

PRINCIPAL COMPONENT REGRESSION MODELS FOR THERMO-
MECHANICAL RELIABILITY OF PLASTIC BALL GRID
ARRAYS ON CU-CORE AND NO CU-CORE PCB
ASSEMBLIES IN HARSH ENVIRONMENTS

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Aniket Shirgaokar

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VITA

Aniket Shirgaokar, son of Mr. Jeevan Shirgaokar and Smt. Anuja Shirgaokar was born on March 29, 1985 in Aurangabad, Maharashtra, India. He graduated in 2006 with a Bachelor of Engineering degree in Production Engineering from Shivaji University, Maharashtra, India. In the pursuit of enhancing his academic qualification he joined the M.S. Program at Auburn University in the Department of Mechanical Engineering in Spring, 2007. Since then, he has worked for Center for Advanced Vehicle Electronics (CAVE) as a Graduate Research Assistant in the area of harsh environment electronic packaging reliability.

THESIS ABSTRACT

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In the current work, Goldmann constants and Norris-Landzberg acceleration factors have been developed for eutectic Tin Lead and Lead free solders (SAC 305) with the help of statistical tools including Principal Component Regression for reliability prediction and part selection of Plastic Ball grid array packages. Two types of PCB assemblies including PCBs with integral copper core and PCBs with no integral copper core have been tested. The models have been developed based on thermo-mechanical reliability data acquired on packages subjected to several different thermal cycling conditions. The thermal cycling conditions differ in temperature range, dwell times, maximum temperature, minimum temperature to enable development of constants needed for life prediction and assessment of acceleration factors.

Goldmann constants and the Norris-Landzberg acceleration factors have been benchmarked against previously published values. In addition, model predictors have been validated against validation datasets which have not been used for model development. Convergence of statistical models with experimental data has been demonstrated using a single factor design of experiment study for individual factors including temperature cycle magnitude, relative coefficient of thermal expansion, solder volume, diagonal length of chip, etc. The predicted and measured acceleration factors have also been computed and correlated. The correlations achieved are of a good accuracy for different parameters examined. Statistics based log transformed models have been presented to show their power dependencies. Box – Tidwell power law modeling has been demonstrated. The presented methodology is valuable in development of fatigue damage constants for the application specific accelerated test data-sets and provide a method to develop institutional learning based on prior accelerated test data.

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CHAPTER 1

INTRODUCTION

The increasing pressure for developing small, reliable and cheap packages on the microelectronics industry have lead to the use of area array packages. After their wide spread use in the commercial field, PBGAs are now implemented in aerospace and military applications [Ghaffarian 2005]. Considering the various factors like geometric parameters, material properties, thermal cycling conditions which govern the reliability of electronic packages, statistical models have been developed for the data obtained by accelerated life cycling of different boards with Cu core and No core PCB substrates. Principal component regression models are used for life prediction of these packages which are subjected to harsh environments.

It is very important to understand the underlying physics and the mechanical failure theories which govern the failure of the solder joint. The mismatch between the coefficient of thermal expansion between the chip and the module due to the thermal cycling which the chip undergoes, results in shear strains in the solder joint. Thus the mechanical strain along with the time and temperature factors has to be taken into consideration while evaluating the fatigue behavior of solder interconnections under accelerated conditions. Previously researchers have studied the behavior of the solder and developed life predictions for Eutectic Tin Lead solder. With the Electronic industry

going Lead free, there have been many challenges for the researchers to predict the behavior of the solder and thus their failure.

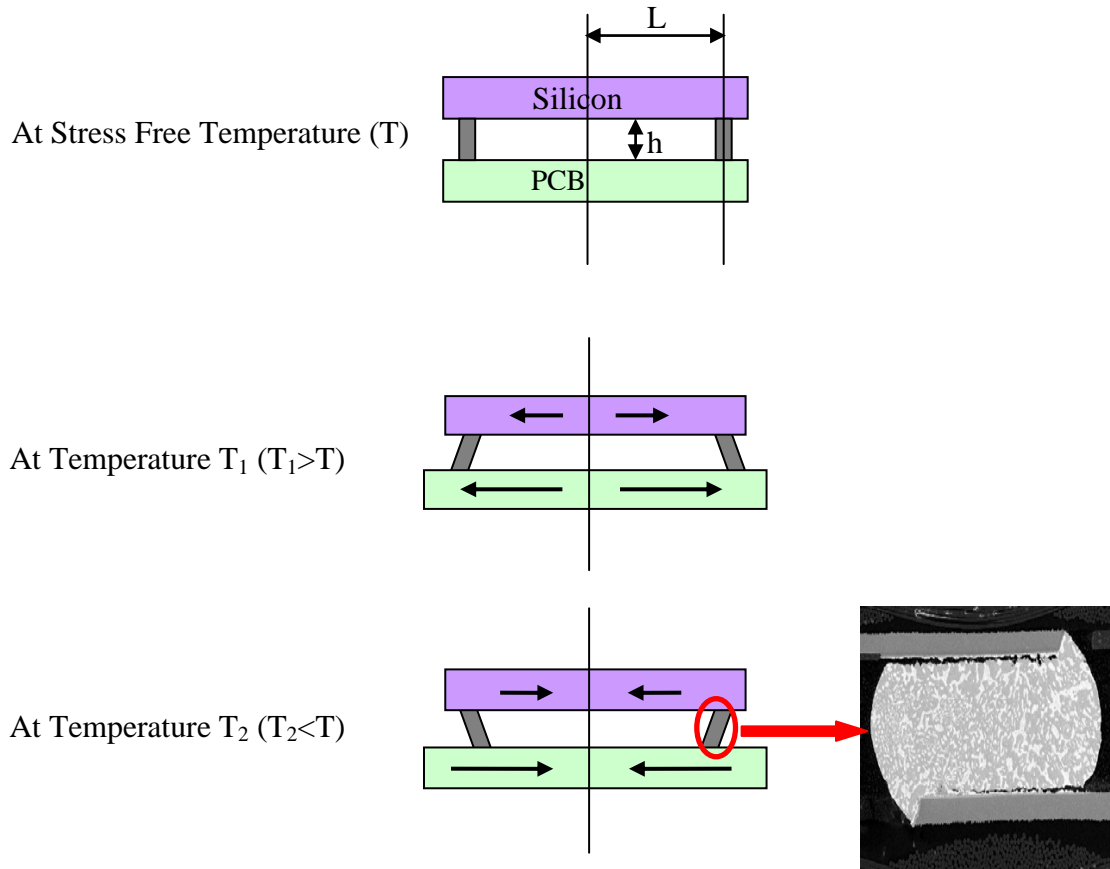


Figure 1.1: Solder joint fatigue failure due to thermal cycling

When the package under goes thermal cycling, may it be an accelerated one or one in the field, the PCB which has a higher coefficient of thermal expansion heats up and expands more than the silicon. When the temperature decreases, due to cessation of the operation or environment, the PCB will contract faster. The expansion and contraction introduces shear strains and shear stresses in the solder joint. High shear stress can cause delamination of various interfaces like UBM/intermetallic, solder/underfill etc. Apart from delamination, the repeated heating and cooling can

eventually cause fatigue of the solder joints. The high shear stresses would enhance the fatigue initiation making solder interconnect more susceptible to such fatigue failures as shown in Figure 1.1 [Singh 2006] represents the same. Hence evaluation of stresses at the joints has become critical to predict the reliability of the assembly.

The Classical Coffin Manson's Equation which related the plastic strain that develops due to the difference in coefficient of thermal expansion is given in the equation 1.1 below:

$$N(\Delta\varepsilon_p)^n = C \qquad \text{Eqn 1.1}$$

Where,

$\Delta\varepsilon_p$ is the plastic strain,

N is number of cycles to failure,

n is empirical constant observed to be 2 for nearly all metals,

C is the proportionality factor.

Goldmann developed his form of the Coffin Manson which is given in Equation 1.2 below

Goldmann`s Equation:

$$N_f = K_T \left[\left(\frac{T_u \Pi r_f^2}{A} \right) \left(\frac{h^{1+\beta}}{V} \right) \right]^{\frac{m}{\beta}} \left(\frac{1}{\delta} \right)^m \quad \text{Eqn 1.2}$$

Where,

K_T is a constant which is a function only of parameters of the testing cycle,

T_u is ultimate shear strength of the critical interface,

r_f is the radius of the critical interface,

A and β are constants in the stress strain relationship,

h is the height of the solder joint,

V is the volume of the solder joint,

δ is the shear deformation of the joint

m is an empirical constant.

The Norris Landzberg Model is given in the following equation 1.3 below:

$$AF = \frac{N_U}{N_A} = \left(\frac{f_U}{f_A} \right)^{\frac{1}{3}} \left(\frac{\Delta T_A}{\Delta T_U} \right)^2 \Phi(T_{\max}) \quad \text{Eqn 1.3}$$

Where,

AF is the Acceleration factor.

Subscript U stands for use-conditions and Subscript A is used for accelerated-test conditions

N_U and N_A are the lives of the packages f_U and f_A are the frequencies

T_A and T_U are the temperature excursions

T_{\max} is the maximum temperature of the cycle in Kelvin

The Equation is often in used in the form [Lau 1997] given by Equation 1.4 below

$$AF = \frac{N_u}{N_A} = \left(\frac{f_U}{f_A} \right)^{\frac{1}{3}} \left(\frac{\Delta T_A}{\Delta T_U} \right)^2 \exp \left(1414 \left(\frac{1}{T_{\max,U}} - \frac{1}{T_{\max,A}} \right) \right) \quad \text{Eqn 1.4}$$

Principal Component Regression is used to formulate these equations in the statistical model. In case of the Goldmann Equation, the Number of cycles to failure is taken as the response variable and the terms on the right hand side of the equation like the ultimate shear strength of the critical interface, height of the solder joint, volume of the solder joint are taken as the response variables. In case of the Norris Landzberg's model the Acceleration Factor which is the ratio of the lives of the package is taken as the response variable and the parameters on the right hand side of the equation like the ratio of the frequencies and the ratio of temperature excursions are taken as the predictor variables.

Principal Component Regression is a method to overcome the multi-collinearity in a regression model by transforming the original predictor variables to a new dataset with the help of Eigen vectors and then transforming the original variables back after the regression is done. It will be discussed in details in the further chapters.

CHAPTER 2

LITERATURE REVIEW

2.1 Experimental Techniques

Various experimental tests, such as Accelerated Thermal Cycling (ATC), Thermal Shock, HAST (highly accelerated stress test) and vibration test, have been used by the researchers to analyze the solder joint fatigue life for qualifying the components for different applications. ATC exposes the packages to a series of low and high temperatures usually in a single air chamber in which the temperature ramp can be controlled carefully. Thus accelerating the failure modes caused by cyclic stresses. Thermal shock testing is a liquid-liquid test in which two liquid chambers at different temperatures are used. Thermal shock tests generate very high ramp rates.

Darveaux, et al. [2000] conducted several board level thermal cycle reliability tests, the packages used included Flex-BGA, Tape Array Ball Grid Array, PBGA and Micro-BGA. He tested wide range of package and board variables and reported findings about life of the package by changing dies size, package size thickness of test boards etc. He also reported 1.6X acceleration factor between -40°C to 125°C and 0°C to 100°C temperature cycling ranges.

Mercado, et al. [2000] conducted test on flip chip PBGA package for FSRAM (Fast Static RAM) application in order to analyze the effect of pad size and substrate thickness on the solder joint reliability. It was reported that C5 solder joints with larger

solder pad and thicker substrates demonstrated higher reliability. Hung, et al. [2000] investigated the effect of chip size, surface finish, Au plating thickness, epoxy thickness, polyimide thickness and underfilling on the interconnect thermal cyclic fatigue life by conducting experimental test on Flex-BGA packages. Chip size, polyimide thickness and underfilling were found to have significant impacts on the joint fatigue life, epoxy thickness was found to have little effect on the joint fatigue life.

Suhling, et.al. [2004] presented research on the thermal cycling reliability of lead free solder joints for use in the automotive industry. Four solder compounds were tested: 95.5Sn3.8Ag0.7Cu and three variations of lead free SAC solder that incorporate small additions of bismuth and indium to enhance fatigue resistance. These solder joint compounds were thermally cycled under two test conditions: -40 C to 125 C, and -40 C to 150 C. Results from this study showed that the eutectic SAC alloy 95.5Sn3.8Ag0.7Cu gave comparable reliability results to standard 63Sn37Pb solder alloy during the -40 C to 125 C temperature condition, but differed greatly, demonstrating much lower reliability relative to the 63Sn37Pb alloy, when subjected to the more harsh -40 C to 150 C temperature range. It was also shown that adding trace amounts of bismuth and indium can enhance the -40 to 150 C thermal cycling fatigue resistance relative to 95.5Sn3.8Ag0.7Cu.

2.2 Physics of failure based models

Manson and Coffin [1965, 1954] developed an equation that related plastic strain $\Delta\varepsilon_p$, with number of cycles to failure. Goldmann [1969] analyzed a controlled collapse joint with spherical dimensions for developing an equation that related the plastic strain of a joint with relative thermal expansion coefficients of chip to substrate, distance from chip neutral point to substrate, height of the solder, volume of solder, radius of the cross section under consideration and exponent from plastic shear stress strain relationship. The plastic strain obtained from Goldmann formulation can be substituted in Coffin- Manson equation for predicting the number of cycles for fatigue failure.

Norris and Landzberg [1969] studied the effect of cycling frequency and maximum temperature of cycling on fatigue failure of solder joints and added an empirical correction factor for time dependent and temperature dependent effects for the thermal fatigue model.

Solomon [1986] analyzed the fatigue failure of 60Sn/40Pb solder for various temperatures and developed an isothermal low cycle fatigue equation that correlated the number of cycles to failure with applied shear strain range. He also studied the influence of frequency, and temperature changes and added corrections that account for temperature changes, cycling wave shape and joint geometries.

Engelmaier [1990] developed a surface mount solder joint reliability prediction model containing all the parameters influencing the shear fatigue life of a solder joint due to shear displacement caused by thermal expansion mismatch between component and substrate. Engelmaier developed separate equation for stiff solder joints and compliant solder joints. The parameters of the model include effective solder joint area, solder joint

height, diagonal flexural stiffness, distance from neutral point and thermal coefficient mismatch thermal cycling conditions, degree of completeness of stress relaxation and slope of weibull distribution.

Knecht and Fox [1991] developed a strain based model using creep shear strain as damage metric to determine the number of cycles to failure. The creep shear strain included creep of component due to matrix creep alone ignoring the plastic work. The equation was applicable to both 60Sn40Pb and 63Sn37Pb solder joints.

Vandevelde [1998] developed thermo-mechanical models for evaluating the solder joint forces and stresses. Barker et al [2002] synthesized the Vandevelde models for calculating the solder joint shear forces in ceramic and plastic ball grid array packages.

Clech [1996] developed a solder reliability solutions model for leadless and leaded eutectic solder assemblies and extended it to area array and CSP packages. Clech obtained the inelastic strain energy density from area of solder joint hysteresis loop and developed a prediction equation correlating inelastic strain energy density with number of cycles to failure.

Singh [2006] developed failure mechanics based models for solder joint life prediction of ball array and flip chip packages. He calculated the maximum shear strain using a simplified DNP formula which was then used for initiating a hysteresis loop iteration for both global and local thermal mismatch. Inelastic strain energy was then calculated from the area of the hysteresis loop for both the thermal mismatch cases. The number of cycles for failure was determined using Lall [2003] model.

2.3 Statistical Analysis

Researchers have used different statistical methods for the analysis of the experimental test failure data, the most common being regression analysis and Weibull distribution. Clech, et al. [1994] presented statistical analysis of thermo-mechanical wear out failure data from 26 accelerated tests and tested the goodness-of-fit using two and three parameter Weibull and log-normal distributions. It was concluded that the three parameter Weibull treatment provides more accurate reliability projections and failure free time prediction, potentially qualifying component assemblies that would be rated marginal or unacceptable based on conservative two parameter Weibull or log-normal analysis.

Stoyanov [2002] used a design of experiments and response surface modeling methodology for building a quadratic equation that related underfill modulus, underfill CTE, stand off height and substrate thickness with number of cycles to failure for a flip chip package. The data for model building was collected from a finite element analysis of a flip chip package. Residual analysis, analysis of variance and statistical efficiency measure were used for validating the models. Taguchi optimization technique was used by Lai [2005] for optimizing the thermo-mechanical reliability of a package on package for various design parameters. The package parameters considered for optimization included die thickness, package size, mold thickness, substrate thickness and solder joint stand off.

Muncy, et al. [2003, 2004] conducted thermal reliability test including air-to-air thermal cycling (AATC) and liquid-to-liquid thermal shock (LLTS) on various configurations of flip-chip on board (FCOB) packages. The failure data was then

analyzed using multiple linear regression and ANOVA (analysis of variance) to determine the parameters that had influence on the reliability performance of the components in accelerated life testing, the input parameters investigated included, substrate metallization, substrate mask opening area versus the UBM area of the flip chip bump, die size, perimeter or area array flip chip interconnect pattern, underfill material, location of the die on the test board, frequency of cycling, number of I/O, and percent area voiding. A model based on regression analysis was developed in order to quantify the effect of process and design decisions on the reliability of a flip chip on board assembly.

Perkins [2004] developed a multiple linear regression based polynomial equation for correlating fatigue life of a ceramic package with its design parameters. A data matrix was formulated using a full factorial design of simulation study for the five design parameters including substrate size, substrate thickness, CTE mismatch between substrate and board, board thickness and solder ball pitch with two levels each. Simulations were run for each data point using a finite element analysis and the fatigue life was extracted. Interactions between the predictor variables were studied and a regression model with both main terms and interaction terms was built.

Iyer [2005] correlated the reliability of a flip chip package with its properties of underfill and flux using a regression and back propagation neural networks based models. Data from accelerated life testing of flip chip package with 95 different underfill flux combinations was used for model building. The underfill parameters for model building included modulus of elasticity, coefficient of thermal expansion, glass transition temperature and filler content. The flux parameters studied include acid number and

viscosity. The regression models and the neural network models were validated using a test data set and the neural networks model was found to outperform the regression model owing to minimum residual mean square errors.

Singh [2006] developed multivariate regression based models for life prediction of BGA packages. The input data for model building was collected from published literature and accelerated test reliability database based on the harsh environment testing of BGA packages by the researchers at the NSF Center for Advanced Vehicle Electronics (CAVE). The predictor variables considered for model building included die, die to body ratio, ball count, ball diameter, solder mask definition, printed circuit board surface finish printed circuit board thickness, encapsulant mold compound filler content and deltaT. Dummy variables were used for categorical variables like board finish, encapsulant mold compound filler content and solder mask definition. Linear, modified linear and non-linear models were developed using regression analysis and analysis of variance and validated with experimental data.

Hariharan [2007] developed MLR and PCR model for Predicting the reliability of various Ball Grid Array Packages including Flex-BGA, CBGA, CCGA and Flip Chip Packages. He also demonstrated the power law dependencies of the various parameters in the regression model with Box Tidwell Power law modeling.

CHAPTER 3

DATASET AND THERMAL CYCLING CONDITIONS

The table 3.1 gives a brief idea of the scope of the packages and the range of the data which was tested for accelerated life and the failure data was utilized for statistical analysis. The database is fairly diverse in terms of materials and geometry parameters. The dataset used for model building has been accumulated from an extensive accelerated test reliability database of plastic ball-grid array (PBGA) and chip-array ball-grid array (CABGA) devices based on the harsh environment accelerated test database developed by the researchers at the NSF Center for Advanced Vehicle Electronics. Each data point in the database is based on the Weibull-Parameters including the time to one-percent failure, characteristic life, and the shape parameter for the area array devices of a given configuration tested under harsh thermal cycling or thermal shock conditions. The material properties and the geometric parameters investigated include die thickness, die size, die to body ratio, substrate thickness, ball count, ball pitch, board finish, solder joint height, solder joint volume, bump size, weight of the package and printed circuit board thickness.

Table 3.1 Scope of the Test Dataset

| Package type | Array type | I/O Pitch (mm) | I/O Count Range | Solder alloy | Package size (mm) | Die Size Range | Package to Die size ratio |
|--------------------|------------|----------------|-----------------|-----------------|-------------------|----------------|---------------------------|
| PBGA | Full | 0.5 - 1.00 | 49 - 900 | Pb-free SAC 305 | 7.0 - 31.0 | 4.00 - 24.0 | 1.00 - 3.94 |
| | Perimeter | | | | | | |
| | Mixed | | | | | | |
| FC-PBGA | Full | 0.8-1.00 | 532 - 1508 | Pb-free SAC 305 | 23.0 - 40.0 | | |
| | Perimeter | | | | | | |
| | | | | | | | |
| MCM-PBGA | Full | 1.00 | 128 - 324 | | 22.0 | | |
| | Perimeter | | | | | | |
| | | | | | | | |
| Hi-TCE CBA | Full | 1.27 | 360 | Pb-free SAC 305 | 25.0 | | |
| | | | | | | | |
| | | | | | | | |
| CBGA | Full | 1.27 | 483 | | 29.0 | | |
| | | | | | | | |
| | | | | | | | |
| CSP | Full | 0.5 - 0.8 | 132 - 228 | Pb-free SAC 305 | 7.0 - 12.0 | | |
| | Perimeter | | | | | | |
| | | | | | | | |
| Flip Chip | Full | 0.25 - 0.45 | 48 - 317 | Pb-free SAC 306 | 5.08 - 6.35 | | |
| | Perimeter | | | | | | |
| | | | | | | | |
| Micro - Lead Frame | | 0.40 - 0.65 | 44 - 100 | Pb-free SAC 307 | 9.0 - 12.0 | | |
| | Perimeter | | | | | | |
| | | | | | | | |
| QFP / LQFP | | 0.4 - 0.5 | 100 - 176 | Pb-free SAC 308 | 14.0 - 20.0 | | |
| | Perimeter | | | | | | |

The figure 3.1 and Figure 3.2 shows the individual packages which were mounted two different types of boards viz. PCB with integral Copper Core and PCB without integral Copper Core for thermal cycling.

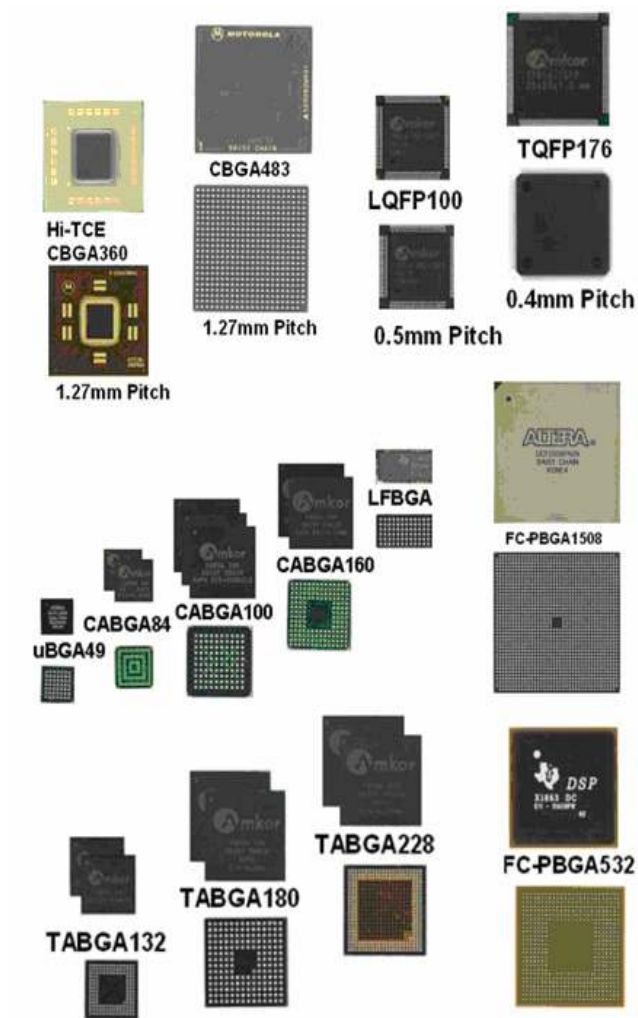


Figure 3.1 Individual Packages tested for ATC

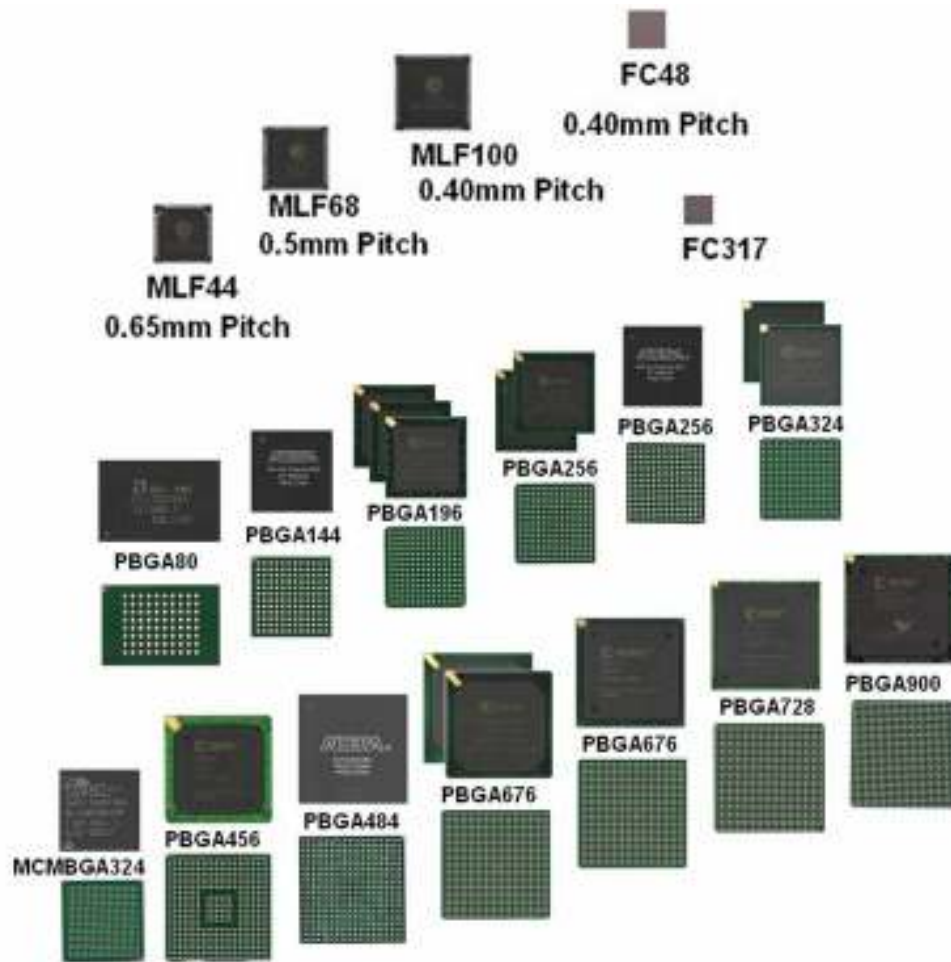


Figure 3.2: Representative List of Different Package Architectures

Table 3.2 shows the temperature ranges, dwell times, and ramp rates for the four thermal cycling profiles labeled as TC1, TC1, TC3 and TC4.

Table 3.2: Thermal Cycling Conditions

| Profile | Low Temp (°C) | High Temp (°C) | Low Dwell (min) | High Dwell (min) | Ramp Rate (°C/min) |
|---------|---------------|----------------|-----------------|------------------|--------------------|
| TC1 | -40 | 95 | 30 | 30 | 3 |
| TC2 | -55 | 125 | 30 | 30 | 3 |
| TC3 | 3 | 100 | 30 | 30 | 3 |
| TC4 | -20 | 60 | 30 | 30 | 3 |
| TC5 | -20 | 80 | 30 | 30 | 3 |
| TC6 | 0 | 100 | 15 | 15 | 3 |
| TC7 | 0 | 100 | 10 | 10 | 3 |
| TC8 | -55 | 125 | 15 | 15 | 3 |
| TC9 | -40 | 125 | 15 | 15 | 3 |

Pictures of the boards which were subjected to thermal cycles are shown in Figures 3.3 to Figure 3.8 below:



Figure 3.3 Front Side of test board CCA 091-099 (TC2: -55C to 125C).

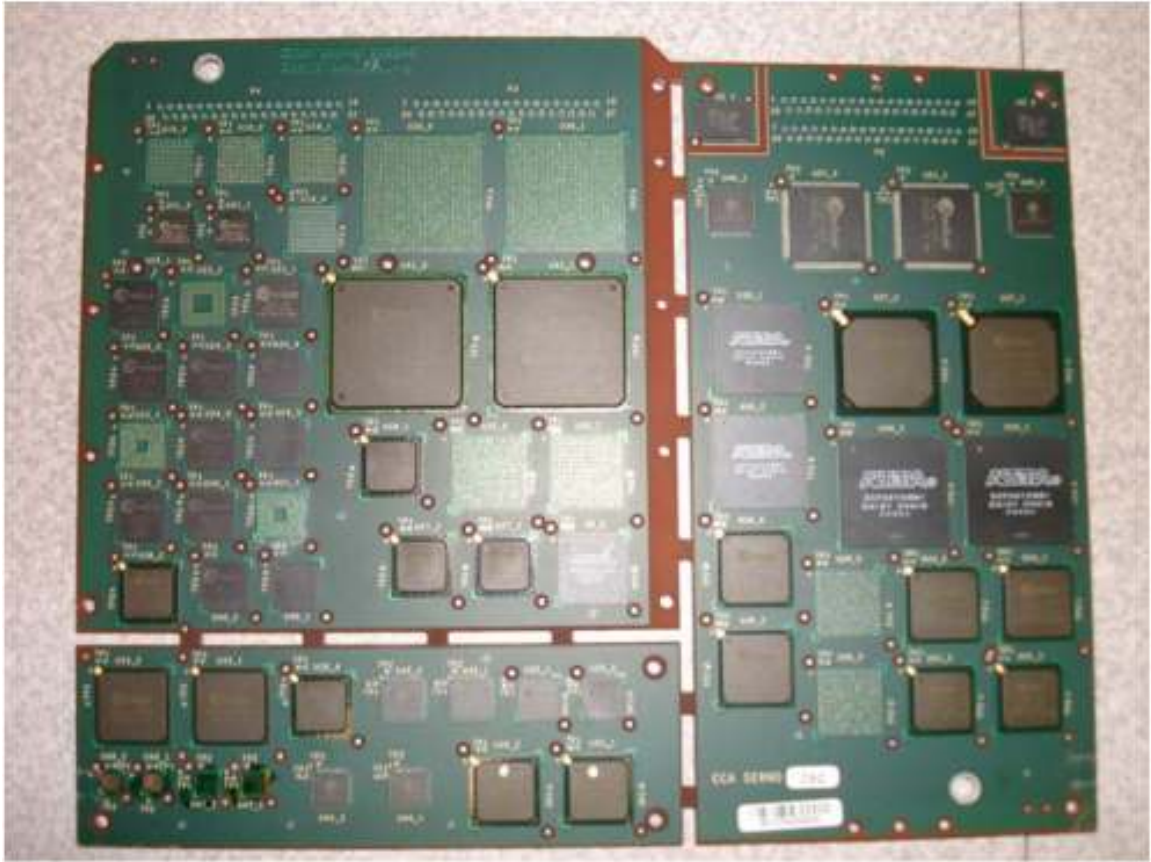


Figure 3.4 Back Side of test board CCA 091-099 (TC2: -55C to 125C).

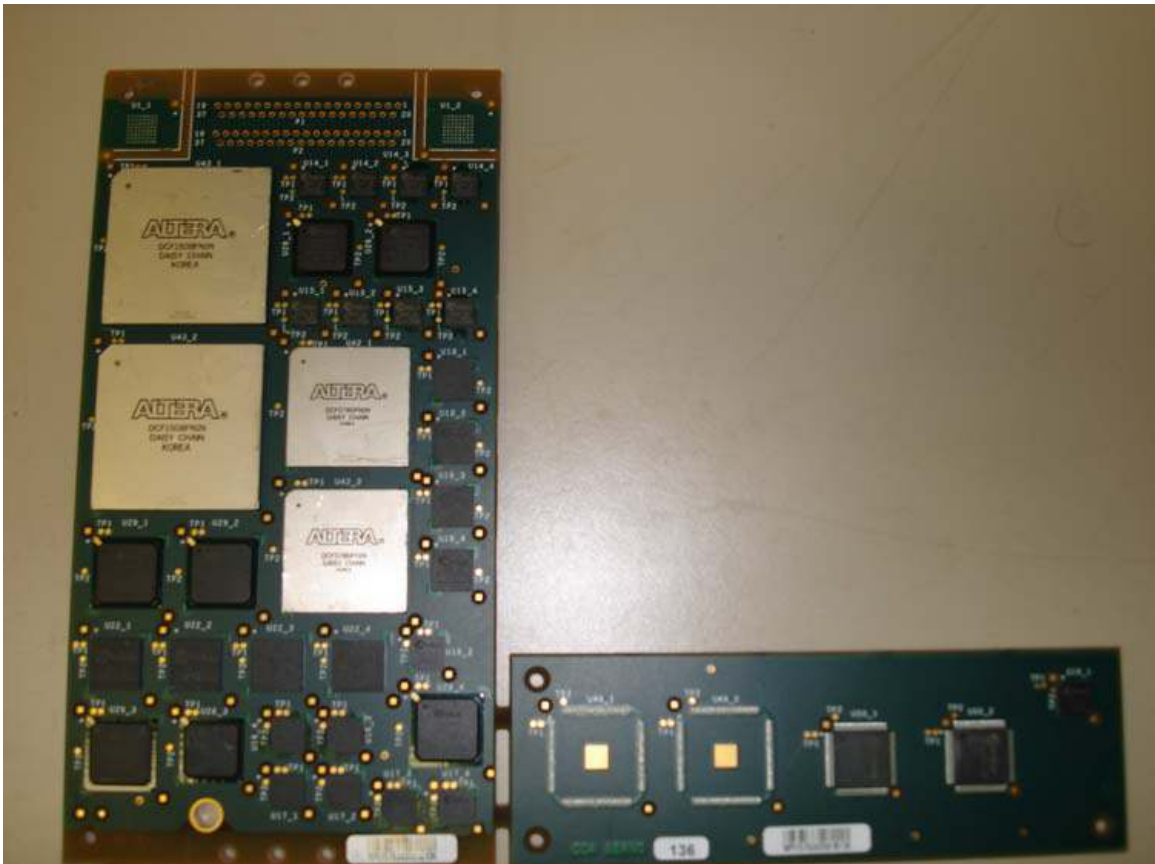


Figure 3.5 Front Side of test board CCA 136-144 (TC3: 3C to 100C)



Figure 3.6 Back Side of Test Board CCA 136-144 (TC3: 3C to 100C).

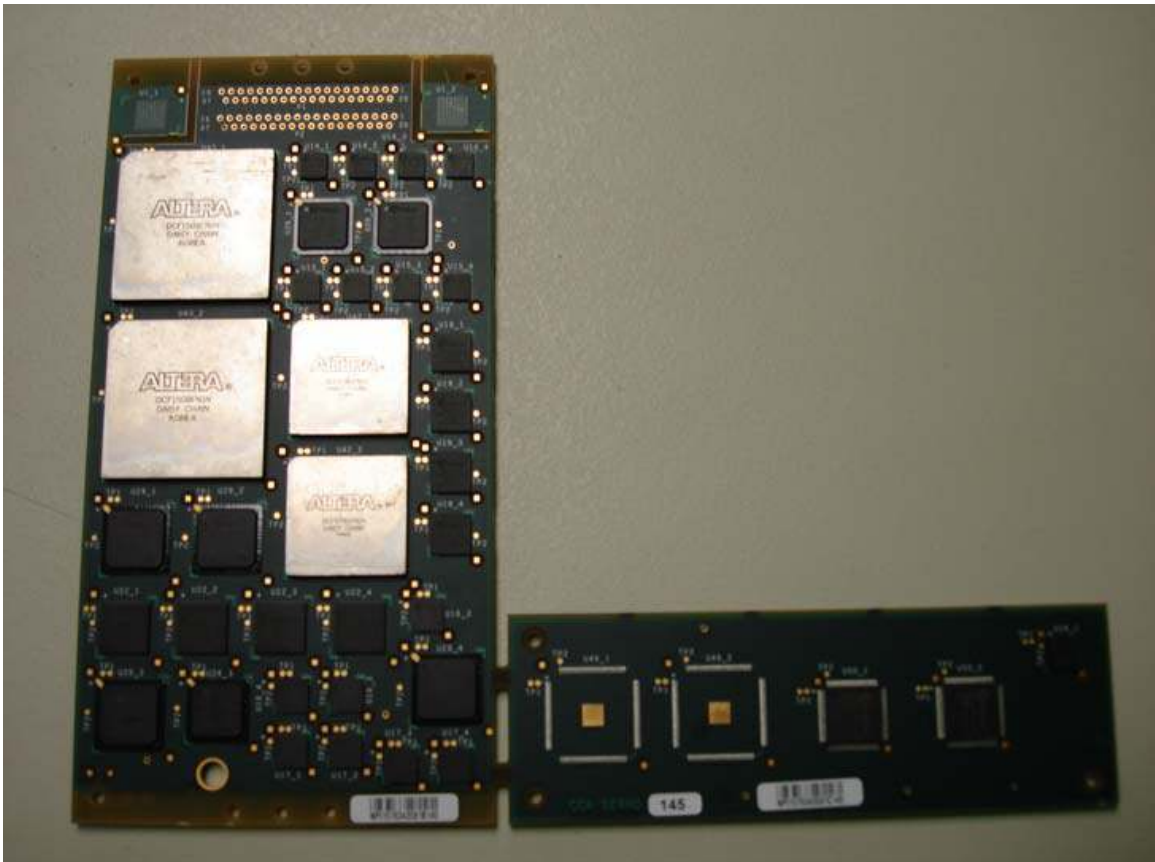


Figure 3.7 Front Side of test board CCA 145-154 (TC4: -20C to 60C).



Figure 3.8 Back Side of test board CCA 145-154 (TC4: -20C to 60C).

CHAPTER 4
APPROACH AND PROCEDURE FOR PRINCIPAL COMPONENT
REGRESSION

Multiple linear regression methods assume the predictor variables to be independent of each other. Linearly dependent variables result in multi-collinearity, instability and variability of the regression coefficients [Cook et al. 1977]. Principal components models have been used for dealing with multi-collinearity and producing stable and meaningful estimates for regression coefficients [Fritts et al 1971]. The Figure 4.1 shows the modeling methodology and procedure for developing the PCR models. The different parameters like part architecture and geometry, thermal cycling environment have been used to formulate the mission requirements using the different statistical techniques like Principal Component Regression, Box Tidwell Transformation. Models have been validated using the other reliability database by comparing the results with failure mechanics models. The effects of the output design parameters and acceleration factors have been presented.

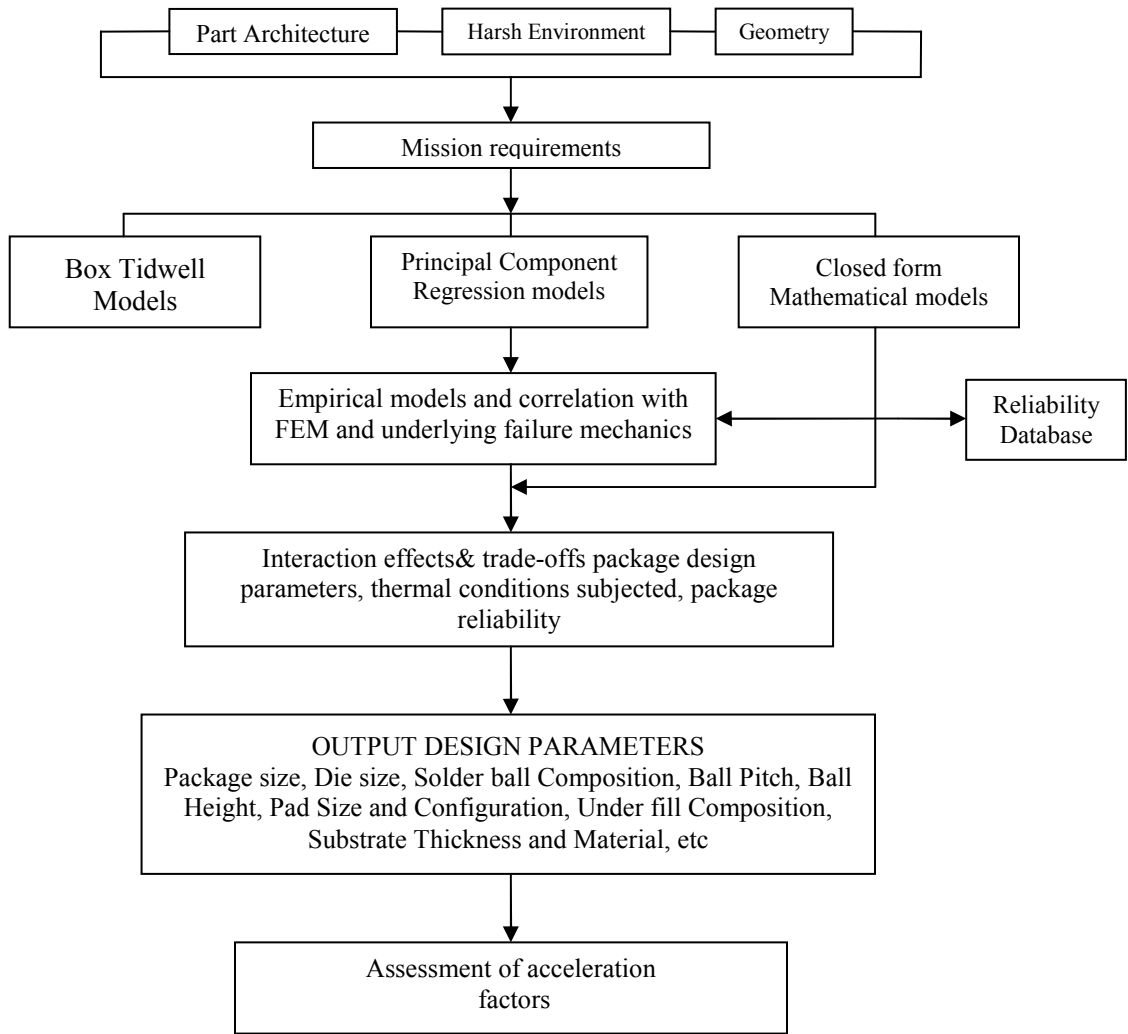


Figure 4.1 Flow Chart for Modeling Methodology

Methodology for developing a Principal Component Regression Model is presented here:

Matrix Notation for the model is given in the Eqn 4.1 below:

$$\{y\} = [X]\{\beta\} + \{\varepsilon\} \quad \text{Eqn 4.1}$$

Where,

$$\{y\} = \begin{Bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ y_n \end{Bmatrix}, X = \begin{matrix} \text{k_predictor_variables} \\ \left[\begin{array}{cccccc} 1 & x_{11} & x_{21} & \cdot & \cdot & \cdot & x_{k1} \\ 1 & x_{12} & x_{22} & \cdot & \cdot & \cdot & x_{k2} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & x_{1n} & x_{2n} & \cdot & \cdot & \cdot & x_{kn} \end{array} \right] \end{matrix} \left. \vphantom{\begin{matrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{matrix}} \right\} \text{n_data_sets}$$

$$\{\beta\} = \begin{Bmatrix} \beta_0 \\ \beta_1 \\ \cdot \\ \cdot \\ \beta_n \end{Bmatrix} \quad \text{and} \quad \{\varepsilon\} = \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_1 \\ \cdot \\ \cdot \\ \varepsilon_n \end{Bmatrix}$$

Multi-collinearity of predictor variables may cause, large variance and co-variance of individual regression coefficients, high standard error of individual regression coefficients in spite of high R^2 values, instable regression models fluctuating in magnitude and sign of regression coefficients for small changes in the specification, and wider confidence intervals of regression coefficients. Previously the problem of multi-collinearity has been overcome by removing one of the variables which resulted in loss of

some influential parameters. The principal components technique determines a linear transformation for transforming the set of X predictor variables into new set Z predictor variables known as the principal components. The set of new Z variables are uncorrelated with each other and together account for much of variation in X. The principal components correspond to the principal axes of the ellipsoid formed by scatter of simple points in the n dimensional space having X as a basis. The principal component transformation is thus a rotation from the original x coordinate system to the system defined by the principal axes of this ellipsoid. The principal component transformation is used to rank the new orthogonal principal components in the order of their importance.

Multiple linear regression is then performed with the original response variable and reduced set of principal components. The principal components estimators are then transformed back to original predictor variables using the same linear transformation. Since the ordinary least square method is used on principal components, which are pair wise independent, the new set of predictor coefficients are more reliable. The Pearson's Co-relation matrix is calculated to check for the multicollinearity in the matrix X. And the Eigen values are used in transforming the original predictor variables in the new Z variables. Scree plots, Eigen values and proportion of total variance explained by each principal component are then used to eliminate the least important principal components.

The Equation for calculation of the Eigen values and the Eigen vector is given in the Eqn 4.2 below:

$$([C] - \lambda[I])[V] \quad \text{Eqn 4.2}$$

$$|[X^*]^T[X^*] - \lambda[I]| = 0 \quad \text{Eqn 4.3}$$

Where λ is the eigen value and V is the eigen vector matrix. The original set of predictors has been transformed (matrix A) to a new set of predictor variables (matrix Z) called the principal components. The principal component matrix Z contains exactly the same information as the original centered and scaled matrix A , except that the data are arranged into a set of new variables which are completely uncorrelated with one another and which can be ordered or ranked with respect to the magnitude of their Eigen values (Draper and Smith 1981, Myers 1986).

$$Z_j = [x_1^* \ x_2^* \ \dots \ x_3^*] \quad \left\{ \begin{array}{c} V_{1j} \\ V_{2j} \\ \cdot \\ \cdot \\ \cdot \\ V_{kj} \end{array} \right\} \quad \text{Eqn 4.4}$$

Eigen_Vector_associated_with_λ_j

MLR has been performed with the transformed predictor variables and the original response variable. The coefficients obtained as a result of this regression model are stored in a variable named alpha. Matrix notation for the same is given by the Equation 4.5:

$$\{\alpha\}_{k*1} = [V]_{k*k}^T \{\beta^*\}_{k*1} \quad \text{Eqn 4.5}$$

The Principal Components have been transformed back to the Original variables. To eliminate the principal components the coefficients are transformed back to the original ones by using the reverse transformation given in the Equation 4.6 below.

$$\{\beta\}_{k*1} = [V]_{k*k} \{\alpha\}_{k*1} \quad \text{Eqn 4.6}$$

The overall adequacy of the model is tested using ANOVA table. Small P value of the ANOVA table rejects the null hypothesis and proves the overall adequacy of the

model. Individual T tests on the coefficients of regression of principal components yielded very small P values indicate the statistical significance of all the predictor variables.

The individual T test values of principal components regression components are then used for conducting individual T test on the coefficients of regression of original variables. The test statistic proposed by Mansfield et al.[1997] and Gunst et al. [1980] for obtaining the significance of coefficients of regression of original variables is given in the Equation 4.7 below:

$$t = \frac{b_{j,pc}}{\left[\text{MSE} \times \left(\sum_{m=1}^l \lambda_m^{-1} v_{jm}^2 \right) \right]^{\frac{1}{2}}} \quad \text{Eqn 4.7}$$

Where $b_{j,pc}$ is the coefficient of regression of the j^{th} principal component, MSE is the mean square error of the regression model with l principal components as its predictor variables, v_{jm} is the j^{th} element of the Eigen vector v_m and λ_m is its corresponding Eigen value. M takes the values from 1 to l , where l is the number of principal components in the model. The test statistic follows a students T distribution with $(n-k-1)$ degrees of freedom. The P values of individual T tests retaining values < 0.05 prove the statistical significance of individual regression coefficients of original predictor variables at a 95 % confidence.

CHAPTER 5

PRINCIPAL COMPONENT REGRESSION ON COPPER CORE ASSEMBLIES

A superset of predictor variables including Area of the chip, Board finish, Die length, Die to body ratio, Ball count, Ball Pitch, Solder ball diameter, Weight of the package, Solder ball height, Solder Volume, Package pad area, and Thermal cycling conditions has been created. The predictor variables have then been checked for being correlated to each other since independence of predictor variables is one of the most important assumptions of a linear regression model. Predictor variables with very strong correlation for e.g. die length and area of the die, which have a correlation factor of almost 1 have been tackled by eliminating one of the two as they convey more or less the same information from analysis point of view. Predictor variables that are needed for model building are then selected through stepwise regression and method of best subsets using the following criteria: Maximization of Coefficient of determination R^2 , Maximization of Adjusted R^2 and Minimization of Residual Errors. Predictor variables which contribute significantly with a confidence level of 95 % and more are retained in the model. The procedure for Principal component regression which is discussed in Chapter 4 in details is then followed to construct the model.

A check for determining the presence of multi-collinearity was done. The Pearson's co-relation matrix and the Variance Inflation Factors have been used to gauge

the intensity of the multi-colinearity. The VIF values in the table 5.1 below are more than 10 and confirm the presence of Multi-colinearity in the model.

Table 5.1 Checking the VIF values

| Predictor | Coef | SE Coef | T | P | VIF |
|------------------|-------------|----------------|----------|----------|--------------|
| Constant | 24370 | 5479 | 4.45 | 0 | |
| BrdFinis | 66.69 | 28.6 | 2.33 | 0.026 | 1.1 |
| DieLenMM | -227.75 | 57.26 | -3.98 | 0 | 89.2 |
| DieToBod | -254.6 | 264 | -0.96 | 0.342 | 4.6 |
| BallCoun | 3.314 | 2.315 | 1.43 | 0.162 | 28 |
| BallPtch | -4745 | 1296 | -3.66 | 0.001 | 167.7 |
| BallHgtM | 10628 | 2412 | 4.41 | 0 | 246.2 |
| SdrVol | 0.08249 | 0.0892 | 0.92 | 0.362 | 5.7 |
| 1/TmeanK | -5855583 | 1481712 | -3.95 | 0 | 36.2 |
| DeltaT | -20.564 | 3.363 | -6.12 | 0 | 37.1 |

The Pearson's correlation matrix in the Table 5.2 below also shows many values greater than 0.8 which suggest the same.

Table 5.2 Pearson's Correlation Matrix

| | BF | DLmm | DTB | BC | BaPtmm | PPdDmm | PWtgm | BHgtmm | SdrVol | DeltaT |
|---------------|-----------|-------------|------------|-------------|---------------|---------------|--------------|---------------|---------------|---------------|
| BF | 1 | -0.01 | -0.01 | 0.01 | 0.02 | 0.02 | -0.00 | 0.00 | -0.00 | -0.01 |
| DLmm | -0.01 | 1 | 0.64 | 0.89 | 0.78 | 0.78 | 0.92 | 0.84 | 0.71 | -0.05 |
| DTB | -0.01 | 0.64 | 1 | 0.67 | 0.14 | 0.14 | 0.63 | 0.20 | 0.19 | -0.09 |
| BC | -0.01 | 0.89 | 0.68 | 1 | 0.62 | 0.62 | 0.99 | 0.63 | 0.52 | -0.08 |
| BaPtmm | 0.01 | 0.78 | 0.14 | 0.62 | 1 | 1 | 0.72 | 0.98 | 0.86 | -0.01 |
| PPdDmm | 0.01 | 0.78 | 0.14 | 0.62 | 1 | 1 | 0.72 | 0.98 | 0.86 | -0.01 |
| PWtgm | -0.00 | 0.92 | 0.63 | 0.99 | 0.72 | 0.72 | 1 | 0.72 | 0.59 | -0.08 |
| BHgtmm | 0.00 | 0.84 | 0.20 | 0.63 | 0.98 | 0.98 | 0.72 | 1 | 0.85 | 0.00 |
| SdrVol | -0.00 | 0.71 | 0.19 | 0.52 | 0.86 | 0.86 | 0.59 | 0.86 | 1 | -0.03 |
| DeltaT | -0.01 | -0.05 | -0.09 | -0.08 | -0.02 | -0.01 | -0.08 | 0.00 | -0.00 | 1 |

Figure 5.1 is the plot of Principal component on X-axis and the Cumulative % contribution of the Eigen value on the Y-axis

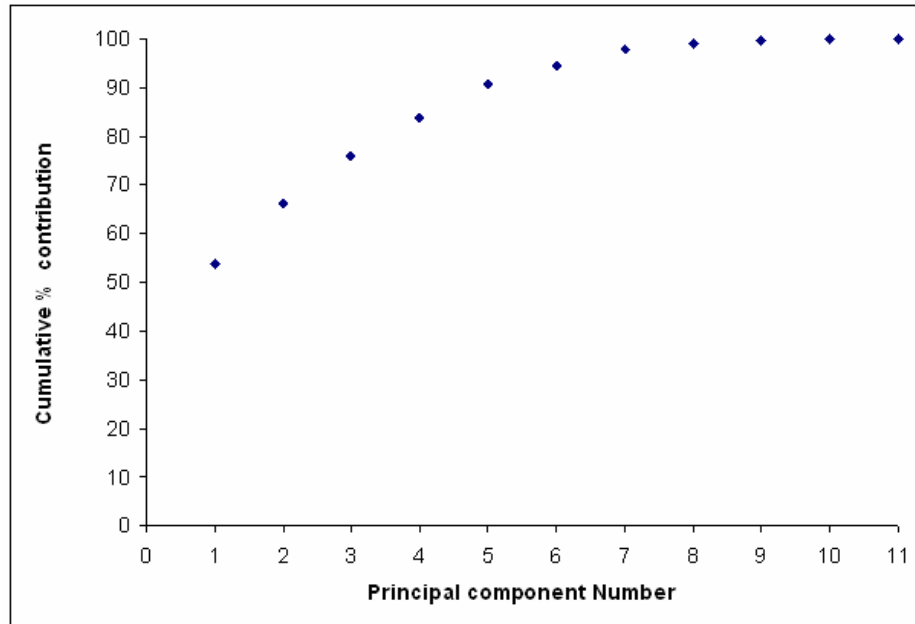


Figure 5.1 Contribution of each Principal Component

The variable selection was done based on the stepwise regression procedure and the partial F-tests which help in selecting the variables which contribute significantly to the linear regression model. One of the tests for ball pitch is demonstrated below:

Partial F test:

Hypothesis: $H_0: \beta_5 = 0$, where β_5 is the slope for ball count

Test statistic:

$$F_0 = \frac{\frac{SS_{Res, Full} - SS_{Res, Reduced}}{Df_{Full} - Df_{Reduced}}}{\frac{SS_{Res, Reduced}}{Df_{Reduced}}} \quad \text{Eqn 5.1}$$

$$\begin{aligned}
& \frac{2866013 - 2856834}{94 - 93} \\
&= \frac{9179}{2856834} \\
&= \frac{9179}{30718.64516} = 0.298808
\end{aligned}$$

As $F_0 = 0.2988 < F_{0.05,1,93} = 3.94$,

We Accept $H_0: \beta_4 = 0$ which implies that the parameter ball count does not contribute significantly to form a linear model. Similarly, all the other variables viz. board finish, area of the chip, Solder Volume and Solder Ball Diameter which fail to contribute significantly to form a linear regression model have been eliminated. A regression model with the rest of the predictor variables is then developed. Results and discussion of the same has been presented below.

The regression equation obtained by regressing the Z predictor variables against the N1% life of the package is given by Eqn 5.2 below:

$$\begin{aligned}
N1\% = & 2859.3 - 390.4 * Z1 - 1075.2 * Z2 + 766.9 * Z3 - 1042.5 * Z4 \\
& + 303.38 * Z5 - 847.1 * Z6 + 382.1 * Z7
\end{aligned} \tag{Eqn 5.2}$$

The Table 5.3 shown below represents a detailed result of the regression of principal components against the N1% life of the packages. The P-values of all the predictors are less than 0.05 suggesting the statistical significance of the 7 predictors with 95% confidence.

Table 5.3 : Transformed Z variable regression for Cu Core Assemblies

| Predictor | Coef | SE Coef | T | P |
|-----------|---------|---------|-------|---|
| Constant | 2859.3 | 237.7 | 12.03 | 0 |
| Z1 | -390.4 | 100.6 | -3.88 | 0 |
| Z2 | -1075.2 | 201.9 | -5.33 | 0 |
| Z3 | 766.9 | 196.5 | 3.9 | 0 |
| Z4 | 1042.5 | 270.1 | 3.86 | 0 |
| Z5 | 303.38 | 62.8 | 4.83 | 0 |
| Z6 | -847.1 | 176.6 | -4.8 | 0 |
| Z7 | 382.1 | 80.86 | 4.73 | 0 |

The Table 5.4 below represents the Analysis of Variance used initially to prove that the predictors have a linear relationship with the response variable N1%

Table 5.4 : ANOVA table for Cu Core Assemblies

| Source | DF | SS | MS | F | P |
|----------------|----|---------|--------|-------|---|
| Regression | 7 | 5156023 | 736575 | 32.56 | 0 |
| Residual Error | 90 | 2035939 | 22622 | | |
| Total | 97 | 7191962 | | | |

Regression equation for original variables is given by Eqn 5.3 below

$$\begin{aligned}
 \text{N1\%} = & 2859.06 + 6.17 * \text{ChipAreaSQMM} - 55.66 * \text{DiagLenMM} \\
 & - 1319.61 * \text{DiatoBodyRatio} - 632.53 * \text{PitchMM} + 1301.55 * \\
 & \text{PkgPdAreaSQMM} - 312.49 * \text{PkgWtGM} - 6.95 * \text{DeltaTDEGC}
 \end{aligned}
 \tag{Eqn 5.3}$$

The Table 5.5 below gives a detailed result of the regression between the transformed original variables and N1%

Table 5.5: Transforming Z back to Original Variables in the Cu Core Assemblies

| Predictors | Coeff | SE | T Value | P-Value |
|----------------|----------|--------|---------|---------|
| (a0, fk) | (bk) | Coeff | | |
| Constant | 2859.06 | 237.66 | 12.03 | 0 |
| ChipAreaSQMM | 6.17 | 1.59 | 3.88 | 0 |
| DiagLengthMM | -55.66 | 10.44 | -5.33 | 0 |
| DietoBodyRatio | -1319.61 | 338.36 | -3.9 | 0 |
| PitchMM. | -632.53 | 163.87 | -3.86 | 0 |
| PkgPdAreaSQMM | 1301.55 | 269.47 | 4.83 | 0 |
| PkgWgtGM | -312.49 | 65.102 | -4.8 | 0 |
| DeltaTDEGC | -6.95 | 1.47 | -4.73 | 0 |

Model Adequacy Checking:

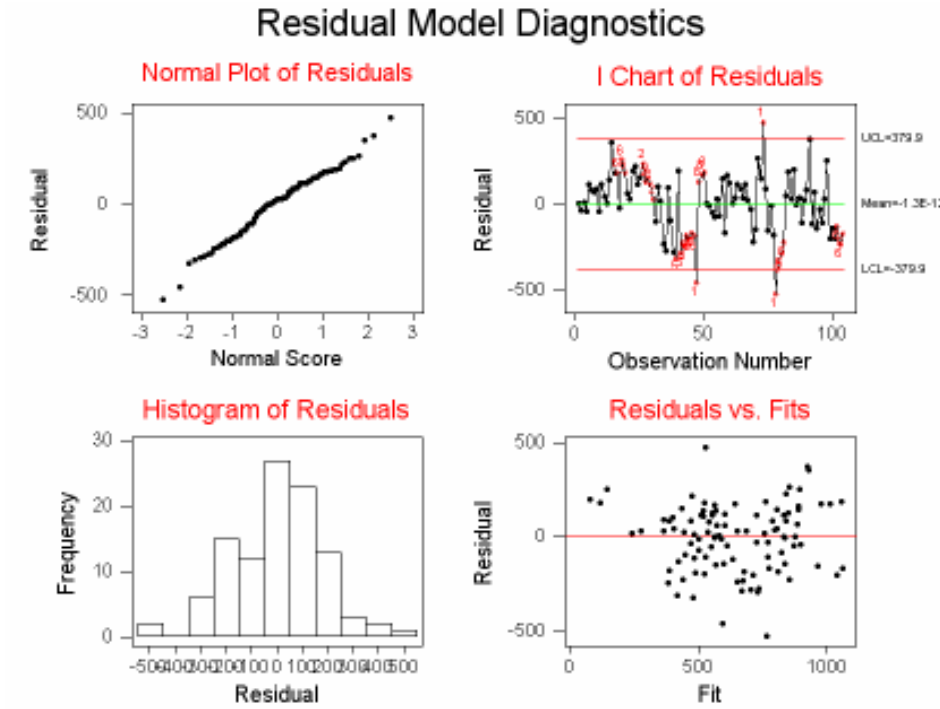


Figure 5.2: Residual Analysis for PCR on Cu Core Assemblies

From the above Figure 5.2, the plot of residuals Vs Fits we do not observe any specific pattern which implies that the linearity assumptions are met.

The plot does not show any signs of the scatter increasing with the fitted values which implies that the constant variance assumptions are satisfied.

Normality Test:

The Shapiro –Wilk test was performed to check if the normality assumptions are satisfied. A P-value of 0.3076 which is > 0.05 confirms the normality assumptions the dataset. The results for the same are shown in Table 5.6 below

Table 5.6: Shapiro Wilk Test

| Test | Test Statistic (W) | P-value |
|--------------|--------------------|---------|
| Shapiro-Wilk | 0.985305 | 0.3076 |

The Figure 5.3, plot of studentized residuals Vs Normal Quantiles also produces points close to a straight line suggesting that the normality assumptions are met same.

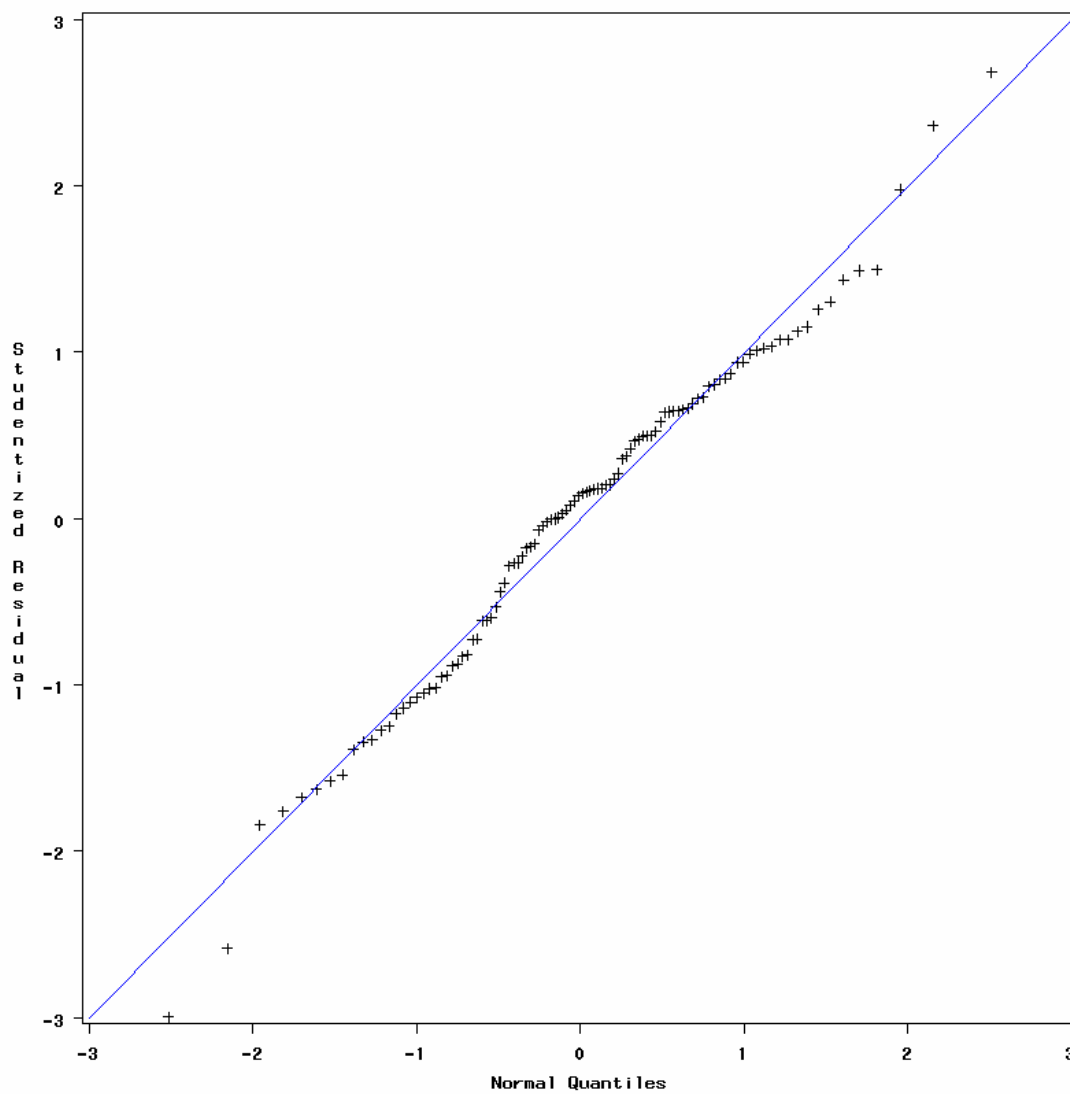


Figure 5.3: Plot of Studentized residuals Vs Normal Quantiles

Model Validation:

Figure 5.4 below shows the correlation of the actual N1% life obtained from the experimentation and predicted N1% life obtained from the PCR Model.

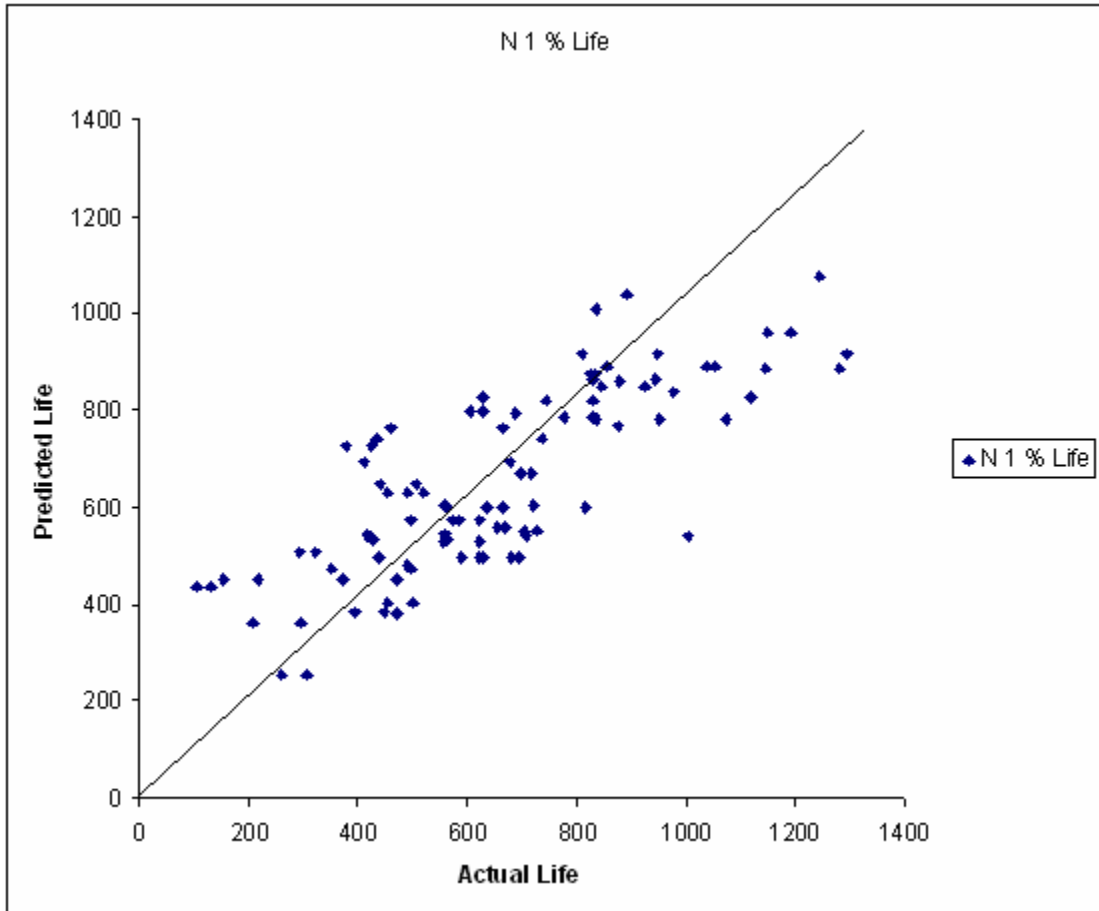


Figure 5.4 Plot of Actual Vs the Predicted Life for the PCR Model for Cu Core Assemblies

CHAPTER 6

PRINCIPAL COMPONENT REGRESSION ON NO COPPER CORE

ASSEMBLIES

An approach similar to the one discussed in Chapter 7 for the Ball Grid Array Packages on copper core Assemblies is used for the assemblies with no Copper core PCBs. A log transformation is done on all the predictor variables to have a better fit to the dataset. Principal Component Regression is used to overcome the Multi-collinearity which exists between the predictor variables. Different Predictor variables like Area of the chip, Chip to Package Ratio, Ball Count, Board finish, Die length, Die to body ratio, Ball Pitch, Solder ball diameter, Weight of the package, Package Pad Diameter, Delta T and Solder Volume have been selected as input variables in the model. The original Matrix X of predictor variable has been transformed to a new matrix Z by multiplying it with the Eigen vector matrix of the correlation coefficients. The contribution of the individual variables has been checked for a 95 % level of significance and only those variables which contribute significantly to form a linear model have been retained. The figure below shows the contribution of individual principal components to the model. The main aim for implementation of Principal Components here is to overcome the multi-collinearity and not dimensional reduction. The following Figure 6.1 is the plot of % cumulative contribution of each Eigen value:

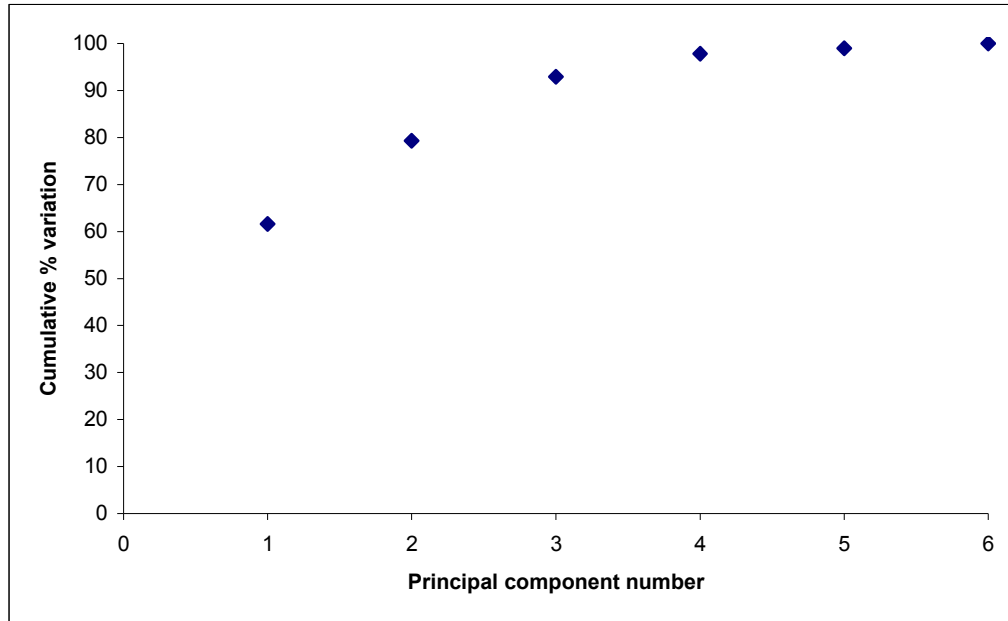


Figure 6.1: Contribution of each Principal Component for PCR of Cu Core Assemblies

Stepwise regression led to the elimination of variables which do not contribute significantly to form a linear regression model with 95 % confidence. The variables which got eliminated in this process are: Board finish, Solder Ball diameter, Package Weight, Ball Pitch and Package length.

Regression equation for Z predictor variables:

The regression equation obtained by regressing the Z predictor variables against the Log transformed N1% life of the package is given by equation 6.1 below:

$$\begin{aligned} \text{LnN1\%} = & 24.492 + 2.856 * Z1 - 4.857 * Z2 - 3.1781 * Z3 \\ & + 1.685 * Z4 + 0.7727 * Z5 + 2.2462 * Z6 \end{aligned} \quad \text{Eqn 6.1}$$

The Table 6.1 shown below represents a detailed result of the regression of 6 principal components against the Ln N1% life of the packages. The P-values of all the predictors are less than 0.05 suggesting the statistical significance of the 6 predictors with 95% confidence.

Table 6.1: Transformed Z variable regression for PCR on No Cu Core Assemblies

| Predictor | Coef | SE Coef | T | P |
|-----------|---------|---------|-------|-------|
| Constant | 24.492 | 2.714 | 9.03 | 0 |
| Z1 | 2.856 | 0.7961 | 3.59 | 0.001 |
| Z2 | -4.5857 | 0.8566 | -5.35 | 0 |
| Z3 | -3.1781 | 0.5787 | -5.49 | 0 |
| Z4 | 1.685 | 0.433 | 3.89 | 0 |
| Z5 | 0.7727 | 0.3517 | 2.2 | 0.033 |
| Z6 | 2.2462 | 0.5403 | 4.16 | 0 |

The Table 6.2 below represents the Analysis of Variance used initially to prove that the predictors have a linear relationship with the response variable N1%. The P-value of < 0.05 suggests that at least one predictor has a significant linear relationship with the response variable.

Table 6.2: Analysis of Variance for PCR on No Cu Core Assemblies

| Source | DF | SS | MS | F | P |
|----------------|----|-------|------|-------|---|
| Regression | 6 | 19.51 | 3.22 | 21.32 | 0 |
| Residual Error | 51 | 7.77 | 0.15 | | |
| Total | 57 | 27.29 | | | |

Table 6.3: Transforming Z back to Original Variables in the N-L Model for Cu Core Assemblies

| Predictor | Coef | SE Coef | T | P |
|------------------|-------|---------|-------|-------|
| Constant | 24.49 | 2.71 | 9.03 | 0 |
| LnChipAreaSQMM | -1.23 | 0.34 | -3.59 | 0.001 |
| LnChipToPkgRatio | 0.038 | 0.0071 | 5.35 | 0 |
| LnBallCount | 0.095 | 0.017 | 5.49 | 0 |
| LnPkgPadDiaMM | 6.54 | 1.68 | 3.89 | 0 |
| LnDeltaTDEG C | -1.79 | 0.815 | -2.2 | 0.033 |
| LnSolderVolCUMM | -0.38 | 0.091 | -4.16 | 0 |

The Principal Components are then transformed back to the original variables using the same back transformation. Table 6.3 above gives a detailed result of the regression between the log transformed original predictors and log N1% Life of the package.

Regression equation for original predictor variables is given in the Equation 6.2 below:

$$\begin{aligned} \text{LnN1\%} = & 24.49 - 1.23 * \text{LnChipAreaSQMM} + 0.038 * \text{LnChipToPkgRatio} \\ & + 0.095 * \text{LnBallCount} + 6.54 * \text{LnPkgPdDiaMM} - 1.79 * \text{LnDeltaTDEGC} \\ & - 0.38 * \text{LnSdrVolCUMM} \end{aligned} \quad \text{Eqn 6.2}$$

Model Adequacy Checking:

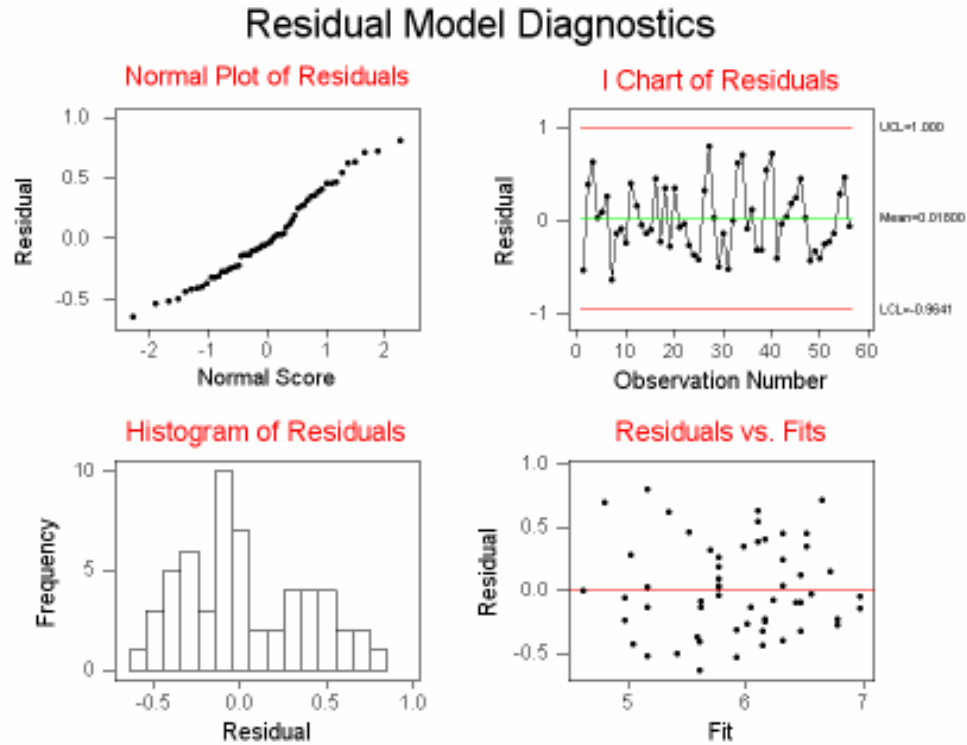


Figure 6.2 Analysis of Residuals for PCR on No Cu Core Assemblies

From the above Figure 6.2, Plot of residuals Vs Fits we do not observe any specific pattern which implies that the linearity assumptions are met.

The plot does not show any signs of the scatter increasing with the fitted values which implies that the constant variance assumptions are satisfied.

Normality test:

The Shapiro–Wilk test was performed to check if the normality assumptions are satisfied.

A P-value of 0.4838 which is > 0.05 confirms the normality assumptions. Table 6.4 below gives details of the test.

Table 6.4 Results for the Shapiro Wilk test on No Cu Core Assemblies

| Test | Test Statistic (W) | P-value |
|--------------|--------------------|---------|
| Shapiro-Wilk | 0.980468 | 0.4838 |

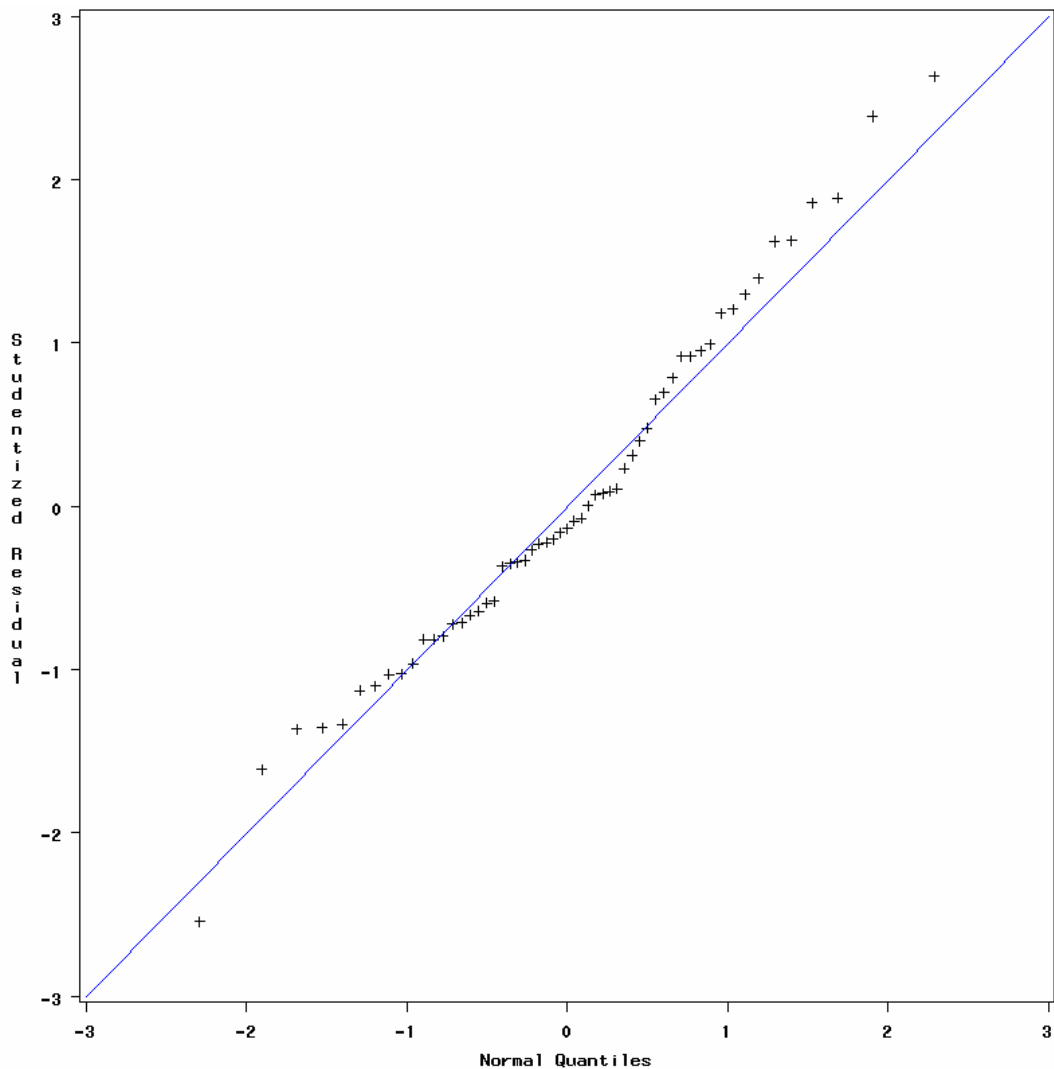


Figure 6.3 Plot of studentized residuals Vs Normal Quantiles

The figure 6.3 above, plot of studentized residuals Vs Normal Quantiles also produces points close to a straight line suggesting that the normality assumptions are met same.

Model Validation:

Figure 6.4 below shows the correlation of the actual N1% life obtained from the experimentation and predicted N1% life obtained from the PCR Model on No Cu Core Assemblies

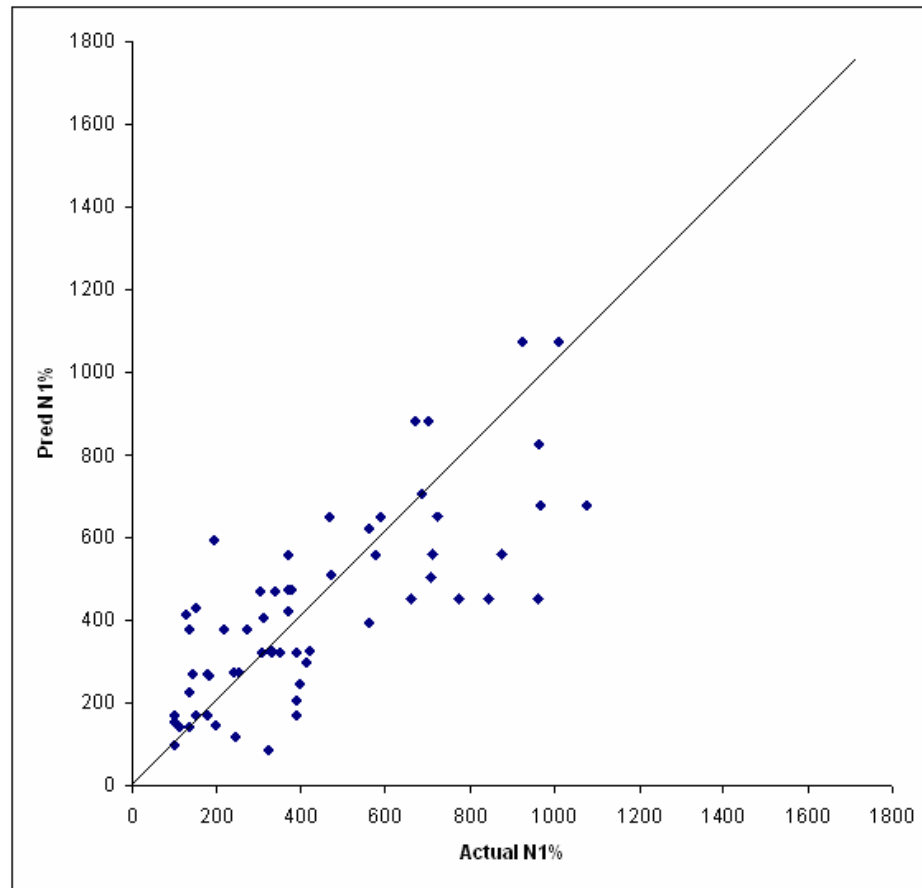


Figure 6.4 Plot of Actual Vs the Predicted Life for the PCR Model for No Cu Core Assemblies

CHAPTER 7

POWER LAW DEPENDENCY OF PREDICTOR VARIABLES:

Power law relationship of predictor variables with N1% life have been developed for various area array packages including PBGAs, flip chip BGA and CABGA packages. These power law relationships form the basis of reliability models in determining the appropriate family of transformations for linearizing the predictor variables for building robust multiple linear regression models that describe the data more efficiently. The power law relationship also help determining the appropriate transformation of predictor variables for coping with multi-collinearity, non normality and hetro-skedasticity. The power law dependence of predictor variables have been obtained using Box-Tidwell power law modeling and compared with traditional failure mechanics values.

BOX TIDWELL POWER LAW MODELLING:

Box-Tidwell power law model attempts to model the power law dependence between predictor variable and a response variable. The relationship is expressed as an equation that predicts a response variable from a function of predictor variables and parameters. The parameter is adjusted so that residual sum of squares is minimized. The prediction equation is of the form given by the equation 7.1 below

$$N_{1\%} = a_0 \prod_{k=1}^n (f_k)^{\lambda_k} \quad \text{Eqn 7.1}$$

Where, parameter $N_{1\%}$ on the left hand side of the equation represents the 1 percent failures of three-parameter Weibull distribution for the PBGA packages when subjected to accelerated thermo-mechanical stresses. The parameters on the right hand side of the equation are the predictor variables or the various parameters that influence the reliability of the package and the parameter λ_k is the power law value obtained from box Tidwell method.

The Box-Tidwell method has been used to identify a transformation from the family of power transformations on predictor variables. Box, et. al. [1962] described an analytical procedure for determining the form of the transformation on regressor variables, so that the relation between the response and the transformed regressor variables can be determined. Assume that the response variable t , is related to a power of the regressor,

$$E(t) = f(\xi, \beta_0, \beta_1) = \beta_0 + \beta_1 \xi \quad \text{Eqn 7.2}$$

Where,

$$\xi = \begin{cases} x^\alpha, & \alpha \neq 0 \\ \ln x, & \alpha = 0 \end{cases}$$

and β_0, β_1, α are unknown parameters. Suppose that α_0 is the initial guess of the constant α . Usually the first guess is $\alpha_0 = 1$, so that $\xi_0 = x^{\alpha_0} = x$, or that no transformation at all is applied in the first iteration. Expanding about the initial uses in Taylor series,

$$E(t) = f(\xi, \beta_0, \beta_1) + (\alpha - \alpha_0) \left(\frac{df(\xi, \beta_0, \beta)}{d\alpha} \right)_{\substack{\xi=\xi_0 \\ \alpha=\alpha_0}} + \frac{(\alpha - \alpha_0)^2}{2!} \left(\frac{d^2f(\xi, \beta_0, \beta)}{d^2\alpha} \right)_{\substack{\xi=\xi_0 \\ \alpha=\alpha_0}} +$$

$$\frac{(\alpha - \alpha_0)^3}{3!} \left(\frac{d^3 f(\xi, \beta_0, \beta)}{d^3 \alpha} \right)_{\substack{\xi = \xi_0 \\ \alpha = \alpha_0}} + \dots + \frac{(\alpha - \alpha_0)^n}{n!} \left(\frac{d^n f(\xi, \beta_0, \beta)}{d^n \alpha} \right)_{\substack{\xi = \xi_0 \\ \alpha = \alpha_0}} \quad \text{Eqn 7.3}$$

and ignoring terms of higher than first order gives the Equation 7.2 below,

$$\begin{aligned} E(t) &= f(\xi, \beta_0, \beta_1) + (\alpha - \alpha_0) \left(\frac{df(\xi, \beta_0, \beta)}{d\alpha} \right)_{\substack{\xi = \xi_0 \\ \alpha = \alpha_0}} \\ &= \beta_0 + \beta_1 x + (\alpha - 1) \left(\frac{df(\xi, \beta_0, \beta)}{d\alpha} \right)_{\substack{\xi = \xi_0 \\ \alpha = \alpha_0}} \end{aligned} \quad \text{Eqn 7.4}$$

Now if the terms in braces in Equation 7.3 were known, it could be treated as an additional regressor variable, and it would be possible to estimate the parameters β_0 , β_1 , and α by method of least squares. This way the value necessary to linearize the regressor variable can be determined.

This procedure has been carried out for both the Copper core as well as no core PBGAs for each of its predictor variable and the results are tabulated and compared with power law dependence values obtained from failure mechanics method. The power law dependence values obtained from Box-Tidwell method are found be very close to the power law dependence values obtained from failure mechanics models. Table 7.1 below shows the comparison of these values:

Table 7.1: Comparison of Power Law Dependence values

| Parameter | Box-Tidwell | | A | B | C |
|------------|---------------|---------------|------|------|----|
| | Cu Core PBGAs | No Core PBGAs | | | |
| Die Length | -2.7 | -1.2 | -2 | -2.3 | -2 |
| Delta T | -1.6 | -7.8 | -2.3 | -2 | -2 |

INTERACTION EFFECT MODEL:

Predictor variables for model building have been selected by developing a super-set of variables that are known to influence the characteristic life of an area array package and then selecting the potentially important variables using stepwise regression and method of best subsets. Coefficient of multiple determination, adjusted R^2 , residual mean squares and induced bias has been used as criteria for variable selection. Coefficient of multiple determinations (R^2 which measures the overall adequacy of the regression model and variables that create a significant increase in coefficient of multiple determination are retained in the model. As coefficient of multiple determination increases marginally for every newly added variable, adjusted R^2 has been used for studying the overall adequacy of the model and variables that create significant increase in adjusted R^2 are retained in the model. A PCR model with the interaction term between Delta T and Half Diagonal Length along with the original predictor variables has been developed. The

shows the results for regression between the transformed Z variables as predictors and N1% life as the response variable

Table 7.2: PCR Model for Cu Core Assemblies with the Interaction Effect between Delta T and Half Diagonal Length

| Predictor | Coef | SE Coef | T | P |
|-----------|---------|---------|-------|-------|
| Constant | 2081.1 | 423.3 | 4.92 | 0 |
| Z1 | -954.5 | 392.9 | -2.43 | 0.017 |
| Z2 | 3908 | 1617 | 2.42 | 0.018 |
| Z3 | 4587 | 1833 | 2.5 | 0.014 |
| Z4 | -5355 | 2227 | -2.4 | 0.018 |
| Z5 | 3189 | 1464 | 2.18 | 0.032 |
| Z6 | -84.39 | 50.62 | -1.67 | 0.099 |
| Z7 | -1362.6 | 863.9 | -1.58 | 0.118 |
| Z8 | -2906 | 1389 | -2.09 | 0.039 |

The Analysis of Variance given below is used to check if a linear relationship exists between the response variable and at least one of the predictor variables.

Table 7.3: ANOVA Table for Interaction Effect Model

| Source | DF | SS | MS | F | P |
|----------------|----|---------|--------|-------|---|
| Regression | 8 | 4541750 | 567719 | 18.08 | 0 |
| Residual Error | 91 | 2858166 | 31408 | | |
| Total | 99 | 7399916 | | | |

To establish the relationship between the Response variable and the original predictor variables, the Principal components have to be back transformed using the same back transformation which was used to convert them into Principal components. The table below shows the relation between the response variable and the original predictor variables.

Table 7.4: Transforming the Z's Back to the Original Variables in the Interaction Effect Model

| Predictor variable | PCR Coeffs. | S.E. Coeffs | T Statistic | P-Value |
|---------------------------|-------------|-------------|-------------|---------|
| Constant | 2081 | 422.96 | 4.92 | 0 |
| HalfdiaglenMM | -68.1 | 28.02 | -2.43 | 0.017 |
| DieToBodyRatio | -642.31 | 265.41 | -2.42 | 0.018 |
| BallCount | -0.5569 | 0.22 | -2.5 | 0.014 |
| PkgPdArSQMM | 1671.2 | 696.33 | 2.4 | 0.018 |
| PkgWtGM | 2.1949 | 1.00 | 2.18 | 0.032 |
| Delta T DegC | -8.4376 | 5.05 | -1.67 | 0.099 |
| Halfdialen* DeltaTMM°C | 0.2965 | 0.18 | 1.58 | 0.118 |
| SdrVolCUMM. | -9107.4 | 4357.6 | -2.09 | 0.039 |

The regression equation is given below:

$$\begin{aligned}
 \text{N1\%} = & 2081 - 68.1 * \text{HalfDiaLenMM} - 624.31 * \text{DieToBodyRatio} \\
 & - 0.5569 * \text{BallCount} + 1671.2 * \text{PkgPdArSQMM} + 2.1949 * \\
 & \text{PkgWtGM} - 8.44 * \text{DeltaT}^\circ\text{C} + 0.2965 * \text{HalfDiaLen} * \text{DeltaT} \\
 & - 9107.4 * \text{SdrVolMM}^3
 \end{aligned}
 \tag{Eqn 7.5}$$

Residual Model Diagnostics:

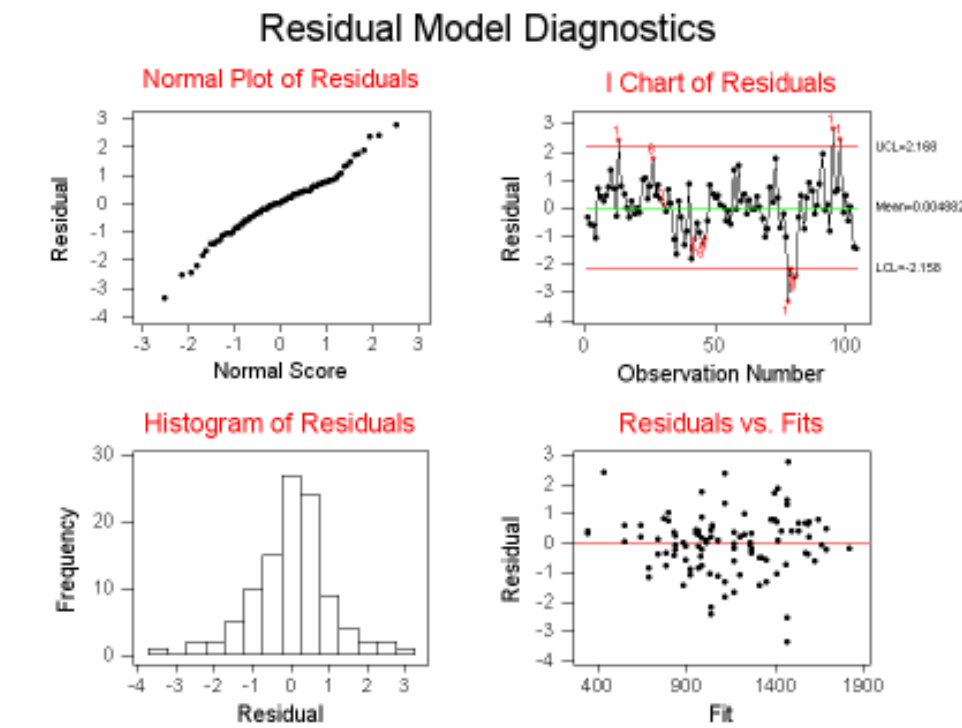


Figure 7.1 Model Adequacy Checking for Interaction effect model

From the Figure 7.1, the Plot of residuals Vs Normal Quantiles shows almost straight line. The histogram is also more like a bell shape suggesting that the normality assumptions are met. From the plot of residuals Vs Fits we do not observe any specific pattern which implies that the linearity assumptions are met. The plot does not show any

signs of the scatter increasing with the fitted values which implies that the constant variance assumptions are satisfied.

A Box – Tidwell Transformation was done on the interaction term as the predictor variable and the N1% Life of the package as the response variable to estimate the Power of the interaction term. The power retained by using SAS is -1.42 whereas the Classical models (Norris-Landzberg's and Goldmann's Equation) suggest a power transformation of -2.

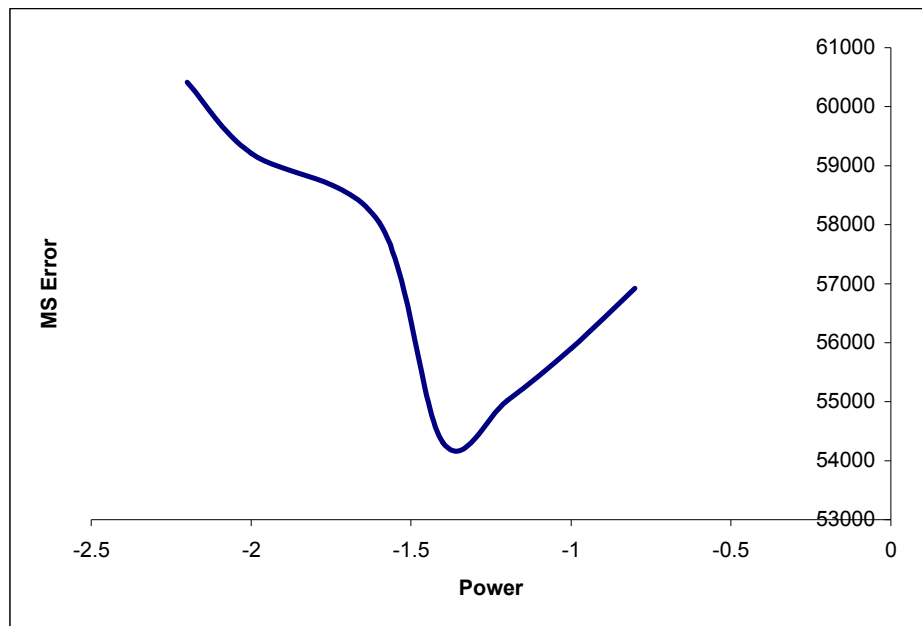


Figure 7.2: Plot of Mean Square Error Vs Power Transposed

The figure 7.2 above shows the change in values of the Mean Square Residual with the change in Power of the response variable. The value of the mean square error is lowest at the power transformation value of about -1.5 which is consistent with our value of -1.42

CHAPTER 8

STATISTICAL FORM OF THE NORRIS LANDZBERGS MODEL

The Norris-Landzberg Equation is based on the Coffin Mansion Equation and the Goldmann Equation. It provides a way of calculating the acceleration factor for Controlled Collapse Interconnections [Norris, Landzberg 1969]. The equation 8.1 below represents the same

$$AF = \frac{N_U}{N_A} = \left(\frac{f_U}{f_A} \right)^{\frac{1}{3}} \left(\frac{\Delta T_A}{\Delta T_U} \right)^2 \phi(T_{\max}) \quad \text{Eqn 8.1}$$

Where,

AF is the Acceleration factor.

N_U and N_A are the lives of the packages f_U and f_A are the frequencies

ΔT_A and ΔT_U are the temperature excursions

T_{\max} is the maximum temperature of the cycle in Kelvin

This Equation is often used in the form given below [Lau 1997]

$$AF = \frac{N_u}{N_A} = \left(\frac{f_U}{f_A} \right)^{\frac{1}{3}} \left(\frac{\Delta T_A}{\Delta T_U} \right)^2 \exp \left(1414 \left(\frac{1}{T_{\max,U}} - \frac{1}{T_{\max,A}} \right) \right) \quad \text{Eqn 8.2}$$

The Equation can be transformed by computing the natural Log format as follows:

$$\text{Ln}(AF) = C_1 \text{Ln} \left(\frac{f_U}{f_A} \right) + C_2 \text{Ln} \left(\frac{\Delta T_A}{\Delta T_U} \right) + C_3 \left(\frac{1}{T_{\max,U}} - \frac{1}{T_{\max,A}} \right) \quad \text{Eqn 8.3}$$

This Model was initially developed by Norris and Landzberg [1969] of IBM for controlled collapse chip interconnects for 5-95 Sn-Pb solder composition on ceramic substrate which had silver-palladium paste, tinned with 10-90 Sn-Pb solder deposition.

Now we model the above equation into a regression model with ratio of cyclic frequencies, Temperature cycle magnitude and the difference of inverse of maximum temperatures as the independent predictor variables and the Acceleration factor as the response variable. The Solder composition used for this model is lead free SAC 305.

Due to the presence of Multi-colinearity Principal Component Regression is implemented. Regression results of the transformed Principal Components against the Acceleration Factor are given in the Table 8.1 below:

Table 8.1: Transformed Z variable regression for N-L model

| Predictor | Coef | SE Coef | T | P |
|-----------|-----------|-----------|--------|--------|
| Constant | 0.7448 | 0.1161 | 6.4123 | 0 |
| Z1 | 3589.0768 | 1354.5949 | 2.6496 | 0.0095 |
| Z2 | 285.8296 | 107.7056 | 2.6538 | 0.0094 |
| Z3 | 2802.1627 | 1057.2824 | 2.6503 | 0.0095 |

The ANOVA Table 8.2 below is used to check the presence of a linear relationship between the predictor variables and any response variables. P-value less than 0.05 confirm the presence of a linear relationship between the response variable and atleast one predictor variable.

Table 8.2: ANOVA Table for Z transformed Variables of N-L model

| Source | DF | SS | MS | F | P |
|----------------|----|---------|--------|------|-------|
| Regression | 3 | 2.136 | 0.712 | 5.82 | 0.001 |
| Residual Error | 90 | 11.0016 | 0.1222 | | |
| Total | 93 | 13.1375 | | | |

To get the relationship between the original variables and the response variable, we need to back transform the Principal Components using the same back transformation. Regression results for the same are given in the Table 8.3 below,

Table 8.3: Transforming Z back to Original Variables in the N-L Model for Cu Core Assemblies

| Predictor | Coef | SE Coef | T | P |
|-------------------------|-----------|---------|--------|--------|
| Constant | 0.7448 | 0.1161 | 6.4123 | 0 |
| Ln(Fu/Fa) | 0.3035 | 0.1145 | 2.6496 | 0.0095 |
| Ln(Delta Ta / Delta Tu) | 2.3149 | 0.8722 | 2.6538 | 0.0094 |
| (1/Tu-1/Ta) | 4562.3767 | 1721.45 | 2.6503 | 0.0095 |

The regression is given as follows

$$AF = 0.7448 + 0.3035 * \ln\left(\frac{F_U}{F_A}\right) + 2.3149 * \ln\left(\frac{\Delta T_U}{\Delta T_A}\right) + 4562.3767 * \ln\left(\frac{1}{T_U} - \frac{1}{T_A}\right)$$

Eqn 8.4

The N-L model is given by [Lau 1997]:

$$AF = \frac{N_u}{N_A} = \left(\frac{f_U}{f_A} \right)^{\frac{1}{3}} \left(\frac{\Delta T_A}{\Delta T_U} \right)^2 \exp \left(1414 \left(\frac{1}{T_{\max,U}} - \frac{1}{T_{\max,A}} \right) \right) \quad \text{Eqn 8.5}$$

Writing the equation in the form of the NL equation:

$$AF = \frac{N_u}{N_A} = \left(\frac{f_U}{f_A} \right)^{0.3} \left(\frac{\Delta T_A}{\Delta T_U} \right)^{2.31} \exp \left(4562 \left(\frac{1}{T_{\max,U}} - \frac{1}{T_{\max,A}} \right) \right) \quad \text{Eqn 8.6}$$

The differences in the values of the constants are justified by the difference in the solder joint composition of the two models. The original model [Norris, Landzberg 1969] was developed for 5-95 Sn-Pb Solder on ceramic substrates whereas, the model which we have developed is for Lead Free SAC 305 solder composition for Plastic substrates. The type of PCB in our study has an integral copper core which may also be one of the factors for the difference in the values of the constants retained.

Residual Model Diagnostics

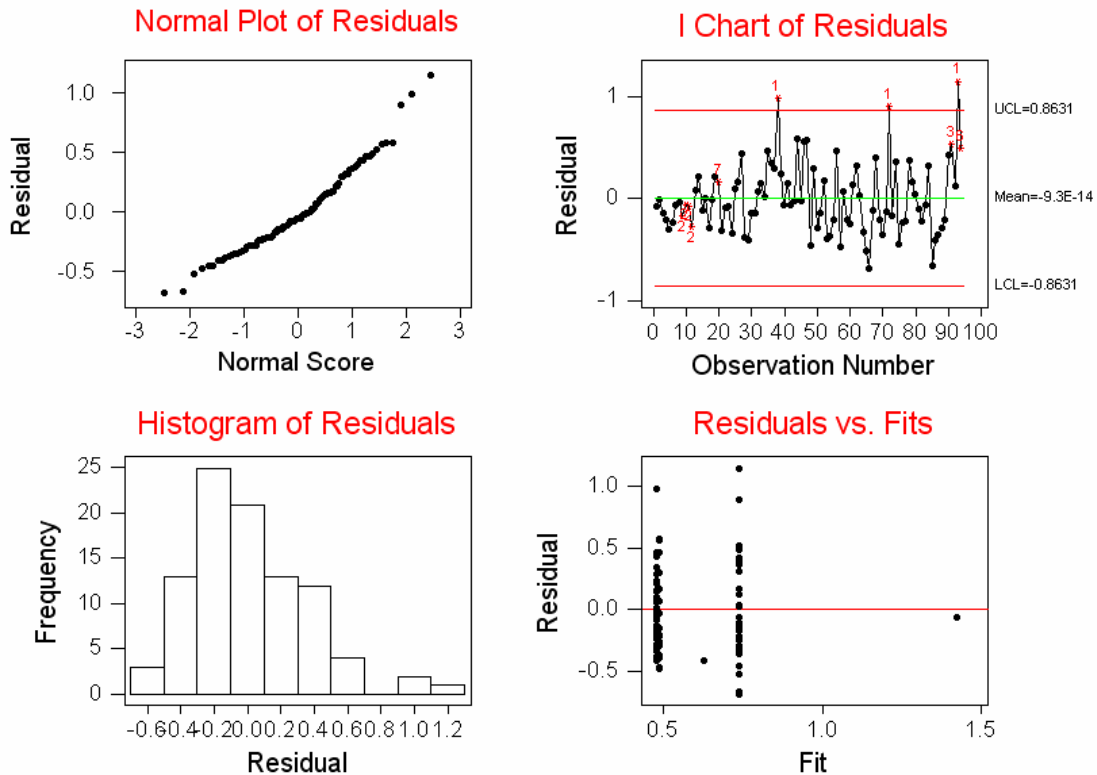


Figure 8.1 Model Adequacy checking for N-L Model

From the Figure 8.1, the Plot of residuals Vs Normal Quantiles shows almost straight line. The histogram is also more like a bell shape suggesting that the normality assumptions are met. From the plot of residuals Vs Fits we do not observe any specific pattern which implies that the linearity assumptions are met. The plot does not show any signs of the scatter increasing with the fitted values which implies that the constant variance assumptions are satisfied.

CHAPTER 9

STATISTICAL FORM OF GOLDMANN'S MODEL

L.S. Goldman of IBM presented his work in mechanical reliability of controlled collapse solder joints in May 1969. His main emphasis was on design variability and how the shape and dimensions of solder joint and chip affect reliability. He presented a systematic technique to optimize pad dimensions. His life prediction equation is developed based on the Coffin Manson equation. He used the local shear strain as the determinant parameter. The critical parameters like Difference in coefficients of thermal expansion, Distance from chip neutral point to interconnections, Temperature excursion of the cycle, Volume of the solder, radius and height of the solder ball, are included in the equation.

$$N_f = K_T \left[\left(\frac{\tau_u \pi r_f^2}{A} \right) \left(\frac{h^{1+\beta}}{V} \right) \right]^{m/\beta} \left(\frac{1}{\delta} \right)^m \quad [\text{Goldmann 1969}] \quad \text{Eqn 9.1}$$

$$\delta = d \cdot \alpha_{\text{rel}} \cdot \Delta T$$

Where,

N_f is number of cycles to failure,

τ_u is the ultimate shear strength of the critical interface.

α_{rel} is the relative thermal expansion of the chip to substrate,

d is the distance from chip neutral point to interconnection,

ΔT is the temperature excursion of the cycle,

V is the volume of solder joint,

r is the radius of cross section under consideration,

h is the height of solder,

A and β are constants from plastic shear stress-shear strain relationship

m is empirical constant in Coffin Manson Equation

The equation is rearranged as per our convenience and the values of the exponents for 5-95 Sn-Pb solder are given by Equation 9.2 below:

$$N_f = C \left[(\alpha_{rel})^{-1.9} (d)^{-1.9} (\Delta T)^{-1.9} \left(\frac{\pi r^2 h}{V} \right)^{3.275} (h)^{1.9} \right] \quad \text{Eqn 9.2}$$

The Figure below represents all the terms involved in the Goldmann's Equation:

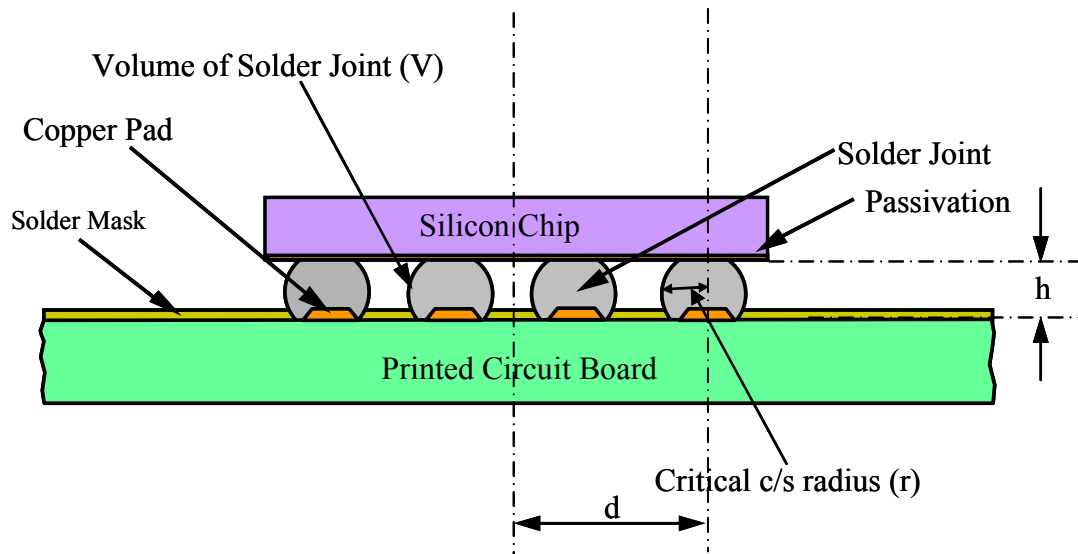


Figure 9.1: Different predictor variables in the Goldmann's model

Using these parameters as predictor variables, we model the Goldmann's Equation in the form of a log transformed Principal Component Regression model for PBGAs assembled on Cu Core PCB:

A Log transformed X matrix is created using the original predictor variables.

The X matrix is given by:

$$[x] = \begin{bmatrix} -12.2061 & 2.2911 & 4.9053 & -0.4721 & 0.1117 \\ -12.2061 & 2.2911 & 4.9053 & -0.2833 & 0.1117 \\ -12.061 & 2.2911 & 5.1930 & -0.4721 & 0.1117 \\ \dots & \dots & \dots & \dots & \dots \\ -12.5627 & 1.9708 & 4.9053 & -0.2145 & 0.3559 \end{bmatrix}$$

The Pearson's Co-relation matrix is calculated to check for the multicollinearity in the matrix X. And the Eigen values are used in transforming the original predictor variables in the new Z variables. Scree plots, eigen values and proportion of total variance explained by each principal component are then used to eliminate the least important principal components.

The Equation for calculation of the eigen values and the eigen vector is:

$$([C] - \lambda[I])[V] \quad \text{Eqn 9.3}$$

$$[C] - \lambda[I] = 0, \text{ or} \quad \text{Eqn 9.4}$$

$$|[X^*]^T [X^*] - \lambda[I]| = 0 \quad \text{Eqn 9.5}$$

Where λ the Eigen value and V is is the matrix of Eigen vectors.

The transformation matrix V of Eigen vectors of the correlation matrix is given by:

$$[V] = \begin{bmatrix} -0.5062 & 0.4823 & 0.2728 & -0.4497 & -0.4841 \\ 0.0549 & 0.797 & -0.2981 & 0.0527 & 0.5197 \\ -0.0074 & 0.2095 & 0.7386 & 0.6396 & 0.0384 \\ 0.4849 & -0.0182 & 0.5233 & -0.613 & 0.339 \\ -0.711 & -0.2966 & 0.1319 & -0.1005 & 0.6157 \end{bmatrix}$$

The principal component matrix Z contains exactly the same information as the original matrix, except that the data are arranged into a set of new variables which are completely uncorrelated with one another and which can be ordered or ranked with respect to the magnitude of their Eigen values (Draper and Smith 1981, Myers 1986).

The principal components matrix Z is obtained using the transformation:

$$[Z] = [X] * [V] \quad \text{Eqn 9.6}$$

MLR is performed with the transformed predictor variables and the original response variable. The coefficients obtained as a result of this regression model are stored in a variable named alpha. Matrix notation for the same is given as:

$$\{\alpha\}_{k*1} = [V]_{k*k}^T \{\beta^*\}_{k*1} \quad \text{Eqn 9.7}$$

Regressing the transformed Z variables against the N1% life of the packages, we get the following results as shown in Table 9.1

Table 9.1 Transformed Z variable regression for Goldmann`s model of Cu Core Assemblies

| Predictor | Coef | SE Coef | T | P |
|-----------|---------|---------|-------|-------|
| Constant | 17.014 | 8.375 | 2.03 | 0.047 |
| Z1 | 0.8251 | 0.4777 | 1.73 | 0.09 |
| Z2 | 0.703 | 0.4837 | 1.45 | 0.152 |
| Z3 | -1.8552 | 0.3743 | -4.96 | 0 |
| Z4 | 1.167 | 0.193 | 6.05 | 0 |
| Z5 | 0.8535 | 0.3332 | 2.56 | 0.013 |

The overall adequacy of the model has been tested using ANOVA table given by Table 9.1 above. Small P value of the ANOVA table rejects the null hypothesis proving the overall adequacy of the model. Individual T tests on the coefficients of regression of

principal components yielded very small P values indicating the statistical significance of all the five variables.

The Table 9.2 below shows the Analysis of variance in the statistical form of the Goldmann's model

Table 9.2: ANOVA Table for Z transformed Variables of Cu Core Assemblies

| Source | DF | SS | MS | F | P |
|----------------|----|-------|--------|-------|---|
| Regression | 7 | 9.61 | 1.3721 | 21.77 | 0 |
| Residual Error | 90 | 5.67 | 0.063 | | |
| Total | 97 | 15.27 | | | |

In order to obtain the relationship between the N1% life and original predictor variables the Z transformed variables are transformed back using the same back transformation

$$\{\beta\}_{k*1} = [V]_{k*k} \{\alpha\}_{k*1} \quad \text{Eqn 9.8}$$

The individual T test values of principal components regression components are then used for conducting individual T test on the coefficients of regression of original variables. The test statistic proposed by Mansfield et al.[1997] and Gunst et al. [1980] for obtaining the significance of coefficients of regression of original variables is given in the equation 9.9 below:

$$t = \frac{b_{j,pc}}{\left[\text{MSE} \times \left(\sum_{m=1}^l \lambda_m^{-1} v_{jm}^2 \right) \right]^{\frac{1}{2}}} \quad \text{Eqn 9.9}$$

Where $b_{j,pc}$ is the coefficient of regression of the j^{th} principal component, MSE is the mean square error of the regression model with l principal components as its predictor

variables, v_{jm} is the j^{th} element of the Eigen vector v_m and λ_m is its corresponding Eigen value. M takes the values from 1 to l , where l is the number of principal components in the model. The test statistic follows a students T distribution with $(n-k-1)$ degrees of freedom. The P values of individual T tests given by Table 9.3 below are < 0.05 proving the statistical significance of individual regression coefficients of original predictor variables at a 95 % confidence.

Table 9.3: Transforming Z back to Original Variables in the Goldmann's Model for Cu Core Assemblies

| Predictor | Coef | SE Coef | T | P |
|---|---------|---------|-------|-------|
| Constant | -2.651 | 4.014 | -0.66 | 0.511 |
| $\ln\left(\frac{\pi r_f^2 h}{V}\right)$ | 0.0495 | 0.0171 | 2.89 | 0.005 |
| $\ln(h)$ | 0.4121 | 0.054 | 7.64 | 0 |
| $\ln(d)$ | -0.3705 | 0.0476 | -7.77 | 0 |
| $\ln(\alpha_{rel})$ | -1.3721 | 0.4369 | -3.14 | 0.002 |
| $\ln(\Delta T)$ | -1.56 | 1.068 | -1.46 | 0.148 |

The regression equation between the N1% Life and the original predictors is given by equation 9.10 below:

$$\begin{aligned}
 \text{N1\%Life} = & -2.65 + 0.0495 * \ln\left(\frac{\pi r_f^2 h}{V}\right) + 0.4121 * \ln(\text{BallHt}) \\
 & - 0.3705 * \ln(\text{HalfDiagLen}) - 1.3721 * \ln(\text{Alpha Re I PPM}/^\circ \text{C}) \\
 & - 1.56 * \ln(\Delta T)^\circ \text{C}
 \end{aligned}
 \tag{Eqn 9.10}$$

We write the model in equation format to compare the values of constants obtained from the PCR model with standard values for Cu Core Assemblies.

Following are the two models:

Goldmann`s Model:

$$N_f = C \left((\alpha_{rel})^{-1.9} (d)^{-1.9} (\Delta T)^{-1.9} \left(\frac{\pi r_f^2 h}{V} \right)^{3.275} (h)^{1.9} \right) \quad \text{Eqn 9.11}$$

Statistical form based on PCR for Goldmann`s Model for Copper Core assemblies is given by Equation 9.12 below:

$$N_{1\%} = C \left[\alpha_{rel}^{-1.37} d^{-0.37} \Delta T^{-1.56} \left(\frac{\pi r_f^2 h}{V} \right)^{0.05} h^{0.41} \right] \quad \text{Eqn 9.12}$$

The differences in the values of the constants are justified by the difference in the solder joint composition of the two models. The original model was developed for 5-95 Sn-Pb Solder whereas, the model which we have developed is for Lead Free SAC 305 solder composition. The type of PCB in our study has a integral copper core which may also be one of the factors for the difference in the values of the constants retained.

Residual Model Diagnostics

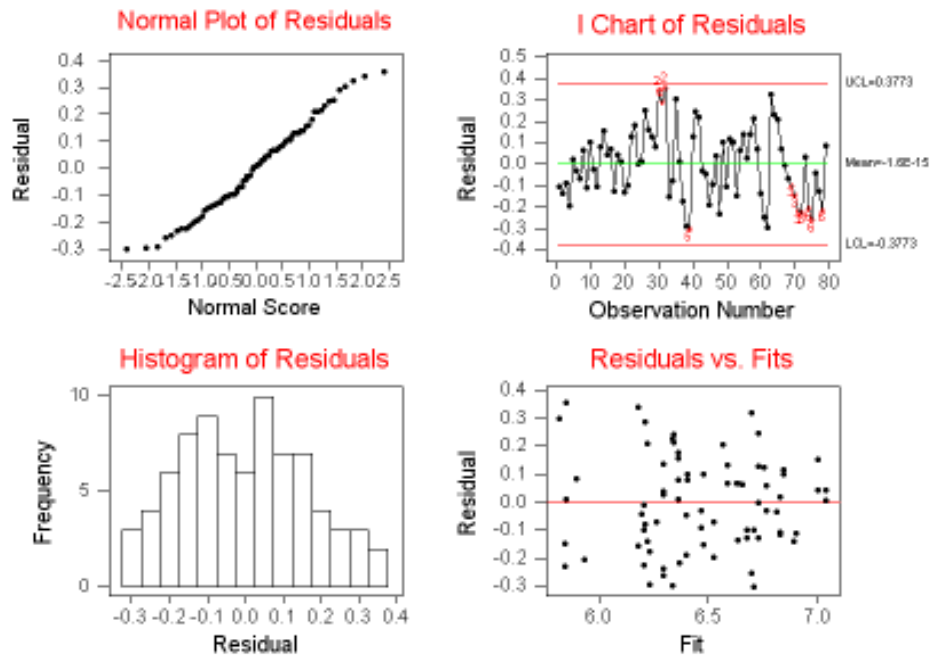


Figure 9.2 Model Adequacy Checking for Goldmann model on Cu Core Assemblies

From the above 9.2, the Plot of residuals Vs Normal Quantiles shows almost straight line. The histogram is also more like a bell shape suggesting that the normality assumptions are met. From the plot of residuals Vs Fits we do not observe any specific pattern which implies that the linearity assumptions are met. The plot does not show any signs of the scatter increasing with the fitted values which implies that the constant variance assumptions are satisfied.

Results for No Cu Core Assemblies:

A model similar to one developed for the Cu Core Assemblies is also developed for the No Cu Core Assemblies. The critical parameters like Difference in coefficients of thermal expansion, Distance from chip neutral point to interconnections, Temperature excursion of the cycle, Volume of the solder, radius and height of the solder ball, are

included in the Goldmann equation. Using these parameters as predictor variables, we model the Goldmann's Equation in the form of a log transformed Principal Component Regression model for PBGAs assembled on No Cu Core PCB:

The procedure for PCR described in Chapter 4 is used to develop the model. The results for regression of the transformed Z variables and the Predictor variable are given in the Table 9.4 below:

Table 9.4 Regression of Z variables against N1% life in Goldmanns Equation for No Cu Core Dataset

| Predictor | Coef | SE Coef | T | P |
|-----------|---------|---------|-------|-------|
| Constant | -2.54 | 10.96 | -0.23 | 0.818 |
| Z1 | 1.2882 | 0.8402 | 1.53 | 0.131 |
| Z2 | 0.4325 | 0.6319 | 0.68 | 0.497 |
| Z3 | 1.7433 | 0.4698 | 3.71 | 0 |
| Z4 | 0.8542 | 0.2402 | 3.56 | 0.001 |
| Z5 | -0.4093 | 0.2628 | -1.56 | 0.125 |

The overall adequacy of the model has been tested using ANOVA table given by Table 9.5 below. Small P value of the ANOVA table rejects the null hypothesis proving the overall adequacy of the model. Individual T tests on the coefficients of regression of principal components yielded very small P values indicating the statistical significance of all the five variables.

Table 9.5: ANOVA Table for Goldmanns Equation on No Cu Core Assemblies

| Source | DF | SS | MS | F | P |
|----------------|----|---------|--------|-------|---|
| Regression | 5 | 14.5313 | 2.9063 | 11.01 | 0 |
| Residual Error | 55 | 14.5236 | 0.2641 | | |
| Total | 60 | 29.0549 | | | |

The Principal Components are then transformed back to the original variables using the same back transformation. Table 9.6 below gives the detailed results of the regression between the log transformed original predictors and log N1% Life of the package.

Table 9.6: Transforming back to the original variables in the Goldmann equation for No Core Assemblies

| Predictor | Coef | SE Coef | T | P |
|---|---------|---------|-------|-------|
| Constant | -2.54 | 10.96 | -0.23 | 0.818 |
| $\ln\left(\frac{\pi r_f^2 h}{V}\right)$ | -0.3733 | 0.244 | -1.53 | 0.131 |
| $\ln(h)$ | 0.3109 | 0.457 | -0.68 | 0.497 |
| $\ln(d)$ | -1.2119 | 0.327 | -3.71 | 0 |
| $\ln(\alpha_{rel})$ | -1.2825 | 0.36 | -3.56 | 0.001 |
| $\ln(\Delta T)$ | -1.5592 | 0.999 | -1.56 | 0.125 |

The regression equation for the model with its original predictors is given in the equation 9.13 below

$$\begin{aligned}
 \text{N1\%Life} = & -2.54 - 0.3733 * \ln\left(\frac{\pi r_f^2 h}{V}\right) + 0.3109 * \ln(\text{BallHt}) \\
 & - 1.2119 * \ln(\text{HalfDiagLen}) - 1.2825 * \ln(\text{Alpha Re I PPM} / ^\circ \text{C}) \\
 & - 1.56 * \ln(\text{DeltaT} / ^\circ \text{C})
 \end{aligned}
 \tag{Eqn 9.13}$$

We write the model in equation format to compare the values of constants obtained from the PCR model with standard values for No Cu Core Assemblies.

Following are the two models:

Goldmann's Model:

$$N \left((\alpha_{rel})^{-2} (L)^{-2} (\Delta T)^{-2} \left(\frac{V}{\pi r_f^2 h^{1+\beta}} \right)^{-0.152} \right) = C \quad \text{Eqn 9.14}$$

Statistical model based on PCR for Goldmann's Model:

$$N_{1\%} = C \left[\alpha_{rel}^{-1.3} d^{-1.2} \Delta T^{-1.6} \left(\frac{\pi r_f^2 h}{V} \right)^{-0.37} h^{0.3} \right] \quad \text{Eqn 9.15}$$

The differences in the values of the constants are justified by the difference in the solder joint composition of the two models. The original model was developed for 5-95 Sn-Pb Solder where as, the model which we have developed is for Lead Free SAC 305 solder composition.

Now we check if the assumptions of the linear regression model are satisfied,

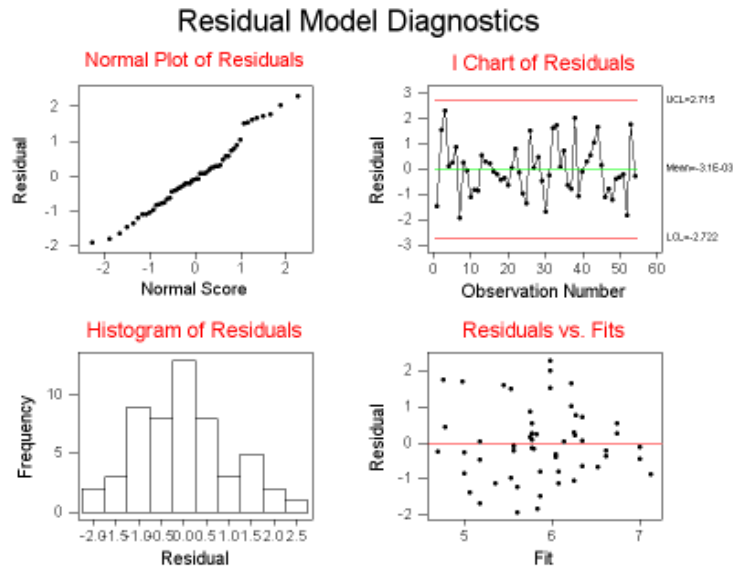


Figure 9.3 Model Adequacy Checking for No Cu Core Goldmann Model

From the above Figure 9.3, the Plot of residuals Vs Normal Quantiles shows almost straight line. The histogram is also more like a bell shape suggesting that the normality assumptions are met. From the plot of residuals Vs Fits we do not observe any specific pattern which implies that the linearity assumptions are met. The plot does not show any signs of the scatter increasing with the fitted values which implies that the constant variance assumptions are satisfied.

CHAPTER 10

MODEL VALIDATION

In order to determine the effect of individual design parameters on the thermo-mechanical reliability of the Cu Core PBGAs, the life of various packages was studied and the effect of each parameter was measured by keeping all other parameters at a constant level and varying just the parameter under consideration. The effect of individual parameter which is gauged by the sensitivity factor is of a great help to build confidence in trade-off decisions. Results obtained from the statistical analysis using the Principal Component Regression models were used to predict the life of the packages. The convergence of the predicted values of life with the experimental data has been demonstrated in this section.

Delta T:

A negative sensitivity factor for Delta T from the PCR models implies that the thermo-mechanical reliability of Cu-Core PBGA packages reduces with increase in the temperature range of ATC. The life obtained from the experimental data and PCR models have been plotted against temperature differences of 180 and 135 deg C. The predicted values from the prediction model follow the experimental values quite accurately and show the same trend, as in Figure 10.1.

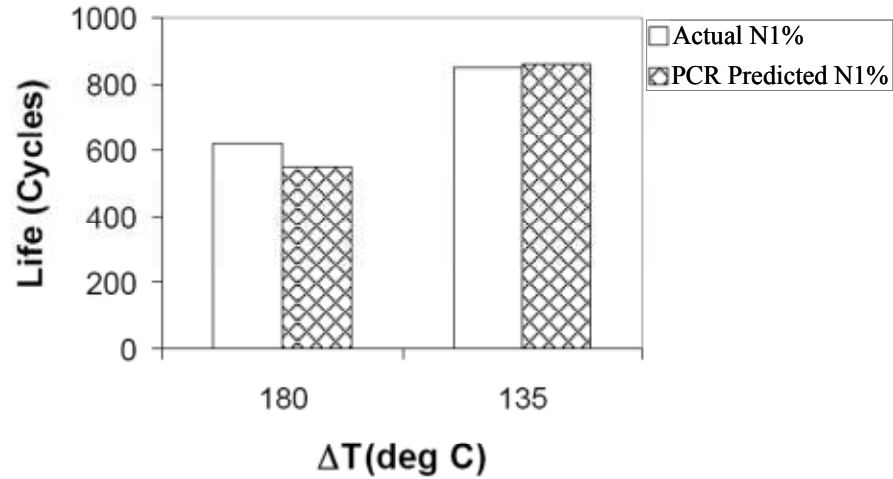


Figure 10.1: Effect of Delta T on N1% Life of the Packages assembled on Cu Core PCBs

Solder Volume:

A negative sensitivity factor for Solder Volume from the PCR models implies that the thermo-mechanical reliability of Cu-Core PBGA packages reduces with increase in the solder volume. The life obtained from the experimental data and PCR models have been plotted against the Solder volumes of 1200 and 720 MM³. The predicted values from the prediction model follow the experimental values quite accurately and show the same trend represented in Figure 10.2 This trend is supported by failure mechanics theory as, increasing the solder volume would make the solder joint very stiff leading to increased stress conditions resulting in higher hysteresis loops with more dissipated energy per cycle.

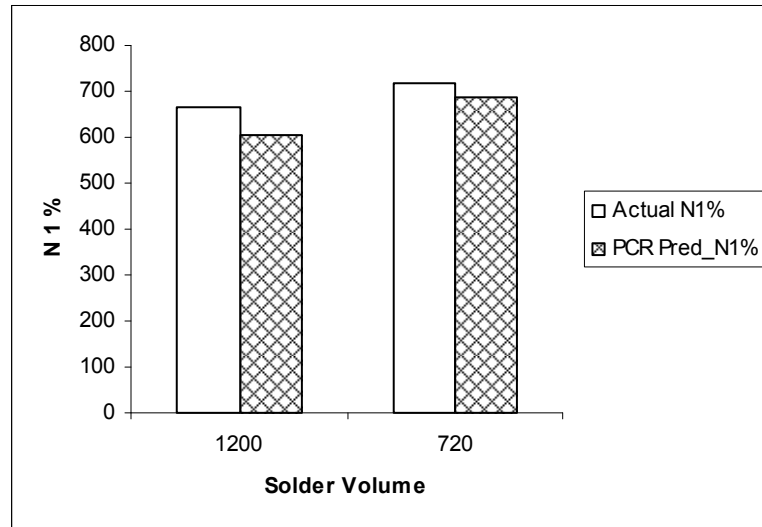


Figure 10.2 Effect of Solder Volume on Life of the Package assembled on Cu Core PCBs

Die to Body Ratio:

A negative sensitivity factor for Die to body ratio from the PCR models implies that the thermo-mechanical reliability of No Core PBGA packages reduces with increase in Die to body ratio. The life obtained from the experimental data and PCR models have been plotted against the Die to body ratio of 0.5 and 0.7407. The predicted values from the prediction model shown in Figure 10.3 follow the experimental values quite accurately and show the same trend. This is also consistent with the failure mechanics standpoint, as the Die to body ratio increases the solder balls in the vicinity of the die shadow region undergo much higher strains and are bound to fail faster.

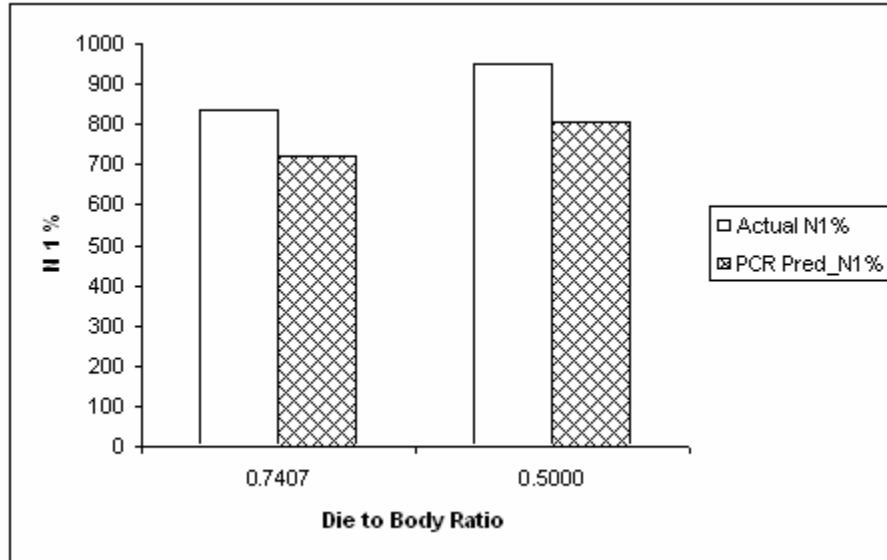


Figure 10.3: Effect of Die to body ratio on the life of Package assembled on No Cu Core PCBs

Half Diagonal Length:

The thermo-mechanical reliability of packages generally decreases with increase in the half diagonal length. This effect has been demonstrated for Goldmanns model and Cu Core Assemblies used to develop the same. The predicted values from the prediction model follow the experimental values quite accurately and show the same trend. This trend is also consistent from the failure mechanics standpoint, as the solder joints with larger die length are subjected to much higher strains due to the increased distance from the neutral point, thus having lower reliability. The figure 10.4 represents shows the variation in the life with the variation of half diagonal length.

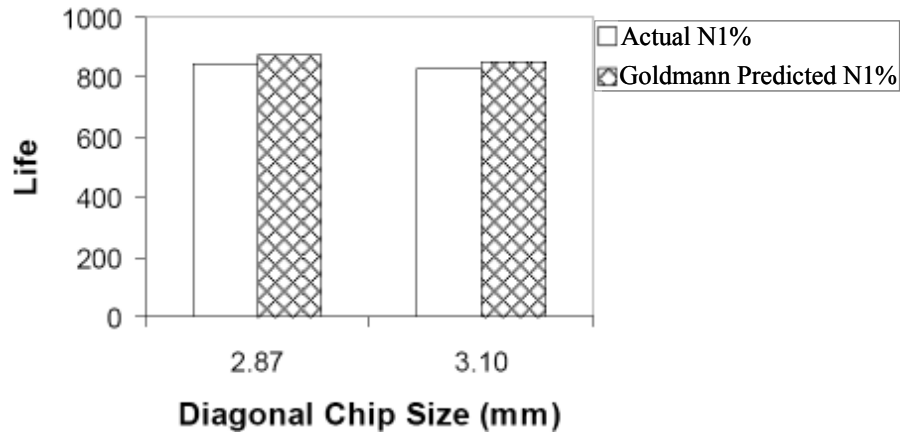


Figure 10.4: Effect of Diagonal Length on Life of Package Assembled on Cu Core PCBs for Goldmanns Model

Model Validation plots for Norris Landzbergs PCR model:

In this section, the effect of individual parameters on the acceleration factor predicted by N-L Model for SAC305 area array assemblies has been validated. The acceleration factor varies with a 0.3-power with increase in the ratio of frequencies. Model predictions agree with the experimental data. In addition, acceleration factor has been shown to vary with a 2.3-power of the temperature cycle magnitudes. The figures 10.5 and 10.6 given below represent the same.

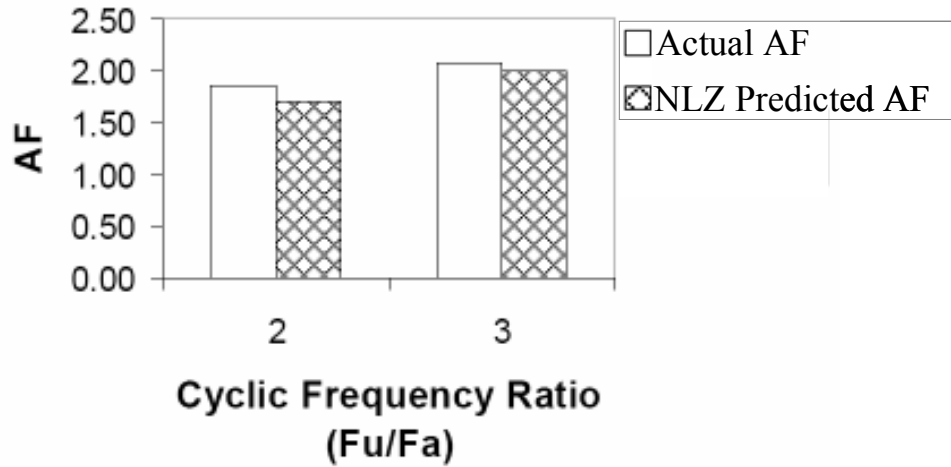


Figure 10.5: Effect of cyclic frequency on Acceleration Factor

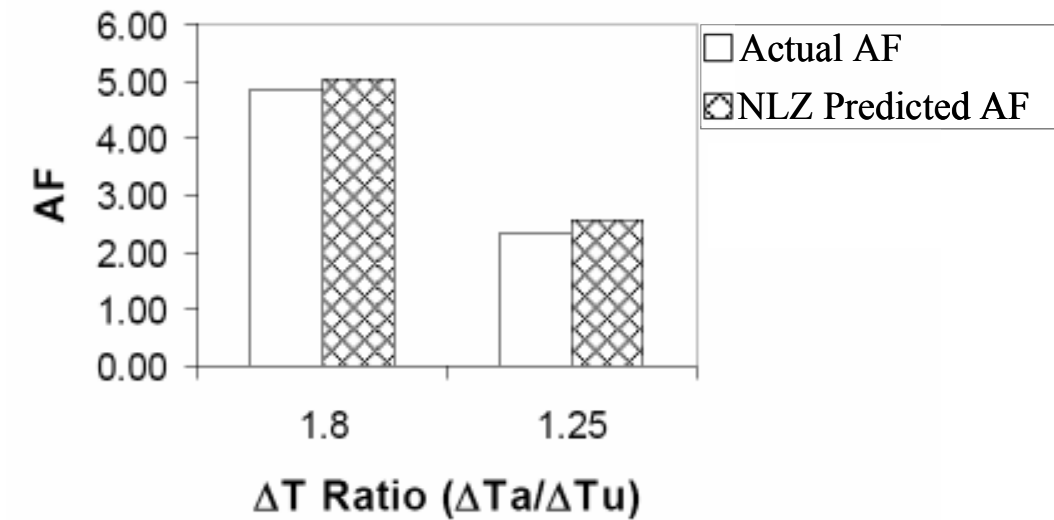


Figure 10.6: Effect of temperature cycle magnitude on Acceleration Factor

CHAPTER 11

SUMMARY AND CONCLUSION

A perturbation modeling methodology based on multiple linear regression, principal components regression and power law modeling has been presented in this research. The method provides an extremely cost effective and time effective solution for doing trade-offs and the thermo-mechanical reliability assessment of various Plastic BGA packages, CABGA, Flip-chip BGA subjected to extreme environments. This methodology also allows the user to understand the relative impact of the various geometric parameters, material properties and thermal environment on the thermo-mechanical reliability of the different configurations of BGA packages with leaded as well as lead-free solder joints.

The model predictions from both statistics and failure mechanics based models have been validated with the actual ATC test failure data. The convergence between experimental results and the model predictions with higher order of accuracy than achieved by any first order closed form models has been demonstrated, which develops the confidence for the application of the models for comparing the reliability of the different BGA packages for various parametric variations. The current approach allows the user to analyze independent as well as coupled effects of the various parameters on the package reliability under harsh environment. It is recommended to use these models

for analyzing the relative influence of the parametric variations on the thermo-mechanical reliability of the package instead of using them for absolute life calculations.

Power law relationship of predictor variables with 63 % characteristic life have been developed for various area array Packages. Interaction effects between different parameters which are often overlooked are also presented in this work. These power law relationships form the basis of reliability models in determining the appropriate family of transformations for linearizing the predictor variables for building robust multiple linear regression models that describe the data more efficiently. The power law values show good conformance with failure mechanics values for most of the variables. Advanced power law models can then be developed by transforming each predictor variable with its appropriate power law transformation and then conducting a linear regression analysis. Such power law transformed linear regression models can describe the data more efficiently and resulting in better prediction models. Also, the power law lamda values can be used for adding correction factors to existing first order failure mechanics models and building power law based models.

Development of the classical failure mechanics equations like the Norris Landzberg's and the Goldmann equation in the statistical form has been presented. Log transformation and has been used to convert the original multiplicative model to additive model and the power values of the various terms involved in these equations are compared to the ones obtained by statistical PCR model.

BIBLIOGRAPHY

- Anand, L., "Constitutive Equations for Hot-Working of Metals", *International Journal of Plasticity*, Vol. 1, pp. 213-231, 1985.
- Anghel, L., Saleh, V., S., Deswaertes, S., Moucary, A.E., Preliminary Validation of an Approach Dealing with Processor Obsolescence, *Proceedings of the 18th IEEE International Symposium on Defect and Fault Tolerance in VLSI Systems*, pp. 493, 2003.
- Amagai, M., Watanabe, M. Omiya, M. Kishimoto, K and Shibuya, T., "Mechanical Characterization of Sn-Ag Based Lead-Free Solders", *Transactions on Microelectronics Reliability*, Vol. 42, pp. 951-966, 2002.
- Amagai, M., Nakao, M., "Ball Grid Array (BGA) Packages with the Copper Core Solder Balls" Proceedings of *Electronic Components and Technology Conference*, Seattle, WA, pp. 692-701, May 25-28, 1998.
- Banks, D. R., Burnette, T. E. Gerke, R.D. Mammo, E. Mattay, S., "Reliability Comparison of Two Metallurgies for Ceramic Ball Grid Array", *IEEE Transactions on Components, Packaging and Manufacturing Technology, Part B*, Vol. 18, No. 1, February 1995.
- Barker, D. B., Mager, B. M. And Osterman, M. D., "Analytic Characterization of Area Array Interconnect Shear Force Bahavios", *Proceedings of ASME International*

- Mechanical Engineering Congress and Exposition*, New Orleans, LA, pp. 1-8, November 17-22, 2002.
- Bedinger, J. M., “Microwave Flip Chip and BGA Technology”, *IEEE MTT-S International Microwave Symposium Digest*, v 2, pp 713-716, 2000.
- Box, G. E. P., Cox, D, R., “An analysis of transformations revisited, rebutted”, *Journal of American Statistical Association*, vol. 77, no. 377, pp. 209-210, March 1982.
- Box, G. E. P., Tidwell, P. W., “Transformation of the independent variables” *Technometrics*, vol. 4, no. 4, pp.531-550, Nov.1962.
- Braun, T., Becker, K.F. Sommer, J.P. Löher, T. Schottenloher, K. Kohl, R. Pufall, R. Bader, V. Koch, M. Aschenbrenner, R.Reichl, H.,” High Temperature Potential of Flip Chip Assemblies for Automotive Applications” *Proceedings of the 55th Electronic Components and Technology Conference*, Orlando, Florida, pp. 376-383, May 31-June 3, 2005.
- Brooks, S. P., N. Friel, R. King, Classical Model Selection via Simulated Annealing, *Journal of the Royal Statistical Society*, Vol. 65, No. 2, pp. 503, May 2003.
- Brown, S. B., Kim, K. H. and Anand, L., “An Internal Variable Constitutive Model for Hot Working of Metals”, *International Journal of Plasticity*, Vol. 5, pp. 95-130, 1989.
- Burnettel, T., Johnson, Z. Koschmieder, T. and Oyler, W., “Underfilled BGAs for Ceramic BGA Packages and Board-Level Reliability”, *Proceedings of the 50th Electronic and Components Technology Conference*, Las Vegas, NV, pp. 1221-1226, May 23-26, 2000.

- Busso, E. P., and Kitano, M., "A Visco-Plastic Constitutive Model for 60/40 Tin-Lead Solder Used in IC Package Joints," *ASME Journal of Engineering Material Technology*, Vol. 114, pp. 331-337, 1992.
- Cheng, Z., "Lifetime of Solder Joint and Delamination in Flip Chip Assemblies", *Proceedings of 2004 International Conference on the Business of Electronic Product Reliability and Liability*, Shanghai, China, pp. 174- 186, April 27-30, 2005.
- Clech, Jean-Paul, "Solder Reliability Solutions: A PC based design-for-reliability tool", *Proceedings of Surface Mount International Conference*, San Jose, CA, pp. 136-151, Sept. 8-12, 1996.
- Clech, Jean-Paul, "Tools to Assess the Attachment Reliability of Modern Soldered Assemblies", *Proceedings of NEPCON West '96*, Anaheim, CA, pp.35-45, February 23-27, 1997
- Clech, Jean-Paul, "Flip-Chip/CSP Assembly Reliability and Solder Volume Effects", *Proceedings of Surface Mount International Conference*, San Jose, CA, pp. 315-324, August 23-27, 1998.
- Coffin, L. F., "A Study of the Effects of Cyclic Thermal Stresses on a Ductile Metal", *Transactions of ASME*, Vol. 76, pp. 931-950, 1954.
- Corbin, J.S., "Finite element analysis for Solder Ball Connect (SBC) structural design Optimization", *IBM Journal of Research Development*, Vol. 37, No. 5 pp. 585-596, 1991.

- Darveaux, R., and Banerji, K., "Fatigue Analysis Of Flip Chip Assemblies Using Thermal Stress Simulations and Coffin-Manson Relation" *Proceedings of 41st Electronic Components and technology Conference*, pp. 797-805, 1991.
- Darveaux, R., "How to use Finite Element Analysis to Predict Solder Joint Fatigue Life", *Proceedings of the VIII International Congress on Experimental Mechanics*, Nashville, Tennessee, June 10-13, pp. 41-42, 1996.
- Darveaux, R., and Banerji, K., "Constitutive Relations for Tin-Based Solder Joints," *IEEE Transactions on Components, Hybrids and Manufacturing Technology*, Vol. 15, No. 6, pp. 1013-1024, 1992.
- Darveaux, R., Banerji, K., Mawer, A., and Dody, G., "Reliability of Plastic Ball Grid Array Assembly", *Ball Grid Array Technology*, J. Lau, ed., McGraw-Hill, Inc. New York, pp. 379-442, 1995.
- Darveaux, R., "Effect of Simulation Methodology on Solder Joint Crack Growth Correlation," *Proceedings of the 50th Electronic Components and Technology Conference*, Las Vegas, Nevada, pp.1048-1058, May 21-24, 2000.
- Darveaux, R., and Banerji, K., "Constitutive Relations for Tin-Based Solder Joints," *IEEE Transactions on Components, Hybrids and Manufacturing Technology*, Vol. 15, No. 6, pp. 1013-1024, 1992.
- Dougherty, D., Fusaro, J. and Culbertson, D., " Reliability Model For Micro Miniature Electronic Packages" *Proceedings of International Symposium On Microelectronics*, Singapore, pp. 604-611, 23-26 June 1997.

- Drake J.L. “Thermo-Mechanical Reliability models for life prediction of ball grid arrays on Cu-Core PCBs in Extreme Environments”. *Masters Dissertation, Auburn University, Auburn, AL, August 2007.*
- Duan, Z., He, J., Ning, Y. and Dong, Z., “Strain Energy Partitioning Approach and Its Application to Low-Cycle Fatigue Life Prediction for Some Heat-Resistant Alloys,” *Low-Cycle Fatigue, ASTM STP 942*, H. D. Solomon, G. R. Halford, L. R. Kaisand, and B. N. Leis, Eds., *ASME*, Philadelphia, pp.1133-1143, 1988.
- Dwinnell, W., Modeling Methodology, *PCAI Magazine*, Vol. 12, No. 1, pp. 23-26, Jan. 1998.
- Engelmaier, W., “Fatigue life of leadless chip carrier solder joints during power cycling,” *IEEE Transactions on Components, Hybrids, Manufacturing Technology*, Vol. 6, pp. 52–57, September, 1983.
- Engelmaier, W., “Functional Cycles and Surface Mounting Attachment Reliability”, *ISHM Technical Monograph Series*, pp. 87-114, 1984.
- Engelmaier, W., “The Use Environments of Electronic Assemblies and Their Impact on Surface Mount Solder Attachment Reliability” *IEEE Transactions on Components, Hybrids, and Manufacturing Technology*, Vol. 13, No. 4, pp. 903-908, December 1990.
- Farooq, M., Gold, L. Martin, G. Goldsmith, C. Bergeron, C., “Thermo-Mechanical Fatigue Reliability of Pb-Free Ceramic Ball Grid Arrays: Experimental Data and Lifetime Prediction Modeling”, *Proceedings of the 52nd Electronic Components and Technology Conference*, New Orleans, LA, pp. 827-833, May 27-30, 2003.

- Fusaro, J. M., and Darveaux, R., “Reliability of Copper Base-plate High Current Power Modules”, *Int. Journal Of Microcircuits Electronic Packaging*, Vol. 20, No. 2, pp. 81–88, 1997.
- Garofalo, F., *Fundamentals of Creep and Creep-Rupture in Metals*, The Macmillan Company, New York, NY, 1965.
- Gerke, R.D., Kromann, G.B., “Solder Joint Reliability of High I/O Ceramic-Ball-Grid Arrays and Ceramic Quad-Flat-Packs in Computer Environments: The PowerPC 603™ and PowerPC 604™ Microprocessors”, *IEEE Transactions on Components and Packaging Technology*, Vol. 22, No. 4, December 1999.
- Goetz, M., Zahn, B.A., “ Solder Joint Failure Analysis Using FEM Techniques of a Silicon Based System-In-Package”, *Proceedings of the 25th IEEE/CPMT International Electronics Manufacturing Symposium*, pp. 70-75 October 2000.
- Goldmann, L.S., “Geometric Optimization of Controlled Collapse Interconnections”, *IBM Journal of Research Development*, Vol. 13, pp. 251-265, 1969.
- Gonzalez, M., Vandeveld, M. Vanfleteren, J. and Manassis, D., “Thermo-Mechanical FEM Analysis of Lead Free and Lead Containing Solder for Flip Chip Applications” *Proceedings of 15th European Microelectronics and Packaging Conference*, Brugge, Belgium, pp. 440-445, June 12-15, 2005.
- Hariharan, G.M., “Models for Thermo-mechanical reliability trade-offs for Ball Grid Array and Flip Chip Packages in extreme environments” *Masters Thesis, Auburn University*, Auburn, AL, May 2007.

- Hong, B.Z., Yuan, T.D, “Integrated Flow-Thermo-mechanical and Reliability Analysis of a Densely Packed C4/CBGA Assembly” *Proceedings of 1998 Inter Society Conference on Thermal Phenomena*, Seattle, WA, pp. 220-228, May 27-30, 1998.
- Hong, B.Z., “Thermal Fatigue Analysis of a CBGA Package with Lead-free Solder Fillets”, *Proceedings of 1998 Inter Society Conference on Thermal Phenomena*, Seattle, WA, pp. 205-211, May 27-30, 1998
- Hou, Z., Tian, G. Hatcher, C. Johnson, R.W., “Lead-Free Solder Flip Chip-on-Laminate Assembly and Reliability”, *IEEE Transactions on Components and Packaging Technology*, Vol. 24, No. 4, pp. 282-292, October 2001.
- Howard, M. A., “Component Obsolescence – It’s not just for electronics anymore”, *Proceedings of FAA/DoD/NASA Aging Aircraft Conf.*, San Francisco, CA, Sept. 2002.
- Ingalls, E.M., Cole, M. Jozwiak, J. Milkovich, C. Stack, J., “Improvement in Reliability with CCGA Column Density Increase to Imm Pitch”, *Proceedings of the 48th Electronic and Components Technology Conference*, Seattle, WA, pp. 1298-1304, May 25-28, 1998.
- Interrante, M., Coffin, J. Cole, M. Sousa, I.D. Farooq, M. Goldmann, L., “Lead Free Package Interconnections for Ceramic Grid Arrays”, *Proceedings of IEEE/CPMT/SEMI 28th International Electronics Manufacturing Technology Symposium*, San Jose, CA, pp. 1-8, July 16-18, 2003.
- Iyer, S., Nagarur, N. Damodaran, P., “Model Based Approaches For Selecting Reliable Underfill Flux Combinations for Flip- Chip Packages”, *Proceedings Of 2005 Surface Mount Technology Association (SMTA 05)*, Rosemont, IL, Sep. 25-29 2005, pp. 488-493.

- Jagarkal, S.G., M. M.Hossain, D. Agouafer, “Design Optimization and Reliability of PWB Level Electronic Package” Proceedings of *2004 Inter Society Conference on Thermal Phenomena*, Las Vegas, NV, p.p. 368-376, June 1-4,2004.
- Johnson, Z., “Implementation of and Extension to Darveaux’s Approach to Finite Element Simulation of BGA Solder Joint Reliability”, Proceedings Of 49th *Electronic Components and Technology Conference*”, pp. 1190-1195, June 1999
- Ju, S.H., Kuskowski, S. Sandor, B. and Plesha, M.E., “Creep- Fatigue Damage Analysis of Solder Joints”, *Proceedings of Fatigue of Electronic Materials, ASTM STP 1153, American Society for Testing and Materials*, Philadelphia, PA, pp. 1-21, 1994.
- Jung, E. Heinrich, K. Kloeser, J. Aschenbrenner, R. Reichl, H., Alternative Solders for Flip Chip Applications in the *Automotive Environment, IEMT-Europe*, Berlin, Germany, pp.82-91, 1998.
- Kang, S.K., Lauro, P. Shish, D.Y., “Evaluation of Thermal Fatigue Life and Failure Mechanisms of Sn-Ag-Cu Solder Joints with Reduced Ag Contents”, *Proceedings of 54th Electronic Components & Technology Conference*, Las Vegas, NV, pp. 661-667, June 1-4, 2004.
- Karnezos, M., M. Goetz, F. Dong, A. Ciaschi and N. Chidambaram, “Flex Tape Ball Grid Array”, *Proceedings of the 46th Electronic and Components Technology Conference*, Orlando, FL, pp. 1271-1276, May 28-31, 1996.
- King, J. R., D. A. Jackson, Variable selection in large environmental data sets using principal component analysis, *Environmetrics Magazine*, Vol 10, No. 1, pp. 66-77, Feb. 1999

- Kitchenham, B., E. Mendes, Further comparison of cross-company and within- company effort estimation models for web Applications, *10th International Symposium on Software Metrics*, Chicago, IL, USA, pp. 348-357, Sep 14-16, 2004
- Knecht, S., and L. Fox, “Integrated matrix creep: application to accelerated testing and lifetime prediction”, *Chapter 16, Solder Joint Reliability: Theory and Applications*, ed. J. H. Lau, Van Nostrand Reinhold, pp. 508-544, 1991.
- Lai, Y.S., T.H Wang, C.C.Wang, C.L.Yeh, “Optimal Design in Enhancing Board-level Thermomechanical and Drop Reliability of Package-on-Package Stacking Assembly”, *Proceedings of 2005 Electronics Packaging Technology Conference*, Singapore, p.p. 335-341, December 7-9 2005.
- Lall, P., G. Hariharan, A. Shirgaokar, J. Suhling, M. Strickland, J. Blanche, Thermo-Mechanical Reliability Based Part Selection Models for Addressing Part Obsolescence in CBGA, CCGA, FLEXBGA, and Flip-Chip Packages, *ASME InterPACK Conference*, Vancouver, British Columbia, Canada, IPACK2007-33832, pp. 1-18, July 8-12, 2007.
- Lall, P., N. Islam, J. Suhling and R. Darveaux, “Model for BGA and CSP Reliability in Automotive Underhood Applications”, *Proceedings of 53rd Electronic Components and Technology Conference*, New Orleans, LA, pp.189 –196, May 27-30, 2003.
- Lall, P.; Islam, M. N. , Singh, N.; Suhling, J.C.; Darveaux, R., “Model for BGA and CSP Reliability in Automotive Underhood Applications”, *IEEE Transactions on Components and Packaging Technologies*, Vol. 27, No. 3, p 585-593, September 2004.

Lau, J. H. and Dauksher, W., “Reliability of an 1657CCGA (Ceramic Column Grid Array) Package with Lead-Free Solder Paste on Lead-Free PCBs (Printed Circuit Boards)”, *Proceedings of the 54th Electronic and Components Technology Conference*, Las Vegas, NV, pp. 718-725, June 1-4, 2004.

Lau, J. H., *Ball Grid Array Technology*, McGraw-Hill, New York, 1995.

Lau, J. H., Shangguan, D., Lau, D. C. Y., Kung, T. T. W. and Lee, S. W. R., “Thermal-Fatigue Life Prediction Equation for Wafer-Level Chip Scale Package (WLCSP) Lead-Free Solder Joints on Lead-Free Printed Circuit Board (PCB)”, *Proceedings of 54th Electronic Components & Technology Conference*, IEEE, Las Vegas, NV, pp. 1563-1569, June 1-4, 2004.

Manson, S.S. and Hirschberg, M.H., *Fatigue: An Interdisciplinary Approach*, Syracuse University Press, Syracuse, NY, pp. 133, 1964.

Master, R. N., and T. P. Dolbear, “Ceramic Ball Grid Array for AMD K6 Microprocessor Application”, *Proceedings of the 48th Electronic and Components Technology Conference*, Seattle, WA, pp. 702-706, May 25-28, 1998

Master, R. N., Cole, M.S. Martin, G.B., “Ceramic Column Grid Array for Flip Chip Application”, *Proceedings of the Electronic and Components Technology Conference*, pp. 925-929, May 1995.

Malthouse, E. C., Performance Based Variable Selection for Scoring Models, *Journal Of Interactive Marketing*, Vol. 16, No. 4, pp. 37-50, Oct. 2002.

McCray, A. T., J. McNames, D. Abercromble, Stepwise Regression for Identifying Sources of Variation in a Semiconductor Manufacturing Process, *Advanced*

Semiconductor Manufacturing Conference, Boston, MA, USA, pp. 448-452, May 4-6, 2004.

Meiri, R., J. Zahavi , And the Winner is Stepwise Regression, *Tel Aviv University*, Urban Science Application.

Mendes, E., N. Mosley, Further Investigation into the use of CBR and Stepwise Regression to Predict Development Effort for Web Hypermedia Applications, *International Symposium on Empirical Software Engineering*, Nara, Japan, pp. 69-78, Oct 3-4, 2002.

Meng, H.H., Eng, O.K., Hua, W.E., beng, L.T., “Application of Moire Interferometry in Electronics Packaging”, *IEEE Proceedings of Electronic Packaging and Technology Conference*, pp. 277-282, October 8-10, 1997.

Moore T.D. “Area-Array package reliability models for No-Core PCB assemblies in extreme thermo-mechanical environments.” *Masters Dissertation, Auburn University*, Auburn, AL, August 2007.

Muncy, J. V. and Baldwin, D. F., “A Component Level Predictive Reliability Modeling Methodology”, *Proceedings of 2004 SMTA International Conference*, Chicago, IL, pp. 482-490, September 26-30, 2004.

Muncy, J. V., Lazarakis, T. and Baldwin, D. F., “Predictive Failure Model of Flip Chip on Board Component Level Assemblies”, *Proceedings of 53rd Electronic Components & Technology Conference*, IEEE, New Orleans, LA, May 27-30, 2003.

- Muncy, J. V., Predictive Failure Model For Flip Chip On Board Component Level Assemblies, *Ph. D. Dissertation, Georgia Institute of Technology, Atlanta, GA, January, 2004*
- Norris, K.C., Landzberg, A.H, “Reliability of Controlled Collapse Interconnections”, *IBM Journal of Research Development*, Vol. 13, pp. 266-271, 1969.
- Ostergren, W., and Krempl, E., “A Uniaxial Damage Accumulation Law for Time-Varying Loading Including Creep-Fatigue Interaction,” *Transactions of ASME, Journal of Pressure Vessel Technology*, Vol. 101, pp. 118-124, 1979.
- Pang, H. L. J., Kowk, Y.T. and SeeToh, C. W., “Temperature Cycling Fatigue Analysis of Fine Pitch Solder Joints”, *Proceedings of the Pacific Rim/ASME International Intersociety Electronic and Photonic Packaging Conference, INTERPack '97*, Vol. 2, pp. 1495-1500, 1997.
- Pang, J. H. L., Prakash, K. H. And Low, T. H., “Isothermal and Thermal Cycling Aging on IMC Growth Rate in Pb-Free and Pb-Based Solder Interfaces”, *Proceedings of 2004 Inter Society Conference on Thermal Phenomena*, Las Vegas, NV, pp. 109-115, June 1-4, 2004.
- Pang, J. H. L., Chong, D. Y. R, “Flip Chip on Board Solder Joint Reliability Analysis Using 2-D and 3-D FEA Models”, *IEEE Transactions On Advanced Packaging*, Vol. 24, No. 4, pp. 499-506, November 2001.
- Pang, J. H. L., Xiong, B. S. and Che, F. X., “Modeling Stress Strain Curves for Lead-Free 95.5Sn-3.8Ag-0.7Cu Solder”, *Proceedings of 5th International Conference on*

- Thermal and Mechanical Simulation and Experiments in Microelectronics and Microsystems*, pp. 449-453, 2004.
- Pang, J. H. L., Xiong, B. S. and Low, T. H., “Creep and Fatigue Characterization of Lead Free 95.5Sn-3.8Ag-0.7Cu Solder”, *Proceedings of 2004 Inter Society Conference on Thermal Phenomena*, Las Vegas, NV, pp. 1333-1337, June 1-4, 2004
- Pascariu G., Cronin P, Crowley D, “Next-generation Electronics Packaging Using Flip Chip Technology”, *Advanced Packaging*, Nov.2003.
- Peng, C.T., Liu, C.M. Lin, J.C. Cheng, H.C., “Reliability Analysis and Design for the Fine-Pitch Flip Chip BGA Packaging”, *IEEE Transactions on Components and Packaging Technology*, Vol. 27, No. 4, pp. 684-693, December 2004.
- Pendse, R., Afshari, B. Butel, N. Leibovitz, J. “New CBGA Package with Improved 2nd Level Reliability” *Proceedings of the 50th Electronic Components and Technology Conference*, Las Vegas, Nevada, pp.1189-1197, May 21-24, 2000.
- Perkins, A., and Sitaraman, S. K., “Predictive Fatigue Life Equations for CBGA Electronic Packages Based on Design Parameters”, *Proceedings of 2004 Inter Society Conference on Thermal Phenomena*, Las Vegas, NV, pp. 253-258, June 1-4, 2004.
- Perkins, A., and Sitaraman, S.K., “Thermo-Mechanical Failure Comparison and Evaluation of CCGA and CBGA Electronic Packages” *Proceedings of the 52nd Electronic Components and Technology Conference*, New Orleans, LA, pp. 422-430, May 27-30, 2003.

- Pitarresi, J.M., Sethuraman, S. and Nandagopal, B., “ Reliability Modelling Of Chip Scale Packages”, Proceedings of 25th *IEEE/CPMT International Electronics Manufacturing Technology Symposium*, pp. 60-69, October 2000.
- Qian, Z., and Liu, S., “A Unified Viscoplastic Constitutive Model for Tin-Lead Solder Joints,” *Advances in Electronic Packaging*, ASME EEP-Vol.192, pp. 1599–1604, 1997.
- Ray, S.K., Quinones, H., Iruvanti, S., Atwood, E., Walls, L., “Ceramic Column Grid Array (CCGA) Module for a High Performance Workstation Application”, Proceedings - *Electronic Components and Technology Conference*, pp 319-324, 1997.
- Riebling, J., “Finite Element Modelling Of Ball Grid Array Components”, *Masters Thesis, Binghamton University*, Binghamton, NY, 1996.
- Shi, X. Q., Pang, H. L. J., Zhou, W. and Wang, Z. P., “A Modified Energy-Based Low Cycle Fatigue Model for Eutectic Solder Alloy”, *Journal of Scripta Material*, Vol. 41, No. 3, pp. 289-296, 1999.
- Shi, X. Q., Pang, H. L. J., Zhou, W. and Wang, Z. P., “Low Cycle Fatigue Analysis of Temperature and Frequency Effects in Eutectic Solder Alloy”, *International Journal of Fatigue*, pp. 217-228, 2000
- Sillanpaa, M., Okura, J.H., “Flip chip on board: assessment of reliability in cellular phone application”, *IEEE-CPMT* Vol.27, Issue:3, pp. 461 – 467, Sept.2004.
- Singh, N.C., “Thermo-Mechanical Reliability Models for Life Prediction Of Area Array Packages”, *Masters Dissertation, Auburn University*, Auburn, AL, May 2006.

- Skipor, A. F., et al., "On the Constitutive Response of 63/37 Sn/Pb Eutectic Solder," *ASME Journal of Engineering Material Technology*, 118, pp. 1-11, 1996.
- Solomon, H.D., "Fatigue of 60/40 Solder", *IEEE Transactions on Components, Hybrids, and Manufacturing Technology*", Vol. No. 4, pp. 423-432, December 1986.
- Stogdill, R. C., Dealing with Obsolete Parts, *IEEE Design and Test of Computers*, Vol. 16, No. 2, pp. 17-25, Apr-Jun, 1999
- Stoyanov, S., C. Bailey, M. Cross, "Optimisation Modelling for Flip-Chip Solder Joint Reliability", *Journal of Soldering & Surface Mount Technology*, Vol. 14, No 1, p.p. 49-58, 2002.
- Suhir, E., "Microelectronics and Photonics-the Future", *Proceedings of 22nd International Conference On Microelectronics (MIEL 2000)*, Vol 1, NIS, SERBIA, pp. 3-17, 14- 17 MAY, 2000.
- Swanson, N. R., H. White, A Model Selection Approach to Real-Time Macroeconomic Forecasting Using Linear Models and Artificial Neural Networks, *International Symposium on Forecastors*, Stockholm, Sweden, pp. 232-246, Mar. 1994.
- Syed, A. R., "Thermal Fatigue Reliability Enhancement of Plastic Ball Grid Array (PBGA) Packages", *Proceedings of the 46th Electronic Components and Technology Conference*, Orlando, FL, pp. 1211-1216 May 28-31, 1996.
- Syed, A. R., "Thermal Fatigue Reliability Enhancement of Plastic Ball Grid Array (PBGA) Packages", *Proceedings of the 46th Electronic Components and Technology Conference*, Orlando, FL, pp. 1211-1216, May 28-31, 1996.

- Syed, A., "Factors Affecting Creep-Fatigue Interaction in Eutectic Sn/Pb Solder Joints", *Proceedings of the Pacific Rim/ASME International Intersociety Electronic and Photonic Packaging Conference, INTERPack '97*, Vol. 2, pp. 1535-1542, 1997.
- Syed, A., "Predicting Solder Joint Reliability for Thermal, Power and Bend Cycle within 25% Accuracy", *Proceedings of 51st Electronic Components & Technology Conference*, IEEE, Orlando, FL, pp. 255-263, May 29-June 1, 2001.
- Syed, A., "Accumulated Creep Strain and Energy Density Based Thermal Fatigue Life Prediction Models for SnAgCu Solder Joints", *Proceedings of 54th Electronic Components & Technology Conference*, Las Vegas, NV, pp. 737-746, June 1-4, 2004.
- Teo, P.S., Huang, Y.W. Tung, C.H. Marks, M.R. Lim, T.B. "Investigation of Under Bump Metallization Systems for Flip-Chip Assemblies", *Proceedings of the 50th Electronic Components and Technology Conference*, Las Vegas, Nevada, pp.33-39, May 21-24, 2000.
- Teng, S.Y., Brillhart, M., "Reliability Assessment of a High CTE CBGA for high Availability Systems", *Proceedings of 52nd Electronic and Components Technology Conference*, San Diego, CA, pp. 611-616, May 28-31, 2002.
- Tummala, R. R., Rymaszewski, E. J. and Klopfenstein, A. G., *Microelectronics Packaging Handbook Technology Drivers Part 1*, Chapman and Hall, New York, 1997.
- Tunga, K.R., "Experimental and Theoretical Assessment of PBGA Reliability in Conjunction With Field Use Conditions", *Masters Dissertation, Georgia Institute of Technology*, Atlanta, GA, April, 2004.

- Van den Crommenacker, J., “The System-in-Package Approach”, *IEEE Communications Engineer*, Vol 1, No. 3, pp. 24-25, June/July, 2003.
- Vandevelde, B., Christiaens F., Beyne, Eric., Roggen, J., Peeters, J., Allaert, K., Vandepitte, D. and Bergmans, J., “Thermomechanical Models for Leadless Solder Interconnections in Flip Chip Assemblies”, *IEEE Transactions on Components, Packaging and Manufacturing Technology*, Part A, Vol.21, No. 1, pp.177-185, March 1998.
- Vandevelde, B., Gonzalez, M., Beyne, E., Zhang, G. Q. and Caers, J., “Optimal Choice of the FEM Damage Volumes for Estimation of the Solder Joint Reliability for Electronic Package Assemblies”, *Proceedings of 53rd Electronic Components and Technology Conference*, New Orleans, LA, pp.189 –196, May 27-30, 2003.
- Vandevelde, B, Beyne, E., Zhang, K. G. Q., Caers, J. F. J. M., Vandepitte, D. and Baelmans, M., “Solder Parameter Sensitivity for CSP Life-Time Prediction Using Simulation-Based Optimization Method”, *IEEE Transactions on Electronic Packaging Manufacturing*, Vol. 25, No. 4, pp. 318-325, October 2002.
- Vayman, S., “Energy Based Methodology for The fatigue Life Prediction Of Solder Materials”, *IEEE Transactions on Components, Hybrids, and Manufacturing Technology*, Vol. 16, No. 3, pp. 317-322, 1993.
- Wang, G.Z., Cheng, Z.N. Becker, K. Wilde. J., “Applying Anand Model to Represent the Viscoplastic Deformation Behavior of Solder Alloys”, *ASME Journal Of Electronic Packaging*, Vol. 123, pp. 247-253, September 2003

- Wang, L., Kang, S.K. Li, H., “Evaluation of Reworkable Underfilms for Area Array Packaging Encapsulation”, *International Symposium on Advanced Packaging Materials, Braselton, GA*, pp. 29-36, March -11-14, 2001
- Warner, M., Parry, J., Bailey, C. and Lu, H., “Solder Life Prediction in a Thermal Analysis Software Environment”, *Proceedings of 2004 Inter Society Conference on Thermal Phenomena*, Las Vegas, NV, pp. 391-396, June 1-4, 2004
- Yi, S., Luo, G. Chian, K.S., “A Viscoplastic Constitutive Model for 63Sn37Pb Eutectic Solders”, *ASME Journal Of Electronic Packaging*, Vol. 24, pp. 90 -96, June 2002.
- Zahn, B.A., “Comprehensive Solder Fatigue and Thermal Characterization of a Silicon Based Multi-Chip Module Package Utilizing Finite Element Analysis Methodologies”, *Proceedings of the 9th International Ansys Conference and Exhibition*, pp. 274 -284, August 2000.
- Zhang, C., Lin, J.K. Li, L., “Thermal Fatigue Properties of Lead-free Solders on Cu and NiP”, *Proceedings of 51st Electronic Components & Technology Conference*, IEEE, Orlando, FL, pp. 464-470, May 29-June 1, 2001.
- Zhu, J., Zou, D. Liu, S., “High Temperature Deformation of Area Array Packages by Moire Interferometry/FEM Hybrid Method”, *Proceedings of Electronic Components and Technology Conference*, pp. 444-452, May 18-21, 1997.

APPENDIX

List of Symbols

| | |
|---------------|--|
| α | Coefficient of Thermal Expansion |
| β | Coefficient of regression |
| ΔT | Temperature Cycle Magnitude |
| ε | Model random error |
| ξ_0 | Predictor Variable after Power-Law Transformation ($\xi_0 = X^{\alpha_0}$) |
| 1/TmeanK | Inverse of the mean temperature in Kelvin |
| AF | Acceleration Factor |
| [A] | Matrix of Predictor Variables, of full column rank |
| 1/TmeanK | Inverse of the mean temperature in Kelvin |
| AlphaRelPPMC | Difference in CTE between part and PCB in ppm/C |
| BGA | Ball Grid Array |
| BallCount | Number of solder balls in the package |
| BallDiaMM | Diameter of the solder ball in millimeters |
| BallHtMM | Height of the solder ball in millimeters |
| ChipAreaSQMM | Area of the chip in Sq. millimeters |
| CABGA | Chip array BGA |
| Coef | Coefficient |
| Cu | Copper |

| | |
|----------------|--|
| DeltaTdegC | Temperature cycle range in degree centigrade |
| DieLengthMM | Chip Length in millimeters |
| DietoBodyRatio | Ratio of the length of the chip to the length of the package |
| ENIG | Electroless Nickel Immersion Gold |
| f_u | frequency of temperature cycle under use conditions |
| f_a | frequency of temperature cycle under accelerated test conditions |
| h | Solder Joint Height |
| HalfDiagLenMM | Half Diagonal Length of chip in mm. |
| HASL | Hot Air Solder Leveling |
| k | number of predictors |
| m | Empirical Constant in Coffin-Manson Equation |
| MS_{res} | Mean Square of residuals |
| n | number of data points |
| N_U | Life under Use Conditions |
| N_A | Life under Accelerated Test Conditions |
| p | number of variables |
| PitchMM | Solder Ball Pitch in millimeters |
| Prefix Ln | Natural logarithm |
| PBGA | Plastic Ball Grid Array |
| PCB | Printed Circuit Board |
| PCR | Principal Component Regression |
| PkgPadDiaMM | Diameter of the package pad in millimeters |
| PkgPdAreaSQMM | Area of the Package Pad in sq. millimeters |

| | |
|---------------|--|
| PkgWtGM. | Weight of the package in grams |
| R^2 | Multiple coefficient of determination |
| R_j^2 | Adjusted R Square |
| s | Standard Deviation |
| SolderVolCUMM | Volume of the solder in cubic mm |
| SS_{res} | Sum of Squares of residuals |
| $T_{max,U}$ | Maximum Use Temperature |
| $T_{max,A}$ | Maximum Accelerated Test Temperature |
| ΔT_U | Use Temperature Excursion |
| ΔT_A | Accelerated test temperature Excursion |
| V | Volume of Solder Joint |
| [V] | The k x k eigenvector matrix consisting of normalized eigenvectors |
| VIF | Variance Inflation Factor |
| X | Predictor Variable |
| [X] | Scaled and Centered Predictor Variable Matrix |
| Y | Regressor Variable |
| [Z] | The n x k matrix of principal components |