

**A MARKOV CHAIN DYNAMIC SOCIAL NETWORK MODELING AND SIMULATION TO ASSESS THE
IMPACT OF OUT-OF-SCHOOL SUSPENSIONS ON DROPOUTS AND TRUANCY**

by

Kim Y. Peters

A dissertation submitted to the Graduate Faculty of
Auburn University
in partial fulfillment of the
requirements for the Degree of
Doctor of Philosophy

Auburn, Alabama
August 9, 2010

Keywords: Stochastic Process, Markov Chain, Modeling, Social Networking, Learning
Coefficient, Dynamic Connectivity, Student Dropouts, Truancy, Juvenile Delinquency

Copyright 2010 by Kim Y. Peters

Approved by

Jorge Valenzuela, Chair, Associate Professor of Industrial and Systems Engineering
Chan S. Park, Samuel Ginn Distinguished Professor of Industrial and Systems Engineering
Robert Thomas, Emeritus Professor of Industrial and Systems Engineering

ABSTRACT

Generally, a social network is mainly concerned with actors, the actions of actors, and the interactivities among actors in a community. Employing graph theory to represent overall social network structure, this study develops a Student School Attendance System (SSAS) evaluation model to provide a frame work in order to understand and assess the impacts of unsupervised Out-of-School (OOS) Suspensions on dropout, truancy, and the overall attendance performance of students and proposes a new attendance category, supervised out-of-school suspension. In recent years, there has been a dramatic increase in juvenile delinquency and student dropout rates. This research examines the impacts of a current OOS suspension program which is devoid of any form of supervision or monitoring system. In brief, the school transfers the problem student from the school to the street without any support mechanism. To examine and assess OOS, this research develops a model that represents current student attendance states paradigms considering student interaction by assimilating learning coefficients and threshold values for connectivity between peers. The study further examines student attendance for supervised versus unsupervised OOS.

This research employs a Stochastic Markov Chain Process that considers the random events of the student school attendance phenomenon. The SSAS model represents a comprehensive state diagram of student school attendance categories based on graph theory and defines relevant parameters and variables. The model process begins with formulation of the initial transition probability and calculation of cumulative and steady state probabilities. The

student school attendance states are determined based on random numbers and ranges of cumulative probability distributions applying the Markov Chain Memory-Less property. The next day student attendance probability computation considers the influence of friends by integrating a threshold value for dynamic connectivity and the fitness of the learning coefficient as the interaction influential factor among students.

In addition, the SSAS model explores in three ways the development of the initial transition probability. The first model employs a pure random process by generating the initial transition probability of the student school attendance in a random selection scheme. The second model provides a controlling mechanism by providing the initial transition probability distribution for student school attendance via a pre-arranged input value data table. The third model provides a controlling mechanism as well as the stability in the system by implementing a pre-arranged output value look-up table which is formulated based on attendance data to generate the initial transition probability. Furthermore, the model also exploits the connectivity between the peers in three different ways: Median Connectivity, Virtual Connectivity, and Hybrid Connectivity.

As a result of the SSAS model simulation, the Pre-Arranged Output Value model generated the most stable system among all three models with 1.4% error. In terms of connectivity, Virtual Connectivity produced the most connections between peers. In the SSAS model student school attendance assessment there was about 22% improvement in the number of students promoted to the next grade/graduated from high school under supervised OOS suspensions in comparison to unsupervised OOS suspensions. Among three types of connectivity, Virtual Connectivity produced the most improvement in the number of students

succeeding at school. In conclusion, the SSAS model successfully rejects the null hypothesis “Implementation of Supervised Out-of-School Suspensions will have no effect on students’ attendance and graduation rate.”

ACKNOWLEDGMENTS

It is my privilege to acknowledge the contributions in directing this research mainly to Dr. Jorge Valenzuela who is Chair for providing the fundamental framework of the Stochastic Process Markov Chain modeling techniques, Operations Management, and Information Technology, Dr. Chan S. Park for assisting in quality assurance management and risk analysis in Economic Engineering, Dr. Robert Thomas for supporting the overall perspectives of Human Behavioral Theory of social networking and relevant issues involved in Ergonomics, and the Dr. Cheryl Seals who served as an outsider reader for my dissertation. My sincere appreciation extends to Dr. Alice Smith, the Department Chair who has been advocator and motivator throughout my graduate study.

I have the utmost gratitude and admiration to my husband, Judge Bobby G. Peters, Superior Court of Chattahoochee Circuit, Columbus Georgia, for his unconditional love and encouragement in all aspects. In addition, my ultimate source of strength, my sons David and Jonathan for their love and understanding during my study.

TABLE OF CONTENTS

ABSTRACT	ii
ACKNOWLEDGMENTS	v
LIST OF TABLES	x
LIST OF FIGURES	xi
LIST OF ABBREVIATIONS	xii
1. INTRODUCTION	1
1.1. Background	4
1.2. Motivation	5
2. STATEMENT OF RESEARCH OBJECTIVES	8
2.1. Goals and Problem Statement	9
2.2. Objectives and Contributions	10
3. LITERATURE REVIEW	13
3.1. Introduction	13
3.2. Social Networking	14
3.3. Basic Graph Theory	17
3.4. Stochastic Procedures	19
3.4.1. Property of Random Event Process	20
3.4.2. Stochastic Process and Exponential Distribution	20
3.5. Markov Chain Phenomenon	21

3.5.1.	Characteristics of the Markov Chain	22
3.5.2.	The Long-Run Behavior of Finite-State Markov Chain	23
3.6.	Chapter Remarks and Summary	25
4.	MODEL FORMULATION	27
4.1.	Introduction	27
4.2.	Model Assumptions	29
4.3.	Student School Attendance Model	31
4.3.1.	Background Introduction	31
4.3.2.	State Diagram of the Student Attendance Paradigms	33
4.3.3.	State Probability Distributions	34
4.4.	The Student Interaction Model	36
4.4.1.	The Graph Representation of Student Interactions	37
4.4.2.	Assessment of the Interaction between Students	39
4.4.2.1.	Initial Student Attendance State Probabilities	39
4.4.2.2.	Distribution of the Student Attendance States	39
4.4.2.3.	Interactivity of the Students and Learning Coefficient	40
4.4.3.	Three Dynamic Modes of the Connectivity between the Peers	41
4.4.4.	Interaction Probability Computation	43
4.4.4.1.	Initial Student Attendance State Probabilities	43
4.4.4.2.	Determination of the Student Attendance States	43
4.5.	The Student School Attendance Simulation (SSAS) Model	44
4.5.1.	Introduction	44

4.5.2. The Model	45
4.6. Chapter Remarks and Summary	53
5. METHODS AND PROCEDURES OF SSAS MODEL	55
5.1. Introduction	55
5.2. Procedures, Algorithms, and Model Validations	57
5.2.1. Development of Initial Transition Probability Distribution Table	57
5.2.2. Cumulative Probability and the Range of Attendance States	60
5.2.3. Transition from Current State to Next State	63
5.2.4. Formation of the Connectivity between the Peers	65
5.2.5. Interaction Probability Computation	70
5.4. Chapter Remarks and Summary	72
6. THREE DIFFERENT TYPES OF THE SSAS MODEL	74
6.1. Introduction	74
6.2. Pure Input Output Value (PIOV) Dynamic Model	75
6.2.1. Procedures and Methods of PIOV Model	76
6.2.2. Algorithms of PIOV Model	77
6.3. Pre-Arranged Input Value (PAIV) Dynamic Model	78
6.3.1. Procedures and Methods of PAIV Model	79
6.3.2. Algorithms of PAIV Model	80
6.4. Pre-Arranged Output Value (PAOV) Model	82
6.4.1. Procedures and Methods of PAOV Model	82
6.4.2. Algorithms of PAOV Model	85
6.5. Chapter Remarks and Summary	87

7. EXPLOITATION RESULTS AND ANALYSIS	90
7.1. Introduction	90
7.2. Simulation Results of the SSAS Models	90
7.3. The Connectivity in between Three Types of Connections	93
7.4. Attendance Assessment between Supervised and Unsupervised OOS	96
7.5. Correlation of Supervised and Unsupervised OOS and Dropouts	99
7.6. Chapter Remarks and Summary	100
8. CONCLUSION AND FUTURE RESEARCH	102
8.1. Remarks and Recommendations	103
8.2. Future Research	104
REFERENCES	108
Appendix 1. THE SSAS MATLAB CODE	111
Appendix 2. THE ATTENDANCE STATES OF THE STUDENTS	121

LIST OF TABLES

Table 1	The Matrix Representation of the Relationships of 6 Students	38
Table 2	Procedures of Transition Probability Computation	49
Table 3	Example of Input Data Table for the Initial Transition Probability Distribution	58
Table 4	Transition Probability Distribution of the Attendance States for Student 2	59
Table 5	Examples of Cumulative Probability Distribution Table	60
Table 6	Cumulative Probability Distribution Table for Student 1 and 2	61
Table 7	The Current State and Next State Allocations	64
Table 8	The Rationale in Formulation of the Connectivity among the Students	66
Table 9	Pre-Arranged Output Probability Distribution Look-Up Table.....	83
Table 10	Simulation Results of the SSAS Models	92
Table 11	Measurements of the Errors and Correctness of the SSAS Models	92
Table 12	The Number of Connectivity in a Five-Day Interval for School Year	94
Table 13	Student Attendance Assessment Using Median Connectivity	97
Table 14	Student Attendance Assessment Using Virtual Connectivity	97
Table 15	Student Attendance Assessment Using Hybrid Connectivity	98
Table 16	The Difference in Number of Student Graduates in Three Types of Connectivity.	100

LIST OF FIGURES

Figure 1	Examples of Undirected Graphs of Social Network	18
Figure 2	Overview of Inputs, Outputs, and Parameters of the SSAS	28
Figure 3	State Diagram Representation of Student School Attendances	33
Figure 4	Undirected Graph Representation of 6 Students and their Relationships	38
Figure 5	Procedures of the Next Day Transition Probability Computation	50
Figure 6	The Pseudocode in Determining the Attendance States	65
Figure 7	Initial Iteration for the Connectivity among 6 Students	67
Figure 8	Pseudocode for Median Connectivity	69
Figure 9	Flow Chart in Computing the Interaction Probability among the Students	71
Figure 10	Process Overview of SSAS Model	75
Figure 11	Procedures of the PIOV SSAS Model	76
Figure 12	Pseudocode of the PIOV Model	78
Figure 13	Procedures of the PAIV SSAS Model	79
Figure 14	Pseudocode of the PAIV Model	81
Figure 15	Procedures of the PAOV SSAS Model	84
Figure 16	Pseudocode of the PAOV Model	86
Figure 17	Comparison of the PIOV, PAIV, and PAOV Models	88
Figure 18	The Average Number of Connectivity in SSAS Models	96
Figure 19	The Number of Student Graduates of Supervised and Unsupervised OOS	99

List of Abbreviations

E (Exe)	Excused Absence
HC	Hybrid Connectivity
ITP	Initial Transition Probability
IP	Interaction Probability
IS	In-School Suspension
MC	Median Connectivity
MCP	Markov Chain Process
MCS	Monte Carlo Simulation
MCMCS	Markov Chain Monte Carlo Simulation
OS	Out-of-School Suspension
PAIV	Pre-Arranged Input Value
PAOV	Pre-Arranged Output Value
PIOV	Pure Input Output Value
SOS	Out-of-School Suspension with Supervision
SSAS	Student School Attendance System
SSP	Steady State Probability
T (Tru)	Truancy, unexcused absence
TP	Transition Probability
VC	Virtual Connectivity

1. INTRODUCTION

Nowadays, cutting edge technology and the pervasiveness and ubiquitous access to the Internet affect all aspects of our society. The younger generation in particular is sculptured with new revolutionary technologies, impetuously establishing a new era of culture. Quite often, it seems having our young children in-tune with their cell phone or I-pod in a general public place is a part of our every day life style. This new life style of our younger generation brings enormous ongoing challenges to parents, educators, and lawmakers. Daily, each school district faces the immense task of trying to meet those challenges. Overwhelmingly, in spite of efforts to accommodate those challenges, according to the national statistics, the student attendance rate in middle school and high school is at an all time low [37, 40]. Moreover, in recent years, there has been a dramatic increase in juvenile crime and the high school dropout rate [38]. In response to the alarming statistical data and intricate issues surrounding this young population, the U.S. Department of Justice established the Office of Juvenile Justice and Delinquency Prevention (OJJDP). Since 1974, it has provided a support system to decrease the incidence of juvenile delinquency of young students [38]. The main mission of the OJJDP is to provide various resources to protect children and prevent juvenile delinquency. The OJJDP website reads in part: “OJJDP supports states and communities in their efforts to develop and implement effective and coordinated prevention and intervention programs and to improve the juvenile justice system so that it protects public safety, holds offenders accountable, and provides treatment and rehabilitative services tailored to the needs of juveniles and their families” [38]. In addition,

national data based on the Statistical Briefing Book (SBB) reflects the magnitude of the problem and the seriousness related to juvenile crime [39]. According to the SBB report, in the year 2004, there were 1,571,230 juvenile related crimes and 1,593,661 for year 2005. The SBB data synopsis indicates that among the age group from 12 to 17 the most prevalent age for juvenile crime is at age 16, which accounted for approximately 394,000 crimes, the quarter percentile of the total number. Furthermore, the data from the Department of Education shows the national level for the high school graduation rate is at a record low, which is around 73 percent [40, 41]. The high school graduation rate for the State of Georgia and the City of Columbus, Muscogee County is around 70 percent [35, 36]. Consequently, across the board, each school district faces the monumental responsibility to improve student behavior and academic performance of the students in an increasingly complex and multi-faceted school environment.

Subsequently, those students identified within the at-risk ages of 12 to 17, were found to have a high drop-out rate, low achievement in school performance, and a high incidence of juvenile delinquency. This realization is evidenced by the SBB statistics, where the enormous numbers of young children are committing crimes, and fail to graduate from high school according to the Department of Education. The negative impact of students' unsuccessful school experience and the consequences of students' not attending class extend much further than merely failing the class or missing instruction. A student's not attending class could result in his or her becoming a dropout and increase in the possibility of involvement in juvenile delinquency. This research examines the problem associated with the current out-of-school suspension program. In general, the current out-of-school suspension program does not require monitoring of the out-of-school suspended student to ascertain whether or not he or she keeps up with school assignments or the class work during out-of-school suspension period [36]. Out-of-school suspension is usually

considered an unexcused absence and the student is not entitled to “make up” the missed grades during the suspension period. Therefore, school is neither obligated nor mandated to provide academic materials to the students who are subject to out-of-school suspension. Consequently, these at-risk students fall further behind in school as a result of an out-of-school suspension. To examine the correlation between the high dropout rates and the number of out-of-school suspensions, this research develops a model that describes the various student attendance states and identifies relevant parameters and variables. In addition, the model integrates the threshold value for the connectivity and learning coefficient in accommodation of the interactions with their peers during the school day.

Comprehensively, this research examines the complicated dynamic student school attendance phenomenon in regard to interactions among peer paradigms. The main objective of this research is to develop an evaluation model for student school attendance performance considering the social interactions with their friends at the school. The model distinguishes student attendance categories and employs a stochastic process with the Markov Chain’s memory-less property assuming that the possibility of the student’s coming to school tomorrow is solely based on today’s activity. The interactions of the students with their peers are modeled using social network and graph theory. Overall, this research presents a Student School Attendance System (SSAS) model which can be used to evaluate the student school attendance paradigms.

In particular, this research focuses on assessment of the impacts of the current out-of-school suspension program which is devoid of any form of supervision or monitoring program by the school system. Currently, the school transfers the problem student from the school to the street

without any support or monitoring mechanism. To address the inevitable and yet often ignored catastrophic problem resulting from suspending the student from school without any support system, the underlying impact must be identified and assessed. The main premise of the research in order to address the complexity of the student school social network is: the state of the student school attendance and the interactions with their peers in accommodation of the influential factor.

The subsequent chapters are organized as follows: Chapter 2 presents the statement of research objectives. Chapter 3 includes a review of the literature and back ground study. Chapter 4 introduces the formal representation of the SSAS model. Chapter 5 explains the methods and procedures of the model. Chapter 6 illustrates the three different types of the SSAS models. Chapter 7 provides the results and the data analysis. Chapter 8 draws the conclusions and future research areas.

1.1. Background

In January 2009, the City of Columbus, Georgia, established the “Mayor’s Commission on Crime Prevention” in order to establish a baseline to reduce the crime rate with the goal of making the city a safer place. The main initiatives of the commission were outlined by the Chief Executive of the City to research the underlying issues associated with criminal activity, study the possible enhancement of existing programs currently addressing these issues, and evaluate new alternatives or new systems to reduce and/or prevent deviant and illegal activity throughout the community. Members of the commission represented various segments of the city and had diverse backgrounds and experiences, such as a notable State Representative in the General Assembly, bankers, lawyers, educators, pastors, sports director, social workers, and law

enforcement officers. Such diversity allowed the commission to be more creative and to openly brainstorm ideas and suggestions in order to reach a consensus on which ideas and objectives should be adopted to meet the main goal to “reduce the crime rate in the city.” The Mayor mandated total cooperation from the Police Chief, the Sheriff, and the Marshall. Each department provided factual statistical data to map out the areas of the city with the highest crime rates, identify the most frequent times for particular crimes, and demographics of the offenders. The Commission discovered that the age groups most at-risk in criminal activity are young offenders between ages 16 to 23, with most deviant behavior being committed after school hours between 3:00pm and 2:00am. Specifically, age 16 was the prevalent age of the offender. Based on these statistics, each commission member was charged to submit a proposal and initiative that would address the underlying causes of juvenile crimes and reduce the opportunities for young people to become involved in criminal activity. Based on these statistics, the essential attention in juvenile crime is substantial because one way to prevent future adult offenders is to prevent juvenile crimes.

1.2. Motivation

As a member of the Mayor’s Commission on Crime Prevention, and in response to the Mayor’s charge to the commission, I evaluated and submitted a proposal to aim toward the high-risk group, which is closely linked to truancy and is also a high indicator of juvenile delinquency. This group involves students enrolled in the public school system who are subject to discipline through the “Out-Of-School Suspension Program.” These students, subjected to out-of-school suspension, are usually marginal students to begin with. In most cases, they are simply sent home without supervision or any structured follow-up procedures by the school system. The

student attendance data from the Muscogee County School District in Columbus, Georgia, provided the appalling realization that there were about 28,000 out-of-school suspension student days that took place during year 2008 between the months of January through December. Of 180 student school days, the maximum acceptable absentee days a student can miss per year is 15 [36]. The out-of-school suspension period is considered as an absence albeit an involuntary action of the student not coming to school. In general, the privilege of coming to school is taken from these students usually due to their misconduct. When the student reaches the magic number of 15 absences, he or she fails the grade and is unable to be promoted. The statistics from the School District of Muscogee County also indicated that the most out-of-school suspensions took place with students enrolled in the 9th grade level at ages 15 and 16. This is the transition period from middle to high school where a young person is most vulnerable. These data correlate with national statistics, SBB of juvenile delinquency where the most crime took place at age 16 [39]. As a result, the Commission, School Board, and Mayor's Office became very alarmed and motivated to provide a reinforcement system for the problems associated with the out-of-school suspensions.

Generally, students who are subjected to out-of-school suspensions are at home without supervision possibly wandering around the neighborhood or local mall, become idle, bored, and easily influenced by others on the street. Perhaps, they may join gangs, become unruly at home, run away, and eventually may commit deviant behavior or other serious juvenile offenses. Nowadays, 47 percent of our children live within a single parent home according to Families and Lives by the U.S. Census Bureau [41]. Many of these children living with a single parent are left alone and vulnerable to mischievous conduct since the mother or father is working elsewhere for long hours to support the family instead of being home with the children. In addition, some

parents lack parenting skills or the education to make sound decisions, especially for those impressionable at-risk youth, much less make decisions for a whole family [41]. Regrettably, there is a lack of mechanism to examine the living situation or follow-up the progress of the students who are subjected to the current out-of-school suspension program. Therefore, the students are sent home or to the street with no evaluation of their living environment or the conditions thereof, or any method to measure the impacts of such suspensions on their future school performance, or means to measure the percentage of juvenile delinquency cases while on out-of-school suspensions.

Considering the current 70 percent high school graduation rate in the state of Georgia and the staggering number of the days in out-of-school suspension, the underlying impact of Out-Of-School suspension is detrimental [36]. How many of those students just give up and never come back or fall so far behind in their studies that they fail to graduate? In basic terms, the 70 percent high school graduation rate reflects the 30 percent failure rate, and the loss of 30 percent of the resources [36]. This overwhelming number of a 30 percent high school failure rate causes apprehension for parents, educators, and lawmakers. In society as a whole, the responsibility lies upon everyone to identify the causes of such a high percentage of student school dropout rate and juvenile delinquency. It is everyone's obligation to examine the problem and endeavor to find solutions to maximize the student school success level and reduce the student population committing crimes at an early age, thereby preventing them from circulating through the judicial system for the rest of their lives.

2. STATEMENT OF RESEARCH OBJECTIVES

School absenteeism is detrimental to students' self-esteem and reduces the opportunity for graduation or being promoted to the next grade. It significantly affects their overall achievement in their personal and professional endeavors, in addition to hampering their future employment potential. Subsequently, students will fall behind their peers in the classroom if they continue to miss school voluntarily or involuntarily such as a truancy or out-of-school suspension. Each absence could affect the students unconstructively damaging both their grade point average and their self-esteem. In addition, such a situation could result in an antagonistic environment due to lack of self-esteem or could cause the students to be skeptical about self-worth which will increase the likelihood that students may eventually give up, drop out, and/or be expelled from school entirely. The current school system is neither mandated nor required to evaluate the consequences or the impacts on the students due to out-of-school suspensions. Therefore, the main theme of this research is to develop a model that can be used to evaluate the impacts of the current out-of-school suspension program.

The model examines student school attendance patterns to assess the impacts of out-of-school suspensions in relation to the dropout rate and overall student failure rate. This model can be used as an evaluation tool to examine and investigate the overall impacts of the existing out-of-school suspension system and identify a more positive direction which would promote the success of the student in the future. Furthermore, studying the impacts of out-of-school suspensions is necessary and vital for the juvenile criminal justice system also, since there are so

many young students on the street during the school day without any supervision, possibly as a result of the existing OOS suspension program. Many of those students suspended may have committed crimes during their out-of-school suspension period. The model can estimate and measure the associated impacts of the out-of-school suspension in an attempt to improve the program for the young population and to reduce juvenile delinquency and/or enhance the rehabilitation program for young offenders. This research develops the evaluation model that measures the impacts of unsupervised out-of-school suspensions in terms of student attendance and success at school in order to examine the ongoing challenges of having high dropout rates and criminal activities among the young population.

2.1. Goals and Problem Statement

The main goal of this research is to develop a student school attendance model in an attempt to examine the impact of out-of-school suspensions, for the existing suspension program in particular. The model can be used as a decision support system to a) measure the student school attendance paradigms such as the in-school suspensions, out-of-school suspensions, and truancy and b) evaluate the relationship between the out-of-school suspensions, the dropout rate, graduation rate, and truancy. The students with high absentee records will most likely fail to graduate and dissipate all the resources allocated for that child. The ultimate goal of this dissertation is to provide a tool that can be used to evaluate the student school attendance performances and overall impacts of out-of-school suspensions. In addition, the output of the model can be used to encourage programs that reduce the student failure rate and the number of juvenile delinquents, and subsequently guide the students toward a positive and productive future at an early age.

2.2. Objectives and Contributions

The objective of this research is to develop an evaluation model which can be used to quantify student school attendance paradigms and thereby examine the student failure rate as a result of out-of-school suspensions and the related impact that may negatively affect society as a whole. Significantly, the outcome of this research can be used to help prevent at-risk students from falling into the exclusive category of low achievers and high involvement in criminal activity which may lead them to become life-long repeat offenders circulating through the criminal justice system. The main purpose for developing SSAS is to respond to the need for analytical tools that can be used to evaluate student school attendance paradigms and therefore facilitate amendments to the malfunctioning programs and, hopefully, to reduce the number of our young student population that may become victims of this unproductive system.

An impact on one child can have an impact on the whole. Providing a practical model to evaluate the student school attendance paradigms would be a significant asset for school administrators in amending programs to promote a higher graduation rate, ensure a successful school year, enhance education, and facilitate a brighter future for the child. The fundamental objectives of this research include:

- 1) Develop a simulation model that represents the student school attendance phenomenon and the interrelationships of attendance states which include: in-school suspensions, out-of-school suspensions without supervision, out-of-school suspensions with supervision, truancy, and excused absences. The main purpose of this model is to simulate and quantify student school attendance categories and assess the performance correlation

between attendance states during the school year. The associated attendance states are quantified through a transition probability matrix which is used in estimating the distribution of the student attendance states. Each student school attendance state is represented by probabilities associated with student school attendance paradigms.

- 2) Model the student interactions with their friends based on ad hoc mobile networking concepts. The model considers the random environment and the influence among the students in the development process. The friendships are exhibited as the connections between the students, and the influences of the peers are implemented through a learning coefficient. The model provides a learning coefficient to integrate the influential factor resulting from the student interactions. The dynamics of the connections between the students are assimilated based on a random number and threshold value.
- 3) Evaluate the effects of supervised out-of-school suspensions and unsupervised out-of-school suspensions, and their impacts on dropout rates and overall student school attendances. The SSAS model examines and analyzes the overall student dropout rate due to a result of the unsupervised out-of-school suspensions. This model can assist school officials and community leaders in measuring and assessing the relationship between supervised out-of-school suspensions and unsupervised out-of-school suspensions in terms of the student attendance rate and high school graduation rate. The model can assist in re-evaluating the current suspension programs, particularly the out-of-school suspension program, which may contribute to the high student drop out rate, truancy, or juvenile criminal activity.

4) Verify and validate the model. Sensitivity analysis and heuristic testing are used to verify the procedures and validate the data. The verification method for this research employs the normal standard error procedure. To validate the model, this research applies heuristic normal testing which compares the simulation results with actual student attendance data from the Muscogee County School District, Columbus Georgia. The model validation and verification process considers a) the student school attendance information such as normal attendances, in-school-suspensions, out-of-school suspensions, unexcused and excused absences, and make up days if any, b) the internal dropout rates, and c) the high school graduation rate.

3. LITERATURE REVIEW

3.1. Introduction

The social network represents the interactive behavioral structure between numerous actors such as individuals, organizations, or other social entities. The social network involves interactions through friendships, kinships, professional associates, and many others. Generally, the social network studies actors and their interactivities and relationships in a given community. The network is equivalent to complexity theory and systems theory which are interdisciplinary areas where many traditional fields correspond. Examples include: a) anthropology that accentuates social relationships, b) mathematics which provides structured algorithms and c) psychology which concentrates on our social structure based on an individual's perception [21]. The typical social network analysis is the study of social structure and its effects. It conceives social structure as a social network, that is, a set of actors (nodes) and a set of relationships (edges) connecting pairs of these actors. Its basic premise is that knowledge about the structure of social relationships enriches explanations based only on knowledge about the attributes of actors. Overall, the social network consists of two major components: actors and relationships between them. In order to understand the social network, many researchers have studied a pattern of social relationships in a graphical representation to analyze and quantify them. The graph or mathematical representation was used to better understand and evaluate behavioral patterns of a given community. The actors are described by the node which also represents various entities such as groups, individuals, or organizations. The edge between the nodes, which is the line that

connects the node, represents the relationship between the actors where it can describe dependence, information exchange platform, control, cooperation, and competition [6, 21, 30].

The students' school attendance performances are the result of daily influences from multiple factors. Whether the student comes to school each day is a purely random and unpredictable discrete event. Events, whether the student comes to school and is subject to suspension in-school or out-of-school, are purely discrete outcomes. This research considers the unique properties of the random events and employs the stochastic process which effectively represents the characteristics of randomness in the given environment. In brief, stochastic is a synonym for random which involves probabilistic behavior and conjecture or guesswork. The process of stochastic is to index the unknown parameters into real numbers using a probability distribution. Furthermore, the possibility of the student coming to school tomorrow is based on today's activities. Therefore, this study applies the Markov Chain memory-less process to present the independence from past history of student school attendance paradigms during the school year. The attempt to evaluate student school attendance is merely based on guess work, especially concerning the complexity that is associated with influences from peers. This chapter introduces the relevant topics in modeling techniques for the student school attendance phenomenon of the SSAS. The topics included in this chapter are: 1) social networking, 2) a brief description of graph theory, 3) the stochastic process, and 4) the Markov Chain memory-less properties.

3.2. Social Networking

Social network theory is based on the intuitive notion that the patterns of social interactions are essential elements to each individual who exhibits certain behavior. It can be stated that the influence and behavior of an individual heavily depends on how he or she interacts with others

and how each is tied to a specific social network. In addition to individual behavior, network theorists believe that the success or failure of societies or organizations often depends on the internal interaction patterns among actors. Besides the theoretical essence, social network theory is also characterized as a distinctive methodology encircling the pattern of certain behavior [6]. Hence, the social network encompasses the world which consists of actors and the connections between them. The actors can be represented as nodes and the connectivity from one node to another is the relationship between them. This connectivity between the actors and the ties between the nodes represent a type of relationship such as kinship or friendship between the actors, and they describe a particular pattern of action that an actor performs. Subsequently, the different types of ties will generate different degrees of social networks and represent the different types of functionality [19, 21].

The study of social networking begins with an understanding of the structure which is the way the society is organized and mapped into rather predictable relationships. Mainly, the aspect of a social structure considers three components: a) actors, b) the actions of the actor, and c) the interactions of the actors. Social actors represent the static concept relatively with emphasis on issues such as position, roles, and status. The actors, as individuals, are intermingled in a particular social platform, hence, their actions and activities are heavily influenced by the connections among them. Generally, social interaction can be defined as the way in which people respond to one another. To some degree, the patterns of these interactions are independent of the individual who creates the pattern of certain behavior [19, 21, 30].

In the past, most research on social interaction mainly focused on how individuals actually communicated with one another in a given specific environment. These studies concentrate on

such issues as the patterns of interaction, the rules and ethics to guide the interaction, the motivation for the way of interaction, and the effect of those interaction patterns which impact the behavior as an individual and the performance as the group. These issues begin by questioning what might be the interaction pattern in a specific social setting, thereby addressing one of the main themes of this research: understanding the dynamics of social interaction [19].

Social network activity analysis incorporates the social dimensions and the challenges of the adequate behavioral underlying theory. The paradigm of the social network mainly concerns understanding on how social structures facilitate and constrain opportunities, behaviors, and cognitions. Therefore, it involves designing and implementing a reliable and quantifiable data collection method, and performing an informative analysis of the main behavioral elements [6]. Theoretically, the core values of social networks are the attributes and the structural knowledge of social interrelationships and extended rationalizations solely based on information about the characteristics of actors. Behavior is defined and addressed by attributes, constraints, and opportunities which have a personal as well as a social aspect. Hence, the structure of social characteristics which represents the interactivities among the members in a given social network is the core source of addressing behavioral patterns.

Furthermore, the social network analysis consists of the methods and techniques for data collection, mathematical and statistical analysis, and a visual and graphical description. Usually, this process is referred to as the social network analysis, an essential procedure when studying the cohesiveness of the social network. Since the main characteristic of social activity constitutes the social dimension, an analysis of its performance is based on the vitality of the personal interactions and the one-on-one connections among actors. The complementary

perspective element for the social network analysis extends to the attractiveness between the actors at their destined social activity patterns. Conceptually, this interaction among the actors generates the accessibility to one another embedding the influential characteristics. There are intrinsic challenges in terms of the social network analysis due to unspecified social dimensions and intangible human behavior. The stochastic process effectively formalizes the social interactions by its unique properties; therefore, the behavior or the certain structure of a particular social network can be quantified and assessed [19, 21, 29].

3.3. Basic Graph Theory

In order to understand the social network, many researchers studied patterns of social relationships in a graphical representation to analyze and quantify. The graph or mathematical representation was used to better understand and forecast such patterns for unknown futuristic probability. In most cases, the social network is described in either a directed or undirected graph. Using the graph, $G (Node, Edge)$ the node represents the social context or community which has the embedded entities. The edge represents the contact, interaction, influence, or communication from one entity to another entity within the given community. In turn, social scientists study and analyze the corresponding graph to forecast and understand the social patterns, property, and behavior of the community. The size of the social network, therefore, can be determined by the number of nodes which can be very large representing a massive community such as the Internet or very few representing a small circle of friends. Each social network also has its own property embedded in the node that represents the application area or the specific type of the social community. The first application of digraph to describe the social community was proposed by Leinhardt in 1977. This idea of graphical representation became

popular in other research areas such as e-learning modelers and network designers [19, 29, 30, 34].

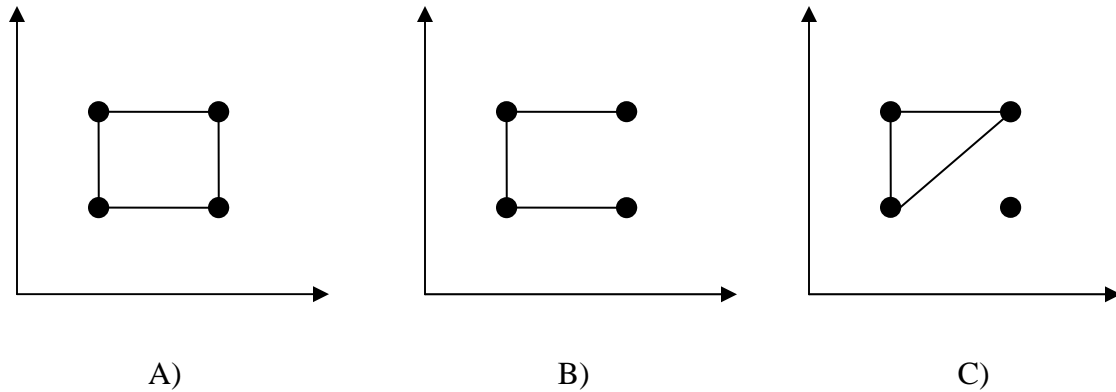


Figure 1. Examples of Undirected Graphs of Social Network: A) Complete connections, B) Partial connections, and C) Hybrid connections

Overall, a social network graphical representation provides specific fundamental structural properties which distinguish it from other types of networks or graphs. Designing the graph leads to an understanding of real life in a community which in turn can be used to study the correlation and overall changes in a given period. The graphs included in Figure 1 are the basic types of graphs which describe the connectivity among the nodes and the characteristics of the given community through the line which reflects the linkage between the nodes: A) represents the community that has complete ties among the actors, B) represents partial connections and the partial separations between the actors, and C) the actor completely isolated from the community. Graph C features the isolated node which could be considered in a situation where a new member enters into a community social network. In this case, as presented in the graph, an actor may not have any connection within the community and is noted as an isolated node.

By examining these structural properties of the graph, the overall patterns of the particular social network can be analyzed and evaluated. Most often, the graph theory can be used to compare the two distinct communities to predict and study the better performance. One of the main properties is the type of connectedness between the actors such as centrality of actors having dense degree, connectivity between the pair indicating reciprocity, links between the source and actor stating reach-ability, and the sub-division of actor having a strongly connected component. Mitra presented the extensive form structural definition and the study on cycles and nested cycles in his paper “Design of a Data Model for Social Network Application.” Mitra stated that to study the degree of cohesiveness in a community, the sub-division of actors into a cycle of nested cycles is very vital for a structure of the community because the size, number, and connections among the sub-division in a community are key factors for understanding the characteristics of the network as a whole and their likely behavior. In the paper “The dynamics of social balance,” Moody stated, “While we revel in the diversity of social settings, a primary motivation for social theory is to explain common features across settings and account for social differentiation endogenously” [20]. Generally, social scientists focus on the connectivity of the network and how the structure is molded into smaller groups over an identified period. This fundamental idea has been applied to other settings including the referral system and web community [21].

3.4. Stochastic Procedures

Stochastic is a synonym for random which involves probabilistic behavior and conjecture of guesswork for the entity or the system. Therefore, the stochastic process is known as a random process that deals with possibilities and probabilities to study how future process might be

revolutionized in a given time using a probability distribution. Application of statistical probability models has been widely used by many researchers to understand the nature of social network structures since the late 1930s [33]. The stochastic statistical model has been very popular as a research method in computer science and the fields of engineering, healthcare, and social study areas because of its unique characteristic which can be used to simulate the performance of a dynamic system that is subject to uncertainty.

3.4.1. Property of Random Event Process

By definition, Heyman stated “A stochastic process is a collection of random variables that are all defined on the same probability space and indexed by a real parameter.” The similar random variables that are defined in a given sample space are collected over time in stochastic process. Stochastic process is formally described as $\{X(t); t \in T\}$, where the $X(t)$ is X -process, the valued random variables at time t which indicates time parameter and T indicates the range of times parameter set. When T is finite or countable, T represents discrete-time stochastic process with $\{0, 1, \dots\}$; otherwise T is continuous-time stochastic process with $[0, \infty)$ and the formal description is $\{X(t), t \geq 0\}$. State space, S , takes all the possible values of the random variable that the $X(t)$ can undertake. The process is discrete stochastic when S contains countable finite numbers $S = \{0, 1, \dots, i\}$ and the T is a set of integers X_1, X_2, \dots, X_n ; otherwise the process is continuous stochastic. If $X(t) = i$ for any time $t \in T$ and for any state $i \in S$, then the process is in state i at time t [2, 9, 13, 18, 22]. The detailed properties of discrete and continuous stochastic processes are outlined in the following sections.

3.4.2. Stochastic Process and Exponential Distribution

The student attendance events, whether or not the student coming to school, are discrete events. In order to represent the school attendance state of a student over a given period, a Markov Chain is suitable to model the process.

3.5. Markov Chain Phenomenon

Markov Chain was developed by a Russian mathematician Andrey Andreyevich Markov who is well known for work on the theory of stochastic processes. Markov Chain method has been widely used because it satisfies the property of a linear equation system which is amenable to numerical solution methods based on the finite and limited probability distributions [4]. A discrete Markov Chain is a stochastic process with a discrete state and discrete time that has a probability distribution of the next state of the process which depends solely on the last state of the process. Therefore, the future state only depends on the current state and the process does not consider the past history. The stochastic process $\{X(t); t \in T\}$ is a Markov process with given integer i with time intervals of T where $t_1 < t_2 < \dots < t_n < t_{n+1}$, and the set of states S where the formal representation is,

$$Pr(X_{n+1} = x / X_n = x_n, \dots, X_1 = n_1) = Pr(X_{n+1} = x / X_n = x_n).$$

The sequence of random variables $X_1, X_2, X_3, \dots, X_n$ is represented with the Markov unique property where the given present state along with the future and past states are independent of each other since the past history of the process is completely summarized by the present state. In other words, the student's attendance tomorrow solely depends on today's activity and is not compounded by a whole week's or month's activities. The set of state space can be formulated by the possible values of S_i which also can be described through graph theory applying the

probability from one state to the other state on the edge [1, 13]. Markov Chains with time-homogeneous transition probabilities are formally defined by

$$Pr (X_{n+1} = x / X_n = y) = Pr (X_n = x / X_{n-1} = y) \text{ for all } n.$$

3.5.1. Characteristics of the Markov Chain

The Markov Chain process provides unique properties that enable the classification of the states which include irreducibility, periodicity, and ergodicity. The irreducibility of the Markov Chain Process is where the state space is a communicating component. In an irreducible Markov Chain, for example, one can get to any state from any other state. If a process started in state i that has some chance of probability to move into state j , then the state j is accessible from a state i ($i \rightarrow j$) formally $Pr (X_n = j / X_0 = i) = p^{(n)}_{ij} > 0$ where $n \geq 0$. For some instances, if $n=0$, every state is accessible from itself where one can move into all other states. These phenomena provide the ability for a state i to communicate with state j and if every pair of states in a given sample domain communicates with each other, the states are called the communicating class. There are three properties for communicating states: a) a self-communicate where $P^0_{ij} = P\{X_0 = i / X_0 = i\} = 1$; b) if state i communicates with j , then j also communicates with state i where the exchange of information is possible from one to another; c) if state i communicates with j and j communicates with k , then i communicates with k also creating triangular connectivity and information exchange platform.

The state could be a recurrent state if the process can go back to the original state. For example, if state i is accessible to state j and j is also accessible to i where the process can go back to state i , then state i is a recurrent state that the process can revisit. However, the state is transient if state i is accessible to j but j is not accessible to i where the process can not go back to state i ,

and state i is transient because the process can not be revisited. The formal representation is to set T_i as the random variable that first return time to state i then, $T_i = \inf \{ n \geq 1: X_n = i / X_0 = i \}$ with $Pr(T_i = \infty) > 0$, state i is the transient state. The state is called an absorbing state if the process never leaves the state. The state is recurrent if $\sum_{n=0}^{\infty} p^{(n)}_{ii} = \infty$ and if state i is unable to leave this state, then the state i is an absorbing state *iff* $p_{ii} = 1$ and $p_{ij} = 0$ for $i \neq j$. In any instance, every state is either transient or recurrent and not all states can be transient in the finite-state Markov Chain process. In the finite-state Markov Chain, if all states are irreducible then all the states are also recurrent. The sufficient scenario for the Markov Chain is to have all states to be accessible to each other communicating where $p_{ij} > 0$ for all i and j . However in the real world there is no guarantee such a system exists.

A Markov Chain also provides the periodicity where state i has period t if any return to state i takes place in t time steps. Periodicity is formally represented as $t = gcd \{ n: Pr (X_n = i / X_0 = i) > 0 \}$ where *gcd* refers to “greatest common denominator.” In a given period t , the process may be unable to reach the state in t steps, thus the resulting state is called aperiodic which occurs when $t=1$, otherwise the state is called periodic where $t > 1$. Every state within a communicating class should have the same period. Ergodicity occurs when state i is aperiodic and positively recurrent. The finite state irreducible Markov Chain is ergodic if the states are aperiodic [1, 2, 18, 22, 31].

3.5.2. The Long-Run Behavior of Finite-State Markov Chain

In addition to those properties presented in the previous section, Markov Chain provides another significant feature, steady state probability. Remarkably, the Markov Chain process produces an

identical probability distribution after a large number of iterations. This is also called limiting probability where the system remains in a certain state after a large number of iterations which is independent of the initial probability, p_0 . The unique property of a long-run finite state of Markov Chain is formally stated as

$$\check{\delta}_j = \sum_{i=0}^M \check{\delta}_i p_{ij} \quad \text{for } j = 0, 1, \dots, M \quad \text{where } \sum_{j=0}^M \check{\delta}_j = 1.$$

Here $\check{\delta}_j$ are the Markov Chain steady-state probabilities which are also called the stationary probabilities. After a rather large number of iterations of the transition probability, the system will be in particular state producing the identical probability distributions, $\check{\delta}_j$ that are independent of the initial transition probability distribution.

The steady state has $M + 2$ equations for $M + 1$ unknowns in order to satisfy the unique solution. Having one extra equation, the system can utilize it as redundant which can be disregarded leaving the final equation to produce the appropriate probability distribution. A simple illustration of steady state equation for four unknowns using five equations can be expressed as follows:

$$\begin{aligned} \check{\delta}_0 &= \check{\delta}_0 p_{00} + \check{\delta}_1 p_{10} + \check{\delta}_2 p_{20} + \check{\delta}_3 p_{30} \\ \check{\delta}_1 &= \check{\delta}_0 p_{01} + \check{\delta}_1 p_{11} + \check{\delta}_2 p_{21} + \check{\delta}_3 p_{31} \\ \check{\delta}_2 &= \check{\delta}_0 p_{02} + \check{\delta}_1 p_{12} + \check{\delta}_2 p_{22} + \check{\delta}_3 p_{32} \\ \check{\delta}_3 &= \check{\delta}_0 p_{03} + \check{\delta}_1 p_{13} + \check{\delta}_2 p_{23} + \check{\delta}_3 p_{33} \\ 1 &= \check{\delta}_0 + \check{\delta}_1 + \check{\delta}_2 + \check{\delta}_3 \end{aligned}$$

In this example, the $\check{\delta}_0$, $\check{\delta}_1$, $\check{\delta}_2$, and $\check{\delta}_3$ are the four unknowns having five equations. Solving the above equations, one can obtain the values for $\check{\delta}_0$, $\check{\delta}_1$, $\check{\delta}_2$, and $\check{\delta}_3$ rather easily. One important aspect of the steady-state probability is that if i and j are a recurrent state but belong to different

classes, the probability of finding the process in state j is zero; formally $p^{(n)}_{ij} = 0$ for all n [18, 22, 33].

The steady state probability δ_j can be used to compute the long-run average value per unit of time called expected average value per given unit. If there is the value function X associated with a given example, the expected average value can be computed by using the equation

$$E(x) = \sum_{i=0}^M \delta_j X(j).$$

For more complicated problems where the state associated with a value that is also affected by the random variables, then the equation must consider an additional parameter y where $X(A_{t-1}, S_i)$, A_{t-1} represent the number of items, t for day, and S is the sale demand, for example. The total value for day t is a function of A_{t-1} and S where $X(A_{t-1}, S_i)$. Expected value can be computed by using the following equation:

$$E(x) = \sum_{i=0}^M y_j X(j) \quad \text{where } y_j = E[X(j, S_t)].$$

Therefore, the central concentration of the given distribution can be measured by computing the expectation value $E(x)$ to evaluate and analyze the problem and understand the characteristics for the given domain or system [9, 10].

3.6. Chapter Remarks and Summary

In a social network, actors represent the static concept with emphasis on issues such as position, roles, and status relatively. The actors are intermingled in the social network setting and their actions and activities are heavily influenced by the characteristics of the connections and the density of connectivity among them. Generally, social interaction can be identified as the way in which people respond to one another and it describes the characteristics of influence upon their

neighbors. The dynamics of the interaction among the actors also needs to be considered, albeit the environment encompasses randomness, by means of probability distribution with a learning coefficient fitness value. Nowadays, the social network represents every day life from the small circle of colleagues to the large group web community where there is a massive multiplayer group thereby forming the connections together.

A stochastic process can effectively model a social network over time since it comprehends the randomness of the system. Generally, a stochastic process describes the social network through a mathematical representation formally. The attendance state whether the student comes to school or not, is affected by random events. Therefore, a stochastic process can be used to represent student school attendance over a given period. The connections between actors are also affected by random events. The connectivity can be integrated dynamically based on a fitness value and a random number. The stochastic Markov Chain considers the discrete state and discrete time processing through linearly enabling remarkable properties such as irreducibility, periodicity, and ergodicity. The Markov Chain process is one of the most powerful tools available to construct a model that consists of a discrete stochastic process. The Markov Chain properties are very useful because they could yield results for both the steady state of the system and time dependent evolution of the system. Therefore, the Markov Chain process effectively assimilates the random variation for student school attendance states. Overall, this chapter provides supporting materials in rationalization of the modeling techniques for the student school attendance phenomenon.

4. MODEL FORMULATION

In brief, the SSAS model considers the complexity of the social network and attempts to describe the student school social network; it applies the properties of graph theory to represent the school attendance states of the student. The modeling procedure considers the random environment and therefore employs the stochastic characteristics and the memory-less property of the Markov Chain to assimilate the student school attendance paradigms. In essence, the model rationalizes the random environment and the intricate patterns of the student-attending-school phenomenon. This chapter provides the modeling techniques and the formal representations of the student school attendance paradigms that include: the model assumptions, parameters and variables, attendance state diagrams, mathematical representations, computation method for the student attendance probabilities, dynamic connectivity, and the interactivities of the students.

4.1. Introduction

In the development of the SSAS, the procedure mainly concentrated on two areas: a) the student attendance state and b) the interactions with their peers. This research investigates the out-of-school suspension state closely and further categorizes it into two distinct states: a) supervised out-of-school suspensions and b) unsupervised out-of-school suspensions. Therefore, the SSAS model provides six different students' school attendance classifications: 1) normal class attendance, 2) in-school suspension, 3) supervised out-of-school suspension, 4) unsupervised out-of-school suspension, 5) truancy the unexcused absence, and 6) an excused absence. Figure

2 below describes the general overview of associated input or output values and parameters of SSAS model.

In order to model the multi-dimensional student school network, the SSAS considers undirected edge where the direction of the influence is disregarded among the peers.

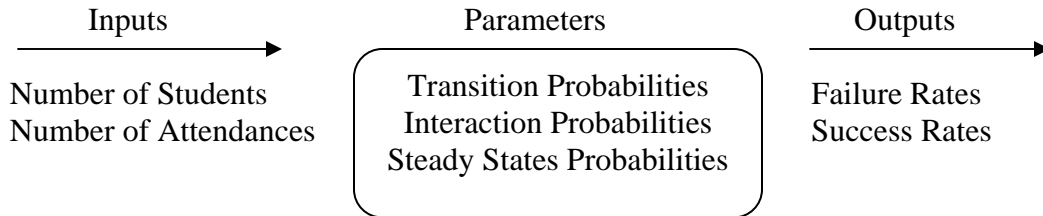


Figure 2. Overview of Inputs, Outputs, and Parameters of the SSAS

The model assumption in the subsequent chapter introduces the detailed reasoning behind this concept. Applying the graph theory, the model presents nodes which describe each student with his or her own attendance probability distributions coupled with learning coefficient and threshold value for connectivity. The nodes are able to communicate with each other in the school social network, since the students are not prohibited from having contacts with their peers at their school, and therefore they have knowledge about each other. Friendships are described in the model as the connections between the students.

Furthermore, the SSAS considers a learning coefficient in realization of the influences through friendships among the students that may affect the overall behavior of the student including school attendance. The SSAS model employs peers' influences by providing a fitness value for the learning coefficient. The initial probability distributions of the student school attendance are either provided or generated according to the input or the output probability distribution table. In order to update the next day probability of the student attendance, the model calculates the next

day attendance probability based on four factors: 1) the student's current probability, 2) the student's learning coefficient, 3) the connected peers' probability, and 4) the learning coefficient of the connected friends. Subsequently, the next day student attendance state is constructed based on the memory-less property of the Markov Chain applying today's student attendance state as tomorrow's student attendance state.

4.2. Model Assumptions

A multi-faceted social network encapsulates perplexing human behavior and therefore it is very difficult to understand since the subject matter is intangible, whether it represents a small circle of friends or the large Web community. In an attempt to understand and assess the student school attendance paradigms, this research selects two main areas to overcome the complexity of the student school social network: a) the state of the student school attendance and b) the interaction with their peers accommodating the peers' influential factor. In general, young children are more prone to develop friendships with other children without considering the other person's class, per se. They tend to look at other people as to who they are and not what they are. The status and the background of the person seem not to matter. This can be seen easily on any play ground or in any nursery. Most of the time, young children will play with other children who are physically closer to them regardless of their status. In comparison, adults typically pursue a relationship based on a personal agenda or some type of motivation for a connection with others, mostly for their own benefit, comfort, similarity, and many other reasons. Therefore, there will be an apparent direction between the relationships whether it is one-way or both-ways and this direction can be described either positively or negatively. However, generally, young children do not have an agenda in mind for pursuing a relationship.

Young children have a tendency to develop relationships with others in the surrounding area and can be easily influenced by those people who are closer to them, especially from closer friends. The direction of the relationship, therefore whether one way or both ways or negative or positive, is rather ambiguous. For this rationalization, the development of SSAS applies the undirected graph theory in describing the relationship between peers as presented in Figure 1. Nonetheless, the relationship between the school attendance states, the direction from one attendance state to the other, can be easily identified as to whether or not a student can go from normal class state to in-school-suspension or vice versa. Therefore, the SSAS applies the directed graph theory to describe the relationship of the student school attendance states and undirected graph theory for the connectivity with their peers. The directed graph procedure between the attendance states is presented in section 4.3.1.

As introduced previously, in order to model the state of student school attendance paradigms and the random nature of the friendships among peers, this research employs the Markov Chain process to generate the discrete attendance states and integrate learning coefficients to accommodate the influence among peers. The model also employs the random number with a threshold value in connection range to determine whether or not connectivity exists between the peers. Overall, the development procedure of the SSAS model considers the unique characteristics and properties of the Stochastic Markov Chain Process and assumes the following:

- At the first day ($t=1$), every student starts at the current state S_0 .
- The student can only be at one state at a time considering the discrete state and discrete time events.
- The length of student's attendance or absence from the class is mutually independent.

- After an absence, the student returns to the normal class with a certain probability.
- The true underlying process of the length of student absences is exponentially distributed.
- The connectivity between the nodes is determined based on probability distribution and the learning coefficient.
- The student school attendance states are governed by a Markov Chain.
- The students are connected with their peers through an undirected graph.
- The connectivity between students is determined based on probability distributions and the learning coefficient.
- The student school attendance state and the interactions with their peers are the main premise.

4.3. Student School Attendance Model

4.3.1. Background Introduction

Most of the current school systems categorize student school attendance and absentee characteristics into five areas or classifications: 1) normal class attendance, 2) in-school suspension, 3) out-of-school suspension, 4) unexcused absence, and 5) excused absence. The unexcused absence represents truancy where the student willfully misses school without any valid reason or excuse. As presented, the SSAS model further indentifies student school attendance categories into six classifications (S_i), dividing the current out-of-school suspensions into two areas: supervised and unsupervised. The SSAS model classifies a total of six student school attendance states as:

S_0 : student with a normal in-class status (Normal),

- S_1 : student with in-school suspension (IS),
- S_2 : student with out-of-school suspension with supervision (SOS),
- S_3 : student with out-of-school suspension without supervision (OS),
- S_4 : student with unexcused absence (Truancy, T), and
- S_5 : student with an excused absence (Exe, E).

The existing out-of-school suspension program removes from the student the privilege of coming to school without any support mechanism for their academic work or follow-up procedure by the school or affiliations. Apparently, the students subjected to out-of-school suspension will fall further behind under the current suspension program. In an attempt to evaluate the impacts of this apparent problem, the SSAS model distinguishes between the unsupervised out-of-school suspensions (OS) and proposes a new attendance category, the supervised out-of-school suspensions (SOS).

On any given school day, an ordinary student has the possibility of being absent from school, either voluntarily or involuntarily. In general, the most typical involuntary absence or excused absence could be the result of natural causes such as sickness, personal issues, field trips with the class, a death in the family, and many others. Conventional and habitual causes of voluntary absences, students who have unexcused absences, might include students trying to avoid school work, peer pressure concerning their appearance or relationships, conflicts with a teacher, or simple laziness, which all fall into the category of truancy, being absent from school without a valid excuse. The SSAS provides a state diagram in the following section that describes the interrelationships of the student attendance states presenting the six distinct categories of student school attendance paradigms.

4.3.2. State Transition Diagram of the Student Attendance Paradigms

In the graphical representation in Figure 3, the six numbers of nodes indicate six possible student attendance states adapting the five states from the current school system and employing a new proposed state, the supervised out-of-school suspensions.

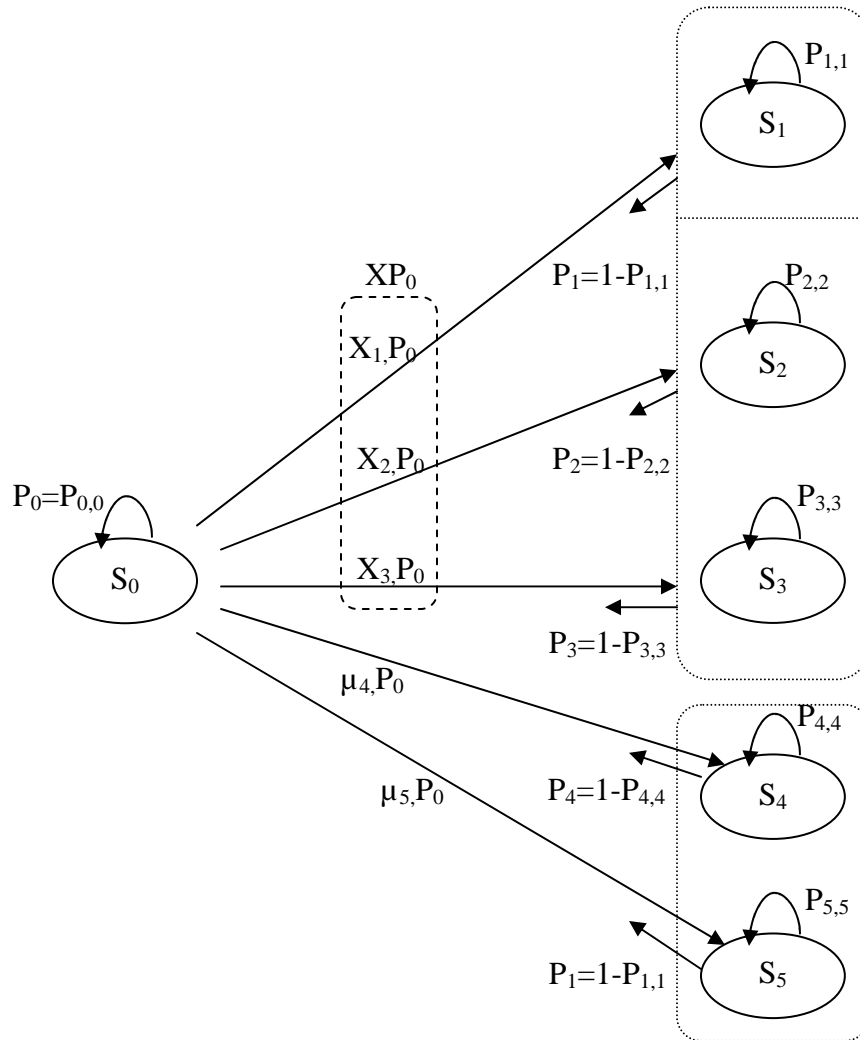


Figure 3. State Diagram Representation of Student School Attendances

As noted in the assumptions, any student can fall into any other state on any given day throughout the school year. Application of the directed graph for attendance shows the overall performance of the student attendance can be evaluated and understood by examining the characteristics such as the patterns of the students' school attendance. In Figure 3 below, the graph displays a corresponding directed graph that explains the interrelationship between the six student school attendance states. Each node represents each attendance state and contains the state probability for each student. The directed edge between the nodes describes the flow of attendance between the states. In this figure, the process begins with state S_0 , the initial state supporting the assumption that the students will come to school on the very first day of school. Also this is the only state which has the access to all the other five states as the arrow line illustrates.

4.3.3. State Probability Distribution

State S_0 in Figure 3 represents a typical student attending a normal class at school. Today, the students in this state attending class may also continue to be in this state tomorrow which is represented by $P_{0,0}$. In general, the students in state S_0 attend classes and conduct a satisfactory behavior by continuing the normal school performance. Nonetheless, students who attend school on a regular basis possibly could also fall into one of the five other states on any given school day. The interrelationships among six attendance states are illustrated in Figure 3 by the directed arrow line going from state S_0 to all the other five attendance states. The top three states S_1 , S_2 , and S_3 in Figure 3 that are enclosed in the small dotted box, are subject to suspensions either in-school or out-of-school. In these three states, the student is subject to the decisions (X) by the school administrators concerning his or her suspension status: the duration or the types of the

suspension. When students are suspended, they fall in among three states S_1 , S_2 , or S_3 with probability X_1P_0 , X_2P_0 , and X_3P_0 respectively. The X_i denotes the decisions that are made by the school officials in determining the suspensions state S_i where $i= 1$ (IS), 2 (SOS), and 3(OS). The next day, as illustrated in Figure 3, students in these three categories can either remain in their suspension state with probabilities of $P_{IS,IS}$, $P_{SOS,SOS}$, $P_{OS,OS}$, respectively, or go back to their original class, the state S_0 upon the completion of their suspensions. The value of X_i is dependent on school officials to determine the suspensions: the X_1 for the in-school suspensions to compute X_1P_{IS} , X_2 for the out-of-school suspensions with supervision to compute X_2P_{SOS} , and X_3 for the out-of-school suspensions without supervision to compute X_3P_{OS} .

Furthermore, the students can also fall into the last two states, the S_4 and S_5 which refer to truancy, the unexcused absence and excused absence. The S_4 state is the truancy state where the student is absent from school without any excuse. The S_5 state is the state where the student has a valid excuse for the absence. Most of these student school attendances depend on their own circumstantial preferences where the μ_4 represents the student making the decision for truancy with the probability μ_5P_T . The μ_5P_E represents the probability that the student is absent from the school with a valid excuses. The students subjected to states S_4 and S_5 also have the probability of not returning to school for an extended period after a prolonged suspension period and make excuses or simply avoid coming to school based on individual choices. Therefore the $P_{4,4}$ ($P_{T,T}$) represents the probability of the student repeating the truancy the next day and $P_{5,5}$ ($P_{E,E}$) for the student continuing the excused absence the next day.

In SSAS model, each student is represented by a transition probability matrix \mathbf{P} . The transition probability matrix \mathbf{P} between the transient states is represented by setting state S to be a Markov

Chain process in the sample space $X = \{S_i: S_0, S_1, S_2, S_3, S_4, S_5\}$. The stochastic process $X = \{S_{(n)}: n \geq 0\}$ represents the discrete time process with discrete state spaces $\{S_i: S_0, S_1, S_2, S_3, S_4, S_5\}$ describing the six distinct student attendance state paradigms. The discrete time process with time index $t = 0, 1, 2, \dots$, represents the number of days. The transition probability matrix P , is

$$P = \begin{bmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,j} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,j} \\ \vdots & \vdots & \ddots & \vdots \\ p_{i,1} & p_{i,2} & \cdots & p_{i,j} \end{bmatrix} \geq 0, \text{ where the row } i \text{ represents the current state and the}$$

column j represents the state corresponding to the next day in time t . Each row has the property

$$\sum_{j \in X} p_{ij} = 1.$$

The student's attendance state for tomorrow is a consequence of today's attendance state. The matrix of transition probability at time t is $P(t)$ which represents that the probability P is a function of time t . For example, the students' out-of-school suspensions state will not remain for more than a specific period. The probability of students' changing their state from out-of-school suspension to attending class will vary everyday through the school year for all students. In addition, a factor of influence from peers which affects the student attendance varies over the time during school year. Therefore, the events of student school attendance are functions of the time given school days.

4.4. The Student Interaction Model

As stated in the assumption, the model considers that all students will come to school on the first day of the school year. In general, students will connect with one another developing friendships that may affect the outlook of the student, including school attendance. For this reason, the SSAS model implements the connections between the peers implying that the students will be in

contact with one another once they come to school. In general, once the students develop friendships, those friendships will continue for some reasonable time which raises a concern in applying the same value for the connectivity throughout school days from the very first day of the school to the very last day of the school year. Instantiating the connection between peers on the first day of the school will not be applicable to the remaining days of the school year, since the students have developed friendships which may continue for sometime. Therefore, in this chapter, the SSAS provides three different approaches to investigate this phenomenon.

Another concern is the interactivity between the peers and the resulting influences thereof. Once students develop friendships with one another, there will be influence factoring between them. As explained in the previous chapter, the direction of the influence between students has been disregarded based on the assumption that children tend to develop friendships with closer people regardless of their background. An attempt to accommodate the influence between the peers, the SSAS presents mechanisms to compute the future transition probabilities of student school attendance paradigms employing a learning coefficient.

4.4.1. The Graph Representation of Student Interactions

The SSAS model assumes the undirected graph theory for the student interactions, not specifying the directions of the relationship based on the ambiguous nature of characterizing their friendships. In addition, the student relationships are described using the matrix representation. For example, the following table includes 6 students and explains their relationships. Illustrations in Table 1 matrix, the values 1, 0, -1 indicate connections, no connections, and self-connections respectively among 6 students. Referring to student number 4, this student is

connected with 3 other students, student number 1, 3, and 6 which is notated with 1 but not connected with the other 2 students, student number 2 and 5 which is notated with 0.

Student	1	2	3	4	5	6
1	-1	1	1	1	0	1
2	1	-1	1	0	0	1
3	1	1	-1	1	0	1
4	1	0	1	-1	0	1
5	0	0	0	0	-1	0
6	1	1	1	1	0	-1

Table 1. The Matrix Representation of the Relationships of 6 Students

Student number 5 represents the isolated student who is not connected to any other students in this example. The matrix is illustrated using the undirected graph in Figure 4 below.

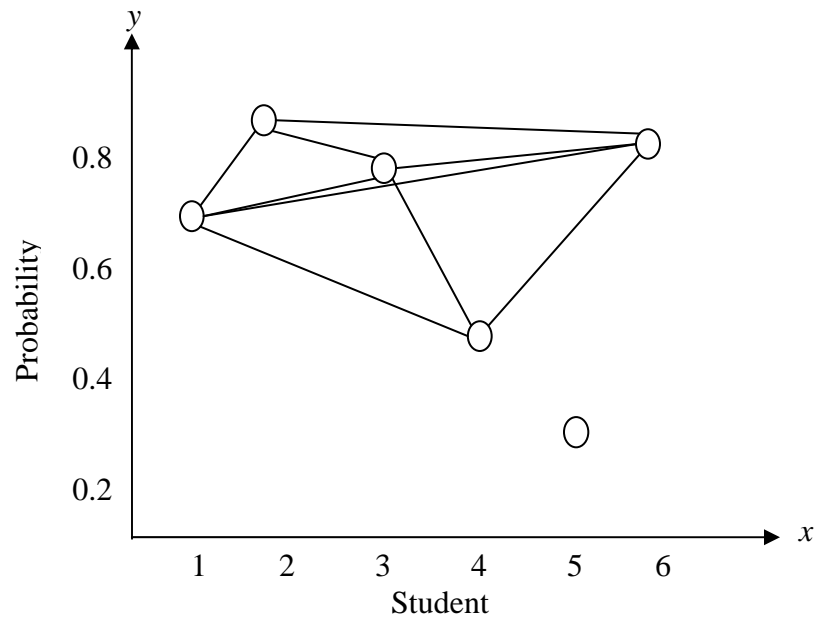


Figure 4. Undirected Graph Representation of 6 Students and their Relationships

In Figure 4, x indicates the student and y is the corresponding probability P_0 of the student.

4.4.2. Assessment of the Interaction between Students

4.4.2.1. Initial Student Attendance State Probabilities

The model formulates the initial probability distributions of the student school attendance states in two ways: 1) utilize the pre-arranged input probability distribution table or 2) generate the probability based on the pre-arranged output lookup table. The SSAS imports the external file that contains the pre-arranged input probability distribution or to simulate the relevant attendance probability distribution based on the pre-arranged probability output lookup table.

4.4.2.2. Distribution of the Student Attendance States

As stated, SSAS employs the one-way data table to store the subsequent values throughout the process. In determining the attendance state of the student, the model generates the random number and compares that number to the first row of the cumulative probability distribution. As presented in Table 1 that explains how to compute the transition probabilities, the very first state S_0 is the only state that is reachable from any other state. The reasoning behind the assumption is that students, whether they are suspended (S_1, S_2, S_3) or absent (S_3, S_4, S_5) from school, will go back to their original class upon returning to school from suspension or absence. For example, if the student is suspended, either in-school suspension or out-of-school suspension, the student has the right to go back to the same class upon completion. Moreover, if a student is absent, he or she will attend their original class once they decide to come back to school instead of going straight to suspension. Therefore, the SSAS determines the student attendance state considering the random number and the cumulative probability distribution of the student.

4.4.2.3. Interactivity of the Students and Learning Coefficient

In calculating the next day attendance probability, the model considers four significant areas: 1) the student's current probability, 2) the student's learning coefficient, 3) the connected peers' probability, and 4) the learning coefficient of the connected friends. Therefore, influential factors between the peers are implemented throughout the simulation. The information about each student, their current attending state probability distributions and the learning coefficients of the students are assumed known. The following equation provides the mechanism to compute the next day attendance probability of the student k where α is the learning coefficient and represents the realization of the influences through friendships among the students which may affect the overall behavior of the students including their school attendance.

$$P_{0}^{k}(t+1) = (\alpha * P_{0}^{k}(t)) + (1 - \alpha) * (\text{Average of } P_{0}^{k}(t) \text{ of connected peers})$$

Specifically, the next day probability P_{0} of the student attendance state S_{0} depends on the current state probability with the learning coefficient subtracted (or added) by the average probability P_{0} of the linked friends. The model computes the weighted-average probability of linked friends as $(1 - \alpha) * (\text{Average of } P_{0}^{k}(t) \text{ of connected peers})$. And if their weighted-average probability is greater than the student's current probability $\alpha * P_{0}^{k}$, which is interpreted as a positive influence, then the student's current state probability will increase. However, if the average probability of peers is less than the student's current state probability which reflects a negative influence, then the student's current probability will decrease.

Overall, student friendships are represented as undirected vertices and the links between the nodes are instantiated dynamically based on a random selection. The node represents a student

and characteristics associated with that student's school attendance. The undirected link between the nodes represents the inter-relationship with other students in the school social network. Formally, it is defined as $G_t = (N_t, L_t)$ where G refers to graph, N for node (the actor) and L for the link between the nodes. SSAS employs the learning coefficient (α) to model the level of influence between peers.

4.4.3. Three Dynamic Modes of the Connectivity between the Peers

The linkages between students are instantiated based on the random selection with a threshold value, $\beta= 0.5$ specifically for this research. SSAS generates random numbers through the Monte Carlo Simulation to determine the connectivity between students. In essence, the connections are instantiated based on 1 or 0, either yes or no respectively. Taking into account the initial connectivity between the students on the first day of school, SSAS generates the random number and provides the threshold to instantiate the connectivity between the peers. The SSAS model assumes that, if the random number is greater or equal to 0.5, there will be a connection between the students which will be displayed in the matrix as 1 and 0 for the disconnection. Throughout the model simulation process, the resulting connections of the students are presented in two different ways: a) displayed on the chart and b) displayed in the matrix form to verify and validate the correctness of the connectivity among students. Fundamentally, the link or connectivity between students exists if the random number is greater than or equal to 0.5. Otherwise they are disconnected and there is no link which will be displayed on the chart between the nodes. SSAS produces the charts and matrices to illustrate and verify the connections between the peers throughout the simulation process of the student school attendance phenomenon.

The next day connection between the peers considers the stronghold and bond as a result of friendships. In general, if students have a tight friendship today, to a greater extent they will have that strong relationship tomorrow. Subsequently, the connectivity no longer can be rationalized with a threshold value equal to 0.5. Therefore, the SSAS updates the connectivity in three different modes. The three modes for those dynamics are: a) Median Connectivity, b) Virtual Connectivity, and c) Hybrid Connectivity.

a) The median connectivity refers to the procedures introduced previously which is the main linkage process of the SSAS model. This method generates random numbers and generates the link between the nodes that are determined by the threshold value β formally as,

$$l_{(i,j)} = \begin{cases} 1 & \text{if } \beta \geq 0.5, \dots\dots\dots \text{the connectivity between student } i \text{ and } j \text{ exists} \\ 0 & \text{otherwise, } \dots\dots\dots \text{the connectivity does not exist.} \end{cases}$$

b) The virtual connectivity targets the connections among peers at $\beta = 0.95$ from the very first day of school to the end of the school year. Since the fitness point for this scheme is rather high at 0.95, it virtually links everyone in the network. The virtual connectivity can be presented formally as,

$$l_{(i,j)} = \begin{cases} 1 & \text{if } \beta \geq 0.95, \dots\dots\dots \text{the connectivity between student } i \text{ and } j \text{ exists} \\ 0 & \text{otherwise, } \dots\dots\dots \text{the connectivity does not exist.} \end{cases}$$

c) The hybrid connectivity combines both methods, median and virtual connectivity. Connections between peers are implemented using both processes. The first day of school student connectivity proceeds through the median connectivity scheme. Subsequently, if there is a connection between two students, the virtual connectivity scheme is applied. Hybrid

connectivity promotes the idea of preserving the existing friendships between the students. If students are connected, the friendships will continue and there would be little possibility of breaking up daily.

4.4.4. Interaction Probability Computation

4.4.4.1. Initial Student Attendance State Probabilities

As introduced, the SSAS considers the learning coefficient in realization of the influences through friendships among the students which may affect the overall behavior of the students including their school attendance. The SSAS model employs the peers' influences in regard to attending school by providing the threshold value for the learning coefficient at $a = 0.7$. The model formulates the initial probability distributions of the student school attendance states in two ways: 1) read the pre-arranged input probability distribution table or 2) generate the probability based on the pre-arranged output lookup table. The SSAS provides the algorithm to read the external file that contains the pre-arranged input probability distribution or to simulate the relevant attendance probability distribution based on the pre-arranged probability output lookup table. The process stores those values in a separate data table for future reference.

4.4.4.2. Determination of the Student Attendance States

SSAS modeling employs the one-way data table to store the subsequent values throughout the process. In determining the attendance state of the student, the model generates the random number through the Monte Carlo Simulation, and compares that number to the first row of the cumulative probability distribution. The very first state S_0 is the only state that undergoes revolution and the other five states, S_1 , S_2 , S_3 , S_4 , and S_5 , are fixed and, therefore, remain the

same. The reasoning behind this assumption is that students, whether they are suspended (S_1, S_2, S_3) or absent (S_3, S_4, S_5) from school, will go back to their original class upon returning to school from suspension or absence. For example, if the student is suspended either in-school suspension or out-of-school suspension, the student has the right to go back to their same class upon completion. Additionally, if a student is absent, he or she will attend the original class once they decided to come back to school instead of going straight to suspension from absence. Table 2 explains how to compute the transition probabilities in detail. Overall, the SSAS determines the student attendance state considering the random number and the cumulative probability distribution of the student.

4.5. The Student School Attendance Simulation (SSAS) Model

4.5.1. Introduction

The SSAS model considers the pre-set initial connectivity which undergoes dynamic changes during the simulation based on the value of β . In SSAS, each student is represented as a node as described previously. Briefly, the connections between the students are instantiated by generating a random number during simulation to determine connectivity based on the value of β . SSAS creates the connections between the students dynamically considering the random number and the specified connection range within the interval $[0, 1]$. As stated in the model assumption, the students can be subjected to any attendance states on any given school day. Additionally, this research selects two main areas to overcome the complexity of the student school social network: a) the state of the student school attendance and b) the interactions with their peers to accommodate the influential factor. Overall, the SSAS model considers the influence among peers by integrating the threshold value of the learning coefficient and random

selection connections. The next day student attendance probability is computed based on their own probability and the connected peers' probability coupling with the learning coefficient.

4.5.2. The Model

In the development procedure in formalizing the stochastic process of the student school attendance phenomenon, as introduced, the SSAS model rationalizes the complexity of the problem by focusing on two main areas: a) the student school attendance states and b) the interactions with their peers. This chapter introduces the relevant nomenclature of the model.

The SSAS model parameters include:

N_t is the set of nodes (students) at day t .

n_t is the total number of nodes at day t , $|N_t|$.

S_{kt} is the state school attendance of student k at day t .

L_t is the set of links (friendship) between nodes at day t .

$l_{(i,j)t}$ is the link between node i and j at day t (1 if the link exist, 0 otherwise).

α is the learning coefficient.

β is the threshold to determine the connectivity among peers. The SSAS model sets the initial connectivity threshold at $\beta = 0.5$ and $\beta = 0.95$ for the subsequent days of the connections between the students.

$P^k_{i,j,t}$ is the transition probability of state i to j of the student k at day t where $p_{ijt} = P[S(t+1) = j | S(t) = i]$ for all i , and $j \in X$.

P^K_{IS} is the probability of the state S_I where the student k has in-school suspension (IS).

P_{SOS}^K is the probability of the state S_2 where the student k has out-of-school suspension with supervision (SOS).

P_{OS}^K is the probability of the state S_3 where the student k has out-of-school suspension without supervision (OS).

P_T^K is the probability of the state S_4 where the student k has an unexcused absence, (Truancy, T).

P_E^K is the probability of the state S_5 where the student k has an excused absence (E).

e^k is an epoch, a particular day at which student k returned to attend a normal class earning grades following an absence.

el^K is the number of days that have elapsed since e for student k .

q is the number of school days the student must attend to be promoted or graduate which is the number required by the current school system, $q=165$.

γ is the possibility that a student is suspended from the school.

μ is the possibility that a student is absent but not suspended from the school,

$$\mu = \gamma - 1.$$

The SSAS model includes the following variables in order to evaluate and assess the student school attendance states during the school year:

t is the time in days for school year (1, 2, ..., 180).

na_t^k is the total number of days a student k was absent in time t .

ma_t is the summation of number of days of absence for all students in a given period t .

nt_n^k is the number of times student k started his/her absence period.

P_{0}^K is the probability of the state S_0 where a normal class attending student k is going to other state j .

As introduced, the SSAS model considers 5 different transition probabilities ($P_{0,IS}^K, P_{0,SOS}^K, P_{0,OS}^K, P_{0,T}^K, P_{0,E}^K$) where the state 0 (S_0) is the only state that the student transits to the other 5 states and undergoes changes during the simulation process. In Figure 3, there are 2 distinct areas where the student transitions from the normal class attendance state S_0 with probability P_0 . The top portion of the dotted box represents all types of suspensions ($P_{0,IS}^K, P_{0,SOS}^K, P_{0,OS}^K$) and the dotted box at the bottom represents the absent students ($P_{0,T}^K, P_{0,E}^K$) who decided to stay out of school (μ). It is assumed that the proportion of students who move from state 0 to any of the suspended states is γ . When a student is suspended, the school officials decide whether the student is suspended in state IS, SOS, or OS. The proportion of students that are suspended in state IS, SOS, and OS is denoted by the variables $X_1, X_2,$ and X_3 respectively. Similarly, a student who is absent could be in state T or E. The variables μ_4 and μ_5 denote the proportion of students with an unexcused absence, truancy and excused absence respectively. SSAS distinguishes the absence state from the suspension state because the sole dependency of the suspension is by the school officials and considers the following decision variables:

X_1 represents the probability that a suspended student is in state IS. This value is determined by school officials.

X_2 represents the probability that a suspended student is in state SOS. This value is determined by school officials.

X_3 represents the probability that a suspended student is in state OS. This value is determined by school officials where $X_3= 1-X_1-X_2$ which implies $X_1+X_2+X_3= 1$.

Accordingly, in order to compute the next day probability of student school attendance, the SSAS model implements the following equations:

- $P_{0,0}^K(t) = P_{0,0}^K(t+1) = (\alpha * P_{0,0}^K(t)) + (1 - \alpha) * (\text{Average of } P_{0,0}^K(t) \text{ of connected peers}) \dots (4.1)$

Equation 4.1 computes the next day probability of the student if he or she remains in state S_0 which indicates the student attending normal class continuously. Computation of this equation applies the same as introduced in the previous section.

- $P_{0,IS}^K(t) = (1 - P_{0,0}^K(t)) * \gamma X_1 \dots (4.2)$

Equation 4.2 computes the probability where the student changes attendance state from normal attendance S_0 with probability P_0 to in-school suspension S_{IS} with probability P_{IS} . X_1 represents the value determined by the school official in terms of in-school suspension of the students.

- $P_{0,SOS}^K(t) = (1 - P_{0,0}^K(t)) * \gamma X_2 \dots (4.3)$

Equation 4.3 computes the probability where the student changes attendance state from the normal attendance S_0 to an out-of-school suspension with supervision S_{SOS} with probability P_{SOS} . X_2 represents the value determined by the school official in terms of supervised out-of-school suspension of the students.

- $P_{0,OS}^K(t) = (1 - P_{0,0}^K(t)) * \gamma X_3 \dots (4.4)$

Equation 4.4 computes the probability where the student changes attendance state from normal attendance S_0 to an out-of-school suspension without supervision S_{OS} with probability P_{OS} . X_3 represents the value determined by the school official in terms of supervised out-of-school suspension of the students.

- $P_{0,T}^K(t) = (1 - P_{0,0}^K(t)) * (1 - \gamma) * \mu_4 \dots (4.5)$

Equation 4.5 computes the probability where the student changes attendance state from normal attendance S_0 to truancy which is an unexcused absent S_T with probability P_T . The μ_4 represents the student's preference in not attending school without any valid excuse. SSAS provides fixed value for μ_4 in order to provide coherency during the simulation.

- $P_{0,E}^k(t) = (1-P_{0,0}^k(t))*(1-\gamma)*\mu_5$ (4.6)

Equation 4.6 computes the probability where the student changes attendance state from normal attendance S_0 to an excused absence S_E : The μ_4 represents the student being absent from school with valid excuse. SSAS provides fixed value for μ_5 in order to provide coherency during the simulation.

The procedure of computing the transition probability distributions of the six school attendance states are presented in Table 2 below. The state 0 is the only state which undergoes changes as explained earlier. The state from 1 to 5 has two options for each student: either remain in the same state or go back to a normal class.

State	0 (Po)	1 (IS)	2 (SOS)	3 (OS)	4 (T, Tru)	5 (E, Exe)
0	$P_{0,0}^k(t)$	$(1-P_{0,0}^k(t))\gamma X_1$	$(1-P_{0,0}^k(t))\gamma X_2$	$(1-P_{0,0}^k(t))\gamma X_3$	$(1-P_{0,0}^k(t))(1-\gamma)\mu_4$	$(1-P_{0,0}^k(t))(1-\gamma)\mu_4$
1	$1-P_{SI}^k$	$P_{IS,IS}^k$	-	-	-	-
2	$1-P_{SOS}^k$	-	$P_{SOS,SOS}^k$	-	-	-
3	$1-P_{OS}^k$	-	-	$P_{OS,OS}^k$	-	-
4	$1-P_T^k$	-	-	-	$P_{T,T}^k$	-
5	$1-P_E^k$	-	-	-	-	$P_{E,E}^k$

Table 2. Procedures of Transition Probability Computation

These values are no longer time dependent since the determination whether to return to their class is made by the school official regardless of the school days or the peer's influences.

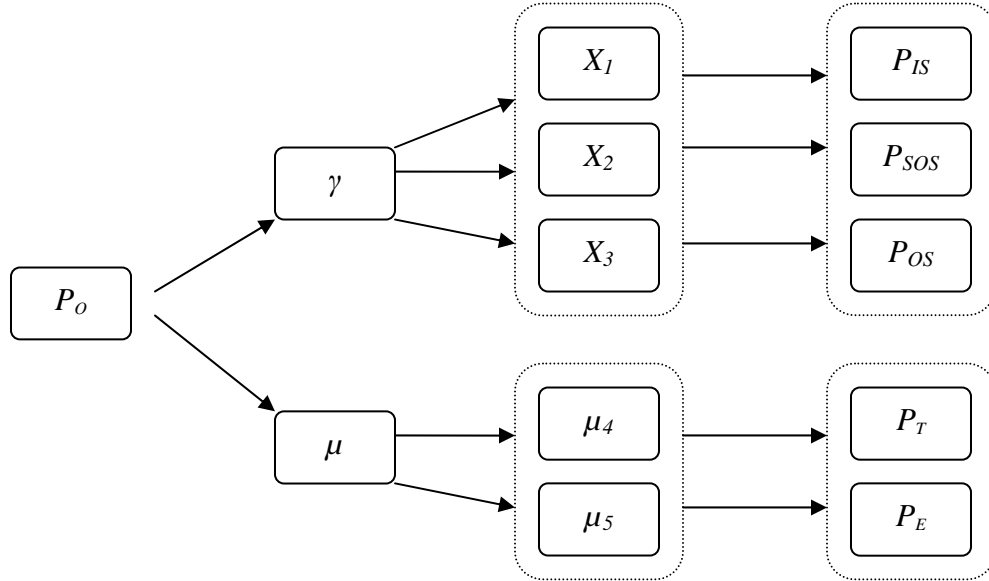


Figure 5. Procedures of the Next Day Transition Probability Computation

To illustrate the computation procedures of the next day student school attendance probabilities, the diagrams in Figure 5 present the relevance of the probability distribution between the states. The SSAS process generates the relevant probability distributions of the student throughout the given period for each student and records those values in a separate file for future reference.

As an example, consider the following one-step transition probability matrix P where the process change from state i to state j in one time step has been introduced as $p_{ijt} = P[X(t+1) = j | X(t) = i]$ and the transition probability defines the row vector as $p = (p_i; i \in X)$ where $i \in X$, $\sum_{i \in S} p_i = 1$.

Sequentially, the student's attendance state for tomorrow is a consequence of today's attendance state where the matrix of transition probability P at stage t is $P_i(t) = p_{ijt}$ with $p_{ijt} \geq 0$.

$$P = \begin{pmatrix} 0.90 & 0.03 & 0.015 & 0.015 & 0.03 & 0.01 \\ 0.70 & 0.3 & 0 & 0 & 0 & 0 \\ 0.70 & 0 & 0.3 & 0 & 0 & 0 \\ 0.70 & 0 & 0 & 0.3 & 0 & 0 \\ 0.70 & 0 & 0 & 0 & 0.3 & 0 \\ 0.99 & 0 & 0 & 0 & 0 & 0.01 \end{pmatrix}$$

If the next day probability $P_{0,0}^K(t)$ changes from 0.9 to 0.85 (i.e. $P_{0,0}^K(t+1) = 0.85$) with given value of $\gamma = 0.6$, $X_1 = 0.6$, $X_2 = 0.2$, $X_3 = 1 - X_1 - X_2 = 0.2$, $\mu_4 = 0.6$, and $\mu_5 = 1 - \mu_4 = 0.4$, the next day transition probabilities are updated as follows:

$$P_{0,SI}^K(t+1) = (1 - P_{0,0}^K(t+1)) * \gamma X_1 = (1 - 0.85) \times (0.6) * (0.6) = 0.054,$$

$$P_{0,SOS}^K(t+1) = (1 - P_{0,0}^K(t+1)) * \gamma X_2 = (1 - 0.85) \times (0.6) * (0.2) = 0.018,$$

$$P_{0,SO}^K(t+1) = (1 - P_{0,0}^K(t+1)) * \gamma X_3 = (1 - 0.85) \times (0.6) * (0.2) = 0.018,$$

$$P_{0,I}^K(t+1) = (1 - P_{0,0}^K(t+1)) (1 - \gamma) \mu_4 = (1 - 0.85) \times (1 - 0.6) * (0.6) = 0.036,$$

$$P_{0,E}^K(t+1) = (1 - P_{0,0}^K(t+1)) (1 - \gamma) \mu_5 = (1 - 0.85) \times (1 - 0.6) * (0.4) = 0.024.$$

Notice that these probabilities are the sum to 0.15. A new one-step transition probability matrix

P of the next day, $t+1$ is

$$P = \begin{pmatrix} 0.85 & 0.054 & 0.018 & 0.018 & 0.036 & 0.024 \\ 0.70 & 0.3 & 0 & 0 & 0 & 0 \\ 0.70 & 0 & 0.3 & 0 & 0 & 0 \\ 0.70 & 0 & 0 & 0.3 & 0 & 0 \\ 0.70 & 0 & 0 & 0 & 0.3 & 0 \\ 0.99 & 0 & 0 & 0 & 0 & 0.01 \end{pmatrix}.$$

The simulation of SSAS will yield the following output matrices:

- s represents the number of successful students during the school year. The model computes s by adding promoted/graduated and makeup/repeating students as the passing students during the school year.
- f represents the number of failing students.
- v represents the number of promoted students.
- y represents the number of students who completed the makeup or repeat in order to meet $q=165$.
- z represents the number of dropout students.

As introduced, the student coming to school tomorrow is mainly based on today's activity. Therefore the model applies the memory-less property of the Markov Chain process to generate the next day attendance state applying the concept that today's attendance state is the tomorrow's attendance state. The model simulates the stochastic function of student attendance over time and examines the correlations, especially the impacts of the supervised or unsupervised out-of-school suspension, and measures the overall student failure as the result of the unsupervised out-of-school suspension.

The following chapter introduces the procedure in determining the connections between the students and computing the subsequent probability distribution of the student attendance. In addition, in the SSAS development process, the model configures in several different ways to implement the best fitted procedure for the interactions and the connections between the peers. The relevant variations of the modeling techniques of the SSAS are presented in the Chapter 6.

4.6. Chapter Remarks and Summary

The SSAS model considers the complexity of the social networking aspects of school social network and concentrates on two main areas in the modeling process: a) the student attendance phenomenon and b) the interactions with their peers. The development of the SSAS model begins with constructing the comprehensive state diagram that represents the current existing school system's five attendance states (S_i), and the model proposes a new category, supervised out-of-school suspensions. Students' social networking in regard to the school attendance is strategized considering the fundamentals of graph theory properties: the node that represents students as the actors and their relationships with others applying the edges between the nodes. The state of students attending school are presented with six different options: 1) student with a normal in-class status (Normal, Po), 2) student with in-school suspension (IS), 3) student with out-of-school suspension with supervision (SOS), 4) student with out-of-school suspension without supervision (OS), 5) student with unexcused absence (Truancy, T), and 6) student with an excused absence (Exe, E).

The model considers the random environment and applies the stochastic process of Markov Chain properties to integrate the random and discrete events of the student school attendance phenomenon. The students' coming to school and participating in the attendance states are discrete and mutually exclusive events. The implementation of random environment is the rationalization of understanding the unpredictable students' school attendance from day to day.

The student school attendance events for each day are modeled as a discrete state and discrete time process where the student can be present in only one state either absent or present at any time. The SSAS model presents the attendance of each student in a distribution pattern by

computing the transition probability. Therefore, each state of the student school attendance is organized into a transition probability and calculates the steady state probabilities accordingly.

The SSAS assumes that the students will develop friendships while attending school. The friendships between students are considered through the implementation of the learning coefficient. In addition, the model considers the dynamic interactions among students using connectivity threshold values. Overall, each student attending school categories are presented in distributions through the transition probability and the cumulative probability. The state of the student attendance is determined based on those probabilities and the random number. The SSAS employs the memory-less property of Markov Chain where the current state is based entirely on yesterday's value and not on the previous data.

Overall, the SSAS model presents the function of each student's probability (P^k_i) and associated influences of their friends that may affect overall student attendance. The model applies the stochastic process to consider a randomness environment and employs the learning coefficient α to integrate the influential factors between the peers. In order to create the connectivity between the students, the model generates a random number with a definite threshold range for connectivity. Once the students exhibit connectivity, the probability of student attendance considers differentiated threshold β . This chapter provides the formal mathematical representation of the SSAS and the relevant assumptions, parameters and equations.

5. METHODS AND PROCEDURES OF SSAS MODEL

5.1. Introduction

This chapter provides detailed modeling procedures of SSAS implementation. Mainly, the SSAS algorithm consists of two significant parts: 1) the simulation of the student school attendance phenomenon and 2) the assessment of student school attendances. The first part of the model algorithm includes multiple processes in simulating the overall procedures presented in the previous chapters:

- a) initialize the number of students and the number of school days,
- b) set parameters such as learning coefficient and decision variables,
- c) set matrix notation for the connectivity,
- d) read the input or output pre-arranged file from external source,
- e) get random number and evaluate against the pre-arranged table,
- f) allocate appropriate probability distribution range accordingly,
- g) calculate the transition probability distribution table for all S_i for each student, and save those values in a separate file,
- h) calculate the cumulative probability distribution table for all S_i for each student based on the transition probability distribution, and save those values in a separate file,
- i) determine the corresponding current student attendance states, S_i accordingly, and save those values in a separate file,

- j) formulate the connections between the peers, l_{ij} , using random number and fitness, summarize those results, save the data in a separate file,
- k) calculate steady state probability, save the values in a separate file,
- l) display the connections in the Matrix and the Chart for all N , save those data in a separate file,
- m) get the random number and evaluate it against the first row of the cumulative probability distributions,
- n) calculate the next day attendance probability $P_{0(t+1)}^k$, considering correlated learning coefficient and the average probability of the connected friends coupling with their learning coefficient, save those values in a separate file,
- o) determine the next day state of the student attendance, $S_i(t+1)$ based on today's attendance state, save those values in a separate file, and
- p) repeat the process for the all school days for each student.

The second part of the SSAS algorithm is designed to assess attendance data from the first part of the simulation which includes:

- q) set the counters,
- r) define the absent and present states,
- s) set the makeup attendance parameters,
- t) define the student status such as promoted or dropout,
- u) count each attendance states for all N , save the values in a separate file,
- v) summarize the probabilities for the students, save the values in a separate file,
- w) calculate present and absent days, save the values in a separate file,

- x) calculate promoted, dropout, or makeup number of students, save the values in a separate file, and
- y) summarize the success and failure of the students, output the values in a separate file.

The corresponding pseudocode for the simulation is presented in this chapter.

5.2. Procedures, Algorithms, and Model Verifications

5.2.1. Development of Initial Transition Probability Distribution Table

Implementation of the SSAS model begins with the probability distribution table that describes the probability for each student attendance state to generate the initial transition probability. For example, the conceptual data and the probability distribution of the students are described in the following Table 2. The purpose of constructing this distribution table is to provide the baseline to compute the transition probabilities for each student for the remaining school days. Table 3 below describes six students with six distinct categories of their school attendance states and its distribution of the probabilities for each student. The table presents six classifications of the student school attendance paradigms: 1) normal class attendance, Po, 2) in-school suspension, IS, 3) supervised out-of-school suspension, SOS, 4) unsupervised out-of-school suspension, OS, 5) unexcused absence, Truancy Tru, and 6) an excused absence, Exe. Examining the Student 1 probability distribution, the student coming to school and attending the normal class (N) is 90% for this particular example.

Student 1 can also have a chance of being in school but suspended (IS) for 3% of the time, suspended out-of-school with supervision for 1.5% of the time, suspended out-of-school with no supervision for 1.5% of the time, not showing up at the school without any valid reason for 1%

of the time, and excused absence for 3% of the time. Again, this is a conceptual distribution and for this particular example, this student comes to school and attends class for 90% which is a very good student compared to Student 6 who has a value of 65% for the same category.

Name of States	States	Students					
		Student 1	Student 2	Student 3	Student 4	Student 5	Student 6
Po	0	0.900	0.850	0.800	0.750	0.700	0.650
IS	1	0.030	0.040	0.050	0.060	0.070	0.080
SOS	2	0.015	0.030	0.030	0.040	0.050	0.060
OS	3	0.015	0.030	0.030	0.040	0.050	0.060
Tru	4	0.010	0.020	0.030	0.040	0.050	0.060
Exe	5	0.030	0.030	0.060	0.070	0.080	0.090
Total	6	1	1	1	1	1	1

Table 3. Example of an Input Data Table for the Initial Transition Probability Distribution

Applying the probability distribution guidelines presented in Figure 2 in arranging the initial transition probability, the distribution of the student attendance state probabilities are organized based on the following assumptions:

- The state S_0 is accessible to itself and is the only state that has the accessibility to all the other five states.
- The state S_1 has the accessibility to itself (S_1) and to the state S_0 .
- The state S_2 has the accessibility to itself, (S_2) and to the state S_0 .
- The state S_3 has the accessibility to itself (S_3) and to the state S_0 .
- The state S_4 has the accessibility to itself (S_4) and to the state S_0 .
- The state S_5 has the accessibility to itself (S_5) and to the state S_0 .

For example, in reference to Student 2 from the Table 3, the probability distribution of Student 2's school attendance can be arranged as in Table 4 below which is the exact scheme explained in matrix P in the previous chapter.

States	0 (Po)	1 (IS)	2 (SOS)	3 (OS)	4 (T, Tru)	5 (E, Exe)
0	0.85	0.04	0.03	0.03	0.02	0.03
1	0.7	0.3	-	-	-	-
2	0.7	-	0.3	-	-	-
3	0.7	-	-	0.3	-	-
4	0.7	-	-	-	0.3	-
5	0.9	-	-	-	-	0.1

Table 4. Transition Probability Distribution of the Attendance States for Student 2

As illustrated in Figure 2 state diagram, the state 0 represents a Student 2 attending a normal class which is 85% for this case, indicates the he or she is relatively a good student in terms of attendance where the probability of $P_{0,0} = 0.85$. In reference to the state diagram, the state S_0 could fall into five states and those five states can only come back to a normal class, S_0 or repeat their status. The four states, $S_1, S_2, S_3,$ and S_4 which represent IS, SOS, OS, and Tru respectively contain the same fixed probability. All four states are fixed with 0.7 probability of returning to a normal classroom, S_0 and 0.3 for repeating their state status, $S_1, S_2, S_3,$ and S_4 . A student with an excused absence, the state S_5 has 0.9 probability of returning to a normal classroom and 0.1 possibility of remaining an excused absence. The outcome of four state students returning to their normal classroom upon the completion of suspensions or absence remains on these fixed probabilities for each state. Therefore, the state from S_1 to S_5 applies the same probability distributions and the state S_0 is the only state to undergo changes throughout the simulation for every student in this school social network.

The SSAS model presents three different ways to construct the initial transition probability: 1) PIOV model constructs transition probability based on random number purely, 2) PAIV model employs the pre-arranged input data table to generate the transition probability, and 3) PAOV mode which composes the transition probability according to the pre-arranged output table. Chapter 6 is devoted to explaining these three methods in detail including the algorithms for each type.

5.2.2. Cumulative Probability and the Range of Attendance States

In Table 3, the initial transition probability distributions for students can be transformed into the cumulative probability distribution for each student. In particular, Table 5 describes the cumulative distribution probabilities for student 1, 2, 3, and 4.

States	Student 1		Student 2		Student 3		Student 4	
Po, 0	0	0.9	0	0.85	0	0.8	0	0.75
IS, 1	0.9	0.93	0.85	0.89	0.8	0.85	0.75	0.81
SOS, 2	0.93	0.945	0.89	0.92	0.85	0.88	0.81	0.85
OS, 3	0.945	0.96	0.92	0.95	0.88	0.91	0.85	0.89
Tru, 4	0.96	0.97	0.95	0.97	0.91	0.94	0.89	0.93
Exe, 5	0.97	1	0.97	1	0.94	1	0.93	1

Table 5. Examples of Cumulative Probability Distribution Table

The same procedure and method in formulating the cumulative probability according to the transition probability applies for every student in the network. Table 6 includes the cumulative probabilities that are derived from Table 3 for Student 1 and 2. As the table illustrates, for each student, the probability begins with 0 and ends with 1 and reveals the range for each attendance state between the interval [0, 1] and organizes the subsequent attendance state for each interval.

The SSAS simulates this procedure of constructing the cumulative probability distribution according to the transition probability distribution for each student throughout the process. The following tables explain the corresponding format of the cumulative probability computation that the SSAS generates.

Student 1						
States	0	1	2	3	4	5
0	0.9	0.93	0.945	0.96	0.97	1
1	0.7	1	1	1	1	1
2	0.7	0.7	1	1	1	1
3	0.7	0.7	0.7	1	1	1
4	0.7	0.7	0.7	0.7	1	1
5	0.99	0.99	0.99	0.99	0.99	1

Student 2						
States	0	1	2	3	4	5
0	0.85	0.89	0.92	0.95	0.97	1
1	0.7	1	1	1	1	1
2	0.7	0.7	1	1	1	1
3	0.7	0.7	0.7	1	1	1
4	0.7	0.7	0.7	0.7	1	1
5	0.99	0.99	0.99	0.99	0.99	1

Table 6. Cumulative Probability Distribution Table for Student 1 and 2

The examples of Student 1 and 2 in Table 6 show the cumulative probability distribution format that the SSAS produces. The table clearly defines the states S_1 , S_2 , S_3 , S_4 , and S_5 according to Table 2 that illustrates how to compute transition probabilities for each state during the simulation process. Overall, the transition probability from state S_0 is the only probability that changes throughout the process presented on the very first row of the matrix in Table 6.

For this research, constructing the appropriate cumulative probability distribution is a very significant procedure. It provides the definite probability distribution range, and therefore, the interval for each attendance state. The main functionality of the cumulative probability is to provide the guideline to allocate the appropriate attendance state throughout the iterations. The stochastic process effectively models the recognized random variation within the given environment sample space. This random variation is included in the modeling process by generating the random numbers. The generated random numbers are evaluated against the cumulative distribution as the lookup table and allocate its proper range or interval in order to determine the corresponding state. Specifically, the random number is measured up to the very first row of the cumulative distribution as introduced in Table 5. Generally, in determining the attendance state for students, the process selects the very first row of the cumulative probability distributions of each student. Overall, the SSAS constructs a cumulative probability distribution table according to the transition probability using the computation procedures of Table 2 that defines how to calculate the transition probability of the student during the simulation process. The determination process of student attendance state is accomplished through random number generation and those random numbers are evaluated against the first row of the cumulative probability distributions. For instance, if the random number is 0.87, the corresponding attendance state for Student 1 is 0 which refers to the student who attends a normal class (P_0), S_0 and attendance state for the Student 2 is 1 which refers to a student who is in the in-school-suspension (IS) the S_2 category. Determination of the next day student-school-attendance state is a more complicated process than the initial day of the school. The following chapter explains the detailed procedures.

5.2.3. Transition from Current State to Next State

The unique property of the Markov Chain features discrete state and discrete time processes. Applying this concept, the student can only be in one state at a time and the attendance states are mutually exclusive. Basically, the student can either be absent or present at any time and the student only can participate in one attendance state at a time. Therefore, student attendances are mutually exclusive and discrete events. Furthermore, the Markov Chain provides the memory-less property. The past history of the process is no longer considered nor has any effect on the next event. The only event that affects the next event is the current event and not the events from a week ago or month before. This notion allows the next day student attendance state to be dependent of just today's current attendance state.

An example for this concept is illustrated in Table 6. In this table, "CState" refers to current state (today's attendance state) and "NState" refers to the next state (next day attendance state) for each student. Additionally, the "Rand ()" is for random number and "Days" expresses the number of school days. In determining a future attendance state, the model generates the random numbers which can be seen in the second column in the table and selects the first row of the cumulative probability distribution table for each student and allocates the corresponding attendance state. In examining Table 7, consider Table 5 that describes cumulative probability distribution for four students; Student 1, 2, 3, and 4 ("CState" = 0) which supports the assumption of having every student coming to school on the very first day of school. The next day state ("NState") of student attendance state is derived from today's current attendance state. To explain in detail, for Student 1 on school day 16, the "NState" is determined as 1 and examining the cumulative distribution table of this student, the range of the probability

distribution between 0.9 (90%) to 0.93 (93%) falls in this state, the in-school suspension state (SI). The generated random number for this day is 0.916 therefore, on day 16, the Student 1 is determined to be at state “1” which verifies the procedure.

Days	Rand ()	Student 1		Student 2		Student 3		Student 4	
		CState	NState	CState	NState	CState	NState	CState	NState
1	0.2537232	0	0	0	0	0	0	0	0
2	0.7249950	0	0	0	0	0	0	0	0
3	0.1142639	0	0	0	0	0	0	0	0
4	0.7962289	0	0	0	0	0	0	0	1
5	0.5666119	0	0	0	0	0	0	1	0
6	0.8353732	0	0	0	1	0	1	0	2
7	0.4690057	0	0	1	0	1	0	2	0
8	0.7372688	0	0	0	0	0	0	0	0
9	0.3610402	0	0	0	0	0	0	0	0
10	0.7353362	0	0	0	0	0	0	0	0
11	0.8518764	0	0	0	1	0	2	0	3
12	0.1922290	0	0	1	0	2	0	3	0
13	0.5153358	0	0	0	0	0	0	0	0
14	0.8400221	0	0	0	0	0	1	0	2
15	0.5534624	0	0	0	0	1	0	2	0
16	0.9160283	0	1	0	2	0	4	0	4
17	0.0046494	1	0	2	0	4	0	4	0
18	0.1388602	0	0	0	0	0	0	0	0
19	0.3402275	0	0	0	0	0	0	0	0
20	0.6339770	0	0	0	0	0	0	0	0
21	0.1430599	0	0	0	0	0	0	0	0
22	0.0597933	0	0	0	0	0	0	0	0
23	0.4045422	0	0	0	0	0	0	0	0
24	0.9356192	0	2	0	3	0	5	0	5
25	0.7857365	2	0	3	0	5	0	5	1

Table 7. The Current State and Next State Allocations

The exact methods and procedures apply to the rest of the entire student population throughout the simulation process of the SSAS. The SSAS uses the transition probabilities and cumulative probabilities to determine the next day state of the student attendance. Table 7 clearly illustrates that each student starts with state S_0 . Figure 6 provides the pseudocode to determine the next day of the student school attendance state.

```

The Pseudocode in Determining the Attendance States

..... //upon calculation of cumulative probability

for set the first current state to 0;
end

get randNum = ( );           //get random number to be evaluated
set tempRow = ( );           //select the row to be used to

for i = 6 to 1
    { if randNum < cumStudent
      state 0;
      else randNum < cumStudent(i)
        state i, i= 6-1;
      } end for loop

write current state to an external file ( );

```

Figure 6. The Pseudocode in Determining the Attendance States

The following chapter provides the relevant procedures in computing the next day attendance probability distributions for the students.

5.2.4. Formation of the Connectivity between the Peers

Up to this point, the computation process has been rather simple because the majority of the procedure was focused on how to initialize the process appropriately and compute the first day outlook of the student attendance paradigms correspondingly. Once the students come to school, the SSAS model assumes that they will intermingle with their peers and therefore make friends which are described as the connections between the students. In addition, the model assumes there will be an influential factor β playing a roll in the phenomenon of the student school attendance. Therefore, the model attempts to create the connections between the students that may affect the overall aspects of the student school attendance.

Students	Random #	Logic	Matrix	Connectivity
1	0.702646	0.703>0.5	1	Yes
2	0.934999	0.935>0.5	1	Yes
3	0.881752	0.882>0.5	1	Yes
4	0.500214	0.500=0.5	1	Yes
5	0.355652	0.356<0.5	0	No
6	0.786445	0.786>0.5	1	Yes

Table 8. The Rationale in Formulation of the Connectivity among the Students

The next day connection between the peers considers the stronghold and bond as a result of friendships. Initially, the SSAS generates the random numbers and provides the threshold value for the connectivity. The model sets the connection threshold value to $\beta= 0.5$. Fundamentally, the SSAS forms the connections between the students if the random number is greater than or equal to 0.5; otherwise they are disconnected and there is no link exhibited between the nodes. Table 8 describes the rationale in how to formulate the connectivity between the students. In Table 8, the second column consists of the random numbers that are assigned to each of six students. At third column in the table, the “Logic” presents the reasoning that the SSAS employs

in the modeling process which provides the threshold value of the connectivity at $\beta= 0.5$ and the process evaluates the random numbers accordingly. The model generates the matrix for the connectivity and exhibit 1 for the student who is eligible, for instance since $0.7 > 0.5$, to be connected with other students in the network illustrated in the fourth column of Table 8. In addition, the model forms the line on the chart between the students who exhibit 1 in the matrix. Refer to Table 8 and Figure 6 for 6 students' connectivity analysis. In this table and figure, Student 1, 2, 3, 4, and 6 are connected but not student 5; this student is isolated from the network. The SSAS provides the corresponding chart to verify the matrix of the connectivity between the students. This chart displays the connections between the students described in Table 8 and presented in Figure 6 below. For chart in Figure 6, x -axis represents the student number and y -axis the attendance probability. As stated, the connection is exhibited between Student 1, 2, 3, 4, and 6 but not with Student 5. Student 5 is isolated from the network in this example.

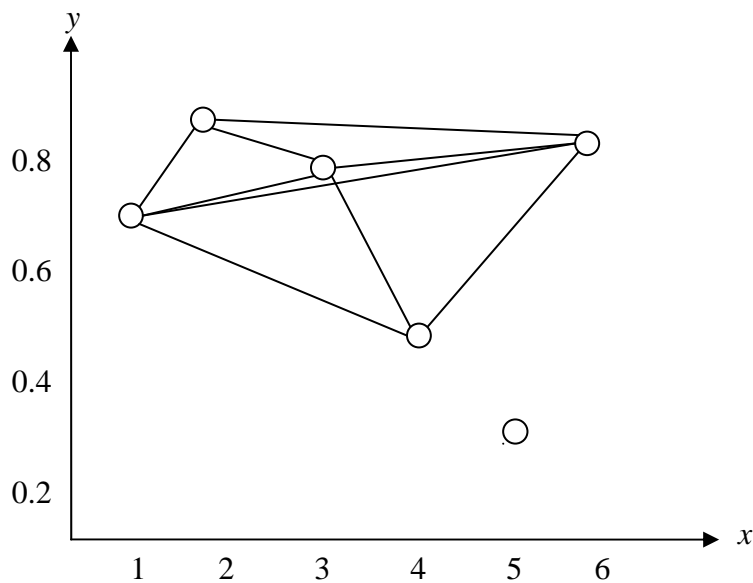


Figure 7. Initial Iteration for the Connectivity among 6 Students

The edges between the nodes are undirected assuming everyone is influencing one another through their connectivity. Nonetheless, $\beta = 0.5$ is only applicable for the connectivity between the students on the first day of school because the SSAS integrates the influential factor α between the students the next day of school. SSAS model provides modification in creating the connection between peers and several schemes in applying the connectivity at different parts of the simulation.

Subsequently, the next day connection between peers considers the stronghold and bond as a result of the friendships between the students. In general, if students have a tight friendship today, they will probably continue that relationship tomorrow also. Therefore, the connectivity no longer can be rationalized with $\beta \geq 0.5$ which may create instant breakup daily. Overall, the SSAS provides a dynamic connectivity in three different ways: a) Median Connectivity, b) Virtual Connectivity, and c) Hybrid Connectivity.

First, the median connectivity refers to the procedures introduced previously where the connectivity is determined by the following guidelines:

$$l_{(i,j)} = \begin{cases} 1 & \text{if } \beta \geq 0.5, \dots\dots\dots \text{the connectivity between the student } i \text{ to } j \text{ exists} \\ 0 & \text{otherwise, } \dots\dots\dots \text{the connectivity does not exist.} \end{cases}$$

The median connectivity name comes from having the threshold value at half point between the interval $[0, 1]$. The pseudocode for median connectivity in Figure 7 contains the only part that differs from the other connectivity.

The Pseudocode for Median Connectivity

```

..... //upon the determination of the next state
def peers;
sn = s+1;
int matrix average = 0;
int sum of neighbor student's probability = 0;
int number of neighbor students = 0;

for (counterSN = 1: NumberOfStudents -1) //
{   { if sn > NumberOfStudents
        sn = sn -NumberOfStudents;
    } end
    tempRand = rand( ); // get random number for connectivity
    { if n == 1
        { if conn(s, sn) < 0 && tempRand >= 0.5 //initial day connection
            conn (s, sn) = 1; // connection sets to 1 in the matrix
            conn (sn, s) = 1;
        elseif conn (s, sn) conn(s, sn) < 0 &&< tempRand < 0.5
            conn (s, sn) = 0; // connection sets to 0 in the matrix
            conn (sn, s) = 0;
        } end
    elseif n > 1
        { if conn(s, sn) < 0 && tempRand < 0.5 //following days connection
            conn (s, sn) = 1; //connection sets to 1 in the matrix
            conn (sn, s) = 1;
        elseif conn (s, sn) conn(s, sn) < 0 &&< tempRand < 0.0
            conn (s, sn) = 0; //connection sets to 0 in the matrix
            conn (sn, s) = 0;
        } end
    } end
    sn = sn +1;
} end for loop ..... //continue remaining simulation

```

Figure 8. Pseudocode for Median Connectivity

Second, virtual connectivity forms the connections at $\beta \geq 0.95$. Since the threshold point for this mode is much higher than the median mode with $\beta \geq 0.5$, it virtually connects everyone in the network. This virtual connectivity can be presented formally as:

$$l_{(i,j)} = \begin{cases} 1 & \text{if } \beta \geq 0.95 \quad \dots\dots \text{the connectivity between the student } i \text{ to } j \text{ exists} \\ 0 & \text{otherwise,} \quad \dots\dots \text{the connectivity does not exist.} \end{cases}$$

For example referencing Table 8, all 6 students are connected in this connectivity, whereas, there were only 5 students connected in the median mode. Therefore, this mode virtually connects everyone in the network and has only 0.05 possibility of disconnecting the students.

Finally, the hybrid connectivity method combines both the median and virtual connectivity methods. Therefore the connections between the peers are implemented using both modes. Initially, the hybrid connectivity mode applies the median connectivity which is on the first day of school. After the first day of school, if connections exist between the students, the virtual connectivity scheme is applied. Overall, the hybrid connectivity recognizes the existing friendships between the students and the influential factors thereof. In essence, if the students are connected and are friends today, they will be friends tomorrow also and there would be little possibility of breaking up daily. The SSAS model considers this theory and applies the hybrid connectivity mode to accommodate the ongoing friendships between the students.

5.2.5. Interaction Probability Computation

In calculating the next day attendance probability, the SSAS simulation integrates the influential factor to compute interaction probability once the students are connected to each other which it applies from the second day of the school year. As presented earlier, the next day probability consists of four elements: 1) the student's current probability, 2) the student's learning coefficient, 3) the connected peers' probability, and 4) the learning coefficient of the connected friends. The model assumes the information about each student; the probabilities and the

learning coefficients of the students are known. The mechanism to compute interaction probability is presented in equation 5.1.

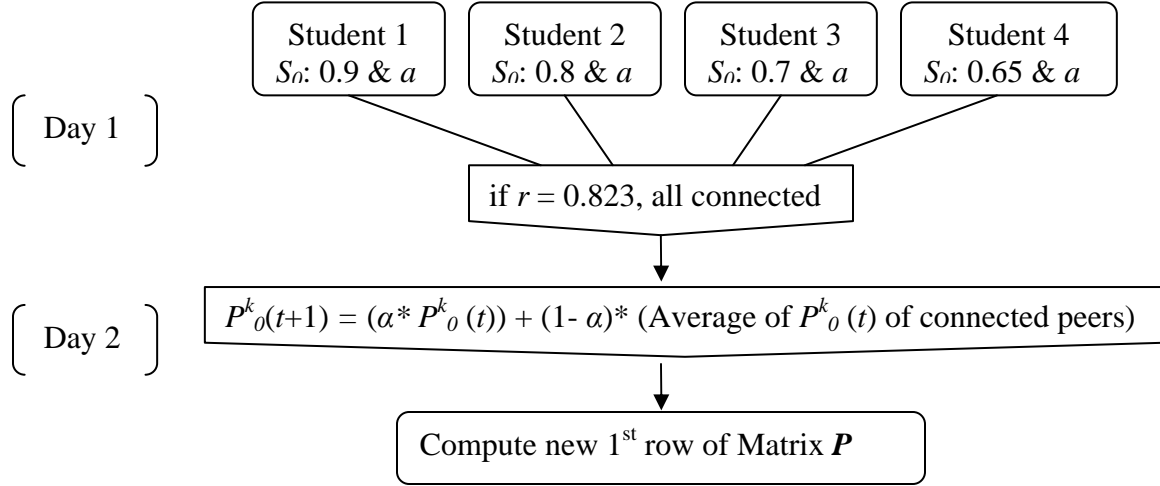


Figure 9. Flow Chart in Computing the Interaction Probability among the Students

The next day probability of the student, $P_o^k(t+1)$ is the difference of the probability between the student probability and the average of the linked peers' probability coupled with the learning coefficient. If linked peers' probability is greater than the student's current probability, $\alpha * P_o^k(t)$ then there will be an increase in the student's current state probability, $P_o^k(t+1) = (\alpha * P_o^k(t)) + (1 - \alpha) * (\text{Average of } P_o^k(t) \text{ of connected peers})$. However, if the average probability of peers is less than the student's current state probability, then the student's current probability will decrease. The flow chart in Figure 9 illustrates the procedure using four students (N_4) in estimating the next day attendance probability of the students. As stated, the SSAS uses the fitness value of the learning coefficient at $\alpha = 0.7$. For example using the data in Figure 9, in calculating the next day probability for the Student 1, the first step is to find the difference between Student 1 and other students where $(-0.1 -0.2 -0.25) = -0.55$, $(0.8 -0.9 = -0.1$, $0.7 -0.9 =$

- 0.2, and $0.65 - 0.9 = - 0.25$). Since the calculation results in a negative number, the process will decrease Student 1's probability applying $(\alpha * P_o^k(t)) + (1 - \alpha) * (\text{Average of } P_o^k(t) \text{ of connected peers})$ where $(0.9 * 0.7) + ((0.8 + 0.7 + 0.65) / 3) * 0.3 = 0.63 + 0.2155 = 0.8455$. Applying the same method, the next day probability calculation for Student 4, the difference between Student 4 and other students is 0.45, therefore the process will increase the peers' probability of Student 4 which results in 0.695, $(0.65 * 0.7) + ((0.9 + 0.8 + 0.7) / 3) * 0.3 = 0.695$.

The detailed development procedures are included in the previous section throughout the model simulation process. The transition probability and the cumulative probability are presented in the tables. The SSAS produces the charts and matrices to illustrate and verify the connections between the peers throughout the process. The model verifies the connectivity between the students by comparing the connectivity in the chart and the matrix notations represented by 1 or 0. Initially, the link or connectivity between the students exists if the random number is greater than or equal to 0.5; otherwise they are disconnected and no link will be displayed on the chart between the nodes.

5.4. Chapter Remarks and Summary

School attendance states are discrete and mutually exclusive events. The SSAS model considers two main areas in the modeling process: the student attendance phenomenon and interactions with their peers. The model constructs the comprehensive state diagram to represent the current existing school system's five attendance states (S_i) and a new category, the supervised out-of-school suspensions. The model presents the graph with the node that represents students and their relationship with others with edges between the nodes. In addition, the model considers the random environment and generates the random numbers through the Monte Carlo Simulation.

The SSAS model presents the attendance of the each student in a distribution pattern by computing the transition probability. Therefore, each state of the student school attendance is organized into a transition probability and calculates the cumulative transition probability distribution and steady state probabilities accordingly. Furthermore, the model considers the friendships between the students and implements a learning coefficient. The connections between the students are created dynamically using connectivity fitness values. Each student-attending-school category is presented in distributions through the transition probability and the cumulative probability. The state of the student attendance is determined based on those probabilities and the random number. The SSAS employs the Memory-less property of the Markov Chain where the current state is a result based solely on yesterday's value and not on the previous data.

Overall, the SSAS model presents the function of each student's probability (P_t^k) and associated influences of their friends (αP^k) that may affect overall student attendance. The model applies the stochastic process to consider randomness environment and employs the learning coefficient to integrate the influential factors between the peers. In order to create the connectivity between the students, the model generates a random number with a definite threshold range for the connectivity. Once the students exhibit the connectivity, the probability of student attendance considers the influential factor. This chapter provides the mathematical formal representation of the SSAS and the relevant assumptions, parameters and constraints.

6. THREE DIFFERENT TYPES OF THE SSAS MODEL

6.1. Introduction

The stochastic process comprehends random events, thereby selecting an actor, action of the actor, and the interactivities of the actors in an arbitrary method. In the previous chapter introduction of the procedures and methods, the model begins with the computation of the initial transition probability distribution table. The SSAS explores the alternative ways to develop the evaluation model and presents three different approaches: 1) PIOV that applies a pure random selection, 2) PAIV that uses a pre-arranged input table, and 3) the PAOV that uses a pre-arranged output look-up table. A Pure Input Output Value model considers the pure randomness of the given community where students' relationships with their peers are random events combined with unknown outlook in regard to school attendance. The PAIV model incorporates the pre-arranged probability distribution table that can be used as the lookup table to allocate the range of the distribution, thereby determining the corresponding student attendance state categories. In pre-arranged probability distributions for each of the student attendance states, the range of the state can be organized through the cumulative distribution table. The random events of students' attending states and the interaction with their peers are considered in the model as explained in the introduction of SSAS. The overall processes in relationship to probabilities are presented in Figure 10.

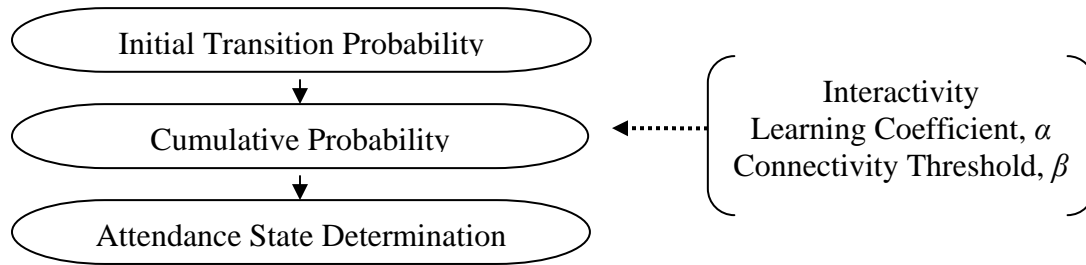


Figure 10. Process Overview of SSAS Model

In the PAOV model, the formulation of probability distributions look-up table is based on the actual students' attendance data set. The previous PIOV model implementation is solely based on random selection, and the PAIV model provides stability through the pre-arranged lookup table of the probability distribution as the input values. The PAOV model differs in that the initial probability distribution and depiction of the current student attendance state are based on the pre-arranged output look-up table designed from the actual existing school attendance data set. Therefore, the PAOV is most useful and feasible if the student attendance data are known and available. However, the pre-arranged output probability distribution look-up table also can be designed arbitrarily if the actual student attendance data set is unavailable. The following three sections of this chapter describe the detailed procedures for each model.

6.2. Pure Input Output Value (PIOV) Dynamic Model

The SSAS PIOV model precedes the probability of the student and the interactions with peers by purely a random selection process without a preset value of the student attendance probability thereby selecting the transition probability randomly. Note that in the probability computation of state 0, S_0 is the only state which undergoes changes throughout the simulation and the state S_1 ,

$S_2, S_3, S_4,$ and S_5 remains the same as presented in Table 4, following the procedure introduced in Table 2. The process of PIOV is introduced in detail in the following section.

6.2.1. Procedures and Methods of PIOV Model

Initially, the PIOV model generates the random number using $rand ()$ function and the generated random number becomes the value of current S_0 probability of the student.

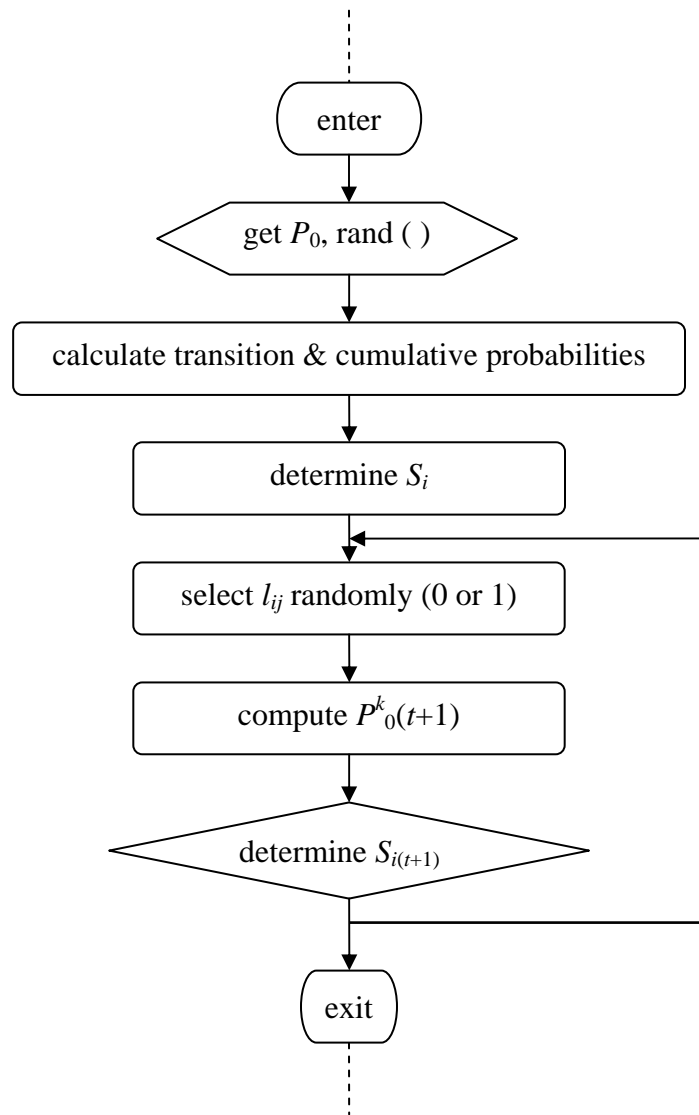


Figure 11. Procedures of the PIOV SSAS Model

For example, if the random number is 0.8521, the student has a current probability of 0.852 (85%) for state S_0 of the student attendance. This number is used as the first day, the current probability of the student, $P_{0,0}$. The initial attendance transition distribution probability is then arranged based on the input file that defines the ratio for the five remaining states.

The cumulative probability and the next day transition probability computations are the same as presented in the introduction. The learning coefficient also uses the exact value, $a=0.7$. Briefly, the flow chart in Figure 11 describes the overall procedures of the PIOV simulation method.

6.2.2. Algorithms of PIOV Model

In order to simplify, the algorithm included in this chapter is limited to the section that has been modified from the other types of the model and the subsequent algorithms can be found in the basic algorithm for the SSAS mode. The process introduced here includes the main areas where the PIOV model is differentiated and the subsequent relevant 7 processes:

- a) generate a random number and set it as the first day probability, P_0 ,
- b) read the fixed state distribution input file that sets the range for the other states,
- c) calculate transition probability and cumulative probability for all N ,
- d) determine the current states based on first row of cumulative probability, S_i ,
- e) create the linkage among peers, l_{ij} , based on threshold, $\beta \geq 0.5$,
- f) compute the $P^k_{0(t+1)}$ with correlated learning coefficient $\alpha=0.7$, and
- g) determine the next day attendance state, $S_{i(t+1)}$.

Mainly, the PIOV future outcome is solely based on a random selection process.

```

The Modified Portion of Pseudocode for PIOV Model

def school days, number of student, parameters, and chart style;
set external input file to be read and external file to output;

for each student n
write studentID into the external file;
end

read fixed state probabilities;
read preset range for the state;

for each day
get random number;
set random number as initial probability;
determine the appropriate state based on the preset range for the state;
for each num in cumulative probability (1-nth)
    if rand < first num in cumprob
        get state 0
    else compare to the nth # in cumprob n: 1-6
        if rand > nth # in cumprob
            state = n
        end
    end
end
end
..... //continue remaining simulation

```

Figure 12. Pseudocode of the PIOV Model

6.3. Pre-Arranged Input Value (PAIV) Dynamic Model

The PAIV model generates the student attendance state based on the input data set which is the pre-arranged probability distribution table describing the pre-set attendance data range for the student attendances. The students’ interactions with peers are determined by a similar method as described in the previous chapters. The calculation procedure includes the corresponding

transition probability and cumulative probability distributions coupled with the associated learning coefficient.

6.3.1. Procedures and Methods of PAIV Model

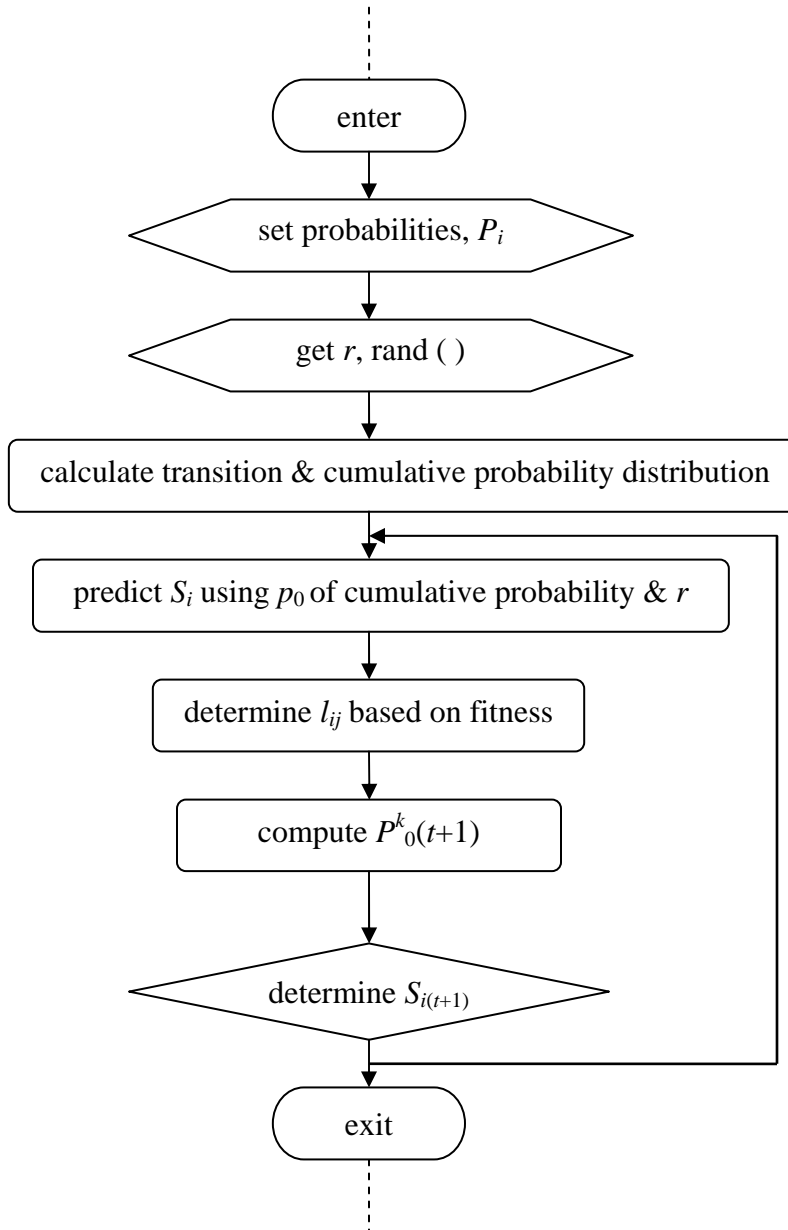


Figure 13. Procedures of the PAIV SSAS Model

Unlike the PIOV model, the PAIV model generates the student attendance status by the pre-arranged input probability distribution table for every student. This is a similar procedure presented in the introduction of the SSAS implementation procedures. In this development process, the distribution table is pre-composed in a separate file and the processor reads those pre-arranged values during the simulation in generating the initial probability distribution of the student's school attendance. Similarly, the PAIV model also considers random events and generates the random number which compares with a pre-arranged probability distribution to allocate the appropriate probability distribution range, thereby determining the current state of the student attendance.

Therefore, the PAIV model exploits the student school attendance category based on the pre-arranged probability distribution table as input data. This model is designed to provide a mechanism to control the input probability distribution and the range of probability dispersion. The flow chart in Figure 13 presents the overall process of the PAIV dynamic procedures. PAIV is a more constructive model in that the pre-arranged probability distribution for the attendance state provides some stability in determining the current state of the student. The next day probability, $P^k_0(t+1)$ of the student is computed using a similar method as described in the introduction. The exact value of learning coefficient also applies in the PAIV model.

6.3.2. Algorithms of PAIV Model

The procedures and algorithm of the PAIV model presented in this section are the portions that differ from other models and the process related to the modification which includes:

- a) set the probability distribution table for all S_i for each student, and save it in a separate file,
- b) generate the random number,
- c) read the file,
- d) calculate the transition probability and cumulative probability for all N ,
- d) determine the current states based on first row of cumulative probability, S_i ,

```

The Modified Portion of Pseudocode for PAIV Model

def school days, number of student, parameters, and chart style;
set external input file to be read and external file to output;

for each student n
    write studentID into the external file;
end

set the attendance probabilities for the external file ;

read the probability from input file;
for each student
    generate random #;
    for each num in cumprob (1-nth)
        if rand < first num in cumprob
            get state 0;
        else compare to the nth# in cumpob
            if rand > nth# in cumprob
                state = n;
            end
        end
    end
end
end
end
//continue remaining simulation

```

Figure 14. Pseudocode of the PAIV Model

- e) create the linkage among peers, l_{ij} , based on threshold, $\beta \geq 0.5$,

- f) compute the $P_0^k(t+1)$ with correlated learning coefficient $\alpha=0.7$, and
- g) determine the next day attendance state, $S_{i(t+1)}$.

The modified area of the PAIV algorithm is presented in Figure 14.

6.4. Pre-Arranged Output Value (PAOV) Dynamic Model

The PAOV process calculates the transition probability and cumulative probability distributions corresponding to the pre-arranged output probability distribution as a look-up table. The interactivities of the students are calculated in a similar manner as in the POIV and PAIV models using $\beta = 0.5$ and $a = 0.7$ as the threshold value and the learning coefficient respectively.

6.4.1. Procedures and Methods of the PAOV Model

The PAOV process begins with defining the attendance probability distribution look-up table based on the student attendance data set from the specific school district. Therefore, the initial probability, S_0 is determined by the pre-arranged probability distribution look-up table. The process estimates the probability of student school attendance by evaluating the random number against this pre-arranged look-up table which is designed based on the attendance data set. The student attendance state is also allocated accordingly as the pre-arranged output look-up table distribution range. See Table 9 for the pre-arranged probability distributions as the output look-up values. For instance, if the student comes to school for 100% to 95% of the time and attends a normal class, the student can be in the state of truancy for 1%, an excused absence for 4%, and 0% for in-school suspension or out-of-school suspension. Based on Table 9, in case the student comes to school 70% of the school days and attends class, the student possibly could be in in-

school suspension for 5%, out-of-school suspension for 4%, truancy for 7%, and excused absence for 14%.

P_0	IS	OS	Truancy	Excused
0	1	3	4	5
0.95	0.00	0.00	0.01	0.04
0.90	0.01	0.00	0.03	0.06
0.85	0.02	0.01	0.04	0.08
0.80	0.03	0.02	0.05	0.10
0.75	0.04	0.03	0.06	0.12
0.70	0.05	0.04	0.07	0.14
0.65	0.06	0.05	0.08	0.16
0.60	0.07	0.06	0.09	0.18
0.55	0.08	0.07	0.10	0.20
0.50	0.09	0.08	0.11	0.22
0.45	0.1	0.09	0.12	0.24
0.40	0.11	0.10	0.13	0.26
0.35	0.12	0.11	0.14	0.28
0.30	0.13	0.12	0.15	0.30
0.25	0.14	0.13	0.16	0.32
0.20	0.15	0.14	0.17	0.34

Table 9. Pre-Arranged Output Probability Distribution Look-Up Table

Notice that Table 9 omits student school attendance state 2, the supervised out-of-school suspension (SOS), because this category does not exist in the current school system. The supervised out-of-school suspension state is a new category which this study proposes in order to reduce the failure rate of students due to excessive absences. The attendance assessment between supervised and unsupervised out-of-school suspensions and their impacts on students is presented in Chapter 7.4.4 in detail.

As described in Figure 15, the process begins with formulating the output pre-arranged probability distribution table. Using the actual students' attendance data set, the table is

organized for each student attendance category. This table is used as a look-up table in setting the initial transition probability and allocating the corresponding attendance state.

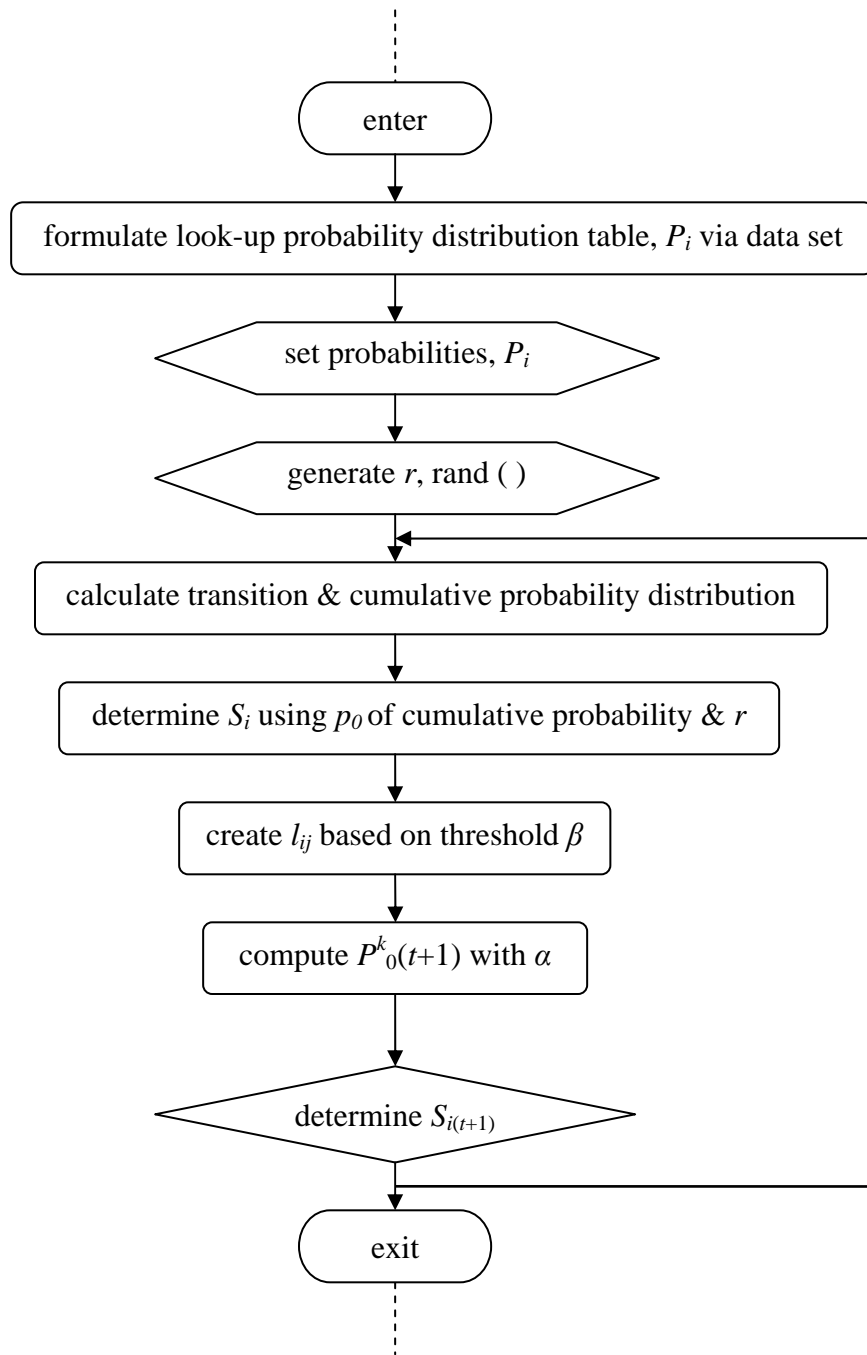


Figure 15. Procedures of the PAOV SSAS Model

The computations of interactivities among peers proceed as the procedure in the PIOV and PAIV models. In addition, the PAOV model applies the same value as the other models for the learning coefficient, $\alpha= 0.7$, and threshold value, $\beta= 0.5$ for connectivity. The transition probability and the cumulative probability computation procedures are also the same as in the PIOV and PAIV models.

6.4.2. Algorithms of the PAOV Model

The modified part of the PAOV model algorithm and the relevant calculation thereof consists of 8 major processes:

- a) formulate the probability distribution look-up table for all S_i for each student based on the data set, and save it in a separate file,
- b) read the file,
- c) generate random number and allocate appropriate range,
- d) calculate initial transition probability and cumulative probability for all N ,
- e) determine the corresponding current student attendance states, S_i accordingly based on first row of cumulative probability and output probability distribution look-up table,
- e) create the linkage among peers, l_{ij} , based on threshold, $\beta \geq 0.5$,
- f) compute the $P^k_{0(t+1)}$ with correlated learning coefficient $\alpha= 0.7$, and
- g) determine the next day attendance state, $S_{i(t+1)}$.

The pseudocode for PAOV model is explained in Figure 16.

The Modified Portion of Pseudocode for PAOV Model

```
def school days, number of student, parameters, and chart style;
set external input file to be read and external file to output;

for each student n
write studentID into the external file;
end

formulate the attendance probability look-up table AttendDistribution;

read external file, AttendDistribution file;
get the size for the AttendDistribution;
read the FixedState from input file;

for each student
    generate random #;
    for each num in cumprob (1-nth)
        if rand < first num in cumprob
            get state 0;
        else compare to the nth# in cumpob
            if rand > nth# in cumprob
                state = n;
            end
        end
    end
    for (s=1:NumOfStudents)
        { InitialProb{s}=0;
          InClassProb=NumOfAttend(s)/Period;
        }
        for (i=1:row )
            get other initial probabilities;
        end
    end
end
end

for each student n
    write studentID into the external file;
end //continue simulation
```

Figure 16. Pseudocode of the PAOV Model

6.5. Chapter Remarks and Summary

The SSAS model provides three modifications in simulating the student school attendance phenomenon: PIOV, PAIV, and PAOV. Considering the random environment of the student school attendance and the interaction with peers at school, the PIOV implements the unique attributes of the stochastic process very efficiently by using a random number to generate the initial transition probability distribution. In the PIOV development process, the simulation focuses on the main concept, the random selection process for the initial probability, P_0 for the state S_0 of the student school attendance. The connections between the peers, $l_{(i, j)}$ are selected randomly also with threshold value, $\beta= 0.5$. The next day probability and the attendance state of the students are then determined by considering the learning coefficient, $\alpha= 0.7$ and the calculation procedures introduced in Table 2 and Table 4. The overall procedure and algorithms of the PIOV model are presented in Figure 11 and Figure 12.

Understanding the lack of stability in the random environment of the student school social networking, the PAIV model considers a controlling mechanism by providing the pre-arranged initial transition probability distribution for the student school attendance states. The pre-arranged initial transition probability distribution table provides significant stability in determining the attendance state of students since it sets the pre-arranged attendance range for each state for every student. A similar procedure applies in calculating the transition probability and cumulative probability as described in Table 2 and Table 4. The next day probability and the application of threshold value, $\beta= 0.5$ for the connectivity and the learning coefficient, $\alpha= 0.7$ for the interactivity, are also applied in a similar manner as in the previous model. The PAIV model

flow chart in Figure 13 presents the overall procedure and Figure 14 describes the algorithms for the model.

The main concept of the PAOV features the use of the pre-arranged output probability distribution look-up table that is formulated based on student school attendance data. The initial transition probability is generated based on the distributions in the look-up table and calculates the cumulative probability thereof. Subsequently, the corresponding student attendance state is determined accordingly also. The learning coefficient, $\alpha= 0.7$ for interactions and threshold value, and $\beta= 0.5$ for connectivity among peers, are considered in the same manner as in the PIOV and PAIV models.

Models	Similarities	Differences
PIOV	Learning coefficient threshold, $\alpha = 0.7$ Transition & cumulative computation States determination of S_i Interaction computation with $\beta \geq 0.5$	randomly selects initial P_0 &
PAIV	same as above	pre-arranged input table for P_0 & controlling mechanism for ITP
PAOV	same as above	output look-up table for P_0 & stability for ITP

Figure 17. Comparison of the PIOV, PAIV, and PAOV Models

The PAOV model is most useful when the student school attendance data set is available. The PAOV model provides significant stability by employing a pre-arranged output probability

distributions look-up table for each student attendance state. Furthermore, this modeling is applicable in the case where the attendance data set is unavailable by composing an arbitrary output probability distribution look-up table and generating initial transition probability based on the range that look-up table provides. An overall summary is provided in Figure 16 comparing all three models of the SSAS: PIOV, PAIV, and PAOV.

Exploiting the different procedures to accommodate the random environment, the stochastic process provides an effective tool to index the parameters using statistical probability distributions. Chapter 6 introduces three distinct implementations in arrangement of the initial transition probability for the SSAS. It begins with the PIOV model, the most vulnerable method where the initial transition probability is determined purely based on random events, followed by the PAIV model that provides a controlling mechanism by employing an input table which consists of pre-arranged initial transition probability distributions. Lastly, in the PAOV model, the formulation of a pre-arranged output look-up table has the probability distribution based on a data set which provides a controlling mechanism as well as significant stability in estimating the student school attendance disseminations. In brief, Figure 17 describes the similarities and differences of three models of the SSAS.

7. EXPLOITATION RESULTS AND ANALYSIS

7.1. Introduction

As stated previously, in the development of the SSAS, the model consists of multiple processes: random selection attachment; computation of transition, cumulative, and steady state probability distribution; determination of the students' school attendance states; creation of dynamic connectivity, learning coefficient integration, and attendance assessments. This research also presents three models of the SSAS: PIOV, PAIV, and PAOV which are differentiated in generation of the initial transition probabilities for the student school attendance states. For all three models, the next day probability, $P^k_0(t+1)$ of the student attendance state is determined by computing the difference of probability between the current student and the average of the linked peers probabilities considering positive and negative influences as well as the threshold value of the connectivity and learning coefficient for interactions.

The connectivity between students is constructed based on the threshold value of the connectivity, $\beta = 0.5$. In addition, the SSAS model also provides three methods for creating the connectivity between students: Median Connectivity, Virtual Connectivity, and Hybrid Connectivity. This chapter presents results from the three SSAS models and the three dynamic connections between the students.

7.2. Simulation Results of the SSAS Models

The simulation of the SSAS model evaluates 1000 students for the 180-day school year. Table 10 includes overall results from all three SSAS models: PIOV, PAIV, and PAOV. Examining the results according to data in Table 10 indicates that the PIOV model generated the least number of students for the attendance category N, the student attending normal class with 642 students out of 1000 students.

Attendance States	PIOV		PAIV		PAOV	
	Number	%	Number	%	Number	%
N	642	0.64	711	0.71	702	0.70
IS	82	0.08	67	0.07	68	0.07
OS	67	0.07	54	0.05	56	0.06
Tru	101	0.10	89	0.09	91	0.09
Exe	108	0.11	79	0.08	83	0.08
Total	1000	1	1000	1	1000	1

Table 10. Simulation Results of the SSAS Models

However, the PIOV model generated the highest number of students for all the other categories: 82 students for in-school-suspension (IS), 67 students for out-of-school suspension (OS), 101 students for unexcused absences (Tru), and 108 students for excused absence (Exe). On the other hand, the PAIV generated the highest number of students attending class, category N with 711 students and the least number of students for all the other five categories: 67 students for in-school-suspension (IS), 54 students for out-of-school suspension (OS), 89 students for unexcused absences (Tru), and 79 students for excused absences (Exe). Interestingly, the PAOV model generated the middle range between the POIV and PAIV models for all five categories: 702 students for normal attendance (N), 68 students for in-school-suspension (IS), 56 students for out-of-school suspension (OS), 91 students for unexcused absences (Tru), and 83 students for excused absences (Exe).

Further examination of the results in Table 11 presents an assessment in terms of error or the correctness of the data. Standard errors are calculated by subtracting the resulting data from the original data and dividing it by the original data formally, $m-n/m$. For example, where the N original data was 699 and the resulting data was 642, the error can be measured by $(699-642)/699$. Based on this table, the resultant errors are the PIOV model with 16.97 %, PAIV model with 3.57%, and PAOV model with 1.4%. Consequently, the correctness of the system is 83.03% for the PIOV model, 96.43% for the PAIV model, and 98.6% for the PAOV model. This research applies the standard acceptable range for error from 0 to 0.5 which is used by most testing procedures. In other words, the acceptable system should not produce error at higher than 5% and the correctness of the system should be more than 95%.

Attendance States, Number of Students	PIOV	PAIV	PAOV
	Error %	Error %	Error %
N, 699	8.155	1.717	0.429
IS, 69	18.841	2.899	1.449
OS, 57	17.544	5.263	1.754
Tru, 93	8.602	4.301	2.151
Exe, 82	31.707	3.659	1.220
Average Error, %	16.970	3.568	1.401
Correctness, %	83.030	96.432	98.599

Table 11. Measurements of the Errors and the Correctness of the SSAS Models

Applying this range, the POIV model is an unacceptable model since it produced the most error out of all three models at 0.1697%. This model is only feasible at about 83% which is much less than the acceptable norm of 95%. PAIV and PAOV resulted in 96.4% and 98.6% correctness respectively which are better than the 95% of acceptable correctness of the system. Therefore,

the data results and the assessments thereof are focused on only two models, PAIV and PAOV of the SSAS in the following chapters.

7.3. The Connectivity in between Three Types of Connections

As introduced, the SSAS simulation exploits three different ways to implement the dynamic connectivity between the peers: a) Median Connectivity, b) Virtual Connectivity, and c) Hybrid Connectivity. These three types of connectivity are tested using the PAOV model since this model resulted in significant stability and produced the best result with 98.6% correctness. The results of three connectivity types are included in the table below. In this table, the school days are organized in intervals of five days for the 180-day school year. For easy assessment of connections among the students, the resulting data in the table are also divided into two areas: the number of students per class size of 30 and the number of students per grade size of 1000. These numbers for the size of class and grade are arbitrary figures and apply to this table only. The table also includes the average number, median of connections and the mode, the most frequently appearing numbers of connections.

In examining the results, the following reasoning is based on a class size of 30 students for simplicity. To begin, the second column of Table 12 depicts the number of connections of the median connectivity (MC). In this connectivity, the process generates the random number and the connection between the students is determined at $\beta \geq 0.5$ as introduced earlier. The median connectivity forms the link between the students with this premise throughout the process. The very first day of the link created starts out with 16 connections and ends with 15 connections. The mean number of connections was 14, with 15 connections appearing most frequently.

Days	Class Size of 30			Grade Size of 1000		
	Number of Friends			Number of Friends		
	MC	VC	HC	MC	VC	HC
1	16	28	17	533	933	567
5	17	28	19	567	932	633
10	13	27	18	432	899	601
15	19	25	20	632	831	667
20	13	27	19	433	901	634
25	15	29	20	501	967	667
30	15	29	20	499	966	665
35	12	27	22	399	901	733
40	15	29	21	500	967	700
45	15	29	20	499	967	667
50	18	25	20	602	833	665
55	11	29	21	367	967	701
60	11	29	22	365	967	733
75	11	27	22	366	900	732
80	14	27	21	467	901	701
85	10	29	21	333	967	700
90	15	27	23	500	900	767
95	9	28	21	300	933	700
100	13	27	20	433	900	667
105	15	27	20	501	901	666
110	15	27	24	503	900	800
115	12	28	25	401	933	833
120	17	29	25	567	967	833
125	19	29	23	633	967	767
130	16	27	21	533	900	701
140	16	28	19	532	933	633
145	11	27	19	367	900	632
150	16	28	20	533	933	667
155	17	26	20	567	867	666
160	16	26	18	531	867	599
165	16	25	17	533	833	567
170	15	27	17	531	901	565
175	13	27	18	530	897	606
180	15	26	19	532	933	533
Median	14	27	20	486	917	676
Mode	15	27	20	533	967	667

Table 12. The Number of Connectivity in a Five-Day Interval for School Year

The third column of the table depicts the result from the virtual connectivity simulation which targets the connections among peers at $\beta \geq 0.95$ throughout the process. This connectivity began with 28 connections initially and ended with 26, 2 connections less than initial connections. The average and the mode for this connectivity were 27 out of 30 students virtually having all students connected. Lastly, the hybrid connectivity combines both methods of the median and virtual connectivity. Connections between peers are implemented using both processes. The first school day of student connectivity proceeds through the median connectivity scheme which uses $\beta \geq 0.5$ and $\beta \geq 0.95$ for those students who are already connected. Hybrid connectivity promotes the idea of continuing existing friendships. If students are connected and are friends, more than likely the friendships will continue with very little possibility of an immediate breakup. The hybrid connectivity started with 17 connections and ended with 19 connections. The median and mode of the connectivity were each 20 students.

Overall, there were fewer connections resulting in the median and virtual of connectivity compared to the initial connection: 1 connection less resulted in median connectivity ($16-15 = 1$) and 2 connections less resulted in virtual connectivity ($28-26 = 2$) and the hybrid connectivity generated 2 connections more ($17-19 = -2$). The initial expectation of the median connectivity was that it would produce about half the number of connections, virtual connectivity would connect almost everyone, and hybrid would have more than half the connections but fewer than virtual connectivity. In conclusion, the SSAS simulation resulted as expected having MC with 14 connections out of 30 and 486 connections out of 1000 which is almost half the connectivity among the population; VC with 27 connections out of 30 and 917 connections out of 1000 which is almost all of the population; and HC with 20 connections out of 30 and 676 connections out of

1000 which is more than half but fewer than in Virtual Connectivity. Figure 18 below illustrates the difference in average number of connections among the three types of connectivity.

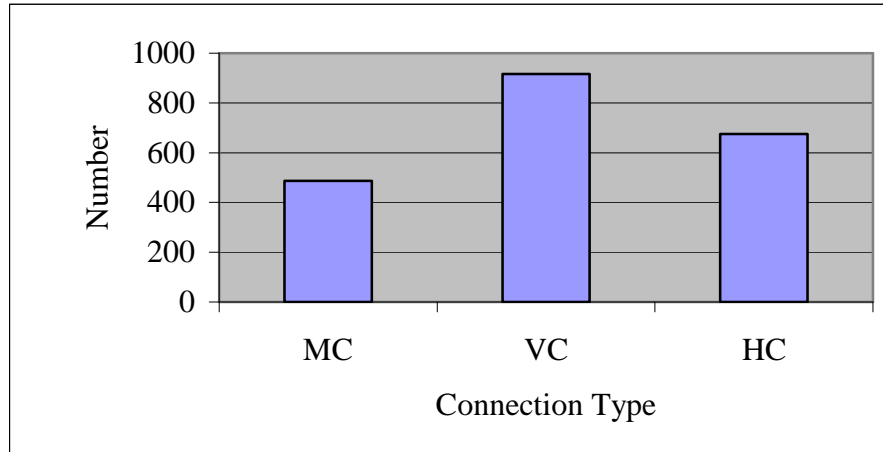


Figure18. The Average Number of Connectivity in SSAS Models

7.4. Attendance Assessment between Supervised and Unsupervised OOS

The SSAS model attempts to assess the overall student school attendance in all three types of connectivity using the PAOV model which includes: Table 13 with Median Connectivity, Table 14 with Virtual Connectivity, and Table 15 with Hybrid Connectivity. For the first simulation, we consider the student attendance assessment using Median Connectivity. Under the unsupervised out-of-school suspension program, there were 637 students promoted, 160 repeated, and 203 dropped out from the group of 1000 students. Hypothetically, if we use this 1000 student group as the senior year in high school and consider promoted and repeated categories as successful students, there were 203 students failed and 797 students graduated from high school. Under the supervised out-of-school suspension program, there were 897 students promoted, 47 repeated, and 57 dropped out. Interpreting this data using the group of 1000

students in high school, 943 students graduated and 57 students were unable to graduate from high school.

Status of Attendance	Unsupervised OOS		Supervised OOS	
	Number	%	Number	%
Promoted	637	0.6367	870	0.8700
Repeated	160	0.1600	47	0.0467
Drop-Out	203	0.2033	57	0.0567
Total	1000	1	1000	1
Status of Student	Unsupervised OOS		Supervised OOS	
	Number	%	Number	%
Graduate	797	0.7967	943	0.9433
Failed	203	0.2033	57	0.0567
Total	1000	1	1000	1

Table 13. Student Attendance Assessment Using Median Connectivity

Second, considering student attendance assessment using Virtual Connectivity, under the unsupervised out-of-school suspensions program, there were 493 students promoted, 71 repeated, and 436 dropped out from group of 1000 students.

Status of Attendance	Unsupervised OOS		Supervised OOS	
	Number	%	Number	%
Promoted	493	0.4933	844	0.8443
Repeated	71	0.0700	53	0.0533
Drop-Out	436	0.4367	102	0.1023
Total	1000	1	1000	1
Status of Student	Unsupervised OOS		Supervised OOS	
	Number	%	Number	%
Graduate	564	0.5643	898	0.8977
Failed	436	0.4357	102	0.1023
Total	1000	1	1000	1

Table 14. Student Attendance Assessment Using Virtual Connectivity

Examining this data for the senior year with 1000 students, 564 students graduated and 436 students were unable to graduate from high school. Under the supervised out-of-school suspension program, there were 844 students promoted, 53 repeated, and 102 dropped out. Applying this data to the senior year of high school with 1000 students, 898 students graduated and 102 students were unable to graduate from high school.

Third, the simulation considers the student attendance assessment using Hybrid Connectivity. Under the unsupervised out-of-school suspension program, there were 528 students promoted, 113 repeated, and 359 dropped out from the total of 1000 students. Considering this data for the senior year of high school with 1000 students, 641 students graduated and 359 students were unable to graduate. Under the supervised out-of-school suspension program, there were 786 students promoted, 61 repeated, and 153 dropped out. Once more, applying this data for the senior year of high school with a group of 1000 students, 847 students graduated and 153 students were unable to graduate.

Status of Attendance	Unsupervised OOS		Supervised OOS	
	Number	%	Number	%
Promoted	528	0.5267	786	0.7857
Repeated	113	0.1133	61	0.0610
Drop-Out	359	0.3600	153	0.1533
Total	1000	1	1000	1
Status of Student	Unsupervised OOS		Supervised OOS	
	Number	%	Number	%
Graduate	641	0.6400	847	0.8467
Failed	359	0.3600	153	0.1533
Total	1000	1	1000	1

Table 15. Student Attendance Assessment Using Hybrid Connectivity

The difference in graduates between unsupervised OOS and supervised OOS in three types of connectivity is illustrated in Figure 19.

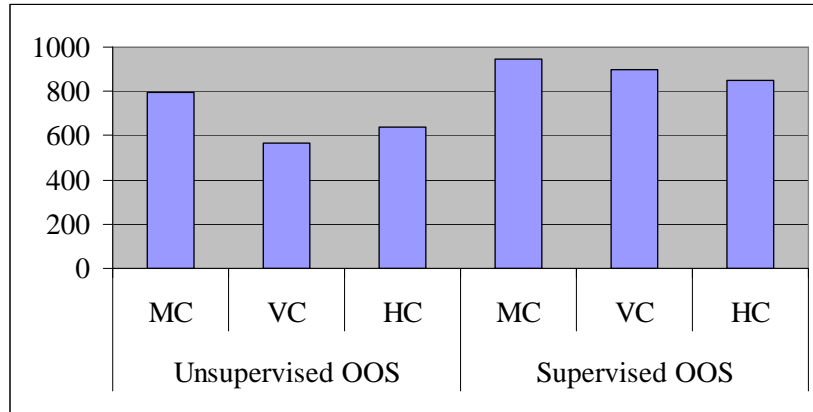


Figure 19. The Number of Student Graduates of Supervised and Unsupervised OOS

7.5. Correlation of Supervised and Unsupervised Out-of-School Suspensions and Dropouts

In an attempt to evaluate the success and failure rates of the student school attendance, Table 16 includes the difference in the number of students having supervised out-of-school suspension that is considered as class attendance *N* status instead of absence, as the current school system applies. The difference in the table below is the number of students in unsupervised out-of-school suspension subtracted from the number in supervised out-of-school suspension. The negative number indicates the decrease in the number and the positive number indicates an increase.

Under supervised out-of-school suspension, there was an average of 284 more students promoted, 56 fewer that repeated, and 229 fewer that dropped out. In addition, Table 16 indicates that there were 229 more students graduated on average. Therefore, the highest student

success rate was achieved with supervision being provided during an out-of-school suspension period.

Status of Attendance	Difference in Number of Students				
	Median Co.	Virtual Co.	Hybrid Co.	Average	% Average
Promoted	241	351	260	284	0.284
Repeated	-36	-18	-113	-56	-0.056
Drop-Out	-206	-333	-147	-229	-0.229
Change in Graduate	147	333	206	229	0.229

Table 16. The Difference in Number of Student Graduates in Three Types of Connectivity

7.6. Chapter Remarks and Summary

This chapter presents the results of three types of SSAS model simulations. The simulation consists of random selection attachment, computation of transition, cumulative, and steady state probability distribution, determination of the students' school attendance states, connectivity integration, learning coefficient integration, and attendance assessments. In addition, this research presents three different models, PIOV, PAIV, and PAOV, in generating the initial probabilities for student school attendance. The next day probability, $P^k_{0(t+1)}$ of the student attendance state is determined by computing the difference of probability between students and the average probabilities of the linked peers which reflect a positive and negative influence coupled with the threshold value for the connectivity and the learning coefficient for interactions between peers.

Furthermore, the SSAS model provides three different ways to create the connection dynamically between students. The SSAS simulation forms the connectivity between the students based on the random number evaluating against the threshold value of the connectivity

at $\beta \geq 0.5$ for the initial connectivity. The subsequent connectivity considers $\beta \geq 0.5$ and/or $\beta \geq 0.95$. The Median Connectivity uses threshold of $\beta \geq 0.5$ throughout the process, Virtual Connectivity uses $\beta \geq 0.95$ throughout the process, and Hybrid Connectivity uses $\beta \geq 0.5$ for the initial connections and $\beta \geq 0.95$ for the connected peers thereof.

This chapter also provides an assessment of student attendance. Assessments between the unsupervised and supervised out-of-school suspensions are compared to estimate the overall success or failure of the student in reference to graduating from high school.

8. CONCLUSIONS AND FUTURE RESEARCH

Based on the low percentage of 70% to 73% national and local high school graduation rates, the notion is that if action is not taken for our younger generation and/or their educational environment, especially for at-risk children, history will continue to repeat itself with higher dropout, truancy, and juvenile delinquency rates, causing a negative impact on the student individually and on society as a whole. Considering this appalling circumstance, this research presents the SSAS model that can be used to evaluate the student school attendance phenomenon. In particular, this research rigorously investigates the option of implementing a better program to decrease the dropout and truancy rates. In doing so, the SSAS presents a new approach, “Supervised Out-of-School Suspension,” instead of continuing the unsupervised out-of-school suspension program which is currently administered by the school system. The SSAS model presents the student attendance phenomenon in order to assess the student school success rates, which have been low as a result of having only unsupervised out-of-school suspensions. The model presents three different ways to compose the transition probability distributions of the student school attendance phenomenon.

The development of the SSAS explored the different methods of organizing and calculating the initial transition probability: 1) The PIOV model employs the pure random selection by generating the initial transition probability of the student school attendance in a random process. 2) The PAIV model provides a controlling mechanism by providing the initial transition probability of the student school attendance through a pre-arranged input data table. 3) The

PAOV model provides controlling mechanism as well as the stability of the system by implementing the pre-arranged output look-up table to generate the initial transition probability. The model forms the connectivity between the students both dynamically and randomly. Furthermore, the connectivity between the students is explored in three different ways: 1) Median Connectivity that creates the link between the peers with $\beta \geq 0.5$, 2) Virtual Connectivity which forms the connection at $\beta \geq 0.95$, and 3) Hybrid Connectivity that applies $\beta \geq 0.5$ for initial connection and $\beta \geq 0.95$ for connected students thereof. The simulation results are described in the following section.

8.1. Remarks and Recommendations

In the results of the SSAS model simulation, the PAOV model generated the most stable system among all three models with 1.4% error; the POIV was the most unstable which resulted in 17% error; and PAIV model exhibited 3.6% error. In terms of connectivity, first Median Connectivity created 486 connections, Virtual Connectivity created the 917 links, and Hybrid Connectivity created 676 connections out of 1000. The simulation explores the connectivity for a class size of 30 where Median Connectivity resulted in 14, Virtual Connectivity 27, and Hybrid Connectivity 20 connections out of 30. In terms of student school attendance assessment, under supervised out-of-school suspensions, Virtual Connectivity produced improvement in number of students graduating from high school with 333 students, 206 students from Hybrid Connectivity, and 147 students from Median Connectivity. Overall, there were 229 more students graduated with supervised OOS in comparison to unsupervised OOS.

There are several major issues directly resulting from the current unsupervised out-of-school suspension. First, unsupervised suspension leads to student failure with students misbehaving

further in an attempt to mask their inability to keep up, and finally convinced they will never be able to catch up. Obviously, this is a vicious cycle where too many of our at-risk students find themselves caught at a very young age. Then, the lack of supervision leads the child to the streets. As the national report from police department statements, juvenile crime spikes when students are not in school, making a case for finding new and creative ways to engage our youth in constructive activities. Additionally, drop outs lead to complete loss of financial resources and negative impact on the economy. In the current school system, the student failure rate is not taken into consideration during the budget process. The budget is based on the total number of students at full enrollment. Economic analysis related to the out-of-school suspension is addressed in a following section.

8.2. Future Research

First, this research can be further advanced by optimizing the use of the SSAS model. Young children are pre-assigned to a specific class or activities. The location of the student is well defined at school or at home. Therefore, the SSAS model can be explored further by concentrating on the position of the student. The particle swarm optimization algorithm is a stochastic process used to optimize the population-based discrete and continuous problem. It is a method of swarm intelligence techniques which is based on social and psychological principles in order to provide the social behavior insights in terms of engineer perspectives [15]. In a given situation, the particle swarm simulates social problems to evaluate a proposed solution dynamically. A social network is defined by assigning interaction with neighbors for each individual. Groups of these individuals form the population and they are defined as a random probability in a given sample space with positions. The optimal candidate solution is achieved

by studying these individuals. These individuals are known as the particles. Then, the motion is specified as best fitness throughout the iterative process to improve these particles candidate solutions. During the iterative evaluation process, the particles remember the point at which they had the best of the candidate successful solutions which refers to particle best or local best. This information is available to their neighbors and thus each particle has access to neighbors' information whether they have a successful solution or not. Ultimately, the number of these successful solutions controls the flow of the movement in the search space of the social setting. Each particle refers to an individual containing its position and velocity creating the swarm population and moving dynamically through out the hyperspace, R . A particle contains two pieces of critical information: 1) its best position memory and 2) the neighbor's or global best position. Therefore, each particle searches for better positions in the swarm hyperspace by communicating good positions to each other and adjusting their own position and velocity based on that information. This optimization is applicable for the SSAS model since it deals with a similar stochastic process environment and the discrete nature of student school attendance phenomena.

Second, the SSAS model can be expanded to analyze the economic impact of students failing from school. The current school system formulates its annual budget based on the total number of school age children available to enroll into the public school system [8, 11, 26]. However, the student's school attendance performance is not factored into the budgeting process or used as any indicator of future resource requirements or those which might be misdirected to failing academic procedures. The decision variables, X_1 , X_2 , and X_3 and the output variables that SSAS model provides in this study along with the steady state probability that enable us to compute the expected cost can be used to study the expenditure correlation between the supervised and

unsupervised Out-of-School suspensions. Providing additional expenditures for alternatives to putting the student unsupervised out-of-school and to maintaining some type of educational setting during suspension is still a positive approach economically. This process will also decrease juvenile crime if the supervised out-of-school suspension is in place. The student is then able to return to the school current on course work without any difficulties. In addition, more resources for court related expenditures are in the future if the students end up in trouble on the street due to out-of-school suspension. A model can be used to evaluate the financial loss due to unsupervised out-of-school suspension for the city leaders and school system officials in examining the current unsupervised out-of-school suspension instead of continuing the program without supervision, structure, and substance. Therefore, an economic cost-effect analysis would be very useful in estimating and measuring the impact of the out-of-suspension on the school budget and local economy.

There is no silver bullet to eradicate student truancy or dropouts. However, supervised out-of-school suspension will reduce the drop-out rate and the number of juvenile crimes which in turn will save innumerable dollars. Understanding the consequences of unsupervised out-of-school suspension used for years throughout the U.S. and the subsequent financial loss, this research provides a baseline model to evaluate the impact of unsupervised out-of-school suspension. The results of the SSAS model indicated that there would be a significant improvement and impacts on the problem related to student school attendance if the OOS is structured and supervised. Also, the results of the model simulation indicated that the current unsupervised out-of-school suspensions produce a higher dropout rate and failure rate in comparison to a “Supervised Out-of-School Suspension.” Therefore, SSAS provides an evaluation model for school official, administrators, and others involved in the decision making process in our schools and

communities in improving the student success rate in terms of attendance. Overall, the SSAS model provide a frame work for understanding and assessing the impacts of unsupervised out-of-school suspensions and to propose a new attendance category, supervised out-of-school suspensions, which would reduce the dropout and overall student failure rate. In addition, failure from school at an early age greatly diminishes the potential earning capacity in later years resulting in society as a whole being affected by the early failures of the few. Optimistically, the SSAS model would provide opportunities to encourage a different approach to address the issue related to OOS, thereby bringing new action to an ongoing problem, to ensure the success of each student instead of continuing the past failing policies and practices.

REFERENCES

- [1] Andrieu, C. et all. An Introduction to MCMC for Machine Learning. Kluwer Academic Publishers. Machine Learning 50: 5-43. 2003.
- [2] Banks, J. et all. Discrete-Event System Simulation. Prentice Hall. ISBN: 0131446797. 4th Edition. 2004.
- [3] Bartlett, L. et all. "Absences: A Model Policy and Rule." Iowa State Department of Public Instruction, Des Moines. ED 162 433. 1978.
- [4] Bazaraa, M. et all. Linear Programming and Network Flows. John Wiley & Sons Inc. ISBN:0471485993. 2002.
- [5] Carrasco, J and Miller, E. The Social Dimension in Action: A Multilevel, Personal Networks Model of Social Activity Frequency between Individuals. Science Direct: Transportation Research Part A 43: 90–104. 2009.
- [6] Eugene. At-Risk Youth in Crisis: A Handbook for Collaboration Between Schools and Social Services. Volume 5: Attendance Services. ED 347 621. July 1992.
- [7] Fenner, T. et all. A Stochastic Evolutionary Growth Model for Social Networks. Science Direct: Computer Networks 51: 4586-4595. 2007.
- [8] Garry, E. Truancy: First Step to a Lifetime of Problems. Washington, DC: Office of Juvenile Justice and Delinquency Prevention. ED 408 666. October 1996.
- [9] Geo, E. et all. Behavior Distance Measurement Using Hidden Markov Models. RAID, LNCS 409, pp19-40. 2006
- [10] Geyer, C. Markov Chain Monte Carlo Maximum Likelihood. School of Statistics, University Minnesota.
- [11] Gullatt, E. and Lemoine, D. "Truancy: What's a Principal to Do?" American Secondary Education 1: 7-12. EJ 527 489. September 1997.
- [12] Hovelaque, V et all. Analysis of Linear Structure Systems Using a Primal-Dual Algorithm. Elsevier Science: Stsystems & Control Letters 27: 73-85. 1996.

- [13] Hiller, R. and Lieberman, G. Introduction to Operations Research. McGraw-Hill. ISBN: 0072321695. 7th Edition. 2001.
- [14] Ingersoll, S. and LeBoeuf, D. "Reaching Out to Youth Out of the Educational Mainstream." Washington, DC: Office of Juvenile Justice and Delinquency Prevention. February 1997.
- [15] Kenney, J and Eberhart, R. Particle Swarm Optimization. IEEE. 0-7803-2768-3. 1995.
- [16] Kennedy, J. and Mendes, R. Neighborhood Topologies in Fully Informed and Best-of-Neighborhood Particles Swarms. IEEE. 2006.
- [17] Kennedy, J. and Mendes, R. Population Structure and Particle Swarm Performance.
- [18] Law, A. and Kelton W. Simulation Modeling and Analysis. McGraw-Hill International. ISBN: 0070592926. 3rd Edition. 2000.
- [19] Long, Y. and Siau, K. Social Network Structures in Open Source Software Development Teams. USA Journal of Database Management, 18(2), 2 -40, April-June 2007.
- [20] Madar, J. et all. Interactive Particle Swarm Optimization. IEEE: ISDA 05. 2005.
- [21] Mitra, S. et all. Design of a Data Model for Social Network Applications. IGI Publishing. Journal of Database Management. Vol:18. Issue 4. 2007.
- [22] Nelson, Barry L. Stochastic Modeling Analysis and Simulation. McGraw-Hill Inc. ISBN: 0070462135. 1995.
- [23] Naylor, Michele. Reducing the Dropout Rate through Career and Vocational Education. Overview. ERIC Digest Series. Columbus, Ohio: Clearinghouse on Adult, Career, and Vocational Education. ED 282 094. 1987.
- [24] Park, C . Contemporary Engineering Economics. Prentice Hall. ISBN:0130893102. 2002.
- [25] Park, J et all. A Particle Swarm Optimization for Economic Dispatch with Nonsmooth Cost Functions. IEEE: Power Systems. Vol: 20, No 1. 2005.
- [26] Robins, L. and Ratcliff, K. Long Range Outcomes Associated with School Truancy. Washington, DC: Public Health Service. ED 152 893. 1978.
- [27] Rohrman, D. "Combating Truancy in Our Schools: A Community Effort." NASSP Bulletin: 76 40-45. EJ 457 251. Jan 1993.
- [28] Shi, Y. and Eberhart, R. Population Diversity of Particle Swarms. IEEE 978-1-4244-1823-7/08. 2008.

- [29] Skyrms, B and Pemantle, R. A Dynamic Model of Social Network Formation. National Academy of Sciences. Vol: 97, No 16. 2000.
- [30] Toivonen, R. et al. A Model for Social Networks. Science Direct. Physica: 851-860. 2006.
- [31] Troutt, M. et al. Linear Programming System Identification: The general nonnegative parameters case. European Journal of Operational Research 185: 63-75. 2007.
- [32] Wilson, K. "Tough on Truants." American School Board Journal 180: 43, 46. EJ 461 151. April 1993.
- [33] Winston, Wayne L. Data Analysis and Business Modeling, Microsoft Excel. Microsoft Press. ISBN: 0735619018. 2004.
- [34] Yucesan, E. Analysis of Markov Chains Using Simulation Graph Models. Proceedings of the Winter Simulation Conference. 1990.
- [35] Georgia. Understating High School Graduation Rates in Georgia. <http://www.all4ed.org>
- [36] Georgia Department of Education. <http://www.doe.k12.ga.us>
- [37] National Dropout Prevention Center. <http://www.dropoutprevention.org>
- [38] Office of Juvenile Justice and Delinquency Prevention. <http://ojjdp.ncjrs.gov>
- [39] Statistical Briefing Book. <http://ojjdp.ncjrs.gov>
- [40] Student Truancy. Kids Source. <http://www.kidsource.com/education/student.truancy>
- [41] U.S. Census Bureau. Families and Living Arrangements. <http://www.census.gov>
- [42] U.S. Department of Education, National Center for Education. <http://nces.ed.gov>

Appendix 1. SSAS MATLAB CODE WITH HYBRID CONNECTIVITY

```
clear all
clc

aMin=0;           //define parameters
aMax=0.7;
afix=0.999;
g=0.6;
m=1-g;
m4=0.6;
m5=1-m4;
x1=0.6;
x2=1-x1;

MatrixLine=1;

Year=180;
Semester=90;
Week=5;

Period=180;
startCol=2;
iterCol=3;

color=['b','g','r','c','m','y','w']; //set colors for first seven students to be display on the chart

NumOfStudents=input('Number of students: ');

mycell={'Unsupervised'};
for i=1:NumOfStudents
    xlswrite('result',mycell,'Present&Absent',[char(xlsColNum2Str(4*(i-1)+2)),char(num2str(2))]);
end

mycell={'Supervised'};
for i=1:NumOfStudents
    xlswrite('result',mycell,'Present&Absent',[char(xlsColNum2Str(4*(i-1)+4)),char(num2str(2))]);
end
```

```

mycell={'Present'};
    xlswrite('result',mycell,'Present&Absent',[char('A'),char(num2str(2))]);

mycell={'Absent'};
    xlswrite('result',mycell,'Present&Absent',[char('A'),char(num2str(8))]);

mycell={'Total'};
    xlswrite('result',mycell,'Present&Absent',[char('A'),char(num2str(7))]);
    xlswrite('result',mycell,'Present&Absent',[char('A'),char(num2str(11))]);

mycell={'In Class'};
    xlswrite('result',mycell,'Present&Absent',[char('A'),char(num2str(3))]);

mycell={'ISS'};
    xlswrite('result',mycell,'Present&Absent',[char('A'),char(num2str(4))]);

mycell={'OOS'};
    xlswrite('result',mycell,'Present&Absent',[char('A'),char(num2str(6))]);
    xlswrite('result',mycell,'Present&Absent',[char('A'),char(num2str(9))]);

mycell={'Truancy'};
    xlswrite('result',mycell,'Present&Absent',[char('A'),char(num2str(10))]);

mycell={'Excused'};
    xlswrite('result',mycell,'Present&Absent',[char('A'),char(num2str(5))]);

%user inputs attendance

for i=1:NumOfStudents
    NumOfAttend(i)=input(['Number of attendances for student ',num2str(i),' : ']);
    mycell={ ['Student',num2str(i)]};
    xlswrite('result',mycell,'mresult',[char(xlsColNum2Str(3*(i-1)+2)),char(num2str(1))]);
    xlswrite('result',mycell,'Attendances',[char(xlsColNum2Str(3*(i-1)+1)),char(num2str(1))]);
    xlswrite('result',mycell,'Present&Absent',[char(xlsColNum2Str(4*(i-1)+2)),char(num2str(1))]);
end

AttenDist=xlsread('ModelStudent.xls','AttendDistribution','B8:F27');

[row,column]=size(AttenDist);

FixState=xlsread('ModelStudent.xls','DataInOutput','F28:F38');

for s=1:NumOfStudents

```

```

AbsencesProb{s}=0; % AbsencesProb is the Interaction Probability
InClassProb=NumOfAttend(s)/Period;

for i=1:row % get the first line of the Probability
    if InClassProb-AttenDist(i,1)<0.0501&&InClassProb-AttenDist(i,1)>0
        AbsencesProb{s}(1,1)=InClassProb;
        AbsencesProb{s}(1,2)=(1-InClassProb)*AttenDist(row,2)/sum(AttenDist(row,2:5));
        AbsencesProb{s}(1,3)=(1-InClassProb)*AttenDist(row,3)/sum(AttenDist(row,2:5));
        AbsencesProb{s}(1,4)=(1-InClassProb)*AttenDist(row,4)/sum(AttenDist(row,2:5));
        AbsencesProb{s}(1,5)=(1-InClassProb)*AttenDist(row,5)/sum(AttenDist(row,2:5));
        break;
    end
end

for i=2:5 % get the other lines probabilities
    for j=[1,i]
        if j==1
            AbsencesProb{s}(i,j)=FixState((i-2)*3+1);
        else
            AbsencesProb{s}(i,j)=FixState((i-2)*3+2);
        end
    end
end
end

for s=1:NumOfStudents % set the first current state to 0
    currentState(s)=0;
end

student=AbsencesProb; % get a copy of AbsencesProb

for i=1:NumOfStudents % calculate the steady state probability
    steadyState{i}=student{i}^100;
    xlswrite('result',steadyState{i},'Steady State',[char(xlsColNum2Str(6*(i-1)+1)),char(num2str(2))]);
end

% get user input for the days, NumOfDays=input('Number of School days: ');
NumOfDays=180;

for n=1:NumOfDays % set the # of Day in the result set
    result(n,1)=n;
    for s=1:NumOfStudents
        graphProbability(s)=student{s}(1,1);
    end
end

```

```

if n<=20||mod(n,5)==0
    figure();
    hold on;
    c=1;
    title(['Day ',num2str(n)]);
    xlabel('Student');
    ylabel('Probability');

    for s=1:NumOfStudents
        if c>7
            text(s,graphProbability(s),num2str(s));
        else

plot(s,graphProbability(s),'rs','LineWidth',2,'MarkerEdgeColor','k','MarkerFaceColor',color(c),'M
arkerSize',5);
        c=c+1;
        end
    end
end

        %initialize the connectivity of students to -1 which means not calculated yet
for i=1:NumOfStudents
    for j=1:NumOfStudents
        conn(i,j)=-1;
    end
end

        %every student using the studentCopy{s}(1,1)same rand number in the result set
randNum=rand(1);
for s=1:NumOfStudents
    result(n,startCol+iterCol*(s-1))=randNum;
    result(n,startCol+iterCol*(s-1)+1)=currentState(s);
    xlswrite('result',student{s},'InteractionProbability',[char(xlsColNum2Str((s-
1)*6+1)),char(num2str(MatrixLine))]);

    cumStudent=student{s};
    for i=1:5
        for cumState=2:5
            cumStudent(i,cumState)=cumStudent(i,cumState-1)+student{s}(i,cumState);
        end
    end
    xlswrite('result',cumStudent,'CumulativeProbability',[char(xlsColNum2Str((s-
1)*6+1)),char(num2str(MatrixLine))]);

        %determine the next state and change the current state according to the matrix
tempRow=result(n,startCol+iterCol*(s-1)+1)+1;
for i=1:5
    if randNum<cumStudent(tempRow,1)

```

```

    result(n,startCol+iterCol*(s-1)+2)=0;
    currentState(s)=result(n,startCol+iterCol*(s-1)+2);
    break;
end
if result(n,startCol+iterCol*(s-1))>cumStudent(tempRow,6-i)
    if i==1
        result(n,startCol+iterCol*(s-1)+2)=4;
    else
        result(n,startCol+iterCol*(s-1)+2)=6-i;
    end
    currentState(s)=result(n,startCol+iterCol*(s-1)+2);
    break;
end
end
end

sn=s+1; %s neighbor student: sn
matrixAverage=0;
sumNP=0;
numOfNP=0;
for counterSN=1:NumOfStudents-1
    if sn>NumOfStudents
        sn=sn-NumOfStudents;
    end
    tempRand=rand(1);
    if n==1
        if conn(s,sn)<0&&tempRand>=0.5
            conn(s,sn)=1;
            conn(sn,s)=1;
            if n<=20||mod(n,5)==0
                plot([s,sn],[graphProbability(s),graphProbability(sn)]);
            end
        elseif conn(s,sn)<0&&tempRand<0.5
            conn(s,sn)=0;
            conn(sn,s)=0;
        end
    elseif n>1
        if conn(s,sn)<0&&tempRand>0.05
            conn(s,sn)=1;
            conn(sn,s)=1;
            if n<=20||mod(n,5)==0
                plot([s,sn],[graphProbability(s),graphProbability(sn)]);
            end
        elseif conn(s,sn)<0&&tempRand<=0.05
            conn(s,sn)=0;
            conn(sn,s)=0;
        end
    end
end

```

```

end

if conn(s,sn)==1
    matrixAverage=matrixAverage+student{sn}(1,1)-student{s}(1,1);
    sumNP=sumNP+student{sn}(1,1);
    numOfNP=numOfNP+1;
end
sn=sn+1;
end

studentCopy{s}=student{s};

if numOfNP==0
    studentCopy{s}(1,1)=afix*student{s}(1,1);
elseif matrixAverage>=0
    studentCopy{s}(1,1)=afix*student{s}(1,1)+(1-afix)*sumNP/numOfNP/n;
else
    studentCopy{s}(1,1)=afix*student{s}(1,1)-(1-afix)*sumNP/numOfNP/n;
end

TempStudent1=studentCopy{s}(1,1);           % using look up table

for i=1:row                                 % get the first line of the Probability
    if TempStudent1-AttenDist(i,1)<0.05
        studentCopy{s}(1,2)=(1-TempStudent1)*g*x1;
        studentCopy{s}(1,3)=(1-TempStudent1)*g*x2;
        studentCopy{s}(1,4)=(1-TempStudent1)*(1-g)*m4;
        studentCopy{s}(1,5)=(1-TempStudent1)*1-sum(studentCopy{s}(1,1:4));
        break;
    end
end

disp([num2str(s),' ',num2str(n),' sumNP numOfNP MatrixAverage ']);
disp([num2str(sumNP),' ',num2str(numOfNP),' ',num2str(matrixAverage)]);
end

disp(conn);                                % print the connectivity table for each day

student=studentCopy;

% close the figure configuration

if n<=20||mod(n,5)==0
    hold off;
end
MatrixLine=MatrixLine+8;
end

% output the total counts for each state

```



```

                                %start from column 3 which is the current state
for s=3:iterCol:startCol+iterCol*(NumOfStudents-1)+3

```

```

    counter0=0;
    counter1=0;
    counter2=0;
    counter3=0;
    counter4=0;
    for j=1:NumOfDays
        if result(j,s)==0
            counter0=counter0+1;
        elseif result(j,s)==1
            counter1=counter1+1;
        elseif result(j,s)==2
            counter2=counter2+1;
        elseif result(j,s)==3
            counter3=counter3+1;
        else
            counter4=counter4+1;
        end
    end
    studentID=(s-3)/iterCol+1;
    StateCouter(1,(studentID-1)*3+1)=counter0;
    StateCouter(2,(studentID-1)*3+1)=counter1;
    StateCouter(3,(studentID-1)*3+1)=counter2;
    StateCouter(4,(studentID-1)*3+1)=counter3;
    StateCouter(5,(studentID-1)*3+1)=counter4;
    StateCouter(6,(studentID-1)*3+1)=sum(StateCouter(1:5,(studentID-1)*3+1));

    StateCouter(1,(studentID-1)*3+2)=counter0/NumOfDays;
    StateCouter(2,(studentID-1)*3+2)=counter1/NumOfDays;
    StateCouter(3,(studentID-1)*3+2)=counter2/NumOfDays;
    StateCouter(4,(studentID-1)*3+2)=counter3/NumOfDays;
    StateCouter(5,(studentID-1)*3+2)=counter4/NumOfDays;
    StateCouter(6,(studentID-1)*3+2)=sum(StateCouter(1:5,(studentID-1)*3+2));

    StateCouter(1,(studentID-1)*3+3)=AbsencesProb{studentID}(1,1);
    StateCouter(2,(studentID-1)*3+3)=AbsencesProb{studentID}(1,2);
    StateCouter(3,(studentID-1)*3+3)=AbsencesProb{studentID}(1,3);
    StateCouter(4,(studentID-1)*3+3)=AbsencesProb{studentID}(1,4);
    StateCouter(5,(studentID-1)*3+3)=AbsencesProb{studentID}(1,5);
    StateCouter(6,(studentID-1)*3+3)=sum(StateCouter(1:5,(studentID-1)*3+3));
end

```

```

                                %Calculate Present and Absent number of days as below

```

```

for s=1:NumOfStudents

```

```

Present(1,(s-1)*4+1)=StateCouter(1,(s-1)*3+1);
Present(1,(s-1)*4+2)=StateCouter(1,(s-1)*3+2);
Present(1,(s-1)*4+3)=StateCouter(1,(s-1)*3+1);
Present(1,(s-1)*4+4)=StateCouter(1,(s-1)*3+2);
Present(2,(s-1)*4+1)=StateCouter(2,(s-1)*3+1);
Present(2,(s-1)*4+2)=StateCouter(2,(s-1)*3+2);
Present(2,(s-1)*4+3)=StateCouter(2,(s-1)*3+1);
Present(2,(s-1)*4+4)=StateCouter(2,(s-1)*3+2);
Present(3,(s-1)*4+1)=StateCouter(5,(s-1)*3+1);
Present(3,(s-1)*4+2)=StateCouter(5,(s-1)*3+2);
Present(3,(s-1)*4+3)=StateCouter(5,(s-1)*3+1);
Present(3,(s-1)*4+4)=StateCouter(5,(s-1)*3+2);
Present(4,(s-1)*4+1)=0;
Present(4,(s-1)*4+2)=0;
Present(4,(s-1)*4+3)=StateCouter(3,(s-1)*3+1);
Present(4,(s-1)*4+4)=StateCouter(3,(s-1)*3+2);
Present(5,(s-1)*4+1)=sum(Present(1:4,(s-1)*4+1));
Present(5,(s-1)*4+2)=sum(Present(1:4,(s-1)*4+2));
Present(5,(s-1)*4+3)=sum(Present(1:4,(s-1)*4+3));
Present(5,(s-1)*4+4)=sum(Present(1:4,(s-1)*4+4));

```

```

Absent(1,(s-1)*4+1)=StateCouter(3,(s-1)*3+1);
Absent(1,(s-1)*4+2)=StateCouter(3,(s-1)*3+2);
Absent(1,(s-1)*4+3)=0;
Absent(1,(s-1)*4+4)=0;
Absent(2,(s-1)*4+1)=StateCouter(4,(s-1)*3+1);
Absent(2,(s-1)*4+2)=StateCouter(4,(s-1)*3+2);
Absent(2,(s-1)*4+3)=StateCouter(4,(s-1)*3+1);
Absent(2,(s-1)*4+4)=StateCouter(4,(s-1)*3+2);
Absent(3,(s-1)*4+1)=sum(Absent(1:2,(s-1)*4+1));
Absent(3,(s-1)*4+2)=sum(Absent(1:2,(s-1)*4+2));
Absent(3,(s-1)*4+3)=sum(Absent(1:2,(s-1)*4+3));
Absent(3,(s-1)*4+4)=sum(Absent(1:2,(s-1)*4+4));

```

end

```
if NumOfDays==180
```

```
    %read the Make-Up lookup table
```

```
    MakeUpTable=xlsread('ModelStudent','AttendDistribution','D59:D108');
```

```
    mycell={'Unsupervised'};
    xlswrite('result',mycell,'AttendAssmt','B1');
    xlswrite('result',mycell,'AttendAssmt','H1');
```

```
    mycell={'Supervised'};
    xlswrite('result',mycell,'AttendAssmt','D1');
    xlswrite('result',mycell,'AttendAssmt','J1');
```

```

mycell={'student ID'};
xlswrite('result',mycell,'AttendAssmt','A2');

mycell={'status'};
xlswrite('result',mycell,'AttendAssmt','B2');
xlswrite('result',mycell,'AttendAssmt','D2');
xlswrite('result',mycell,'AttendAssmt','G2');

mycell={'Promoted'};
xlswrite('result',mycell,'AttendAssmt','C2');
xlswrite('result',mycell,'AttendAssmt','E2');

xlswrite('result',{'Number'},'AttendAssmt','H2');
xlswrite('result','%','AttendAssmt','I2');
xlswrite('result',{'Number'},'AttendAssmt','J2');
xlswrite('result','%','AttendAssmt','K2');

xlswrite('result',{'Promoted'},'AttendAssmt','G3');
xlswrite('result',{'Repeated'},'AttendAssmt','G4');
xlswrite('result',{'Drop-Out'},'AttendAssmt','G5');
xlswrite('result',{'Total Promoted'},'AttendAssmt','G6');

                                %initial the counter for both unsupervised and supervised
UnsuperPromoted=0;
UnsuperMakeup=0;
UnsuperDropout=0;

SuperPromoted=0;
SuperMakeup=0;
SuperDropout=0;

for i=1:NumOfStudents
    xlswrite('result',{'Student ',char(num2str(i))},'AttendAssmt',['A',char(num2str(i+2))]);
    TempRandom=rand(1);
    if Present(5,(i-1)*4+1)>=165
        UnsuperPromoted=UnsuperPromoted+1;
        xlswrite('result',{'Promoted'},'AttendAssmt',['B',char(num2str(i+2))]);
        xlswrite('result',{'Yes'},'AttendAssmt',['C',char(num2str(i+2))]);
    else
        if TempRandom<=MakeUpTable(Absent(3,(i-1)*4+1)-15,1)*7/40
            UnsuperMakeup=UnsuperMakeup+1;
            xlswrite('result',{'Make Up'},'AttendAssmt',['B',char(num2str(i+2))]);
            xlswrite('result',{'Yes'},'AttendAssmt',['C',char(num2str(i+2))]);
        else
            UnsuperDropout=UnsuperDropout+1;
            xlswrite('result',{'Drop Out'},'AttendAssmt',['B',char(num2str(i+2))]);
        end
    end
end

```

```

        xlswrite('result',{ 'No'},'AttendAssmt',['C',char(num2str(i+2))]);
    end
end
if Present(5,(i-1)*4+3)>=165
    SuperPromoted=SuperPromoted+1;
    xlswrite('result',{ 'Promoted'},'AttendAssmt',['D',char(num2str(i+2))]);
    xlswrite('result',{ 'Yes'},'AttendAssmt',['E',char(num2str(i+2))]);
else
    if TempRandom<=MakeUpTable(Absent(3,(i-1)*4+3)-15,1)*7/40
        SuperMakeup=SuperMakeup+1;
        xlswrite('result',{ 'Make Up'},'AttendAssmt',['D',char(num2str(i+2))]);
        xlswrite('result',{ 'Yes'},'AttendAssmt',['E',char(num2str(i+2))]);
    else
        SuperDropout=SuperDropout+1;
        xlswrite('result',{ 'Drop Out'},'AttendAssmt',['D',char(num2str(i+2))]);
        xlswrite('result',{ 'No'},'AttendAssmt',['E',char(num2str(i+2))]);
    end
end
end
UnsuperGraduate=UnsuperPromoted+UnsuperMakeup;
SuperGraduate=SuperPromoted+SuperMakeup;

AssmtVector(1,1)=UnsuperPromoted;
AssmtVector(1,2)=UnsuperPromoted/NumOfStudents;
AssmtVector(2,1)=UnsuperMakeup;
AssmtVector(2,2)=UnsuperMakeup/NumOfStudents;
AssmtVector(3,1)=UnsuperDropout;
AssmtVector(3,2)=UnsuperDropout/NumOfStudents;
AssmtVector(4,1)=UnsuperGraduate;
AssmtVector(4,2)=UnsuperGraduate/NumOfStudents;

AssmtVector(1,3)=SuperPromoted;
AssmtVector(1,4)=SuperPromoted/NumOfStudents;
AssmtVector(2,3)=SuperMakeup;
AssmtVector(2,4)=SuperMakeup/NumOfStudents;
AssmtVector(3,3)=SuperDropout;
AssmtVector(3,4)=SuperDropout/NumOfStudents;
AssmtVector(4,3)=SuperGraduate;
AssmtVector(4,4)=SuperGraduate/NumOfStudents;

xlswrite('result',AssmtVector,'AttendAssmt','H3');
end
xlswrite('result',result,'mresult','A2');
xlswrite('result',StateCouter,'Attendances','A2');
xlswrite('result',Present,'Present&Absent','B3');
xlswrite('result',Absent,'Present&Absent','B9'); .....end

```

Appendix 2. THE ATTENDANCE STATES OF THE STUDENTS

Days	Random #	Stud1	Stud2	Stud3	Stud4	Stud5	Stud6	Stud7	Stud8	Stud9
1	0.814724	0	4	0	0	0	0	0	0	0
2	0.00782	4	0	0	0	0	0	0	0	0
3	0.186571	0	1	0	0	0	0	0	0	0
4	0.74849	1	1	0	0	0	0	0	0	0
5	0.377512	1	0	0	0	0	0	0	0	0
6	0.962959	0	4	0	0	1	0	3	0	4
7	0.789256	4	0	0	0	1	1	3	3	4
8	0.87345	0	4	0	0	1	1	3	3	0
9	0.935234	4	0	0	0	1	1	3	3	0
10	0.971035	0	4	0	0	1	1	3	3	2
11	0.944107	4	0	0	0	1	1	3	3	2
12	0.981283	0	4	0	3	1	1	3	3	2
13	0.194648	4	0	3	0	1	0	3	0	2
14	0.884512	0	4	0	0	0	0	0	0	0
15	0.323787	4	0	0	0	0	0	0	0	0
16	0.781287	0	4	0	0	0	0	0	0	0
17	0.874201	4	0	0	0	0	0	0	0	0
18	0.250516	0	2	0	0	0	0	0	0	0
19	0.499074	2	0	0	0	0	0	0	0	0
20	0.364042	0	2	0	0	0	0	0	0	0
21	0.242964	2	0	0	0	0	0	0	0	0
22	0.501671	0	3	0	0	0	0	0	0	0
23	0.642184	3	0	0	0	0	0	0	0	0
24	0.171214	0	1	0	0	0	0	0	0	0
25	0.813498	1	1	0	0	0	0	0	0	0
26	0.102277	1	0	0	0	0	0	0	0	0
27	0.178597	0	1	0	0	0	0	0	0	0
28	0.342104	1	0	0	0	0	0	0	0	0
29	0.61555	0	4	0	0	0	0	0	0	0
30	0.594426	4	0	0	0	0	0	0	0	0
31	0.114232	0	1	0	0	0	0	0	0	0
32	0.293887	1	0	0	0	0	0	0	0	0
33	0.700893	0	4	0	0	0	0	0	0	0
34	0.407163	4	0	0	0	0	0	0	0	0
35	0.972325	0	4	0	3	0	4	0	4	0

Days	Random #	Stud1	Stud2	Stud3	Stud4	Stud5	Stud6	Stud7	Stud8	Stud9									
36	0.574937	4	0	3	0	4	0	4	0	4	0	3	0	4	0	4	0		
37	0.743245	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
38	0.333447	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
39	0.06737	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
40	0.09843	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
41	0.92828	0	4	0	0	0	1	0	2	0	3	0	0	0	2	0	2	0	3
42	0.125912	4	0	0	0	1	0	2	0	3	0	0	0	2	0	2	0	3	0
43	0.718208	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
44	0.281619	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	0.800691	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
46	0.009753	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	0.075959	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	0.28207	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	0.147308	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0.691719	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
51	0.231021	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
52	0.311761	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
53	0.970524	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4
54	0.823455	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0
55	0.974895	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4
56	0.5405	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0
57	0.963649	0	4	0	3	0	4	0	4	0	4	0	3	0	4	0	4	0	4
58	0.835781	4	0	3	3	4	0	4	0	4	0	3	3	4	0	4	0	4	0
59	0.644164	0	4	3	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0
60	0.929142	4	0	0	1	0	2	0	3	0	3	0	1	0	2	0	3	0	4
61	0.999258	0	4	1	1	2	2	3	3	3	3	1	1	2	2	3	3	4	4
62	0.199395	4	0	1	0	2	0	3	0	3	0	1	0	2	0	3	0	4	0
63	0.595685	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
64	0.734683	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
65	0.061514	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
66	0.132409	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
67	0.500203	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
68	0.656837	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
69	0.962317	0	4	0	3	0	4	0	4	0	4	0	4	0	4	0	4	0	4
70	0.718891	4	0	3	3	4	0	4	0	4	0	4	0	4	0	4	0	4	0

Days	Random #	Stud1	Stud2	Stud3	Stud4	Stud5	Stud6	Stud7	Stud8	Stud9							
71	0.972249	0	4	3	3	0	4	0	4	0	4	0	4	0	4	0	4
72	0.100181	4	0	3	0	4	0	4	0	4	0	4	0	4	0	4	0
73	0.74716	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
74	0.566045	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
75	0.134025	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
76	0.803425	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
77	0.954566	1	1	0	3	0	4	0	4	0	4	0	3	0	4	0	4
78	0.255384	1	0	3	0	4	0	4	0	4	0	3	0	4	0	4	0
79	0.421467	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
80	0.343712	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
81	0.178318	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
82	0.94905	1	1	0	3	0	3	0	4	0	4	0	3	0	4	0	4
83	0.117084	1	0	3	0	3	0	4	0	4	0	3	0	4	0	4	0
84	0.971946	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4
85	0.360879	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0
86	0.917657	0	4	0	1	0	2	0	3	0	3	0	2	0	3	0	3
87	0.132365	4	0	1	0	2	0	3	0	3	0	2	0	3	0	3	0
88	0.471583	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
89	0.382117	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
90	0.970866	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4
91	0.136015	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0
92	0.712946	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
93	0.204704	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
94	0.81021	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	1
95	0.437507	4	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
96	0.043786	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
97	0.03557	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
98	0.925134	0	4	0	2	0	3	0	3	0	3	0	2	0	3	0	4
99	0.79837	4	0	2	2	3	3	3	3	3	2	2	3	3	3	4	0
100	0.039324	0	1	2	0	3	0	3	0	3	0	2	0	3	0	3	0
101	0.858034	1	1	0	0	0	0	1	0	1	0	0	0	1	0	1	2
102	0.889536	1	1	0	1	0	1	1	1	1	0	1	1	1	1	2	2
103	0.611741	1	0	1	0	1	0	1	0	1	0	1	0	1	0	2	0
104	0.636304	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
105	0.477704	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Days	Random #	Stud1	Stud2	Stud3	Stud4	Stud5	Stud6	Stud7	Stud8	Stud9							
106	0.951487	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4
107	0.991735	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
108	0.747589	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0
109	0.740532	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
110	0.329241	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
111	0.921097	0	4	0	2	0	3	0	3	0	3	0	3	0	3	0	4
112	0.630423	4	0	2	0	3	0	3	0	3	0	3	0	3	0	4	0
113	0.36277	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
114	0.512032	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
115	0.781006	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
116	0.957533	4	0	0	4	0	4	0	4	0	4	0	4	0	4	0	4
117	0.718802	0	4	4	0	4	0	4	0	4	0	4	0	4	0	4	0
118	0.001877	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
119	0.422026	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
120	0.346543	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
121	0.099366	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
122	0.685273	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
123	0.891401	0	4	0	1	0	2	0	3	0	3	0	2	0	2	0	3
124	0.456547	4	0	1	0	2	0	3	0	3	0	2	0	2	0	3	0
125	0.174784	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
126	0.3021	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
127	0.202489	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
128	0.049169	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
129	0.993881	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4
130	0.537953	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0
131	0.10516	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
132	0.15506	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
133	0.228617	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
134	0.526155	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
135	0.920498	0	4	0	3	0	3	0	3	0	4	0	3	0	3	0	4
136	0.397261	4	0	3	0	3	0	3	0	4	0	3	0	3	0	4	0
137	0.287911	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
138	0.237313	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
139	0.054675	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
140	0.940114	1	1	0	4	0	4	0	4	0	4	0	4	0	4	0	4

Days	Random #	Stud1	Stud2	Stud3	Stud4	Stud5	Stud6	Stud7	Stud8	Stud9									
141	0.029825	1	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0		
142	0.470704	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
143	0.82201	3	3	0	0	0	0	0	1	0	1	0	0	0	1	0	1	0	2
144	0.441015	3	0	0	0	0	0	1	0	1	0	0	0	1	0	1	0	2	0
145	0.676189	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
146	0.512393	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
147	0.245535	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
148	0.151448	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
149	0.573305	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
150	0.102441	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
151	0.070791	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
152	0.447446	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
153	0.060653	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
154	0.110286	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
155	0.956313	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4
156	0.446454	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0
157	0.48195	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
158	0.14739	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
159	0.621687	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
160	0.773824	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
161	0.455521	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
162	0.09305	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
163	0.540615	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
164	0.974811	3	3	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4
165	0.387845	3	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0
166	0.766563	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
167	0.297282	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
168	0.388117	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
169	0.25183	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
170	0.957281	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4
171	0.091749	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0
172	0.490297	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
173	0.58907	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
174	0.494992	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
175	0.792579	3	3	0	0	0	0	0	1	0	1	0	0	0	0	0	1	0	2

Days	Random #	Stud1		Stud2		Stud3		Stud4		Stud5		Stud6		Stud7		Stud8		Stud9	
176	0.866647	3	3	0	2	0	2	1	1	1	1	0	2	0	2	1	1	2	2
177	0.678189	3	0	2	0	2	0	1	0	1	0	2	0	2	0	1	0	2	0
178	0.589848	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
179	0.220915	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
180	0.167721	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0