Emergency Management Strategies for the Retail Industry

by

Kandace N. Ballard

A dissertation submitted to the Graduate Faculty of
Auburn University
in partial fulfillment of the
requirements for the Degree of
Doctor of Philosophy

Auburn, Alabama
December 12, 2011

Keywords: Inventory, minimax, minimax regret, disruption management, stochastic programming

Approved by:

Emmett J. Lodree, Jr., Chair, Assistant Professor of Industrial and Systems Engineering
Chan Park, Daniel F. and Josephine Breeden Professor of Industrial and Systems Engineering
Ming Liao, Professor of Mathematics and Statistics

Abstract

Uncertain disruptions complicate inventory management for retailers because it is difficult to determine when and how to adjust order quantities to ensure high service levels before and after a possible disruption. In order to assist retailers with their decision-making, three different models have been developed to determine the importance and effectiveness of considering proactive approaches to disruptions, specifically, forecasted storms. The first model addresses the time horizon when a storm is approaching and it is not known whether a demand surge for emergency items will occur. Minimax decision criterion highlighted the circumstances that constitute increasing the order quantity or encouraging the retailer to adopt a proactive approach instead of the current strategy of "wait-and-see". Ordering strategies also affect the response efforts as shown in the second model, which is comprised of two uncertain disruptions. Minimax and minimax regret decision criteria were used to evaluate the model and provided insight into the inventory management decisions during and after the storm. Minimax regret decision criterion supported holding inventory during the storm to ensure a higher service level after the storm while minimax decision criterion supported the opposite. However, both criteria advocated holding the same order quantity during the second disruption or demand surge after the storm. Lastly, a stochastic programming model was developed to determine pre-positioned quantities for a network of retailers in addition to post-storm shipments between retailers and the manufacturer to alleviate shortages. The results from the commercial software and solution methodology supported pre-positioning items at most retailers despite the manufacturer's location. Overall, the research presented in this dissertation illustrates the importance and cost effectiveness of incorporating inventory strategies into management decisions when a storm is on the horizon.

Acknowledgments

I would first and foremost like to thank my Lord and Savior Jesus Christ because without Him, I would not have completed the process of obtaining a Ph.D. He refined me and showed me that I can do anything with Him. Secondly, I want to thank my husband, Eric, for being so encouraging and supportive thoughout my studies. He continued to push me when I wanted to give up and listened to all of the issues I ran into even when it was not interesting. I am so grateful and thankful for him. I am also eternally grateful for my advisor, Dr. Emmett Lodree, for continuing to guide my dissertation from afar. He continued to encourage, support, and guide me even when it was uncertain where my dissertation was going. In addition, I would like to thank my committee, Dr. Chan Park and Dr. Ming Liao, for their support in the dissertation process. I would also like to thank my parents, Don and Diane, for always teaching me to strive to do my best in all things. I also appreciate my second set of parents, my in-laws, Randy and Sandy for their continued prayers, support, and understanding (especially during the holidays). Lastly, I hope that I can be an example to my daughter, Kya, to show her that she can do anything with God on her side, hard work, and perseverance.

Table of Contents

List of Illustrations

List of Tables

Chapter 1

Introduction

Recent natural disasters such as the terrorist attacks of 9/11, Hurricane Katrina, and the flooding along the Mississippi River during the spring of 2011, have revealed the shortcomings of the preparation and response procedures of public and private sectors. As a result, a multitude of academic disciplines have begun to explore ways to mitigate risks, prepare in advance for impending disasters, respond to supply disruptions caused by disasters, and rebuild socio-economic structures affected by disasters.

Private sector organizations' decisions regarding emergency management affect residents' abilities to prepare for a possible storm. Their decisions also affect government and non-governmental organizations' execution of their policies to provide necessary supplies. Retailers face the decision of whether or not they should order additional supplies to prepare for a storm, and if the storm does not materialize, they will incur holding costs or be forced to sell goods at discounted prices. On the other hand, if they are not prepared, they will hinder the preparation and response processes and forfeit additional revenue. The uncertainty of events such as hurricanes make decisions regarding emergency management for the private sector challenging.

On a personal note, my parents evacuated for Hurricane Ike in 2008, and it made me consider how to make the process of preparing for a storm seem less daunting. They, like many others, struggled to find the necessary supplies and food to sustain them until the retailers reopened. Through my research, I am thankful that I can utilize my knowledge about inventory and operations research to solve meaningful problems that may alleviate the stress for coastal residents in the event that they have to evacuate or prepare to ride out a storm.

1.1 Overview of Proposal

1.1.1 Minimax EOQ Policies with Demand Surge, Lead-Time and Lost Sales

Chapter 2 addresses an inventory control problem that seeks to determine an ordering strategy for an uncertain demand disruption considering emergency supplies. The model extends the model presented in Lodree [16] by including lead-time and lost sales. The demand disruption that is explored in this paper is a possible sudden surge in demand or an increase in the demand rate before a possible disaster. Retailers must decide if they will adopt a proactive approach meaning they will modify their batch size before they are certain the demand surge will occur, or a reactive approach which means that they will not modify their batch size until they are certain the demand surge will occur. If retailers choose a reactive ordering strategy, they may incur lost sales as a consequence of the lead-time. The typical approach for handling demand uncertainty is determining an optimal order quantity using long run approximations, which does not adequately address the problem because each storm has different characteristics that will affect the strategy selection. In order to effectively address the problem, minimax decision criterion is used to evaluate the ordering strategy decision. Many papers have addressed risk mitigation associated with supply or demand disruptions by determining optimal order quantities during disruptions caused by various events (e.g., [46], [23] and [65]) and predicting the supplier's availability to determine optimal order quantities (e.g., [47], [47], and [55]). Within the demand disruption literature, Lodree and Taskin [35] and Taskin and Lodree [62] allowed lost sales for single-period problems when a demand surge occurs. Other papers in demand disruption literature address lead-time, but not lost sales because the disruption is defined as a period of no demand (e.g., [51]). As previously discussed, this paper investigates ordering strategies for uncertain demand surges while considering lead-time and allowing lost sales. Minimax decision criterion has been utilized in the literature considering optimal ordering quantities (e.g., [49], [19], [28], and [18]), but minimax decision criterion has been applied to a disruption problem in only one other paper ([16]) to the knowledge of the authors. We develop a framework to assist retailers with their decision-making before an uncertain demand surge using minimax decision criterion.

This paper provides the following contributions:

- Closed form solutions are determined for the number of orders and order quantities for items with non-zero lead-time during a demand disruption over a finite horizon.
- An ordering strategy (proactive or reactive) is determined by applying minimax decision criterion to inventory disruption problems.

1.1.2 Minimax and Minimax Regret Inventory Control Policies Regarding Demand Disruptions and Damaged Inventory

The model presented in Chapter 3 considers the time horizon that begins at the end of the demand disruption that is explored in Chapter 2. When the time horizon in Chapter 3 begins, a second and third demand disruption may occur and last for a known amount of time. The disruptions explored in Chapter 3 are a period of no demand where the retailer might be closed as a result of an uncertain event such as a hurricane and the demand surge that occurs after the effects of the storm have subsided. The surge in Chapter 2 captures the preparation for the event, while Chapter 3 considers the time period during and after the storm. When retailers are faced with an uncertain event, they must decide if they will keep inventory on-hand to be prepared for a demand surge after the disruption is over. An additional source of uncertainty that is considered in this chapter is whether the inventory will be damaged during the disruption. Retailers may either hold inventory, which also includes the decision about the quantity to hold, or they may not hold inventory assuming that all of the inventory will be damaged.

In addition to minimax decision criterion, minimax regret decision criterion will be utilized to determine the best strategy. Savage [57] first explored the minimax regret decision rule to demonstrate its uses when making optimal decisions. Several authors have used minimax regret decision criterion when considering the distribution free newsvendor problem (e.g., [43], [32], [64], and [69]). Perakis and Roels [49] found solutions for the optimal order quantity that minimize the maximum regret for not choosing optimally when the demand distribution has partial information. Jammernegg and Kischka [29] considered two other risk factors in their objective function to determine a robust solution for the order quantity. With regard to our model, the absolute regret will be minimized considering each inventory decision.

This paper provides similar contributions to Chapter 2 and they are the following:

- A model is developed for two possible disruptions that occur after the demand surge presented in Chapter 2.
- A more general class of possible disruption cases with two sources of uncertainty are considered.
- An ordering strategy is determined by applying minimax and minimax regret decision criteria to an inventory disruption problem.

1.1.3 Pre-Positioning Hurricane Supplies in a Commercial Supply Chain

The fourth chapter considers the pre-positioning problem from a profit-driven perspective for a supply chain comprised of one manufacturer and a network of retailers. The manufacturer must make decisions regarding the location and quantity of emergency items that will be pre-positioned across a network of retailers when an observed storm might make landfall at one or more of the retailers. After the storm comes ashore, transshipments from other retailers and shipments directly from the manufacturer alleviate shortages at retailer locations affected by the storm. The problem is formulated as a two-stage stochastic programming model, which is illustrated by a numerical example and case study scenarios derived by Rawls and Turnquist [53]. Commercial software and a solution approach that reduces the model to the well-known transportation problem are employed to solve the first and second stage decisions, which involve the pre-positioned quantities and post-storm shipments including transshipments and direct shipments from the manufacturer. As a result of several devastating hurricanes, earthquakes, and other disasters, pre-positioning has been researched extensively for governmental and non-governmental agencies. Location, inventory and distribution comprise the three aspects of pre-positioning mentioned in Richardson et al. [54], and only a few studies integrate all three into a two-stage stochastic programming model (e.g., [14], [38], [53], and [56]). The model presented in this chapter considers the inventory and distribution decisions similar to Barbarosoğlu and Arda [6].

This paper provides the following contributions to the pre-positioning literature:

- A two-stage stochastic programming model is developed considering the manufacturer's perspective.
- The model takes into account the additional production costs incurred during the first and second stages, transshipments from other retailers in addition to direct shipments from the manufacturer.

Chapter 2

Minimax EOQ Policies with Demand Surge, Lead-Time and Lost Sales

2.1 Introduction

Many organizations such as FEMA and the Red Cross have been instrumental in providing disaster relief assistance to residents that have been affected by flooding, hurricanes, and other catastrophic storms. These organizations set up shelters and pre-position supplies to respond to the affected area as soon as possible after a storm has passed through. While these organizations have provided assistance to millions of disaster victims, the success of disaster relief efforts also depends heavily upon local retailers.

Retailers are an important part of the humanitarian relief supply chain because they stockpile items that are consumed directly by victims. Consider that organizations such as FEMA recommend that coastal residents prepare a survival kit at the beginning of the hurricane season ([1]), which begins June 1st. However, most coastal residents do not gather these items until they are certain an observed storm will make landfall. In 2010, the National Hurricane Survival Initiative surveyed residents from the coast of Virginia to Texas and discovered that 47 percent of the polled residents did not have a hurricane survival kit, which is three day's worth of water, food, and medicines for each family member [27]. Coastal residents or consumers typically wait until the last minute to gather supplies, and unfortunately, many retailers adopt the same attitude regarding their preparation for a possible emergency supply demand surge. As a result, residents are more susceptible to the risks posed by severe storms or other hazards. Additionally, stockout of items penalize retailers in terms of lost sales costs and possibly long term effects related to loss of customer goodwill. While preparation is the residents' responsibility, they can only prepare as well as the retailers that provide the supplies.

In order to improve preparation efforts for a possible spike in demand for emergency supplies that often accompanies disaster relief operations, this paper presents an approach that allows the retailer to compare reactive and proactive inventory strategies. We consider reactive and proactive inventory control policies for commodities such as food, water, and over the counter medicines and supplies such as batteries and flashlights. Since these types of items are typically characterized by a steady demand rate with minimal variation under normal conditions, we assume an underlying continuous review inventory control policy, particularly the economic order quantity. We do not specifically address demand for gas-powered generators, which are slower moving items on a day-to-day basis. A newsvendor or periodic review policy would be more appropriate for this purpose. We also evaluate the effectiveness of the continuous review inventory control policy under storm uncertainty according to the minimax decision criterion as opposed to the more mainstream expected value criterion. Disruption management strategies based on the expected value criterion are appropriate when planning for frequently occurring disruptions in which meaningful forecasts could be generated based on sufficient historical data. However, our focus in this paper is severe storms, namely hurricanes or tropical storms, as well as winter storms. For these and other extreme events, forecasts are less meaningful compared to more frequent disruption events. Interestingly, distribution free approaches to proactive disruption management planning are a rare occurrence in the research literature. Existing proactive approaches to disruption management are almost exclusively based on expected value criterion, which is not appropriate for rare disruption events as described above. On the other hand, existing approaches to disruption management that do apply to rare events are almost exclusively reactive strategies. This paper investigates both reactive and proactive disruption management policies for rare events that do not depend on unreliable forecasts. We extend Lodree's [16] work by incorporating lost sales and a non-zero lead-time into the model, and we seek to answer the following questions:

Question 2.1 *How does the lead-time and lost sales affect the strategy decision?*

Question 2.2 *What circumstances dictate the strategy decision?*

Question 2.3 *How does each parameter affect the strategy decision?*

It should be noted that this paper does not intend to show how the retailers can always meet their customers' needs during a crisis, but we demonstrate that there are circumstances when it is beneficial for retailers to be prepared for a possible demand surge before a storm affects their area. As a result, it might improve consumers' ability to obtain the supplies that they need to prepare for a storm. The paper is organized as follows: literature review, model formulation, results, and summary and future extensions.

2.2 Literature Review

Supply and demand disruptions caused by events such as strikes, natural disasters, unreliable suppliers, and machine breakdowns within the context of production, operations, and supply chain management have been extensively researched in the literature. As a result of disruptions, lost sales or backorders are common occurrences depending on the length of the disruption.

Supply disruptions are characterized as an interruption in a supplier's operations, which means that the supplier becomes unavailable or unable to fill orders for various reasons from strikes to machinery breakdowns. One of the earliest papers to address disruptions and lost sales computed the percentage of unmet demand in a production system as a result of a disruption [39]. Several papers have also concentrated on supply disruptions within a production system and determined periodic review (e.g., [42], [34]) and continuous review inventory policies ([41]). In relation to our model, Parlar and Berkin [46], Berk and Arreola-Risa [9], Snyder [60], and Heimann and Waage [23] developed economic order quantity inventory models considering supply disruptions. Recently, Qi et al. [50] found an optimal order quantity when the retailer and supplier both face disruptions where the retailer's disruption is a loss of inventory. The focus of this literature review will mainly be on inventory models that consider demand disruptions, lead-time, and lost sales as well as a brief review of minimax inventory models.

2.2.1 Demand Disruptions

Demand disruptions have been addressed as periods of no demand ([65]), fluctuating demand ([61], [11]), and a surge in demand ([16], [35], [55]). Brill and Chaouch [11] found results to support their claim that an adjustment should be made to an EOQ policy when a major disruption occurs, but if the disruption is small, no adjustment is necessary. Ross et al. [55] developed time-dependent policies for disruptions caused by a surge in demand. Within the disaster management literature, Lodree and Taskin [35] compared the newsvendor solution to the optimal inventory level they determined for a one-time decision for retailers that were considering stocking up before a storm. However, the model can be applied to any type of demand disruption that has a forecast associated with it. Using optimal control, Lodree and Taskin [35] determined when retailers should increase their order quantity of emergency supplies. In addition, Taskin and Lodree [62] determined how a retailer should adjust their inventory level in anticipation of a demand surge that will occur during hurricane season based on estimated demand that is a convolution of current demand and an estimate of demand for hurricane season. Lodree [16] presented a model to determine the retailers' ordering strategy for emergency supplies when they are faced with an uncertain demand surge caused by a forecasted storm. He used minimax decision criterion to determine the best ordering strategy, which is contrary to the approach in the demand and supply disruption literature. Typically, long run approximations are used to determine order quantities in order to alleviate the effects of a demand or supply disruption (e.g., [46], [23]). Similar to the model presented in Lodree [16], we are considering infrequent, unpredictable storms, and it is not appropriate to use these types of approximations as previously mentioned. In addition, each storm is unique with different characteristics such as the severity of the storm, how long people may potentially be confined to their homes, etc. Therefore, the same approximation for an order quantity cannot be applied to each storm. We handle this issue by using Lodree's [16] approach of employing minimax decision criterion to aid the retailer in determining whether they should consider adopting a reactive (maintain order quantity) or proactive (increase order quantity) strategy regarding inventory control policies before an uncertain demand surge.

2.2.2 Minimax Decision Criterion

Lastly, we will briefly review a sample of minimax models in the inventory control literature. Scarf [58] developed the first distribution free model by determining the demand distribution with a known mean and variance that would maximize the minimum profit. Several authors have extended Scarf's research by increasing the number of observed periods ([2]), proving Scarf's ordering rule is optimal for the newsboy problem ([19]), determining optimal (Q, r) policies ([28], [18]), and determining the ordering policy using linear programming ([20]). By applying minimax decision criterion, Yu [68] solved the EOQ problem with a random demand rate, order cost, and holding cost using an efficient linear time algorithm. He demonstrated the many advantages of the robustness approach compared to the stochastic approach. With respect to our model, the only other paper known to have used minimax decision criterion within the context of disruption management is Lodree [16].

The main contribution of this paper is to provide a general decision framework that retailers can utilize to determine when it is advantageous to increase their order quantity before an uncertain demand surge. We provide an ordering strategy supported by minimax decision criterion instead of a specific inventory policy, which is contrary to other literature addressing demand disruptions and lost sales. If the retailer's objective is to minimize the cost for the worst outcome, the results given by minimax decision criterion will be more useful than determining an inventory policy that minimizes average total cost.

2.3 Model Formulation

2.3.1 Model Assumptions

The inventory model presented in this paper extends the model presented in Lodree [16], which is based on an underlying continuous review model, and more specifically, the EOQ model. We assume that if the disruption does not occur, the retailer employs an EOQ ordering policy. The EOQ model is extended to consider how a change in the order quantity will improve the retailers' costs if a disruption occurs. If the disruption occurs, we determine the conditions that promote increasing the current order quantity (EOQ) to accommodate a surge in demand by using minimax decision criterion. In addition, lead-time is considered in the model, and we assume that retailer will choose a reorder point that will minimize lost sales. However, it should be noted that we are not focused on finding an optimal reorder point or quantity, but determining the strategy that minimizes the retailers' maximum cost.

The retailer may decide to adopt a reactive, R, inventory control strategy meaning that the current order quantity will not be changed in response to learning about a potential demand disruption or surge. After the demand surge begins, the retailer will have the opportunity to modify the order quantity, which will also alter the reorder point. On the other hand, she may choose to be proactive, P, where the order quantity is increased in anticipation of a demand surge. If the disruption does not materialize, she will decrease the order quantity to the normal order quantity (EOQ) after it is certain the demand surge will not occur. It should be noted that the order quantity received before the demand disruption occurs is based on the strategy selected before the first cycle begins. Please refer to Table 2.3.1 for the list of model assumptions in addition to the following EOQ model assumptions: constant lead-time, known demand for disruption and normal conditions, constant demand, and constant ordering cost.

A few of the model assumptions generate further explanation, which we will address in this paragraph. We assume that the lead-time is short because the types of items that

Table 2.3.1: Chapter 2 Model Assumptions

1. Single, fast-moving, non-perishable item is considered.
2. Lead-time is short.
3. Backorders are not allowed.
4. Order quantity under normal conditions is EOQ.
5. Reliable and adaptable supplier
6. Demand surge occurs during first inventory cycle.
7. Time horizon for disruption is known.
8. Time horizon for demand surge is long.
9. Retailers will not incur lost sales if they are proactive.
10. Order quantity (q) and number of orders (m) are non-integer valued.
11. Reordering decisions are based on strategy selection.
12. The one source of uncertainty is the occurrence of a disruption.

are considered are fast-moving items such as bottled water. Backorders are not allowed because customers will not wait for the next order to arrive. In addition, we assume that the demand surge occurs during the first inventory cycle meaning retailers make a decision beforehand to receive either a modified order quantity or the EOQ at the beginning ($t = 0$) of the cycle containing the possible demand surge. The time horizon considered is longer than a usual inventory cycle because retailers would not be concerned about a negligible disruption. An additional note about reordering is that the retailer may have to reorder before the disruption is expected to occur. Therefore, when a retailer selects the proactive strategy, she will reorder based on the assumption that the disruption will occur. On the other hand, if the retailer decides to maintain her order quantity, she will reorder based on the assumption that the disruption will not occur. This will be discussed further in the sections regarding the strategies.

2.3.2 General Cost Equation and Model Notation

In order to determine the strategy supported by minimax decision criterion, cost equations were developed for all of the possible cases considering the model assumptions. The cost equations have the following notation (Please refer to Table 2.3.2 for the model notation):

$$TC_S^D(\vec{q}) = A\vec{m} + h\overline{I}_S^D + sY_S^D. \tag{2.3.1}$$

Table 2.3.2: Chapter 2 Notation

Parameter	**Description**
$D \in \{0, 1\}$	0, no disruption occurs or 1, disruption occurs
$S \in \{R, P\}$	R, reactive strategy or P, proactive strategy
$\vec{q} \in \{q_i, q_{i+1}, ..., q_M\}$	ordering policy
$q_i \in \{q_E, q_R, q_P\}$	order quantity i
q_E	economic order quantity
q_R	disruption order quantity for R
q_P	disruption order quantity for P
m_i	sum of the number of orders for each q_i
A	ordering cost
h	holding cost (per unit/per unit time)
$\overline{I}$	average inventory level
s	lost sales cost (per unit)
Y	number of lost sales units
T_1	time when disruption begins
T_2	time when disruption ends
λ	normal demand
λ_D	surge demand

If the retailer chooses a reactive strategy and a demand surge does not occur, the retailer's cost will be $TC_R^0(\vec{q})$; if a disruption does occur, the cost is $TC_{R_j}^1(\vec{q})$. If a reactive strategy is selected, there are four different cost equations for the outcome, $D = 1$, which are dependent upon the lead-time denoted by $j \in \{1, 2, 3, 4\}$. Similarly, the cost functions for the proactive strategy are $TC_P^1(\vec{q})$ and $TC_{P_k}^0(\vec{q})$ where $k \in \{1, 2, 3, 4\}$.

2.3.3 Minimax Decision Criterion

We apply minimax decision criterion to the cost equations developed for each combination of strategy and outcome to determine the strategy that minimizes the maximum cost.

The general minimax formulation is shown in Eq. (2.3.2).

$$\min_{S\in\{R,P\}} \max_{D\in\{0,1\}} \min_{\vec{q}\in\mathbb{Q}} TC_S^D(\vec{q}), \tag{2.3.2}$$

where

$$\mathbb{Q} = \sum_{i=1}^{M} q_i m_i = D_T \text{ and } I(t) \geq 0 \; \forall \, t \in [0, T_2]$$

- $\mathbb{Q}$ = solution space for inventory policies
- q_i = order quantity
- m_i = number of orders of q_i
- M = number of times q_i changes
- D_T = total demand fulfilled during time horizon

First, the inventory policies that minimize the cost for each of the cases are selected by finding $\vec{q}$, which is based on the strategy. Then, the cost function for the outcome that maximizes the cost for each strategy is selected meaning that the choices are narrowed down to two, a maximum cost equation for the reactive strategy and a maximum cost equation for the proactive strategy. It should be noted that the cost equation used for the comparison between the reactive disruption case (j) and proactive no disruption case (k) is based on the parameters meaning two cost equations are compared for each strategy, which are no disruption and disruption. Lastly, the maximum cost for each strategy is compared, and the strategy with the smallest maximum cost is selected.

2.3.4 Reactive Strategy

In this section, we develop cost equations for each possible case, give the optimal disruption order quantity, and determine the maximum cost equation for the reactive strategy. Four demand disruption cases are presented and the costs are compared to the no disruption case, which is a finite horizon EOQ model.

Disruption Cases

Based on our assumptions, the lead-time (L) and the time when the disruption begins (T_1) create four cases when a demand surge occurs. In addition, it is assumed that the time horizon for the demand surge is long (i.e., $T_1 + L \geq T_2$), so we only consider cases that require the retailer to place at least one order of the disruption order quantity meaning the lead-time is shorter than an inventory cycle regardless of whether the disruption occurs.

Case 1: $\frac{q_E-\lambda T_1}{\lambda_D} \leq L \leq \frac{q_E-\lambda T_1}{\lambda}$

The inventory level and order quantities over the time horizon are shown in Figure 2.3.1 for Case 1. The inequality for the values of L gives the range of lead-time values that create the situation where the retailer normally reorders after the disruption has begun and the lead-time causes lost sales to occur. The lead-time is longer than the time between T_1 and t_{1R} (time when the order is depleted). In a typical EOQ model, the retailer would reorder when the inventory level is λL, and in this case, the reorder point will occur after the disruption has begun indicating a short lead-time. We assume that the retailer places her order at T_1 instead of waiting until the inventory level reaches λL. If the retailer waits to place her next order when the inventory level reaches λL, she will incur higher lost sales costs, which is why we assume the order is placed at T_1. The retailer will increase her order quantity to q_R at T_1 in order to meet demand during the remainder of the disruption. When the first order of q_R arrives at $T_1 + L$, it will be after the inventory has been depleted at t_{1R} and the retailer will experience lost sales.

The ordering policy consists of two components, which are the order quantity received at $t = 0$ ($q_1 = q_E$), and the order quantity received after the disruption has begun ($q_2 = q_R$). After the retailer receives the order of q_R, she will not incur lost sales for the remainder of the disruption. We determined the number of orders, m_i, by determining the total demand that will be met by the retailer (excluding lost sales) and dividing by the order quantity, q_i. The values of m are as follows: $m_1 = \frac{\lambda T_1+\lambda_D(t_{1R}-T_1)}{q_E}$ and $m_2 = \frac{\lambda_D(T_2-T_1-L)}{q_R}$. After inserting

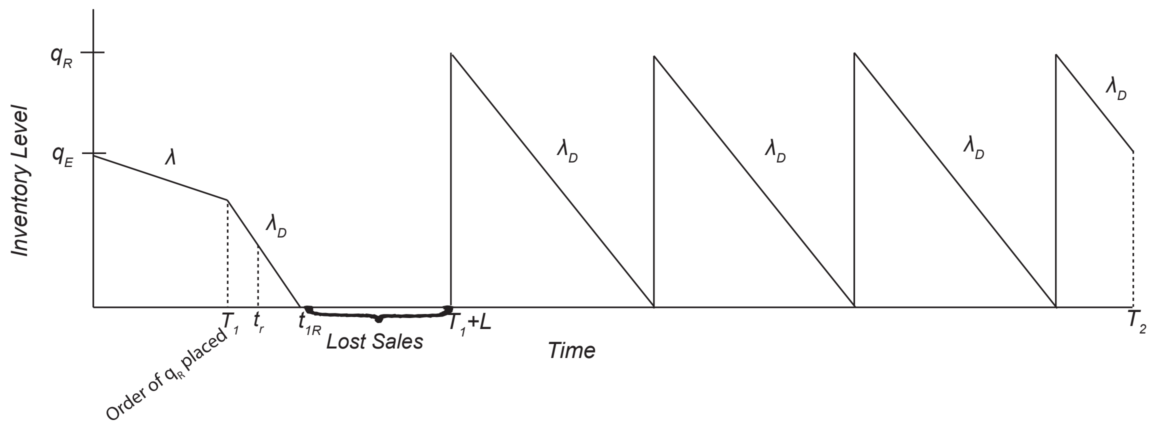


Figure 2.3.1: Reactive Strategy with Disruption: Case 1

the values for $\vec{q}$ and m into Eq. (2.3.2), we derived Eq. (2.3.3) in Appendix A.1.

$$TC^1_{R_1}(q_E, q_R) = A\left[\frac{\lambda T_1}{q_E} + \frac{\lambda_D(t_{1R} - T_1)}{q_E} + \frac{\lambda_D(T_2 - T_1 - L)}{q_R}\right] + h\left(\frac{q_E t_{1R}}{2} + \frac{q_R(T_2 - T_1 - L)}{2}\right) + s\lambda_D(T_1 + L - t_{1R}) \tag{2.3.3}$$

Case 2: $L \leq \frac{q_E - \lambda T_1}{\lambda_D}$

In Case 2, the lead-time is such that the retailer does not experience lost sales, and her typical reorder point occurs after T_1 meaning the lead-time is shorter than the lead-time in Case 1. Again, we assume that the retailer will place an order at T_1 and receive it when the first order has been depleted at t_{1R}. This case is similar to Case 1 as shown in Figure 2.3.1 with the exception of no lost sales. Eq. (2.3.4) gives the cost equation for Case 2.

$$TC^1_{R_2}(q_E, q_R) = A\left(\frac{\lambda T_1}{q_E} + \frac{\lambda_D(t_{1R} - T_1)}{q_E} + \frac{\lambda_D(T_2 - t_{1R})}{q_R}\right) + h\left(\frac{q_E t_{1R}}{2} + \frac{q_R(T_2 - t_{1R})}{2}\right) \tag{2.3.4}$$

Case 3: $L \geq \frac{q_E}{\lambda_D} + \frac{q_E}{\lambda} - T_1$

The third case represents when the lead-time causes the time to reorder to occur before T_1. We assume that the retailer places another order of q_E at her normal time to reorder because she has selected a reactive strategy. She maintains her order quantity because she is ordering before it is known that the disruption will occur as shown in Figure 2.3.2. In addition, we assume that the retailer places an order of q_R at T_1, and she will receive it at $T_1 + L$, which is after the second order has been depleted. This creates a second time period of lost sales. Once she receives her order of q_R, she will not incur lost sales for the remainder of the disruption. The cost equation for Case 3 is shown in Eq. 2.3.5.

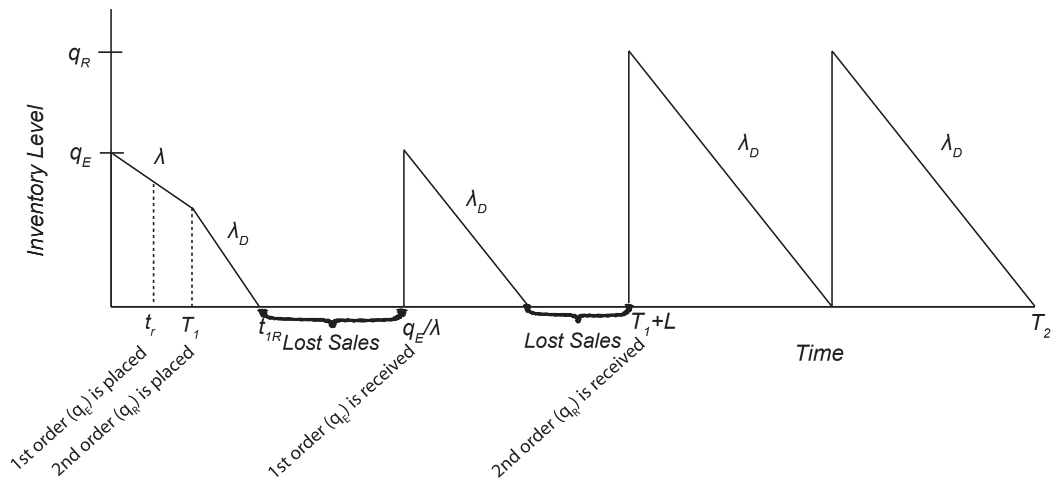


Figure 2.3.2: Reactive Strategy with Disruption: Case 3

$$
\begin{aligned}
TC_{R_3}^{1}(q_E, q_R) \quad = \quad & A\left(\frac{\lambda T_1}{q_E} + \frac{\lambda_D(t_{1R} - T_1)}{q_E} + 1 + \frac{\lambda_D(T_2 - T_1 - L)}{q_R}\right) \\
& + h\left(\frac{q_E t_{1R}}{2} + \frac{{q_E}^2}{2\lambda_D} + \frac{q_R(T_2 - T_1 - L)}{2}\right) + s\lambda_D\left(T_1 + L - \frac{q_E}{\lambda_D} - t_{1R}\right)
\end{aligned}
\qquad (2.3.5)
$$

Case 4: $\frac{q_E}{\lambda} - T_1 \leq L \leq \frac{q_E}{\lambda_D} + \frac{q_E}{\lambda} - T_1$

The fourth and final case that we consider for the reactive strategy with a disruption is similar to the third case because the retailer places her second order of q_E before T_1. However, the lead-time is such that lost sales are not incurred after the second order is received. We assume that the retailer chooses to order q_R so that she receives it when the second order is depleted at $\frac{q_E}{\lambda_D} + \frac{q_E}{\lambda}$ meaning the order will be placed after T_1. Please refer to Eq. 2.3.6 for the cost equation for Case 4.

$$TC^1_{R_4}(q_E, q_R) = A\left(\frac{\lambda T_1}{q_E} + \frac{\lambda_D(t_{1R} - T_1)}{q_E} + 1 + \frac{\lambda_D(T_2 - \frac{q_E}{\lambda_D} - \frac{q_E}{\lambda})}{q_R}\right) + h\left(\frac{q_E t_{1R}}{2} + \frac{{q_E}^2}{2\lambda_D} + \frac{q_R(T_2 - \frac{q_E}{\lambda_D} - \frac{q_E}{\lambda})}{2}\right) + s\lambda_D\left(\frac{q_E}{\lambda} - t_{1R}\right) \tag{2.3.6}$$

We determine the optimal order quantity during the disruption by applying the first order condition to each cost equation. The disruption order quantity is the same for each case, and it is similar to the EOQ (q_E) with the disruption demand rate, λ_D, replacing λ as shown in Eq. (2.3.7).

$$q_R = \sqrt{\frac{2A\lambda_D}{h}} \tag{2.3.7}$$

No Disruption Case

If the disruption does not occur, we have a finite horizon EOQ model. This means that the disruption does not occur at T_1 and the demand rate is λ during the time horizon $[0, T_2]$. Please refer to Eq. 2.3.8 for the cost equation.

$$TC^0_R(q_E) = A\left(\frac{\lambda T_2}{q_E}\right) + h\left(\frac{T_2 q_E}{2}\right) \tag{2.3.8}$$

For the following proposition, the proofs are provided in Appendix A.2.

Proposition 1 *When $S = R$,*

$$\max_{D\in\{0,1\}} \min_{\vec{q}\in\mathbb{Q}} TC^D_S(\vec{q}) = TC^1_{R_j}(\vec{q})\ \forall\, j \in \{2, 3, 4\}$$

.

Proposition 1 states that when the reactive strategy is selected, the maximum cost is incurred when a disruption occurs. There are a few conditions to this statement, though. For the disruption cases, $j = \{2, 3, 4\}$, it is always clear that the cost is greater than if there is no disruption. If the disruption case $j = 1$ occurs, we are able to find that the cost is greater than the case when there is no disruption if the lost sales cost satisfy the inequality, $s \geq \frac{hq_R}{\lambda_D}$. Overall, retailers can expect to incur the maximum cost if a disruption occurs, and a reactive strategy is selected.

2.3.5 Proactive Strategy

Disruption Case

When the retailer selects a proactive strategy and the demand surge occurs, there is only one case to consider. Regardless of whether the time to reorder is before or after T_1, the retailer will order q_P, which is the optimal order quantity during the disruption if a proactive strategy is selected. The retailer will base the ordering decision on the assumption that the disruption will occur. Please refer to Figure 2.3.3 for the representation of the disruption case for the proactive strategy.

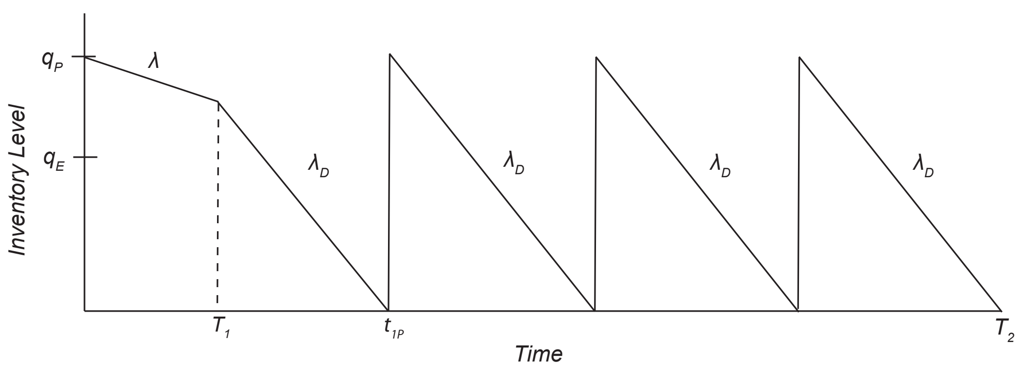


Figure 2.3.3: Proactive Strategy with Disruption

Eq. (2.3.9) gives the cost equation for the proactive strategy with a demand surge.

$$TC_P^1(q_P) = A\left(\frac{D_T}{q_P}\right) + h\left(\frac{T_2 q_P}{2}\right), \tag{2.3.9}$$

where:

$D_T = \lambda T_1 + \lambda_D(T_2 - T_1)$

By applying the first order condition to Eq. (2.3.9), we are able to determine the optimal order quantity for the proactive strategy given in Eq. (2.3.10).

$$q_P = \sqrt{\frac{2AD_T}{hT_2}} \tag{2.3.10}$$

No Disruption Cases

If no disruption occurs, four cases are associated with the proactive strategy. The four cases are dependent upon the lead-time, T_1, and T_2.

Case 1: $L \geq \frac{q_P - \lambda T_1}{\lambda_D}$ **and** $T_2 \geq \frac{2q_P}{\lambda}$

In this case, the lead-time is longer than the time between when the disruption begins and the first order is depleted, which means that the retailer must reorder before being certain that the demand surge will occur. Because the retailer has selected a proactive strategy, she will reorder based on the assumption that the demand surge will occur as shown by the two orders of q_P in Figure 2.3.4. In addition, the retailer may delay the order to be received at $\frac{q_P}{\lambda}$ to decrease holding costs. We assume this change can be made at no penalty because the supplier is adaptable and reliable. However, the supplier is not be able to rush orders to prevent lost sales in previous cases. Also, the time horizon is such that the retailer will switch her order quantity after T_1 to q_E. The cost equation for Case 1 is given in Eq. (2.3.11).

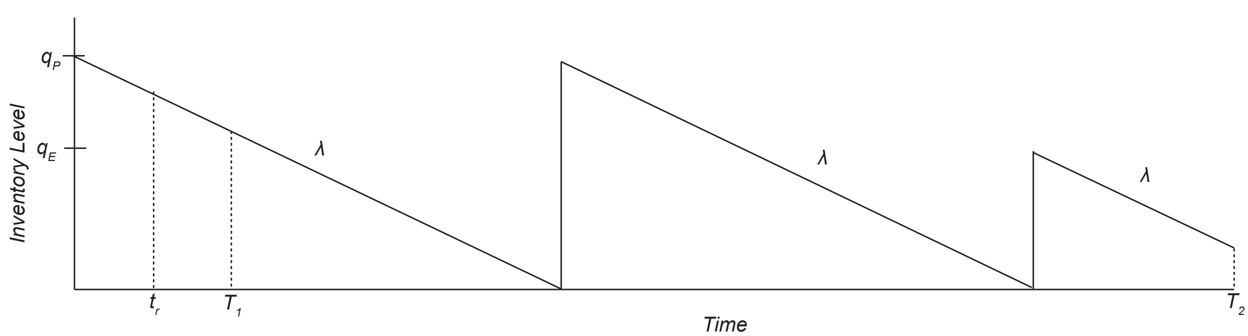


Figure 2.3.4: Proactive Strategy with No Disruption: Case 1

$$TC^0_{P_1}(q_P, q_E) = A\left(2 + \frac{\lambda T_2 - 2q_P}{q_E}\right) + h\left(\frac{{q_P}^2}{\lambda} + \frac{q_E\left(T_2 - \frac{2q_P}{\lambda}\right)}{2}\right) \tag{2.3.11}$$

Case 2: $L \geq \frac{q_P - \lambda T_1}{\lambda_D}$ **and** $T_2 \leq \frac{2q_P}{\lambda}$

Case 2 is similar to Case 1 with the exception that the time horizon ends before the retailer receives an order of q_E. Essentially, the retailer receives at most two orders of q_P. The cost equation for Case 2 is given in Eq. (2.3.12).

$$TC^0_{P_2}(q_P) = A\left(\frac{\lambda T_2}{q_P}\right) + h\left(\frac{q_P T_2}{2}\right) \tag{2.3.12}$$

Case 3: $L \leq \frac{q_P - \lambda T_1}{\lambda_D}$ **and** $T_2 \geq \frac{q_P}{\lambda}$

In the third case, the lead-time causes the time to reorder to occur after T_1. This leads the retailer to change her order quantity to q_E, and she will reorder so that she will receive the order of q_E at $\frac{q_P}{\lambda}$. Figure 2.3.5 illustrates Case 3, and the cost equation is given in Eq. (2.3.13).

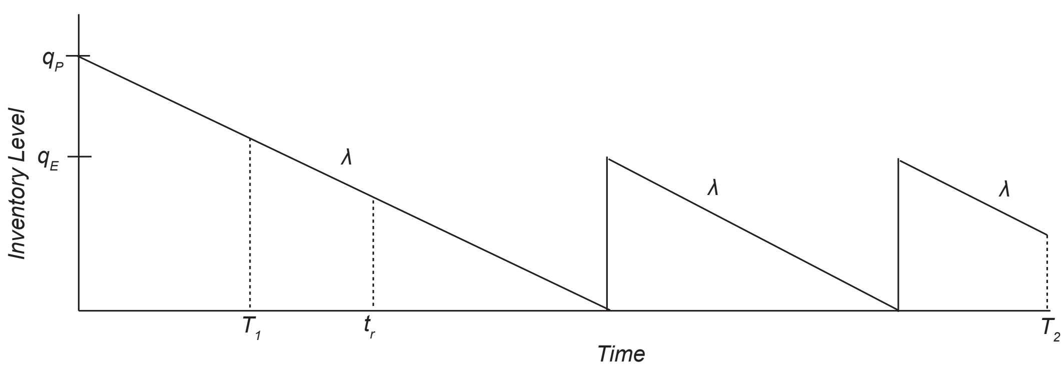


Figure 2.3.5: Proactive Strategy with No Disruption: Case 3

$$TC^0_{P_3}(q_P, q_E) = A\left(1 + \frac{\lambda T_2 - q_P}{q_E}\right) + h\left(\frac{{q_P}^2}{2\lambda} + \frac{q_E\left(T_2 - \frac{q_P}{\lambda}\right)}{2}\right) \tag{2.3.13}$$

Case 4: $L \leq \frac{q_P - \lambda T_1}{\lambda_D}$ **and** $T_2 \leq \frac{q_P}{\lambda}$

Case 4 is similar to Case 3 except there is only one order of q_P because the disruption ends before the first order is depleted. Eq. (2.3.12) can be used to determine the cost for Case 4.

Proposition 2 *When $S = P$,*

$$\max_{D \in \{0,1\}} \min_{\vec{q} \in \mathbb{Q}} TC_S^D(\vec{q}) = TC_P^1(\vec{q})$$

.

Similar to the reactive strategy, we are able to find that the maximum cost equation is always the disruption cost equation when it is compared to the no disruption cost equations. Please see Appendix A.3 for the proof of Proposition 2.

2.4 Results

It is impossible to analytically show that one strategy is always preferred over the other based on minimax decision criterion, so we performed a numerical experiment and sensitivity analysis. Our analysis enabled us to better understand the conditions when it is advantageous to select either the reactive or proactive strategy when faced with a possible demand disruption.

2.4.1 Numerical Experiment

In order to determine the conditions associated with each strategy selection, a 2^8 experimental design was created. Table 2.4.1 contains the set of values tested for each of the eight parameters. We ensured that the values tested did not violate any of the previously mentioned assumptions. For example, we checked that the lead-time was less than each type of inventory cycle (depending on the demand rate and order quantity) and ensured that T_2 was greater than the time when the first order is depleted (t_{1R} and t_{1P}), which depends on the strategy selected. In Table 2.4.2, we provide a sample of our results. The table contains

Table 2.4.1: Values Tested for 2^8 Experiment

Parameter	**Low**	**High**
λ_D	50	100
T_2	6	12
h	1	2
A	100	200
s	10	20
L	0.5	1
λ	10	15
T_1	2	2.5

information regarding the description of the value (low or high) for each parameter, the cost for each outcome associated with the strategy, and the best strategy for the given conditions. When we performed the numerical experiment with the values in Table 2.4.1, there

Table 2.4.2: Sample Results for 2^8 Experiment

λ_D	T_2	h	A	L	s	λ	T_1	**Decision**
L	L	L	L	L	L	L	L	REACTIVE
L	L	L	L	L	L	L	High	PROACTIVE
L	L	L	L	L	L	High	L	REACTIVE
L	L	L	L	L	L	High	High	PROACTIVE
L	L	L	L	L	High	L	L	REACTIVE
L	L	L	L	L	High	L	High	PROACTIVE
L	L	L	L	L	High	High	L	REACTIVE
L	L	L	L	L	High	High	High	PROACTIVE

were cases with lost sales and no lost sales. The reactive strategy was selected for about 8% of the tested combinations with lost sales. We found the reactive strategy combinations to have the following characteristics if lost sales were incurred:

- High lost sales cost were allowed
- Short lead-time
- Small λ_D and T_1
- $t_r \geq T_1$

After analyzing the characteristics of the reactive strategy combinations, we explored the explanations for each of the characteristics. Surprisingly, a high lost sales cost (s) did not automatically mean that a proactive strategy was more cost effective. If the reactive strategy was selected when s was at its high value, the amount of lost sales was minimal. In addition, the reactive strategy was predominantly chosen if the surge demand was at its smallest value, which resulted in less lost sales. As expected, if the lead-time was short, the time to reorder (t_r) was after T_1 supporting the reactive strategy. As a result, the retailer was able to change her order quantity to q_R at T_1 and ensure no additional lost sales after she received the next order.

When no lost sales were incurred, the proactive strategy was selected almost 50% of the time. The ordering costs affected the selection of the proactive strategy in these circumstances. In our experiment, the reactive strategy always yielded higher ordering costs than the proactive strategy while the proactive strategy had higher holding costs. With the values tested, the reactive strategy was preferred when the difference in the number of orders was small meaning there was a small difference in the ordering costs. The ordering costs affected the strategy selection more than the holding costs, which is to be expected because $A \geq\geq h$. In addition, when lost sales were not incurred, the time to reorder was after T_1, which also means that the lead-time (L) was short.

2.4.2 Sensitivity Analysis

In order to better understand the effect of each parameter on the strategy selection, we performed a sensitivity analysis. The parameter in question was varied so that it did not violate the model assumptions while the other parameters were kept at their low values given in Table 2.4.1. We will first review the results of the parameters that incurred lost sales for all values tested.

Parameters that Incurred Lost Sales Only

When T_2 and s were tested, the reactive strategy was preferred for every tested case. Through the results, we concluded that the lost sales cost and the length of the disruption do not have a significant effect on the strategy selection.

The time when the disruption begins (T_1), the normal demand (λ), and the disruption demand (λ_D) yielded reactive strategies at the lowest values tested. Intuitively, one would expect the reactive strategy to be selected if the demand and disruption demand are both small because the retailer would experience a smaller number of lost sales. In addition, if T_1 is small, this means that $t_R \geq T_1$, so the retailer will be able to change her order quantity to q_R and thus, only incur lost sales for one time period.

Parameters that Incurred Lost Sales and No Lost Sales

When the lead-time was tested at smaller values, there were no lost sales and minimax decision criterion supported a reactive strategy. On the other hand, when the retailer incurred lost sales, the reactive strategy was selected if the lead-time was small meaning that the number of lost sales was small. Otherwise, as the lead-time increases, the proactive strategy is favored.

The lost sales cases were separated from lost sales in Figure 2.4.1 in order to show the values of A where lost sales are present, which is when A is less than 100. In addition, there is a greater difference between the total cost when lost sales are incurred, and we wanted to highlight this finding. According to our model, the retailer experiences lost sales when the order cost is small (or less than 100). As shown in Figure 2.4.1, the retailer should choose a proactive strategy until the order cost reaches 100. Then, the retailer should select a reactive strategy for a small range of values when lost sales are incurred. For these values of A, the number of lost sales is very small, which is why the retailer may select a reactive strategy. As the order cost increases, the retailer does not incur lost sales because the order quantities increase while the lead-time remains the same. In general, one would expect for the retailer to always choose a reactive strategy when they do not anticipate incurring lost sales. However, we found that as the order cost increases, the proactive strategy is more cost effective. It is apparent that the difference in the costs for each strategy is very small for most values of A as shown in Figure 2.4.1, so the retailer should select the strategy that will

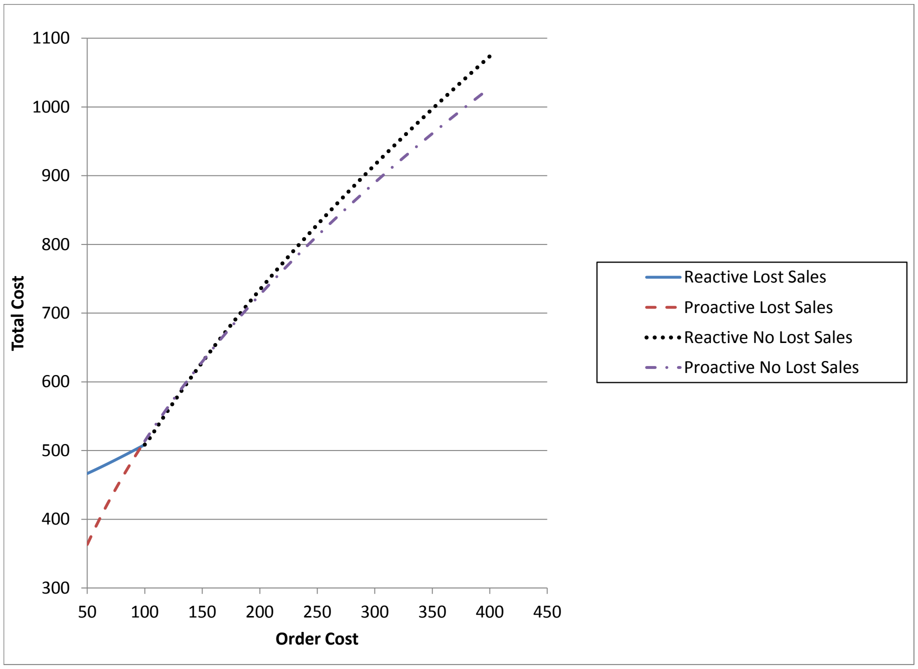


Figure 2.4.1: The Effect of the Order Cost, A

best suit the circumstances. In all of the cases when the retailer does not incur lost sales, the reactive strategy always has a larger order cost than the proactive strategy. Figure 2.4.2 illustrates that as the difference between the number of orders increases, it is better to select the proactive strategy when there are no lost sales.

In contrast to the order cost, there were no lost sales incurred when the holding cost was small, but as the holding cost increased, lost sales were incurred. When there were no lost sales, the proactive strategy was preferred with the exception of a small range of h values that yielded a reactive strategy. When the value of h causes a switch from no lost sales to lost sales, there is a small range of h values that lead to a reactive strategy as shown in Figure 2.4.3. As h increases, the chosen strategy is proactive. The amount of lost sales increases because the order quantities decrease, thus, the proactive strategy is selected. The lost sales cost is more expensive than the holding cost, so the difference in cost continues

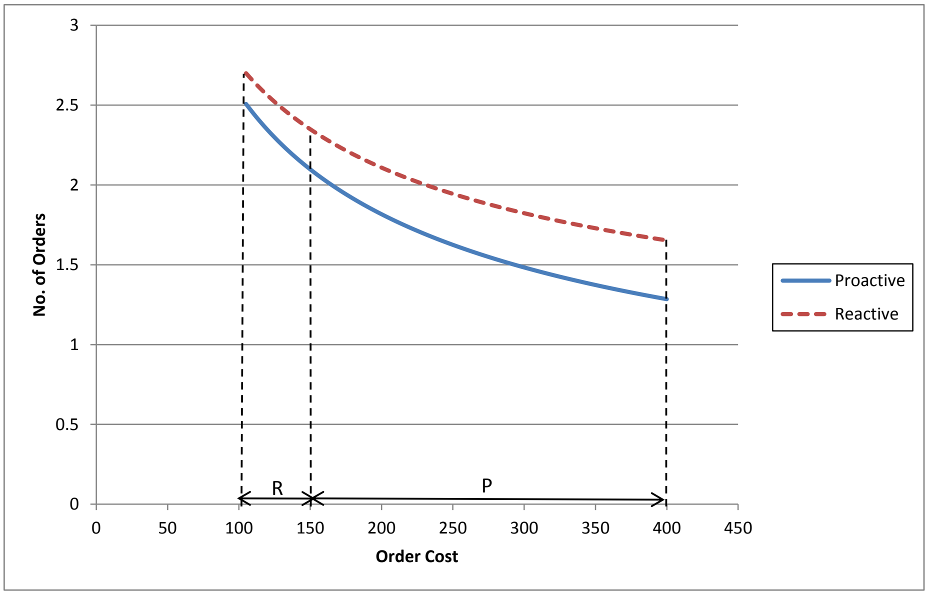


Figure 2.4.2: Strategy Selection Considering No. of Orders with No Lost Sales

to increase until $t_r \leq T_1$, and then, it begins to decline as shown in Figure 2.4.3. When $t_r \leq T_1$, the retailer will reorder based on her strategy and if she is reactive, she might incur two periods of lost sales. However, the values we tested did not lead to two time periods of lost sales. The decrease in the difference in cost is attributed to the decrease in holding costs because with a reactive strategy, the retailer will hold two orders of q_E, and then, a smaller number of q_R orders than if she had only placed one order of q_E. Plus, during the time period of lost sales, holding costs will not be incurred.

2.5 Summary and Future Extensions

The numerical experiment and sensitivity analysis provided us with valuable conclusions about the conditions that warrant each strategy selection. The following is a summarized list of our findings:

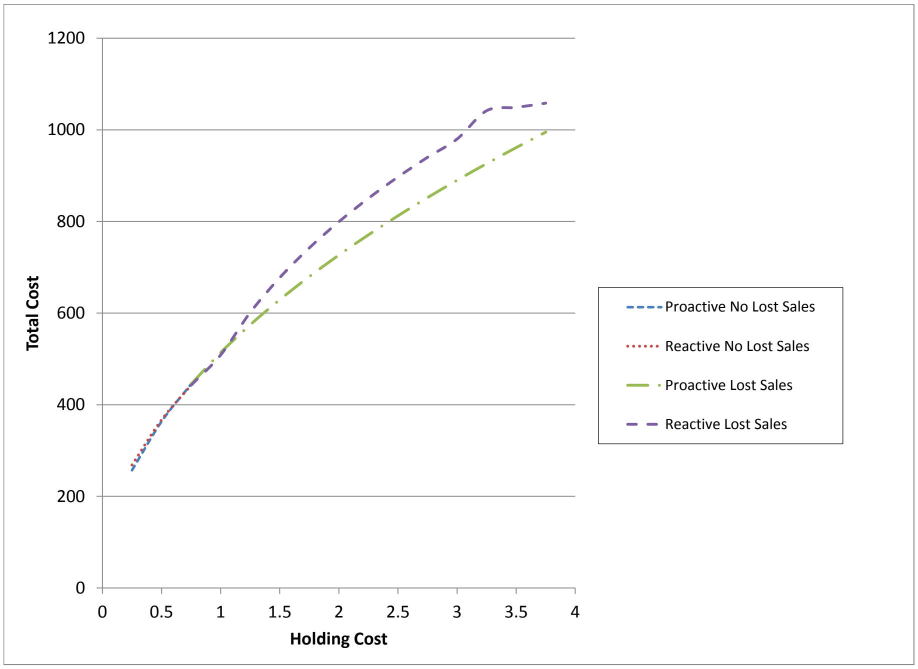


Figure 2.4.3: The Effect of the Holding Cost, h

1. Lead-time has a greater effect on strategy selection than lost sales cost.
2. It is possible to select the reactive strategy despite lost sales when lead-time is short.
3. It is possible to select the proactive strategy when there are no lost sales.

Perhaps the most interesting finding is that the lead-time has a greater effect than the lost sales cost on the strategy selection, which was proven in the sensitivity analysis. The values of T_2 and s are arbitrary compared to the lead-time. As a practical application, the retailer can determine which strategy she should choose based on the lead-time. We also found that the proactive strategy is favored for most circumstances even when there are no lost sales. A retailer should select the proactive strategy if her holding or ordering costs are high regardless of whether lost sales occur. In summary, the retailer should adopt a proactive strategy unless the lead-time is short and the other parameters have small values.

In order to capture the essence of a disruption, the model could be extended in several ways such as including a variable lead-time. During the demand surge, some suppliers may experience many orders from other retailers, thus decreasing their ability to fulfill orders in a timely manner. In addition, we could explore optimizing the reorder point because we assume that it is optimal for the retailer to not incur lost sales if possible. Lastly, an ordering strategy based on a retailer's target service level could be determined.

Retailers play a key role in helping communities prepare for a forecasted storm, and their preparation is crucial. Through our research, we were able to show situations where it is beneficial for retailers to adopt a proactive strategy as well as a reactive strategy. In most cases, it is better for them to select a proactive strategy, which will in turn assist the community in their preparations because the retailers will be able to provide the necessary supplies. However, there are situations when it is best to select the reactive strategy, and if lost sales are incurred, it is a very small percentage, so most customers' needs will be met. All in all, our approach provides a practical tool for retailers to use when they are faced with an uncertain demand surge.

Chapter 3

Minimax and Minimax Regret Inventory Control Policies Regarding Demand Disruptions and Damaged Inventory

3.1 Introduction

On September 13, 2008, residents in Galveston, Texas, assessed the damage that Hurricane Ike left in its wake. Many parts of Galveston were unrecognizable, and power outages were widespread on the island. The residents who stayed behind despite warnings to evacuate were running low on basic commodities such as gas for generators and non-perishable food. Most retailers were unable to reopen quickly with the exception of one Kroger store that opened it doors 3.5 days later ([48]). The purpose of this paper is to assist retailers with inventory policy decisions during and after a forecasted storm.

In this paper, we focus on two disruptions that occur consecutively: a period of no demand leading into a surge in demand for emergency items. Chapter 2 captures the disruption before the storm and provides insight into the conditions that support increasing the order quantity before the disruption occurs using minimax decision criterion. In Chapter 3, we evaluate two uncertain disruptions that occur during and after the storm by applying minimax and minimax regret decision criteria to provide retailers with an inventory policy depending on the objectives they strive to meet. Figure 3.1.1 illustrates the time horizon considered for Chapters 2 and 3. Chapter 2 covers the time horizon between $t = 0$ and $t = T_2$, while Chapter 3 covers the time horizon between $t = T_2$ to $t = T_4$. Please note that the demand rate for the disruption between T_3 and T_4 is greater than the demand rate for the disruption between T_1 and T_2. It is assumed that all customers will not stock up on emergency supplies before a possible storm, but if the storm occurs, all customers will be in need of emergency supplies. In this paper, we will refer to the disruption between T_2 and

T_3 as the first disruption while the second disruption refers to the demand surge between T_3 and T_4. We assume that $T_2 = 0$ because we are considering the last two disruptions separate from the disruption between T_1 and T_2.

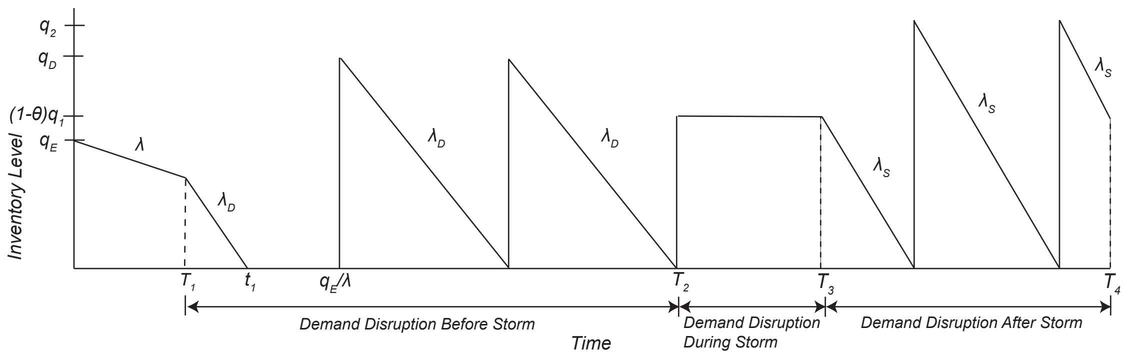


Figure 3.1.1: Time Horizon for Demand Disruptions

In the supply disruption literature, it has been shown that retailers should take disruptions into account when determining inventory control policies (e.g., [46], [9], [50]). For the time horizon considered in this paper, there are key factors that must be examined to determine inventory policies: the likelihood of the forecasted storm affecting the area and causing the retailer to close, the likelihood of damaged inventory caused by a storm, the length of time the retailer will be closed, the lead-time for emergency items, and the length of time the demand surge will last after the storm. Our intention is to provide retailers with an inventory policy based on two different criteria: minimax and minimax regret. If the retailer decides to hold inventory during the first disruption, the inventory policy will be determined based on the amount that the retailer assumes will be damaged. On the other hand, if the retailer anticipates that all of the inventory will be lost, she will not hold inventory during the first disruption. We seek to answer the following research questions using minimax and minimax regret:

Question 3.1 *Under what conditions should retailers hold inventory if they expect a storm to affect their area?*

Question 3.2 *If the retailer should hold inventory, how much inventory should be held considering a percentage may be damaged?*

Question 3.3 *How does each parameter affect the inventory policy?*

In the next section, we will review relevant disruption literature in addition to a brief summary of minimax and minimax regret literature. After the literature review, the general model is presented followed by the cost equations for each outcome associated with the inventory policies. Then, the results of the numerical experiment and sensitivity analysis give insight into the inventory policy supported by each decision criterion. In closing, a brief summary of the findings and possible extensions are provided.

3.2 Literature Review

Supply and demand disruptions caused by events such as strikes, natural disasters, unreliable suppliers, and machine breakdowns within the context of production, operations, and supply chain management have been extensively researched in the literature. As a result of disruptions, lost sales or backorders are common occurrences depending on the length of the disruption.

Supply disruptions are characterized as an interruption in a supplier's operations, which means that the supplier becomes unavailable or unable to fill orders for various reasons from strikes to machinery breakdowns. One of the earliest papers to address disruptions and lost sales computed the percentage of unmet demand in a production system as a result of a disruption ([39]). Several papers have also concentrated on supply disruptions within a production system and determined periodic review (e.g., [42], [34]) and continuous review inventory policies ([41]). In relation to our model, Parlar and Berkin [46], Berk and Arreola-Risa [9], Snyder [60], and Heimann and Waage [23] developed economic order quantity inventory models considering supply disruptions. Recently, Qi et al. [50] found an optimal order quantity when the retailer and supplier both face disruptions where the retailer's disruption is a loss of inventory. The focus of this literature review will mainly be on inventory models that consider demand disruptions, lead-time, and lost sales as well as a brief review of minimax and minimax regret inventory models.

3.2.1 Demand Disruptions

Demand disruptions have been addressed as periods of no demand ([65]), fluctuating demand ([61], [11]), and a surge in demand ([16], [35], [55]). Brill and Chaouch [11] found results to support their claim that an adjustment should be made to an EOQ policy when a major disruption occurs, but if the disruption is small, no adjustment is necessary. Ross et al. [55] developed time-dependent policies for disruptions caused by a surge in demand. Within the disaster management literature, Lodree and Taskin [35] compared the newsvendor solution to the optimal inventory level they determined for a one-time decision for retailers that were considering stocking up before a storm. However, the model can be applied to any type of demand disruption that has a forecast associated with it. Using optimal control, Lodree and Taskin [35] determined when retailers should increase their order quantity of emergency supplies. In addition, Taskin and Lodree [62] determined how a retailer should adjust their inventory level in anticipation of a demand surge that will occur during hurricane season based on estimated demand that is a convolution of current demand and an estimate of demand for hurricane season. Lodree [16] presented a model to determine the retailers' ordering strategy for emergency supplies when they are faced with an uncertain demand surge caused by a forecasted storm. He used minimax decision criterion to determine the best ordering strategy, which is contrary to the approach in the demand and supply disruption literature. Typically, long run approximations are used to determine order quantities in order to alleviate the effects of a demand or supply disruption ([46], [23]). Similar to the model presented in Lodree [16], we are considering infrequent, unpredictable storms, and it is not appropriate to use these types of approximations. In addition, each storm is unique with different characteristics such as the severity of the storm, how long people may potentially be confined to their homes, etc. Therefore, the same approximation for an order quantity cannot be applied to each storm. We handle this issue by adopting the approach of employing minimax decision criterion in addition to minimax regret to aid the retailer in determining an inventory policy for two possible disruptions with two sources of uncertainty similar to the method presented in Lodree [16].

3.2.2 Minimax Decision Criterion

Scarf [58] developed the first distribution free model by determining the demand distribution with a known mean and variance that would maximize the minimum profit. Several authors have extended Scarf's research by increasing the number of observed periods ([2]), proving Scarf's ordering rule is optimal for the newsboy problem ([19]), determining optimal (Q, r) policies ([28], [18]), and determining the ordering policy using linear programming ([20]). By applying minimax decision criterion, Yu [68] solved the EOQ problem with a random demand rate, order cost, and holding cost using an efficient linear time algorithm. He demonstrated the many advantages of the robustness approach compared to the stochastic approach. With respect to our model, the only other paper known to have used minimax decision criterion within the context of disruption management is Lodree [16].

3.2.3 Minimax Regret Decision Criterion

In addition to using minimax decision criterion, we will also apply minimax regret decision criterion to the model. Minimax regret finds the minimum regret among the worst case scenarios for each decision. Savage [57] first explored the minimax regret decision rule to demonstrate its uses when making optimal decisions. Several authors used minimax regret decision criterion when considering the distribution free newsvendor problem ([43], [32], [64], [69]). Perakis and Roels [49] found solutions for the optimal order quantity that minimize the maximum regret for not choosing optimally when the demand distribution has partial information. Jammernegg and Kischka [29] considered two other risk factors in their objective function to determine a robust solution for the order quantity. With respect to our model, minimax regret decision criterion is applied to the objective function to determine the ordering strategy that minimizes the maximum regret. The contributions of this paper to the reviewed literature include: a general class of possible disruption cases with two sources of uncertainty are considered and an ordering strategy is determined by applying minimax and minimax regret decision criteria to an inventory disruption problem. In the next section, we present the inventory model that will be evaluated using the two decision criteria previously reviewed.

3.3 Model Formulation

3.3.1 General Model

The model presented in this paper closely resembles the EOQ model in that many underlying assumptions for the EOQ model are included in the model presented in this section. If no disruption occurs, the retailer's inventory policy is the EOQ. However, the model contains two sources of uncertainty that differentiate it from the EOQ model, which are the occurrence of the two disruptions and the percentage of damaged inventory as a result of the first disruption. In addition, the model includes lost sales if the retailer underestimates the percentage of damaged inventory or decides not to hold inventory during the first disruption.

3.3.2 Model Assumptions and Notation

In this section, the notation shown in Table 3.3.1 will be discussed in addition to the model assumptions. $\vec{S}$, which is given in Eq. (3.3.1), represents the inventory policy selected by the retailer and contains two decisions: the decision to hold inventory, and the amount of inventory that will be held during the first disruption with assumptions made about the amount of damaged inventory. S denotes the decision to hold (H) or not hold (NH) inventory, while α represents the amount of inventory the retailer assumes will be damaged during the first disruption. The list of possible decisions for $\vec{S}$ is given by Ω. If $\alpha < 1$,

Table 3.3.1: Chapter 3 Notation

Parameter	**Description**
$\vec{S}$	inventory policy
$\vec{D}$	outcome
$\vec{q} \in \{q_1, q_2\}$	ordering policy
q_1	order quantity during first disruption
q_2	order quantity during second disruption
q_E	economic order quantity
$\vec{n} \in \{n_1, n_2\}$	number of orders of $\vec{q}$
A	order cost (per order)
h	holding cost (per unit/per unit time)
$\bar{I}$	average on-hand inventory
z	lost sales cost (per unit)
X	number of lost sales units
y	cost per unit of damaged inventory (purchase price)
W	number of units of damaged inventory
θ	percentage of damaged inventory
α	percentage of inventory retailer assumes will be damaged
T_3	time when first disruption ends
T_4	time when second disruption ends
L	lead-time
λ	normal demand rate
λ_S	surge demand rate

$S = H$, meaning the retailer assumes that all of the inventory will not be damaged, an order quantity will be determined to attempt to combat lost sales. The retailer will not hold (NH) inventory if $\alpha = 1$ because it means that the retailer assumes that all of the inventory will be damaged.

$$\vec{S} \in \{S, \alpha\} \tag{3.3.1}$$

$$S \in \{H, NH\} \tag{3.3.2}$$

$$\alpha \in [0, 1] \tag{3.3.3}$$

$$\Omega = \{(H, 0), (H, \alpha), ..., (NH, 1)\} \tag{3.3.4}$$

The notation for the first disruption is represented by $\vec{D}$. The vector, $\vec{D}$, has two entities, D and $\Theta(D)$. D represents the occurrence of the disruption and the outcome space is

$D \in \{d_0, d_1\}$. If no disruption occurs, $D = d_0$, and if a disruption occurs, $D = d_1$. $\Theta(D)$ denotes the percentage of inventory that is damaged as a result of the disruption. Eqs. (3.3.5)-(3.3.8) represent the outcome space for $\Theta(D)$ and Δ.

$$\Theta(D) \in \{\Theta(d_0), \Theta(d_1)\} \tag{3.3.5}$$

$$\Theta(d_0) = 0 \tag{3.3.6}$$

$$\Theta(d_1) = \{0, \theta, ..., 1\} \tag{3.3.7}$$

$$\Delta = \{(d_0, 0), (d_1, 0), (d_1, \theta), ..., (d_1, 1)\} \tag{3.3.8}$$

Please also note that the ordering policy consists of two order quantities as shown in Table 3.3.1. We allow for different order quantities for each disruption if necessary. If a disruption occurs and $\alpha < 1$, we will explore two policies, $q_1 = q_2$ and $q_1 \neq q_2$. If $\alpha = 1$, the retailer will not hold inventory, so $q_1 \neq q_2$. As previously mentioned, if there is no disruption, we assume that the retailer will place an order for the economic order quantity, q_E, to be received after it is certain that the first disruption will not occur meaning $q_1 > 0$ or $q_1 = 0$ and $q_2 = q_E$. The number of orders of each quantity is given by $\vec{n}$ and is determined by the demand and order quantity; therefore, it is not a decision variable. The first entity, n_1, will either be 0 or 1 depending on the value for q_1, and we allow n_2 to be non-integer because it accounts for the number of orders of q_2 between the time when q_1 is depleted or the retailer receives q_2 at L (depending on the inventory policy) and T_4. The average on-hand inventory, $\overline{I}$, is determined by the number of orders of each order quantity. The number of lost sales units given by X is determined by the length of time when the inventory level is zero and the demand rate is greater than zero. Lost sales will occur if $q_1 = 0$ or $\alpha < \theta$ meaning the retailer underestimated θ. The number of damaged units, W, is determined by q_1 and the percentage of units damaged, which is given by θ. The normal demand rate, λ, represents the demand rate when the disruptions do not occur while λ_S represents the demand rate during the second disruption.

Table 3.3.2: Chapter 3 Model Assumptions

EOQ Assumptions
1. Lead-time is constant.
2. The retailer's ordering policy does not include lost sales if $q_1 > 0$.
3. Demand rate for surge is known.
4. All costs are constant.
Additional Assumptions
5. Single, fast-moving, non-perishable item is considered.
6. Supplier is reliable and adaptable.
7. The retailer will not be charged a holding cost and damaged inventory cost.
8. No backorders will be allowed.
9. The second disruption only occurs if the first one occurs.
10. If a disruption occurs, the retailer cannot place an order until $t = T_3$.
11. The retailer will determine q_2 after the first disruption begins.
12. Order quantity under normal conditions is EOQ.
13. First and second disruption time horizon is known.
14. Time horizon for demand surge is long ($T_4 \geq T_3 + L$).

Table 3.3.2 provides a list of the assumptions for the model with the EOQ assumptions listed first followed by additional model assumptions. The EOQ model assumes all parameters are known and lost sales are not allowed. If the retailer selects α based on the assumption that all of the inventory will not be lost, she determines q_1 such that lost sales will not be incurred if $\theta = \alpha$. This means that the retailer will order more than the anticipated demand to account for a percentage of the inventory being damaged during the storm. We also assume that customers will be stocking up on fast-moving items such as bottled water and canned goods to sustain them during the recovery period when drinking water may be unsafe or electricity lines may be down. Also, if any or all of the inventory is damaged, the retailer will not be charged a holding cost for the unusable items. Instead, she will be charged the purchase price, y, for the items that were damaged if $q_1 > 0$. We will assume that the purchase price is greater than the holding cost. In addition, we do not allow backorders if lost sales are incurred because customers will need items immediately after the disaster is over, so if the items are unavailable, they will go to another retailer. Moreover, the second disruption, which is characterized as a demand surge, will only occur if the first disruption occurs. It is also assumed that if the first disruption occurs, the retailer will not

be able to place an order of q_2 until T_3 because the retailer may not know the extent of the damage or be able to place orders to the supplier until the disruption is over. Along the same lines, Assumption 11 assumes that the retailer will determine q_2 after the first disruption begins because the retailer will know how to adjust q_2, so that lost sales are not incurred for the remainder of the time horizon. Lastly, the time horizon is assumed to be longer than the time it takes to receive the first order of q_2. Otherwise, the second disruption would be negligible.

3.3.3 General Cost Equation

The general cost equation given in Eq. (3.3.9) is comprised of the ordering, holding, damaged inventory, and lost sales costs.

$$TC_{\vec{S}}^{\vec{D}}(\vec{q}) = A\vec{n} + h\overline{I}_{\vec{S}}^{\vec{D}} + yW_{\vec{S}}^{\vec{D}} + zX_{\vec{S}}^{\vec{D}} \tag{3.3.9}$$

where

$$\begin{aligned} W &= \theta q_1 && q_1 \geq 0 \\ X &= \max[0, \lambda_S L - (1-\theta)q_1] && q_1 \geq 0 \end{aligned}$$

Before applying either of the decision criterion, we will find the inventory policy that minimizes Eq. (3.3.9), and it is shown in Eq. (3.3.10).

$$MinTC_{\vec{S}}^{\vec{D}}(\vec{q}) = \min_{(\vec{q})\in\Pi} TC_{\vec{S}}^{\vec{D}}(\vec{q}) \tag{3.3.10}$$

where

$$\begin{aligned} (\vec{q}) &\in \Pi, \quad \text{and} \\ \Pi &= \{(\vec{q}) : n_1q_1 + n_2q_2 \leq \Gamma\Lambda \ \forall\ \pi,\ \ \Lambda \in \{\lambda, \lambda_S\},\ \ \Gamma \in \{T_4, T_4 - T_3\}\} \end{aligned}$$

Γ represents the amount of time that the demand rate is greater than zero while Λ represents the demand rate during Γ. If the disruption occurs, $\Gamma = T_4 - T_3$ and $\Lambda = \lambda_S$, and if the disruption does not occur, $\Gamma = T_4$ and $\Lambda = \lambda$. The sum of the amount ordered during the time horizon, $n_1 q_1 + n_2 q_2$, may be less than the total demand, which is given by $\Gamma\Lambda$. The solution space accounts for the decision to hold or not hold inventory during the first disruption. If the retailer decides not to hold inventory, the sum of the amount ordered will be less than the total demand during the time horizon, and the retailer will experience lost sales. After the solution that minimizes the cost equation for each combination of outcome and inventory policy is determined, minimax and minimax regret decision criteria are applied.

The general minimax formulation is presented first and is represented by Eq. (3.3.11). First, the maximum cost equation is determined for each policy considering the outcomes in Δ. Then, the minimum cost equation is found considering the maximum cost for each policy.

$$\min_{\vec{S}\in\Omega} \max_{\vec{D}\in\Delta} MinTC_{\vec{S}}^{\vec{D}}(\vec{q}) \tag{3.3.11}$$

Next, the minimax regret formulation is presented. The first step is to create a matrix with columns representing the possible outcomes and rows representing the possible decisions for q_1 and q_2 that minimizes the cost equation for each possible disruption outcome. If the retailer decides to hold inventory, the amount held will be a quantity associated with a disruption outcome meaning that $q_1 \neq q_E$. The maximum regret formulation is given in Eq. (3.3.12). Let $Min^{*}TC_{\vec{S}}^{\vec{D}}$ represent the minimum cost for each outcome, and please note that the maximum regret is chosen for each decision in $\vec{S}$. Please also note that Eq. (3.3.12) finds the absolute value of the difference because the largest regret is determined by the largest negative value. However, in order to dispel any confusion, we took the maximum of the absolute value of the difference.

$$MaxMinTC_{\vec{S}}^{\vec{D}}(\vec{q}) = \max_{\vec{S}} \mid Min^{*}TC_{\vec{S}}^{\vec{D}}(\vec{q}) - MinTC_{\vec{S}}^{\vec{D}}(\vec{q}) \mid \tag{3.3.12}$$

Lastly, the minimum regret is selected among the maximum regrets for each policy in Γ as shown in Eq. (3.3.13).

$$\min_{\vec{S}\in\Omega} MaxMinTC_{\vec{S}}^{\vec{D}}(\vec{q}) \tag{3.3.13}$$

3.3.4 Inventory Policy: Hold Inventory During 1st Disruption

The retailer bases her decision to hold inventory on the percentage of inventory that is expected to be damaged by the storm. If the retailer underestimates the percentage of damaged inventory, lost sales will be incurred. On the other hand, if the retailer has sufficient on-hand inventory after the storm, she will meet customer demand and aid in the recovery efforts.

Disruption Cases

If the retailer decides to hold inventory, there are three cases to consider if the disruption occurs and two cases to consider if the disruption does not occur. The first case describes the outcome where no lost sales are incurred, which occurs when $(1-\theta)q_1 \geq q_L$ and $q_L = \lambda_S L$. The retailer will not incur lost sales if the on-hand inventory at T_3 is at least q_L to meet the demand until the first order of q_2 arrives. In this case, the retailer has an adequate amount of on-hand inventory to meet demand after the first disruption ends and the second disruption immediately begins as shown in Figure 3.3.1. The cost equation is given in Eq. (3.3.14).

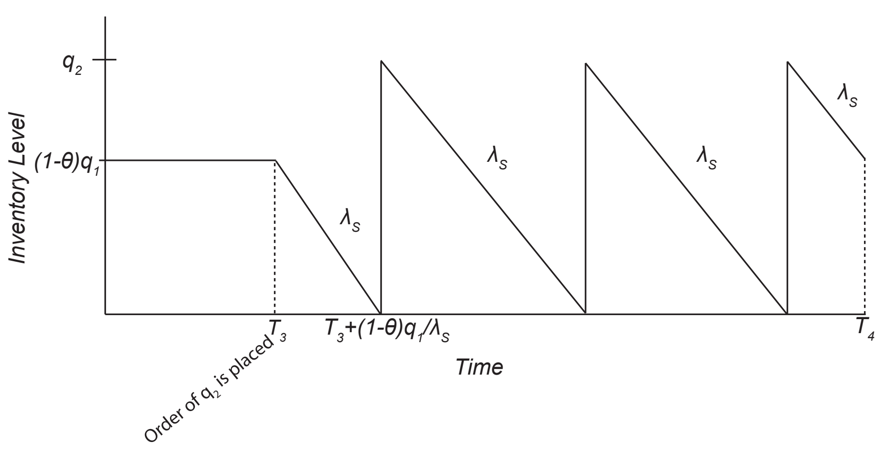


Figure 3.3.1: Hold Inventory with Disruption: Case 1

$$TC1^{(d_1,\theta)}_{(H,\alpha)}(q_1,q_2) = A\left(1+\frac{\lambda_S\left(T_4-T_3-\frac{(1-\theta)q_1}{\lambda_S}\right)}{q_2}\right)+h(1-\theta)q_1T_3 \quad (3.3.14)$$
$$+h\left(\frac{(1-\theta)^2{q_1}^2}{2\lambda_S}+\frac{q_2\left(T_4-T_3-\frac{(1-\theta)q_1}{\lambda_S}\right)}{2}\right)+y\theta q_1$$

The second case addresses the outcome in which lost sales are incurred where $(1-\theta)q_1 \leq q_L$. The main difference between Case 1 and Case 2 is the inclusion of lost sales, which is caused by the retailer ordering under the assumption that a smaller percentage of inventory will be damaged ($\alpha < \theta$). Figure 3.3.2 illustrates the inventory levels for Case 2 when lost sales are incurred. The cost equation for Case 2 is given in Eq. (3.3.15).

Figure 3.3.2: Hold Inventory with Disruption: Case 2

$$TC2^{(d_1,\theta)}_{(H,\alpha)}(q_1,q_2) = A\left(1+\frac{\lambda_S(T_4-T_3-L)}{q_2}\right)+h\left((1-\theta)q_1T_3\right) \quad (3.3.15)$$
$$+h\left(\frac{(1-\theta)^2{q_1}^2}{2\lambda_S}+\frac{q_2(T_4-T_3-L)}{2}\right)+y\theta q_1+z\left(L-\frac{(1-\theta)q_1}{\lambda_S}\right)$$

The third and final case is shown in Figure 3.3.3. If the retailer anticipates that a large percentage of her inventory will be damaged, she might order a very large order quantity. However, the actual percentage of inventory lost may be much smaller than α causing the retailer not to place another order before the second disruption is over as shown in Figure

3.3.3. The cost equation for Case 3 is given in Eq. (3.3.16). In this case, q_2 is not a decision

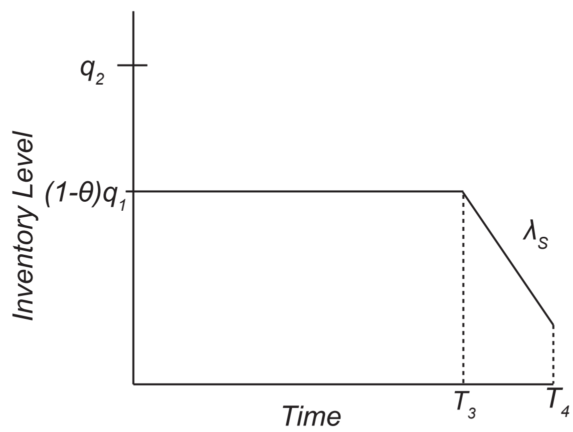


Figure 3.3.3: Hold Inventory with Disruption: Case 3

variable because the retailer will revert back to the EOQ inventory policy when the second disruption is over at T_4.

$$TC3^{(d_1,\theta)}_{(H,\alpha)}(q_1) = A + h\left((1-\theta)q_1T_3 + \frac{(1-\theta)q_1(T_4-T_3)}{2}\right) \tag{3.3.16}$$

In order to utilize minimax and minimax regret decision criteria, the optimal inventory policy associated with each outcome for θ where $\alpha = \theta$ must first be determined. As previously mentioned, it is assumed that the retailer either places an order for the quantity necessary to satisfy demand if a disruption occurs or does not place an order. Two different policies are considered for the special case $\{d_1, 0\}$: $q_1 = q_2$ and $q_1 \neq q_2$. Eq. (3.3.17) gives the cost equation for the special case $\{d_1, 0\}$ where $q = q_1 = q_2$.

$$TC^{(d_1,0)}_{(H,0)}(q,q) = A\left(\frac{\lambda_S(T_4-T_3)}{q}\right) + h\left(T_3q + \frac{q(T_4-T_3)}{2}\right) \tag{3.3.17}$$

By applying the first order condition, the closed form solution for q is derived and given in Eq. (3.3.18).

$$q^* = \sqrt{\frac{2A\lambda_S(T_4-T_3)}{h(T_4+T_3)}} \tag{3.3.18}$$

Even though we are able to find a closed form solution similar to the EOQ, we are able to prove that the total cost can be improved if $q_1 \neq q_2$. However, the optimal values for q_1 and q_2, considering the assumptions, must first be found before it can be explained that the cost can be minimized by adopting an inventory policy where $q_1 \neq q_2$. In order to determine the optimal values for q_1 and q_2, the optimal value for one variable is found and plugged into the cost equation before solving for the other variable. The optimal value for q_2, which is similar to the result that was found in Chapter 2, is given in Eq. (3.3.19). We will refer to q_2 as q_S for the remainder of this section.

$$q_2{}^* = \sqrt{\frac{2A\lambda_S}{h}} \tag{3.3.19}$$

In order to solve for q_1, we employed the use of commercial software since a closed form solution that guaranteed no lost sales could not be found. When the constraints for no lost sales and non-negativity were included, the same solution was consistently returned, and it is given in Eq. (3.3.20). Eq. (3.3.20) is the least amount that the retailer can hold to ensure no lost sales until the first order can be received after the second disruption begins if $\alpha = \theta = 0$.

$$q_1{}^* = \lambda_S L \tag{3.3.20}$$

Eq. (3.3.20) can be generalized to include other decisions for α, not including $\alpha = 1$, which is shown in Eq. (3.3.21). In the results section, we will give insight into the conditions when Eq. (3.3.21) is the best ordering policy for q_1.

$$q_1{}^* = \frac{\lambda_S L}{1-\alpha} \tag{3.3.21}$$

If the cost equations when $q = q_1 = q_2$ and $q_1 \neq q_2$ are compared, the value of q that does not allow lost sales has to be considered. Therefore, we employed commercial software to compare the answer for q^* to ensure that it did not allow lost sales if the amount of damaged inventory is known. Again, we considered the case $(d_1, 0)$ to solve for q and compared the answer to the total cost for $q_1 \neq q_2$. We are able to prove that if $q_1 = q_L$ ($q_L = \lambda_S L$) and $q_2 = q_S$, the cost is less than if $q = q_1 = q_2$ as long as $q \geq q_L$. This proof is shown in Appendix B. In addition, 64 combinations of eight parameters were tested to compare $q_1 = q_L$ and $q_2 = q_S$ to $q = q_1 = q_2$. The constraint $q \geq q_L$ was used to ensure no lost sales, and the results showed that $q = q_L$ if $q^* \leq q_L$, and $q = q^*$ if $q^* \geq q_L$. The proof in Appendix B shows that it is cheaper to hold two different quantities for q_1 and q_2 even when q^* was the optimal value for q.

No Disruption Cases

The retailer must consider the possibility that no disruption will occur if she decides to hold inventory. If the disruption does not materialize, the retailer will incur more expensive holding costs if $q_1 \geq\geq q_E$. There are two possible cases to consider if the retailer holds inventory during the first disruption and the disruption does not occur. The first case is illustrated in Figure 3.3.4, and represents the situation when the retailer places multiple orders before T_4. The cost equation for Case 1 is given in Eq. (3.3.22) where q_1 is based on

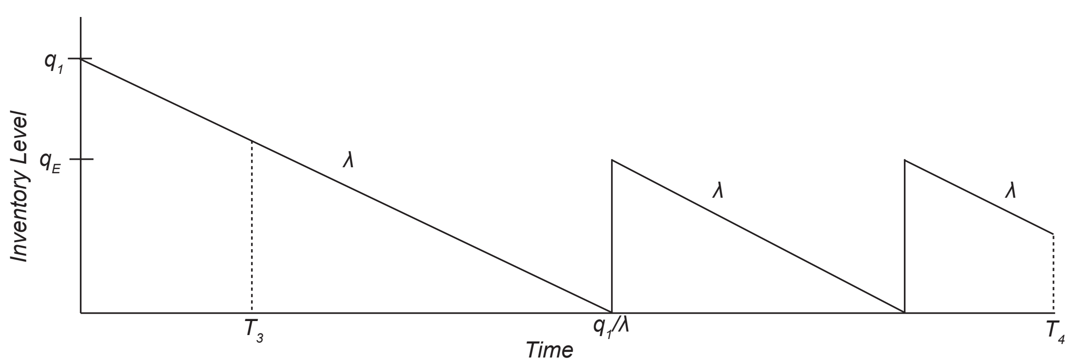


Figure 3.3.4: Hold Inventory with No Disruption: Case 1

(H, α) and $q_2 = q_E$.

$$TC4_{(H,\alpha)}^{(d_0,0)}(q_1, q_2) = A\left(1 + \frac{\lambda\left(T_4 - \frac{q_1}{\lambda}\right)}{q_E}\right) + h\left(\frac{{q_1}^2}{2\lambda} + \frac{q_E\left(T_4 - \frac{q_1}{\lambda}\right)}{2}\right) \tag{3.3.22}$$

The fifth and final outcome associated with holding inventory during the first disruption does not require any additional orders before T_4 occurs. The figure is not provided because it is similar to Figure 3.3.4 with the exception that there is only one order, q_1. The cost equation is given in Eq. (3.3.23).

$$TC5_{(H,\alpha)}^{(d_0,0)}(q_1) = A + \frac{hq_1T_4}{2} \tag{3.3.23}$$

3.3.5 Inventory Policy: Do Not Hold Inventory During 1st Disruption

Based on forecasts, retailers may decide that it is better to not hold inventory during the first disruption if they expect the storm to severely damage their property. However, forecasts are typically not accurate, so retailers will risk lost sales and dissatisfied customers if the disruption does not occur or if the disruption is not as extreme as predicted.

Disruption Case

If the disruption occurs, the retailer will experience lost sales until an order can be received at $T_3 + L$ as shown in Figure 3.3.5. As previously mentioned, it is assumed that the retailer is unable to place an order until the disruption is over at T_3, which is why the first order of q_2 will not be received until $T_3 + L$. The cost equation associated with Figure 3.3.5 is given

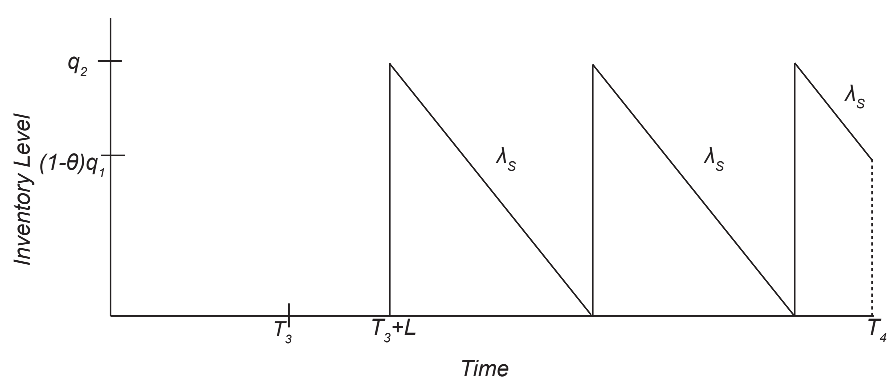


Figure 3.3.5: Do Not Hold Inventory with Disruption

in Eq. (3.3.24). Please note that if $\theta = 1$, it would be better for the retailer to not hold inventory because the retailer would not incur the damaged inventory cost, y, or ordering cost, A, for the damaged inventory if Eq. (3.3.24) is compared to Eq. (3.3.14).

$$TC_{(NH,1)}^{(d_1,\theta)}(q_1, q_2) = A\left(\frac{\lambda_S(T_4 - T_3 - L)}{q_2}\right) + h\left(\frac{q_2(T_4 - T_3 - L)}{2}\right) + z\lambda_S L \qquad (3.3.24)$$

Regarding the inventory policy, it is assumed that $q_1 = 0$, and if the first order condition is applied to Eq. (3.3.24), $q_2 = q_S$, which is the same result if the retailer decides to hold inventory during the first disruption. Therefore, we can conclude that the retailer's decision for q_2 will be the same regardless of the decision for q_1.

No Disruption Case

If no disruption occurs, the retailer will incur lost sales until she is able to receive an order at $t = L$. It is assumed that the retailer will place an order of q_E at $t = 0$ after it is known that the disruption will not occur as shown in Figure 3.3.6. Depending on the holding costs resulting from $q_1 \geq\geq q_E$, it might be beneficial from a cost perspective to incur lost sales.

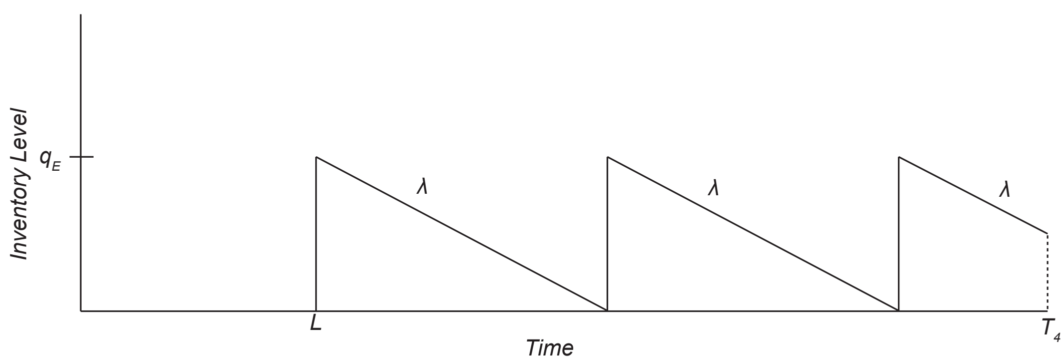


Figure 3.3.6: Do Not Hold Inventory with No Disruption

The cost equation for the not holding inventory when no disruption occurs is given in Eq. (3.3.25). Please note that $q_1 = 0$ and $q_2 = q_E$.

$$TC_{(NH,1)}^{(d_0,\theta)}(q_1, q_2) = A\left(\frac{\lambda\,(T_4 - L)}{q_E}\right) + h\left(\frac{q_E(T_4 - L)}{2}\right) + z\lambda L \qquad (3.3.25)$$

After identifying all of the possible outcomes considering the assumptions, we performed a numerical experiment to determine the conditions that support each inventory policy. The results of the numerical experiment and sensitivity analysis are provided in Section 3.4.

3.4 Results

A 2^9 experimental design was created to determine the inventory policy supported by minimax and minimax regret decision criteria. Both decision criteria were applied because different retailers may approach uncertain disruptions with contrasting objectives, and we wanted to determine if the decision criteria would provide contradicting decisions.

Table 3.4.1 gives the values tested in the numerical experiment. One constraint was considered when the values were selected, and it is based on the inequality, $T_3 + L \leq T_4$. The constraint was necessary because retailers may consider the second disruption negligible if the first shipment is received after the demand rate has returned to λ. It should be noted that the lead-time values are longer than the tested values in Chapter 2 because the supplier's lead-time may increase after a disruption.

Table 3.4.1: Values Tested for 2^9 Experiment

Parameter	**Low**	**High**
λ_S	100	200
λ	10	15
T_3	2	4
T_4	8	16
h	1	2
A	100	200
L	1	2
z	10	20
y	4	8

3.4.1 First Experiment: $\theta = \{0, 1\}$

For the first experiment, we assumed that only three outcomes might occur: no disruption, disruption and all of the inventory was damaged, or disruption and none of the inventory was damaged. Minimax decision criterion supported the decision not to hold inventory if a possible disruption may interfere with the retailer's operations for all tested cases, and this result can be proven analytically. The maximum cost equation for the decision to hold inventory is always the cost equation associated with the outcome where all of the inventory is damaged, $\theta = 1$. By referring back to Eqs. (3.3.14) and (3.3.24), it is obvious that the difference between the two maximum cost equations is yq_L, the cost of damaged inventory, and the order cost for q_1, A. This leads us to the conclusion that minimax decision criterion supports the decision not to hold inventory for an uncertain disruption.

When we performed the numerical experiment using the values from Table 3.4.1 for the special case $\theta = \{0, 1\}$, we found that $q_1 = q_L$ when minimax regret decision criterion is applied. This is contrary to our results found using minimax decision criterion. Therefore, we can assume that there are conditions when it is favorable to hold inventory during the first disruption if the retailer seeks to minimize the maximum regret regarding costs.

3.4.2 Second Experiment: $\theta \in [0, 1]$

In order to understand the impact of θ on the retailers ordering decision, a numerical experiment including additional values of θ was performed. When five different values of $\theta \in [0, 1]$ were tested, minimax decision criterion advocated not holding inventory, which is the same result as the first experiment. The total cost for each decision of q_1 was compared considering all outcomes (θ), and the maximum cost for each decision occurred when $\theta = 1$. After the maximum cost was found for each q_1, the minimum cost considering all of the order quantities associated with the maximum cost was found to be $q_1 = 0$. This result is intuitive because $q_1 = 0$ minimizes the cost equation when $\theta = 1$, which always gives the maximum cost for each decision. An example is given in Table 3.4.2 for five values of θ and five values of q_1 based on Eq. (3.3.21). When we determined the order quantity, q_1, associated with each value of θ, we assumed that the retailer ordered more than q_L to accommodate the damaged inventory caused by the first disruption meaning the retailer would have q_L on-hand to satisfy demand after the disruption. It should be noted that when $\theta = 0.75$, the minimum cost occurred when $q_1 = 0$ not $q_1 = \frac{q_L}{1-0.75}$. We will explore the selection for q_1 to minimize costs by considering a threshold value for θ later in this section. In Table 3.4.2, the bolded values represent the maximum cost for each decision for q_1, and the minimum maximum cost occurred when $q_1 = 0$ with a total cost of $1,707.

Table 3.4.2: Minimax Costs

q_1	$\theta = 0$	$\theta = 0.25$	$\theta = 0.50$	$\theta = 0.75$	$\theta = 1$	**No Disruption**
q_L	$1,057	$1,335	$1,620	$1,910	**$2,207**	$500
$\frac{q_L}{1-0.25}$	$1,116	$1,190	$1,563	$1,946	**$2,340**	$633
$\frac{q_L}{1-0.50}$	$1,266	$1,349	$1,457	$2,020	**$2,607**	$900
$\frac{q_L}{1-0.75}$	$1,983	$1,974	$2,066	$2,257	**$3,407**	$1,700
0	**$1,707**	**$1,707**	**$1,707**	**$1,707**	**$1,707**	$413

In order to gain more insight about the effect of θ on the ordering decision, we compared two outcomes: one value of θ and no disruption. Again, we determined the possible solutions for q_1 based on Eq. (3.3.21) for each value of $\theta < 1$. The experiment demonstrated that there is a threshold value of θ when it is best not to hold inventory even if all of the inventory is not damaged. This means $q_1 = \frac{q_L}{1-\alpha}$ is not optimal for some problem instances even when $\alpha = \theta$. We compared the total cost of not holding inventory and holding inventory such that q_L would be available at T_3 regardless of θ. The threshold value of θ was most sensitive to changes in the lost sales cost. As expected, if the lost sales cost was increased, the threshold value of θ increased. When the low values for each parameter were tested, the value of θ caused the decision to switch from $q_1 = \frac{q_L}{1-\theta}$ to $q_1 = 0$ when at least 61% of the inventory was lost. This explains the result we found for the minimum cost for $\theta = 0.75$ in Table 3.4.2. If $z = 40$, or ten times the purchase price, y, the threshold value was 90%. We were able to find a value of z that supports the threshold value as shown in Eq. 3.4.1. If z upholds the inequality in Eq. (3.4.1), it is best to hold $q_1 = \frac{q_L}{1-\alpha}$ when $\alpha = \theta$.

$$z \geq hT_3 + \frac{hL(\theta - 2)}{2} + y\left(\frac{1}{1-\theta} - 1\right) \tag{3.4.1}$$

Next, we applied minimax regret decision criterion considering the same five values for θ. First, we found the lowest cost for each outcome (column), which is shown in Table 3.4.3, considering all of the combinations in the 2^9 experiment. Please note that $q_1 = 0$ was a minimum cost alternative for every outcome. Also, the value of q_1 that ensured no lost sales for each value of θ was also a minimum cost alternative for combinations in the experiment. When the lost sales cost was less than 2.5 times the purchase price, y, minimax decision criterion supported the decision to hold less inventory, thus resulting in lost sales in some situations when the holding cost and T_3 were at their highest values. As expected, if the disruption damaged all of the inventory, the retailer should not hold inventory, which is the same result supported by minimax decision criterion.

Table 3.4.3: Possible Minimum Cost Decisions for Each Outcome

Outcome	**Possible Minimum Cost Decisions**
$\theta = 0$	$q_1 = 0$, $q_1 = q_L$
$\theta = 0.25$	$q_1 = 0$, $q_1 = q_L$, $q_1 = \frac{q_L}{1-0.25}$
$\theta = 0.50$	$q_1 = 0$, $q_1 = q_L$, $q_1 = \frac{q_L}{1-0.50}$
$\theta = 0.75$	$q_1 = 0$, $q_1 = \frac{q_L}{1-0.75}$
$\theta = 1$	$q_1 = 0$
No Disruption	$q_1 = 0$, $q_1 = q_L$

In order to test the sensitivity of the decisions based on z, we performed the same experiment, but increased the values of z to be five, ten and twenty times the purchase price, y, to see the effect on the minimum cost. We found that the minimum cost occurred when $q_1 = \frac{q_L}{1-\theta}$ for $\theta < 1$ and $q_1 = 0$ for $\theta = 1$ if we assumed θ to be known. These results matched previous results when we found the value of q_1 that minimized the cost equation. If no disruption occurred, the minimum cost was almost split evenly between $q_1 = 0$ and $q_1 = q_L$. The difference between the decision depended on the lead-time, lost sales cost, holding cost, and order cost. However, if the disruption does not occur, the retailer should not hold more than q_L in order to minimize costs.

We then compared the maximum regret for each row (decision) to determine the decision that supported the smallest maximum regret. In Table 3.4.4, the regret matrix is shown for the costs given in Table 3.4.2, and the bolded values represent the maximum regret for each decision. The largest negative value in each row represents the largest regret for each decision.

Table 3.4.4: Regret Matrix

q_1	$\theta = 0$	$\theta = 0.25$	$\theta = 0.50$	$\theta = 0.75$	$\theta = 1$	**No Disruption**
q_L	\$0	-\$145	-\$163	-\$203	**-\$500**	-\$87
$\frac{q_L}{1-0.25}$	-\$59	\$0	-\$106	-\$239	**-\$633**	-\$220
$\frac{q_L}{1-0.50}$	-\$209	-\$159	\$0	-\$313	**-\$900**	-\$487
$\frac{q_L}{1-0.75}$	-\$926	-\$784	-\$609	-\$550	**-\$1,700**	-\$1,287
0	**-\$650**	-\$517	-\$250	\$0	\$0	\$0

Table 3.4.5 gives the possible outcomes that resulted in the maximum regret for each decision. As expected, the possible maximum regret outcomes were associated with the extreme scenarios (e.g., $\theta = 0$) with the exclusion of $q_1 = q_L$.

Table 3.4.5: Possible Maximum Regret Outcomes for Each Decision

q_1	**Possible Maximum Regret Outcomes**
q_L	No Disruption, $\theta = 50\%$, $\theta = 1$
$\frac{q_L}{1-0.25}$	No Disruption, $\theta = 1$
$\frac{q_L}{1-0.50}$	No Disruption, $\theta = 0$, $\theta = 1$
$\frac{q_L}{1-0.75}$	No Disruption, $\theta = 0$, $\theta = 1$
0	$\theta = 0$

When we evaluated the eleven different possibilities for maximum regret, only six gave the minimum maximum regret for the 512 combinations tested. The decision, outcome, and percentage of occurrence is given in Table 3.4.6. It was best to hold q_L for more than half of the combinations. On the other hand, 45% of the combinations tested supported the decision not to hold inventory.

3.4.3 Sensitivity Analysis

In the sensitivity analysis, we tested the same values for θ that were used in the numerical experiment for $\theta = [0, 1]$. All of the parameters were kept at the lowest value given in Table 3.4.1, and the parameter in question was increased or decreased. Table 3.4.7 gives the lowest

Table 3.4.6: Minimax Regret Outcomes

q_1	**Outcome**	**Percentage of Occurrence**
q_L	No Disruption	9%
q_L	$\theta = 0.5$	9%
q_L	$\theta = 1$	33%
$\frac{q_L}{1-0.25}$	No Disruption	0.2%
$\frac{q_L}{1-0.25}$	$\theta = 1$	4%
0	$\theta = 0$	45%

and highest values tested for each parameter. We did not allow any of the parameters to equal zero because that would not be realistic, and if $T_3 = 0$, there would be no disruption. For T_3 and L, we did not test values that would cause $T_3 + L > T_4$ because the second disruption would be negligible. For h and y, we did not test values that were higher than z because the holding cost and purchase price would most likely not be greater than the lost sales cost. In addition, the values of z were all higher than y because the lost sales cost would probably be at least the purchase price.

Table 3.4.7: Sensitivity Analysis Values

Parameter	**Lowest Value**	**Highest Value**
λ_D	25	500
λ	5	100
A	25	500
h	0.25	4
y	1	10
s	4	80
T_3	1	6
T_4	3	20
L	1	6

We discovered that z was the only cost that changed the decision given by minimax decision criterion if the lost sales cost was about 1.5 times y. The lost sales cost often encompasses the retail price of the item and loss of customer goodwill while y represents the purchase price, which in most cases, will be less than the retail price. Therefore, we can still conclude that minimax decision criterion will always support the inventory policy to not hold inventory during the first disruption.

On the other hand, the sensitivity analysis for minimax regret decision criterion provided many insights into the effect of each parameter. In the next four subsections, we separate the parameters according to how they affect q_1.

$q_1 = q_L \Rightarrow 0$

Four of the parameters, A, h, T_3, and y, showed a change in the decision for q_1 from q_L to 0 as the parameters were increased. This result is expected because a low ordering cost, holding cost, and purchase price as well as a short first disruption will constitute holding inventory when minimax regret decision criterion is applied. However, as the costs and duration of the first disruption increase, it becomes more cost effective not to hold inventory during the first disruption, especially if the retailer expects a large percentage of the inventory to be damaged.

$q_1 = 0 \Rightarrow q_L$

There were only two parameters, L and λ_S, that caused a change in q_1 from 0 to q_L. As the lead-time and disruption demand rate were increased, the total cost increased as well. A short lead-time allows the retailer to order at T_3 and incur minimal lost sales. In some cases, the total lost sales may be cheaper than the order and holding costs incurred for holding inventory during the first disruption. If the disruption demand rate, λ_S, is small, then the cost of lost sales will also be minimal similar to the result for the short lead-time.

No Change

Two parameters, λ and T_4, supported holding q_L during the first disruption for all tested parameter values. Therefore, we can conclude that the normal demand and the length of second disruption do not have an affect the decision for q_1. This result is intuitive because neither parameter affects lost sales or q_1.

Multiple Changes in q_1

The lost sales cost, z, altered the value of q_1 four times for the tested values meaning all of the values of q_1 beginning with $q_1 = 0$ were utilized as z increased. The value for z was increased in increments of four with the first value equaling y. If $z \geq 50$ or more than 12 times the purchase price, the retailer will hold $\frac{q_L}{0.25}$ ($\theta = 0.75$), which is the largest amount of inventory we tested to be held by the retailer. This result indicates that the decision is very sensitive to changes in the lost sales cost as expected. Retailers must seriously consider how they will value lost sales because it will affect their ordering decisions, particularly if they implement minimax regret decision criterion.

3.5 Summary and Future Extensions

Minimax and minimax regret decision criteria provided different solutions in our numerical experiment, and the retailer must prioritize the objectives of the company when making decisions regarding inventory policies for possible disruptions caused by forecasted storms. The following is a summarized list of our findings:

1. It is best to hold a different quantity for each disruption ($q_1 \neq q_2$).
2. Minimax decision criterion always yielded $q_1 = 0$ unless the lost sales cost was slightly higher than the holding cost.
3. Both decision criteria supported $q_2 = q_S$.
4. A threshold value of θ exists indicating that even if all of the inventory is not damaged, it is better to not hold inventory depending on the lost sales cost, z.
5. The minimax regret decision is very sensitive to changes in the lost sales cost, but it is also sensitive to changes in all other parameters except λ and T_4.

The results showed the importance of the retailer accurately estimating the lost sales cost because it will drastically change the decision for q_1. However, it is difficult to assess the ramifications of not meeting the customers' needs. In addition, the forecast may play a significant role in the retailers' inventory policy. If the storm is expected to be catastrophic, it is probably in the best interests of the retailer not to hold any inventory. On the other hand, most property structures for retailers such as grocers are able to withstand the effects of storms. Based on the results, our recommendation is that the retailer carefully consider how lost sales are valued when selecting an inventory policy. Our results indicated that if the lost sales cost is estimated to be five times or more than the purchase price, then the retailer should hold at least q_L. Otherwise, the retailer should hold nothing.

The model could be extended to include the disruption discussed in Chapter 2. It seems that if minimax decision criterion is used to solve the extended model, q_1 would either be 0 or the amount of inventory at the retailer at T_2. On the other hand, it might be worth exploring the problem using minimax regret decision criterion. In addition, a variable lead-time and flexible reorder point could also be included to make the model more realistic.

Customers are often in search of items such as bread, milk, and fruit after a storm, and the model could be extended to include perishable items. We did not consider perishable items because we assumed that the retailer would not stock up on such items because of the uncertainty surrounding the length of the disruption between T_2 and T_3. Also, if perishable items were included, the cost for the decision to hold inventory would only increase, resulting in the decision not to hold inventory. This is the same decision we found for minimax decision criterion considering non-perishable items. We also predict that the decision not to hold inventory would increase among the situations we tested considering minimax regret decision criterion because the total cost for holding inventory during the first disruption would increase.

In closing, our results indicated the importance of estimating cost parameters as well as selecting decision criterion to determine an inventory policy for disruptions. However, if retailers do not consider the impact of their inventory decisions before a disruption, they will not be able to capitalize on an opportunity to generate additional profit as well as foster positive customer relationships during a time of crisis.

Chapter 4

Pre-Positioning Hurricane Supplies in a Commercial Supply Chain

4.1 Background

4.1.1 Disastrous Hurricanes and Emergency Management

Hurricane Katrina was a daunting reminder that hurricanes represent one of Nature's most destructive forces. This catastrophic storm, which left a trail of destruction along the United States Gulf Coast in August 2005, was responsible for more than 1,800 casualties, 250,000 displaced residents, and $125 billion in expenditures making it the costliest disaster in United States history [21]. Although the significant environmental, socio-economic, and cultural repercussions of Hurricane Katrina are evident, disastrous hurricanes are certainly not a new phenomenon. For example, the 18th century storm known as the "Great Hurricane" claimed 22,000 lives throughout the Caribbean and is the deadliest hurricane of recorded history [40]. The deadliest hurricane in United States history was responsible for an estimated 8,000 to 12,000 casualties in Galveston, Texas during the year 1900 [26]. More recently, hurricanes Ike, Gustav, and Dolly in 2008 and Dennis, Katrina, Rita, and Wilma in 2005 each accounted for over $1 billion in expenditures and numerous casualties [37].

The field of Emergency Management outlines a framework for protecting civilization from the adverse effects of natural and man-made disasters caused by hazards such as hurricanes, tornadoes, earthquakes, drought, and terrorist attacks. This framework entails the following functional areas: (i) mitigation, (ii) preparedness, (iii) response, and (iv) recovery. *Mitigation* refers to "a sustained action to reduce or eliminate risk to people and property from hazards and their effects" [22]. Mitigation activities focus on long-term solutions such as construction design and structural control (e.g., strengthening levee systems), and tend to be the most expensive emergency management options. In contrast to mitigation, *preparedness* targets short term activities directly related to response and recovery such as preparing large-scale evacuation plans or securing emergency supply items. The objective of the *response* function is to provide immediate short term relief following disaster by saving lives, protecting property, and meeting basic human needs. Activities include search and rescue, medical assistance, law and order, and immediate food and shelter. The final stage of the disaster-relief lifecycle is the *recovery* effort, which seeks to restore communities to pre-disaster conditions, and also to curtail future vulnerability. Examples of recovery activities include debris removal, rebuilding residential properties, and restoring businesses.

4.1.2 Pre-Positioning

The focus of this paper will be strategic positioning of emergency supplies in anticipation of a threatening hurricane or other predictable hazard, which can be considered a cross-section of the preparedness and response activities of the Emergency Management framework described in Section 4.1.1. Pre-positioning is defined as the "stockpiling of equipment and supplies at, or near the point of planned use" [15]. Although originally applied within military contexts (e.g., [8], [31]), public and private sector organizations have adopted pre-positioning strategies to prepare for civilian response and recovery operations. For example, the U.S. Federal Emergency Management Agency (FEMA) oversees a variety of permanent and temporary staging locations for pre-positioning commodities, equipment, and personnel. Permanent facilities include nine FEMA Logistics Centers, which are traditional distribution centers for commodities geographically dispersed throughout the Continental United States, as well as a number of commercial storage sites owned and operated by private firms [59]. On a smaller scale, locating ambulances and fire stations can also be considered a form of public sector pre-positioning (e.g., [3], [52]).

Private sector firms implemented a most noteworthy application of temporary staging and pre-positioning in response to Hurricane Katrina. In particular, The Home Depot and

Wal-Mart independently pre-positioned commodities at temporary staging locations near New Orleans prior to Katrina's landfall in 2005 (e.g., [24]). As a result, emergency supplies, equipment, and personnel were readily available for initial response efforts in contrast to the government's slow and uncoordinated response. The logistics response of Wal-Mart and The Home Depot to Hurricane Katrina accentuates the pivotal role of private sector firms in facilitating effective disaster relief operations, which is a perspective that has been acknowledged in the humanitarian logistics research literature.

4.2 Problem Description: Commercial Pre-Positioning of Commodities

In an effort to encourage a critical mass of private sector firms to follow the lead of Wal-Mart and The Home Depot in responding to domestic disasters as described in Section 4.1, this study examines a humanitarian logistics problem from the perspective of a traditional profit-driven private sector firm. Specifically, the problem presented in this paper is motivated by a real-world supply chain scenario consisting of one manufacturer, multiple retailers, and one or more observed storms that threaten the consumers served by these retailers. The manufacturer produces items such as bottled water, non-perishable foods, or portable electronic devices that are in high demand during the hurricane season, especially in the presence of an ominous hurricane or tropical storm. In particular, it is common for retailers to experience a spike in demand for these and other emergency supply items during the inventory cycles that precede an observed storm's probable landfall. Such "pre-storm demand surge" is predominantly driven by consumers who are preparing for emergency evacuation, or by consumers planning to "ride out the storm" and risk the inconvenience of potentially pervasive power outages. A temporary spike in demand can also be observed subsequent the actual landfall of a major storm as a result of disaster relief and recovery activities.

Based on conversations between one of the authors and supply chain managers from a manufacturing firm that encounters potential post-storm demand surge as described above, orders for emergency supplies are typically initiated by retailers in response to a realized pre- or post-storm demand spike. In other words, the retailers served by this manufacturer exhibit a propensity for adopting reactive inventory policies that respond to an actual demand surge as opposed to proactive inventory policies that anticipate a likely demand surge. Such reactive approaches to stock control coupled with the delivery lead-times associated with order fulfillment often lead to widespread stockout of emergency supplies during peak demand surge periods, which in turn exacerbates the vulnerability of the populace affected by an approaching storm, inhibits post-storm response and recovery efforts, and proliferates lost sales and/or backorders for the above-mentioned supplier-retailer supply chain.

In order to alleviate the negative effects of supply shortages that often occur during pre- and post-storm demand surge periods, this paper will explore inventory pre-positioning strategies from the standpoint of the manufacturing firm described at the beginning of this section. The proposed pre-positioning strategy is characterized by a manufacturer that proactively pushes inventory upon geographically dispersed retailers in anticipation of pre- or post-storm demand surge, which is a strategy that contrasts the reactive wait-and-see approach indicative of current practice. After the storm subsides and actual demands become known, inventory is then transhipped among the retailers and manufacturer.

The proposed pre-positioning strategy assumes that the manufacturer bears all the costs and risks of pre-positioning, similar to vendor-managed inventory (VMI) systems. These costs may be attributed to production at the manufacturing facility, material handling at the manufacturing facility, and transportation of products from the manufacturer to the retailers. On the other hand, risks are driven by the uncertainty of which retailers will experience a demand surge for emergency supplies as a result of a storm making landfall at their location. These risks include the expected costs due to inventory shortages or excess inventory at each retailer location, and the possibility of additional transportation costs for redistributing improperly pre-positioned supplies. If there is a shortage of inventory at a retailer location, the items are backordered, and the manufacturer will incur a shortage cost representing loss of customer goodwill for not providing the item immediately after the effects of the storm have subsided. Additionally, conversations with the representative manufacturer also revealed the risk of a prohibitively constrained post-storm logistics system characterized by a scarcity of available third-party logistics providers (3PLs). This environment inhibits post-storm redistribution efforts, or significantly increases the cost of doing so. Within the above-mentioned context, this investigation will be driven by the following research questions.

Question 4.1 *Given the possibility of a post-storm demand surge for an emergency supply item, how should the manufacturer optimally pre-position the commodity among a network of geographically dispersed retailers? In particular, what quantity should be stockpiled at each retailer prior to a potential post-storm demand surge?*

Question 4.2 *How beneficial is pre-positioning relative to the existing wait-and-see approach currently used in practice?*

Note that Question 1 includes the possibility that the pre-position quantities for any one or more retailers could be zero, which suggests that pre-positioning will occur for a subset of the retailers. Also note that the manufacturer's interests relative to these research questions will be represented solely in terms of expected cost, which is consistent with the commercial perspective of this study.

The remainder of this manuscript is organized as follows. First, Section 4.3 surveys the relevant academic literature. Next, Section 4.4 introduces a stochastic programming based methodology. Specifically, the manufacturer's problem is represented as a two-stage stochastic programming model with recourse where the first stage decision is the quantity of supplies to stockpile at each retailer situated in the projected path of an observed storm, and the recourse decision entails the redistribution of supplies throughout the supply chain network after all demand surge information (i.e., quantities and locations) is known with certainty. In addition to the stochastic programming methodology, an alternative solution methodology is applied to the pre-positioning problem, which utilizes a heuristic to compute the pre-positioned quantities and reduces the model to the well-known transportation problem. The stochastic programming and alternative solution methodology are illustrated via numerical examples, including a case study based on real-world data in Section 4.5. The examples are solved using the scenario driven stochastic programming capability of the commercial optimization software, What's Best™. Then, the optimal solution found by What's Best™ and the solution approach are compared. A sensitivity analysis is also provided to illustrate the effects of the cost parameters on the expected benefit of pre-positioning. The final section of the paper, Section 4.6, summarizes key ideas and findings, discusses limitations, and outlines an agenda for future research.

4.3 Literature Review

As previously mentioned, the model presented in this paper resembles a VMI (vendor-managed inventory) system because the manufacturer assumes a majority of the costs and risk associated with the production and distribution of an item. VMI models also consider various costs associated with holding, transportation, shortage, and ordering costs incurred by the vendor that are similar to the costs considered in our model ([33], [67]). Moreover, many models within the VMI literature determine the optimal shipment quantity to the retailer(s) (e.g., [13], [17]), which is also determined in our model. However, the model presented in this paper differs from a VMI system in several ways. Current VMI models assume a multi-period or infinite planning horizon ([13],[33]) in contrast to our model, which considers a short planning horizon with a one-time decision regarding inventory quantities at retailers. In addition, our model allows for transshipments between retailers if a shortage is experienced at a retailer, which is not characteristic of VMI models. Lastly, we consider an increased unit transportation cost after the extreme event (i.e., hurricane) has occurred. Our model is formulated as a two-stage stochastic program to determine the quantity that should be pre-positioned at each retailer in the network, which is similar to the models presented in the pre-positioning literature. Therefore, the focus of the literature review will be pre-positioning models that are closely related to the model presented in this paper.

The definition of pre-positioning presented in [15] as the "stockpiling of equipment and supplies at, or near the point of planned use" suggests that pre-positioning activities entail both location and inventory related choices. In fact, the more recent definition proposed in Richardson et al. [54] implies that pre-positioning encompasses location, inventory, and distribution decisions, which spans the preparation and response dimensions of the emergency management framework. More specifically, pre-positioning in preparation for potential disaster relief operations involves determining the number, location, and capacity of permanent or temporary warehouses used to coordinate distribution of emergency supplies to disaster areas following a catastrophic event. In addition, pre-positioning also concerns inventory decisions for these facilities as well as transportation policies (e.g., shipping and routing) for distributing commodities from storage facilities to affected communities. From this perspective, the academic literature related to inventory pre-positioning can be characterized as a cross-section among the facility location, inventory control, and distribution / routing literatures. Indeed, several studies address some of these areas individually within the context of disaster relief. For instance, Jia et al. [30] and Huang et al. [25] consider humanitarian relief chain design in terms of facility location with no explicit inventory or routing decisions, while inventory models related to disaster relief with no network design or distribution decisions include Beamon and Kotleba [7], Lodree and Taskin [36], and Taskin and Lodree [63]. Papers that focus exclusively on "last-mile distribution" with no location or inventory decisions include Özdamar et al. [45] and Balçik et al. [4]. A few studies also integrate the location, inventory, and distribution aspects of pre-positioning described in Richardson et al. [54], namely Chang et al. [14], Mete and Zabinsky [38], Rawls and Turnquist [53], and Salmerón and Apte [56]. Nevertheless, the focus of this review is a series of papers that consider at least two components of pre-positioning including location and inventory ([5],

[12]), inventory and distribution ([6]), and the above-mentioned papers [14], [38], [53], and [56].

One of the first papers to investigate pre-positioning of commodities in preparation for potential disaster relief activities is Barbarosoğlu and Arda [6], who address inventory and distribution decisions on an existing humanitarian logistics network. In particular, Barbarosoğlu and Arda [6] formulate a two-stage stochastic programming model where the first stage represents a multi-commodity, multi-modal network flow problem in which commodities are transported from a given set of supply nodes to a set of potential demand nodes with uncertain requirements. During the second stage, demand locations and requirements become known. Then, redistribution during the second stage based on known demand information and the pre-positioning arrangement generated in the first stage is optimized by solving another multi-commodity, multi-modal network flow problem. The model also accounts for vulnerability of supply and demand nodes as well as uncertain arc capacities, both of which become known in the second stage. The objective function of the stochastic programming model minimizes first stage transportation cost and the expected transportation and inventory costs of the second stage. Commercial software was used to solve a numerical example based on data from an August 1999 earthquake that occurred in Turkey.

Balçik and Beamon [5] and Campbell and Jones [12] consider both humanitarian relief chain design and inventory decisions. Balçik and Beamon [5] develop a variation of the maximal covering problem in which the number of distribution centers (DCs) and their locations are chosen such that the total expected demand covered by the distribution centers across a set of probabilistic scenarios is maximized. Similar to Barbarosoğlu and Arda [6], Balçik and Beamon [5] illustrate their model by generating data sets that correspond to historical earthquake scenarios and use commercial software to conduct computational experiments. On the other hand, Campbell and Jones [12] introduce the first scenario-free pre-positioning study. They develop an analytic framework with newsboy results and sensitivity analyses for a network with one supply node and one demand node, where the supply node faces the possibility of being destroyed. Using results from their single supply / single demand node model, Campbell and Jones [12] also describes a procedure for optimally selecting a supply location from among a set of potential locations. In addition, a heuristic approach to locating multiple supply points for multiple demand locations is also described and explored in a computational experiment.

Chang et al. [14], Rawls and Turnquist [53], Salmerón and Apte [56], and Mete and Zabinsky [38] incorporate all three aspects of pre-positioning into their models. Chang et al. [14] seems to be the first study that location, inventory, and distribution decisions are considered explicitly. In addition, the two-stage stochastic programming model is predicated on a preliminary stochastic model with chance constraints that partitions a multi-group, multi-echelon supply chain network into zones. The solution approach is a multi-step procedure that entails sample average approximations of the stochastic problems. Rawls and Turnquist [53] also integrate network design, inventory, and distribution decisions for disaster relief purposes. In Rawls and Turnquist [53], a two-stage stochastic mixed integer programming model is presented in which DC locations, DC capacities, and inventory levels are determined during the first stage based on probabilistic information concerning demand locations and requirements. The second stage decision concerns the distribution of commodities from the DC locations established during the first stage to disaster areas which are revealed at the beginning of the second stage. Rawls and Turnquist [53] also design a heuristic solution approach called the Lagrangian L-shaped method intended for large-scale problem instances, which is then illustrated in terms of a case study where disaster scenarios are generated based on historical hurricane data. Further details of the aforementioned case study are discussed in Section 4.5 of this paper. Moreover, the focus of Salmerón and Apte [56] is more on people as opposed to facilities and commodities, which is unlike other pre-positioning studies. The two-stage stochastic programming formulation is actually a hierarchical bi-objective model that minimizes casualties within the critical and stay-back populations, and minimizes unmet demand for the transfer population. Lastly, Mete and

Zabinsky [38] investigate location, inventory, and distribution decisions within the context of pre-positioning medical supplies. In the first stage of their two-stage stochastic programming methodology, warehouse locations are determined. Then, after demands throughout a network of hospitals are revealed, medical supplies are transported from warehouses to hospitals. The second stage distribution problem is a mixed integer programming (MIP) model that represents a pseudo vehicle routing problem in which vehicles are assigned to predetermined routes. To illustrate the model, a case study based on earthquake scenarios in the Seattle Washington area is presented. For this case problem, the integrated stochastic programming / MIP models are solved using commercial software.

It is evident from this review that the research literature has produced several frameworks that could be useful to government and relief organizations in a variety of real-world disaster relief environments. However, none of the existing studies specifically address the manufacturer's pre-positioning problem introduced in Section 4.2. For instance, since the manufacturer's pre-positioning problem involves an existing network of retailers, there is no need to consider facility location decisions as in references [14], [38], [53], [56], [5], and [12]. From this perspective, our paper is similar to Barbarosoğlu and Arda [6], who also address inventory and distribution decisions on an existing network. Similar to Barbarosoğlu and Arda [6], the two-stage stochastic programming methodology described in Section 4.4 of this paper considers the transportation costs of pre-positioning commodities during the first stage and the expected cost of transportation, inventory shortages, and excess inventory during the second stage. But in addition to the transportation and inventory costs considered in Barbarosoğlu and Arda [6], the model described in Section 4.4 also takes into account the production costs incurred by the manufacturing facility during both the first and second stages. Another difference is that the underlying model in Barbarosoğlu and Arda [6] is a multi-commodity, multi-modal network flow problem, whereas the underlying model presented in this paper is a transportation problem with possible transshipments among retailers. Finally, the case study data applied to the model in this paper is derived from the hurricane scenarios generated by Rawls and Turnquist [53].

4.4 Methodology

4.4.1 Stochastic Programming Model

This section describes a two-stage stochastic programming model for the manufacturer's inventory pre-positioning problem introduced in Section 4.2. The fundamental logic of the two-stage stochastic programming methodology entails a first stage decision $\vec{x} \in \mathbb{R}^n$ in the presence of uncertain parameters $\vec{D} \in \mathbb{R}^n$, followed by a second stage (or recourse) decision $\vec{y} \in \mathbb{R}^n$ after the realization $\vec{\xi} \in \mathbb{R}^n$ of $\vec{D}$ has occurred (a complete list of notations is shown in Table 4.4.1). The general form of a two-stage stochastic program can be expressed as follows (e.g., [10]):

$$\text{Minimize:} \quad \vec{c}^{\,T}\vec{x} + \mathbb{E}\left[Q\left(\vec{x}, \vec{D}\right)\right] \tag{4.4.1}$$

$$\text{Subject to:} \quad A\vec{x} = \vec{b} \tag{4.4.2}$$

$$\vec{x} \geq \vec{0}, \tag{4.4.3}$$

where the *recourse function* is given by

$$\mathbb{E}\left[Q\left(\vec{x}, \vec{D}\right)\right] = \min\left\{\vec{q}^{\,T}\vec{y} \mid W\vec{y} = h - T\vec{x}, \quad \vec{y} \geq \vec{0}\right\}. \tag{4.4.4}$$

The objective function (4.4.1) minimizes the cost of the first stage decision ($\vec{c} \in \mathbb{R}^n$ represents objective function coefficients or unit costs, and $\vec{c}^{\,T}$ is the transpose of $\vec{c}$) plus the expected cost of the second stage decision, which is the recourse function. The recourse decision $\vec{y}$ is optimized based on both the first-stage decision $\vec{x}$ and the observations $\vec{\xi}$. W represents the fixed recourse matrix, and T represents the technology matrix while h refers to the right-hand side in the second stage constraints. T and h become known when $\vec{\xi}$ is realized. Decisions at each stage have their respective constraints, (4.4.2) and (4.4.3) for the first stage and (4.4.4) for the second.

Stage 1

Applying this framework to the manufacturer's pre-positioning problem, the first stage decision $\vec{x} = (x_1, x_2, \ldots, x_n)$ represents the quantity of supplies that the manufacturer should pre-position across the network of retailers. That is, x_i is the amount of inventory sent to retailer i prior to the landfall of an observed storm, where the demand at each retailer is a random variable $D_i \in \vec{D}$. The demands, D_i, are interpreted as *additional* demand at each retailer i that emerge because of the storm's presence. It is assumed that $c_i = c$ for each $c_i \in \vec{c}$ and $i = 1, 2, \ldots, n$, where c is the cost attributed to producing and preparing one unit of inventory for shipment. The first stage decision also incurs the transportation cost $c_v d_i x_i$ for each retailer location $i = 1, 2, \ldots, n$, where c_v is the unit cost of transportation (\$ per mile) and d_i is the distance (in miles) between the manufacturer and retailer i. Therefore, the total first stage cost is

$$\text{Production cost} + \text{pre-storm transportation cost} = \sum_{i=1}^{n} c x_i + \sum_{i=1}^{n} c_v d_i x_i. \tag{4.4.5}$$

Stage 2

The second stage cost is comprised of holding costs due to excess inventory at certain retailer locations, shortage costs due to inventory shortages (backorders) at other retailer locations, and the cost of redistributing inventories throughout the supply chain network. More precisely, the recourse function optimizes the recourse decision $\vec{y}$ in terms of the *expected* costs due to excess inventory, shortages, and redistribution. In general, the stochastic programming methodology handles the uncertain parameters $\vec{D} = (D_1, D_2, \ldots, D_n)$ using *scenarios* as opposed to assuming a conventional probability distribution for each D_i. In particular, suppose there are k scenarios such that $\vec{D} = \vec{\xi}_t$ under scenario $t \in \{1, 2, \ldots, k\}$,

Category	Symbol	Description
Decision variables	x_i	Quantity shipped to retailer i before hurricane
	y_{lmt}	Quantity transshipped from retailer l to retailer m in scenario t
	$y_{\theta mt}$	Quantity transshipped from manufacturer to retailer m in scenario t
Random variable	D_i	Demand at retailer i
Costs	c	Unit production cost
	c_v	Pre-hurricane transportation cost/mile
	c_w	Post-hurricane transportation cost/mile
	h	Unit holding cost
	s	Unit shortage cost
	$\widehat{E}_{PP}$(Benefit)	Expected benefit of PDSA solution
	E^*_{PP}(Benefit)	Expected benefit of optimal pre-positioning strategy
	$\widehat{EC}_{PP}$	Expected cost of PDSA solution
	EC^*_{PP}	Expected cost of optimal pre-positioning strategy
	EC_{WS}	Expected cost of wait-and-see strategy
Other notations	i	Retailer locations, $i = 1, ..., n$
	t	Scenarios, $t = 1, ..., k$
	ξ_{it}	Observed demand at retailer i if scenario t occurs
	I^+	Set of retailers with excess inventory
	l	Retailers with excess inventory, $l \in I^+$
	I^-	Set of retailers with an inventory shortage
	m	Retailers with an inventory shortage, $m \in I^-$
	θ	Manufacturer location index
	Θ	Manufacturer location
	d_i	Distance between manufacturer and retailer i
	d_{lm}	Distance between retailer l and retailer m
	$d_{\theta m}$	Distance between manufacturer and shortage location
	P_t	Probability of scenario t occurring

Table 4.4.1: Summary of notations for stochastic programming model.

where scenario t occurs with probability P_t. That is, ξ_{it} represents the actual demand for retailer i if scenario t occurs. Then the scenario-based probability distribution for demand at retailer i is $\Pr\{D_i = \xi_{it}\} = P_t$ for each $i = 1, 2, \ldots, n$ and $t = 1, 2, \ldots, k$.

Now if $x_i - \xi_{it} > 0$ for a particular scenario t, then retailer i has excess inventory under scenario t. Similarly, $x_j - \xi_{jt} < 0$ suggests that a shortage has occurred at retailer j under scenario t. Clearly, two disjoint sets of retailers emerge for each scenario t for a given first-stage pre-positioning arrangement: the set of retailers with excess inventory, denoted I^+, and the set of retailers with inventory shortages, denoted I^-. Let h be the unit holding cost associated with excess inventory and s the unit cost of an inventory shortage, both of which are presumed to be homogeneous among the retailers. Since scenario t occurs with probability P_t, the expected cost due to inventory overages and shortages from the manufacturer's perspective is

$$\text{Expected cost (holding + shortage)} = \sum_{t=1}^{k} P_t \left[\sum_{l \in I^+} h(x_l - \xi_{lt}) + \sum_{m \in I^-} s(\xi_{mt} - x_m) \right]. \tag{4.4.6}$$

Note that it is convenient to think of s as the unit cost associated with the loss of customer goodwill. In general, the components of shortage costs when backorders are considered include the cost of processing one or more additional orders (possibly at an increased rate) and the loss of customer goodwill (e.g., [44]). In this paper, the redistribution costs described above can be interpreted as the former component of the conventional shortage cost, and s represents the latter.

In addition to the expected holding and shortage costs shown in Eq. (4.4.6), second stage costs are also incurred for post-storm transportation and production activities. The transportation cost between a location $l \in I^+$ with excess inventory and a location $m \in I^-$ with inventory shortages is given by $c_w d_{lm} y_{lmt}$. Here, $c_w > c_v$ is the unit cost of transportation during the post-storm redistribution stage, d_{lm} is the distance between retailers l and m, and y_{lmt} is the quantity of supplies transhipped from retailer l to retailer m if the observed post-storm demand scenario is t. Note that the inequality $c_w > c_v$ is assumed in order to reflect the scarcity of 3PLs available for post-storm redistribution relative to pre-storm pre-positioning as described in Section 4.2. Also note that y_{lmt} for all $l \in I^+$, $m \in I^-$, and $t = 1, 2, \ldots, k$ represent the recourse decision variables, which are optimized for each scenario t subject to the pre-positioned quantities x_i and observed demands ξ_{it}.

So far, we have assumed that all shortages (i.e., backorders) can be resolved through transshipments using excess inventory from the retailers $l \in I^+$. However, direct shipments from the manufacturer for the purpose of backorder replenishments will be necessary if the total network shortage associated with retailers $m \in I^-$ exceeds the total amount of excess inventory across the retailers $l \in I^+$. Furthermore, direct shipments from the manufacturer may sometimes be more cost effective than transshipments from retailers. In order to account for direct replenishments from the manufacturer, let $d_{\theta m}$ denote the distance between the manufacturer and shortage location $m \in I^-$ and $y_{\theta mt}$ represent the quantity shipped from the manufacturer directly to retailer $m \in I^-$ under scenario t. Then the expected cost of transportation during the second stage, which includes both transshipments from retailers and direct shipments from the manufacturer, is given by

$$\text{Expected cost (transportation)} = \sum_{t=1}^{k} P_t \left[\sum_{l \in I^+ \cup \Theta} \sum_{m \in I^-} c_w d_{lm} y_{lmt} \right]. \tag{4.4.7}$$

Finally, all products shipped directly from the manufacturer to shortage locations $m \in I^-$ during the transshipment stage incur production costs in addition to the transportation cost shown in Eq. (4.4.7). Assuming the unit cost of production, c, is preserved between the first and second stages, the expected cost due to production during the second stage is given by

$$\text{Expected cost (production)} = \sum_{t=1}^{k} P_t \left[\sum_{m \in I^-} c y_{\theta mt} \right]. \tag{4.4.8}$$

Model Formulation

Equations (4.4.5), (4.4.6), (4.4.7), and (4.4.8) will be combined to formulate the objective function of the stochastic programming model. In addition, constraints related to overages and shortages will also be incorporated. The resulting stochastic programming model is as follows:

$$\text{Minimize} \quad \sum_{i=1}^{n} cx_i + \sum_{i=1}^{n} c_v d_i x_i + E\left[Q\left(\vec{x}, \vec{D}\right)\right] \tag{4.4.9}$$

$$\text{where,} \quad E\left[Q\left(\vec{x}, \vec{D}\right)\right] = \min_{y_{lmt}} \Bigg\{ \sum_{t=1}^{k} P_t \Bigg[\sum_{l \in I^+} h(x_l - \xi_{lt}) \tag{4.4.10}$$

$$+ \sum_{m \in I^-} s(\xi_{mt} - x_m) + \sum_{l \in I^+ \cup \Theta} \sum_{m \in I^-} c_w d_{lm} y_{lmt} + \sum_{m \in I^-} c y_{\theta mt} \Bigg] \Bigg\}$$

$$\text{Subject to:} \quad x_i \geq 0 \;\; \forall \;\; i \tag{4.4.11}$$

$$x_l - \xi_{lt} \geq \sum_{m \in I^-} y_{lmt}, \qquad \forall \;\; l \in I^+ \tag{4.4.12}$$

$$\xi_{mt} - x_m = \sum_{l \in I^+ \cup \Theta} y_{lmt}, \qquad \forall \;\; m \in I^- \tag{4.4.13}$$

$$x_l - \xi_{lt}, \;\; \xi_{mt} - x_m, \;\; y_{lmt} \;\geq 0 \;\; \forall \;\; l \in I^+, m \in I^-, \text{ and } t = 1, .., k \tag{4.4.14}$$

Constraints (4.4.11) and (4.4.14) are non-negativity constraints. Specifically, constraints (4.4.11) are non-negativity constraints for pre-positioned quantities during the first stage, and constraints (4.4.14) ensure that overage, shortage, and transshipment quantities for the second stage are non-negative. Constraints (4.4.12) reflect the fact that it is impossible for any retailer with excess inventory to transship more than her surplus to other retailers during the post-storm redistribution stage. Lastly, constraints (4.4.13) ensure that all shortages in the network are replenished during the second stage either from retailers with excess inventory or directly from the manufacturing facility.

4.4.2 PDSA Solution Approach

The computational effort associated with solving the stochastic programming model defined by equations (4.4.9) - (4.4.14) may become prohibitive for large scale problems, which turned out to be the case for the sensitivity analysis experiments presented in Section 4.5.4. In particular, computation times of over 30 hours were reported for problem instances solved used the commercial software What's Best™. In order to address this issue, we introduce an efficient solution approach that reduces the stochastic programming model to the well-known transportation problem (e.g., [66]), which can solve each scenario in seconds using Excel Solver. For the remainder of the paper, we refer to this solution approach as the *Percentage of Demand Scenarios solution Approach* or *PDSA*. The proposed approach entails two stages. In the first stage, the pre-positioning quantities x_i are determined. After demand realizations occur, the second stage is simply a deterministic transportation problem in which supply nodes are retailers with excess inventory as well as the manufacturer, and demand nodes are retailers with inventory shortages.

The rationale behind the first stage of PDSA is as follows. If the expected unit cost of excess inventory for no-demand scenarios is greater than the expected unit shortage cost for demand scenarios, then the pre-positioning strategy should be conservative. In this case, there is more risk associated with excess inventory compared to inventory shortages. Therefore, excess inventory should be avoided, which can be achieved by pre-positioning small quantities. Similarly, a more aggressive pre-positioning strategy would be appropriate if the expected unit cost of excess inventory for no-demand scenarios is less than the expected unit shortage cost of demand scenarios.

In order to develop expressions for the expected unit cost of excess inventory for no-demand scenarios described above as well as the expected unit shortage cost for non-zero demand scenarios, it is necessary to partition the set of scenarios $S = \{1, 2, \dots\ k\}$ into two mutually exclusive and collectively exhaustive sets for each retailer. Let $\mathbb{N}_i \subset S$ denote the non-zero demand scenarios for retailer i, and $\mathbb{Z}_i \subset S$ the scenarios in which retailer i has

Symbol	Description
$S = \{1, \ldots,\ k\}$:	Set of demand scenarios.
$\mathbb{N}_i \subset S$:	Non-zero demand scenarios for retailer i.
$\mathbb{Z}_i \subset S$:	Scenarios with zero demand for retailer i.
$D_i = \{\xi_{i1}, \xi_{i2}, \ldots\ \xi_{ik}\}$:	Demand values for retailer i for each scenario $t \in S$.
$\mathbb{D}_i \subset D_i$:	Non-zero demand values for retailer i over all scenarios.
$\xi_i^{\min} = \inf \mathbb{D}_i$:	Minimum value (infimum) of the set $\mathbb{D}_i$.
$t_{\min}$:	Scenario that corresponds to $\xi_i^{\min}$.
$\mathbb{N}_i^{\min} = \{t_{\min}\}$:	The set that contains $t_{\min}$.

Table 4.4.2: Notations for PDSA.

zero demand, where $\mathbb{N}_i \cup \mathbb{Z}_i = S$ (note that additional notations used for PDSA are shown in Table 4.4.2). Then

$$E\,(\text{Holding Cost} \mid \text{Zero Demand Scenarios}) = \sum_{t \in \mathbb{Z}_i} hP_t \tag{4.4.15}$$

$$E\,(\text{Shortage Cost} \mid \text{Non-Zero Demand Scenarios}) = \sum_{t \in \mathbb{N}_i} sP_t, \tag{4.4.16}$$

where $\sum_{t \in \mathbb{Z}_i} P_t$ and $\sum_{t \in \mathbb{N}_i} P_t$ are the percentage of zero-demand and non-zero demand scenarios for retailer i, respectively.

PDSA will also utilize the minimum non-zero demand value and its corresponding scenario to determine the actual pre-positioning quantities. Let $D_i = \{\xi_{i1}, \xi_{i2}, \ldots\ \xi_{ik}\}$ denote the demand values for retailer i under each scenario $t \in S$, and $\mathbb{D}_i \subset D_i$ denote the non-zero demand values for retailer i. Then the minimum non-zero demand value is $\inf \mathbb{D}_i$, denoted $\xi_i^{\min}$. Define $t_{\min}$ as the scenario that corresponds to $\xi_i^{\min}$ and $\mathbb{N}_i^{\min} = \{t_{\min}\}$ as the set that contains $t_{\min}$. Then PDSA calculates the pre-positioning quantities x_i as follows:

<u>Case 1:</u> $\sum_{t \in \mathbb{Z}_i} hP_t > \sum_{t \in \mathbb{N}_i} sP_t$.

Case 1 corresponds to the case in which the expected unit cost of excess inventory for no demand scenarios is greater than the expected unit shortage cost of non-zero demand scenarios (see Equations 4.4.15 and 4.4.16). For this case, a conservative strategy is warranted and the PDSA pre-positioning quantities are given by

$$x_i = \begin{cases} \xi_i^{\min} = \inf \mathbb{D}_i, & \text{if} \quad \sum_{t \in \mathbb{Z}_i} P_t < \sum_{t \in \mathbb{N}_i} P_t \\ 0, & \text{if} \quad \sum_{t \in \mathbb{Z}_i} P_t \geq \sum_{t \in \mathbb{N}_i} P_t. \end{cases} \tag{4.4.17}$$

If the total probability associated with non-zero demand scenarios is greater than the total probability of zero-demand scenarios, then Equation (4.4.17) pre-positions the minimum non-zero demand for that retailer. Otherwise, the PDSA pre-positioning quantity is zero. Case 1 is further divided to consider the total probability of non-zero and no-demand scenarios, which is why the minimum amount demanded is the recommended amount when it accounts for a larger percentage of the demand scenarios. In Case 1, the expected unit holding cost is more expensive, therefore, the manufacturer would only want to pre-position the minimum amount of items if any at all.

Case 2: $\sum_{t \in \mathbb{Z}_i} hP_t \leq \sum_{t \in \mathbb{N}_i} sP_t$.

Case 2 corresponds to the case in which the expected unit cost of excess inventory for zero-demand scenarios is less than the expected unit shortage cost of non-zero demand scenarios, which warrants a more aggressive pre-positioning strategy relative to Case 1. For this case, the PDSA pre-positioning quantities are determined by

$$x_i = \begin{cases} \sum_{t \in \mathbb{N}_i / \mathbb{N}_i^{\min}} \xi_{it} P_t, & \text{if} \quad \sum_{t \in \mathbb{N}_i^{\min}} P_t < \sum_{t \in \mathbb{N}_i / \mathbb{N}_i^{\min}} P_t \\ \xi_i^{\min} = \inf \mathbb{D}_i, & \text{if} \quad \sum_{t \in \mathbb{N}_i^{\min}} P_t \geq \sum_{t \in \mathbb{N}_i / \mathbb{N}_i^{\min}} P_t. \end{cases} \tag{4.4.18}$$

The quantity $\sum_{t \in \mathbb{N}_i / \mathbb{N}_i^{\min}} \xi_{it} P_t$ in Equation (4.4.18) represents the expected demand for retailer i with respect to non-zero demand scenarios, but excluding the minimum non-zero demand scenario, $t_{\min}$. This value is the PDSA pre-positioning quantity if total probability associated with non-zero demand scenarios (not including $t_{\min}$) is larger than the probability that scenario $t_{\min}$ occurs. Otherwise, the PDSA pre-positioning quantity is the minimum non-zero demand, $\xi_i^{\min}$. The non-zero demand scenarios are only considered for Case 2 because it is more expensive to incur shortages according to the calculated expected unit costs. As a result, the manufacturer should pre-position a non-zero quantity at the retailer location. Case 2 is further divided to consider the total probability of the smallest non-zero demand. For this reason, the recommended amount is the smallest non-zero demand when it accounts for a majority of the non-zero demand scenarios.

4.4.3 Numerical Example

In this section, a numerical example is introduced to illustrate the stochastic programming formulation given by Eqs. (4.4.9) through (4.4.14) as well as the solution approach, PDSA, presented in Section 4.4.2. Hypothetical input data for a supply chain network consisting of $n = 5$ retailers and $k = 3$ possible demand scenarios is shown in tables 4.4.3 and 4.4.4. Observe that each scenario is assumed to be equally likely for this example. Also note that demands are interpreted as requests for *additional* emergency supplies caused by an observed storm's presence. Therefore, a zero in Table 4.4.4 indicates that location i is not affected by the storm under scenario t.

Factor	Notation	Value	Distance	θ	1	2	3	4	5
Unit trans. cost (pre-storm)	c_v	2	θ	0	8	9	5	7	11
Unit trans. cost (post-storm)	c_w	4	1	8	0	6	9	19	14
Unit holding cost	h	4	2	9	6	0	6	12	15
Unit shortage cost	s	5	3	5	9	6	0	6	5
Unit production/handling cost	c	6	4	7	19	12	6	0	7
			5	11	14	15	5	7	0

Table 4.4.3: Input data for numerical example.

Scenario	Probability	Retailer				
t	P_t	1	2	3	4	5
1	1/3	15	150	200	0	0
2	1/3	0	150	200	50	0
3	1/3	0	0	200	50	90

Table 4.4.4: Example demand scenarios.

Retailer	1	2	3	4	5
x_i	0	150	200	50	0

Table 4.4.5: Optimal pre-positioning strategy for numerical example.

Optimal Solution using Commercial Software

The solution to the example problem that corresponds to the data shown in Table 4.4.3 and Table 4.4.4 was generated using the stochastic programming option of the commercial software program, What's Best™. According to Table 4.4.5, the optimal strategy pre-positions enough inventory to cover retailers 2, 3, and 4 for all three demand surge scenarios. Consequently, the second-stage redistribution quantities among retailers 2, 3, and 4 are zero for all three scenarios. That is, the recourse decision variables are $y_{lmt} = 0$ for $l = \theta, 2, 3, 4$; $m = 2, 3, 4$; and $t = 1, 2, 3$. However, retailer 1 would experience an inventory shortage of 15 units if scenario $t = 1$ occurs (see tables 4.4.4 and 4.4.5) and would require a post-storm replenishment. Similarly, a shortage of 90 units would occur at retailer 5 if scenario $t = 3$ occurs, which would also warrant a post-storm replenishment. Thus the only nonzero recourse quantities in the optimal solution are $y_{\theta 11} = 15$ and $y_{\theta 53} = 90$. Note that the post-storm replenishments for retailers 1 and 5 under scenarios 1 and 3, respectively, would be direct shipments from the manufacturer. All other recourse variables y_{lmt} for retailers $l, m = 1, 2, 3, 4, 5$, the manufacturer $l = \theta$, and scenarios $t = 1, 2, 3$ are zero in the optimal solution.

The optimal solution also leads to excess inventory under scenarios 1 and 3. Specifically, retailer 4 will have 50 units of excess inventory if scenario 1 occurs and retailer 2 will have 150 units of excess inventory if scenario 3 occurs. Using our notational convention, we have

$I^+ = \{4\}$ and $I^- = \{1\}$ if scenario $t = 1$ occurs, $I^+ = I^- = \emptyset$ if scenario $t = 2$ occurs, and $I^+ = \{2\}$ and $I^- = \{5\}$ if scenario $t = 3$ occurs.

The above information can be used to compute the expected cost of inventory overages and shortages. Recalling from Table 4.4.3 that the unit costs for overages and shortages are $h = 4$ and $s = 5$, respectively, and that each scenario is equally likely according to Table 4.4.4, the expected cost due to excess and shortage inventories is

$$\frac{1}{3} \cdot [(\$5)(15) + (\$4)(50)] + \frac{1}{3} \cdot 0 + \frac{1}{3} \cdot [(\$4)(150) + (\$5)(90)] = \$441.67. \tag{4.4.19}$$

Using Eq. (4.4.7), the expected transportation cost associated with post-storm redistribution is

$$\begin{aligned} \text{Expected cost (transportation)} &= \frac{1}{3} \cdot c_w d_{\theta 1} y_{\theta 11} + \frac{1}{3} \cdot c_w d_{\theta 5} y_{\theta 53} \\ &= \frac{1}{3} \cdot (\$4)(8)(15) + \frac{1}{3} \cdot (\$4)(11)(90) \\ &= \$1,480. \end{aligned} \tag{4.4.20}$$

From Eq. (4.4.8), the expected production cost for direct shipments from the manufacturer during post-storm redistribution is

$$\text{Expected cost (production)} = \frac{1}{3} \cdot c y_{\theta 11} + \frac{1}{3} \cdot c y_{\theta 53} = \frac{(\$6)(15)}{3} + \frac{(\$6)(90)}{3} = \$210 \tag{4.4.21}$$

The sum of Eq. (4.4.19), Eq. (4.4.20), and Eq. (4.4.21) is the expected cost of the recourse function:

$$\mathbb{E}\left[Q\left(\vec{x}, \vec{D}\right)\right] = \$441.76 + \$1,480 + \$210 = \$2,131.67. \tag{4.4.22}$$

From Eq. (4.4.5), Table 4.4.3 and Table 4.4.5, the actual cost of pre-storm pre-positioning can be computed as

$$c(0 + 150 + 200 + 50 + 0) + c_v \Big[0 + (9)(150) + (5)(200) + (7)(50) + 0\Big]$$
$$= (\$6)(400) + (\$2)(2700) = \$7,800. \qquad (4.4.23)$$

The sum of Eq. (4.4.22) and Eq. (4.4.23) gives the expected cost of optimal pre-positioning:

$$\text{Expected cost (pre-positioning strategy)} = \$2,131.67 + \$7,800 = \$9,931.67. \qquad (4.4.24)$$

The Expected Benefit of Pre-Positioning

The solution to the two-stage stochastic programming model given by equations (4.4.9) through (4.4.14) in Section 4.4.1 and then illustrated via the numerical example presented in Section 4.4.3 answers the first of two research questions introduced in Section 4.2. The focus of this section is the second of those two questions, which investigates the value of the proposed pre-positioning strategy relative to the reactive wait-and-see approach currently used in practice. First, an expression for the expected cost associated with the reactive wait-and-see approach is developed. Then the expected cost of the reactive strategy is calculated for the numerical example shown in Section 4.4.3. The expected benefit of pre-positioning is the difference between these two expected costs, which will be positive if pre-positioning is beneficial and negative (or zero) otherwise.

If no inventory has been pre-positioned prior to the observed storm, then all post-storm requests for emergency supplies will be direct shipments from the manufacturer to the affected retailers. Consequently, all shipments will incur both production and post-storm transportation costs. Also recall that the ξ_{it}'s are interpreted as additional demands necessarily induced by the storm, which suggests that ξ_{it} also represents the amount of inventory shortage at retailer i under scenario t whenever no inventory has been pre-positioned. The expected cost of the wait-and-see strategy can then be expressed as

$$\text{Expected cost (prod, shortage, trans)} = \sum_{t=1}^{k} P_t \left[\sum_{i=1}^{n} (c+s)\,\xi_{it} + \sum_{i=1}^{n} c_w d_i \xi_{it} \right]. \qquad (4.4.25)$$

From the example data in Table 4.4.3, we have $c = 6$, $c_w = 4$, $s = 5$, and the distances d_i for $i = 1, 2, \ldots, 5$ are $(8, 9, 5, 7, 11)$. The total demands $\sum \xi_{it}$ for each scenario $t = 1, 2, 3$ can be obtained by summing across the rows of Table 4.4.4 to obtain $(365, 400, 340)$. Incorporating these values, the demands in Table 4.4.4, and $P_t = 1/3$ for each t into Eq. (4.4.25) leads to

$$\text{Expected cost (reactive)} = \$4,631.67 + \$5,066.67 + \$4,366.67 = \$14,065. \qquad (4.4.26)$$

The difference between Eq. (4.4.26) and Eq. (4.4.24) yields the value of pre-positioning:

$$\text{Expected benefit of pre-positioning} = \$14,065 - \$9,931.67 = \$4,133.33. \qquad (4.4.27)$$

Since the quantity in Eq. (4.4.27) is greater than zero, it would be in the manufacturer's best interest to implement the proposed pre-positioning strategy as opposed to the current reactive strategy. Specifically, the manufacturer is $\$4,133.33$ better off with pre-positioning relative to the wait-and-see approach. In general, the expected benefit of pre-positioning can be obtained by subtracting Eq. (4.4.9) from Eq. (4.4.25).

PDSA Solution

Based on the example data shown in tables 4.4.3 and 4.4.4, PDSA generated the optimal solution shown in Table 4.4.5. Therefore, the expected cost and benefit will not be reproduced in this section. However, for illustration, PDSA calculations for determining the pre-positioning quantity for *Retailer 2* will be summarized. First, the scenarios are partitioned as $\mathbb{N}_2 = \{1, 2\}$ and $\mathbb{Z}_2 = \{3\}$. Since each scenario occurs with probability 1/3 as shown in Table 4.4.4, we have

$$\begin{aligned}
\sum_{t\in\mathbb{Z}_2} hP_t &= 4\sum_{t=3}^{3}\frac{1}{3} = \frac{4}{3} \\
\sum_{t\in\mathbb{N}_2} sP_t &= 5\sum_{t=1}^{2}P_t = 5\left(\frac{1}{3}+\frac{1}{3}\right) = \frac{10}{3}.
\end{aligned}$$

The above calculations show that *Case 2* of PDSA applies, which implies that the PDSA pre-positioning quantities are determined according to Equation (4.4.18). The minimum non-zero demand is $\xi_2^{\min} = 150$ with $t_{\min} = 1$ or 2. Without loss of generality, let $t_{\min} = 1$ so that $\mathbb{N}_2^{\min} = \{1\}$. Then $\sum_{t\in\mathbb{N}_2^{\min}} P_t = P_1 = 1/3$ and $\sum_{t\in\mathbb{N}_2/\mathbb{N}_2^{\min}} P_t = P_2 = 1/3$. Thus $x_2 = \xi_2^{\min} = 150$, which is consistent with the optimal solution. PDSA pre-positioning quantity calculations for the other retailers are similar.

4.5 Case Study

This section examines the proposed pre-positioning strategy in a more realistic setting than the illustrative example presented in Section 4.4.3 by considering more representative hurricane information, a larger number of scenarios, and a more extensive network of retailers. The scenario data is derived from the case study introduced in Rawls and Turnquist [53], and 30 retailers are assumed to be situated according to the network depicted in Figure 4.5.1. Specifically, a macro level view of the manufacturer's pre-positioning problem is assumed where each city in the network represents a retailer location or collection of local retailers.

In order to reflect the necessary input data for the stochastic programming model presented in Section 4.4.1, the case study data in [53] is modified as follows. First, observe that the network shown in Figure 4.5.1 represents potential DC locations in [53]. However, this study interprets each node in the network as a retailer location, where exactly one of the nodes is assumed to be the location of the manufacturing facility. Next, recall that the

Figure 4.5.1: *Source: Rawls and Turnquist [53].* Network in southeast United States used for case study.

model in [53] accommodates multiple commodity types (i.e., water, food, and medical kits) whereas our model assumes a single commodity type. Therefore, this paper considers one of the product types (water) from [53] for the case study. Moreover, consider that the scenarios generated in [53] include demand locations and quantities as well as nodes and arcs that are destroyed by a hurricane or tropical storm. Since the pre-positioning model presented in this paper does not consider network reliability, all scenario information presented in [53] related to unusable nodes and arcs is ignored for the purposes of this study. Finally, the model in [53] does not include production costs or inflated post-storm transportation costs. Hence, this case study introduces hypothetical values for these costs to accompany the illustrative cost data described in [53]. A summary of the cost data used in this study is shown in Table 4.5.1. Note that one unit corresponds to 1,000 gallons of water, which explains the high cost of production and inventory shortages. Also, the post-storm transportation cost is assumed to be twice the pre-storm transportation cost.

Notation	Description	Value
c	Production cost	\$323.85 / unit
c_v	Pre-hurricane transportation cost	\$0.30 / mile
c_w	Post-hurricane transportation cost	\$0.60 / mile
h	Holding cost for excess inventory	\$161.93 / unit
s	Shortage cost	\$6,477 / unit

Table 4.5.1: Cost data for case study.

4.5.1 Demand Scenarios

The case study in [53] uses 15 historical hurricanes that have impacted the southeastern United States to create scenarios. For each of these 15 recorded hurricanes, aggregate demands for three commodities (water, food, and medical kits), affected cities, and damaged roadways are documented in Table 3 of [53]. Since the stochastic programming model introduced in Section 4.4.1 is based on a single product type as mentioned previously, only the aggregate demands for water shown in Table 4.5.2[1] are considered for the purposes of this study.

Rawls and Turnquist [53] generated 50 scenarios using the 15 hurricanes shown in Table 4.5.2 and combinations of these hurricanes. An additional scenario in which none of the nodes experience demand surge is also accounted for so that the total number of scenarios is actually 51. The probabilities associated with each scenario are "...based on approximately matching aggregate historical characteristics of hurricanes in the region, but should be treated as simply illustrative values" [53]. For scenarios that represent a combination of two hurricanes, it is assumed that the hurricanes strike within a short time frame. Scenarios based on one hurricane have a combined probability of 75% and the scenarios with a combination of two hurricanes account for the other 25%. Table 4 in [53] lists the aggregate demands for each commodity for each of the 51 scenarios along with the associated probabilities.

[1] A "-" in the landfall node column of Table 4.5.2 indicates no actual landfall for the observed hurricane. However, the referenced storm still managed to induce demand surge.

Hurricane	Category	Landfall Node	Water Demand (1000 gallons)
1	3	5	350
2	5	14	560
3	2	22	861
4	2	22	9000
5	4	11,29	7500
6	3	15	1000
7	2	21	600
8	1	11	1500
9	5	13,29	1040
10	2	-	2250
11	3	21	5000
12	3	-	18000
13	3	-	2818
14	4	14,30	2239
15	4	22	4400

Table 4.5.2: Demand information for each sample hurricane, adapted from [53].

4.5.2 Case Study Results

The optimal pre-positioning strategy for the network shown in Figure 4.5.1, the cost data shown in Table 4.5.1, and the 51 demand scenarios derived in [53] was determined using the stochastic programming model presented in Section 4.4.1 and the stochastic programming solver of the commercial software package What's Best™. In addition, the PDSA was applied to the pre-positioning problem and compared to the optimal solution. For illustrative purposes, it was assumed that the location of the manufacturing facility was Birmingham Alabama.

Manufacturer Location in Birmingham, Alabama

Within the 51 scenarios, demand for water was reported at 26 of the 30 nodes shown in Figure 4.5.1. The optimal strategy pre-positioned inventories at all but one of these nodes (Brownsville, Texas). This is likely explained by the large distance between Birmingham and Brownsville relative to the other cities in the network. In addition, the expected demand for water in Brownsville across all scenarios is comparatively small. On the other hand, the

PDSA pre-positioned items at every location. The exorbitant shortage cost caused the PDSA to classify all of the locations as Case 2, meaning all of the locations will hold inventory.

In some cases, pre-positioned quantities were sufficient in terms of satisfying storm demands (more detailed results related to service level are discussed in Section 4.5.2) when the optimal solution was applied. However, most scenarios required post-storm transshipments. For example, the scenario that represents Hurricane Emily created demand for water in Wilmington NC that exceeded the pre-positioned quantity. Consequently, transshipments from Charlotte NC, Charleston SC, and Columbia SC were necessary to fulfill the unmet demand. As another example, consider Hurricane Fran which generated a demand surge for water in North Carolina (Charlotte and Wilmington), South Carolina (Charleston and Columbia), and Savannah GA. For this scenario, shortages materialized in Charleston and Savannah based on the optimal pre-positioning strategy. Transshipments from Wilmington NC and Orlando FL were used to replenish inventory in Charleston while the unfulfilled demand in Savannah was replenished with a transshipment from Orlando. Similar results were found when the PDSA was applied to the problem, but about 1,000 less items were transshipped considering all 51 scenarios. This can be explained by the larger quantity of total pre-positioned items computed by the PDSA relative to the optimal solution. The PDSA focuses on the holding and shortage costs without regard to the cost of transportation, which most likely explains the difference between the amount pre-positioned and transshipped. In Section 4.5.3, the PDSA solution is compared to the optimal solution based on expected costs.

Expected Benefit of Pre-positioning

The benefit of pre-positioning relative to the reactive wait-and-see approach currently used in practice was quite dramatic. The expected cost of the reactive policy with no pre-positioning was $28.0 million and the expected cost of the pre-positioning optimal strategy was $16.5 million while the expected cost of the PDSA solution was $17.1 million. Thus the corresponding expected benefit was approximately \$11.5 million and \$10.9 million, which was the average benefit of the optimal pre-positioning and PDSA strategies over all 51 scenarios. However, the wait-and-see approach was actually more cost effective for about half of the scenarios considering the optimal solution. The scenarios that seemed conducive to the wait-and-see approach were characterized by a small number of retailers with nonzero demand. At the other extreme, the cost increase for not pre-positioning when a major storm impacts many large cities was astronomical, especially for the scenarios that represent two landfall hurricanes within a short period of time. In some cases, the benefit was more than \$50 million. When the PDSA approach is compared to the wait-and-see approach, it is more cost effective for approximately 88% of the 51 scenarios and the associated probabilities. This can be attributed to lower shortage costs incurred by the PDSA approach. In comparison to the optimal solution, the maximum benefit was \$30 million less (\$20 million). The decrease in expected benefit is associated with the additional holding costs incurred by the PDSA. In addition, smaller quantities were pre-positioned at high impact retailers, in particular, Charleston and Columbia. Since the expected benefits of pre-positioning for both strategies are positive for the case data, the pre-positioning strategy is recommended in lieu of the wait-and-see strategy if the manufacturer is located in Birmingham AL.

Other Manufacturer Locations

The expected benefit of pre-positioning also turned out to be positive if the manufacturer was situated in a city other than Birmingham. This claim is based on solving nine variations of the case problem in which the manufacturer location was a different city for each problem instance.

The PDSA approach pre-positioned the same amount of items regardless of manufacturer location because shortage and holding costs are only considered to determine the amounts, so it will not be discussed in this section. However, it should be mentioned that shortages were met by overages at other retailers first, and then, the manufacturer. This finding was consistent with all manufacturer locations.

Pre-positioning and transshipment policies were quite different depending on the manufacturer's location in the network for the optimal solution. For example, a manufacturer located in San Antonio TX, which is far removed from the majority of the other nodes in the network, would pre-position small quantities. Consequently, the majority of shortages replenished during the second stage would be direct shipments from the manufacturer as opposed to transshipments from other retailers. At the other extreme, a manufacturer located in Tallahassee FL would pre-position larger quantities with few post-storm direct shipments. These somewhat counterintuitive findings can be explained as follows. If the manufacturer is in close proximity to high impact cities, then the definitive upfront cost of pre-positioning is low in comparison to the expected shortage and redistribution costs. On the other hand, a manufacturer far removed from high impact cities would be subjected to a prohibitive and certain pre-positioning expense compared to an excessive but uncertain shortage and direct shipment expense. In summary, the location of the manufacturer does not affect the decision of whether or not a pre-positioning strategy should be used, but it does affect decisions related to pre-storm pre-positioning quantities and post-storm transshipment policies.

Service Level

In addition to expected costs and benefits, service level is also an important performance metric for inventory systems, particularly within the context of disaster relief activities. In this paper, *service level* refers to the percentage of demand that is filled on time from pre-positioned quantities. Shortages replenished during post-storm transshipments from other retailers or direct shipments from the manufacturer are considered late, and consequently incur shortage penalty costs. This section reports the service level performance associated with our numerical experimentation.

The optimal pre-positioning strategies associated with 10 variations of the case problem, each of which corresponds to a distinct manufacturer location, induced service levels of at least 80%. Specifically, for any given scenario, less than 20% of retailers required post storm replenishment even when other manufacturer locations were considered. This result can most likely be attributed to the exorbitant shortage costs shown in Table 4.5.1. The PDSA solution approach produced a slightly higher service level of 83% compared to the optimal solution. However, at most 35% of locations required post storm shipments for a given scenario, which can be explained by the smaller pre-positioned amounts at high demand retailer locations in Charleston and Columbia.

The proposed pre-positioning strategy also represents an improvement in service level compared to the wait-and-see strategy. This observation is a direct consequence of interpreting retailer demands as the *additional* inventory requests due to the presence of a threatening storm (see the beginning of Section 4.4.3). From this perspective, the wait-and-see service level is 0%. Therefore, any pre-positioning effort would be an improvement.

4.5.3 PDSA Results

On average, PDSA (Section 4.4.2) produced results that were within 3.6% of the optimal solution as shown in Table 4.5.3. The maximum deviation was less than 5% (Charleston SC) and the minimum was just under 3% (Mobile AL). The performance of PDSA appears to be influenced by the location of the manufacturer relative to high impact retailers. In particular, smaller deviations from optimality seemed to occur whenever the manufacturer was situated in close proximity to high impact retailers (e.g., manufacturer in Mobile AL). On the other hand, differences between optimal and PDSA solutions were slightly more pronounced for larger distances between the manufacturer and high impact retailers (e.g., manufacturer in Charleston SC). Deviations between the PDSA and optimal solutions were most likely due to the fact that the first stage of PDSA does not account for transportation costs. Thus, the performance of PDSA will deteriorate as the manufacturer's distance from other retailers increases. Computation times for PDSA were essentially negligible. The methodology for determining PDSA pre-positioning quantities outlined in Section 4.4.2 was instantaneous using a spreadsheet. Similarly, the resulting transportation problems associated with post-storm redistribution were solved instantaneously using Excel Solver™. Computation times for obtaining optimal solutions are discussed in the next section.

Manufacturer Location	**% Increase in Expected Cost**
Atlanta	3.71%
Baton Rouge	3.28%
Birmingham	3.31%
Charleston	4.71%
Jackson	3.20%
Little Rock	3.41%
Mobile	2.97%
Nashville	3.62%
San Antonio	4.39%
Tallahassee	3.53%
AVERAGE	3.61%

Table 4.5.3: PDSA performance relative to the optimal solution for each problem instance.

4.5.4 Sensitivity Analysis

This section further investigates Question 2 (see Section 4.2) by examining the effects that various cost parameters have on the expected benefit of pre-positioning determined by the optimal solution and the PDSA. The solution approaches were also compared in terms of the expected benefit with respect to the wait-and-see strategy.

Regarding the optimal solution, the commercial software What's Best™ generated globally optimal solutions within 2 to 15 minutes for a majority of the computational experiments. However, for some problem instances in which shortage cost was varied, the solver was manually interrupted before finding an optimal solution after several hours of computation, sometimes as many as 30 hours. In these cases, What's Best™ managed to find a feasible solution, which we used as a proxy for the optimal solution in our sensitivity analysis experiments. On the other hand, the PDSA was able to provide a solution for the instances that resulted in a prolonged computation time. All results presented in this section assume that the manufacturer location is Birmingham, Alabama.

The cost parameters that were reflected in the sensitivity analysis include (i) the unit cost of production, (ii) the post-storm unit cost of transportation, and (iii) the unit cost for inventory shortages (the notations for these parameters are c, c_w, and s, respectively). Two of the cost parameters actually had no effect on EC_{WS}. In particular, EC_{WS} was not influenced by changes in the pre-storm transportation cost c_v since there is no pre-storm transportation process associated with the wait-and-see approach. Additionally, the wait-and-see strategy does not produce any excess pre-positioned inventory after landfall, which suggests that h also does not affect EC_{WS}. Therefore, c_v and h will not be included in the sensitivity analysis experiments. The forthcoming analyses will examine the following measures with respect to changes in c, c_w, and s: (i) the expected cost of the optimal pre-positioning solution, EC_{PP}; (ii) the expected cost of the wait-and-see strategy, EC_{WS}; (iii) the expected cost of the PDSA solution, EC_{PD}; (iv) the expected benefit of pre-positioning, E(Benefit), for both solution approaches; and (v) the percentage increase in expected cost associated with the wait-and-see solution relative to the optimal pre-positioning and PDSA solutions.

Observation 1 E(Benefit) *decreases as* c *increases.*

Table 4.5.4 and Figure 4.5.2 illustrate the effects of increasing production costs. In Figure 4.5.2, EC_{WS} and $\widehat{EC}_{PP}$ increase linearly while EC^*_{PP} appears to be a concave function. However, as c increases, the slopes of EC^*_{PP} and $\widehat{EC}_{PP}$ are greater than the slope of EC_{WS}, which results in a decrease in E(Benefit) for both strategies. Table 4.5.4 also shows that the expected benefit of pre-positioning for the optimal solution (E^*_{PP}(Benefit)) is extraordinarily significant for smaller production costs: over \$13 million if the nominal production cost is cut in half and over \$15 million if production is free, which represent 94% and 148% increases, respectively. The PDSA produces an expected benefit of \$12.5 million if the production cost is cut in half and \$14 million if the production is free, representing an increase of 85% and

116%, respectively. It should also be noted that as c increases, the performance of the PDSA improves until c is equal to the hypothetical production value in the Case Study.

The findings associated with Observation 1 are a result of the fact that the manufacturer's total production quantity associated with pre-positioning is usually larger than the production quantity associated with wait-and-see. In particular, the production quantity for pre-positioning is determined based on probabilistic demand information, which often leads to the production of additional inventory for the purpose of avoiding expensive shortage costs. On the other hand, the wait-and-see production quantity corresponds to exact demand requirements and zero safety stock. Thus, as c increases, EC^*_{PP} and $\widehat{EC}_{PP}$ increase faster than EC_{WS} because of the corresponding production quantities.

c	E^*_{PP}(Benefit)	$\widehat{E}_{PP}$(Benefit)	Diff in E(Benefit)	% Cost Inc. Opt.	% Cost Inc. PDSA
\$0.00	\$15,962,733	\$14,345,336	\$1,617,397	148.31%	115.87%
\$161.93	\$13,254,631	\$12,595,650	\$658,981	93.92%	85.27%
\$323.85	\$11,438,729	\$10,890,449	\$548,280	69.04%	63.62%
\$485.78	\$9,941,756	\$9,190,852	\$750,904	53.15%	47.24%
\$647.70	\$8,713,214	\$7,491,403	\$1,221,812	42.35%	34.37%

Table 4.5.4: The effect of the unit production cost, c.

Observation 2 E(Benefit) *increases as* c_w *increases.*

The benefit of pre-positioning increases as post-storm transportation costs increase. In this case, it becomes more cost effective to avoid post-storm transportation as c_w increases, which can be accomplished by more aggressive pre-positioning. Table 4.5.5 and Figure 4.5.3 also show that the effect of c_w is not as dramatic as the effect of c. In particular, E(Benefit) and the corresponding percentage cost increases do not fluctuate as much for changing values of c_w as they do for changing values of c. When the optimal solution and PDSA solution are compared, it is evident that the PDSA performance remains consistent in relation to the optimal solution. The PDSA is based solely on the expected unit shortage and expected unit holding costs, so its performance should not be greatly affected by changes in c_w.

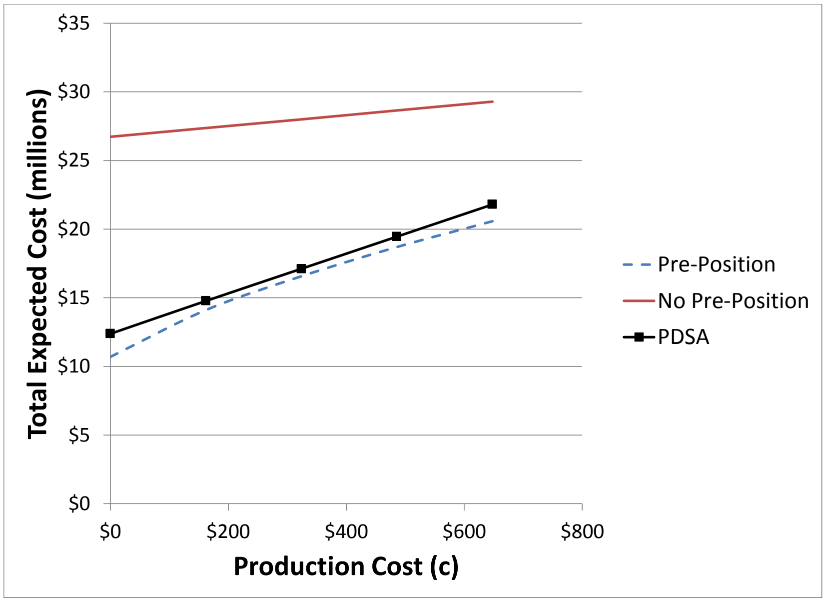


Figure 4.5.2: The effect of the unit production cost, c.

c_w	E^*_{PP}(Benefit)	$\widehat{E}_{PP}$(Benefit)	Diff in E(Benefit)	% Cost Inc. Opt.	% Cost Inc. PDSA
\$0.30	\$11,031,237	\$10,398,386	\$632,850	67.15%	60.95%
\$0.45	\$11,234,743	\$10,604,436	\$630,306	68.10%	61.91%
\$0.60	\$11,438,729	\$10,810,487	\$628,242	69.04%	62.86%
\$0.75	\$11,643,005	\$11,016,539	\$626,466	69.97%	63.81%
\$0.90	\$11,848,971	\$11,222,592	\$626,379	70.92%	64.74%
\$1.05	\$12,059,257	\$11,428,636	\$630,621	71.90%	65.67%

Table 4.5.5: The effect of the post-hurricane transportation cost, c_w.

Observation 3 E^*_{PP}(Benefit) *increases as s increases, and is most sensitive to s.*

Pre-positioning reduces the risk of shortages relative to wait-and-see. In fact, all demands incur shortage penalty costs based on the wait-and-see policy. In Figure 4.5.4, it can be observed that EC_{WS} increases linearly as s increases and that EC^*_{PP} increases as s increases. However, the rates of change in the increase of EC^*_{PP} are decreasing (i.e., EC^*_{PP} appears concave in s) so that the distance between EC_{WS} and the optimal pre-positioning strategy increases in s.

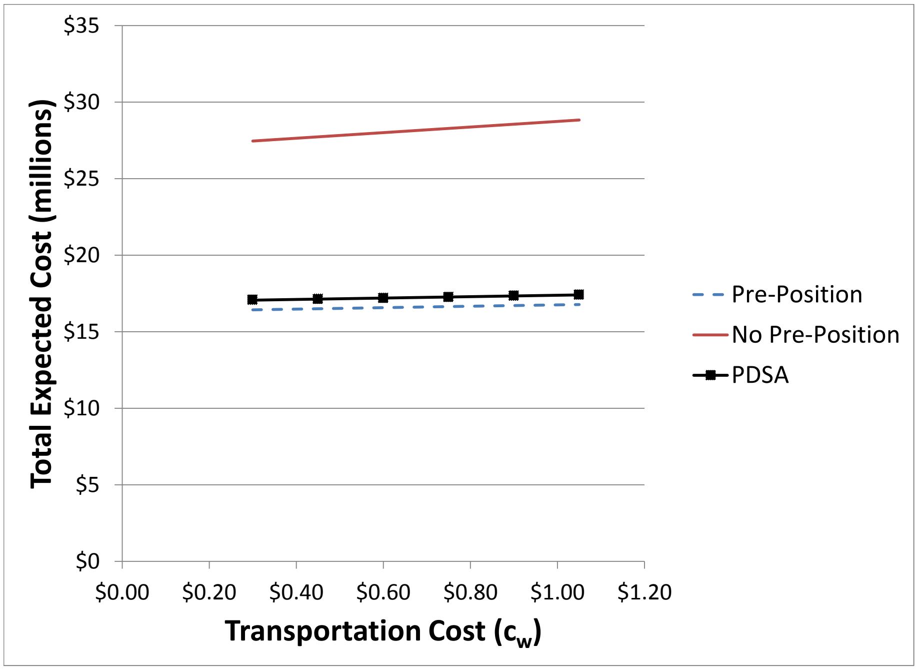


Figure 4.5.3: The effect of the post-hurricane transportation cost, c_w.

Table 4.5.6 and Figure 4.5.4 also show that the benefit of pre-positioning is insignificant for small values of s relative to the other values of s and the other cost parameters. In particular, the smallest value of s in Table 4.5.6 represents the minimum expected benefit for the optimal strategy (just over \$1.2 million) and corresponding cost increase (14.39%) for all cost parameters and problem instances considered in the case study. On the other hand, E(Benefit) and the percentage cost increase rapidly as s increases, which is likely a result of extremely large shortage cost values relative to the other cost parameters.

Observation 4 *The performance of PDSA deteriorates as s decreases.*

When the PDSA is compared to the wait-and-see approach, the $\widehat{E}_{PP}(Benefit)$ is negative for small values of the shortage cost as shown in Table 4.5.6. The large amount of pre-positioned items cause expensive holding costs resulting in $\widehat{EC}_{PP} > EC_{WS}$ for these values. However, when the shortage cost is increased, the PDSA provides a solution relatively close to the optimal as shown in Table 4.5.6. It should be noted that the optimal solution should be employed when the value of s is similar to the purchase price. The PDSA provides a negative expected benefit for these smaller values of s leading the decision maker to choose the wait-and-see approach, which is contradictory to the optimal solution.

As mentioned at the beginning of this section, What's Best™ was unable to obtain globally optimal solutions for sensitivity analysis experiments that involved varying the shortage cost, s. These problem instances resulted in prohibitive computation times of up to 30 hours for reasons that are not apparent. For these problem instances, the solver was manually interrupted and a feasible solution was used for the purposes of this analysis. Specifically, a feasible solution was used as a proxy for the optimal solution in Table 4.5.6 and Figure 4.5.4 whenever s was 5 times c or less. If the actual optimal values for EC^*_{PP} were available, then a more dramatic increase in E^*_{PP}(Benefit) and departure of the curves shown in Figure 4.5.4 would be observed. On the other hand, the PDSA was able to provide solutions for these problem instances, and it should be noted that in practice, this is the advantage of using PDSA instead of commercial software.

s	E^*_{PP}(Benefit)	$\widehat{E}_{PP}$(Benefit)	Diff in E(Benefit)	% Cost Inc. Opt.	% Cost Inc. PDSA
\$323.85	\$73,069	-\$5,854,090	\$5,927,159	2.04%	-61.52%
\$647.70	\$208,743	-\$4,977,007	\$5,185,750	4.41%	-50.17%
\$1,295.40	\$656,918	-\$3,222,841	\$3,879,759	9.59%	-30.04%
\$1,943.10	\$1,266,876	-\$1,468,674	\$2,735,551	14.39%	-12.73%
\$2,590.80	\$2,052,275	\$285,492	\$1,766,784	19.40%	2.31%
\$3,238.50	\$3,309,799	\$2,039,658	\$1,270,141	27.85%	15.51%
\$3,886.20	\$4,723,495	\$3,793,824	\$929,671	36.24%	27.17%
\$4,533.90	\$6,325,856	\$5,547,990	\$777,866	45.21%	37.56%
\$5,181.60	\$7,947,087	\$7,302,156	\$644,931	53.21%	46.87%
\$5,829.30	\$9,669,318	\$9,056,322	\$612,996	61.29%	55.26%
\$6,477.00	\$11,438,731	\$10,810,488	\$628,242	69.04%	62.86%

Table 4.5.6: The effect of the unit shortage cost, s.

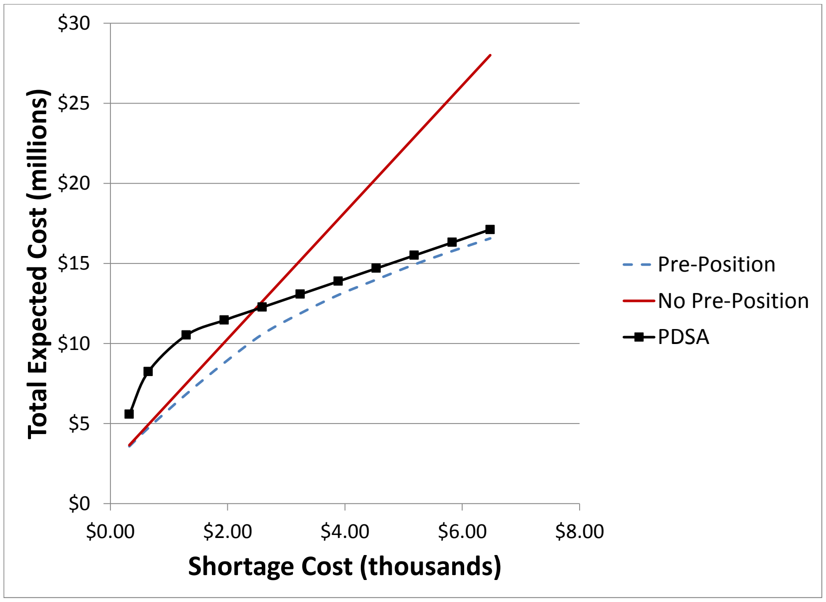


Figure 4.5.4: The effect of the unit shortage cost, s.

Observation 5 *For all of the problem instances solved optimally, the expected benefit of pre-positioning relative to the wait-and-see approach was quite significant.*

For the optimal solution, the smallest expected benefit and corresponding percentage cost increase were \$73,069 and 2.04%, respectively, which corresponds to the problem instance where the shortage cost and production cost were equal. On the other hand, the largest benefit and percentage increase (excluding the case when production was free) were \$13.25 million and 93.92%, respectively. The largest benefit occurred for the problem instance with the smallest production cost, c. The PDSA produced a negative benefit when the shortage cost was small, but it performed well for all other problem instances with the largest benefit, \$12.5 million, occurring when the production cost was half of the tested production cost. Overall, this study shows that the optimal pre-positioning always outperforms the wait-and-see approach currently used in practice whenever probabilistic information regarding landfall of an observed hurricane is available.

4.6 Summary and Future Directions

This paper is motivated by the problem that emerges when a commercial manufacturer responds to orders for emergency supply items placed by retailers situated in the path of an observed hurricane or tropical storm. In practice, retailers often postpone the decision to initiate emergency orders from manufacturers until the demand requirements associated with pre-storm demand surge or post-storm disaster relief activities are known with certainty. A typical consequence of this reactive "wait-and-see" policy is widespread stockout of emergency supplies and equipment, which compromises the effectiveness of disaster relief operations and proliferates lost sales for the commercial supply chain. In an effort to encourage higher service levels from the private sector in support of disaster relief efforts, this paper proposes a proactive inventory pre-positioning strategy from the perspective of the manufacturer. The proposed pre-positioning strategy is characterized by a single manufacturer that pushes inventory across a network of retailers prior to the landfall of an observed storm, and assumes responsibility for the resulting costs and risks similar to vendor managed inventory systems. The manufacturer's pre-positioning problem is represented as a two-stage stochastic programming model. During the first stage, supplies are distributed from the manufacturer to selected retailer locations within the supply chain network. The corresponding second stage is characterized by transshipment of inventories among retailers, including possible direct shipments from the manufacturer, after demand information becomes known with certainty. To illustrate the model, the case study introduced in reference [53] which is derived from historical hurricane data, was solved. Solutions were generated using the stochastic programming capability of the commercial software program, What's Best™. In addition, a heuristic solution methodology (PDSA) was introduced for the purpose of reducing the computational effort associated with solving large-scale problems. The proposed approach produced solutions within 3% - 5% of optimality and was executed with negligible computation time in a spreadsheet. Computational results and sensitivity analysis based on the case study suggest that the optimal pre-positioning strategy is always more cost effective than the wait-and-see approach often used in practice whenever the landfall locations and demand requirements associated with an observed storm are uncertain.

The stochastic programming model presented in this study can be extended in several ways to reflect additional issues that may arise when commercial manufacturers attempt pre-positioning strategies in practice. In fact, some of these issues have been addressed in the pre-positioning literature based on the perspectives of government and non-government organizations and humanitarian relief supply chains. For instance, network reliability in which pre-positioned inventories and roadways are partially or completely destroyed could be considered as in [53], [12], and [6]. Also, it would be useful from a practical standpoint to expand the model to include multiple product types as in [14], [38], [53], [5], and [6].

There are also extensions which have not been considered in the literature that would be particularly relevant to the commercial pre-positioning problem introduced in this study. Before describing these extensions, first recall that one of the unique features of the pre-positioning model developed in Section 4.4.1 is that it represents the manufacturer's perspective and cost structure. However, a limitation of the model from a production standpoint is that infinite production capacity is assumed and that only the variable cost of production is considered. It would therefore be of interest to expand the model to include more details regarding the manufacturer's production process and cost measures. For example, a finite production rate could be incorporated into the model as opposed to assuming that production is instantaneous. Similarly, the model could also be extended such that production rate is a decision variable and the corresponding production cost is a function of the selected rate. Another useful extension would be a multiple product model as previously mentioned. This would introduce opportunities for scheduling production to reduce the costs associated with changeovers. Differentiating pre- and post-storm production costs might also be relevant to the proposed pre-positioning strategy in practice. In a post-storm environment, constrained resources may lead to overtime labor and other inflated costs pertaining to production. In terms of distribution, only the variable cost of transportation is considered. Therefore, it would be interesting to examine the effect of fixed transportation costs in a future effort.

The model introduced in this paper characterizes the role of the manufacturer in a short-term pre-positioning environment. This basic model, along with the above-mentioned extensions related to the manufacturer's problem within the context of pre-positioning, represents a promising new direction for disaster relief research.

Chapter 5

Summary

In closing, the proposed research exhibits the potential to provide meaningful strategies that can be applied to the private sector with an emphasis on the retail industry when faced with an uncertain disruption such as a hurricane. This dissertation stresses the importance of considering the effects of disasters when retailers make decisions regarding how they will approach inventory management for emergency supplies. The private sector can benefit from employing the strategies discussed in this dissertation by providing better customer service and minimizing costs. Additionally, the strategies presented can assist residents by ensuring that they are able to obtain the necessary supplies before and after the storm.

5.1 Minimax EOQ Policies with Demand Surge, Lead-Time and Lost Sales

The model presented in Chapter 2 provides a solution for how retailers can determine an ordering strategy when faced with an uncertain demand surge or demand disruption. Retailers may select a reactive strategy meaning they will maintain their order quantity until it is certain that the surge will occur, or they may select a proactive strategy, which means they will modify the order quantity before it is certain that the surge will occur. A closed form solution was found for the optimal order quantity during the disruption. Minimax decision criterion was used to determine the best ordering strategy considering a 2^8 experimental design. In the numerical experiment, most of the tested cases constituted selecting the proactive strategy, which is contrary to the approach employed by most retailers when faced with a forecasted storm. On the other hand, minimax decision criterion supported the reactive strategy despite lost sales when lead-time was short. The sensitivity analysis revealed that the lead-time has a greater effect on the strategy selection than the lost sales cost. All of these findings illustrate that the retailer must strongly consider the lead-time and the objectives (i.e., minimizing maximum costs) when selecting an inventory ordering strategy for handling uncertain disruptions.

5.2 Minimax and Minimax Regret Inventory Control Policies Regarding Demand Disruptions and Damaged Inventory

The model presented in Chapter 3 is an extension of the model presented in Chapter 2 by considering a different time horizon with two disruptions and two sources of uncertainty. The first disruption represents a possible period of no demand caused by a storm while the second disruption symbolizes the surge in demand for emergency supplies after the storm. The first source of uncertainty is whether the disruption, a period of no demand, will occur while the second source is whether the inventory will be damaged if the first disruption occurs. Minimax and minimax regret decision criteria were applied to the model to determine the quantity that the retailer should hold during the first and second disruptions. Minimax decision criterion did not support holding inventory during the first disruption while minimax regret advocated holding inventory during 55% of the tested cases. However, the order quantity for the second disruption was the same for both decision criteria. All in all, the retailer should consider the decision criterion that best suits the cost objectives when selecting a strategy for the time horizon during and after the storm.

5.3 Pre-Positioning Hurricane Supplies in a Commercial Supply Chain

Pre-positioning has not been considered in the literature from a profit-driven perspective, and Chapter 4 provides a model that minimizes the costs of the manufacturer when a hurricane has been observed. A stochastic programming model was developed to determine the quantity of emergency supplies that should be pre-positioned at each retailer as well as the necessary transshipments and direct shipments from the manufacturer after the storm to satisfy shortages. In addition, a solution methodology termed PDSA reduces the stochastic programming model to the transportation problem by employing a heuristic to solve the first stage decisions (i.e., pre-positioned quantities). The commercial software package What's Best™ was utilized to solve the stochastic programming model, and the optimal solution revealed that it was best to pre-position supplies at most retailers regardless of the manufacturer's location. The manufacturer's location did affect the amount pre-positioned at each retailer meaning the retailers located closest to the manufacturer generally held larger amounts of pre-positioned quantities. In most cases, at least 80% of the retailers' demand for emergency items was met by the pre-positioned amounts illustrating that the retailers did not rely solely on post-storm shipments and the pre-positioning strategy is more cost effective considering the numerical example and case study data from ([53]). The sensitivity analysis indicated that the lost sales cost had the greatest effect on the decision to pre-position. In addition, the PDSA methodology should not be applied to the problem if the shortage costs are small in relation to the other parameters. Overall, the pre-positioning strategy is more cost effective than the wait-and-see approach currently used in practice when the expected cost is considered.

Bibliography

[1] Federal Emergency Management Agency. Hurricane preparedness for people with disabilities, 2006. http://www.fema.gov/news/newsrelease.fema?id=24487.

[2] N. Agin. A min-max inventory model. *Management Science*, 12(7):517–529, 1966.

[3] Othman Ibraheem Alsalloum and Graham K. Rand. Extensions to emergency vehicle location models. *Computers & Operations Research*, 33(9):2725 – 2743, 2006.

[4] B. Balçik, B.M. Beamon, and K. Smilowitz. Last mile distribution in humanitarian relief. *Journal of Intelligent Transportation Systems*, 12(2):51 – 63, 2008.

[5] Burcu Balçik and Benita M. Beamon. Facility location in humanitarian relief. *International Journal of Logistics: Research and Applications*, 11(2):101 – 121, 2008.

[6] G. Barbarosoğlu and Y. Arda. A two-stage stochastic programming framework for transportation planning in disaster response. *Journal of the Operational Research Society*, 55(1):43 – 53, 2004.

[7] Benita M. Beamon and Stephen A. Kotleba. Inventory modelling for complex emergencies in humanitarian relief operations. *International Journal of Logistics: Research and Applications*, 9(1):1–18, 2006.

[8] John E. Bell. *A simulated annealing approach for the composite facility location and resource allocation problem: a study of strategic positioning of US Air Force munitions.* PhD thesis, Auburn University, 2003.

[9] E. Berk and A. Arreola-Risa. A note on "future uncertainty in EOQ models". *Naval Research Logistics*, 41(1):129–132, 1994.

[10] John R. Birge and François Louveaux. *Introduction to Stochastic Programming.* Springer, New York, 1997.

[11] Percy H. Brill and Ben A. Chaouch. An EOQ model with random variations in demand. *Management Science*, 41(5):927–936, 1995.

[12] Ann Melissa Campbell and Philip C. Jones. Prepositioning supplies in preparation for disasters. *European Journal of Operational Research*, 209:156–165, 2011.

[13] S. Çetinkaya and C-Y. Lee. Stock replenishment and shipment scheduling for vendor-managed inventory systems. *Management Science*, 46(2):217 – 232, 2000.

[14] M-S. Chang, Y-L. Tseng, and J-W. Chen. A scenario planning approach for the flood emergency logistics preparation problem under uncertainty. *Transportation Research Part E: Logistics and Transportation Review*, 43(6):737 – 754, 2007.

[15] Department of the Air Force. Compendium of authenticated systems and logistics terms, definitions, and acronyms, 1981.

[16] Jr. E.J. Lodree. Pre-storm emergency supplies inventory planning. *Journal of Humanitarian Logistics and Supply Chain Management*, 1(1):50–77, 2011.

[17] M.J. Fry, R. Kapuscinski, and T.L. Olsen. Coordinating production and delivery under a (z,z)-type vendor-managed inventory contract. *MSOM*, 3(2):151–173, 2001.

[18] G. Gallego. A minimax distribution free procedure for the (Q,R) inventory. *Operations Research Letters*, 11(1):55–60, 1992.

[19] G. Gallego and I. Moon. The distribution free newsboy problem: Review and extensions. *Journal of the Operational Research Society*, 44(8):825–834, 1993.

[20] G. Gallego, J. K. Ryan, and D. Simchi-Levi. Minimax analysis for finite-horizon inventory models. *IIE Transactions*, 33(10):861–874, 2001.

[21] Axel Graumann, Tamara Houston, Jay Lawrimore, David Levinson, Neal Lott, Sam McCown, Scott Stephens, and David Wuertz. Hurricane Katrina: A climatological Perspective. Technical Report 2005-01, NOAA National Climatic Data Center, 2005.

[22] George D. Haddow and Jane A. Bullock. *Introduction to Emergency Management*. Elsevier, Oxford,UK, 2nd edition, 2006.

[23] D. I. Heimann and F. Waage. A closed-form approximation solution for an inventory model with supply disruptions and non-ZIO reorder policy. *Manufacturing & Service Operations Management*, 2005. Submitted for publication.

[24] Steven Horwitz. Making hurricane response more effective: Lessons from the private sector and the Coast Guard during Katrina. In *Mercatus Policy Series*. George Mason University Mercatus Center, 2008.

[25] Rongbing Huang, Seokjin Kim, and Mozart Menezes. Facility location for large-scale emergencies. *Annals of Operations Research*, 181:271–286, 2010.

[26] P. Hughes. The great Galveston hurricane. *Weatherwise*, 43(4):190–198, 1990.

[27] National Hurricane Survival Initiative. New poll shows coastal residents complacent after a quiet 2009 hurricane season, 2010. http://www.hurricanesafety.org/newpoll2010.shtml.

[28] R. Jagannathan. A minimax ordering policy for the infinite stage dynamic inventory problem. *Management Science*, 24(11):1138–1149, 1978.

[29] W. Jammernegg and P. Kischka. Risk preferences and robust inventory decisions. *International Journal of Production Economics*, 118(1):269–274, 2009.

[30] H. Jia, F. Ordóñez, and M. Dessouky. A modeling framework for facility location of medical services for large-scale emergencies. 39:41 – 55, 2007.

[31] D. P. Johnstone, R. R. Hill, and J. T. Moore. Mathematically modeling munitions prepositioning and movement. *Mathematical and Computer Modelling*, 39(6-8):759 – 772, 2004.

[32] H. Kasugi and T. Kasegai. Note on minimax regret ordering policy-static and dynamic solutions and a comparison to maximin policy. *Journal of Operational Research Society Japan*, 3:155–169, 1961.

[33] A.J. Kleywegt, V.S. Nori, and M.W.P. Savelsbergh. The stochastic inventory routing problem. *Transportation Science*, 36(1):94–118, 2002.

[34] Zhaolin Li, Susan H. Xu, and Jack Hayya. A periodic-review inventory system with supply interruptions. *Probability in the Engineering and Informational Sciences*, 18:33–53, 2004.

[35] Emmett J. Lodree and Selda Taskin. An insurance risk management framework for supply chain disruption and disaster relief inventory planning. *Journal of the Operational Research Society*, 59(5):674–684, 2008.

[36] Emmett J. Lodree and Selda Taskin. Supply chain planning for hurricane response with wind speed information updates. *Computers & Operations Research*, 36(1):2–15, 2009.

[37] N. Lott, T. Ross, T. Houston, and A. Smith. Billion dollar U.S. weather disasters, 1980 - 2010. Technical report, NOAA National Climatic Data Center, Asheville, NC, 2010.

[38] Huseyin Onur Mete and Zelda B. Zabinsky. Stochastic optimization of medical supply location and distribution in disaster management. *International Journal of Production Economics*, 126:76 – 84, 2010.

[39] R. R. Meyer, M. H. Rothkopf, and S. A. Smith. Reliability and inventory in a production-storage system. *Management Science*, 25(8):799–807, 1979.

[40] J.C. Millás. *Hurricanes of the Caribbean and adjacent regions, 1492-1800*. Academy of the Arts and Sciences of the Americas, Miami, FL, 1968.

[41] Esmail Mohebbi. Supply interruptions in a lost-sales inventory system with random lead-time. *Computers & Operations Research*, 30(3):411–426, 2003.

[42] K. Moinzadeh and P. Aggarwal. Analysis of a production/inventory system subject to random disruptions. *Management Science*, 43:1577–1588, 1997.

[43] W. T. Morris. Inventorying for unknown demand. *The Journal of Industrial Engineering*, 10(4):299–302, 1959.

[44] S. Nahmias. *Production and Operations Analysis*. McGraw-Hill, New York, NY, 6th edition, 2008.

[45] L. Özdamar, E. Ekinci, and B. Küçükyazici. Emergency logistics planning in natural disasters. *Annals of Operations Research*, 129:217 – 245, 2004.

[46] M. Parlar and D. Berkin. Future supply uncertainty in EOQ models. *Naval Research Logistics*, 38(1), 1991.

[47] M. Parlar, W. Wang, and Y. Gerchak. A periodic review inventory model with markovian availability. *International Journal of Production Economics*, 42:131–136, 1995.

[48] A. Patton. Quick reopening of supermarket served galveston citizens, Nov. 2008.

[49] G. Perakis and G. Roels. Regret in the newsvendor model with partial information. *Operations Research*, 56(1):188–203, 2008.

[50] L. Qi, Z-J. Shen, and L. V. Snyder. A continuous-review inventory model with disruptions at both supplier and retailer. *Production and Operations Management*, 18(5):516–532, 2009.

[51] Xiangtong Qi, Jonathan F. Bard, and Gang Yu. Supply chain coordination with demand disruptions. *Omega: International Journal of Management Science*, 32:301–312, 2004.

[52] Hari K. Rajagopalan, Cem Saydam, and Jing Xiao. A multiperiod set covering location model for dynamic redeployment of ambulances. *Computers & Operations Research*, 35(3):814 – 826, 2008.

[53] C.G. Rawls and M.A. Turnquist. Pre-positioning of emergency supplies for disaster response. *Transportation Research Part B: Methodological*, 44(4):521 – 534, 2010.

[54] Delia Richardson, Sander de Leeuw, and Iris Vis. Conceptualising inventory prepositioning in the humanitarian sector. In Luis Camarinha-Matos, Xavier Boucher, and Hamideh Afsarmanesh, editors, *Collaborative Networks for a Sustainable World*, volume 336 of *IFIP Advances in Information and Communication Technology*, pages 149–156. Springer, Boston, 2010.

[55] A. M. Ross, Y. Rong, and L. V. Snyder. Supply disruptions with time-dependent parameters. *Computers & Operations Research*, 35(11):3504–3529, 2008.

[56] Javier Salmerón and Aruna Apte. Stochastic optimization for natural disaster asset prepositioning. *Production and Operations Management*, 19(5):561 – 574, 2010.

[57] L. J. Savage. The theory of statistical decision. *Journal of the American Statistical Association*, 46(253):55–67, 1951.

[58] H.E. Scarf. A min-max solution to an inventory problem. In K.J. Arrow, S. Karlin, and H.E. Scarf, editors, *Studies in the Mathematical Theory of Inventory and Production*, pages 201–209. Stanford University Press, 1958.

[59] E. Smith. Emergency logistics management: Transforming the delivery of disaster relief for the 21st century, September 2010. Accessed: May 25, 2011.

[60] L. V. Snyder. A tight approximation for an EOQ model with supplier disruptions. Working paper. Lehigh University, Bethlehem, PA, 2008.

[61] Jing-Sheng Song and Paul Zipkin. Inventory control in a fluctuating demand environment. *Operations Research*, 41(2):351–370, 1993.

[62] Selda Taskin and Emmett J. Lodree. Inventory decisions for emergency supplies based on hurricane count predictions. *International Journal of Production Economics*, 126:66–75, 2010.

[63] Selda Taskin and Emmett J. Lodree. A Bayesian decision model with hurricane forecast updates for emergency supplies inventory management. *Journal of the Operational Research Society*, 62:1098–1108, 2011.

[64] G.L. Vairaktarakis. Robust multi-item newsboy models with a budget constraint. *International Journal of Production Economics*, 66(3):213–226, 2000.

[65] H. J. Weiss and E.C. Rosenthal. Optimal ordering policies when anticipating disruption in supply or demand. *European Journal of Operational Research*, 59:370–382, 1992.

[66] W.L. Winston. *Operations Research: Applications and Algorithms.* Duxbury Press, Ontario, Canada, 4th edition, 2004.

[67] Y. Yao, P.T. Evers, and M.E. Dresner. Supply chain integration in vendor-managed inventory. *Decision Support Systems*, 43(2):663–674, 2007.

[68] G. Yu. Robust economic order quantity models. *European Journal of Operational Research*, 100(3):482–493, 1997.

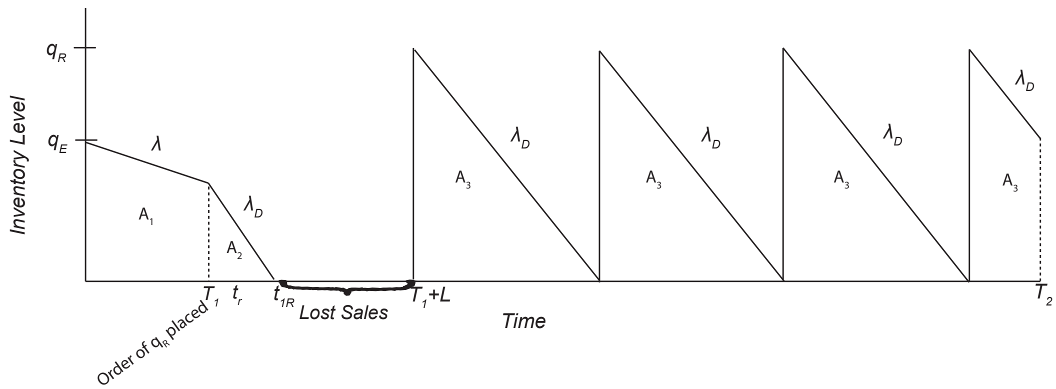
[69] J. Yue, B. Chen, and M.C. Wang. Expected value of distribution information for the newsvendor problem. *Operations Research*, 54(6):1128–1136, 2006.

Appendix A

Chapter 2

A.1 Derivation of Cost Equation

Figure A.1.1: Reactive Strategy with Disruption: Case 1 Appendix



The number of orders given in Chapter 2 are m_1 orders of q_E and m_2 orders of q_R, which can be found by dividing the total demand during the time period by the order quantity. The m_1 orders of q_E are found using A_1 and A_2 while the m_2 orders encompass the m_2 orders of q_R. The total number of orders is given in Eq. A.1.1.

$$m_1 + m_2 = \frac{\lambda T_1 + \lambda_D(t_{1R} - T_1)}{q_E} + \frac{\lambda_D(T_2 - T_1 - L)}{q_R} \tag{A.1.1}$$

The holding cost is determined by computing the average inventory level for A_1, A_2 and A_3. The average inventory level for A_1 and A_2 is found by Eq. (A.1.2) where the number of orders of q_E is multiplied by the area of A_1 and A_2.

$$\overline{I_1} = \left(\frac{\lambda T_1}{q_E}\right) * \left(\frac{{q_E}^2}{\lambda}\right) + \left(\frac{\lambda_D(t_{1R} - T_1)}{q_E}\right) * \left(\frac{{q_E}^2}{\lambda_D}\right) = \frac{q_E t_{1R}}{2} \tag{A.1.2}$$

The average inventory level is computed in Eq. (A.1.3), which entails multiplying m_2 by the area for A_3.

$$\overline{I_2} = m_2 * (\text{Area of } A_3) = \frac{\lambda_D(T_2 - T_1 - L)}{q_R} * \left(\frac{{q_R}^2}{\lambda_D}\right) = \frac{q_R(T_2 - T_1 - L)}{2} \tag{A.1.3}$$

The total number of lost sales is found by multiplying the demand rate by the time horizon when the inventory level is zero. In the case considered here, Eq. (A.1.4) gives the total number of lost sales (L).

$$L = \lambda_D(T_1 + L - t_{1R}) \tag{A.1.4}$$

The cost equation presented in Eq. (2.3.3) is the sum of Eqs. (A.1.1)-(A.1.4) with each part multiplied by its respective cost.

A.2 Reactive Strategy

Case 1

Consider the following assumptions: $T_2 \geq T_1 + L$, $T_1 \leq \frac{q_E}{\lambda} - L$, $L \geq \frac{q_E - \lambda T_1}{\lambda_D}$, and $s \geq \frac{q_R}{\lambda_D}$.

$$TC^1_{R_1}(q_E, q_R) - TC^0_R(q_E) \geq 0$$

$$A\left(\frac{\lambda T_1}{q_E} + \frac{\lambda_D(t_{1R} - T_1)}{q_E} + \frac{\lambda_D(T_2 - T_1 - L)}{q_R}\right) + h\left(\frac{q_E t_{1R}}{2} + \frac{q_R(T_2 - T_1 - L)}{2}\right)$$
$$+ s\lambda_D\left(T_1 + L - t_{1R}\right) - A\left(\frac{\lambda T_2}{q_E}\right) - h\left(\frac{T_2 q_E}{2}\right) \geq 0$$

Substitute $t_{1R} = \frac{q_E - \lambda T_1}{\lambda_D} + T_1$, $q_E = \sqrt{\frac{2A\lambda}{h}}$, $q_R = \sqrt{\frac{2A\lambda_D}{h}}$ and simplify.

$$A\left(1 + \frac{\lambda}{\lambda_D}\right) + \sqrt{2hA\lambda_D}(T_2 - T_1 - L) - T_2\sqrt{2hA\lambda} + T_1\sqrt{\frac{hA\lambda}{2}}\left(1 - \frac{\lambda}{\lambda_D}\right)$$
$$+ s\lambda_D\left(L - \frac{\sqrt{\frac{2A\lambda}{h}} - \lambda T_1}{\lambda_D}\right) \geq 0$$

Substitute $T_2 = T_1 + L$ and $L = \frac{\sqrt{\frac{2A\lambda}{h}} - \lambda T_1}{\lambda_D}$.

$$A\left(1 + \frac{\lambda}{\lambda_D}\right) - \sqrt{2hA\lambda}\left(T_1 + \frac{\sqrt{\frac{2A\lambda}{h}} - \lambda T_1}{\lambda_D}\right) + T_1\sqrt{\frac{hA\lambda}{2}}\left(1 - \frac{\lambda}{\lambda_D}\right) \geq 0$$

$$A\left(1 + \frac{\lambda}{\lambda_D}\right) - T_1\sqrt{2hA\lambda}\left(1 - \frac{\lambda}{\lambda_D}\right) - 2A\left(\frac{\lambda}{\lambda_D}\right) + T_1\sqrt{\frac{hA\lambda}{2}}\left(1 - \frac{\lambda}{\lambda_D}\right) \geq 0$$

$$A\left(1 - \frac{\lambda}{\lambda_D}\right) - T_1\sqrt{\frac{hA\lambda}{2}}\left(1 - \frac{\lambda}{\lambda_D}\right) \geq 0$$

$$A \geq T_1\sqrt{\frac{hA\lambda}{2}}$$

$$\sqrt{\frac{2A}{h\lambda}} \geq T_1$$

$$\frac{q_E}{\lambda} \geq T_1$$

We can say that $TC^1_{R_1}(q_E, q_R) - TC^0_R(q_E) \geq 0$ because $T_1 \leq \frac{q_E}{\lambda}$.

Case 2

Consider the following assumptions: $T_2 \geq t_{1R}$, $T_1 \leq \frac{q_E}{\lambda}$, and $t_{1R} = T_1 + \frac{q_E - \lambda T_1}{\lambda_D}$

$$TC^1_{R_2}(q_E, q_R) - TC^0_R(q_E) \geq 0$$

$$A\left(\frac{\lambda T_1}{q_E} + \frac{\lambda_D(t_{1R} - T_1)}{q_E} + \frac{\lambda_D(T_2 - t_{1R})}{q_R}\right) + h\left(\frac{q_E t_{1R}}{2} + \frac{q_R(T_2 - t_{1R})}{2}\right)$$
$$-A\left(\frac{\lambda T_2}{q_E}\right) - h\left(\frac{T_2 q_E}{2}\right) \geq 0$$

Substitute $t_{1R} = \frac{q_E - \lambda T_1}{\lambda_D} + T_1$, $q_E = \sqrt{\frac{2A\lambda}{h}}$, $q_R = \sqrt{\frac{2A\lambda_D}{h}}$ and simplify.

$$A\left(1+\frac{\lambda}{\lambda_D}-2\sqrt{\frac{\lambda}{\lambda_D}}\right)+T_2\sqrt{2hA\lambda_D}+T_1\sqrt{\frac{hA\lambda}{h}}\left(1-\frac{\lambda}{\lambda_D}\right)-T_1\sqrt{2hA\lambda_D}\left(1-\frac{\lambda}{\lambda_D}\right)\geq 0$$

Substitute $T_2 = T_1 + \frac{q_E - \lambda T_1}{\lambda_D}$ and $q_E = \sqrt{\frac{2A\lambda}{h}}$.

$$\begin{aligned}
&A\left(1+\frac{\lambda}{\lambda_D}-2\sqrt{\frac{\lambda}{\lambda_D}}\right)+T_1\sqrt{2hA\lambda_D}\left(1-\frac{\lambda}{\lambda_D}\right)-T_1\sqrt{2hA\lambda}\left(1-\frac{\lambda}{\lambda_D}\right)\\
&+2A\left(\sqrt{\frac{\lambda}{\lambda_D}}\right)-2A\frac{\lambda}{\lambda_D}-T_1\sqrt{2hA\lambda_D}\left(1-\frac{\lambda}{\lambda_D}\right)+T_1\sqrt{2hA\lambda}\left(1-\frac{\lambda}{\lambda_D}\right)\geq 0
\end{aligned}$$

$$A\left(1-\frac{\lambda}{\lambda_D}\right)-T_1\left(\sqrt{2hA\lambda}-\sqrt{\frac{hA\lambda}{2}}\right)\left(1-\frac{\lambda}{\lambda_D}\right)\geq 0$$

Substitute $T_1 = \frac{q_E}{\lambda}$ with $q_E = \sqrt{\frac{2A\lambda}{h}}$.

$$\begin{aligned}
&A\left(1-\frac{\lambda}{\lambda_D}\right)-\sqrt{\frac{2A}{h\lambda}}\left(\sqrt{2hA\lambda}-\sqrt{\frac{hA\lambda}{2}}\right)\left(1-\frac{\lambda}{\lambda_D}\right)\geq 0\\
&A\left(1-\frac{\lambda}{\lambda_D}\right)-A\left(1-\frac{\lambda}{\lambda_D}\right)=0
\end{aligned}$$

Since we substituted the smallest value for T_2 and largest value for T_1, the inequality should be ≥ 0 thus proving that $TC^1_{R_2}(q_E, q_R) - TC^0_R(q_E) \geq 0$.

Case 3

Consider the following assumptions: $T_1 + L \geq \frac{q_E}{\lambda_D} + \frac{q_E}{\lambda}$, $T_2 \geq T_1 + L$.

$$TC^1_{R_3}(q_E, q_R) - TC^0_R(q_E) \geq 0$$

$$A\left(\frac{\lambda T_1}{q_E}+\frac{\lambda_D(t_{1R}-T_1)}{q_E}+1+\frac{\lambda_D(T_2-T_1-L)}{q_R}\right)+h\left(\frac{q_E t_{1R}}{2}+\frac{{q_E}^2}{2\lambda_D}+\frac{q_R(T_2-T_1-L)}{2}\right)$$
$$+s\lambda_D\left(T_1+L-\frac{q_E}{\lambda_D}-t_{1R}\right)-A\left(\frac{\lambda T_2}{q_E}\right)-h\left(\frac{T_2 q_E}{2}\right)\geq 0$$

Substitute $t_{1R}=\frac{q_E-\lambda T_1}{\lambda_D}+T_1$, $q_E=\sqrt{\frac{2A\lambda}{h}}$, $q_R=\sqrt{\frac{2A\lambda_D}{h}}$ and simplify.

$$2A\left(1+\frac{\lambda}{\lambda_D}\right)+\sqrt{2hA\lambda_D}(T_2-T_1-L)-T_2\sqrt{2hA\lambda}+T_1\sqrt{\frac{hA\lambda}{2}}\left(1-\frac{\lambda}{\lambda_D}\right)$$
$$+s\lambda_D\left(L-\frac{2q_E-\lambda T_1}{\lambda_D}\right)\geq 0$$

Substitute $T_2=T_1+L$, $T_1=\frac{q_E}{\lambda}+\frac{q_E}{\lambda_D}-L$, and $M=L-\frac{2q_E-\lambda T_1}{\lambda_D}$ and simplify.

$$2A\left(1+\frac{\lambda}{\lambda_D}\right)-\left(\frac{q_E}{\lambda}+\frac{q_E}{\lambda_D}\right)\sqrt{2hA\lambda}+\left(\frac{q_E}{\lambda}+\frac{q_E}{\lambda_D}-L\right)\sqrt{\frac{hA\lambda}{2}}\left(1-\frac{\lambda}{\lambda_D}\right)+s\lambda_D M\geq 0$$

$$2A\left(1+\frac{\lambda}{\lambda_D}\right)-2A\left(1+\frac{\lambda}{\lambda_D}\right)+\left(\frac{A\lambda}{\lambda_D}+A-L\sqrt{\frac{hA\lambda}{2}}\right)\left(1-\frac{\lambda}{\lambda_D}\right)+s\lambda_D M\geq 0$$

$$\left(\frac{A\lambda}{\lambda_D}+A-L\sqrt{\frac{hA\lambda}{2}}\right)\left(1-\frac{\lambda}{\lambda_D}\right)+s\lambda_D M\geq 0.$$

This inequality holds because $\frac{A\lambda}{\lambda_D}+A-L\sqrt{\frac{hA\lambda}{2}}\geq 0$ and $M\geq 0$.

Case 4

Consider the following assumptions: $T_2\geq\frac{q_E}{\lambda_D}+\frac{q_E}{\lambda}$ and $\frac{q_E}{\lambda}-T_1\leq L\leq\frac{q_E}{\lambda_D}+\frac{q_E}{\lambda}-T_1$.

$$TC^1_{R_4}(q_E,q_R)-TC^0_R(q_E)\geq 0$$

$$A\left(\frac{\lambda T_1+\lambda_D(t_{1R}-T_1)}{q_E}+1+\frac{\lambda_D(T_2-\frac{q_E}{\lambda_D}+\frac{q_E}{\lambda})}{q_R}\right)+h\left(\frac{q_E t_{1R}}{2}+\frac{{q_E}^2}{2\lambda_D}+\frac{q_R(\frac{q_E}{\lambda_D}+\frac{q_E}{\lambda})}{2}\right)$$
$$+s\lambda_D\left(\frac{q_E}{\lambda}-t_{1R}\right)-A\left(\frac{\lambda T_2}{q_E}\right)-h\left(\frac{T_2 q_E}{2}\right)\geq 0$$

Substitute $t_{1R}=\frac{q_E-\lambda T_1}{\lambda_D}+T_1$, $q_E=\sqrt{\frac{2A\lambda}{h}}$, $q_R=\sqrt{\frac{2A\lambda_D}{h}}$ and simplify.

$$2A\left(1+\frac{\lambda}{\lambda_D}\right)-T_2\sqrt{2hA\lambda}+\sqrt{2hA\lambda_D}\left(T_2-\sqrt{\frac{2A\lambda}{h{\lambda_D}^2}}-\sqrt{\frac{2A}{h\lambda}}\right)+T_1\sqrt{\frac{hA\lambda}{2}}\left(1-\frac{\lambda}{\lambda_D}\right)$$
$$+s\lambda_D\left(\sqrt{\frac{2A}{h\lambda}}-T_1\left(1-\frac{\lambda}{\lambda_D}\right)-\sqrt{\frac{2A\lambda}{h{\lambda_D}^2}}\right)\geq 0$$

Substitute $T_2=\frac{q_E}{\lambda_D}+\frac{q_E}{\lambda}$.

$$2A\left(1+\frac{\lambda}{\lambda_D}\right)-\sqrt{2hA\lambda}\left(\sqrt{\frac{2A\lambda}{h{\lambda_D}^2}}+\sqrt{\frac{2A}{h\lambda}}\right)$$
$$+\sqrt{2hA\lambda_D}\left(\sqrt{\frac{2A\lambda}{h{\lambda_D}^2}}+\sqrt{\frac{2A}{h\lambda}}-\sqrt{\frac{2A\lambda}{h{\lambda_D}^2}}-\sqrt{\frac{2A}{h\lambda}}\right)+T_1\sqrt{\frac{hA\lambda}{2}}\left(1-\frac{\lambda}{\lambda_D}\right)$$
$$+s\lambda_D\left(\sqrt{\frac{2A}{h\lambda}}-T_1\left(1-\frac{\lambda}{\lambda_D}\right)-\sqrt{\frac{2A\lambda}{h{\lambda_D}^2}}\right)\geq 0$$

$$2A\left(1+\frac{\lambda}{\lambda_D}\right)-\sqrt{2hA\lambda}\left(\sqrt{\frac{2A\lambda}{h{\lambda_D}^2}}+\sqrt{\frac{2A}{h\lambda}}\right)+T_1\sqrt{\frac{hA\lambda}{2}}\left(1-\frac{\lambda}{\lambda_D}\right)$$
$$+s\lambda_D\left(\sqrt{\frac{2A}{h\lambda}}-T_1\left(1-\frac{\lambda}{\lambda_D}\right)-\sqrt{\frac{2A\lambda}{h{\lambda_D}^2}}\right)\geq 0$$

$$2A\left(1+\frac{\lambda}{\lambda_D}\right)-2A\left(1+\frac{\lambda}{\lambda_D}\right)+T_1\sqrt{\frac{hA\lambda}{2}}\left(1-\frac{\lambda}{\lambda_D}\right)$$
$$+s\lambda_D\left(\sqrt{\frac{2A}{h\lambda}}-T_1\left(1-\frac{\lambda}{\lambda_D}\right)-\sqrt{\frac{2A\lambda}{h{\lambda_D}^2}}\right)\geq 0$$

$$T_1\sqrt{\frac{hA\lambda}{2}}\left(1-\frac{\lambda}{\lambda_D}\right)+s\lambda_D\left(\sqrt{\frac{2A}{h\lambda}}-T_1\left(1-\frac{\lambda}{\lambda_D}\right)-\sqrt{\frac{2A\lambda}{h{\lambda_D}^2}}\right)\geq 0$$

We know that the lost sales are positive, so $\left(\sqrt{\frac{2A}{h\lambda}} - T_1\left(1 - \frac{\lambda}{\lambda_D}\right) - \sqrt{\frac{2A\lambda}{h{\lambda_D}^2}}\right) \geq 0$. Also, $T_1\sqrt{\frac{hA\lambda}{2}}\left(1 - \frac{\lambda}{\lambda_D}\right) \geq 0$.

A.3 Proactive Strategy

Case 1

$$TC_P^1(q_P) - TC_{P_1}^0(q_P, q_E) \geq 0$$

$$A\left(\frac{D_T}{q_P}\right) + h\left(\frac{T_2 q_P}{2}\right) - A\left(2 + \frac{\lambda T_2 - 2q_P}{q_E}\right) - h\left(\frac{{q_P}^2}{\lambda} + \frac{q_E\left(T_2 - \frac{2q_P}{\lambda}\right)}{2}\right) \geq 0$$

Substitute $q_E = \sqrt{\frac{2A\lambda}{h}}$ and $q_P = \sqrt{\frac{2AD_T}{hT_2}}$ and simplify.

$$\begin{aligned}
&2A\left(2\sqrt{\frac{D_T}{\lambda T_2}} - 1 - \frac{D_T}{\lambda T_2}\right) + \sqrt{2hA}\left(\sqrt{D_T T_2} - T_2\sqrt{\lambda}\right) \geq 0 \\
&\sqrt{2hA}\left(\sqrt{D_T T_2} - T_2\sqrt{\lambda}\right) - 2A\left(\sqrt{\frac{D_T}{\lambda T_2}} - 1\right)^2 \geq 0 \\
&\sqrt{2hA}\left(\sqrt{D_T T_2} - T_2\sqrt{\lambda}\right) \geq 2A\left(\sqrt{\frac{D_T}{\lambda T_2}} - 1\right)^2 \\
&T_2\sqrt{2hA\lambda}\left(\sqrt{\frac{D_T}{\lambda T_2}} - 1\right) \geq 2A\left(\sqrt{\frac{D_T}{\lambda T_2}} - 1\right)^2 \\
&T_2\sqrt{\frac{h\lambda}{2A}} \geq \left(\sqrt{\frac{D_T}{\lambda T_2}} - 1\right)
\end{aligned}$$

To simplify the inequality, change the left side of the inequality to $\sqrt{\frac{D_T}{\lambda T_2}}$ because it is greater than $\left(\sqrt{\frac{D_T}{\lambda T_2}} - 1\right)$.

$$\begin{aligned}
&\lambda T_2\sqrt{\frac{hT_2}{2AD_T}} \geq 1 \\
&\lambda T_2\left(\frac{1}{q_P}\right) \geq 1 \\
&T_2 \geq \frac{q_P}{\lambda}
\end{aligned}$$

We know that $T_2 \geq \frac{q_P}{\lambda}$ because the assumption for this case is that $T_2 \geq \frac{2q_P}{\lambda}$.

Case 2 and Case 4

$$TC_P^1(q_P) - TC_{P_2}^0(q_P, q_E) \geq 0$$

$$\begin{aligned}
&A\left(\frac{D_T}{q_P}\right) + h\left(\frac{q_P T_2}{2}\right) - A\left(\frac{\lambda T_2}{q_P}\right) - h\left(\frac{q_P T_2}{2}\right) \geq 0 \\
&\frac{A\lambda_D}{q_P}(T_2 - T_1) - \frac{A\lambda}{q_P}(T_2 - T_1) \geq 0 \\
&\lambda_D - \lambda \geq 0
\end{aligned}$$

Because we know that $\lambda_D \geq \lambda$, $TC_P^1(q_P) \geq TC_{P_2}^0(q_P, q_E)$. The same is true for Case 4.

Case 3

$$TC_P^1(q_P) - TC_{P_3}^0(q_P, q_E) \geq 0$$

The proof for Case 3 is very similar to the proof for Case 1 with the exception that only one order of q_P is placed. The inequality reduces to the following:

$$\begin{aligned}
&\sqrt{2hA}\left(\sqrt{D_T T_2} - T_2\sqrt{\lambda}\right) - A\left(\sqrt{\frac{D_T}{\lambda T_2}} - 1\right)^2 \geq 0 \\
&T_2\sqrt{2hA\lambda}\left(\sqrt{\frac{D_T}{\lambda T_2}} - 1\right) \geq A\left(\sqrt{\frac{D_T}{\lambda T_2}} - 1\right)^2 \\
&T_2\sqrt{\frac{2h\lambda}{A}} \geq \left(\sqrt{\frac{D_T}{\lambda T_2}} - 1\right)
\end{aligned}$$

To simplify the inequality, change the left side of the inequality to $\sqrt{\frac{D_T}{\lambda T_2}}$ because it is greater than $\left(\sqrt{\frac{D_T}{\lambda T_2}}-1\right)$.

$$\lambda T_2\sqrt{\frac{2hT_2}{AD_T}} \geq 1$$
$$\lambda T_2\left(\frac{2}{q_P}\right) \geq 1$$
$$T_2 \geq \frac{q_P}{2\lambda}$$

We know that $T_2 \geq \frac{q_P}{2\lambda}$ because the assumption for this case is that $T_2 \geq \frac{q_P}{\lambda}$.

Appendix B

Chapter 3

Consider the cost equation where $q_1 = q_L$, $q_2 = q_S$, $\vec{D} = (d_1, 0)$, and $\vec{S} = (H, 0)$.

$$TC^{(d_1,0)}_{(H,0)}(q_L, q_S) \quad = \quad A\left(1 + \frac{\lambda_S\left(T_4 - T_3 - \frac{q_L}{\lambda_S}\right)}{q_S}\right) + hq_LT_3 + \frac{hq_L{}^2}{2\lambda_S} + \frac{hq_S\left(T_4 - T_3 - \frac{q_L}{\lambda_S}\right)}{2}$$

Consider the cost equation where $q_1 = q_2 = q^*$, $\vec{D} = (d_1, 0)$, $\vec{S} = (H, 0)$, and $q^* \geq \lambda_S L$.

$$TC^{(d_1,0)}_{(H,0)}(q^*, q^*) \quad = \quad A\left(1 + \frac{\lambda_S\left(T_4 - T_3 - \frac{q^*}{\lambda_S}\right)}{q^*}\right) + hq^*T_3 + \frac{hq^{*2}}{2\lambda_S} + \frac{hq^*\left(T_4 - T_3 - \frac{q^*}{\lambda_S}\right)}{2}$$

Now, we will show the following:

$$TC^{(d_1,0)}_{(H,0)}(q^*, q_S) \leq TC^{(d_1,0)}_{(H,0)}(q^*, q^*)$$

For the proof, we will substitute $q^* = q_L$, which is the smallest value of q^*. If we simplify the inequality using the cost equations, we have:

$$\begin{aligned}
&\frac{A\lambda_S}{q_S} + \frac{hq_S}{2} \leq \frac{A\lambda_S}{q_L} + \frac{hq_L}{2} \\
&\frac{h}{2}(q_S - q_L) \leq A\lambda_S\left(\frac{1}{q_L} - \frac{1}{q_S}\right) \\
&\frac{h}{2}(q_S - q_L) \leq A\lambda_S\left(\frac{q_S - q_L}{q_Lq_S}\right) \\
&\frac{h}{2} \leq \frac{A\lambda_S}{q_Lq_S} \\
&q_Lq_S \leq \frac{2A\lambda_S}{h} \\
&q_Lq_S \leq q_S{}^2
\end{aligned}$$

We know this to be true because $q_S \geq q_L$, and if this is true for the smallest value, then it will be true for all values of q^* because $q^* \leq q_S$ and $q^* \geq q_L$.