Rich Instruction of Mathematical Academic Vocabulary to Enhance Mathematics Achievement of Elementary School Students

by

Carol Taylor Tarpley

A dissertation submitted to the Graduate Faculty of
Auburn University
in partial fulfillment of the
requirements for the Degree of
Doctor of Philosophy

Auburn, Alabama
May 9, 2015

Approved by

Charles Eick, Chair, Associate Professor of Curriculum and Teaching
Megan Burton, Associate Professor of Curriculum and Teaching
Bruce Murray, Associate Professor of Curriculum and Teaching
David Shannon, Humana-Germany-Sherman Distinguished Professor of Educational Foundations, Leadership, and Technology

Abstract

Mathematics scores on national and international tests for students in the United States have indicated that students in the United States have procedural understanding of mathematics but lack conceptual understanding (TIMMS, 2012; U.S. Department of Education, 2013). Therefore, the purpose of this study was to determine if an added enrichment of rich instruction of mathematical vocabulary would result in greater improvement of conceptual understanding in mathematics than the enrichment of non-digital games. A total of 134 students in the fourth grade provided complete data for this study. Sixty-three students received approximately 15 minutes of daily mathematics vocabulary instruction while 71 students spent a comparable amount of time playing non-digital mathematical games. Data consisted of pre- and post-tests scores for both mathematics and vocabulary tests. The findings suggested that the variable of group, vocabulary or game, was not significant. This study also investigated how the mean mathematics unit test scores of the vocabulary group differed for students designated as achieving or underachieving in mathematics and in reading using Aspire (ACT, 2014) test benchmarks. Results suggested that the increase in mean scores for both the students considered achieving and underachieving in math were statistically significant with achieving students making twice the gains of the underachieving students. The increases in mean scores for both the students considered achieving and underachieving in reading were also statistically significant but there was not a statistically significant difference between the groups. The results of this study can help mathematics educators, school administrators, and policy makers interested in mathematics reform understand the possible supplemental supports for effective mathematics instruction that may benefit students' increased mathematics achievement. These findings provide information specifically for educators focused on providing supplemental instruction for differentiated instruction for students in mathematics.

Acknowledgements

I would like to express my deepest appreciation to my committee chair, Dr. Charles Eick, for his full support, understanding, and encouragement throughout my study and research, and Dr. David Shannon for his assistance with statistical analyses. I would also like to thank Dr. Megan Burton and Dr. Bruce Murray for serving as committee members and for their help and support as I worked to focus this research. I would also like to thank Dr. Jamie Carney for serving as the outside reader.

In addition, I would like to thank my husband, children, grandchildren and friends for their unconditional love, support, and encouragement during the last few years. They all served as wonderful cheerleaders throughout the dissertation process.

Table of Contents

List of Tables

CHAPTER 1. INTRODUCTION

Comparisons of students' mathematical achievement in the United States to other industrial nations continues to be a concern (Peterson, Woessmann, Hanushek, & Lastra-Anadon, 2011). American students have participated in the Program for International Student Assessment (PISA) (Organization for Economic Cooperation and Development, 2013), The Trends in Mathematics and Science Study ([TIMMS] TIMMS, 2012), and the National Assessment of Educational Progress ([NAEP] NAEP, 2013). The purpose of the PISA and TIMMS assessments has been to provide information to educators, researchers, and other stakeholders concerning how the achievement of students in the United States in key subject areas compares to students in more than 60 foreign countries, while the NAEP reports similar results on the national level (Neidorf, Binkley, Grattis, & Nohara, 2006).

Results from the 2011 TIMMS (Mullis, Martin, Foy, & Arora, 2012) and 2012 PISA (2013) reported that American students as a whole scored in the middle ranges but above the international average in mathematics, while the 2013 NAEP (2013) reported that average mathematics scores for U.S. students increased slightly from the previous testing. Because each of the three tests reported scores on levels that progressed from low to high, clearer pictures of what U.S. students knew and were able to do can be gleaned from comparing descriptions of earned categories to descriptions of the higher level categories for each assessment.

Results for the 2012 PISA (2013) in mathematics were reported for process skills and content skills. The U.S. was not statistically significantly different from average on the process skill of formulating situations mathematically, but was statistically below average on employing concepts, facts, procedures, and reasoning; and on interpreting, applying, and evaluating mathematics outcomes. The U.S. was statistically significantly below average on two content categories, quantity and space and shape, but not statistically significantly different on uncertainty and data and change and relationships. Overall PISA (U.S. Department of Education, 2003) mathematics scores were reported on 6 proficiency levels, with 6 being the highest and 1 the lowest. Twenty-seven percent of U.S. students scored at or above level 4, while 73% scored in the lower levels. Performance descriptors for the higher levels required students to apply mathematical concepts and operations and to communicate how they arrived at solutions. The lower levels required students to complete tasks in sequential processes.

The 2011 TIMMS (2012) assessed students according to content and cognitive domains with performance scores reported on one of four international benchmarks: advanced, high, intermediate, or low. Results for both U.S. fourth- and eighth-graders were in the intermediate range. According to TIMMS (2012), these students could apply their mathematical knowledge but were not able to reason, explain or justify conclusions, or make generalizations. Overall, average NAEP (2013) mathematics scores for grades four and eight in 2013 were one point higher than in 2011, and 28 and 22 points higher respectively in comparison to the first assessment year. Scores for Alabama students in grades four and eight were 2 and 0 points higher respectively for 2013. When 2013 scores were broken down into one of three levels (advanced, proficient, and basic), 58% of fourth graders and 65% of eighth graders in the United States overall scored in the basic range or below, compared to 70% of Alabama fourth graders and 80% of eighth graders (National Center for Educational Statistics, 2013a). According to NAEP (2013) descriptions for the three levels, students in the basic range and below were unable to use problem-solving strategies using appropriate information; supply written solutions that contained supporting inferences, explanations, and examples in order to defend their ideas; or to communicate the reasoning processes underlying their conclusions clearly and concisely.

These national and international tests indicated that U.S. students knew the procedures for solving problems but, because they had difficulty recognizing connections between mathematical concepts, they were not able to explain, defend, or communicate their conclusions sufficiently. The knowledge that U.S. students exhibited on these tests has been referred to as procedural knowledge (Rittle-Johnson & Alibali, 1999; Skemp, 1987) while the mathematics understanding required to score in higher levels has been referred to as conceptual knowledge (Rittle-Johnson & Alibali, 1999). The National Research Council defined conceptual knowledge as “comprehension of mathematical concepts, operations, and relations” (Kilpatrick et al., 2001, p. 5). Star (2005) stated that “the term conceptual knowledge has come to encompass not only what is known (knowledge of concepts) but also is one way that concepts can be known (e.g. deeply and with rich connections)” (p. 408). Conceptual knowledge is sometimes called conceptual understanding. Procedural knowledge entails knowledge of a series of steps, or actions, done to accomplish a goal. Procedural knowledge is sometimes called procedural understanding (Rittle-Johnson, Siegler, & Alibali, 2001). Star (2005) noted that sometimes “the term procedural knowledge indicates not only what is known (knowledge of procedures) but also one way that procedures (algorithms) can be known (e.g. superficially and without rich connections)” (p. 408). Measures of conceptual knowledge usually involve providing definitions and explanations, explaining why a procedure works, or drawing. Measures of procedural knowledge almost always involve ways to solve problems with the outcome measure based on the accuracy of the answer (Rittle-Johnson et al., 2001).

Because procedural knowledge and conceptual knowledge are both important for understanding mathematics (Rittle-Johnson & Siegler, 1998; Star, 2005), U.S. mathematics instruction should continue to include instruction for building procedural knowledge but increase the focus on conceptual understanding. In order for U.S. students to meet the challenges of national and international tests, they must be able to recall rules and formulas but they also must know when to apply them. They must be able to use the language of mathematics to explain their thinking, consider alternative strategies when faced with problems to solve, monitor their own thinking, question the reasonableness of solutions, and transfer knowledge to new tasks (NCTM, 2000).

Mathematics Education Reform in the U.S.

The average results reported by international and national tests, especially the mathematical achievement gap between U.S. gains compared to gains made by students from other countries, have been met with calls for educational reform. Educators Peterson, Hanushek, Woessmann, and Riddell (2011) suggested that the U.S. set higher educational goals to close the gap. Kornell (2012) and Ball, Hill, and Bass (2005) urged the U.S. to adopt instructional systems driven by strong standards like those found in top-ranked countries that foster key skills such as critical thinking. These calls for reform were based on beliefs that the mediocre performance of U.S. students was due to a lack of having been taught the "conceptual basis for understanding mathematics that could support flexible transfer and generalization" (Richland, Stigler, & Holyoak, 2012, p. 189). In addition, Ball et al. (2005) stressed their belief that U.S. students would continue to score in the average or proficient ranges rather than in the advanced ranges on national and international tests unless they have the abilities to apply what they know to solve problems, think critically, and communicate effectively.

Hiebert et al. (2005) investigated the results of the TIMMS 1999 Video Study to compare the mathematical instructional systems actually used in classrooms of the United States and six higher-achieving countries participating in TIMMS. Initial comparisons of 75 individual features of eighth-grade mathematics instruction suggested that classroom teaching in the United States appeared very similar to that of the other countries. However, when a comparison was made that focused on the interactions of teachers and students, the researchers reported that students from the U.S. spent more time practicing or reviewing procedural skills than developing conceptual understanding. As a result, Hiebert et al. (2005) proposed that the United States consider adopting several characteristics of the higher-achieving countries, including emphasizing conceptual understanding rather than basic skills and procedures. In an earlier report concerning the findings of the TIMMS 1999 Video Study conducted by Hiebert et al. (2005), Stigler and Hiebert (1999) reported that "American mathematics teaching is extremely limited, focused for the most part on a very narrow band of procedural skills" (Stigler & Hiebert, 1999, p. 10). They explained that the differences in the American teaching system for mathematics compared to higher-achieving countries was a reflection of a teaching gap, a term they coined to describe the differences in teaching methods across cultures.

Michael McGill (2013), Superintendent of Schools for Scarsdale, New York, answered the call for reform in an editorial in the New York Times. There he reminded the public that meaningful reform based on thoughtful analyses, rather than premature judgments, is a complex process that should depend on what has been proven to work, or evidence-based practices (Slavin, 2008) and what has not been proven to work. The U.S. Department of Education, Institute of Education Sciences and National Center for Education Evaluation and Regional Assistance (2003) described unproven educational practices as those "introduced with great fanfare as being able to produce dramatic gains…yielding little in the way of positive and lasting change – a perception confirmed by the flat achievement results over the past 30 years in the National Assessment of Educational Progress' long-term trend" (p. iii).

According to Mehta (2013), effective educational reform must include teachers hired, trained, and retrained extensively so that they have knowledge of the subjects they teach, how to teach, and how children learn. In addition, reform must include standards internal to the profession that guide everyday work and provide time for teachers to work together, discuss lessons, reflect on their students' performance, and develop new and better approaches in an effort to build a shared knowledge base for education as a whole. The result would be expert teachers no longer working in isolation, and the time local, state, and federal overseers spent previously on teacher accountability would be better used to "assist in the creation of curricula, invest in research and development, screen teacher resumes, and provide expert technical assistance" (p.115). Mehta (2013) stated that the good news about educational reform lies in the serious academic research conducted to determine effective teaching practices. Slavin (2008) agreed that evidence-based reform would result in positive results, for:

> [M]illions of children, especially those who are least well served by the current system, the teachers who yearn for more effective tools to help them do their job well, and the whole society, which would come to expect progress in education as confidently as it currently expects progress in other fields. (p. 127)

Building Conceptual and Procedural Understanding

In an effort to determine evidence-based approaches for building student conceptual and procedural understanding, Hiebert and Grouws (2007) synthesized current research on best practices and deduced that both of these aspects of mathematics ability are enhanced by two overarching best practices they called *work and talk* and *work and wrestle*. Both practices involved classroom discourse, with the first centered on important mathematical relationships between facts and procedures, why procedures work the way they do, and why they do not work in other ways. The second practice, *work and wrestle*, was centered on the validity of student solutions and possible alternative solutions.

The value of classroom discourse in mathematics is evident in the National Council of Teachers of Mathematics Principles to Action (NCTM, 2014), Principles and Standards for School Mathematics (NCTM, 2000), and the Common Core State Standards (NGA Center and CCSSO, 2010). These documents stress that students learn mathematics best when they speak about mathematics using the language of mathematics, defined by Jamison (2000) as the vocabulary necessary to express the procedural elements of mathematics including the precision with which the vocabulary is used. Standards 3 and 6 of the Common Core Standards (NGA Center and CCSSO, 2010) specifically attend to student discourse and the language of mathematics. Standard 3, *Construct viable arguments and critique the reasoning of others*, states that "proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments" (p. 6). Teachers who wish to contribute to the development of their students' abilities to construct arguments and critique the reasoning of others must engage their students in active mathematics discussions, or discourse. Standard 6, *Attend to precision*, states that "mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussions with others and in their own reasoning" (p. 7). Promoting the development of academic language and engaging students in discourse for justification support students' learning of mathematics (National Research Council, 2001). Therefore, mathematics instruction should include an emphasis on the language of mathematics in order to enable students to move from their everyday informal mathematical vocabulary toward the use of academic mathematical vocabulary.

Academic Vocabulary Instruction

Marzano and Pickering (2005) referred to specialized vocabulary as academic vocabulary, defined as the words traditionally used in academic discussion and learning that are critical to understanding content taught, as well as vocabulary often found in directions included in instruction. Because this vocabulary is necessary for student understanding of content, especially for students who do not come from academically advantaged backgrounds (Marzano & Pickering, 2005), it must be taught explicitly using strategies that focus on word recognition and word use in meaningful contexts (Bintz, 2011).

The extent to which students need to understand word meanings has been greatly discussed and researched (Beck, McKeown, & Omanson, 1987; Cronback, 1942; Dale, 1965) with most agreeing that understanding the meaning of words deeply is a complex on-going process. Cronback (as cited in Beck, McKeown, & Kucan, 2002, p. 10) divided this process into five dimensions beginning with the ability to define a word, recognizing situations when the word could be used appropriately, understanding multiple meanings of the word, recognizing inappropriate use of the word, and finally, using the word in thoughts and discourse.

To support students' progress through the process of understanding word meanings, researchers (Beck et al., 2002; Marzano & Pickering, 2005) have devised routines for vocabulary instruction that include steps for introducing new terms and activities to review the terms. To introduce new terms a description, explanation or example of the term is supplied by the teacher. Students are then asked to restate the description, explanation, or example in their own words, and to construct a graphic for the term. Activities to review terms provide situations in which students interact with terms by responding to and explaining examples as well as creating their own examples. Beck et al. (2002) refer to instruction of this type as rich or robust instruction, "mean[ing] instruction that goes beyond definitional information to get students actively involved in using and thinking about word meanings and creating lots of associations among words" (p. 73). Nagy and Townsend (2012) conducted a review of research on instruction in academic vocabulary and the importance of academic vocabulary for student success in school. The researchers reported that:

> Rich instruction has been applied to academic vocabulary and found to produce gains in knowledge of the instructed words and, in several cases, gains in the ability to use the instructed words in writing and comprehending text. We argue that the success of such instruction derives in large part from the fact that it treats words as tools rather than as facts to be memorized. That is, the richness of rich vocabulary instruction consists largely in providing students with multiple opportunities to use the instructed words, both receptively and productively, generally in the context of discussion about academic content. (p. 101)

Several studies (August, Branum-Martin, Cardenas-Hagan, & Francis, 2009; Brown, Ryoo, & Rodriguez, 2010; Vaughn et al., 2009) have been conducted that involved academic vocabulary in science and social studies and included students whose first language was not English. These studies reported that students in treatment groups who received rich vocabulary instruction that included videos, discussion, graphic organizers, and other activities designed to help students build conceptual understanding outperformed students in control groups who did not experience those type activities.

Academic Vocabulary for Mathematics

The U.S. is not alone in the quest for approaches to increase conceptual understanding of mathematics. In 2008 the Council of Australian Governments commissioned a report of synthesized national and international research on effective teaching in order to support the country's goal of improving mathematics understanding of Australian students. The resulting document, The National Numeracy Review Report (Council of Australian Governments, 2008), addressed the role of language in mathematics learning in Australia. This report suggested that the language of mathematics be explicitly taught in recognition that language can be a formidable barrier to understanding mathematical concepts and assessment items for determining mathematical understanding.

Schleppegrell (2007) synthesized research conducted by applied linguist and mathematics educators to determine strategies to help students move from the informal everyday language of mathematics to language that is more precise and technical. In her review she stressed the role that teachers play in helping students develop their mathematical vocabulary and concluded her synthesis by suggesting that teachers encourage students to use technical mathematical vocabulary in classroom discourse, completion of word problems, and writing about mathematical concepts. To inform teachers about the differences between vocabulary development for mathematics and general vocabulary development, Meiers (2010) compiled a list of especially problematic issues relating to mathematical vocabulary. This list included specialized expressions of mathematics, the use of everyday terms that have different mathematical meanings, and language factors that come into play as students work to solve word problems. Kouba (1989) summed up the characteristics of mathematical vocabulary when he cautioned teachers that vocabulary for this discipline has proven especially difficult for students because it is “complex, content-bound, and largely abstract” (p. 266).

In an effort to aid teachers’ decisions about what words to teach and how to teach them, Monroe and Panchyshyn (1997) classified the mathematical vocabulary for instruction into four categories: technical, subtechnical, general, and symbolic. Their category for technical vocabulary included words usually viewed as purely mathematical, such as *integer*. Subtechnical vocabulary words, such as *volume*, included words that have more than one meaning, varying from one content area to another or from a content area to common everyday usage. For example, students might find it difficult to conceptualize the difference between the volume of a cylinder and the volume control on a computer. Some subtechnical words also have multiple meanings within mathematics, such as *degree*, as used with degrees of an angle and degrees of temperature. For Monroe and Panchyshyn (1997) general mathematics vocabulary words included words usually a part of common language, such as *gallon,* while symbolic vocabulary consisted of numerals and symbols, such as $\neq$, $<$, and $>$.

Challenges with academic vocabulary become even greater for students for whom English is a second language (Beimiller & Boote, 2006). However, research suggests that vocabulary instruction that works for non-ELLs works as well if not better for ELLs (August & Shanahan, 2006; Carlo et al., 2004; Collins, 2005; Silverman, 2007). Educators and researchers (Blachowicz, Watts-Taffe, & Fisher, 2005; Sweeny & Mason, 2011) suggest that the answer to meeting the vocabulary needs of all students would be an intentional, multi-dimensional, school-wide vocabulary program consistent across all grades and subject areas. Multi-dimensional instructional programs are often referred to as being rich or robust (Beck et al., 2002) and include explanations of word meanings using student friendly terms, multiple exposures to words, and activities that engage students in using words in conversations, writing, and representations.

According to the National Council of Teachers of Mathematics (2000), instruction on mathematical vocabulary should begin with addressing how a word is introduced, explained and used. In addition, mathematics teachers must consider the teaching sequence of mathematical concepts and the vocabulary that gives names to those concepts. When determining a sequence for the teaching of any new topic or concept, mathematics teachers should first select problems that require students to connect and use information they already know. Vacca and Vacca (2005) suggest that once problems have been selected and discussion and exploration planned, the teacher should categorize the vocabulary as either words to be addressed before students begin their exploration of the mathematical concept or words that should be introduced following exploration. Words for the first category would be those previously introduced but that need review in order to aid students in their discussions. Next, students should have opportunities to reason, use representations, and explain their understanding informally as they explore new concepts. Once students have solidified their ideas and have conceptual understanding, the remaining new vocabulary should be introduced in order to give names to newly formed generalizations that resulted from discussion, exploration, and explanations. These vocabulary words eventually become review words as the sequence continues in a cyclical fashion (Vacca & Vacca, 2005).

Bay-Williams and Livers (2009) addressed the how and when aspects of instruction of mathematics vocabulary and suggested that teachers think about each word during lesson planning. These educators (2009) differentiated between vocabulary for lesson context and vocabulary for lesson concept. When considering the context of a lesson or task, teachers should be aware of students' prior knowledge, language skills, and interests in order to determine whether or not the context is meaningful to all students. For example, problems involving feed requirements for cattle may not be culturally relevant for some students. Concept considerations address how and when vocabulary for mathematical concepts should be introduced and reviewed. For example, when deciding if new vocabulary should be introduced before or after students are asked to solve a problem, teachers should consider if the problem could be solved without knowing the vocabulary. If the problem could be solved without the vocabulary, students should be allowed to solve the problem before the vocabulary is introduced. However, if the problem or task cannot be solved without knowledge of the vocabulary, it should be carefully previewed so that students will have enough knowledge of the vocabulary to support their new learning.

Academic Vocabulary for Mathematics and Achievement

Research to determine the effects of mathematical vocabulary instruction on mathematical achievement has varied according to interventions applied, concepts explored, and instruments used for assessing achievement. Gifford and Gore (2010) conducted one of the few research studies that focused on the effects of rich mathematical vocabulary instruction on student achievement in mathematics. These researchers worked with 175 sixth-grade mathematics students to determine the effects of Marzano's (2009) academic vocabulary program. This group of students included male, female, African American, Hispanic, Caucasian, special education, and economically disadvantaged students in Tennessee. The researchers reported a 93 percent increase in normal curve equivalent scores (NCE) on the Tennessee Comprehensive Assessment Program standardized achievement test. This multiple-choice test is composed of questions written to test student performance on state content standards (Tennessee

Department of Education, 2012). Likewise, Jackson and Phillips (1983) completed a study of 213 seventh-grade students in three suburban schools in Florida using a posttest-only design during a four-week unit on ratio and proportion. The researchers reported that one school was located in a high socioeconomic area, one in a moderate socioeconomic area, and one in a low socioeconomic area. Students and teachers were randomly assigned to one of two treatment groups. Treatment group students received vocabulary activities for five to ten minutes each day, while control group students spent the time working computational problems. According to the researchers, vocabulary activities emphasized recognition and identification of terms and symbols, attaching literal meaning to terms and symbols, categorizing terms and symbols by inclusion and exclusion, and identifying examples and nonexamples of concepts represented by terms or symbols. Data from three existing measures were collected from school records. The first and second sources of data were students' scores from two sections of the Metropolitan Achievement Test, mathematics computation and math comprehension, administered at the beginning of the school year. The third measure was students' letter grades earned in seventh-grade mathematics prior to the experiment. A chapter test developed by the researchers in conjunction with the cooperating teachers was used as the posttest. This test included a set of computational items and a set of verbal items. These two sets of test items plus the three existing sets of data were assessed simultaneously using the general linear model analysis of variance procedure. The researchers concluded that an emphasis on academic vocabulary associated with ratio and proportion significantly improved students' achievement as measured by mean scores for verbal and computational problems on the chapter test. The verbal score was the number of items correct out of 10 vocabulary items and 1 word problem. The computational score was the number of items correct out of 15 computational items. Students in the experimental treatment scored significantly higher on both verbal and computation than students in the control group. The researchers reported internal consistency coefficients for both scores of .51 and .75 respectively and deemed these adequate for inferences about performance of the group for each score.

Despite few studies like the one above linking vocabulary instruction and mathematics achievement directly, a linear relationship between vocabulary in academic discourse, conceptual understanding, and achievement has been supported by research. Piccolo, Harbaugh, Carter, Capraro, and Capraro (2008) studied the relationship between classroom discourse, including mathematical academic language, and the development of conceptual understanding of mathematics. For three years the researchers videotaped 48 teachers and their sixth through eighth-grade students in five school districts described as diverse. Teachers received a one-week training prior to the study that focused on discourse strategies that would be investigated. Video was transcribed and coded for interactive discussions and questioning according to three indicators representing student understanding of mathematical concepts. The researchers concluded that rich classroom interaction that included discussion resulting from both teacher-generated and student-generated questions led to a level of discourse perceived to be at a deeper conceptual level. In addition, they suggested that teachers should adopt the goal of student conceptual understanding of mathematical content with the ability to converse and articulate their own understanding using rich mathematical discourse. They stated that it is "reasonable to extrapolate that when this goal is attained, student achievement in advanced academics will follow" (Piccolo et al., 2005, p. 404).

Research conducted by Rittle-Johnson et al. (2001) supports the last link in the linear relationship of vocabulary in academic discourse which is conceptual understanding and achievement. These researchers conducted a study that included two experiments with 74 fifth and sixth grade mathematics students from two rural public elementary schools serving a range of socioeconomic levels. In the first experiment, students completed conceptual and procedural knowledge pretests in their classrooms prior to participating in an individual intervention session that lasted approximately 40 minutes. This session included one of four brief lessons on decimal fractions that focused on either conceptual knowledge relevant to the problems, a procedure for solving the problems, both, or neither. According to the researchers, the lessons involved repeated practice and feedback associated with having students place decimal fractions on a number line. The students then played a computer game designed by the researchers before being post-tested using a conceptual knowledge post-test, a procedural knowledge post-test, and a transfer test. Student ability to complete tasks was used as a measure of procedural understanding. Student ability to apply the procedure to novel activities not presented during the intervention was used to measure conceptual understanding.

The second experiment was similar to the first but treatment included researcher provided prompts and number lines divided into 10 equal sections. Participants for this study included 117 fifth and sixth graders in two parochial schools located in a predominantly white urban or suburban neighborhood. The mathematics subtest of the Iowa Test of Basic Skills was used to assess general achievement as well as procedural and conceptual knowledge assessments similar to those used in the first experiment. The researchers reported that the use of prompts and marked number lines led to larger gains than expected for either one used independently. Overall, the researchers reported that students' increased conceptual knowledge improved their procedural knowledge. An additional interesting finding was that the students' conceptual and procedural knowledge appeared to develop simultaneously.

Statement of Purpose

The purpose of this study was to investigate whether evidence-based rich instruction of mathematical academic vocabulary following the routines designed by Beck et al. (2002) would result in greater improvement in fourth-grade students' mathematics test scores on grade level units and vocabulary tests than the use of non-digital mathematics games. Student achievement was based on student pre- and post-test scores on two unit tests accompanying the commercially developed mathematics program called Investigations® (Russell, Tierney, Mokros, Goodrow, & Murray, 2012) and two researcher-devised unit vocabulary tests. According to the Nation's Report Card (National Center for Educational Statistics, 2011) fourth grade students scoring at the advanced level should be able to solve story problems using mathematical operations. Scott Foresman® Investigations unit tests include solving story problems as well as asking students to construct story problems.

Educators (Peterson et al., 2011) concerned with the mediocre mathematics achievement of U.S. students on national and international tests have proposed educational reform of mathematics instruction to increase student achievement. In response to these calls for educational reform, researchers such as Slavin (2008) urged educators to base reform efforts on evidence-based practices. Rich vocabulary instruction (Beck, Perfetti, & McKeown, 1982; McKeown, Beck, Omanson, & Perfetti, 1983; McKeown, Beck, Omanson & Pople, 1985) including activities such as those mentioned previously, has been found to increase student deep or conceptual understanding of academic mathematical vocabulary. In turn, increased conceptual understanding of mathematical concepts has been found to increase mathematical achievement on mathematics assessments (Kulm, Capraro, & Capraro, 2007). It was the author's hope that the results of this study would supply teachers of mathematics with an easy to implement approach and enrichment to help students develop a deeper understanding of mathematical vocabulary that would contribute to improved mathematics test scores on the local, state, national, and international levels.

Research Questions

The questions addressed by this study were:

1. Will rich instruction of mathematical academic vocabulary result in greater improvement in elementary students' mathematics achievement as assessed by unit tests from two consecutive mathematics units of study than use of non-digital mathematics games?

2. Will rich instruction of mathematical academic vocabulary result in greater improvement in elementary students' achievement on a researcher devised vocabulary test than use of non-digital mathematic games?

3. After receiving rich instruction of mathematical academic vocabulary, will there be a significant difference between unit mathematics mean test scores for underachieving and achieving students in mathematics?

4. After receiving rich instruction of mathematical academic vocabulary, will there be a significant difference between unit mathematics mean test scores for underachieving and achieving students in reading?

Definitions of Terms

Academic vocabulary: Specialized vocabulary traditionally used in academic discussion and learning that are critical to understanding content taught, as well as vocabulary often found in directions included in instruction (Marzano & Pickering, 2005).

Conceptual knowledge: Comprehension of mathematical concepts, operations, and relations (National Research Council, 2001).

Content skills: The math skills students should know and be able to use; includes knowledge of numbers and operations, algebra, geometry, measurement, and data analysis and probability (Kennedy, Tipps, & Johnson, 2008).

Direct or explicit vocabulary instruction: Teaching students the meanings of words, techniques to determine word meanings from context, and the meanings of word roots and affixes (Shanahan & August, 2006).

Discourse: Whole-class discussions in which students talk about mathematics in such a way that they reveal their understanding of concepts and reasoning why a particular method was chosen, and learn to critique their own and others' ideas and seek out efficient mathematical solutions (Maguire & Neill, 2006).

Evidence-based reform: A movement that is intended to use high-quality evidence from rigorous experiments to guide educational policies and practices. Proponents of evidence-based reform hold that true progress will take place in education only when educators and policymakers have a broad set of programs and practices with a strong evidence of effectiveness available and when government policies support the use of well-evaluated programs as well as the development and evaluation of new, untested programs (Slavin, 2008).

Indirect or implicit vocabulary instruction: Classroom activities such as reading to children or encouraging them to read with the benefit of increasing student vocabulary (Shanahan & August, 2006).

Procedural knowledge: The knowledge of the steps, skills, or strategies required to attain various goals (Byrnes &Wasik, 1991).

Read-aloud: When a teacher reads a picture or chapter book, a poem, a letter, the wall, a sentence chart, etc., orally to a group of students. While the teacher reads, the students listen, engage in the material, and comprehend what they hear (Trelease, 2006).

Rich vocabulary instruction: Instruction that includes child-friendly explanations of the meaning of words along with thought provoking, playful, and interactive follow up activities in which students engage in conversations, readings, and writings reusing the words. Explanations may be developed and presented to students by the teacher or may be constructed with student input (Beck et al., 2002).

Vocabulary: The knowledge of words and word meanings, specifically words that students must know to read increasingly demanding text with comprehension (Kamil & Hiebert, 2005).

CHAPTER II. REVIEW OF LITERATURE

Introduction

American students' mathematical achievement on national and international tests has been a cause of concern despite past reform efforts in mathematics. Some researchers believe the slow improvement in raising scores on these assessments has been the result of mathematics reform that more often consisted of greater emphasis on curriculum development than on a focus on improved student conceptual understanding of mathematics (Ball et al., 2005; Hiebert et al., 2005). Research (Piccolo et al., 2008) has indicated that rich classroom discourse using the vocabulary of mathematics resulted in increased conceptual understanding. In addition, research (Rittle-Johnson et al., 2001) has suggested that students' increased conceptual understanding contributed to increases in achievement on mathematics assessments on state mandated achievement tests. Therefore, educators (Miller, 1992; Monroe & Orme, 2002; Thompson & Rubenstein, 2000) have agreed that a key component in understanding mathematics is the vocabulary of mathematics.

Current research concerning academic vocabulary instruction and intervention supports the premise that students with strong academic vocabulary understanding are more prepared to think and communicate about disciplinary content (Nagy & Townsend, 2012). Systematic and direct vocabulary instruction has been shown to help students build strong academic vocabulary skills by supporting students' development of deep understanding of those words. Instruction with the goal of building a deep understanding of words is often referred to as being rich or robust (Beck et al., 2002). Fennema et al. (1996) observed that when students built on their thinking through quality classroom discourse that included usage of important academic vocabulary, their achievement in problem solving and conceptual understanding increased. Yet, K–12 academic vocabulary instruction often lacks the quality necessary to help students develop strong academic vocabulary skills. Instead, observations of teachers have demonstrated that instruction in most classrooms differs qualitatively from that which has been supported to be effective.

This review of research examines conceptual and procedural understanding in mathematics, how students learn mathematics, and the role of academic mathematical vocabulary in the development of conceptual understanding. This review will support the hypothesis that increased understanding of academic mathematics vocabulary will contribute to increased mathematics achievement assessed by school-level unit tests. Because few studies exist that investigated mathematics vocabulary specifically, this review also includes studies that reported effective vocabulary instruction in other content areas.

Conceptual and Procedural Understanding of Mathematics

Researchers such as Rittle-Johnson et al. (2001) have acknowledged four different viewpoints on the relationship between conceptual knowledge and procedural knowledge in mathematics learning. Concepts-first posits that concepts are developed first, with procedural knowledge built as students practice solving problems. Procedures-first posits that procedures are learned first, followed by conceptual knowledge developed through processes to aid abstraction. The inactivation view contends that conceptual knowledge and procedural knowledge develop independently. And last, the most well accepted view is the iterative view which holds that conceptual knowledge and procedural knowledge both increase gradually in relation to the other over time (Rittle-Johnson et al., 2001). Research (Cowan et al., 2011) has shown a predictive relationship between conceptual knowledge and procedural knowledge and that experimentally manipulating one type of knowledge can lead to increases in the other (Rittle-Johnson et al., 2001). One study has shown that conceptual knowledge instruction has had a stronger influence on procedural knowledge than procedural has had on conceptual knowledge (Rittle-Johnson et al., 2001)

Hallett, Nunes, and Bryant (2010) conducted a study in reaction to conflicting results of previous research conducted to investigate the connection between conceptual and procedural understanding (Byrnes & Wasik, 1991; Rittle-Johnson et al., 2001). Hallett et al. (2000) theorized that individual differences in how students rely on conceptual and procedural understanding were responsible for the dissimilar results. For example, they wanted to explore the possibility that some students focus on learning procedures, some focus on learning concepts, while still others focus on learning both in tandem. However, unlike Rittle-Johnson et al. (2001), Hallet et al. (2000) did not assume that one of the types of learning necessarily leads to the other. Three hundred eighteen fourth and fifth grade students from eight elementary schools participated in this study. Students completed an assessment of fraction knowledge composed of 40 items assigned as either conceptual or procedural. Items for assessing conceptual understanding required understanding of fraction equivalence, and items for procedural understanding required application of a procedure or rule to solve the problems. The researchers used cluster analysis in an effort to determine if conceptual and procedural learning reflect individual differences or developmental processes. When student responses were divided into five clusters, the results challenged the assumption of previous research that all children follow a uniform sequence in gaining conceptual and procedural understanding. For example, there were some students who learned procedures for solving fraction problems without having the expected level of conceptual knowledge. However, results indicated that the students who relied on conceptual knowledge had an advantage over students who relied on procedural knowledge, suggesting that conceptual approaches alone for mathematical instruction are more successful than procedural approaches alone.

Research to determine procedures for increasing conceptual knowledge of mathematics has identified effective approaches such as having students compare different correct procedures for solving the same problem and then reflect on them individually (Star & Rittle-Johnson, 2009), prompting students to self-explain solution procedures (Berthold & Renkl, 2009), and having students explore problems before instruction (Hiebert & Grouws, 2007). Additional approaches found to be successful included having students use multiple representations of mathematical concepts (Rillero & Padgett, 2014), manipulatives (Berthold & Renkl, 2009), analogies and explanations (Nokes & Ross, 2007), discourse (Michaels, O'Connor, & Resnick, 2008; Moschkovich, 1999), and student invented strategies prior to procedural instruction (Carpenter, Franke, Jacobs, Fennema, & Empson, 1998). According to the NCTM (2000) communication standard, students involved in these approaches should be encouraged to increase their ability to express their mathematical ideas precisely using the language of mathematics.

Panasuk (2010) conducted a multi-year study with 443 seventh and eighth graders in one underperforming urban school district in order to determine how to recognize when students have achieved conceptual understanding. This knowledge is important to educators who want to understand more about how to facilitate development of conceptual understanding as well as how to plan instruction accordingly. Students were classified by achievement levels based on the mathematics portion of the statewide Comprehensive Assessment System (sCAS). They were then asked to complete surveys in order to assess their ability to recognize and solve problems that involved the same linear relationship structurally but were presented in different modalities. The surveys of 15 students were selected due to their potential for providing information concerning their ability to recognize structurally the same linear relationship and solve problems. Of these 15 students, nine agreed to being interviewed. These nine students were later divided into low tier and upper tier categories based on their ability to recognize and explain representations used in problems. The researchers declared the upper tier students to have conceptual understanding and reported that conceptual understanding of algebra “is manifested by competency in reading, writing, and manipulating both number symbols and algebraic symbols… [and] fluency in the language of algebra” (p. 237).

To determine if teaching and learning with understanding correlated with achievement on high-stakes multiple-choice tests, Kulm et al. (2007) conducted a yearlong study with 105 sixth-grade mathematics students. The researchers were also interested in determining if levels of achievement would be comparable for the variable of at-risk, gender, ethnicity, and socio-economic status. While the primary quantitative data for the study were scores from the Texas Assessment of Academic Skills (TAAS), the researchers also obtained qualitative data from classroom observations. Test scores for the mathematics portion of the TAAS indicated that student performance improved significantly with an effect size of .82. The at-risk group gained on average 10.16 points as compared to the not at-risk group that gained on average 1.74 points. Gains for gender and gender at risk were not significant. Kulm et al. (2007) reported that only two students failed to pass the TAAS mathematics test compared to previous years’ passing results of only 60 to 75% of minority, disadvantaged, and at-risk students. Therefore, the researchers concluded that “teaching and learning with understanding is compatible with achievement on high stakes multiple-choice tests” (p. 44).

Learning in Mathematics

Understanding how children learn mathematics is necessary for improving mathematics education, a goal of many schools and school systems due to the emphasis now placed on standardized test scores. While students are still expected to learn basic computation skills, educators realize the need for more time and energy to be devoted to working with students to enable them to solve problems with understanding, evidenced by their ability to explain their reasoning and justify their approaches (Baxter, Woodward, & Olson, 2001). According to the National Council for Teachers of Mathematics (NCTM) (2000), students who fail to develop mathematical understanding struggle in subsequent work in mathematics and related subjects. In addition, understanding of mathematics is necessary for increased competence in everyday tasks and future employment. To assist mathematics teachers in their effort to help all students develop their mathematical understanding, Ball et al. (2005) identified three fundamental needs for mathematics students: 1) proficiency with computational procedures, 2) ability to communicate mathematical ideas using mathematical vocabulary, and 3) ability to formulate and solve problems.

According to Carpenter and Lehrer (1999), student mathematical understanding develops as the result of constructing relationships, extending and applying mathematical knowledge, reflecting on experiences, talking about what one knows, and making mathematical knowledge one’s own. Because students construct meaning for new mathematical concepts by relating the new to what they already understand, instruction in mathematics should begin on the students’ level with problems and activities selected so that students build their mathematical understanding on their current knowledge and interests, rather than on their weaknesses (Carpenter, Fennema, & Franke, 1996). Activities or strategies found to be successful in determining students' knowledge levels include student discussions of problem solutions (Cobb, 1988) and observing students' lack of understanding when they inappropriately apply procedures in new contexts (Hiebert & Wearne, 1993). Saxe, Gearhart, and Seltzer (1999) conducted a study to determine the effects of analysis of students' prior understanding on pre- and post-test achievement data related to achievement in problem solving. When data for students in nineteen classrooms was compared, the researchers found that students in classrooms where teachers based instruction on students' prior understanding of fractions performed better than students in classrooms where students' levels of understanding were not considered prior to instruction.

Once students have the ability to build and connect new mathematical concepts to previous experiences they should begin applying their new understanding to solve new and unfamiliar problems. Students should understand that it is more important to reflect on their thinking and learn from their mistakes than it is to get correct answers without understanding. Therefore, the classroom climate must support students as they become competent in their ability to tackle and persevere when tasks are challenging (NCTM, 2000).

Stipek, et al. (1998) conducted a study to assess effects of mathematical instruction in classrooms where teachers emphasized effort and learning over performance. Students and teachers in 24 fourth- through sixth-grade classrooms were observed, completed questionnaires, and students were pre- and post-tested using a researcher devised assessment composed of problems described by the researchers as being procedurally and conceptually oriented. Students in classrooms where the teachers encouraged students to extend and apply their mathematical knowledge made substantially greater gains on items of the test that required conceptual understanding. Based on findings of their research, the researchers suggested that students should have opportunities to struggle with mathematics in order to focus on learning, develop self-confidence and a willingness to take risks. In addition, teachers should incorporate inadequate solutions into classroom instruction so students learn that errors are part of the learning process.

Reasoning and proof, one of the NCTM (2000) process standards, addresses the need for students to look for patterns in mathematics and question if they are accidental or there for a reason. In addition, the standard includes the need for students to make and investigate ideas and develop mathematical arguments and proofs as part of their reasoning and justification. Hiebert et al. (2000) stated that "[c]ommunication works together with reflection to produce new relationships and connections. Students who reflect on what they do and communicate with others about it are in the best position to build useful connections in mathematics" (p. 6). Research conducted by Yackel, Cobb, and Wood (1991) reported the ability of students as young as second grade to engage in sophisticated forms of explanation and justification and that their understanding advanced with time. Reflection may take multiple formats based on the NCTM (2000) Principles and Standards for School Mathematics which stated, "Students who have opportunities, encouragement, and support for speaking, writing, reading, and listening in mathematics classes reap dual benefits: they communicate to learn mathematics, and they learn to communicate mathematically" (p. 60).

Discourse and Learning Mathematics

Hiebert and Wearne (1993) investigated effects of classroom instruction for 135 second-grade students that included reducing the number of problems, spending more time with each problem, and asking students more questions requiring them to describe and explain strategies for solving problems. The researchers were particularly interested in the instructional tasks presented to the students and the nature of the classroom discourse. Classroom questions and discourse were coded, and students were assessed three times during the nine-month study using researcher-devised tests on place value and multi-digit addition and subtraction. The researchers reported that students who had opportunities to engage in reflective thought and self-expression had higher levels of performance on assessments indicating that instructional tasks and discourse encouraged more productive ways of thinking.

Chapin, O'Connor, and Anderson (2009) explored the effects of a discussion-based teaching format on student learning in a four-year study in a low-income school district. Mathematics lessons included daily logic-problem warm-ups and weekly quizzes that emphasized discussion with explanations for student reasoning. Student learning, based on results of the California Achievement Test, indicated that students involved in the study scored better than 70 percent of a national sample. When the same students were tested again two years later they scored better, on average, than 91 percent of a national sample. After three years in the program, 90 percent of the sixth-grade students from the first group of students scored in the top two categories on the state assessment in mathematics. According to the researchers, discourse supported mathematical thinking and learning for these students because it allowed teachers and students to address misconceptions, contributed to increase in students' ability to reason, allowed students to become involved in their thinking and problem solving, and motivated them to become interested in mathematics.

Wood, Williams, and McNeal (2006) conducted a study in two classes of seven to eight year old students to explore the relationship between mathematical thinking and two types of classroom interactions they called the strategy/reporting classroom and the inquiry/argument classroom. In the strategy/reporting classroom, students reported their use of strategies for problems solved. Students in inquiry/argument classes offered their solution methods but were also asked to provide reasons for their thinking. The researchers analyzed and coded 42 videotaped lessons according to interaction patterns and students' mathematical thinking, and reported that the interaction pattern that required greater student involvement and personal investment was related to higher levels of expressed mathematical thinking by the students. Wood et al. (2006) discussed the importance of teacher support for students as they work through problem solving and the need to allow students to pull together ideas for making judgments and identifying flaws in arguments.

Results of a study conducted by Gooding and Stacey in 1993 supports the NCTM (2000) call for students to use the language of mathematics to express mathematical ideas precisely for effective student discourse. The study was conducted with one class of 28 fifth- and sixth-grade students working in groups of four. Students were pre-tested one week prior to the activity and post-tested three weeks after the activity using a nine-item test. Students were videotaped as they worked on a mathematical task that required them to correctly order cards representing steps of division story problems. Once groups had placed the cards in order, they were shown a corrected set of cards, complete except for one set of cards. Students were asked to talk about how they placed their cards and to discuss any misconceptions they had with their placement or the placement of the corrected set of cards, especially the cards that were not correct. Sessions were videotaped and coded for type and content of discourse. Data from the pre- and post-test was used to compare the number of correct items and to determine students who were termed either improvers or non-improvers. Groups were then designated effective if two or more of the students were improvers. Other groups were designated as ineffective. The researchers reported that the use of specific mathematical vocabulary in "intellectual interaction" (Gooding & Stacey, p. 67), the reading aloud of example cards, explanations with evidence, and repeating other's statements were characteristics of effective student discourse as evidenced by transcribed video tapes and post-test scores for the effective students and groups versus the non-effective students and groups.

Writing and Learning Mathematics

Miller (1992) conducted a qualitative study to determine the effects of timed, in-class impromptu writing as a part of learning mathematics in three algebra classes serving 85 students in eighth grade through twelfth grade. Writing prompts were presented to students that required them to address a specific question or problem and students were given five minutes to read the prompt, formulate, and write their response. Prompts sometimes asked students to write a clear expression of their understanding of a mathematical concept, skill, or generalization within a meaningful context. Some prompts were more specific while others dealt more with how the students felt about mathematics and their mathematics class. Miller (1992) stated that his study, which viewed writing as a part of the mathematics class, differed from previous studies that investigated writing as an enrichment activity. According to the researcher, an interpretive research methodology was used to collect and analyze students' writing that looked for recurring patterns that might identify positive ways impromptu writing enhanced the mathematics learning of the students as well as the mathematics instruction of the teacher. Results indicated that students' understanding of mathematics improved over the course of the study. In addition, Miller (1992) reported that participating teachers found the students' writing useful in determining their understanding of students' algebra knowledge. Field notes and writings produced by both students and teachers indicated that the teachers became more attentive to the misunderstandings and problems the students experienced and adjusted their instruction accordingly.

Porter and Masingila (2000) experimented with 33 students in two introductory calculus classes to determine if writing to learn mathematics (WTLM) would benefit their conceptual and procedural understanding. The researchers defined WTLM as any type of writing used to help students learn mathematics. All students in the study were engaged in activities that required them to think about and discuss mathematical ideas, but only half of the students were also asked to complete a writing assignment in addition to the thinking and discussion activities. When errors from in-class and final tests were categorized and analyzed using a researcher devised classification system for errors in calculus, the researchers reported that there were no significant differences between the writing and non-writing students. They suggested that the benefit from writing may not be in the actual writing activity, but rather the fact that such activities required the students to struggle to understand mathematical concepts well enough to communicate their understanding to others.

Borasi, Siegel, Fonzi, and Smith (1998) studied 82 secondary mathematics students to determine the effectiveness of having students write, draw, and talk about mathematical ideas in mathematical text. Strategies used by the researchers required pairs of students to read text and discuss, write, or draw in response to the text. Students were asked to address their confusion, understanding, questions, and feelings at certain points in their reading, while drawings were to represent the text. Activities were assessed using a unit of analysis the researchers called the reading to learn mathematics (RLM) episode, which focused on what was read, why it was read, and how it was read. Analysis of a total of 18 RLMs indicated that students actively participated in all episodes and were engaged in making sense of the mathematics in all but two RLMs. The researchers suggested that instructional practices drawn from the field of reading, such as writing, drawing, and talking about ideas, supported the students' mathematical sense-making and understanding. They also stressed the need for teachers to plan appropriate follow-up experiences that invite students to discuss and share their ideas and questions. In addition, they suggested that teachers should make the reasons for using different ways of reading mathematical text explicit to the students.

Games and Learning Mathematics

There have been numerous studies conducted to investigate the effectiveness of games for mathematical instruction. Divjak and Tomic (2011) completed a literature review of 32 quantitative and qualitative research studies to summarize the effects of mathematical digital or electronic games on students' mathematical achievement. Of these studies, approximately 73% were conducted with primary and elementary school students. Divijak and Tomic (2011) reported that using digital math games increased student motivation and math knowledge when compared to students taught without the use of digital games.

Research studies conducted to determine the effects of using non-digital games as part of mathematics instruction have produced mixed findings. Mustafa, Khan, and Ullah (2004) used a game called *Guess and Tell* to teach the mathematical concepts of mean and mode to 39 eighth-grade students in Pakistan. For two weeks, students spent 45 to 50 minutes per day in game play. According to the researchers, students were pre- and post-tested using researcher devised instruments constructed to evaluate the students' math achievement. The pre-test was developed using the textbook and the teacher's evaluation of the students' prior knowledge. The post-test was compiled from questions selected from the chapter taught to the students through game-based teaching. The researchers reported a statistically significant increase in student test scores as a group, but there was not a statistically significant difference between the scores for males and for females. This study did not incorporate a control group which the researchers confessed was a limitation of their study. In addition, no description of the game was supplied other than to note that the game was similar to a game played by children in Pakistan.

Over a period of two weeks, researchers Ramani and Siegler (2008) worked with 124 low-income preschoolers randomly assigned to two groups. Each group participated in four game sessions of 15–20 minutes each playing a researcher-created board game. One game utilized consecutively numbered squares while the other used colored squares with no numbers. Pre- and post-tests evaluated the students' skills of identifying numerals, comparing numbers, counting, and number line estimation, with a follow-up test nine weeks later. The researchers reported that the students who played the numbered game board made substantial improvements on all four tasks that were assessed, outperforming the students who played the colored game board.

Bragg (2012) investigated the effectiveness of non-digital games as a tool to increase mathematical learning. This research was conducted with 112 10–12 year old students in eight classrooms in Australia. Each class had two treatment sessions each week for four weeks. On the other days, students received traditional math instruction. However, no teaching on these days focused on the topic of the games and activities, multiplication and division of decimals. Seventy-six students played games either with or without formal teacher-led discussion. Of these 76 students, 32 students played games for approximately 35 minutes with no planned teacher-led discussion while 44 played games with 15 minutes of teacher led-discussion. The remaining 36 students participated in approximately 35 minutes of non-game activities designed to teach the same content. Teacher-led discussion was not controlled for these students.

Students were pre-, post-, and delayed post-tested using a researcher-constructed test. All three treatments, games classrooms without discussion, games classrooms with discussion, and activity classrooms, resulted in improved scores for the students. However, the activities group experienced the greatest gains, followed by the games without discussion group, and the games with discussion group.

In 1985 Bright, Harvey, and Wheeler reported on 11 studies they had completed to investigate the effects of using non-digital games as part of mathematics instruction. Participants for all studies were from grades five to ten and each study included two grade levels with at least two classes per grade. Each study included pre- and post-testing that included tests to measure content taught as well as prerequisite content, but some of the 11 studies also included other forms of measurement. All students took a test of formal operations. Treatment time varied by study but there was a minimum of seven days for 15 to 20 minutes per day. When data from all 11 studies was considered, the researchers reported that the use of non-digital games was effective for helping students retain, develop, and improve basic mathematics skills, especially when used with other methods of instruction.

Koran and McLaughlin (1990) conducted a study with 28 fifth grade students randomly assigned to two groups in order to compare the effectiveness of two approaches for teaching basic multiplication, drill and non-digital games. After six days of basic instruction, one group played a game while the other group was involved in drill for 10 additional days of instruction. The students concluded the treatment with an additional 10 days of instruction. The researchers found both drill and games to be equally effective in teaching basic multiplication facts and increasing the students' achievement scores.

Importance of Vocabulary Instruction

The communication standard (NCTM, 2000) emphasizes that students should recognize and use connections between mathematical ideas, and suggested that students discuss, write about, and create visual representations to help them realize these connections. However, the NCTM (2000) also recognizes that as students advance in grades, they are met with more abstract concepts and the need to use formal mathematical language more often in their discourse and mathematical writing. The NCTM (2000) made suggestions for teachers as they assist their students in speaking and writing as mathematicians. For example, teachers should encourage oral communication and model writing in mathematics before asking students to write. Teachers of young children should encourage students to describe their ideas in their own words until they understand concepts. Once understanding is in place, the formal mathematical vocabulary should be supplied. However, the NCTM (2000) cautioned that the point of readiness for students to move from invented to standard vocabulary will vary. In addition, they suggested that teachers supply a wide variety of experiences in order for students to develop a deep understanding of the mathematical concepts that are expressed by vocabulary. As students acquire formal mathematical language, their communications will become much more precise and they will be more likely to communicate their thinking effectively with their teachers and classmates.

Educators Pierce and Fontaine (2009) have proposed that the principles of vocabulary instruction found to be successful in the language arts be applied to the instruction of content area vocabulary, especially mathematics. The importance of vocabulary for reading comprehension and general school success has been well documented (Butler, et al., 2010; National Reading Panel, 2000). Becker (1977) was one of the first educators to link vocabulary size to academic achievement when he asserted that vocabulary deficiencies were the primary cause of academic failure of disadvantaged children in grades 3 and up. Becker (1977) reported results from a 1967 study conducted to evaluate several educational programs designed for disadvantaged students. In 1977 evaluations of these programs were completed and one program, the University of Oregon's Direct Instruction Model, had produced significant gains in measures of skills and conceptual reasoning as measured by the Wide Range Achievement Test (WRAT), the Metropolitan Achievement Test, and the Slossen Intelligence Test. One aspect of this program was the scripted presentation of lessons that controlled the selection of vocabulary and examples in order to assure that terms unknown to low-achieving students were not introduced initially. According to Becker (1977), schools have often failed to provide sufficient instruction in words and their referents, especially for children from homes without strong adult support for building vocabulary. More recent research that focused on diverse learners such as students with learning disabilities, high achievers, and culturally disadvantaged students, was synthesized by Baker, Simmons, and Kame'enui (1995). Among several themes that emerged from this review was the contrast between the number of known vocabulary words of high-achieving students and low-achieving students. Baker et al. (1995) concluded that this difference significantly limited academic success in basic skills areas as well as in the content areas. This difference, or gap, begins in the primary grades and continues throughout the school years (Blachowicz, Fisher, Ogle, & Watts-Taffe, 2006). Because this achievement gap widens over time (Biemiller & Slonim, 2001) and because there is a strong link between early vocabulary knowledge and early reading achievement (Coyne, Simmons, & Kame'enui, 2004), vocabulary instruction should be an important focus of classroom instruction.

Methods of Vocabulary Instruction

The National Reading Panel (2000) concluded that there is no single best research-based method for teaching vocabulary. However, the panel did recommend that teachers use a variety of direct, or explicit, instruction and indirect, or incidental, methods of instruction. Several indirect components of effective classroom vocabulary instruction have been supported by research. For example, Biemiller (2003) suggested that students learn many new words incidentally in classrooms having a language and word-rich environment that fosters word consciousness. Classrooms of this caliber provide students with many opportunities to read, use, and discuss new vocabulary. Dickinson and Smith (1994) conducted a year-long study of 25 four-year old children to determine the effects of naturally occurring analytical discourse during book reading on children's vocabulary and story comprehension. The researchers found that discourse helped create a strong conceptual base for student vocabularies while providing occasions for students to use low-frequency words. In addition, they reported that components found to enhance learning, frequency of exposure to vocabulary, deep processing of word meaning, and provision of sufficient information to clarify word meanings, were all incorporated into the analytical discussions. These findings imply that classroom book reading with analytical discussion, one example of indirect or incident vocabulary instruction, promotes student growth in vocabulary and comprehension without requiring major adjustments in instruction.

Armbruster, Lehr, and Osborn (2001) described direct or explicit instruction for vocabulary as "providing students with specific word instruction and teaching students word-learning strategies" (p. 30). The National Reading Panel (2000) report suggested that direct instruction is needed for words when meanings are necessary for specific texts to be read or when the words are part of a lesson. Strategies for direct vocabulary instruction found to be effective included pre-instruction of words, repetition and multiple exposures to target words, use of context clues, active engagement, computer technology (National Reading Panel, 2000; Armbruster et al., 2001), and student created illustrations (Cohen & Johnson, 2011).

Research on Multiple Approach Use for Vocabulary Instruction

In 1988, Nagy reviewed available research on effective vocabulary approaches and summarized the research by stating that all effective approaches included opportunities for students to connect new vocabulary to prior knowledge, encounter words multiple times, and have multiple opportunities to use new words in reading, writing and discussion. A more current review of research, conducted by Blachowicz and Fisher (2011) resulted in the researchers compiling a list of what they termed research-based essentials for vocabulary instruction. Their list consisted of a considerable amount of reading, writing, and meaningful talking; teaching of individual words; instruction in word learning strategies; and promoting word consciousness.

Multiple method procedures that include researched-based vocabulary approaches have been proposed by researchers (Beck et al., 2002; Graves, 1984; Stahl & Nagy, 2006). Because the routine suggested by Beck et al. (2002) included approaches found to be effective, these researchers described their multiple methods program as being rich and robust. In addition, the researchers proposed that teachers select words for instruction by categorizing them according to three tiers. According to Beck et al. (2002), tier one words are the most basic words that rarely require instruction. Tier two words are high-frequency words found across a variety of domains. These words are important to student comprehension and it is here that instruction is most effective for general reading. Tier three words are those words whose frequency of use is quite low and often associated with specific subject areas. The researchers suggested that a deep understanding of these words is beneficial for most learners but instruction should be as need arises. Beck et al. (2002) also suggested that teachers introduce new vocabulary by presenting students with student-friendly definitions, which they defined as definitions that “characterize the word and how it is typically used and explained in everyday language” (p. 35).

McKeown et al. (1985) conducted research with the goal of determining the effectiveness of traditional, rich and extended rich vocabulary instruction as well as the effects of high and low frequency encounters. Participants were 82 fourth-grade students from two urban schools in a lower socioeconomic neighborhood. These students received one of three types of instruction. Students who received rich instruction participated in activities that explored word meanings, word relationships, and application of words in various contexts. Two frequency conditions were used with this group called high and low. In high frequency, six words were presented in twelve instructional encounters, and in low frequency, six words were presented in four encounters, both over a seven-day cycle. Students in the extended-rich group received instruction equal to rich instruction with the addition of Word Wizard, an activity designed to encourage students to use, look, and listen for target words outside the classroom. A third treatment group, the traditional group, participated in rich activities (same format and number of encounters per word) but instruction consisted only of words and their definitions or synonyms. Pre- and post-test assessment included a vocabulary knowledge test and a story comprehension test. In addition, at the conclusion of the study, students were also assessed using a semantic decision task and a context interpretation task. Results within treatment comparisons revealed that students in the extended rich treatment had equal or greater gains than either the rich or traditional groups. Results for the rich group indicated that their gains were equal to or greater than gains in the traditional group. While the researchers did not report means for the groups, they did state that results indicated that students benefitted most from frequent encounters with vocabulary and, as a result, they exhibited increased reaction times, comprehension, and fluency resulting in greater learning gains.

Graves (2006) outlined a comprehensive vocabulary program consisting of four components: rich and varied language experiences, teaching individual words, teaching word-learning strategies, and fostering word consciousness. Bauman, Ware, and Edwards (2007) conducted a year-long formative experiment to determine the effectiveness of Grave's four components on the vocabulary development of 20 fifth grade students. Participants were exposed to children's literature vocabulary through multiple experiences that included read-alouds, weekly dialogue journals, word walls, and the use of graphic organizers. Results indicated that the students' expressive, or speaking, vocabulary grew more than expected as measured by the Expressive Vocabulary test and that students initially below average in vocabulary may have benefited more than students initially above average. Qualitative findings revealed three themes: 1) students used more sophisticated and challenging words, 2) their interest and attitudes toward vocabulary learning increased, and 3) they demonstrated use of word-learning tools and strategies independently.

Townsend and Collins (2009) conducted research to determine the effectiveness of instruction that included meaningful exposures and opportunities to practice academic vocabulary. In this study, the researchers conducted an intervention program for middle school English language learners. This program, termed Language Workshop, was modeled after the principles of rich vocabulary instruction advocated by Beck et al. (2002) with goals of building depth of academic vocabulary and breadth of general word knowledge. Over the course of 15 weeks, students were exposed to rich vocabulary instruction on 60 academic vocabulary words in a language-rich environment. The number of exposures for each target word was tracked daily revealing an average number of five exposures per word. Results showed significant growth that occurred immediately following intervention with an effect size of +0.74, which indicated that a program using research-based strategies was effective for teaching academic vocabulary to English language learners. The researchers also concluded from testing that the students with greater English proficiency made greater gains during the program and that younger students showed more growth than older students.

Snow, Lawrence, and White (2009) conducted a quasi-experimental study of a cross-content area vocabulary intervention program called Word Generation in the Boston Public Schools in response to concerns about the vocabulary skills of the system's students. Students from sixth to eighth grades participated in a 24-week-long program that focused on all-purpose academic words during language arts, science, mathematics, and social studies classes. Instructional features of Word Generation were based on instructional factors that previous research had found to be successful (Beck, Perfetti, & McKeown, 1982; Beck, McKeown, & Kucan, 2002; Graves, 2006; McKeown, Beck, Omanson & Perfetti, 1983; McKeown, Beck, Omanson & Pople, 1985; National Reading Panel, 2000; Stahl & Fairbanks, 1986; Stahl & Nagy, 2006). These factors included presenting target words in rich contexts, multiple exposures in varied contexts, opportunities to use words orally and in writing, explicit instruction in word meanings, and explicit instruction in word learning strategies. Pre- and post-tests, composed of 44 multiple choice items based on two of the five words taught each week, were identical except for the order of the items. The researchers presented their analysis in terms of total number of words learned. When totals for the treatment schools were compared to totals for the comparison schools, treatment students learned approximately twice the number of target words, even though students in the comparison schools performed at a higher level at the start of the study. In addition, within the treatment group, language minority students averaged learning 4.9 words compared to 4.0 words learned by English-only students.

Timing of Vocabulary Instruction

Whether the instructional method used is direct, indirect, or a mixture, research suggests that vocabulary instruction should begin in the primary grades (Sanacore & Palumbo, 2009). Beck and McKeown (2007) found that low-income kindergarten and first grade students were capable of learning sophisticated tier-2 vocabulary. Ninety-eight students participated in a research project based on read-alouds of stories selected because they were conceptually challenging and included ideas, events, or topics that were complex or unfamiliar to the students, as well as sophisticated vocabulary that students could use in their daily lives. Following the reading of each weekly story, rich vocabulary instruction was used for three words per story. While achievement gains for these young students increased, the researchers believed their gains were hampered by the limited prior knowledge of the students. To determine the effects of more frequent instruction for a longer duration, 40 students from a different school, also low-income and either kindergarten or first grade, were provided with the same rich vocabulary for six words per week, plus additional instruction for a subset of three words, which the researchers called More Rich. In addition, there were two review cycles which gave students multiple exposures to the focus words. Students who participated in this study experienced significantly greater gains. Both studies indicated that it is important that vocabulary instruction provide students with opportunities to encounter words repeatedly and in multiple contexts.

Puhalla (2011) conducted a study with at risk first graders to determine if explicit instruction might reduce or eliminate the vocabulary gap between students at risk and their average-achieving peers. Students received instruction that included multiple exposures, modeling, corrective feedback, guided practice, and cumulative review while using narrative and expository storybooks with open-ended questions and conversations during read-alouds. The researcher reported that results of the study supported explicit teaching of sophisticated vocabulary to primary students at risk of reading failure is not only possible, but can help students acquire deeper vocabulary knowledge.

Academic Vocabulary

Academic literacy has been defined as “the kind of reading proficiency required to construct the meaning of content-area texts and literature encountered in school. It also encompasses the kind of reading proficiencies typically assessed on state-level accountability measures” (Torgesen et al., 2007, p. 3). The vocabulary used in content-area texts and literature, known as academic vocabulary, is often more conceptually complex and difficult to learn than words learned and used in everyday life (Townsend, 2009). However, according to the National Institute for Literacy (2007), a student’s depth of knowledge of the academic vocabulary for a given content area relates to the student’s success in that area. For that reason, content area teachers have been urged to teach the vocabularies of their subject areas.

Understanding the need for students to acquire academic vocabularies for success in content areas, teachers are faced with making decisions about what words to teach and how they should be taught. According to Blachowicz and Fisher (2000) teachers have found “making the leap from theory to practice” (p. 227) to be very difficult despite research that has indicated general instructional recommendations for effective academic vocabulary instruction. For example, Scott, Jamieson-Noel, and Asselin (2003) conducted research in 23 ethnically diverse classrooms to compare the time spent by teachers on academic vocabulary to that spent on general vocabulary. They found that teachers spent 6% of school time on vocabulary development; however, this percentage was reduced to 1.4% when they observed instruction in core academic subjects. The researchers reported that the instruction they observed in the latter setting involved mentioning and assigning vocabulary rather than teaching vocabulary. Even more distressing were the results when Blachowicz and Fisher (2000) asked vocabulary researchers to reflect on how much research on vocabulary instruction had affected classroom practice. The researchers' collective answer was "not much" (p. 509).

Believing that teachers might be more receptive to academic vocabulary instruction if they had a framework to aid them in selecting words and methods that matched their students, Blachowicz and Fisher (2000) devised a four-level instructional framework for vocabulary instruction. The first level of the framework included words essential to understanding a passage and that represented concepts students needed to understand in-depth before reading. Level 2 words were concept words new to the students that represented both familiar and unfamiliar concepts. Because these words included those not familiar to students, they were introduced before reading but returned to after reading in order to explore them in greater depth. Level 3 words represented concepts not necessary to initial reading but that could be dealt with either during or after reading. This level also included high-utility words likely to be encountered in other texts, such as academic reading. Level 4 words included words the researchers deemed not worthy of valuable instructional time. The researchers reported that incorporating a framework helped teachers explore their goals or purposes for teaching words and enabled them to consider both the words and strategies to use at different points of instruction. Similarly, Bauman and Graves (2010) devised an academic vocabulary classification process based on academic vocabulary research which included "five types of words and conceptual representations: 1) domain-specific academic vocabulary, 2) general academic vocabulary, 3) literary vocabulary, 4) meta-language, and 5) symbols" (p. 9) with the goal of providing teachers an initial way for them to select words for various instructional purposes.

In addition to routines for selecting and classifying academic vocabulary, several researchers have devised routines to guide instruction for these words. For example, Fisher and Frey (2008) devised a school-wide vocabulary initiative that included intentional word selection and instruction, intentional teacher modeling of word learning, activities such as open word sorting, and collaborative learning. Marzano (2005) devised a six-step process for teaching academic vocabulary. These steps included providing an explanation or example for each new word, asking students to restate the explanation or example in their own words, and provide a representation of the word. Students were also engaged in activities to help them add to their knowledge of words, such as listening for words outside of school, discussing words, and playing games with words.

In 2005, Marzano reported on findings from a 2004–2005 evaluation study of the Building Academic Vocabulary (BAV) program based on the assumption that teaching standards-based academic vocabulary using the specific six-step process could not only enhance students' content area comprehension but also help them improve their academic performance. Five school districts, 11 schools, 118 teachers and 2,683 students from pre-kindergarten to grade nine participated with participating treatment teachers trained on the BAV protocols. Students were assessed using researcher-devised tests for mathematics, science, and general literacy. Results indicated that the mean score for the experimental group was significantly greater than the mean score for the control group for all measures. Marzano (2009) reported being involved in more than 50 studies that involved the six-step process utilizing control and treatment groups. He admitted that some studies showed greater gains than others while some showed negative gains. While Marzano (2009) did not specify why negative gains were experienced, he did conclude from these studies that all six steps of his vocabulary routine should be utilized carefully. For example, allowing students to copy a teacher's explanation of a word is not as powerful as having students devise their own explanations based on their lived experiences. Marzano (2009) also stressed the importance of having students supply a representation of the term by drawing a picture or other symbolic representation.

Kelley, Lesaux, Kieffer, and Fuller (2010) designed and evaluated an academic vocabulary program for 476 sixth grade readers in seven participating schools. Three hundred forty-six of the students were language minority learners and 130 were native English speakers. The researchers developed an 18-week academic vocabulary program that included eight two-week units and two review weeks. Each unit included an eight-day lesson cycle of 45 minutes sessions, four days a week. Units revolved around informational text from which eight or nine academic words were selected. Students were exposed to each word on three days between two and five times, and subgroups of those words were used each of the eight unit days. Assessments used for pre- and post-testing were described by the researchers as a multiple-choice test of academic words, a curriculum-based measure of deep knowledge of the words taught, and a test of students' ability to break down words into parts. Students in the treatment group experienced improvement equal to about eight to nine months of extra growth in reading comprehension. Observations of teachers trained to deliver instruction in the treatment group were "better than control teachers at providing students with multiple opportunities to use words, posting visual resources for learning words, affirming correct use of words, using personal anecdotes to give examples for words, supporting students' writing, and facilitating student talk" (p. 8). Based on results from the study, the researchers suggested that teachers limit the number of high-utility words they teach and to take twice as long to teach them, incorporate structured discussions among students, repeat targeted words in different contexts, talk about interesting words or different uses of words studied in class, and have students include new vocabulary in their writing. Activities such as these enabled students to have a deeper understanding of vocabulary, which in turn increased their achievement.

Academic Vocabulary Instruction in Mathematics

To determine the effects of a kindergarten mathematics program that included specific vocabulary instruction, Clark et al. (2011) conducted a study with 64 classrooms and more than 1,300 students using a program the researchers called Early Learning in Mathematics (ELM). The ELM program provided vocabulary instruction embedded within the studies of number operations, geometry, and measurement and was "designed to increase the amount of math discourse and use of critical mathematics vocabulary" (p. 566). Instruction included intensive teacher modeling, use of different representations of mathematical concepts, and students engaged in verbalizing their thinking as they solved mathematical problems. In addition, frequent cumulative review was provided within and across lessons. Five measures were used for both pre- and post-test. The Early Numeracy-Curriculum Based Measurement included four subtests, oral counting, number identification, quantity discrimination, and missing number. The fifth measure was the Test of Early Mathematics Ability (TEMA-3). Data for student achievement indicted that gains for at-risk treatment students were significantly greater than for control students. Also, the gains for at-risk students surpassed those of students considered not at risk, resulting in a reduction in the achievement gap for those students.

Although the amount of research addressing mathematical vocabulary and mathematics achievement is low, the benefits for students who have the ability to use vocabulary confidently have been well supported (Benjamin, 2011; Biemiller, 2003; National Reading Panel, 2000). In addition, the importance of vocabulary instruction in mathematics and its role in the development of conceptual understanding have recently become more urgent for teachers (Benjamin, 2007) due to calls for mathematics education reform because, in part, of low achievement scores from national and international tests. Given the standards based learning environment that is expected in mathematics classrooms today, students must be able to communicate their mathematical ideas during class discussions, writing, and problem solving. In order to do this successfully, research has indicated that they must be supported with a deep understanding of the appropriate mathematical vocabulary (Nagy & Townsend, 2012).

Pierce and Fontaine (2009) contended that "proficiency in mathematics has increasingly hinged upon a child's ability to understand and use two kinds of math vocabulary words: math specific words and ambiguous, multiple-meaning words with math denotations" (p. 242). Pierce and Fontaine (2009) urged teachers to design lessons around both kinds of words that include research based vocabulary improvement activities such as those that include intentional instruction, repeated and multiple exposures, and active student engagement in a variety of experiences. Activities of this type require students to interact with words and their meanings to help them use the language of mathematics to improve their mathematics achievement.

Further research is needed to determine the effects of rich and robust vocabulary strategies on the mathematics achievement of students as measured by school-level, state, and national mathematics tests. Research has shown that the procedures found in rich and robust instruction have promoted deep understanding of academic vocabulary in social studies and science (Brown, et al., 2010; Cohen & Johnson, 2011; Harmon, et al., 2005; Marzano, 2005). Using the same rich, elaborated direct instruction of words in mathematics may be a promising way to promote greater conceptual understanding of mathematics, and therefore, enhance the mathematics achievement of students.

CHAPTER III. METHODS

Increased understanding of mathematical vocabulary has been reported by research as a valuable method to increase student conceptual understanding of mathematics and subsequent scores on school and state achievement tests, particularly for lower achieving students. This study was based on this research in order to investigate the effectiveness of vocabulary instruction as an enrichment using the routines of Beck et al. (2002) on elementary mathematics students' achievement as compared to use of non-digital math games. A pre- and post-test control group design (Campbell & Stanley, 1963) was used to test the hypothesis of greater achievement in utilizing vocabulary routines over non-digital math games to increase students' achievement on unit mathematics tests and on a researcher-designed vocabulary test. The dependent variables were the unit mathematics test scores and vocabulary test scores, while the independent variable was group, vocabulary instruction and game time. The primary hypothesis was that there would be a significant difference between the unit five and unit one mathematics mean test scores and the unit five and unit one vocabulary mean test scores for the vocabulary group over the game group. These two units were taught consecutively, with unit five serving as a review of addition, subtraction, and place value, followed by unit one with a focus on multiplication.

In addition, the study hypothesized that the achievement gap for underachieving and achieving students in mathematics and reading would be reduced as a result of the vocabulary treatment. The following research questions guided the study:

1. Will rich instruction of mathematical academic vocabulary result in greater improvement in elementary students' mathematics achievement as assessed by unit tests from two consecutive mathematics units of study than use of non-digital mathematics games?

2. Will rich instruction of mathematical academic vocabulary result in greater improvement in elementary students' achievement on a researcher-designed vocabulary test than use of non-digital mathematics games?

3. After receiving rich instruction of mathematical academic vocabulary, will there be a significant difference between unit mathematics mean test scores for underachieving and achieving students in mathematics?

4. After receiving rich instruction of mathematical academic vocabulary, will there be a significant difference between math mean test scores for underachieving and achieving students in reading?

Participants

This study took place in the fall of 2014 in one elementary school located in a rural town in southeast Alabama serving students in grades three and four only. The study was conducted with all fourth-grade mathematics students and all three fourth-grade mathematics teachers. Each fourth-grade mathematics teacher taught three classes of students each day in mathematics for approximately 110 minutes per class, and each class contained approximately 25 students. The school currently serves 469 students composed of 59% Caucasian, 39% African American, 1% American Indian/Alaskan, and 1% Asian students, with 60% of students eligible for free or reduced lunch. In 2012, the percentages of sub-groups of fourth graders in this school who met or exceeded standards for mathematics as assessed by the Alabama Reading and Mathematics Tests were: 91% of Caucasian students, 82% of African American students, 87% of female students, 88% of male students, 83% of poverty students, and 94% of non-poverty students. Overall, 88% of the school's fourth graders met or exceeded standards for mathematics as assessed by the state test (Great Schools, 2014). Alabama fourth graders also participated in the 2013 NAEP testing. The percentage of students in Alabama who performed at or above the NAEP Basic level for mathematics was 75%. Table 1 reports average scores and percentages of students for each achievement level for these students by sub-group (NAEP, 2013).

Table 1

2013 NAEP Results for Alabama Fourth Graders

Subgroups	Average Scaled Score	Below Basic	At or Above Basic	At or Above Proficient	At Advanced
White	242	14	86	40	5
Black	215	47	53	9	0
Hispanic	228	30	70	23	3
Male	233	25	75	31	4
Female	232	25	75	28	3
Eligible for Free/Reduced School Lunch	223	36	64	17	1
Not Eligible for Free/ Reduced School Lunch	246	10	90	47	7
English Language Learners	209	60	40	7	0
Not English Language Learners	233	24	76	30	3

According to the NAEP (2013) report for Alabama fourth grade students, the average score for black students was 26 points lower than that for White students. Students eligible for free/reduced price lunch, an indicator of low-family income, had an average score 23 points lower than students not eligible for free/reduced price lunch. The performance gaps for both sub-groups were not significantly different from that in 1992 and 1996 NAEP scores respectively.

Research Design

The design of this study was similar to a study conducted by Jackson and Phillips (1983) to determine if mathematics unit achievement would be increased with the addition of daily vocabulary instruction. Jackson and Phillips (1983) completed their research with seventh grade mathematics students randomly assigned to two groups receiving either five to ten minutes of daily vocabulary instruction or additional time working computational problems. Similarities between the study reported here and the Jackson and Phillips (1983) research included the time spent in vocabulary instruction, the focus of the vocabulary instruction, and the content area, mathematics. Students participating in the Jackson and Phillips (1983) study spent five to 10 minutes in daily vocabulary instruction, where students in this study spent 10 to 15 minutes in vocabulary instruction or playing non-digital math games. Jackson and Phillips described their vocabulary instruction as emphasizing student recognition, categorization, and identification of examples and nonexamples of terms and symbols associated with ratio and percent. The study described here also focused vocabulary instruction on recognition and identification of examples and nonexamples, but also included opportunities for students to discuss relationships between focus words. A comparison of vocabulary activities used in the two studies revealed that both studies asked students to justify their reasoning when asked to make comparisons or select the correct term for a concept.

Differences between the two studies include length of study, number of vocabulary words included in the study, and research design. The Jackson and Phillips (1983) study was conducted over four weeks, utilized a post-test only design, and focused on six terms and five symbols, where the study reported here was conducted for seven weeks, utilized a pre- and post-test design over two units, and focused on 22 vocabulary words. In addition, students not receiving rich vocabulary instruction spent a comparable amount of time playing non-digital mathematics games. Jackson and Phillips (1983) based their selection of terms and symbols on a list composed by mathematics educators of terms and symbols considered essential for ratio and proportion. The vocabulary addressed during the present study was selected from words highlighted within instruction found in the teachers' editions, as well as words used in unit test instructions. An original vocabulary list was reduced in consultation with the participating teachers who identified the words they believed to be most important for the topics being taught during the research study. The post-test administered in the Jackson and Phillips (1983) study was developed by one of the researchers. No examples of the test were supplied but the researchers did state that the test was modeled after the textbook chapter test except for the inclusion of vocabulary-oriented items. The study presented here utilized the unit tests provided by the math program as well as vocabulary tests developed by the researcher.

The study conducted for this research was completed in the classrooms of three participating teachers. Teacher A incorporated the vocabulary treatment into the daily mathematics instruction in two classes. This teacher's third class used a comparable amount of time playing non-vocabulary, non-digital mathematics academic games. Teachers B and C incorporated the vocabulary treatment into the daily instruction of one class each and used a comparable amount of time playing mathematics games with the remaining two classes. The use of games for students not receiving vocabulary instruction was at the request of the superintendent of the school district. This procedure resulted in four vocabulary classes with a total of 63 students and five game classes with a total of 71 students. Data for 75 students were incomplete and were not used. Missing data resulted from student absences on the days pre- or post-tests were administered and for transfer students who did not take the Aspire test (ACT, 2014). In addition, the parents of one student opted out of the study. The vocabulary routines for the vocabulary group were provided by the mathematics teachers as an enrichment for part of the daily instructional time and were completed at the end of each class period for approximately 10 to 15 minutes. These routines included direct vocabulary instruction that required the students to think about and use the words in examples, sentences, and verbal activities designed to help students explore relationships among words (see Appendix A).

The study was conducted during the instruction of unit five, Landmarks and Large Numbers: Addition, Subtraction, and the Number System, and unit one, Factors, Multiples, and Arrays: Multiplication and Division, of Scott Foresman's Investigations® mathematics program for grade four (Russell et al., 2012). This mathematical program is currently the program adopted and used by the school system. Each Investigations® teacher's edition includes suggestions for supporting learners experiencing difficulty with program content, especially students whose first language is not English. Important words assumed to be part of the working vocabulary for the unit are listed, and activities are included in the appendix for inclusion in students' second language work either before or during the unit. There are no specific activities or procedures supplied by Investigations® for the general introduction or review of vocabulary except through discussion and instruction suggested as part of daily instruction.

Seventeen days of instruction were devoted for each unit. Unit tests for units five and one were used for pre-test and post-test data (see Appendix B). In addition, students completed pre- and post-vocabulary tests for both units designed by the researcher (see Appendix C).

Questions on this test included focused questions and generative questions (Feldman & Kinsella, 2005) using vocabulary from units five and one of the Investigations® mathematics program for fourth grade. Both the unit pre-tests and the vocabulary pre-tests were administered within three instructional days prior to the start of each unit, while the unit post-tests and the vocabulary post-tests were administered within three days following the conclusion of each unit.

The researcher prepared all materials for the vocabulary treatment and delivered them to the office of the principal of the school on Monday of each week of the study. These materials included scripts for conducting daily vocabulary routines and vocabulary cards for activities where they were necessary (see Appendix A). While the teachers were not required to read the scripts verbatim for fear of hindering teacher-student or student-student discourse, treatment teachers were asked to complete all the activities with each class in order to maintain consistent treatment in all classes. Because the scripts were lengthy, review days were incorporated in the schedule in order to provide time to address all material eventually. Vocabulary words included in the study were words suggested by the mathematics program as well as words found in mathematical tasks and instructions from mathematics units five and one. This list was reviewed by the participating teachers prior to the start of the study in order to identify the words they believed to be most important for understanding the unit concepts. The researcher also worked with the three participating teachers to identify appropriate academic games for the game group (see Appendix D). Because the first unit of instruction reviewed addition, subtraction, and place value, games for unit five focused on these concepts while games for unit one focused on multiplication. All games were intended for use by students playing in pairs, in small groups, or as a class. Close to Zero (Learn With Math Games, 2013) is an example of one game played by the game groups during the first unit. This game had students compete to make the number closest to zero using numeral cards. None of the games were vocabulary based, but because the games supported classroom instruction and involved students interacting with their classmates, it was possible that the students would have the opportunity to use vocabulary from their lessons while playing. Games included for use in the study were selected from resources designated as free for classroom use, and were duplicated and supplied to the participating teachers by the researcher.

During the month prior to the beginning of the study, the researcher observed the classes of each participating teacher in order to complete an observation checklist to determine each teacher's approach to mathematical vocabulary instruction (see Appendix E). The checklist used during these observations was developed by the researcher based on vocabulary activities, strategies, and procedures identified as being effective during the completion of the literature review for this research. The researcher wished to use information from these observations to plan the training for participating teachers so that new learning about vocabulary instruction might be built on the participating teachers' existing practices. These observations were completed on one day during one section of each teacher's mathematics instruction. On this day all three teachers were working with their students to complete personal place value booklets. The physical environments of the three classrooms included no word walls or other postings of academic vocabulary. Math manipulatives, such as base ten blocks, were not in evidence except for one classroom where they were crated and placed on a bookcase. No items on the checklist pertaining to vocabulary introduction were observed. The only checklist item observed was vocabulary review provided by the teachers in the form of descriptions and explanations of vocabulary dealing with the base-ten value system.

Two weeks prior to the beginning of the study, participating teachers attended two training sessions led by the researcher (see Appendix F). During the first meeting, participating teachers randomly selected a color-coded file folder to determine which classes would receive vocabulary instruction and which would play mathematical games. Inside the file folders was either an A, A, B or an A, B, B assignment, with A being a vocabulary instruction class and B being a math game class. The remainder of the first meeting was comprised of a one-hour training session concerning how student information and confidentiality would be maintained during the study. The second training session consisted of two hours of training on the importance of vocabulary instruction, vocabulary research, academic vocabulary, and robust vocabulary instruction. Based on the classroom observations, the researcher anticipated that the introductory discussion of the training pertaining to the importance of vocabulary instruction in mathematics would need to be more basic than originally planned. This proved to be true. For example, when the vocabulary words *numeral* and *number* were discussed during training, two of the three teachers said they thought the words were synonyms. The researcher observed that the teachers became uncomfortable as discussion continued about the need for vocabulary instruction, with one teacher stating that she was embarrassed about her knowledge of the importance of vocabulary instruction and the meanings of certain mathematics vocabulary in particular. Because it was not the researcher's goal to make the teachers uncomfortable, the teachers were assured that their knowledge of vocabulary development was not unusual, especially for mathematics. This second training session also included explanations and modeling of vocabulary activities and strategies identified as being part of robust instruction. The researcher discussed each type of vocabulary routine that would be used in the study, provided a handout of samples of the routines using vocabulary other than that planned for this study, and modeled a portion of the routines for the teachers. Time was provided originally in the training agenda for the teachers to practice vocabulary routines using these scripts. However, this was omitted from the training based on the teachers' hesitation to participate at that time. The teachers stated that they did not anticipate problems providing the vocabulary routines given more time to prepare. Each teacher was given a copy of *Bringing Words to Life* by Beck et al. (2002) for additional information. Each of the three teachers received a $100 gift card at the completion of the study in return for their participation in this research study.

Daily vocabulary routines designed and used in the study varied according to whether or not the vocabulary had been previously presented to students. According to the participating teachers, most of the mathematical content and the associated academic vocabulary included in the study had been introduced in previous grades or at the beginning of the current school year, especially for unit five. For example, the students had seen and heard words such as *sum*, *quantity*, and *algorithm,* but the teachers did not believe their students had a true understanding of the meanings of the words. Therefore, these words were scheduled for instruction according to planned lesson content. There were words new to students, especially from unit one, such as *prime, composite, multiples*, and *factors*. For these words, vocabulary instruction was held until the students had associated the words with corresponding math concepts during regular mathematics instruction.

Most of the 10 to 15 minute vocabulary sessions addressed a single vocabulary word. Students were supplied with index cards which they divided into quadrants (see Appendix G) as well as plastic bags in which to keep completed cards for reference. The teacher began by writing and pronouncing the word and then having students write the word on their cards. The teacher then presented a child-friendly definition of the word which the students wrote in the first quadrant. As students became more familiar with the vocabulary routines they were asked to participate in a sentence completion activity for the words. The teacher then modeled the word or provided examples or representations of the meaning of the word before having student complete the second quadrant by supplying an example of their choice. Students provided a drawing or symbol for the word in the third quadrant and a sentence using the word in the fourth quadrant. Daily scripts included at least one activity requiring students to compare relationships between words and concepts as outlined by Beck et al. (2002) (see Appendix A). Once vocabulary instruction began it became apparent that the 10 to 15 minute time allotted for the treatment was not sufficient to complete a quadrant card and more than one activity. Therefore, a make-up day was held after the instruction of every three to five words which provided additional time for the completion of the suggested activities. In addition, cumulative review was provided throughout the study.

The researcher rotated vocabulary for review to insure that all words were reviewed multiple times in order to provide students with multiple exposures to each word. To help insure fidelity to treatment, the researcher conducted seven weekly visits to observe vocabulary instruction. During these observations the researcher observed instruction but did not participate in instruction. The teachers often asked for and received feedback following observations concerning their vocabulary instruction, especially during the first weeks of the study. Weekly observations also helped the researcher adjust the scripts as the study progressed. For example, cards with examples were added to scripts after the researcher noted that the teachers struggled to provide visual examples for their students. Three observations were cancelled upon the arrival of the researcher due to school events that interfered with classroom instruction. All other observations were completed as scheduled with the teachers providing vocabulary instruction from the scripts; however, no observation included all instruction as outlined by the scripts due to lack of instructional time. Because the participating teachers were not asked to provide documentation of daily vocabulary instruction, no information is available concerning instruction on days the researcher did not observe.

Data Collection

Participating teachers gathered and maintained information on students in both the vocabulary and game classes throughout the course of the study. This information included gender, 2014 Aspire math and reading scores, pre- and post-test scores for Investigations® units five and one, and pre- and post-test scores on the vocabulary test (see Appendix J). This information was coded, put in manila envelopes, and turned in to the school office upon completion of the study. The principal designated a mail box for the researcher where the data was deposited for pick up. All data collected was identified with a number for each student and letters to identify each of the three teachers. The school principal designated the letters to represent the teachers. The researcher then determined the achievement level of each student based on 2014 Aspire Reading and Math scores. According to ACT (2014), third grade benchmark scores for Reading and Mathematics were 415 and 413 respectively. Students scoring below the benchmarks were considered to be underachieving while students scoring on or above were considered to be achieving. No student names or otherwise identifying information was supplied to the researcher. Letters were sent home with each student to notify parents of the study and to give parents the option to request their child not participate.

The unit tests for Scott Foresman® Investigations (Russell, 2004) administered in this study included three to seven items per test that addressed particular benchmarks covered by instruction in the units. Items included both procedural and conceptual items. For example, on the unit one test the first part of the first item was computational, the second asked students to provide a representation of a problem, and the third asked students to write a story problem to go with a given equation. Students were then asked to solve the problem and to show their work (see Appendix B).

The vocabulary unit tests were devised by the researcher to include multiple choice items as well as generative activities as suggested by Feldman & Kinsella (2005). Generative questions included completion activities and yes-no-why activities. Completion activities asked students to finish a sentence in order to demonstrate their understanding. An example of a completion activity would be: I know seven is a prime number because… Yes-no-why activities required students to read a sentence and write *yes* if the sentence made sense or *no* if the sentence did not make sense. The students were then asked to explain why. These generative items required students to go beyond simple memorization to demonstrate a deeper level of thinking and understanding (see Appendix C).

All unit tests and vocabulary tests were scored by the researcher. Scoring for the unit tests was based on suggestions and examples of possible student responses found in the Investigations® teacher's guides and on observed teaching approaches used by the participating teachers. These suggestions, examples, and approaches were converted by the researcher into scoring guides for each problem (see Appendix H). The resulting raw score was noted at the top of each test paper and returned to the school for recording by the participating teachers. The researcher also developed scoring guides for the vocabulary tests for all items except multiple choice items (see Appendix I). Each student's raw score was written at the top of each test before returning the tests to the teachers for recording.

Data Analysis

Data from this study was analyzed using a mixed-ANOVA (Analysis of Variance). In this study all participating students were given pre-tests and post-tests consisting of researcher-designed vocabulary tests and mathematics tests from two units of the math program used by the school district. These two data sources served as within-subjects factors. Analysis included data from all participating students having both pre- and post-test scores for units five and one and pre- and post-test vocabulary test scores for units five and one. This analysis was conducted in order to determine any significant differences in the learning outcomes of the vocabulary group and the game group as measured by the unit and vocabulary tests. In addition, mean test scores from math unit five and unit one tests were compared for students in the game group to determine if there would be a significant difference between scores for achieving and underachieving students. More specifically, scores were compared for students considered achieving and underachieving in math as well as for students considered achieving and underachieving in reading.

The researcher designed vocabulary test format was piloted by the researcher in the spring of 2014 to insure validity. The researcher provided the daily math instruction for one class of fourth grade mathematics students for five days using the fourth grade data and probability unit of Investigations®. This unit of instruction was not scheduled to be taught by the regular mathematics teachers. During instruction, the students completed log sheets and participated in rich vocabulary instruction for six words from the unit. A vocabulary test was then administered as a pilot test on the fifth day of instruction. The test included four sentence completion questions, nine multiple-choice questions, and nine Yes-No-Why questions for a total of 22 items. Completed tests were reviewed by the researcher in an effort to determine if certain types of questions were particularly difficult for the students. A discussion was also held with the students following the administration of the test. During this discussion the students reported that the sentence completion and Yes-No-Why questions were hard for them because they were not accustomed to answering those type questions. Based on the teaching and pilot test experiences, the researcher decided to utilize sentence completion questions and Yes-No-Why questions along with multiple-choice questions in the pre- and post-vocabulary tests for the study. The researcher expected that the opportunities for discussion and the inclusion of similarly formatted items within the vocabulary routines would help students become more comfortable with these formats. In addition, because the Yes-No-Why questions required so much effort from the students, it was decided to reduce the number of this type question on the pre- and post-test tests used in the study.

CHAPTER 4. RESULTS

The first three chapters of this study provided an introduction to the research problem, an overview of the purpose and significance of this study, a review of literature and research describing the connection between vocabulary, conceptual understanding of mathematics and increased math achievement, and the methods and procedures used to collect and analyze the data. This chapter reports the analysis of quantitative data collected to respond to each research question. SPSS Statistics program version 21 was used to evaluate the data gathered.

Research Question One

Will rich instruction of mathematical academic vocabulary result in greater improvement in elementary students' math achievement as assessed by unit tests from two consecutive mathematics units of study than use of non-digital games?

The dependent variable was the mean scores from two mathematics unit tests. The independent variable was the treatment, rich vocabulary instruction or game time. Both groups of students increased their math unit scores. More specifically, the vocabulary group increased their scores on math unit five from 4.14 to 5.84 and from 1.83 to 4.10 on math unit one, while the game group increased their scores on math unit five from 4.27 to 5.93 and from 1.87 to 3.65 on math unit one. The overall change for unit five and the overall change for unit one were both statistically significant (see Table 2).

Table 2

Descriptive Statistics for Unit Mathematics Pre- and Post-Tests by Group

Group	Pre-Test Unit 5 Mean (SD)	Post-Test Unit 5 Mean (SD)	Pre-Post Change Mean	Pre-Test Unit 1 Mean (SD)	Post-Test Unit 1 Mean (SD)	Pre-Post Change Mean
Vocabulary n = 63	4.14	5.84	1.70	1.83	4.10	2.27
	(2.917)	(2.377)		(1.120)	(1.965)	
Game n = 71	4.27	5.93	1.76	1.87	3.65	1.78
	(3.056)	(3.006)		(.999)	(2.306)	
Overall Sample	4.21	5.89	1.68*	1.85	3.86	2.022*

*The mean difference is significant at the .05 level.

The results of the 2 X (2) mixed ANOVA are summarized in Table 3. The overall differences between the vocabulary group and the game group were not statistically significant for unit five [F(.06), p = .810, η^2 = .000] or unit one [F(.665), p = .416, η^2 = .005]. The results of within subjects effects revealed that overall changes in both groups' mean scores were statistically significant over time for both unit five and unit one, but there was not a statistically significant interaction between group and time on either unit five [F(.007), p=.935, η^2 =.000] or unit one [F(2.335), p =.129, η^2 = .017] mean math scores.

Table 3

Test of Between and Within Subject Effects

	Mathematics Test, Unit 5				Mathematics Test, Unit 1			
	df	F	Sig.	Effect Size	df	F	Sig.	Effect Size
Between Subjects								
Group	1	.058	.810	.000	1	.665	.416	.005
Error	132				132			
Within Subjects								
Time	1	57.115	<.001	.302	1	155.738	<.001	.541
Group X Time	1	.007	.935	.000	1	2.335	.129	.017
Error	132				132			

Research Question Two

Will rich instruction of mathematical academic vocabulary result in greater improvement in elementary students' mathematics achievement on a researcher devised vocabulary tests than use of non-digital games?

The dependent variable was the mean scores from two unit vocabulary tests. The independent variable was the treatment, either rich vocabulary instruction or game time. Both groups of students increased their vocabulary scores. More specifically, the vocabulary group increased their scores on math unit five from 6.38 to 7.54 and from 5.30 to 8.44 on math unit one, while the game group increased their scores on math unit five from 6.77 to 7.39 and from 5.69 to 8.07 on math unit one. The overall change for unit five and the overall change for unit one were both statistically significant (see Table 4).

Table 4

Descriptive Statistics for Unit Vocabulary Pre- and Post-Tests by Group

Group	Pre-Test Unit 5 Mean (SD)	Post-Test Unit 5 Mean (SD)	Pre-Post Change Mean	Pre-Test Unit 1 Mean (SD)	Post-Test Unit 1 Mean (SD)	Pre-Post Change Mean
Vocabulary n = 63	6.38 (3.018)	7.54 (2.950)	1.16	5.30 (2.407)	8.44 (2.657)	3.14
Game n = 71	6.77 (3.062)	7.39 (2.930)	0.62	5.69 (2.539)	8.07 (2.582)	2.38
Overall Sample	6.59	7.46	.889*	5.51	8.25	2.74*

*The mean difference is significant at the .05 level.

The results of the 2 X (2) mixed ANOVA are summarized in Table 5. The overall differences between the vocabulary group and the game group were not statistically significant for unit five [F(.071), p = .790, η^2 =.001] or unit one [F(.000), p = .985, η^2 =.000]. The results of within subjects effects revealed that overall changes in both groups' mean scores were statistically significant over time for both unit five and unit one, but there was not a statistically significant interaction between group and time on either unit five [F(1.424), p = .235, η^2 = .011] or unit one [F(3.343), p = .070, η^2 = .025] mean math scores.

Table 5

Test of Between and Within Subject Effects

	Vocabulary Test, Unit 5				Vocabulary Test, Unit 1			
	df	F	Sig.	Effect Size	df	F	Sig.	Effect Size
Between Subjects								
Group	1	.071	.790	.001	1	.000	.985	.000
Error	132				132			
Within Subjects								
Time	1	15.500	<.001	.105	1	175.371	<.001	.571
Group X Time	1	1.424	.235	.011	1	3.343	.070	.025
Error	132				132			

Research Question Three

After receiving rich instruction of mathematical academic vocabulary, will there be a significant difference between unit mathematics mean test scores for underachieving and achieving students in math?

The dependent variable was the mean scores from two mathematics unit tests. The independent variable was the achievement level of students from the vocabulary group as determined by the Aspire Mathematics test. Students scoring below the benchmark were considered to be underachieving, while students scoring on or above the benchmark were considered to be achieving. Both groups of students increased their math unit scores (see Table 6). More specifically, the underachieving group increased their scores on math unit five from

2.41 to 4.66 and from 1.56 to 3.19 on math unit one, while the achieving group increased their scores on math unit five from 5.94 to 7.06 and from 2.10 to 5.03 on math unit one.

Table 6

Descriptive Statistics for Mathematics Pre- and Post-Tests by Group

Group	Pre-Test Unit 5 Mean (SD)	Post-Test Unit 5 Mean (SD)	Pre-Post Change Mean	Pre-Test Unit 1 Mean (SD)	Post-Test Unit 1 Mean (SD)	Pre-Post Change Mean
Underachieving	2.41	4.66	2.25	1.56	3.19	1.63
Aspire Math n = 32	(2.241)	(2.336)		(.759)	(1.533)	
Achieving Aspire	5.94	7.06	1.12	2.10	5.03	2.93
Math n = 31	(2.421)	(1.731)		(1.375)	(1.941)	
Overall Sample	4.14	5.84	1.69*	1.83	4.10	2.27*

*The mean difference is significant at the .05 level.

The results of the 2 X (2) mixed ANOVA are summarized in Table 7. The overall differences between the achieving and underachieving group were statistically significant for unit five [*F*(39.564), $p = .000$, $\eta^2 = .393$] and unit one [*F*(15.555), $p = .000$, $\eta^2 = .203$]. The results of within subjects effects (see Table 7) revealed that overall changes in both groups' mean scores were statistically significant. While there was not a statistically significant interaction between group and time on unit five [*F*(3.708), $p = .059$, $\eta^2 = .356$], there was a statistically significant interaction between group and time on unit one mean math scores [*F*(9.626), $p = .003$, $\eta^2 = .036$]. To determine if the means for the mathematics pre- and post-test for unit one for the underachieving group and the achieving group differ, a paired-samples t-test was conducted. For the underachieving group there was a significant difference in the pre-test mean scores (M = 1.56, SD = .759) and the post-test mean scores (M = 3.19, SD = 1.533); t(31) = -6.053, p = < .001. There was also a significant difference in scores for the achieving group's pre-test scores (M = 2.10, SD = 1/375) and their post-test scores (M = 5.03, SD = 1.941; t(31) = -8.958, p = < .001. When the overall change for the achieving group was compared to the overall change for underachieving group, the change for the achieving group was almost twice as large.

Table 7

Test of Between and Within Subjects Effects

	Mathematics Test, Unit 5				Mathematics Test, Unit 1			
	df	F	Sig.	Effect Size	df	F	Sig.	Effect Size
Between Subjects								
Group	1	39.564	<.001	.393	1	15.555	<.001	.203
Error	61				61			
Within Subjects								
Time	1	33.694	<.001	.356	1	116.570	<.001	.656
Group X Time	1	3.708	.059	.057	1	9.626	.003	.036
Error	61				61			

Changes over two units. To investigate whether or not there was a statistically significant difference in mean scores on the mathematics tests over the entire length of the study for the underachieving math and achieving math groups, students' pre- and post-test unit math scores were converted to z-scores. Student mean scores from the pre-test for unit five were compared to mean scores for the post-test for unit one. The underachieving group's grand mean was below average on the pre-test for unit five but above average on the post-test for unit one while the achieving group's grand mean was above average on the pre-test for unit five but below average on the post-test for unit one (see Table 8).

Table 8

Descriptive Statistics for Pre-test Unit 5 to Post-test Unit 1

	Pre-Test Math Unit 5 Mean (SD)	Post-Test Math Unit 1 Mean (SD)
Underachieving Benchmark Aspire Math n = 63	-.0221707 (.97857304)	.1099324 (-.0975457)
Achieving Benchmark Aspire Math n = 71	.096726 (1.02518315)	-.0975457 (1.06945280)

The results of the 2 x (2) mixed ANOVA are summarized in Table 9. The overall differences between the underachieving and the achieving groups were not statistically significant [F(3.17, p = .574, η^2 = .002]. The group by time interaction effect was not significant [F(1.852), p = .176, η^2 = .014].

Table 9

Test of Between and Within Subjects Effects

	df	F	Sig.	Effect Size
Between Subjects				
Group	1	3.17	.574	.002
Error	132			
Within Subjects				
Time	1	.007	.935	<.001
Group X time	1	1.852	.176	.014
Error	132			

Research Question Four

After receiving rich instruction of mathematical academic vocabulary, will there be a significant difference between unit mathematics mean test scores for underachieving and achieving students in reading?

The dependent variable was the mean scores from two mathematics unit tests. The independent variable was the achievement level of students as determined by the Aspire Reading test. Students scoring below the benchmark were considered to be underachieving, while students scoring on or above the benchmark were considered to be achieving. Both groups of students increased their math unit scores. More specifically, the underachieving group increased their scores on math unit five from 3.34 to 5.12 and from 1.61 to 3.68 on math unit one, while the achieving group increased their scores on math unit five from 5.64 to 7.18 and from 2.23 to

4.86 on math unit one. The overall change for the achieving in reading students and the underachieving in reading students were both statistically significant (see Table 10).

Table 10

Descriptive Statistics for Mathematics Pre- and Post-Tests by Group

Group	Pre-Test Unit 5 Mean (SD)	Post-Test Unit 5 Mean (SD)	Pre-Post Mean Change	Pre-Test Unit 1 Mean (SD)	Post-Test Unit 1 Mean (SD)	Pre- Post Mean Change
Underachieving Aspire Reading n = 41	3.34 (2.834)	5.12 (2.379)	1.78	1.61 (.919)	3.68 (1.890)	2.07
Achieving Aspire Reading n = 22	5.64 (2.498)	7.18 (1.736)	1.54	2.23 (1.378)	4.86 (1.910)	2.63
Overall Sample	4.14	5.84	1.663*	1.83	4.10	2.355*

*The mean difference is significant at the .05 level.

The results of the 2 X (2) mixed ANOVA are summarized in Table 11. The overall differences between the underachieving group and the achieving group were statistically significant for unit five [F(14.535), p = .000, η^2 = .192] and unit one [F(7.199), p = .009, η^2 = .106]. The results of within subjects effects revealed that overall changes in both groups' mean scores were statistically significant for both unit five and unit one tests. There was not a statistically significant interaction between group and time on unit five [F(.140), p = .710, η^2 = .002] or on unit one [F(1.429), p = .237, η^2 = .023] mean math scores.

Table 11

Test of Between and Within Subject Effects

	Mathematics Test, Unit 5				Mathematics Test, Unit 1			
	df	F	Sig.	Effect Size	df	F	Sig.	Effect Size
Between Subjects								
Group	1	14.535	<.001	.192	1	7.199	.009	.106
Error	61				61			
Within Subjects								
Time	1	28.045	<.001	.315	1	99.917	<.001	.621
Group X Time	1	.140	.710	.002	1	1.429	.237	.023
Error	61				61			

Changes over two units. To investigate whether or not there was a statistically significant difference in mean scores on the mathematics tests over the entire length of the study for the underachieving reading and the achieving reading groups, students' pre- and post-test math scores were converted to z-scores. Student mean scores from the pre-test for unit five were then compared to their mean scores for the post-test for unit one. The underachieving group's grand mean was below average on the pre-test for unit five but above average on the post-test for unit one while the achieving group's grand mean was above average on the pre-test for unit five but below average on the post-test for unit one (see Table 12). The change by group over time was not significant [$F(2.882)$, $p = .092$, $\eta^2 = .021$] (see Table 13).

Table 12

Descriptive Statistics for Pre-test Unit 5 to Post-test Unit 1

	Pre-Test Math Unit 5 Mean (SD)	Post-Test Math Unit 1 Mean (SD)
Underachieving Aspire Reading n = 63	-.0686941 (.99398247)	.0758097 (1.01623686)
Achieving Aspire Reading n = 71	.0609539 (1.00841352)	-.0672678 (.98768810)

Table 13

Test of Between and Within Subjects Effects

	df	F	Sig.	Effect Size
Between Subjects				
Group	1	.002	.965	<.001
Error	132			
Within Subjects				
Time	1	.010	.919	<.001
Group X time	1	2.882	.092	.021
Error	132			

Summary

The purpose of research questions one and two was to examine math mean score differences and vocabulary mean score differences for students receiving the enrichment of rich vocabulary instruction or game time. Research question one focused on differences between mean scores for unit math tests while research question two focused on differences between mean scores for unit vocabulary tests. Data analysis suggested that overall gains made by both the vocabulary and the game groups for both the unit tests and vocabulary tests for units five and one were statistically significant, but the difference in the two group means was not statistically significant. These findings suggest that the rich vocabulary treatment was not more effective than game playing for increasing either unit five or unit one mean math scores or unit five or unit one mean vocabulary scores.

The purpose of research questions three and four was to examine math mean score differences for students who received the rich vocabulary instruction and were categorized as achieving or underachieving by either the Aspire math or Aspire reading test. Data indicated that overall gains made by both the achieving and underachieving students in math and achieving and underachieving students in reading were significant. However, when unit means over time for the achieving in math were compared to unit means over time for the underachieving in math, and unit means over time for the achieving in reading were compared to unit means over time for the underachieving in reading, the only difference that was statistically significant was for the achieving and underachieving math students on the second unit of study, unit one. A follow-up t-test and a comparison of overall change for each group, achieving in math and underachieving in math, indicated that the achieving group experienced the greater increase.

Two one-way ANOVAs were conducted to investigate whether or not there were significant changes in math mean scores over the entire length of the study between the achieving and underachieving students in both math and reading. Student mean scores from the pre-test for unit five math were compared to mean scores for the post-test for unit one math. In addition, mean scores from the unit five pre-test for vocabulary were compared to mean scores for the post-test for unit one vocabulary. No significant changes by group were reported.

CHAPTER V. CONCLUSIONS AND IMPLICATIONS

Introduction

This research study examined the effect of the enrichments of rich instruction of mathematics academic vocabulary and the playing of non-digital mathematics games on students' mathematics achievement on unit tests and unit vocabulary tests. Students were pre- and post-tested using unit mathematics tests for units one and five as well as with pre- and post-tests for the vocabulary from instruction for these units. It was hypothesized that the mean math unit test scores and the mean vocabulary test scores of the students who received rich vocabulary instruction would be significantly greater from the mean test scores of the students who played mathematical games. In addition, the students who received rich vocabulary instruction were designated as achieving or underachieving according to the students' 2013 Aspire scores for mathematics and for reading in order to investigate whether or not the vocabulary instruction might affect the achievement gap between these two groups.

The results of this study contribute to the understanding of the association between conceptual and procedural understanding of mathematics and the potential role of academic vocabulary and the use of non-digital games on student achievement in mathematics. This chapter presents an overview of this experimental study and a summary of the important conclusions drawn from the analysis of data. Also provided is a discussion of the implications for action and recommendations for further research.

Research Question 1

Research question one examined the relationship between rich vocabulary instruction, game time, and increased achievement on unit mathematics tests. The results of this study did not detect a significant difference between student mean math test scores for the vocabulary and game groups over time. Therefore, it appears that neither the vocabulary instruction nor the opportunity for students to play non-digital mathematics games contributed more than the other to the students' increased unit test mean scores. However, overall mean scores for the unit tests did increase significantly for both the vocabulary group and the game group. While both rich vocabulary instruction (Gifford & Gore, 2010) and playing non-digital games (Bragg, 2012; Bright, Harvey, & Wheeler, 1985; Koran & McLaughlin, 1990; Randel, Morris, Wetzel, & Whitehill, 1992) especially with quality discourse (Hufferd-Ackles, Fuson, & Sherin, 2004) have been found to be effective for increasing students' math achievement, and while both these strategies may have potentially affected student scores in this study, it is more likely that the use of the Scott Foresman Investigations® program contributed to the significant increases for both the vocabulary and game groups. The Investigations® program, funded by the National Science Foundation (Russell et al., 2012), is considered a standards-based mathematics curriculum. Many research studies have been conducted to determine the effectiveness of standards-based math programs including that of Riodan and Noyce (2001). These researchers used matched groups to compare math achievement of sixth through eighth-grade students' in 107 schools using one of two standards-based programs to students using traditional math programs. The researchers reported that students using the standards-based programs significantly outscored students from traditional programs, and that the longer students used standards-based programs, the greater student scores increased. According to these researchers, standards-based programs were successful due to their emphasis on problem solving, reasoning, making connections between math concepts, communicating math ideas, and providing opportunities for all students to learn.

Research Question 2

Research question two examined the relationship between rich vocabulary instruction, game time, and increased achievement on unit vocabulary tests. Analysis of data indicated that there was not a significant difference between student mean unit vocabulary test scores for the vocabulary and game groups over time. Therefore, it is likely that neither the vocabulary instruction nor the opportunity for students to play non-digital mathematics games contributed greatly to the students' increased vocabulary test mean scores. However, the overall mean scores for the unit vocabulary tests increased significantly for both the vocabulary and the game group.

Investigations® and Vocabulary Development

Just as for research question one, it seems that the significant increases for both the vocabulary and game groups were likely due to the nature of the standards-based Investigations® program. This program does not include specific vocabulary activities but integrates vocabulary with instruction. For example, in the fourth grade teacher's edition for unit five, Landmarks and Large Numbers, teachers are given lists of focus points suggested for incorporation into instruction for the unit. Examples of these points are developing arguments about why two addition expressions are equivalent and using story contexts and representations to support explanations about related subtraction expressions (Russell et al., 2004, pg. 11). The points provided for this unit include a focus on six of the 14 words used for rich vocabulary instruction for this unit. In addition, throughout the teacher's edition, suggestions are made for teacher-led discussions that include specific questions for students using highlighted math vocabulary.

Therefore, it is likely that the integrated vocabulary instruction found in the Investigations® program and used by the participating teachers during regular math instruction of all students was sufficient to result in significant increases in overall vocabulary test mean scores for both the vocabulary and the game groups.

Effective Vocabulary Instruction

The lack of fidelity of all of the teachers providing the rich vocabulary instruction is one possible reason why the vocabulary instruction did not result in greater gains than playing games. Two issues come into play when teachers are asked to incorporate activities for effective mathematics instruction (Ball et al., 2005), including effective rich vocabulary instruction. The first deals with whether or not the teachers have the math understanding necessary to enable them to provide appropriate mathematical instruction. The second deals with how teachers learn to incorporate questions, activities, and discussions into their instruction. Weekly observations of vocabulary instruction delivered for this study indicated that all three participating teachers had a good mathematical background procedurally. However, during training for this study, two of the teachers appeared to be uncomfortable with their knowledge of the relationships between some of the vocabulary words selected for instruction. Discomfort continued as they worked to incorporate questions and discussions without relying entirely on the vocabulary routine scripts. Although their discomfort decreased, these teachers never appeared comfortable to the point of modifying instruction in response to student needs. In contrast, the third teacher was especially adept at understanding the relationships between the vocabulary words selected for instruction. This was apparent by the questions she asked and her ability to deliver the vocabulary routines without following them word-for-word. In addition, this teacher was able to use her students' responses to lead her to additional questions so that learning was built in a series of steps. The instruction provided by this third teacher was an example of the type of instruction that helps students develop conceptual understanding of mathematics (Ball et al., 2005).

The Importance of Prior Knowledge

The lack of prior mathematical knowledge of the students in both the vocabulary and game groups is a second reason why the vocabulary instruction likely resulted in less than expected gains on the vocabulary tests. An indication of students' overall levels of prior knowledge may be their relatively low pre-test means compared to the total scores possible for each test (see Table 14).

Table 14

Vocabulary Pre-Test Means and Changes Compared to Maximum Possible

Group	Pre-Test Unit 5 Maximum 15	Mean Change	Pre-Test Unit 1 Maximum 12	Mean Change
Vocabulary	6.38	1.16	5.30	3.14
Game	6.77	0.62	5.69	2.38
Overall Sample	6.59	.889*	5.51	2.74*

*The mean difference is significant at the .05 level.

To be successful in math, students must have adequate prior knowledge and conceptual understanding in order to access concepts from various perspectives. They must also be able to practice core functions, such as addition and subtraction, with speed and accuracy. And last, students must be able to apply their math knowledge, an ability dependent upon their level of conceptual understanding (Star, 2005). According to Carpenter, Fennema and Franke (1996) students construct mathematical meaning by relating new concepts to what they already understand. Therefore, instruction should begin on the students' level. If the level of the participating students was basically procedural, similar to the knowledge base of U.S. mathematics students revealed by Hiebert et al. (2005), it is possible that they were restricted to that knowledge as they attempted to integrate learning from rich vocabulary instruction. Learning conceptually takes time for students to relate new learning to what they already know. Therefore, it is likely that the vocabulary treatment provided as part of regular daily math instruction was not more effective than games because the length of the treatment was not sufficient for students to make needed connections.

Vocabulary Test Format

The format of the vocabulary test is a third reason why the vocabulary gains might not have been greater. After the study was completed, several of the students described the sentence completion and Yes-No-Why items as being "hard" for them to answer. These test items required the students to either elaborate or compare math concept vocabulary and to justify their reasoning. When asked to explain why they felt these items were difficult, they said they had trouble writing explanations of their thinking. In 1988, Sweller developed the cognitive load theory which states that all instruction has an inherent difficulty associated with it and when this difficulty involves great effort, it is possible for a student's cognitive load to exceed his or her capacity to process the information in their working memories. This increased effort, or increased cognitive load, is strongly influenced by the number of elements in the student's working memory interacting with each other, especially if several of the elements are interacting simultaneously. Sweller (1988) described three types of cognitive load, germane, extraneous, and intrinsic. Germane cognitive load results from students constructing and using new background information. If cognitive load is generated by the way information is presented, known as extraneous cognitive load, learning may be negatively affected. An example of an activity that might increase extraneous cognitive load would be teachers describing concepts without providing examples or representations for the concepts. The examples and representations would be preferable because the students would not have to deal with extraneous, unnecessary information in order to build knowledge. The third type of cognitive load, intrinsic, is associated with the effort imposed by a learning task. When intrinsic cognitive load becomes too great for students, teachers should teach chunks of information in isolation before bringing the parts back together as a combined whole. To combat increasing cognitive load of students, teachers should focus on helping students build background knowledge and require the students to successfully apply their knowledge repeatedly. Therefore, it is possible that the demands of the vocabulary test items coupled with the relatively low level of the vocabulary and game group students' prior knowledge increased their cognitive load resulting in neither group performing as well as expected.

Novelty of Instruction

A final reason why the vocabulary instruction might not have had a differential effect was the novelty of the instruction coupled with the short duration of the study. The J-curve theory (Moran, 2011) postulates that learners go through stages when learning new educational routines. The learners' performance in the initial stage of learning is represented by a plateau with no disruption. However, the new routine soon produces anxiousness and performance dips. With proper instructional practice over time, the learners master the routine and performance increases. The amount of time required for mastery varies depending on the students' abilities and disabilities (Steedly, Dragoo, Arefeh, & Luck, 2008). Had the study been longer, the students would have had more time to become comfortable with the routines used in the rich vocabulary instruction, and perhaps their mean scores would have been significant.

Research Question 3

Research question three investigated the effects of rich vocabulary instruction on students considered to be achieving and underachieving in mathematics. Analysis of data indicated that overall both achieving and underachieving students in mathematics who received rich vocabulary instruction experienced significant changes in unit test scores. The only statistically significant results for group by time were the mean test scores for both the achieving in math students and the underachieving in math students for unit one, with the achieving students experiencing almost twice the increase as the underachieving students.

There are a couple of possible explanations for why a difference between groups was found in unit one and not unit five. First, because instruction for unit one followed that for unit five, both the teachers and students had been involved in rich vocabulary instruction for four weeks and had time to increase their comfort levels. It is also possible that the significant results for unit one support the iterative view of the relationship between conceptual and procedural understanding (Rittle-Johnson et al., 2001). This view contends that conceptual knowledge and procedural knowledge are intertwined and increase gradually in relation to the other over time. The iterative view may be congruent with the Matthew Effect. Walberg and Tsai (1983) and Stanovich (1986) used the term Matthew Effect to describe how students who start well in reading tend to continue to do well, while students who do not start well in reading are unlikely to catch up. This gap develops because relatively small differences in reading in the early grades develop into large differences in achievement in the later grades. In 2007, Bahr (2007) declared that the Matthew Effect is not only applicable to the acquisition of reading skills, but also to skills in general, including math.

If both the underachieving and achieving students in math had sufficient conceptual and procedural knowledge to promote an iterative increase in both areas over the time of this study, it is possible that this increase contributed to the significant difference in their unit one pre- and post-test mean scores. More importantly, when the mean change of 1.63 for the achieving students was compared to the mean change of 2.93 for the underachieving students, it appears that the rich vocabulary instruction had the greater impact on the achieving students. This result is contrary to results of some research studies such as that conducted by Kulm et al. (2007) and Clark et al. (2011) where underachieving students experienced more significant gains than students considered to be achieving. However, the results of this study were similar to research conducted by Baxter, Woodard, and Olson (2001) who found that many activities requiring students to articulate their math thinking and verify their ideas presented verbal challenges to low achieving students. These researchers reported that low achieving students were often minimally involved in lessons, rarely speaking and often distracted.

Research Question 4

Research question four investigated the effects of rich vocabulary instruction on students considered to be achieving and underachieving in reading. Analysis of data indicated that overall both achieving and underachieving students in reading who received rich vocabulary instruction experienced significant changes in unit math test scores (see Table 15).

With a total possible score of nine on both the unit five test and the unit one test, both the underachieving and achieving students began with low to moderate pre-test means, and while their changes were significant, there was not a statistically significant interaction between group and time on unit five or unit one mean math scores. Meaning, the effect of the rich vocabulary instruction on mathematics mean test scores did not differ for the two groups, achieving in reading and underachieving in reading.

Table 15

Math Unit Pre-Test Means and Changes Compared to Maximum Possible

Group	Pre-Test Unit 5 Mean Possible 9	Mean Change	Pre-Test Unit 1 Mean Possible 9	Mean Change
Underachieving Reading	3.34	1.78	1.61	2.07
Achieving Reading	5.64	1.54	2.23	2.63
Overall Sample	4.14	1.66*	1.83	2.36*

*The mean difference is significant at the .05 level.

Results for this research question must be compared to results for the third research question. For research question 3, a difference was found between the achieving and underachieving in math students with results indicating that the rich vocabulary treatment was more effective for the achieving in math students. However, when the same group of students was divided into subgroups based on their reading achievement, results indicated that the rich vocabulary instruction did not have a differential effect for the achieving or underachieving students in reading. The question then must focus on why a vocabulary treatment would be more beneficial for a group of students when the students are designated by mathematical ability than by reading ability. Relatively little research has focused on the relationship between reading ability, as measured on standardized reading assessments, and level of math achievement, as measured on standardized tests (Larwin, 2010). According to the U.S. Department of Education (1996), most standardized math assessments use questions presented in a word problem format. Therefore, reading ability would be expected to affect students' math achievement. However, on the unit five and unit one Investigations® tests, only two out of eight questions were story problems. Therefore, students' reading ability, whether achieving in reading or underachieving in reading, would only come into play for 25% of the test items. Ultimately, it appears that students' prior math achievement had more bearing on the unit math achievement than prior reading achievement.

Implications

This study provided a few important implications for teachers and researchers studying the use of enrichments for increasing mathematics achievement such as rich academic vocabulary instruction and non-digital games. First, it seems that the vocabulary instruction embedded in the Investigations math program was more effective for increasing student achievement on unit math tests and unit vocabulary tests than either rich vocabulary instruction or use of non-digital math games. According to Pearson Education, Inc. (2012), the Investigations program was intentionally designed to encourage teachers to constantly ask students what they know about math concepts. Teachers are also encouraged to provide opportunities for students to discuss, represent, and justify their generalizations about math. In so doing, students and teachers use the language of math as they express their ideas in words, with variables, and with representations. In addition, the program often has students record their thinking and use of strategies so that all students can analyze, view, and use their classmates' ideas. Through these approaches, important math vocabulary is used throughout instruction by verbally saying the words and physically representing them, giving students multiple exposures to the words as they interact with them in different mathematical contexts.

The second implication of this study has to do with the importance of student prior knowledge. Results indicated that rich vocabulary instruction was more effective for increasing math achievement for students achieving in math than for students underachieving in math. Therefore, this study may support the belief that students with high levels of mathematical prior knowledge are more able to learn new mathematical information. Marzano (2004) stated that what students already know about a subject or content is one of the strongest indicators of how well they will learn new information relative to the content. He cautioned that because levels of prior knowledge can create advantages for some students and disadvantages for other students, providing opportunities for students to build their prior knowledge should be a goal of any program designed to increase student achievement.

A third implication of this study was that rich vocabulary instruction benefitted all students despite their level of reading achievement. In this study, both the students who were considered achievers in reading and those considered underachievers in reading made significant gains on the unit math tests. This finding is similar to research conducted by Snow, Lawrence, and White (2009) and Puhalla (2011) that reported that rich vocabulary instruction might reduce or eliminate the vocabulary gap between students at risk and their average-achieving peers.

Limitations of Study

There were limitations of the current study. First, despite all three participating teachers' experience teaching mathematics, the group had little prior knowledge of vocabulary instruction. This limitation was apparent from the first training session when all three teachers expressed some concern about providing the vocabulary routines. Two of the three teachers struggled initially to provide the routines as evidenced by their reliance on the scripts during instruction. While their instruction did improve over the course of the study, valuable instructional time was lost by the teachers continually referring to their notes. During the length of the study only one of the three teachers was observed providing rich vocabulary instruction independent of the scripts. Although the routines were vocabulary based, the types of activities included in the routines represented the types of questions, activities, and discussions suggested for instruction designed to help students develop their conceptual understanding of mathematics (Hiebert et al., 1993; Wood, 1999).

The complexity of the scripts provided for the participating teachers was an additional limitation of this study. Scripts were lengthy due to the inclusion of background information for the teachers, the number of words included in the study, examples provided for use with the students, and suggestions for activities to be used during instruction. Although instructional time was provided in the schedule for teachers to cover activities not addressed during class due to lack of time, the amount of information included in the scripts exceeded the amount of time allotted for rich vocabulary instruction.

A third limitation of this study was the format and scoring of the vocabulary tests. These tests included multiple choice, sentence completion, and yes-no-why test items in an attempt to include questions that required students to demonstrate understanding (Feldman & Kinsella, 2005). However, the items may have been more appropriate for assessing definitional knowledge rather than ownership, defined by Beck et al. (2002) as knowing how to use a word correctly and understanding it when it is heard or seen in various contexts. According to Beck et al. (2002) when students have definitional knowledge of a word rather than ownership, they are unable to use their knowledge of the word to bring meaning to a context. In addition, for this study the sentence completion and yes-no-why test items were scored as either acceptable or unacceptable (see Appendix I). Because there were multiple acceptable responses for these questions, it might have been advantageous to have scored the items using a rubric. Brookhart (2013) suggests using acceptable or unacceptable when items have one clear, correct answer and using rubrics with scoring item responses that have degrees of quality. Had rubrics been used for grading the sentence completion and yes-no-why items, it would have been possible to award credit in relation to students' levels of understanding which might have increased student raw scores.

A fourth limitation concerned study fidelity. The researcher completed seven weekly visits to one class of each of the three participating teachers to observe rich vocabulary instruction. However, no additional documentation was provided by the teachers concerning what activities were taught or omitted from instruction for the classes not observed by the researcher. Therefore, it is possible that the fidelity of the treatment on these non-observed days was lacking.

Further Research

There are several possible avenues for further research when considering remaining questions and limitations of this research. The first deals with the training of the participating teachers. For further research, either teachers with some working knowledge of rich vocabulary instruction would be recruited for study participation or else teacher training would be more extensive. It would be preferable if teachers could be recruited that already had an understanding of the need for increased conceptual understanding of students and the possible effects of rich vocabulary instruction on conceptual knowledge. For these teachers, training time would be reduced to time spent helping the teachers learn to use activities commonly found in rich vocabulary instructional routines. However, for teachers without that prior knowledge, it would be advantageous to supply sufficient training for them to learn the importance of conceptual understanding and the value of rich vocabulary instruction in addition to learning to supply quality activities. Training for these teachers would be provided ideally during the school year prior to the scheduled study. This would allow time for the teachers to attend multiple training sessions while having the opportunity to hone their skills and build their comfort levels with their current students. The multiple training sessions would also be designed to include time for teachers to discuss their successes and struggles with rich vocabulary instruction among themselves and the researcher in a professional learning community (Darling-Hammond & Richardson, 2009).

An additional option would entail recruiting teachers who use a commercially prepared mathematics program such as Go Math! (Dixon, Larson, Leiva & Adams, 2011) rather than a program developed with funding from the National Science Foundation and favored by the National Council of Teachers of Mathematics (Garelick, 2005). Commercially prepared mathematics programs often include specific suggestions for developing vocabulary. For example, the Go Math! program supplies teachers with lists of vocabulary by chapter, highlights each vocabulary word in the student and teacher editions, and incorporates activities for each chapter of instruction specifically designed to develop students' math language. Teachers accustomed to having access to vocabulary support for their mathematics instruction might have a greater knowledge of the potential benefits of learning academic vocabulary for mathematics, as well as knowledge of the types of vocabulary activities that can be utilized in instruction, and be more comfortable with delivering the instruction.

A second area of change for further research would involve refinement of the scripts used for rich vocabulary instruction. This refinement would include reducing the number of words for instruction as well as reducing the number of activities provided for each instructional period. It is possible that the teachers in this study did not have enough time to complete all of the suggested activities, even with extra time provided through the inclusion of scheduled review days. If that was the case, it is also possible that an imbalance of material and time affected the quality of instruction as a result of either rushing through instruction or skipping activities.

And finally, further research could be conducted with a group of students with overall higher levels of reading and math proficiency. According to the benchmarks for the Aspire (2014) achievement test, approximately 50% of the students in this study were below benchmark for mathematics and 64% were below benchmark for reading. Since it is possible that the effects of the rich vocabulary and game treatments were affected by the low levels of prior vocabulary knowledge of the participating students, it would be interesting to explore the effects on a population with overall higher levels of achievement.

REFERENCES

ACT, Inc. (2014). Understanding your ACT Aspire score. Retrieved from http://www.discoveractaspire.org/pdf/2014_actaspire_UnderstandingAspireResults.pdf

Armbruster, B. B., Lehr, F., & Osborn, J., (2001). *Put reading first: The research building blocks for teaching children to read.* Washington, DC: U.S. Government Printing Office. Retrieved from https://www.nichd.nih.gov/publications/Pages/pubs_details.aspx?pubs_id=226

August, D., Branum-Martin, L., Cardenas-Hagan, E., & Francis, D. J. (2009). The impact of an instructional intervention on the science and language learning of middle grade English language learners. *Journal of Research on Educational Differences, 2*(4), 345–376.

August, D., & Shanahan, T. (2006). *Executive summary, Developing literacy in second-language learners:* Report of the national literacy panel on language-minority children and youth. Mahwah, NJ: Lawrence Erlbaum Associates.

Bahr, P. R. (2007). Double jeopardy: Testing the effects of multiple basic skill deficiencies on successful remediation. *Research in Higher Education*, *4*(6), 695–725.

Baker, S. K., Simmons, D. C., & Kame'enui, E. J. (1995). *Vocabulary acquisition: Synthesis of the research.* (Tech. Rep. No. 13). Eugene, OR: University of Oregon, National Center to Improve the Tools of Educators.

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade and how can we decide? *American Educator, 29*(1), 14–17, 20–22, 43–46.

Baumann, J. F., & Graves, M. F. (2010). What is academic vocabulary? *Journal of Adolescent & Adult Literacy, 54*(1), 4–12.

Baumann, J. F., Ware, D., & Edwards, E. C. (2007). Bumping into spicy, tasty words that catch your tongue: A formative experiment on vocabulary instruction. *The Reading Teacher, 61*(2), 108–122.

Baxter, J. A., Woodward, J., & Olson, D. (2000). Effects of reform-based mathematics instruction on low achievers in five third-grade classrooms. *The Elementary School Journal, 101*(5), 529–547.

Bay-Williams, J., & Livers, S. (2009). Supporting math vocabulary acquisition. *Teaching Children Mathematics, 16*(4), 238–245.

Beck, I. L., & McKeown, M. G. (2007). Increasing young low-income children's oral vocabulary repertoires through rich and focused instruction. *The Elementary School Journal, 107*(3), 251–271.

Beck, I. L., McKeown, M. G., & Kucan, L. (2002). *Bringing words to life.* New York: Guilford.

Beck, I. L., McKeown, M. G., & Omanson, R.C. (1987). The effects and uses of diverse vocabulary instructional techniques. In M.G. McKeown & M.E. Curtis (Eds.), *The nature of vocabulary acquisition*. Hillsdale, NJ: Erlbaum.

Beck, I. L., Perfetti, C., & McKeown, M. G. (1982). The effects of long-term vocabulary instruction on lexical access and reading comprehension. *Journal of Educational Psychology, 74*(4), 506-521.doi:10.1037/0022-0663.74.4.506

Becker, W. C. (1977). Teaching reading and language to the disadvantaged: What we have learned from research. *Harvard Educational Review, 47*(4), 31–52.

Benjamin, A. (2007). *But I'm not a reading teacher: Strategies for literacy instruction in the content Areas.* Larchmont, NY: Eye on Education, Inc.

Benjamin, A. (2011). *Math in plain English: Literacy strategies for the mathematics classroom.* Larchmont, NY: Eye on Education, Inc.

Berthold, K., & Renkl, A. (2009). Instructional aids to support a conceptual understanding of multiple representations. *Journal of Educational Psychology, 111*(1), 70–87.

Biemiller, A. (2003). Vocabulary: Needed if more children are to read well. *Reading Psychology, 24*, 323–335.

Beimiller, A., & Boote, C. (2006). An effective method for building meaning vocabulary in primary grades. *Journal of Educational Psychology, 98*(1), 44–62.

Biemiller, A., & Slonim, N. (2001). Estimating root word vocabulary growth in normative and advantaged populations: Evidence for a common sequence of vocabulary acquisition. *Journal of Educational Psychology, 93*(3), 498–520.

Bintz, W. P. (2011). Teaching vocabulary across the curriculum. *Middle School Journal, 42*(4), 44–53.

Blachowicz, C. L. Z., & Fisher, P. (2000). Vocabulary instruction. In M. I. Kamil, P. B. Mosenthal, P. D. Pearson, & R. Barr (Eds.), *Handbook of reading research* (Vol. 3, pp. 503–523). Mahwah, NJ: Erlbaum.

Blachowicz, C. L. Z., & Fisher, P. (2006). *Teaching vocabulary in all classrooms* (2nd ed.). Columbus, OH: Prentice-Hall.

Blachowicz, C. L. Z., & Fisher, P. (2011). What students need to learn: A word for the words. *Educational Leadership, 68*(6). Retrieved from http://www.ascd.org/publications/educational-leadership/mar11/vol68/num06/A-Word-for-the-Words.aspx

Blachowicz, C. L. Z., Fisher, P. J. L., Ogle, D., & Watts-Taffe, S. (2006). Vocabulary: Questions from the classroom. *Reading Research Quarterly, 41*(4), 524–539.

Blachowicz, C. L. Z., Watts-Taffe, S., & Fisher, P. (2005). *Integrated vocabulary instruction: Meeting the needs of diverse learners in grades 1-5.* Naperville, IL: Learning Point Associates.

Borasi, R., Siegel, M., Fonzi, J., & Smith, C. F. (1998). Using transactional reading strategies to support sense-making and discussion in mathematics classrooms: An exploratory study. *Journal for Research in Mathematics Education*, *29*(3), 275-305.

Bragg, L. A. (2012). Testing the effectiveness of mathematical games as a pedagogical tool for children's learning. *International Journal of Science and Mathematics Education, 10*(6), 1445–1467.

Bright, G. W., Harvey, J. G., & Wheeler, M. M. (1985). Learning and mathematics games. *Journal for Research in Mathematics Education, Monograph* 1, 1–189.

Brookhart, S. M., (2013). *How to create and use rubric for formative assessment and grading*. Alexandria, VA: Association for Supervision and Curriculum Development.

Brown, B. A., Ryoo, K., & Rodriguez, J. (2010). Pathway towards fluency: Using 'disaggregate instruction' to promote science literacy. *International Journal of Science Education, 32*(11), 1465–1493.

Butler, S., Urrutia, K., Buenger, A., Gonzalez, N., Hunt, M., & Eisenhart, C. (2010). A review of the current research on vocabulary instruction. National Reading Technical Assistance Center. Retrieved from http://www2.ed.gov/programs/readingfirst/support/rmcfinal1.pdf

Byrnes, J. P., & Wasik, B. A. (1991). Role of conceptual knowledge in mathematical procedural learning. *Developmental Psychology, 27*(5), 777–786.

Campbell, D.T., & Stanley, J.C. (1963). *Experimental and quasi-experimental designs for research*. Chicago: Rand-McNally.

Carlo, M. S., August, D., McLaughlin, B., Snow, C., Dressler, C., Lippman, D. N., Lively, T. J., & White, C. E. (2004). Closing the gap: Addressing the vocabulary needs of English-language learners in bilingual and mainstream classrooms. *Reading Research Quarterly, 39*(2), 188–215.

Carpenter, T. P., Fennema, E., & Franke, M. (1996). Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal, 97*(1), 3–20.

Carpenter, T. P., Franke, M. L., Jacobs, V. R., Fennema, E., & Empson, S. B. (1998). A longitudinal study of invention and understanding in children's multidigit addition and subtraction. *Journal for Research in Mathematics Education, 29*(1), 3–20.

Carpenter, T. P., & Lehrer, R. (1999). Teaching and learning mathematics with understanding. In E. Fennema & T. A. Romberg (Eds.), *Classrooms that promote mathematical understanding* (pp. 19-32). Mahwah, NJ: Erlbaum.

Center for Elementary Mathematics and Science Education. (n.d.). Everyday mathematics. Retrieved from http://everydaymath.uchicago.edu/about/understanding-em/games/name-that-number.html

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn*. Sausalito, CA: Scholastic, Math Solutions.

Clark, B., Smolkowski, K., Baker, S. K., Fien, H., Doabler, C. T., & Chard, D. J. (2011).The impact of a comprehensive tier I core kindergarten program on the achievement of students at risk in mathematics. *The Elementary School Journal*, *111*(4), 561–584.

Cobb, P. (1988). The tension between theories of learning and instruction in mathematics education. *Educational Psychologist, 23*(2), 87–103.

Cohen, M., & Johnson, H. (2011). Improving acquisition of novel vocabulary through the use of imagery interventions. *Early Childhood Education Journal*, *38*(5), 357–366.

Collins, M. F., (2005). ESL preschools' English vocabulary acquisition from storybook reading. *Reading Research Quarterly, 40*(4), 406-408.

Council of Australian Governments, (2008). *National Numeracy Review Report.* Canberra: Commonwealth of Australia. Retrieved from http://www.coag.gov.au/sites/default/files/national_numeracy_review.pdf

Cowan, R., Donlan, C., Shepherd, D.L., Cole-Fletcher, R., Saxton, M., & Hurry, J. (2011). Basic calculation proficiency and mathematics achievement in elementary school children. *Journal of Educational Psychology, 103,* 786–803.

Coyne, M. D., Simmons, D. C., & Kame'enui, E. J. (2004). Vocabulary instruction for young children at risk of experiencing reading difficulties: Teaching word meanings during shared storybook reading. In J. F. Baumann & E. J. Kame'enui (Eds.), *Vocabulary instruction: Research to practice* (pp. 41–58). New York: Guilford.

Cronback , L. J. **(**1942). An analysis of techniques for diagnostic vocabulary testing. *Journal of Educational Research, 36*, 206–217.

Dale, E. (1965).Vocabulary measurement: Techniques and major findings. *Elementary English, 42*, 82–88.

Darling-Hammond, L., & Richardson, N. (2009). Research Review/Teachers Learning: What Matters? *How Teachers Learn, 66*(5), 46–53.

Dickenson, D., & Smith, M. (1994). Long-term effects of preschool teachers' book readings on low-income children's vocabulary and story comprehension. *Reading Research Quarterly*, *29*(2), 104–122.

Divjak, B., & Tomic, D. (2011). The impact of game-playing on the achievement of learning goals and motivation for learning mathematics: Literature review. *The Journal of International Organizations Studies, 35*(1), 15–30.

Dixon, J. K., Larson, M., Leiva, M. A., & Adams, T. L. (2011). *Go math!* Houghton Mifflin Harcourt, Orlando, FL.

Feldman, K., & Kinsella, K. (2005). Narrowing the language gap: The case for explicit vocabulary instruction. New York, NY: Scholastic.

Fennema, C., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education*, *27*(4), 403-434.

Fisher, D., & Frey, N. (2008). Word wise and content rich: Five essential steps to teaching academic vocabulary. Portsmouth, NH: Heinemann.

Garick, B. (2005). The new amazing approach to math: A mathematician with a child learns some politics. *Education Next, 5*(2), 29–36.

Gifford, M., & Gore, S. (2008). The effects of focused academic vocabulary instruction on underperforming math students. ASCD Report. Retrieved from https://www.ascd.org/

ASCD/pdf/Building%20Academic%20Vocabulary/academic_vocabulary_math_white_paper_web.pdf

Gooding, A., & Stacey, K. (1993). Characteristics of small group discussion reducing misconceptions. *Mathematics Education Research Journal*, *5*(1), 60–73.

Graves, M. F. (1984). Selecting vocabulary to teach in the intermediate and secondary grades. In J. Flood (Ed.), *Promoting reading comprehension* (pp. 245–260). Newark, DE: International Reading Association.

Graves, M. F. (2006). *The vocabulary book: Learning and instruction*. New York: Teachers College Press.

Great Schools. (2014). School overview: Nathaniel H. Stephens elementary school. Retrieved from: http://www.greatschools.org/alabama/alexander-city/52-Nathaniel-H-Stephens-Elementary-School/

Hallett, D., Nunes, T., & Bryant, P. (2010). Individual difference in conceptual and procedural knowledge when learning fractions. *Journal of Educational Psychology, 102*(2), 395–406.

Harmon, J. M., Hedrick, W. B**.,** & Wood, K. D. (2005). Research on vocabulary instruction in the content areas: Implications for struggling readers. *Reading and Writing Quarterly, 21,* 261–280.

Hiebert, J., & Grouws, D. A.(2007). The effects of classroom mathematics teaching on student learning. In F. Lester (Ed.), Second *Handbook of Research on Mathematics Teaching and Learning*. Retrieved from http://www.carnegiefoundation.org/sites/default/files/Hiebert_Grouws.pdf

Hiebert, J., Stigler, J. W., Jacobs, J. K., Givvin, K. B., Garnieer, H., Smith, M., Hollingsworth, H., Manaster, A., Wearne, D., & Gallimore, R. (2005). Mathematics teaching in the United States today (and tomorrow): Results from the TIMSS 1999 video study. *Educational Evaluation and Policy Analysis*, *27*(2), 111–132.

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American Educational Research Journal*, *30*(2), 393–425.

Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for Research in Mathematics Education*, *35*(5), 330–351.

Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education, 35*(2), 81–116.

Jackson, M. B., & Phillips, E. R. (1983). Vocabulary instruction in ratio and proportion for seventh-graders. *Journal for Research in Mathematics Education*, *14*(5), 337–343.

Jamison, R. E. (2000). Learning the language of mathematics. *Language and Learning Across the Disciplines*, *4*(1), 45–54.

Kamil, M., & Hiebert, E. (2005). Teaching and learning vocabulary: Perspectives and persistent issues. In E. H. Hiebert & M. L. Kamil (Eds.), *Teaching and learning vocabulary: Bringing research to practice.* Mahwah, NJ: Lawrence Erlbaum.

Kawas, T. (2010). Factor blaster. Retrieved from http://www.mathwire.com/games/multgames.html

Kelley, J. G., Lesaux, N. K., Kieffer, M. J., & Fuller, S. E. (2010). Effective academic vocabulary instruction in the urban middle school. *The Reading Teacher*, *64*(1), 5–14.

Kennedy, L. M., Tipps, S., & Johnson, A. (2008). *Guiding children's learning of mathematics*. Belmont, CA: Thomson Learning, Inc.

Kilpatrick, J., Swafford, J. O., & Findell, B. (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.

Koran, L. J., & McLaughlin, T. F. (1990). Games for drill: Increasing the multiplication skills of students. *Journal of Instructional Psychology, 17*(4), 222–230.

Kornell, N. (2012).U. S. achievement: How bad is it? *Psychology Today*. Retrieved from http://www.psychologytoday.com/blog/everybody-is-stupid-except-you/201211/us-math-achievement-how-bad-is-it

Kouba, V. L. (1989). Common and uncommon ground in mathematics and science terminology. School Science and Mathematics, 89(7), 598.

Kulm, G., Capraro, R. M., & Capraro, M. M. (2007). Teaching and learning middle grades mathematics with understanding. *Middle Grades Research Journal, 2*(1), 23–48.

Larwin, K. H. (2010). Reading is fundamental in predicting math achievement in 10th graders. *International Electronic Journal of Mathematics Education, 5*(3), 131–145.

Learn With Math Games. (2013). Retrieved from http://www.learn-with-math-games.com

Maguire, T., & Neill, A. (2006). *Classroom discourse (mathematics).* Assessment Resource Banks. Retrieved from http://arb.nzcer.org.nz/strategies/mcd.php

Marzano, R. J. (2004). Building background knowledge. Alexandria, VA: Association of Supervision and Curriculum Development.

Marzano, R. J. (2005). *Preliminary report on the 2004-05 evaluation study of the ASCD program for building academic vocabulary.* Alexandria, VA: Association of Supervision and

Curriculum Development. Retrieved from www.ascd.org/ASCD/pdf/Building%20Academic$Vocabulary%Report.pdf

Marzano, R. J. (2009). Six steps to better vocabulary instruction. *Educational Leadership, 67*(1), 83–84.

Marzano, R. J., & Pickering, D. J. (2005). *Building academic vocabulary: Teacher's manual.* Alexandria, VA: Association for Supervision and Curriculum Development.

McGill, M. (2013).Caution on education. *New York Times*. Retrieved from http://www.nytimes.com/2013/11/29/opinion/caution-on-education.html?_r=0

McKeown, M. G., Beck, I. L., Omanson, R. C., & Perfetti, C. (1983). The effects of long-term vocabulary instruction on reading comprehension: A replication. *Journal of Reading Behavior*, *15*(1), 3–18.

McKeown, M. G., Beck, I. L., Omanson, R. C., & Pople, M. T. (1985).Some effects of the nature and frequency of vocabulary instruction on the knowledge and use of words. *Reading Research Quarterly*, *20*(5), 522–535.

Mehta, J. (2013). Why American education fails. *Foreign Affairs, 92*(3), 105–116.

Meiers, M. (2010). Language in the mathematics classroom. *QCT Research Digest*, *7*. Retrieved from http://www.trb.tas.gov.au/Shared%20Documents/Language%20in%20the%20mathematics%20classroom.pdf

Michaels, A., O'Connon, M. C., & Resnick, L. (2008). Deliberate discourse idealized and realized: Accountable talk in the classroom and in civic life. Studies in Philosophy and Education, 27(4), 283–297.

Miller, L. D. (1992). Teacher benefits from using impromptu writing prompts in algebra classes. *Journal for Research in Mathematics*, *23*(4), 329–340.

Monroe, E., & Orme, M. P. (2002). Developing mathematical vocabulary. *Preventing School Failure, 46*(3), 139–142.

Monroe, E., & Panchyshyn, R. (1997).Vocabulary considerations for teaching mathematics. *Childhood Education, 72*, 80–83.

Moran, P. (2011). Access to the j-curve: A universal concept of learning. Retrieved from https:connectedsuperintendent.wordpress.com/2011?12?13?access-to-the-jcurve-a-universal-concept-of-learning/

Morgan, P. L., Farkas, G., & Maczuga, S. (2014). Which instructional practices most help first-grade students with and without mathematics difficulties? *Educational Evaluation and Policy Analysis, 20*(10), 1–22. DOI: 10.3102/0162373714536608.

Moschkovich, J. (1999). Supporting the participation of English language learners in mathematical discussions. *For the Learning of Mathematics, 19*(1), 11–19.

Mullis, I. V. S., Martin, M. O., Fox, P., & Arora, A. (2012). TIMSS 2011 International Results in Mathematics. TIMSS and PIRLS International Study Center, Chestnut Hill, MA.

Mustafa, J., Khan, A., & Ullah, A. (2011). Investigating student’s achievement in mathematics through non technological games based teaching. *International Journal of Scientific Research in Education, 4*(3-4), 151–164.

Nagy, W. (1988). *Teaching Vocabulary to Improve Reading Comprehension.* Newark, DE: International Reading Association.

Nagy, W., & Townsend, D. (2012). Words as tools: Learning academic vocabulary as language acquisition. *Reading Research Quarterly, 47*(1), 91–108.

National Assessment of Educational Progress (NAEP). (2013).U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics. Retrieved from http://nces.ed.gov/nationsreportcard/

National Center for Educational Statistics (2011). *The Nation's Report Card: Mathematics 2011* (NCES 2012–458). National Center for Education Statistics, Institute of Education Sciences, U.S. Department of Education, Washington, DC.

National Center for Educational Statistics (2013a). *The Nations Report Card: Mathematics 2013, State Snapshot Report, Alabama Grade 8, Public Schools.* Retrieved from http://nces.ed.gov/nationsreportcard/stt2013/20144658.aspx

National Center for Educational Statistics (2013b). *The Nations Report Card: Mathematics 2013, State Snapshot Report, Alabama Grade 4, Public Schools*. Retrieved from http://nces.gov/nationsreportcard/subject/publications/stt2013/pdf/2014465AL4.pdf

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics.* Retrieved from http://www.nctm.org/standards/content.aspx?id=3436

National Council of Teachers of Mathematics (NCTM). (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA.

National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards.* National Governors Association Center for Best Practices, Council of Chief State School Officers, Washington DC.

National Institute for Literacy. (2007). *What content area teachers should know about adolescent literacy*. Washington, DC: National Institute of Child Health and Human Development.

National Reading Panel (The). (2000). *Report of the National Reading Panel-Teaching children to read: An evidence-based assessment of the scientific research literature on reading and its implications for reading instruction.* Washington, DC: National Institute of Child Health and Human Development.

Neidorf, T. S., Binkley, M., Grattis, K., & Nohara, D. (2006). Comparing mathematics content in the National Assessment of Educational Progress (NAEP), Trends in International Mathematics and Science Study (TIMMS), and Program for International Student Assessment (PISA) 2003 Assessment. U.S. Department of Education. Washington, DC: National Center for Education Statistics. Retrieved from http://nces.ed.gov/pubsearch

Nokes, T. J., & Ross, B. H. (2007). Facilitating conceptual learning through analogy and explanation. *2007 Physics Education Research Conference.*

Organization for Economic Cooperation and Development (OECD), Program for International Student Assessment (PISA) (2013). PISA 2012 Results in Focus. Paris: OECD. Retrieved from: http://www.oecd.org/pisa/keyfindings/pisa-2012-results-overview.pdf

Panasuk, R. M. (2010). Three phase ranking framework for assessing conceptual understanding in algebra using multiple representations. *Education*, *131*(2), 235–257.

Pearson Education, Inc. (2012). *Investigations and the Common Core standards*. Alexandria, VA: Association for Supervision and Curriculum Development.

Perry, R. R., & Lewis, C. C. (2011). Improving the mathematical content base of lesson study summary of results. Retrieved from http://www.lessonresearch.net/IESAbstract10.pdf

Peterson, P. E., Hanushek, E. A., Woessman, L., & Riddell, J. B. (2011). Student achievement gains in U.S. fail to close international achievement gap. *Education Next*. Retrieved from http://educationnext.org/files/ednext_20124_petersonhanushek_PR.pdf

Peterson, P. E., Woessmann, L., Hanushek, E. A., & Lastra-Anadon, C. X., (2011). Globally challenged: Are U.S. students ready to compete? Harvard Kennedy School. Retrieved from http://www.hks.harvard.edu/pepg/PDF/Papers?PEPG11-03_GloballyChallenged.pdf

Piccolo, D. I., Harbaugh, A. P, Carter, T. A., Capraro, M. M., & Capraro, R. M. (2008). Quality of instruction: Examining discourse in middle school mathematics instruction. *Journal of Advanced Academics, 19*(3), 376–410.

Pierce, M. E., & Fontaine, L. M. (2009). Designing vocabulary instruction in mathematics. *The Reading Teacher, 63*(3), 239–243.

Porter, M. K., & Masingila, J. O. (2000). Examining the effects of writing on conceptual and procedural knowledge in calculus. *Educational Studies in Mathematics, 42*(2), 165–177.

Puhalla, E. M. (2011). Enhancing the vocabulary knowledge of first-grade children with supplemental booster instruction. *Remedial and Special Education, 32*(6), 471–481.

Ramtani, G., & Siegler, R. (2008). Promoting broad and stable improvements in low-income children's numerical knowledge through playing with number board games. *Child Development, 79*(2), 375–394.

Randel, J. M., Morris, B. A., Wetzel, C. D., & Whitehill, B. V. (1992). The effectiveness of games for educational purposes: A review of recent research. *Simulation and Gaming, 23*(3), 261–276.

Richland, L. R., Stigler, J. W., & Holyoak, K. J. (2012). Teaching the conceptual structure of mathematics. *Educational Psychologist, 47*(3), 189–203.

Rillero, P., & Padgett, H. (2014). Supporting deep conceptual understanding. *THE Journal, 39(*9), 37–40.

Riordan, J. E., & Noyce, P. E. (2001). The impact of two standards-based mathematics curricula on student achievement in Massachusetts. *Journal for Research in Mathematics Education, 32*(4), 368–398.

Rittle-Johnson, B., & Alibali, M. S. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other. *Journal of Educational Psychology, 91*(1), 175–189.

Rittle-Johnson, B., & Siegler, R. S. (1998). The relations between conceptual and procedural knowledge in learning mathematics: A review. In C. Donlan (Ed.), *The development of mathematical skill.* Hove, England: Psychology Press.

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. S. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology, 93*(2), 346–374.

Russell, S. J., Tierney, C., Mokros, J., Goodrow, A., & Murray, M. (2004). *Investigations in number, data, and space.* Glenview, IL: Scott Foresman.

Sanacore, J., & Palumbo, A. (2009). Understanding the fourth-grade slump: Our point of view. *The Education Forum, 73*, 67–74.

Saxe, G. B., Gearhart, M., & Seltzer, M. (1999). Relations between classroom practices and student learning in the domain of fractions. *Cognition and Instruction, 17*(1), 1–24.

Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning. A research review. *Reading & Writing Quarterly, 23*(2), 139–159.

Scott, J., Jamieson-Noel, D., & Asselin, M. (2003). Vocabulary instruction throughout the day in twenty-three Canadian upper-elementary classrooms. *Elementary School Journal, 103,* 269–286.

Shanahan, T., & August, D. (2006). *Report of the National Literacy Panel: Research on teaching reading to English language learners.* Mahwah, NJ: Erlbaum.

Silverman, R. (2007). A comparison of three methods of vocabulary instruction during read-alouds in kindergarten. *The Elementary School Journal, 108*(2), 97–113.

Skemp, R. (1987). *The psychology of learning mathematics.* Hillsdale, NJ: Erlbaum.

Slavin, R. C. (2008). Evidence-based reform in education: What will it take? *European Educational Research Journal, 7*(1), 124–128.

Snow, C. E., Lawrence, J. F., & White, C. (2009). Generating knowledge of academic language among urban middle school students. *Journal of Research on Educational Effectiveness, 2*, 325–344.

Stahl, S. A., & Fairbanks, M. (1986). The effects of vocabulary instruction: A model-based meta-analysis. *Review of Educational Research, 56*(1), 72–110.

Stahl, S. A., & Nagy, W. E. (2006). *Teaching word meanings*. Mahwah, NJ: Erlbaum.

Stanovich, K. E. (1986). Matthew effects in reading: Some consequences of individual differences in the acquisition of literacy. *Reading Research Quarterly*, 21, 360–406.

Star, J. R. (2005). Reconceptualizing procedural knowledge in mathematics. *Journal for Research in Mathematics Education, 36,* 404–411.

Star, J. R., & Rittle-Johnson, B. (2009). It pays to compare: An experimental study on computational estimation. *Journal of Experimental Child Psychology*, *101*, 408–426.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap*: *Best ideas from the world's teachers for improving education in the classroom.* New York: Free Press.

Stipek, D., Salmon, J. M., Givvin, K. B., Kazemi, E., Saxe, G., & MacGyvers, V. L. (1998). The value (and convergence) of practices suggested by motivation research and promoted by mathematics education reformers. *Journal for Research in Mathematics Education, 29*(4), 465–488.

Sweeny, S. M., & Mason, P. A. (2011). Research-based practices in vocabulary instruction: An analysis of what works in grades pre K–12. Studies & Research Committee of the Massachusetts Reading Association. Retrieved from http://massreading.org/wp-content/uploads/2013/08/vocpaper.pdf

Sweller, J. (1988). Cognitive load during problem solving: Effects on learning. Cognitive Science, 12, 257–285.

Tennessee Department of Education. (2012). *Tennessee Comprehensive Assessment Program Achievement Test*. Retrieved from https://www.tn.gov/education/assessment/doc/ACH_2012_PracticeTest_Gr3_Feb13.pdf

Thompson, D., & Rubenstein, R. (2000). Learning mathematics vocabulary: Potential pitfalls and instructional strategies. *The Mathematics Teacher, 93*(7), 568–574.

Torgesen, J. K., Houston, D. D., Rissman, L. M., Decker, S. M., Roberts, G., Vaughn, S., et al. (2007). Academic literacy instruction for adolescents: A guidance document from the Center on Instruction. Retrieved from http://opi.mt.gov/pub/rti/EssentialComponents/

RBCurric/Reading/RTIResources/Academic%20Literacy%20Instruction%20for%20Adolescents.pdf

Townsend, D. (2009). Building academic vocabulary in after-school settings: Games for growth with middle school English-language learners. *Journal of Adolescent and Adult Literacy, 53*(3), 242–251. doi:10.1598/JAAL.53.3.5.

Townsend, D., & Collins, P. (2009). Academic vocabulary and middle school English learners: An intervention study. *Reading Writing, 25*(9), 993–1019.

Trelease, J. (2006). *The read aloud handbook.* New York, NY: Penguin Press.

U.S. Department of Education. (1996). Reading literacy in the United States: Findings from the IEA reading literacy study. National Center for Education Statistics, Washington, D.C.

U.S. Department of Education, Institute of Education Sciences and National Center for Education Evaluation and Regional Assistance. (2003). *Identifying and implementing educational practices supported by rigorous evidence: A user friendly guide*. Retrieved from U.S. Department of Education via Ed.gov Access: http://www.ed.gov/rschstat/research/pubs/rigorousevid/index.html

U.S. Department of Education. (2012). *Highlights from TIMSS 2011: Mathematics and science achievement of U.S. fourth and eighth-grade students on an international context.* Retrieved from http://nces.ed.gov/pubs2013.2013009_1.pdf

Vacca, R. T., & Vacca, J. L. (2005). *Content area reading: Literacy and learning across the curriculum* (6th ed.), Menlo Park, CA: Longman.

Vaughn, S., Martinez, L. R., Linan-Thompson, S., Reutebuch, C. K., Carlson, C. D., & Francis, D. J. (2009). Enhancing social studies vocabulary and comprehension for seventh-grade

English language learners: Findings from two experimental studies. *Journal of Research in Educational Effectiveness*, *2*(4), 297–324.

Walberg, H. J., & Tsai, S. L. (1983). Matthew effects in education. *American Educational Research Journal*, *20*(3), 359–373.

Wood, T. (1999). Creating a context for arguments in mathematics class. *Journal for Research in Mathematics Education*, *30*(2), 171–191.

Wood, T., Williams, G., & McNeal, B. (2006). Children's mathematical thinking in different classroom cultures. *Journal for Research in Mathematics Education*, *37*(3), 222–255.

Yackel, E., Cobb, P., & Wood, T. (1991). Small group interactions as a source of learning opportunities in second-grade mathematics. *Journal for Research in Mathematics Education*, *22*(5), 390–408.

APPENDIX A

VOCABULARY ROUTINE SCRIPTS

Script 1: Vocabulary word for today and tomorrow: *concise*

Word Card: Work with your students until they understand how to divide an index card into fourths:

Definition	Sentence
Example	Drawing or Symbol

- Post the word for students to see.
- Pronounce the word. Pronounce it a second time before asking students to repeat the word.
- Write the following child-friendly definition on the board:
  Brief and to the point
- Ask students to write the word and the definition in the top left quadrant of a card.
- Discuss and give examples of *concise:* For example,
  --If your teacher wants you to answer a question, he or she may want you to be *concise*. That means you would answer the question using only words that would be necessary. You would not include a lot of unnecessary words.
  --If your teacher wants you to answer a math problem, he or she wants you to show your work using only number that would be necessary. You would not include a lot of unnecessary numbers.
  Feel free to include any explanations or examples that you feel are relevant to your students. Discuss the word and its meaning until you feel that your students understand.
- Since this is the first vocabulary card, I suggest you work with your students to develop a sentence that all students will then write in the top right quadrant. Later, when students get accustomed to filling out their cards, you can start having them write their own

sentences. Be sure to ask volunteers to read their sentences once they begin to write their own, just to be sure the word is used correctly.

- Ask students to complete the example quadrant. Ask them to think of a time when being *concise* would be better than not being *concise.* An example of a correct response might be: *I want my mother to be concise when she gives me directions to complete a chore.* After allowing time for students to complete their examples, ask for volunteers to read their examples and tell why being *concise* would be good. Spend several minutes in discussion.
- Last, have students add a drawing or symbol for the word. It doesn't matter what they draw as long as they can explain HOW it relates to being *concise*. Again, ask for volunteers to share and explain their drawings.
- Review Activities:
  Concise or Not Concise?
  Read each of the following example pairs. After each, ask, "Is this an example of concise?" Ask the students to explain why or why not.
  1. A grocery list of 5 items.
     A 5-page grocery list with a map of the store and descriptions of all the items on the list.
  2. A definition containing 230 words.
     A definition containing 15 words.
  3. 7 x 8 = 56
     7 x 8 = For example, you have 7 bags and you have 8 marbles in each bag. Each bag has 8 marbles. All the bags have the same number. When you pour them all out and count them, you have 56 marbles. There would be 56 marbles in all.

  Sentence Completion
  Post the following sentence stem:
  *Bobby's mother was really mad at him. Bobby knew she was about to get on to him and as she bent over and looked him eye to eye, he knew she was going to tell him why he had been bad. He hoped she would be concise because*______________________.

Script 2: Vocabulary word: *notation*

- Post the word for students to see.
- Pronounce the word. Pronounce it a second time before asking students to repeat the word.
- Write the following child-friendly definition on the board:
  Written symbols used to represent special things.
- Ask students to write the word and the definition in the top left quadrant of a card.
- Discuss and give examples of *notation:* For example,

--Write the following addition problem on the board: 14 + 6 = 20.
When you have a problem, such as 14 plus 6, and you are expected to answer the problem, your answer (20) is a special thing, it is a *notation*. The number 20 represents the answer to the problem.
--If you are asked to write down your birthday, you might write: June 6, 2005. June 6, 2005 is an example of a *notation*. The date represents the day you were born.

Feel free to include any explanations or examples that you feel are relevant to your students. Discuss the word and its meaning until you feel that your students understand.

- Use your own judgment whether you have your class work together to write a sentence with *notation* or if you want each student to write their own. If you decide they are ready to write their own sentences, follow up by asking volunteers to read and discuss their sentences.
- Ask students to complete the example quadrant. Ask them to think of an example of a *notation* that is different from the examples already supplied. Possible answers include their name written at the top of a paper, their address, a grocery list, etc. Again, ask volunteers to share and explain their examples.
- Last, have students add a drawing or symbol for the word. Ask for volunteers to share and explain their drawings.

Review Activities: These activities will review both words, *concise* and *notation*.

HAVE YOU EVER…? Tell me a time when you….

* Tell me a time when you wrote a *notation* in math class. (After a student shares a response, ask, "Was your *notation concise?* Why or why not?
How concise should an answer be? Why would a teacher want a concise answer to a problem? Why should YOU want to give a concise answer, or notation?"
* "Tell me a time when you wrote a *notation* that was <u>not</u> *concise*.
Why would someone write a notation that is not *concise*?"
* "Tell me a time when you wished your parent had been *concise*. Explain why."
*"Tell me a time when you had to read a notation that was not *concise*. For example, have you ever tried to read and follow directions that seem long and hard to read? How did you feel?
Script 3: Vocabulary Word: *representation*

Teacher Background Information: some information about *representation* has been included in this packet if you would like to see what the Principles and Standards for School Math has to say on the topic.

- Post the word
- Pronounce the word. Pronounce it a second time before asking students to repeat the word.

- Write the child-friendly definition on the board:
  A way to show a math idea, such as a number sentence, a display of manipulative materials, a drawing, a graph, etc.
- Ask students to write the word and the definition in the definition section of an index card.
- Discuss and give examples of *representation*:
  1. Ask students if they can name any representations used in their math. Discuss. Using the set of cards supplied for this lesson, display one at a time (leave the 5 card to last.) Ask what each represents. Ask students again to see if they can add any other examples. (FYI: the 5 card represents a child's age. When asked to write her age, a little girl, who was 5 and a half, represented her age with a whole numeral 5 and what she thought was half of another 5.)
  2. In addition to the examples on the cards, include your verbal pronunciation of 17. Be sure students understand that the spoken word is also a *representation*. This is an important concept that they should understand.
  3. Feel free to include any explanations or examples that you feel are relevant to your students. Discuss the word and its meaning until you feel your students understand.
- Ask students to write a sentence with *representation*. Ask volunteers to read and discuss their sentences.
- Ask students to add an example for *representation*. Ask them to try to come up with an example that has not already been used but accept anything that the students can explain.
- Last, have students add a drawing or symbol (a representation) for *representation*. This may be confusing to them. I suggest you allow them to use a second example.

Review Activities:
Questions, Reasons, Examples: Students should be asked to explain their answers.

- Why would a math teacher ask you to make a *representation* for the problem 6 x 8?
  Goal: the students include in their discussion the fact that a correct representation would prove that the students understand the problem and why 48 is the answer.
- Why are base ten blocks good *representations* of the 10s place value system?
  Goal: the students include in their discussion the fact that 1 unit cube represents one, 10 units represent ten, etc.
- Which do you think is a better *representation* of 10: 10 pieces of candy or the word *diez* (this is Spanish for ten.)
  Goal: the students would include in their discussion that fact that *diez* might not be the better representation unless a lot of their classmates know Spanish.

Word Relationships (a review): Students should be asked to explain their answers.

- Should a *representation* be *concise*?
  Probably: that would make them easier to understand, take less time to make, etc.

- Are all *representations* also *notations*?
  No. Saying the word *triangle* is an example of a *representation*. Notations are written.
- Are all *notations* also *representations*?
  Yes. Our child-friendly definition for notation was: written symbols used to represent special things.

REPRESENTATIONS:

According to the Principles and Standards for School Math, instructional programs from prekindergarten through grade 12 should enable all students to— create and use representations to organize, record, and communicate mathematical ideas; select, apply, and translate among mathematical representations to solve problems; use representations to model and interpret physical, social, and mathematical phenomena.

> The term *representation* refers both to process and to product—in other words, to the act of capturing a mathematical concept or relationship in some form and to the form itself. There are eight widely used representational systems used in the teaching and learning of mathematics:
> 1. Written mathematical symbols (Symbolic) – these can include numbers, mathematical expressions, i.e. x + 2, <, etc.
> 2. Descriptive written words: For example, instead of writing 2 x 3, we might write "two groups of three" or "three repeated two times"
> 3. Pictures or diagrams – figures that may represent a mathematical concept or a specific manipulative model
> 4. Concrete models/Manipulatives – like Base-10 blocks, counters, etc., where the built-in relationships within and between the models serve to represent mathematical ideas;
> 5. Concrete / Realia: where the objects represent themselves; for example, candies that are being used to count or to graph. The candies themselves are not representing anything other than candies.
> 6. Spoken languages / Oral representations – i.e. the teacher saying the number one hundred thirty-two is quite different from the teacher writing the number 132 on the board for students to see;
> 7. Experience-based – or real world problems, drawn from life experiences, where their context facilitates the solution;
> 8. School word problems: "If Mary is three years older than Carl, and Mary will be 34 next year, how old is Carl now?"

Script 4: Vocabulary Word: *expression*

Teacher Background Information: An *equation* uses an *equal sign* between two *expressions*.
Example: $8 + 4 = 12$

- Post the word
- Pronounce the word. Pronounce it a second time before asking students to repeat the word.
- Write the child-friendly definition on the board: (FYI-Please note that expressions do not include = .)
  Numbers, symbols and operators (such as + and ×) grouped together that show the value of something.
- Ask students to write the word and the definition in the definition section of an index card.
- Discuss and give examples of *expression*:
  1. Write 2×3 on the board. Tell your students that 2 x 3 is an expression. It is a group that contains numbers and a symbol (x) to show the value of 6. Discuss. Ask if students can give you an addition expression; a subtraction expression
  2. Share the chart of phrases and expressions. Explain that what we can say, for example *"the sum of 9 and 8",* can be expressed as *9 + 8*. Read the remaining phrases and ask students to supply the corresponding expressions. Complete the chart for each expression.
  3. Feel free to include any explanations or examples that you feel are relevant to your students. Discuss the word and its meaning until you feel your students understand.
- Ask students to write a sentence with *expression*. Ask volunteers to read and discuss their sentences.
- Ask students to add an example for *expression*. Ask them to try to come up with an example that has not already been used but accept anything that the students can explain.
- Last, have students add a drawing or symbol for *expression*. Ask volunteers to share and explain their symbols.

Review Activities:
Yes, No, Why? Ask students to answer yes or no to each statement. Be sure to always ask *why?*

1. Story problems can be written as *expressions*.
   Yes: You might need to use an example with your students: *Mary had 14 sacks and 28 pieces of candy. How many pieces of candy will Mary put in each sack so that all sacks have the same amount?* This can be expressed as 28 ÷ 14
2. (Read) The following *expression* is correct for the difference between 56 and 23: (Write on the board) 23 – 56
   No. Ask why and discuss student responses.

3. (Read) *The following expression is correct for the amount of allowance Jerry will receive in a week if he gets $1 a day:*
   (Write on the board) 7 x 1
   Yes. Ask why and discuss student responses.

Word Relationships: (have students pull their index cards for their words to date)

1. Can an *expression* also be a *representation*?
   Yes. Why? They both are ways to showing something. An expression shows an amount and a representation shows an idea, for example, x for multiplication.
2. Should expressions be *concise*?
   Yes. Why? (This might be hard for your students to understand. If so, write the following expression on the board:
   1+ 1 + 1 + 5 + 1 + 4 + 1 + 1 + 1 + 1
   Ask, would this be a *concise* expression for three more than seventeen? Then follow with a discussion of the value of being *concise* (being concise takes less time and effort).

Phrase	Expression
The sum of nine and eight	9 + 8
6 less than 15	
Ten more than eight	
Ten divided by 2	
Three multiplied by four	
The difference between 10 and 5	

Script 5: Vocabulary word: *solution*

- Post the word
- Pronounce the word. Pronounce it a second time before asking students to repeat the word.
- Write the child-friendly definition on the board:
  Answer to a problem
- Ask students to write the word and the definition in the definition section of an index card.
- Discuss and give examples of *solution*:
  1. Answers to number problems, story problems, mathematical tasks, etc.
  2. Be sure that students understand that *solutions* should be correct. An incorrect answer is not a *solution*.

3. Feel free to include any explanations or examples that you feel are relevant to your students. Discuss the word and its meaning until you feel your students understand.

- Ask students to write a sentence with *solution*. Ask volunteers to read and discuss their sentences.
- Ask students to add an example for *solution*. Ask them to try to come up with an example that has not already been used but accept anything that the students can explain.
- Last, have students add a drawing or symbol for *solution*. Ask volunteers to share and explain their symbols.

Idea Completion:

Share the sentence stem on the next page with your class.
Ask them to complete the sentence on paper.
After giving students sufficient time, ask students to share and explain.

Good completions should indicate that the student understands that *solution* and *answer* mean the same thing. Good completions may also include reference to the fact that the teacher wanted to know if the students could supply <u>correct</u> answers.

Anna's math teacher asked her students to give a *solution* for each problem on their test because Anna's teacher wanted to know if her students

____
____

Script 6: Cumulative Vocabulary Review

Review Activities for *concise, notation, representation, expression, solution*

Have students pull their index cards for reference for all the review activities.

A. Word Association:

1. Look at your word cards: Which word means about the same as *notation*? *Solution*
   Be sure to ask for an explanation and follow with discussion.
   (FYI: notations are written; solution can be either written or verbal)
2. Which word goes with *short*? *Concise* Why?
3. Which word goes with *answer*? NOTE: there are two words can could answer this question: *solution* and *notation.* You may need to extend discussion in order to get both answers.
4. Draw a square on the board:
   Ask, which word goes with my drawing? *Representation* Why?
5. Write on the board: 5 x 3

Ask, which word goes with this? NOTE: there are two words can could answer this question: *expression* and *representation.* You may need to extend discussion in order to get both answers.

B. Yes, No, Why?

1. Can a *representation* also be a *solution*? Why?
   Ask students for examples.

Yes. For example, if a teacher asked students to show 7 x 7, a student might draw an array for 7x7. The array would be a *representation* for the *solution*, 49.

2. *Representations* help me understand math. Why?
   Ask students to answer and then explain by giving examples.

C. Example/Nonexample: Display the table on page 10. Using another piece of paper as a cover, uncover each item as it is addressed. Ask students to differentiate between two descriptions. Answers:

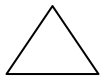
- Concise: b
- Notation: a (notations are written)
- Representation: b (In math, the vocabulary word *representations* should connect to math ideas or concepts.)
- Expression: b (an expression contains numbers, symbols and operators (such as + and ×) grouped together that show the value of something.)
- Solution: b (A is an expression)

D. Idea Completion

Display the sentence stem on page 12. Ask the students to complete the sentence on a piece of paper. Have students share and explain their completions.

Example or Nonexample?

Concise	a. A square has four sides. All four sides are equal. That means they are the same. They are the same length. Each side, when measured, is the same as the other three sides. The four corners are right angles. That means all angles are the same, 90 degrees. Therefore, the sides stand up and are not slanted. b. A 4-sided flat shape with straight sides where all sides have equal length, and every angle is a right angle (90°).
Notation	a. The student wrote: 17

	b. The student said: “seventeen”
Representation	a.  b. 
Expression	a. 17 b. 8 + 11
Solution	For 8 x 11 = a. 8 x 11 b. 88

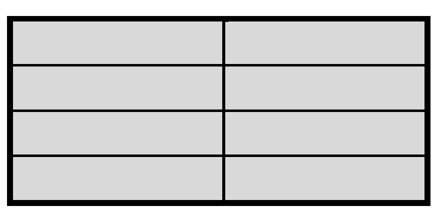
The teacher gave Mark the following problem:

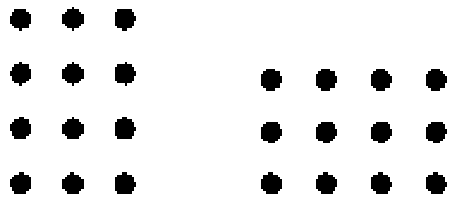
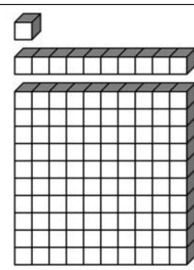
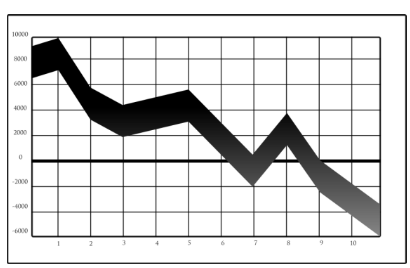
$$\begin{array}{r} 896 \\ \underline{-\ 146} \end{array}$$

The teacher asked that he be *concise* when he gave a *solution* because

.

3 x 3



5
 4×3   3×4
<
+

fifty


$$\frac{\mathbf{2}}{\mathbf{3}}$$

Script 7: Vocabulary Word: *equivalent*

- Post the word
- Pronounce the word. Pronounce it a second time before asking students to repeat the word.
- Write the child-friendly definition on the board: *Two things are equivalent if they have the same value or amount.*
- Ask students to write the word and the definition in the definition section of an index card.

  Discuss and give examples of *equivalent* using the cards supplied. Ask students to explain how the two items on each card are equivalent. Ask what mathematical symbol is often used to represent *equivalence*. Feel free to add examples of your own.
- Write the following sentence stem on the board. Work with the whole class to complete the stem. When the class has decided on a good completion for the stem, have the students copy the stem and its completion on their card in the sentence quadrant.

  *3 x 4 is equivalent to 2 x 6 because*___________________________________
- Ask students to add an example for equivalent. Ask them to try to come up with an example that has not already been used but accept anything the students can explain.
- Last, have students add a drawing or symbol for *equivalent*. (Hopefully they will think to use =) Ask volunteers to share and explain their symbols.

Review Activities:

1. Have you ever….?

Ask students to tell about a time when they shared something with another person (for example, candy or a pizza) but their "share" wasn't *equivalent* to their friend's share.

2. Making Choices

Ask students to listen carefully as you read two things out loud. Ask them to say "*equivalent*" if the two items are equivalent, but to say nothing if the two items are not equivalent.

a. <u>3 cans of Pepsi</u> and <u>half a six-pack of Pepsi</u>

b. <u>one week</u> and <u>10 days</u> (Ask students to explain why these 2 are not equivalent.)

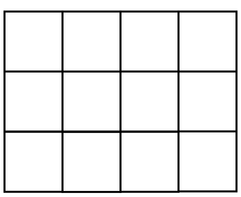
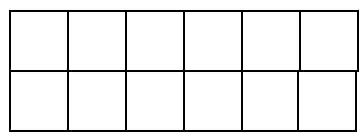
c. <u>one year</u> and <u>8 months</u> (Ask students to explain why these 2 are not equivalent.)

d. <u>a dozen cookies</u> and <u>12 cookies</u>

e. <u>100</u> and <u>2 x 50</u>

f. <u>a boy 6 and ½ years old</u> and a <u>boy 6 years and 6 months old</u>

g. <u>a large coke from McDonalds</u> and <u>a medium coke from McDonalds</u> (Ask a student to explain why these 2 are not equivalent.)

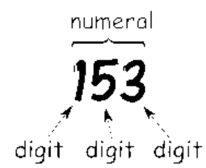
2 + 7 is equivalent to 5 + 4
4 x 5 is equivalent to 2 x 10
12 – 6 + 1 is equivalent to 10 – 4 + 1
 is equivalent to 

Script 8: Vocabulary Word: digit
Teacher Background Information: Numbers, Numerals and Digits
Number:

- A number is a count or measurement that is really an idea in our minds.
- We write or talk about numbers using numerals such as "5" or "five". We could also hold up 5 fingers, or tap the table 5 times.
- These are all different ways of referring to the same number.

Numeral:

- A numeral is a symbol or name that represents a number.
- Examples: 3, 49 and twelve are all numerals.
- So the number is an idea, the numeral is how we write it.



Digit:

- A digit is a single symbol used to make numbers.
- 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are the ten digits we use in everyday numbers.

Also, please note: For our fourth graders, we will not be distinguishing between number and numeral in this lesson.

Vocabulary Word: *digit*

- Post the word
- Pronounce the word. Pronounce it a second time before asking students to repeat the word.
- Write the child-friendly definition on the board: *A digit is a single symbol used to represent a number. We use ten digits to make all numbers.*
- Ask students to write the word and the definition in the definition section of an index card.
- Ask the students to name the 10 digits that are used to make all numbers as you write them on the board.
- Have each student think of a 3 digit number to write in the example section of his/her card. Ask the students to circle each digit in the number they wrote. Ask them how many circles they have? Write 333 on the board and ask how many circles it would take to circle each digit. (3--We want to be sure they understand that a digit used more than once is counted each time it is used.)
  Ask: How many digits would you need to write a number in the thousands?
  How many digits would you need to write for a number in the ten thousands? In the hundred thousands? In a million?
- Write the following sentence stem on the board. Work with the whole class to complete the stem. When the class has decided on a good completion for the stem, have the students copy the stem and its completion on their card in the sentence quadrant.
  *Digits can be combined to write*_______________(big or large numbers)**.**
  (You may have to discuss what it means to combine.)
- Last, have students add a drawing or symbol for *digit*. Ask volunteers to share and explain their symbols.

Review Activity

Write these two numbers on the board: 268 862

Ask:

- What *digits* were used to make these two numbers? (2, 6, and 8)
- Do the two numbers *represent* the same amount? For example, would you rather have 268 dollars or 862 dollars? Why?
- So, would you say that the two numbers are or are not *equivalent*? Why not?
  They both have a 2, a 6, and an 8, so why aren't they *equivalent*? (Our goal here to review place value so that students are reminded that a digit's location within a number determines the "worth" of the digit.)
- Is there a *digit* that has the same value in both 268 and 862?
- What value does the 2 represent in 268?
- What value does the 2 represent in 862?
- So, a digit's value is determined by what? Its place in the number.

Script 9: Vocabulary Word: *array*

- Post the word
- Pronounce the word. Pronounce it a second time before asking students to repeat the word.
- Write the child-friendly definition on the board: *An array is made by arranging items in rows and columns.*
- Ask students to write the word and the definition in the definition section of an index card.
- Use the cards provided for this lesson to show share examples of arrays. Ask the students how they have used arrays in math before.
  (Hopefully your discussion with them will lead them to remember that multiplying the number of rows and the number of items in each row is an easier and faster way to figure out "how many" than counting each item in the array.)
- Have the students add an example of *array* to their index card that is different from the examples you used. Ask for volunteers to share. Discuss.
- Write the following sentence stem on the board. Work with the whole class to complete the stem. When the class has decided on a good completion for the stem, have the students copy the stem and its completion on their card in the sentence quadrant.
  *It is easy to figure out how many items are in an array because I can just*_____________.

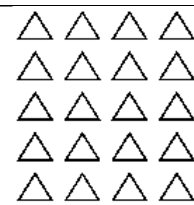
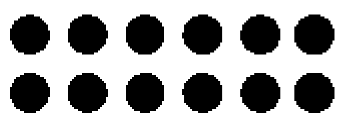
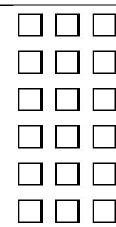
Review: *expression, array, representation, equivalent*

- Pull the example array cards. Displaying one card at a time, ask: What *expression* does this *array represent*?

☆☆☆
☆☆☆
☆☆☆
For example: ☆☆☆ 4 x 3 (4 rows x 3 in each row)

Continue with each of the remaining cards.

- To complete the symbol quadrant for *array*, ask the students to write an *expression* for the example array they drew on their card. Remind them that their *expression* is a *representation* of the value of their array. Ask volunteers to share and explain their examples and *expressions*.
    - Display all 6 of the example cards. Ask: Are any of these cards *equivalent;* do any *represent* the same amount? (12 dots-2x6 and 12 stars-4x3)
    - Take the other cards away leaving just the dot array and the star array. Ask: Can you design an array that is *equivalent* to these 2 arrays but has a different arrangement for the number 12?
      Give them time to work on this before asking for volunteers. (There are four more possible arrays: 6 x 2, 3 x 4, 1 x 12, and 12 x 1.)

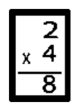
	
	
 Box of chocolates	 Crate of oranges

Script 10: Vocabulary Word: *multiplication*

- Post the word

- Pronounce the word. Pronounce it a second time before asking students to repeat the word.
- Write the child-friendly definition on the board: *Multiplication is the same as adding a number over and over.*
- Ask students to write the word and the definition in the definition section of an index card.
- Discuss and give examples of *multiplication* using the cards supplied.
  <u>Card A</u> (4 x 3 = 3 + 3 + 3 + 3): Ask:
    - 4 x 3 is the same as what number added repeatedly? 3
    - How many times 3 being added? 4

  <u>Card B</u> (Apple Array): Ask:
    - What number is being added repeatedly? 5 (Theoretically, this array pictures 3 sets of 5, so 5 is being added repeatedly, or 5 + 5 + 5.)
    - What would be the multiplication *expression* that would *represent* this problem? 3 x 5
    - What shape does the array make? rectangle

  <u>Card C</u>: Say:
    - Here is a *multiplication* flash card. 

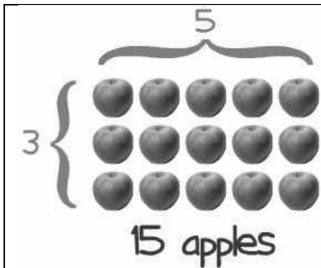
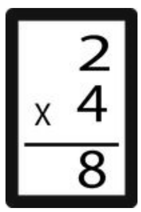
    - Think of this problem as repeated addition and tell me what number is being repeatedly added? (Theoretically, the 4 represents the number of sets and the 2 represents the number of items in each set. However, accept answers of both 2 and 4.)
    - Is 2 x 4 *equivalent to* 4 x 2? Yes (At least for our purposes today.)

  <u>Card D</u> (Sunglasses): Say:
    - Look at this *array*.
    - It represents 3 sets with how many in each set? 1
    - So, what number is being added repeatedly? 1 or 1 + 1 + 1
    - What would be the *multiplication expression* that would *represent* this problem? 3 x 1 (3 sets with 1 pair of glasses in each set.)

  <u>Card E</u> (Eggs): Look at this card.
    - How many "sets" of eggs are shown? 2
    - How many eggs are in each "set"? 6
    - Can you tell me what number is being added repeatedly? 6
    - How many times should 6 be added? 2
    - Write an *expression* for this problem? 2 x 6
- Write the following sentence stem on the board. Work with the whole class to complete the stem. When the class has decided on a good completion for the stem, have the students copy the stem and its completion on their card in the sentence quadrant.

I use repeated addition to solve 5 x 7 when I ________________________________.
Possible endings: …add 7 five times.
…write 7 + 7 + 7 + 7 + 7

- Ask students to add an example to their index card for *multiplication*. Ask them to try to come up with an example that has not already been used but accept anything the students can explain.
- Last, have students add a drawing or symbol for *multiplication*. Ask for volunteers to share and discuss.

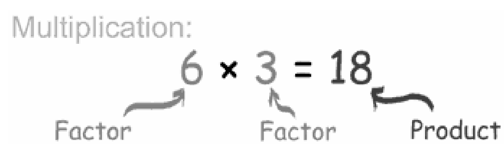
	**4 x 3 = 3 + 3 + 3 + 3**
	
	

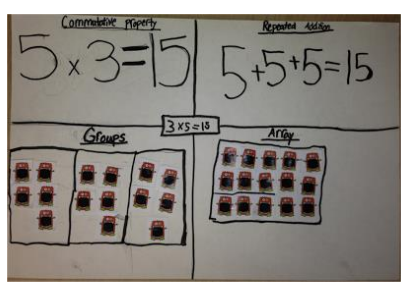
Script 11: Vocabulary Word: *product*

- Post the word
- Pronounce the word. Pronounce it a second time before asking students to repeat the word.
- Write the child-friendly definition on the board: *The product is the answer when numbers are multiplied together.*
- Ask students to write the word and the definition in the definition section of an index card.

- Discuss and give examples of *product* using the cards supplied
  Card A: (This is a good example showing the vocabulary for the problem.)
  Card B: Say:
    - Look at the work one student did. This student was asked to prove the answer to the problem 3 x 5. What was the *product*? (15)

  Card C: Say: look at this *equation.*
    - Ask: What is the *product*?
    - What symbol shows that the *expression* 7 x 9 and the *product* are *equivalent*? =
- Write the following sentence stem on the board. Work with the whole class to complete the stem. When the class has decided on a good completion for the stem, have the students copy the stem and its completion on their card in the sentence quadrant.
  The teacher asked her students to find the product of a problem because
  ____.
  Possible endings: …it was a multiplication problem.
  …she wanted them to multiply the two numbers. *etc*
- Ask students to add an example to their index card for *product.* Ask them to try to come up with an example that has not already been used but accept anything the students can explain.
- Last, have students add a drawing or symbol for *product*. Ask for volunteers to share and discuss.





7 x 9 = 63

Script 12: Review

A. *Which word?* Post the word cards supplied so that the students may use them to answer the following questions.

- Which would you probably use if your teacher wanted you to <u>prove</u> an answer to a multiplication problem? (array)
  How could an array prove your answer was correct?
  Why doesn't just writing down a *product* prove it is correct?
- Which is *represented* by equal marks? (Equivalent or equivalence)
- Which is a symbol for a number? (digit)
- Which is represented by an *array*? (multiplication)
- Which word means the same as *answer*? (product)
- In 1, 229 there are four ______? (digits)

B. Word Relationships This activity pairs words so that students have to consider how meanings interact in order to respond to the questions. *<u>Remember to always ask students to explain their answer</u>*. Their explanations are the most important part of this activity because it requires them to explicitly think through how the word fits the choices in order to express the relationship between the words.

Post the word cards so that your struggling readers have another opportunity to see the words being discussed.

Display the card: 5 x 9 = 45

- In 5 x 9 = 45, is the *product* also an *equivalent*?
  (Yes. The *product* 45 is *equivalent* to the *expression* 5 x 9. Both 45 and 5 x9 have equal values.)
- Could an *array* be used to represent this *multiplication* problem?
  (Yes, it would have 5 rows with 9 items in each row.)
- Could an *array* be used to represent every *multiplication* problem?
  (Yes, it is theoretically possible.)
  Can you think of a multiplication problem that might be hard to draw an array? (Once numbers become large, it would be very hard to draw an *array*. For example, 2, 184 x 398 would require drawing 2,184 rows with 398 items in each row.)
- If you had a *multiplication* story problem for homework, would you use *digits* to find a *product*?
  (Yes. We would use *digits* to *represent* the amounts in the story problem. Then we would *multiply* them to get an answer, or *product*.)
- If your class was asked to draw an *array* to represent the *product* 12, would you expect everyone's array to be alike?

(No. Because 12 can be represented by several arrays: 1 x 12, 12 x 1, 2 x 6, 6 x 2, 3 x 4, and 4 x 3)

- In the problem on the card, is the value of the *digit* 5 (in 5 x 9) *equivalent* to the value of the digit 5 in 45?
  (Yes. The digit 5 in 45 is in the one's place so its value is 5, just as the 5 used in the expression, 5 x 9.)
- Is *the multiplication combination* 5 x 9 *equivalent* to the *multiplication combinations* of 2 x 9 + 3 x 9?
  (Yes. An array for 5 x 9 could be separated so that the first two rows represent 2 x 9 and the other 3 rows represent 3 x 9.)

C. Children Create Examples

Have your students discuss when they might be asked to do the following. Ask student volunteers to write examples on the board so the whole class can discuss them.

- Find a *product*.
- Use an *array*.
- Use *multiplication* rather than subtraction.
- Look carefully at a *digit* in a larger number, such as 2, 709.
- Decide if two things or amounts are *equivalent*.

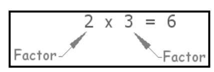
Multiplication	Digit
Array	Equivalent
Product	5 x 9 = 45

Script 13: Vocabulary Word: *multiples*

- Post the word
- Pronounce the word. Pronounce it a second time before asking students to repeat the word.
- Write the child-friendly definition on the board: *Multiples are the numbers landed on when skip counting on a number line.*
- Ask students to write the word and the definition in the definition section of an index card.
- Discuss and give examples of *multiples* using the card supplied below.
  This card shows the multiples of 3 and the multiples of 5.

- Write the following sentence stem on the board. Work with the whole class to complete the stem. When the class has decided on a good completion for the stem, have the students copy the stem and its completion on their card in the sentence quadrant.
  *I know that 8 and 12 are multiples of 4 because*_______________________________.
  Possible answer: …when I skip count by 4, I will land on 8 and 12.
- Ask students to add an example to their index card for *multiples*. Ask them to try to come up with an example that has not already been used but accept anything the students can explain.
- Last, have students add a drawing or symbol for *multiples*. Ask for volunteers to share and discuss.

Script 14: Vocabulary Word: *factor*

- Post the word
- Pronounce the word. Pronounce it a second time before asking students to repeat the word.
- Write the child-friendly definition on the board: *Factors are numbers you can multiply to get another number.*
- Ask students to write the word and the definition in the definition section of an index card.
- Discuss *factor* using the card supplied below. 

  Discuss:

  A number can have MANY *factors*!
  Ask: What are the *factors* of 12?
  3 and 4 are *factors* of 12, because 3 × 4 = 12.
  Also 2 × 6 = 12 so 2 and 6 are also *factors* of 12.
  And 1 × 12 = 12 so 1 and 12 are *factors* of 12 as well.
  So 1, 2, 3, 4, 6 and 12 are all *factors* of 12.

  (Now, relate these *factors* of 12 to skip counting using the cards with number lines, writing on the number lines as you go. The purpose here is for the students to see that skip counting on a number line is the same as finding 2 *factors* for each number on which they land. And remember from yesterday's word--the number on which they land is a *multiple*.)

  - For the first example, say: Let's skip count by 3 to 12. How many times did we "hop"? 4 So, 3 x 4 = 12
    3 and 4 are *factors* and each number we landed on as we hopped to 12 is a *multiple* of 3
  - For the next example, say: Let's skip count by 2 to 12. How many times did we "hop"? 6 So, 2 x 6 = 12

2 and 6 are *factors* of 12 and each number we landed on as we hopped to 12 is a *multiple* of 2

- For the last example, say: Let's skip count by 1 to 12. How many times did we "hop"? 12 So, 1 x 12 = 12
  1 and 12 are *factors* of 12 and each number we landed on as we hopped to 12 is a *multiple* of 1.

- Write the following sentence stem on the board. Work with the whole class to complete the stem. When the class has decided on a good completion for the stem, have the students copy the stem and its completion on their card in the sentence quadrant.
  I know that 5 is a factor of 20 because…….
  Possible endings: …5 x 4 is 20.
  ...I can skip count to 20 by 5s.
  ...20 is a multiple of 5.
- Ask students to add an example to their index card for *factor.* Ask them to try to come up with an example that has not already been used but accept anything the students can explain.
- Last, have students add a drawing or symbol for *factor*. Ask for volunteers to share and discuss.

Script 15: Vocabulary Word: *prime*

Background information: 2 is the only even number that is prime.

- Post the word
- Pronounce the word. Pronounce it a second time before asking students to repeat the word.
- Write the child-friendly definition on the board: *A prime number can only be divided evenly by 1 and itself.*
- Ask students to write the word and the definition in the definition section of an index card.
- Discuss and give examples of *prime.*
  Example: 5 can only be divided evenly by 1 or 5, so it is a *prime* number.
  Ask: Can you name other *prime* numbers?
  Then use 6 as a non-example: 6 can be divided evenly by 1, 2, 3 and 6 so it is NOT a prime number (it is a composite number, which we will talk about tomorrow).
  Ask: How many arrays do you think can be made for each *prime* number? (Allow them to discuss but do not give away the answer. Instead, use unit cubes or draw squares on the board to model arrays for a few prime numbers until students realize that there would only be 2 ways to model each prime number.)

- Write the following sentence stem on the board. Work with the whole class to complete the stem. When the class has decided on a good completion for the stem, have the students copy the stem and its completion on their card in the sentence quadrant.
  *I know that 7 is a prime number because*____________________________________
  Possible answer: …there are only 2 ways to make an array for 7: 1 x 7, or one row with 7 in the row, or 7 x 1, 7 rows with 1 in each row.
- Ask students to add an example to their index card for *prime*. Ask them to try to come up with an example that has not already been used but accept anything the students can explain.
- Last, have students add a drawing or symbol for *prime.* Ask for volunteers to share and discuss.

Script 16: Vocabulary Word: *composite*

- Post the word
- Pronounce the word. Pronounce it a second time before asking students to repeat the word.
- Write the child-friendly definition on the board: *A composite number can be divided evenly by numbers other than 1 and itself.*
- Example: 9 can be divided evenly by 3 (as well as 1 and 9), so 9 is a composite number. But 7 cannot be divided evenly (except by 1 and 7), so is NOT a composite number (it is a prime number).
  Whole numbers above 1 are either composite or prime.
- Ask students to write the word and the definition in the definition section of an index card.
- Discuss and give examples of *composite.*
  Example: 9 can be divided evenly by 3, as well as by 1 and 9, so 9 is a composite number.
  Ask: Can you name other *composite* numbers?
  Then use 7 as a non-example: 7 cannot be divided evenly except by 1 and 7, so 7 is NOT a *composite* number, it is what kind of a number? *prime*
  Ask: How could we use arrays to help prove which numbers are *composite*? (Allow them to discuss but do not give away the answer. Instead, use unit cubes or draw squares on the board to model arrays for a few composite numbers until students realize that there would be more than 2 ways to model each composite number.)
- Write the following sentence stem on the board. Work with the whole class to complete the stem. When the class has decided on a good completion for the stem, have the students copy the stem and its completion on their card in the sentence quadrant.
  *I know that 12 is a composite number because*____________________________________

Possible answer: …there are more than 2 ways to make an array for 12: 1 x 12, 2 x 6, 3 x 4, 12 x 1, 6 x 2, and 4 x 3

- Ask students to add an example to their index card for *composite*. Ask them to try to come up with an example that has not already been used but accept anything the students can explain.
- Last, have students add a drawing or symbol for *composite.* Ask for volunteers to share and discuss.

Script 17: Review of *multiples*, *factor*, *prime, composite*
Say: We have talked about 4 words this week, *multiples, factor, prime*, and *composite*. Let's think about them some more.

Display the first question card and give the students time to think about their answer before opening up a discussion. Follow with the remaining cards.

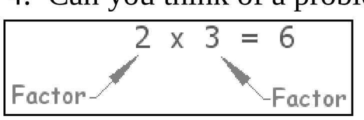
Q Card 1: How could we use what we know about *factors* to help us figure out if a number is *prime* or *composite*?
(Hopefully they will suggest that numbers with more factors than 1 and the number are prime while numbers with more factors than 1 and the number are composite.)

Q Card 2: If you earned a piece of candy for each *factor* in a number, would you rather find *factors* for 20 or 21? Explain why.
(Let them use paper and pencil, if needed.
20: 1 x 20, 2 x 10, 4 x 5, or 6 factors
21: 1 x 21, 3 x 7, or 4 factors)
After determining that 20 has more *factors*, ask: What kind of numbers are 20 and 21, *prime* or *composite*? Composite, because each has more factors than 1 and itself.

Q Card 3: Can you think of an example or a problem where 6 is a *multiple*?
Possible answers:

- When skip counting by 3 we count 3 and then 6. Therefore, 6 is a multiple of 3.
- They might put the same thoughts into equations, such as 3 x 2 = 6 or 2 x 3 = 6. But, ask for an explanation. We want them to realize that 3 x 2 can also mean skip counting by 3 and making 2 hops.

Q Card 4: Can you think of a problem where 6 is a *factor*?



Question Card 1 How could we use what we know about *factors* to help us figure out if a number is *prime* or *composite*?
Question Card 2 If you earned a piece of candy for each *factor* in a number, would you rather find *factors* for 20 or 21? Explain why.
Question Card 3 Can you think of an example or a problem where 6 is a *multiple*?
Question Card 4 Can you think of a problem where 6 is a *factor*?

Script 18: Vocabulary Word: *square numbers*

- Do not have students complete an index card today. Instead, lead a discussion to compare *prime*, *composite,* and *square* numbers by drawing arrays for an example of each such as 7, 12, and 16.
- Share the attached chart, walking through each number asking students if the number is *prime* (have them say <u>yes</u> or <u>no</u>), and if so, asking them **how** they know. Continue on with discussing if the number is *composite* and/or a *square*. The explanation component of this exercise is the important part. Record their answers on the chart.

Number	Prime? Why or Why Not?	Composite? Why or Why Not?	Square? Why or Why Not?
9			
15			
19			
20			

Number	Prime?	Composite?	Square?
9	• No: It has factors other than 1 and itself. 1 x 9 3 x 3 9 x 1 (Be *careful here: all prime numbers are odd but not all odd numbers are prime.)*	• Yes: It has factors other than 1 and itself (1, 3, 9)	• Yes: It is possible to make an array for 9 that has the same number of rows as the number in each row, or a 3 x 3 square.
15	• No: It has factors other than 1 and itself. 1 x 15 3 x 5 15 x 1	• Yes: It has factors other than 1 and itself (1, 3, 5, 15) • Yes: Numbers that end with 5 and are greater than 5 are composite.	• No: there is no possible array for 15 that has the same number of rows as the number in each row.
19	• Yes: Its only factors are 1 and itself.	• No: It does not have any other factors than 1 and itself.	• No: There is no possible array for 19 that has the same number of rows as the number in each row.
20	• No: It has factors other than 1 and itself. 1 x 20 2 x 10 4 x 5, etc.	• Yes: It has factors other than 1 and itself • Yes: It is an even number and all even numbers are divisible by 2, in addition to 1 and itself	• No: There is no possible array for 20 that has the same number of rows as the number in each row

Script 19:
Cumulative Review Activities for *equivalent, digit, array, multiplication, product, multiples, factor, prime, composite, square number*

- Share each statement, one at a time.

- Ask students to use paper and pencil to answer each question *with yes or no*, and an *explanation of their answer.* (Accept explanations in the forms of sentences, example problems, or drawings.)
- Once adequate time has been allowed, ask students to share their answers for all students to see.
- In addition: Feel free to use any parts of previous scripts for these words that you did not have time to address earlier.

12 and 15 have an *equivalent* number of *factors*.
An *array* can be used to prove when a number is a *square number*.
20 is a multiple of 5
An *array* can be used to find a *product*.
A product is also a multiple.

APPENDIX B

SCOTT FORESMAN® UNIT TEST MATERIALS

Name Date

Landmarks and Large Numbers

End-of-Unit Assessment

Solve the following problems. Show your solution with clear and concise notation.

1.
$$\begin{array}{r} 1{,}405 \\ -\ \ 619 \\ \hline \end{array}$$

2. Yuki is saving to buy a new bicycle. He is keeping track of how much money he saves each month on a chart. This is how much he has saved so far.

January	$28.85
February	$52.00
March	$36.54

a. How much has Yuki saved altogether?

Show your solution below with clear and concise notation.

b. A new bicycle costs $149.95. How much more does Yuki need to buy the bicycle? Show your solution below with clear and concise notation.

Name Date

Factors, Multiples, and Arrays

End-of-Unit Assessment (page 1 of 2)

Problem 1

A. Solve this multiplication combination.

$$6 \times 9 =$$

Did you use another multiplication combination to help you get the answer? If you did, explain what combination you used and how it helped you find the product of 6×9.

B. Draw a picture of either arrays, objects, or cubes to show that your answer is correct.

C. Write a story to go with the problem 6×9.

Name Date

Factors, Multiples, and Arrays

End-of-Unit Assessment (page 2 of 2)

Problem 2

You have 36 cans of juice.

A. Show all the ways you can arrange these cans into arrays. You may either draw arrays in the space below or use grid paper.

B. List all the factors of 36.

APPENDIX C

VOCABULARY TESTS

Unit 5

Number___________________________

Finish Each Sentence

1. The math teacher asked his students to be ***concise*** when solving a problem because…

2. When Sam's math teacher asked him to solve a problem and show his ***solution***, he knew she wanted him to…

3. Max ***estimated*** the number of beans in a jar because…

4. Betty had a math story problem for homework. After she read the problem she wrote an ***expression*** because…

5. 20 is a ***multiple*** of 5 because

6. The ***place value*** of 3 in 13, 472 is 3,000 because

7. _____Which of the following is a ***numeral***?

a. +
b. <
c. 7
d. -

8. _____Which word goes with ***equivalent***?

a. Different

b. Same

c. More

d. Less

9. _____Which word goes with ***representation***?

a. Chart

b. Pencil

c. Notebook

d. Eraser

10. _____Which word goes with ***quantity***?

a. Method

b. Line

c. Amount

d. Round

11. _____Which word goes with ***algorithm***?

a. Chart

b. More

c. Amount

d. Method

Yes-No-Why: Read each sentence. If the sentence makes sense, write yes and explain why it makes sense. If it does not make sense, write no and explain why it does not make sense.

12. 187 ***rounded*** to the nearest hundred would be 100.

____

____

13. In 267, 6 is the ***digit*** in the tens place.

____

____

14. The ***sum*** of 60 and 20 is 40.

____

____

15. A ***number line*** shows distance between numbers.

____

____

Unit 1

Number____________________________

Finish Each Sentence

1. Instead of counting the number of boxes in an array, Sara used ***multiplication*** because she knew

____

____

2. Sara's math class has been making arrays. The class decided that 25 is a ***square number*** because

____

____

Multiple Choice

3. ____The word ***array*** means
a. subtraction
b. rectangular arrangement
c. pair
d. addition
4. ____ 2, 4, 6, 8, 10 are all ***multiples*** of
a. 1
b. 2
c. 3
d. 4

5. ____In a multiplication problem, the ***product*** is the
a. factor
b. sum
c. answer
d. divisor

6. ___ ***Equivalent*** means:
a. different
b. more
c. less
d. same

7. ____A ***symbol*** used to show a number:
a. comma
b. ÷
c. digit
d. >

Yes-No-Why: Read each sentence. If the sentence makes sense, write yes and explain why it makes sense. If it does not make sense, write no and explain why it does not make sense.

8. 17 is a ***prime*** number.

9. 16 is a ***composite*** number.

10. 3 is a ***factor*** of 9.

11. 20 is the ***product*** of 4 x 7.

12. 2 x 6 is ***equivalent*** to 3 x 4.

APPENDIX D

MATHEMATICS GAMES

Close to Zero (Learn With Math Games, 2013)

Materials:

- Deck of Numeral Cards and Wild Cards per group
- Close to 0 Score Sheet for each player

Players: 3 in a group—2 players and one game leader

Directions:

1. The game leader should shuffle the cards and then deal out 3 cards to each of the other 2 players.
2. All three players work together to make two 3-digit numbers whose difference is as close to zero as possible. These two numbers and their difference should be written on a score sheet.
3. Repeat 5 times. When all cards have been used, reshuffle them and start a new pile.
4. To determine a final score, the 5 differences should be added.
5. When all groups have completed their score sheet, the group with the lowest score wins!

	Close to Zero	Score
Round 1	____ ____ ____ - ____ ____ ____ =	____ ____ ____
Round 2	____ ____ ____ - ____ ____ ____ =	____ ____ ____
Round 3	____ ____ ____ - ____ ____ ____ =	____ ____ ____
Round 4	____ ____ ____ - ____ ____ ____ =	____ ____ ____
Round 5	____ ____ ____ - ____ ____ ____ =	____ ____ ____

Total:

Make the Smallest Number/Make the Largest Number (Kennedy, Tipps, & Johnson, 2008)

Materials: For the teacher: 1 set of number disks, 0 to 9

For students: scratch paper and pencil

Object of the Game: Students try to place randomly drawn numbers in order to make the smallest 3-digit number.

Directions:

The teacher asks students to draw 3 short lines on their paper to represent the ones, tens, and hundreds places. For example: _____ _____ _____

The teacher shuffles the number disks and draws out a number.

Students must write that number in one of their three spaces. Once the number is written it cannot be changed. Do not return this number to the draw pile--each number will be used only one time per game.

The teacher continues by drawing a second number and allowing students time to write that number in one of the two remaining spaces. Finally, a third number is drawn.

For example, if 7, 2, and 0 are drawn, possible 3-digit numbers would be 720, 702, 270, 207, 072, and 027. The winning number would be 27.

Each student with the smallest possible number awards themselves one point.

Play continues until a pre-determined score is reached. That player (or players) is the winner.

After playing a few rounds, stop and discuss possible strategies for winning the game.

Variations:

- Make the Largest Number

  Play is the same except that students try to make the largest number possible using the drawn number disks. Remember to discuss possible winning strategies after playing the first couple of rounds.

- Make Larger Numbers

  Determine the size of number you wish students to make and have students draw the appropriate number of spaces on their paper. For example, for numbers in the thousands, have them draw ____, ____ ____ ____. For numbers in the ten-thousands, have them draw

  ____ ____, ____ ____ ____.

  FYI: Use two sets of number disks for 6-digit and larger numbers. Remember: do not return drawn number to the draw pile.

Name That Number (Center for Elementary Mathematics and Science Education, (n.d.)

Materials: Number Cards 0-20

Players: 4

Directions:

1. Shuffle the deck.
2. Place 5 cards face up on the table. Leave the rest of the deck facedown.
3. Turn over the top card of the deck and lay the card down. The number on this card is the number to be “named”. Call this number the “target number”.
4. Players take turns. When it is your turn, try to name the target number. You can name the target number by adding or subtracting the numbers on 2 of the 5 cards that are face up.
5. If you can name the target number, take the 2 cards you used to name it. Also take the target-number card. Then replace all 3 cards by drawing from the top of the deck.
6. If you cannot name the target number, your turn is over.
7. Turn over the top card of the deck and lay it down. The number on this card becomes the new target number to be named.
8. Play continues until all of the cards in the deck have been turned over.
9. The player who has taken the most cards wins.

What's the Difference? (Learn With Math Games, 2013)

Skill--subtracting 3-digit numbers

Players--2 or more, or teacher *vs.* the entire group

Materials--2 sets of number cards, 0-9, per pair of students

The object of this game is to make the smallest difference. The teacher should first model the game by playing against the entire class. When students understand the game, allow pairs of students to play against one another.

- Place the deck face down in the middle of the playing area.
- Have students draw the following “game board” on paper: (This represents a 3-digit subtraction problem.)

- A player draws a card from the deck and places the drawn number card face up. Each player selects a space on their game board and writes the number of the card on that space.
- The player then draws five more cards, allowing time for the other players to write each number in a space. Once numbers are written in a space they cannot be changed. Example --The first card turned over is a 7. The second card turned over is a 2. The other cards turned over were 3, 6, 5, and another 2. Completed game boards could look like this:

Player 1	Player 2
522	657
-367	-232
155	425

Once players have filled in all spaces on their game boards, they complete the subtraction. The player with the smallest difference is the winner for the round and scores 1 point. In the example above, Player 1 has the smallest difference and is the winner of this round.

In the event of a tie, each player receives 1 point. Any negative difference causes that player to strike out for that round.

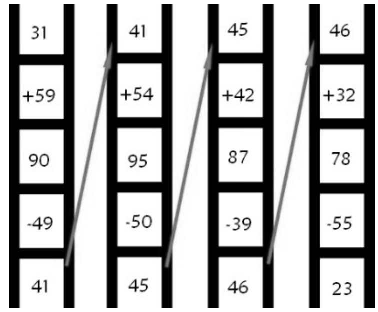
The winner of the game is the player with the most points after a set number of rounds or a set time limit.

As players gain more experience with this game, they will develop strategies to maximize their chances of winning.

Variation--Vary the number of cards to modify the level of difficulty. For example, draw 7 cards to create a 4-digit minus 3-digit number, such as

$$\begin{array}{r} 4,285 \\ \underline{-\ \ 107} \end{array}$$

Adder Ladders (Learn With Math Games, 2013)



Skill: 2 and 3-digit addition and subtraction

Number of players: 2

Object of the game:

Be the first player to correctly complete the game card.

Supplies:

- Copies of game cards
- A calculator

Directions for Teacher:

- Give each player a game card.
- Assign a range of addends for players to use, such as 1-50, or 600-800, etc., based on the level of addition and subtraction problems appropriate for their age/grade.
- Write the range on the board. Ask students to copy the range numbers in the box on the right side of their game card.
- Instruct each player to write any 7 numbers within the range in the box on the right of his or her opponent's game board. Return game boards to partners.

To Play:

1. Play should begin when both players have their own game board returned. Both players race to be the first to complete their ladders. Players add and subtract up and down the ladders, using as many of the numbers from the box as possible. A number may not be

repeated within a ladder. See the sample completed ladder and explanation on the sample page.

2. When the first player finishes, he calls out, “Done!” The second player should keep working to finish his game card.
3. When both players are done, each one checks his opponent’s work using the calculator. If the first finisher’s card has mistakes, then the other player receives the points for finishing first.
4. Points: 3 points for being the first to correctly complete the game card; 1 point for each number used from the box
5. The winner is the player with highest point total.

Factor Blaster (Kawas, 2010)

Rules

•Play as two teams. Team A chooses a number. That number is eliminated from the playing board and moved into the Team A column. Team A gets that many points.

• Team B identifies the factors of the number Team A chose. Any of these factors that are still on the playing board are moved into the Team B column. When they have moved all factors, Team B must say “no more factors.”

• If Team A finds a factor that has not been moved, they may steal that factor for their Team A column. If there are no more factors, Team A must say “no more factors.”

• Team B now chooses any number remaining on the playing board and moves that number into the Team B column.

• Team A must identify all of the factors of the number Team B chose. Any of these factors that are still on the playing board are moved into the Team A column. When they have moved all factors, Team A must say “no more factors.”

• If Team B finds a factor that has not been moved, they may steal that factor for their Team B column. If there are no more factors, Team B must say “no more factors.”

• Play continues in this way until all numbers have been removed from the playing board.

• The team with the most points wins.

FACTOR BLASTER
Game Mat

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

APPENDIX E

VOCABULARY OBSERVATION CHECKLIST

Classroom Physical Environment

Check or Tally		Notes
	Word wall present	
	List of word learning strategies posted	
	Specific vocabulary word list evident	
	Word activity projects posted	
	Manipulatives available for student use i.e., magnetic letters, magnetic words, etc	

Teacher indicates that vocabulary knowledge is important

Check or Tally		Notes
	Sense of excitement about word learning	
	Encourages students to monitor their attempts to use new words in daily conversations and/or writing	
	Encourages students to use rich vocabulary in class discussions	
	Discusses need for vocabulary improvement	

Teacher models strategies for exploring and practicing words

Check or Tally		Notes
	Contextual Analysis	
	Morphemic Analysis	
	Use of dictionary	
	Teacher reminds students to apply skills when reading on their own	

Vocabulary Introduction Activities

Check or Tally		Notes
	Present/assign words with little or no explanation	
	Teach word before teaching concept	
	Read aloud with context for new words	
	Discussion of interesting words	
	Word relationships and connections made evident	
	Description, explanation, or examples provided for new words	
	Students asked to restate the description, explanation, or example in own words	
	Use of realia, pictures, etc. to teach new words	

Direct Review/Extension Activities

Check or Tally		Notes
	Multiple exposures; different text but same words	
	Multiple exposures; activities to reinforce definitional or contextual information (Beck-like activities)	
	Word sorts	
	Word family work	
	Use of graphic organizers: word maps, etc.	
	Students asked to discuss concepts or terms with one another	
	Students encouraged to create images, sketches, or models (as with clay or paper) to support word meaning	
	Acting out concepts or using song or story.	
	ELL students reminded to connect English words to native language counterparts	
	Using digital resources such as electronic libraries, desktop publishing, word games, and word processing.	
	Word card games (commercial or teacher-made)	
	Commercial games like Scrabble, Scrabble Junior, or Boggle.	
	Worksheet "games" (word finds, crossword puzzles, acrostics, etc.)	
	Worksheet of vocabulary words; fill-in-blank, matching, etc.	
	Cloze activities	

	Vocabulary journal	
	Structure for students to collect words they find interesting	
	Dictionary, thesaurus work	
	Writing (stories, poems, etc. as long as required to incorporate new vocabulary)	
	Homework: look up words and/or write sentences as initial activity	
	Making models to represent vocabulary/concepts	

Indirect Review/Extension Activities

Check or Tally		Notes
	Independent reading time	
	Morphemic Analysis	
	Use of dictionary	
	Reference to word(s) previously taught	
	Bulletin Board	
	Casual questions or references to prior discussion/teaching	

APPENDIX F

TRAINING AGENDA

Session 1:

I. Overview of Study and Selection of Folders to Determine Assignment (15 minutes)

II. Procedures for Data Collection (45 minutes)

Session II:

III. Training: Importance of Vocabulary (15 minutes)

Vocabulary Research

Common Core Standards and Vocabulary

NCTM Principles and Standards and Vocabulary

IV. Training: Academic Vocabulary (15 minutes)

What is academic vocabulary?

How does instruction for academic vocabulary differ from vocabulary for reading?

V. Training: Rich Instruction for Vocabulary Instruction (1 hour)

What comprises “rich” instruction?

Introduction to Vocabulary Routines

Researcher Modeling of Routines

VI. Participating Teacher Practice Using Vocabulary Routines (20 minutes)

VII. Closing Remarks and Questions (10 minutes)

APPENDIX G

VOCABULARY QUADRANT CARD

Word:________________________________

Definition	Example
Drawing or Symbol	Sentence

APPENDIX H

UNIT MATH TEST SCORING GUIDE

Unit 5

1. Solve and show solution with clear and concise notation.

3 pts	2 pts	1 pt	0
Traditional algorithm Correct Answer:	Computational error in one place	Computational errors in 2 places	Computation not close and/or answer more than 1405

2. Yuki is saving to buy a new bicycle. He is keeping track….this is how much he has saved thus far (chart included for 3 months)

a. How much has Yuki saved altogether? Show solution with clear and concise….

3 pts	2 pts	1 pt	0
Traditional algorithm Correct answer: $117.39	Computational error in one place Or Missing $ sign	Computational errors in 2 places (including missing $ sign)	Not addition Or Computation errors in 3 places Or answer not reasonable Or Solution not given

b. A new bicycle costs $149.95. How much more does Yuki need to buy the bicycle? Show your solution below with clear and concise…

3 pts	2 pts	1 pt	0
Traditional algorithm:	Correct total from 2a, with one subtraction error or no $ sign Or Incorrect total from 2a but subtraction is correct	Correct total from 2a with computational errors in 2 places, $ sign present Or Correct total from 2a with one subtraction error and missing $ sign Or Incorrect total from 2a but subtraction is correct; missing $ sign	Not subtraction

Unit 1

Problem 1

A. 6 x 9 =

Acceptable	Not acceptable
• Correct answer of 54	• Anything other than 54

B. Draw a picture of either arrays, objects, or cubes to show that your answer is correct.

Acceptable	Not acceptable
• An array depicting either 6 sets or 9 or 9 sets of 6 • 6 sets with 9 in each set (or the opposite)	• Anything other than an array, groups, or cubes. Ex: tally marks in groups of 5

C. Write a story to go with the problem 6 x 9.

Acceptable	Not acceptable
• Must include 6 sets of 9 or 9 sets of 6 • Must include either a question (ex: how many in all?) or a statement that the answer is 54 • Problem where a person had 6 of some object while another person had 9 times as many. Accept this as long as there is a question or a statement concerning how many the second person has (54)	• In reference to the third item to the left: Asking how many more…..

Problem 2

You have 36 cans of juice.

A. Show all the ways you can arrange these cans into arrays. You may either draw arrays in the space below or use grid paper.

Acceptable	Not acceptable
• One point for each of the possible 5 arrays 1. 1 x 36 2. 2 x 18 3. 3 x 12 4. 4 x 9 5. 6 x 6	• Anything equaling other than 36

B. List all the factors of 36.

Acceptable	Not acceptable
• To get 1 point, all factors must be listed with no incorrect factors. 1, 2, 3, 4, 6, 9, 12, 18, 36	• Anything other than the 9 correct answers

APPENDIX I

VOCABULARY TEST SCORING GUIDES

Unit 5
Number__________________________
Finish Each Sentence

1. The math teacher asked his students to be ***concise*** when solving a problem because…

Acceptable:	Unacceptable:
• Teacher did not want any unnecessary words • Teacher did not want any unnecessary numbers • Teacher wanted work to be clear	• Be sure your answer is right/correct.

2. When Sam's math teacher asked him to solve a problem and show his ***solution***, he knew she wanted him to…

Acceptable:	Unacceptable:
• Show his work • Show his answer to the problem • Work/solve the problem	• Be sure your answer is right/correct.

3. Max ***estimated*** the number of beans in a jar because…

Acceptable:	Unacceptable:
• He couldn't count them • He didn't know so he guessed • He didn't know how many there were • To be sure there was enough for everybody • As a 1st step of PS "He is trying to figure out the answer"	• Be sure your answer is right/correct.

4. Betty had a math story problem for homework. After she read the problem she wrote an ***expression*** because…

Acceptable:	Unacceptable:
• She wanted to show how she got the answer • She had to solve it/wanted to solve it • She showed the problem to be solved ex: ten divided by two would be 10 ÷ 2 • It showed the information from the problem • It helped her know what to do next (how to solve the problem)	• Be sure your answer is right/correct.

5. 20 is a ***multiple*** of 5 because

Acceptable:	Unacceptable:
• 5 x 4 = 20 • Reference to skip counting by 5	• because it is an even number (because 2, 4, 6, and 8 are even numbers but they are not multiples of 5)

6. The ***place value*** of 3 in 13, 472 is 3,000 because

Acceptable:	Unacceptable:
• Must have reference to "thousand's place" • 3 x 1,000 • It is in the thousand's place • That is the expanded form,	• Be sure your answer is right/correct.

7. __C___Which of the following is a ***numeral***?

a. +
b. <
c. 7
d. -

8. __B_Which word goes with ***equivalent***?

a. a. Different
b. Same
c. More
d. Less

9. __A___Which word goes with ***representation***?

a. Chart
b. Pencil
c. Notebook
d. Eraser

10. __C___Which word goes with ***quantity***?

a. Method
b. Line
c. Amount
d. Round

11. ___D__Which word goes with ***algorithm***?

a. Chart
b. More
c. Amount
d. Method

Yes-No-Why: Read each sentence. If the sentence makes sense, write yes and explain why it makes sense. If it does not make sense, write no and explain why it does not make sense. EACH MUST HAVE ANSWER OF YES OR NO OR AN EQUIVALENT

12. 187 ***rounded*** to the nearest hundred would be 100.

Acceptable:	Unacceptable:
• No • It is not closer to 100 • It is closer to 200 • If the number is a 1, 2, 3, or 4, go down	• Yes

13. In 267, 6 is the ***digit*** in the tens place.

Acceptable:	Unacceptable:
• Reference to 10's place/location or place value chart • It's value is 60 • the 2 is in the 100s, 7 in ones, and 6 in 10s	• If you draw a chart it will be in the ten's place. (Saying without drawing is just repeating what the stem stated.) • The ten's place is in the middle (true for a 3-digit number only.)

• drawing of place value chart with numbers in the chart	• The 2 is in the hundred's place and the 7 is in the one's place (made no reference to where thee 6 was located or to its value)

14. The ***sum*** of 60 and 20 is 40.

Acceptable:	Unacceptable:
• NO • The sum would be 80 • Sum means to add and they subtracted • 40 isn't the sum because it isn't larger than 60	• Yes

15. A ***number line*** shows distance between numbers.

Acceptable:	Unacceptable:
• YES • a drawing or reference to equal distances between numbers • shows how far apart numbers are • used the word "divided" • used the phrase "skip counting" • OK if they compared it to miles	• No • Using the same phrase as used in the statement: "shows distance between numbers"

Unit 1

Finish Each Sentence

1. Instead of counting the number of boxes in an ***array***, Sara used ***multiplication*** because she knew

Acceptable	Not Acceptable
• She could multiply the number of rows by the number in each row • It is easier/quicker/good strategy • It would give her the same answer/number • An array is a multiplication picture	• Multiplication is repeated addition • Any reference to skip counting

2. Sara's math class has been making ***arrays.*** The class decided that 25 is a ***square*** number because

Acceptable	Not Acceptable
• The array makes a square • 5 x 5=25	• Multiplication is repeated addition • Any reference to skip counting

Multiple Choice

3. __B__The word ***array*** means
a. subtraction
b. rectangular arrangement
c. pair
d. addition

4. __B__ 2, 4, 6, 8, 10 are all ***multiples*** of
a. 1
b. 2
c. 3
d. 4

5. __C__In a multiplication problem, the ***product*** is the
a. factor
b. sum
c. answer
d. divisor

6. __D_ ***equivalent*** means:
a. different
b. more
c. less
d. same
7. __C__A symbol used to show a number:
a. comma
b. ÷
c. digit
d. >

Yes-No-Why: Read each sentence. If the sentence makes sense, write yes and explain why it makes sense. If it does not make sense, write no and explain why it does not make sense.

8. 17 is a ***prime*** number.

Acceptable	Not Acceptable
• It is an odd number and all odd numbers are prime • Only divisible by 1 and itself (or only factor pair is 1 and 16)	• Only has one factor (when they mean only 1 factor PAIR)

9. 16 is a ***composite*** number.

Acceptable	Not Acceptable
• It is an even number • Has more factors than 1 and itself • 1, 2, 4, 8, 16 (if they list the factors)	• has two factors (when they mean 2 factor pairs) • 4 x 4 = 16 (and they don't mention 1 and the number itself)

10. 3 is a ***factor*** of 9.

Acceptable---YES	Not Acceptable
• skip count by 3s will come to 9 • 3 x 3 = 9	• No as an answer

11. 20 is the ***product*** of 4 x 7.

Acceptable---NO	Not Acceptable
• 4 x 7 is 28 (or it is not 20) • 20 is not the product of 4 x 7	• Yes as an answer

12. 2 x 6 is ***equivalent*** to 3 x 4.

Acceptable---YES	Not Acceptable
• Both equal 12 • Both have the same answer/value • 2 x 6 = 12 and 3 x 4 = 12	• It is the same (when the student doesn't define "it".) • Repeating verbiage of the stem, for ex: yes, 2 x 6 is equivalent to 3 x 4

APPENDIX J

DATA COLLECTION FORM

Stud't No. & Letter	Gender	2014 Aspire Math Score	2014 Aspire Rding Score	Pre-Test Unit 1	Post-Test Unit 1	Pre-Test Unit 5	Post-Test Unit 5	Pre-Test Vocab Unit 1	Post-Test Vocab Unit 1	Pre-Test Vocab Unit 5	Post-Test Vocab Unit 5