COURNOT COMPETITION UNDER UNCERTAINTY

IN POWER MARKETS

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COURNOT COMPETITION UNDER UNCERTAINTY

IN POWER MARKETS

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DISSERTATION ABSTRACT

COURNOT COMPETITION UNDER UNCERTAINTY

IN POWER MARKETS

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As deregulation of the electric industry has come into effect in many parts of the world, the price of electricity is no longer determined by regulatory agencies. In contrast, price is determined by market demand, supply conditions, load elasticity, and strategic behavior. Firms, nowadays, face much greater risks and have become more responsible for their own economic decisions in deregulated power markets. Therefore, decision-support models can help firms fulfill these new requirements.

This research is aimed at developing analytical models for long-term markets to assess the effect of uncertainties on electricity market prices. A multi-period Cournot model was developed for this purpose. Specifically, two significant uncertainty factors, the availability of the generating units and fuel price uncertainty, are considered in this model. An impact analysis of these two factors on firms' expected profits is also carried out. Finally, a sensitivity analysis is performed to determine the parameters that have the most significant impact on the Nash-equilibrium solutions.

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CHAPTER I

INTRODUCTION

Power supply systems have been changing their economic modes of operation to systems based on a market mechanism as deregulation of electric industries has become the norm in many parts of the world since the 1970s (Fujii, Okamura, Inagaki, and Yamaji, 2004). The deregulation of the electricity industry is essentially changing the way in which suppliers do business. A firm's decisions now depend, to a large extent, on market electricity prices. The price of electricity under deregulation is determined by market demand, supply conditions, load elasticity, and strategic behavior. It also depends on physical factors such as production cost, load, unit commitment, and transmission constraint (Valenzuela and Mazumda, 2005). Moreover, considerations of uncertainty factors such as generator reliability, demand uncertainty, and fuel price volatility are inevitable when making decisions. In general, most companies handle uncertainty in power markets by making hasty decisions about sensitivity, comparing scenarios, performing worst-case analyses, etc. This is, however, not an effective way to cope with uncertainties (Krukanont and Tezuka, 2006). Electricity is different from other products because it has yet to become efficiently storable. Therefore, its demand and supply must be matched every second, and failure to do so may result in a costly system collapse.

Firms face much more risk and become responsible for their own economic decisions in deregulated power markets. Those firms, hence, need decision-support models that fulfill these new requirements. In other words, decision-support models need to incorporate the uncertainties and other factors involved in deregulated power markets.

Recent attempts to model the structure of deregulated electricity markets via utility system production simulation models have continued to rely on models used in the past for planning and regulatory purposes (Kahn, Bailey, and Pando, 1996), but many analysts believe that the Cournot model is better able to represent the electricity market as it has evolved (Borenstein and Bushnell, 1999). However, in the existing literature the Cournot model usually assumes perfect information about the salient factors such as generator outages and fuel cost uncertainty. This is difficult and poses risk for decision makers, especially in long-term analyses that involve large uncertainties in the decision-making process (Krukanont and Tezuka, 2006).

This dissertation develops models for the long-term markets to assess the effect of uncertainties (generator failure and fuel price uncertainty) on electricity market prices under Cournot competition. Specifically, the stochastic single-period model is extended to a multi-period model. Uncertainty factors, the availability of the generating units and fuel price uncertainty, are then added to the model as well as their sensitivity analysis, and the model shows their effects on market prices. Moreover, the effects of those factors on a firm’s expected profits are studied in this research. Transmission congestion and demand uncertainty are not considered in this dissertation.

1.1. Contributions

The main contribution of this dissertation is within the study of market prices in the long-term power markets when uncertainties take place. In particular, this research focuses on developing a stochastic Cournot model to evaluate the effects of vital uncertainty factors on electricity market prices. In reality, most companies cope with uncertainty by performing simple methods such as sensitivity and worst-case analysis. Those techniques, however, could lead to inaccurate results. As the electricity prices in deregulation have major effects on firms' profits, companies are responsible for their own economic decisions. The development of an analytical model in this research which incorporates uncertainty and other crucial factors in deregulated power markets will help power companies make the precise decisions which they need to operate.

Another contribution of this dissertation is an approach used to cope with generator failures. In the existing literature, the approach used to consider generator outages into production cost models is to derate plant capacity. However, this method could lead to inaccurate results (Valenzuela and Mazumda, 2007). In this research, the expected production cost function, including generator outages, is modeled. This approach yields more accurate results when power producers consider the uncertainty of generator availability. When incorporating generator outages in the model, the expected cost function becomes a piecewise linear function. The piecewise linear function in some cases generates a large number of slopes which has a direct link to computational complexity. The algorithm to reduce a number of slopes of a piecewise linear function is implemented. The small number of slopes means less computational complexity, as the number of slopes of an expected cost function grows exponentially with the number of generators. In general, the algorithm developed in this research can reduce the number of slopes efficiently and ease computational complexity which will help support other research.

This dissertation contributes to the optimization theory and applications. The tolerance approach to sensitivity analysis in a linear complementarity problem is here implemented and applied to the stochastic Cournot model. It can be broadly applied to numerous applications of the linear complementarity problem such as game theory and equilibrium problems. In addition, power companies can employ this approach to study the effect of input data on the output results. It might be useful for companies if they can detect which input data are sensitive and have a significant impact on firms' optimal strategic planning and operations.

Finally, this research also provides a valuable new tool for all participants in power markets. The tolerance approach to sensitivity analysis is applied to determine whether the new input data affects the optimal solutions. If the perturbed problems still have the same index set of solutions, the new optimal solutions can be calculated without directly solving a linear complementarity problem. Since solving a large scale linear complementarity problem may take long computational time and the input data such as the fuel price may change every minute, power producers can make a decision swiftly with this approach, as new input data are obtained.

1.2. Dissertation Organization

The remainder of this dissertation is organized as follows: Chapter 2 provides a brief history of the development of the power market. The structure, volume, and market concentration of the capacity market are also discussed as is the basic Cournot model. Finally, a background and review of the significant literature concerning the Cournot model, mixed complementarity problems (MCP), and linear complementarity problems (LCP), which are a subset of mixed complementarity problems, are provided.

Chapter 3 presents the multi-period deterministic Cournot model in the long term market which is extended from a single period model. The time value of money is also considered in the model while demand and fuel cost are assumed to be constant. Moreover, the availability of generating units is ignored in this chapter. The Nash-equilibrium quantities are calculated by combining the KKT first order optimality conditions of the extended model. The KKT conditions of the deterministic Cournot model are considered as an LCP. Finally, the market prices and each firm's expected profit are calculated. These results are used as standard results to show the effect of uncertainties in power markets when we consider those uncertainties in the model.

Chapter 4 presents an approach to determining market prices when generator outages are taken into consideration. Specifically, the expected cost function including generator availability is developed. This expected cost function yields a large number of slopes. Each slope represents one marginal cost and maximum capacity including generator availability which is used to compute the Nash-equilibrium quantities. To consider all of them would take long computational time. Therefore, an algorithm to reduce a number of slopes without losing precision is developed. Finally, the effect of generator outages on the market prices is analyzed.

Chapter 5 provides an approach for determining the price of electricity when generator outages and fuel price uncertainty are in effect. Generally, each power company owns capacity resources in different fuel technologies. Four types of fuel technology including oil, coal, hydro, and nuclear are assumed in this dissertation. The most recent data on fuel prices obtained from reliable sources are used to generate the distribution for each generator's marginal cost. The effects of generator availability and fuel price uncertainty are investigated.

Chapter 6 describes the tolerance approach to sensitivity analysis in the stochastic Cournot model is proposed. Moreover, an algorithm to compute new optimal solutions when all parameters on the right-hand side vary simultaneously without directly solving the LCP is also presented in this chapter. The maximum allowable range which does not affect firms' optimal strategy for each marginal cost and maximum capacity is computed in order to detect the sensitive parameters.

Chapter 7 summarizes the study, discusses the conclusion of this research, and suggests directions for future research.

CHAPTER II

LITERATURE REVIEW

This chapter presents the review of the current literature in four sections. The first section presents the history and current state of deregulation in the United States. The second section provides information about deregulation and the basic structure and operation of the capacity market as well as the current state of the capacity market. The third section presents a Cournot model, originally developed by Augustin Cournot along with a literature review of how the Cournot model is applied to the power markets. The fourth section focuses on the literature review of mixed complementarity problems. The general form of a model and the available software which can be used to solve mixed complementarity problems are discussed. Finally, a linear complementarity problem as well as its general form is highlighted. Some crucial algorithms, which are able to solve the linear complemetarity problem, are also presented.

2.1. Deregulation in the United States

The report prepared by the Energy Information Administration (1997) gives an interesting perspective on the history of the topic from the beginning of power markets in the United States until deregulation. In 1882, Thomas Edison's Pearl Street Station began supply electricity to 85 customers for the first time in New York City. By 1916, 33 states had established regulatory agencies to organize the utilities in their jurisdictions, with the authority to franchise utilities, regulate their rates, financing, and service, and establish utility accounting systems. State regulation provided protection to consumers from the possibility of monopolistic practices by the utilities and ensured the reliability of electricity supplies. Moreover, they allowed utilities to receive a fair rate of return but there was debate at that period of time whether state regulation of electric power emerged to protect the consumers or to protect the profits of the electric utilities.

By the early 1930s, the price of electricity had fallen and service had been extended to two-thirds of the U.S. population which meant the demand for electricity increased. Consequently, ownership of operating companies was centralized under holding companies which facilitated access to the capital required for expansion and for exploiting economies of scale. As many states regulated local operating companies, there was no effective regulation of the increasingly expansive holding companies. As a result, when the worldwide economic downturn called Great Depression arrived in the early 1930s, many holding companies failed because of high-leverage, unsecured financing, and investments in business unrelated to energy services. In return, Congress passed the Public Utility Holding Company Act of 1935 (PUCHA). The purpose of this legislation was to give control of electricity service to local operating companies. In other words, PUCHA restricted the electric power generating business to local utilities which built and operated power plants to serve service territories. This meant that there was no competition in supplying electricity under PUCHA. Title II of PUCHA granted the Federal Government explicit authority over most interstate wholesale electric power sales. Therefore, by the end of 1935, nearly all electric power transactions in the U.S. were regulated under a scheme called "rate of return." This scheme facilitated the production and sale of low-cost, reliable electricity in the U.S. for about 50 years.

Before deregulation, there was a belief that electricity is a national asset. Therefore, electric sectors in most regions of the country were subject to full regulation. The generation, transmission, distribution and retail segments were controlled by state governments playing a dual role as electricity service providers and as regulators. As economic growth became more and more dependent on sufficient electricity suppliers, the importance of electricity increased tremendously. Consequently, many governments have started to realize that this growth may be impeded by full regulation because of suppliers' slow response to technological progress in electricity operations. Furthermore, the successful deregulation in oil and gas supports the belief that electricity is a service and it can be accomplished by deregulation (Yao, 2006).

Thus, the first step of restructuring the market began in 1978 when congress passed the Public Utility Regulatory Policies Act of 1978 (PURPA). PURPA not only opened the door to competition in the U.S. but also promoted greater use of renewable energy. This law created a market for non-utility electric power producers by forcing electric utilities to purchase power from these firms at the "avoided cost" rate, which is determined by bids from non-utility electric power producers. This is the first time in the United States that organizations other than public utilities were allowed to sell electric power.

The Federal Energy Regulatory Commission (FERC) was given the responsibility by Congress of implementing open transmission access under the Energy Policy Act of 1992 (EPACT) in order to spur competition in the wholesale electricity market. On April 24, 1996, the Federal Energy Regulatory Commission issued order 888 establishing a guideline to provide open access to transmission lines. This policy removed restrictions on ownership of power generation facilities, which allowed non-utility electric power producers to access transmission lines. This was a major step toward electricity deregulation in the United States.

The main purpose of deregulation is to reduce operational cost, to increase efficiency, and to encourage competition among electricity suppliers with the medium and long-term goal of combating high prices. Deregulation gives consumers more alternatives because consumers are not held to only one service provider. Availability of power from various suppliers guarantees supply reliability in case of a peak demand or unexpected outages. In other words, the more there is available, the greater the competition will be to produce and sell power in an efficient way, leading to lower prices and more energy efficiency. Moreover, deregulation is believed to provide better economic incentives and opportunities to both consumers and suppliers because the existence of a large number of consumers and suppliers reduces market power which prevents a firm from dominating a market. Consequently, it enables a company to enter or exit the markets which allows competitors to take advantage of any economic opportunity.

2.2. Capacity Market

Capacity markets have proven to be one of the most contentious elements of electricity restructuring. However, Cramton and Stoft (2005) argue that a capacity market is needed in most restructured electricity markets.

In competitive markets, consumers can easily decrease their demand in response to prices and other market signals for most products. In this case, extra capacity is not necessary because prices give consumers the right signals when supplies are tight (NEPA, 2008). In power markets, however, there is little demand response to price, primarily because load neither sees nor pays the real-time price. Real-time meters and demand management control systems are not yet in place for most electricity consumers. This absence prevents consumers' willingness to limit demand during times of supply scarcity. As a result, the price can reach extreme values. Price caps are set by market administrators in order to limit the peak prices, which occur at peak demand periods or with unexpected outages. As the price caps are in effect, the investors do not see the opportunity for them to make an investment in new resources. The imperfectly competitive market structure is the other issue in power markets. Consequently, there are instances when one or more suppliers have substantial market power, especially at peak times or during an outage of a large generator or transmission line. Addressing these two issues typically results in price peaks that are too infrequent and insufficient to motivate efficient investment in new capacity (Cramton and Stoft, 2005) which leads to the failure of electricity markets.

Moreover, the economic consequences of running out of electric generating capacity are severe. The best example is an event of the blackout that took place in the US East coast in August 2003 which shows that the value of the losses caused by a system imbalance may be substantially large (Creti and Fabra 2004). Fixed cost is also an important factor in electricity markets. Since power cannot be stored, there has to be significant extra capacity available to meet both peak demand periods, and because some generators will not be available due to outages and maintenance. To stay reliable, a power system has to have this built-in reserve margin. Some source of money is necessary to cover the cost of the extra plants required for this purpose, which is not very often. Normal revenues in the energy market will not cover the costs of this extra reserve margin, especially with prices subject to various regulatory controls (NEPA, 2008).

For those reasons, the capacity markets must be introduced to the restructured electricity markets. In capacity markets, each retail supplier is required to produce its share of the responsibility for ensuring there is sufficient generating capacity in the region. Retail suppliers can purchase capacity either from generators that meet certain standards of availability or on the spot market to meet their requirements. The main purpose of introducing capacity markets into the restructured electricity markets is to ensure that sufficient generating capacity will be available to meet peak demands while providing investment incentives for power suppliers. In addition, capacity markets will cover the cost to keep adequate generation available. The capacity market payments reflect the costs of keeping sufficient capacity (plus the reserve) ready and available to the region. In other words, they represent the option to call on generators as and when needed (NEPA, 2008).

Creti and Fabra (2004) discussed in their paper that capacity markets can be classified as either price based or quantity based. In price based systems, the capacity availability is paid either via lump-sum payments or increase to energy payments depending on the probability of outages. The price based systems are not working as well as expected because producers are able to increase capacity payments by making fewer capacity resources available instead of increasing capacity resources. This increases the probability of shortages. Quantity based systems have been introduced in several power markets in the U.S. such as New England Power Generators Association Inc., New York ISO, and PJM. Nowadays, installed capacity markets, which are one of the quantity based systems, are the focus of the policy debate in the United States. The purpose of introducing the installed capacity markets is to ensure that adequate capacity is committed on a daily or seasonal basis to meet system loads and reserve requirements. The Load Serving Entities (LSEs) that sell electricity to end-user consumers must satisfy the expected peak loads plus a reserve margin. LSEs can buy through internal transactions, bilateral transactions, or capacity markets in the event of shortages. The equilibrium price in the capacity markets should be related to the overall capacity in the system.

2.3. Cournot Model

The Cournot model was named after Antoine Augustin Cournot, the nineteenth-century French mathematician, who first examined its implications. Augustin Cournot was born in 1801. His book, *Researches into the Mathematical Principles of the Theory of Wealth*, was published in 1838. The basic Cournot Model is a one-period model in which each firm has to forecast the other firm's output decision. Given its forecast, each firm then chooses a profit-maximizing output for itself (Varian, 2006). Cournot competition, in other words, is a form of quantity competition which means firms must choose profit-maximizing output levels instead of prices in the belief that each competing firm maximizes its expected profits. It also assumes that the quantities supplied by other producers are fixed and do not react to price changes. The competition then seeks equilibrium. The Cournot equilibrium refers to a situation where each firm finds its beliefs about the other firms to be confirmed. The Nash equilibrium solution for the optimum quantities to be generated by each producer is provided by Cournot model. The market price is determined by the Nash equilibrium solution given the price elasticity of demand.

The following is the standard Cournot model that will be used later as a fundamental model in this dissertation. Daughety (2005) discussed the basic one-stage Cournot model for an industry comprised of *n* firms. Each firm chooses its output level. Firm *i*'s output level is denoted as q_i where $i = 1, \ldots, n$ and let the vector of firm outputs be denoted as $\boldsymbol{q} \equiv (q_1, q_2, ..., q_n)$. Let Q refers to the aggregate industry output level (i.e., $Q = \sum_{i=1}^{n} q_i$). We will refer the $(n-1)$-vector of output levels chosen by other firms as $\boldsymbol{q}_{-i}$. Thus, $(\boldsymbol{q}_{-i,} q_i)$ also denotes to the vector of firm outputs, $\boldsymbol{q}$. The inverse market demand is denoted as $p(Q)$. Furthermore, firm i's cost function can be denoted as $c_i(q_i)$. Therefore, firm i's profit function can be written as $\pi_i(\boldsymbol{q}) = p(Q)q_i - c_i(q_i)$. The Cournot equilibrium consists of a vector of output levels, $\boldsymbol{q}^{CE}$, such that no firm wishes to change its output level when other firms produce the output levels assigned to them in the equilibrium. A Cournot equilibrium can be alternatively called a Nash-Cournot equilibrium because it is a Nash equilibrium with quantities as strategies chosen from a compact space. Hence, $\boldsymbol{q}^{\boldsymbol{CE}}$ is a Cournot equilibrium if $\pi_i(\boldsymbol{q}^{\boldsymbol{CE}}) \geq \pi_i(q_{-i}^{CE}, q_i)$ for all values of q_i, for $i = 1, 2, \ldots, n$.

An immense effort has been made to design several models and tools that specifically represent the electricity market behavior. The Cournot model has been one of the theoretical frameworks most widely used to model strategic behavior in electricity markets. In this research, our main goal is to model electricity markets under competition conditions and several uncertain factors. Our approach considers a market in which firms compete in quantity as in the Cournot model.

Browning and Browning (1989) discussed the definition of the Cournot model, an excellent way to introduce the nature of oligopolistic interdependence. Each firm takes into account how price changes as the firm or its competitors change quantity and choose their quantity to maximize their profits given the quantity that their competitor is producing. The Cournot model shows how uncoordinated output decisions between rival firms could interact to produce an outcome that lies between the competitive and monopolistic equilibria. However, the final equilibrium reflects their interdependence although each firm explicitly ignores the other. In the last several years, the topic of strategic behavior in electricity markets has received a great deal of attention. Several oligopoly models have been proposed, notable among which is the Cournot model. Many analysts believe that the Cournot model is better able to represent the electricity bilateral market than most other models. Borenstein and Bushnell (1999) give the arguments about various approaches and explain why the Cournot model is the best approach for market power in electricity markets. In addition, competing firms have long-term commitments to capacity although they may compete on price in the short term. The perfect competition model is based on the assumption that any firm can capture the entire market by pricing below other suppliers and supplying the entire demand but generating capacity constraints and increasing marginal costs make this assumption invalid. Thus, the centralized pricing mechanism and the capacity constraints support the case for adopting the Cournot model. The Cournot model also enables the analysis of situations in which producers unilaterally decide to withhold supply from the market by declaring some of their generators to be unavailable. Furthermore, The Cournot model leads to a simple analytical expression for the market price that renders itself easily to analytical manipulations.

Restructured power markets take a wide variety of forms. A wide range of models have been proposed for simulating the interaction of competing power generation companies. Benjamin Hobbs (2001) presents two Cournot models of imperfect competition among electricity producers. The first model presents the producers' and grid owner's optimization problems. It includes a congestion pricing scheme for transmission. After combining their KKT conditions with the market clearing condition, they yield a mixed linear complementarity problem. The second model differs from the first model in that the first one has no arbitrage between nodes of the network, while in the other model, arbitragers erase any non-cost-based differences in price. In other words, power generators recognize that marketers will buy and resell power where price differences exceed the cost of transmission. Finally, a simple example is presented to illustrate their application.

Most previous Nash-Cournot models of competition among electricity suppliers have assumed smooth demand functions. However, nonsmooth demand functions are an important feature of real power markets due to many factors such as transmission constraints. Pang and Hobb (2005) developed the complementarity-based model of Nash-Counot oligopolistic electric power markets to include concave demand functions that are piecewise linear. These models also include linear joint constraints within generator profit maximization problems. Furthermore, they begin with a multivalued complementarity formulation of the equilibrium problem, from which an equivalent single-valued linear complementarity problem formulation is derived. They mentioned in the paper that this new model is computationally challenging. For instance, they immediately invalidate the solution methods employed for the previous models. However, they successfully solved this problem by using a specific algorithm. As mentioned earlier, transmission constraints are another important factor in real power markets. Yuan, Liu, Jiang, and Hou (2005) proposed the Cournot model taking into account transmission constraints based on DC power flow. They analyzed the effects of simple two-bus network and three-bus network transmission constraints on the pure strategies of suppliers. The results show that there may exist different pure strategy equilibriums if transmission constraints are considered. Cunningham, Baldick, and Baughman (2002) also investigated perfect competition equilibrium and Cournot equilibrium in a simple example, triangular connection. Both cases are examined on transmission unconstrained and transmission constrained in order to compare the results.

Results show that a pure strategy equilibrium can break down even when a transmission constraint exceeds the value of the unconstrained Cournot equilibrium line flow.

One of the major problems in the power markets is assuring that generators, which independently decide about their outputs, will not produce more than the available transmission capacity. Willems (2000) developed a model for the power markets of generators when transmission capacity is scarce. The purpose of this model is to apply the different models to a simple electricity market with one transmission line. They apply different Cournot concepts and explain the implicit assumptions about the behavior of the system operator. They also show that these implicit assumptions are not realistic. According to implicit assumptions, they formulate some alternative assumptions for the behavior of the System Operator and examine the results.

2.4. Mixed Complementarity Problems

Complementarity problems are a natural format for expressing a variety of economic models and arise frequently in the general equilibrium theory of economics (Ferris and Kanzow, 1998 and Rutherford, 2002). Optimization may be viewed as a special case of complementarity problems, since the standard optimality conditions for linear and smooth nonlinear optimization are complementarity problems (AMPL, 2008). Many computable general equilibrium models are used for various aspects of policy design and analysis, including carbon abatement, trade reform, and game theory (Ferris and Kanzow, 1998). One example of this area is the deregulation of electricity markets.

According to Rutherford (2002), the mixed complementarity problem is defined as

Given: $F : R^N \to R^N, \quad l,\ u\ \in\ R^N$

Find: $z,\ w,\ v\ \in\ R^N$

s.t. $F(z) - w + v = 0$

$$l \le z \le u, \quad w \ge 0, \quad v \ge 0$$

$$w^T(z - l) = 0, \quad v^T(u - z) = 0$$

in which $-\infty \le l \le u \le +\infty$.

Complementarity problems consist of complementarity conditions and each of them requires that the product of two or more decision variables be zero. Michael Ferris and Munson (1998) present how these problems are modeled within the GAMS modeling language and provide details about the PATH solver for finding a solution. Specifically, they develop the complementarity framework by looking at the transportation model. The transportation model is a simple linear program where demands for a single product must be satisfied by suppliers at minimal transportation cost. In other words, they show how to convert a linear program into a linear complementarity problem which can be recognized as the complementary slackness conditions of the linear program. One popular solver for this problem is called PATH which can be found in GAMS and AMPL. Like AMPL, GAMS is a high-level modeling system for mathematical programming and optimization.

Ferris and Munson show how to implement a linear complementarity problem in GAMS and discuss the available options and output of the PATH solver. Finally, some extensions of complementarity problems and additional uses of the solver are given.

Another is Billups and Murty (1999). They provide an introduction to complementarity problems. Various forms of complementarity problems are described along with a few sample applications such as: piecewise linear equations; an application of a small size convex QP model; obstacle with free boundary problems; and traffic equilibrium. The important algorithms are presented with a discussion of when they can be used effectively. They also present other interesting algorithms for solving linear complementarity problem. The first one is Pivotal Method which tries to obtain a basic feasible complementary vector through a series of pivot steps. The next algorithm is the interior point method. This algorithm follows a path in an effort to reduce the constraints to zero. Furthermore, they provide a brief introduction to the study of matrix classes and their relation to linear complementarity problems.

2.4.1. Linear Complementarity Problem

A linear complemetarity problem (LCP) is a subset of mixed complementarity problems. Linear complementarity problems are problems where a given $m \times m$ matrix $\boldsymbol{M}$ and a compatible vector $\boldsymbol{q}$ are given. The task is to find the value of a vector $\boldsymbol{z}$ that satisfies a set of constraints. The general form of LCP can be written as

Given: $\boldsymbol{M} \in \boldsymbol{R}^{m \times m}, \quad \boldsymbol{q} \in \boldsymbol{R}^{m}$

Find: $\boldsymbol{z} \in \boldsymbol{R}^{m}$

s.t. $\boldsymbol{Mz} + \boldsymbol{q} \geq \boldsymbol{0}$

$\boldsymbol{z} \geq \boldsymbol{0}$

and $\boldsymbol{z}^{T}(\boldsymbol{Mz} + \boldsymbol{q}) = \boldsymbol{0}.$

LCP is truly nonlinear and tools of nonlinear analysis can be successfully applied to LCP (Stewart, 2008 and Thomas, 2002). They can be represented in various ways in terms of nonlinear systems of equations such as quadratic programs.

Many algorithms which can be used to solve linear complementarity problem have been proposed including one by He, Li, and Pan (2005). They developed a self-adjusting interior point algorithm for linear complementarity problems. This algorithm is based on constructing a new proximity measure function instead of using the primal-dual interior point methods. As a result, they get a new centering equation with a set of parameters which play the important role of being self-adjusting in this algorithm. Numerical comparison is made between the proposed algorithm and the primal-dual interior point methods. Results show that the proposed algorithm has the efficiency as well as some other advantages. For example, the number of iterations increases very slowly as the number of variables increases.

Li and Dai (2007) introduced a generalized AOR method for solving a linear complementarity problem whose general case is reduced to a generalized SOR method.

These two methods are considered to be an iterative algorithm. Some computational results are presented in this paper.

Liao and Wang (2003) proposed a self-adaptive projection and contraction method for linear complementarity problems. They claim their algorithm is better than He's algorithm (1992) in the sense that their algorithm improves the practical performance of the modified projection method. The proof of global convergence of this new method is also included in the end of their paper.

Sun and Huang (2006) developed a smoothing Newton algorithm for the LCP with a sufficient matrix. First, they applied a smoothing function to LCP which leads to a new formulation called a parameterized smooth equation. Then a Newton method with a projection type testing procedure is used to solve that equation. They also show that this algorithm will terminate in a finite number of steps as long as the LCP has a solution.

CHAPTER III

COURNOT MODEL IGNORING UNCERTAINTY

Abstract --- Since deregulation of the electric industry has become the norm in many parts of the world, the price of electricity under deregulation is no longer determined by regulatory agencies but by market demand, supply conditions, load elasticity, and strategic behavior. Firms now face much more risk and are responsible for their own economic decisions. The firms, therefore, need decision-support models that support these new requirements. In this research, we have developed the multi-period deterministic Cournot model for the long-term market which is extended from a single period model. However, demand uncertainty, generator outages, and fuel price uncertainty are ignored in this chapter. The Nash-equilibrium quantities are calculated by combining the KKT first order optimality conditions of the extended model. The KKT conditions of the deterministic Cournot model are considered as linear complementarity problems (LCP). The market prices and each firm's profit are calculated. Results in this chapter are used as the standard results to show the effect of uncertainties in power markets when we consider them in the model.

3.1. NOMENCLATURE

The notation used in this chapter is given below for reference.

n	Number of firms
N	Total number of generators
N_f	Number of generators of firm f
$P_{fj}^{\max}$	The capacity of the j^{th} unit of firm f (MW)
c_{fj}	The marginal cost of the j^{th} unit of firm f (\$/MWh)
s_f^*	The Nash-equilibrium quantities (MWh)
S^*	The Nash-equilibrium total bid (MWh)
p^*	The Cournot price (\$/MWh)
K	Nominal demand (MWh)
h	Number of contracted hours
t_0	The beginning of a contract
t	The present time
i	Interest rate (%)
δ	Compound-amount factor
π_f	Profit of firm f (\$)
ξ	Slope parameter for demand

3.2. INTRODUCTION

Before deregulation, there was a belief that electricity is a national asset. Therefore, electric sectors in most countries were subject to full regulation. The generation, transmission, distribution and retail segments were controlled by state governments which played a dual role as electricity service providers and regulators. As economic growth became more and more dependent on sufficient electricity suppliers, the importance of electricity increased tremendously. Consequently, many governments started to realize that this growth may be impeded by being fully regulated because of the regulators' slow response to technological progress in electricity operations. In addition, the successful deregulation in oil and gas established the belief that electricity is a service which can also be improved by deregulation (Yao, 2006).

Deregulation in the United States took place at both the federal and state levels in 1996. The Federal Energy Regulatory Commission (FERC) encouraged deregulation of the electricity market by issuing order 888 and establishing guidelines to provide open access to transmission lines. This policy removed restrictions on ownership of power generation facilities which allowed non-utility electric power producers to access transmission lines.

The main purpose of deregulation is to reduce operational cost, to increase efficiency, and to encourage competition among electricity suppliers with the medium and long-term goal of combating high prices. Deregulation gives consumers more choices because they are then not held to only one power provider. Availability of power from diverse suppliers ensures supply reliability throughout the operation in case of a peak demand or unexpected outages. It can be said that the greater the availability, the greater the competition will be to produce and sell power in an efficient way, leading to lower prices and more energy efficiency. Moreover, deregulation is believed to provide better economic incentives and opportunities to both consumers and suppliers because the existence of a large number of consumers and suppliers reduces market power in which a firm is prevented from dominating a market. Consequently, it enables any company to enter or exit the market which in turn allows competitors to take advantage of any economic opportunity.

The deregulation of the electricity industry is significantly changing the way in which suppliers do their business. Firms' optimal decisions will now be dependent on market electricity prices. The price of electricity under deregulation is determined by market demand, supply conditions, load elasticity, and strategic behavior. It also depends on physical factors such as production cost, load, unit commitment, and transmission constraint (Valenzuela and Mazumda, 2005). Electricity is different from other products because it has yet to become efficiently storable. Therefore, its demand and supply must be matched every second. Otherwise, a costly system collapse may result. Firms, therefore, are faced with much more risk, and they become greatly responsible for their own economic decisions in deregulated power markets. Hence, these firms need decision-support models that fulfill these new requirements. The decision-support models need to incorporate the uncertainties and other important factors involved in deregulated power markets.

Recent attempts have been made to model the structure of deregulated electricity markets via utility system production simulation models that have been used in the past for planning and regulatory purposes (Kahn, Bailey, and Pando, 1996) but many analysts believe that the Cournot model is better able to represent the electricity market as it has evolved (Borenstein and Bushnell, 1999).

In this chapter, the multi-period deterministic model which is extended from a single period model is presented. Long-term power agreements and the Cournot competition is assumed for firms' bidding structure in the market. The time value of money is also considered in the model. Demand and fuel cost are assumed to be constant. Moreover, the availability of generating units is ignored in this chapter.

The remainder of this chapter is organized as follows: Section 3 provides a methodology to develop the multi-period deterministic Cournot model in the long term markets. In section 4, a numerical example, Nash-equilibrium quantities, market prices and, each firm's profits are presented. The conclusions are given in section 5.

3.3. MODEL DESCRIPTION

In this section, model assumptions and a methodology to develop the multi-period deterministic Cournot model in the long term markets are presented.

3.3.1. Model Assumptions

In the restructured wholesale market, power generators can trade power in both short-term and long-term markets. Short-term refers to a day or hours, while long-term refers to weeks or years. In the long-term market, generators and consumers agree in private at time 0 through a central exchange on the delivery of specified power quantities at some specific time in the future (at time t_0) through a long term power purchase agreement. In this research, the main focus is to model the generators in long-term power agreements and study their effects on market prices and firms' profits. The price is set at the time of the agreement and remains unchanged for the period of the contract in the long-term market. The following additional assumptions apply to the model:

3.3.1.1. Power Producer

A total of n competing asymmetric firms with firm f having a set of $N_f - 1$ units available for production at time 0 are assumed. The total number of units available in the market at time 0 is denoted by N. In reality, there are other available power sources. Instead of producing its own electricity, a firm has the ability to purchase energy at a higher price from outside sources. Therefore, the last unit of production or a unit N_f^{th} represents the available power sources which are considered to have infinite capacity and to be always available. The following are assumptions related to each unit:

- The capacity of the j^{th} unit of firm f is represented by $P_{fj}^{\max}$ (MW).
- Each firm's units are dispatched according to an ascending order of their marginal costs, which is denoted by c_{fj}.
- The unit commitment and transmission constraints are ignored.
- Fuel cost is assumed to be a constant and generator outages are ignored.

3.3.1.2. Demand (load)

To represent the behavior of the load-serving entities, a linear inverse demand function is assumed. In Figure 1, the curve shows a general demand function for the long-term market and indicates the price responsiveness of consumers. The quantity K is the nominal demand and is assumed to be a constant in this model. The actual realized demand function, which is denoted by L, is affected by the price elasticity of demand $\xi \geq 0$, which is also known by the producers. p is the electricity price ($/MWh). Thus, the actual demand of the system is represented by the following linear relationship:

$$L = K - \xi p \tag{3.1}$$

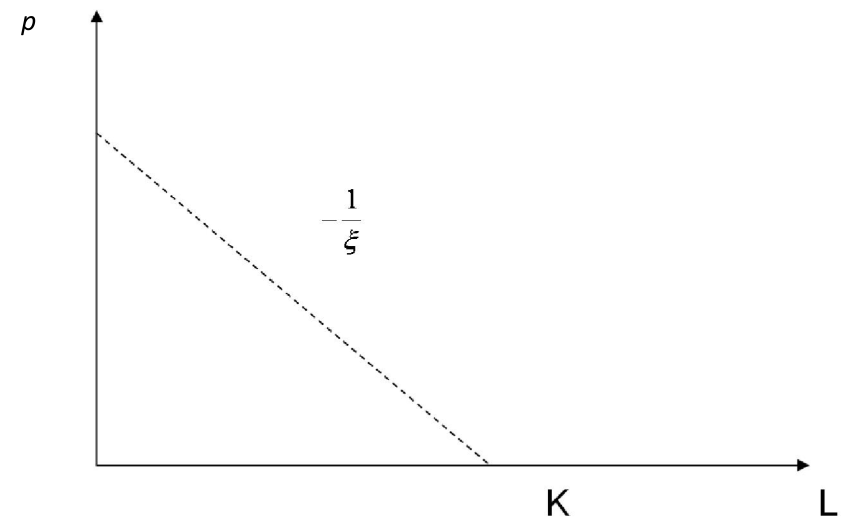

Figure 1. Consumer Demand Curve

3.3.1.3. Market Operation

As mentioned earlier, generators and consumers sign a long-term contract at time 0. Therefore, the production amounts are determined at time 0 and the actual generation will occur at time t_0. This scenario is depicted in Figure 2.

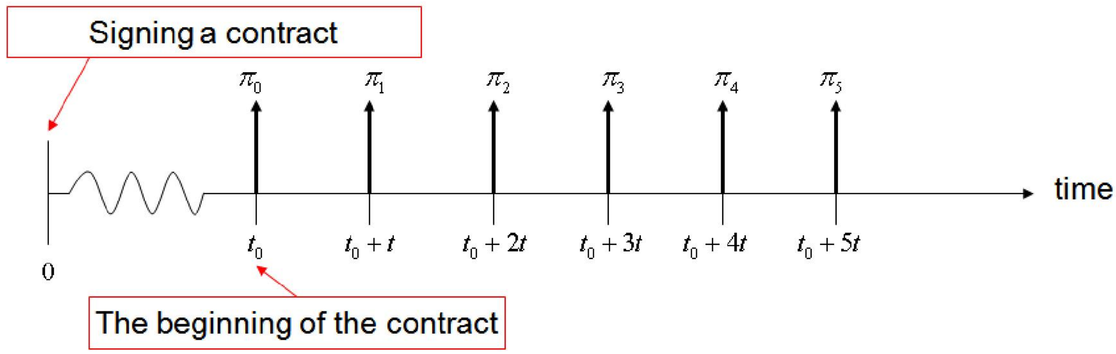

Figure 2. Market operation in the deterministic model

3.3.2. Mathematical Model

To model the Cournot competition, each supplier simultaneously determines a quantity s_f that it is willing to supply at each period t. Suppliers choose their quantities in order to maximize their total profit at each period over the duration of the contract, assuming that the total of other firms' bids s_{-f} is known. Hence, the profit of firm f at hour t_0+t can be written as

$$\pi_{f,t_0+t}(s_f \mid s_{-f}) = s_f p(K, s_f + s_{-f}) - Cost_{f,t_0+t}(s_f) \tag{3.2}$$

where t_0 is time at the beginning of the contract, $Cost_{f,t_0+t}(s_f)$ is the cost at hour t_0+t for supplier f to produce the quantity s_f and $p(K, s_f + s_{-f})$ refers to the non-negative price at this period when the nominal demand is K, and the total market supply is the Nash-equilibrium total bid (S^*). Note that t can be any number between 0 and the number of contracted hours ($t = 0,1,2,...,h$). The cost function, $Cost_{f,t_0+t}(s_f)$, is not a random variable as fuel cost is assumed to be a constant and generator outages are ignored in this chapter.

As we know, the value of money now is different from the value of money in the future, the profit at every hour t must take the time value of money into consideration before calculating the total profit. Therefore, the net present value must be applied to the profit at every hour t in order to obtain the total profit at time t_0. The total profit function after applying the net present value is shown below:

$$\underset{s_f}{Max}\ \pi_f = \left[\sum_{t \in D} \pi_{f,t_0+t}(s_f | s_{-f}) \times \delta_t \right] \tag{3.3}$$

where $\delta_t = \dfrac{1}{(1+i)^t}$,

D is the set of hours at which the supplier will deliver the contracted quantity s_f (MW) in the future, δ is compound amount factor, and i is the discount rate (%).

Considering that after the production amounts are determined at time 0 and the actual generation will occur at time t_0. In a deterministic model, the hourly profit remains the same at every hour. Hence, the total profit function can be written as follows:

$$\begin{aligned} \underset{s_f}{Max}\ \pi_f &= \left[\pi_f(s_f | s_{-f}) \sum_{t \in D} \delta_t \right] \\ &= \delta \left[\pi_f(s_f | s_{-f}) \right] \end{aligned}$$

where $\delta = \sum_{t \in D} \delta_t$.

Therefore, the total profit function (3.3) is shown below:

$$\underset{s_f}{Max}\ \pi_f = \delta \left[s_f p(K, s_f + s_{-f}) - Cost_f(s_f) \right]. \tag{3.4}$$

When the Cournot decision maker assumes that generators are always available and fuel costs remain unchanged over a period, the function $p(K, s_f + s_{-f})$ in (3.4) based on (3.1) can be written as $\frac{1}{\xi}(K - s_f - s_{-f})$. Note that K is assumed to be a constant due to constant demand. The cost function in (3.4) is represented by $C_f(s_f)$ when generator outages are ignored and fuel prices are assumed to remain unchanged. Thus, the total profit function for each firm becomes

$$\begin{aligned} &\underset{s_f}{Max} \quad \pi_f = \delta R_f(s_f; s_{-f}) - \delta C_f(s_f) \\ &\text{where} \\ &R_f(s_f; s_{-f}) = \frac{1}{\xi} s_f (K - s_f - s_{-f}) \end{aligned} \tag{3.5}$$

where h is the contracted number of hours, the cardinality of D, and $\delta = \sum_{t=1}^{h} \delta_t$. The function $C_f(s_f)$ is the production cost curve of supplier f assuming all generators are available. The production cost curve of supplier f can be calculated by

$$C_f(s_f) = \sum_{j=1}^{N_f} c_{fj} g_{fj}$$

where c_{fj} is the variable cost of the j^{th} unit of firm f in \$/MWh and g_{fj} is the power generated by the j^{th} generator of firm f in MWh.

Therefore, the optimization problem given by (3.5) can be written as the following programming model:

$$
\begin{aligned}
&\underset{s_f}{Max}\, \pi_f = \delta R_f(s_f; s_{-f}) - \delta \sum_{j=1}^{N_f} c_{fj} g_{fj} \\
&\text{subject to} \\
&\sum_{j=1}^{N_f} g_{fj} - s_f \geq 0 \qquad (\lambda_f) \\
&g_{fj} \leq P_{fj}^{\max} \qquad (\alpha_{fj}) \quad \text{for } j = 1, \ldots.., N_f \\
&\forall g_{fj}, s_f \geq 0
\end{aligned}
\tag{3.6}
$$

Where the values of λ_f and α_{fj} are the dual variables of the corresponding constraints. The above model (3.6) is a quadratic programming model.

The Nash-equilibrium quantities that solve the set of problems s_f ($f = 1, \ldots, n$) can be computed by combining the KKT first order optimality conditions of system equation (3.6) of all suppliers. The KKT conditions of optimization problem (3.6) can be written as the following equations:

$$
\begin{aligned}
&\text{for } f = 1, \ldots, n \\
&\lambda_f - \frac{\delta}{\xi}(K - 2s_f - s_{-f}) \geq 0 \quad \perp \quad s_f \geq 0 \\
&\sum_{j=1}^{N_f} g_{fj} - s_f \geq 0 \quad \perp \quad \lambda_f \geq 0 \\
&\text{for } f = 1, \ldots, n \text{ and } j = 1, \ldots, N_f \\
&\delta c_{fj} + \alpha_{fj} - \lambda_f \geq 0 \quad \perp \quad g_{fj} \geq 0 \\
&P_{fj}^{\max} - g_{fj} \geq 0 \quad \perp \quad \alpha_{fj} \geq 0
\end{aligned}
\tag{3.7}
$$

The result of these KKT optimality conditions is a linear complementary problem (LCP). By using available software to solve it as a complementarity problem, the Nash-equilibrium quantity for each firm in MWh, s_f^*, is obtained. The total demand can be calculated as $S^* = \sum_{f=1}^{n} s_f^*$. The demand relationship (3.1) is used to determine the Cournot price, p^*, which becomes

$$p^* = \frac{K - S^*}{\xi}. \tag{3.8}$$

3.4. EXPERIMENTAL RESULTS

For a numerical illustration of results, we consider a market that consists of three firms. The composition of each firm is given in Table 1 which includes the capacity, marginal cost, and net plant heat rate. The characteristics of the unit types in Table 1 are taken from the IEEE reliability test system (Grigg, 1996). Firms 1, 2, and 3 have 11, 7, and 9 generators respectively.

The last row of each firm in the table corresponds to the assumptions that the N_f^{th} unit of generator for firm f has infinite capacity and is perfectly reliable due to other available sources in the markets. Each firm is assumed to operate 12 hours per day (off-peak hour) and 30 days per month. The annual percentage rate (APR) is assumed to be 7% for all firms. It is also assumed each firm and the consumers agree through a central exchange on the delivery of specified power quantities for 2 months and they receive the payment from the customers at the end of each month. Furthermore, the values of parameters ξ and K are assumed to be 15 and 3000 respectively. Based on the given numerical data, the value of the discount coefficient (δ) in (3.5) is 713.75.

Table 1. Market composition and generating unit Data

Firm	Unit	Fuel Type	Capacity (MW)	Net Plant Heat Rate (Mbtu/MWh)	Marginal Cost ($/MWh)
Firm 1	1	Coal	350	9.5	52.45
	2	Coal	350	9.5	52.45
	3	Coal	155	9.72	52.76
	4	Coal	155	9.72	52.76
	5	Coal	76	11.9	55.86
	6	Coal	76	11.9	55.86
	7	Gas	48	10.23	62.56
	8	Gas	48	10.23	62.56
	9	Gas	78	11.63	68.41
	10	Gas	78	11.63	68.41
	11	Gas	149	12.87	73.59
	12	-	∞	-	999
Firm 2	1	Hydro	50	-	0.07
	2	Hydro	50	-	0.07
	3	Coal	155	9.72	52.76
	4	Coal	76	11.9	55.86
	5	Gas	48	10.23	62.56
	6	Gas	48	10.23	62.56
	7	Gas	78	11.63	68.41
	8	-	∞	-	999
Firm 3	1	Uranium	400	-	0.017
	2	Hydro	50	-	0.07
	3	Hydro	50	-	0.07
	4	Coal	350	9.5	52.45
	5	Coal	76	11.9	55.86
	6	Gas	48	10.23	62.56
	7	Gas	78	11.63	68.41
	8	Gas	149	12.87	73.59
	9	Gas	149	12.87	73.59
	10	-	∞	-	999

The Nash-equilibrium quantities are obtained by solving the linear complementarity problem in (3.7) using PATH solver called by AMPL. The Cournot price is calculated according to (3.8). The results are shown in Table 2.

Table 2. The results of Nash-equilibrium ignoring uncertainty

Firm 1 (MWh)	Firm 2 (MWh)	Firm 3 (MWh)	Total (MWh)	p^* ($)
595.42	427	595.42	1,617.84	92.14

The total profit for each firm can be computed by substituting the Nash-equilibrium quantities and the power generated by each generator into (3.5). The result is shown in Table 3.

Table 3. The total profit when ignoring uncertainty

Firm	Total Profit ($)
1	16,869,300
2	14,924,400
3	35,577,600

These results will be used to show the effect of uncertainties in power markets when we consider those uncertainties in chapters 4 and 5.

3.5. CONCLUSIONS

The multi-period deterministic Cournot model for the long-term market was developed in this research. The deterministic Cournot model in this chapter belongs to a class of a quadratic programming. The Nash-equilibrium quantities were calculated by combining the KKT first order optimality conditions of the extended model. The KKT conditions of the deterministic Cournot model are considered as linear complementarity problems (LCP). The linear complementarity problem can be solved by using the PATH solver in AMPL. The market prices and each firm's profit were calculated by substituting the Nash-equilibrium quantities and the power generated by each generator into the profit function. The deterministic model and results in this chapter will be used to show the effect of uncertainties in power markets when we consider these uncertainties in the next two chapters.

CHAPTER IV

STOCHASTIC COURNOT MODEL INCLUDING GENERATOR OUTAGES

Abstract --- The uncertainty in generator availability is a crucial factor which should be considered in a medium-term or long-term planning process. An approach to determining market prices considering generator outages is proposed in this chapter. The multi-period Cournot model in the previous chapter is modified by replacing its cost function with the expected cost function. Specifically, the expected cost function in terms of generator availability is developed. The expected cost curve is a piece-wise linear function with a large number of slopes. Each slope represents the marginal cost and capacity of a hypothetical generator. Since considering all slopes could take long computational time and make the problem difficult to solve, an algorithm for reducing the number of slopes without losing accuracy is developed. In addition, the effect of generator outages on the firms' expected profits is analyzed.

4.1. NOMENCLATURE

The notation used in this chapter is given below for reference.

n	Number of firms
N	Total number of generators
N_f	Number of generators of firm f
$\overline{P}_{fj}^{\max}$	The capacity of the j^{th} hypothetical unit of firm f considering outages (MW)
$\overline{c}_{fj}$	The marginal cost of the j^{th} hypothetical unit of firm f including generator outages ($/MWh)
s_f^*	The Nash-equilibrium quantities (MWh)
S^*	The Nash-equilibrium total bid (MWh)
p^*	The Cournot price ($/MWh)
K	Nominal demand
h	Number of contracted hours
i	Interest rate (%)
δ	Compound-amount factor
π_f	Profit of firm f ($)
ξ	Slope parameter for demand
t	The present time
μ_{fj}	The failure rate of the j^{th} unit of firm f per hour
λ_{fj}	The repair rate of the j^{th} unit of firm f per hour

4.2. INTRODUCTION

Power outages play an important role in both developing and industrialized countries. When the outages take place, they can be momentary or last for several days affecting only a few small areas or entire cities. Although most of the power outages are man-made, there are some outages caused by nature such as hurricanes, flooding, and earthquakes.

Unplanned power outages are the situation that power plants in both developing and industrialized countries try to avoid because the economic consequences of electric power outages are severe. Blackouts affect not only economics but also our daily life. The best example took place in the Northeastern United States in August 2003, which was the worst blackout in U.S. history.

The blackout in August 2003 started shortly after 4 PM EDT and resulted in the loss of 61,800 MW of electric load that served more than 50 million people in the U.S. and Canada including large urban centers that are heavily industrialized and important financial centers (e.g., New York City and Toronto, Ontario). Nearly half the Canadian economy is located in Ontario and was affected by the blackout. Most areas were fully restored within two days but parts of Ontario took more than a week before power was restored (Electricity Consumers Resource Council (ELCON), 2004). In addition, the blackout also affected industry as well as infrastructure such as water supply, transportation, and communication. All manufacturers in the Northeastern United States reporting indicated that the blackout caused a complete shutdown in operation. The Ohio manufacturers' Association estimated the direct cost of the blackout on Ohio manufacturers to be $1.08 billion (Electricity Consumers Resource Council (ELCON), 2004). The U.S. Department of Energy (2008) has also published a total economic cost estimate of about $6 billion due to the blackout. Electricity is a very special product in the sense that it is not storable over extended periods of time. It is generally consumed in less than a second after being produced. The power generation and demand must be matched every second in the power supply networks. The mismatch between supply and demand, either overload of a power line or underload/overload of a generator, can cause severe damage to a network component which may lead to a cascading failure of a large section. The reality is that generators are not always available. They may fail any second and the next cheapest available generators will replace them in order to meet the demand. Generator failure is believed to be one of the common causes for long duration outages in power markets. In addition, analysis has shown that the outage of a generator is the initiating event of cascading faults which may rapidly lead to a catastrophic failure, i.e. a major blackout (Genesi, Granelli, Innorta, Marannino, Montagna, and Zanellini, 2007). Since only a few minutes of blackout can cost millions of dollars to the whole system, uncertainty of generator availability is one of the significant factors in power markets which must be considered in a decision-support model.

The main purpose of this chapter is to develop a model to assess the effects of generator outages on electricity prices under Cournot competition. The extension of the deterministic version of the Cournot model in chapter 3 is developed by including the reliability of the generating units. Fuel price uncertainty is ignored in this chapter. The method generally used in the literature to deal with the issue of generator failure is to derate plant capacities. This approach, however, may lead to inaccurate results (Valenzuela and Mazumda, 2007). Some literature simply ignores this factor. Unlike other literature, the expected production cost function including generator failure is implicitly modeled in this chapter rather than the cost of the expected production quantities obtained after the generators are derated. When incorporating generator outages, the cost function is modified by introducing the operating state of each generator which will lead to a new expected cost function.

This chapter is organized as follows: A stochastic Cournot model in the long term markets including generator outages is developed in section 3. In section 4, numerical results as well as the comparison of both firm's expected profit considering generator failure and ignoring outages are presented. The conclusions are outlined in section 5.

4.3. MODEL DESCRIPTION

In this section, a procedure for computing the market prices taking generator failure in a deregulated long-term power market into consideration is described. Instead of derating plant capacities, the expected production cost function including generator outages is developed.

4.3.1. Model Assumptions

The following assumptions apply to this model:

4.3.1.1. Market Operation

In a long-term market, the price is set at the time of the agreement (time 0) and remains unchanged for the period of the contract. In other words, the production amounts are determined at time 0 and the actual generation will occur long after this time (at time

t_0). The Markov process is assumed to reach its steady state when the contracted amount will be generated. This scenario is illustrated in Figure 3.

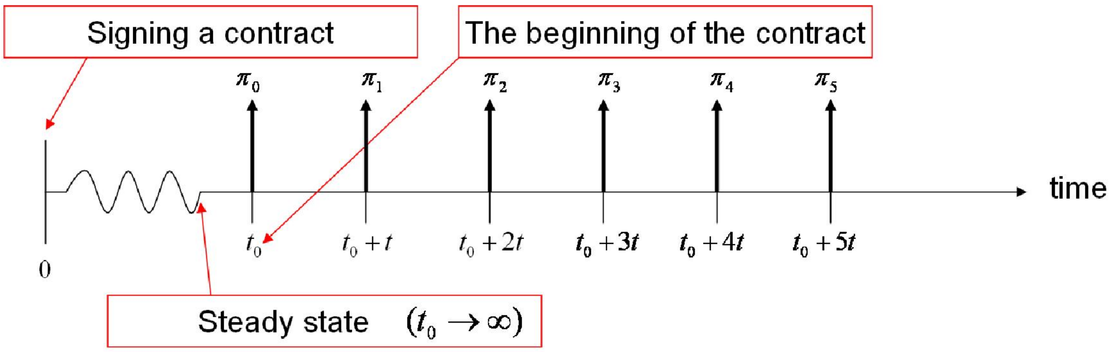

Figure 3. Market operation in the stochastic model

4.3.1.2. Power Producer

A total of n competing asymmetric firms are considered in this chapter, in which all firms make their decisions according to the Cournot model. That each firm f possesses a set of $N_f - 1$ units available for production at time 0 is assumed. In addition, the last unit of production or a unit N_f^{th} represents the available power sources. In this chapter, the power sources are considered to have infinite capacity and to be always available. The following are assumptions related to each unit:

- The capacity of the j^{th} unit of firm f is represented by $P_{fj}^{\max}$ (MW).
- Each firm's units are dispatched according to an ascending order of their marginal costs, which is denoted by c_{fj}.
- Generator outages are assumed in this chapter.

- The unit commitment and transmission constraints are ignored and fuel price is assumed to be a constant.

4.3.2. Mathematical Model

4.3.2.1. Demand (load)

To represent the behavior of the load-serving entities, a linear inverse demand function is assumed. The actual realized demand function of the system, denoted by S^*, is affected by the price elasticity of demand $\xi \geq 0$, which is known by the power producers. The actual demand of the system is represented by the following linear relationship:

$$L = K - \xi p . \tag{4.1}$$

The quantity K stands for the nominal demand and is assumed to be a constant in this section. The generation quantity for firm f (MWh), s_f, is the sum of generation quantities of all generators of firm f ($f = 1, 2, ..., n$) and is calculated as follows:

$$s_f = \sum_{j=1}^{N_f} g_{fj} \quad .$$

The actual demand of the system is equal to the total of all firms' generation quantities and it can be written as follows:

$$S^* = \sum_{f=1}^{n} s_f^* .$$

4.3.2.2. Profit Function

Power producers that are bidding in the market own a set of generators. When those generators are not always available, the cost function becomes a random variable. Therefore, the total profit function from the previous chapter becomes the expected total profit function in this chapter. After considering the time value of money and generator outages, the expected total profit function, based on (3.3), becomes

$$\underset{s_f}{Max}\, \pi_f = E\left[\sum_{t\in D} \pi_{f,t_0+t}(s_f | s_{-f}) \times \delta_t\right]$$

where $\delta_t = \dfrac{1}{(1+i)^t}$.

Considering that after the production amounts are determined at time 0 and the actual generation will occur long after time t_0, the Markov process of the unit availability is assumed to reach the steady state (see Figure 3). In the steady state ($t_0 \to \infty$), hourly expected profits remain the same at every hour. The total expected profit function in the steady state can be simplified as follows:

$$\begin{aligned}\underset{s_f}{Max}\, \pi_f &= \underset{t_0\to\infty}{E}\left[\sum_{t\in D} \pi_{f,t_0+t}(s_f | s_{-f}) * \delta_t\right] \\ &= \underset{t_0\to\infty}{E}\left[\pi_f(s_f | s_{-f})\right]\left[\sum_{t\in D} \delta_t\right] \\ &= \delta \underset{t_0\to\infty}{E}\left[\pi_f(s_f | s_{-f})\right]\end{aligned}$$

where $\delta = \sum_{t\in D} \delta_t$ and $\underset{t_0\to\infty}{E}$ represents the expected value considering that the stochastic process that reigns the unit availability is in the steady state.

Hence, the expected total profit function for supplier f, in the steady state, can be rewritten as

$$\delta \underset{t_0 \to \infty}{E}\left[\pi_f(s_f \mid s_{-f})\right] = \delta R_f(s_f; s_{-f}) - \delta \underset{t_0 \to \infty}{E}\left[Cost_f(s_f)\right].$$

In Cournot competition, firm f assumes the total quantity, produced by other firms$\left(s_{-f}\right)$, and each firm f then determines the quantity s_f in order to maximize its total expected profit at each period over the duration of the contract. Thus, the total expected profit function for supplier f in the steady state can be calculated as follows:

$$\pi_f = \delta\left\{s_f p(K, s_f + s_{-f}) \ - \ E\left[Cost_f(s_f)\right]\right\}. \tag{4.2}$$

4.3.2.3. Reliability

To introduce the operating state of a generator j of firm f, we begin by defining a two-state continuous-time Markov process which is 1 if unit j of firm f is available at time t and 0 for otherwise. The Markov process has a failure rate per hour $\lambda_{f,j}$ and a repair rate per hour $\mu_{f,j}$. Since in our model the production amounts are determined at time 0 and the actual generation will occur long after this time (at time t_0), the Markov process is assumed to reach its steady state when the contracted amount is generated. The steady state probability that the generator j of firm f will be available is denoted by

$$r_{f,j} = \frac{\mu_{f,j}}{\mu_{f,j} + \lambda_{f,j}} \tag{4.3}$$

and, the steady state probability that generator j of firm f will not be available is denoted by

$$q_{f,j} = 1 - r_{f,j}\,. \tag{4.4}$$

4.3.2.4. Cost function

To compute the expected cost function, we first define $\overline{C}_{f,j}(s)$ as the expected cost function of supplier f producing s units of energy when generators 1, 2, ... , $j-1$ are not available. The expected production cost function including generator outages is developed below.

For unit $j = 1,\ 2, ...,\ N_f - 1$, the following recursive relationship is used:

$$\overline{C}_{f,j}(s) = \begin{cases} r_{f,j} c_{f,j} s + q_{f,j} \overline{C}_{f,j+1}(s) & \text{for } 0 \le s \le P_{f,j}^{Max} \\ \\ r_{f,j} \left\{ c_{f,j} P_{f,j}^{Max} + \overline{C}_{f,j+1}(s - P_{f,j}^{Max}) \right\} + q_{f,j} \overline{C}_{f,j+1}(s) & \text{for } s > P_{f,j}^{Max} \end{cases}$$

The last unit of generator (N_f), which is always available, can be represented by the following relationship:

$$\overline{C}_{f,N_f}(s) = s \times c_{f,N_f}\,.$$

Owing to the recursive relationship, the calculation starts from the first unit of generator to the last unit (N_f) in order to obtain the total expected cost. Therefore, the expected production cost function considering generator outages can be written as follows:

$$E\left[Cost_f(s_f) \right] = \overline{C}_{f,1}(s_f)$$

where the expected cost of firm f , given by $\overline{C}_{f,1}(s_f)$, is a piecewise linear function with respect to s_f . The slopes of this piecewise linear function are always increasing, as each firm's units are dispatched according to an ascending order of their marginal costs. Each slope represents a marginal cost and capacity of a generator including outages. The set of all combinations of unit capacities determines where the function changes its slope. For example, one firm has three units with a capacity of 12, 20, and 50 MW. There are $2^3 - 1$ combinations in total with these three generators which mean the expected cost function changes its slope at 12, 20, 32, 50, 62, 70, and 82 MW. In other words, this firm actually owns a set of seven hypothetical generators with the maximum capacity of 12, 20, 32, 50, 62, 70, and 82 MW respectively when considering generator outages. In the aspect of complexity, this may not seem to be much different from assuming that generators are always available. However, if a firm owns a total of n generators, it could have $2^n - 1$ different slopes in the worst case which means this firm has to consider a set of $2^n - 1$ different generators and maximum capacities. Since this is an exponential function, when n is a big number, it will make a huge difference in computational complexity. An example of the expected cost curve when a power company has 12 units of generators is depicted in Figure 4.

In Figure 4, the graph seems to show only 3 or 4 different slopes but the expected cost curve actually contains roughly $2^{12} - 1$ different slopes. The reason that the graph displays only 3 to 4 slopes is some slopes may differ from others by a small amount. Considering the fact that some slopes may differ from others by a small amount, it is better to combine them and represent the cost function with a lesser number of slopes without losing accuracy. Moreover, the complexity of calculations can be eased with a smaller number of slopes.

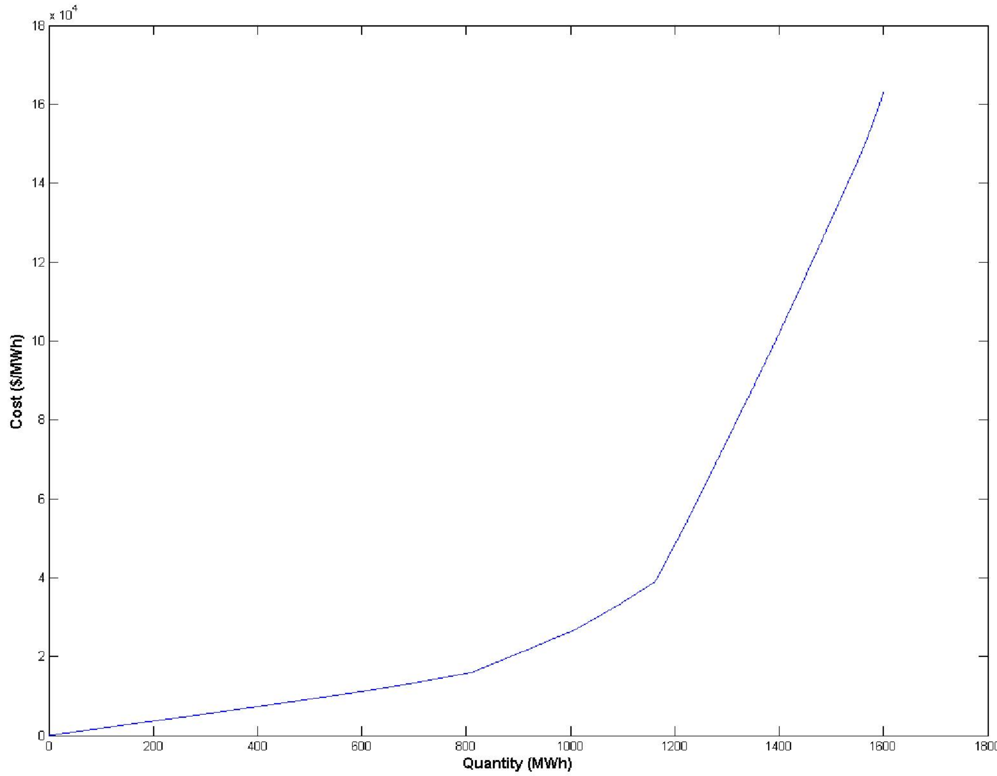

Figure 4. The expected cost curve of 12 generating units

To reduce the number of slopes, we define Δs and γ to be the increasing amount of capacity and the difference between slopes, respectively. Both are constant values and must be set at the beginning. Let N^0 be the maximum range in which we want to combine slopes such that $N^0 > \sum_{j=1}^{N_f} P_{f,j}^{\max}$ for each firm *f*.

The following pseudo-code describes the slope reduction algorithm for firm f:

FOR $l = 2$ to N^0
    Set $X_l = l \times \Delta s$, and $C_l = \overline{C}_{f,1}(X_l)$
END FOR
Set $r = 1,\ K = 2,\ C_1 = 0,$ and $X_1 = 0$

WHILE $K < N^0$

$j =$ Min i such $K \le i \le N^0$ and $(C_{i+1} - 2C_i + C_{i-1}) > \gamma$

$$m_{f,r} = \frac{\sum_{l=K-1}^{j} X_l C_l}{\sum_{l=K-1}^{j} X_l^2}$$

$NewP_{f,r}^{Max} = X_j$ and $Temp = X_j$

FOR $l = j$ to N^0

$X_l = X_l - Temp$

$C_l = C_l - m_{f,r} \times Temp$

END FOR

$K = j + 1$

$r = r + 1$

END

The slope reduction algorithm produces an estimation of the expected cost function which is used to generate estimated expected cost curve. A slope in estimated expected cost curve is associated with each hypothetical generator (*r*) which has a maximum capacity ($NewP_{f,r}^{Max}$) and marginal cost $(m_{f,r})$. Let $r^{\max}$ represent a value of *r* when the slope reduction algorithm terminates. The values of $m_{f,r}$ and $NewP_{f,r}^{Max}$ are similar in nature to those $c_{f,j}$ and $P_{f,j}^{\max}$ described in chapter 3 but they take generator outages into account. In order to simplify the presentation, the values of marginal cost $(m_{f,r})$ and maximum capacity ($NewP_{f,r}^{Max}$) will be represented as $\bar{c}_{f,j}$ and $\bar{P}_{f,j}^{\max}$ where ($j = 1, 2, \ldots, r^{\max}$).

An estimation of the total expected cost for each firm f, $\bar{C}_{f,1}(s_f)$, can be calculated as follows:

$$\overline{C}_{f,1}(s_f) \approx \sum_{j=1}^{N_f} \overline{c}_{f,j} g_{f,j}$$

where $\overline{c}_{f,j}$ is the marginal cost obtained from the slope reduction algorithm.

4.3.2.5. Modeling Competition

When the Cournot decision maker assumes demand remains unchanged over a period of time, the function $p(K, s_f + s_{-f})$ in (4.4) based on (4.1) can be written as

$$p(K, s_f + s_{-f}) = \frac{1}{\xi}(K - s_f - s_{-f}).$$

Thus, each supplier f solves the following programming model:

$$\underset{s_f}{Max}\, \pi_f = \delta R_f(s_f; s_{-f}) - \delta \overline{C}_{f,1}(s_f)$$

subject to

$$\sum_{j=1}^{N_f} g_{fj} - s_f \geq 0 \qquad (\lambda_f)$$

$$g_{fj} \leq \overline{P}_{fj}^{\max} \qquad (\alpha_{fj}) \qquad \text{for } j = 1, \ldots\ldots, r^{\max} \qquad (4.5)$$

$$\forall\, g_{fj}, s_f \geq 0$$

where

$$R_f(s_f; s_{-f}) = \frac{1}{\xi} s_f (K - s_f - s_{-f})$$

and $\quad \delta = \sum_{t=1}^{h} \delta_t.$

The Nash-equilibrium quantities s_f ($f = 1, 2, \ldots, n$) that solve the set of problems can be computed by combining the KKT first order optimality conditions of system equation (4.5) of all firms. The KKT conditions of optimization problem (4.5) can be written as the following equations:

$$
\begin{aligned}
&\text{for } f = 1,\dots,n \\
&\lambda_f - \frac{\delta}{\xi}(K - 2s_f - s_{-f}) \geq 0 \quad \perp \quad s_f \geq 0 \\
&\sum_{j=1}^{N_f} g_{fj} - s_f \geq 0 \quad \perp \quad \lambda_f \geq 0 \\
&\text{for } f = 1,\dots,n \text{ and } j = 1,\dots,r^{\max} \\
&\delta \bar{c}_{fj} + \alpha_{fj} - \lambda_f \geq 0 \quad \perp \quad g_{fj} \geq 0 \\
&\bar{P}_{fj}^{\max} - g_{fj} \geq 0 \quad \perp \quad \alpha_{fj} \geq 0
\end{aligned}
\tag{4.6}
$$

These KKT conditions are still considered as a linear complementary problem (LCP). The Nash-equilibrium quantity for each firm in MWh (s_f^*) is obtained by using available software to solve those optimality conditions. As mentioned earlier, the actual demand of the system or total demand can be calculated as $S^* = \sum_{f=1}^{n} s_f^*$. The linear relationship in demand (4.1) is used to determine the Cournot price, p^*, which becomes

$$
p^* = \frac{K - S^*}{\xi}. \tag{4.7}
$$

4.4. EXPERIMENTAL RESULTS

In this section, the methodology explained in section 3 is implemented for a market that consists of three firms. For a numerical illustration of results, the composition of each firm is given in Table 4 including the capacity, marginal cost, net plant heat rate, and availability of each generator. The characteristics of the unit types in Table 4 are taken from the IEEE reliability test system (Grigg, 1996). Firms 1, 2, and 3 have 11, 7, and 9 generators respectively. The last row of each firm in the table corresponds to the assumption that the N_f^{th} generating unit of firm f has infinite capacity and is assumed to be perfectly reliable due to other available sources in the markets. The available generator types are coal, oil, hydro, and nuclear. The marginal cost of one type of technology is calculated based on its heat rate. Although generators consume the same type of fuel, they do not always have the same marginal cost. Furthermore, the values of parameters ξ and K are assumed to be 15 and 3000 respectively. Each firm is assumed to operate 12 hours per day (off-peak hours) and 30 days per month. It is also assumed that all firms and consumers agree in private through a central exchange on the delivery of specified power quantities for 2 months and the firms receive the payment from customers at the end of each month. The annual percentage rate (APR) is assumed to be 7% for all firms. Next, the discount coefficient in (4.5), δ, can be computed in order to consider time value of money in the model. Based on the given APR, the value of the discount coefficient is 713.75.

Based on the numerical data in Table 4, the expected cost function of each firm contains a large number of slopes. The slope reduction algorithm plays a vital role in reducing the number of slopes (those marginal costs and maximum capacities). The value of Δs is assumed to be 1 in this section. Moreover, a value of the difference between slopes γ is chosen by determining the smallest number that yields the value of estimated cost as close as the original value of expected cost. The greater the value of γ, the more inaccurate the results will be. The value of γ can be simply judged by comparing the expected cost curve and the estimated cost curve. If those two graphs are similar, the value of γ can be used without making the final results imprecise. After applying the slope reduction algorithm to the numerical data in Table 4, the expected cost curve and the estimated cost curve for firms 1, 2, and 3 are shown in Figures 5, 6, and 7, respectively.

The value of γ for firms 1, 2, and 3 is chosen to be 6, 3, and 5, respectively. Comparing the expected cost curve and the estimated cost curve in each of Figures 5, 6, and 7, the two graphs are almost identical. Therefore, it can be said that this new set of hypothetical generators can be used to compute the equilibrium quantities without losing accuracy. The curve of the normal cost function is also included in Figures 5, 6, and 7. Note that for a given quantity, a value of the expected cost is higher than the normal cost described in chapter 3 because of generator outages.

Marginal costs and maximum capacities of the hypothetical generators for firms 1, 2, and 3 are shown in Tables 5, 6, and 7, respectively.

Table 4. Market composition and generating unit data including unit availability

Firm	Unit	Fuel Type	Capacity (MW)	Net Plant Heat Rate (Mbtu/MWh)	Marginal Cost ($/MWh)	Availability (hours)	
						1/λ	1/μ
Firm 1	1	Coal	350	9.5	52.45	1150	100
	2	Coal	350	9.5	52.45	1150	100
	3	Coal	155	9.72	52.76	960	40
	4	Coal	155	9.72	52.76	960	40
	5	Coal	76	11.9	55.86	1960	40
	6	Coal	76	11.9	55.86	1960	40
	7	Gas	48	10.23	62.56	1340	26
	8	Gas	48	10.23	62.56	1340	26
	9	Gas	78	11.63	68.41	1720	30
	10	Gas	78	11.63	68.41	1720	30
	11	Gas	149	12.87	73.59	1505	42
	12	-	∞	-	999	-	-
Firm 2	1	Hydro	50	-	0.07	1980	20
	2	Hydro	50	-	0.07	1980	20
	3	Coal	155	9.72	52.76	960	40
	4	Coal	76	11.9	55.86	1960	40
	5	Gas	48	10.23	62.56	1340	26
	6	Gas	48	10.23	62.56	1340	26
	7	Gas	78	11.63	68.41	1720	30
	8	-	∞	-	999	-	-
Firm 3	1	Uranium	400	-	0.017	1100	150
	2	Hydro	50	-	0.07	1980	20
	3	Hydro	50	-	0.07	1980	20
	4	Coal	350	9.5	52.45	1150	100
	5	Coal	76	11.9	55.86	1960	40
	6	Gas	48	10.23	62.56	1340	26
	7	Gas	78	11.63	68.41	1720	30
	8	Gas	149	12.87	73.59	1505	42
	9	Gas	149	12.87	73.59	1505	42
	10	-	∞	-	999	-	-

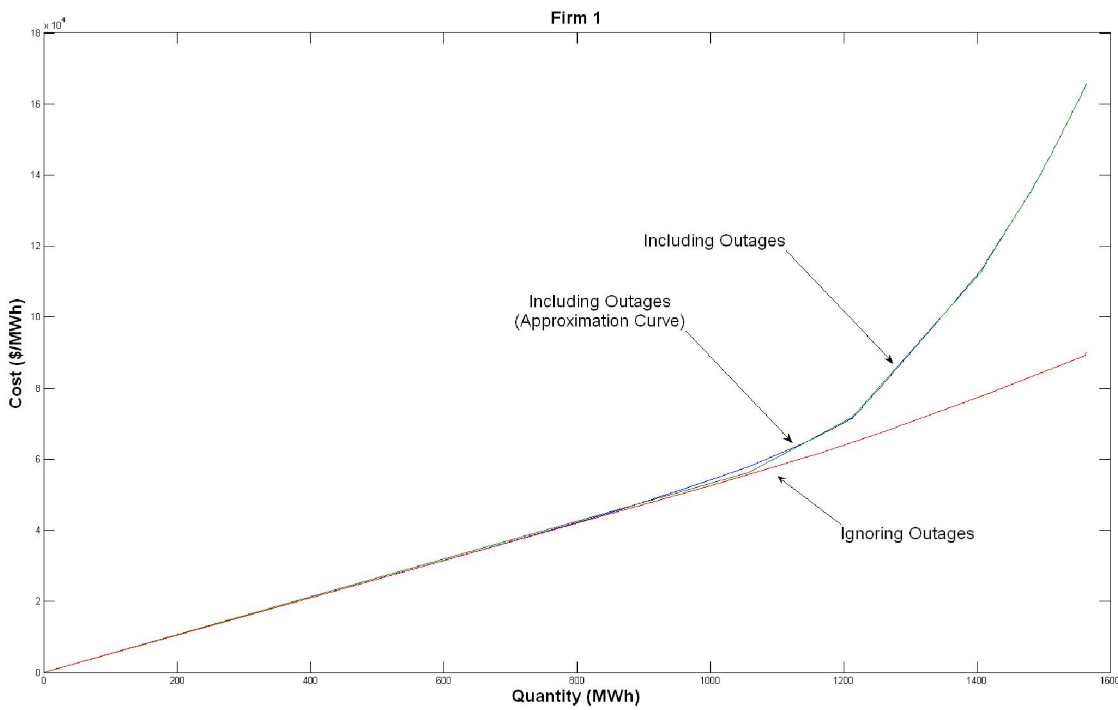

Figure 5. Cost curves of supplier 1

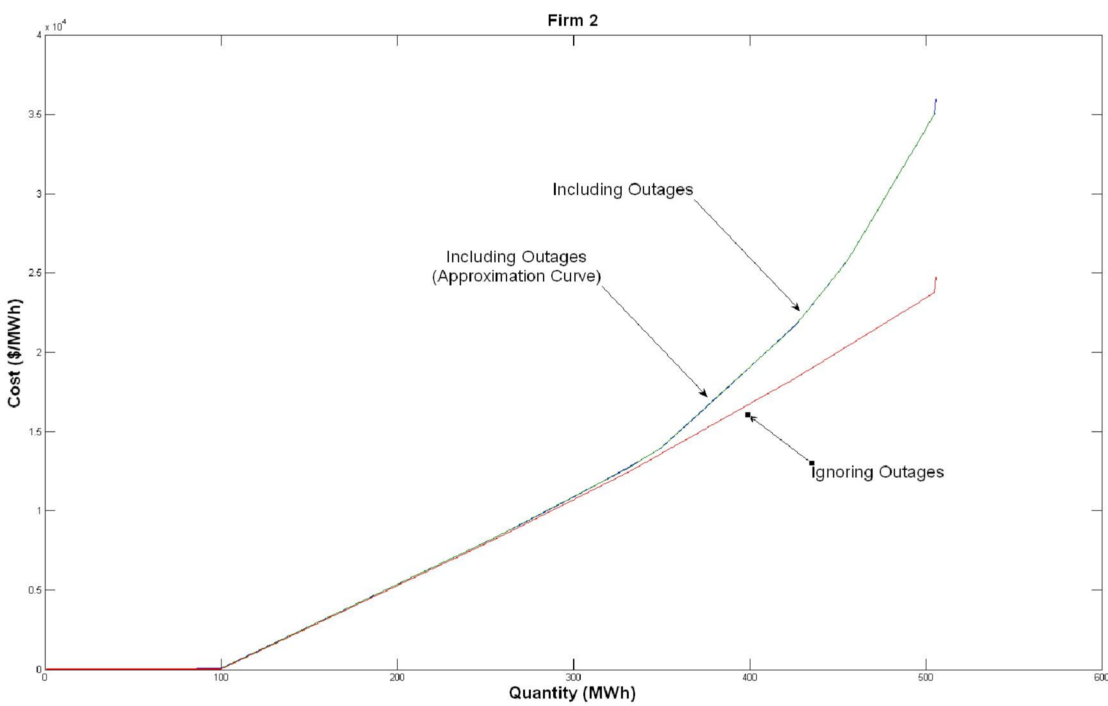

Figure 6. Cost curves of supplier 2

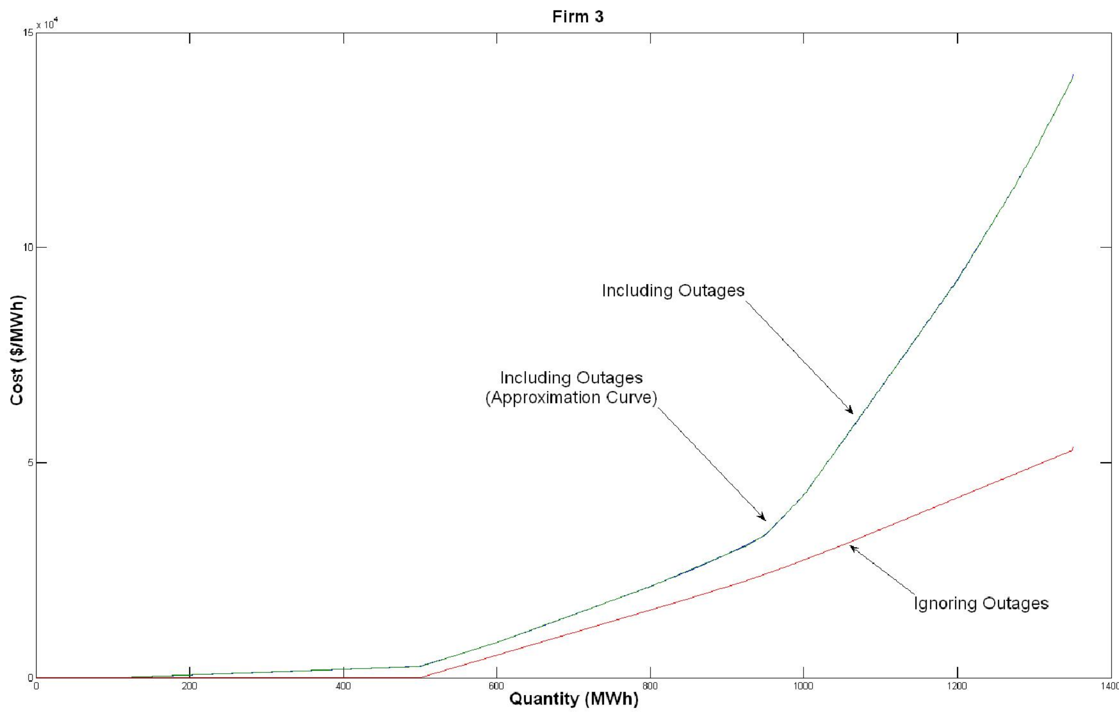

Figure 7. Cost curves of supplier 3

Table 5. The data list of hypothetical generating units for firm 1

Unit	Capacity (MW)	Marginal Cost ($/MWh)
1	1058	53.19
2	155	99.85
3	195	211.47
4	71	294.29
5	28	346.29
6	48	372.44
7	2	403.11
8	6	415.63
9	∞	999

Table 6. The data of hypothetical generating units for firm 2

Unit	Capacity (MW)	Marginal Cost ($/MWh)
1	100	0.41
2	155	53.18
3	76	57.91
4	19	69.47
5	77	101.18
6	26	139.00
7	2	152.01
8	2	158.68
9	48	187.40
10	∞	999

Table 7. The data of hypothetical generating units for firm 3

Unit	Capacity (MW)	Marginal Cost ($/MWh)
1	100	0.06
2	400	6.50
3	100	55.95
4	201	64.93
5	125	75.68
6	24	107.94
7	50	181.97
8	201	249.86
9	2	264.23
10	71	294.10
11	26	317.07
12	2	325.95
13	48	342.38
14	∞	999

The Nash-equilibrium quantities or firms' quantity bids are calculated according to the KKT first order optimality conditions in (4.6) using the PATH solver. The Cournot price is calculated according to (4.7) assuming a linear relationship. The results of Nash-equilibrium quantities and market prices are shown in Table 8.

Table 8. The results of Nash-equilibrium including generator outages

Firm 1 (MWh)	Firm 2 (MWh)	Firm 3 (MWh)	Total (MWh)	p^* ($)
631.16	350	589.78	1570.94	95.27

4.4.1. Effect of generator availability

For ease of explaning, the deterministic model described in chapter 3 is called Model A and the stochastic model concerning outages is called Model B. To evaluate whether the availability of generators has an effect on market prices, first the model profit and adjusted profit ignoring uncertainty (Model A) for each firm are compared. Next, the adjusted profits of Model A and the expected profits of Model B are compared.

For model A, the model profit is the amount that firms believe they will make when ignoring outages, while the adjusted expected profit is the amount that firms would obtain in reality because generator outages do occur.

The model profits of Model A are computed by substituting the Nash-equilibrium quantities and the power generated by each generator into the objective function of Model A (3.5). Let s_f^{*A} represent the Nash-equilibrium quantities of Model A. Model A's profit can be written as follows:

$$\pi_f = \delta\left[R_f(s_f^{*A}; s_{-f}^{*A}) - C_f(s_f^{*A})\right]$$

where $R_f(s_f^{*A}; s_{-f}^{*A}) = \dfrac{1}{\xi} s_f^{*A}(K - s_f^{*A} - s_{-f}^{*A})$.

Unlike the model profit, the adjusted expected profit of Model A is calculated from the difference between the revenue function and the expected cost function including outages, $\overline{C}_{f,1}(s_f^*)$. For Model A, the adjusted profit can be calculated as follows:

$$E[\pi_f] = \delta\left\{R_f(s_f^{*A}; s_{-f}^{*A}) - \overline{C}_{f,1}(s_f^{*A})\right\}$$

where s_f^{*A} stands for the Nash-equilibrium quantities obtained from Model A.

The model expected profit and expected profit of Model B are the same because the model considers outages. The model expected profit and expected profit when considering generator failures can be computed as follows:

$$\pi_f = \delta\left\{R_f(s_f^{*B}; s_{-f}^{*B}) - \overline{C}_{f,1}(s_f^{*B})\right\}$$

where s_f^{*B} represents the Nash-equilibrium quantities obtained from Model B.

Results of the model profits and expected profits for each firm in both cases are shown in Table 9.

Table 9. Expected profits of firms when ignoring uncertainty and including outages

	Model A Ignoring Uncertainty ($/Hour)		Model B Including Outages ($/Hour)	
Firm	Model Profit	Adjusted Expected Profit	Model Expected Profit	Expected Profit
1	23,634.67	23,194.05	26,557.68	26,557.68
2	20,909.78	17,549.88	19,340.47	19,340.47
3	49,845.87	46,919.70	48,560.83	48,560.83

According to Table 9, the adjusted profits and the model profits of firm 1 for model A are comparable. For firm 2 and 3, however, the adjusted profit is less than the model profit, which means, for example, firm 3 believes that it will make $49,845.87/hour, but in reality it would make only $46,919.70/hour on average.

When comparing the expected profits using Model B firms 1, 2, and 3 make more profit than Model A. These results indicate market participants, who make decisions without considering uncertainty in generator availability, could be led to false decision-making and an inaccurate planning process.

4.5. CONCLUSIONS

An approach to determining market prices that considers generator outages was developed in this chapter. The multi-period Cournot model in the previous chapter was modified by replacing its cost function with the expected cost function. Specifically, the expected cost function in regard to generator availability was developed by defining a two-state continuous-time Markov process. The expected cost function generated a large number of slopes. Each slope represents one value of marginal cost and maximum capacity which includes the uncertainty in generator availability. To ease computational complexity, the slope reduction algorithm was developed.

Results showed that the slope reduction algorithm can efficiently reduce a number of slopes and aid computational complexity. In addition, the model and expected profits were computed to evaluate whether generator outages have an effect on firms’ profit. Results indicated that generator availability is a crucial factor, as it has effects on both market prices and firms’ profit.

CHAPTER V

STOCHASTIC COURNOT MODEL INCLUDING GENERATOR OUTAGES AND FUEL PRICE UNCERTAINTY

Abstract --- Nowadays, the volatility associated with generation and fuel prices places a new emphasis on modeling uncertainties in power markets. It is essential for all companies to account for uncertainty. Each firm operates a set of generators which use different types of fuels to produce electricity. The fluctuation in fuel costs significantly impacts the firm's long-term operation. In this chapter, the stochastic Cournot model is extended to consider not only the availability of generators but also fuel price uncertainty. Since each generator consumes a fuel type whose price is not known in advance, the marginal cost of each generator becomes a random variable. A convenient way to calculate the Nash-equilibrium quantities when considering the randomness in fuel price is the use of a Monte Carlo simulation. A numerical example is given, where the market prices and firms' expected profits are computed. In addition, the effects of generator outages and fuel price uncertainty on power markets are analyzed.

5.1. NOMENCLATURE

The notation used in this chapter is given below for reference.

n	Number of firms
N	Total number of generators
N_f	Number of generators of firm f
$\overline{\overline{P}}_{fj}^{\max}$	The capacity of the j^{th} hypothetical unit of firm f considering outages and fuel price uncertainty (MW)
$\overline{\overline{c}}_{fj}$	The marginal cost of the j^{th} hypothetical unit of firm f including generator outages and fuel price uncertainty (\$/MWh)
s_f^*	The Nash-equilibrium quantities (MWh)
S^*	The Nash-equilibrium total bid (MWh)
p^*	The Cournot price (\$/MWh)
K	Nominal demand
h	Number of contracted hours
i	Interest rate (%)
δ	Compound-amount factor
π_f	Profit of firm f (\$)
ξ	Slope parameter for demand
μ_{fj}	The failure rate of the j^{th} unit of firm f per hour
λ_{fj}	The repair rate of the j^{th} unit of firm f per hour

5.2. INTRODUCTION

It is widely accepted that an increase in fuel price volatility has a great impact on the whole of economic activity and creates an uncertain situation for power producers, consumers, investors, and legislators. It may slow down economic growth. It may delay producers' decisions on making new investments which may result in lost market opportunities and inefficient long-run resource allocations. Daily or hourly fluctuations in wholesale prices may be almost irrelevant to the consumers, but it is vital to power trading companies since increase or decrease in prices can change the way companies do their business. Moreover, fuel price uncertainty may create pressures for regulatory intervention which can bias the power markets and penalize market participants by generating wide and unpredictable revenue swings (Henning, Sloan, and Leon, 2003). Hence, volatility in fuel costs has become a new issue that power companies must be able to handle in order to guarantee appropriate power system planning and operation.

The price of gasoline is the best example to show why fuel price is so difficult to forecast. In October 2006, the average retail price for a gallon of gasoline in the U.S. was around $2.20 per gallon. Since then, the price of gasoline has risen dramatically. The maximum average price per gallon was over $4. This trend continued until late 2008. This situation is depicted in Figure 8 (EIA, 2008). In August 2008, the average price of gasoline decreased sharply. The average price decreased from $4.10 to $1.60 in only 4 months. Demand is not the only factor that affects the price of gasoline; other factors include worldwide economics and politics. Thus, the price of gasoline is considered highly volatile similar to those of natural gas and electricity.

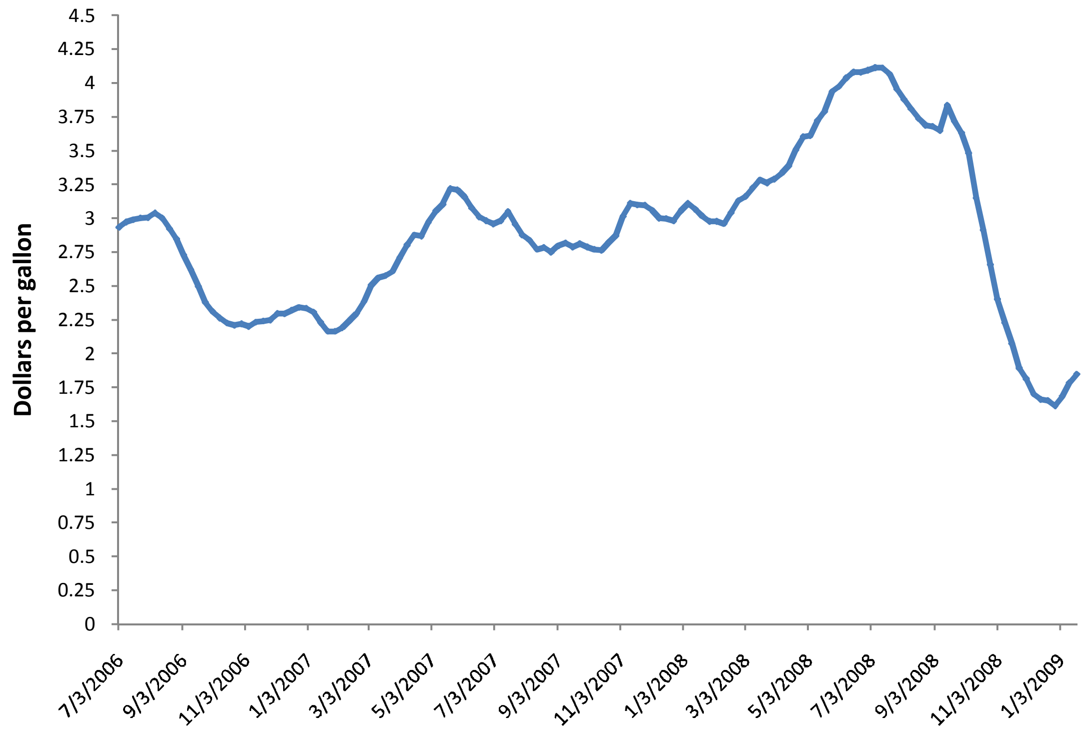

Figure 8. U.S. retail gasoline prices, Regular grade (Source: EIA 2009)

Most electricity in U.S. is generated by rotating turbines which are most commonly driven by steam. Steam is typically produced from water that is boiled by burning coal, natural gas, or petroleum. In 2008, coal-fired plants contributed 48.3% of the U.S electric power. Nuclear plants contributed 19.3%, while 21.5% was generated at natural gas-fired plants. Hydroelectric provided 6.7% of the total while petroleum and renewable energy generated the remaining electric power (EIA, 2008). Owing to lower fuel costs, nuclear plants are operating at a much higher utilization and they supply the base load in general.

The Energy Information Administration (EIA) forecasts in the Energy Outlook 2009 early release that the price of petroleum, natural gas, and coal in U.S. will increase about 80.55%, 33.8%, and 14.4% respectively in the next 20 years. Moreover, EIA also predicts in the annual energy outlook that the price of electricity will only increase about 13%. Power producers have to sell electricity cheaper due to severe competition, and they have to generate electricity cheaper than other firms despite the increase in the price of fuel. For a thermal generation-based electric utility, fuel costs comprise approximately 80% of the system operating cost. Consequently, a small percentage savings in fuel costs represents significant monetary value. In the era of intense competition in power markets, a small savings may be crucial (Lee, Liao, and Breipohl, 1992). As it appears fuel costs will be volatile in the future and fuel savings will be vital to stay in the business; power producers in the U.S. are facing a new set of challenges. In the last decades, uncertainty in fuel costs and generator availability have become a structural element in this new environment that all power companies must be able to cope with in order to guarantee appropriate power system planning and operation as well as their economical liquidity (Gomes, Saraiva, and Neves, 2008). In addition, company profits are also influenced by the fluctuation of market prices of electricity that varies concomitantly depending on fuel market prices (Bannai and Tomita, 2005).

In this research, the primary aim is to develop a model to evaluate the effects of the uncertainty of fuel costs and generator availability on firms' profits. The determination of market price is achieved by the Cournot competition of firms in the market. Each firm operates a set of generators which use different types of fuels whose marginal costs are subject to uncertainty. Thus, the marginal costs become random variables which affect the expected cost function. When considering both generator outages and fuel price uncertainty, a convenient way to calculate the Nash-equilibrium quantities is to use a Monte Carlo simulation (MCS) based technique. The major advantage in an MCS-based approach is that it can capture the spatial distribution of the uncertainties of generation, which is very important where marginal cost differentials play a leading role (Wijayatunga and Cory, 2003).

The remainder of this chapter is organized as follows: Section 3 provides a methodology to develop the stochastic Cournot model considering generator outages and fuel cost uncertainty. In section 4, numerical example, Nash-equilibrium quantities, market prices and each firm's expected profit are presented. The conclusions are given in section 5.

5.3. MODEL DESCRIPTION

In this section, model assumptions and a methodology to develop the stochastic Cournot model including generator availability and fuel price uncertainty are presented.

5.3.1. Model Assumptions

The main focus of this chapter is to model the generators in long-term power agreements and to study the effects of uncertainty on market price and firms' profits. Hence, the price of electricity is set at the time of the agreement and remains unchanged for the period of the contract in the long-term market. A total of n competing firms with firm f having a set of $N_f - 1$ units available for production at time 0 is assumed, and the last unit of production or a unit N_f^{th} represents the available power sources and is considered to have infinite capacity and to be always available. The total number of units available in the market at time 0 is denoted by N. The capacity of the j^{th} unit of firm f

is represented by $P_{fj}^{\max}$ (MW). Each firm's units are dispatched according to an ascending order of their marginal costs, which is denoted by c_{fj}. Unit commitment and transmission constraints are ignored. Fluctuation of fuel costs and generator outages are assumed in this chapter.

5.3.2. Mathematical Model

5.3.2.1. Reliability

To introduce the operating state of a generator j of firm f, we begin by defining a two-state continuous-time Markov process, which is 1, if unit j of firm f is available at time t and 0 for otherwise. The Markov process is assumed to reach its steady state when the contracted amount is generated. The Markov process has a failure rate per hour $\lambda_{f,j}$ and repair rate per hour $\mu_{f,j}$. The steady state probability that generator j of firm f will be available is denoted by

$$r_{f,j} = \frac{\mu_{f,j}}{\mu_{f,j} + \lambda_{f,j}},$$

and the steady state probability that the generator j of firm f will not be available is denoted by

$$q_{f,j} = 1 - r_{f,j}.$$

5.3.2.2. System load

To represent the behavior of the load-serving entities, a linear inverse demand function is assumed. The actual demand of the system is represented by the following linear relationship:

$$L = K - \xi p \tag{5.1}$$

The actual realized demand function of the system, which is denoted by L, is affected by the price elasticity of demand $\xi \geq 0$, which is also known by the power producers. The quantity K stands for the nominal demand (MWh), and it is assumed to be a constant in this section. The generation quantity for firm f is s_f, which is the sum of generation quantities of all generators of firm f ($f = 1, 2, \dots, n$) in MWh and it is calculated as follows:

$$s_f = \sum_{j=1}^{N_f} g_{fj} \ .$$

The actual demand of the system is equal to the total of all firms' generation quantities and it can be written as follows:

$$S^* = \sum_{f=1}^{n} s_f \ .$$

5.3.2.3. Profit function

Power companies that are bidding in the market operate a set of generators. When those generators are not always available and their marginal costs are subject to uncertainty because of fuel price fluctuation, the cost function is a random variable. The revenue function, however, is not affected by those uncertainties. Owing to generator outages and fuel price uncertainty, the expected total profit function for supplier f in the steady state can be written as follows:

$$\pi_f = \delta \left\{ s_f p(K, s_f + s_{-f}) - E\left[Cost_f(s_f) \right] \right\} \tag{5.2}$$

where $s_f p(K, s_f + s_{-f})$ is the revenue function and is not affected by generator availability and fuel cost uncertainty. When the Cournot decision maker assumes demand remains unchanged over a period of time, the function $p(K, s_f + s_{-f})$ based on relationship in (5.1) can be written as $p(K, s_f + s_{-f}) = \frac{1}{\xi}(K - s_f - s_{-f})$.

5.3.2.4. Cost function

The expected production cost function including only generator outages of supplier f producing s units of energy (MWh), when generating units 1, 2, ... , $j-1$ are not available, can be written as follows:

For unit $j = 1,\ 2,...,\ N_f - 1$, the following recursive relationship is used:

$$\overline{C}_{f,j}(s) = \begin{cases} r_{f,j} c_{f,j} s + q_{f,j} \overline{C}_{f,j+1}(s) & \text{for } 0 \le s \le P_{f,j}^{Max} \\ \\ r_{f,j} \left\{ c_{f,j} P_{f,j}^{Max} + \overline{C}_{f,j+1}(s - P_{f,j}^{Max}) \right\} + q_{f,j} \overline{C}_{f,j+1}(s) & \text{for } s > P_{f,j}^{Max} \end{cases} \tag{5.3}$$

The last unit of generator (N_f), which is always available, can be represented by the following relationship:

$$\overline{C}_{f,N_f}(s) = s \times c_{f,N_f}\,. \tag{5.4}$$

When the price of fuel becomes volatile, the marginal costs of generators are subject to uncertainty. In other words, marginal costs of all generators ($c_{f,j}$) are random variables except for the last unit (N_f) whose marginal cost is assumed to be a constant. It is assumed that each marginal cost is a continuous random variable which has an associated probability density function. Since the marginal costs in (5.3) are random variables, it becomes difficult to analytically compute the expected production cost.

A convenient way to calculate Nash-equilibrium quantities, when considering fuel price uncertainty in the expected cost function, is to use a Monte Carlo simulation based technique.

The following procedure describes the algorithm to compute a Monte-Carlo estimate of the expected cost function for firm f (see Fig. 9):

Step 1) Sample the costs of fuel and calculate the marginal cost of each unit.

It is assumed that the fuel costs are continuous random variables with associated probability density functions. The Monte Carlo simulation algorithm is employed to sample the cost of each fuel type. When the values of all fuel types are generated, one sample is obtained. Each generator is assumed to use only a specific fuel type. The marginal cost for each generator is obtained by the multiplication of a sample value of the fuel cost ($/MBTU) and its net plant heat rate (MBTU/MWh). Note that the net plant heat rate is a given constant value. The maximum number of samples is denoted by R. The parameter z is used to count the number of samples ($z = 1, 2, \ ..., R$).

Step 2) Sort the values of marginal costs in an ascending order.

After randomly generating the values of marginal costs, the generators are dispatched according to an ascending order of their marginal costs.

Step 3) Calculate the expected cost.

Since the values of marginal costs are known after step 1 and sorted in step 2, the expected cost can be calculated by using (5.3) and (5.4) with s in increments of Δ MWh. A single expected cost curve is obtained after completing step 3.

Step 4) Set $z = z + 1$. Repeat step 1, 2, and 3 until $z > R$.

Step 5) Calculate the mean value of all expected costs.

R sampled expected cost curves were obtained in step 4. Next, the average cost of production s (in increments of Δ MWh) is computed. The result is an estimate of the expected cost curve. Note that the values of costs after computing the average include both the generator outages and the fuel price uncertainty. However, the problem of a large number of slopes of the estimated expected cost curve as experienced in the previous chapter arises.

Step 6) Apply the slope reduction algorithm.

The pseudo-code below describes the slope reduction algorithm used to reduce the number of slopes of the expected cost curve for firm f:

FOR $l = 2$ to N^0

  Set $X_l = l \times \Delta s$, and $C_l = \overline{C}_{f,1}(X_l)$

END FOR

Set $r = 1$, $K = 2$, $C_1 = 0$, and $X_1 = 0$

WHILE $K < N^0$

  $j =$ Min i such $K \le i \le N^0$ and $(C_{i+1} - 2C_i + C_{i-1}) > \gamma$

$$m_{f,r} = \frac{\sum_{l=K-1}^{j} X_l C_l}{\sum_{l=K-1}^{j} X_l^2}$$

$NewP_{f,r}^{Max} = X_j$ and $Temp = X_j$

FOR $l = j$ to N^0

  $X_l = X_l - Temp$

  $C_l = C_l - m_{f,r} \times Temp$

END FOR

$K = j + 1$

$r = r + 1$

END

where Δs and γ are the increasing amount of capacity and the difference between slopes, respectively. Both are constant values and must be set at the beginning. Let N^0 be the maximum range in which we want to combine slopes such that $N^0 > \sum_{j=1}^{N_f} P_{f,j}^{\max}$ for each firm *f*.

Step 7) Obtain an estimation of the expected cost function.

The slope reduction algorithm produces an estimation of the expected cost function which is used to generate the estimated expected cost curve. A slope in the estimated expected cost curve is associated with each hypothetical generator (*r*) which has a maximum capacity ($NewP_{f,r}^{Max}$) and marginal cost ($m_{f,r}$). Let $r^{\max}$ represent a value of *r* when the slope reduction algorithm terminates. The values of $m_{f,r}$ and $NewP_{f,r}^{Max}$ are similar in nature to those $\bar{c}_{f,j}$ and $\overline{P}_{f,j}^{\max}$ described in chapter 4, but they take the generator outages and the fuel price fluctuation into account. In order to clarify the presentation, the values of marginal cost ($m_{f,r}$) and maximum capacity ($NewP_{f,r}^{Max}$) will be represented as $\overline{\overline{c}}_{f,j}$ and $\overline{\overline{P}}_{f,j}^{\max}$ where $(j = 1, 2, \ldots, r^{\max})$.

Thus, the approximation of a total expected cost considering generator availability and fuel price uncertainty for each firm f, which is denoted by $\overline{\overline{C}}_f(s_f)$, can be calculated as follows:

$$\hat{E}\left[Cost_f(s_f)\right] \approx \overline{\overline{C}}_f(s_f) = \sum_{j=1}^{N_f} \overline{\overline{c}}_{f,j} g_{f,j}$$

where $\overline{\overline{c}}_{f,j}$ is the marginal cost obtained from the slope reduction algorithm in step 6.

5.3.2.5. Cournot Model

Each firm f solves the following programming model:

$$\underset{s_f}{Max}\, \pi_f = \delta R_f(s_f; s_{-f}) - \delta \overline{\overline{C}}_f(s_f)$$

subject to

$$\sum_{j=1}^{N_f} g_{fj} - s_f \geq 0 \qquad (\lambda_f)$$

$$g_{fj} \leq \overline{\overline{P}}_{fj}^{\max} \qquad (\alpha_{fj}) \quad \text{for } j = 1, \ldots\ldots, r^{\max} \tag{5.5}$$

$$\forall g_{fj}, s_f \geq 0$$

where

$$R_f(s_f; s_{-f}) = \frac{1}{\xi} s_f (K - s_f - s_{-f})$$

and $\delta = \sum_{t=1}^{h} \delta_t$.

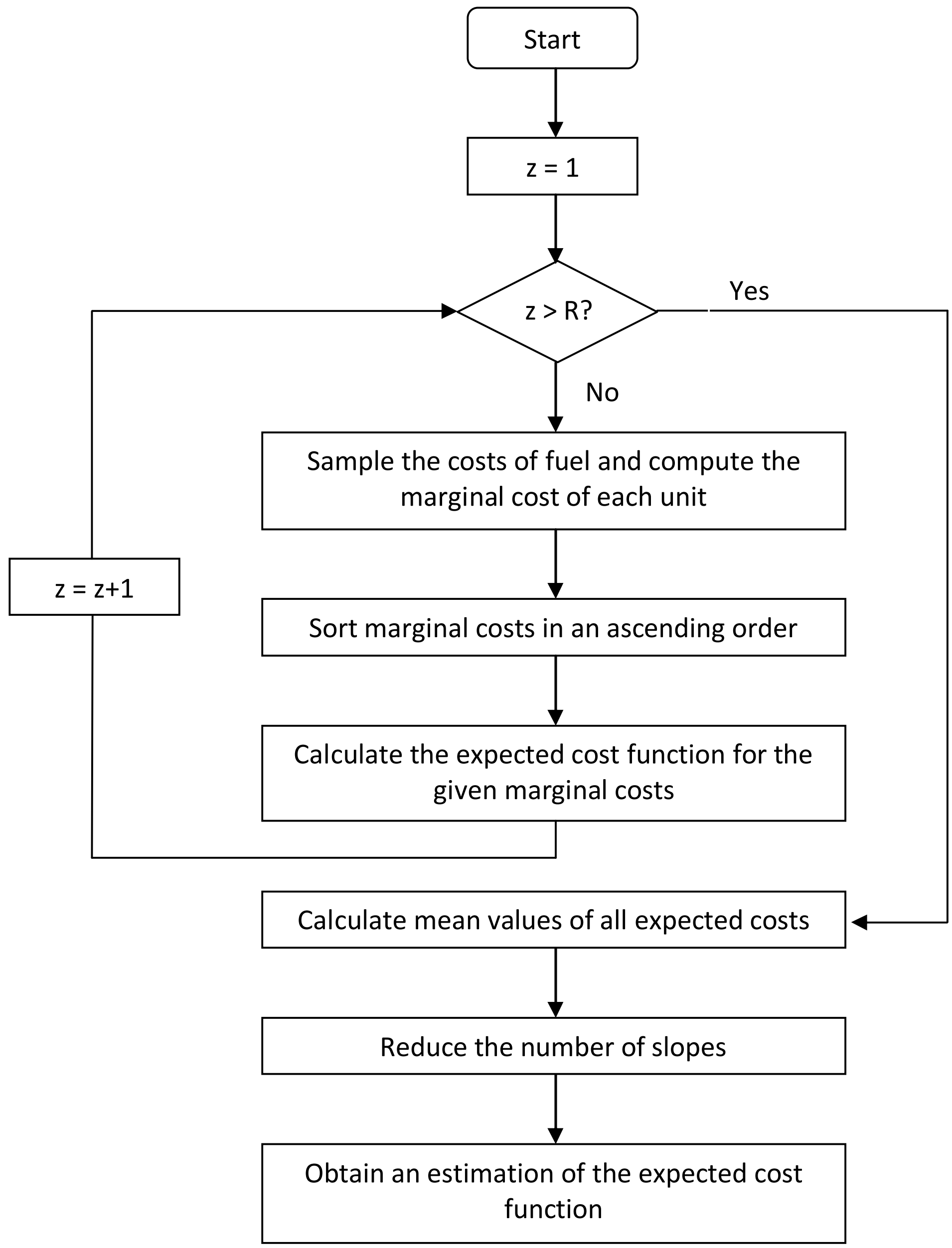

Figure 9. Flowchart of the algorithm used to calculate an approximation of the expected cost function

The Nash-equilibrium quantities s_f ($f = 1, 2, \ldots, n$) that solve the set of problems can be computed by combining the KKT first order optimality conditions of system equation (5.5) of all firms. The KKT conditions of the optimization problem above can be written as the following equations:

$$
\begin{aligned}
&\text{for } f = 1,\ldots,n \\
&\lambda_f - \frac{\delta}{\xi}(K - 2s_f - s_{-f}) \geq 0 \quad \perp \quad s_f \geq 0 \\
&\sum_{j=1}^{N_f} g_{fj} - s_f \geq 0 \quad \perp \quad \lambda_f \geq 0 \\
&\text{for } f = 1,\ldots,n \text{ and } j = 1,\ldots,r^{\max} \\
&\delta \overset{=}{c}_{f,j} + \alpha_{fj} - \lambda_f \geq 0 \quad \perp \quad g_{fj} \geq 0 \\
&\overset{=}{P}{}_{fj}^{\max} - g_{fj} \geq 0 \quad \perp \quad \alpha_{fj} \geq 0
\end{aligned}
\tag{5.6}
$$

These KKT conditions are considered as a linear complementary problem (LCP). The Nash-equilibrium quantity for each firm in MWh (s_f^*) is obtained by using available software to solve those optimality conditions. As mentioned earlier, the actual demand of the system or total demand can be calculated as $S^* = \sum_{f=1}^{n} s_f^*$. The linear relationship in demand (3.1) is used to determine the Cournot price, p^*, which becomes

$$
p^* = \frac{K - S^*}{\xi}. \tag{5.7}
$$

5.4. EXPERIMENTAL RESULTS

A market that consists of three firms is considered in this section. For a numerical explanation of results, the composition of each firm is given in Table 11 including the capacity, net plant heat rate, and availability of each generator. The characteristics of the unit types in table 11 are taken from the IEEE reliability test system (Grigg, 1996). Firms 1, 2 and 3 have 11, 7, and 9 generators, respectively. The last row of each firm in the table corresponds to the assumption that the N_f^{th} generator for firm f has infinite capacity and is perfectly reliable due to other available sources in the markets. Furthermore, the values of parameters ξ and K are assumed to be 15 and 3000 respectively. Each firm is assumed to work 12 hours per day (off-peak hours) and 30 days per month. The annual percentage rate (APR) is assumed to be 7% for all firms. It is also assumed that all firms and consumers agree in private through a central exchange on the delivery of specified power quantities for 2 months and that the firms receive the payment from customers at the end of each month. The discount coefficient, δ, in (5.5) is computed in order to assess the time value of money in the model. Based on a given APR, it can be shown that the value of the discount coefficient is 713.75.

The fuel sources of these generators are coal, oil, hydro, and nuclear. Since fuel price uncertainty is assumed in this chapter, the fuel costs are continuous random variables which have an associated probability density function. The probability density function for each fuel type is determined by collecting the daily price ($/MBTU) of each fuel for 3 months. All daily marginal costs in 3 months for each fuel type are then fit into a distribution. The distribution is selected by choosing the minimum value of the Chisquare goodness of fit test. As a result, a lognormal distribution is selected for coal-type generators, and a uniform distribution is selected for oil-type generators. The means and standard deviations of each fuel type are displayed in Table 10. Generators which use the same fuel type do not always have the same marginal cost, as each may have a different heat rate. If generators, however, use the same fuel type and have the same net plant heat rate, they are assumed to have the same marginal costs. The capture costs of CO_2, which will be added to the marginal costs, are assumed to be \$19.77/MWh and \$38.96/MWh for gas and coal, respectively. Since the marginal cost of uranium and hydro are not notably volatile within 3 months, they are assumed to be constant. The marginal costs of uranium and hydro are assumed to be 0.0168 and 0.07 \$/MWh respectively.

Table 10. The distribution of oil and coal price (\$/MBTU)

Type	Distribution
Gas	Lognormal (μ = 1.30, σ = 0.51)
Coal	Lognormal (μ = 0.34, σ = 0.15)

After the distribution of marginal costs for all generators are defined, the Monte-Carlo-simulation algorithm flowcharted in Figure 9 is performed to sample the values of fuel costs. A value of the expected cost function can then be computed in increments of Δ. The value of R is set to 5,000 in this experiment, which means we perform the simulation algorithm defined in the previous section with 5,000 sample sets. As a result, the total of 5,000 cost curves is obtained after completing step 4 for each firm as shown in Figures 10, 11, and 12.

Table 11. Market composition and generating unit data including unit availability

Firm	Unit	Fuel Type	Capacity (MW)	Marginal Cost ($/MWh)	Net Plant Heat Rate (Mbtu/MWh)	Availability (hours)	
						1/λ	1/μ
Firm 1	1	Coal	350	9.5	-	1150	100
	2	Coal	350	9.5	-	1150	100
	3	Coal	155	9.72	-	960	40
	4	Coal	155	9.72	-	960	40
	5	Coal	76	11.9	-	1960	40
	6	Coal	76	11.9	-	1960	40
	7	Gas	48	10.23	-	1340	26
	8	Gas	48	10.23	-	1340	26
	9	Gas	78	11.63	-	1720	30
	10	Gas	78	11.63	-	1720	30
	11	Gas	149	12.87	-	1505	42
	12	-	∞	-	999	-	-
Firm 2	1	Hydro	50	-	0.07	1980	20
	2	Hydro	50	-	0.07	1980	20
	3	Coal	155	9.72	-	960	40
	4	Coal	76	11.9	-	1960	40
	5	Gas	48	10.23	-	1340	26
	6	Gas	48	10.23	-	1340	26
	7	Gas	78	11.63	-	1720	30
	8	-	∞	-	999	-	-
Firm 3	1	Uranium	400	-	0.017	1100	150
	2	Hydro	50	-	0.07	1980	20
	3	Hydro	50	-	0.07	1980	20
	4	Coal	350	9.5	-	1150	100
	5	Coal	76	11.9	-	1960	40
	6	Gas	48	10.23	-	1340	26
	7	Gas	78	11.63	-	1720	30
	8	Gas	149	12.87	-	1505	42
	9	Gas	149	12.87	-	1505	42
	10	-	∞	-	999	-	-

Because of generator outages and the fluctuation of fuel prices, the estimated cost curve has a large number of slopes upon completion of step 5. To reduce the number of slopes, the slope reduction algorithm in step 6 is applied. The value of Δs is assumed to be 1 in this section. The value of γ for firms 1, 2, and 3 is selected to be 6, 3, and 5 respectively. The estimated cost curves before and after applying the slope reduction algorithm for firms 1, 2, and 3 are illustrated in Figures 13, 14, and 15, respectively.

By analyzing Figures 13, 14, and 15, a new set of hypothetical generators can be used to calculate the Nash-equilibrium quantities without losing accuracy. Specifically, the estimated cost curves before and after applying the slope reduction algorithm are nearly indistinguishable. Based on the estimated production cost curves, the lists of the new set of hypothetical generators which have a smaller number of marginal costs and maximum capacities for firms 1, 2, and 3 are shown in Tables 12, 13, and 14, respectively.

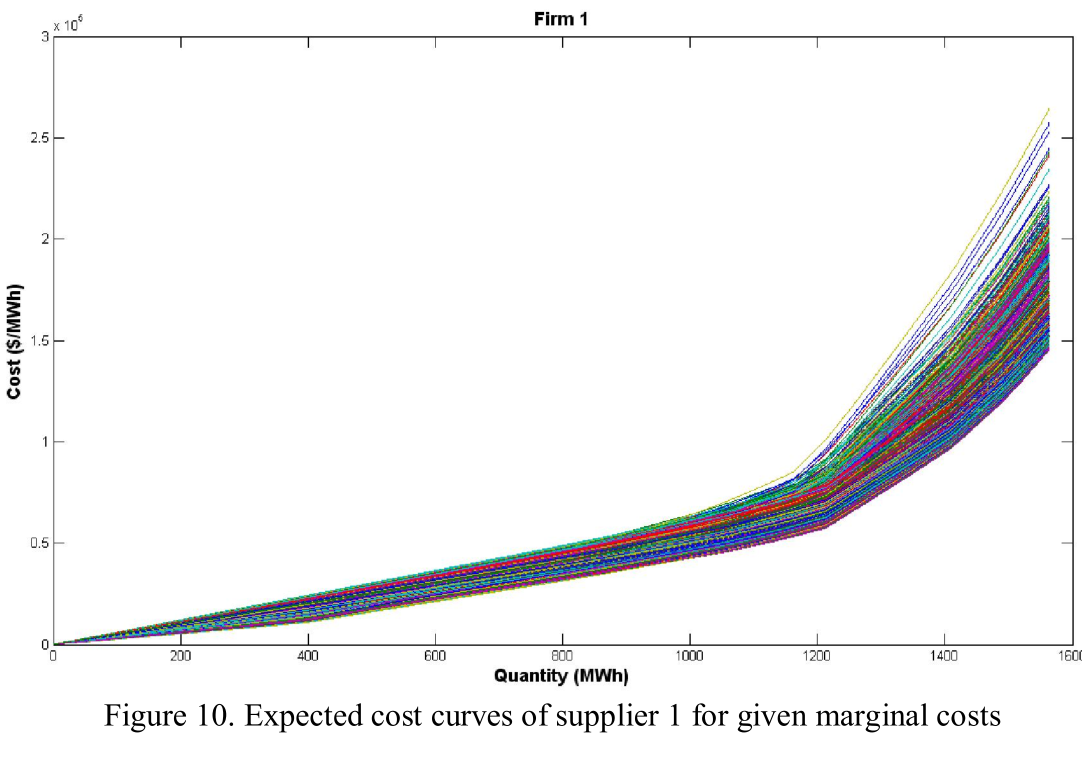

Figure 10. Expected cost curves of supplier 1 for given marginal costs

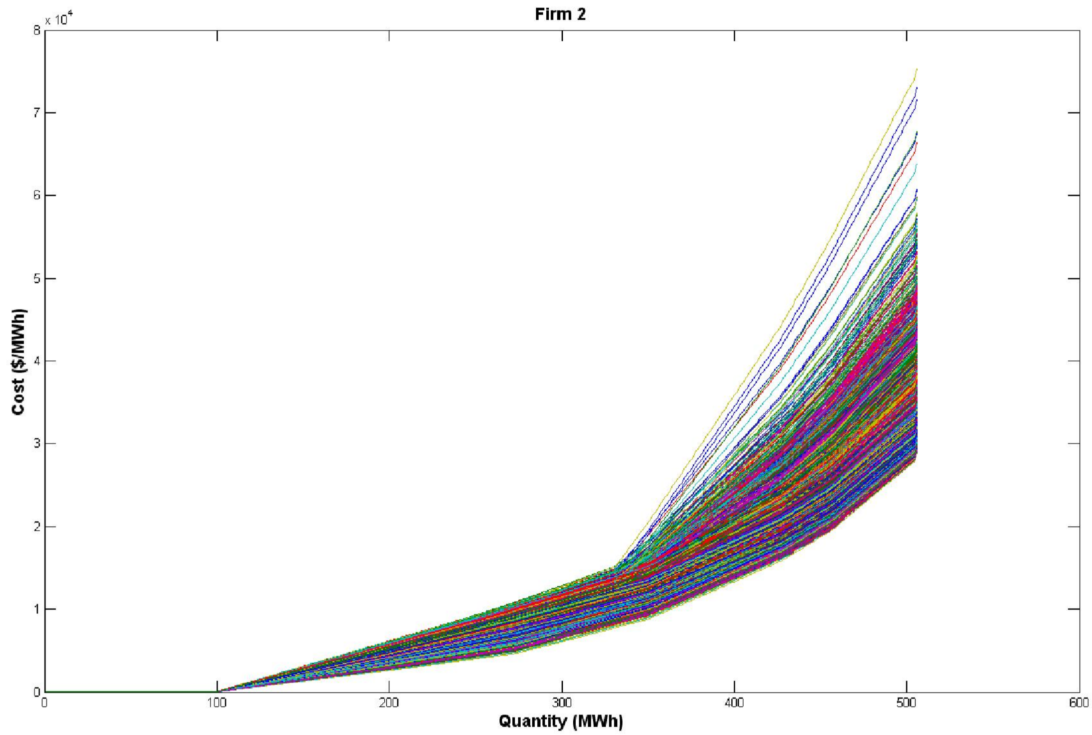

Figure 11. Expected cost curves of supplier 2 for given marginal costs

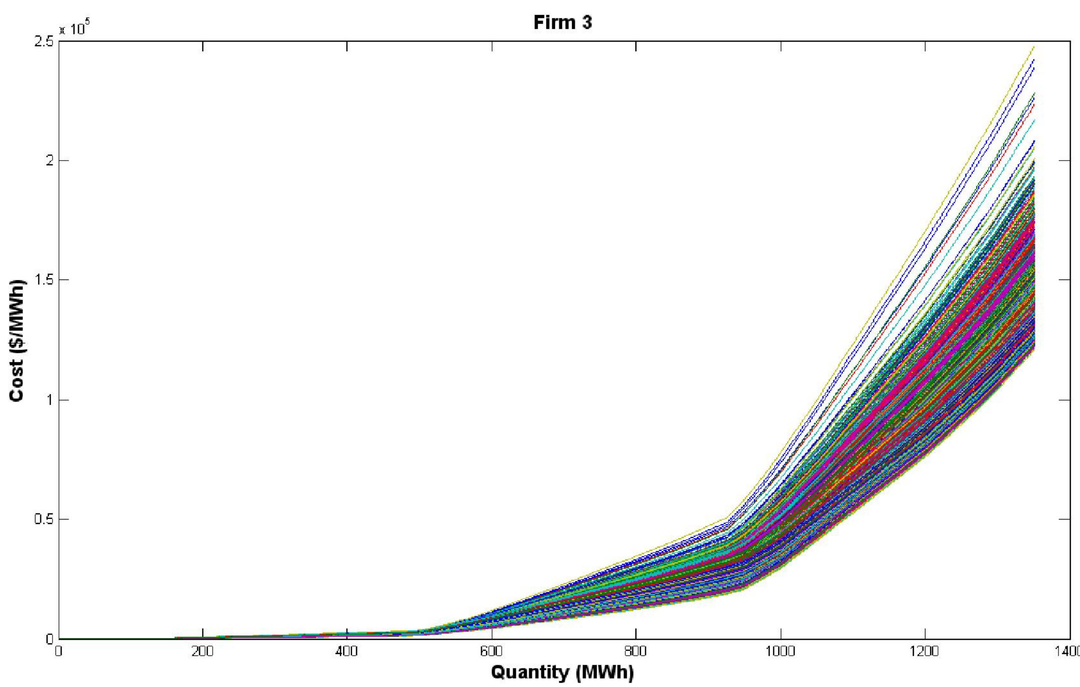

Figure 12. Expected cost curves of supplier 3 for given marginal costs

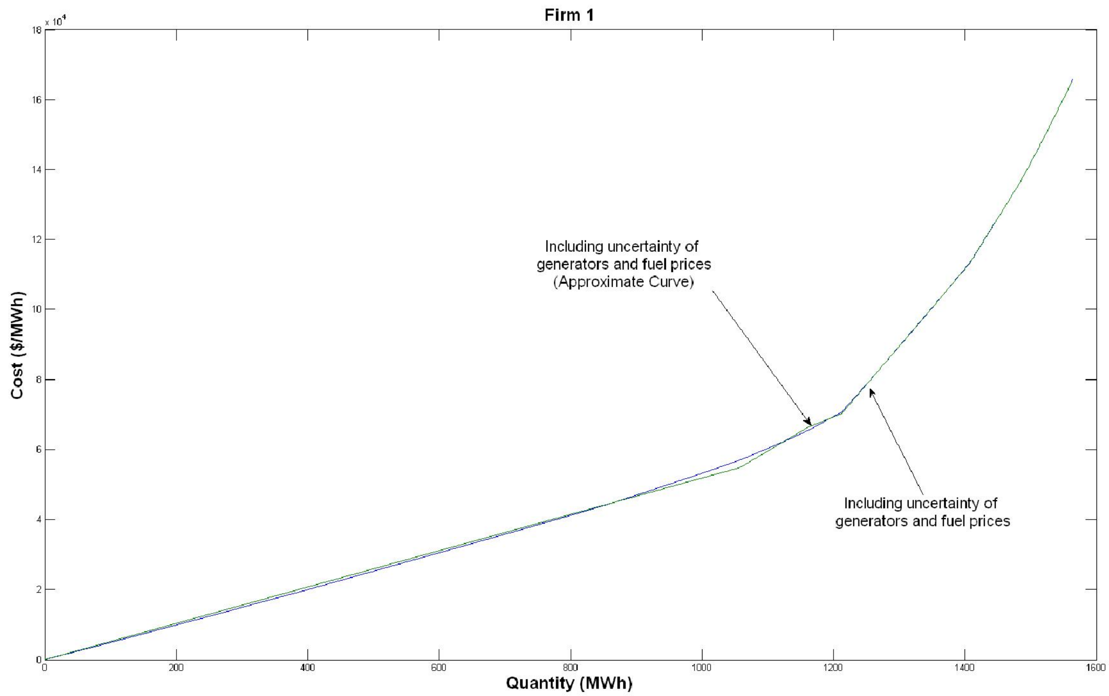

Figure 13. Estimate of expected cost curves before and after applying the slope reduction algorithm for firm 1

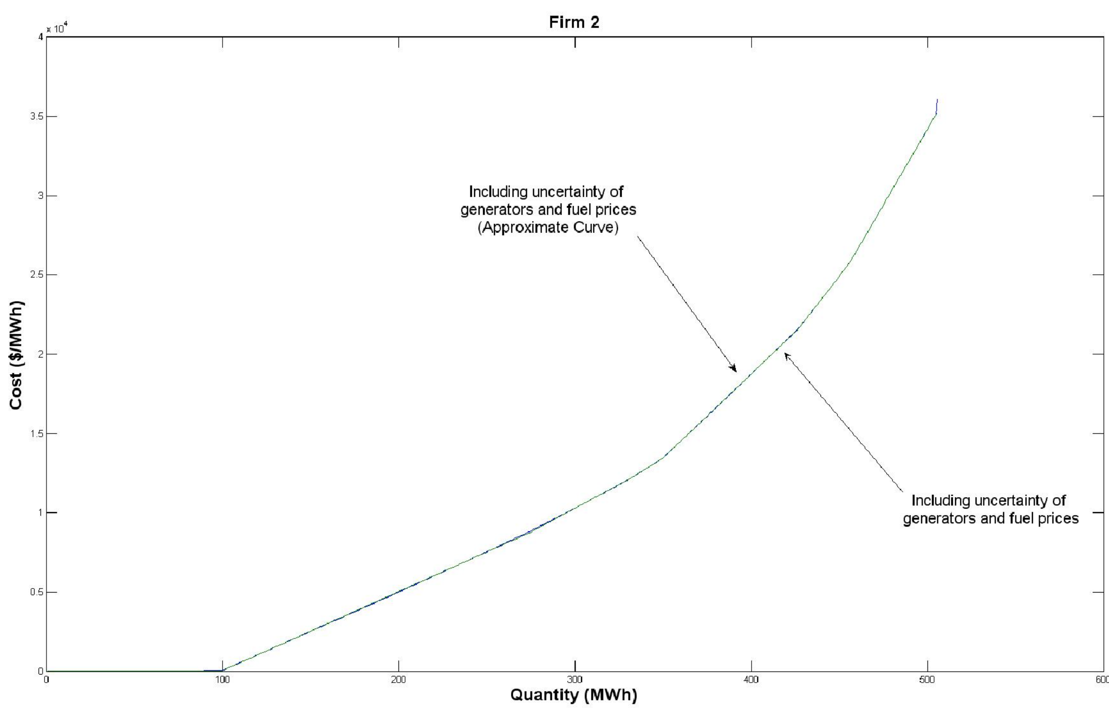

Figure 14. Estimate of expected cost curves before and after applying the slope reduction algorithm for firm 2

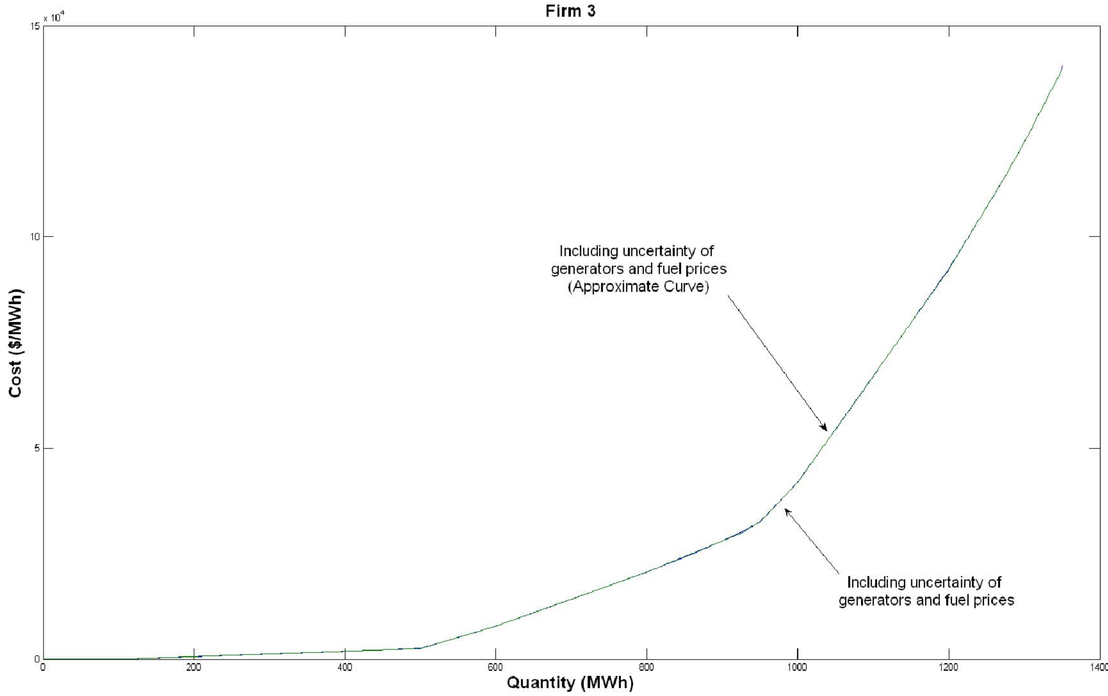

Figure 15. Estimate of expected cost curves before and after applying the slope reduction algorithm for firm 3

Table 12. Data for the hypothetical generating units of firm 1

Unit	Capacity (MW)	Marginal Cost ($/MWh)
1	1058	51.84
2	51	75.75
3	104	110.94
4	195	220.26
5	71	299.98
6	6	329.62
7	28	349.45
8	2	361.91
9	48	374.52
10	∞	999

Table 13. Data for the hypothetical generating units of firm 2

Unit	Capacity (MW)	Marginal Cost ($/MWh)
1	100	0.38
2	174	49.89
3	57	59.91
4	19	68.98
5	77	105.96
6	26	143.03
7	2	146.76
8	2	161.70
9	48	191.08
10	∞	999

Table 14. Data for the hypothetical generating unit of firm 3

Unit	Capacity (MW)	Marginal Cost ($/MWh)
1	100	0.06
2	400	6.23
3	100	53.32
4	201	63.93
5	125	74.03
6	24	111.71
7	50	185.42
8	201	252.04
9	2	269.02
10	71	295.83
11	26	319.52
12	2	328.68
13	48	344.79
14	∞	999

Note that each firm still owns the original set of generators listed in Table 11, but each firm in reality pays the amount of money given in Tables 12, 13, and 14, when generator outages and fuel price uncertainty are considered.

The Nash-equilibrium quantities or firms' quantity bids are calculated according to the KKT first order optimality conditions in (5.6) using the PATH solver. The market price is calculated according to (5.7) assuming a linear relationship. The results of the Nash-equilibrium quantities and market prices are displayed in Table 15.

Table 15. Results of Nash-equilibrium including generator outages and fuel price uncertainty

Firm 1 (MWh)	Firm 2 (MWh)	Firm 3 (MWh)	Total (MWh)	p^* ($)
636.20	350	600	1586.2	94.25

5.4.1. Effect of uncertainty of generator availability and fuel price

In order to simplify the presentation, the model in each case is named as follows:

Model A: the deterministic model described in chapter 3

Model B: the stochastic model considering outages described in chapter 4

Model C: the stochastic model including outages and fuel price uncertainty described in this chapter

For model A, the model profit is the amount firms believe they will make when ignoring outages, while the adjusted expected profit is the amount that firms will make because generator outages and fuel price uncertainty do occur.

The model profits of Model A are computed by substituting the Nash-equilibrium quantities and the power generated by each generator into the objective function of Model A (3.5). Let s_f^{*A} represent the Nash-equilibrium quantities of Model A. The Model A's profit can be written as follows:

$$\pi_f = \delta\left[R_f(s_f^{*A}; s_{-f}^{*A}) - C_f(s_f^{*A})\right]$$

where $R_f(s_f^{*A}; s_{-f}^{*A}) = \frac{1}{\xi} s_f^{*A}(K - s_f^{*A} - s_{-f}^{*A})$.

Unlike the model profit, the adjusted expected profit of Model A is calculated from the difference between the revenue function and the estimation of a total expected cost function including outages and fuel price uncertainty, $\overline{\overline{C}}_f(s_f^*)$. For Model A, the expected profit can be calculated as follows:

$$E[\pi_f] = \delta\left\{R_f(s_f^{*A}; s_{-f}^{*A}) - \overline{\overline{C}}_f(s_f^{*A})\right\}$$

where s_f^{*A} stands for the Nash-equilibrium quantities obtained from Model A.

For model B, the model expected profits are calculated by substituting the Nash-equilibrium quantities and the power generated by each generator into the objective function of Model B (4.5). Let s_f^{*B} represent the Nash-equilibrium quantities of Model B. Then Model B's profit can be written as follows:

$$E[\pi_f] = \delta\left[R_f(s_f^{*B}; s_{-f}^{*B}) - \overline{C}_{f,1}(s_f^{*B})\right]$$

where $R_f(s_f^{*B}; s_{-f}^{*B}) = \frac{1}{\xi} s_f^{*B}(K - s_f^{*B} - s_{-f}^{*B})$.

The adjusted expected profit of Model B is computed from the difference between the revenue function and an estimation of a total expected cost function including outages and fuel price uncertainty, $\overline{\overline{C}}_f(s_f^*)$. For Model B, the adjusted expected profit can be calculated as follows:

$$E[\pi_f] = \delta\left\{R_f(s_f^{*B}; s_{-f}^{*B}) - \overline{\overline{C}}_f(s_f^{*B})\right\}$$

where s_f^{*B} stands for the Nash-equilibrium quantities obtained from Model A.

The model expected profit and expected profit of Model C are the same because the model considers uncertainty in generator availability and fuel prices. The model profit and expected profit when considering generator failures can be computed as follows:

$$E[\pi_f] = \delta\left\{R_f(s_f^{*C}; s_{-f}^{*C}) - \overline{\overline{C}}_f(s_f^{*C})\right\}$$

where s_f^{*C} represents the Nash-equilibrium quantities obtained from Model C.

Results of the model profit and expected profit in all three cases for each firm are shown in Table 16.

Table 16. A comparison of firm expected profits when ignoring the uncertainty of fuel prices and outages and when including the uncertainty of generator and fuel costs

	Model A Ignoring Uncertainty ($/Hour)		Model B Including Outages ($/Hour)		Model C Including Outages & Fuel Cost Uncertainty ($/Hour)	
Firm	Model Profit	Adjusted Expected Profit	Model Expected Profit	Adjusted Expected Profit	Model Expected Profit	Expected Profit
1	23,634.67	23,997.59	26,557.54	27,411.56	26,983.03	26,983.03
2	20,909.78	17,742.00	19,340.41	19,899.82	19,543.76	19,543.76
3	49,845.87	47,279.01	48,560.82	48,903.96	48,722.51	48,722.51

According to Table 16, the adjusted expected profits and the model profits of firm 1 for Model A are similar. For firms 2 and 3, however, the adjusted expected profit is significantly less than the model profit, which means, for example, firm 3 anticipates making $49,845.87/hour, but in reality it would make only $47,279.01/hour on average.

For Model B, the model expected profits of all firms are slightly less than the adjusted expected profits.

When comparing the adjusted expected profit of Model A with Model C firms 1, 2, and 3 make more profits when using Model C than when using Model A. However, all firms make fewer profits when the adjusted expected profit of Model B is compared with Model C.

These results indicate that making decisions without considering the uncertainties in generator availability and fuel prices could lead to bad decision-making and an inaccurate planning process.

5.5. CONCLUSIONS

In this chapter, the stochastic Cournot model was extended to consider not only the availability of generators but also fuel price uncertainty. Some generators used a type of fuel whose price was unpredictable thereby affecting the cost function. The Monte Carlo simulation based technique was used to calculate an estimate of the expected cost function. Then the Nash-equilibrium quantities, market prices, and firms' expected profits were computed.

The model expected profits and expected profits were computed for models A, B, and C to evaluate whether the availability of generators and the volatility of fuel prices have an effect on the expected total profit for all firms. Results indicate that both generator outages and fuel price uncertainty are vital factors in power markets and they must be considered in both the power system planning and operation.

CHAPTER VI

TOLERANCE APPROACH TO SENSITIVITY ANALYSIS IN THE STOCHASTIC COURNOT MODEL

Abstract --- The Cournot model is used to describe the behavior of generating companies in power markets. Two major uncertainty factors, generator outages and fuel price uncertainty, are considered in the model. One way to compute the Nash-equilibrium quantities when considering both factors is the use of a Monte Carlo simulation based technique. Due to its random processes, this simulation technique yields slightly different results each time it is run. Accordingly, it is doubtful that companies should make a decision based on those results. In addition, running the simulation several times in order to certify the results may take considerable computational time. Therefore, a sensitivity analysis is performed to determine which parameter is having a significant impact on the Nash-equilibrium quantities. Since the KKT conditions of the Cournot model represent a linear complementarity problem (LCP), the theory of tolerance approach to sensitivity analysis in LCP is applied. The maximum tolerance gives the maximum allowable fluctuation of marginal costs and capacities without affecting the firms' strategic planning and operation.

6.1. NOMENCLATURE

The notation used in this chapter is given below for reference.

n	Number of firms
N	Total number of generators
N_f	Number of generators of firm f
$\overline{\overline{P}}_{fj}^{\max}$	The capacity of the j^{th} hypothetical unit of firm f considering outages and fuel price uncertainty (MW)
$\overline{\overline{c}}_{fj}$	The marginal cost of the j^{th} hypothetical unit of firm f including generator outages and fuel price uncertainty (\$/MWh)
s_f^*	The Nash-equilibrium quantities (MWh)
S^*	The Nash-equilibrium total bid (MWh)
p^*	The Cournot price (\$/MWh)
K	Nominal demand
h	Number of contracted hours
i	Interest rate (%)
δ	Compound-amount factor
π_f	Profit of firm f (\$)
ξ	Slope parameter for demand

6.2. INTRODUCTION

The stochastic Cournot model described in chapter 5 considers uncertainty in generator availability and fuel price. The Monte Carlo simulation based technique was used to estimate the expected cost function. A set of marginal costs ($\overline{\overline{c}}_{f,j}$) and the maximum capacity ($\overline{\overline{P}}^{\max}_{f,j}$) for each generator j were obtained after performing the simulation method. These results were used to compute the Nash-equilibrium quantities. However, the simulation technique yielded slightly different results each time it was run. Due to the fluctuation in simulation output, it is doubtful that firms should make a decision based on those results. Moreover, it is also difficult for a company to run the simulation several times in order to certify those results due to the long computational time.

Therefore, a sensitivity analysis is performed to determine which parameters have the most significant impact on the optimal solutions. The company can then concentrate on acquiring accurate data for those sensitive parameters. Sensitivity analysis will help a company determine whether the optimal solution is sensitive to small changes in some of the input data used in the simulation so that the company can use the results with confidence.

The main goal of this chapter is to find the maximum tolerance on the crucial parameters of the stochastic Cournot model. Since the stochastic Cournot model considering generator outages and fuel price uncertainty in chapter 5 is a linear complementarity problem, the theory of tolerance approach to sensitivity analysis in linear complementarity problem is used. The tolerance approach is mainly applied to the marginal costs and maximum capacities as they are believed to have significant impacts on the Nash-equilibrium quantities. The maximum tolerance gives the maximum allowable fluctuation of marginal costs and capacities without affecting the firms' strategic planning and operation. Moreover, it also specifies which generators have significant impact on the Nash-equilibrium quantities.

This chapter is organized as follows: The tolerance approach to sensitivity analysis in LCP is developed in section 3. In section 4, numerical results and the analysis of the maximum tolerance on each parameter are presented. The conclusions are outlined in section 5.

6.3. MODEL DESCRIPTION

This section begins with an introduction to the concept of the linear complementarity problem (LCP). The theory of tolerance approach to sensitivity analysis in LCP is then developed to find the maximum tolerance such that the perturbed problems have the same index set of nonzero elements as the original problems.

6.3.1. Linear Complementarity Problem (LCP)

The linear complementarity problem is to find the value of a vector $\boldsymbol{z}$ that satisfies a set of constraints for a given $m \times m$ matrix $\boldsymbol{M}$ and compatible vector $\boldsymbol{q}$. The general form of an LCP can be written as (Stewart, 2008 and Thomas, 2002):

Given: $\boldsymbol{M} \in \boldsymbol{R}^{m \times m}, \quad \boldsymbol{q} \in \boldsymbol{R}^{m}$

Find: $\boldsymbol{z} \in \boldsymbol{R}^{m}$

Subject to $\boldsymbol{Mz} + \boldsymbol{q} \geq \boldsymbol{0}$

$\boldsymbol{z} \geq \boldsymbol{0}$

and $\boldsymbol{z}^{T}(\boldsymbol{Mz} + \boldsymbol{q}) = \boldsymbol{0}.$

Throughout this chapter, we shall refer to this problem as LCP(***M, q***).

6.3.2. Cournot Model

Since matrices ***M*** and ***q*** play an important role in developing the tolerance approach to sensitivity analysis in LCP, the KKT conditions of the stochastic Cournot model described in chapter 5 must be transformed into the matrix form. For the sake of completeness, the stochastic Cournot model described in chapter 5 is shown below.

$$\underset{s_f}{Max}\, \pi_f = \delta R_f(s_f; s_{-f}) - \delta \overline{\overline{C}}_f(s_f)$$

subject to

$$\sum_{j=1}^{N_f} g_{fj} - s_f \geq 0 \qquad (\lambda_f)$$

$$g_{fj} \leq \overline{\overline{P}}_{fj}^{\max} \qquad (\alpha_{fj}) \quad \text{for } j = 1, \ldots, N_f$$

$$\forall g_{fj}, s_f \geq 0$$

where

$$R_f(s_f; s_{-f}) = \frac{1}{\xi} s_f (K - s_f - s_{-f}) \text{ and } \quad \delta = \sum_{t=1}^{h} \delta_t.$$

The KKT first optimality conditions of this optimization problem are:

for $f = 1,...,n$

$$\lambda_f - \frac{\delta}{\xi}(K - 2s_f - s_{-f}) \geq 0 \quad \perp \quad s_f \geq 0 \tag{6.1}$$

$$\sum_{j=1}^{N_f} g_{fj} - s_f \geq 0 \quad \perp \quad \lambda_f \geq 0 \tag{6.2}$$

for $f = 1,...,n$ and $j = 1,...,N_f$

$$\delta \overset{=}{c}_{f,j} + \alpha_{fj} - \lambda_f \geq 0 \quad \perp \quad g_{fj} \geq 0 \tag{6.3}$$

$$\overset{=\max}{P_{fj}} - g_{fj} \geq 0 \quad \perp \quad \alpha_{fj} \geq 0. \tag{6.4}$$

These KKT conditions (6.1-.6.4) are represented in the matrix form as follows:

$$\boldsymbol{z}^T = \begin{bmatrix} s_1 & \ldots & s_f & \lambda_1 & \ldots & \lambda_f & g_{11} & \ldots & g_{fj} & \alpha_{11} & \ldots & \alpha_{fj} \end{bmatrix}$$

$$\boldsymbol{q}^T = \Bigg[\underbrace{-\frac{\delta K}{\xi} \quad \ldots \quad -\frac{\delta K}{\xi}}_{n} \quad \underbrace{0 \quad \ldots \quad 0}_{n} \quad \underbrace{\delta \overset{=}{c}_{11} \quad \ldots \quad \delta \overset{=}{c}_{fj}}_{n*N} \quad \underbrace{\overset{=\max}{P_{11}} \quad \ldots \quad \overset{=\max}{P_{fj}}}_{n*N} \Bigg]$$

Due to the hefty size of matrix $\boldsymbol{M}$, this matrix is separated into five parts for ease of presentation. Each part is associated with a constraint in the KKT conditions. All of the components of the matrix $\boldsymbol{M}$ are summarized below.

(Equation 5.1)

$$\boldsymbol{M}_{(5.1)} = \left[\begin{array}{cccc|cccc|ccccc} \frac{2\delta}{\xi} & \frac{\delta}{\xi} & \cdots & \frac{\delta}{\xi} & 1 & 0 & \cdots & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & \frac{2\delta}{\xi} & & \frac{\delta}{\xi} & 0 & 1 & & 0 & \vdots & \ddots & & & \vdots \\ \vdots & & \ddots & \vdots & \vdots & & \ddots & \vdots & \vdots & & \ddots & & \vdots \\ \frac{\delta}{\xi} & \frac{\delta}{\xi} & \cdots & \frac{2\delta}{\xi} & 0 & 0 & \cdots & 1 & 0 & \cdots & \cdots & \cdots & 0 \end{array}\right]$$

(Equation 5.2)

$$
\boldsymbol{M}_{(5.2)} = \left[\begin{array}{cccc|cccc|ccccccccc|cccc}
-1 & 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 & 1 & \cdots & 1 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 \\
0 & -1 & & 0 & \vdots & \ddots & & \vdots & 0 & \cdots & 0 & 1 & \cdots & 1 & 0 & \cdots & 0 & \vdots & \ddots & & \vdots \\
\vdots & & \ddots & \vdots & \vdots & & \ddots & \vdots & \vdots & & \vdots & \vdots & \ddots & \ddots & \vdots & & \vdots & \vdots & & \ddots & \vdots \\
0 & 0 & \cdots & -1 & 0 & \cdots & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 1 & \cdots & 1 & 0 & \cdots & \cdots & 0
\end{array}\right]
$$

(Equation 5.3)

$$
\boldsymbol{M}_{(5.3)} = \left[\begin{array}{ccc|cccc|ccc|ccc}
0 & \cdots & 0 & -1 & 0 & \cdots & 0 & 0 & \cdots & 0 & 1 & \cdots & 0 \\
\vdots & & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\vdots & & \vdots & -1 & 0 & \cdots & 0 & \vdots & & \vdots & \vdots & & \vdots \\
\vdots & & \vdots & 0 & -1 & 0 & 0 & \vdots & & \vdots & \vdots & & \vdots \\
\vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\vdots & & \vdots & 0 & -1 & 0 & 0 & \vdots & & \vdots & \vdots & & \vdots \\
\vdots & & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\vdots & & \vdots & 0 & 0 & \cdots & -1 & \vdots & & \vdots & \vdots & & \vdots \\
\vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
0 & \cdots & 0 & 0 & 0 & \cdots & -1 & 0 & \cdots & 0 & 0 & \cdots & 1
\end{array}\right]
$$

(Equation 5.4)

$$
\boldsymbol{M}_{(5.4)} = \left[\begin{array}{ccc|ccc|ccc|ccc}
0 & \cdots & 0 & 0 & \cdots & 0 & -1 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & -1 & 0 & \cdots & 0
\end{array}\right]
$$

Hence, the matrix $\boldsymbol{M}$ is equivalent to

$$
\boldsymbol{M} = \begin{bmatrix} \boldsymbol{M}_{(5.1)} \\ \boldsymbol{M}_{(5.2)} \\ \boldsymbol{M}_{(5.3)} \\ \boldsymbol{M}_{(5.4)} \end{bmatrix}.
$$

6.3.3. Tolerance Approach to Sensitivity Analysis in the Stochastic Cournot model

The results obtained from the simulation will be used as an input to the LCP model (6.1-6.4) in order to calculate the Nash-equilibrium quantities. One of the drawbacks of the simulation technique is that it yields slightly different results each time it is run. Since those results will be used as an input to the LCP, it would be best to know the effects of input data perturbation on the optimal solutions. Because ***M*** does not affect firms' strategic planning and operation, the main focus of this section is to discover whether marginal costs ($\overset{=}{c}_{f,j}$) and capacities ($\overset{=}{P}{}^{\max}_{f,j}$) in vector ***q*** have a major impact on the Nash-equilibrium quantities. The algorithm developed in (Ha and Narula, 1992) is modified in order to calculate the range within which an entry value of vector ***q*** can vary independently such that the perturbed problem has the same index set of nonzero elements as the original problem.

Let $\boldsymbol{z}^*$ be a solution of the linear complementarity problem, LCP*(**M**, **q**)*. Assume that $\boldsymbol{z}^*$ is locally unique and a nondegenerate solution. Note that the size of matrix ***M*** is $m \times m$. According to the definition of a nodegenerate LCP,

either $z_i^* > 0$ or $x_i^* = \left(Mz^* + q\right)_i > 0$.

Define the sets ***B*** and ***N*** as follows:

$$\boldsymbol{B} := \left\{ i \middle| z_i^* > 0 \right\} \quad \text{and} \quad \boldsymbol{N} := \left\{ i \middle| x_i^* > 0 \right\}$$

where the cardinality of sets ***B*** and ***N*** are assumed to be *u* and *v*, respectively. Define vector $\boldsymbol{z_B}$ as the vector whose components are the components z_i of vector ***z***, for $i \in \boldsymbol{B}$. In addition, define matrix $\boldsymbol{M}_{\boldsymbol{BN}}$ as the submatrix of $\boldsymbol{M}$ whose entries are m_{ij}, for $i \in \boldsymbol{B}$ and $j \in \boldsymbol{N}$. Other vectors and matrices are defined in the same way.

The problem LCP*(**M**, **q**),* using the index sets ***B*** and ***N,*** can be written as

$$\boldsymbol{x}_{\boldsymbol{B}} = \boldsymbol{M}_{\boldsymbol{BB}}\boldsymbol{z}_{\boldsymbol{B}} + \boldsymbol{M}_{\boldsymbol{BN}}\boldsymbol{z}_{\boldsymbol{N}} + \boldsymbol{q}_{\boldsymbol{B}} \geq \mathbf{0},$$

$$\boldsymbol{x}_{\boldsymbol{N}} = \boldsymbol{M}_{\boldsymbol{NB}}\boldsymbol{z}_{\boldsymbol{B}} + \boldsymbol{M}_{\boldsymbol{NN}}\boldsymbol{z}_{\boldsymbol{N}} + \boldsymbol{q}_{\boldsymbol{N}} \geq \mathbf{0},$$

$$\boldsymbol{z}_{\boldsymbol{B}} \geq \mathbf{0}, \quad \boldsymbol{z}_{\boldsymbol{N}} \geq \mathbf{0}, \quad \boldsymbol{z}^{\boldsymbol{T}}\boldsymbol{x} = \mathbf{0}.$$

By the definition of sets ***B*** and ***N***, at the solution $\boldsymbol{z}^*$ we know

$$\boldsymbol{z}_{\boldsymbol{B}}^* > \mathbf{0} \text{ and } \boldsymbol{z}_{\boldsymbol{N}}^* = \mathbf{0}. \tag{6.5}$$

Thus, from (6.5)

$$\boldsymbol{x}_{\boldsymbol{B}}^* = \boldsymbol{M}_{\boldsymbol{BB}}\boldsymbol{z}_{\boldsymbol{B}}^* + \boldsymbol{q}_{\boldsymbol{B}} = \mathbf{0}, \tag{6.6}$$

$$\boldsymbol{x}_{\boldsymbol{N}}^* = \boldsymbol{M}_{\boldsymbol{NB}}\boldsymbol{z}_{\boldsymbol{B}}^* + \boldsymbol{q}_{\boldsymbol{N}} > \mathbf{0}. \tag{6.7}$$

The equation (6.6) and (6.7) can be rewritten as

$$\boldsymbol{z}_{\boldsymbol{B}}^* = -\boldsymbol{M}_{\boldsymbol{BB}}^{-1}\boldsymbol{q}_{\boldsymbol{B}}, \tag{6.8}$$

$$\boldsymbol{x}_{\boldsymbol{N}}^* = -\boldsymbol{M}_{\boldsymbol{NB}}\boldsymbol{M}_{\boldsymbol{BB}}^{-1}\boldsymbol{q}_{\boldsymbol{B}} + \boldsymbol{q}_{\boldsymbol{N}}. \tag{6.9}$$

Note that (6.8) and (6.9) are used later in this section when the tolerance approach is implemented.

We introduce the perturbed linear complementarity problem in order to develop the tolerance approach in LCP(***M**, **q***). As mentioned earlier, only vector ***q*** is disturbed. Therefore, the perturbed linear complementarity problem can be written as follows:

$$\boldsymbol{Mz} + \boldsymbol{q} + \Delta \geq \mathbf{0},$$

$$\boldsymbol{z} \geq \mathbf{0}, \tag{6.10}$$

$$\boldsymbol{z}^T(\boldsymbol{Mz}+\boldsymbol{q}+\Delta),$$

where Δ is the parameter vector in $\boldsymbol{R}^m$.

6.3.3.1. A single parameter q_k $(k \in \boldsymbol{B})$ is perturbed.

This section begins by developing the tolerance approach to sensitivity analysis in the LCP in which a single parameter in vector $\boldsymbol{q}$ is perturbed. As marginal costs and maximum capacities have significant impacts on the firms' operations, it would be beneficial for a company to know which parameters are more sensitive. To perform the sensitivity analysis of a single parameter in vector $\boldsymbol{q}$ by the tolerance approach, we first define Δ_k (size $u \times 1$) to be the parameter vector which has a value ω_k in the k^{th} position ($\omega_k \neq 0$) and the value of 0 in all other positions. For example, the parameter vector Δ_k, when $k=3$, can be written as

$$\Delta_3 = \begin{bmatrix} 0 \\ 0 \\ \omega_3 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{u\times 1}.$$

Using the index sets $\boldsymbol{B}$ and $\boldsymbol{N}$, the perturbed problem (6.10) can be rewritten as

$$\begin{aligned} &\boldsymbol{x_B} = \boldsymbol{M_{BB}z_B} + \boldsymbol{M_{BN}z_N} + \boldsymbol{q_B} + \Delta_k \geq \boldsymbol{0}, \\ &\boldsymbol{x_N} = \boldsymbol{M_{NB}z_B} + \boldsymbol{M_{NN}z_N} + \boldsymbol{q_N} \geq \boldsymbol{0}, \qquad (6.11) \\ &\boldsymbol{z_B} \geq \boldsymbol{0}, \quad \boldsymbol{z_N} \geq \boldsymbol{0}, \quad \boldsymbol{z_B^T x_B} = \boldsymbol{0}, \quad \boldsymbol{z_N^T x_N} = \boldsymbol{0}. \end{aligned}$$

The parameter vector Δ_k is allowable if problem (6.11) has a solution $\hat{z}$ such that

$$\widehat{\boldsymbol{z}_B} \geq \boldsymbol{0}, \quad \widehat{\boldsymbol{z}_N} = \boldsymbol{0}, \quad \widehat{\boldsymbol{x}_B} = \boldsymbol{0}, \text{and} \quad \widehat{\boldsymbol{x}_N} \geq \boldsymbol{0}.$$

Thus, if Δ_k is allowable, then (6.11) becomes

$$\widehat{\boldsymbol{x}_B} = \boldsymbol{M}_{BB}\widehat{\boldsymbol{z}_B} + \boldsymbol{q}_B + \Delta_k = \boldsymbol{0}, \tag{6.12}$$

$$\widehat{\boldsymbol{x}_N} = \boldsymbol{M}_{NB}\widehat{\boldsymbol{z}_B} + \boldsymbol{q}_N \geq \boldsymbol{0}, \tag{6.13}$$

$$\widehat{\boldsymbol{z}_B} \geq \boldsymbol{0}.$$

We can solve for $\widehat{\boldsymbol{z}_B}$ in (6.12) as we did in (6.8). It can be written as

$$\widehat{\boldsymbol{z}_B} = -\boldsymbol{M}_{BB}^{-1}(\boldsymbol{q}_B + \Delta_k) \geq \boldsymbol{0},$$

or

$$\boldsymbol{M}_{BB}^{-1}\Delta_k \leq -\boldsymbol{M}_{BB}^{-1}\boldsymbol{q}_B. \tag{6.14}$$

Substituting this value of $\widehat{z_B}$ in (6.13), equation (6.13) becomes

$$-\boldsymbol{M}_{NB}\boldsymbol{M}_{BB}^{-1}(\boldsymbol{q}_B + \Delta_k) + \boldsymbol{q}_N \geq \boldsymbol{0},$$

or

$$\boldsymbol{M}_{NB}\boldsymbol{M}_{BB}^{-1}\Delta_k \leq -\boldsymbol{M}_{NB}\boldsymbol{M}_{BB}^{-1}\boldsymbol{q}_B + \boldsymbol{q}_N. \tag{6.15}$$

Using the expression in (6.8) and (6.9), equation (6.14) and (6.15) can be rewritten in terms of z_B^* and x_N^* as follows:

$$\boldsymbol{M}_{BB}^{-1}\Delta_k \leq \boldsymbol{z}_B^*, \tag{6.16}$$

$$\boldsymbol{M}_{\boldsymbol{NB}}\boldsymbol{M}_{\boldsymbol{BB}}^{-1}\Delta_k \le \boldsymbol{x}_N^*. \tag{6.17}$$

Equation (6.16) and (6.17) can be combined and written as linear inequality system,

$$\boldsymbol{A}\Delta_k \le \boldsymbol{b}, \tag{6.18}$$

where $\boldsymbol{A}$ and $\boldsymbol{b}$ are given by

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{M}_{\boldsymbol{BB}}^{-1} \\ \boldsymbol{M}_{\boldsymbol{NB}}\boldsymbol{M}_{\boldsymbol{BB}}^{-1} \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} \boldsymbol{z}_{\boldsymbol{B}}^* \\ \boldsymbol{x}_N^* \end{bmatrix}.$$

Note that $\boldsymbol{b}$ is the vector in $\boldsymbol{R}^{|m|}$ whose components are b_i, for $i \in \{1, 2, ..., m\}$. Matrix $\boldsymbol{A}$ and vector $\boldsymbol{b}$ are outlined as follows:

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1u} \\ a_{21} & a_{22} & & a_{2u} \\ \vdots & & \ddots & \vdots \\ a_{v1} & a_{v2} & \cdots & a_{vu} \end{bmatrix}, \qquad \boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_{u+v} \end{bmatrix}.$$

Define UB_k as an upper bound of the maximum allowable range in ω_k and LB_k as a lower bound of the maximum allowable range in ω_k. In other words,

$$LB_k \le \omega_k \le UB_k$$

where $k \in \boldsymbol{B}$.

The values of UB_k and LB_k are obtained by solving a linear inequality system (6.18) for each value of k $(k = 1,\ 2,\ ...,\ u)$ and they can be determined by the following equations:

$$UB_k = \underset{1 \le i \le m}{minimum}\left\{\frac{b_i}{a_{ik}} : a_{ik} > 0\right\}, \tag{6.19}$$

$$LB_k = \underset{1 \le i \le m}{maximum} \left\{ \frac{b_i}{a_{ik}} : a_{ik} < 0 \right\}. \tag{6.20}$$

6.3.3.2. A single parameter q_k $(k \in \boldsymbol{N})$ is perturbed.

In this section, the main focus is to perform sensitivity analysis on a single parameter q_k in set $\boldsymbol{N}$. Thus, we define Δ_k to be the parameter vector which has a value ω_k in the k^{th} position ($\omega_k \neq 0$) and the value of 0 in all other positions. The size of the vector Δ_k is $v \times 1$. Note that the cardinality of set $\boldsymbol{N}$ is v.

Using the index sets $\boldsymbol{B}$ and $\boldsymbol{N}$**,** the perturbed problem (6.10) is equivalent to the following systems:

$$\begin{aligned}
&\boldsymbol{x_B} = \boldsymbol{M_{BB}} \boldsymbol{z_B} + \boldsymbol{M_{BN}} \boldsymbol{z_N} + \boldsymbol{q_B} \geq \mathbf{0}, \\
&\boldsymbol{x_N} = \boldsymbol{M_{NB}} \boldsymbol{z_B} + \boldsymbol{M_{NN}} \boldsymbol{z_N} + \boldsymbol{q_N} + \boldsymbol{\Delta_k} \geq \mathbf{0}, \\
&\boldsymbol{z_B} \geq \mathbf{0}, \quad \boldsymbol{z_N} \geq \mathbf{0}, \quad \boldsymbol{z_B^T x_B} = \mathbf{0}, \quad \boldsymbol{z_N^T x_N} = \mathbf{0}.
\end{aligned} \tag{6.21}$$

if Δ_k is said to be allowable, then (6.21) becomes

$$\widehat{\boldsymbol{x_B}} = \boldsymbol{M_{BB}} \widehat{\boldsymbol{z_B}} + \boldsymbol{q_B} = \mathbf{0}, \tag{6.22}$$

$$\widehat{\boldsymbol{x_N}} = \boldsymbol{M_{NB}} \widehat{\boldsymbol{z_B}} + \boldsymbol{q_N} + \boldsymbol{\Delta_k} \geq \mathbf{0}, \tag{6.23}$$

$$\widehat{\boldsymbol{z_B}} \geq \mathbf{0}.$$

We can solve for $\widehat{\boldsymbol{z_B}}$ in (6.12) as we did in (6.8). It can be written as

$$\widehat{\boldsymbol{z_B}} = -\boldsymbol{M_{BB}^{-1}} \boldsymbol{q_B}. \tag{6.24}$$

By substituting this value of $\widehat{z_B}$ in (6.23), equation (6.23) becomes

$$-\boldsymbol{M}_{NB}\boldsymbol{M}_{BB}^{-1}\boldsymbol{q}_B + \boldsymbol{q}_N + \Delta_k \geq \mathbf{0},$$

or

$$\Delta_k \geq \boldsymbol{M}_{NB}\boldsymbol{M}_{BB}^{-1}\boldsymbol{q}_B - \boldsymbol{q}_N. \qquad (6.25)$$

Using the expression in (6.9), equation (6.25) can be rewritten as follows:

$$\Delta_k \geq -\boldsymbol{x}_N^*. \qquad (6.26)$$

Since the values of all components in x_N^* are always positive, a lower bound of the maximum allowable range in Δ_k (or ω_k) can be instantly determined by the value of x_k^* in equation (6.26), for $k \in \boldsymbol{N}$. According to (6.26), the upper bound of the maximum allowable range in Δ_k will be ∞. In other words, the maximum allowable range for each index *k* can be written as

$$x_k^* \leq \omega_k \leq \infty \qquad (6.27)$$

where $k \in \boldsymbol{N}$.

6.3.3.3. Numerical Example

The main purpose of this section is to perturb one parameter at a time in either set ***B*** or ***N***. If the parameter is in set ***B***, equations (6.19) and (6.20) are used to determine the maximum allowable range of that parameter. Equation (6.27) is used, if the parameter is in set ***N***. As mentioned earlier, only marginal costs ($\overset{=}{c}_{f,j}$) and capacities ($\overset{=\max}{P}_{f,j}$) in vector ***q*** are perturbed.

The optimal solutions, obtained from solving the stochastic Cournot model using data in Table 17, are used to perform the sensitivity analysis to the LCP and shown in Table 18.

Table 17. Market composition and generating unit data

Firm	Unit	Capacity (MW)	Marginal Cost ($/MWh)
Firm 1	1	812	17.06
	2	195	52.99
	3	79	60.97
	4	76	77.61
	5	44	215.80
	6	24	227.08
	7	20	232.94
	8	133	242.57
	9	173	264.04
	10	24	328.35
	11	20	345.67
	12	∞	400
Firm 2	1	100	0.23
	2	26	25.31
	3	50	28.66
	4	24	222.45
	5	76	229.10
	6	24	277.24
	7	20	338.82
	8	∞	400
Firm 3	1	176	0.80
	2	324	25.87
	3	76	48.52
	4	194	214.90
	5	400	242.69
	6	∞	400

The table includes values of the dual variables in the variable vector ***z***. Due to a large number of variables, only the optimal solutions in set ***B*** are shown. All other variables, based on the definition of the LCP, have a value of 0. The values of the parameters ξ and K are assumed to be 38.5 and 1972, respectively. The APR is assumed to be 7%. The value of δ is calculated to be 713.75. It is assumed that the first six values in the parameter vector ***q*** do not change.

The maximum allowable ranges of all parameters in vector ***q*** are shown in Tables 19 and 20.

Table 18. The solutions associated with set ***B*** ($\boldsymbol{z}_B^*$)

Variable	Value	Variable	Value
s_1^*	506.54	$g_{2,1}$	100
s_2^*	126	$g_{2,2}$	26
s_3^*	176	$g_{3,1}$	176
λ_1	12178.81	$\alpha_{2,1}$	19070.62
λ_2	19233.57	$\alpha_{2,2}$	1170.82
λ_3	18306.62	$\alpha_{3,1}$	17734.89
$g_{1,1}$	506.54	-	-

Table 19. The maximum allowable range of each marginal cost ($\overline{\overline{c}}_{f,j}$)

Parameter	Set ***B***	Current Value	Δ-	Δ+	%Δ-	%Δ+	LB_k	UB_k
$c_{1,1}$	Yes	17.06	-3.28	0.44	-19.23	2.61	13.78	17.51
$c_{1,2}$		52.99	-35.93	∞	-67.80	∞	17.06	∞
$c_{1,3}$		60.97	-43.91	∞	-72.01	∞	17.06	∞
$c_{1,4}$		77.61	-60.54	∞	-78.01	∞	17.06	∞
$c_{1,5}$		215.80	-198.74	∞	-92.09	∞	17.06	∞
$c_{1,6}$		227.08	-210.01	∞	-92.49	∞	17.06	∞
$c_{1,7}$		232.94	-215.88	∞	-92.68	∞	17.06	∞
$c_{1,8}$		242.57	-225.51	∞	-92.97	∞	17.06	∞
$c_{1,9}$		264.04	-246.97	∞	-93.54	∞	17.06	∞
$c_{1,10}$		328.35	-311.28	∞	-94.80	∞	17.06	∞
$c_{1,11}$		345.67	-328.61	∞	-95.06	∞	17.06	∞
$c_{1,12}$		400.00	-382.94	∞	-95.73	∞	17.06	∞
$c_{2,1}$	Yes	0.23	-∞	26.72	-∞	11703.77	0.00	26.95
$c_{2,2}$	Yes	25.31	-∞	1.64	-∞	6.48	0.00	26.95
$c_{2,3}$		28.66	-1.72	∞	-5.99	∞	26.95	∞
$c_{2,4}$		222.45	-195.50	∞	-87.89	∞	26.95	∞
$c_{2,5}$		229.10	-202.15	∞	-88.24	∞	26.95	∞
$c_{2,6}$		277.24	-250.29	∞	-90.28	∞	26.95	∞
$c_{2,7}$		338.82	-311.87	∞	-92.05	∞	26.95	∞
$c_{2,8}$		400.00	-373.05	∞	-93.26	∞	26.95	∞
$c_{3,1}$	Yes	0.80	-∞	24.85	-∞	3102.00	0.00	25.65
$c_{3,2}$		25.87	-0.22	∞	-0.86	∞	25.65	∞
$c_{3,3}$		48.52	-22.88	∞	-47.14	∞	25.65	∞
$c_{3,4}$		214.90	-189.25	∞	-88.06	∞	25.65	∞
$c_{3,5}$		242.69	-217.04	∞	-89.43	∞	25.65	∞
$c_{3,6}$		400.00	-374.35	∞	-93.59	∞	25.65	∞

Table 20. The maximum allowable range of each capacity ($\overline{\overline{P}}_{f,j}^{\max}$)

Parameter	set ***B***	Current Value	Δ-	Δ+	%Δ-	%Δ+	LB_k	UB_k
$P_{1,1}^{Max}$		812.00	-305.46	∞	-37.62	∞	506.54	∞
$P_{1,2}^{Max}$		195.00	-195.00	∞	-100.00	∞	0.00	∞
$P_{1,3}^{Max}$		79.00	-79.00	∞	-100.00	∞	0.00	∞
$P_{1,4}^{Max}$		76.00	-76.00	∞	-100.00	∞	0.00	∞
$P_{1,5}^{Max}$		44.00	-44.00	∞	-100.00	∞	0.00	∞
$P_{1,6}^{Max}$		24.00	-24.00	∞	-100.00	∞	0.00	∞
$P_{1,7}^{Max}$		20.00	-20.00	∞	-100.00	∞	0.00	∞
$P_{1,8}^{Max}$		133.00	-133.00	∞	-100.00	∞	0.00	∞
$P_{1,9}^{Max}$		173.00	-173.00	∞	-100.00	∞	0.00	∞
$P_{1,10}^{Max}$		24.00	-24.00	∞	-100.00	∞	0.00	∞
$P_{1,11}^{Max}$		20.00	-20.00	∞	-100.00	∞	0.00	∞
$P_{1,12}^{Max}$		30000.00	-30000.00	∞	-100.00	∞	0.00	∞
$P_{2,1}^{Max}$	Yes	100.00	-17.12	42.10	-17.12	42.10	82.88	142.10
$P_{2,2}^{Max}$	Yes	26.00	-17.12	42.10	-65.86	161.94	8.88	68.10
$P_{2,3}^{Max}$		50.00	-50.00	∞	-100.00	∞	0.00	∞
$P_{2,4}^{Max}$		24.00	-24.00	∞	-100.00	∞	0.00	∞
$P_{2,5}^{Max}$		76.00	-76.00	∞	-100.00	∞	0.00	∞
$P_{2,6}^{Max}$		24.00	-24.00	∞	-100.00	∞	0.00	∞
$P_{2,7}^{Max}$		20.00	-20.00	∞	-100.00	∞	0.00	∞
$P_{2,8}^{Max}$		30000.00	-30000.00	∞	-100.00	∞	0.00	∞
$P_{3,1}^{Max}$	Yes	176.00	-5.71	126.31	-3.24	139.34	170.29	302.31
$P_{3,2}^{Max}$		324.00	-324.00	∞	-100.00	∞	0.00	∞
$P_{3,3}^{Max}$		76.00	-76.00	∞	-100.00	∞	0.00	∞
$P_{3,4}^{Max}$		194.00	-194.00	∞	-100.00	∞	0.00	∞
$P_{3,5}^{Max}$		400.00	-400.00	∞	-100.00	∞	0.00	∞
$P_{3,6}^{Max}$		30000.00	-30000.00	∞	-100.00	∞	0.00	∞

6.3.4. Tolerance approach to sensitivity analysis when all values of $\boldsymbol{q}$ vary simultaneously

The major drawback in running a simulation is the computational time. One of the most important factors that affect the simulation time is the sample size. In this research, we sample the prices of fuel and compute the expected cost curve based on those prices. Since the results from the simulation will be used as input data to the LCP to compute the Nash-equilibrium quantities, the simulation results obtained from a large sample size are preferred. However, a large sample size may a take long computational time because of the process of calculating the expected cost curves. Furthermore, a linear complemetarity problem (LCP) is equivalent to quadratic programming. The linear complementarity problem belongs to a class of NP-complete problems (Murty, 2008). There are several methods for solving an LCP, such as iterative methods and pivoting methods, but solving the LCP in polynomial time is not expected in these algorithms.

The tolerance approach to sensitivity analysis in the LCP is applied to determine whether the perturbed problem has the same index set of nonzero elements as the original problem. If both problems still have the same index set of nonzero elements, then the new solutions can be calculated without directly solving the LCP.

6.3.4.1. Algorithm

Since the main interest of this section is to determine whether the perturbed vector $\boldsymbol{q}$ has effects on an index set of the optimal solutions, we define $\boldsymbol{q}^{new}$ to be the perturbed vector and $\boldsymbol{q}$ to be the original vector.

Based on (6.10), $\boldsymbol{W}$ is defined as the perturbed amount vector where

$$\boldsymbol{W} = \boldsymbol{q}^{new} - \boldsymbol{q}. \tag{6.28}$$

The elements of vector $\boldsymbol{W}$ are

$$\boldsymbol{W} = \begin{bmatrix} \omega_1 = q_1^{new} - q_1 \\ \omega_2 = q_2^{new} - q_2 \\ \vdots \\ \omega_m = q_m^{new} - q_m \end{bmatrix}_{m\times 1}.$$

Note that the size of vector $\boldsymbol{W}$ is $m \times 1$. Thus, the perturbed problem (6.10) using the index sets $\boldsymbol{B}$ and $\boldsymbol{N}$ can be written in the following equivalent forms:

$$\boldsymbol{x_B} = \boldsymbol{M_{BB}z_B} + \boldsymbol{M_{BN}z_N} + \boldsymbol{q_B} + \boldsymbol{W_B} \geq \boldsymbol{0},$$

$$\boldsymbol{x_N} = \boldsymbol{M_{NB}z_B} + \boldsymbol{M_{NN}z_N} + \boldsymbol{q_N} + \boldsymbol{W_N} \geq \boldsymbol{0}, \tag{6.29}$$

$$\boldsymbol{z_B} \geq \boldsymbol{0}, \quad \boldsymbol{z_N} \geq \boldsymbol{0}, \quad \boldsymbol{z_B^T x_B} = \boldsymbol{0}, \quad \boldsymbol{z_N^T x_N} = \boldsymbol{0}.$$

The vector $\boldsymbol{W}$ is said to be allowable if problem (6.11) has a solution $\hat{z}$ such that

$$\widehat{\boldsymbol{z_B}} \geq \boldsymbol{0}, \quad \widehat{\boldsymbol{z_N}} = \boldsymbol{0}, \quad \widehat{\boldsymbol{x_B}} = \boldsymbol{0}, \text{and} \quad \widehat{\boldsymbol{x_N}} \geq \boldsymbol{0}.$$

Thus, if Δ is allowable, then (6.29) becomes

$$\widehat{\boldsymbol{x_B}} = \boldsymbol{M_{BB}}\widehat{\boldsymbol{z_B}} + \boldsymbol{q_B} + \boldsymbol{W_B} = \boldsymbol{0}, \tag{6.30}$$

$$\widehat{\boldsymbol{x_N}} = \boldsymbol{M_{NB}}\widehat{\boldsymbol{z_B}} + \boldsymbol{q_N} + \boldsymbol{W_N} \geq \boldsymbol{0}, \tag{6.31}$$

$$\widehat{\boldsymbol{z_B}} \geq \boldsymbol{0}.$$

We can solve for $\widehat{z_B}$ in (6.30) which is

$$\widehat{\boldsymbol{z_B}} = -\boldsymbol{M_{BB}^{-1}}(\boldsymbol{q_B} + \boldsymbol{W_B}) \geq \boldsymbol{0},$$

or

$$\boldsymbol{M}_{BB}^{-1}\Delta_{B} \leq -\boldsymbol{M}_{BB}^{-1}\boldsymbol{q}_{B}. \tag{6.32}$$

Substituting this value of $\widehat{\boldsymbol{z}_{B}}$ in (6.31), equation (6.31) can be written as

$$-\boldsymbol{M}_{NB}\boldsymbol{M}_{BB}^{-1}(\boldsymbol{q}_{B}+\boldsymbol{W}_{B})+\boldsymbol{q}_{N}+\boldsymbol{W}_{N} \geq \boldsymbol{0},$$

or

$$\boldsymbol{M}_{NB}\boldsymbol{M}_{BB}^{-1}\boldsymbol{W}_{B}-\boldsymbol{W}_{N} \leq -\boldsymbol{M}_{NB}\boldsymbol{M}_{BB}^{-1}\boldsymbol{q}_{B}+\boldsymbol{q}_{N}. \tag{6.33}$$

Using the expressions in (6.8) and (6.9), equations (6.32) and (6.33) can be rewritten in terms of $\boldsymbol{z}_{B}^{*}$ and $\boldsymbol{x}_{N}^{*}$ as follows:

$$\boldsymbol{M}_{BB}^{-1}\boldsymbol{W}_{B} \leq \boldsymbol{z}_{B}^{*}, \tag{6.34}$$

$$\boldsymbol{M}_{NB}\boldsymbol{M}_{BB}^{-1}\boldsymbol{W}_{B}-\boldsymbol{W}_{N} \leq \boldsymbol{x}_{N}^{*}. \tag{6.35}$$

Equations (6.34) and (6.35) can be written as the following linear inequality system,

$$\boldsymbol{C}\boldsymbol{W} \leq \boldsymbol{b}, \tag{6.36}$$

where $\boldsymbol{C}$ and $\boldsymbol{b}$ are given by

$$\boldsymbol{C}=\begin{bmatrix} \boldsymbol{M}_{BB}^{-1} & \boldsymbol{0} \\ \boldsymbol{M}_{NB}\boldsymbol{M}_{BB}^{-1} & -\boldsymbol{I} \end{bmatrix}, \quad \boldsymbol{b}=\begin{bmatrix} \boldsymbol{z}_{B}^{*} \\ \boldsymbol{x}_{N}^{*} \end{bmatrix},$$

where $\boldsymbol{I}$ is the identity matrix of size $v \times v$.

To determine effects of $\boldsymbol{q}^{new}$ on the optimal solutions, the following procedure is applied:

START

Given $\boldsymbol{M}, \boldsymbol{q},$ and $\boldsymbol{z}^*$.

Sets $\boldsymbol{B}$ and $\boldsymbol{N}$ are defined as $\boldsymbol{B} := \{i | z_i^* > 0\}$ and $\boldsymbol{N} := \{i | x_i^* > 0\}$ where

$\boldsymbol{x}^* = (\boldsymbol{M}\boldsymbol{z}^* + \boldsymbol{q})$.

$\boldsymbol{z}_{\boldsymbol{B}}^* = -\boldsymbol{M}_{\boldsymbol{BB}}^{-1}\boldsymbol{q}_{\boldsymbol{B}}$ and $\boldsymbol{x}_N^* = -\boldsymbol{M}_{NB}\boldsymbol{M}_{\boldsymbol{BB}}^{-1}\boldsymbol{q}_{\boldsymbol{B}} + \boldsymbol{q}_N$.

$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{M}_{\boldsymbol{BB}}^{-1} & \boldsymbol{0} \\ \boldsymbol{M}_{NB}\boldsymbol{M}_{\boldsymbol{BB}}^{-1} & -\boldsymbol{I} \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} \boldsymbol{z}_{\boldsymbol{B}}^* \\ \boldsymbol{x}_N^* \end{bmatrix}$.

IF $\boldsymbol{CW} \le \boldsymbol{b}$ THEN

The new optimal solutions can be calculated as

$$\widehat{\boldsymbol{z}_{\boldsymbol{B}}^{\boldsymbol{new}}} = -\boldsymbol{M}_{\boldsymbol{BB}}^{-1}\left(\boldsymbol{q}_{\boldsymbol{B}}^{new}\right), \tag{6.37a}$$

$$\widehat{\boldsymbol{z}_{N}^{\boldsymbol{new}}} = \boldsymbol{0}. \tag{6.37b}$$

ELSE

The index set of nonzero elements has changed and the LCP needs to be resolved with the value $\boldsymbol{q}^{new}$.

END

END

6.3.4.2. Numerical Example

This numerical example shows how the algorithm described in this section is applied. The main objective is to determine whether a small sample size can be used to compute the Nash-equilibrium quantities. For the numerical example, data in Table 17 are used. The values given in those tables correspond to an estimation of the expected cost curve and are associated with the vector $\boldsymbol{q}$. These values were obtained by using a sample size equal to 10,000. It took roughly 2 weeks, 15 hours, and 3 days to obtain the results in those three tables, respectively. Running a simulation this long is not practical since in real operation the decisions must be made on a daily basis. However, sampling with a small sample size yields volatile results each time the simulation is run. When using a small sample size, the tolerance approach to sensitivity analysis developed in this section is employed to determine if a smaller sample is practical.

The results obtained from simulation using a sample size equal to 10,000 are compared with the results using a sample size equal to 100. Results obtained from the simulation with sample size equal to 100 are displayed in Tables 21, 22, and 23. The procedure to determine effects of $\boldsymbol{q}^{new}$ on the optimal solutions described in section 3.4.1 is then employed. The Nash-equilibrium quantities using $\boldsymbol{q}^{new}$ associated with set $\boldsymbol{B}$ are shown in Table 24. The process of running the simulation and determining the effect of $\boldsymbol{q}^{new}$ is repeated 10 times.

The results in Table 24 indicate that the results obtained from the simulation with sample size equal to 100 break the optimal condition 3 times out of 10.

Table 21. The list of hypothetical generating unit data associated with estimated expected cost function for firm 1

Firm 1													
Run	Parameter	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8	Unit 9	Unit 10	Unit 11	Unit 12
1	c_{1j}	17.06	52.99	60.97	77.60	215.80	227.08	232.94	242.57	264.03	328.34	345.67	400.00
	P_{1j}^{Max}	812	195	79	76	44	24	20	133	173	24	20	∞
2	c_{1j}	17.02	52.45	60.39	76.89	213.01	224.16	230.01	240.12	261.82	326.51	342.61	400.00
	P_{1j}^{Max}	812	195	79	76	44	24	20	133	173	24	20	∞
3	c_{1j}	17.05	53.24	61.24	77.95	217.29	228.62	234.50	243.87	265.21	329.32	347.30	400.00
	P_{1j}^{Max}	812	195	79	76	44	24	20	133	173	24	20	∞
4	c_{1j}	17.09	53.09	61.08	77.73	216.23	227.53	233.40	242.95	264.38	328.63	346.14	400.00
	P_{1j}^{Max}	812	195	79	76	44	24	20	133	173	24	20	∞
5	c_{1j}	17.01	52.80	60.77	77.37	215.01	226.25	232.11	241.87	263.41	327.83	344.80	400.00
	P_{1j}^{Max}	812	195	79	76	44	24	20	133	173	24	20	∞
6	c_{1j}	17.06	52.99	60.98	77.61	215.86	227.14	233.01	242.62	264.08	328.39	345.74	400.00
	P_{1j}^{Max}	812	195	79	76	44	24	20	133	173	24	20	∞
7	c_{1j}	17.11	52.65	60.60	77.14	213.64	224.82	230.67	240.68	262.32	326.93	343.30	400.00
	P_{1j}^{Max}	812	195	79	76	44	24	20	133	173	24	20	∞
8	c_{1j}	17.07	53.25	61.25	77.96	217.26	228.60	234.48	243.85	265.19	329.30	347.27	400.00
	P_{1j}^{Max}	812	195	79	76	44	24	20	133	173	24	20	∞
9	c_{1j}	17.14	52.69	60.64	77.18	213.74	224.93	230.78	240.76	262.40	326.99	343.41	400.00
	P_{1j}^{Max}	812	195	79	76	44	24	20	133	173	24	20	∞
10	c_{1j}	17.01	52.77	60.73	77.32	214.83	226.06	231.92	241.71	263.26	327.71	344.60	400.00
	P_{1j}^{Max}	812	195	79	76	44	24	20	133	173	24	20	∞

Table 22. The list of hypothetical generating unit data associated with estimated expected cost function for firm 2

Firm 2									
Run	Parameter	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8
1	c_{2j}	0.23	25.17	28.49	220.00	226.71	274.71	335.79	400.00
	P_{2j}^{Max}	100	26	50	24	76	24	20	∞
2	c_{2j}	0.23	25.29	28.69	224.65	231.24	279.51	341.54	400.00
	P_{2j}^{Max}	100	26	50	24	76	24	20	∞
3	c_{2j}	0.23	25.33	28.70	223.50	230.12	278.32	340.11	400.00
	P_{2j}^{Max}	100	26	50	24	76	24	20	∞
4	c_{2j}	0.23	25.19	28.55	222.01	228.67	276.79	338.27	400.00
	P_{2j}^{Max}	100	26	50	24	76	24	20	∞
5	c_{2j}	0.23	25.31	28.69	223.77	230.39	278.60	340.45	400.00
	P_{2j}^{Max}	100	26	50	24	76	24	20	∞
6	c_{2j}	0.23	25.29	28.61	220.42	227.13	275.15	336.31	400.00
	P_{2j}^{Max}	100	26	50	24	76	24	20	∞
7	c_{2j}	0.23	25.32	28.71	224.46	231.06	279.31	341.30	400.00
	P_{2j}^{Max}	100	26	50	24	76	24	20	∞
8	c_{2j}	0.23	25.33	28.65	220.69	227.38	275.42	336.63	400.00
	P_{2j}^{Max}	100	26	50	24	76	24	20	∞
9	c_{2j}	0.23	25.20	28.56	222.09	228.76	276.87	338.38	400.00
	P_{2j}^{Max}	100	26	50	24	76	24	20	∞
10	c_{2j}	0.23	25.18	28.49	219.90	226.61	274.61	335.66	400.00
	P_{2j}^{Max}	100	26	50	24	76	24	20	∞

Table 23. The list of hypothetical generating unit data associated with estimated expected cost function for firm 3

Firm 3							
Run	Parameter	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6
1	c_{3j}	0.80	26.13	48.79	217.03	244.57	400.00
	P_{3j}^{Max}	176	324	76	194	400	∞
2	c_{3j}	0.80	25.59	48.12	212.58	240.65	400.00
	P_{3j}^{Max}	176	324	76	194	400	∞
3	c_{3j}	0.80	25.66	48.28	213.18	241.18	400.00
	P_{3j}^{Max}	176	324	76	194	400	∞
4	c_{3j}	0.80	26.00	48.67	215.94	243.61	400.00
	P_{3j}^{Max}	176	324	76	194	400	∞
5	c_{3j}	0.80	25.84	48.41	214.66	242.48	400.00
	P_{3j}^{Max}	176	324	76	194	400	∞
6	c_{3j}	0.80	25.95	48.58	215.55	243.27	400.00
	P_{3j}^{Max}	176	324	76	194	400	∞
7	c_{3j}	0.80	25.67	48.31	213.24	241.23	400.00
	P_{3j}^{Max}	176	324	76	194	400	∞
8	c_{3j}	0.80	26.13	48.80	217.01	244.54	400.00
	P_{3j}^{Max}	176	324	76	194	400	∞
9	c_{3j}	0.80	25.68	48.35	213.35	241.33	400.00
	P_{3j}^{Max}	176	324	76	194	400	∞
10	c_{3j}	0.80	25.72	48.35	213.64	241.58	400.00
	P_{3j}^{Max}	176	324	76	194	400	∞

Table 24. The new optimal solutions using $\boldsymbol{q}^{new}$ associated with set $\boldsymbol{B}$

	Parameter													
Run	s_1^*	s_2^*	s_3^*	λ_1	λ_2	λ_3	$g_{1,1}$	$g_{2,1}$	$g_{2,2}$	$g_{3,1}$	$\alpha_{2,1}$	$\alpha_{2,2}$	$\alpha_{3,1}$	$CW \le b$
1	505.49	126.00	176.00	12217.59	19252.96	18326.01	505.49	100.00	26.00	176.00	19090.62	1286.14	17754.04	Yes
2	-	-	-	-	-	-	-	-	-	-	-	-	-	No
3	506.71	126.00	176.00	12172.26	19230.29	18303.34	506.71	100.00	26.00	176.00	19067.25	1153.53	17732.54	Yes
4	506.07	126.00	176.00	12196.06	19242.20	18315.24	506.07	100.00	26.00	176.00	19079.76	1261.23	17743.02	Yes
5	507.51	126.00	176.00	12142.76	19215.54	18288.59	507.51	100.00	26.00	176.00	19052.59	1153.59	17719.00	Yes
6	506.67	126.00	176.00	12173.83	19231.08	18304.13	506.67	100.00	26.00	176.00	19068.22	1181.11	17732.93	Yes
7	-	-	-	-	-	-	-	-	-	-	-	-	-	No
8	506.46	126.00	176.00	12181.57	19234.95	18308.00	506.46	100.00	26.00	176.00	19071.91	1155.96	17735.71	Yes
9	-	-	-	-	-	-	-	-	-	-	-	-	-	No
10	507.55	126.00	176.00	12141.31	19214.82	18287.87	507.55	100.00	26.00	176.00	19052.44	1241.15	17716.65	Yes

6.4. CONCLUSIONS

In this chapter, the theory of tolerance approach to sensitivity analysis in LCP was applied. The maximum tolerance within which the right-hand side (vector $\boldsymbol{q}$) of the problem can vary independently and simultaneously such that the perturbed problems have the same index set of nonzero elements as the original problems was established.

An algorithm was developed to find the maximum tolerance, in which the right-hand side of the problem is perturbed independently. As the major parameters within the right-hand side are marginal costs and maximum capacities, the maximum tolerance on those two factors was evaluated. The maximum tolerance indicates the maximum range of marginal costs and maximum capacities that can be perturbed without affecting the firms' strategic planning and operation.

In addition, the algorithm to find the maximum allowable range when the right-hand sides of the problems vary simultaneously was developed. This algorithm also can be used to determine whether the new input data within vector $\boldsymbol{q}$ affect the index set of nonzero elements. If the perturbed problems still have the same index set of nonzero elements, the new optimal solutions can be calculated without directly solving LCP.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

7.1. Conclusions

This research proposed a model and solution for evaluating the effects of uncertainty in deregulated electricity markets. Two essential factors in power markets, generator outages and fuel price uncertainty, were considered in this research. First, a multi-period deterministic Cournot model was developed to resemble the structure of long-term deregulated electricity markets. The Cournot model then incorporated generator outages by replacing the cost functions with the expected cost functions. The Cournot model then becomes a stochastic model that considers the availability of generators. Next, both generator outages and fuel price uncertainties were included in the Cournot model. Finally, the tolerance approach to sensitivity analysis was implemented to determine the sensitive parameters of the stochastic Cournot model when considering both uncertainty factors.

In the deterministic model, uncertainty is disregarded, but the model takes the time value of money into account. The model belongs to a class of quadratic programming models. The KKT first order optimality conditions of the model were considered as a linear complementarity problem (LCP). The Nash-equilibrium quantities were computed by combining the KKT first order optimality conditions.

To incorporate generator outages into the model, the expected production cost function that includes the availability of generators was developed. The resulting expected production cost function was a piecewise linear function. It was shown that the set of all combinations of unit capacities determines where the expected production cost function changes its slope. The number of slopes grows exponentially as the number of generating units increases. This issue has a direct link to the computational complexity of the problem. Hence, an algorithm to reduce the number of slopes without losing computational accuracy was devised. The results showed that the proposed algorithm is able to reduce the number of slopes effectively and thus simplifies the computations. The algorithm produced a set of hypothetical generators with a smaller number of units when taking generator outages into account. The results also showed that generator outages have an important effect on firms' expected profits and that they should be considered in any medium-term or long-term planning process.

The consideration of stochastic fuel costs in the stochastic Cournot model provided more accurate decisions to power producers as the fluctuation of fuel costs significantly impacts a firm's long-term operation. Each firm was assumed to operate a set of generators which used different types of fuels whose marginal costs are subject to uncertainty. Therefore, the marginal costs were considered random variables. The Monte Carlo simulation based technique was employed to sample the cost of each fuel type. The slope reduction algorithm was applied in order to aid the computational complexity as the estimated production cost curve contains a large number of slopes. The Nash-equilibrium quantities were then calculated. The model expected profits and expected profits of all three cases (models A, B, and C) were computed to show the effects of both factors. The results showed that the availability of generators and the volatility of fuel prices have a significant impact on firms' expected profits and that they should be considered in the planning and operation of a power system.

The theory of tolerance approach to sensitivity analysis in LCP was applied to the stochastic Cournot model. Specifically, a method was devised to find the maximum tolerance within the right-hand side of the problem that can vary independently or simultaneously such that the perturbed problems have the same index set of nonzero elements as the original problems. The maximum tolerance indicates the maximum range of each parameter in the right-hand side that can be perturbed without affecting firms' strategic planning and operation.

The algorithm to find the maximum range when the right-hand sides of the problems vary simultaneously can be used to determine whether the new input data affects the index set of nonzero elements. If the perturbed problems still have the same index set of nonzero elements, the new optimal solutions can be calculated without directly solving a linear complementarity problem. This approach is intended to lessen computational complexity in a large-scale linear complementarity problem. It is also useful for market participants when they have a new set of input data or the input data in the right-hand side are perturbed.

7.2. Directions for future research

This research proposed a Cournot model to evaluate two major uncertainty factors in power markets. However, there are many other factors which affect the price of electricity and market participants, and among them are transmission constraints. The capacity of transmission lines is restricted by technical constraints. As a result, power market trading can be limited and controlled by transmission constraints. An extension of this dissertation could be to investigate the effect of network configuration on market prices and firms' profits as the constraints could limit the competition because of congestion.

Another research opportunity is the study of the effects of demand uncertainty on electricity prices. In general, the demand of electricity follows daily or seasonal cycles. Demand also depends on the lifestyle of the consumers and weather conditions. The price of electricity, however, does not follow the same pattern as demand. Thus, the effect of demand uncertainty on market prices and the behavior of market participants are two interesting topics worth investigating.

This research is one of the very first attempts to apply the tolerance approach to sensitivity analysis to the stochastic Cournot model. Future research in developing the theory of tolerance approach is still wide open. Since all significant parameters in the proposed model are in the right-hand side of the problem formation, the algorithm developed in this research considered only the perturbation of the right hand side of the equation. One major improvement opportunity to the algorithm is the perturbation of the other parameters of the problem as models may have some vital parameters in the left-hand side.

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