Estimating Elasticities of Substitution in African Crops

by

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Abstract

This paper examines cross price elasticities of substitution for labor, capital, and fertilizer for the agricultural sector in Africa with a translog production function. The data are from the Food Agricultural Organization and World Development Indicators Database. The elastisities are calculated directly from the estimated parameters of a production function. Those elastisities include the Allen Substitution Elasticity (AES), factor price substitution elasticity, and Hicks elasticity of complementary (HEC).

Allen elastiticies of substitution of all inputs are positive. Results show that only labor and capital are complements among the inputs in HEC and factor price substitution elasticities. Own price elasticities for labor, capital, and fertilizer are inelastic. Labor is the most sensitive of the inputs to input prices. Furthermore, HEC shows that fertilizer demand is elastic with respect to the capital price and inelastic with respect to the wage.

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CHAPTER 1

INTRODUCTION

1.1 Background and Motivation

Agriculture dates back as far as 9500 BC. At that time farmers within Southwest Asia commenced to cultivate special crops. By the middle ages, industry came and irrigation and crop rotation started. Finally, by the middle of nineteen century and early twenty century, technology was applied leading to agricultural production growth through increasing crop yields. The technological improvements enabled farmers to meet the demand for food until the late 20th century, particularly in Asia and Africa.

From 1973, the African continent has relied heavily on food imports and food aid. Even though Africa possesses large portions of arable land, food insecurity and malnutrition affect more than half of the population according to the International Food Policy Research Institute (2008). One of the most critical challenges facing Africa today is how to increase agricultural production to meet increasing domestic demand arising from increased population pressure (Pretty, 1995). Despite these circumstances, the aim of agriculture production is to feed the population, assure food security, and ease poverty in the region.

This thesis will concentrate on the African continent. Table 1 depicts all the countries included in this study. Agriculture is a crucial economic activity in this region. Agricultural activities employs about 70% of the population and contributes to about 30% of the continent's gross domestic product, comprises of 50% of export value, brings a substantial amount of exchange rate, generates 34% of income, and serves the basis for many industries.

The motivation of this study is to analyze and estimate the African agricultural production function. There have only been a few studies on this matter. However, numerous studies of production function and its elasticities have been done in different regions with a cross sectional and time series data.

1.2 Objective of the Study

The objective of this study is to develop and analyze the structure of agricultural crops in Africa from 1961 to 2007. This thesis will define, derive, and provide a comprehensive understanding of the African agricultural production function. Investigation into the roles of capital, labor, fertilizer, and output will be included so that the production function will be estimated.

The results will be used to assess and highlight differences within substitution possibilities between capital, labor, and fertilizer. In order to achieve good policy in this matter, the elasticity of substitution is an appropriate method. It is necessary to estimate the elasticity of substitution between inputs, and own and cross price elasticities of inputs. The empirical results of the estimation of the production function and elasticities of substitution will be useful for policy makers and future research.

1.3 Approach and Methods

In order to achieve the above objective, the thesis includes all crops in the region for 47 countries in an annual panel data from 1961-2007 to examine the production function with capital, labor, and fertilizer as inputs.

1.4 Organization of the Study

The thesis has been divided into five chapters. The next chapter will provide an overview of the literature and will contain a detailed discussion of the African continent and its agriculture. Chapter 3 will set up the production function, define the various elasticities to be estimated, and describe the methodological approach. The econometric procedures and empirical results from the estimation of the model and its elasticities are presented in Chapter 4. Chapter 5 consists of conclusions, highlights some remarks, and offers some suggestions to assist in creating better growth in agricultural production.

CHAPTER 2

REVIEW OF LITERATURE

The African continent covers over 30 million square kilometers (11.58 square miles). Africa is considered the second largest continent after Asia. In addition it surrounded by Mediterranean Sea, Atlantic Ocean, Indian Ocean, and Red Sea. It climate is mostly tropical with the majority (60%) of its land are deserts and dry land. The African continent is seen as a complicated and diverse continent containing more than one billion of the world's population and more than 1000 different ethnic groups. (United States Department of State, 2009).

2.1 African Agriculture

In 1960 the independence of many African countries, the Continent was looked as self-sufficient in its agricultural production. In contrast, the Asia continent began to endure food crisis until the birth of the Green Revolution which came in the middle of the 1960s; a revolution that primarily motivates the adaption of high-yielding seeds. Beginning in 1973, several African nations began to depend on food imports. One of the most critical challenges facing Africa today is how to scale up agricultural production to meet increasing domestic demand arising from population pressure (Hunt, 2011).

Lack of rain fall has made agriculture difficult for many African nations. With areas that get unreliable rainfall amounts, farmers are ambivalent of what kinds of crops to use. To survive these severe conditions, farmers were obliged to swap their strategies frequently; on the other hand, other areas are receiving too much rain that exceedingly destroying nourishing substances. Another common issue is droughts. It is more frequent in the region than any elsewhere in the world. As a result, crop yield is diminishing greatly and food insecurity is increasing significantly. Climate variation is another impediment to African agriculture. Precipitation sometimes delayed or advanced, leading to catastrophic consequences such as hunger, and malnutrition. Another constraint to agriculture development is water shortage that is the outcome from bad management of the environmental resources that are ruined regularly by wars and mining usage. Water shortages in reservoirs cause lakes, rivers, and streams to dry out in the warmer seasons which create water stress. Today many African countries have water shortages, and they cannot meet their demands. Since water is a vital component to agriculture, soil has a poor capability to keep or emit moisture. Soil depletion affects most areas in Africa with nutrient losses estimated from 30kg to 60 kg per hectare each year (Henao and Baanante, 2006). Irrigated area is less than 5 million hectares which represent 5 percent of the total cultivated land.

Further complication to African agriculture is Soil fertility. Soil is low in magnesium, phosphorous, zinc, and nitrogen. The fast growing of the population has worsened the problem because it decreases the length and the amount of fallow lands. Therefore, most of the countries hinge on imported fertilizer in which most farmers can’t afford due to high price. Overall, the average use of fertilizer in the African Continent still inferior than any other place.

Agriculture remains one of the most important issues in the region. Rural infrastructure and change in policy is a must; a situation that urges full attention from all parties.

2.2 Related Studies

Over the last thirty years, many studies studied elasticity of substitution between capital and labor. However, the reported estimates differ considerably depending on cross sectional and time series studies.

Most of elasticity of substitution has been done in industry while there is only limited amount of study has been conducted to agriculture in comparison to other sectors.

In a study of productivity and economic growth in Tunisian agriculture, Dhehibi (2006) showed that labor and capital are substitutes. His elasticities show that an increase in labor increases demand for capital by (1.63) while an increase in the capital increase demand for labor by (0.23).

Chandharry, Khan, and Nqair (1998) in a study on "estimates of farm output supply and input demand elasticities" in Pakistan agriculture, found that an increase in capital increases demand for labor and fertilizer by 0.69 and 0.64 respectively.

The study of Lianos (1971) on production function in United States from 1950-1989 found that the short run wages elasticity was -1.04 while the long run was -3.51.

The study of Vincent (1979) of factor substitution in Australian agriculture from 1920 to 1970 used land, labor, and capital as factors inputs and the value added as output using both Cobb Douglas and constant elasticity of substitution production functions. The estimation was done through full information maximum likelihood and the major finding was that there is very low elasticity of substitution between factor inputs.

Binswanger (1973) conducted a study on the measurement of technical change biases with many factors of production for the years, 1949, 1954, 1959, and 1964 including 39 states of United States. He used translog cost function with five inputs land, labor, machinery, fertilizer, and all others. His results showed elasticity of substitution between land-labor, land-machinery, labor-machinery, land-fertilizer were substitutes while labor-fertilizer and machinery-fertilizer were complements.

Thrisk (1973) examined factor substitution in Colombian agriculture using two estimation methods: ordinary and generalized least squares estimation techniques. He used cross sectional data over major crops in the region. The crops included in his study were rice, cotton, corn, wheat, barley, sesame, soybeans, and sorghum. He distinguished land, labor, and machinery as factors inputs. The results of elasticity of substitution estimates were similar in both approaches with 1.5 for labor-machinery.

Mupondwa (2005) in a study called induced technological change in Canadian agriculture field crops-canola and wheat: 1962-2003, he used constant elasticity of substitution production function with four inputs: land, machinery, labor, and fertilizer. The major finding was that the long run elasticity of substitution in the wheat for fertilizer-land and machinery-labor was -2.08 and -.346 respectively while canola had an elasticity of substitution for fertilizer-land and machinery-land of -1.06 and -1.53 respectively.

Arrow and Solow (1961) analyzed capital-labor substitution and economic efficiency for the years 1949-1955 for manufacturing. They included the following countries in their study: United States, Canada, New Zealand, Australia, Denmark, Norway, Puerto Rico, United Kingdom, Colombia, Ireland, Mexico, Argentina, Japan, El Salvador, Brazil, S. Rhodesia, Ceylon, India, and Iraq. Their results showed that the bilateral elasticity of substitution between capital and labor were less than unity.

In a study of the translog cost function analysis of U.S agriculture from 1939-77, Ray (1982) based his analysis on neoclassical duality theory to derive the elasticity of substitution.

The model was based on the estimation of crops and livestocks as two separate outputs while his inputs variables were limited to hired labor, farm capital, fertilizer, and miscellaneous input. The results suggest a decline in substitutability between labor and capital while price elasticity increased in all inputs.

CHAPTER 3

METHODOLOGY

3.1 Production Theory

The production function is a focal point in the economy. It deals with the maximum amount of output from optimal inputs. The objective is to develop a production function for African crops. The study focuses on three important inputs in African agriculture.

The production function will be:

$Q = Q (K, L, F)$

where

Q is the crop output (all crops are measured in quantity tons)

K is the capital (combine harvesters, threshers, and agriculture tractors in thousands)

L is labor (agricultural labor force in thousands)

F is fertilizer (fertilizer consumption in metric tons of nutrient in thousands)

3.2 Assumptions

Certain assumptions are made of production functions to assure technical validity and viability of an economic optimum involving the following restrictions. (Chambers, 1988):

- The production is monoperiodic meaning that the production is autonomous from the previous and subsequent period.

- The production function is homogeneous of degree one.
- The production is monotonic in X. that is, an increase in X (ceteris paribus) can never diminish output Y.
- The production is quasi-concave (convexity of the isoquant and diminishing marginal rate of technical substitution).
- The production is twice continuously differentiable.

3.3 Empirical Model Specification

Production can take numerous forms such as: linear functional, polynomial functional, Cobb Douglas, translog, and CES. Various studies have been conducted with Cobb Douglas production function due to its linearity in logarithms; however, its elasticity is constant and the elasticity of substitution is unity.

This thesis uses translog production function to study productive behavior. The translog function is more flexible in that it imposes few assumptions on the function and its elasticities. The transcendental logarithmic function was introduced by Christensen and Lau (1973) and has been used to analyze factors inputs and their substitution. The model is specified as follows:

$$lnY_{it} = \beta_0 + \beta_L \, lnL + \beta_K \, lnK + \beta_F \, lnF + \frac{1}{2}\beta_{LL} ln\, L^2 + \frac{1}{2}\, \beta_{KK} \, ln\, K^2 + \frac{1}{2}\, \beta_{FF} F^2 + \quad (1)$$

$$\beta_{LK} \, lnK \, lnL + \beta_{LF} \, lnL \, lnF + \beta_{KF} \, lnK \, lnF + \beta_t t + \frac{1}{2}\, \beta_{tt} t^2 + \beta_{Lt} lnLt +$$

$$\beta_{Kt} \, lnKt + \beta_{ft} \, lnFt + \varepsilon_{it}$$

where

$\beta_L, \beta_K, \beta_F, \beta_t$, and β_{tt} represent the first-order parameters.

$\beta_{LL}, \beta_{LK}, \beta_{KF}, \beta_{LF}, \beta_{KK}, \beta_{FF}, \beta_{Lt}, \beta_{Kt},$ and β_{ft} represent second-order parameters. The variable t is a time trend perhaps capturing technical change and ε_{it} are errors which are assumed to be independently and identically distributed and have N (0, σ_v^2) distribution.

Assuming perfect competition, the monotonocity condition necessitates positive marginal physical product (MP) which can be derived from translog production. Mathematically,

$$MP = f_i = \frac{\partial Y}{\partial X_i} = \frac{\partial lnY}{\partial lnX_i} \times \frac{Y}{X_i} \tag{2}$$

$$f_i = \frac{Y}{X_i}\left(\beta_i + \sum_{j=1}^{n} \beta_{ij}\ lnX_j\right) > 0 \tag{3}$$

Diminishing marginal productivity entails:

$$f_{ii} = \frac{Y}{X_i^2}\left[\beta_{ii} + \left(\beta_i - 1 + \sum_{j=1}^{n} \beta_{ij}\, lnX_j\right)\left(\beta_i + \sum_{j=1}^{n} \beta_{ij}\, lnX_j\right)\right] < 0 \tag{4}$$

$$f_{ij} = \frac{Y}{X_i X_j}\left[\beta_{ij} + \left(\beta_i + \sum_{j=1}^{n} \beta_{ij}\ lnX_j\right)\left(\beta_j + \sum_{i=1}^{n} \beta_{ij}\, lnX_i\right)\right] < 0 \tag{5}$$

Using these conditions, the factor share for each input will be:

$$\begin{aligned}
\frac{\partial lnY}{\partial lnL} &= \theta_L = \beta_L + \beta_{LL}\, lnL + \beta_{LK}\, lnK + \beta_{LF}\, lnF \\
\frac{\partial lnY}{\partial lnK} &= \theta_K = \beta_K + \beta_{KL}\, lnL + \beta_{KK}\, lnK + \beta_{KF}\, lnF \\
\frac{\partial lnY}{\partial lnL} &= \theta_F = \beta_F + \beta_{FL}\, lnL + \beta_{FK}\, lnK + \beta_{FF}\, lnF
\end{aligned} \tag{6}$$

Young's theorem imposes the following constraints:

$$\beta_{KL} = \beta_{LK}; \quad \beta_{LF} = \beta_{FL}; \quad \beta_{KF} = \beta_{FK}$$

The output elasticity can be calculated using equation (6).

Elasticities

This study will focus on Allen elasticity of substitution (AES), cross price elasticities, and the Hicks elasticity of complementary (HEC).

AES is prevalent in the literature especially for production function; it measures the changes in relative input as their relative price changes. Furthermore, the bigger the value of the elasticity, the higher the substitution between the inputs. Cross price elasticity measures the percentage changes in an input in response to a price change. HEC, on contrast, measures the variation in the price as the quantity of the input change. The advantage of HEC is that it can be applied even when the input price are falsified or are missing while the elasticity of substitution is pertinent only when the price and the quantity are clearly stated. HEC involves specification and estimation of production function when the quantity is exogenous and the price is endogenous. (Kim, 2000)

A positive sign of the elasticity means that the inputs are substitutes while a negative sign means that they are complements. The Allen elasticity can be calculated from

$$\sigma_{ij} = \left(\frac{\sum_i f_i X_i}{X_i X_j}\right)\left(\frac{|F_{ij}|}{|F|}\right) \qquad i \neq j \qquad (7)$$

where

$i \; and \; j$ are the inputs (L, K, and F)

$X_i \; and \; X_j$ are inputs levels

f_i is marginal product of inputs

$|F_{ij}|$ is the cofactor in F_{ij}

$|F|$ is the determinant of the bordered Hessian matrix

$$|F| = \begin{pmatrix} 0 & f_L & f_K & f_F \\ f_L & f_{LL} & f_{LK} & f_{LF} \\ f_K & f_{KL} & f_{KK} & f_{KF} \\ f_F & f_{FL} & f_{FK} & F_{FF} \end{pmatrix}$$

And f_{ii} and f_{ij} are as defined in the equations (4) and (5)

The necessary and sufficient condition for the convexity of the isoquant requires the bordered Hessian matrix to be negative semi-definite. That is, the determinants of all principal minors of ($|F_1|, |F_2|, and\ |F_3|$) must alternate in sign starting with negative.

The price elasticity can be calculated now using (6)

$$\varepsilon_{ij} = \frac{\beta_{ij}}{\theta_i} + \theta_j \tag{8}$$

$$\varepsilon_{ii} = \frac{\beta_{ii}}{\theta_i} + \theta_i - 1$$

The Hicks elasticity of complementary (HEC) is as follow:

$$\alpha_{ij} = \frac{\beta_{ij}}{\theta_i\ \theta_j} + 1 \tag{9}$$

$$\alpha_{ii} = \frac{\beta_{ii}}{\theta_i^2} + 1 - \frac{1}{\theta_i}$$

Separability

Separability is a standard assumption in empirical work and it is necessary for checking economic aggregation. They are two types of separability:

“Weak separabiliy: it must be satisfied that the marginal rate of substitution between two goods from a group is independent of the quantities of the two goods which do not belong to that group.”

“Strong separability: it is satisfied when the marginal rate of substitution between two goods belonging to different groups does not depend on the quantities of the goods which do not depend belong to any those groups” (Millan, 2003 p.11)

Berndt and Christensen (1973) argue that in translog function with three inputs $(i, j, and\ k)$, $i\ and\ j$ are separable from the input k when $\sigma_{ik} = \sigma_{jk} = 1$ which can hold only if $f_i \beta_{jk} - f_j \beta_{ik} = 0$.

Therefore separability in this translog production requires satisfaction of the following conditions:

$$f_L \beta_{KF} - f_K \beta_{LF} = 0 \qquad (\sigma_{LF} = \sigma_{KF})$$
$$f_K \beta_{LF} - f_F \beta_{LK} = 0 \qquad (\sigma_{LK} = \sigma_{LF}) \qquad (10)$$
$$f_L \beta_{KF} - f_F \beta_{LK} = 0 \qquad (\sigma_{LK} = \sigma_{KF})$$

The above equations represent weak separability. The test will be implemented to linear and non-linear separability. Starting with linear separability, if one of the above conditions is satisfied, no further tests needed. If the test is rejected, an inquiry of one of the others types is required until the test is accepted by any of them. If this hypothesis is rejected as well, then none of these separability conditions is holds for this particular translog production function. The first equation in the linear separability above will test if the capital and labor inputs are separable from the fertilizer input. The second equation will test if the fertilizer and capital inputs are separable from the labor input. The last equation will test if the fertilizer and labor inputs are separable from the capital input. The purpose of testing separability is to see if any input in production can be aggregated.

Consequently, the estimation of the translog production function will go under certain tests. First, a test for symmetry conditions is obligatory by economic theory to assure that the estimated model meets the restriction. Second, monotonicity necessitates that the marginal product is positive. Finally, the concavity condition of the translog production function has to be checked. The Hessian determinant built on the parameters estimation must be negative definite.

3.4 Data and Variables

There are abundant studies of translog production function conducted by cross-sectional or time series but very limited ones that treat panel data. This study used annual panel data from 1961-2007 to estimate the parameters of the translog production function. The study focuses on Africa and covers 47 countries while the other missing countries are excluded due to data unavailability. The list of the countries is provided earlier in Table 1. All tables and figures are presented in the Appendix.

Since the price data (wage of labor, rent of capital, and cost of fertilizer) in African countries is unobtainable, physical quantity is utilized and the estimation comes from the production function. Therefore, data include all African crops as agricultural output and labor, capital, and fertilizer as agricultural inputs. In addition, trend variable (t) is employed as proxy to catch technical change.

Table 2 presents a detailed explanation of the variables used in this research. Summary statistics and other details of the data are reported in table 3.

The average labor in agricultural sector is close to 2.5 million which is superior than any other inputs. Figure 2 shows the trend of agricultural labor. The average of fertilizer use is almost 61 thousands metrics tons per year while the capital has the smallest portion of approximately of 12 thousands which implies that labor and animal are the most performers of agricultural work (Figure 3 and 4).

The entire data are divided by the mean before transformed to logarithms following Friedlaender and Spady (1980).

Finally, different sources of data are used for this study. Output and input production data conducted by the Food and Agricultural Organization's online database and World Development Indicators Database.

CHAPTER 4

RESULTS AND DISCUSSION

This chapter is divided into six sections. The model discussed in the previous chapter is estimated with fixed effect regression. The first section starts by introducing the specification and estimation of the model followed by several tests and its discussions. Further the chapter reports the estimation results of the translog production function and its elasiticies with their interpretation in the last section.

4.1 Specification and Estimation of the Model

Fixed effect

In the fixed effect (FE), the intercept can vary across individuals but the individuals intercept terms cannot differ over the time and the coefficients of the independents variable does not allow fluctuating across individuals or over time. (Gujarati, 2004).

Let consider the model below:

$$y_{it} = \alpha_i + x_{it}\beta + \varepsilon_{it} \tag{11}$$

where $u_{it} = \alpha_i + \varepsilon_{it}$ and y_{it} is the dependent variable; x_{it} are independents variables; ε_{it} is the error; and α_i are “random individual-specific effects”. (Cameron and Trivedi, 2005). The α_i are allowed to be correlated with x_{it} and x_{it} is uncorrelated with ε_{it}.

The estimation of fixed effect can be obtained by mean differencing of (7), in which it removes the α_i .

$$(y_{it} - \bar{y}_i) = (x_{it} - \bar{x}_i)' \beta + (\varepsilon_{it} - \bar{\varepsilon}_i) \quad (12)$$

Since the α_i have been excluded, OLS estimates will be consistent. Kumbhakar and Lovell (2000) establish that fixed effect models are simple to implement and its consistency does not rely on the independence or distribution of the regressors.

Random effect

In the random effect (RE) model, the α_i are random and uncorrelated with the independent variables. The estimation can be conducted with feasible generalized least square (FGLS) or transformed OLS.

Assuming α_i and ε_{it} are independently and identically distributed with variance of σ_α^2 and σ_ε^2 respectively. Since $u_{it} = \alpha_i + \varepsilon_{it}$ then variance $(u_{it}) = \sigma_\alpha^2 + \sigma_\varepsilon^2$ and covariance (u_{it}, u_{is}) $= \sigma_\alpha^2, s \neq t$; so the correlation is

$$Cor(u_{it}, u_{is}) = \sigma_\alpha^2 / (\sigma_\alpha^2 + \sigma_\varepsilon^2)$$

Then the random effect estimator can be calculated as follow:

$$(y_{it} - \hat{\theta}_i \bar{y}_i) = (1 - \hat{\theta}_i)\alpha + (x_{it} - \hat{\theta}_i \bar{x}_i)'\beta + \{(1 - \hat{\theta}_i)\alpha_i + (\varepsilon_{it} - \hat{\theta}_i \bar{\varepsilon}_i)\} \quad (13)$$

Where

$$\theta_i = 1 - \sqrt{\left(\frac{\sigma_\varepsilon^2}{T_i \sigma_\alpha^2 + \sigma_\varepsilon^2}\right)}$$

The advantage of the random effect model is that it produces estimates for all coefficients including the time invariant ones and uses few degrees of freedom.

4.2 Hausman test

The Hausman test (1978) chooses the appropriate model between FE and RE. The null hypothesis is that no correlation exists (RE) meaning that there is no differences between the estimators since both of them are consistent. While the alternative suggests that both estimators are dissimilar. If the null hypothesis is rejected then the FE is the appropriate model.

The Hausman Test (H) is

$$H = \left(\hat{\theta}_r - \hat{\theta}_e\right)' \left[\widehat{Var}\left(\hat{\theta}_r - \hat{\theta}_e\right)\right]^{-1} \left(\hat{\theta}_r - \hat{\theta}_e\right)$$

where

$\hat{\theta}_r$ *and* $\hat{\theta}_e$ are the vector of RE and FE estimates.

4.3 Heteroskedasticity Test

Since this analysis is panel data, several problems arise. Heteroskedasticity arises when the variance of the errors are not constant. However, it does not generate bias but the standard errors estimated are underestimated leading to high t values. Hypotheses testing may lead to reject the right null and accept the wrong null, leading to non-reliable results. Thus, this is a strong incentive to check heteroskedasticity. The modified Wald test is one among others. Greene (2000) defines it as

$$W = \sum_{i=1}^{N} \frac{(\hat{\sigma}_i^2 - \sigma^2)}{V_i}$$

where

$\hat{\sigma}_i^2$ is the estimated variance.

$$V_i = T_i^{-1}(T_i - 1)^{-1} \sum_{t=1}^{T}(e_{it}^2 - \hat{\sigma}_i^2)^2$$

The solution can be by correcting the bias of the standard errors, or using generalized least squares.

4.4 Autocorrelation Test

The second common issue that panel data encounter is autocorrelation. It is defined as a correlation between successive values and statistically as the error in one period correlated with the error in the next period.

The consequences of autocorrelation are the same as heteroskedasticity. Autocorrelation does not generate bias however the standard errors are underestimated leading to high t values. Thus hypotheses testing may lead to the failure to reject the null when it is false. This is a strong incentive to check if the errors are related.

However, detecting autocorrelation is not an easy task because the error cannot be observed. The model must be estimated to get the residuals. This latter reflects the pattern in the error term. A number of tests can diagnose this problem but the most recent one is the Wooldridge test (2002). This test is simple and imposes few assumptions.

From equation (12)

$$y_{it} - y_{it-1} = (x_{it} - x_{it-1})\beta_1 + \varepsilon_{it} - \varepsilon_{it-1}$$

$$\Delta y_{it} = \Delta x_{it}\beta_1 + \Delta\varepsilon_{it}$$

The test is based on residuals e_{it}. Wooldridge (2002) examined first ε_{it}. If they are not correlated, then the correlation of the first differences (Δ) of ε_{it} and ε_{it-1} equal to - 0.5. Using this technique, Wooldridge proceed with the regression of e_{it} on e_{it-1} and test of whether the coefficient is equal to -0.5 or not to determine if serial correlation exists.

The problem of autocorrelation can be solved by either correcting the bias in the standard errors, or transforming the data and using generalized least squares or feasible generalized least squares.

4.5 Estimation and Hypothesis Test

Panel data analysis will be implemented in this study. One step is to choose the appropriate functional form. Many factors are taken in consideration when making that decision to choose the functional form that obeys certain restrictions such as concavity and non-negativity of the input and output. Cobb Douglas, translog, and quadratic functional forms are most widely used in agricultural production functions. Translog production function is implemented, evaluated, and tested in this analysis.

Before the estimation of equation (1), a test for the functional form of the production models must be conducted using the below Cobb Douglas and translog equations.

$$lnY_{it} = \beta_0 + \beta_L\, lnL + \beta_K\, lnK + \beta_F\, lnF + \beta_t t + \varepsilon_{it}$$

$$lnY_{it} = \beta_0 + \beta_L\, lnL + \beta_K\, lnK + \beta_F\, lnF + \frac{1}{2}\beta_{LL} ln\, L^2 + \frac{1}{2}\beta_{KK}\, ln\, K^2 + \frac{1}{2}\beta_{FF}F^2 +$$

$$\beta_{LK}\, lnK\, lnL + \beta_{LF}\, lnL\, lnF + \beta_{KF}\, lnK\, lnF + \beta_t t + \frac{1}{2}\beta_{tt}t^2 + \beta_{Lt} lnLt +$$

$$\beta_{Kt}\, lnKt + \beta_{ft}\, lnFt + \varepsilon_{it}$$

where The variables and the parameters are as defined before in equation (1).

The estimation of the Cobb Douglas and translog function were conducted with the Wald test estimator using STATA. A hypothesis test was performed to choose the best functional form. The results are presented in Table 4.

The first test is that the null hypothesis is a Cobb Douglas function which strongly rejected implying that the translog is the adequate function for this analysis. The second test is that there is no technical change in African agriculture over time. This test is also rejected.

4.6 Results of the Empirical Estimation

After the functional form has been chosen, the next step is to apply the adequate econometric estimation to get the parameters for the model. The estimation of the empirical model is based on fixed effects since the Hausman test rejects random effects regression. The results are presented in Table 5.

Before proceeding with any conclusions to the analysis, a few tests are applied in order to ensure reliable results. As mentioned in the last section, heteroskedasticity, autocorrelation, and cross sectional dependence are common in panel data. The test used for heteroskedasticity is Wald test indicating its existence. The test for serial correlation is Lagrangian-multiplier test which also confirms its existence. A further test of cross-sectional dependence is specified in this analysis using Breusch-Pagan Lagranger Multiplier, the results shows no correlation of residuals.

The monotocity test is performed (Table7) showing that the marginal products of the three inputs are positive. A concavity test is also adapted in this translog production function. To make sure that this condition holds as imposed by economic theory, the bordered Hessian matrix is checked and confirmed that it is negative semi definite. Another test of symmetry is performed, the results from the model shows that the p value was 0.23 meaning that the null hypothesis of symmetry is not rejected.

The last test is the separability test (Table 6), a complete test is performed, and the null hypothesis is rejected at 5% level indicating continuing testing until one of the conditions is accepted. As results, investigating linear separability, the null hypothesis is rejected also at 5 percent level for (LK-F) and (LF-K) indicating that the optimum ratio of (LK) and (LF) is influenced by the level and the price of F and K respectively. On the other hand, the null hypothesis is accepted for (KF-L) meaning that capital and fertilizer are independent from the level and the price of labor. Since one of the separability tests is accepted no further tests (non-linear separability) can hold.

After the tests are implemented to the model and the corrections are made when necessary, the standards errors are robust for analysis. Table 5 shows that most of the coefficients are statistically significant supporting the relationship between the inputs/output and the first order parameters are well behaved since their estimate fall between zero and one. The three inputs are significantly different from zero. The output elasticities are calculated for each input at the mean level. The output elasticities with respect to labor, fertilizer, and capital, are (0.19), (0.10), and (0.07).

Labor appears to be the most important factors in African agricultural growth. Fertilizer has low effect due to the average intensity of its use has been stagnant since 1980s and then declined sharply (50%) in 1990s. Africa remains the lowest of any developing countries in using fertilizer (8 kilograms (kg) per hectare (h)) compared to 86 kg/h in Latin America and 123kg/h in Asia (FAO, 2002). The low estimate of capital maybe related to the use of archaic cultivation techniques. Human and animal powers are still cultivating the majority of the total area.

The sum of the elasticities is less than one, indicating that African agricultural production is experiencing decreasing returns to scale at the sample mean (Table 6). The coefficient of time is 0.014 suggests that the output has been increasing by 1.4 % per annum due to technical change while the estimate of time squared means that technical progress rate increases by 0.2 %. These estimates are statistically significant at the 1 and10 percent level.

The elasticities are calculated directly from the parameter estimation of the production function. The AES are reported in Table 8 (Appendix). From the results, the Allen elastiticies of substitution of all inputs are positive. Capital appears to be more substitutable with respect to labor than to fertilizer. On the other hand, fertilizer appears to be more substitutable to labor than to capital.

Factor price substitution is presented in Table 9. The own price elasticities are negative as expected with labor is more elastic (-0.82) followed by capital (-0.80) while the fertilizer input has the smallest own elasticity (-0.64).

The sign of the estimated cross price elasticities for labor and capital are negative indicating that they are complements while the cross price elasticities of capital and fertilizer and fertilizer and labor are positive indicating that they are substitutes. The demand for fertilizer is sensitive to the price change of capital (0.12) and the demand for labor is sensitive to the price change of fertilizer by the same amount. The demand for capital and labor is not as sensitive to price change of labor and capital with (-0.02) and (-0.05) respectively. Furthermore, a 10 percent increase in the price of labor will have positive effect on demand for fertilizer of about 7 percent and an increase of 10 percent in the price of fertilizer will have increase demand for capital of about 8 percent.

The HEC are reported in Table 10. The results show that only labor and capital are complements as factor price substitution elasticities found earlier. A rise in capital will lead the wage of labor to fall but the impact is opposite on the fertilizer. A rise in the fertilizer will raise the price of both the capital and labor.

CHAPTER 5

CONCLUSION

The objective of this study is to estimate cross price elasticities of substitution for labor, capital, and fertilizer for African agriculture. To conduct such an analysis, all crops in the region are estimated with annual panel data from 1961-2007. The model is specified as a translog production function estimated with a fixed effect regression. In order to get reliable results, several tests and corrections are implemented. The symmetry condition is imposed on the model.

To gain insights of the relation between the production function and its inputs, the Allen substitution elasticity, factor price substitution elasticity, and Hicks elasticity of complementary are computed. The Allen elastiticies of substitution of all inputs are positive. The results suggest that the substitution is low for capital and fertilizer. Capital appears to be more substitutable with respect to labor than to fertilizer. On the other hand, fertilizer appears to be more substitutable for labor than capital.

Own price elasticities for labor, capital, and fertilizer are less than one, implying inelastic demand with the labor is the most inelastic. It is reasonable to expect this result knowing that developing countries in Africa rely heavily on labor input in agricultural production.

The own price elasticity for labor and capital are sensitive to the price change to almost the same degree. Fertilizer is the least sensitive of the three inputs. Elasticity of substitution results of factor price elasticity and HEC suggests that labor and capital are the only complements among the inputs. Government should promote investment in agricultural mechanization. Fertilizer use needs to be encouraged also by different motivations. Such as efforts will help African agriculture.

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Figure1. Trend of fertilizer Consumption in Africa in Thousands

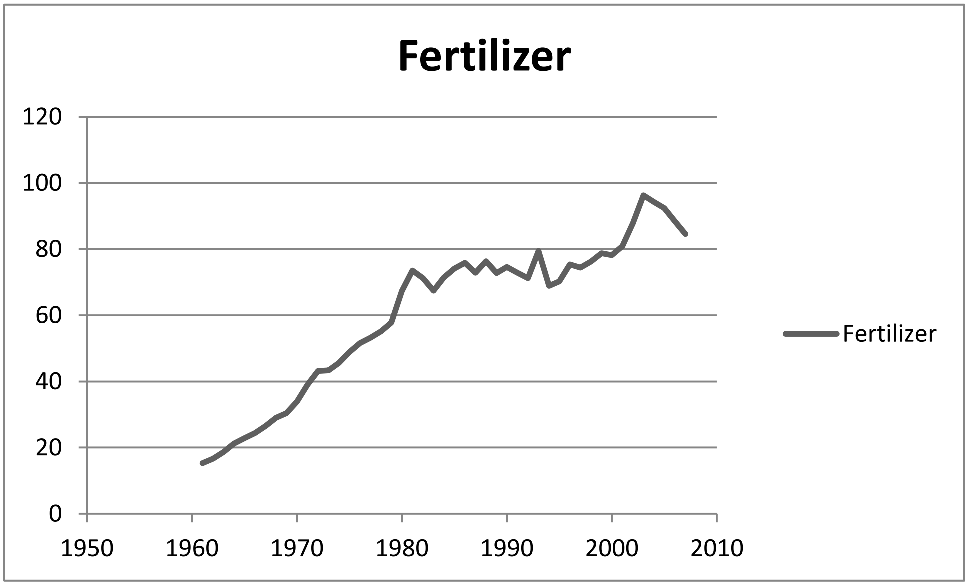

Figure2. Trend of African Agricultural Labor in Thousands

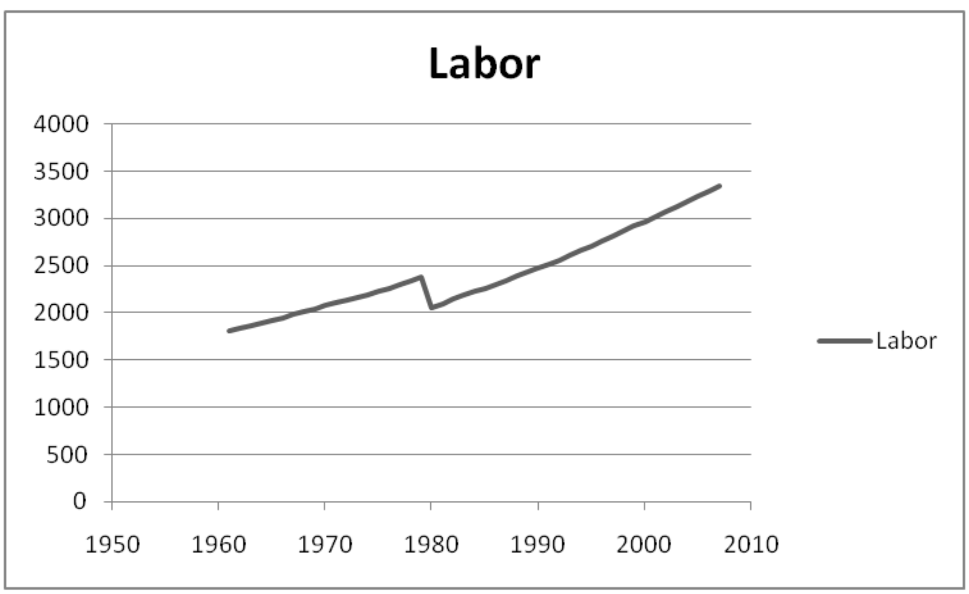

Figure3. Capital Use in African Agriculture in Thousands

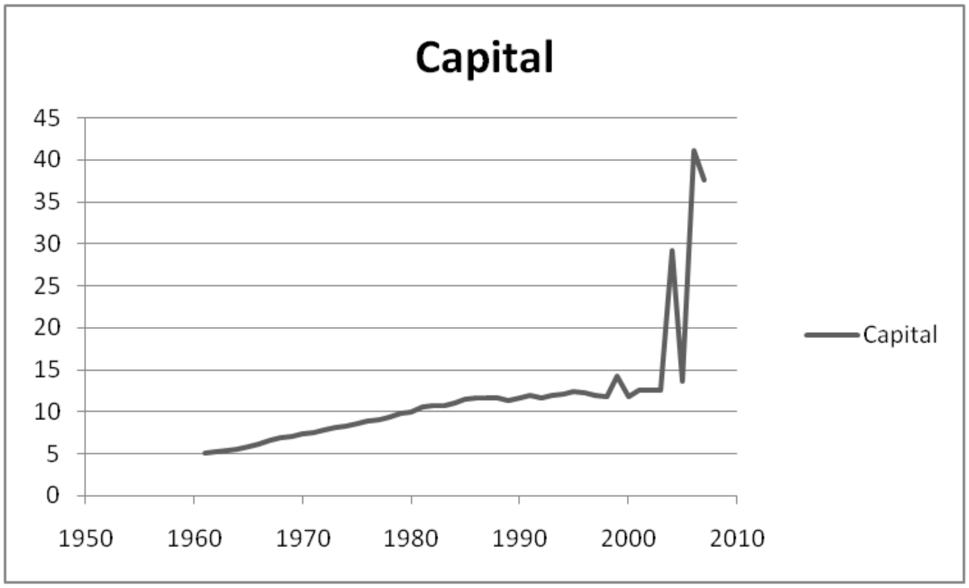

Table1. The Share of Agriculture in GDP

Country	Share of Agriculture in GDP (%)	Country	Share of Agriculture in GDP
Algeria	8	Madagascar	26
Angola	8	Malawi	30
Benin	33	Mali	37
Botswana	2	Mauritania	19
Burkina Faso	33	Mauritius	5
Burundi	35	Morocco	14
Cameroon	9	Mozambique	28
Cape Verde	9	Namibia	9
Central African	54	Niger	40
Chad	13	Nigeria	33
Comoros	45	Reunion	-
Cote d'Ivoire	24	Rwanda	36
Democratic Congo	42	Senegal	13
Congo	4	Sierra Le	50
Egypt	14	Somalia	-
Gabon	5	South Africa	3
Gambia	29	Sudan	28
Ghana	29	Swaziland	7
Guinea	25	Togo	44
G-B	62	Tunisia	30
Kenya	20	Uganda	24
Lesotho	8	Zambia	22
Liberia	55	Zimbabwe	23
Libya	2		

Source: 2011 World Bank Indicator database

Table2: Description of Variables Used in the Analysis

Variable	Description
Production frontier variables:	
Output	includes aggregate detailed crops measured as quantity in millions tons
Inputs	
Labor	agricultural labor force in thousands, farmers and employers
Capital	combines harvesters, threshers, and agriculture tractors
Fertilizer	measured as consumption of nitrogen, phosphate, and potash in metric tons

Table3. Summary Statistics of the Variables

Variable	Means	Std. Dev	Min	Max
Output	7389242	1.42e+0.7	30809	1.61e+0.8
Labor	2430078	2747816	8000	1.45e+07
Capital	11562.81	45358.09	1	1095478
Fertilizer	60518.55	1742523	0	1824243

Table4. Hypothesis Tests

Null Hypothesis	F statistic	Critical $F_{k,n}$	Decision at 5%
$H_0 : \beta_{ij} = 0, i, j = 1, \dots 4$	168.26	$F_{11,2148} \cong 1.75$	Reject H_0
$H_0 : \beta_4 = \beta_{4i} = \cdots = 0$	122.08	$F_{5,2148} = 2.21$	Reject H_0

Note: The critical values come from table 4 (Griffiths, Hill, and Judge, 1993).

Table5. Parameter Estimation of Translog Production Function

Serial no	Parameters	Coefficients	Estimates	T-ratio
0	Constant	β_0	0.967***	-16.28
1	*ln* L	β_1	0.107**	2.40
2	*ln* K	β_2	0.056***	2.88
3	*ln* F	β_3	0.134**	7.49
4	T	β_t	0.014***	-0.08
5	*ln* L * *ln* L	β_{11}	-0.001	2.27
6	*ln* K * *ln* K	β_{22}	0.009**	6.06
7	*ln* F * *ln* F	β_{33}	0.026***	-2.25
8	*ln* K * *ln* L	β_{21}	-0.016**	-1.40
9	*ln* K * *ln* F	β_{23}	0.001	0.36
10	*ln* F * *ln* L	β_{31}	-0.007	7.56
11	*ln* L * T	β_{1t}	0.001***	2.58
12	*ln* K *T	β_{2t}	0.001**	2.94
13	*ln* F * T	β_{3t}	0.001**	2.08
14	TT	β_{tt}	0.002	2.05
R^2			0.72	
N of obs			2209	

*** is significant at 1%
** is significant at 5 %
*is significant at 10%

Table 6: Separability Tests

Type	Restrictions	Number of restriction	Decision
Symmetry	β_{LK} = β_{KL} β_{LM} = β_{ML} β_{FK} =β_{KF}	3	Fail to reject
Complete Separability	β_{LK} = 0 β_{KF} = 0 β_{LF} = 0	3	Fail to reject
Linear			
LK- F separability	β_{LF} = 0 β_{KF} = 0	2	Rejected
LF- K separability	β_{LK} = 0 β_{KF} = 0	2	Rejected
KF-L separability	β_{LF} = 0 β_{LK} = 0	2	Fail to reject

Table 7: Estimated marginal product and output elasticites for African crops

Inputs	Output Elasticities	Marginal Products
Labor	0.187	0.568
Capital	0.068	43.379
Fertilizer	0.102	12.504

Table8. Estimated Allen Substitution Elasticity

Allen Elasticities of Substitution	
Elasticies	Estimates
σ_{Lk}	1.078
σ_{LF}	1.032
σ_{KF}	0.852

Table9. Cross Price Substitution Elasticity

Factor Price Elasticities					
Elasticies	Estimates	Elasticies	Estimates	Elasticies	Estimates
ε_{LL}	-0.818	ε_{LK}	-0.018	ε_{LF}	0.065
ε_{KL}	-0.048	ε_{KK}	-0.800	ε_{KF}	0.117
ε_{FL}	0.118	ε_{FK}	0.078	ε_{FF}	-0.643

Table10. Hicks Elasticity of Complementary (HEC)

$\alpha_{ij} = \alpha_{ji}$	Hicks Elasticity of Complementary (HEC)				
Elasticies	Estimates	Elasticies	Estimates	Elasticies	Estimates
α_{LL}	-4.291	α_{LK}	-0.203	α_{LF}	0.632
α_{KL}	-0.203	α_{KK}	-11.449	α_{KF}	1.143
α_{FL}	0.632	α_{FK}	1.143	α_{FF}	-6.400