Pre-service Teachers' Computational Knowledge, Efficacy, and Number Sense Skills

by

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Abstract

The National Council of Teachers of Mathematics (NCTM) Curriculum Focal Points (2006) suggested heavy emphasis on instruction in whole numbers for young elementary students. Any intervention curriculum for students who are at-risk for mathematic difficulties should not be oversimplified. Number sense was defined by Berch (1998) as a developing construct that referred to: (a) children's fluidity and flexibility with numbers, (b) the sense of what numbers mean, and (c) the ability to perform mental mathematics and look at the world and make comparisons. Additionally, the Glenn Report (US Dept. of Education, 2000) outlined the need for improving teacher preparation in mathematics and science. Currently, there is scarce research about students with disabilities' number sense skills or teachers' aptitude regarding number sense skills. Therefore, this study surveyed elementary, special education and general education teachers' mathematical content and pedagogical knowledge with an added focus on number sense. Specifically, special education and general education teachers' computational knowledge, efficacy to teach mathematics, and their approach to calculate math problems was explored.

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CHAPTER I. INTRODUCTION

To solve the dilemma of low student achievement, the *No Child Left Behind Act* (NCLB; 2002) held schools accountable for the adequate achievement of all students. This included students with disabilities. The NCLB Act proposed that student achievement would improve considerably and consistently so that all students (including students with disabilities) are proficient in reading and mathematics no later than 2013–2014. To confirm that all students demonstrate proficiency, each state has defined “adequate yearly progress” (AYP) to measure its schools’ achievement. These AYP standards were primarily based on state assessment results, but include high school graduation rates and school attendance rates. Evaluations of AYP must contain the progress of the majority (95%) of students with disabilities included in a school district’s AYP assessment. The NCLB Act was landmark legislation in that it recognized the needs of students with disabilities and mandated schools make changes to improve their academic achievement. In addition to the directives of NCLB (2002), teachers serving students with disabilities must adhere to the regulations of the *Individuals with Disabilities Education Improvement Act* (IDEIA, 2004). This means that special education teachers must ensure that students with disabilities receive access to the general education curriculum as well as meet standards for AYP. Researchers argued that teachers’ content knowledge, methodological training, and education were critical features in promoting student achievement (Kamil, 2003). Student achievement was higher when teachers were certified and when teachers possessed content area knowledge in the subject area taught through either a college major or minor (Kaplan & Owings, 2003; Laczko-Kerr & Berliner, 2002).

Mathematics Instruction

The National Council of Teachers of Mathematics (NCTM) Curriculum Focal Points (2006) suggested heavy emphasis on instruction in whole numbers for young elementary students. This position was strengthened by the 2008 report of the National Mathematics Advisory Panel (NMAP), which provided detailed benchmarks and emphasized in-depth coverage of key topics involving whole numbers as crucial for all students. Therefore, Milgram and Wu (2005) suggested an intervention curriculum for students who are at-risk for mathematic difficulties should not be oversimplified and that in-depth coverage of key concepts involving whole numbers was critical for success in mathematics. Fuchs, Fuchs, and Hollenbeck (2007) supported Milgram and Wu's proposition that research was needed with regard to effective curriculum and interventions to improve math achievement. In addition, Gersten et al. (2009) recommended that interventions for students with mathematical difficulties provide intensive instruction of whole numbers. Such a recommendation was important because, with increased competency in basic addition or subtraction facts, children develop or fail to develop number sense (Gersten & Chard, 1999). Number sense was defined as "moving from the initial development of basic counting techniques to more sophisticated understandings of the size of numbers, number relationships, patterns, operations, and place value" (National Council of Teachers of Mathematics [NCTM], 2000, p. 79). In addition, it was defined as a developing construct that refers to: (a) children's fluidity and flexibility with numbers, (b) the sense of what numbers mean, and (c) the ability to perform mental mathematics and look at the world and make comparisons (Berch, 1998). Research suggested number sense led to the automatic use of math information and was a key ingredient in solving basic arithmetic computations (Gersten & Chard, 1999). Therefore, Griffin, Case, and Siegler (1994) propose number sense as a necessary ingredient for learning formal arithmetic in the early elementary grades.

Teacher Mathematical Knowledge and Preparation

Mathematics instruction required understanding mathematical concepts and knowledge about how children acquire and apply mathematical skills. Therefore, teacher preparation as well as mathematical knowledge must be examined. A report to the nation from the National Commission on Mathematics and Science Teaching for the 21st Century, also known as the Glenn Report (US Dept. of Education, 2000) delineated the need for improving teacher preparation in mathematics and science. The report used findings from the Third International Mathematics and Science Study (TIMSS; 1995) and the National Assessment of Educational Progress (NAEP; 1996) to inspect the mathematics and science achievement of students in the United States compared to their peers in other countries. Glenn et al. (2000) stated the performance of students in the United States from both the TIMSS and NAEP studies was unacceptable. In response to students' low achievement, the Glenn Report set three goals. First, establish an ongoing system to improve the quality of mathematics and science teaching in grades K–12. Second, significantly increase the number of mathematics and science teachers and improve the quality of their preparation. Third, improve the working environment and make the teaching profession more attractive for K–12 mathematics and science teachers.

In addition to the Glenn Report's findings, research has shown that the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics* may not be effectively utilized by special education teachers (Maccini, Gagnon, & Calvin, 2002). The authors found that a significant number of special education teachers were not familiar with the goals of the NCTM Standards. The authors also expressed concern regarding the method of special education instruction in the area of mathematics and the mathematical content knowledge and pedagogical knowledge of special education teachers. Maccini et al.'s (2002) findings were disheartening considering the NCTM standards were a critical component of standards-driven instruction, and bearing in mind the goal that students with disabilities were to make adequate academic progress, thus achieving at grade level within the next decade.

Statement of the Research Problem

To date, there was a lack of information about pre-service educators' number sense abilities and preparing them to teach number sense skills to students with disabilities (Gersten et al., 2009). Therefore, the focus of this study was to explore elementary, special education and general education teachers' mathematical content and pedagogical knowledge with an added focus on number sense.

Justification for the Study

Farmer, Gerretson, and Lassak (2003) stated better teaching was the lever for change and effective professional development was the indispensible foundation for high quality teaching. Moreover, Griffin, Case, and Siegler (1994) suggested number sense as a necessary ingredient for learning formal arithmetic in the early elementary grades. Difficulties with numeracy interfere with acquisition of math skills later in childhood (Van Luit & Schoman, 2000). Mazzocco and Thompson (2005) analyzed test items on a psychoeducational test battery and found that subsets of items involving number sense (e.g., reading numerals, magnitude judgments, mental addition of one-digit numbers) accurately predicted students who would later develop math disabilities. In addition, Clarke and Shinn (2004) found aspects of number sense, such as magnitude comparisons and quantity discrimination, correlate with math achievement.

Researchers need to know educators' aptitude regarding number sense if pre-service teachers are to be prepared to provide the intensive mathematics instruction recommended by the NCTM and the NMAP. This is especially important for special education teachers because they are responsible for providing quality instruction that is not oversimplified and allows the in-depth coverage of key concepts involving whole numbers which is critical for success in mathematics (Milgram & Wu, 2005).

Purpose of the Study

The purpose of this study was to investigate special education and general education teachers' mathematical content and pedagogical knowledge with an added focus on number sense. This was accomplished through examining special education and general education teachers' computational knowledge, efficacy to teach mathematics, and their approach to calculate math problems by using number sense strategies, rule based strategies, or neither.

Research Questions

For this study the following research questions were developed:

1. To what extent is there a difference in the computational knowledge of special education teachers and general education teachers at the elementary level?

2. To what extent is there a difference in elementary level special education teachers' and general education teachers' personal efficacy to teach mathematical knowledge?

3. To what extent is there a difference in the mathematical outcome expectations of special education teachers and general education teachers at the elementary level?

4. In what ways do special education teachers and general education teachers approach calculating numbers, operations, and their relationships as defined by using number sense strategies, rule based strategies, or neither to solve computational problems?

Definition of Terms

In order to avoid any ambiguity, confusion, or misunderstanding in the usage of terms, a definition section has been added to this study.

AB Design: A single-subject design that includes a treatment phase and a phase where treatment is withdrawn to determine if there is a functional relation (Kennedy, 2005).

Adequate Yearly Progress (AYP): The means of establishing whether a school as well as a school district meets proficiency standards determined by the state (Yell, 2006).

Arithmetic: Mathematics that addresses addition, subtraction, multiplication, and division with integers, rational and real numbers and includes measurement, geometry and base ten (Jordan, Kaplan, Locuniak, & Ramineni 2007).

Concrete Representational Abstract (CRA) Sequence: An instructional sequence used in mathematics that provides instruction first through the use of objects, then through the use of pictures, and finally through the use of symbols (Mercer & Miller,1992).

Cuisenaire: Manipulatives shaped as rods of different sizes used to teach students place value and relationships among numbers.

Curriculum Based Measures: A way of monitoring a student's progress through timed assessments.

Direct Instruction: A method of providing instruction that is structured, systematic, and eliminates misinterpretations through clear expectations. It can be a scripted program with a step-by-step format and is usually fast-paced. It includes continuous modeling by teachers, followed by more limited teacher involvement and then fading teacher involvement as students demonstrate mastery of the material (Montague & Bos, 1986).

Discovery Oriented Instruction: A method of instruction which is inquiry based, and is shaped by a student's natural motivation to learn (Gardner, 1990).

DRAW: A mnemonic device that helps remind students the procedure for completing mathematics problems. The DRAW strategy has four steps: (a) discover the sign, (b) read the problem, (c) answer with a conceptual representation, and (d) write the answer (Mercer & Miller, 1992).

Emotional Behavior Disability: A category of special education eligibility which is defined as a condition exhibiting one or more of the following characteristics over a long period of time and to a marked degree that adversely affects a child's educational performance:

(A) An inability to learn that cannot be explained by intellectual, sensory, or health factors.

(B) An inability to build or maintain satisfactory interpersonal relationships with peers and teachers.

(C) Inappropriate types of behavior or feelings under normal circumstances.

(D) A general pervasive mood of unhappiness or depression.

(E) A tendency to develop physical symptoms or fears associated with personal or school problems (IDEIA, 2004).

Explicit Instruction: Teacher directed instruction that is very organized and task oriented. Concepts are presented in a clear, direct manner. Students respond to instruction and receive immediate feedback (Miller, 2009).

FAST DRAW: A mnemonic device that helps remind students the procedure for completing mathematics problems. The steps of FAST DRAW are as follows: (a) read the problem out loud; (b) find, highlight the question, then write the label; (c) ask what are the parts of the problem, then circle the numbers needed; (d) set up the problem by writing and labeling the numbers; (e) re-read the problem and decide if addition or subtraction is required, (f) discover the sign by rechecking the operation; (g) read the number problem; (h) answer the problem and; (i) write the answer and make sure the answer makes sense. (Cassel & Reid, 1996)

Individuals with Disabilities Education Improvement Act (IDEIA): The Individuals with Disabilities Education Improvement Act (IDEA) is a law ensuring a free, appropriate public education to children with disabilities throughout the nation. This act protects people from birth to age 21 who qualify under the 13 categories established by the federal government and must show evidence the disability impacts educational performance. Examples of the 13 categories are specific learning disability, other health impairment, emotional behavioral disturbed, mild, moderate, or profound intellectual disability, deaf/blind, autism, visual impairment and blind, deaf or hard of hearing, traumatic brain injury, or speech language. There are six principles of the law. They are zero reject, individualized educational program, parent participation, least restrictive environment, due process, and testing safeguards (Yell, 2006).

Maintenance Data: Data which are taken using single subject methodology that measures whether a student has mastered a skill or concept being taught (Kennedy, 2005).

Mild Intellectual Disability: A category of eligibility defined as a significantly subaverage general intellectual functioning, existing concurrently with deficits in adaptive behavior and manifested during the developmental period, that adversely affects a child's educational performance (IDEIA, 2004).

Mnemonic Device: A small phrase or rhyme used as a memory tool.

Montessori Instruction: Instruction in which children teach themselves through a prepared environment that enables students to freely choose from a number of developmentally appropriate activities. Students receiving instruction learn at their own, individual pace (Olaf**,** 2011).

Multiple Baseline Design: A single subject research design which examines an effect of an intervention through replicating a change in behavior at least three times across different people, settings, or behaviors. This design is used when a behavior cannot be unlearned (Kennedy, 2005).

Multisensory Instruction: Instruction that involves more than one sense (visual, auditory, kinesthetic or seeing, hearing, touching) simultaneously in order to enhance memory and learning (Orton, 1996).

No Child Left Behind Act: The No Child Left Behind Act (NCLB) is a federal law that mandates educational policies which strive for all children to meet state academic achievement standards. Guiding principles are accountability for results and student progress, evidence based practices, highly qualified teachers, more flexibility at the local and state levels, and more choices for parents. Schools show evidence of progress for all students through statewide assessment tests. A school must make adequately yearly progress which means that all students have demonstrated progress for the school year. Evidence based practices means that professionals use educational practices that are research based. Highly qualified teachers means that teachers must show competency in the areas in which he/she teaches. Flexibility at the state and local levels deals with reducing the "red tape" involved in providing educational services and the administration procedures involved. More choices for parents mean that if a school does not make adequate yearly progress then the parents of children who attend that particular school may chose to move the child to a different school that did make adequate yearly progress (Yell, 2006)

Number Sense: Number sense is moving from the initial development of basic counting techniques to more sophisticated understandings of the size of numbers, number relationships, patterns, operations, and place value. Number sense is a developing construct that refers to: (a) children's fluidity and flexibility with numbers, (b) the sense of what numbers mean, and (c) the ability to perform mental mathematics and look at the world and make comparisons (Berch, 1998; National Council of Teachers of Mathematics, 2006).

Numeracy: A synonym for number sense.

Problem Solving: Solving mathematical problems that involves ignoring extraneous information, organizing a strategy to solve the problem, completing steps required to solve the problem, representing the word problem using number equations, and computing basic facts (Jitendra et al., 1998).

Probe: An assessment that monitors a student's progress toward a behavior goal.

Schema Approach: Instruction that involves a strategy of classifying different types of word problems with graphic representations. Students define the problem by characteristics, features and facts and represent the situation described in the problem. Then students select the procedure (e.g., counting, adding, subtracting). Last, students use the procedure to reach the solution (Jitendra & Hoff, 1996).

Social Validity Data: Data taken that examine the social importance and practicality of an intervention as well as the intervention outcomes.

Specific Learning Disability: A category of special education eligibility defined as a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, that may manifest itself in the imperfect ability to listen, think, speak, read, write, spell, or to do mathematical calculations, including conditions such as perceptual disabilities, brain injury, minimal brain dysfunction, dyslexia, and developmental aphasia (IDEIA, 2004).

Supplemental Instruction*:* Instruction that is provided in addition to the curriculum used to teach content knowledge.

Verbalization Strategies: A strategy that involves having students verbalize and plan a solution to a given situation. Students solve math problems by verbalizing the problem, justifying the decision chosen to solve the problem, and explaining why other options are not correct (Naglieri & Das, 1997).

Limitations of the Study

The study surveyed elementary general and special education pre-service teachers' mathematical computation skills, personal efficacy in providing mathematic instruction, outcome expectancies, and number sense strategies to solve problems. The majority of the sample came from one geographical region of the country; thus, the results may not be representative of the whole country. Also the distribution of respondents was not even in that 70 percent of participants were in a general education program and 30 percent of the respondents were in a special education program. A limitation in the examination of number sense strategies was the amount of pre-service educators who left open ended questions blank. A large amount of teachers did not answer how they solved the problem. Yang used an interview format which was better suited to probe pre-service educators' reasons for answering questions and gathered more descriptive detail that enriches the study. This study addressed only skills and number sense strategies of pre-service educators from kindergarten through the sixth grade.

Summary

There was little research about the number sense skills of students with disabilities, or educators' aptitude regarding number sense (Gersten et al., 2009). Additionally, the Glenn Report (US Dept. of Education, 2000) expressed the need for improving teacher preparation in mathematics and science. Therefore, teacher preparation as well as mathematical knowledge must be examined. Researchers need to know educators' utilization of number sense strategies to prepare pre-service teachers in providing the intensive mathematics instruction recommended by the NCTM and the NMAP. This study surveyed special education and general education teachers' mathematical content and pedagogical knowledge with an added focus on number sense. This research provided information for preparation of pre-service special education teachers in furnishing access to mathematical instruction and raising the mathematical achievement of students with disabilities.

CHAPTER II. LITERATURE REVIEW

Introduction

The Glenn Report set three goals for improving teacher preparation in mathematics and science. First, establish an ongoing system to improve the quality of mathematics and science teaching in grades K–12. Second, significantly increase the number of mathematics and science teachers and improve the quality of their preparation. Third, improve the working environment and make the teaching profession more attractive for K–12 mathematics and science teachers. This literature review examines research that contributes to improving the quality of teacher preparation in mathematics instruction for elementary students who have a disability or mathematical difficulty with a particular focus on number sense. This review is broken into two major parts. First, a general examination of research on instructional interventions for students with disabilities or mathematical difficulties recommended by Gersten et al. (2009), the National Council of Teachers of Mathematics (NCTM; 2006), and the National Mathematics Advisory Panel (NMAP; 2008) is inspected. The criteria for selecting intervention studies were as follows: (a) the research was applicable to national benchmarks in early math curricula, (b) the research examined an intervention for students at risk or students with disabilities, (c) the research was conducted on or after 1990, and (d) the research focused on student performance outcomes as a dependent variable. The second major component of this review was an analysis of research that investigated teacher preparedness regarding elementary mathematics. The criteria for selecting teacher preparation studies were as follows: (a) the research was applicable to national benchmarks in early math curricula, (b) the research examined teacher knowledge or development pertinent to improving instruction for students at risk or students with disabilities, and (c) the research was conducted on or after 1990.

Research Regarding Problem Solving Instruction

Problem solving involves ignoring extraneous information, organizing a strategy to solve the problem, completing steps required to solve the problem, representing the word problem using number equations, and computing basic facts (Jitendra et al., 1998). Studies on interventions that teach word problem skills employ group and single subject research methods. The group designs utilized pretest and posttest measures to compare means, and the single subject design demonstrated functional relations using multiple baseline designs across subjects and behaviors. These studies are important for pre-service educators to learn because these interventions can be formed to assist students with math problem solving difficulties. Research suggests direct instruction, self-regulated strategies coupled with explicit instruction, and schema training that involved direct instruction improved students' word problem skills (Case, Harris, & Graham, 1992; Cassel & Reid, 1996; Jitendra & Hoff, 1996; Wilson & Sindelar, 1991).

Case, Harris, and Graham (1992) extended problem solving strategy instruction (Leon & Pepe, 1983; Montague & Bos, 1986) by focusing on word problem errors due to students choosing the wrong operation. The researchers examined effects of the self-regulated strategy intervention on the problem solving performance of fifth and sixth graders with Specific Learning Disability (SLD). The participants demonstrated Intellectual Quotient (IQ) scores between 75 and 125 on the *Wechsler Intelligence Scale for Children-Revised* (WISC-R; Wechsler, 1974) and achievement at least 2 years below age/grade level in one or more academic areas as measured by the *Woodcock-Johnson Psychoeducational Battery* (WJ; Woodcock, 1978).

The students learned the following strategy: (a) read the problem aloud, (b) look for important words, (c) draw pictures to tell what is happening, (d) write the math sentence, and (e) write down the answer. Instruction in this strategy consisted of the following components: (a) conferencing where performance and instructional goal was discussed, (b) discussion of the strategy using charts, (c) modeling of the strategy and self instruction, (d) mastery of the strategy steps, (e) collaborative practice, (f) independent performance, and (g) generalization and maintenance. Instruction was provided on an individual basis in the students' school. In addition to the strategy, Case et al. (1992) explicitly taught mathematics vocabulary, in which instructors demonstrated words using manipulatives and instruction continued until the students identified the words with 100% accuracy.

Measurements included twenty-five probes, each containing seven addition and seven subtraction problems. Addition consisted of four joining problems and three combining problems. Subtraction consisted of one separate problem, two comparison problems, two joining missing addend problems, and two combining problems.

The design was multiple baseline across subjects and across two behaviors. During baseline, the researchers assessed students' response rates on producing correct equations and correctly solving addition and subtraction word problems. During the intervention phases, the researchers assessed student's progress toward producing correct equations and correctly solving addition and subtraction word problems.

Case and colleagues (1992) reported successive increases in problem solving behaviors across students after instruction, thus demonstrated a functional relation. Students wrote the correct equation and correct answer for slightly over half (56%) of the 14 addition and subtraction items. Most of the baseline word problems completed correctly were addition (82%).

During the addition condition, immediately after learning to use the strategy, students wrote the correct equation, and obtained the correct answer for 95% of these types of problems. For two students, learning the strategy for addition problems had a positive carryover effect for subtraction problems. On probes immediately administered after the first instructional phase, errors for subtraction problems decreased from 61% to 39% for one student, and from 76% to 43% for the other. Conversely, two students demonstrated a decrement in performance where they overgeneralized and added all problems. During the subtraction condition, students wrote the correct equation and correct answer for 82% of the subtraction problems compared to only 30% during baseline. Maintenance data were collected 8 to 13 weeks after instruction and students wrote the correct equation and answer on 88% of the problems. Social validity data were collected via student and teacher interviews. Students spoke positively about the importance of the strategy steps, instruction helped them concentrate better, and they valued sharing what they were doing with their teacher. The teacher indicated she wanted to use the strategy for the upcoming year and students' concentration during seatwork improved.

Rather than focusing on the specific mathematical operations involved in word problems, Wilson and Sindelar (1991) taught students to differentiate between types of word problems. Wilson and Sindelar expanded research that suggested problem solving instruction must include sequencing, adequate practice, cognitive strategies, explicit instruction and generalization techniques (Darch, Camine, & Gersten, 1984; Fleischner & O'Loughlin, 1985; Jones, Krouse, Feorene, & Saferstein, 1985; Montague & Bos, 1986). Wilson and Sindelar used a general strategy that taught students that problems could be classified and provided explicit instruction in how to solve each type of problem. They incorporated a direct instruction strategy and sequencing practice of word problems as a means of improving students with learning disabilities' performance on addition and subtraction word problems. To summarize, Wilson and Sindelar (1991) incorporated direct instruction that taught students a general strategy in classifying types of math problems.

There were 62 participants with learning disabilities from nine elementary schools and the criteria for participation were as follows: (a) participants were labeled SLD according to district criteria, (b) participants attended a special education math program, (c) participants scored at 80% or better on a test of basic addition and subtraction skills, (d) participants read on at least a 1.5 grade level, and (e) participants were identified by their teacher as needing instruction in word problem solving. Students were provided instruction in groups of three to five and it took place in classrooms, office space, the media center, and school cafeteria. Each lesson was 30 minutes and intervention lasted approximately one month.

The design of the study was a pretest and posttest design that compared three intervention groups: strategy only, sequence only, and strategy and sequence. The intervention consisted of detailed, scripted lesson plans. All groups received the same number and identical sets of word problems. There were 216 word problems used in a pretest, posttest, and follow up test. Word problems were divided into four main types: simple action problems, classification problems, complex problems, and comparison problems. The students were administered a two-part pretest. The first part was a test of basic addition and subtraction facts that involved single digits, and was used to select students to participate in the study. The second part of the pretest consisted of 24 randomly selected word problems that included 3 addition and 3 subtraction of each of the four problem types. The post test and follow up test were equivalent forms of the word problem part of the pretest. The posttest was given to all students at the end of the 3-week instructional period and the follow-up test was administered 2 weeks after the conclusion of the study. On all three of the tests, students were assessed on the ability to write the correct algorithm, but were not scored on the accuracy of computing answers. Procedural reliability measures were taken to ensure fidelity of treatment. Each instructor was observed twice during the course of the study. Instructors in the strategy-plus sequence and strategy-only conditions were judged on the presence or absence of six elements of their scripted lessons; instructors in the sequence-only condition were judged on seven elements. The criterion for reliable administration of the lessons was at least 80% "yes" responses. The reliability scores averaged 89% and ranged from 83% to 97%. Analysis of covariance (ANCOVA) was used to examine data, with the pretest serving as the covariate. Separate ANCOVAs were conducted on the posttest and the follow-up tests. The analysis of the data indicated significant differences among the groups on both the posttest and the follow-up test. Students in the strategy-plus sequence group scored significantly higher on both tests than did students in the sequence-only group. Students in the strategy-only group also scored significantly higher on both the posttest and the follow-up test than did students in the sequence-only group. On the posttest, the difference between the strategy-plus sequence and strategy-only groups was not significant; yet on the follow-up test, the strategy-plus sequence group scored significantly higher.

Cassel and Reid (1996) incorporated the self-regulated strategy, Concrete Representational Abstract strategies, and explicit instruction for word problem solving (Case et al., 1992; Marzola, 1987; Mercer & Miller, 1992; Montague & Bos, 1986). Students were taught the "FAST DRAW" mnemonic method to solve problems. The steps of FAST DRAW were as follows: (a) read the problem out loud; (b) find, highlight the question, then write the label; (c) ask what are the parts of the problem then circle the numbers needed; (d) set up the problem by writing and labeling the numbers; (e) re-read the problem and decide if addition or subtraction is required, (f) discover the sign by rechecking the operation; (g) read the number problem; (h) answer the problem and; (i) write the answer and make sure the answer makes sense. The FAST DRAW technique was coupled with self-speech in which students generated statements to help solve the problem and recorded them on a strategy check off sheet. Examples of these self instructions were: (a) to find the question, look for the sentence ending with a question mark; (b) when setting up the problem, remember to write the large number on top; (c) to tie down the sign ask, "Am I putting together so my answer will be larger than the others, or am I taking apart so my answer will be smaller than the largest number?" The FAST DRAW technique and self instruction strategy were taught through explicit instruction that provided modeling, guided practice, practice to mastery, and independent practice.

Participants using FAST DRAW were two third and fourth grade students who received special education services for mild intellectual disability. Before the study, each student completed a 20-problem math test that included addition and subtraction facts as well as one-step addition and subtraction word problems. All students were able to answer 80% of the pretest facts, yet were not able to answer the word problems. Ninety-six percent of the errors made were due to selecting the wrong operation to solve the word problem.

First the instructor taught students how to identify a relationship between subtraction and addition problems using manipulatives. Students manipulated objects while verbally explaining each relationship until a mastery criterion of 80% was reached for each concept. After mastery, a conference was held with each student to discuss the student's performance and baseline probes. A bar graph was used to provide a visual representation of performance. The student and instructor discussed the goal of instruction, and the student signed a contract.

Measurements included post-instruction probes. Data for each student were collected following successful mastery of a strategy for each pair of problem types, change/equalize (Phase 1) and combine/compare (Phase 2). Two maintenance checks were taken 6 to 8 weeks after completion of post instruction Phase 2. The dependent measures were the number of problems that each equation and answer was correctly derived and the number of correct addition and subtraction problems. Each was graphed separately. Interobserver agreement was calculated on 40% of the data by dividing the total number of agreements by the total number of agreements plus disagreements. Agreement of the number of correct change/equalize equations, number of correct combine/compare equations, the number of correct change/equalize equations and answers, and the number of correct combine/compare equations and answers was .98, .97, .96, and .97 respectively. In addition, each problem was examined to determine if students performed strategy steps. This included whether students highlighted the question, circled numbers, placed the larger number at the top of the problem, and wrote a label. Percent of strategy step was determined by the number of times in a phase the step was used divided by the total number of problems in the phase.

The design of the study was multiple baseline across subjects. During baseline, pre-instructional rates on producing correct equations and correctly solving word problems were established. Response rates were established by collecting multiple probes spread across the baseline periods. Phase 1 intervention started with the first student after a stable baseline was established and continued until she demonstrated independent mastery of the strategy on change/equalize problems. Identical procedures introducing and terminating instruction were followed with the other students. Procedures for introducing and terminating instruction in Phase 2 were identical to those in Phase 1. Once again following the instruction phase, word problem probes were administered following strategy instruction with combine/compare problems. Maintenance probes were administered 6 and 8 weeks after the completion of Phase 2.

Following completion of instruction on change/equalize problems, all students showed an immediate increase in performance levels. After instruction in combine/compare problems, mean levels of correct equations and answers increased for all students. The students increase in performance was similar or greater than the increases demonstrated in the previous phase. A functional relation was demonstrated with replication of increased performance across students and problem solving behaviors. Both maintenance levels for correct equations and answers for phases were stable and remained at levels consistent with treatment phases. The students' use of strategy steps either increased or remained steady in the first post-instructional phase. Use of a label and putting the larger number on top remained consistent for all students across the remainder of the study. For two students, the use of highlighting the question and circling the number decreased while the use of the strategy remained consistent for the other students.

Jitendra and Hoff (1996) extended the research on strategy based instruction by combining instruction in problem type with a schema approach, classifying different types of word problems with graphic representations. Participants were students enrolled in third and fourth grades, selected based on their teachers' judgments that they possessed adequate addition and subtraction computational skills but were poor word-problem solvers. Students had to meet three additional criteria for participation: (a) completion of addition and subtraction problems with 90% accuracy; (b) performance on simple action problems that involved phrase-by-phrase translation adopted from Silbert, Carnine, and Stein (1990) had to be at or above 90% accuracy;

and (c) scores on a criterion test of word-problem solving were required to be below 50%. The test consisted of 5 change, 5 group, and 5 compare word problems.

The essential elements of the intervention were categorized as problem schemata, action schemata and strategy knowledge. Three steps led to the solution of the word problem. First, the student defined the problem which involved processing schemata. The problem was defined by characteristics, features and facts, and one has to recognize and represent the situation described in the problem. The second step, action schemata, required the student to select the action procedure (e.g., counting, adding, subtracting). One used information present in the problem to select the appropriate action procedures. Therefore, arithmetic operation was selected based on which part of the problem situation was unknown and which of the critical elements in the problem structure represented the total. Third, strategic knowledge comprised a set of procedures, rules, or algorithms that can be used to reach the solution (Marshall, 1990, 1993).

Measurements that Jitendra and Hoff (1996) took were sets of simple, one-step story situations and word problems probes that included three problem types (change, group, and compare) based on Riley, Greeno, and Heller's (1983) categorization scheme. The only difference in the sets was the context and numerical values used which were randomly generated using two digit numbers between 10 and 90. The experimental design was a multiple baseline across students to assess the effectiveness of the schema strategy. The design began with a probe condition (PI) in which all students concurrently completed three probes across 3 days. Next, the problem schemata training phase was initiated with the three students in a small group session. When students reached 100% mastery in identifying and representing problem schemata, they completed a single probe (P2). Following the probe, the schema-based direct instruction strategy was introduced to teach word problems; participants were introduced to the strategy one at a time (intervention condition). Once a student reached criterion (100% correct for 2 consecutive days), the probe condition (P3) was repeated. The intervention with the second student was then implemented and followed by the probe condition sequence. The same sequence continued with the third student. The design ended with a maintenance condition for all students. All sessions were 40 to 45 minutes in length. Written responses were judged as correct or incorrect based on correct operation only because the study was limited to word problem solving rather than computational ability.

For Subject 1, performance in Probe Condition 1 (baseline) and 2 (following problem schemata training) was below 50% correct. Following the intervention, the mean scores for Subject 1 were 93%, 93%, and 80%, respectively. Maintenance probes were conducted 2 weeks after the last probe, yielding a score of 73% correct on Maintenance Probe 1 and 67% correct on Maintenance Probe *2.*

For Subject 2, performance in Probe Condition 1 (baseline) and 2 (following problem schemata training) was below 50% correct. Subject 2 also demonstrated improved performance on word-problem solving. Following intervention, the student's scores were 87%, 53%, and 93%, respectively. Two of these three scores show that the student improved her performance in condition 3. Maintenance probes conducted 3 weeks after the last probe yielded the following scores: 80% and 93% correct.

For Subject 3, performance in Probe Condition 1 (baseline) and 2 (following problem schemata training) was below 50% correct. Subject 3, improved performance on word-problem solving was seen from the first and second probe conditions to the intervention condition, with scores of 87%, 87%, and 93% correct, respectively. In addition, maintenance probes conducted

2 weeks later indicated a score of 73% correct on Maintenance Probe 1 and 87% correct on Maintenance Probe 2.

The researchers demonstrated a functional relation between the problem solving intervention and student performance. Increases in student performance were replicated across three students and each maintained increases in performance following instruction.

Social validity data were collected using student interviews. The students reported that the strategy was very useful. A checklist of instructional steps was used to ensure fidelity of implementation in which the instructor was rated according to whether or not he/she followed the scripted lesson. Independent-variable agreement was computed as 100 x agreements (agreements + disagreements).

About 20% of the lessons were observed for treatment integrity. During the problem schemata training condition, all aspects of the procedure were at 100%, except for using appropriate signals for student responding (95.1%; range = 92.3% to 100%). During the intervention condition, all were at 100% except for asking students to identify where the question mark (or unknown) was placed in the schema diagram (94.3%; range = 84.6% to 100%).

Jitendra et al. (1998) replicated the previous study but compared schema training to traditional word problem solving and used a group design instead of a single subject design. Schema instruction used representations that delineated the relationships among the key components of word problems. It entailed identifying if the problem was a change, group, or compare problem. Direct instruction was used to explain the principles that involved teacher lead demonstrations, modeling, and frequent student exchanges to identify the critical elements of the problem. Later the instructor would explain how to find the amount based on the information provided in the problem. Traditional instruction was derived from the *Addison-*

Wesley Mathematics (Eicholz, O'Daffer, & Fleenor, 1985) basal mathematics program. It consisted of a checklist procedure to solve word problems. The procedure consisted of: (a) understanding the problem, (b) finding the needed data given in the problem, (c) planning what to do, (d) finding the answer, and (e) checking the answer by deciding if it is reasonable. All instruction consisted of 40- to 45-minute training sessions with small groups of 3 to 6 students in a quiet room in the school building. Scripted formats were used for each instructional condition.

There were 58 elementary students from four public school classrooms total that were included in the study. Twenty-four students who were typically achieving served as a control group and a normative sample to compare results. This group received traditional math instruction. The other participants (25 with high incidence disabilities), who were to receive the schema instruction, met the following criteria: (a) adequate addition and subtraction computation skills as demonstrated by completing problems with 90% accuracy, (b) solve simple action problems with 90% accuracy, and (c) performance of 60% on a 15-item criterion pretest of one step word problems.

The design of the study was group comparisons of students receiving schema intervention and students receiving traditional math instruction. Pre-test and post-test measurements consisted of researcher-designed curriculum-based problem solving tests including five of each problem type (i.e., change, group, compare). The forms were randomly generated by using two-digit numbers. To assess generalization of the strategy to word problems that were different from the controlled ones used during intervention, word problems from Grades 4 and 5 were selected from a basal mathematics program not used in the study. Differential effects of the schema and traditional strategy were examined as well as generalization of the content that was taught.

Pretest scores for the two conditions were analyzed using an analysis of variance (ANOVA) and no significant differences were found. A 2 X 2 analyses of covariance (ANCOVAs) with repeated measures was used to test for treatment effects and a significant main effect was obtained for group only. No differences for time or an interaction of group and test were found. Both groups showed an increase in scores from pretest to posttest and maintained their use of problem-solving skills. ANOVA was used to assess differences between groups from pretest to generalization test. Significant main effects for group, test time, and group X test interaction were found. Both groups made gains, but the schema group made more gains from pretest to generalization than the traditional group.

In summary, Jitendra et al. (1998) explain problem solving involves ignoring extraneous information, organizing a strategy to solve the problem, completing steps required to solve the problem, representing the word problem using number equations, and computing basic facts (see Table 1). Research literature suggests direct instruction, self regulated strategies coupled with explicit instruction, and schema training that involved direct instruction improved students' word problem skills. These studies are important for pre-service educators to learn because these interventions can be formed to assist students with math problem solving difficulties. The next section of this paper reviews research literature related to interventions that assist students with arithmetic difficulties.

Table 1

Problem Solving Studies

Authors and Date	Intervention	Participants	Design	Outcomes Measured
Case, Harris, & Graham (1992)	Self Regulation Strategy	4 students identified as SLD	Multiple Baseline across subjects and behaviors	Addition or subtraction operation chosen based on joining missing addend, comparisons, or combining amounts
Wilson & Sindelar (1991)	General strategy to classify problems with explicit instruction	62 students identified as SLD	Pretest/posttest comparison	Addition or subtraction operation chosen based on simplification, classification, comparison or complex problems
Cassel & Reid (1996)	CRA sequence, self regulation, and explicit instruction	4 students identified as MID	Multiple Baseline across subjects to criterion	Addition or subtraction operation chosen based on joining missing addend, comparisons, or combining amounts
Jitendra & Hoff (1996)	Classification of problem type with schema strategy	3 students identified as at-risk	Multiple baseline across subjects to criterion	Addition or subtraction operation chosen based on change, group, or comparison of amounts
Jitendra & Hoff (1998)	Classification of problem type with schema strategy	25 students identified with mild disabilities and 9 students identified as at-risk	Group comparison of schema strategy to traditional basal instruction	Addition or subtraction operation chosen based on change, group, or comparison of amounts.

Research Regarding Arithmetic Instruction

Jordan, Kaplan, Locuniak, and Ramineni (2007) found that deficits in fact mastery are highly persistent and appear to be independent of reading and language abilities. Deficits related to mastery of arithmetic facts are a key characteristic of students with mathematics difficulties (Jordan, Hanich, & Kaplan, 2003). Studies on interventions that teach arithmetic consisted of group and single subject research methods. The group designs utilized pretest and posttest measures to compare means, and the single subject designs utilized a baseline intervention method across subjects. One study correlated outcomes to procedures (Ho & Cheng, 1997). These studies involved interventions that are relevant for the teaching repertoire of pre-service educators as they assist students with arithmetic difficulties. This research related to arithmetic intervention suggests that instruction in place value and cognitive processes improved students' performance.

Mercer and Miller (1992) field tested the *Strategic Math Series* (SMS) curriculum that incorporated problem solving and basic arithmetic that involved subtraction and multiplication. This 1992 investigation expanded a pervious study that involved multiplication facts (Mercer & Miller, 1991). Miller and Mercer built upon prior research on Concrete Representational Abstract (CRA) sequence (Hudson, Petersen, Mercer, & McLeod, 1988; Petersen, Mercer, O'Shea, 1988) that demonstrated CRA as an effective way to teach place value and basic math facts to students with disabilities.

Mercer and Miller combined CRA with systematic instruction that combines fact retrieval and problem solving. This curriculum was called the *Strategic Math Series* (SMS). The SMS curriculum was divided into seven phases with 21 basic lessons (Table 2). Student completion of all 21 lessons was important for two reasons: (a) the lessons were sequenced and build upon each other in terms of complexity; and (b) although most students acquired the respective computation skill (e.g., multiplication facts) when they reached the posttest, they needed additional practice to maintain their knowledge and skills, to increase their fluency, and to ensure further development of their problem-solving skills.

Table 2

Phases of SMS instruction

Phases	Instruction	Procedures
Phase 1	Pretest	Contract
Phase 2	Concrete Application	Lessons 1–3
Phase 3	Representational Application	Lessons 4–6
Phase 4	DRAW Strategy Introduction	Student Masters Steps
Phase 5	Abstract Application	Lessons 8–10
Phase 6	Posttest	Repeat Phase 5 if below 90%
Phase 7	Practice to Fluency	Lessons 11–21

Phase 1 of the SMS curriculum was the pretest. During this instructional phase a pretest was administered to the student to determine whether instruction is needed. Before the pretest, a rationale for assessing the respective basic facts was discussed with the student. If his or her score on the pretest falls below the mastery criterion (i.e., 80%), the student was informed that he or she needed to work on the targeted basic facts. The need for instruction was discussed, and a commitment to learn was obtained from the student by a signed contract.

Phase 2 was teaching the concrete application. The concrete phase of instruction included Lessons 1 through 3. For each lesson, a sample script and learning sheets guided the teacher through the instructional sequence. During these lessons, students manipulated concrete objects to solve basic facts on their learning sheets. A separate curriculum manual was used for each skill area (i.e., addition, subtraction, multiplication, or division). Students solved word problems in which the numbers were vertically aligned and then wrote the name of the object on the space provided to the student. These concrete lessons acted as a spring board for learning facts at representational and abstract levels.

Phase 3 was teaching the representational application. This phase included lessons 4 through 6. A sample script and learning sheets guided the teacher through the instructional sequence. Students in this phase used drawings and tallies to solve basic facts where numbers were vertically aligned. Then students filled in blanks after the numbers with the name of the drawing.

Phase 4 was the introduction of the "DRAW" strategy which has four steps:

1. <u>D</u>iscover the sign
2. <u>R</u>ead the problem
3. <u>A</u>nswer with a conceptual representation
4. <u>W</u>rite the answer

Phase 5 was teaching the abstract application. This phase of instruction started in lesson 8 and continued to lesson 10. A scripted guide was used through the instructional sequence. Students used the DRAW strategy to solve abstract level problems when they were unable to recall an answer and they began to solve word problems in which numbers were vertically aligned. Students included the names of common objects and phrases after the numbers instead of blank spaces provided.

Phase 6 was a posttest to determine whether the student has learned basic facts and was ready to proceed to the next phase of instruction that built fluency skills. If the student's score was below 90%, he/she repeated one or more lessons in Phase 5.

Phase 7 was practice to fluency. Practice took place in lessons 11 through 21. Each lesson was scripted and students worked on three primary areas: (a) solving word problems, (b) increasing computation rate, and (c) discriminating previously learned facts from new facts with accuracy. Problems were presented in sentence form instead of vertically aligned. As lessons progressed, students filtered extraneous information and created their own word problems. A 1-minute timed probe was given during selected lessons to help students increase their rate of fluency. In addition, during selected lessons students received a fact review page that contained two or more types of facts. This practice helped students to discriminate between types of facts and provided practice of previously learned facts.

Mercer and Miller (1992) field tested SMS with 109 elementary students of whom 102 had a learning disability, 5 students had an emotional behavior disorder, and 2 were at risk. Field testing took place in small group (less than 7 students) and large group (7 to 18 students) instructional arrangements. The study had a pretest/posttest design. The pretest was conducted prior to Lesson 1, and the posttest was administered within 1 to 5 days after Lesson 21 was completed. The posttest included two problems with extraneous information and two problems without extraneous information, and it required the students to create two word problems and solve them. The posttest was conducted by examiners whom the students did not know. For problem solving, the initial rate data were collected after Lesson 8 (the first abstract lesson), and the posttest rate data were collected after Lesson 21. The follow-up data were gathered by examiners whom the students did not know.

Social validity data were gathered through teacher and student questionnaires. Of the 22 teachers who participated, 21 (96%) indicated they would use the SMS curriculum again. Of the 75 students who were asked to complete follow-up questionnaires, 60% rated SMS as better than other math instruction and 51 % rated it as equal to other math instruction. Thus, 90% rated the curriculum as equal to or better than other math instruction. Although teachers did give suggestions to make the curriculum more "user friendly", the educators concluded that SMS has positive consumer satisfaction.

Students were able to acquire the respective facts within Lessons 1 though 10. Total mean scores demonstrate that the average gain across skills was 59%. Moreover, the findings reveal that the students in the subtraction and multiplication groups were able to apply the DRAW strategy to solve computation problems that they were not taught. Across all skills, the mean rate improvement was 132 % after Lessons 9 through 21. The mean weekly percentage increase was 51% across skills, with a range of 31% to 69%. In multiplication, the students significantly reduced their error rates and increased their digits-correct rates. Miller and Mercer (1992) conclude that overall, the field test data indicated that students with learning problems were able to (a) acquire computational skills across facts, (b) solve word problems with and without extraneous information, (c) create word problems involving facts, (d) apply a mnemonic strategy to difficult problems, (e) increase rate of computation, and (f) generalize math skills across examiners, settings, and tasks.

Flores (2009) expanded on Miller and Mercer's research and investigated the use of visual representation to teach students how to subtract with regrouping. Flores demonstrated that the use of the concrete-representational-abstract (CRA) sequence was effective for teaching mathematics involving regrouping to students who struggle with learning mathematics. This instructional sequence involved three phases. First instruction involves using manipulatives to demonstrate the meaning of a particular concept. The second phase involved illustration of the mathematical process by using pictures to represent numbers. In a transition from representation to abstraction, students learned to use a mnemonic strategy to aid the computation and problem-solving process. The last phase involved memorization and continued until the students learned the operation or procedure automatically.

Participants were 6 third-grade students who were all failing mathematics in terms of grades and performance on district wide mandated benchmark assessments. In addition, none of the students wrote more than seven correct digits on a curriculum-based measure. The students received instruction during a regularly scheduled time in which teachers provided math practice and remediation in the general education classroom. Intervention was provided outside of the general education classroom in a conference room. During the concrete level, instruction involved manipulative objects. After three lessons with 80% accuracy or better, instruction progressed to the representational level. Instead of using Base-10 blocks, drawings were used. After at least three lessons and 80% accuracy, the DRAW strategy was introduced. The students moved to the next phase of instruction when they solved problems with at least 80% accuracy and could recite the DRAW steps accurately. The next phase of instruction was the abstract level, in which students were encouraged to answer problems from memory rather than by using drawings. They could use the DRAW strategy. After Lesson 10, instruction involved timed fluency activities. Maintenance measures were implemented 4 weeks after instruction had ended.

Treatment integrity was conducted during 30% of the lessons, using digital video. A treatment checklist for the intervention was used to ensure that procedures were implemented correctly. A graduate assistant was trained in using the treatment integrity checklists through demonstration and practice. When the graduate assistant completed a checklist with 100% accuracy, treatment integrity checks began. Treatment integrity was calculated at 100% for the study.

The study used a multiple probe design replicated across groups to evaluate the efficacy of CRA instruction for teaching subtraction with regrouping. The multiple probe across groups design was used to show a functional relation between the CRA intervention and a behavior that could not be reversed or unlearned. The data were interpreted by visual inspection, and the author noted the following data characteristics: overlap between each baseline and treatment, slope of each treatment data path, and number of data points from the beginning of each treatment to criterion. A functional relation was demonstrated between CRA and subtraction skills. All 5 students met the criterion of writing 20 digits on three consecutive 2 minute curriculum-based measures. There were no overlapping data points, and there was an immediate change in student performance between baseline and treatment conditions across all students. Of the students, 4 of 5 maintained their performance 4 weeks after the end of instruction.

Ho and Cheng (1997) investigated instruction similar to the CRA strategy for place-value concepts for arithmetic in Hong Kong. Place-value intervention employed counting skills, and activities of grouping, regrouping, and trading straws. Session 1 focused on reviewing and consolidating the children's oral and object counting skills through simple counting exercises. In Session 2, the children were presented with some straws as counting objects. They were asked to count and tie 10 straws into a bundle, and to put 10 bundles of straws into a glass. When counting the straws in bundles, they counted in tens, and when counting the straws in glasses, they counted in hundreds. After counting and grouping the straws, the researchers taught regrouping and trading concepts by giving more straws to the children or asking the children to give away some straws. The next session consisted of written numerals to indicate the quantity of objects. Straws and cards were used for this session, in which some cards had a decade number (i.e., “10,” “20,” . . . , “90”) while other cards had a unit number (i.e., “1,” “2,” . . . , “9”). Students used the cards to represent the bundle of straws. For example, the card “20” would represent two bundles of straws. Instructors would demonstrate that when the “3” card over the “20” card a new number “23” is made.

First grade students participated. They were grouped by skill level into three groups of 15 students, one performing above average and two performing below average. All students were assessed on intelligence, place-value understanding, and addition and subtraction skills. All three groups were matched in age, IQ scores, and gender ratio. Place-value intervention was administered to only one of the groups whose performance was below average. The groups were Control Group One, Control Group Two, and the Intervention Group.

Measures were taken before the study to establish a baseline for comparison with posttraining performance. Measures included the *Raven’s Standard Progressive Matrices* (Raven, Court, & Raven, 1995) as a measure of intelligence. Place-value understanding was assessed using local school curricula, and past research in place-value (Fuson & Briars, 1990; Fuson & Kwon, 1991, 1992; Sinclair, Garin, & Tieche-Christinat, 1992). Addition and subtraction probes were given that consisted of two digit and one digit, two digit and two digit, and three digit and two digit problems. Intervention consisted of five weekly 1hour sessions after school. Each session included direct instruction, demonstrations, games, class work, and homework. The place-value, addition and subtraction assessments were administered again postintervention.

Analysis was conducted using the Pearson correlation coefficients among the test scores for the students in the first assessment. IQ scores were found to correlate significantly with addition, and marginally significantly with subtraction. Correlations were still significant after controlling for effects of IQ. The findings suggest a significant relationship between place-value understanding and arithmetic skills. ANOVAs were used to analyze comparisons among group performance. Control Group Two did not differ significantly in any arithmetic tasks. The Intervention Group performance was significantly higher than preintervention scores in place-value understanding and addition. Subtraction performance for the Intervention Group was not significant. Control Group One showed some improvement in arithmetic tasks. Only performance in addition was significantly higher than preintervention scores. However, it needs to be noted that students in this group were fairly proficient in place-value tasks prior to instruction and Control Group One performed significantly better in all tasks than the other two groups. Still, the Intervention Group showed greater improvement in place-value understanding and addition after training in place-value concepts than did the two control groups.

Fuchs et al. (2005, 2007) broadened research on CRA by combining it with computer drills for math facts. Fuchs et al. examined effects of preventative tutoring based on the CRA construct for first grade math instruction. Tutoring involved small group instruction and computer practice. Small group tutoring was based on the concrete-representational-abstract (CRA) method (Butler, Miller, Crehan, Babbitt, & Price, 2003; Cass, Cates, Smith, & Jackson, 2003; Mercer, Jordan, & Miller, 1996). The CRA method relied on concrete objects to promote conceptual learning. Lessons followed a sequence of 17 scripted topics, and each topic included activities that relied on worksheets and manipulatives. During the final 10 minutes of each intervention session, students used the software *Math FLASH* (Fuchs, Hamlett, & Powell, 2003b), designed to promote automatic retrieval of math facts. Participants included students that were randomly assigned to four conditions. The four conditions were: (a) 69 students at risk for math failure as a control group, (b) 70 students at risk for math failure receiving tutoring, (c) 180 students designated as not at risk for math failure who were tested individually, and (d) 348 students designated as not at risk that were tested as a group. Due to students moving to other schools, the size of the groups changed to 63, 64, 145, and 292 respectively.

Seven measures were administered at pre and posttest. Curriculum-Based Measurement (CBM) Computation, Addition Fact Fluency, Subtraction Fact Fluency, First Grade Concept/ Applications, and Story Problems were used weekly to measure baseline and student progress (Fuchs, Hamlett, & Powell, 2003a). After week 7, CBM assessments were conducted every two weeks. In addition, the *Woodcock-Johnson III* (WJ III; Woodcock, McGrew, & Mather, 2001) Applied Problems and Computation subtests were used. At pretest reading skills, intelligence, language, nonverbal problem solving, phonological processing, processing speed, induction ability, and working memory were measured. Reading skills were assessed using the *Woodcock Reading Mastery Test-Revised* (WRMT-R; Woodcock, 1998) Word Identification subtest. The four-subtest *Wechsler Abbreviated Scale of Intelligence* (WASI; Psychological Corporation, 1999) was used to obtain Verbal, Performance, and Full Scale intelligence scores. WASI Vocabulary measures were used to assess expressive and receptive language, and the *Woodcock Diagnostic Reading Battery-Listening Comprehension* (WDRB; Woodcock, 1997) assesses the ability to understand sentences or passages. WASI Block Design and Matrix Reasoning was used to index nonverbal problem solving. Two assessments were administered to measure phonological processing: the *Comprehensive Test of Phonological Processing* (CTOPP; Torgesen et al., 2001) and *Rapid Digit Naming* (Wagner, Torgesen, & Rashotte, 1999). Process Speed was assessed using Cross Out from the WJ-R. Induction ability was measured using the WJ III Cognitive-Concept Formation subtest and working memory was assessed using the *Working Memory Test Battery for Children-Listening Recall* (WMTB; Pickering & Gathercole, 2001).

One-way analyses of variance (ANOVAs) were applied to pretest, posttest, and improvement scores on the seven mathematics dependent variables, using condition as the factor. On each measure, pretreatment performance differed by condition with students not at risk performing consistently higher. The improvements of students who received tutoring and were at risk were differentially high. On Story Problems, the improvement of students at risk who received tutoring reliably exceeded students at risk who did not receive tutoring. However, scores on Story Problems were still lower than students who were not at risk.

Naglieri and Gottling (1995, 1997) examined a different approach to math intervention. Instead of using visual representation to learn math concepts, they utilized verbal skills. Naglieri and Gottling accomplished this by investigating the effects of intervention using the PASS cognitive processes (Naglieri, 1989; Naglieri & Das, 1990) for math instruction in subtraction and multiplication. Pass cognitive processes involved having students verbalize and plan a solution to a given situation. The PASS intervention instructed students to solve math problems by verbalizing the problem, justifying the decision chosen to solve the problem, and explaining why each of the other options provided were not correct.

The participants were 12 students (6 girls and 6 boys) struggling in mathematics who attended a school that specialized in teaching students with learning disabilities. At the beginning of the study, students were given the *Cognitive Assessment System* (CAS; Naglieri & Das, 1997a, 1997b; Naglieri, Das, & Jarman, 1990) to assess competence in planning, attention, simultaneous, and successive processes. Results from the CAS were used to sort the participants according to their level of planning to obtain contrast groups. The low group had planning scores of 85 and below while the high group had planning scores of 100 and above. Each intervention session lasted 30 minutes and was provided two to three times a week. During the first 10 minutes, students were given the worksheet and were instructed to get as many correct as possible. The next 10 minutes were dedicated to discussion, and the last 10 minutes was designated as a working period. All students were given 21 sessions during the intervention phase and in each phase students solved 54 mathematics problems within 10 minutes.

The study was a descriptive in which contrast groups were organized to compare results of an intervention based on planning process scores. Probes consisted of 28 subtraction and 28 multiplication worksheets, which included one to three digits subtraction with and without regrouping and two-digit multiplication with 10 to 99, with and without carrying. Data analyses included calculating the number correct for each student's 28 math worksheets. The data show improvement for most students. The students with low planning scores improved from 44% to 205% over baseline and those with high planning scores improved from 6% to 159% over baseline.

Differences between contrast groups according to successive and attention scores were inconsistent with those obtained when planning scores defined the groups. Instead, those with low scores on successive processing showed a 40% less gain over baseline than those with high scores. Similarly, when contrast groups were defined by attention scores, the overall differences between baseline and intervention sessions were 57%.

Dev, Doyle, and Valente (2002) extended the use of visual and verbal representations to arithmetic through a multisensory method. They investigated the *Orton-Gillingham* and *TouchMath* systems of instruction to improve language and mathematics skills for 6- and 7-year olds. The *Orton-Gillingham* system was a multisensory approach to reading instruction (Orton, 1966; Sheffield, 1991). This approach incorporated the visual, auditory, and kinesthetic modalities; and teaches letter sounds as well as sound blends. The *TouchMath* system was also a multisensory approach to math instruction (Scott, 1993). This approach incorporated visual, kinesthetic, and tactile modalities. Number concepts were learned with the help of dots and circles printed on numerical symbols (Bullock, Pierce, & McClelland, 1989; Bullock & Walentas, 1989). Students learned basic math facts and mathematical operations. Students were gradually taught to solve mathematical problems without depending on the dots and circles.

Eleven participants were selected for the study based on teachers' referrals for evaluation of students in their kindergarten year. The participants were given the *Wide Range Achievement Test III* (WRAT III; Wilkinson, Stone, & Jastak, 1995) and scored below average in the basic areas of reading, spelling, and mathematics. The students in this study were taught according to the *TouchMath* system in the general education classroom for 25 to 55 min every day while they were in first grade. Instructions of the *TouchMath* system were followed as outlined in the manual that came with the kit (Bullock & Walentas, 1989). Those who needed a reminder reviewed *TouchMath* strategies in the second grade, but Dev at al. (2002) state none of the students required reteaching. Seven students were at the pre-first grade level in arithmetic, 3 students were at the early first-grade level, and 1 was at the intermediate first grade level. The 11 students were administered the WRAT-III again at the end of second grade in 1996 after the interventions. Eight of the eleven participants scored above the 2nd grade level in arithmetic. One student scored below grade level in arithmetic.

In summary, Jordan, Hanich, and Kaplan (2003) stated deficits related to mastery of arithmetic facts were a key characteristic of students with mathematics difficulties. Research literature suggested instruction in place value and cognitive processes improved students with math difficulties' arithmetic skills (see Table 3). These studies involved interventions that are relevant for the teaching repertoire of pre-service educators because students who struggle in mathematics have arithmetic difficulties. The next section of this review included studies of number sense interventions that assist students with number sense which was also referred to as numeracy.

Table 3

Arithmetic Studies

Authors and Date	Intervention	Participants	Design	Outcomes Measured
Mercer & Miller (1992)	CRA sequence combined with systematic instruction	102 students identified as SLD and 5 students identified as EBD	Pretest/posttest comparison	Basic addition and subtraction fluency coupled with problem solving
Flores (2009)	CRA sequence combined with systematic instruction	6 students identified as at-risk	Multiple baseline across subjects to criterion	Addition and subtraction fluency that required regrouping coupled with problem solving
Ho & Cheng (1997)	Method similar to CRA sequence	45 students that included students identified as at-risk	Pearson correlation and pretest/posttest comparisons	Place value coupled with 1-3 digit addition and subtraction fluency
Fuchs et al., (2005)	CRA sequence with computer drills	682 students that included students identified as at-risk	Pretest/posttest comparison	Computation of numbers Addition and subtraction fluency Concept applications (1st grade level)

				Story Problems that involve addition and subtraction operations without regrouping

Table 3 (continued)

Authors and Date	Intervention	Participants	Design	Outcomes Measured
Naglieri & Gottling (1997)	PASS verbalization method	12 students identified as SLD	Descriptive study with group baseline/ intervention comparisons	Basic subtraction and multiplication computation
Dev, Doyle, & Valente (2002)	Multisensory method	11 students identified as at-risk	Pretest/posttest comparison	Basic addition and subtraction computation

Definition of Number Sense

Number sense or numeracy is defined as "moving from the initial development of basic counting techniques to more sophisticated understandings of the size of numbers, number relationships, patterns, operations, and place value" (National Council of Teachers of Mathematics [NCTM], 2000, p. 79). Difficulties with numeracy interfere with acquisition of math skills later in childhood (Van Luit & Schoman, 2000). Mazzocco and Thompson (2005) analyzed test items on a psychoeducational test battery and found that subsets of items involving number sense (e.g., reading numerals, magnitude judgments, mental addition of one-digit numbers) accurately predicted students who would later develop math disabilities. In addition, Clarke and Shinn (2004) found aspects of number sense, such as magnitude comparisons and quantity discrimination, correlate with math achievement.

Howell and Kemp (2009) surveyed expert opinion on numeracy to identify exactly what skills and concepts might be indicative of number sense in young children and how they could best be measured. Howell and Kemp (2009) expanded on a previous investigation of number sense components (Howell & Kemp, 2005, 2006) and used a modified Delphi procedure to establish a consensus on tasks proposed to assess components of number sense that are essential for early mathematics success by a broad range of academics with expertise in the area of early mathematics. The participants were academics with recent published work in the area of early mathematics and/or ‘number sense’ identified via a literature search using the Education Resources Information Center (ERIC) and PsycINFO databases. Of the academics identified, 20 were invited to participate in the previous study and 27 were invited to participate in the current study. Of the 13 academics who agreed to participate in the previous study, 12 returned the first questionnaire. Of the 9 academics who agreed to participate in the current study, 6 returned the first questionnaire. Data was drawn from a total of 18 responses. There were 11 participants from Australia, 1 from New Zealand, 1 from the Netherlands, 3 from the UK, and 2 from the USA.

Tasks included as measures of number sense components were based on assessment tasks developed by early mathematics researchers. Participants responded to questionnaires by rating, on a five-point scale, their agreement with each task, with 5 indicating *strongly agree*, 4 indicating *agree,* 3 indicating *neutral,* 2 indicating *disagree* and 1 indicating *strongly disagree*. The *neutral* option was provided to save participants from a forced choice, which could reduce the accuracy of the survey. Howell and Kemp (2009) reported the responses to the first questionnaire only. Participants were also invited to include comments on each task and were given the opportunity to add clearly defined tasks that they felt should be included to assess the components of number sense presented in the questionnaire. A mean rating of 3.75 for a component of number sense was used as the criterion for inclusion of the component. Analysis of responses revealed that 19 proposed components of number sense reached the criterion of a mean rating of 3.75 but no component was rated as *strongly agree* or *agree* by all respondents.

Howell and Kemp (2009) concluded a relationship between numeracy and mathematics must be established before teachers can be confident that number sense is a prerequisite to successful mathematics performance. Once links between specific components of number sense and future achievement become established, instructional practices can be developed for teaching children who may have mathematics difficulties. In addition, differentiated instruction provided to children at risk of mathematics difficulties should address all the early mathematics competencies, including number sense.

Researchers are not sure of the best approach to teach number sense (Gersten & Chard, 1999). Berch (2005) explained researchers have not come to a consensus of the definition of number sense. Studies examined classification, seriation and conservation as broad categories of number sense. However, Jordan, Kaplan, Olah, and Locuniak (2006) identified key elements of number sense as counting, number knowledge, number transformation, estimation, and number patterns.

Research Regarding Number Sense Instruction

Intervention studies that teach numeracy concepts consisted of group methods, utilizing pretest and posttest measures to compare means. Pasnak, Holt, Campbell and McCutcheon (1991) employed a step-wise discriminate function analysis, while Bryant, Bryant, Gersten, Scammacca, and Chavez (2008) used regression discontinuity analyses. These intervention components that build pre-service educators' teaching repertoires are important because they assist students with number sense difficulties. Research suggests instruction in classification, seriation, conservation improved students with math difficulties' numeracy skills. Instruction was direct, discovery oriented, and involved guided practice and modeling techniques.

Pasnak and colleagues (1991) built upon the Piacceleration method (Gagne', 1968; Gelman, 1969; Klahr & Wallace, 1973) that had been shown to teach classification, seriation and number conservation to students with disabilities (Campbell, McCutcheon, Perry, & Pasnak, 1988; Lebron & Pasnak, 1977; Lopata & Pasnak, 1988; McCormick, Campbell, Pasnak, & Perry, 1990; Pasnak, Campbell, Perry, & McCormick, 1989). The Piacceleration method was a constructivist curriculum that used sets of objects to teach students how to classify, put numbers in sequence and conserve numbers. Pasnak et al. (1991) compared the effects of the Piacceleration curriculum to the standard curriculum for kindergarten students who were at risk. The curriculum consisted of 160 sets of items used to teach classification, seriation, and conservation concepts. Half of the items were oddity (classification) problems in which four items were similar and one was different based on size, shape, orientation, etc. Here were also 65 seriation problems, consisting of items that could be ordered based on a particular characteristic such as length, height, width or overall size. Intervention consisted of 40 lessons over a period of three months.

Sixty-five students participated in the study, thirty-five in the experimental group and thirty in the control group that received traditional math instruction. Measurements included the Classification, Omnibus, and the School Ability Index sections of the *Otis-Lennon School Ability Test* (O-LSAT; Otis & Lennon, 1989). The Mathematics and Listening to Stories (verbal comprehension) portions of the *Stanford Early School Achievement Test* (SESAT; Madden, Gardner, & Collins, 1982) were also used to measure academic achievement.

Results were analyzed by a step-wise discriminate function analysis. The O-LSAT components were most strongly affected by the two curricula. Classification was affected the most, followed by ability index and Omnibus. Univariate F tests confirm the Piacceleration curriculum was significantly better for classification and ability index subsets. No significant difference was found for Omnibus. An overall effect of the two curricula on the Mathematics scale was found. Univariate *F* tests confirm a significant difference favoring the Piacceleration curricula for the Concepts part of the Mathematics scale. Lastly, an overall effect of the two curricula on the Words and Stories subtest was found. Univariate *F* tests confirm a significant difference favoring the Piacceleration curricula for the Comprehension subscale.

Pasnak, Hansbarger, Dodson, Hart, and Blaha (1996) replicated the 1991 examination of the Piacceleration curriculum with two different samples of kindergarten students. Sixty-four participants from two different schools were identified by their teachers as being one of the eight having math difficulties and no identifiable cognitive deficits. Students were randomly assigned to a group receiving instruction on materials recommended by their teacher, and a group to receive instruction from the Piacceleration curriculum.

Assessments included the nonverbal pictorial and figural reasoning as well as verbal reasoning and comprehension assessment of the 0-LSAT, Level A, Form 1. The Mathematics and Listening to Stories (verbal comprehension) portions of the SESAT were used to measure academic achievement.

A group design was used to compare effects of the Piacceleration curriculum and traditional teaching groups. *A priori* t-tests employed the pooled mean square error to contrast experimental and control children within each school. Data were analyzed using 2 X 2 ANOVAs. Schools and groups (experimental versus control) were treated as independent variables. Effects were found for students' scores in the experimental group on all measures. In addition, there were no significant interactions between schools and the experimental/control condition.

Van de Rijt and Van Luit (1998) expanded on intervention that involved Piagetian operations and combined the method with guided or explicit instruction. They investigated the effectiveness of the *Additional Early Mathematics* (AEM) program for teaching students early mathematics using guiding or structured instruction. Guiding instruction was used in which the teacher observes and provides feedback to students as they engage in the problem solving process. Based on observations, the teachers chose the materials and activities that fit with the abilities of the student. Structured instruction was explicit and the teacher made suggestions and modeled solving the problem. The AEM program covered the Piagetian operations and counting skills with an emphasis on the development and knowledge of using counting skills. The program consisted of 26 lessons that last 30-min, and involves the numbers 1 to 20. Lessons were divided into several themes. The themes provided the children with a familiar background in which the activities became meaningful and useful.

Participants were 136 students in the 4–7 age range who scored below criterion of 45% correct on the first *Early Mathematical Competence Scale* (Van de Rijt & Van Luit, 1994). The students were divided into four groups, two experimental and two control groups. One experimental group received the AEM program using the guiding instruction (EPG-group). The other experimental group received the AEM program using the structured instruction (EPS-group). One control group received instruction based on the common Dutch arithmetic methods (CWM-group). The other control group received instruction based on textbook information like Montesorri and Cuisenaire (CNM-group). Each group consisted of 34 students with the mean age of each group being 71 months.

Measures included the *Early Mathematical Competence Scale* that consists of eight parts: Concepts of comparison, Classification, Correspondence, Seriation, Using counting words, Structured counting, Resultative counting and General knowledge of numbers. The study was a pretest and posttest design. Maintenance tests were administered to examine long term effects of the AEM program. Analysis between the four groups was conducted using 4 x 3 (research group by moment of measurement) multivariate analysis of variance (MANOVA) with repeated measurements on early mathematical competence scores. The scores were obtained during the pretest (T1), posttest (T2), and follow up test (T3). One-way ANOVA showed no statistical significant differences between the two experimental groups on the pretest. The mean early mathematical competence scores on T2 and T3 of the experimental group with the guiding instruction are not significantly different from the mean early mathematical competence scores of the experimental group with the structuring instruction. The mean early mathematical competence scores of both experimental groups on T2 differs significantly from the mean early mathematical competence score of both control groups. In addition, on T3 there is a significant difference between the mean scores of both experimental groups and both control groups. The authors concluded results indicated a positive effect of the AEM program on the early mathematical competence scores for both experimental groups.

Van Luit and Schopman (2000) extended research on guided verses explicit instruction for number sense in which the instructor chose the approach best suited to the student. They examined early numeracy based on perceptual gestalt theory using the program *Young Children with Special Education Needs Count Too* (Van Luit & Schopman, 1998). The program consisted of 20 lessons with complete instructional plans and materials to assist students in learning to count. Both learning by doing and structured learning was incorporated in the program and the teacher chooses which type of instruction was the best fit for the student’s needs. The student connected new information with prior knowledge by repeating, organizing, and arranging information. Realistic math problems were posed to make the math skills and problem-solving meaningful, in which the reason for using a particular strategy was explained. Initially students needed instruction in a structured way with lots of repetition. After 2 to 3 months, some students were able to receive instruction using the learning-by-doing principles of the math program.

Participants were 124 kindergarten students between the ages of 5 and 7 with special education needs. Participants were selected based on performance comparable to the lowest 25% of a normative group on a 40-item standardized early numeracy test. The participants were assigned to one of two groups by matching for gender, age, and early numeracy performance. Students received instruction for 6 months in groups of three. There were two 30-min sessions per week.

Measures include the Utrecht Test for Number Sense (Van Luit, Van de Rijt, & Pennings, 1994) that assessed counting skills and math prerequisites skills. The test consisted of eight parts: Concepts of comparison, Classification, Correspondence, Seriation, Using counting words, Structured counting, Resultative counting, and General knowledge of numbers. A transfer test was administered one week after intervention. The test included 14 simple items drawn from the Student Monitoring System: Mathematics 1 for Grade 3 (Janssen, Bokhove, & Kraemer, 1992). A log book was used to ensure treatment fidelity and monitor student progress.

The study was a pretest and posttest design that analyzed results from the number sense and transfer measures. The test data were analyzed using *t* tests. A significant difference was found between the experimental and comparison group with effect sizes of 1.44 for the experimental group and .68 for the comparison group. A significant difference was found between the two groups for comparison, but not classification, serration, or correspondence. A significant difference was found for all three of the counting skills that favored the experimental group. A significant difference was found favoring the experimental group for general understanding of numbers; however, no significant differences were found between groups for transfer task performance.

Bryant, Bryant, Gersten, Scammacca, and Chavez (2008) expanded research on number sense by using explicit instruction and combining it with the CRA method. Bryant et al. investigated Tier I (not-at-risk students) and Tier II (at-risk students) interventions using the CRA approach. A median of 64 15-minute sessions for first graders and a median of 15-min sessions for second graders were conducted across 18 weeks for Tier I intervention. Tier II intervention ranged from 45 to 60 minutes of instruction based on number, operation, and quantitative-reasoning skills. The CRA approach (Mercer & Miller, 1992; Mercer & Sams, 1996) was used to teach number concept and relationships, base 10 and place value, and addition and subtraction combinations. Lesson procedures including modeling, "thinking aloud," guided practice, pacing, and error correction were used to deliver the scripted lessons. All tutoring sessions were implemented in small group settings of students within the same ability and grade level.

Participants in this study included 126 students in first grade and 140 students in second grade. Students who scored at or below the 25th percentile (total standard score of 90 or below) at the start of the school year on the *Texas Early Mathematics Inventories–Progress Monitoring* (TEMI-PM; University of Texas System/Texas Education Agency, 2006) were assigned to the treatment group. Students who scored above 90 on the TEMI-PM at the start of the school year received no intervention but did take the TEMI-PM posttest measure at the same time as the intervention students.

The TEMI-PM was administered in the fall (September), winter (January), and spring (late April–early May). This assessment has four subtests: Magnitude Comparison (MC), Number Sequences (NS), Place Value (PV), and Addition/Subtraction Combinations (ASC). The mathematics subtests from the *Stanford Achievement Test–10th Edition* (SAT-10; Harcourt Assessment, 2003) were also administered in the spring of 2006. First grade students were given the Primary I level, consisting of Mathematics Procedures (MP), Mathematics Problem Solving (MPS) and a Total Mathematics score (TMS). Second grade students were administered the Primary II which consisted of the same assessments as in Primary I.

The design was a pretest and posttest design that employed regression discontinuity (RD) analyses. For first grade, pretest and posttest data were available for 100 students who did not qualify for intervention and did not receive intervention, and for 26 students who qualified for and received intervention. There is little to no discontinuity between the regression line for the Tier 2 (at-risk) group and the Tier 1 (not-at-risk) group. Therefore, RD analysis revealed that no significant effect was observed among first-grade students.

For second grade, pretest and posttest data were available for 115 students who did not qualify for intervention and did not receive intervention and for 25 students who qualified for and received intervention. There is a discontinuity between the regression line for the Tier 2 (at-risk) group and the Tier 1 (not-at-risk) group. This discontinuity demonstrates the positive significant effect of the program for at-risk students in Grade two. RD analyses at the subtest level indicated that a significant main effect existed for the Addition/Subtraction subtest, however results for the remaining three subtests showed no significant effects.

In summary, number sense is defined as the development of more sophisticated understandings of the size of numbers, number relationships, patterns, operations, and place value" (NCTM, 2000). Difficulties with number sense interfere with learning math skills later in childhood (Van Luit & Schoman, 2000). Gersten and Chard (1999) explain researchers are not sure of the best approach to teach numeracy. Research literature suggests instruction in classification, seriation, and conservation improves students with math difficulties' numeracy skills (see Table 4). Instruction on these concepts involve direct and discovery oriented, and as well as guided practice and modeling techniques. These studies are important for pre-service teachers because they can use this knowledge to provide interventions to assist young learners with number sense difficulties.

Table 4

Number Sense Studies

Authors and Date	Intervention	Participants	Design	Outcomes Measured
Pasnak, Holt, Campbell, & McCutcheon (1991)	Piacceleration method	65 students identified as at-risk	Group comparison of Piacceleration method to traditional basal instruction	Number Classification, seriation, and conservation
Pasnak, Hansbarger, Dodson, Hart, & Blaha (1996)	Piacceleration method	64 students identified as at-risk	Group comparison of Piacceleration method to traditional basal instruction	Number Classification, seriation, and conservation
Van de Rijt & Van Luit (1998)	Piagetian Operations that included guided or explicit instruction	136 students identified as at-risk	Group comparison of guided verse explicit instruction	Counting Skills
Van Luit & Schopman (2000)	Piagetian Operations that included guided and explicit instruction	124 students identified with special needs	Pretest/posttest comparison	Counting Skills
Bryant, Bryant,	CRA sequence	266 students	Pretest/posttest	Number concept

Gersten, Scammacca, & Chavez (2008)	with explicit instruction	identified as at-risk	comparison with regression discontinuity analysis	relationships, base ten place value, and addition and subtraction combinations

Teacher Preparedness Regarding Mathematics

Investigating special and general education teachers' ability to provide math instruction, Flores, Patterson, Shippen, Hinton, and Franklin (in press) surveyed special education and general education teachers' mathematical knowledge and perception of competence to teach mathematics. Participants included pre-service and in-service graduate and undergraduate students enrolled in elementary (K–6) or middle (4–8) level teacher preparation programs. A 3 X 3 Multivariate Analysis of Variance (MANOVA) was conducted. The independent variables were: (a) *grade level*, representing elementary and middle level; (b) *certification,* representing general and special education; and (c) *competence in teaching mathematics skills*, representing "yes" or "no" to a question regading their competence in teaching mathematics to children with disabilities. The dependent variables were the percent correct scores on the computational and problem solving portions of a curriculum-based assessment that included kindergarten through sixth grade content completed by the participants and the total percent correct. Middle level teachers reported a higher level of perceived teaching competence than their elementary level peers. Middle level teachers also showed higher scores in computation skills with no differences between elementary and middle level teachers for problem solving skills. Participants who had higher efficacy scores in teaching mathematics showed higher scores in problem solving skills with no differences between those who denoted competence in teaching mathematics and their computation scores. No differences were found between general education and special education teachers across mathematics skills and efficacy scores.

Approaching teacher mathematical preparation from a different angle, Farmer et al. (2003) conducted a qualitative study investigating the *Enhancing Mathematics in the Elementary School* (EMES) project in which two, week-long seminars provided professional development regarding mathematics instruction. Every session included the following (a) mathematical communication, (b) discussion of mathematical concepts and principles, (c) analysis of the learning process, (d) reflection on pedagogical principles (especially those which challenge traditional views of teaching and learning) and, (e) discussion of implications for classroom instruction and/or planning/debriefing implementation activities.

Farmer et al. (2003) stated three questions were asked to gain an understanding of what makes for successful mathematics professional development. They were as follows:

(a) What does an inquiry stance toward mathematics teaching look like?

(b) How does this stance develop?

(c) What was the role of the EMES project in its development?

Three teachers were chosen to participate in the study. The authors report that teachers were selected who (a) had been involved from the beginning or near the beginning of the project, (b) appeared to be interested and enthusiastic in some way, (c) appeared to be learning from the project, and (d) were willing to allow us to interview them and observe their classes. In addition, the three educators chosen were at different places in their careers and teach at different schools. The teachers and their students vary in ethnicity. Two interviews, two classroom observations, analyses of each participant's emails, analyses of each participant's final written report, and analyses of daily reflections from each participant about the seminars were conducted.

Each participant was given a pseudonym in the interest of confidentiality. The names were Donna, Eva, and Vera. Farmer and colleagues explained Donna had been teaching elementary school for three years. She focused on promoting and improving written communication in her classroom and wanted her students to gain a better understanding of decimals. Donna also indicated that she used many ideas and specific activities from the EMES workshops during her third year of teaching. In an early interview, Donna said the EMES project taught her how to use manipulatives and cooperative grouping for instruction. During training, she observed that written and oral communications are closely linked, and realized that communication promotes conceptual understanding. After professional development, Donna stated that her students were empowered by what they were learning and were now able to connect their new knowledge to other subject areas. In addition, she began to move away from the textbook and use explorations with other standards-based teaching techniques. She believed her students benefited from inquiry-driven experiences.

Eva had been teaching elementary school for 13 years. Two events helped Eva solidify her commitment to change the first year. First, students who were struggling in math showed improvement when she tried new methods of instruction, and second Eva was given a high ability group of students who grew bored with the traditional means of providing mathematic instruction. She placed her instructional focus on place value and mathematical journaling. Eva worked to implement the idea of creating good questions, and spent considerable time in class asking students to explain their responses. Eva viewed her decisions to implement her own ideas and give her students sufficient time to work on the activities as not exactly in line with current policies. Before the EMES project, she viewed herself as an inadequate mathematics teacher. After professional development, Eva described herself has having a math philosophy and getting better at teaching math.

Vera had been teaching for 25 years. She had been through mathematics professional development workshops the previously and felt that she had learned a lot, but wanted more knowledge. Three major themes emerged. First, Vera became empowerment as a mathematics learner due to her changed views of the nature of mathematics and deeper understanding of mathematical concepts. Second, Vera developed a strong desire to empower her students mathematically. Third, she focused on teaching students how to pose appropriate problems, ask good questions, and pay close attention to their thinking. Vera found herself revisiting concepts that she had learned many years before, gaining new understanding and insight.

Probing how teachers solve mathematical problems, Yang (2009) examined the math strategies used by pre-service educators in Taiwan. Of fifteen participants, five were in elementary education, five majored in math education, and five in language education. All participants completed a basic mathematics course and a mathematics teaching course. Yang conducted interviews with each participant. All interviews took place at the school and were videotaped for data analysis purposes. The interview instrument included 12 items that included four number sense components (3 items in each component). The four components are (a) understanding the meaning of numbers, operations, and their relationships, (b) recognizing relative number size, (c) developing and using benchmarks appropriately, and (d) judging the reasonableness of a computational result by using the strategies of estimation. Before interviewing participants, three mathematical educators reviewed the items and all agreed the items were appropriate. A pilot study was then conducted with two pre-service teachers who were comparable in general academic background to the participants. The pilot study was conducted to ensure the items were clear and appropriate and that the time limit for each item was reasonable. Yang conducted all interviews with participants individually in a quiet room. Participants were discouraged from using written computation at the beginning of the interview.

The pre-service teachers' responses were examined and scored and Yang coded the responses into three categories. The categories were number sense based in which strategies utilized at least one of the four components of number sense, rule base in which the explanation of the strategy was associated only with standard written algorithms and not beyond, or neither in which the participant could not provide an appropriate explanation of the strategy used. Yang found that most participants (2/3) relied on rule based strategies to answer the items while fewer (1/3) of the participants relied on number sense strategies. Yang also explained that these findings were consistent with findings of earlier studies (Reys & Yang, 1998; Yang & Reys, 2002; Yang, 2003) that fifth, sixth, and eighth grade students in Taiwan relied heavily on written algorithms when responding to number sense related questions.

In summary, studies that investigate special and/or general education teachers' ability to provide elementary mathematics instruction survey teacher mathematical knowledge, use of strategies, and professional development experiences that enhance mathematical knowledge. Studies consist of surveys, achievement assessments, interviews, observations, and artifact analysis. Flores et al. (in press) examined both general and special education teachers' mathematical knowledge and perception of competence to teach mathematics. Middle level teachers reported greater perceived teaching competence than their elementary level peers. In addition, middle level teachers showed higher scores in computation skills with no significant differences between elementary and middle level teachers for problem solving skills. Participants indicating higher levels of competence in teaching mathematics showed higher scores in problem solving skills with no significant differences between those who denoted competence in teaching mathematics and their computation scores. No significant differences were found between general education and special education teachers across mathematics skills and perceived competence. Farmer et al. (2003) conducted a qualitative study exploring professional development regarding positive, reform-oriented mathematics instruction. After the professional development experience, one participant stated that her students were empowered by what they were learning and were now able to connect their new knowledge to other subject areas, another participant described herself has having a math philosophy and getting better at teaching math. A further participant found herself revisiting concepts that she had learned many years before, gaining new understanding and insight. Yang (2009) examined math strategies used by pre-service educators. Pre-service teachers' responses were examined and scored and Yang coded the responses into three categories: number sense based in which strategies utilized at least one of the four components of number sense, rule base in which the explanation of the strategy was associated only with standard written algorithms and not beyond, or neither in which the participant could not provide an appropriate explanation of the strategy used. Yang found that most participants (2/3) relied on rule based strategies to answer the items while fewer (1/3) of the participants relied on number sense strategies. In addition, Yang's study was consistent with findings of earlier studies (Reys & Yang, 1998; Yang & Reys, 2002; Yang, 2003) that fifth, sixth, and eighth grade students relied heavily on written algorithms when responding to number sense related questions.

Conclusion

The National Council of Teachers of Mathematics (NCTM; 2006) and the National Mathematics Advisory Panel (NMAP; 2008) recommend providing intensive instruction in mathematics that focus on rigorous comprehensive treatment of whole numbers. Such a recommendation is tied to the concept of number sense which leads to automatic use of math information. It is necessary for learning math in the early grades and Gersten et al. (2009), Fuchs et al., (2007), and Milgram and Wu (2005) emphasized in depth coverage of such concepts for students with disabilities or mathematical difficulties. Therefore, mathematic interventions must include development of new skills that thoroughly cover the use of whole numbers which are critical to success in math.

Research regarding early math curricula involves problem solving, arithmetic, and number sense. Research on problem solving has demonstrated that effective methods include explicit instruction in self regulated strategies (Case, Harris, & Graham, 1992; Wilson & Sindelar, 1991), CRA coupled with direct instruction that utilized mnemonic devices (Cassel & Reid, 1996), or schema training (Jitendra & Hoff, 1996). Arithmetic intervention research has shown that effective instruction includes CRA methods coupled with direct instruction that utilized mnemonic devices (Flores, 2009; Ho & Cheng, 1997; Mercer & Miller, 1992), CRA methods combined with computer drills (Fuchs et al., 2005, 2007), verbalization strategies (PASS; Naglieri & Gottling, 1997), multisensory methods (Dev, Doyle, & Valente, 2002), modeling, guided practice, independent work, and student contracts (Flores, 2009; Fuchs et al., 2005, 2007; Mercer & Miller, 1992). Number sense research suggests that effective instruction includes direct and discovery oriented methods (Pasnak et al., 1996; Pasnak et al., 1991; Van Luit & Schopman, 2000; Van de Rijt & Van Luit, 1998), and involved guided practice and modeling techniques (Bryant, et al., 2008; Van Luit & Schopman, 2000; Van de Rijt & Van Luit, 1998). Studies in problem solving, arithmetic, and number sense consisted of group and single subject research methods. Single subject designs utilize multiple baseline designs across participants and behaviors, and AB designs that contrasted groups based on performance on the

Cognitive Assessment System (measurements of planning, attention, simultaneous, and successive processes). Group designs employ pretest and post test measures, group comparison, correlation, and regression discontinuity analysis. These studies are important for pre-service educators to learn because these interventions can be formed to assist young learners at risk for failure. Numeracy is especially important because it is a critical pre-requisite to higher order mathematical concepts (Van Luit & Schoman, 2000). In addition, aspects of number sense, such as magnitude comparisons and quantity discrimination, correlate with math achievement, and accurately predict students who would later develop math disabilities (Clarke & Shinn, 2004; Mazzocco & Thompson (2005).

Research regarding educators' preparation to teach mathematics instruction involved quantitative and qualitative studies that survey teacher mathematical knowledge, use of strategies, and professional development experiences that enhance mathematical knowledge. Studies consisted of surveys, achievement assessments, interviews, observations, and artifact analysis. Flores et al. (in press) examined both general and special education teachers' mathematical knowledge and perception of competence to teach mathematics. Participants indicating higher levels of competence in teaching mathematics showed higher scores in problem solving skills with no significant differences between those who denoted competence in teaching mathematics and their computation scores. In addition, no significant differences were found between general education and special education teachers across mathematics skills and perceived competence. Farmer et al. (2003) conducted a qualitative study exploring professional development regarding positive, reform-oriented mathematics instruction. All participants were empowered by what they learned and one participant found herself revisiting concepts that she had learned many years before, gaining new understanding and insight. Yang (2009) examined math strategies used by pre-service educators. Participants responses were coded into three categories: number sense based in which strategies utilized at least one of the four components of number sense, rule base in which the explanation of the strategy was associated only with standard written algorithms and not beyond, or neither in which the participant could not provide an appropriate explanation of the strategy used. Yang found that most participants (2/3) relied on rule based strategies to answer the items while fewer (1/3) of the participants relied on number sense strategies.

Researchers have not come to consensus about the best approach to teaching number sense (Berch, 2005; Gersten, & Chard, 1999), but the literature included studies regarding classification, seriation, number conservation (Pasnak et al., 1996; Pasnak et al., 1991), number relationships and base ten place value (Bryant, et al., 2008), and counting skills (Van Luit & Schopman, 2000; Van de Rijt & Van Luit, 1998).

A relationship between the various components thought to be the building blocks of future success and mathematics achievement must be established to ensure teachers can be confident that the advice they receive about teaching number sense will lead to improved mathematics performance (Howell & Kemp, 2009). Once links between specific components of number sense, mathematical needs of pre-service educators, and student achievement are established, future educators can be empowered to provide the recommended intensive instruction for children at risk of mathematics difficulties. Therefore, teachers would provide the in-depth instruction recommended by Gersten et al. (2009), Fuchs et al. (2007), Milgram and Wu (2005), the NCTM, and NMAP which furnishes all the competencies children need to succeed in early mathematics — including number sense.

CHAPTER III. PRE-SERVICE TEACHERS' COMPUTATIONAL KNOWLEDGE, EFFICACY, AND NUMBER SENSE SKILLS

Introduction

To solve the dilemma of low student achievement, the *No Child Left Behind Act* (NCLB; 2002) held schools accountable for the adequate achievement of all students. This included students with disabilities. The NCLB Act proposed that student achievement will improve considerably and consistently so that all students (including students with disabilities) are proficient in reading and mathematics no later than 2013–2014. To confirm that all students demonstrate proficiency, each state has defined "adequate yearly progress" (AYP) to measure its schools' achievement. These AYP standards were primarily based on state assessment results, but include high school graduation rates and school attendance rates. Evaluations of AYP must contain the progress of the majority (95%) of students with disabilities included in a school district's AYP assessment. The NCLB Act was landmark legislation in that it recognized the needs of students with disabilities and mandated schools make changes to improve their academic achievement. In addition to the directives of NCLB (2002), teachers serving students with disabilities must adhere to the regulations of the *Individuals with Disabilities Education Improvement Act* (IDEIA, 2004). This means that special education teachers must ensure that students with disabilities receive access to the general education curriculum as well as meet standards for AYP. Researchers argued that teachers' content knowledge, methodological training, and education were critical features in promoting student achievement (Kamil, 2003).

Student achievement was higher when teachers were certified and when teachers possessed content area knowledge in the subject area taught through either a college major or minor (Kaplan & Owings, 2003; Laczko-Kerr & Berliner, 2002).

Mathematics Instruction

The National Council of Teachers of Mathematics (NCTM) Curriculum Focal Points (2006) suggested heavy emphasis on instruction in whole numbers for young elementary students. This position was strengthened by the 2008 report of the National Mathematics Advisory Panel (NMAP), which provided detailed benchmarks and emphasized in-depth coverage of key topics involving whole numbers as crucial for all students. Therefore, Milgram and Wu (2005) suggested an intervention curriculum for students who are at-risk for mathematic difficulties should not be oversimplified and that in-depth coverage of key concepts involving whole numbers was critical for success in mathematics. Fuchs et al. (2007) supported Milgram and Wu’s proposition saying research is needed that incorporates major components of math curriculum, interventions that develop new skills, and the efficacy of interventions to improve math achievement. In addition, Gersten et al. (2009) recommended interventions for students with mathematical difficulties provide intensive instruction of whole numbers. Such a recommendation was important because, with increased competency in basic addition or subtraction facts, children develop or fail to develop number sense (Gersten & Chard, 1999). Number sense was defined as “moving from the initial development of basic counting techniques to more sophisticated understandings of the size of numbers, number relationships, patterns, operations, and place value” (National Council of Teachers of Mathematics [NCTM], 2000, p. 79). This developing construct refers to children’s fluidity and flexibility with numbers, the sense of what numbers mean, and the ability to perform mental mathematics and look at the world and make comparisons (Berch, 1998). Number sense is a key component in basic arithmetic computation because it leads to the automatic use of math information (Gersten & Chard, 1999). Griffin, Case, and Siegler (1994) suggest number sense is a necessary ingredient for learning formal arithmetic in the early elementary grades.

Teacher Mathematical Knowledge and Preparation

Since mathematics instruction requires an understanding of mathematical concepts and knowledge about how children acquire and apply mathematical skills, teacher preparation as well as pre-service educators' mathematical knowledge must be examined. A report to the nation from the National Commission on Mathematics and Science Teaching for the 21st Century, also known as the Glenn Report (US Dept. of Education, 2000) delineated the need for improving teacher preparation in mathematics and science. The report used findings from the Third International Mathematics and Science Study (TIMSS; 1995) and the National Assessment of Educational Progress (NAEP; 1996) to inspect the mathematics and science achievement of students in the United States compared to their peers in other countries. Glenn et al. (2000) stated the performance of students in the United States from both the TIMSS and NAEP studies was unacceptable. In response to students' low achievement, the Glenn Report set three goals. First, establish an ongoing system to improve the quality of mathematics and science teaching in grades K–12. Second, significantly increase the number of mathematics and science teachers and improve the quality of their preparation. Third, improve the working environment and make the teaching profession more attractive for K–12 mathematics and science teachers.

In addition to the Glenn Report's findings, research has shown that the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics* may not be effectively utilized by special education teachers (Maccini, Gagnon, & Calvin, 2002). The authors found that a significant number of special education teachers were not familiar with the goals of the NCTM Standards. The authors also expressed concern regarding the method of special education instruction in the area of mathematics and the mathematical content knowledge and pedagogical knowledge of special education teachers. Maccini et al.'s (2002) findings were disheartening considering the NCTM standards are a critical component of standards-driven instruction, and bearing in mind the goal that students with disabilities will make adequate academic progress, thus achieving at grade level within the next decade.

Investigating special and general education teachers' ability to provide math instruction, Flores, Patterson, Shippen, Hinton, and Franklin (in press) surveyed special education and general education teachers' mathematical knowledge and perception of competence to teach mathematics. Participants included pre-service and in-service graduate and undergraduate students enrolled in elementary (K–6) or middle (4–8) level teacher preparation programs. A 3 X 3 Multivariate Analysis of Variance (MANOVA) was conducted. The independent variables were: (a) *grade level*, representing elementary and middle level; (b) *certification,* representing general and special education; and (c) *competence in teaching mathematics skills,* representing "yes" or "no" to a question regarding their competence in teaching mathematics to children with disabilities. The dependent variables were the percent correct scores on the computational and problem solving portions of a curriculum-based assessment that included kindergarten through sixth grade content. Middle level teachers reported a higher level of perceived teaching competence than their elementary level peers. Middle level teachers also showed higher scores in computation skills with no differences between elementary and middle level teachers for problem solving skills. Participants who had higher efficacy scores in teaching mathematics showed higher scores in problem solving skills with no differences between those who denoted competence in teaching mathematics and their computation scores. No differences were found between general education and special education teachers across mathematics skills and efficacy scores.

Yang (2009) took a different approach and examined math strategies used by pre-service educators in Taiwan. Of fifteen participants, five were in elementary education, five majored in math education, and five in language education. All participants completed a basic mathematics course and a mathematics teaching course. Yang conducted interviews with each participant. All interviews took place at the school and were videotaped for data analysis purposes. The interview instrument included 12 items that included four number sense components (3 items in each component). The four components are (a) understanding the meaning of numbers, operations, and their relationships, (b) recognizing relative number size, (c) developing and using benchmarks appropriately, and (d) judging the reasonableness of a computational result by using the strategies of estimation. Before interviewing participants, three mathematical educators reviewed the items and all agreed the items were appropriate. A pilot study was then conducted with two pre-service teachers who were comparable in general academic background to the participants. The pilot study was conducted to ensure the items were clear and appropriate and that the time limit for each item was reasonable. Yang conducted all interviews with participants individually in a quiet room. Participants were discouraged from using written computation at the beginning of the interview.

The pre-service teachers' responses were examined and scored and Yang coded the responses into three categories: number sense, rule base, and neither. The categories were number sense based in which strategies utilized at least one of the four components of number sense, rule base in which the explanation of the strategy was associated only with standard written algorithms and not beyond, or neither in which the participant could not provide an appropriate explanation of the strategy used. Yang found that most participants (2/3) relied on rule based strategies to answer the items while fewer (1/3) of the participants relied on number sense strategies. Yang also explained that these findings were consistent with findings of earlier studies (Reys & Yang, 1998; Yang & Reys, 2002; Yang 2003) in which fifth, sixth, and eighth grade students in Taiwan relied heavily on written algorithms when responding to number sense related questions.

Statement of the Problem

To date, there was a lack of information about pre-service educators' number sense abilities and preparing them to teach number sense skills to students with disabilities (Gersten et al., 2009). Therefore, the focus of this study was to explore elementary, special education and general education teachers' mathematical content and pedagogical knowledge with an added focus on number sense.

Participants

A total of 113 pre-service and in-service graduate and undergraduate students enrolled in either a general or special education program participated in the study. The criteria in which participants were chosen was (a) participants had to be enrolled in a general education or special education program, (b) participants had to have less than one year of experience, and (c) participants indicated that they planned to teach elementary aged students (K-Grade 5) once they graduated from their college program. Pre-service teachers were recruited through both graduate and undergraduate instructional methods classes at a large university in the southeast region. Three hundred surveys were given out and 230 surveys were completed. Of the 240 surveys that were completed, 113 met the criteria set by the researcher. One hundred and seventeen surveys were discarded due to not meeting the criteria set forth at the beginning of the study or the participant not filling out the survey. The largest demographic group was White females enrolled in an undergraduate general education program and under the age of 29. Table 5 outlined the characteristics of the participant sample.

Table 5

Participant Characteristics

Gender		Culture		Age		Degree Program	
Male	4	African American	9	18–20	42	UG General	80
Female	109	Latino/a	0	21–29	72	UG Special	20
		White	104	30–39	2	M-Alt General	1
		Asian	0	40–49	0	M-Alt Special	3
						Masters General	1
						Masters Special	8

Note. UG = undergraduate program, M-Alt = alternative 5th year graduate program

Method

This study expanded previous research (Flores et al., in press; Yang, 2009). Special education and general education teachers' computational knowledge was surveyed, their efficacy to teach mathematics was investigated, and special and general education teachers' approach to calculate math problems by using number sense strategies, rule based strategies, or neither explored. The following research questions guided this study.

1. To what extent is there a difference in the computational knowledge of special education teachers and general education teachers at the elementary level?

2. To what extent is there a difference in the personal efficacy to teach mathematical knowledge of elementary level pre-service special education teachers' and general education teachers'?

3. To what extent is there a difference in the mathematical outcome expectations of pre-service special education teachers and general education teachers at the elementary level?

4. In what ways do pre-service special education teachers and general education teachers approach calculating numbers, operations, and their relationships as defined by using number sense strategies, rule based strategies, or neither to solve computational problems?

The following null hypotheses were formulated to respond to the first three research questions.

(a) There are no statistically significant differences in the computational knowledge of pre-service special education teachers and general education teachers at the elementary level.

(b) There are no statistically significant differences in the personal efficacy to teach mathematical knowledge of pre-service elementary level special education teachers and general education teachers.

(c) There are no statistically significant differences in mathematical outcome expectations of pre-service special education teachers and general education teachers at the elementary level?

In addition, the fourth research question was examined by calculating the frequency and percent of responses to an open-ended questionnaire developed using Yang's (2009) interview instrument. This questionnaire explored teachers' understanding of numerical operations, and their relationships as defined by using number sense strategies, rule based strategies, or neither.

The independent variable was *certification* representing general education or special education. The dependent variables were *standard scores* on the calculation subtest of the *Woodcock Johnson III* (WJIII; Woodcock, McGrew, & Mather, 2001) instrument, and *teacher* *efficacy scores (Personal Efficacy and Outcome Expectancy)* on the *Mathematics Teaching Efficacy Beliefs Instrument* (MTEBl; Enochs, Smith, & Huinker, 2000). The percentage of correct scores from the open-ended questionnaire developed using Yang's (2009) interview instrument was calculated to examine strategies used by educators to solve problems based on the four number sense components. The four components were (a) understanding the meaning of numbers, operations, and their relationships, (b) recognizing relative number size, (c) developing and using benchmarks appropriately, and (d) judging the reasonableness of a computational result by using the strategies of estimation.

Survey Instruments

Special education and general education teachers' computational knowledge was surveyed using the calculation subtest of the *Woodcock Johnson III* (WJIII; Woodcock et al., 2001). The calculation subtest measured one's ability to perform mathematical computations. Woodcock et al. (2001) stated the initial test items require one to write single numbers. The remaining items required the individual to perform addition, subtraction, multiplication, division, combinations of the basic operations, and geometric, trigonometric, logarithmic, and calculus operations. Calculations involved negative numbers, percents, decimals, fractions and whole numbers. Calculations were presented in a traditional problem format in the *Subject Response Booklet* and the individual was not required to make decisions about what operations to use or what data to include (Woodcock et al., 2001). The reliability for the math calculation was calculated using the split-half procedure. The calculation of the split-half coefficients used data provided by odd and even test items. All split-half coefficients were corrected for length of the published test using the Spearman-Brown correction formula (Woodcock et al., 2001).

Woodcock et al. (2001) reported a math calculation subtest median reliability score of .89 in the adult range.

Perceived competence of special education and general education teachers' to teach mathematical knowledge was measured using The *Mathematics Teaching Efficacy Beliefs Instrument* (MTEBl; Enochs et al., 2000). The MTEBl consists of 21 items: thirteen on the Personal Mathematics Teaching Efficacy subscale and 8 on the Mathematics Teaching Outcome Expectancy subscale. The Personal Mathematics Teaching Efficacy subscale addressed the pre-service teachers' beliefs in their individual capabilities to be effective mathematics teachers (Enochs et al., 2000). The Mathematics Teaching Outcome Expectancy subscale addressed teachers' beliefs that effective teaching can bring about student learning of mathematics regardless of external factors (Enochs et al., 2000). The instrument used a Likert scale with five response categories including strongly agree, agree, uncertain, disagree, and strongly disagree. Possible scores on the Personal Mathematics Teaching Efficacy subscale range from 13 to 65; Mathematics Teaching Outcome Expectancy subscale scores range from 8 to 40. Reliability analysis produced an alpha coefficient of .88 for the Personal Mathematics Teaching Efficacy subscale and an alpha coefficient of .75 for the Mathematics Teaching Outcome Expectancy subscale (n = 324). Confirmatory factor analysis indicated that the two subscales were independent (Enochs et al., 2000). An example of the *Mathematics Teaching Efficacy Beliefs Instrument* is available in Appendix 1.

To investigate the types of strategies that educators used to solve mathematical computation problems, an open ended questionnaire was created using Yang's (2009) interview instrument that examined strategies used by adults to solve problems based on the four number sense components. The four components were (a) understanding the meaning of numbers, operations, and their relationships, (b) recognizing relative number size, (c) developing and using benchmarks appropriately, and (d) judging the reasonableness of a computational result by using the strategies of estimation. There were 12 number sense questions in which there were 3 questions for each component (see Appendix 2 for the questionnaire). Respondents were asked to choose an answer to the problem and then state how they solved the problem.

The survey packet also included a questionnaire eliciting demographic information and perception of competence. Participants were asked to identify the following: (a) age; (b) cultural background; (c) number of years of teaching experience; (d) area of current or future certification; and (e) grade level at which they taught or would teach. See Appendix 3 for the questionnaire that elicited demographic information.

Procedures

The surveys and questionnaires were distributed by the researcher in the form of a survey packet and completed by graduate and undergraduate students enrolled in general education and special education courses specific to methods within each major. All surveys and questionnaires were numbered and participants did not write their name on any items provided to ensure confidentiality. The participants volunteered for the study and the researcher provided a letter explaining the risks to the participants as well as their rights and who to contact regarding any questions or concerns (see Appendix 4 for the letter that detailed the study's risks and rights of the participants). The students who volunteered filled out the survey in the classroom. Packets were handed out to all individuals in the class. Participants chose to complete or not complete the packet. The background questionnaire was completed first so that the mathematics tasks within the survey did not interfere with the participants' answers. Participants completed the math knowledge survey, perceived competence survey, and number sense questionnaire using pencil and paper. All participants completed the math efficacy survey first. No time limit was assigned and the order of the math knowledge and number sense portions of the survey packet was counterbalanced to limit any one variable receiving higher scores due to participant fatigue or order effects. Therefore, 1/2 of participants received the math knowledge portion first then the number sense questionnaire, and 1/2 of participants received the number sense questionnaire, then math knowledge survey. All participants whether they completed a survey or not, sealed the survey packet and place them in a bin provided by the researcher.

Data Analysis

Computation, teaching efficacy, and outcome efficacy data were analyzed with a Multivariate Analysis of Variance (MANOVA). A MANOVA tests for differences between groups as defined by one independent categorical variable. A MANOVA is used to simultaneously test two or more related dependent variables while controlling for the correlations among the dependent variables (Mertler & Vannatta, 2005). The independent variable in this study was *certification* categorized as general and special education. The dependent variables were *the computational scores* on the mathematical knowledge survey instrument completed by the participants, *the perceived teaching competence scores, and the outcome expectancy scores* on the efficacy scale. Two participants did not answer questions regarding personal math efficacy and outcome expectancy. In addition, two cases were deleted as outliers because the scores were extremely high. Outliers are data that have extreme values on one variable or a combination of variables in that results can be distorted (Mertler & Vannatta, 2005). Then data were transformed to meet the assumption of a normal distribution for the outcome expectancy scores. Outcome expectancy scores had a substantial negative skew and were transformed using reflect and logarithm. Results from the open-ended questionnaire were examined by calculating the frequency and percent of responses exploring how teachers calculate numbers, teachers' understanding of numerical operations, and their relationships as defined by using number sense strategies, rule based strategies, or neither.

Results

A MANOVA statistical procedure was conducted to ascertain if a statistically significant difference existed between a pre-service teacher's certification categorized as general education or special education and his/her calculation skills, personal math efficacy, and outcome expectancy. The results indicate that there were no statistically significant differences regarding the computational knowledge, personal efficacy, and outcome expectancy of special education teachers and general education teachers at the elementary level, Wilk's $\Lambda = .980, F(3, 105) = .725, p = .539$.

The percentage of correct scores from the open-ended questionnaire developed using Yang's (2009) interview instrument was calculated to examine strategies used by adults to solve problems based on the four number sense components. The four components were (a) understanding the meaning of numbers, operations, and their relationships, (b) recognizing relative number size, (c) developing and using benchmarks appropriately, and (d) judging the reasonableness of a computational result by using the strategies of estimation. Strategies that were used to solve the calculation problems were grouped into three categories. These categories were: (a) based on four number sense components, (b) rule based in which the explanation of solving the problem relied on standard written algorithms, or (c) neither in which an explanation was not given or did not include a number sense component or algorithm. Examples of answers that were classified as number sense strategies were as follows:

> *"36/37 is closer to 1 because 1/37 is smaller than 1/31. Therefore, if something is missing 1/37 is closer to the whole than if it were 1/31."*

".4975 is almost .5 and 4690.828 is what I guessed to be half of 9428.8"

"Both numbers rounded to the nearest whole number are close to 50 and 50 X 50 = 2500"

Examples of answers that were classified as rule based strategies were as follows:

"Adding together decimals, converting fractions, and adding and subtracting those"

"I cross multiplied"

"See work above"

Examples of answers that were classified as neither or left blank were as follows:

" no idea"

"1/500 is a greater number than 1/1000 therefore it is closer to 0."

"guessed"

The first three questions involved the component of understanding the meaning of numbers, operations, and their relationships. Table 6 outlined the questions and responses. Question one was as follows:

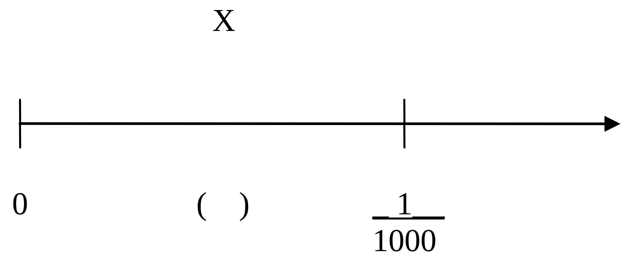
Circle the best choice to fill in the blank (a, b, or c)?

$$\frac{174 \times 10000}{9999} \quad \underline{\qquad} \quad 174$$

(a) $<$ (b) $>$ (c) $=$

Of the 113 respondents, 59 participants (52%) answered question one correctly, 50 participants (44%) answered question one incorrectly, and four participants (4%) did not answer question one. Twenty-seven participants (24%) used a number sense strategy, 23 participants (20%) used a rule base strategy, and 63 participants (56%) used a strategy categorized as neither. Question two was as follows:

Circle the best answer that represents the letter X on this number line.



(a) $\frac{2}{1000}$ (b) $\frac{1}{500}$ (c) $\frac{1}{2000}$ (d) $\frac{5}{1000}$

Fifty-nine participants (52%) answered question two correctly, 47 participants (42%) answered question two incorrectly, and seven participants (6%) did not answer question two. Fifty participants (44%) used a number sense strategy, one participant (1%) used a rule base strategy, and 60 participants (53%) used a strategy categorized as neither. Question three was as follows:

Without calculating, circle the best estimate for 103 X 48.

(a) 100 x 50 (b) 103 x 50 (c) 100 x 48

Seventy-two participants (64%) answered question three correctly, 40 participants (35%) answered question three incorrectly, and one participant (1%) did not answer question three. Forty-three participants (38%) used a number sense strategy, 12 participants (11%) used a rule base strategy, and 58 participants (51%) used a strategy categorized as neither.

Table 6

Understanding Numbers and Their Relationships

	Number Sense		Rule Based		Neither		Correct		Not Correct		Blank	
	N	%	N	%	N	%	N	%	N	%	N	%
Question One	27	24	23	20	63	56	59	52	50	44	4	4
Question Two	50	44	1	1	60	53	59	52	47	42	7	6
Question Three	43	38	12	11	58	51	72	64	40	35	1	1

The next set of three questions (4–6) involved the component of recognizing relative number size. Table 7 outlined the questions and responses. Question four was as follows: Without calculating, write two numbers from: 4, 7, 9, 13, 15, to form a fraction closest to ½. Thirty-seven participants (33%) answered question four correctly, 67 participants (59%) answered question four incorrectly, and nine participants (8%) did not answer question four. Twenty-seven participants (24%) used a number sense strategy, six participants (5%) used a rule base strategy, and 80 participants (71%) used a strategy categorized as neither.

Table 7

Recognizing Number Size

	Number Sense		Rule Based		Neither		Correct		Not Correct		Blank	
	N	%	N	%	N	%	N	%	N	%	N	%
Question Four	27	24	6	5	80	71	26	22	67	59	9	8
Question Five	36	32	1	1	76	67	62	55	45	40	6	5
Question Six	16	14	8	7	89	79	29	26	64	57	20	18

Question five was as follows:

Without calculating, which fraction

$\frac{30}{31}$ or $\frac{36}{37}$ is closest to 1?

Sixty-two participants (55%) answered question five correctly, 45 participants (40%) answered question five incorrectly, and six participants (5%) did not answer question five. Thirty-six participants (32%) used a number sense strategy, one participant (1%) used a rule base strategy, and 76 participants (67%) used a strategy categorized as neither. Question six was as follows:

Without Calculating, order the following numbers from smallest to largest:

$\frac{13}{38}$, 0.966, $\frac{7}{29}$, 0.4828, $\frac{17}{16}$, $\frac{8}{15}$

Twenty-nine participants (26%) answered question six correctly, 64 participants (57%) answered question five incorrectly, and 20 participants (18%) did not answer question six. Sixteen participants (14%) used a number sense strategy, eight participants (7%) used a rule base strategy, and 89 participants (79%) used a strategy categorized as neither.

Questions seven through nine involved the component of developing and using benchmarks appropriately. Table 8 outlined the questions and responses. Question seven was as follows:

Which sum is larger than 1?

(a) $\frac{3}{11} + \frac{29}{61}$ (b) $\frac{13}{21} + \frac{37}{71}$ (c) $\frac{13}{31} + \frac{4}{9}$ (d) $\frac{6}{17} + \frac{1}{2}$

Table 8
Using Benchmarks

	Number Sense		Rule Based		Neither		Correct		Not Correct		Blank	
	N	%	N	%	N	%	N	%	N	%	N	%
Question Seven	11	10	12	11	90	80	30	27	48	42	35	31
Question Eight	2	2	59	52	52	46	5	4	88	78	20	18
Question Nine	2	2	36	32	75	66	34	30	49	43	30	27

Thirty participants (27%) answered question seven correctly, 48 participants (42%) answered question seven incorrectly, and 35 participants (31%) did not answer question seven. Eleven participants (10%) used a number sense strategy, 12 participants (11%) used a rule base strategy, and 90 participants (80%) used a strategy categorized as neither. Question eight was as follows:

Place the decimal point for the product of 0.4975 x 9428.8 = 4690828

Five participants (4%) answered question eight correctly, 88 participants (78%) answered question eight incorrectly, and 20 participants (18%) did not answer question eight. Two participants (2%) used a number sense strategy, 59 participants (52%) used a rule base strategy, and 52 participants (46%) used a strategy categorized as neither. Question nine was as follows:

The product of $\frac{17}{29} \times \frac{6}{13}$ is

(a) >1/2 (b) = 1/2 (c) < 1/2 (d) Can't decide

Thirty-four participants (30%) answered question nine correctly, 49 participants (43%) answered question nine incorrectly, and 30 participants (27%) did not answer question nine.

Two participants (2%) used a number sense strategy, 36 participants (32%) used a rule base strategy, and 75 participants (66%) used a strategy categorized as neither.

The last set of three questions involved the component of judging the reasonableness of a computational result by using the strategies of estimation. Table 9 outlined the questions and responses. Question ten was as follows:

What's the reasonable estimate of 61027 divided by 33.275?

Table 9
Using Strategies of Estimation

	Number Sense		Rule Based		Neither		Correct		Not Correct		Blank	
	N	%	N	%	N	%	N	%	N	%	N	%
Question Ten	4	4	28	25	81	72	7	6	62	55	44	39
Question Eleven	8	7	18	16	87	77	9	8	47	42	57	50
Question Twelve	2	2	23	20	88	78	5	4	80	71	28	25

Seven participants (6%) answered question ten correctly, 62 participants (55%) answered question ten incorrectly, and 44 participants (39%) did not answer question ten. Four participants (4%) used a number sense strategy, 28 participants (25%) used a rule base strategy, and 81 participants (72%) used a strategy categorized as neither. Question eleven was as follows:

What's the reasonable estimate of

$0.495 + \underline{37} + 2.875 - \underline{27}$

18	110

Nine participants (8%) answered question eleven correctly, 47 participants (42%) answered question eleven incorrectly, and 57 participants (50%) did not answer question eleven. Eight participants (7%) used a number sense strategy, 18 participants (16%) used a rule base strategy, and 87 participants (77%) used a strategy categorized as neither. Question twelve was as follows:

Which of the following choices is the closest to 2500?

(a) 241+425+504+855 (b) 41719.178 divided by 19.295

(c) 48.775 x 58.985 (d) 623.97 divided by 0.2499

Five participants (4%) answered question twelve correctly, 80 participants (71%) answered question twelve incorrectly, and 28 participants (25%) did not answer question twelve. Two participants (2%) used a number sense strategy, 23 participants (20%) used a rule base strategy, and 88 participants (78%) used a strategy categorized as neither.

Discussion

The purpose of this study was to gather information about special education and general education teachers' mathematical knowledge at the pre-service level to investigate computational knowledge along with their perceptions of competence to teach mathematics and outcome expectancies. In addition, pre-service teachers' utilization of computational strategies was examined.

Findings Related to Certification

The results were consistent with Flores et al. (in press), which indicated there were no statistically significant differences in general education and special education teachers' calculation knowledge. As Flores and colleagues (in press) state finding no differences was important and has implications for teacher preparation because despite differences in preparation and highly qualified status, both groups performed at the same level. It must be noted, however, that it was unknown whether the current changes made in teacher certification had an impact on this finding or if teachers' mathematics skills were similar prior to the changes in the law. Additionally, there was no statistically significant difference in general education and special education teachers' perceived competence or outcome expectancies in teaching mathematics. This appeared to be a reasonable finding since both groups performed similarly.

Findings Related to Mathematical Strategies

Results regarding pre-service teachers' use of mathematical strategies were consistent with Yang's (2009) study which explored teachers' understanding of numerical operations and their relationships. Most participants relied on rule based strategies or did not give an explanation for the strategy chosen while fewer participants relied on number sense strategies to solve the mathematical problems. Pre-service teachers performed better regarding problems which involved the first two number sense components: (a) understanding the meaning of numbers, operations, and their relationships; and (b) recognizing relative number size. However, the majority of pre-service educators struggled with problems that required the use of the last two components: (c) developing and using benchmarks appropriately, and (d) judging the reasonableness of a computational result by using the strategies of estimation.

Such findings were consistent with earlier studies (Reys & Yang, 1998; Yang & Reys, 2002; Yang, 2003) that found fifth, sixth, and eighth grade students relied heavily on rule based strategies to solve mathematical problems, which were similar to the strategies teachers used and taught. These findings seem logical given that previous studies (Flores, Houchins, & Shippen, 2006; Montague & Van Garderen, 2003; Yang, 2009) found the types of mathematical difficulties educators display were similar to those demonstrated by children in schools. It needs to be noted that it was unknown whether there was a connection to teachers' knowledge or level of comfort with these types of mathematical tasks.

Limitations and Suggestions for Future Research

Results of the current study do have limitations. The majority of the sample came from one geographical region of the country; thus, the results might not be representative of the whole country. Also the distribution of respondents was not even in which 70 percent were students in a general education program and 30 percent of the respondents were in a special education program. Two participants did not answer questions regarding personal math efficacy and outcome expectancy. In addition, two cases were deleted as outliers. Then data were transformed to meet the assumption of a normal distribution for the outcome expectancy scores. Outcome expectancy scores had a substantial negative skew and were transformed using reflect and logarithm. A limitation in the examination of number sense strategies was the amount of pre-service educators who left open-ended questions blank. A large number of teachers did not answer how they solved the problem. This could be due to testing fatigue. Yang used an interview format which was better suited to probe pre-service educators' reasons for answering questions and elect more descriptive detail that enriches the study.

The diversity in the types of teacher certification, and the experiences that were a part of the various types of programs, could potentially reveal differences in ways strategies were chosen and in pre-service teachers' perceived competence and performance. For example, as the number of alternatively certified teachers' increase, it may be beneficial to note their perceptions regarding mathematics instruction and use of strategies to solve problems. Such perceptions and the types of strategies utilized should be considered in future investigations.

This study addressed only skills and number sense strategies of pre-service educators from kindergarten through the sixth grade. Continued investigation is needed in higher level mathematics instruction as well. For example, secondary general and special education teachers could be surveyed with consideration given to their computational knowledge and use of strategies in subjects such as algebra, geometry, and other areas of mathematics included in high school content standards. It should be noted that teacher certification standards have changed significantly for special education teachers at the secondary level. The effects of these changes currently are unknown.

CHAPTER IV. CONCLUSIONS AND RECOMMENDATIONS

The *No Child Left Behind Act* (2002) held schools accountable for adequate achievement of all students including students with disabilities. The *Individuals with Disabilities Education Improvement Act* (2004) required that special education teachers ensure students with disabilities receive access to the general education curriculum as well as meet standards for adequate yearly progress which was a measurement achievement. Teachers' content knowledge, methodological training, and education were critical features in promoting student achievement (Kamil, 2003). In consideration of mathematics achievement, the National Council of Teachers of Mathematics (NCTM) Curriculum Focal Points (2006) suggested heavy emphasis on instruction in whole numbers for young elementary students. Therefore, any intervention curriculum for students who are at-risk for mathematic difficulties should not be oversimplified and that in-depth coverage of key concepts involving whole numbers was critical for success in mathematics (Fuchs et al., 2007; Milgram & Wu, 2005). Furthermore, number sense was defined as a developing construct that referred to: (a) children's fluidity and flexibility with numbers, (b) the sense of what numbers mean, and (c) the ability to perform mental mathematics and look at the world and make comparisons (Berch, 1998).

The Glenn Report (U.S. Department of Education, 2000) outlined the need for improving teacher preparation in mathematics and science. In addition to the Glenn Report's findings, research has shown special education teachers were not familiar with the goals of the National

Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics* (Maccini, Gagnon, & Calvin, 2002).

There was scarce research about students with disabilities' number sense skills or teachers' aptitude regarding number sense skills (Gersten et al., 2009; Yang, 2009). This study explored elementary, special education and general education teachers' mathematical content and pedagogical knowledge with an added focus on number sense. Therefore, the current study examined special education and general education teachers' (a) computational knowledge, (b) efficacy to teach mathematics, and (c) their approach to calculate math problems by using number sense strategies, rule based strategies, or neither. The following research questions were developed:

1. To what extent is there a difference in the computational knowledge of special education teachers and general education teachers at the elementary level?

2. To what extent is there a difference in elementary level special education teachers' and general education teachers' personal efficacy to teach mathematical knowledge?

3. To what extent is there a difference in the mathematical outcome expectations of special education teachers and general education teachers at the elementary level?

4. In what ways do special education teachers and general education teachers approach calculating numbers, operations, and their relationships as defined by using number sense strategies, rule based strategies, or neither to solve computational problems?

The following null hypotheses were formulated for the first three research questions.

a) There are no statistically significant differences in the computational knowledge of pre-service special education teachers and general education teachers at the elementary level.

b) There are no statistically significant differences in the personal efficacy to teach mathematical knowledge of pre-service elementary level special education teachers and general education teachers.

c) There are no statistically significant differences in mathematical outcome expectations of pre-service special education teachers and general education teachers at the elementary level.

The fourth research question was examined by calculating the frequency and percent of responses to an open-ended questionnaire developed using Yang's (2009) interview instrument. Yang's questionnaire investigated teachers' understanding of strategies used by educators to solve problems based on four number sense components: (a) understanding the meaning of numbers, operations, and their relationships, (b) recognizing relative number size, (c) developing and using benchmarks appropriately, and (d) judging the reasonableness of a computational result by using the strategies of estimation. Responses were classified into three categories: number sense strategies, rule based strategies, or neither.

The independent variable in this study was *certification* representing general and special education. The dependent variables were *the percentage of correct scores* on the mathematical knowledge survey instrument completed by the participants, *the perceived competence scores, and the outcome expectancy scores* on the efficacy scale. MANOVA results indicate that there were no significant differences regarding the computational knowledge, personal efficacy, and outcome expectancy of special education teachers and general education teachers at the elementary level, Wilk's $\Lambda = .980$, $F(3, 105) = .725$, $p = .539$. These findings were consistent with a previous study conducted by Flores et al. (in press). Therefore, pre-service elementary general and special education teachers demonstrated the same level of computational skill, perceived competency, and outcome expectations.

Results from the open-ended questionnaire were examined by calculating the frequency and percent of responses exploring how teachers calculate numbers, teachers understanding of numerical operations, and their relationships as defined by using number sense strategies, rule based strategies, or neither. The findings were consistent with previous research in which most participants relied on rule based strategies or did not give an explanation for the strategy chosen while fewer participants relied on number sense strategies to solve the mathematical problems. These results indicate two possibilities that impact instruction provided to students. First, pre-service educators demonstrated difficulties with math computations that mirror problems students presently display in the classrooms. These findings were consistent with previous research (Flores et al., 2006; Montague & Van Garderen, 2003; Yang & Reys, 2002; Yang, 2003). Therefore, intensive instruction in mathematics might be limited due to the fact future elementary teachers might have the same difficulties in math. This is problematic in that educators are to teach students strategies in solving math problems as well as the mathematical concepts for students to at least achieve at a proficient level in mathematics. Not to mention, the mandates of NCLB (2002) and IDEIA (2004) that every student will perform on grade level will not be met, thus schools will increasingly struggle to make AYP. Therefore, programs that prepare pre-service educators may need to offer courses that specifically target such difficulties, and teach content that entails understanding numbers and their relationships. Second, the majority of pre-service educators had difficulty conceptually explaining how they arrived at an answer to computation problems that involved using benchmarks, and judging the reasonableness of a computational result through the use of estimation. This might hinder intensive intervention in mathematics given that future teachers might have problems articulating conceptual understanding when providing feedback to elementary students. Additionally, students must have an understanding and effectively use benchmarks and judgment of a reasonable number if they are to excel in higher order mathematical problems. Therefore, programs that prepare teachers might need to address this concern through intensive methodological courses in which pre-service teachers must articulate their conceptual understanding of the content. Results from this study are preliminary, and future research is warranted. However, once links between specific components of number sense, mathematical needs of pre-service educators, and student achievement are established, future educators can be empowered to provide the recommended intensive instruction for children at risk of mathematics difficulties.

It was recommended that perceptions and the types of mathematical strategies utilized should be considered in future investigations due to the variation in types of teacher certification programs that may lead to different experiences in mathematical instruction preparation. Moreover, continued investigation is needed in higher level mathematics instruction with consideration to computational knowledge and use of strategies in subjects such as algebra, geometry, and other areas of mathematics included in high school content standards. Therefore, teachers would be empowered to provide the in-depth instruction recommended by Gersten et al. (2009), Fuchs et al. (2007), Milgram and Wu (2005); and the NCTM, and NMAP which furnishes the competencies students with disabilities need to succeed in the mathematics curriculum.

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Appendix 1

Efficacy Scale

Please indicate the degree to which you agree or disagree with each statement below by circling the appropriate letters to the right of each statement.

SA	A	UN	D	SD
Strongly agree	Agree	Undecided	Disagree	Strongly Disagree

1	When a student does better than usual in mathematics, it is often because the teacher exerted a little extra effort.	**SA**	**A**	**UN**	**D**	**SD**
2	I will continually find better ways to teach mathematics.	**SA**	**A**	**UN**	**D**	**SD**
3	Even if I try hard, I will not teach mathematics as well as I will most subjects.	**SA**	**A**	**UN**	**D**	**SD**
4	When the mathematics grades of students improve, it is often due to their teacher having found a more effective teaching approach.	**SA**	**A**	**UN**	**D**	**SD**
5	I know how to teach mathematics concepts effectively.	**SA**	**A**	**UN**	**D**	**SD**
6	I will not be very effective in monitoring mathematics activities.	**SA**	**A**	**UN**	**D**	**SD**
7	If students are underachieving in mathematics, it is most likely due to ineffective mathematics teaching.	**SA**	**A**	**UN**	**D**	**SD**
8	I will generally teach mathematics ineffectively.	**SA**	**A**	**UN**	**D**	**SD**
9	The inadequacies of a students' mathematics background can overcome by good teaching.	**SA**	**A**	**UN**	**D**	**SD**
10	When a low achieving child progresses in mathematics, it is usually due to extra attention given by a teacher.	**SA**	**A**	**UN**	**D**	**SD**
11	I understand mathematics concepts well enough to be effective in teaching elementary mathematics.	**SA**	**A**	**UN**	**D**	**SD**
12	The teacher is generally responsible for the achievement of students in mathematics.	**SA**	**A**	**UN**	**D**	**SD**
13	Students' achievement in mathematics is directly related to their teacher's effectiveness in mathematics teaching.	**SA**	**A**	**UN**	**D**	**SD**
14	If parents comment that their child is showing more interest in mathematics at school, it is probably due to the performance of the child's teacher.	**SA**	**A**	**UN**	**D**	**SD**
15	I will find it difficult to use manipulatives to explain to students why mathematics works.	**SA**	**A**	**UN**	**D**	**SD**

16	I will typically be able to answer students' questions.	**SA**	**A**	**UN**	**D**	**SD**
17	I wonder if I will have the necessary skills to teach mathematics.	**SA**	**A**	**UN**	**D**	**SD**
18	Given a choice, I will not invite the principal to evaluate my mathematics teaching.	**SA**	**A**	**UN**	**D**	**SD**
19	When a student has difficulty understanding a mathematics concept, I will usually be at a loss to how to help the student understand it better.	**SA**	**A**	**UN**	**D**	**SD**
20	When teaching mathematics, I will usually welcome student questions.	**SA**	**A**	**UN**	**D**	**SD**
21	I do not know what to do to turn students on to mathematics.	**SA**	**A**	**UN**	**D**	**SD**

Appendix 2

Number Sense Questionnaire

Please answer the following:

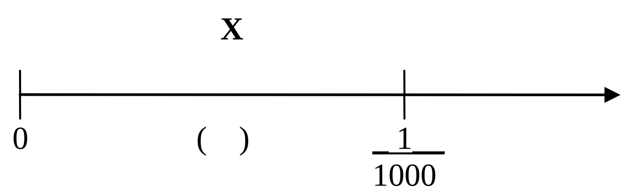
1. Circle the best choice to fill in the blank (a, b, or c)?

$\frac{174 \times 10000}{9999}$ ______ 174

(a) $<$ (b) $>$ (c) $=$

Briefly Explain How You Got Your Answer

2. Circle the best answer that represents the letter X on this number line.



(a) $\frac{2}{1000}$ (b) $\frac{1}{500}$ (c) $\frac{1}{2000}$ (d) $\frac{5}{1000}$

Briefly Explain How You Got Your Answer

3. Without calculating, circle the best estimate for 103 X 48.

   (a) 100 x 50 (b) 103 x 50 (c) 100 x 48

Briefly Explain How You Got Your Answer

4. Without calculating, write two numbers from: 4, 7, 9, 13, 15, to form a fraction closest to ½.

Briefly Explain How You Got Your Answer

5. Without calculating, which fraction

   $\frac{30}{31}$ or $\frac{36}{37}$ is closest to 1?

Briefly Explain How You Got Your Answer

6. Without Calculating, order the following numbers from smallest to largest:

   $\frac{13}{38}$, 0.966, $\frac{7}{29}$, 0.4828, $\frac{17}{16}$, $\frac{8}{15}$

Briefly Explain How You Got Your Answer

____

____

____

7. Which sum is larger than 1?

(a) $\frac{3}{11} + \frac{29}{61}$ (b) $\frac{13}{21} + \frac{37}{71}$ (c) $\frac{13}{31} + \frac{4}{9}$ (d) $\frac{6}{17} + \frac{1}{2}$

Briefly Explain How You Got Your Answer

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____

8. Place the decimal point for the product of 0.4975 x 9428.8 = 4690828

Briefly Explain How You Got Your Answer

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____

9. The product of $\frac{17}{29} \times \frac{6}{13}$ is

(a) >1/2 (b) = ½ (c) < ½ (d) Can’t decide

Briefly Explain How You Got Your Answer

____

____

____

10. What’s the reasonable estimate of 61027 divided by 33.275?

____

Briefly Explain How You Got Your Answer

____

11. What's the reasonable estimate of

$$0.495 + \frac{37}{18} + 2.875 - \frac{27}{110}$$

Briefly Explain How You Got Your Answer

12. Which of the following choices is the closest to 2500?

(b) 241+425+504+855 (b) 41719.178 divided by 19.295

(c) 48.775 x 58.985 (d) 623.97 divided by 0.2499

Briefly Explain How You Got Your Answer

Appendix 3
Teacher Background Information

Please answer the following questions if applicable:

What is your birth date?___________________

Gender: Female_______ Male________

Cultural Background:		**What grade level do you plan to teach:**	
African-American:	________	Pre K	________
Hispanic:	________	K-3	________
White:	________	4-5	________
Asian:	________	6-8	________
Other:	________	9-12	________

Type of Teaching Certificate Currently Held:

None yet	________	
Elementary	________	
Special Education (collaborative)	________	
Special Education (early childhood)	________	
Reading	________	
Middle School	________	
High School	________	
Other	________	Explain____________________

Teacher Education Program Enrolled in:

Undergraduate (general education, K-12)	________
Undergraduate (special education)	________
Master's (initial certification, general education, K-12)	________
Master's (initial certification, special education)	________
Master's (general education)	________
Master's (special Education)	________

Number of courses in K-3, 4-5, 6-8, 9-12 education <u>math</u> methodologies if any:

0	________
1	________
2	________
3	________

4 ________
5+ ________

What is the highest level mathematics course that you successfully completed?

Algebra I ________
Geometry ________
Algebra II/College Algebra ________
Trigonometry ________
Pre-Calculus ________
Calculus ________

Are you majoring in Math Education:
Yes________ No________

Do you feel competent to teach math to the level of students you will be instructing?
Yes________ Sort Of ________ No________

Appendix 4

INFORMATION LETTER
for a Research Study entitled
"Pre-service teachers' computational knowledge, efficacy, and number sense skills"

You are invited to participate in a research study to investigate pre-service teachers' mathematics computational knowledge, number sense skills, and efficacy. The study is being conducted by Vanessa Hinton, graduate student in the Auburn University Department of Special Education, Rehabilitation, Counseling/School Psychology. You were selected as a possible participant because you are a pre-service teacher and are age 19 or older.

What will be involved if you participate? If you decide to participate in this research study, you will be asked to complete a questionnaire about your background, complete an efficacy rating scale, complete 45 mathematics computation problems with problems written on the Kindergarten through adult level, and complete a questionnaire with 12 number sense problems written on an adult level. Your total time commitment will be approximately 60 minutes.

Are there any risks or discomforts? The risks associated with participating in this study are coercion to participate and psychological stress due to mathematics anxiety. To minimize risk of coercion, I will provide every student with an informational letter and survey packet so that the choice to participate or not participate is not obvious to others. To minimize risks associated with anxiety, I emphasize that participation is voluntary with no penalty for nonparticipation or for withdrawing your consent to participate. Your responses will be anonymous, meaning that there will be no way for the researcher to connect you to your information.

Are there any benefits to yourself or others? If you participate in this study, you can expect to feel personal gratification that your responses will inform the field of teacher preparation and shape future teachers' preparation in the area of mathematics. I cannot promise you that you will receive any or all of the benefits described.

Will you receive compensation for participating? There is no compensation, but I thank you for your time.

Are there any costs? If you decide to participate, there will be no costs to you.

If you change your mind about participating, you can withdraw at any time during the study. Your participation is completely voluntary. If you change your mind about participating, you can withdraw at any time, even after submitting the survey. There will be a code number written on the top of the first page of the survey. Keep a record of that number, and if you want to withdraw your survey after submitting it, you only will need to give us that number (no name) so that we can locate the survey and shred it. Your name will never be associated with that number in order to keep your responses anonymous. Your decision about whether or not to participate or to stop participating will not jeopardize your future relations with Auburn University, the Department of Special Education, Rehabilitation, Counseling/School Psychology.

Any data obtained in connection with this study will remain anonymous. The researcher will have no way to connect you to your information that you provide.

If you have questions about this study, *please ask them now or* contact **Vanessa Hinton** at 334-707-1494 or **Margaret Flores** at **844-2107**. You may also email any questions to Vanessa Hinton at vmh0002@auburn.edu or Margaret Flores at mmf0010@auburn.edu. A copy of this document will be given to you to keep.
If you have questions about your rights as a research participant, you may contact the Auburn University Office of Human Subjects Research or the Institutional Review Board by phone (334)-844-5966 or e-mail at hsubjec@auburn.edu or IRBChair@auburn.edu.

HAVING READ THE INFORMATION PROVIDED, YOU MUST DECIDE IF YOU WANT TO PARTICIPATE IN THIS RESEARCH PROJECT. IF YOU DECIDE TO PARTICIPATE, THE DATA YOU PROVIDE WILL SERVE AS YOUR AGREEMENT TO DO SO. THIS LETTER IS YOURS TO KEEP.

________________________________ ___Vanessa Hinton__________

Investigator's signature Date Print Name