Ridge Regression Based Development of Acceleration Factors and Closed Form Life Prediction Models for Lead-free Packaging

by

Dinesh Kumar Arunachalam

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Approved by

Pradeep Lall, Chair, Thomas Walter Professor of Mechanical Engineering
Jeffrey C Suhling, Quina Distinguished Professor of Mechanical Engineering
Peng Zeng, Associate Professor of Mathematics and Statistics

Abstract

The thermo-mechanical mismatch caused by the difference in the coefficient of thermal expansion between the electronic part and the printed circuit board results in shear strains in the solder interconnects during thermal excursions. Widely used life prediction models include the Manson [1966] and Coffin Model [1954, 1963] which correlates plastic strain amplitude, Goldmann Model [1969] which correlates the geometry and material parameters with cyclic life, $\Delta\varepsilon p$, with fatigue life, Norris-Landzberg`s Model [1969] which correlates the thermo-mechanical material and geometry parameters with cyclic life. Norris-Landzberg acceleration factors for lead-free solders have been developed based on ridge regression models (RR) and on PCR for reliability prediction and part selection of area-array packaging architectures under thermo-mechanical loads. The principal component transformation has been used to rank the new orthogonal principal components in the order of their importance. Scree plots, Eigen values and proportion of total variance explained by each principal component are then used to eliminate the least important principal components. Multiple linear regressions have been performed with the original response variable and reduced set of principal components. Ridge regression adds a small positive bias to the diagonal of the covariance matrix to prevent high sensitivity to variables that are correlated. The proposed procedure proves to be a better tool for prediction than multiple-linear regression models. Models have been developed in conjunction with Stepwise Regression

Methods for identification of the main effects. Package architectures studied include, BGA packages mounted on copper-core and no-core printed circuit assemblies in harsh environments. The models have been developed based on thermo-mechanical reliability data acquired on copper-core and no-core assemblies in four different thermal cycling conditions. Packages with Sn3Ag0.5Cu solder alloy interconnects have been examined. The models have been developed based on perturbation of accelerated test thermo-mechanical failure data. Data has been gathered on nine different thermal cycle conditions with SAC305 alloys. The thermal cycle conditions differ in temperature range, dwell times, maximum temperature and minimum temperature to enable development of constants needed for the life prediction and assessment of acceleration factors. Norris-Landzberg acceleration factors have been benchmarked against previously published values. In addition, model predictions have been validated against validation data-sets which have not been used for model development. Convergence of statistical models with experimental data has been demonstrated using a single factor design of experiment study for individual factors including temperature cycle magnitude, relative coefficient of thermal expansion, and diagonal length of the chip. Life prediction models have been developed over the years trying to assess the influence of the different geometrical, material and thermo-cycling parameters on the life of an electronic package. In this study the influence of silver content on packages based on SAC alloys have been investigated. Along with silver content, the solder ball configuration parameters such as ball pitch, ball count, ball height, ball diameter and cycle conditions such as dwell time and delta T have been considered .An assortment of packages such as CBGAs, PBGAs, flips chips based on a variety of SAC alloys with a set of different silver contents were considered for the analysis.

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I would like to thank my parents Mr. Arunachalam M and Mrs. Shanthi A and my sister Divya Senthil for having faith in me and providing endearing love, encouragement and moral support. I would also like to thank all my friends and colleagues for their priceless friendship and camaraderie all throughout my graduate studies.

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Chapter 1

Introduction

1.1 Overview

The fast pace developments and improvements in the field of electronics demands a faster method of determination of reliability and improvements in electronic packaging. Institutional learning is a proven method of understanding the behavioral pattern of electronics and hence knowing beforehand the mode of failure and the variables and parameters responsible in contributing to failure. Understanding the behavioral pattern helps one to be ready for what is going to happen when an electronic component is deployed in a given operating condition. Improving reliability could be based on changing or improving the parameter that is most contributing to the failure, given that parameter can be learnt by understanding the behavior of the package to a given condition with given geometrical and material configuration. Knowing the acceleration factor for a thermo-mechanical process is critical in understanding the used and available thermo-mechanical life of a package. Several Models to predict the Acceleration factors of an electronic package have been proposed, proven and used over the years.

1.2 Life Prediction

The mismatch between the coefficient of thermal expansion between the chip and the module due to the thermal cycling which the chip undergoes, results in shear strains in the solder joint. Thus the mechanical strain along with the time and temperature factors has to be taken into consideration while evaluating the fatigue behavior of solder interconnections under accelerated conditions.

Manson and Coffin [1965, 1954] developed an equation that related plastic strain $\Delta\varepsilon_p$, with number of cycles to failure N. Following Goldmann`s analysis and assuming the interconnection to be a spherical segment, the plastic strain amplitude at any cross section is given by the Norris-Landzberg`s Model [1969] for controlled collapse interconnections.

Engelmaier [1990] developed a surface mount solder joint reliability prediction model containing all the parameters influencing the shear fatigue life of a solder joint due to shear displacement caused by thermal expansion mismatch between component and substrate. Engelmaier developed separate equation for stiff solder joints and compliant solder joints. The parameters of the model include effective solder joint area, solder joint height, diagonal flexural stiffness, distance from neutral point and thermal coefficient mismatch thermal cycling conditions, degree of completeness of stress relaxation and slope of weibull distribution.

Knecht and Fox [1991] developed a strain based model using creep shear strain as damage metric to determine the number of cycles to failure. The creep shear strain included creep of component due to matrix creep alone ignoring the plastic work. The equation was applicable to both 60Sn40Pb and 63Sn37Pb solder joints.

Vandevelde [1998] developed thermo-mechanical models for evaluating the solder joint forces and stresses. Barker [2002] synthesized the Vandevelde models for calculating the solder joint shear forces in ceramic and plastic ball grid array packages.

Clech [1996] developed a solder reliability solutions model for leadless and leaded eutectic solder assemblies and extended it to area array and CSP packages. Clech obtained the inelastic strain energy density from area of solder joint hysteresis loop and developed a prediction equation correlating inelastic strain energy density with number of cycles to failure.

Singh [2006] developed failure mechanics based models for solder joint life prediction of ball array and flip chip packages. He calculated the maximum shear strain a using a simplified DNP formula which was then used for initiating a hysteresis loop iteration for both global and local thermal mismatch. Inelastic strain energy was then calculated from the area of the hysteresis loop for both the thermal mismatch cases. The number of cycles for failure was determined using Lall [2003] model.

Previously Hariharan [2006] demonstrated the power law dependencies of various design parameters for flip-chip, CBGA and CCGA packages using Box-Tidwell transformation. He compared the values obtained with those in the table.

Table 1: NLZ Constants Comparison

	Model	
Variable	Norris-Landzberg	Goldmann
Delta T	-2	-2
Ball Height	2.7	2
Solder Volume	-0.152	-0.175
Solder ball radius	4	5.44

1.3 Regression Analysis

Regression is an effective life prediction tool that predicts the life of packages based on historical behavior of the packages. Multiple linear regression attempts to model the relationship between two or more explanatory variables and a response variable by fitting a linear equation to observed data. Every value of the independent variable x is associated with a value of the dependent variable y. The population regression line for p explanatory variables $x_1, x_2, \ldots, x_p$ is defined to be $\mu_y = \beta_0 + \beta_1 x1 + \beta_2 x_2 + \ldots + \beta_p x_p$. This line describes how the mean response μ_y changes with the explanatory variables. The observed values for y vary about their means μ_y and are assumed to have the same standard deviation σ. The fitted values $b_0, b_1 \ldots b_p$ estimate the parameters $\beta_0, \beta_1 \ldots \beta_p$ of the population regression line.

Since the observed values for y vary about their means μ_y, the multiple regression model includes a term for this variation. In words, the model is expressed as DATA = FIT + RESIDUAL, where the "FIT" term represents the expression $\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots \beta_p x_p$. The "RESIDUAL" term represents the deviations of the observed values y from their means μ_y, which is normally distributed with mean 0 and variance σ. The notation for the model deviations is ε. In common terms the matrix 'Y' is called the response matrix and the matrix 'X' is called the predictor matrix.

The most predominant method of solving a regression problem uses the co-relation matrix X'X matrix and is given by the formula $\hat{\beta} = (X'X)^{-1} X'Y$. This method is good if and only if the determinant of the X'X matrix is nearly one. However the case may not always be true as the determinant of the X'X matrix tends to move toward zero if the columns of the X matrix are related to each other. In a lot of engineering applications, some of the columns are derived functions of the other. In other words, the factors that significantly contribute to the response can be derived functions of each other. In such cases the regular solution to multiple linear regression, $\hat{\beta} = (X'X)^{-1}X'Y$ would fail. There have been a lot of techniques developed over the years to circumvent the resulting numerical snag. Since the determinant is approaching zero, the method fails the co-efficient tends to infinity losing the actual meaning and failing to explain the actual significance of the variable.

The predominant methods are using Principal component Analysis and Ridge Regression. The principal component method transforms the predictors into their principal components and hence reduces the dimensions of the predictors and hence nulls the effect of the co-relation. The method is effective for curve fitting and for low dimension data and is also useful for making life prediction models.

The other most commonly used technique is called Ridge Regression. The method tries to circumvent the numerical snag by adding a small positive bias to the diagonal of the matrix and hence avoiding the determinant to tend toward zero. The method visibly reduces the mean square errors and is a lot flexible. The method is more of a prediction tool than a curve fitting tool as we manually handle the flow of the process of regression.

Both methods have been used to re- create the Norris Landzberg and Goldmann Models for SAC alloys exposed to thermo-mechanical loading. The methods have been validated against a validation data set to assess the prediction accuracy of the models and their dependability.

Additionally the dependency of Life on the Silver content of the SAC alloy has been investigated taking to account the other variables that are critical in deciding the life of the electronic package. The other variables looked at include Ball Parameter, Geometric parameters and the test conditions like Dwell time and Delta T. The results have also been validated and checked for dependability.

1.4 Scope of Data

Accelerated test data is accumulated from the tests that conducted at the CAVE[3] and from publications. The accumulated data is used to run the regression analysis. The data accumulated is based on various package types and various material and geometric configurations.

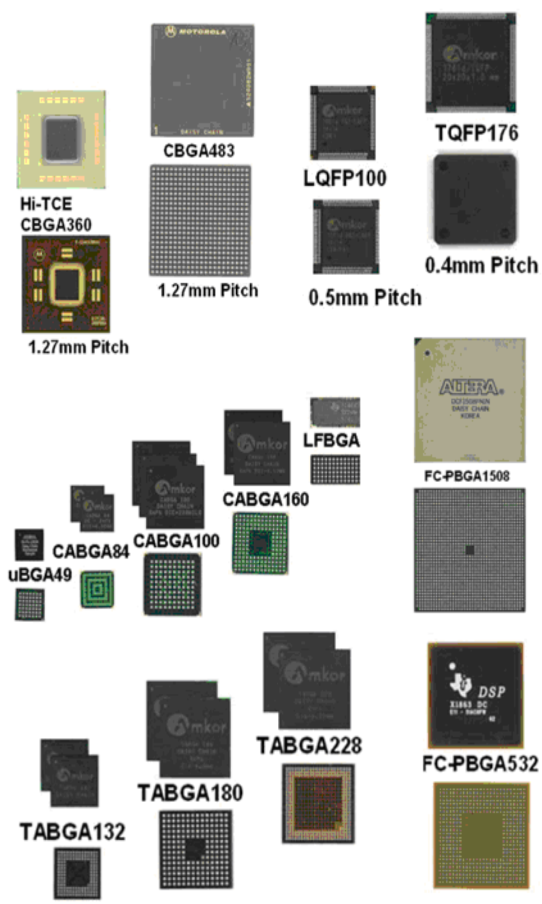


Figure 1 Package Configurations

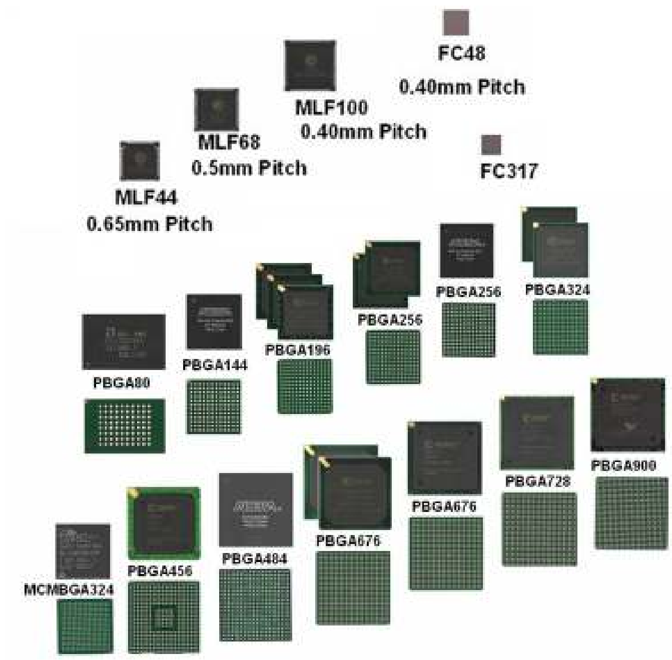


Figure 2 More representation of Scope of packages

There is a wide scope of architecture made available through test conducted previously at the CAVE3 and through publications; a small representation of the available material is shown below.

Table 2: Scope of Packaging Architectures

<table>
<tr><th>Package type</th><th>Array type</th><th>I/O Pitch (mm)</th><th>I/O Count Range</th><th>Solder alloy</th><th>Package size (mm)</th><th>Die Size Range</th><th>Package to Die size ratio</th></tr>
<tr><td rowspan="3">PBGA</td><td>Full</td><td rowspan="3">0.5 - 1.00</td><td rowspan="3">49 – 900</td><td rowspan="3">Pb-free SAC 305</td><td rowspan="3">7.0 - 31.0</td><td rowspan="27">4.00 - 24.0</td><td rowspan="27">1.00 - 3.94</td></tr>
<tr><td>Perimeter</td></tr>
<tr><td>Mixed</td></tr>
<tr><td rowspan="3">FC-PBGA</td><td>Full</td><td rowspan="3">0.8- 1.00</td><td rowspan="3">532 – 1508</td><td rowspan="3">Pb-free SAC 305</td><td rowspan="3">23.0 - 40.0</td></tr>
<tr><td>Perimeter</td></tr>
<tr><td></td></tr>
<tr><td rowspan="3">MCM-PBGA</td><td>Full</td><td rowspan="3">1.00</td><td rowspan="3">128 – 324</td><td rowspan="3"></td><td rowspan="3">22.0</td></tr>
<tr><td>Perimeter</td></tr>
<tr><td></td></tr>
<tr><td rowspan="3">Hi-TCE CBA</td><td>Full</td><td rowspan="3">1.27</td><td rowspan="3">360</td><td rowspan="3">Pb-free SAC 305</td><td rowspan="3">25.0</td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
<tr><td rowspan="3">CBGA</td><td>Full</td><td rowspan="3">1.27</td><td rowspan="3">483</td><td rowspan="3"></td><td rowspan="3">29.0</td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
<tr><td rowspan="3">CSP</td><td>Full</td><td rowspan="3">0.5 - 0.8</td><td rowspan="3">132 – 228</td><td rowspan="3">Pb-free SAC 305</td><td rowspan="3">7.0 - 12.0</td></tr>
<tr><td>Perimeter</td></tr>
<tr><td></td></tr>
<tr><td rowspan="3">Flip Chip</td><td>Full</td><td rowspan="3">0.25 - 0.45</td><td rowspan="3">48 – 317</td><td rowspan="3">Pb-free SAC 306</td><td rowspan="3">5.08 - 6.35</td></tr>
<tr><td>Perimeter</td></tr>
<tr><td></td></tr>
<tr><td rowspan="3">Micro - Lead Frame</td><td></td><td rowspan="3">0.40 - 0.65</td><td rowspan="3">44 – 100</td><td rowspan="3">Pb-free SAC 307</td><td rowspan="3">9.0 - 12.0</td></tr>
<tr><td>Perimeter</td></tr>
<tr><td></td></tr>
<tr><td rowspan="3">QFP / LQFP</td><td></td><td rowspan="3">0.4 - 0.5</td><td rowspan="3">100 – 176</td><td rowspan="3">Pb-free SAC 308</td><td rowspan="3">14.0 - 20.0</td></tr>
<tr><td>Perimeter</td></tr>
<tr><td></td></tr>
</table>

The center has been publishing prediction models (Lall 04, 07, 08, 09) previously based on regression techniques. Hence the behavioral properties of certain variables are known for certain given conditions are known and hence it gives an approximate idea of what to expect from the regression analysis in terms of positive or negative dependence. The models were checked to see if they comply with the underlying physics of the package.

Chapter 2

Literature Review

The rapid minimization of electronics and the fast paced change of technology demand a faster method of determining the reliability of packages. Accelerated tests, reliability models and life predictions are useful tools and hence making these tools more accurate and reliable is critical. A reliability assessment numerical model that could take into account the geometric and material properties of the widely operating conditions could be of great help in obtaining the failure modes such as die cracking, solder joint fatigue failure, de-lamination and total failure. Solder joint fatigue failure being a dominant failure mode contributing 90% of all structural and electrical failures [Tummala 1997] demands greater focus for improving the mechanical reliability of the package. In this section, traditional approaches for solder joint reliability prediction, including physics of failure based models, statistical models, finite element models and experimental techniques have been discussed. Elaboration of the two methods that are mainly used, PCR and Ridge regression is also done.

2.1 Statistical Prediction Models

Statistical prediction models developed include cumulative failure distribution functions for expressing the experimental failure data as a probability function of time to failure for any failure distribution. Weibull distribution and Log normal distribution have been most widely used failure distribution functions. Log normal distributions [Muncy

2004] have widely been used for modeling failure due to slow degradation such as chemical reactions and other corrosions and weibull distributions have been used for modeling failures due to weak link propagations such as solder joint failure.

Regression analysis and analysis of variance have been widely used by researchers for correlating the reliability of a package with its geometic attributes, material properties and operating conditions. Muncy [2004] conducted air to air thermal cycling and liquid to liquid to liquid thermal shock tests on a flip chip package for 1200 test boards with four different die sizes, eight board configurations, two underfill materials and two substrate metallization. The predictor variables considered for model building include substrate metallization, substrate mask opening area versus the UBM area of the flip chip bump, die size, perimeter or full area array flip chip interconnect pattern, underfill material properties, location of the die on the test board, frequency of cycling, number of interconnects, and percent area voiding. Multiple linear regression modeling and regression with life data modeling methodologies were used for obtaining the parameters of regression.

The variables of interest on a regression analysis for an electronic package are often closely related to each despite the fact that they significantly contribute to the life of the package by themselves. Multi-co linearity implies near linear dependence among predictors. Multi- co linearity can seriously disturb the least squares fit and in some cases render the regression model almost useless. In some cases, the regression co-efficients can have the wrong sign or many of the predictors are not statistically significant when the overall F-test is highly significant. We make such intuitions from physical significances and prior learning. Thus, when a model includes more than one predictor, it is important to asses whether strong co-relations exist among the predictors. Several techniques have been proposed for detecting multi-co linearity. When the method of least squares is applied to collinear data, very poor estimates of regression coefficients can be obtained. The variance of the least square estimates of regression co-efficient may considerably inflate in the presence of near linear dependencies among predictors. This implies that the least squares estimate of regression co-efficient are very stable, that is, their magnitude and signs may change considerably given a different sample.

The problem with the least squares estimation method is the requirement that $\hat{\beta}$ be an unbiased estimate of β. Though ordinary least squares gives unbiased estimates and indeed enjoy minimum variance of all linear unbiased estimates, there is no upper limit bound on the variance of the estimates and the presence of multi co linearity may produce large variances. As a result, one can visualize that, under the condition of multi co linearity, a huge price is paid for un-biasedness property that one achieves by using ordinary least squares. One way to alleviate this problem is to drop the requirement that the estimate β be unbiased. Biased estimation is used to attain a substantial reduction in variance with accomplished increase in stability of the regression co-efficients become biased and , simply put, if one is successful, the reduction in variance is of greater magnitude than he bias induced in the estimates.

2.1.1 Ridge regression

Consider the standard model for multiple linear regression, $Y=X\beta+\varepsilon$, where X is the matrix of predictors and Y is the matrix of the response. β is the regression co-efficient matrix and is at this point unknown. The usual procedure of determining the values is called the Gauss-Markov linear functions.

Let B be the estimate of any vector β. The residual sum of squares can be written as,

$$F(\beta) = \phi = (Y - X\beta)'(Y - X\beta)$$
$$= Y'Y + \beta' X' X\beta - 2\beta' X' Y$$

The estimate has been expressed as the difference between the observed and fitted values. In an ideal case we want the value to be minimum; hence to find the minimum, we differentiate the above equation and equate it to zero.

$$\frac{\partial F(\beta)}{\partial \beta} = X' X\beta - X' Y = 0$$
$$X' X\beta = X' Y$$
$$\beta = (X' X)^{-1} X' Y$$

This method is good if and only if the determinant of the X'X matrix is nearly one. However the case may not always be true as the determinant of the X'X matrix tends to move toward zero if the columns of the X matrix are related to each other. In a lot of engineering applications, some of the columns are derived functions of the other. In other words, the factors that significantly contribute to the response can be derived functions of each other. In such cases the regular solution to multiple linear regression, $\hat{\beta} = (X' X)^{-1} X' Y$ would fail. There have been a lot of techniques developed over the years to circumvent the resulting numerical snag. Since the determinant is approaching zero, the method fails the co-efficient tends to infinity losing the actual meaning and failing to explain the actual significance of the variable.

The predominant methods are using Principal component Analysis and Ridge Regression. The principal component method transforms the predictors into their principal components and hence reduces the dimensions of the predictors and hence nulls the effect of the co-relation. The method is effective for curve fitting and for low dimension data. It loses its accountability when the size of the dataset is large. It is more a curve fitting tool than a prediction tool. Ridge regression on the other side uses a small positive bias to keep the X'X from tending to infinity. The method is effective no matter how big of a dataset we have.

For data where X matrix is not specified to be near co linearity, the dispersion is expressed as,

$$D(\beta) = \sigma^2 (X'X)^{-1}$$

The trace of the dispersion matrix is the total variance, thus,

$$TrD(\beta) = \sigma^2 \sum_{i=1}^{s} \frac{1}{\lambda_i} \tag{1}$$

Where the λ_i are the non-zero eigen values of X'X,

It is clear that if one or more of the Eigen values are low, the variance inflation is going to be high.

The ridge estimator as suggested by Hoerl and Kennard as a possible remedy for the inflation is obtained by adding the scalar matrix kI to X'X in the least square estimator. Thus the regression equation takes the form

$$\hat{\beta} = (X'X + kI)^{-1} X'Y$$

Upon calculating the dispersion and variance for the above equation as we did before, we get,

$$TV(\hat{\beta}) = \sigma^2 \sum_{i=1}^{s} \frac{\lambda_i}{(\lambda_i + k)^2}$$

It is clear that variance inflation will be lesser in the eqn (2) in the event if low Eigen values.

The value of the ridge estimator 'k' was originally obtained by finding the point on the ellipsoid centered at the LS estimator β. The hyper ellipsoid is formed by the residual sum of squares. The objective as we understand is to reduce the residual sum of squares when there is inflation in the actual values. Let B be any estimate of the actual vector β. The residual sum of squares in that case will be.

$$\begin{aligned} F(B) &= \phi = (Y - XB)'(Y - XB) \\ &= (Y - X\hat{\beta})'(Y - X\hat{\beta}) + (B - \hat{\beta})' X' X (B - \hat{\beta}) \\ &= \phi_{\min} + \phi(B) \end{aligned}$$

As we understand, β has inflated to B and hence φ(B) is the residual that has added because of the inflation. Hence we try to reduce that term.

$$\phi(B) = F = (B - \hat{\beta})' X' X (B - \hat{\beta})$$

The ridge trace can be shown to be following a path through sum of the squares so that for a fixed φ a single value of B is chosen and that is the one with the minimum length.

Minimize B'B

Subject to $(B - \hat{\beta})' X' X (B - \hat{\beta}) = \phi_0$

Solving it using a lagrangian multiplier 'k', Minimize, $F = B'B + (\frac{1}{k})[(B - \hat{\beta})' X' X (B - \hat{\beta})' - \phi_0]$

Where (1/k) is the lagrangian multiplier,

$$\frac{\delta F}{\delta B} = 2B + (\frac{1}{k})[2(X'X)B - 2(X'X)\hat{\beta}] = 0$$

Hence it reduces to

B= [X'X+kI]$^{-1}$X'Y [A.E Hoerl, 1970]

The value of 'k' is chosen such that k > 0 and then φ_o is computed. In terms of $\boldsymbol{\beta^*}$**,** the residual sum of squares becomes,

$$\phi^*(k) = (Y - X\hat{\beta}^*)'(Y - X\hat{\beta}^*) = \phi_{min} + k^2\hat{\beta}^{*\prime}(X'X)^{-1}\hat{\beta}^*$$

If the squared length of the regression vector B is fixed at R^2, then $\hat{\beta}^*$ is the value of B that gives a minimum sum of squares. Hence $\hat{\beta}^*$ is the value of B that minimizes the function,

$$F = (Y-XB)'(Y-XB) + (1/k)(B'B - R^2)$$

The equation B= [X'X+kI]$^{-1}$X'Y is used to perform Ridge Regression, and as seen it is just a modification of the original Multiple Linear Regression equation, $\hat{\beta} = (X'X)^{-1} X'Y$, except that we introduce a small positive bias term, 'k' which is added to the diagonal of the variance-covariance portion of the Regression equation .In essence, the original data is retained as we observe that there is no change to the second part of the equation , rather the variance which is a derived property of the actual data undergoes a small bias addition. The entire flow of the Ridge Regression process can be explained by the flow chart shown in fig(3).

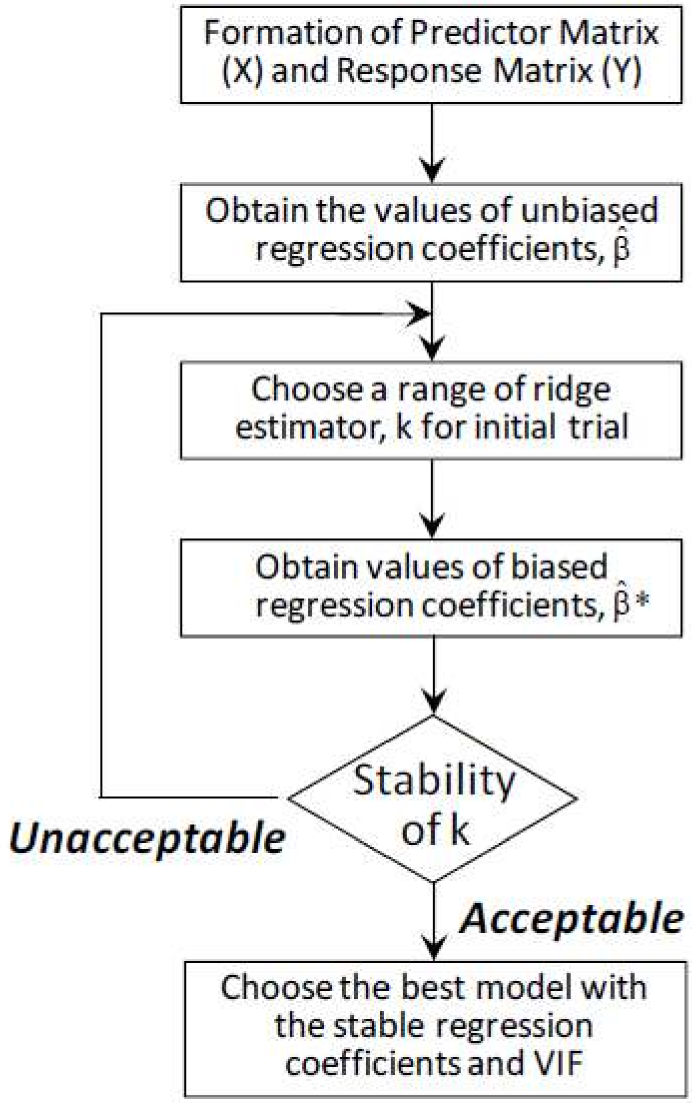


Figure 3 Ridge Process

The process is started like how a regular regression process is started by forming a set of Predictor(X) and Response Variables (Y).An initial range of the Bias Parameters ‘k’ is chosen . The equation (5) is solved with the range of bias and in each attempt the stability is looked for. The stability of the bias can be determined by a few methods mentioned later. Care has to be taken to make sure that the model is neither over-biased nor under-biased. A perfect bias will be at a point where both the Regression co-efficients and the VIF values remain stable or show minimal change upon further biasing. The chosen k is taken and the results of the equation (5) for the chosen k value will be the results of the Ridge Regression process.

The overall adequacy of the model is tested using ANOVA table. Small P value of the ANOVA table rejects the null hypothesis proving the overall adequacy of the model. Individual T tests on the coefficients of regression of variables should yield very small P values indicating the statistical significance of all the predictor variables.

The individual T test values of variables are then used for conducting individual T test on the coefficients of regression of original variables. The test statistic follows a students' T distribution with (n-k-1) degrees of freedom. The P values of individual T tests given by the 'p' values table are < 0.05 proving the statistical significance of individual regression coefficients of original predictor variables at a 95 % confidence.

2.1.2 Principal Component Regression

Principal components models have been used for dealing with multi-collinearity and producing stable and meaningful estimates for regression coefficients [Fritts et al 1971]. Methodology for developing a Principal Component Regression Model is presented here:

Matrix Notation for the model:

$$\{y\} = [X]\{\beta\} + \{\varepsilon\}$$

Where,

$$\{y\} = \begin{Bmatrix} y_1 \\ y_2 \\ . \\ . \\ . \\ y_n \end{Bmatrix}, \quad X = \left.\overbrace{\begin{bmatrix} 1 & x_{11} & x_{21} & . & . & . & x_{k1} \\ 1 & x_{12} & x_{22} & . & . & . & x_{k2} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 1 & x_{1n} & x_{2n} & . & . & . & x_{kn} \end{bmatrix}}^{k\_predictor\_variables}\right\} n\_data\_sets$$

$$\{\beta\} = \begin{Bmatrix} \beta_0 \\ \beta_1 \\ . \\ . \\ . \\ \beta_n \end{Bmatrix} \text{ and } \{\varepsilon\} = \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_1 \\ . \\ . \\ . \\ \varepsilon_n \end{Bmatrix}$$

Multi-collinearity of predictor variables may cause large variance and co-variance of individual regression coefficients, high standard error of individual regression coefficients in spite of high R^2 values, instable regression models fluctuating in magnitude and sign of regression coefficients for small changes in the specification, and wider confidence intervals of regression coefficients. Previously the problem of multi-collinearity has been overcome by removing one of the variables which resulted in loss of some influential parameters. The principal components technique determines a linear transformation for transforming the set of X predictor variables into new set Z predictor variables known as the principal components. The set of new Z variables are uncorrelated with each other and together account for much of variation in X. The principal components correspond to the principal axes of the ellipsoid formed by scatter of simple points in the n dimensional space having X as a basis. The principal component transformation is thus a rotation from the original x coordinate system to the system defined by the principal axes of this ellipsoid. The principal component transformation is used to rank the new orthogonal principal components in the order of their importance.

Multiple linear regression is then performed with the original response variable and reduced set of principal components. The principal components estimators are then transformed back to original predictor variables using the same linear transformation. Since the ordinary least square method is used on principal components, which are pair wise independent, the new set of predictor coefficients are more reliable. The Pearson's Co-relation matrix is calculated to check for the multi-colinearity in the matrix X. And the Eigen values are used in transforming the original predictor variables in the new Z variables. Scree plots, Eigen values and proportion of total variance explained by each principal component are then used to eliminate the least important principal components. The Equation for calculation of the Eigen values and the Eigen vector is:

$$([C]-\lambda[I])[V]$$

$$\left|[X^*]^T[X^*]-\lambda[I]\right|=0$$

Where λ is the eigen value and V is the eigen vector matrix. The original set of predictors has been transformed (matrix A) to a new set of predictor variables (matrix Z) called the principal components. The principal component matrix **Z** contains exactly the same information as the original centered and scaled matrix A, except that the data are arranged into a set of new variables which are completely uncorrelated with one another and which can be ordered or ranked with respect to the magnitude of their Eigen values (Draper and Smith 1981, Myers 1986).

$$Z_j = [\ x_1^* \ x_2^* \ \ldots\ldots.. \ x_3^* \ ] \qquad \underbrace{\begin{Bmatrix} V_{1j} \\ V_{2j} \\ \cdot \\ \cdot \\ \cdot \\ V_{kj} \end{Bmatrix}}_{Eigen\_Vector\_associated\_with\_\lambda_j}$$

MLR has been performed with the transformed predictor variables and the original response variable. The coefficients obtained as a result of this regression model are stored in a variable named alpha. Matrix notation for the same is given as:

$$\{\alpha\}_{k*1} = [V]^T{}_{k*k}\{\beta^*\}_{k*1}$$

The Principal Components have been transformed back to the Original variables. To eliminate the principal components the coefficients are transformed back to the original ones by using the reverse transformation.

$$\{\beta\}_{k*1} = [V]_{k*k}\{\alpha\}_{k*1}$$

The individual T test values of principal components regression components are then used for conducting individual T test on the coefficients of regression of original variables. The test statistic proposed by Mansfield et al.[1997] and Gunst et al. [1980] for obtaining the significance of coefficients of regression of original variables is given in the equation below:

$$t = \frac{b_{j,pc}}{\left[MSE \times \left(\sum_{m=1}^{l} \lambda_m^{-1} v_{jm}^2 \right) \right]^{\frac{1}{2}}}$$

Where $b_{j,pc}$ is the coefficient of regression of the j^{th} principal component, MSE is the mean square error of the regression model with l principal components as its predictor variables, v_{jm} is the j^{th} element of the Eigen vector v_m and λ_m is its corresponding Eigen value. M takes the values from 1 to l, where l is the number of principal components in the model. The test statistic follows a students' T distribution with (n-k-1) degrees of freedom.

2.2 Physics of Failure Based Models

Manson and Coffin [1965, 1954] developed an equation that related plastic strain $\Delta\varepsilon_{p,}$ with number of cycles to failure N. Goldmann [1969] analyzed a controlled collapse joint with spherical dimensions for developing an equation that related the plastic strain of a joint with relative thermal expansion coefficients of chip to substrate, distance from chip neutral point to substrate, height of the solder, volume of solder, radius of the cross section under consideration and exponent from plastic shear stress strain relationship. The plastic strain obtained from Goldmann formulation can be substituted in Coffin- Manson equation for predicting the number of cycles for fatigue failure. Norris and Landzberg [1969] studied the effect of cycling frequency and maximum temperature of cycling on fatigue failure of solder joints and added an empirical correction factor for time dependent and temperature dependent effects for the thermal fatigue model.

Solomon [1986] analyzed the fatigue failure of 60Sn/40Pb solder for various temperatures and developed an isothermal low cycle fatigue equation that correlated the number of cycles to failure with applied shear strain range. Solomon also studied the influence of frequency, and temperature changes and added corrections that account for temperature changes, cycling wave shape and joint geometries.

Engelmaier [1990] developed a surface mount solder joint reliability prediction model containing all the parameters influencing the shear fatigue life of a solder joint due to shear displacement caused by thermal expansion mismatch between component and substrate. Engelmaier developed separate equation for stiff solder joints and compliant solder joints. The parameters of the model include effective solder joint area, solder joint height, diagonal flexural stiffness, distance from neutral point and thermal coefficient mismatch thermal cycling conditions, degree of completeness of stress relaxation and slope of weibull distribution.

Knecht and Fox [1991] developed a strain based model using creep shear strain as damage metric to determine the number of cycles to failure. The creep shear strain included creep of component due to matrix creep alone ignoring the plastic work. The equation was applicable to both 60Sn40Pb and 63Sn37Pb solder joints.

Vandevelde [1998] developed thermo-mechanical models for evaluating the solder joint forces and stresses. Barker et al [2002] synthesized the Vandevelde models for calculating the solder joint shear forces in ceramic and plastic ball grid array packages. Clech [1996] developed a solder reliability solutions model for leadless and leaded eutectic solder assemblies and extended it to area array and CSP packages. Clech obtained the inelastic strain energy density from area of solder joint hysteresis loop and developed a prediction equation correlating inelastic strain energy density with number of cycles to failure.

2.3 Experimental Methods

Temperature cycling is a widely method for solder joint reliability predictions. In this method the component is exposed to a series of low and high temperatures accelerating the failure modes caused by cyclic stresses. The thermal cycling uses a single air chamber in which the temperature ramp can be controlled carefully. Thermal shock tests like thermal cycling are used for accelerated life testing of solder joints. Thermal shock testing is a liquid-liquid test in which two liquid chambers at different temperatures are used. Thermal shock tests generate very high ramp rates.

Master, et al. [1998] conducted accelerated thermal cycling tests on CBGA packages for various body size and assembly parameters to study the effect of package thickness and card pad design on reliability of the package. Master, et al. [1995] studied the effect of column length on fatigue life of solder joint for two different thermal profiles using accelerated thermal cycling tests. Gerke, et al. [1995] studied the reliability of high I/O CBGA packages used in computer environment using accelerated thermal cycling tests for two different thermal profiles. Kang [2004] evaluated the thermal fatigue life and failure mechanism of Sn-Ag-Cu solder joints with reduced Ag contents for a CBGA package. Hong [1998] predicted the mean fatigue life of CBGA packages with lead free (Sb5-Sn95, Ag3.5-Sn96.5, Zn9-Sn91) solder fillets and found the lead free joints outperform the leaded ones. Ingallas [1998] conducted accelerated thermal cycling tests on CCGA packages for two different ball pitch, to study the effect of solder ball pitch on solder joint reliability of the package .Zhang, et al. [2001] evaluated the reliability of SnCu0.7, SnAg3.8Cu0.7 and SnAg3.5 solder joints on both NiP and Cu under bump metallurgies for flip-chip application. Peng, et al. [2004] analyzed the sensitivity of reliability of flip chip package to solder joint geometric parameters such as stand-off height, lower and upper contact angles, and solder joint profile using accelerated thermal cycling tests. Wang, et al. [2001] assessed the reliability of flip chip packages with no flow underfills using liquid to liquid thermal shock tests. Hou, et al. [2001] conducted liquid to liquid thermal shock tests for reliability assessment of flip chip packages with SnAgAu joints. He found the leaded solder joints perform better than the lead free ones. Teo, et al. [2000] conduated accelerated thermal cycling tests for investigating the effect of under bump metallurgy solder joint reliability of flip chip packages. Braun, et al. [2005] studied the high temperature potential of flip chip assemblies for automotive applications. Darveaux, et al [2000] studied the impact of design and material choice on solder joint fatigue life of various BGA packages including PBGA, FlexBGA, tape array BGA and mBGA.

Moire interferometry is an optical method which provides whole field contour maps of in-plane displacements with sensitivity as low as 0.417µm [Tunga 2004]. Moire Interferometry technique has been increasing employed in mapping thermally induced deformation of electronic packages. Meng [1997] applied this technique for solder joint reliability prediction of BGA, CSP and flip chip packages. He subjected the packages to a temperature cycling and extracted the accumulated thermal deformations for reliability predictions. Zhu [1997] studied the reliability of OMPAC BGA and a flip chip BGA using moiré interferometry technique. Zhu also studied the effect of bonding, encapsulation, soldering and geometry on the reliability of both the packages and using the same technique.

Chapter 3

Extended Life Prediction Model

Life prediction models have been developed over the years trying to assess the influence of the different geometrical, material and thermo-cycling parameters on the life of an electronic package. In this study the influence of silver content on packages based on SAC alloys have been investigated.

3.1 Scope of Data:

The dataset includes accelerated test reliability data for a variety of packaging architectures including, of plastic ball-grid array (PBGA), flip-chip ball-grid array (FC-BGA), chip-array ball-grid array (CABGA) devices, MCM-PBGA, Hi-CTE ball grid array, Flip-Chip, ceramic ball-grid array (CBGA), metal lead frame (MLF), and quad-flat packs (QFP). The electronic components have been assembled on copper core and no-core printed circuit boards. Data gathered on test assemblies from nine temperature cycle conditions including, TC1 (-40 to 95^{o}C, 30 min dwell), TC2 (-55 to 125^{o}C, 30 min dwell), TC3 (3 to 100^{o}C, 30 min dwell), TC4 (-20 to 60^{o}C, 30 min dwell), TC5 (-20 to 80^{o}C, 30 min dwell), TC6 (0 to 100^{o}C, 15 min dwell), TC7 (0 to 100^{o}C, 10 min dwell), TC8 (-55 to 125^{o}C, 15 min dwell), TC9 (-40 to 125^{o}C, 15 min dwell). The chambers were profiled with full-load, and the temperatures measured at various locations on the test boards in the stack, to ensure that the packages experience uniform temperature exposure. All packages are daisy-chained and the resistance monitored to identify failures.

3.2 Model Development

Along with silver content, the solder ball configuration parameters such as ball pitch, ball count, ball height, ball diameter and cycle conditions such as dwell time and delta T have been considered .An assortment of packages such as CBGAs, PBGAs, flips chips based on a variety of SAC alloys with a set of different silver contents were considered for the analysis. Initial model diagnosis revealed that the data set had correlated variables and hence was causing Variance inflation.

From the table below, it is clear that there are a few variables with high co-relation coefficients and hence the regression will yield high variance inflation and in-accurate regression coefficients. Below is the table of Pearson's correlation co-efficient, the correlated variables have co-efficient values of more than 0.5.

Table 3 Pearson's Correlation Matrix

	Die Length	Ball Count	Ball pitch	Ball Dia	Ball H	% of AG	Dwell Time	Delta T
Die Length	1.00	0.86	0.61	-0.19	0.42	0.55	0.51	-0.66
Ball Count	0.86	1.00	0.53	-0.09	0.43	0.69	0.37	-0.62
Ball pitch	0.61	0.53	1.00	0.03	0.88	0.75	0.37	-0.60
Ball Dia	-0.19	-0.09	0.03	1.00	-0.03	-0.11	-0.05	-0.07
Ball H	0.42	0.43	0.88	-0.03	1.00	0.64	0.38	-0.49
% AG	0.55	0.69	0.75	-0.11	0.64	1.00	0.12	-0.50
Dwell Time	0.51	0.37	0.37	-0.05	0.38	0.12	1.00	-0.50
Del T	-0.66	-0.62	-0.60	-0.07	-0.49	-0.50	-0.50	1.00

3.3 Selection of Ridge Parameter ‘K’:

The regular multiple linear regression model with the above data will yield a model with inaccuracies and high variance inflations. Below is the result of the multiple linear regression performed on the above data.

Table 4 MLR Results

Variable	Estimate	Error	T	P	VIF
Intercept	24.51	1.77	13.84	0.00	0.00
die_length	-0.83	0.22	-3.71	0.00	9.63
ball_count	-0.29	0.17	-1.72	0.09	8.51
ball_dia	-1.13	0.46	-2.46	0.02	12.65
ball_h	1.37	0.24	5.62	0.00	1.46
ball_pitch	0.41	0.21	1.93	0.06	6.81
Ag	0.77	0.18	4.25	0.00	5.02
dwell_time	-0.27	0.09	-3.04	0.00	1.85
delta_T	-2.71	0.29	-9.46	0.00	2.29

It can be seen that despite the fact that the VIF is high for a few variables, the corresponding ‘p’ values a really low explaining the importance of the variable to the model and so those variables cannot be dropped off.

Ridge regression is applied to the problem to circumvent the numerical snag. A small positive bias k is introduced to try and reduce the variance and hence the MSE. Different bias values from 0 to 0.1 in increments of 0.001 are tried to see if the regression co-efficients and the variance stabilize. In case no stability is observed, the upper limit of the bias is increased until a range which records the stability of the parameters is reached at. The biased models are made by using the k value in the equation (5).The process is carried out as mentioned in fig (1).The regression co-efficients and the VIF are recorded in each case. These values are observed to see if there is any stability. Stabilization of ridge parameter can be determined in a lot of ways. In most cases the requirements of the model play a vital role. Since the objective is prediction we have to make sure the model chosen to be with stable bias has good prediction accuracy and complied with the physical interpretations of the case .The most common method is observing the ‘ridge plot’. The ridge plot is plot of the bias parameter on the X-axis against parameters like ‘VIF’ or the β co-efficients themselves. An ideal bias parameter is determined based on where the plots seem to stabilize. In some cases the VIF and co-efficients stabilize at different points. In those cases we try to choose the bias in such a way the model is neither ‘over-biased’ nor ‘under-biased’. The trade-off can be based on prediction accuracy of the model in each case. Below is a ridge plot of the model for assessing the influence of silver content based on the parameters mentioned above.

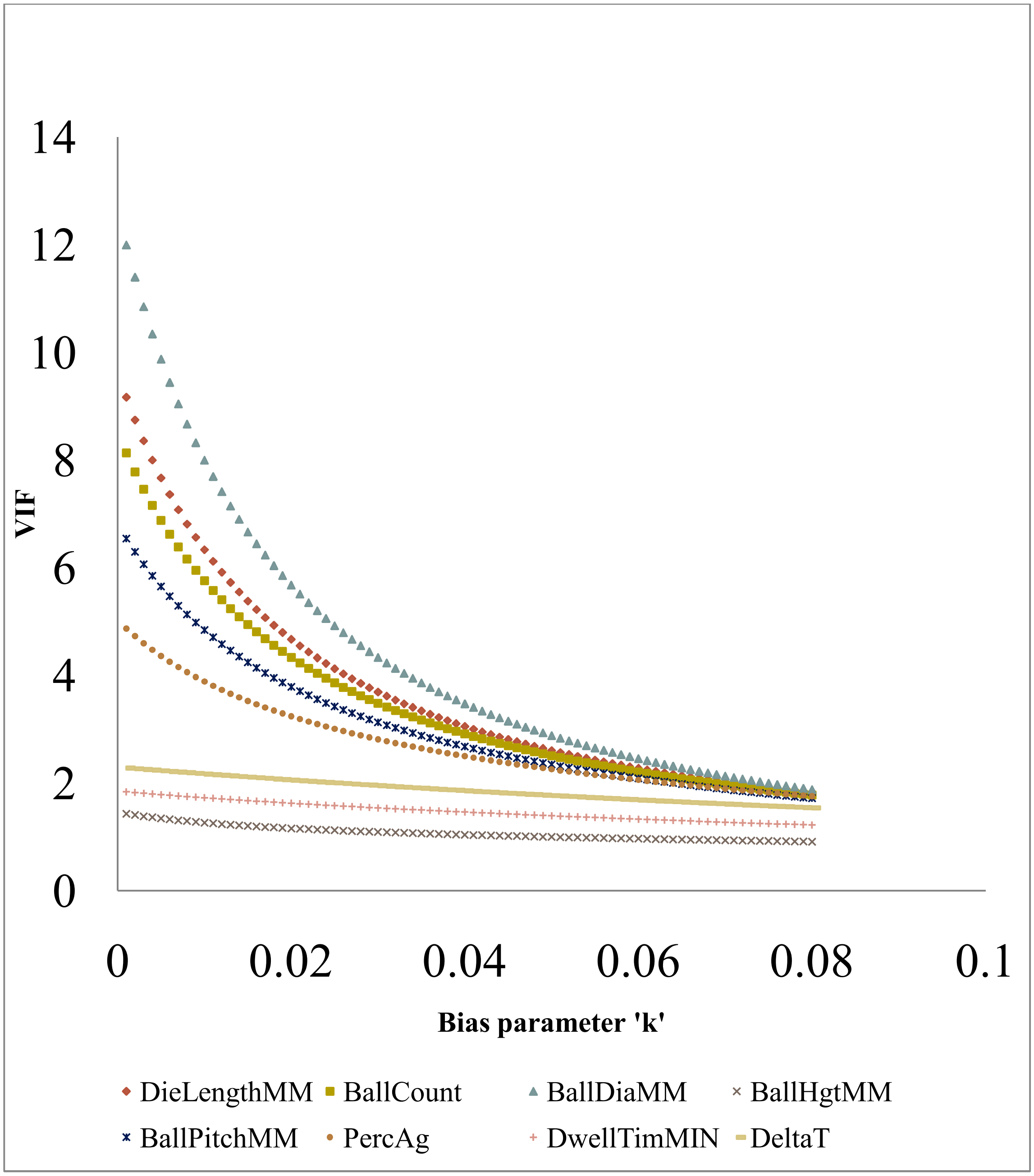


Figure 4 Ridge Plot (based on VIF)

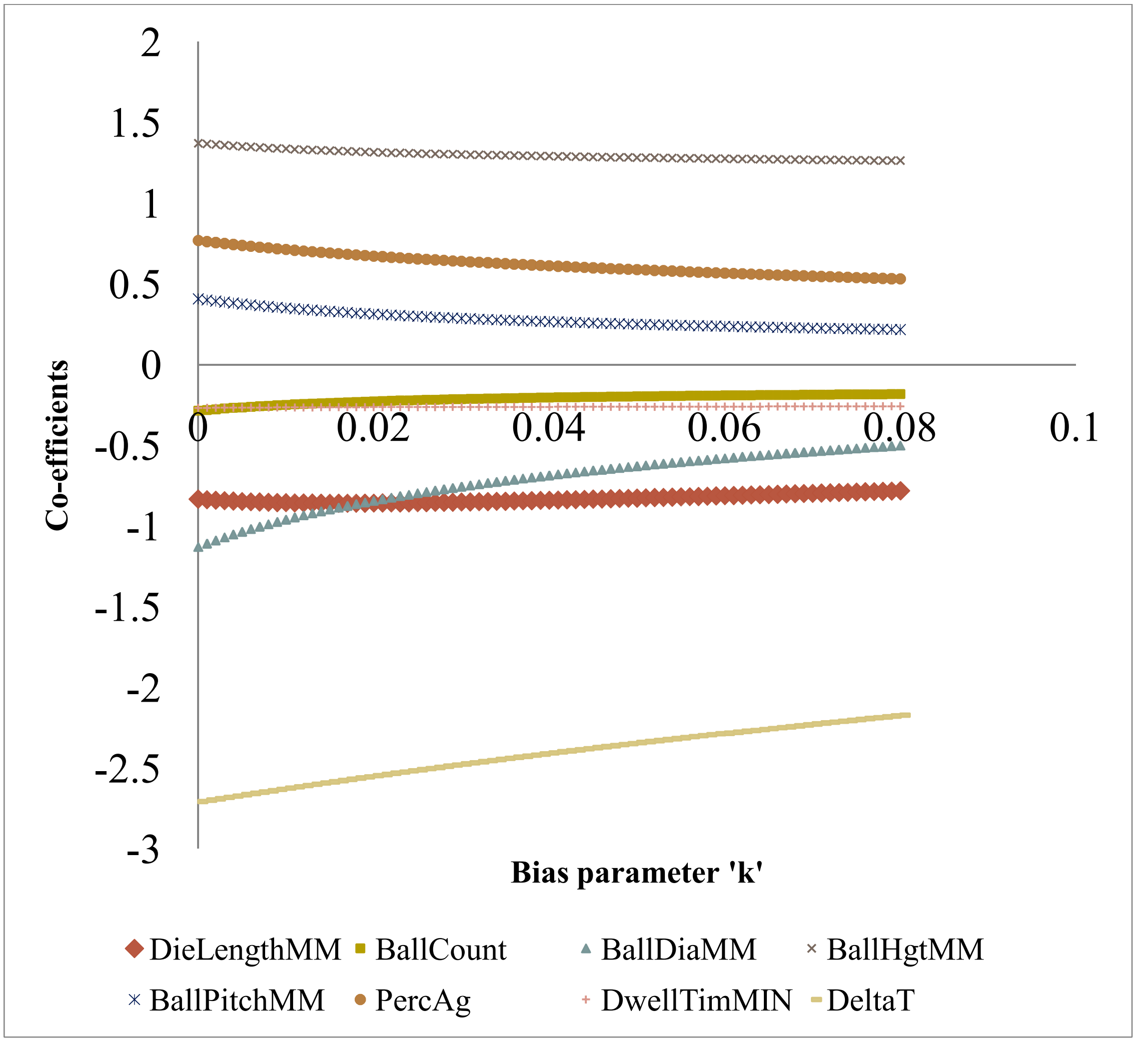


Figure 5 Ridge Plot 2 (based on Co-efficients)

In the initial attempt to regress the data, it was seen that a few variables had high variance inflations. These variables are seen to stabilize with increase in bias parameter. A bias at which a stability of both the co-efficients and the VIF is seen is chosen to the biasing parameter. In the above case, it is seen that the VIF of all the variables attain stability at about 0.06 and after. Upon closely observing the second plot, we see that all the variables stabilize at about 0.076. Hence the bias parameter is chosen to be 0.076. The model corresponding to k=0.076 is given below:

Table 5 RR Results

parameter	Constant	Die Length	Ball Count	Ball Pitch
B	21.151	-0.788	-0.183	-0.509
Ball Dia	Ball Height	% of Ag	Dwell Time	Delta T
1.2646	0.2818	0.5358	-0.258	-2.11

Hence we have,

$$Ln(char\_life) = 21.151 - 0.788\,Ln(L_D) - 0.18294\,Ln(C_B) - 0.50908\,Ln(P_B) + 1.2646\,Ln(D_B) + 0.2818\,Ln(H_B) + 0.5358\,Ln(\%ofAg) - 0.2585\,Ln(T_D) - 2.11\,Ln(\Delta T) \tag{6}$$

The model reduces to,

$$CharLife = 21.151(L_D)^{-0.788}(C_B)^{-0.18294}(P_B)^{-0.50908}(D_B)^{1.264}(H_B)^{0.28}(\%ofAg)^{0.53}(T_D)^{-0.2585}(\Delta T)^{-2.11} \tag{7}$$

Where, L_D is the Die Length, C_B is the ball count, P_B is the ball pitch, D_B is the ball diameter, H_B is ball height, T_D is the dwell time and ΔT is the temperature cycle range.

As an inference, we have, die length, ball count, ball pitch, dwell time and Delta T have negative dependencies on life and parameters like ball height, % of AG and ball diameter have positive influence on life. The inferences are further assessed and confirmed when the model is validated. An Analysis of variance is performed to verify that the results of the Regression process are significant.

ANOVA

Table 6 ANOVA

Source	DF	Sum of squares	Mean Square	F	P
Model	8	64.8702	8.1088	34.2	0.00010
Error	83	19.68	0.2371		
Total	91	84.5504			

3.4 Model Validation:

The Validation is done using data outside of the data set used for regression. A few data are set aside from the original dataset for the purpose of validation. Since the objective of the regression is prediction, a validation done by predicting the life of packages that were not included in the regression will give a better validation the model. A plot with the predicted and actual life of package would give an idea of the reliability of the model.

3.4.1 Dwell Time:

Dwell time is a critical contributor to the life of the solder ball and as the model suggests, the increase in dwell time decreases life. The plot below re-confirms the same.

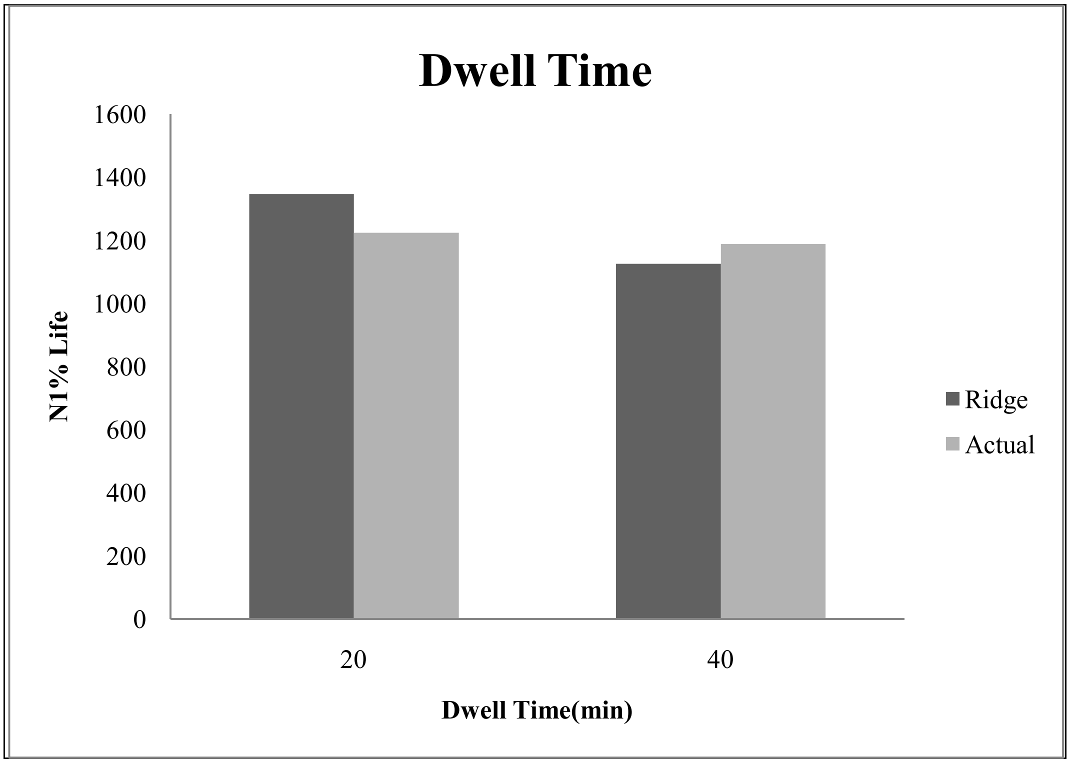


Figure 6 Validation of Dwell Time

3.4.2 Percentage of Silver:

The model suggests that the increase in the silver content increases life. Two similar packages with the only difference being silver content (2.1 and 2.3) are considered and the plot shows that there is a increase in life both actual and predicted with increase in Silver content.

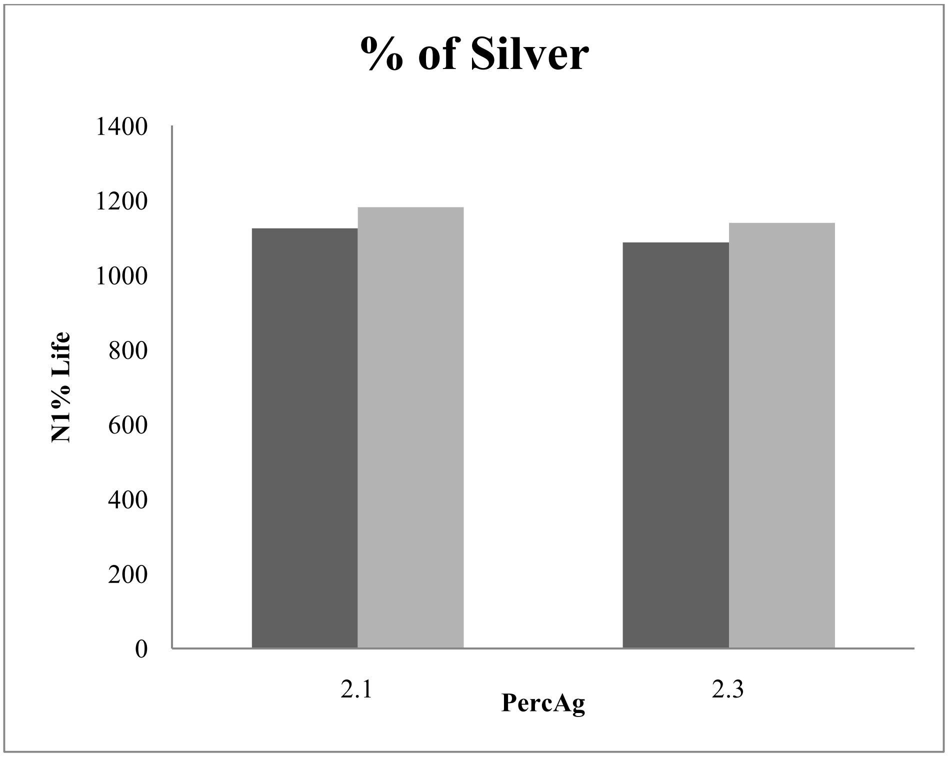


Figure 7 Validation of Silver Content

3.4.3 Delta T:

Temperature difference is the most significant parameter in thermo-cycling. The model predicts square negative influence on life. The plot below shows that there is a significant decrease in life with increase in Delta T.

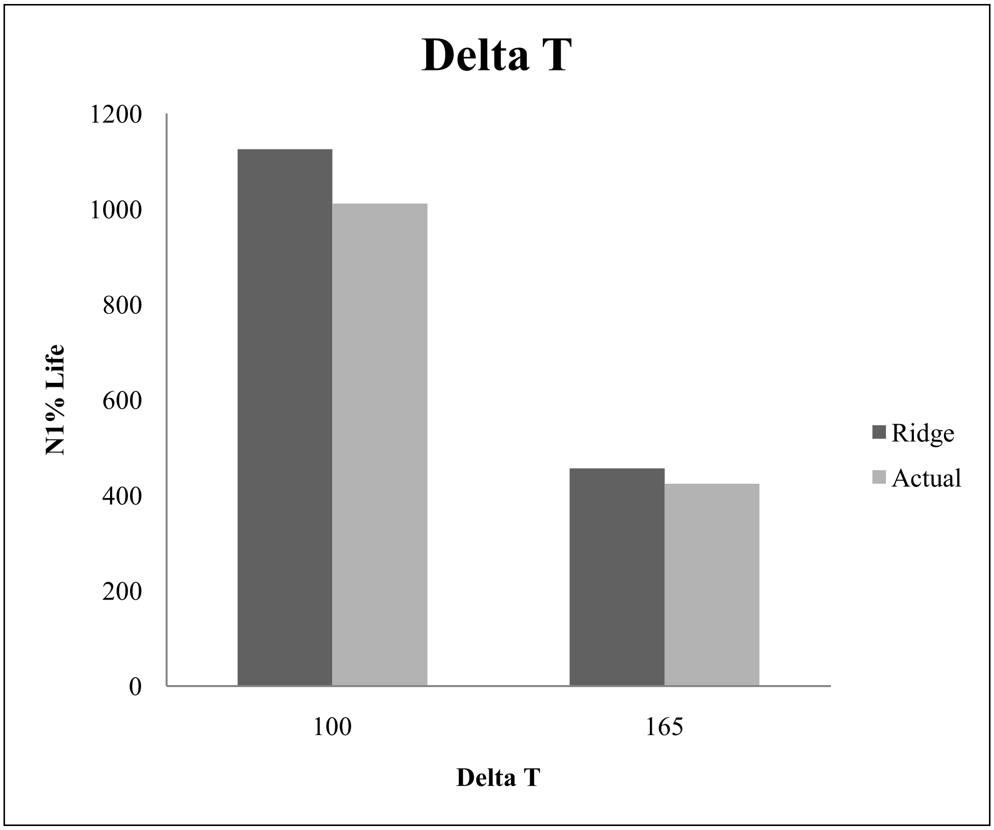


Figure 8 Validation of Delta T

3.4.4 Ball Diameter:

The solder ball diameter has a positive co-efficient and the plot shows increase in life with increase in Diameter. Hence we infer that considering two packages with similar material and Geometric properties except for the ball diameter, we can say that the package with the higher ball diameter will have better life amongst the two.

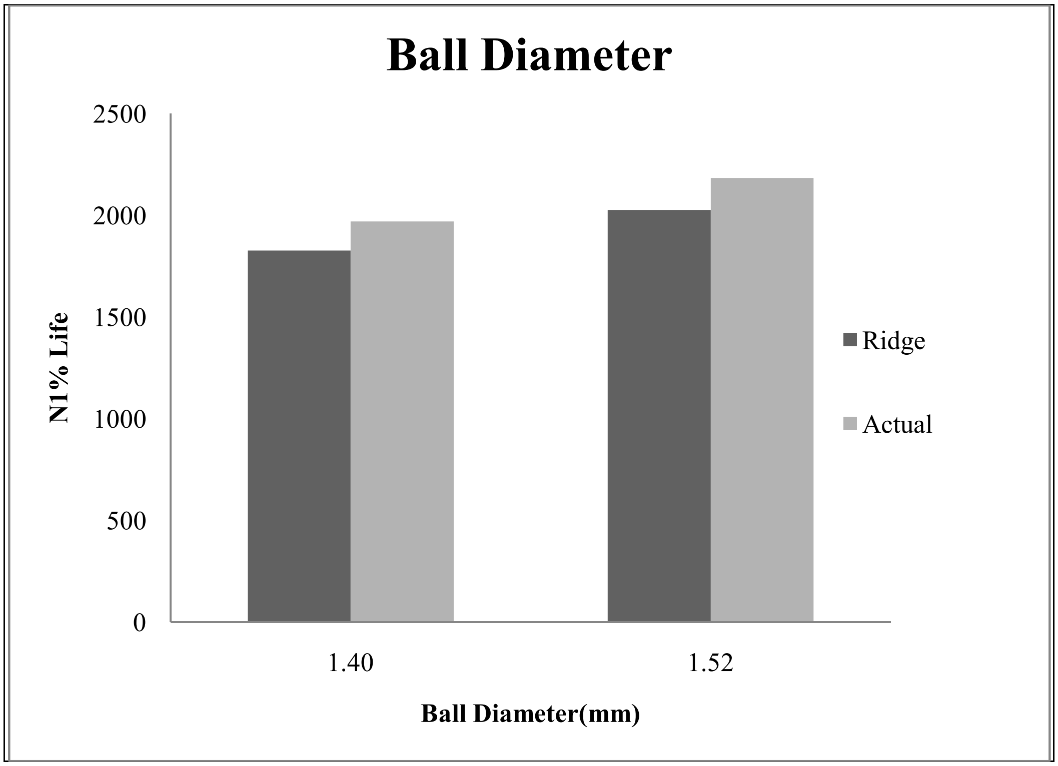


Figure 9 Validation of Ball Diameter

3.4 Model Prediction:

The model is used to predict life and the model predictions are plotted against the actual life to see the accuracy of the model. The dotted lines represent the 90% interval. As seen from the plot, most of the model predictions fall in the interval.

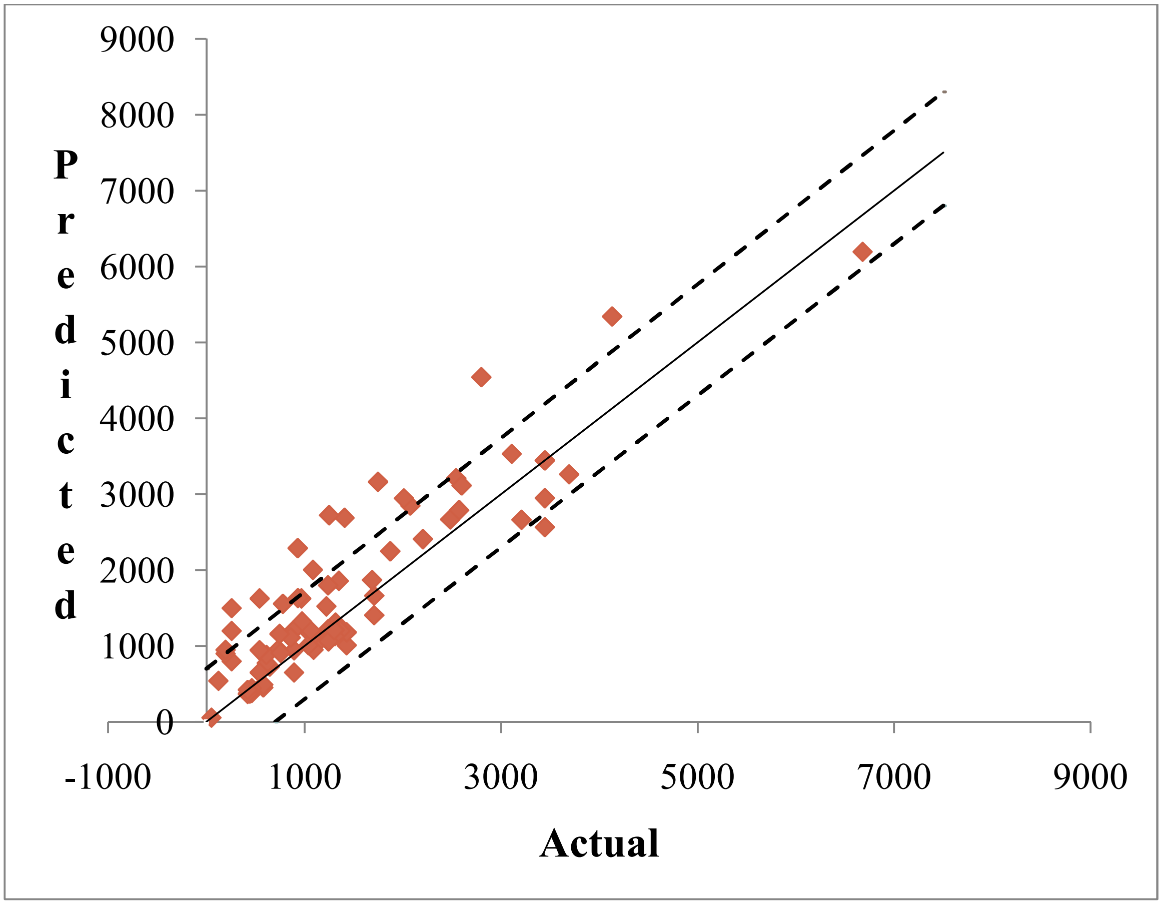


Figure 10 Model Prediction for Silver content model

Chapter 4

Norris Landzberg Constants Development

The Norris-Landzberg Equation is based on the Coffin Mansion Equation and the Goldmann Equation. It provides a way of calculating the acceleration factor for Controlled Collapse Interconnections [Norris 1969]. The equation is given by:

$$AF = \frac{N_U}{N_A} = \left(\frac{f_U}{f_A}\right)^{\frac{1}{3}} \left(\frac{\Delta T_A}{\Delta T_U}\right)^2 \Phi\left(T_{\max}\right)$$

Where,

AF is the Acceleration factor.

N_U and N_A are the lives of the packages f_U and f_A are the frequencies

T_A and T_U are the temperature excursions

Tmax is the maximum temperature of the cycle in Kelvin

This Equation is often used in the form [Lau 1997]

$$AF = \frac{N_u}{N_A} = \left(\frac{f_U}{f_A}\right)^{\frac{1}{3}} \left(\frac{\Delta T_A}{\Delta T_U}\right)^2 \exp\left(1414\left(\frac{1}{T_{\max,U}} - \frac{1}{T_{\max,A}}\right)\right)$$

The Equation can be transformed by computing the natural Log format as follows:

$$Ln(AF) = C_1 Ln\left(\frac{f_U}{f_A}\right) + C_2 Ln\left(\frac{\Delta T_A}{\Delta T_U}\right) + C3\left(\frac{1}{T_{\max,U}} - \frac{1}{T_{\max,A}}\right)$$

Now we model the above equation into a regression model with ratio of cyclic frequencies, Temperature cycle magnitude and the difference of inverse of maximum temperatures as the independent variables and the Acceleration factor as the response variable.

Due to the presence of Multi-colinearity Principal Component Regression is implemented. Regression of the transformed Principal Components against the Acceleration Factor is given in the table below:

Table 7 PCR Results (Transformed)

Predictor	Coef	SE Coef	T	P
Constant	0.7448	0.1161	6.4123	0
Z1	3589.0768	1354.5949	2.6496	0.0095
Z2	285.8296	107.7056	2.6538	0.0094
Z3	2802.1627	1057.2824	2.6503	0.0095

The ANOVA table is used to check the presence of a linear relationship between the predictor variables and the response variables.

Table 8 ANOVA for PCR NLZ

Source	DF	SS	MS	F	P
Regression	3	2.136	0.712	5.82	0.001
Residual Error	90	11.0016	0.1222		
Total	93	13.1375			

To get the relationship between the original variables and the response variable, we need to back transform the Principal Components using the same back transformation. Regression results for the same are:

Table 9 PCR Results Transformed back

Predictor	Coef	SE Coef	T	P
Constant	0.7448	0.1161	6.4123	0
Ln(Fu/Fa)	0.3035	0.1145	2.6496	0.0095
Ln(Delta Tu / Delta Ta)	2.3149	0.8722	2.6538	0.0094
(1/Tu-1/Ta)	4562.3767	1721.45	2.6503	0.0095

The regression equation is given by:

$$AF = 0.7448 + 0.3035 * Ln\left(\frac{F_U}{F_A}\right) + 2.3149 * Ln\left(\frac{Delta\_T_U}{Delta\_T_A}\right) + 4562.3767 * Ln\left(\frac{1}{T_U} - \frac{1}{T_A}\right)$$

Writing the equation in the form of the NL equation:

$$AF = \frac{N_u}{N_A} = \left(\frac{f_U}{f_A}\right)^{0.3} \left(\frac{\Delta T_A}{\Delta T_U}\right)^{2.31} \exp\left(4562\left(\frac{1}{T_{\max,U}} - \frac{1}{T_{\max,A}}\right)\right)$$

The PCR model that is shown above has been shown to predict the acceleration factor of PBGAs with SAC305 alloys with good prediction accuracy. The same three variables are taken for regression and with the same dataset using ridge regression.

4.1 Ridge Regression

An initial attempt to model the case using Multiple Linear regression was made and the model was found to have very high values for regression co-efficient and high MSE and VIF due to a high variance. $\hat{\beta} = (X'X)^{-1}X'y$. A biasing parameter k was introduced to the least squares formula and a significant reduction of variance and a more meaningful co-efficient matrix was seen. The value of ‘k’ was varied with small increments to choose the best value of it.

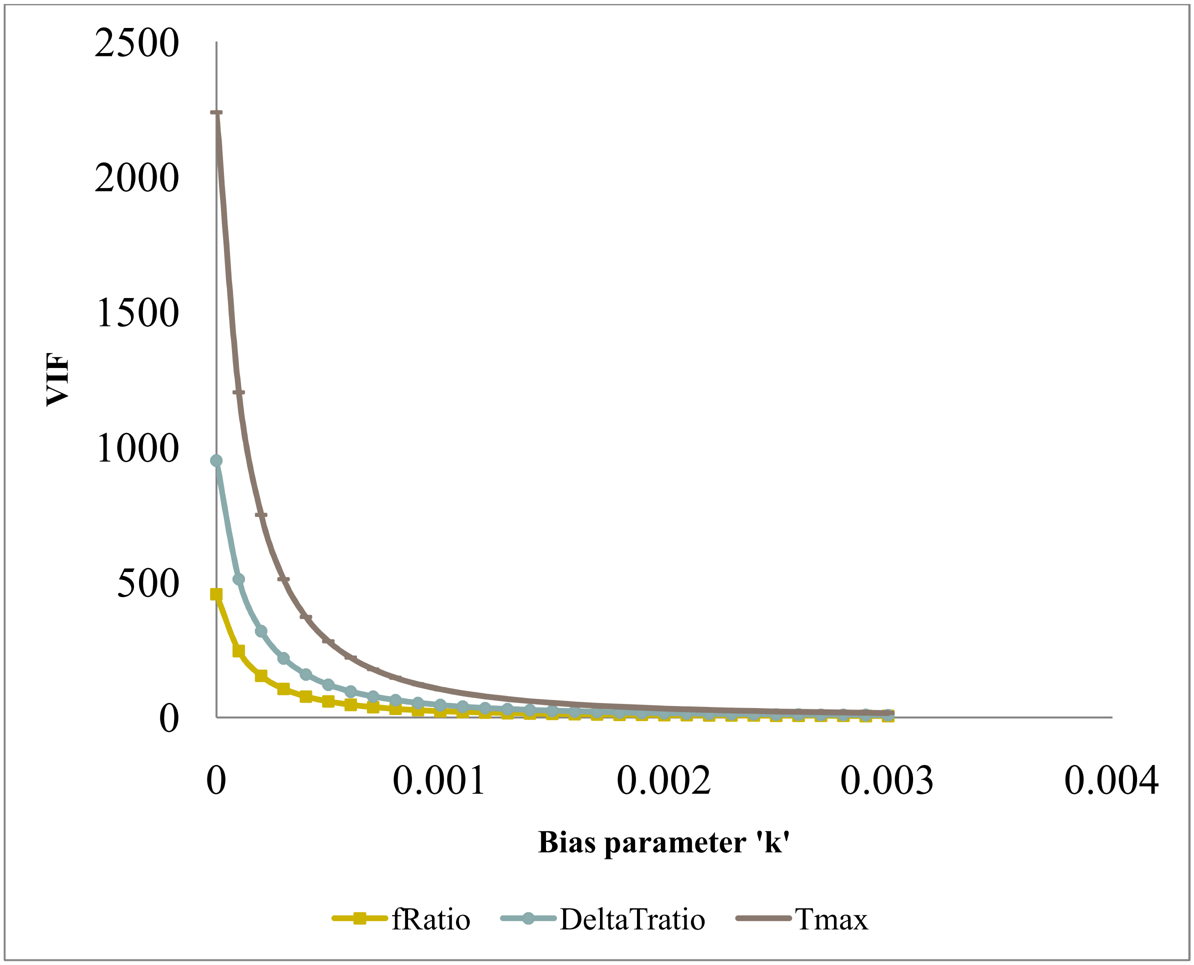
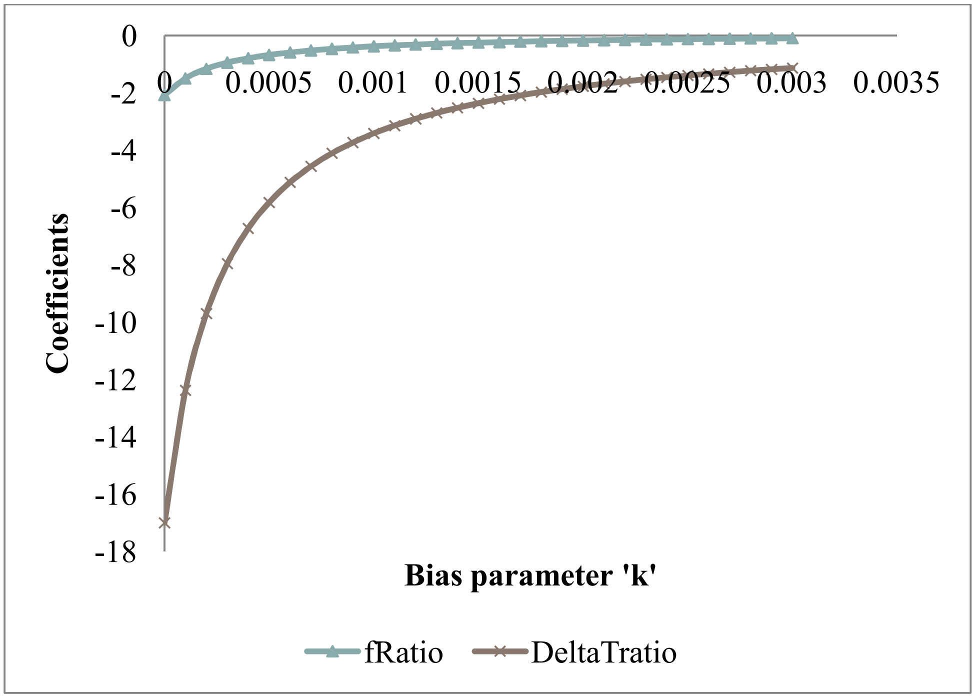


Figure 11 Ridge Plot - VIF



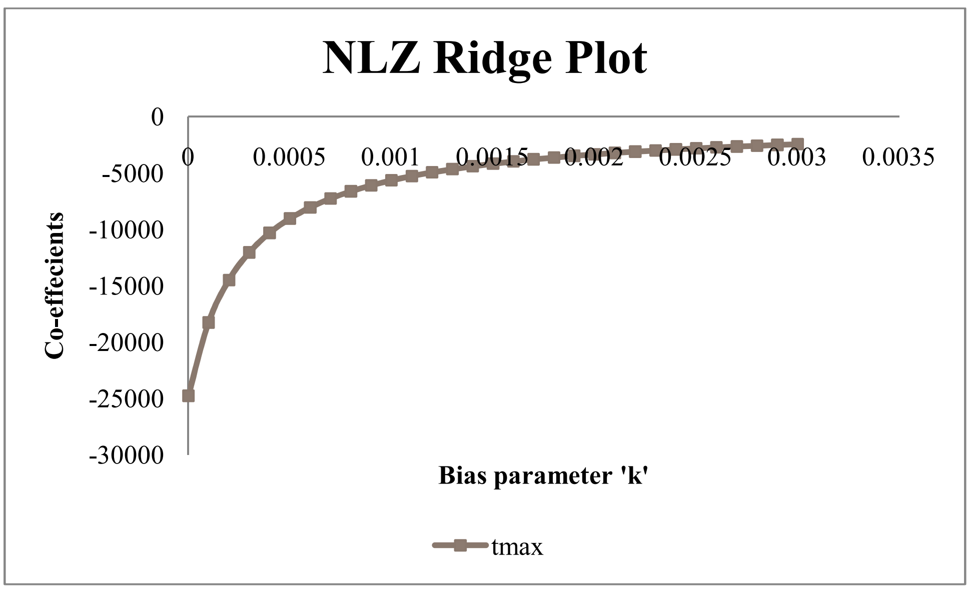


Figure 12 Ridge Plot Regression Co-efficients

At one point of the biasing parameter, the ridge plots become stable and the VIF also stabilizes. That model is checked to see if it fits accurately. A trial with k values between 0 and 0. 003 with increments of 0.0001 is done and in each case, the k is plugged into the equation $\hat{\beta}_R = (X'X + kI)^{-1}(X'X)\beta$ and the corresponding regression coefficients are calculated. In each case, the corresponding variation inflation factor is calculated. With each trial of the regression and increment of 'k', the stability of the VIF and the regression co-efficients are closely watched for. If the case does not stabilize within the given range of values for k, the range is increased and the same procedure is followed to move toward a perfect biasing factor. The above graph, also called the 'ridge trace' plots the stabilization of the regression co-efficients with the increase in bias. Based on the three plots, it is seen that all of them stabilize just about when k =0.0017.

Taking the regression co-effiecients at k=0.0017, the Norris Landzberg equation,

Table 10 NLZ Results (Ridge)

Constant	F Ratio	Delta T Ratio	T Max
0.74	0.201	2.086	3802

$$AF = \frac{N_u}{N_A} = \left(\frac{f_U}{f_A}\right)^{0.201} \left(\frac{\Delta T_A}{\Delta T_U}\right)^{2.086} \exp\left(3802\left(\frac{1}{T_{\max,U}} - \frac{1}{T_{\max,A}}\right)\right) \quad (11)$$

ANOVA Table

Table 11 ANOVA NLZ (Ridge)

Source	DF	Sum of squares	Mean Square	F	P
Model	3	1.313	0.437	5.36	0.002
Error	84	6.856	0.08162		
Total	87	8.1698			

4.2 Model Comparison

Norris-Landzberg Model Constants (SnPb)

$$AF = \frac{N_u}{N_A} = \left(\frac{f_U}{f_A}\right)^{\frac{1}{3}}\left(\frac{\Delta T_A}{\Delta T_U}\right)^{2} \exp\left(1414\left(\frac{1}{T_{max,\,U}} - \frac{1}{T_{max,\,A}}\right)\right)$$

PCR Based Norris-Landzberg Model Constants (SAC 305)

$$AF = \frac{N_u}{N_A} = \left(\frac{f_U}{f_A}\right)^{0.3}\left(\frac{\Delta T_A}{\Delta T_U}\right)^{2.31} \exp\left(4562\left(\frac{1}{T_{max,U}} - \frac{1}{T_{max,A}}\right)\right)$$

Ridge Regression Based Norris-Landzberg Model Constants (SAC 305)

$$AF = \frac{N_u}{N_A} = \left(\frac{f_U}{f_A}\right)^{0.201}\left(\frac{\Delta T_A}{\Delta T_U}\right)^{2.086} \exp\left(3802\left(\frac{1}{T_{max,U}} - \frac{1}{T_{max,A}}\right)\right)$$

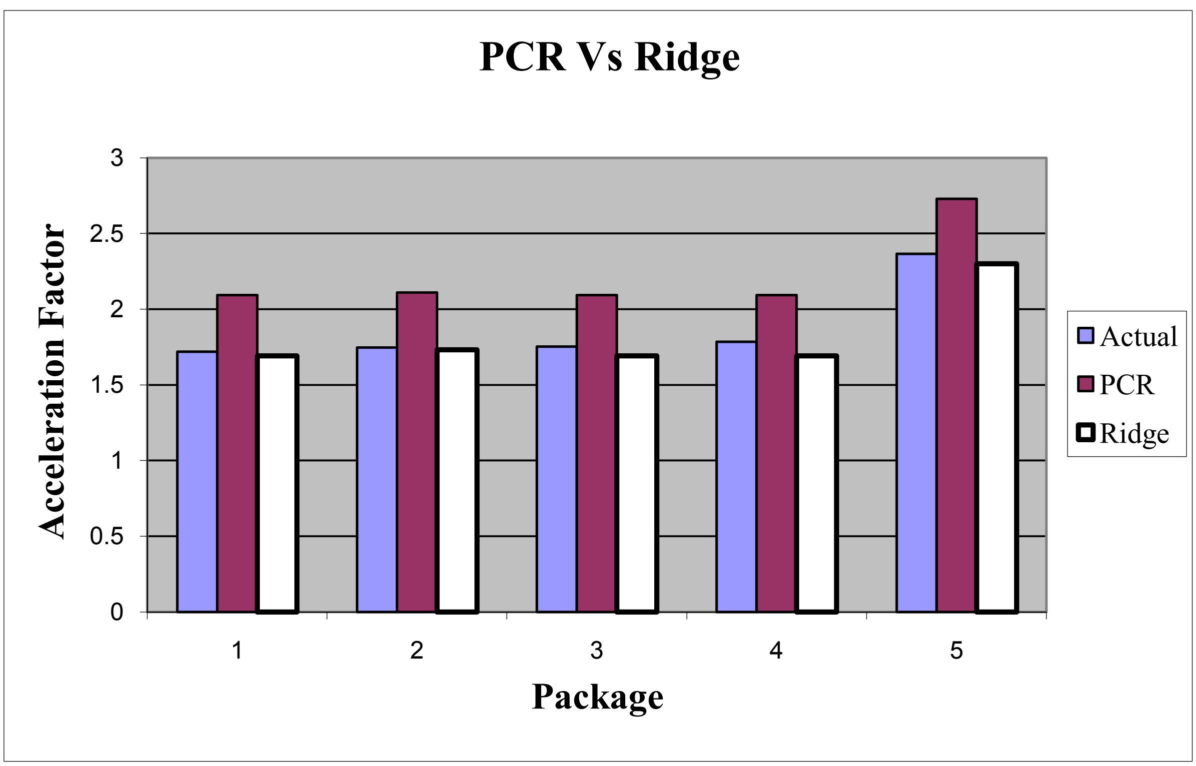
4.3 Model Validation

The Validation is done using data outside of the data set used for regression. A few data are set aside from the original dataset for the purpose of validation. Since the objective of the regression is prediction, a validation done by predicting the life of packages that were not included in the regression will give a better validation the model. A plot with the predicted and actual life of package would give an idea of the reliability of the model.

Table 12 Validation dataset NLZ

Package No.	1	2	3	4	5
Frequencies	0.01667	0.01667	0.01667	0.01667	0.03333
	0.01667	0.0333	0.01667	0.01667	0.01667
Delta T	135	135	165	135	165
	180	165	180	180	180
Tmax	368	368	368	368	398
	398	398	398	398	398

Figure 13 Model Validation for NLZ (PCR Vs Ridge)



4.3.1 Delta T

The temperature excursion or DeltaT is as known the most critical parameter in thermo-cycling. The model predicts that the increase in Delta T decreases the Acceleration factor. The plot shows the same trend.

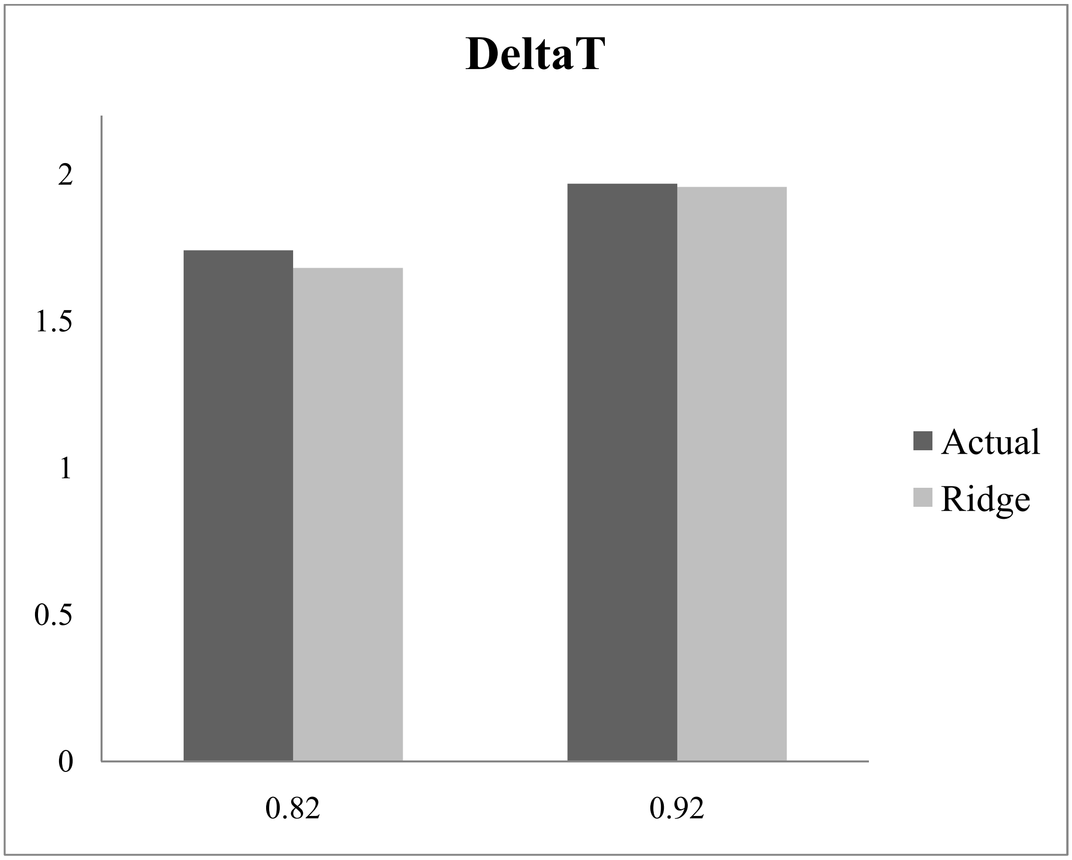


Figure 14 Validation of DeltaT (NLZ)

4.3.2 Frequency Ratio:

The increase in frequency ratio decreases the acceleration factor as explained by the plot.

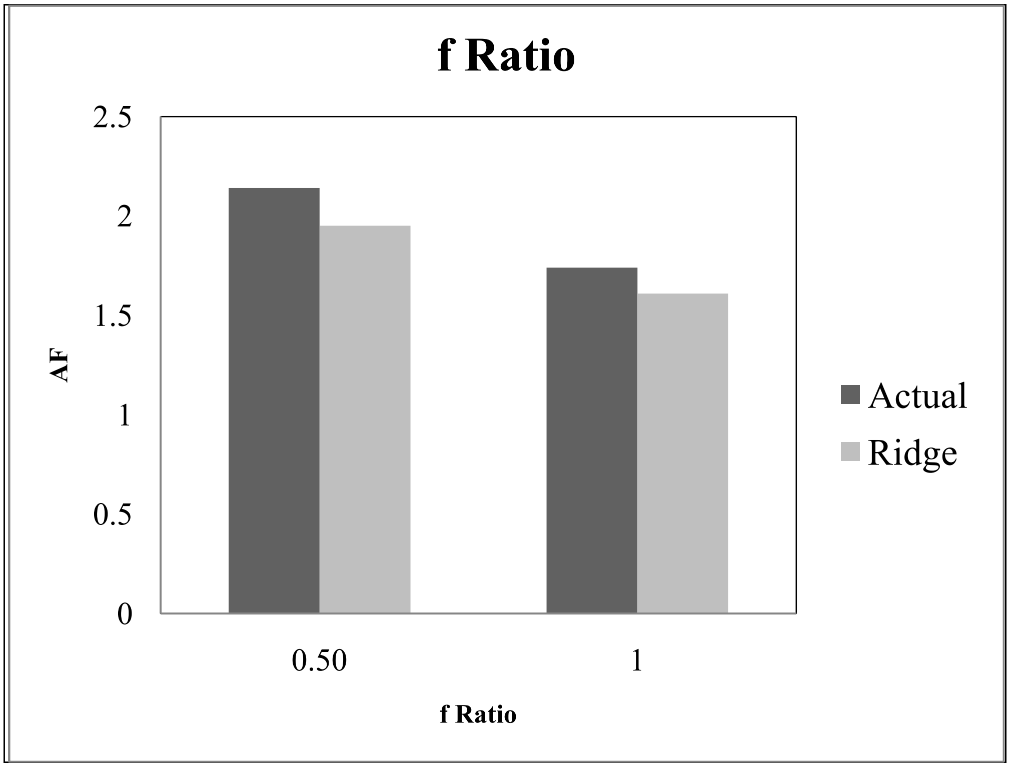


Figure 15 Validation of f Ratio (NLZ)

Chapter 5

Goldmann Constants Development

The Goldmann Model [1969] relates the coefficients of thermal expansion, distance from chip neutral point to interconnections, temperature excursion of the cycle, volume of the solder, radius and height of the solder ball, with the thermo-mechanical reliability of components. The Goldmann Model [1969] has the form,

$$N_f = K_T\left[\left(\frac{\tau_u \pi r_f^2}{A}\right)\left(\frac{h^{1+\beta}}{V}\right)\right]^{m/\beta}\left(\frac{1}{\delta}\right)^m$$

$$\delta = d.\alpha_{rel}.\Delta T$$

Where N_f is the number of cycles to failure, K_T is the Constant in applied Coffin-Manson Equation, τ_u is the ultimate shear strength of the critical joint interface, i.e., chip-pad interface or land-substrate interface, rf is the radius of critical interface, h is the solder joint height, A is the constant in stress-strain relationship $\tau = A\gamma^\beta$, V is the solder volume of joint, δ is the shear deformation of joint, d is the distance of the solder joint from neutral point of the chip, α_{rel} is the relative coefficient of thermal expansion, ΔT is change in temperature. The equation has been re-arranged as follows,

$$N_f = \left(K_T \left(\frac{\tau_u}{A} \right)^{m/\beta} \right) \left[\left(\frac{\pi r_f^2 h}{V} \right) \left(h^{\beta} \right) \right]^{m/\beta} \left(\frac{1}{d.\alpha_{rel}.\Delta T} \right)^{m}$$

$$N_f = \left(K_T \left(\frac{\tau_u}{A} \right)^{m/\beta} \right) \left[\left(\frac{\pi r_f^2 h}{V} \right)^{m/\beta} h^{m} d^{-m} \alpha_{rel}^{-m} \Delta T^{-m} \right]$$

Since K_T, τ_u and A are material and damage constants, these have been combined into one constant, C. The equation has been modified as follows,

$$N_f = C \left[\left(\frac{\pi r_f^2 h}{V} \right)^{m/\beta} h^{m} d^{-m} \alpha_{rel}^{-m} \Delta T^{-m} \right]$$

Substituting m = 1.9, β = 0.58 into the equation,

$$N_f = C \left[\left(\frac{\pi r_f^2 h}{V} \right)^{3.275} h^{1.9} d^{-1.9} \alpha_{rel}^{-1.9} \Delta T^{-1.9} \right]$$

The equation has been transformed by computing a natural logarithm of equation.

$$\ln(N_f) = \ln(C) + \left(\frac{m}{\beta}\right) + m.\ln(h) + m.\ln(d) + m.\ln(\alpha_{rel}) + m.\ln(\Delta T)$$

5.1 Principal Component Regression

The critical parameters like Difference in coefficients of thermal expansion, Distance from chip neutral point to interconnections, Temperature excursion of the cycle, Volume of the solder, radius and height of the solder ball, are included in the Goldmann equation. Using these parameters as predictor variables, we model the Goldmann`s Equation in the form of a log transformed Principal Component Regression model for PBGAs assembled on Copper Core BGAs

The principal components matrix Z is obtained using the transformation:

$[Z] = [X]*[V]$

MLR is performed with the transformed predictor variables and the original response variable. The coefficients obtained as a result of this regression model are stored in a variable named alpha. Matrix notation for the same is given as:

$$\{\alpha\}_{k*1} = [V]^{T}{}_{k*k}\{\beta^{*}\}_{k*1}$$

Regressing the transformed Z variables against the N1% life of the packages, we get the following results.

Table 13 Transformed Z variable regression for Goldmann`s model of Cu Core Assemblies

Predictor	Coef	SE Coef	T	P
Constant	-2.651	4.014	-0.66	0.511
Z1	-0.8412	0.2914	-2.89	0.005
Z2	1.3919	0.1823	7.64	0
Z3	1.3075	0.1682	7.77	0
Z4	-0.4962	0.1579	-3.14	0.002
Z5	0.1626	0.1114	1.46	0.148

The overall adequacy of the model has been tested using ANOVA table. Small P value of the ANOVA table rejects the null hypothesis proving the overall adequacy of the model. Individual T tests on the coefficients of regression of principal components yielded very small P values indicating the statistical significance of all the five variables

ANOVA

Table 14 ANOVA Goldmann PCR

Source	DF	Sum of squares	Mean Square	F	P
Model	5	11.5452	2.3090	15.08	0.0001
Error	98	15.003	0.1530		
Total	103	26.5482			

Table 15 Transforming Z back to Original Variables in the Goldmann`s Model for Copper Core

Predictor	**Coef**	**SE Coef**	**T**	**P**
Constant	-2.651	4.014	-0.66	0.511
IntTerm	0.0495	0.0171	2.89	0.005
Ln(BallhgtMM)	0.4121	0.054	7.64	0
Ln(HalfDLengthMM)	-0.3705	0.0476	-7.77	0
Ln(CTEppmC)	-1.3721	0.4369	-3.14	0.002
Ln(DeltaT)	-1.56	1.068	-1.46	0.148

We write the model in equation format to compare the values of constants obtained from the PCR model with standard values for Cu Core Assemblies. Following are the two models:

Goldmann`s Model:

$$N\left((\alpha_{rel})^{-2}(L)^{-2}(\Delta T)^{-2}\left(\frac{V}{\pi r^2 h}\right)^{-0.152}\left(\frac{1}{h^{1.32}}\right)^{-0.152}\right)=C$$

Statistical form based on PCR for Goldmann`s Model:

$$N_f = C\left[\left(\frac{\pi r_f^2 h}{V}\right)^{0.0495} h^{0.4121} d^{-0.3705} \alpha_{rel}^{-1.3721} \Delta T^{-1.56}\right]$$

5.2 Ridge Regression:

The initial bias range was set to be 0 and 0.1 with intervals of 0.001. The stability of bias is looked for in the VIF and regression co-efficient ridge plot.

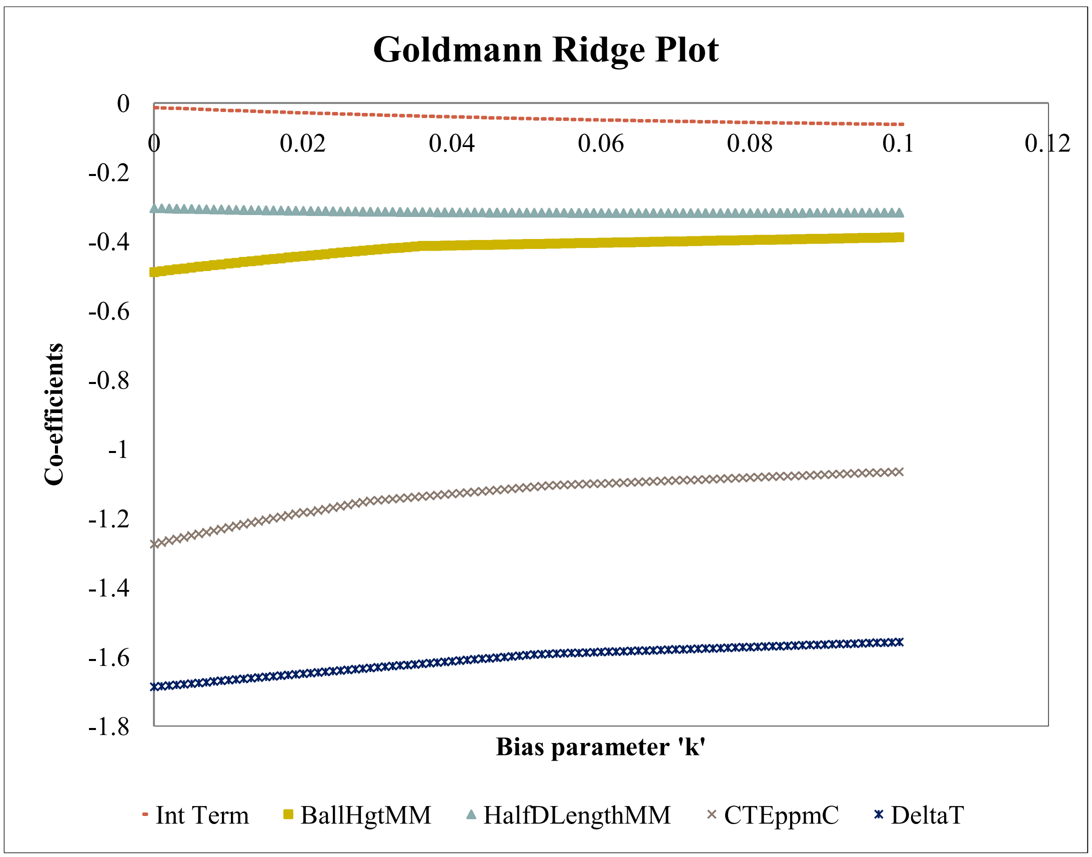


Figure 16 Ridge plot (co-efficients)

Stabilization of VIF

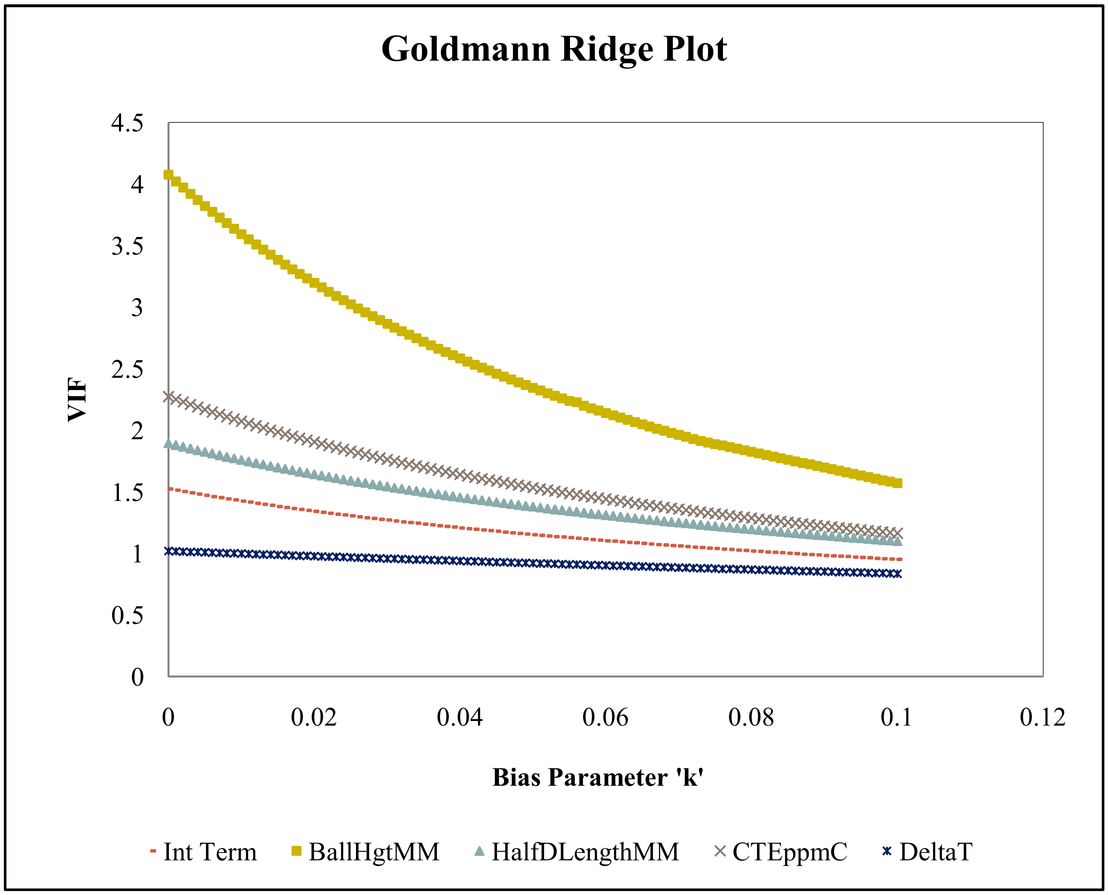


Figure 17 Ridge plot (co-efficients)

Upon close examination of the results and of the plot, stability is attained at different point for different variables but at k =0.062, all the variables are stable. The results at this particular bias factor are taken to be best bias. The results are as follows:

Table 16 Ridge Results (Goldmann Model)

Predictor	C	Int term	Ln (BallHgtMM)	Ln(Half DLengthMM)	Ln (CTEppmC)	Ln(DeltaT)
Co-eff	1.64	0.050	0.402	-0.31	-1.09	-1.584

ANOVA

Table 17 ANOVA - Ridge (Goldmann)

Source	DF	Sum of squares	Mean Square	F	P
Model	5	11.5452	2.3090	15.08	0.0001
Error	98	15.003	0.1530		
Total	103	26.5482			

Upon plugging in the above values on to equation and taking anti-log, we end up with,

$$N_{1\%} = C\left[\alpha_{rel}^{-1.09} d^{-0.3174} \Delta T^{-1.58} \left(\frac{\pi r_f^2 h}{V}\right)^{0.05} h^{0.4}\right]$$

5.3 Model Comparison

Goldmann`s Model

$$N_f = C\left((\alpha_{rel})^{-1.9} (d)^{-1.9} (\Delta T)^{-1.9} \left(\frac{\pi r_f^2 h}{V}\right)^{3.275} (h)^{1.9}\right)$$

PCR Model

$$N_{1\%} = C\left[\alpha_{rel}^{-1.4} d^{-0.3} \Delta T^{-1.6} \left(\frac{\pi r_f^2 h}{V}\right)^{0.05} h^{0.4}\right]$$

Ridge Model

$$N_{1\%} = C\left[\alpha_{rel}^{-1.09} d^{-0.3174} \Delta T^{-1.58} \left(\frac{\pi r_f^2 h}{V}\right)^{0.05} h^{0.4}\right]$$

The above equations are in close comparison. The equations 15 and 16 were created using accelerated data of SAC alloy using two different methods and show close resemblance. Model validation would reveal which model would work better.

5.4 Model Validation

The Validation is done using data outside of the data set used for regression. A plot with the predicted and actual life of package would give an idea of the reliability of the model.

Table 18 Model Validation Dataset (Goldmann)

Package No	$\left(\frac{\pi r_f^2 h}{V}\right)$	Ball Height	Diagonal Length	Diff in CTE	ΔT
1	3.31	0.36	3.95	3.50E-06	135
2	5.52	0.36	3.95	3.50E-06	180
3	2.12	0.19	3.10	5.00E-06	180
4	5.52	0.36	3.95	3.50E-06	135
5	2.12	0.19	3.10	5.00E-06	135

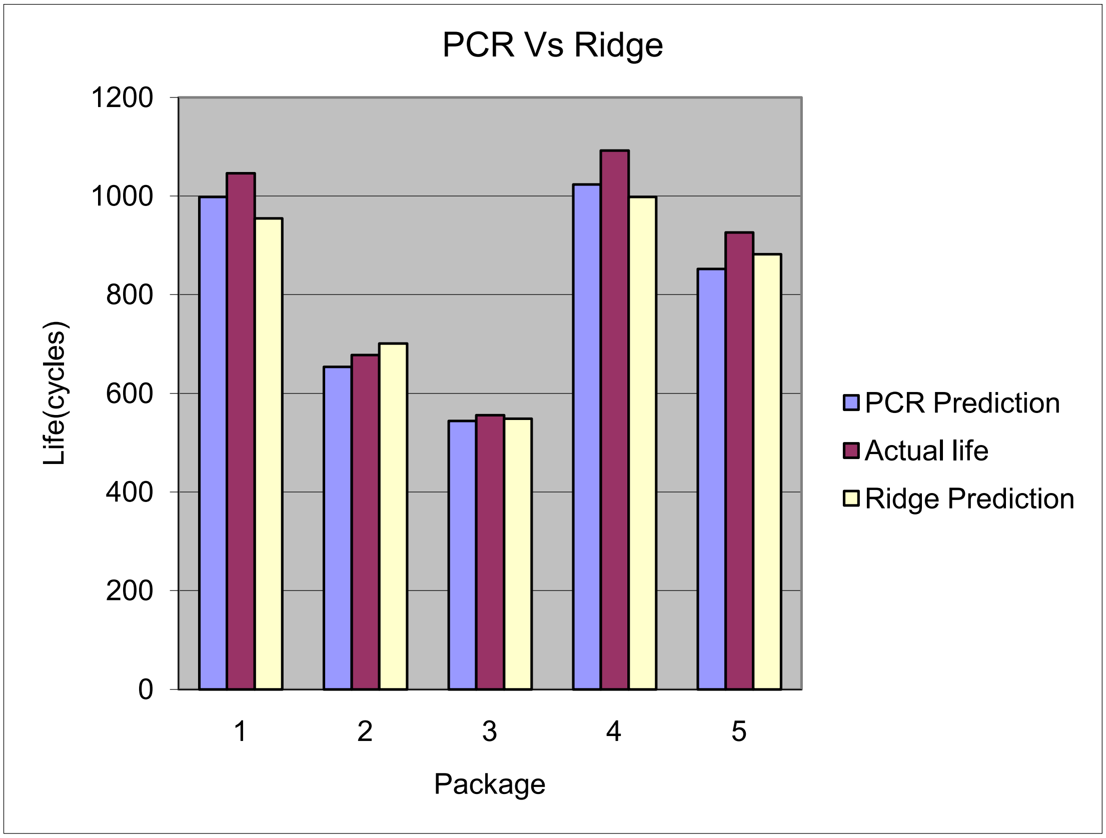


Figure 18 Model Validation (PCR Vs Ridge)

5.4 Model Validation (Parameters)

The Validation is done using data outside of the data set used for regression. A plot with the predicted and actual life of package would give an idea of the reliability of the model.

5.4.1 Half Diagonal Length

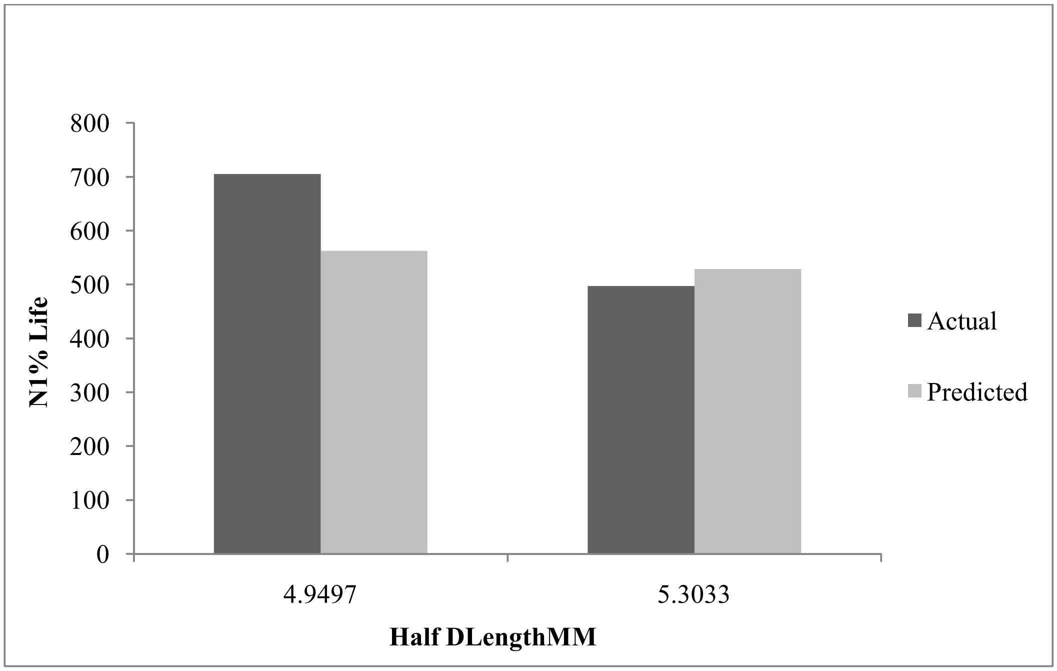


Figure 19 Effect of Die Length

The distance of the solder ball from the neutral axis of the chip increases the stress induced in the solder ball. The increase in diagonal length means more number of solder balls away from the neutral axis. Hence the half diagonal length has negative influence on life. The plot shows the same trend.

5.4.2 Delta T

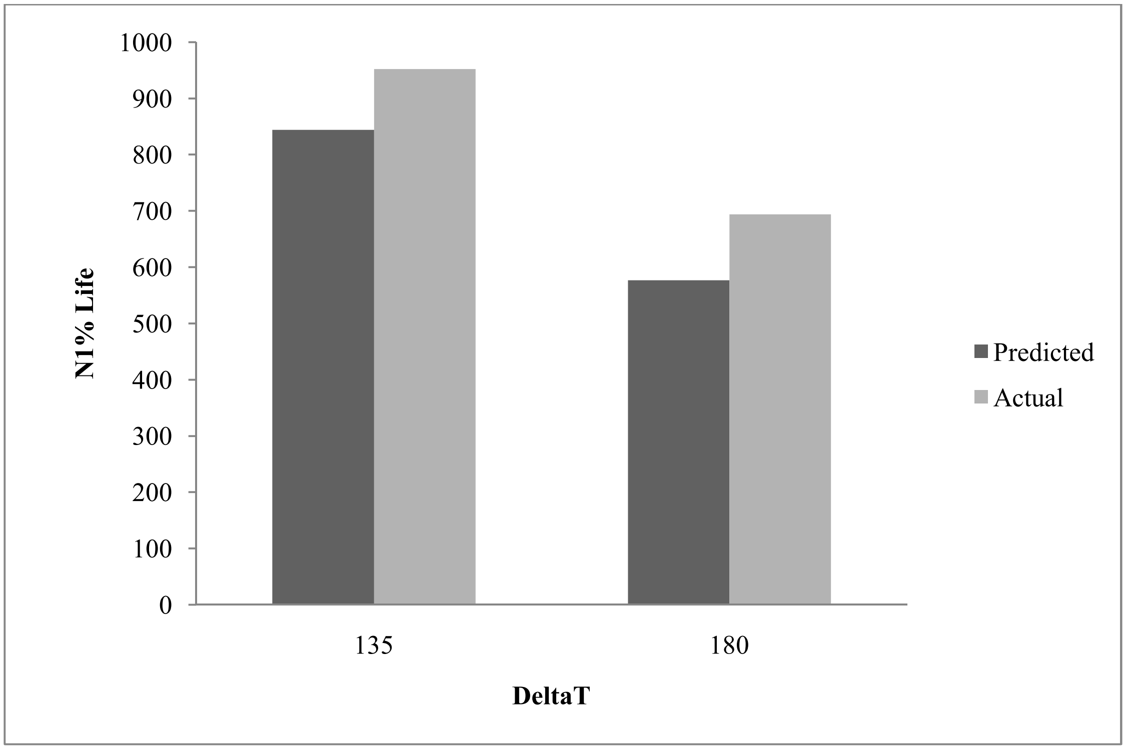


Figure 20 Delta T Validation

The temperature difference between the minimum and maximum temperatures is the most significant variable on a thermal cycle. The temperature excursion (Delta T) has square dependency on life. The plot shows the same trend. The packages looked at have the same dimensions and test conditions but undergo different thermal cycle. The package undergoing lesser temperature excursion has better life.

5.5 Model Predictions

The model is used to predict life and the model predictions are plotted against the actual life to see the accuracy of the model. The dotted lines represent the 90% interval. As seen from the plot, most of the model predictions fall in the interval.

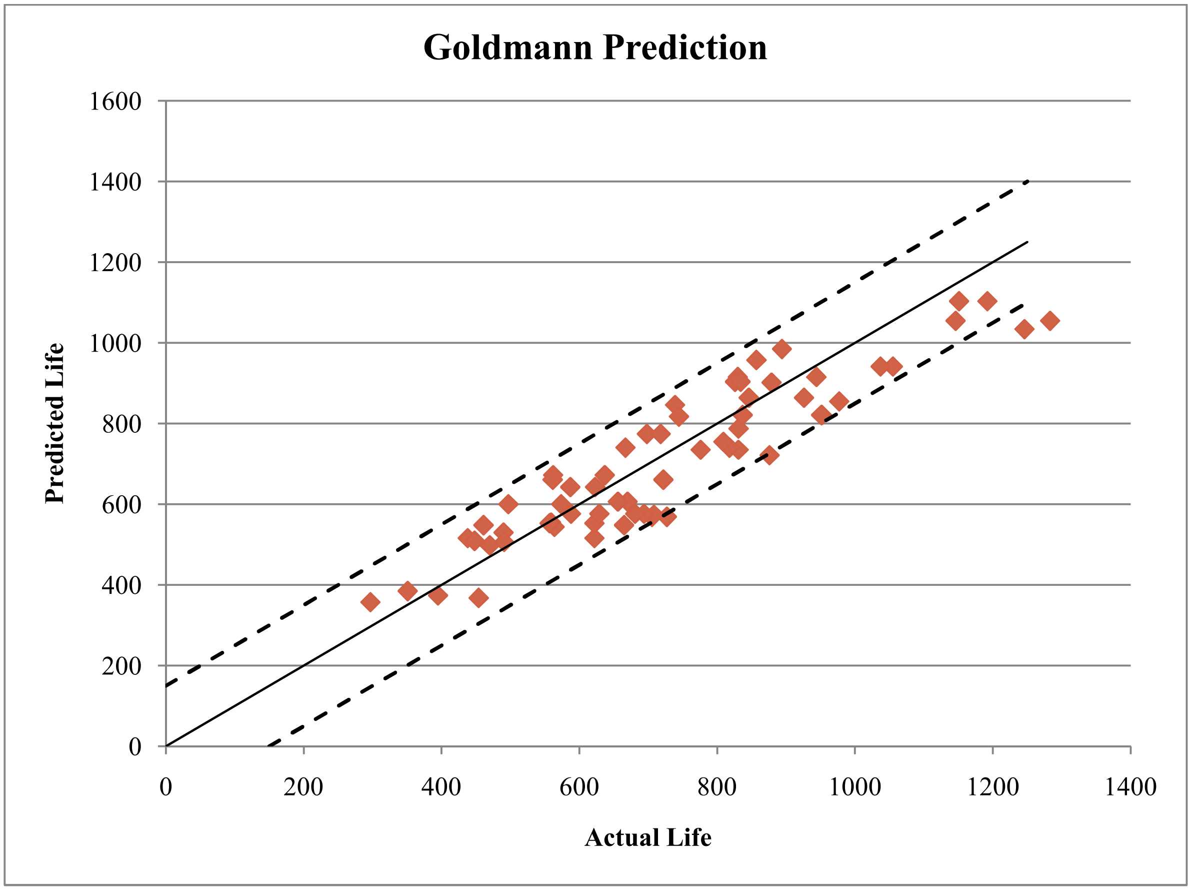


Figure 21 Goldmann Prediction

Chapter 6

Summary and Conclusions

Statistics-based modeling methodologies have been presented in this work. The methods have been used for development of Norris-Landzberg Acceleration Factors and Goldmann Constants for area-array packages with Sn3Ag0.5Cu solder alloy interconnects. The methodology is based on accelerated test data. Test-boards with various package architectures, tested under different temperature ranges, dwell times, maximum temperature, and minimum temperatures have been included in the dataset. The time to one percent failure and characteristic life of the weibull distribution has been used for the model development. The models developed have been validated with experimental data in a number of different ways. Life has been predicted for a completely different data-set and the error in the model predictions quantified. In addition, change in thermo- mechanical fatigue life versus individual parameter variations has been studied for a number of test cases. The model predictions in each case have been correlated with experimental data and Weibull distributions presented for each case. The presented approach provides a method for institutional learning based on databases of accelerated test data developed for product specific applications. The closed form models are a time effective solution for doing trade-offs and the thermo- mechanical reliability assessment of the area-array packages on PCB assemblies subjected to extreme environments. The developed methodology also allows the user to understand the relative impact of the various geometric parameters, material properties and thermal environment on the thermo-mechanical reliability of the different configurations of area array devices with leaded as well as lead-free solder joints. The influence of silver content on the life of SAC alloy based packages has been assessed and the model has been validated. A wide range of SAC alloys like SAC 105,305,405, 387 and other not very common SAC alloys have also been used for the model, making the model dependable. The convergence between experimental results and the model predictions with higher order of accuracy than achieved by any first order closed form models has been demonstrated, which develops the confidence for the application of the models for comparing the reliability of the different BGA packages for various parametric variations. The current approach allows the user to analyze independent as well as coupled effects of the various parameters on the package reliability under harsh environment.

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