Stein, 1996; Weiss et al., 2003). High quality instruction is shown to invite students to be engaged and interact with the content (Boaler, 1998; Carpenter, Fennema, & Franke, 1996; Silva et al., 1990; Silver & Stein, 1996; Weiss et al., 2003).

2) *Discourse* is defined as ways students learn through representing, thinking, talking, and agreeing and disagreeing about mathematics (NCTM, 1991). Teachers need to create classroom environments that are rich in mathematical activity that supports the development of understanding of mathematical ideas. The studies show that creating these kinds of classrooms helps students' performance on tasks that require mathematical understanding, competence in reasoning and problem solving, and fluency in communicating mathematical justifications and explanations (Seymour & Lehrer, 2006).

3) *Equity* is defined as ensuring that *all* students are learning important mathematical content. The studies demonstrate that all children, regardless of background, can learn mathematics, if they have access to high quality instructional programs that support their learning (Boaler, 1998; Carpenter, Fennema, & Franke, 1996; Silva et al., 1990; Silver & Stein, 1996). Students must be given opportunities to learn mathematics in several ways.

4) *Connections* is defined as making sense of mathematics in contexts within the discipline itself *and* contexts outside of mathematics. Students are better prepared for the real world by sense making and making connections within and outside the context of mathematics (Boaler, 1998; Carpenter, Fennema, & Franke, 1996; Silva et al., 1990; Silver & Stein, 1996; Weiss et al., 2003).

The framework of effective teaching was drawn from several comprehensive studies that examined effective teaching practices. The rest of this chapter will provide a review of literature specific to each of the four components in the framework for this study. Each of the four components will be defined and discussed, referring back to the studies drawn from that provided the framework for effective teaching. A brief discussion of the role of the students will also be included in the introduction of each section to establish a vision of student learning. The focus in each of the following sections will be predominantly on the roles of the teacher in the mathematics classroom. “Roles” is used to encompass what the teacher and students do in the classroom in terms of making teaching and learning effective.

Content

The first component in my model of effective teaching is mathematical content. Content is discussed in terms of how teachers use their content knowledge to teach mathematics lessons that are meaningful and worthwhile.

Describing effective content. The studies discussed previously are based on the premise that meaningful learning occurs when instruction involves tasks that require students to construct meaning and/or relate important mathematical concepts (Boaler, 1998; Carpenter, Fennema, & Franke, 1996; Doerr & English, 2006; Seymour & Lehrer, 2006; Silva et al., 1990; Silver & Stein, 1996; Weiss et al., 2003). A worthwhile task can be characterized as being sound and significant mathematically, but should also be based on students' understandings, interests, and experiences (NCTM, 1991). To explain this, it is necessary to look at how tasks can "invite" the learner into purposeful interactions with the mathematics content (Weiss et al., 2003). A worthwhile task engages students because it challenges them to find something. For example, here are two different tasks designed to help students learn about perimeter and area. Compare the two tasks.

> Task 1: Find the area and perimeter of each rectangle (the dimensions of each rectangle are given)
>
> Task 2: Suppose you had 64 meters of fence in which you were going to build a pen for your large dog, Bones. What are some different pens you can make if you use all the fencing? What is the pen with the least play space? The biggest pen you can make? Which is the best for running? (NCTM, 1991)

Task 1 asks for little more than recalling what area and perimeter are and plugging some numbers into a formula. There is limited potential for higher ordered thinking about the relationship between perimeter and volume, and this task does not require problem solving or reasoning by the students (NCTM, 1991). Task 2 can engage students because they are required to look for something. It can be approached in multiple ways and would require the students to justify their solution and how they reached that solution. It lends itself to discussions and lets students develop their understanding about the relationship between area and perimeter (NCTM, 1991).

Ascribing to the core ideas of constructivist learning theories, the role of the mathematics student is to engage in mathematical activities such as exploring, justifying, proving, generalizing, and reflecting on ideas, representations, and procedures of their strategies for solving a mathematical problem (Fosnot, 1996; Lampert, 1990; Simon & Schifter, 1991). The student must be engaged intellectually with the mathematics content (NCTM, 2000). The following vignette takes place in a ninth grade Algebra classroom. The teacher probably knew and understood the content, but it was presented in such a way that did nothing to actively engage the students, so virtually no learning was taking place (Weiss et al., 2003).

> In a 9th grade teacher's efforts to help his students better understand how to solve equations and inequalities, he asked them to remember and repeat procedures he had demonstrated in the beginning of the class. The teacher's presentation of the content included questions and comments such as, "there's the variable, what's the opposite?" and "tell me the steps to do." He did very little to engage students with the content; two students slept through the teacher's entire presentation, and one read a magazine. Other students contributed very little, spending most of the time asking

about the particulars of an upcoming assignment (Weiss et al., 2003, p. 46).

The teacher's role in content. NCTM's *Principles and Standards* states that "teachers must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks (NCTM, 2000, p. 17). The founders of CGI conclude teachers also play a key role in identifying and selecting problems that the students can solve and the strategies used in solving them (Carpenter, Fennema, & Franke, 1996). Teachers must choose mathematical tasks that engage the students' intellect, develop students' mathematical understandings and skills, stimulate students to make connections, use reasoning and problem solving skills, and represent mathematics as relevant to all students (NCTM, 1991). Weiss et al. (2003) found that one of the most important aspects, if not the most important, of effective mathematics lessons is that the lesson needs to provide the students with important content.

The disconnect between content knowledge and teaching is an issue of growing concern. Analyses of teachers' knowledge have become a central concern for understanding the process of teaching as shown in three aforementioned major studies (Carpenter, Fennema, & Franke, 1996; Doerr & English, 2006; Seymour & Lehrer, 2006). "Although some teachers have important understandings of the content, they often do not know it in ways that help them hear students, select good tasks, or help all their students learn" (Ball, 2000; Ball & Bass, 2000). Often, teachers understand the concepts they are teaching, but they cannot convey this understanding to their students because they cannot "hear flexibly, represent ideas in multiple ways, connect content to contexts effectively, and think about things in ways other than their own" (Ball, 2000, p. 243). There is a distinction between knowing how to "do the math" and doing it in ways students can use (Ball, 2005).

Teachers must have the content knowledge needed to build deep mathematical experiences (Ball, 2005). Despite the significant gain in attention content knowledge has received from teachers needing to be "highly qualified", how content knowledge relates to student achievement has been poorly specified. Researchers have predominantly measured teachers' knowledge using teacher characteristic variables, such as courses taken, degrees attained, or results of basic skills tests (Wayne & Youngs, 2003). This is quite different from other studies that show that teacher effects on student achievement are driven by teachers' ability to understand and use subject-matter knowledge to teach (Hill, Rowan, & Ball, 2005). Doerr and English (2006) acknowledge that teachers must not only change the math content of their teaching, they must change the way they implement this content.

Hill, Rowan, and Ball (2005) examined the relationship between teachers' content knowledge for teaching and students' gains in achievement. A key feature of this study was the measurement of the knowledge teachers use in the classroom, rather than just the teachers' general mathematical knowledge. This was done by designing measurement tasks that determined proficiency at providing mathematical explanations and representations and working with unusual methods to solutions. For example, one of the items on the teacher questionnaire was for the teachers to determine the value x in $10^x = 1$. This requires common content knowledge, but not necessarily the kind of knowledge needed for teaching. Another type of problem, however, shows three different approaches to solving 25×35. The teachers had to assess whether the students' approaches would work with any multi-digit multiplication problem. The purpose of this task was to see if teachers could measure whether the approach was indeed a "method" to solve the problem, whether it made sense, and if it could be used as a method in general to work similar problems. "Teachers must be able to size up and evaluate the mathematics of these alternatives—often swiftly and on the spot" (Hill, Rowan, & Ball, 2005, p. 388).

Hill, Rowan, and Ball (2005) found a positive effect of teachers' mathematical knowledge for teaching and gains in student achievement. An important feature to note is the positive effect knowledge for teaching had on student gains in the first grade implying that teachers' content knowledge is important even at the early elementary age (Hill, Rowan, & Hill, 2005).

Another study that focused teacher's content and pedagogical knowledge and worthwhile tasks was based on the Problem-Solving Cycle (PSC), a model of professional development designed to assist teachers in supporting students' mathematical reasoning and developing of content and pedagogical content knowledge (Koellner, et al., 2007). Each PSC was organized around a rich mathematical task similar to the ones discussed in the study by Doerr and English (2006). The teachers shared a common mathematical and pedagogical experience through a series of three interrated workshops (Koellner et al, 2007). Specifically, the researchers developed a model of mathematical professional development to provide opportunities for teachers to enhance their professional knowledge and develop new instructional practices by examining mathematical problems, pedagogical practices, and student thinking (Koellner et al., 2007).

During the first PSC, teachers collaboratively solved a rich mathematical problem and developed plans for teaching the problem to their own students (Koellner et al., 2007). In the second workshop, the major emphasis was on the role the teacher played in implementing the problem. A critical examination of students' mathematical reasoning was the focus of workshop three. Teachers used and expanded their common content knowledge and specialized content knowledge as they worked on the featured mathematical problem (Koellner et al., 2007).

The authors of the study concluded that when teachers effectively engage and draw from multiple knowledge domains in the planning, implementation, and reflection of rich mathematical tasks in their classrooms, they are more likely to make more informed instructional decisions which leads to the production of more capable students (Koellner et al., 2007).

Llinares (2007) conducted a study to examine the relationship between a secondary mathematics teacher's pedagogical content knowledge and the dilemmas posed when dealing with students' conceptions, preconceptions, and misconceptions about specific topics in algebra (Llinares, 2000). The purpose of the study was similar to the Seymour and Lehrer (2006) study in that the research looked at the development of teacher pedagogical knowledge as a means of examining how teacher knowledge relates to teaching practice. Specifically, the study was done to analyze the interrelationship between teacher knowledge about the way students understand the concept of a function and the generation of teaching dilemmas in his/her teaching practice (Llinares, 2000).

Several interviews, along with recordings of classes, and several tasks were completed by the teachers. The tasks included classifying textbook problems, analyzing problems, and analyzing hypothetical problems.

The study supports the results of previous studies that suggest that pedagogical content knowledge is the integration of different components: knowledge of mathematics, knowledge of modes of representation, and knowledge about students (Ball, 2000; Fennema & Franke, 1992; Llinares, 2000). The study also highlighted the central role played by teacher knowledge about student understanding of a function. Furthermore, it illustrated that teachers' pedagogical content knowledge determines the particular content they teach (Llinares, 2000).

Another study of teacher knowledge, specific to algebra, involved the collaboration of a seventh grade teacher and a university faculty member to develop students' mathematical thinking (Pape, Bell, & Yetkin, 2003). The study was conducted at a middle school in a down town area of a medium sized city in the U. S. It was typical of an urban school district where the population consisted of 58% Black students, 41% White, and 1% Hispanic, American Indian, or Asian students (Pape, Bell, & Yetkin, 2003). One of the classes was a seventh grade pre-algebra class. The students had above average mathematical experience and ability and were studying Pre-Algebra as to take Algebra I in the eighth grade. The other class in the study was a seventh grade class engaged in the regular seventh grade mathematics curriculum (Pape, Bell, & Yetkin 2003).

The year long teaching experiment led to the emergence of several principles that are critical to student learning. One of these features of the classroom instruction that relates to this learning is the use of multiple representations and rich mathematical tasks (Pape, Bell, & Yetkin, 2003).

Conclusion. It is important for teachers to not only know the mathematics content, but also to be able to teach the content in a way that engages and challenges the students intellectually. In effective teaching, the content must be meaningful and worthwhile. Teachers should provide the students with rich mathematical tasks that develop students' mathematical understandings and skills, stimulate students to use reasoning and problem solving skills.

Discourse

The second component of my model of effective teaching is discourse. Discourse refers to the ways students learn through representing, thinking, talking, and agreeing and disagreeing about mathematics (NCTM, 1991).

Describing effective discourse. The student's role in discourse is to listen, respond, and question the teacher and each another. He/she should be able to reason, make connections, solve problems and communicate. The student should initiate problems and questions, and defend his/her position on these problems by presenting his/her solutions, exploring examples and counter-examples, and relying on mathematical evidence to convince himself and others (NCTM, 1991). Opportunities to make conjectures, explain, and defend, can extend skill, knowledge, and understanding. Discourse-oriented teaching is designed so that the student will discuss mathematics and, in turn, focus on the meaning mathematics has for him, which will result in the mathematics making sense.

Discourse plays a critical role in promoting the kind of teaching and learning that is valued in the mathematics classroom today. The studies aforementioned in the beginning of this chapter reiterate the importance of the students planning and regulating their own thought processes, and explaining and justifying their strategies to themselves and others (Boaler, 1998; Doerr & English, 2006; Carpenter, Fennema, & Franke, 1996; Seymour & Lehrer, 2006; Silva et al., 1990; Silver & Stein, 1996; Weiss et al., 2003). The teachers as well as the students are respectful to each others thoughts. One particular example that stood out to an observer in the Weiss et al. (2003) study was when a student offered a solution that was slightly off and confusing to others in the class. The teacher offered "right idea, let's clean it up a bit." The class remained supportive and offered ideas rather than totally dismissing the idea. The classroom was an environment of mutual respect in which the students assisted and benefited from their colleagues. The rigor of the lesson was high and the questions the teacher asked helped the students really think about the mathematics (Weiss et al., 2003).

McNair (2000) agreed that good mathematics classroom discussions provide an opportunity for ideas to be shared and developed, but stated that not all mathematical discussions produce these opportunities. The subject of a mathematical lesson may determine what the discussion is about, but it does not explain why the students are having the discussion. It is suggested that if students stop short of discussing the procedures used to solve their problems, they might not learn the mathematics that is contained in those procedures. In short, classroom discussions that maximize student learning must have a mathematical subject, a purpose of adding structure and understanding in reasoning, and the students must use mathematics to accomplish mathematical goals (McNair, 2000).

The teacher's role in discourse. The teacher plays the major role in orchestrating classroom discourse. The teacher must establish a classroom from a teacher focused setting to one that is centered on student thinking and reasoning. This requires pre-planning on the teacher's part. In order for this to happen, teachers must create a classroom atmosphere of mutual respect and trust. The students should feel comfortable in critiquing their work and others. Also, teachers should select activities that provide for taking different positions on the issue and then encourage the students to defend their position with mathematical evidence (Stein, 2001). Stein (2001) and several colleagues analyzed and explained the teacher success by the teacher by:

> recruiting attention and participation from the class and by aligning students with positions through rephrasing their contributions; highlighting their positions through repetition; and pointing out implicit but important aspects of their explanations through expansion. (p. 112)

Effective teachers develop practices that are tuned to mathematical interpretations by the students, but not mere repetition of student responses (Forman & Ansell, 2002). Some teachers implement discourse attempts by repeating portions of what students say or by expanding on what students say. These tactics are known as *revoicing* and can include shaping everyday conversation into mathematical argument and can support student identity (Forman & Ansell, 2002; O'Conner & Michaels, 1996; Strom et al., 2001). Classrooms promote understanding by assessing how closely the student is participating in mathematical discourse, not just repeating the correct answer (Lampert, 2001). Effective teachers understand mathematics' central conceptual structures and forms of argument (Yackel & Cobb, 1996) and effective teachers also develop the practices of mathematics and support student's participations in these practices (Lampert, 2001). Yet, as shown in the research done by Seymour and Lehrer (2006) creating these classrooms is a complicated matter.

One teacher who was involved in the QUASAR project was dedicated and energetic about learning mathematics in a new way. She was devoted to helping her students learn mathematics by sharing their ideas. The researchers found the teacher "consistently describing teaching and learning in terms of students producing their own knowledge.… Through class discussions, the teacher saw herself and her students building their own understanding of mathematics" (Williams & Baxter, 1996, p. 29). The teacher was engaged in discourse-oriented teaching. She gave rewards for student questions and asked effective questions of her own to spark discussion. In small groups, they were encouraged to share ideas, agree and disagree about problem solving strategies, and explain their thinking to one another. Finally, the whole group would come back together for presentations and discussion.

One of the best methods used for discourse-oriented teaching is to ask the students directly to explain their thoughts (Williams & Baxter, 1996). The idea is to get the students to know why talking about mathematical ideas is important. The following vignette shows the importance of this concept. The students had measured the area of a region and decided the area was 17.5 square centimeters. The teacher asked them what the area would be in square millimeters. The two solutions offered were 175 square millimeters and 1750 square millimeters. The teacher called on a student to go to the board and explain his answer.

> *Larry*: 17.5 square centimeters. And in millimeters that's 17.5 times 10 or 175.
>
> *Teacher:* Why did you multiply by 10?
>
> *Larry*: For every centimeter there's 10 millimeters.
>
> *Teacher*: Did everybody get that? If you got something else your hand should be up to ask a question.
>
> *Eva*: How did you get that 10?
>
> *Larry*: For every centimeter there's 10 millimeters
>
> *Eva:* The dimensions [of a square centimeter] in millimeters are 10 by 10. So that makes every centimeter worth a 100 square millimeters right? So if you add them all up, each centimeter being 100, you get 1750 millimeters
>
> *Tom*: You move the decimal over one. So it's 175.
>
> *Naomi*: This is like one little square (showing grid on projector of 100 square-centimeter square and a centimeter square). One centimeter by 1 centimeter. If you blew this up to get millimeters.… This is 1 centimeter by 1 centimeter, then 10 millimeters by 10 millimeters, so 100 millimeters are in here. So is this blown up. It's 100 times bigger.… So if 17.5 centimeters, then 1750 millimeters squared. (Williams & Baxter, 1996)

Although Tom was viewed as the brightest student in the class, he did not like to explain things. Both girls challenged his thinking and made their own convincing arguments. Each constructed an understanding that made sense to her. The "rule" of moving the decimal place over did not convince them. The teacher went on to talk about the area and ask the questions needed to lead Tom to see that area moves the decimal one place for length of one side and one place for the other side, so you move the decimal two places (Williams & Baxter, 1996).

A study done by Sherin (2002) addressed the tensions that arose from creating and maintaining a classroom environment of doing and talking about mathematics. The tension arises from teachers having to encourage students to share mathematical ideas, but at the same time, make sure these discussions are mathematically productive (Sherin, 2002). The purpose of the study was to characterize how the tension played out in one teacher's classroom. The study contrasted the teacher's focus on the *process* of mathematical discourse with the *content* of mathematical discourse. Process refers to the way the teacher and students participate in classroom discussion which involves questions and comments and through what means the class agrees. The content refers to the mathematical substance of the comments, questions, and responses that arise (Sherin, 2002).

The teacher in this study taught middle school mathematics in an upper middle suburban school. He had planned to focus on developing a "mathematical discourse community". He imagined his classroom in which students were excited about sharing their thoughts and ideas with their classmates and where they could agree and disagree with each other's ideas. Data was collected through classroom observations and videotapes, field notes, a written journal, kept by the teacher, and interviews (Sherin, 2002). Questions that arose after a preliminary analysis included: What happened in the lessons in which the teachers focused on both process and content? Was the teacher able to use the students' ideas to discuss the key mathematical concept in the lesson? And what affected whether and how this was achieved? Why did the teacher focus sometimes on either process or content, but not both? And how did those lessons play out in class (Sherin, 2002)?

Initially, the teacher's interest was in process. For example, on the first day of class the students were explaining and comparing ideas, but the ideas were not mathematical in nature. When the class came together the teacher focused on discussing working as a group rather than the different shapes the students had made and why (Sherin, 2002). The teacher was explicit in his journal by stating "we didn't discuss too much mathematics today." After a few weeks, the teacher began to prompt the students to talk about the mathematics, and the students responded accordingly as seen in the excerpt explaining their group's method to determining for estimating the number of dots placed randomly in a 9 x14 cm rectangle (adapted from Lappan, Fey, Fitzgerald, Frile & Phillips, 1997):

> Student 1: We divided it up by one centimeter by one centimeter … and then we'd have 126 little squares. So we counted the dots in one of the little squares and there'd be about 17 little dots in there. So then we multiplied 17 by 126.
>
> Teacher: Okay. What do people think about this group's method?
>
> Student 2: I think it's a good idea but bigger squares would have been more accurate.
>
> Teacher: Why do you say that?
>
> Student 2: Because … there may be a bunch of dots packed into a small area. In just that particular area. Or, there might be not a lot of dots.

Student 3: I agree … because there are not the same amount of dots in the same place.

Teacher: And why would that make a difference?

Throughout the course of the study, the teacher shifted from a balance of process and content to having some lessons high on content and low on process to some that were high on process and low on content. This continual shift suggested that discourse poses a problem for the learning of content. On one hand, students are expected to learn specific content, but they are also supposed to share ideas that direct the discussion (Sherin, 2002).

One way of supporting both the process and the content is an approach called *filtering.* In this approach, multiple ideas are solicited from the students. Students are encouraged to elaborate their ideas and then compare and evaluate their ideas with others. The teacher then focuses on the subset of ideas the students have raised or may introduce new ideas to consider the focused content (Sherin, 2002). In terms of process, students have the opportunity to share ideas and thinking. Yet at the same time, the teacher has some control over the mathematical direction of the lesson. The teacher is able to orchestrate discussions that are based on student ideas and that are also mathematically productive and worthwhile (Sherin, 2002).

Students need to be encouraged to take responsibility for their own thinking and listening carefully to others. This requires a classroom where a teacher’s rapport with her students and students’ rapport with each other is exceptional. Discussions should no longer focus on “What is the answer?” but “why?” and “how do you know?” Teachers have to step up to the challenge and then guide their students to do the same as to become life-long mathematical thinkers.

Conclusion. Teachers establish the environment in their classrooms. If students are encouraged to think about mathematics and explore their thought processes through questions and discussion, the teacher is responsible for creating a classroom where students are expected to elaborate their ideas and then compare and evaluate their ideas with others. Students need to be encouraged to take responsibility for their own thinking and listening carefully to others.

Equity

Equity is the third component in my model of effective teaching. Effective lessons have to ensure that *all* students are learning important mathematical content (Weiss et al., 2003).

Describing effective equity. Creating an equitable mathematics-learning environment is a growing concern, and The No Child Left Behind Act (U. S. Department of Education, 2005) does not discriminate when it comes to who is expected to achieve. Equity does not mean mere equality. It means entitling all students to a quality education, taking into consideration the diversity around us, and accounting for those differences. One of the biggest obstacles in education is meeting the needs of all students. QUASAR and the Algebra Project support the proposition that low performance in mathematics is most likely due to lack of providing high-quality mathematics learning opportunities (Silver & Stein, 1996; Silva et al., 1990).

One of the Principles in the National Council of Teachers of Mathematics *Principles and Standards of School Mathematics* is the equity principle (NCTM, 2000).

The equity principle states “that all students regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study—and support to learn—mathematics (NCTM, 2000, p. 12). Teachers who practice pedagogy that is equitable have a vast understanding of both pedagogy and cultural experiences that enable them to teach effectively (Banks & Banks, 1995). These messages rely on two notions. One, teachers should provide all students with the highest opportunity to successfully learn mathematics. Second, teachers should understand the strengths and needs of their students from diverse backgrounds and ethnicities (Banks & Banks, 1995). Students have to engage with learning tasks. One observer of a seventh grade mathematics classroom in the Weiss et al. (2003) study noted that the teacher “made no adjustments in instruction to accommodate the diverse needs of his students.” The teacher had described the students in the class as having a variety of ability levels with some retaining information very quickly and others having a low level of attainment. The lesson was designed as a “one size fits all” (Weiss et al., 2003). This shows how equity was not demonstrated.

Another type of equitable practice is culturally relevant teaching Culturally relevant teaching necessitates that students sustain academic excellence in addition to their own cultural integrity. This type of teaching was developed in hopes of allowing the students to “make sense” of the mathematics around them by appealing to what they identify to. “Equity pedagogy creates an environment in which students can acquire, interrogate, and produce knowledge and envision new possibilities for the use of that knowledge” (Banks & Banks, 2001). Students “who are empowered by their interactions with educators experience a sense of control over their own lives and they develop the ability, confidence, and motivation to succeed academically (Tate, 2005, p. 4).

The teacher's role in equity. Equity pedagogy requires teachers to facilitate the learning process (Means & Knapp, 1991). Students generate knowledge and create new understanding rather than rote memorization. Students are encouraged to look for multiple solutions rather than a single answer. They are encouraged to problem solve in contextualized topics (Means & Knapp, 1991). Effective teaching of equity pedagogy requires the teacher to have content knowledge, pedagogical knowledge, and multicultural knowledge (Means & Knapp, 1991).

The "pedagogy of poverty" is a term used by Haberman (1991) to describe a mindset of what teachers, students, parents, and the community assumes teaching is. Students in the pedagogy of poverty achieve neither minimum levels of life skills nor what they are capable of learning and teachers burn out because of the emotional and physical energy that they must expend to maintain their authority every hour of every day (Haberman, 1991). "Essentially it is the pedagogy in which learners can succeed without becoming either involved or thoughtful" (Haberman, 1991, p. 308). Unfortunately, this is what is going on in our classrooms today, and those who are living in poverty are at even more of a disadvantage. Lack of resources, such as a rich curriculum and qualified teachers as well as low expectations stunt any opportunities for students to become actively engaged in learning and disabilities achievement.

Students come from a variety of racial, cultural, linguistic and economic backgrounds. Teachers must be willing and prepared to work with children from backgrounds other than their own. According to Banks (1992), "We need to create a school environment that is equitable and just, then, in our discussions and classrooms, honestly try to search for a balance of views, and present them as fairly as possible." Strategies to benefit students from all backgrounds include: helping students develop a relational understanding of concepts, helping students develop number sense, expressing a deep belief in the capabilities of the students, enabling students to use mathematics as a tool in society, and creating classrooms environments where students ideas and thoughts are expressed and valued (Strutchens, 2000).

Flores (2007) presents data to show differences among standardized test scores in students who differ in ethnicity and socioeconomic levels. Evidence is presented that demonstrates that the opportunity to learn is not accessible to all students. Furthermore, African American, Latino, and low income students are more likely to face low expectations and have less qualified and less experienced teachers. They are also unlikely to receive equitable per student funding (Flores, 2007). The statistics presented showed that African American and Latino twelfth graders perform at the same level as white eighth graders.

In order to address why such disparities exist Flores (2007) examined what students were actually experiencing in schools. One of the disparities pointed at the lack of access to qualified and experienced teachers. It was found that schools that served mostly minority schools were twice as likely to be taught by inexperienced teachers (Flores, 2007). Also, more out-of-field teachers, or teachers who do not have at least a minor in the subject they teach, teach in high poverty schools.

Another disparity was found in the learning expectations of African American and Latino students. Students in poor schools receive A's for work that would earn C's in a more affluent school (Flores, 2007). The findings also showed that only about a fourth of Latino and African American high school graduates were enrolled in college track courses. Less than half of the Latinos and African American high school students had taken prealgebra or algebra in the 8th grade compared to 68% of European Americans (Flores, 2007). Funding in high poverty areas also shows inequities of opportunity. There are huge differences in average teacher salaries from one school to another. Also, in some cases, the per student spending in a low-minority district is twice as much as in districts with large numbers of African American and Latino students (Flores, 2007).

The solution is framed as what is known as the opportunity to learn. Opportunity-to -learn (OTL) as used in this context was defined by Carroll as the “amount of time allocated to the learner for the learning of a specific task” (Tate, 2005). “Qualified teachers who are committed to the learning of their students are the single most important factor for student success” (Flores, 2007, p. 38). Teachers should have high expectations for all their students and offer sustained support for these students. Students need the opportunity with committed and qualified mathematics teachers. Students need challenging mathematical content and high level instruction that focuses on sense making and problem solving. Students need to develop empowerment (Flores, 2007).

Studies show that disadvantaged students get less instruction in higher-order skills than the advantaged students (Means & Knapp, 1991). Tate (1995) gave an example of this based on a fourth grade teacher in a large urban school. The teacher had a class of “slow learners”. She drilled the basics, gave short lectures, and assigned worksheets and textbook problems. The students were disengaged and had no focus. Not surprisingly, students were expected to not achieve, were not given opportunities to learn (Tate, 1995).

A summary of critiques showed that teachers teaching at-risk students tend to 1) underestimate what students are capable of doing; 2) postpone more challenging and interesting work for too long-in some cases, forever; 3) deprive students of a meaningful or motivating context for learning or for employing the skills that are taught (Means & Knapp, 1991). Instead of problem solving through investigation, formulating questions, and verbally, numerically, or graphically representing situations, students solved routine, well-defined problems. Instead of communicating multiple solution methods and effective discourse, students answered yes or no questions. Instead of using reasoning skills, students relied on the textbook or the teacher. Instead of connecting mathematics with real-world situations and developing mathematical literacy, students memorize rules and learn skills out of context (Tate, 1995). “Once we say that some children are not capable thinkers or problem solvers, we have all but, guaranteed that they will not be so. If, instead, we recognize that students gain confidence in themselves through experience with problem solving, we will be surprised by what our ‘low achievers’ accomplish” (Robert, 2002, p. 294).

In more recent years, educators have taken Carroll’s model and designed their own framework for OTL (Tate, 2005). In his monograph, he identified three variables that form the OTL framework: 1) content exposure and coverage variables 2) content emphasis variables and 3) quality of instructional delivery variables (Tate, 2005).

Content exposure and coverage variables measure two things. First, they measure the amount of time students spend on a topic and the richness of the instruction provided for that topic. And secondly, these variables measure whether or not important and correct content is covered for a specific grade or discipline (Tate, 2005). A significant amount of time should be allotted for the learning of mathematics and sufficient time for developing and understanding key concepts and procedures (Tate, 2005). One key influence in mathematics achievement is course-taking. Studies show that black, Hispanic, and low SES students are less likely to be enrolled in high-level mathematics courses than middle class White students (Tate, 2005).

Studies done in the past show that school tracking practices have created mathematics programs that restrict the educational opportunities and outcomes of certain children (Oakes, 1985). Oakes used the responses of teachers from six racially mixed high schools in an attempt to create alternatives to traditional ability grouping and to increase student achievement and create equity. The overall concept was, instead of separating the students identified as lower achieving or not college-bound into “low track” classes with a low level of curriculum, give all students high quality opportunities to learn and increase the vigor of mathematics teaching and learning (Oakes, 1985). Although this study showed that all were not in favor of “detracking”, those who were most likely supported reform practices. Most of all, detracking requires that teachers rethink and challenge the existing notions of who can and who cannot be successful in mathematics. The goal is to bridge mathematics teaching and learning and equity (Oakes, 1985).

The second variable in Tate’s monograph is described as the content emphasis variable. These variables affect what topics are selected within the implemented curriculum and what students are selected for higher order skills instruction (Tate, 2005). This can be taken back to tracking, and also to the “educationally disadvantaged.” Classroom studies show that disadvantaged students get less instruction in higher-order skills than their advantaged peers (Means & Knapp, 1991). Approaches to teaching at-risk students tend to: 1) underestimate what students are capable of doing, 2) postpone or never get to challenging or interesting activities, and 3) deprive students of meaningful or worthwhile tasks for learning (Means & Knapp, 1991). Instead of taking a deficit view, a new set of curricular principles should be provided to all students to focus on complex, meaningful problems, have embedded basic skills instruction in the context of more real-world tasks, and make more connections with students' out-of-school experience and cultures. Supporting this curriculum, teaching methods should 1) model powerful thinking strategies, 2) encourage multiple approaches to academic tasks, 3) provide scaffolding to enable students to accomplish complex tasks, and 4) make dialogue the central medium for teaching and learning (Means & Knapp, 1991).

The last variable Tate identifies is the quality of instructions delivery variables. This reveals how classroom pedagogical strategies affect students' academic achievement. Teachers orchestrate the classroom environment. Banks and Banks (1995) define equity pedagogy as teaching strategies and classroom environments " that help students from diverse racial, ethnic, and cultural groups attain the knowledge, skills, and attitudes needed to function effectively within, and help create, and perpetuate, a just, humane, and democratic society" (p. 152). In a recent study, Gutierrez (2007) addressed *context* as key for attending to equity. *Context* is a way of moving away from thinking that mathematics teaching is reduced to a list of basic skills and strategies, and focuses on the fact that learning is interwoven in the contexts in which it occurs (Gutierrez, 2007).

Gutierrez (2007) presents the definition of equity along four dimensions: 1) access, 2) achievement, 3) identity, and 4) power. He then concludes by exploring how teaching and learning contexts might play a role in future research on equity. Equity in this study is defined as fairness or “the inability to predict mathematics achievement and participation based solely on student characteristics such as race, class, ethnicity, sex, beliefs, and proficiency in the dominant language” (Gutierrez, 2002, p.153).

- *Access* relates to resources that students have available to them. For example, high quality teachers, technology and supplies in the classroom, a curriculum that is not watered down, and a classroom environment that is conducive to learning (Gutierrez, 2007).
- *Achievement* is measured by student results at all levels of mathematics. This involves participation in class as well as standardized test scores, and course taking (Gutierrez, 2007).
- *Identity* is identified as more than one’s culture. Students need to have opportunities to identify themselves within their curriculum as well as the real world. Students should be able to “be themselves and better themselves” (Gutierrez, 2007, p. 4).
- *Power* is the fourth piece of equity and encompasses many levels of social transformation. Power can be measured by who decides the curriculum, who speaks up in the classroom, and opportunities for students to use math in society (Gutierrez, 2007).

Gutierrez (2007) looked at nine US. Schools serving mostly Latino and African American, and/or working class students. Four of the schools had clear gains in student success, four had little or no signs of gains in students success, and one school was chosen to represent somewhere in between. Gutierrez used the entire mathematics department as the contextual frame in the four schools that was excelling in mathematics. There are several components that distinguished the effective mathematic departments from the ineffective ones. They had a rigorous curriculum and pushed students towards higher level courses. The four schools also had a strong commitment to students and innovative instructional practices. These instructional practices moved beyond worksheets and basic skills by offering projects that were relevant in the students' lives. Technology was prominent. They were a collective enterprise and regarded themselves as a community of practice, learning within and from colleagues (Gutierrez, 2007). From this aspect, Gutierrez (2007) pointed out that 3 of the 4 dimensions of equity were highlighted: access, achievement, and identity.

The next part of this study focused not only what the nature of a successful mathematic department is, but also how was this effective community created and sustained. This particular math department served over 80% Latino and African American students, 98% qualifying for free lunch. These students took more than the required math courses while in high school with a large number in calculus. The teachers supported the students and collaborated with one another. This department moved beyond access and achievement to include issues of identity and power.

The focus of the study shifted to teacher education. She wanted to know how to develop individual's knowledge to teach effective mathematics. These teachers were mostly white middle class females, strong in content knowledge. By the end of the study, teachers had begun to see issues of identity and power. They realized that teaching involves giving students opportunities to see themselves in the curriculum and analyze the world around them. The study revealed the importance of successful contexts, not only for students but also teachers. Teachers need to see outcomes for students in ways that address their identity and power in society.

Conclusion. All students can learn when given the opportunity. Teachers must realize that all students should have access to high quality instruction. If students are not expected to learn, they won't. Teachers should have high expectations for all their students and offer sustained support for these students. Students need challenging mathematical content and high level instruction that focuses on sense making and problem solving (Hiebert & Grouws, 1996; Gutierrez, 2007; Flores, 2007; Tate, 2005).

Connections

The final component in my model of effective teaching is connections. Connections will be defined here as making sense of mathematics in contexts within the discipline itself *and* contexts outside of mathematics, particularly the real-world.

Describing effective connections. The connection standard describes how "students should recognize and use connections among mathematical ideas; understand how mathematical ideas interconnect and build on each other to produce a coherent whole; recognize and apply mathematics in contexts outside of mathematics" (NCTM, 2000, p. 354). Student success is the ability to problem solve by building on prior mathematical understanding. They should be able to connect mathematics to their lives in and outside the classroom.

If students are verbalizing "when will I ever use this again," chances are it means nothing to them nor do they intend to ever understand it. Because everyday living requires mathematical thinking and problem solving, not just procedural knowledge, but true understanding, this education should be available to all students whether they intend to further their education or simply need to make everyday purchasing decisions. In the Boaler (1998) study, students in the traditional school spoke very strongly of their inability to connect school mathematics with anything done outside of the classroom. Students at the other school reported no real difference in the mathematics needed inside the classroom and the mathematics needed outside of school (Boaler, 1998). When asked if he/she would be able to use what you're learning now or do you think you will make something up, the student replied "No, I think I'll remember. When I'm out of school now, I can connect back to what I done in class so I know what I'm doing" (Boaler, 1998, p. 58).

The mathematics must also make sense. An observer in the Weiss et al. (2003) study of a sixth grade class wrote the teacher did not seem to care if the "big ideas" made sense to the students or not. In another mathematics class, the students were not encouraged to make sense, just follow directions. They were not asked to explain anything—mathematics was simply a set of rules and procedures (Weiss et al., 2003). Silver & Stein (1996) reiterated that "in mathematics…, [meaningful learning] involves the use of tasks that require students to construct meaning and/or relate important mathematical concepts to symbols, rules, and procedures" (p. 481).

The teacher's role in connections. Teachers can help students make a connection within the mathematical context, by emphasizing connections rather than presenting mathematics as a set of disconnected, isolated concepts and skills (NCTM, 2000). Problem selection is important because students are unlikely to make connections unless the problems have the potential for such connections. The following is an example of a teacher's inadequate use of sense making in the classroom.

A teacher in an Algebra I classroom asked a student to put the equation $6x+7=-14y$ in standard form on the board. The student explained that she subtracted $6x$ from both sides and ended up with $-6x-14y=7$. The other students seemed confused and ask the teacher if it was right. The teacher agreed that it was, then solved it a different way by first moving the *y* term and ending up with $6x+14y=-7$. The teacher concluded "so you can have two different answers", never mentioning that the two answers are mathematically equivalent (Weiss et al., 2003).

Rich problems, on the other hand, can allow for multiple approaches and solutions and encourage students to reflect on and compare their solutions as a means of making connections (NCTM, 2000). The following is a hypothetical example from *Principals and Standards of School Mathematics* (NCTM, 2000) that highlights connections within the mathematics and the real-world. The teacher starts class with the story:

> I have a dilemma. As you may know, I have a faithful dog and a yard shaped like a right triangle. When I go away for short periods of time, I want Fido to guard the yard. Because I don't want him to get loose, I want to put him on a leash and secure the lease somewhere on the lot. I want to use the shortest lease possible, but wherever I secure the lease, I need to make sure the dog can reach every corner of the lot. Where should I secure the leash? (NCTM, 2000, p. 354)

This problem supports a number of interesting explorations and exploration of the properties of triangles and circles as well. The teacher encouraged the students to follow various leads which led to new ideas and connections (NCTM, 2000). This approach is aligned with the teaching philosophy that teachers should encounter a need to use mathematics in situations that are realistic and meaningful to them (Boaler, 1998).

One study looked at communication as a means of building mathematical connections (Uptegrove & Maher, 2005). The researchers' view is that the process of communication ideas and providing support for those ideas lead to making suitable connections between problems of equivalent structure (Uptegrove & Maher, 2005). Four high school students explored the addition rule for Pascal's triangle and how it relates to two combinatorics problems. One was the pizza problem which asks how many pizzas is it possible to make when there are n toppings to choose from? The second problem known as the towers problem asks how many towers n cubes tall can be built from unifix cubes when choosing from two colors (Uptegrove & Maher, 2005)? The students had explored both the pizza problem and the towers problem in depth. They had observed that the solutions to both problems could be found in Pascal's Triangle. They were also aware of the relationship between Pascal's Triangle and the binomial coefficients (Uptegrove & Maher, 2005).

The students looked at the relationship between Pascal's Triangle, binomial coefficients, and the pizza and towers problems. They worked together to form connections between mathematical ideas through discourse. Discourse in this study is defined as arguing, asking questions, and anticipating feedback (Uptegrove & Maher, 2005). Episodes conducted in the study were: 1) connecting the towers problem to the binomial expansion and Pascal's Triangle, 2) connecting the pizza problem and Pascal's Triangle, and 3) connecting the towers problem and the pizza problem. The researchers believe that all four students participated in the first episode and could describe how the answers to the pizza problem could be found in Pascal's Triangle. In the last episode, they described the connection to the two problems by building on earlier discussions. The researchers concluded that through communicating their ideas and supporting these ideas, the students were able to make connections between problems of equivalent structure and to build their understanding of that isomorphic relationship (Uptegrove & Maher, 2005).

Another study analyzed a teacher's efforts at problem solving in groups and her ability to see and make connections in the problems (Davis & McGowen, 2001). The goal was to

> change what students value in mathematics … from severely procedural orientation to mathematics focused on 'correct answers' that prospective teachers have learned to value above all. How can we explicitly emphasize connections, and assist students to construct relationships between parts of mathematics that they see as different. (Davis & McGowen, 2001, p. 1)

The study was conducted with a class of pre-service elementary teachers. The focus of the course was on making connections between different combinatorial problems and on multiple ways of interpreting answers. The study was built on the theory that there are several distinctions in memory for mathematical facts: 1) memories of labels, customs, and conventions — prime numbers are whole numbers with exacting 2 factors, 2) factual memories of things done-the proportion of prime numbers less that 500 is 19%, 3) Memories of things believed — there are infinitely prime numbers, 4) memories of explanations — a proof that there are infinitely prime numbers (Davis & McGowen, 2001). Students worked in groups on the problems and then explained the connections in their homework. Opportunities were also given throughout the course to make connections with their earlier work.

One of the students in the class did not immediately see the connections but through class discussions and insight from others, she was able to use patterns and her own insights to solve two further problems. She articulates the value of seeing connections:

> When I joined this class in August, I thought of math as a series of formulas, each of which should be followed in order to find an answer. It was working on the tower building investigation and traveling through tunnels that I discovered how each relates.… My original approach to the tower building revealed that instead of looking at the small picture (i.e., What do I do with what I have in front of me? What is it I'm trying to solve?), I just dove in expecting multiple problems. When our class finally concluded that the towers, tunnels, grids, and Pascal's Triangle were all about 'choices', everything seemed to fall into place … my perspective of mathematics changed over this semester. The changes occurred due to learning that my mathematical understanding was instrumental and not relational. I had to relearn basic math in order to eventually teach it to children (Davis & McGowen, 2001, p. 7).

Another study focused on the importance of mathematical connections by investigating the mathematical connections students form and use on non-routine problems (Shroeder, 1993). Tenth grade students were interviewed while solving either an algebra problem involving internal connections or a geometry problem using external connections. *Internal connections* in this study are defined as connections across mathematical topics and *external connections* are defined as connections between mathematics and its application in other fields or in the real world (Shroeder, 1993). The first problem was presented.

> A textbook is opened at random. The product of the numbers of the facing pages is 3192. To what pages is the book open?

The students were given a copy of the problem, paper, and a scientific calculator. The anticipated approaches to the solution were guess and check, factorization, an algebraic method, or a method using square roots. The students were interviewed individually as they worked the problem. 50% of the students solved the problem on their own, and 50% solved the problem with help from the interviewer. The total length in time for each interview ranged from 12 to 45 minutes and the time it took to solve the problem ranged from 2 to 32 minutes. One student having found the solution by guess and check was asked if she could solve it another way. She had used 2500 and 3600 to establish a search interval. The interviewer asked “could you do the same with 3192”, in which the student proceeded to say “yes…I should be able to extract 56 x 57”, but shortly afterward stated” I don’t know why I’m doing this” (Shroeder, 1993).

In the second problem, the task was presented orally by the interviewer who explained the problem with photographs. Students were told that a waterfront restaurant has a dock, one part that rises and falls with the tide. Access from the fixed part of the dock to the floating part is a ramp that is steep at low tide, but less steep at high tide. The owners of the dock wish to mount wheels on the lower end of the dock to prevent any damage to the surface because the ramp has been causing damage when it rises and falls. The problem is to determine how long the track needs to be. Data provided are that 1) the ramp is 18m long, 2) when the tide is at its highest, the floating dock is 1 m below the fixed dock, and 3) when the tide is at its lowest, the floating dock is 6 m below the fixed dock (Shroeder, 1993). It was anticipated that the students would use the Pythagorean Theorem to determine the unknown sides then subtract. 33% of the students solved the problem on their own and 67% solved it with help from the interviewer (Shroeder, 1993). Time spent in the interview and solving the problem ranged from 22 to 55 minutes and 9 to 50 minutes, respectively. This suggests that the students didn't need direction about what to do, but encouragement and help thinking about their plans for proceeding. Overall, the study seems to imply that many mathematical connections are not obvious to most students, even when given substantial hints (Shroeder, 1993) and substantial amounts of time are needed to ponder them (Shroeder, 1993).

Another study was conducted to show the importance of mathematical connections. Pre-service teachers at Mississippi State University developed lesson plans by using children's literature to deepen secondary students' mathematical understanding and make important mathematical connections. It was the belief of these teachers that secondary students are not routinely shown the importance of the mathematics they are learning in the context of the real world (Pomykal & Pope, 2005). Therefore these students have difficulty applying and understanding mathematics in real world situations (NCTM, 2000). One book that can be used to make mathematical connections is *The Water Hole* (Base, 2001). The story in the book is as the number of animals coming to a water hole increases, the water supply decreases. *The Water Hole* can be used for teaching indirect and direct variations and functions (Pomykal & Pope, 2005). The book demonstrates a real world scenario of inverse variation. The Water Hole can also be used to integrate math to science and social science by expanding on research of animals presented in the book as well as integrated research projects between water conservation, water consumption and the water cycle (Pomykal & Pope, 2005).

Conclusion. In order for students to "recognize and use connections among mathematical ideas; understand how mathematical ideas interconnect and build on each other to produce a coherent whole; recognize and apply mathematics in contexts outside of mathematics" (NCTM, 2000), teachers must present mathematical ideas in a way that enables students to make these connections. Teachers should provide mathematical opportunities for the students to encounter a need to use mathematics in situations that are realistic and meaningful to them.

Conclusion

Any mathematics classroom, in a sense, is a community of practice. Classroom communities differ, however, in the kinds of teaching and learning practices that become accepted by teachers and students (Boaler, 1998). Classrooms that use a more traditional approach are usually led by the textbook instruction. The teacher demonstrates mathematical procedures and fosters learning mathematics by memorization and repeated practice. The student participation involves listening, watching, and perhaps demonstrating what the teacher has done, but mathematical understanding is limited to "rules without reason" or as Skemp (1978) defined, *instrumental understanding.* In contrast, classrooms with more open approaches value discussion and collaboration in an intellectual environment. Students are expected to propose and defend mathematical ideas, without relying on the teacher as the dominant giver of knowledge (Bishop, 1988;

Brophy & Good, 1970; Forman, Minick, & Stone, 2003). In these classrooms, students develop *relational understanding*, knowing both what to do and why (Skemp, 1978).

Quality teachers possess subject knowledge, but they must know how to get the students to use it and understand it. “Students’ understanding of mathematics, their ability to use it to solve problems, and their confidence in, and disposition toward, mathematics are all shaped by the teaching they encounter in school. The improvement of mathematics education for all students requires effective mathematics teaching in all classrooms” (NCTM, 2000, pp. 16–17). A more recent study done by Boaler (2008) demonstrated that teachers do play a role in higher achievement. The teachers at Railside High worked collaboratively to plan and design curricula, discussing decisions and actions. Students were asked to represent their ideas in different ways, using words, graphs, tables, and symbols. Connections between the mathematics courses were also emphasized. This demand for high levels of mathematical work led to students not only achieving at higher levels, but differences in attainment between student from diverse ethnicities were reduced in all cases and disappeared in some (Boaler, 2008).

A vision of effective instruction can be characterized by emphasizing the need for active learning by the students with meaningful content; creating a learning environment that is conducive to learning by promoting discourse; making mathematical understanding accessible to *all* students; and having mathematics “make sense” to the learner (NCTM, 2000). By drawing from several comprehensive studies, a framework for effective teaching was introduced. These four components were used to explore the roles of the teacher and the student in the mathematics classroom. Specifically, what was the role of the teacher in implementing content, discourse, equity, and connections in the

Algebra I classroom? Also addressed were how these roles affect the role of the student and student learning?

III. METHODOLOGY

This study was designed to gain an understanding of effective teaching and its impact on learning by providing a deep, detailed analysis of the role of the teacher inside the Algebra I classroom. This analysis attempts to provide insight into interactions in the data rather than making prior assumptions about the Algebra I classroom. This study examined the roles of the teacher and how they impacted student learning. In this study, qualitative research strategies were used in order to seek to understand the multiple interactions that emerge from the data by using thick description rather than testing a hypothesis.

In alignment with qualitative research strategies, this study attempted to understand what was going on in a real-world setting, with no attempt to manipulate, control, or alter it (Patton, 2001). More specifically, this study takes a qualitative approach to answer how the teacher implemented his or her role in the classroom and how this implementation affected student learning and achievement.

Qualitative research, broadly defined, means the kind of research that produces findings from real-world settings where the "phenomenon of interest unfold naturally" (Patton, 2001, p. 39). Qualitative research seeks to understand phenomena in a context-specific or real-world setting using a naturalistic approach where the researcher does not attempt to manipulate the phenomenon of interest (Patton, 2001). Qualitative researchers stress socially constructed issues. They seek answers to questions that stress *how* social experience is created and given meaning (Denzin & Lincoln, 2000).

Researchers select methods that provide educators with detailed information about educational practices (Suter, 2000). "Qualitative research" alone does not provide an indication of the perspective of the researcher. Schwandt (1989) stated

> Our constructions of the world, our values, and our ideas about how to inquire into those constructions, are mutually self-reinforcing. We conduct inquiry via a particular paradigm because it embodies assumptions about the world we believe and the values that we hold, and because we hold those assumptions and values we conduct inquiry according to the precepts of that paradigm. (p. 399)

Reliability and Validity

The terms reliability and validity cannot be viewed separately in qualitative research and have been replaced with a parallel concept of "trustworthiness" (Guba & Lincoln, 1981). Seale (1999) stated that the "trustworthiness of a research report lies at the heart of issues conventionally discussed as validity and reliability" (p. 266). Trustworthiness contains four aspects: credibility, transferability, dependability, and confirmability (Guba & Lincoln, 1981). Methods for ensuring credibility, transferability, dependability, and confirmability were considered during the design and implementation of this study and also during the analysis of this study. Each will be briefly discussed, but will be more detailed in the use for this study in their respective sections.

One way the credibility of this study was established is by the use of triangulation. Triangulation is defined as "a validity procedure where researchers search for convergence among multiple and different sources of information to form themes or categories in a study" (Creswell & Miller, 2000, p. 126). This use of multiple data-collection methods contributes to the trustworthiness of the data (Glesne & Peshkin, 1992). The data-gathering techniques used in this study were questionnaires, interviews, and observations and will be discussed in detail in the *instrument* section of this paper

Transferability refers to the generalizability of the results of the study. In other words, can the conclusions of this study be transferred to other contexts. Ways to ensure transferability are through the sample selection and the characteristics of the samples. This will be discussed in more detail in the section *subject selection*. Another way is by providing a "thick description" of the findings for the readers to assess the potential transferability appropriate to their own settings (Miles & Huberman, 1994).

An underlying issue in any research is whether the process of the study is consistent and reasonably stable over time. This is referred to as dependability. One way of ensuring dependability is through triangulation, as defined earlier. Another way of establishing dependability is through inter-rater reliability. This will be explained in the *analysis* section of this paper.

Confirmability can be defined as objectivity. Do the conclusions depend on "the subjects and conditions of the inquiry, rather than the inquirer" (Guba & Lincoln, 1981)? The sequence of how data were collected, processed, and transformed for drawing specific conclusions can be followed in the *analysis* section of this chapter. Any researcher bias will be discussed in the limitations of this study in Chapter 5.

Design of the Study

Because this study was designed to emphasize exploration rather than prescription or prediction, the researcher chose case studies to discover and address issues related to the research question. Case studies were chosen to allow for observations, questions and interaction with the research participants (Patton, 2001). In addition, case studies allow the researcher to begin with broad questions and narrow the focus of the study rather than attempt to predict every possible outcome. By seeking to understand as much as possible about a single subject or small group of subjects, case studies offer a thick description of what is happening in the study.

This study consisted of five individual case studies. Data collected on all the cases focused on the roles the teacher plays in the classroom by looking through the lenses of content, discourse, equity, and connections. Furthermore, the cases incorporated attention to how these roles interacted and how these roles affected student learning. Data was analyzed and used to develop a descriptive model that includes the factors related to the cases in the classroom (Bogdan & Biklen, 1998). In addition, by doing multiple case studies, the individual cases were able to be compared and contrasted. Multiple cases enhance transferability as defined previously, or the relevance of our findings to other similar settings (Miles & Huberman, 1994). A cross-case comparison deepens understanding and explanation.

Subject Selection

In order to contribute to the credibility of this study, how the samples were chosen and the characteristics of the samples will be described in the following sections in order to permit adequate comparisons with other samples (Miles & Huberman, 1994). Note that pseudonyms will be used throughout for autonomy.

Project Math

My initial thought was to select subjects for this study that would provide variability in teaching practices. Most of the surrounding school districts are currently participating in a systemic change initiative which I will call Project Math. However, the level of involvement and implementation varies across teachers and schools greatly, thus a variability in teaching practices might be expected.

Project Math is a partnership between two universities and fifteen school districts in East Alabama. Some characteristics of the vision of Project Math include meeting the needs of all students, engaging students in making sense of mathematics by taking a more inquiry based approach, and focusing on the usefulness of mathematics. The mission statement is "to enable all students to understand, utilize, communicate, and appreciate mathematics as a tool in everyday situations in order to become life-long learners and productive citizens" (Project Math, 2003). These goals will be met by aligning the curriculum K–12, ensuring consistency in teaching, providing professional development, and redesigning preparation of new teachers (Project Math, 2003).

The instructional philosophy of Project Math is in alignment with the instructional practices discussed in the review of literature. Project Math equips teachers to ask more questions, get students involved in exploring and making sense of mathematics, and engaging in meaningful mathematics (Project Math, 2003). The Project Math initiative is committed to ensuring that all students in the region receive an equitable mathematics education by participating in a rigorous curriculum that is taught by highly qualified teachers who use a variety of instructional practices designed to promote student learning and understanding (Project Math, 2003). Students need to know more than mathematical facts and procedures, they need to be able to apply their knowledge to solve problems in mathematics and in real life. Students need to understand not just how to do mathematics, but why it works. To accomplish this goal, Project Math seeks to expand the teachers' mathematical knowledge, as well as their range of instructional tools, so that they can increase the learning of all students. Rather than relying on "show and tell", teachers will help students to become more autonomous learners of mathematics through the use of engaging problems and innovative instructional practices, including reading and writing in the mathematics classroom and working collaboratively (Project Math, 2003).

The comprehensive professional development provided by Project Math begins with a two-week summer institute, introducing teachers to best practices based on research in mathematics education. The teachers then return for a one-week follow up training the following summer. Teachers also attend half-day follow-up meetings on four Saturdays throughout the year. Teachers are encouraged to complete a total of 160 hours of professional development during their involvement with Project Math (Project Math, 2003). Project Math provides professional development district-level and school-level teacher leaders who coordinate on-going collaborative planning and conduct workshops for the teachers in the schools. Teacher leaders are those who coordinate events at the schools or district and act as agents of change to both individual teachers and groups of teachers (Project Math, 2003). Project Math also developed a common curriculum guide based on a common selection of textbooks. This guide provides additional help for teachers to plan instruction, beyond the basic requirements set forth in the Alabama Course of Study (Project Math, 2003).

The School

The school was purposefully chosen because it was a part of Project Math., but as stated previously, the teachers were involved in Project Math at different levels. Also, all of the teachers were following the same curriculum, but it was expected that their implementation would vary. It was also convenient for me because the school was in a local suburban school district.

After deciding which school, I needed permission to conduct the research. After deciding on the school, the researcher needed permission to conduct the research. At the beginning of Spring Semester 2007, the principal was emailed to set up an initial meeting. After being given permission by the principal to work in the school, it was suggested that the teacher leader of the school be contacted. A teacher leader serves as a liaison to Project Math and is responsible for coordinating activities at their level.
The teacher leader subsequently arranged a meeting with all the subjects to be included in the study. An overview and purpose of the study was presented, and all the teachers of the second semester of Algebra I agreed to participate. An information letter was given to the principal and all of the teachers involved (see Appendix A).

The next step was to meet with the teachers individually, to answer any questions they might have and to schedule each observation. Each teacher signed a consent form agreeing to participate in the study. A copy of this consent form may be found in Appendix B.

This school's curricular implementation is based on the standards in the Alabama Course of Study which serves as a minimum requirement for the students. These standards are based on national standards and research based expectations for student learning. To ensure effective implementation, curriculum and pacing guides are aligned with the state course of study. The National Council of the Teachers of Mathematics Standards serves as a framework in the area of mathematics. A compilation of measurable assessment objectives from high stakes tests is also used to guide instruction (Eastside, 2007). Academic achievement levels define how well students are mastering the state's academic content standards at grade level. The results of the Alabama Reading and Mathematics Test (ARMT) are reported in four academic achievement levels: Level IV-Exceeds Academic Content Standards; Level III- Meets Academic Content Standards; Level II-Partially Meets Academic Content Standards; and Level I- Does Not Meet Academic Content Standards. The district reports that 83% of the 8^{th} graders at this school score at levels III and IV in mathematics on the ARMT (Alabama Department of Education, 2005).

The vision of this school district is to inspire all students to achieve their potential, educate all students to use and evaluate knowledge, and empower all students to be responsible, productive citizens. This school system reported having standards that help achieve this vision. One of the standards reported is:

> implementing a curriculum based on clear and measurable expectations for student learning that provides opportunities for all students to acquire requisite

> knowledge, skills, and attitudes. Teachers use proven instructional practices that actively engage students in the learning process. Teachers provide opportunities for students to apply their knowledge and skills to real world situations. Teachers give students feedback to improve their performance. (Eastside, 2007, p. 4.10)

The mathematics department meets regularly to discuss the curriculum and cover the same topics throughout the semester. This particular school utilizes the Project Math curriculum guide. The recommended Basal textbook, as well as the integrated curriculum are implemented. Demographics of the school in this study are shown in Table 1 (Eastside, 2007).

Table 1

School Demographics

Grades Serviced	Total Population	Student/ Teacher Ratio	Students Eligible for Free or Reduced-Price Lunch (entire school system)	Spending per student (entire school system)	Racial Background
8–9	848	19:1	27%	$9014	White–62% African American–29% Asian–6% Hispanic–2% American Indian–1%

As stated previously, the general purpose of this study was to explore the role of the teacher in the Algebra I classroom and to examine the potential impact of these roles on student learning. This particular school offers Algebra I over the course of the entire year for ninety minute blocks. The study was conducted during the Spring Semester of 2007.

Teachers

After gaining verbal approval from the principal, all of the teachers at the school teaching ninth grade Algebra I were contacted, and gave the written consent. Teachers from ninth grade Algebra I classes were chosen purposefully because of the importance of Algebra as discussed in the first chapter and lack of research on effective teaching at the secondary level. All of the teachers at the school who were teaching the second part of Algebra were chosen to obtain variability. It was anticipated that only some of the teachers would be implementing the practices outlined in the review of literature based on the teacher's variable involvement in Project Math. Table 2 provides the characteristics of the five teachers that participated on in this study, including the teacher's professional development hours.

Table 2

Teacher Characteristics

	Highly Qualified	Years of Teaching Experience at Time of Study	Degree Held	Professional Development Hours from Project Math	Ethnicity and Gender
Ms. Adams	yes	2	BS in Math Ed	Less than 20 hours	White female
Ms. Cook	yes	22	Masters +15 hours	More than 60 hours	White female
Mr. King	yes	3	Masters in Math Ed	Between 20–60 hours	Black male
Ms. Parker	yes	6	Masters in Math	More than 60 hours	White female
Ms. Johnson	yes	3	BS in Math Ed	Less than 20 hours	White female

Student Population

The reported student population for the 2006-2007 school year was 848 eighth and ninth graders. According to the District Accreditation Guided Self Study, the district allocated $9014 per student. Ethnicity in the district at the time of the study was reported as follows: 63% Caucasian, 28% African American, 6% Asian, 2% Hispanic, and less than 1% American Indian. The district also reported that the average class size in the school selected for this study was 19 students (Eastside, 2007).

Most of the teachers in the study were teaching more than one section of ninth grade and the teachers were allowed to choose the class block observed in the study.

Since Algebra I is also offered in the seventh and eighth grades at the school, the students in these ninth-grade classes were considered by school staff “at grade level”, which implies these students were average or below in achievement, since the more advanced students would have taken Algebra I at an earlier grade. Both the principal and the teachers confirmed that students were assigned teachers randomly.

Parental consent forms were sent out at the beginning of the semester to grant permission to use any data collected from the students. The initial return of these consent forms was low, so the consent forms were sent out again which resulted in a higher return. Refer to Appendix C for a copy of the student consent form.

Instrumentation

Instrumentation comprises specific methods for collecting data (Miles & Huberman, 1994). This use of multiple data-collection methods, or triangulation, contributes to the trustworthiness of the data (Glesne, 1999). This study incorporated the use of interviews, questionnaires, observations, and a content test to collect the data. Each will be described in the following sections.

Teacher Questionnaire

Initially, a questionnaire was given to all the Algebra I teachers participating in the research, as a self report on their teaching practices in their classroom. The questionnaire was designed to ask general questions about their perceptions and beliefs inside the Algebra I classroom that relate to each of the components that emerged from the literature including: 1) teacher content knowledge, 2) classroom environment, 3) lessons that are accessible to all students, and 4) mathematical connections. Refer to Appendix D for a copy of the teacher questionnaire.

Observation Protocol

Data was collected during a minimum of five observations in each of the five subjects' classrooms using the observation protocol. The school is on block scheduling, so each observation lasted approximately ninety minutes and included notes taken from the observer and an audio recording. An observation protocol (see Appendix E) was adopted by combining the observation protocol from Horizon's Research, Inc. (Weiss et al, 2003) and the *Reformed Teaching Observation Protocol (RTOP)* (Arizona Board of Regents, 2000) used in observing the teachers participating in Project Math. The RTOP was developed as an observation instrument to provide a standardized means for detecting the degree to which K–20 classroom instruction in mathematics inquiry based (Arizona Board of Regents, 2000). The items the observation protocol contains were designed to measure the four components of effective teaching identified in the review of literature. The observation protocol was completed after the classroom observations to provide me another data source when analyzing each teacher with respect to content, discourse, equity, and connections. On the observation protocol, a score of 1 represents "never" and 5 represents "to a great extent" (AzTEC, 2002). The scores were averaged to provide a picture of the degree each teacher implemented the four components.

In addition to a rating scale for each of the four components, the Equity component of the observation protocol had a seating chart of the room with individual student characteristics listed and notes of teacher/student interaction was recorded.

Similarly, the discourse component of the observation protocol had a seating chart to record teacher/student questions and responses.

A pilot observation was done in an Algebra I classroom to gain a feel for how effective the protocol would be and to see if the protocol accurately represented the components as discussed in the review of literature. This was done for dependability purposes.

Teacher Interviews

A teacher interview was given to each teacher at one point during the study in search of opinions, perceptions, and attitudes towards teaching and learning in the mathematics classroom. The interview was given to clarify inferences made by the researcher that were related to the teacher's role in the classroom and the student's role in the classroom during an Algebra lesson and to further validate any findings. The interview was given at a convenient time, chosen by the teacher, following an observation. The questions addressed the role of the teacher and student in that specific lesson. All the teachers were interviewed once throughout the study; see Appendix F for the protocol.

Student Content Instrument

In order to link teaching and learning to achievement growth, all Algebra I students of these teachers were given a test of the content of Algebra I at the beginning of the semester and again at the end of the semester. Some items were selected from Glencoe *Algebra I* (McGraw-Hill, 2004). Some of the items were also selected from a question bank *ExamView Pro* (McGraw-Hill, 2003) and edited to be short answer. The items are multiple choice, short answer, and open-ended. Each item is aligned with the

Alabama Course of Study. The multiple choice items are worth 1 point each, short-answer are worth 2 points each, and the open-ended questions are worth 3 points each. The format and grading designed were similar to the Alabama Reading and Math Test (ARMT) (Alabama Department of Education, 2005a) and the Alabama High School Graduation Exam (AHSGE) (Alabama Department of Education, 2005b).

In order to attend to the dependability of this instrument, the test has been reviewed by a mathematician, a mathematics educator, and mathematics teacher and graduate student in mathematics education. The instrument was also piloted in an Algebra classroom to check for initial misunderstandings in the wording of the questions and to get an initial response of how students would react to the test. Refer to Appendix G for a copy of the content instrument.

Student Questionnaire

A student questionnaire was developed based on the four components of the observation protocol and modified so the students could respond to a five point rating scale, 1 being Strongly Disagree and 5 being Strongly Agree. An example question was "I feel respected in the Algebra I classroom". The purpose was to ask the students what they thought was happening in their classroom and to see if the students perceptions of the Algebra I classroom matched with my observations of the classroom (see Appendix H). The student questionnaire was given at the end of the course along with the post- test.

Additional Data Sources

The researcher also maintained a research journal which was used as an additional data source. Journal entries included the researcher's comments on all facets of the data collection process. The researcher's journal included comments on the administration of the written instruments, classroom observations, and interviews with teachers. Field notes were taken during classroom observations.

Procedure

The formal data collection process began with the content instrument which was given as a pre-test to all students in the Algebra I classes chosen for this study at the start of the semester. Based on the Project Math teacher leader's advice, all the surveys were given to her. She then distributed the tests to each of the teachers, and they returned the completed tests to her. Consent forms for the students and parents were also given at this time.

All of the teachers were given the questionnaire about their current teaching practices that encompassed the four main components of this study as outlined in the review of literature. Then during the course of the semester, each classroom was observed five times. The observation protocol was used to obtain a general picture of content, discourse, equity, and connections in each classroom. Each teacher was also interviewed once over the course of the semester following one of the observations. All observations were audio taped and field notes were taken. The audio tapes of the observations, the teacher questionnaire, and the teacher interview were transcribed during the course of the semester and the following months.

The data collection process concluded by giving the students the content instrument given as a post- test. The student questionnaire was also given at this time. The content instrument which was given as the pre- and post- tests were all graded by hand using a key and rubric that is included in Appendix G.

Data Analysis

The data-gathering techniques used in this study, as stated previously, were questionnaires, interviews, and observations, and a content test. All interviews, questionnaires, and observations were audio-taped and transcribed. The transcriptions were then loaded as primary document into Atlas-ti (Muhr, 2004). Atlas-ti is a qualitative analysis tool designed to handle large bodies of data. Each primary document was then coded in Atlas-ti according to an initial set of codes set up a priori based on the four components of the classroom as identified from the research. Any case categories other than these four components that emerged from this study were then coded. Emergent themes and commonalities were then analyzed to get a description of the role of each of the teachers in this study. The components of this section are organized as follows. First, I describe the coding process by outlining each phase of the analysis. This is done for confirmability purposes as discussed earlier in this chapter. This shows how the analysis helped the researcher develop the four main themes in the individual cases and how the analysis lays the groundwork for cross-case comparisons (Miles & Huberman, 1994).

Coding

"Codes" are tags or labels for assigning units of meaning to the descriptive or inferential information compiled during a study (Miles & Huberman, 1994). Codes may be several paragraphs, several sentences, a phrase, or a just a word. Codes are used to organize data collected and then retrieve it. Codes pull the data together, thus permitting

analysis. A data analysis ladder is shown in Figure 1 to give an overview of the progression of the coding process. A complete coding guide can be found in Appendix I.

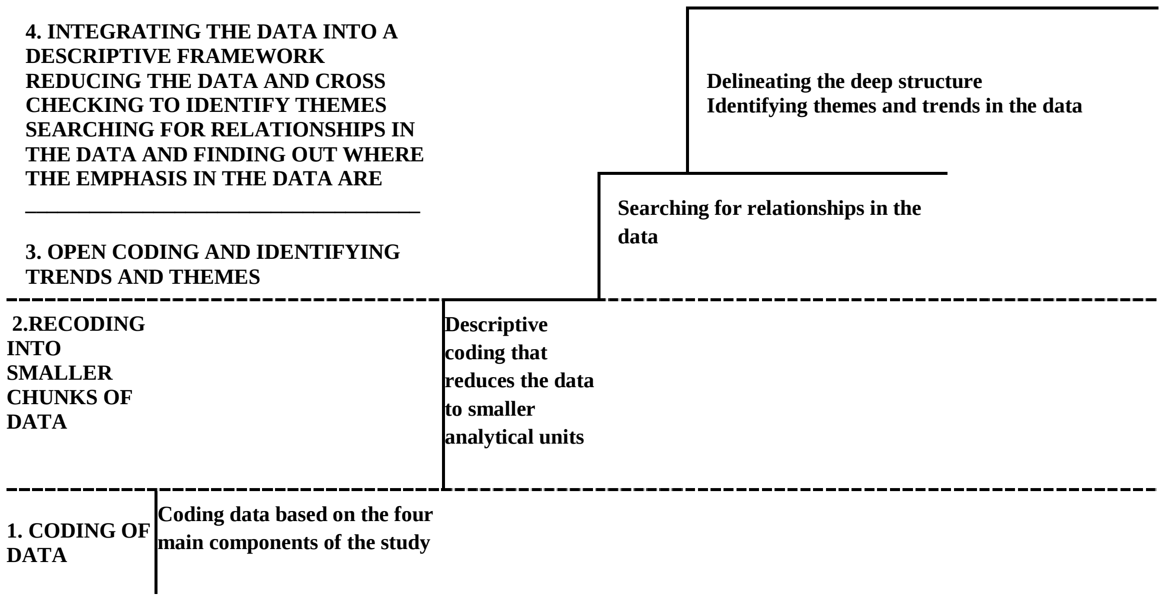

Figure 1. Data Analysis Ladder (adapted from Carney, 1990)

Phase 1

Data reduction and analysis began by coding each of the primary documents . One method of creating codes is to create start codes or codes a priori to any data collection (Miles & Huberman, 1994). The starting list for the study came from the conceptual framework of the study and the research questions addressed in the study (Miles & Huberman, 1994). These codes are descriptive and entail little interpretation. The data can have multiple codes. The function of this coding was to facilitate the development of the four main components: 1) content, 2) discourse, 3) equity, and 4) connections.

Phase II

This phase in the coding is still descriptive. These codes allowed the larger chunks of data to be reduced into smaller analytical units (Miles & Huberman, 1994). This list of codes comes from the conceptual framework from the review of literature (Miles & Huberman, 1994). The codes were given names closest to the concept it is describing. These codes include: 1) connections within content, 2) real world connections, 3) sense making, 4) engaging students, 5) questions for understanding, 6) teacher leading, 7) questions for explanation, 8) trying to get the students involved, 9) student relevant response, 10) verification, 11) student uncertainty, 12) student explanation, and 13) opportunity to learn.

Phase III

The third phase in my analysis began the grounded theory approach. Grounded theory refers to theory developed inductively from data (Strauss & Corbin, 1998). The basis for grounded theory is to read and re-read the data, and discover categories and concepts that are interrelated (Strauss & Corbin, 1998). Grounded theory utilizes three types of coding, open-coding, axial coding, and selective coding (Strauss & Corbin, 1998). Open coding refers to the part of the data analysis that labels and categorizes the phenomena. Data are broken down by asking simple questions such as what, how, when, etc., and then data are compared (Miles & Huberman, 1994). Similar phenomena are grouped together and given the same label (Pandit, 1996). Open coding is a way of developing codes more inductively. Codes emerge progressively, rather than the researcher trying to force the data into pre-existing codes (Miles & Huberman, 1994). The next part of the analysis involves axial coding. Axial coding is relating codes to each other through a combination of deductive and inductive thinking (Strauss & Corbin, 1998). The codes that emerged from the data in this process were: 1) connection attempt, 2) discourse attempt, 3) equity attempt, 4) emphasis on mathematical language, 5) intimidation environment, 6) missed opportunity, 7) off topic, 8) question for sake of question, and 9) teacher explanation.

During this phase, the researcher also involved two fellow graduate assistants to conduct *interrater reliability* checks. Interrater reliability is used to assess the degree to which different raters/observers give consistent estimates of the same phenomenon, or in this particular study, transcripts from the classroom observations. The researcher gave both of the assistants an excerpt from a classroom observation, along with a complete coding guide. The codes were briefly explained and examples given of some of the codes for clarification. Reliability on the 42 fragments was .85 and .90 within the four main codes of content, discourse, equity, and connections. Any disagreements or conflicts seemed minor. For example, one of the coders labeled several of the phrases "teacher leading" whereas the researcher had used the code "discourse attempt". After some discussion, an agreement was reached, because both the researcher and the assistant viewed the codes as an attempt for the teacher to initiate student discussion, but the teacher was rephrasing questions and asking questions in such a way that the students were "led" in any responses given.

Phase IV

Strauss (1987) suggests that coding and recoding are over when the analysis seems to have run its course. After open and axial coding was completed, a selected code could produce all of the quotations associated with that code as well as the source document of the quotation. The next step was to identify themes and trends by searching for prevalent codes in the data (Miles & Huberman, 1994). At this point in the analysis, the descriptive codes as well as any codes that had emerged from the data in the open and axial coding process could be structured under the four main components of effective teaching. This is known as selective coding. Selective coding is choosing one category, then relating all categories to that category (Strauss & Corbin, 1998). For example, one code that emerged from the analysis was the code *question for the sake of question*. This code was used when a teacher asked a question, but allowed no wait time for the student to respond. This code is associated with discourse as discussed in the review of literature in the sense that the teacher is going through the motions of asking questions to promote student response, but is not implementing the strategy effectively. Figure 2 provides a visual format of how the codes were connected to the four main themes. Each type of code is in a different shaped node in the network. Note the hierarchical levels of the analysis. For example, *discourse* is an a priori code that was used in the first phase of coding. *Student explanation* is a code that was created from the review of literature and used to break the data into smaller chunks. The arrow goes from *discourse* to *student explanation* indicating that "student explanation" is a descriptive code that can be defined directly from discourse. The next phase in coding was to open-code the data. These codes were crucial for getting to the next level above the flow of events in the data (Miles & Huberman, 1994). For example, *communication* emerged in the coding process, and then could be associated with *discourse*. These codes were developed and then applied to the overall structure of the data. The network shows how the arrows link the open codes to the four main components.

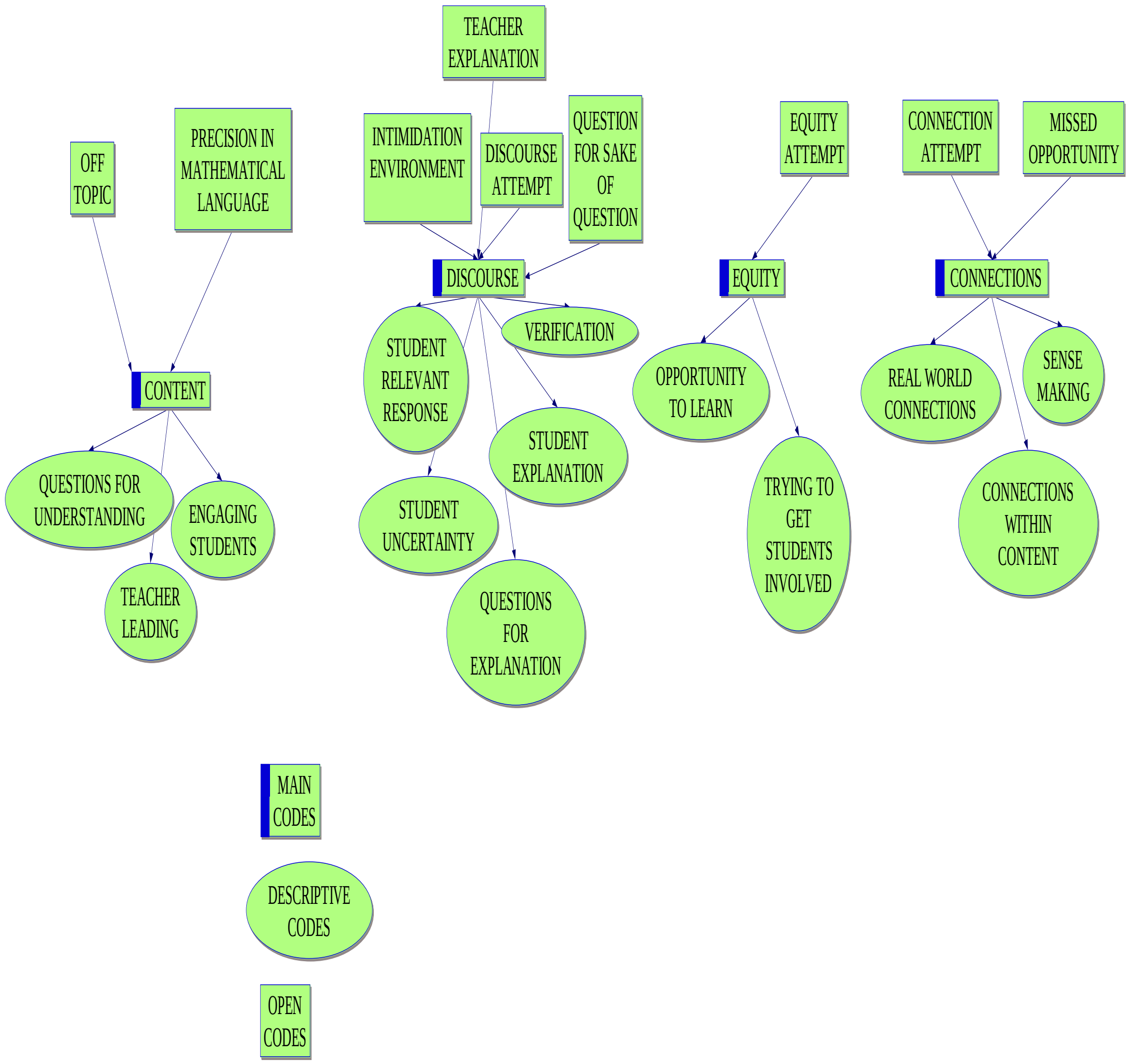

Figure 2. Phase Four of the Analysis

IV. FINDINGS

It was my intent to gain insights into the components of effective teaching in the Algebra I classroom and how they affect student roles and learning. The review of literature framed effective teaching and learning through four main components: content, discourse, equity, and connections by looking at the roles of the teacher and the roles of the students. As noted earlier, the purpose of this study was to explore what the role of the teacher is in implementing these four components in the mathematics classroom and discuss the potential impact the role of the teacher has on student learning. Multiple individual case studies were utilized to examine the following questions:

1. What is the role of the teacher in implementing content, discourse, equity, and connections in the Algebra I classroom?
2. How do these roles affect the role of the student and student learning?

The cases presented in the following will look at how the teacher orchestrates his/her roles in content, discourse, equity, and connections in the Algebra I classroom. Also, each case will examine how the role of the teacher affects the role of the student and student learning. Excerpts from lesson observations will be used to illustrate the findings. In the final section, a comparison of the cases within each of the four main components will be presented.

Cases

Description means “making complicated things understandable by reducing them to their component parts” (Bernard, 1988). Qualitative studies are designed to explore a new area and to build a theory about that area (Miles & Huberman, 1994). The data is condensed, clustered, sorted, and linked over time in a process known as “data transformation” (Gherardi & Turner, 1987). As was shown in figure 1, one of the steps in the last phase in this process is integrating the data into a descriptive framework from relationships and themes that emerge from the data. As stated previously, these themes came from prevalent codes in the data. I then took these themes to provide descriptions that were indicative of the cases in this study. Rein and Schon (1977) suggest first telling a story about a specific situation. The following cases provide a “story” of what happened in each of the classrooms by drawing from the prevalent codes that emerged from the data and any other sources of data that support these findings. Additional sources of data include the observation protocol, the student questionnaire, and the pretest and post test data.

The cases include vignettes and illustrations that offered an opportunity to focus on descriptions that are representative or typical in each case. In each case, a brief description of the teacher’s classroom is provided, along with a table that shows an overall summary of each case. Each case is then described within the components of content, discourse, equity, and connections, including an assessment of effective teaching. The following table summarizes an assessment of effective teaching drawn from the literature, similar to the ratings of the lessons assessed in *Inside the Classroom*

(Weiss et al., 2003) and will provide a framework for the assessment of effective teaching in all four components of effective teaching.

Table 3

Effective Teaching Rating Scale

Rating	
Low	Unlikely to enhance students' understanding of important mathematics or provide students with opportunities to engage or participate.
Medium	Beginning stages of lessons that are purposeful and include some elements of effective practices, but also include weaknesses that may limit potential for students.
High	Provide opportunities for students to interact purposefully with mathematics and are focused on learning goals.

Tables 4–7 summarize the assessments specific to each of the four components of effective teaching. These assessments are based on a similar rating scale as Inside the Classroom (Weiss et al., 2003), and are drawn from the characteristics of effective content, discourse, equity, and connections.

All of the students in these classrooms were considered "average", and the description of equity is based on the observation data and questionnaires. Thus, equity in this context is defined as how active participation and success was encouraged and valued

Finally, the pre test and post test are used to examine student achievement. Also included are the scores broken down into each specific type of question. A twenty-point scale was used because it was the maximum score across the cases.

Table 4

Content Rating Scale

Rating	
Low	Teacher does not provide students with opportunities to engage or participate.
Medium	At times students are engaged, but there are weaknesses.
High	Teacher provides content that is meaningful and worthwhile. Students are engaged in the lesson and are provided with the challenge to find something.

Table 5

Discourse Rating Scale

Rating	
Low	Teacher dominated, students do not participate or interact.
Medium	Teacher has begun to elicit mathematical discussion and thinking but still weaknesses. Some components of effective discourse, but still in early stages.
High	Safe and respectful environment where students discuss and explore. Exceptional rapport. Students are independent thinkers.

Table 6

Equity Rating Scale

Rating	
Low	No or few students participate. Teacher does not provide opportunities for all students to learn. Low expectations.
Medium	Beginning stages of equitable teaching, but also include weaknesses that may limit potential for all students.
High	High expectations and high quality instruction for all students. Provide opportunities for all students to interact purposefully with mathematics.

Table 7

Connections Rating Scale

Rating	
Low	Topics presented as isolated facts. No coherence or sense making.
Medium	Beginning stages of lessons that are interconnected within mathematics and real world. Student are beginning to make sense of the mathematics.
High	Students encounter mathematics that is realistic and makes sense. Prior knowledge is utilized to build new knowledge. Connections are made within mathematics and the real-world.

Setting

This study was conducted at one school in a suburban school district located in southeastern Alabama. The school is located in a mid-sized city. The school offers

Algebra I over the course of the entire year. The five teachers selected for this study were all the ninth grade teachers teaching the second part of Algebra I in the spring semester of 2007.

All of the teachers in this study were located on the first two floors of the same hall in a new building. The building is clean and during the course of data collection, a mosaic tile wall display of the school's motto was completed. Students were talkative, yet calm during the change of classes. At the time of the study, the students held a record of over eighty days without a fight in the school. The students received a big incentive for this.

The mathematics teachers would go in the hallway during change of class, but most had work for the students to start on as they entered the classroom. All teachers in this school system have a desktop or a laptop computer with Internet access. The school selected for this study is participating in a technology initiative where all the students are provided a laptop, and the classrooms are equipped with interactive white boards for instruction. Teachers are expected to integrate technological resources effectively in teaching, communication and data.

Ms. Adams

Ms. Adams is a white young female who was a second year teacher. She holds a Bachelor of Science degree in Secondary Mathematics Education. She dressed professionally, but was somewhat soft spoken in the classroom. As a fairly new teacher, Ms. Adams was not yet completely confident in her ability to teach the mathematics. This was reflected in the item, *The teacher appeared confident in her ability to teach mathematics,* which was included in the observation protocol and the student questionnaire. The scores were 3 and 3.1, on a five point scale, respectively, which indicated an average score on the observation protocol, but was a low score relative to other scores given on the student questionnaire. Refer to Table 8 for Ms. Adam's average scores.

The classroom was made up of twenty students: three black females, three black males, nine white females, and five white males. The desks in this classroom were set up in both quads and rows. The classroom was decorated neatly with personal pictures and typical math posters. Classroom rules were also posted. The class was not dismissed until any and all trash was picked up.

Each of the lessons in this classroom started off with a "bell ringer." A "bell ringer" is a problem or problems that students are expected to start working on as soon as they come into class. Ms. Adam's bell ringer typically included problems that revisited topics already taught. The students were expected to be in their seats working the problems while the teacher typically checked the student's homework from the night before. Most of the class was then spent going over the bell ringer problems and the homework. The students usually led the discussions as the teacher asked several questions to guide them in their thought process. She was very conscientious about not just talking *to* the students about the mathematics, but wanting them to think about the mathematics and explore the mathematical concepts for themselves. Ms. Adams asked questions which led the students to think and figure out the mathematics for themselves. Table 8 provides a case summary of Ms. Adams.

Table 8

Ms. Adams — Case Summary

	CONTENT	DISCOURSE	EQUITY	CONNECTIONS
Prevalent codes	"engaging students" "questions for understanding"	"discourse attempt" "questions for explanation"	"trying to get students involved" "equity attempt"	"sense making"
Observation protocol averages	3.1	3.3	3.1	3.1
Student survey averages	3.6	3.6	3.7	3.4
Overall assessment	medium	medium	medium	medium

Content. Questions for understanding was the prevalent code associated with content for Ms. Adams, and the code *engaging students* was also present. When asked to describe herself as a teacher, Ms. Adam's response was

> As a teacher, I like to try to get the students engaged in the mathematics. I try to ask the students several questions that provoke their thinking. I feel as though I do not need to be front and center. I want the students to have power in their learning.

Most of the students in this classroom were engaged in the content and participated in discussions, but a series of questions was usually asked to prompt the students or lead them to the answer. The students on some days volunteered to present the problems they had worked on the board and on other days were assigned to work a given problem.

Typically, the students were told to explain their work to the other students, but this too required prompting by teacher questions. Two of the lowest scores from the student questionnaire in content were the items, *I feel engaged in the lessons in Algebra I* and *The topics covered in this Algebra I class are significant and worthwhile.* This suggests that while Ms. Adams was trying to engage the students in worthwhile mathematics, she had just begun to ask the types of questions that develop students' mathematical understanding and skills and to stimulate students to use reasoning and problem solving skills.

The following is a typical example of how this teacher used questioning as a strategy to engage the students in the content, but it usually required a series of questions in order to get the students to respond. This lesson was introducing characteristics of a graph of a quadratic function. The students had investigated how the coefficient a in $f(x) = ax^2 + bx + c$ affects the characteristics of parabolas by using a dynamic geometry program on their laptops. This exchange then followed:

> Teacher: So how did the vertex change direction?
>
> Student 1: Over the negative number.
>
> Teacher: So a negative flips over downward? So is the vertex actually moving to the top or the bottom?
>
> Student 1: No
>
> Teacher: Let's see what McKenzie just said, if you have a negative A, then it flips over right? Does anybody else notice anything else they want to share about when you change the A value?
>
> Teacher: What did you notice with one half?
>
> Teacher: We only have whole numbers on here right?

Teacher: I’m asking you what did the graph do? What did it look like?

Teacher: So what value of A do you think would start to grow fatter?

Teacher: So we started off this x squared right? This is like the parent graph, this is where all these other graphs come from right? All of them came from x squared. You see the benefits coming inward?

Student 2: Yes.

Teacher: One, two, three, four, five. What’s going to happen with this one?

Student 2: Your numbers get bigger

Teacher: Okay, you’re telling me when my numbers get bigger, they’re more narrow?

Student 2: Yes

Teacher: So half number would be what kind of number?

Teacher: So what do you all think? Did ya’ll look at the parent graph?

Teacher: What would have worked as well?

Teacher: My question to you is, why are the numbers getting bigger instead of narrow?

Teacher: So as they get smaller do they get wider? Are we going to say negative numbers are smaller than positive numbers?

Teacher: Okay, do we agree with Martel? Can everybody hear what he says? So what is really the mid point, or the breaking point as they get larger or smaller?

Student 3: One half

Teacher: What would be a smaller fraction?

Student 3: One fourth

Teacher: What is the smaller?

Student 3: One sixteenth

Teacher: When it actually get closer to zero what happens? It never equals zero right? Do we see what happens when A equals zero? Is that kind of coming together why we said A equals zero?

Student 3: Falls flat

Teacher: What would that be – zero times x squared? Is that a problem anyone?

Student 4: Zero

Teacher: What kind of problem would this be?

Student 4: Linear

Teacher: Linear. Are we working with linear when we talk about quadratics? We all see A opens upwards when A is positive and moving downward when A is negative and when does it start getting more narrow?

Student 3: As you increase the numbers.

Overall, I assessed Ms. Adams a “medium” in content as shown in Table 8. Ms. Adams did ask a lot of questions to engage the students in the content, but as the vignette shows, she had to ask several questions before any student response was given. Ms. Adams knew the mathematics content, but she was just beginning to develop strategies to teach the content in a way that engaged and challenged the students intellectually. Questions had to be rephrased or asked in way that led the students to a response.

Discourse. The teacher and the students seemed to have mutual respect for one another. It was evident that this particular teacher’s goal for this class was to orchestrate discourse. Discourse was the dominant theme for this teacher and the component that received the highest score in the observation protocol for Ms. Adams. Codes that frequently appeared with Ms. Adams were *student explanation* and *questions for explanation*. The teacher allowed students the opportunity for challenging each other’s thought or adding their own thoughts by asking questions such as, “Do you want to put more with my definition?", "Do you want to explain your work?", "Did anyone do something else?", "Did everybody do it the exact same way?", "Any questions for (name) right now?", "I heard yes and no, do you have an idea?" Ms. Adams received a 4 on the item, *The set up of the lesson was encouraged to generate ideas, questions, conjectures, and or propositions* on the observation protocol.

However, even though questions for explanation and discussion were frequently asked, student responses were typically minimal. The code *discourse attempt* was recurrent. *Discourse attempt* reflects an attempt to get the students involved in agreeing or disagreeing with mathematical questions or responses, but no wait time or response occurred. This is also reinforced by the student responses in the student questionnaire. The item, *The teacher expects me to participate in the Algebra I classroom*, received the highest average score from the student questionnaire, while the item, *I actively participate in the Algebra I classroom*, received the lowest score. The following excerpt is typical of how this teacher encouraged thinking and discussion, but had to rephrase and continue to ask questions to get response from the students.

> Teacher: You are telling me that the vertex helps define more points?
>
> Teacher: The question was, you told me that the vertex helps me find four corresponding points
>
> Student 1: Yes.
>
> Teacher: It does? Why?
>
> (no response)
>
> Teacher: All right let me ask you this - why do we even have corresponding points in the parabola?
>
> Student 1: Makes a U shape.

Teacher: I know how many points it takes to make a U, but why can we say well all right this point has got a corresponding point?

Student 2: What were you saying?

Teacher: What about the parabola will give me the right to say well this point has this.… We talked about it.

Student 1: Same distance from the axis.

Teacher: All right, Martez said because the distance from the axis is….. The same way that you take to fill up the left side you can fill in the right side … does this make sense? The problem is special because it has symmetry.

Teacher: So, what tells me that this point has that point?

Student 3: Because you fold the diagram.

Teacher: On what?

Student 3: On y

Teacher: So there's a Y that tell me about the four starting points. What tells me where my four starting points are? What allows me to find four starting points? All right the original question I asked when we graphed the vertex was what?

Student 3: It'll always be there.

Student 4: All right, so now we have to find the axis of symmetry

Teacher: All right, do we agree with that?

Ms. Adams received a "medium" in discourse as shown in Table 8. Characteristics of discourse oriented teaching as discussed in the review of literature were being implemented. Students listened, responded, and sometimes questioned the teacher and each other, but all the problems and the questions were initiated by the teacher. The students did not rely on mathematical evidence to convince themselves, rather still on the teacher to revoice or repeat questions and thoughts.

Equity. Ms. Adams scored an average of 3 on all the items from the equity section of the observation protocol, except the item, *indicate that teaching is "digging knowledge out"of students by asking appropriate questions*, with which she scored an average of 4. Ms. Adams frequently asked questions like, "Does everyone agree?" and "What do the rest of you think?" However, these items were coded, *trying to get students involved* and *equity attempt*, because of the short wait time allowed after posing the questions. Equity attempt refers to an attempt to get all the students involved or allow for different responses, but the teacher moved on quickly if no immediate response was given. For example, in one class the lesson had focused on simplifying square roots with variables in the expressions. Ms. Adams had shown a couple of examples and had written on the board, "need absolute values when you start with even and get odd." She then asked, "Are there any other questions? Is everybody clear about the variables? Does everybody understand why we need absolute values? All right, so let's go ahead and do the homework."

Ms. Adams seemed aware of trying to get the entire class involved and participating. After the first two observations, however, it was apparent that one student liked to dominate classroom discussions and volunteered for most questions. The teacher decided to remedy the situation by putting all the student's names in a bag. Each problem on this particular day had two parts. After the teacher drew a name out of the bag, the student was allowed to pick what part of the problem he/she would work and picked another student to complete the other part of the problem.

On average, the students' perceptions indicate they felt Ms. Adams expected them to be successful. The items, *My teacher believes I can succeed in the Algebra I classroom* and *My teacher expects me to do well in the Algebra I classroom,* received the average scores of 4 and 4.4, respectively.

Overall, Ms. Adams received a “medium” in equity. She did implement strategies to involve the students and the students felt like Ms. Adams wanted them to succeed. She asked questions to elicit student participation and used different techniques to ensure the students were involved in the lessons. Such as, randomly drawing names out of a bag to make sure more than a few were participating. However, even though Ms. Adam’s questions indicated an attempt to keep all the students involved, her techniques reflected lack of effectiveness because little wait time was given to allow student respond or questions.

Connections. Ms. Adams asked a lot of questions in order to get the students to think more deeply about the mathematics, and it was clear she wanted the mathematics to make sense to the students. *Sense making* was a code that appeared frequently in the analysis process for this teacher. A piece from a previously mentioned vignette illustrates this. The students were using corresponding points as a method of finding enough points to graph on the parabola. This method used the symmetric property of parabolas to find the coordinating point of a point already found. After a student had graphed the functions on the board, the teacher asked “let me ask you this — why do we even have corresponding points in the parabola?” Ms. Adams revoiced the student’s response. “All right, Martez said because the distance from the axis is the same…The same way that you take to fill up the left side you can fill in the right side ….. does this make sense? The parabola is special because it has symmetry.” Ms. Adams was trying to help the students make sense of the methods they were using to connect it to the characteristics of a parabola. “Sense making” in this classroom is also evident from the students response on their questionnaire. *The teacher helps me “make sense” of Algebra I* and *Algebra I makes sense to me* were the items that received the highest average score under connections.

In another lesson, the code *connections within the content* recurred. Ms. Adams led the students to think of the “big idea” of the problem, and then led the students to work a simpler problem, which helped them apply what they already knew and use it to work a problem they were having difficulty starting. The students had the opportunity to make connections because the teacher constantly asked questions that made them think about how the mathematics made sense. The students were given the simpler problem to work, but the students came up with two different answers. The following vignette shows the interactions between the teacher and the students and illustrates how connections were being made.

> Teacher: What’s the big idea of the problem? What is basically happening with these fractions? What’s the big idea with adding fractions?
>
> Student 1: Common denominators
>
> Teacher: Could that maybe have started the problem off?
> What do you think? Let’s look at a simpler problem. $\frac{1}{2}+\frac{3}{4}$
>
> Teacher: It wasn’t easy to think about those easy fractions one-half and three-fourths. Can we think about it kind of like money? One half would be what?
>
> Student 1: Fifty-cents.
>
> Teacher: Three fourths would be what?
>
> Student 1: Seventy-five.
>
> Teacher: If you have fifty cents you’re adding seventy-five to it, what do you get?
>
> Student 1: You would get $1.25.

Teacher: And which ones do those match?

Student 1: The first one

As Table 8 shows, I assessed Ms. Adams as “medium” in connections. Ms. Adams tried to help the students make sense of the mathematics by asking questions that led to the students making mathematical connections. One of the characteristics of effective discourse is students recognizing the connections within the mathematics. The students had not developed this recognition, but Ms. Adams asked questions that showed connections within the mathematics.

Results from the pre- and post-test. The results from the pre- and post test scores indicate that some learning was taking place. Ms. Adam’s class had an increase in score (see Figure 3) from the beginning of the semester to end of the semester that was statistically significant. Scores from each type of question also supported the fact that some type of meaningful mathematics teaching occurred in this classroom. There was a fairly large increase in the multiple choice type questions, and also in the open middle questions. This is indicative that Ms. Adams questioning had stimulated higher order thinking and problem solving. Table 9 shows the breakdown of the percentages of questions answered correctly in Ms. Adams class.

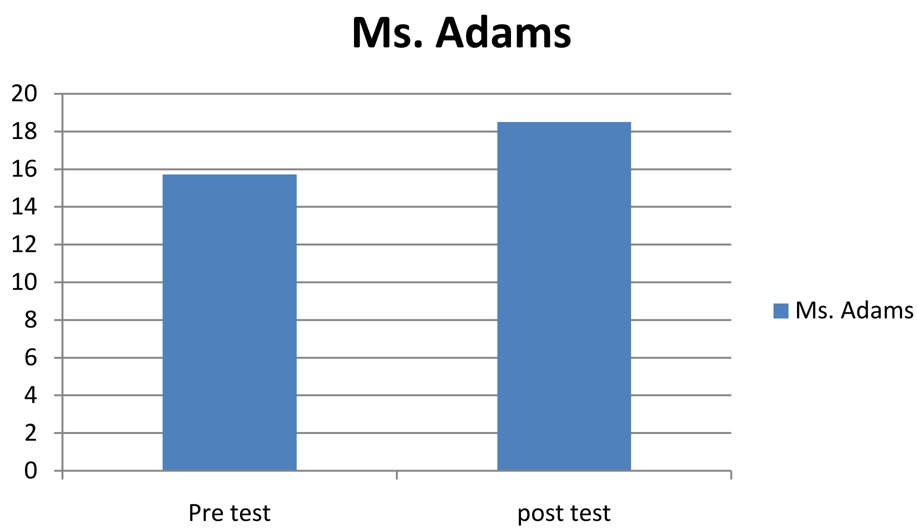

Figure 3. Ms. Adams Pre and Post-Test Scores.

Table 9

Ms. Adams — Percentage of Correct Answers from Each Type of Question

Types of questions	Pre test	Post test
Multiple choice	59.74%	73.21%
Short answer	47.73%	48.61%
Open middle	10.91%	16.11%

Summary. Overall, it seemed Ms. Adams was conscientious of implementing the effective teaching practices as discussed in the review of literature. It was apparent she was a fairly new teacher and was just beginning to implement strategies needed to establish her role in content, discourse, equity, and connections effectively. She used higher order questioning in an attempt to engage the students with the mathematics. She also used these types of questions in an attempt to involve the students in rich mathematical discussions. Her classroom was equitable in terms of trying to involve all of the students. Ms. Adams questions also made it evident she wanted the mathematics to make sense to the students. Although her questioning in each of the four components had not yet elicited the response of the students desired in effective teaching, it is fair to characterize Ms. Adams as on the right track within the components of effective teaching.

Ms. Cook

A veteran teacher who had taught more than 20 years, Ms. Cook is a tall, white female who always dressed very professionally. Ms. Cook never raised her voice in the classroom but had a strong presence in the classroom. She received a perfect score on both the student questionnaire and observation protocol item, *The teacher appeared confident in her ability to teach mathematics.*

This classroom was made up of only eleven students. The majority of the class was white males. There were two white females and one black male. The classroom was very organized. There were few decorations on the walls. Graphing calculators were available in the back of the room if needed. Students sat in rows, but had been assigned groups. If group work was used the students moved the desks themselves.

This class was first block. The teacher did not have a "bell ringer" but always had all of the problems the students were going to work displayed by her computer. The students would be instructed to open their laptops and log on to the dynamic geometry program, if needed for the lesson, and then instructed to put the laptops down to listen to the morning announcements. As soon as housekeeping was completed, the teacher would have her first problem on the screen. The teacher in this classroom had the classroom set up to where the students would first try to help each other and answer each others questions and then ask the teacher if help was still needed. Table 10 provides a case summary for Ms. Cook.

Table 10

Ms. Cook — Case Summary

	CONTENT	DISCOURSE	EQUITY	CONNECTIONS
Prevalent codes	"engaging students" "questions for understanding" "precision in mathematical language"	"question for the sake of question"	"equity attempt"	"connections within content"
Observation protocol averages	4.3	3.1	3	3.4
Student Survey Averages	4.2	4.1	4.2	3.8
Overall assessment	high	medium	medium	medium

Content. Typically, the class started with a problem on the screen for the students to solve. The students were then given time to work the problem independently, then, the students were instructed to check their answers with the students in their group. The problems from homework were gone over only if a student had a question. When the discussion was complete, the teacher then called on the students who had worked the problem correctly and wrote exactly what the student said. The teacher taught new concepts in a similar way; she gave the problem to the students, and then had the students call out how to work the problem. In this way, the students were engaged in important mathematical terms and the precision of mathematical language was stressed. Ms. Cook's highest score in the observation protocol averages was in content. Her strategy, not only got the students to think about what they were saying and how they said it, but it also checked for mathematical understanding which was reinforced by the frequent code, *questions for understanding*. This was in alignment with the above average score on the students' response on the student survey item, *I understand the mathematics in Algebra I.* Another code, *precision in mathematical language*, emerged from the analysis of Ms. Cook's transcripts and remained prominent. This code emerged because of the emphasis put on mathematical terminology and the communication of mathematical concepts and ideas. If the students were not giving clear responses to a question or demonstrating understanding, Ms. Cook typically asked the students to write responses. For example, **"**Okay, this is what I want you to do right now. Everybody in this classroom, I want you to explain to me on paper when you get the direction to *simplify*. What does it mean and what you are going to do? Explain it in a complete sentence—subject verb agreement. Have your group member read your sentence and see if it makes sense." The students read their responses aloud, and the responses were referred to throughout the rest of the lesson.

Another code that frequently appeared was *engaging students*. Students were engaged in mathematical content due to teacher questioning and getting them involved in explanations throughout the lesson. They were also accountable to one another by having to check each others' work.

Overall, I assessed Ms. Cook “high” in content. She conducted her lesson in such a way that the students were engaged in the mathematics. She was also very attentive to precise mathematical communication and understanding the terminology in mathematics. Ms. Cook kept the students engaged by frequently having them write what the mathematics meant. Ms. Cook not only knew the mathematics content, but she taught the content in a way that engaged and challenged the students intellectually.

Discourse. Ms. Cook seemed to want the students to be involved, and there was a welcoming environment. The focus was on learning, and the teacher constantly asked questions like, “Do ya’ll agree or disagree?”, “What do you want me to write?”, and “Is everybody okay?”, but seemed content with yes or no responses. These questions almost seemed routine, and very little wait time was given after each question was posed. This is supported by the two main codes, *discourse attempt* and *question for the sake of question*. The teacher called on various students throughout the lesson and most of the students appeared to be involved in the lesson. One of the highest scores from the student questionnaire was from the item, *The teacher expects me to participate in the mathematics,* which received a score of 4.9. If an incorrect response was given, other students were called on to help out, but usually explanations were procedural. The students did interact with one another quite frequently which is supported with a score of 4 on the observation protocol item, *Interactions reflected students working together and talking to each other about the lesson*. However, new ideas and thoughts were rarely posed which is also reflected in the item, *The focus and direction of the lesson was determined by ideas originating with the teacher.* Instead, they usually just checked one another’s work with little discussion other than procedures Ms. Cook received all 3’s on the observation protocol questions in discourse mentioned above. The following illustrates the environment in this classroom.

Teacher: Give me another irrational number. Give me another one.

Student: $\sqrt{9}$

Teacher: The square root nine would be your rational.

Teacher: Give me another irrational.

Teacher: Any other irrationals?

Student: Thirty one.

Teacher: Thirty one is irrational. This is the set notation.
Are you okay? Are you sure?

Teacher: Is this rational or irrational?

Student: Rational

Teacher: All right what's this one — is it rational or irrational?

Student: Irrational.

Teacher: Irrational? Why?

Student: Because it's not a square.

Teacher: Because it's not a perfect square.

Teacher: What about this one? $\sqrt{12}$ Is it irrational or rational?

Teacher: Four is the perfect square. Is six a perfect square? Two is a perfect square? So four is a perfect square? If four is a perfect square, is four rational or irrational?

Student: Rational

Teacher: What is the other factor of twelve besides four?

Student: Three.

Teacher: Is three a perfect square?

Student: No.

Teacher: Is three rational or irrational?

Student: Irrational.

Teacher: So is my product rational or irrational?

Student: Irrational.

Overall, I assessed Ms. Cook a “medium” in discourse as Table 10 shows. The students did interact with each other in small groups, but the discussions usually focused on procedures. As shown in the excerpt above, Ms. Cook’s questions were not higher order questions, and therefore, elicited little discussion. The environment in this classroom was teacher led.

Equity. Another code that frequently appeared was *equity*. Ms. Cook constantly asked questions like, “Does everybody agree?” or “Is everybody okay?” In one of the observations, questions like these were posed fifteen times, thus the code *equity attempt* emerged. Although questions like these do indicate on the surface that all students are expected to participate, if the students aren’t allowed time to respond, the questions become nothing more than a routine. Ms. Cook scored a 3 on every item in equity on the observation protocol, but the students scored her extremely high on the student questionnaire which indicated the students thought Ms. Cook had high expectations for them.

One way Ms. Cook guaranteed everyone involved was by having them all write a paragraph or sentence describing key mathematical terms or concepts. Here is a typical example: The students in the class did not seem engaged, so Ms. Cook had everyone take out a sheet of paper and answer the following question: "Is $\frac{\sqrt{5}}{4}$ in simplest form? Explain."

Another source of data that reflects Ms. Cook's implementation of equity was a seating chart in which teacher/student interactions were recorded. In this particular observation, *all* of the students in this classroom participated and contributed to the class.

In terms of equity, I assessed Ms. Cook a "medium" overall. Frequently, she did make all the students write a complete sentence about the mathematics she was teaching. However, Ms. Cook's constantly asking questions like "Is everyone okay?" without allowing any wait time indicated these questions were more of a routine than monitoring all of the student's involvement in the lesson.

Connections. The item *The teacher helps me "make sense" of Algebra I* received a score of 4.6 on the student questionnaire, but the item, *I can apply the Algebra I skills and concepts to real-world situation*, received one of the lowest scores. The dominant code for connections in Ms. Cook's classroom was *connections within content*. The following vignette shows how Ms. Cook made connections within the content. This particular lesson was on simplifying radicals and the teacher was trying to lead the students into seeing how absolute values were important when simplifying radicals with variables in them. The teacher used the knowledge the students already knew to answer questions by building on prior knowledge and then using that understanding to apply it to new concepts.

Teacher : $\sqrt{x \cdot x}$ So what is the square root of x squared?

Student: x

Teacher: When I take the square root of x squared and I get x. Let's take the square root of 16 okay? What is the square root of sixteen?

Student: Four

Teacher: Four. What if I have the square root of 4 squared, what would that be?

Student: Four

Teacher: Four, right. Ya'll with me? Suppose I have the square root of negative four squared?

Teacher: Negative four squared is sixteen. What's the square root of sixteen?
If I take the square root of negative four squared or if I take the square root of four squared, I know that I got a positive four. Ya'll understand what I'm saying?
Watch this, suppose that I have this, what would that be? $\sqrt{-36}$

Teacher: No real number right? Are you with me?

Teacher: Suppose I had this, what would this be? $\sqrt{(-5)^2}$

Student: Five.

Teacher: What about this? $\sqrt{(-3)^2}$

Student: Three

Teacher: Okay, what about this? $\sqrt{3^2}$

Student: Three

Teacher: Okay, what about this? $\sqrt{(-7)^2}$

Student: 7

Teacher: Hang on, what about this? $\sqrt{(x)^2}$

Student: x

Teacher: So that means when I take the square root of that squared, when I write x is my answer going to positive or negative?

Student: positive

Teacher: So if I write $|x|$ and I guarantee the x is positive, I put the absolute value sign.

Student: I've a question.

Teacher: Okay, what's your question?

Student: Are we trying to figure out if the numbers are rational or irrational?

Teacher: No, we already figured that out.

Teacher: See right here how I took the square root of three squared and I got three. I took the square root of negative three squared and this is negative and the factor is negative and the square is positive right? So if I were to put negative seven right here, will we get seven? Seven. So I just can't say it's negative, I have to say it's positive. Because you know when you have a positive like this, you can make a positive.

Teacher: Now what about this? $\sqrt{(-4)^2}$

Student: Four

Teacher: What is the square root of four squared?

Student: 4

Teacher: See how important these are here because this is square root of seven squared whereas this one says the square root of negative seven squared.

Overall, I assessed Ms. Cook a "medium" in connections. Ms. Cook used student's previous knowledge and then applied this knowledge to the new concepts she was trying to teach. These connections were always made within the mathematics content as shown in the previous vignette. In this way, Ms. Cook showed how the concepts in mathematics build on each other, but there was little sense making or real world contexts.

Results from the pre- and post-test. Ms. Cook had an increase in scores from the pretest given at the beginning of the semester to the posttest given at the end of the semester. However, the gain was not statistically significant. The scores increased in all three types of question. Ms. Cook's emphasis on content explains the overall gain in the multiple choice and short answer questions. There was also a fair gain in the open middle questions which indicates that being able to understand and communicate mathematically is important.

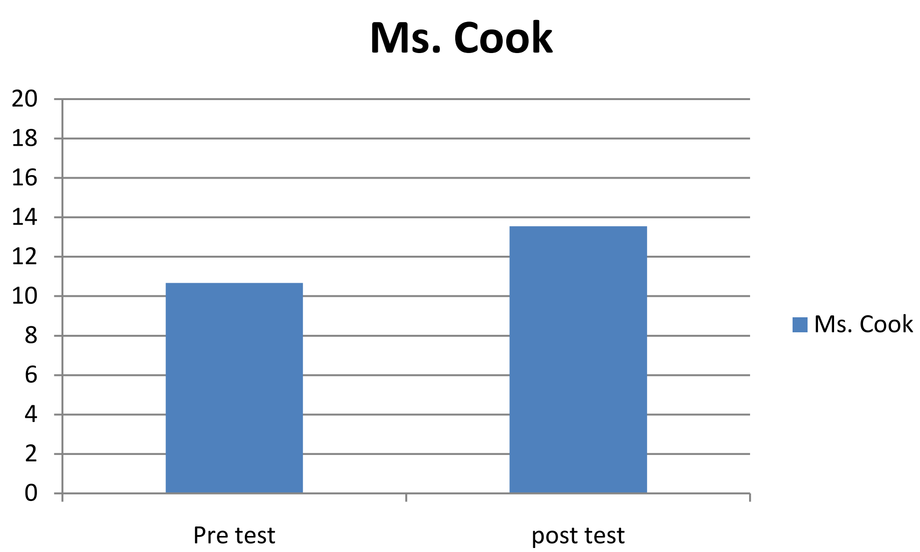

Figure 4. Ms. Cook Pre- and Post-Test Scores

Table 11

Ms. Cook — Percentage of Correct Answers from Each Type of Question

Types of questions	Pre test	Post test
Multiple choice	52.38%	57.94%
Short answer	22.22%	32.41%
Open middle	4.44%	10.37%

Summary. Ms. Cook received the highest score in content as shown in her case summary in Table 11. Her emphasis on precise mathematical language contributed to her implementing the content effectively. Ms. Cook was a veteran teacher who was obviously aware of effective strategies, but sometimes was going through the routine of this implementation rather that making sure the students truly were engaged and understanding. This was apparent in both the components of discourse and equity. Ms. Cook's helped the students make connections within the content by building on their previous knowledge. Ms. Cook can be characterized as content consistent.

Mr. King

Mr. King has a Master's Degree in Mathematics Education and had been teaching three years. He was a young black male who always dressed professionally and had a commanding presence in his classroom.

This Algebra I classroom consisted of nineteen students total. There were eight White males, four White females, two Black males, three Black females and two Indian females. This teacher was by far the most rigid in terms of classroom management. The desks were set up in rows. There were no decorations on the walls, but objectives were always written on the board. They were the high school graduation exam objectives and only the objective number was given.

The students were called by their last name and not expected to do any talking unless called on. The teacher in this classroom described himself as a teacher that gives guided practice, then, allows the students to work problems very similar to the guided practice. Most of the lessons were set up for the students to work quietly and independently the majority of class time, then some time was spent procedurally working some examples with the class emphasizing the sequence of steps to be done. Any remaining class time was spent working a worksheet or other problems assigned for homework. Table 12 gives the case summary for Mr. King.

Table 12

Mr. King — Case Summary

	CONTENT	DISCOURSE	EQUITY	CONNECTIONS
Prevalent codes	"teacher leading"	"intimidation environment"		"missed opportunity"
Observation protocol averages	2.9	2.1	2.4	2.3
Student Survey Averages	3.9	3.7	4	3.8
Overall Assessment	Low	low	low	low

Content. The teacher in this classroom appeared very confident in his ability to teach mathematics as was indicated by a 4 on the observation protocol and a 4.7 on the student questionnaire. However, there was little or nothing designed to help the students engage in the content other than asking a few questions to help them recite rules or procedures the teacher had demonstrated throughout the lesson. A score of 3.3 on the item, *I feel engaged in the mathematics* on the student questionnaire, was one of Mr. King's lowest scores in content. The most prevalent code that arose from the coding process was *teacher leading*. The following is a typical excerpt from this class which illustrates how Mr. King basically dominated over the lessons.

> Teacher: Any time B is positive you're going to get a negative. Any time B is negative, you're going to get a positive. How many of you understand that? All right you ready? Ya'll see what you're doing right? All right, Buddy do you know the formula?
>
> Student: (reads quadratic formula)
>
> Teacher: Now tell me what's A, what's B and what's C.
>
> Student: a = 2, b = 4, c = -5
>
> Teacher: Right.
>
> Teacher: All right, I'll tell you exactly what I want to see. I want to see the numbers plugged in the formula; not only that, I will count off. I want you to list A, B and C. Got it?

As shown in Table 12, I assessed Mr. King a "low" in content. Although, he knew the content, the students in this class were given little opportunity to be engaged. Even when the task could have potentially engaged the students, the students were guided to just do the procedures. Mr. King dominated the classroom. He worked examples, asked very few questions other than facts about the formula or rules, and then instructed the students to work independently.

Discourse. The culture of this classroom was one of an authoritarian teacher. The teacher focused on control of the class and did very little to involve the class in any discussion or discourse. Mr. King received a score of 2 on every item on the observation protocol except the item, *The focus and direction of the lesson was determined by ideas originating with the teacher*, which was a score of 5. Most questions did not require any student thinking and were typically a statement with "right?" at the end. The students were not allowed to sleep and were expected to work, but the work was mainly rote memorization with little understanding. This may explain why the scores on the student questionnaire are so high in discourse, because the students were expected to be working at all times, and they may have interpreted this as participation and thinking in the Algebra I classroom. Some statements that were typical of this teacher are: “You know what to do right? And if you didn’t that’s your problem. Can we do anything else?” and “All right moving right along. If I don’t call your name, I don’t want to hear your voice anymore, okay?” “Do you understand that?” “Are there any questions?” The code, *intimidation environment* emerged often in the analysis of this teacher. This code was used to identify any classroom environment that was intimidating or authoritarian in nature.

Overall, I assessed Mr. King a “low” in discourse as shown in Table 12. The students in this classroom worked independently and quietly most of the time. Some time was spent going over problems and sometimes students were allowed to go to the board and work a problem for the class. The students responded to questions asked by the teacher by reciting rules or answering computational questions. The environment in this classroom was authoritarian and intimidating.

Equity. The teacher in this classroom viewed the teacher role as the “instructor” and the student’s role as a “student” as indicated in his response in the interview. This is also reflected in Mr. King’s average score of 2.4 in equity from the observation protocol. Very little participation was expected from the students other than to give back facts or formulas. Occasionally, the students would be asked to find the mistake of another student’s work, but the error would be computational and usually students were called on for classroom management purposes. When asked if he believed all students can succeed in Algebra I, Mr. King’s response was “No, some have the belief that they can’t do it, so they will not try" One the responses on the student questionnaire also reinforces this authoritarian environment; on the item, *I feel respected in the Algebra I classroom*, one student crossed out the scale and wrote in "$-\infty$."

Overall, I assessed Mr. King received a "low" in equity. If students are not treated with respect, they cannot possibly feel as though they are expected to succeed. Mr. King also made it clear he did not expect all the students to succeed, but he did expect them to keep their mouths shut. Mr. King showed little respect for his students.

Connections. Most of class time was spent working problems from a worksheet or the book independently, and few attempts were made to develop connections. The students were instructed to follow directions and attention was given to rules and procedures rather than trying to make sense of the mathematics. *Missed opportunity* was the code used frequently, because sometimes real-world problems were used in class, but then a formula would be given and procedures on how to plug in numbers to solve the problem would follow. This is reinforced by the students response to the item, *I can apply the Algebra I skills and concepts to real-world situation,* which received the lowest score on the student questionnaire. This code was used to identify times where the lesson or class discussion had the potential to have implemented one of the four main components of content, discourse, equity, or connections, but was not developed or utilized. In this case, Mr. King missed the opportunity to make real world connections. This is also reflected in Mr. King's response to the interview question "Do you think this lesson made appropriate connections to real-world contexts? He replied "Yes, we use the formula for kinetic energy of a moving object to solve for the velocity of an object. The following illustrates this.

A roofer tosses a piece of roofing tile from a roof onto the ground 30 feet below. He tosses the tile with an initial downward velocity of 10 ft. per second. Write an equation to find out how long it takes to hit the ground. Use the model for vertical motion H = where $-16t^2 + vt + h$ H is the height of an object after t seconds, v is the initial velocity, and h is the initial height. How long does it take for the tile to hit the ground?

The teacher had a student read the problem and then went on to tell the students this was an example of the quadratic equation and simply wrote down what a, b, and c were and plugged them into the quadratic formula and worked the problem.

As shown in Table 12, I assessed Mr. King a “low” in connections. Mr. King used real world problems in his classroom more than any other teacher, but never used them in a way to make real world connections. He simply took the problem and plugged the numbers into a formula. Procedures were followed and formulas memorized.

Results of the pre- and post-test. Mr. King’s scores decreased, but not at a statistically significant level. Based on the overall assessment of Mr. King, it was not surprising that his test scores shown in Figure 5, went down over the semester. However, based on his style of teaching, it was not expected that the scores from the open middle questions to increase, although slightly. Table 13 shows the percentages of correct scores from each type of question.

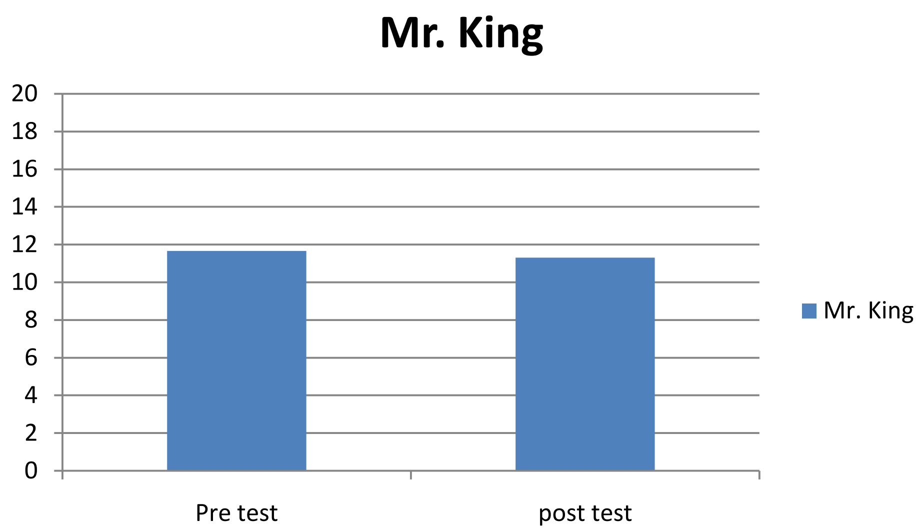

Figure 5. Mr. King — Pre and Post-Test Scores

Table 13

Mr. King — Percentages of Correct Answers from Each Type of Question

Types of questions	Pre test	Post test
Multiple choice	52.98%	45.98%
Short answer	31.25%	32.81%
Open middle	3.33%	6.25%

Summary. As shown in Table 13, Mr. King was not implementing the four components of effective teaching. His classroom was teacher dominated and the students contributed very little unless called on to reiterate procedures and rules. The content revolved around being shown examples and then the students were expected to work independently and quietly. There were no mathematical discussions in this classroom. Mr. King called on students occasionally and expected them to spit out a procedure or computation. This class was not considered equitable because the teacher was authoritarian. Even though some of the problems used had the potential to make connections, they never did. Mr. King can be characterized by his name in the fact he was an authoritarian.

Ms. Parker

Ms. Parker had been teaching for six years and held a Master's degree in mathematics. Ms. Parker is a White female dressed casually most of the time and did not have a strong presence in her classroom.

This Algebra class had nineteen students. There were five White females, seven White males, four Black females, and three Black males. The desks are in rows, and there was a seating arrangement, but the teacher doesn't seem to enforce it. There were some standard math posters hanging on the walls and piles of paper all around the room. There was a lot of clutter in Ms. Parker's classroom. There was also a lot of extraneous talking going on. Table 14 shows the case summary for Ms. Parker.

Table 14

Ms. Parker — Case Summary

	CONTENT	DISCOURSE	EQUITY	CONNECTIONS
Prevalent codes	"off topic"	none	none	none
Observation protocol averages	2.5	2.2	2.1	2.4
Student Survey Averages	3.8	3.4	3.7	3.7
Overall Assessment	low	low	low	low

Content. This classroom was by far the least structured of the five classrooms observed in this study. The teacher had little control over the students, and therefore, very few were engaged in the lessons. Most of the time the teacher gave some problems, and then one or two students asked questions and the teacher worked the problems on the board, almost as if the teacher was just working the problems for her own benefit. Ms. Parker scored a 2 on the item *The teacher appeared confident in her ability to teach mathematics* on the observation protocol. The lessons seemed to be planned out and may have had potential for meaningful mathematics, but the students usually got the teacher off track and were rarely involved or paying attention. The student response on the student questionnaire however, did not support this. In fact the highest score from the student questionnaire was the item *I feel engaged in the Algebra I classroom*. But from the researchers view, it was engaged in anything but the mathematics. For example, the following lesson was mostly teacher led and only one student was participating:

Teacher: I want to make this one on one I really need to be careful. This is a parabola. This is the basic one, called the parent graph notice: what is A in this case? Positive one. Let's make it three. This is the one up here that's a graph. Multiply by three. So when I come up with the shape of your parabola, I want you to always be careful. Right now we're trying to find the shape of it. When I talk about the shape of your parabola, I want you to always be careful how you handle it. The thing that's going to shape it, we'll talk about that. Right now we're going to talk about the shape. We're going to shape this graph and make it $25\ x^2$. How does the shape change?

Student 1: Got skinnier

Teacher: What's the function — a hundred times, narrower, narrower multiply by a hundred. How do you think I could make it narrower? Maybe I could make it shallow? Just like this — see what I mean?

Student 1: You could try a negative.

Teacher: Okay, let's try a negative. Negative — put a number in there? Is it narrower?

Student 1: I don't know.

Teacher: What did we say the negative do?

Student 1: Opens down.

Teacher: The negative makes it open down. J. T., do you have any idea how to make it narrower?

Student 1: No.

Teacher: We went up from one to three to twenty-five, to one hundred

Overall, I assessed Ms. Parker "low" in content as shown in Table 14. The students did not seem to be engaged in the content in this classroom. There was very little order or management, and the students were successful in getting the teacher and themselves off task. Ms. Parker knew the content, but most of the time was talking to herself because the students remained distracted and disruptive.

Discourse. The environment in this classroom was mostly chaotic. The teacher was openly friendly and seemed to genuinely care about the students, but because there was so much outside talking going on, very few students were engaged in learning. The teacher seemed to want to focus on learning, but the lack of control usually meant one or two students were somewhat involved in the lesson, while the others stayed disinterested. The teacher usually was undisturbed by the inappropriate talking, but every once and awhile, would attempt to pull students in by telling them "this will be on your test"

Several students played games on their laptops or had individual conversations. The students sat in rows and had assigned seating, but this was not enforced. Most of the questions were asked by the students, and the teacher answered them. Typical questions were "How do I do that?" or "Where did that come from?" The lowest score from the student questionnaire was on the item, *I am comfortable sharing ideas, questions, and contributions in the Algebra I classroom,* which emphasizes an environment in which there was little contribution from the students .

Overall, I assessed Ms. Parker a "low" in discourse. There was a lot of discussion going on, but none of the discussion was relevant to the mathematics and very few students contributed any thoughts whatsoever to the lessons. The environment in this classroom was disorderly and there was not enough classroom management to have meaningful mathematical discussions.

Equity. The teacher in this classroom seemed to want involvement and participation in this classroom and was willing to answer any of the student's questions. However, very few questions were asked by the teacher. In one observation in which all the teacher questions were recorded, Ms. Parker only asked one question, "Will that always work?", which elicited no response. The teacher mainly worked problems and gave notes. Few students ever contributed in this classroom. It was believed by both the researcher and the students that Ms. Parker wanted the students to succeed as indicated in the scores of a 3 and a 3.9 on the observation protocol and the student questionnaire, respectively. These are relatively high scores in comparison with the overall averages on these instruments.

Table 14 shows I assessed Ms. Parker a "low" in equity. It appeared that Ms. Parker cared about her students but did not maintain a classroom that provided an opportunity for all to get involved. In this way the classroom was not equitable, because even if students wanted to participate, the distractions around them prevented involvement in the lessons.

Connections. There was little evidence of connections being made in this classroom. The teacher presented most of the lessons as isolated facts, but the lessons had potential to be tied in together. One example was in graphing quadratic functions. The first day's lesson was simply making a t-chart and graphing enough points until a clear shape could be determined. There was no discussion on the characteristics of a parabola in this first lesson. However, in a follow up observation, the teacher used plotting points to begin to bring certain characteristics out. The students had been instructed to graph a minimum of seven points in the first lesson, but as the lessons went on, symmetry emerged. The student's role in this is minimal, but there was a hint of a connection being made. This may explain the consistency in the student's response to the two items addressing sense making.

> Teacher: The next one is one, two three. The third thing I need to know is the axis of symmetry. You were doing this yesterday J. C. and you were telling me that it turned. Remember doing this table? Take a look at this parabola. I have this one turning point. This parabola opens up right? This is similar with the table. This is similar with the table, and this is the turning point. I go over one and up one, back one and up one. Those two points kind of backed up. If I go over… If I were to take this parabola and say fold it right along with this turning point here.
> Here we go here's the parabola, now look this is the turning point. I go over one to get to this point (inaudible) how do I find this one from the turning point I go over. How far do I go up? So from this turning point from the last two, go up four? Where would the next point on this side be? They have a symmetry about them. All parabolas have symmetry.

I assessed Ms. Parker a "low" in connections as well. It was difficult to get anything accomplished in a chaotic environment. It is believed the lessons were planned and did have some potential to make connections within the content, but classroom management ruined any attempts at this.

Results of the pre- and post-test. Figure 6 shows the results of the pre- and post-test given in to Ms. Parker's class at the beginning and the end of the semester. Her scores minimally increased but the gain was not significantly significant. Table 15 shows the results in each type of question. The open middle score by the end of the semester was less than 1%, and it is believed this was a direct reflection of the lack of discipline in this room.

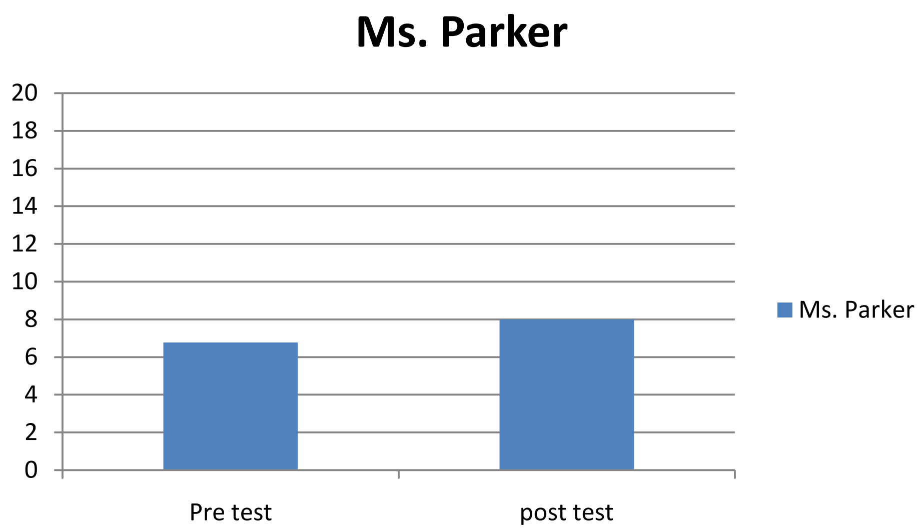

Figure 6. Ms. Parker — Pre- and Post-Test Scores

Table 15

Ms. Parker — Percentage of Correct Answers from Each Type of Question

Types of Questions	Pre-Test	Post-Test
Multiple choice	38.10%	44.44%
Short answer	9.26%	14.81%
Open middle	2.96%	.74%

Summary. As shown in Table 14, Ms. Parker received very low scores on all of the components in the classroom. The dominant code in her classroom was “off topic.” This code was used because of conversations or statements that came up that had nothing to do with the mathematics lesson that day. Any attempt at effective teaching strategies were overridden be the lack of classroom management. For the most part, this classroom appeared to have a teacher who talked to herself. Ms. Parker can be characterized as "poor scores due to poor management."

Ms. Johnson

Ms. Johnson is a young White female who was in her third year of teaching. She dressed nicely and she had a respected presence in her classroom. There were five White females, six White males, two Black females, one Black male, and one biracial male for a total of fifteen students in this class. This classroom always smelled fresh and was decorated according to the season. Desks were in neat rows.

This was also a first block class. The students had "do now" work to do at the beginning of class, but it was turned in instead of going over the problems at that time. The teacher then worked any problems from the homework by asking key questions and writing down the solution as the students called it out. Notes were given on new material, and then problems were worked in whole class. The lessons seemed procedural, but the teacher asked a lot of questions to get the students thinking and making connections. There was a great dynamic in this classroom because both the teacher and students seemed enthusiastic about learning. Table 16 shows the case summary for Ms. Johnson.

Table 16

Ms. Johnson — Case Summary

	CONTENT	DISCOURSE	EQUITY	CONNECTIONS
Prevalent codes	"engaging students" "questions for understanding"	"verification" "student relevant response"	"trying to get students involved"	"sense making"
Observation protocol averages	4	3.7	3.9	3.7
Student Survey Averages	3.8	3.8	4	3.4
Overall Assessment	medium	medium	medium	medium

Content. On the surface, this teacher used very traditional teaching techniques, but the presentation strategies seemed to have the students engaged in the content, even though their response on the item, *I feel engaged in the Algebra I classroom,* was low relative to the other items on the student questionnaire. In the observations, all the students except one or two were usually engaged in the lesson. Most were taking notes and participating in class discussions or asking questions when they were unclear about something that Ms. Johnson had said. *Questions for understanding* was the code that surfaced the most with Ms. Johnson. The somewhat high average in *content* from the observation protocol also supports Ms. Johnson's effective implementation of this component. Her highest scores on the student questionnaire were the items: *The teacher carefully plans and organizes the Algebra I lessons, The teacher appears confident in* *his/her ability to teach Algebra I,* and *The teacher displays an understanding of Algebra I.*

The following is a vignette from this classroom which illustrates how Ms. Johnson and students are involved with the content:

> Teacher: What's the very first step that we always do when you're doing quadratics functions?
>
> Student 1: Find the axis of symmetry
>
> Teacher: Find the axis of symmetry and how do I find my axis of symmetry?
>
> Student 1: X = Negative B
>
> Teacher: X = Negative B over 2 A. Now is B negative?
>
> Student 1: No.
>
> Teacher: No, the negative comes with your equation, it will always be there. Okay, I have X = negative. What's my B?
>
> Teacher: Step two is to find my vertex.
>
> Student 2: Find the vertex.
>
> Teacher: Step two is to find my Vertex. How on earth am I going to find my vertex?
>
> Student 3: Put it in
>
> Teacher: Put it where?
>
> Student 3: In the problem.
>
> Teacher: What problem?
>
> Student 3: The original problem.
>
> Teacher: My original problem. I have $y = x^2$ + 8x+15, all right but my x is negative four.
>
> Teacher: I have a negative four squared will give you what?

Student 4: Sixteen

Teacher: Sixteen, minus thirty-two, plus fifteen. Sixteen plus sixteen is thirty two. So my thirty two is negative…. one. Is negative one my vertex Taylor?

Student 5: It's a point

Teacher: How do I write a point?

Student 5: x and y

Teacher: What's that called? What's the vertex called?

Student 5: An ordered pair

Teacher: A negative four and a negative one. What does the vertex mean? Can anyone tell me what that means?

Overall, I assessed Ms. Johnson "medium" in content as shown in Table 16. She engaged the students with the content by constantly asking them questions and keeping them involved in the lesson as the vignette above illustrates. However, the questions were not higher-order.

Discourse. The environment in this classroom was one where the teacher was very enthusiastic and warm to the students. There was a feeling of mutual respect in this classroom. The lesson itself is procedural, but the teacher asked questions like "Are you good?" and "Does everybody agree?" She got the students engaged in thinking, even though overall, the lessons were teacher led. All the students in this classroom were involved in the lesson. Most lessons were set up as whole class, but the students were allowed to interact with one another and most seemed comfortable. Overall, Ms. Johnson received high scores on both the observation protocol and the student questionnaire. Codes that frequently appeared in the analysis were, *questions for explanation, student relevant response*, and *verification*. The code *student relevant response* means a student responded to a question asked by the teacher and the response was on topic or related to the mathematics being discussed. The code *verification* was used when the teacher repeated what a student said in order to make sure they were saying and thinking the same thing. The following example illustrates the type of discourse typical of this classroom:

> Teacher: What am I going to do to show the triangle ABC is similar to DEF. You must justify your answer.
>
> Student 1: The corresponding angles are the same
>
> Teacher: So you're telling me that if I know that angle D is the same as B, and angle C is the same as angle F that the triangles are similar.
>
> Student 2: yes
>
> Teacher: It worked, did anyone do it differently?
>
> Student 2: Add up the angles
>
> Teacher: Like this one, 180 minus 100 equals 80 right? Then minus 50 is equal to angle C is 30 degrees?
>
> Teacher: Okay are they similar?
>
> Student 2: Yes.
>
> Teacher: How do you know that?
>
> Teacher: You're on the right track. Corresponding angle or degree?
>
> Student 2: Angles
>
> Teacher: Okay, because I have corresponding angle measuring the same, I can say that the two triangles are similar.

Overall, I assessed Ms. Johnson "medium" in discourse. She kept the students involved in the lessons, but Ms. Johnson led most of the discussion. There was an overall comfortable environment in this classroom. Ms. Johnson had good rapport with her students. The students seemed relaxed and were willing to discuss the mathematics.

Equity. Ms. Johnson constantly asked questions to encourage participation by the students. In one lesson, Ms. Johnson asked sixty-six questions, the majority of which were directed to the class as a whole. The frequent code *trying to get the students involved* also supported this. The teacher did not monitor who contributed in any systematic way, but most of the students seemed engaged in the lessons. The teacher usually led the class in whole group discussions. Specific students were not usually called on, but when a student answered, the teacher would ask “is everybody good?,” waited for a response, and then moved on. Students were typically attentive throughout the lessons. Students answered without having to be called on, and would ask questions for explanation or clarification. High scores from the student questionnaire and observation protocol are indicative of this.

As shown in Table 16, I assessed assessment of Ms. Johnson “medium” in equity. It was clear Ms. Johnson expected her students to be successful and conducted her classroom in such a way that kept the students involved. Ms. Johnson allowed time for students to respond to questions such as “is everybody good,” but the responses of students were still part of a routine for the most part.

Connections. Ms. Johnson presented most of the mathematics in a very traditional way, but did want the students to be able to make sense of it. Usually this was accomplished by questioning. For example, when looking at maximums and minimums in quadratic functions, all the graphs had opened up and had given a minimum. The teacher on this day had graphed some quadratics that opened down. Questions like, “What is different about the graphs today?” and “What is different about my equation?” led the students to make connections about what was happening with the coefficient a in the function and if there would be a maximum or minimum depending on which way the graph opened. The code that supported this was *sense making*. Students in this classroom were expected to make sense out of the mathematics. This was also supported by the student response on the items, *The teacher helps me “make sense” of Algebra I* and *Algebra I makes sense to me,* which received the highest scores on the student questionnaire. The following dialogue illustrates the discussion about the maximum and minimum.

Teacher: How do you know it’s a minimum?

Student: Because you can’t go any lower.

Teacher: You can’t go any lower. If I take my pencil and trace the graph it will not get any lower. Now I’ll have to label my graph, my vertex is at negative four, negative one, my axis of symmetry is at negative four and then is there a minimum or maximum?

Student: Minimum

Teacher: It opens up, think about that. Anytime my graph opens up will my vertex ever be a maximum right here?

Student: No.

Teacher: No and anytime my graph opens down can I go any higher?

Student: No.

Overall, I assessed Ms. Johnson “medium” in connections. The questions Ms. Johnson asked showed the students how to make the mathematics make sense. Ms.

Johnson also showed how mathematical concepts build on one another, but this was all teacher initiated and there were few real-world connections made.

Results from the pre- and post-tests. Results from the pre-test, given at the beginning of the semester, and the post-test, given at the end of the semester decreased although not at a statistically significant level, as shown in Figure 7. The only increase was in the short answer types of questions and the open middle scores showed a fairly large decrease as shown in Table 17. Ms. Johnson's questioning style might attribute to why this happened. She constantly asked questions, but the responses were usually short answer and rarely required the students to do higher order thinking.

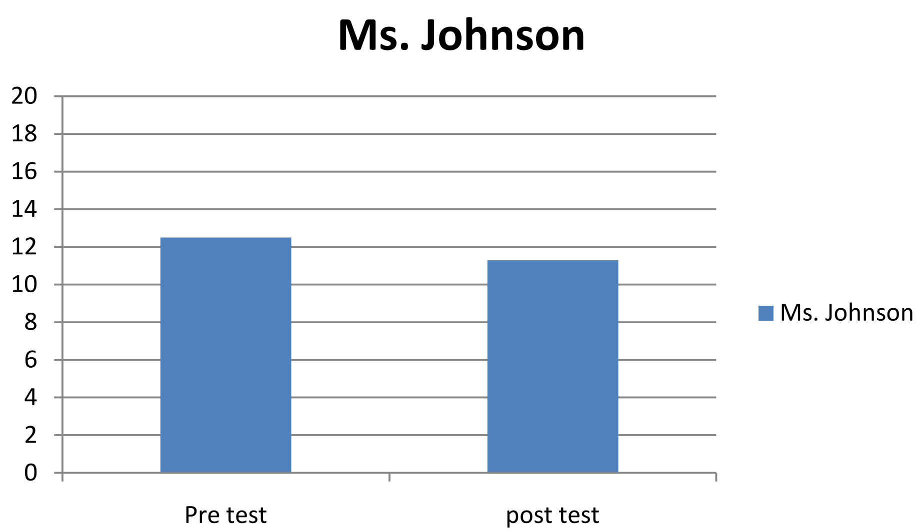

Figure 7. Ms. Johnson — *Pre- and Post-Test Scores*

Table 17

Ms. Johnson — Percentage of Correct Answers from Each Type of Question

Types of Questions	Pre-Test	Post-Test
Multiple choice	45.92%	45.60%
Short answer	23.81%	28.85%
Open Middle	16.19%	11.79%

Summary. Ms. Johnson received all “medium” assessments in the four components of effective teaching. Ms. Johnson had a great rapport with the students and most of them seemed comfortable and engaged in the mathematics. Ms. Johnson helped the students make sense of the mathematics by her questioning. Overall, Ms. Johnson seemed to be implementing the components of effective teaching. However, this was not reflected in the scores on the pre- and post- tests. On the surface, it appeared that Ms. Johnson was implementing some of the characteristics of effective teaching, but based on the scores from the pre- and post-tests, there was no higher order thinking and students were not independent thinkers. Ms. Johnson can be characterized as teacher dependency.

Comparison of the Cases

The following discussion will describe what happened across the cases by looking at the similarities and differences among the cases. The cases will be compared in the same way they were presented individually, within the four main components of content, discourse, equity, and connections. Table 18 is a summary of the cases in implementation of content, discourse, equity, and connections. This assessment was pooled from the student questionnaire and observation data and coding analysis.

Table 18

Summary of the Cases

	Content	Discourse	Equity	Connections
Ms. Adams	Medium "engaging students" "questions for understanding"	Medium "discourse attempt" "questions for explanation"	Medium "trying to get students involved" "equity attempt"	Medium "sense making"
Ms. Cook	High "engaging students" "questions for understanding" "precision in mathematical language"	Medium "question for the sake of question"	Medium "equity attempt"	Medium "connections within content"
Mr. King	Low "teacher leading"	Low "intimidation environment"	Low	Low "missed opportunity"
Ms. Parker	Low "off topic"	Low	Low	Low
Ms. Johnson	High "engaging students" "questions for understanding"	High "verification" "student relevant response"	High "trying to get students involved"	High "sense making"

Content

Certainly one of the most important aspects of a mathematics lesson is that the content is meaningful and worthwhile. “Meaningful and worthwhile” in this case meaning, not only do the teachers have a clear understanding of each lesson, but that the students also see a purpose to the instruction. Content was discussed in the review of literature in terms of how teachers used their content knowledge to teach mathematics lessons that were meaningful and worthwhile. All of the studies discussed, were based on the premise that meaningful learning occurs when instruction involved tasks that required students to construct meaning and/or relate important mathematical concepts (Boaler, 1998; Carpenter, Fennema, & Franke, 1996; Doerr & English, 2006; Seymour & Lehrer, 2006; Silva et al., 1990; Silver & Stein, 1996; Weiss et al., 2003;). A worthwhile task was characterized as being sound and significant mathematically, but was based on students’ understandings, interests, and experiences (NCTM, 2000).

It was clear that all the teachers in this study knew the mathematical content of Algebra I. There were few, if any, errors in content by the teachers. However, as stated previously, the content must not only be accurate, it must be presented in a way that actively engages the students. Overall, the teachers in this study did not provide the students with rich mathematical tasks that developed students’ mathematical understandings and skills, stimulated students to use reasoning and problem solving skills, rather student engagement was encouraged predominately by teacher questioning.

As shown in Table 19, some of the teachers engaged the students, where others failed to engage the students with the mathematics content. Ms. Cook and Ms. Johnson received the highest scores in this area. This was attributed to their questioning. Even though both of these classrooms were mostly teacher-led, the constant questioning strategies kept the students engaged in the content. Ms. Adams also had a fairly high score in content and asked several questions, but since initial student response was often low, it was hard to tell if the students were truly engaged. Mr. King presented the content in a procedural manner that rarely got the students engaged in the lesson and Ms. Parker's classroom was too chaotic for the students to be engaged.

Table 19

Content Observation Protocol Averages for All Teachers

	Ms. Adams	Ms. Cook	Mr. King	Ms. Parker	Ms. Johnson
Content	3.1	4.3	2.9	2.5	4

Discourse

Discourse refers to the ways students learn through representing, thinking, talking, and agreeing and disagreeing about mathematics (NCTM, 1991). Teachers play a key role in orchestrating discourse by moving the classroom culture from one that is teacher focused to one that is centered on student learning. The most effective way to do this is by asking questions that elicit higher order thinking which can lead to rich mathematical discussions. However, in order for this to happen, the classroom must have a climate of mutual respect and trust. The students must feel safe in their mathematical explanations. The classroom must have an environment that is conducive to learning (NCTM, 1991). Effective teachers develop practices that are tuned to mathematical interpretations by the students, but not mere repetition (Forman & Ansell, 2002). Some teachers often implement discourse attempts by repeating portions of what students say or by expanding on what students say. These tactics are known as *revoicing* and can include shaping everyday conversation into mathematical argument and can support student identity (Forman & Ansell, 2002; O'Conner & Michaels, 1996; Strom et al., 2001). It was apparent there were several different types of learning environments in this study, those that lent themselves to discourse and those that were predominately or completely teacher led.

Refer to Table 20 for the discourse summary for all the teachers. Ms. Adams highest score was in discourse. It was clear that she was trying to establish a discourse-oriented classroom, but was still struggling with getting the students to respond. She was trying to establish a student centered classroom by asking higher order questions, but the students gave minimal responses. Ms. Adams mostly revoiced any contributions, rather than rich mathematical discussions taking place. Ms. Parker and Mr. King did very little to promote any mathematical discussions. Mr. King wanted a quiet, teacher dominated classroom and Ms. Parker did not have enough control of her classroom to have any effective discourse. Ms. Johnson kept the class involved by asking the right questions at the right time and implementing problems that provided for discussion, but they were very procedural and the students remained dependent on the teacher for any mathematical thought. Ms. Cook focused more on individuals than on whole-group discussions.

Table 20

Discourse Observation Protocol Averages for All Teachers

	Ms. Adams	Ms. Cook	Mr. King	Ms. Parker	Ms. Johnson
Discourse	3.3	3.1	2.1	2.2	3.7

Equity

Effective teaching and learning requires that all students have the opportunity to learn important mathematical concepts. There were never times that the researcher observed deliberate unequal treatment of students because of personal characteristics. Because all of the students in these classrooms were considered “average”, equity in this context will be how active participation was encouraged and valued.

Access relates to resources that students have available to them. For example, high quality teachers, technology and supplies in the classroom, a curriculum that is not watered down, and a classroom environment that is conducive to learning (Gutierrez, 2007). All of the classrooms in this study had access as defined here except Ms. Parker. Because of her classroom management, there was not an environment conducive to learning. Ms. Parker did little to ensure all were involved because of the chaos in her classroom.

Three teachers in this study appeared to have high expectations, however none used content that challenged the students to find knowledge. In this way, none of the students were empowered (Flores, 2007).

Ms. Johnson received a medium rating because her classroom reflected students being actively involved in the lessons. However, the students were never empowered

(Flores, 2007) because of their dependence on Ms. Johnson. Ms. Adams was aware of trying to get all students involved. After realizing that particular students tried to monopolize class discussions, she drew names out of a bag to provide for fairness and promoting active participation by all. Ms. Cook got all involved by having everyone in the class write on occasion.

Students expected to not achieve, were not given opportunities to learn (Tate, 1995). This was the case for Mr. King's classroom. He wanted the students to repeat his procedures. Mr. King's class did not reflect equity because of the authoritarian atmosphere, and he did not seem to value any student input. Table 21 shows the equity summary for the five teachers.

Table 21

Equity Observation Protocol Averages for All Teachers

	Ms. Adams	Ms. Cook	Mr. King	Ms. Parker	Ms. Johnson
Equity	3.1	3	2.41	2.1	3.9

Connections

Connections was defined in the review of literature as "making sense of mathematics in contexts within the discipline itself *and* contexts outside of mathematics, particularly the real-world." *Internal connections* in this study were defined as connections across mathematical topics and *external connections* were defined as connections between mathematics and its application in other fields or in the real world (Shroeder, 1993).

There were very few attempts in any of the classrooms to make connections in the mathematics with real world situations, but there were lessons that demonstrated an effort to enhance mathematical understanding by including appropriate sense making and connections within the content. Ms. Johnson had the highest score in connections as shown in Table 22. This is again because of the questions she asked. Ms. Adams and Ms. Cook mostly made connections within the content itself. Ms. Adams emphasized sense making within the mathematics, while Ms. Cook presented the content in a way to show how mathematical concepts build on each other. As said before, all the connections made were internal connections (Shroeder, 1993). There was a slight attempt to make a connection in one of the lessons taught by Ms. Parker and the researcher saw no attempts in making connections in Mr. King's class, even though he used problems that would have emphasized real world connections.

"Students should recognize and use connections among mathematical ideas; understand how mathematical ideas interconnect and build on each other to produce a coherent whole; recognize and apply mathematics in contexts outside of mathematics" (NCTM, 2000, p. 354). This was never seen in this study. If any connections were made, they were teacher initiated.

Table 22

Connections Observation Protocol Averages for All Teachers

	Ms. Adams	Ms. Cook	Mr. King	Ms. Parker	Ms. Johnson
Connections	3.1	3.4	2.3	2.4	3.7

Summary

In looking at the characteristics of what made the instruction effective in the review of literature, several common themes recurred. These themes were synthesized into four main components of effective teaching: 1) content- how teachers use their content knowledge to teach mathematics lessons that are meaningful and worthwhile, 2) discourse- ways students learn through representing, thinking, talking, and agreeing and disagreeing about mathematics, 3) equity- ensuring that *all* students are learning important mathematical content, and 4) connections- making sense of mathematics in contexts within the discipline itself *and* contexts outside of mathematics. The vision of effective instruction can be characterized by emphasizing the need for active learning by the students with meaningful content; creating a learning environment that is conducive to learning by promoting discourse; making mathematical understanding accessible to *all* students; and having mathematics "make sense" to the learner (NCTM, 2000).

In comparing the cases, it is apparent that the effective teaching and learning in this context varied inside each classroom. It is safe to say that all the teachers had sufficient content knowledge; however, the presentation of this content differed from case to case. Ms. Cook and Ms. Johnson engaged the students most effectively in the content by asking appropriate questions. All five of the classrooms could be considered teacher-led, but Ms. Adam's classroom stood out as being more discourse oriented. Ms. Adams frequently asked higher-order questions in an attempt to engage the students in mathematical thinking and discussion, and was on the right track of implementing discourse effectively. Ms. Johnson also implemented characteristics of effective discourse due to a classroom environment that was conducive to student participation.

Ms. Adams and Ms. Cook seemed aware of making sure all the students were involved in the lessons, but had somehow fallen into more of a ritualized trap of asking general question to “all”, but allowing little time for response. Ms. Johnson asked the same types of questions and the students did respond, but their responses also seemed to be routine. The overall dynamic in her classroom also attributed to equity being effectively implemented in her classroom, but as stated previously, participation was initiated by the students, rather than the students themselves. There were little real-world connections seen in any of the classrooms, but connections within the mathematics content was evident in the classrooms of Ms. Adams, Ms. Cook, and Ms. Johnson. Ms. Adams emphasized making sense of the mathematics, Ms. Cook showed how the mathematics builds by connecting the student’s previous knowledge to new concepts, and Ms. Johnson did a little of both. Ms. Parker’s and Mr. King’s classrooms did very little to implement any of the four main components of the study. However, Ms. Parker’s lack of effective teaching was attributed to poor class management, while Mr. King’s was because of teacher dominance.

As shown in Figure 8, all of the scores reflect the assessment given to each teacher but Ms. Johnson’s, who although received “medium” assessments, scores decreased. Ms. Adam’s and Ms. Cook’s increased as expected, given their “high” and “medium” assessments in the four components of effective teaching. Mr. King showed no improvement and Ms. Parker had very low scores which was indicative of seeing few characteristics of the effective teaching components in their classrooms. Implications of this will be discussed in the following chapter.

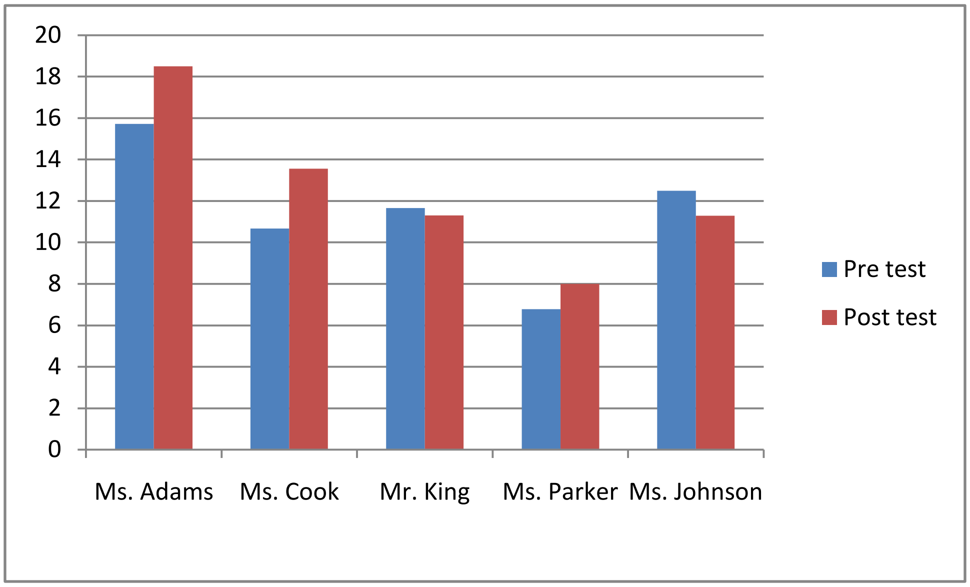

Figure 8. Comparison Scores on the Pre-Test and Post-Test

V. CONCLUSION

The final chapter will begin with a discussion of the limitations of the study. Then, the following section will set forth the conclusions of the study, as they are organized by the two research questions. Finally, the chapter will close with implications that can be drawn from the study, possible use for future studies, and concluding remarks.

Limitations

Prior to any discussion of conclusions and implications of the study, a brief summary of limitations of this research is included. With respect to the school and the participants, the school and participants in this study were purposefully selected because of the involvement in systemic change project in the Southeast. The number of professional development hours varied for each participant, and there is no way to determine the influence of the professional development, or lack there of, had on each of the participants and their teaching practices. Participants were included on a voluntary basis, and the findings are not necessarily representative of all Algebra I classrooms in the nation or even the region.

Many students at the school in the study take Algebra I in the eighth or even seventh grade. Since this study only observed ninth grade students, these students were judged average or below. There is no way to determine if the teachers would have taught the “advanced” students differently.

In terms of longevity, this investigation was limited to one semester in the school year. Instead of examining any longitudinal aspects of teaching and learning, I was confined to more of a snapshot view of what was happening in the Algebra I classroom.

Finally, while I did not actively participate in the lessons, it is difficult to determine if my presence influenced the lessons in any way. Also, due to the nature of the study, the findings and analysis are subject to my own interpretations and biases of the events.

Conclusions

Many documents describe a vision for a classroom that is very different from the one commonly seen in mathematics classrooms (NCTM, 1989, 1991, 2000). The traditional mathematics classroom is characterized by a teacher-dominated classroom in which the "teacher tells" and the "student listens" (Marlowe & Page, 2005). The traditional practices are seen not only as passive and controlling, but also dysfunctional in relation to individual, democratic, and societal needs (Marlowe & Page, 2005). Non-traditional teachers see traditional teaching as stifling student's "creativity, autonomy, independent thinking, competence, confidence, and self-esteem and as making students dependent, conforming, and non-thinking" (Marlowe & Page, 2005, p. 10). Current reform efforts call for a new vision in teaching and learning. This vision emphasizes: high expectations and strong support for all students, focus on important mathematics, creating an environment to understand what students know and need to learn, and student learning mathematics with understanding, actively building new knowledge and connecting it with prior knowledge (NCTM, 2000). Effective teachers not only possess subject knowledge, but they also know how to get the students to use it and understand it.

> Students' understanding of mathematics, their ability to use it to solve problems, and their confidence in, and disposition toward, mathematics are all shaped by the teaching they encounter in school. The improvement of mathematics education for all students requires effective mathematics teaching in all classrooms. (NCTM, 2000, pp. 16–17)

A vision of effective instruction can be characterized by emphasizing the need for active learning by the students with meaningful content; creating a learning environment that is conducive to learning by promoting discourse; making mathematical understanding accessible to all students; and having mathematics "make sense" to the learner.

This study was designed to examine and describe what is happening in the Algebra I classroom with reference to teaching and learning. This involved looking at the teacher role in the classroom, as well as its effects on student achievement, as measured by a test given at the beginning of the semester and at the conclusion of the semester. The study was designed to answer the following questions: What is the role of the teacher in the mathematics classroom? How do these roles, and the interactions of these roles, affect teaching and learning in the Algebra I classroom, when looking through the lenses of content, discourse, equity, and connections within that classroom? The previous chapter summarized the results of the findings. The following sections will attempt to answer the research questions by addressing the implications of these findings.

What is the Role of the Teacher in Implementing Content, Discourse, Equity, and Connections in the Algebra I Classroom?

The role of the teacher was examined using a framework of effective teaching drawn from the review of literature. The four components of effective teaching examined in this study were content, discourse, equity, and connections. These are discussed in the following sections.

Content. While the teachers in this study had an adequate knowledge of the mathematics content they were teaching, content knowledge in isolation is not enough to promote effective teaching and learning. Teachers also need expertise in helping students develop an understanding of that content, including knowing how to determine what students typically think about particular concepts. For the purposes of this study, content was defined as how teachers use their content knowledge to teach mathematics lessons that are meaningful and worthwhile (NCTM, 2000). A "high" assessment for content was based on teachers providing content that was meaningful and worthwhile. Students are engaged in the lesson and are provided with the challenge to find something. The teachers in this study who were assessed a "medium" or "high" in content did use questioning to engage the students in the content, but none used worthwhile tasks or model-eliciting tasks as defined in the review of literature.

Three of the teachers in this study presented the content in such a way that most of the students were engaged in the problems. Two of the teachers, however, just presented the material with little student involvement. Ms. Adams, Ms. Cook, and Ms. Johnson not only demonstrated sufficient content knowledge, but through questioning-also demonstrated the pedagogical content knowledge (Hill, Rowan, & Ball, 2005) needed for effective teaching.

Discourse. It is the role of the teacher to promote effective discourse by creating an environment that is conducive to learning and by asking appropriate questions in order to get the students to think about mathematics in a meaningful way (NCTM, 2000). A safe environment is one where the ideas of the teacher and the students are mutually respected. Ideas are generated by the teacher, but ideas are also introduced by the students. A teacher has to know how to determine what a student is thinking about a mathematical idea. Discourse refers to the ways students learn through representing, thinking, talking, and agreeing and disagreeing about mathematics. A "high" assessment for discourse was given when the teacher created and maintained a safe and respectful environment where students discussed and explored mathematics, where there was exceptional rapport among all in the classrooms and the students are independent thinkers. The findings in this study suggest that teachers have very different views on what the environment in the classroom should be. Some of the instruction was teacher-led instruction while some was centered more around student involvement. Some teachers were beginning to show the beginning stages of effective discourse. Teachers had begun to lead discussions, and tactics such as "revoicing" (Forman & Ansell, 2002) were utilized. However, there were few, if any, student led discussions. Characteristics of effective discourse such as the teacher orchestrating discussions based on student ideas that were also mathematically productive and worthwhile (Sherin, 2002) were not evident.

Ms. Adams had a classroom that seemed to promote the highest discourse-oriented teaching (Williams & Tate, 2003). However, she was still struggling with having to ask multiple questions and leading the students to respond, it was apparent that she valued discourse but was still grappling with how to implement discourse effectively. She mostly *revoiced* student response as discussed in the study by Forman and Ansell (2002), focusing on what Sherin (2002) referred to as *process*. Ms. Cook focused on individual questions and did very little to promote whole group discussions. Ms. Johnson asked questions, but unlike Ms. Adams, focused mainly on *content* as defined in the study by Sherin (2002), meaning she focused on mathematical questions and responses. Unlike the others, Mr. King and Ms. Parker did very little to promote any mathematical discussions. Mr. King wanted a quiet teacher dominated classroom, and Ms. Parker did not have enough control of her classroom to have any effective discourse.

Equity. It is the role of the teacher to have a classroom in which lessons are accessible to all students and ensure that all students are actively participating in the mathematics. Most of the students in this study seemed to be involved in the lessons in some way, but steps have to be taken to ensure that all students are given an opportunity to learn, meaning students are provided circumstances to engage in relevant mathematics (National Research Council, 2005). Studies show that inequitable practices include tracking students on a non- algebra track or having low expectations for students by not providing learning opportunities adequate for learning mathematics in a meaningful way (Haberman, 1991; Means & Knapp, 1991; Silva & Moses, 2000; Tate, 2005). The focus in this study was on what Tate (2005) called content exposure and coverage variables, the amount of time students spend on a topic and the richness of the instruction provided for that topic (Tate, 2005).

Equity ensures that all students were learning important mathematical content. In this study, equity was assessed “high” when the teacher exhibited high expectations and high quality instruction for all students. The teacher provided opportunities for all students to interact purposefully with mathematics. Access relates to resources the students had available to them (Gutierrez, 2007). In these classrooms, technology and supplies were available to the students. The curriculum was not watered down and the environments in the classrooms were conducive to learning. However, I observed a lack of *active* engagement most of the time by the student in the classrooms in this study. Most participation was initiated by the teacher and the students did not have ownership of their mathematics knowledge. In other words, the students’ learning was teacher-led. Research suggests that students need challenging mathematical content and high level instruction that focuses on sense making and problem solving, relevant to all students (Flores, 2007). This was not evident in the classrooms in this study.

Ms. Adams was aware of trying to get all students involved in mathematics in her classroom. After realizing that particular students tried to monopolize class discussions, she drew names out of a bag to provide for fairness and promoting active participation by all. Ms. Cook got all involved by having everyone in the class write on occasion. Ms. Johnson also seemed to have a classroom where all the students were actively involved in the lessons, but did not develop identity for themselves.

Mr. King’s class did not reflect equity because of the authoritarian atmosphere, and he did not seem to value any student input. Instead of problem solving through investigation, formulating questions, and verbally, numerically, or graphically representing situations, students solved routine, well-defined problems (Tate, 1995). Ms. Parker did little to ensure all were involved because of the chaos in her classroom.

Connections. In effective instruction, it is the teacher's role to make sense of the mathematics and to help students make appropriate connections within and outside of the content. While, most of the teachers in this study did attempt to make connections within the content, none of the findings demonstrated connections being made to the real world or *external* connections as defined by Shroeder (1993). Students could have a grasp on knowing mathematics within the context of the school, but do they apply this knowledge in the real-world? A "high" assessment for connections was given when students encountered mathematics that was realistic and made sense. Prior knowledge was utilized to build new knowledge. Connections were made within mathematics and the real-world. The findings from this study showed that some of the teachers effectively made connections across mathematical topics and some sense making was evident. The findings, however, did not show connections being made between mathematics and its application in other fields or in the real world (Shroeder, 1993).

Ms. Johnson had the highest score in connections. This is again because of the questions she asked. Ms. Adams and Ms. Cook mostly made connections within the content itself or *internal* connections (Shroeder, 1993). There was a slight attempt to make a connection in one of the lessons taught by Ms. Parker, and the researcher saw no attempts to make connections in Mr. King's class.

Conclusion. The teachers of this study determined what and how each of the four components of effective teaching, as defined in the review of literature, was utilized in his/her classroom. The teachers in this study implemented the components of effective teaching on different levels. Some implemented little, or none of the characteristics of effective teaching, while others utilized at least some of the components. For the teachers that did utilize some of the effective teaching components, questioning was a common theme throughout. Teachers asked questions related to the content, but most were procedural instead of higher-order. Teachers asked questions to promote effective discourse, but had to keep asking questions to lead the students, rather than empowering the students to initiate any thoughtful discussions on their own. The teachers called on students to get the students involved and participating in the lessons, but this had become a routine of the same types of questions with minimal wait time, which limited student's empowerment as independent thinkers. Teachers asked students questions to help them make connections within the content, but never helped the students make real-world connections. The teacher played an essential role in implementing, or in many of these cases, not implementing, the four components of effective teaching.

How Do These Roles Affect the Role of the Student and Student Learning?

In drawing from the review of literature, the student plays an important role in effective mathematical teaching and learning. The role of the mathematics student is to engage in mathematical activities such as exploring, justifying, proving, generalizing, and reflecting on ideas, representations, and procedures of their strategies for solving a mathematical problem (Fosnot, 1996; Lampert, 1990; Simon & Schifter, 1991). The student must be engaged intellectually with the mathematics content. If students are verbalizing "when will I ever use this again," chances are it means nothing to them nor do they intend to ever understand. Haberman (1991) identifies characteristics of good teaching that emphasize how both the student and the teacher cooperate. The teachers help the students to see major concepts, big ideas, and general principles and are not merely engaged in the pursuits of isolated facts. The students are involved with issues they regard as vital concerns. Students are asked to think about an idea in a way that questions common sense or a widely accepted assumption that relates new ideas to ones learned previously, or that applies an idea to the problems of living. Students are actively involved in a real-life experience. Students are involved in planning what they will be doing.

In effective instruction, the student's role is to listen, respond, and question the teacher and each another. He/she should be able to reason, make connections, solve problems and communicate. The student should initiate problems and questions, and defend his/her position on these problems by presenting his/her solutions, exploring examples and counter-examples, and relying on mathematical evidence to convince himself and others (NCTM, 1991).

Although the focus of this study was on the role of the teacher, the role of the student should have complemented any effective teaching and learning (Bruner, 1990; Cobb & Yackel, 1996). The role of the student is largely dependent on how the teacher implements his or her role in the mathematics classroom. In reflecting on the teacher's roles in content, discourse, equity, and connections, it could be said that only some of the teachers even attempted to implement these strategies effectively. It would naturally follow that the role of the students would tend to be minimal, if it existed at all. In three out of five of the classes, the students were engaged in the mathematical content. Although the tasks were not always rich enough to elicit multiple representation, the students did seem to understand the importance of the mathematics. However, in two of the classes, students were either completely tuned out or were simply going through the motions and procedures.

Also, the students in one particular class did seem to be involved in more of a discourse oriented class (Williams & Tate, 2003) than the other four. Even though the teacher had a long way to go in creating an effective discourse environment, the students had begun to respond and were learning that mathematical discussions are an important part of their learning. All of the other classes students contributed mainly when specifically called on and little or no classroom discussions took place.

The student’s role in equity seemed generally positive because students were actively involved in the lessons in three of the classrooms. Students in these classrooms seemed to value the importance of mathematics. Unlike these classrooms, the students in the other two classes did not seem valued at all because the teacher either dominated the classroom, or the environment in the classroom prohibited involvement.

The student’s role in making connections within the content seemed apparent only when the teacher asked appropriate questions. At no time did students voluntarily make connections on their own. There were also few if any, connections being made by the students to the real world.

The final answer to this question may be inferred from the results from the pretest and post test given in each class. Based on the assessments of each of the teachers in the four components, it appears that Ms. Cook, Ms. Adams, and Ms. Johnson should have the biggest gain and highest scores on the posttest. Ms. Adams and Ms. Cook did have the highest scores and improvement which correlates with both of these teachers implementing some of the components of effective teaching. This, however, was not the case with Ms. Johnson. In fact, Ms. Johnson's scores decreased over the semester, which seems to suggest that although, on the surface, some characteristics of effective teaching were seen, this classroom may still be characterized by teacher direction, and the students were thus dependent on her for any mathematical thinking. Since there were few, if any, of the effective teaching characteristics seen in Mr. King's and Ms. Parker's classrooms, the results of the tests in both of these classrooms were not surprising.

Synthesis of Findings

Despite the calls for a shift from isolated teaching skills to more coherent teaching strategies (Ball, 2000; Pape, Bell, & Yetkin, 2003), all available evidence suggests that classroom practice on the whole has changed little in the past 100 years (Stigler & Hiebert, 2004). While I can conclude that some of the teachers in this study were implementing some of the components of effective teaching in the classroom. Overall there was a lack of effective instruction pertaining to content, discourse, equity, and connections, as defined in the framework for the study.

In reflecting on the teacher's roles in content, discourse, equity, and connections, it could be said that only some of the teachers even attempted to implement these strategies effectively, but none encompassed all the components that defined effective content, discourse, equity, and connections. As stated earlier, the role of the student is largely dependent on how the teacher implements the role of the teacher in the mathematics classroom. This study affirms that, not only the teachers, but also the students, play key roles in the learning of mathematics.

Implications for Practitioners

This study involved five teachers and their students in the Algebra I classroom. It was my goal not to bring outside variables into the study such as teacher and student demographics, parental influence, and so forth, but to solely concentrate on what was happening in the classroom and to look at how classroom practices influenced student achievement. It could be assumed that the teachers in my cases had attained various numbers of professional development hours in a systemic change initiative, their classroom practices would vary. In fact, the descriptions of the classrooms show that the teaching and learning that took place was very different. Some of the teachers had begun implementing components of effective teaching; however, none of the classrooms exhibited *all* the characteristics of these effective teaching practices. The role of the teacher in the classroom is essential in effective teaching. Throughout this study, a number of implications for teachers, administrators, and teacher educators are evident.

Implications for Teachers

From an instructional viewpoint, a shift from teacher-dominated classrooms towards classrooms emphasizing student-constructed knowledge and understanding can be developed when the teacher implements effective teaching practices. Recommended effective teaching practices include content that is meaningful and worthwhile, an environment that promotes discourse, ensuring that *all* students are learning mathematics, and understanding how mathematical ideas interconnect and build on each other within mathematics and in contexts outside of mathematics. Knowing and doing are not the same thing. Participating in professional development is meaningless if it does not show up in their classroom practice. Moreover, if teachers only implement some effective practices in bits and pieces, the characteristics of effective teaching are fragmented and become essentially ineffective. Teachers must continue to move forward in their role of incorporating effective teaching practices to launch the role of the student. These cases suggest questioning plays a key role. Questions can determine whether or not students become engaged in their role in learning mathematics.

A classroom that has no management can not be effective, regardless of intentions, as was seen in the case of Ms. Parker. Teachers must provide students with an environment that is conducive to learning. The students must feel safe, respected, and confident. Otherwise, nothing productive can occur.

Finally, effective teaching practices must be aligned with student assessment. Clearly, concerns about accountability play a role in classroom practices. Assessment has to be more than a test given at the end of instruction to measure student performance under certain conditions. Assessment must be integrated into instruction to help inform and guide instructional decisions. Research shows that assessment that reflects the teaching and learning components of the classroom, leads to improved students' learning (NCTM, 2000).

Implications for Administrators

Teachers may be covering the same objectives, but as this study shows, this can look quite different in different classrooms. Administrators need to be aware of the practices used in their teacher's classrooms. They should support effective teaching practices. Teachers should be allowed the freedom to implement effective practices in their classrooms without the pressure of an administrator contradicting or prohibiting these teaching practices. In fact this should be expected.

Also, teachers must be provided opportunities to sustained professional development focused on improving instruction. Implementing effective teaching practices is hard work and does not happen overnight. Professional development must be considered a long-term investment (Fonzi & Borasi, 2002). Administrators should allow their teachers to attend supportive workshops offered throughout the year. If needed, coaches should be brought in the classrooms to collaborate and reflect on classroom practices. Administrators should also provide resources, as well as professional development on how to utilize those resources, to help teachers implement effective practices. Administrators must support and encourage their teachers throughout the process of change.

Implications for Teacher Educators

Despite the fact that research suggests little overall progress has been made in teaching practices at the high school level, the teachers in this study were beginning to acquire the tools needed to begin to make a change. Teacher educators need to be aware that these changes do not happen overnight (Fonzi & Borasi, 2002). Simply attending a workshop does not ensure that teachers are equipped to make such changes. Research in teacher change reveals that a one-time workshop or seminar is unlikely to result in significant long term change. Rather, change requires multiple opportunities to learn, practice and reinforce new behaviors. The best workshops, seminars, and institutes are designed to include a variety of modes through which learners can process information. These include journal writing, analysis of case studies, role playing, small group discussions, modeling lessons, engaging in problem solving, and creating classroom materials (Fonzi & Borasi, 2002).

Finally, rather than having to constantly retrain teachers to incorporate effective teaching practices, teacher educators should develop teacher educations programs that incorporate these effective teaching practices. These practices should be implemented as the new teachers enter their own classrooms rather than having to "unlearn" ineffective practices.

Implications for Further Research

While this study begins to address steps that can be taken towards implementing effective teaching in the classroom, several additional directions for research are discussed.

First, the students in the classrooms in this study were ninth graders taking Algebra I. Since the students at this school in this study could take Algebra I in the seventh or eighth grades, the students in the ninth grade classrooms that I studied were considered "grade level", which actually means they were average or below in terms of achievement. Further research should explore whether students in more advanced classes may perhaps get more effective teaching. Do teachers lower their expectations in these classrooms and act differently than they would in a seventh or eighth grade classroom? (O'Neil, 1992).

When I began designing this study, my original intent purpose was to explore how classroom practices influenced student achievement. However, given the design of the study, I found it very difficult to draw any conclusions about student achievement. This was due at least in part to the content test I used, which covered all the objectives for Algebra I, as stated in the course of study. Further research could explore the characteristics of effective teaching more deeply by specifically focusing on a single unit and then testing only those objectives covered in that unit to make more conclusive inferences about the impact teaching practices have on student learning.

The study showed that only some of the teachers were implementing the effective teaching components in their classrooms. Even the teachers that utilized these practices, only implemented some of the characteristics of effective teaching, or had just begun laying the foundations for using these effective practices. Greater attention needs to be given to the progress teachers make in becoming effective teachers or to determine if they just continue implementing fragments of effective teaching components. Future studies could include longitudinal research to measure growth over a period of time, or follow up observations could investigate these classrooms.

Finally, if the teacher determines what is going on in his/her classroom, what are the attitudes, beliefs, and concerns of the teacher that influences this teacher's instruction? If a teacher believes that learning is constructed knowledge, then the teacher should be inclined to provide opportunities for students to explore, investigate, use a variety of problem solving strategies. Beliefs about teaching and learning mathematics by direct instruction should be portrayed by teaching by telling, and using memorization of rules, formulas, and procedures to solve problems. Further research could explore the alignment of teacher beliefs and teacher practices.

Concluding Thoughts

Algebra is considered central in improving mathematical achievement. Teachers are the key in this process. "Qualified teachers who are committed to the learning of their students are the single most important factor for student success" (Flores, 2007, p. 38). This study suggests that role of the teacher is essential in shaping students' learning opportunities. Students construct their own knowledge, but the role of the teacher is the vehicle to this construction of knowledge. NCTM (2000) asserts "students' understanding of mathematics, their ability to solve problems, and their confidence in, and disposition toward, mathematics are all shaped by the teaching they encounter in school" (pp. 16-17). The role of the teacher is to enable students' learning. This study demonstrates that some teachers are beginning to implement at least some aspects of effective instruction practices that influence student learning. Effective teaching can be characterized by emphasizing the need for active learning by the students with meaningful content; creating a learning environment that is conducive to learning by promoting discourse; making mathematical understanding accessible to *all* students; and having mathematics "make sense" to the learner (NCTM, 2000). This process of change, however, is not simple, nor does it happen overnight. Nonetheless, we need to maintain our commitment to finding ways of ensuring teachers are implementing effective teaching practices in their classrooms, which in turn affects the learning of students and provides them all the opportunities they all deserve.

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APPENDICES

APPENDIX A

INFORMATION LETTER TO PRINCIPAL AND TEACHERS

One of the goals of Project Math is for students to be able to apply their knowledge to solve problems in mathematics and in real life. Students need to understand not just how to do mathematics, but why it works. To accomplish this goal, Project Math seeks to expand the teachers' mathematical knowledge, as well as their range of instructional tools, so that they can increase the learning of all students. Rather than relying on "show and tell", teachers will help students to become more autonomous learners of mathematics through the use of engaging problems and innovative instructional practices.

My research interest is getting a description of the Algebra I classroom. The methodology used in this research will be several case studies that focus on the roles the teacher and students play in the classroom by looking through the lenses of content, discourse, equity, and connections. Furthermore, the cases will incorporate attention to how these roles interact and what happens in student learning. Also, a diagnostic test will be given to the students near the beginning of the course and towards the end of the course to assess how students are performing. Permission will be sought from the teachers, students, and parents/guardians to conduct the observations, interviews, and testing. This will all be done with minimal intrusion to the class.

Thank you and your teachers for allowing me to conduct this research. I will be happy to share the results with you and the teachers.

Lora Merchant Joseph

APPENDIX B

CONSENT FORM FOR TEACHERS

INFORMED CONSENT BY TEACHERS

You are being invited to participate in a research study related to a project whose goal is to improve mathematics education in East Alabama. The study is being conducted by____________ , Professor in the Department of Curriculum and Teaching at and director of the project, along with faculty members at________. The project is being conducted by a partnership of__________, and fifteen school districts in the area, including the district in which you are currently teaching. The project's goal is to improve students' mathematics achievement and learning through changes in educational policies and practices. You were selected as a possible participant because you are a teacher in a school which has agreed to participate in initial data collection.

If you decide to participate, we will ask you to complete a questionnaire about your beliefs, knowledge, and attitudes related to the teaching and learning of mathematics. Completion of the questionnaire should take approximately 45 minutes. You may also be chosen to participate in an interview providing additional information about your knowledge of mathematics. Participation in the interview will take approximately 30 minutes. A trained investigator may also observe you teaching a mathematics classroom in order to assess the pedagogical methods used. This, however, should not require that you take any additional time. This data collection will be repeated on an annual basis for the next six years. As such, if you agree to participate we will be recontacting you each of the following years.

Any information obtained in connection with this study that can be identified with you will remain confidential. No information will be shared with anyone who has supervisory responsibilities over you nor will it be shared with any of your colleagues. To minimize the potential risk that any information gathered will be inadvertently divulged, we will use a unique code to identify you, and any identifying information will be removed as the information is transcribed. The original documents containing identifying information will be stored in a secure location, and the key linking codes with identifying information will be stored in a separate, secure location. Information collected through your participation may be used to meet dissertation requirements of graduate students associated with the project, published in a professional journal, and/or presented at a professional meeting. If so, none of your identifiable information will be included. All data that might identify you, including the list of codes, will be destroyed one year after the conclusion of the study.

Your decision whether or not to participate will not jeopardize your future relations with __________.Note that you may withdraw from participation at any time, without penalty, and that any data which has been collected may be withdrawn, as long as that data is identifiable.

As a result of your participation in this project, you may experience increased effectiveness in carrying out your duties related to mathematics teaching and learning, resulting in increased mathematics achievement and learning by your students. Moreover, if the model developed in this project is successful, it may also benefit teachers and students in other parts of the state. I cannot, however, promise that you will receive any or all of the benefits described. No additional compensation will be offered for participating in the research study, although stipends may be offered for participation in other selected activities of the project.

By agreeing to participate in this study, you will be provided professional development designed to improve your effectiveness as a teacher. However, this professional development may occur at different times, depending on your school's assignment to one of five cohorts that will begin participation in the project in the following six years. All teachers who agree to participate in the study will have an opportunity to participate in the project's activities at some time in the following six years.

For more information regarding your rights as a research participant you may contact the _________Office of Human Subjects Research or the Institutional Review Board by phone

HAVING READ THE INFORMATION PROVIDED, YOU MUST DECIDE WHETHER OR NOT YOU WISH TO PARTICIPATE IN THIS RESEARCH STUDY. YOUR SIGNATURE INDICATES YOUR WILLINGNESS TO PARTICIPATE.

Participant's signature Date	Principal investigator's signature Date
Print Name	Print Name
School Name	

Interviews and classroom observations may be audio or videotaped. The tapes will only be used for research purposes, allowing qualified researchers to review the event after the fact, or for educational purposes, such as professional development. In no case will a tape be used for any commercial enterprise, disseminated beyond the______, or used in anyway designed to cause a negative perception. Please sign below if you agree to allow audio and videotaping.

Participant's signature

APPENDIX C

CONSENT FORM FOR STUDENTS

INFORMED CONSENT BY PARENTS/GUARDIANS OF STUDENTS

Your child has been invited to participate in a research study related to a project whose goal is to improve mathematics education in East Alabama. The study is being conducted by________, Professor in the Department of Curriculum and Teaching at _______ and director of the project, along with faculty members at _________. The project is being conducted by a partnership of __________ and fifteen school districts in the area, including the district in which your child is enrolled. The project's goal is to improve students' mathematics achievement and learning through changes in educational policies and practices. Your child was selected as a possible participant because she or he is a student of a teacher who is participating in the project.

If you decide to allow your child to participate, we will ask your child to complete a questionnaire about his or her beliefs and attitudes related to the teaching and learning of mathematics. Completion of the questionnaire will take 30-45 minutes and will be conducted as a part of the regular school day. In addition, she or he may be asked to participate in an interview providing additional information about her or his knowledge of mathematics. Participation in the interview will take approximately 30 minutes and will be scheduled with your child's teacher as a part of the regular school day. We may also gather additional information (such as transcripts) about your child from school records. This data collection will be repeated on an annual basis for the next six years. As such, if you agree to let your child participate we will be re-contacting you each of the following years.

Any information obtained in connection with this study that can be identified with your child will remain confidential. No information will be shared with your child's teacher. To minimize the potential risk that any information gathered will be inadvertently divulged, we will use a unique code to identify your child, and any identifying information will be removed as the information is transcribed. The original documents containing identifying information will be stored in a secure location, and the linking codes with identifying information will be stored in a separate, secure location. Information collected through your child's participation may be used to meet dissertation requirements of graduate students associated with the project, published in a professional journal, and/or presented at a professional meeting. If so, none of your child's identifiable information will be included. All data that might identify your child, including the list of codes, will be destroyed one year after the conclusion of the study.

Your decision whether or not to participate will not jeopardize your future relations, or your child's future relations, with __________. Note that your child may withdraw from participation at any time, without penalty, and that any data which has been collected may be withdrawn, as long as that data is identifiable.

As a result of your child's participation in this project, his or her teachers may become more effective in teaching mathematics instruction, resulting in improved achievement for your child and other children. Moreover, if the model developed in this project is successful, it may also benefit teachers and students in other parts of the state and nation. I cannot, however, promise that your child will receive any or all of the benefits described. No compensation will be offered for participating in the research study.

For more information regarding your rights as a research participant you may contact the __________ Office of Human Subjects Research or the Institutional Review Board by phone
.

HAVING READ THE INFORMATION PROVIDED, YOU MUST DECIDE WHETHER OR NOT YOU WISH TO ALLOW YOUR CHILD TO PARTICIPATE IN THIS RESEARCH STUDY. YOUR SIGNATURE INDICATES YOUR WILLINGNESS TO ALLOW YOUR CHILD TO PARTICIPATE.

____
Parent's signature Date

____
Principal investigator's signature Date

____
Print Name

____
Print Name

____
Print Student's Name

____
School Name

____
Math Teacher's Name

Interviews and classroom observations may be audio or videotaped. The tapes will only be used for research purposes, allowing qualified researchers to review the event after the fact, or for educational purposes, such as professional development. In no case will a tape be used for any commercial enterprise, disseminated beyond the ______ project, or used in anyway designed to cause a negative perception. Please sign below if you agree to allow audio and videotaping.

____
Parent's signature

APPENDIX D

TEACHER QUESTIONNAIRE

I. Teacher Questionnaire

1. Describe yourself as a teacher.

2. Describe your typical Algebra I class. How do you view your role? How do you view your students' roles?

3. Describe a successful Algebra I lesson.

4. How would you describe your confidence level in your ability to teach Algebra I?

5. Do you think the content in Algebra I is appropriate for the developmental levels of all students?

6. Is Algebra I useful in the real-world? In what ways?

7. What types of questions do you typically ask in the Algebra I classroom?

8. Realistically, do you believe all students can succeed in Algebra I? Explain.

9. What kind of student do you prefer to teach and interact with?

10. Do you expect all of the students to be actively engaged in the Algebra I classroom? If so, how do you get them engaged?

APPENDIX E

OBSERVATION PROTOCOLS

Observation Protocol Content

Name:
School:
Class and Time

For each item identified below, circle the number
to the right that best fits your judgment of its quality.
Use the scale above to select the quality number.

CONTENT	Scale				
	never				To a great extent
TEACHER					
1. The mathematics was significant and worthwhile	1	2	3	4	5
2. The mathematics content was appropriate for the developmental levels of the students in this class	1	2	3	4	5
3. The teacher appeared confident in his/her ability to teach mathematics	1	2	3	4	5
4. The teacher provided accurate content	1	2	3	4	5
5. The teacher displayed an understanding of mathematic concepts	1	2	3	4	5
6. Mathematics was portrayed as a dynamic body of knowledge continually enriched by conjecture, investigation analysis, and/or proof/justification	1	2	3	4	5
7. Elements of mathematical abstraction were included when it was important to do so	1	2	3	4	5
8. Appropriate connections were made to other areas of mathematics, to other disciplines, and/or real-world contexts.	1	2	3	4	5
9. The degree of "sense making" of mathematics content within the lesson was appropriate for the developmental levels/needs of the students and the purposes of the lesson	1	2	3	4	5
10. The design of the lesson reflected careful planning and organization	1	2	3	4	5
STUDENT					
11. Students were intellectually engaged with important ideas relevant to the focus of the lesson	1	2	3	4	5
12. The student displayed an understanding of mathematic concepts	1	2	3	4	5
13. Students were involved in conjecture, investigation analysis, and/or proof/justification	1	2	3	4	5
14. Appropriate connections were made to other areas of mathematics, to other disciplines, and/or real-world contexts	1	2	3	4	5
15. Students made sense of the mathematics	1	2	3	4	5

Narrative:

Observation protocol

Name:
School:
Class and Time

For each item identified below, circle the number
to the right that best fits your judgment of its quality.
Use the scale above to select the quality number.

ENVIRONMENT	Scale				
	never				To a great extent
TEACHER					
16. Active participation from all students was expected, encouraged, and valued.	1	2	3	4	5
17. There was a climate of respect for the students' ideas, questions, and contributions.	1	2	3	4	5
18. The lesson was designed to engage students as members of a learning community	1	2	3	4	5
19. The focus and direction of the lesson was determined by ideas originating with teachers	1	2	3	4	5
20. The set up of the lesson was encouraged to generate ideas, questions, conjectures, and/or propositions	1	2	3	4	5
21. Intellectual rigor, constructive criticism, and the challenging of ideas were evident	1	2	3	4	5
22. The teacher's classroom management style/strategies enhanced the quality of the lesson	1	2	3	4	5
23. The design of the lesson encouraged collaboration among the students	1	2	3	4	5
24. The pace of the lesson was appropriate for the developmental needs/levels of the students and the purposes of the lesson.	1	2	3	4	5
25. The teacher was able to "read" the students' levels of understanding and adjusted instruction accordingly	1	2	3	4	5
26. The teacher's questions triggered divergent modes of thinking	1	2	3	4	5
27. The teacher's questioning strategies were likely to enhance the development of conceptual understanding/problem solving	1	2	3	4	5
STUDENT					
28. Students actively participated	1	2	3	4	5
29. Students were comfortable sharing ideas, questions, and contributions	1	2	3	4	5
30. The students acted as members of a learning community	1	2	3	4	5
31. The focus and direction of the lesson was determined by ideas originating with students	1	2	3	4	5
32. Interactions reflected students working together and talking to each other about the lesson	1	2	3	4	5

Observation protocol

33. Students generated ideas, questions, conjectures, and/or propositions	1	2	3	4	5
34. Intellectual rigor, constructive criticism, and the challenging of ideas were evident.	1	2	3	4	5
35. Students collaborated	1	2	3	4	5
36. Students responded to questions of higher order	1	2	3	4	5

NARRATIVE:
The observer will make a written record of each question asked by the teacher. If the teacher's statement is asked in a questioning manner or has the intent of a question, it will be included in the manuscript. Questions will be categorized into:

a. Factual or procedural
b. Cognitive or conceptual

Observation protocol

Name:
School:
Class and Time

For each item identified below, circle the number
to the right that best fits your judgment of its quality.
Use the scale above to select the quality number.

EQUITY	Scale				
	never				To a great extent
TEACHER					
37. Active participation was expected, encouraged, and valued from all students	1	2	3	4	5
38. The instructional strategies and activities reflected attention to issues of access, equity, and diversity for students.	1	2	3	4	5
39. The instructional strategies and activities used in this lesson reflected attention to students' experience, preparedness, prior knowledge, and/or learning styles.	1	2	3	4	5
40. The mathematics content was appropriate for the developmental levels of the students in this class	1	2	3	4	5
41. The teacher's interactions indicate that the teacher sees himself/herself as part of the community, sees teaching as giving back to the community, and encourages his/her students to do the same	1	2	3	4	5

Observation protocol

42. Actions indicate a belief that all students can succeed	1	2	3	4	5
43. Help students make connections between their community, national, and global identities	1	2	3	4	5
44. Indicate that teaching is “digging knowledge out” of students by asking appropriate questions	1	2	3	4	5
STUDENT					
45. Students used a variety of means to represent phenomena	1	2	3	4	5
46. Students had high self-esteem and a high regard for others	1	2	3	4	5
47. Students indicate they believe they can succeed	1	2	3	4	5
48. Students see themselves as contributors in society indicated by participation in the lesson.	1	2	3	4	5

Seating Chart:

The first step is to sketch a seating chart with the students’ characteristics (name, race, gender, low-, high-, average-achiever) in each box. Pre-conference with the teacher will determine student characteristics. Code each interaction:

Q = teacher question
P = teacher praise
C = teacher criticism
Q = student question
r = student volunteered response relevant or correct
x = student volunteered response irrelevant or incorrect

Narrative:

Observation protocol

Name:
School:
Class and Time

For each item identified below, circle the number
to the right that best fits your judgment of its quality.
Use the scale above to select the quality number.

CONNECTIONS	Scale				
	never				To a great extent
TEACHER					
1. Appropriate connections were made to other areas of mathematics, to other disciplines, and/or real-world contexts	1	2	3	4	5
2. The degree of “sense making” of mathematics content within the lesson was appropriate for the developmental levels/needs of the students and the purposes of the lesson	1	2	3	4	5
3. The instructional strategies were consistent with investigative mathematics.	1	2	3	4	5
4. The design of the lesson reflected careful planning and organization	1	2	3	4	5
5. The design of the lesson encouraged collaboration among the students	1	2	3	4	5
6. Adequate time and structure were provided for wrap-up	1	2	3	4	5
7. STUDENT					
8. Appropriate connections were made to other areas of mathematics, to other disciplines, and/or real-world contexts	1	2	3	4	5
9. The degree of “sense making” of mathematics content within the lesson was appropriate for the developmental levels/needs of the students and the purposes of the lesson	1	2	3	4	5
10. The students collaborated with one another	1	2	3	4	5
11. Students were able to apply or generalize skills and concepts to other areas of mathematics, other disciplines, and/ or real-life situations	1	2	3	4	5
12. Students are involved in activities with a clear sense of purpose and/or clear link to conceptual development rather than an activity for activity’s sake.	1	2	3	4	5

Narrative:

APPENDIX F

TEACHER INTERVIEW

Interview with teacher

1. What is the name/title of this course?
2. Can I have a copy of the lesson plan?
3. Tell me about the ability level of students in this class?
4. Where does this lesson fit in the sequence of the unit you are working on? What have the students experienced prior to today's lesson?
5. What was the purpose of today's lesson?
6. Why did you teach the mathematics topics/concepts/skills in this lesson?
7. How do you think the lesson went? What did the students gain?
8. How do you feel about teaching this topic?
9. Do you think this lesson encouraged connections in mathematics? How so?
10. Do you think this lesson made appropriate connections to real-world contexts? How so?
11. What was your role in today's lesson?
12. What was the students' role in today's lesson?

APPENDIX G

CONTENT INSTRUMENT

Multiple Choice

Identify the letter of the choice that best completes the statement or answers the question.

____ 1. Simplify: $(x - 4)(x + 4)$

a. x^2 c. $x^2 + 8x + 16$

b. $x^2 - 8x - 16$ d. $x^2 - 16$

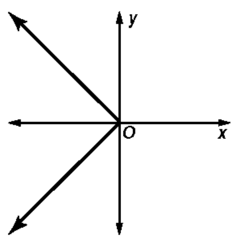
____ 2. Factor: $25x^2 - 25$

a. $25(x - 1)$ c. $25(x + 1)$

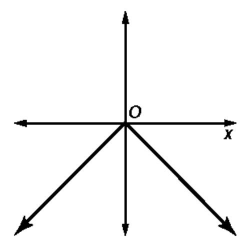
b. $25(x - 1)(x + 1)$ d. $25(x - 1)(x - 1)$

____ 3. Which of these graphs represents a function?

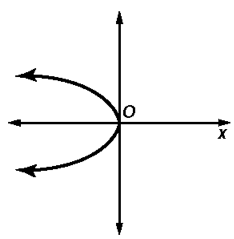
a.



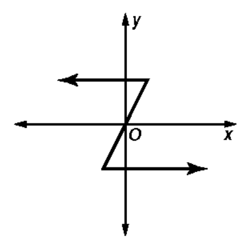
c.



b.



d.



____ 4. Simplify: $3x - 6\left(\frac{1}{2}x - \frac{1}{12}y\right) - \frac{2}{3}y$

a. $-\frac{1}{6}y$ c. $-\frac{3}{5}y$

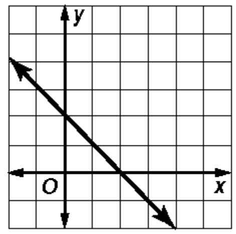
b. $6x - \frac{7}{6}y$ d. $6x - y$

____ 5. Factor: $4p^2 + 16p + 15$

a. $(4p + 3)(p + 5)$ c. $(2p + 3)(2p + 5)$

b. $(4p - 3)(p - 5)$ d. $(2p - 3)(2p - 5)$

_____ 6. What is the equation of the line shown in the graph?



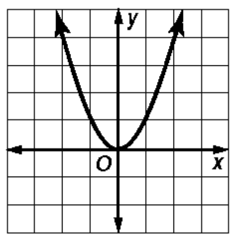
a. $y = x - 2$

b. $y = -x - 2$

c. $y = -x + 2$

d. $y = x + 2$

_____ 7. Which of these equations represents the graph below?



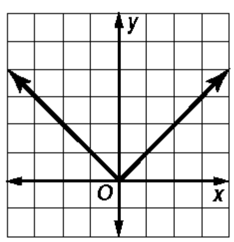
a. $y = |x|$

b. $y = x$

c. $y = \sqrt{x}$

d. $y = x^2$

_____ 8. Which of these equations represents the graph below?



a. $y = x$

b. $y = |x|$

c. $y = -x$

d. $y = x^2$

_____ 9. A model car is built to a scale of 1:24. If the length of the model is 4 inches, what is the length of the actual car?

a. 6 feet

b. 8 feet

c. 12 feet

d. 96 feet

______ 10. Solve: $7x^2 + 10x = 8$

a. $\frac{4}{7}, -2$

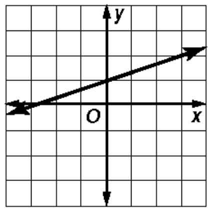
b. $\frac{1}{7}, -8$

c. $-\frac{1}{7}, 8$

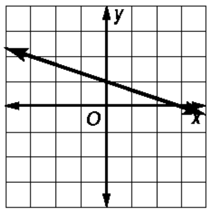
d. $\frac{4}{7}, 2$

_____ 11. Which of these graphs represents the equation $y = -\frac{1}{3}x + 1$?

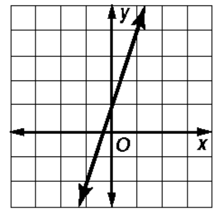
a.



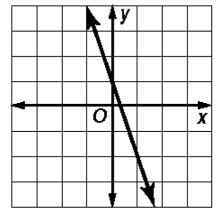
b.



c.

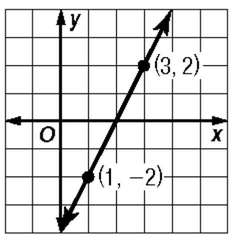


d.



_____ 12. What is the slope of the line shown in the graph?

Slope Formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$



a. 2

b. $\frac{1}{2}$

c. –2

d. $-\frac{1}{2}$

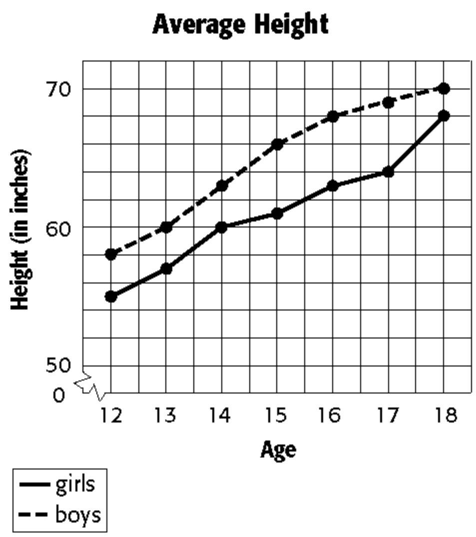
_____ 13. The table shows the distribution of family members at a campout. To select the planning committee for the next campout, each person's name is written on a slip of paper.

FAMILY MEMBERS AT REUNION	
Last Name	**Number of Members**
Richardson	10
Clark	4
Morris	5
Mudersbach	7
Souder	9
Total Attending	35

If one slip of paper is drawn at random, what is the probability of selecting someone with a last name of Richardson or Morris?

a. $\frac{3}{7}$
b. $\frac{4}{7}$
c. $\frac{1}{2}$
d. $\frac{2}{49}$

_____ 14. The double-line graph shown below compares the average heights of the girls and boys measured at Dr. Moore's office at different ages. At which ages is the difference between the average heights about 3 inches?



a. 12, 13, and 14
b. 13, 14, and 15
c. 14, 15, and 16
d. 15, 16, and 17

Short Answer. Please show any work and write your answer in the space provided.

15. Solve: $-y + 3 = -2y + 15$

16. Simplify $x^{-2} \bullet x^{-3}$

17. Solve the equation $x^2 - 7x - 30 = 0$

18. Find the range of $f(x) = x^2 + 2x - 3$ when given the domain $\{-2, 0, 2\}$

19. Find the radius of a circle with area of 300 square inches. Round to the nearest whole number.

20. Solve the following system of inequalities graphically: $4x + 2y \geq 8$ and $y < x$. Name 3 points in the solution set.

These problems require you to show your work and/or explain your reasoning. You may use drawings, words, and/or numbers in your answer. Your answer should be written so that another person could read it and understand your reasoning. It is important to show all your work for each part in the space provided.

21. Quentin bought a used car for $11,700. Each year the value of the car decreases by $1250.

    a. Write an equation modeling the value of the car over time. Let x represent the number of years Quentin owns the car, and let y represent the value of the car in dollars.
    b. What is the slope and what does it mean in the context of the problem?
    c. What is the y-intercept and what does it mean in the context of the problem?

22. Anna turned in the following work in her Algebra class. Should the teacher give her full credit for her solution? Explain why or why not.

Solve this system:

$$y = x - 5$$
$$3y + 2x = 5$$

Solution:

$$3(x-5) + 2x = 5$$
$$3x - 15 + 2x = 5$$
$$5x = 20$$
$$x = 4$$

23. A box of ceramic tiles provides 90 square feet of floor covering. Find the length in whole feet of the largest square room that can be covered with one box of the ceramic tiles.

24. Are the graphs of $y=(x-2)^2$ and $y=x^2-2$ the same? Explain your answer and sketch the graphs.

25. Plot the points A(0,2), B(0,-2) and C(3,1) on a coordinate plane. What is the perimeter of triangle ABC?

Content Instrument
Answer Section and Alignment

Question	ALCOS OBJECTIVE	AHSGE OBJECTIVE	ANSWER	DESCRIPTION
1	5	I-3	D	Perform operations on polynomial expressions
2	6	I-4	B	Factor difference of squares
3	3	III-1	C	Determine whether the graph is a function
4	5	I-2	A	Perform operations on polynomial expressions
5	6	I-4	C	Factor trinomials
6	2,4	V-1,V-4	C	Determine linear equation from a graph
7	4	V-2	D	Represent graphically common relations
8	4	V-2	B	Represent graphically common relations
9	7	VII-2	B	Model real-world problem involving direct variation
10	9	II-2	A	Solve quadratic equations
11	2,4	V-1,V-4	B	Graphing 2-variable linear equations
12	2	IV-2	A	Finding the slope using the slope formula
13	15	VII-6	A	Estimate probability given data in lists
14	12		A	Compare various methods of data reporting including scatterplots
15	7	II-1	y=12	Solve multi-step equations
16	1	I-1	$1/x^5$	Apply laws of exponents
17	9	III-2	x=10,-3	Solve quadratic equation using zero-product property
18	11	IV-1	{-3,5}	Finding the range of a function when given it's domain
19	2	V-1,V-4	≈ 9 or 10	Solve problems that involve area of a circle
20	8	V-3, II-4	answers vary	Graphing the solution of an inequality
21	7	II-1	see rubric	Model real world problems by developing and solving equations
22	8	II-3	see rubric	Solve systems of linear equations
23	1	n/a	see rubric	Estimating square roots
24	3	n/a	see rubric	Determine the characteristics of functions
25	10,11	n/a	see rubric	Calculate length and find perimeter of polygon

Open-ended response rubrics

#21. Score point	Response attribute
3	All are correct
2	The equation is correct. Slope and y-int. identified, but no meaning
1	Part a) **OR** b) **OR** c) is correct
0	No parts are correct (also blanks, illegibles, off tasks)

#22. Score point	Response attribute
3	All are correct
2	Response correct with explanation to x coordinate but not y coordinate
1	Response correct with no explanation
0	No parts are correct (also blanks, illegibles, off tasks)

#23. Score point	Response attribute
3	All are correct
2	Correct explanation but incorrect solution
1	Correct solution but no or incorrect explanation
0	No parts are correct (also blanks, illegibles, off tasks)

#24. Score point	Response attribute
3	All are correct
2	Correct graphs with explanations without characteristics of parabolas (shifts, etc.) **OR** correct explanations with incorrect graphs
1	Correct graphs with no explanation
0	No parts are correct (also blanks, illegibles, off tasks)

#25. Score point	Response attribute
3	All are correct
2	Plotted points correctly and found lengths of all 3 sides
1	Plotted points correctly and found at least 2 side lengths
0	No parts are correct (also blanks, illegibles, off tasks)

Question	Points possible
1	1
2	1
3	1
4	1
5	1
6	1
7	1
8	1
9	1
10	1
11	1
12	1
13	1
14	1
15	2
16	2
17	2
18	2
19	2
20	2
21	3
22	3
23	3
24	3
25	3
Total points possible	41

APPENDIX H

STUDENT SURVEY

Name:
School:
Class and Time

For each item identified below, circle the number
to the right that best fits your judgment of its quality.
Use the scale above to select the quality number.

STUDENT QUESTIONNAIRE	**Scale** Strongly disagree				Strongly agree
1. Algebra I is useful in the real-world.	1	2	3	4	5
2. I need Algebra I to be successful in life.	1	2	3	4	5
3. The teacher expects me to do well in Algebra I.	1	2	3	4	5
4. The teacher calls on me even if I don't raise my hand.	1	2	3	4	5
5. I actively participate in the Algebra I classroom.	1	2	3	4	5
6. The topics covered in this Algebra I class are significant and worthwhile.	1	2	3	4	5
7. The teacher displays an understanding of Algebra I.	1	2	3	4	5
8. The teacher helps me "make sense" of Algebra I.	1	2	3	4	5
9. I feel engaged in the lessons in Algebra I.	1	2	3	4	5
10. I understand the mathematics in Algebra I.	1	2	3	4	5
11. The teacher expects me to participate in the Algebra I classroom.	1	2	3	4	5
12. I feel like I'm part of a learning community in the Algebra I classroom.	1	2	3	4	5
13. I am comfortable sharing ideas, questions, and contributions in the Algebra I classroom.	1	2	3	4	5
14. The teacher asks me questions that make me think in the Algebra I classroom.	1	2	3	4	5
15. I work and talk together with my peers in the Algebra I classroom.	1	2	3	4	5
16. My teacher believes I can succeed in the Algebra I classroom.	1	2	3	4	5
17. I have high-self esteem in the Algebra I classroom.	1	2	3	4	5
18. The teacher carefully plans and organizes the Algebra I lessons.	1	2	3	4	5
19. I can apply the Algebra I skills and concepts to real-world situations.	1	2	3	4	5

STUDENT QUESTIONNAIRE	Scale Strongly disagree				Strongly agree
20. The students collaborate with each other in the Algebra I classroom.	1	2	3	4	5
21. The teacher provides adequate time for wrap-up in the Algebra I classroom.	1	2	3	4	5
22. The teacher appears confident in his/her ability to teach Algebra I.	1	2	3	4	5
23. Algebra I makes sense to me.	1	2	3	4	5
24. Algebra I is important.	1	2	3	4	5
25. I feel respected in the Algebra I classroom	1	2	3	4	5

APPENDIX I

CODING GUIDE

Four main codes, description, and example

Code Name	Description	Example
Content	This code is used to identify any mathematical content found in the data. In the literature, the component of mathematical content is identified as meaningful and worthwhile. For this first level of analysis, any mathematical content was labeled.	T: All right, you got a negative and a negative, so positive. 144 right? Any number squared gives us a positive 144 right. Any number squared will give us a positive answer right? T: If your discriminant is> 0, you will have 2 rational or irrational roots. If the discriminant = 0, then 1 real, rational root. If discriminant < 0, then you get no solution.
Discourse	Discourse refers to the ways students learn through representing, thinking, talking, and agreeing and disagreeing about mathematics. This code was used to label the teacher orchestrating discourse or student involvement in discourse.	T: Do you want to show us what you're talking about? If you need help you can ask for it. S1: I was wondering why we have a negative four at the beginning. S2: Because of the sign in the middle T: Have you talked about the middle negative the very first thing? Why don't we check our factors really quick to make sure we're on the right track. T: Are you positive about the factors? Student: Yes. T: Do we agree? S2: Okay the first thing is distribute T: Okay, do we agree with what Corey did? S3: yes T: Did anybody do it a different way.

Code Name	Description	Example
Equity	This code is used to identify any segments that show all students learning and/or the opportunity to learn.	T: Does anybody else have a question? T: Do you all agree with this so far?
Connections	This code was used when making sense of the mathematics in contexts within the discipline itself and contexts outside of mathematics, particularly the real-world.	T: What do you think? Let's look at a simpler problem. T: Do ya'll remember anything we've done with quadratics so far?

Descriptive codes and examples

Code name	Example
Connections within content	T: That's one way. So the main idea of using the discriminant is what? T: Open GSP on your laptop. What did we look at yesterday? What value were we looking at?
Real world connections	T: Three fourths would be what? S: Seventy-five. T: If you have fifty cents you're adding seventy-five to it, what do you get? S: You would get $1.25.
Sense making	T: All right, Martez said because the distance from the axis is ….. The same way that you take to fill up the left side you can fill in the right side ….. does this make sense? The problem is special because it has symmetry.
Engaging students	T: I don't think completing the squares is a favorite for a lot of you. Do ya'll like completing the squares?

Code name	Example
Questions for understanding	T: So Christine how did you know that you didn't need to find an X intercept? T: What does the main purpose of the equal sign serve in an equation if you're thinking about working with the equation?
Teacher leading	T: So he could have used quadratics but it is easier to factor it. Factoring or the quadratic formula all will give you the same thing right? They all give you the value of what?
Questions for explanation	T: How do we know we don't have any? S: Because of negative numbers. T: So any time I have negative numbers I won't have any zeros? T: Okay, which means what? T: Is there any way we can check our work?
Trying to get students involved	T: The negative makes it open down. J.T. do you have any idea how to make it narrower? T: You guys need to know about a graph using a table.
Student relevant response	S: It has to be zero in order for it to cross the y axis.
Verification	T: Do ya'll remembering hearing that? So what did we just say the square root was?
Student uncertainty	S: Which way is the right way? S: I've got one. How do you get100?
Student explanation	S: Square root 49 and I got minus 3 over 2. I'm going to write it all the way out plus minus 7 over 2 minus 3 over 2
Opportunity to learn	T: All right it's going to be the same exact page. I do not want you to read the directions, I want you to go over it however way you think it needs to be solved in a most efficient way.

Open codes, description, and examples

Code name	Description	Example
Connection attempt	This code was when a connection was attempted to be made, but the teacher just said it instead of letting the students think about it.	T: So if a solution and an x-intercept mean the same thing then would it be the same thing after you have found them?
Discourse attempt	This is attempt to get the students involved in agreeing or disagreeing, but no wait time or response occurred.	T: All right, do we agree with that? T: Does everybody agree?
Equity attempt	This was an attempt to get all the students involved or allow for different responses, but the teacher just moved on.	T: Does everybody agree? T: Let's hear what some other people think. T:"Did somebody get anything different?
Emphasis on mathematics language	This code emerged because of the emphasis put on mathematical terminology and the communication of mathematical concepts and ideas.	T: Do you understand the words coming out of my mouth? T: Everybody in this classroom, I want you to explain to me on paper when you get the direction to *simplify*. What does it mean and what you are going to do. Explain it in a complete sentence--subject verb agreement. S: Because it's not a square. T: Because it's not a *perfect* square

Code name	Description	Example
Intimidation environment	This code was used to identify any classroom environment that was intimidating or authoritarian in nature.	T: You know what to do right? And if you didn't that's your problem. Can we do anything else? T: If I don't call your name, I don't won't to hear your voice anymore, okay? Do you understand that?
Missed opportunity	This code was used to identify times where the lesson or class discussion had the potential to have implemented one of the four main components of content, discourse, equity, or connections, but was not developed or utilized.	T: Basically all you have to do is plug in the formula. What's A, what's B. Negative B and what's C? T: Any time B is positive you're going to get a negative. Any time B is negative, you're going to get a positive. How many of you understand that? All right you ready? Ya'll see what you're doing right? T: In a negative two, Y would be…? Any other number would not fit in the graph.
Off topic	This code was use because of conversations or statements that came up that had nothing to do with the mathematics lesson that day.	T: We're going to go over the math session, history and language arts and science. We'll take the math section in the fall. The only section you can take now is the math. S: I want to be a dentist.
Question for sake of question	This code emerged when the teacher would throw out a question but had no wait time for the students to respond.	T: Any questions about this problem here? All right, let's move on.

Code name	Description	Example
Teacher explanation	This code was used when the teacher did the explanation of the mathematical concept wih little or no student input.	T: What times seven gives you what? Take a look at B. Okay Y times 8 times two that gives you sixteen. Y times two is 2 Y right? In order to get rid of the squares you have. If you do one side you have to do the other side. You're going to end up with 8 equals (inaudible). That's it. Are there any questions? T: Six times 4A times five equals ….good. So if it works out you know you've got a perfect square.

Codes	*Frequency*
Content	418
Discourse	364
Equity	53
Connections	148