The Effects of Concrete-Representational-Abstract Sequence and a Mnemonic Strategy on Algebra Skills of Students Who Struggle in Math

by

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A dissertation submitted to the Graduate Faculty of
Auburn University
in partial fulfillment of the
requirements for the Degree of
Doctor of Philosophy

Auburn, Alabama
August 4, 2012

Keywords: concrete-representational-abstract sequence (CRA), SUMLOWS, mnemonic strategy, algebraic equations, algebra, multiple-step equations

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Abstract

The National Council of Teachers of Mathematics (2012) and the Council for Exceptional Children (CEC; 2012) have stressed the importance of all students learning algebra. CEC stated that there was a crucial need for algebra instruction and intervention that can help students with disabilities in the general education classroom. Algebra is difficult to understand because of the abstractness of the concept. The use of the concrete-representational-abstract (CRA) sequence has been an effective instructional tool for students with disabilities in learning many math concepts. In order to assist with the abstractness of algebra, the CRA sequence assists students by using concrete manipulatives, drawings, and then numbers. Mnemonics have also been effective as a memory technique for students with disabilities. Currently, there was little research on CRA and SUMLOWS instruction for middle school students who struggle in solving algebraic equations. Therefore, this study investigated the effects of CRA and SUMLOWS on algebraic skills of middle school students who struggle in math. Specifically, students were taught one-step, two-step, and multiple-step equations with the distributive property. A multiple probe-across-behaviors design was used to demonstrate a functional relation between CRA and SUMLOWS instruction and the solving of equations across all behaviors.

Acknowledgments

My deepest appreciation and thanks are given to Dr. Margaret M. Flores, my major professor, whose wisdom, support, and encouragement guided me through each step of this journey. I am humbled by the time, guidance, and sincere interest she gave to me, and the genuine desire she had to see me grow as a professional and a person. She was always available for questions and provided endless feedback. She instantly saw qualities in me that I did not know existed until now. Thank you Dr. Flores.

I give thanks to my dissertation committee. Dr. Margaret “Peggy” Shippen, I am sincerely grateful for the many opportunities and encouraging words she has bestowed upon me over the many years. Thank you for continuously checking on me and for always saying kind and encouraging words when I needed to hear them. Dr. Jill Meyer, thank you for agreeing to take this journey with me and offering your assistance, guidance and feedback. To Drs. Dunn, Reilly, Rabren, Martin, Darch, and Curtis, I want to say thank you for the knowledge of special education content, grant writing, and special education law.

I hold a special appreciation and respect for Dr. Vanessa Hinton. She was a constant support as I proceeded though the last phase of this journey. She provided me with strength, wisdom, and friendship as I neared the end.

This study would not have been possible without the students that I have taught over the years. Thank you for the motivation to make math easier for you. My former students were always my inspiration and guiding force behind the creation of the SULMOWS mnemonic strategy for solving algebraic equations.

Finally, I give honor to my parents Clinton and Sharon Strozier, for whom my participants in the study are named after, for being my biggest fans. From birth, you all have always encouraged me to continue on any journey I wanted to pursue regardless of the obstacles or fears that were there. To my sisters, Shantal and Chantrice, I want to say thank you for providing listening ears and motivating me to continue my education. I am grateful for my nephew, Clinton, for understanding that "Auntie" could not come home because she had to do homework. Thank you for the sprints from one side of the room to me to give me gigantic hugs whenever I came home. To other family members and friends, thank you for your unconditional love, support, patience, and understanding during these many years.

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CHAPTER 1. INTRODUCTION

In the past 30 years, the educational setting for students with disabilities has changed significantly. It is no longer acceptable to exclude students with disabilities from their right to a free appropriate public education or least restrictive environment, which may include the general education classroom. The number of students with disabilities educated for the majority of the school day in the general education classroom is growing each year (U. S. Department of Education, 1996). This educational shift from exclusion to inclusion of students with disabilities was because of laws such as the Education of All Handicapped Children Act of 1975 (later renamed Individuals with Disability Education Act [IDEA] in 1990). This law and other similar to it gave students with disabilities certain rights. Under IDEA students with disabilities had the right to a free appropriate public education. Although IDEA and its amendments gave students with disabilities access to school and the general education, they did not ensure participation in the general education curriculum. The expectation, with regard to what students with disabilities achieve, was addressed by the 1997 Amendments to the IDEA, which emphasized students' participation and progress in the general education curriculum. In addition to IDEA, the standards outlined in the No Child Left Behind Act of 2001 (NCLB; 2002) required states to annually assess the reading and mathematics skills of all students in third through eighth grade, including those with disabilities. The reauthorization of the Individuals with Disabilities Education Improvement Act (IDEIA; 2004) reinforced these standards by requiring students with disabilities, despite their intellectual, emotional, or physical challenges, to meet the same math standards as their grade-level peers. Meeting these standards presents a significant challenge for students who struggle in mathematics, and the challenge is especially daunting for students with disabilities (Scheuermann, Deshler, & Shumaker, 2009). Prompted by these requirements, educators are searching for ways to instruct students who struggle with mathematics, specifically those performing below the 25^{th} percentile on standardized measurements and who may be identified as students with mathematics learning disabilities (MLD) (Scheuermann, Deshler, & Shumaker).

Not only have IDEIA and NCLB changed the curriculum expectation for students with disabilities, the National Council of Teachers of Mathematics (NCTM) has also altered their core curriculum. The Principles and Standards for School Mathematics (first published in 1989 and revised in 2000) are the reason for this change, with focus on conceptual understanding and problem solving rather than perceptual knowledge or rule driven computation. Most states and districts have used the NCTM Standards to some degree in revamping their mathematics curricula. Due to these higher standards to receive a high school diploma, NCTM (2000) emphasized the need to prepare all students for algebra. As students progress in school, the advanced courses are dependent upon the successful completion of prerequisite courses, such as algebra (National Center for Education Statistics, 2010). Successful completion of algebra in middle school has been shown to lead to improved performance in high school as well as understanding of higher concepts. A strong foundation of algebra concepts should be in place by the end of eighth grade (National Council of Teachers of Mathematics, 2012; Wang & Goldschmidt, 2003). Although higher-level math skills are vital to their future, many students with disabilities experience difficulty with secondary math concepts, such as algebra (Maccini, McNaughton, & Ruhl, 1999).

In early 2012, the Council for Exceptional Children stated that there was a critical need for instruction and interventions that go beyond the traditional abstract-only instruction (CEC Webinars, 2012). Because of the abstractness of algebra, many students struggle with the concepts and the skill becomes overwhelming for many students with disabilities. Now that states require all students to adhere to the same graduation standards, introducing high-stakes assessments, algebra performance is a great concern for students with disabilities (Witzel, Mercer, & Miller, 2003). Devlin (2002) stated that for students to understand abstract concepts more easily, it is important for them to learn precursor concepts in a concrete manner first. One way to simplify students' understanding of abstract concepts is to transform such complex concepts into concrete manipulative and pictorial representations.

Statement of the Research Problem

While research has demonstrated that mnemonic strategies and CRA have been successful in teaching math skills, very few studies used a specific mnemonic for solving one-step, two-step, and multiple-step equations. Additionally, the study that used a specific algebraic mnemonic included high school participants (Strozier, Hinton, Flores, & Shippen, 2012). Furthermore, none of the current literature regarding mnemonics and CRA algebra instruction has addressed multiple step equations using the distributive property as the first step to solving the equation. In addition, none of the current literature analyzed common mistakes that students made in solving equations. These results led to the development of this study, which focuses on implementation of a mnemonic strategy while teaching algebraic equations using CRA and SUMLOWS mnemonic strategy to solve one-step, two-step and multiple-step equations using the distributive property for middle school students. Another focus of this study was to investigate the types of errors students make while solving algebraic equations.

Justification for the Study

The Council of Exceptional Children (2012), NCTM, (2012), and NCTC (2000) agree that algebra was an important concept for all students to understand. The need for students to successfully complete algebra has become increasingly apparent over the decade. Due to the rigor of graduation standards, algebra performance is a great concern for students with disabilities (Witzel, Mercer, & Miller, 2003).

Algebra is abstract in nature and many educators have struggled to help students comprehend initial algebra instruction (CEC Webinars, 2012; Witzel, Mercer, & Miller, 2003). One way to simplify students' understanding of abstract concepts is to transform such complex concepts into concrete manipulative and pictorial representations. Concrete-Representational-Abstract Sequence (CRA) has been proven to be an effective instructional sequence for many math concepts, such as addition, subtraction, (Miller & Mercer, 1993), multiplication (Harris, Miller, & Mercer, 1995; Morrin & Miller, 1998; Strozier, Hinton, Terry, & Flores, 2012), fractions (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Flores, 2010), integers (Maccini & Hughes, 2000; Maccini & Ruhl, 2000), and solving equations (Strozier, Hinton, Flores, & Shippen, 2012; Witzel, Mercer, & Miller, 2003). In addition to using the CRA sequence, mnemonics have proven to be effective in aiding students through the mathematical learning process (Strozier, Hinton, Flores, & Shippen, 2012; Kaffar & Miller, 2011; Cassel & Reid, 1996; Mercer & Miller, 1992). Examples of mnemonics that have been used in research were Mercer and Miller's (1992) DRAW that stands for (a) discover the sign, (b) read the problem, (c) answer with a conceptual representation, and (d) write the answer; and Cassel and Reid's (1996) mnemonic FASTDRAW which stands for (a) read the problem out loud; (b) find, highlight the question, then write the label; (c) ask what are the parts of the problem then circle the numbers needed; (d) set up the problem by writing and labeling the numbers; (e) re-read the problem and decide if addition or subtraction is required, (f) discover the sign by rechecking the operation; (g) read the number problem; (h) answer the problem and; (i) write the answer and make sure the answer makes sense. Other mnemonics include RENAME which stands for (a) read the problem, (b) examine the ones, (c) note the ones, (d) address the tens column, (e) mark the tens column, and (f) examine and not the hundreds and exit with a quick check; and SUMLOWS that stands for (S) Separate the sides, (U) Unite Like Terms, (M) Modify the new equation, (L) Loop (circle) around the coefficient, (O) Opposite sign (inverse operation), (W) What you do to one side, you must do to the other, and (S) Substitute the solution for the variable and check (Strozier, Hinton, Flores, & Shippen, 2012). With the many steps involved in solving equations, students need specific steps that will aid them in solving difficult algebraic equations, including those with the distributive property. Furthermore, by identifying students' common error patterns, algebra interventions can be developed or modified to address these common student mistakes.

Purpose of the Study

The purpose of this study was to investigate the use of CRA with SUMLOWS mnemonic strategy to teach algebra to middle school students who struggle in math. This was accomplished by examining one-step, two-step, and multiple-step probes given before each lesson. The probes were graded as correct or incorrect and then analyzed for types of errors that students made while completing probes.

Research Questions

For this study, the following research questions were developed:

1. What are the effects of CRA and SUMLOWS on solving one-step equations?

2. What are the effects of CRA and SUMLOWS on solving two-step equations?

3. What are the effects of CRA and SUMLOWS on solving multiple-step equations?

4. What are the effects of CRA and SUMLOWS on students' maintenance of solving equations two weeks after instruction ended?

5. What are the common mistakes that students made on probes?

Definitions of Terms

In order to avoid any ambiguity, confusion, or misunderstanding in the usage of terms, a definition section has been added to this study.

Absolute Value*:* The distance a number is from zero on the number line.

Algebra: A branch of mathematics dealing with symbols

Algebra Tiles: Mathematical manipulatives that allow students to better understand ways of algebraic thinking and the concepts of algebra. These tiles have proven to provide concrete models for students while learning algebra.

Algebraic Equation: An equation that contains sums and/or products of variables and numbers.

Algebraic Expression: An expression that contains sums and/or products of variables and numbers.

Coefficient: The numerical part of a term that contains the variable.

Concrete-Representational-Abstract (CRA) Sequence: An instructional sequence used in mathematics that provides instruction first through the use of objects (concrete phase), then through the use of pictures (representational phase), and finally through the use of numbers and symbols only (abstract phase).

Constant: A terms without a variable, such 2 in the expression 3X + 2

Distributive Property (sometimes referred to as the Distributive Law): A property indicating a special way in which multiplication is applied to addition of two or more numbers in which each term inside a set of parentheses can be multiplied by a factor outside the parentheses, such as 2(4X – 5).

DRAW: A mnemonic device that helps remind students the procedure for completing mathematics problems. The DRAW strategy has four steps: (a) discover the sign, (b) read the problem, (c) answer with a conceptual representation, and (d) write the answer (Mercer & Miller, 1992).

Equation: A mathematical sentence that contains an equal sign.

FAST DRAW: A mnemonic device that helps remind students the procedure for completing mathematics problems. The steps of FAST DRAW are as follows: (a) read the problem out loud; (b) find, highlight the question, then write the label; (c) ask what are the parts of the problem then circle the numbers needed; (d) set up the problem by writing and labeling the numbers; (e) re-read the problem and decide if addition or subtraction is required, (f) discover the sign by rechecking the operation; (g) read the number problem; (h) answer the problem and; (i) write the answer and make sure the answer makes sense. (Cassel & Reid, 1996).

FOIL Method: To multiply two binomials, find the sum of the products of the First terms, the Outer terms, the Inner terms, and the Last terms.

Integers: The whole numbers and their opposites (i.e., -3,-2, -1, 0,1,2,3).

Inverse operation: Operation the undoes another, such as addition and subtraction; multiplication and division

Like Terms (also known as combining or uniting like terms): Expressions that contain the same variables to the same power, such as 2n and 5n or $6xy^2$ and $4xy^2$.

Maintenance Data: Data which are taken using single subject methodology that measures whether a student has mastered a skill or concept being taught.

Mild Intellectual Disability: A category of eligibility defined as a significantly subaverage general intellectual functioning, existing concurrently with deficits in adaptive behavior and manifested during the developmental period, that adversely affects a child's educational performance.

Mnemonic Device: A learning technique that aids memory and improve long term memory.

Multiple Baseline Design: A single subject research design which examines an effect of an intervention through replicating a change in behavior at least three times across different people, settings, or behaviors. This design is used when a behavior cannot be unlearned.

Multiple-step Equations: An equation that consist of more than two steps to find the solution.

Negative Number*:* A number less than zero

Please Excuse My Dear Aunt Sally (also known as PEMDAS): A mnemonic device for solving order of operations expressions. The rule is used to identify the order in which procedures should be performed when solving mathematical expressions. The steps of PEMDAS are as follows: P) Parenthesis, E) Exponents, MD) Multiplication and Division (whichever comes first from left to right), and AS) Addition and Subtraction (whichever comes first from left to right).

One-step Equation*:* Equations that only take one operation to isolate the variable.

Opposites: Two numbers with the same absolute value but have different signs, such as -3 and +3.

Positive Number: Any number that is greater than zero.

Probe: An assessment that monitors a student's progress toward a behavior goal.

Simplify the Expression: To use distribution to combine like terms.

Specific Learning Disability: A category of special education eligibility defined as a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, that may manifest itself in the imperfect ability to listen, think, speak, read, write, spell, or to do mathematical calculations, including conditions such as perceptual disabilities, brain injury, minimal brain dysfunction, dyslexia, and developmental aphasia.

Solution: A value for the variable that makes an equation true. For x + 7 = 19, the solution is 12.

Social Validity Data: Data taken that examines the social importance and practicality of an intervention as well as the intervention outcomes.

Solving an Equation: The process of finding a solution to an equation.

Substitute the Solution: A checking mechanism used algebra. Once the solution is found, it is inserted into to the original equation in place of the variable. If both sides of the equation are equal, the solution is correct.

SUMLOWS: A specific mnemonic used for solving algebraic equations. The SUMLOWS are as follows: (S) Separate the sides, (U) Unite Like Terms, (M) Modify the new equation, (L) Loop (circle) around the coefficient, (O) Opposite sign (inverse operation), (W) What you do to one side, you must do to the other, and (S) Substitute the solution for the variable and check (Strozier, Hinton, Flores, & Shippen, 2012).

Term: When addition or subtraction signs separate an algebraic expression into parts, each part is a terms.

Two-step Equation: An equation that contains two operations, such as 2X + 4 = 12.

Variable: A variable is a placeholder for any value, such as X, can be used as a variable.

What you do to one side, you must do to the other: This saying is used when solving equations to keep each side of the equation balanced.

Limitations of the Study

The study participants were two students who demonstrated substantial improvement with the CRA and SUMLOWS instruction for solving one-step, two-step, and multiple-step algebraic equations. Due to the number of participants, results cannot be generalized. CRA and SUMLOWS instruction was not compared with any other mathematics programs. Therefore, there may be other instructional programs or interventions as or more effective. A limitation in the examination of CRA and SUMLOWS was the lack of representation from other disability subgroups; thus the results may not be representative to other students with disabilities. Also, generalizability of results across setting is a limitation because instruction was provided in one school. The setting of this study involved students in small group during tier 2 instruction. It is unclear if the same effects would be replicated in a larger group in a general education classroom. In addition, the researcher assisted with implementation of the majority of the instruction. It is unclear if a classroom teacher would have yielded the same results. Lastly, the analysis of common errors were assess from the probes of two students in this study. Results of the probe error analysis cannot generalize potential errors that other students may make while solving algebraic equations.

Summary

While research has demonstrated that mnemonic strategies and CRA have been successful in teaching math skills, very few studies used a specific mnemonic for solving one-step, two-step, and multiple-step algebraic equations. Furthermore, none of the current literature regarding mnemonics and CRA algebra has addressed multiple step equations using the distributive property as the first step to solving the equation. In addition, none of the current literature analyzed common mistakes that students made when solving equations. Additionally, NCTM has expressed a need and a great concern that all students need to learn to solve algebraic equations before entering high school. CEC Webinars (2012) stated that there was a great need for algebra instruction and interventions that go beyond the typical approach. Therefore, the CRA and SUMLOWS instruction must be examined. Researchers and classroom teachers need to know that students can learn to solve algebraic equations, including students with disabilities, with the use of CRA and SUMLOWS instruction. This study examined the effects of CRA and SUMLOWS instructional sequence on students that struggle with math. This research provided information for classroom algebra teachers with regard to students learning to solve one-step, two-step, and multiple-step equations while providing a specific strategy for solving equations. Additionally, this research provided students with access to effective algebra instruction and raised the mathematical achievement of students who struggle in math.

CHAPTER 2. REVIEW OF LITERATURE

Introduction

Meeting grade level math standards presents a significant challenge for students who struggle in math and the challenge is especially daunting for students with disabilities (Scheuerman, Deshler, & Shumaker, 2009). Students with disabilities are required to meet the same math standards as their grade-level peers. *No Child Left Behind* (2002) requires states to annually assess math skills of all students in third grade through eighth grade, including students with disabilities. Thus, teachers in the general education classroom are expected to teach students with disabilities the same grade level content. In order to meet math standards, educators are searching for ways to instruct students who struggle with math.

As students' progress in school, the advanced skills are dependent upon the successful completion of prerequisite skills. Successful completion of algebra in middle school has been shown to lead to improved performance in high school. Therefore, a strong understanding of algebra concepts should be in place by the end of eighth grade (Matthews & Farmer, 2008). Although higher-level math concepts are vital to their future, many students with disabilities experience difficulty with math concepts, such as algebra (Maccini, McNaughton, & Ruhl, 1999). Due to the abstract nature of algebra, many students with disabilities become overwhelmed with algebra. CEC Webinars (2012) stated that there was a critical need for algebra instruction and interventions for students with disabilities that goes beyond the typical teaching approach. Today's secondary general and special education teachers must be prepared to educate students with disabilities to achieve high math standards. Therefore, math teachers must utilize a variety of different instructional practices to ensure that students at-risk and students with disabilities are successful in meeting these rigorous standards. This chapter examines research that contributes to improving the algebra skills for secondary school students who struggle in math. This review is presented in three major sections: overview of students with disabilities, mnemonics use in math, and CRA for math. The CRA component is further divided into two sections: CRA for basic math skills and CRA for algebra.

Students with Learning Disabilities

Rigorous academic expectations, as required by current laws, may be challenging for many secondary school students, particularly those with learning disabilities (LD), given their unique learner characteristics. Specifically, research suggests that 5–8 percent of school-aged students experience some sort of mathematics LD (Geary, 2004). Students with LD commonly have difficulties identifying key information, connecting relationships between broad concepts and details, and strategically approaching the solving of mathematical word problems. Students with LD may also lack fluency with mathematical facts and have problems with basic mathematical procedures (Maccini & Ruhl, 2000; Mercer, 1997).

However, given the difficulties experienced by many students with LD in math, it is imperative to incorporate instructional practices that are both effective and efficient for helping secondary students with LD access the general education math curriculum in a meaningful way (Maccini, Mulcacahy, & Wilson, 2007). Many students with disabilities and those at risk for educational failure exhibit problems remembering academic material (Mastropieri & Scruggs, 1998). One intervention that has been successful in helping students remember procedural steps is the use of mnemonics (Bottege, 1999; Cade & Gunner, 2002; Greene, 1999; Manalo, Bunnell,

& Stillman, 2000; Steele, 2002). Mnemonic strategy refers to “a word, sentence, or picture device or technique for improving or strethening memory” (Test & Ellis, 2005, p. 12). Mnemonics can be classified as fact mnemonics (i.e., used to help students recall facts) or process mnemonics (i.e., used to help remember steps, processes, or rules; Manalo, Bunnell, & Stillman, 2000; Test & Ellis, 2005). With respect to mathematics, intervention research studies have included the areas of basic mathematics computation (VanDerHeyden & Witt, 2005), strategy instruction for problem solving and the use of the concrete-representational- abstract (CRA) sequence to teach place value, geometry, and fractions (Fuchs, Fuchs, & Hollenbeck, 2007). Concerns regarding the poor performance of students with disabilities have increased due to several federal laws. Because of these laws, students with disabilities will receive instruction on the same important concepts as their grade-level peers without disabilities. These concepts include high-level conceptual understanding and problem-solving. These higher-level skills are challenging for many students, but especially for students with disabilities (Butler, Miller, Crehan, Babbitt, & Pierce, 2003). The remaining major sections will review research studies in the area of math mnemonics and the CRA sequence of instruction for students with disabilities.

Research Regarding Mnemonic Strategies in Mathematics

For more than 20 years, mnemonic strategies, such as FOIL Method and PEMDAS, have been demonstrated to be highly effective with students with disabilities as well as students without disabilities (Mastropieri, Sweda, & Scruggs, 2000). Mnemonic strategies such as reconstructive elaborations, the key word, and peg word methods have been successfully employed by researchers to improve recall and learning in students with LD (Green, 1994; Mastropier, & Scruggs, 1991). Scruggs and Mastropieri (1990) conducted a meta-analysis of 34 experiments involving the use of mnemonic strategies with students with mild disabilities (LD, behavioral disorders, mild intellectual disabilities). The overall effect size was 1.62 (cf. Mastropieri & Scruggs, 1989), a very high level of effectiveness (Forness, Kavale, Blum, & Lloyd, 1997).

The effectiveness of mnemonic strategies was demonstrated to be consistent across a variety of subject areas, age, and disability areas of participants, and instructional settings. Mnemonic strategies have been found to be effective in experimental "laboratory" investigations, classroom implementations, and teacher-directed applications (Scruggs & Mastropieri, 1990). In earlier investigations, mnemonic strategies have been effective in promoting verbal memory objectives without detracting from other important learning objectives.

Two studies (Manalo, Bunnel, & Stillman, 2000; Mercer, Jordan, & Miller, 1994; Test & Ellis, 2005) incorporated some type of mnemonic strategy to help learners remember information. In the first study, Test and Ellis (2005) analyzed the effectiveness of the LAP strategy for helping students solve fraction problems with unlike denominators. Six students, including three students with LD, participated in the delayed multiple-probe design across three pairs of students. Students were paired according to their abilities and compatibility and each pair was exposed to the intervention after the previous pair obtained criterion. The strategy consisted of a first-letter mnemonic, LAP, for solving adding and subtracting fractions: "**L**ook at the denominator and sign; **A**sk yourself the question, "Will the smallest denominator divide into the largest denominator an even number of times?"; **P**ick your fraction type' (Test & Ellis, 2005). The LAP fraction strategy instruction included: (a) teacher modeling of the strategy, (b) guided practice of the steps with the teacher and students restating the steps, (c) individual practice of the strategy steps, and (d) pair practice using games and flash cards to recall the strategy steps and types of fractions.

The second math mnemonic strategy is FAST DRAW (Cassel & Reid, 1996; Mercer & Miller, 1992) cues students to solve math problems involving computational tasks. The strategy was used to help elementary students with learning problems to compute math facts and solve word problems. In addition to explicit teacher strategy modeling, their intervention included guided practice, independent practice, corrective feedback, student generated word problems, mnemonics, mastery learning, and graphing of student progress (shown in Table 1). FAST stands for **F**ind what you're looking for, **A**sk yourself, "What are the parts of the problem?" **S**et up the numbers, and **T**ie down the sign. DRAW in the mnemonic strategy stands for **D**iscover the sign, **R**ead the problem. **A**nswer or DRAW a conceptual representation of the problem using lines and tallies, and check, and **W**rite the answer and check. Results of this study indicated that the students made substantial gains in math achievement. Moreover, they acquired, maintained, and generalized the strategies to help students represent and solve word problems (Mercer, Jordan, & Miller, 1994). In addition to mnemonics strategies, CRA research have been found to be effective for teaching students with disabilities or at-risk for failure in math.

Table 1

Steps of Fast Draw

Steps	
Step 1	**F**ind what you're solving for.
Step 2	**A**sk yourself, "What are the parts of the problem?"
Step 3	**S**et up the numbers
Step 4	**T**ie down the sign
Step 4	**D**iscover the sign
Step 5	**R**ead the problem
Step 6	**A**nswer or DRAW a conceptual representation of the problem using lines and tallies, and check,
Step 7	**W**rite the answer and check

Research Regarding CRA Instruction

Mastropieri, Scruggs, and Shiah (1991) reviewed 30 research studies related to the provision of mathematics instruction for students with learning disabilities. Their findings document the effectiveness of behavioral interventions (e.g., use of manipulatives and pictures) and cognitive interventions (e.g., self-instruction) for teaching computation and problem-solving skills. Using manipulatives and pictures, various studies used a three sequence strategy, concrete, representational, and abstract (CRA), to teach place values (Mercer & Miller, 1992; Miller & Mercer, 1993; Peterson, Mercer, & O'Shea, 1988) to students with math disabilities. At the concrete level, the concept of place value was taught using three dimensional objects. At the representational level, pictorial presentations were used. At the abstract level, only numbers were used. The results showed that students who received the three sequence strategy outperformed students who received traditional instruction using numbers only.

The initial research of CRA was studied by Peterson, Mercer, and O'Shea (1988). This study investigated the effectiveness of teaching students with learning disabilities place value through a conceptual sequence including three levels of understanding (i.e., concrete, representational , and abstract) compared to the effectiveness of teaching the same skill at the abstract level without manipulatives or pictorial representations. They assessed the effect on these two procedures on initial skill acquisition, maintenance, retention, and generalization to a higher skill.

The participants for this study were elementary and middle school students with LD. There were 20 males and 4 females ranging in age from 8 to 13. All participants received mathematics instruction in special education classrooms in Florida. Of the 24 subjects, 19 were in self-contained classrooms, 4 were temporarily placed in a self-contained diagnostic classroom, and 1 was in a resource classroom.

The study was divided into three phases. Phase one was a training period for teachers involved in the study. Phase two involved 9 days of direct instruction in an effort to maximize performance. The final phase consisted of posttreatment skill acquisition, maintenance, retention, and generalization measurement.

During phase one, the five teachers attended training workshops. These teachers were introduced to the direct instruction model, effective teaching research, and the concrete-to-abstract teaching sequence. Examples and nonexamples were also provided to the teachers. Next, one of the 18 lessons was demonstrated for the teachers. The teacher role was modeled by the workshop leader. During modeling, the workshop participants acted as the students receiving instruction. Teachers were given the scripts and materials of all 18 lessons. After practicing lessons on students not involved in the lessons, teachers were ask to demonstrate one lesson to the group. Teachers had to demonstrate mastery before beginning the actual treatment with participants in the study.

Phase two included actual direct instruction for participants in the study. Participants with LD served in both the experimental and control groups. During this phase, participants were taught to identify how many ones or tens were in a double-digit number using a six-step direct instruction model. The steps were: (a) provide an advance organizer, (b) demonstrate and model the skill, (c) provide guided practice, (d) provide independent practice, (e) give posttest and (f) provide feedback.

Table 2

Steps of Direct Instruction Model

Steps	
Step 1	Provide an advance organizer
Step 2	Demonstrate and model the skill
Step 3	Provide guided practice
Step 4	Provide independent practice
Step 5	Give posttest
Step 6	Provide feedback

The advance organizer phase ensures students have mastered the required pre-requisite skills, states what the students will learn, and builds relevance for the concept. The demonstration phase is when the teacher presents the concept and step by step strategies that detail how to solve the problem. In the guided practice phase, students and the teacher solve the problem together which allows students to practice with teacher input and feedback. Finally, the independent practice phase students solve problems without teacher assistance.

The CRA sequence involved three levels of understanding to teach a mathematical concept. Each level of instruction is taught explicitly through the use of an advance organizer, teacher demonstration, guided practice, independent practice and a post organizer. The first level is the concrete level in which manipulatives are used to promote conceptual understanding. The second level of understanding is the representative phase of instruction. It involves an instructor teaching the same concept and type of mathematical problems by representing the concept using pictures or drawings instead of manipulating objects. This level of instruction is similar to the concrete level; however, the student draws pictures to solve the mathematical problems. The representational level of instruction is a bridge to the abstract level in which a student uses solely numbers to solve mathematical problems. The abstract phase is the last level of instruction and students use numbers only in completing the mathematical task.

Each of these nine lessons for both groups was scripted to last between 10 and 15 minutes. Total time-on-task was held constant for both groups and both groups received the same amount of instruction on a given day. Verbal praise was included in the teacher scripts for both groups. If a student in a group did not meet criterion, the lesson was retaught on a one-to-one basis with the teacher. However, no participants needed reteaching. The only difference between the groups were three lessons using concrete manipulatives, three lessons using representational instruction, and then three lessons with abstract instruction. The control group received all nine lessons at the abstract level.

The third and final phase included postteatment. Posttreatment was conducted after the instructional implementation was complete. During this phase, a posttest was administered by a person independent from the instructional process. Posttests were administered one-on-one to each participant to measure skill acquisition. In addition, each participant's ability to generalize newly learned information to a different stimulus (i.e., three-and four digit numbers) was tested. The following week, skill acquisition and generalization were again measured for maintenance. Three weeks after instruction ceased, retention and generalization were again tested.

The study used a multivariate analysis of variance (MANOVA). The MANOVA yielded a significant effect for instructional treatment, $F(2, 21) = 4.49$, $p < .05$. Therefore, two follow-up univariate analysis of variance were calculated, one for the acquisition and one for the generalization measure. There was a significant main effect for the instruction treatment variable found on the acquisition measure, $F(1, 22) = 8.79$, $p < .01$. Participants who received the concrete-to-abstract instructional sequence identified ones and tens in double-digit numbers significantly better than the students who received only abstract-level instruction. No main effect difference for the instructional treatment was found on the generalization measure, $F(1, 22) = 1.66$, $p > .05$. Participants in the experimental and control groups were statistically similar when performing the untaught skill of identifying ones and tens in three-and four digit numbers.

This study found that the concrete to abstract teaching sequence is more effective than only abstract teaching with students with learning disabilities who acquire initial place value skills. The use of manipulatives and pictorial representations affects skill acquisition, maintenance, and retention positively. This study suggested that further research was needed to validate the concrete-to-abstract teaching sequence with other math skills and to examine in greater detail the effects of this sequence on generalization skills of students with LD.

The initial study of CRA was later extended to include a number of basic skills. Miller and Mercer (1991) compiled many basic skills into a series of CRA lessons. The curriculum is called the Strategic Math Series (SMS). Strategic Math Series is comprised of seven manuals. Each manual at this level is designed to teach one of the following mathematical skills: (a) addition facts 0 to 9, (b) subtraction facts 0 to 9, (c) place value, (d) addition facts 10 to 18, (e) subtraction 10 to 18, (f) multiplication facts 0 to 81, and (g) division facts 0 to 81. Each level of instruction has its own manual. Each manual provides a systematic method for teaching problems that only involves numbers no greater than the sum listed. All lessons to teach the facts are included with in the manual. Included in the manuals are numbered lessons that feature the name that describes the major instruction within that lesson. It also includes pretest and post test lessons that are not numbered. Manuals also include evaluation guidelines and answers keys for the tests. In addition to lessons and assessments, games and student materials are also included in manuals. CRA studies are shown in Table 3

Table 3

CRA Studies with Basic Concepts

Authors and Date	Intervention	Participants	Design	Outcomes Measured
Peterson, S. K., Mercer, C. D., & O'Shea, L (1998)	CRA sequence	24 (20 males & 4 females) ranging in age from 8 to 13 with LD	Group comparison of CRA sequence to abstract only	Place value
Mercer, C. D. & Miller, S. P (1992)	Strategic Math Series	CRA place value: 40 students (18 –at risk, 4 - ED, 15- LD, and 3- SED) Basic facts: 109 elementary school students (102 with LD, 5- ED, and 2 – at risk)		Place value and basic facts

(table continues)

Table 3 (continued)

Authors and Date	Intervention	Participants	Design	Outcomes Measured
Miller, S. P. & Mercer, C. D. (1993)	CRA sequence	9 students (5 w/ LD, 3 at risk, & 1 ID)	Single-subject – multiple baseline across subjects	Basic facts and coin sums
Harris, C. A., Miller, S. P., & Mercer, C. D. (1995)	CRA sequence with DRAW and FAST DRAW	13 second graders (12 with LD and 1 with an emotional disability)	Single-subject – multiple baseline across subjects	Multiplication
Morin, V. A., & Miller, S. P (1998)	CRA sequence with DRAW and FAST DRAW	3 – 7^{th} grade students with ID	Single-subject – multiple baseline across subjects	Multiplication and related word problems
Butler, F. M., Miller, S. P., Crehan, K., Babbitt, B., & Pierce, T. (2003)	CRA sequence	50 students with mild-moderate disabilities in 6-8 grades (42- SLD, 8 – other disabilities)	Group comparison of CRA sequence to RA teaching sequence	Fractions
Flores, Margaret (2009)	CRA sequence	6- 3^{rd} graders (4- SLD)	Single-subject – multiple baseline across subjects	Subtractions with regrouping
Flores, Margaret (2010)	CRA sequence	6- 3^{rd} graders	Single-subject – multiple baseline across subjects	Subtractions with regrouping to the tens and hundreds place
Kaffar, B. J. & Miller, S. P. (2011)	CRA Sequence and RENAME Strategy	11 students (8- at risk and 3 with disabilities)	Group comparison of CRA sequence to abstract	Subtraction with regrouping
Strozier, S.D., Hinton, V., Terry, L., & Flores, M. (2012)	CRA sequence	3 students with Autism Spectrum Disorder	Single-subject – multiple baseline across behaviors	Addition with regrouping, subtraction with regrouping, and multiplication facts from 0-5

Research Regarding CRA with Mnemonic Device

Mercer and Miller (1992) field-tested the Strategic Math Series (SMS) curriculum. This 1992 investigation expanded the previous study of Peterson, Mercer, and O'Shea (1988). This study built upon prior research on Concrete-Representational-Abstract sequence as an effective way to teach students with learning disabilities place value. This study evaluated the effectiveness of SMS curriculum on acquiring, understanding, and applying basic math facts.

Not only did this study evaluate the effectiveness of the CRA teaching strategy on place value, it also extended the CRA sequence on other basic facts and included the DRAW strategy (shown in Table 3) in phase 4 of the Strategic Math Series.

The SMS provided a systematic means of CRA instruction. According to the CRA sequence, instruction begins at the concrete level, where the student uses three dimensional objects to solve computation problems. After successfully solving several problems at the concrete level, students proceeds to the representational level. At this level, drawings are used to solve problems. The third and final level is the abstract level. At the abstract level, students looked at the computation problem and tried to solve it without using objects or drawings.

The instructional sequence of SMS curriculum is divided into seven phases with 21 basic lessons. Student completion of all 21 lessons is important for two reasons. First, the lessons are sequenced and build upon each other in terms of complexity. Second, although most students acquire the respective computation skill (e.g., multiplication facts) when they reach the posttest, they need additional practice to maintain their knowledge and skills, to increase their fluency, to ensure further development of their problem-solving skills.

Phase 1 is the pretest. During this instructional phase, a pretest is administered to the student to determine whether instruction is needed. Before the pretest, a rationale for assessing the respective basic facts is discussed with the student. If his or her score on the pretest falls below the mastery criterion (i.e., 80%), the student is informed that he or she needs to work on the targeted basic facts. The need for instruction is discussed, and a commitment to learn is obtained from the student by a signed contract.

Phase 2 is teaching the concrete application. The concrete phase of instruction includes Lessons 1 through 3. For each lesson, a sample script and learning sheets guide the teacher through the instructional sequence. During these lessons, students manipulate concrete objects to solve basic facts on their learning sheets. A separate curriculum manual is used for each skill area (i.e., addition, subtraction, multiplication, or division). Students solve word problems in which the numbers are vertically aligned and then write the name of the object on the space provided to the student. These concrete lessons act as a spring board for learning facts at representational and abstract levels.

Phase 3 is teaching the representational application. This phase includes Lessons 4 through 6. A sample script and learning sheets guide the teacher through the instructional sequence. During this phase, students use drawings and tallies to solve basic facts where numbers are vertically aligned. Students will then fill in blanks after the numbers with the name of the drawing. Representational lessons help students understand the respective facts as they more toward the abstract level.

Phase 4 is the introduction of the "DRAW" strategy. This strategy helps students solve facts at the abstract level. Each letter of DRAW cues students to perform certain procedures. The four steps of the DRAW strategy are shown in Table 4.

Table 4

Steps of Draw

Steps	
Step 1	**D**iscover the sign
Step 2	**R**ead the problem
Step 3	**A**nswer or DRAW a conceptual representation of the problem using lines and tallies, and check,
Step 4	**W**rite the answer and check

Phase 5 is teaching the abstract application. This phase of instruction starts in lesson 8 and continues to lesson 10. Each lesson has a scripted guide to use through the instructional sequence. Throughout these lessons, students use the DRAW strategy to solve abstract level problems when they are unable to recall an answer. Students begin to solve word problems in which numbers are vertically aligned. During this time, students also include the names of common objects and phrases after the numbers instead of blank spaces provided.

Phase 6 is a posttest. The posttest is administered to determine whether the student has learned basic facts and is ready to proceed to the next phase of instruction. The next level of instruction is designed to build fluency skills and further develop problem solving skills. If the student's score is below 90%, he or she repeats one or more lessons in the abstract phase.

Fluency is practiced in Phase 7. Practice takes place in lessons 11 through 21. Each lesson has a scripted guide to assist the teacher through the sequence. During this phase, students work on three primary areas: (a) solving word problems, (b) increasing computation rate, and (c) discriminating previously learned facts from new facts with accuracy. To increase difficulty, problems are presented in sentence form instead of vertically aligned. As lessons progress, students create their own word problems as they learn to filter extraneous information. As a way to increase the rate of computation, a 1-minute timed probe is given during selected lessons. To help students discriminate between facts and practice previously learned facts, students receive a fact review page that contains two or more types of facts. These pages are given to students during selected lessons throughout this phase. Phases of the strategic math series are included in Table 5.

Table 5

Phases of the Strategic Math Series

Phases	Purpose	Lessons
Phase 1	Pretest	Pretest Lesson
Phase 2	Concrete Application	Lessons 1-3
Phase 3	Representational Application	Lessons 4-6
Phase 4	Introduce DRAW Strategy	Lesson 7
Phase 5	Abstract Application	Lessons 8-10
Phase 6	Posttest	Repeat Phase 5 if below 90%
Phase 7	Practice to Fluency	Lessons 11-21

In order to evaluate the effectiveness of the SMS curriculum, Mercer and Miller (1992) field-tested the program with 109 elementary students. Out of these students, 102 students had a learning disability, 5 students had an emotional behavior disorder, and 2 were at risk. Field testing took place in special education classrooms in 7 school districts in Florida. Small groups were categorized as having less than 7 students and large groups consisted of 7 to 18 students. The lessons were used to teach place value (one and tens) and basic facts (addition, subtraction, multiplication, and division).

The study had a pretest/posttest design. The pretest was given prior to Lesson 1 and the posttest was administered within 1 to 5 days following to Lesson 21 by examiners whom the students did not know. The posttest contained two problems with extraneous information and two problems without extraneous information. This test required students to create two word problems to solve them. Five to ten school days after the termination of instruction, a retention test was given to students. The retention test examiner was also unknown to the students.

Teacher and student questionnaires were gathered to assess social validity. Data from the questionnaires indicated that of the 22 teachers who participated, 21 (96%) indicated they would use the SMS curriculum again. Of the 75 students who were asked to complete follow-up questionnaires, 60% rated SMS as better than other math instruction and 51 % rated it as equal to other math instruction. Consequently, 90% rated the SMS curriculum as equal to or as better as other math instruction. Overall, the educators concluded that SMS has positive consumer satisfaction.

Results from the pretest and posttest indicated that students were able to acquire the respective facts within Lessons 1 through 10. The total mean scores revealed that the average gain across skills was 59%. Furthermore, the findings revealed that the students in the subtraction and multiplication groups were able to apply the DRAW strategy to solve problems that were not previously taught. After Lessons 9 through 21, the mean rate improvement, across all skills, was 132%. Given that 15% to 25% weekly improvement is considered an excellent criterion, these data were very positive. With a range of 31% to 69%, the mean weekly percentage increase was 51 across skills. In multiplication, the students significantly reduced their error rates and increased their digits-correct rates. Overall, Miller and Mercer (1992) concluded that the field test data indicated that students with learning difficulty were able to (a) acquire computational skills across facts, (b) solve word problems with and without extraneous information, (c) create word problems involving facts, (d) apply a mnemonic strategy to difficult problems, (e) increase rate of computation, and (f) generalize math skills across examiners, settings, and tasks. CRA continues to hold promise for teaching place value and basic facts.

In 1993, Miller and Mercer continued to investigate the CRA teaching sequence. In this study, they wanted to know whether students with math disabilities generalized from the instructional lessons to abstract-level probes as evidenced by the "crossover effect." Of the 9 students in this study, there were 5 students with LD, 3 students who were classified as at risk for a program for LA, and 1 student who were intellectually impaired. The three students who were classified as at risk for a program for LD received their instruction in the general education classroom. The six students who were receiving special education received their math instruction in the resource rooms.

The students in this study were taught the CRA sequence for three elementary math skills. These skills included: (a) addition facts with sums ranging from 10 to 18, (b) division facts with quotients ranging from 0 to 9, and (c) coin sums up to 50 cents. There were three abstract-level probe sheets (one for each skill). The addition probe sheet contained 60 problems in vertical format. The division probe contained 80 problems written in the traditional horizontal format. The coin probe sheet contained 34 problems written in a horizontal format. Each investigation contained the same three phases. Phase 1 involved the collection of baseline data.

Phase 2 involved the implementation of the CRA teaching sequence, and Phase 3 consisted of posttreatment measures. Throughout the phases, acquisition and retention were assessed.

During the baseline phase (Phase 1), the 1 minute abstract-level probes were administered to each subject on a daily basis. Feedback from the teacher concerning the student performance was withheld. Baseline data (i.e., the rate of correct and incorrect responses per minute) were obtained for a minimum of three days. As a criterion for participation in this study, students had to have more incorrect responses than correct responses on the probes throughout baseline. These procedure was done to ensure that the subjects did not have prior knowledge or understanding of the skills taught.

Throughout the treatment phase (Phase 2), teachers used scripted (20 minute) lessons. Every lesson included four instruction steps (see Table 2). The teacher began each lesson by providing a verbal advance organizer. During the advance organizer, the teacher related the current lesson to previous learning, provided a brief description of the skill being taught, asked students for a rationale for learning the skills, and showed the students the materials that would be used for the lesson.

After completing the advance organizer, the teacher demonstrated the skill being taught in the lesson. Most of the lessons contained three problems to be demonstrated. After the demonstration, students were given an opportunity to model the skill. Two basic procedures were used during the demonstration portion of the lessons. First while demonstrating how to compute one of the problems, the teacher would ask and answer the questions aloud. The second procedure used in the demonstration portion of the lesson involved the teacher asking the students questions while solving the problems. Therefore, the student became more actively involved in the learning process.

The next part of the lesson involved guided practice. With the teacher providing prompts and cues as needed, students were given several problems to complete. During this part of the lesson, the teacher watched the student's performance carefully and immediately provided assistance if errors were made. The goal was to have the student complete the problems successfully with minimal teacher assistance. The remainder of each lesson involved independent practice. During independent practice, students were asked to complete 10 problems independently without any teacher feedback. While giving directions for the independent practice, the teacher would remind students to use previously learned strategies to complete problems.

During the beginning of the treatment phase, three dimensional objects were used to solve problems. Different manipulatives were used during each lesson to promote generalization abject objects. At the representational level of instruction, drawings were used to solve the math problems. By the end of the representational level instruction, students were taught to draw tallies rather than pictures of objects. To save time, tallies were encouraged. After successfully completing lessons at the representational level, teachers presented the abstract-level lessons to the students. At this level, students were taught to look at the problem and try to solve it without using objects or drawings. They were expectant to begin memorizing the facts and to use drawings only for problems not yet committed to memory. After completing every lesson at all three levels, teacher administered a 1 minute assessment probe involving the skill that was taught. One week after completing of all CRA lessons, the posttreatment phase (Phase 3) was implemented. During this phase, teachers administered the 1 minute probe again to determine whether the students retained the skill without any additional instruction or review activities.

A multiple baseline across subjects was used for the three investigations. Each investigation involved different teachers, students, and skills. In other words, one teacher taught three students addition facts. Another teacher taught division facts to three different students, and a third teacher taught a set of different three students how to add coins.

Results from this study indicated that four students experienced the crossover effect during the concrete instruction and five students experienced it during representational instruction. The number of lessons required prior to crossover ranged from three to seven. Once the crossover effect occurred, student's rates for correct responses continued to increase, and their rate for incorrect responses either decreased or remained very low. Students in this study were able to remember skills for at least seven days without intermittent instruction or review. This study by Miller and Mercer (1993) continued to demonstrate that the CRA instructional sequence, implemented via the four-step lesson-plan format was effective for acquisition and short-term retention of basic math facts. This study suggested that CRA is an effective way to teach acquisition and short-term retention of coin sums. Specifically, this study demonstrated when, during the CRA sequence and after how many lessons, students made the association between their concrete or representational instruction.

Harris, Miller, and Mercer (1995) evaluated the effectiveness of teaching multiplication skills to elementary students with disabilities in the general education classrooms. The procedures for teaching multiplication using CRA (see Table 5) had previously been validated in small group instructional arrangements (primarily in resource and self-contained classrooms for students with disabilities) by special education classrooms. This study was designed to extend previous studies to determine how students with disabilities would perform if CRA was implemented in a general education setting in larger group setting along with the general education population.

Participants of this study included 13 second graders. Of the participants in the study, there were 12 students with LD and 1 student with an emotional disability. There were eight male and five female participants and all of the participants were white. Also included in this study were 99 second grades without disabilities. Of these 99 students, 51 were female and 48 were male. There were 83 students who were white, 15 students who were black, and 1 student who was Hispanic. All participants were selected for participation using two criteria. First, parental or guardian permission was required for reporting results. The second criterion for subject selection was the students' ability to pass an experimenter-design screening instrument that was based on prerequisite skills.

There were four measurement instruments used during the investigation. The first instruments used in this study were 1 minute timed probe sheets. Each probe sheet contained six rows of nine multiplication problems with products up to 81. Alternate forms of these probes were created to prevent students from simply memorizing answers to the first few problems. The second and third instruments used were pretest and posttest. These tests contained 20 single-digit multiplication facts. Daily worksheets served as the fourth instrument. A different work sheet was used during each lesson. The daily worksheets contained more multiplication problems than the 10 independent practice problems. These additional problems were used as teacher demonstration problems. However, during Lesson 7, the daily worksheet did not contain any multiplication problems. Instead, it listed steps to the mnemonic device taught in lesson 7.

Teachers involved in this study were obligated to attend a training session to familiarize themselves with the procedures and materials they would be using to implement the lesson. During baseline, students were given basic multiplication problems for several days. At this time, students did not receive any feedback from teachers. Once baseline was established, teachers administered the pretest. Students were asked to make a written commitment to learn multiplication. Throughout the intervention, the teacher taught the first 10 scripted lessons (see Table 5). Three of the lessons at concrete level, 3 lessons at representation level, 1 lesson on the use of a mnemonic strategy to solve the unknown problems, and 3 lessons at the abstract level.

Concrete lessons involve manipulatives and two multiplication rules (i.e., any number times zero equals zero and any number times one equals the original number). After the concrete level lessons, the students received three representational lessons. These lessons involved teaching the students to use pictures of objects and tallies. Basic word problems were included in both the abstract and representational levels. During lesson 7, students were taught the DRAW strategy (see Table 4). After learning the DRAW strategy, the next three lessons were at the abstract level. At this level, students were encouraged to begin memorizing facts. However, they were encouraged to use the draw strategy when attempting to solve unknown facts.

For independent practice, students solved 10 problems on the daily worksheet. There were no time restrictions for completing the sheets. After students completed the worksheets, students were given immediate feedback. Reteaching was necessary for students who did not receive a score of 80% or higher on the daily worksheets. During reteaching, the same material was used. Following Lesson 10, teachers administered the posttest. The mastery criterion for moving to Lesson 11 was 90% on the posttest. Students who failed to reach the criterion had to repeat lesson 10 and the posttest. Beginning with Lesson 11, student learned to set up and solve word problems using a strategy called FAST DRAW (see Table 1). Then students used the previously learned DRAW strategy to solve the problem. After this lesson, teacher will no longer describe and model procedures to solve problems. Students are independently working using strategies taught.

A single-subject multiple baseline design across three functionally independent, yet similar, classrooms was used to demonstrate experimental control and document the effects of the intervention. Results from the pretest confirmed the need for instruction in multiplication. During Lessons 1–10, all subjects completed the entire CRA sequence. No subject repeated more than one lessons. This study concluded that all 6 subjects improved from pretest to posttest. The amount of improvement ranged from 25–75 percentage points. With regard to rate, the range of baseline of baseline median scores was 0 to 10.5 correct digits and 4–54 incorrect digits per minute. Across the 6 subjects, the average median performance under timed conditions during baseline was 2.7 correct digits and 23.4 incorrect digits per minute. The range of median scores for these same subjects during the final phase of the investigation was 5–19 correct digits and 0–6 incorrect digits per minute. Across the 6 subjects, the average median performance under timed conditions during the final phase was 11.1 correct digits and 2.1 incorrect digits per minute. Comparison of median rate scores during baseline and the final phase of instruction show that the average amount of change for the 6 subjects was an increase of 8.4 correct digits and a decrease of 21.3 incorrect digits per minute. The performance of subjects with disabilities was most similar to the performance of their normally achieving peers during the phase of instruction that required demonstration of conceptual understanding of the multiplication process. With a strategy for counting objects within groups, subjects with disabilities were as successful as their peers at reaching correct answers on the independent practice sections of the daily worksheets.

The findings of this study suggest that there are teaching methods in the area of mathematics that can be implemented within general education classes which will result in achievement for all students. As evident in this study, multiplication using CRA was a successful sequence to use in teaching multiplication to students with learning disabilities. Implications, in this study, concluded that teachers who are expected to introduce multiplication to large groups of students with varying ability and skills must first obtain pretest prior to instruction. Second, mastery the control movement within the curriculum provide teachers with data-based information for planning instruction and feedback. Third, students with learning difficulty who demonstrate the prerequisite skill for multiplication can perform similarly to their peers when given the use of appropriate curriculum materials.

Morin and Miller (1998) extended the study of Harris, Miller, and Mercer (1995), using multiplication using the CRA teaching sequence. The purpose of this study was to evaluate the effectiveness of teaching multiplication facts and related word problems using the CRA teaching sequence to middle school students with intellectual disabilities. Participants of this study included three seventh grade students (1 Black female, 1 White female and 1 black male). Of the participants, two received services in the self-contained classroom for multiple disabilities and the third subject received services in a self-contained classroom for students with intellectual disabilities. The participant’s ages ranged from 15 to 16 years old. All participants were selected based on four criteria. First criteria included at referral by the teacher. Second criteria for participation was based on the student’s ability to count to 81, work out addition problems with sums to 18, and add single-column sums to 18. In order to be included in this study, participants had to have a score of at least 90% on these skills. The third criterion for participation was the student's performance on the pretest. The pretest included 20 single-digit multiplication facts with products from 0 to 81. Lastly, the participant's parent or guardian had to give permission to participate in the study.

Four instruments were used during the investigation. The first two instruments used in the investigation was pretest and pretest probes. The pretest probes contained 20 single-digit multiplication facts arranged in five rows with four problems in each row. The probes used during the baseline condition were similar to the pretest. The third instrument used in this investigation was posttest. This test also contained 20 single-digit multiplication facts. The fourth and final instrument used was a daily worksheet. Each of the 21 lessons included a different daily worksheet. The only exception to this procedure was Lesson 7. This daily worksheet contained the DRAW mnemonic strategy (see Table 4). There were no multiplication problems to be scored in lesson 7.

In order to familiarize the special educator in this investigation with the materials and procedures, a training session was provided for the teachers. Following the pretesting and baseline condition for each participant, students were given feedback on their performance. Next, the teacher taught the first 10 scripted lessons. These lessons were taught using systematic instructional procedures and the CRA sequence. There were 3 lessons at the concrete level, 3 lessons at the representational level and 1 lesson on the use of the DRAW mnemonic device, and 3 lessons at the abstract level.

Each lesson began with an advance organizer. After introducing the advanced organizer, the teacher described and modeled the skill or strategy being taught in the lesson. The scripted lessons included two procedures. Procedure 1 included that the teacher asked and answered questions while demonstrating how to compute answers for one or two problems on the daily worksheet. Procedure 2 included that the teacher demonstrated how to solve one or two problems. In this procedure, the teacher asked questions, but students solicited the responses. As a result, the teacher and students answered problems together. The next part of each lesson involved guided practice and independent practice.

Once baseline was established, teachers administered the pretest. Students were asked to make a written commitment to learn multiplication. During the intervention, the teacher taught the first 10 scripted lessons. The concrete lessons involve manipulatives and two multiplication rules (i.e., any number times zero equals zero and any number times one equals the original number). After the concrete level lessons, the students received three representational lessons. These lessons involved teaching the students to use pictures of objects and tallies. Basic word problems were included in both the abstract and representational levels. During lesson 7, students were taught the DRAW strategy (see Table 4). After learning the DRAW strategy, the next three lessons integrated problems at the abstract level. At this level, students were encouraged to begin memorizing facts. However, they were encouraged to use the draw strategy when attempting to solve unknown facts.

For independent practice, students solved 10 problems on the daily worksheet. There were no time restrictions for completing the sheets. After students completed the worksheets, students were given immediate feedback. Reteaching was necessary for students who did not receive a score of 80% or higher on the daily worksheets. During reteaching, the same material was used. Following Lesson 10, teachers administered the posttest. The mastery criterion for moving to Lesson 11 was 90% on the posttest. Students who failed to reach the criterion repeated lesson 10 and the posttest. Beginning with Lesson 11, student learned to set up and solve word problems using a strategy called FAST DRAW (see Table 1). During Lesson 12, students were taught to solve word problems that contained extraneous information. Students began independent practice immediately after the teacher provided the advanced organizer.

A single-subject multiple baseline design across the three subjects was used to demonstrate experimental control and document the effects of the intervention. Baseline and intervention data along with pre-and posttest scores reflected excellent progress for all three subjects. Out of the 63 lessons assessed, only four occurrences were below 80%. On the pretest and posttest, Subject 1 earned 50 percent on her pretest and 90 percent on her posttest. Subject 2 earned 70 percent on the pretest and90 percent on the posttest. Subject 3 earn 20 percent on the pretest and 90 percent on the posttest. The percent gain scores for Subjects 1, 2, 3 were 40, 20, and 70 points respectively.

The findings of this investigation suggest that students with intellectual disabilities can learn to solve multiplication facts and related word problems using the CRA teaching sequence and systematic instructional procedures. Moreover, these students learned to use mnemonic devices to cue specific cognitive functions involved in solving both computation and word problems. These results of this investigation extended previous research related to the use of cognitive math strategies. The appropriateness of teaching cognitive strategies to students with learning disabilities has been reported (e.g. Harris, Miller, & Mercer, 1995), but this study expanded the line of research to the use of these strategies with students with intellectual disabilities.

Butler, Miller, Crehan, Babbitt, and Pierce (2003) extended CRA research to include higher order problems. This study taught middle school students with a disability equivalent fraction concepts and procedures comparing the effects of the CRA instructional sequence and the representation- abstract (RA) instructional sequence. This extension of the literature was significant because the researchers investigated the need for the concrete level of instruction within the CRA sequence. Participants in this study were 50 students with mild-moderate disabilities. These participants ranged in age from eleven to fifteen and were enrolled in grades 6, 7, and 8th grade. Participants in this study were split into two treatment groups. Twenty-six students formed the CRA group, and 24 students formed the RA group. The majority of these students (i.e., 42) were identified with a specific learning disability in mathematics. All students received mathematics instruction in a resource room setting. In this study, an additional 65 students enrolled in the general education class took the post assessment only. The information obtained from this comparison group was used was used to give an estimate of what a student without a disability understands about fractions by the end of the eighth grade year.

Pretests and posttests were used as instruments in this study, consisting of five subtests from the Brigance Comprehensive Inventory of Basic Skills-Revised (CIBS-R) (Brigance, 1999) which included concepts such as quantity fractions, area fractions, and abstract fractions. Two additional subtests were designed by the investigator in order to measure skills not assessed in the CIBS-R, such as fraction word problems and improper fractions; they had significant correlations with the subtests taken from the CIBS-R ($p < .01$). Also, students' attitude toward instruction was measured by an investigator-constructed questionnaire using a three-point Likert scale.

A total of four phases were used in this study. Phase one was teacher training. At the two hour training, two certified special educators were given materials for the study. The materials included 10 scripted lessons. Each lesson contained an advanced organizer, a teacher demonstration, guided practice, independent practice, problem-solving practice, and a feedback routine. There were ten student learning sheets (one per lesson). These learning sheets contained the items for guided practice, independent practice, and problem-solving practice. Cue cards were used which included definitions and step-by-step procedures for solving fractions. Teachers participated in training which involved observations of the following five criteria: (a) using consistent strategies, (b) providing appropriate feedback, (c) guided practice and independent monitoring, (d) following the order of the lesson, and (e) allocating time for each activity.

Phase two was the preassessment. The participants independently completed the pretest-posttest in one 50-minute regularly scheduled mathematics period. The preassessment contained questions designed to measure understanding fractions concepts, procedures, and applications. The participants were prompted to do their best. In addition an attitude questionnaire was given the day before beginning Lesson 1.

During phase three, the intervention phase, daily lessons followed the format of the Strategic Math Series (see Table 5). Each lesson was scripted with the advanced organizer, teacher demonstration, guided practice, independent practice, problem-solving practice, and a feedback routine (as discussed in the other CRA studies). The cue cards and notes were used in both treatment groups. These were provided just in case participants forgot the steps for solving a problem. Participants placed the cards and notes in their individual notebooks for easy access during class.

For the CRA group, concrete manipulatives were used to introduce the concept of fractions equivalence. In Lesson 2, beans were used to show proportional reasoning. In the next lesson, students used folded construction paper as the manipulative device in order to promote generalization to various types of objects. Concept development continued in lessons 4–6. In

Lesson 7, participants began to solve fractions using the abstract algorithm for computing fractions. Practice of abstract problems, containing proper fractions, improper fractions, and mixed numbers, were continued to Lesson 10. Lessons 4–10 were the same for both groups (CRA and RA).

Phase four, the post-assessment, consisted of the same subtests and attitude questionnaire used in the pre-assessment. The post-assessment was administered after Lesson 10 was completed. The pre-assessment and post-assessment were given to the comparison group, 65 eighth grade students who were enrolled in the general education math class.

Paired sample t-tests were used to show differences between the pre-assessment and post-assessment scores for both groups. Results showed significant improvements ($p < .05$) on all measures for measures except for the CRA group's difference on Area Fractions subtest. A Multivariate analysis of covariance statistical procedure (MANCOVA) was used to test for treatment effects for the subtests. The difference between the two groups were statistically significant ($F(5, 49) = 2.81$, $p = .029$, using the Wilk's criterion, eta squared was 0.265). Follow-up univariate tests revealed a statistically significant difference for only the Quality Fraction subtest, which favored the CRA group ($F(1, 43) = 14 = 14.759$, $p < .0005$). The adjusted raw score was higher for the CRA treatment on all five measures.

When comparing the treatment group with the traditional group, the MANOVA revealed statistically significant difference for the set of dependent variable between the groups ($F(5, 109) = 9.003$, $p < .0005$). This indicates a moderate association between the independent and dependent variables. Between subject effects univariate follow-up tests found significant differences between the groups favoring the treatment group on the improper fractions $F(1, 113) = 18.642$, $p < .0005$) and on the word problem subtest ($F(1, 1113) = 6.833$, $p = .010$).

Attitudes between the CRA and RA group were compared using an analysis of covariance (ANCOVA). The ANCOVA found no significant difference between the groups. The paired samples t-tests revealed statistically significant attitudes between preassessment and postassessment. The CRA group had a mean difference of 2.62, (t (25) = 2.768, p = .01), while RA group mean difference of 2.29, (t (23) = 2.277, p= .03).

The results of this study revealed that CRA and RA sequences are both effective for teaching students with disabilities. However, CRA produced slightly higher means. Therefore, concrete manipulatives are useful for teaching students with disabilities. This study was the initial study to show that CRA can be an effective strategy to teach fractions for students with learning disabilities. It also added evidence to the argument that the concrete phase of the instruction is helpful for students.

Flores (2009) extended the CRA line of research to include subtraction with regrouping. This study investigated the effectiveness of the CRA sequence on the computation performance of students with specific learning disabilities and students identified as at risk for failure in mathematics. The participants in this study were 6 third graders who were failing mathematics in terms of grades and performance on benchmark assessments. Four of these students had a learning disability. The participants names used in this study were Art, Juan, Beto, Ed, Mari, and May.

Materials for this study contained a probe with thirty (two-digit minus two digits numbers) subtraction problems, that required regrouping in the tens place, were used as probes for this study. Other materials used were student contracts, Base-10 blocks made out of foam, and 8.5 X11 learning sheets with target problems written vertically. The learning sheets were divided into multiple sections: model (three problems), guided practice (three problems) and independent problems (six problems). The student also had sheets with the mnemonic DRAW strategy printed (see Table 4). Later, lesson learning sheets had few problems for the model section.

The lessons were implemented according to the Miller and Mercer (1992) structure (see Table 5). Concrete manipulatives (i.e., Base-10 blocks) were used in Lessons 1–3. Lessons 4–6 were the representational lessons which contained the drawings instead of the Base-10 blocks. The DRAW strategy (see Table 4) was used in Lesson 7. In Lessons 8–10, the abstract level, students were encouraged to answer problems from memory rather than using drawings.

A multiple-probe across participants design was used to evaluate the efficacy of the CRA for teaching subtraction with regrouping. From the results, Art reached criterion after 15 probes. There was an immediate change in the level of performance between baseline and CRA with no overlapping data points. Art showed steady improvement over time. Beto reached criterion after 11 probes. He also had an immediate change with no overlapping of data points, with steady improvement. Ed, Mari, and Juan showed steady improvements and reached criterion after 11, 10, and 10 probes, respectively. May reached criterion after 11 probes. However, some variability was seen in the first four points after the CRA phase, but showed an upward path for the remaining points. After 4 weeks without instruction, 5 out of 6 participants maintained performance at or above the criterion level. The results of this study provided evidence that there is a functional relation between CRA and the students' performance on subtraction skills. This study extended the line of CRA research to include regrouping.

Flores (2010) further validated her previous CRA research (Flores, 2009) to include subtraction with regrouping with more complex computation. This study investigated the effectiveness of the CRA sequence in teaching subtraction with regrouping to the tens and hundreds place. The participants of this study were 6 third graders. Participation eligibility requirements were: current failure in mathematics, lack of skills in subtraction with regrouping, defined as less than 10 digits written correctly on a curriculum-based measure. The students in this study were named Ray, Al, Walt, Ron, Joe, and Ann.

Materials for this study contained probes with thirty (three-digit minus three digits numbers) subtraction problems, that required regrouping in the tens place. Other materials used were student contracts, Base-10 blocks made out of foam, and 8.5 X11 learning sheets with target problems written vertically. The learning sheets were divided into multiple sections: model (three problems), guided practice (three problems) and independent problems (six problems). The student also had sheets with the mnemonic DRAW strategy printed (see Table 3). Later, lesson learning sheets had few problems for the model section.

The lessons were implemented according to the Miller and Mercer's Strategic Math Series Place Value (1992) structure (see Table 5). Concrete manipulatives (i.e., Base-10 blocks) were used in Lessons 1–3. Lessons 4–6 were the representational lessons which contained the drawings instead of the Base-10 blocks. The DRAW strategy (see Table 4) was used in Lesson 7. In Lessons 8-10, the abstract level, students were encouraged to answer problems from memory rather than using drawings.

A multiple-probe across participants design was used to evaluate the efficacy of the CRA for teaching subtraction with regrouping to the tens and hundreds place. At the tens place, results indicate that Ray reached criterion after 6 probes and had immediate change between baseline and CRA. AL and Walt reached criterion after 9 and 7 probes, respectively. Both showed immediate change between baseline and CRA. Ron and Joe reached criteria after 13 and 11 probes, respectively. Ann reached criterion for reducing at the tens place after 5 probes.

From the tens place to the hundreds place, Ray reached criterion after 6 probes. Al and Walt reached criterion, for the hundreds, after 5 and 4 respectively. At the same place value, Ron and Joe reached criterion after 6 probes and 8 probes. Ann reached criterion after 5 probes. Each participant showed an upward path, indicating steady improvement.

The results of this study provided evidence that there is a functional relation between CRA and the students' performance on subtraction with regrouping to the tens and hundreds place. It further validated that CRA is an effective instruction sequence for teaching students with learning disabilities and those at risk subtraction with regrouping to the tens and hundreds place. This study extended previous study and included more complex computation.

Similar to Flores (2010, 2009), Kaffar and Miller (2011) investigated the effects of CRA instruction but included the mnemonic titled "RENAME" for subtraction with regrouping. The "RENAME' mnemonic was made up of six steps. They are (a) read the problem, (b) examine the ones, (c) note the ones, (d) address the tens column, (e) mark the tens column, and (f) examine and note the hundreds and exit with a quick check. Steps of the RENAME strategy are included in Table 6.

Table 6

Steps of RENAME Strategy

Steps	Description of RENAME
Step 1	**R**ead the problem,
Step 2	**E**xamine the ones
Step 3	**N**ote the ones
Step 4	**A**ddress the tens column
Step 5	**M**ark the tens column,
Step 6	**E**xamine and not the hundreds and exit with a quick check

Participants of this study received CRA sequence with the embedded RENAME mnemonic. There were a total of 24 students in this study. Of the 24 students, twelve students who were in the treatment group received the CRA and RENAME instruction for subtraction with regrouping. In the treatment group, there were eight students with math difficulties and three students with disabilities. The remaining twelve students were in the control group and received the basal program for instruction on subtraction with regrouping. Results indicated that both groups made gains in subtraction with regrouping; however, the gains made by the students who received instruction with the CRA sequence and RENAME were greater.

Strozier, Hinton, Terry, and Flores (2012) investigated CRA and extended previous research of CRA as an effective instructional sequence for students with Autism Spectrum Disorders (ASD). Participants in this study included three elementary male students with ASD. A multiple-baseline design across three basic mathematical computation behaviors was utilized to evaluate the effects of CRA for students with ASD. These three behaviors included addition with regrouping (Mercer & Miller, 1992), subtraction with regrouping (Kaffar & Miller, 2011), and multiplication facts 0-5 (Miller & Mercer, (1993). Results of this study showed a functional relation between CRA instruction and the behaviors of addition and subtraction with regrouping and multiplication facts zero to five with three participants with ASD. CRA studies with mnemonics are shown in Table 3. CRA has been effective for many basic concepts; however researchers expanded the use of CRA instruction to include more difficult mathematical concepts such as algebra.

Algebra

Algebra is the branch of mathematics that symbolizes general number relations and properties and includes such topics such as signs of operations, solutions of equations, and polynomials (Kieran, 1992). This branch of mathematics is considered the "gatekeeper" to educational and occupational opportunities; most secondary schools now require all students take higher level mathematics to graduate (Chambers, 1994). The need for students to successfully complete algebra has become increasingly apparent over the decade. Now that states require all students to adhere to the same graduation standards, introducing high-stakes assessments, algebra performance is a great concern for students with disabilities (Witzel, Mercer, & Miller, 2003).

Due to its abstract nature, educators have struggled to help students comprehend initial algebra instruction (CEC Webinars, 2012; Witzel, Mercer, & Miller, 2003). Devlin (2002) stated that for students to understand abstract concepts more easily, it is important for them to learn precursor concepts in a concrete manner first. One way to simplify students' understanding of abstract concepts is to transform such complex concepts into concrete manipulative and pictorial representations.

Research Regarding CRA and Algebra

Maccini and Ruhl (2000) pilot tested an instructional strategy that combined CRA with the STAR strategy for solving subtraction of integers. The participants in this study were three students with learning disabilities. To be eligible to participate in this study, students were: (a) in a secondary school, (b) diagnosed as having a learning disability, and (c) lacking knowledge in the targeted subtraction tasks. All three male participants meet the criteria for the study. IQ scores for the individuals ranged from 70–104.

The treatment for this study consisted of the algebra problem-solving strategy, STAR (Maccini, 1998). Included in this strategy are (a) CRA instruction sequence, (b) general problem-solving strategies, and (c) self-monitoring strategies. STAR incorporated several phases: (a) pretest, (b) concrete application, (c) representational application, and (d) abstract application. Instructional phases include teaching the first letter of the mnemonic, STAR to cue students to perform steps. The steps to STAR are located in Table 7.

Table 7

Steps and Sub-steps of STAR

	Steps	Sub-steps
Concrete & Representational		
Step 1	Search the word problem	a) Read the problem carefully
		b) Ask yourself questions: "What facts do I know?" "What do I need to find?"
		c) Write down facts
Step 2	Translate the words into an equation in picture form	a) Choose a variable
		b) Identify the operation(s)
		c) Draw a picture of the representation
Abstract		
Step 1	Search the word problem	a) Read the problem carefully
		b) Ask yourself questions: "What facts do I know?" "What do I need to find?"
		c) Write down facts
Step 2	Translate the words into a mathematical equation	a) Choose a variable
		b) Identify the operation(s)
		c) Draw a picture of the representation
Step 3	Answer the problem	Subtraction of Integers Rule – Add the opposite of the second term
		Addition of Integer Rule – Same signs – add the #'s and keep the sign
		Different signs – find difference of #'s & keep sign of # farthest from zero
Step 4	Review the solution	a) Reread the problem
		b) Answer question, "Does the answer make sense? Why?"
		c) Check answer

Algebra Lab Gear™ was also used in this study. Algebra Lab Gear is a comprehensive program incorporating manipulatives to build conceptual understanding of algebraic terms, integers, expressions, and equations.

The instructional procedures used to teach STAR were adapted from the Strategic Math Series (see Table 4). The procedures included six elements used in each lesson: (a) provided an advance organizer, (b) describe and model with think aloud prompts, (c) conduct guided practice, (d) conduct independent practice, (e) give posttest, and (f) provide feedback. Corrected feedback consisted of five steps: (a) The researcher and student documented student performance, (b) incorrect responses and error patterns were targeted, (c) the researcher modeled or retaught error correction procedures using a similar problem or problems, and (d) the research closed the session with positive feedback.

The study's dependent measures were (a) percent of strategy use, (b) percent correct on problem representation, (c) percent correct on problem solution and answer, (d) generalization, and (e) social validation. Separate tools were used for each measure. Social validity determined the participants' opinion of the intervention. Participants answered questions in a Likert-scale format (1 to 5) about intervention effectiveness, efficiency, and acceptability. A multiple probe across three students was utilized to determine the effectiveness of the intervention. Four probes were used intermittently during baseline to determine their current status and stability of the behavior under investigation as well as the need for intervention.

Results indicated that participants with learning disabilities can learn to successfully represent and solve word problems involving subtraction of integers. All participants increased their percent of strategy use from baseline to instructional phases in subtraction of integers. For Shane, there were 46 percentage points increase. Angel had an increase of 26 percentage points, while Eric had 13 percentage points increase. Further, all students increased percent of strategy use from baseline to representational instructional phase. All students increased their mean percent accuracy on problems representation from baseline to concrete instruction (differences of 67.5, 66.25, and 46.25 percentage point for Shane, Angel, and Eric, respectively). All participants increased their mean percent accuracy on problem solution from baseline to the concrete instructional phase (increases of 58.5, 29, and 64.5 percentage points for Shane, Angel, and Eric, respectively). The average score of participants indicated for effectiveness of STAR was 4.67. Overall, students agreed or strongly agreed that the strategy helped them to (a) remember problems-solving steps, (b) learn about subtracting integers and word problems, and (c) identify when it is necessary to subtract integer numbers when solving word problems. They also agreed that manipulatives helped them understand what it means to subtract integers as well as to solve problems involving integers, and that the intervention was worth their time.

Results of this study provided initial evidence that students with learning disabilities can learn to represent word problems word problems involving integers with the use of concrete manipulatives and pictorial displays. The STAR strategy (see Table 7) was proven to be a useful tool to help students attend to critical features of word problems. This study also contributes to the field by (a) teaching higher mathematical skills to students with learning disabilities, (b) addressing conceptual understanding of mathematics concepts, and (c) providing self-regulation training with the use of the STAR strategy.

Maccini and Hughes (2000) extended previous research by investigating the effects of the STAR strategy using the CRA with integers. This study replicates the study of Maccini and Ruhl (2000). The participants of this study were six students with learning disabilities from a public school in central Pennsylvania. Participants received part-time support in a resource class. Three participants were enrolled in a modified introductory algebra course in preparation for higher level mathematics (i.e. Algebra 1) as a graduation requirement. Participants were in grades 9-12. Math achievement scores were collected to indicate that participants were functioning more than 2 years below grade level.

The lessons in this study incorporated computation of integer numbers and problem solving involving integer numbers. The treatment consisted of the algebra problem-solving strategy, STAR, (see Table 7) within graduated instruction (i.e. CRA), teaching through concrete, representational, and abstract applications. The students applied the STAR strategy during the instructional phases. Students were given a sheet that consisted of steps and substeps and a workmat which contained positive and negative area. These steps are slightly different from the steps in the Maccini & Ruhl (2000) study. The steps and substeps of the STAR strategy used within this study were:

1. Search the word problem
    a) Read the problem carefully
    b) Ask yourself questions: "What facts do I know?" "What do I need to find?"
    c) Write down facts
2. Translate the words into an equation in picture form
    a) Choose a variable
    b) Identify the operation(s)
    c) Represent the problem with the Algebra Lab Gear (CONCRETE APPLICATION)

       Draw a picture of the representation (REPRESENTATIONAL APPLICATION)

       Write an algebraic equation (ABSTRATE APPLICATION)

3. Answer the problem

Addition

Same signs, add #s & keep sign

Different signs, find difference of #s & keep sign of # farthest from zero

Subtraction

Add the opposite of the second term

Multiplication/Division

Same signs – positive

Different signs – negative

4. Review the solution

a) Reread the problem

b) Answer question, “Does the answer make sense? Why?”

c) Check answer

Procedures used to teach STAR was adapted from the Strategic Math Series (see Table 5). Six elements of the direct instruction model (see Table 2) were included each lesson: Corrected feedback consisted of five steps: (a) The researcher and student documented student performance, (b) incorrect responses and error patterns were targeted, (c) the researcher modeled or retaught error correction procedures using a similar problem or problems, and (e) the research closed the session with positive feedback.

This study used percentage correct for problem representation, percentage correct for problem solution and answer and percentage of strategies were the dependent measures. A multiple-probe design across subjects was used to investigate individual performance across time. The first set of students began the CRA instructional sequence when the participants consistently performed below 80% accuracy in problem representation, solution, and percentage of strategy-use. The treatment was then applied serially across the remaining students after the preceding student demonstrated 80% or higher on two consecutive probes.

Social validity data were collected to determine participants' and teachers' opinions of the intervention. Students were asked questions about usefulness, practicality, and effectiveness of the intervention using a Likert-type scale (ranging from 1 to 5). Open-ended questions were also asked concerning what they liked the best and least and about how they use algebra in everyday life and improvement suggestions. Results from the social validity evaluation indicated that most participants indicated that they were interested in learning about algebra concepts and skills. Most participants agreed that the intervention was worth their time, that it helped them understand mathematics concepts. Participants also stated that the intervention helped them feel better about their introductory algebra skills.

The results also indicated that all participants learned to represent and solve addition word problems involving integer numbers. Five of the six participants learned to solve subtraction, multiplication, and division word problems involving integer numbers. Participants also demonstrated increases in their percentage of strategy-use across instructional phases. Categories for marked improvement included: (a) searching for word problem, (b) translating the word problem into an equation, (c) identifying the correct operation or operations, (d) drawing a picture of the problem, (e) writing a correct equation, and (f) answering the problems. Results of this study provided initial evidence that students with learning disabilities can be taught to represent word problems involving integer numbers via concrete manipulatives and pictorial displays.

In an effort to extend the Maccini and Hughes (2000) study to include more algebra concepts, Witzel, Mercer, and Miller (2003) investigated the effectiveness of a new explicit CRA model that was designed to represent more complex equations. Participants of this study were 358 sixth and seventh graders. Of these students, 34 students with disabilities or at risk for algebra difficulty in the treatment group were matched with 34 students with similar characteristics (i.e., statewide achievement test, age within one year, grade level, pretest accuracy within one level, at risk or disability) across the same teacher's classes in the comparison group. To qualify as at risk for algebra difficulties, the student met the following criteria: (a) performed below average in the classroom, (b) scored below the 50th percentile in mathematics on most of their statewide assessments, and (c) was from a low socioeconomic background.

The lead researcher developed a test instrument, from a question pool, to measure the acquisition and maintenance of knowledge of single-variable equation and solving for a single variable in multiple-variable equations. The 27 item test was used as the assessment instrument. The same test will be used for the pretest, posttest, and follow-up test. The pretest scores were obtained one week prior to implementation of the treatment. Posttest score were obtained upon completion of the last day of treatment and follow-up scores were obtained three weeks after treatment ended.

A five-step 19 lesson sequence of algebra equations was used. Skill areas included: (a) reducing expressions, (b) solving inverse operations, (c) solving inverse operations with negative and divisor variables, (d) performing and solving transformations with multiple variables on one side of the equal sign, and (e) performing and solving transformations with multiple variables on both sides of the equal sign. The sequence of lessons followed that of major algebra textbooks by Houghton Mifflin, Scott Foresman-Addison Wesley and Saxon. Both groups received three lessons on reducing fractions and four lessons on inverse operations, solving for negative and divisor variables, solving for single variables distributed on the same side of the equal sign, and solving for single variables distributed on both sides of the equal sign.

A pre-post-follow-up design with random assignment of clusters was employed for this study. The teacher taught the comparison groups according to explicit instruction, following modeling, guided practice and independent practice strategies. The teachers also taught the treatment group using explicit instruction techniques but with addition of CRA components of instruction. A repeated measure analysis of variance statistical procedure was used to determine if any significant differences existed between the instructional groups on posttreatment and maintenance measures. Follow-up univariate analysis of variance and t-tests were computed for acquisition and maintenance.

A repeated measure of analysis of variance was performed on two levels of instruction (CRA vs. abstract) and three levels of occasions (pretest, posttest, and follow-up). Results showed that both groups had significant improvements in answering single-variable algebraic equations form the pretest to the posttest and from the pretest to the follow-up. However, students who participated in the CRA instruction outperformed their traditional abstract instruction peers on both the posttest and follow-up test.

In a pilot study, Strozier, Hinton, Flores, and Shippen (2012) extended CRA and algebra to more complex algebraic equations. This study added a specific mnemonic to teach algebra along with the CRA sequence. The SUMLOWS mnemonic strategy aided students with steps to solving one-step, two-step, and multiple-step algebraic equations. The steps to SUMLOWS are located in Table 8.

Table 8

SUMLOWS mnemonic

Steps	Description of SUMLOWS
Step 1	(S) **S**eparate the sides,
Step 2	(U) **U**nite the like terms,
Step 3	(M) **M**odify (rewrite) the new equation,
Step 4	(L) **L**oop (circle) around the coefficient,
Step 5	(O) **O**pposite sign (inverse operation),
Step 6	(W) "**W**hat you do to one side, you must do to the other,"
Step 7	(S) **S**ubstitute the solution for the variable and check.

Three ninth graders participated in this study. Of the 3 participants, two students were identified as SLD and one as at-risk. A multiple-baseline across students with embedded behaviors (one-step, two-step, and multiple-step equations) showed a functional relation across three students for one-step and two-step behaviors. The results of this study showed that CRA and SUMLOWS instruction could be effective in teaching equations to students with disabilities. Consistent with the research of Witzel et al. (2003), the results of this study also showed the benefits of including concrete and pictorial representations when teaching algebra. The CRA algebra model proved effective for students with disabilities and those at risk for math difficulties in the general education classroom. CRA and algebra studies are shown in Table 9.

Table 9

Algebra and CRA Studies

Authors and Date	Intervention	Participants	Design	Outcomes Measured
Maccini, P. & Ruhl, K. L. (2000)	CRA sequence with STAR Strategy	Three 8^{th} grade males with LD	Single-subject– multiple baseline across subjects	Word problems involving integers
Maccini, P. & Hughes, C. (2000)	CRA sequence & STAR Strategy	Six students w/LD	Single-subject– multiple baseline across subjects	Word problems involving integers
Witzel, B. S., Mercer, C. D., & Miller, M. D (2003)	CRA Sequence	358 sixth and seventh graders (34 SWD or at risk)	Group comparison of CRA sequence to abstract	Algebra
Strozier, S. D., Hinton, V., Flores, M., & Shippen, M. (2012)	CRA sequence and SUMLOWS mnemonic strategy	Three 9^{th} grade students (2 with LD and 1 at risk)	Single-Subject– multiple baseline across students with embedded behaviors	One-step, two-step, and multiple step algebraic equations and a specific mnemonic for solving equation

The results of these studies demonstrated that CRA can be effective in teaching equations. The results also showed that teachers need to use concrete and pictorial representations for teaching algebra. The CRA algebra model proved effective for students with disabilities and those at risk for math difficulties in the general classroom.

CHAPTER 3. EFFECTS OF CRA AND SUMLOWS WITH ALGEBRAIC EQUATIONS

Introduction

A major goal of *No Child Left Behind Act* (NCLB; 2002) was for all students, including students with disabilities to be proficient in reading and mathematics. To measure academic achievement, the act required states to annually assess the reading and mathematics abilities of all students (including students with disabilities). The reauthorization of the *Individuals with Disabilities Education Improvement Act* (IDEIA; 2004) reinforced the educational goals of NCLB by requiring that students with disabilities meet the same achievement standards as their peers without disabilities; despite any intellectual, emotional, or physical challenges. Scheuermann, Deshler, and Shumaker (2009) explain achieving the current mathematical academic standards poses a great challenge for students who struggle in mathematics, and an even greater challenge for students with disabilities. Therefore, educators along with researchers continue to explore ways of providing effective instruction for students who struggle with mathematical content, specifically those performing below the 25^{th} percentile on standardized measurements and who may be identified as students with mathematics learning disabilities (MLD) (CEC Webinars, 2012; Scheuermann, Deshler, & Shumaker, 2009).

Mnemonics Strategies in Mathematics

Many students with disabilities and those at risk for educational failure exhibit problems remembering academic material (Mastropieri & Scruggs, 1998). One intervention that has been successful in helping students to remember steps or a series of information is the use of mnemonic strategies (Bottege, 1999; Cade & Gunner, 2002; Greene, 1999; Manalo, Bunnell, & Stillman, 2000; Steele, 2002). Mnemonic strategy refers to “a word, sentence, or picture device or technique for improving or strengthening memory” (Test & Ellis, 2005, page 12). Mnemonics can be classified as fact mnemonics (i.e., used to help students recall facts) or process mnemonics (i.e., used to help remember steps, processes, or rules) (Manalo, Bunnell, & Stillman, 2000; Test & Ellis, 2005).

For more than 20 years, mnemonic strategies have been demonstrated to be highly effective with students with disabilities as well as normally achieving students (Mastropieri, Sweda, & Scruggs, 2000). The effectiveness of mnemonic strategies was demonstrated to be consistent across a variety of subject math areas, age, and disability status of participants, and instructional settings. Mnemonic strategies have been found to be effective in experimental “laboratory” investigations, classroom implementations, and teacher-directed applications (Scruggs & Mastropieri, 1990). Mnemonic strategies have been effective in promoting verbal memory objectives without detracting from other important learning objectives.

While research has demonstrated that mnemonic strategies have been successful in teaching certain math skills, few were designed to teach algebra skills (Maccini & Hughes, 2000). Due to its abstract nature, educators have struggled to help students comprehend initial algebra instruction (Witzel, Mercer, & Miller, 2003). Devlin (2002) stated that for students to understand abstract concepts more easily, it is important that they first learn precursor concepts in a concrete manner. One way to simplify students’ understanding of abstract concepts is to transform such complex concepts into concrete manipulative and pictorial representations.

Concrete-Representational-Abstract (CRA) Sequence Overview

In addition to mnemonics, Mastropieri, Scruggs, and Shiah's (1991) review documented the effectiveness of the concrete-to-abstract teaching sequence in teaching students with disabilities. Within the CRA sequence, the teacher uses explicit instruction within the CRA sequence. With explicit instruction, the lessons should be organized, clear and direct to enhance student learning and engagement. During lessons, students are encouraged to actively participate and receive teacher immediate feedback (Peterson, Mercer, & O'Shea, 1988; Strozier, Hinton, Flores, & Shippen, 2012; Witzel, Mercer, & Miller, 2003). There are four phases of the explicit teaching sequence: advance organizer, demonstration, guided practice, and independent practice. The advance organizer phase ensures students have mastered the required pre-requisite skills, states what the students will learn, and builds relevance for the concept. In the demonstration phase, the teacher presents the concept and step by step strategies that detail how to solve the problem. During this phase, the teacher uses self-talk to model thinking. In the guided practice phase, students and the teacher solve the problem together which allows students to practice with teacher input and feedback. Finally, in the independent practice phase, students solve problems without teacher assistance.

In addition to explicit instruction, CRA instruction involves concrete, representational, and abstract levels of instruction. At the concrete level, the mathematical concept is taught using three dimensional objects. At the representational level, pictorial presentations are used. At the abstract level, only numbers are used. Research has shown that students who received the three sequence strategy outperformed students who received traditional instruction using numbers only (Mercer & Miller, 1992; Miller & Mercer, 1993; Paterson, Mercer, & O'Shea, 1988).

The initial CRA research was conducted by Peterson, Mercer, and O'Shea (1988). This study investigated the effectiveness of teaching students with learning disabilities place value through a conceptual sequence including three levels of understanding (i.e., concrete, representational, and abstract) compared to the effectiveness of teaching the same skill at the abstract level without manipulatives or pictorial representations. They assessed the effect on these two procedures on initial skill acquisition, maintenance, retention, and generalization to a more complex skill.

Since the initial CRA research, other researchers have investigated an array of other math skills. These skills include addition facts 0–9, subtraction facts 0–9, addition facts 10–18, subtraction 10–18, multiplication 0–81, and division 0–81 (Miller & Mercer, 1991, 1993), coins up to 50 cents (Miller & Mercer, 1993), higher multiplication (Harris, Miller, & Mercer, 1995), multiplication with word problems (Morin & Miller, 1998), subtraction with regrouping (Flores, 2009; Kaffar & Miller, 2011), subtraction with regrouping with more complex computation (Flores, 2010) and fractions (Butler, Miller, Crehan, Babbitt, & Pierce, 2003). CRA has been shown to be an effective intervention for many students with different disabilities, including students with autism spectrum disorders (Strozier, Hinton, Terry, & Flores, 2012).

Algebra and CRA

Algebra is important for preparing students for success in high school and beyond (Matthews & Farmer, 2008). With the higher standards to receive a high school diploma, NCTC (2000) emphasized the need to prepare all students for algebra. As students' progress in school, the advanced courses are dependent upon the successful completion of prerequisite courses, such as algebra (National Center for Education Statistics, 2010). Successful completion of algebra in middle school has been shown to lead to improved performance in high school as well as understanding of higher concepts. A strong foundation of algebra concepts should be in place by the end of eighth grade (National Council of Teachers of Mathematics, 2012; Wang & Goldschmidt, 2003). Although higher-level math skills are vital to their future, many students with disabilities experience difficulty with secondary math concepts, such as algebra (Maccini, McNaughton, & Ruhl, 1999).

The Council for Exceptional Children (CEC) highlighted the critical need for mathematical instruction to move beyond traditional "abstract- only" implementation (CEC Webinars, 2012). Due to the higher order thinking, many students struggle with algebraic concepts and tasks become frustrating (Witzel, Mercer, & Miller, 2003). Legal mandates that presently require students with disabilities adhere to the same graduation standards and proficiency on high-stakes assessments, make algebra performance a great concern in meeting the needs of students with disabilities (Witzel, Mercer, & Miller, 2003). Teaching foundational concepts in a concrete manner is an effective method of building an understanding of abstract concepts (Devlin, 2002). Therefore, demonstrating higher order math concepts using manipulatives and pictorial representations can build algebraic knowledge for students with disabilities.

Since Mercer and Miller's (1992) work, instructional procedures combining CRA and mnemonic devices have been utilized to teach higher order mathematical concepts (Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Witzel, Mercer, & Miller, 2003). Maccini and Ruhl investigated the CRA sequence combined with the algebra problem solving strategy "STAR" through a multiple-probe research design across subjects. The students in the study were in secondary school and diagnosed with a specific learning disability (SLD). The mnemonic device "STAR" prompted students to perform steps to solve subtraction of integers. The steps of

“STAR” strategy were (a) search the word problem, (b) translate words into a mathematical equation, (c) answer the problem, and (d) review the solution. Instructional procedures were adapted from Mercer and Miller (1992). Maccini and Ruhl examined students’ percent of strategy use, percent correct on problem representation, percent correct on problem solution and answer, and generalization. All participants increased their percent of strategy use from baseline to instructional phases in subtraction of integers.

Maccini and Hughes (2000) extended Maccini and Ruhl’s study and applied CRA instruction combined with the “STAR” strategy to teaching subtraction, multiplication, and division word problems involving integer numbers. The researchers examined improvements in (a) searching the word problem, (b) translating the word problem into an equation, (c) identifying the correct operation or operations, (d) drawing a picture of the problem, (e) writing a correct equation, and (f) answering the problems. Results of this study provided initial evidence that students with learning disabilities can be taught to represent word problems involving integer numbers through manipulatives and pictorial representations.

Researchers expanded the use of CRA instruction in teaching more difficult mathematical concepts. For example, Witzel and colleagues (2003) investigated the effectiveness of the CRA model for the acquisition and maintenance of knowledge of single-variable equations and solving for a single variable in multiple-variable equations. There were 358 sixth and seventh grade students who participated in the study. Of the 358 participants, 34 students with disabilities or at risk for algebra difficulty were in the treatment group and matched with 34 students with similar characteristics (i.e., statewide achievement test, age within one year, grade level, pretest accuracy within one level, at risk or disability) in the comparison group.

A pre-post-follow-up design with random assignment of clusters was employed for this study. Participants in the comparison group received intervention that included explicit instruction techniques while the treatment group received explicit instruction along with CRA. Both groups had significant improvements in answering single-variable algebraic equations from the pretest to the posttest and from the pretest to the follow-up. However, students who participated in the CRA instruction outperformed students who received traditional instruction.

Research has demonstrated that explicit instruction that incorporates the CRA sequence and mnemonic strategies have been successful in teaching certain mathematical skills. CRA and mnemonic devices have been shown effective in teaching higher order mathematical concepts (Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Witzel, Mercer, & Miller, 2003). However, there is a lack of research regarding instruction in solving complex algebra equations.

In a pilot study, Strozier, Hinton, Flores, and Shippen (2012) extended CRA and algebra to more complex algebraic equations. This study added a specific mnemonic to teach algebra along with the CRA sequence. The SUMLOWS mnemonic strategy aided students with steps to solving one-step, two-step, and multiple-step algebraic equations. A multiple-baseline across students with embedded behaviors showed a functional relation across three students for one-step and two-step behaviors. The results of this study showed that CRA and SUMLOWS instruction could be effective in teaching equations to students with disabilities. Consistent with the research of Witzel et al. (2003), the results of this study also showed the benefits of including concrete and pictorial representations when teaching algebra. The CRA algebra model proved effective for students with disabilities and those at risk for math difficulties in the general education classroom.

While research has demonstrated that mnemonic strategies and CRA have been successful in teaching math skills, few studies used a specific mnemonic for solving one-step, two-step, and multiple-step equations. Furthermore, none of the current literature regarding mnemonics and CRA algebra has addressed multiple step equations using the distributive property as the first step to solving the equation. In addition, none of the current literature analyzed common mistakes that students experience in solving equations. These results led to the development of this study, which focuses on implementation of a mnemonic strategy while teaching equations using CRA and SUMLOWS mnemonic strategy to solve one-step, two-step and multiple-step equations using the distributive property. Therefore, the purpose of this study was to investigate the use of CRA and SUMLOWS mnemonic strategy to teach middle school students who are identified as students who struggle in math. The research questions were as followed: What are the effects of CRA and SUMLOWS on solving one-step equations? What are the effects of CRA and SUMLOWS on solving two-step equations? What are the effects of CRA and SUMLOWS on solving multiple step equations? What are the effects of CRA and SUMLOWS on students' maintenance of solving equations two weeks after instruction? What were the common mistakes that students make while solving algebraic equations?

Method

Participants

Two eighth grade students, Sharon and Clint, participated in the study. The criteria for participation were: (a) a need for targeted math skills (i.e., solving one-step, two-step, and multiple-step equations) as defined by less than 50% correct on a pretest, a curriculum-based assessment; (b) below average performance according to the classroom teacher; (c) difficulty with mathematics achievement as demonstrated by a standard score of below 85 on the total test and a scale score of below 7 for the algebra subtest (Connolly, 2007); and (d) parent permission to participate in the study. The Kaufman Brief Intelligence Test, Second Edition (KBIT; Kaufman & Kaufman, 2004) was used to assess the students' cognitive functioning. Permission forms and flyer are included in Appendices 1, 2, 3, and 4.

Sharon received instruction in the general education classroom for math. Sharon qualified for special education services under the category of specific learning disability in math calculation and math reasoning. Her cognitive functioning was within the average range; she demonstrated a total standard score of 91 on the K-BIT (Kaufman & Kaufman, 2004). Sharon's mathematics achievement was below the average range; she demonstrated a standard score of 73 for total Key-Math III (Connolly, 2007) test composite and scaled score of 4 for the algebra subtest of the KeyMath. For post-intervention achievement, Sharon's scores on the algebra subtest improved with a scaled score of 6. According to KeyMath III, the grade equivalent of Sharon's total math achievement was 3.8 and the grade equivalent for the algebra subtest was 3.2. Sharon's deficits in mathematics included: a) adding and subtracting decimals, b) multiplying and dividing numbers over 5 at 100% accuracy, c) multiplying two-digit numbers, and d) adding, subtracting, multiplying, and dividing integers.

Clint received instruction in the general education classroom for math instruction. Clint did not qualify for special education services; however he was receiving Tier 2 instruction for low mathematics performance on high-stakes testing and local benchmark testing in math. His cognitive functioning was below the average range; he demonstrated a total standard score of 76 on the K-BIT (Kaufman & Kaufman, 2004). Clint's mathematics achievement was below the average range; he demonstrated a standard score of 66 for total Key-Math III (Connolly, 2007) test composite and scaled score of 4 for the algebra subtest of the KeyMath. For post-intervention achievement, Clint's scores on the algebra subtest improved with a scaled score of 5. According to KeyMath III, the grade equivalent of Clint's total math achievement was 2.8 and the grade equivalent for the algebra subtest was 3.2. Clint's deficits in mathematics included: a) adding and subtracting decimals, b) subtract two digits by two digits with regrouping, c) multiplying two-digit numbers, and d) adding, subtracting, multiplying, and dividing integers. The participants' characteristics are included in Table 10.

Table 10

Participant Characteristics

Name	Age	Grade	Eligibility	Cultural Background	Cognitive Ability[a]	Math Achievement[b]
Sharon	13	8	SLD	White	91	73
Clint	13	8	At-Risk	African American	76	66

a. Standard score on the Kaufman Brief Intelligence Test, Second Edition (Kaufman & Kaufman, 2004).

b. Total Test Composite Standard Score on the KeyMath3 (Connolly, 2007).

Setting

The study took place in a Title I middle school within a rural district in the Southeast. The majority of the enrolled students received free lunch (51%) and reduced-price lunch (9%). The cultural background of the students was as follows: 57% White, 41% African American, 2% Latino, and less than 1% Asian.

Materials

The general education teacher was provided and trained to use the CRA Manual. The manual included 27 scripted lessons (9 one-step lessons, 9 two-step lessons, and 9 multiple-step lessons). The CRA intervention materials included algebra tiles and learning sheets in which problems were written vertically and divided into three instructional sections: (a) model, (b) guided practice, and (c) independence practice. Learning sheets are available in Appendices 5, 6, and 7. Also included in the manual were probes for one-step, two-step, and multiple equations with answer keys. The SUMLOWS mnemonic was placed on an 8.5 X 11 sheet of paper with detailed steps of the mnemonic. The steps were: (S) Separate the sides, (U) Unite Like Terms, (M) Modify the new equation, (L) Loop (circle) around the coefficient, (O) Opposite sign (inverse operation), (W) What you do to one side, you must do to the other, and (S) Substitute the solution for the variable and check. The SUMLOWS strategy is available in Appendix 8.

The progress monitoring probes were sheets of paper with ten problems written vertically in boxes. There were four different alternate probes for each behavior: one-step, two-step, and multiple-step behaviors. The probes involved adding, subtracting, multiplying, and dividing integers. Sporadically, problems were written with the variable on the right side instead of the left as traditionally written. The distributive property was required for two-step and multiple-step behaviors along with variables on both sides of the equal sign. Probes are available in Appendices 9, 10, and 11.

The curriculum-based pretest included 30 questions of one-step, two-step, and multiple-step equations (10 questions of each behavior using positive and negative numbers). The pretest also contained variables on the left side, right side, and both sides of the equal sign. The pretest is available in Appendix 12.

The CRA manual consisted of scripted lessons for solving algebraic equations using CRA and SUMLOWS. The manual included three sections: one-step, two-step, and multiple-step equation. The materials for the concrete phase (lessons 1–3) of each lesson included algebra tiles, algebra work mat, the SUMLOWS mnemonic and learning sheets for targeted algebraic equations. The materials for the representational phase (lessons 4–6) of each behavior involved the SUMLOWS mnemonic, algebra work mat, and learning sheets with the targeted equation problems. The last phase of each instructional section was the abstract phase (lessons 7–9). The materials for the abstract phase included learning sheets with targeted equation problems. Neither the algebra work mat nor mnemonic were given during the abstract instructional phase. For each lesson, learning sheets with model problems, guided practice problems, and independence problems were included. The CRA manual is included in appendix 13. Additionally, the *KeyMath III* (Connolly, 2007) and *Kaufman Brief Intelligence Test II* (KBIT; Kaufman & Kaufman, 2004) were administered to assess mathematics achievement and students' cognitive functioning.

Assessment Tools

From the middle school math curriculum material, the researcher developed three pools of 60 math problems representing one-step, two-step, and multiple-step algebraic equations. The questions consisted of positive and negative integers that required the student to add, subtract, multiply, and divide to find the solution to the equation. The questions consisted of variables on the left side, right side, and both sides of the variable. Equations that required the use of the distributive property were also included in the pool for two-step and multiple-step questions.

For content validity, the pools of questions were distributed to four algebra teachers for expert review. All four of the middle school experts had at least a master's degree from an accredited university with more than 8 years in teaching pre-algebra/algebra to students. The reviewers were asked to score each algebraic equation as its relevance to the content using a 4-point scale to avoid having neural and ambivalent midpoint. The item relevance continuum used was 1= not relevant, 2= somewhat relevant, 3= quite relevant, and 4= highly relevant. Then for each item, the item-level content validity index (I-CVI) was computed as the number of experts giving a rating of 3 or 4 (thus dichotomizing the ordinal scale into relevant and not relevant), divided by the total number of experts (Lynn, 1986). The I-CVI for this pool items was calculated as 1.00. All expert reviewers rated the questions as being relevant for solving algebraic equations.

To further examine the items, the experts were asked to rate the same pool questions as being easy, medium, or difficult for the average middle school student. The difficulty level of the questions required consistency of 3 of the 4 raters on each question. Problems with less than the majority of consistency were thrown out and were not used in the probes or the pretest. Based on these results, the researcher kept 56 one-step questions, 57 two-step questions, and 53 multiple-step equations.

To assess the reliability of the probes, the researcher gave three twenty question assessments to 19 college level students from a major four-year university. The assessments were untimed and completely voluntary. Results from the internal consistency using SPSS software revealed Cronbach's Alpha Coefficient of $r = .75$ for one-step, $r = .75$ for two-step, $r = .79$ for multiple-steps, and $r = .91$ for total test.

Research Design

A multiple probe across three behaviors was utilized to evaluate the effects of CRA and SUMLOWS for students who struggle with algebraic equations. Mathematics instruction was provided on a daily basis. Data were collected across the behaviors of one-step, two-step, and multiple-step through probes. Students completed the probes before daily instruction began for that day. The untimed probes were scored based on the number of problems answered correctly, including both digits and appropriate signs for symbols. For example, if the student did not add the negative sign, the question was marked as incorrect.

This design was used to show a functional relation between CRA and SUMLOWS and three problem solving behaviors (i.e., one-step, two-step, and multiple-step equations). The criterion for phase change was 80% of problems correct or better on two consecutive probes. Prior to intervention, baseline data were taken for all behaviors (i.e., one-step, two-step, and multiple-step algebraic equations). Once three stable data points were established, instruction began for one-step. A stable baseline was defined as three consecutive data points in which the points varied no more than 20% from the average. Once the student met criterion of 80% correct on two consecutive probes for one-step problems, instruction on two-step began. Once the student reached criterion for two-step equations, instruction for multiple-step algebraic equations began. Maintenance data were taken two weeks after CRA and SUMLOWS instruction ended.

The intervention consisted of CRA and SUMLOWS mnemonics strategy in order to answer this study's research questions: What are the effects of CRA and SUMLOWS on solving one-step equations? What are the effects of CRA and SUMLOWS on solving two-step equations? What are the effects of CRA and SUMLOWS on solving multiple step equations? What are the effects of CRA and SUMLOWS on students' maintenance of solving equations two weeks after instruction? What were the common mistakes that students make while solving algebraic equations? The SUMLOWS mnemonic represented steps to solving one-step, two-step, and multiple-step algebraic equations.

The CRA instructional sequence was used to represent the problems in three different ways. First, teachers and students used three-dimensional objects, such as algebra tiles to assist with solving the equations. Later, the teacher and students used drawing instead of algebra tiles to solve algebraic equations. Lastly, the teachers and students used numbers only to solve the equations. In each lesson, the SUMLOWS mnemonic was embedded in the CRA sequence. The SUMLOWS mnemonic was a remembering device that aided the students in following the problem solving steps. Each letter of SUMLOWS instructed the students in the problem solving steps, which are: (a) Separate the two sides, (b) Unite like terms, (c) Rewrite the equation, (d) Loop (circle) the coefficient (e) Opposite Operation, (f) What you do to one side, you must do to the other side, and (g) Substitute the solution into the equation and check. The dependent variable was the number of correct answers on untimed probes consisting of 10 problems of each behavior (one-step, two-step, and multiple-step algebraic equations.

Teacher Training

The lead researcher trained the classroom teacher to implement instruction. The training used examples and nonexamples. The classroom teacher was encouraged to ask questions to clarify instruction. The researcher began the training by showing the classroom teacher how to use the algebra tiles and the drawings. The researcher explained what the shapes and colors represented. The researcher demonstrated three scripted lessons (one concrete, one representation, and the last abstract) for the classroom teacher. After the researcher demonstration, the classroom teacher was asked to demonstrate three different lessons. During the classroom teacher demonstration, the researcher and a trained colleague used the treatment integrity checklist for the fidelity of the intervention. The classroom teacher demonstrated 100%

of the teaching behaviors for the three lessons before instruction began. The checklist is available in Appendix 14.

Instructional Procedures

Instructional sessions lasted for 60 minutes, scheduled for 5 days a week. In addition to regular math in the general education class, students received instruction during a regularly scheduled connection/exploratory math time, in which Tier 2 math remediation was provided by the general education teacher. The study lasted for 8 weeks.

Concrete Phase

Lessons one through three of solving one-step algebraic equations used two-sided rectangular and square tiles as concrete manipulatives. The rectangular tiles were used for coefficients of the variable (i.e., 5X, X/3, etc.). The squares were used for the numbers (i.e., 2, 6, etc.). On both types of tiles, the red side was used for negative numbers. For positive coefficients of X, the tile was turned to the cream side. For positive numbers, the tile was turned to the green side. Below are examples on how to set up algebraic problems using algebra tiles during the concrete phase of the CRA sequence. Examples of how to use the algebra tiles are available on Table 11.

Table 11

Examples Using Algebra Tiles in the Concrete Phase

Problem	Tiles on left side	Tiles on right side
X – 5 = -4	1 green rectangle and 5 red squares	4 red squares
-X + 3 = 8	1 red rectangle and 3 cream squares	8 cream squares
-4X = 12	4 red rectangles	12 cream squares
X/2 = -6	X over 2 green rectangles	6 red squares

One-step Algebraic Equations

For each lesson**,** prior to instruction, probes were administered. Instruction began when the first student demonstrated a stable baseline. The probes were given once daily before the lesson began. Each student was given untimed probes which contained ten problems. The first lesson involved instruction using the phases of instruction which were adopted from Maccini and Hughes (2000). The first phase of instruction involved a pretest in which instructional need was established and discussed with the student. The researcher also obtained a commitment from the student in the form of a contract. The written contract was dated and signed by the student and teachers. The contract lists items that the students do during the course of the intervention.

The second phase of the intervention involved instruction in solving algebra equations. The first lesson involved instruction in using the mnemonic strategy (SUMLOWS) were: (a) Separate the two sides, (b) Unite like terms, (c) Rewrite the equation, (d) Loop (circle) the coefficient (e) Opposite operation, (f) What you do to one side, you must do to the other side, and (g) Substitute the solution into the equation and check will be on transparency. Using the explicit teaching sequence, the teacher provided an advance organizer, demonstrated the strategy, guided the students in reciting the strategy and asked the students to recite the strategy independently. After the students had memorized the strategy, lessons 1–3 involved instruction at the concrete level using algebra tiles in which the teacher demonstrated the concept and skills involved in solving one-step algebra equations using the SUMLOWS strategy. One-step algebraic equations involves equations that involved one-step to isolate the variable, such as $X - 6 = -3$, $X + 5 = 2$, $15 = 3X$, or $X/5 = 3$.

During one-step, the teacher used the SUMLOWS mnemonic, a sample script, learning sheets that guided the teacher through the instructional sequence and the e CRA sequence. During the teacher demonstration, the teacher modeled how to use the steps of SUMLOWS to solve the problem. In the steps of SUMLOWS for solving one-step algebraic equations, two of the steps of SUMLOWS (unite like terms and modify the equation) are not used; however the student learned to look for like terms. The teacher talked aloud as she worked out the problems. For example for the problem X – 4 = -12 in the concrete phase, the teacher would think out loud and say " I have X – 4, so I will need a rectangle to represent my coefficient for X. Since the X is positive, the rectangle is green. The next part of the problem is minus 4. This is a regular number unit so I must use the squares instead of the rectangles. I am going to put 4 squares on my mat. Because the four is negative, the squares need to be red. Lastly, I have negative 12. The 12 is a number unit, so I will need to use 12 squares. Since the 12 is negative, they will need to be red.

After the teacher set up the algebra tiles, she uses SUMLOWS to solve the equation. The first step of SUMLOWS is to separate the two sides. The teacher draws a line threw the equal sign. The teacher states that this is to let her know that there are two different sides of equations, a left side and a right side. The next step of SUMLOWS is to unite like terms. In order for the terms to be a like, they need to be the same shape on the same side of the equal sign. The teacher examines the left side and asks aloud whether there are any shapes that are the same that are not already together. The teacher tells the students that there are rectangles and squares, different shapes. So, the teacher tells the students that there are no like terms on the left side. Next, to the teacher demonstrates the same process for the other side of the equation, examining the right side of the equation for like terms. On the right side of the equation, the teacher notices that there are shapes that are the same, squares. The teacher says, “I have squares and they are already together. So, I do not have any like terms to unite.” The next step of SUMLOWS is to modify the equation. Since there are no like terms, the teacher tells the students that she will skip this step. The next letter of SUMLOWS is “L” and it stands for loop the coefficient. The teacher tells the students that she will circle the rectangles because the rectangle represents “X.” The teacher explains why she circles the rectangle, telling the students that this is the shape that wants to be by itself or the number that will be moved last. The teacher asks what is on the side with the circle. The teacher answers her question, stating that there are four red squares on the side with the circle. The teacher tells the students that she will move the four red squares because the circle wants to be by itself.

The next step is to do the opposite sign. The teacher tells the students that in order to move the six red squares, she must add 4 cream squares. The teacher tells the students that what she does to one side, she must do to the other. Since she added 4 cream squares on the left, she must add 4 cream squares to the right. The teacher explains that the red squares and the cream squares on the left side are opposites and will equal zero. The teacher takes all of the squares off of the mat. The teacher explains the tiles on the right side, twelve red squares and four cream squares which are the same shape. The teacher tells the students that she must unite them and since they are different colors, she will subtract (four from twelve). The teacher demonstrates by pairing a red square with a cream square until there are no more matches. The teacher shows the students that there are eight squares remaining. The teacher asks what she has on the left side and answers her question, telling the students that she has a green rectangle. The teacher concludes that the answer to the problem is X = -8. The teacher demonstrates the last step of SUMLOWS by plugging the solution into the problem. The teacher shows the students this step by going back to the original problem (X – 4 = -12) and replacing X with -8 using tiles. The teacher puts eight red tiles in the place of the X, four red squares in place of -4, and twelve red squares in place of -12. The teacher examines the equation, asking how many squares are on each side. The teacher points out that there are twelve red squares on each side of the equation and this indicates that the correct solution has been found.

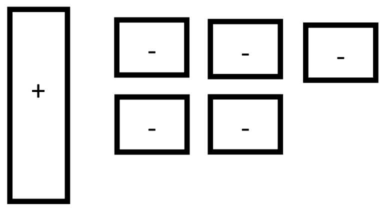
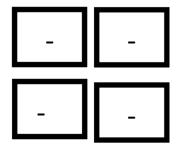
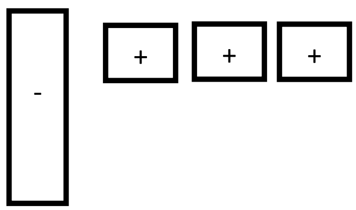
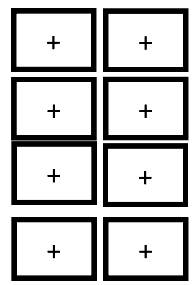
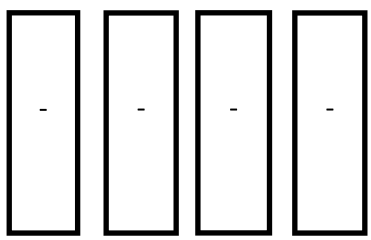
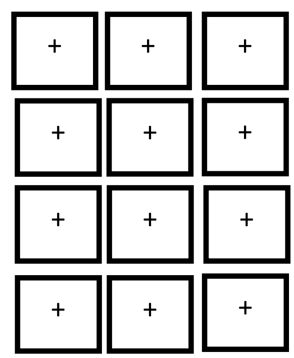
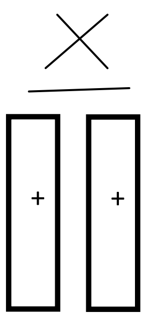
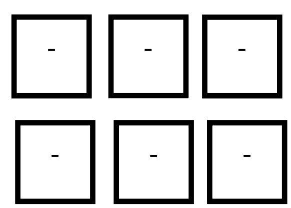
After demonstration, the teacher guides the student in solving one-step equations. During guided practice, the students and teacher do problems together. The teacher used prompted the students during each step of the problem solving process. After guided practice, students worked equations independently, and then teacher provided the students with feedback. Instruction continued throughout the concrete, representational, and abstract phases of the CRA sequence. After the criterion for phase change was reached (80% or better on two consecutive probes), the student began instruction on two-step algebraic equations. If lessons remained, the student continued lessons for one-step equations after instruction for two-step equations began. At times, instruction in more than one behavior occurred within the same instructional session.

Representational Phase

The next phase was the representational application. This phase included lessons four through six. A sample script and learning sheets guided the teacher through the instructional sequence. Instead of using algebra tiles to solve for and demonstrate the algebra concept, the teacher and students used drawings. The coefficients (the number in front of the variable (i.e., 5X, X/3, etc.) were represented by long rectangles. Within the rectangles, students drew a plus (+) when the number is positive and a minus (-) when the number was negative. The unit number was represented by a small square (i.e., 2, 6, etc.). On both types of shapes, a plus sign were used if the number was positive and a minus sign in the shape was used if the number was negative. For example, if the unit number was positive, the student will draw a plus (+) sign inside the square. If the unit number was negative, the student drew a minus (-) sign inside the square. For positive coefficients of X, the teacher and student drew a rectangle with a plus (+) sign in it. For negative coefficient of X, the teacher and student drew a rectangle with a minus (-) sign in it. See Table 12 for examples of drawings in the representation phase. Instruction at the representational level included advance organizers, teacher demonstration, guided practice, independent practice, and post organizers.

Table 12

Examples of Representational Phase in One-step

Problem	Tiles on left side	Tiles on right side
X – 5 = -4		
-X + 3 = 8		
-4X = 12		
X/2 = -6		

Abstract Phase

The next phase involved teaching the abstract application. This phase started in lesson eight and continued to lesson ten. The teacher used the SUMLOWS mnemonic and used numbers only to solve problems at the abstract level. Similar to the concrete and representational phases, the teacher taught using the same explicit instruction procedures.

Two-step Algebraic Equations

Once the student reached criterion for one-step equations (80% on two consecutive probes), the student began instruction in two-step equations. Two-step equation instruction was similar to one-step; however, it involved two steps to isolate the variable. The two steps can consist of, but is not limited to, combining like terms or distributive property and then multiplying or adding. While solving two-step equations, students used the SUMLOWS mnemonic strategy and concrete manipulatives for the first level (lessons 1–3; see examples in Table 13), and then moved to representational level using drawings (lessons 4–6; see examples in Table 14), and then the abstract level (lessons 7–9), in which only numbers were used.

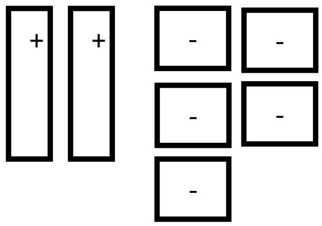
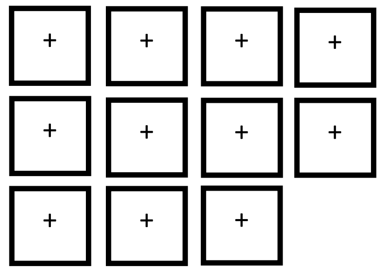
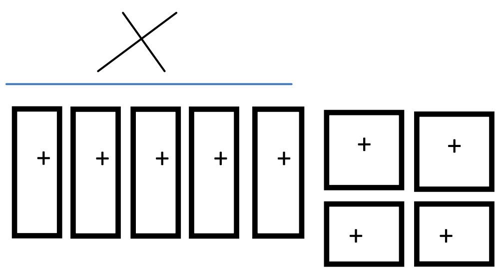
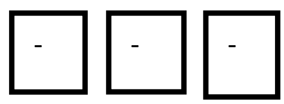
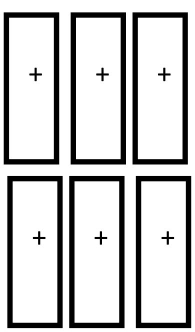
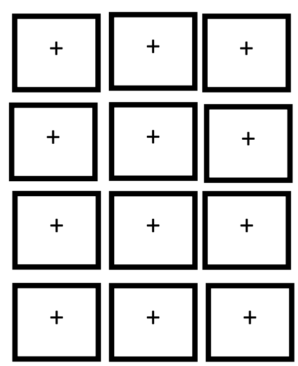
Table 13

Two-step Examples Using Algebra Tiles in the Concrete Phase

Problem	Tiles on left side	Tiles on right side
2X – 5= 11	2 green rectangles & 5 red squares	11 cream squares
X/5 + 4 = -3	X over 5 green rectangles & 4 cream squares	3 red squares
2(3X) = 12	3 green rectangles	12 cream squares
	<u>3 green rectangles</u>	
	6 green rectangles	

Table 14

Examples of Representational Phase in Two-step

Problem	Tiles on left side	Tiles on right side
$2X - 5 = 11$		
$X/5 + 4 = -3$		
$2(3X) = 12$		

Multiple-step Algebraic Equations

Multiple-step equation involved at least three steps to solve the equation. The same procedures apply to multiple-step as in one-step and two step equations. While solving two-step equations, students used the SUMLOWS mnemonic strategy and concrete manipulatives for the first level (lessons 1–3; see examples in Table 15), and then moved to representational level using drawings (lessons 4–6; see examples in Table 16), and then the abstract level (lessons 7– 9).

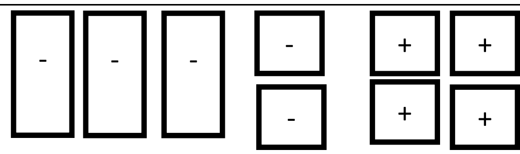
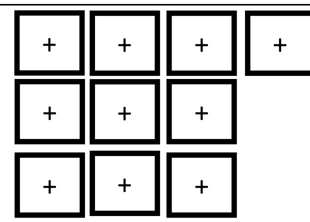
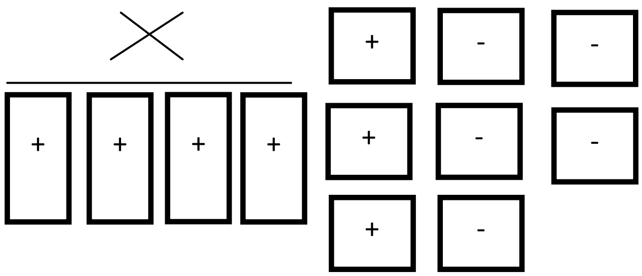
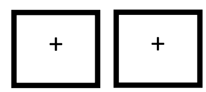
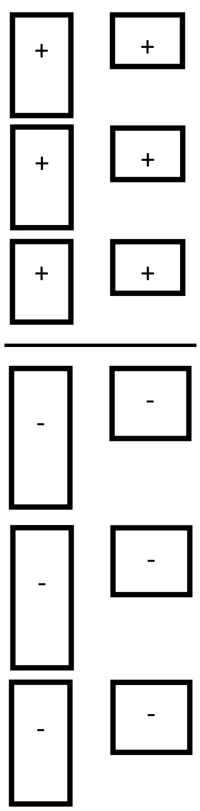
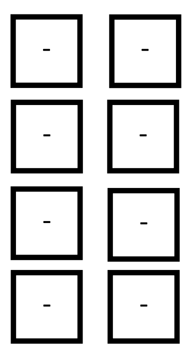
Table 15

Examples of Concrete Phase for Multiple-Step Equations

Problem	Tiles on left side	Tiles on right side
-3X – 2 + 4 =10	3 red rectangles, 2 red squares, & 4 cream squares	10 cream squares
X/4 + 3 – 5 = 2	X over 4 cream rectangles, 3 cream squares, & 5 red squares	2 cream squares
-3(X + 1) = -8	1 green rectangle & 1 cream square	8 red squares
	1 green rectangle & 1 cream square	
	1 green rectangle & 1 cream square	
	3 green rectangles & 3 cream squares	
	Since the 3 is negative, the tiles flip	
	3 red rectangles & 3 red squares	

Table 16

Examples of Representational Phase for Multiple-Step Equations

Problem	Tiles on left side	Tiles on right side
-3X – 2 + 4 =10		
X/4 + 3 – 5 = 2		
-3(X + 1) = -8		Since the 3 is negative, you must flip the sign 

Treatment Integrity and Inter-Observer Agreement

A treatment checklist was used to ensure treatment fidelity. The checklist is available in Appendix 14. A researcher completed the checklist. Treatment integrity was conducted during 55% of the lessons pertaining to solving one-step equations, 55% of the lessons pertaining to solving two-step equations, and 67% of lessons pertaining to multiple-step equations. Instructional lessons were recorded with digital video. In order to calculate integrity, the researcher took the total number of agreements between both the researchers and divided the number by the total number of observations; then multiplied by 100 (Poling, Method, and LeSage, 1995). Treatment integrity was calculated as 100% for solving one-step equations, 100% for solving two-step equations, and100% for solving multiple-step equations.

Additionally, inter-observer agreement was conducted for 100% of the probes administered. Data were collected before each instructional lesson. Probes were scored by the researcher implementing instruction and later scored by another researcher. To calculate inter-rater reliability, the total number of agreements between both the researchers were divided by the total number of observations; then multiplied by 100 (Poling, Method, & LeSage, 1995). Inter-rater reliability was 100% for all behaviors for both students.

Social Validity

Social validity was addressed through a closed and open-ended questionnaire before and after the study. The Pre-intervention questionnaire is available in Appendix 15. The general education teacher answered questions before the intervention regarding the need for the intervention. The teacher indicated that algebra is very important in the eighth grade standards. The teacher also indicated that students often missed the necessary procedural steps when solving two-step and multiple-step equations. Multiple-step equations are difficult for many eighth grade students to understand. The teacher stated that they were not currently using maniplutives to teach algebraic equations to their students. The teacher indicated that she would like to learn a strategy to aid her students in solving equations.

The questionnaire after the intervention addressed the efficacy of the interventions and recommendations for the intervention. The post intervention questionnaire is available in Appendix 16. The teacher's feedback indicated that the use of algebra helped students visualize the concept. The reduction of concrete, representational to abstract helped students understand equations better and overall improved their algebra skills. The teacher liked that the tiles assisted the students with understanding numbers. The teacher indicated that she would use the CRA and SUMLOWS and also recommend this strategy to other teachers.

Social validity was also addressed through a closed and open-ended student questionnaire before and after the study. The pre-Intervention questionnaire is available in Appendix 17. Sharon and Clint indicated that it was difficult for them to remember steps to solve equations. Both students wanted to learn a strategy to make solving algebraic equation easier. Sharon and Clint wanted to do be better in algebra. For the open ended questions, Sharon and Clint said that they would like to change their algebra grade and they were not using anything that helped them remember steps.

The questionnaire after the intervention addressed the efficacy of the interventions and recommendations for the intervention from the student's perspective. The post intervention questionnaire is available in Appendix 18. Sharon and Clint indicated that CRA and SUMLOWS helped them complete their algebra work. They believed that the CRA and SUMLOWS mnemonic strategy helped their math skills. They would use the strategy again. Both agree that it was very true that the algebra tiles helped them solve algebraic equations better and the SUMLOWS mnemonic assisted them in remembering what step came next. When asked about what they liked about the CRA and SUMLOWS strategy, Clint stated that he enjoyed using the algebra tiles and drawings; they made it easier. Sharon stated it helped her improved her math skills. She continued to add that she went home and taught her mother how to solve equations. She added that her mother stated that she was getting better with solving algebraic equations. When asked what they would change about the CRA and SUMLOWS strategy, both students said nothing. They both stated that they "like it the way it is." Additionally, both students added that they used to think that solving equations were hard, but now it was fun and easy.

Results

A multiple probe across behaviors was utilized to evaluate the effects of CRA and SUMLOWS for students who struggle with solving algebraic equations. Data were interpreted by visual inspection and the following were noted: overlap between baseline and treatment, slope of each treatment data path, and number of data points from the beginning of treatment to criterion. Results for Sharon and Clint are summarized in Figures 1 and 2.

Baseline Data

Before intervention began, students were given baseline probes on different days. Sharon's baseline data were stable across all behaviors, zero correct problems on all probes. Clint's baseline performance was also stable across all behaviors. The baseline probes indicated zero correct problems on all probes. Once baseline was established, the classroom teacher began one-step intervention using CRA sequence and SUMLOWS mnemonic strategy.

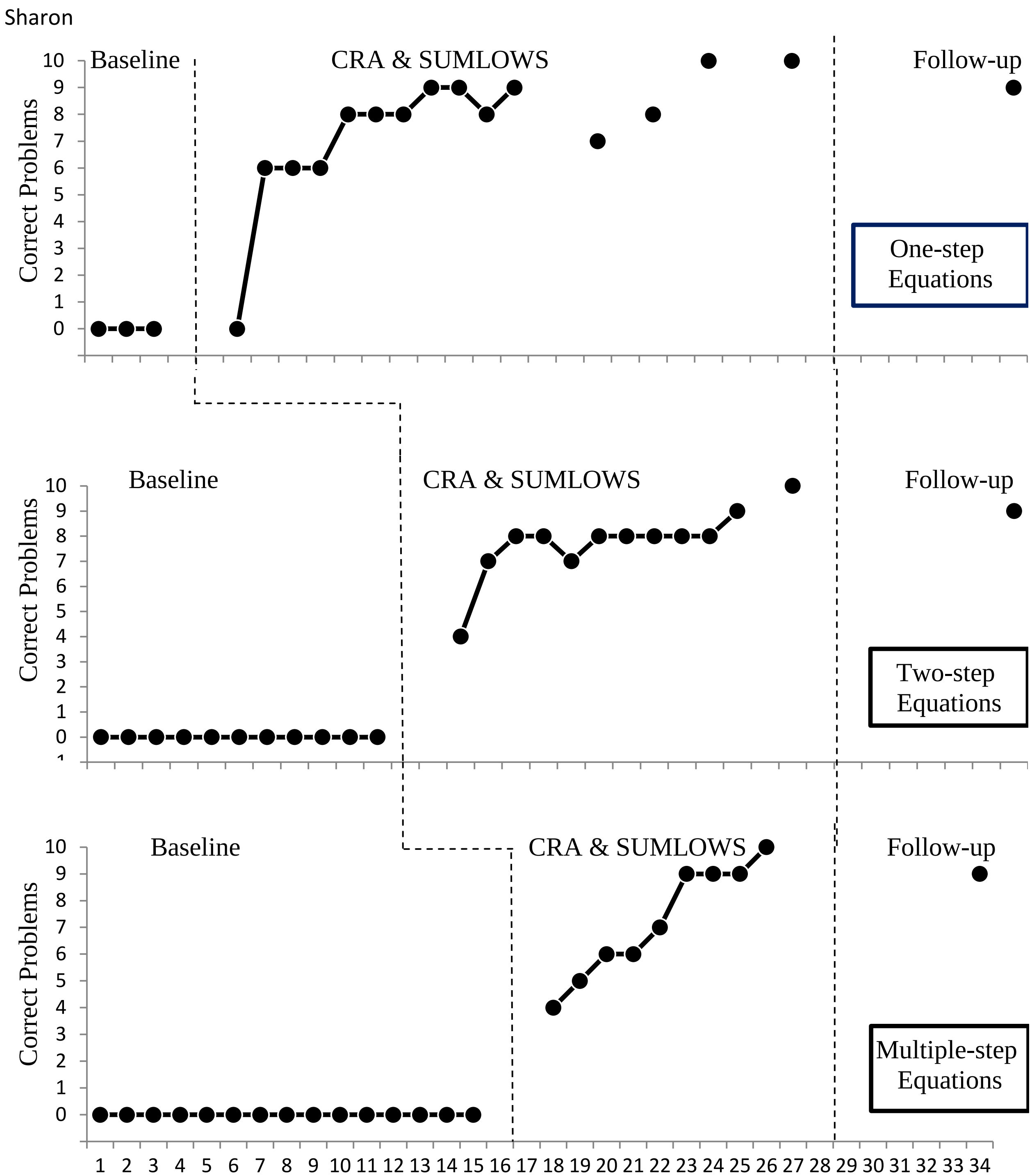


Figure 1. Sharon Results

Performance after Instruction

Sharon. Sharon reached criterion for one-step equations after six probes (80% or better on two consecutive probes). There was a change in performance level; however the first data point overlapped her baseline performance. The data points continued in an upward path throughout intervention. Sharon completed all the nine lessons for one-step. Throughout the intervention, Sharon was given 15 one-step probes. The range of problems correct for one-step equations were from zero to ten. The mode for the number of correct probes was eight. The average correct problems from the one-step probes were 7.5. For the second behavior, Sharon reached criterion for two-step equations after four probes. There was an immediate change in performance between baseline and CRA and SUMLOWS instructional phase and no overlapping data points between the baseline and instructional phases. The data points continued in an upward path throughout intervention. She received nine lessons for solving two-step equations. Throughout the intervention, Sharon was given 12 two-step probes. The range of problems correct for two-step equations were from four to ten. The mode for the number of correct probes was eight. The average correct problem from the two-step probes was 8.4. She reached criterion for multiple-step equations after seven probes. There was an immediate change in performance between baseline and CRA and SUMLOWS instruction and no overlapping data points between the baseline and instructional phases. The data points continued in an upward path throughout intervention. She received nine lessons for solving multiple step equations. Throughout the intervention, Sharon was given nine multiple step probes. The range of problems correct for multiple-step equations were from four to ten. The mode for the number of correct probes was nine. The average correct problem from the multiple-step probes was 7.2.

Clint

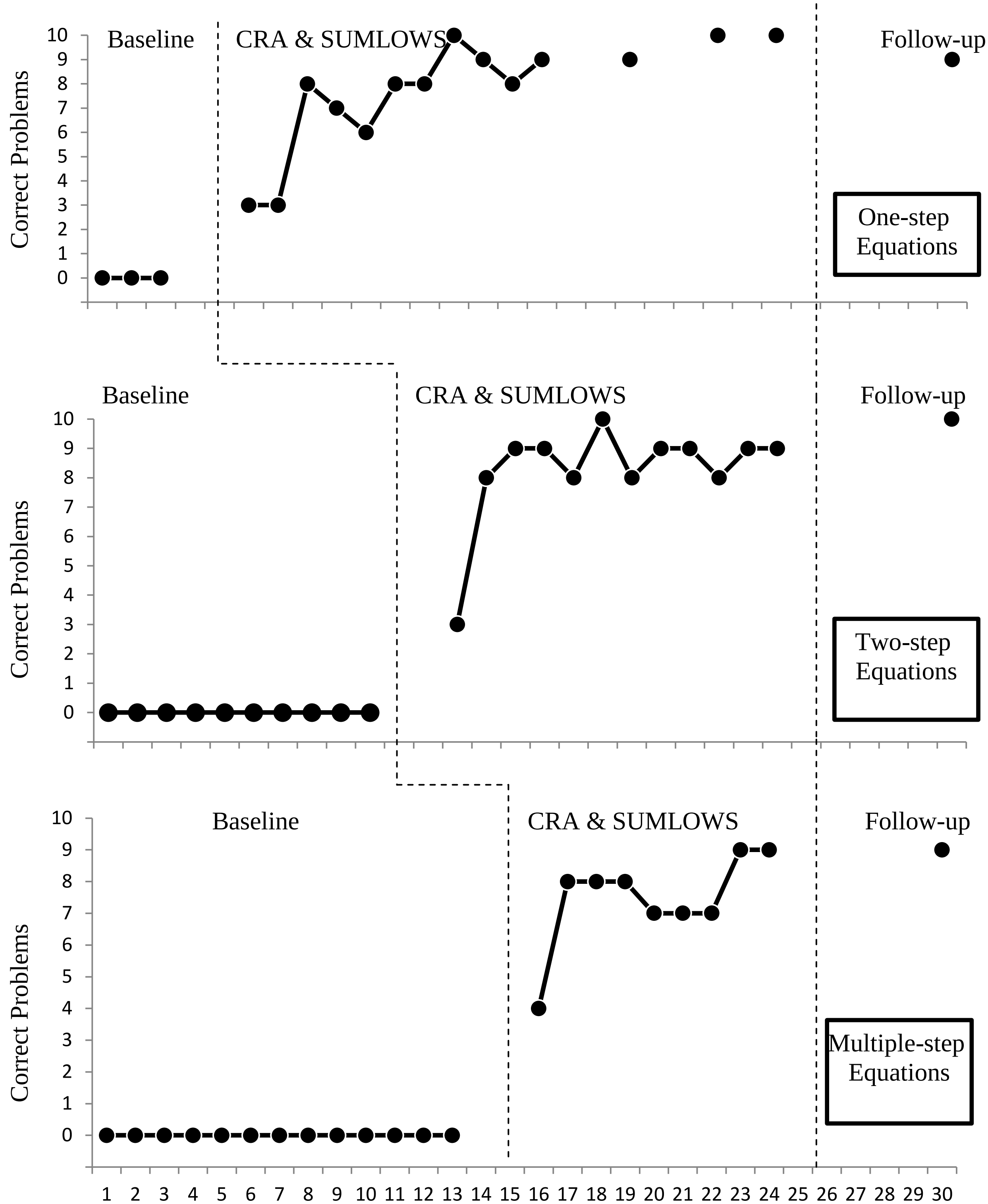


Figure 2. Clint Results

Clint. Clint reached criterion for one-step equations after seven probes (80% or better on two consecutive probes). There was an immediate change in performance between baseline and CRA and SUMLOWS instructional phase and no overlapping data points between the baseline and instructional phases. The data points continued in an upward path throughout intervention. Clint completed all the nine lessons for one-step algebraic equations. Throughout the intervention, Clint was given 14 one-step probes. The range of problems correct for one-step equations were from three to ten. The mode for the number of correct probes was 8. The average correct problem from the one-step probes was 7.7. For the second behavior, he reached criterion for two-step equations after three probes. There was an immediate change in performance between baseline and CRA and SUMLOWS instructional phase and no overlapping data points between the baseline and instructional phases. The data points continued in an upward path throughout intervention. He received nine lessons for solving two-step equations. Throughout the intervention, Clint was given 12 two-step probes. The range of problems correct for two-step equations were from three to ten. The mode for the number of correct probes was seven. The average correct problem from the two-step probes was 8.3. Clint reached criterion for multiple-step equations after three probes. There was an immediate change in performance between baseline and CRA and SUMLOWS instruction and no overlapping data points between the baseline and instructional phases. The data points continued in an upward path throughout intervention. He received nine lessons for solving multiple step equations. Throughout the intervention, Overall, Clint was given nine multiple step probes. The range of problems correct for multiple-step equations were from four to nine. The modes for the number of correct probes were seven and eight. The average correct problem from the multiple-step probes was 7.4.

Analysis of Probes after Intervention

After intervention, the researcher analyzed the missed problems on the probes given during intervention. The probes were evaluated for the first mistake that the student made that caused the answer to be incorrect. The researcher scored the mistakes as an integer mistake (1), calculation mistake (2), distributive property mistake (3), procedure/steps mistake (4), or left blank (5). The results from Sharon's probes indicated that 46% of her missed problems were from integer mistakes. Integer mistakes consisted of having the wrong sign in the answer, having wrong sign when adding or subtracting, or adding when she should have subtracted. Twenty-nine percent of Sharon's missed problems were from leaving problems blank. Thirteen percent of problems were missed because of procedures or steps. The most common of the procedural mistakes included canceling out the variable that was already on the opposite sign of the variable. Other mistakes included adding or subtracting integers instead of multiplying or dividing them. Simple calculation errors were the cause of eight percent of the incorrect problems. Lastly for Sharon, four percent of the errors were caused by mistakes in the distributive property steps.

The results from Clint's probes indicated that forty percent of his missed problems were from integer mistakes. Integer mistakes consisted of having the wrong sign in the answer, having the wrong sign when adding or subtracting, or adding when he should have subtracted. Twenty-six percent of problems were missed because of procedures or steps. The most common of the procedural mistakes included canceling out the variable that was already on the opposite sign of the variable. Other mistakes included adding or subtracting integers instead of multiplying or dividing them. Simple calculation errors were the cause of fifteen percent of the incorrect problems. Four percent of the errors were caused by mistakes in the distributive property steps. Lastly for Clint, nine percent of missed problems were from leaving problems blank. Analysis of problems missed on probes is included in Table 17.

Table 17

Analysis of Missed Problems

Participant	Integer	Calculation	Distributive	Procedure	Blank
Sharon	46%	8%	4%	13%	29%
Clint	40%	15%	10%	26%	9%

Maintenance Performance

Maintenance data were collected two weeks after instruction ended. Sharon was given a probe for each of the three behaviors. Sharon scored 90% correct for one-step equations. She scored 90% correct for two-step equations and 90% for multiple step equations. Clint was given a maintenance probe for all three behaviors. He scored 90% correct for one-step equations, 100% correct for two-step equations, and 90% correct for multiple step algebraic equations.

Discussion

The purpose of this study was to investigate the effectiveness of CRA and SUMLOWS mnemonic strategy for teaching one-step, two-step, and multiple step algebraic equations. The research design was a multiple probe across three behaviors. A functional relation was demonstrated between CRA and SUMLOWS instruction and the behaviors of one-step, two-step, and multiple step algebraic equations across two participants. Both students met criterion for all three behaviors. Sharon and Clint demonstrated steady progress across all three behaviors.

Findings Related to CRA with SUMLOWS and Algebra

With the higher standards to receive a high school diploma, algebra has become a more significant challenge for students with disabilities and their teachers. Although higher-level math skills are vital to their future, many students with disabilities experience difficulty with secondary math concepts, such as algebra (Maccini, McNaughton, & Ruhl, 1999). The results of this study are significant because classroom teachers are in need of research-based instruction and interventions to assist students with disabilities in the general education classroom. CRA and SUMLOWS instruction gives the teachers a way to eliminate the abstractness of algebra and a specific mnemonics for solving algebraic equations.

Many students struggle with algebra because of the abstractness of the concept; therefore the concepts and the skill become overwhelming for many students with disabilities. Concrete manipulative and pictorial representations that are used in the CRA sequence have been shown to increase students' understanding of these abstract concepts. Consistent with previous research (Maccini & Hughes, 2000; Maccini & Ruhl, 2000, Witzel et al., 20003, Strozier et al., (2012)) the effects of the CRA instructional sequence and SUMLOWS mnemonic resulted in mathematical gains in algebra. The students' pretest and baseline data indicated a lack of knowledge regarding one-step, two-step, and multiple-step algebraic equations. The CRA and SUMLOWS provided students with clear expectations and scaffolding which led to increased understanding of the concepts behind solving algebraic equations.

The SUMLOWS mnemonic strategy provided students with procedural knowledge to complete algebraic problems. This mnemonic provided students who struggle with math a memory strategy to use while solving equations. This study affirms and extends the studies of Maccini and Ruhl (2000), Maccini and Hughes (2000), along with Witzel et al. (2003) by investigating the effects of the CRA instructional sequence with regard to algebraic equations. Additionally, this study extends previous research by Strozier et al. ((2012)) in which CRA and SUMLOWS demonstrated effectiveness for solving one- step and two-step algebraic equations. This study extends that research to a different population, middle school students with disabilities. Extending research to middle school is important because general education standards states that a strong foundation of algebra concepts should be in place by the end of eighth grade (National Council of Teachers of Mathematics, 2012). The current study also extends the line of research, showing that this methodology is effective for multiple-step equations, two-step and multiple-step equations with the distributive property.

The researcher analyzed the students' missed problems. Forty-four percent of the problems missed were due to integer mistakes. Maccini and Ruhl (2000) and Maccini and Hughes (2000) found that CRA was effective for learning integers. Even though the algebra manual incorporated integers during the self-talk of the scripted lessons, students struggled with deciding when a number is positive or negative when multiplying and dividing integers. Future research might address this by making fluency with integers a focus before teaching equations. This can be done through extending and replicating research for Maccini and Ruhl (2000) and Maccini and Hughes (2000). Additionally, the CRA manual needs to incorporate concrete, representational, and abstract lessons on solving integers before teaching one-step equations. The current findings have implications for practitioners; CRA and SUMLOWS can be beneficial for the classroom teacher. The study was conducted in a small group during tier 2 instruction. General educators and special educators can use the CRA and SUMLOWS instruction during tier 2 and tier 3 instructions. The study lasted for eight weeks for an hour a day; therefore it can be used during a typical six period classes or an hour and a half block schedule. For the block schedule, students would receive additional time for feedback and time to correct mistakes made during independent practice.

Limitations and Suggestions for Future Research

The research design presents a limitation to the present study because the CRA and SUMLOWS instruction was not compared with another mathematics program. Therefore, there may be other instructional programs or interventions as or more effective. In addition, the researcher assisted with implementation of the instruction. Thus, replication is warranted to validate these current results. Two students demonstrated substantial improvement with the instruction, but these results cannot be generalized. Therefore, additional research is needed to generalize the result for students solving one-step, two-step, and multiple-step algebraic equations. Also, generalizability of results across settings limited, as instruction was provided in one school. The setting of this study involved students in small group. It is unclear if the same effects would be replicated in a larger group in a general education classroom.

This study needs to be replicated by different researchers, with different teachers implementing the CRA and SUMLOWS instruction. Researchers need to investigate the effects of the intervention with larger number of participants as well as investigate the effects of intervention in an inclusive setting for student with different disabilities. Lastly, research should be conducted to compare the effects the CRA and SUMLOWS with other mathematical programs.

CHAPTER 4. CONCLUSIONS AND RECOMMENDATIONS

The expectation for students with disabilities' achievement was increased by the 1997 Amendments to the Individuals with Disability Education Act (IDEA), which emphasized students' participation and progress in the general education curriculum. In addition to IDEA, the standards outlined in the No Child Left Behind Act of 2001 (NCLB; 2002) required states to annually assess the reading and mathematics skills of all students in third through eighth grade, including those with disabilities. The reauthorization of the Individuals with Disabilities Education Improvement Act (IDEIA; 2004) reinforced these standards by requiring students with disabilities to meet the same standards as their peers without disabilities despite their intellectual, emotional, or physical challenges. Meeting these standards presents a significant challenge for students who struggle in mathematics, and the challenge is especially daunting for students with disabilities (Scheuermann, Deshler, & Shumaker, 2009).

Secondary content, such as algebra may be overwhelming to many students and teachers. Because of the abstractness of algebra, many students find it difficult to understand related or specific concepts (CEC Webinars, 2012). The CEC stated that there was a critical need for instruction and intervention that goes beyond "typical" instruction of abstract concept only. Therefore the current study examined the CRA and SUMLOWS instruction for students who struggled in algebra. The following research questions were developed:

a) What are the effects of CRA and SUMLOWS on solving one-step equations?

b) What are the effects of CRA and SUMLOWS on solving two-step equations?

c) What are the effects of CRA and SUMLOWS on solving multiple step equations?

d) What are the effects of CRA and SUMLOWS on students' maintenance of solving equations two-week instruction?

The independent variable in this study was CRA and SUMLOWS instruction. The dependent variables were the number of problems correct on the one-step, two-step, and multiple-step probes. A multiple-baseline across behaviors design was utilized to evaluate the effect of CRA and SUMLOWS with students who struggle in algebra. A functional relation was demonstrated between the CRA and SUMLOWS instruction and the behaviors of one-step, two-step, and multiple-step equations with two participants. Both students met criterion for all three behaviors.

Results from the CRA and SUMLOWS instruction were examined by the student's responses on the probes. The problems within the probes were graded as correct or incorrect. The findings were consistent with previous CRA research in which CRA was found to be effective for teaching math concepts such as addition, subtraction, multiplication, fractions, integers, and solving algebraic equations.

Additionally, the probes were analyzed further to examine the students' mistakes. Results indicated that the majority of students' errors were caused by integer mistakes. The lack of understanding of integers and their rules may hinder students' understanding of algebraic equations.

It was recommended that teachers give a pre-test before teaching algebraic equations. The pre-test should assess mathematical skills such as addition, subtraction, multiplication, and divisions. The pre-test should also assess integer skills. If students do not possess the skills, classroom teachers should re-teach these skills using other CRA materials before algebraic instruction. Moreover, it is recommended that the researcher add a pretest and integer lessons in the Algebraic CRA Manual to decrease the number of mistakes caused by integers. Additionally, it was recommended that further research continue for secondary math concepts using CRA instructional sequence. Therefore, teachers will be empowered to provide adequate instruction for secondary concepts for students who have disabilities at the secondary level who need to succeed in the mathematics curriculum.

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Appendix 1

Parental Permission/Consent Letter

AUBURN UNIVERSITY

DEPARTMENT OF SPECIAL EDUCATION, REHABILITATION, AND COUNSELING

The Auburn University Institutional Review Board has approved this document for use from 2/21/12 to 3/8/13
Protocol # 11-063 EP 1103

NOTE: DO NOT AGREE TO PARTICIPATE UNLESS AN APPROVAL STAMP WITH CURRENT DATES HAS BEEN APPLIED TO THIS DOCUMENT

PARENTAL PERMISSION/CONSENT
For a Research Study entitled
"Effects of a Mnemonic Strategy on Solving Equations"

Your child is invited to participate in a research study to examine the effects of a mnemonic strategy to enhance student's performance in solving equations. The study is being conducted by Shaunita Strozier and Cindy Head, doctoral students, under the direction of Dr. Margaret Flores, Assistant Professor in the Auburn University Department of Special Education, Rehabilitation, and Counseling. Your child was selected as a possible participant because he or she is enrolled in the pre-algebra class. Since your child is age 18 or younger, we must have your permission to include him/her in the study.

If you decide to allow your child to participate in this research study, your child's total time commitment will be approximately eight weeks. Instruction will take place in your child's regular classroom during regular math time. Instruction will be provided by Shaunita Strozier. Instruction will involve solving equations using objects, pictures and numbers. During this time, your child will be asked to memorize a mnemonic strategy and use the strategy to solve these equations.

The risks associated with participating in this study are minimal risk or discomfort. To minimize these risks, we will look for signs of increased anxiety or discomfort and the student will be removed from the activity if such signs are observed. Discomfort will be minimized by preparing students prior to daily instruction, providing verbal cues and manipulatives about the instructional activities included in the lesson.

If your child participates in this study, your child can expect to improve his/her math skills in solving equations and use a mnemonic strategy to help remember steps. We cannot promise you that your child will receive any or all of the benefits described.

Your child will not receive any compensation for participation. To thank you child for participating, your child will receive knowledge in algebra development.

If you decide to allow your child to participate, there will not be a cost to you or the student. The service is free.

Parent/Guardian Initials______

Page 1 of 2

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www.auburn.edu/serc

AUBURN UNIVERSITY

DEPARTMENT OF
SPECIAL EDUCATION,
REHABILITATION, AND COUNSELING

NOTE: DO NOT AGREE TO PARTICIPATE UNLESS AN APPROVAL STAMP WITH CURRENT DATES HAS BEEN APPLIED TO THIS DOCUMENT

If you change your mind about your child's participation, your child can be withdrawn from the study at any time. Your child's participation is completely voluntary. If you choose to withdraw your child, your child's data can be withdrawn as long as it is identifiable. Your decision about whether or not to allow your child to participate or to stop participating will not jeopardize you or your child's future relations with Auburn University, the Department of Special Education, Rehabilitation, Counseling/School Psychology or Troup County School System.

Any information obtained in connection with this study will remain confidential. The data collected will be protected by Cindy Head and Shaunita Strozier. Finding from this study may be published in an educational journal or presented at a conference. Your child will not be identified personally.

If you have questions about your child's rights as a research participant, you may contact the Auburn University Office of Human Subjects Research or the Institutional Review Board by phone (334-844-5966 or email at hsubjec@auburn.edu or IRBChair@auburn.edu.

HAVING READ THE INFORMATION PROVIDED, YOU MUST DECIDE WHETHER OR NOT YOU WISH FOR YOUR CHILD TO PARTICIPATE IN THIS RESEARCH STUDY. YOUR SIGNATURE INDICATES YOUR WILLINGNESS TO ALLOW YOUR CHILD TO PARTICIPATE.

____________________ Parent/Guardian Signature	____________________ Investigator obtaining consent Date
____________________ Printed Name	____________________ Printed Name
____________ Date	

Child's Name ____________________________

The Auburn University Institutional Review Board has approved this document for use from 2/21/12 to 3/8/13
Protocol # 11-063 EP 1103

____
Co-Investigator Date

____
Printed Name

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Appendix 2

Student Assent Form

AUBURN UNIVERSITY

DEPARTMENT OF
SPECIAL EDUCATION,
REHABILITATION, AND COUNSELING

The Auburn University Institutional Review Board has approved this document for use from 2/21/12 to 3/8/13
Protocol # 11-063EP1103

MINOR ASSENT
For a research study entitled
"Effects of a Mnemonic Strategy on Solving Equations"

You and your parents or guardian(s) are invited to be in a research study to help us understand how some children can learn a mnemonic strategy and it will help them solve equations.

If you decide to participate in this research study, your total time commitment will be approximately eight weeks. During this time, you will be asked to memorize a mnemonic strategy and use the strategy to solve equations. At the beginning, the lessons will use manipulatives called algebra tiles to help you with the solve equations. Eventually, you will be able to solve the equations without anything manipulatives. The lessons will take place during your regular math time and in your regular classroom.

Some of the time that you are working in the classroom working with the researcher on solving equations, we will have a video camera on, taking a video of you. We need the video to study later, after you go home. We can only make the video if you and your parent(s) or guardians give us permission to do that.

You can stop at any time. Just tell your parents or your teacher if you don't want to work anymore. No one will be angry with you if you stop wanting to learn to solve equations in this way. You will go back to the regular group instead. Stopping participation in this study will not affect your grade in your class and your teacher will not be mad at you. Participation in this study is voluntary.

If you have any questions about what you will do or what will happen, please ask your parents or guardian or ask Ms. Strozier now. If you have any questions while you are working we want you to ask us.

If you have decided to help us, please sign or print your name on the line below.

Child's Signature	Print Name	Date
Parent/Guardian Signature	Print Name	Date

(Parent/Guardian must also sign Parent/Guardian Permission form!)

Investigator obtaining consent	Print Name	Date

Page 1 of 1

2084 Haley Center, Auburn, AL 36849-5222; Telephone: 334-844-7676; Fax: 334-844-7677
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Appendix 3

Video Release Permission Form

AUBURN UNIVERSITY

DEPARTMENT OF
SPECIAL EDUCATION,
REHABILITATION, AND COUNSELING

Video Release – Minor

During your child's participation in this research study, **"Effects of a Mnemonic Strategy on Solving Equations"** your child will be videotaped. Your signature on the Informed Consent gives us permission to do so.

Your signature on this document gives us permission to use the videotape(s) for the additional purposes of publication and training beyond the immediate needs of this study. These videotapes will not be destroyed at the end of this research but will be retained until the final manuscript has been accepted for publication.

I give my permission for videotapes produced in the study, "Effects of a Mnemonic Strategy on Solving Equations," which contain images of my child, to be used for the purposes listed above, and to also be retained until the final manuscript has been accepted for publication.

Parent/Guardian's Signature Date

Investigator's Signature Date

Parent/Guardian Printed Name

Investigator Printed Name

Minor's Signature Date

Co-investigator Signature Date

Minor's Printed Name

Printed Name

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Protocol # 11-063EP1103

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Appendix 4

Flyer

AUBURN UNIVERSITY
DEPARTMENT OF
SPECIAL EDUCATION,
REHABILITATION, AND COUNSELING

Learn to Solve Equations

Who?
Shaunita Strozier, a doctoral student at Auburn University, is recruiting students who want to learn to solve equations.

What?
The study will investigate a new mnemonic (help you remember) strategy for solving equations.

Where and For how long?
The study will take place at your school for 8 weeks during math time.

Who can you participate?
Students who struggle in math or have trouble learning new concepts easily can be eligible to participate.

What are the Procedures?
Brief testing to see if you qualify.
Pre-test
Introduction of the mnemonic strategy
Lessons
Post-test
Follow-up

What are the Benefits?
Learning to solve equations with a mnemonic strategy

If interested, please contact your math teacher or have your parent/guardian to contact Shaunita Strozier at 2084 Haley Center Auburn University, AL 36849 (706)594-0976 or sds0003@auburn.edu

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www.auburn.edu/serc

Appendix 5

One-step Learning Sheets

Learning Sheet 1

One-Step
Concrete
Code Number _________________ Date _____________
SUMLOWS

Describe and Model

1) $X - 3 = 6$

2) $x + 5 = -8$

3) $3X = 9$

4) $\frac{X}{2} = 3$

Guided Practice

1) $X - 4 = 9$

2) $X + 2 = 7$

3) $-X = 5$

4) $-4X = 12$

5) $\frac{X}{4} = -2$

6) $-3X = 12$

Independent Practice

$X - 2 = -5$

$\frac{X}{2} = -4$

$X + 8 = -5$

$-4X = -16$

$\frac{X}{3} = 4$

$-10 = X - 5$

Learning Sheet 2

One-Step
Concrete
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) X- 5 = -4

2) -3X = 9

3) $\frac{X}{2} = 4$

Guided Practice

1) X + 4 = -2

2) –X = 2

3) -2X = 4

4) $\frac{X}{3} = -2$

Independent Practice

$\frac{X}{3} = -4$

X + 6 = -5

-3X = -15

$\frac{X}{2} = 5$

-8 = X - 4

-X= 6

Learning Sheet 3

One-Step
Concrete
Code Number ________________ Date _____________
SUMLOWS

Describe and Model

1) $-4X = 8$

2) $\frac{X}{5} = -2$

Guided Practice

1) $-2X = 8$

2) $\frac{X}{2} = -8$

3) $-5X= 15$

Independent Practice

$X - 6 = -7$

$\frac{X}{2} = -3$

$X + 4 = -6$

$-4X = -16$

$\frac{X}{2} = 10$

$-9 = X - 5$

$-6X = -24$

$-X= 4$

$X -4 = 10$

$X + 2 = -6$

$-X = 6$

Learning Sheet 4

One-Step
Representational
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) $X+3 = -2$

2) $X - 5 = -8$

3) $-2X = 8$

4) $\frac{X}{5} = -4$

Guided Practice

1) $X - 6 = -9$

2) $X + 5 = -7$

3) $-X = 3$

4) $-5X = 15$

5) $\frac{X}{3} = 6$

6) $-5X = 10$

Independent Practice

$X - 2 = -5$

$\frac{X}{2} = -4$

$X + 8 = -5$

$-4X = -16$

$\underline{X} = 4$

$-10 = X - 5$

$- 2X = -8$

$-X = 4$

Learning Sheet 5

One-Step
Representational
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) X- 1 = -5

2) -4X = 8

3) $\frac{X}{5} = -2$

Guided Practice

1) X + 2 = -6

2) –X = 6

3) -2X = 8
2

4) X = -8

Independent Practice

X – 6 = -7
2

X = -3

X + 4 = -6

-4X = -16

$\frac{X}{2} = 10$

-9 = X - 5

- 6X = -24

-X= 4

Learning Sheet 6

One-Step
Representational
Code Number ________________ Date _____________
SUMLOWS

Describe and Model

1) $-3X = 9$

2) $\frac{X}{2} = 4$

Guided Practice

1) $X + 4 = -2$

2) $-X = 2$

3) $\frac{X}{3} = -2$

4) $-5X = 10$

Independent Practice

$X - 4 = -7$

$\frac{X}{3} = -4$

$X + 6 = -5$

$-3X = -15$

$\frac{X}{4} = 5$

$-8 = X - 4$

$-7X = -14$

$-X = 6$

Learning Sheet 7

One-Step
Abstract
Code Number _________________ Date _____________
SUMLOWS

Describe and Model

1) $X+8 = -2$

2) $X - 4 = -8$

3) $14 = -7X$

4) $\frac{X}{3} = -4$

Guided Practice

1) $X - 8 = -6$

2) $-5 = X+3$

3) $-X = 2$

4) $-3X = 15$

5) $-\frac{X}{3} = -2$

6) $-\frac{X}{2} = 4$

Independent Practice

$X - 2 = -5$

$\frac{X}{2} = -4$

$X + 8 = -5$

$-16 = -4X$

$\frac{X}{3} = -4$

$-10 = X - 5$

Learning Sheet 8

One-Step
Abstract
Code Number ________________ Date _____________
SUMLOWS

Describe and Model

1) $X + 3 = -7$

2) $\frac{X}{5} = -2$

Guided Practice

1) $X - 4 = 10$

2) $X + 2 = -6$

3) $-X = 6$

4) $-2X = 8$

5) $\frac{X}{2} = -8$

Independent Practice

$X - 6 = -7$

$\frac{X}{2} = -3$

$X + 4 = -6$

$-4X = -16$

$\frac{X}{2} = 10$

$-9 = X - 5$

$-6X = -24$

$-X = 7$

Learning Sheet 9

One-Step
Abstract
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) $X + 1 = -6$ 2) $-3X = 9$

Guided Practice

1) $X - 6 = 9$ 2) $-X = 2$

Independent Practice

$X - 4 = -7$ $\frac{X}{3} = -4$ $X + 6 = -5$ $-3X = -15$

$\frac{X}{2} = 5$ $-8 = X - 4$ $-7X = -14$ $-X = 6$

Appendix 6

Two-Step Learning Sheets

Learning Sheet 1

Two-Step
Concrete
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) 3X+1 = 13

2) 3X + 5X = -16

3) 3(X+2) = 3

4) $5 = \frac{X}{2} + 10$

Guided Practice

1) -2X +6 = 10

2) 4X – 2X = 8

3) -2(X +3) = 10

4) $10 + \frac{X}{2} = 12$

5) $\frac{X}{4} + 2 = -2$

6) -X-7= 8

Independent Practice

4X – 1 = 11

$\frac{X}{2} + 3 = -4$

-X + 8 = -5

-4(X+1) = -16

$\frac{X}{3} - 2 = 4$

-10 = -2(X – 3)

- 2X+ 4X = -8

7X-2X = 20

Learning Sheet 2

Two-Step
Concrete
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) $-4X+2 = 10$

2) $-2(X+2) = 10$

3) $3 = \frac{X}{2} - 6$

Guided Practice

1) $-2X + 6 = 12$

2) $-5X - 2X = 14$

3) $2(X + 2) = 10$

4) $1 + \frac{X}{2} = -10$

5) $\frac{X}{3} + 2 = -4$

Independent Practice

$3X - 1 = 11$

$\frac{X}{2} + 5 = -2$

$-X + 6 = -5$

$-3(X+1) = 15$

$\frac{X}{3} - 3 = -5$

$4 = -2(X - 3)$

$-3X + 6X = -18$

Learning Sheet 3

Two-Step
Concrete
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) -2(2X+2) = 12

2) $5 = \frac{X}{4} - 3$

Guided Practice

1) -6X -6 = 12

2) -2(3X +1) = 10

3) $3 + \frac{X}{4} = -1$

Independent Practice

4X – 5 = 11

$\frac{X}{2} + 3 = -5$

-X +8 = -5

-3(2X+1) = 15

$\frac{X}{3} - 2 = -4$

-3 = -3(X – 3)

- 2X- 6X = -16

Learning Sheet 4

Two-Step
Representational
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) $-2X+5 = 11$

2) $6X - 3X = 6$

3) $-3(2X+1) = 9$

4) $5 = \frac{X}{4} - 2$

Guided Practice

1) $-3X + 6 = 12$

2) $-4X - 3X = 14$

3) $-2(X + 2) = 10$

4) $3 + \frac{X}{4} = -1$

5) $\frac{X}{5} + 2 = -3$

Independent Practice

$5X - 1 = 9$

$\frac{X}{4} + 1 = -2$

$-X + 5 = -8$

$-4(X+2) = 8$

$\frac{X}{3} - 4 = -5$

$12 = -4(2X - 1)$

$-2X + 6X = -12$

Learning Sheet 5

Two-Step
Representational
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) -5X – 3X = 16

2) -2(3X+1) = 4

3) $4 = \frac{X}{2} - 1$

Guided Practice

1) -7X +1 = -13

2) -2X – 3X = 15

3) -3(X -2) = -3

4) $5 + \frac{X}{3} = -2$

Independent Practice

4X – 3 = 5

$\frac{X}{2} + 4 = 2$

-X -6 = -4

-2(3X-2) = -8

$\frac{X}{5} - 1 = -2$

-3 = -3(X – 2)

-4X+ 6X = -8

Learning Sheet 6

Two-Step
Representational
Code Number _________________ Date _____________
SUMLOWS

Describe and Model

1) $-5(2X+1) = 15$

Guided Practice

1) $-5X - 3 = 7$

2) $-3(X - 3) = 12$

Independent Practice

$3X - 3 = 8$

$\frac{X}{3} + 2 = 5$

$-X - 8 = -2$

$-4(X-2) = 12$

$\frac{X}{3} - 4 = 2$

$-6 = 3(X + 2)$

$-3X - 3X = 12$

$\frac{X}{2} + 3 = 4$

Learning Sheet 7

Two-Step
Abstract
Code Number _________________ Date _____________
SUMLOWS

Describe and Model

1) -5X +5=15

2) 9X – X = 24

3) 2(4X – 2) = 4

4) 4= $\frac{X}{5}$ + 2

Guided Practice

1) -X -3 = 15

2) -7X – X = 16

3) 2(-X – 3) = 12

4) 8= $\frac{X}{4}$ +3

Independent Practice

3(X+2) = 3

-X + 4X = 6

5= $\frac{X}{2}$ +10

-2X +6 = 10

4X – 2X = 8

-2(X +3) = 10

Learning Sheet 8

Two-Step
Abstract
Code Number __________________ Date ______________

SUMLOWS

Describe and Model

1) $4X - 1 = 11$

2) $\frac{X}{2} + 3 = -4$

3) $-4(X+1) = -16$

Guided Practice

1) $\frac{X}{2} - 2 = 4$

2) $-10 = -2(X - 3)$

3) $-2X + 4X = -8$

4) $-2X + 6 = 10$

Independent Practice

$7X - 2X = 20$

$3(X+2) = 3$

$5 = \frac{X}{2} + 10$

$-2X + 6 = 10$

$4X - 2X = 8$

$-2(X + 3) = 10$

Learning Sheet 9

Two-Step
Abstract
Code Number _________________ Date _____________
SUMLOWS

Describe and Model

1) -3(2X+1) = 9

Guided Practice

1) -4X – 3X = 14

2) -2(X +2) = 10

Independent Practice

-2X+5 = 11

6X – 3X = 6

-2(X +3) = 10

4) $2 + \frac{X}{4} = -1$

5) $\frac{X}{5} +2= -3$

3(X+2) = 3

Appendix 7

Multiple-step Learning Sheets

Learning Sheet 1

Multiple-Step
Concrete
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) $5X+10-5 = 15$

2) $-2X - 5 + 4X = 13$

3) $3(X+2) + 2X = 16$

4) $-\frac{X}{2} + 3 - 4 = -6$

Guided Practice

1) $-X + 6 - 3 = 10$

2) $3X - X - 4 = 8$

3) $-3(X+1) + 2X = 10$

4) $-5 = \frac{X}{6} + 2 - 4$

5) $\frac{X}{4} + 2 - 3 = -2$

6) $-2(X+1) - 4 = 6$

Independent Practice

$20 = 3(3X) + 2$

$5 - X + 3X = 9$

$-6 = \frac{X}{2} + 3 - 1$

$-4(X+1) + 2 = -10$

$-2X + 4 = 4 + 6$

$-10 = -2(X - 3) + 2$

$-2X + 4X - 2 = -8$

$7X - 2X - 5 = 20$

Learning Sheet 2

Multiple-Step
Concrete
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) $2X + 10 - 6 = 10$

2) $6 = -\frac{X}{6} + 3 - 4$

3) $-2(2X + 4) + 2 = 2$

Guided Practice

1) $-16 = -2(2+2X)+2X$

2) $4X - X - 3 = 9$

3) $-4(X + 1) + 2X = 8$

4) $-5 = \frac{X}{6} - 5 - 4$

5) $\frac{X}{3} + 5 - 1 = -5$

6) $-2(X + 1) - 4 = 6$

Independent Practice

$5 = 3(3X - 2) + 2$

$3 - X + 4X = 9$

$-5 = \frac{X}{2} + 2 - 5$

$4(X+1) + 2 = -2$

Learning Sheet 3

Multiple-Step
Concrete
Code Number _________________ Date _____________
SUMLOWS

Describe and Model

1) $3X + 2X - 5 = 15$

2) $3 = -\frac{X}{2} + 3 - 4$

3) $-2(4X + 1) + 3 = -7$

Guided Practice

1) $-18 = -3(X + 2X) + 2$

2) $4X - 2X - 3 = 9$

3) $-4(X + 1) + 2X = 8$

4) $-5 = \frac{X}{6} - 5 - 2$

Independent Practice

$12 = 2(3X + 2) + 2$

$5 - 2X + 4X = -5$

$3 + 3X - 8 = 4$

$-2 = \frac{X}{3} + 2 - 5$

$-4(X + 2) + 3 = 3$

$\frac{X}{2} - 6 + 3 = 4$

Learning Sheet 4

Multiple-Step
Representational
Code Number ________________ Date _____________
SUMLOWS

Describe and Model

1) $2X+6 + 2 = 12$

2) $-2X – 5 +4X = 13$

3) $3(X+2) +2X = 16$

4) $-\frac{X}{2} +3 -4 = -6$

Guided Practice

1) $-X +6- 3 = 10$

2) $3X – X- 4 = 8$

3) $-3(X +1)+2X = 10$

4) $-5= \frac{X}{6} +2 - 4$

5) $\frac{X}{4} +2 -3= -2$

6) $-2(X +1) – 4 = 6$

Independent Practice

$20= 3(3X) + 2$

$5- X +3X = 9$

$-6= \frac{X}{2} +3-1$

$-4(X+1) +2= -10$

$-2X +4 = 4+6$

$-10 = -2(X – 3) +2$

$- 2X+ 4X -2= -8$

$7X-2X -5= 20$

Learning Sheet 5

Multiple-Step
Representational
Code Number _________________ Date _____________
SUMLOWS

Describe and Model

1) $-3X + 6 - 4 = -13$

2) $5 = \frac{X}{3} + 2 - 4$

3) $-2(3X + 2) + 2 = 10$

Guided Practice

1) $-4 = -3(X+3)+2X$

2) $-X + 3X - 2 = 8$

3) $-2(2X + 1) - 2X = -8$

4) $-4 = \frac{X}{2} - 3 - 4$

Independent Practice

$6 = 4(3X - 2) + 2$

$2 - X + 5X = -6$

$2(X + 1) - 4 = 6$

$3 = \frac{X}{4} + 1 - 5$

$6(X+1) - 2 = -8$

$\frac{X}{4} + 6 - 1 = -5$

Learning Sheet 6

Multiple-Step
Representational
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) 3X + 2X - 5= 15

2) $3 = -\frac{X}{2} + 3 - 4$

Guided Practice

1) -16 = -3(X+2X)+2

2) 4X – 2X- 3 = 9

3) -4(X +1)+2X = 8

Independent Practice

12= 2(3X+ 2) + 2

5 - 2X +4X = -5

3+ 3X -8 = 4

$-5 = \frac{X}{2} + 2 - 5$

-4(X+2) +3= 3

$\frac{X}{2} - 6 + 3 = 4$

Learning Sheet 7

Multiple-Step
Abstract
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) 5X+10-5 = 15

2) -2X – 5 +4X = 13

3) 3(X+2) +2X = 16

4) $-\frac{X}{2} + 3 - 4 = -6$

Guided Practice

1) -X +6- 3 = 10

2) 3X – X- 4 = 8

3) -3(X +1)+2X = 8 = 10

4) $-5 = \frac{X}{6} + 2 - 4$

5) $\frac{X}{4} + 2 - 3 = -2$

6) -2(X +1) – 4 = 6

Independent Practice

20= 3(3X) + 2

5- X +3X = 9

$-6 = \frac{X}{2} + 3 - 1$

-4(X+1) +2= -10

-2X +4 = 4+6

-10 = -2(X – 3) +2

- 2X+ 4X -2= -8

7X-2X -5= 20

Learning Sheet 8

Multiple-Step
Abstract
Code Number _________________ Date _____________
SUMLOWS

Describe and Model

1) 6 +2X+ 4 = 12

Guided Practice

1) 1= 3X -7 +X 2) -2(-3+3X) = -18

Independence Practice

-X- 4 + 2X = 12 12= 3X - 4 + X 20 = -2(-4X + 2)

-2(-3+3X) = -18 -20 = -2(X +6) 20 = -2(-4X + 2)

$\frac{X}{3}$ - 12 = 3 – 8 8) - $\frac{X}{2}$ – 5 -4 = -1

Learning Sheet 9

Multiple -Step
Abstract
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) $-2(-3 + 3X) = -18$

Guided Practice

1) $-3X - 7 - 3 = 13$

Independent Practice

$2X + 2 - 4 = 10$ $1 = 3X - 7 + X$ $-19 = -9 - 3X - 7X$

$\frac{X}{4} - 10 - 3 = -2$ $-5(-X - 2) = -20$ $-\frac{X}{2} + 7 + 3 = 3$

Appendix 8

Mnemonic Strategy

"SUM LOWS"

(S) **S**eparate the sides,

(U) **U**nite the like terms,

(M) **M**odify (rewrite) the new equation,

(L) **L**oop (circle) around the coefficient,

(O) **O**pposite sign (inverse operation),

(W) "**W**hat you do to one side, you must do to the other,"

(S) **S**ubstitute the solution for the variable and check.

Appendix 9

One-step Probes

Equation 1

Untimed Probe

Code _____________ Date____________

1) $7x = 49$	2) $X + 10 = 15$	3) $2 = \frac{X}{5}$	4) $5 + X = 8$	5) $50 = -10x$
6) $-12 = X - 7$	7) $\frac{X}{2} = 4$	8) $-X = 10$	9) $X - 8 = 10$	10) $-11 = 6 + X$

Equation 1

Untimed Probe

Code _____________ Date___________

1) -5 =X – 4	2) $-5 = \frac{X}{6}$	3) X - 4 = -12	4) 8 = 12 + X	5) -3X = 15
6) 5= -X	7) 10 = X - 6	8) $\frac{X}{6} = 3$	9) $\frac{X}{2} = -8$	10) 10 = -2X

Equation 1

Untimed probe

Code__________ Date_______________

1) 12= X - 6	2) 5 + X = - 3	3) -X =- 4	4) -7 = $\frac{X}{3}$	5) 2X = 12
6) 10 + X = 8	7) 20 = 4X	8) $\frac{X}{2}$ = 14	9) -12 = X +5	10) $\frac{X}{4}$ = 2

Equation 1

Untimed Probe

Code ______________ Date____________

1) $\frac{X}{3} = -6$	2) -9X = 18	3) X -10 = 30	4) 11 = 3 + X	5) 2X = 12
6) -12= X – 7	7) $\frac{X}{2} = 4$	8) –5X = 10	9) X - 8 = 10	10) -11= 6 + X

Appendix 10

Two-step Probes

Equation 2

Untimed probe

Code____________ Date____________

1) 3X + 1 = 13	2) 6X – 4 = 20	3) 4X – 1 = 11	4) 11 = 3 + 2X	5) 30 = -X - 10
6) 5X – 7X = 4	7) 32 = 4(2X)	8) -12 = 3(4X)	9) -2(4X) = -24	10) $5 = \frac{X}{2} + 10$

Equation 2

Untimed probe

Code____________ Date____________

1) $2t - 1 = 9$	2) $-p-11=20$	3) $10 + \frac{a}{2} = 12$	4) $- X - 7 = 8$	5) $-f + 6 = 10$
6) $5q - 2= 8$	7) $15 = -5X + 5$	8) $\frac{X}{4} - 6 = 9$	9) $-X + 3X = 10$	10) $4r = 12 - 4$

Equation 2

Untimed Probe

Code ___________ Date____________

1) $5w - 7w = 4$	2) $32 = -4(2x)$	3) $-12 = -3(4X)$	4) $-2(-4X) = -24$	5) $5 = \frac{X}{2} + 10$
6) $5t - 2 = -12$	7) $-X-8=20$	8) $8 + \frac{a}{2} = 12$	9) $- 2X - 4 = 8$	10) $-3X + 6 = 12$

Equation 2

Untimed Probes

Code ___________ Date___________

1) 5q – 2= 8	2) 15 = -5X + 5	3) $\frac{X}{4}$ – 6 = 9	4) -X + 3X = 10	5) 4r = 12 – 4
6) -2t – 1 = 9	7) -3p-9 = 9	8) -6 + $\frac{a}{2}$ = 12	9) - X – 6 = 10	10) -4f -2 = 10

Appendix 11

Multi-step Probes

Equation 3

Untimed Probe

Code_______________ Date______________

1) 5X + 10 - 5= 15	2) 5 – X+ 3X = 9	3) -16 = 2(3X – 2)	4) -6 = $\frac{X}{-6}$ + 3 – 4
5) 20= 3(3x)+ 2	6) 3X – 10 +8 = -11	7) -2X +4 -6 = 12	8) -16 = -2(2X+2)
9) 20 = -2x – 6 + 4	10) $\frac{X}{5}$ - 5 + 4 = -2		

Equation 3

Untimed Probe

Code_______________ Date______________

1) 16 = 4X + 3X+ 2	2) 10 = 4X +X -5	3) 6 +2X+ 4 = 12	4) 1= 3X -7 +X
5) -4x + 6X +5 = -5	6) 5X + 3X -3 =5	7) -5 = $\frac{X}{6}$ + 2 – 4	8) -12 = -3(X +3)
9) $\frac{X}{2}$ + 10 - 2 = 4	10) -19 = -9 - 3X - 7X		

Equation 3

Untimed Probe

Code_______________ Date______________

1) 2 + 4X +6X = 12	2) $\frac{X}{2}$ + 5 - 1 = 6	3) -2X - 5+ 4X = 13	4) 40) -3x + 5x -2 = 10
5) 12= 3X - 4 + X	6) 20 = -2(-4X + 2)	7) 5X - 6X + 4 = -7	8) -X- 4 + 2X = 12
9) 2 = 2X - 6 + 2X	10) -3 (-2X +1) = 15		

Equation 3

Untimed Probe

Code_______________ Date______________

1) $20 = -2x - 6 + 4$	2) $\frac{X}{5} - 5 + 4 = -2$	3) $\frac{X}{2} + 10 - 2 = 4$	4) $-19 = -9 - 3X - 7X$
5) $16 = 4X + 3X + 2$	6) $10 = 4X + X - 5$	7) $6 + 2X + 4 = 12$	8) $1 = 3X - 7 + X$
9) $-5 = \frac{X}{6} + 2 - 4$	10) $-12 = -3(X + 3)$		

Appendix 12

Pre-test

Equation Pre-test

Name ____________________________ Date _______________

1) $7x = 49$	2) $X + 10 = 15$	3) $2 = \frac{X}{5}$	4) $5 + X = 8$
5) $-12 = X - 7$	6) $\frac{X}{2} = 4$	7) $-2X = 10$	8) $X - 8 = 10$
9) $5X - 7X = 4$	10) $32 = 4(2X)$	11) $-12 = 3(4X)$	12) $11 = 3 + 2X$
13) $16 = 4X + 3X + 2$	14) $10 = 4X + X - 5$	15) $6 + 2X + 4 = 12$	16) $1 = 3X - 7 + X$

17) 20= 3(3x)+ 2	18) 3X – 10 = 2x	19) -2X +4 = 4 + 6	20) -16 = -(2+2X)
21) 3X + 1 = 13	22) 6X – 4 = 20	23) 4X – 1 = 11	24) 2X -4 = 16
25) 16 = -2(2+2X)	26) $\frac{X}{5}$ - 5 + 4 = -2	27) -3X = 30	28) -20 = 4X - 4
29) 30 = -X - 10	30) $\frac{X}{3}$ = -6		

Appendix 13

CRA Manual

Appendix 13

Solving Algebraic Equations With CRA & SUMLOWS Manual

Shaunita D. Strozier
Auburn University
Spring 2012

Dr. Margaret M. Flores, Chair

Lessons 1-3
One-step Equations

Concrete Method

Materials:

- Chalk-board/white board/easel
- Algebra tiles
- Learning sheet 1
- workmat/mini dry erase board
- SUMLOWS mnemonic on index cards

Advance Organizer

- Tell students what they will be doing and why
  - Remind students about the commitment they made to learn to solve equations. We will work hard to teach them and they will work hard to learn a new way. They will learn to solve equations using algebra tiles.

Demonstrate

- Give students SUMLOWS mnemonic and learning sheets. Wait to pass out the manipulative so that students do not become distracted.
- Begin with the first problem in the model section. Tell the student that we will show them how to solve the problem and that they will have a chance to solve problems also. State the expectations for behavior and attention to the demonstration.
  - Begin with the first problem and think out aloud
  - I Read the problem.
  - This problem is X – 3 = 6. First, I take out my workmat or dry erase board and my mnemonic index card.
  - Next, I set out the algebra tiles. On the left side, I place green rectangle tile (that means I have 1X). We do not see a number in front of the X. It is an invisible 1. Then we will put down 3 red squares (red means we have a negative number and the cream side means that we have a positive number). Then on the right side, we will place 6 cream square tiles down (the six is positive that is why the tiles are on the cream side).
  - The first step In the SUMLOWS is to separate the two sides. Draw a line through the equal sign.
  - The "U" is for unite like terms. Look on the left side. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.

- The next step is Loop the variable. The variable is the rectangular tile. This is the number that we want to get by itself and want to move last. So, what is on the side with the variable (the rectangle)? The 3 red squares. We have to get rid of those because the rectangle should be by itself.
- In order to move the 3 red squares, we have to do the next step which is the opposite sign. We have 3 red squares. To get rid of the 3 red squares we must add 3 cream squares. Opposites cancel each other out.
- Next step is what we do to one side we must do to the other. We added 3 cream squares to the left. We must add 3 cream squares to the right. On the left side, each cream square cancel out each red square (move them off the board or mat). On the right side, the squares are the same size and color. We combine those.
- Now we have X = 9. Is the X by itself? Yes. Now we do the last step.
- Substitute the solution for the variable and check. Go up to the original problem. X – 3 = 6.
- Instead of a green rectangle for X, put in 9 cream squares (X = 9), as well as 3 red squares on the left side. On the right side, we place 6 cream tiles. Combine the tiles on the left side. Remember, cream tiles and red tiles cancel each other out. Three red tiles will cancel three cream tiles (when they cancel themselves out, move them from the board or mat). How many cream tiles are on the left side? 6. How many cream tiles are on the right side? Six. The answer is correct.

Model problem #2

- I read the next problem. X + 5 = -8
- I make sure my mat or board is clear
- On the mat, place 1 green rectangular tile (green is positive and red is negative) on the left side. I also place 5 cream square tiles on the left side (beside the green rectangular tile). On the right side, place 8 red square tiles (cream is positive and red is negative).
  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
  - Loop around the variable. Draw a circle around the 1 green rectangular tiles. That is the number that should be by itself.
  - Is there anything else on the side with the circle? Yes! There are 5 cream square tiles. We must move those 5 cream tiles.
  - Next step is opposite sign. In order to move the 5 cream square tiles, we must do the opposite. The opposite of "plus 5" is "minus 5." So, we must place 5 red tiles on the left side of the equal sign.
  - What we do to one side, we must do to the other. Because we placed 5 red tiles on the left side, we must place 5 red squares on the right side of the equal sign. The square tiles on the left side cancel each other out (+5 – 5 = 0).
  - Combine like terms. On the right side, we have 8 red squares and 5 red squares. Since they are the same shape, we should combine them. These squares are ALL red, so we will add them together. We have 13 red squares. On the left side of the equal sign, we have 1 green rectangle tile. On the right side, we have 13 red squares. Now

we have X = -13 (the square tiles are red, therefore they are negative). Is the X by itself? Yes. Now we do the last step.

    - Substitute the solution for the variable and check. Go up to the original problem. X +5 = -8.Instead of a green rectangle for X, put in 13 red squares (X = -13) on the left side, as well as 5 cream squares (represent +5). Place 8 red squares on the left side. Combine the tiles on the left side. Remember, cream tiles and red tiles cancel each other out. Pair one cream tile and one red tile and move them off the mat. Five cream tiles will cancel out five red tiles (when they cancel themselves out, move them from the board or mat). How many red tiles are on the left side? 8. How many red tiles are on the right side? 8. The answer is correct.

Model problem #3

- I read the next problem. 3X = 9
- I make sure my mat or board is clear
- On the mat, place 3 green rectangular tiles (green is positive and red is negative) on the left side. On the right side, place 9 cream square tiles (cream is positive and red is negative).
    - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
    - Unite like terms. Are there any tiles the same that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
    - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
    - Loop around the variable. Draw a circle around the 3 green rectangular tiles.
    - Is there anything else on the side with the circle? No! So, we have to see what 1 green rectangle is equal to?
    - Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw division sign and put three green rectangles beside the other green rectangles.
    - What we do to one side, we must do to the other. Draw a division sign and put three green rectangles beside the 9 cream squares. The left side cancels each other out. On the right side, separate the three green rectangles. Give each green rectangle a cream square until they are gone. One green rectangle is equal to how many cream squares? 3. Therefore, x = 3
    - Last step is to substitute the solution 3 in for X in the original problem. Original problem is 3X = 9. On the left side put 3 cream squares for each X. There are three x, so put 9 cream squares on the left. On the right, put 9 cream squares. Are both sides equal? Yes. The answer is correct.

Model problem #4

- I read the problem: $\frac{X}{2} = 3$
- Make sure the algebra mat is clear

- On the left side of the equal sign, draw a line and place 2 green rectangular tiles (green is positive and red is negative) under it. On the right side, place 3 cream squares (cream is positive and red is negative).
  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any tiles the same that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
  - Loop around the variable. Draw a circle around the variable. The variable is the green rectangular tiles.
  - Is there anything else on the side with the circle? No! So, we have to see what 1 green rectangle is equal to?
  - Next step is opposite sign. A number under a line means to divide. The opposite of division is multiplication. Draw a multiplication sign beside the three green rectangles and place 2 more green rectangles.
  - What we do to one side, we must do to the other. Since we drew a multiplication sign and placed 2 green rectangles on the left, we must draw a multiplication sign and place 3 green rectangles on the right. The left side cancel each other out. On the right side, separate the two green rectangles. Give all the squares to one of the rectangles. Then add 6 more cream squares so they will have the same amount. How many squares are in all? 6. Both shapes represent positive numbers; therefore the answer is +12. X= 6.
  - Last step is to substitute the solution 6 in for X in the original problem. Original problem is X/2 = 6. On the right side, put 6 cream tiles. On the left side, put 12 cream square tiles in for X. Draw a line under the cream squares and put 2 green rectangles under the line. Separate the rectangles and give each rectangle a cream square until they are all gone. How many does each rectangle have? 6. How many cream tiles are on the right side? 6. The numbers are the same. Are both sides equal? Yes. The answer is correct.

Guided Practice

- Make sure students have their algebra tiles
- Direct students to the “Guide” section of the learning sheet
- Tell students to touch the first problem and that we will do this problem together, using their algebra tiles and the SUMLOWS mnemonic.
- Let’s read the problem. This problem is X – 4 = -9.
- On our workmat, what do we put on the left? 1 green rectangular tile and 4 red square tiles.
- What do we place on the right? 9 red tiles
  - What is the first step in SUMLOWS? Separate the sides. We are going to draw a line through the equal sign.
  - What does the “U” stand for in SUMLOWS? Unite like terms. Are there any tiles the same shape that are not already together on the same side of the equal sign? NO.

Look on the right side. Are there any tiles the same shape that are not already together? No. We go on to step three.

  - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
  - What is the next step? Loop around the variable. Which shape is the variable? The green rectangle. Draw a circle around the 1 green rectangular tile. That is the number that should be by itself.
  - Is there anything else on the side with the circle? Yes! There are 4 red square tiles. We must move those 4 red tiles.
  - What is the next step? Next step is opposite sign. In order to move the 4 red square tiles, we must do the opposite. The opposite of "minus 4" is "plus 4." So, we must place 4 cream tiles on the left side of the equal sign.
  - What is the next step? What we do to one side, we must do to the other. Because we placed 4 cream tiles on the left side, we must place 4 cream squares on the right side of the equal sign. The square tiles on the left side cancel each other out (-4 +4 = 0).
  - Combine like terms. On the right side, we have 9 red squares and 4 cream squares. Since they are the same shape, we should combine them. These squares are different colors, so we will subtract (or pair them up and move them off the workmat). What are left on the right side of the equal sign? 5 red squares.
  - On the left side of the equal sign, we have 1 green rectangle tile. On the right side, we have 5 red squares. Now we have X = -5 (the square tiles are red, therefore they are negative). Is the X by itself? Yes. Now we do the last step.
  - Substitute the solution for the variable and check. Go up to the original problem. X - 4 = -9. Instead of a green rectangle for X, put in 5 red squares (X = -5) on the left side, as well as 4 red squares (represent -4). Place 9 red squares on the right side. Combine the tiles on the left side. Remember, cream tiles and red tiles cancel each other out. Since all the square tiles are red, add (combine them). How many red tiles are on the left side? -9. How many red tiles are on the right side? -9. The answer is correct.

Guided Practice #2

- Locate problem # 2 in the Guided practice section.
- Let's read the problem. This problem is X+ 2 = 7.
- On our workmat, what do we put on the left? 1 green rectangular tile and 2 cream square tiles.
- What do we place on the right? 7 cream tiles
  - What is the first step in SUMLOWS? Separate the sides. We are going to draw a line through the equal sign.
  - What does the "U" stand for in SUMLOWS? Unite like terms. Are there any tiles the same shape that are not already together on the same side of the equal sign? NO. Look on the right side. Are there any tiles the same shape that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.

- What is the next step? Loop around the variable. Which shape is the variable? The green rectangle. Draw a circle around the 1 green rectangular tile. That is the number that should be by itself.
- Is there anything else on the side with the circle? Yes! There are 4 red square tiles. We must move those 4 red tiles.
- What is the next step? Next step is Opposite sign. In order to move the 2 cream square tiles, we must do the opposite. The opposite of "plus 2" is "minus 2." So, we must place 2 red tiles on the left side of the equal sign.
- What is the next step? What we do to one side, we must do to the other. Because we placed 2 red tiles on the left side, we must place 2 red squares tiles on the right side of the equal sign. The square tiles on the left side cancel each other out (+2 -2 = 0).
- Combine like terms. On the right side, we have 7 cream squares tiles and 2 red squares tiles. Since they are the same shape, we should combine them. These squares are different colors, so we will subtract (or pair them up and move them off the workmat). What are left on the right side of the equal sign? 5 cream square tiles.
- On the left side of the equal sign, we have 1 green rectangle tile. On the right side, we have 5 cream squares. Now we have X = 5 (the square tiles are cream, therefore they are positive). Is the X by itself? Yes. Now we do the last step.
- Substitute the solution for the variable and check. Go up to the original problem. X + 2 = 7. Instead of a green rectangle for X, put in 5 cream squares (X = 5) on the left side, as well as 2 cream squares (represent +2). Place 7 cream squares on the right side. Combine the tiles on the left side. Remember, cream tiles and red tiles cancel each other out. Since all the square tiles are cream, add (combine them). How many red tiles are on the left side? 7 cream tiles. How many red tiles are on the right side? 7 cream tiles. The answer is correct.

Guided Practice #3

- Locate problem # 3 on the Guided practice section.
- Let's read the problem. This problem is –X = 5.
- On our workmat, what do we put on the left? 1 red rectangular tile
- What do we place on the right? 5 cream tiles
  - What is the first step in SUMLOWS? Separate the sides. We are going to draw a line through the equal sign.
  - What does the "U" stand for in SUMLOWS? Unite like terms. Are there any tiles the same shape that are not already together on the same side of the equal sign? NO. Look on the right side. Are there any tiles the same shape that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
  - What is the next step? Loop around the variable. Which shape is the variable? The red rectangle. Draw a circle around the 1 red rectangular tile. That is the number that should be by itself.
  - Is there anything else on the side with the circle? No. Is the X by itself? Yes. Is it positive? No. It is negative. We must move the -1 (the1 is invisible).

- What is the next step? Next step is opposite sign. In order to move the negative one, we must do the opposite. The opposite of "minus 1" is "plus one." So, we flip the tile to the green side.
- What is the next step? What we do to one side, we must do to the other. Because we flipped the left tile, we must flip the tiles on the right side.
- On the left side of the equal sign, we have 1 green rectangle tile. On the right side, we have 5 red squares. Now we have X =- 5 (the square tiles are cream, therefore they are positive). Is the X by itself? Yes. Is it positive? Yes. Now we do the last step.
- Substitute the solution for the variable and check. Go up to the original problem. – X=5. Instead of a red triangle for X, put in 5 cream squares (X =- 5) on the left side. The tiles are cream because both numbers have the same sign, therefore it's positive. On the right side, put down 5 cream tiles. The tiles are the same on both sides. The answer is correct.

Guided problem #4

- Locate problem #4 in the Guided practice section
- Let's read the problem: -4X = 12
- On our workmat, what do we put on the left? 4 red rectangular tiles
- What do we place on the right? 12 cream tiles
    - What is the first step in SUMLOWS? Separate the sides. We are going to draw a line through the equal sign.
    - What does the "U" stand for in SUMLOWS? Unite like terms. Are there any tiles the same shape that are not already together on the same side of the equal sign? NO. Look on the right side. Are there any tiles the same shape that are not already together? No. We go on to step three.
    - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
    - What is the next step? Loop around the variable. Which shape is the variable? The red rectangle. Draw a circle around the 4 red rectangular tiles. That is the number that should be by itself.
    - Is there anything else on the side with the circle? No. So, we have to see what 1 red rectangle is equal to?
    - Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw division sign and put four red rectangles beside the other red rectangles.
    - What we do to one side, we must do to the other. Draw a division sign and put four red rectangles beside the 12 cream squares. The left side cancels each other out.
    - On the right side, separate the 4 red rectangle tiles. Give each rectangle tile a cream square until they are gone. One green rectangle is equal to how many cream squares? 3. In multiplication and division, if the tiles are different colors, the answer is negative. If the tiles are the same color, the sign is positive. Therefore, x = -3
    - Last step is to substitute the solution -3 in for X in the original problem. Original problem is -4X = 12. On the left side put 4 red squares for each X. There are four x, so put 3 red squares for each X on the left. So, put down 12 cream squares on the left

and we have 12 cream squares (both numbers are a negative). On the right, put 12 cream squares. Are both sides equal? Yes. The answer is correct.

Guided practice #5

- I read the problem: $\frac{X}{4} = -2$
- make sure the algebra mat is clear
- On the left side of the equal sign, draw a line and place 4 green rectangular tiles (green is positive and red is negative) under it. On the right side, place 2 red squares (cream is positive and red is negative).
  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any tiles the same that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
  - Loop around the variable. Draw a circle around the variable. The variable is the green rectangular tiles.
  - Is there anything else on the side with the circle? No! So, we have to see what the green rectangle is equal to?
  - Next step is opposite sign. A number under a line means to divide. The opposite of division is multiplication. Draw a multiplication sign beside the 4 green rectangles and place 4 more green rectangles.
  - What we do to one side, we must do to the other. Since we drew a multiplication sign and placed 4 green rectangles on the left, we must draw a multiplication sign and place 4 green rectangles on the right. The left side cancel each other out. On the right side, separate the 4 green rectangles. Give all the squares to one of the rectangles. Then add 2 more red to each rectangle so they will have the same amount. How many squares are in all? 8. The colors represent different signs, therefore the answer is negative, X = -8.
  - Last step is to substitute the solution -8 in for X in the original problem. Original problem is X/4 = -2. On the right side, put 2 red square tiles. On the left side, put 8 red tiles in for X. Draw a line under the red squares and put 4 green rectangles under the line. Separate the rectangles and give each rectangle a red square until they are all gone. How many does each rectangle have? 2 red tiles. How many red tiles are on the right side? 2. The numbers are the same. Are both sides equal? Yes. The answer is correct.

Repeat the same interactive process with the remaining problems

Independent Practice

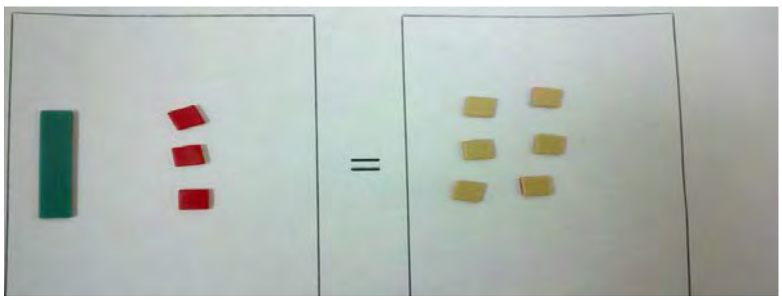
- Direct the students to the "Independent Practice" section. Read the first problem together and direct students to complete the problems without us.
  When students finish problems, provide immediate corrective feedback for errors.

Graphing

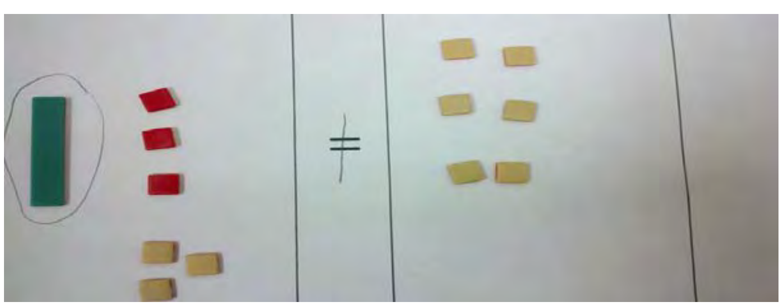
Steps for Concrete Level

Model 1: X -3 = 6

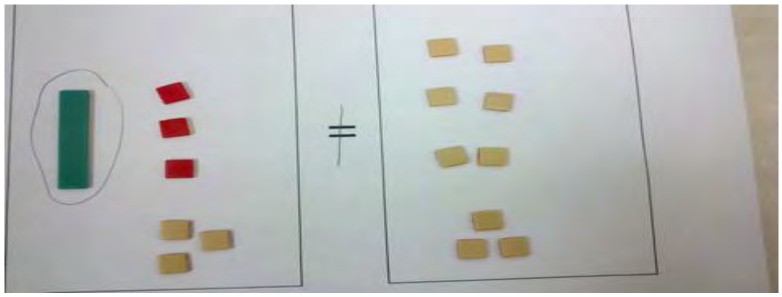
- I read the problem.
- This problem is X – 3 = 6. First, I take out my workmat or dry erase board and my mnemonic index card.
- Next, I set out the algebra tiles. On the left side, I place green rectangle tile (that means I have 1X). We do not see a number in front of the X. It is an invisible 1. Then we will put down 3 red squares (red means we have a negative number and the cream side means that we have a positive number). Then on the right side, we will place 6 cream square tiles down (the six is positive that is why the tiles are on the cream side).

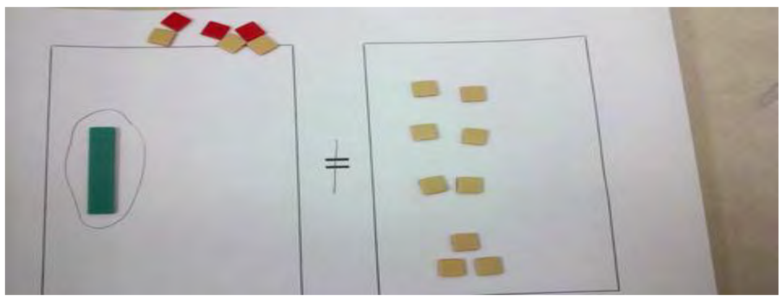


- The first step In the SUMLOWS is to separate the two sides. Draw a line through the equal sign.
- The "U" is for unite like terms. Look on the left side. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
- The next step is Loop the variable. The variable is the rectangular tile. This is the number that we want to get by itself and want to move last. So what is on the side with the variable (the rectangle)? The 3 red squares. We have to get rid of those because the rectangle should be by itself.
- In order to move the 3 red squares, we have to do the next step which is the opposite sign. We have 3 red squares. To get rid of the 3 red squares we must add 3 cream squares. Opposites cancel each other out.

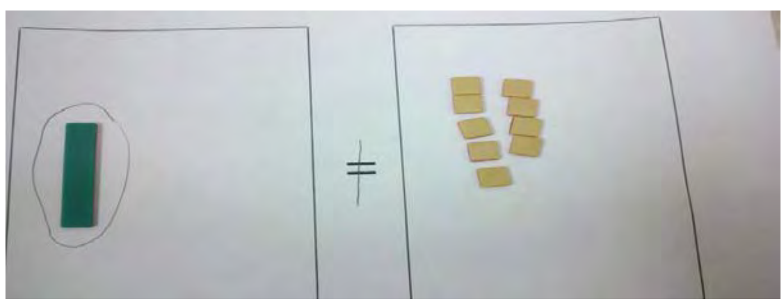


- Next step is what we do to one side we must do to the other. We added 3 cream squares to the left. We must add 3 cream squares to the right. On the left side, each cream square cancel out each red square (move them off the board or mat). On the right side, the squares are the same size and color. We combine those.

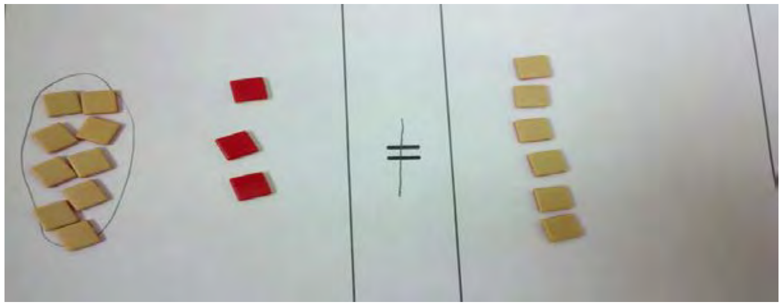


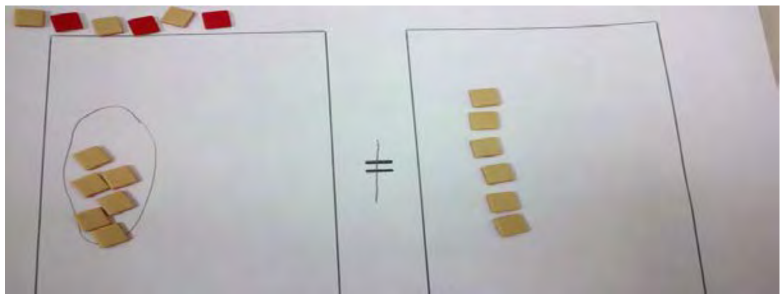


- Now we have X = 9. Is the X by itself? Yes. Now, we do the last step.



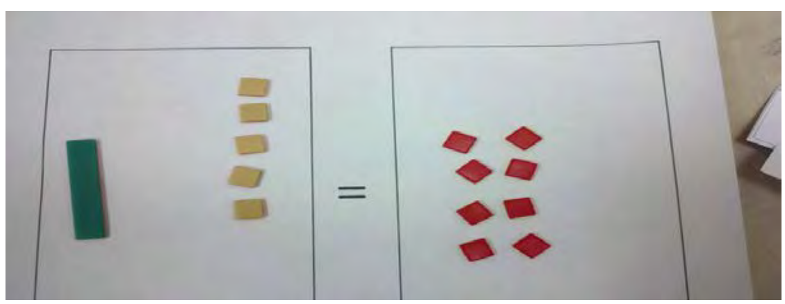
- Substitute the solution for the variable and check. Go up to the original problem. X – 3 = 6. Instead of a green rectangle for X, put in 9 cream squares (X = 9), as well as 3 red squares on the left side. On the right side, we place 6 cream tiles. Combine the tiles on the left side. Remember, cream tiles and red tiles cancel each other out. Three red tiles will cancel three cream tiles (when they cancel themselves out, move them from the board or mat). How many cream tiles are on the left side? 6. How many cream tiles are on the right side? Six. The answer is correct.



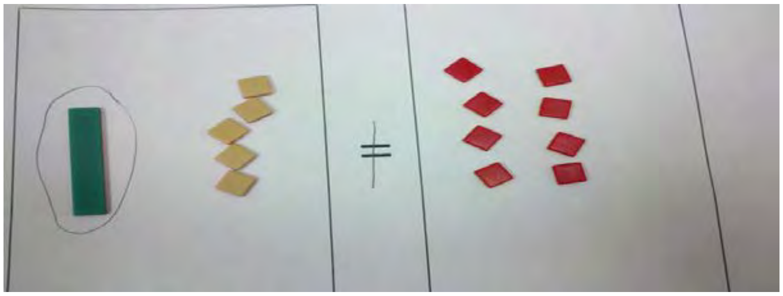


Model #2 X + 5 = -8

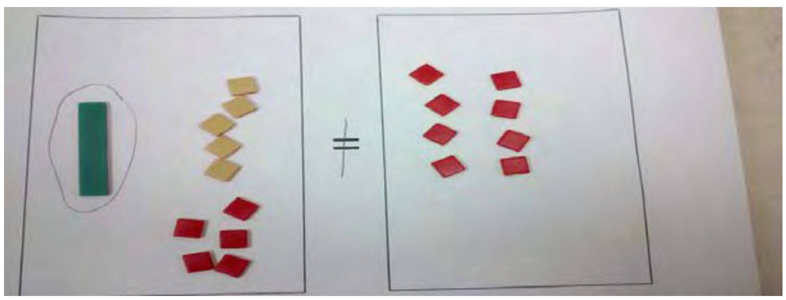
- I make sure my mat or board is clear
- On the mat, place 1 green rectangular tile (green is positive and red is negative) on the left side. I also place 5 cream square tiles on the left side (beside the green rectangular tile). On the right side, place 8 red square tiles (cream is positive and red is negative).



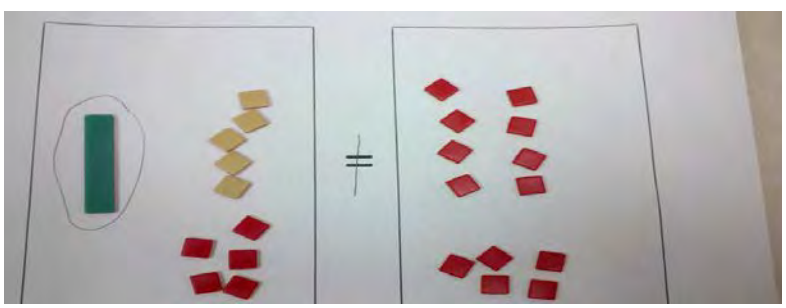
- Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
- Unite like terms. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
- Loop around the variable. Draw a circle around the 1 green rectangular tiles. That is the number that should be by itself.



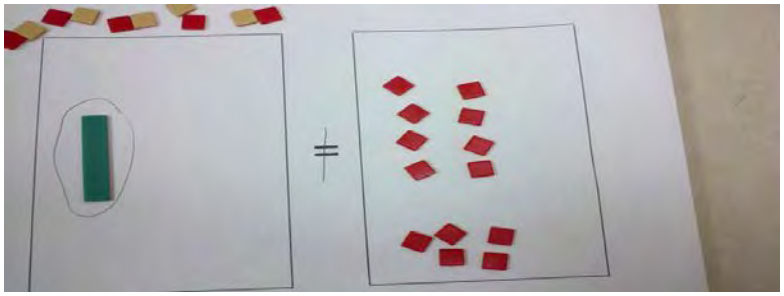
- Is there anything else on the side with the circle? Yes! There are 5 cream square tiles. We must move those 5 cream tiles.
- Next step is opposite sign. In order to move the 5 cream square tiles, we must do the opposite. The opposite of “plus 5” is “minus 5.” So, we must place 5 red tiles on the left side of the equal sign.



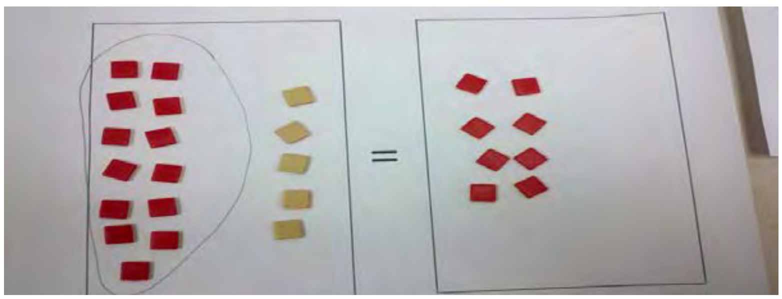
- What we do to one side, we must do to the other. Because we placed 5 red tiles on the left side, we must place 5 red squares on the right side of the equal sign. The square tiles on the left side cancel each other out (+5 – 5 = 0).



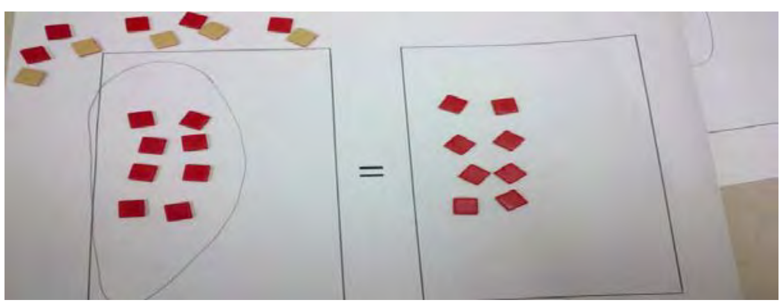
- Combine like terms. On the right side, we have 8 red squares and 5 red squares. Since they are the same shape, we should combine them. These squares are ALL red, so we will add them together. We have 13 red squares. On the left side of the equal sign, we have 1 green rectangle tile. On the right side, we have 13 red squares. Now we have X = -13 (the square tiles are red, therefore they are negative). Is the X by itself? Yes. Now we do the last step.



- Substitute the solution for the variable and check. Go up to the original problem. X +5 = -8. Instead of a green rectangle for X, put in 13 red squares (X = -13) on the left side, as well as 5 cream squares (represent +5). Place 8 red squares on the left side. Combine the tiles on the left side.

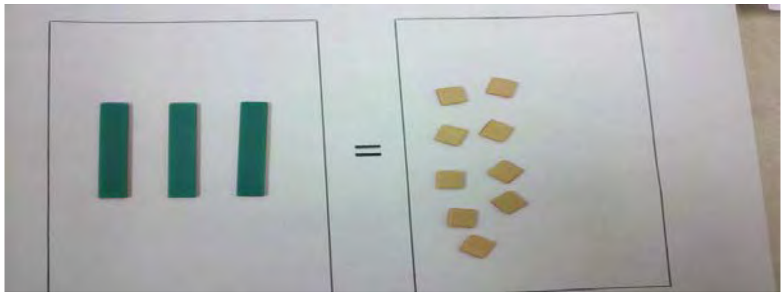


- Remember, cream tiles and red tiles cancel each other out. Pair one cream tile and one red tile and move them off the mat. Five cream tiles will cancel out five red tiles (when they cancel themselves out, move them from the board or mat). How many red tiles are on the left side? 8. How many red tiles are on the right side? 8. The answer is correct.

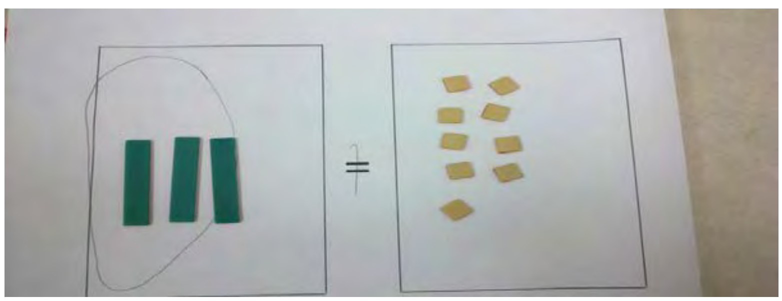


Model # 3 3X = 9

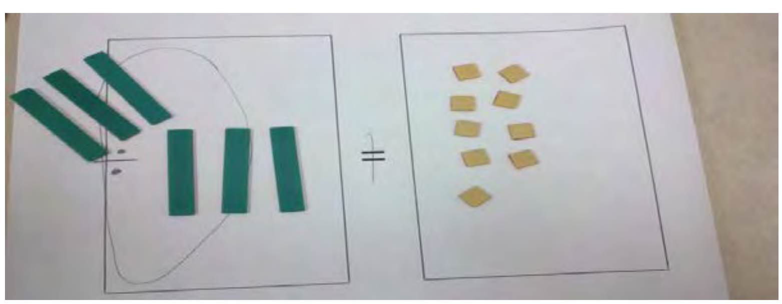
- I make sure my mat or board is clear
- On the mat, place 3 green rectangular tiles (green is positive and red is negative) on the left side. On the right side, place 9 cream square tiles (cream is positive and red is negative).



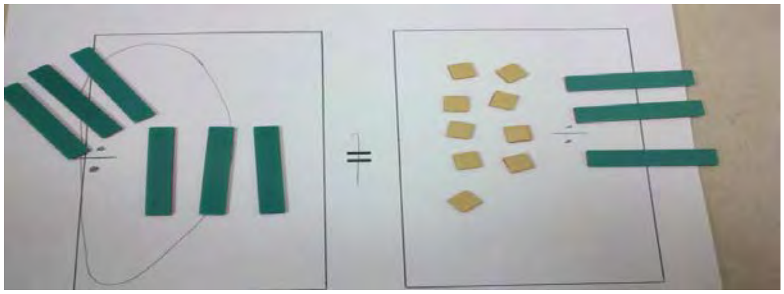
- Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
- Unite like terms. Are there any tiles the same that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
- Loop around the variable. Draw a circle around the 3 green rectangular tiles.
- Is there anything else on the side with the circle? No! So, we have to see what 1 green rectangle is equal to?



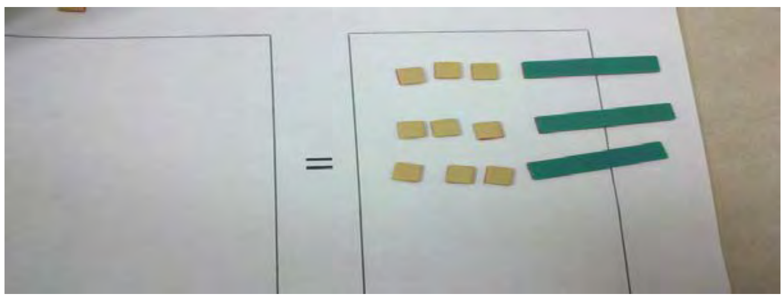
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw division sign and put three green rectangles beside the other green rectangles.

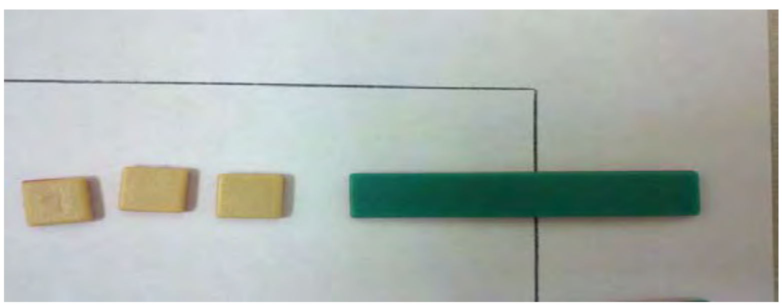


- What we do to one side, we must do to the other. Draw a division sign and put three green rectangles beside the 9 cream squares.

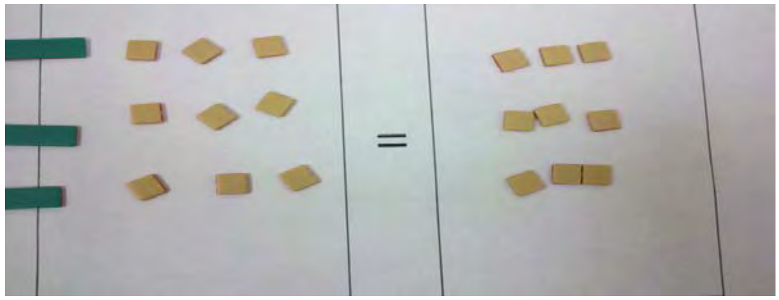


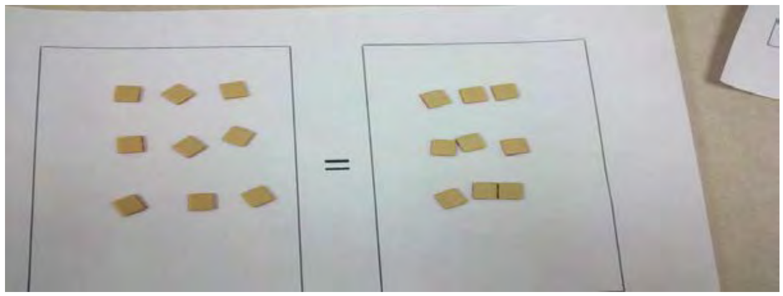
- The left side cancels each other out. On the right side, separate the three green rectangles. Give each green rectangle a cream square until they are gone. One green rectangle is equal to how many cream squares? 3. Therefore, x = 3.





- Last step is to substitute the solution 3 in for X in the original problem. Original problem is 3X = 9. On the left side put 3 cream squares for each X. There are three x, so put 9 cream squares on the left. On the right, put 9 cream squares. Are both sides equal? Yes. The answer is correct.





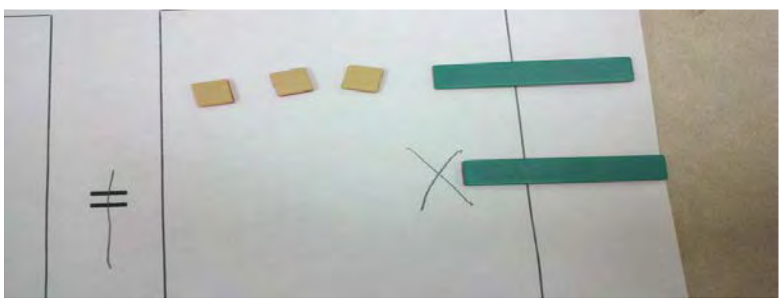
Model problem #4 $\frac{X}{2} = 3$

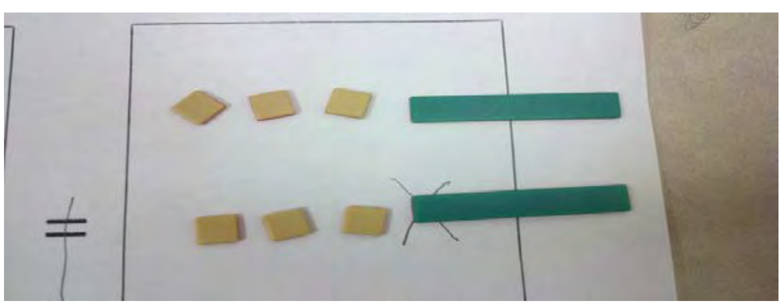
- Make sure the algebra mat is clear
- On the left side of the equal sign, draw a line and place 2 green rectangular tiles (green is positive and red is negative) under it. On the right side, place 3 cream squares (cream is positive and red is negative).

- Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
- Unite like terms. Are there any tiles the same that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
- Loop around the variable. Draw a circle around the variable. The variable is the green rectangular tiles.

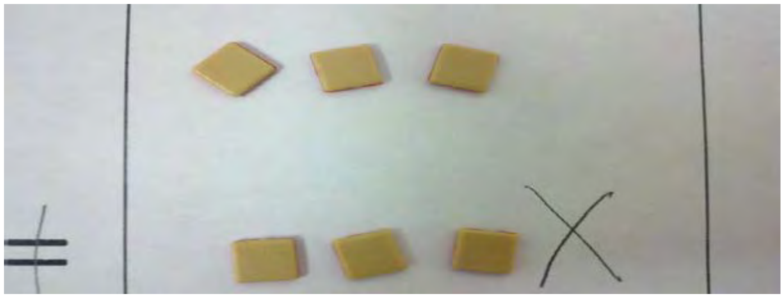
- Is there anything else on the side with the circle? No! So, we have to see what 1 green rectangle is equal to?
- Next step is opposite sign. A number under a line means to divide. The opposite of division is multiplication. Draw a multiplication sign beside the three green rectangles and place 2 more green rectangles.
- What we do to one side, we must do to the other. Since we drew a multiplication sign and placed 2 green rectangles on the left, we must draw a multiplication sign and place 2 green rectangles on the right.

- The left side cancel each other out. On the right side, separate the two green rectangles. Give all the squares to one of the rectangles. Then add 3 more cream squares so they will have the same amount.

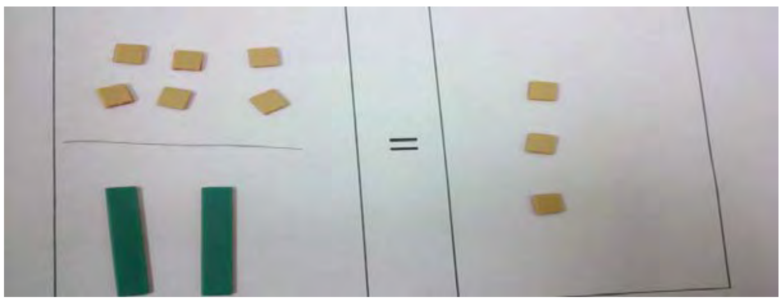




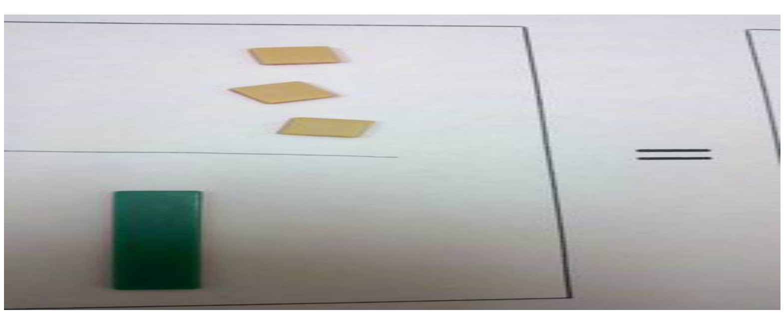
- How many squares are in all? 6. Both shapes represent positive numbers; therefore the answer is 6. X= 6



- Last step is to substitute the solution 6 in for X in the original problem. Original problem is X/2 = 3. On the right side, put 6 cream tiles. On the left side, put 6 cream square tiles in for X. Draw a line under the cream squares and put 2 green rectangles under the line.



- Separate the rectangles and give each rectangle a cream square until they are all gone. How many does each rectangle have? 6. How many cream tiles are on the right side? 6. The numbers are the same. Are both sides equal? Yes. The answer is correct.



Learning Sheet 1

One-Step
Concrete
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) X- 3 = 6

2) X + 5 = -8

3) 3X = 9

4) $\frac{X}{2} = 3$

Guided Practice

1) X -4 = 9

2) X + 2 = 7

3) –X = 5

4) -4X = 12

5) $\frac{X}{4} = -2$

Independent Practice

X – 2 = -5

$\frac{X}{2} = -4$

X + 8 = -5

-4X = -16

$\frac{X}{3} = 4$

-10 = X - 5

- 2X = -8

-X= 4

Learning Sheet 2

One-Step
Concrete
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) $X - 5 = -4$

2) $-3X = 9$

3) $\frac{X}{2} = 4$

Guided Practice

1) $X + 4 = -2$

2) $-X = 2$

3) $-2X = 4$

4) $\frac{X}{3} = -2$

Independent Practice

$\frac{X}{3} = -4$

$X + 6 = -5$

$-3X = -15$

$\frac{X}{2} = 5$

$-8 = X - 4$

$-X = 6$

Learning Sheet 3

One-Step
Concrete
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) $-4X = 8$

2) $\frac{X}{5} = -2$

Guided Practice

1) $-2X = 8$

2) $\frac{X}{2} = -8$

3) $-5X= 15$

Independent Practice

$X - 6 = -7$

$\frac{X}{2} = -3$

$X + 4 = -6$

$-4X = -16$

$\frac{X}{2} = 10$

$-9 = X - 5$

$- 6X = -24$

$-X= 4$

$X -4 = 10$

$X + 2 = -6$

$-X = 6$

Lessons 4-6

One-step Equations

Representational Method

Materials

- chalkboard/whiteboard/easel
- learning sheets
- progress chart

Advance Organizer

- Tell students what they will be doing and why
- Remind students about the commitment they made to learn to solve equations. We will work hard to teach them and they will work hard to learn a new way to solve equations. Tell student that they will solve one step equations a new way. They already know how to solve equations using algebra tiles, but today they will learn how to draw pictures. The pictures will help them solve problems and they can use pictures anytime they need to solve problems because sometimes algebra tiles are not available.

Demonstrate

- Give students learning sheets
- Begin with the first problem in the "Model" section. Tell the students that we will show them how to solve the problem and that they will have a chance to solve problems also. State the expectation for behavior and attention to the demonstration.
- Begin with the first problem and think out loud (see problem model)
- First we read the problem. X + 3 = -2
- Next, I take out my workmat or dry erase board.
- In the representational method, I am going to draw rectangles for my variables (X number) and draw squares for my ones number. For positive numbers, I am going to draw a "+" plus sign in the rectangle or square and a "-"minus sign for negative numbers.
- In this problem, I am going to draw a rectangle that represents the X. I am going to draw a "+" plus sign in the rectangle. Also on the left side I am going to draw 2 squares with plus signs in them. On the right side, I am going to draw 2 squares with negative or minus signs in them.
    - The first step in SUMLOWS is to separate the two sides. Draw a line through the equal sign.
    - The "U is for unite like terms. Look on the left side; are there any tiles that are the same shape that are not already together? No. Look on the right side. Are there any tiles the same shape that are not already together? No. We go to the next step.
    - Modify the new equation. There is not anything to unite or combine, so skip this step too.
    - The next step is Loop the variable. The variable is the rectangular drawing. This is the number that should be by itself and want to move last. So what is on the side with

the coefficient or rectangle? 3 squares with plus signs in them. We have to get rid of those because the rectangle should be by itself.

- o In order to move the 3 squares with plus signs in them, we have to draw three squares with minus signs in them. Opposites cancel each other out. put an X over each pair of plus and minus pairs (all of the squares on the left side should have an "X" over them)
- o The next step is "what we do to one side, we must do to the other" We added 3 squares with a minus sign in them, so we must add 3 squares with a minus signs in them on the right side.
- o Combine like terms. On the right side, we have 2 squares with minus signs in them and 3 squares with minus signs in them. Since they are the same shape, we must combine them. Since all the squares have a minus sign in them, we must add them together. We have a total of 5 squares with a minus sign in them. This is a -5. So X = -5.
- o Is the X by itself? Yes. Now we do the last step.
- o Substitute the solution for the variable and check. Go up to the original problem. X + 3=-2
- o For the rectangle, draw 5 squares with minus signs in them (X = -5) on the left side. Also draw 3 squares with plus signs in them on the left side. On the right side, draw 2 squares with minus signs in them.
- o Combine the tiles on the left side. Remember, squares with plus and minus cancel each other out. Three squares with plus in them will cancel out three squares with minus signs in them. How many squares are on the left side? 2 squares with minus in them. How many squares are on the right side? 2 squares with minus signs in them. They are equal. The answer is correct.

Model problem #2

- I read the next problem. X - 5 = -8
- I make sure my mat or board is clear
- On the mat, draw 1 rectangle with a minus sign in it on the left side. I also draw 5 squares with minus signs on them on the left side (beside the rectangle with the plus sign on it). On the right side, draw 8 squares with minus signs in them.
  - o Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - o Unite like terms. Are there any shapes that are not already together on the left side? NO. Look on the right side. Are there any shapes that are not already together? No. We go on to step three.
  - o Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
  - o Loop around the variable. Draw a circle around the 1 rectangle with the plus in it. That is the number that should be by itself.
  - o Is there anything else on the side with the circle? Yes! There are 5 squares with minus signs in them. We must move those 5 squares with the minus signs.
  - o Next step is opposite sign. In order to move the 5 squares with the minus in them, we must do the opposite. The opposite of "minus 5" is "plus 5." So, we must draw 5 squares with plus signs in them on the left side of the equal sign.

- What we do to one side, we must do to the other. Because we drew 5 squares with plus signs in them on the left side, we must draw 5 squares with plus signs in them on the right side of the equal sign. The square tiles on the left side cancel each other out (+5 – 5 = 0). Pair up one square with a minus sign with one square with a plus sign. Draw an X for each pair. All the squares on the left side should have an X over them. On the right side, we have 8 squares with minus signs in them and 5 squares with plus signs in them.
- Combine like terms. On the right side, we have 8 squares with minus signs in them and 5 squares with plus signs in them. Since they are the same shape, we should combine them. These squares have different signs in them, so we will subtract them or pair them with the opposite sign. Put an X on one square with a plus sign and a square with a minus sign until there are no more pairs. What are left on the right side? We have 3 squares with a minus sign in them.
- On the left side of the equal sign, we have 1 rectangle with a plus sign in it. On the right side, we have 3 squares. Now we have X = -3
- Is the X by itself? Yes. Is it positive? Yes. Now we do the last step.
- Substitute the solution for the variable and check. Go up to the original problem. X - 5 = -8.
- Instead of a rectangle for X, draw in 3 squares with minus signs in them (X = -3) on the left side. Also draw 5 squares with minus signs in them (represent -5). Draw 8 squares with minus signs in them on the right side. Combine the tiles on the left side. Remember, squares with minus signs and squares with plus signs cancel each other out. Since all the squares have a minus sign, we will add them. How many squares are on the left side? 8 squares with a minus sign. How many squares are on the right side? 8 squares with a minus sign. We have the same number and sign in them. The answer is correct.

Model problem #3

- I read the next problem. -2X = 8
- I make sure my mat or board is clear
- On the mat, draw 2 rectangles with a negative on the left side. On the right side, draw 8 squares with a plus sign in them.
  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any shapes on the left side that are not already together? NO. Look on the right side. Are there any shapes that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
  - Loop around the variable. Draw a circle around the 2 rectangles with the minus sign in them.
  - Is there anything else on the side with the circle? No! So, we have to see what 1 rectangle is equal to?
  - Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw division sign and draw two rectangles with negative signs in them beside the other rectangles with the negative signs in them.

- What we do to one side, we must do to the other. Draw a division sign and draw two rectangles with negative signs in them beside the 8 squares with the plus sign in them. The left side cancels each other out. On the right side, pair up the two rectangles with the 8 squares with plus signs in them. Give each rectangle a square until they are all with a rectangle. One rectangle is equal to how many squares? 4. Do they have the same sign in them? Yes, the answer is positive. If no, the answer is negative. They have different signs, therefore, x = -4
- Last step is to substitute the solution -4 in for X in the original problem. Original problem is -2X = 8. On the left side draw 4 squares with a minus sign in them for each X (we have two). There are two Xs, so draw 8 squares plus signs on them on the left. The squares are positive because both numbers (-2 and -4) are negative, therefore it's positive. On the right, draw 8 squares with the plus sign in them. Are both sides equal? Yes. The answer is correct.

Model problem # 4

- I read the problem: $\frac{X}{5} = -4$
- Make sure the algebra mat is clear
- On the left side of the equal sign, draw a line and draw 5 rectangles with plus signs in them under it. On the right side, draw 4 squares with minus signs in them.
  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any shapes that are the same that are not already together? NO. Look on the right side. Are there any shapes that are the same that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
  - Loop around the variable. Draw a circle around the variable. The variable is the rectangles.
  - Is there anything else on the side with the circle? No! So, we have to see what the rectangle is equal to?
  - Next step is opposite sign. A number under a line means to divide. The opposite of division is multiplication. Draw a multiplication sign beside the 5 green rectangles and draw 5 more rectangles with plus signs in them.
  - What we do to one side, we must do to the other. Since we drew a multiplication sign and drew 5 rectangles with plus sign in them on the left, we must draw a multiplication sign and draw 5 rectangles with plus signs in them on the right. The left side cancels each other out. On the right side, separate the 5 rectangles. Give all the squares to one of the rectangles. Then add 4 more squares with minus signs in them to each rectangle, so they all will have the same amount. How many squares are in all? 20. The signs in the rectangles and squares are different; therefore the answer is negative 20. X= -20.
  - Last step is to substitute the solution -20 in for X in the original problem. Original problem is X/5 = -4. On the right side, draw 4 squares with minus signs in them. On the left side, draw 20 squares with minus signs in them in for X. Draw a line under the squares and draw 5 rectangles with plus signs in them under the line. Separate the

rectangles and give each rectangle a square until they are all gone. How many squares does each rectangle have? 4 red squares. How many squares are on the right side? 4 red squares. The numbers are the same. Are both sides equal? Yes. The answer is correct.

Guided Practice

- Direct students to the "Guide" section of the learning sheet
- Tell students to touch the first problem and that we will do this problem together, using their drawings and the SUMLOWS mnemonic.
- Let's read the problem. This problem is X – 6 = -9.
- On our workmat, what do we draw on the left? 1 rectangle with a plus in it and 6 squares with minus sign in them.
    - What do we draw on the right side? 9 squares with minus signs in them
    - What is the first step in SUMLOWS? Separate the sides. We are going to draw a line through the equal sign.
    - What does the "U" stand for in SUMLOWS? Unite like terms. Are there any shapes on the same side that are not already together? NO. Look on the right side. Are there any shapes on the same side? No. We go on to step three.
    - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
    - What is the next step? Loop around the variable. Which shape is the variable? The green rectangle. Draw a circle around the 1 rectangle with the plus sign in it. That is the number that should be by itself.
    - Is there anything else on the side with the circle? Yes! There are 6 square with minus signs in them. We must move those 6 squares with the minus signs in them.
    - What is the next step? Next step is opposite sign. In order to move the 6 squares with the minus signs in them, we must do the opposite. The opposite of "minus 6" is "plus 6." So, we must draw 6 squares with the plus sign in them to the left side of the equal sign.
    - What is the next step? What we do to one side, we must do to the other. Because we drew 6 squares with plus signs on them on the left side, we must draw 6 squares with plus signs in them on the right side of the equal sign. The square tiles on the left side cancel each other out (-6 +6 = 0). On the left side, pair up a square with a minus sign with a square with a plus sign. Put an X over each pair. All squares on the left side should have an X over them.
    - Combine like terms. On the right side, we have 9 squares with a minus sign in them and 6 squares with a plus sign in them. Since they are the same shape, we should combine them. These squares have different signs in them, so we will subtract. Pair them up and put an X on each pair until there are no more pairs. What are left on the right side of the equal sign? 3 squares with minus signs in them.
    - On the left side of the equal sign, we have 1 rectangle with a plus sign. On the right side, we have 3 squares with minus signs in them. Now we have X = -3. Is the X by itself? Yes. Does the rectangle have a plus sign in it? Yes. Now we do the last step.

- Substitute the solution for the variable and check. Go up to the original problem. X - 6 = -9. Instead of a rectangle for X, draw 3 squares with minus signs in them (X = -3) on the left side. Also draw 6 squares with minus signs in them (represent -6). Draw 9 squares with minus signs in them on the right side. Combine the tiles on the left side. Since all of the squares on the left side have the same sign in them, add (combine them). How many squares are on the left side? -9. How many squares are on the right side? -9. They are the same; therefore the answer is correct.

Guided Practice #2

- Locate problem # 2 in the Guided practice section.
- Let's read the problem. This problem is X+ 5 = -7.
- On our workmat, what do we draw on the left? 1 rectangle with a plus on it and 5 squares with plus signs in them.
- What do we draw on the right side? 7 squares with a minus sign on them.
  - What is the first step in SUMLOWS? Separate the sides. We are going to draw a line through the equal sign.
  - What does the "U" stand for in SUMLOWS? Unite like terms. Are there any shapes that are the same that are not already together on the same side of the equal sign? NO. Look on the right side. Are there any shapes that are the same that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
  - What is the next step? Loop around the variable. Which shape is the variable? The rectangle with the plus sign in it. Draw a circle around rectangle. That is the number that should be by itself.
  - Is there anything else on the side with the circle? Yes! There are 5 squares with plus signs in them. We must move those 5 squares.
  - What is the next step? Next step is opposite sign. In order to move the 5 squares with the plus sign in them, we must do the opposite. The opposite of "plus 5" is "minus 5." So, we must draw 5 squares with minus signs in them on the left side of the equal sign.
  - What is the next step? What we do to one side, we must do to the other. Because we drew 5 squares with minus signs in them on the left side, we must draw 5 squares with minus signs in them on the on the right side of the equal sign. The square on the left side cancel each other out (+5 -5= 0).
  - Combine like terms. On the right side, we have 7 squares with minus signs in them and 5 squares with minus signs in them. Since they are the same shape, we should combine them. These squares have the same sign in them, so we will add them. What are left on the right side of the equal sign? 12 squares with minus signs in them.
  - On the left side of the equal sign, we have 1 rectangle with a plus sign in them. On the right side, we have 12 squares with minus signs in them. Now we have X = -12. Is the X by itself? Yes. Does the rectangle have a plus sign in it? Yes. Now we do the last step.

- Substitute the solution for the variable and check. Go up to the original problem. X +5 = -7. Instead of a rectangle for X, draw 12 squares with minus signs in them (X = -12) on the left side. Also draw 5 squares with plus signs in them (represent +5). Place 7 squares with minus signs on the right side. Combine the squares on the left side. Remember, squares with minus signs in them and squares with plus signs in them cancel each other out. Since we have squares with plus signs and minus signs in them, we subtract or pair them up and draw an X for each pair. How many squares are on the left side without an X on them? 7 squares with minus signs in them. How many squares are on the right side? 7 squares with a minus sign in them. They are the same amount and with the same sign in them. The answer is correct.

Guided Practice #3

- Locate problem # 3 on the Guided practice section.
- Let's read the problem. This problem is –X = 3.
- On our workmat, what do we draw on the left? 1 rectangle with a minus sign in it.
- What do we draw on the right? 3 square tiles with plus sign in them.
  - What is the first step in SUMLOWS? Separate the sides. We are going to draw a line through the equal sign.
  - What does the "U" stand for in SUMLOWS? Unite like terms. Are there any tiles the same shape that are not already together on the same side of the equal sign? NO. Look on the right side. Are there any tiles the same shape that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
  - What is the next step? Loop around the variable. Which shape is the variable? The rectangle. Draw a circle around the 1 rectangle with the minus sign in it. That is the number that should be by itself.
  - Is there anything else on the side with the circle? No. Is the X by itself? Yes. Is it positive? No. It is negative. We must make the rectangle positive.
  - What is the next step? Next step is opposite sign. In order to move the negative one, we must do the opposite. The opposite of "minus 1" is "plus 1." So, we flip/change the rectangle to the positive sign. Cross out the minus sign on the rectangle and draw a plus sign.
  - What is the next step? What we do to one side, we must do to the other. Because we changed the sign of the rectangle on the left side, we must change the signs of the squares on the right side. Draw an X over the plus signs and draw minus signs on the squares.
  - On the left side of the equal sign, we have 1 rectangle with a plus sign on it. On the right side, we have 3 squares with the negative sign on them. Now we have X =- 3. Is the X by itself? Yes. Is it positive? Yes. Now we do the last step.
  - Substitute the solution for the variable and check. Go up to the original problem. –X=3. Instead of a red triangle for X, put in 3 cream squares (X =- 3) on the left side. The tiles are cream because both numbers have the same sign, therefore it's positive. On the right side, put down 3 cream tiles. The tiles are the same on both sides. The answer is correct.

Guided problem #4

- Locate problem #4 in the Guided practice section
- Let's read the problem: -5X = 15
- On our workmat, what do we draw on the left? 5 rectangles with minus signs in them
- What do we draw on the right? 15 squares with plus signs in them
  - What is the first step in SUMLOWS? Separate the sides. We are going to draw a line through the equal sign.
  - What does the "U" stand for in SUMLOWS? Unite like terms. Are there any shapes on the same side of the equal sign that are not already together? NO. Look on the right side. Are there any shapes on the same side of the equal sign that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
  - What is the next step? Loop around the variable. Which shape is the variable? The rectangle. Draw a circle around the five rectangles with the minus signs in them. That is the number that should be by itself.
  - Is there anything else on the side with the circle? No. So, we have to see what 1 rectangle is equal to?
  - Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw division sign and draw five rectangles with minus signs in them beside the 5 rectangles with the minus signs in them.
  - What we do to one side, we must do to the other. Draw a division sign and draw 5 rectangles beside the 15 squares with the plus signs in them. The left side cancels each other out.
  - On the right side, separate the 5 rectangle with the minus signs in them. Give each rectangle a square until they are all gone. One rectangle is equal to how many squares? 5. In multiplication and division, if the shapes have different signs in them, it's negative. If the signs are the same, the sign is positive. Therefore, x = -3.
  - Last step is to substitute the solution -3 in for X in the original problem. Original problem is -5X = 15. On the left side draw 5 squares for each X. There are five Xs, so draw 15 squares with plus signs in them for each X on the left. The squares are positive because both number (-5 and -3) are negative. If the signs are the same, it's a positive number. On the right, draw 15 squares with plus signs in them. Are both sides equal? Yes. The answer is correct.

Guided practice problem #5

- I read the problem: $\frac{X}{3} = 6$
- Make sure the algebra mat is clear
- On the left side of the equal sign, draw a line and draw 3 rectangles with plus signs in them under it. On the right side, draw 6 squares with plus signs in them.
  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.

- Unite like terms. Are there any shapes that are the same that are not already together? NO. Look on the right side. Are there any shapes that are the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
- Loop around the variable. Draw a circle around the variable. The variable is the rectangles.
- Is there anything else on the side with the circle? No! So, we have to see what the rectangle is equal to?
- Next step is opposite sign. A number under a line means to divide. The opposite of division is multiplication. Draw a multiplication sign beside the 3 rectangles with plus signs in them and draw 3 more rectangles with plus signs in them.
- What we do to one side, we must do to the other. Since we drew a multiplication sign and drew 3 rectangles with plus sign in them on the left, we must draw a multiplication sign and draw 3 rectangles with plus signs in them on the right. The left side cancels each other out. On the right side, separate the 3 rectangles. Give all the squares to one of the rectangles. Then add 6 more squares with plus signs in them to each rectangle, so they all will have the same amount. How many squares are in all? 18. The signs in the rectangles and squares are the same; therefore the answer is positive 18. X= 18.
- Last step is to substitute the solution 18 in for X in the original problem. Original problem is X/3 = 6. On the right side, draw 6 squares with plus signs in them. On the left side, draw 18 squares with plus signs in them in for X. Draw a line under the squares and draw 3 rectangles with plus signs in them under the line. Separate the rectangles and give each rectangle a square until they are all gone. How many squares does each rectangle have? 6 squares with plus signs in them. How many squares are on the right side? 6 squares with plus signs in them. The numbers are the same. Are both sides equal? Yes. The answer is correct.

Repeat the same interactive process with the remaining problems

Independent Practice #5

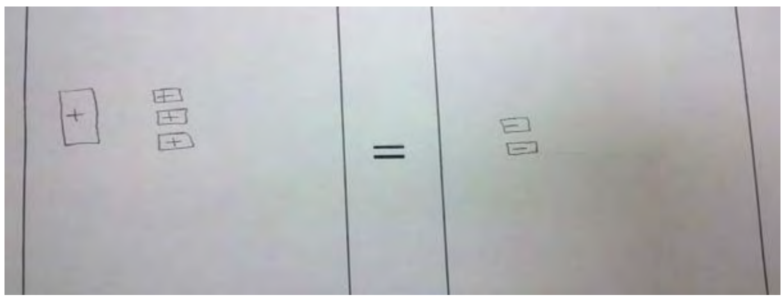
- Direct the students to the "Independent Practice" section. Read the first problem together and direct students to complete the problems without you. When students finish problems, provide immediate corrective feedback for errors.

Graphing

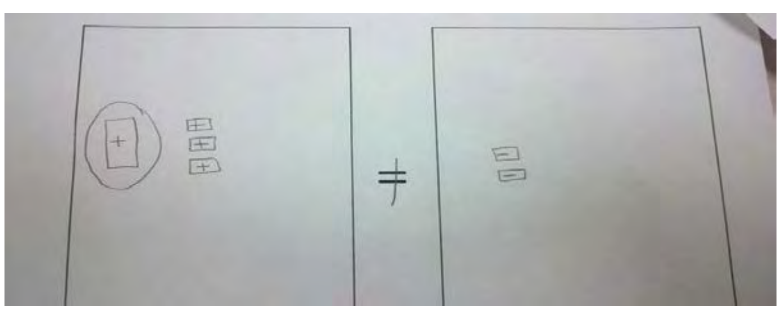
Steps for the Representational Level

Model #1: X + 3 = -2

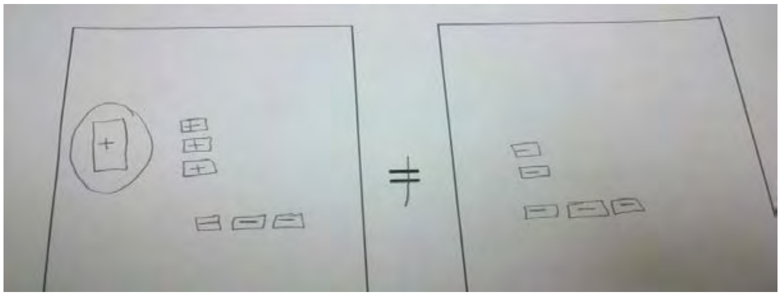
- Next, I take out my workmat or dry erase board.
- In the representational method, I am going to draw rectangles for my variables (X number) and draw squares for my ones number. For positive numbers, I am going to draw a "+" plus sign in the rectangle or square and a "-"minus sign for negative numbers.
- In this problem, I am going to draw a rectangle that represents the X. I am going to draw a "+" plus sign in the rectangle. Also on the left side I am going to draw 2 squares with plus signs in them. On the right side, I am going to draw 2 squares with negative or minus signs in them.



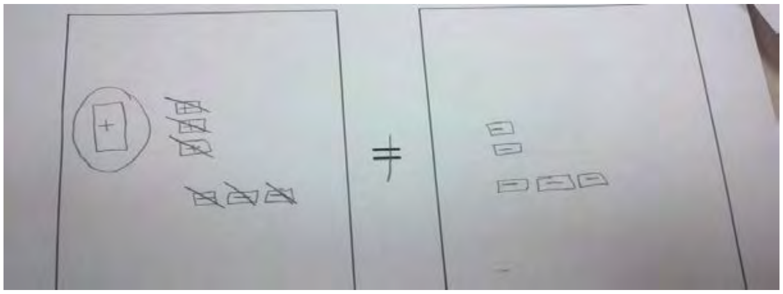
  - The first step in SUMLOWS is to separate the two sides. Draw a line through the equal sign.
  - The "U is for unite like terms. Look on the left side; are there any tiles that are the same shape that are not already together? No. Look on the right side. Are there any tiles the same shape that are not already together? No. We go to the next step.
  - Modify the new equation. There is not anything to unite or combine, so skip this step too.
  - The next step is Loop the variable. The variable is the rectangular drawing. This is the number that should be by itself and want to move last. So what is on the side with the coefficient or rectangle? 3 squares with plus signs in them. We have to get rid of those because the rectangle should be by itself.



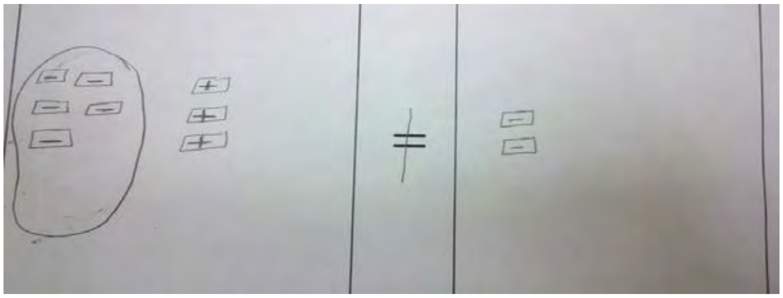
- In order to move the 3 squares with plus signs in them, we have to draw three squares with minus signs in them. Opposites cancel each other out. put an X over each pair of plus and minus pairs (all of the squares on the left side should have an "X" over them).
- The next step is "what we do to one side, we must do to the other" We added 3 squares with a minus sign in them, so we must add 3 squares with a minus signs in them on the right side.



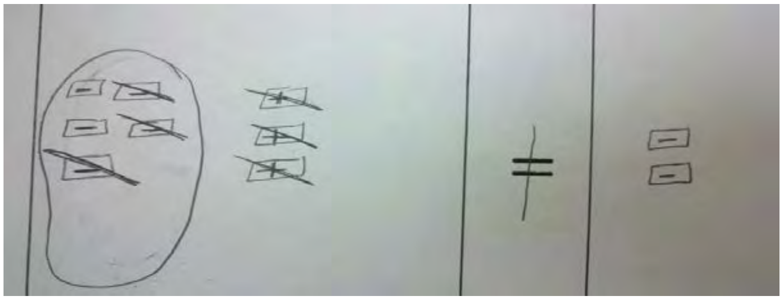
- Combine like terms. On the right side, we have 2 squares with minus signs in them and 3 squares with minus signs in them. Since they are the same shape, we must combine them. Since all the squares have a minus sign in them, we must add them together. We have a total of 5 squares with a minus sign in them. This is a -5. X= -5.



- Is the X by itself? Yes. Now we do the last step. Substitute the solution for the variable and check. Go up to the original problem X + 3=-2.
- For the rectangle, draw 5 squares with minus signs in them (X = -5) on the left side. Also draw 3 squares with plus signs in them on the left side. On the right side, draw 2 squares with minus signs in them.

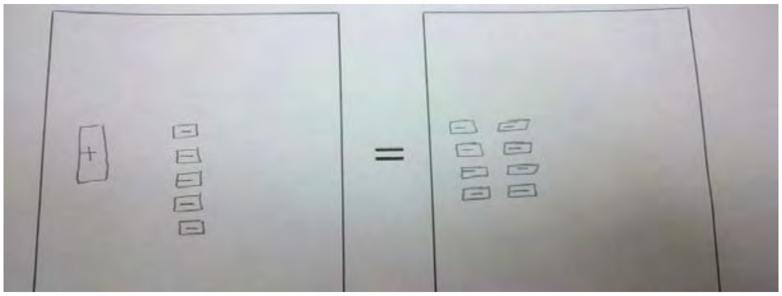


- Combine the tiles on the left side. Remember, squares with plus and minus cancel each other out. Three squares with plus in them will cancel out three squares with minus signs in them. How many squares are on the left side? 2 squares with minus in them. How many squares are on the right side? 2 squares with minus signs in them. They are equal. The answer is correct.



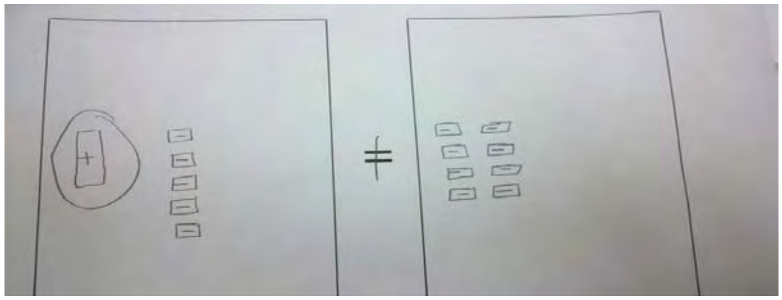
Model problem 2: X - 5 = -8

- I make sure my mat or board is clear
- On the mat, draw 1 rectangle with a minus sign in it on the left side. I also draw 5 squares with minus signs on them on the left side (beside the rectangle with the plus sign on it). On the right side, draw 8 squares with minus signs in them.

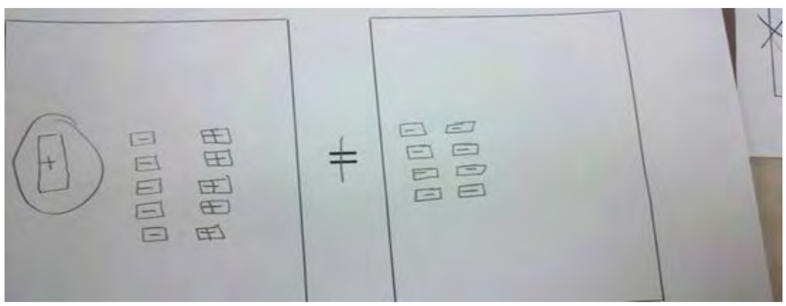


  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any shapes that are not already together on the left side? NO. Look on the right side. Are there any shapes that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.

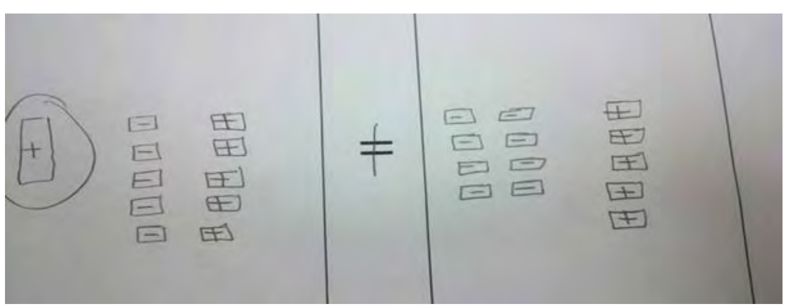
- Loop around the variable. Draw a circle around the 1 rectangle with the plus in it. That is the number that should be by itself.



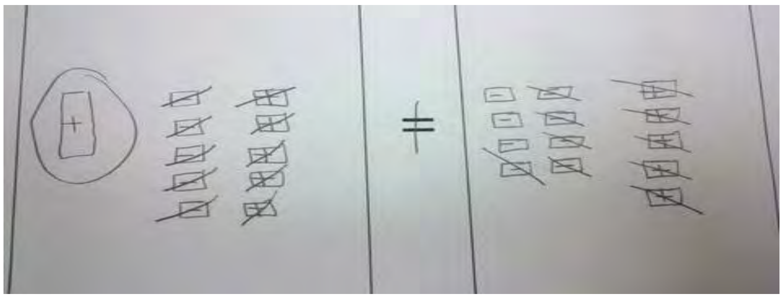
- Is there anything else on the side with the circle? Yes! There are 5 squares with minus signs in them. We must move those 5 squares with the minus signs.
- Next step is opposite sign. In order to move the 5 squares with the minus in them, we must do the opposite.   The opposite of “minus 5” is “plus 5.” So, we must draw 5 squares with plus signs in them on the left side of the equal sign.



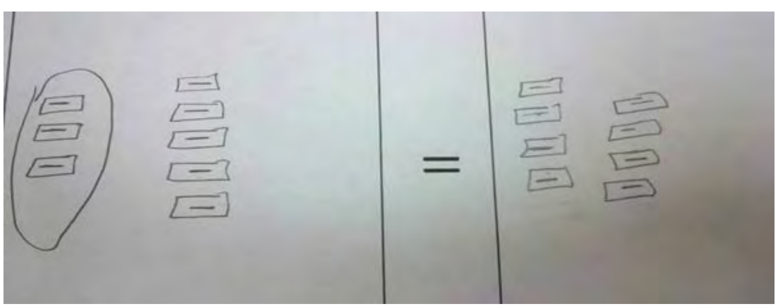
- What we do to one side, we must do to the other. Because we drew 5 squares with plus signs in them on the left side, we must draw 5 squares with plus signs in them on the right side of the equal sign. The square tiles on the left side cancel each other out (+5 – 5 = 0). Pair up one square with a minus sign with one square with a plus sign. Draw an X for each pair. All the squares on the left side should have an X over them. On the right side, we have 8 squares with minus signs in them and 5 squares with plus signs in them.



- Combine like terms. On the right side, we have 8 squares with minus signs in them and 5 squares with plus signs in them. Since they are the same shape, we should combine them. These squares have different signs in them, so we will subtract them or pair them with the opposite sign. Put an X on one square with a plus sign and a square with a minus sign until there are no more pairs. What are left on the right side? We have 3 squares with a minus sign in them.
- On the left side of the equal sign, we have 1 rectangle with a plus sign in it. On the right side, we have 3 squares. Now we have X = -3
- Is the X by itself? Yes. Is it positive? Yes. Now we do the last step.

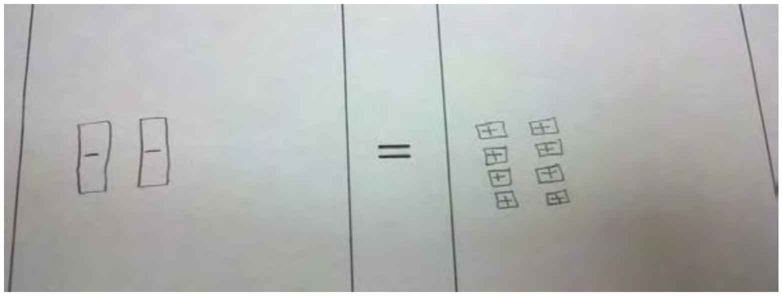


- Substitute the solution for the variable and check. Go up to the original problem. X - 5 = -8. Instead of a rectangle for X, draw in 3 squares with minus signs in them (X = -3) on the left side. Also draw 5 squares with minus signs in them (represent -5). Draw 8 squares with minus signs in them on the right side. Combine the tiles on the left side. Remember, squares with minus signs and squares with plus signs cancel each other out. Since all the squares have a minus sign, we will add them. How many squares are on the left side? 8 squares with a minus sign. How many squares are on the right side? 8 squares with a minus sign. We have the same number and sign in them. The answer is correct.

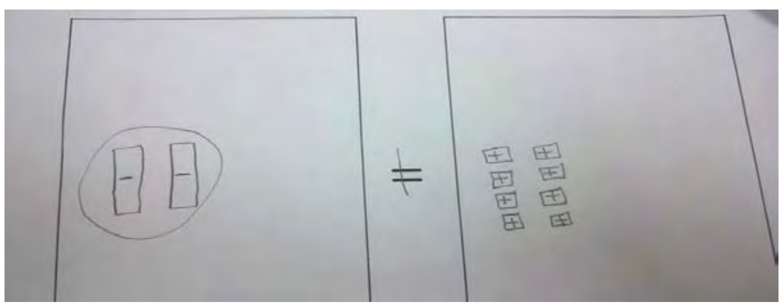


Model problem #3 -2X = 8

- I make sure my mat or board is clear
- On the mat, draw 2 rectangle with a negative on the left side. On the right side, draw 8 squares with a plus sign in them.

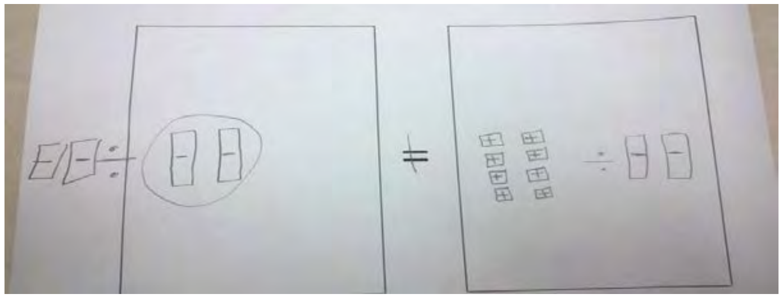


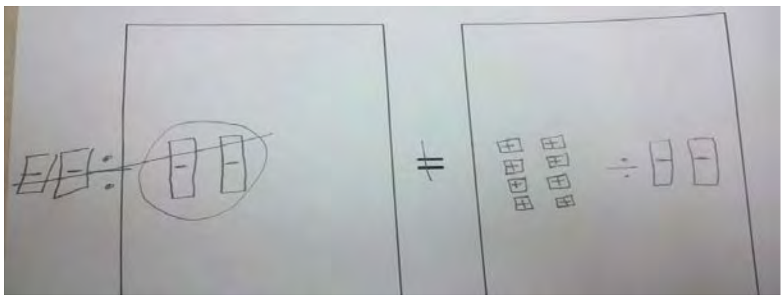
  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any shapes on the left side that are not already together? NO. Look on the right side. Are there any shapes that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
  - Loop around the variable. Draw a circle around the 2 rectangles with the minus sign in them.
  - Is there anything else on the side with the circle? No! So, we have to see what 1 rectangle is equal to.



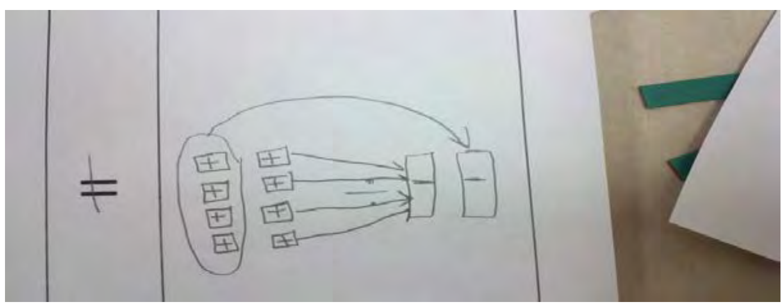
  - Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw division sign and draw two rectangles with negative signs in them beside the other rectangles with the negative signs in them.

- What we do to one side, we must do to the other. Draw a division sign and draw two rectangles with negative signs in them beside the 8 squares with the plus sign in them. The left side cancels each other out.
- On the right side, pair up the two rectangles with the 8 squares with plus signs in them. Give each rectangle a square until they are all with a rectangle.
- One rectangle is equal to how many squares? 4. Do they have the same sign in them?

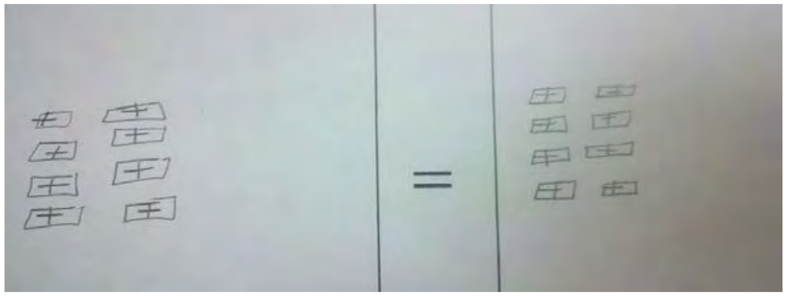




Yes, the answer is positive. If no, the answer is negative. They have different signs, therefore, x = -4.

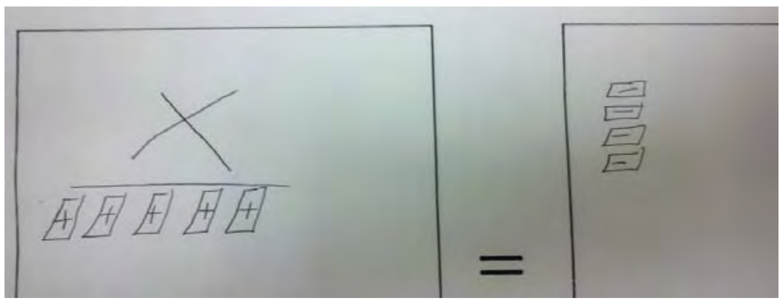


- Last step is to substitute the solution -4 in for X in the original problem. Original problem is -2X = 8. On the left side draw 4 squares with a minus sign in them for each X (we have two). There are two Xs, so draw 8 squares plus signs on them on the left. The squares are positive because both numbers (-2 and -4) are negative, therefore it's positive. On the right, draw 8 squares with the plus sign in them. Are both sides equal? Yes. The answer is correct.

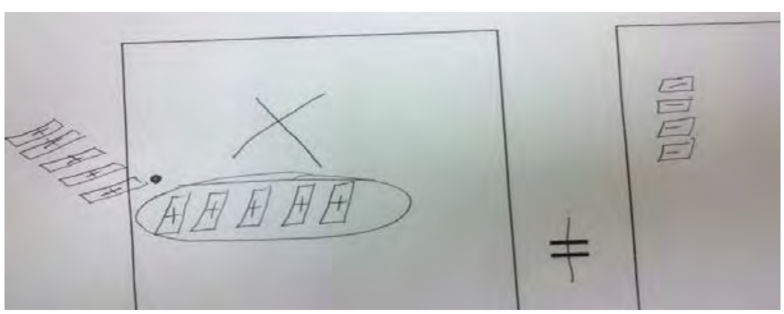


Model problem # 4 $\frac{X}{5} = -4$

- Make sure the algebra mat is clear
- On the left side of the equal sign, draw a line and draw 5 rectangles with plus signs in them under it. On the right side, draw 4 squares with minus signs in them.

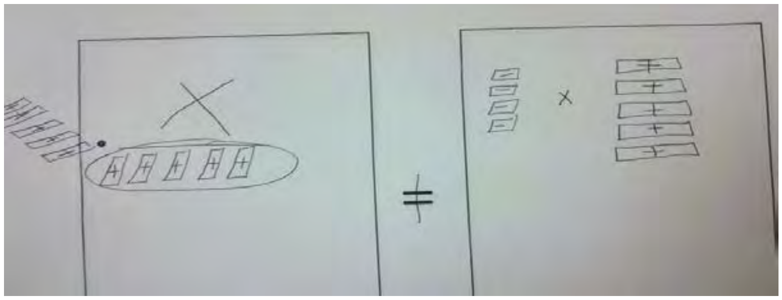


  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any shapes that are the same that are not already together? NO. Look on the right side. Are there any shapes that are the same that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
  - Loop around the variable. Draw a circle around the variable. The variable is the rectangles.

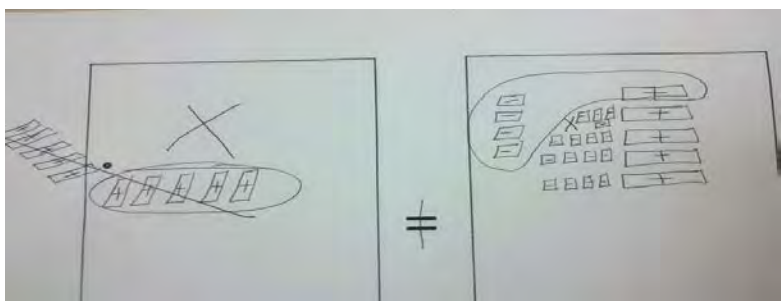


  - Is there anything else on the side with the circle? No. So, we have to see what the rectangle is equal to?
  - Next step is opposite sign. A number under a line means to divide. The opposite of division is multiplication. Draw a multiplication sign beside the 5 green rectangles and draw 5 more rectangles with plus signs in them.

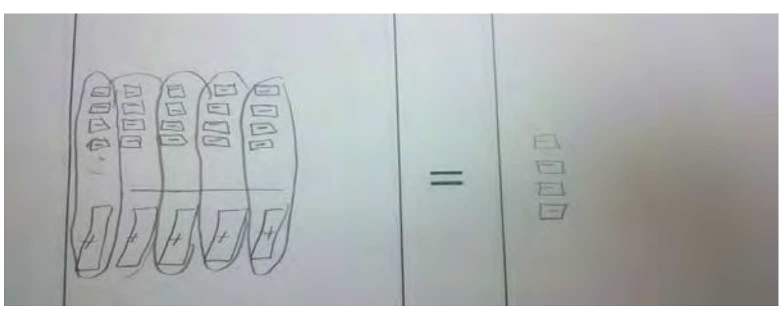
- What we do to one side, we must do to the other. Since we drew a multiplication sign and drew 5 rectangles with plus sign in them on the left, we must draw a multiplication sign and draw 5 rectangles with plus signs in them on the right. The left side cancels each other out.



- On the right side, separate the 5 rectangles. Give all the squares to one of the rectangles. Then add 4 more squares with minus signs in them to each rectangle, so they all will have the same amount. How many squares are in all? 20. The signs in the rectangles and squares are different; therefore the answer is negative 20. X= -25.



- Last step is to substitute the solution -20 in for X in the original problem.   Original problem is X/5 = -4. On the right side, draw 4 squares with minus signs in them. On the left side, draw 20 squares with minus signs in them in for X. Draw a line under the squares and draw 5 rectangles with plus signs in them under the line. Separate the rectangles and give each rectangle a square until they are all gone. How many squares does each rectangle have? 4 red squares. How many squares are on the right side? 4 red squares. The numbers are the same. Are both sides equal? Yes. The answer is correct.



Learning Sheet 4

One-Step
Representational
Code Number __________________ Date ______________
SUMLOWS

Describe and Model

1) $X+3 = -2$

2) $X - 5 = -8$

3) $-2X = 8$

4) $\frac{X}{5} = -4$

Guided Practice

1) $X - 6 = -9$

2) $X + 5 = -7$

3) $-X = 3$

4) $-5X = 15$

5) $\frac{X}{3} = 6$

Independent Practice

$X - 2 = -5$

$\frac{X}{2} = -4$

$X + 8 = -5$

$-4X = -16$

$\frac{X}{3} = 4$

$-10 = X - 5$

$- 2X = -8$

$-X= 4$

Learning Sheet 5

One-Step
Representational
Code Number _________________ Date _____________
SUMLOWS

Describe and Model

1) $X - 1 = -5$

2) $-4X = 8$

3) $\frac{X}{5} = -2$

Guided Practice

1) $X + 2 = -6$

2) $-X = 6$

3) $-2X = 8$

4) $\frac{X}{2} = -8$

Independent Practice

$X - 6 = -7$

$\frac{X}{2} = -3$

$X + 4 = -6$

$-4X = -16$

$\frac{X}{2} = 10$

$-9 = X - 5$

$-6X = -24$

$-X = 4$

Learning Sheet 6

One-Step
Representational
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) -3X = 9

2) $\frac{X}{2} = 4$

Guided Practice

1) X + 4 = -2

2) –X = 2

3) $\frac{X}{3} = -2$

4) -5X= 10

Independent Practice

X – 4 = -7

$\frac{X}{3} = -4$

X + 6 = -5

-3X = -15

$\frac{X}{2} = 5$

-8 = X - 4

- 7X = -14

-X= 6

Lessons 7-9
One-step

Abstract Level

Materials

- Chalkboard/whiteboard/easel
- learning sheets 7-9
- progress chart

Advance Organizer

- Tell students what they will be doing and why
- Remind students about the commitment they made to learn to solve equations. We will work hard to teach them and they will work hard to learn a new way to solve equations. They learned to solve one-step equations with algebra tiles and with drawing. Now they will practice solving equations without using tiles or drawings. Remind students of the SUMLOWS mnemonic and tell them that this mnemonic can be used to help them solve equations if they do not remember the steps.

Demonstrate

- Give students learning sheets 7-9
- Begin with the first problem in the "Model" section. Tell the students that we will show them how to solve the problem using numbers only.
- First we First we read the problem. X + 8 = -2
- Next, I take out my workmat or dry erase board.
- In the abstract method, I am going to use symbols and numbers. For positive numbers, I am going to use a "+" plus sign or no sign at all. It depends on where the number or symbol is in the problem. For negative numbers, I am going to use "-"minus signs.
- In this problem, I am going to write out the problem like I see it. On the left, I am going to write X + 8. On the right side, I am going to write -2.
    - The first step in SUMLOWS is to separate the two sides. Draw a line through the equal sign.
    - The "U is for unite like terms. Look on the left side, are there any like terms or numbers that not already together? No. Look on the right side. Are there any like terms that are not together? No. We go to the next step.
    - Modify the new equation. There is not anything to unite or combine, so skip this step too.
    - The next step is Loop the variable. The variable is the X. This is the number that should be by itself and want to move last. So what is on the side with the variable or X? +8. We have to get rid of +8 because X should be by itself.
    - In order to move the +8, we have write -8 under the +8. Opposites cancel each other out. Put a slash mark through -8 and 8. This equals 0 and cancels each other out.

- The next step is “what we do to one side, we must do to the other” We subtracted 8 on the left side, so we must subtract 8 on the right side. Write -8 under the negative 2.
- Combine like terms. On the right side, we have -2 and -8. Since both numbers are both negative, we must combine them by adding them together. Two plus eight is 10. Since they are both negative. The answer is negative. So X = -10
- Is the X by itself? Yes. Is the X positive? Yes. Now we do the last step.
- Substitute the solution for the variable and check. Go up to the original problem X + 8=-2. Instead of writing an X, write -10. Also, write +8 on the left side. On the right side, write -2. Combine the tiles on the left side. Remember, if the numbers have the same sign add. If the signs are different, we are going to subtract. Then, we take the sign of the larger number. Negative 10 and positive 8 have different signs, so we should subtract. -10 plus 8 = 2. The larger number is negative; therefore the answer is negative 2. On the left side, we have -2. On the right side, we have -2. They are equal. The answer is correct.

Model problem #2

- I read the next problem. X -4 = -8
- On the left side of the mat, we are going write X – 4. On the right side, write -8.
  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any like terms or numbers that can be combined that are not already together on the left side? NO. Look on the right side. Are there any like terms or numbers that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
  - Loop around the variable. Draw a circle around the coefficient or X. That is the number that should be by itself.
  - Is there anything else on the side with the circle? Yes! There is a -4. We must move the -4.
  - Next step is opposite sign. In order to move-4, we must do the opposite. The opposite of “minus 4” is “plus 4.” So, we must write +4 under the -4 on the left side.
  - What we do to one side, we must do to the other. Because we wrote +4 on the left side, we write +4 on the right side of the equal sign under the -8. On the left side, the numbers -4 and +4 cancel each other out (-4 +4 = 0). Draw a line through it.
  - On the right side, we have -8 and +4. Remember if the signs are the same we add. If the signs are different, we will add and take the sign of the larger (absolute value) number. These signs are different, so we are going to subtract the number and take the sign of the largest number (absolute value). 8 minus 4 is 4. Eight is larger than 4, so the answer is -4.
  - On the left side of the equal sign, we have X. On the right side, we have -4. Now we have X = -4
  - Is the X by itself? Yes. Is it positive? Yes. Now we do the last step.
  - Substitute the solution for the variable and check. Go up to the original problem X -4 = -8. Instead of writing X, write -4 (X = 4) on the left side. Also on the left side, write -4. On the right side, write -8. We are going to combine the tiles on the left side. Remember if the numbers have the same sign, we are going to add. If the signs are

different we are going to subtract. Lastly, take the sign of the largest number (absolute value). Since both signs are negative, we are going to add. Four plus four is 8. The biggest number is negative, so the answer is negative. On the left side, we have -8. We have -8 on the right side. These numbers are the same. The answer is correct.

Model problem #3

- I read the next problem. 14 = -7X
- On the left side of the mat, I am going to write 14. On the right side of the mat, I am going to write 7X. This particular problem looks a little funny because we usually like the variable or the X on the left side. We can work the problem the same way or we can turn it around. If we decide to turn it around, we make sure we take the negative signs with the number. For this model, we will work the problem out like it is.
    - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
    - Unite like terms. Are there any like terms or numbers that are not already together? NO. Look on the right side. Are there any like terms or numbers that are not already together? No. We go on to step three.
    - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
    - Loop around the variable. Draw a circle around- 7X.
    - Is there anything else on the side with the circle? No! So, we have to see what 1X is equal to?
    - Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a line under the -7 (drawing a line means to divide) and write -7 under the line.
    - What we do to one side, we must do to the other. On the left side, draw a line under 14. Write -7 under the line. This means to divide.
    - The right side cancels each other out. On the left side, we are going to divide. What is 14 divided by 7? 2. In multiplication and division, if the signs are the same, the number is positive. If the numbers are different, it is negative. Therefore, the solution is -2. X = -2. Is the X by itself? Yes. Is the X positive? Yes. We go to the last step.
    - Last step is to substitute the solution -2 in for X in the original problem. Original problem is 14 = -7X. On the left side, write 14. On the right side, write-7. Then write -2 in parenthesis in place of X. A number beside a letter mean multiplication. What is 7 times 2 equals? 14. Are the signs same or different? They are the same. Therefore, the answer is positive. On the right side, we have 14 and on the right side we have 14. Are both sides equal? Yes. The answer is correct.

Model problem # 4

- I read the problem: $\frac{X}{3} = -4$
- Make sure the algebra mat is clear.
- On the left side of the equal sign, write X over 3 (X/3). On the right side, write -4.

- Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
- Unite like terms. Are there any shapes that are the same that are not already together? NO. Look on the right side. Are there any shapes that are the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
- Loop around the variable. Draw a circle around the variable. The variable is the X.
- Is there anything else on the side with the circle? Yes. The three is on the side with the X.
- Next step is opposite sign. A number under a line means to divide. The opposite of division is multiplication. Draw a multiplication sign beside the 3 and write 3.
- What we do to one side, we must do to the other. Since we drew a multiplication sign and wrote 3 on the left, we must draw a multiplication sign and write 3 on the right. The left side cancels each other out. On the right side, multiply-4 times 3? 12. The signs squares are different; therefore the answer is negative 12. X= -12.
- Last step is to substitute the solution -12 in for X in the original problem. Original problem is X/3 = -4. On the right side, write 12. On the left side, write -12 instead of X. Draw a line under the -12 and write 3under the line. What is 12 divided by 3? 4. The signs are different; therefore the answer is negative -4. We have -4 on the left and right sides. The numbers are the same. Are both sides equal? Yes. The answer is correct.

Guided Practice

- Direct students to the "Guide" section of the learning sheet
- Tell students to touch the first problem and that we will do this problem together, using numbers, letters and the SUMLOWS mnemonic.
- Let's read the problem. This problem is X -8 = -6.
- On our workmat, what do we write on the left side? X – 8
- What do we write on the right side? -6
  - What is the first step in SUMLOWS? Separate the sides. We are going to draw a line through the equal sign.
  - What does the "U" stand for in SUMLOWS? Unite like terms. Are there any terms or numbers on the same side that are not already together? NO. Look on the right side. Are there any like terms or numbers on the same side that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
  - What is the next step? Loop around the variable. What letter is the variable? X. Draw a circle around the letter. That is the number that should be by itself.
  - Is there anything else on the side with the circle? Yes! -8. We must move -8.
  - What is the next step? Next step is opposite sign. In order to move -6, we must do the opposite. The opposite of "minus 8" is "plus 8." So, we write +8 under the -8.
  - What is the next step? What we do to one side, we must do to the other. Because we wrote +8 under the -8 on the left side, we must write +8 under the -6 on the right side of the equal sign. The -8 and +8 on the left side cancel each other out (-8 +8 = 0). Put

a line through the _ 8 and + 8.   On the right side, we have -6 and + 8. They are both numbers; therefore we have to combine like terms.

- Combine like terms. On the right side, we -6 and + 8. Remember if the signs are the same, we add and take the sign of the larger number (absolute value). If the signs are different, we subtract and take the sign of the largest number (absolute value).   Since the signs are different, we are going to subtract. Eight minus six equals 2. Eight is larger than 6, and 8 is positive. Therefore, our answer is positive 2.
- On the left side of the equal sign, we have X. On the right side, we have 2. Now we have   X = 2. Is the X by itself? Yes. Is the X positive? Yes. Now we do the last step.
- Substitute the solution for the variable and check. Go up to the original problem X - 8 = -6. Instead of writing X, we are going to write 2 (X = 2) on the left side. Also write -8. On the right side, we write -6.   Combine the like terms on the left side. Remember if the signs are the same add and take the signs of the largest number (absolute value). If the signs are different, we are going to subtract and take the sign of the largest number (absolute value). We have positive 2 and -8. These signs are different, so we subtract them and take the sign of the largest number. Eight minus two equals 6. Eight is larger than six and the sight is negative, therefore the answer is -6.   What number are on the left side? -6. What number are on the right side? -6. They are the same; therefore the answer is correct.

Guided Practice #2

- Locate problem # 2 in the Guided practice section.
- Let's read the problem. This problem is -5 = X +3
- On our workmat, what do we draw on the left? -5
- What do we draw on the right side? X + 3.
    - What is the first step in SUMLOWS? Separate the sides. We are going to draw a line through the equal sign.
    - What does the "U" stand for in SUMLOWS? Unite like terms. Are there any like terms or numbers that are the same that are not already together on the left side of the equal sign? NO. Look on the right side. Are there any like terms or numbers that are the same that are not already together on the right side of the equal sign? No. We go on to step three.
    - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
    - What is the next step? Loop around the variable. Which letter is the variable? The X is the variable. Draw a circle around the X. That is the number that should be by itself.
    - Is there anything else on the side with the circle? Yes! + 3. We must move.
    - What is the next step? Next step is opposite sign. In order to move the +3, we must do the opposite.   The opposite of "plus 3" is "plus3." So, we write -3 under the +3.
    - What is the next step? What we do to one side, we must do to the other. Because we wrote -3 under +3, we must write -3 under -5 one the right side of the equal sign. The numbers on the right side (+3 and -3) cancel each other out (+3 -3= 0).

- Combine like terms. On the left side, we have -5 and -3. If the signs are the same, we add and take the sign of the larger number (absolute value). If the signs are different, we subtract and take the sign of the largest number (absolute value). Since the signs are the same, we add them and take the sign of the largest number. Five plus three equals 8. The sign is negative.
- On the left side of the equal sign, we have-8. On the right side, we have -5. Now we have X = -8. Is the X by itself? Yes. Is the X positive? Yes. Now we do the last step.
- Substitute the solution for the variable and check. Go up to the original problem. -5 = X + 3. Instead of X, we are going to -8 (X = -8) on the right side. Also write +3. Write -5 on the left side. Combine the squares on the right side. Remember if the signs are the same, we add and take the sign of the larger number (absolute value). If the signs are different, we are going to subtract and take the sign of the larger number (absolute value). Since the signs are different, subtract the numbers and take the sign of the larger number (absolute value). Eight minus three equals five. Eight is larger and is negative, therefore our answer is negative. On the left side, we have -5 and on the right side we have -5. They are the same number and with the same sign in them. The answer is correct.

Guided Practice #3

- Locate problem # 3 on the Guided practice section.
- Let's read the problem. This problem is –X = 2.
- On our workmat, what do we put on the left? -X
- What do we place on the right? 2
    - What is the first step in SUMLOWS? Separate the sides. We are going to draw a line through the equal sign.
    - What does the "U" stand for in SUMLOWS? Unite like terms. Are there any tiles the same shape that are not already together on the same side of the equal sign? NO. Look on the right side. Are there any tiles the same shape that are not already together? No. We go on to step three.
    - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
    - What is the next step? Loop around the variable. Which shape is the variable? The X is the variable. Draw a circle around the X. That is the number that should be by itself.
    - Is there anything else on the side with the circle? Yes! -1 (the 1 is invisible). We must move the negative.
    - What is the next step? Next step is opposite sign. In order to move the -1, we must do the opposite. . A number beside a letter means to multiply. The opposite of multiplication is division. Draw a line under -1 on the left side.
    - What we do to one side, we must do to the other. Draw a line under 2 and write 2 under it. The left side cancels each other out.
    - On the right side, what is 2 divided by 1? 2. Are the signs the same or different? They are different. Therefore, the answer is negative. X = -2.

- Last step is to substitute the solution -2 in for X in the original problem. Original problem is -X = 2. On the left side, write – (-2). This is a positive 2. On the right, write 2. On the left side, we have 2. On the right side, we 2. They are the same. The answer is correct.

Guided problem #4

- Locate problem #4 in the Guided practice section
- Let's read the problem: -3X = 15
- On our workmat, what we write on the left? -3X
- What do we write on the right? 15
    - What is the first step in SUMLOWS? Separate the sides. We are going to draw a line through the equal sign.
    - What does the "U" stand for in SUMLOWS? Unite like terms. Are there any like terms or numbers on the same side of the equal sign that are not already together? NO. Look on the right side. Are there any like terms or numbers on the same side of the equal sign that are not already together? No. We go on to step three.
    - Modify the new equation. There is not anything to unite or combine. So, skip this step too.
    - What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the X. That is the number that should be by itself.
    - Is there anything else on the side with the circle? No. So, we have to see what 1 X is equal to?
    - Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a line under -3 on the left side.
    - What we do to one side, we must do to the other. Draw a line under 15 and write -3 under it. The left side cancels each other out.
    - On the right side, what is 15 divided by 3? 5. Are the signs the same or different? They are different. Therefore, the answer is negative. X = -5.
    - Last step is to substitute the solution -5 in for X in the original problem. Original problem is -3X = 15. On the left side, write -3 and substitute. -5 for X. Write -5 with parenthesis around it. On the right, write 15. On the left, what is 3 times 5? 15. Is it positive or negative? Positive because the signs are the same. On the left side, we have 15. On the right side, we have 15. They are the same. The answer is correct.

Guided problem #5

- I read the problem: $-\frac{X}{3} = -2$
- Make sure the algebra mat is clear
- On the left side of the equal sign, write -X over 3 (-X/3). On the right side, write -2.
    - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.

- Unite like terms. Are there any shapes that are the same that are not already together? NO. Look on the right side. Are there any shapes that are the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- Loop around the variable. Draw a circle around the variable. The variable is the X.
- Is there anything else on the side with the circle? Yes. The three is on the side with the X.
- Next step is opposite sign. A number under a line means to divide. The opposite of division is multiplication. Write a multiplication sign beside the -3 and write -3 (bring the negative sign in front of the X with the 3).
- What we do to one side, we must do to the other. Since we wrote a multiplication sign and wrote -3 on the left, we must write a multiplication sign and write -3 on the right. The left side cancels each other out. On the right side, multiply-2 times -3? 12. The signs squares are the same; therefore the answer is positive 6. X= 6.
- Last step is to substitute the solution 6 in for X in the original problem. Original problem is -X/3 = -2. On the right side, write -2. On the left side, write – (6) instead of X. A – (6) turns into a -6. Draw a line under the- 6 and write 3 under the line. What is -6 divided by 3? 2. The signs are different; therefore the answer is negative -2. We have -2 on the left and right sides. The numbers are the same. Are both sides equal? Yes. The answer is correct.

Repeat the same interactive process with the remaining problems

Independent Practice #5

- Direct the students to the "Independent Practice" section. Read the first problem together and direct students to complete the problems without you.

When students finish problems, provide immediate corrective feedback for errors.

Graphing

Abstract for one-step equations

Model 1: X + 8 = -2

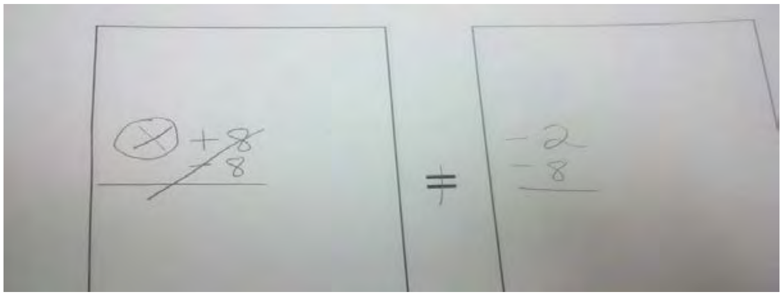
- Next, I take out my workmat or dry erase board.
- In the abstract method, I am going to use symbols and numbers. For positive numbers, I am going to use a "+" plus sign or no sign at all. It depends on where the number or symbol is in the problem. For negative numbers, I am going to use "-"minus signs.
- In this problem, I am going to write out the problem like I see it. On the left, I am going to write X + 8. On the right side, I am going to write -2.

    - The first step in SUMLOWS is to separate the two sides. Draw a line through the equal sign.

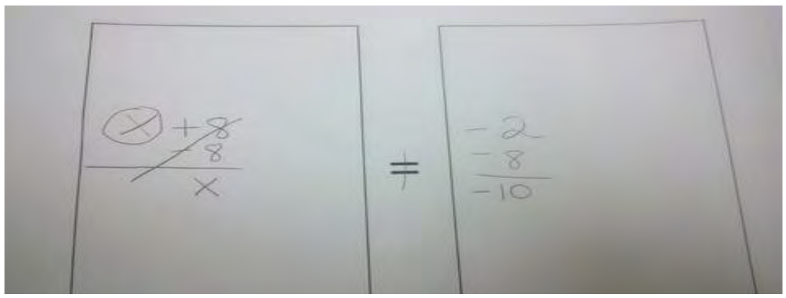
    - The "U is for unite like terms. Look on the left side, are there any like terms or numbers that not already together? No. Look on the right side. Are there any like terms that are not together? No. We go to the next step.
    - Modify the new equation. There is not anything to unite or combine, so skip this step too.
    - The next step is Loop the variable. The variable is the X. This is the number that should be by itself and want to move last. So what is on the side with the variable or X? +8. We have to get rid of +8 because X should be by itself.

    - In order to move the +8, we have write -8 under the +8. Opposites cancel each other out. Put a slash mark through -8 and 8. This equals 0 and cancels each other out.

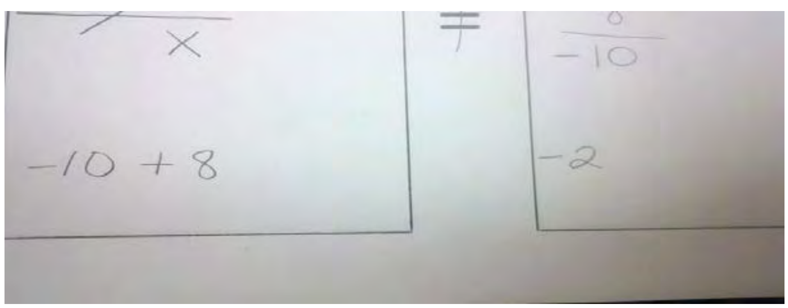
    - The next step is "what we do to one side, we must do to the other" We subtracted 8 on the left side, so we must subtract 8 on the right side. Write -8 under the negative 2.

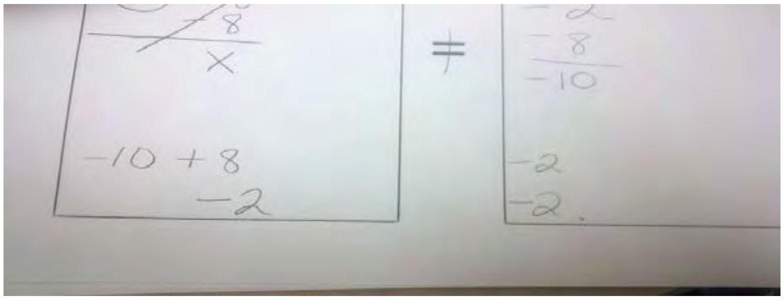


- Combine like terms. On the right side, we have -2 and -8. Since both numbers are both negative, we must combine them by adding them together. Two plus eight is 10. Since they are both negative. The answer is negative. So X = -10
- Is the X by itself? Yes. Is the X positive? Yes. Now, we do the last step.



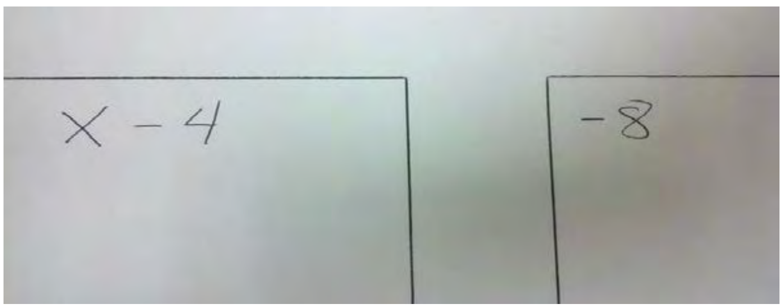
- Substitute the solution for the variable and check. Go up to the original problem X + 8=-2. Instead of writing an X, we are going to write -10. Also, we are going to write +8 on the left side. On the right side, we are going to write -2. Combine the tiles on the left side. Remember, if the numbers have the same sign add. If the signs are different, we are going to subtract. Then, we take the sign of the larger number. Negative 10 and positive 8 have different signs, so we should subtract. -10 plus 8 = 2. The larger number is negative; therefore the answer is negative 2. On the left side, we have -2. On the right side, we have -2. They are equal. The answer is correct.



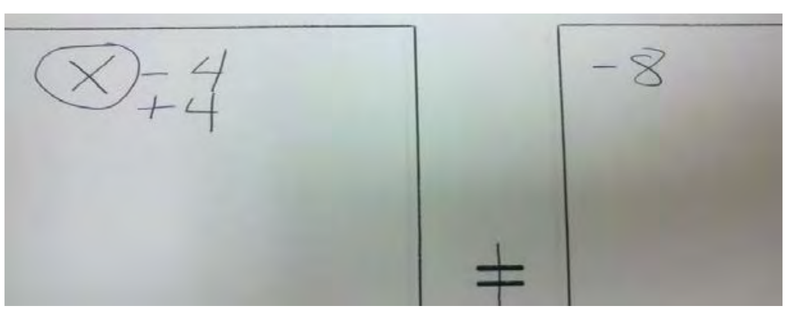


Model problem #2

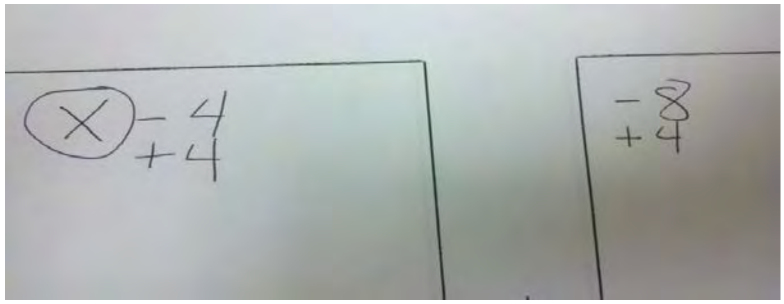
- I read the next problem. X -4 = -8
- On the left side of the mat, we are going write X – 4. On the right side, write -8.



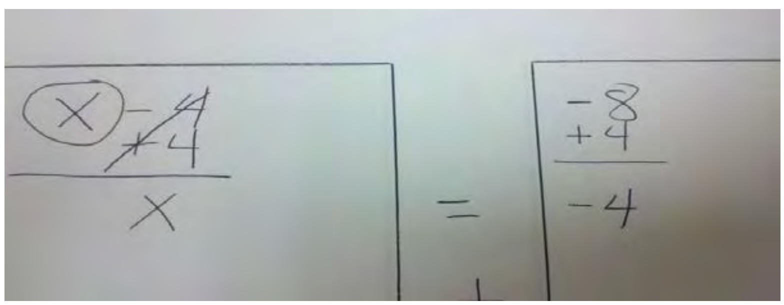
  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any like terms or numbers that can be combined that are not already together on the left side? NO. Look on the right side. Are there any like terms or numbers that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
  - Loop around the variable. Draw a circle around the coefficient or X. That is the number that should be by itself.
  - Is there anything else on the side with the circle? Yes! There is a -4. We must move the -4.
  - Next step is opposite sign. In order to move-4, we must do the opposite.   The opposite of “minus 4” is “plus 4.” So, we must write +4 under the -4 on the left side.



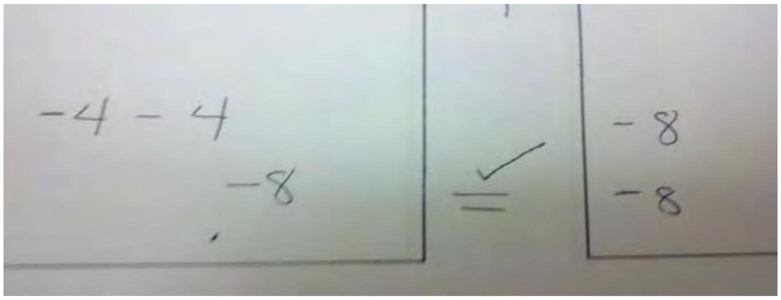
- What we do to one side, we must do to the other. Because we wrote +4 on the left side, we write +4 on the right side of the equal sign under the -8. On the left side, the numbers -4 and +4 cancel each other out (-4 +4 = 0). Draw a line through it.



- On the right side, we have -8 and +4. Remember if the signs are the same we add. If the signs are different, we will add and take the sign of the larger (absolute value) number. These signs are different, so we are going to subtract the number and take the sign of the largest number (absolute value). 8 minus 4 is 4. Eight is larger than 4, so the answer is -4.
- On the left side of the equal sign, we have X. On the right side, we have -4. Now we have X = -4

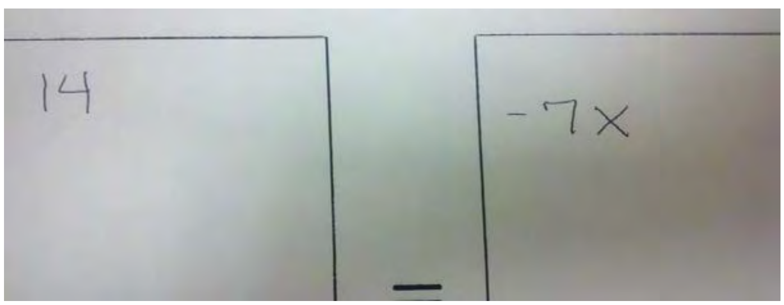


- Is the X by itself? Yes. Is it positive? Yes. Now we do the last step.
- Substitute the solution for the variable and check. Go up to the original problem X -4 = -8. Instead of writing X, write -4 (X = 4) on the left side. Also on the left side, write -4. On the right side, write -8. We are going to combine the tiles on the left side. Remember if the numbers have the same sign, we are going to add. If the signs are different we are going to subtract. Lastly, take the sign of the largest number (absolute value). Since both signs are negative, we are going to add. Four plus four is 8. The biggest number is negative, so the answer is negative. On the left side, we have -8. We have -8 on the right side. These numbers are the same. The answer is correct.

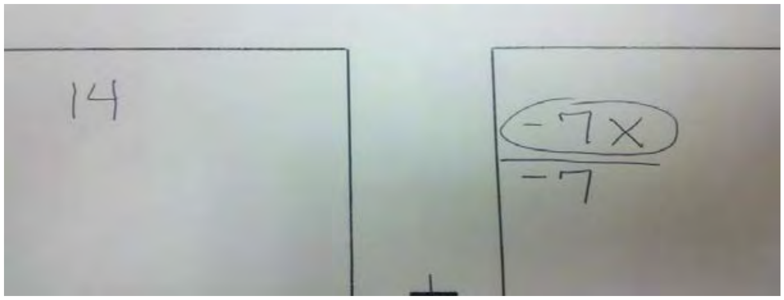


Model problem #3

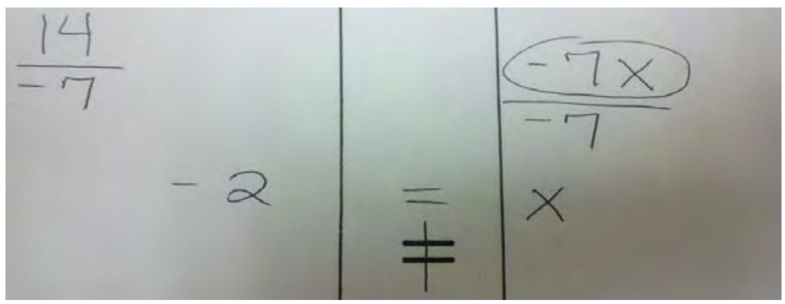
- I read the next problem. 14 = -7X
- On the left side of the mat, I am going to write 14. On the right side of the mat, I am going to write 7X. This particular problem looks a little funny because we usually like the variable or the X on the left side. We can work the problem the same way or we can turn it around.   If we decide to turn it around, we make sure we take the negative signs with the number. For this model, we will work the problem out like it is.



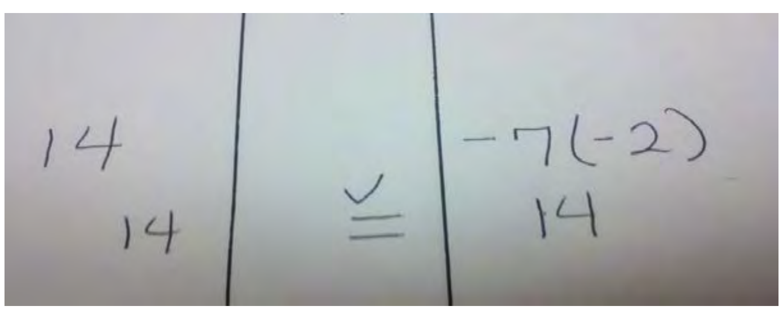
  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any like terms or numbers that are not already together? NO. Look on the right side. Are there any like terms or numbers that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
  - Loop around the variable. Draw a circle around- 7X.
  - Is there anything else on the side with the circle? No! So, we have to see what 1X is equal to?
  - Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a line under the -7 (drawing a line means to divide) and write -7 under the line.



- What we do to one side, we must do to the other. On the left side, draw a line under 14. Write -7 under the line. This means to divide.
- The right side cancels each other out. On the left side, we are going to divide. What is 14 divided by 7? 2. In multiplication and division, if the signs are the same, the number is positive. If the numbers are different, it is negative. Therefore, the solution is -2. X = -2. Is the X by itself? Yes. Is the X positive? Yes. We go to the last step.



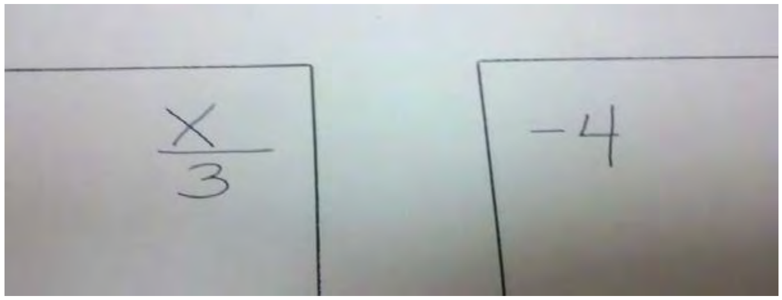
- Last step is to substitute the solution -2 in for X in the original problem. Original problem is 14 = -7X. On the left side, write 14. On the right side, write-7. Then write -2 in parenthesis in place of X. A number beside a letter mean multiplication. What is 7 times 2 equals? 14. Are the signs same or different? They are the same. Therefore, the answer is positive. On the right side, we have 14 and on the right side we have 14. Are both sides equal? Yes. The answer is correct.



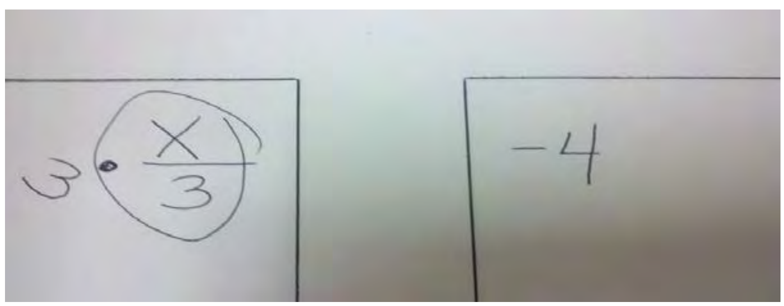
Model problem # 4

- I read the problem: $\frac{X}{3} = -4$
- make sure the algebra mat is clear

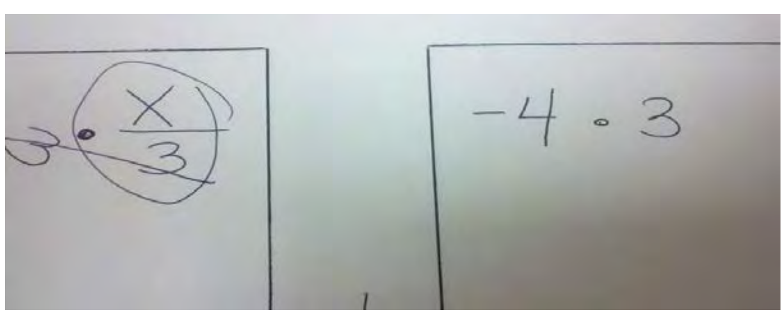
- On the left side of the equal sign, write X over 3 (X/3). On the right side, write -4.

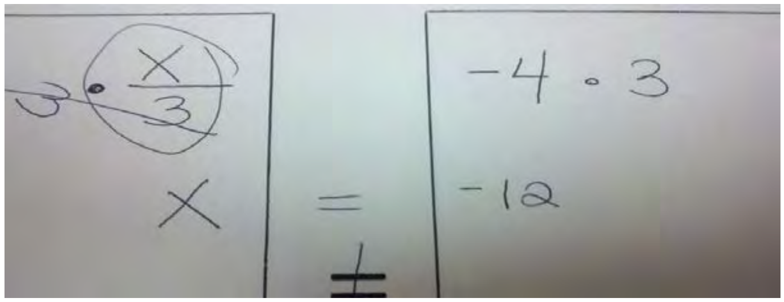


  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any shapes that are the same that are not already together? NO. Look on the right side. Are there any shapes that are the same that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So we will skip this step too.
  - Loop around the variable. Draw a circle around the variable. The variable is the X.
  - Is there anything else on the side with the circle? Yes. The three is on the side with the X.
  - Next step is opposite sign. A number under a line means to divide. The opposite of division is multiplication. Draw a multiplication sign beside the 3 and write 3.

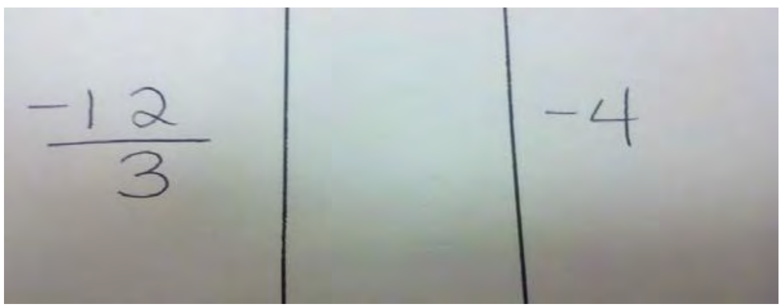


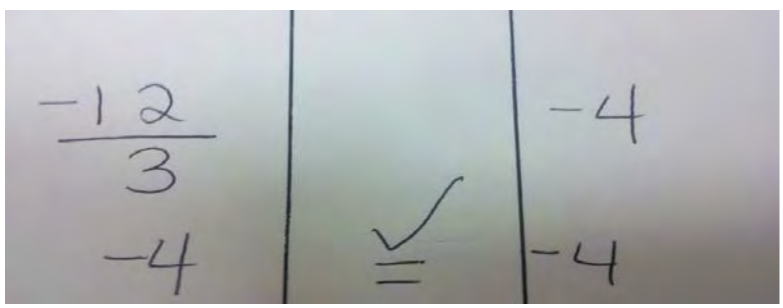
  - What we do to one side, we must do to the other. Since we drew a multiplication sign and wrote 3 on the left, we must draw a multiplication sign and write 3 on the right. The left side cancels each other out. On the right side, multiply-4 times 3? 12. The signs squares are different; therefore the answer is negative 12. X= -12.





- Last step is to substitute the solution -12 in for X in the original problem. Original problem is X/3 = -4. On the right side, write 12. On the left side, write -12 instead of X. Draw a line under the -12 and write 3under the line. What is 12 divided by 3? 4. The signs are different; therefore the answer is negative -4. We have -4 on the left and right sides. The numbers are the same. Are both sides equal? Yes. The answer is correct.





Learning Sheet 7

One-Step
Abstract
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) $X+8 = -2$

2) $X -4 = -8$

3) $14 = -7X$

4) $\frac{X}{3} = -4$

Guided Practice

1) $X - 8 = -6$

2) $-5 = X+3$

3) $-X = 2$

4) $-3X = 15$

5) $-\frac{X}{3} = -2$

Independent Practice

$X - 2 = -5$

$\frac{X}{2} = -4$

$X + 8 = -5$

$-16= -4X$

$\frac{X}{3} =- 4$

$-10 = X - 5$

$- 2X = -8$

$-X= 4$

Learning Sheet 8

One-Step
Abstract
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) $X + 3 = -7$

2) $\frac{X}{5} = -2$

Guided Practice

1) $X - 4 = 10$

2) $X + 2 = -6$

3) $-X = 6$

4) $-2X = 8$

5) $\frac{X}{2} = -8$

6) $-5X = 15$

Independent Practice

$X - 6 = -7$

$\frac{X}{2} = -3$

$X + 4 = -6$

$-4X = -16$

$\frac{X}{2} = 10$

$-9 = X - 5$

$-6X = -24$

$-X = 4$

Learning Sheet 9

One-Step
Abstract
Code Number ________________ Date _____________
SUMLOWS

Describe and Model

1) $X + 1 = -6$

2) $-3X = 9$

Guided Practice

1) $X - 6 = 9$

2) $-X = 2$

Independent Practice

$X - 4 = -7$

$\frac{X}{3} = -4$

$X + 6 = -5$

$-3X = -15$

$\frac{X}{4} = 5$

$-8 = X - 4$

$-7X = -14$

$-X = 6$

Lessons 1-3
Two-step

Concrete Method

Materials

- chalkboard/whiteboard/easel
- algebra tiles to be used for concrete objects
- learning sheet 1
- progress chart

Advance Organizer

- Tell students what they will be doing and why
- Remind students about the commitment they made to learn to solve equations. We will work hard to teach them and they will work hard to learn a new way. They will learn to solve two step equations and the algebra tiles will help.

Demonstrate

- Give students SUMLOWS mnemonic and learning sheets. Wait to pass out the manipulative so that students do not become distracted.
- Begin with the first problem in the model section. Tell the student that we will show them how to solve the problem and that they will have a chance to solve problems also. State the expectations for behavior and attention to the demonstration.
- Begin with the first problem and think out aloud (see problem **model #1**)
- I read the problem.
- This problem is 3X + 1 =13. First, I take out my workmat or dry erase board and my mnemonic index card.
- Next, I set out the algebra tiles. On the left side, I place 3 green rectangle tiles (that means I have 3X). Then we will put down 1 cream square (red means we have a negative number and the cream side means that we have a positive number). Then on the right side, we will place 13 cream square tiles down (the 13 is positive that is why the tiles are on the cream side).
    - The first step In the SUMLOWS is to separate the two sides. Draw a line through the equal sign.
    - The "U" is for unite like terms. Look on the left side. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
    - Modify the new equation. There is not anything to unite or combine. So, skip this step too.
    - The next step is Loop the variable. The variable is the rectangular tile. This is the number that we want to get by itself and want to move last. So what is on the side with the variable (the rectangle)? The1 cream square. We have to get rid of the 1 cream square because the rectangle should be by itself.
    - In order to move the 1 cream square, we have to do the next step which is the opposite sign. We have 1 cream square. To get rid of the 1 cream squares, we must add 1 red squares. Opposites cancel each other out.

- Next step is what we do to one side we must do to the other. We added 1 square to the left. We must add 1 red square to the right. On the left side, each cream square cancel out each red square (move them off the board or mat). On the right side, we have 13 cream squares and 1 red square. These are the same shape but different color; therefore we must subtract or pair a cream tile with a red tile. Pair the red tile with the cream tile until there are no more pairs to make. When we pair them up, move them from the board. How many tiles are left on the right side? We have 12 cream tiles.
- Now we have 3 X = 12. Is the X by itself? No. The X has a 3 with it. We must move the 3 from the X because the X should be by itself.
- I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The "U" is for unite like terms. Look on the left side. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the X. That is the number that should be by itself.
- Is there anything else on the side with the circle? No. So, we have to see what 1 X is equal to?
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a line under 3 on the left side.
- What we do to one side, we must do to the other. Draw a line under 12 and write 3 under it. The left side cancels each other out.
- On the right side, what is 12 divided by 3? 4. Are the signs the same or different? They are the same. Therefore, the answer is positive X = 4. Is the variable by itself? Yes. Is the X positive? Yes. Go to the last step.
- Last step is to substitute the solution 4 in for X in the original problem. Original problem is 3X + 1 = 13. On the left side, write 3 and substitute 4 for X. Write 4 with parenthesis around it. On the right, write 13. On the left, what is 3 times 4? 12. Is it positive or negative? Positive because the signs are the same. On the left side, we have 12. On the right side, we have 12. They are the same. The answer is correct.

Model problem #2

- I read the next problem. 3X +5X = -16
- I make sure my mat or board is clear
- On the mat, place 3 green rectangular tile (green is positive and red is negative) on the left side. Also place 5 green rectangular tiles. On the right side, place 16 red square tiles (cream is positive and red is negative).
  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any tiles the same shape that are not already together? Yes. We must combine the 3 green rectangular tiles and the 5 green rectangular tiles.

Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.

- Modify the new equation. Since we combined the left side, our new equation is 8X = 16. We should have 8 green rectangular tiles together on the left side and 16 red square tiles on the right side.
- Loop around the variable. Draw a circle around the 8 green rectangular tiles. That is the number that should be by itself.
- Is there anything else on the side with the circle? No. So, we have to see what 1 X is equal to?
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a line under 8 on the left side.
- What we do to one side, we must do to the other. Draw a line under -16 and 8 write under it. The left side cancels each other out.
- On the right side, what is 16 divided by 8? 2. Are the signs the same or different? They are the different. Therefore, the answer is negative X =-2. Is the variable by itself? Yes. Is the X positive? Yes. Go to the last step.
- Last step is to substitute the solution -2 in for X in the original problem. Original problem is 3X + 5X = -16. On the left side, write 3 and substitute -2 for X. Write -2 with parenthesis around it. Also write 5 and substitute -2 for X. Write -2 with parenthesis around it. On the right, write 16. On the left, what is 3 times 2? 6. Is it positive or negative? Negative because the signs are the same. What is 5 times 2? 10. Is it positive and negative? Negative. On the left side, we have -6 plus -10. These signs are the same; therefore we add them and take the sign of the larger number which is -16. On the right side, we have -16. They are the same. The answer is correct.

Model problem #3

- I read the next problem. 3(X + 2) =-3
- I make sure my mat or board is clear
- In the problem, notice the parenthesis ( ). This problem requires us to get rid of the parenthesis before we start SUMLOWS.
- What is in the parenthesis? X + 2. What is outside the parenthesis? 3. On the mat, we are going to place tiles that represent X + 2. We are going to place a 1 green rectangular tile and 2 cream square tiles. The number outside the parenthesis tells us how many times we need to put X + 2 on the mat. In this case, we put X + 2 on the mat 3 times.
- How many rectangles are? 3 green rectangles
- How many squares are? 6.
- Now we are ready to solve the equation using SUMLOWS
- On the mat, we should have 3 green rectangles (green is positive and red is negative) and 6 cream squares on the left side (cream is positive and red is negative). On the right side, we have 3 red squares.
    - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
    - Unite like terms. Are there any tiles the same that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.

- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- Loop around the variable. Draw a circle around the 3 green rectangular tiles.
- Is there anything else on the side with the circle? Yes, there are 6 cream squares. We have to get rid of the 6 cream squares because the rectangle should be by itself.
- In order to move the 6 cream squares, we have to do the next step which is the opposite sign. To get rid of the 6 cream squares, we must add 6 red squares. Opposites cancel each other out (+6-6 =0).
- Next step is what we do to one side we must do to the other. We added 6 red squares to the left. We must add 6 red squares to the right. On the left side, each cream square cancel out each red square (move them off the board or mat). On the right side, we have 3 red squares and 6 red squares. These are the same shape and the same color; therefore we must add or combine them. How many tiles are left on the right side? We have 9 red square tiles.
- Now we have 3X = -9. Is the X by itself? No. The X has a 3 with it. We must move the 3 from the X because the X should be by itself. We must find out what 1 X is equal to.
- I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The "U" is for unite like terms. Look on the left side. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the X. That is the number that should be by itself.
- Is there anything else on the side with the circle? No. So, we have to see what 1 X is equal to?
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division sign beside the 3 green rectangles on the left side and put 3 more green tiles.
- What we do to one side, we must do to the other. Draw a division sign beside the 9 red squares and put 3 green rectangular tiles beside it. The left side cancels each other out.
- On the right side, separate the 3 green rectangle tiles. Give each rectangle tile a red square until they are gone. One green rectangle is equal to how many red squares? 3. In multiplication and division, if the tiles are different colors, the answer is negative. If the tiles are the same color, the sign is positive. Therefore, x = -3
- Last step is to substitute the solution -3 in for X in the original problem. Original problem is 3(X + 2) = -3. On the right side, put 3 red squares. On the left side put 3 red squares and 2 cream squares. Because the number outside the parenthesis tell us how many time we need to put it down, we put 3 red squares and 2 cream squares three times. Before we combine like terms, we should have 9 red squares and 6 cream squares. They are different colors; therefore we have to subtract or pair them up and move them off the mat. What are left on the left side? 3 red squares. What

color are on the right? 3 red squares. They are the same. Are both sides equal? Yes. The answer is correct.

Model problem # 4

- I read the problem: $5 = \frac{X}{2} + 10$.
- The student can also flip the question so that it is $\frac{X}{2} + 10 = 5$ (if we decide to flip the equation, the steps will be opposite of the step written below).
- I make sure the algebra mat is clear
- On the left side of the mat, place 5 green square tiles (green is positive and red is negative). On the right side, place a line and put 2 green rectangular tiles under the line. On top of the line draw an X. Also on the right side, put 10 cream tiles down (cream is positive and red is negative).
  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any tiles the same shape that are not already together? No. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
  - Modify the new equation. Since we did not combined any like terms, we skip this step.
  - Loop around the variable. Draw a circle around the 2 green rectangular tiles. That is the number that should be by itself.
  - Is there anything else on the side with the circle? Yes! There are 10 cream squares. We have to get rid of the 10 cream squares.
  - Next step is opposite sign. The opposite of positive (plus) 10 is negative (minus) 10. We are going to add 10 red squares under the 10 cream squares.
  - What we do to one side, we must do to the other. Since we put 10 red squares on the right side of the equation, we must place 10 red squares on the left side of the equation. The right side squares cancels each other out. On the left side, we have 5 cream squares and 10 red squares. We are going to add like terms. Take one cream and one red square and move them off the mat until there are no more pairs to make. On the left side, we should have 5 red squares left.
  - On the algebra mat, we should have 5 red squares on the left side. On the right side, we should have X over 2 green rectangles. The rectangles should have a circle around it.
  - We have already separated the sides, united like terms, modified the equation, and looped the coefficient. The next step is opposite sign.
  - When we have an X over a number, which means to divide. What is the opposite of division? It is multiplication. We are going to write a multiplication sign and place 2 green rectangles beside it.
  - Next step is what we do to one side; we must do to the other. Since we drew a multiplication sign and placed two green rectangles on the right side, we must do the same on the left side. On the left side we are going to draw a multiplication sign and place 2 green rectangles.

- We are going to give all 5 red squares to the first green rectangle. We are going to add 5 red squares for the 2nd rectangle. How many squares are there? We have 10 red squares. So, X = -10.
- Last step is to substitute the solution -10 in for X in the original problem. Original problem is 5= X/2 + 10. On the left side, put 5 green squares. On the right side, put 10 red squares for X. Draw a line under the squares and place 2 green rectangles under the line. Separate the rectangles and give each rectangle a square until they are all gone. How many squares does each rectangle have? 5 red squares. Move both rectangles and 5 red squares off the mat (because -10 divided 2 is -5. On the mat, we should have 5 cream squares on the left side. On the right side, we should have 5 red squares and 10 cream squares. On the right side, combine the squares. Match up a cream square with a red square until there are not more pairs and move them off the mat. How many squares are on the right side? 5 cream squares. How many squares are on the left side? 5 cream squares. The numbers are the same. Are both sides equal? Yes. The answer is correct.

Guided Practice

- Direct students to the "Guide" section of the learning sheet
- Tell students to touch the first problem and that we will do this problem together, using numbers, letters and the SUMLOWS mnemonic.
- Let's read the problem. This problem is -2X + 6 = 10.
- On our workmat, what do place on the left side? 2 red rectangles and 6 cream squares
- What do we write on the right side? 10 cream squares
  - What is the first step in SUMLOWS? Separate the sides. We are going to draw a line through the equal sign.
  - What does the "U" stand for in SUMLOWS? Unite like terms. Are there any shapes on the same side that are not already together? NO. Look on the right side. Are there any shapes on the same side that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So, skip this step too.
  - What is the next step? Loop around the variable. What letter is the variable? X. Draw a circle around the two red rectangles. That is the number that should be by itself and move last.
  - Is there anything else on the side with the circle? Yes! 6 cream squares. We must move the 6 cream squares.
  - What is the next step? Next step is opposite sign. In order to move the 6 cream squares, we must do the opposite. The opposite of "plus 6" is "minus 6." So, we place 6 red squares under the 6 cream squares.
  - What is the next step? What we do to one side, we must do to the other. Because we placed 6 red squares under the 6 cream squares on the left side, we must place 6 red square tiles under the 10 cream tiles on the right side of the equal sign. The 6 cream tiles and the 6 red tiles on the left side cancel each other out (6 -6 = 0). Pair them up and move them off the mat. On the right side, we have 10 cream tiles and 6 red tiles. They are both the same shape; therefore we have to combine like terms.

- Combine like terms. On the right side, we have 10 cream tiles and 6 red tiles. Since they are different colors, we are going to subtract or pair them up and move them off the mat. How many are left on the right side that doesn't have a pair? 4. What color are the square tiles? Since they are cream, the answer is positive 4.
- On the left side of the equal sign, we have 2 red rectangular tiles. On the right side, we have 4 cream tiles. Now we have -2 X = 4. Is there only 1 rectangle? No., there are two rectangles; therefore we must determine what 1 X is equal to. I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The "U" is for unite like terms. Look on the left side. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the X. That is the number that should be by itself.
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division sign and place 2 red rectangular tiles by the other two rectangular tiles.
- What we do to one side, we must do to the other. On the right side, draw a division sign beside the 4 cream squares and place 2 rectangular tiles. The left side cancels each other out. On the right side, separate the 2 red rectangles. Give each red rectangle a cream square until they are gone. One green rectangle is equal to how many cream squares? 2. The colors are different, so the number is negative. Therefore, x = -2
- Last step is to substitute the solution 3 in for X in the original problem. Original problem is -2X +6 + 10. On the left side put 2 red squares for each X. There are 2 Xs, so put 4 cream squares on the left (the squares are cream because both numbers are negative), as well as 6 cream squares on the left. On the left, we have 10 cream squares. On the right, put 10 cream squares. Are both sides equal? Yes. The answer is correct.

Guided Practice #2

- Locate problem # 2 in the Guided practice section.
- Let's read the problem. This problem is 4X – 2X = 8
- On our workmat, what do we place on the left side? 4 green rectangular tiles and 2 red rectangular tiles.
- What do we place on the right side? 8 cream square tiles
    - What is the first step in SUMLOWS? Separate the sides. We are going to draw a line through the equal sign.
    - What does the "U" stand for in SUMLOWS? Unite like terms. Are there any shapes that are the same that are not already together on the left side of the equal sign? Yes. Look on the right side. Are there any like shapes that are the same that are not already together on the right side of the equal sign? No. Because there are shapes

that are not together on the left side of the equal sign, we must combine like terms. We have 4 green rectangular tiles and 2 red rectangular tiles. These tiles are the same shape and should be combined. If they are the same color, add them. If they are different colors, subtract or pair them up and move them off the mat. Since they are different colors, we are going to subtract or pair them up. One green rectangular tile is matched with one red rectangular tile until there are no more pairs.

- Modify the new equation. How many rectangular tiles are on the mat? 2 green rectangular tiles.
- What is the next step? Loop around the variable. Which shape is the variable? The 2 green rectangular tiles represent the variable. Draw a circle around the rectangular tiles. That is the number that should be by itself and we move last.
- Is there anything else on the side with the circle? No.
- Now we have 2 X =8. Is there only 1 rectangle? No., there are two rectangles; therefore we must determine what 1 rectangle is equal to. I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The "U" is for unite like terms. Look on the left side. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the green rectangular tiles. That is the number that should be by itself.
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division sign and place 2 green rectangular tiles by the other two green rectangular tiles.
- What we do to one side, we must do to the other. On the right side, draw a division sign beside the 8 cream squares and place 2 green rectangular tiles. The left side cancels each other out. On the right side, separate the 2 green rectangular tiles. Give each green rectangular tile a cream square until they are gone. One green rectangle is equal to how many cream squares? 4. The colors mean positive, so the number is positive. Therefore, x =4
- Last step is to substitute the solution 4 in for X in the original problem. Original problem is 4X – 2X = 8. On the left side put 4 green squares for each X. There are 4 Xs, so put 16 cream squares on the left, as well as 8 red squares on the left (the numbers are different, so the number is negative). After combining like terms, we have 8 cream squares on the left side. On the right, put 10 cream squares. Are both sides equal? Yes. The answer is correct.

Guided practice problem #3

- I read the next problem. -2(X + 3) =10
- I make sure my mat or board is clear
- In the problem, notice the parenthesis ( ). This problem requires us to get rid of the parenthesis before we start SUMLOWS.

- What is in the parenthesis? X +3. What is outside the parenthesis? -2. The number outside the parenthesis tells us how many times we need to put X + 3 on the mat. Since this number is negative (-2), we must flip the tiles over, therefore we are putting down –X -2 two times. On the mat, we are going to place tiles that represent -X -3. We are going to place a 1 red rectangular tile and 2 red square tiles. In this case, we should put- X -3 on the mat 2 times.
- How many rectangles are? 2 red rectangles
- How many squares are? 6 red squares.
- Now we are ready to solve the equation using SUMLOWS
- On the mat, we should have 2 red rectangles (green is positive and red is negative) and 6 red squares on the left side (cream is positive and red is negative). On the right side, we have 10 cream squares.
  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any tiles the same that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So, skip this step too.
  - Loop around the variable. Draw a circle around the 2 red rectangular tiles.
  - Is there anything else on the side with the circle? Yes, there are 6 red squares. We have to get rid of the 6 red squares because the rectangle should be by itself.
  - In order to move the 6 red squares, we have to do the next step which is the opposite sign. To get rid of the 6 red squares, we must add 6 cream squares. Opposites cancel each other out (-6+6 =0).
  - Next step is what we do to one side we must do to the other. We added 6 cream squares to the left. We must add 6 cream squares to the right. On the left side, each red square cancel out each cream square (move them off the board or mat). On the right side, we have 10 cream squares and 6 cream squares. These are the same shape and the same color; therefore we must add or combine them. How many tiles are on the right side? We have 16 cream square tiles.
  - Now we have -2X =16. Is the X by itself? No. The X has a -2 with it. We must move the -2 from the X because the X should be by itself. We must find out what 1 X is equal to.
  - I start back at the front of SUMLOWS mnemonic.
  - I have already separated the sides.
  - The "U" is for unite like terms. Look on the left side. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So, skip this step too.
  - What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the X. That is the number that should be by itself.
  - Is there anything else on the side with the circle? No. So, we have to see what 1 X is equal to?

- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division sign beside the 2 red rectangles on the left side and put 2 more red tiles.
- What we do to one side, we must do to the other. Draw a division sign beside the 16 cream squares and put 2 red rectangular tiles beside it. The left side cancels each other out.
- On the right side, separate the 2 red rectangle tiles. Give each rectangle tile a cream square until they are gone. One red rectangle is equal to how many cream squares? 8. In multiplication and division, if the tiles are different colors, the answer is negative. If the tiles are the same color, the sign is positive. Therefore, x = -8
- Last step is to substitute the solution -8 in for X in the original problem. Original problem is -2(X + 3) =10. On the right side, put 10 cream squares. On the left side put 8 cream squares and 3 red squares. Because the number outside is negative, we must flip the squares to the opposite side. The number outside the parenthesis tell us how many time we need to put it down, we put 8 cream squares and 3 red squares two times. Before we combine like terms, we should have 16 cream squares and 6 red squares. They are different colors; therefore we have to subtract or pair them up and move them off the mat. What are left on the left side? 10 cream squares. What are on the right? 10 cream squares. They are the same. Are both sides equal? Yes. The answer is correct.

Guided Practice problem # 4

- I read the problem: $12 = \frac{X}{2} + 10$
- If the students flip the equation, the steps would be the opposite side.
- I make sure the algebra mat is clear
- On the left side of the mat, place 12 green square tiles (green is positive and red is negative). On the right side, draw an X with a line under it and put 2 green rectangular tiles under the line. On top of the line draw an X. Also on the right side, put 10 cream tiles down (cream is positive and red is negative).
  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any tiles the same shape that are not already together? No. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
  - Modify the new equation. Since we did not combined any like terms, we skip this step.
  - Loop around the variable. Draw a circle around the 2 green rectangular tiles. That is the number that should be by itself.
  - Is there anything else on the side with the circle? Yes! There are 10 cream squares. We have to get rid of the 10 cream squares.
  - Next step is opposite sign. The opposite of positive (plus) 10 is negative (minus) 10. We are going to add 10 red squares under the 10 cream squares.
  - What we do to one side, we must do to the other. Since we put 10 red squares on the right side of the equation, we must place 10 red squares on the left side of the equation. The right side squares cancels each other out. On the left side, we have 12

cream squares and 10 red squares. We are going to add like terms. Take one cream and one red square and move them off the mat until there are no more pairs to make. On the left side, we should have 2 cream squares left.

- On the algebra mat, we should have 2 cream squares on the left side. On the right side, we should have X over 2 green rectangles. The rectangles should have a circle around it.
- We have already separated the sides, united like terms, modified the equation, and looped the coefficient. The next step is opposite sign.
- When we have an X over a number, which means to divide. What is the opposite of division? It is multiplication. We are going to write a multiplication sign and place 2 green rectangles beside it.
- Next step is what we do to one side; we must do to the other. Since we drew a multiplication sign and placed two green rectangles on the right side, we must do the same on the left side. On the left side we are going to draw a multiplication sign and place 2 green rectangles.
- We are going to give both 2 cream squares to the first green rectangle. We are going to add 2 cream squares for the 2^{nd} rectangle. How many squares are there? We have 4 cream squares. So, X = 4.
- Last step is to substitute the solution 4 in for X in the original problem.   Original problem is 12= X/2 + 10. On the left side, put 12 cream squares. On the right side, put 4 cream squares for X. Draw a line under the squares and place 2 green rectangles under the line. Separate the rectangles and give each rectangle a square until they are all gone. How many squares does each rectangle have? 2cream squares. Move both rectangles and 2 cream squares off the mat (because 4 divided 2 is 2.   On the mat, we should have 12 cream squares on the left side. On the right side, we should have 2 cream squares and 10 cream squares. On the right side, combine the squares. All the squares are cream, so we combine them. How many squares are on the right side? 12 cream squares. How many squares are on the left side? 12 cream squares. The numbers are the same. Are both sides equal? Yes. The answer is correct.

Independent Practice

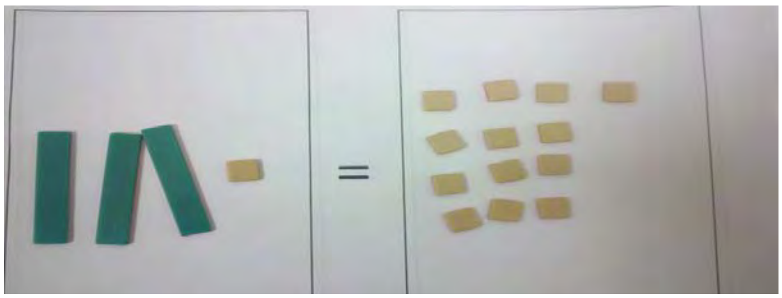
- Direct the students to the "Independent Practice" section. Read the first problem together and direct students to complete the problems without you.
  When students finish problems, provide immediate corrective feedback for errors.

Graphing

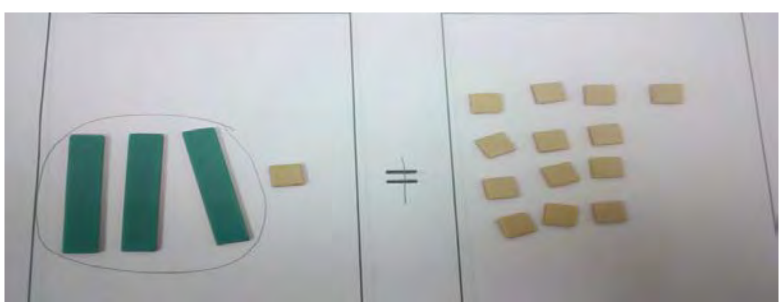
Steps for Concrete Level

Model #1   3X + 1 =13.

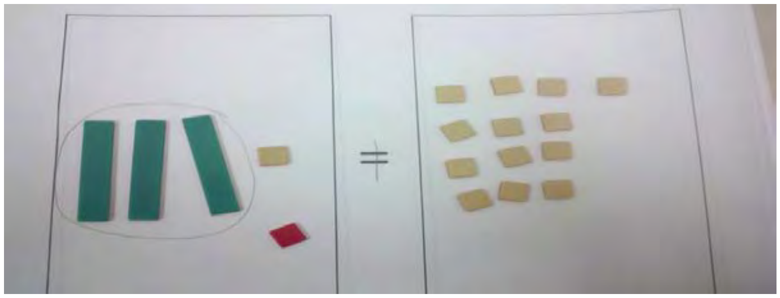
- First, I take out my workmat or dry erase board and my mnemonic index card.
- Next, I set out the algebra tiles. On the left side, I place 3 green rectangle tiles (that means I have 3X). Then we will put down 1 cream square (red means we have a negative number and the cream side means that we have a positive number). Then on the right side, we will place 13 cream square tiles down (the 13 is positive that is why the tiles are on the cream side).



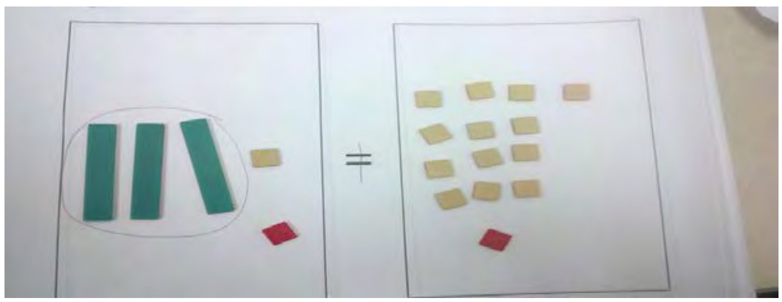
  - The first step In the SUMLOWS is to separate the two sides. Draw a line through the equal sign.
  - The "U" is for unite like terms. Look on the left side. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So, skip this step too.
  - The next step is Loop the variable. The variable is the rectangular tile. This is the number that we want to get by itself and want to move last. So what is on the side with the variable (the rectangle)? The1 cream square. We have to get rid of the 1 cream square because the rectangle should be by itself.



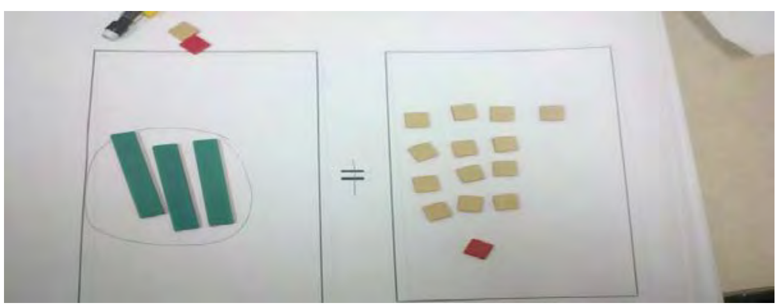
  - In order to move the 1 cream square, we have to do the next step which is the opposite sign. We have 1 cream square. To get rid of the 1 cream squares, we must add 1 red squares. Opposites cancel each other out.



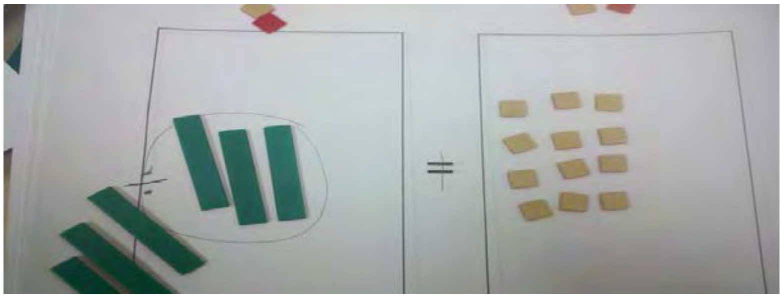
- Next step is what we do to one side we must do to the other. We added 1 square to the left. We must add 1 red square to the right. On the left side, each cream square cancel out each red square (move them off the board or mat).



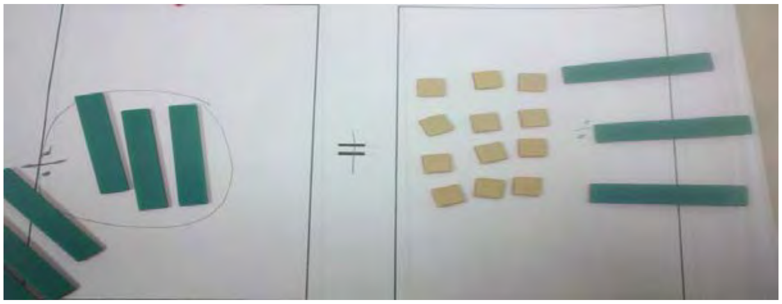
  - On the right side, we have 13 cream squares and 1 red square. These are the same shape but different color; therefore we must subtract or pair a cream tile with a red tile. Pair the red tile with the cream tile until there are no more pairs to make. When we pair them up, move them from the board.



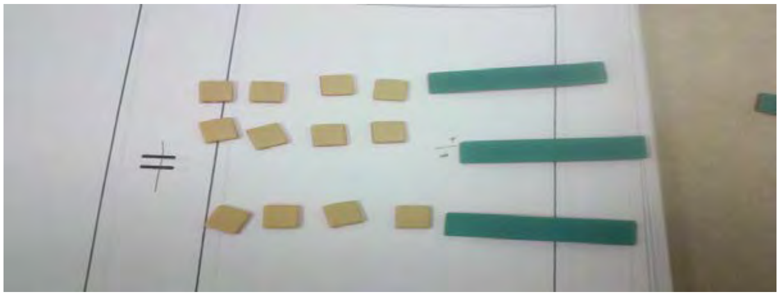
- How many tiles are left on the right side? We have 12 cream tiles.
- Now we have 3 X = 12. Is the X by itself? No. The X has a 3 with it. We must move the 3 from the X because the X should be by itself.
- I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The "U" is for unite like terms. Look on the left side. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the X. That is the number that should be by itself.
- Is there anything else on the side with the circle? No. So, we have to see what 1 X is equal to?
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a line under 3 on the left side.



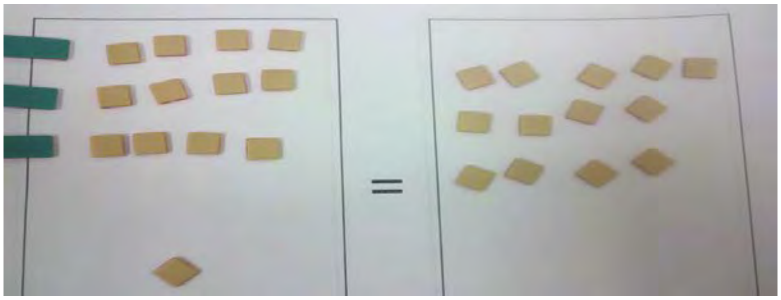
- What we do to one side, we must do to the other. Draw a line under 12 and write 3 under it. The left side cancels each other out.

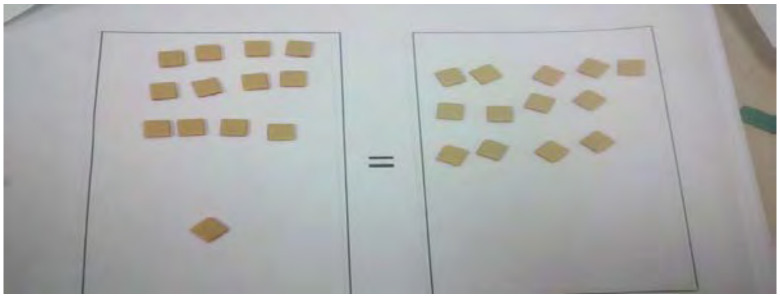


- On the right side, what is 12 divided by 3? 4. Are the signs the same or different? They are the same. Therefore, the answer is positive X = 4. Is the variable by itself? Yes. Is the X positive? Yes. Go to the last step.



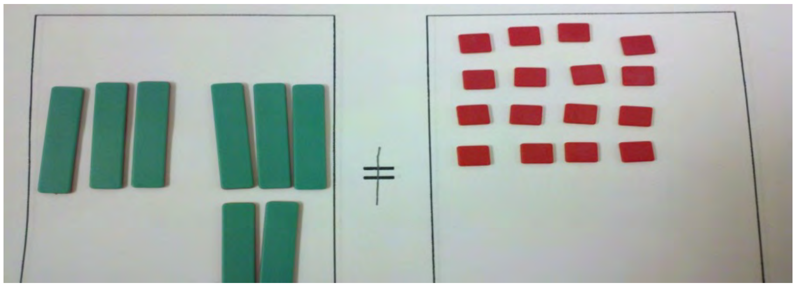
- Last step is to substitute the solution 4 in for X in the original problem.   Original problem is 3X + 1 = 13. On the left side, write 3 and substitute 4 for X. Write 4 with parenthesis around it. On the right, write 13. On the left, what is 3 times 4? 12. Is it positive or negative? Positive because the signs are the same. On the left side, we have 12. On the right side, we have 12. They are the same. The answer is correct.



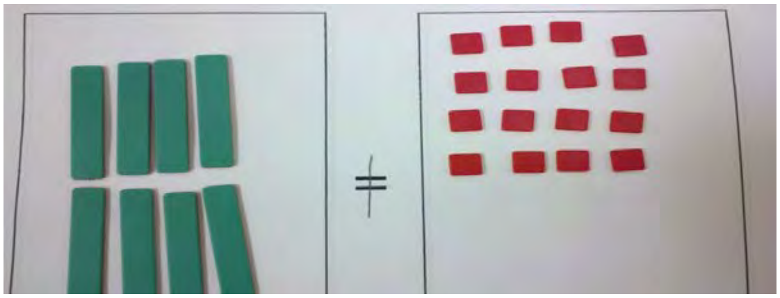


Model problem #2

- I read the next problem. 3X +5X = -16
- I make sure my mat or board is clear
- On the mat, place 3 green rectangular tile (green is positive and red is negative) on the left side. Also place 5 green rectangular tiles. On the right side, place 16 red square tiles (cream is positive and red is negative).
  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.

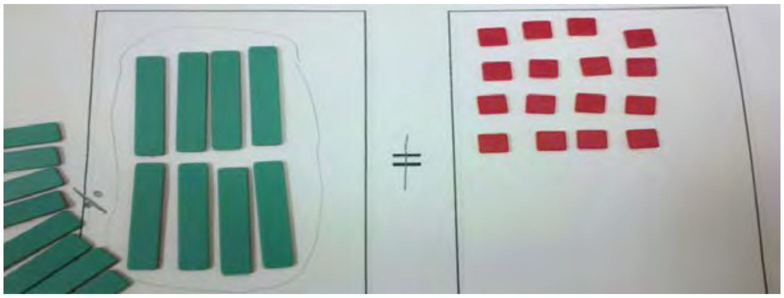


  - Unite like terms. Are there any tiles the same shape that are not already together? Yes. We must combine the 3 green rectangular tiles and the 5 green rectangular tiles. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
  - Modify the new equation. Since we combined the left side, our new equation is 8X = 16. We should have 8 green rectangular tiles together on the left side and 16 red square tiles on the right side.

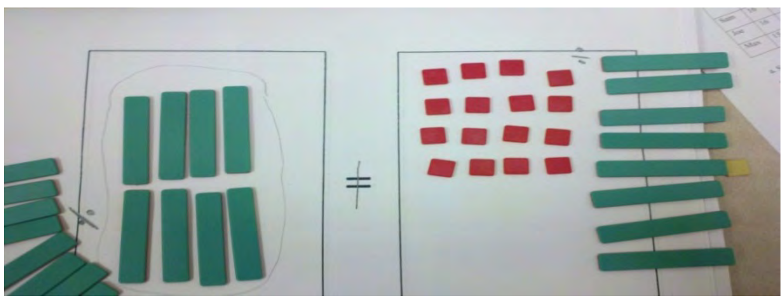


  - Loop around the variable. Draw a circle around the 8 green rectangular tiles. That is the number that should be by itself.

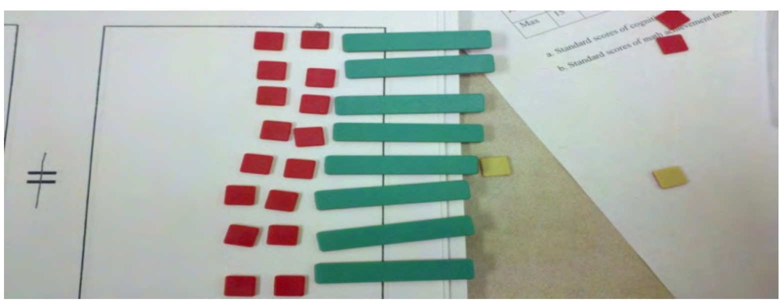
- Is there anything else on the side with the circle? No. So, we have to see what 1 X is equal to?
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a line under 8 on the left side.

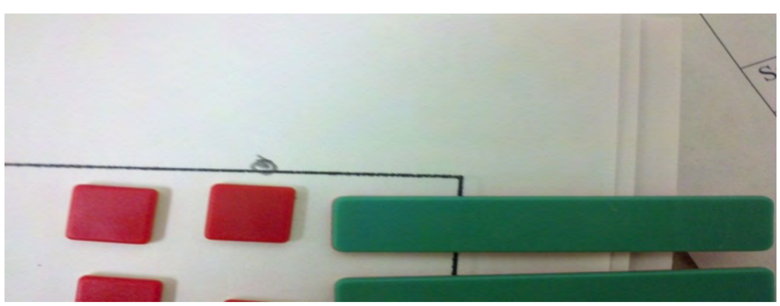


- What we do to one side, we must do to the other. Draw a line under -16 and 8 write under it. The left side cancels each other out.

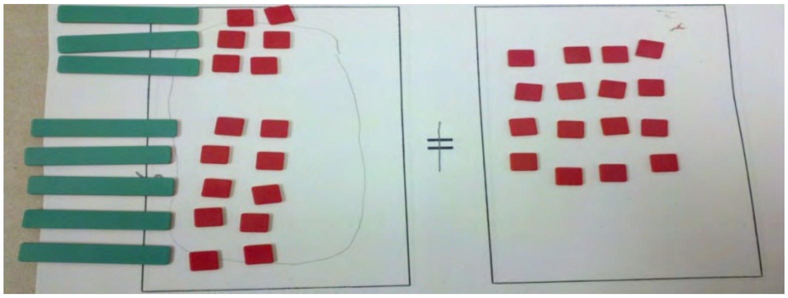


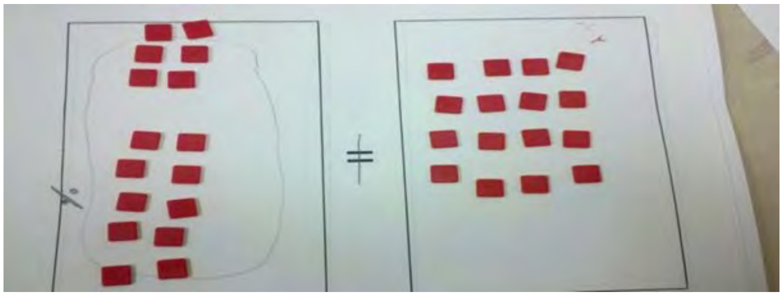
- On the right side, what is 16 divided by 8? 2. Are the signs the same or different? They are the different. Therefore, the answer is negative X =-2. Is the variable by itself? Yes. Is the X positive? Yes. Go to the last step.





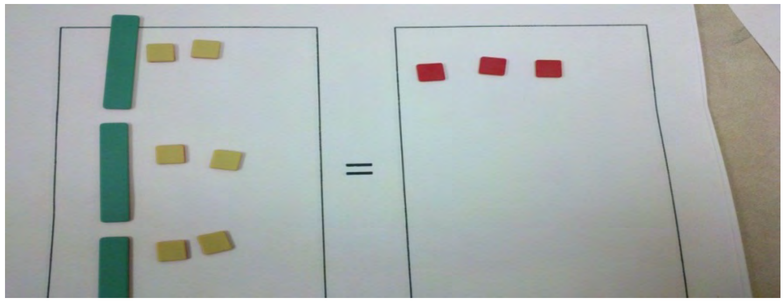
- Last step is to substitute the solution -2 in for X in the original problem. Original problem is 3X + 5X = -16. On the left side, write 3 and substitute -2 for X. Write -2 with parenthesis around it. Also write 5 and substitute -2 for X. Write -2 with parenthesis around it. On the right, write 16. On the left, what is 3 times 2? 6. Is it positive or negative? Negative because the signs are the same. What is 5 times 2? 10. Is it positive and negative? Negative. On the left side, we have -6 plus -10. These signs are the same; therefore we add them and take the sign of the larger number which is -16. On the right side, we have -16. They are the same. The answer is correct.



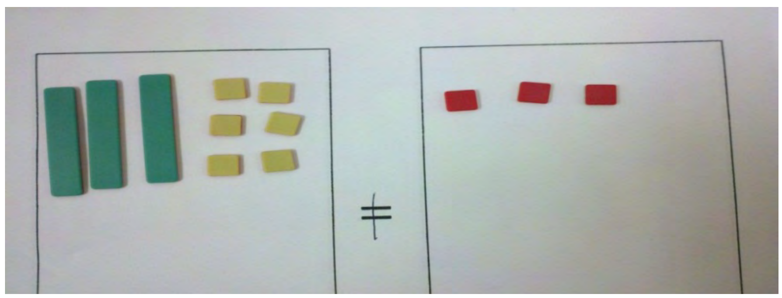


Model problem #3

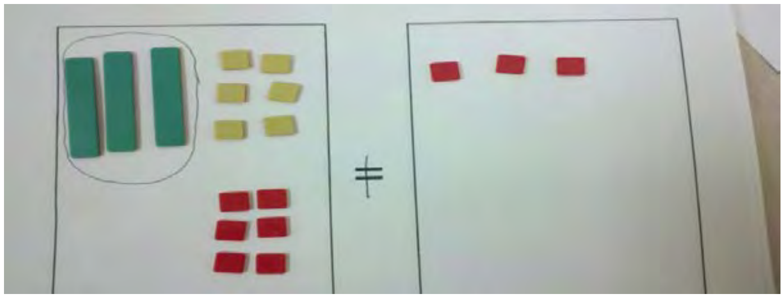
- I read the next problem. 3(X + 2) =-3
- I make sure my mat or board is clear
- In the problem, notice the parenthesis ( ). This problem requires us to get rid of the parenthesis before we start SUMLOWS.
- What is in the parenthesis? X + 2. What is outside the parenthesis? 3. On the mat, we are going to place tiles that represent X + 2. We are going to place a 1 green rectangular tile and 2 cream square tiles. The number outside the parenthesis tells us how many times we need to put X + 2 on the mat. In this case, we should put X + 2 on the mat 3 times.



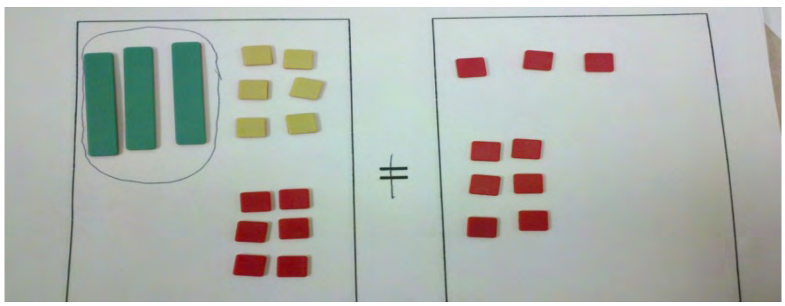
- How many rectangles are? 3 green rectangles
- How many squares are? 6.
- Now we are ready to solve the equation using SUMLOWS
- On the mat, we should have 3 green rectangles (green is positive and red is negative) and 6 cream squares on the left side (cream is positive and red is negative). On the right side, we have 3 red squares.



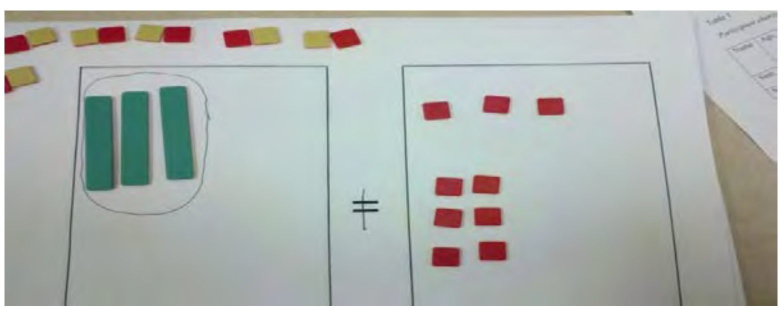
  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any tiles the same that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So, skip this step too.
  - Loop around the variable. Draw a circle around the 3 green rectangular tiles.
  - Is there anything else on the side with the circle? Yes, there are 6 cream squares. We have to get rid of the 6 cream squares because the rectangle should be by itself.
  - In order to move the 6 cream squares, we have to do the next step which is the opposite sign. To get rid of the 6 cream squares, we must add 6 red squares. Opposites cancel each other out (+6-6 =0).



- Next step is what we do to one side we must do to the other. We added 6 red squares to the left. We must add 6 red squares to the right. On the left side, each cream square cancel out each red square (move them off the board or mat). On the right side, we have 3 red squares and 6 red squares. These are the same shape and the same color; therefore we must add or combine them. How many tiles are left on the right side? We have 9 red square tiles.



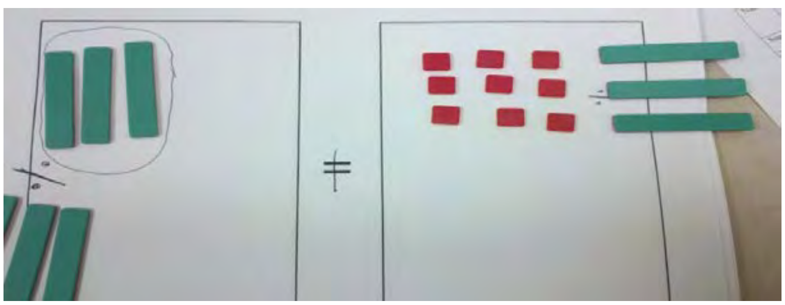
- Now we have 3X = -9. Is the X by itself? No. The X has a 3 with it. We must move the 3 from the X because the X should be by itself. We must find out what 1 X is equal to.



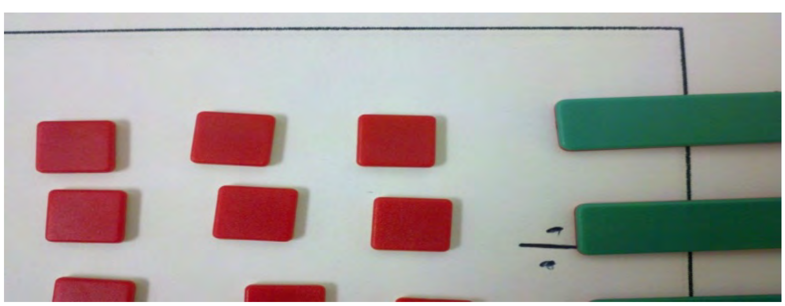
- I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The "U" is for unite like terms. Look on the left side. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.

- What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the X. That is the number that should be by itself.
- Is there anything else on the side with the circle? No. So, we have to see what 1 X is equal to?
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division sign beside the 3 green rectangles on the left side and put 3 more green tiles.

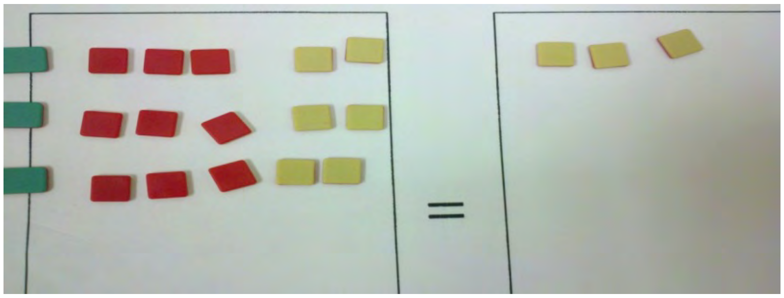
- What we do to one side, we must do to the other. Draw a division sign beside the 9 red squares and put 3 green rectangular tiles beside it. The left side cancels each other out.

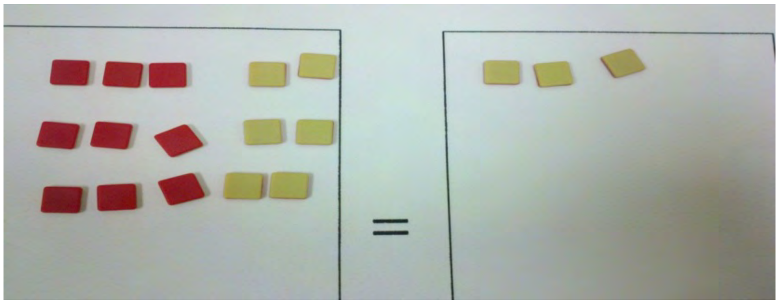


- On the right side, separate the 3 green rectangle tiles. Give each rectangle tile a red square until they are gone. One green rectangle is equal to how many red squares? 3. In multiplication and division, if the tiles are different colors, the answer is negative. If the tiles are the same color, the sign is positive. Therefore, x = -3.



  - Last step is to substitute the solution -3 in for X in the original problem. Original problem is 3(X + 2) = -3. On the right side, put 3 red squares. On the left side put 3 red squares and 2 cream squares. Because the number outside the parenthesis tell us how many time we need to put it down, we put 3 red squares and 2 cream squares three times. Before we combine like terms, we should have 9 red squares and 6 cream squares. They are different colors; therefore we have to subtract or pair them up and move them off the mat. What are left on the left side? 3 red squares. What color are on the right? 3 red squares. They are the same. Are both sides equal? Yes. The answer is correct.

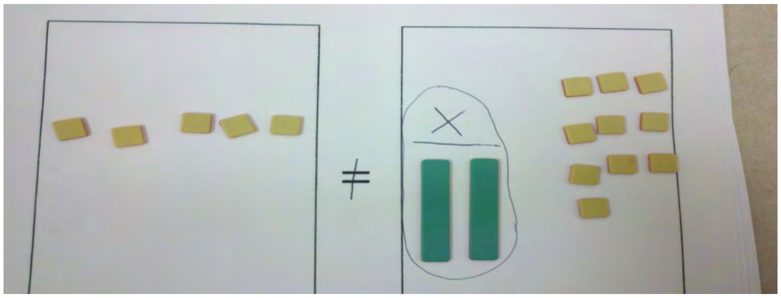




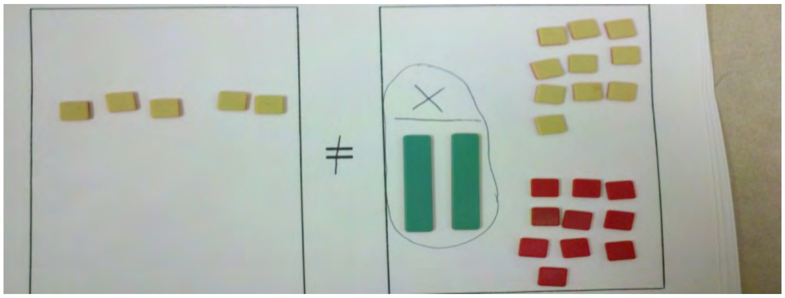
Model problem # 4

- I read the problem: $5 = \frac{X}{2} + 10$.
- The student can also flip the equation so that it is $\frac{X}{2} + 10 = 5$ (if we decide to flip the equation, the steps will be opposite of the step written below.
- I make sure the algebra mat is clear
- On the left side of the mat, place 5 green square tiles (green is positive and red is negative). On the right side, draw an X with a line under it and put 2 green rectangular tiles under the line. On top of the line draw an X. Also on the right side, put 10 cream tiles down (cream is positive and red is negative).

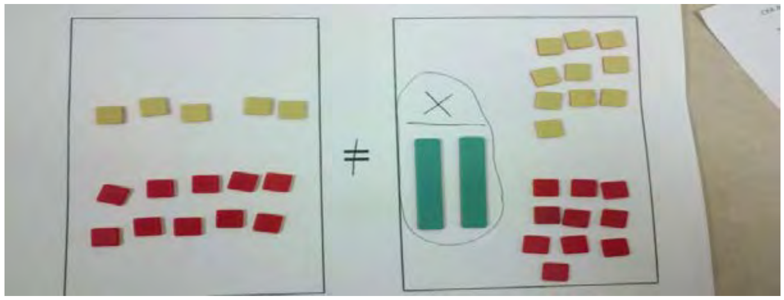
- Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
- Unite like terms. Are there any tiles the same shape that are not already together? No. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
- Modify the new equation. Since we did not combined any like terms, we skip this step.
- Loop around the variable. Draw a circle around the 2 green rectangular tiles. That is the number that should be by itself.



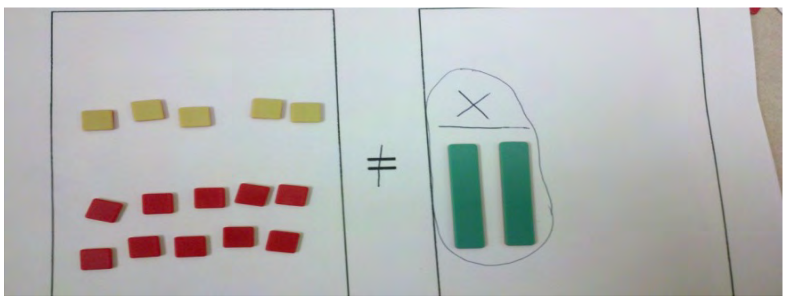
- Is there anything else on the side with the circle? Yes! There are 10 cream squares. We have to get rid of the 10 cream squares.
- Next step is opposite sign. The opposite of positive (plus) 10 is negative (minus) 10. We are going to add 10 red squares under the 10 cream squares.

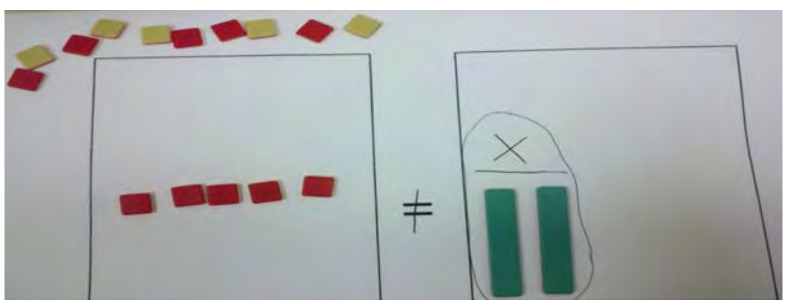


- What we do to one side, we must do to the other. Since we put 10 red squares on the right side of the equation, we must place 10 red squares on the left side of the equation. The right side squares cancels each other out. On the left side, we have 5 cream squares and 10 red squares.

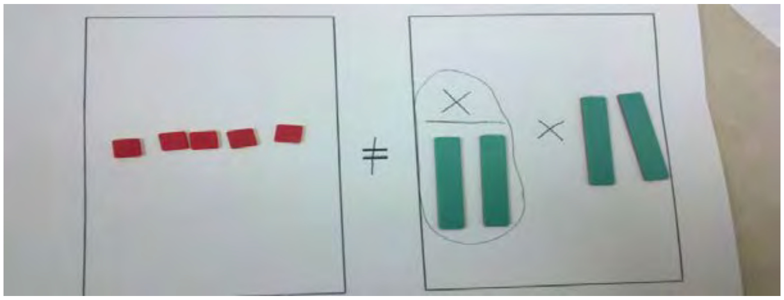


- We are going to add like terms. Take one cream and one red square and move them off the mat until there are no more pairs to make. On the left side, we should have 5 red squares left.
- On the algebra mat, we should have 5 red squares on the left side. On the right side, we should have X over 2 green rectangles. The rectangles should have a circle around it.

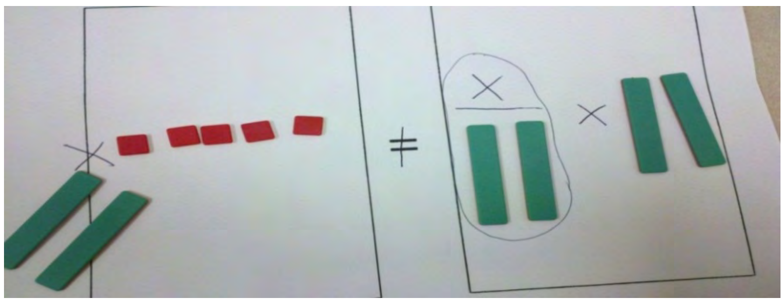




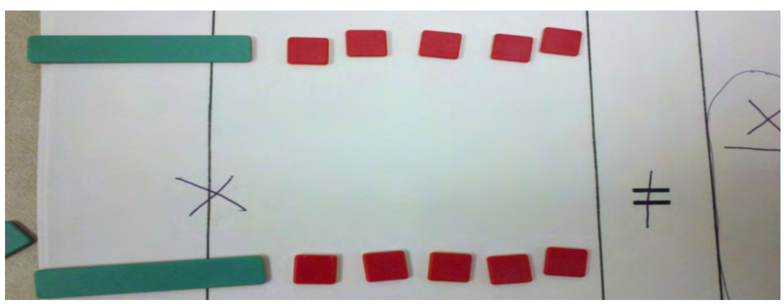
- We have already separated the sides, united like terms, modified the equation, and looped the coefficient. The next step is opposite sign.
- When we have an X over a number, which means to divide. What is the opposite of division? It is multiplication. We are going to write a multiplication sign and place 2 green rectangles beside it.

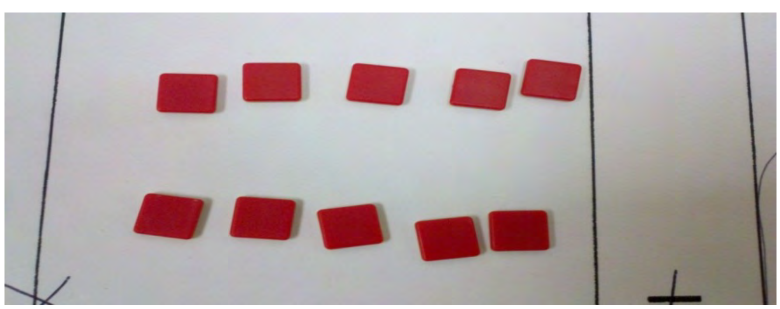


- Next step is what we do to one side; we must do to the other. Since we drew a multiplication sign and placed two green rectangles on the right side, we must do the same on the left side. On the left side we are going to draw a multiplication sign and place 2 green rectangles.



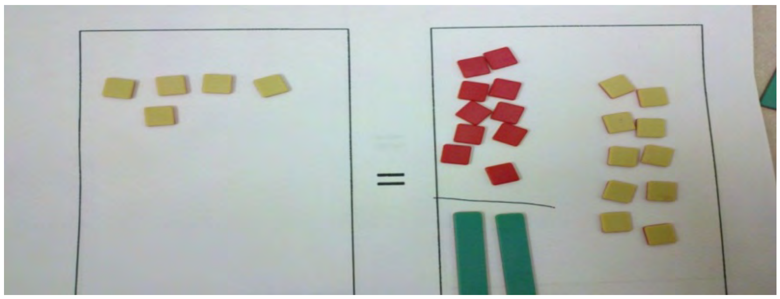
- We are going to give all 5 red squares to the first green rectangle. We are going to add 5 red squares for the 2nd rectangle. How many squares are there? We have 10 red squares. So, X = -10.

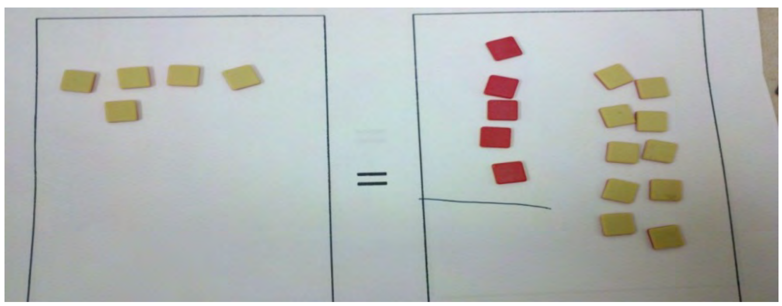


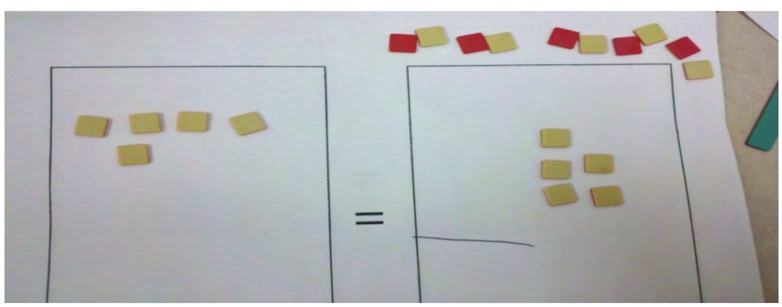


- Last step is to substitute the solution -10 in for X in the original problem. Original problem is 5= X/2 + 10. On the left side, put 5 green squares. On the right side, put

10 red squares for X. Draw a line under the squares and place 2 green rectangles under the line. Separate the rectangles and give each rectangle a square until they are all gone. How many squares does each rectangle have? 5 red squares. Move both rectangles and 5 red squares off the mat (because -10 divided 2 is -5.   On the mat, we should have 5 cream squares on the left side. On the right side, we should have 5 red squares and 10 cream squares. On the right side, combine the squares. Match up a cream square with a red square until there are not more pairs and move them off the mat. How many squares are on the right side? 5 cream squares. How many squares are on the left side? 5 cream squares. The numbers are the same. Are both sides equal? Yes. The answer is correct.







Learning Sheet 1

Two-Step
Concrete
Code Number ____________________ Date _______________
SUMLOWS

Describe and Model

1) $3X+1 = 13$

2) $3X + 5X = -16$

3) $3(X+2) = 3$

4) $5= \frac{X}{2} +10$

Guided Practice

1) $-2X +6 = 10$

2) $4X – 2X = 8$

3) $-2(X +3) = 10$

4) $10 + \frac{X}{2} = 12$

5) $\frac{X}{4} +2= -2$

6) $-X-7= 8$

Independent Practice

$4X – 1 = 11$

$\frac{X}{2} + 3= -4$

$-X + 8 = -5$

$-4(X+1) = -16$

$\frac{X}{3} -2 = 4$

$-10 = -2(X – 3)$

$- 2X+ 4X = -8$

$7X-2X = 20$

Learning Sheet 2

Two-Step
Concrete
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) -4X+2 = 10

2) -2(X+2) = 10

3) $3 = \frac{X}{2} - 6$

Guided Practice

1) -2X +6 = 12

2) -5X – 2X = 14

3) 2(X +2) = 10

4) $1 + \frac{X}{2} = -10$

5) $\frac{X}{3} + 2 = -4$

Independent Practice

3X – 1 = 11

$\frac{X}{2} + 5 = -2$

-X + 6 = -5

-3(X+1) = 15

$\frac{X}{3} - 3 = -5$

4 = -2(X – 3)

- 3X+ 6X = -18

Learning Sheet 3

Two-Step
Concrete
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) $-2(2X+2) = 12$

2) $5 = \frac{X}{4} - 3$

Guided Practice

1) $-6X - 6 = 12$

2) $-2(3X + 1) = 10$

3) $3 + \frac{X}{4} = -1$

Independent Practice

$4X - 5 = 11$

$\frac{X}{2} + 3 = -5$

$-X + 8 = -5$

$-3(2X+1) = 15$

$\frac{X}{3} - 2 = -4$

$-3 = -3(X - 3)$

$-2X - 6X = -16$

Lessons 4-6
Two step

Representational Method

Materials

- chalkboard/whiteboard/easel
- learning sheets
- progress chart

Advance Organizer

- Tell students what they will be doing and why
- Remind students about the commitment they made to learn to solve equations. We will work hard to teach them and they will work hard to learn a new way to solve equations. Tell student that they will solve one step equations a new way. They already know how to solve equations using algebra tiles, but today they will learn how to draw pictures. The pictures will help them solve problems and they can use pictures anytime they need to solve problems because sometimes algebra tiles are not available.

Demonstrate

- Give students learning sheets
- Begin with the first problem in the "Model" section. Tell the students that we will show them how to solve the problem and that they will have a chance to solve problems also. State the expectation for behavior and attention to the demonstration.
- Begin with the first problem and think out loud (see problem model)
  - First we read the problem. $2X + 3 = 11$
  - Next, I take out my workmat or dry erase board.
  - In the representational method, I am going to draw rectangles for my variables (X number) and draw squares for my ones number. For positive numbers, I am going to draw a "+" plus sign in the rectangle or square and a "-"minus sign for negative numbers.
  - In this problem, I am going to draw two rectangles that represent the X. I am going to draw a "+" plus sign in the rectangle. Also on the left side I am going to draw 3 squares with plus signs in them. On the right side, I am going to draw 11 squares with plus signs in them.
  - The first step in SUMLOWS is to separate the two sides. Draw a line through the equal sign.
  - The "U is for unite like terms. Look on the left side. Are there any shapes that are the same shape that are not already together? No. Look on the right side. Are there any shapes that are the same shape that are not already together? No. We go to the next step.

- Modify the new equation. There is not anything to unite or combine, so skip this step.
- The next step is Loop the variable. The variable is the rectangular drawings. This is the number that should be by itself and want to move last. So what is on the side with the variable or rectangles? 3 squares with plus signs in them. We have to get rid of those because the rectangles want to be by itself.
- What is the next step? Next step is opposite sign. In order to move the 3 squares with plus signs in them, we must do the opposite.   The opposite of “plus 3” is “minus 3.” So, we draw 3 squares with minus signs in them under the 3 squares with plus signs in them.
- What is the next step? What we do to one side, we must do to the other. Because we drew 3 squares with minus signs in them under the 3 squares with plus signs in them on the left side, we must draw 3 squares with minus signs in them under the 11 squares with plus signs in them on the right side of the equal sign. The 3 squares with the plus signs and the 3 squares with the minus on the left side cancel each other out (3-3 = 0). Draw a line through them. On the right side, we have 11 squares with plus signs in them and 3 squares with minus signs in them. They are both the same shape; therefore we have to combine like terms.
- Combine like terms. On the right side, we have 11 squares with plus sign and 3 squares with minus signs in them. Since they have different signs in them, we are going to subtract or pair them up and cross out each pair. How many are left on the right side that doesn’t have a pair? 8. What sign are in the squares? They have positive signs in them, so the answer is positive 8.
- On the left side of the equal sign, we have 2 rectangles with the plus sign in them. On the right side, we have8 squares with the plus sign in the. Now we have 2 X =8. Is there only 1 rectangle? No, there are two rectangles; therefore we must determine what 1 rectangle is equal to. I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The “U” is for unite like terms. Look on the left side. Are there any shapes that are the same shape that are not already together? NO. Look on the right side. Are there any shapes that are the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step.
- What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the rectangles. That is the number that should be by itself.
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division sign and draw 2 rectangles with plus signs in them by the other two rectangles.
- What we do to one side, we must do to the other. On the right side, draw a division sign beside the 8 squares with the plus sign in them and draw 2 rectangles with plus signs in them. The left side cancels each other out. On the right side, separate the 2 rectangles with plus sign in them. Give each rectangle a square with a plus sign in them until they are gone. One rectangle is equal to how many squares? 4. Both colors mean positive, so the number is positive. Therefore, x =4

- Last step is to substitute the solution 4 in for X in the original problem. Original problem is 2X + 3 = 11. On the left side draw 4 squares for each X. There are 2 Xs, so draw 8 squares with plus signs in them on the left. Also draw 3 squares with plus signs in them (represent +3) on the left. After combing the like terms, we have 11 squares with the plus sign in them. On the right, draw 11 squares with plus signs in them. Are both sides equal? Yes. The answer is correct.

Model problem #2

- Locate problem # 2 in the "Model" practice section.
- Let's read the problem. This problem is -5X – 2X = -14
- On our workmat, we draw 5 rectangles with minus signs and 2 rectangles with minus signs in them. On the right side, we draw 14 squares with minus signs in them.
  - What is the first step in SUMLOWS? Separate the sides. We are going to draw a line through the equal sign.
  - What does the "U" stand for in SUMLOWS? Unite like terms. Are there any shapes that are the same that are not already together on the left side of the equal sign? Yes. Look on the right side. Are there any like shapes that are the same that are not already together on the right side of the equal sign? No. Because there are shapes that are not together on the left side of the equal sign, we must unite (combine) like terms. We have 5 rectangles with minus signs in them and 3 rectangles with minus signs in them. These shapes are the same and we should combine them. If the signs are the same in them, we add them. If the signs are different, we subtract or pair them up and draw a line through them. Since they have the same sign, we are going to add them. Five rectangles plus two rectangles equal 7 rectangles with minus signs in them.
  - Modify the new equation. We have 7 rectangles with minus signs in them on the left side. On the right side, we have 14 squares with minus signs in them.
  - What is the next step? Loop around the variable. Which shape is the variable? The 7 rectangles with minus signs in them represent the variable. Draw a circle around the rectangular tiles. That is the number that should be by itself and we move last.
  - Is there anything else on the side with the circle? No.
  - Now we have -7X =-14. Is there only 1 rectangle? No., there are 7 rectangles; therefore we must determine what 1 rectangle is equal to. I start back at the front of SUMLOWS mnemonic.
  - I have already separated the sides.
  - The "U" is for unite like terms. Look on the left side. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So, skip this step too.
  - What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the rectangles. That is the number that should be by itself.

- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division sign and draw 7 rectangles with minus signs in them by the other 7 rectangles with minus signs in them.
- What we do to one side, we must do to the other. On the right side, draw a division sign beside the 14 squares with minus signs in them and draw 7 rectangles with minus signs in them. The left side cancels each other out. On the right side, separate the 7 rectangles. Give each rectangle a square until they are gone. One rectangle is equal to how many squares? 2. The signs are the same, so the number is positive. Therefore, x =2.
- Last step is to substitute the solution 4 in for X in the original problem. Original problem is -5X – 2X = -14. On the left side draw 2 squares for each X. There are 5 Xs , so draw 10 squares with minus signs in them (the signs are different so it is negative. Also for -2X, draw 4 squares with minus signs in them (the signs are different, so it's negative). After combining the like terms, we have 14 squares with minus signs in them on the left side. On the right, draw 14 squares with minus signs in them. Are both sides equal? Yes. The answer is correct.

Model problem #3

- I read the next problem. -3(X - 1) =9
- I make sure my mat or board is clear
- In the problem, notice the parenthesis ( ). This problem requires we to get rid of the parenthesis before we start SUMLOWS.
- What is in the parenthesis? X -1. What is outside the parenthesis? -3. The number outside the parenthesis tells us how many times we need to put X -1 on the mat. Since this number is negative (-3), we must flip the signs (positive become negative), therefore we are putting down –X +1 three times. On the mat, we are going to draw shapes that represent -X +1. We are going to draw a 1 rectangle with a minus sign and 1 square with a plus sign in it. In this case, we draw - X +1 on the mat 3 times.
- How many rectangles are? 3 rectangles with minus signs in them
- How many squares are? 3 squares with plus signs in them
- Now we are ready to solve the equation using SUMLOWS
- On the mat, we should have 3 rectangles with minus signs in them and 3 squares with plus signs in them on the left side. On the right side, we have 9 squares with plus sign in them.
    - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
    - Unite like terms. Are there any shapes that are the same that are not already together? NO. Look on the right side. Are there any shapes that are the same that are not already together? No. We go on to step three.
    - Modify the new equation. There is not anything to unite or combine. So, skip this step too.
    - Loop around the variable. Draw a circle around the 3 rectangles with minus signs in them.
    - Is there anything else on the side with the circle? Yes, there are 3 squares with plus signs in them. We have to get rid of the 3 squares with plus signs in them because the rectangle should be by itself.

- In order to move the 3 squares with the plus signs in them, we have to do the next step which is the opposite sign. To get rid of the 3 squares with the plus sign in them, we must add 3 squares with minus signs in them. Opposites cancel each other out (3-3=0).
- Next step is what we do to one side we must do to the other. We added 3 squares with minus signs in them to the left. We must add 3 squares with the plus sign in them to the right. On the left side, square with a plus sign cancel out each square with a minus sign. Put slash marks through each pair. On the right side, we have 9 squares with plus signs in them and 3 squares with minus signs in them. These are the same shape, but have different signs in them. Because of the different signs, we must subtract or pair them up and put slash marks through them. How many squares are on the right side? We have 6 squares with plus signs in them.
- Now we have -3X =6. Is the X by itself? No. The X has a -3 with it. We must move the -3 from the X because the X should be by itself. We must find out what 1 X is equal to.
- I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The "U" is for unite like terms. Look on the left side. Are there any shapes that are the same shape that are not already together? NO. Look on the right side. Are there any shape that are the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the X. That is the number that should be by itself.
- Is there anything else on the side with the circle? No. So, we have to see what 1 X is equal to?
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division sign beside the 3 rectangles with the minus signs in them on the left side and draw 3 more rectangles with the minus signs in them.
- What we do to one side, we must do to the other. Draw a division sign beside the 6 squares with the plus signs in them and draw 3 rectangles with the minus sign in them. The left side cancels each other out.
- On the right side, separate the 3 rectangles with the minus signs in them. Give each rectangle a square with a plus sign in them until they are gone. One red rectangle is equal to how many cream squares? 2. In multiplication and division, if the signs are different colors, the answer is negative. If the signs are the same color, the sign is positive. Therefore, x = -2
- Last step is to substitute the solution -2 in for X in the original problem. Original problem is -3(X - 1) =9. On the right side, draw squares with plus signs in them. On the left side, draw 2 squares minus signs and 1square with minus sign in them. Because the number outside is negative, we must change the signs in the squares to the opposite side. The number outside the parenthesis tell us how many time we need to put it down, we put 2 squares with plus signs and 1square with plus sign three

times. Before we combine like terms, we should have 9 cream squares. They have the same sign; therefore we have to combine them. What are left on the left side? 9 squares with the plus sign in them. What are on the right? 9 squares with the plus sign. They are the same. Are both sides equal? Yes. The answer is correct.

Model problem # 4

- I read the problem: $15 = \frac{X}{2} + 10$
- If the students flip the equation, the steps would be the opposite side.
- I make sure the algebra mat is clear
- On the left side of the mat, draw 15 squares with plus signs in them. On the right side, draw a line and draw 2 rectangles with plus signs under the line. On top of the line draw an X. Also on the right side, draw 10 squares with plus signs in them.
  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any tiles the same shape that are not already together? No. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
  - Modify the new equation. Since we did not combined any like terms, we skip this step.
  - Loop around the variable. Draw a circle around the 2 rectangles with plus signs in them. That is the number that should be by itself.
  - Is there anything else on the side with the circle? Yes! There are 10 squares with plus signs in them. We have to get rid of the 10 squares with plus signs in them.
  - Next step is opposite sign. The opposite of positive (plus) 10 is negative (minus) 10. We are going to add 10 squares with minus signs under the 10 squares with plus signs in them.
  - What we do to one side, we must do to the other. Since we drew 10 squares with minus signs on the right side of the equation, we must draw10 squares with minus signs on the left side of the equation. The right side squares cancels each other out (draw a slash mark in them). On the left side, we have 15 squares with plus signs and 10 squares with minus sign. We are going to add like terms. Put a slash mark through each pair of a square with a plus sign and a square with a minus sign until there are no more pairs to make. On the left side, we should have 5 squares with plus signs left without a slash through them.
  - On the algebra mat, we should have 5 squares with a plus sign in them on the left side. On the right side, we should have X over 2 rectangles with plus signs in them. The rectangles should have a circle around it.
  - We have already separated the sides, united like terms, modified the equation, and looped the coefficient. The next step is opposite sign.
  - When we have an X over a number, which means to divide. What is the opposite of division? It is multiplication. We are going to write a multiplication sign and draw 2 rectangles with plus signs in them beside it.
  - Next step is what we do to one side; we must do to the other. Since we drew a multiplication sign and drew two rectangles with plus signs in them on the right side,

we must do the same on the left side. On the left side we are going to draw a multiplication sign and draw 2 rectangles with plus signs in them.

- We are going to give all of the 5 squares with a plus sign in them to the first green rectangle. We are going to draw 5 more squares with plus signs in them for the 2nd rectangle. How many squares are there? We have 10 squares with plus signs in them. So, X = 10.
- Last step is to substitute the solution 10 in for X in the original problem. Original problem is 15= X/2 + 10. On the left side, draw 15 squares with plus signs. On the right side, draw 10 squares with plus signs in them for X. Draw a line under the squares and draw 2 rectangles with plus signs in them under the line. Separate Give each rectangle a square until they are all gone. How many squares does each rectangle have? 5 squares with plus signs in them. Plus a slash mark through both rectangles and 5 of the 10 squares with plus signs in them (because 10 divided 2 is 5. On the mat, we should have 15 squares with plus signs in them on the left side. On the right side, we should have 5 squares and 10 squares all with plus signs in them. On the right side, combine the squares. All the squares have plus signs in them, so we combine them. How many squares are on the right side? 15 squares with plus signs. How many squares are on the left side? 15 squares with plus signs in them. The numbers are the same. Are both sides equal? Yes. The answer is correct.

Guided Practice

- Direct students to the "Guide" section of the learning sheet
- Tell students to touch the first problem and that we will do this problem together, using numbers, letters and the SUMLOWS mnemonic.
- Let's read the problem. This problem is -6X - 3 = 15.
- On our workmat, what do draw on the left side? 6 rectangles with minus sign in them and 3 squares with minus signs in them.
- What do we draw on the right side? 15 squares with plus signs in them.
    - What is the first step in SUMLOWS? Separate the sides. We are going to draw a line through the equal sign.
    - What does the "U" stand for in SUMLOWS? Unite like terms. Are there any shapes on the same side that are not already together? NO. Look on the right side. Are there any shapes on the same side that are not already together? No. We go on to step three.
    - Modify the new equation. There is not anything to unite or combine. So, skip this step too.
    - What is the next step? Loop around the variable. What shape is the variable? Rectangle. Draw a circle around the six rectangles. That is the number that should be by itself and move last.
    - Is there anything else on the side with the circle? Yes! Three squares with minus signs in them. We must move the 3 squares with minus signs in them.
    - What is the next step? Next step is opposite sign. In order to move the 3 squares with minus signs in them, we must do the opposite. The opposite of "minus 3" is "plus 3." So, we draw 3 squares with plus signs under the 3 squares with minus signs.

- What is the next step? What we do to one side, we must do to the other. Because we drew 3 squares with plus signs under the 3squares with minus signs on the left side, we must draw 3 squares with plus signs under the 15 squares with plus signs in them on the right side of the equal sign. The 3 squares with minus signs in them and the 3 squares with plus signs in them on the left side cancel each other out (-3+3 = 0). Draw a line through them. On the right side, we have 15 squares with plus signs in them and 3 squares with plus signs in them. They are both the same shape; therefore we have to combine like terms.
- Combine like terms. On the right side, we have 15 squares with plus signs in them and 3 squares with plus signs in them. Since they both have the same sign in them, we are going to add them up. How many squares are when we add them? 18. What signs are in the shapes? Plus sign. Since they have plus signs in them, the answer is positive 18.
- On the left side of the equal sign, we have 6 rectangles with minus signs in them. On the right side, we have 18 squares with plus signs in them. Now we have -6 X = 18. Is there only 1 rectangle? No., there are six rectangles; therefore we must determine what 1 rectangle is equal to. I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The "U" is for unite like terms. Look on the left side. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the X. That is the number that should be by itself.
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division sign and draw 6 rectangles with minus signs by the other 6 rectangles with minus signs in them.
- What we do to one side, we must do to the other. On the right side, draw a division sign beside the 18 squares with plus signs in them and draw 6 rectangles with minus signs in them. The left side cancels each other out. On the right side, separate the 6 rectangles. Give each rectangle a square until they are gone. One rectangle is equal to how many squares? 3. The signs in them are different, so the number is negative. Therefore, x = -3.
- Last step is to substitute the solution -3 in for X in the original problem. Original problem is -6X - 3= 15. On the left side draw 3 squares with plus signs in them for each X. There are 6 Xs, so draw 18 squares with plus signs in them on the left. The squares are positive because both numbers are the same, therefore the squares are positive, as well as 3 squares with minus signs on the left. After combing like terms, we have 15 cream squares on the left. On the right, put 15 cream squares. Are both sides equal? Yes. The answer is correct.

Guided Practice number #2

- Locate problem # 2 in the “Guided practice” section.
- Let’s read the problem. This problem is -4X+ X = -9
- On our workmat, we draw 4 rectangles with minus signs and 1 rectangle with a plus sign in it (there is an invisible 1 in front of the X). On the right side, we draw 9 squares with minus signs in them.
    - What is the first step in SUMLOWS? Separate the sides. We are going to draw a line through the equal sign.
    - What does the “U” stand for in SUMLOWS? Unite like terms. Are there any shapes that are the same that are not already together on the left side of the equal sign? Yes. Look on the right side. Are there any like shapes that are the same that are not already together on the right side of the equal sign? No. Because there are shapes that are not together on the left side of the equal sign, we must unite (combine) like terms. We have 4 rectangles with minus signs in them and 1 rectangle with a plus sign in it. These shapes are different, so we need subtract them or pair them up and draw a line through the pairs. Pair up a rectangle with a plus sign with a rectangle with a minus sign until there are no more pairs.
    - Modify the new equation. We have 3 rectangles with minus signs in them on the left side. On the right side, we have 9 squares with minus signs in them.
    - What is the next step? Loop around the variable. Which shape is the variable? The 3 rectangles with minus signs in them represent the variable. Draw a circle around the rectangles. That is the number that should be by itself and we move last.
    - Is there anything else on the side with the circle? No.
    - Now we have -3X =9. Is there only 1 rectangle? No., there are 3 rectangles; therefore we must determine what 1 rectangle is equal to. I start back at the front of SUMLOWS mnemonic.
    - I have already separated the sides.
    - The “U” is for unite like terms. Look on the left side. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
    - Modify the new equation. There is not anything to unite or combine. So, skip this step too.
    - What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the rectangles. That is the number that should be by itself.
    - Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division sign and draw 3 rectangles with minus signs in them by the other 3 rectangles with minus signs in them.
    - What we do to one side, we must do to the other. On the right side, draw a division sign beside the 9 squares with plus signs in them and draw 3 rectangles with minus signs in them. The left side cancels each other out. On the right side, separate the 3 rectangles. Give each rectangle a square until they are gone. One rectangle is equal to how many squares? 3. The signs are the different, so the number is negative. Therefore, x =-3.
    - Last step is to substitute the solution -3 in for X in the original problem. Original problem is -4X +X = 9. For -4X, draw 3 squares with a plus sign for each X . There are four of them, so draw 12 squares with plus signs in the. For X, draw 3 squares

with minus signs in them on the left. After combing like terms, we have 9 squares with plus signs in them on the left side. On the right, draw 9 squares with plus signs in them. Are both sides equal? Yes. The answer is correct.

Guided practice problem #3

- I read the next problem. -2(2X + 1) = 6
- I make sure my mat or board is clear
- In the problem, notice the parenthesis ( ). This problem requires we to get rid of the parenthesis before we start SUMLOWS.
- What is in the parenthesis? 2X +1. What is outside the parenthesis? -2. The number outside the parenthesis tells us how many times we need to put 2X +1 on the mat. Since this number is negative (-2), we must flip the signs (positive become negative), therefore we are putting down –2X -1 two times. On the mat, we are going to draw shapes that represent -2X -1. We are going to draw 2 rectangles with a minus signs and 1 square with a plus sign in it. In this case, we draw - 2X -1 on the mat 2 times.
- How many rectangles are? 4 rectangles with minus signs in them
- How many squares are? 2 squares with minus signs in them
- Now we are ready to solve the equation using SUMLOWS
- On the mat, we should have 4 rectangles with minus signs in them and 2 squares with minus signs in them on the left side. On the right side, we have six squares with plus sign in them.
    - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
    - Unite like terms. Are there any shapes that are the same that are not already together? NO. Look on the right side. Are there any shapes that are the same that are not already together? No. We go on to step three.
    - Modify the new equation. There is not anything to unite or combine. So, skip this step too.
    - Loop around the variable. Draw a circle around the 4 rectangles with minus signs in them.
    - Is there anything else on the side with the circle? Yes, there are 2 squares with minus signs in them. We have to get rid of the 2 squares with minus signs in them because the rectangle should be by itself.
    - In order to move the 2 squares with the minus signs in them, we have to do the next step which is the opposite sign. To get rid of the 2 squares with the minus signs in them, we must add 2 squares with plus signs in them. Opposites cancel each other out (-2+2=0).
    - Next step is what we do to one side we must do to the other. We added 2 squares with plus signs in them to the left. We must add 2 squares with the plus sign in them to the right. On the left side, square with a plus sign cancel out each square with a minus sign. Put slash marks through each pair. On the right side, we have 6 squares with plus signs in them and 2 squares with plus signs in them. These are the same shape and have the same sign. Because of the same signs, we must add or combined the squares. How many squares are on the right side? We have 8 squares with plus signs in them.

- Now we have -4X =8. Is the X by itself? No. The X has a -4 with it. We must move the -4 from the X because the X should be by itself. We must find out what 1 X is equal to.
- I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The "U" is for unite like terms. Look on the left side. Are there any shapes that are the same shape that are not already together? NO. Look on the right side. Are there any shape that are the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the X. That is the number that should be by itself.
- Is there anything else on the side with the circle? No. So, we have to see what 1 X is equal to?
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division sign beside the 4 rectangles with the minus signs in them on the left side and draw 4 more rectangles with the minus signs in them.
- What we do to one side, we must do to the other. Draw a division sign beside the 8 squares with the plus signs in them and draw 4 rectangles with the minus sign in them. The left side cancels each other out.
- On the right side, separate the 4 rectangles with the minus signs in them. Give each rectangle a square with a plus sign in them until they are gone. One red rectangle is equal to how many cream squares? 2. In multiplication and division, if the signs are different colors, the answer is negative. If the signs are the same color, the sign is positive. Therefore, x = -2
- Last step is to substitute the solution -2 in for X in the original problem. Original problem is -2(2X +1) =6. On the right side, draw 6 squares with plus signs in them. For the 2X, draw 4 squares with minus signs in them because 2 times 2 is 4. The signs are different, so it's negative. Also draw 1 square with a plus sign in them. Since the number outside the parenthesis is negative, change the signs in the shapes. Therefore draw 4 squares with plus signs in them and 1 square with minus sign in it. The number outside the parenthesis tell us how many time we need to put it down, we draw 4 squares with plus signs in them and 1 square with minus sign two times. Before we combine like terms, we should have 8 squares with plus signs and 2 with minus signs. They have different signs; therefore we have to pair them up and draw slash marks for each pair. What are left on the left side? 6 squares with the plus sign in them. What are on the right? 6 squares with the plus sign. They are the same. Are both sides equal? Yes. The answer is correct.

Guided Practice problem # 4

- I read the problem: $3= \frac{X}{4} + 7$

- If the students flip the equation, the steps would be the opposite side.
- I make sure the algebra mat is clear
- On the left side of the mat, draw 3 squares with plus signs in them. On the right side, draw a line and draw 4 rectangles with plus signs under the line. On top of the line draw an X. Also on the right side, draw 7 squares with plus signs in them.
  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any tiles the same shape that are not already together? No. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
  - Modify the new equation. Since we did not combined any like terms, we skip this step.
  - Loop around the variable. Draw a circle around the 4 rectangles with plus signs in them. That is the number that should be by itself.
  - Is there anything else on the side with the circle? Yes! There are 7 squares with plus signs in them. We have to get rid of the 7 squares with plus signs in them.
  - Next step is opposite sign. The opposite of positive (plus) 7 is negative (minus) 7. Draw 7 squares with minus signs under the 7 squares with plus signs in them.
  - What we do to one side, we must do to the other. Since we drew 7 squares with minus signs on the right side of the equation, we must draw7 squares with minus signs on the left side of the equation. The right side squares cancels each other out (draw a slash mark in them). On the left side, we have 3 squares with plus signs and 7 squares with minus sign. We are going to add like terms. Put a slash mark through each pair of a square with a plus sign and a square with a minus sign until there are no more pairs to make. On the left side, we should have 4 squares with minus signs left without a slash through them.
  - On the algebra mat, we should have 4 squares with minus sign sin them on the left side. On the right side, we should have X over 4 rectangles with plus signs in them. The rectangles should have a circle around it.
  - We have already separated the sides, united like terms, modified the equation, and looped the coefficient. The next step is opposite sign.
  - When we have an X over a number, which means to divide. What is the opposite of division? It is multiplication. We are going to write a multiplication sign and draw 4 rectangles with plus signs in them beside it.
  - Next step is what we do to one side; we must do to the other. Since we drew a multiplication sign and drew four rectangles with plus signs in them on the right side, we must do the same on the left side. On the left side, draw a multiplication sign and draw four rectangles with plus signs in them.
  - We are going to give all of the 4 squares with minus signs in them to the first green rectangle. We are going to draw 4 more squares with minus signs in them for the 2nd rectangle. We are going to draw 4 more squares with minus signs in them for the 3 rectangles and the 4th rectangle. How many squares are there? We have 12 squares with minus signs in them. So, X = -12.
  - Last step is to substitute the solution -12 in for X in the original problem. Original problem is 3= X/4 + 7. On the left side, draw 3 squares with plus signs. On the right side, draw 12 squares with minus signs in them for X. Draw a line under the squares and draw 4 rectangles with plus signs in them under the line. Separate Give each

rectangle a square until they are all gone. How many squares does each rectangle have? 3 squares with minus signs in them. Plus a slash mark through 4 rectangles and 9 of the 12 squares with minus signs in them (because 12 divided 4 is 3.   On the mat, we should have 3 squares with plus signs in them on the left side. On the right side, we should have 3 squares with minus signs in them and 7 squares with plus signs in them.   On the right side, combine the squares. Put a slash mark for each pair of square with a plus sign and a minus sign in them until there are not more pairs to make. How many squares are on the right side? 3 squares with plus signs. How many squares are on the left side? 3 squares with plus signs in them. The numbers are the same. Are both sides equal? Yes. The answer is correct.

Independent Practice

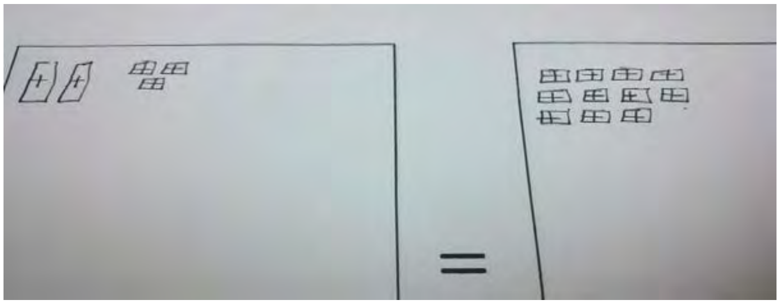
- Direct the students to the "Independent Practice" section. Read the first problem together and direct students to complete the problems without you.
  When students finish problems, provide immediate corrective feedback for errors.

Graphing

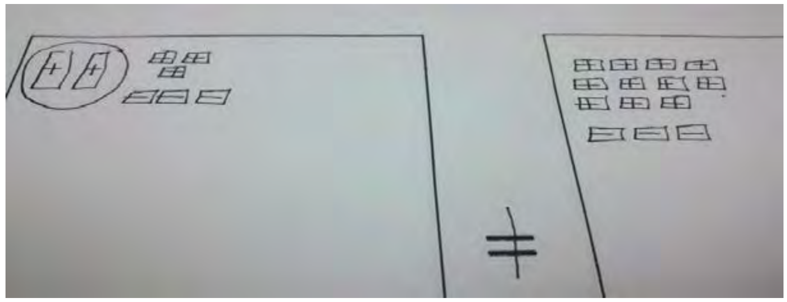
Representational

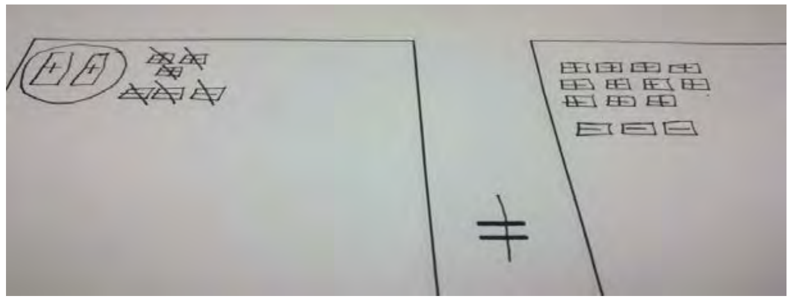
Model Problem #1
2X+3 = 11

- In the representational method, I am going to draw rectangles for my variables (X number) and draw squares for my ones number. For positive numbers, I am going to draw a "+" plus sign in the rectangle or square and a "-"minus sign for negative numbers. In this problem, I am going to draw two rectangles that represent the X. I am going to draw a "+" plus sign in the rectangle. Also on the left side I am going to draw 3 squares with plus signs in them. On the right side, I am going to draw 11 squares with plus signs in them.

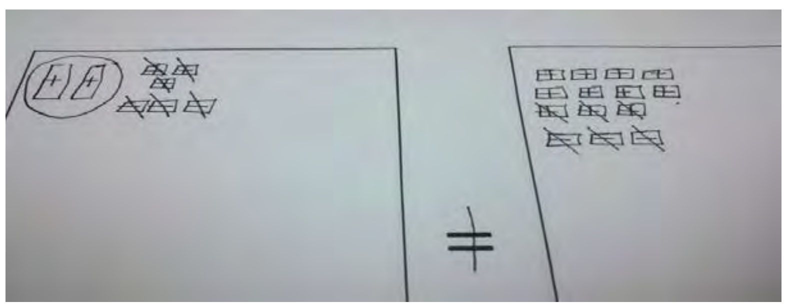


  - The first step in SUMLOWS is to separate the two sides. Draw a line through the equal sign.
  - The "U is for unite like terms. Look on the left side. Are there any shapes that are the same shape that are not already together? No. Look on the right side. Are there any shapes that are the same shape that are not already together? No. We go to the next step.
  - Modify the new equation. There is not anything to unite or combine, so skip this step.
  - The next step is Loop the variable. The variable is the rectangular drawings. This is the number that should be by itself and want to move last. So what is on the side with the variable or rectangles? 3 squares with plus signs in them. We have to get rid of those because the rectangles want to be by itself.
  - What is the next step? Next step is opposite sign. In order to move the 3 squares with plus signs in them, we must do the opposite. The opposite of "plus 3" is "minus 3." So, we draw 3 squares with minus signs in them under the 3 squares with plus signs in them.
  - What is the next step? What we do to one side, we must do to the other. Because we drew 3 squares with minus signs in them under the 3 squares with plus signs in them on the left side, we must draw 3 squares with minus signs in them under the 11 squares with plus signs in them on the right side of the equal sign. The 3 squares with the plus signs and the 3 squares with the minus on the left side cancel each other out (3-3 = 0). Draw a line through them. On the right side, we have 11 squares with plus signs in them and 3 squares with minus signs in them. They are both the same shape; therefore we have to combine like terms.



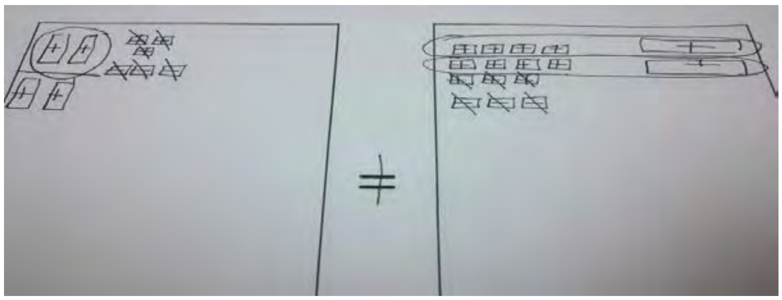


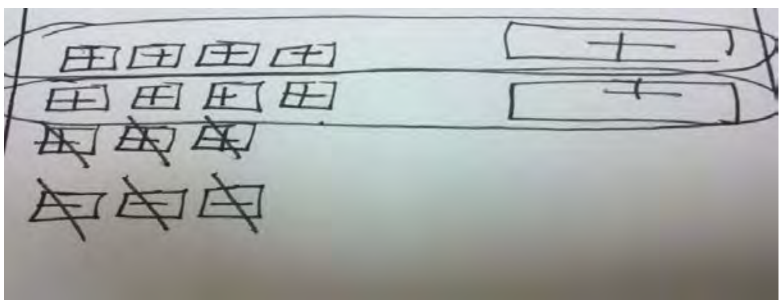
- Combine like terms. On the right side, we have 11 squares with plus sign and 3 squares with minus signs in them. Since they have different signs in them, subtract or pair them up and cross out each pair. How many are left on the right side that doesn't have a pair? 8. What sign are in the squares? They have positive signs in them, so the answer is positive 8.



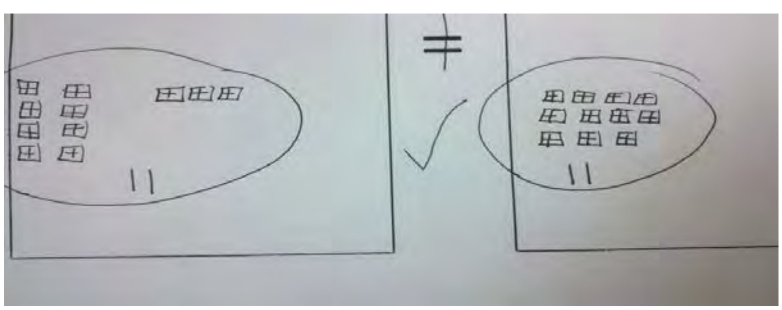
- On the left side of the equal sign, we have 2 rectangles with the plus sign in them. On the right side, we have8 squares with the plus sign in the. Now we have 2 X =8. Is there only 1 rectangle? No, there are two rectangles; therefore we must determine what 1 rectangle is equal to. I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The "U" is for unite like terms. Look on the left side. Are there any shapes that are the same shape that are not already together? NO. Look on the right side. Are there any shapes that are the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step.
- What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the rectangles. That is the number that should be by itself.

- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division sign and draw 2 rectangles with plus signs in them by the other two rectangles.
- What we do to one side, we must do to the other. On the right side, draw a division sign beside the 8 squares with the plus sign in them and draw 2 rectangles with plus signs in them. The left side cancels each other out. On the right side, separate the 2 rectangles with plus sign in them. Give each rectangle a square with a plus sign in them until they are gone. One rectangle is equal to how many squares? 4. Both colors mean positive, so the number is positive. Therefore, x =4





- Last step is to substitute the solution 4 in for X in the original problem. Original problem is 2X + 3 = 11. On the left side draw 4 squares for each X. There are 2 Xs, so draw 8 squares with plus signs in them on the left. Also draw 3 squares with plus signs in them (represent +3) on the left. After combing the like terms, we have 11 squares with the plus sign in them. On the right, draw 11 squares with plus signs in them. Are both sides equal? Yes. The answer is correct.

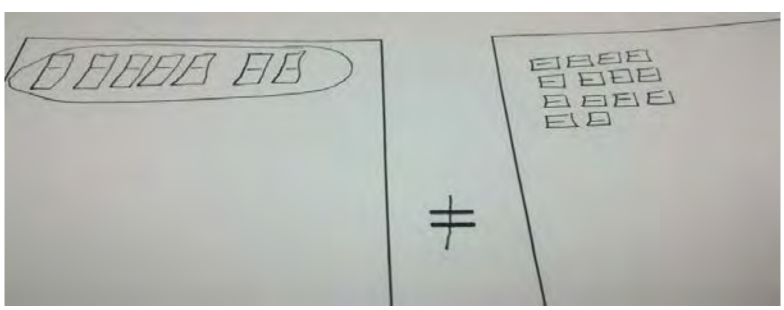


Model Problem #2

-5X – 2X = 14

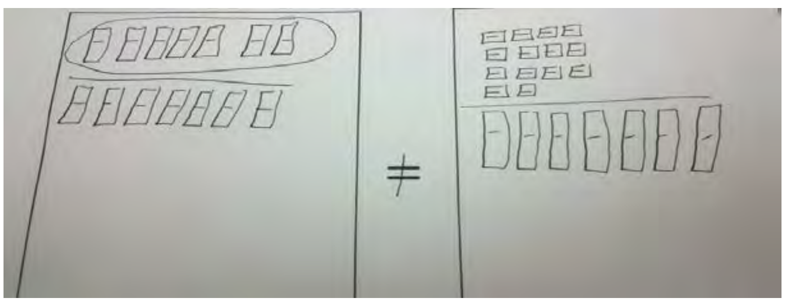
- On our workmat, we draw 5 rectangles with minus signs and 2 rectangles with minus signs in them. On the right side, we draw 14 squares with minus signs in them.

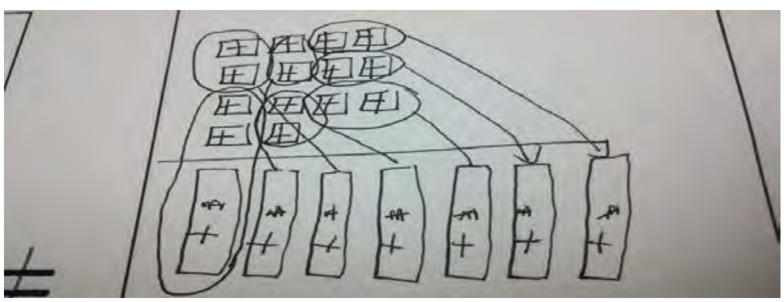
  - What is the first step in SUMLOWS? Separate the sides. We are going to draw a line through the equal sign.
  - What does the “U” stand for in SUMLOWS? Unite like terms. Are there any shapes that are the same that are not already together on the left side of the equal sign? Yes. Look on the right side. Are there any like shapes that are the same that are not already together on the right side of the equal sign? No. Because there are shapes that are not together on the left side of the equal sign, we must unite (combine) like terms. We have 5 rectangles with minus signs in them and 3 rectangles with minus signs in them. These shapes are the same and we combine them. If the signs are the same in them, we add them. If the signs are different, we subtract or pair them up and draw a line through them. Since they have the same sign, add them. Five rectangles plus two rectangles equal 7 rectangles with minus signs in them.
  - Modify the new equation. We have 7 rectangles with minus signs in them on the left side. On the right side, we have 14 squares with minus signs in them.
  - What is the next step? Loop around the variable. Which shape is the variable? The 7 rectangles with minus signs in them represent the variable. Draw a circle around the rectangular tiles. That is the number that should be by itself and we move last.



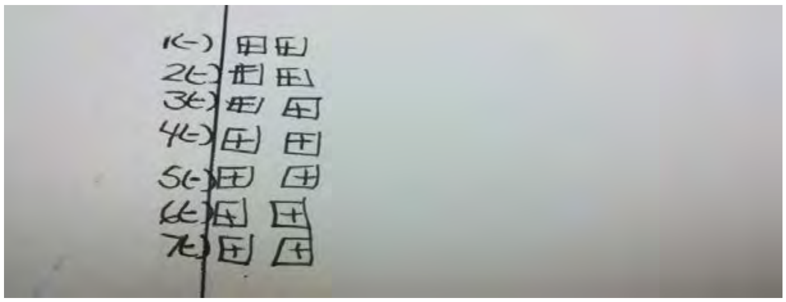
  - Is there anything else on the side with the circle? No.
  - Now we have -7X =-14. Is there only 1 rectangle? No., there are 7 rectangles; therefore we must determine what 1 rectangle is equal to. I start back at the front of SUMLOWS mnemonic.
  - I have already separated the sides.

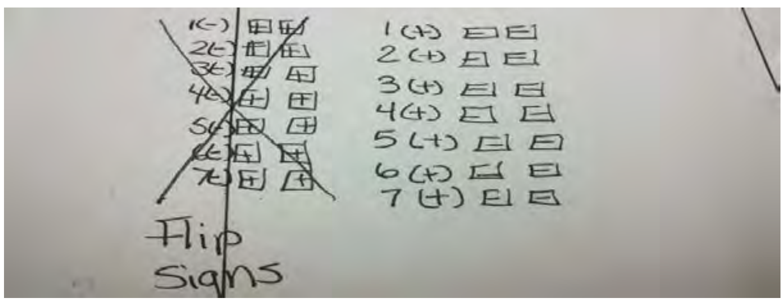
- The “U” is for unite like terms. Look on the left side. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the rectangles. That is the number that should be by itself
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division sign and draw 7 rectangles with minus signs in them by the other 7 rectangles with minus signs in them.
- What we do to one side, we must do to the other. On the right side, draw a division sign beside the 14 squares with minus signs in them and draw 7 rectangles with minus signs in them. The left side cancels each other out. On the right side, separate the 7 rectangles. Give each rectangle a square until they are gone. One rectangle is equal to how many squares? 2. The signs are the same, so the number is positive. Therefore, x =2.

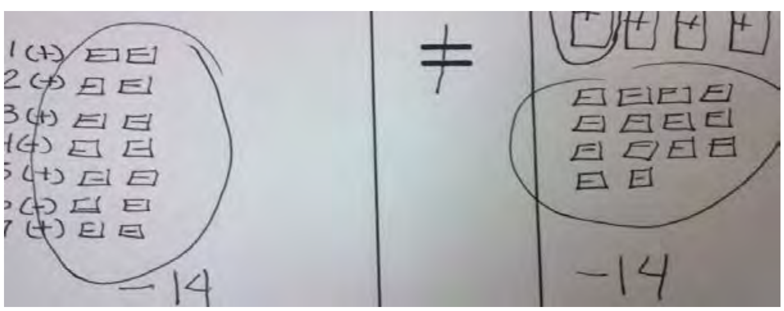




- Last step is to substitute the solution 4 in for X in the original problem. Original problem is -5X – 2X = -14. On the left side draw 2 squares for each X. There are 5 Xs , so draw 10 squares with minus signs in them (the signs are different so it is negative. Also for -2X, draw 4 squares with minus signs in them (the signs are different, so it’s negative). After combining the like terms, we have 14 squares with minus signs in them on the left side. On the right, draw 14 squares with minus signs in them. Are both sides equal? Yes. The answer is correct.





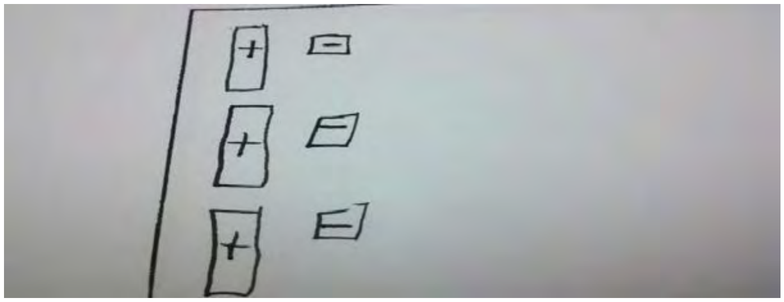


Model Practice #3

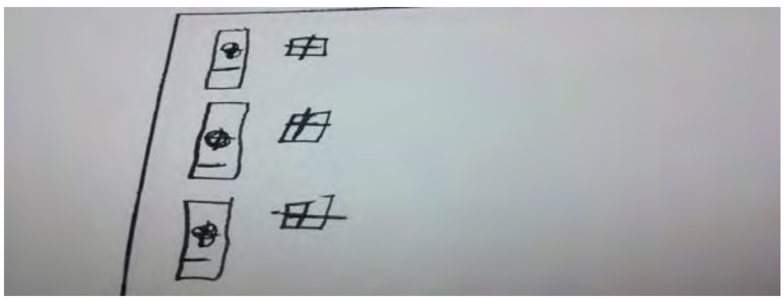
-3(X - 1) =9

- I make sure my mat or board is clear
- In the problem, notice the parenthesis ( ). This problem requires we to get rid of the parenthesis before we start SUMLOWS.
- What is in the parenthesis? X -1. What is outside the parenthesis? -3. The number outside the parenthesis tells us how many times we need to put X -1 on the mat. Since this number is negative (-3), we must flip the signs (positive become negative), therefore we are putting down X -1 three times. On the mat, draw shapes that represent X -1. We

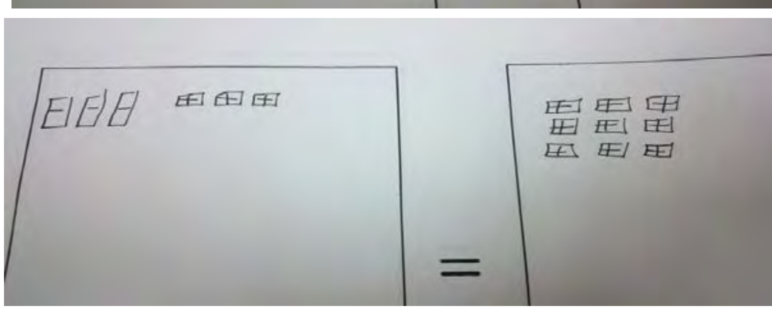
are going to draw a 1 rectangle with a minus sign and 1 square with a plus sign in it. In this case, we draw X -1 on the mat 3 times.



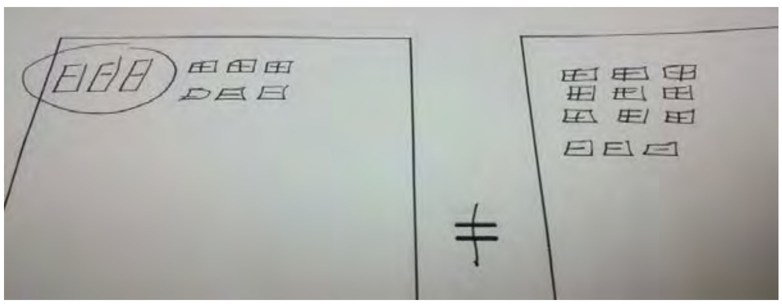
- Because there the 3 is negative, we will flip all the signs to its opposite.
- How many rectangles are? 3 rectangles with minus signs in them
- How many squares are? 3 squares with plus signs in them.



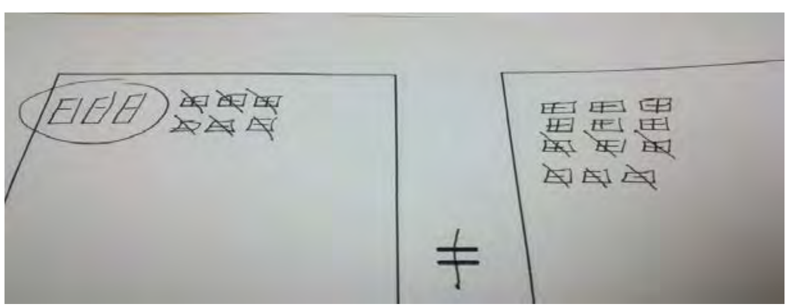
- Now we are ready to solve the equation using SUMLOWS
- On the mat, we should have 3 rectangles with minus signs in them and 3 squares with plus signs in them on the left side. On the right side, we have 9 squares with plus sign in them.



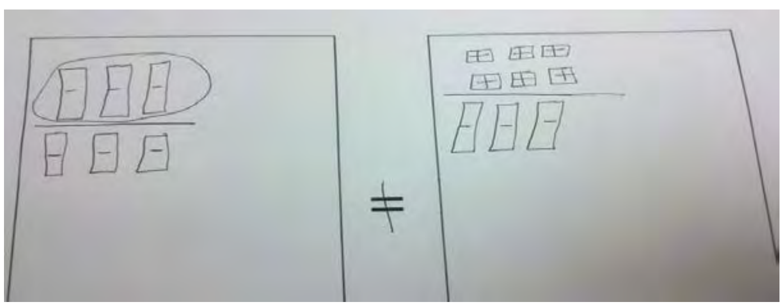
- Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
- Unite like terms. Are there any shapes that are the same that are not already together? NO. Look on the right side. Are there any shapes that are the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- Loop around the variable. Draw a circle around the 3 rectangles with minus signs in them.
- Is there anything else on the side with the circle? Yes, there are 3 squares with plus signs in them. We have to get rid of the 3 squares with plus signs in them because the rectangle should be by itself.
- In order to move the 3 squares with the plus signs in them, we have to do the next step which is the opposite sign. To get rid of the 3 squares with the plus sign in them, we must add 3 squares with minus signs in them. Opposites cancel each other out (3-3=0).



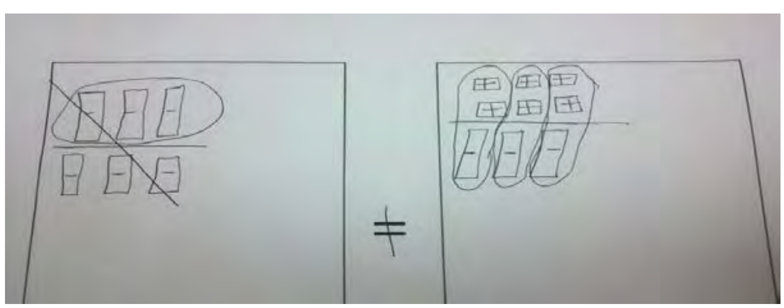
- Next step is what we do to one side we must do to the other. We added 3 squares with minus signs in them to the left. We must add 3 squares with the plus sign in them to the right. On the left side, square with a plus sign cancel out each square with a minus sign. Put slash marks through each pair. On the right side, we have 9 squares with plus signs in them and 3 squares with minus signs in them. These are the same shape, but have different signs in them. Because of the different signs, we must subtract or pair them up and put slash marks through them. How many squares are on the right side? We have 6 squares with plus signs in them.
- Now we have -3X =6. Is the X by itself? No. The X has a -3 with it. We must move the -3 from the X because the X should be by itself. We must find out what 1 X is equal to.



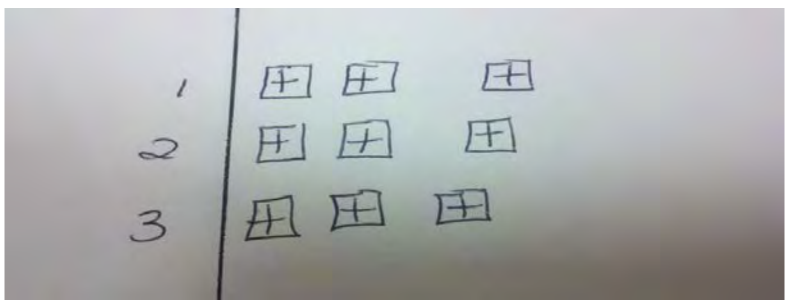
- I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The “U” is for unite like terms. Look on the left side. Are there any shapes that are the same shape that are not already together? NO. Look on the right side. Are there any shapes that are the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the X. That is the number that should be by itself.
- Is there anything else on the side with the circle? No. So, we have to see what 1 X is equal to?
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division sign beside the 3 rectangles with the minus signs in them on the left side and draw 3 more rectangles with the minus signs in them.
- What we do to one side, we must do to the other. Draw a division sign beside the 6 squares with the plus signs in them and draw 3 rectangles with the minus sign in them. The left side cancels each other out.

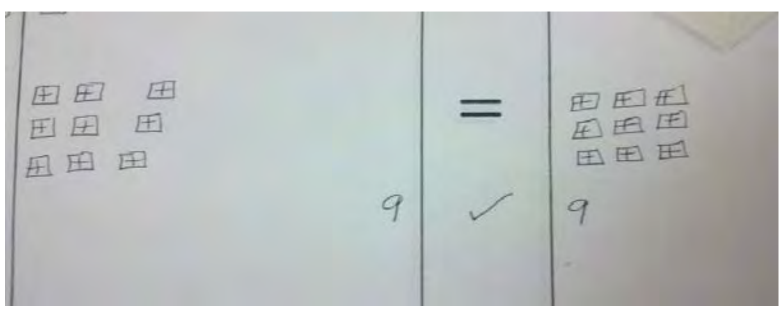


- On the right side, separate the 3 rectangles with the minus signs in them. Give each rectangle a square with a plus sign in them until they are gone. One red rectangle is equal to how many cream squares? 2. In multiplication and division, if the signs are different colors, the answer is negative. If the signs are the same color, the sign is positive. Therefore, x = -2.



- Last step is to substitute the solution -2 in for X in the original problem.   Original problem is -3(X - 1) =9. On the right side, draw squares with plus signs in them. On the left side, draw 2 squares minus signs and 1square with minus sign in them. Because the number outside is negative, we must change the signs in the squares to the opposite side. The number outside the parenthesis tell us how many time we need to put it down, we put 2 squares with plus signs and 1square with plus sign three times. Before we combine like terms, we should have 9 cream squares. They have the same sign; therefore we have to combine them. What are left on the left side? 9 squares with the plus sign in them. What are on the right? 9 squares with the plus sign. They are the same. Are both sides equal? Yes. The answer is correct.





Learning Sheet 4

Two-Step
Representational
Code Number _________________ Date _____________
SUMLOWS

Describe and Model

1) $-2X+5 = 11$

2) $6X - 3X = 6$

3) $-3(2X+1) = 9$

4) $5 = \frac{X}{4} - 2$

Guided Practice

1) $-3X + 6 = 12$

2) $-4X - 3X = 14$

3) $-2(X + 2) = 10$

4) $3 + \frac{X}{4} = -1$

5) $\frac{X}{5} + 2 = -3$

Independent Practice

$5X - 1 = 9$

$\frac{X}{4} + 1 = -2$

$-X + 5 = -8$

$-4(X+2) = 8$

$\frac{X}{3} - 4 = -5$

$12 = -4(2X - 1)$

$-2X + 6X = -12$

Learning Sheet 5

Two-Step
Representational
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) $-5X - 3X = 16$

2) $-2(3X+1) = 4$

3) $4 = \frac{X}{2} - 1$

Guided Practice

1) $-7X + 1 = -13$

2) $-2X - 3X = 15$

3) $-3(X - 2) = -3$

4) $5 + \frac{X}{3} = -2$

Independent Practice

$4X - 3 = 5$

$\frac{X}{2} + 4 = 2$

$-X - 6 = -4$

$-2(3X-2) = -8$

$\frac{X}{5} - 1 = -2$

$-3 = -3(X - 2)$

$-4X + 6X = -8$

Learning Sheet 6

Two-Step
Representational
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) $-5(2X+1) = 15$

Guided Practice

1) $-5X - 3 = 7$

2) $-3(X - 3) = 12$

Independent Practice

$3X - 3 = 8$

$\frac{X}{3} + 2 = 5$

$-X - 8 = -2$

$-4(X-2) = 12$

$\frac{X}{3} - 4 = 2$

$-6 = 3(X + 2)$

$-3X - 3X = 12$

$\frac{X}{2} + 3 = 4$

Lessons 7-9
Two steps

Abstract Level

Materials

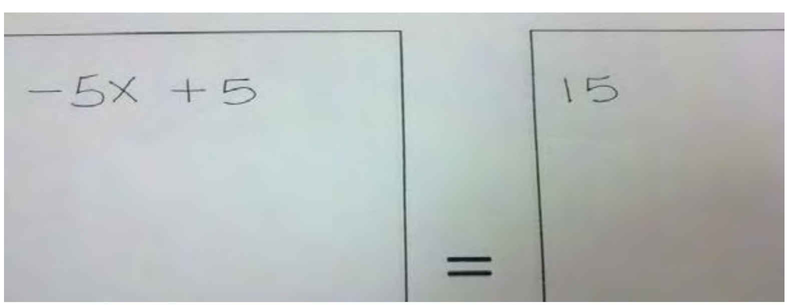
- Chalkboard/whiteboard/easel
- learning sheets 7-9
- progress chart

Advance Organizer

- Tell students what they will be doing and why
  Remind students about the commitment they made to learn to solve equations. We will work hard to teach them and they will work hard to learn a new way to solve equations. They learned to solve one-step equations with algebra tiles and with drawing. Now they will practice solving equations without using tiles or drawings. Remind students of the SUMLOWS mnemonic and tell them that this mnemonic can be used to help them solve equations if they do not remember the steps.

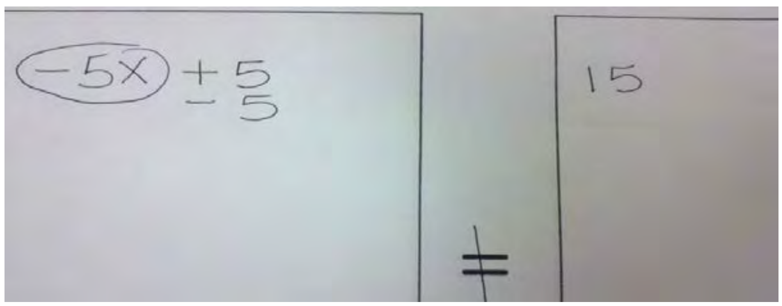
Demonstrate

- Give students learning sheets 7-9
- Begin with the first problem in the "Model" section. Tell the students that we will show them how to solve the problem using numbers only.
- First we First we read the problem. -5X + 5 =15
- Next, I take out my workmat or dry erase board.
- In the abstract method, I am going to use symbols and numbers. For positive numbers, I am going to use a "+" plus sign or no sign at all. It depends on where the number or symbol is in the problem. For negative numbers, I am going to use "-"minus signs.
- In this problem, I am going to write out the problem like I see it. On the left, I am going to write -5 X + 5. On the right side, I am going to write 15.

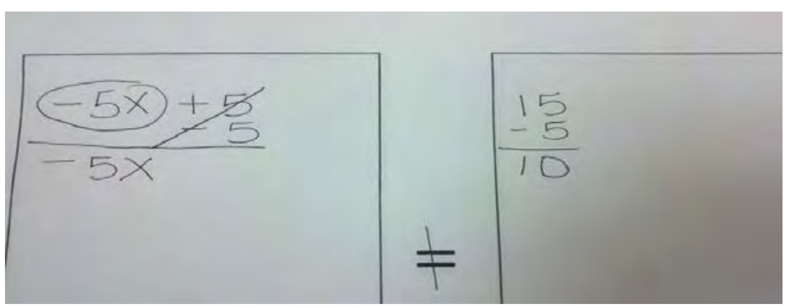


  - The first step in SUMLOWS is to separate the two sides. Draw a line through the equal sign.

- The “U is for unite like terms. Look on the left side, are there any like terms or numbers that not already together? No. Look on the right side. Are there any like terms that are not together? No. We go to the next step.
- Modify the new equation. There is not anything to unite or combine, so skip this step too.
- The next step is Loop the variable. The variable is the X. This is the number that should be by itself and want to move last. Draw a circle around the -5X. What is on the side with the variable or X? +5. We have to get rid of +5 because X should be by itself.
- In order to move the +5, we have write -5 under the +5. Opposites cancel each other out. Put a slash mark through +5 and -5. This equals 0 and cancels each other out.

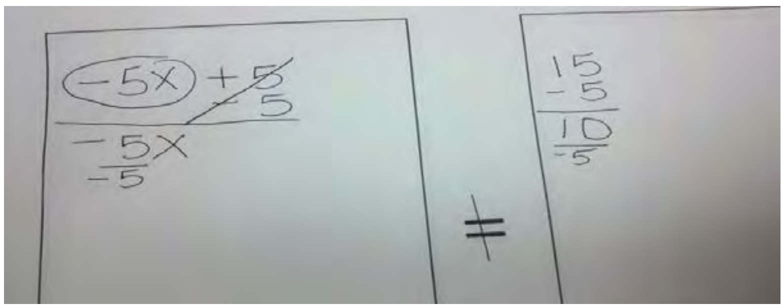


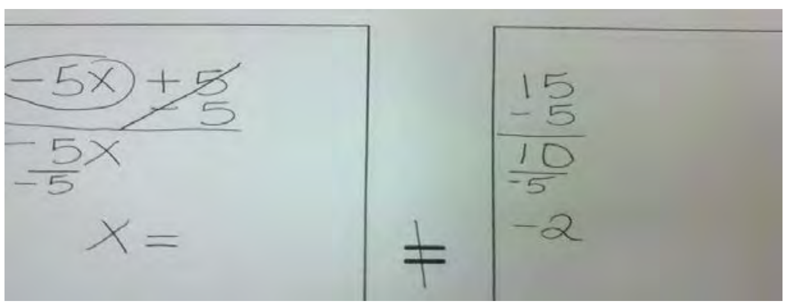
- The next step is “what we do to one side, we must do to the other” We subtracted 5 on the left side, so we must subtract 5 on the right side. Write -5 under the 15.
- Combine like terms. On the right side, we have 15 and -5. Since the numbers have different signs, we must subtract them and take the sign of the larger number (absolute value).   Fifteen minus five is 10. Since the larger number (absolute value) is 15 and it’s positive, the answer is positive. So -5X = 10.



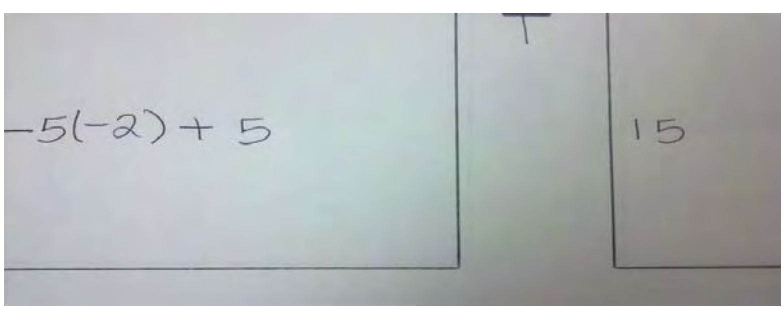
- Is there anything else on the side with the circle? No.
- Now we have -5X =10. Is there only 1X? No., there five; therefore we must determine what 1X is equal to. I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The “U” is for unite like terms. Look on the left side. Are there any like terms or numbers that can be combined that are not already together? NO. Look on the right side. Are there any like terms or numbers that can be combined that are not already together? No. We go on to step three.

- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the X. That is the number that should be by itself.
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a line under the -5 sign and write -5 under it.
- What we do to one side, we must do to the other. On the right side, draw a line under the 10 and write -5 under it. The left side cancels each other out. On the right side, we divide 15 divided by -5. It is 3. The signs are the different, so the number is negative. Therefore, x =-2.



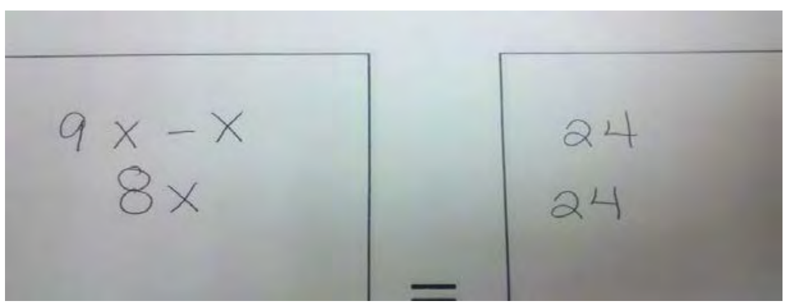


- Last step is to substitute the solution -3 in for X in the original problem. Original problem is -5X +2 = 1. On the left side, write -5(-2) because X = -2. Also write +5. On the right, write 15. On the right, what is 5 times 2? It is 10. The signs are the same; therefore the answer is positive. Positive 10 plus five equals 15. Are both sides equal? Yes. The answer is correct.



Model problem #2

- I read the next problem. 9X -X = 24
- On the left side of the mat, we are going write 9X – X. On the right side, write -24.
    - The first step in SUMLOWS is to separate the two sides. Draw a line through the equal sign.
    - What does the "U" stand for in SUMLOWS? Unite like terms. Are there any like terms or numbers that are the same that are not already together on the left side of the equal sign? Yes. Look on the right side. Are there any like terms or numbers that are the same that are not already together on the right side of the equal sign? No. Because there are like terms that are not together on the left side of the equal sign, we must unite (combine) like terms. We have 9X and -X. The signs are different, so we need subtract them. 9-1 (the- 1 in front of the X is invisible).Nine minus one is eight.
    - Modify the new equation. The new equation after combining the like terms is 8X = 24.
    - What is the next step? Loop around the variable. What is the variable? X. Draw a circle around the X. That is the number that should be by itself.
    - Is there anything else on the side with the circle? yes.



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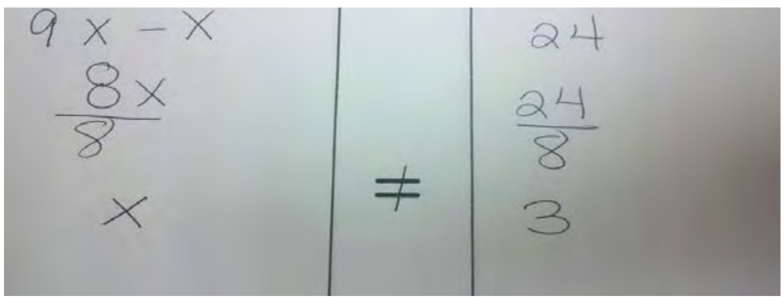
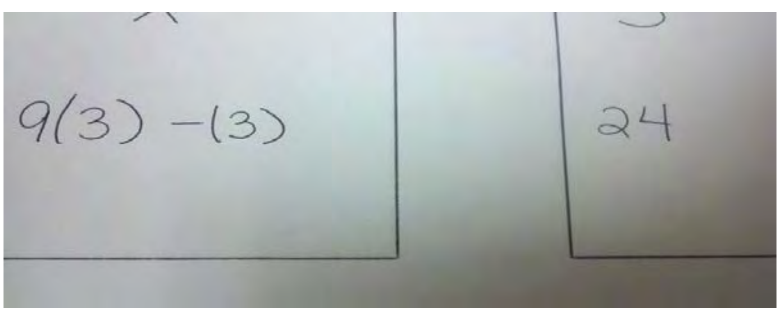
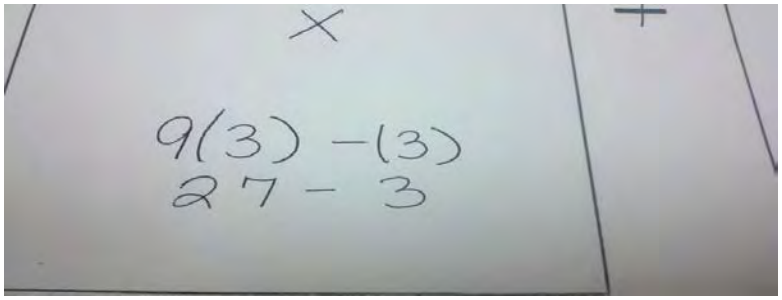
    - Now we have 8X =24. Is there only 1X? No., there are 8X; therefore we must determine what 1X equal to. I start back at the front of SUMLOWS mnemonic.
    - I have already separated the sides.
    - The "U" is for unite like terms. Look on the left side. Are there any like terms the same that are not already together? NO. Look on the right side. Are there any like terms the same that are not already together? No. We go on to step three.
    - Modify the new equation. There is not anything to unite or combine. So, skip this step too.

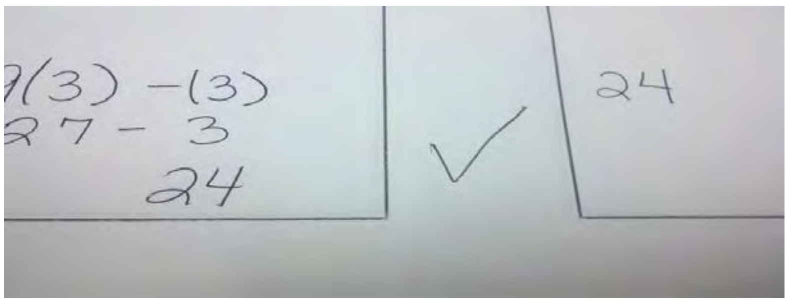
- What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the X. The variable should be by itself.
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a line under the 8 and write 8 under it.
- What we do to one side, we must do to the other. On the right side, draw a line under 24 and write 8 under it. On the right side, divide 8 into 24. Twenty-four divided by eight is 3. Both numbers are positive, therefore the answer is positive. X = 3.



- Last step is to substitute the solution 3 in for X in the original problem. Original problem is 9X -X = 24. On the left side, write 9(3) –3 because X = -3. On the right, write 24. On the right, what is 9 times 3? It is 29. What is 27 -3? 24. Are both sides equal? Yes. The answer is correct.

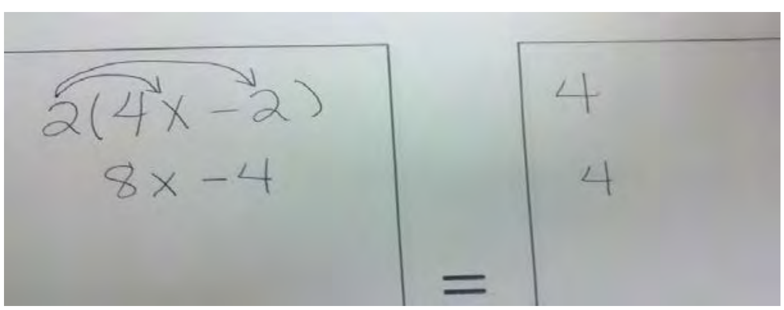




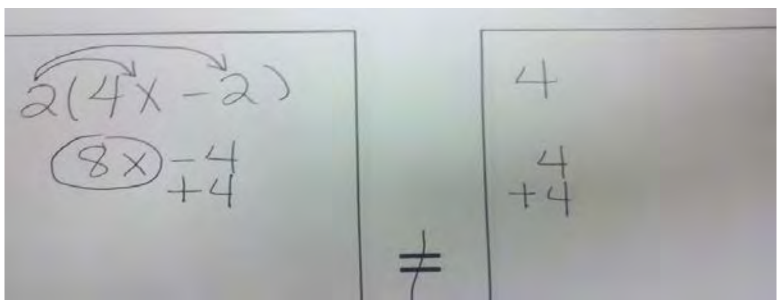


Model problem #3

- I read the next problem. 2(4X - 2) =4
- I make sure my mat or board is clear
- In the problem, notice the parenthesis ( ). This problem requires we to get rid of the parenthesis before we start SUMLOWS.
- What is in the parenthesis? 4X -2. What is outside the parenthesis? 2. This is the distributive property. What's outside the parenthesis has to be multiplied by everything inside the parenthesis. 2(4X) = 8X and 2(-2)= -4
- After the distributive property, we have 8X -4 on the left side and 4 on the right side.

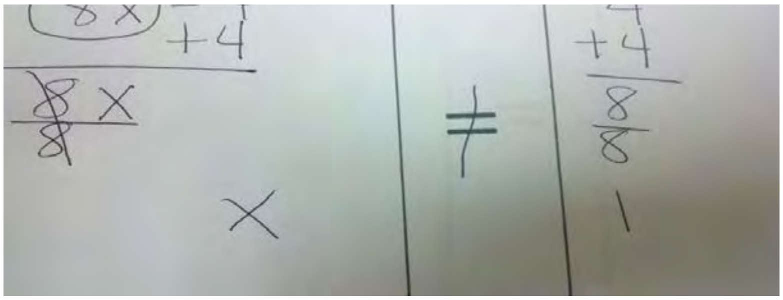


- Now we are ready to solve the equation using SUMLOWS
  - The first step in SUMLOWS is to separate the two sides. Draw a line through the equal sign.
  - The "U is for unite like terms. Look on the left side, are there any like terms or numbers that not already together? No. Look on the right side. Are there any like terms that are not together? No. We go to the next step.
  - Modify the new equation. There is not anything to unite or combine, so skip this step too.
  - The next step is Loop the variable. The variable is the X. This is the number that should be by itself and want to move last. Draw a circle around the 8X. What is on the side with the variable or X? -4. We have to get rid of -4 because X should be by itself.
  - In order to move the -4, we have write +4 under the -4. Opposites cancel each other out. Put a slash mark through -4 and +4. This equals 0 and cancels each other out.
  - The next step is "what we do to one side, we must do to the other" We added 4 on the left side, so we must add 4 on the right side. Write +4 under the 4.

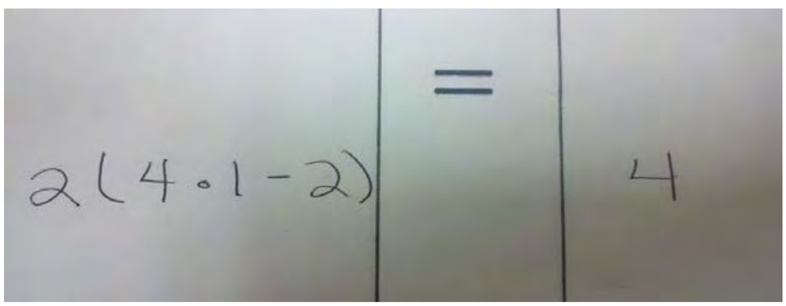


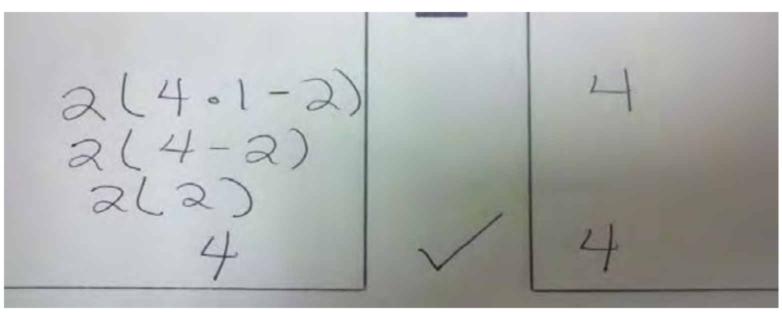
  - Combine like terms. On the right side, we have 4 and 4. Since both numbers are positive, we must add them. Four plus 4 is 8. The answer is positive. So 8X = 8.
  - Is there anything else on the side with the circle? No.
  - Now we have 8X =8. Is there only 1X? No., there are 8; therefore we must determine what 1X is equal to. I start back at the front of SUMLOWS mnemonic.
  - I have already separated the sides.
  - The "U" is for unite like terms. Look on the left side. Are there any like terms or numbers that can be combined that are not already together? NO. Look on the right side. Are there any like terms or numbers that can be combined that are not already together? No. We go on to step three.
  - Modify the new equation. There is not anything to unite or combine. So, skip this step too.
  - What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the X. That is the number that should be by itself.
  - Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a line under the 8 sign and write 8 under it.

- What we do to one side, we must do to the other. On the right side, draw a line under the 8 and write 8 under it. The left side cancels each other out. On the right side, we divide 8 divided by8. It is 1. The signs are the same, so the number is positive. Therefore, x =1.



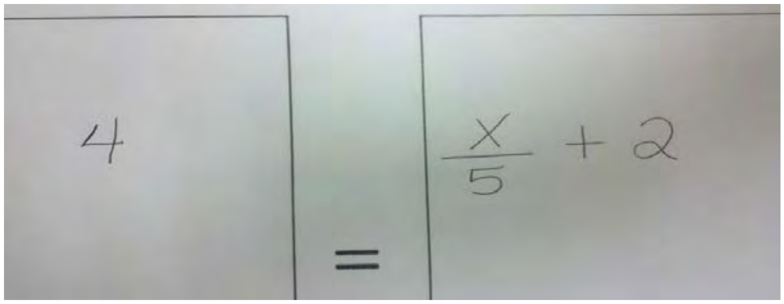
- Last step is to substitute the solution 1 in for X in the original problem. Original problem is 2(4X -2) = 4. On the left side, write 2(4(1)-2 because X = 1. We have to do what is in the parenthesis first. What is 4 times 1? 4. What is 4 minus 2? 2. What is 2 times 2? 4. On the left and right, we have 4. Are both sides equal? Yes. The answer is correct.



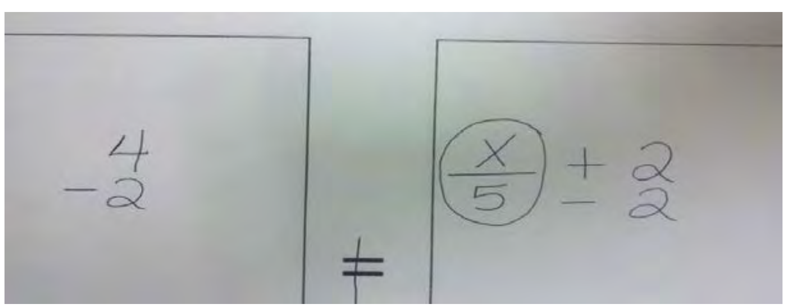


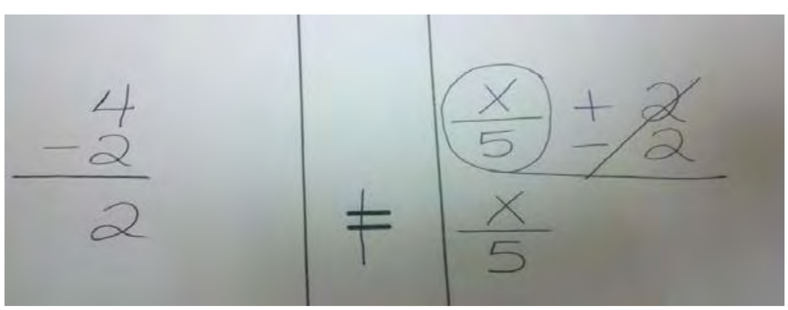
Model problem # 4

- I read the problem: $4 = \frac{X}{5} + 2$
- If the students flip the equation, the steps would be the opposite.
- I make sure the algebra mat is clear
- On the left side of the mat, write 4. On the right side, write X/5 + 2.

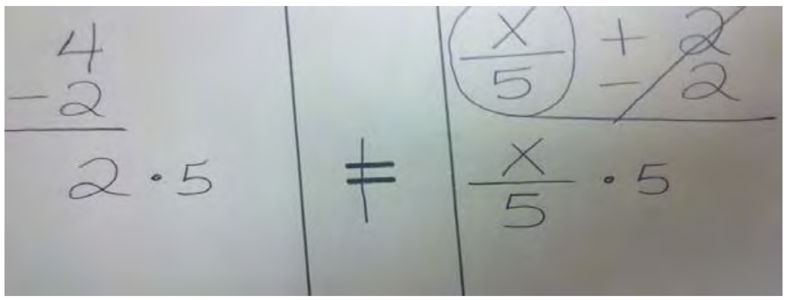


- Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
- Unite like terms. Are there any tiles the same shape that are not already together? No. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
- Modify the new equation. Since we did not combined any like terms, we skip this step.
- Loop around the variable. Draw a circle around the X/5. That is the number that should be by itself.
- Is there anything else on the side with the circle? Yes! There is +2. We have to get rid of the +2.
- Next step is opposite sign. The opposite of positive (plus) 2 is negative (minus) 2. We are going write -2 under the +2.
- What we do to one side, we must do to the other. Since we wrote -2 on the right side of the equation, we must write -2 on the left side of the equation. The right side squares cancels each other out (draw a slash mark in them). On the left side, we have 4 and -2 under it. We are going to combine like terms. These signs are different. Therefore, we have to subtract and take the sign of the greatest number (absolute value). Four minus 2 equals +2.

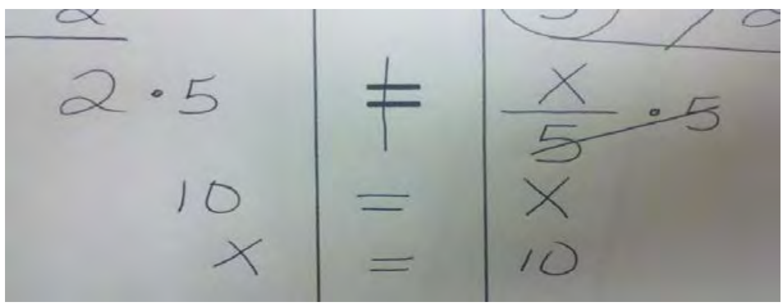




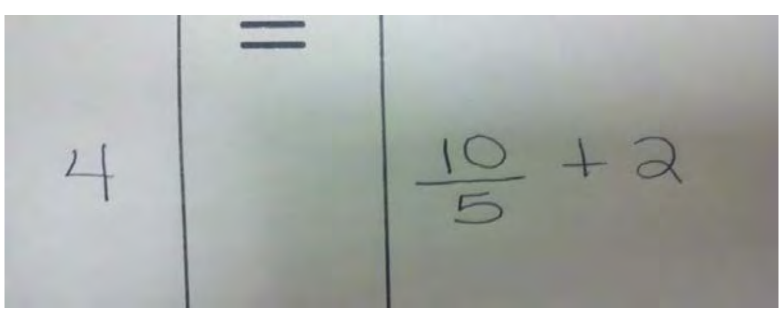
- On the algebra mat, we should have +2 on the left side. On the right side, we should have X over 5. The rectangles should have a circle around it.
- We have already separated the sides, united like terms, modified the equation, and looped the coefficient. The next step is opposite sign.
- When we have an X over a number, which means to divide. What is the opposite of division? It is multiplication. We are going to write a multiplication sign and write 5 beside it.
- Next step is what we do to one side; we must do to the other. Since we drew a multiplication sign and wrote 5 on the right side, we must do the same on the left side. On the left side, draw a multiplication sign and write 5.

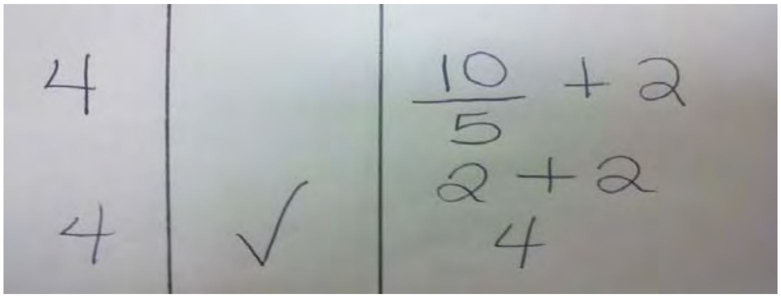


- The right side cancels each other out. On the left side, we are going multiply 2 times 5. So, X = 10.



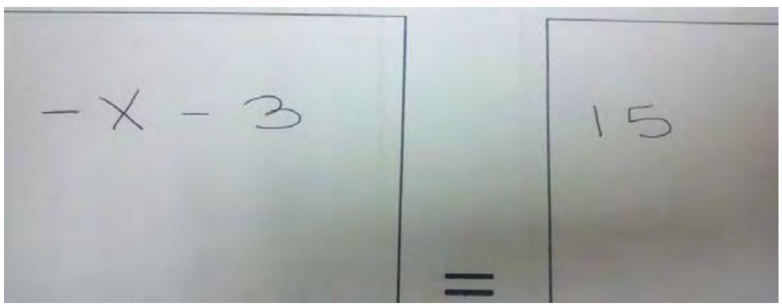
- Last step is to substitute the solution 10 in for X in the original problem. Original problem is 4= X/5 + 2. On the left side, write 4. On the right side, write 10 in them for X. Draw a line under 10 and write 5. Then write plus 2. What is 10 divided by 5? 2. What is 2 plus 2? 4. We have 4 on the left side and 4 on the right side. Are both sides equal? Yes. The answer is correct.



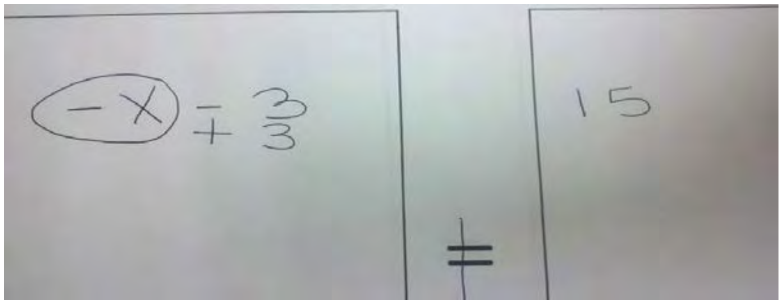


Guided Practice

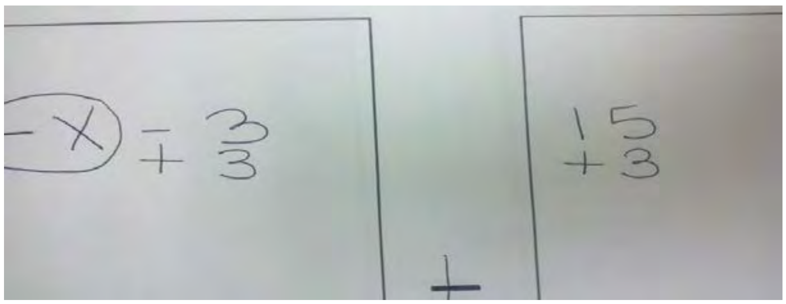
- Direct students to the "Guide" section of the learning sheet
- Tell students to touch the first problem and that we will do this problem together, using numbers, letters and the SUMLOWS mnemonic.
- Let's read the problem. This problem is -X - 3 = 15.
- On our workmat, what do write on the left side? -X -3.
- What do we write on the right side? 15.



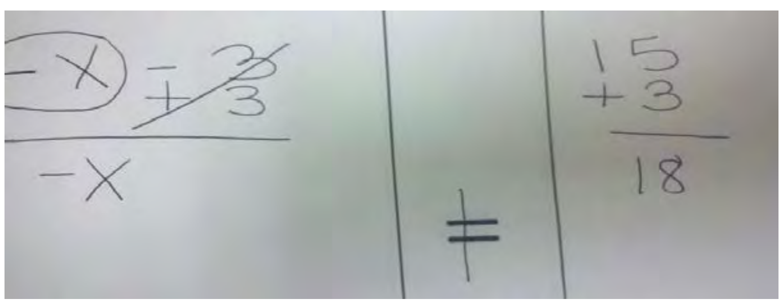
  - What is the first step in SUMLOWS? Separate the sides. We are going to draw a line through the equal sign.
  - The "U is for unite like terms. Look on the left side, are there any like terms or numbers that not already together? No. Look on the right side. Are there any like terms that are not together? No. We go to the next step.
  - Modify the new equation. There is not anything to unite or combine, so skip this step too.
  - The next step is Loop the variable. The variable is the X. This is the number that should be by itself and want to move last. Draw a circle around the -X. What is on the side with the variable? -3. We have to get rid of -3 because X should be by itself.
  - In order to move the-3, we have to write +3 under the -3. Opposites cancel each other out. Put a slash mark through -3 and +3. This equals 0 and cancels each other out.



- The next step is “what we do to one side, we must do to the other.” We added 3 on the left side, so we must add 3 on the right side. Write +3 under the 15.

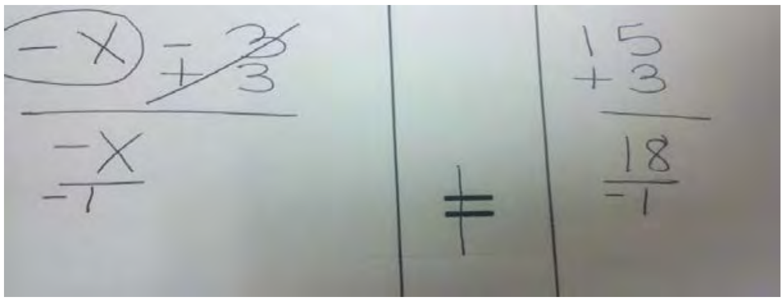


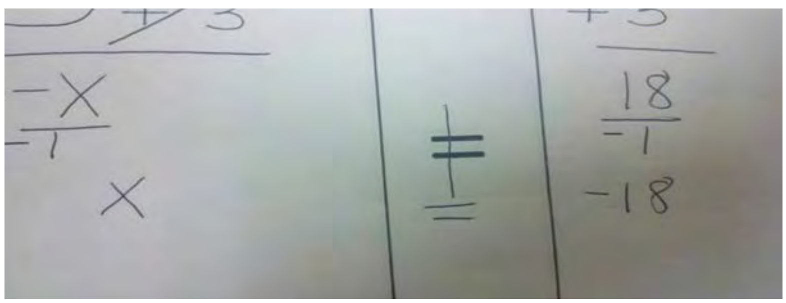
- Combine like terms. On the right side, we have15 and +3. Since both numbers have the same sign, we add the numbers and take the sign of the larger number (absolute value).   Fifteen plus three equals eighteen.   Since the larger number (absolute value) is 18 and it’s positive, the answer is positive. So -X = 18.



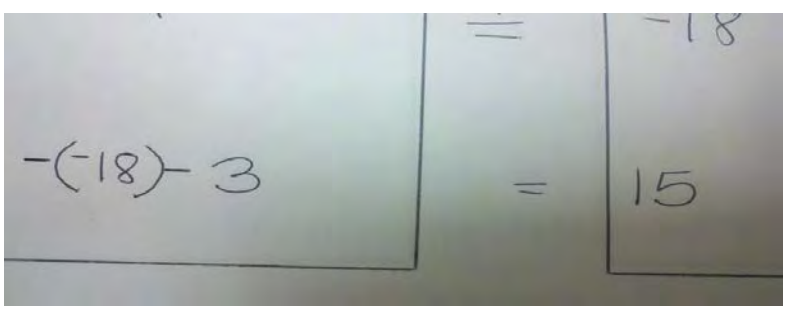
- Is there anything else on the side with the circle? Yes, -1 (the 1 is invisible).
- Now we have -X =18. Is there only 1X? Yes? Is the X positive? No, therefore we must determine what 1X is equal to. I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The “U” is for unite like terms. Look on the left side. Are there any like terms or numbers that can be combined that are not already together? NO. Look on the right side. Are there any like terms or numbers that can be combined that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.

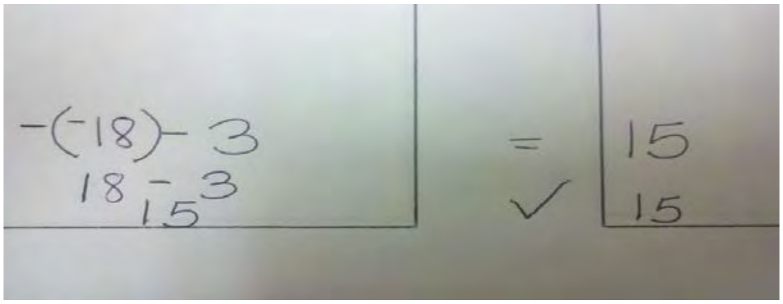
- What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the X. That is the number that should be by itself.
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a line under the -1 and write -1 under it.
- What we do to one side, we must do to the other. On the right side, draw a line under the 18 and write -1 under it. The left side cancels each other out. On the right side, we divide 18 divided by 1. It is 18. The signs are the different, so the number is negative. Therefore, x =-18





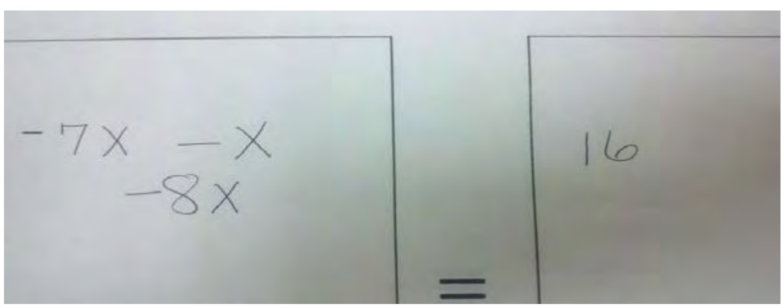
- Last step is to substitute the solution -18 in for X in the original problem. Original problem is –X -3 = 15. On the left side, write -(-18) because X = -18. Also write -3. On the right, write 15. On the left, what is -1 times -18? It is 18. The signs are the same; therefore the answer is positive. Positive 18 minus 3 equals 15. Are both sides equal? Yes. The answer is correct.





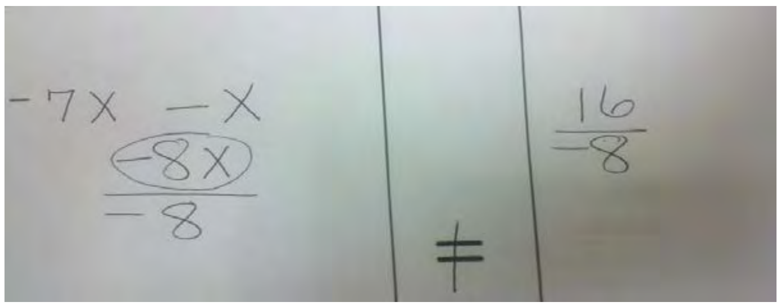
Guided problem # 2

- I read the next problem. -7X -X = 16
- On the left side of the mat, what are we are going write? -7 – X. On the right side, what we are going to write? -16.
    - The first step in SUMLOWS is to separate the two sides. Draw a line through the equal sign.
    - What does the “U” stand for in SUMLOWS? Unite like terms. Are there any like terms or numbers that are the same that are not already together on the left side of the equal sign? Yes. Look on the right side. Are there any like terms or numbers that are the same that are not already together on the right side of the equal sign? No. Because there are like terms that are not together on the left side of the equal sign, we must unite (combine) like terms. We have -7X and -X. The signs are the same, so we need add them. Seven plus one (the- 1 in front of the X is invisible) is eight. The 7 is negative, therefore the answer is negative.
    - Modify the new equation. The new equation after combining the like terms is -8X = 16.

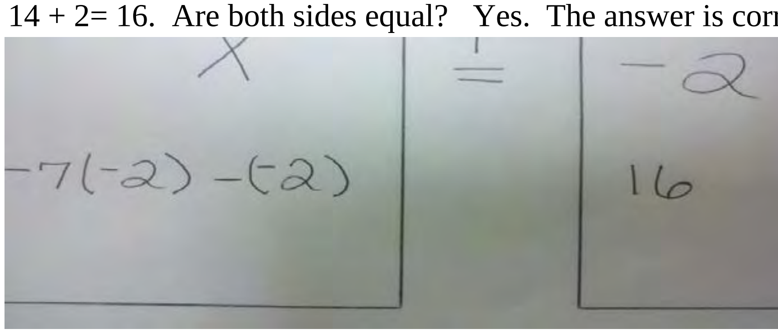


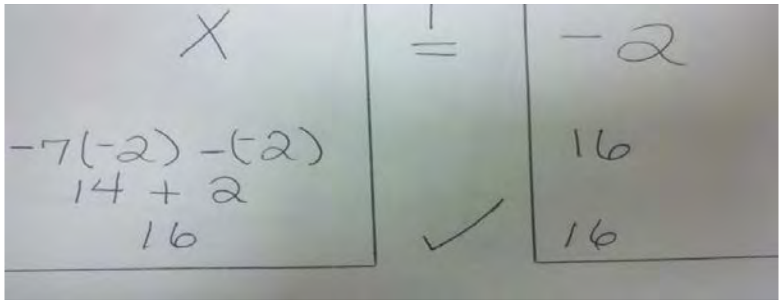
- What is the next step? Loop around the variable. What is the variable? X. Draw a circle around the X. That is the number that should be by itself.
- Is there anything else on the side with the circle? yes
- Now we have 8X =16. Is there only 1X? No., there are -8X; therefore we must determine what 1X equal to. I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.

- The “U” is for unite like terms. Look on the left side. Are there any like terms the same that are not already together? NO. Look on the right side. Are there any like terms the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the X. The variable should be by itself.
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a line under the -8 and write -8 under it.
- What we do to one side, we must do to the other. On the right side, draw a line under 16 and write -8 under it. On the right side, divide -8 into 16. Sixteen divided by eight is two. The numbers have different signs (one is positive and the other is negative), therefore the answer is negative. X = -2.



- Last step is to substitute the solution -2 in for X in the original problem. Original problem is -7X-X = 16. On the left side, write -7(-2) – (-2) because X = -2. On the right, write 16. On the right, what is 7 times 2? It is 14. They have the same signs, so it’s positive 14. When we minus a negative number, it makes a positive number. What is 14 + 2= 16. Are both sides equal? Yes. The answer is correct.

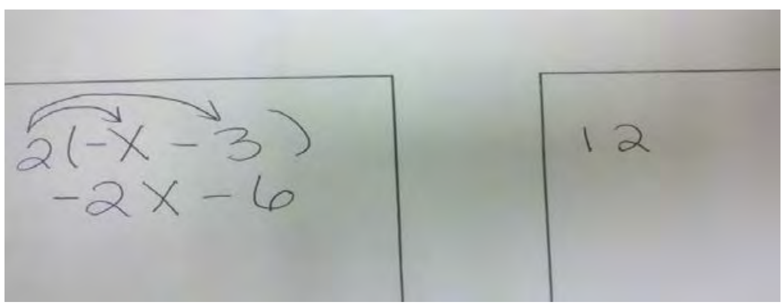




Guided problem #3

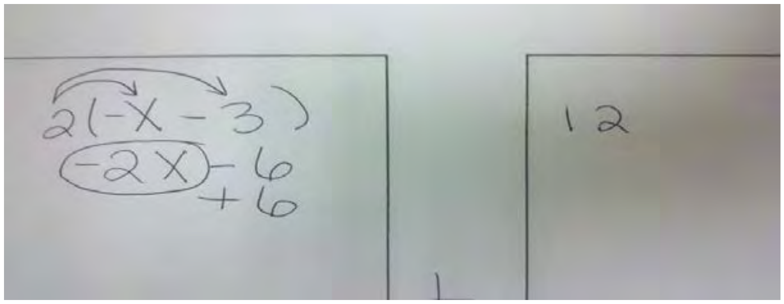
- I read the next problem. 2(-X - 3) =12
- I make sure my mat or board is clear
- In the problem, notice the parenthesis ( ). This problem requires we to get rid of the parenthesis before we start SUMLOWS.

- What is in the parenthesis? -X -3. What is outside the parenthesis? 2. This is the distributive property. What's outside the parenthesis has to be multiplied by everything inside the parenthesis. 2(-X) = -2X and 2(-3)= -6

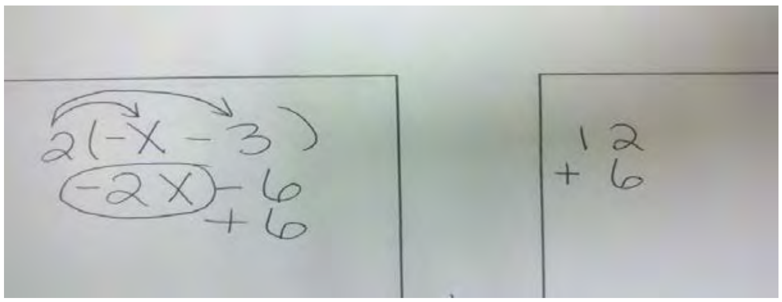


- After the distributive property, we have -2X -6 on the left side and 12 on the right side.
- Now we are ready to solve the equation using SUMLOWS

    - The first step in SUMLOWS is to separate the two sides. Draw a line through the equal sign.
    - The "U is for unite like terms. Look on the left side, are there any like terms or numbers that not already together? No. Look on the right side. Are there any like terms that are not together? No. We go to the next step.
    - Modify the new equation. There is not anything to unite or combine, so skip this step too.
    - The next step is Loop the variable. The variable is the X. This is the number that should be by itself and want to move last. Draw a circle around the -2X. What is on the side with the variable or X? -6. We have to get rid of -6 because X should be by itself.

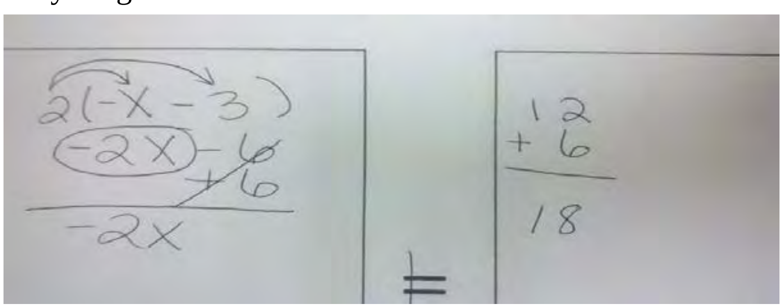


- In order to move the -6, we have write +6 under the -6. Opposites cancel each other out. Put a slash mark through -6 and +6. This equals 0 and cancels each other out.



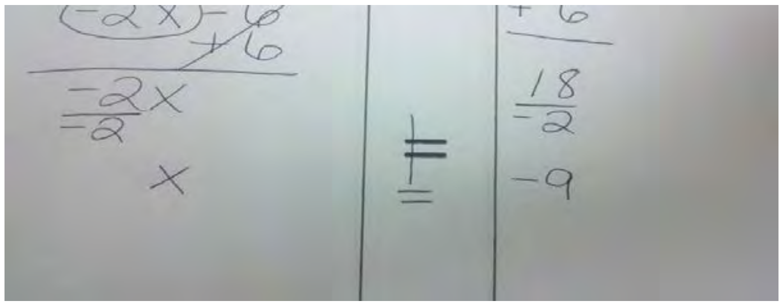
- The next step is "what we do to one side, we must do to the other" We added 6 on the left side, so we must add 6 on the right side. Write +6 under the 12.

- Combine like terms. On the right side, we have 12 and 6. Since both numbers are positive, we must add them. Twelve plus six is 20. The answer is positive. So -2X = 18.
- Is there anything else on the side with the circle? No.

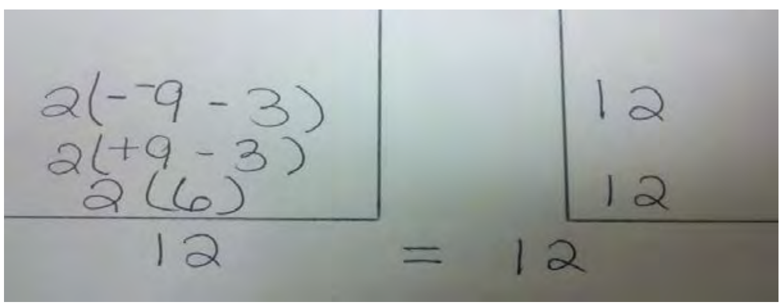


- Now we have -2X =18. Is there only 1X? No., there are -2; therefore we must determine what 1X is equal to. I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The "U" is for unite like terms. Look on the left side. Are there any like terms or numbers that can be combined that are not already together? NO. Look on the right side. Are there any like terms or numbers that can be combined that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.

- What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the X. That is the number that should be by itself.
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a line under the -2 sign and write -2 under it.
- What we do to one side, we must do to the other. On the right side, draw a line under the 18 and write -2 under it. The left side cancels each other out. On the right side, we divide 18 divided by -2. It is 9. The signs are different, so the number is negative. Therefore, x =-9.

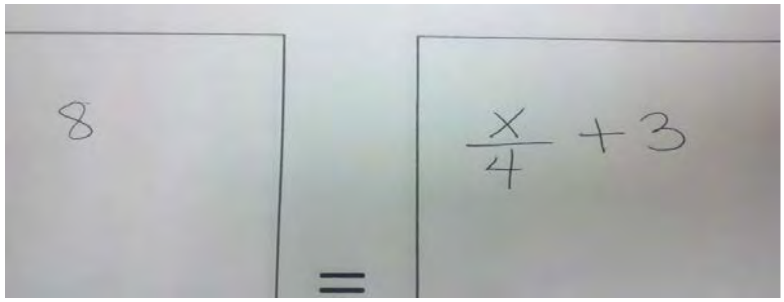


- Last step is to substitute the solution -9 in for X in the original problem. Original problem is 2(-X-3) = 12. On the left side, write 2[-(-9)-3]= 12. We have to do what is in the parenthesis first. Positive nine minus 3 equals 6. What is 2 times 6? 12. On the left and right, we have 12. Are both sides equal? Yes. The answer is correct.

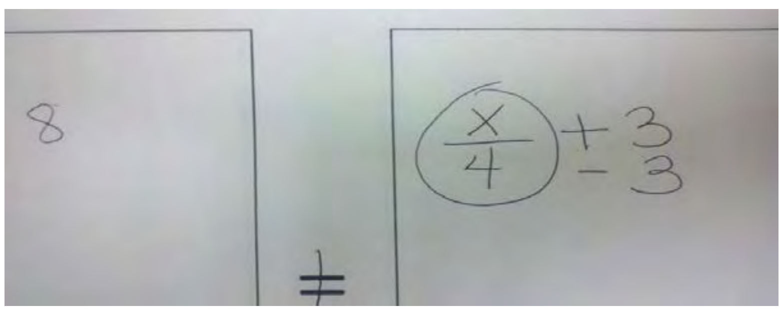


Guided problem # 4

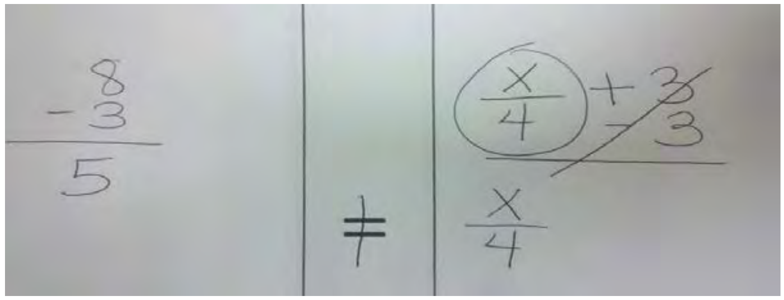
- I read the problem: $8 = \frac{X}{4} + 3$
- If the students flip the equation, the steps would be the opposite side.
- I make sure the algebra mat is clear
- On the left side of the mat, write 8. On the right side, write X/4 + 3.



- Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
- Unite like terms. Are there any tiles the same shape that are not already together? No. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
- Modify the new equation. Since we did not combined any like terms, we skip this step.
- Loop around the variable. Draw a circle around the X/4. That is the number that should be by itself.
- Is there anything else on the side with the circle? Yes! There is +3. We have to get rid of the +3.
- Next step is opposite sign. The opposite of positive (plus) 3 is negative (minus) 3. We are going write -3 under the +3.

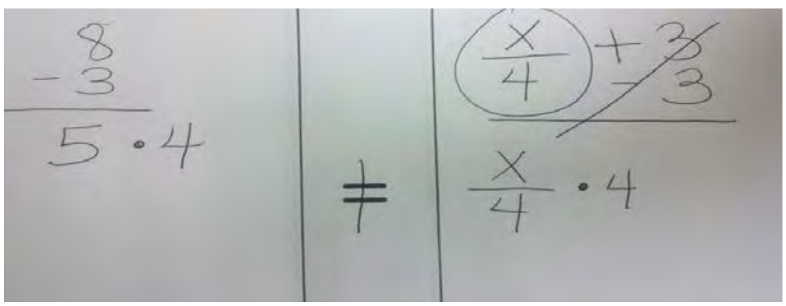


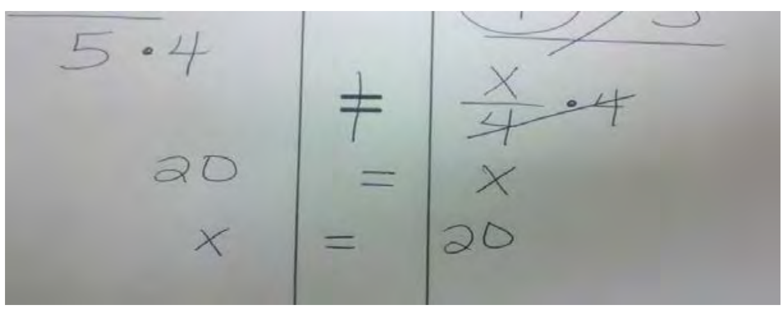
- What we do to one side, we must do to the other. Since we wrote -3 on the right side of the equation, we must write -3 on the left side of the equation. The right side squares cancels each other out (draw a slash mark in them). On the left side, we have 8 and -3 under it. We are going to combine like terms. These signs are different. Therefore, we have to subtract and take the sign of the greatest number (absolute value). Eight minus 3 equals +5.
- On the algebra mat, we should have +5 on the left side. On the right side, we should have X over 4. The rectangles should have a circle around it.
- We have already separated the sides, united like terms, modified the equation, and looped the coefficient. The next step is opposite sign.



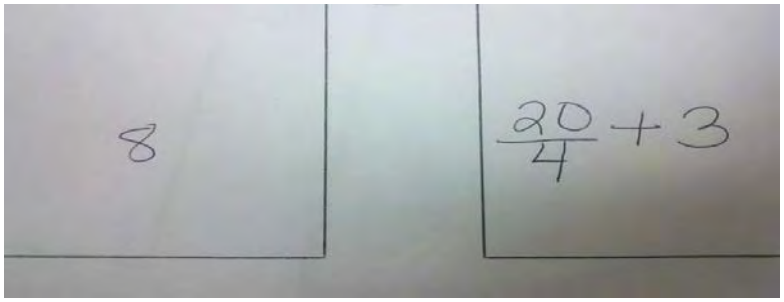
- When we have an X over a number, which means to divide. What is the opposite of division? It is multiplication. We are going to write a multiplication sign and write 4 beside it.
- Next step is what we do to one side; we must do to the other. Since we drew a multiplication sign and wrote 4 on the right side, we must do the same on the left side. On the left side, draw a multiplication sign and write 4.

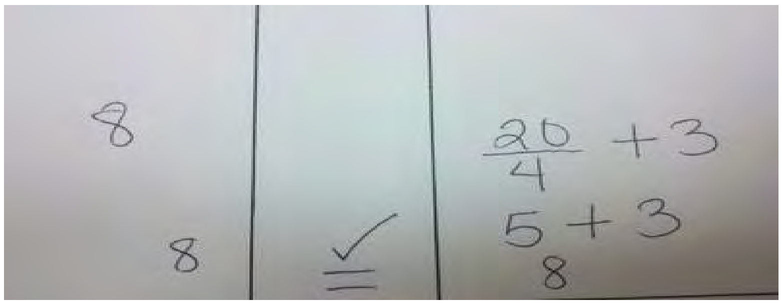
- The right side cancels each other out. On the left side, we are going multiply 5 times 4. So, X = 20.





- Last step is to substitute the solution 20 in for X in the original problem. Original problem is 8= X/4 + 3. On the left side, write 8. On the right side, write 20 in them for X. Draw a line under 20 and write 4. Then write plus 3. What is 20 divided by 4? 5. What is 5 plus 3? 8. We have 8 on the left side and 8 on the right side. Are both sides equal? Yes. The answer is correct.





Independent Practice

- Direct the students to the "Independent Practice" section. Read the first problem together and direct students to complete the problems without you.
  When students finish problems, provide immediate corrective feedback for errors.

Graphing

Learning Sheet 7

Two-Step
Abstract
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) -5X +5=15

2) 9X – X = 24

3) 2(4X – 2) = 4

4) $4 = \frac{X}{5} + 2$

Guided Practice

1) -X -3 = 15

2) -7X – X = 16

3) 2(-X – 3) = 12

4) $8 = \frac{X}{4} + 3$

Independent Practice

3(X+2) = 3

-X + 4X = 6

$5 = \frac{X}{2} + 10$

-2X +6 = 10

4X – 2X = 8

-2(X +3) = 10

Learning Sheet 8

Two-Step
Abstract
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) $4X - 1 = 11$

2) $\frac{X}{2} + 3 = -4$

3) $-4(X+1) = -16$

Guided Practice

1) $\frac{X}{2} - 2 = 4$

2) $-10 = -2(X - 3)$

3) $-2X + 4X = -8$

4) $-2X + 6 = 10$

Independent Practice

$7X - 2X = 20$

$3(X+2) = 3$

$5 = \frac{X}{2} + 10$

$-2X + 6 = 10$

$4X - 2X = 8$

$-2(X + 3) = 10$

Learning Sheet 9

Two-Step
Abstract
Code Number __________________ Date ______________
SUMLOWS

Describe and Model

1) -3(2X+1) = 9

Guided Practice

1) -4X – 3X = 14

2) -2(X +2) = 10

Independent Practice

-2X+5 = 11

6X – 3X = 6

-2(X +3) = 10

4) $2 + \frac{X}{4} = -1$

5) $\frac{X}{5} + 2 = -3$

3(X+2) = 3

Lessons 1-3
Multiple-Step

Concrete Level

Materials

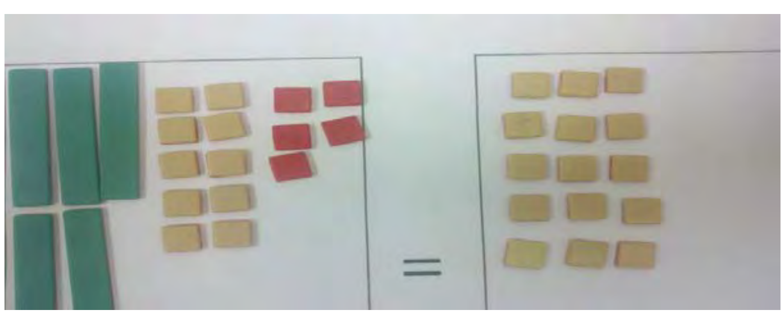
- o chalkboard/whiteboard/easel
- o algebra tiles to be used for concrete objects
- o learning sheet 1
- o progress chart

Advance Organizer

- Tell students what they will be doing and why
- Remind students about the commitment they made to learn to solve equations. We will work hard to teach them and they will work hard to learn a new way. They will learn to solve multiple step equations and the algebra tiles will help.

Demonstrate

- Give students SUMLOWS mnemonic and learning sheets. Wait to pass out the manipulative so that students do not become distracted.
- Begin with the first problem in the model section. Tell the student that we will show them how to solve the problem and that they will have a chance to solve problems also. State the expectations for behavior and attention to the demonstration.
- Begin with the first problem and think out aloud (see problem **model #1**)
  - o I Read the problem.
  - o This problem is 5X + 10 – 5 = 15. First, I take out my workmat or dry erase board and my mnemonic index card.
  - o Next, I set out the algebra tiles. On the left side, I place 5 green rectangle tiles (that means I have 5X). Then we will put down 10 cream squares and 5 red square tiles (red means we have a negative number and the cream side means that we have a positive number). Then on the right side, we will place 15 cream square tiles down (the 15 is positive that is why the tiles are on the cream side).

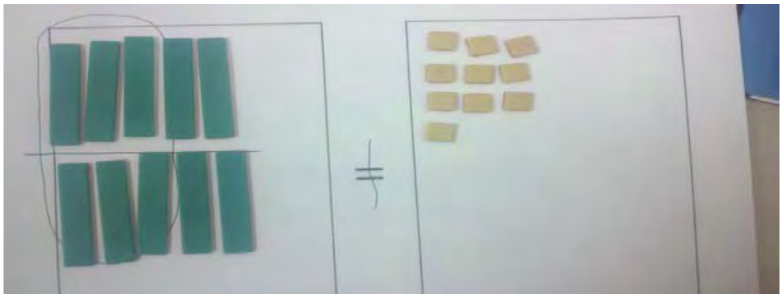


- The first step In the SUMLOWS is to separate the two sides. Draw a line through the equal sign.
- The "U" is for unite like terms. Look on the left side. Are there any tiles the same shape that are not already together? Yes. Look on the right side. Are there any tiles the same that are not already together? No. Because there are shapes that are not together, we must combine like terms. We have 10 cream squares and 5 red squares. They are not the same color, so we will have to subtract or pair them up and move them off the mat. Take one cream square and one red square and move them off the mat until there are no more pairs to make.
- Modify the new equation. How many squares are left on the left side of the equation? 5. What color are they? They are cream, therefore the number is positive. We are going to modify or rewrite the new equation 5X + 5 = 15.

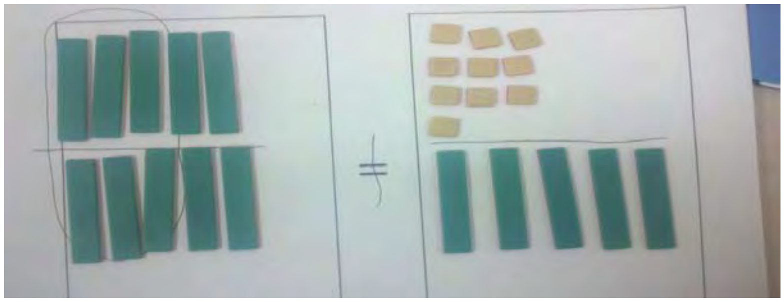
- The next step is Loop the variable. The variable is the rectangular tile. This is the number that we want to get by itself and want to move last. So what is on the side with the variable (the rectangle)? 5 red squares. We have to get rid of the 5 cream squares because the rectangles want to be by itself.
- In order to move the 5 cream squares, we have to do the next step which is the opposite sign. To get rid of the 5 cream squares, we must add 5 red squares to the left side of the equal sign. Opposites cancel each other out (+5 – 5 = 0).

- Next step is what we do to one side we must do to the other. We added 5 red squares to the left. We must add 5 red squares to the right side. On the left side, each cream square cancel out each red square (move them off the board or mat). On the right side, we have 15 cream squares and 5 red squares. These are the same shape but different colors; therefore we must subtract or pair a cream tile with a red tile. Pair the red tiles with the cream tiles until there are no more pairs to make. When we pair them up, move them from the board. How many tiles are left on the right side? We have 10 cream tiles.

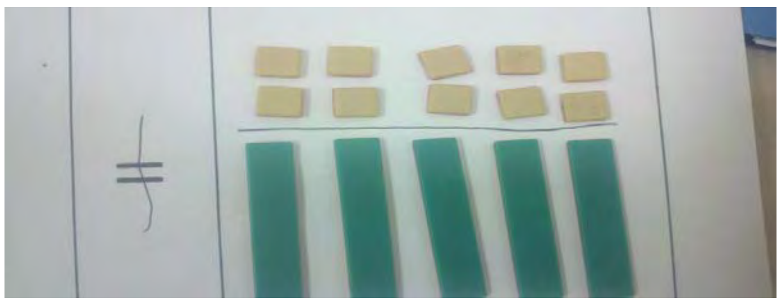
- Now we have 5X = 10. Is the X by itself? No. The X has a 5 with it. We must move the 5 from the X because the X should be by itself.
- I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The “U” is for unite like terms. Look on the left side. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- The next step is Loop the variable. The variable is the rectangular tile. This is the number that we want to get by itself and want to move last.
- Is there anything else on the side with the circle? No. So, we have to see what 1 X is equal to?
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division symbol beside the 5 green rectangles on the left side.



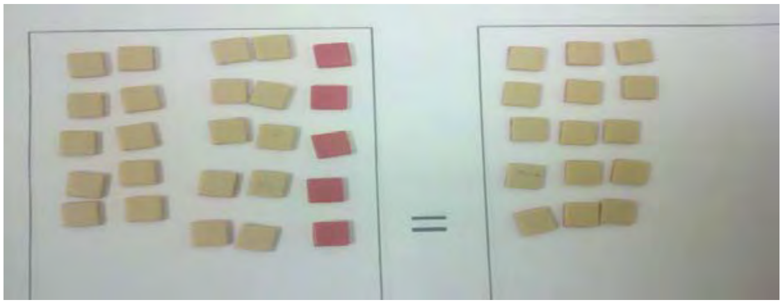
- What we do to one side, we must do to the other. Draw a division symbol beside the 10 cream tiles on the right side. The left side cancels each other out.



- On the right side, separate the rectangles. Give each rectangle a square until there are no more squares to give. How many squares does each rectangle have? 2. The color symbols are both positive, therefore it's a positive 2. X = 2. Is the variable by itself? Yes. Is it positive? Go to the last step.

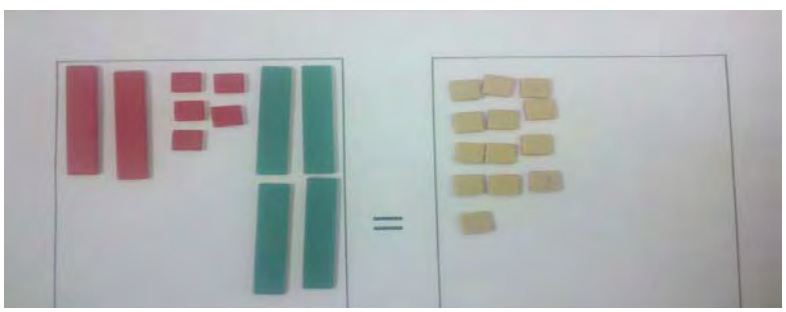


- Last step is to substitute the solution 2 in for X in the original problem. Original problem is 5X + 10 - 5 =15. On the left side, put down 3 cream tiles for every X. There are 5 X, so we will put down 10 cream tiles. Also on the left side we will put 10 cream tiles and 5 red tiles. On the right side, put down 15 cream tiles. Pair up the squares on the left side until we cannot make any more pairs. What is left? We have 15 cream tiles on the left side. We also have 15 cream tiles on the right side. They are the same. The answer is correct.



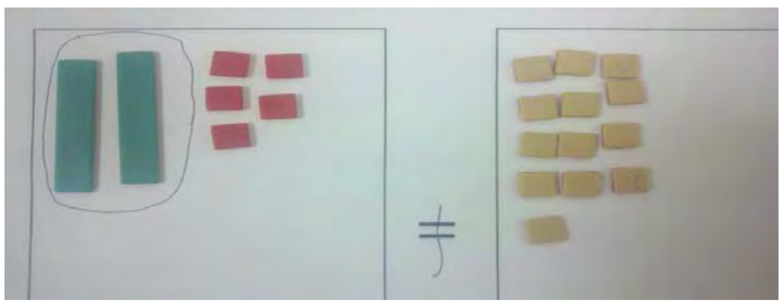
Model problem #2

- I read the next problem. -2x – 5 + 4X = 13
- I make sure my mat or board is clear
- On the mat, place 2 red rectangular tile (green is positive and red is negative) on the left side. Also place 5 red squares and 4 rectangular tiles. On the right side, place 13 cream square tiles (cream is positive and red is negative).

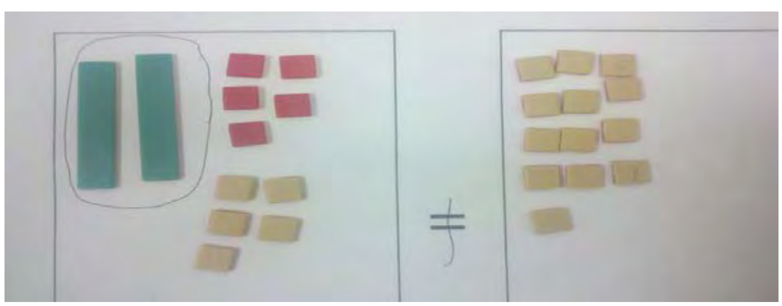


- Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
- Unite like terms. Are there any tiles the same shape that are not already together? Yes. We must combine the 2 red rectangular tiles and the 4 green rectangular tiles. The colors are different, so we must subtract or pair them up and move them off the mat. Look on the right side. Are there any tiles the same that are not already together? No. Since there are shapes on We go on to step three.
- Modify the new equation. Since we combined the left side, how many rectangles are left? 2. What color are they? They are cream, so they are positive. Our new equation is 2X -5 =13. We should have 2 green rectangular tiles and 5 red squares on the left side. We have 13 cream square tiles on the right side.

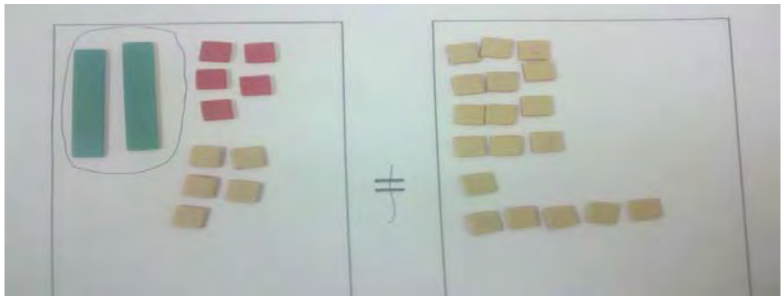
- Loop around the variable. Draw a circle around the 2 green rectangular tiles. That is the number that should be by itself.
- Is there anything else on the side with the circle? Yes, there are 5 red squares. We are going to have to move the 5 red squares because the variable should be by itself.



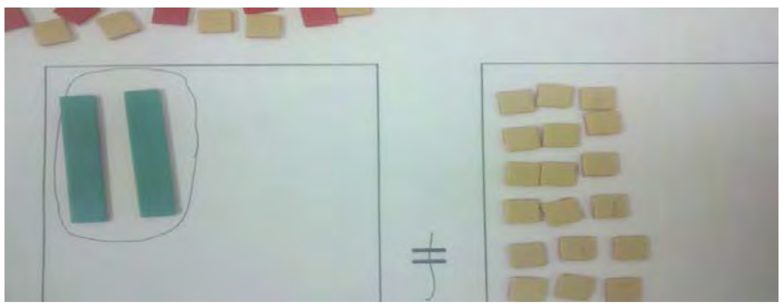
- Next step is opposite sign. In order to move the 5 red squares, we must add 5 cream squares to the left side.



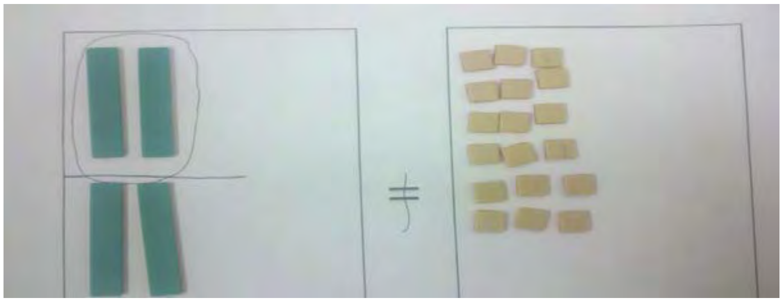
- What we do to one side, we must do to the other. The left side cancels each other out (-5 + 5 = 0). Since we added 5 cream squares on the left side, we must add 5 cream squares to the right side.



- On the right side, we have 13 cream squares and 5 cream squares. These are the same shape and the same color, therefore we must add (combine) the tiles. When we add them. How many tiles are on the right side? We have 18 cream tiles.
- Now we have 2X = 18. Is there only 1 rectangle? No. There are 2 rectangles. We must find out what 1 rectangle is equal to?



- I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The “U” is for unite like terms. Look on the left side. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- The next step is Loop the variable. The variable is the rectangular tile. This is the number that we want to get by itself and want to move last.
- Is there anything else on the side with the circle? No. So, we have to see what 1 X is equal to?
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division symbol beside the2 green rectangles on the left side.



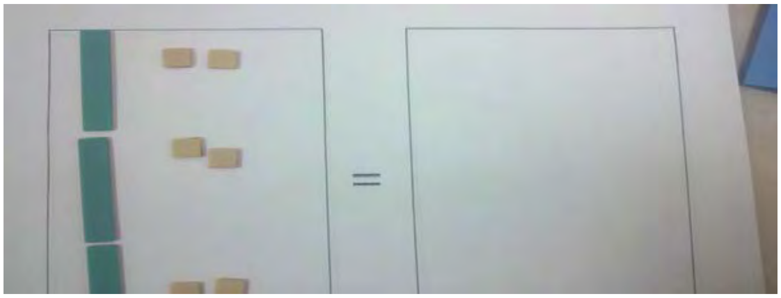
- What we do to one side, we must do to the other. Draw a division symbol beside the 18 cream tiles on the right side. The left side cancels each other out.

- On the right side, separate the rectangles. Give each rectangle a square until there are no more squares to give. How many squares does each rectangle have? 9. The color symbols are both positive, therefore the answer is positive 9. X =9. Is the variable by itself? Yes. Is it positive? Go to the last step.

- Last step is to substitute the solution 9 in for X in the original problem. Original problem is -2X -5 + 4X = 13. For -2X, put down 18 red tiles. The tiles are red because the numbers (-2 and 9) have different signs. Also on the left side we will put 5 red tiles. For 4X, put down 36 cream squares (two squares for every X). These tiles are cream because both numbers (4 and 9 are the same, therefore the number is positive. Pair up the squares on the left side until we cannot make any more pairs. What is left? We have 13 cream tiles on the left side. On the right side, put down 13 cream tiles. They are the same. The answer is correct.

Model problem #3

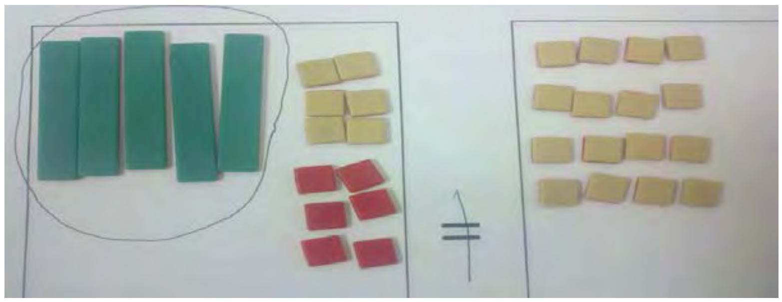
- I read the next problem. 3(X + 2) +2X =16
- I make sure my mat or board is clear
- In the problem, notice the parenthesis ( ). This problem requires we to get rid of the parenthesis before we start SUMLOWS.
- What is in the parenthesis? X + 2. What is outside the parenthesis? 3. On the mat, place tiles that represent X + 2. We are going to place a 1 green rectangular tile and 2 cream square tiles. The number outside the parenthesis tells us how many times we need to put X + 2 on the mat. In this case, we put X + 2 on the mat 3 times.



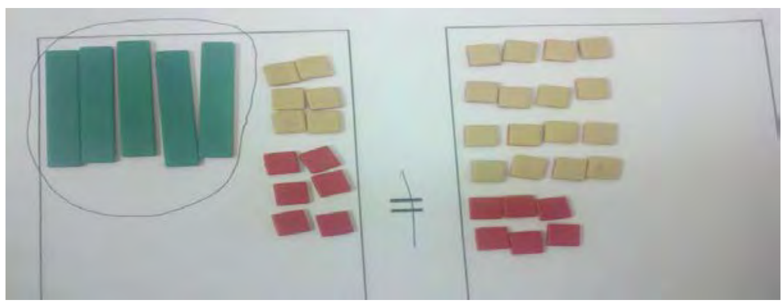
- How many rectangles are? 3 green rectangles
- How many squares are? 6 cream squares

- Now we are ready to solve the equation using SUMLOWS
- In addition to the tiles that we already have on the mat, place 2 green rectangular tiles (green is positive and red is negative) on the left side. On the right side, place 16 cream square tiles (cream is positive and red is negative). On the left side, we should have 3 green rectangles, 6 green squares, and 2 more green rectangles.
  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any tiles the same shape that are not already together? Yes. We must combine the 3 green rectangular tiles and the 2 green rectangular tiles. The colors are the same, so we must add or combine them. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
  - Modify the new equation. Since we combined the left side, how many rectangles are there? 5. What color are they? They are green, so they are positive. Our new equation is 5X +6 =10. We should have 5 green rectangular tiles and 6 cream squares on the left side. We have 10 cream square tiles on the right side.

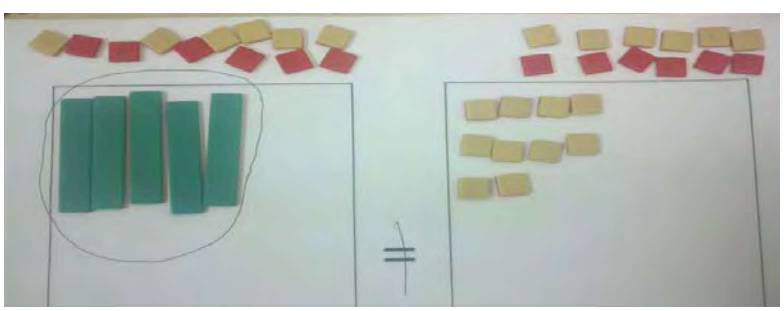
- Loop around the variable. Draw a circle around the 5 green rectangular tiles. That is the number that should be by itself.
- Is there anything else on the side with the circle? Yes, there are 6 cream squares. We are going to have to move the 6 cream squares because the variable should be by itself.
- Next step is opposite sign. In order to move the 6 cream squares, we must add 6 red squares to the left side.



- What we do to one side, we must do to the other. The left side cancels each other out (+6 -6 = 0). Since we added 6 red squares on the left side, we must add 6 red squares to the right side.

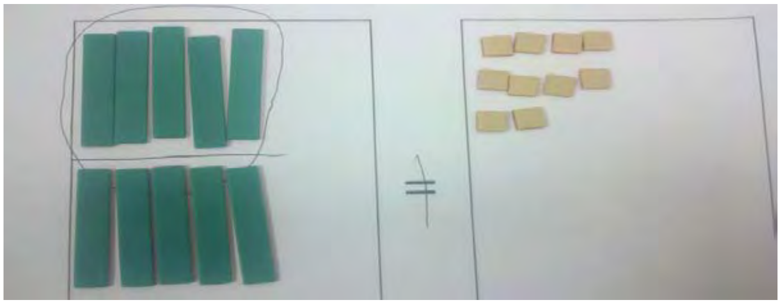


- On the right side, we have 16 cream squares and 6 red squares. These are the same shape but a different color; therefore we must subtract or pair the tiles and move them off the mat. How many tiles do we left on the right side? We have 10 cream tiles.
- Now we have 5X = 10. Is there only 1 rectangle? No. There are 5 rectangles. We must find out what 1 rectangle is equal to?

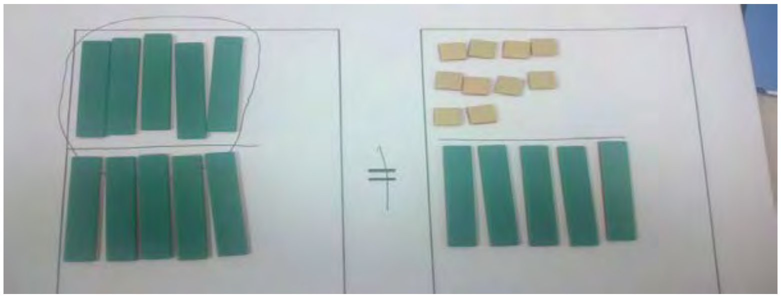


- I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.

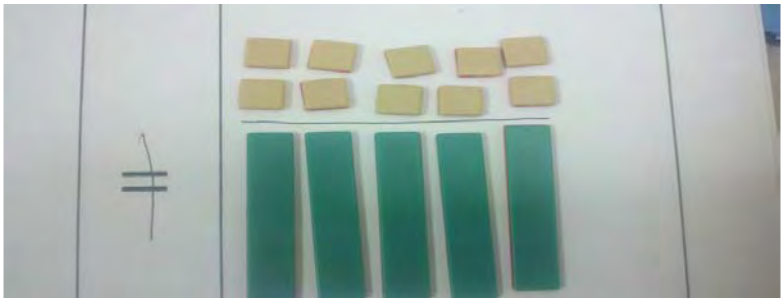
- The “U” is for unite like terms. Look on the left side. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- The next step is Loop the variable. The variable is the rectangular tile. This is the number that we want to get by itself and want to move last.
- Is there anything else on the side with the circle? No. So, we have to see what 1 X is equal to?
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division symbol beside the 5 green rectangles on the left side.



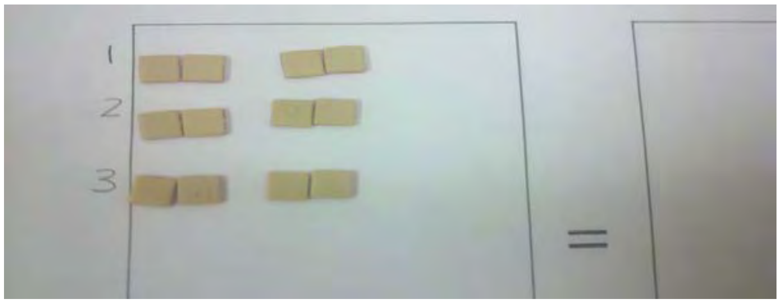
- What we do to one side, we must do to the other. Draw a division symbol beside the 10 cream tiles on the right side and add 5 green rectangles. The left side cancels each other out.

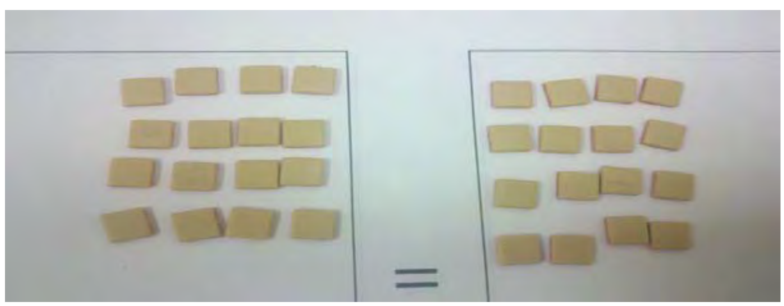


- On the right side, separate the rectangles. Give each rectangle a square until there are no more squares to give. How many squares does each rectangle have? 2. The color symbols are both positive, therefore the answer is positive 2. X =2. Is the variable by itself? Yes. Is it positive? Go to the last step.



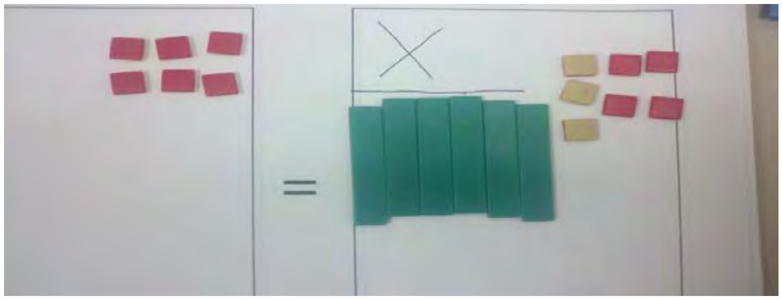
- Last step is to substitute the solution 2 in for X in the original problem. Original problem is 3(X +2) + 2X = 16. For X+2, put 4 cream squares down (2 times 2 = 4). We are going to put these 4 cream squares down 3 times for a total of 12 cream tiles. For 2x, put down 4 more cream square (2 X2 = 4). What is on the left side? We have 16 cream tiles on the left side. On the right side, put down 16 cream tiles. They are the same. The answer is correct.



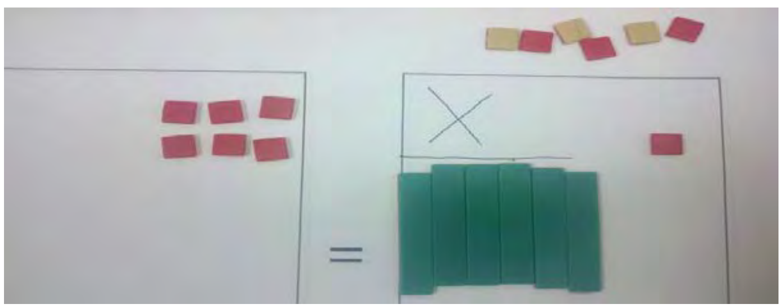


Model problem # 4

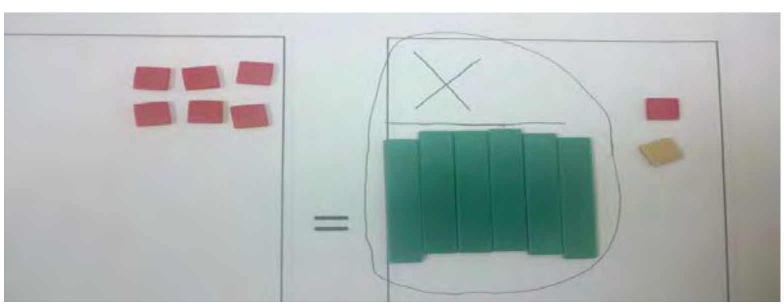
- I read the problem: $-6= \frac{X}{6} +3- 4$
- If the students flip the equation, the steps would be the opposite side.
- I make sure the algebra mat is clear
- On the left side of the mat, place 6 red square tiles (green is positive and red is negative). On the right side, draw an X with a line under it and put 6 green rectangular tiles under the line. On top of the line draw an X. Also on the right side, put 3 cream tiles and 4 red tiles down (cream is positive and red is negative).



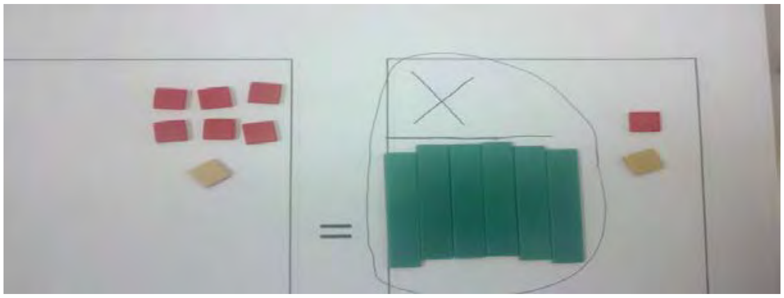
- Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
- Unite like terms. Are there any tiles the same shape that are not already together on the left side? No.   Look on the right side. Are there any tiles the same that are not already together? Yes. We have 3 cream tiles and 4 red tiles. We will pair them up (one cream with one red) and move them off the mat.
- Modify the new equation. How many tiles are left on the right side? 1 red square tile.

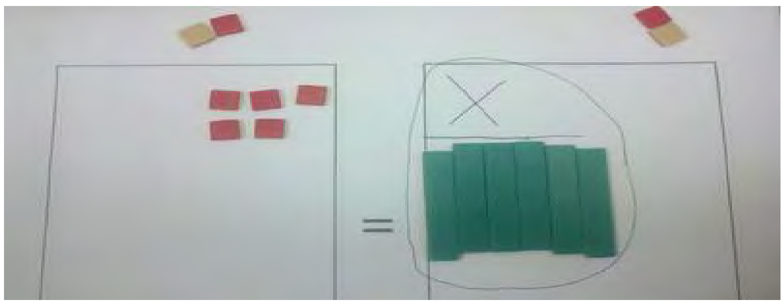


- Loop around the variable. Draw a circle around the 6 green rectangular tiles. That is the number that should be by itself.
- Is there anything else on the side with the circle? Yes! There is 1 red square. We have to get rid of the 1 red square.
- Next step is opposite sign. The opposite of negative (minus) 1 is positive (plus) 1. We are going to add 1 cream square under the 1 red square.

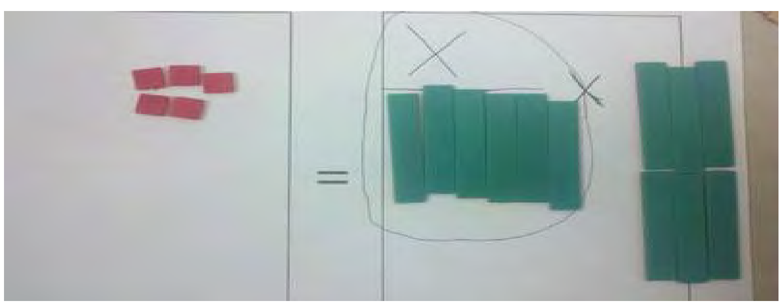


- What we do to one side, we must do to the other. Since we put 1cream square on the right side of the equation, we must place 1 cream square on the left side of the equation. The right side squares cancels each other out. On the left side, we have 1cream square and 6 red squares. We are going to add like terms. Take one cream and one red square and move them off the mat until there are no more pairs to make. On the left side, we should have 5 red squares left.

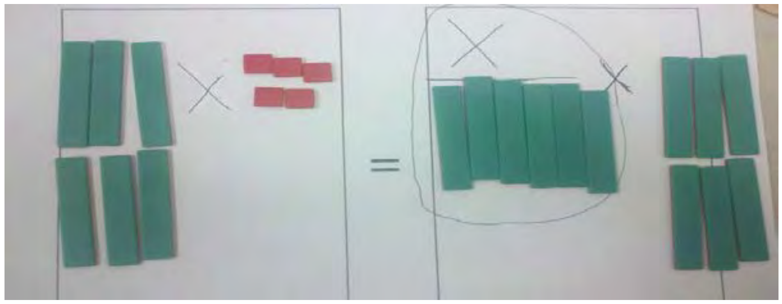




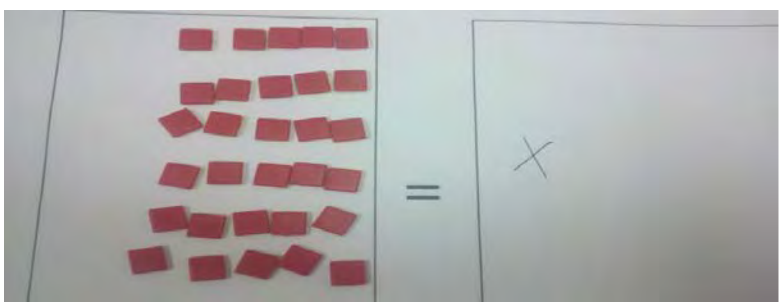
- On the algebra mat, we should have 5 red squares on the left side. On the right side, we should have X over 6 green rectangles. The rectangles should have a circle around it.
- We have already separated the sides, united like terms, modified the equation, and looped the coefficient. The next step is opposite sign.
- When we have an X over a number, which means to divide. What is the opposite of division? It is multiplication. We are going to write a multiplication sign and place 6 green rectangles beside it.



- Next step is what we do to one side; we must do to the other. Since we drew a multiplication sign and placed six green rectangles on the right side, we must do the same on the left side. On the left side, draw a multiplication sign and place 6 green rectangles.



- We are going to give all of the 5 red squares to the first green rectangle. Next, add 5 red squares for the 2nd, 3rd, 4th, 5th, and 6th rectangles. How many squares are all together? We have 30 red squares. So, X = -30.

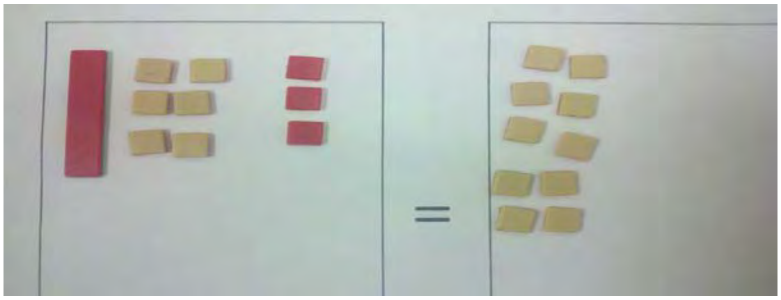


- Last step is to substitute the solution -30 in for X in the original problem. Original problem is -6= X/6 +3-4. On the left side, put 6 red squares. On the right side, put30 red squares for X. Draw a line under the squares and place 6 green rectangles under the line. Separate the rectangles and give each rectangle a square until they are all gone. How many squares does each rectangle have? 5 red squares. Move the 6 rectangles and 25 red squares off the mat (because 30 divided 6=5). On the mat, we should have 6 red squares on the left side. On the right side, we should have 5 red squares, 3 cream squares, and 4 red squares. On the right side, combine the squares. Pair up a cream square with a red square and move them off the mat until there are no more pairs to make. How many squares are on the right side? 6 red squares. How many squares are on the left side? 6 red squares. The numbers are the same. Are both sides equal? Yes. The answer is correct.

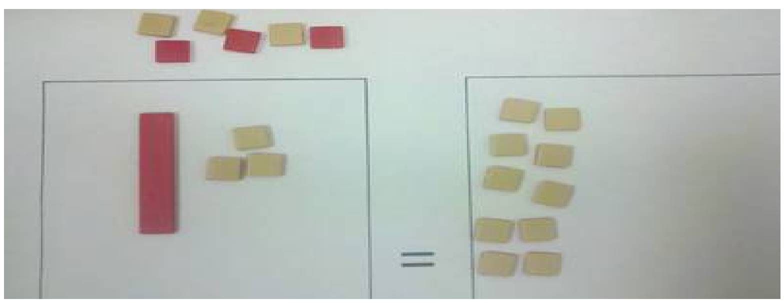
Guided Practice

- Direct students to the "Guide" section of the learning sheet
- Tell students to touch the first problem and that we will do this problem together, using numbers, letters and the SUMLOWS mnemonic.
- Let's read the problem. This problem is -X +6 - 3 = 10

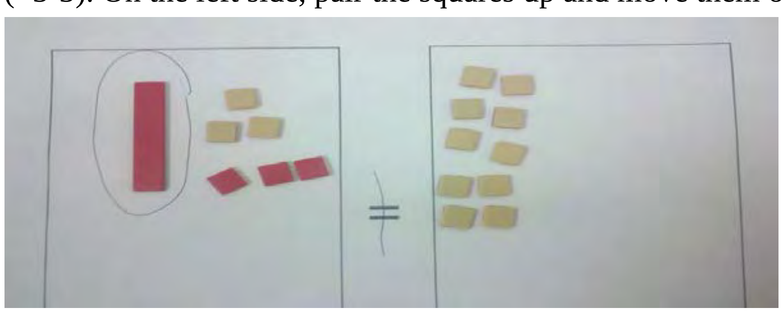
- On our workmat, what do we place on the left side? 1 red rectangular tile, 6 cream tiles, and 3 red tiles.
- What do we place on the right side? 10 cream tiles.



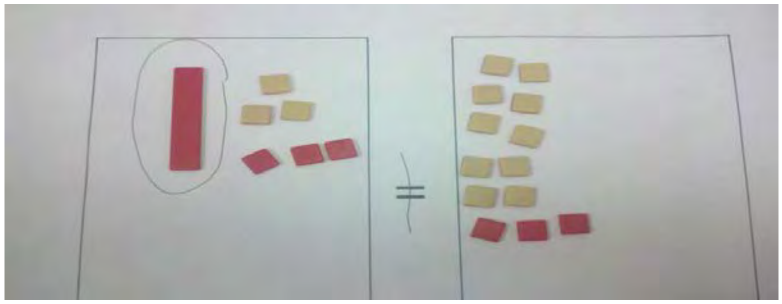
  - What is the first step in SUMLOWS? Separate the sides. We are going to draw a line through the equal sign.
  - The "U is for unite like terms. Look on the left side, are there any shapes that not already together? Yes. We have six cream squares and 3 red squares that can be combined. They are the same shape but not the same color. Since they are not the same color, we subtract or pair the tiles up and move them off the mat. How many tiles are left on the left side? 3 cream tiles. Look on the right side. Are there any shapes that can be combined together? No. We go to the next step.



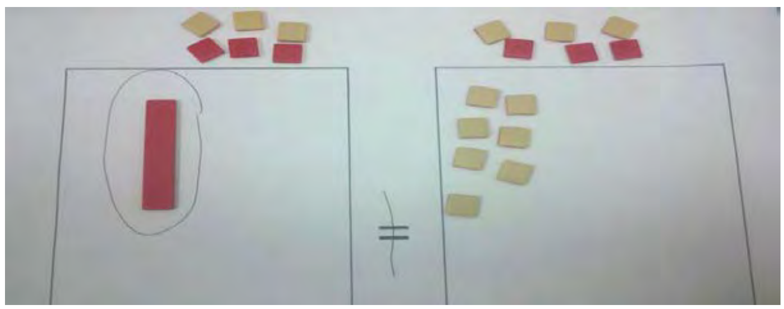
  - Modify the new equation. On the mat, we have 1 red rectangle and 3 cream tiles on the left side. On the right side, we have 10 cream tiles.
  - The next step is Loop the variable. The variable is the X. This is the number that should be by itself and want to move last. Draw a circle around the rectangle. What is on the side with the variable? 3 cream squares. We have to move the 3 cream tiles.
  - In order to move the3 cream tiles, we have to add 3 red tiles. Opposites cancel each other out (+3-3). On the left side, pair the squares up and move them off the mat.



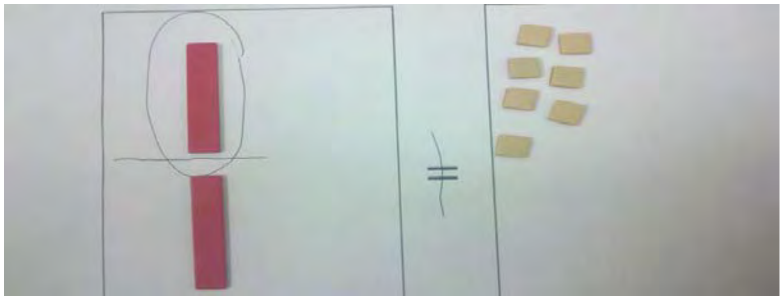
- The next step is "what we do to one side, we must do to the other." We added 3 red squares on the left side, so we must add 3 red squares on the right side.



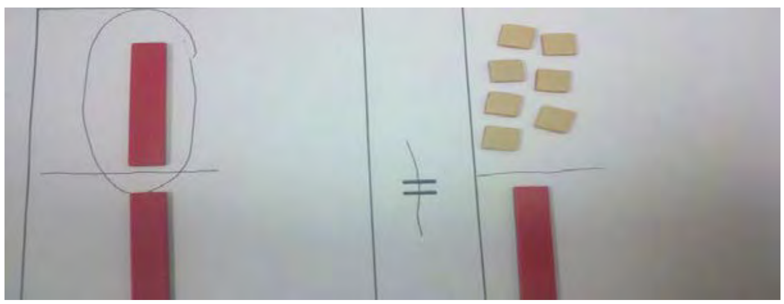
- Combine like terms. On the right side, we have10 cream squares and 3 red squares. Since the squares are different colors, we subtract or pair them up and move them off the mat. How many squares are left on the right side? 7. What color are they? Cream (positive). So -X = 7.

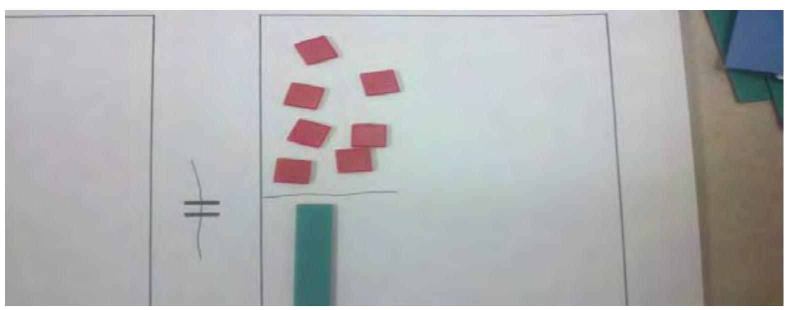


- Is there anything else on the side with the circle? Yes, -1 (the 1 is invisible).
- Now we have -X =1. Is there only 1X? Yes? Is the X positive? No, therefore we must determine what a green rectangle is equal to. I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The "U" is for unite like terms. Look on the left side. Are there any like terms or numbers that can be combined that are not already together? NO. Look on the right side. Are there any like terms or numbers that can be combined that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- What is the next step? Loop around the variable. Draw a circle around the variable. That is the number that should be by itself
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division symbol beside the -1 and add a red rectangle.

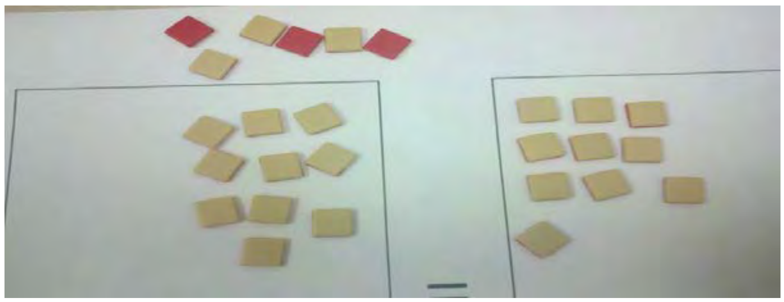


- What we do to one side, we must do to the other. On the right side, draw a division symbol beside the 7 cream tiles and add a red rectangle. The left side cancels each other out, which leaves the variable positive. On the right side, what does 1 rectangle equal to? 7. Do they have the same color symbol? No. Therefore, the answer is negative, x =-7.





- Last step is to substitute the solution -7 in for X in the original problem. Original problem is -X +6 - 3 = 10. On the left side, put down 7 cream tiles (two negatives equals a positive).Also place 6 cream tiles and 3 red tiles. On the right side, place 10 cream tiles. Pair up the tiles on the left side. How many tiles are left? 10 cream tiles. How many are on the right side? 7 cream tiles. They are the same, so the answer is correct.

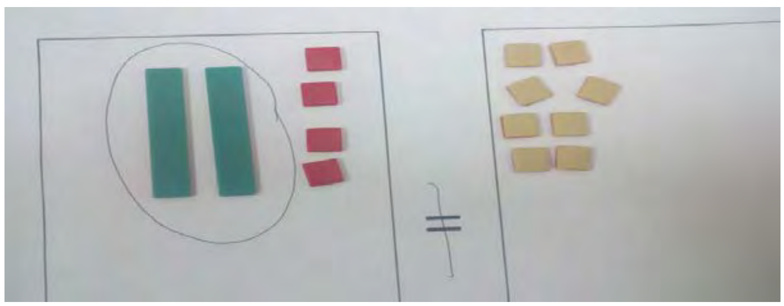


Guided Practice #2

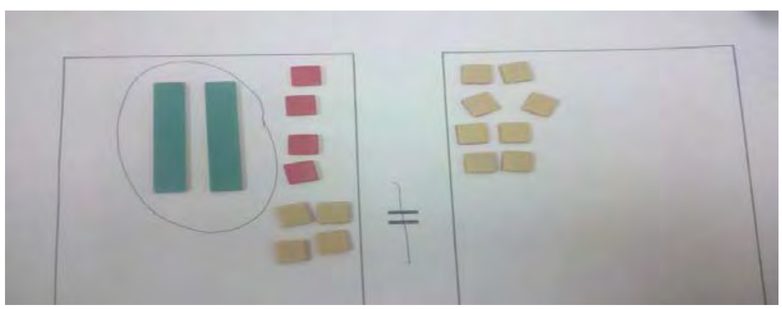
- I read the next problem. 3X -X -4 = 8
- I make sure my mat or board is clear
- On the mat, place 3 green rectangular tile (green is positive and red is negative) on the left side. Also place 1 green rectangle and 4 cream square tiles. On the right side, place 8 cream square tiles (cream is positive and red is negative).

- o Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
- o Unite like terms. Are there any tiles the same shape that are not already together? Yes. We must combine the 3 green rectangular tiles and the red rectangular tile. The colors are different, so we must subtract or pair them up and move them off the mat. Look on the right side. Are there any tiles the same that are not already together? No. Since there are shapes on We go on to step three.
- o Modify the new equation. Since we combined the left side, how many rectangles are left? 2. What color are they? They are cream, so they are positive. Our new equation is 2X -4 =8. We should have 2 green rectangular tiles and 4 red squares on the left side. We have 8 cream square tiles on the right side.

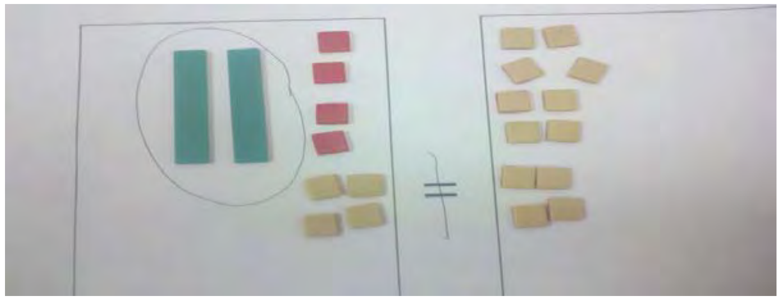
- Loop around the variable. Draw a circle around the 2 green rectangular tiles. That is the number that should be by itself.
- Is there anything else on the side with the circle? Yes, there are 4 red squares. We are going to have to move the 4 red squares because the variable should be by itself.

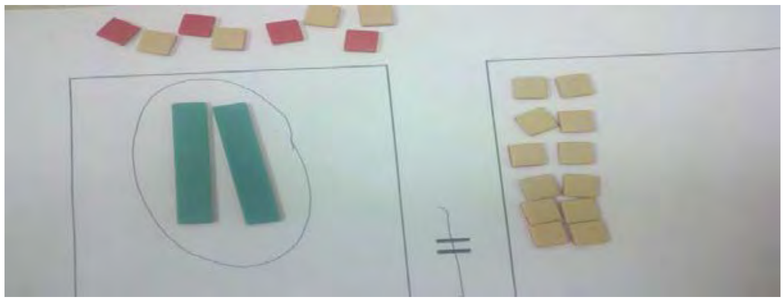


- Next step is opposite sign. In order to move the 4 red squares, we must add 4 cream squares to the left side.

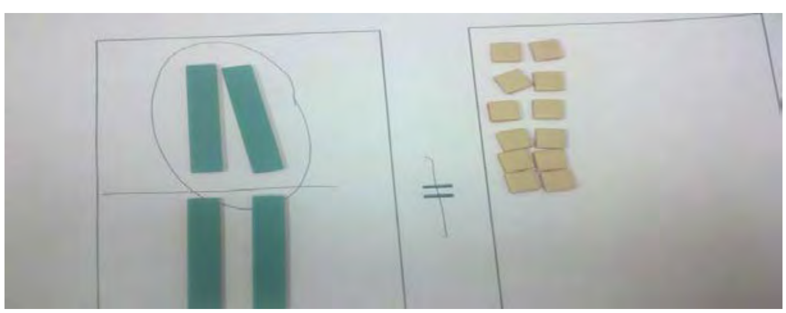


- What we do to one side, we must do to the other. The left side cancels each other out (-4 + 4 = 0). Since we added 4 cream squares on the left side, we must add 4 cream squares to the right side.
- On the right side, we have 8 cream squares and 4 cream squares. These are the same shape and the same color, therefore we must add (combine) the tiles. When we add them, how many tiles are on the right side? We have 12 cream tiles.

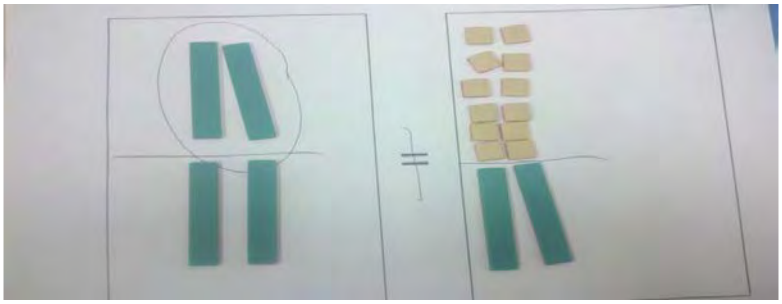




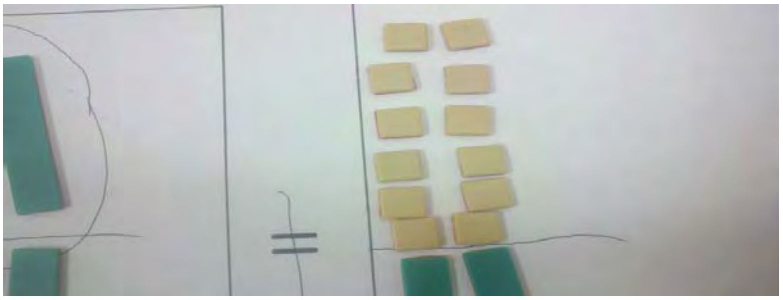
- Now we have 2X = 12. Is there only 1 rectangle? No. There are 2 rectangles. We must find out what 1 rectangle is equal to?
- I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The “U” is for unite like terms. Look on the left side. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- The next step is Loop the variable. The variable is the rectangular tile. This is the number that we want to get by itself and want to move last.
- Is there anything else on the side with the circle? No. So, we have to see what 1 X is equal to?
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division symbol beside the2 green rectangles on the left side.



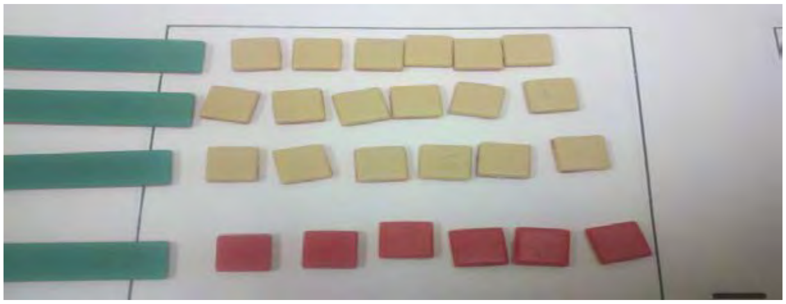
- What we do to one side, we must do to the other. Draw a division symbol beside the 12 cream tiles on the right side. The left side cancels each other out.

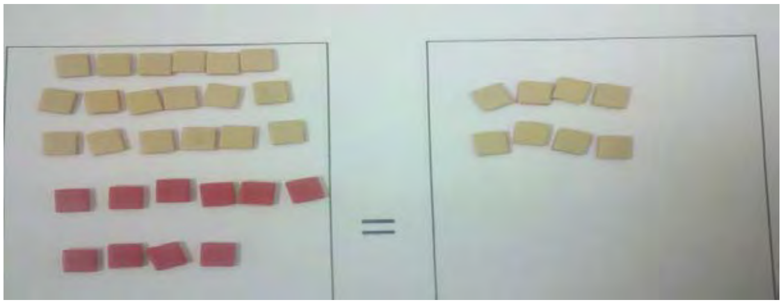


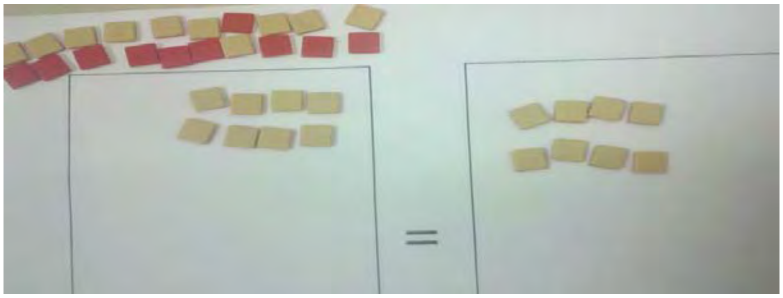
- On the right side, separate the rectangles. Give each rectangle a square until there are no more squares to give. How many squares does each rectangle have? 6. The color symbols are both positive, therefore the answer is positive2. X =6. Is the variable by itself? Yes. Is it positive? Go to the last step.



- Last step is to substitute the solution 6 in for X in the original problem.   Original problem is 3X –X - 4 = 8. On the left side, put down 6 cream tiles for every X. There are3 X, so we will put down 18 cream tiles. Also on the left side we will put 6 red tiles (X is negative) and 4 red squares. On the right side, put down 8 cream tiles. Pair up the squares on the left side until we cannot make any more pairs. What is left? We have 8 cream tiles on the left side. We also have 8 cream tiles on the right side. They are the same. The answer is correct.







Guided practice #3

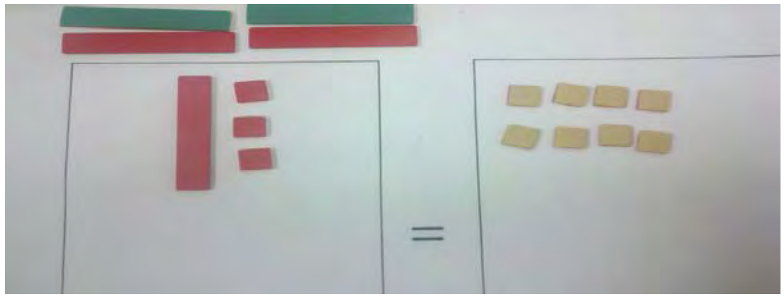
- I read the next problem. -3(X +1) +2X =8
- I make sure my mat or board is clear
- In the problem, notice the parenthesis ( ). This problem requires us to get rid of the parenthesis before we start SUMLOWS.
- What is in the parenthesis? X+1. What is outside the parenthesis? -3. On the mat, place tiles that represent X +1. Because the number outside the parenthesis is negative, the tiles are going to flip to its opposite side. Therefore, put down tiles for –X - 1. We are

going to put these tiles down 3 times. We are going to place a 1 red rectangular tile and 1 red square tile. The number outside the parenthesis tells us how many times we need to put -X -1 on the mat. In this case, we put -X -1 on the mat 3 times.

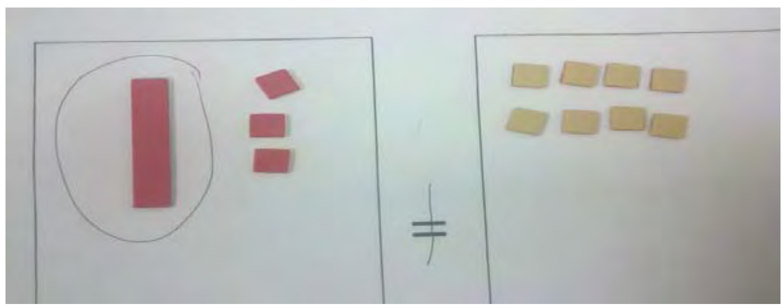
- How many rectangles are? 3 red rectangles
- How many squares are? 3 red squares
- Now we are ready to solve the equation using SUMLOWS.
- In addition to the tiles that we already have on the mat, place 2 green rectangular tiles (green is positive and red is negative) on the left side. On the right side, place 8 cream square tiles (cream is positive and red is negative). On the left side, we should have 3 red rectangles, 3 red squares, and 2 green rectangular tiles.

  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any tiles the same shape that are not already together? Yes. We must combine the 3 red rectangular tiles and the 2 green rectangular tiles. The colors are different, so we must subtract or pair them up and move them off the mat. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
  - Modify the new equation. Since we combined the left side, how many rectangles are there? 1 red rectangular tile. It's red, so it is negative. Our new equation is –X -3 =

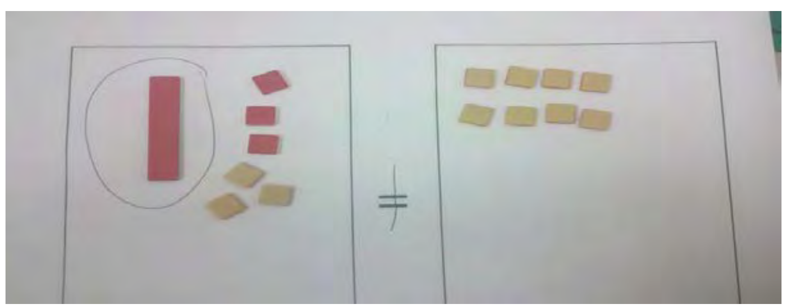
8. We should have 1 red rectangular tile and 3 red squares on the left side. We have 8 cream square tiles on the right side.



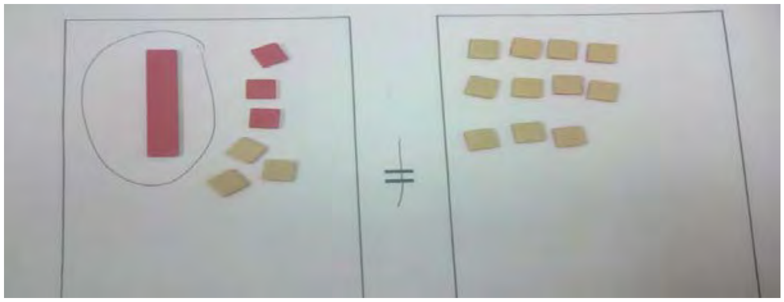
- Loop around the variable. Draw a circle around the 1 red rectangular tile. That is the number that should be by itself.
- Is there anything else on the side with the circle? Yes, there are 3 red squares. We are going to have to move the 3 red squares because the variable should be by itself.



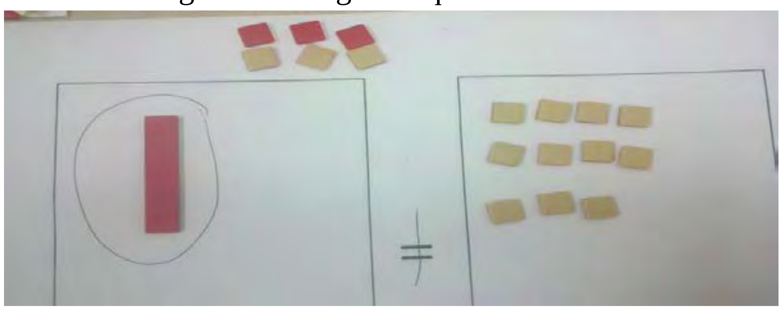
- Next step is opposite sign. In order to move the 3 red squares, we must add 3 cream squares to the left side.



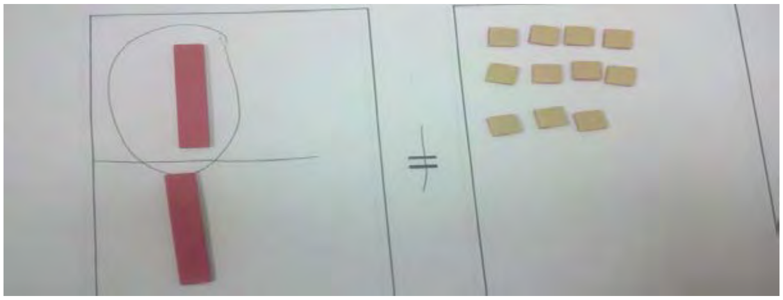
- What we do to one side, we must do to the other. The left side cancels each other out (-3+3 = 0). Since we added 3 cream squares on the left side, we must add 3 cream squares to the right side.



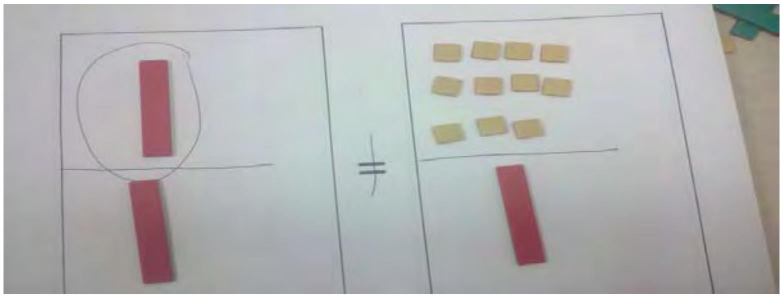
- On the right side, we have 8 cream squares and 3 cream squares. These are the same shape and same color; therefore we must add or combine the tiles. How many tiles are on the right side? We have 11 cream tiles.
- Now we have -X = 11. Is there only 1 rectangle? Yes. Is the X positive? No. We must find out what is a green rectangle is equal to?

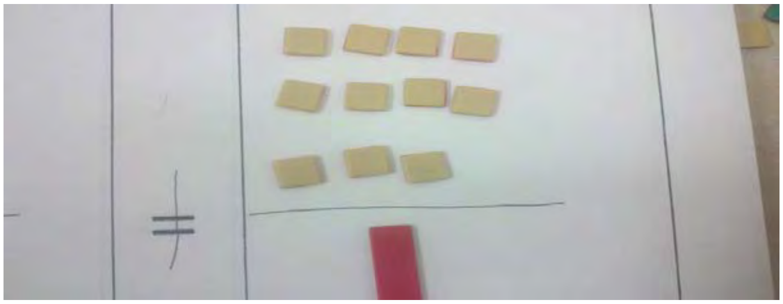


- I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The "U" is for unite like terms. Look on the left side. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- The next step is Loop the variable. The variable is the rectangular tile. This is the number that we want to get by itself and want to move last.
- Is there anything else on the side with the circle? No. Is the X by itself? Yes. Is it positive? No. It is negative. We must move the -1 (the1 is invisible).
- What is the next step? Next step is opposite sign. In order to move the negative one, we must do the opposite. The opposite of multiplication is division. We must divide by 1 red rectangle.

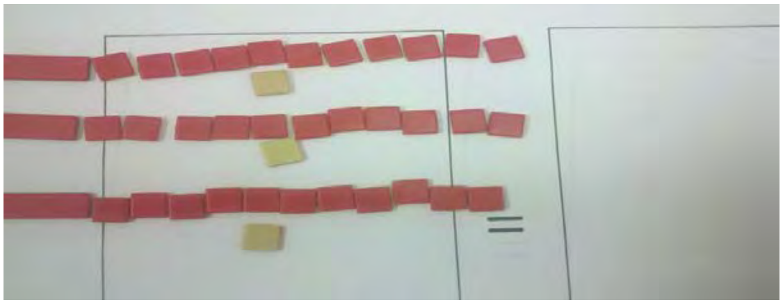


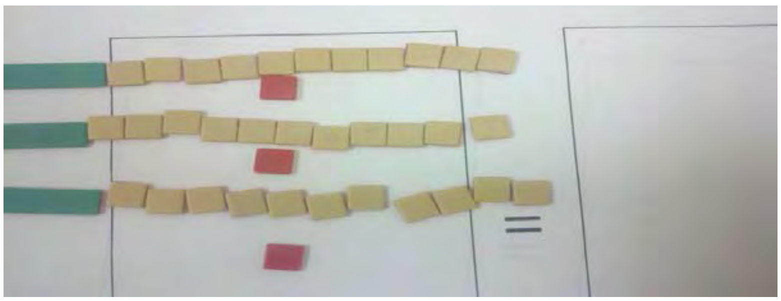
- What is the next step? What we do to one side, we must do to the other. Because we divided by -1 on the left side, we must divide by -1 on the right side and then flipped the tiles to make the rectangle tile positive.
- Now we have X =-11 (the square tiles are red, therefore they are negative). Is the X by itself? Yes. Is it positive? Yes. Now we do the last step.

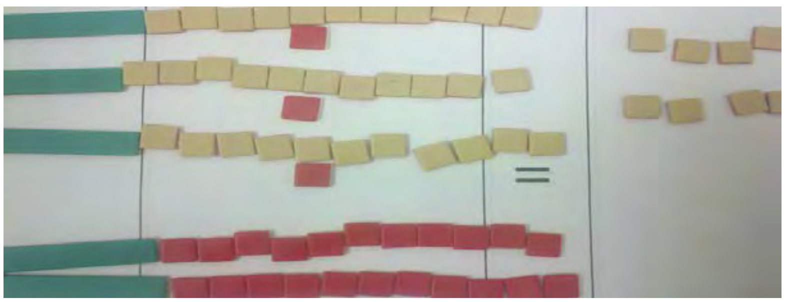




- Last step is to substitute the solution 2 in for X in the original problem. Original problem is -3(X +1) + 2X = 8. For X+1, put 11 red squares and 1 cream square down on the mat three times. Because the X is negative, flip the tiles. The 11 tiles are cream because the number outside the parenthesis is negative, so we have to flip the tiles to its opposite side, as well as two sets of 11 red tiles to represent the 2X. The tiles are different colors, so we put pair them up and move them off the mat. What is left side? We have 8 cream tiles on the left side. On the right side, put down 8 cream tiles. They are the same. The answer is correct.

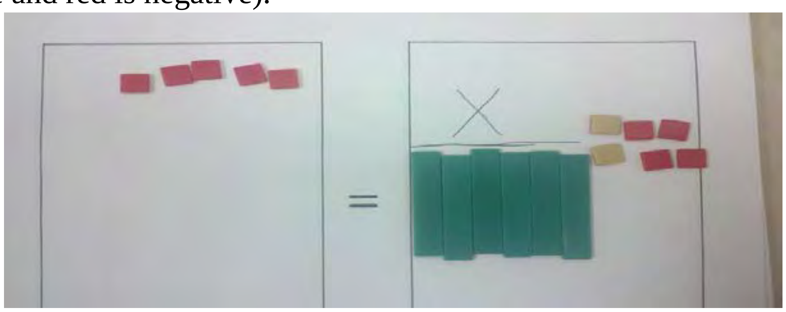




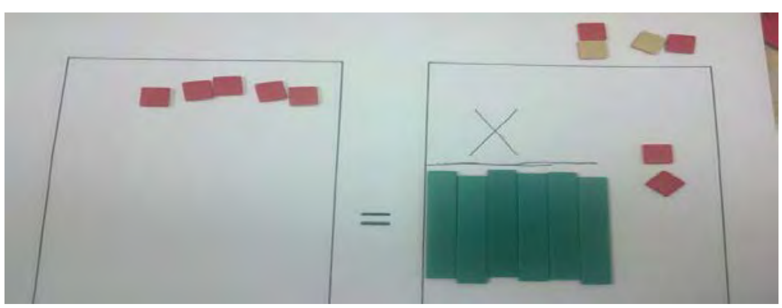


Guided Practice problem # 4

- I read the problem: $-5=\frac{X}{6}+2-4$
- If the students flip the equation, the steps would be the opposite side.
- I make sure the algebra mat is clear
- On the left side of the mat, place 5 red square tiles (green is positive and red is negative). On the right side, place a line and put 6 green rectangular tiles under the line. On top of the line draw an X. Also on the right side, put 2 cream tiles and 4 red tiles down (cream is positive and red is negative).



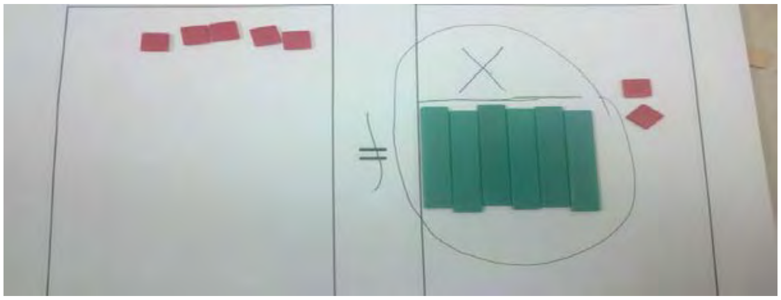
  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any tiles the same shape that are not already together on the left side? No. Look on the right side. Are there any tiles the same that are not already together? Yes. We have 2 cream tiles and 4 red tiles. We will pair them up (one cream with one red) and move them off the mat.

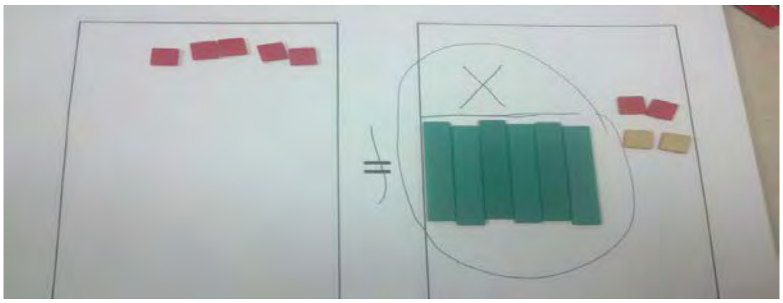


  - Modify the new equation. How many tiles are left on the right side? 2 red square tile.

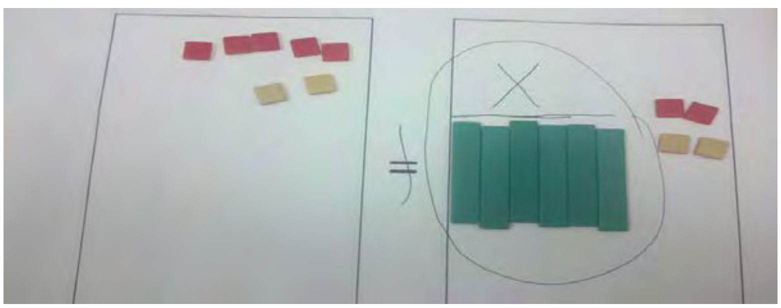
- Loop around the variable. Draw a circle around the 6 green rectangular tiles. That is the number that should be by itself.

- Is there anything else on the side with the circle? Yes. There are 2 red squares. We have to get rid of the 2 red squares.
- Next step is opposite sign. The opposite of negative (minus) 2 is positive (plus) 2. We are going to add 2 cream squares under the 2 red squares.

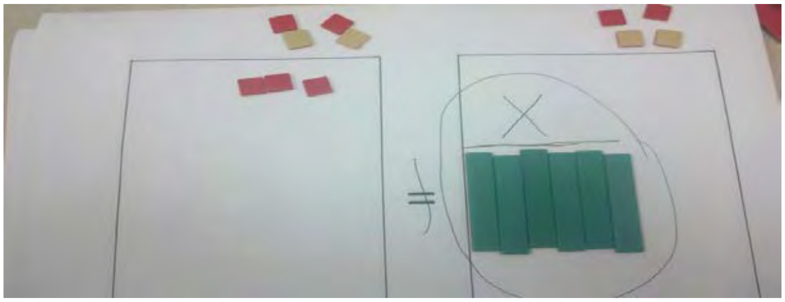




- What we do to one side, we must do to the other. Since we put 2 cream squares on the right side of the equation, we must place 2 cream squares on the left side of the equation. The right side squares cancels each other out. On the left side, we have 2 cream square and 6 red squares. We are going to add like terms. Take one cream and one red square and move them off the mat until there are no more pairs to make. On the left side, we should have 3 red squares left.

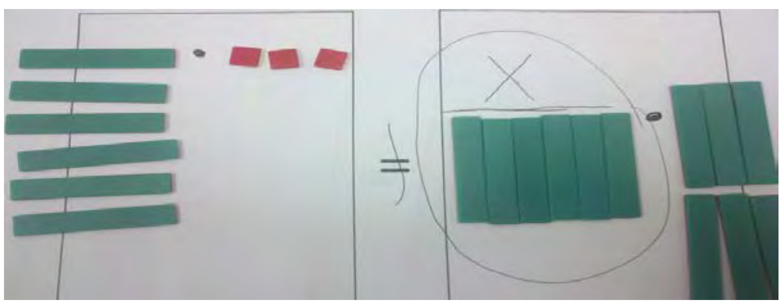


- On the algebra mat, we should have 3 red squares on the left side. On the right side, we should have X over 6 green rectangles. The rectangles should have a circle around it.

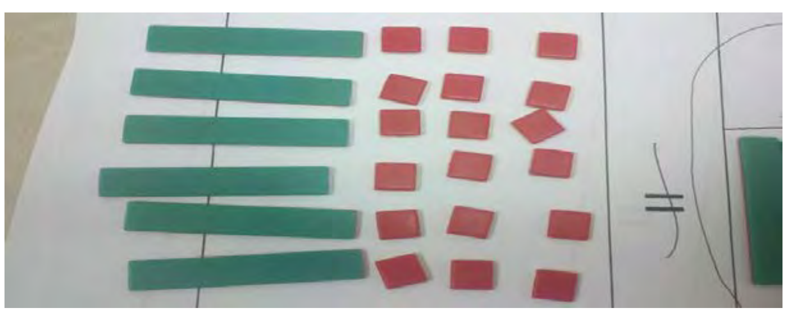


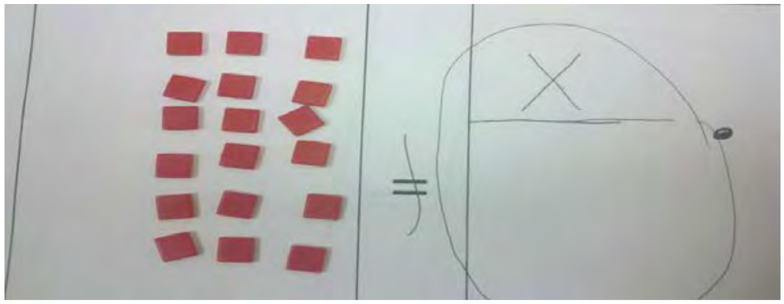
- We have already separated the sides, united like terms, modified the equation, and looped the coefficient. The next step is opposite sign.
- When we have an X over a number, which means to divide. What is the opposite of division? It is multiplication. We are going to write a multiplication sign and place 6 green rectangles beside it.

- Next step is what we do to one side; we must do to the other. Since we drew a multiplication sign and placed six green rectangles on the right side, we must do the same on the left side. On the left side, draw a multiplication sign and place 6 green rectangles.

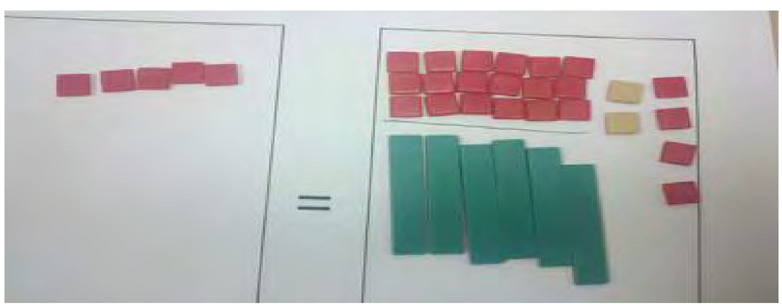


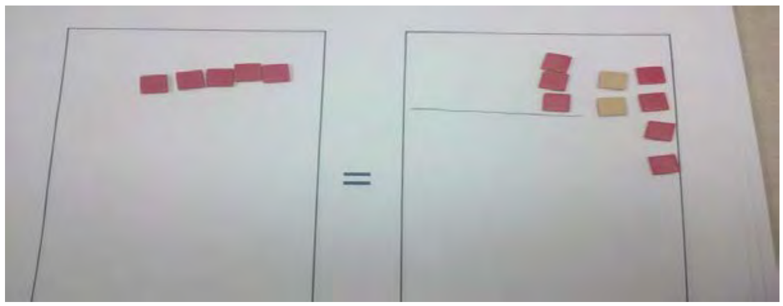
- We are going to give all of the 3 red squares to the first green rectangle. We are going to add 3 red squares for the 2$^{\text{nd, 3rd, 4th, 5th, and 6th}}$ rectangles. How many squares are all together? We have 18 red squares. So, X = -18.

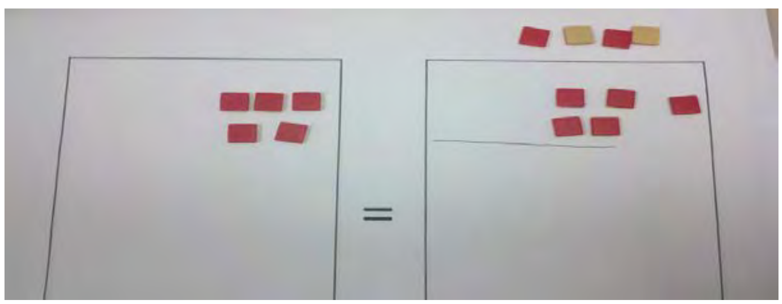




- Last step is to substitute the solution -30 in for X in the original problem. Original problem is -6= X/6 +2-4. On the left side, put 6 red squares. On the right side, put 18 red squares for X. Draw a line under the squares and place 6 green rectangles under the line. Separate the rectangles and give each rectangle a square until they are all gone. How many squares does each rectangle have? 3 red squares. Move the 6 rectangles and 15 red squares off the mat (because 18 divided 6=3). On the mat, we should have 6 red squares on the left side. On the right side, we should have 3 red squares, 2 cream squares, and 4 red squares. On the right side, combine the squares. Pair up a cream square with a red square and move them off the mat until there are no more pairs to make. How many squares are on the right side? 5 red squares. How many squares are on the left side? 5 red squares. The numbers are the same. Are both sides equal? Yes. The answer is correct.







Independent Practice

- Direct the students to the "Independent Practice" section. Read the first problem together and direct students to complete the problems without you.
  When students finish problems, provide immediate corrective feedback for errors.

Graphing

Learning Sheet 1

Multiple-Step
Concrete
Code Number __________________ Date ______________
SUMLOWS

Describe and Model

1) 5X+10-5 = 15

2) -2X – 5 +4X = 13

3) 3(X+2) +2X = 16

4) $-\frac{X}{2} + 3 - 4 = -6$

Guided Practice

1) -X +6- 3 = 10

2) 3X – X- 4 = 8

3) -3(X +1)+2X = 10

4) $-5 = \frac{X}{6} + 2 - 4$

5) $\frac{X}{4} + 2 - 3 = -2$

6) -2(X +1) – 4 = 6

Independent Practice

20= 3(3X) + 2

5- X +3X = 9

$-6 = \frac{X}{2} + 3 - 1$

-4(X+1) +2= -10

-2X +4 = 4+6

-10 = -2(X – 3) +2

- 2X+ 4X -2= -8

7X-2X -5= 20

Learning Sheet 2

Multiple-Step
Concrete
Code Number _________________ Date _____________
SUMLOWS

Describe and Model

1) $2X + 10 - 6 = 10$

2) $6 = -\frac{X}{6} + 3 - 4$

3) $-2(2X + 4) + 2 = 2$

Guided Practice

1) $-16 = -2(2+2X)+2X$

2) $4X - X - 3 = 9$

3) $-4(X + 1) + 2X = 8$

4) $-5 = \frac{X}{6} - 5 - 4$

5) $\frac{X}{3} + 5 - 1 = -5$

6) $-2(X + 1) - 4 = 6$

Independent Practice

$5 = 3(3X - 2) + 2$

$3 - X + 4X = 9$

$-5 = \frac{X}{2} + 2 - 5$

$4(X+1) + 2 = -2$

Learning Sheet 3

Multiple-Step
Concrete
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) $3X + 2X - 5 = 15$

2) $3 = -\frac{X}{2} + 3 - 4$

3) $-2(4X + 1) + 3 = -7$

Guided Practice

1) $-18 = -3(X+2X)+2$

2) $4X - 2X - 3 = 9$

3) $-4(X + 1) + 2X = 8$

4) $-5 = \frac{X}{6} - 5 - 2$

Independent Practice

$12 = 2(3X + 2) + 2$

$5 - 2X + 4X = -5$

$3 + 3X - 8 = 4$

$-2 = \frac{X}{3} + 2 - 5$

$-4(X+2) + 3 = 3$

$\frac{X}{2} - 6 + 3 = 4$

Lessons 4-6
Multiple-step

Representational Level

Materials

- chalkboard/whiteboard/easel
- learning sheets
- progress chart

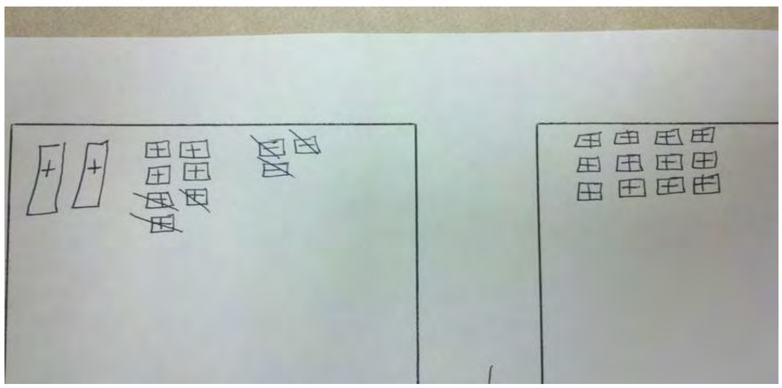
Advance Organizer

- Tell students what they will be doing and why
- Remind students about the commitment they made to learn to solve equations. We will work hard to teach them and they will work hard to learn a new way to solve equations. Tell student that they will solve multiple- step equations a new way. They already know how to solve equations using algebra tiles, but today they will learn how to draw pictures. The pictures will help them solve problems and they can use pictures anytime they need to solve problems because sometimes algebra tiles are not available.

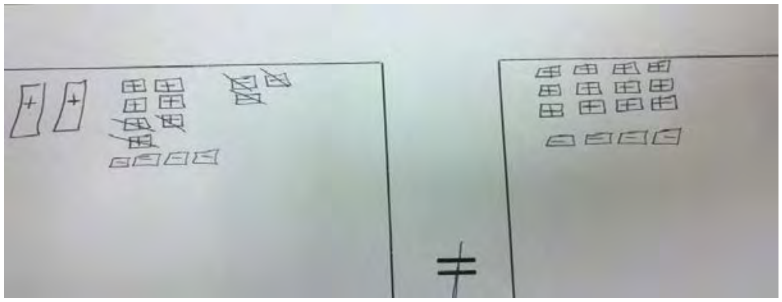
Demonstrate

- Give students learning sheets
- Begin with the first problem in the "Model" section. Tell the students that we will show them how to solve the problem and that they will have a chance to solve problems also. State the expectation for behavior and attention to the demonstration.
- Begin with the first problem and think out loud (see problem model)
- First we read the problem. 2X +7 - 3 =12
- Next, I take out my workmat or dry erase board.
- In the representational method, I am going to draw rectangles for my variables (X number) and draw squares for my ones number. For positive numbers, I am going to draw a "+" plus sign in the rectangle or square and a "-"minus sign for negative numbers.
- In this problem, I am going to draw two rectangles that represent the X. I am going to draw "+" plus signs in the rectangles. Also on the left side I am going to draw 7 squares with plus signs in them and 3 squares with minus signs in them. On the right side, I am going to draw 12 squares with plus signs in them.

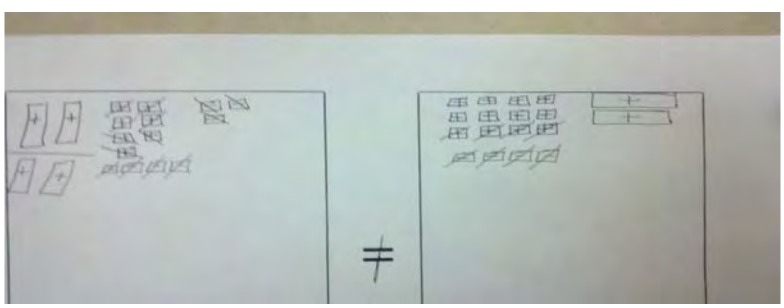
- The first step In the SUMLOWS is to separate the two sides. Draw a line through the equal sign.
- The "U" is for unite like terms. Look on the left side. Are there any shapes that are the same that are not together? Yes. We have 7 squares with a plus sign in them and 3 squares with a minus sign in them. They are the same shape, but they have different signs. We have to pair a square with a positive and a square with a negative and draw an X through each pair. Look on the right side. Are there any shapes that are the same that are not together?
- Modify the new equation. How many squares are left on the left side of the equation when we paired them up? 4. What color are they? They are cream, therefore the number is positive. We are going to modify or rewrite the new equation 2X + 4 = 12.



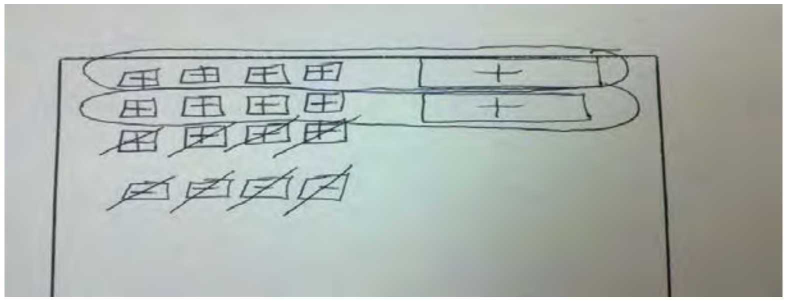
- The next step is Loop the variable. The variable is the rectangular tile. This is the number that we want to get by itself and want to move last. So what is on the side with the variable (the rectangle)? 4 cream squares. We have to get rid of the 4 cream squares because the rectangles want to be by themselves.
- In order to move the 4 cream squares, we have to do the next step which is the opposite sign. To get rid of the 4 cream squares, we must add 4 red squares to the left side of the equal sign. Opposites cancel each other out (+4 – 4 = 0).
- Next step is what we do to one side we must do to the other. We added 4 red squares to the left. We must add 4 red squares to the right side. On the left side, each cream square cancel out each red square (put a slash mark through the squares). On the right side, we have 12 cream squares and 4 red squares. These are the same shape but different colors; therefore we must subtract or pair a cream tile with a red tile. Pair the red tiles with the cream tiles until there are no more pairs to make. When we pair them up, draw a slash mark through them on the mat. How many squares are left on the right side? We have 8 squares with plus sign in them.



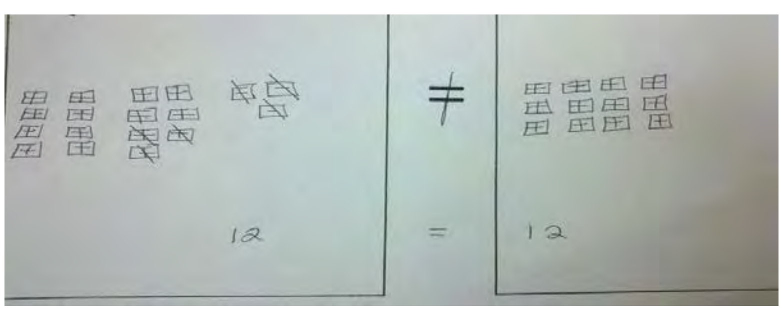
- Now we have 2X = 8. Is their only 1 rectangle? No, there are two rectangles. We must find out what 1 rectangle is equal to.
- I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The “U” is for unite like terms. Look on the left side. Are there any shapes that are the same that are not already together? NO. Look on the right side. Are there any shapes that are the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- The next step is Loop the variable. The variable is the rectangular tile. This is the number that we want to get by itself and want to move last.
- Is there anything else on the side with the circle? No. So, we have to see what 1 X is equal to?
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a line and draw 2 rectangles under the other two rectangles on the left side.
- What we do to one side, we must do to the other. Draw a division symbol beside the 8 squares on the right side and draw two rectangles on the right side. The left side cancels each other out.



- On the right side, separate the rectangles. Give each rectangle a square until there are no more squares to give. How many squares does each rectangle have? 4. The color symbols are both positive, therefore the answer is positive 4. X = 4. Is the variable by itself? Yes. Is it positive? Go to the last step.

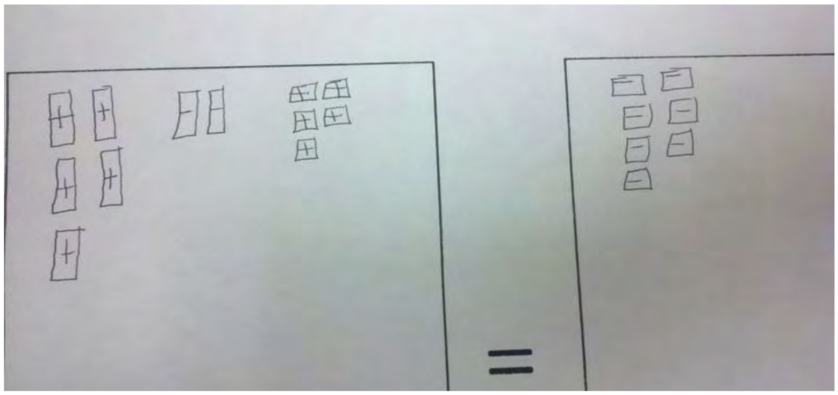


- Last step is to substitute the solution 4 in for X in the original problem. Original problem is 2X +7 -3 =12. On the left side, draw 4 squares with plus signs in them for every X. There are 2 X, so draw 8 squares with plus signs in them. Also on the left side, draw 7 squares with plus signs in them and 3 squares with minus signs in them. On the right side, draw 8 squares. Pair up the squares on the left side until we cannot make any more pairs. Draw a slash mark through every pair. What is left? We have 12 squares with plus signs in them. We also have 12 squares with the plus sign on the right side. They are the same. The answer is correct.

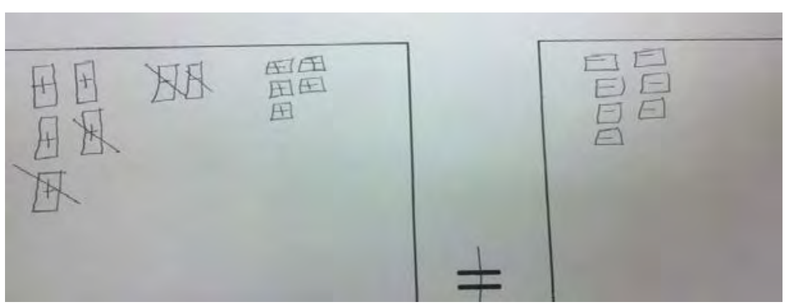


Model problem #2

- I read the next problem. 5X –2X + 5 = -7
- I make sure my mat or board is clear
- On the mat, we are draw 5 rectangles with plus signs in them and two red triangles with minus signs in them. Also, we draw 5 squares with a plus sign in them. On the right side, we draw 7 squares with a minus sign in them.

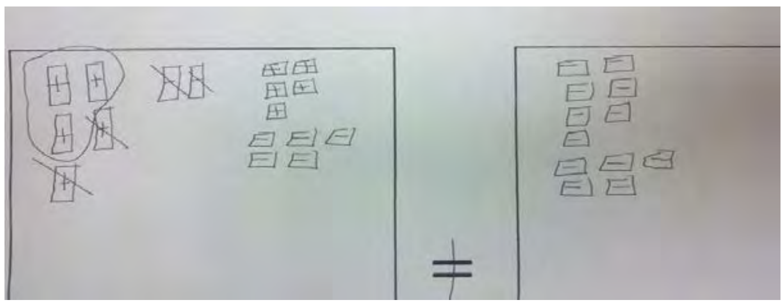


  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any shapes that are the same that are not already together? Yes. We must combine the 5 rectangles with the plus sign in them and the 2 red rectangles with the minus sign in them. They are the same shape, but they do not have the same sign. The signs are different, so we must subtract or pair them up and draw a slash mark through them. Look on the right side. Are there any shapes that are the same that are not already together? No. We go on to step three.
  - Modify the new equation. Since we combined the left side, how many rectangles are that do not have a slash mark through them? 3 rectangles with a plus sign in them. Our new equation is 3X + 5 =-7. We should have 3 rectangles with plus signs in them and 5 squares with the plus sign in them on the left side. We have 7 squares with minus signs in them on the right side.

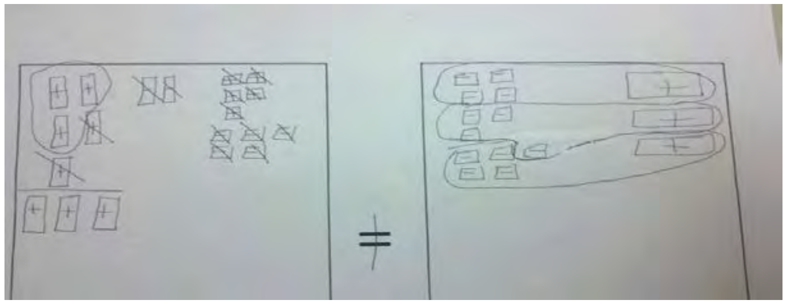


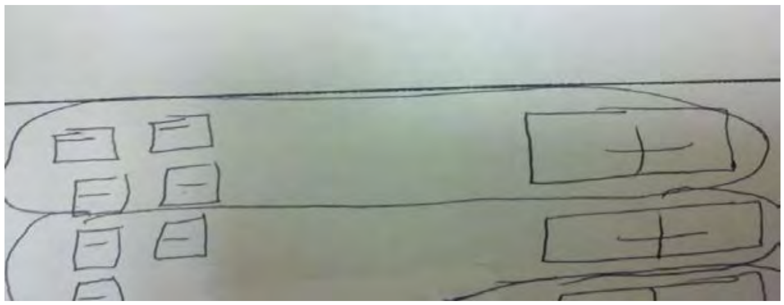
  - Loop around the variable. Draw a circle around the 2 rectangles. That is the number that should be by itself.
  - Is there anything else on the side with the circle? Yes, there are 5 squares with plus signs in them. We are going to have to move the 5 squares because the variable should be by itself.

- Next step is opposite sign. In order to move the 5 squares with the plus signs in them, we must draw 5 squares with minus signs in them on the left side.
- What we do to one side, we must do to the other. The left side cancels each other out (-5 + 5 = 0). Put a slash marks through the 5 squares with the plus signs and 5 squares with the minus signs. Since we drew 5 squares with minus signs in them on the left side, we must draw 5 squares with minus signs in them on the right side.
- On the right side, we have 7 squares with minus signs in them and 5 squares with minus signs in them. These are the same shape and have the same sign in them, therefore we must add (combine) the squares. When we add them, how many squares are on the right side? We have 12 squares with minus signs in them.
- Now we have 3X =-12. Is there only 1 rectangle? No. There are 3 rectangles. We must find out what 1 rectangle is equal to?

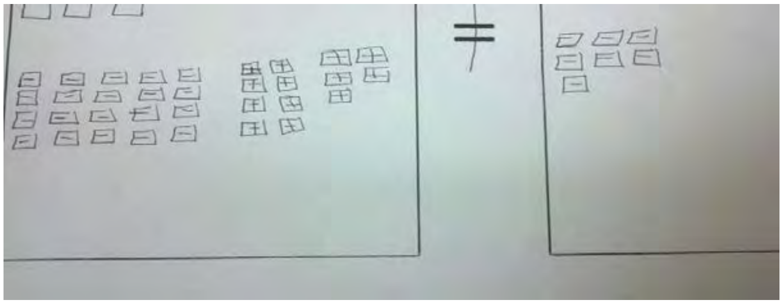


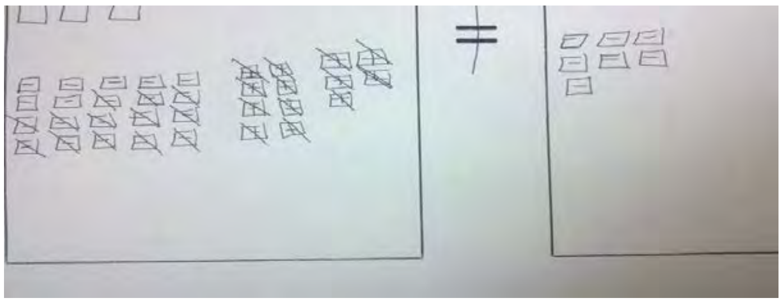
- I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The "U" is for unite like terms. Look on the left side. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- The next step is Loop the variable. The variable is the rectangular tile. This is the number that we want to get by itself and want to move last.
- Is there anything else on the side with the circle? No. So, we have to see what 1 X is equal to?
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division symbol beside the 3 rectangles with the plus sign in them on the left side.
- What we do to one side, we must do to the other. Draw a division symbol beside the 12 squares with the minus sign in them on the right side. The left side cancels each other out. Draw a line through the rectangles on the left side.
- On the right side, separate the rectangles. Give each rectangle a square until there are no more squares to give. How many squares does each rectangle have? 4. Do the rectangle and squares have the same sign in them? No, they have different signs, therefore the answer is negative, X =-4. Is the variable by itself? Yes. Is it positive? Go to the last step.





- Last step is to substitute the solution -4 in for X in the original problem. Original problem is 5X -2X + 5 = -7. For 5X, draw 20 squares with minus signs in them. They are negative because the two numbers (5 and -4) are different. For -2X, draw 8 squares with plus signs in them. These square are positive because the numbers (-2 and -4 have the same sign, so the number is positive. Also, draw 5 squares with plus signs. Pair the squares up and put an X on each pair until there are no more pairs. After pairing the squares up, we have 7 squares with minus signs in them on the left. On the right, draw 7 squares with minus sign in them. They are the same. The answer is correct.

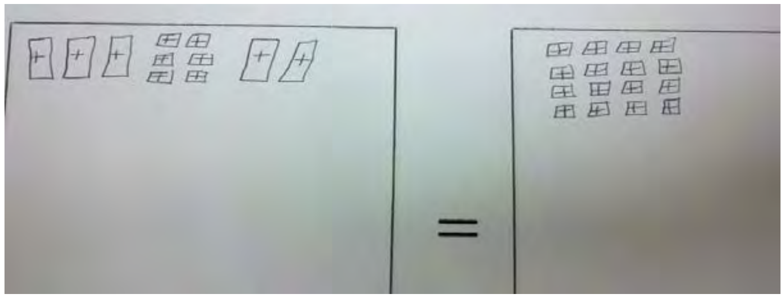




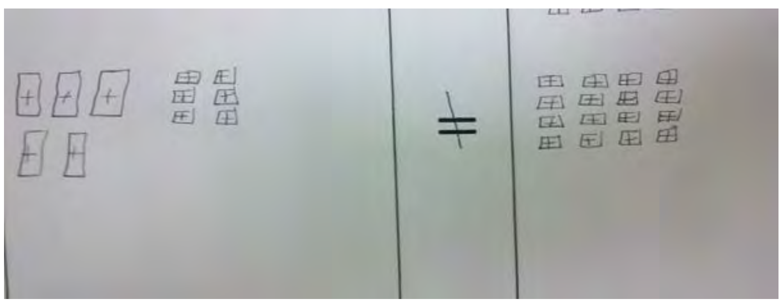
Model problem #3

- I read the next problem. 3(X + 2) +2X =16
- I make sure my mat or board is clear
- In the problem, notice the parenthesis ( ). This problem requires us to get rid of the parenthesis before we start SUMLOWS.
- What is in the parenthesis? X + 2. What is outside the parenthesis? 3. On the mat, place tiles that represent X + 2. We are going to draw a 1 rectangle with a plus sign and 2 squares with plus signs in them. The number outside the parenthesis tells us how many times we need to put X + 2 on the mat. In this case, we put X + 2 on the mat 3 times.
- How many rectangles are? 3 rectangles with plus signs
- How many squares are? 6 squares with plus signs

- Now we are ready to solve the equation using SUMLOWS
- In addition to the tiles that we already have on the mat, place 2 rectangles with plus signs on the left side. On the right side, place 16 squares with plus signs in them. On the left side, we should have 3 rectangles with plus signs, 6 squares with plus signs, and 2 more rectangles with plus signs.



- Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
- Unite like terms. Are there any tiles the same shape that are not already together? Yes. We must combine the 3 rectangles with plus signs and the 2 rectangles with plus signs. The signs are the same, so we must add or combine them. Look on the right side. Are there any shapes the same that are not already together? No. We go on to step three.
- Modify the new equation. Since we combined the left side, how many rectangles are there? 5 rectangles with plus signs in them. Our new equation is 5X +6 =16. We should have 5 rectangles with plus signs and 6 squares with plus signs in them on the left side. We have 16 squares with plus signs in them on the right side.

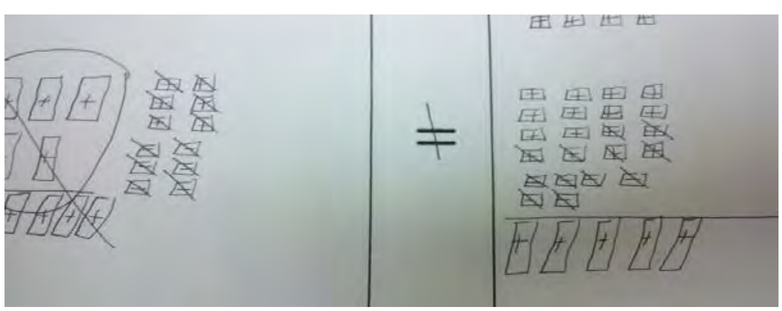


- Loop around the variable. Draw a circle around the 5 rectangles with plus signs in them. That is the number that should be by itself.
- Is there anything else on the side with the circle? Yes, there are 6 squares with the plus signs. We are going to have to move the 6 squares with plus signs because the variable should be by itself.
- Next step is opposite sign. In order to move the 6 squares with plus signs, we must add 6 squares with minus to the left side.

- What we do to one side, we must do to the other. The left side cancels each other out (+6 -6 = 0). Since we added 6 squares with minus signs on the left side, we must add 6 squares with minus signs on them to the right side.
- On the right side, we have 16 squares with positive sign on them and 6 squares with minus signs. They have different signs; therefore we must subtract or pair the sign and put a slash mark through them. How many squares do we left on the right side? We have 10 squares on the plus signs in them.

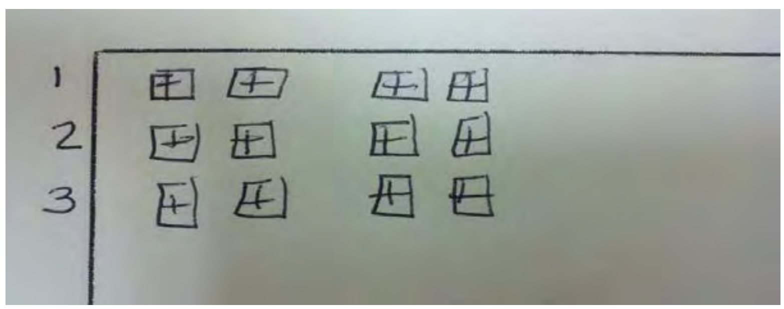
- Now we have 5X = 10. Is there only 1 rectangle? No. There are 5 rectangles. We must find out what 1 rectangle is equal to?

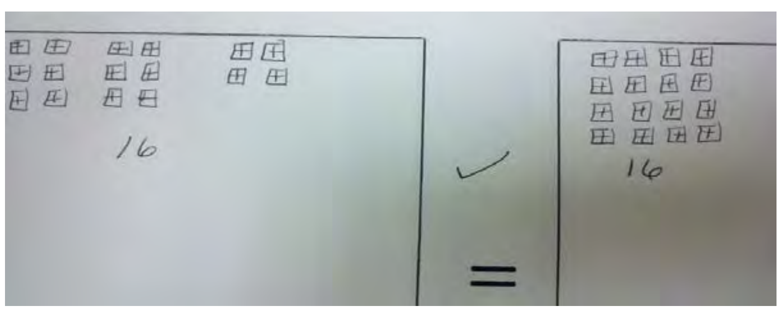
- I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The “U” is for unite like terms. Look on the left side. Are there any shapes that the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- The next step is Loop the variable. The variable is the rectangular tile. This is the number that we want to get by itself and want to move last.
- Is there anything else on the side with the circle? No. So, we have to see what 1X is equal to?
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division symbol beside the 5 rectangles with plus sign on the left side.
- What we do to one side, we must do to the other. Draw a division symbol beside the 10 squares with plus signs in them on the right side and add 5 rectangles with plus signs in them. The left side cancels each other out.



- On the right side, separate the rectangles. Give each rectangle a square until there are no more squares to give. How many squares does each rectangle have? 2. The signs are the same; therefore the answer is positive 2. X =2. Is the variable by itself? Yes. Is it positive? Go to the last step.

- Last step is to substitute the solution 2 in for X in the original problem. Original problem is 3(X +2) + 2X = 16. For X+2, draw 4 squares with plus signs (2 times 2 = 4). We are going to draw 4 squares with plus signs in them 3 times for a total of 12 squares with plus signs in them. For 2x, draw 4 square with plus signs in them (2 X2 = 4). What is left side? We have 16 squares with plus signs in them on the left side. On the right side, draw 16 squares with plus signs in them. They are the same. The answer is correct.

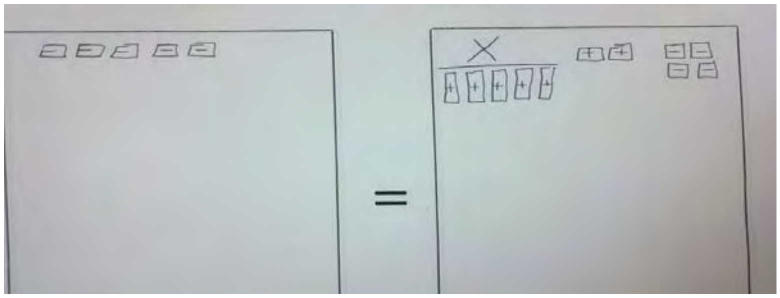




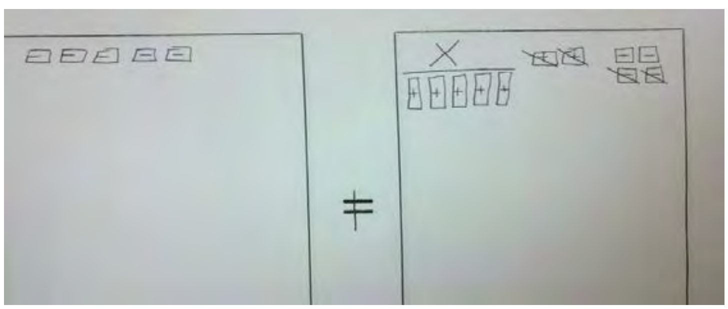
Model problem # 4

- I read the problem: $-5=\frac{X}{5}+2-4$
- If the students flip the equation, the steps would be the opposite side.
- I make sure the algebra mat is clear

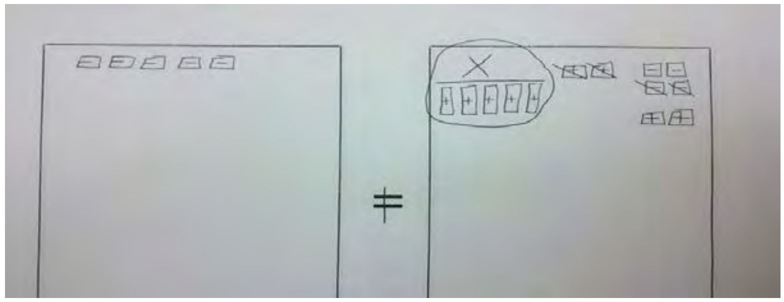
- On the left side of the mat, draw 5 squares a negative sign in them. On the right side, draw an X and under it draw a line and draw 5 rectangles with a plus sign in them under the line. On top of the line draw an X. Also on the right side, draw 2 squares with plus sign in them and 4 squares with minus signs in them.



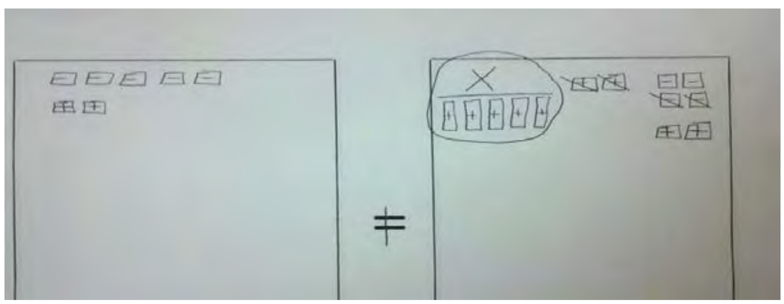
  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any tiles the same shape that are not already together on the left side? No. Look on the right side. Are there any tiles the same that are not already together? Yes. We have 2 squares with plus sign in them and 4 squares with minus signs in them. We will combine these like terms. Draw a line through one square with a minus sign and one square with a plus sign until there are not more pairs to cross out.
  - Modify the new equation. How many tiles are left on the right side? 2 square with minus signs in them.



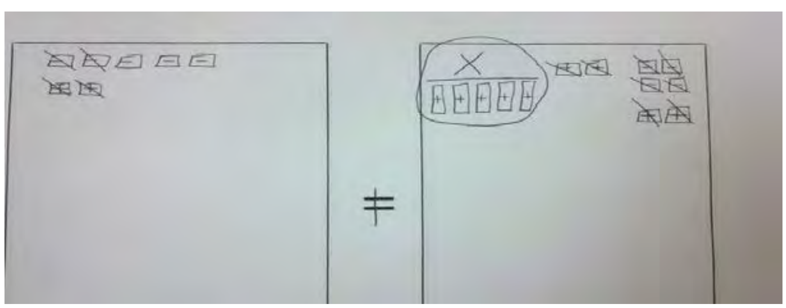
  - Loop around the variable. Draw a circle around the 5 rectangles with plus signs in them. That is the number that should be by itself.
  - Is there anything else on the side with the circle? Yes! There are 2 squares with minus signs in them. We have to get rid of the 2 squares with the minus signs.
  - Next step is opposite sign. The opposite of negative (minus) 2 is positive (plus) 2. We are going to draw 2 squares with plus signs in them under the 2 squares with the minus signs in them.



- What we do to one side, we must do to the other. Since we drew 2 squares with plus signs in them on the right side of the equation, we must draw 2 squares with plus signs in them on the left side of the equation. The right side squares cancels each other out. On the left side, we have 2 squares with plus signs in them and 5 squares with minus signs in them. We are going to combine like terms. Draw a line through one square with a plus sign and one square with a minus sign until they are no more pairs to make. On the left side, we should have 3 squares with minus signs left.

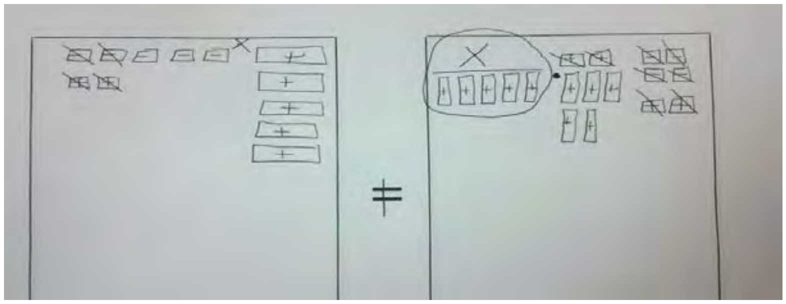


- On the algebra mat, we should have 3 squares with on the left side. On the right side, we should have X over 5 rectangles with plus sign in them. The rectangles should have a circle around it.

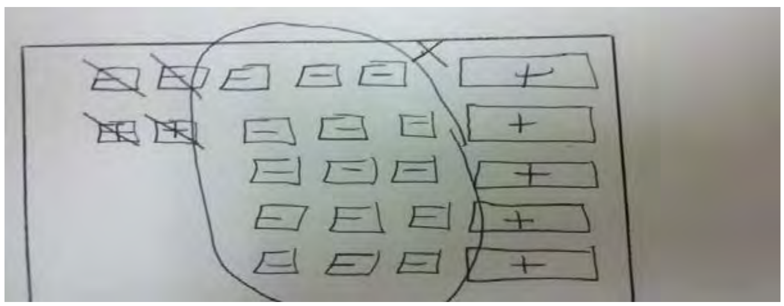


- We have already separated the sides, united like terms, modified the equation, and looped the coefficient. The next step is opposite sign.
- When we have an X over a number, which means to divide. What is the opposite of division? It is multiplication. We are going to write a multiplication sign and draw 5 rectangles with plus sign beside it.
- Next step is what we do to one side; we must do to the other. Since we drew a multiplication sign and drew six rectangles with plus signs on the right side, we must

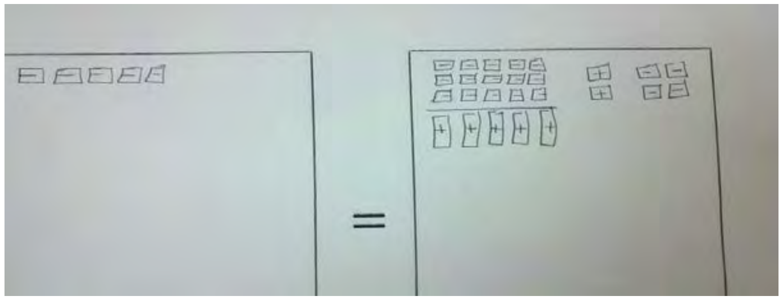
do the same on the left side. On the left side, draw a multiplication sign and draw 5 rectangles with plus signs in them.

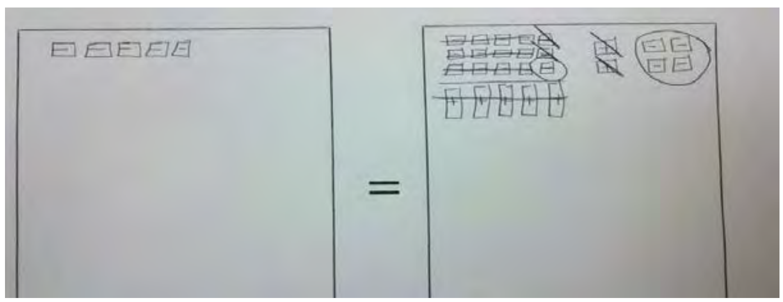


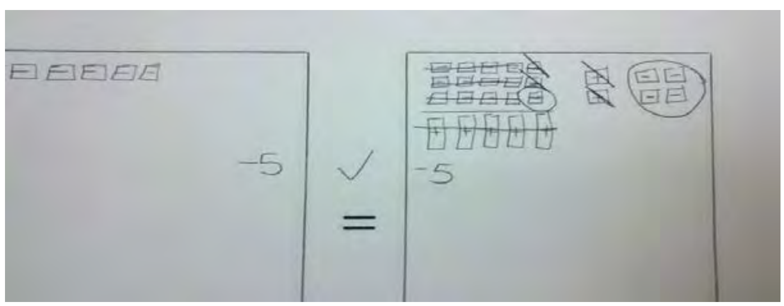
- We are going to give all of the 3 squares with minus signs to the first green rectangle. We are going to add 3 squares with minus signs for the 2nd, 3rd, 4th, and 5th rectangles. How many squares are all together? We have 15 squares with minus signs. So, X = -15.



- Last step is to substitute the solution -15 in for X in the original problem.   Original problem is -5= X/5 +2-4. On the left side, draw 5 squares with minus signs. On the right side, draw 15 squares with minus signs in them for X. Draw a line under the squares and then draw 5 rectangles with plus signs in them under the line. Separate the rectangles and give each rectangle a square until they are all gone. How many squares does each rectangle have? 3 squares with minus signs. Cross out the 5 rectangles and 12 squares with minus signs (because 15 divided 5=3).   On the mat, we should have 5 squares with minus signs on the left side. On the right side, we should have 3 squares with minus sings, 2 squares with plus sign, and 4 squares with minus signs. On the right side, combine the squares. Cross out a square with a minus sign and a square with a plus sign until there are no more pairs to make. How many squares are on the right side? 5 squares with minus signs. How many squares are on the left side? 5 squares with minus signs. The numbers are the same. Are both sides equal? Yes. The answer is correct.

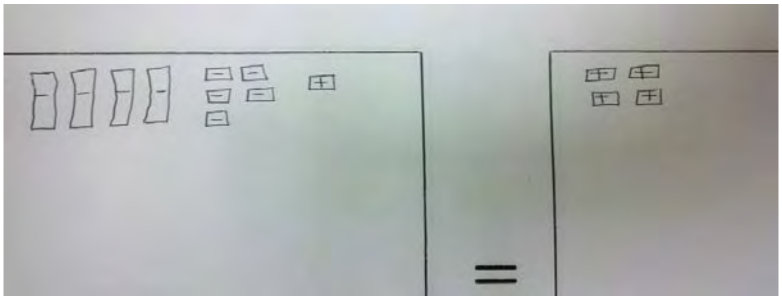




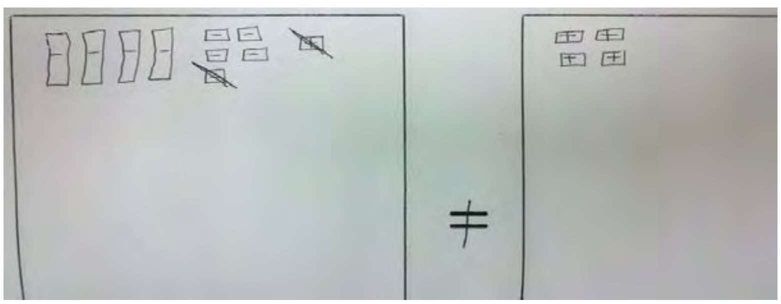


Guided Practice

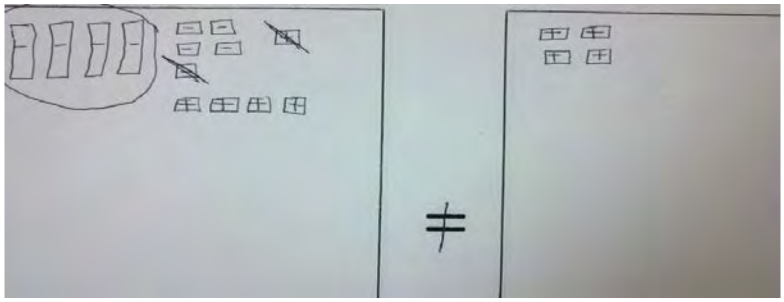
- Direct students to the “Guide” section of the learning sheet
- Tell students to touch the first problem and that we will do this problem together, using numbers, letters and the SUMLOWS mnemonic.
- Let’s read the problem. This problem is -4X -5 +1 = 4
- On our workmat, what do we draw on the left side? 4 rectangles with a minus sign in them, 5, squares with minus signs in them, and 1 square with a plus sign in it.
- What do we draw on the right side? 4 squares with plus signs in them.



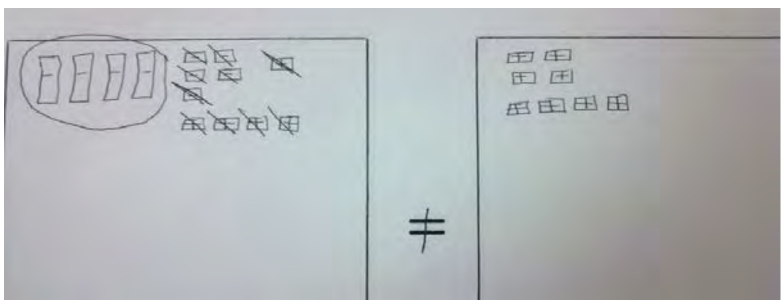
- What is the first step in SUMLOWS? Separate the sides. We are going to draw a line through the equal sign.
- The "U is for unite like terms. Look on the left side, are there any shapes that not already together? Yes. We have 5 squares with minus signs in them and 1 square with a plus sign in it that can be combined. They are the same shape but do not have the same sign in them. Since they do not have the same sign in them, we subtract or pair up the shapes and draw a slash mark through each pair. After drawing the slash marks, how many squares are left on the left side? 4 squares with minus signs in them. Look on the right side. Are there any shapes that can be combined together? No. We go to the next step.
- Modify the new equation. On the mat, we have 4 rectangle with minus signs in them and 4 squares with minus signs in them on the left side. On the right side, we have 4 squares with plus signs in them.



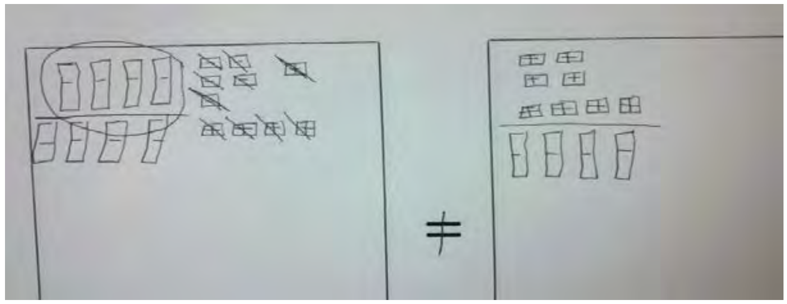
- The next step is Loop the variable. The variable is the X. This is the number that should be by itself and want to move last. Draw a circle around the rectangles. What is on the side with the variable? 4 squares with the minus sign in them. We have to move the 4 squares with the minus signs in them.
- In order to move the 4 squares with the minus sign in them, we have to draw 4 squares with plus signs in them. Opposites cancel each other out (-4+4=0). On the left side, pair the squares up and draw slash marks on each pair.



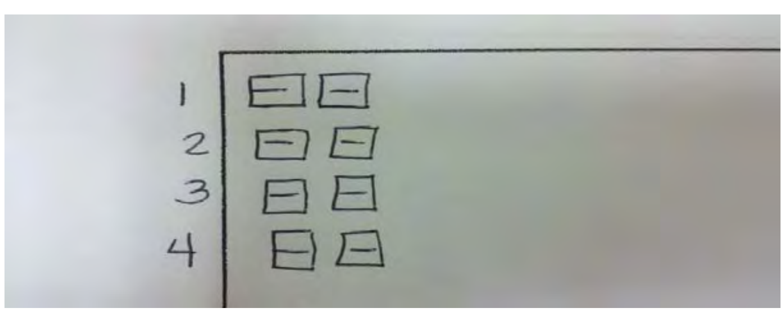
- The next step is “what we do to one side, we must do to the other.” We drew 4 squares with plus signs in them, so we must draw 4 squares with plus signs in them on the right side.
- Combine like terms. On the right side, we have 4 squares with plus signs in them and 4 more squares with plus signs in them.   Since the squares have the same sign in them, we add or combine the squares. How many squares are on the right side? 8 squares with the plus sign in them. So -4X =8.

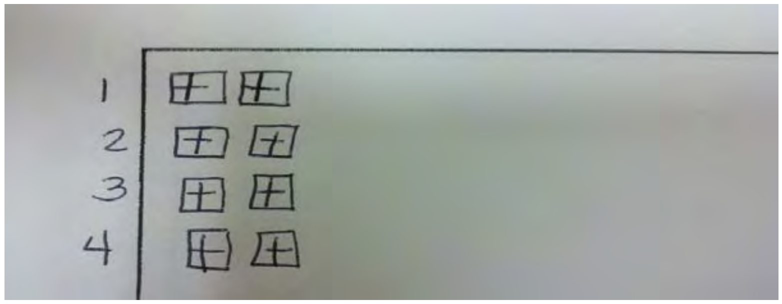


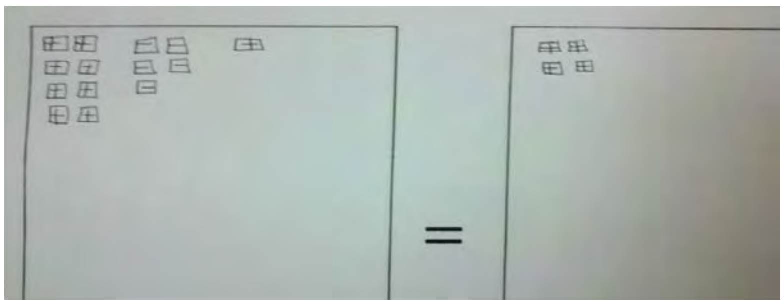
- Is there anything else on the side with the circle? No. Are only one rectangle? No, we have 4 rectangles with minus signs in them. So, we must determine what 1 rectangle is equal to. I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The “U” is for unite like terms. Look on the left side. Are there any like terms or numbers that can be combined that are not already together? NO. Look on the right side. Are there any like terms or numbers that can be combined that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- What is the next step? Loop around the variable.   Draw a circle around the variable. That is the number that should be by itself.
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division symbol beside the 4 rectangles with the minus sign in them and draw 4 more rectangles with minus signs in them.
- What we do to one side, we must do to the other. On the right side, draw a division symbol beside the 8 squares with the plus signs in them and draw 4 rectangles with minus signs in them. The left side cancels each other out. On the right side, give a square to each rectangle until they are all gone. How many does each rectangle have? 2. Do they have the same sign? No. Therefore, the answer is negative, X =-2

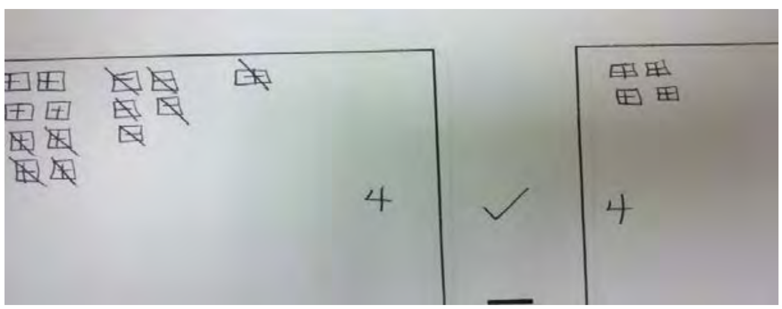


- Last step is to substitute the solution -2 in for X in the original problem.   Original problem is -4X-5+1 = 4. On the left side, draw 2 squares for each X. There are 4X. So we are going to draw 8 squares with plus signs in them (they have plus signs because -4 X -2 = a positive 8). Also draw 5 squares with minus signs in them and 1 square with a plus sign in it. On the right side, draw 4 squares with plus sign in them. Pair up the squares on the left side and put slash marks through each pair. How many squares are left? 4 squares with plus signs in them. How many are on the right side? 4 squares with plus signs in them. They are the same, so the answer is correct.



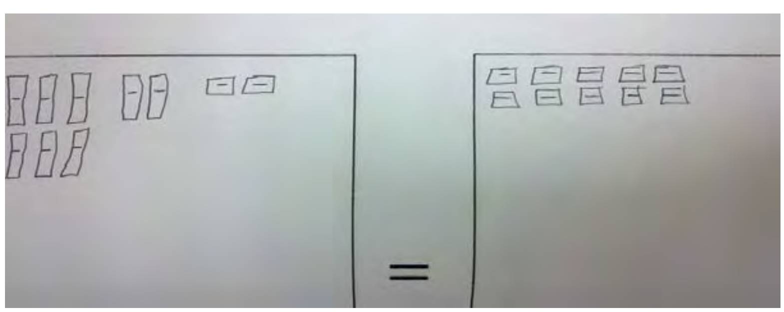






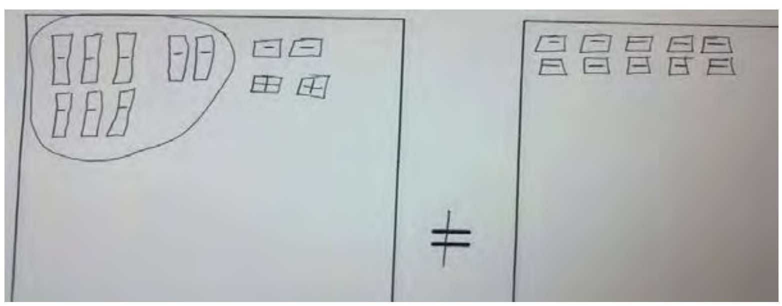
Guided Practice #2

- I read the next problem. -6X -2X -2 = -10
- I make sure my mat or board is clear
- On the mat, we are draw 6 rectangles with minus signs in them and two triangles with minus signs in them. Also, we draw 2 squares with minus signs in them. On the right side, we draw 10 squares with a minus sign in them.

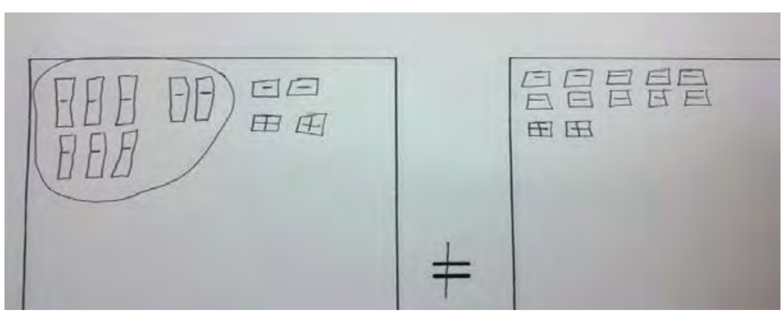


  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.

- Unite like terms. Are there any shapes that are the same that are not already together? Yes. We must combine the 6 rectangles with the minus signs in them and the 2 rectangles with minus signs in them. They are the same shape and they have the same sign. The signs are the same, so we add or combine the rectangles. Look on the right side. Are there any shapes that are the same that are not already together? No. We go on to step three.
- Modify the new equation. Since we combined the left side, how many rectangles are when we combined the rectangles? 8 rectangles with minus signs in them. Our new equation is -8X-2 =-10. We should have 8 rectangles with minus signs in them and 2 squares with the minus sign in them on the left side. We have 10 squares with minus signs in them on the right side.
- Loop around the variable. Draw a circle around the 8 rectangles. That is the number that should be by itself.
- Is there anything else on the side with the circle? Yes, there are 2 squares with minus signs in them. We are going to have to move the 2 squares because the variable should be by itself.
- Next step is opposite sign. In order to move the 2 squares with the minus signs in them, we must draw 2 squares with plus signs in them on the left side.

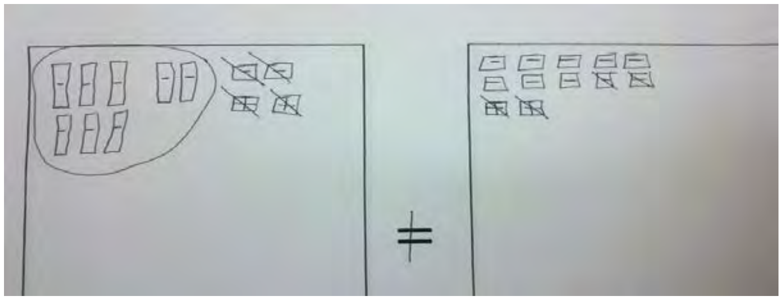


- What we do to one side, we must do to the other. The left side cancels each other out (-2 + 2 = 0). Put a slash marks through the 2 squares with the minus signs and 2 squares with the plus signs. Since we drew 2 squares with plus signs in them on the left side, we must draw 2 squares with plus signs in them on the right side.

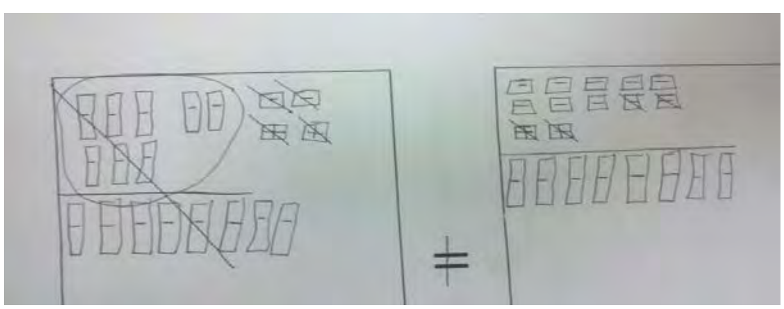


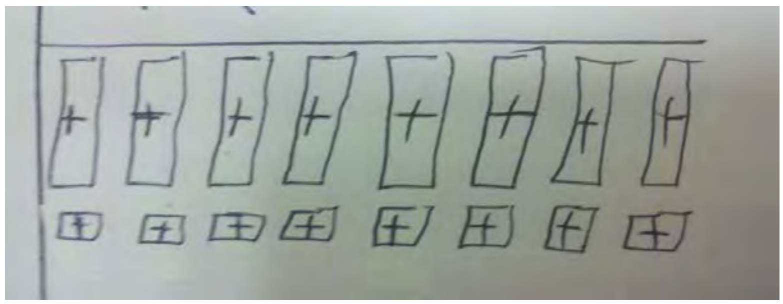
- On the right side, we have 10 squares with minus signs in them and 2 squares with plus signs in them. These are the same shape but have different signs in them, we must subtract or pair up squares and put slash marks through each pair. After drawing slash marks, how many squares are left on the right side? We have 8 squares with minus signs in them.

- Now we have -8X =-8. Is there only 1 rectangle? No. There are 8 rectangles. We must find out what 1 rectangle is equal to?

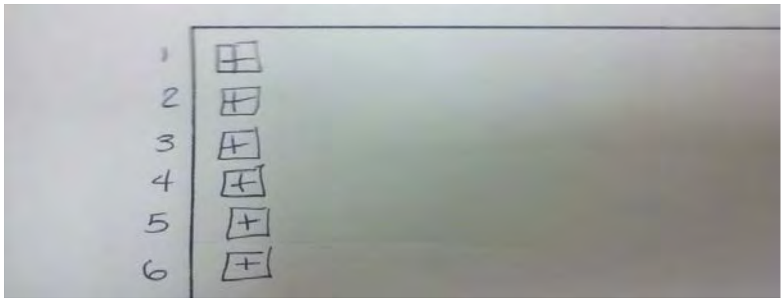


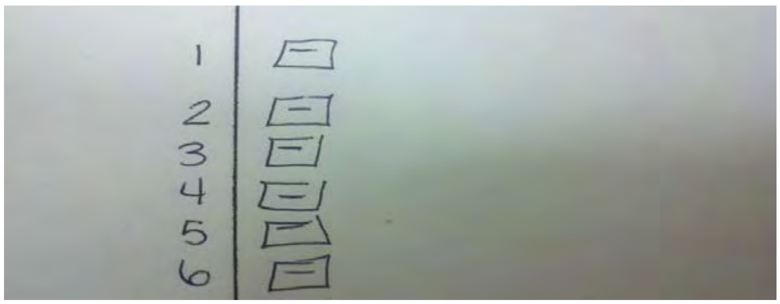
- I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The "U" is for unite like terms. Look on the left side. Are there any tiles the same shape that are not already together? NO. Look on the right side. Are there any tiles the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- The next step is Loop the variable. The variable is the rectangular tile. This is the number that we want to get by itself and want to move last.
  - Is there anything else on the side with the circle? No. So, we have to see what 1 X is equal to?
  - Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division symbol beside the 8 rectangles with the minus signs in them on the left side.
  - What we do to one side, we must do to the other. Draw a division symbol beside the 8 squares with the minus sign in them on the right side. The left side cancels each other out. Draw a line through the rectangles on the left side.
  - On the right side, separate the rectangles. Give each rectangle a square until there are no more squares to give. How many squares does each rectangle have? 1. Do the rectangle and squares have the same sign in them? Yes, they have the same signs, therefore the answer is positive, X =1. Is the variable by itself? Yes. Is it positive? Go to the last step.

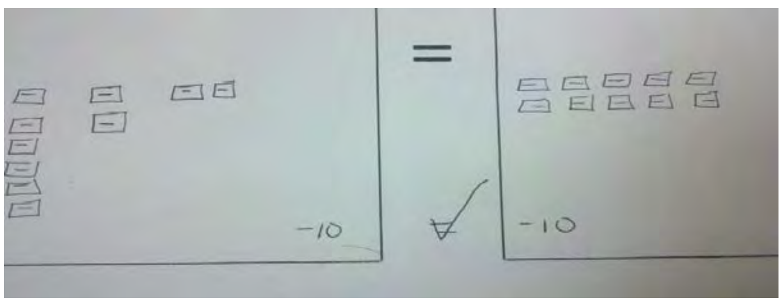




- Last step is to substitute the solution 1 in for X in the original problem. Original problem is -6X -2X - 2 = -10. On the left side, draw 1 square for every X. There are -6X, so draw 6 squares with minus signs in them (they are negative because the numbers have different signs). For -2X, draw 1 square for every X. Since there are -2X, we will draw 2 squares with the minus signs in them (the numbers have different signs). Also, draw 2 squares with negative signs in them. On the right side, draw 10 squares with the minus sign in them. Pair up the squares on the left side until we cannot make any more pairs. Put slash marks for each pair. What is left? We have 10 squares with minus signs in them on the left side. We also have10 squares with the minus sign in them on the right side. They are the same. The answer is correct.

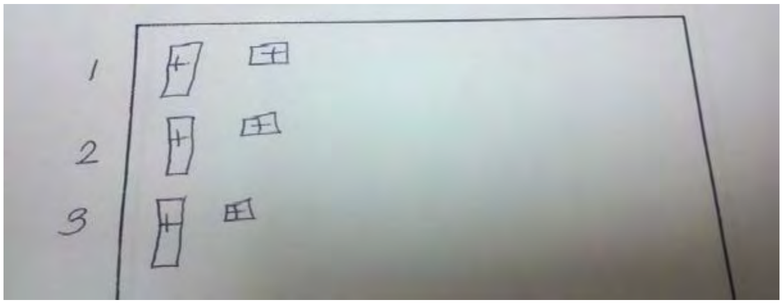


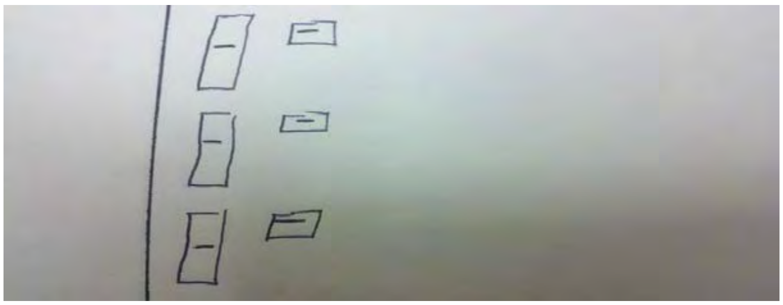


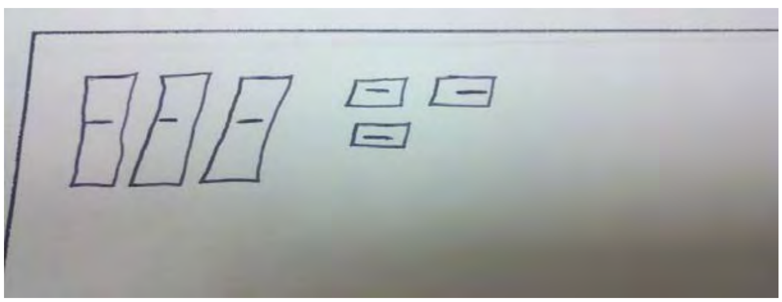


Guided practice problem #3

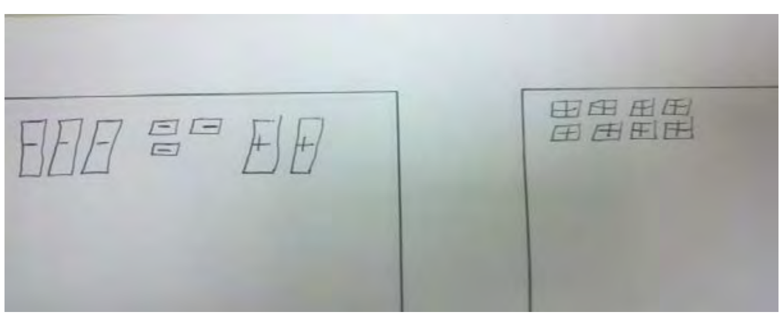
- I read the next problem. -3(X +1) +2X =8
- I make sure my mat or board is clear
- In the problem, notice the parenthesis ( ). This problem requires us to get rid of the parenthesis before we start SUMLOWS.
- What is in the parenthesis? X+1. What is outside the parenthesis? -3. On the mat, draw squares that represent X +1. Because the number outside the parenthesis is negative, the signs are going to flip to its opposite side. Therefore, draw shapes for –X - 1. We are going to draw these tiles down 3 times. We are going to draw a 1 rectangle with a minus sign and 1 square with a minus sign in it. The number outside the parenthesis tells us how many times we need to put -X -1 on the mat. In this case, we draw -X -1 on the mat 3 times.
- How many rectangles are there? 3 rectangles with minus signs in them.
- How many squares are there? 3 squares with minus signs





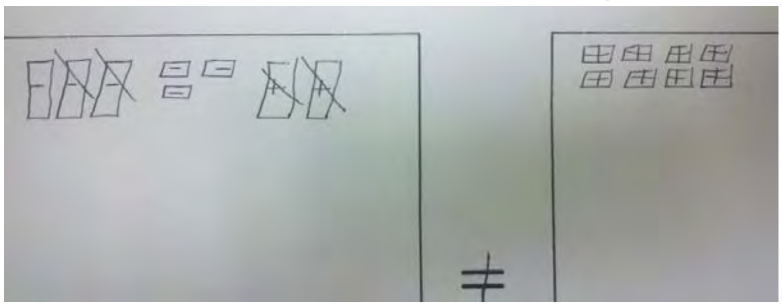


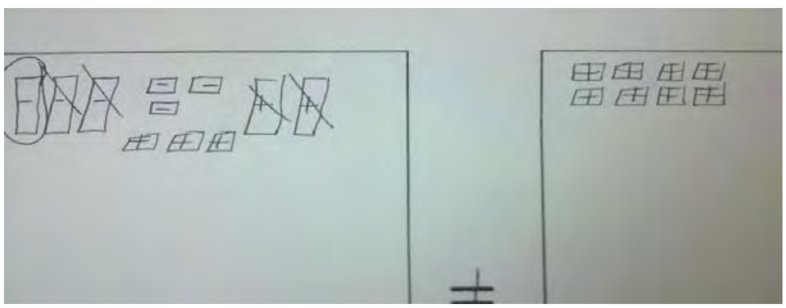
- Now we are ready to solve the equation using SUMLOWS.
- In addition to the shapes that we already have on the mat, draw 2 rectangles with plus signs on the left side. On the right side, draw 8 squares with plus signs in them. On the left side, we should have 3 rectangles with minus signs in them, 3 squares with minus signs in them and 2 rectangular with plus signs in them.



- Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.

- Unite like terms. Are there any shapes the same shape that are not already together? Yes. We must combine the 3 rectangles with minus signs in them and the 2 rectangles with plus signs in them. The signs are different, so we must subtract or pair them up and draw a slash mark through them. Look on the right side. Are there any shapes the same that are not already together? No. We go on to step three.
- Modify the new equation. Since we combined the left side, how many rectangles are there? 1 rectangle with minus sign in it. Our new equation is –X -3 = 8. We should have 1 rectangle with minus sign in it and 3 squares with minus signs in them on the left side. We have 8 squares with plus signs in them on the right side.

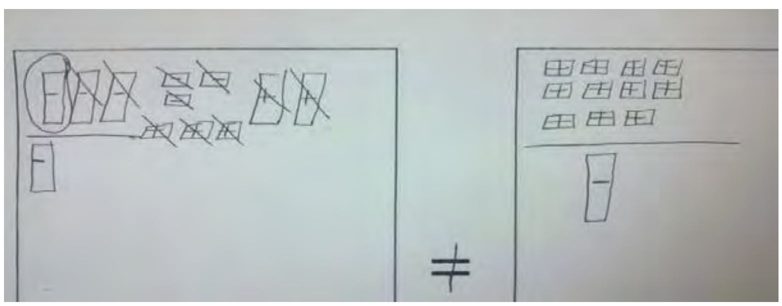




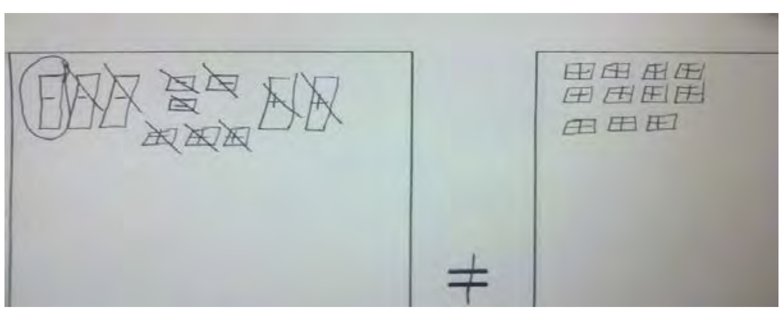
- Loop around the variable. Draw a circle around the 1 rectangle with minus sign in it. That is the number that should be by itself.
- Is there anything else on the side with the circle? Yes, there are 3 squares with minus signs in them. We are going to have to move the 3 squares with minus signs in them because the variable should be by itself.
- Next step is opposite sign. In order to move the 3 squares with minus signs in them, we must add 3 squares with plus sign in them to the left side.

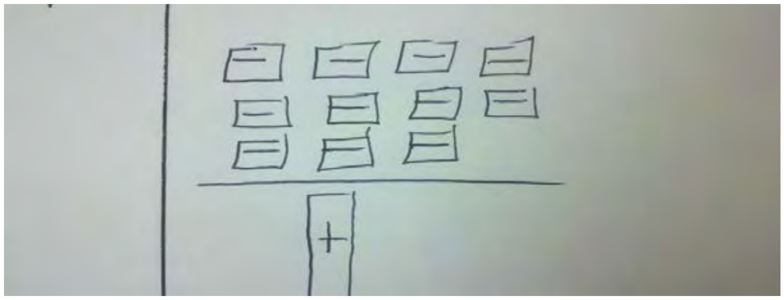
- What we do to one side, we must do to the other. The left side cancels each other out (-3+3 = 0). Since we added 3 squares with plus signs on the left side, we must add 3 squares with plus signs in them to the right side.
- On the right side, we have 8 squares with plus signs in them and 3 squares with plus sign in them. These are the same shape and have the same sign, therefore we must add or combine the tiles. How many shapes are on the right side? We have 11 squares with plus signs in them.

- Now we have -X = 11. Is there only 1 rectangle? Yes. Is the X positive? No. We must find out what a rectangle with a plus sign is equal to?
- I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The “U” is for unite like terms. Look on the left side. Are there any shapes the same shape that are not already together? NO. Look on the right side. Are there any shapes the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- The next step is Loop the variable. The variable is the rectangle. This is the number that we want to get by itself and want to move last.
- Is there anything else on the side with the circle? No. Is the X by itself? Yes. Is it positive? No. It is negative. We must move the -1 (the1 is invisible).
- What is the next step? Next step is opposite sign. In order to move the negative one, we must do the opposite. The opposite of “minus 1” is “plus one.” So, we flip the sign to the green side.
- What is the next step? What we do to one side, we must do to the other. Because we flipped sign on the left side, we must flip the sign on the right side.

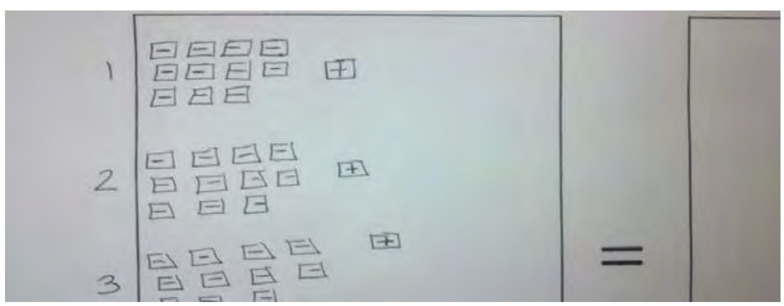


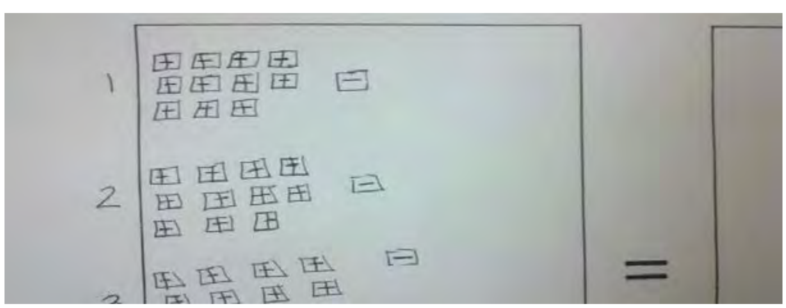
- On the left side of the equal sign, we have 1 rectangle with a plus sign. On the right side, we have 11squares with minus signs. Now we have X =-11. Is the X by itself? Yes. Is it positive? Yes. Now we do the last step.

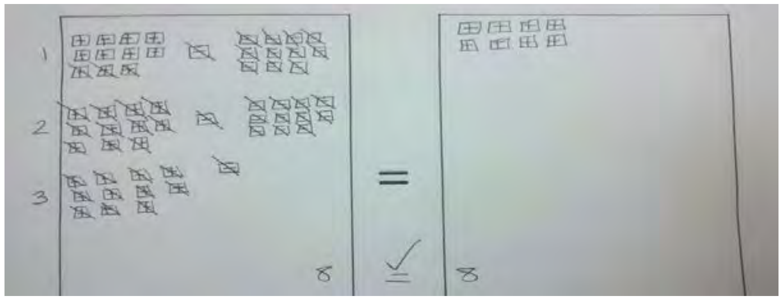




- Last step is to substitute the solution 2 in for X in the original problem. Original problem is -3(X +1) + 2X = 8. For X+1, write 10 (-11+1=-10) squares with plus signs in them. The squares have a plus sign because the number outside the parenthesis is negative; so we have to flip the signs to its opposite side. Also draw 22 square with minus signs in them (-11 X2 = -22). The signs are different, so we put pair them up and put a slash mark through the pairs. What is left side? We have 8 squares with plus signs in them on the left side. On the right side, we have 8 squares with plus signs in them. They are the same. The answer is correct.

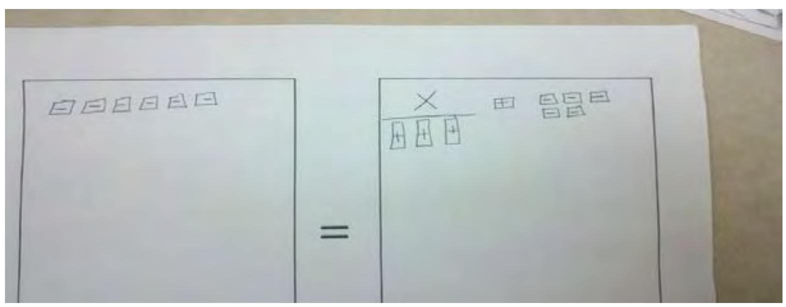




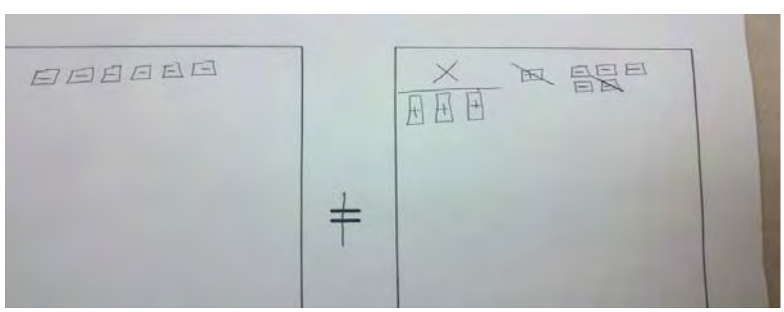


Guided Practice# 4

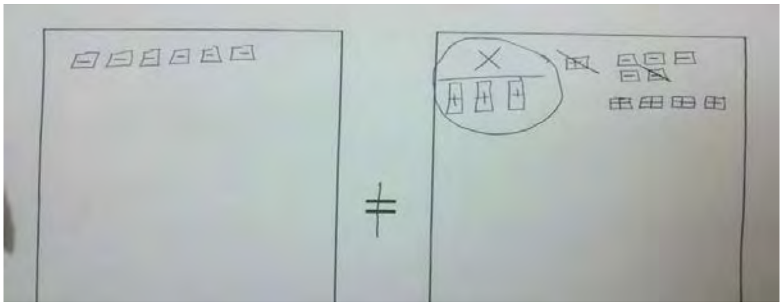
- I read the problem: $-6=\frac{X}{3}+1-5$
- If the students flip the equation, the steps would be the opposite side.
- I make sure the algebra mat is clear
- On the left side of the mat, draw 6 squares a negative sign in them). On the right side, draw a line and draw 3 rectangles with a plus sign in them under the line. On top of the line draw an X. Also on the right side, draw 1 square with plus sign in them and 5 squares with minus signs in them.



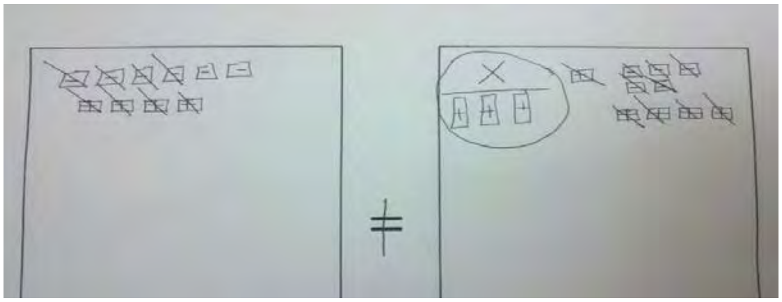
- Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
- Unite like terms. Are there any tiles the same shape that are not already together on the left side? No. Look on the right side. Are there any tiles the same that are not already together? Yes. We have 1 square with plus sign in them and 5 squares with minus signs in them. We will combine these like terms. Draw a line through one square with a minus sign and one square with a plus sign until there are not more pairs to cross out.
- Modify the new equation. How many tiles are left on the right side? 4 square with minus signs in them.



- Loop around the variable. Draw a circle around the 3 rectangles with plus signs in them. That is the number that should be by itself.
- Is there anything else on the side with the circle? Yes! There are 4 squares with minus signs in them. We have to get rid of the 4 squares with the minus signs.
- Next step is opposite sign. The opposite of negative (minus) 4 is positive (plus) 4. We are going to draw 4 squares with plus signs in them under the 4 squares with the minus signs in them.



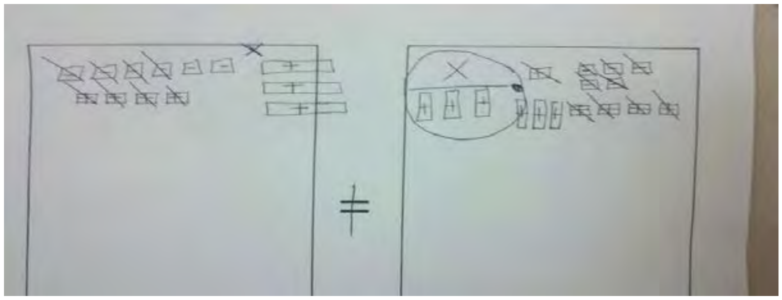
- What we do to one side, we must do to the other. Since we drew 4 squares with plus signs in them on the right side of the equation, we must draw 4 squares with plus signs in them on the left side of the equation. The right side squares cancels each other out. On the left side, we have 4 squares with plus signs in them and 6 squares with minus signs in them. We are going to combine like terms. Draw a line through one square with a plus sign and one square with a minus sign until they are no more pairs to make. On the left side, we should have 2 squares with minus signs left.



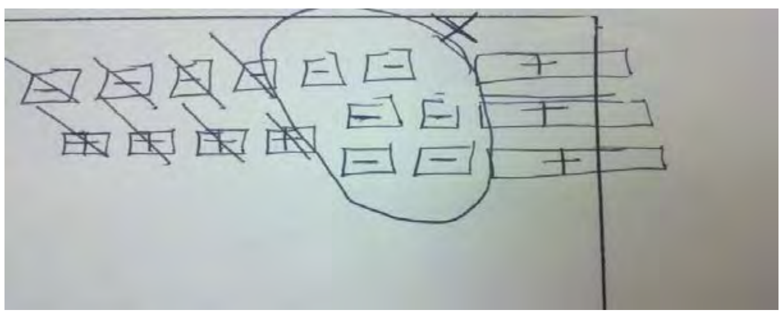
- On the algebra mat, we should have 2 squares with minus sign in them on the left side. On the right side, we should have X over 3 rectangles with plus sign. The rectangles should have a circle around it.

- We have already separated the sides, united like terms, modified the equation, and looped the coefficient. The next step is opposite sign.
- When we have an X over a number, which means to divide. What is the opposite of division? It is multiplication. We are going to write a multiplication sign and draw 3 rectangles with plus sign beside it.
- Next step is what we do to one side; we must do to the other. Since we drew a multiplication sign and drew six rectangles with plus signs on the right side, we

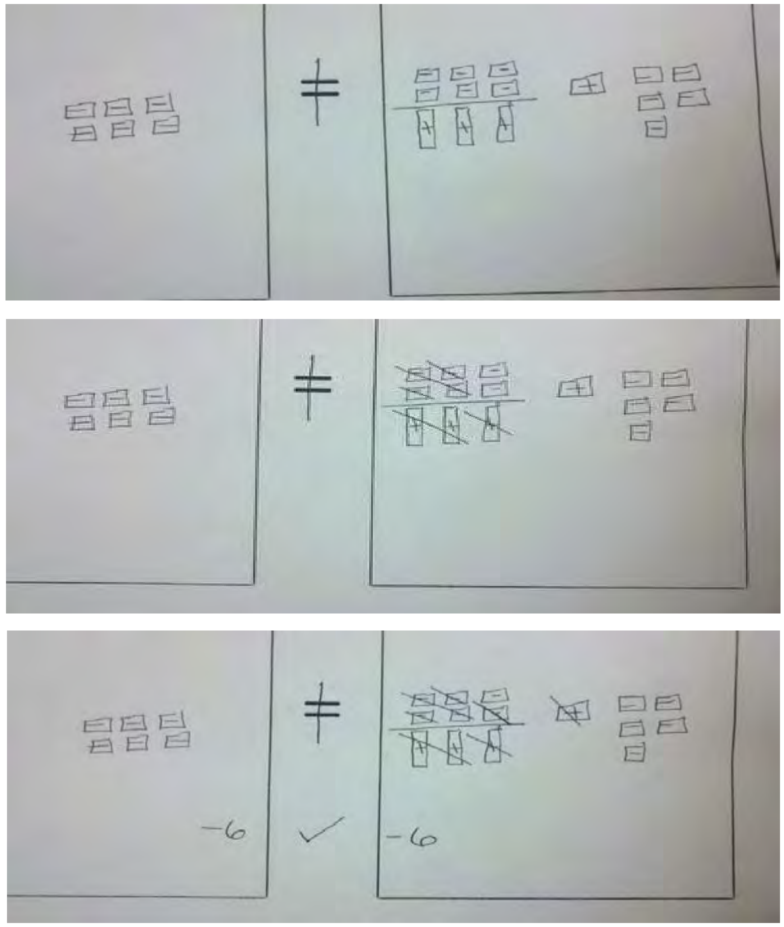
must do the same on the left side. On the left side, draw a multiplication sign and draw 3 rectangles with plus signs in them.



- We are going to give all of the 2 squares with minus signs to the first green rectangle. We are going to draw 2 squares with minus signs for the 2nd, and 3rd, rectangles. How many squares are all together? We have 6 squares with minus signs. So, X = -6.



- Last step is to substitute the solution -6 in for X in the original problem.   Original problem is -6= X/3 +1-5. On the left side, draw 6 squares with minus signs. On the right side, draw 6 squares with minus signs in them for X. Draw a line under the squares and then draw 3 rectangles with plus signs in them under the line. Separate the rectangles and give each rectangle a square until they are all gone. How many squares does each rectangle have? 2 squares with minus signs. Cross out the 3 rectangles and 4 squares with minus signs (because 6 divided 3=2).   On the mat, we should have 6 squares with minus signs on the left side. On the right side, we should have 2 squares with minus sings, 1 square with plus sign, and 5 squares with minus signs. On the right side, combine the squares. Cross out a square with a minus sign and a square with a plus sign until there are no more pairs to make. How many squares are on the right side? 6 squares with minus signs. How many squares are on the left side? 6 squares with minus signs. The numbers are the same. Are both sides equal? Yes. The answer is correct.



Independent Practice

- Direct the students to the "Independent Practice" section. Read the first problem together and direct students to complete the problems without you.
  When students finish problems, provide immediate corrective feedback for errors.

Graphing

Learning Sheet 4

Multiple-Step
Representational
Code Number ___________________ Date ______________
SUMLOWS

Describe and Model

1) $2X+6 + 2 = 12$

2) $-2X - 5 + 4X = 13$

3) $3(X+2) + 2X = 16$

4) $-\frac{X}{2} + 3 - 4 = -6$

Guided Practice

1) $-X + 6 - 3 = 10$

2) $3X - X - 4 = 8$

3) $-3(X+1) + 2X = 10$

4) $-5 = \frac{X}{6} + 2 - 4$

5) $\frac{X}{4} + 2 - 3 = -2$

6) $-2(X+1) - 4 = 6$

Independent Practice

$20 = 3(3X) + 2$

$5 - X + 3X = 9$

$-6 = \frac{X}{2} + 3 - 1$

$-4(X+1) + 2 = -10$

$-2X + 4 = 4 + 6$

$-10 = -2(X - 3) + 2$

$-2X + 4X - 2 = -8$

$7X - 2X - 5 = 20$

Learning Sheet 5

Multiple-Step
Representational
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) $-3X + 6 - 4 = -13$

2) $5 = \frac{X}{3} + 2 - 4$

3) $-2(3X + 2) + 2 = 10$

Guided Practice

1) $-4 = -3(X+3)+2X$

2) $-X + 3X - 2 = 8$

3) $-2(2X + 1) - 2X = -8$

4) $-4 = \frac{X}{2} - 3 - 4$

Independent Practice

$6 = 4(3X - 2) + 2$

$2 - X + 5X = -6$

$2(X + 1) - 4 = 6$

$3 = \frac{X}{4} + 1 - 5$

$6(X+1) - 2 = -8$

$\frac{X}{4} + 6 - 1 = -5$

Learning Sheet 6

Multiple-Step
Representational
Code Number _________________ Date _____________
SUMLOWS

Describe and Model

1) $3X + 2X - 5 = 15$

2) $3 = -\frac{X}{2} + 3 - 4$

Guided Practice

1) $-16 = -3(X+2X)+2$

2) $4X - 2X - 3 = 9$

3) $-4(X+1)+2X = 8$

Independent Practice

$12 = 2(3X+2) + 2$

$5 - 2X + 4X = -5$

$3 + 3X - 8 = 4$

$-5 = \frac{X}{2} + 2 - 5$

$-4(X+2) + 3 = 3$

$\frac{X}{2} - 6 + 3 = 4$

Lessons 7-9

Multiple-step

Abstract Level

Materials

- Chalkboard/whiteboard/easel
- learning sheets 7-9
- progress chart

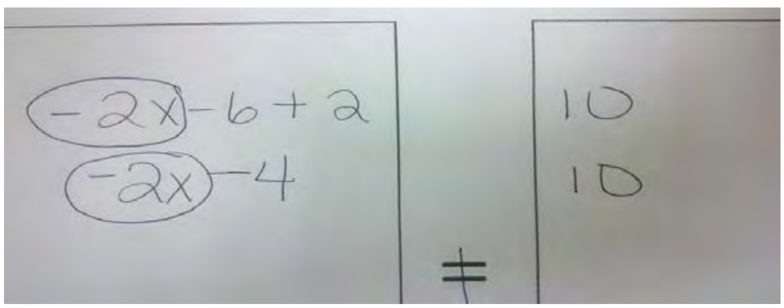
Advance Organizer

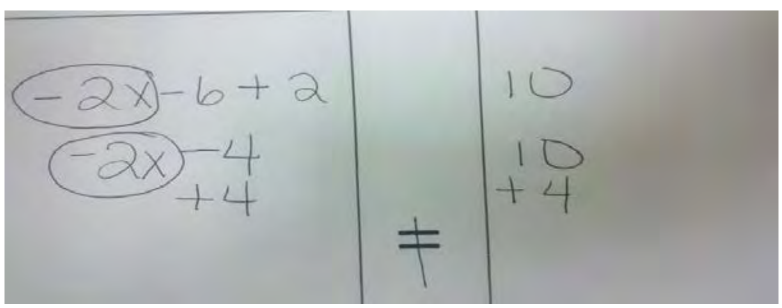
- Tell students what they will be doing and why
  Remind students about the commitment they made to learn to solve equations. We will work hard to teach them and they will work hard to learn a new way to solve equations. They learned to solve multiple equations with algebra tiles and with drawing. Now they will practice solving equations without using tiles or drawings. Remind students of the SUMLOWS mnemonic and tell them that this mnemonic can be used to help them solve equations if they do not remember the steps.

Demonstrate

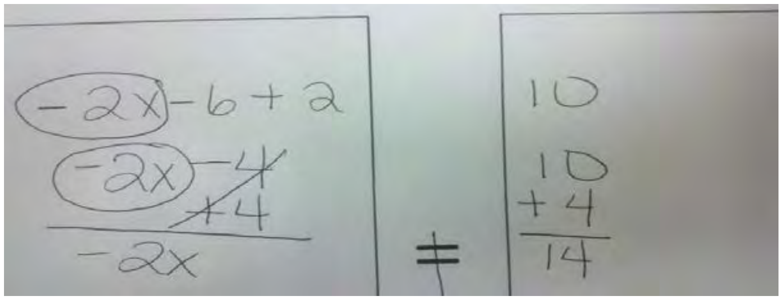
- Give students learning sheets 7-9.
- Begin with the first problem in the "Model" section. Tell the students that we will show them how to solve the problem using numbers only.
- First we First we read the problem. -2X - 6+ 2 =10
- Next, I take out my workmat or dry erase board.
- In the abstract method, I am going to use symbols and numbers. For positive numbers, I am going to use a "+" plus sign or no sign at all. It depends on where the number or symbol is in the problem. For negative numbers, I am going to use "-"minus signs.
- In this problem, I am going to write out the problem like I see it. On the left, I am going to write-2X – 6 +2. On the right side, I am going to write 10.
  - The first step in SUMLOWS is to separate the two sides. Draw a line through the equal sign.
  - The "U is for unite like terms. Look on the left side, are there any like terms or numbers that not already together? Yes. Negative six and positive two are like terms and can be combined together. Look on the right side. Are there any like terms that are not together? No. We go to the next step.

- Modify the new equation. Negative six and positive two are like terms. They have different signs, so we must subtract and take the sign of the larger number (absolute value). 6-2 = 4. Six is larger than two. So, our answer is -4. Our new equation is -2X – 4 =10.
- The next step is Loop the variable. The variable is the X. This is the number that should be by itself and want to move last. Draw a circle around the X. What is on the side with the variable or X? -4. We have to get rid of -4 because X should be by itself.
- In order to move the -4, we have to write +4 under the -4. Opposites cancel each other out (-4 +4 =0). Put a slash mark through -4 and +4. This equals 0 and cancels each other out.
- The next step is "what we do to one side, we must do to the other" We added 4 on the left side, so we must add 4 on the right side. Write +4 under the 10.

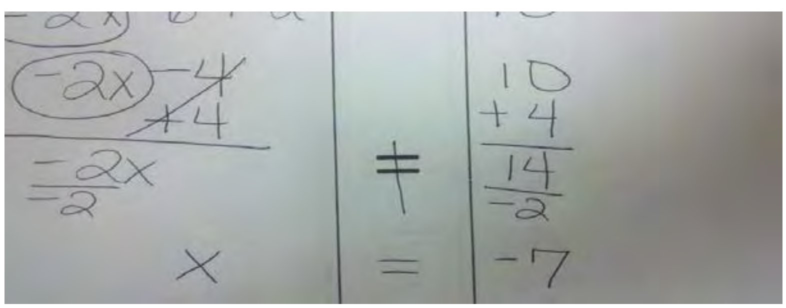




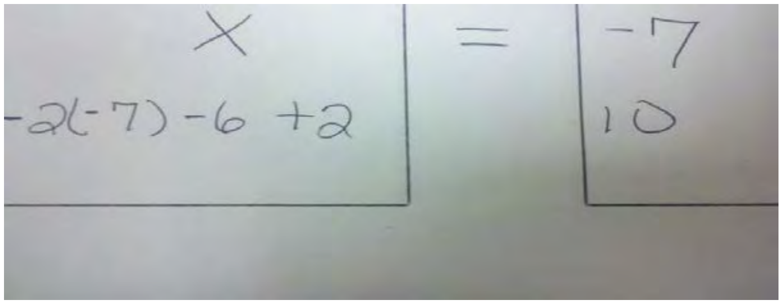
- Combine like terms. On the right side, we have 10 and +4. Since the numbers have the same signs, we must add them and take the sign of the larger number (absolute value). Ten plus 4 is 14. Since the larger number (absolute value) is 14 and it's positive, the answer is positive. So -2X = 14.
- Is there anything else on the side with the circle? Yes, we have -2.



- Now we have -2X =14. Is there only 1X? No., there are two; therefore we must determine what 1X is equal to. I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The “U” is for unite like terms. Look on the left side. Are there any like terms or numbers that can be combined that are not already together? NO. Look on the right side. Are there any like terms or numbers that can be combined that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the X. That is the number that should be by itself.
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a line under the -2 sign and write -2 under it.
- What we do to one side, we must do to the other. On the right side, draw a line under the 14 and write -2 under it. The left side cancels each other out. Draw a slash mark on the lefts side.   On the right side, we divide 14 divided by -2. It is 7. The signs are the different, so the number is negative. Therefore, x =-7.

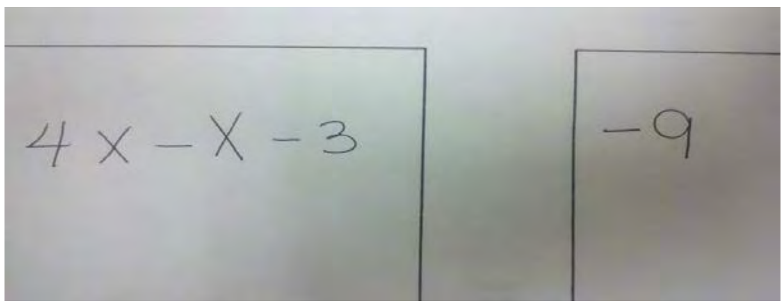


- Last step is to substitute the solution -7 in for X in the original problem.   Original problem is -2X -6 +2 = 10.   On the left side, write -2(-7) because X = -7. Also write -6 + 2. On the right, write 10. On the right, what is 2 times 7? It is 14. The signs are the same; therefore the answer is positive. Positive 14 minus 6 equals 8 plus 2 equals 10 on the left side. Are both sides equal?   Yes. The answer is correct.

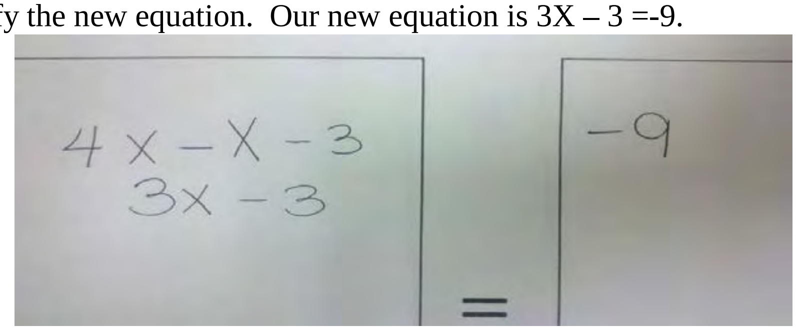


Model problem # 2

- I read the next problem. 4X –X - 3 = -9
- On the left side of the mat, we are going write 4X – X- 3. On the right side, write -9.

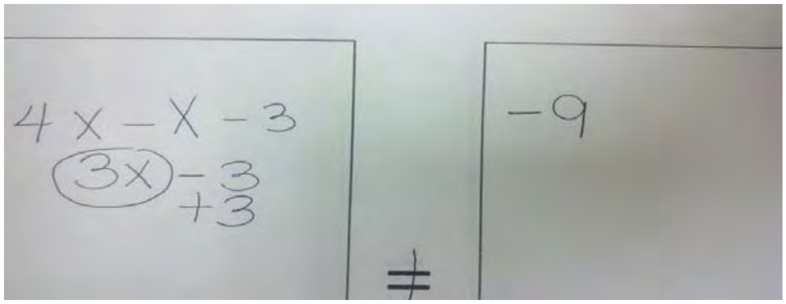


  - The first step in SUMLOWS is to separate the two sides. Draw a line through the equal sign.
  - What does the "U" stand for in SUMLOWS? Unite like terms. Are there any like terms or numbers that are the same that are not already together on the left side of the equal sign? Yes. Because there are like terms that are not together on the left side of the equal sign, we must unite (combine) like terms. We have 4X and -X. The signs are different, so we need subtract them. 4-1 (the- 1 in front of the X is invisible). Four minus one is 3.
  - Modify the new equation. Our new equation is 3X – 3 =-9.

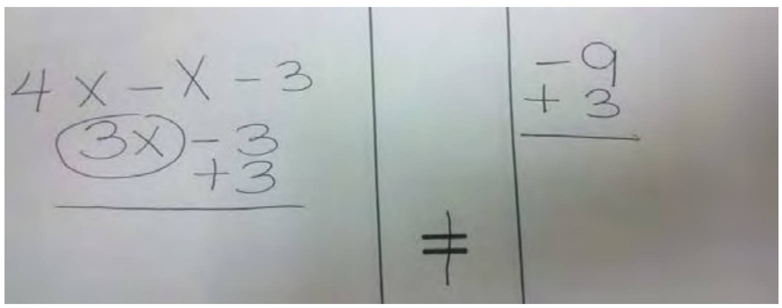


  - The next step is Loop the variable. The variable is the X. This is the number that should be by itself and want to move last. Draw a circle around the X. What is on the side with the variable or X? -3. We have to get rid of -3 because X should be by itself.

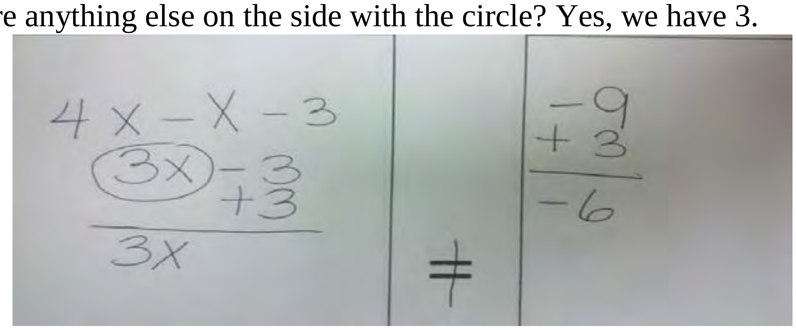
- In order to move the -3, we have to write +3 under the -3. Opposites cancel each other out (-3+3 =0). Put a slash mark through -3 and +3. This equals 0 and cancels each other out.



- The next step is "what we do to one side, we must do to the other" We added 3 on the left side, so we must add 3 on the right side. Write +3 under the-9.

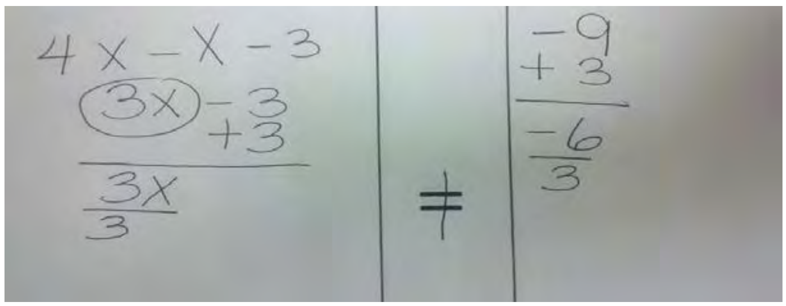


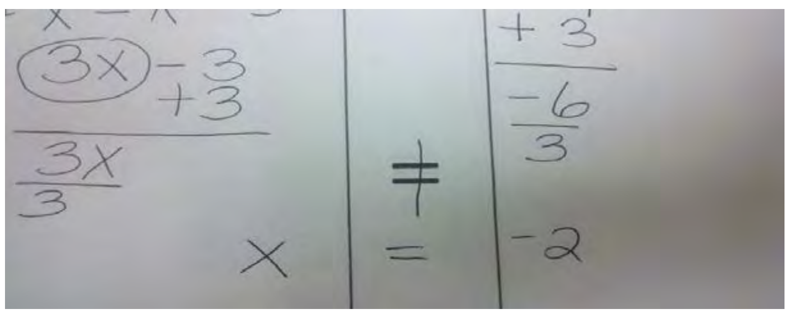
- Combine like terms. On the right side, we have -9 and +3. Since the numbers have different signs, we must subtract them and take the sign of the larger number (absolute value). Nine minus three equals six. Since the larger number (absolute value) is 9 and it's negative, so 3X =-6.
- Is there anything else on the side with the circle? Yes, we have 3.



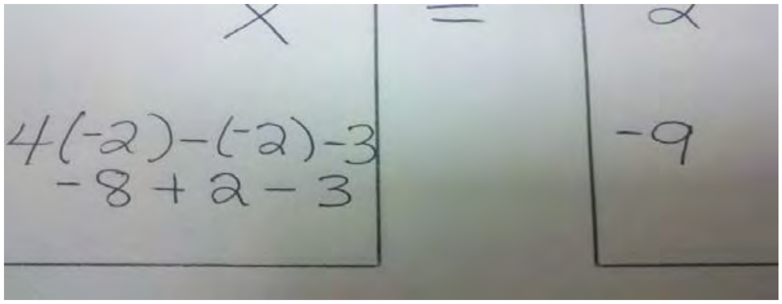
- Now we have 3X =-6. Is there only 1X? No, there are three; therefore we must determine what 1X is equal to.
- I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The "U" is for unite like terms. Look on the left side. Are there any like terms or numbers that can be combined that are not already together? NO. Look on the right side. Are there any like terms or numbers that can be combined that are not already together? No. We go on to step three.

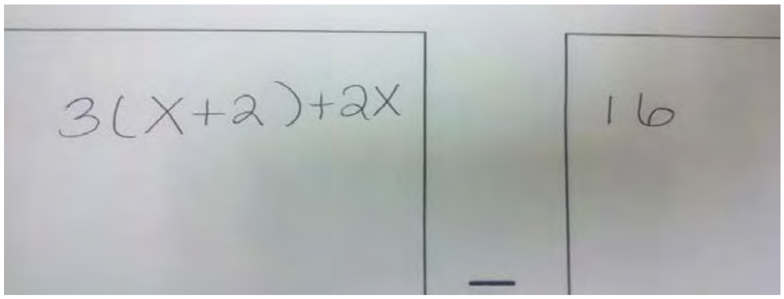
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the X. That is the number that should be by itself.
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a line under the 3 sign and write 3 under it.
- What we do to one side, we must do to the other. On the right side, draw a line under the 9 and write 3 under it. The left side cancels each other out. Draw a slash mark on the lefts side. On the right side, we divide 6 by 3. It is 2. The signs are the different, so the number is negative. Therefore, x =-2.





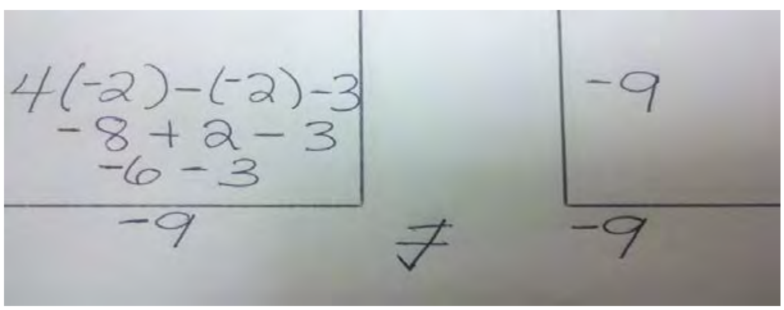
- Last step is to substitute the solution -2 in for X in the original problem. Original problem is 4X –X -3+ -9. On the left side, write 4(-2) because X = -2. For –X, write –(-2), two negatives equal a positive. Also, write -3. On the right, write -9. On the left, what is 4 times 2? It is 8. The signs are the different; therefore the answer is negative. Negative 8 plus 2 equal -6. Negative 6 minus 3 equals negative 9. We have -9 on the left side and the right side. Are both sides equal? Yes. The answer is correct.



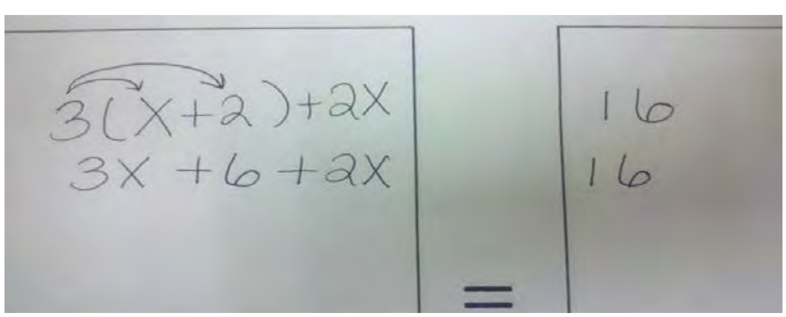


Model problem #3

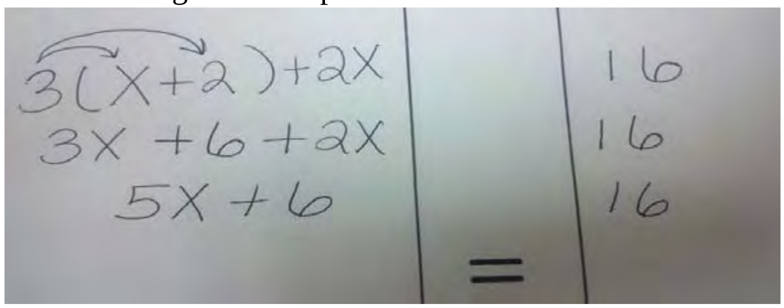
- I read the next problem. 3(X + 2) +2X =16
- I make sure my mat or board is clear
- In the problem, notice the parenthesis ( ). This problem requires we to get rid of the parenthesis before we start SUMLOWS.



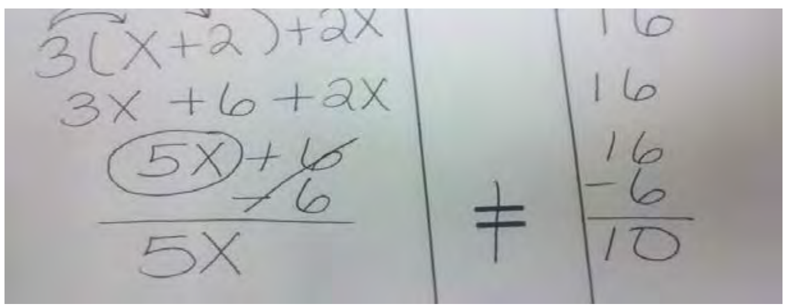
- What is in the parenthesis? X + 2. What is outside the parenthesis? 3. On The mat, three has to be multiplied by everything that is inside the parenthesis. What is 3 times X? 3X. What is three times 2? 6. We have 3X+6 +2X = 16.



- Now, we are ready to solve the equation using SUMLOWS.
  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any like terms or numbers that are not already together? Yes. We must combine the 3X and 2X. The signs are the same, so we must add or combine them. Look on the right side. Are there any like terms that are not already together? No. We go on to step three.

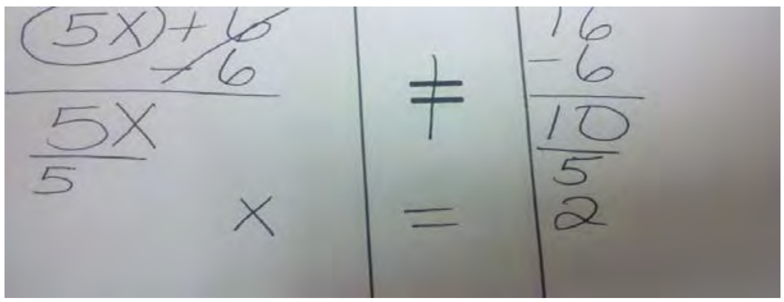


  - Modify the new equation. Since we combined the left side, our new equation is 5X +6 =10.

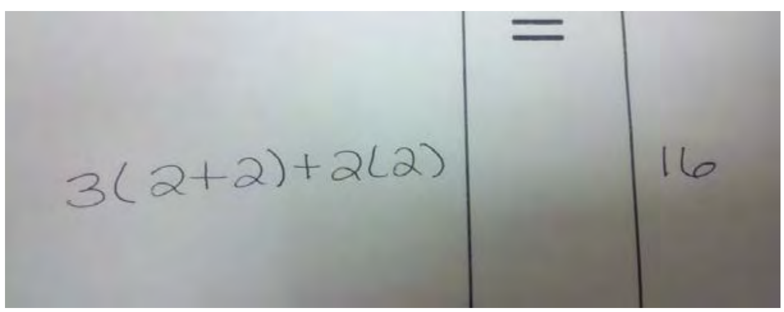


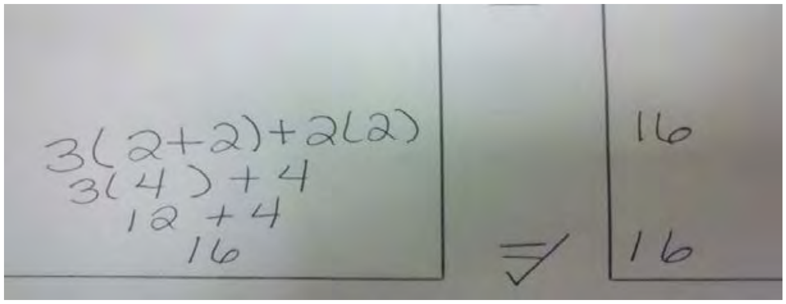
  - Loop around the variable. Draw a circle around 5X. That is the number that should be by itself.
  - Is there anything else on the side with the circle? Yes, there is +6. We are going to have to move +6 because the variable should be by itself.
  - Next step is opposite sign. In order to move +6, we - 6 red squares to the left side.
  - What we do to one side, we must do to the other. The left side cancels each other out (+6 -6 = 0). Since we - 6 on the left side, we must -6 on the right side.
  - On the right side, we have 16 and -6. These numbers have different signs, therefore we must subtract the numbers and take the sign of the larger number (absolute value). What is 16-6? 10. Ten is larger, so the answer is positive 10.
    Now we have 5X = 10. Is there only 1 rectangle? No. There is a 5. We must find out what 1 rectangle is equal to?
  - I start back at the front of SUMLOWS mnemonic.
  - I have already separated the sides.
  - The "U" is for unite like terms. Look on the left side. Are there any like terms that are not already together? NO. Look on the right side. Are there any like terms that are not already together? No. We go on to step three.

- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- The next step is Loop the variable. The variable is the X. This is the number that we want to get by itself and want to move last.
- Is there anything else on the side with the circle? No. So, we have to see what 1 X is equal to?
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a division line under the 5 and write 5 under the line.
- What we do to one side, we must do to the other. Draw a division line under the 10 and write 5 under the line. The left side cancels each other out.
- On the right side, what is 10 divided by 5? Both numbers have the same sign, so it's positive. X =2. Is the variable by itself? Yes. Is it positive? Go to the last step.



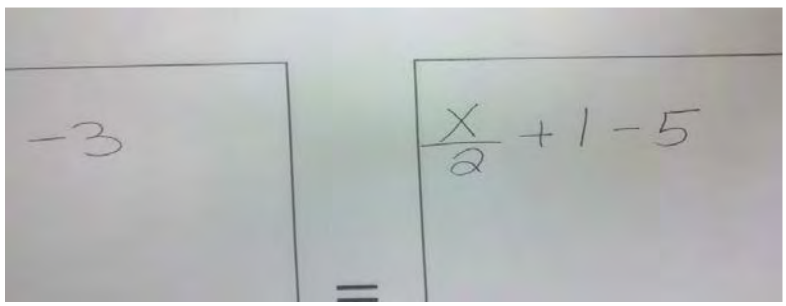
- Last step is to substitute the solution 2 in for X in the original problem. Original problem is 3(X +2) + 2X = 16. Write 3[(2) + 2] + 2(2) = 16. Do what's in the parenthesis first. Two plus two is 4. Three times 4 is 12. Twelve plus 4 is 16. Both sides equal 16. They are the same. The answer is correct.



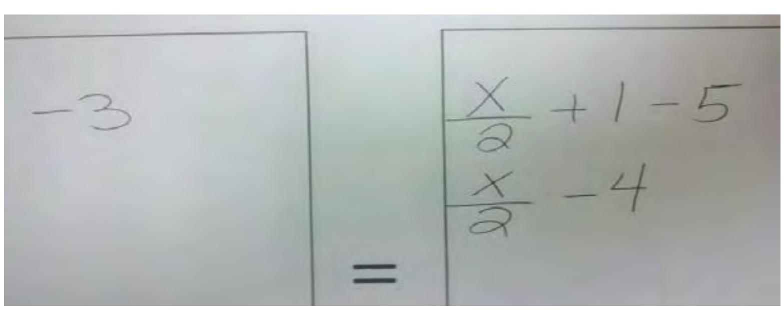


Model problem # 4

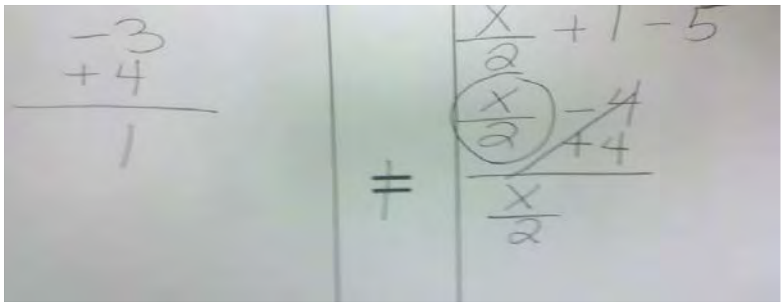
- I read the problem: $-3=\frac{X}{2}+1-5$
- If the students flip the equation, the steps would be the opposite side.
- I make sure the algebra mat is clear
- On the left side of the mat, write -6. On the right side, write X/2. Also on the right side, write +1 and -5.



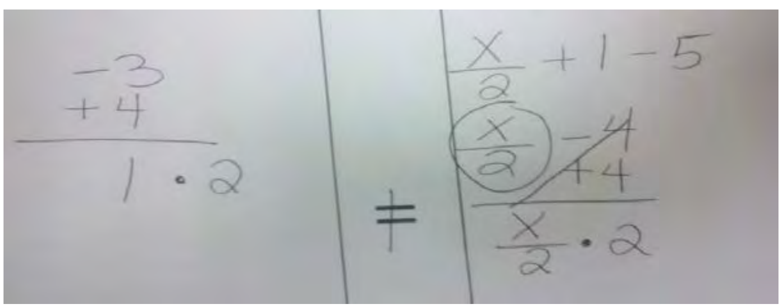
- Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
- Unite like terms. Are there any like terms that are not already together on the left side? No. Look on the right side. Are there any like terms that are not already together? Yes. We have plus 1 and -5 that are like terms. We will combine these like terms.
- Modify the new equation. What is positive 1 minus 5? These numbers do not have the same sign, so we have to subtract them and take the sign of the larger number (absolute value). One minus five is four. Five has a larger absolute value, so our answer when we combine these like terms is -4.



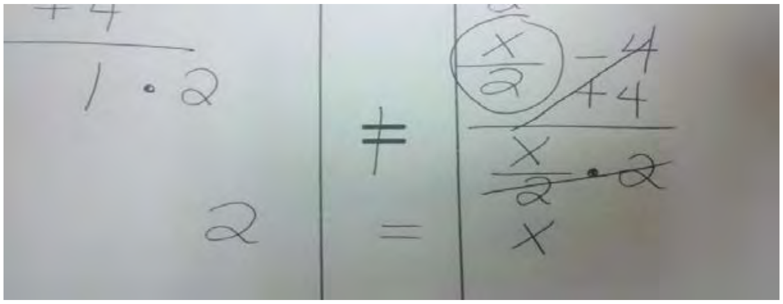
- Loop around the variable. Draw a circle around the X/2. That is the number that should be by itself.
- Is there anything else on the side with the circle? Yes! There are -4 on the same side. We have to get rid of the -4.
- Next step is opposite sign. The opposite of negative (minus) 4 is positive (plus) 4. We are going to write + 4 squares under the -4.
- What we do to one side, we must do to the other. Since we wrote +4 on the right side of the equation, we must write + 4 on the left side of the equation. The right side squares cancels each other out. On the left side, we have +4 and -3. We are going to combine like terms. Since these terms are alike but do not have the same sign, subtract them and take the sign of the largest number (absolute value). Four minus three is 1. The four is positive and larger; so our answer is +1.



- On the algebra mat, we should have +1. On the right side, we should have X over 2.
- We have already separated the sides, united like terms, modified the equation, and looped the coefficient. The next step is opposite sign.
- When we have an X over a number, which means to divide. What is the opposite of division? It is multiplication. We are going to write a multiplication sign and write 2 beside it.



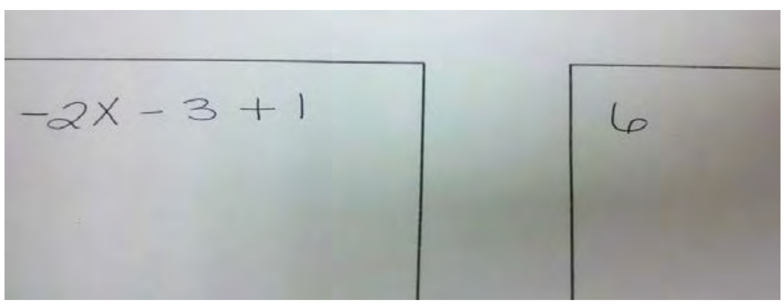
- Next step is what we do to one side; we must do to the other. Since we wrote a multiplication sign and wrote 2 on the right side, we must do the same on the left side. On the left side, write a multiplication sign and write 2 beside the +1.
- We are going multiply +1 times positive 2. So, X = 2.



- Last step is to substitute the solution 2 in for X in the original problem. Original problem is -3-= X/2 +1-5. On the left side, write -3. On the right side, write 2/2 +1-5. Two divided by two is one. Positive one plus one is two. Positive two minus 5 is negative 3. What number are on the left side? -3. What number are on the right side? -3. The numbers are the same. Are both sides equal? Yes. The answer is correct.

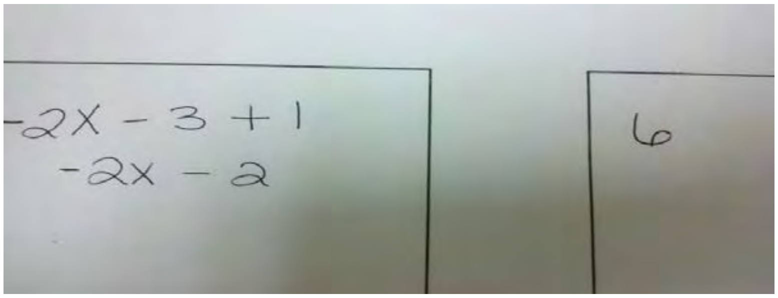
Guided Practice

- Direct students to the “Guide” section of the learning sheet
- Tell students to touch the first problem and that we will do this problem together, using numbers, letters and the SUMLOWS mnemonic.
- Let’s read the problem. This problem is -2X -3 +1 = 6.
- On our workmat, what do we write on the left side? -2X -3 + 1
- What do we write on the right side? 6

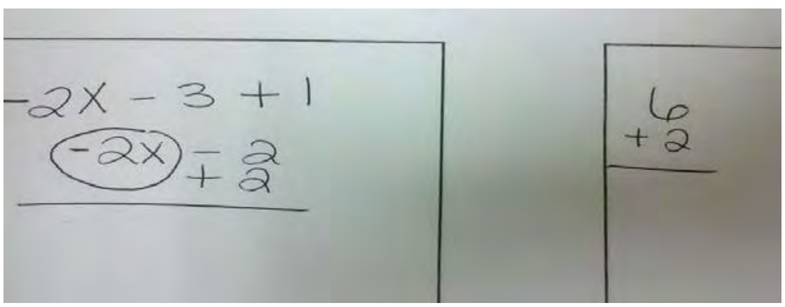


- What is the first step in SUMLOWS? Separate the sides. We are going to draw a line through the equal sign.
- The “U is for unite like terms. Look on the left side, are there any like terms that are not already together? Yes. We have negative 3 and positive 1 that can be combined. They are like terms but do not have the same sign in them. Since they do not have the same sign with them, we subtract them and take the sign of the largest number (absolute value). Three minus 1 equal two. Three is larger than 1, so the answer is negative. Look on the right side. Are there any shapes that can be combined together? No. We go to the next step.
- Modify the new equation. Our new equation is -2X -2 = 6.

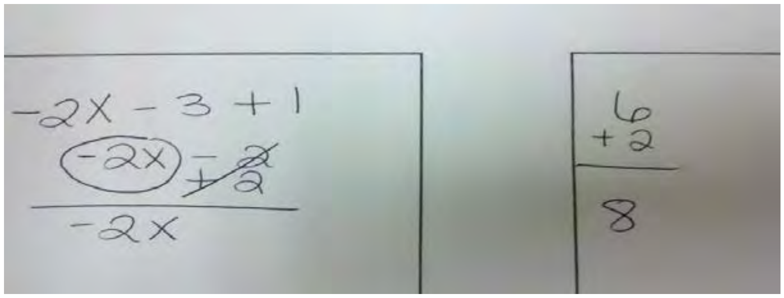
- The next step is Loop the variable. The variable is the X. This is the number that should be by itself and want to move last. Draw a circle around the X . What is on the side with the variable? -2. We have to move the -2.



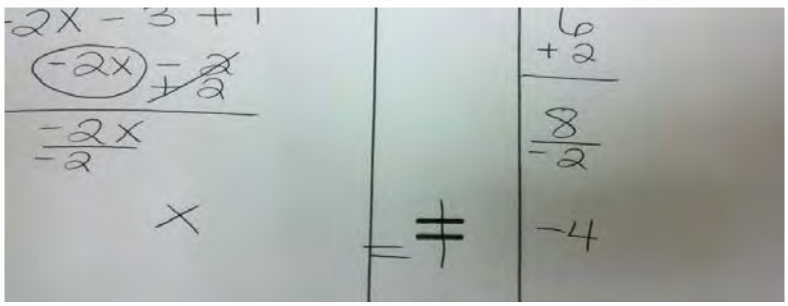
- In order to move the -2, we have to do the opposite. The opposite of -2 is +2. Opposites cancel each other out (-2+2=0). Write +2 under the -2. On the left side, draw slash marks through -2 and +2. .
- The next step is “what we do to one side, we must do to the other.” We added 2 to the left side; therefore we must add 2 to the right side. Write +2 under 6 on the right side.



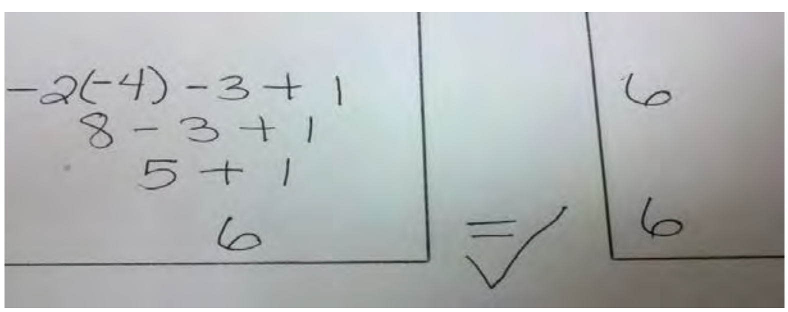
- Combine like terms. On the right side, we have 6 and +2. Since they are like terms, we have to combine them. They both are positive numbers, therefore we add the like terms and take the sign of the larger number (absolute value). Six plus two is 8. Six is larger, so the answer is positive. So -2X =8.
- Is there anything else on the side with the variable? Yes. What else is on the side with the variable? -2. So, we must determine what 1 rectangle is equal to. I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The “U” is for unite like terms. Look on the left side. Are there any like terms or numbers that can be combined that are not already together? NO. Look on the right side. Are there any like terms or numbers that can be combined that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
-



- What is the next step? Loop around the variable. Draw a circle around the variable. That is the symbol that should be by itself.
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a line under the -2 and write -2 below the line.
- What we do to one side, we must do to the other. On the right side, draw a line under the 8 and write -2 under it. The left side cancels each other out. On the right side, divide -2 into 8. How many times will 2 go into 8? 4. Do they have the same sign? No. Therefore, the answer is negative, x =-4

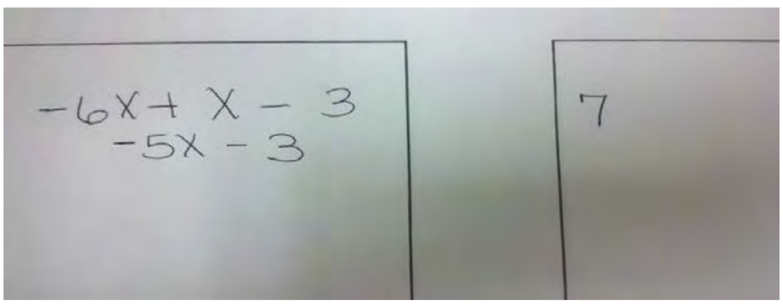


- Last step is to substitute the solution -4 in for X in the original problem. Original problem is -2X-3+1 = 6. On the left side, write -2(-4) -3 + 1. What is 2 times 4? 8. Are the signs the same? Yes. So the answer is +8. What is 8 – 3? 5. What is 5 +1? 6. We have 6 on the left and right sides. They are the same, so the answer is correct.

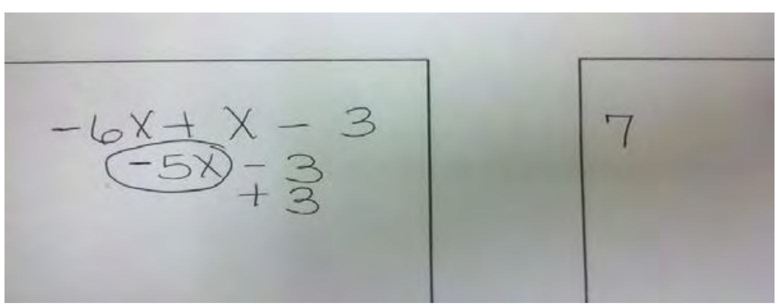


Guided practice problem # 2

- I read the next problem. -6X +X -3 = 7
- On the left side of the mat, we are going write -6X +X -3. On the right side, write 7.
    - The first step in SUMLOWS is to separate the two sides. Draw a line through the equal sign.
    - What does the “U” stand for in SUMLOWS? Unite like terms. Are there any like terms or numbers that are the same that are not already together on the left side of the equal sign? Yes. Because there are like terms that are not together on the left side of the equal sign, we must unite (combine) like terms. We have -6X and X. The signs are different, so we need subtract them. -6+1 (the 1 in front of the X is invisible). Negative seven plus one is -6. The seven is larger (absolute value, therefore the answer is negative).
    - Modify the new equation. Our new equation is -5X -3 = 7.

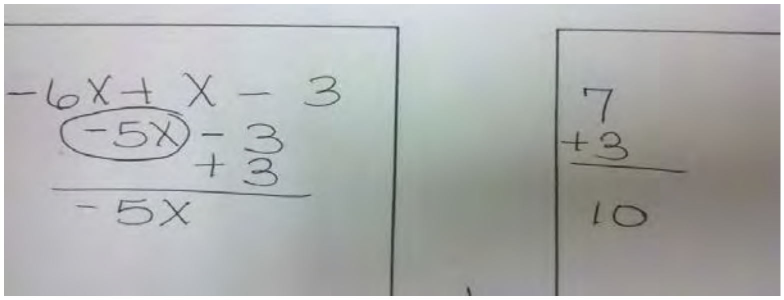


    - The next step is Loop the variable. The variable is the X. This is the number that should be by itself and want to move last. Draw a circle around the X. What is on the side with the variable or X? -3. We have to get rid of -3 because X should be by itself.
    - In order to move the -3, we have to write +3 under the -3. Opposites cancel each other out (-3+3 =0). Put a slash mark through -3 and +3. This equals 0 and cancels each other out.

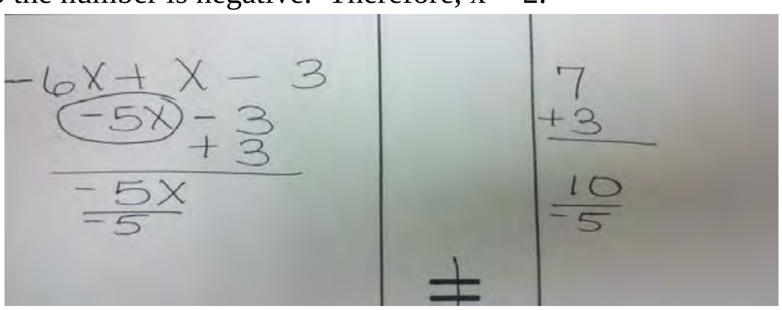


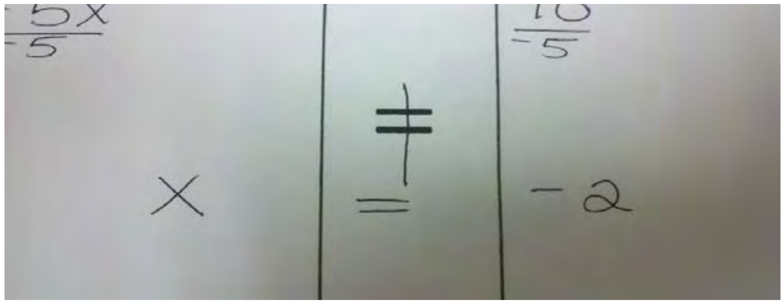
    - The next step is “what we do to one side, we must do to the other” We added 3 on the left side, so we must add 3 on the right side. Write +3 under the 7.
    - Combine like terms. On the right side, we have 7 and +3. Since the numbers have the same signs, we must add them and take the sign of the larger number (absolute value). Seven plus three equals ten. Since the larger number (absolute value) is 7 and it’s positive, so -5X =10.

- Is there anything else on the side with the circle? Yes, we have-5.
- Now we have -5X =10. We must determine what 1X is equal to.

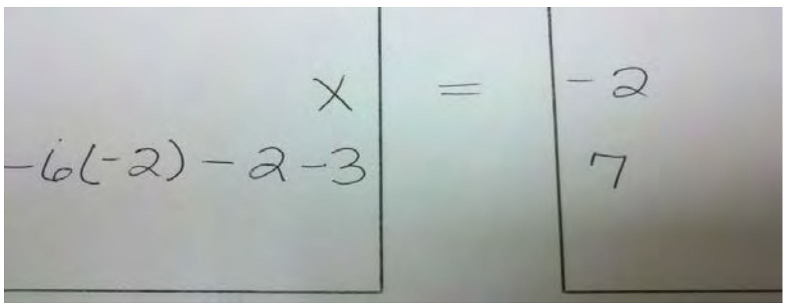


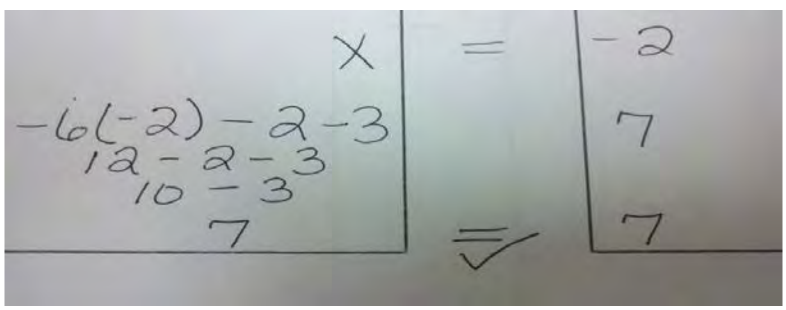
- I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The "U" is for unite like terms. Look on the left side. Are there any like terms or numbers that can be combined that are not already together? NO. Look on the right side. Are there any like terms or numbers that can be combined that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- What is the next step? Loop around the variable. What letter is the variable? The X is the variable. Draw a circle around the X. That is the number that should be by itself.
- Next step is opposite sign. A number beside a letter means to multiply. The opposite of multiplication is division. Draw a line under the -5 and write -5 under it.
- What we do to one side, we must do to the other. On the right side, draw a line under the 10 and write -5 under it. The left side cancels each other out. Draw a slash mark on the lefts side. On the right side, we divide 10 by 5. It is 2. The signs are the different, so the number is negative. Therefore, x =-2.





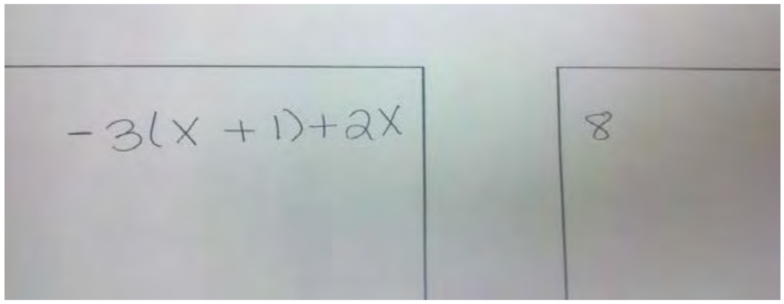
- Last step is to substitute the solution -2 in for X in the original problem. Original problem is -6X +X -3= 7. On the left side, write -6(-2) because X = -2. For X, write (-2), two negatives equal a positive. Also write -3. On the right, write 7. On the left, what is 6 times 2? It is 12. The signs are the same; therefore the answer is positive. Twelve minus two equals 10. Ten minus three is 7. We have 7 on the left side and the right side. Are both sides equal? Yes. The answer is correct.



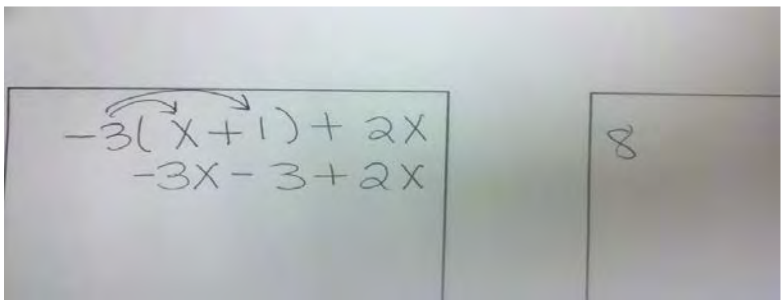


Guided practice problem #3

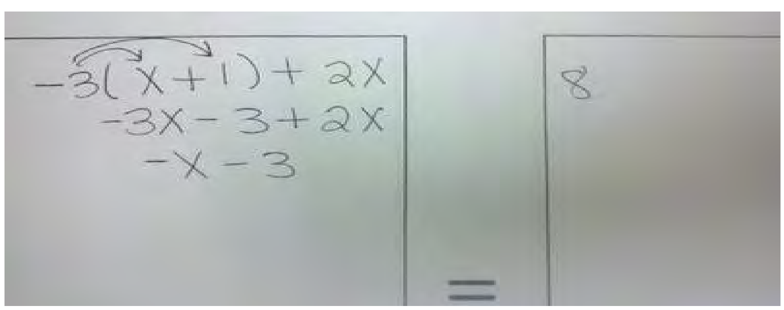
- I read the next problem. -3(X +1) +2X =8
- I make sure my mat or board is clear
- In the problem, notice the parenthesis ( ). This problem requires we to get rid of the parenthesis before we start SUMLOWS.



- What is in the parenthesis? X+1. What is outside the parenthesis? -3. Negative 3 has to be multiplies by everything that is in the parenthesis. -3(X) = -3X and -3 X 1 = -3. So we have -3X -3 + 2X = 8.

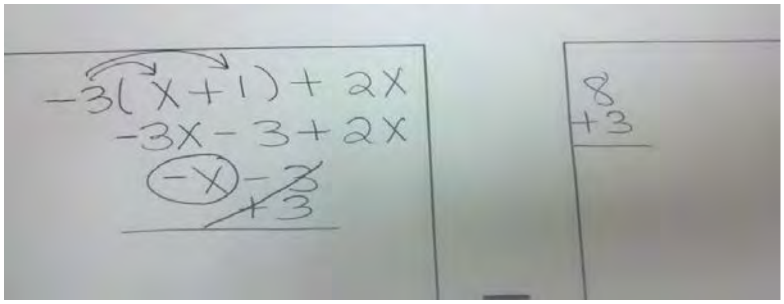


- Now we are ready to solve the equation using SUMLOWS.
  - Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
  - Unite like terms. Are there any like terms or numbers that are not already together? Yes. We must combine -3X and 2X. They have different signs, so we must subtract and take sign of larger number (absolute value). Three minus 2 is 1. Three is larger so the answer is -1X or –X. Look on the right side. Are there any like terms that are not already together? No. We go on to step three.
  - Modify the new equation. Out new equations is -X -3 = 8.

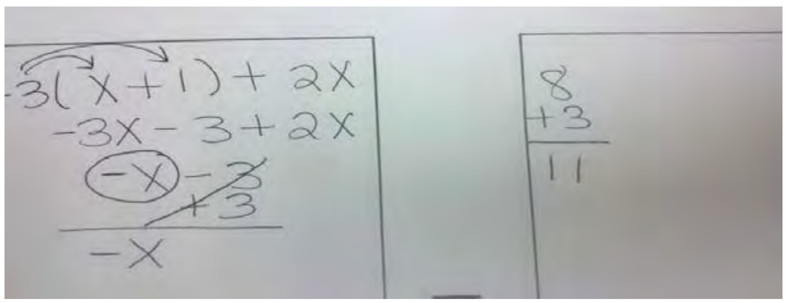


  - Loop around the variable. Draw a circle around the –X. That is the number that should be by itself.
  - Is there anything else on the side with the circle? Yes, there is a -3. We are going to have to move the -3 because the variable should be by itself.
  - Next step is opposite sign. In order to move the -3, we must +3 to the left side. Write + 3 under the -3.

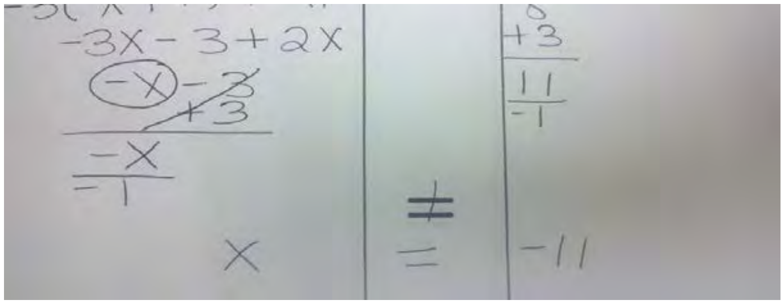
- What we do to one side, we must do to the other. The left side cancels each other out (-3+3 = 0). Since we +3 squares on the left side, we must +3 to the right side. Write +3 under the 8.



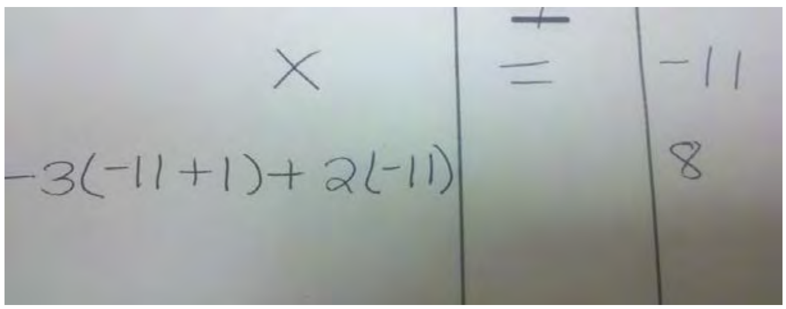
- On the right side, we have 8 and 3. Both numbers are positive; therefore we must add or combine the tiles. Eight plus 3 = 11.
- Now we have -X = 11.

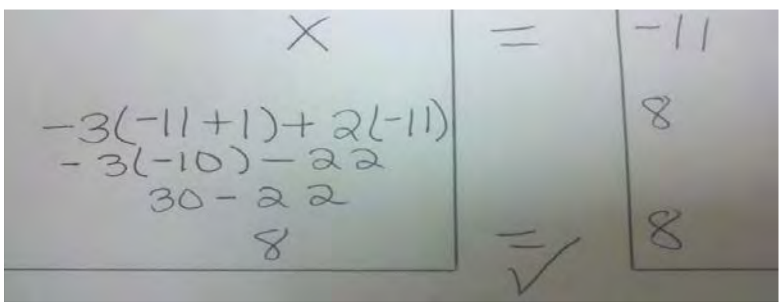


- I start back at the front of SUMLOWS mnemonic.
- I have already separated the sides.
- The “U” is for unite like terms. Look on the left side. Are there any shapes the same shape that are not already together? No. Look on the right side. Are there any shapes the same that are not already together? No. We go on to step three.
- Modify the new equation. There is not anything to unite or combine. So, skip this step too.
- The next step is Loop the variable. The variable is the rectangle. This is the number that we want to get by itself and want to move last.
- Is there anything else on the side with the circle? No. Is the X by itself? Yes. Is it positive? No. It is negative. We must move the -1 (the1 is invisible).
- What is the next step? Next step is opposite sign. In order to move the negative one, we must do the opposite. The opposite of “minus 1” is “plus one.” So, we divide each side by -1. Write a division line under the -1 on the left side. This cancels the left side out.
- What is the next step? What we do to one side, we must do to the other. Because we wrote a division line on the left side, we must add a division line under the 11 and write -1 under it. Eleven divided by -1 is -11. X= -11.



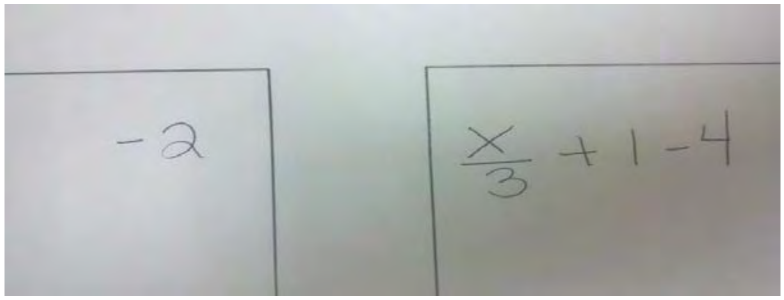
- Last step is to substitute the solution 2 in for X in the original problem. Original problem is -3(X +1) + 2X = 8. Write -3(-11 + 1) + 2(-11) =8. Do that is in the parenthesis first. Negative 11 plus 1 is -10. Negative 10 times -3 is 30. Two times -11 is -22. Thirty minus 22 is 8. What is left side? We have 8 on both sides. They are the same. The answer is correct.



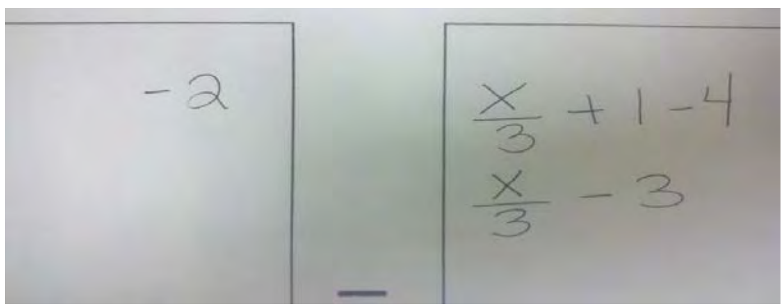


Guidance problem # 4

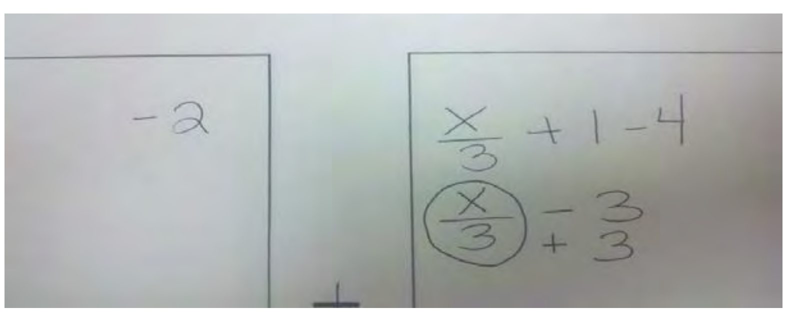
- I read the problem: $-2=\frac{X}{3}+1-4$
- If the students flip the equation, the steps would be the opposite side.
- I make sure the algebra mat is clear
- On the left side of the mat, write -2. On the right side, write X/3. Also on the right side, write +1 and -4.



- Step one of SUMLOWS is separate the sides. Draw a line through the equal sign.
- Unite like terms. Are there any like terms that are not already together on the left side? No. Look on the right side. Are there any like terms that are not already together? Yes. We have plus 1 and -4 that are like terms. We will combine these like terms.
- Modify the new equation. What is positive 1 minus 4? These numbers do not have the same sign, so we have to subtract them and take the sign of the larger number (absolute value). One minus four is three. Four has a larger absolute value, so our answer when we combine these like terms is -3.

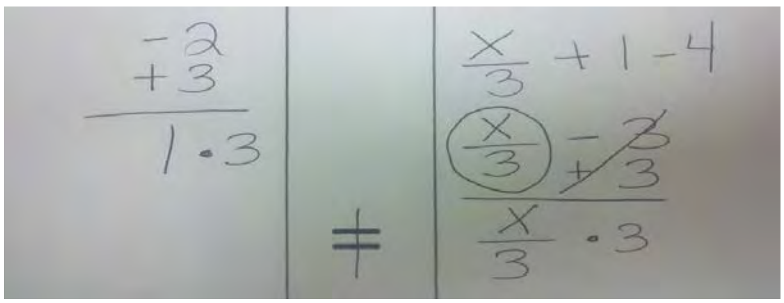


- Loop around the variable. Draw a circle around the X/3. That is the number that should be by itself.
- Is there anything else on the side with the circle? Yes! There are -3 on the same side. We have to get rid of the -3.
- Next step is opposite sign. The opposite of negative (minus) 3 is positive (plus) 3. We are going to write + 3 under the -3.

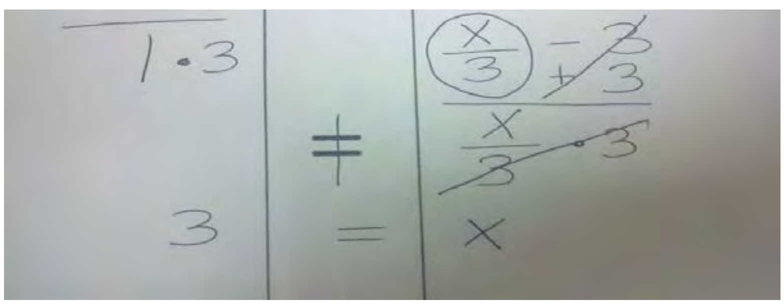


- What we do to one side, we must do to the other. Since we wrote +3 on the right side of the equation, we must write + 3 on the left side of the equation. The right side cancels each other out. On the left side, we have +3 and -2. We are going to combine like terms. Since these terms are alike but do not have the same sign, subtract them and take the sign of the largest number (absolute value). Three minus two is 1. The three is positive and larger; so our answer is +1.

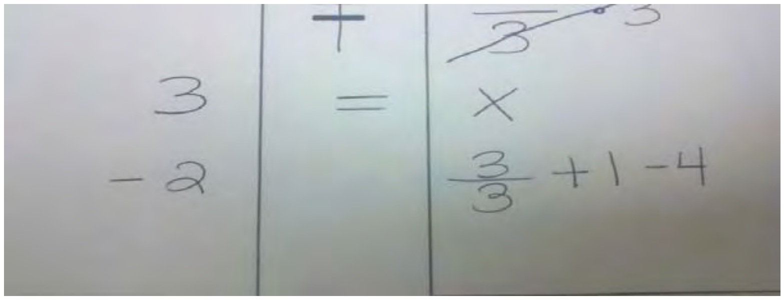
- On the algebra mat, we should have +1. On the right side, we should have X over 3.
- We have already separated the sides, united like terms, modified the equation, and looped the coefficient. The next step is opposite sign.
- When we have an X over a number, which means to divide. What is the opposite of division? It is multiplication. We are going to write a multiplication sign and write 3 beside it.
- Next step is what we do to one side; we must do to the other. Since we wrote a multiplication sign and wrote 3 on the right side, we must do the same on the left side. On the left side, write a multiplication sign and write 3 beside the +1.

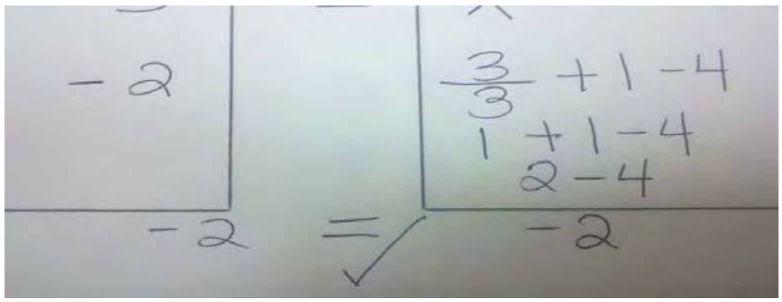


- We are going multiply +1 times positive 3.   So, X = 3.



- Last step is to substitute the solution 2 in for X in the original problem.   Original problem is -2-= X/3 +1-4. On the left side, write -2. On the right side, write 3/3 +1-4.   Three divided by three is one. Positive one plus one is two. Positive two minus 4 is negative 2.   What number are on the left side? -2. What number are on the right side? -2. The numbers are the same. Are both sides equal? Yes. The answer is correct.





Independent Practice

- Direct the students to the "Independent Practice" section. Read the first problem together and direct students to complete the problems without you.
  When students finish problems, provide immediate corrective feedback for errors.

Graphing

Learning Sheet 7

Multiple-Step
Abstract
Code Number _________________ Date _____________
SUMLOWS

Describe and Model

1) 5X+10-5 = 15

2) -2X – 5 +4X = 13

3) 3(X+2) +2X = 16

4) $-\frac{X}{2} + 3 - 4 = -6$

Guided Practice

1) -X +6- 3 = 10

2) 3X – X- 4 = 8

3) -3(X +1)+2X = 8 = 10

4) $-5 = \frac{X}{6} + 2 - 4$

5) $\frac{X}{4} + 2 - 3 = -2$

6) -2(X +1) – 4 = 6

Independent Practice

20= 3(3X) + 2

5- X +3X = 9

$-6 = \frac{X}{2} + 3 - 1$

-4(X+1) +2= -10

-2X +4 = 4+6

-10 = -2(X – 3) +2

- 2X+ 4X -2= -8

7X-2X -5= 20

Learning Sheet 8

Multiple-Step
Abstract
Code Number _________________ Date _____________
SUMLOWS

Describe and Model

1) $6 + 2X + 4 = 12$

Guided Practice

1) $1 = 3X - 7 + X$

2) $-2(-3+3X) = -18$

Independence Practice

$-X - 4 + 2X = 12$

$12 = 3X - 4 + X$

$20 = -2(-4X + 2)$

$-2(-3+3X) = -18$

$-20 = -2(X + 6)$

$20 = -2(-4X + 2)$

$\frac{X}{3} - 12 = 3 - 8$

8) $-\frac{X}{2} - 5 - 4 = -1$

Learning Sheet 9

Multiple -Step
Abstract
Code Number ________________ Date ____________
SUMLOWS

Describe and Model

1) $-2(-3 + 3X) = -18$

Guided Practice

1) $-3X - 7 - 3 = 13$

Independent Practice

$2X + 2 - 4 = 10$ $1 = 3X - 7 + X$ $-19 = -9 - 3X - 7X$

$\frac{X}{4} - 10 - 3 = -2$ $-5(-X - 2) = -20$ $-\frac{X}{2} + 7 + 3 = 3$

Appendix 14

Integrity Checklist

Name ____ Date ___________________________

Observer ____________________________________

	Instructor Behavior	**Yes**	**No**
1	Instructor gives student a blank probe sheet and instructs him/her to complete as man y problems as he /she can do.		
2	Gives an advance organizer telling student what he/she will be doing and why.		
3	Demonstrates how to compute problems with manipulative objects, pictures, or numbers .		
4	Prompts student to solve problems. Student and instructor solve problems together during model and guided practice.		
5	Instructs student to solve problems without guidance. Provides verbal prompts if the student has difficulty.		
6	Monitors student work while he/she solves problems independently. Does not offer the answers.		
7	Collects student's paper and provides feedback regarding responses.		
8	Closes with a positive statement about student's performance in the feedback process and mentions future expectations.		

Appendix 15

Social Validity – CRA and SUMLOWS

Teacher Pre-Intervention

CRA
C – Concrete stage – algebra tiles
R – Representational – Drawing
A – Abstract – Numbers Only

Read each statement and indicate the extent to which it is true by checking one of the boxes to the right.	Very True	Partly True	Not True
Solving equation is important for my students to understand			
Solving equations is difficult for students to understand.			
Do you need manipulatives to teach equations?			
Students often miss steps in solving equations.			

I am currently using an effective strategy to help my students who struggle with algebraic equations? Yes or No
If yes, what are you currently using? ____

I would like to learn a strategy that would help my students who struggle with algebra instruction?
Yes or no

Why is solving equations difficult for some students to understand?

Appendix 16

Social Validity – CRA and SUMLOWS

Teacher Post- Intervention

CRA
C – Concrete stage – algebra tiles
R – Representational – Drawing
A – Abstract – Numbers Only

Read each statement and indicate the extent to which it is true by checking one of the boxes to the right.	Very True	Partly True	Not True
As a result of the CRA and SUMLOWS strategy, my students were able to complete computation problems.			
Using the CRA and SUMLOWS strategy improved my students' algebra skills.			
I would use this strategy again.			
I would recommend this strategy to other teachers.			
The results of the CRA and SUMLOWS strategy were worth the time and resources spent on instruction.			

What did you like about the CRA and SUMLOWS strategy?

What would you change about the CRA and SUMLOWS strategy?

Appendix 17

Social Validity CRA and SUMLOWS

Student Pre- Intervention

CRA
C – Concrete stage – algebra tiles
R – Representational – Drawing
A – Abstract – Numbers Only

Read each statement and indicate the extent to which it is true by checking one of the boxes to the right.	Very True	Partly True	Not True
I like algebra			
It is easy to remember steps in math.			
Solving equations is easy for me.			
I would like to learn a strategy to make algebra easier.			
I learn better with hand-on objects			
I would like to do better in algebra			

What would you change about your algebra class?

What do you currently use to help you remember steps?

Appendix 18

Social Validity CRA and SUMLOWS

Student Post-Intervention

CRA
C – Concrete stage – algebra tiles
R – Representational – Drawing
A – Abstract – Numbers Only

Read each statement and indicate the extent to which it is true by checking one of the boxes to the right.	Very True	Partly True	Not True
As a result of the CRA and SUMLOWS strategy, I was able to complete my work.			
Using the CRA and SUMLOWS strategy improved my algebra skills.			
I would use this strategy again.			
The CRA and SUMLOWS strategy was easier to remember.			
The algebra tiles helped me solve the problems easier.			
The SUMLOWS mnemonic helped me remember what step came next.			

What did you like about the CRA and SUMLOWS strategy?

What would you change about the CRA and SUMLOWS strategy?