New Warehouse Designs: Angled Aisles and Their Effects on Travel Distance

by

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Abstract

In supply chain and logistics systems, warehouses and third-party logistics centers have become the backbone of distribution networks that store and deliver goods. However, these facilities are still built to look much as they have been for the past fifty years (Gue and Meller, 2009). This dissertation builds on some recent research that challenges traditional ways of organizing picking aisles and cross aisles in unit-load warehouses. The designs in this dissertation, and the models that produced them, offer to the research and practicing communities a new and fundamentally different way to extract cost and improve the performance of industrial warehouses.

For warehouses with one, two and three cross aisles, we develop closed form, single-command travel distance functions in a continuous warehouse space. We use these functions to minimize expected travel distance in a unit-load warehouse with a centrally located pickup and deposit (P&D) point under randomized storage. We propose three optimal unit-load warehouses with respect to the inserted cross aisles, the Chevron, the Leaf and the Butterfly. In order to provide a more accurate model and to analyze the wasted space for angled aisle designs, we build a discrete model of aisles and pallet locations that better represents warehouse designs.

We develop a general warehouse design tool that models aisles, pallet locations and dock doors as discrete objects. The tool implements a network representation of the warehouse, a shortest path algorithm to determine travel distances, and a particle swarm meta-heuristic to search for the best arrangement of aisles. To illustrate the use of model, we define five design problems that are differentiated by the locations of multiple P&D points. We propose improved non-traditional aisle designs for each design problem with one and two inserted cross aisles.

Last, we introduce the idea of robustness in non-traditional aisle designs with respect to a varying number of P&D points. For two commonly found flow patterns in industry, we use our models to determine optimal designs for an increasingly large block of P&D points to find the point at which the structure “breaks.” The results suggest that the Chevron design, in particular, is robust over a wide range of P&D points.

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I dedicate this dissertation to my parents. I aspire to guide my daughter.

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Chapter 1

Warehousing

1.1 Introduction

In a global economy, supply chain management plays a critical role in maintaining the flow of goods and services. A well-managed supply chain transfers the right items to customers at the right time in the right quantity at a low cost. Warehouses are important links in this chain, and therefore their efficient design and operation is essential to effective supply chain processes. In a world where customer satisfaction and quick delivery are two of the most important competitive advantages of companies, warehouses become even more important. For example, if a company produces goods in China, then assembles them in the US for sales in Europe, the company needs multiple warehouses to deliver items quickly and to get the benefit of economies scale in transportation.

Because of long supplier lead times, many companies serve customers from inventory rather than making to order. Relentless efforts to reduce supply chain and inventory costs have led to a documented reduction in shipment size and its concomitant increase in material handling cost. According to the Bureau of Transportation Statistics, smaller sized shipments have increased by around 56% since 1993, based on the Commodity Flow Survey shipments in 2002 (RITA, 2009). Also, during times of economic growth or stability, stocks must be processed daily or weekly instead of monthly or quarterly, because some warehouses offer same-day or next-day shipping to customers from inventory (Baker, 2004). For example, Amazon offers same-day delivery if an eligible item is ordered before 10 a.m. or 1 p.m. depending on the location. Therefore, quick receiving, storing, retrieving and shipping have become important to respond quickly to customer orders and increase customer satisfaction.

On another front, manufacturers returning to their core competencies or unwilling to use space to store inventory has led to an increase of 3rd party warehousing. In the future, other small and medium manufacturers as well as big companies are expected to continue this trend of utilizing 3rd party logistic centers. Many smaller warehouses are also being replaced by fewer large warehouses so as to get the benefits of economies of scale. In addition to the trend of using warehousing, a dramatic increase in imported goods, approximately 140% more in 2006 than a decade earlier, is leading industry to build larger warehouses and distribution centers in the US (Hudgins, 2006). Hudgins noted that 300,000 square feet or more was the definition of a large warehouse almost a decade ago, but today it is upwards of 1 million square feet. As the size of the warehouse increases, costs related to managing warehouses or distribution centers, such as operation and maintenance costs, also increase. Of all operational costs, labor cost, which is caused by traveling to store and retrieve items, is very significant in most warehouses. Additionally, since 3rd party logistics (3PLs) centers pay their forklift drivers for work-hours and bill their customers for two handles (receiving/storing and retrieving/shipping) for each pallet, both reducing required receiving and retrieving time for a pallet and increasing the number of handles per hour are two of the cornerstones of running a warehouse efficiently and profitably (Bartholdi and Hackman, 2008). Therefore, our purpose in this dissertation is to explore opportunities for reducing operational costs, in terms of travel cost, for a widely encountered class of warehouses.

Although the importance of warehouses and travel distances to store and retrieve items in warehouses is known, warehouses are still built to look much as they have been for the past fifty years (Gue and Meller, 2009). In our experience, the most common layout in these facilities, both implemented in industry and studied in academia, is comprised of a number of parallel aisles with an orthogonal cross

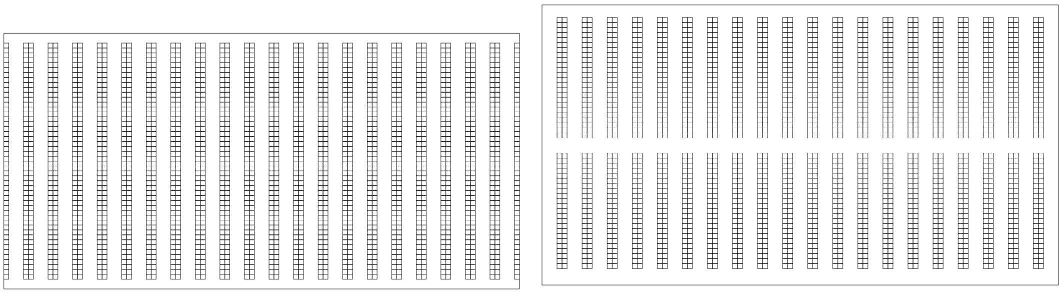


Figure 1.1: Traditional warehouse designs.

aisle to allow more efficient travel between storage locations and pickup and deposit (P&D) points (Figure 1.1).

In this dissertation, we focus on unit-load warehouses because nearly every industrial warehouse has a significant portion of its available space devoted to pallet storage, and many warehouses store and handle pallets exclusively. Because pallet operations are also labor-intensive, much of a worker's time is spent traveling its aisles. Therefore, in our research, our main purpose is to reduce traveling time in aisles to make workers more productive. Our main approach is to design cross and picking aisles for given locations of P&D points in order to reduce travel distances for workers. In the next section, we discuss the main problems of the dissertation.

1.2 Problem Statement

Our dissertation considers ways in which aisles can be oriented to facilitate material flow between established pickup-and-deposit (P&D) points and storage locations, in order to minimize expected travel distance to visit a location.

Problem 1. *What is the optimal angle of aisles for minimizing expected travel distance in unit-load warehouses with a centrally located P&D point?*

We develop a closed form travel distance function, which we use to search for the optimal aisle angles by minimizing expected travel distance in a unit-load warehouse with a centrally located P&D point under randomized storage and single-command operations. We consider both the one and two cross aisle cases. Next, we address another problem: what percentage of space is wasted in new aisle designs due to angled aisles, or how much additional space do we need to have in new aisle designs so as to provide the same number of storage locations as in traditional designs? In order to analyze the wasted space for angled aisle designs, we develop a more accurate model in which discrete locations of available pallet positions and the P&D point are represented in a network.

Problem 2. *What is the best arrangement of aisles in unit-load warehouses with multiple P&D points?*

Our discussions with warehouse and distribution center managers triggered our consideration of several alternative locations for multiple P&D points in unit-load warehouses. Consequently, we look for the best possible arrangement of aisles in a warehouse with pre-specified P&D points in order to minimize expected travel distance. In order to model a warehouse with multiple P&D points and angled aisles, we develop a warehouse network model and a shortest path algorithm, which finds the shortest distance between a storage location and a P&D point. We use a constructive meta-heuristic algorithm, Particle Swarm Optimization (PSO), to search for aisle angles in the warehouse space in order to find the best aisle structure for given P&D locations.

Problem 3. *How robust are non-traditional aisle designs with respect to a varying number of P&D points?*

To answer this question, we first focus on two configurations of P&D points: (1) P&D points are placed at the bottom, and (2) P&D points are placed both at the bottom and the top sides of the warehouse. We considered the non-traditional aisle designs developed for centrally located P&D point(s) in these configurations as primary designs, and we show the changes in improvement of the primary designs as the number of P&D points increases. We then increase the number of P&D

points in these configurations gradually and search for improved designs with one and two cross aisles. The primary designs are compared to the improved designs in order to investigate their robustness in terms of changes in aisle structure and improvement. Hence, we show specific improved designs and the effectiveness of the primary designs when the material flow is distributed along the side(s) of the warehouse.

1.3 Organization of the Dissertation

The remainder of this dissertation is organized as follows. In the next chapter, we discuss the main operations encountered in warehouses and present the underlying reasons behind our model assumptions. In Chapter 3, we review traditional and non-traditional warehouse designs and their similarities with some natural structures and material flow systems. In Chapter 4, we introduce three optimal designs, the Chevron, the Leaf, and the Butterfly, for unit-load warehouses with a single P&D point. In Chapter 5, we present a warehouse network model and PSO algorithm for finding the best aisle structures in a unit-load warehouse with pre-located P&D points. In Chapter 6, we introduce the concept of grouping a number of P&D points and representing them with a centrally located P&D point. Lastly, we offer conclusions and plans for future work in Chapter 7.

Chapter 2

Warehouse Operations

2.1 Introduction

Manufacturing plants often have a warehouse to store raw material, work-in-process items or finished goods. The term *distribution center* is often used to differentiate a place that is a transshipment node in a supply chain network. De Koster et al. (2007) used the term *warehouse* as a place "for storing or buffering products" and *distribution center* as a place "for transshipment or cross-docking" in addition to storage. Rouwenhorst et al. (2000) and Frazelle (2002) used the term warehouse in a wider sense to address a place handling transshipment as well as storage. Hence, in a broader sense, we need warehouses: (1) to buffer products for a certain time to better match supply with customer demand in a shorter time, (2) to consolidate or accumulate shipments from suppliers into combined shipments for downstream customers, and (3) to provide value-added activities such as pricing, product customization, labeling or kitting (Bartholdi and Hackman, 2008). In addition to these reasons, Lambert et al. (1998) mentioned that the advantage of purchase discounts, transportation economies and supporting just-in-time programs of suppliers and customers are other reasons why warehouses exist.

The basic operations in warehouses are to receive items from suppliers, stow them in storage locations, retrieve them to fulfill customers' orders, sort them if required, and ship the completed orders to customers. Receiving starts with an arrival truck to a dock. Products usually arrive in larger units, such as pallets, and sometimes in cases. Even though the warehouse receives pallets, it might need to break pallets out into separate cartons. After products are unloaded to a pickup and deposit point or input/output point, they are taken by workers to be put away in storage locations. The storage area might be comprised of two main sections: a reserve area and a forward area. Whereas stock keep units (SKUs) are stored as pallets or unit-loads in a reserve area, they are stored in smaller units such as cases in a forward picking area. Pallets are moved in a single trip from and to the reserve area. However, in a forward picking area, orderpickers perform multiple picks in a trip to fulfill customer orders. On the other hand, while many warehouses allocate an important portion of their space to a reserve storage area, there are also some warehouses that only handle pallets at a time such as 3PLs centers and import warehouses. Once picking is completed, customers' orders are gathered together at the pickup and deposit point, then packed and loaded into trucks to ship them to customers.

2.2 Receiving and Shipping

The first process encountered in a warehouse is receiving, which is initiated by the notification of the arrival of goods. After a truck or an internal transport (in a manufacturing plan) arrives to the docks, products are unloaded and transported to a place for inspection or depalletizing before being put away. In our research, we call this place a "pickup and deposit (P&D) point."

Once an order arrives to the warehouse management system, products are received from storage locations and transferred to the P&D points in the shipping area. After a customer order is sorted and palletized, if necessary, it is loaded to trucks at the docks for shipping them to the customers. In some warehouses, docks are assigned for both receiving and shipping operations; for others, they are separate. Hence, a P&D point in a warehouse could be a place that comprises an inspection area, a palletizing or shrink wrap machine, or that provides instructions for workers. In our research, we assume that travel starts and ends at P&D points.

The literature on receiving and shipping has mainly focused on truck-to-dock assignment policies or problems for cross-docking warehouses (see Gu et al., 2007, for details). In a cross-docking warehouse, arriving products to the warehouse are sorted and transferred to the shipment area without being put away to storage locations. Generally, dock doors in the shipping area are specifically assigned to customers. Hence, the assignment problem is to specify which incoming trucks and outgoing trucks are scheduled for which dock doors. Additionally, Gue et al. (2010) considered multiple dock doors on one side of the warehouse and presented different aisle orientations to reduce travel distance from docks.

2.3 Storage

Storing and picking are the most time consuming activities in most warehouses (Tompkins et al., 2003). Storage locations can be distinguished as unit-load storage and less-than unit-load storage. In unit-load storage, large products are stored either as block stacking or in pallet racks. Pallet racks can be single deep or multi-deep. We focus on unit-load storage areas in our research, because they are almost ubiquitous in warehousing. For example, most retail distribution centers include a reserve area in which items are moved in pallets, and replenishment is done in unit-loads from reserve areas to forward picking areas. Additionally, unit-load warehousing is very common in warehouses such as 3rd party transshipment warehouses and import warehouses.

The main storage policies used to allocate pallets to the storage locations are randomized, closest-open-location, dedicated, class-based, and full turnover-based. In a randomized storage policy, each open storage location has an equal probability of being selected to store pallets. Because tracking which locations are available for storage requires computerized management, this policy can only work with an appropriate technology. The randomized storage policy is widely used in industry because of its simplicity and higher utilization of storage locations (Petersen, 1999;

Petersen and Aase, 2004). In the closest-open-location policy, open locations are tracked by a management system and once a pallet arrives, it is assigned to the available storage location closest to the P&D point. This method is also widely used in industry. Hausman et al. (1976), Schwarz et al. (1978), and Graves et al. (1977) used the randomized storage policy to approximate the closest-open-location policy.

If storage locations are reserved for specific products, the policy is called dedicated storage. Order pickers know where to store pallets in advance, hence they become familiar with their locations in time, and this decreases searching time for locations. Dedicated storage has the disadvantage of low utilization of storage locations. Bartholdi and Hackman (2008) note that on average, storage utilization is only 50% in dedicated storage. This policy might be useful when applied to forward pick areas whereas randomized storage is applied to the reserve area (De Koster et al., 2007). Class-based storage is a hybrid between randomized and dedicated storage. In this policy, products are partitioned into classes based on their turnover rate, and each class occupies some set of dedicated storage locations. Higher turnover items are assigned to pallet locations closest to the P&D point. Pallets are stored randomly within each class. The advantage of this policy is that fast moving items are stored close to the P&D point, and space utilization is relatively high because of randomized storage in a class. Last, in full turnover-based storage, the highest-turnover item is assigned to the location closest to the P&D point. The next higher-turnover item is assigned to the next closest location and so on. Hence, this policy represents the limit of class-based storage policy to obtain a bound for the improvement of potential of class-based storage (Schwarz et al., 1978). One of the most well known implementations of full turnover-based storage is cube-per-order index (COI). The COI of a product is the ratio of the product's total required space to the number of trips required to satisfy its demand per period. The lower the COI of a product, the closer to the P&D point. The detailed literature about COI and other storage policies are seen in De Koster et al. (2007) and Gu et al. (2007).

The literature about storage policies that we have reviewed up to this point has focused exclusively on conventional warehouse designs. However, some recent non-traditional aisle design studies open a new area in the warehouse literature. Gue and Meller (2009) presented two non-traditional aisle designs in unit-load warehouses under the randomized storage policy. Pohl et al. (2009b) studied one of the proposed designs by Gue and Meller (2009). They also assumed a randomized storage policy because of its advantage of high space utilization. Pohl et al. (2010) also investigated the effect of turnover-based storage on the travel in non-traditional unit-load warehouses. They concluded that the non-traditional aisle designs developed for randomized storage still offer improvement in expected travel distance for turnover-based storage.

Because new aisle designs require larger warehouses than traditional warehouses, we assume the randomized storage policy in our models. Hence, the randomized storage policy is expected to reduce the effect of losing some storage locations in new aisle designs, because it offers higher storage utilization than other storage policies. With this policy, pallets are approximately stored in the closest available location that leads to the most efficient use of storage space. Also, if there is inadequate information about the characteristics of arriving items, which is needed to manage dedicated and class-based storage policies, a randomized storage policy can be easily implemented. Hence, a warehouse management system can easily direct workers to appropriate locations.

2.4 Picking

Picking of items in a warehouse involves traveling to the storage location(s), extracting items, and taking them for shipping to customers. Because these activities are time consuming and require a large workforce in many warehouses, picking is one of the major cost components among operational costs (Tompkins et al., 2003). There are two main picking systems that involve workers in different roles in a warehouse: part-to-picker and picker-to-part systems. In part-to-picker systems, workers are not involved in the picking actively; instead, automated storage and retrieval systems (AS/RS) or carousels, are used to extract items from storage locations. Then, workers handle the remaining operations such as sorting (if necessary), inspection and transferring to the next station. In picker-to-part systems, workers are actively involved in picking from traveling and finding items to extracting and transferring them to the next station. De Koster (2004) mentioned that the picker-to-part systems are the most common systems where order pickers spend most of their time on walking or driving along the aisles. De Koster (2004) also pointed out that 80% of all order-picking systems in Western Europe, as well as the majority of the warehouses worldwide, are comprised of picker-to-part systems. Therefore, we consider picker-to-part systems in our research. See Gu et al. (2007) and Gu et al. (2010) for detailed literature on picking activities and travel models in AS/RS systems or carousels.

A picking activity can be done in a single trip, or done in multiple trips. In a picking environment, as well as a storing environment, single-command operation is the simplest way to transfer items to and from storage locations. In single-command operations, one item, usually a pallet, is picked or stored in one trip by an operator. Therefore, workers travel empty either when they return to a P&D point or go to a storage location for picking (dead-heading). This operation is common in unit-load warehousing and unit-load replenishment in a reserve area (van den Berg et al., 1998). Francis (1967a) and Bassan et al. (1980) are two of the earliest studies that have presented a single-command travel distance model in conventional warehouses. Most studies have mainly focused on modeling expected travel time for single-command operations in AS/RS systems. For example, Hausman et al. (1976) modeled single-command travel in unit-load AS/RS for randomized and dedicated storage policies. Goetschalckx and Ratliff (1990) also simulated single-command operations in a unit-load warehouse for shared and dedicated storage policies.

In order to eliminate the dead-heading in single-command operations, some unit-load warehouses work with dual-command operations. In a dual-command operation, an operator handles two transactions in one trip, stowing and picking. On the trip from a P&D point to a storage location, an operator performs a deposit transaction. Then, the operator performs a withdrawal transaction and returns to the P&D point. Hence, the dual-command operation minimizes the empty travel in manual picking and dead-head travel of the crane in AS/RS. However, in this operation, matching storage and retrieval locations might require a level of computerized management. One of the earliest papers on modeling dual-command travel in conventional warehouses is Malmborg and Krishnakumar (1987). Some other research has focused on dual-command operations in AS/RS systems. We refer Gu et al. (2010) for a list of references about single-command and dual-command operations in unit-load AS/RS.

If a number of goods are retrieved from a number of predetermined storage locations in one trip, this operation called multi-command operation or order-picking. Order-pickers visit multiple aisles to pick items given as a list that shows the SKUs, their required quantities and locations. According to Bartholdi and Hackman (2008), travel time in aisles is a non-value added activity in order-picking operations, and it has the first priority to be improved. De Koster et al. (2007) discussed that travel is the most dominant component of order-picking time, even though some other activities might have an effect on it. Hence, minimization of the travel distance in a tour has been extensively considered in order-picking. Kunder and Gudehus (1975) and Jarvis and McDowell (1991) developed a model for expected tour length using a traversal strategy under randomized storage in a single-block warehouse. Jarvis and McDowell used their model to optimally allocate products to storage locations in symmetric and non-symmetric warehouses. Hall (1993) considered randomized storage policy in a single-block warehouse with an even number of aisles to derive travel models. He applied several routing algorithms in their models such as traversal, midpoint, largest-gap and double traversal. Caron et al. (1998) proposed travel models considering COI-based storage policies in a two-block warehouse. Roodbergen and Vis (2006) also derived an average travel distance as a function of a number of layout elements in a similar way to Kunder and Gudehus (1975) and Hall (1993). Ratliff and Rosenthal (1983) formulated a picking tour as a Traveling Salesman Problem (TSP) and developed a dynamic programming based optimal algorithm to solve the TSP problem in polynomial time. Goetschalckx and Ratliff (1988a,b), De Koster and van der Poort (1998) and Roodbergen and de Koster (2001) also proposed an optimal algorithm to solve the sequence of the pick lists under some varieties of order-picking policies. For a detailed review of order picking, we refer to De Koster et al. (2007).

The references that we have mentioned so far focused on conventional warehouse layouts. Recently, some studies also focused on picking in non-traditional unit-load warehouses. Gue and Meller (2009) developed a single-command expected travel distance expression for two non-traditional layouts, the Flying-V and the Fishbone, in order to optimize some design parameters. Pohl et al. (2009a) developed expected travel distance equations in traditional designs with a middle cross aisle for dual-command operations. Pohl et al. (2009b) also modeled a dual-command expected travel distance expression in Fishbone designs. Gue et al. (2010) derived single-command expected travel distance expressions from multiple P&D points in two non-traditional aisle designs, the modified Flying-V and the Inverted-V.

In this dissertation, we model our problems for single-command operations, because it is common in unit-load warehouses and bulk storage areas, as well as in crossdocking operations. Although dual-command operations reduce dead-head travel, single-command operations are still appropriate and common in warehouses where required information systems to manage dual-command operations are inadequate. Gu et al. (2010) mentioned that single-command operation is the most common assumption for storage department layout problems in warehouse design.

Chapter 3

Designing Warehouses

3.1 Introduction

The adoption of new technologies and management philosophies bring both opportunities for warehouses to improve shipment accuracy and to reduce inventory and lead times. For example, lean warehousing, Just-In-Time, information technologies such as labeling, bar coding, radio frequency communications, and warehouse management systems provide opportunities to improve warehouse operations. Also, operational efficiency in warehouses is strongly affected by design decisions, although these can be expensive or impossible to change once the warehouse is built (Gu et al., 2010). However, Gu et al. also note that most warehouse design research has focused on analysis instead of synthesis, which might be accomplished by combining, say, layout problems with operational problems, such as receiving or order picking.

Analyzing a warehouse design is quantifying the effectiveness of its layout, storage policies, order picking strategies, etc. Designing a warehouse is an act of creation, in which the task is to select and arrange available or new technologies to accomplish some operational objective. Gu et al. (2010) divide warehouse design decisions into five main categories: selecting an aisle structure and orientation, a number of aisles, door locations, storage policies and sizes and dimensions of departments. Most of the existing research on warehouse design focuses on some variations of the design in Figure 3.1, in which storage racks are arranged to form straight and parallel picking aisles, and cross aisles (if present) are perpendicular to the picking aisles. The variations of this design mainly include one middle cross aisle (see in Figure 3.2).

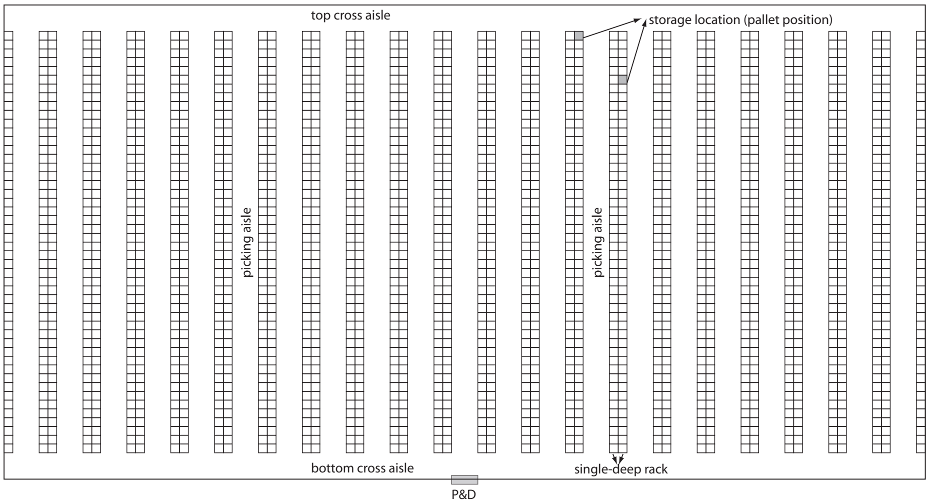
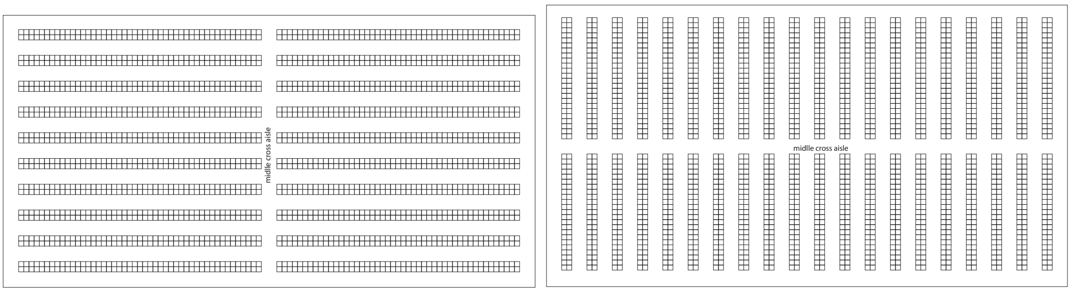


Figure 3.1: A traditional warehouse layout with design elements.



(a) (b)

Figure 3.2: Two main variations of traditional warehouses.

Gue and Meller (2009) mentioned two unspoken design rules for these traditional warehouses:

- All picking aisles must be straight and parallel to each other.
- Cross aisles must be orthogonal and must form a right angle with picking aisles.

Interestingly, to the best of our knowledge, most warehouses in industry and also in warehouse literature meet these rules. In traditional warehouses, researchers used different combinations of P&D locations. Kunder and Gudehus (1975) and Hall (1993) considered a centrally located P&D point to derive their travel models in manual order-picking. Roodbergen and Vis (2006) considered two different P&D

locations for different order-picking problems with one middle cross aisle, at the lower-left and in the middle of the bottom cross aisle. Roodbergen and Vis also showed that the middle P&D point is the best location for multi-command operations. Caron et al. (1998) also proposed travel models in a warehouse with a middle cross aisle and a centrally located P&D point. On the other hand, some studies located a P&D point at the corner of the warehouse instead of placing it centrally. Chew and Tang (1999), Goetschalckx and Ratliff (1990) and Hausman et al. (1976) located a single P&D point at the lower-left corner of the warehouse. Hsieh and Tsai (2006) considered two different locations of input and output points, and they are located at the lower-left and the lower-right corner of the warehouse. Hence, the picker starts traveling from the input point and finishes it at the output point.

Because Roodbergen and Vis (2006) showed that the middle P&D point is the best, we first consider a centrally located P&D point in our research. Because the locations of P&D points play an important role on the travel distance, we consider multiple P&D points at several predefined locations of different sides of the warehouse. We then search for better aisle designs than traditional designs for unit-load warehouses for given P&D locations, if present.

Although many studies have focused on a single-block layout without any cross aisles, some studies have also considered one or multiple middle cross aisles in warehouses. Petersen (2002) investigated the effects of a number of aisles and the aisle length on the average travel distance of a tour. Guenov and Raeside (1992) investigated the optimum aisle width in an AS/RS system for multi-command operations. Roodbergen and de Koster (2001) compared the average travel time in traditional layouts with and without a middle cross aisle, and proved that the warehouse with a cross aisle has a shorter travel in order-picking. Vaughan and Petersen (1999) investigated the effects of cross aisles on order picking efficiency. They inserted a number of lateral cross aisles into a warehouse that consists of one bottom and one top cross aisle. They showed that increasing the number of cross aisles reduces the average picking distance in order-picking. In our research, we also investigate the effects of additional inserted cross aisles on expected travel distance for single-command operations, as well as the wasted space by cross aisles in unit-load warehouses.

Up to now, we have reviewed studies on design characteristics in traditional warehouse layouts. In the next section, we also examine non-traditional warehouse designs and some of their design characteristics.

3.2 Non-Traditional Aisle Designs

Non-traditional aisle designs propose a different aisle structure or aisle orientation than that in the traditional layouts so that the new designs offer improvement in travel distance or some other objectives. For example, Moder and Thornton (1965) introduced a variable, which is “slant angle of the pallets” in order to measure the floor space utilization. They developed a mathematical model to evaluate how floor space efficiency changes with the angle of placement of the pallets and width of the aisle. Francis (1967a,b) investigated the shape of optimal warehouse designs with a single dock. They considered rectilinear travel along “the presupposed orthogonal network of aisles parallel to the x and y axis”, between the dock and the storage space. Berry (1968) proposed two types of warehouse layouts to investigate space utilization and traveling cost of handling a unit. The first layout assumes that there are rectangular pallet blocks with the same depth arranged around a main orthogonal gangway. The second layout assumes that floor-stored pallets are arranged in different depths around a “single diagonal gangway providing access to all stacks.” Although these studies seemed to introduce the idea of placement of cross aisles differently, we think White (1972) is the first study that shows the impact of angled aisles on travel distance.

White (1972) proposed “radial aisles” in order to reduce travel distance in a non-rectangular warehouse design. White showed that travel distance from a P&D point to any point in a storage area gets closer to the Euclidean distance, when a

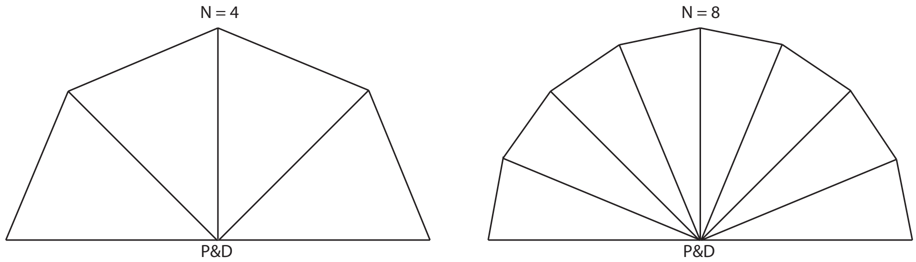
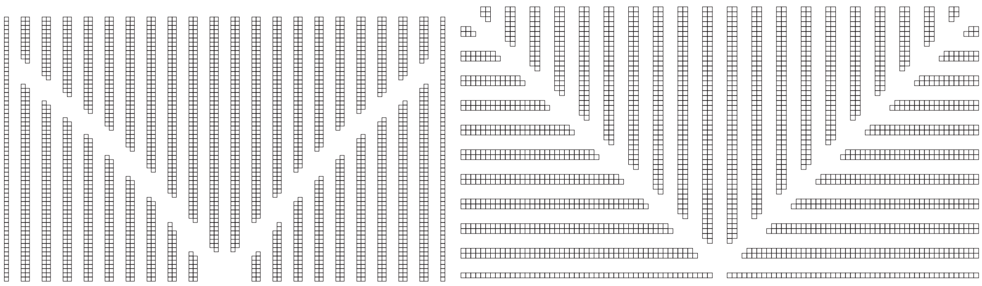


Figure 3.3: Non-rectangular warehouses with radial aisles (White, 1972). N is the number of regions that are defined by radial cross aisles. Travel in these regions is along the cross aisles first and then parallel to either x or y axis.

number of radial aisles increases (see Figure 3.3). White modeled the warehouse as a continuous space. Gue and Meller (2009) extended this idea to propose two non-traditional warehouse designs for unit-load operations using single-command operations (Figure 4.2). They designed their model in a discrete warehouse space with continuous picking on aisles. In the Flying-V design, they inserted two nonlinear cross aisle segments into a traditional layout with vertical picking aisles. (Gue and Meller refer to the Flying-V and Fishbone designs as having one cross aisle. For clarity of exposition, we describe them as having two cross aisle *segments*.) The Fishbone design features picking aisles at different angles with two diagonal cross aisle segments. The authors showed that the Flying-V design offers about 10% improvement over the traditional design; the Fishbone offers about 20%. In the Fishbone design, Gue and Meller identified and (partially) relaxed the conventional rule of parallel picking aisles, but the Fishbone design had its own design rule that picking aisles should be horizontal or vertical.

Pohl et al. (2009a) showed that the optimal placement of a "middle" cross aisle in traditional designs should be slightly aft of the middle. Pohl et al. (2009b) examined the Fishbone design for dual-command operations (also called task interleaving). They showed that Fishbone designs offer a decrease in expected travel distance, but require slightly more space compared to several common traditional designs. They also offered a modified Fishbone design for dual-command operations



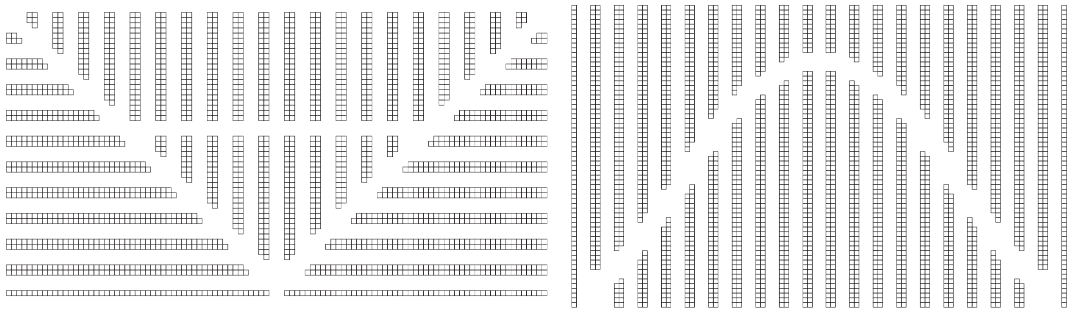
(a) The Flying-V (b) The Fishbone

Figure 3.4: Proposed designs for a single P&D point by Gue and Meller (2009).

called Fishbone-Triangle (see in Figure 3.5a). In this design, Pohl et al. (2009b) improved average travel-between distance in Fishbone designs by inserting an additional cross aisle segment between two diagonal cross aisles for dual-command operations. Pohl et al. (2010) analyzed Fishbone, Flying-V and three traditional designs under turnover-based storage policy and both single- and dual-command operations. Gue et al. (2010) investigated the expected travel distance from multiple P&D points on the bottom side of the warehouse, and showed that the Flying-V still has some benefit, but not as much as with a centrally located, single P&D point. They also developed another non-traditional aisle design called the Inverted-V (see in Figure 3.5b) to investigate the efficiency of single-command operation from multiple P&D points; however, this design offers much less benefit than the Flying-V. In their designs, they adhered to their own design rules:(1) P&D points are at fixed locations and placed on one side; (2) picking aisles are straight and parallel to each other.

In this dissertation, we take these non-traditional aisle design ideas even further with the goal of developing new aisle designs in unit-load warehouses that are operated under randomized storage and single-command operations with either a single P&D point or multiple P&D points. Additionally, we help to identify the magnitude of the impact of newly developed aisle designs by relaxing design rules such that:

- Picking and cross aisles can take any angle.



(a) The Fishbone-Triangle (b) The Inverted-V

Figure 3.5: Proposed designs for multiple P&D points at the bottom of the warehouse by Gue et al. (2010).

- Cross aisles can originate from any location along the side of the warehouse.
- P&D points can be placed at any location on the periphery of the warehouse.

We acknowledge our own design constraints, such as rectangular shape of the warehouse and P&D points on the periphery.

3.3 Material Flow Designs in Nature and Other Structures

Warehouses are not the only places where materials flow in the world. From veins in our body to river basins, they all have a main common point, flow of materials. While these materials are sometimes tangible goods and products, they are sometimes liquid or gas. Bejan (1996) discusses the common structures in material flow with his "constructal theory." Constructal theory is the view that the generation of flow structures in nature is a physics phenomenon, which is described by a constructal law (Bejan and Lorente, 2008): "For a finite-size flow system to persist in time (to live), its configuration must change in time such that it provides easier and easier access to its currents (fluid, energy, species, etc.)." Bejan and Lorente (2008) show how to formulate the flow structures we see in nature with constructal theory, such as circulatory systems in the body, lungs, river basins and leaves as well as mechanical and fluid flow systems.

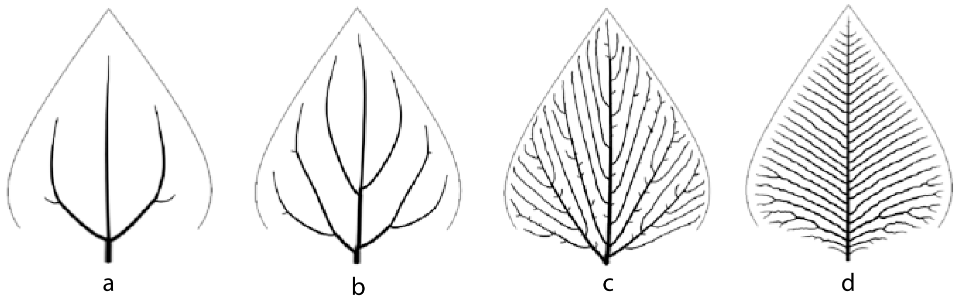


Figure 3.6: A continuum of veins in a leaf that is developed and represented by Runions et al. (2005).

Runions et al. (2005) studied on generating leaf venation patterns in order to investigate the development of veins corresponding to several leaf patterns and sources. Runions et al. introduced a biologically-motivated algorithm that relies on Voronoi diagrams. Using this algorithm, they simulated how vein patterns change with a given hormone source to the leaf blade. In Figure 3.6, leaves (a), (b) and (c) show different vein patterns generated for different kill distances (defining the length of sub-veins or branches) according to a single source node at the bottom of the leaf with a same growth rate. When the growth rate of veins is slow, the leaf generates a vein pattern seen in (d) in Figure 3.6. The similarities between these leaf patterns and warehouse designs are the single source node (a P&D point) and kill distances (different aisle lengths). These patterns also show some similar structures with Chevron aisle designs that we develop in Chapter 4. When the main shape of the leaf is changed, the model generates two main veins for each side and auxiliary veins from each of them (see Figure 3.7) . This also looks like Leaf aisle designs we develop in Chapter 4.

It is not surprising to find literature about flows in street and parking lot designs. Arlinghaus and Nystuen (1991) separated flows of cars, pedestrians and bicycles in an urban street design. Arlinghaus and Nystuen introduced diagonal elements for travel in consideration of square-block street structures including several obstacles. Figure 3.8a might be an interesting example for street designs such that it resembles three cross aisles from a single P&D point as in Butterfly aisle designs we develop

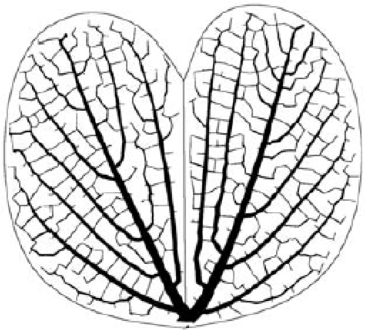
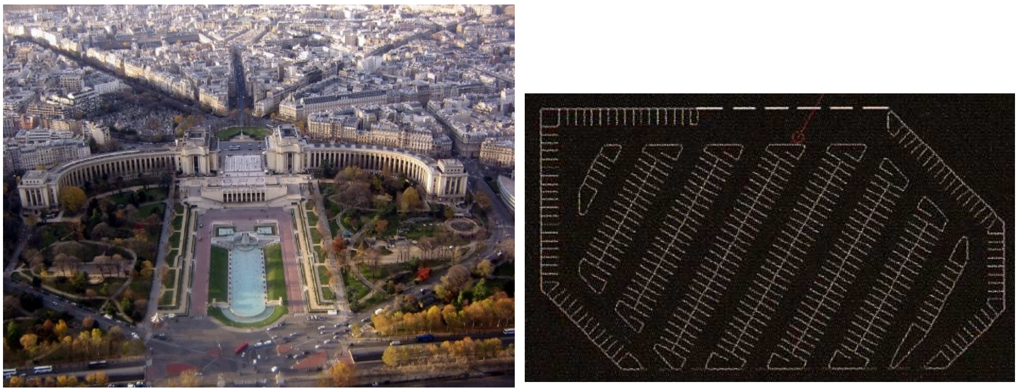


Figure 3.7: A developed orchid leaf by Runions et al. (2005).



(a) Streets in Paris. (b) A parking lot design.

Figure 3.8: (a) The picture shows the Palis de Chailot and Jardins du Trocadero. It is taken from the Eiffel Tower by "http://www.planetware.com/picture/paris-palais-de-chaillot-f-f1037.htm." (b) By Rolland et al. (2011).

in Chapter 4. Rolland et al. (2011) discussed how to design the most efficient parking lot considering traffic circulation or an its overall capacity based on several design constraints using a market software. The parking lot design they developed in Figure 3.8b also shows some similar structures with non-traditional aisle designs such as angled aisles with storage locations.

Airport design is also another flow system that involves the movement of passengers and baggages from one location to another location. Robustè (1991) and Robustè and Daganzo (1991) modeled average walking distance for several airport-hub designs assuming that each passenger has the same probability to go to a random gate. Then, Robustè and Daganzo (1991) proposed optimal shapes for airport-hub designs that minimize total passenger walking distance and baggage travel costs.

Bandara and Wirasinghe (1992a,b) focused on determining the configuration of an airport terminal that minimizes average passenger walking distance. They have considered three different airport configurations: centralized radial, centralized parallel, and semi-centralized parallel. Airport designs generally model average travel distance either between a hub and a gate (single-command) or between hubs (dual-command) that show some similarities with travel in warehouses.

As a result, generating the best structures for flow systems in different environments reflects some similarities with material flows in warehouses. Therefore, a unit-load warehouse is also such a flow system that acts to facilitate the flow of pallets from an open space to a pickup and deposit point via picking and cross aisles.

Chapter 4

Optimal Unit-Load Warehouse Designs for Single Command Operations

The contents of this chapter have been accepted for publication in IIE Transactions, with co-authors Kevin R. Gue and Russell D. Meller.

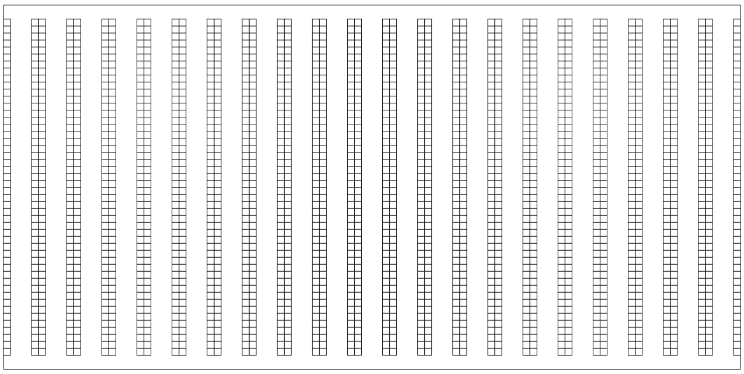


Figure 4.1: Traditional rectangular warehouse.

4.1 Introduction

Unit-load warehouses, such as import distribution centers and third-party transshipment warehouses, are widespread in industry and integral to the global economy. Unit-load operations are also common in many warehouses in the form of bulk storage areas. A dramatic increase in imported goods, approximately 140% more in 2006 than that of a decade earlier, is leading industry to build larger industrial warehouses and distribution centers (Hudgins, 2006). Total storage space of the top-20 North American third-party logistics (3PLs) reached 528 million square feet in 2009 (Rogers, 2009). Hudgins (2006) says that "300, 000 square feet was the definition of a large warehouse almost a decade ago, but today it is upwards of 1 million square feet."

Larger warehouses, of course, impose greater travel distances for the items inside, and greater travel distances lead to higher labor costs. Unit-load warehouses can, for slow-moving items, be rarely visited, but many have several workers actively storing and picking pallets. The largest operations, such as in import distribution centers, employ dozens of workers in these spaces. All to say, the labor cost associated with unit-load warehousing can be significant, especially for large retailers and other importers.

In a traditional warehouse design, storage racks are arranged to form parallel picking aisles, with a cross aisle along the bottom (Figure 4.1). If there are additional cross aisles in a traditional design to facilitate travel between picking aisles, they are arranged at right angles to the picking aisles. Cross aisles are appropriate for order picking operations, in which more than one location is visited per trip, but are inappropriate for the single-command operations we consider in this paper (Roodbergen and de Koster, 2001). Due to the aisle structure in the traditional design, workers travel rectilinear paths to store and receive pallets. White (1972) proposed "radial aisles" to reduce travel distance in a non-rectangular warehouse design. He showed that when the number of radial aisles increases, travel distance from the pickup and deposit (P&D) point to any point in the storage area is close to the Euclidean distance. His model assumed the warehouse as a continuous space.

Gue and Meller (2009) extended this idea to propose two non-traditional warehouse designs for unit-load operations using single-command cycles (Figure 4.2). In the Flying-V design, they inserted two nonlinear cross aisle segments into a traditional layout with vertical picking aisles. (Gue and Meller refer to the Flying-V and Fishbone designs as having one cross aisle. For clarity of exposition, we describe them as having two cross aisle *segments*.) The Fishbone design features picking aisles at different angles. The authors show that the Flying-V design offers about 10% improvement over the traditional design; the Fishbone offers about 20%. In the Fishbone design, Gue and Meller identified and (partially) relaxed the conventional rule of parallel picking aisles, but the fishbone design had its own design rule: that picking aisles should be horizontal or vertical.

In this study we make three improvements to the work of Gue and Meller (2009). First, we allow picking aisles to take on any angle, in an effort to facilitate more direct travel than that in the Fishbone design. Second, Gue and Meller restricted their models to consider two inserted cross aisle segments. We consider the one and three cross aisle cases as well, and show that there is a continuum of designs appropriate

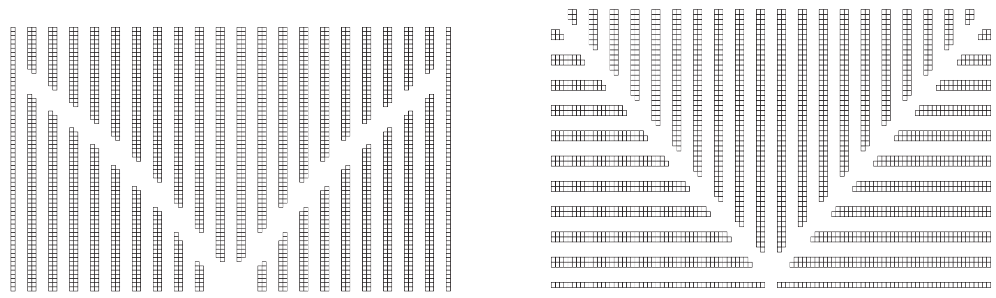


Figure 4.2: The Flying-V (Left) and Fishbone (Right).

for increasingly large warehouses. Last, we develop a more precise model of expected distance by assuming discrete pallet locations. The model allows better estimates of lost space due to inserted cross aisles. The main contribution of our work is to propose to the academic and practicing communities optimal warehouse designs for operations conforming to three main assumptions: uniform picking activity, a single pickup and deposit point, and single-command cycles.

In the next section we review the literature in non-traditional warehouse designs. In Sections 4.3–4.5 we introduce three optimal, non-traditional warehouse designs in continuous space, which we call the Chevron, Leaf, and Butterfly. Section 4.6 presents a comparison of these proposed designs with equivalent traditional warehouses. In Section 4.7 we develop a discrete model to show how much space is lost in implementations of these designs. In Section 4.8 we discuss implications for practice, including a brief description of two warehouses that have implemented non-traditional designs.

4.2 Related Literature

Moder and Thornton (1965) evaluated how floor space utilization is affected by some dependent and independent variables. One of the independent variables is the "slant angle of the pallets." They developed a mathematical model to evaluate how floor space efficiency changes with the angle of placement of the pallets and width of the aisle. Francis (1967a,b) investigated the shape of optimal warehouse designs with a single dock considering rectilinear travel along “the presupposed orthogonal network of aisles parallel to the x and y axis” between the dock and storage space. Berry (1968) proposed two types of warehouse layouts to investigate space utilization and the traveling cost of handling a unit. The first layout assumes that there are rectangular pallet blocks with the same depth arranged around a main orthogonal gangway. The second layout assumes that floor-stored pallets are arranged in different depths around a “single diagonal gangway providing access to all stacks.”

Pohl et al. (2009a) showed that the optimal placement of a “middle” cross aisle in traditional designs should be slightly aft of the middle. Pohl et al. (2009b) examined the Fishbone design for dual-command operations (also called task interleaving). They showed that Fishbone designs offer a decrease in expected travel distance over several common traditional designs. They evaluated the changes in storage area of the Fishbone design over equivalent traditional designs and showed that Fishbone designs require approximately 5% more space. They also offered a modified Fishbone design for dual-command operations. Pohl et al. (2010) analyzed three traditional designs, the Fishbone, and the Flying-V designs under turnover-based storage policies and both single- and dual-command operations. Gue et al. (2010) investigated the expected travel distance from multiple P&D points on one side of the warehouse, and showed that the Flying-V still has some benefit, but not as much as with a centrally-placed, single P&D point.

In addition to these studies, we would like to point out a conceptual relationship between our work and the “constructal theory” developed by Bejan (1996). Bejan’s work concerns heat flow and other mechanical and fluid flow systems, but the similarities with material flows in a warehouse are striking (see Bejan and Lorente, 2008, especially in Chapter 8).

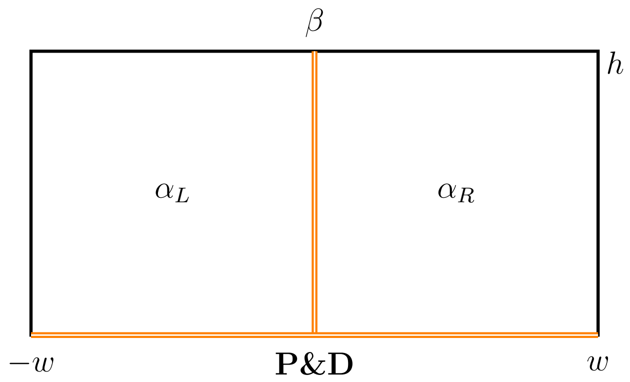


Figure 4.3: One cross aisle warehouse design.

4.3 One Cross Aisle Warehouse Designs

Suppose we have a rectangular warehouse space with one bottom cross aisle and an additional inserted cross aisle segment passing through a single, centrally-located P&D point, as in Figure 4.3. We assume a symmetric design, and therefore the inserted cross aisle has angle $\beta = 90°$. The bottom cross aisle has angle $0°$. All travel in this warehouse is from the single P&D point to any point in the warehouse, either along the angled cross aisle or bottom cross aisle and then along the appropriate picking aisle. The question is, what are the angles of the picking aisles on the right and left side of the angled cross aisle to obtain the lowest expected travel distance in this design?

Because we assume there is a bottom cross aisle available in all our designs, we call the design with one inserted cross aisle segment a "one cross aisle" warehouse. In our models, we assume only one pallet is carried at a time in a single-command cycle. The P&D point could be a place that has a palletizing or shrink wrap machine, or that provides instructions for workers.

In our models, we assume a randomized storage policy for the same reasons described in Gue and Meller (2009). The randomized storage policy is widely used in industry because of its simplicity and higher utilization of storage locations (Petersen, 1999). In this policy, the probability of picking or storing any pallet in the warehouse is the same. We assume that picking aisles within the same region are parallel to each other, and that cross aisles and picking aisles have zero width. Although a continuous storage space is not an especially good model of storage locations in a real warehouse, it is close enough for our purposes, especially when considering large warehouse spaces typically found in industry. In later sections, we investigate the effect of space lost by aisles required in an implementation of our designs.

4.3.1 Model

In this section, we determine the angles of picking aisles that minimize expected travel distance in a rectangular warehouse. Picking aisles can take on any angle between 0° and 180°. Parameters and variables for the one cross aisle warehouse model are shown in Figure 4.3 and described as follows.

Parameters:	
P&D	single pickup and deposit point placed at (0,0)
β	angle of the cross aisle, which is 90°
w	half width of the warehouse
h	height of the warehouse
(x, y)	coordinates of a randomly chosen storage location in the warehouse space, $-w \leq x \leq w$ and $0 \leq y \leq h$
Variables:	
α_R, α_L	angles of picking aisles on the right and left sides.

Because a 90° cross aisle naturally divides the warehouse into two equal sides and we assume symmetry in our designs, we need to model only the right side of the warehouse.

Proposition 4.1. *The optimal α_R is less than 90°.*

Proof. (See Figure 4.4.) Assume that O represents the single P&D point. Point $W = (w, 0)$ is on the right boundary of the warehouse and point $X = (x, y)$ is a random storage location in the right half of the warehouse. Let $B = (x, 0)$ and $D = (0, y)$ represent points of access to vertical and horizontal picking aisles, such

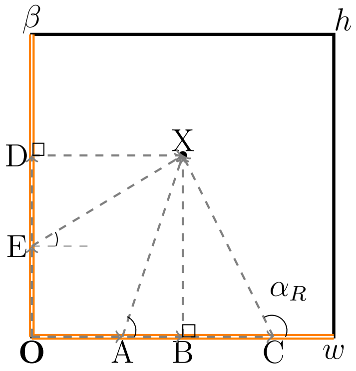


Figure 4.4: Example travel paths based on some possible α_R's.

that $\angle OBX = \angle ODX = 90°$. Let $A = \{(z, 0) : 0 \leq z < x\}$, $E = \{(0, z) : 0 \leq z < y\}$ and $C = \{(z, 0) : x < z \leq w\}$ be points of access such that $\angle WAX < 90°$, $(\angle OEX - 90°) < 90°$ and $\angle WCX > 90°$. By the triangle inequality,

$$|OA| + |AX| < |OB| + |BX| = |OD| + |DX| < |OC| + |CX|.$$

The picking aisles associated with paths OEX and OAX are both less than 90°, therefore the optimal $\alpha_R < 90°$. $\square$

We divide the optimization problem into two cases based on the possible angles of picking aisles. Define angle $\gamma_R = \arctan \frac{h}{w}$, which is the angle defined by the bottom aisle and a diagonal line to the upper right corner. There are two cases: $0° \leq \alpha_R \leq \gamma_R$ and $\gamma_R \leq \alpha_R < 90°$. We include γ_R in both cases because the optimal result, which we are about to show, is obtained at the diagonal for square half-warehouses.

Figure 4.5 illustrates two picking regions constructed by the two cases, along with example travel paths. The single solid line in each figure represent the angle of picking aisles on the right side of the warehouse. We refer to these lines originating from the P&D point as "median picking aisles." For example, region A is the area between the bottom cross aisle and the median picking aisle on the right. Travel to

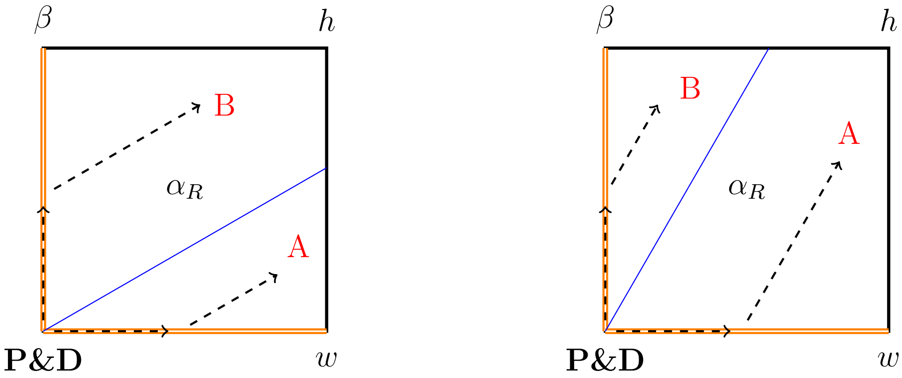


Figure 4.5: The representation of case 1 (left) and case 2 (right) for the one cross aisle model.

any point in this region has the same routing rule: travel along the bottom cross aisle, then travel along a picking aisle with angle α_R.

The travel distance functions from the P&D point to a point (x, y) on the right side are

$$T_A(x, y) = (x - y \cot \alpha_R) + \frac{y}{\sin \alpha_R}, \tag{4.1}$$
$$T_B(x, y) = (y - x \tan \alpha_R) + \frac{x}{\cos \alpha_R}. \tag{4.2}$$

The first terms in (4.1) and (4.2) represent the travel distance along the bottom cross aisle; the second terms, the distance along the picking aisle. These expressions are similar to those in White (1972).

Expected travel distance is the ratio of total travel distance to every available location in a region to its total storage area. So, the expected travel distance to make a pick on the right side ($E[R]$), which is the sum of expected travel distance in regions A and B with the probability of picking in these regions, is:

$$\begin{aligned} E[R] &= p_A E[A] + p_B E[B] \\ &= p_A \frac{1}{m_A} \int_x \int_y T_A(x, y)\, dydx + p_B \frac{1}{m_B} \int_x \int_y T_B(x, y)\, dydx, \end{aligned}$$

where m_A and m_B are the areas of picking regions A and B, respectively. Because we assume that picking is uniformly distributed throughout the space, the probabilities of choosing one of two regions are,

$$\begin{aligned} p_A &= \frac{m_A}{hw}, \\ p_B &= \frac{m_B}{hw}. \end{aligned}$$

Hence, expected travel distances to reach a location on the right side for case 1 and case 2 are

$$\begin{aligned} E[R_1] &= \frac{1}{wh}\left(\int_0^w \int_0^{x\tan\alpha_R} T_A(x,y) \quad dydx + \int_0^w \int_{x\tan\alpha_R}^h T_B(x,y) \quad dydx\right) \\ &= \frac{1}{6h}\left(3h^2 + w\sec\alpha_R(3h - w\tan\alpha_R) + w\tan\alpha_R(-3h + w + w\tan\alpha_R)\right), \\ E[R_2] &= \frac{1}{wh}\left(\int_0^h \int_{y/\tan\alpha_R}^w T_A(x,y) \quad dxdy + \int_0^h \int_0^{y/\tan\alpha_R} T_B(x,y) \quad dxdy\right) \\ &= \frac{1}{6w}\left(h^2\cot^2\alpha_R + h\cot\alpha_R(h - 3w - h\csc\alpha_R) + 3w(w + h\csc\alpha_R)\right). \end{aligned}$$

We obtained the results of the integrations with MATHEMATICA. The optimization problem is to choose α_R such that $E[R] = \min\{E[R_1], E[R_2]\}$ is a minimum, which we can solve by taking derivatives and setting them equal to zero.

$$\begin{aligned} \frac{\mathrm{d}E[R_1]}{\mathrm{d}\alpha_R} &= \frac{w\sec^2\alpha_R}{12h}\left(w\sec\alpha_R\left(-3 + \cos(2\alpha_R)\right)\right) \\ &\quad + \frac{w\sec^2\alpha_R}{12h}\left(2\left(-3h + w + 3h\sin\alpha_R + 2w\tan\alpha_R\right)\right) = 0 \\ \frac{\mathrm{d}E[R_2]}{\mathrm{d}\alpha_R} &= \frac{h\csc\alpha_R}{6w}\left(\cot\alpha_R\left(-3w + h\cot\alpha_R\right)\right) \\ &\quad - \frac{h\csc\alpha_R}{6w}\left(\csc\alpha_R\left(h - 3w + 2h\cot\alpha_R\right) + h\csc^2\alpha\right) = 0 \end{aligned}$$

When $h = w$ for square half-warehouses, the single solution (critical point) to these equations is the same: $\alpha_R = -2\arctan(1 - \sqrt{2})$, which is 45°. The second partial derivatives are positive at this critical point; therefore, the critical point is

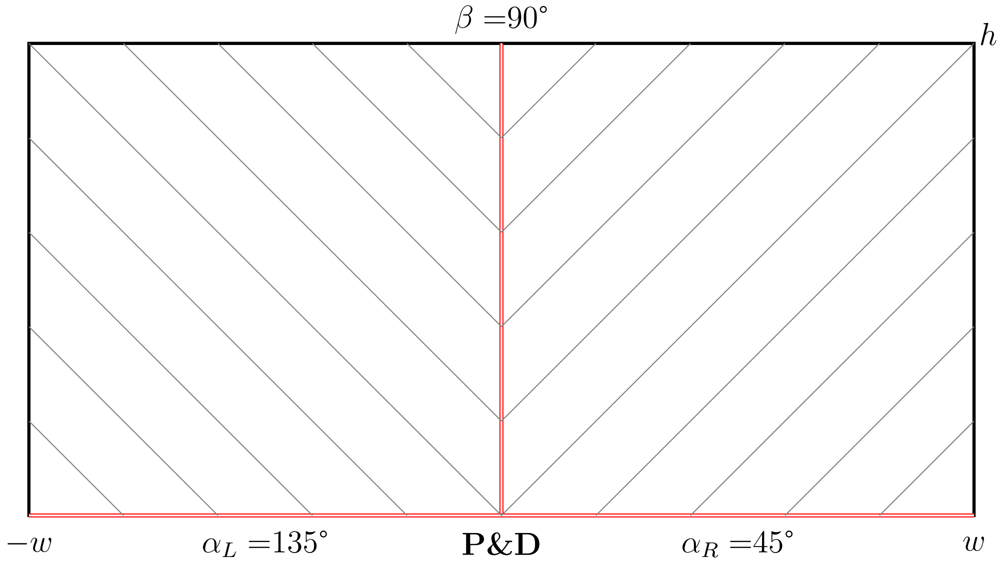


Figure 4.6: The optimal one cross aisle design.

a local minimum. At the boundaries α_R =0° and α_R =90°, $E[R_1]$ and $E[R_2]$ are greater than the local minimum; therefore, the critical point is the global minimum. As expected,

Proposition 4.2. *The optimal* $\alpha_R = 45^\circ$ *in a square half-warehouse.*

Figure 4.6 illustrates the optimal one cross aisle design, which we call the Chevron.

4.3.2 One Cross Aisle Designs for Non-Square Half-Warehouses

When $h \neq w$, should we expect the optimal angles of picking aisles to equal the angles of the diagonals? Figure 4.7 shows how the optimal α_R (computed numerically, with MATHEMATICA) changes when the ratio of h/w increases. The figure shows that the optimal α_R increases as the ratio increases, but that it is always less than or equal to γ_R, the angle of the diagonal. For warehouses in which $w > h$, the optimal α_R decreases as w/h increases (the warehouse gets wider), but it remains above the diagonal.

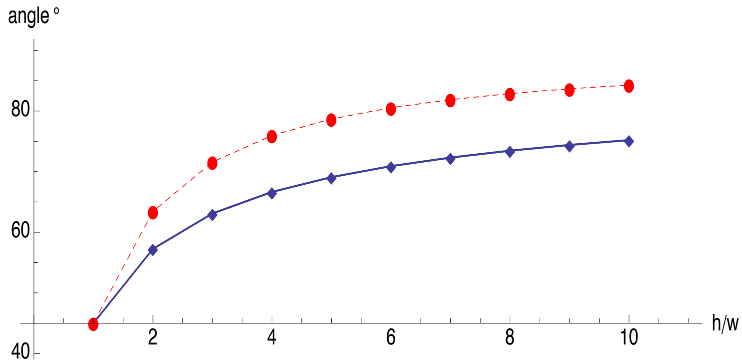


Figure 4.7: Change in optimal α_R for non-square half-warehouses. Solid lines represent optimal angles of picking aisles; dashed lines with circle markers represent angles of the diagonal on the right side.

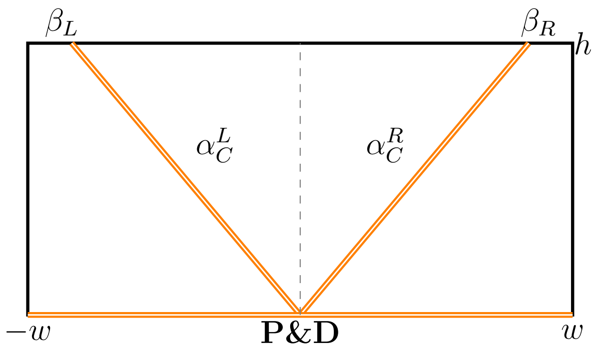


Figure 4.8: A two cross aisle warehouse.

4.4 A Two Cross Aisle Design

In this section we insert into the warehouse two angled cross aisles, in addition to the bottom cross aisle. We call this design a two cross aisle warehouse.

We make the same assumptions discussed in the one cross aisle model. Therefore, the two angled cross aisles are placed on each side, symmetric with respect to a vertical axis drawn through the P&D point. This axis divides the warehouse into two equal parts. We use the following variables depicted Figure 4.8.

Variables:	
β_R, β_L	angles of the right and left cross aisles
α_R, α_L	angles of picking aisles in the rightmost and leftmost sections
α_C^R, α_C^L	angles of the central picking aisles on the right and left sides

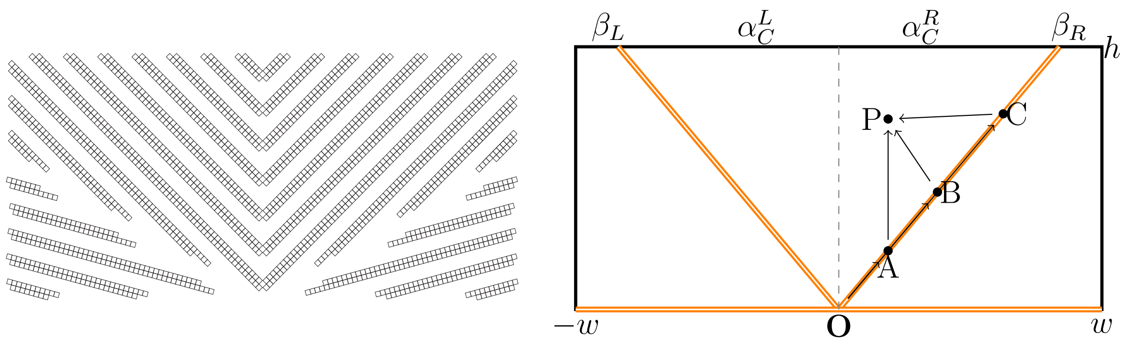
Because the feasible space for angles of picking and cross aisles has many cases to consider, we prove two propositions to limit the search space. Additionally, it is enough to search for the optimal α_C^R, because α_C^R and α_C^L are symmetric.

Proposition 4.3. *In an optimal two cross aisle design,* $\alpha_C^L = \alpha_C^R = 90^\circ$*.*

Proof. When α_C^R <90°, we have an infeasible design because some locations in the central region are unreachable (see Figure 4.9a). Now consider $\alpha_C^R \geq 90°$ in Figure 4.9b, and let point P be a random storage location on the right side of the central region. Point O is the P&D point. Let A, B, and C be points of access to the picking aisle containing point P corresponding to $\alpha_C^R = 90°$, $90° < \alpha_C^R < 180°$, $\alpha_C^R = 180°$ respectively. Path OA is common travel for all three paths in Figure 4.9b. From the triangle inequality,

$$|AP| < |AB| + |BP| < |AC| + |CP|.$$

Therefore $\alpha_C^L = \alpha_C^R = 90°$. □



(a) (b)

Figure 4.9: (a) An infeasible symmetric warehouse design with two cross aisles when $\alpha_C^R < 90°$, (b) sample travel paths to the point P with different α_C^R.

Proposition 4.4. *In an optimal two cross aisle design,* $0^\circ \leq \alpha_R \leq \beta_R$*.*

The proof is analogous to the proof of Proposition 4.3.

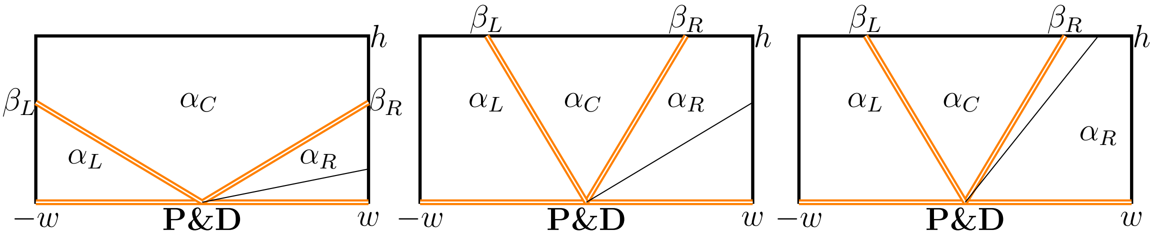


Figure 4.10: Cases in the two cross aisle model.

Because the warehouse design is assumed to be symmetric, the optimization problem is to minimize expected travel in the right half of the warehouse. There are three cases determined by possible ranges of β_R and α_R (see Figure 4.10): (1) $0° \le \beta_R \le \gamma_R$ and $0° \le \alpha_R \le \beta_R$, (2) $\gamma_R \le \beta_R \le 90°$ and $0° \le \alpha_R \le \gamma_R$, and (3) $\gamma_R \le \beta_R \le 90°$ and $\gamma_R \le \alpha_R \le \beta_R$.

Define A, B and C as regions on the right side of the warehouse (see Figure 4.11). Regions A and B are divided by the median picking aisle with an angle α_R. These regions specify that the travel path to reach any point in the defined region is the same. For example, to reach any point in region A, a picker should first go along the bottom cross aisle, then along the appropriate picking aisle. Let T_A, T_B and T_C be the travel distance functions to random points (x,y) in these regions.

$$
\begin{aligned}
T_A(x,y) &= (x - y\cot\alpha_R) + \frac{y}{\sin\alpha_R}, && (4.3)\\
T_B(x,y) &= \sqrt{x^2+y^2}\left(\cos\left(\beta_R - \arctan\frac{y}{x}\right) - \sin\left(\beta_R - \arctan\frac{y}{x}\right)\cot(\beta_R-\alpha_R)\right) \\
&+ \sqrt{x^2+y^2}\left(\frac{\sin\left(\beta_R - \arctan\frac{y}{x}\right)}{\sin(\beta_R-\alpha_R)}\right), \text{ and} && (4.4)\\
T_C(x,y) &= (y - x\tan\beta_R) + \frac{x}{\cos\beta_R}. && (4.5)
\end{aligned}
$$

The first terms in these equations are the travel distances along the cross aisle; the second terms are the distances along the appropriate picking aisles. $E[WR_1]$, $E[WR_2]$ and $E[WR_3]$ are the expected travel distances on the right half of the

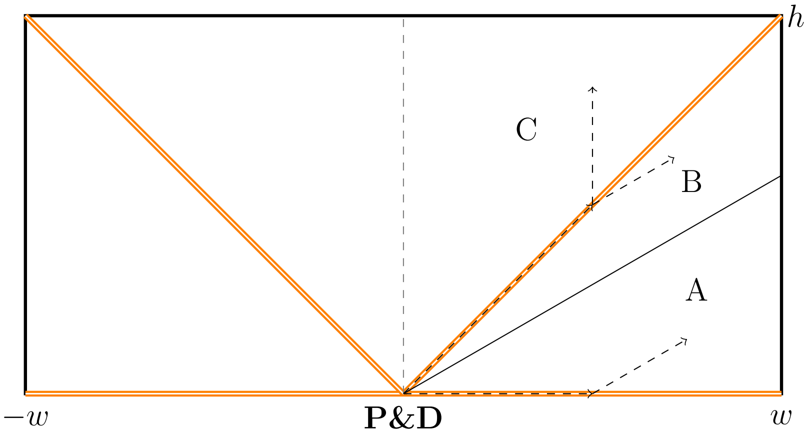


Figure 4.11: Travel paths according to the defined areas in two cross aisle design..

warehouse for the specified cases.

$$\begin{aligned}
E[WR_1] &= \frac{1}{wh}\left(\int_0^w \int_0^{x\tan\alpha_R} T_A(x,y) \quad dydx + \int_0^w \int_{x\tan\alpha_R}^{x\tan\beta_R} T_B(x,y) \quad dydx\right) \\
&+ \frac{1}{wh}\left(\int_0^w \int_{x\tan\beta_R}^{h} T_C(x,y) \quad dydx\right), \\
E[WR_2] &= \frac{1}{wh}\left(\int_0^w \int_0^{x\tan\alpha_R} T_A(x,y) \quad dydx + \int_0^{h/\tan\beta_R} \int_{x\tan\alpha_R}^{x\tan\beta_R} T_B(x,y) \quad dydx\right) \\
&+ \frac{1}{wh}\left(\int_{h/\tan\beta_R}^{w} \int_{x\tan\alpha_R}^{h} T_B(x,y) \quad dydx\right) \\
&+ \frac{1}{wh}\left(\int_0^{h/\tan\beta_R} \int_{x\tan\beta_R}^{h} T_C(x,y) \quad dydx\right), \text{and} \\
E[WR_3] &= \frac{1}{wh}\left(\int_0^h \int_{y/\tan\alpha_R}^{w} T_A(x,y) \quad dxdy + \int_0^h \int_{y/\tan\beta_R}^{y/\tan\alpha_R} T_B(x,y) \quad dxdy\right) \\
&+ \frac{1}{wh}\left(\int_0^{h/\tan\beta_R} \int_{x\tan\beta_R}^{h} T_C(x,y) \quad dydx\right).
\end{aligned}$$

Closed form expressions are in Appendix A.1. The optimization problem is to find α_R and β_R that minimizes

$$E[WR] = \min\{E[WR_1], E[WR_2], E[WR_3]\}.$$

Theorem 4.1. *An optimal two cross aisle warehouse has* $\beta_R = \pi/2 - \left(\arccos \frac{6+\sqrt{6}}{10}\right)$ *and* $\alpha_R = \left(\arccos \frac{6+\sqrt{6}}{10}\right)$.

Proof. Consider case 2 first. Functions f_{α_R} and f_{β_R} are the first partial derivatives of $E[WR_2]$ with respect to α_R and β_R, respectively.

$$f_{\alpha_R}(\alpha_R, \beta_R) = \frac{\partial E[WR_2]}{\partial \alpha_R} = 0, \tag{4.6}$$

$$f_{\beta_R}(\alpha_R, \beta_R) = \frac{\partial E[WR_2]}{\partial \beta_R} = 0. \tag{4.7}$$

The solution to (4.6) and (4.7) (solved with MATHEMATICA) has only one root within the defined ranges of variables: $\beta_R = \pi/2 - \left(\arccos \frac{6+\sqrt{6}}{10}\right)$ and $\alpha_R = \left(\arccos \frac{6+\sqrt{6}}{10}\right)$. To show that this root is a global minimum, it is sufficient to show that it is a local minimum, and that it has a functional value lower than the extreme points. The critical point is a local minimum if

$$\left| H_{E[WR_2]}\left(\arccos \frac{6+\sqrt{6}}{10}, \pi/2 - \arccos \frac{6+\sqrt{6}}{10}\right) \right| > 0,$$
$$\frac{\partial^2 E[WR_2]}{\partial \alpha_R^2}\left(\arccos \frac{6+\sqrt{6}}{10}, \pi/2 - \arccos \frac{6+\sqrt{6}}{10}\right) > 0, \text{and}$$
$$\frac{\partial^2 E[WR_2]}{\partial \beta_R^2}\left(\arccos \frac{6+\sqrt{6}}{10}, \pi/2 - \arccos \frac{6+\sqrt{6}}{10}\right) > 0.$$

where H is the Hessian matrix of $E[WR_2]$. We verified these inequalities in MATHEMATICA. Because the determinant of H is not always greater than or equal to 0 for all possible values of α_R and β_R in their defined ranges, $E[WR_2]$ is not positive semi-definite and not convex. However, we applied the Extreme Value Theorem to check the boundaries of $E[WR_2]$. For this, we solve $f_{\alpha_R}(\alpha_R, \beta_R) = 0$ and $f_{\beta_R}(\alpha_R, \beta_R) = 0$ simultaneously by fixing one of the boundary conditions ($\alpha_R = 0°$, $\alpha_R = 45°$, $\beta_R = 45°$ and $\beta_R = 90°$) and solving for the other variable. Based on the four solutions, we see that the values of $E[WR_2]$ at its boundaries are greater than

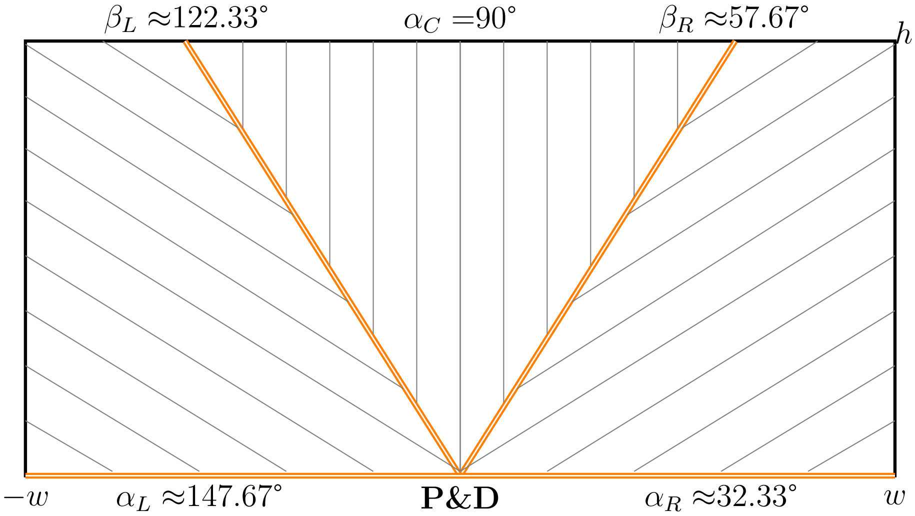


Figure 4.12: The Leaf: Optimal two cross aisle design for square half-warehouses.

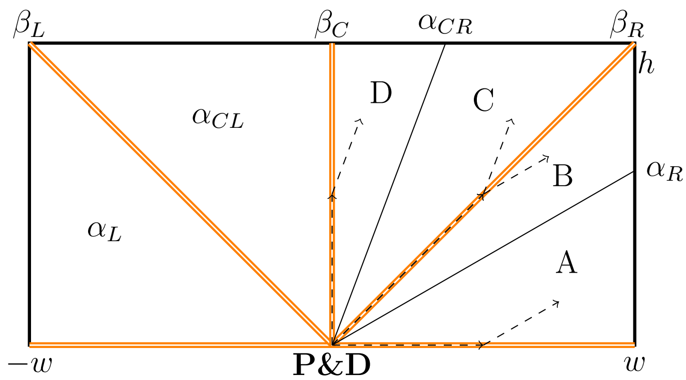


Figure 4.13: Travel paths in regions on the right side of the three cross aisle design.

the local optimum. Similar analysis for case 1 and case 3 also yields local optima greater than the local optimum for case 2. The single root for case 1 is: $\beta = \pi/4$ and $\alpha_R = \arcsin(\sqrt{2} - 1)$. For case 3, $\beta = \pi/2$ and $\alpha_R = \pi/4$. Hence, although $E[WR_2]$ is not convex, $\left(\arccos \frac{6+\sqrt{6}}{10}, \pi/2 - \arccos \frac{6+\sqrt{6}}{10}\right)$ is the global minimum because there is an unique root, it is the local optimum, and the extreme values are larger. □

Figure 4.12 illustrates the optimal two cross aisle design, which we call the Leaf because of its veiny structure and organic appearance.

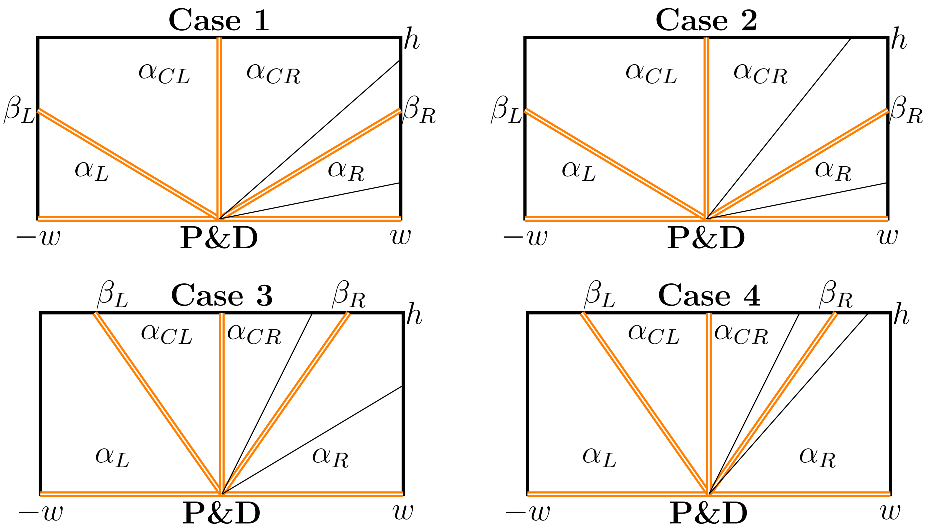


Figure 4.14: Cases in the three cross aisle model.

4.5 Three Cross Aisle Design

Development of the three cross aisle model is as before. There are three inserted cross aisles, in addition to the bottom cross aisle (see Figure 4.13). Due to the assumption of symmetry, $\beta_C = 90°$, and this vertical cross aisle divides the warehouse into two equal parts. Therefore, we can focus on finding the optimal angles of aisles on the right half because the left half is a mirror image.

Define A, B, C and D as regions on the right side of the warehouse (see Figure 4.13). Travel in regions A and B is the same as travel to the same regions in two cross aisle designs (see Figure 4.11). Therefore, travel distance functions in regions A (T_A) and B (T_B) are defined by equations (4.3) and (4.4) given in Section 4.4. Travel in region D is also the same as travel in region B in a one cross aisle design (see Figure 4.5), and the travel function in this region (T_D) is defined by equation (4.2) given in Section 4.3. Therefore, we need only develop a travel distance function (T_C) to a random point (x, y) in region C. The first part is the travel distance along the cross aisle; the second part is along the appropriate picking aisle.

$$
\begin{aligned}
T_C(x,y) = {} & \sqrt{x^2+y^2}\left(\cos\left(\arctan\frac{y}{x}-\beta_R\right)-\sin\left(\arctan\frac{y}{x}-\beta_R\right)\cot(\alpha_R-\beta_R)\right) \\
& + \sqrt{x^2+y^2}\left(\frac{\sin\left(\arctan\frac{y}{x}-\beta_R\right)}{\sin(\alpha_R-\beta_R)}\right).
\end{aligned}
$$

There are four cases determined by possible orientations of the cross and picking aisles on the right side of the warehouse (see Figure 4.14). According to Proposition 4.4, these cases are

1. $0° < \beta_R \leq \gamma_R$, $0° < \alpha_R \leq \beta_R$ and $\beta_R \leq \alpha_{CR} \leq \gamma_R$,

2. $0° < \beta_R \leq \gamma_R$, $0° < \alpha_R \leq \beta_R$ and $\beta_R \leq \alpha_{CR} < 90°$,

3. $\gamma_R \leq \beta_R < 90°$, $0° < \alpha_R \leq \gamma_R$ and $\beta_R \leq \alpha_{CR} < 90°$, and

4. $\gamma_R \leq \beta_R < 90°$, $\gamma_R < \alpha_R \leq \beta_R$ and $\beta_R \leq \alpha_{CR} < 90°$,

where γ_R is the angle of the diagonal on the right side of the warehouse. $E[WR_1]$, $E[WR_2]$, $E[WR_3]$ and $E[WR_4]$ are the associated expected travel distances. Closed form expressions are in Appendix A.3.

$$
\begin{aligned}
E[WR_1] &= \frac{1}{wh}\left(\int_0^w\int_0^{x\tan\alpha_R} T_A(x,y)\quad dydx + \int_0^w\int_{x\tan\alpha_R}^{x\tan\beta_R} T_B(x,y)\quad dydx\right) \\
&+ \frac{1}{wh}\left(\int_0^w\int_{x\tan\beta_R}^{x\tan\alpha_{CR}} T_C(x,y)\quad dydx\right) \\
&+ \frac{1}{wh}\left(\int_0^w\int_{x\tan\alpha_{CR}}^{h} T_D(x,y)\quad dydx\right), \qquad (4.8)
\end{aligned}
$$

$$
\begin{aligned}
E[WR_2] &= \frac{1}{wh}\left(\int_0^w \int_0^{x\tan\alpha_R} T_A(x,y) \quad dydx + \int_0^w \int_{x\tan\alpha_R}^{x\tan\beta_R} T_B(x,y) \quad dydx\right) \\
&+ \frac{1}{wh}\left(\int_0^{h/\tan\alpha_{CR}} \int_{x\tan\beta_R}^{x\tan\alpha_{CR}} T_C(x,y) \quad dydx\right) \\
&+ \frac{1}{wh}\left(\int_{h/\tan\alpha_{CR}}^{w} \int_{x\tan\beta_R}^{h} T_C(x,y) \quad dydx\right) \\
&+ \frac{1}{wh}\left(\int_0^{h/\tan\alpha_{CR}} \int_{x\tan\alpha_{CR}}^{h} T_D(x,y) \quad dydx\right), \qquad (4.9) \\
E[WR_3] &= \frac{1}{wh}\left(\int_0^w \int_0^{x\tan\alpha_R} T_A(x,y) \quad dydx + \int_{h/\tan\beta_R}^w \int_{x\tan\alpha_R}^{h} T_B(x,y) \quad dydx\right) \\
&+ \frac{1}{wh}\left(\int_0^{h/\tan\beta_R} \int_{x\tan\alpha_R}^{x\tan\beta_R} T_B(x,y) \quad dydx\right) \\
&+ \frac{1}{wh}\left(\int_0^h \int_{y/\tan\alpha_{CR}}^{y/\tan\beta_R} T_C(x,y) \quad dxdy\right) \\
&+ \frac{1}{wh}\left(\int_0^h \int_0^{y/\tan\alpha_{CR}} T_D(x,y) \quad dxdy\right), \text{and} \qquad (4.10) \\
E[WR_4] &= \frac{1}{wh}\left(\int_0^h \int_{y/\tan\alpha_R}^{w} T_A(x,y) \quad dxdy + \int_0^h \int_{y/\tan\beta_R}^{y/\tan\alpha_R} T_B(x,y) \quad dxdy\right) \\
&+ \frac{1}{wh}\left(\int_0^h \int_{y/\tan\alpha_{CR}}^{y/\tan\beta_R} T_C(x,y) \quad dxdy\right) \\
&+ \frac{1}{wh}\left(\int_0^h \int_0^{y/\tan\alpha_{CR}} T_D(x,y) \quad dxdy\right). \qquad (4.11)
\end{aligned}
$$

Proposition 4.5. *An optimal three cross aisle, square half-warehouse has* $\beta_R = \pi/4$ *and* $\alpha_{CR} = 90^\circ - \alpha_R$.

Proof. Follows from a simple argument of symmetry. □

The proposition allows us to focus only on finding the optimal α_R. We derive an expected travel distance function only on the right side of the warehouse according to cases 2 and 3, because the optimal design occurs in these cases for square half-warehouses. $E[AB]$ is the expected travel distance in regions A and B in these cases, when $\beta_R = \pi/4$.

$$E[AB] = \frac{1}{wh}\left(\int_0^w \int_0^{x\tan\alpha_R} T_A(x,y) \quad dydx + \int_0^w \int_{x\tan\alpha_R}^{x\tan\pi/4} T_B(x,y) \quad dydx\right)$$
$$= \frac{w^3}{6}\left(\tan\alpha_R + \sec\alpha_R\left(1+\sqrt{2}\sec\alpha_R - 2\sin\left(\alpha_R+\frac{\pi}{4}\right)\tan\alpha_R\right)\right).$$

The first order condition is

$$\frac{\mathrm{d}\ E[AB]}{\mathrm{d}\alpha_R} = \frac{w^3\left(-2+\sqrt{2}+\sqrt{2}\sin\alpha_R\right)}{6\sqrt{2}\cos^2\alpha_R} = 0.$$

The solution produces only one root: $\alpha_R = \arcsin(\sqrt{2}-1)$. The second order conditions show that $E[AB]$ is convex in the range $0 \leq \alpha_R < 90°$ (see Proposition 4.1).

$$\frac{\mathrm{d}^2 E[AB]}{\mathrm{d}\alpha_R^2} = \frac{-w^3}{12\cos^3\alpha_R}\left(-3+\cos(2\alpha_R)+4(-1+\sqrt{2})\sin\alpha_R\right). \tag{4.12}$$

That $\frac{\mathrm{d}^2 E[AB]}{\mathrm{d}\alpha_R^2}$ is strictly positive can be seen by setting $\cos(2\alpha_R)$ and $\sin\alpha_R$ to their maximum values of 1, and noting that $\cos^3\alpha_R$ is positive. Therefore, $E[AB]$ is convex and the solution is a global minimum. In an optimal three cross aisle, square half-warehouse design: $\beta_R = \pi/4$, $\beta_C = \pi/2$, $\alpha_R = \arcsin(\sqrt{2}-1)$ and $\alpha_{CR} = \pi/2 - \arcsin(\sqrt{2}-1)$. We call this design the "Butterfly" (Figure 4.15).

4.6 Comparison of Continuous Warehouse Designs

Gue and Meller (2009) give a lower bound on expected travel distance for a continuous warehouse with a single P&D point. We call this model "Travel by Flight." In terms of our model parameters, the lower bound for Travel-by-Flight is

$$E[TF] = \frac{1}{6wh}\left(2hw\sqrt{h^2+w^2} + w^3\ln\left(\frac{h+\sqrt{h^2+w^2}}{w}\right) + h^3\ln\left(\frac{w+\sqrt{h^2+w^2}}{h}\right)\right).$$

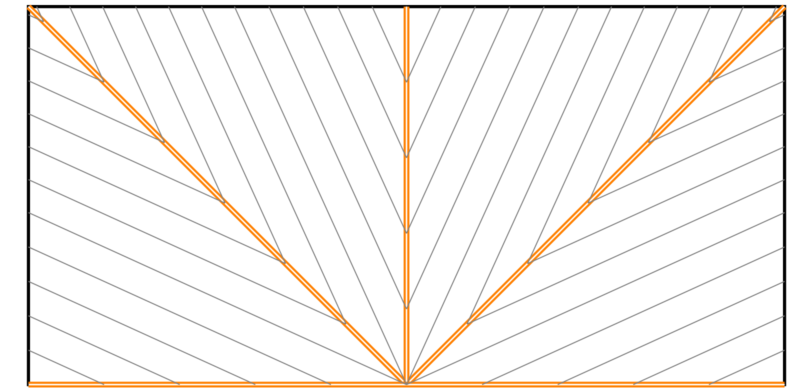


Figure 4.15: Representation of the Butterfly design.

Christofides and Eilon (1969) and Stone (1991) develop similar expressions for Travel by Flight in square and rectangular areas, respectively.

If travel is rectilinear, as in a traditional warehouse, the expected travel distance for a warehouse with a single, centrally located P&D point defines a reasonable upper bound. In terms of our model parameters, expected value for rectilinear travel TR is

$$E[TR] = \frac{(w+h)}{2}.$$

Table 4.1 shows the relative cost performance of several designs, where the cost of a traditional design serves as the base 100%. The Chevron and Fishbone designs offer 19.53% less expected travel distance than an equivalent traditional warehouse. The Leaf design offers a 21.72% savings and is only 1.76% worse than the Travel-by-Flight design. The Butterfly design reduces expected travel distance by 22.52% and is 0.96% worse than Travel-by-Flight. All of these results are for the continuous space representation of a warehouse.

Observation 1. *In an optimal two cross aisle design for a square half-warehouse,* $\alpha_R + \beta_R = 90^\circ$.

Table 4.1: The relative expected travel distance comparison among equivalent warehouse designs based on the continuous model.

Warehouse	**Performance %**
Traditional	100
Chevron	80.47
Fishbone	80.47
Leaf	78.28
Butterfly	77.48
Travel-by-Flight	76.52

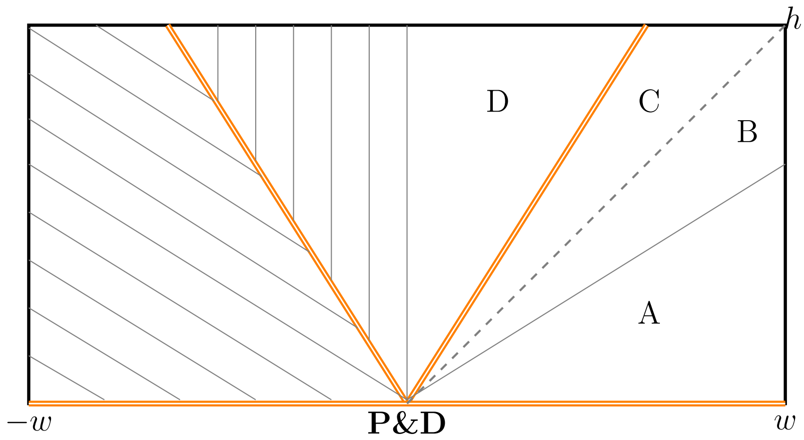


Figure 4.16: Equally distributed imperfections in designs.

The observation follows directly from the optimal values in Theorem 4.1. Unfortunately, we are unable to interpret it in terms of warehousing. However, this observation is likely related to,

Observation 2. *In the optimal two cross aisle design for a square half-warehouse, the expected travel distance in the region below the median picking aisle ($E[A]$ in Figure 4.16) is the same as expected travel distance in the central region ($E[D]$). The expected travel distance in regions above and below the diagonal in the half space are also the same ($E[A] + E[B] = E[C] + E[D]$).*

See Appendix A.2 for a proof. The observation suggests that optimal solutions exhibit a sort of spatial uniformity of costs, similar in spirit to the "optimal distribution of imperfection" in Bejan (1996).

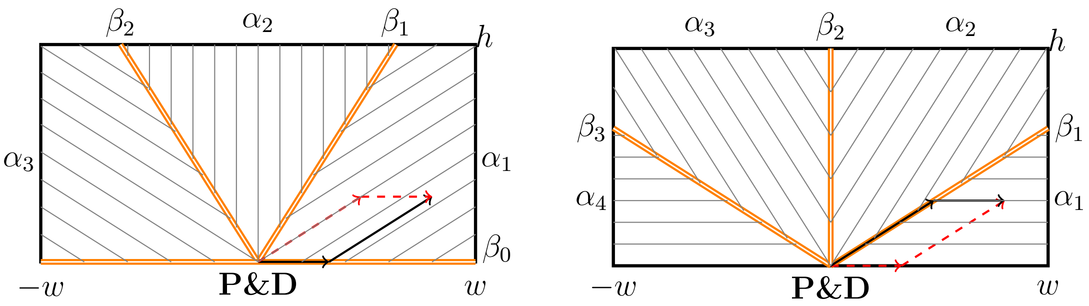


Figure 4.17: Equal travel distance in dual designs shown via parallelogram.

Observation 3. *For any warehouse with an even number k of angled cross aisles and one bottom cross aisle, there is a "dual" warehouse with $k+1$ angled cross aisles having the same expected travel distance, such that the angles of the picking (cross) aisles in one can be determined directly from the cross (picking) aisles in the other.*

The observation can be shown by noting equivalent parallelograms in dual designs (see Figure 4.17) and by an inductive argument. Note also that the Chevron and Fishbone are dual designs, and that both offer the same savings over traditional designs. Costs in dual warehouses are not the same in practice, where aisles occupy space and pallet positions are discrete. We explain why in the next section.

4.7 A Continuum of Designs

With a continuous model of warehouse space, we have shown that two cross aisles is better than one, and three is better than two. Why not four, five, or ten? The answer, of course, is that the continuous model fails to account for the space lost to aisles. The greater the number of cross aisles, the more space is lost to those aisles, and the farther pallet locations are pushed away from the P&D point. Moreover, because cross aisles meet at the P&D point, increasing their number eliminates more and more of the best locations, which are those nearest the P&D point. We have, then, a tradeoff: increasing the number of properly placed cross aisles tends to decrease expected distance to locations, but it also tends to push

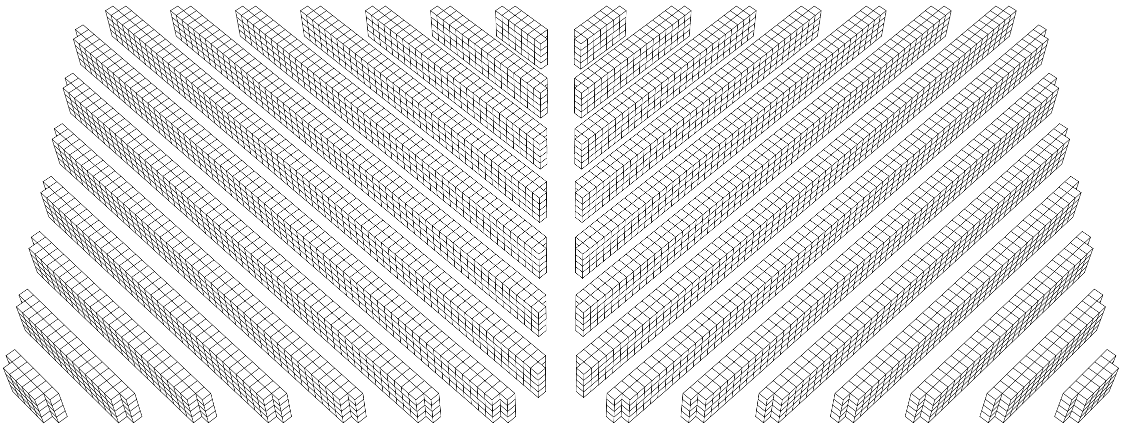


Figure 4.18: The Chevron design.

those locations farther away. How many cross aisles is best, and for what range of warehouse sizes?

To investigate these questions, we build a discrete model of aisles and pallet locations that better represents designs that could be implemented in practice (see Figures 4.18–4.20). In the discrete model, we assume pallet locations are square, that picking and cross aisles are the width of three pallets (about 12 feet), and that there is a bottom cross aisle. We measure travel distance from the P&D point to the location in a picking aisle from which the pallet can be accessed. The expected travel distance for a design is the total distance to all pallet locations divided by the number of locations in that design.

Is it possible to transfer the results of the continuous model directly into discrete space? That is, is a discrete design with angles of cross aisles and picking aisles taken from the optimal continuous solution also optimal in discrete space? To assess the robustness of optimal solutions from the continuous space model, we perturbed slightly these optimal angles in the discrete design to look for lower expected distance. In some cases, we were able to improve very slightly on the continuous result, but the improvements were always less than 0.01%. Part of the discrepancy is likely due to our method of calculating expected cost in a discrete design, in which

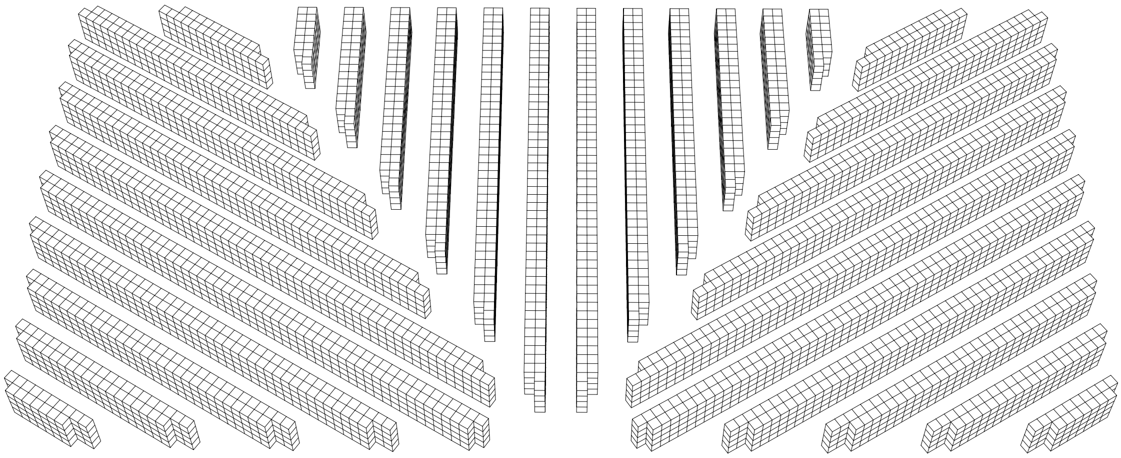


Figure 4.19: The Leaf design.

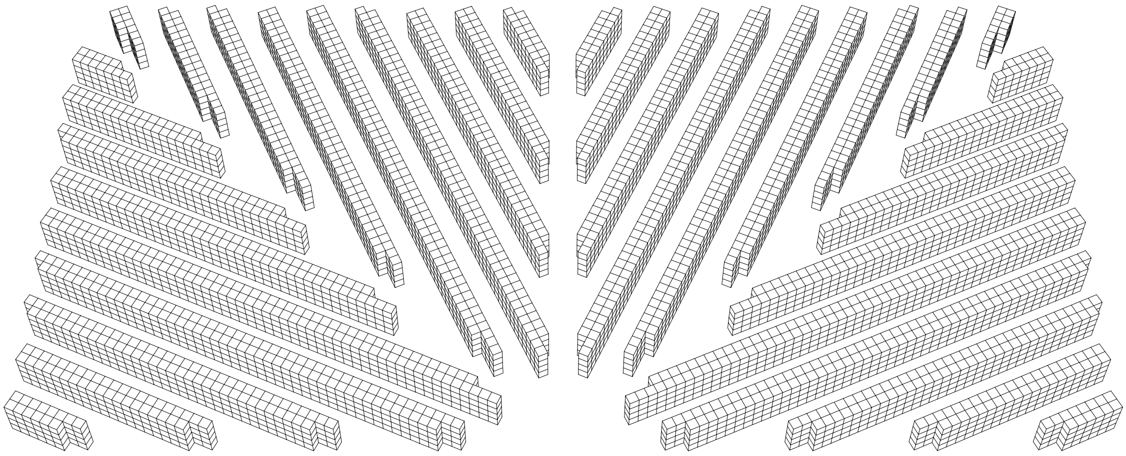


Figure 4.20: The Butterfly design.

a very slight change of angle could allow a single distant pallet location to fit into a crevice that previously could not quite contain it (a careful look at Figure 4.20 should make this clear). In any real implementation, of course, there would be many reasons why neither of our models would be exact: aisle widths may vary because of vehicle types, pallet locations may not be exactly square, there is clearance between racks, and so on. Nevertheless, we contend that direct transfer of results from the continuous model to the discrete model is appropriate, and for our purposes the resulting discrete designs are "optimal."

Figure 4.21 shows the expected distances for designs having approximately a 2:1 width to depth ratio. (We say "approximately," because the discrete nature of the designs precludes maintaining an exact ratio.) All designs offer more than approximately 13% improvement in expected travel distance over equivalent traditional warehouses of different sizes (data for the number of pallet locations in these warehouses are in Appendix A.4.) Also, as the size of the warehouse increases, the advantage of inserting angled cross aisles increases, and the percentage of additional space required for angled aisles decreases.

Among all non-traditional designs, Chevron is the best design for warehouses with 27 or fewer aisles. As the size of the Chevron increases from 19 aisle-widths to 27 aisle-widths, savings increase from approximately 16.1% to 17.1% and additional space requirements decrease from 11.2% to 7.3%. Chevron always has less expected travel distance than the equivalent traditional design, even if the warehouse is small (see Figure 4.22). Although the Chevron and the Fishbone offer the same benefit in continuous space, the Chevron has slightly lower expected travel distance in a discrete design. Nevertheless, the concept of duality does explain why, even in a real layout, the costs of a Chevron and Fishbone are so close.

For warehouses with more than 27 aisles, the Leaf offers more savings than the Chevron, but requires slightly more space due to having two inserted cross aisles. As the size of the warehouse increases, the additional savings of the Leaf increase

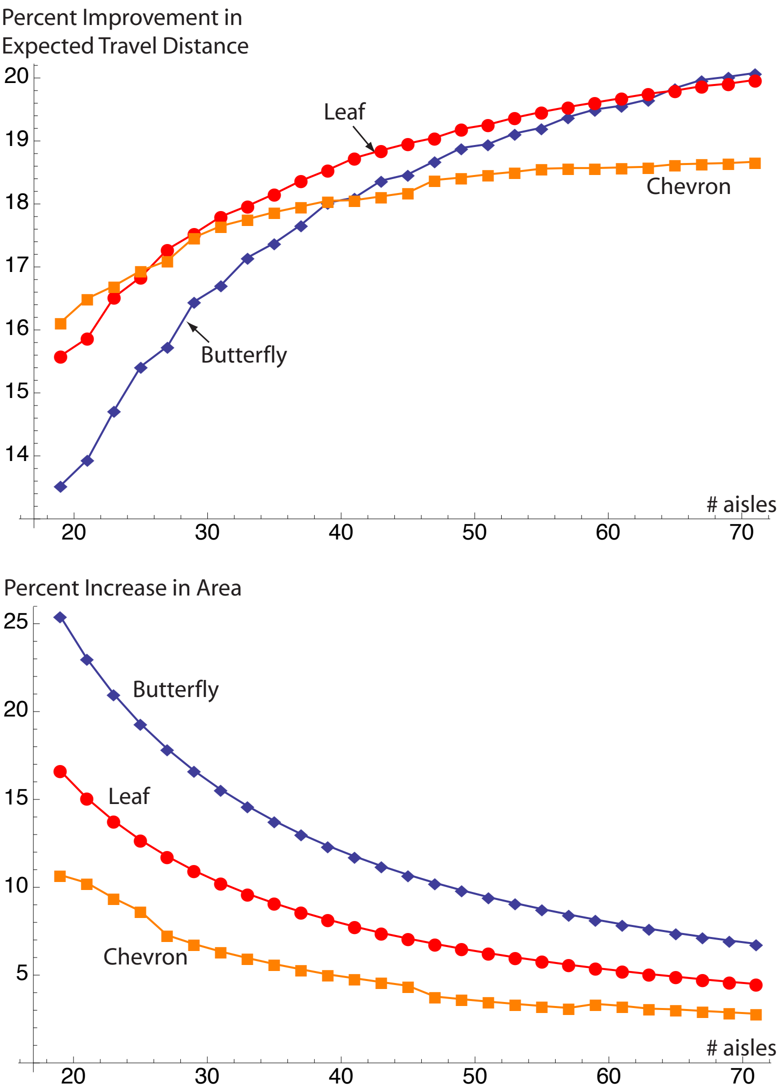


Figure 4.21: Comparison of several approximately square half-warehouses with an equivalent traditional warehouse. (Data for the plots are in Appendix A.4.)

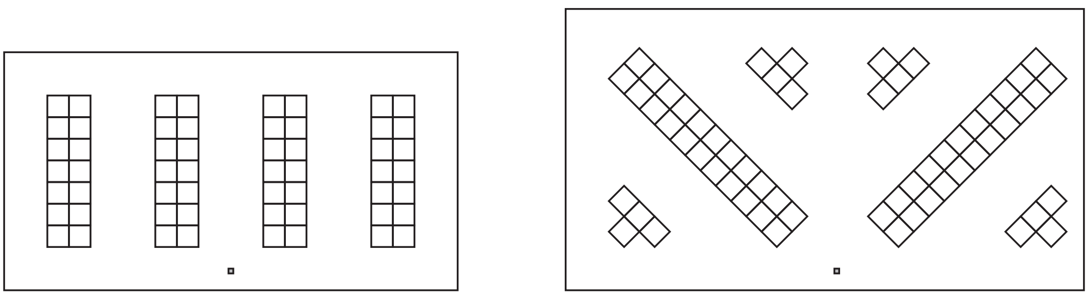


Figure 4.22: Chevron versus Traditional in an equivalent small size warehouse: Chevron has lower expected distance, but requires more space.

and the additional space requirement decreases compared to the equivalent Chevron and traditional designs. Even for a huge warehouse with 63 aisles, the Leaf provides more benefit than the Butterfly and requires less space. For a large warehouse with 51 aisles, the Leaf has 19.3% improvement over an equivalent traditional design and uses 6% more space.

Although the Butterfly offers more improvement than the Leaf in the continuous space model, in a discrete design it has less savings for most warehouse sizes because of the wasted space due to more cross aisles. However, the Butterfly offers slightly more improvement than the Leaf for warehouses larger than 65 aisles.

4.8 Implications for Practice

We have shown that there are new ways to arrange aisles in a unit-load warehouse such that expected distance to store and retrieve pallets is significantly lower. The goal, of course, is to reduce the labor costs for such operations, but as Figure 4.21 illustrates, reducing expected travel distances comes at a cost of lower storage density, or equivalently, of larger facilities. This tradeoff suggests that firms facing the task of designing unit-load storage areas, whether for exclusive pallet-handling operations or for bulk reserve operations in support of order picking, should weigh the fixed cost of a larger space against the operational savings of lower travel distances. The details of such problems will vary with the operational situation, of course.

Our models are based on several important assumptions. First, we have assumed uniform picking density, which is roughly equivalent to the randomized storage policy (Schwarz et al., 1978) used in many bulk storage areas. Pohl et al. (2010) showed that the Fishbone design, which is similar to the designs we describe here, offers significant benefit even under turnover-based storage, so we expect similar results would be found for the Chevron, Leaf, and Butterfly. A second important assumption is that travel begins and ends at a single pickup and deposit point, which is more restrictive. In practice, this assumption would be appropriate when, for example, picks must pass through a single stretch-wrap machine, or when received pallets must pass through a processing station (we describe examples below). Our final important assumption is that all travel is for single-command cycles. This is true for many operations in practice, but even operations capable of executing dual-commands must execute single commands when there are no stows (picks) in the queue. Furthermore, as Pohl et al. (2009b) show, designs that reduce single-command travel also perform well for dual commands because two of the three legs in a dual command cycle are essentially single command distances.

Our computational results show that Chevron is the best design for warehouses with fewer than 27 aisles, which, in our experience, is the majority of unit-load warehouses in practice. The Leaf offers slightly lower expected travel distances for larger warehouses, but the penalty cost in terms of space is high. For example, the Leaf is 0.15% better than Chevron for a 31 aisle-width warehouse, but requires 4% more space. It is hard to imagine such a small operational benefit overcoming the fixed cost of such additional space. Even for warehouses equivalent to a traditional design with 57 aisles (an unrealistic size, in our experience), the Chevron is within one percent of the benefit offered by the Leaf. The Butterfly offers no improvement over the Leaf for warehouses smaller than 65 aisles, and is simply too space-inefficient to be a viable candidate design. For these reasons, we believe the Chevron design is the best choice for any realistic scenario that might be encountered in industry.

But will distributors actually use these ideas? There is now evidence that, under the right conditions, they will. At the time of writing, the authors knew of five implementations of non-traditional aisles, each motivated by the results in Gue and Meller (2009). Generac Power Systems in 2007 implemented a modified Fishbone design at its finished goods warehouse in Whitewater, Wisconsin (Figure 4.23). The lower left and right triangles in the design contain floor stacked pallets and pallet flow rack, whereas the central region is single-deep pallet rack. Pallets are taken by counterbalanced fork truck from receiving doors to a central P&D point, where

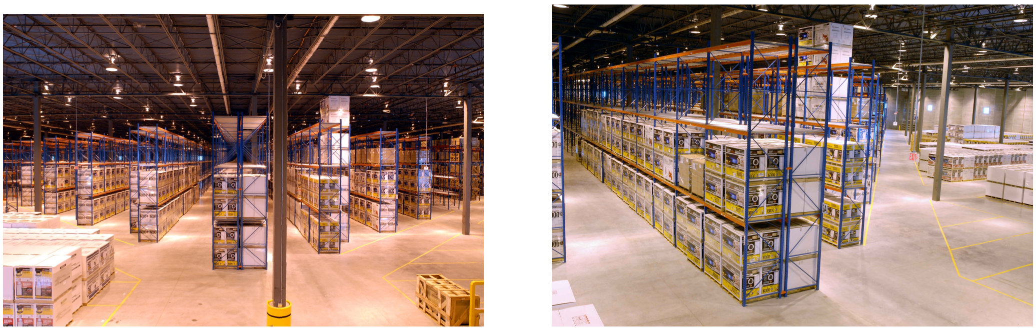


Figure 4.23: An implementation of non-traditional aisles at Generac Power Systems.

they are transferred to a second, more expensive vehicle capable of vertically lifting them to their storage locations. A second implementation, in Florida, uses almost exclusively floor stacked pallets (goods are extremely heavy, making pallet rack infeasible due to floor loading constraints). All picks are taken to a single stretch-wrap machine, thus satisfying the single P&D requirement. Unfortunately, for different reasons, neither of these warehouses could establish a before-and-after comparison of performance, but both have reported satisfaction with the designs.

Of course, there were unanticipated details in the implementations, some good, some not as good. For example, even though forklift drivers rarely need to make a 135° turn when coming out of a picking aisle, still they must look both ways before entering the cross aisle, and that sometimes requires looking back 135°. Semi-hemispherical mirrors were installed to improve safety at these intersections. On the other hand, workers at one warehouse reported that they liked the design because they were "able to take [the 45°] turns at full throttle," a benefit that had not occurred to us.

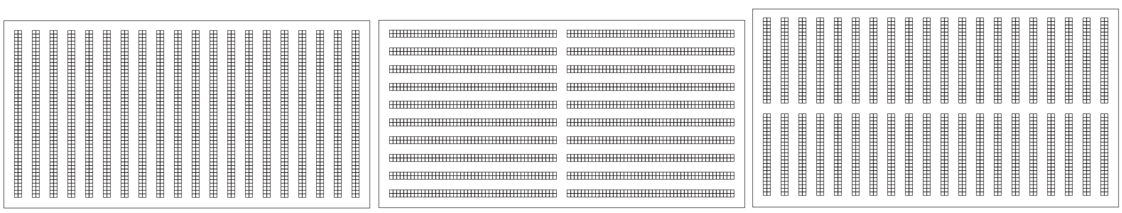
Non-traditional aisle designs are not right for every operation. The benefit for small warehouses, or for those that do not consume much labor, likely would not overcome the fixed cost of needing a larger storage space. Such warehouses should use conventional designs, for which storage density is highest. Labor-intensive operations, however, should consider the long-term benefits of the Chevron design.

Chapter 5

Non-Traditional Unit-Load Warehouse Designs with Multiple Pickup-and-Deposit Points

5.1 Introduction

In global logistics and transportation systems, warehousing plays a critical role in assuring high levels of customer service and overall supply chain performance. Among warehouses in a supply chain network, unit-load receiving and retrieving, where one item or pallet is carried at a time, often makes up a considerable portion of warehouse operations, especially in 3rd party distribution centers. According to Rogers (2009), during times of economic downturn, in addition to increasing customer and supplier satisfaction, many warehouses are also looking internally for ways to cut costs. Among operations costs in a warehouse, labor cost is significant because workers in distribution centers spend their time traveling to store or retrieve items (sometimes traveling empty). Also, because 3rd party distribution centers pay their forklift drivers for work-hours and bill their customers for two movements (receiving and retrieving) for each pallet, reducing travel time and increasing the number of handles per hour are two of the cornerstones of running a warehouse efficiently and profitably (Bartholdi and Hackman, 2008).



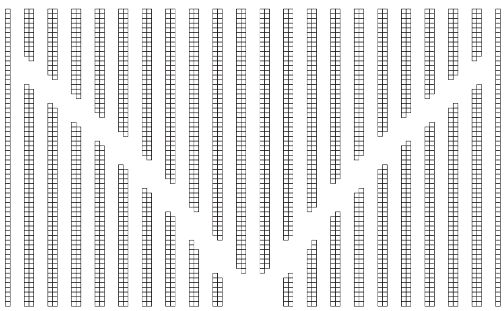
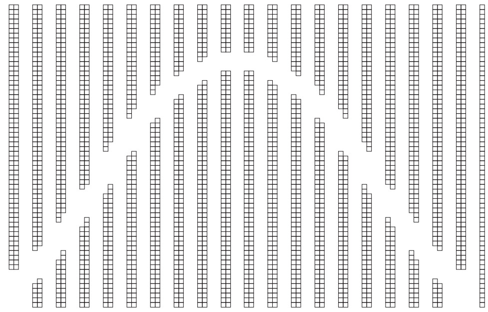
(a) Trad-A (b) Trad-B (c) Trad-C

Figure 5.1: Traditional rectangular warehouses.

In traditional warehouses, forklift drivers travel along the aisles, where cross aisles are arranged at right angles to parallel picking aisles, to reach a storage location (see Figure 5.1). Because of the rectilinear travel paths in traditional unit-load warehouses, inserting an orthogonal cross aisle offers no advantage in travel distance to and from a pickup and deposit point when it is placed at the bottom or top of the warehouse (Roodbergen and de Koster, 2001). However, if there is more than one P&D location and at least one of them is placed on the right or left side of the warehouse, a cross aisle is needed to facilitate travel from P&D points in traditional warehouses (see Figure 5.1b and 5.1c). Additionally, it is not uncommon that warehouses in industry have multiple entry points to access the storage area. For example, while one side of the warehouse has receiving docks, the other side might have shipping docks. Or, receiving and shipping docks might be on the same side of the warehouse.

Gue et al. (2010) took the idea of non-traditional warehouse designs and investigated unit-load warehouses for multiple P&D points on one side. They inserted two angled cross aisles that form "modified Flying-V" and "Inverted-V" designs, into the traditional warehouses (Figure 5.2). They showed that the modified Flying-V design can reduce travel by 3-6%, and the Inverted-V offers less than 1% in savings compared to a traditional warehouse. Additionally, they suggested that if the inserted P&D points are concentrated toward the center of the bottom of the warehouse, the benefit of the Flying-V would be increased.

Although Gue et al. (2010) considered multiple P&D points for non-traditional aisles, the resulting designs still assume that picking aisles should be vertical and parallel to each other. In Chapter 4, by inserting a number of cross aisles, we showed that the expected travel distance between a centrally located pickup and deposit (P&D) point and a storage location can be reduced by 15% to 20%. We also showed that allowing picking aisles and cross aisles to take any angle provides additional reduction in expected travel distances in warehouses. In this chapter we

(a) The Modified Flying-V (b) The Inverted-V

Figure 5.2: The Modified Flying-V and the Inverted-V designs for multiple P&D points. From Gue et al. (2010).

take these ideas further and propose new aisle designs close to the optimal for unit-load storage spaces that have multiple access locations. Additionally, whereas Gue et al. (2010) focused on multiple P&D points on only one side of the warehouse, we consider multiple P&D points both on one side and on different sides of the warehouse to cover more practical cases seen in industry.

In the next section, we review relevant literature in non-traditional warehouse designs. In Section 5.3, we discuss underlying assumptions of a general warehouse network model we present in Section 5.4. Then, in Section 5.5, we discuss our implementation of Particle Swarm Optimization. In Section 5.6, we introduce new warehouse designs to some specific cases differentiated by the locations of P&D points. In the final section, we summarize our findings.

5.2 Previous Research

The first discussions of non-orthogonal aisle designs came from Moder and Thornton (1965), Francis (1967a,b), Berry (1968) and White (1972). White (1972) investigated "Euclidean efficiencies" by inserting radial aisles into a continuous warehouse space. Then, Gue and Meller (2009) proposed two non-traditional designs, the Flying-V and the Fishbone. Pohl et al. (2009a) showed that for dual command operations the optimal placement of a "middle" orthogonal cross aisle in traditional designs is slightly beyond the middle. Pohl et al. (2009b) studied the Fishbone design for dual-command operations (also called task interleaving). They showed that the Fishbone design offers a reduction in expected travel distance compared to several common traditional designs. They also offered a modified Fishbone design for dual-command operations. Pohl et al. (2010) analyzed three traditional designs, as well as Fishbone and Flying-V designs, under turnover-based storage policies and both single- and dual-command operations.

Some research has been done on multiple depots and their different configurations in a warehouse. Randhawa et al. (1991) evaluated an unit-load AS/RS for two different configurations of dual-dock layouts on performance such as throughput and waiting time. Yano et al. (1998) discussed decentralized receiving in a manufacturing assembly facility. They developed an optimization-based procedure to determine some variables including multiple access points to the building so as to minimize the cost. Eisenstein (2008) considered dual depots in a discrete order picking system to minimize required walking distance. In addition to the warehousing literature, Garces-Perez et al. (1996) examined facility layout problems for pre-specified I/O locations for departments by using Dijkstra's shortest path algorithm with genetic programming. Also, Kim and Kim (2000) focused on the facility layout problem with predetermined shape and input and output points. They placed one input and one output point on different sides of a facility represented by four different shapes.

5.3 Assumptions

We assume a randomized storage policy, which is common in unit-load warehouses, for the same reasons described in Chapter 4. With this policy, pallets are stored in the closest available locations, as opposed to reserved locations in a dedicated storage policy. Thus, pallets may be stored at any location in the warehouse that increases the space utilization. In this policy, the probability of picking or

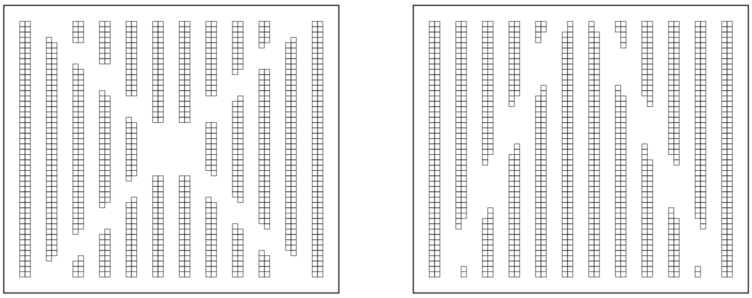


Figure 5.3: The intersecting and non-intersecting cross aisles in a warehouse.

storing any pallet in the warehouse is the same. For easier representation, our storage locations have a square-shaped footprint and one pallet length is the adopted measure of distance.

From the perspective of warehouse design, we assume that picking aisles, which are within regions determined by cross aisles, are parallel to each other. Picking aisles and cross aisles can take on any angle. Contrary to our model in Chapter 4 and the model of White (1972), we take into account aisle width and its effect on travel distances in our new model. Each picking and cross aisle is assumed to have a width of 3 pallet locations. We assume standard, single-deep pallet rack, although our models could easily be modified for double-deep rack or deeper lane storage. The model allows for no more than two angled cross aisles. Cross aisles are not allowed to intersect, but they can originate from the same point. This limitation is serious, but is necessary for computations reasons. The number of cases to consider for the case of intersecting aisles is much greater than the number of cases we describe below.

To allow for P&D points along the periphery of the warehouse, we assume that there are no storage racks along the sides of the warehouse. We call these available spaces on the periphery of the warehouse “side cross aisles” so that they can facilitate travel between locations. In a warehouse, P&D points could be places which have palletizing or shrink-wrap machines, or pick lists for workers. All travel is from the P&D points located on any side of the warehouse to any location in the warehouse (i.e. there are no zones). We assume only one pallet is carried at a time in a single

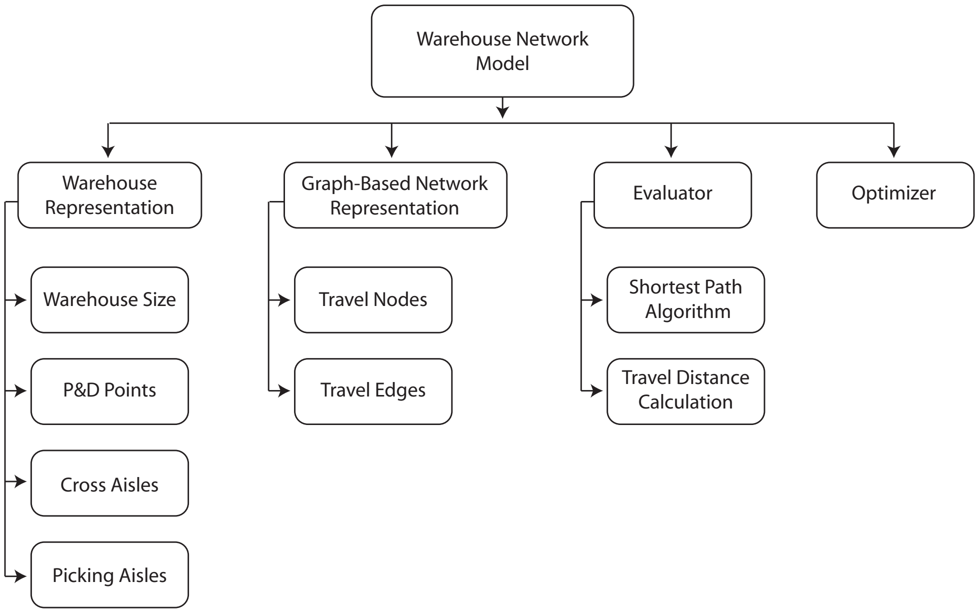


Figure 5.4: Description of the warehouse design tool

command cycle, comprised of travel to a random storage location and back to the P&D point.

Because of the single-command cycle, the same distances are traveled into the storage space and out of it, which enables us to consider only the half of the actual distance traveled. Hence, travel distance in our model is the shortest distance from the P&D point to the location on the picking aisle that accesses the center of a storage location. Picking aisles can also be traversed to reach a location if the travel path through that aisle has the shortest distance. The expected travel distance to pick (store) an item is the sum of the one-way shortest distances to all storage locations from all P&D points divided by the total number of storage locations and the number of P&D points in that design. Therefore, we assume that each P&D point has the same probability of being visited or used.

5.4 The Warehouse Network Model

Our warehouse network model consists of four main modules (Figure 5.4). These modules are designed as the fundamental classes of our conceptual software.

Warehouse representation: In this module, the warehouse as an entity is defined the width (W) and length (H) of the space, the width of the cross and picking aisles (a), the location of P&D points, the number of cross aisles, and the angles of cross and picking aisles in a suitable coordinate system (see Figure 5.6 for an example warehouse). P&D points are placed along the side cross aisles that provide direct travel to the storage area based on the angles of aisles. Based on the aisle width, cross aisles are defined by a straight line going through the storage space and whose start and end points are defined on different sides of warehouse. Single-deep racks are constructed in a space divided by cross aisles based on the angle and width of related picking aisles. Picking aisles are automatically arranged in accordance with valid positions of the pallet racks. Hence, in each pallet rack, storage locations and their access points on aisles are determined by the described procedure in Figure 5.5. A sample design is given in Figure 5.6.

Graph-based network representation: We use the warehouse representation to construct a graph representing the whole warehouse as a network (Figure 5.6). The nodes of the network include all of the potential P&D points, all intersections of the aisles, and an access node for each of the storage locations. P&D points are placed on the center lines of the side cross aisles. The network is constructed with non-negative edge lengths that represent the distances between the connected nodes. An access node for a storage location is placed on the center line of the appropriate picking aisle and to provide access to the center of the corresponding storage locations. The edge length for two connected neighbor access nodes, representing storage locations, in the same rack is one pallet (unit) length. Two access nodes on opposite racks, representing opposite storage locations, are connected and the edge length is zero, because they are actually served from the same coordinate.

Evaluator: In this module, we calculate the distance between a P&D point and a node by computing the shortest paths with Dijkstra's one-to-many algorithm. Dijkstra's algorithm searches the graph to find the path from the source node (P&D)

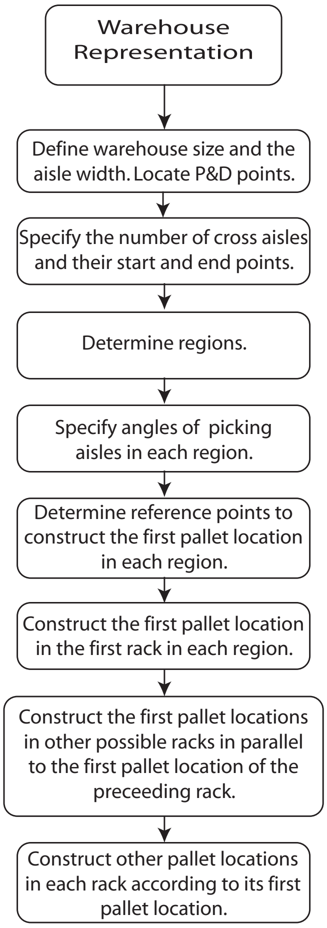


Figure 5.5: The procedure to represent a discrete space warehouse design for given set of parameters.

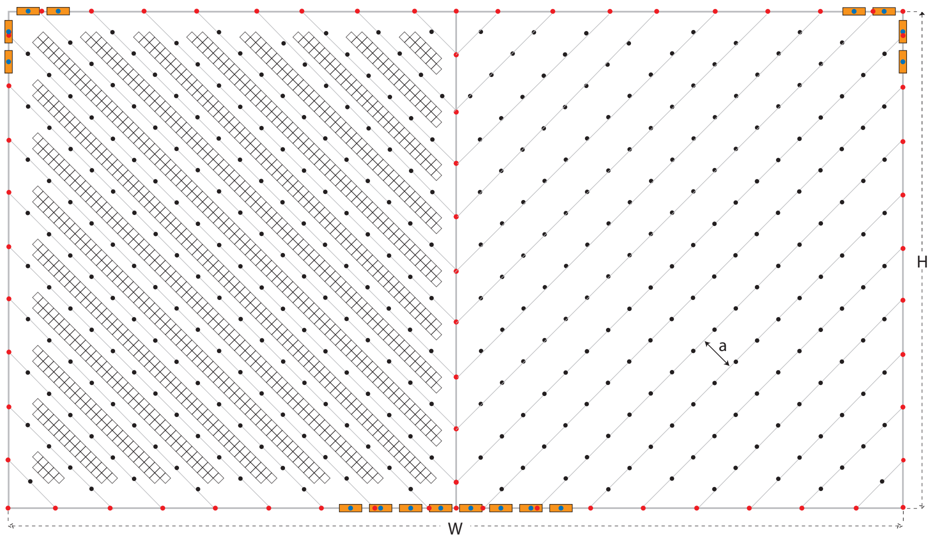


Figure 5.6: Network representation of a small, example warehouse. (Each storage location is represented by an access node in a real network.)

to the target node (storage location) with the lowest cost (distance). The worst case running time for the algorithm on a graph with n nodes and m edges is $O(n^2)$ (Schrijver, 2005).

The expected travel distance to pick an item in a warehouse with k P&D points, n total storage locations and the shortest distance between i^{th} P&D point and j^{th} storage location (d_{ij}) is

$$E[C] = \sum_{i=1}^{k}\sum_{j=1}^{n} \frac{d_{ij}\, p_i}{n} = \frac{1}{kn}\sum_{i=1}^{k}\sum_{j=1}^{n} d_{ij},$$

where $p_i = 1/k$ is the probability of choosing P&D point, because we assume each P&D has the same probability to be utilized.

Proposition 5.1. *E[C] is the same as two-way expected travel distance if the returning P&D point is selected randomly with equal probability.*

Proof. Let I be a set of k P&D points. p_i is the probability of choosing the i^{th} P&D point as a source node. $p(r|i)$ is the probability of choosing the returning P&D point r when the source i^{th} P&D point is known. The expected travel distance to pick an item in this system is

$$E[RC] = \sum_{i=1}^{k}\sum_{r=1}^{k}\sum_{j=1}^{n} \frac{1}{n}\left(\frac{d_{ij} + d_{rj}}{2}\right) p_i\, p(r|i),$$

where $p_i = 1/k$ and $p(r|i) = 1/k$, because returning P&D point might be the same as the source node. Hence, we can write

$$\begin{aligned} E[RC] &= \frac{1}{2nkk}\sum_{j=1}^{n}\left(\sum_{i=1}^{k}\sum_{r=1}^{k} d_{ij} + d_{rj}\right) \\ &= \frac{1}{2nk^2}\sum_{j=1}^{n}\left(\sum_{i=1}^{k}\left(kd_{ij} + \sum_{r=1}^{k} d_{rj}\right)\right) \\ &= \frac{1}{2nk^2}\sum_{j=1}^{n}\left(\sum_{i=1}^{k}\left(kd_{ij} + \sum_{i=1}^{k} d_{ij}\right)\right), \end{aligned}$$

where $\sum_{i=1}^{k} d_{ij} = \sum_{r=1}^{k} d_{rj}$ due to the same probability of visiting P&D points. Finally,

$$
\begin{aligned}
E[RC] &= \frac{1}{2nk^2} \sum_{j=1}^{n} \left(k \sum_{i=1}^{k} d_{ij} + \sum_{i=1}^{k} \sum_{i=1}^{k} d_{ij} \right) \\
&= \frac{1}{2nk^2} \sum_{j=1}^{n} \left(k \sum_{i=1}^{k} d_{ij} + k \sum_{i=1}^{k} d_{ij} \right) \\
&= \frac{2k}{2nk^2} \sum_{i=1}^{k} \sum_{j=1}^{n} d_{ij} = \frac{1}{kn} \sum_{i=1}^{k} \sum_{j=1}^{n} d_{ij},
\end{aligned}
$$

and is equal to E[C].

□

Therefore, in our model we only calculate one-way expected travel distance to pick an item from each P&D point.

Optimizer: We use Particle Swarm Optimization algorithm to minimize our objective function to find optimal designs. We discuss the details in the next section.

5.5 Optimization Methodology: Particle Swarm Optimization (PSO)

Because start and end points of cross aisles can be on any side of the warehouse, there are 12 two cross aisle cases without considering where the P&D points are located (see in Appendix B.1). Considering possible angles of picking aisles in each region increases total cases to approximately 192. Even though each case can be solved, when different start and end points of cross aisles and different locations of P&D points are considered, the problem becomes intractable. Additionally, simply evaluating the cost of a given design requires solving multiple one-to-many shortest path problems from multiple P&D points, we can not derive a closed form distance expressions as an objective function need to be minimized. Therefore, we choose a meta-heuristic solution procedure to search for the optimal values of variables that minimizes the expected travel distance. Specifically, we use an implementation of Particle Swarm Optimization (PSO).

PSO, which was first introduced to optimize continuous nonlinear functions by Kennedy and Eberhard (1995), is one of the latest evolutionary algorithms inspired by nature. Its development was based on observations of social interaction and communication involved in bird flocking and fish schooling. In PSO, each member is called a particle and each particle searches the multi-dimensional function space with a specific velocity. Two cognitive aspects of this initial metaphor, individual learning and learning from a social group, refer to local and global search, respectively (Banks et al., 2008). By individual learning, each particle can memorize its best previous position, called personal best, and move towards it in its restricted neighborhood. Then, the particles share all the information about their personal best points that they have received so far. Hence, to accomplish global search, each particle also moves towards the best particle, called global best, that has the best point among all particles in the whole swarm. By using personal and global bests, each particle adjusts direction and the magnitude of its velocity for movement in the space. Hence, the whole population of particles moves toward the global best as a flock of birds or school of fish. As with other meta-heuristics, PSO is also designed to find a unique, possibly optimum solution.

Advantages of a basic PSO algorithm include its simple structure, ease of implementation, speed in solving nonlinear, non-differentiable multi-model problems, and robustness (Tasgetiren et al., 2007). Additionally, its ability to handle continuous variables is another advantage for our problem.

5.5.1 Formulation of the problem

We follow three steps to formulate our problem and to search the solution space quickly. The first step is "encoding," which in our case is a simple vector of real numbers representing a warehouse. It can be easily translated back into a warehouse design. Such an encoding allows rapid search of the solution space. The elements of this vector are:

- A linear cross aisle is represented with its start (S) and end (E) points, which define the angle, in a given suitable coordinate system. Hence, a set of cross aisles can be represented as a list of such pairs.

- Picking aisles in any resulting region can be represented simply by their angle $0 \leq \alpha \leq \pi$ in radians.

To simplify the encoding, start and end points of a cross aisle are represented by real numbers in a single dimensional coordinate system. The origin is at the upper-left corner and is defined as 0 and 4 in order to make a closed loop for the perimeter a rectangular warehouse. Lower-left, lower-right and upper-right corners are defined as 1, 2 and 3, respectively (see also in Figure 5.7). Hence, if the appropriate point of the cross aisle, S or E, is located on the left side $\in [0, 1)$, on the bottom side $\in [1, 2)$, on the right $\in [2, 3)$ and on the top $\in [3, 4]$. When evaluating the cost of a design, we convert the encoding of the cross aisle to an x, y point in the coordinate system. Additionally, P&D points are pre-located at their given positions in the coordinate system, which are along the perimeter of the warehouse rectangle.

To narrow the search space, some inappropriate cases, such as having the start and end points of a cross aisle on the same side of the warehouse, and duplication (switching the start and end) are avoided. Hence, for the one cross aisle model, there are mainly 6 cases representing the possible orientations of the cross aisle (Figure 5.7). Because cases 3, 4 and 6 are symmetric cases of case 1, we only consider cases 1, 2 and 5 as subproblems. In these subproblems, $S \in [0, 1)$ and $E \in (1, 2]$, $S \in [0, 1)$ and $E \in [2, 3]$, $S \in [1, 2]$ and $E \in [3, 4]$, respectively.

For the two cross aisle model, there are 34 cases (see in Appendix B.1). Twelve of them are defined as main cases and others are symmetric. We divide these 12 main cases into 3 subproblems as described in Figure 5.8 to narrow the search space

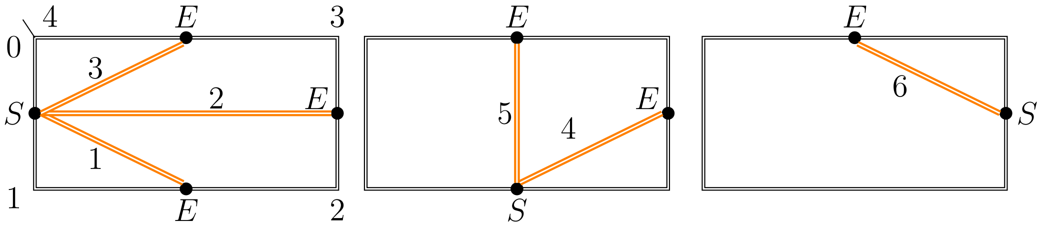


Figure 5.7: Possible arrangements of one cross aisle. (Numbers on the cross aisles represent cases in the one cross aisle model.)

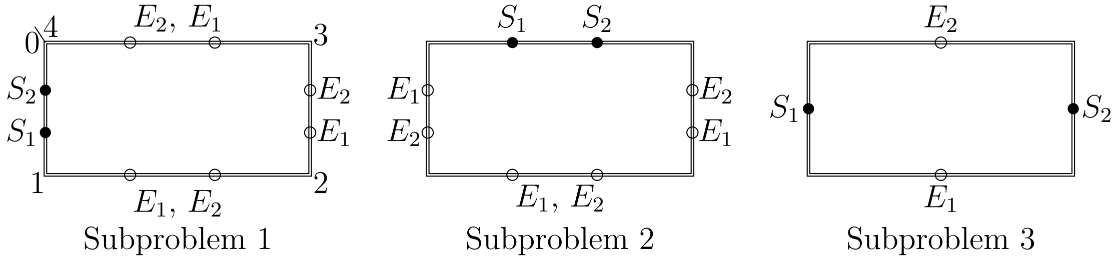


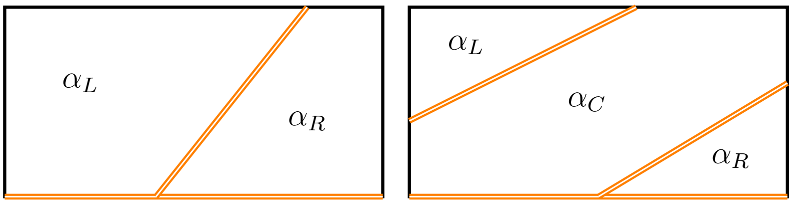
Figure 5.8: Possible directions of cross aisles

and control the movement of the cross aisles. In the two cross aisle model, S_1 and E_1 are the start and end points of the first cross aisle, as well as S_2 and E_2 of the second cross aisle. Because of the assumption of non-intersecting cross aisles, we define upper and lower limits for start and end points of cross aisles (see Table 5.1).

Additionally, because our model assumes at most two cross aisles and we assume non-intersecting cross aisles, there are at most three regions (Figure 5.9). Angles α_R, α_L and α_C (if present) represent the angles of picking aisles in the right, left and central regions. For an example, the encoding for a two cross aisle model is $\{\alpha_L, \alpha_C, \alpha_R, S_1, E_1, S_2, E_2\}$.

	S_1	S_2	E_1	E_2
Subproblem 1	(0,1)	(0, S_1)	(1, E_2)	(1,4)
Subproblem 2	(3,4)	(3, S_1)	(0, E_2)	(0,3)
Subproblem 3	(0,1)	(2, 3)	(1, 2)	(3,4)

Table 5.1: Upper and lower limits of start and end points of cross aisles in the two cross aisle model.



(a) One cross aisle design (b) Two cross aisle design

Figure 5.9: Possible region definitions and angles of aisles in these region.

The second step is "evaluation," in which we evaluate the performance of a design represented by a specific encoding (a vector of design parameters) on expected travel distance of a pick or stow. For this, we use the graph-based network model of the warehouse discussed in the previous section, and our objective function is to minimize the expected travel distance $E[C]$.

The third step is to "search" encodings to find the lowest possible $E[C]$ for each defined subproblem in the one and two cross aisle models. For this task, we use the PSO algorithm. Finally, we select the aisle design that has the lowest $E[C]$ among solutions of the subproblems with respect to the one and two cross aisle models.

5.5.2 PSO Algorithm

In the simple PSO algorithm, each particle represents a solution and moves toward its previous best position and the global best position found in the population so far. After initializing the parameters and generating the initial population randomly, each particle is evaluated by the fitness function. After evaluation, each particle is updated with its position, velocity and fitness value. If there is an improvement in its fitness value, it updates its personal best. Also, the best particle in the population is used to update the global best. Next, the velocity of the particle is updated by using its previous velocity, personal best and the global best to move the particle to a better place in the search space. In doing these steps repeatedly, the algorithm searches the space until it is terminated by a stopping criterion.

The structure of our PSO algorithm is basically the same as the modified PSO by Shi and Eberhart (1998a,b). Hence, the basic elements of our algorithm are particles, population, particle velocity, personal best, global best, inertia weights, coefficients and maximum velocity.

Population and Subproblem: F is the set of s subproblems, $F = \{I_1, I_2, ..., I_s\}$. I_s is also the set of i particles at iteration t, $I_s^t = \{X_{1s}^t, X_{2s}^t, ..., X_{is}^t\}$.

Particles: X_{is}^t denotes the i^{th} particle in the s^{th} subproblem at iteration t. It is represented by d dimensions: $X_{is}^t = \{x_{is1}^t, x_{is2}^t, ..., x_{isd}^t\}$, where x_{isd}^t is the position or the value of i^{th} particle in the s^{th} subproblem for the d^{th} dimension. Dimensions can be thought of as model variables or parameters.

Velocity of particles: V_{is}^t is the set of d velocities of particle i in the s^{th} subproblem at iteration t, $V_{is}^t = \{v_{is1}^t, v_{is2}^t, ..., v_{isd}^t\}$. Each dimension of each particle moves in the search space with a distance v_{isd}^t at each iteration.

Personal best: P_{is} is the best previous position, which gives the best fitness value, of the i^{th} particle in the s^{th} subproblem, and $P_{is} = \{p_{is1}, p_{is2}, ..., p_{isd}\}$ where p_{isd} is the best value of dimension d of the i^{th} particle in the s^{th} subproblem so far.

Global best: $G_s = \{g_{s1}, g_{s2}, ..., g_{sd}\}$ is the best particle among all the particles in the s^{th} subproblem, where g_{sd} is the best value of dimension d in G_s.

Inertia weights: Shi and Eberhart (1998a,b) introduced an inertia weight to the algorithm. It is denoted as w and is used to balance global and local searches. It can be a positive constant or a function of time or iteration. A larger w favors the global search, while a smaller w favors the local.

Coefficients and maximum velocity: Coefficients c_1 and c_2 are the weights for "cognitive" and "social" parts in the velocity update equation given in (5.1). To control the explosion of velocities and provide stability, there are two other mechanisms: maximum velocity V_{max} and a constriction coefficient (K). For the quality of the search, Shi and Eberhart (1998a) showed the effect of V_{max} that determines the maximum change one particle can undergo in its positional coordinates during an iteration. Clerc and Kennedy (2002) showed that the following value of K provides rapid convergence:

$$K = \frac{2}{|2 - \phi - \sqrt{\phi^2 - 4\phi}|},$$
$$\phi = c_1 + c_2 > 4.$$

Also, r_1 and r_2 are uniformly distributed random numbers [0,1] that influence the movement toward personal or global best. Thus, the particles are updated based on the equations,

$$v_{isd}^t = K\left(w^{t-1}v_{isd}^{t-1} + c_1r_1(p_{isd} - x_{isd}^{t-1}) + c_2r_2(g_{sd} - x_{isd}^{t-1})\right) \tag{5.1}$$
$$x_{isd}^t = v_{isd}^t x_{isd}^{t-1}. \tag{5.2}$$

Implementation Details

In accordance with the one and two cross aisle models, we have 3 subproblems for each model ($s = 3$), and each subproblem is searched by 7 particles. Note that there is no interaction or communication among particles in these different subproblems. Picking aisles can take any angle between the lower bound of 0° and the upper bound of 180°. Because each subproblem is defined based on the cases of cross aisles, start and end points of cross aisles in each subproblem can take any real numbers between their lower and upper bounds. In order to keep the variables in their boundary, we need to adjust the velocity of particles based on their bounds in addition to V_{max}. Therefore, the applied velocity of the d^{th} dimension for the i^{th} particle in the s^{th} subproblem (m_{isd}) is used to update the positions of the particle in our algorithm. Generally, m_{isd} is calculated based on the issues shown in Figure 5.10 and 5.11, and described as the following

$$m_{isd}^t = \begin{cases} -\xi(U_d - L_d) - (x_{isd}^t - U_d), & \text{if Issue 1 or 5 or 6 occurs} \\ \xi(U_d - x_{isd}^t), & \text{if Issue 2 occurs} \\ \xi(U_d - L_d) + (L_d - x_{isd}^t), & \text{if Issue 3 or 4 or 8 occurs} \\ -\xi(x_{isd}^t - L_d), & \text{if Issue 7 occurs} \\ -V_{max}, & \text{otherwise if } v_{isd}^t < -V_{max} \\ V_{max}, & \text{otherwise if } V_{max} < v_{isd}^t \end{cases}$$

$$x_{isd}^t = m_{isd}^t x_{isd}^{t-1},$$

where L_d and U_d are lower and upper bounds of the d^{th} dimension, respectively. Parameter ξ is a uniformly distributed random variable between [0,1] that specifies the magnitude of the movement toward the specified bound. Additionally, we set V_{max} equal to one third of the length of the warehouse. This value allows enough space for exploration, but not so much that particles do not converge (Kennedy, 2007).

For the two cross aisle model, in the encoding, the dimensions corresponding to the start and end points of cross aisles move according to a rule such that: S_1 moves first, S_2 second, E_2 third and E_1 last. Thus, we can maintain their movement within their limits defined in Table 5.1 using m_{isd}. For example, as soon as S_1 moves, if it moves to a lower value than the current value of S_2 in subproblem 1 for two cross aisle model, then the movement of S_2 might be adjusted based on issues 1, 5 or 6.

After moving cross aisles with an applied distance m_{isd}^t, cross aisles might get so close to each other that there is not enough space for having storage locations in a region between cross aisles because of the aisle width. If this happens, we select the most appropriate cross aisle that has more available space to be moved than the other cross aisle. We assign λ, which equals V_{max} in our algorithm, to the velocity of

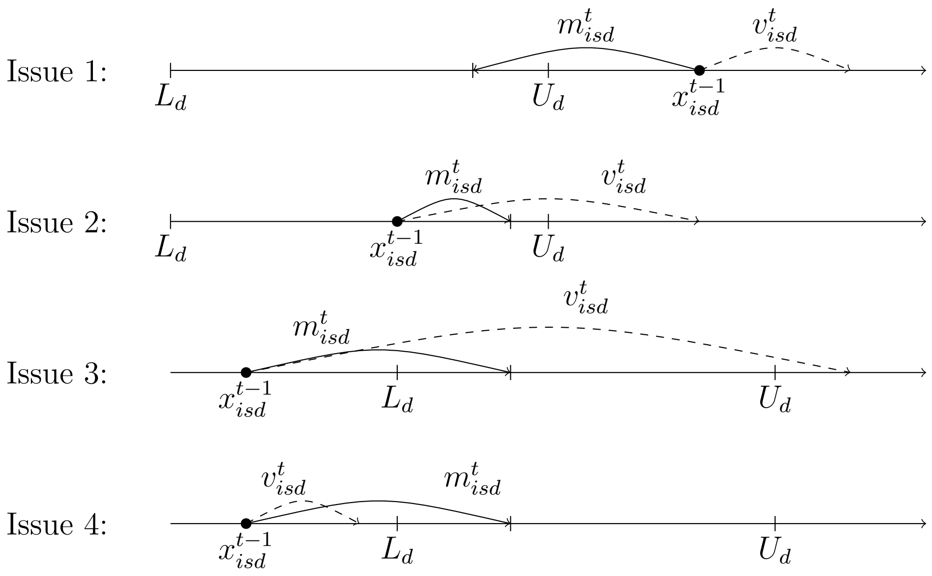


Figure 5.10: Issues show that the variable goes out of its boundary if v^t_{isd} is applied, when $0 < v^t_{isd}$.

the selected cross aisle's start and end points and move them away by recalculating m^t_{isd}. Thus, this repair function is used to separate cross aisles from each other so that we can have reasonable warehouse designs for the two cross aisle model.

We define inertia weight as a linear function of the number of iterations: $w^t = (-t/k) + q$, where t is the iteration number, q is the allowable maximum value of w^t, and k determines the minimum value of w^t. In our algorithm, we chose $q = 1.2$ in order to increase global search at the beginning of the iterations and then reduced it to around 0.8 at the end, because Shi and Eberhart (1998a) mentioned that PSO had the best chance of finding the global optimum when $0.8 < w < 1.2$. Clerc and Kennedy (2002) recommended that $c_1 + c_2 = 4.1$, sol we selected $c_1 = c_2 = 2.05$. As a termination criterion, we both use the total number of iterations and number iterations the global best solution for each subproblem was not improved. Hence, the algorithm runs for the longer time. In our models, if the solutions cannot be improved in the last 100 iterations, we terminate the search after 1000 iterations. Pseudo-code for the PSO algorithm is given in Figure 5.12.

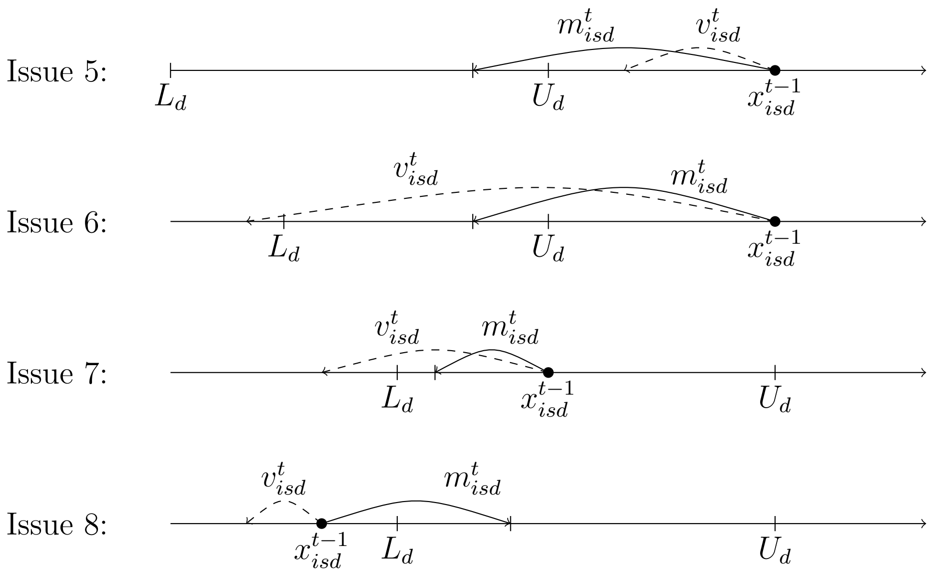


Figure 5.11: Issues show that the variable goes out of its boundary if v_{isd}^t is applied, when $v_{isd}^t < 0$

5.6 Optimization of Layouts for Unit-Load Warehouses

To validate the PSO algorithm, we applied it to a warehouse with a single, centrally located P&D point and one cross aisle. The warehouse has a width of 100 and height of 50 units. The model terminated after 1000 iterations. As expected, the model proposed a design that looks like the Chevron, which is the optimal one cross aisle design with a centrally located P&D point (see Chapter 4). In the proposed design, while the cross aisle is vertical and originated from the P&D point, the angles of the picking aisles are slightly different from the optimal angles in the Chevron. The angle of the picking aisles in the right region is approximately 44.4°, instead of 45°, as in the Chevron. The expected travel distance to pick an item in this design is approximately 0.2% greater than in the Chevron. Therefore, there might be a slightly better design than the proposed design, and it could be obtained by slightly modifying the aisle angles. This is because a few pallet locations might either disappear or appear in a discrete warehouse space by a slight change in the angles of the aisles (see in Figure 5.13.) Hence, we might accept to modify the

Initialize x_{isd} randomly between its bounds (L_d, U_d)
Initialize v_{isd} randomly within $[-V_{max}, V_{max}]$
Convert encoding of start and end points of cross aisles to (x,y) points
Evaluate the cost of design for X_{is} by Evaluator module
Do{ $\forall s \in F$
    Do { $\forall i \in I_s$
        Find personal best P_{is}
        Find global best G_s
        Calculate v_{isd} and m_{isd}
        If needed, use repair function to separate cross aisles
        Convert encoding of start and end points of cross aisles to (x,y) points
        Update the particle X_{is}
        Evaluate the cost of design for X_{is} by Evaluator module
    }
} Until termination criterion is met.

Figure 5.12: Pseudo-code for the PSO algorithm

angles of aisles slightly for exploring better design than the proposed design. In the next sections, we will present several example modified designs and discuss their performance, comparing to the proposed designs.

In this section, we look for the optimal aisle structures for warehouse designs with different locations of P&D points. Figure 5.14 shows the locations of the P&D points for five different design problems. These problems are nominal, in the sense that they are intended to explore optimal designs for different flow patterns, rather than represent actual industrial problems. Nevertheless, the solutions to these problems do suggest the potential benefit of alternative aisle designs for actual industrial problems.

We search for the best aisle angles in these design problems for three different warehouse sizes each having a 2:1 width to depth ratio. The small-sized warehouse is equivalent to a 19-aisle width traditional design, and it has a width of 100 and height of 50 pallet units. The medium-sized warehouse is equivalent to a 29-aisle width traditional design that has a width of 150 and height of 75 units. The larger one is equivalent to a 39 aisle-width, 200-pallet width and a 100-pallet depth traditional design. We insert one and two angled cross aisles into warehouses with multiple

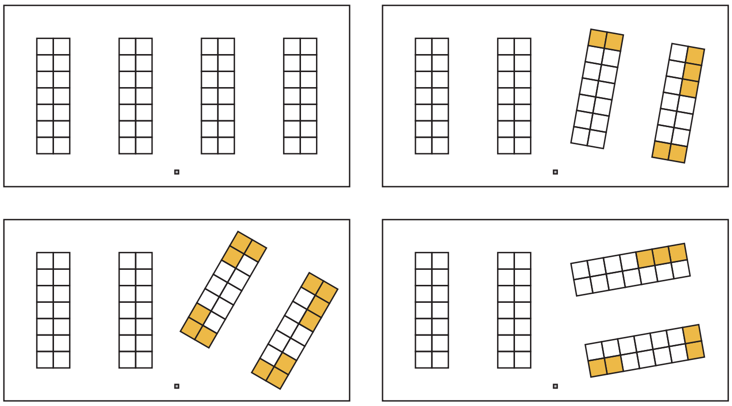


Figure 5.13: Changing number of available storage locations in a discrete warehouse space by angle of aisles. The dark squares represent the disappearing pallet locations because they overlap at the appropriate cross aisle.

P&D points that are defined in Figure 5.14. After running our algorithm three times, we select the design that has the lowest expected travel distance.

Although meta-heuristics do not guarantee optimality, they do offer feasible and good solutions. The designs we propose in the next sections are the best designs we have found so far for the specified design problems with an appropriate number of cross aisles. Hereafter, we call them "improved" designs. We compare the improved designs with appropriate traditional designs with respect to reductions in expected travel distance to pick an item in equally sized warehouses. Because the improved designs often propose angled aisles and include inserted cross aisles, they provide less storage capacity than an equivalent traditional design. Therefore, we expand the warehouse space while preserving the aisle structure and shape ratio so that they provide approximately the same storage capacity as an equivalent traditional design. We then compare them with respect to the improvement in expected travel distance to pick an item and the additional space requirement to provide an equal capacity over traditional designs.

5.6.1 Design Problem A

In design problem A, we locate two P&D points at the 1/3 and 2/3 point of the bottom side of the warehouse. For an accurate comparison, we take the design

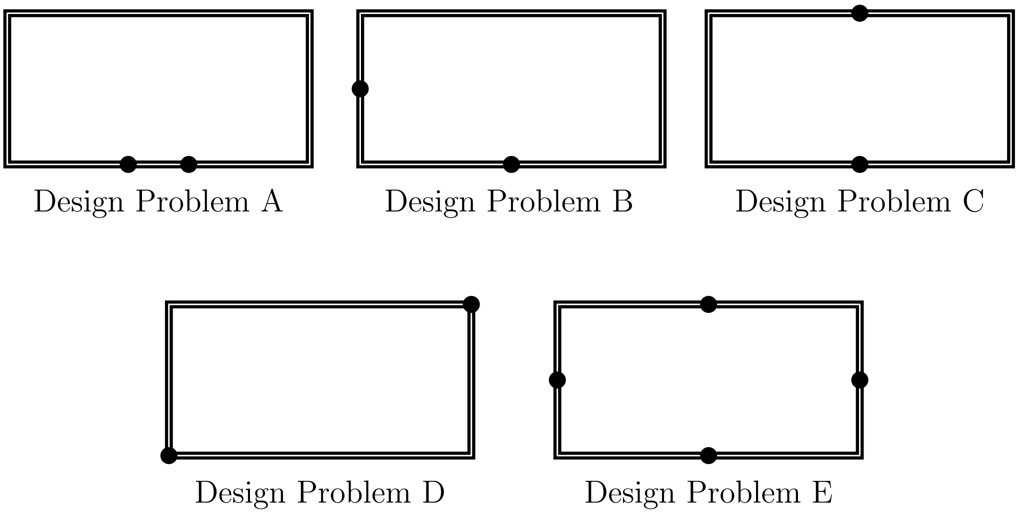


Figure 5.14: Design problems differentiated by the locations of P&D points.

Warehouse	$E[C]$	# Pallets
Small (19 aisles)	528.3	1880
Medium (29 aisles)	791.7	4320
Large (39 aisles)	1055.8	7760

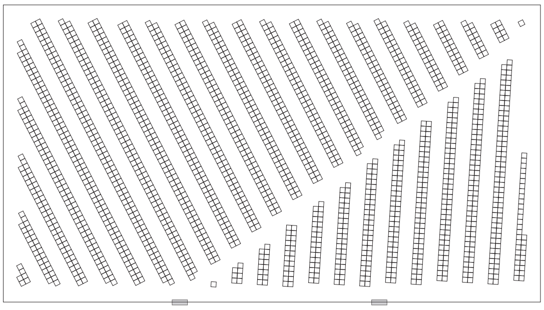
Table 5.2: Performance of Trad-A when P&D points are placed at the 1/3 and 2/3 point of the bottom side of the warehouse.

Trad-A as a base in this problem because it is the most compact traditional design, while it allows direct travel from both of the P&D points to the storage area. The expected travel distance and the number of storage locations in the Trad-A design for small-, medium- and large-sized warehouses are given in Table 5.2. In the next sections, we look for the best aisle designs with one and two inserted cross aisles for this problem and compare them with the Trad-A.

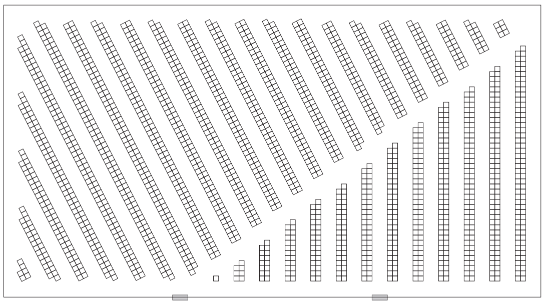
Design A with One Cross Aisle

The improved aisle design proposed by the heuristic model, with one inserted cross aisle in design problem A, is shown in Figure 5.15a. We call this "Design A1" (see Appendix B.2.1 for detailed computational results).

For a medium-sized warehouse, the cross aisle originates from the left P&D point to slightly below the upper-right corner with an angle (β) of approximately 33°. The model seems to select one of the P&D points and originates the cross aisle



(a) Design A1



(b) Modified Design A1

Figure 5.15: Designs with one cross aisle in design problem A.

from that P&D point. The angles of the picking aisles in the right (α_R) and left (α_L) regions are 88° and 117°, respectively. α_R is slightly angled so that it can reduce the expected travel distance to the right region from the left P&D point, in two ways: (1) it presents slightly better travel paths than rectilinear travel, and (2) it reduces the number of storage locations due to the angled aisles. α_R also improves travel from the right P&D point to some storage locations in the right region; however, it also slightly worsens for some other locations in the same region. For example, only travel to the right side of the right P&D point to reach the right storage area has a slightly better travel than the rectilinear travel. Travel from the left P&D point to the left region has a better travel path than rectilinear travel because the difference in the α_L and β is less than 90°. Travel from the right P&D point to the left region is also slightly improved for most storage locations in that region. For this travel, the picking aisles in the right region are utilized as cross aisles in order to provide the shortest paths to the furthest locations from the right P&D point.

In Table 5.3, we present the best solutions that we have obtained in each run for small-, medium- and large-sized warehouses. For medium-sized warehouses, Design A1 offers 9.6% reduction in expected travel distance, but it provides a 9.1% lower number of storage locations than Trad-A (see Figure 5.16b). The lost storage capacity in equally sized, non-traditional aisle designs is caused by the inserted cross aisles and the angled aisles. In order to have a fair comparison, we expand the size

	Run 1		Run 2		Run 3	
Warehouse	$E[C]$	# Pallets	$E[C]$	# Pallets	$E[C]$	# Pallets
Small	475.9	1617	477.8	1620	477.2	1615
Medium	715.8	3910	715.7	3927	717.1	3927
Large	955.3	7232	954.8	7204	957.2	7240

Table 5.3: Performance of Design A1 for different warehouse sizes.

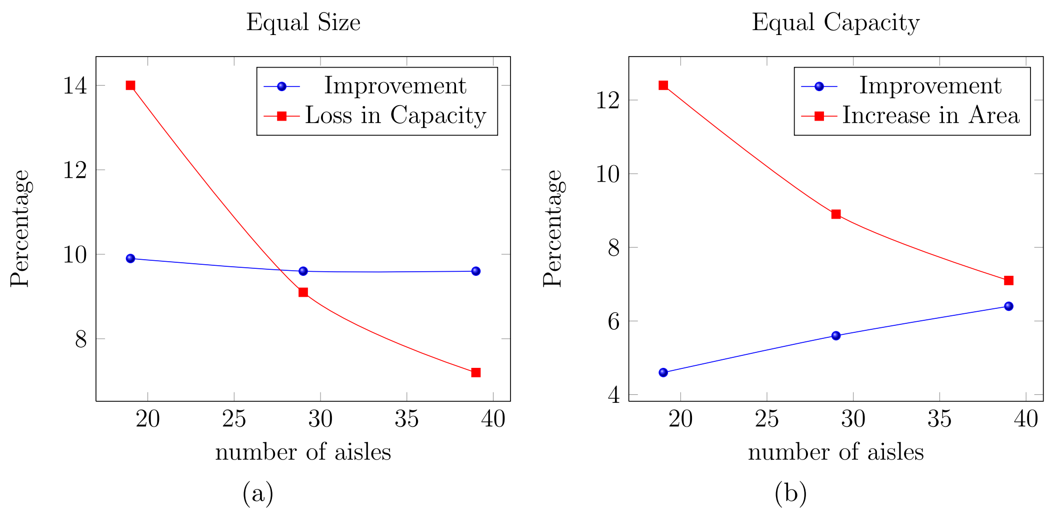


Figure 5.16: Comparison of Design A1 and Trad-A.

of Design A1, without changing its aisle structure, so that it has approximately the same storage capacity as Trad-A. For medium-sized, equally capacitated warehouses, Design A1 presents 5.6% savings in expected travel distance, but requires a 8.9% larger space compared to Trad-A (see Figure 5.16b and Table B.4 in the Appendix for details).

As the warehouse size increases, the new, additional storage locations are placed at the furthest locations to the P&D points. The distance between the P&D points and these storage locations increases. Therefore, because the expected travel distance to newly added storage locations is greater than the expected travel distance in a regular size warehouse, the improvement decreases.

In Design A1, the picking aisles in the right region are slightly slanted by approximately 2°, compared to vertical aisles. Because of this and the fact that 90° aisles provide more compact storage than angled aisles, we modify α_R to 90° in order

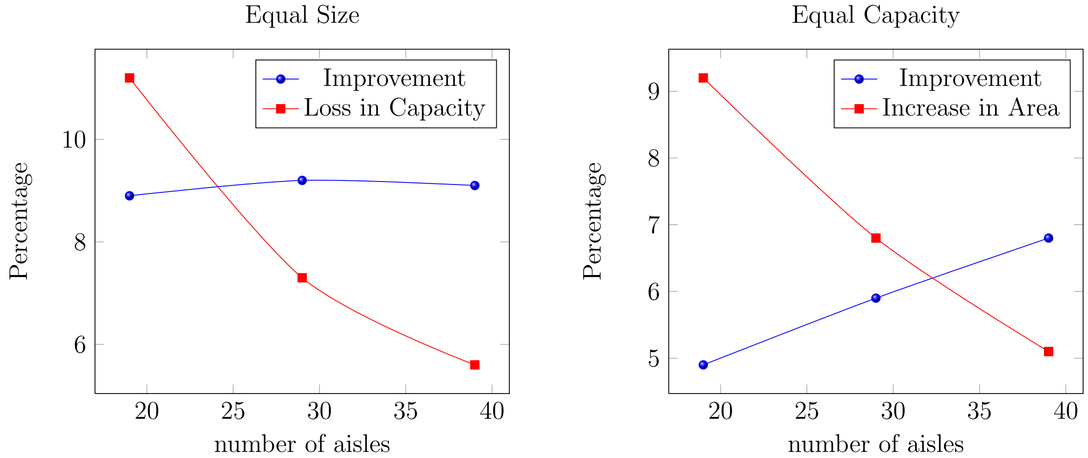


Figure 5.17: Comparison of modified Design A1 and Trad-A.

to explore the change in the performance. We call the resulting design "modified Design A1" (see Figure 5.15b).

The expected travel distances and the number of pallets in modified Design A1 are given in Table B.5 in the Appendix. For an equal, medium-sized warehouse, modified Design A1 offers 0.5% less improvement than than Design A1. This is an expected result because Design A1 is the best design we have found so far for equally sized warehouses. However, for equally capacitated, medium warehouses, modified Design A1 offers an additional 0.3% improvement and requires 2.1% less warehouse space than Design A1 (see Figure 5.17 for the performance of modified Design A1).

Design A with Two Cross Aisles

As shown in Chapter 4, improvement in expected travel distance increases with the number of inserted cross aisles. Therefore, in this section, we insert two angled cross aisles into a warehouse space that represents the problem design A, and we search for the best aisle design. The improved aisle design proposed by the heuristic model for this problem is shown in Figure 5.18. We call this design "Design A2."

Design A2 is an approximately symmetric design with respect to a vertical axis going through the center of the warehouse. The angles of the picking aisles in the right (α_R) and the central (α_C) regions are 53° and 180°, respectively. Because there

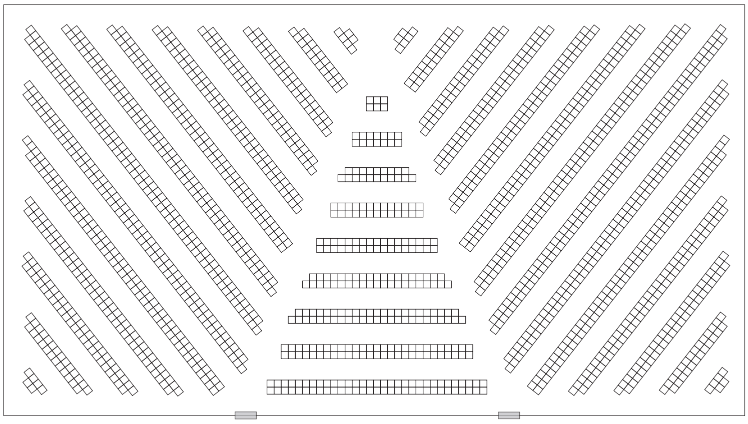


Figure 5.18: Design A2.

are two P&D points and two inserted cross aisles, the model originates each cross aisle from each P&D point in order to facilitate travel to the storage locations from both of the P&D points equally. The cross aisles also intersect at the middle of the top side of the warehouse so that: (1) they can create a fishbone-like travel in the central region with an appropriate angle of picking aisles, and (2) they can increase the right and left storage areas where they present a better travel path than the rectilinear travel, with appropriate angles of picking aisles. Therefore, travel from both P&D points is better than rectilinear travel.

Table 5.4 presents the best solutions that we have obtained in each run for small-, medium- and large-sized warehouses. Based on the best result for a medium-sized warehouse, Design A2 offers 14.8% reduction in expected travel distance, but it supplies a 9.6% lower number of storage locations compared to Trad-A (see Figure 5.19a). While preserving the same aisle structure in Design A2, we expand its size with an approximate 2:1 width to depth ratio so that it has the same storage capacity with the equivalent Trad-A. Thus, Design A2 presents an 11% improvement in travel, but it requires a 9.6% larger space compared to an equally capacitated, medium-sized Trad-A design (see Figure 5.19b and Appendix B.2.2 for details).

In comparison to Design A1, as expected, Design A2 increases travel efficiency, but requires more space, mainly due to the additional cross aisle. For an equally

	Run 1		Run 2		Run 3	
Warehouse	$E[C]$	# Pallets	$E[C]$	# Pallets	$E[C]$	# Pallets
Small	448.2	1610	455.4	1586	452.6	1616
Medium	674.4	3905	674.5	3906	676.5	3906
Large	899.8	7209	898.4	7204	904.7	7199

Table 5.4: Performance of Design A2 for different warehouse sizes.

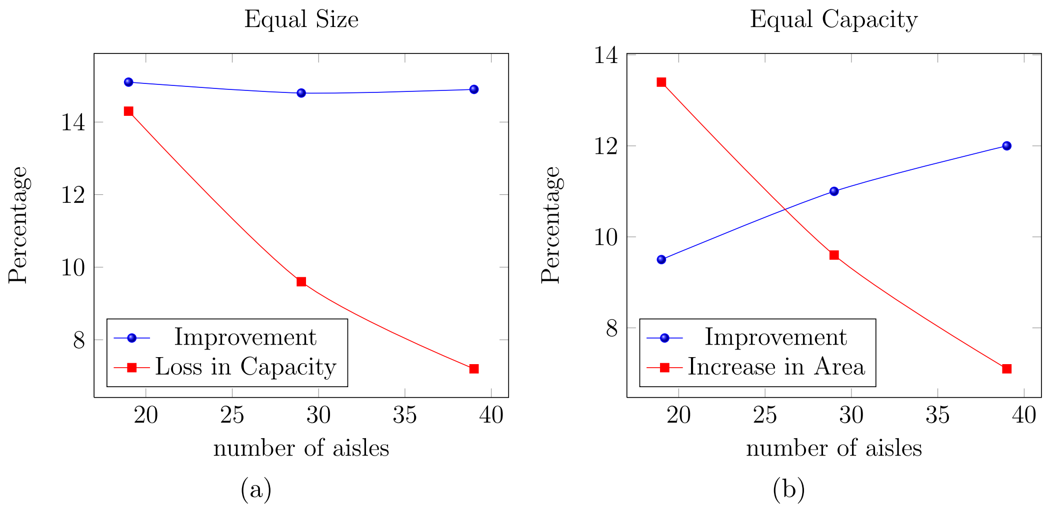


Figure 5.19: Comparison of Design A2 and Trad-A.

capacitated, medium-sized warehouse, while the modified Design A1 offers a 5.9% improvement, Design A2 offers an 11% improvement in expected travel distance. On the other hand, while modified Design A1 requires a 6.8% larger warehouse, Design A2 requires a 9.6% larger warehouse than an equivalent, medium-sized Trad-A.

Gue et al. (2010) also considered multiple P&D points at the bottom of the warehouse in a Flying-V design. The Flying-V has two nonlinear cross aisle segments that are symmetric with respect to a vertical axis passing through the central P&D point. In order to compare Design A2 with the Flying-V, we slightly modify the cross aisles in the Flying-V to be linear because our model can only accommodate linear cross aisles. To find the points of intersection on the cross aisles and the sides of the warehouse, we used a central P&D point and used a manual local search to find the best point. Then, we insert two P&D points at the 1/3 and 2/3 point of the bottom in order to compare with Design A2 (Figure 5.20). For equally capacitated

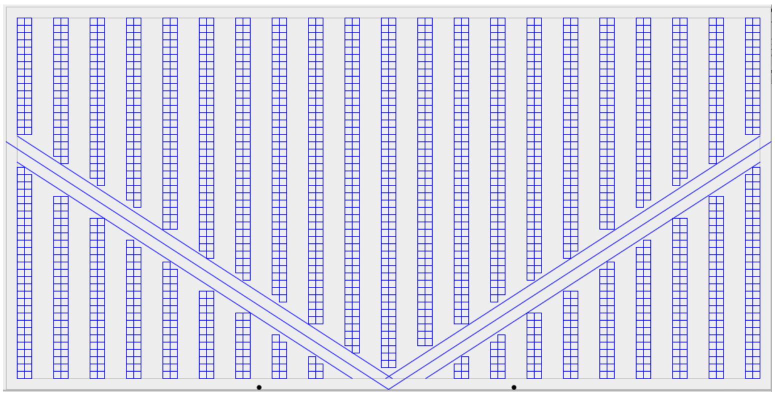


Figure 5.20: The modified Flying-V design with two, inserted linear cross aisles.

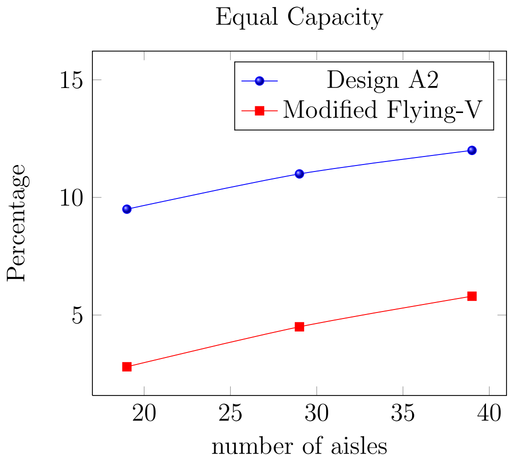


Figure 5.21: Comparison of Design A2 and the modified Flying-V. (See Appendix B.10 for detailed results.)

warehouses, Figure 5.21 shows the comparison of Design A2 and the modified Flying-V. Because Flying-V is specifically developed for a central P&D point, Design A2 dramatically offers more benefit than the modified Flying-V when P&D points are placed at the 1/3 and 2/3 of the bottom.

5.6.2 Design Problem B

In design problem B, the two P&D locations are placed to the middle of the bottom and the left sides of the warehouse. For this problem, Trad-A does not allow direct travel from the left P&D point to the storage area. Therefore, Trad-B and Trad-C are the alternative designs for our comparison (see Figure 5.1). Because Trad-C requires slightly more space than Trad-B due to the horizontal cross aisle,

Warehouse	$E[C]$	# Pallets
Small (19 aisles)	562.5	1880
Medium (29 aisles)	844.2	4320
Large (39 aisles)	1125	7760

Table 5.5: Performance of Trad-B when P&D points are placed at the mid-left and mid-bottom.

we take design Trad-B as a base for comparisons with the proposed designs in this problem. The performance of Trad-B for small-, medium- and large-sized warehouses are given in Table 5.5. In the next sections, we look for optimal designs with one and two inserted cross aisles for this problem and compare them with Trad-B.

Design B with One Cross Aisle

The improved aisle design proposed by the heuristic model, with one inserted cross aisle in design problem B, is shown in Figure 5.22. We call this "Design B1" and describe its angles of aisles in a medium-sized warehouse as the following (see Appendix B.3.1 for the detailed computational results). In Design B1, the cross aisle originates from the bottom P&D point and intersects with the top side of the warehouse with an angle of approximately 123°. The model seems to save some pallet locations closer to the left P&D point by placing the cross aisle further away from the upper-left corner. The picking aisles in the right region have a right angle with the cross aisle. This results in improved travel from the left P&D point to the right storage area, while the worst travel path from the bottom P&D point to the right is still rectilinear travel. The angle of the picking aisles in the left region is approximately 180°. Hence, while the travel from the bottom P&D point to the left region enjoys fishbone-like travel, the worst travel path from the left P&D point to the left storage area is still rectilinear travel. As a result, the model tries to balance the improvement in travel paths from both of the P&D points such that there is a rectilinear travel path for some storage locations and an improved path for some other locations. We present the best solutions that we have obtained in each run for

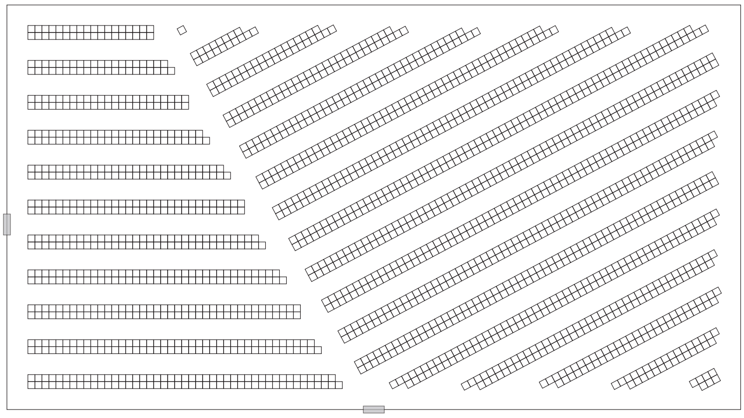


Figure 5.22: Design B1.

	Run 1		Run 2		Run 3	
Warehouse	$E[C]$	# Pallets	$E[C]$	# Pallets	$E[C]$	# Pallets
Small	533.8	1714	534.9	1717	531.9	1726
Medium	805.6	4072	803.3	4074	804.5	4068
Large	1067.6	7437	1069.0	7423	1070.7	7454

Table 5.6: The lowest heuristic results of one cross aisle Design B for equal sizes of Trad-B designs.

small-, medium- and large-sized warehouses in the Table 5.6 (see Appendix B.3.1 for details).

For an equal, medium-sized warehouse, Design B1 offers a 4.9% improvement in expected travel distance, but it provides 5.7% fewer storage locations than Trad-B. When its storage area is expanded in order to provide an equal storage capacity with Trad-B, the improvement reduces to 1.5% for an equally capacitated, medium-sized warehouse, and it requires 5.3% additional warehouse space. Figure 5.23 shows the performance of Design B1 over Trad-B for equally sized and equally capacitated warehouses. Table B.14 in the Appendix also gives the details for the expanded Design B1.

Design B with Two Cross Aisles

The improved aisle design (Design B2) proposed by the heuristic model, with two inserted cross aisles in design problem B, is shown in Figure 5.24a. The angles

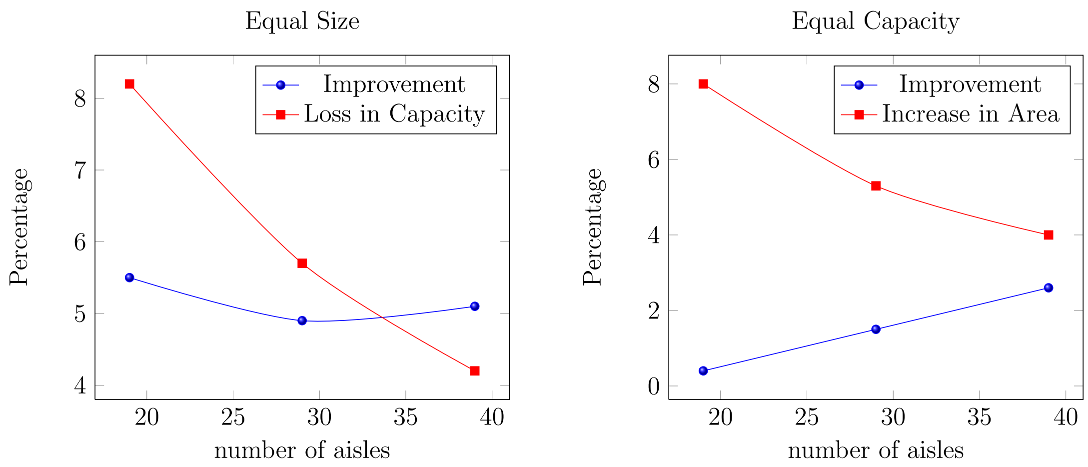
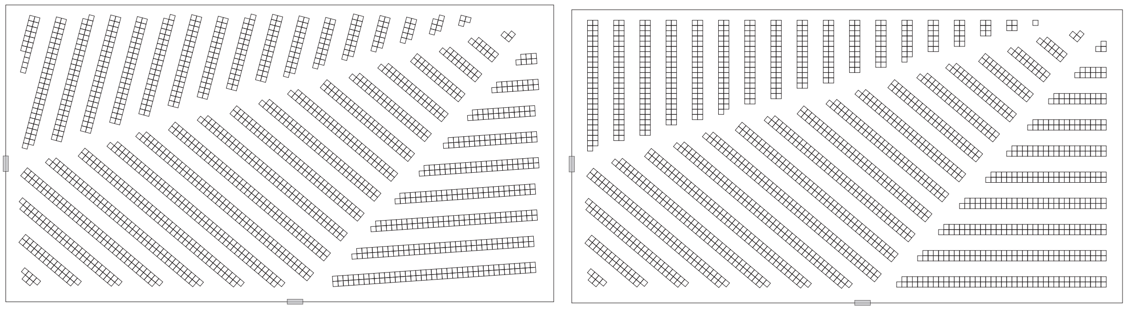


Figure 5.23: Comparison of Design B1 and Trad-B.

of the picking aisles in the right (α_R) and left (α_L) regions are approximately 71° and 8°, respectively. In Design B2, each cross aisle originates from each of the P&D points, and both intersect at the upper-right corner. This occurs because of the similar reasons that we discussed for Design A2: (1) they can create a leaf-like travel in the right and left regions from the bottom and the left P&D points, respectively, and (2) increase the right and the left storage areas where travel is better than the rectilinear travel. The angle of the picking aisles in the central (α_C) region is approximately 136° that forms a right angle with the cross aisle originated from the bottom P&D point. Thus, (1) while travel from the bottom P&D point to some portion of the central storage area is improved, travel to the other portion has a rectilinear travel path, and (2) travel from the left P&D point to the central region is slightly improved. On the other hand, for travel from the left P&D point to the right region and travel from the bottom P&D point to the left region, central picking aisles are utilized as cross aisles in order to provide the shortest paths to the furthest locations from the P&D points. We might also say that this travel worsens if α_R increases or α_L decreases. We present the best solutions that we have obtained in each run for small-, medium- and large-sized warehouses in Table 5.7.

Design B2 offers an 11.6% savings in expected travel distance, but provides a 13.2% fewer storage locations over an equal, medium-sized Trad-B design. When



(a) Design B2 (b) Modified Design B2

Figure 5.24: Designs with two cross aisles in design problem B.

	Run 1		Run 2		Run 3	
Warehouse	$E[C]$	# Pallets	$E[C]$	# Pallets	$E[C]$	# Pallets
Small	492.6	1501	495.3	1501	493.5	1491
Medium	746.9	3725	748.2	3716	746.5	3752
Large	999.6	6957	1001.6	6947	999.9	6945

Table 5.7: Performance of Design B2 for different warehouse sizes.

we enlarge the storage area in Design B2 by preserving its aisle structure and shape ratio, the savings reduce to 6.1% with a cost of 14.4% larger space than an equivalent, medium-sized Trad-B. Figure 5.25 shows the performance of Design B2 over equally sized and equally capacitated Trad-B designs.

Figure 5.24b shows the more compact modified Design B2. As expected, the modified Design B2 has less improvement in expected travel distance than Design

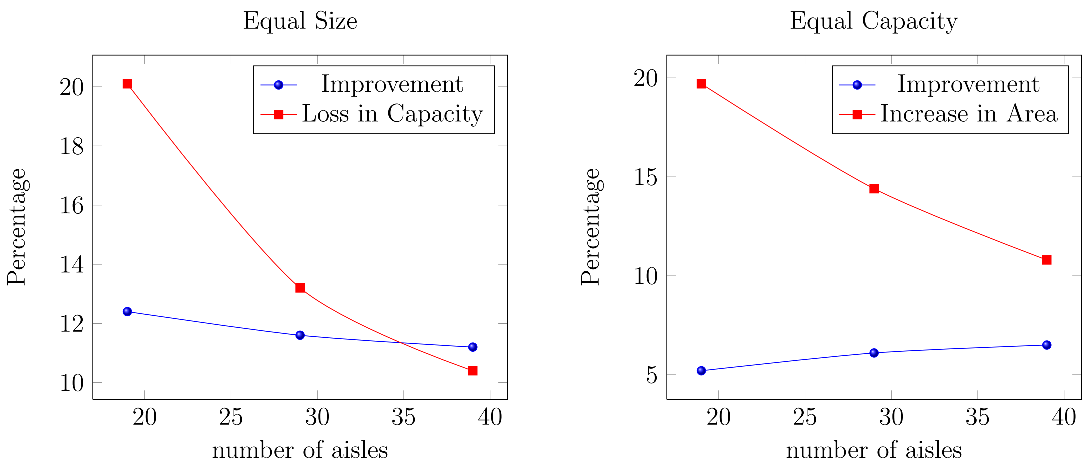


Figure 5.25: Comparison of Design B2 and Trad-B.

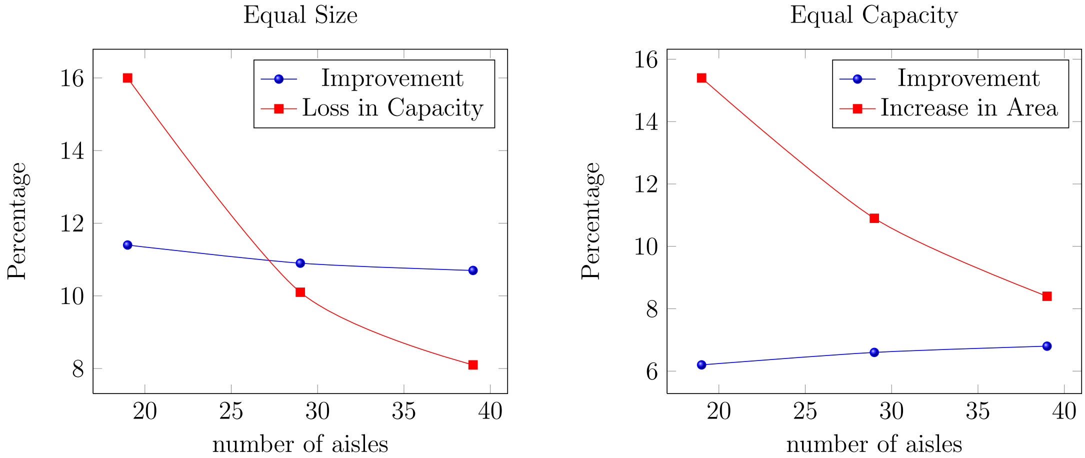


Figure 5.26: Comparison of modified Design B2 and Trad-B.

B2 when their warehouse sizes are equal. However, when their warehouse spaces are expanded, modified Design B2 offers an additional 0.5% improvement and necessitates 3.5% less space than the equally capacitated, medium-sized Design B2 (see Table B.19 in the Appendix for details). The modified Design B2 offers a little more savings in expected travel distance than Design B2 because decreasing necessity of additional space brings some of the storage locations closer to the P&D points. The performance of modified Design B2 over the equivalent Trad-B is given in Figure 5.26.

For an equally capacitated, medium-sized warehouse, modified Design B2 offers approximately a 7% improvement in expected travel distance, while Design B1 offers an 1.5% improvement. However, as expected, modified Design B2 also increases the additional space requirement to 11% from 5.3% due to an additional cross aisle.

5.6.3 Design Problem C

One of the most likely encountered P&D locations in industry has P&D locations at the bottom and the top sides of the warehouse. Therefore, in this section we locate one P&D point at the center of the bottom and the top sides of the warehouse. In order to have an accurate comparison with the proposed designs, we take Trad-A as a base design in this section because of its high storage capacity and allowance

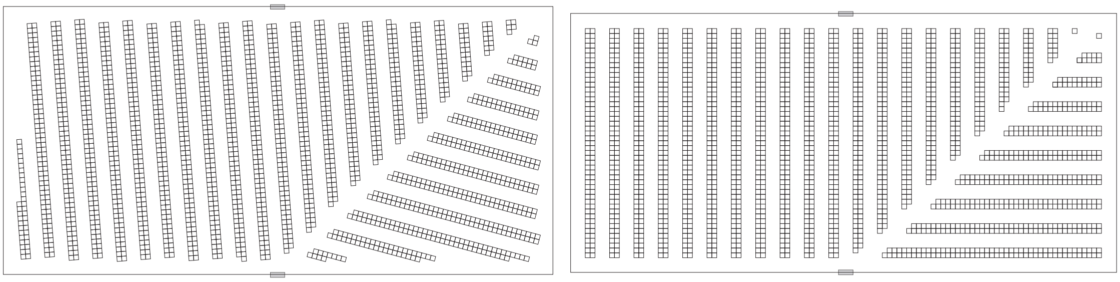
Warehouse	$E[C]$	# Pallets
Small (19 aisles)	500	1880
Medium (29 aisles)	750	4320
Large (39 aisles)	1000	7760

Table 5.8: Performance of Trad-A when P&D points are placed at the mid-bottom and mid-top.

of direct travel from both of the P&D points. The performance of Trad-A for this P&D point configuration in small-, medium- and large-sized warehouses are given in Table 5.8. In the next sections, we insert one and two cross aisles into a warehouse space that represents the design problem C consecutively, and we search for optimal designs for this problem.

Design C with One Cross Aisle

The improved aisle design (Design C1) is shown in Figure 5.27a. The cross aisle originates from the bottom P&D point through the upper-right corner with an angle of 45°. The angles of the picking aisles on the right and left sides of the cross aisle are approximately 165° and 98°, respectively. Because our model calculates the shortest travel path to the storage locations, there are two potential travel paths to reach the right storage area from the bottom P&D point: (1) travel along the angled cross aisle, then an appropriate picking aisle, (2) travel along the bottom cross aisle with an appropriate picking aisle. Hence, there is an indifference point in each picking aisle such that it has the same travel distance to be reached from both of these travel paths (Gue and Meller, 2009). Even though the first travel path offers a little improvement for some storage locations in an aisle, the second one proposes a worse travel than a rectilinear travel path to the other locations in the same aisle. That is why the angle of aisles in the right region seem not to offer fully improved travel paths from the bottom P&D point to the storage locations in the right region. Travel from the top P&D point to the right storage area is also better than rectilinear travel.



(a) Design C1 (b) Modified Design C1

Figure 5.27: Designs with one cross aisle in design problem C.

	Best Solution 1		Best Solution 2		Best Solution 3	
Warehosue	$E[C]$	# Pallets	$E[C]$	# Pallets	$E[C]$	# Pallets
Small	464.7	1646	465.1	1646	464.9	1642
Medium	701.5	3979	700.5	3968	701.2	3965
Large	937.2	7281	938.8	7295	935.8	7279

Table 5.9: Performance of Design C1 for different warehouse sizes.

We present the best solutions that we have obtained in each run for different warehouse sizes in Table 5.9, and details are given in Appendix B.4.1. Compared to an equal, medium-sized Trad-A, Design C1 offers 6.5% reduction in expected travel distance, but provides 8.2% fewer storage capacity due to the angled aisles. In a warehouse with equal storage capacity, Design C1 presents 2.9% improvement in expected travel distance, but requires 8.2% more space than a medium-sized Trad-A (see Table B.23). The performance measures of Design C1 for equally sized and equally capacitated warehouses are plotted in Figure 5.28.

For the same reasons discussed in Section 5.6.2, we modified the angles of picking aisles in Design C1 as seen in Figure 5.27a. The rectilinear travel path from the top P&D point to the right region, diminishes the total benefit of the design because the cross aisle pushes storage locations in the right region a little further away from the P&D point. All travel paths from both of the P&D points to the left half of the warehouse are rectilinear. Hence, we might expect that this design offers at most 25% of what Fishbone designs offer because only half of the warehouse is a Fishbone design for only one P&D point. For an equally capacitated, medium

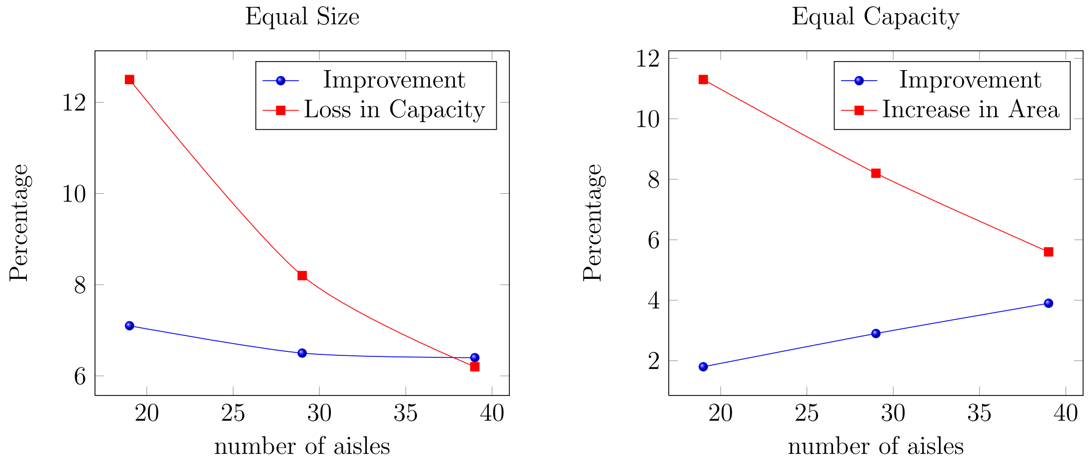


Figure 5.28: Comparison of Design C1 and Trad-A.

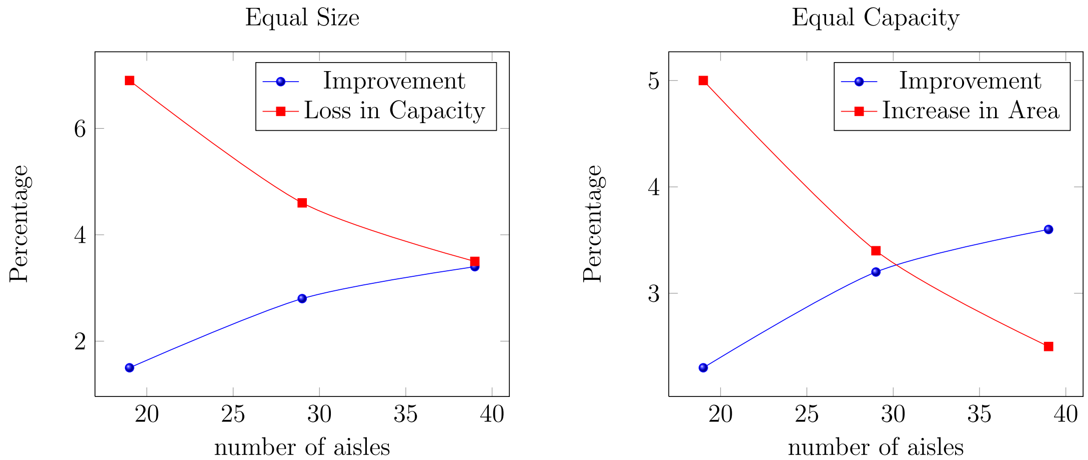
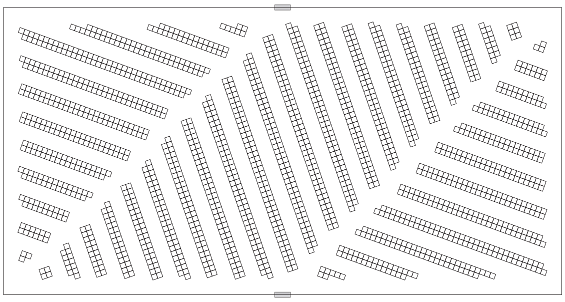


Figure 5.29: Comparison of modified Design C1 and Trad-A.

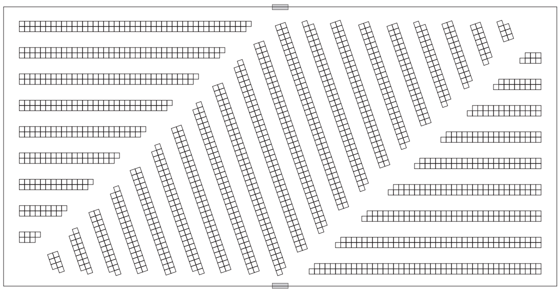
warehouse, modified Design C1 offers an additional 0.3% reduction in expected travel distance and also requires 4.8% less space than Design C1. Although the additional improvement is inconsiderable in modified Design C1, it provides great savings in the additional space requirement. The performance of the modified Design C1 is plotted in Figure 5.29, and details are given in Table B.24 in the Appendix.

Design C with Two Cross Aisles

Figure 5.30a shows the improved aisle design (Design C2) proposed by the heuristic model, with two inserted cross aisles in design problem C. As we would expect, each cross aisle originates from each of the P&D points. Furthermore, Design



(a) Design C2



(b) Modified Design C2

Figure 5.30: Designs with two cross aisles in design problem C.

C2 is approximately symmetric to the diagonal which passes through the lower-left corner to the upper-right corner. We might have expected this, because the locations of the two P&D points, centrally located on the opposite sides of the warehouse, raise the idea of equal usage of the two cross aisles and the square-half warehouse space. Because of symmetry, we now focus on the right half of the warehouse. The angle of the picking aisles in the right region (α_R) is approximately 161°. Travel from the bottom P&D point to the right storage area is the same as in Design C1 that we discussed in Section 5.6.3. Additionally, the angle of picking aisles in the right region seems to facilitate travel from the top P&D point to the right storage area through the picking aisles in the central region. Also, note that the storage locations in the right region are the furthest locations to the top P&D point. Hence, while the model loses a little improvement from the bottom P&D point to its closest locations in the right region, it actually increases the improvement in travel from the top P&D point to these locations with an appropriate angle in the central (α_C) region. The angle of picking aisles in the central region is approximately 108°. Hence, travel to the central region from both of the P&D points is also improved by these angles. The angle of aisles in the left (α_L) region is approximately the same as α_R, due to symmetry in the design. We present the best solutions that we have obtained in Table 5.10 and Appendix B.4.2.

	Run 1		Run 2		Run 3	
Warehouse	$E[C]$	# Pallets	$E[C]$	# Pallets	$E[C]$	# Pallets
Small	423.1	1539	425.3	1539	423.4	1541
Medium	639.1	3790	639.8	3788	639.9	3802
Large	855.1	7034	855.9	7052	857.8	7043

Table 5.10: Performance of Design C2 for different warehouse sizes.

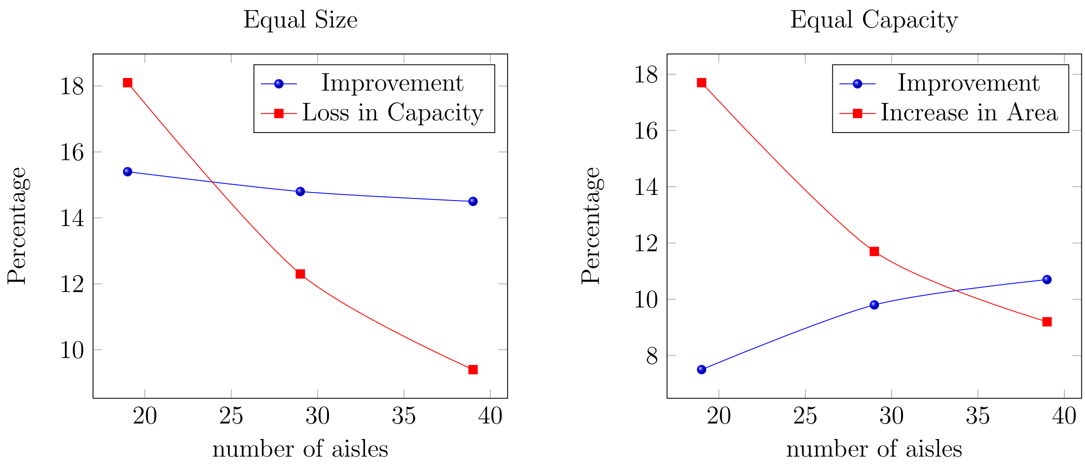


Figure 5.31: Comparison of Design C2 and Trad-A.

For a, medium-sized warehouse, Design C2 reduces expected travel distance by 14.8%. However, it provides 12.3% less storage capacity than the equivalent medium-sized Trad-A design. Design C2 with an equal capacity offers a 9.8% improvement in expected travel distance, but requires an 11.7% more space than an equivalent Trad-A design.

In the modified Design C2, we make the angle of picking aisles 180° in the right and the left regions. We preserve the angle of aisles in the central region as appears in Design C2. In equal space, the modified Design C2 reduces expected travel distance by 12.3%, but provides a 9.1% fewer storage locations over an equal, medium-sized Trad-A. An equal capacity design offers 8.6% savings in expected travel distance, but requires an 8.9% larger space (Figure 5.32 and Table B.29 in the Appendix).

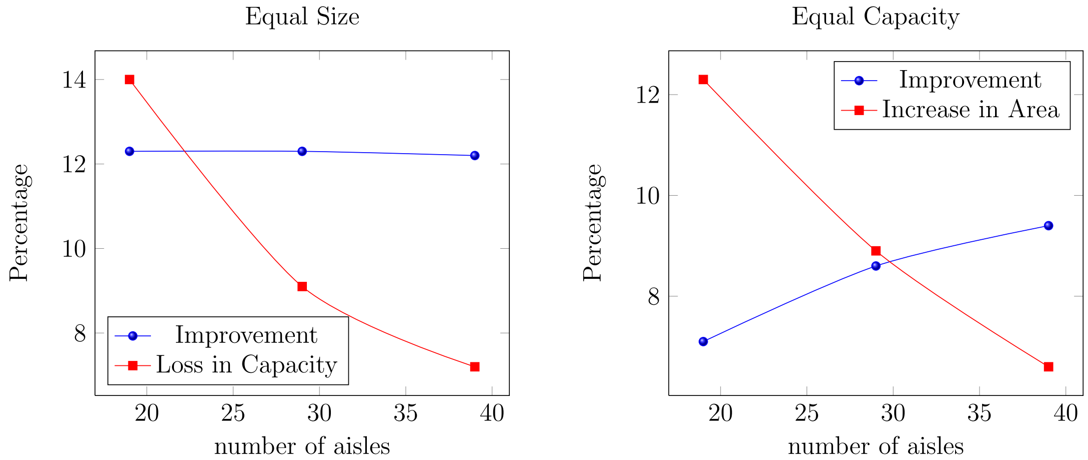


Figure 5.32: Comparison of modified Design C2 and Trad-A.

Warehouse	$E[C]$	# Pallets
Small (19 aisles)	750	1880
Medium (29 aisles)	1125	4320
Large (39 aisles)	1500	7760

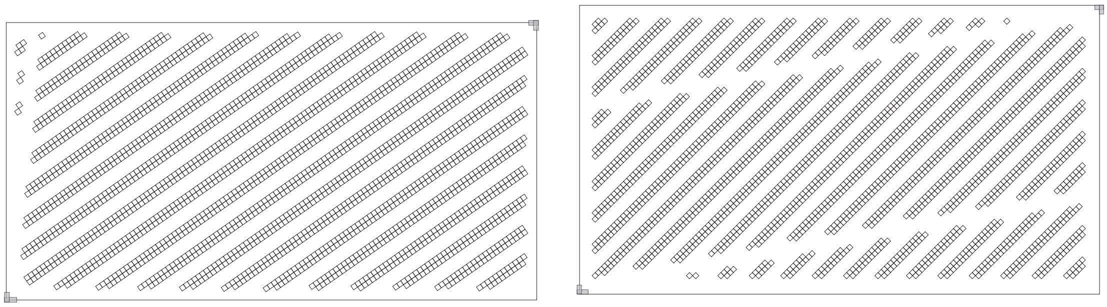
Table 5.11: Performance of Trad-A when P&D points are placed at the lower-left and upper-right corners.

5.6.4 Design Problem D

In this section, we search for optimal aisle structures in a warehouse that has one P&D point at the lower-left and one at the upper-right corner. For this problem, we take Trad-A as a base for our comparisons with the proposed designs. The expected travel distances and the number of pallets in the Trad-A with these P&D points are given in Table 5.11. In the next section, we look for the best aisle designs with one and two inserted cross aisles for this problem and compare them with the Trad-A.

Designs D with One and Two Cross Aisles

We first insert one angled cross aisle into a warehouse that represents the design problem D. Design D1 is the improved aisle design proposed by the heuristic model (see Figure 5.33a). In Design D1, the model originates the cross aisle from the lower-left corner and places it as close as to the the left side of the warehouse. The angles of the picking aisles in the right and the left regions are approximately 33°. It



(a) Design D1 (b) Design D2

Figure 5.33: Designs with one and two cross aisles in design problem D.

seems that the main role of the cross aisle is to reduce the storage capacity, rather than facilitating travel from P&D points to the storage area. Therefore, a single cross aisle seems to be redundant in this problem. However, for future comparisons, the expected travel distance in a medium-sized Design D1 is 921.9 units, and there are 4048 pallet locations. When the size of Design D1 is expanded, it offers an 15.4% improvement in expected travel distance, and it requires 6.1% more space than Trad-A.

We then insert two cross aisles into a warehouse that represents the design problem D. Figure 5.33b presents the improved aisle design (Design D2) proposed by the heuristic model (see Appendix B.5 for details). In Design D2, as expected, each cross aisle originates from each of the P&D points. Design D2 is approximately symmetric with respect to the diagonal that passes through the lower-left and upper-right corners. The angles of the picking aisles in the right (α_R), central (α_C) and the left (α_L) regions are approximately 33°, 32° and 34°. Thus, cross aisles facilitate travel from P&D points to the storage area.

Design D2 offers an 18.5% improvement, but it provides a 14.7% fewer storage locations than an equally sized Trad-A design (see details in Appendix B.5). For equally capacitated warehouses, Design D2 offers a 12.2% improvement in expected travel distance with a cost of 15.2% larger space than Trad-A. (see Table B.35 in the Appendix). Design D2 presents worse performance than Design D1, with respect to improvement in expected travel distance and additional space requirement. Interestingly, this is the first time that a non-traditional aisle design does not increase the improvement in expected travel distance with an additional cross aisle. Because of this and Design A1 implies a design without any cross aisles, in the next section, we look for a better aisle design than Design D1 and Design D2, without inserting any cross aisles.

Design D without Any Cross Aisles

Figure 5.34 shows the improved aisle design (Design D0) proposed by the heuristic model without inserting any additional cross aisles. The angle of the picking aisles (α) in this design is approximately 33° (see Appendix B.5 for details). Travel from the P&D points to the storage locations is facilitated by the side cross aisles with angled picking aisles. Hence, Design D0 enjoys with chevron-like travel paths. We also can see the whole space in the Design D0 as a half space in the Chevron. Therefore, the Design D becomes a non-square, half Chevron design design with an additional P&D point at the upper-right corner. Therefore, as in a non-square half-Chevron design, the angle of the picking aisles in Design D0 is greater than the angle of the diagonal that passes through the lower-left and upper-right corners, because the half-width of the warehouse is greater than its height.

Table 5.12 presents the best solutions that we have obtained for Design D0. Design D0 offers a 17.8% reduction in expected travel distance with a 4.6% loss in the storage capacity compared to a medium-sized Trad-A (see Figure 5.35). For equally capacitated warehouses, Design D0 offers a 16.1% improvement, but requires a 4% larger space than Trad-A (see Figure 5.35 and Table B.31 in the Appendix). As expected, Design D0 dominates Design D1. Therefore, the best design in design problem D is Design D0.

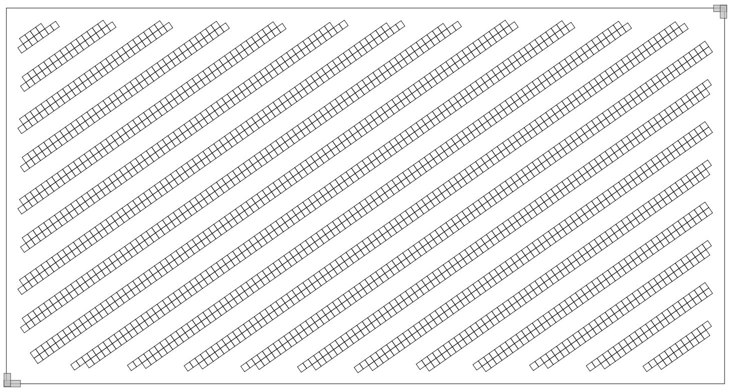


Figure 5.34: Design D0.

	Run 1		Run 2		Run 3	
Warehouse	$E[C]$	# Pallets	$E[C]$	# Pallets	$E[C]$	# Pallets
Small	615.8	1748	616.1	1751	615.7	1747
Medium	925.2	4122	925.6	4125	925.5	4119
Large	1234.7	7492	1234.9	7494	1234.9	7493

Table 5.12: Performance of Design D0 for different warehouse sizes.

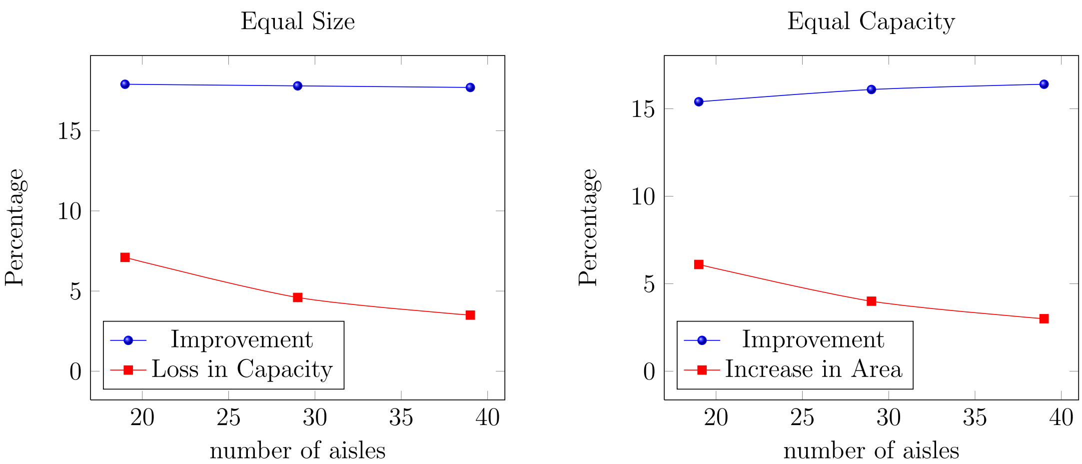


Figure 5.35: Comparison of Design D0 and Trad-A.

Warehouse	$E[C]$	# Pallets
Small (19 aisles)	562.5	1880
Medium (29 aisles)	844.2	4320
Large (39 aisles)	1125.0	7760

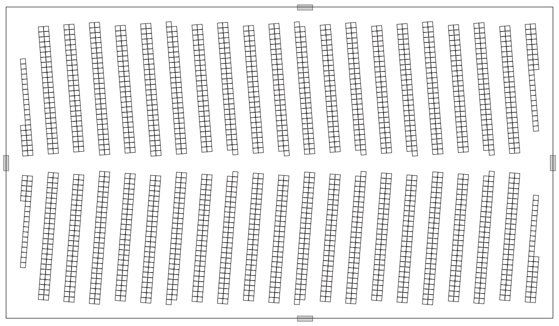
Table 5.13: Performance of Trad-B when P&D points are placed in the middle of each side of the warehouse.

5.6.5 Design Problem E

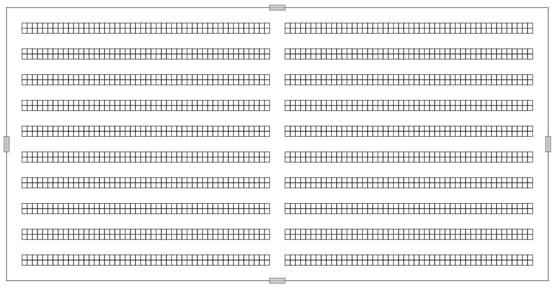
In this section, we locate four P&D points at the periphery of a unit-load, square-half warehouse. Each of the P&D points are placed in the middle of the each side of the warehouse. For this arrangement of P&D points, we take design Trad-B as a base because design Trad-C requires slightly more space than Trad-B, and Trad-B provides direct travel from the P&D points to the storage area. The performance of Trad-B for different size warehouses with these P&D points is given in Table 5.13.

Design E with One Cross Aisle

Figure 5.36a shows the aisle design (Design E1) proposed by the heuristic model. The angles of picking aisles in the bottom and the top region are approximately 85° and 95°. It seems that these picking aisles are slightly slanted so that the model reduces the expected travel distance by both slightly improving the travel path and reducing the storage capacity. Design E1 proposes 0.5% reduction in expected travel distance with a 9.3% loss in the storage capacity compared to an equal, medium-sized Trad-B. In order to recover the loss capacity in Design E1, we expand its storage area. Thus, Design E1 does not offer any savings in expected travel distance over an equally capacitated Trad-B. This is because the travel efficiency gained by the angled aisles does not compensate the increase in travel distances due to an increasing warehouse space. The detailed computational results for Design E1 is given in the Appendix B.6.1.



(a) Design E1



(b) Trad-B

Figure 5.36: Designs with one cross aisle in design problem E.

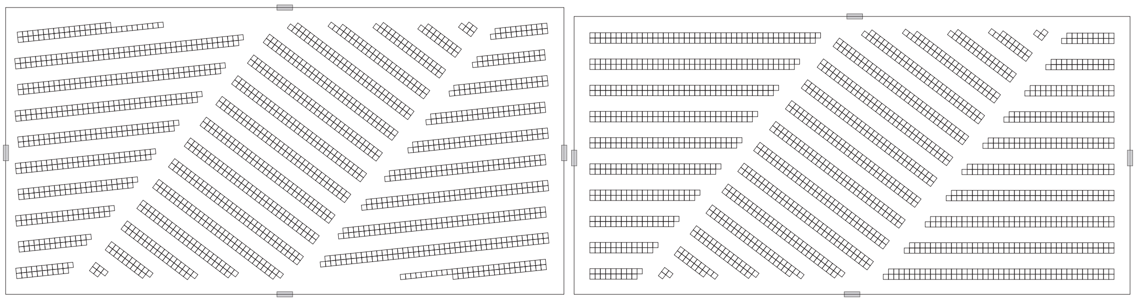
	Run 1		Run 2		Run 3	
Warehouse	$E[C]$	# Pallets	$E[C]$	# Pallets	$E[C]$	# Pallets
Small	559.1	1623	559.4	1634	559.6	1629
Medium	840.2	3917	840.4	3931	841.9	3906
Large	1121.1	7220	1121.9	7224	1122.6	7208

Table 5.14: Performance of Design E1 for different warehouse sizes.

Design E with Two Cross Aisles

An improved design proposed with two cross aisles (Design E2) is Figure 5.37a. Design E2 is approximately symmetric with respect to the diagonal that passes through the lower-left and the upper-right corners. The angles of the picking aisles in the right (α_R) and left (α_L) regions are approximately 6°. The angle of the central (α_C) picking aisles is approximately 138°. Because of the symmetry in the design, we focus on the travel paths from the bottom and the right P&D points.

Travel from the bottom P&D point to the right storage area is improved by approximately fishbone-like travels. The worst travel path from the bottom P&D point to the central region is approximately rectilinear travel because α_C forms a right angle with the cross aisle originated from the bottom P&D point. Travel from the bottom P&D point to the left region seems to be worse than rectilinear travel. Travel from the right P&D point to most storage locations in the design has worse travel than the rectilinear because, (1) it does not utilize the cross aisles, and (2) α_C does not offer an improved transit travel to the left region.



(a) Design E2 (b) Modified Design E2

Figure 5.37: Designs with two cross aisles in design problem E.

	Run 1		Run 2		Run 3	
Warehouse	$E[C]$	# Pallets	$E[C]$	# Pallets	$E[C]$	# Pallets
Small	543.7	1554	544.7	1548	545.0	1548
Medium	818.7	3825	819.1	3773	819.7	3790
Large	1089.1	7079	1091.1	7111	1090.5	7121

Table 5.15: Performance of Design E2 for different warehouse sizes.

Table 5.15 shows the performance of Design E2 (see Appendix B.6.2 for details). Design E2 offers a 3% savings in expected travel distance, but it provides an 11.5% fewer storage locations than an equal, medium-sized Trad-B. An equal capacity version of Design E2 does not offer any savings in expected travel distance compared to an equivalent Trad-B (see Figure 5.38 and Table B.42 in the Appendix for details).

A modified Design E2 (Figure 5.37b) increases storage efficiency, but it still does not offer any improvement in expected travel distance over an equally capacitated

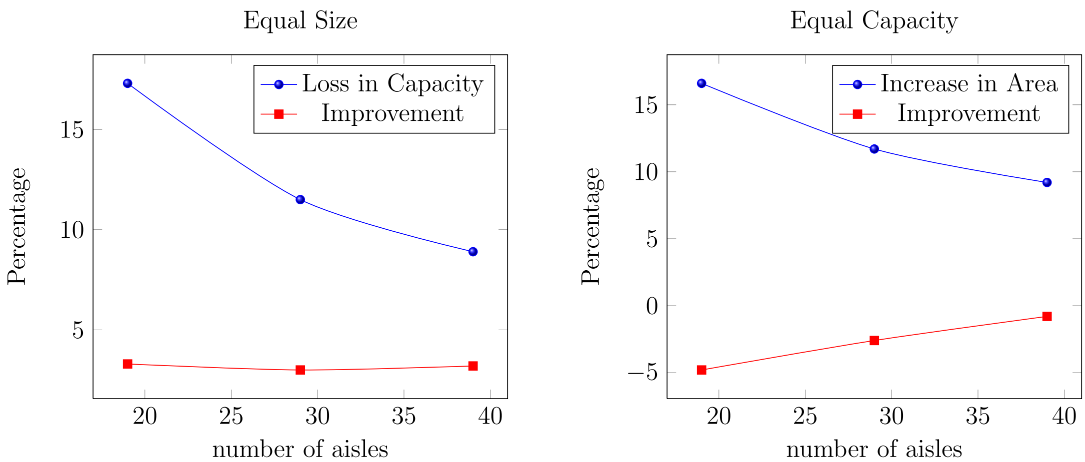


Figure 5.38: Comparison of Design E2 and Trad-B.

Trad-B for small- and medium-sized warehouses. However, modified Design E2 offers 0.3% improvement in expected travel distance for large-sized warehouses with a cost of 9% more space than Trad-B. Therefore, even modified design E2 is not a good design for design problem E. This result shows that it is hard to obtain a better design than the traditional designs by inserting one and two non-intersecting cross aisles, when the P&D points are distributed all around the warehouse.

5.7 Computational Complexity

We evaluated the shortest path distance from each P&D point to all storage locations with Dijkstra's one-to-many algorithm. According to Schrijver (2005), the worst case running time for Dijkstra's algorithm is $O(n^2)$, where n is the number of nodes. As the number of P&D points increases, the running time increases approximately linearly.

We run the model on two computers, both running Mac OS X version 10.6.7. The first has 4GB 1067 MHz RAM and a 2.66 GHz Intel Core 2 Duo processor. The second has a 2GB 667 MHz RAM and 1.83 GHz Intel Core Duo processor. The average computational time in three runs for small, medium and large size warehouses in each design problem is given in Table 5.16. In these runs, the best objective function values obtained in each solution are relatively close to one other, even though the angles in these solutions of aisles are slightly different in these solutions.

5.8 Conclusion

We have proposed new aisle designs for different flow patterns in unit-load warehouses such that the arrangement of aisles suggests a potential benefit in decreasing expected travel distance. The purpose is to decrease the travel distance to store and pick pallets in order to make workers more productive. However, reducing travel distance by angled aisles causes an additional space requirement in warehouses in

Design Problem	Inserted cross aisles	Average Running Time (min)		
		Small	Medium	Large
A	1	1592.1	4751.4	11765.9
	2	1573.6	4589.2	11276.8
B	1	1702.4	4942.5	11878.9
	2	1477.5	4249.3	10635.1
C	1	1659.2	5119.2	12381.5
	2	1507.1	4663.1	11559.6
D	0	1762.3	5303.5	12935.1
	2	1392.3	4413.9	10926.9
E	1	2782.9	8619.4	20624.4
	2	2590.0	7929.7	19775.3

Table 5.16: Average running time in design problems for different warehouse sizes.

order to provide an equal storage capacity as in an equivalent traditional design. Therefore, there is a tradeoff between reducing variable operational cost by decreasing expected travel distance, or increasing fixed cost by increasing warehouse size and number of storage racks. The decision of improving one of these conflicting purposes is based on operational characteristics in a specific warehouse in practice.

We have considered different locations of multiple P&D points in order to represent possible patterns of material flows in warehouses. For this representation, we place a single P&D point on different sides of a warehouse. A single P&D point on one side would be appropriate, when there is a single stretch-wrap machine for all outgoing pallets, or all incoming pallets need to be processed at a place before they are stored. We also place the single P&D point at the center of an appropriate side of the warehouse, because, material flow is most likely to be dense at the center in practice, and Gue et al. (2010) showed that concentrated P&D points toward center presents more benefit than distributed P&D points along one side of the warehouse.

We have proposed improved designs with one and two cross aisles in the defined design problems A, B and C. In these problems, the improved designs with two cross aisles offer considerable amount of savings in expected travel distance compared to one cross aisle designs. This is because each of the cross aisles in two cross aisle designs facilitate travel from each of the P&D points. However, in design problem D, the improved design does not require any additional cross aisles because travel from P&D points is facilitated by Chevron-like travel. Additionally, when we place one P&D point at each side of the warehouse, we observe that it is hard to obtain a better design than traditional designs with one or two non-intersecting cross aisles. Improved designs offer better travel paths from P&D points to most storage locations; however, occasionally they have travel paths worse than rectilinear for a small number of locations. The maximum turn angle in such cases is approximately $98°$.

Our improved designs are developed under two main assumptions, randomized storage and single command operation. Randomized storage is widely used in unit-load warehouses in industry because of its higher utilization of storage locations (Petersen, 1999; Petersen and Aase, 2004). Therefore, randomized storage is expected to alleviate the negative impact of angled aisles on storage density. The second important assumption is that all travel is for single-command operations. This operation is common in unit-load warehousing and unit-load replenishment in a reserve area (van den Berg et al., 1998). The improved designs in this chapter are expected to perform well for dual command operations because dual-command travel is comprised of two times single-command travel distance and one times travel-between distance.

Because every warehouse is different, the improved designs are not right for every unit-load warehouses. For example, warehouses where storage density is important do not utilize angled aisles because of the wasted space. The improved designs also perform well where material flow is concentrated at the center of the sides of warehouses.

Chapter 6

Robustness of Non-Traditional Aisle Designs with respect to a Varying Number of P&D Points

6.1 Introduction

It is not uncommon that warehouses in industry have multiple entry points to access a storage area. In Chapter 5, we have simplified this case, and we mainly placed centrally located P&D points at different sides of the periphery of a warehouse. In doing so, we assume that incoming and outgoing materials flow through the center of a side. In this chapter, we relax this assumption and diffuse more P&D points on the sides of a warehouse in order to investigate the change in aisle designs for non-traditional unit-load warehouses.

In Chapter 5, we reviewed some literature that addressed multiple P&D points. Here, we also acknowledge some facility layout studies that include the location of I/O points. Several researchers have studied the optimal placement of I/O locations by considering either a block or a detailed layout. Kane and Nagi (1997) solved the placement of I/O locations for a given automatic guided vehicle network. Benson and Foote (1997) determined both the locations of I/O points and the placement of aisles, after they solved the location of departments and the general aisle structure for the layout. Kado et al. (1995) found the optimal locations of I/O points in an integrated facility layout. Arapoglu et al. (2001) found out the best location of an I/O station for each department in a given block facility layout in order to minimize material handling costs. Deb and Bahttacharyya (2005) considered a manufacturing environment with an open field rectangular shape facility and pickup and drop-off locations along the periphery.

To the best of our knowledge, Gue et al. (2010) is the only study that considers a varying number of P&D points in warehouses. However, they consider P&D points only at the bottom of the warehouse with a vertical picking aisles. In this chapter, we also consider P&D points at the bottom and the top by allowing cross aisles to take any angles. Hence, we seek answers to the following questions:

- How good are previously proposed designs when the number of P&D points increases?
- How robust are their aisle structures for a varying number of P&D points?

6.2 Model and Assumptions

Industrial warehouses often have many dock doors along an exterior side. For some operations, a single P&D point is an appropriate assumption because travel begins and ends at a stretch-wrap or palletizing machine, but for many others this assumption is inappropriate.

In this chapter, we consider two common different configurations: (1) P&D points are placed at the bottom, and (2) P&D points are placed both at the bottom and the top sides of the warehouse. These configurations represent the majority of industrial warehouses. We use the following primary designs in our comparisons: the Chevron, Leaf, Design C1, and Design C2, which were all developed for centrally located P&D point(s). In our investigation, we will permit the aisle design to change as the number of P&D point increases and we will compare the performance of these new designs with the appropriate primary design.

In order to seek answers for our questions, we increase the number of P&D points per side by placing two additional P&D points incrementally. These additional P&D points are equally distributed around the central one. The distance between two consecutive P&D points is the distance between center lines of two consecutive picking aisles in a traditional design. Hence, in our models, this distance is equal to five pallet locations. We also assume an equal flow from each P&D point. However, when considering the utilization of dock doors in practice, we expect that the designs developed for centrally located P&D point(s) would perform even better. This is because, in practice, all P&D points are not occupied all the time because of the different number of arrival trucks to the warehouse. Thus, central P&D points are most likely to be utilized compared to the P&D points farthest from the center. We also have the same assumptions that we discussed in Chapter 4 and Chapter 5, except for the locations of P&D points.

In our first question, we increase the number of P&D points in primary designs and evaluate the change in improvement in expected travel distance compared to equivalent traditional designs.

In the second question, we first increase the number of P&D points per side in the defined P&D configurations. We then search for the best aisle designs for each change in number of P&D points inserting one and two cross aisles. We use our warehouse design tool and the PSO algorithm to search for the best aisle designs. Hence, we call the best designs we found so far "improved designs." Next, we compare our primary designs with the improved designs in order to investigate the robustness of our primary designs with respect to a varying number of P&D points. The robustness of designs in our problem is assessed both by the visual similarity in aisle structures, allowing for slightly different angles of aisles, and the similarity of the improvements in expected travel distance between primary and improved designs with respect to a number of P&D points. For the latter criterion, we chose that 2 – 3% loss in improvement in the primary design over an improved design.

We also consider small-sized, square-half warehouses in our evaluations, because smaller warehouses require less computational effort than the larger warehouses due to the number of storage locations that are needed to be evaluated by Dijkstra's shortest path algorithm. (A small-sized warehouse has a width of 100 and height of 50 pallet locations. A small-sized, traditional design is comprised of 19 aisles.) Small-sized warehouses also have other advantages:

- Small changes in the coordinates of the start and end points of a cross aisle reflect bigger variances in its angle for smaller warehouses than that of larger warehouses. Hence, smaller warehouses provide easy track in the change of aisle structure.
- The differences in expected travel distance due to variances in angles of picking aisles are larger for smaller size warehouses compared to that of larger size warehouses. This is because smaller size warehouses have a lower number of storage locations than that of larger warehouses.
- Based on our experience with the proposed non-traditional aisle designs, the orientation of aisles does not change with the size of the warehouse even though there might be a slight change in the angles of aisles.

As we showed in Chapter 4 and Chapter 5, the improvement in expected travel distance increases in non-traditional aisle designs as the size of the warehouse increases. Because we consider small size warehouses in this chapter, we expect that the primary and the improved designs with a varying number of P&D points will most likely provide more improvements for larger warehouses than that of smaller warehouses.

6.3 Unit-Load Warehouse Designs with P&D Points at the Bottom of the Warehouse

In this section, we increase the number of P&D points at the bottom of the warehouse and search for the best designs. We consider both the one and two cross aisle cases. We then compare the improved designs with the Chevron and Leaf.

6.3.1 One Cross Aisle Designs

We insert an angled cross aisle into a warehouse such that it can take on any angle and can be oriented from any point on the periphery of the warehouse. We then increase the number of P&D points from 3 to 19 at the bottom on either side of the center point, incrementally. We run our model three times to search for the best aisle structure for each problem. We also use the same parameters that we used in Chapter 5 for the warehouse network model and the PSO algorithm. The improved aisle structures that we have obtained so far for the nine design problems with one cross aisle are shown in Figure 6.1.

Figure 6.1 shows that the model generates Chevron-like designs for warehouses that have less than or equal to 13 P&D points. In these Chevron-like designs, the cross aisle is approximately vertical, originates from the central P&D point and divides the warehouse into approximately two equal spaces. The angles of picking aisles are approximately symmetric to each other with respect to the vertical axis going through the central P&D point. For the improved designs with more than 13 P&D points, the model generates a completely different aisle structure with one cross aisle. In this aisle structure, the cross aisle is not vertical anymore; instead, it originates from a P&D point on the left half of the warehouse and approximately intersects with the upper-right corner of the warehouse. The picking aisles in the right region are slightly angled, at approximately 87°. The angle of the picking aisles on the left is approximately 114°. Figure 6.2 shows the percentage improvement in expected travel distance of the Chevron and the improved designs for equally sized warehouses (see Appendix C.1 for details).

Because we assume a vertical cross aisle in the Chevron and the improved designs generate Chevron-like designs, we insert a fixed vertical cross aisle and search for the best angles of picking aisles. Figure 6.3a shows that the improved designs with a floating and a fixed cross aisle are nearly identical for less than or equal to 13 P&D points. Because of this and the aisle structure changes for warehouses with

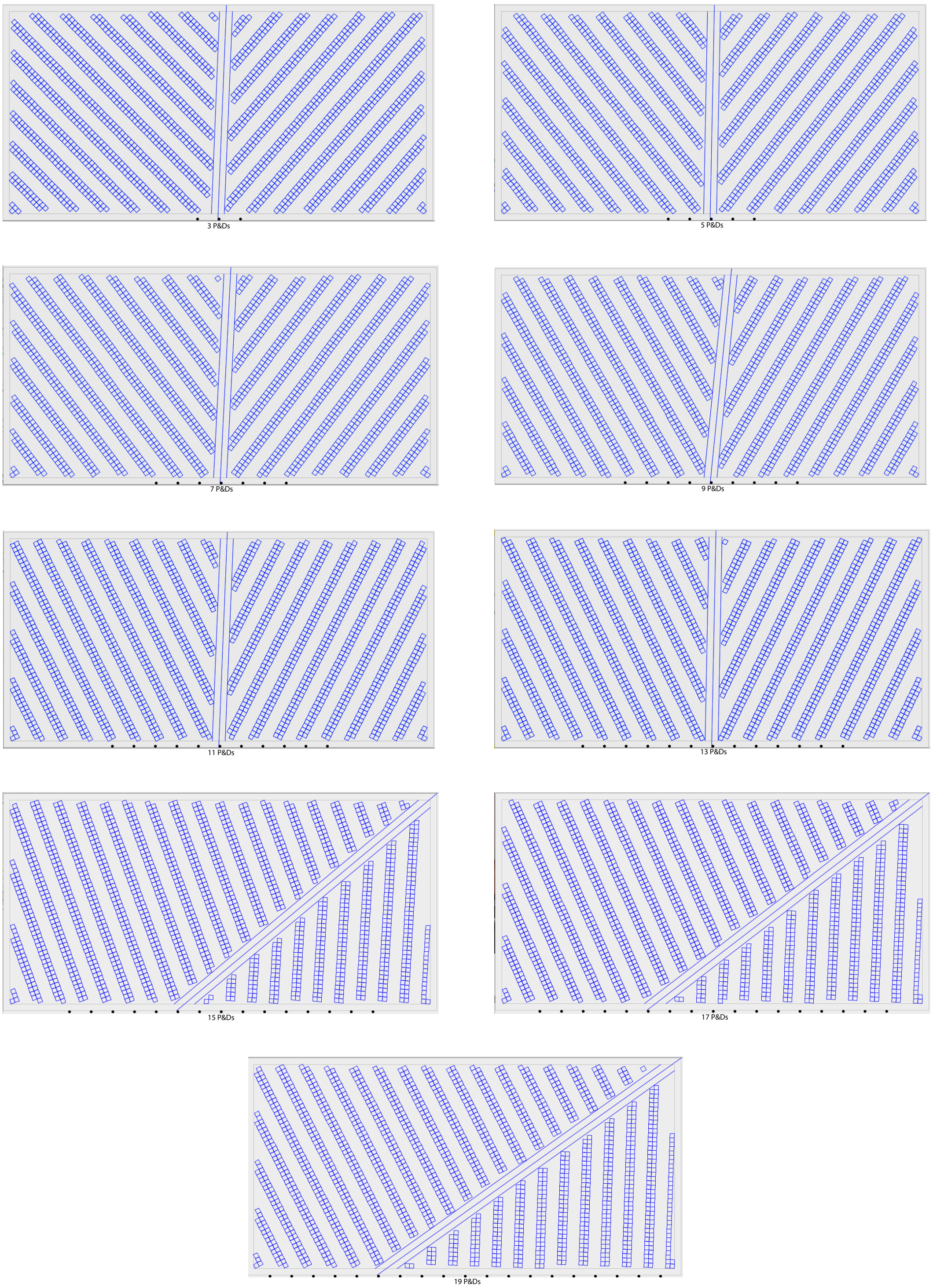


Figure 6.1: One cross aisle designs with a varying number of P&D points at the bottom.

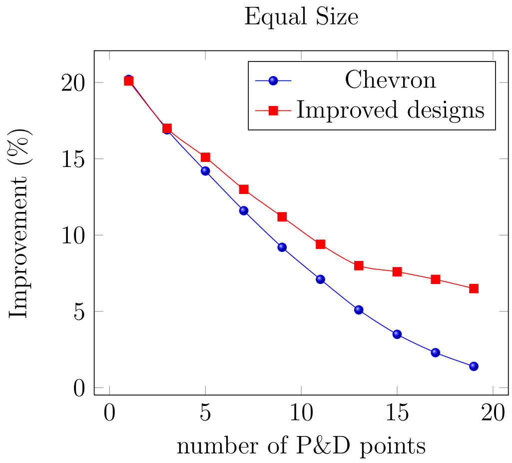
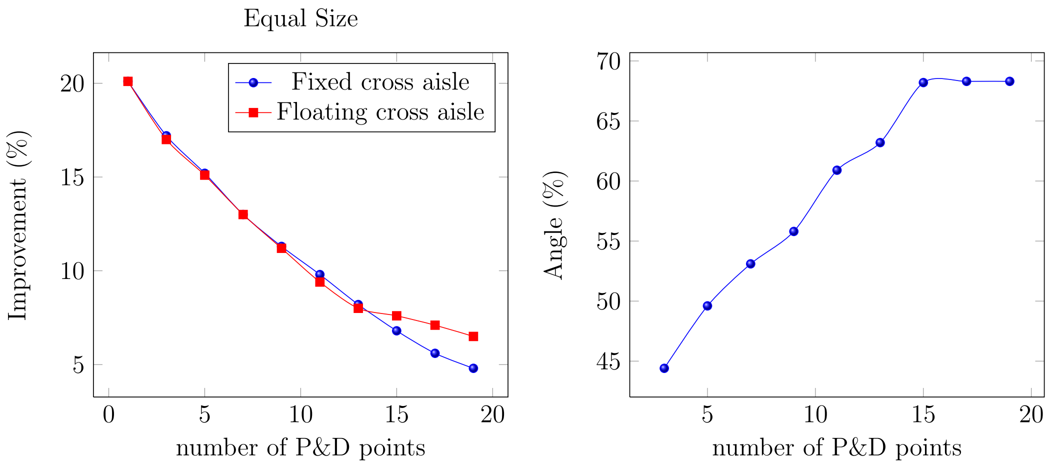


Figure 6.2: Comparison of the Chevron and the improved designs.

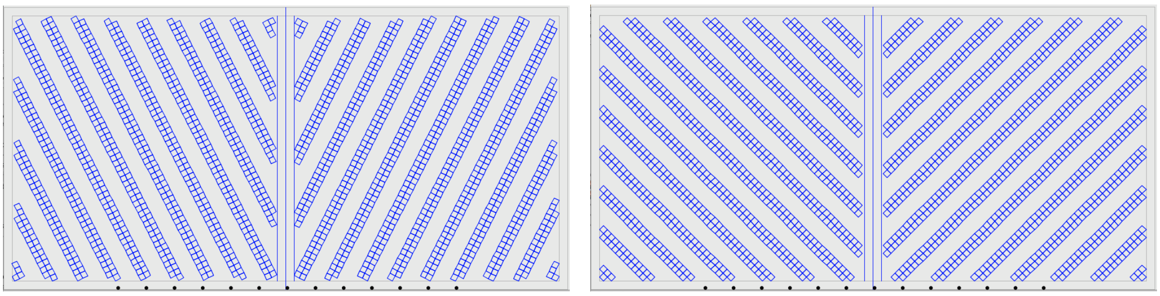


(a) (b)

Figure 6.3: (a) Comparison of improved designs with a floating and a fixed cross aisle. (b) Angles of picking aisles on the right side of improved designs with a fixed cross aisle.

more than 15 P&D points, hereafter, we use designs with the fixed cross aisle as a basis for comparison. Figure 6.4 shows the aisle structures in the Chevron and the improved design with 13 P&D points.

We expand the size of the Chevron and the improved designs so that they can have the equal storage capacity with a small-sized traditional design. Figure 6.5 shows that the 45° aisles in the Chevron loses their power to facilitate travel, as the number of P&D points increases at the bottom. However, the Chevron design is robust with respect to a maximum number of 13 P&D points, because the improved



(a) The improved design (b) The Chevron

Figure 6.4: The Chevron and the improved design with 13 P&D points at the bottom.

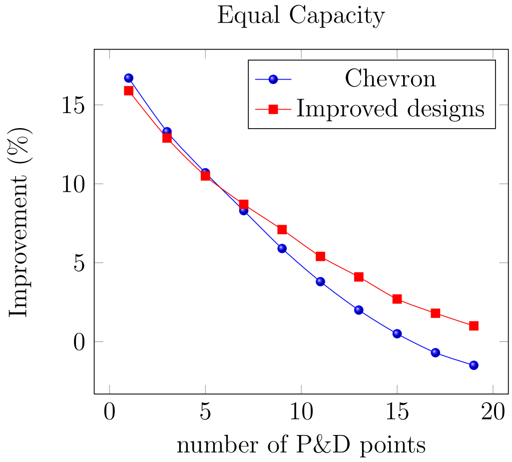


Figure 6.5: Comparison of the Chevron and improved designs for equally capacitated warehouses.

designs maintain Chevron-like aisle structure and the difference in improvement between the improved and Chevron design is, at most, 2% for equally capacitated, small-sized warehouses.

6.3.2 Two Cross Aisle Designs

In this section, we locate a number of P&D points at the bottom of the warehouse with two cross aisles in order to investigate changes in the Leaf aisle structure. We assume that cross aisles are symmetric with respect to the vertical axis going through the central P&D point. However, we allow cross aisles to originate from any sides of the warehouse and take on any angle. Hence, we consider the subproblems defined in Chapter 5 for two cross aisle models. Here, we also consider small-sized, square-half warehouses that are comprised of 19 aisles. We solve nine design problems, each with a different number of P&D points that starts from 3 and ends at 19 P&D points with an increment of two. These two P&D points are equally distributed around the central P&D point.

The improved aisle structures for these nine design problems are shown in Figure 6.6 (see Appendix C.2 for details). The model generates Leaf-like designs for warehouses with 13 P&D points or fewer. Even though the cross aisles are not originated from the central P&D points and their angles are different from the angles of the cross aisles in the Leaf design, their orientations are still similar to the Leaf. In the Leaf design, the angle of picking aisles on the right (α_R) must be less than or equal to the angle of the cross aisle on the right (β_R). Hence, the cross aisle is utilized to reach some storage locations in the right region with a shorter distance than the rectilinear travel. However, α_R is greater than β_R in the improved designs with more than 7 P&D points, in contrast to the Leaf aisle structure. Additionally, α_R increases as the P&D points are placed further away from the center.

For warehouses with more than 13 P&D points, the solutions have approximately vertical picking aisles in all regions with angled cross aisles originated from the central P&D point, similar to the Flying-V in Gue et al. (2010). This is a surprising result, because the "modified Flying-V" developed by Gue et al. (2010) was never claimed to be the best structure; the authors assumed vertical picking aisles, for example. Our results suggest that the Flying-V is the best design *if cross aisles are not allowed to intersect*. Designs with intersecting cross aisles might be investigated for potential benefits in travel distance.

Figure 6.7 shows improvements in expected travel distance that the Leaf and the improved designs offer for equally sized and equally capacitated warehouses. Because the Leaf is specifically developed for a single P&D point, it loses its efficiency as P&D

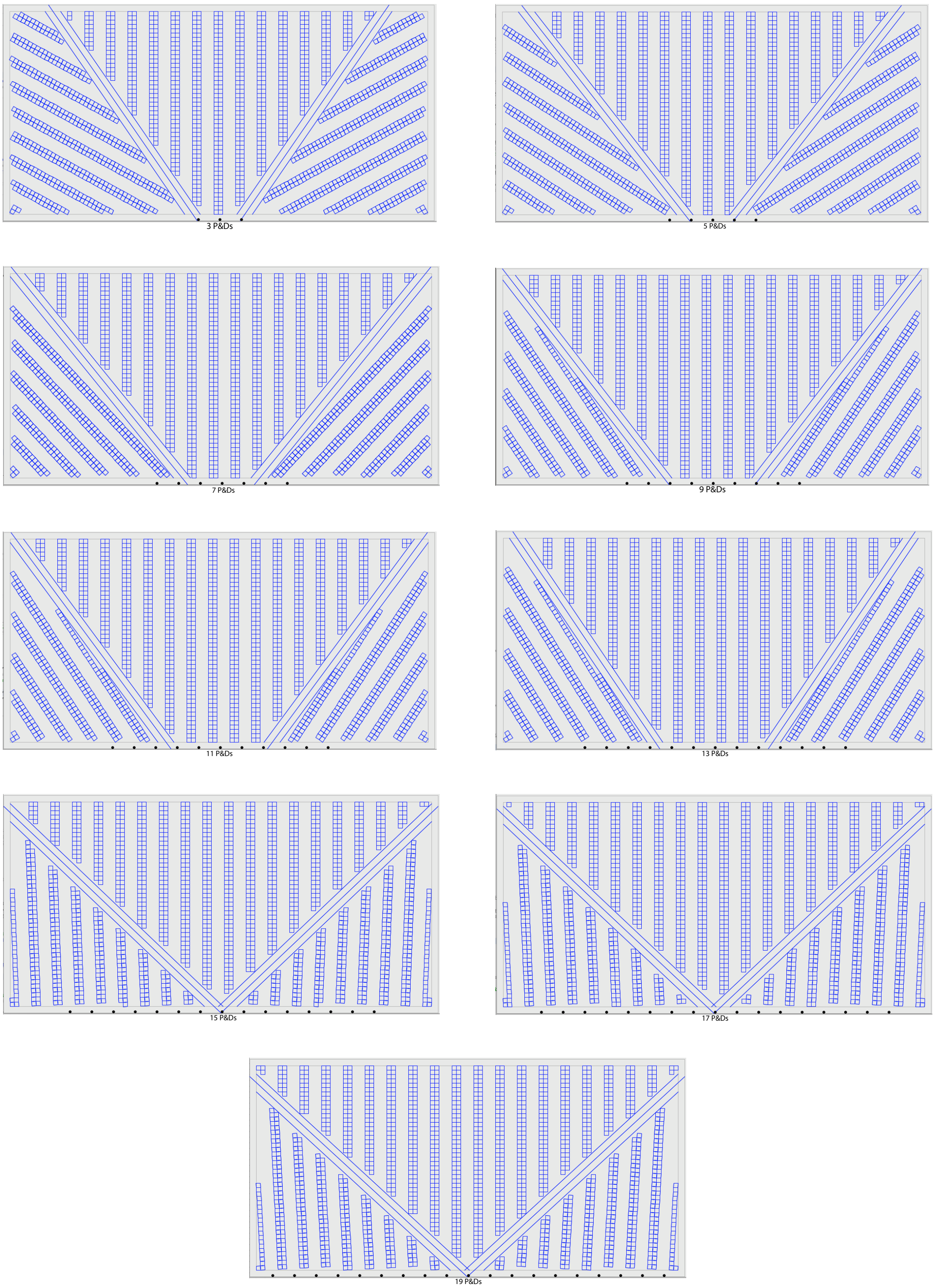


Figure 6.6: Two cross aisle designs with a varying number of P&D points at the bottom.

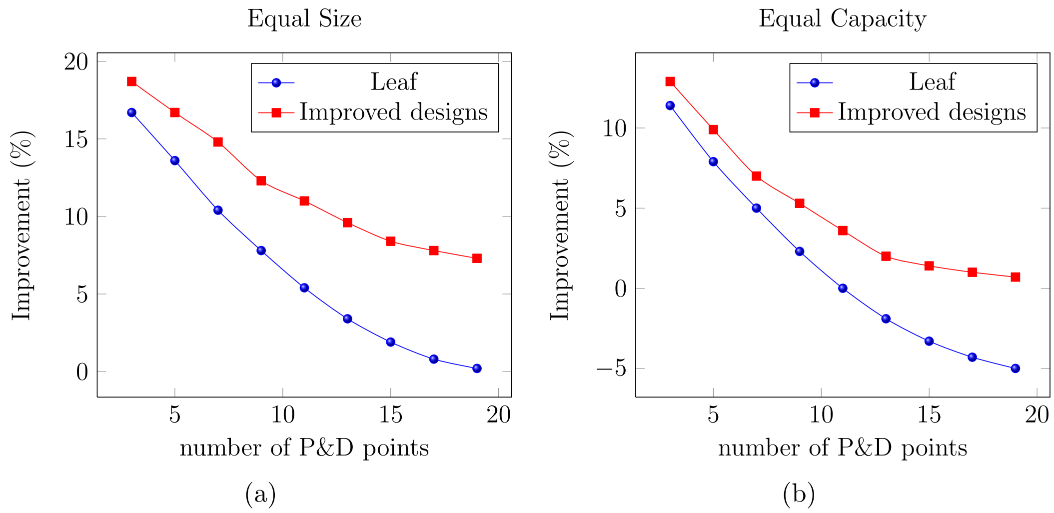


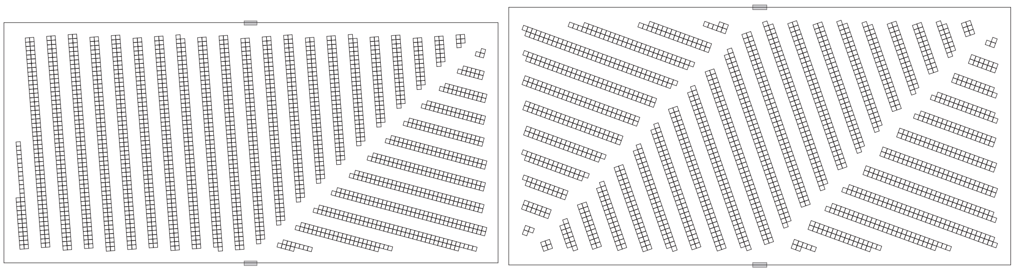
Figure 6.7: Comparisons of the Leaf and the improved designs over equally sized and equally capacitated warehouses.

points move away from the center. For more than 9 P&D points, it does not offer any savings over an equivalent, small-sized traditional warehouse. However, as improved designs suggest, it is possible to increase the benefit of the Leaf by moving cross aisles away from the center and increasing the angle of picking aisles on the right (decreasing on the left). Therefore, the Leaf is not as robust as the Chevron. The Leaf is robust with respect to the maximum of 7 P&D points such that: (1) the improved designs also provide a similar aisle structure to the Leaf, and (2) the Leaf offers, at most, 2% less improvement than the equivalent improved design.

For a large number of P&D points, the Flying-V-like designs seem to present less savings compared to the results in Gue et al. (2010). This is because;

- The improved designs require more warehouse space than the Flying-V because of slightly angled aisles.

- The improvements of the modified Flying-V in Gue et al. (2010) is investigated for larger warehouses. However, we work on small-sized warehouses.



(a) Design C1 (b) Design C2

Figure 6.8: Designs with centrally located P&D points at the bottom and the top of the warehouse.

6.4 Unit-Load Warehouse Designs with P&D Points at the Bottom and the Top of the Warehouse

In this section, we locate a number of P&D points both at the bottom and the top sides of the warehouse. We then search for the best aisle designs, inserting one and two cross aisles to investigate the robustness of Design C1 and Design C2 (see in Figure 6.8).

6.4.1 One Cross Aisle Designs

In this section, we insert one angled cross aisle and increase the number of P&D points by placing two additional P&D points both at the bottom and the top of the warehouse. These P&D points are equally distributed around the central P&D points to preserve a symmetric distribution. Figure 6.9 shows the improved designs for a varying number of P&D points. The improved designs with up to 9 P&D points per side have similar aisle structure with Design C1. In these designs, the angles of aisles in the right and left regions are approximately 168° and 93°, respectively. However, when the number of P&D points increases, a different structure emerges, especially in the right region. For more than 9 P&D points per side, the angle of picking aisles on the right becomes approximately vertical so that the picking aisles allow direct travel to the storage area from the P&D points that are further away from both the central P&D point and the inserted cross aisle.

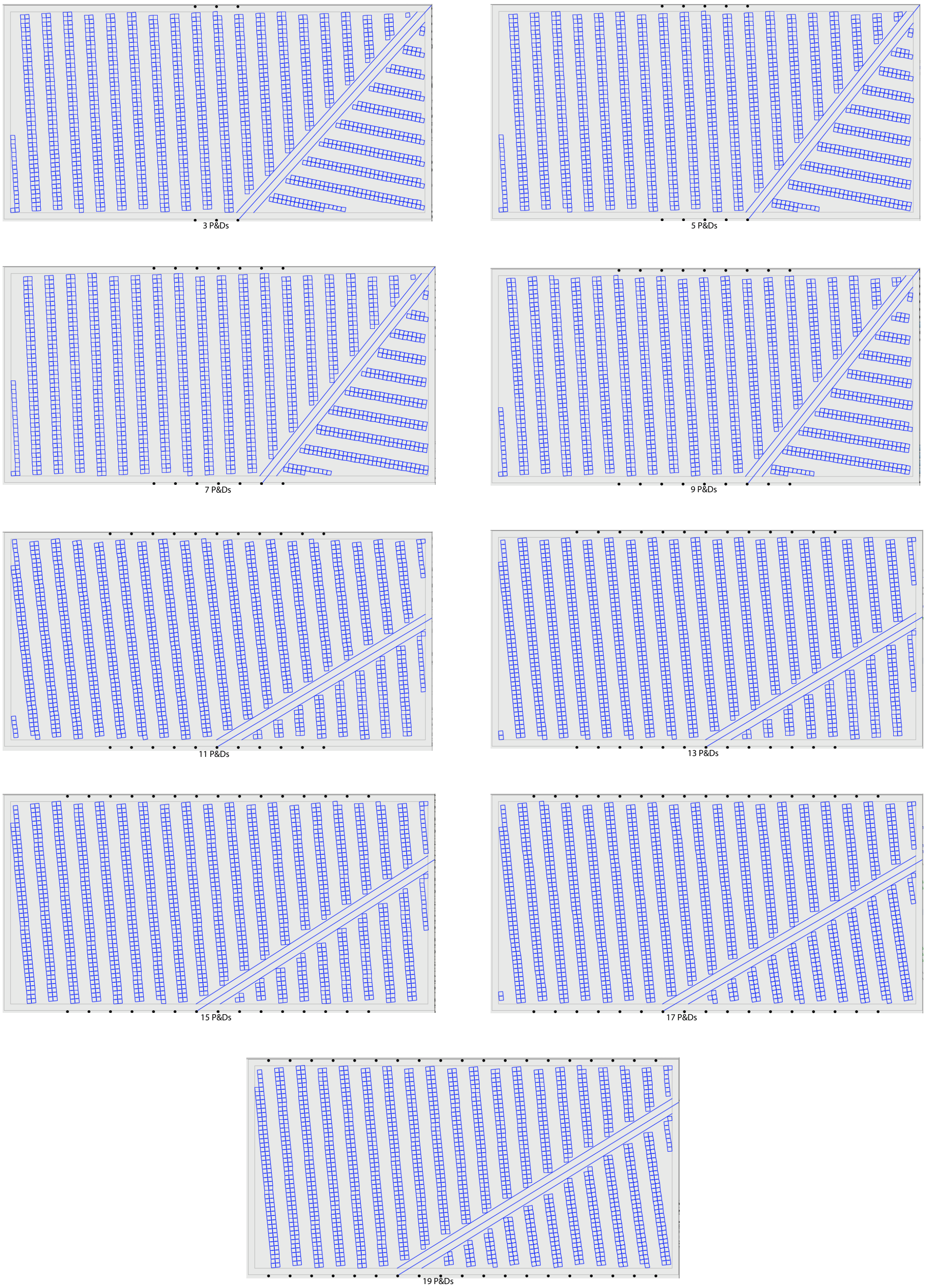
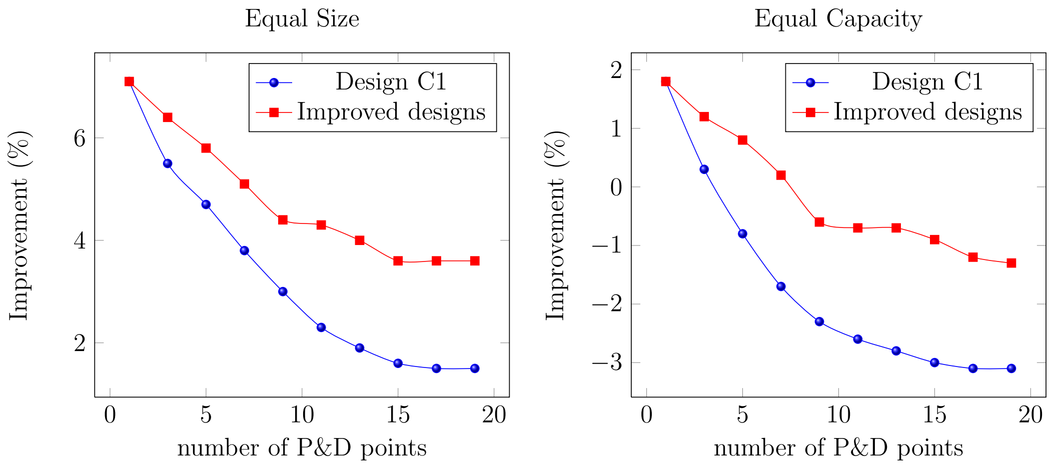


Figure 6.9: Improved designs with one cross aisle and P&D points at the bottom and the top.



(a) (b)

Figure 6.10: Comparison of Design C1 and improved designs with one cross aisle and multiple P&D points at the bottom and the top.

Figure 6.10 shows the improvements in expected travel distance that the improved designs and Design C1 have over equally sized and equally capacitated warehouses. Design C1 does not provide any improvement for more than 3 P&D points per side, and the improved designs do not offer any savings in travel for more than 7 P&D points per side. Even though the improved designs have similar aisle structure with Design C1, the improved designs offer more improvement. This is because the improved designs place the cross aisle away from the center in order to allow newly located P&D points to better utilize the cross aisle (see Appendix C.3 for details).

Design C1 is a robust design with respect to a maximum of 9 P&D points per side such that the aisle structures in improved designs are similar to the aisle structure in Design C1. Additionally, for 9 P&D points per side, even though the improved design and Design C1 have worse expected travel distance than the equivalent traditional design, Design C1 provides approximately 1.5% less expected travel distance than the improved design.

6.4.2 Two Cross Aisle Designs

Figure 6.11 shows the improved two cross aisle designs with P&D points at the bottom and the top. Designs with less than or equal to 9 P&D points per side have similar aisle structure with Design C2. These designs are also approximately symmetric with respect to the diagonal that passes through the lower-left and the upper-right corners. For more than 11 P&D points per side, the model generates different aisle structures.

In improved designs with 9 P&D points or fewer, the angles of picking aisles are similar to the the angles in Design C2, because the model moves the cross aisles away from the central P&D points instead of changing angles of picking aisles. Thus, travel is facilitated from the P&D points further away from the central one. For designs with more than 9 P&D points, the angles of picking aisles in regions, either right or left regions and the central region, are approximately the same because these picking aisles facilitate travel from the P&D points to the storage locations further away from them.

Figure 6.12 shows the comparison of Design C2 and the improved designs with equally sized and equally capacitated traditional warehouses (see details in Appendix C.4). The aisle structure in Design C2 is robust for warehouses with up to 9 P&D points per side, because of the similarity in aisle structures of Design C2 and the improved designs. However, for equally capacitated warehouses, the improved design with 9 P&D points per side proposes 3.2% more improvement than Design C2. Even for 3 P&D points per side, the improved design has an additional 2% more benefit than Design C2. Therefore, up to 9 P&D points per side, we can conclude that Design C2 is a robust design, and it offers similar benefit to improved designs, if cross aisles are slightly moved away from the center.

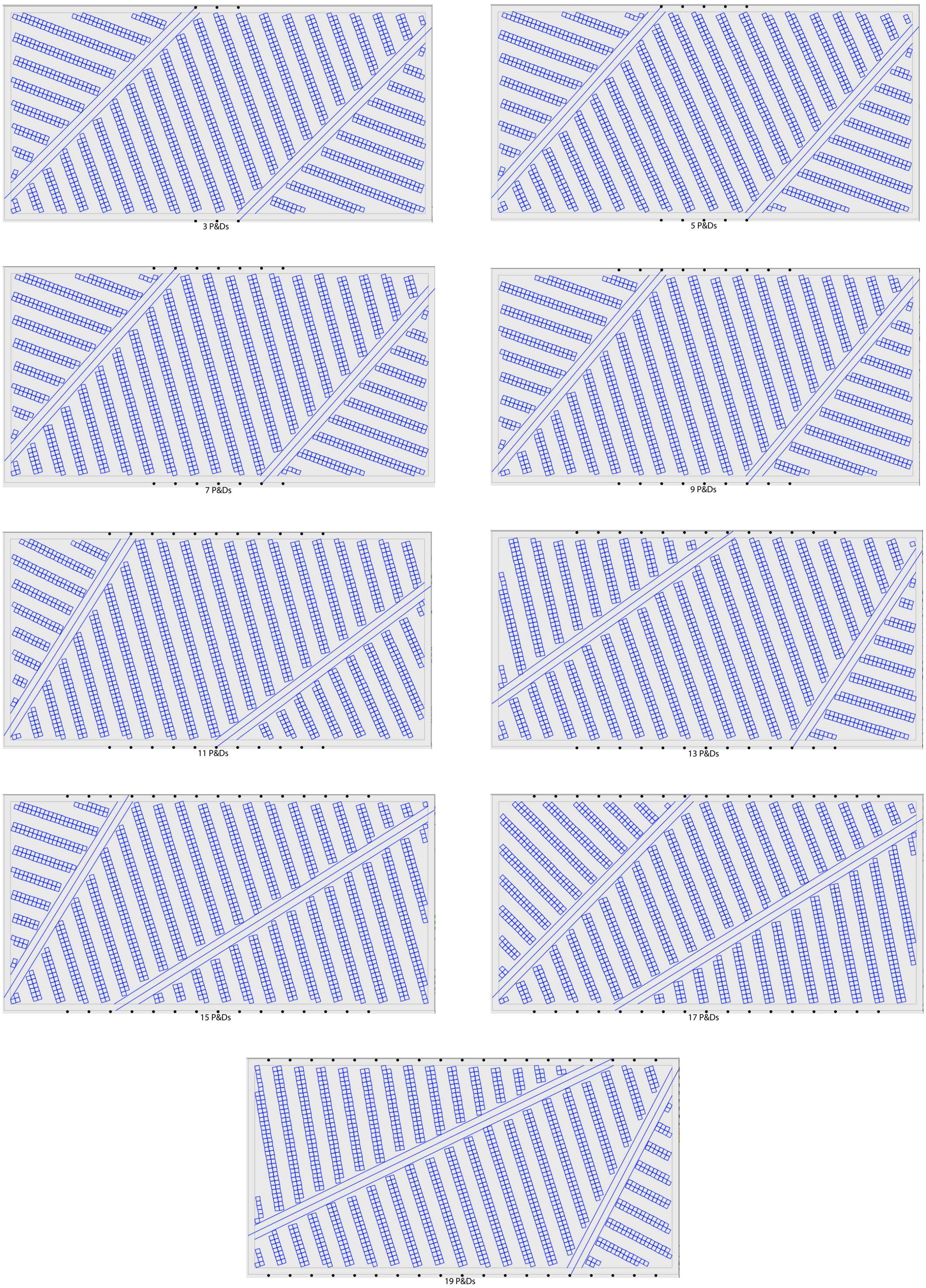


Figure 6.11: Improved designs with two cross aisles and multiple P&D points at the bottom and the top.

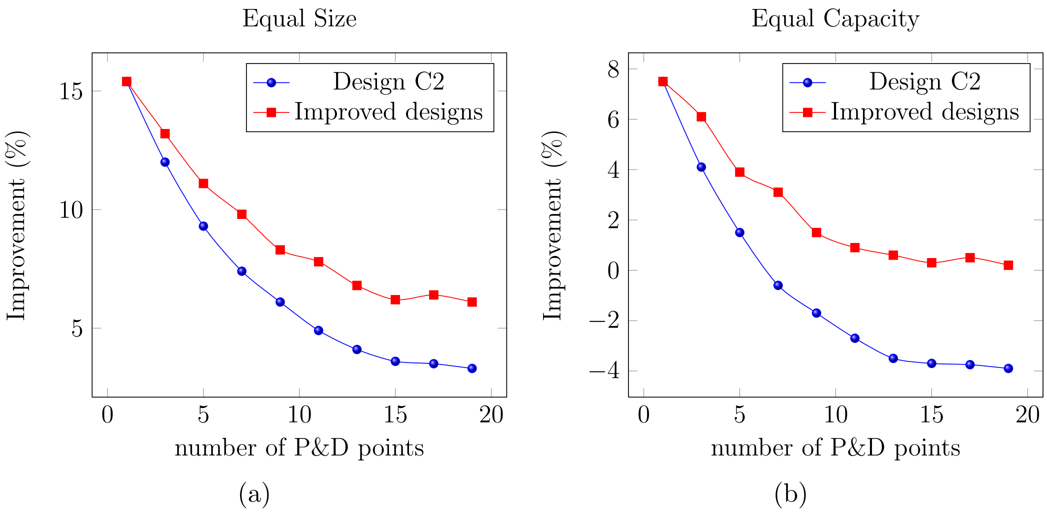


Figure 6.12: Comparison of Design C2 and improved designs with two cross aisles and multiple P&D points at the bottom and the top.

6.5 Conclusion

Many warehouses spaces have multiple P&D points along their sides, we have investigated non-traditional aisle designs developed for centrally located P&D point(s) and found them to be relatively robust with respect to a varying number of P&D points. We first investigated changes in performance of the Chevron and Leaf as the number of P&D points increases at the bottom. Whereas the improvement in the Chevron is 14% for 3 P&D points, it loses all of its efficiency when there are 15 P&D points at the bottom of a small-sized warehouse. The Chevron is also very robust with respect to a maximum of 13 P&D points compared to the improved designs.

When we insert two cross aisles into a warehouse with bottom P&D points, we have observed that the Leaf preserves its aisle structure for less than or equal to 7 P&D points, and it offers more than 5% savings in expected travel distance over an equivalent, small-sized traditional design. Therefore, the Leaf only has, at most, 2% less improvement than the equivalent improved designs with 7 P&D points or fewer. Surprisingly, the model generates designs that look like the Flying-V for warehouses with two cross aisles and more than 13 P&D points at the bottom. Hence, if there is almost one P&D point at the beginning of every aisle in a small-sized warehouse, the best design we found is the Flying-V (with non-intersecting cross aisles).

We have also considered warehouses with P&D points both at the bottom and the top sides of the warehouse. Design C1 provides positive improvement in expected travel distance for less than 5 P&D points per side over equivalent, small-sized traditional warehouses. The improved designs with one cross aisle propose very similar aisle structure with Design C1 for 9 P&D points per side or less. Design C1 also has 1.5% higher expected travel distance than the improved design with 9 P&D points per side. Even though Design C1 offers worse expected travel distance than traditional designs for more than 3 P&D points per side, in terms of robustness, it is a robust design for up to 9 P&D points per side.

When two cross aisles are inserted with P&D points at the bottom and the top, we have shown that Design C2 is a robust design with respect to change in its aisle structure for a maximum of 9 P&D points per side. However, the improved designs offer more improvement than Design C2; for example, 3.2% more improvement for 9 P&D points. Hence, the improved designs suggest that the cross aisles in Design C2 need to be moved slightly away from the center by preserving their orientation, as the number of P&D points increases. For more than 9 P&D points per side, different aisle structure emerges, because of the spreading P&D points per side.

We have assumed an equal flow from each P&D point. However, P&D points close to the center likely would be more utilized than more distant P&D points. This is because a number of trucks is sometimes less than the number of available doors; therefore, P&D points farther from the center are not always active. Thus, we can expect that the improvement from primary designs would be higher than these results indicate. Additionally, because non-traditional aisle designs offer an increasing improvement in expected travel distance as the size of warehouses increase, we expect that primary designs would yield better performance for larger warehouses.

Chapter 7

Conclusions and Future Research

7.1 Conclusions

In this research, we extended the idea of non-traditional aisle designs and proposed new layouts for a variety of warehouse design problems. We also show their robustness with respect to a varying number of P&D points.

Contribution 1. *We proposed three optimal aisle designs for unit load warehouses with a single P&D point.*

Although Gue and Meller (2009) relaxed some established rules of warehouse design, they did adhere to one rule: *Picking aisles must be either horizontal or vertical.* We relaxed this assumption by allowing cross aisles and picking aisles to take any angle in our model. We presented travel distance functions in continuous warehouse space for a given half-width and height of the warehouse. By solving our model, we introduced an optimal one cross aisle design, called the *Chevron*, in square-half, unit-load warehouses. We presented the optimal two cross aisle design, called the *Leaf*, which has symmetric cross aisles in square-half, unit-load warehouses. Additionally, we proposed an optimal three cross aisle design, called the *Butterfly*. We showed that the maximum savings of the Chevron, the Leaf and the Butterfly are 19.53%, 21.72% and 22.52%, respectively.

In order to assess the real benefit of these designs and to analyze the wasted space due to angled aisles, we developed a discrete cost analysis that considers travel to every available storage location. We showed that the Chevron is the best design for square-half warehouses with a width of 27 aisles or fewer. As the size of the Chevron increases from 19-aisles to 27-aisles, its savings increase from approximately

16.1% to 17.1%, and additional space requirements decrease from 11.2% to 7.3%. The Chevron always has less expected travel distance than the equivalent traditional design, even if the warehouse is small. For warehouses with more than 27 aisles, the Leaf offers more savings than the Chevron, but requires slightly more space due to having two inserted cross aisles. For a large warehouse with 51 aisles, the Leaf has 19.3% improvement over an equivalent traditional design and uses 6% more space. The Butterfly offers similar improvement as the Leaf for warehouses larger than 65 aisles, but the Leaf requires less space than the Butterfly. Therefore, we concluded that the Chevron is the best design for the most warehouse sizes in industry with a centrally located P&D point.

Contribution 2. *We developed better aisle designs than traditional for unit-load warehouses with multiple P&D points.*

We allowed picking and cross aisles to take any angle, and the cross aisles to be oriented freely as long as their start and end points are on different sides of the warehouse. We also permitted multiple predefined P&D points to be on different sides of the warehouse.

We developed a warehouse network model and used a shortest path algorithm to efficiently evaluate travel distance between storage locations and P&D points. Next, we used a Particle Swarm Optimization algorithm to search for aisle angles in a given warehouse with pre-located P&D points.

We differentiated five design problems based on the location of P&D points. We inserted one and two cross aisles incrementally, and searched for the best aisle structures for each of the design problems.

In comparison with an equivalent, medium-sized traditional warehouse, the Design A2 offers approximately 11% savings with a cost of 9.6% larger space. The Design B2 provides approximately 6% reduction in expected travel distance but requires 14.4% more space. The Design C2 has approximately 10% less savings but necessitates a 11.7% bigger warehouse. In design problem D, the best design we found so far does not need any additional cross aisle because of the locations of P&D points. Hence, it offers 16% savings with a cost of 4% larger space. When we distribute one P&D point to each side of the warehouse we could not get any solution better than the traditional design with a vertical cross aisle.

Last, we modified several of these best or nearly best designs in order to increase their storage capacity as well as to decrease their additional space requirements. We modified their angles of picking aisles to 90° or 180°. Some of these modified designs slightly increase the improvement in expected travel distance for equally capacitated warehouses. They offer a considerable reduction in additional warehouse space required to provide a capacity equal to non-traditional designs, 3% on average.

Contribution 3. *We investigated the robustness of aisle designs with respect to a varying number of P&D points.*

In some warehouses in industry, incoming and outgoing material flows to and from a warehouse are distributed along a number of dock doors. Therefore, we first investigated to see how much improvement is presented by non-traditional aisle designs, the Chevron, Leaf, Design C1, and Design C2, that are specifically developed for centrally located P&D point(s), as the number of P&D points increases.

Next, we search for better aisle designs than these designs by increasing their number of P&D points. The improved designs show that the Chevron is very robust for a large number of P&D points, such that its aisle structure is similar to the aisle structures in improved designs. However, the Leaf is not as robust as the Chevron. As the P&D points move away from the center, the Leaf loses its power because of the angle of picking aisles in the right and left regions. Design C1 and Design C2 are also robust designs for warehouses with 9 P&D points or fewer. Hence, the aisle structure in improved designs and these designs are similar for that number of P&D points. Because improvements in Design C2 are slightly different from the results in improved designs, the model suggests to move the cross aisles away from the central

P&D points. Hence, robustness of non-traditional aisle designs is assessed according to the change in the aisle structure and the change improvement.

Interestingly, we have also showed that the improved designs we obtain with two cross aisles and a large number of P&D points at the bottom look like Flying-V designs. Hence, the Flying-V is the best design we found if there is one P&D point at the beginning of every aisle. with non-intersecting cross aisles.

7.2 Future Research

This research is one of the first attempts to propose a general warehouse design tool and optimization approach for designing aisles to minimize expected travel distance to visit locations in a warehouse. However, the tool and the designs we developed rely on operational and design assumptions: randomized storage, single-command operations, non-intersecting cross aisles and a rectangular shape warehouse.

There is a potential for savings in travel distance for other common types of operations, such as dual-command and order picking. Existing non-traditional aisle designs could be evaluated for dual-command and order picking efficiency, or entirely new aisle designs could be investigated. Aisle designs for order picking is an especially important problem.

We assume in our models that each storage location has the same probability of being visited by workers. This assumption refers to the randomized storage policy and is also used to approximate the closest-open-location storage policy (Graves et al., 1977). Pohl et al. (2010) evaluated the performance of Fishbone and Flying-V designs under turnover-based storage policy and showed that these designs still offer improvement over traditional designs under turnover-based storage. Therefore, even though randomized storage policy is common in industry, the performance of our existing aisle designs could be evaluated for other common storage policies, such as class-based and turnover-based storage policies.

Our existing aisle designs that are developed for multiple, pre-located P&D points consider travel to all available storage locations from all P&D points with an equal probability. However, some P&D points might be utilized intensively comparing to some others. Or, the storage space could be divided into zones or could be partitioned based on customers. Then, P&D points could be distributed to zones or partitions such that travel is restricted from specific P&D points to storage locations in a specific area. New aisle designs could be developed for this type of problem.

Even though intersecting cross aisles seem to waste more space than the non-intersecting cross aisles, they might still offer benefits compared to non-intersecting aisles, especially when travel between storage locations becomes important. Hence, intersecting aisles might be allowed in warehouse design models in order to develop new aisle designs for new problems. Because we assume a rectangular shape warehouse, non-rectangular shape warehouses might also be considered with a new aisle designs in order to minimize expected travel distance.

7.2.1 Dynamics of inserted P&D points

As shown in Section 6.3.1, the model generates Chevron-like designs up to 13 P&D points and approximately symmetric designs with a fixed cross aisle. We observe that the model generates a different aisle structure after inserting 13 P&D points, because the cross aisle needs to be appropriately placed in order to facilitate travel from P&D points furthest to the center. Therefore, here we quantify the cost of inserting P&D points and open a discussion for future research about what kind of dynamics there are between inserting additional P&D points and the change in aisle structure. In order to investigate the cost of each inserted P&D location in improved designs, their locations are numbered according to their closeness to the center (see Figure 7.1). Because P&D points are symmetrically distributed around the center, P&D points with a same distance to the center on each side have the same expected travel distance as long as the design is symmetric.

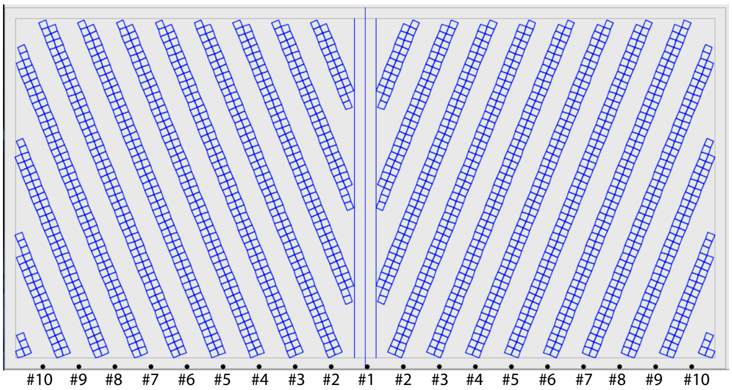


Figure 7.1: An improved design with a fixed cross aisle and the locations of P&D points are marked based on the closeness to the center.

We then evaluate the expected travel distance from each numbered location in improved designs that are developed for a specific number of P&D points (see Figure 7.1). For example, the black circled line presents the expected travel distance of location #1 in improved designs specifically developed with 1 – 19 P&D points. Thus, the cost of locations closer to the center increases as the inserted number of P&D points increases in improved designs, especially the locations #1 and #2 because the designs are not specifically developed for central P&D points. In Locations after location #4, the expected travel distance decreases as the inserted number of P&D points increases because the improved designs are specifically developed for a larger number of P&D points.

The left plot in Figure 7.3 also shows how much cost an inserted P&D point has at a specific location if that P&D point is the last inserted one in the design. For example, as the cost of one P&D point at location #1 is 398 if there is only one P&D point in the improved design; or, the cost of inserted one P&D point at location #4 is 479.62 if the improved design is developed for 7 P&D points at the bottom. Therefore, the right plot in Figure 7.3 shows that the model does not change the aisle structure until some point that has a big marginal cost of inserting one P&D point at a location further to the center. When the model provides a different aisle

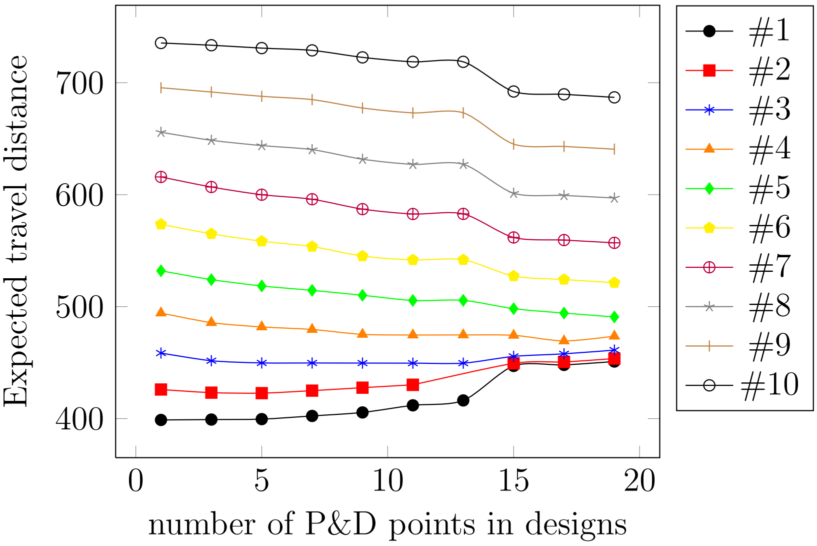


Figure 7.2: Costs of P&D points at different location in different designs developed for a specific number of P&D points.

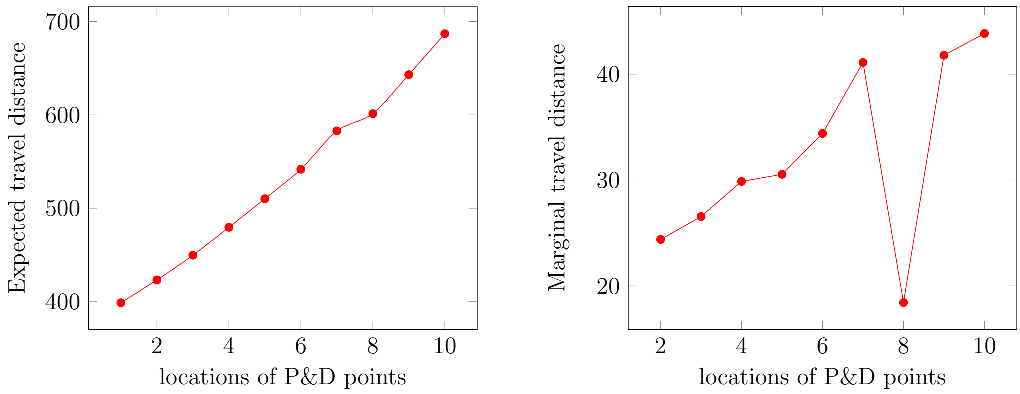


Figure 7.3: Expected travel distance and marginal cost of an inserted P&D point at locations in improved designs.

structure, the marginal cost of inserting the last P&D point decreases dramatically, then it starts to increase again for newly inserted P&D points.

As a result, quantifying the cost of inserting a P&D locations might be also helpful for practitioners to determine the efficiency of dock doors. To find an inflection point of non-traditional warehouses in terms of number of P&D locations also could be a good research topic.

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Appendices

Appendix A

A.1 Closed form expressions for $E[WR]$

$$
\begin{aligned}
E[WR_1] = {} & \frac{h}{2} + \frac{w^2 \tan\alpha_R \left(1+\sec\alpha_R\right)}{6h} - \frac{w^2 \sec\alpha_R \tan\alpha_R \sec^2\beta_R \cos^2\left(\frac{\alpha_R-\beta_R}{2}\right)}{3h} \\
& + \frac{w^2 \tan^2\beta_R}{6h} - \frac{w\tan\beta_R}{2} \\
& + \frac{w \sec\beta_R \left(3h + w\tan\beta_R \sec^2\alpha_R \left(-\cos\left(2\alpha_R\right) + \cos\left(\alpha_R-\beta_R\right)\right)\right)}{6h} \\
E[WR_2] = {} & \frac{h^2 \cot\beta_R \left(1+\csc\beta_R\right)}{6w} + \frac{\sec\left(\frac{\alpha_R-\beta_R}{2}\right)\left(w^2 - h^2\cot^2\beta_R\right)\cos\left(\frac{\alpha_R+\beta_R}{2}\right)}{2w} \\
& + \frac{h\sec\left(\frac{\alpha_R-\beta_R}{2}\right)\left(w - h\cot\beta_R\right)\sin\left(\frac{\alpha_R+\beta_R}{2}\right)}{2w} + \frac{w^2\tan\alpha_R\left(1+\sec\alpha_R\right)}{6h} \\
& - \frac{\sec\alpha_R \tan\alpha_R \left(w^3 - h^3\cot^3\beta_R\right)\left(3 + \cos\left(\frac{3\alpha_R+\beta_R}{2}\right)\sec\left(\frac{\alpha_R-\beta_R}{2}\right)\right)}{12hw} \\
& + \frac{h^2\cot\beta_R \csc\beta_R \sec\alpha_R \cos^2\left(\frac{\alpha_R-\beta_R}{2}\right)\left(\sec\alpha_R - \csc\beta_R\tan\alpha_R\right)}{3w} \\
E[WR_3] = {} & \frac{w}{2} + \frac{h^2\cot^2\alpha_R}{6w} + \frac{h\csc\alpha_R}{2w} \\
& + \frac{h^2\cot\beta_R\left(1 - \csc\beta_R\left(-1 + \csc^2\alpha_R\left(1+\cos\left(\alpha_R-\beta_R\right)\right)\right)\right)}{6w} \\
& + \frac{h\cot\alpha_R\left(-3w + h\csc\alpha_R\csc^2\beta_R\left(\cos\left(\alpha_R-\beta_R\right) + \cos\left(2\beta_R\right)\right)\right)}{6w}
\end{aligned}
$$

A.2 Proof of Observation 2

Let us first show $E[A] = E[D]$ by using travel distance functions T_A and T_C defined in (4.3) and (4.5) (see also Figure 4.11).

$$E[A] = \frac{2}{w^2 \tan \alpha_R} \int_0^w \int_0^{x \tan \alpha_R} T_A(x, y) \quad dydx$$
$$E[D] = \frac{2}{h^2 \cot \beta_R} \int_0^{h/\tan \beta_R} \int_{x \tan \beta_R}^{h} T_C(x, y) \quad dydx$$

Because $w = h$ and $\alpha_R + \beta_R = 90°$ in the optimal design (see Observation 1), we can substitute and solve in MATHEMATICA. We find that

$$E[A] = E[D] = \frac{1}{6} w^3 \tan \alpha_R \left(1 + \sec \alpha_R\right).$$

The travel distance function for regions B and C in Figure 4.16 is T_B which is described in (4.4). $E[B]$ and $E[C]$ are

$$E[B] = \frac{2}{w(h - w \tan \alpha_R)} \int_0^w \int_{x \tan \alpha_R}^{x \tan \pi/4} T_C(x, y) \quad dydx,$$
$$E[C] = \frac{2}{h(w - h \cot \beta_R)} \int_0^h \int_{y/\tan \beta_R}^{y/\tan \pi/4} T_C(x, y) \quad dxdy.$$

The solution, obtained by MATHEMATICA, produces

$$E[B] = E[C] = -\frac{w^3 \left(-3 + 2 \tan \beta_R + \tan(\alpha_R)^2\right)}{6 \left(\cos \alpha_R + \sin \alpha_R\right)}.$$

A.3 Expected travel distances in a three cross aisle model

The following expressions were obtained with MATHEMATICA, as solutions to equations 4.8–4.11.

$$E[WR_1] = \frac{h + w\sec\alpha_{CR} - w\tan\alpha_{CR}}{2} - \frac{w^2\sec\beta_R\sin^2(\beta_R/2)\tan\alpha_R}{3h}$$
$$+ \frac{w^2\sec\alpha_{CR}\tan\alpha_{CR}}{6h} + \frac{w^2\sec\beta_R\tan\alpha_{CR}}{6h} + \frac{w^2\tan^2\alpha_{CR}}{6h} + \frac{w^2\sec\alpha_R\tan\beta_R}{6h}$$
$$+ \frac{w^2\sec\alpha_{CR}\tan\beta_R}{6h}$$
$$E[WR_2] = \frac{h^2\cot\alpha_{CR}\left(1+\csc\alpha_{CR}+2\cos^2\left(\frac{\alpha_{CR}-\beta_R}{2}\right)\csc\alpha_{CR}\sec^2\beta_R\right)}{6w}$$
$$+ \frac{1}{2}\cos\left(\frac{\alpha_{CR}+\beta_R}{2}\right)\sec\left(\frac{\beta_R-\alpha_{CR}}{2}\right)\left(w - \frac{h^2\cot\alpha_{CR}}{w}\right)$$
$$+ \frac{h}{2}\sin\left(\frac{\alpha_{CR}+\beta_R}{2}\right)\sec\left(\frac{\beta_R-\alpha_{CR}}{2}\right)\left(1 - \frac{h\cot\alpha_{CR}}{w}\right)$$
$$+ \frac{w^2\tan\alpha_R}{6h}\left(1+\sec\alpha_R - 2\sec\alpha_R\sec^2\beta_R\cos\left(\frac{\alpha_R-\beta_R}{2}\right)\right)$$
$$+ \frac{\sec\beta_R\tan\beta_R}{4}\left(\frac{w^2}{h} + \frac{h^2\cot^3\alpha_{CR}}{w} - \frac{4h^2\cot\alpha_{CR}\csc^2\alpha_{CR}\cos^2\left(\frac{\alpha_{CR}-\beta_R}{2}\right)}{3w}\right)$$
$$+ \frac{w^2\sec\beta_R\tan\beta_R}{3h}\left(\sec^2\alpha_R\cos^2\left(\frac{\alpha_R-\beta_R}{2}\right) - \frac{\cos\left(\frac{\alpha_{CR}+3\beta_R}{2}\right)\sec\left(\frac{\beta_R-\alpha_{CR}}{2}\right)}{4}\right)$$
$$+ \frac{h^2}{12w}\tan\beta_R\sec\beta_R\cot^3\alpha_{CR}\cos\left(\frac{\alpha_{CR}+3\beta_R}{2}\right)\sec\left(\frac{\beta_R-\alpha_{CR}}{2}\right)$$
$$E[WR_3] = \frac{h^2\cot\alpha_{CR}}{6w}(1+\csc\alpha_{CR}) + \frac{w^2\tan\alpha_R}{6h}\left(1 - \frac{\sec\alpha_R}{2}\right)$$
$$+ \frac{h^2\cos^2\left(\frac{\alpha_{CR}-\beta_R}{2}\right)\csc\beta_R}{3w}\left(\cot\beta_R\csc^2\alpha_{CR} - \cot\alpha_{CR}\csc\alpha_{CR}\csc\beta_R + \cot\beta_R\sec^2\alpha_R\right)$$
$$+ \frac{1}{2}\cos\left(\frac{\alpha_R+\beta_R}{2}\right)\sec\left(\frac{\alpha_R-\beta_R}{2}\right)\left(w - \frac{h^2\cot^2\beta_R}{w}\right)$$
$$+ \frac{1}{2}\sin\left(\frac{\alpha_R+\beta_R}{2}\right)\sec\left(\frac{\alpha_R-\beta_R}{2}\right)\left(h - \frac{h^2\cot^2\beta_R}{w}\right)$$
$$+ \frac{h^2\cot\beta_R\sec\alpha_R\tan\alpha_R}{12w}\left(3\cot^2\beta_R - 4\csc^2\beta_R\cos^2\left(\frac{\alpha_R-\beta_R}{2}\right)\right)$$
$$- \frac{1}{12}\cos\left(\frac{3\alpha_R+\beta_R}{2}\right)\sec\left(\frac{\alpha_R-\beta_R}{2}\right)\sec\alpha_R\tan\alpha_R\left(\frac{w^2}{h} - \frac{h^2\cot^3\beta_R}{w}\right)$$

$$E[WR_4] = \frac{1}{6w}\left(h^2\cot^2\alpha_R + 3w(w + h\csc\alpha_R) - h\cot\alpha_R(3w + 2h\csc\alpha_R)\right)$$
$$+ \frac{h^2}{6w}\cot\alpha_{CR}(1 + \csc\alpha_{CR})$$
$$+ \frac{2h^2}{6w}\cos^2\left(\frac{\alpha_R - \beta_R}{2}\right)\csc^2\alpha_R\csc^2\beta_R(\cos\alpha_R - \cos\beta_R)$$
$$+ \frac{2h^2}{6w}\cos^2\left(\frac{\alpha_{CR} - \beta_R}{2}\right)\csc^2\alpha_{CR}\csc^2\beta_R(\cos\beta_R - \cos\alpha_{CR})$$

A.4 Data

	Chevron		Leaf		Butterfly	
# Aisles	Improvement	Area	Improvement	Area	Improvement	Area
19	16.12	11.30	15.59	16.64	13.53	25.44
21	16.50	10.25	15.87	15.07	13.94	23.01
23	16.70	9.38	16.52	13.78	14.72	21.00
25	16.94	8.64	16.84	12.69	15.42	19.31
27	17.10	7.27	17.28	11.76	15.74	17.88
29	17.47	6.77	17.53	10.95	16.45	16.64
31	17.65	6.34	17.80	10.25	16.71	15.56
33	17.76	5.97	17.97	9.63	17.15	14.62
35	17.87	5.63	18.16	9.09	17.38	13.78
37	17.96	5.33	18.37	8.60	17.67	13.03
39	18.05	4.04	18.54	8.16	18.02	12.36
41	18.06	4.82	18.73	7.76	18.10	11.76
43	18.12	4.60	18.85	7.40	18.37	11.21
45	18.18	4.39	18.96	7.08	18.47	10.71
47	18.38	3.78	19.05	6.78	18.68	10.25
49	18.42	3.63	19.19	6.50	18.89	9.83
51	18.47	3.49	19.26	6.25	18.95	9.44
53	18.51	3.36	19.37	6.01	19.12	9.09
55	18.56	3.24	19.46	5.80	19.21	8.76
57	18.59	3.13	19.54	5.59	19.38	8.45
59	18.54	3.36	19.61	5.40	19.50	8.16
61	18.57	3.25	19.68	5.23	19.56	7.89
63	18.59	3.79	19.75	5.06	19.66	7.64
65	18.63	3.05	19.80	4.91	19.84	7.40
67	18.62	2.96	19.87	4.76	19.97	7.18
69	18.65	2.88	19.91	4.62	20.02	6.97
71	18.67	2.80	19.97	4.49	20.08	6.78

Table A.1: Improvements in expected travel distance over an equivalent traditional design.

	# Pallet Locations			
# Aisles	Traditional	Chevron	Leaf	Butterfly
19	1880	1904	1868	1900
21	2288	2306	2272	2304
23	2736	2760	2726	2756
25	3224	3240	3208	3248
27	3752	3746	3732	3764
29	4320	4328	4300	4348
31	4928	4928	4908	4948
33	5576	5568	5554	5604
35	6264	6248	6246	6284
37	6992	6972	6974	7004
39	7760	7760	7734	7792
41	8568	8580	8548	8592
43	9416	9428	9390	9456
45	10304	10304	10278	10328
47	11232	11220	11204	11260
49	12200	12180	12174	12240
51	13208	13178	13174	13240
53	14256	14212	14230	14300
55	15344	15284	15308	15368
57	16472	16406	16438	16508
59	17640	17640	17598	17680
61	18848	18832	18816	18884
63	20096	20072	20058	20144
65	21384	21344	21344	21356
67	22712	22752	22670	22695
69	24080	24104	24034	23995
71	25488	25506	25444	25460

Table A.2: Number of storage locations in the proposed designs and in the equivalent traditional design when the shape ratio is approximately 2:1. “Locations” refers to two-dimensional space—the number of columns of pallet rack or stacks of pallets.

Appendix B

B.1 Two Cross Aisle Model Cases

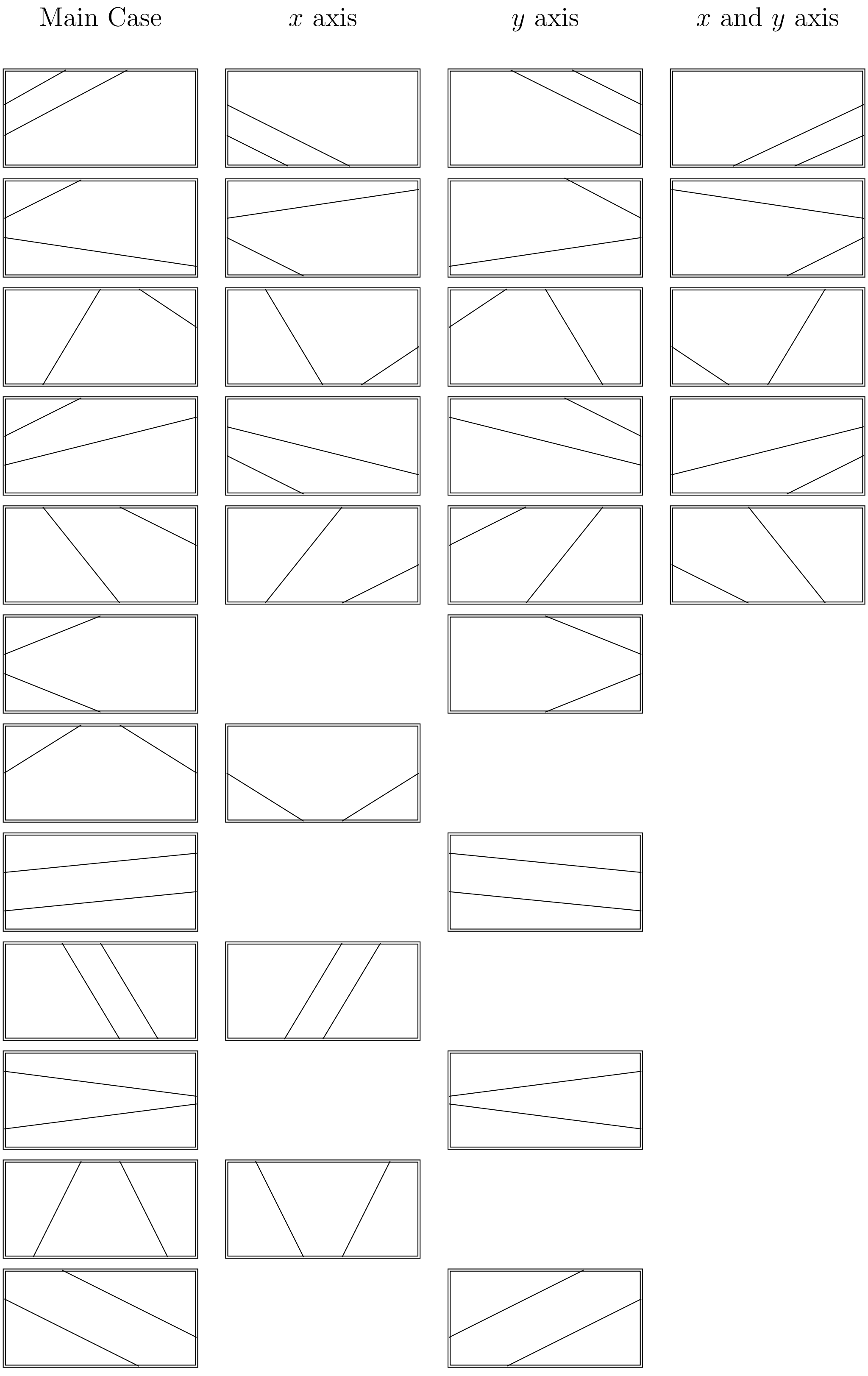

Figure B.1: The defined main cases and their transformations for two cross aisle model.

B.2 Computational Results in Design Problem A

B.2.1 Results for Design A1

	Run	S	E	α_L (°)	α_R (°)	# Pallets	$E[C]$
Subproblem 1	1	(334,500)	(1000,25)	116.40	87.18	1617	475.92
	2	(332,500)	(1000,35)	120.99	88.90	1638	480.33
	3	(330,500)	(1000,5)	115.83	85.46	1621	477.71
Subproblem 5	1	(332,500)	(970,0)	115.26	84.32	1630	479.38
	2	(335,500)	(1000,0)	118.12	85.46	1620	477.77
	3	(333,500)	(990,0)	114.11	86.03	1615	477.23
Subproblem 2	1	(0,437)	(1000,73)	114.11	95.78	1603	503.41
	2	(0,480)	(1000,25)	117.55	87.18	1575	496.53
	3	(0,450)	(1000,0)	116.40	93.48	1605	501.76

Table B.1: Computational results for Design A1 in a small-sized warehouse (W=100, H=50).

	Run	S	E	α_L (°)	α_R (°)	# Pallets	$E[C]$
	1	(500,750)	(1500,25)	116.97	87.18	3910	715.75
Subproblem 1	2	(500,750)	(1500,95)	116.40	88.33	3927	715.68
	3	(500,750)	(1500,85)	120.41	84.32	3942	719.19
	1	(500 ,750)	(1485,0)	114.68	86.61	3930	717.53
Subproblem 5	2	(500,750)	(1465,0)	111.25	83.74	3928	719.54
	3	(500,750)	(1447,0)	115.83	87.18	3927	717.14
	1	(0,722)	(1500,53)	115.83	88.90	3874	746.15
Subproblem 2	2	(0,738)	(1500,15)	117.55	86.03	3868	748.22
	3	(0,695)	(1500,35)	107.23	87.75	3879	752.09

Table B.2: Computational results for Design A1 in a medium-sized warehouse (W=150, H=75).

	Run	S	E	α_L (°)	α_R (°)	# Pallets	$E[C]$
	1	(666,1000)	(2000,105)	118.12	88.90	7232	955.33
Subproblem 1	2	(667,1000)	(2000,50)	114.11	88.33	7204	954.80
	3	(666,1000)	(2000,88)	121.56	86.61	7242	958.23
	1	(666,1000)	(1953,0)	117.55	87.75	7229	956.59
Subproblem 5	2	(666,1000)	(1935,0)	115.26	86.03	7241	958.43
	3	(667,1000)	(1975,0)	116.40	87.18	7240	957.23
	1	(0,948)	(2000,16)	115.26	88.33	7135	997.98
Subproblem 2	2	(0,830)	(2000,34)	116.40	94.63	7177	1010.66
	3	(0,882)	(2000,54)	118.69	86.03	7164	1003.73

Table B.3: Computational results for Design A1 in a large-sized warehouse (W=200, H=100).

	W	H	S	E	α_L (°)	α_R (°)	# Pallets	$E[C]$
Small	106	53	(353,530)	(1000,27)	116.40	87.18	1871	503.67
Medium	157	78	(523,780)	(1570,100)	116.4	88.33	4316	747.70
Large	207	103.5	(690,1035)	(2070,52)	114.11	88.33	7746	988.45

Table B.4: Performance of the expanded Design A1 for different warehouse sizes.

	W	H	S	E	α_L (°)	α_R (°)	# Pallets	$E[C]$
Small	100	50	(334,500)	(1000,25)	116.4	90	1670	481.05
	105	52	(350,500)	(1000,26)	116.4	90	1874	502.05
Medium	150	75	(500,750)	(1000,95)	116.4	90	4005	719.28
	156	77	(520,770)	(1560,89)	116.4	90	4312	744.81
Large	200	100	(667,1000)	(2000,50)	114.11	90	7324	960.00
	206	102	(687,1020)	(2060,42)	114.11	90	7755	984.73

Table B.5: Performance of modified Design A1 for different warehouse sizes.

B.2.2 Results for Design A2

	Run	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_C(^\circ)$	$\alpha_R(^\circ)$	# Pallets	$E[C]$
Subproblem 2	1	(333 ,500)	(500,0)	(667,500)	(500,0)	127.52	180.00	52.46	1610	448.24
	2	(333,500)	(483, 0)	(667,500)	(504,0)	129.58	174.75	47.65	1586	455.39
	3	(333,500)	(465,0)	(667,500)	(478,0)	122.13	180.00	56.81	1616	452.56
Subproblem 1	1	(333 ,500)	(0,80)	(667,500)	(0, 15)	9.36	86.61	100.36	1576	471.62
	2	(333,500)	(0,148)	(667,500)	(0,13)	14.42	87.75	113.54	1562	464.24
	3	(333,500)	(0,126)	(667,500)	(0,49)	171.98	84.89	121.56	1558	469.03
Subproblem 3	1	(333 ,500)	(0,46)	(1000,117)	(924,0)	155.36	88.90	100.93	1664	489.28
	2	(333,500)	(0,12)	(1000,97)	(916,0)	158.80	87.18	94.63	1655	486.63
	3	(333,500)	(0,21)	(1000,13)	(935 ,0)	157.65	84.89	98.07	1655	484.48

Table B.6: Computational results for Design A2 in a small-sized warehouse (W=100, H=50).

	Run	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_C(^\circ)$	$\alpha_R(^\circ)$	# Pallets	$E[C]$
Subproblem 2	1	(500 ,750)	(750,0)	(1000,750)	(753,0)	127.52	180.00	52.46	3905	674.35
	2	(500,750)	(733,0)	(1000,750)	(742,0)	133.59	179.34	47.07	3906	674.45
	3	(500,750)	(737,0)	(1000,750)	(750,0)	123.28	180.00	57.96	3906	676.49
Subproblem 1	1	(500 ,750)	(0,59)	(1000,750)	(0,45)	10.40	83.74	107.23	3800	701.30
	2	(500,750)	(0,127)	(1000,750)	(0,62)	18.43	85.46	103.80	3809	703.14
	3	(500,750)	(0,91)	(1000,750)	(0,26)	21.86	86.03	110.67	3808	695.52
Subproblem 3	1	(500 ,750)	(0,5)	(1500,142)	(1449,0)	156.51	84.32	180.00	3979	732.02
	2	(500,750)	(0,9)	(1500,82)	(1433,0)	158.23	87.18	0.66	3981	735.55
	3	(500,750)	(0,2)	(1500 ,95)	(1427,0)	151.93	82.02	8.11	3983	729.23

Table B.7: Computational results for Design A2 in a medium-sized warehouse (W=150, H=75).

	Run	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_C(^\circ)$	$\alpha_R(^\circ)$	# Pallets	$E[C]$
Subproblem 2	1	(667 ,1000)	(988,0)	(1333, 1000)	(996,0)	125.57	180	49.37	7209	899.85
	2	(667,1000)	(1005,0)	(1333,1000)	(1009 ,0)	129.58	180	50.51	7204	898.38
	3	(667,1000)	(1018,0)	(1333,1000)	(1019,0)	127.29	179.57	62.54	7199	904.67
Subproblem 1	1	(667 ,1000)	(0,60)	(1333,1000)	(0 ,48)	26.45	83.17	98.64	7055	945.96
	2	(667,1000)	(0,102)	(1333,1000)	(0,8)	23.58	87.18	107.23	7043	927.88
	3	(667,1000)	(0,43)	(1333,1000)	(0,2)	31.60	88.90	100.93	7033	935.86
Subproblem 3	1	(667,1000)	(0,13)	(2000,108)	(1941,0)	154.79	79.73	2.38	7303	973.04
	2	(667,1000)	(0,28)	(2000,119)	(1943,0)	157.65465	86.03492	75.14872	7318	983.64
	3	(667,1000)	(0,22)	(2000,98)	(1939,0)	156.50873	78.58647	90.04563	7317	971.54

Table B.8: Computational results for Design A2 in a large-sized warehouse (W=200, H=100).

	W	H	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_C(^\circ)$	$\alpha_R(^\circ)$	Pallet	$E[C]$
Small	105	54	(350,540)	(525,0)	(700,540)	(525,0)	127.52	180	52.46	1875	478.26
Medium	156	79	(520,790)	(708,0)	(1040,790)	(888,0)	127.52	180	52.46	4317	705.24
Large	206	104	(687,1040)	(959,0)	(1373,1040)	(1138,0)	129.58	180	10.51	7758	929.58

Table B.9: Performance of the expanded Design A2 for different warehouse sizes.

	W	H	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_C(^\circ)$	$\alpha_R(^\circ)$	Pallet	$E[C]$
Small	105	52.5	(525,525)	(0,185)	(525,525)	(1050,185)	90	90	90	1875	513.57
Medium	156	77.5	(780,775)	(0,275)	(780,775)	(1560,275)	90	90	90	4329	756.76
Large	206	102	(1030,1020)	(0,400)	(1030,1020)	(2060,400)	90	90	90	7739	994.52

Table B.10: Performance of the expanded, our modified Flying-V design for different warehouse sizes.

B.3 Computational Results in Design Problem B

B.3.1 Results for Design B1

	Run	S	E	α_L (°)	α_R (°)	# Pallets	$E[C]$
Subproblem 5	1	(500,500)	(130,0)	179.81	41.92	1714	533.88
	2	(500,500)	(158,0)	179.67	44.21	1717	534.87
	3	(500,500)	(201,0)	179.92	33.32	1726	531.87
Subproblem 1	1	(0,250)	(500,500)	128.43	63.69	1683	562.36
	2	(0,263)	(500 ,500)	122.13	65.98	1673	568.88
	3	(0,255)	(500,500)	115.26	61.40	1669	561.92
Subproblem 2	1	(0,250)	(1000,45)	85.46	95.20	1601	567.77
	2	(0,250)	(1000,255)	53.38	90.05	1643	551.80
	3	(0,250)	(1000,262)	46.50	93.48	1610	552.54

Table B.11: Computational results for Design B1 in a small-sized warehouse (W=100, H=50).

	Run	S	E	α_L (°)	α_R (°)	# Pallets	$E[C]$
Subproblem 5	1	(750,750)	(233,0)	179.88	29.31	4072	805.62
	2	(750,750)	(257, 0)	179.92	34.47	4074	803.31
	3	(750,750)	(198,0)	179.75	37.33	4068	804.47
Subproblem 1	1	(0,384)	(750,750)	111.25	59.11	3985	834.47
	2	(0, 375)	(763,750)	117.55	65.41	4001	844.42
	3	(0,375)	(750,750)	119.84	52.23	3986	833.65
Subproblem 2	1	(0,375)	(1500,405)	43.64	94.63	3887	830.98
	2	(0,375)	(1500,379)	40.77	90.15	3988	840.09
	3	(0,375)	(1500,394)	44.78	90.05	3975	833.35

Table B.12: Computational results for Design B1 in a medium-size warehouse (W=150, H=75).

	Run	S	E	α_L (°)	α_R (°)	# Pallets	$E[C]$
Subproblem 5	1	(1000,1000)	(326 ,0)	179.62	33.95	7437	1067.59
	2	(1000,1000)	(282,0)	179.81	39.63	7423	1069.04
	3	(1000,1000)	(506 ,0)	179.86	37.33	7454	1070.73
Subproblem 1	1	(0,504)	(1016,1000)	110.67	49.94	7307	1113.35
	2	(0,501)	(1000,1000)	115.83	62.54	7321	1114.93
	3	(0,503)	(1007,1000)	118.12	57.96	7323	1111.37
Subproblem 2	1	(0,500)	(2000,1055)	49.37	92.91	7184	1092.18
	2	(0,500)	(2000,1126)	47.65	90.05	7302	1093.89
	3	(0,500)	(2000,1072)	45.93	95.20	7204	1094.81

Table B.13: Computational results for Design B1 in a large-sized warehouse (W=200, H=100).

	W	H	S	E	α_L (°)	α_R (°)	# Pallets	$E[C]$
Small	108	50	(540,500)	(241,0)	180	33.32	1881	560.45
Medium	158	75	(790,750)	(297,0)	180	34.47	4314	831.97
Large	208	100	(1040,1000)	(366,0)	180	33.95	7756	1096.22

Table B.14: Performance of the expanded Design B1 for different warehouse sizes.

Results for the Design B2

	Run	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_C(^\circ)$	$\alpha_R(^\circ)$	# Pallets	$E[C]$
Subproblem 2	1	(500,500)	(1000,0)	(0,250)	(1000,0)	71.14	135.02	8.11	1501	492.62
	2	(500,500)	(1000,33)	(0,250)	(975,0)	76.20	134.65	8.02	1501	495.26
	3	(500,500)	(1000,51)	(0,250)	(982,0)	77.92	132.93	10.89	1494	497.29
Subproblem 1	1	(500,500)	(985,0)	(0,250)	(982,0)	107.72	135.79	23.49	1488	496.49
	2	(500,500)	(984,0)	(0,250)	(976,0)	84.22	134.65	20.05	1498	495.33
	3	(500,500)	(1000,0)	(0,250)	(1000,0)	69.99	134.16	6.39	1491	493.47
Subproblem 3	2	(0,250)	(1000,260)	(500,500)	(0,250)	170.74	63.03	50.42	1520	529.93
	1	(0,250)	(1000,251)	(500,500)	(0,250)	164.44	65.89	53.29	1514	528.45
	3	(0,250)	(1000,150)	(500,500)	(0,250)	159.86	60.16	52.14	1508	531.30

Table B.15: Computational results for Design B2 in a small-sized warehouse (W=100, H=50).

	Run	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_C(^\circ)$	$\alpha_R(^\circ)$	# Pallets	$E[C]$
Subproblem 2	1	(750,750)	(1500,33)	(0,375)	(971,0)	110.58	131.21	5.16	3733	753.85
	2	(750,750)	(1500,22)	(0,375)	(982,0)	84.22	134.07	6.88	3731	750.08
	3	(750,750)	(1500,0)	(0,375)	(1000,0)	70.57	135.88	8.11	3717	746.65
Subproblem 1	1	(750 ,750)	(1481,0)	(0,375)	(1478,0)	79.64	138.08	18.33	3725	746.98
	2	(750,750)	(1483,0)	(0,375)	(1480,0)	76.20	134.07	22.35	3716	748.20
	3	(750,750)	(1500,0)	(0,375)	(1000,0)	74.00	135.02	7.54	3752	746.50
Subproblem 3	2	(0,375)	(1500,295)	(750,750)	(0,375)	161.00	63.60	55.00	3763	788.12
	1	(0,375)	(1500,370)	(750,750)	(0,375)	156.42	63.03	48.13	3774	786.92
	3	(0,375)	(1500,345)	(750,750)	(0,375)	166.16	64.74	52.71	3754	789.20

Table B.16: Computational results for Design B2 in a medium-sized warehouse (W=150, H=75).

	Run	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_C(^\circ)$	$\alpha_R(^\circ)$	# Pallets	$E[C]$
Subproblem 2	1	(1000,1000)	(2000,0)	(0,500)	(2000,0)	74.00	135.02	8.11	6957	999.63
	2	(1000,1000)	(2000,19)	(0,500)	(1980,0)	80.79	136.36	10.31	6947	1001.57
	3	(1000,1000)	(2000,17)	(0,500)	(1976,0)	107.72	134.07	11.46	6968	1007.57
Subproblem 1	1	(1000,1000)	(1484,0)	(0,500)	(1481,0)	96.26	136.94	17.19	6956	1002.08
	2	(1000,1000)	(1480,0)	(0,500)	(1477,0)	81.93	133.50	21.20	6951	1002.36
	3	(1000,1000)	(2000,0)	(0,500)	(2000,0)	72.28	134.74	9.26	6945	999.89
Subproblem 3	2	(0,500)	(2000,503)	(1000,1000)	(0,500)	169.02	63.03	59.01	6993	1048.03
	1	(0,500)	(2000,465)	(1000,1000)	(0,500)	165.58	64.17	54.43	6997	1047.58
	3	(0,500)	(2000,497)	(1000,1000)	(0,500)	163.29	65.89	50.99	7002	1047.40

Table B.17: Computational results for Design B2 in a large-sized warehouse (W=200, H=100).

	W	H	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_C(^\circ)$	$\alpha_R(^\circ)$	Pallet	$E[C]$
Small	105	57	(525,570)	(1050,0)	(0,525)	(1050,0)	71.14	135.02	8.11	1868	533.00
Medium	157	82	(785,820)	(1570,0)	(0,410)	(1570,0)	74.00	135.02	7.54	4314	793.00
Large	209	106	(1045,1060)	(2090,0)	(0,530)	(2060,0)	74.00	135.02	8.11	7751	1051.59

Table B.18: Performance of the expanded Design B2 for different warehouse sizes.

	W	H	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_C(^\circ)$	$\alpha_R(^\circ)$	# Pallets	$E[C]$
Small	100	50	(500,500)	(1000,0)	(0,250)	(1000,0)	90	135.02	180	1579	498.12
	103	56	(515,560)	(1030,0)	(0,515)	(1050,0)	90	135.02	180	1877	528.09
Medium	150	75	(750,750)	(1500,0)	(0,375)	(1000,0)	90	135.02	180	3884	752.27
	156	80	(780,800)	(1560,0)	(0,400)	(1560,0)	90	135.02	180	4321	789.22
Large	200	100	(1000,1000)	(2000,0)	(0,500)	(2000,0)	90	135.02	180	7129	1005.11
	208.5	104	(1042.5,1040)	(2085,0)	(0,520)	(2085,0)	90	135.02	180	7758	1048.31

Table B.19: Performance of modified Design B2 for different warehouse sizes.

B.4 Computational Results in Design Problem C

B.4.1 Results for Design C1

	Run	S	E	α_L (°)	α_R (°)	# Pallets	$E[C]$
Subproblem 1	1	(500 ,500)	(1000,0)	93.48	165.10	1646	464.66
	2	(500,500)	(1000,8)	94.63	165.68	1646	465.14
	3	(500,500)	(1000,15)	92.91	163.96	1642	464.94
Subproblem 5	1	(500 ,500)	(983,0)	91.76	135.88	1641	470.00
	2	(500,500)	(997,0)	87.75	5.25	1642	472.32
	3	(500,500)	(980,0)	92.34	6.97	1643	468.69
Subproblem 2	1	(0,256)	(1000,253)	92.34	88.33	1645	494.33
	2	(0,305)	(1000,168)	87.75	91.76	1643	489.07
	3	(0,150)	(1000,265)	88.33	92.91	1634	489.15

Table B.20: Computational results for Design C1 in a small-sized warehouse (W=100, H=50).

	Run	S	E	α_L (°)	α_R (°)	# Pallets	$E[C]$
Subproblem 1	1	(750,750)	(1500,50)	94.06	165.10	3979	701.53
	2	(750,750)	(1500,0)	98.07	164.53	3968	700.45
	3	(750,750)	(1500,3)	96.92	167.39	3965	701.17
Subproblem 5	1	(750 ,750)	(1480,0)	92.91	6.39	3954	707.22
	2	(750,750)	(1488,0)	92.34	8.69	3954	707.83
	3	(750,750)	(1494,0)	93.48	7.54	3963	707.67
Subproblem 2	1	(0,445)	(1500,190)	92.34	87.18	3935	732.83
	2	(0,285)	(1500,605)	86.61	94.06	3903	730.36
	3	(0,225)	(1500,580)	87.75	92.34	3919	731.00

Table B.21: Computational results for Design C1 in a medium-sized warehouse (W=150, H=75).

	Run	S	E	α_L (°)	α_R (°)	# Pallets	$E[C]$
Subproblem 1	1	(1000,1000)	(2000,2)	94.06	165.10	7281	937.24
	2	(1000,1000)	(2000,45)	93.48	163.38	7295	938.80
	3	(1000,1000)	(2000,6)	96.92	165.68	7279	935.77
Subproblem 5	1	(1000,1000)	(1990,0)	93.48	3.53	7293	942.49
	2	(1000,1000)	(1998,0)	94.06	7.54	7278	945.27
	3	(1000,1000)	(1975,0)	92.34	5.82	7272	945.49
Subproblem 2	1	(0,845)	(2000,260)	91.76	87.18	7211	975.32
	2	(0,720)	(2000,195)	93.48	88.33	7187	973.49
	3	(0,330)	(2000,780)	86.61	92.91	7185	974.04

Table B.22: Computational results for Design C1 in a large-sized warehouse (W=200, H=100).

	W	H	S	E	α_L (°)	α_R (°)	# Pallets	$E[C]$
Small	105	53	(525,530)	(1050,0)	93.48	165.10	1865	490.96
Medium	156	78	(780,780)	(1560,3)	96.92	167.39	4328	728.48
Large	205	103	(1025,1030)	(2050,6)	96.92	165.68	7755	961.30

Table B.23: Performance of the expanded Design C1 for different warehouse sizes.

	W	H	S	E	α_L (°)	α_R (°)	# Pallets	$E[C]$
Small	100	50	(500,500)	(1000,0)	90	180	1750	492.34
	101	52	(505,520)	(1010,0)	90	180	1875	488.40
Medium	150	75	(750,750)	(1500,3)	90	180	4120	728.86
	151	77	(755,770)	(1510,3)	90	180	4119	726.08
Large	200	100	(1000,1000)	(2000,6)	90	180	7490	966.01
	201	102	(1005,1020)	(2010,6)	90	180	7758	963.81

Table B.24: Performance of modified Design C1 for different warehouse sizes.

B.4.2 Results for Design C2

	Run	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_C(^\circ)$	$\alpha_R(^\circ)$	# Pallets	$E[C]$
Subproblem 2	1	(500 ,500)	(1000,0)	(500, 0)	(0,500)	160.43	108.86	160.43	1539	423.07
	2	(500,500)	(1000,7)	(500 ,0)	(0,495)	162.15	109.43	163.29	1538	426.41
	3	(500,500)	(1000,483)	(500,0)	(0,485)	163.29	110.58	164.44	1539	425.29
Subproblem 1	1	(500,500)	(995,0)	(500 ,0)	(0,492)	165.58	110.01	159.28	1544	425.99
	2	(500,500)	(1000,0)	(500,0)	(0,500)	162.72	108.29	159.86	1541	423.44
	3	(500,500)	(996,0)	(500,0)	(0,380)	164.44	110.58	162.72	1552	427.36
Subproblem 3	1	(0,300)	(500,500)	(0,280)	(500,0)	21.77	87.66	151.83	1584	456.28
	2	(0,250)	(500,500)	(0,245)	(500,0)	24.06	92.25	155.27	1585	455.66
	3	(0,265)	(500,500)	(0,234)	(500,0)	21.77	86.52	153.55	1580	457.83

Table B.25: Computational results for Design C2 in a small-sized warehouse (W=100, H=50).

	Run	S_1	E_1	S_2	E_2	α_L	α_C	α_R	# Pallets	$E[C]$
Subproblem 2	1	(750 ,750)	(1500,5)	(750,0)	(0,750)	161.57	107.72	160.43	3790	639.13
	2	(750,750)	(1500,0)	(750,0)	(0,750)	160.43	108.86	159.86	3790	640.60
	3	(750,750)	(1500,10)	(750,0)	(0,735)	159.86	112.30	155.27	3788	639.79
Subproblem 1	1	(750 ,750)	(1490,0)	(750,0)	(0,735)	160.43	108.86	161.00	3793	640.96
	2	(750,750)	(1465,0)	(750,0)	(0,750)	161.57	109.43	163.87	3802	639.98
	3	(750,750)	(1485,0)	(750,0)	(0,725)	163.29	110.58	162.15	3802	640.59
Subproblem 3	1	(0,375)	(750,750)	(0,370)	(750,0)	22.35	87.09	157.56	3867	688.89
	2	(0,400)	(750,750)	(0,395)	(750,0)	25.21	85.37	161.00	3875	691.52
	3	(0,360)	(750,750)	(0,360)	(750,0)	23.49	85.94	158.71	3874	690.91

Table B.26: Computational results for Design C2 in a medium-sized warehouse (W=150, H=75).

	Run	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_C(^\circ)$	$\alpha_R(^\circ)$	# Pallets	$E[C]$
Subproblem 2	1	(1000,1000)	(2000,0)	(1000,0)	(0,1000)	158.71	111.73	157.56	7034	855.07
	2	(1000,1000)	(2000,45)	(1000,0)	(0,1000)	156.99	108.29	155.84	7053	858.72
	3	(1000,1000)	(2000,30)	(1000,0)	(0,990)	158.14	110.01	160.43	7052	855.98
Subproblem 1	1	(1000,1000)	(2000,0)	(1000,0)	(0,990)	161.00	110.01	159.86	7039	858.29
	2	(1000,1000)	(1980,0)	(1000,0)	(0,1000)	163.87	112.87	166.16	7043	857.75
	3	(1000,1000)	(1995,0)	(1000,0)	(0, 995)	166.16	107.72	166.73	7047	857.72
Subproblem 3	1	(0,525)	(1000,1000)	(0,515)	(1000,0)	25.78	87.09	156.42	7153	921.89
	2	(0,500)	(1000,1000)	(0,500)	(1000,0)	22.92	87.66	158.14	7144	921.05
	3	(0,490)	(1000,1000)	(0,485)	(1000,0)	24.06	86.52	156.99	7160	922.17

Table B.27: Computational results for Design C2 in a large-sized warehouse (W=200, H=100).

	W	H	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_C(^\circ)$	$\alpha_R(^\circ)$	# Pallets	$E[C]$
Small	107	55	(535,550)	(1070,0)	(535,0)	(0,550)	160.43	108.86	160.43	1876	462.68
Medium	159	79	(795,790)	(1590,5)	(795,0)	(0,790)	160.43	107.72	161.57	4312	676,55
Large	208	105	(1040,1050)	(2080,0)	(1040,0)	(0,1050)	157.56	111.73	158.71	7755	893.42

Table B.28: Performance of the expanded Design C2 for different warehouse sizes.

	W	H	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_C(^\circ)$	$\alpha_R(^\circ)$	# Pallets	$E[C]$
Small	100	50	(500,500)	(1000,0)	(500,0)	(0,500)	180	108.86	180	1616	438.68
	104	54	(525,540)	(1040,0)	(525,0)	(0,540)	180	108.86	180	1875	464.47
Medium	150	75	(750,750)	(1500,5)	(750,0)	(0,750)	180	107.72	180	3928	657.59
	157	78	(785,770)	(1570,5)	(785,0)	(0,770)	180	107.72	180	4319	685.45
Large	200	100	(1000,1000)	(2000,0)	(1000,0)	(0,1000)	180	111.73	180	7202	878.54
	207	103	(1035,1030)	(2060,0)	(1035,0)	(0,1030)	180	111.73	180	7760	906.47

Table B.29: Performance of modified Design C2 for different warehouse sizes.

B.5 Computational Results in Design Problem D

Results for the Design D0				
	Run	$\alpha_R(^\circ)$	# Pallets	$E[C]$
Small	1	32.75	1748	615.77
	2	30.46	1751	616.05
	3	33.90	1747	615.69
Medium	1	32.75	4122	925.2
	2	31.03	4125	925.57
	3	33.90	4119	925.48
Large	1	32.18	7492	1234.68
	2	33.32	7494	1234.95
	3	31.60	7493	1234.97

Table B.30: Computational results of Design D0 for different warehouse sizes.

	W	H	$\alpha_R(^\circ)$	# Pallets	$E[C]$
Small	103	51.5	33.9	1876	634.59
Medium	153	76.5	32.75	4317	943.79
Large	203	101.5	32.18	7758	1253.35

Table B.31: Performance of the expanded Design D0 for different warehouse sizes.

	Run	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_C(^\circ)$	$\alpha_R(^\circ)$	# Pallets	$E[C]$
Subproblem 1	1	(0,500)	(1000,300)	(0,225)	(1000,0)	43.64	45.35	42.34	1449	612.97
	2	(0,500)	(1000,295)	(0,325)	(1000,0)	44.78	43.64	44.06	1479	612.58
	3	(0,500)	(1000,335)	(0,155)	(1000,0)	42.92	35.04	43.06	1450	610.22
Subproblem 2	1	(0,500)	(95,0)	(1000,0)	(920 ,500)	44.69	40.68	39.53	1619	613.6
	2	(0,500)	(140,0)	(1000,0)	(925 ,500)	40.11	38.96	40.68	1619	612.08
	3	(0,500)	(220,0)	(1000,0)	(835,500)	41.83	40.68	42.97	1593	614.57
Subproblem 3	1	(0,150)	(300 ,500)	(1000,400)	(280,0)	33.32	29.31	31.03	1555	618.81
	2	(0,40)	(270,500)	(1000,470)	(350,0)	30.46	27.59	34.47	1561	622.44
	3	(0,95)	(360,500)	(1000,450)	(120,0)	31.60	28.74	30.46	1511	623.18

Table B.32: Computational results for Design D2 in a small-sized warehouse (W=100, H=50).

	Run	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_C(^\circ)$	$\alpha_R(^\circ)$	# Pallets	$E[C]$
Subproblem 1	1	(0,750)	(1500,630)	(0,145)	(1500,0)	37.91	41.92	36.76	3686	920.46
	2	(0,750)	(1500,675)	(0,95)	(1500,0)	36.19	37.91	35.04	3731	917.74
	3	(0,750)	(1500,560)	(0,210)	(1500,0)	34.47	31.60	33.32	3687	916.88
Subproblem 2	1	(0,750)	(110,0)	(1500,0)	(1460,750)	41.25	38.96	44.12	3921	921.75
	2	(0,750)	(60,0)	(1500,0)	(1270,750)	45.84	43.54	44.69	3921	930.5
	3	(0,750)	(180,0)	(1500,0)	(1350,750)	46.41	42.40	46.98	3895	927.39
Subproblem 3	1	(0,130)	(395,750)	(1500,375)	(110,0)	29.31	26.45	28.74	3794	938.42
	2	(0,180)	(450,750)	(1500,405)	(70,0)	25.30	24.16	27.02	3781	946.07
	3	(0,145)	(430,750)	(1500,430)	(95,0)	28.17	25.87	31.60	3773	933.52

Table B.33: Computational results for the Design D2 in a medium-sized warehouse (W=150, H=75).

	Run	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_C(^\circ)$	$\alpha_R(^\circ)$	# Pallets	$E[C]$
Subproblem 1	1	(0,1000)	(2000,690)	(0,270)	(2000,0)	40.06	32.75	39.05	6870	1224.2
	2	(0,1000)	(2000,550)	(0,400)	(2000,0)	37.48	33.90	36.19	6830	1225.1
	3	(0,1000)	(2000,875)	(0,110)	(2000,0)	34.61	32.18	33.32	6965	1224.89
Subproblem 2	1	(0,1000)	(160,0)	(2000,0)	(1810,1000)	47.56	46.41	45.26	7196	1250.22
	2	(0,1000)	(250,0)	(2000,0)	(1900,1000)	43.54	41.83	44.12	7196	1237.66
	3	(0,1000)	(130,0)	(2000,0)	(1905,1000)	44.69	45.84	42.97	7210	1248.49
Subproblem 3	1	(0,100)	(410,1000)	(2000,400)	(80,0)	29.89	28.17	30.46	7030	1241.57
	2	(0,125)	(400,1000)	(2000,360)	(95,0)	26.45	25.30	28.74	7036	1250.38
	3	(0,85)	(425,1000)	(2000,420)	(130,0)	29.31	27.59	31.03	7031	1240.77

Table B.34: Computational results for Design D2 in a large-sized warehouse (W=200, H=100).

	W	H	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_C(^\circ)$	$\alpha_R(^\circ)$	# Pallets	$E[C]$
Small	109	56	(0,560)	(1090,331)	(0,169)	(1090,0)	41.92	35.04	43.06	1870	670.31
Medium	162	80	(0,800)	(1620,703)	(0,281)	(1620,0)	34.47	31.6	33.32	4321	987.73
Large	210	106	(0,1060)	(2100,674)	(0,189)	(2100,0)	43.06	32.75	39.05	7753	1289.04

Table B.35: Performance of the expanded Design D2 for different warehouse sizes.

B.6 Computational Results in Design Problem E

B.6.1 Results for Design E1

	Run	S	E	α_L (°)	α_R (°)	# Pallets	$E[C]$
	1	(0,250)	(1000,250)	94.63	85.46	1623	559.14
Subproblem 2	2	(0,250)	(1000,250)	92.91	86.61	1634	559.36
	3	(0,250)	(1000,250)	94.06	84.32	1629	559.58
	1	(500,0)	(500 ,500)	176.56	2.96	1697	560.58
Subproblem 5	2	(500,0)	(500,500)	171.98	6.39	1694	562.05
	3	(500,0)	(500,500)	5.82	5.25	1689	562.40
	1	(0,250)	(1000,500)	93.48	54.52	1602	598.33
Subproblem 1	2	(0,250)	(935, 500)	92.34	55.67	1609	601.32
	3	(0,250)	(950,500)	94.63	53.38	1602	602.41

Table B.36: Computational results for Design E1 in a small-sized warehouse (W=100, H=50).

B.6.2 Results for Design E2

	Run	S	E	α_L (°)	α_R (°)	# Pallets	$E[C]$
Subproblem 2	1	(0,375)	(1500,375)	94.06	84.89	3917	840.15
	2	(0,375)	(1500,375)	95.78	84.32	3931	840.42
	3	(0,375)	(1500,375)	99.21	82.60	3906	841.94
Subproblem 5	1	(750 ,0)	(750,750)	177.14	2.96	4025	841.25
	2	(750,0)	(750,750)	175.99	1.81	4041	842.22
	3	(750,0)	(750,750)	3.53	1.81	4027	842.71
Subproblem 1	1	(0,375)	(1495,750)	92.34	51.66	3888	897.13
	2	(0,375)	(1495,750)	93.48	55.10	3882	897.33
	3	(0,375)	(1495,750)	94.06	53.95	3886	899.25

Table B.37: Computational results for Design E1 in a medium-sized warehouse (W=150, H=75).

	Run	S	E	α_L (°)	α_R (°)	# Pallets	$E[C]$
Subproblem 2	1	(0,500)	(2000,500)	93.48	86.61	7220	1121.08
	2	(0,500)	(2000,500)	95.20	84.32	7224	1121.94
	3	(0,500)	(2000,500)	96.92	83.74	7208	1122.58
Subproblem 5	1	(1000,0)	(1000,1000)	176.56	3.53	7390	1121.71
	2	(1000,0)	(1000,1000)	177.14	2.38	7369	1121.86
	3	(1000,0)	(1000,1000)	2.96	4.10	7382	1123.60
Subproblem 1	1	(0,500)	(2000,1000)	93.48	54.52	7170	1195.13
	2	(0,500)	(1995,1000)	95.20	52.80	7168	1195.97
	3	(0,500)	(1995,1000)	94.06	49.37	7176	1199.00

Table B.38: Computational results for Design E1 in a large-sized warehouse (W=200, H=100).

	Run	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_C(^\circ)$	$\alpha_R(^\circ)$	# Pallets	$E[C]$
Subproblem 2	1	(500,0)	(50,500)	(450, 0)	(500 ,500)	5.73	137.51	6.30	1554	543.70
	2	(500,0)	(35,500)	(475,0)	(500,500)	6.88	136.36	8.02	1558	545.30
	3	(500,0)	(20,500)	(460, 0)	(500,500)	5.73	140.37	173.03	1548	544.71
Subproblem 3	1	(0,15)	(500, 500)	(1000,470)	(500, 0)	173.70	48.79	173.12	1541	543.80
	2	(0,25)	(500, 500)	(1000,475)	(500, 0)	62.54	48.22	59.68	1548	545.00
	3	(0,250)	(950, 500)	(1000,250)	(75,0)	94.63	42.49	93.48	1459	547.05
Subproblem 1	1	(0,250)	(1000,200)	(0,250)	(1000,325)	85.94	86.52	85.37	1503	565.02
	2	(0,250)	(1000,250)	(0,250)	(1000,385)	86.52	84.80	87.09	1503	556.86
	3	(0,250)	(1000,190)	(0,250)	(1000,405)	87.66	88.81	87.09	1484	566.33

Table B.39: Computational results for Design E2 in a small-sized warehouse (W=100, H=50).

	Run	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_C(^\circ)$	$\alpha_R(^\circ)$	# Pallets	$E[C]$
Subproblem 2	1	(750,0)	(65,750)	(430,0)	(750 ,750)	5.73	138.08	5.73	3825	818.73
	2	(750,0)	(100, 750)	(410, 0)	(750, 750)	174.18	146.10	6.88	3820	823.08
	3	(750,0)	(90,750)	(395, 0)	(750 ,750)	6.30	142.67	7.45	3816	821.07
Subproblem 3	1	(0,55)	(750, 750)	(1500,725)	(750, 0)	173.70	46.50	174.27	3806	820.22
	2	(0,15)	(750, 750)	(1500,740)	(750, 0)	174.84	45.35	172.55	3773	819.09
	3	(0,0)	(750, 750)	(1500,735)	(750, 0)	172.55	45.93	173.12	3790	819.74
Subproblem 1	1	(0,375)	(1500,365)	(0,375)	(1500,500)	86.52	85.94	84.22	3724	839.36
	2	(0,375)	(1500,376)	(0,375)	(1500,450)	85.37	84.80	86.52	3746	906.93
	3	(0,375)	(1500,180)	(0,375)	(1500,350)	85.94	84.80	85.37	3716	841.12

Table B.40: Computational results for Design E2 in a medium-sized warehouse (W=150, H=75).

	Run	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_C(^\circ)$	$\alpha_R(^\circ)$	# Pallets	$E[C]$
Subproblem 2	1	(1000,0)	(110 ,1000)	(900, 0)	(1000,1000)	4.58	136.94	6.30	7079	1089.08
	2	(1000,0)	(195, 1000)	(830 ,0)	(1000,1000)	6.30	138.08	6.88	7111	1091.08
	3	(1000,0)	(245, 1000)	(800 ,0)	(1000,1000)	5.73	140.37	5.16	7121	1090.48
Subproblem 3	1	(0,40)	(1000,1000)	(2000,1000)	(1000,0)	171.98	43.64	173.12	7058	1091.78
	2	(0,20)	(1000,1000)	(2000,990)	(1000,0)	172.55	44.78	171.41	7059	1093.03
	3	(0,500)	(960, 1000)	(2000,500)	(55,0)	94.63	39.05	95.78	6859	1098.88
Subproblem 1	1	(0,500)	(2000,490)	(0,500)	(2000,715)	85.94	85.37	86.52	6930	1112.89
	2	(0,500)	(2000,400)	(0,500)	(2000,660)	87.66	86.52	85.37	6923	1125.6
	3	(0,500)	(2000,515)	(0,500)	(2000,690)	86.52	86.52	84.80	6938	1117.31

Table B.41: Computational results for Design E2 in a large-sized warehouse (W=200, H=100).

	W	H	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_C(^\circ)$	$\alpha_R(^\circ)$	# Pallets	$E[C]$
Small	108	54	(540,0)	(54,540)	(486,0)	(540,540)	5.73	137.51	6.30	1854	589.54
Medium	159	79	(795,0)	(73,790)	(721,0)	(795,790)	5.73	138.08	5.73	4290	866.34
Large	206	106	(1030,0)	(117,1060)	(954,0)	(1060,1030)	4.58	136.94	6.30	7779	1134.35

Table B.42: Performance of the expanded Design E2 for different warehouse sizes.

Appendix C

C.1 One Cross Aisle Designs with P&D Points at the Bottom of the Warehouse

# P&D points	$E[C]$	# Pallets	S	E	$\alpha_L(^\circ)$	$\alpha_R(^\circ)$
1	399.68	1667	(500,0)	(500,500)	135.60	44.40
3	416.39	1667	(500,0)	(520,500)	130.39	45.66
5	428.94	1667	(500,0)	(510,500)	127.49	52.20
7	443.86	1669	(500,0)	(525,500)	126.65	53.37
9	458.89	1674	(500,0)	(550,500)	120.29	60.26
11	475.59	1683	(500,0)	(520,500)	117.00	63.51
13	492.07	1674	(500,0)	(510,500)	116.11	63.89
15	505.24	1623	(400,500)	(1000,0)	110.07	86.60
17	520.14	1618	(350,500)	(1000,0)	113.99	87.48
19	537.80	1619	(300,500)	(1000,0)	116.91	88.06

Table C.1: The heuristic results calculated by the PSO algorithm for small size warehouses with one floating cross aisle and multiple P&D points at the bottom of the warehouse.

# P&D points	Traditional	Chevron:equal size	Chevron:equal capacity
1	500.00	398.81	416.67
3	501.67	416.94	434.82
5	505.00	433.59	451.01
7	510.00	450.95	467.96
9	516.67	468.97	486.05
11	525.00	487.98	504.96
13	535.00	507.66	524.16
15	546.67	527.41	543.86
17	560.00	547.19	563.87
19	575.00	567.01	583.70

Table C.2: The expected travel distance in a small-sized traditional and equally sized and equally capacitated Chevron designs with multiple P&D points at the bottom of the warehouse. The equally capacitated Chevron has a width of 105 and height of 52.

	(W=100, H=50)						(W=105, H=53)	
# P&D points	$E[C]$	# Pallets	S	E	$\alpha_L(^\circ)$	$\alpha_R(^\circ)$	$E[C]$	# Pallets
1	399.68	1667	(500,500)	(500,0)	135.60	44.40	420.33	1880
3	415.33	1670	(500,500)	(500,0)	130.38	49.62	437.11	1876
5	428.38	1672	(500,500)	(500,0)	127.71	53.06	451.86	1877
7	443.75	1669	(500,500)	(500,0)	124.21	55.79	465.61	1876
9	458.49	1680	(500,500)	(500,0)	120.97	60.86	479.96	1886
11	473.78	1684	(500,500)	(500,0)	116.68	63.23	496.53	1872
13	491.13	1674	(500,500)	(500,0)	116.72	63.28	513.09	1872
15	509.46	1682	(500,500)	(500,0)	111.81	68.19	531.78	1900
17	528.77	1676	(500,500)	(500,0)	111.73	68.27	550.06	1895
19	547.44	1680	(500,500)	(500,0)	111.89	68.32	569.53	1890

Table C.3: The heuristic results calculated by the PSO algorithm for small size warehouses with a fixed inserted cross aisle and multiple P&D points at the bottom of the warehouse.

C.2 Two Cross Aisle Designs with P&D Points at the Bottom of the Warehouse

# P&D points	$E[C]$	# Pallets	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_R(^\circ)$	$\alpha_C(^\circ)$
3	407.72	1587	(450,0)	(105,500)	(550,0)	(895,500)	151.32	28.68	90
5	420.61	1571	(450,0)	(60,500)	(550,0)	(940,500)	148.78	32.69	90
7	434.33	1578	(423,0)	(17,500)	(577,0)	(983,500)	134.03	47.11	90
9	452.92	1596	(400,0)	(14,500)	(600,0)	(986,500)	124.36	56.25	90
11	467.18	1590	(388,0)	(17,500)	(612,0)	(983,500)	125.08	56.81	90
13	483.55	1600	(375,0)	(37,500)	(625,0)	(963,500)	120.69	58.18	90
15	500.73	1550	(500,0)	(0,475)	(500,0)	(1000,475)	92.86	87.81	90
17	516.50	1547	(500,0)	(0,470)	(500,0)	(1000,470)	92.54	88.01	90
19	532.86	1548	(500,0)	(0,460)	(500,0)	(1000,460)	93.03	88.13	90

Table C.4: The heuristic results calculated by the PSO algorithm for small size warehouses with two inserted cross aisles and multiple P&D points at the bottom of the warehouse.

	(W=100, H=50)	(W=108, H=54)
# P&D points	$E[C]$	$E[C]$
1	392.46	421.66
3	416.79	444.68
5	436.62	465.02
7	456.86	484.69
9	476.61	504.86
11	496.88	524.90
13	516.68	544.93
15	536.39	564.92
17	555.66	584.31
19	574.13	603.65

Table C.5: The expected travel distances in Leaf designs. They provide 1575 storage locations in a small size, and 1875 storage locations in an expanded small size warehouse.

# P&D points	$E[C]$	# Pallets	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_R(^\circ)$	$\alpha_C(^\circ)$
3	437.17	1885	(490,0)	(105,540)	(590,0)	(975,540)	151.32	28.68	90
5	455.18	1869	(490,0)	(60,540)	(590,0)	(1020,540)	148.78	32.69	90
7	474.44	1887	(463,0)	(17,540)	(617,0)	(1063,540)	134.03	47.11	90
9	489.57	1879	(440,0)	(14,540)	(640,0)	(1066,540)	124.36	56.25	90
11	505.98	1883	(428,0)	(17,540)	(652,0)	(1063,540)	125.08	56.81	90
13	524.44	1876	(425,0)	(37,540)	(655,0)	(1043,540)	120.69	58.18	90
15	539.18	1879	(540,0)	(0,475)	(540,0)	(1040,475)	92.86	87.81	90
17	554.37	1883	(540,0)	(0,470)	(540,0)	(1040,470)	92.54	88.01	90
19	570.75	1890	(540,0)	(0,460)	(540,0)	(1040,460)	93.03	88.13	90

Table C.6: The expected travel distance in the two cross aisle proposed designs for equally capacitated warehouses with multiple P&D points at the bottom of the warehouse (W=108, H=54).

C.3 One Cross Aisle Designs with P&D Points at the Bottom and the Top of the Warehouse

	(W=100, H=50)						(W=105, H=53)	
# P&D points (per side)	$E[C]$	# Pallets	S	E	$\alpha_L(^\circ)$	$\alpha_R(^\circ)$	$E[C]$	# Pallets
1	464.66	1646	(500,500)	(1000,0)	93.47	165.10	490.96	1865
3	469.41	1647	(550,500)	(1000,0)	93.51	168.80	495.76	1871
5	475.53	1658	(600,500)	(1000,0)	93.72	169.43	501.24	1878
7	483.88	1657	(600,500)	(1000,0)	92.79	170.05	509.28	1876
9	493.90	1656	(600,500)	(1000,0)	93.94	170.62	519.68	1873
11	502.39	1669	(500,500)	(1000,200)	96.69	94.28	528.40	1871
13	513.71	1674	(500,500)	(1000,200)	95.81	94.38	538.78	1879
15	526.87	1661	(550,500)	(1000,150)	95.37	94.78	551.67	1878
17	540.08	1656	(600,500)	(1000,150)	95.95	98.60	566.67	1887
19	553.48	1647	(650,500)	(1000,150)	95.43	98.03	582.37	1896

Table C.7: The heuristic results calculated by the PSO algorithm for small size warehouses with two cross aisles and multiple P&D points located both at the bottom and the top sides of the warehouse.

	(W=100, H=50)	(W=105, H=53)
# P&D points (per side)	$E[C]$	$E[C]$
1	464.66	490.96
3	474.07	500.28
5	481.22	509.08
7	490.45	518.73
9	501.09	528.65
11	512.80	538.70
13	524.62	550.22
15	537.69	562.88
17	551.71	576.34
19	566.46	590.42

Table C.8: The expected travel distances in the Design C1 for varying number of P&D points at the bottom and the top sides of the warehouse.

C.4 Two Cross Aisle Designs with P&D Points at the Bottom and the Top of the Warehouse

	(W=100, H=50)									(W=108, H=54)	
# P&D points (per side)	$E[C]$	# Pallets	S_1	E_1	S_2	E_2	$\alpha_L(^\circ)$	$\alpha_R(^\circ)$	$\alpha_C(^\circ)$	$E[C]$	# Pallets
1	423.07	1539	(0,0)	(500,500)	(1000,500)	(500,0)	160.43	160.43	108.86	462.68	1876
3	435.23	1564	(0,50)	(450,500)	(1000,450)	(550,0)	159.17	159.17	109.79	471.35	1878
5	448.99	1572	(0,50)	(400,500)	(1000,450)	(600,0)	159.71	159.71	114.32	485.38	1877
7	459.99	1572	(0,50)	(400,500)	(1000,450)	(600,0)	159.71	159.71	105.49	494.24	1879
9	473.62	1564	(0,25)	(400,500)	(1000,475)	(600,0)	160.79	160.79	106.53	509.24	1874
11	484.15	1569	(0,125)	(500,500)	(1000,475)	(700,0)	114.00	153.82	105.47	520.39	1876
13	498.54	1567	(0,45)	(300,500)	(1000,400)	(440,0)	103.35	162.96	109.17	531.80	1877
15	513.10	1546	(0,35)	(740,500)	(1000,470)	(710,0)	161.39	105.77	107.36	544.97	1867
17	523.99	1537	(0,55)	(715,500)	(1000,470)	(540,0)	134.63	101.42	111.59	557.05	1865
19	540.12	1534	(0,25)	(250,500)	(1000,410)	(155,0)	99.78	152.66	107.46	573.77	1865

Table C.9: The heuristic results calculated by the PSO algorithm for small size warehouses with two inserted cross aisles and multiple P&D points at the bottom and the top sides of the warehouse.

# P&D points (per side)	(W=100, H=50)	(W=107, H=55)
1	423.07	462.68
3	441.49	480.97
5	458.12	497.65
7	472.09	512.89
9	485.05	525.17
11	499.38	539.16
13	513.40	553.52
15	527.02	566.84
17	540.81	580.56
19	554.11	594.03

Table C.10: The expected travel distances in the Design C2 for varying number of P&D points at the bottom and the top sides of the warehouse. They provide 1539 storage locations in a small size, and 1876 storage locations in an expanded small size warehouse.