Lag Order and Critical Values for the RMA Based Augmented Dickey-Fuller Test

by

Zhixiao Liu

A thesis submitted to the Graduate Faculty of
Auburn University
in partial fulfillment of the
requirements for the Degree of
Master of Science

Auburn, Alabama
August 9, 2010

Keywords: Finite and asymptotic critical value; Recursive mean adjustment;
Monte Carlo; Response surface.

Approved by

Hyeongwoo Kim, Chair, Assistant Professor, Department of Economics
John Jackson, Professor, Department of Economics
Randy Beard, Professor, Department of Economics

Abstract

This thesis examines the validity of asymptotic critical values for a Recursive Mean Adjustment (RMA) based Augmented Dickey-Fuller (ADF) unit root test. Cheung and Lai show that critical values for the Ordinary least square (OLS) based ADF test depend substantially on the lag order in finite samples. The present article extends their work to a newly proposed RMA-based unit root test, which is more powerful than the OLS-based test. Our Monte Carlo simulation results show that asymptotic critical values for the test with the deterministic terms are valid only when the lag order is one. When lag order is greater than one, the RMA based test with asymptotic critical values tends to be overall over-sized. I also provide finite sample critical values for an array of lag-order and sample size pairs.

Acknowledgments

First and foremost, I would like to thank my advisor, Dr. Hyeongwoo Kim. Without his direction, his patience and encouragement, this thesis would not have been possible. I sincerely appreciate invaluable academic and personal support I have received from him throughout this thesis.

I would also thank the rest of my thesis committee members: Dr. John Jackson and Dr. Randy Beard for their valuable feedbacks and suggestions that helped me to improve the thesis.

I also respectfully acknowledge Dr. Barry Burkhart, Chair, Department of Economics at AU for giving me his valuable time. I thank Dr. Jackson for admitting me to Master of Science program to pursue higher education.

I thank you all for teaching me and guiding me.

Finally, my further gratitude goes to my family: my husband Xueyi, my son Kaishuo, My parents, and my sister Zhixin. Thanks for your love, support, encouragement and patience.

Table of Contents

List of Tables

List of Figures

List of Abbreviations

ADF	Augmented Dick-Fuller
AR	Autoregressive
CV	Critical Value
DF	Dickey -Fuller
DGP	Data Generating Process
GDP	gross domestic product
OLS	Ordinary Least Squares
RMA	Recursive Mean Adjustment
RSM	Response Surface Methodology
RTA	Recursive Trend Adjustment

Chapter 1

Introduction

The analysis of unit root nonstationarity has been one of the major areas of research in time series econometrics over the last two decades. Stationararity or nonstationarity of macro-economic time series is quite important to investigate statistically because (a) macro economic time series are known to exhibit persistence in their intertemporal behavior, (b) Spurious regression problems can lead to misleading inference, (c) conventional statistical analyses may be invalid when applied to regressions with nonstationary variables. Early motivation for a unit root test was to help determine whether to use forecasting models expressed in differences or levels in particular applications (e.g. Dickey, Bell, and Miller, 1986). Nowadays, unit root tests are useful to test certain hypotheses such as purchasing power parity (e.g. Rogoff, 1996), the efficient market hypothesis (e.g. Balvers et al ,2000), and the natural rate of unemployment or hysteresis hypothesis (e.g. Blanchard and Summers,1987), just to name a few. Generally, the major problem when working with nonstationarity results from the breakdown of conventional asymptotic distribution theory under nonstationarity. Standard statistical inferences become invalid, and many test statistics developed for nonstationarity converge to nonstandard distributions. Therefore, unit root tests are important.

Many methods for unit root tests have been developed. Among them, the Augmented Dick-Fuller (ADF) test is by far the most popular. This test examines the null hypothesis of nonstationarity against stationary alternatives. Asymptotic critical values for the test were tabulated by Dickey-Fuller (1976). Despite of its popularity, it is well known that the ADF test has a low power to find stationarity, especially when the sample size is small.

In order to improve the power of unit root tests, many new methods have been put forward. For example, Zilliott, Rothenberg and Stock (1992) proposed a simple modification of the ADF test, referred to as the DF-GLS test, which is shown to have higher power by Cheung and Lai (1995b). Recently, an ADF unit root test based on recursive mean adjustment (RMA) has been put forth by So and Shin (1999) and Shin and So (2001), which showed significant power improvement according to their Monte Carlo studies.[1] Shin and So (2001) derived the limiting distribution of the test with a constant. Their asymptotic critical values for the test with a constant are tabulated for some sample sizes based solely on AR(1) processes.

Cheung and Lai (1995a) showed that finite sample critical values are determined by lag order in addition to sample size. It is crucial to correcting for the lag order impact in implementing a RMA based ADF test (ADF_{RMA}), for critical values that ignore the dependence of lag order can be misleading. Kim et al (2009) showed that the RMA based ADF test outperformed the DF-GLS and standard ADF tests in their study for G7 stock markets. Despite its power and convenience to implement, this method is largely overlooked in the financial literature.

The purpose of this study is to examine the validity of asymptotic critical values for a Recursive Mean Adjustment based Augmented Dickey-Fuller test. Our Monte Carlo simulation results suggest that asymptotic critical values (e.g. Shin and So(2001)) computed based on k=0) for the test with the deterministic terms are valid only when the lag order is one .When the lag

---

[1] The logic behind RMA method is to use partial mean instead of global mean: $Y_t - \overline{Y}_{t-1} = \alpha(Y_{t-1} - \overline{Y}_{t-1}) + \varepsilon_t$, ε_t is uncorrelated to the recursive mean adjusted regressor $Y_{t-1} - \overline{Y}_{t-1}$,which results in biased reduction RMA estimator, while LS estimator is to estimate $Y_t - \overline{Y} = \alpha(Y_{t-1} - \overline{Y}) + \varepsilon_t$, , ε_t is correlated to regressor $Y_{t-1} - \overline{Y}$,which is biased. See chapter 2 for details.

order is greater than one, the test with asymptotic critical values tends to be overall over-sized even when the sample size is fairly big.[2] .

Response surface analysis has been used by Mackinnon (1991) to obtain approximate finite sample critical values for the traditional ADF unit root test. In his method, lag order is assumed to be fixed and equal to 1 for ADF test. Cheung and Lai (1995a) extended the response surface analysis and showed that although the asymptotical ADF test may not depend on the lag parameter, lag order can be important in finite samples. Employing their ideas by properly accounting for the effect of lag order, our study provides improved estimation of lag-adjusted critical values for the ADF_{RMA} test. Our experimental design generalizes Mackinnon's method (1991) by including lag order but still omits those other nuisance parameters as in Cheung and Lai (1995a). Finite-sample correction for the nuisance parameter, although is desirable, is hard to make, given the potential size of the parameter space of these unknown parameters, it is plausible to omit them.

This thesis is organized as follows: In chapter 2, conventional ordinary least square (OLS) DF and ADF unit root tests (ADF_{ols}) are described, and compared to the RMA based ADF test (ADF_{RMA}). Chapter 3 discusses the methodology of response surface analysis and our experimental design. Chapter 4 reports and analyzes response surface estimation of the critical values of ADF_{RMA} , and provides finite sample critical value Tables for the ADF_{RMA} test. Finally in Chapter 5 we offer conclusions.

[2] A test is oversized when the actual size with asymptotic critical value is greater than the nominal size. That is, such tests tend to reject the null hypothesis too often.

Chapter 2

ADF Unit Root Tests

2.1 OLS-based ADF unit root test

Why people worry about unit root? Most macroeconomic time series are known to exhibit high persistence, possibly nonstationarity, in their intertemporal behavior. Conventional statistical inferences may be invalid when the true data generating process is nonstationary Therefore, unit root tests are important. A widely used unit root is the Augmented Dickey-fuller or ADF test (Dickey and Fuller, 1979). The test typically examines the null hypothesis (random walk without a drift) of nonstationarity against three stationary forms of alternatives.

2.1.1 Autoregressive Unit Root Test

To illustrate the important statistical issues associated with an autoregressive unit root test, we considered the following simple AR (1) model

$$Y_t = \alpha Y_{t-1} + \varepsilon_t \qquad (1)$$

Where ε_t is white noise. The hypotheses of interest are

H_0: $\alpha = 1$ (unit root in θ=0)[3], $Y_t \to I(1)$

H_1: $|\alpha| < 1$, $Y_t \to I(0)$

[3] The AR(1) model may be re-written as $\Delta Y_t = \theta Y_{t-1} + \varepsilon_t$,where $\theta = \alpha - 1$, testing $\alpha = 1$ is equivalent to testing $\theta = 0$, unit root tests are often computed using this alternative regression

One may use a test statistic, $t_{\alpha=1} = \dfrac{\hat{\alpha}-1}{se(\hat{\alpha})}$, where $\hat{\alpha}$ is the least squares estimate and $se(\hat{\alpha})$ is the associated OLS standard error estimate. The test is a one-sided left tail test. Under the alternative hypothesis, $\{Y_t\}$ is stationary ($|\alpha|<1,$), and it can be shown that the following holds.

$$\sqrt{T}(\hat{\alpha}-\alpha) \xrightarrow{d} N(0,(1-\alpha^2))$$

Or $\hat{\alpha} \xrightarrow{A} N(\alpha, \frac{1}{T}(1-\alpha^2))$

Under the null hypothesis, however, the above results give

$$\hat{\alpha} \xrightarrow{A} N(1,0)$$

This clearly doesn't make any sense because it has a degenerating asymptotic distribution. The problem is that under the unit root null hypotheses, $\{y_t\}$ is neither stationary nor ergodic, and the usual sample moments do not converge to fixed constants. Instead, Phillips (1987) showed that the sample moments of $\{Y_t\}$ converge to random function of Brownian motion[4]:

$$T^{\frac{-3}{2}}\sum_{t=1}^{T} Y_{t-1} \xrightarrow{d} \alpha\int_0^1 W(r)dr$$

$$T^{-2}\sum_{t=1}^{T} Y_{t-1}^2 \xrightarrow{d} \alpha^2\int_0^1 W(r)^2 dr$$

$$T^{-1}\sum_{t=1}^{T} Y_{t-1}\varepsilon_t \xrightarrow{d} \alpha^2\int_0^1 W(r)dW(r)$$

where $W(r)$ denotes a standard Brownian motion (Wiener process) defined on the unit interval. Using the above results Phillips derived the asymptotic distributions of the two test statistics under the unit root null $H_0:\alpha=1$

[4] A Wiener process W(.) is continuous-time stochastic process, associating each data $r \in (0,1)$, a scalar random variable that satisfies 1)W(0)=0; (2) any dates. $0 \le t_1 \le \ldots t_k \le 1$, the changes $W(t_2)-W(t_1),\ldots..W(t_k)-W(t_{k-1})$ are independent normal with W(s)-W(t)~N(0,s-t); (3) W(s) is continuous in s.

$$t_{\alpha=1} \xrightarrow{d} \frac{\int_0^1 W(r)dW(r)}{\int_0^1 W(r)^2 d(r)^{1/2}}$$

$$T(\hat{\alpha}-1) \xrightarrow{d} \frac{\int_0^1 W(r)dW(r)}{\int_0^1 W(r)^2 d(r)}$$

The above yields the following results:

(1) $\hat{\alpha}$ is super-consistent; that is $\hat{\alpha} \xrightarrow{p} \alpha$ at rate T instead of usual rate of $T^{1/2}$

(2) $\hat{\alpha}$ is not asymptotically normally distributed and $t_{\alpha=1}$ is not asymptotically standard normal.

(3) The limiting distribution of $t_{\alpha=1}$ is called the Dickey-fuller (DF) distribution and does not have a closed form representation. Therefore, critical values must be computed by approximation or by simulation.

(4) Since $T(\hat{\alpha}-1)$ has a well defined limiting distribution that does not depend on nuisance parameter. It can also be used as a test statistic for null hypothesis H_0: $\alpha = 1$

2.1.2 Three cases under alternative hypothesis

When testing for a unit root, it is important to specify the null and alternative hypotheses. Practically, the common null hypothesis is a random walk without a drift, while alternative hypotheses can be written as the three regression equations below.

Y_t is stationary with no deterministic terms: $Y_t = \alpha Y_{t-1} + \varepsilon_t \quad |\alpha| < 1$ (2)

Y_t is stationary with a constant $Y_t = c + \alpha Y_{t-1} + \varepsilon_t \quad |\alpha| < 1$ (3)

Y_t is stationary with a constant and a time trend. $Y_t = c + \delta t + \alpha Y_{t-1} + \varepsilon_t \quad |\alpha| < 1$ (4)

We should appropriately specify different alternative hypothesis to characterize the trend properties of the data at hand. For instance, if our observed data doesn't show an increasing or decreasing trend (e.g., the real exchange rate), our regression equation alternative hypothesis should reflect this property. If our observed data clearly exhibits an increasing or decreasing trend, (e.g., real GDP), our alternative hypothesis should also reflect it. The trend properties of the data under the alternative hypothesis will determine the form of the test regression used. Moreover, the type of deterministic terms in the test regression will influence the asymptotic distribution of the unit root test statistics. The two most common cases are constant only (3) and constant with a time trend (4). Since most macro variables have non-zero means, the regression (2) is hardly used.

Case I: Constant only

The test regression equation is $Y_t = c + \alpha Y_{t-1} + \varepsilon_t$ (3')

and includes a constant to capture the nonzero mean under the alternative. The hypotheses

H_0: $\alpha = 1$, $Y_t \to I(1)$ without drift.

H_1: $|\alpha| < 1$, $Y_t \to I(0)$ with an intercept.

This formulation is appropriate for non-trending financial series such as the interest rate or exchange rate. The least square estimate $\hat{\alpha}$ is computed from the above regression (3').The test statistic is $t_{\alpha=1} = \dfrac{\hat{\alpha} - 1}{se(\hat{\alpha})}$

Case II: Constant and Time Trend

The test regression is $Y_t = c + \delta t + \alpha Y_{t-1} + \varepsilon_t$ (4')

includes a constant and a deterministic time trend to capture the deterministic trend under the alternative. The hypotheses to be tested are:

H_0: $\alpha = 1$, $Y_t \rightarrow I(1)$ without drift.

H_1: $|\alpha| < 1$, $Y_t \rightarrow I(0)$ with an intercept and deterministic time trend.

This formulation is appropriate for trending time series such as asset prices or level of macroeconomic aggregates such as real GDP. The least square estimate $\hat{\alpha}$ is computed from the above regression (4').The test statistic is $t_{\alpha=1} = \dfrac{\hat{\alpha}-1}{se(\hat{\alpha})}$

2.1.3 Dickey-fuller(ADF) unit root tests

The unit root tests described above are valid if the time series Y_t is well characterized by an AR(1) process with white noise errors only. In practice, many economic variables are better described by AR(P) (where P>1) when the error term ε_t is serially correlated. Consequently, Said and Dickey (1984) developed a test, known as augmented Dickey-Fuller (ADF) test. This test is conducted by "augmenting" the preceding three equations with the lagged values of the differenced dependent variable Y_t. To be specific, we use form (4). The ADF test here consists of estimating the following regression:

$$Y_t = C + \delta t + \alpha Y_{t-1} + \sum_{j=1}^{K} \beta_j \Delta Y_{t-j} + \varepsilon_t \quad (5)$$ [5]

The specification of deterministic terms depends on the assumed behavior of Y_t under the alternative hypothesis of trend stationarity as describe in the previous section. Under the null hypothesis, y_t is I(1), which implies that $\alpha = 1$, The test statistics are based on the least square estimate of (5) and are given by

$$ADF_t = \frac{\hat{\alpha}-1}{se(\hat{\alpha})} \qquad ADF\alpha = \frac{T(\hat{\alpha}-1)}{1-\hat{\beta}_1 - \dots \hat{\beta}_k}$$

[5] Alternatively $\Delta Y_t = C + \delta t + \theta Y_{t-1} + \sum_{j=1}^{K} \beta_j \Delta Y_{t-j} + \varepsilon_t$ can be used, where $\theta = \alpha - 1$ $ADF_t = \dfrac{\hat{\theta}}{se(\hat{\theta})}, ADF\alpha = \dfrac{T(\hat{\theta})}{1-\hat{\beta}_1 - \dots \hat{\beta}_k}$

ADF_t and ADF_α follow the same asymptotic distribution as the Dickey-Fuller tests with white noise error when lag order P is selected appropriately.

It is well-known that LS for autogressive (AR) suffers from serious downward bias in the persistence coefficient when the process includes deterministic. To see the bias, assume that the regression equations follow (3). By the Frisch-Lowell-Waugh theorem, estimating $\hat{\alpha}$ by OLS is equivalent to estimating the following regression with de-meaned terms.

$$Y_t - \overline{Y} = \alpha(Y_{t-1} - \overline{Y}) + \varepsilon_t$$

where $\overline{Y} = T^{-1}\sum_{j=1}^{T} Y_j$. We see that ε_t is correlated with Y_j , for *j=t, t+1,...,T,* thus it is also correlated with $\overline{Y}$. Therefore, the OLS estimator for the AR(1) process with an intercept creates a mean-bias. The bias has an analytical representation, and as Kendall (1954) shows, the OLS estimator is biased downward. It is known that correcting for bias may help enhancing the power of the test. In what follows, we demonstrate that this is also the case for the recursive mean and recursive trend adjusted versions of the ADF unit root tests.

2.2 Recursive mean adjusted (RMA) based ADF test (ADF_{RMA})

The RMA-based unit root test possesses greater power than an ADF_{ols} test. Due to reduced-bias estimation, the left percentile of the null distribution (of the test) shifts to the right, while the asymptotic distribution of RMA and the OLS estimator are identical under the alternative. This leads to an improvement in power over the ADF_{ols} (Shin and So, 2001). We will examine the principle behind the ADF_{RMA} test by reviewing recursive demeaning and detrending procedures.

2.2.1 Recursive mean adjusted based ADF unit root test (ADF_{RMA})

So and Shin (1999) originally introduced recursive mean adjustment in univariate autogression to reduce the small sample bias of the least square estimator, and Shin and So (2001) extended their recursive mean adjustment to a unit root test for the case of an unknown mean .

Shin and So (2001) introduced the concept of recursive mean adjustment by considering the following AR(1) model.

$$Y_t - \mu = \alpha(Y_{t-1} - \mu) + \varepsilon_t, \qquad t = 1,2,\cdots\cdots,T \tag{6}$$

where ε_t is zero mean stationary process. Shin and So (2001) note that when the absolute value of α is less than 1, because μ is unknown, therefore, μ can be replaced by the mean of Y_t

$$\overline{Y} = \frac{1}{T}\sum_{j=1}^{T} Y_j \tag{7}$$

Application of the ADF or DF test to the mean-adjusted observation ($Y_t - \overline{Y}$) is achieved using the following regression

$$Y_t - \overline{Y} = \alpha(Y_{t-1} - \overline{Y}) + \varepsilon_t \tag{8}$$

However, as Shin and So further note, replacing μ with $\overline{Y}$ in (6) leads to correlation between the regressor ($Y_{t-1} - \overline{Y}$) and ε_t. Denoting the OLS estimator as $\hat{\alpha}_0$, the resulting bias of $\hat{\alpha}_0$ has been derived by *inter alia*, Kendall(1954), Tanaka (1984) and Shaman and Stine (1988) as

$$E(\hat{\alpha}_0 - \alpha_0) = -T^{-1}(1 + 3\alpha) + o(T^{-1}) + \varepsilon_t \tag{9}$$

To overcome the problem of correlation between the error term and regressor, Shin and So (2001) propose the use of recursive mean, Y_{t-1}, using the partial mean instead of global mean.

$$\overline{Y}_{t-1} = \frac{1}{t-1}\sum_{i=1}^{t-1} Y_i \qquad t = 2,3,\cdots,T \tag{10}$$

Define $\widetilde{Y}_t = Y_t - \overline{Y}_{t-1}$, and $\widetilde{Y}_{t-1} = Y_{t-1} - \overline{Y}_{t-1}$. The recursive mean-adjusted version of (6) and (8) is then given as

$$\widetilde{Y}_t = \alpha \widetilde{Y}_{t-1} + \varepsilon_t \tag{11}$$

In a nutshell, the logic behind RMA estimator can be seen by defining, $\overline{Y}_{t-1} = \frac{1}{t-1}\sum_{i=1}^{t-1} Y_i$, so ε_t is uncorrelated with the recursive mean adjusted regressor $Y_{t-1} - \overline{Y}_{t-1}$, which results in substantial biased reduction for RMA estimator.

$$\hat{\alpha}_{RMA} = \frac{\sum_{t=2}^{T}(Y_t - \overline{Y}_{t-1})(Y_{t-1} - \overline{Y}_{t-1})}{\sum_{t=2}^{T}(Y_{t-1} - \overline{Y}_{t-1})^2} \tag{12}$$

Similarly, the extending the RMA estimation to higher order autoregressive process AR(p) (where p is greater than 1) is as:

$$\widetilde{Y}_t = \alpha \widetilde{Y}_{t-1} + \sum_{j=1}^{k} \beta_j \Delta Y_{t-j} + \varepsilon_t \tag{13}$$

$\hat{\alpha}_{RMA}$ can be obtained by regression (13). We control for nuisance parameters (β_j) by a method described in Kim et al (2010).

2.2.2 Recursive trend adjusted based ADF unit root tests(ADF$_{RTA}$)

Consider the following model: $Y_t = \gamma_0 + \gamma_1 T + \alpha Y_{t-1} + \varepsilon_t$ (14)

where ε_t is white noise, null hypothesis to be tested is H_0: α =1

The model of interest includes a constant and time trend so that the vector of deterministic variables considered is Z_t=(1,t)', with corresponding vector of parameters to be estimated,

(γ_0, γ_1). In order to consider the recursive trend adjustment, Shin and So (2001) took an OLS based approach whereby the vector of estimators of the deterministic component at time t is given by:

$$\tilde{\gamma}_t = (\sum_{k=1}^{t} Z_k Z_k')^{-1} \sum_{k=1}^{t} Z_k y_k \tag{15}$$

Thus, once the T by 2 vector of parameters of the deterministic component is estimated as in equation (16), following Shin and So (2001), the test regression can be set up using the following recursively adjusted variable,

$$\tilde{Y}_t = y_t - Z_{t-1}'\tilde{\gamma}_{t-1} \tag{16}$$

$$\tilde{Y}_{t-1} = y_{t-1} - Z_{t-1}'\tilde{\gamma}_{t-1} \tag{17}$$

As equations (17), (18) show, only the sample mean of the observations up to time t-1 is considered. Where $Z_{t-1}'\tilde{\gamma}_{t-1}$ is the mean value of recursively trend variable.

We have,

$$\tilde{Y}_t = \alpha \tilde{Y}_{t-1} + \varepsilon_t \tag{18}$$

And the relevant test statistic given as $\tau = \hat{\alpha} - 1 / se(\hat{\alpha})$, where $se(\hat{\alpha})$ is a standard error.

Remark: In order to account for potential autocorrelation, equation model (18) can be augmented with lags of depended variable as in the conventional Augmented DF (ADF) test as

$$\tilde{Y}_t = \alpha \tilde{Y}_{t-1} + \sum_{j=1}^{k} \beta_j \Delta Y_{t-j} + \varepsilon_t \tag{19}$$

see *inter, alia,* Shin and So (2001) and Taylor (2002). We also control for nuisance parameter following Kim et al (2010).

Chapter 3

Experimental Designs for Response Surface Methodology

3.1 Response surface literature review

The response surface methodology (RSM) is important in designing, formulating, developing and analyzing new scientific studies. It is also efficient in improving existing studies. In statistic, the response surface methodology explores the relationship between several explanatory variables and one or more response variables. The method was introduced by Box and Wilson (1951). Their main idea of RSM is to use a sequence of designed experiment to obtain an optimal response. Box and Wilson (1951) suggested using the second degree polynomial model to approximate the response variable. They acknowledged that this model is only an approximation, not exact, but such a model is easy to estimate and apply even when little is known about the process. Response surface methodology has been used in many fields of applied statistics (Myers, Khuri and Cater, 1989) since this method was introduced by Box and Wilson . Researchers have applied the RSM in econometrics fields in 1970.

Early studies that use the response surface methodology in econometrics include Hendry (1979), Hendry and Harrison (1974), and Hendry and Srba (1977); the references to later work were reviewed by Hendry(1984). Cheung and Lai (1993a) estimated finite-sample critical values for reduced-rank integration tests, Cheung and Lai (1995) estimated finite sample critical values for ADF tests by taking into account dependence on the lag order in addition to sample size.

3.2 The response surface methods and experimental design

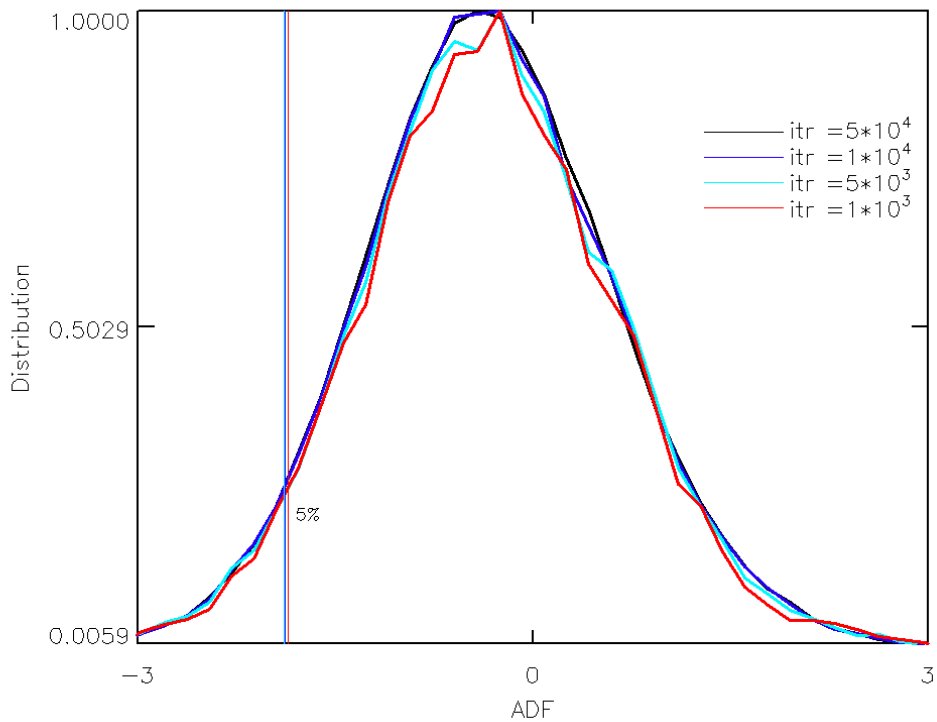


Figure 1 ADF distribution of ADF t-statistic

Note:: itr stands for number of iterations. T is the sample size and k is the lag order parameter. This distribution was obtained from case of T=100, K=0, with iterations are 1000, 5000, 10000 and 50000. The vertical line gives 5% critical values at different *itr*.

Response surface analysis applies to a system where the response of some variables depends on a set of other variables that can be controlled and measured in experiment. Simulations are conducted to evaluate the effect on the response variable of designed change in control variables. A response surface describing the response variables as a function of control variables is then estimated. When there are constraints on the design data, the experimental design has to meet the requirements of the constraints. In general, the response surface changes can be visualized graphically. The graph is helpful to see the shape of the response surface, hence, the function $f(x_1,x_2)$, where x_1,x_2 are control variables can be plotted versus the level of x_1 and x2. The three dimensional graph shows the response from the side is called response surface plot. Sometimes, however, it is easier to see the response surface in two-dimensional graphs, which our study will provide in addition to three dimensional graphs to show how response variables are affected by control variables.

In our analysis, the response variable is the finite sample critical value of the RMA based ADF (ADF_{RMA}), and the control variables are the sample size (T) and the lag order parameter (K). Our design covers 168 different pairing of (T,K), for which T varies from 50 to 700 with an increment 50, and K={0,1,2,3,4,5,6,7,8,9,10,11}. In each experiment for given (T,K), the 1 percent, 5 percent, and 10 percent critical value are computed as corresponding percentiles of the empirical finite-sample distribution based on a same number of iterations. (Figure 1 shows the distribution in the case of T=100 and K=0 with iteration numbers 1000, 5000, 10000, 50000. The vertical lines give the 5% critical value at different iterations. It is found that the distributions are almost saturated at iteration≥ 10000. Therefore, in the following simulation study, the iteration number is chosen 10000.

The data generating process considered in the simulation is a conventional random walk.

$$X_t = X_{t-1} + e_t \qquad (20)$$

where e_t is an independently distributed standard normal innovation. Sample series of X_t are generated by setting an initial value x_0 equal to zero, and creating T+50 observations, of which the first 50 observations are discarded to avoid the problem of initialization. The GAUSS programming language and subroutine RNDN are used to generate random normal innovations.

The regression model given by the equations in section 2.2 is more general than the DGP considered. Higher-order DGP's , for which e_t can be autocorrelated, is allowed for in our tests provided that the lag order parameter K is large enough to capture the dependence. Because if the lag order is too small relative to the true lag order, the error term e_t in the regression will no longer be white noise. In this case, RMA based ADF test can be seriously biased, making estimates of critical values inaccurate.

Selecting the functional form for response surface is not entirely arbitrary and need to be satisfied some restrictions. In our case here, intuitively, with a given sample size T, the choice of lag order parameter can effect on RMA base ADF test by determining the effective number of observation available and number of parameters to be estimated in the test. As the sample size increase to infinity, the effect of K on critical value may be diminished to zero. When sample size goes to infinity, the effect of sample size on critical value should also be diminished to zero. Taking these restrictions into consideration, we adopted the response surface polynomial equation by Cheng and Lai (1995a). This experimental design generalizes Mackinnon's (1991) by including lag order, but omits that nuisance parameter that e_t contains due to autocorrelation. The polynomial equation is the following:

$$CV_{T,K} = \tau_0 + \sum_{i=1}^{r} \tau_i \left(\frac{1}{T}\right)^i + \sum_{j=1}^{s} \varphi_j \left(\frac{K}{T}\right)^j + \varepsilon_t \qquad (21)$$

where $CV_{T,K}$ is the critical value estimate for sample size of T and lag order parameter K, ε_t is error term. r and s are respective polynomial orders for variables 1/T and K/T. The second summation term capture the incremental contribution from the lag order. It is obvious that K/T variable will be diminished to zero as value of T goes to infinity. Since both 1/T and K/T $\rightarrow 0$ as $T \rightarrow \infty$, the intercept term gives an estimate of asymptotic critical value.

In order to find the response surface equation that fits the data well, a range of different value of r and s have been considered, the test values to be found in next chapter.

Chapter 4

Monte Carlo Simulation Results

Considering different values of *r and s* (r = 1,2,1/2, s = 1,2,1/2) in estimating equation (21). For the critical value in the tests of the constant with a trend and without a trend model, It was found that data fits well at r=1 and s=1, the higher orders of polynomial term do not add much power to the explanatory variables. So, the response surface equation can be written as:

$$CV_{T,K} = \tau_0 + \tau_1(\frac{1}{T}) + \varphi_1(\frac{K}{T}) + \varepsilon_t \qquad (22)$$

Table 1 shows the results of response surface regression from equation (22). The tests with and without a trend are conducted at 1%, 5%, and 10% significance levels. (6 response surface regressions were run). τ_0 gives intercepts at three different significance levels ,which are very close to the asymptotic critical values that computed by Shin and So (2001) when the sample size is large. τ_1 is the coefficient of variable 1/T, ϕ_1 is the coefficient of variable K/T. Note that in both cases, variable 1/T showed to be statistically significant in all regressions at all three levels. K/T variable showed up to be even more statistically significant in all regressions at all three levels than the variable 1/T. This implies that the effect of lag order on critical values can be more sensitive than that of the sample size in the finite sample. In other words, lag order in addition to the sample size has a strong effect on finite sample critical values for RMA based test. Various measures of data fit are also computed, including goodness of fit, the standard error of regression and mean absolute error. The results in Table 1 show the ability of the response surface equation (22) to fit the data, not only the intercepts are close to asymptotic critical value, but also in the view of goodness of fit (R squares are high at all three significance level in all regressions), and in the views of standard error and mean absolute error(both measures of standard error and the mean absolute error are fairly small in all six regressions).

Table 1 Response surface estimation of Critical values for the ADF_{RMA} statistic

Coefficient & statistics	Constant no trend			Constant and trend		
	1%	5%	10%	1%	5%	10%
τ_0	-2.51033	-1.86617	-1.53426	-2.51148	-1.8583	-1.5118
	0.00877	0.00602	0.00476	0.00497	0.00387	0.00337
τ_1	4.41123	4.24757	4.13655	7.05625	6.7304	6.17495
	1.99228	1.36615	1.08037	1.12764	0.8787	0.76494
ϕ_1	-21.902	-13.7101	-10.4743	-32.536	-22.3816	-18.2898
	0.27332	0.18742	0.14822	0.1547	0.12055	0.10494
R^2	0.98859	0.98604	0.98479	0.99832	0.99781	0.9975
σ	0.08206	0.05627	0.0445	0.04645	0.03619	0.03151
Mean($\lvert\hat{e}\rvert$)	0.05901	0.0362	0.02924	0.03335	0.02393	0.01971
Max($\lvert\hat{e}\rvert$)	0.58802	0.41714	0.34164	0.15864	0.23551	0.22572

notes: The response surface regression is given by equation (22). The ADF_{RMA} , Corresponding heterorskedasticity-consistent standard errors for coefficient estimates are put in parentheses. δ represents the standard error of the regression. Mean($\lvert\hat{e}\rvert$) gives the mean absolute error of the response surface prediction against estimated critical value from simulations.

Some finite sample critical values were estimated by Shin and So (2001) for RMA based ADF unit root tests based on K=0. It is interesting to compare those estimates directly with the response estimate of critical values obtained here as displayed in Table 2. The estimates provided by Shin and So (2001) are given in the third column. The first column is sample size, The second column is the significance level (1%, 5%, and 10%). The last four columns contain response surface estimates for K=0, K=4, K=7, K=10. Not unexpectedly, when K equals zero, the two estimates are matched very closely. However, if we look at K=4, K=7 and K=10, it is evident that critical values obtained (5-7 column) are different from those obtained by Shin and So (2001). Note that differences in those estimates decrease as sample size increase. Therefore, if lag order is greater than one, using the asymptotic critical values that tabulated based on K=0 can be misleading, which causes one to reject nonstationarity too often.

Table 2a Lag Order and Finite-sample Critical Values

Sample Size	Sig. Level	SS Estimate	Constant no trend			
			K=0	K=4	K=7	K=10
50	10%	-1.54	-1.52061	-2.2064	-2.87791	-3.70069
	5%	-1.88	-1.87178	-2.8269	-3.57357	-4.71707
	1%	-2.57	-2.53395	-4.06249	-5.43865	-6.9005
100	10%	-1.54	-1.52744	-1.8864	-2.17908	-2.49887
	5%	-1.88	-1.89109	-2.3313	-2.76876	-3.11599
	1%	-2.54	-2.55762	-3.32239	-3.8541	-4.52167
250	10%	-1.54	-1.53696	-1.70023	-1.81685	-1.90924
	5%	-1.88	-1.85298	-2.0581	-2.24729	-2.35806
	1%	-2.53	-2.47415	-2.82888	-3.07332	-3.2957
500	10%	-1.54	-1.54083	-1.61355	-1.66143	-1.71053
	5%	-1.88	-1.86064	-1.95929	-2.04771	-2.07821
	1%	-2.53	-2.55082	-2.67263	-2.78071	-2.93049

note: The finite-sample critical values tabulated for the RMA based ADF test. The third column gives the estimated of critical values provided by Shin and so, their critical values are tabulated based on K=0.

By comparing, we note that when lag order is greater than 1 in finite samples, the RMA based test with asymptotic critical values can be oversized even when the sample size is fairly large (e.g., T=500). Table3 and Table 4 contain the size samples for 50, 100, 150, 200, 250, 300, 350, 400, 500 and 700. Lag order parameter for k=0, k=1,k=4, k=7, and k=10, for the constant with a trend and the constant without a trend case respectively.

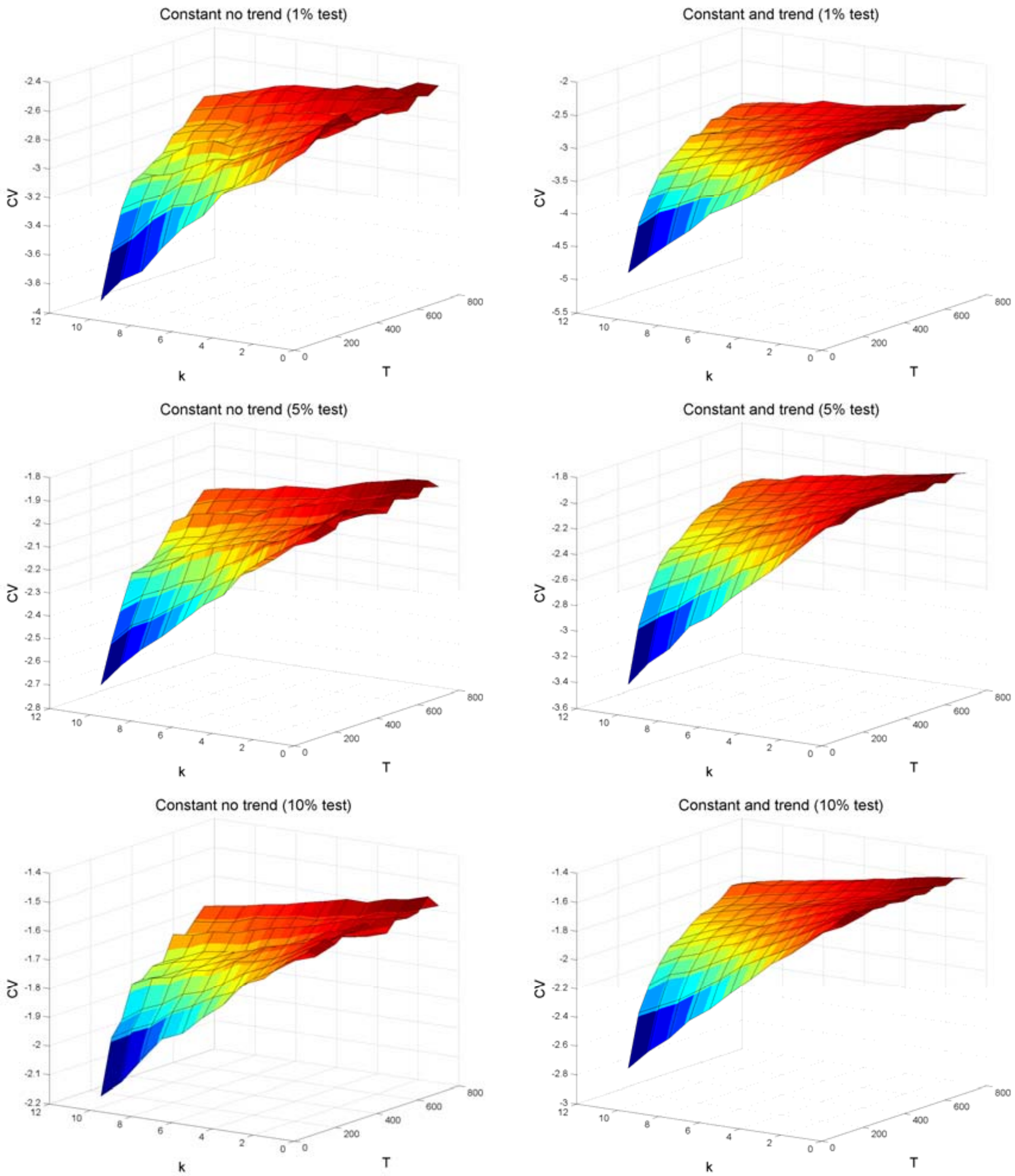
Table 3 Lag Order and Finite-sample Critical Values (with constant only)

Sample Size	Sig. Level	Constant no trend				
		K=0	K=1	K=4	K=7	K=10
50	10%	-1.52061	-1.71129	-2.2064	-2.87791	-3.70069
	5%	-1.87178	-2.09758	-2.8269	-3.57357	-4.71707
	1%	-2.53395	-2.91018	-4.06249	-5.43865	-6.9005
100	10%	-1.52744	-1.61213	-1.8864	-2.17908	-2.49887
	5%	-1.89109	-1.97353	-2.3313	-2.76876	-3.11599
	1%	-2.55762	-2.74225	-3.32239	-3.8541	-4.52167
150	10%	-1.53581	-1.59643	-1.73442	-1.93768	-2.13946
	5%	-1.88011	-1.95934	-2.14825	-2.40002	-2.63186
	1%	-2.51324	-2.71848	-3.01869	-3.37777	-3.80271
200	10%	-1.56443	-1.61965	-1.72762	-1.85436	-1.96858
	5%	-1.92177	-1.98652	-2.15246	-2.31493	-2.47407
	1%	-2.52997	-2.66364	-2.94306	-3.2297	-3.52086
250	10%	-1.53696	-1.58198	-1.70023	-1.81685	-1.90924
	5%	-1.85298	-1.91337	-2.0581	-2.24729	-2.35806
	1%	-2.47415	-2.56988	-2.82888	-3.07332	-3.2957
300	10%	-1.54128	-1.57486	-1.67971	-1.75265	-1.81263
	5%	-1.87465	-1.92166	-2.05973	-2.12496	-2.26618
	1%	-2.58249	-2.62431	-2.80629	-2.96733	-3.1465
350	10%	-1.52718	-1.5624	-1.64502	-1.73164	-1.81761
	5%	-1.87848	-1.91812	-2.03097	-2.14781	-2.23945
	1%	-2.54502	-2.65563	-2.80415	-3.0574	-3.17389
400	10%	-1.57436	-1.59626	-1.67024	-1.71469	-1.78129
	5%	-1.90003	-1.93776	-2.04227	-2.11551	-2.19669
	1%	-2.56224	-2.62814	-2.76469	-2.93333	-3.03889
500	10%	-1.54083	-1.5592	-1.61355	-1.66143	-1.71053
	5%	-1.86064	-1.89132	-1.95929	-2.04771	-2.07821
	1%	-2.55082	-2.60103	-2.67263	-2.78071	-2.93049
700	10%	-1.54923	-1.57325	-1.60717	-1.63943	-1.67771
	5%	-1.88772	-1.90553	-1.96961	-1.99999	-2.06051
	1%	-2.49949	-2.4997	-2.5837	-2.64767	-2.76336

Table 4 Lag Order and Finite-sample Critical Values (with constant and trend)

Sample Size	Sig. Level	Constant no trend				
		K=0	K=1	K=4	K=7	K=10
50	10%	-1.47205	-1.80006	-2.77986	-3.86395	-5.13153
	5%	-1.84173	-2.2253	-3.39948	-4.79138	-6.25507
	1%	-2.52754	-3.10357	-4.81799	-6.86265	-8.92842
100	10%	-1.49502	-1.66452	-2.15989	-2.71169	-3.27288
	5%	-1.84282	-2.06463	-2.65397	-3.34345	-4.01644
	1%	-2.5349	-2.86154	-3.73827	-4.79021	-5.74805
150	10%	-1.47004	-1.60452	-1.92896	-2.30694	-2.66329
	5%	-1.82663	-1.97507	-2.41763	-2.8466	-3.27923
	1%	-2.49265	-2.69174	-3.31183	-3.91968	-4.71777
200	10%	-1.53278	-1.62822	-1.8635	-2.1257	-2.37419
	5%	-1.84629	-1.99704	-2.30306	-2.61474	-2.9213
	1%	-2.4987	-2.66193	-3.11449	-3.68513	-4.04091
250	10%	-1.50982	-1.5751	-1.78759	-1.99093	-2.17857
	5%	-1.83116	-1.92892	-2.21428	-2.43789	-2.69948
	1%	-2.49963	-2.64609	-2.97453	-3.43644	-3.76817
300	10%	-1.51862	-1.5623	-1.73839	-1.9168	-2.07993
	5%	-1.85901	-1.93811	-2.13809	-2.34395	-2.53138
	1%	-2.47546	-2.58012	-2.88916	-3.25563	-3.53404
350	10%	-1.49331	-1.54098	-1.67846	-1.84522	-1.99903
	5%	-1.83554	-1.90016	-2.06524	-2.26431	-2.472
	1%	-2.473	-2.57033	-2.85746	-3.13251	-3.41309
400	10%	-1.52667	-1.56711	-1.69544	-1.84959	-1.95867
	5%	-1.85861	-1.92469	-2.09782	-2.26432	-2.41766
	1%	-2.55818	-2.61636	-2.84022	-3.11207	-3.33235
500	10%	-1.52286	-1.55684	-1.6599	-1.76309	-1.85708
	5%	-1.88443	-1.92506	-2.02402	-2.16568	-2.25709
	1%	-2.54	-2.63973	-2.75189	-2.94264	-3.17992
700	10%	-1.51037	-1.54111	-1.61852	-1.67488	-1.74792
	5%	-1.84951	-1.88258	-1.95933	-2.04298	-2.13594
	1%	-2.4927	-2.5482	-2.69819	-2.77213	-2.93323

Figure2 Plots of Monte Carlo-Estimated critical values



Note: figure 2 is estimated value for various RMA based ADF test, where T is the sample size, and K is the lag order parameter. In each graph, the vertical axis gives the Monte-Carlo estimated values corresponding to different combination of T and K

Finally, the Monte Carlo simulation critical values are plotted as Figure 2 for various ADF_{RMA} tests. Three dimensional graphs provide a sense of the numerical fluctuation in the critical values as a function of the lag order parameter K and sample size T. The 6 graphs are arranged in a 3 by 2 matrix to allow efficient comparison across types of tests and across the test sizes. To see how critical values are affected by sample size T and lag order K more clearly, we also provide two dimensional graphs (Figures 3 and 4).

Three dimensional graphs of Figure 2 show that the presence of a signed the correction to the asymptotic critical value at k=0 and $T \to \infty$. This is consistent with the fact that τ_1, which determines the effect of pure sample size, are positively signed in response surface. The effect of lags is unambiguously signed in all response surfaces. The graphs also show that critical values decrease (while absolute values increase) when k increases. Therefore, the test with asymptotic critical values is overall oversized. A comparison between graphs shows that the similar speeds at which finite-sample critical values approach the asymptotic levels.

Figure 3a and 3b are two dimensional graphs for the constant only case. The color bar in Figure 3a represents different number of k, which varies from 0 to 11. The color bar in Figure 3b represents different sample sizes, which varies from 50 to 700. By observing these graphs, we have found that (1) given a k, critical values increase as sample sizes increase and the signs are consistent with τ_1 in response surface. Finite-sample critical values converge gradually to their asymptotic critical value; (2) given a relatively small sample size, critical values linearly decrease (absolute critical values increase) as k increases, which causes the test with asymptotic critical values to be oversized. When sample size (T) goes to infinity, critical values with k and without k converge to the asymptotic critical value.

Figure 4a and 4b are 2-D plots for the constant with a trend case. It is clear that the pattern of critical value as a function of T and K is very similar to that of the constant only case. The asymptotic critical values for both time trend and no trend case are also nearly the same. However, the variations of critical value in time trend case is greater than that of no trend case, which means the test with asymptotic critical values in constant with time trend becomes more oversized.

In summery, based on those simulation results, we found that asymptotic critical values are valid only when lag order is one.

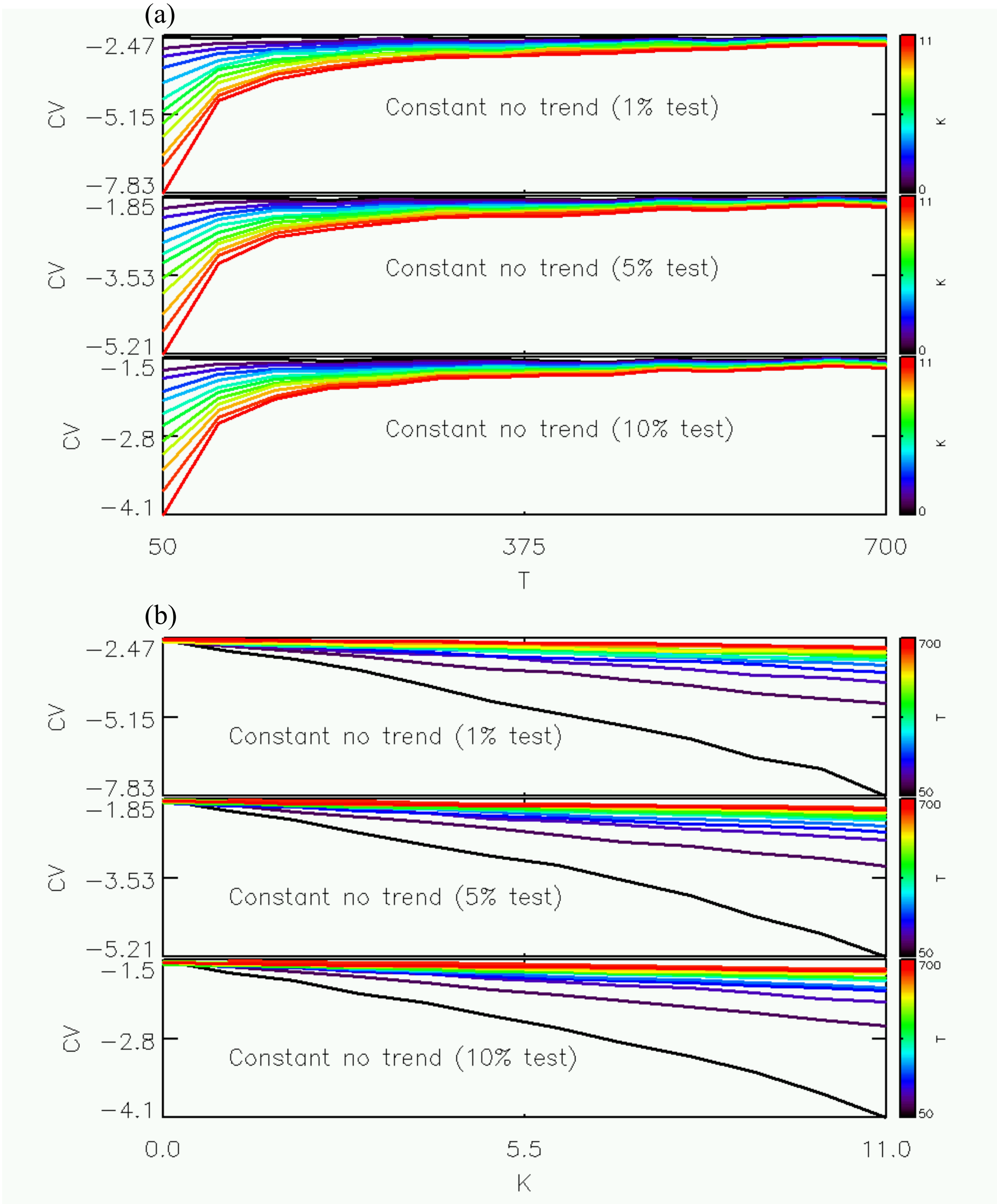


Figure 3 RMA based critical value as a function of (a) T and (b) K in the case of constant without time trend.

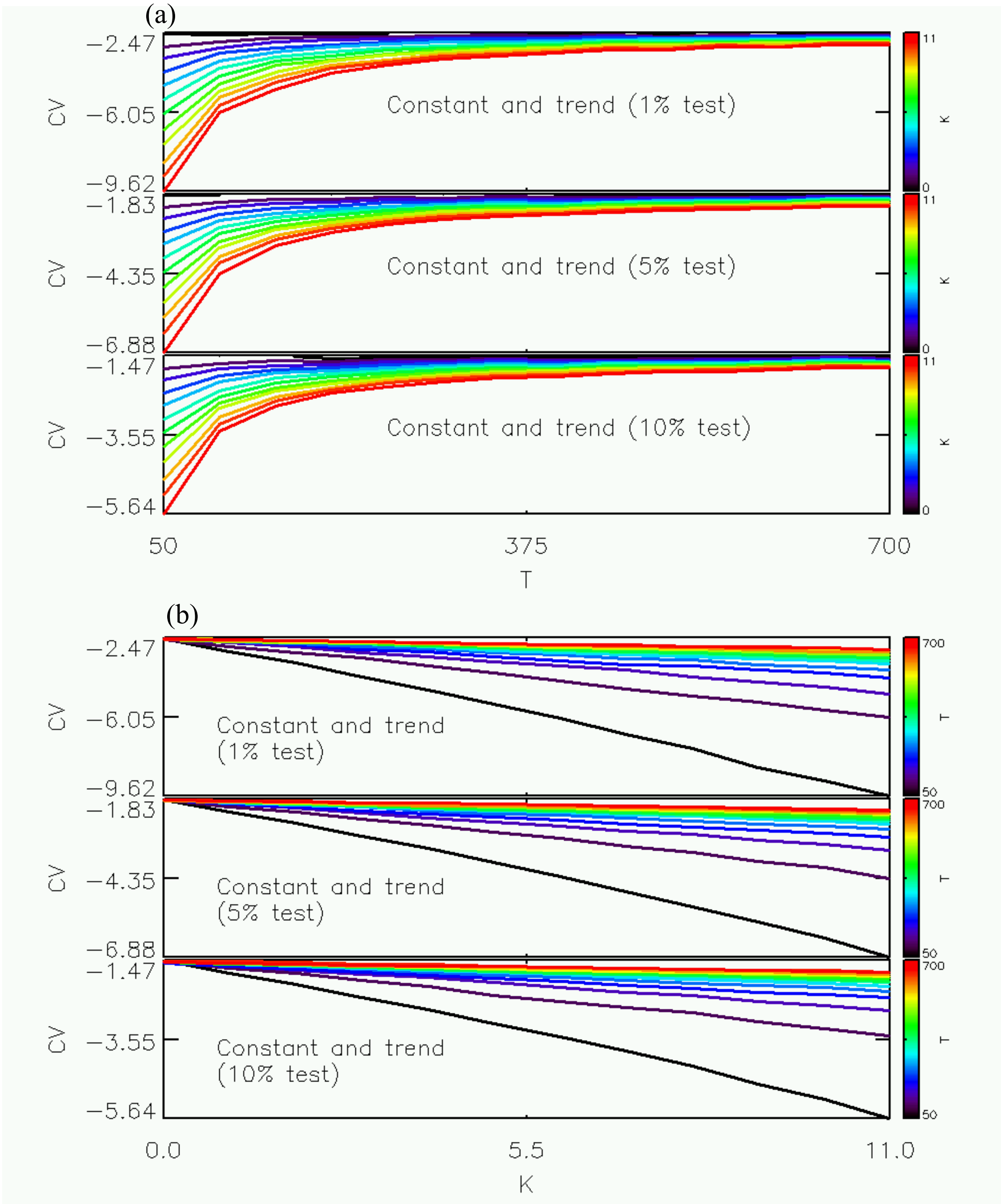


Figure 4 RMA based critical value as a function of (a) T and (b) K in the case of constant and time trend.

Chapter 5

Conclusion

Usually the practice of applying the RMA based unit root test has largely ignored the sensitivity of the lag order, which is often justified by asymptotic results that the limiting distribution of the test is free of the lag order. Even though the lag order may not affect the critical values when T goes to infinity, this practice may not be valid. Cheung and Lai (1995a) showed that critical values for the ordinary least square (OLS) based ADF test depend on the lag order in finite samples. We extend their work here by examining a more powerful RMA based ADF unit root test. Our Monte Carlo simulation results show that asymptotic critical values for the test are valid only when the lag order is one (k=0). When the lag order is greater than one, the RMA based unit root test with asymptotic critical values tends to be overall over-sized when the deterministic terms are allowed.

References

Box, E. P., J.S. Hunter, G. W. Hunter, 2005, Statistics for Experiments, New Jersey: John Willey and Sons, Lnc.

Blanchard,O. and L.H. Summers ,1987 Hysteresis and the European unemployment problem. NBER working paper series. Http://ssrn.com/abstract=227281

Balvers,R.,Y.Wu, and E.Gilliland, 2000, Mean recursive across national stock market and parametric contrarian investment strategies, Journal of Finance,55,745-772.

Cheung, Y. and K .S. Lai, 1995a, Lag order and critical value of augmented Dickey-Fuller test, Journal of Business and Economic Statistics, 13, 277-280.

Cheung, Y. and K.S. Lai, 1993, Finite–sample size of Johansen's likelihood ratio test for cointergration, Bulletin ,55,313-328

Cheng, Y., and K. S. Lai, 1995b, Lag order and critical value of a modified Dickey-Fuller test, Oxford Bulletin of Economics and Statistics, 57, 411-418.

Dickey, A.D., W. R. Bell, and R. B .Miller, 1986, Unit root in time series model: test and implications, The American Statistician, 40, 12-26

Dickey ,D. A. and W. A. Fuller, 1979, Distribution of the Estimator for autogressive time series with a unit root, Journal of American Statistical Association, 74, 427-31.

Eliott, G., T. J. Rothenberg and J. H. Stock,, 1992, Efficient test for an autogressive unit root , National Bureau of Economic Research Technical Working Paper, No,130.

Fuller, W. A., 1976, Introduction to Statistical Time Series, New York, John Wiley.

Hendry, D. F.,1979, The behavior of inconsistent variables estimator in dynamic systems with autocorrelated Error's, Journal of Econometric,9, 295-314.

Hendry,D.F and F. Srba, 1977, The properties of autoregressive instrumental variables estimator in dynamic system, Econometrica, 45, 969-990.

Hendry, D. F., 1984, Monte Carlo experimentation in econometric. In Grillliches,Z.and M.D.Intriligator(eds.), Handbook of Econometrics, 2,north-Holland,Amsterdam, 937-76.

Kendall, M. G, 1954: Note on bias in estimation of autocorrelation Biometrika, 41,403-404.

Kim, H., L. Stern, and M. L. Stern, 2010, Half-life bias correction and G7 stock market, Economics letters, forthcoming.

Kim, H, N. Durmaz, 2009, bias correction and out-of-sample .MPRA paper No, 16780, http: //mpra.ub.uni-muenchen.de, 16780

Mackinnion, J.G, 1991, Critical values for cointegration test, in Long Run Economic Relationship, New York: Oxford University Press ,266-276.

Myers, R.H., A.I. Khuri, and W. H. Carter, 1989, Response surface methodology 1966-1988, technimetrics, 31,.137-153.

Rodrigues, P, M. M., 2006, Properties of recursive trend-adjusted unit root test, Economic Letter, 91, 413-419.

Rogoff, K,1996, The purchasing power parity puzzle. Journal of Economic literature, 34, 2. 647-668

Said, E. S. and D. A. Dickey, 1984, Testing for unit root in autoregression –moving average Model of unknown order，Biometrika, 71, 599-607.

Shaman, P. and R. Sine, 1988, The bias of autoregressive coefficient estimators .J. Amer,statist Assoc., 83,842-848.

Shin D.W and B.S So, 2001, Recursive mean adjustment and test for unit roots, Journal of Time series Analysis, 22, 595-612.

Shin, D.W. and B. S. So, 2002, Recursive mean adjustment and test for nonstationarities, Economic Letters, 75, 203-208.

So, B. S and D.W. Shin, 1999, Recursive mean adjustment in time-series inferences, Statistic & Probability Letter, 43, 65-73.

Tanaka, K,1984, An asymptotic expansion associated with maximum likelihood estimators in ARMA Model. J. Roy Statist Soc., B 46(5), 58-67.

Taylor, R., 2002, Regression-based unit root test with recursive mean adjustment for seasonal and nonseasonal time series, Journal of Business and Economics Statistics, 20, 269-281.

Appendix: Outline for Generating Critical Values

1. Given T, generate N sets of random walk observations, where N=10000. Each series is generated by setting initial value equals zero, and creating T+50 observations, of which the first 50 observations are discarded to avoid problem of initialization. The GAUSS programing language and subrouinte are used to generate random normal innovations.

2. In the case of the constant with no trend, obtain recursively adjusted mean value by equation (10). For the case of constant with trend, equation (15) is calculated and the recursively adjusted trend mean value $Z_{t-1}'\tilde{\gamma}_{t-1}$ is thus obtained.

3. For each parameter K, test regression of equation (11) or (18) for k=0 and equation (13) or (19) for k>0 to estimate $\hat{\alpha}_{RMA}$. Then RMA based ADF t statistics are calculated.

4. From N statistic for each K and for RMA based ADF test statistic, obtain α % percentile, α =1, 5, 10, which gives α % critical value.