Wireless Communication of a Chaotic Waveform

by

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Abstract

Over the past couple of decades, mathematical equations have been developed that successfully model natural phenomena that occur in various fields such as chemistry, biology, and fluid mechanics. These equations, however simple, produce complicated solutions with occasional unpredictable behavior. Analysis and research of these models and other models that exhibit such response has been deemed "chaos theory". One of the requirements for a system to be considered chaotic is that the system must be sensitive to initial conditions. The term "chaos theory" implies that an exact outcome is incomprehensible, leading to a common assumption that an analytic solution is unattainable for chaotic systems. This expectation has since been refuted as an exact solution has been derived for some chaotic systems. This unlocked the potential for a chaotic system to be implemented in practical applications. One such novel technique has arisen in the form of an electronic circuit. This circuit has been designed to oscillate in a chaotic manner and possess an exact solution that can be calculated.

Due to sensitivity intrinsic to all chaotic systems, small perturbations can be used to control the chaotic oscillation. Because the chaotic oscillation can be controlled and its response determined, a circuit can be employed as a form of a modulator in encoding and encryption in communication systems. Common communication systems, subsystems and circuits are discussed along with analog modulation and demodulation techniques. A transmitting and receiving is circuit are detailed that successfully presents the transmission of a chaotic waveform in a wireless medium.

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Chapter 1

Introduction

In 1926, Gilbert Vernam documents one of the first attempts at secure communications by combining plain text with pseudorandom code to create a ciphertext [27]. Decades later this process was revisited by independent research groups from which the first chaotic circuit was realized. Created by Leon Chua, the Chua oscillator was able to prove that synchronized chaos is, in fact, a possible and viable means of secure communication and various applications [1–7].

1.1 Chaos Theory

Chaos theory is the study of deterministic nonlinear dynamical systems exhibiting unstable and aperiodic behavior. An unstable system is unable to achieve and operate in a steady state that can endure small perturbations. Aperiodic behavior is described as the recurrence of variables in an irregular manner. Therefore a system classified as unstable and aperiodic describes a system that is unable to repeat and reacts to the presence of small disturbances. While complex, simple mathematical systems have been shown to produce unstable aperiodic behavior making dynamical systems of interest to chaos theory.

Differentiable dynamical systems are prevalent, characterized by consistent changes in variables. The evolution of these variables can be expressed by a set of differential equations, occasionally referred to as "evolution equations". For dynamical systems expressed mathematically, analytical solutions can be generated to produce immediate, future and, in some cases, past states of system. This provides a straightforward model of the time dependency of actual systems. Until recently, a closed-form solution to nonlinear dynamical systems seemed unattainable or at the very least impractical. During this time, much research drifted away from deriving an exact solution and sought to provide information about the general nature and long-term behavior of chaotic systems. [8]

1.2 From Academia to Application

Considered widely as purely academic, many thought that chaos theory lacked any form of application. It was long believed that chaos was simply an unwanted occurrence that interfered with the normal operations of a system. As a result, the intent of those studying chaos worked toward reducing or even removing chaos from the system designs. However chaos it not easily avoided and can be encountered even in commonly encountered systems. Weather dynamics, for example, is a system that functions at a large-scale, encompassing numerous sources of dynamical systems. On the micro-scale, chaos has been shown to control brain dynamics in mammals. Due to its ubiquitous nature, one has to question if chaos is truly limited to academia or is it possible to influence these chaotic systems externally.

Considering the instability in a chaotic system, one could deduce that due to its sensitivity to initial and operating conditions, a pair of synchronized systems is unfeasible. In fact this challenge has already been attempted and confirmed the original assessment. Two nearly identical Chua oscillators were assembled, but due to small differences, an exact reproduction was unattainable. [6]

1.3 Exactly Solvable Chaotic Oscillator

It was assumed that the complex nature of chaos prevents any form of precise solution [9, 10]. However an innovative chaotic oscillator was constructed that conceded an exact solution despite this common assumption [1–6, 17, 21, 22]. The chaotic oscillator described can be modeled as a continuously differential equation with discrete switching conditions. The circuit can be represented as a second order differential equation joined with finite switching conditions used to control a binary value. This combination of continuous and discrete properties makes the oscillator a hybrid system. Between switching events, the system can be described as a linear system expressed by a set of differential equations containing piecewise constant arguments and providing the "folding" required for chaotic oscillation [12, 14, 19]. As a result, the system is unstable between switching events, critical for the generation of chaos. Through linear convolution of a binary sequence and a fixed basis pulses, this circuit concedes an analytically exact solution [18].

1.3.1 Chaotic Hybrid System

There exist two types of hybrid chaotic systems of this nature. One is constructed with regards to a shift map while the other is made with respect to a folded band map. The hybrid system based on a folded band map was selected due to difficulties executing the encoding process in the other systems.

While achieving an exactly-solvable chaotic oscillator is paramount, a method of controlling the oscillations is required for proper encryption. In order for a chaotic system to operate, two crucial properties must be present: a positive Lyapunov exponent and a guard condition. A positive Lyapunov exponent causes the system to exponentially diverge while a negative exponent will converge to a known state. The magnitude of this exponent determines the rate of convergence or divergence. A guard condition ensures that the system oscillates within certain parameters. The hybrid system can be expressed by the following equations:

$$\ddot{u} - 2\beta\dot{u} + (\omega^2 - \beta^2)(u - s) = 0 \tag{1.1}$$

$$\dot{u}(t) = 0 \Rightarrow s(t) = H[u(t) - 1] \tag{1.2}$$

where:

$$H[x] = \begin{cases} 1 & \text{if } x > 1 \\ 0 & \text{if } x \leq 0 \end{cases} \tag{1.3}$$

where $u(t)$ is the system output and a Heaviside step function, commonly called the unit step function, represented as $H[x]$. In this hybrid topology the variable, beta, is synonymous with the Lyapunov exponent, controlling the systems oscillation. By setting $\beta > 0$ the oscillations become negatively damped and progressively increase the oscillations magnitude. These oscillations occur at one of the two specific points called instantaneous equilibrium points, represented by s(t). The guard condition, previously mentioned, keeps these oscillations from continually growing by switching the oscillations equilibrium point. A demonstration provides a better method at comprehending the discrete-time portion of the hybrid system and its role within the system. The following is an example provided by Beal *et al.* [5]

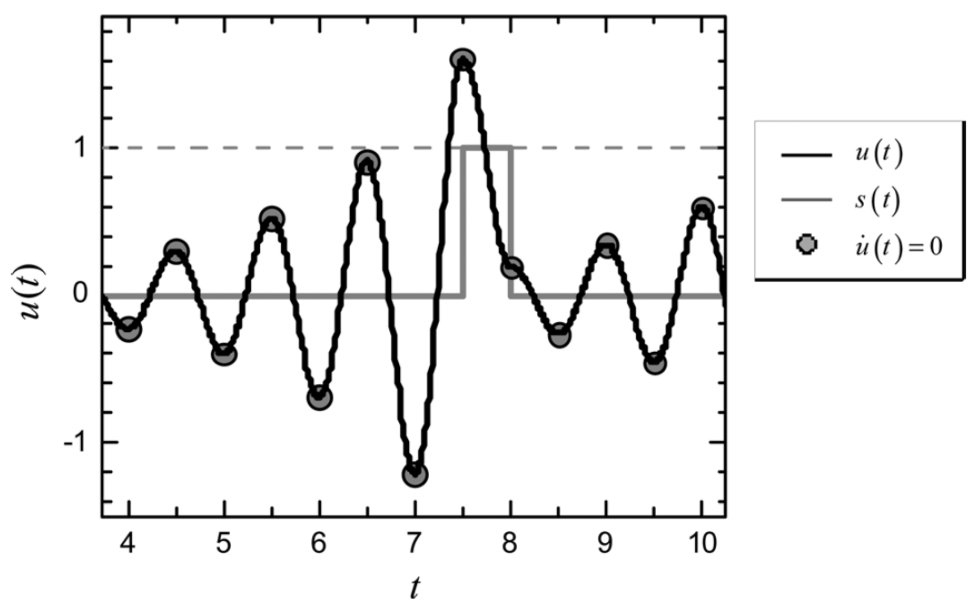

Figure 1.1: Waveform segment illustrating the hybrid dynamics [16].

Assume that the instantaneous equilibrium point, s, is equal to 0 initially. As a result the output, u, begins to oscillate and grow about this point. If beta happened to be negative, the output would settle to 0. Since this is not the case, the output will grow, diverging from this value. The guard condition is prompted whenever the derivative of the output becomes 0, i.e. when the output reaches a local maximum or minimum. When a guard condition occurs and the output, $u(t)$, is greater than 1, the instantaneous equilibrium point, $s(t)$, is set to 1 otherwise the $s(t)$ remains at 0. Illustrated in Figure 1.1, the output is less than 1 for seven guard events. By the eighth guard event the output becomes greater than 1, toggling $s(t)$ to 1. This equilibrium point will only last until the next guard event where as the output has fallen below 1, returning $s(t)$ back to 0, allowing for the output to oscillate about 0 once again. Consequently, $u(t)$ is an exact solution when described as the superposition of weighted deterministic basis pulses $Q(t)$.

$$u(t) = \sum_{m=0}^{\infty} \sigma_m Q(t - t_n - \frac{m}{2}) \tag{1.4}$$

In equation 1.4 the variable σ_m represents the value of $s(t)$ at the guard event that occurs every $^1/_2$ unit of time. By employing the "inverse coding function"

$$u_n = (1 + e^{-\beta/2}) \sum_{m=0}^{\infty} \sigma_m (-e^{-\beta/2})^m, \tag{1.5}$$

one is able to obtain a sequence of σ_m using a stated initial condition, u_n. Conversely an initial condition can be calculated using a specified σ_m series. Therefore by setting the initial state or by adjusting the current state, a desired sequence of symbols can be generated. This process was first proposed by Scott Hayes and as a result is referred to as Hayes-type chaos communication. [7,21,22] Using this approach, minimal effort is needed by a circuit to encode information into the chaotic signal.

Due to the relationship between initial and future parameters, the initial output, u_n, and the initial equilibrium point, s_n, can be used to express the output at the first guard event following the initial state, $u(t_n +^1 /_2)$. As a result, a tent map can be used to show the relationship between u_n, and $u(t_n +^1 /_2)$.

The tent map for this chaotic oscillator can be divided into three sections: A, B and C (see Figure 1.2).

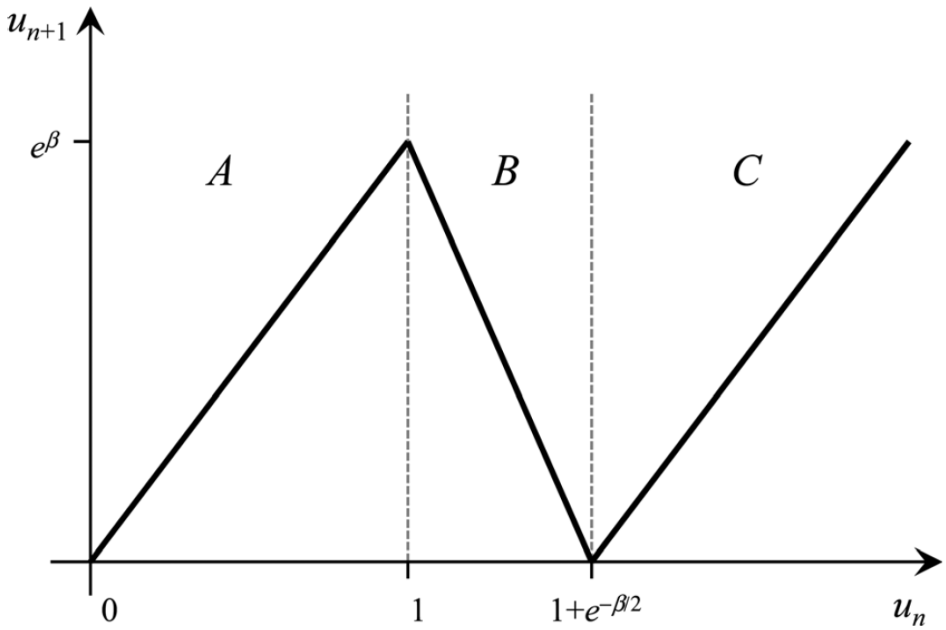

Figure 1.2: Successive maxima return map [16].

Note that the segments are not evenly spaced. Each letter represents a binary sequence of guard events transpired by the instantaneous equilibrium point, $s(t)$: $A = 00$, $B = 100$ and $C = 10$. To simplify the structure of the receiver, the symbol C is not utilized. If, for example, one would like to encode the character string $\{A, A, B, A \dots\}$. By applying the inverse coding function to an equilibrium point weight, σ_m, pattern $\{000010000\dots\}$, the necessary initial condition, u_n, can be calculated. Clearly, the farther ahead in time that the symbols are being scheduled, the smaller the change in the current u_n state required to ensure that they are reliably obtained as the oscillator passively evolves.

With an understanding of the dynamical nature of the system, an equation can be created from equations 1.1, 1.2 and 1.3:

$$\ddot{u} - [2\beta - GH(u - h)]\dot{u} + (\omega^2 - \beta^2)(u - s) = 0 \tag{1.6}$$

Where G is a large positive number and a maximum peak return value is specified by variable h, calculated via equation 1.5. Once the output surpasses this value, a positive damping effect is applied and only removed once the output falls below this threshold.

1.3.2 Matched Filter

In order for the exactly-solvable folded-band chaotic oscillator described in this paper to be used as a viable method of communication, a receiving circuit must include a matched filter. Matched filters are created to maximize signal-to-noise ratio, making them the most important stage in many communication systems. The design of a matched filter allows for a desired signal to be recovered even in the presence of broadband interference, such as white Gaussian noise. For the chaotic oscillator described, the matched filter allows for information encoded within the chaotic signal to be properly extracted. In order for a matched filter to operate, the receiver must be aware of the desired signal's structure, classifying matched filters as coherent signal processors. The impulse response for a matched filter can be expressed as the time-reversed and T-translated waveform of the signal [20]. Just as the oscillator can be expressed as a set of differential equations, the matched filter for the exactly-solvable folded-band chaotic oscillator can be articulated by the following equations:

$$\dot{\eta} = [u(t + \frac{1}{2}) - u(t)] \tag{1.7}$$

$$\ddot{\xi} + 2\beta\dot{\xi} + (\omega^2 + \beta^2)\xi = (\omega^2 + \beta^2)\eta(t) \tag{1.8}$$

The matched filter produces an output, ξ, after a basis pulse, $Q(t)$, is applied to the circuit. Seeing as the match filter is designed to operate in conjunction with the chaotic oscillator, similarities can be made when comparing these equations to that of the oscillator.

While a digital processing approach is possible, the matched filter was implemented using an analog delay circuit followed by an inductor-capacitor circuit.

By increasing the operating frequency to the radio frequency spectrum, the chaotic oscillator becomes more appealing for wireless communication applications. Within this range, existing RF communication methods can benefit from the chaotic oscillators inherent broad spectrum aspect and also achieve acceptable data rate standards.

Chapter 2

Communication Systems

For basic wireless communication, a transmitter and receiver are required. By combining the transmitter and receiver into one unit, a *transceiver* is created, allowing two-way communication. Amateur radio (*ham radio*) and citizens band radio service (*CB radio*) are two examples of the transceiver's noncommercial applications [13].

2.1 Modulation

The mechanical energy carried by sound waves is only able to traverse a short distance. However if this mechanical signal was converted to an audio-frequency AC signal it would be able to travel even further using an antenna. Yet two problem exist that prevent this process.

The first issue is encountered when trying to radiate the waveform from an antenna. The functionality of an antenna is dependent on frequency, specifically the wavelength of the signal being considered. In order for the antenna to radiate effectively, the size of the antenna needs to be designed to a size appropriate for the signal's wavelength. While this size is dependent on the antenna type, its established that: the size of an antenna is proportional to wavelength. For example, in order to transmit a 3 kHz signal, an antenna of roughly 16 miles long is required. [13]

Assuming an antenna was able to properly broadcast a signal with the audio frequency range, an additional problem would be experienced. Suppose multiple stations within an area transmitted different signals but still within the same audible range of frequencies. The signals would overlap, creating an indistinguishable clutter of signals. Through the process of signal modulation, these issues can be overcome. In this instance, modulation would allow for stations to modify different signals to carry information so that information can accurately be extracted. A signal used to carry information is aptly called a carrier signal or simply a "carrier" and the process of positioning information on a carrier is modulation. As the name implies, an aspect of the carrier is modified with respect to a specific information signal [13]. There are two basic approaches when considering analog modulation: amplitude or angle. Exact options can be further divided. Double side-band, single-sideband, and vestigial side-band modulation are all derivatives of amplitude modulation. Phase and frequency modulation are both classified as angle modulation.

2.1.1 Amplitude Modulation

Amplitude modulation is accomplished by changing the magnitude (amplitude) of a high frequency sinusoidal wave with respect to a message signal, which is at a much lower frequency. The sinusoidal wave and message signal are commonly referred to as the carrier wave, $c(t)$, and modulating wave, $m(t)$, respectively. Since the carrier wave is a sinusoidal wave, it can be expressed as

$$c(t) = A_c \cos(2\pi f_c t)$$

where A_c is the peak amplitude of the carrier wave and f_c represents the carrier frequency. In order to produce an AM wave, the modulating wave and carrier wave are, from a numerical perspective, multiplied together. The resulting AM wave, $s(t)$, can be written as

$$s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$

The portion of the AM wave equation, $[1 + k_a m(t)]$, corresponds to the amount of modulation experienced by the carrier and must stay positive. This ensures the resulting AM wave is always positive. If the wave was to ever become negative, information would experience a level of distortion. By performing a Fourier Transform on the AM wave:

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2}[M(f_c - f) + M(f_c + f)]$$

and plotting the resulting equation, the message signal is now positioned with respect to the carrier frequency instead of the baseband frequency of zero, as seen in Figure 2.1. The new created AM signal is then sent through a bandpass filter to remove signals that were inserted or created during the modulation process. The frequencies that are under (fc-fm) and above (fc+fm) are attenuated by the bandpass filter to the point that these signals can be considered as removed from the AM signal. To recover the message embedded in the AM signal, the transmitted signal is mixed yet again with a carrier signal, identical to the signal used to create the AM wave. Once again two signals are produced that are centered with respect to the frequency difference and the summation of the two carrier signals. Because the carrier waves are identical and the difference between the two frequencies is zero, the signal produced by the difference is the initial message reconstructed. To fully retrieve this message, the signal from the second mixer is sent through a low pass filter. The low pass filter attenuates the second signal located at a frequency that is twice that of the carrier signal, leaving the original message.

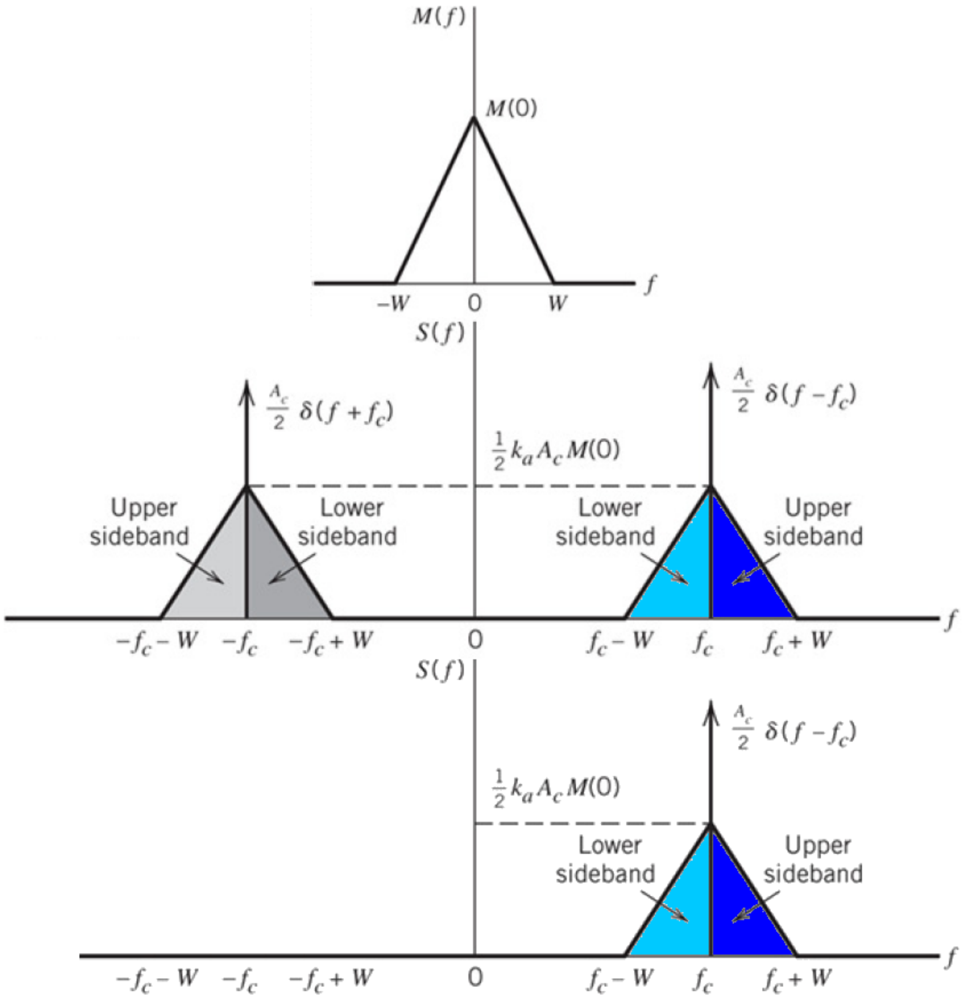

Figure 2.1: The top plot represents an arbitrary message signal within the frequency domain. The middle plot depicts how the message is duplicated and shifted, with half of its power centered on the carrier frequency, fc. Using a bandpass filter the desired information, highlighted in blue, is isolated so that it can be transmitted, illustrated by the bottom plot.

The process of distorting a carrier waveform for amplitude modulation can only occur in nonlinear circuits. After this process occurs it is mathematically and experimental proven that the resulting amplitude modulated waveform contains frequency components at multiple locations within the frequency spectrum. A resulting waveform is comprised of the following [13]:

1. A DC value

2. A portion of the original carrier

3. A portion of the original information signal

4. Carrier harmonics

5. Message harmonics

6. A signal whose frequency is equal to the sum of the carrier and information signal frequencies

7. A signal whose frequency is equal to the difference between the carrier and information signal frequencies

2.1.2 Angle Modulation

Amplitude modulation has many disadvantages. The sidebands created from the modulation process are identical and therefore an inefficient use of the frequency spectrum. AM systems are inherently noisy. Noise induced by external sources or within a radio receiver can distort a desired waveform and hinder the recovery of a desired waveform. “Increased knowledge of the principles of radio led to the development of a method for transmitting information by modulating the frequency of a carrier rather than its amplitude” [13]. With frequency modulation the effects of noise are subdued and it is able to evade the effects of AM-type noise. As a revolutionary concept, systems and applications for amplitude modulation evolved, accommodating needs as they were encountered. Frequency modulation systems, however, were designed with consideration to the implementation. Because ample attention was given during the design process, FM systems are widely accepted as low-noise with a high-fidelity.

Angle modulation refers to the process of changing the frequency or phase of a carrier signal with respect to a message signal. Consider a resulting angle modulated wave, $s(t)$

$$s(t) = A_c \cos[\theta(t)] \tag{2.1}$$

Unlike amplitude modulation, the carrier's amplitude, A_c, remains constant. In an angle modulated waveform, the angular argument, $\theta(t)$, is altered by a desired message signal, $m(t)$. There are numerous ways of performing angle modulation but the two methods are examined: phase modulation and frequency modulation. [23]

Phase modulation (PM) is accomplished when the angular argument, $\theta(t)$, is linearly altered by the message signal, $m(t)$. The angular argument of a PM wave can be written as the sum of the carrier's angular argument, $2\pi f_c t$, and the message waveform.

$$\theta(t) = 2\pi f_c t + k_p m(t) \tag{2.2}$$

Equation 2.2 assumes that the angular value of the unmodulated carrier wave is zero for the initial condition $t = 0$. The message signal is augmented by a constant known as the phase sensitivity, k_p, (rad/V). By inserting equation 2.2 into equation 2.1, a resulting phase-modulated waveform can be expressed as

$$s(t) = A_c \cos[2\pi f_c t + k_p m(t)] \tag{2.3}$$

Frequency modulation (FM) occurs when the message signal, $m(t)$ linearly alters the instantaneous frequency, $f_i(t)$. The instantaneous frequency can be written

$$f_i(t) = f_c + k_f m(t)] \tag{2.4}$$

which is the sum of the carrier frequency and the message signal. Similar to equation 2.2, the message signal is altered by the frequency sensitivity constant k_f (Hz/V). Again assuming that the angular value of the unmodulated carrier wave is zero for the initial condition $t = 0$, the angular argument for an FM signal can be calculated by integrating equation 2.4 with respect to time and multiplying the result by 2π [23]

$$\theta(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(t)dt] \tag{2.5}$$

Combining equation 2.5 and equation 2.1, a time dependent expression of an FM wave is realized.

$$s(t) = A_c \cos[2\pi f_c t + 2\pi k_f \int_0^t m(t)dt] \tag{2.6}$$

An important quality is noticed when examining equation 2.3 and 2.6: an FM or PM wave can be regarded as a PM or FM wave, respectively. An FM wave can be created by integrating the message signal before being sent to a phase modulator. Conversely, a PM wave can be generated by sending a differentiated message through a frequency modulator. For this reason only FM waves will be discussed.

By examining equation 2.6, the message signal, $m(t)$, undergoes a nonlinear process and, as a result, makes the process of frequency modulation nonlinear. Unlike amplitude modulation, the spectral representation of an FM wave is dependent on time. In order to understand this property, the most basic example is considered: single-tone modulation. The message signal, $m(t)$, used to modulate a carrier frequency can be expressed as the following

$$m(t) = A_m \cos(2\pi f_m t) \tag{2.7}$$

After modulation is accomplished, an FM waveform is produced with an instantaneous frequency of

$$\begin{aligned} f_i(t) &= f_c + k_f A_m \cos(2\pi f_m t) \\ &= f_c + \Delta f \cos(2\pi f_m t) \end{aligned} \tag{2.8}$$

where

$$\Delta f = k_f A_m \tag{2.9}$$

The frequency deviation, Δf, represents the change in center frequency of the FM wave and is proportional to the modulating signal's magnitude. The carrier frequency, f_c, determines the resulting frequency modulated signal's fundamental frequency, the instantaneous frequency of the FM wave varies with respect to the information signal. While the amplitude of the message performs a variance from the fundamental frequency, the frequency of an FM wave can be expressed as the rate of deviation, f_m. The rate of deviation can be calculated by the number of times the frequency changes within one second. This change however is relatively small when compared to the fundamental frequency. [13]

The phase argument, $\theta(t)$, of an FM wave could be extracted from equation 2.8

$$\theta(t) = 2\pi \int_0^t f_i(t)dt = 2\pi f_c t + \frac{\Delta f}{f_m}\sin(2\pi f_m t) \tag{2.10}$$

The modulation index for FM waves is calculated by dividing the frequency deviation, Δf, by the modulation frequency, f_m. Variable β is used to represent the modulation index. Equation 2.10 can be restated as

$$\theta(t) = 2\pi f_c t + \beta \sin(2\pi f_m t) \tag{2.11}$$

In this form, it's clear that parameter, β, determines the maximum departure of the angular argument $\theta(t)$ from the angle $2\pi f_c t$ of the unmodulated carrier. It's important to note that the modulation index, β, is not a constant but a function of the message frequency and message amplitude. [23]

When using a sinusoidal waveform, a frequency modulated signal can be expressed as

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)] \tag{2.12}$$

This equation can be expanded using a trigonometric identity

$$s(t) = A_c\cos(2\pi f_c t)\cos[\beta \sin(2\pi f_m t)] - A_c\sin(2\pi f_c t)\sin[\beta \sin(2\pi f_m t)] \tag{2.13}$$

In this form the resulting signal can be expressed as can the difference of two components: in-phase and quadrature.

$$\begin{aligned} s_I(t) &= A_c\cos[\beta \sin(2\pi f_m t)] \\ s_Q(t) &= A_c\sin[\beta \sin(2\pi f_m t)] \end{aligned} \tag{2.14}$$

By adding these components together, an equation is made to describe the complex envelope of an FM signal

$$\begin{aligned} \bar{s}(t) &= s_I(t) + js_Q(t) \\ &= A_c\exp[j\beta \sin(2\pi f_m t)] \end{aligned} \tag{2.15}$$

While the complex envelope does not retain all information about the FM signal, equation 2.15 maintains all information regarding the modulation process. The FM wave, $s(t)$, however can be expressed in terms of the complex envelope, $\bar{s}(t)$.

$$\begin{aligned} s(t) &= \text{Re}[A_c\exp(j2\pi f_c t + j\beta \sin(2\pi f_m t))] \\ &= \text{Re}[\bar{s}(t)\exp(j2\pi f_c t)] \end{aligned} \tag{2.16}$$

In this form, the complex envelope can be seen as a periodic function whose fundamental frequency is the frequency of the modulating waveform, f_m. Returning to the the complex envelope equation, it can be expressed as a complex Fourier series

$$\bar{s}(t) = \sum_{n=-\infty}^{\infty} c_n \text{exp}(j2\pi n f_m t) \tag{2.17}$$

where

$$c_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} \text{exp}[j(\beta \text{sin} x - nx)]dx \tag{2.18}$$

The integral in this equation is recognized as an nth order Bessel function of the first kind. By incorporating the Bessel function, an single-tone FM wave can be assembled in a Fourier series representation.

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \text{cos}[2\pi(f_c + nf_m)t)] \tag{2.19}$$

From this form, a Fourier transform can be performed, producing a spectral representation of an FM wave.

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)] \tag{2.20}$$

Figure 2.2 illustrates the Bessel function, $J_n(\beta)$, with respect to varying values of β.

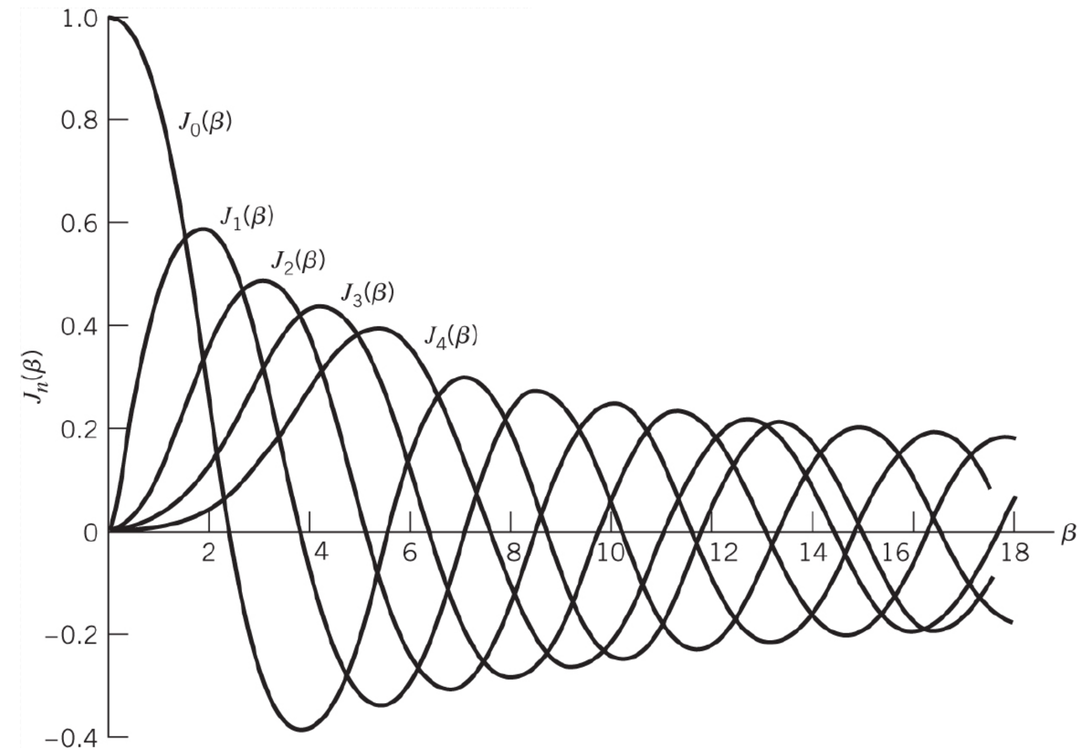

Figure 2.2: Plots of Bessel functions of the first kind [23].

From this plot for a constant value of β, the Bessel function oscillates between positive and negative magnitudes as the value of β increases. Additionally as β approaches infinity, the absolute value of the Bessel function approaches zero. The value of β determines whether the FM wave can be classified as narrow-band or wide-band. If the value of β is small relative to one radian, an FM signal is said to be narrow-band. Narrow-band FM signals are fundamentally composed of the carrier and two symmetric side-band components above and below the carrier frequency. However for large values of β, the carrier signal in an FM signal is accompanied by an infinite number of side-bands, symmetrically positioned above and below the carrier frequency. [13]

Table 2.1.2 lists the amount of significant side frequencies created from a specific modulation index, β. Since side frequencies are generated in pairs, the number of pairs are recorded in Table 2.1.2. The side-bands are located with respect to the center frequency, f_c, while the space between side frequencies is controlled by the modulating frequency, f_m.

Modulation	Number of side
Index	frequencies (pairs)
0.25	1
0.5	2
1.0	3
1.5	4
2.0	4
3.0	6
5.0	8
10.0	14
15.0	16

Any changes in the modulation index will not affect the total power of the FM wave, only the amount of power transported by the carrier. In contrast, the total power of an amplitude modulated waveform fluctuates where power from the message signal is held within the side-bands, leaving the power from the carrier unaffected. In frequency modulation, increases in the modulation index result in power from the carrier being distributed to the side-frequencies. Two scenarios are given that illustrate the shifting distribution of power among the frequency components. The first case considers a set message frequency but changing magnitude. Depicted in Figure 2.3 is the amplitude spectrum of an FM wave with a phase deviation, β, of 1, 2 and 5. As β increases, the spectral bandwidth increases but the distance between spectral lines remains constant.

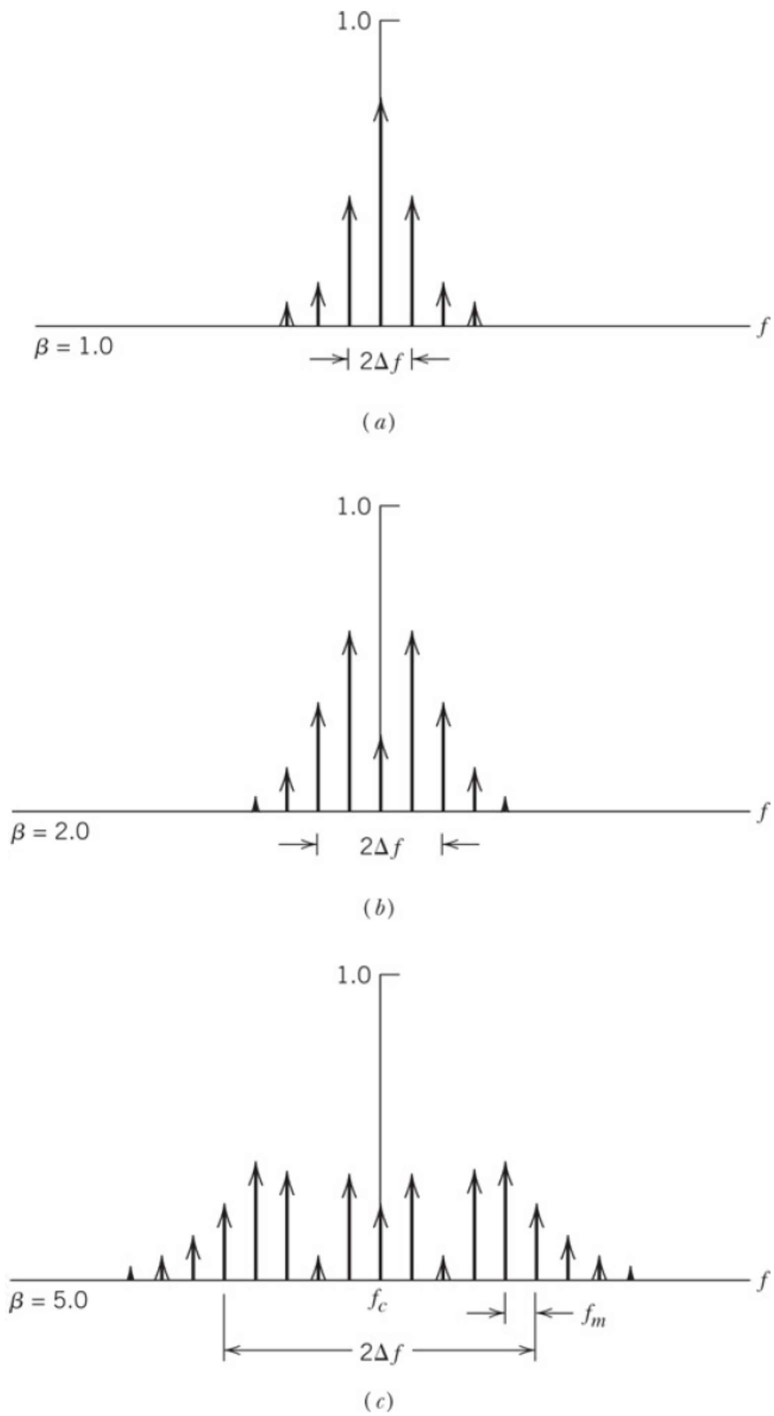

Figure 2.3: Discrete amplitude spectra of an FM signal, normalized with respect to the carrier amplitude, for the case of sinusoidal modulation of fixed frequency and varying amplitude. Only the spectra for positive frequencies are shown [23].

In the second instances, consider a message stable amplitude but with a varying frequency. In Figure 2.3, the spectral bandwidth remains unchanged by β. However, increasing

β, increases the number of spectral lines within a specific frequency interval defined by $f_c - \Delta f < f < f_c + \Delta f$.

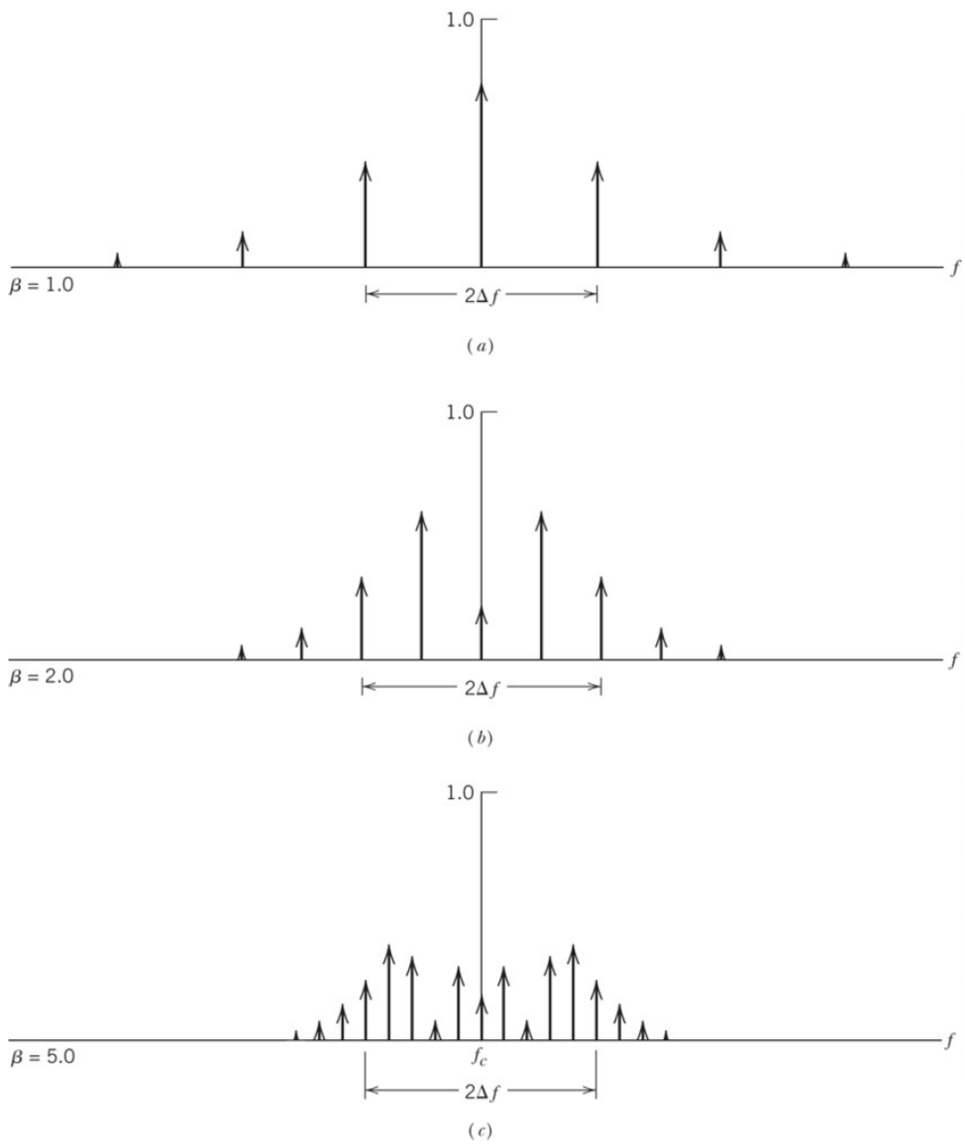

Figure 2.4: Discrete amplitude spectra of an FM signal, normalized with respect to the carrier amplitude, for the case of sinusoidal modulation of varying frequency and fixed amplitude. Only the spectra for positive frequencies are shown [23].

The static nature of FM emission can be beneficial. For an amplitude modulated transmitter, the amount of power the transmitting circuit requires fluctuates with the amount of modulation. However, since the power from a FM signal does not change, an FM transmitter can be created with respect to a known required power level. It was stated previously that

FM systems experience less noise than AM systems. Most noise is created as a result of noise signals mixing with desired signals in nonlinear circuits of an AM receiver to produce signals equivalent of an AM signal, which are detectable. This phenomenon is referred as intermodulation distortion (IM). Frequency modulated receivers are commonly designed with limiting circuits that help remove dissimilarities in the magnitude of an FM carrier signal, prior to signal detection. However, the limiting process is not able to remove all influences to the FM carrier amplitude. Any change in carrier magnitude will generate a phase change in the resulting FM signal, equivalent to frequency modulation. Hence, it can be said that amplitude modulation can generate frequency modulation [13].

2.2 Demodulation and Demodulators

Demodulation is the process of extracting the message signal, $m(t)$, from a modulated waveform, $s(t)$. Accordingly the circuits that perform this method of recovery are called demodulators. After the demodulation process, the recovered message signal can be distorted by noise, transmission and electromagnetic interference. When designing analog receiving circuits, steps are taken in order to reduce these effects, reducing discrepancies between the demodulated waveform and the original modulating waveform.

2.2.1 Amplitude Demodulation

To demodulate the AM wave to recover the message signal, the AM wave can be mixed again with a local oscillator identical in frequency to that of the local oscillator used to make the AM signal. The mixing process produces data at $f_{c2} - f_{c1}$ and $f_{c2} + f_{c1}$, where f_{c1} and f_{c2} are the carrier frequencies of the transmitter and receiver local oscillators, respectively. Since the carrier frequencies are identical ($f_{c2} = f_{c1}$), the mixing process produces information centered about 0 Hz, $f_{c2} + f_{c1}$, and at twice the carrier frequency, $f_{c2} + f_{c1}$. Using a low pass filter, the data at 0 Hz can be isolated by attenuating the redundant data along the frequency spectrum. This process is illustrated in Figure 2.5.

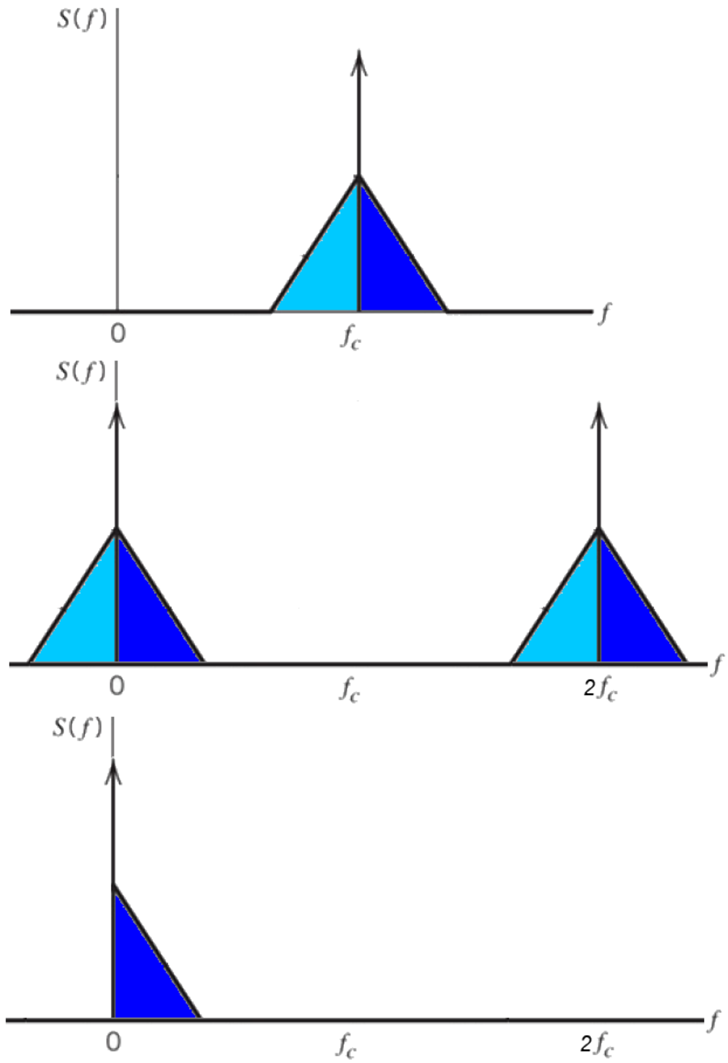

Figure 2.5: The top plot represents the AM wave within the frequency domain. The middle plot depicts the location of the data after mixing the AM wave with a local oscillator that is identical to that used to create the AM wave. The bottom plot is the resulting spectral density of the signal after passing through a low pass filter.

2.2.2 Frequency Demodulation

To perform frequency demodulation, a device is required that generates a signal whose magnitude is directly proportional to the instantaneous frequency of a frequency-modulated RF waveform. This device is called a frequency demodulator and has various implementations. Frequency demodulators can be classified as either direct or indirect. Direct frequency demodulators, such as frequency-discrimincators and zero crossing detectors, are used to recover information contained within a frequency modulated waveform. For indirect frequency demodulators, circuits designed with feedback to react to changes in instantaneous frequency are used. Phase-locked loops (PLL) are an example of indirect frequency demodulation and are used for frequency demodulation in almost all FM receivers [23, 29].

2.3 Receivers

A receiver is a circuit or device used to retrieve information from a specific ac signal received by an antenna. The functionality of a receiver is determined by certain key parameters. Sensitivity, dynamic range and selectivity are the more dominant properties used to characterize a receiver. Sensitivity refers to the capability of the receiver for extracting information from weak signal, therefore, highly sensitive receivers are able to extract the information from a weak signals. The dynamic range is a specific range of frequencies that the receiver is capable of processing. Selectivity is the receiver's ability to isolate a specific signal from the numerous signals received. Additional performance factors of a receiver are size, weight, control capabilities and power consumption [25].

2.3.1 Heterodyne and Homodyne Receivers

Related to the regenerative receiver is the homodyne receiver, also known as a coherent detection receiver. The detector, more commonly called the mixer, produces the sum and difference of two signals. By filtering the signal produced by the detector, these sum and difference frequencies can be isolated. The sum frequency is referred to as the radio frequency. With proper selection, a difference frequency can be created that is located within the audio frequency range. This signal can then be used to detect binary coded signals. Inside heterodyne and homodyne receivers is a beat frequency oscillator (BFO). The operation of heterodyne and homodyne receivers are similar. The labels heterodyne and homodyne refer to the frequencies being mixed. In a heterodyne receiver, an audible beat is generated when the received signal is mixed with the signal produced by the beat frequency oscillator (different frequencies, *hetero-*). A homodyne receiver uses a tunable local oscillator. By tuning the local oscillator to a desired AM signal (equivalent frequencies, *homo-*), the audible tone is removed leaving only the modulated waveform.

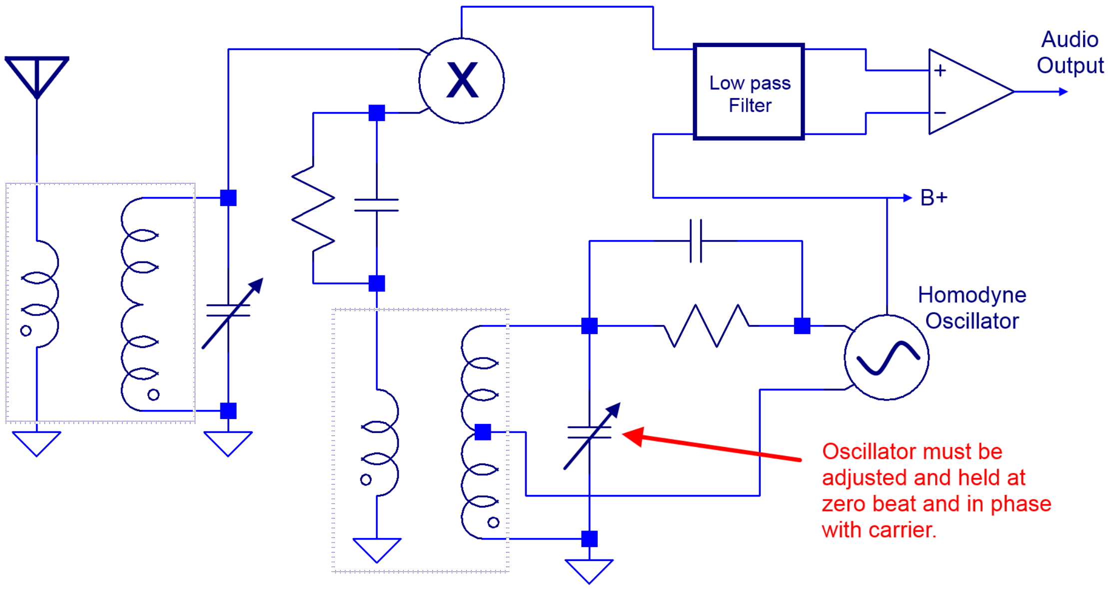

Figure 2.6: Schematic diagram of homodyne receiver [24]

Homodyne receivers act as the forerunners to modern communication receivers that use modulation schemes such as PSK and OAM. For the homodyne receiver depicted in Figure 2.6, a signal is received by an antenna is filtered before being sent to the mixer, generating the audible beat signal. A circuit is included that isolates a desirable beat frequency and amplifies it so that the beat can be heard. Homodyne receivers can offer innovative features at a reduced cost. However, in order for the receiver to operate properly, the local oscillator must precisely match the frequency and phase of the desired received signal to prevent the low-frequency beats from interfering with the detection process. While these systems have limited use it modern communication systems, homodyne receivers can be acceptable in some applications, such as radar [24].

2.3.2 Superheterodyne Receiver

Improving upon the functionality of the homodyne and heterdyne receivers is the superheterodyne receiver. Similar to a heterodyne receiver, the local oscillator signal is mixed with a received signal. In a heterodyne circuit, the mixing process is used to synchronize the two frequencies but in the superheterdyne circuit (see Figure 2.7), the frequency of the local oscillator is offset by a predetermined intermediate frequency from the incoming signal i.e. the frequency of the local oscillator must be higher than the frequency of the carrier of the desired signal. "Because a nonlinear device generates a difference frequency that is identical, if the signal frequency is either above or below the LO frequency (and also a number of other spurious responses), it is necessary to provide sufficient filtering prior to the mixing circuit so that this undesired signal response is substantially suppressed. The frequency of the undesired signal is referred to as an image frequency, and a signal at this frequency as an image. The image frequency is separated from the desired signal frequency by a difference equal to twice the IF, so that the preselection filtering required at the signal frequency is much broader than if the filtering of adjacent channel signals were required. The channel filtering is accomplished at IF. This is a decided advantage when the receiver covers a wide frequency band, since it is much more difficult to maintain constant bandwidth in a tunable filter than in a fixed one. Also, for receiving different signal types, the bandwidth may be changed relatively easily at a fixed frequency by switching filters of different bandwidths. Since the IF at which channel selectivity is provided is often lower than the signal band frequencies, it may be easier, to provide selectivity at IF, even if wide-band RF tuning is not required." [24].

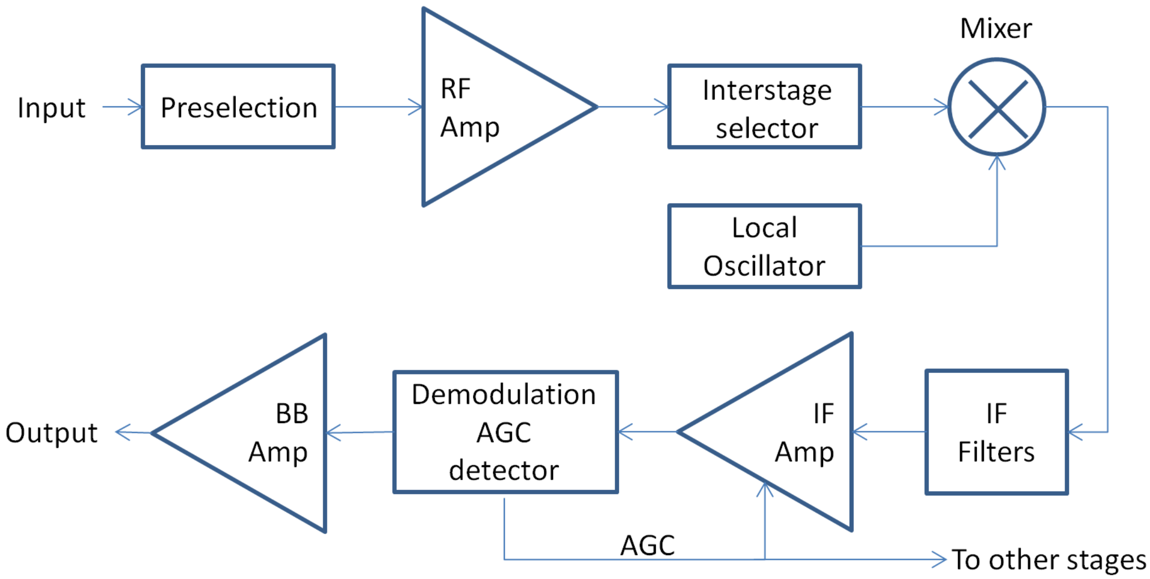

Figure 2.7: Block diagram of a superheterodyne receiver [24]

Despite the involuntary generation of image frequencies, the superheterodyne receiver succeeded previous receiver topologies due to its many advantages. In Figure 2.7, a received signal enters a preselection stage from the antenna. This circuit matches the antenna to the system impedance to maximize power transfer and sensitivity, and filters strong unwanted signals before being amplified. Because superheterodyne receivers generate more image frequencies, this preselection stage is commonly divided into multiple sections designed to filter and amplify specific frequencies in an effort to reduce filter loss. After the preselection stage(s), the signal is mixed with a local oscillator frequency to produce an intermediate frequency. From the mixer, there is a circuit designed to amplify the specific intermediate frequency while rejecting frequencies outside of a specific bandwidth. The final portion of the superheterodyne receiver is designed to condition the desired signal for its intended use and will vary with respect to communication parameters such as modulation schemes, frequency or signal objective [24].

Chapter 3

Communication Subsystems

Numerous communication topologies exist for each communication scheme. While transmitting and receiving circuits vary, there are components and circuits commonly found in each, and are detailed in the following sections.

3.1 Antennas

Every antenna has parameters that determines its application. These parameters are cataloged and described in Table 3.1.

Table 3.1: Antenna Performance Parameters [26]

Radiation Pattern $\mathbf{F}(\theta, \phi)$: Angular variation of radiation around the antenna, including:

Directive, single or multiple narrow beams

Omnidirectional (uniform radiation in one plane)

Shaped main beam

Directivity D: Ratio of power density in the direction of the pattern maximum to the average power density at the same distance from the antenna.

Gain G: Directivity reduced by the losses on the antenna.

Polarization: The figure traced out with time by the instantaneous electric field vector associated with the radiation from an antenna when transmitting. Antenna polarizations: Linear, Circular and Elliptical.

Impedance Z_A: Input impedance at the antenna terminals.

Bandwidth: Range of frequencies over which important performance parameters are acceptable

Scanning: Movement of the radiation pattern in space. Scanning is accomplished by mechanical movement or by electronic means such as adjustment of antenna current phase.

System Considerations: Size, weight, power handling, radar cross section, environmental operating conditions, etc.

When designing antennas, trade-offs are inevitable: increasing the magnitude of one parameter will cause the value of one or more parameters to decrease. The radiation pattern, efficiency, impedance and bandwidth are all determined by the antenna design with respect to operating frequency. For the best antenna performance, the impedance is designed to match the impedance characteristics of the system. In most cases, it's desired to have a purely resistive impedance, where 50 or 75 Ω values are common [24]. Antenna classification can be separated into four groups: electrically small, resonant, broadband and aperture antennas.

3.1.1 Electrically Small Antennas

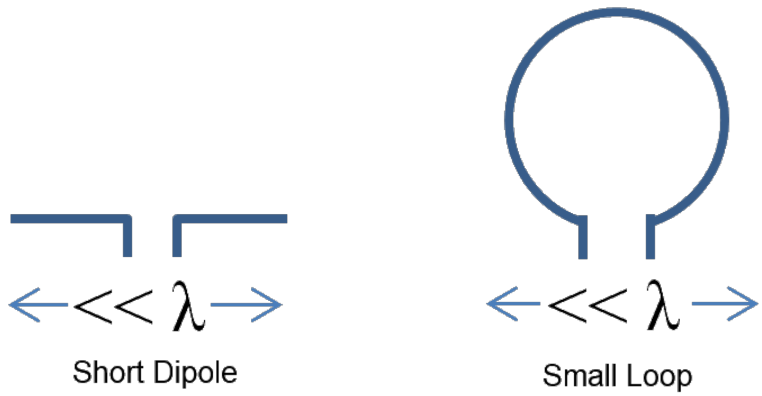

Figure 3.1: Examples of electrically small antennas

Electrically small antennas are used for communication at frequencies below and within the VHF band. These antennas are resilient to deviation design specifics and are easily made due to their simple design. Also the length of electrically small antennas are significantly smaller with respect to these frequencys' wavelengths. Many cars have a monopole installed for AM radio. This electrically small antenna is about 0.003λ long and exhibits an omnidirectional radiation pattern in the horizontal plane. While these properties are advantageous, small antennas are inefficient due to ohmic losses and the input impedance as an unfortunate combination of low resistance and high reactance [24].

3.1.2 Resonant Antennas

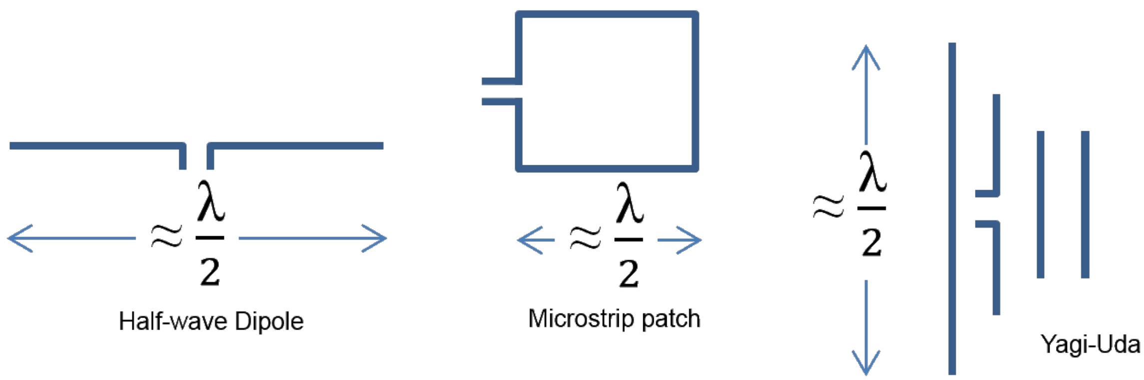

Figure 3.2: Examples of resonant antennas

With a simple structure and respectable input impedance, resonant antennas are a popular choice and used mainly in HF to low GHz frequency range. Its performance is best when used within a narrow band of frequencies. An example of a resonant antenna is the half-wave dipole. As the name implies, the length of the antenna is roughly 1/2 the length of the design wavelength, providing a wide main beam with a modest gain.

3.1.3 Broadband Antennas

Figure 3.3: An example of broadband antennas

While better parameter values can be encountered by other antenna types, the merit of broadband antennas is a result of a wide range of operating frequencies. Broadband antennas are most commonly used for communication in the VHF to middle GHz frequencies, where wide-bandwidth communication is common. Due to how the wave propagates within the antenna, antennas of this variety have an input impedance comprised of only resistance. The majority of power that radiates from a broadband antenna comes from the active region. Because only a portion of the antenna is used for energy transfer, broadband antennas provide little gain. However the gain remains consistent over its range of frequencies which could prove beneficial.

3.1.4 Aperture Antennas

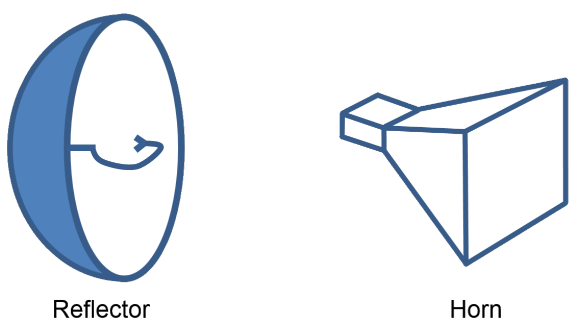

Figure 3.4: Examples of aperture antennas

Used mainly at UHF frequencies and above, aperture antennas function differently than any of the other antenna types. Waveforms enter and leave through the antennas front opening and are then channeled due the funnel-like shape. The shape of these antennas commonly causes a narrow main beam in its radiation pattern. As a result, a high gain is associated with aperture antennas [26].

3.2 Amplifiers

The main purpose of an amplifier is to increase the magnitude of a signal. The extent of this increase is determined by its *gain*. The gain is calculated as a ratio of output to input power, commonly displayed in dB. The operation of an amplifier is primarily governed by its dynamic range, noise figure and VSWR at each port. The dynamic range is the range of frequencies where the amplifier responds linearly with a consistent gain. The noise figure specifies the amount of noise added to the signal by the amplifying circuit, calculated by the ratio of input to output signal-to-noise ratio. Having a low noise figure is important when amplifying small signals. A port's voltage standing wave ratio (VSWR) corresponds to how well the port is matched to a specific impedance, with 50Ω and 75Ω being the most common. Impedance mismatches will cause a portion of the input power to be reflected back to the source. The amount reflected is based of the severity of the mismatch [28].

In receivers, amplifiers are used to increase the magnitude of a small signal (1 μV or less) received by an antenna in order for a desired output to be produced at a reasonable level (on the order of volts). To reduce intermodulated products, linear amplifiers are used. The linearity of an amplifier is inversely proportional to the power created from intermodulation of strong singals. Therefore any substantial gain at a receiver should be limited to the range of operating frequencies to help prevent intermodulation. Amplifiers designed to be used with radio frequency signals are especially susceptible to strong signal interference due to narrow bandwidth limitations [24].

3.3 Mixers

In a receiving circuit, a signal will undergo a frequency conversion and, in some cases, up to three conversions can occur. The circuit responsible for this task is referred to as a mixer or converter. In older literature, first or second detectors are synonymous with first and second mixers. An nth detector refers to a demodulating circuit that performs multiple frequency conversions, where n is equal to the number of frequency conversions. A mixer executes a nonlinear procedure on the received radio frequency (RF) signal using a local oscillator (LO) to produce an intermediate frequency (IF). Ideally only one intermediate frequency is desired from the mixing process, however, in actuality, mixers produce additional outputs that accompany the intermediate frequency. Therefore a filter is required to isolate a specific frequency. A mixer can be created from any device that exhibits nonlinear transfer characteristics. Mixers can be divided into three categories: passive, active or switching. Passive mixers typically employ diodes as nonlinear components to perform the mixing process. Unlike passive mixers, active mixers require power for transistors that are performing the modulation process. A switching mixer, while a distinct classification of mixer, can be made from a passive or an active mixer. Switching mixers use the local oscillator to toggle mixing components within the circuit on and off [24].

3.3.1 Passive Mixers

In the past, passive mixers used thermionic, germanium and silicon diodes for the mixing process. However these diode have since been replaced by hot carrier diodes. As a result passive mixers experienced a significant boost in performance. A single diode could be used to make a passive mixer. Unfortunately the circuit would produces an output containing frequencies from, not only, the input and local oscillator signals, but harmonics and spurious mixing products as well. Additionally, this circuit provides no isolation for the local oscillator. This allows for the local oscillator to "bleed" through to other ports on the mixer.

As an alternative, a double balanced mixer can be used. Pictured in Figure 3.5, this circuit is capable of removing harmonics and provides isolation between ports.

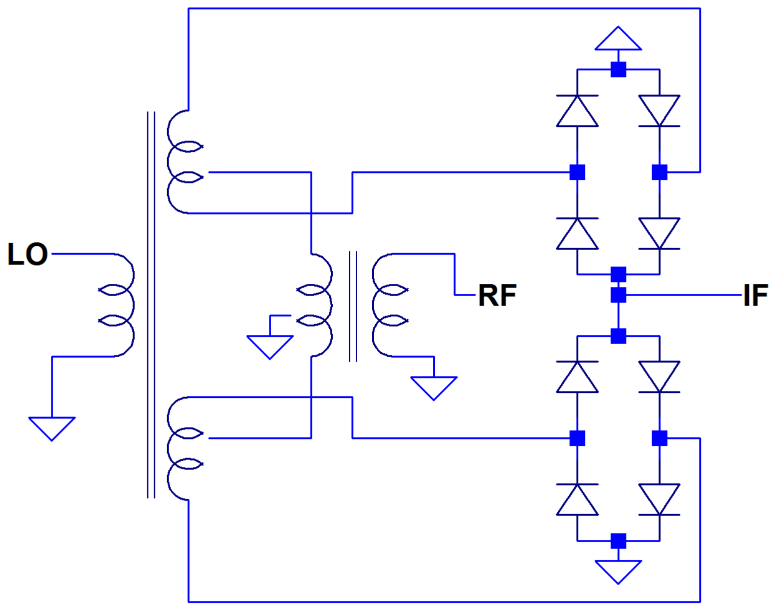

Figure 3.5: Schematic diagram of a double balanced mixer

The operation of this mixer relies on the balance diodes and transformers. Any mismatch in diode and transformer characteristics will impede functionality. Hot carrier diodes can be manufactured uniformly, further supporting the use of hot carrier diodes [24]. Passive mixers have experienced an increase in operating bandwidth from improvements made to transformers and ferrite cores.

The number of diodes determines whether the mixer can be classified as a low-, medium- or high-level mixer. Three common passive mixer topology can be seen in Figures 3.6-3.8.

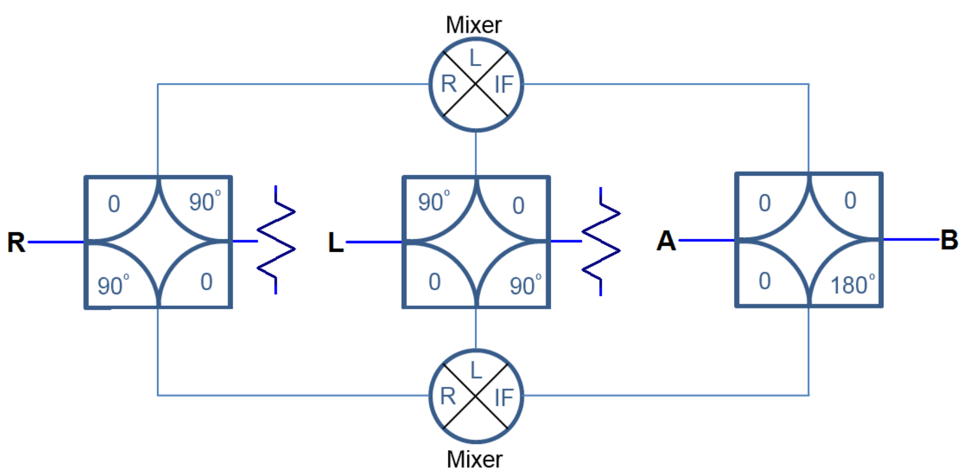

Figure 3.6: Schematic diagram of a SSB mixer

Recall that in amplitude modulation, all information is located in each side-band. A single side-band mixer (SSB) is able to obtain information from only one of ths side-bands. Note that the SSB mixer in Figure 3.6 provides information from the upper side-band from port A and information from the lower side-band from port B.

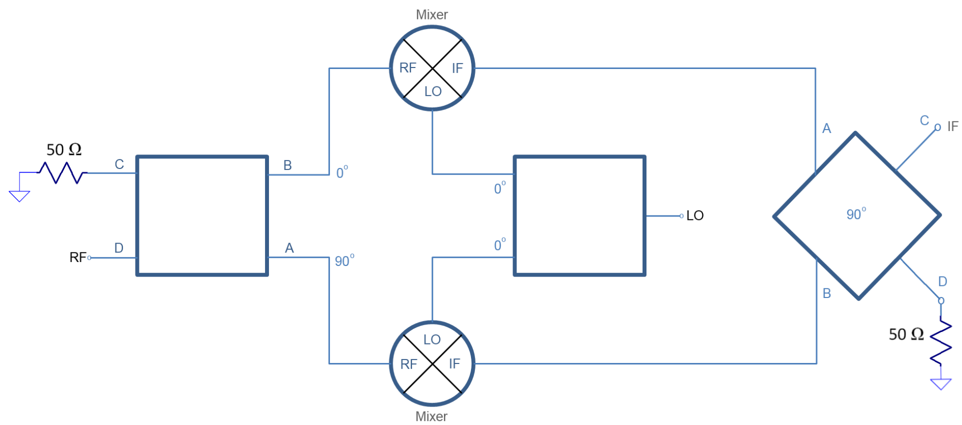

Figure 3.7: Schematic diagram of an image-rejection mixer

Consider the following: an intermediate frequency of 50 MHz is generated when a 75 MHz local oscillator signal and 25 MHz radio signal are mixed. If, however, the radio signal was accompanied by a spurious image frequency at 125 MHz, this image frequency would also produce a 50 MHz intermediate frequency. The image-rejection mixer in Figure 3.7 was designed as a solution. As a derivative of the SSB mixer, the image-rejection mixer provides the desired intermediate frequency by dismissing difference frequencies from radio signals that are higher than the frequency of the local oscillator.

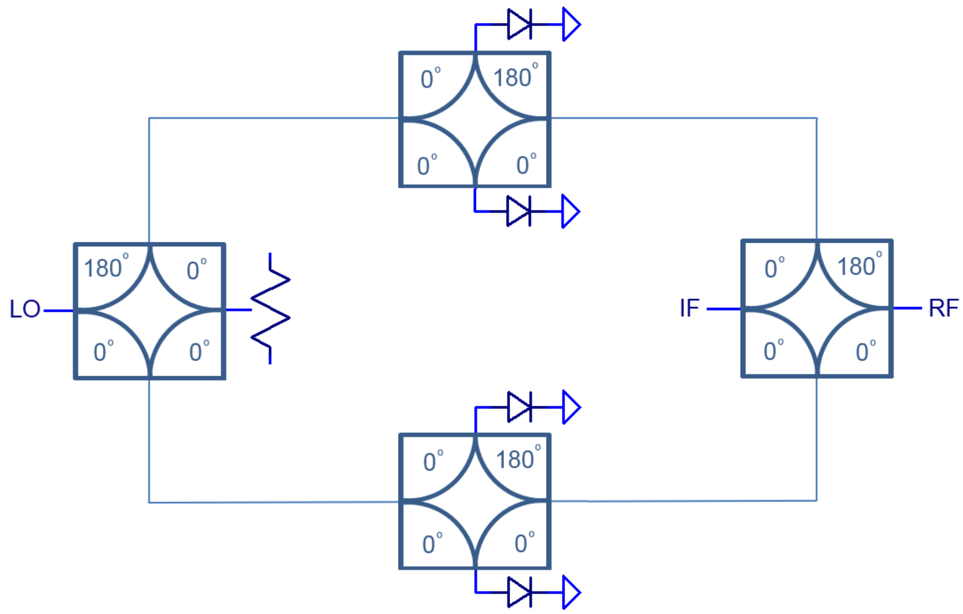

Figure 3.8: Schematic diagram of a termination-insensitive mixer

Shown in Figure 3.8 is an example of the termination-insensitive mixer. Some mixers, such as the double balanced mixer, are extremely sensitive to nonresistive terminations, but termination-insensitive mixers provide reasonable VSWR at its output even under port mismatches.

3.3.2 Active Mixers

Active and passive mixers express similar behavior, such as sensitivity to impedance mismatch which can cause distortion. However, since active mixers are designed using transistors, a signal is subjected to gain during the mixing process. Generally, active mixers can operate correctly with a low local oscillator signal [24]. Figure 3.9 and 3.10 are two examples of active mixing circuits.

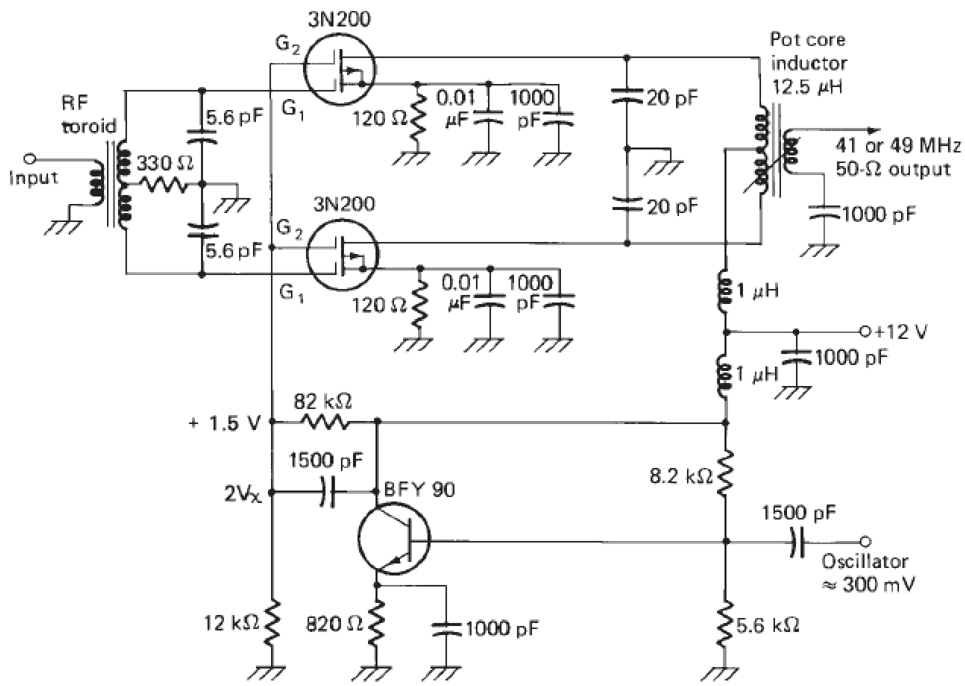

Figure 3.9: Schematic diagram of a push-pull dual gate FET mixer [24]

A push-pull balance FET mixer can be seen in Figure 3.9. Two 3N200 P-channel dual gate FETs are arrange in a push-pull configuration. The first gate (G_1) of the FETs are connected to the input via a balun and an oscillating signal is applied the the second gate (G_2).

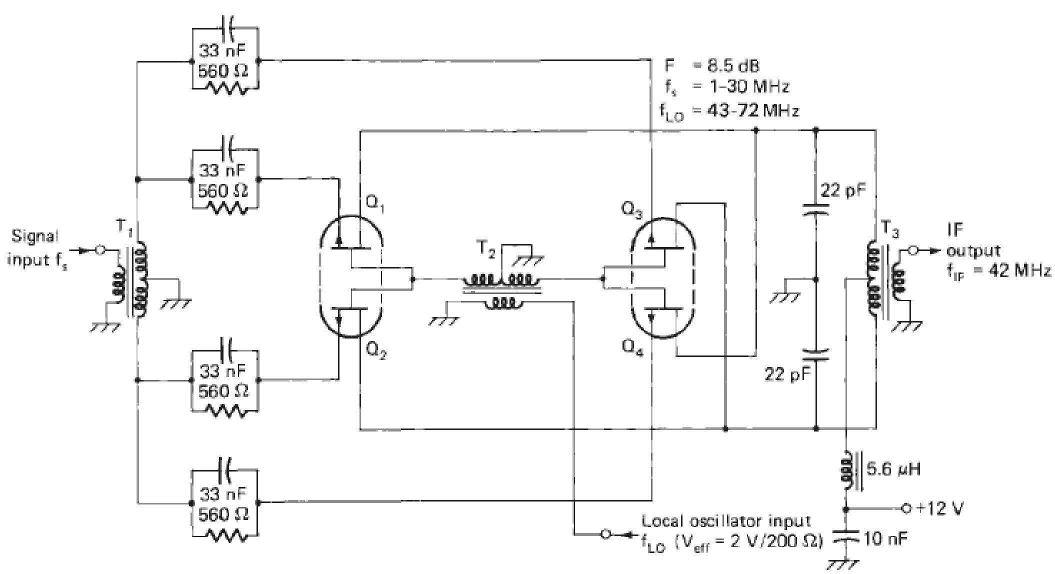

Figure 3.10: Schematic diagram of a double balanced FET mixer [24]

For the double balanced FET mixing circuit in Figure 3.10, four JFETs are used for the mixing process. A local osillator controls the gates in pairs with the input signal connected to the source of the JFETs.

A special type of active mixers use varactor diodes for high frequency up-conversion. Despite exceptional intermodulation distortion and spurious response behavior, issues with termination and drive variations present can be problematic in systems that cover a wide band of radio frequencies [24].

3.3.3 Switching Mixers

Switching mixers are the most common type of mixers in modern radio equipment [25]. By using a local oscillator with either a respectively large oscillating signal or rectangular waveform, a passive or active mixer can be converted to a switching mixer by quickly toggling the diodes or transistors used in the mixer between an on and off state. However, the creation of a switching mixer in this fashion tends to produce additional harmonics that could impede communication [24].

3.4 Local Oscillators

Modern receivers are designed to operate at numerous frequencies. Local oscillators provide a necessary signal source in the modulation and demodulation process and can be created from numerous topologies. In early radio receivers, free-running tunable oscillators were used, but were replaced by oscillators that were more stabile and precise. Fixed-tuned crystal-controlled oscillators are used by superheterdyne receivers in the second and third frequency converting stages where a single-reference frequency does not need to be exact. In synthesizers, a local oscillator can be fabricated from varactor diodes with the ability to be tuned by adjusting a voltage level [24].

The harmonic oscillator is a classification of oscillator that produces a relatively sinusoidal waveform and is used in most RF applications. If the oscillating waveform differs greatly from a sine wave, the oscillator is deemed a relaxation oscillator. Linear approaches can be taken toward the design and analysis of sinusoidal oscillators despite the non-linearity of oscillating circuits.

3.4.1 Variable Frequency Oscillators

For radio equipment using LC oscillators, the circuit is generally designed as variable frequency oscillators (VFO). Early variable frequency oscillators were fundamentally an LC tank circuit, with a tunable capacitor or inductor, and required mechanical adjustments. Air-gap capacitors and coils with powdered iron cores were used as variable capacitors and variable inductors, respectively. Wafer switches were used to select different component values to achieve a different range of frequencies. The mechanical systems have since be surpassed by electrically tuned oscillators. For example, diodes can be arranged to select a desired frequency range while voltage-sensitive capacitors can be used to provide frequency tuning within this band [24].

For decades, there was tremendous effort to create a low-drift VFO. This goal was achieved with the advent of frequency synthesizers. These synthesizers exhibited stability similar to crystal-controlled oscillators at a low-cost. Most VFO circuits offer similar capabilities, therefore, the design, component selection and assembly process determine the performance integrity. Two commonly used VFOs are the Colpitts oscillator and Hartley oscillator [25].

3.4.2 Crystal-Controlled Oscillators

Crystal-controlled oscillators, driven by a piezoelectric quartz crystal, express very high Q and can replace LC circuits where a stable oscillation at a set or narrow-band of frequencies is required. Of the available piezoelectric crystalline materials, quartz crystals offer the greatest temperature stability and are, therefore, preferred. When subjected to a mechanical strain or a voltage, a piezoelectric material will generate an electrical charge. With conscious sculpting, a piezoelectric material can be forced to favor specific resonant frequencies. For crystalline material, the shape, electrode placement and resonator cut determine its piezoelectric vibration. Most crystal piezoelectric materials can have several resonant frequencies and should be considered during implementation.

Chapter 4

Communication System Requirements and Components

Early exactly solvable chaotic circuits exhibited low frequency ranges which limited their application, especially in communication systems. In an effort to increase applicability, an exactly solvable chaotic oscillator of this topology was created to bring the operating frequency from 84 Hz to a level greater than 1 MHz. An exactly solvable chaotic oscillator has been created that operates at a frequency above 1 MHz while remaining compatible to a filter matched to chaotic oscillators of this topology.

An operating frequency of 1 to 2 MHz was selected when redesigning the exactly-solvable chaotic oscillator. At this frequency the oscillator would operate within the RF spectrum and allowed for the use of operational amplifiers that were commercially available. Operational amplifiers were relied on heavily in the low frequency design and were also utilized in the high frequency oscillator in an effort to keep the structures similar. Doing so avoids additional complications during the redesign. By redesigning the chaotic oscillator to operate at 2 MHz, components, commonly used for amateur radio, can be utilized. Additionally, 2 MHz frequency can achieve adequate bit rates supported by modern communications [1–5].

In its current state, the exactly-solvable folded-band chaotic oscillator produces a signal that can be transmitted without the process of modulation or signal conditioning i.e. as a baseband signal. While the chaotic oscillator was able to achieve a frequency of approximately 2 MHz, performing a form of modulation such as AM, FM or PM would allow for the signal to operate at an ever higher frequency and allow for the system to operate in more practical wireless communication configurations [1–5].

Now that symbolic information, encoded using high frequency (1>MHz) chaotic oscillators, can be recovered with precision, this moves exactly solvable chaotic circuits within range to be practicably used in a communication environment. To test the validity of this claim, a system will be designed to wirelessly send a chaotic message to a receiver connected to to a corresponding matched filter that will recover the information encoded by the oscillator.

4.1 Communication Selection

Before a wireless communication system can be created, a modulation procedure must be selected. Amplitude modulation is selected for two reasons. Amplitude modulation is simple, therefore, a substantial starting point for subsequent chaotic communication implementations. The spectral representation presents the second rationale for choosing amplitude modulation. A frequency modulated waveform presents dynamic behavior within the frequency domain whereas the information carried by an AM wave is explicitly known before the modulation occurs. However, there are several AM derivatives from which to chose.

Disregarding noise and image signals, an AM waveform contains two side-bands in addition to the carrier. In this case a simple envelope or square-law detecting circuit can be used to extract the information from the side-bands but is spectrally inefficient. On the other hand, suppressed-carrier systems require less power when transmitting the same amount of information but have more complicated receiving circuits. Due to the absence of a carrier wave, additional circuitry is required to recover a modulating waveform.

Single-sideband modulation reduces the transmission and bandwidth, further making it a common selection for point-to-point communication. Vestigial-sideband modulation can be used to send information that requires large bandwidths. The bandwidth of VSB modulation is larger than single-sideband modulation, while still less that double-sideband suppressed-carrier modulation. Both SSB and VSB modulation are able to reduce the required bandwidth by using the quadrature component to negate one of the sidebands, removing it from the transmitted spectrum. Accordingly, the systems require a coherent detector to the recover the information for the modulated waveform [23]. Of the amplitude modulation schemes discussed, standard amplitude modulation is used for the initial version of the wireless chaotic communication system.

Chapter 5

Communication System Testing

Before pieces of the system are connected, the status and functionality of each component of the system is examined, starting with the message signal source.

5.1 AM Transmitter

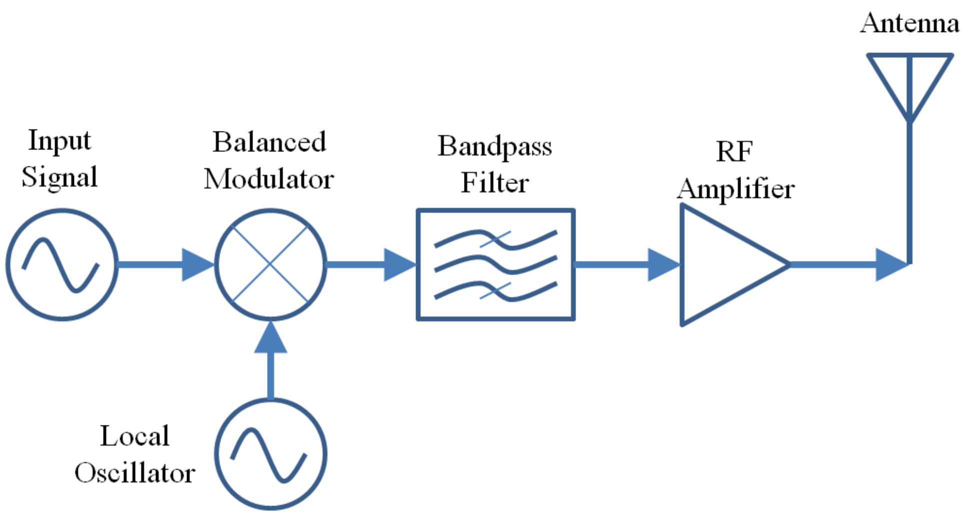

Figure 5.1: Block diagram of the transmitting circuit.

A 500mV sine wave is selected as the input signal, oscillating at 2 MHz with no DC offset voltage, and is acting as a test source to develop the communication system. Using a spectrum analyzer, the frequency domain of this waveform is examined. The sine wave is measured to have a 3.705 dBm power level at 2 MHz, falling within the range of the calculated value of 3.963 dBm.

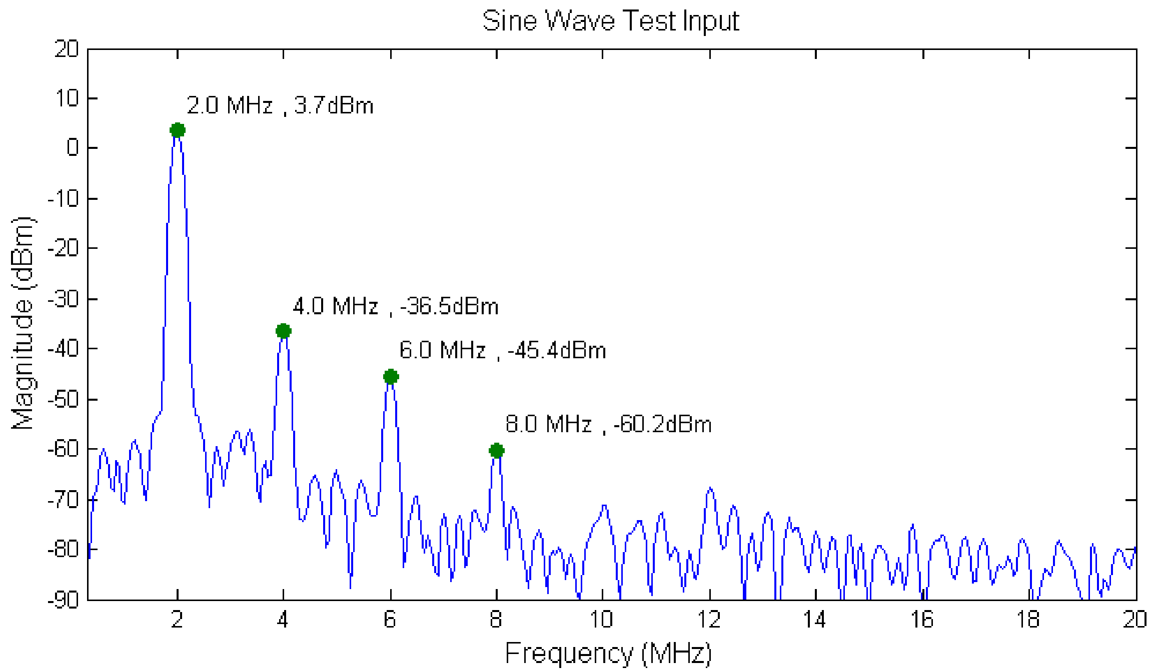

Figure 5.2: A spectral depiction of the test sinusoidal wave and its harmonics.

The spectrum analyzer revealed, illustrated in Figure 5.2, that the message is also accompanied by harmonics, occurring at integer multiples of the base frequency. After inspecting the power level and frequency of the test message source, the ZX95-2536C voltage-controlled oscillator (VCO) operating as the local oscillator is analyzed.

The 2536C offers low phase noise, low pushing and low pulling and is designed with a 5V operating voltage and tuning range to work with phase-locked loop integrated circuits (PLL IC). The VCO reports operating between the 2315 to 2536 MHz, but by connecting the Vtune pin to ground, the frequency of the VCO becomes approximately 2250 MHz (seen Figure 5.3) . While the frequency is lower that the desired local oscillator signal of 2300 MHz, 2250 MHz is sufficient.

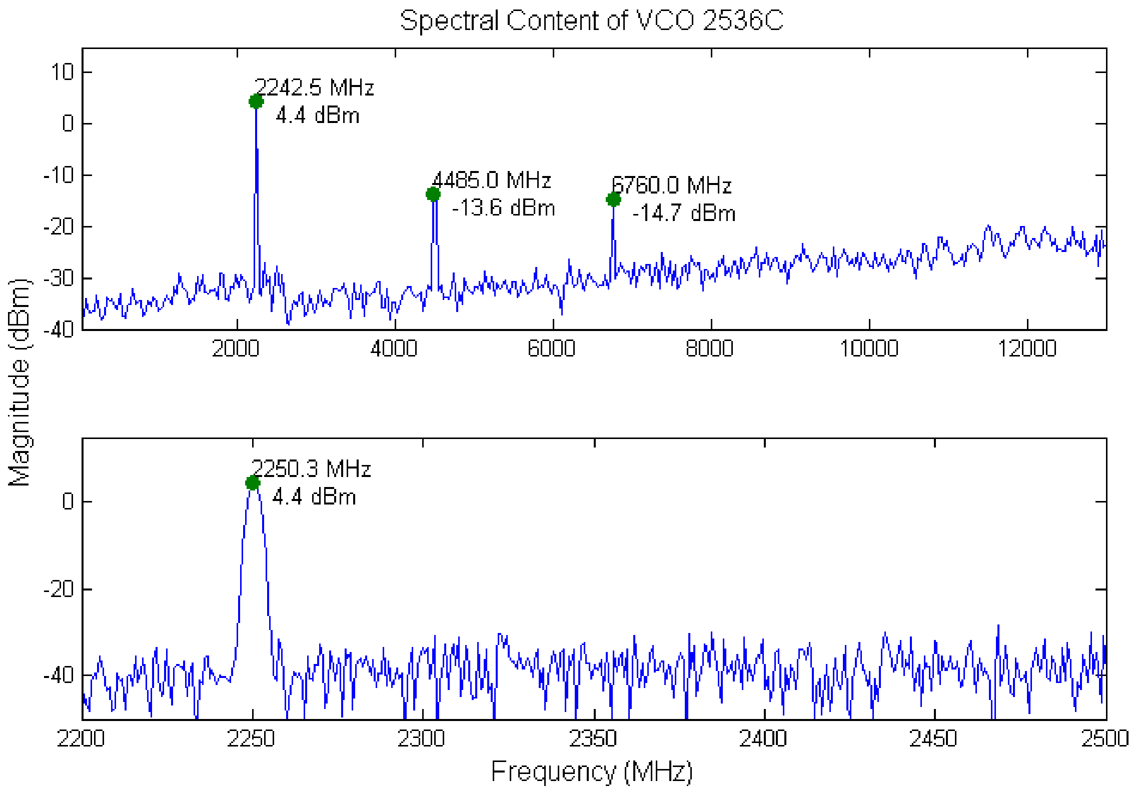

Figure 5.3: The carrier signal and its harmonics as seen in the frequency domain.

Similar to the function generator, the FFT reveals harmonics from the VCO(Figure 5.3), but this is a common phenomenon in a signal generated from a VCO. However by adjusting the frequency range to only examine the tallest peak, the VCO appears to be oscillating at 2.25 GHz with a magnitude of 4.38 dBm.

In order to multiply the signals to produce an AM signal, the message and carrier signals are applied to a mixer. The selected mixer, ZX05-U432H, is passive and utilizes four diodes to perform the multiplication process. Similar to the voltage controlled oscillator, the mixers datasheet contains a table of performance data. Not only is specific information for a 2.3 GHz local oscillator not given, the local oscillator used to produce this chart has a power level larger than what the ZX95-2500 VCO is producing. Even though the local oscillator and input signal differ from what was used to produce the performance data, the amount of conversion loss can still be extrapolated from the chart to give a rough estimate of the loss one can expect. Additionally, even if the loss is greater than what is expected, an amplifier can be used to increase the AM wave before it is transmitted. If, however, the AM wave is not properly constructed, no amount of amplification can resolve the issue and presents a fundamental flaw in the system. For a local oscillator of 2.25 GHz and a message signal of 2 MHz, two frequency spikes are expected to occur at 2248 MHz and 2252 MHz. When the VCO and sine wave are directly connected to the mixer, the spectral result differs from what is expected. After investigation, the problem was found to be the strength of the local oscillator with respect to the input signal. Details of this investigation are described in Appendix A.

Based on the previous research, the input magnitude is set to -3dBm to avoid the generation of harmonics and given a DC offset of 300 mV in order to be compatible with the receiving circuit.

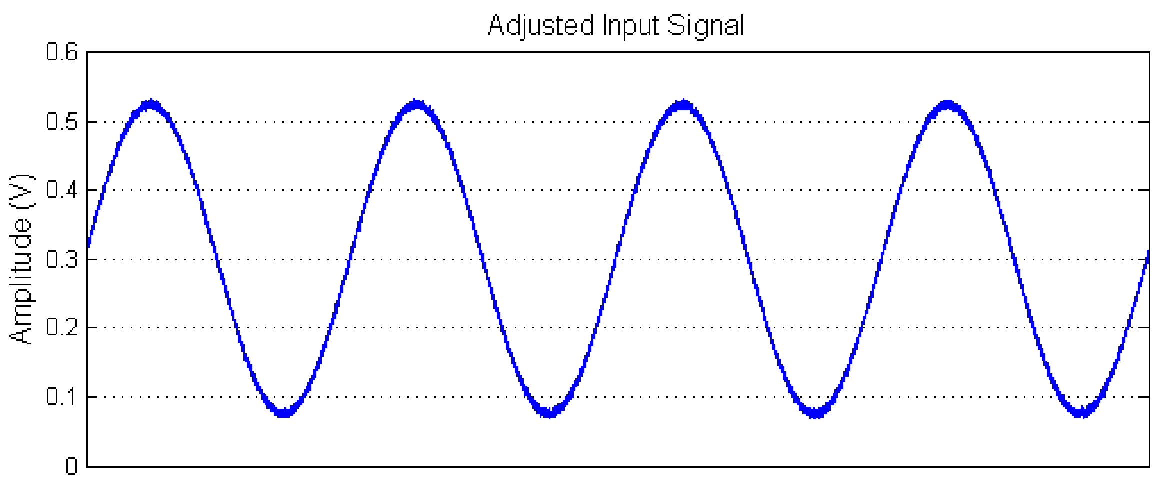

Figure 5.4: The input signal is adjusted to include a DC offset and adjusted to approximately -3dBm.

The ZX05-U432H is a class 17 double balanced mixer and requires a local oscillator power of +17 dBm. A ZX60-272LN low noise amplifier is used to amplify the local oscillator signal to +17.3 dBm which is of reasonable magnitude for the mixer. Figure 5.5 depicts the spectral content of the local oscillator signal after the LNA is inserted.

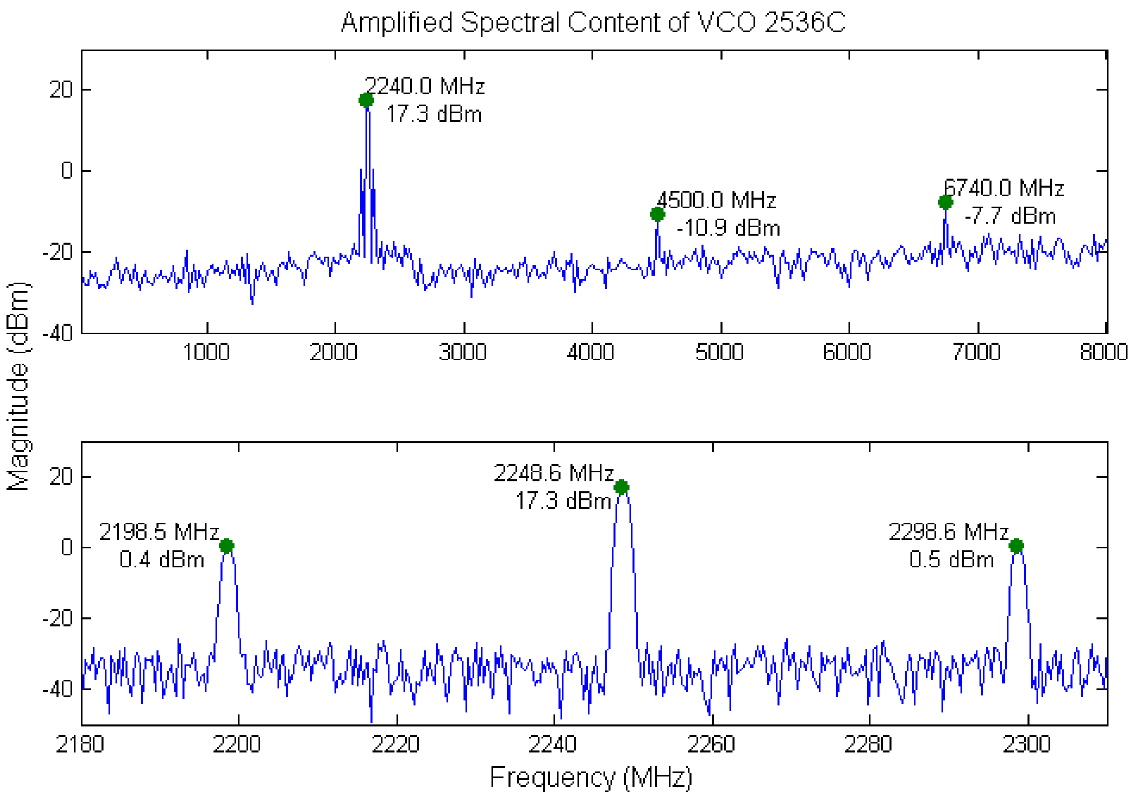

Figure 5.5: The spectral content of the amplified local oscillator signal.

After adjusting the local oscillator strength, the mixer begins to operate as expected. Recorded in Table 5.1 are the sidebands' power levels and the resulting loss from the mixing process.

Local Oscillator Power = 17.3 dBm at 2.25 GHz					
Message (V_{pp})	Input Power (dBm)	Output Power $f_c - f_m$(dBm)	Output Power $f_c + f_m$(dBm)	Conversion Loss $f_c - f_m$(dBm)	Conversion Loss $f_c + f_m$(dBm)
1	3.979	-3.7	-3.6	7.679	7.579
2	10	2.4	2.6	7.6	7.4
3	13.52	5.8	6.1	7.72	7.42
4	16.02	8	8.2	8.02	7.82
5	17.96	9	9.2	8.96	8.76
6	19.54	9.5	9.7	10.04	9.84

Table 5.1: The mixers output power with respect to various input power levels. The message is a sine wave with no DC offset.

After the mixing process, the AM signal enters the VBF-2275 bandpass filter. The bandpass filter suppresses unwanted frequencies that accompany the AM signal before being transmitted from the antenna. The VBF-2275 band pass filter has a pass band from 2170 to 2380 MHz. Ideally a filter allows frequencies within a desired range to pass through unattenuated, however this is not the case in actual filters. The average attenuation within the pass band of this filter is 2 db on average. Using a network analyzer, the amount of attenuation around the carrier frequency is approximately 1.7 dB. The S_{21} scattering parameter measured from 1 to 3 GHz is displayed in Figure 5.6 along with its effect on the generated AM wave.

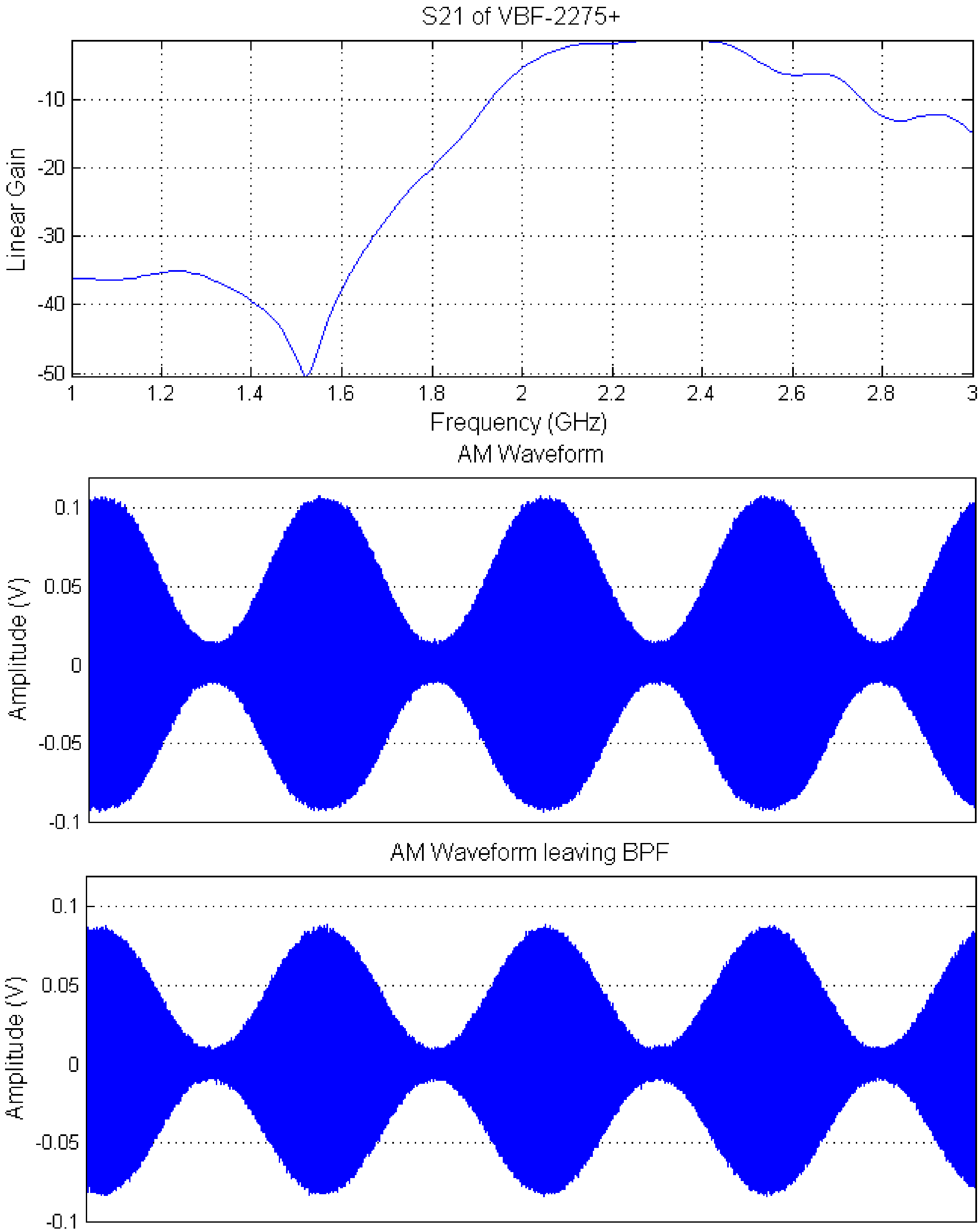

Figure 5.6: The top plot is the S_{21} parameter of the BPF, showing the level of attenuation as a function of frequency. The middle and bottom plot depicts the change in the AM waveform's magnitude due to the BPF.

As previously mentioned, antennas are selected based on the frequency of transmission. Because the AM signal is operating at a frequency close to 2.4 GHz, a WiFi antenna makes for a fitting choice. Unfortunately the majority of WiFi antennas commercially sold use reverse SMA (R-SMA) connectors. Reverse SMA connectors reverses the gender of the connector at the devices interface, making it impossible to utilize the majority of WiFi antennas directly without the aid of an additional adapter. Instead of buying special antennas or R-SMA adapters, WiFi patch antennas from a wireless modem/router were harvested and attached to coaxial cables, terminating to an SMA connector, pictured in Figure 5.7.

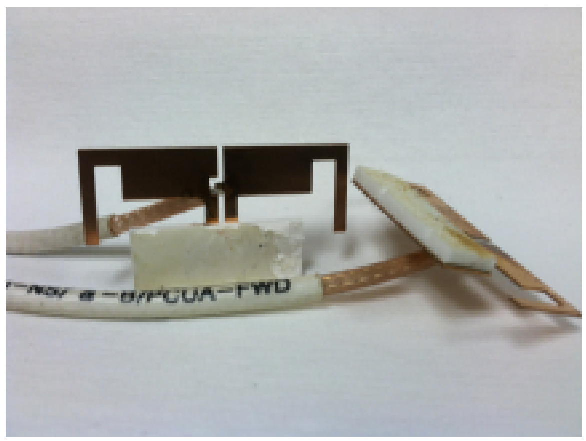

Figure 5.7: Patch antennas

Patch antennas fall under the classification of *resonant antennas*. These types of antennas are designed to operate at a specific frequency or narrow band of frequencies. The impedance can also be designed to match a 50 Ω system. Due to their low gain and directive radiation pattern, resonant antennas are not as suited for long distances or isotropic radiation. However patch antennas are perfectly suited for the communication system at this point in development.

When moving to wireless transmission, free space path loss must be considered. Free space path loss ($FSPL$) is the attenuation that occurs when an electromagnetic wave passes through a free space medium. The amount of loss that occurs is directly related to the distance the electromagnetic wave travels from the transmitter to the receiver, evident by the free space path loss equation:

$$FSPL = (\frac{4\pi Rf}{c})^2, \quad (5.1)$$

where variable f is the frequency of the signal in Hz, R is the distance in meters from transmitter to receiver and c is the speed of light. The desired distance from transmitter to receiver for this system was stated to be approximately 5 meters. For this distance and at a frequency of 2.3 GHz, the FSPL is roughly 54 dB. This is a substantial amount of attenuation. Therefore amplifiers are required so that the recovered message can be properly observed on an oscilloscope. Therefore an amplifier is placed before the transmitting antenna and after the receiving antenna. Ideally a power amplifier would be inserted before the antenna. Power amplifiers boast a high gain, high efficiency and relatively high power compression. Because a power amplifier is not available, a general purpose low noise amplifier is used.

5.2 AM Receiver

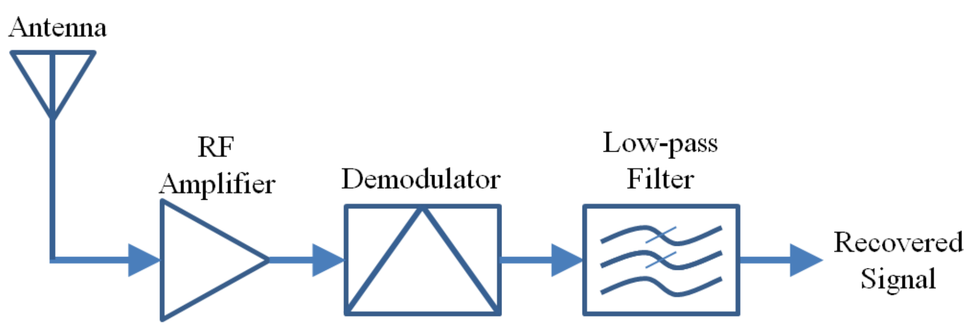

Figure 5.8: Block diagram of the receiving circuit.

The signal transmitted is received by a matching antenna attached to low-noise amplifier in the receiving circuit. Low-noise amplifiers operate by increasing a weak input signal, within a certain frequency band, and exhibit low noise figures. This helps prevent any noise that accompanies an input signal from being significantly amplified in following stages. Once amplified, the signal is demodulated using a diode detecting circuit.

A diode detector is a primitive circuit that can be used to demodulate a standard amplitude modulated waveform, removing the need for a local oscillator in the receiving circuit. Also called an envelope detector, the detector circuit is comprised of a diode, a capacitor and a resistor.

The circuit schematic for an envelope detector is depicted in Figure 5.9.

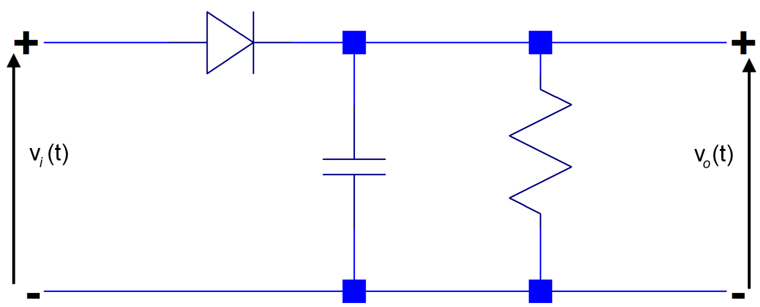

Figure 5.9: Schematic of diode detection circuit.

When the AM wave is applied to the input, the signal will charge the capacitor. The diode only allows current to flow in one direction, therefore only the positive portion of the wave passes through the circuit. Once the signal falls below the diode's turn on voltage, the diode turns off and the capacitor will discharge. The rate at which the capacitor discharges is determined by the capacitor and load resistor, referred to as the RC time constant. The capacitor needs to discharge fast enough so that drastic changes in amplitude can be reproduced, yet still provide enough dampening to remove the higher oscillations from the carrier wave.

When implemented correctly a diode detector is able to recover a modulating waveform exactly, but presents certain disadvantages. The recovered signal from a diode detector will be extremely small and will require amplification. Due to poor sensitivity, a strong desired received signal is required in order for proper demodulation. Poor selectivity allows for signals at similar frequencies to interfere with the recovery process. Additionally, the circuit provides no form of impedance matching, which can present issues when implemented in a receiving circuit.

A few adjustments needed to be made to the circuit in order for it to operate within the desired ultra-high frequency band of around 2.3 GHz. Simple silicon switching diodes such as the 1N4148 diode are not able to switch fast enough for signals at this frequency. Schottky diodes, however, are able to switch faster and offer a low forward voltage drop across its PN junction.

The BAT63 RF Schottky diode made by Infineon was selected as the rectifying diode for the detector circuit. The capacitor and resistor values selected are 1 nF and 100 kΩ, respectively. An additional resistor of 50 Ω was added across the input to the circuit to help match the input impedance to the system impedance. The input and output of the envelope detector circuit are terminated to 50 Ω SMA connectors, allowing for a direct insertion to the communication system.

Due to the high frequency of the signal, a circuit assembled on a breadboard would introduce parasitic effects that would hinder the circuits operation or prevent it from working entirely. The circuit must be implemented via a PCB. The final schematic and PCB created using KiCAD can be seen in Figure 5.10.

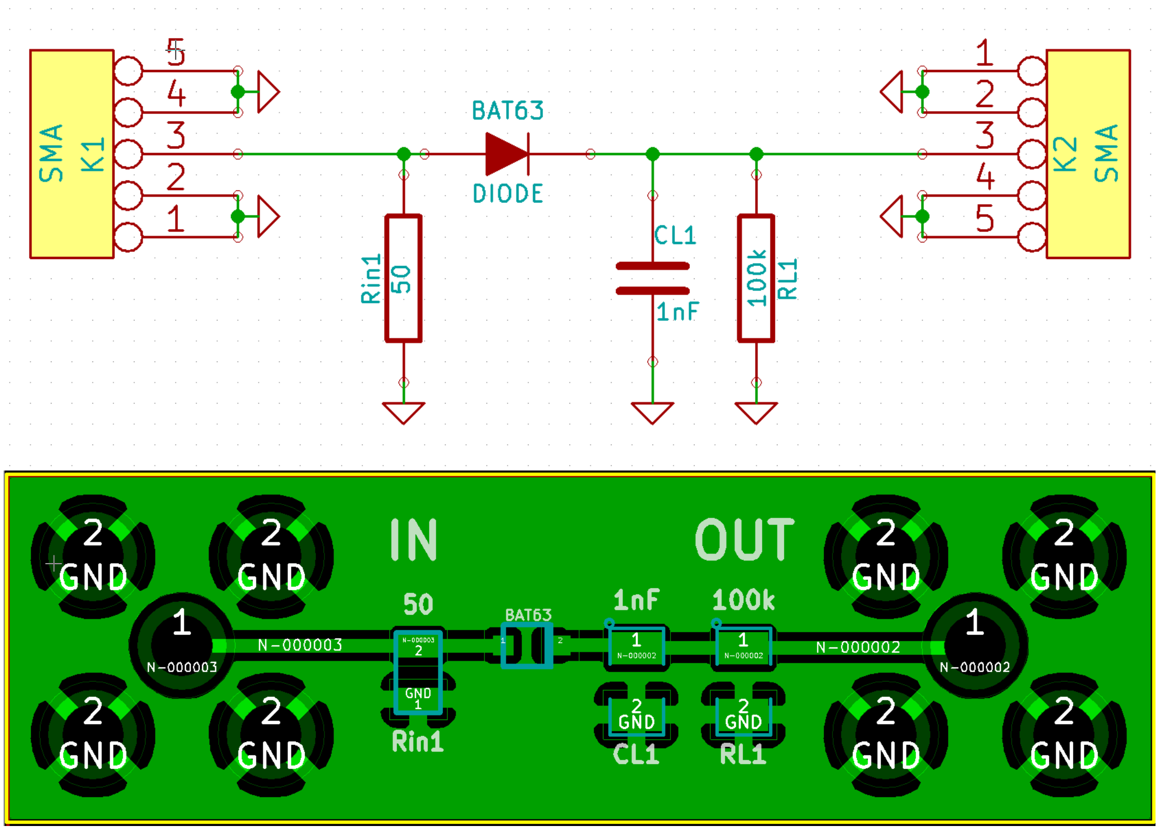

Figure 5.10: (Top) The envelope detector circuit in the form of a KiCAD schematic. (Bottom) The envelope detector circuit implemented on a one-sided PCB.

To test the functionality of the envelope detector, the PCB is attached directly to the transmitter side of the communication system, picture in Figure 5.11, and a sinusoidal waveform is used as the modulating waveform (message). Using a sinusoidal waveform allows for any distortion or clipping that could occur to be visible on the oscilloscope. The output of the envelope detector is attached directly to the oscilloscope.

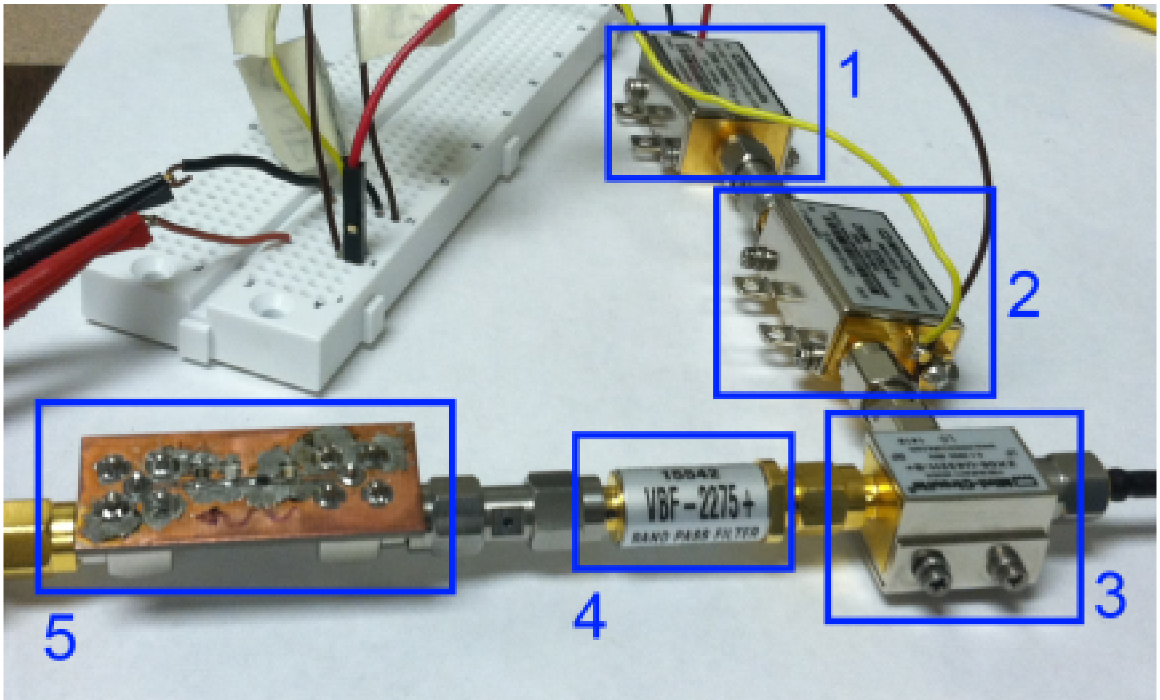

Figure 5.11: The transmitter design with the (1) voltage controlled oscillator, (2) low noise amplifier, (3) frequency mixer, (4) band-pass filter and the (5) envelope detector circuit shown.

The waveform produced by this circuit corresponds to the envelope of the AM wave, however, some of the high frequency carrier wave accompanies the recovered message. A low pass filter follows the demodulation circuit to fully attenuate the carrier wave, leaving only the original message on the output, as seen in Figure 5.12.

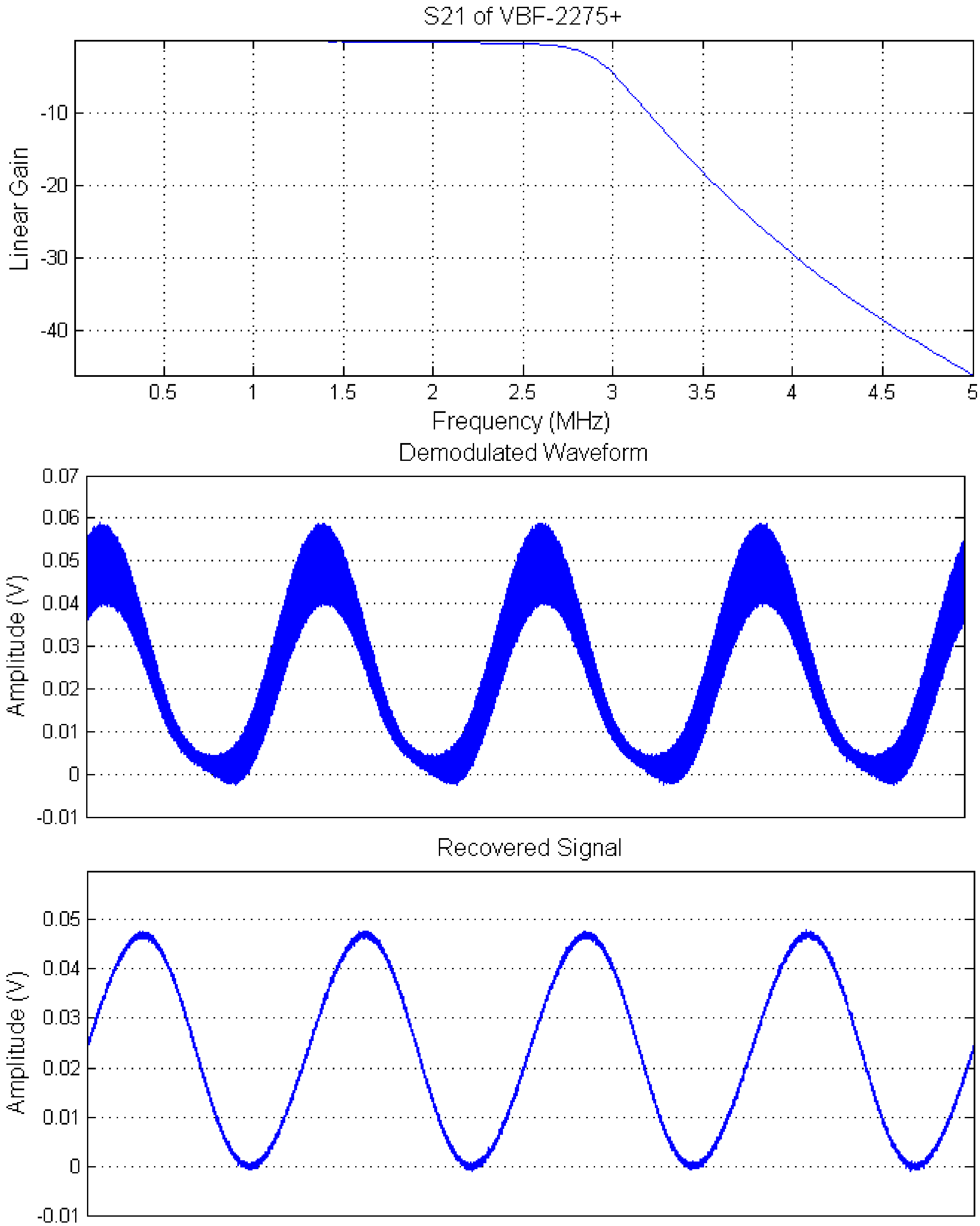

Figure 5.12: Acting as the last step in the demodulation process, a low pass filter is used to recover the original signal. The top plot is the S_{21} parameter of the LPF, showing the level of attenuation as a function of frequency. The middle and bottom plots depicts the effect of the LPF on the demodulated signal.

5.3 Printed Circuit Board Implementation

In an effort to make the system easier to use, a circuit board has been created. By using this circuit board, the communication system can be independently powered by two single AA batteries (one AA battery per circuit board). Additionally, the circuit boards were designed to allow the transmitting and receiving circuit to be mounted next to their respectively battery supply circuits, providing a desirable ground plane for each circuit.

In order for the power supply circuit to produce a 5V rail from an AA battery (generally rated at 1.5 V), the NCP1402 step-up regulator is used. This DC-to-DC converter only requires four additional components in order to provide a 5V rail from a variety of input voltages sources and can supply a maximum 200 mA. A circuit diagram of the power supply circuit is shown in Figure 5.13.

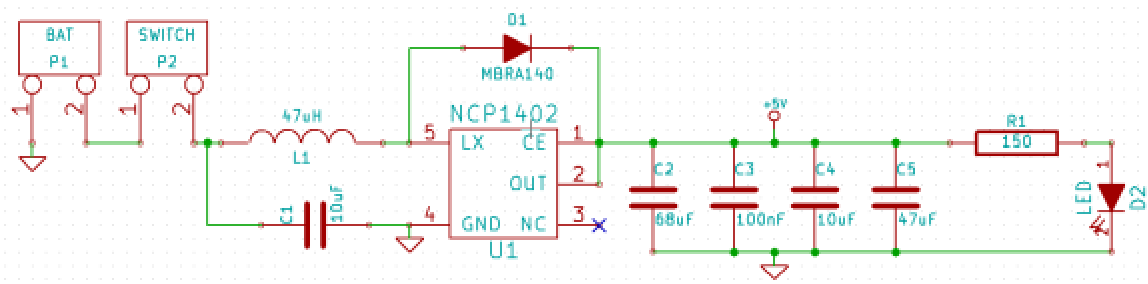

Figure 5.13: The power supply circuit in KiCad Eeschema.

While only C1, C2, L1 and D1 are required, three decoupling capacitors are added to help keep a healthy 5V rail and an LED, in series with a resistor, acts as an indicating light to let the user know that power is being supplied to the circuit. The resulting printed circuit boards for the transmitter and receiver are pictured in Figure 5.15.

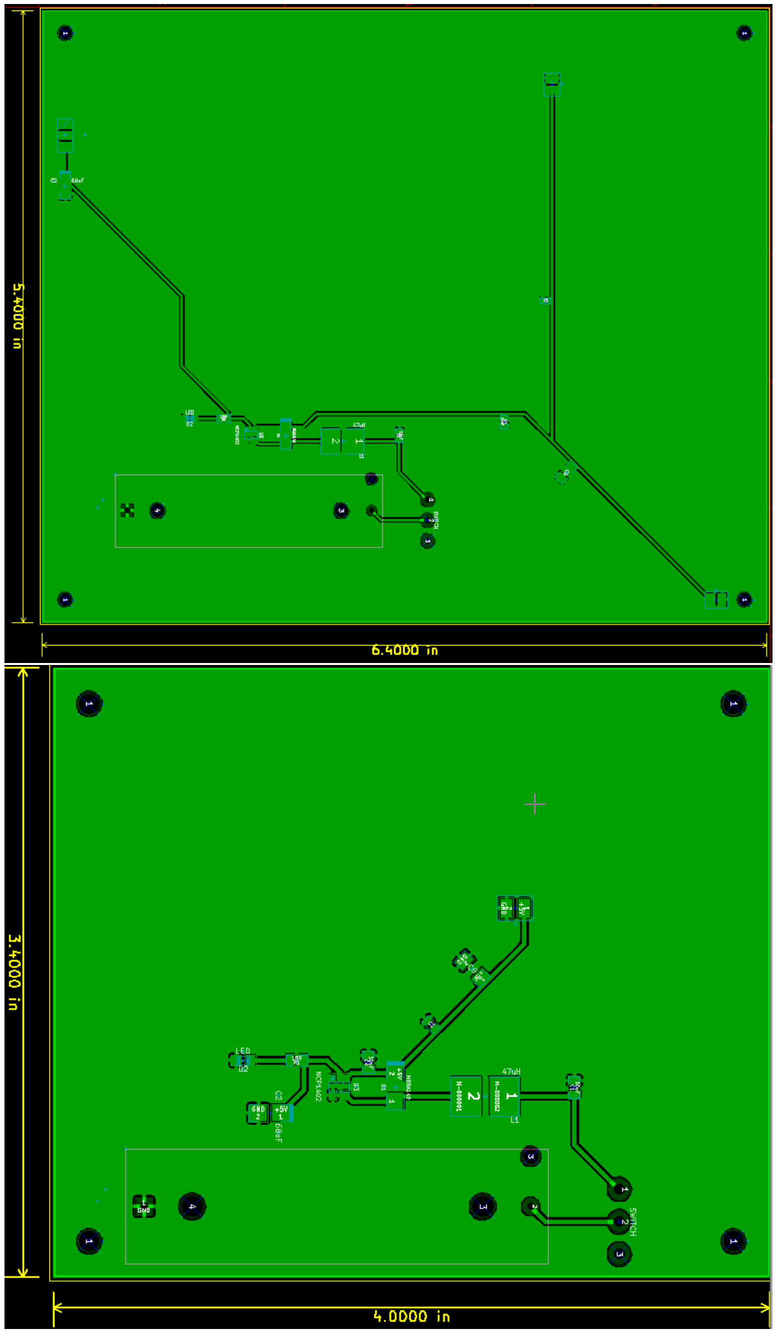

Figure 5.14: The printed circuit boards designed in KiCad PCB Editor.

The circuit boards were produces in house on one-sided copper clad circuit board. Components used for the communication system are in panel-mount packaging, while providing a rugged case, also allows from components to be swapped, moved or even replaced without much effort. To retain this benefit, holes are made to accommodate sockets for the components' leads. As a result, the components can be separated from the circuit board by removing the hardware holding the panel-mount packages and disconnecting the leads from the sockets. Once the etching and drilling were completed, the boards were coated in a tin alloy to prevent corrosion and to aid the soldering process. After the PCBs were populated, the transmitting and receiving circuits were mounted to their respective PCBs (Figure 5.15).

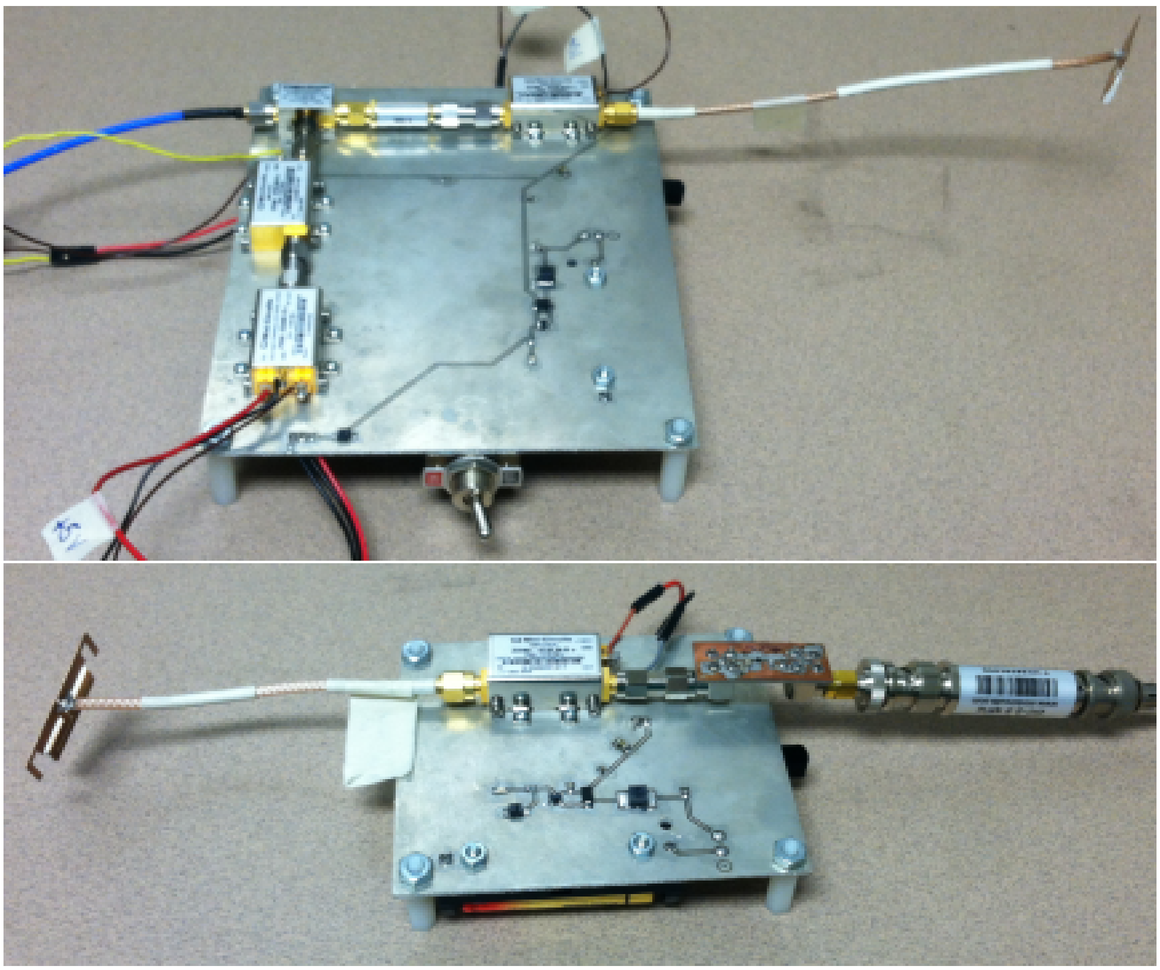

Figure 5.15: The transmitting (top) and receiving (bottom) circuits mounted to their respective printed circuit boards.

5.4 Results

With the communication system functioning as expected, the sinusoidal waveform, that has been used as a placeholder, can be replaced with a chaotic waveform. When examined in the frequency domain, a chaotic signal generates spectral content over a wide band of frequencies. The waveform, generated by the chaotic oscillating circuit, and it's spectrum are presented in Figure 5.16 and 5.17 respectively. This is considered to be a spread spectrum signal. The spectral content of the mixer is shown in Figure 9 when using the chaotic signal from Figure 5.18 as the modulating waveform with a 2.3 GHz LO and the 1.6 MHz chaotic waveform.

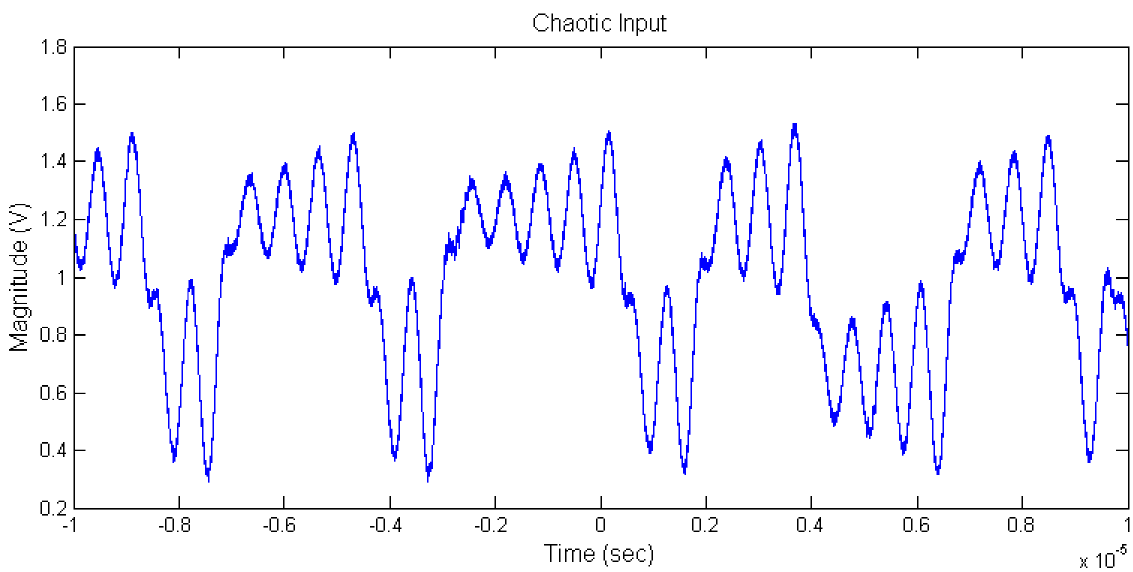

Figure 5.16: A chaotic signal of 1.6 MHz in the time domain

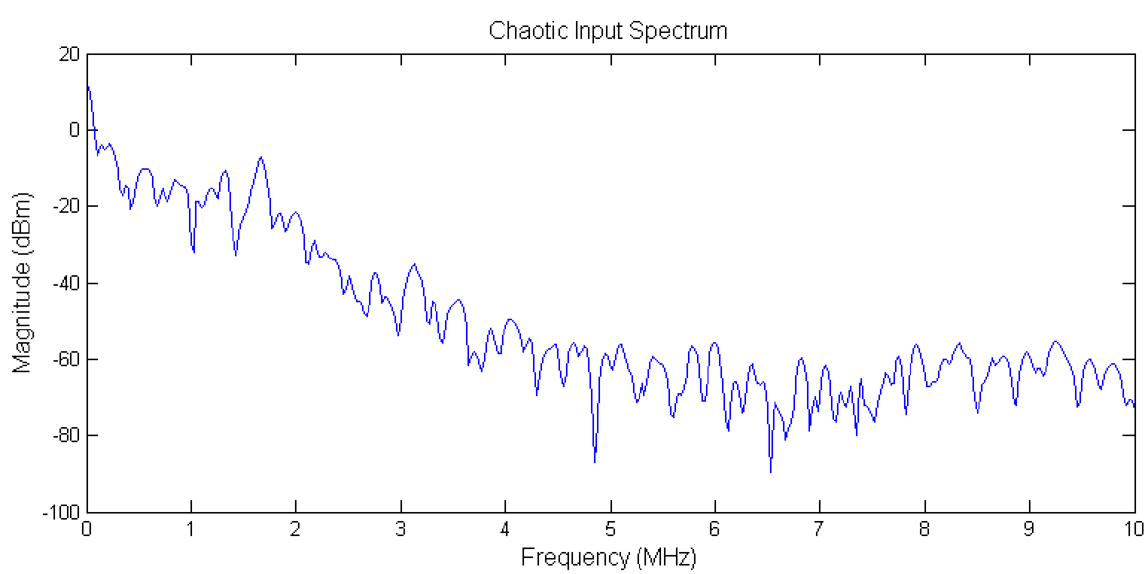

Figure 5.17: Spectral content of the 1.6 MHz chaotic signal

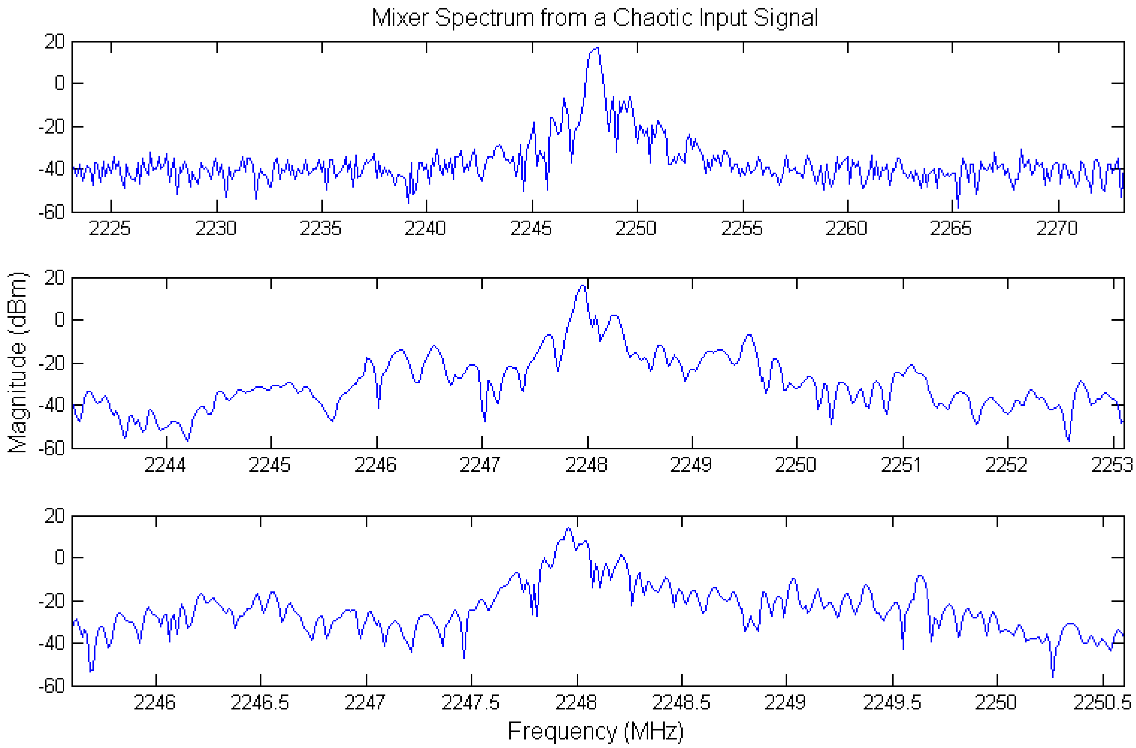

Figure 5.18: The spectral content of an AM waveform at three different frequency spans: 50 MHz (top), 10 MHz (middle) and 5 MHz (bottom). The signal was created by mixing a 1.6 MHz chaotic signal with a +17.3 dBm carrier signal at 2.3 GHz

An adapter board is inserted between the chaotic input and the mixer. This board matches the impedance of the chaotic oscillator to system impedance of 50 Ω. Additionally the adapter board provides signal conditioning such as tunable gain and dc offset. The variable gain is used to keep the signal strength low enough to avoid harmonic distortions that occur during the mixing process yet strong enough to overcome attenuation within the system such as due to impedance mismatch at the antennas, conversion loss by the mixer and insertion loss of the bandpass filter. The transmitter and receiver are situated so that the antennas are approximately 6 inches apart. Below, in Figure 5.19, the chaotic input and the demodulated input are compared.

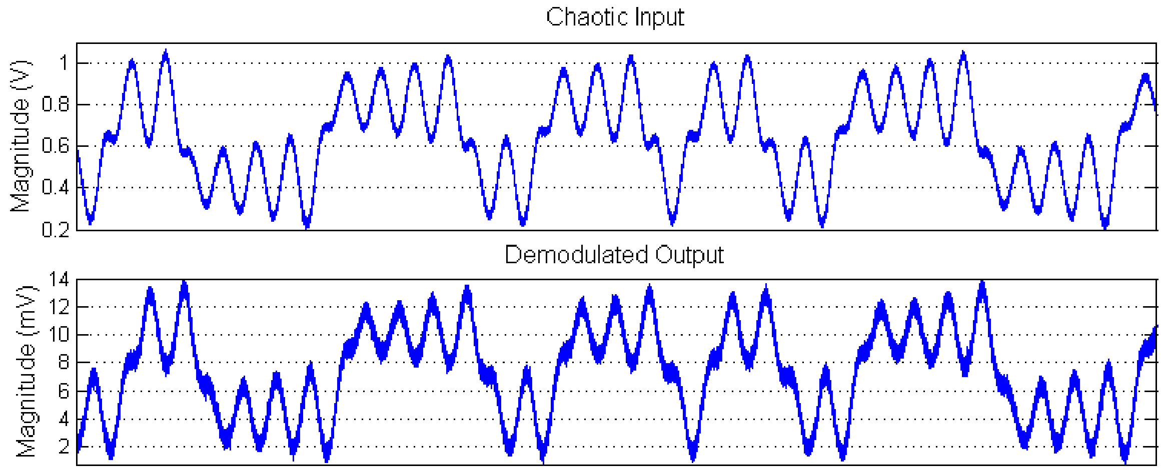

Figure 5.19: The top plot shows the chaotic waveform fed directly into the transmitter's mixer. The waveform in the lower plot is produced from the output of the receiver.

When comparing the input and output waveforms, the chaotic signal experiences slight distortion during the transmission process. Still, the results prove that a chaotic waveform can be successfully transmitted and received, therefore bolstering the notion that chaos theory is no longer limited to purely academic exercises and offers new and innovative approaches to the world of electronics.

Chapter 6

Conclusions

Chaotic oscillators were able to escape their purely academic limitations and present a novel approach to communication systems. An exactly solvable chaotic oscillating circuit refuted the assumption that a precise solution is unattainable for chaotic systems. The circuit can mathematically be expressed by a continuous differential equation with discrete switching conditions and can be, as a result, constructed using analog and digital components. In order for the chaotic circuit to be useful, a matched filter is required to accurately detect a transmitted chaotic waveform. Adjustments were made to a novel chaotic oscillator to increase its frequency of operation from 84 Hz to approximately 2 MHz. At a higher frequency, the chaotic oscillator is more applicable to modern communication systems.

Processes of analog modulation are examined when considering possible wireless communication systems using a 2 MHz exactly solvable chaotic oscillator. The processes of amplitude and frequency modulation are described along with benefits and disadvantages to be considered when making a modulation selection. In order to recover information from an AM signal, receivers are presented to shed light on the various elements used to perform the demodulation process. While circuits and components vary with transmitter and receiver designs, there are circuits commonly found in the world of radio communications.

Antennas come in a variety of shapes and sizes but ultimately can be divided into four groups: electrically small, resonant, broadband and aperture. Each type exhibits benefits and disadvantages. When designing an antenna, trade-offs will be experienced: increasing one parameter causes reductions in other parameter(s). When creating a wireless communication system, designs must account for attenuation from free space path loss. Amplifiers are used to overcome these obstacles and two classifications are reported. Power amplifiers are commonly employed before a transmitting antenna. These amplifiers offer high gain, efficiency and power compression over a wide range of frequencies, but consequently amplify signals and noise, which can be undesirable. Low-noise amplifiers, however, only amplify signals within a specific bandwidth. This amplifier is frequently used in a receiving circuit to help suppress noise and image frequencies that accompany a received signal.

At the heart of a modulating circuit is a mixer. Mixers exist in three forms: passive, active and switching mixers. When the message and carrier signal are fed to a mixer, the mixer performs a nonlinear procedure to produce signals at the sum and difference of the two input signal frequencies. A carrier signal is often generated by a local oscillator. The purpose of a local oscillator is to provide a signal for modulation during the frequency conversion process. Variable frequency and crystal-controlled are two oscillator classification seen in communication systems.

Because novelty is not desired, amplitude modulation is selected for two reasons: simplicity and predictability. An AM transmitter and receiver for point-to-point communication is relatively simple, removes unnecessary complexities and acts as a starting point for future communication systems using chaotic oscillators. A transmitting and receiving circuit is presented that successfully transmits a chaotic waveform wirelessly. The transmitting circuit modulates a message signal using standard amplitude modulation. A diode detecting circuit is used in the receiving circuit to demodulate the AM signal. A 2 MHz sinusoidal waveform is used to represent the chaotic waveform during the design and testing of the communication system. Once the functionality of the circuits are verified, the sinusoidal signal is replaced with a 2 MHz chaotic signal to demonstrate a wireless transmission of a chaotic waveform.

Chapter 7

Future Work

In double-sideband suppressed-carrier (DSBSC) signal, the carrier is reduced, saving power. However, information is still carried simultaneously in the upper and lower sidebands. Since the information in both bands are identical, one of them can be removed to reduce the transmitted bandwidth in half. The resulting system is called a single-sideband suppressed-carrier (SSBSC) or simply SSB. Single-sideband suppressed-carrier signals have exceptional power efficiency compared to other amplitude modulation schemes due to the proportion of modulating signal to transmitted power. For example, if a portion of the modulating waveform is at 0V, fundamentally no power would be transmitted from the antenna.

There are two basic approaches to creating a SSB waveform. The most common approach is the *filter method* and uses a transmitter similar to that depicted in Chapter 5, Figure 5.1. By simply adjusting (or replacing) the filter's pass band, one of sidebands can be also be removed before being transmitted. The second approach is referred to as the *phasing method* and it's block diagram seen in Figure 7.1.

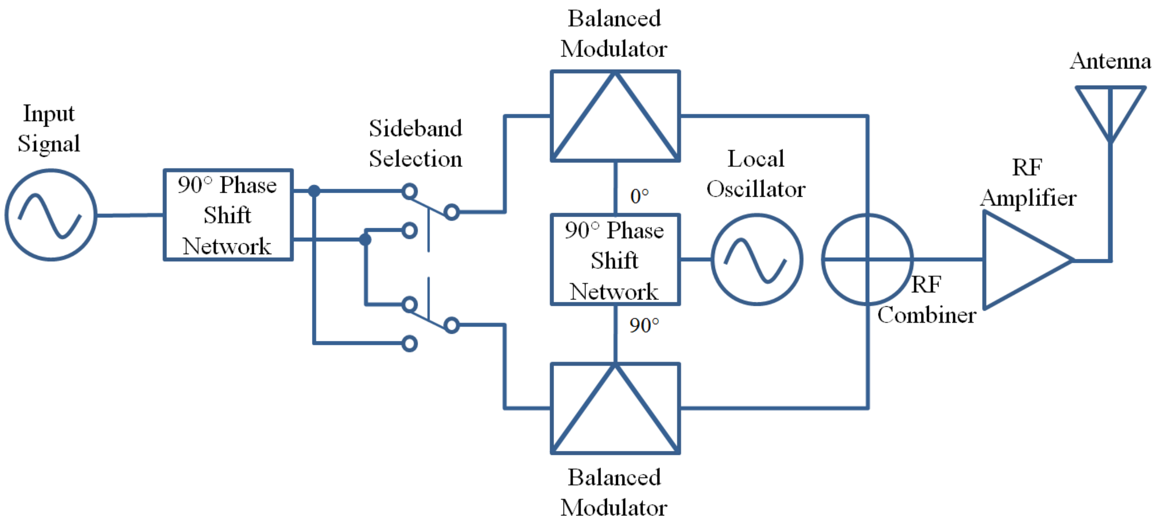

Figure 7.1: Block diagram of a phasing-type SSB transmitter.

Each balanced modulator is given identical information, 90° out of phase. When the outputs from the modulators are added, only one sideband will remain. Many systems have the ability to select which sideband is suppressed by reversing the polarity (inducing a 180° phase shift) of one of the inputs. The phasing method is generally has better quality than SSB signals produced from the filter method.

In the receiving circuit, a synchronous detecting circuit utilizing a phased-lock loop is unable to regenerate the carrier from the SSB signal since only one sideband exists. Therefore the synchronous detecting circuit replaces the phased-lock loop with a *product detector*. As the name implies, a product detector multiplies the SSB waveform with a signal generated by a free-running beat-frequency oscillator. If the suppress-carrier and BFO frequencies differ, the detector produces a composite waveform. A points where both signals have peaks in amplitude, a subsequent peaks (or beats) appear in the composite waveform at a difference frequency of the two original signals. The undesirable beat produced by frequency mismatch can be used to tune a local oscillator to correct carrier frequency, a technique used by superheterodyne receivers [25].

Bibliography

[1] A.N. Beal, B.K. Rhea and R.N. Dean, "A random stimulation source for evaluating MEMS devices using an exact solvable chaotic oscillator," 2015 IMAPS Device Packaging Conference, Scottsdale/Fountain Hills, AZ, March 16-19, 2015.

[2] A.N. Beal, R.N. Dean; "Time-delay of an exact solvable chaotic signal for electronic communications," XXXIV Dynamic Days US 2014, Rice University, Houston, TX, January 9-11, 2015.

[3] J.P. Bailey, A.N. Beal, R.N. Dean, M.C. Hamilton and J.K. Tugnait, "High-frequency reverse-time chaos generation using digital chaotic maps," IET Electronic Letters, Vol. 50, No. 23, Nov. 6, 2014, pp. 1683-1685.

[4] A.N. Beal, J.P. Bailey, S.A. Hale, R.N. Dean, M.C. Hamilton, J.K. Tugnait, D.W. Hahs and N.J. Corron, "Implementing an exactly solvable chaotic oscillator for chaos communications," 2013 SIAM Conference on Applications of Dynamical Systems, Snowbird, UT, May 19-23, 2013.

[5] A. Beal, J. Bailey, S. Hale, R. Dean, M. Hamilton, J. Tugnait, D. Hahs, and N. Corron. Design and simulation of a high frequency exact solvable chaotic oscillator. In *MILITARY COMMUNICATIONS CONFERENCE, 2012 - MILCOM 2012*, pages 1–6, 2012.

[6] M. Ogorzalek. Taming chaos. i. synchronization. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, 40(10):693–699, Oct. 1993.

[7] S. Hayes, C. Grebogi, E. Ott, and A. Mark. Experimental control of chaos for communication. *Physical Review Letters*, 73(13):1781–1784, Sept. 1994.

[8] S. H. Kellert. *In the wake of chaos: unpredictable order in dynamical systems*. University of Chicago Press, Chicago, 1993.

[9] S. Katsura and W. Fukuda. Exactly solvable models showing chaotic behavior. *Physica A: Statistical Mechanics and its Applications*, 130(3):597–605, Apr. 1985.

[10] K. Umeno. Method of constructing exactly solvable chaos. *Physical Review E*, 55(5):5280–5284, May 1997.

[11] N. J. Corron, J. N. Blakely, and M. T. Stahl. A matched filter for chaos. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 20(2):023123–023123–10, June 2010.

[12] M. U. Akhmet. Integral manifolds of differential equations with piecewise constant argument of generalized type. *Nonlinear Analysis: Theory, Methods & Applications*, 66(2):367–383, Jan. 2007.

[13] F. L. Barker. *Communications Electronics: Systems, Circuits, and Devices*. Prentice Hall, Englewood Cliffs, N.J, Jan. 1987.

[14] K. L. Cooke and J. Wiener. Retarded differential equations with piecewise constant delays. *Journal of Mathematical Analysis and Applications*, 99(1):265–297, Mar. 1984.

[15] N. Corron, M. Stahl, and J. Blakely. Exactly solvable chaotic circuit. In *Proceedings of 2010 IEEE International Symposium on Circuits and Systems (ISCAS)*, pages 1356–1359, May 2010.

[16] N. J. Corron and J. N. Blakely. Exact folded-band chaotic oscillator. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 22(2):023113–023113–7, Apr. 2012.

[17] N. J. Corron, J. N. Blakely, and M. T. Stahl. A matched filter for chaos. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 20(2):023123–023123–10, June 2010.

[18] D. Drake and D. Williams. Linear, random representations of chaos. *IEEE Transactions on Signal Processing*, 55(4):1379–1389, Apr. 2007.

[19] K. Gopalsamy. *Stability and Oscillations in Delay Differential Equations of Population Dynamics*. Springer Science & Business Media, Mar. 1992.

[20] T. T. Ha. *Theory and Design of Digital Communication Systems*. Cambridge University Press, New York, Nov. 2010.

[21] S. Hayes, C. Grebogi, and E. Ott. Communicating with chaos. *Physical Review Letters*, 70(20):3031–3034, May 1993.

[22] S. T. Hayes. Chaos from linear systems: implications for communicating with chaos, and the nature of determinism and randomness. *Journal of Physics: Conference Series*, 23(1):215, Jan. 2005.

[23] S. Haykin. *An Introduction to Analog and Digital Communications*. John Wiley, New York, 1st edition, Jan. 1989.

[24] U. L. Rohde and T. T. N. Bucher. *Communications Receivers: Principles and Design*. Mcgraw-Hill, New York, Jan. 1988.

[25] H. W. Silver, editor. *The ARRL 2015 Handbook for Radio Communictions*. Amer Radio Relay League, 92nd edition, Oct. 2014.

[26] G. A. T. W. L. Stutzman. *Antenna Theory and Design:2nd (Second) edition*. Wiley, John & Sons, Incorporated, Dec. 1997.

[27] G. Vernam. Cipher printing telegraph systems: For secret wire and radio telegraphic communications. *A.I.E.E., Journal of the*, 45(2):109–115, Feb. 1926.

[28] S. M. Wentworth. *Applied Electromagnetics : Early Transmission Lines Approach.* Wiley, Hoboken, NJ, 1st edition, Jan. 2007.

[29] D. H. Wolaver. *Phase-Locked Loop Circuit Design.* Prentice Hall, Englewood Cliffs, N.J, 1st edition, Feb. 1991.

Appendices

Appendix A

Diode Double-balanced Mixer Research (Done)

With excellent balance and large dynamic range, the double-balanced mixer (DBM) is common in communication systems, serving as: a simple mixer, modulator, demodulator, limiter, attenuator, switch phase detector or frequency doubler. A basic DBM consists of two or more baluns and a Schottky-diode ring. The level of balance can be determined by how much (if any) of the input signal appears from the output port. An unbalanced mixer does not have RF or LO port suppression, allowing them to accompany the product at the IF port. A single-balanced mixer will suppress one port, either the RF or LO port, while a double-balanced mixer suppresses signals from both the RF and LO ports.

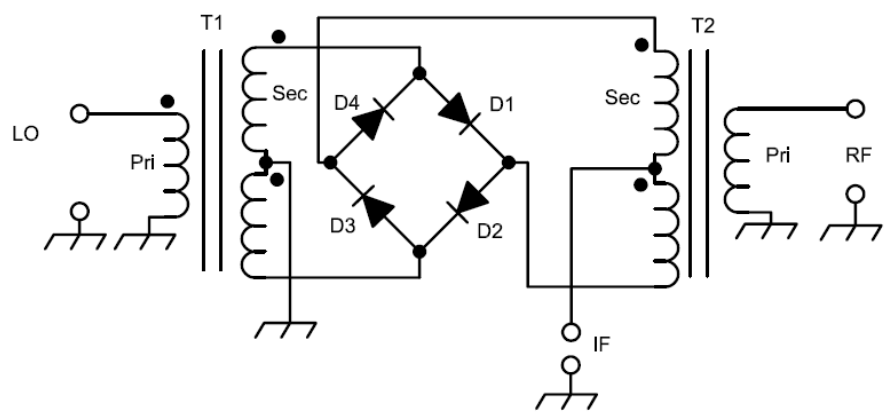

Figure A.1: Double-balanced mixer [25]

In a double-balanced mixer (Figure A.1), the local oscillator port is equally tapped in the center of both baluns resulting in a LO potential at the IF port to be ideally zero. Additionally the secondary winding in the T2 balun creates a null effect of the RF port on the IF port, therefore, the potential between the RF and IF port is also ideally zero when the diodes are off. The diode ring prevents the two baluns from being connected directly, resulting in high RF-IF isolation.

The baluns in a DBM also contributes to the LO drive requirement along with determining the mixer's conversion loss. The losses attributed to the balun can be divided into three types: ferrite core loss, copper loss and impedance mismatch. The lumped impedance components associated with the twisted pair wrapped about the ferrite core sets the high-frequency response of the balun while the the magnetic permeability of the ferrite core, its size, and number of windings about the core govern the lower frequency limit.

When using a mixer, port termination is crucial and must be resistively terminated at the correct system impedance (generally 50 or 75 Ω). Proper port termination can be achieved in the follow ways:

50 Ω resistor or attenuator pad: While that while this does provide sufficient termination, this method should be avoided in receiving circuits due to direct degradation of the system's noise figure.

Amplifier: A low-noise amplifier is a reasonable choice due to its stable resistive input impedance across a wide-band of frequencies.

Diplexer: A diplexer is a type of power splitter with an frequency dependent operation. It electrically appears as a resistive load with two terminals, allowing specific signals to pass through while unwanted signals are dissipated.

By inserting the local oscillator signal to the RF port and keeping IF port dedicated to the modulating signal, a double-balanced mixer becomes classical balanced modulator. The mixer then produces a double-sideband suppressed carrier waveform from the LO port. In addition, the balance of the mixer can be disturbed by the presence of a dc-bias at the IF port, allowing for a portion of the carrier to accompany the waveform leaving the LO port i.e. "partially suppressed carrier".

There exists a fundamental limit of sidebands' magnitude relative to carrier's magnitude in an AM waveform. Pushing the sidebands' magnitude past this point will causing distortion. This limit is referred to as the modulation factor (or index) and must fall between 0 and 1 in order to avoid distortion. The higher the number, the more energy is carried in the sidebands. For a modulation factor of 1 (100% modulation) the amount of power held by each sideband is $^{1}/_{4}$ of the carrier.

The output of a modulator, when used for amplitude modulation, is unable to fall below zero. If, during the modulating process, a modulating waveform forces the carrier signal below zero, over-modulation will occur. While a negatively over-modulated AM wave still carries information in its sidebands, a portion of this energy, however, is used in the generation of harmonics.

The diode rings in DBMs are constructed with symmetrical hot-carrier diodes that have low on-resistances. The mixer's ideal local oscillator drive is dictated by the forward voltage drop across each diode and number of diodes in each ring leg. The local oscillator level requirement is typically specified in dBm. Ensuring that the local oscillator signal, not the RF or IF signals, control the operation of the didoes, it's suggested that the local oscillator be 20 dB greater than the RF and IF signal to achieve proper functionality [25]. This concept minimizes intermodulation distortion and maximizes the dynamic range of the mixer. By keeping the LO drive high enough to switch the diodes on fully and rapidly, the conversion loss, noise figure and intermodulation can be minimized. Note that increasing the LO drive past a device's recommended value will not improve the mixer's performance, only dissipate power produced by the local oscillator.

This proposal is tested by examining the spectral content produced by the mixer under various input signal magnitudes. The spectral content is recorded for an input power level from -23 dBm to +17 dBm with 1 dBm increments. The ranges maximum is selected to match the local oscillators power level of +17 dBm and its minimum value 40 dB below at

-23 dBm. The signal is a sine wave at 2MHz with no dc offset. Pictured in Figure A.2 is the mixers spectrum when the input power level is -23, -11, +2 and +17 dBm.

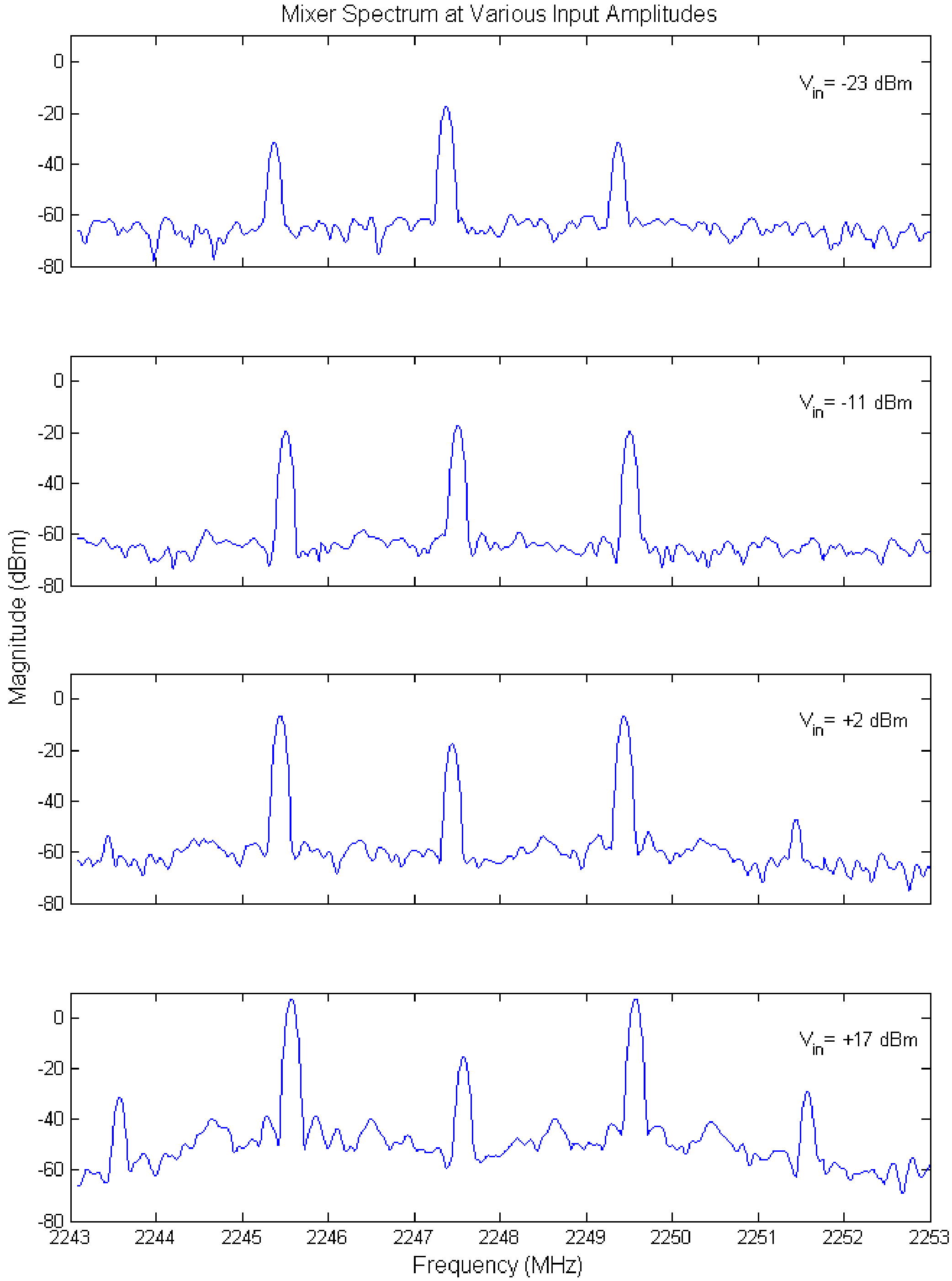

Figure A.2: The mixers spectral output due at various input levels with a local oscillator of +17.3 dBm

From this figure, harmonics are visible in the bottom to graphs. At some point, the input levels becomes too large and leads to the generation of harmonics. In Figure A.3, the power level of the carrier, upper sideband and lower sideband along with the primary harmonic associated with each sideband are presented over the 40 dB range of input power levels.

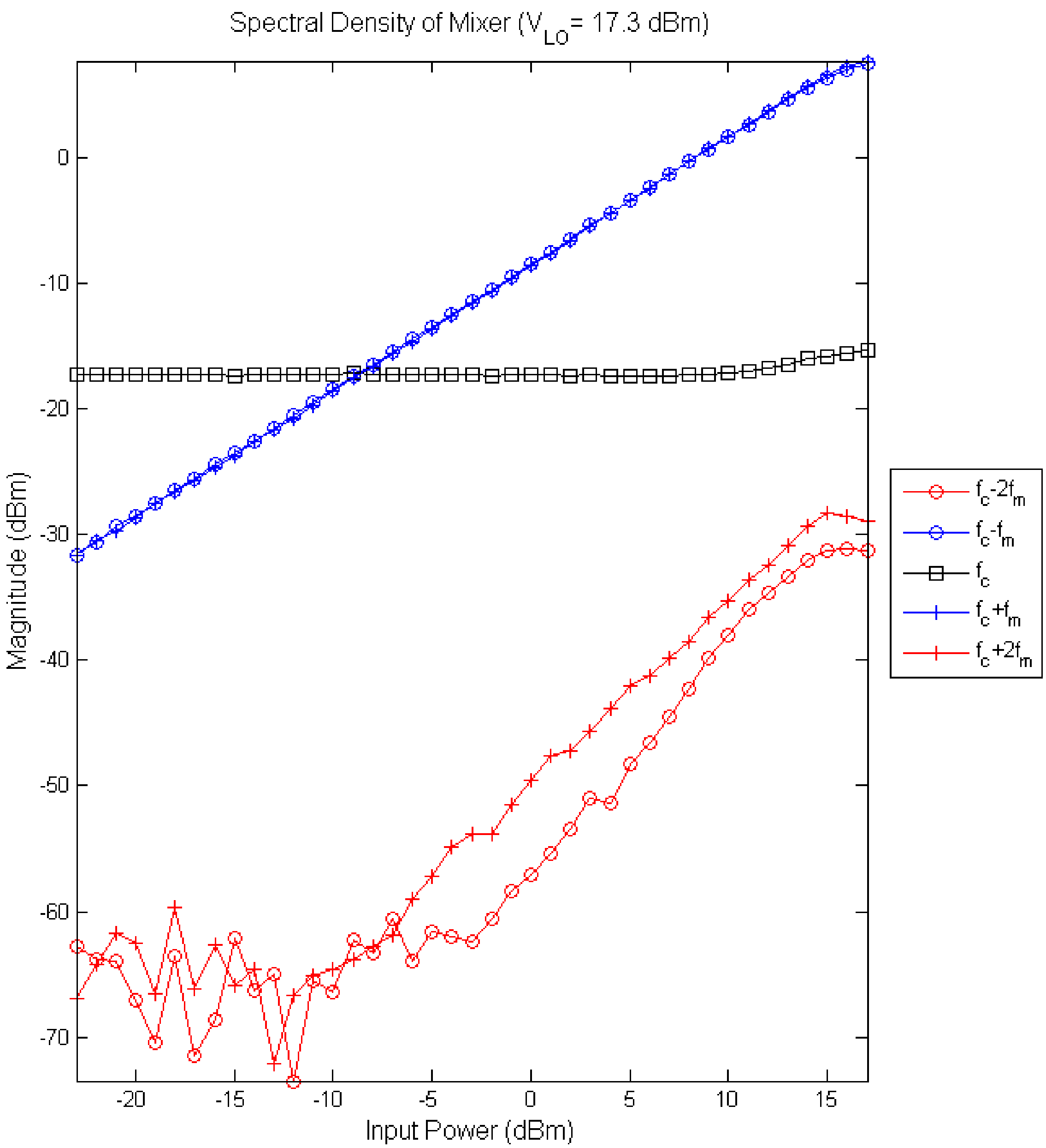

Figure A.3: The power level of the carrier, both sidebands and the primary harmonic associated from each sidebands resulting from an input magnitude from -23 dBm to +17 dBm

Returning to the earlier claim, if the input signal exceeds a level 20 dB lower than the local oscillator, the AM wave would experience harmonic distortion. Therefore given a local oscillator signal of +17.3 dBm, harmonics are expected to appear at input signals greater than -3 dBm. A steady increase in primary harmonics can be seen after for input signals greater than -3 dBm in Figure 3 and confirming the recommendation.

Additionally note that as the sidebands increases, the local oscillators spectral amplitude remains relatively constant at -17 dBm. Referring to the mixers datasheet, the mixer has L-R isolation of roughly 39 dB at 2.3 GHz. For a local oscillator strength of +17.3 dBm and a -17 dBm carrier magnitude measured from the RF port, the isolation is calculate to be 34.3 dB which is within reason.