A Comparison of High School Students' Development of Statistical Reasoning Outcomes in High and Low Statistical Reasoning Learning Environments

by

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A dissertation submitted to the Graduate Faculty of
Auburn University
in partial fulfillment of the
requirements for the Degree of
Doctor of Philosophy

Auburn, Alabama
December 12, 2015

Keywords: reasoning, teaching, learning, statistics, education, constructivism

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Abstract

The purpose of this study was to study the impact of conformity to Statistical Reasoning Learning Environment (SRLE) principles on students' statistical reasoning in Advanced Placement statistics courses. The study encompassed ten different schools in the Southeast United States over a school year. A quasi-experimental design was used to compare teachers with higher and lower levels of conformity to SRLE principles through a matching process used to mitigate the effects of non-random assignment. This matching process resulted in five pairs of similar teachers and schools who differed in self-reported beliefs in the effectiveness of SRLE principles and application of those principles in their instruction.

The impact of teachers' level of conformity to SRLE principles was determined by using an instrument to measure students' statistical reasoning. This instrument was administered as a pre-test after approximately a quarter of the course was completed and re-administered as a post-test at the end of the course. Analysis of covariance with post-hoc analysis was used to compare the development of statistical reasoning between and among teachers' classrooms with self-reported beliefs and practice that were either of low or high conformity to SRLE principles.

Increases in students' statistical reasoning were found at varying levels in both high and low conformity classrooms. Improvements among teachers with low conformity to SRLE principles were less varied and consistent with national averages for improvement by college students. Improvements in classes with high conformity to SRLE principles were more varied, with the students of two teachers with high levels of conformity to SRLE principles showing

large levels of improvement in statistical reasoning in comparison to national averages and those of other teachers within the study. While the comparison between classrooms showing low and high conformity to SRLE principles revealed no statistically significant differences in students' statistical reasoning ability, results from this study suggest that beliefs and practices aligned with SRLE principles show potential to increase students' statistical reasoning at rates above national averages and teachers with similar characteristics. The variation in classrooms with high conformity to SRLE principles suggest the need to further research variables affecting their impact on students' statistical reasoning.

Acknowledgements

Thank you to Jesus for your undeserving love and mercy. By your generosity I live, not to earn my life, but rather to show your love to others (Ephesians 2:8). By your example, others have modeled your character.

Specifically, thank you to Mallory Conway, my Proverbs 31 wife, for your love, understanding, and patience. You model the characteristics of Christ; I love you forever.

Thanks be to my biological parents, in-laws, and grandmother. My life would not be as full without your guidance and compassion.

Thank you to Dr. W. Gary Martin and Dr. Marilyn Strutchens for your mentorship and the opportunities you have provided me. Dr. Martin, thank you for leading by example. Dr. Strutchens, your compassion for all students is magnetic in both your teaching and actions. You both imitate Christ through your servant leadership that I hope to imitate.

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List of Abbreviations

ANCOVA	Analysis of Covariance
CAOS	Comprehensive Assessment of Outcomes in Statistics
CCSS-M	Common Core State Standards for Mathematics
FHSM	Focus in High School Mathematics
GAISE	Guidelines for Instruction and Assessment in Statistics Education
IMP	Interactive Mathematics Program
NAEP	National Assessment of Educational Progress
NCTM	National Council of Teachers of Mathematics
NGSS	Next Generation Science Standard
PBL	Project Based Learning
PCK	Pedagogical Content Knowledge
PISA	Program for International Student Assessment
PSSM	Principles and Standards for School Mathematics
PtA	Principles to Action
SATS-36	Survey of Attitudes Toward Statistics
SRLE	Statistics Reasoning Learning Environment
STI	Statistics Teaching Inventory Survey
TIMSS	Trends in International Mathematics and Science Study
TPCK	Technological, Pedagogical and Content Knowledge

TTLP	Thinking Through a Lesson Protocol

Chapter 1: Introduction

Research in statistics education is a relatively new field (Rossman & Shaughnessy, 2013). Statistics education research has focused in recent years on the increase of statistical reasoning in one cognitive area (Slauson, 2008; Jacobs, 2013; Zieffler & Garfield, 2009; Reading & Reid, 2006; Ben-Zvi, 2004). Joan Garfield (2002) stated that statistical thinking and reasoning should be the desired outcomes for a course; however, “no one has yet demonstrated that a particular set of teaching techniques or methods will lead to the desired outcomes” (p.10). Since Garfield’s discussion (2002), the limited number of studies that have attempted to measure outcomes in statistics education based on teaching methods have had mixed outcomes (Loveland, 2014). Studies that have compared outcomes within statistical courses with the treatment of different teaching methods have used the same teacher using different pedagogical techniques and are at potential risk of errors in treatment effects (Baglin, 2013; Loveland, 2014; Slauson, 2008).

Baglin (2013) and Loveland (2014) both looked at promoting reasoning and thinking holistically, but in one class being taught by the same instructor in two different ways. There has been little to no empirical research in statistics education on what pedagogical strategies improve statistical reasoning and thinking (Baglin, 2013; Loveland, 2014). There has also been little attention to pedagogical strategies within statistics education that promote conceptual understanding, thus the literature review in this dissertation uses mathematics education to help fill these gaps in pedagogical content knowledge needed for statistics. Studies from mathematics education were considered relevant to the study of promoting statistical reasoning because of their similarities but also compared because of their differences (delMas, 2004).

Statistics Education as a Discipline

The *Guidelines for Assessment and Instruction in Statistics Education* (GAISE) written by Franklin et. al. (2005) stated that statistics has "become a key component of the K-12 mathematics curriculum" (p. 3). Society is becoming more and more data driven, and students' fundamental understanding of statistics is integral if they are to be informed decision makers in a democratic society and prepared to enter the new statistical driven workforce (Franklin et. al, 2007; National Mathematics Advisory Panel, 2008). The difficulty students have in dealing with variation and context in real world data makes the attention of statistics education starkly different from mathematics (delMas, 2004). Cobb and Moore (1997) stated that:

> Statistics is a methodological discipline. It exists not for itself, but rather to offer to other fields of study a coherent set of ideas and tools for dealing with data. The need for such a discipline arises from the *omnipresence of variability*.

The understanding of statistics is fundamental; The National Council for Teachers of Mathematics' (NCTM, 2000) *Principles and Standards for School Mathematics* (PSSM) and the National Governor's Association for Best Practices and Council of Chief State School Officers (NGA & CCSSO, 2010) *Common Core State Standards for Mathematics* (CCSS-M) have included data analysis and probability objectives at all grades. Ideas within these documents have pushed many teachers of mathematics and statistics to rethink the curriculum in their courses and their pedagogy. Similar to mathematics reform movements over the last fifty years attempting to shift teachers instructional roles from "a sage on the stage" to "a guide on the side", statistics educators have also emphasized the facilitation of learning rather than the dissemination of content (Chance & Garfield, 2002). Reform movements in statistics education have promoted the use of real world data, cooperative groups, alternative assessments, and technology to develop student understanding of statistics and statistical concepts (Garfield and Ben-Zvi, 2008; Chance, 2002; Chance, 1997; Keeler & Steinhorst, 1997; Mills, 2002; Franklin et. al, 2007; Cobb, 1992).

In addition to changes in pedagogical approaches, many teachers have been emphasizing less pure probability in their courses in order to develop stronger understanding of statistical concepts (Rossman & Chance, 2002; Cobb & Moore, 1997; Shaughnessy, 2003). Others have devoted less time to computation of asymptotic structures of statistics in favor of understanding these structures of data through randomized designs for tests of difference (Tintle, Topliff, Vanderstoep, Holmes, & Swanson, 2012; Cobb & Moore, 1997; Rossman & Chance, 2013; Cobb, 2013). In fact, the newest document for the Statistics Education of Teachers (Franklin et. al, 2015) purposefully does not include asymptotic structures for probability.

Though statistics has been considered a discipline of mathematics for many years, statistical content has continuously shifted in value (Moore, 1993; Cobb & Moore, 1997, McAfee & Brynjolfsson, 2012; Cobb, 2013). Powerful computational capabilities and large amounts of data collected via the internet and through large scale corporations have reshaped the need for statistics on a global level (Moore, 1993; Cobb & Moore, 1997; McAfee & Brynjolfsson, 2012; Cobb, 2013). Random number generation, statistical computation, bootstrapping, big data collection, and reiteration are just a few of the processes once only dreamed of by statisticians approximately three decades ago (Moore, 1993; Cobb & Moore, 1997; Cobb & Moore, 2000; Cobb, 2013). McAfee and Brynjolfsson (2012) and Lohr (2012) discussed recent changes in the collection, use, and analysis of big data to increase business revenue and change managerial responsibilities and qualifications. The power of computational procedures, ease of data collection, and condensed storage space has increased the need for statistics and created new jobs that require the understanding of statistical designs and programming in small and large businesses to become profitable (Konold & Higgins, 2003; McAfee & Brynjolfsson, 2012).

Based on trends in 2004, the American Diploma Project's *Ready or Not: Creating a High School Diploma that Counts* recommended that high schools increase their attention to data analysis, statistics, and their applications to ensure students were ready for more advanced subjects in post-secondary education and prepared for growing jobs in a data rich society. Horton, Baumer, and Wickham (2014) suggested that giving students learning experiences in introductory statistics and math courses that develop reasoning and conceptualization of statistics can help those dealing with statistics understand the differences between "big data" and statistics. Understanding these differences can allow for the use of big data in better business decision making. The problem however lies in the monitoring of conceptualization of statistics at both the national and international level.

The Problem

Our data rich society has increased attention to the need for statistical literacy, reasoning, and thinking by students and teachers. To monitor this need at the national level, the National Assessment of Educational Progress (NAEP), the "Nation's Report Card", began monitoring data analysis, statistics, and probability subscales in 1990 every two years in grades four, eight, and twelve. The Trends in International Mathematics and Science Study (TIMSS) started in 1995 with collection periods every four years in grades four, eight, and twelve. The Program for International Student Assessment (PISA) for fifteen year olds began in 2000 and assesses students on probability and statistics every three years. These assessments have been used to both monitor student and teacher knowledge of statistical literacy, reasoning, and thinking.

International Assessments

In 2007, TIMSS reported that 92% of teachers felt well prepared to teach data analysis and probability; however, in 2011 83% felt well prepared to teach data analysis and probability. Moreover, the 2011 TIMSS reported that “only 62 percent of students on average internationally, had teachers who felt very well prepared to teach the data and chance topics” (Mullis, Martin, Foy, & Arora, 2012, p. 303). The closest rival of all content domains measured on the TIMSS report was geometry in which 85% of students’ teachers felt well prepared. This extreme difference shows approximately a quarter or fewer of teachers felt well prepared to teach content in data and chance as opposed to any other content domain and almost half of current educators do not feel well prepared to teach topics of data and chance. Mullis et. al (2012) also reported that data and chance topics were given the least attention internationally at the eighth grade and had the largest variation between countries on the TIMSS. The introduction of new probability and statistics standards in the United States through large state adoption in 2010 of the CCSS-M has affected many classrooms’ and teachers’ focus in the United States (Franklin & Jacobbe, 2014; Franklin et. al, 2015). These standards may have potentially contributed to the negative trend in teachers’ self-reported preparedness to teach probability and statistics (Franklin et. al, 2015).

Though less teachers feel comfortable with statistics and probability standards, students have reported approximately an 8% increase in their teachers’ focus on data analysis and probability standards. In 2007, 83% of students reported being taught data analysis and probability standards; however, in 2011, 91% reported being taught data analysis and probability standards (U.S. Department of Education, 2014). A larger emphasis on data analysis and probability standards with less confidence seems to be a recipe for disaster in student achievement.

According to TIMSS, the data analysis and probability subscale has been flat for the last 16 years in both grade 4 and grade 8 with the exception of a significant increase from the years 1999 to 2003 in grade 8 (Mullis et. al, 2012). By 2011, however, the grade 8 scaled score was again below that recorded in 2003. Internationally, the United States scored above average; however, this is likely correlated with teacher preparedness at the international level. Teacher preparedness to teach or shifting standards in the CCSS-M may also potentially explain the decrease in the United States subscale from 2007 to 2011 in grade 8. *Figure 1* summarizes the TIMSS data for the U. S. and international averages from the years 1995 to 2011.

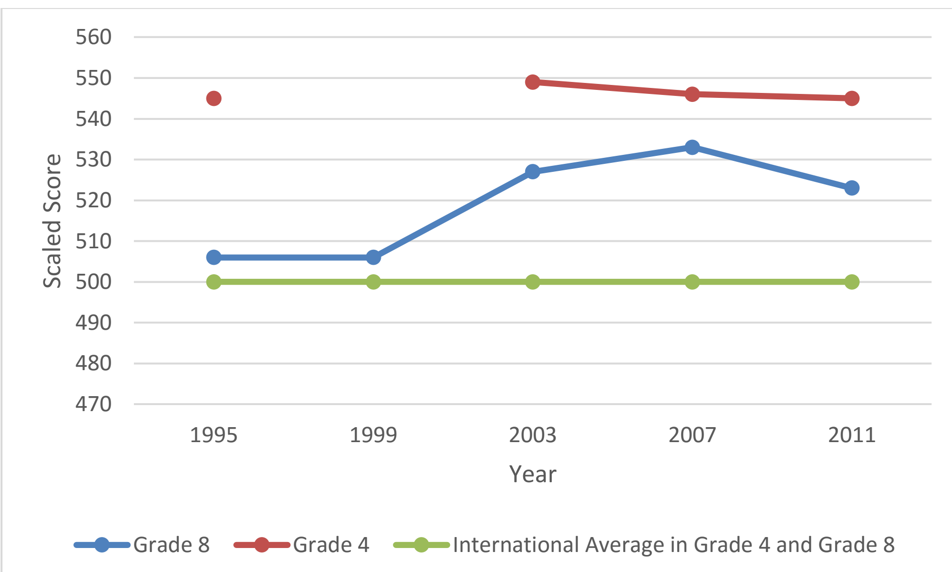

Figure 1. TIMSS data and probability subscale for years 1995 through 2011.

Scoring rubrics from the TIMSS indicated that students on average were at an intermediate stage in which they can apply their mathematical knowledge in basic situations but lack the ability to use their knowledge to solve problems and apply them to complex situations

(Provasnik et. al, 2012). Results from the TIMSS (Provasnik et. al, 2012) also highlighted the stagnation in student achievement in mathematics and in particular data display, data, and chance. The Program for International Assessment (PISA, OECD, 2014) reported that students struggled with tasks involving the creation, use, and interpretation of real world data and the use of reasoning about this data. Results from the 2009 and 2012 PISA indicated that the United States average score was still below the international average (OECD, 2014). These results make apparent the need for increased attention in probability and statistics at the K-12 level that emphasizes the use of real world data and reasoning in the United States.

National Assessments

When the NAEP chose to monitor the content domain of statistics and probability, the proportion of questions addressing this domain increased from 6% in 1986 to 20% in 1996 for grade twelve students (Shaughnessy, 2003). The most recent version given to grade twelve students in 2013 included approximately 25% of problems related to data analysis, statistics, and probability. Similarly, grade 8 problems related to probability and statistics were increased by 15% from 1986 to 1996 for students at the fourth grade level in 1996 (Shaughnessy, 2003). The 2013 version of the NAEP included approximately 15% of problems for grade eight students and 10% of problems for grade four students (U.S. Department of Education, 2014). This data has shown an increase in the monitoring of probability, statistics, and data analysis at the national level, but not necessarily at the classroom level.

Shaughnessy (2003) reported that 12% of twelfth grade students in 1990 had experience with statistics, data analysis, and probability in high school and 20% in 1996 from NAEP data. Unfortunately, the NAEP has not measured students experience with statistics, data analysis, and probability since 1996. The complement to the data in 1996, however, was that approximately

80% of students had no experience with data analysis, statistics, and probability in the secondary school setting. These students left the high school setting to enter a very wide range of professions that called for a very diverse use of statistics, data analysis, and probability. With so few students having experience in their high school careers with statistics, K-12 stakes holders leave a great deal of responsibility on the development of knowledge related to data analysis, statistics, and probability at the tertiary level.

The data analysis, statistics, and probability subscale scores on NAEP have been flat, even more dramatically than international assessments. The largest gains seem to have occurred between the years 1996, 2000, and 2003 in grades 4 and 8. The grade 8 subscale increased from 270 to 275 from 1996 to 2000 and from 275 to 280 in 2003. The grade 4 subscale increased from 228 to 237 from the year 2000 to 2003. From 2003 to 2013, grade 4 subscale scores have only increased by a scaled score of five and grade 8 scaled scores have only increased by a scaled score of 6. Figure 2 summarizes NAEP data for data analysis, statistics, and probability for the years 1990 to 2013 in grades 4, 8, and 12.

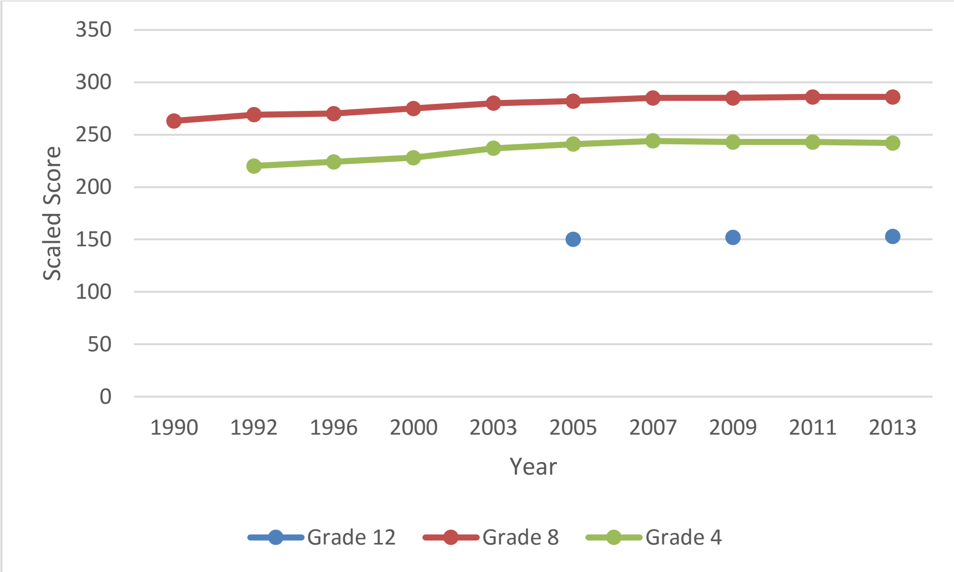

Figure 2. NAEP data analysis, statistics, and probability subscales for 1990-2013 in grades 4, 8, and 12.

Teacher Knowledge

The problem of student achievement in statistics is potentially manifested by the lack of teacher statistical pedagogical and statistical content knowledge. In addition, the lack of experience students have in the secondary setting is also potentially strongly associated with teacher knowledge. In 2007, Jacobbe used NAEP questions to assess three teachers' content knowledge of statistics in the 4^{th} grade. All three teachers answered 90% of problems correctly that required less difficult procedural knowledge but only 37.5%, 62.5%, and 75% of difficult procedural knowledge problems (Jacobbe, 2007). When assessed conceptually, these teachers did not recognize the difference between histograms and bar graphs and had no familiarity with boxplots and histograms (Jacobbe, 2007). Similarly, these three teachers could calculate the mean of a data set, but could not determine the meaning of mean and mode or exhibit the ability to use the mean or median appropriately (Jacobbe, 2007). These three teachers also had difficulty in understanding range, applying the range to a histogram, and understanding spread about a certain location (Jacobbe, 2007).

In 2011, Jacobbe and Carvalho analyzed teachers' knowledge of average and found strong reliance on procedural skills. This fact highlights students' inability to create, use, and interpret real world data and use of reasoning about data in the PISA (OECD, 2014) and use their knowledge to solve problems and apply them to complex situations on the TIMSS (Provasnik et. al, 2012). According to Jacobbe (2007), it seems illogical to ask these teachers to teach students material that exceeds their own level of knowledge. Jacobbe and Carvalho (2011) stated, "A teacher that feels uncomfortable with statistics may have a tendency to reduce or omit statistical topics from their enacted curriculum" (p. 16). This suggests that teachers' lack of understanding of statistics was possibly causally related to the inclusion of statistical content in the enacted curriculum. Jacobbe's studies highlighted the need at the post-secondary level to increase teacher knowledge of statistics education.

Summary

These assessments have highlighted the need for improvement in K-12 statistics education. The United States scored above average internationally in both grade 4 and grade 8 since the beginning of the TIMSS and below average on the PISA. According to the OECD (204), reasons for scoring below average on the PISA related to student use of real world data and the ability to reason about data. With the exception from 1999 to 2003 on the grade eight TIMSS report, little difference has been seen between scaled scores in grades 4 and grade 8 in national and international results. Grade 12 United States scaled data analysis, statistics, and probability scores have similarly shown little improvement in the last decade. At a national level, NAEP results have highlighted an increase in the monitoring of probability and statistical standards. In addition, the NAEP results have shown gradual increases but recent decreases in student achievement in probability and statistics. Assessments and research have highlighted the fact that teacher preparedness is in need of improvement (Jacobbe, 2010). The increase in rigor and number of data analysis and probability standards in both national and international assessments make the teaching of statistics a pressing issue in current education.

Steps to Understanding a Solution

This study was guided by the broad question, “What instructional practices promote statistical reasoning most effectively?” Thus, this study exists to help explore the impact of effective teaching practices to the development of statistical reasoning. Though teaching methods promote reasoning, other factors such as technology, curriculum, and assessment have the ability to promote statistical reasoning as well. For this reason, attention to teaching, technology, curriculum, and assessment are also potentially important components to consider in answering this broad question.

In order for students to learn statistics at deeper levels, they must learn statistics in ways that develop conceptual understanding of important statistical ideas at all levels of education (Pesek & Kirshner, 2000/2002; Feldman, Konold, Coulter, & Conroy, 2000; Mokros & Russell, 1995). NCTM (2014) stated that effective teaching and learning is composed of eight major practices related to learning mathematics and statistics. Table 1 summarizes the Principles to Action’s (PtA) teaching practices and correlates them with related statistics education literature (NCTM, 2014). Teachers should recognize and devote attention to breaking cognitive and affective domain barriers during their teaching.

Table 1

Teaching Practices Related to Selected Statistics Education Literature

Mathematics Teaching Practices (NCTM, 2014)	**Selected Guiding Statistical and Mathematical Literature**
focus on establishing clear statistical goals	Franklin et al, 2007; NCTM, 2000
implementing tasks that promote reasoning and sense making	Shaughnessy, Chance, & Kranendonk, 2009
use and connect representations	Chance & Rossman, 2006
pose purposeful questions	Smith & Stein, 2011; Herbal-Eisenmann & Breyfogle, 2005
build procedural fluency from conceptual understanding	Pesek & Kirshner, 2000/2002; Feldman, Konold, Coulter, & Conroy, 2000; Mokros & Russell, 1995; Konold, 2002
support productive struggle	Warshauer, 2011; Hiebert and Grouws, 2007
orchestrate productive discourse	Smith & Stein, 2011
elicit and use evidence of student thinking	Chance, 2002; Garfield, 2002; delMas, 2002

Teaching that promotes statistical reasoning also incorporates the use of curriculum, technology, and assessment that is aligned with learning goals to promote in-depth learning (Garfield and Ben-Zvi, 2008, 2009; NCTM, 2014, 2000). Curricula should provide progressions that allow students to build upon and develop their own knowledge through high-level tasks (NCTM, 2014). Teachers should use technology to build students' conceptual understanding of statistics and not just as a means for computing statistics or presenting presentations (Chance &

Rossman, 2006). Teachers should use assessments as an instructional tool. This provides a way for students to gain understanding while at the same time allow the teacher to gage student understanding (Petit, Zawojewski, & Lobato, 2010; Chance, 1997; NCTM, 1995). Teachers who align their curriculum to in-depth learning goals, use technology to build conceptual understanding, and use assessment as a learning tool are more likely to promote statistical reasoning and thinking.

This study aimed to understand the extent to which students' statistical reasoning and thinking ability improve in classrooms whose teacher had self-reported beliefs and practices that that had higher or lower levels of alignment with best practices in mathematics and statistics education. These best practices were aligned with research in teaching, learning, curriculum, and assessment in mathematics and statistics education found within Chapter 2. In addition to analyzing the extent to which students improved in reasoning in these classes, this study quantified differences found between these two different types of classrooms.

Chapter 2: Literature Review

To understand the impact of instruction on students' statistical development, the following literature review is subdivided in a similar manner to National Council for Teachers of Mathematics (NCTM) (2014, 2000, 1991) documents that promote a strong vision for mathematics education. The focus of the review highlights five major aspects of statistics education: teaching, learning, technology, curriculum, and assessment. The learning section expounds on statistical learning levels, cognitive barriers to learning, and affective barriers to learning. The teaching section explicates teaching strategies that promote student learning at in-depth levels as categorized by NCTM (2014). The technology section details technology that has been used to teach statistics and research that has been conducted in the area of technology and instruction. The curriculum section reviews important statistical content, programs, and progressions that promote teaching and learning of statistics at in-depth levels. The assessment section highlights the use of assessments in statistics education that have been used to monitor the efforts of statistical achievement. The final section synthesizes this information to describe a learning environment that promotes statistical reasoning and thinking.

Learning of Statistics

NCTM's (2014) *Principles to Action: Ensuring Mathematical Success for All* (PtA) stated that setting clear learning goals sets the stage for everything within mathematics instruction. For this reason, learning was reviewed first in this literature review. NCTM's (2000, 2014) vision for the learning of mathematics requires students to understand and actively build new knowledge from experience and prior knowledge. NCTM (2014) reiterated support for this vision of learning through principles of *Adding It Up* (Kilpatrick, Swafford, & Findell, 2001). This vision of mathematical learning proficiency has five intertwined strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Kilpatrick, Swafford, & Findell, 2001). Each of these strands are interconnected and develop upon and with one another. These strands are all key ingredients in a mathematically proficient student.

Just as each of these strands are essential for mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001; NCTM, 2014), the learning of statistics with literacy, reasoning, and thinking should be critical elements within the statistics classroom. Literacy, reasoning, and thinking are all key components in what makes a statistically proficient student. The following sections look at how the concepts of statistical literacy, reasoning, and thinking defined in statistics education research play a fundamental role in learning experiences of statistics classrooms.

Statistical Learning Levels

An outside observer might interpret the three terms statistical reasoning, statistical thinking, and statistical literacy as synonymous. Indeed many in statistics education have used literacy, reasoning, and thinking interchangeably in the past (delMas, 2002). Though these terms have overlapped one another in research, there have been specific distinct characteristics in statistics education research that encompass each (delMas, 2002; Rumsey, 2002; Garfield, 2002; Chance, 2002). Statistical literacy has been a very broad term that researchers have used to define citizens reading statistical graphs in the news (Rumsey, 2002). Rumsey (2002) also described students interpreting statistical output in classrooms as statistical literacy. Students who reason statistically have the ability to move from the reading and writing of statistics to the understanding behind certain principles like sampling distributions, sampling designs, and testing procedures (Garfield, 2002). Statistical thinkers have the ability to view statistics as an iterative and conjoined process through the four major goals of asking questions, collecting and representing data, analyzing data, and making conclusions (Chance, 2002). The next four sections look into these levels or goals for statistical instructors in more detail and discuss some of the interconnections between each.

Statistical Literacy. Rumsey (2002) wrote that a statistically literate student was able to recognize information they were exposed to on a daily basis, assess its validity, and make adequate decisions based on this information. Many people can claim statistical literacy in this very broad definition. Rumsey's (2002) definition of statistical literacy does not provide a clear distinction of statistical literacy for a classroom, a high school, or a nation. To help provide clarity for this needed distinction, Rumsey (2002) describes statistical literacy as either "statistical citizenship" or "statistical competency".

Many teachers being pushed for time in their classes may potentially teach more towards statistical competency. Rumsey (2002) described statistical competency in greater detail as data awareness, an understanding of certain basic statistical concepts and terminology, knowledge of the basics of collecting data and generating descriptive statistics, basic interpretation skills, and basic communication skills. These skills require statistically competent students to explain results to others, describe results within context, and know and perform certain statistical methodology (Rumsey, 2002). Though all of these concepts are extremely important, they are only marks of what makes a statistically competent individual.

Rumsey (2002) discussed that teachers of statistics should want more from a class that statistical competency, including students understanding of why and how. Students have been rarely pushed in statistical classes to look for reasoning behind statistical tests and formula construction (the why), but rather look for how to complete a procedure and interpret its results correctly (Rumsey, 2002). Pushing statistical students to be statistically literate helps make them be better readers and criticizers of research results. Teaching for statistical literacy also makes grading and instruction easier for teachers through explicit logic to conclusions, procedures to complete tests through software, and complete mathematical calculations that result in one solution. In short, many teachers who teach only for statistical literacy or competency miss broader goals of the reasoning behind statistics and lose the ability to be sophisticated criticizers of research and statistical reports.

Research on Improving Statistical Literacy. Verkoeijen, Imbos, van de Wiel, Berger, & Schmidt (2002) described students who have gained conceptual understanding of statistical knowledge as having the ability to use statistical terms, apply formulas, complete arithmetic procedures, understand the conditions for application of the learned knowledge, interpret outcomes of mathematical calculations, and understand theoretical background knowledge correctly. All of the characteristics that Verkoeijen et al. (2002) used to describe conceptual understanding were those of statistical literacy (Rumsey, 2002; delMas, 2002). Verkoeijen et al. (2002) looked into the effects of increasing literacy using a constructivist-learning environment in health sciences statistics classrooms.

Verkoeijen et. al (2002) subjected 107 students to a four-week constructivist-learning environment, and then used free recall to understand students’ development of knowledge. Drawing on Verkoeijen et al.’s (2002) implementation of the constructivist learning environment, it could be seen that Verkoeijen et. al’s (2002) learning environment was not in complete alignment with this learning theory. Students were introduced to concepts in a lecture format before being allowed to construct their knowledge. This may promote procedural interference with students' statistical reasoning (Pesek & Kirshner, 2000/2002; Feldman, Konold, Coulter, & Conroy, 2000; Mokros & Russell, 1995). Scaffolding which potentially lead to explaining (Verkoeijen et. al, 2002) during group work was also being administered by "advanced undergraduate students" or "staff members" who may have not learned important aspects of good questions and fostering discourse (Stein, Engle, Smith, & Hughes, 2008).

Verkoeijen et al. (2002) used a free recall protocol to understand student conceptions because of the difficulty of tapping into knowledge constructs of students by use of multiple-choice tests. Verkoeijen et. al (2002) used concept maps to identify how much information students were able to "freely recall" correctly and incorrectly related to content covered during the four-week session. As Verkoeijen et al. (2002) suggested, assessment using free recall did not form a seamless technique for students to demonstrate understanding the way it was developed which could misrepresent acquired data. The assessment rubric was also not clearly disseminated to students. The use of rubric dissemination would allow students to share the knowledge they do have and make the assessment technique more open (NCTM, 1995).

Verkoeijen et al. (2002) reported that only 40% of students developed conceptual understanding or Rumsey's (2002) statistical literacy within the course. Students were also highly likely to be unable to recall information taught in the four-week session and recall it accurately (Verkoeijen et al., 2002). Analysis was not judged as a comparison to other learning environments but as meeting a set of standards to be checked off in Verkoeijen et al. (2002) study. This lack of comparison makes it difficult to assess the constructivist learning environments true effectiveness on statistical literacy.

Statistical Reasoning. Past statistics education research using the word statistical "reasoning" has mentioned the incorporation of statistical comprehension, planning and execution of statistical procedures, and evaluation and interpretation of statistical procedures very similar to literacy (Garfield, 2002). Statistical reasoning is more; it is the actual process one must take to understand and be able to implement a statistical procedure in multiple contexts. Garfield (2003), a leading researcher in statistical reasoning, defined statistical reasoning as "the way people reason with statistical ideas and make sense of statistical information" (p.23).

Statistical reasoning has been of extreme importance in the development of curriculum that fosters student development of knowledge (Garfield, 2003). Good statistical reasoning pushes students to give appropriate reasoning to descriptions, judgments, inferences, and opinions about data (Garfield, 2003). For this reason, Garfield (2003) pointed out that paper and pencil exams are extremely hard to measure statistical reasoning, and statistical reasoning is best understood through face-to-face discussions and highly detailed student work.

Shaugnessy, Chance, and Kranendonk (2009) used the following action words to describe habits of mind that illuminate statistical reasoning: describing, analyzing, explaining, looking for structure, critiquing, considering, creating, questioning, evaluating, applying, noticing, understanding, connecting, determining, justifying, determining plausibility, and validating. Shaughnessy, Chance, and Kranendonk (2009) focused on the relationship between student reasoning and formulating questions, designing and employing a plan for collecting data, analyzing and summarizing data, and interpreting the results from the analysis. Projected student discourse was used throughout the text to help teachers foster meaningful classroom small groups and whole class discussions that develop statistical reasoning. Plots were given to help clarify and give statistical justification to student discussions while side box notes for teachers were given to point out objectives of the classroom teacher during their discussions. Although not explicitly stated, Shaugnessy, Chance, and Kranendonk (2009) described the important role discourse has on the development and assessment of statistical reasoning.

Statistical Questions from the Classroom (Shaughnessy & Chance, 2005) promoted statistical literacy by helping teachers and students alike understand certain concepts of statistics that many times seem unexplainable to non-mathematicians. One particular portion was a discussion of why population variance was divided by N, the population size, and sample variance divided by n-1, the sample size minus one. Other concepts of particular interest from this text for middle school teachers focused on the reason why the standard deviation was preferred to the mean absolute deviation found within the standards. Justification of why the average standard deviation was less biased to be divided by n-1 was shown through computations of statistics with n-1 and N with known population parameters to develop statistical reasoning. Justification for the difference between measures of variation was shown similarly through examples and graphs. Shaughnessy and Chance's book was designed to develop teacher reasoning with important statistical content.

NCTM's (2000) *Principles and Standards for School Mathematics* (PSSM) provided multiple examples of reasoning in mathematics education more generally and in probability and statistics more specifically. NCTM (2000) pushed students to make and investigate conjectures, develop and evaluate arguments, communicate thinking to others, organize and consolidate information coherently and in different ways, use appropriate language, make connections between major concepts, apply to the real world, and build new knowledge through problem solving in fostering statistical reasoning. In particular, PSSM (NCTM, 2000) stated that teachers creating activities that push students to explain their reasoning, make sense of problems, clarify thinking, critique one another's arguments, and find new ways to solve problems should intertwine reasoning and proof within mathematics and statistics continually. To encourage reasoning teachers were encouraged to ask why rather than how and push students to justify their thinking process to ensure conceptual understanding.

Though mathematical reasoning and statistical reasoning are very similar, statistical reasoning provides an extra complexity with the inclusion of variation (Garfield, 2003). DelMas (2004) went into detail over the differences and difficulties of mathematical and statistical reasoning. Of particular importance, delMas (2004) discussed how statistics relies on data. Many of the concepts within statistics are abstract in nature making it very difficult to teach (delMas, 2004). In addition, statistics often provides different formal inferences from different viewpoints and the importance of context within statistics can never be disregarded (delMas, 2004; Cobb & Moore, 1997). In mathematics, context may or not matter and may interfere with abstraction during mathematical tasks (delMas, 2004). On this note, delMas (2004) suggested that removing causation from bivariate data was a difficulty for students moving from mathematical to statistical conclusions. Because of this heavy reliance on context and abstraction, statistical reasoning for students can become flawed in many different ways making it harder to understand student misconceptions (delMas, 2004; Cobb & Moore, 1997). Table 2 summarizes some of the differences between teaching mathematics and statistics.

Table 2

A Comparison of Demands in Mathematics and Statistics Education

Mathematics	**Statistics**
Reason with highly abstract concepts and relationships without variation	Reason with highly abstract concepts and relationships with variation
Begin with context for familiarity, then generalize outside the context of a situation	Analyze within context and describe within context
Teacher focuses on mathematical reasoning errors	Teacher must focus on reasoning errors contextual and/or mathematical.
Contextual situations compared case by case to find patterns and apply generalization	Contextual situations generalized with variation in anticipation of errors

Research on Improving Statistical Reasoning. Slauson (2008) investigated the difference between student learning of variation from a standard lecture based format and a format emphasizing experimentation in two different sections of an introductory statistics course at the tertiary level. To measure outcomes of the experiment, Slauson (2008) used a pre and post multiple-choice assessment that focused on statistical reasoning. Results were also gathered through student interview questions related to variation. In analyzing results from the reasoning assessment, Slauson (2008) found improvement in students' statistical reasoning about standard deviation in the hands-on experiment group but not in the lecture based group. There was no improvement found in students reasoning about sampling distributions (Slauson, 2008). The effect size, however, was minimal with approximately only one more questions being answered correctly (Slauson, 2008). For this reason, there was no statistically significant differences found between differences in pretest and posttest for the hands-on experiment learning and lecture based classes.

Difficulties in the study design might help to understand the findings. Slauson (2008) did not completely allow for student exploration of variation in the hand-on experiment class because the first two days were devoted to explaining the formula for standard deviation in a lecture setting. Slauson (2008) also lacked attention to discourse that fosters an active learning approach. Students' prior experience with the lecture based learning environment could be a confounding factor in Slauson's (2008) study. The inclusion of student background variables might have helped in explaining differences in student outcomes.

Loveland (2014) subdivided one section of an introductory statistics course at the university level into two sections taught by the same teacher. One section was taught with entirely lecture based teaching methods and the other with largely active learning methods that included group work, hands on activities to gather data, and teacher facilitation with minimal amount of lecture. Loveland (2014) was investigating whether students developed more conceptually, procedurally, or both using active learning environments. After completing the courses, Loveland (2014) found no difference between students' ability to comprehend statistics or complete statistical procedures in the traditional lecture based and active learning based course. Loveland (2014) used stepwise selection to determine the most significant affects in her study. This procedure found that previous GPA, pre-test points, and attendance were the most contributive predictors of student achievement with whether classes were lecture based or activity based being insignficant.

Statistical Thinking. Beth Chance (2002) described multiple views of statistical thinking in research. The overarching theme from Chance (2002) has suggested that statistical thinking be a global view of the statistical process. Those who think statistically have the ability to critique, evaluate the process in its entirety, and immerse themselves in fine-tuning the process to produce better results. Chance (2002) presented six habits of mind that characterize statistical thinkers:

1. consider how to best obtain meaningful and relevant data to answer the question at hand
2. constant reflection on the variables involved and curiosity for other ways of examining and thinking about the data and problem at hand
3. seeing the complete process with constant revision of each component
4. omnipresent skepticism about the data obtained
5. constant relation of the data to the context of the problem and interpretation of the conclusions in non-statistical terms
6. thinking beyond the textbook

Her analysis of statistical thinking was based on the development of the constant critiquing nature of a statistician. Chance (2002) went on to describe the needed relationship between assessment and instruction: if an instructor wants students to gain the habits of mind of a statistical thinker, they must include follow up questions in assignments and projects that develop this process and hold students accountable for this information.

Research on Improving Statistical Thinking. Baglin (2013), as well as Chance and Garfield (2002), discussed the lack of empirical evidence in statistics education literature to show increases in student reasoning and thinking. Baglin (2013) reported two different attempts to show increases in student thinking and affection of statistics through a Project Based Learning (PBL) environment called *Island*. This environment required students' to make decisions in simulated case studies about whether to conduct experimental studies or use observational data to make inferences. Baglin's (2013) first study gave qualitative evidence for further investigation of the PBL. Surveys administered by Baglin (2013) showed connection between the use of the PBL and the development of statistical thinking and affection.

To more quantitative evidence after completing the first qualitative study, Baglin (2013) taught one section of an introductory statistics course with use of the PBL *Island*. Based on student decisions to proceed on the *Island*, some students completed content related to observational and others experimental studies. For this and ethical reasons, all students had some exposure to the PBL which could have in turn enhanced thinking characteristics on later comparisons of treatments (Baglin, 2013). Baglin hoped to find evidence to support stronger statistical thinking for students who completed specific portions of the PBL. Quantitative analysis found no significant difference between students' statistical thinking of observational and experimental studies using the PBL *Island* (Baglin, 2013).

DelMas (2004) discussed how the relationship between literacy, reasoning, and thinking could be very difficult if not impossible to separate through only the content of a problem. The goals of a statistical task however were much easier to separate into literacy, reasoning, and thinking (delMas, 2002). DelMas (2004) described a person who uses statistical thinking as "a person who knows how and when to apply statistical knowledge" (p. 85). This then creates the justification that statistical thinking and reasoning were not separate in human cognition, but were relevant to the task or problem posed by the instructor. Statistical problems and tasks posed by the instructor should represent goals of the instructor (Garfield & Chance, 2000). DelMas (2002) diagrams in *Figure 3* illustrate the difficulty in separating the concepts of literacy, reasoning, and thinking in statistics education.

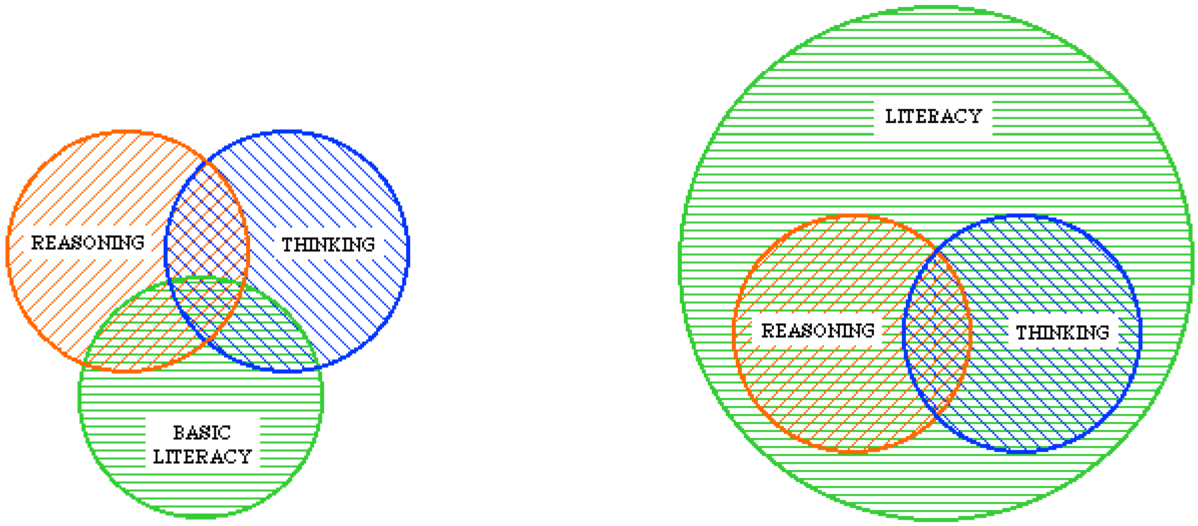

Figure 3. Two different views of the relationship of literacy, reasoning, and thinking as proposed by delMas (2002).

Though delMas (2002) did not propose literacy, reasoning, and thinking to be hierarchal, their descriptions fall in close relationship to Bloom's Taxonomy (Marriott, Davies, & Gibson, 2009). DelMas (2002) suggested that statistical literacy allows students to identify, describe, rephrase, translate, interpret, and read statistics; statistical reasoning allows students to explain the process of statistics such as why and how; and statistical thinking allows students to apply, critique, evaluate, and generalize statistical situations. Similarly, Shaugnessy, Chance, and Kranendonk (2009) used words such as describing, analyzing, explaining, looking for structure, considering, creating, questioning, evaluating, noticing, understanding, connecting, determining, determining plausibility, and validating to describe habits of mind that illuminate statistical reasoning. These views of statistical learning fall in line with Bloom's Taxonomy and Depth of Knowledge (citation) assessment levels of tasks. These levels have been shown to be hierarchical in student understanding but debate has been given over whether these levels are progressive

(Marriott, Davies, & Gibson, 2009). Figure 4 illustrates the relationship between Bloom's taxonomy and statistical literacy, reasoning, and thinking.

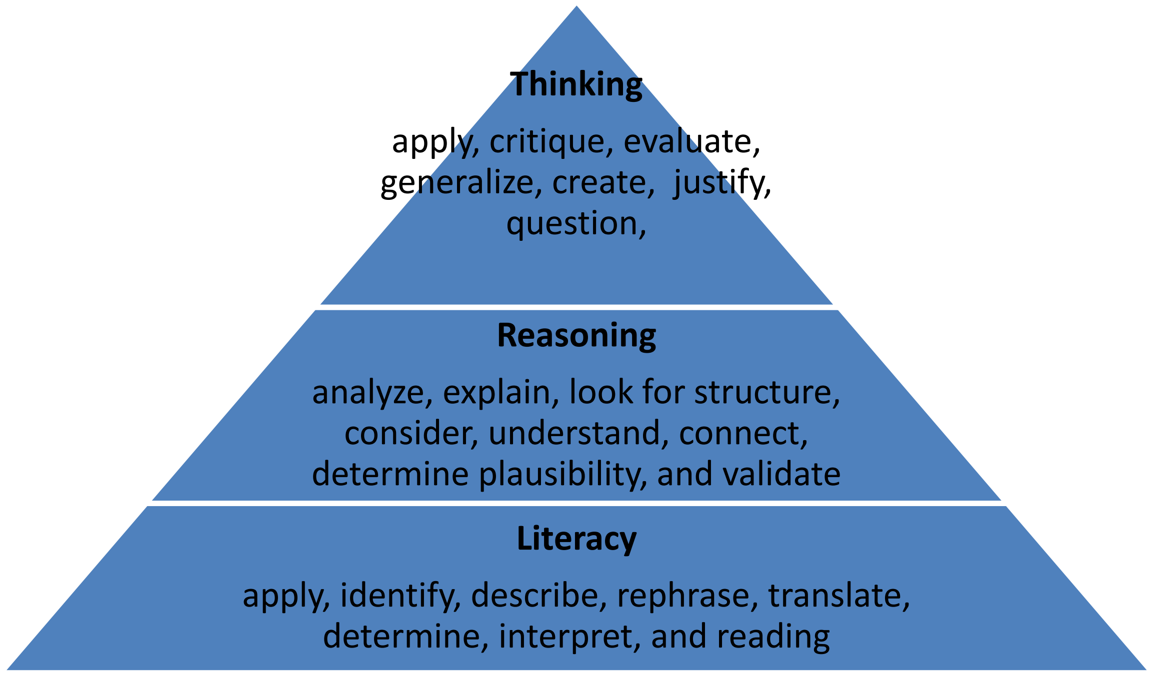

Figure 4. Litearcy, reasoning, and thinking taxonomy triangle.

The relationship of statistical tasks to thinking, reasoning, and literacy is also aligned with many studies observing students' conceptions of statistics (Reid & Petocy, 2002; Bulmer and Rolka, 2005; Bond, Perkins, & Ramirez, 2012); they also align with delMas' (2002) diagrams shown in Figure 3. The proposed model is more hierarchical than linear and relates to the recommendations of the GAISE report (Franklin et. al, 2007). The GAISE report suggested that the development of student conceptions of formulating questions, collecting data, analyzing data, interpreting results, and understanding variability must begin at level A and develop into level C (Franklin et. al, 2007). Though level C was not explicitly described as statistical thinking, guidelines at level A do not require procedures for each process component, but encourage students to observe association in displays, note differences, and display variability (Franklin et al., 2007). Students at level A develop data sense by bringing data into context, generating questions of interest, analyzing with basic tools, and making informal inferences (Franklin et. al, 2007).

Though these criteria for task difficulty were hierarchical in nature, there were conflicting views on whether the development of these cognitive skills is linear (Marriot, Davies, & Gibson, 2009; Krathwohl, 2002). The general view from researchers in mathematics education (Pesek & Kirshner, 2000/2002), as well as statistics education (Garfield & Ben-Zvi, 2008; Chance, 2002; Feldman, Konold, & Coulter, 2000; Mokros & Russell, 1995), was that incorporating tasks at higher levels of difficulty also promotes understanding and limit numbers of misconceptions. Researchers in this area however have been quick to highlight Vygotsky's (1978) zone of proximal development, which requires that tasks or activities introduce limited amounts of new information to students while connecting to prior knowledge in order to ensure cognitive development.

Though literacy, reasoning, and thinking are intertwined, tasks can be categorized by the levels of statistical literacy, reasoning, and thinking (delMas, 2004). It is therefore hypothesized that as teachers' goals shift towards higher levels of student statistical understanding--that is, reasoning and thinking--their learning environments should also shift (Chance, 2002; Garfield & Ben-Zvi, 2008). Introductory statistics classes should move beyond simple literacy, the interpretation of certain statistics correctly by a student, to statistically thinking and providing proper reasoning for abstract statistical ideas. Students who think statistically can make sound judgments about the creation of a statistical experiment from its creation to interpretation and can look backwards and forwards toward improvement. Students who reason statistically can justify their responses and provide responses to solutions that go beyond surface level understanding of statistical concepts.

Cognitive Barriers to Statistical Learning

Though statistical questions, data collection, data analysis, and making conclusions seem to be linear, these standards are interdependent and cyclic in nature (Konold & Higgins, 2003; Marriott, Davies & Gibson, 2009). This interdependency also requires students to use a habit of mind that may be unfamiliar to them (Shaughnessy, Chance, and Kranendonk, 2009; Baglin, 2013). Students who think statistically anticipate the statistical process to pose answerable questions as well as look backwards during the process to monitor digression from the problem, potential problems with earlier steps in the statistical design, and ways to enhance the design in the future (Shaughnessy, Chance, and Kranendonk, 2009; Chance, 2002). These habits of mind in statistics may also create difficulty in student learning, faulty reasoning, or misconceptions at different times during instruction (Baglin, 2013). The following section reviews certain misconceptions students may bring to the table in statistical questions, data collection, data analysis, and making conclusions.

Asking Questions. Many students have difficulty in the development of a statistical question. Students at an early level have been encouraged to collect data and describe the collection of data from a mathematical or descriptive position (Franklin et. al, 2007). For example students may ask, “What percent of the M&Ms are green,” “What percent of the class is male,” or “How many students ride the bus?” These students do not anticipate variability within their questions but rather answer mathematical questions that are then displayed. Moving students away from descriptive questions to statistical questions is important in order to move to the next phase of statistical objectives (Franklin et. al, 2007). Better questions for students may be were there more green candies in a bag of M&Ms than any other color (Froelich, Stephenson & Duckworth, 2008), "Are there a larger percent of males in every class at the school" or "How many students ride the bus on a given day?" These type of questions anticipate variability and look for quantitative comparisons.

Once students have learned and are able to anticipate variability within their questions, difficulty can still arise within the interdependence of the statistical process. "Often in trying to make a question more precise, students lose track of what they want to know in the first place" (Konold & Higgins, 2003, p. 195). Of similar confusion, students and teachers tend to use data to serve as pointers to themselves, friends, or the more complex event creating a case-oriented view (Konold, Higgins, & Russell, 2000 as cited in Konold and Higgins, 2003; Prodromou & Pratt, 2006; Jacobbe, 2007). Connecting data and graph values is important for younger students through pictographs and other elementary content objectives. Similarly, moving students to the understanding that data can become objects that can be manipulated and queried, independent of the actual event, pushes students to think and reason statistically (Konold & Higgins, 2003). Konold and Higgins (2003) stated that students should view the process of asking a question as an opportunity to gain more insight into the world around them rather than representing themselves as a key element to progression within statistical reasoning.

Students who abstract data very well for manipulation may easily fall into the pitfalls of losing the context of the question, however (Lehrer & Romberg, 1996). Lehrer and Romberg (1996) described students analyzing lunch menus for elementary students and those who participate to find association between item choices and preference; however, students failed to look at the relationship between similar items within the data set. Students described by Lehrer and Romberg (1996) abstracted their data so much that they fell away from the context; the students failed to relate similar items in their data set. DelMas (2004) described the importance of context in statistics and the difficulty of removing this context from statistical problems for students during mathematical generalization. For this reason, context within statistics must be handled with care as to help provide mathematical generalization and abstraction but at the same time be considered during the statistical process.

Data Collection. Another important part of developing a statistical question lies in the understanding of the "population of interest" and how to sample that population appropriately. Jacobs (1998/2002) found that nearly a third of her students could not differentiate between biased and unbiased sampling techniques after being confronted with conflicting data results. These students based their decisions on personal experience or the experience of others (Jacobs, 1998/2002). Nonbiased sampling techniques are at the heart of good experimental designs, yet many students have had difficulty accepting ideas that a sample is representative of a population and have been skeptical of using random sampling techniques (Schwartz, Goldman, Vye, & Barron, 1998).

Cobb and Moore (1997) discussed the importance of students understanding of randomness in data collection and experimentation as contributions to causation within studies. In introductory statistics, an important concept for students to differentiate has been the difference between an observational and experimental study (Cobb & Moore, 1997; Prodromou & Pratt, 2006). Baglin (2013) also found student difficulty in students' ability to think statistically about experimental and observational designs. Baglin (2013) suggested the use of Project Based Learning to increase student thinking of observational and experimental designs. Though Baglin's (2013) study did not provide statistical evidence for generalization, qualitative analysis suggested increase in conceptualization of experimental and observational studies.

Prodromou and Pratt (2006) reported that students have a strong attachment to a case-oriented view during data collection. To reduce this case oriented view, Prodromou and Pratt (2006) suggested a data centric perspective devoted to the understanding of variation and shape the through sampling processes. Prodromou and Pratt (2006) used a Microworld simulation of a basketball player shooting baskets into a goal that was controlled by sliders. These sliders controlled the shooters distance from goal, release height, release speed, and release angle (Prodromou & Pratt, 2006). Though each one of these characteristics were interconnected and played a part of having a successful throw, not just one played a key casual role on the successful attempt of making a basket (Prodromou & Pratt, 2006). In addition to these controls for the students, random error was included in the model of ringing the basket, so reiterations of the same shooting paths could have slight variations (Prodromou & Pratt, 2006). Students taking place in this experiment were able to see characteristics of a shooting path that were causal and those that were not. Students were also able to see how these variables related to one another. Prodromou and Pratt (2006) showed how moving students to a data centric environment could help in reducing student misunderstanding of correlation and causation.

Analyzing Data. Once data is collected and analysis begins, many students at the tertiary level recollect methods of analysis they have learned in elementary and secondary schooling (Cobb and Moore, 1997). Konold, Pollatsek, Well, and Gagnon (1997) elaborated the difficulty for introductory statistics students to choose an analysis procedure for comparing distributions because of their reliance on the curricula and teacher instead of basing judgments on exploratory data analysis or the answering of a question. Cobb and Moore (1997) stated that students cannot automatically read and interpret graphs and disaggregate the use of central and spread analysis procedures any better than they can read and write equations in mathematics. For this reason, it is paramount for teachers to build conceptual understanding of statistical analysis procedures.

Often in mathematics as well as statistics, teachers have introduced procedures before students have had a chance to understand conceptually its use, thus interfering with conception (Pesek & Kirshner, 2000/2002). “Educators often encourage students to use plots and summaries before they sufficiently understand them and, by doing so, effectively pull the rug from beneath them” (Feldman, Konold, Coulter, & Conroy, 2000, p. 119). Mokros and Russell (1995) described 2% of third graders, 85% of sixth graders, and 90% of ninth graders referring to the standard mean algorithm when asked about how weather forecasters determine average temperatures. Additionally Mokros and Russell (1995) suggested that those students who were prematurely introduced to the standard algorithm for means generally misunderstand, misinterpret, and misapply it conceptually.

Another important idea for students when analyzing data is choosing graphs and scales to aid in analysis. Shaughnessy, Chance, and Kranendonk (2009) described students analyzing data sets of the geyser, Old Faithful. The interesting aspect of this discussion focuses on how the change in scale presents two different stories. At a smaller scale, students were able to see a bimodal histogram, but at larger scales, the data appears to have only one mode. Konold and Higgins (2003) highlighted the fact that not any one representation is necessarily better than another, but some graphs lead to different analyses of the data.

Shaughnessy, Chance, and Kranendonk (2009) focused student reasoning on the comparison of two different scatter plots of Olympic 200m race times of males and females. When the data were plotted separately and on different scales, students were less likely to make comparing statements between the two predictive models and reasoning for extrapolation errors

(Shaughnessy et. al, 2009). In addition, Konold and Higgins (2003) reported student misconceptions and difficulty on the creation of scales on graphs. These graphs only included actual numbers from the data thus lacked extension or ability to decide on how to graph to justify a question they were intending to answer (Konold & Higgins, 2003). The analysis of data graphically is a key component to extending student reasoning to variation and covariation.

When analyzing spread of data, Konold and Higgins (2003) suggested using tools such as Tinker Plots and Core Math Tools that allow students to place plots side by side and on the same scale. Students were able to relate different graphs to one another and understand that each graph helps tell a different story of center, shape, and spread (Konold & Higgins, 2003). The use of graphs also allow for students to explore covariation between two variables as well. Noss, Pozzi, and Hoyles (1999) discussed how practicing nurses could not readily see a relationship between increase in age and blood pressure; however, when the researchers prompted the nurses to make vertical separations in the data to disaggregate age, nurses perceived a trend showing that the covariability in the data was masking the overall relationship. Noss, Pozzi, and Hoyles' (1999) study demonstrates the importance of students developing understanding of bivariate graphs and their relationship to univariate parameters.

The GAISE report advocated a similar approach to helping students understand the analysis of an r value through a quadrant count ratio (Franklin et. al, 2007). The GAISE report describes the construction of vertical and horizontal dotted lines for the mean values of the x-axis and y-axis (Franklin et. al, 2007). This creates four quadrants within the bivariate scatter plot in which students can count the number of data points that fall in each of the four quadrants (Franklin et. al, 2007). A ratio is then created that produces a perfect 1 relationship if all data points were in the upper right and lower left quadrants showing a positive relationship and a perfect -1 relationship if all points were in the upper left and lower right quadrants (Franklin et. al, 2007). When students complete this activity, they have an opportunity to develop understanding of how the covariation between the two variables relate to the association between the two mean values in the quadrant count ratio and are more able to extend this knowledge to a conceptual understanding of Pearson's R (Franklin et. al, 2007).

Making Conclusions. No one would argue that probability plays a fundamental role in analyzing and making conclusions in statistics; however, the emphasis in introductory statistics courses has been suggested to be limited (Cobb and Moore, 1997; Shaughnessy, 2003). Cobb and Moore (1997) argued that "informal probability was sufficient for a conceptual grasp of inference" and that "probability is conceptually the hardest subject in elementary mathematics" (p. 821). Former NCTM president and leading researcher in the area of probability was questioned about the most important concepts in statistics education. He mentions only long-range relative frequency and the power of simulations to estimate probability as important probabilistic concepts (Rossman & Shaughnessy, 2013). Cobb and Moore (1997) described how the understanding of sampling distributions, the reiteration of many experimental samples, underlie all of the introductory statistics conceptual ideas. Similarly, Chance and Rossman (2006) described the use of technological tools for sample distributions and simulating and making informal inference to help develop conceptual understanding of p-values and confidence intervals. For these reasons, Cobb and Moore (1997) and Shaughnessy (2003) encouraged teachers to focus on the conceptual understanding of sampling distributions, an essential idea of statistics, rather than try to illuminate how formal probability relates to statistics.

When drawing conclusions, the use of null and alternative hypothesizes are extremely important (Cobb & Moore, 1997). Based on this idea and lack of formal probability, sampling distributions play a fundamental role in students making appropriate conclusions when performing statistical tests or creating confidence intervals. Schwarz and Sutherland (1997) described using a capture-recapture experiment to illustrate the concept of a sampling distribution and computer simulation to investigate the estimation of a population parameter. This in essence helps remove some of the “black magic” many students describe during statistics. Similarly, Lunsford, Rowell, and Goodson-Espy (2006) highlighted the importance for students to use these simulation tools to develop understanding of sampling distribution instead of the teacher just demonstrating these simulation tools.

If statistics was seen as climbing a ladder, students’ ability to compare two groups should land somewhere midway on that ladder, according to Konold and Higgins (2003). Students often have difficulty knowing how to compare two groups even when they know a great deal about averages (Konold & Higgins, 2003). As students compare two groups, they tend to move towards comparisons of what was typical using modal clumps, yet lack the ability to look at the difference in typical values from the two groups (Bright & Friel, 1998). Using averages to compare two groups requires students to view averages as a way to represent the entire group and not just parts of it (Konold & Higgins, 2003). Dambolena (1986) along with Gordon and Gordon (1989) encouraged the use of simulation and graphics to look for the differences between two populations; however, not all differences are between two quantitative variables.

When students analyze data in contingency tables, they often only look at the marginal distribution to decide for association in affect only comparing the percent of times two events occur and in essence lack the ability to understand covariation (Konold & Higgins, 2003). Kader and Perry (2007) discussed the role that variability plays in categorical variables and how the use of the coefficient of “unalikability” can be used to develop reasoning behind other statistical tests. Shaughnessy and Chance (2005) provided ideas for students to develop the understanding for degrees of freedom in chi-square tests by having students finish completing contingency tables to develop common formulas found for tests of dependence and homogeneity and the understanding of degrees of freedom in these procedures.

To promote deep statistical learning, it is important for teachers to grapple with certain cognitive misconceptions that may develop or have developed over time. This section has highlighted four major areas in the statistical process conclusions that students have cognitive difficulties in developing learning: asking questions, data collection, analyzing data, and making. However, difficulties in learning of mathematics and statistics are not limited to cognitive difficulties. The next section highlights affective barriers that have played a part in the learning of mathematics and statistics.

Affective barriers to learning

Research has shown that mathematical classrooms have been seen as boring and unimportant to many students' everyday lives (Mitchell, 1993). This negative view has contributed to the decline of mathematics performance in the United States (Mitchell, 1993; National Mathematics Advisory Panel, 2008). The number of students engaging in rigorous mathematics courses and pursuing fields in Science, Technology, Mathematics, and Engineering (STEM) careers were not meeting demands from the contemporary world (National Mathematics Advisory Panel, 2008; Middleton and Jansen, 2011). These discrepancies were even more dramatic when comparing subpopulations by race and gender in the United States (Flores, 2007).

Kilpatrick, Swafford, and Findell (2001) stated that "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own self efficacy" encompasses productive disposition. For this reason, it is important for teachers to not only focus on the learning of statistics in the cognitive domain, but also the affective domain. This section examines at ways in which researchers have chosen to monitor the affective domain or productive disposition within statistics. Explanation may also be found in students' prior knowledge of statistics, pre-conceived notions of statistics, and relationship to student experiences (Griffith, Adams, Gu, Hart, & Nichols-Whitehead, 2012; Schraw, Flowerday, & Lehman, 2001). The discussion begins by highlighting some important definitions in the understanding of the affective domain related to and outside of mathematics education. This is followed by a review of measurement instruments developed to monitor the affective domain in statistics education and research in this area. This section concludes with a discussion of research that has been done conducted on the development of students' attitudes towards mathematics and statistics.

Definitions. Research related to interest has spanned through multiple content domains, but the area of mathematics has been especially targeted by researchers outside the field of mathematics education (Garofalo & Lester, 1985; Zan, Brown, Evans, & Hannula, 2006; Middleton & Jansen, 2011). Middleton and Jansen (2011) suggested that the study of interest has been a focus in the mathematics community by psychologists because of no significant differences being found in the learning of mathematics with other subjects; however, common perceptions of being "good" or "not good" in mathematics are rampant. The study of students' interest in mathematics has led to descriptions of interest as being described as situational or personal.

Situational and personal interest. Situational interest was what many researchers dub as the aspect of interest that can be changed in the classroom by instructional methods. The teacher can promote these changes in students spontaneously, transitorily, or environmentally (Schraw, Flowerday, & Lehman, 2001). Schraw, Flowerday, and Lehman (2001) described situational interest as being different from personal interest. Personal interest was more related to a persons' internal motivation to learn or do mathematics. Mitchell (1993) described situation interest as the single best way to increase student interest in mathematics as though personal interest was purely controlled by the student. The increase in situational interest has been correlated to teachers who increase student autonomy and choice (Schraw, Flowerday, & Lehman, 2001). Similarly, situational interest has been related to student's prior knowledge of the subject and problem (Schraw, Flowerday, & Lehman, 2001). For these reasons, it is not only important to teach students cognitively, but affectively (Middleton & Jansen, 2011). Classroom environments that promote appropriate student discussion, respect students ideas and curiosity, respect and responds to diverse experiences and interests, and encourage full participation by all students are much more likely to provide the means necessary to increase situational interest (Middleton & Jansen, 2011; Smith and Stein, 2011; NCTM, 1991).

Metacognition, mathematical task knowledge, and mathematical strategy knowledge. Metacognition "has two separate but related aspects: (a) knowledge and beliefs and cognitive phenomena, and (b) the regulation and control of cognitive actions" (Garofalo & Lester, 1985). Metacognition therefore directly relates to student interest in subject matter. Metacognition is what controls for students' belief in what they like about, understand of, or affection towards mathematics and statistics. Garofalo and Lester (1985) categorized metacognition into knowledge of cognition and regulation of cognition. Studies in this area have had large criticism because data gathering methods require self-reporting (Garofalo & Lester, 1985). Self-reporting may interfere with the cognitive process, and lacks of clear definitions provide trouble for scientific investigation (Garofalo & Lester, 1985).

Very similar to metacognition in the psychological realm is the term mathematical task knowledge in the mathematics education realm (Garofalo & Lester, 1985). Mathematical task knowledge includes what one believes "about the subject of mathematics as well as beliefs about the nature of mathematical tasks" (Garofalo & Lester, 1985, p. 167). This is different than mathematical strategy knowledge, "a person's awareness of strategies to aid in comprehending problem statements, organizing information or data, planning solution attempts, executing plans, and checking results" (Garofalo & Lester, 1985, p. 168). Metacognition and mathematical task knowledge have significant impact on student motivation and interest (Garofalo & Lester, 1985). Garofalo and Lester (1985) showed that attempts to understand interest and motivation are slippery, and that any attempt should control for factors in students prior mathematical and statistical knowledge.

Intrinsic and extrinsic motivation. Intrinsic and extrinsic motivation is motivational terminology commonly referred to across educational disciplines and psychology. Intrinsic motivation relates to a student's encouragement to complete a task, do a homework assignment, etc. based purely on the students own self-interest or satisfaction (Middleton & Jansen, 2011; Sengondan & Iksan, 2012). Extrinsic motivation arises from teachers, family, other students, the environment, etc. (Middleton & Jansen, 2011; Sengondan & Iksan, 2012). These motivational techniques may come in the form of praise, rewards, gifts, and good grades (Middleton & Jansen, 2011; Sengondan & Iksan, 2012). Studies have shown that extrinsic motivation can work temporarily, but if overused can negatively affect student achievement (Sengondan & Iksan, 2012). In order to encourage student pursuit of and achievement in mathematics and statistics, students' intrinsic motivation must be increased. Middleton and Jansen (2011) suggested that the context of a problem can increase student motivation and more specifically increase student intrinsic motivation.

Measuring interest. The focus and methods of much of mathematical and statistical interest research has been on the creation and monitoring of students by student attitudinal surveys (Zan, Brown, Evans, & Hannula, 2006). The Statistics Attitude Survey, Attitude Towards Statistics, Survey of Attitudes Toward Statistics (SATS36), and Expectant Value Theory scale have been validated as methods to understand students' perceptions of statistics (Ramirez, 2012; Schau, & Emmioglu, 2012). Though many of the studies directly related to interest contain these attitudinal surveys, many qualitative studies also directly deal with students' perceptions of mathematics and its connection to their lives (Boaler, 2002, 2011). The distinction between these two types of research was largely based on the connection of student interest terminology and the focus of studies.

Schau and Emmioglu (2012) explored how introductory statistics courses affect student attitudes in the United States through voluntary sampling of students throughout the United States. Using the SATS36 (VanHoof, Kuppens, Castro Sotos, Verschaffel, & Onghena, 2011) containing 7-point Likert scale items, students in Schau and Emmioglu's (2012) study completed pre and post surveys in their introductory statistics courses. A total of approximately 2200 students and 101 sections completed the survey (Schau & Emmioglu, 2012). Their results showed that introductory statistics courses throughout the United States had no significant effect on attitudes toward affect, cognition, competence, and difficulty of statistics (Schau & Emmioglu, 2012). Value, interest, and effort actually showed decreases at the national level. Internal consistency was critiqued and all correlations with the below measures were larger than .76, showing strong validity in measurement of the phenomenon (Schau & Emmioglu, 2012).

1. *Affect* (6 items) – students' positive and negative feelings concerning statistics – "I am scared by statistics."
2. *Cognitive Competence* (6 items) – students' attitudes about their intellectual knowledge and skills when applied to statistics – "I can learn statistics."
3. *Value* (9 items) – students' attitudes about the usefulness, relevance, and worth of statistics in personal and professional life – "I use statistics in my everyday life."
4. *Difficulty* (7 items) – students' attitudes about the difficulty of statistics as a subject – "Most people have to learn a new way of thinking to do statistics."
5. *Interest* (4 items) – students' level of individual interest in statistics – "I am interested in using statistics."
6. *Effort* (4 items) – amount of work the student expends to learn statistics – "I plan to work hard in my statistics course." (Schau & Emmioglu, 2012)

Carnell (2008) performed a study to determine differences in attitude and achievement using the SATS36 (VanHoof et. al, 2011). Carnell (2008) treated or assigned random classes a group statistical project while leaving other classes untreated. To ensure treatment affects were based solely on the given project, little in-class time was devoted to the project. No significant differences were found between the two classes in end of course evaluations and student interest in statistics.

Bond, Perkins, and Ramirez (2012) explored students' attitudes, conceptualizations, and content knowledge of statistics by using the SATS36 instrument and qualitative analysis of 47 students in an introductory college statistics course. Bond, Perkins, and Ramirez (2012) coded qualitative data in a hierarchical fashion related to content knowledge and conceptualizations. This coding showed significant improvement in students perceived content knowledge and conceptions of statistics after taking a statistics course; however, data from the SATS-36 suggest attitudes did not change (Bond, Perkins, & Ramirez, 2012). Even after coding implied increased conceptualization, usefulness and value strata in the SATS-36 actually lowered from pre to post surveys. Bond, Perkins, and Ramirez (2012) suggested that the fuzziness of the word statistics might affect the validity of attitudinal surveys. This may indeed be true based on students' lack of experience in introductory statistics courses.

Bond, Perkins, and Ramirez (2012) found differences largely in the cognitive ability of students to understand statistics with no differences in the affective domain. Bond, Perkins, and Ramirez (2012) made plausible concern for what does positively affect student interest in mathematics and especially statistics. The difficulty, however, was student's preconceived notions of statistics. Reid and Petocy (2002); Bulmer and Rolka (2005); Bond, Perkins, and Ramirez (2012) described students entering an introductory statistics course with very little adequate conceptions of statistics and finishing courses with a much clearer definition. When students come into classes with very little knowledge of what statistics means, pre and post attitudinal surveys lose validity effectiveness results. With this knowledge, pre and post quantitative assessments of the affective domain must be considered with care.

Research outside of mathematics and mathematics education has also confirmed results that undergraduate students have negative conceptions of statistics and mathematics before and after completing courses (Griffith et. al, 2012). Undergraduate students many times leave statistics as one of their final courses because of the perceived difficulty, reporting that statistics was an obstacle to obtaining their degree (Griffith et. al, 2012). Similarly, many postgraduate students do not feel adequately prepared to report findings from their own research and to understand the research of others. NCTM's publication of *Lessons Learned from Research*

(Sowder & Schappelle, 2002) to make mathematics education research more readable for practicing teachers is a direct example of efforts that have been made to make research more understandable to practitioners.

Increasing Interest. Mitchell (1993) suggested two motivational strategies to encourage students' interest and motivation called catch and hold. The incorporation of techniques such as games, group work, puzzles, the use of technology and computers were methods Mitchell (1993) described as catches. Pedagogical catches are ways to promote interest or stimulate student participation, but do little to affect student intrinsic or personal motivation (Mitchell, 1993). Hold strategies involve creating meaningfulness to a problem and involvement in the process of learning (Mitchell, 1993). These strategies empower students toward an end of accomplishment and make use of the structure and methods being implored (Mitchell, 1993). Gal and Ginsburg (1994) suggested the same approach for improving motivation with statistical instruction. Incorporating the usefulness of statistics to students' everyday lives by diving into questions students are interested about may positively increase student perceptions of statistics.

Research on statistical interest has suggested that situational interest may also relate to students' statistical or mathematical background and students' perceived relatedness to their college major (Griffith et. al, 2012). Though Griffith et. al (2012) included some qualitative aspects pertaining to reasons for lack of statistical interest, pedagogical techniques used by instructors were not controlled for or monitored. Perhaps techniques in the classes in this study did not provide environments that foster reasoning and sense making, respect for ideas by all students, and the chance to build on prior knowledge (NCTM, 1991; Smith & Stein, 2011).

Griffith et. al (2012) suggested providing more context for problems with limited attention to pedagogical techniques. Griffith, et. al (2012) attempted to improve students' interest, but introduced an important confounding factor related to pedagogy. Though context may produce increases in student situational interest, closer attention to teaching in addition to situational interest may have longer lasting effects of students' interest and ability in statistics (NCTM, 1991; Smith & Stein, 2011). Incorporating strategies that develop the context of problems has also been shown to help English Language Learners in introductory statistics courses (Lesser & Winsor, 2009). Boaler (2002, 2011) suggested that allowing students to grapple with problems that encourage practical implications to their everyday lives increases their perception of mathematical task and strategy knowledge, empowers students to use and view mathematics positively, and promotes the use of mathematics in their everyday lives. In addition to not controlling for pedagogical techniques, Griffith et. al (2012) did not control for confounding factors of more mathematically proficient students encompassing specific majors. Any study attempting to understand the phenomena of interest will improve in rigor by including cognitive aspects of the students and pedagogical techniques used by the instructor.

Though statistics and mathematics are different, they complement one another (delMas, 2004). Research involving statistical interest could easily be confounded with mathematical interest (Bond, Perkins, & Ramirez, 2012). This makes the distinction of affection mathematics and statistics very challenging to discriminate. Though they are different, research in these two areas can also be very similar and used to build and strengthen existing research. It has been hypothesized that the use of best teaching practices from mathematics classrooms such as discourse and appropriate tasks to teach statistics can promote student understanding and interest in statistics (Boaler, 2002, 2011; Middleton & Jansen, 2011; Smith & Stein, 2011). Many projects looking into the development of student interest and understanding in statistics do not attempt to teach in a constructivist manner in which students build up their own knowledge from authentic experiences with statistical content. Much of the research to date does little to integrate the effect of teaching and learning methods in statistics with increases in motivation. For this reason, a more detailed look into the influence of pedagogical techniques used during statistical instruction on student interest would be worthwhile.

Summary

This section has highlighted the differences and similarities in statistical literacy, reasoning, and thinking. Each of these plays a fundamental role in students' understanding of statistics. In particular, statistically literate students can read, write, and interpret statistics (Rumsey, 2002). Students who reason about statistics are able to make connections and express these relationships between statistical concepts (Chance, 2002; Chance & Garfield, 2002). Students who have the ability to think statistically are able to critique, question, and develop statistical designs in order to improve their understanding (Chance, 2002; Chance & Garfield, 2002; Garfield, 2002). The goals of literacy, reasoning, and thinking are interconnected (delMas, 2002). In order for students to develop at each of these levels, direct attention must be played to these certain goals.

Barriers for learning at these different levels have also been discussed from both the cognitive and affective domains. Though these barriers exist, they can be overcome. Teachers' attention to common misconceptions in the learning process can help students' develop reasoning and sense making and reduce errors in reasoning before they occur in students (Baglin, 2013; Konold & Higgins, 2003; Loveland, 2014). Teachers can diminish affective barriers by increasing student interest through catch and hold strategies of real world contexts that interest students and provide meaning to their real lives (Mitchell, 1993; Griffth et. al, 2012). The next section highlights effective teaching practices that address learning at deep levels, reducing student misconceptions, and promoting interest towards mathematics and statistics.

Teaching Statistics

The *Principles to Actions* (NCTM, 2014) describes eight mathematics teaching practices that are effective in promoting student learning: Establish mathematics goals to focus learning, implement tasks that promote reasoning and problem solving, use and connection mathematical representations, facilitate meaningful mathematical discourse, pose purposeful questions, build procedural fluency from conceptual understanding, support productive struggle in learning mathematics, and elicit and use evidence of student thinking. This section takes a detailed look at each of these areas in relation to statistics teaching, providing relationships to statistics education research when available.

Establish Mathematics Goals to Focus Learning

PtA stated that “goals should describe what mathematical concepts, ideas, or methods students will understand more deeply as a result of instruction and identify the mathematical practices that students are learning to use more proficiently” (NCTM, 2014, p. 12). This description of goals by the PtA highlighted both goals towards content and practices. Goals of practice highlight characteristics that teachers foster in students during instruction.

Standards. *Probability and Statistics Units for Junior High Schools* (1984) by Chaweewan Sawetamalya attempted to find “a set of potential objectives for the probability and statistics units” that were consistent through current mathematical literature and educators in the 1980s (p. 37). His content review focused on literature in the National Assessment of Educational Progress (NAEP), Priorities in School Mathematics: Executive Summary of PRISM Project, the 1978 Alabama Course of Study, NCTM publications, American Statistical

Association (ASA) publications, mathematics textbooks dealing with probability and statistics in elementary and secondary levels, and surveys of educators. Using a non-parametric rank based statistical technique, Sawetamalya (1984) found junior high school statistics had three focuses. The primary focus of statistics from his study was on interpreting data that were present in forms such as tables, graphs, charts, diagrams, pictographs, maps, and flowcharts (Sawetamalya, 1984, p. 42). Following closely behind this large focus was the objective of defining, finding, and comparing central tendency measurements such as mean, median, and mode (Sawetamalya, 1984). Objectives of spread or dispersion from central tendency measurements were found last in the ranking system. Reviewing Sawetamalya's (1984) work highlights the change of core statistical ideas in the United States, specifically the change in emphasis of variation.

A report by Cobb (1992), published by the Mathematical Association of American (MAA) support, sought to highlight important issues in statistics by setting up an e-mail focus group of MAA members. The meeting of MAA members resulted in three large calls for change in statistics education related to variation. The first call was for an emphasis in statistical thinking that centered on variation (Cobb, 1992). Two of the four major ideas for statistical thinking emphasized the "omnipresence of variability" and the "quantification and explanation of variability" of data in sampling and analysis (Cobb, 1992). This call for change has been seen in statistical documents such as the PSSM and GAISE report including the conceptual understanding of variation throughout their standards. In addition, the CCSS-M introduces variation concepts such as mean absolute deviation as early as sixth grade (NGA & CCSSO, 2010).

Garfield and Ben-Zvi (2008) discussed key components of introductory statistics courses content as being data, distributions, variability, center, randomness, covariation, and sampling.

Shaughnessy, Chance, and Kranendonk (2009) provided six chapters focusing on students' reasoning and sense making in statistics. These chapters require students to formulate questions, design and employ a plan for collecting data, analyze and summarize the data, and interpret the results from the analysis. Gelman and Nolan (2002) summarized their introductory courses into sampling, descriptive statistics, inference, and probability. Though different wording has been used across recent research, there are two fundamental similarities in content objectives (Burrill, Franklin, Godbold, & Young, 2003; Tintle et. al, 2012; Franklin et. al, 2007). The first has been that student creation and understanding of data sets is pivotal to developing understanding of inference and omnipresence of variability. The second has been the need for analyzing these data sets to make useful predictive inferences or summative reports through the understanding of probability and random designs (Burrill, Franklin, Godbold, & Young, 2003; Tintle et al, 2012). Content objectives for introductory statistics largely fall into one of these two rather broad areas.

Processes and Practices. Attention to processes and practices in statistics "can benefit students at all levels by engaging them in doing mathematics in ways that make sense to them" (NCTM, 2014, p. 66). Effective processes and practices help focus student learning at in depth levels such as statistical literacy, reasoning, and thinking. These processes and practices should not be seen as a set of objectives for teachers to check off, but rather tools for students to use to develop mathematical content proficiency.

Principles and Standards for School Mathematics. The PSSM (2000) identified five process standards for students to acquire during mathematics instruction: problem solving, reasoning and proof, communication, connections, and representation. Problem solving requires students to build new mathematical knowledge through problem solving, solve problems that arise in mathematics and in other contexts, apply and adapt a variety of appropriate strategies to solve problems, and monitor and reflect on the process of mathematical problems solving. PSSM (2000) also required students to recognize reasoning and proof as fundamental aspects of mathematics, make and investigate mathematical conjectures, and develop and evaluate mathematical arguments and proofs. PSSM (2000) encouraged students to communicate about mathematics by organizing and consolidating their mathematical thinking precisely and evaluating and analyzing their own and others thinking. Students should develop an ability to draw connections outside of mathematics to make a coherent whole (PSSM, 2000). Lastly students should create and use representations to organize, record, and communicate mathematical ideas, select, apply and translate among mathematical representations to solve problems, and use representations to model and interpret physical social, and mathematical phenomena (PSSM, 2000).

Reasoning and Sense Making Habits of Mind in Statistics. Shaughnessy, Chance, and Kranendonk (2009) described five habits of mind that help develop reasoning and sense making in statistics and probability: analyzing a problem, implementing a strategy, monitoring one's progress, seeking and using connections, and reflecting on one's solution (Shaughnessy, Chance, & Kranendonk, 2009). These five habits of mind help students develop literacy, reasoning, and thinking in statistics.

Students who effectively analyze a problem are able to find hidden structure in data; make preliminary deductions and conjectures; analyze and explain variation; and describe overall patterns (Shaughnessy, Chance, & Kranendonk, 2009). Being able to select and critique data collection based on their questions; create meaningful data displays and summaries; consider the random components behind the data; and draw deductions beyond the data are able to implement strategies effectively (Shaughnessy, Chance, & Kranendonk, 2009). Students who monitor their progress are said to compare various graphical and numerical representations; compare interpretations of data; evaluate the consistency of an observation with a model; question whether the observations make sense in the context of the problem; evaluate the consistency of different components within analysis; and apply the iterative statistical process to the investigation (Shaughnessy, Chance, & Kranendonk, 2009). By seeking and using connections to communicate connections between graphical and numerical representations; recognize common mechanisms of analyses; comprehend the sensitivity of an analysis to numerous components; and link conclusions and interpretations to the context of the problem students are able to develop reasoning and sense making (Shaughnessy, Chance, & Kranendonk, 2009). In order to develop reasoning and sense making students should: reflect on one's solutions; consider and evaluate alternate explanations; understand the allowable scope of conclusions; determine whether a conclusion based on the data is plausible; justify or validate the solution or conclusion by using inferential reasoning; analyzing and account for variability; and look for connections between the context of the situation and the data (Shaughnessy, Chance, & Kranendonk, 2009). Though each of these habits of mind are distinct, they are very much entangled with statistical literacy, reasoning, and thinking.

Common Core State Standards for School Mathematics Practice Standards. CCSS-M's inclusion of rigorous practice standards into states content standards have made many teachers rethink the way they teach mathematics. These practice standards have called for students inside the classroom to make sense of problems and persevere in solving them, reason abstractly and quantitatively, construct viable arguments and critique the reasoning of others, model with mathematics, use appropriate tools strategically, attend to precision, look for and make use of structure, and look for and express regularity in repeated reasoning (NGA & CCSSO, 2010).

These mathematical practice standards were developed from the PSSM (2000) and Kilpatrick, Swafford, and Findell's (2001) report, *Adding It Up: Helping Children Learn Mathematics*. While these practice standards may seem new to some in education, they fall in close relationship to PSSM (NCTM, 2000) and statistical literacy, reasoning, and thinking in statistics education. These mathematical practice standards were developed from the PSSM and the National Research Council's report, *Adding It Up: Helping Children Learn Mathematics* (CCSSO, 2010). Though these practice standards may seem unique to some in education, they fall in close relationship to PSSM (NCTM, 2000) and statistical literacy, reasoning, and thinking, as shown in Table 3.

Table 3

Instructional Practices that Support Development of Statistical Learning

CCSS-M Practice Standards	**PSSM (NCTM, 2000)**	**Habits of Mind in FHSM**	**Helps Develop Statistical…**
1. Make sense of problems and persevere in solving them.	Problem Solving Communication Representation	Analyzing a problem, implementing a strategy, monitoring one's progress, seeking and using connections, and reflecting on one's solution	Reasoning and Literacy
2. Reason abstractly and quantitatively.	Problem Solving Reasoning and Proof	Implementing a strategy and monitoring one's progress	Reasoning and Literacy
3. Construct viable arguments and critique the reasoning of others.	Reasoning and Proof Communication Representation	Reflecting on one's solution and monitoring one's progress	Thinking, Reasoning, and Literacy

4. Model with mathematics.	Problem Solving Reasoning and Proof Connections Representation	Seeking and using connections and implementing a strategy	Reasoning and Literacy
5. Use appropriate tools strategically.	Problem Solving Representation Connections	Implementing a strategy, seeking and using connections, and reflecting on one's solution	Thinking, Reasoning, and Literacy
6. Attend to precision.	Problem Solving Communication	Analyzing a problem	Literacy
7. Look for and make use of structure.	Problem Solving Reasoning and Proof Connections	Analyzing a problem	Reasoning and Literacy
8. Look for and express regularity in repeated reasoning.	Problem Solving Connections		Reasoning and Literacy

Implement Tasks that Promote Reasoning and Sense Making

NCTM's (2014) second practice said that effective teaching engages students in solving and discussing tasks that promote reasoning and problem solving and allow for multiple entry points and varied solution pathways. When students have multiple pathways and entry points to solve problems, lessons become differentiated and have the ability to reach a diverse range of students. Students meeting this second practice are allowed to build on one another's understanding and critique the reasoning of others. In addition, Smith and Stein's (2011) *5 Practices for Orchestrating Productive Mathematics Discourse* relies on setting up cognitively challenging tasks for students.

Erlwanger (1973/2004) suggested that meaningful mathematics should foster opportunities to discover patterns, seek relationships, generalize, and verify mathematical concepts and principals. Erlwanger (1973/2004) took a detailed look at the progress of Benny, a successful and fast tracking student in an Individually Prescribed Instruction (IPI) mathematics program. The IPI program was very similar to remedial math courses taught at the junior colleges in which learners were self-paced on their own needs and work to answer questions in which a specific answer was expected from a practiced algorithm (Erlwanger, 1973/2004). These programs, when used exclusively from teaching, encourage learners to guess and check or essentially be on a "wild goose chase" for answers, as Benny says (Erlwanger, 1973/2004, p. 53).

As a result, Benny created his own rules and algorithms for computing solutions in which many were incorrect and use no sense making (Erlwanger, 1973/2004). His cognition of mathematics was based on rules in which meaning was pointless (Erlwanger, 1973/2004). When confronted with conflicting views of his mathematical sense and algorithms, Benny based his rules on format issues of answers in the program (Erlwanger, 1973/2004). Later discussions with Benny suggested that manipulatives should be used to address his misunderstandings as his understanding of problems lacked any number sense (Erlwanger, 1973/2004). Benny's situation highlights the need for students to develop reasoning and sense making from tasks and hands on manipulatives that build conceptual knowledge rather than use formulas haphazardly.

Froelich, Stephenson, and Duckworth (2008) described a set of tasks involving M&Ms that can be used to encompass the entire introductory statistics course. Froelich et. al (2008) activity started with students understanding the difference between quantitative and qualitative data through actual sampling of M&Ms, next proceeded to measuring weight of the M&Ms to discover the normal distribution and empirical rule, thirdly predicted bag weight based on using the individual weight of candies, and lastly designed an experiment to predict bag weight and individual M&M weight. Froelichs et. al's (2008) activities using a similar experiment can potentially reduce confusion for students changing contexts of problems throughout the class, however this also has drawbacks. Not allowing students to perform experiments from multiple contexts reduces the importance of how important context is in statistics and the chance to build students situational and personal interest.

Focus in High School Mathematics: Reasoning and Sense Making in Statistics and Probability (FHSM) by Shaughnessy, Chance, and Kranendonk (2009) provided six different tasks for teachers to develop reason and sense making in high school statistics and probability. Though each of these tasks focus on certain statistical objectives, they had similarities in structure (Shaughnessy, Chance, Kranendonk, 2009). Each of FHSM's (Shaughnessy, Chance, Kranendonk, 2009) tasks focused on promoting student discourse and building habits of mind. These tasks allow students to analyze a problem, implement a strategy to solve the problem, monitor one's progress in solving the problem, seek and use connections to analyze the problem, and reflect on ones or others solutions (Shaughnessy, Chance, Kranendonk, 2009).

FHSM (Shaughnessy, Chance, & Kranendonk, 2009) used a mock class discussion to highlight the use and comparison of range and interquartile range as a measure of spread at grade 9. The student discussion focused on the ranges inability to truly measure spread. Probably the most strikingly similarity between all of these tasks involves the teacher and student interactions during the discussion (Shaughnessy, Chance, & Kranendonk, 2009). The discussion was largely student centered with little interaction from the teacher. This highlights the fact that good tasks should be student centered and allow for student explorations of important statistical ideas that builds habits of mind towards statistical reasoning and thinking.

Use and Connect Representations

The third teaching practice in PtA stated that effective teaching of mathematics engages students to make connections among different representations (NCTM, 2014). This allows for deeper understanding of concepts and procedures. When students use representations to build deeper understanding, they are able to use representations as tools for problem solving. Statistics education has highlighted representation through the incorporation of statistical software, simulation, and internet applets. In addition to computer based randomized experiments and inference, many educators still use randomized experiments in classes to connect statistical concepts (Chance & Rossman, 2006; Tintle et. al, 2012; Cobb & Moore, 1997; Rossman & Chance, 2013; Rossman & Chance, 2002).

NCTM (2014) elaborated on the interconnection of students' verbal, symbolic, visual, physical, and contextual representations and the use of each of these representations like a lens for students' development of understanding. In particular, NCTM (2014) highlighted the critical role that visual representations play on students building of conceptual knowledge in mathematics and linking relationship between quantities. Through discussion of the interconnection of manipulatives, graphs, charts, symbols, tables, and diagrams in context students are able to better build conceptual knowledge and gain number sense (NCTM, 2014).

Chance and Rossman (2006) have developed online applets to help students make connections between abstract concepts such as long run probability, sampling distributions, biased sampling, sampling distributions of regression lines, sampling distributions of odds ratios, and simulating randomized tests. Chance and Rossman (2006) described the use of applets as a way for students to make abstract concepts more concrete through different representations of the same concept. One of the practical suggestions given by Chance and Rossman (2006) and

Lunsford, Rowell, and Goodson-Espy (2006) described the importance of involvement by the students in the representation process. When the teacher allows for students to be involved in this process, students have a chance to develop ownership, interact to develop understanding, create a link between tactile and computer simulations, be given immediate feedback, test their own reasoning and conjectures, and reinforce key ideas. Teachers who allow students to create these representations are much more likely to help develop statistical reasoning.

Chance, Ben-Zvi, and Medina (2007) discussed the use of statistical software packages that have helped students and largely industries deal with statistical analysis. These software packages have moved to more menu driven features to help in usability; however, educational software such as Fathom, Inspiredata, and Tinkerplots have concentrated on the use of multiple representations of a data set concurrently. Other free software like StatCrunch and NCTM's Core Math Tools also allow students to represent data in different ways, calculate different summary statistics, and perform randomization experiments. Chance, Ben-Zvi, Garfield, and Medina (2007) highlighted the role that technology has played on students metacognitive process to connect different representations and connect data with different representations. The world of technology has fundamentally shifted the way teachers can help students understand statistics through representation (Chance et. al, 2007).

Bridgette Jacob's (2013) dissertation focused on the development of introductory statistics students' informal inferential reasoning and its relationship to their formal inferential reasoning. Questions posed during the study were "Does their informal inferential reasoning develop?" and "If their informal inferential reasoning develops, what are the characteristics of this informal inferential reasoning as it develops?" Jacob (2013) used three subcategories of statistical content to analyze this development: comparing two distributions of data, drawing conclusions based on sampling and estimating a probability, and comparing data from a single sample to the sampling distribution of all such samples. Within these categories, Jacob (2013) paid particular attention to the representations of statistics through different graphical displays. Qualitative analysis of student responses showed strong relationships between students' representations of median and mode to distributional shape, but the lack of relating distributional shape to spread in context. Students also found difficulty in relating distributional representations to statistical intervals and testing procedures (Jacob, 2013)

Pose Purposeful Questions

PtA's fifth teaching practice stated that "Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships" (NCTM, 2014, p. 35). When teachers use effective questioning, students are able to explain and reflect on their thinking through meaningful mathematical discourse. This questioning allows teachers to discern what students do and do not know (NCTM, 2014). Effective mathematics teaching relies on questions that encourage students to explain and reflect on their thinking as an essential component of meaningful mathematical discourse.

Smith and Stein's (2011) first step in *5 Steps to Orchestrating Productive Mathematical Discussion* was posing purposeful questions. Too many times teachers and textbooks were caught in the trap of funneling students to solutions (NCTM, 2014). When teachers step students through a cognitively challenging task, they in effect lower the cognitive difficulty (NCTM, 2014). Good questioning habits by teachers, textbooks, and tasks encourage students to think at a deeper level and be responsible for their own understanding and communication of this understanding (Smith & Stein, 2011).

Herbal-Eisenmann and Breyfogle (2005), Smith and Stein (2011), and NCTM (2014) discussed the misuse of the Initiate, Respond, and Evaluate (IRE) pattern of questioning (Mehan, 1979, as cited in NCTM, 2014) in many classrooms. In this pattern, teachers pose a question, the student responds, and then the teacher evaluates the response as correct or incorrect. To break this cycle, develop deeper level thought processes, and encourage responsibility for learning by students, Herbal-Eisenmann and Breyfogle (2005) encourage teachers to shift from funneling questions to focusing questions. When using focusing questions, the teacher listens to the student's solutions and guides them based on their thoughts (Herbal-Eisenmann & Breyfogle, 2005). When teachers use focusing questions, they help students make connections and articulate their thinking by referring back to common activities within the classroom and student contributions (Herbal-Eisenmann & Breyfogle, 2005). These actions value and draw out student thinking which in turn supports specific goals the teacher may have (Herbal-Eisenmann & Breyfogle, 2005).

Smith and Stein (2011) discussed the use of the Thinking Through a Lesson Protocol (TTLP), another approach to breaking the IRE cycle. Part of the TTLP calls for teachers to work through tasks to understand all the possible ways they could be solved. By doing these teachers have a chance to pinpoint common misconceptions before the task and develop questioning patterns that were not funneling but relate students' prior knowledge to a new task (Baglin, 2013; Stein & Smith, 2011; Herbal-Eisenmann & Breyfogle, 2005). The TTLP also called for teachers to work through high need questioning patterns to ensure students completion of a task: how to help a group get started, how to focus students' direction on key mathematical portions of the task, advance understanding of the key mathematical ideas, how to assess students understanding of these key ideas, and how to hold each student accountable for their own construction of knowledge during the task. The TTLP was essentially a tool for teachers to anticipate how students may go about solving a task in order to guide students to understanding through questioning rather than funnel them to a particular solution.

Though anticipating student responses is essential for good questioning, teachers should also be careful during classroom discussions to hold students accountable for acquiring knowledge during tasks. The teacher, to clarify student responses and questions that were not loud enough for others to hear or present the same conception in a more understandable format, provided the revoicing technique, but revoicing should be used with care so as to not strip meaning or authorship from the student (Smith & Stein, 2011). It is actually better for teachers to call on students to revoice if at all possible, because this reinforces student ownership, provides a means for formative assessment during class, and holds students accountable for understanding others ideas (Smith & Stein, 2011). Smith and Stein (2011) also offered strategies such as wait time to help foster this accountability by students.

NCTM (2014) suggested four types of questions that help to advance student reasoning and sense making about important mathematical content. The first type of question helps direct students toward starting a task. These questions gather information and push students to recall facts, definitions, or procedures. The second type of questioning habit probes student thinking by asking them to elaborate, clarify, or explain their reasoning. The third type of question makes a student make the mathematics visible to others through representation, structures, patterns, or other methods the student may use. The fourth type of question encourages justification, extension, and reflection by the student to generalize their concept. The important part of these questions require students to explain their reasoning which in turn breaks the IRE pattern and requires students to make sense of mathematics discussed by themselves and others. NCTM

(2014) elaborated this idea through two different questioning patterns for relating a dot plot and box plot that funnel students to the relationship between the two plots and those that focus students on the relationship.

Lampert (1990/2004) offered an environment for educating students related to a mathematical community similar to mathematicians. This community fostered collaboration and questioning, but at the same time required confidence and courage to share solutions (Lampert, 1990/2004). Lampert (1990/2004) had purposefully created this environment by entertaining actions and discussion by students and herself that promoted the success of the classroom environment. Students were expected to give solutions and explain their reasoning while at the same time allowing modification of thought processes based on other students' responses. Lampert's (1990/2004) study focused on this environment and the creation of an activity to build laws of exponents when multiplying like bases. Her findings suggested that it was possible for students, particularly in elementary grades, to engage in reasoning and sense making with argumentation. Lampert's (1990/2004) study highlighted the impact that using purposeful questions to foster reasoning can have on students' conceptual understanding of mathematics.

Though statistics education literature does not include much attention to questioning habits during student tasks, Chance (1997) elaborates on questioning in assessments that promote statistical thinking. Chance (1997) highlights the role that teachers' questions play in what was seen as meaningful learning in the classroom. Chance (1997) also highlights the teacher's role in allowing students to question the process of statistics and the difficulties students face when faced when students were asked to question themselves and their work.

Shaughnessy, Chance, and Kranendonk (2009) also highlight the role that teacher questioning plays in part of fostering thinking habits in their six tasks. Though many of the discussions center on student discussion in the statistical tasks, teacher's input were normally questioning in nature and not explanatory (Shaughnessy, Chance, & Kranendonk, 2009). Teachers in the discussions ask probing questions and questions that move students forward like, "Do you think the equation for the gap is a good one?", Is there a pattern over time that we can use to predict the future?", "How are they changing?", "What is the slope of this line, and what does it mean in our context?" (Shaughnessy, Chance, & Kranendonk, 2009, p.48-49).

Build Procedural Fluency from Conceptual Understanding

The sixth teaching practice in PtA stated that mathematics should build procedural fluency on a conceptual underpinning (NCTM, 2014). Students will become over time skillful in using these procedures to solve contextual and mathematical problems (NCTM, 2014). To achieve this goal teaching should focus on both conceptual understanding and procedural fluency (NCTM, 2014). When procedures are connected with their underlying concepts, students will more likely retain these procedures and be more able to apply them to different situations.

With strong emphasis in recent years on standards and assessment, many teachers have been pulled towards teaching instrumentally rather than conceptually (Pesek & Kirshner, 2000/2002); however, this discussion was highlighted much earlier (cf. Brownell, 1947/2002). All students, young and old, and even non-students look for shortcuts and easy methods to solve problems (Pesek & Kirshner, 2000/2002). Instrumental teaching fosters shortcut methods that students and parents enjoy, but interfere with understanding (Pesek & Kirshner, 2000/2002). Many students who "enjoy" this "luxury" of mathematical instruction perform algorithms and apply formulas haphazardly, without thought, and many times outside applicable context (Pesek & Kirshner, 2000/2002). Martin (2009) emphasized that students should be able to know when to apply a specific procedure and what procedure was most productive in a given situation, and what the procedure will accomplish or result in. Methodical application of procedures without conceptual knowledge often leads to results that are strange (Martin, 2009). Students receiving instrumental instruction were also much less likely to understand concepts such as area outside the sphere of the mathematical classroom to real life applications (Pesek & Kirshner, 2000/2002).

Jacobs (1998/2002) discussed possible outcomes for teaching for understanding through invented arithmetic algorithms rather than traditional algorithms. Jacob's (1998/2002) suggest that students gain understanding of place value and become better problem solvers when given the opportunity to build their own algorithms for operations with whole numbers. Findings by Jacob (1998/2002) do not suggest that students invented algorithms are more efficient computationally. As Pesek and Kirshner (2000/2002) suggest, allowing students time to gain relational understanding increases retention and ability to interconnect concepts (Brownell, 1947/2002; Jacob, 1998/2002).

Brownell (1947/202) discussed major implications for the teaching of arithmetic, generalizable to any mathematical concept, meaningfully. He defines the meanings of arithmetic and describes the processes in which teachers may setup tasks of instruction that foster reasoning and sense making (Brownell, 1947/2002). The increase of interest in student understanding of arithmetic Brownell (1947/2002) described as coming from the armed forces evidences and testimonies, teachers experiences above elementary school, and evidence from the broader community of adults and their day-to-day activities. Brownell (1947/2004) also addressed four of the major objections for teaching arithmetic or mathematical topics meaningfully: (1) Does it really matter if you know why something works? (2) Is knowing how something works worth the time it takes to understand it? (3) Is knowing why something works too difficult for students to understand? (4) Do students suffer by creating and using their own algorithms for mathematical concepts?. Brownell (1947/2004) painted clear answers to these questions but has little reference to answering these objections from research. His answers hinge on factors such as stronger retention, functional value, conception producing better results than memorization, and goals of learning and teaching such as reasoning, sense making, and problem solving (Brownell, 1947/2004). These answers have guided research in the preceding years laying strong foundations for research and practice in the mathematical education community.

These same issues have also been discussed in statistics education specifically. The ASA published an article on the importance of teaching statistics more in the light of conception and less of procedural processes titled, *Improving Mr. Miyagi's Coaching Style: Teaching Data Analytics with Interactive Data Visualizations* (Leman & House, 2012). Feldman, Konold, Coulter, and Coyer (2000) discussed the impacts of introducing graphical summaries to students before they were conceptually ready to understand certain characteristics of these graphs. Mokros and Russell (1995) similarly discussed the interference of introduction to the standard mean algorithm before a conceptual underpinning has been established. These studies have highlighted the need to build procedural fluency from conceptual understanding.

Jacob (2013) stated that students “primarily relied on their procedural knowledge for formal statistical inference” (p. 150). Jacob (2013) suggests future research is needed to relate the connection of formal inference and procedural knowledge with representations that build conceptual understanding. It would be suggested however that future studies similar to Jacob’s (2013) study looking to understanding the connection between representations, conceptual knowledge, and procedural knowledge of confidence intervals and hypothesis testing attend to the inference of procedural knowledge with conceptual understanding (Pesek & Kirshner, 2000/2002; Feldman, Konold, Coulter, & Coyer, 2000; Mokros & Russell, 1995; Konold, 2002).

Procedural fluency is very important in mathematics and statistics; however, it has been shown to interfere with student conception of important mathematics and statistics when introduced prematurely (Konold & Higgins, 2003; Pesek & Kirshner, 2000/2002; Feldman, Konold, & Coulter, 2000; Mokros & Russell, 1995). Effective teaching builds fluency with procedures on a foundation of conceptual understanding (NCTM, 2014). By having a strong conceptual foundation before developing procedural skills, students can become skillful in using these procedures flexibly in different contexts and mathematical problems (NCTM, 2014). NCTM (2014) also discussed students' ability to better retain procedural skills when connected to conceptual understanding and the cause of anxiety in mathematics with early incorporation of procedural skills.

Support Productive Struggle

NCTM's (2014) seventh teaching practice stated that student productive struggle is a necessary component of promoting a class that fosters learning mathematics with understanding. In addition to promoting conceptual understanding, NCTM (2014) recognized the fact that students who experience productive struggle receive long-term benefits and the ability to apply their knowledge into multiple contexts. CCSS-M's first mathematical practice standard stated that students should be able to make sense of problems and preserve in solving them (NGA & CCSSO, 2010). Persistence for each individual is different and requires teachers' attention to both the cognitive and affective domain of instruction (Turner et. al, 2002).

Turner et. al (2002) suggested ways that the classroom environment's goal structure, classroom discourse, and students reporting of avoidance strategies may inhibit student learning in the affective domain and discourage avoidance strategies. Classroom goal structures that focus on grades and use grades as motivating factors tend to affect negatively student performance and affection toward mathematics and statistics (Turner et. al, 2002). When the classroom environment promotes discourse free of criticism and the sharing of ideas is still promoted, student cognitive and affective abilities in mathematics increase and avoidance strategies decrease (Turner, et. al, 2002; NCTM, 1991; Smith & Stein, 2011). Teachers who attend to the affective aspects of teaching during instruction have also reported lower amounts of avoidance strategies by students. This in turns help teachers assess and develop tasks that promote student understanding and usefulness for the content (Turner et. al, 2002). Attention to good scaffolding, promotion of discourse and conjecture, and allowing time for the sharing of ideas help develop students persevere in problem solving (Turner et. al, 2002). These techniques have a much stronger chance of increasing student’s ability to understand and find meaning for mathematics and statistics, which provides support for the increase of interest (Turner et. al, 2002).

Unfortunately, teachers and students many times equate student struggle as an indication that the teachers have failed (NCTM, 2014). In order for a classroom to embrace productive struggle, it is imperative that the teacher and student reshape what it means to teach and learn mathematics (NCTM, 2014). Teague et. al (2011) discussed the possible negative view of struggle many advanced students face when confronted with cognitively difficulty tasks. Students who have succeeded in classes that require memorization and the use of repetitive algorithms may have difficulty in problem solving tasks that require application and number sense (Teague et. al, 2011).

Hierbert and Grouws (2007) make two strong arguments for effective teaching in mathematics. Effective teaching should attend explicitly to important concepts as discussed earlier and students' productive struggle. This productive struggle requires students to make sense of the mathematical concepts that are being taught during an activity. Warshauer (2011) focused on this issue in middle school mathematics, finding that teacher responses to student struggle can be categorized as either telling, directed guidance, probing guidance, or affordance. These responses to struggle were then related to their ability to maintain the cognitive level of the activity, provide support for the student, and build on student thinking (Warshauer, 2011). Providing tasks that create struggle for students and handling these struggles appropriately are integral components to building students conceptual understanding in mathematics (Hierbert and Grouws, 2007; Warshauer, 2011).

Though statistics education has been rather silent in the area of productive struggle, Chance (2002) recognized the issue of teaching for statistical thinking being different from many mathematics courses because of the need for understanding errors in the statistical process. DelMas (2004) also recognized many unique struggles teachers may face that are different in statistics from mathematics such as incorporating variability in conclusions. Being able to anticipate these struggles and design lessons that address these areas are important for teachers to progress conceptual understanding in their statistics course. This makes the use of productive struggle with teacher support that maintains the cognitive demand very valuable in both mathematics and statistics education.

Orchestrate Productive Discourse

PtA's fourth teaching practice stated that teachers should encourage discourse among students (NCTM, 2014). Productive discourse allows opportunities for students to build the understanding of mathematical ideas by examining and comparing one another's arguments and methods. When teachers effectively use discourse to advance mathematical learning, students are more likely to develop conceptual understanding and make mathematics meaningful to themselves (NCTM, 2014). Discourse involving mathematics should include purposeful exchanges of ideas between students in verbal, visual, and in written communication (NCTM, 2014). Mathematics classrooms that promote discourse focus on tasks that promote reasoning and problem solving in the classroom (NCTM, 2014).

Analyzing discourse within the classroom can provide strong evidence of what is valued within the classroom. Orchestrating productive mathematical discussions is an art that provides a means for students to build mathematical power and continually assess understanding (Smith & Stein, 2011). Through monitoring, teachers can provide sequenced explanations that build on each student strengths and help build connections between mathematical concepts (Smith & Stein, 2011).

NCTM (1991) made clear that students must talk with one another as well as in response to the teacher. The teacher's role is to initiate and orchestrate discourse that was meaningful and fosters student learning by being perceptive and skillful in analyzing the culture of the classroom, monitoring inequity, dominance, and low expectations of students (NCTM, 1991). In addition, NCTM (1991) advocated that the center for knowledge be on the reasoning and sense making of students, not teacher dissemination of knowledge. To facilitate this type of discourse, NCTM (1991) elaborated that teachers should:

1. Listen carefully to student's ideas
2. Pose questions and tasks that elicit student thinking
3. Ask clarifying and justifying questions orally and in writing
4. Decide what to pursue in depth based on student ideas
5. Decide when is best to introduce certain notation and vocabulary

6. Decide when to appropriately clarify an issue, lead the class, or let students struggle
7. Monitor student participation and decide when and how to encourage student participation

To enhance discourse effectively in the classroom, NCTM (1991) stated that the means for communication must be broad and varied. This meaning that students not only have multiple forms for communicating ideas but also that there are multiple approaches to a problem. Tasks and problems for students should not rely on conventional forms of mathematical symbols and encourage the use of different tools drawings, diagrams, graphs, and analogies. Smith and Stein (2011) suggested this as being the first stage or step zero of orchestrating meaningful discourse in the classroom. Creating and using tasks that promote students to reason and think statistically help enhance productive student discourse.

FHSM (Shaughnessy, Chance, and Kranendonk, 2009) strongly support the idea that classroom discourse is essential for effective classroom teaching, learning, and assessment. FHSM (Shaughnessy, Chance, and Kranendonk, 2009) described the use of student discussion to monitor students' habits of mind through six chapters presented in forms of investigations that secondary students can complete. Sample student discourse was used throughout the text to help teachers foster meaningful classroom discourse in small groups and class discussions and understand common pitfalls in reasoning by students (Shaughnessy, Chance, and Kranendonk, 2009).

Elicit and Use Evidence of Student Thinking.

PtA's eighth teaching practice focused on eliciting and using evidence of student thinking to promote mathematical understanding and adjust instruction (NCTM, 2014). Eliciting and using evidence has been an essential component of formative assessment (NCTM, 2014; Petit, Zawojewski, & Lobato, 2010). PtA also emphasized that teachers should continually look for ways in which they can generate evidence of student learning and use this to adapt their instruction to meet the needs of their students (NCTM, 2014). Student indicators should identify important factors in student thinking, plan for ways to elicit that information, interpret what the evidence means in respect to student thinking, and then make a decision on how to respond based on this information (NCTM, 2014).

Assessments geared toward increasing understanding are commonly referred to as formative assessments. Petit, Zawojewski, and Lobato (2010) described formative assessment as "a way of thinking about gathering, interpreting, and taking action on evidence of learning by both the teachers and students as learning occurs" (p. 68). According to Petit, Zawojewski, and Lobato (2010), formative assessments should improve instruction by clarifying and sharing learning intentions and criteria for success, providing feedback that moves learners forward, and activating students as instructional resources for one another.

Petit, Zawojewski, and Lobato (2010) clarified that formative assessments should activate students as the owners of their learning. Effective forms of formative assessment may entail classroom discussions, questions, and learning tasks that elicit evidence of learning (Petit et. al, 2010). Activating students as owners of their own learning provides openness (NCTM, 1995) within an assessment that is not offered in all mathematical classrooms. Students who complete self-assessment tasks may be asked to create an individual work plan, create problem-solving activities to explore, provide opportunities to evaluate their own performance against their work plan or against jointly created criteria for performance, or identify specific problems they are having (NCTM, 2014; Petit, Zawojewski, & Lobato, 2010; Chance, 1997).

Developing and analyzing formative assessments is potentially enhanced with the use of Petit, Zawojewski, & Lobato (2010) assessment triangle (Figure 5). The assessment triangle's vertices consist of observation, interpretation, and cognition (Petit, Zawojewski, & Lobato, 2010). The cognition vertex refers to the ways students understand, misrepresent, misunderstand, or use prior knowledge that influences this understanding of a particular concept (Petit, Zawojewski, & Lobato, 2010). The observation vertex refers to descriptions from research that produce specific responses (Petit, Zawojewski, & Lobato, 2010). Interpretation in the assessment triangle represents the tools and methods used to reason from the evidence (Petit, Zawojewski, & Lobato, 2010). "To develop effective assessments that lead to sound inferences, each corner of the assessment triangle must connect to the other in a significant way regardless of the level of assessment" (Petit, Zawojewski, & Lobato, 2010, p. 70-71).

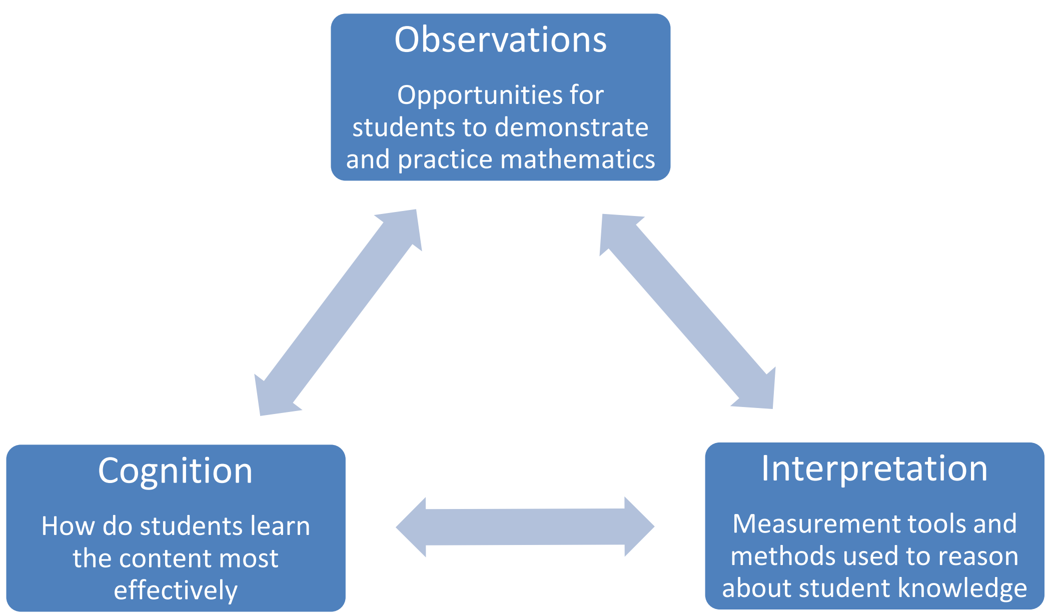

Figure 5. The Assessment Triangle by Petit, Zawojewski, and Lobato (2010).

Assessments in statistics and mathematics should focus on student growth, improvement of instruction, reorganization of student accomplishment, and modification to the class or program (Garfield & Chance, 2000; NCTM, 1991, 1995, 2014; Hubbard, 1997; Garfield, 2003; Chance, 1997). Thus, assessment or eliciting and using student evidence are very similar in both mathematics and statistical literature. Garfield and Chance (2000) described innovative methods of assessment of individual students and instructional practice in statistics such as: individual and group projects, case studies and authentic tasks, portfolios of students' work, concept maps, critiques of statistical ideas or issues in the news, and minute papers. These assessments are encouraged to assess and develop deep conceptual understanding of student. If multiple-choice items are used, these items should focus on matching concepts or appropriate explanations rather than pure procedures (Garfield & Chance, 2000). To use assessment effectively, PSSM (NCTM, 2000, 2014) argued the assessment should support the learning of mathematics and furnish useful information to both teachers and students for improvement.

Garfield (2003) emphasized a reformation that has been happening in statistics assessment to more align with instructional techniques. This type of assessment is used to improve student learning rather than solely evaluate student performance. Assessments in mathematics and statistics should move students forward in their understanding of a concept (NCTM, 2014). Slauson (2008) found that eliciting and using student evidence was helpful in promoting student statistical reasoning. Making expectations and objectives clear helps students achieve satisfactory results by demonstrating how students can achieve these expectations (Petit, Zawojewski, & Lobato, 2010; NCTM, 1995). Teachers who provide feedback to student assessments encourage self-reflection; however, identifying special actions a student can do that can support improvement are even stronger methods that can be used for student improvement (Petit, Zawojewski, & Lobato, 2010).

Chance (1997) provided a summary of assessment techniques used in an introductory statistics course. Assessment techniques emphasized by Chance (1997) were journals, writing lab reports, presenting homework problems, traditional quizzes, take home exam questions, and projects to promote statistical literacy, reasoning, and thinking. Chance's (1997) description of labs seemed to emphasize reasoning and literacy skills while journals and projects emphasize statistical thinking. An important aspect of Chance's (1997) methods for assessment emphasized the importance of understanding students learning in a more formative manner. Chance (1997) acknowledged the importance of measuring student learning and having them assess their own learning as the course progresses. Chance (1997) promoted the idea that students should understand the assessment process and the goals for each activity in order to gain completely from assignments.

Formative assessment is one of the most robust tools teachers can use to improve student learning (Petit, Zawojewski, & Lobato, 2010). Though it may seem like more work for a teacher who has not incorporated these strategies in the past, students engaging in self-reflection and critique actually take the burden away from teacher grading and move toward teacher facilitation and self-assessment. Good instruction uses formative assessment in ways that when summative assessment takes place, there was no surprise. It is imperative that teachers use these tools appropriately to improve student understanding within the classroom.

Summary

This section on teaching has highlighted eight critical areas to consider during the teaching of mathematics and statistics. Building procedural fluency from conceptual understanding has been a key target for much of the recent statistical literature (Jacob, 2013; Leman & House, 2012; Mokros & Russell, 1995; Konold & Higgins, 2003; Feldman, Konold, Coulter, & Coyer, 2000). Literature related to representations in statistics education has pushed towards randomized experiments to help building students informal reasoning to formal (Chance & Rossman, 2006; Tintle et. al, 2012; Rossman & Chance, 2013; Chance & Rossman, 2001; Jacobs, 2012). The development of statistical content should focus on visual representations and computer simulations (Chance & Rossman, 2006; Cobb & Moore, 1997; Chance & Rossman, 2001; Jacob, 2013). Statistics education literature has not explicitly addressed teachers' use of productive discourse in their classrooms and use of purposeful questions. Though some of these areas have not been directly addressed in statistics education literature, the relationship between effective mathematics and statistics instruction can be seen as very similar, thus mathematics education literature was used to fill some of these gaps (Smith & Stein, 2011; NCTM, 2014).

This chapter has also highlighted the interconnection between learning and teaching. In order for students to gain high levels of student learning, teaching techniques must be used that facilitate this (NCTM, 2014). Implementing statistical tasks that promote reasoning must also include productive struggle (NCTM, 2014). Students involved in these tasks are much more likely to need and use discourse within the classroom to facilitate learning (Smith & Stein, 2011). Lastly, assessments used in the classroom should foster learning of statistics by eliciting and using evidence of students reasoning and thinking (Garfield & Chance, 2000; NCTM, 1991, 1997; Hubbard, 1997; Garfield, 2003; Chance, 1997). The difficulty however are the resources many teachers have to help teach effectively and encourage deep learning. For this reason, the next sections looks closer at technology and curriculum related to statistics education.

Technology for Statistics Teaching

PtA stated that excellent mathematics programs integrate "the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking" (NCTM, 2014, p. 78). PSSM (NCTM, 2000) stated that technology is essential in teaching and learning mathematics and influences what is learned. PtA described tools in mathematics such as counters, snap-cubes, base-ten blocks, pattern blocks, and building blocks in the lower grades, and algebra tiles, geoboards, protractors, compasses and straightedges, and geometric models in the upper grades (NCTM, 2014). Current statistical literature has pushed for introductory statistics courses to begin with the use of tools such as dice, playing cards, etc. to understand randomized experiments that can then be carried through multiple iterations using technology (Cobb, 2013).

The technological landscape is rapidly changing. While many classrooms use basic calculators in early grades for basic operation and more advanced calculators for graphing in upper grades, tablets and mobile devices are blurring the lines between computing and educational devices. For this reason, PtA (2014) recognized technology as "interactive whiteboards and a wide range of handheld, tablet, laptop, and desktop-based devices that can be used to help students make sense of mathematics, engage in mathematical reasoning, and communicate mathematically" (NCTM, 2010, p. 78). This makes the platform for technology less important and brings to the forefront what technology is actually being used for (NCTM, 2014).

Chance, Ben-Zvi, Garfield, and Medina (2007) addressed technological changes within statistics education. These changes have largely dealt with the introduction of high power computing and power of re-sampling within statistics itself and education as a whole (Chance et. al. 2007). These changes have also been directed toward change in pedagogical strategies, curriculum, content, and focus (Chance, et. al, 2007). Chance et. al (2007) described some of the most popular statistical software packages as both language and menu driven, however focus was given to software written explicitly for instructional purposes by drag and drop methods like Fathom, Tinkerplots, and InspireData. Other focuses by Chance et. al (2007) were on website applets, graphing calculators, spreadsheet software, multimedia methods, data and material repositories, and ability of this software to aid instruction.

In particular Chance et. al (2007) dealt with recommendations for teachers of statistics to not focus on computational statistics, but emphasize more cooperative learning with technology. Focus has also been given on the use of technology to facilitate and develop conceptual understanding (Chance, et. al, 2007). Chance et. al (2007) also discussed barriers to using technology in statistics including such as: the need to reinvestigate learning goals, lack of awareness and familiarity with available technology, lack of support for teachers, lack of class time that fosters reasoning and sense making, instability of technology, and unclear role of distance learning (Chance et. al, 2007). The integration of technology can help support student learning but it offers complexities that instructors need to anticipate (Chance et al, 2007).

Mishra and Koehler (2009) and Voogt, Fisser, Pareja, Tondeur, and van Braak (2013) described Technological, Pedagogical and Content Knowledge (TPACK) as the use of technology in conjunction with pedagogical and content knowledge. This was an extension of work related to Pedagogical Content Knowledge (PCK), introduced by Shulman (1987). The PCK represented how particular aspects of subject matter were organized, adapted, and represented for instruction in relationship to content and pedagogy. Hill, Ball, and Schilling (2008) extended PCK to describe the mathematical knowledge needed for teaching. The PCK can be used to help distinguish between common and specialized content knowledge.

PCK and TPACK frameworks have shown how the use of technology has benefited classrooms when focus has been given to the integration of content, pedagogy, and technology (Mishra & Koehler, 2009; Hill, Ball, & Schilling, 2008; Voogt et. al, 2013). To use technology appropriately, teachers need to use technologically appropriate tools that enhance instruction and learning while helping students to make connections to previous knowledge (NCTM, 2014). When a teacher uses their content, pedagogical, and technological content knowledge to help students learn, this can be described by the TPACK framework (Mishra & Koehler, 2008).

Similar to Hill, Ball, & Schilling's 2008 article on specialized content knowledge in mathematics, Lee & Hollebrands (2011) developed the Technological Pedagogical Statistical Knowledge (TPSK) framework. This framework characterized the important aspects of knowledge needed to teach statistics with technology into: using the six components of the Statistical Knowledge (SK) in the Wild and Pfannkuch (1999) model; understanding of how to use technology to explore statistical ideas; and understanding of pedagogical issues related to teaching statistics. Teachers who teach with statistical knowledge, pedagogical knowledge, and technological knowledge understand students' learning and thinking of statistical ideas (Lee & Hollebrands, 2011). Teachers who integrate technology according to the TPSK framework should use technology tools and representations to support statistical thinking (Lee & Hollebrands, 2011). The TPSK framework also encourages the utilization of instructional strategies for developing statistics lessons with technology (Lee & Hollebrands, 2011). Lastly, the TPSK framework calls for teachers to evaluate and use curricula materials for teaching statistical ideas with technology (Lee & Hollebrands, 2011). The TPSK framework supports the idea of the integration of technology, statistical content knowledge, and pedagogical knowledge.

Bayés, Meletiou-Mavrotheris, and Paparistodemou (2014) developed a program called EarlyStatistics to help develop early and middle grades teachers Statistics Technological and Pedagogical Content Knowledge (STPACK). The EarlyStatistics course focused on investigation through four components: clarifying and formula a problem answerable by data, designing and employing a plan to answer the question, analyzing data by graphical and numerical techniques, and interpreting results (Bayés et. al, 2014). The STPACK emphasized the integration of statistical content knowledge, pedagogical content knowledge, and statistics technological technology.

Cobb (2013) described the current uses for technology in statistics as: a tool to distribute resources, a tool used in the art of teaching, a tool for curricular content, and a tool as curricular content. Cobb (2013) defined open source classrooms as classrooms where individuals can participate in open source textbooks. These textbooks can be used to share information to help instruct individual or groups of statistical learners. Reform curriculum (ZIG) from Zieffler, Isaak, and Garfield (as cited by Cobb, 2013) was presented as a one of the kind curricula material. ZIG relies on the use of computers to complete applied problems that rely on randomized experimentation and student exploration to develop understanding. Cobb (2013) anticipated hopeful changes in introductory statistics courses content as technology becomes more integrated in the classroom. Cobb (2013) hoped these changes would create a stronger emphasis on bootstrapping and randomization that approximates the normal distribution, increase the use of Bayesian methods through randomization, allow for data mining of large data sets, complete multiple regression and multiple way ANOVAs, and complete generalized linear models with logistic regression.

The following review has highlighted how many of the features of technology have changed the way and what statistics was being taught. For teachers to utilize technology effectively, attention must be played to statistical content knowledge, pedagogical content knowledge, and technological content knowledge. Though tools such as playing cards, dice, etc. are important for beginning insight into randomization, the computers' ability to harness multiple iterations must be used to develop student reasoning (Cobb, 2013). The transition of course content and curricula to randomized methods may require many teachers to develop their statistical content and pedagogical content knowledge more fully (Cobb, 2013).

Statistics Curriculum

When teachers and stakeholders in education use the word curriculum, it is important that this be properly defined. District leaders and teachers may view curriculum as a pacing guide that describes the order in which content objectives are to be taught (Stein, Remillard, and Smith, 2007). Curriculum may also be seen as the set of instruments that are used to instruct which in turn defines many ways how the content is taught (Stein, Remillard, and Smith, 2007). Though standards and curriculum are interwoven, they are different. Stein, Remillard, and Smith (2007) described curriculum as "the substance or content of teaching and learning" (p. 321). Standards may be taught through many different curricula that encompass different methods or approaches to instruction, materials that are used, and order of instruction. Curriculum is essentially the means to meet standards through instructional materials, activities, tasks, units, lessons, and assessments (PtA, 2014).

PSSM (NCTM, 2000) stated that "curriculum is more than a collection of activities; it must be coherent, focused on important mathematics, and well-articulated across the grades" (p. 14). Though many teachers piece together different activities that have worked for them through years of experience (Gelman & Nolan, 2002), a curriculum must be coherent with the teaching and learning goals of the instructor. For this reason, the focus of programs should highlight important goals of introductory statistics and helped students move beyond statistical literacy to statistical reasoning and thinking.

The following report of curriculum reviews statistics education literature related to priorities for instruction, programs, and progressions. This section begins with what is important to teach in statistics at the K-12 setting and in Advanced Placement (AP) statistics according to current literature in mathematics and statistics education. After this, arguments for programs that encourage coherent teaching and learning of important statistics are presented. Lastly, progressions of the K-12 standards and AP statistics courses are discussed from the literature.

Curriculum Priorities

Calls for change in statistics education have emphasized the use of active learning principles with more data and concepts, less theory, and fewer recipes for interpretation (Cobb, 1992; Chance, 1997; Keeler & Steinhorst, 1997). Statistics education literature has pointed statistics more toward a conceptual understanding with less emphasis on procedural computational abilities (Cobb, 1992; Chance, 1997; Keeler & Steinhorst, 1997). To be most comprehensive, the following sections highlight current frameworks for introductory statistics classrooms in the K-12 curriculum that have fostered this call. The PSSM, GAISE report, and CCSS-M focused on the K-12 setting, while the AP statistics program focused on reducing redundancy between the high school and tertiary levels (Klopfenstein & Thomas, 2009). Though these frameworks were created for different levels, there are striking similarities and differences (Rossman & Chance, 2013) discussed. In addition to these set of standards, the Next Generation Science Standards are reviewed and compared to these frameworks for standards. The overall intention of the following sections is to articulate some of these major frameworks for curricula in statistics education in more detail, then describe their similarities and differences.

Principles and Standards for School Mathematics. PSSM was created to make an argument for all students learning important mathematics with understanding and describing ways all students can attain this understanding (NCTM, 2000). PSSM (NCTM, 2000) attempted to address mathematics teachers, teacher leaders, developers of curricula, professional development leaders, educating mathematics teachers, pre-service teachers, schools, sates, administrators, and policy makers. PSSM (NCTM, 2000) (NCTM, 2000) presented six "principles" and five "process standards" that articulated and guided mathematics for the last decade by presenting a coherent vision for mathematics education. A brief discussion follows related to NCTM's (2000) principles and standards.

Principles. The six principles in PSSM (NCTM, 2000) highlighted particular features of high-quality mathematics education. Their overarching principles in PSSM (NCTM, 2000) were equity, curriculum, teaching, learning, assessment, and technology. Though these principles do not address specific mathematics content, they are crucial for effective mathematics instruction.

NCTM's (2000) equity principle has vision for those involved with students to have high expectations and strong support for all students. To ensure equity, PSSM (NCTM, 2000) required the accommodation for differences of individual learners, providing opportunities and resources for all. The curriculum principle stated that curriculum is more than a set of activities, and must be coherent, focused on essential mathematics, and progressed to maximize effectiveness in learning (NCTM, 2000). PSSM's (NCTM, 2000) teaching principle required teachers to understand what students know and need to learn then support them to move forward. Moving students forward requires teachers to provide a challenging and supportive learning environment for students and understand the mathematics students are learning (NCTM, 2000). NCTM's (2000) learning principle required students learn mathematics with understanding by building on new knowledge from experience and prior knowledge (NCTM, 2000). PSSM's (NCTM, 2000) assessment principle required teachers to make assessments that were valuable for making instructional decisions and enhancing student learning. NCTM's (2000) technology principle stated that technology should support effective mathematics instruction and enhance mathematics learning of important mathematics.

Standards. The PSSM (NCTM, 2000) standards were divided into five important content standards: number and operation, algebra, geometry, measurement, and data analysis and probability. In addition to content standards, PSSM's (NCTM, 2000) process standards were also investigated and discussed in detail in the goals of practice section. These categories were explored in relationship to four different grade bands: PreK-2, 3-5, 6-8, and 9-12 (NCTM, 2000). The data analysis and probability standards through these grade bands explicate what students should know and need to know to move students forward.

PSSM's (NCTM, 2000) content standards at each grade band were divided into four overarching subcategories: formulating questions and addressing data collection, organizing, and presenting data, selecting and using appropriate statistical methods for data analysis, developing and evaluating inferences and predictions based on data, and understanding and applying basic concepts of probability. PSSM (NCTM, 2000) provided clear progression of each of these subcategories at each grade band with examples of the content standards, habits of mind that move students forward, and students' demonstration of mastery of standards. The progression of content was clearly intertwined with goals of instruction that promoted PSSM's (NCTM, 2000) guiding principles.

Guidelines for Assessment and Instruction in Statistics Education Report. The GAISE report produced by the ASA provided key concepts, activities that help develop these key concepts, and levels for student understanding within statistics that should be sought in the K-12 curriculum. The GAISE used the PSSM as a foundation for its own framework and did not attempt to replace the PSSM (Franklin et. al, 2007, p. 5). More specifically, the GAISE report created three sequential levels within K-12 education at which students should be able to apply statistical concepts or statistical process components such as formulating questions, collecting data, analyzing data, interpreting results, and understanding variability (Franklin et. al, 2007). The GAISE report did not directly correlate the three sequential levels of statistical understanding with grade levels; however, it detailed process components by looking at the complexity of a question it could entail.

Objectives in the GAISE of level A students required teachers to formulate statistical questions and students distinguish between solutions and fixed answers (Franklin et. al, 2007). Students at level A in the GAISE should conduct census information, understand individual to individual variability, conduct simple experiments with non-random assignment, and understand variability that may be attributed by an experimental condition, induced, measured, or natural (Franklin et. al, 2007). Level A students should begin to understand the idea of a distribution, tools to explore those distributions, and observe the association between two variables while comparing individuals to individual or individuals to a group (Franklin et. al, 2007). Level A students may restrict their analysis to the classroom, but should recognize that their data may show different results in other classes or groups (Franklin et. al, 2007). Generating questions and incorporating tools that potentially answer these questions were characteristics of developing level A students in the GAISE as well (Franklin et. al, 2007).

Unlike level A students, level B students “become more aware of the statistical question distinctions” (Franklin et. al, 2007, p. 35). Level B students develop methods to analyze their questions by selecting measureable variables and determining how to measure them (Franklin et. al, 2007). Tools such as two-way frequency tables, numerical summaries, mean absolute deviation, quadrant count ratios, and graphical displays such as box plots were expanded in the GAISE to help answer more sophisticated questions that involve multiple variables with emphasis on association (Franklin et. al, 2007). Probability should also expanded from level A students to level B students by the introduction of inducing random selection and assignment to conduct experiments and show causation (Franklin et. al, 2007). The introduction of sampling helps build students understanding of variability to sampling during experiments. Level B students were also encouraged to move beyond their classroom models to understanding the uses and misuses of statistics in the real world (Franklin et. al, 2007).

Level C students in the GAISE report should be able to create statistical questions that can be answered by the collection of data while using experimental, observational, survey, and comparative designs appropriately (Franklin et. al, 2007). Once students create these questions, level C students should be able to devise a reasonable plan to sample and an experiment to answer their questions (Franklin et. al, 2007). When this process is completed, level C students should be able to use their data to draw and support their conclusions (Franklin et. al, 2007). Level C students should use tools for conclusions that draw on the use of sampling distributions, correlation coefficients, and statistical tests (Franklin et. al, 2007). Level C students are able to understand the role variability plays in the sampling, measurement, and conclusion process and incorporate the reduction and reporting of this variability in their conclusions (Franklin et. al, 2007).

The GAISE report's levels of instruction also incorporated statistical activities that teachers may use to help develop students' statistical reasoning. The GAISE report's articulation of major content and inclusion of content outside many curriculum standards such as quadrant count ratios and mean absolute deviation made it a unique set of standards for teachers in K-12 education. Though the GAISE report did not tell how to teach, activities found within the GAISE report embed the importance of conceptual understanding, the development of statistical reasoning, and development of CCSS-M.

Common Core State Standards for Mathematics. The CCSS-M is a set of standards formed by the National Governor's Association and Council of Chief State School Officers that was fully implemented in 45 states and the District of Columbia in 2014. The importance of statistics can be found within the CCSS-M as domains of study, Measurement and "Data" and "Statistics and Probability", throughout K-12 instruction (NGA & CCSSO, 2010). NCTM's K-12 standards within PSSM (NCTM, 2000) provided a similar emphasis on measurement, data, statistics, and probability and was used as a road map for the CCSS-M (NGA & CCSSO, 2010).

The CCSS-M standards have increased rigor and cognitive demand in many state curricula in addition to remapping standards to different grade levels (Porter, 2011a, 2011b). The combination of these new demands has provided increased stress for teachers in middle grades who come from varying backgrounds. Though middle grades content objectives have increased in statistics, the CCSS-M placed very little emphasis on data analysis and statistical standards in grades K-5 (Rossman & Chance, 2013). Rossman and Chance (2013) suggested some small adjustments of statistical content in grades K-5 instead of a reduction in standards to barely nothing. Though K-5 standards in statistics are very slim, a sudden shift in student expectations in statistics and probability occurs at the 6th and 7th grade levels with push of understanding of distributional shape, center, and spread. Experts within curriculum development and assessment at a conference in Arlington, VA noted a key recommendation for successful implementation of the CCSS-M was to “focus attention to content changes at the middle grades” (Garfunkel et. al, 2011, p. 12). These expert participants claimed there were “substantial increase in the number and nature of learning goals… at middle grades” (Garfunkel et. al, 2011, p. 12).

Porter (2011a, 2011b) tried to quantify the relationship between changes in standards for many different states after adopting the pre-CCSS-M. Porter (2011a, 2011b) described a shift of needed content knowledge for teachers and new standards for students for many states. The policy workshop, *Gearing Up for the Common Core State Standards in Mathematics* (2011), focused on pivotal standards for grades K-8 professional development (Institute for Mathematics and Education, Center for Science, Mathematics, and Computer Education, & Institute for Research on Mathematics and Science Education, 2011). The CCSS-M’s emphasis in grades six and seven were on ratios and proportional relationships with emphasis on developing statistical critical thinking.

Although little emphasis is placed on probability and statistics at the K-5 level, the emphasis from 6-8 continues to increases at the high school level. Both quantity and depth of knowledge of statistical standards was increased by the adoption of the CCSS-M by most states in the United States (Rossman & Chance, 2013). CCSS-M’s high school standards are divided into four domains: Interpreting categorical and quantitative data, making inferences and justifying conclusions, conditional probability and the rules of probability, and using probability to make decisions (NGA & CCSSO, 2010).

Standards in the CCSS-M place an emphasis on the understanding of variability in context especially in their first two domains. These first two domains also emphasize the understanding of tools that can be used to understand variability both numerically and graphically. Students are required to interpret measures of center and spread while also using them for comparison between different distributions. Attention to the relationship of these measures with the shape of distributions also plays a key role. Similarly, students are required to relate the probability of an event to the normal distribution. The second domain continues in this relationship by having students make decisions and inferences based on random processes. An interesting characteristic of the second domain is an emphasis on random processes to describe events and the use of simulation to help in inference.

The third and fourth domains from statistics and probability in the CCSS-M focus largely on theoretical rules of probability and the understanding of mathematical representations of random processes. Some standards place emphasis on applying a rule, while others place more emphasis on the understanding of a rule. An example of one standard that helps in the understanding and development of the mathematical formulas for conditional and independent events requires students to develop this relationship through the construction of two-way frequency tables. The fourth domain emphasizes the use of mathematical expectations in student decision making and begins to intertwine the first three domains within the statistics and probability conceptual category.

Next Generation Science Standards. The development of the Next Generation Science Standards (NGSS) began in 2011 and went through multiple reviews by the public, state, and critical stakeholders before being released for adoption in 2013. Though it has not been as strongly embraced as the Common Core State Standards for Mathematics (CCSS-M) and English Language Arts, twenty six states lead in its completion and eleven states have adopted these standards as of June 24, 2014 (National Center for Science Education). The NGSS was characterized by disciplinary core ideas of physical science, life science, earth and space science, and engineering, technology, and applications science. The NGSS was also characterized by cross cutting concepts such as patterns; cause and effect; scale, proportion, and quantity; systems and system models, energy and matter, structure and function, stability and change; interdependence of science, engineering, and technology; and influence of engineering, technology, and science on the nature and world.

The third characterization of standards of the NGSS relates to statistics education. The NGSS discussed eight practices of science and engineering that were essential for students to learn (NGSS Lead States, 2013). A blue print for implementing these practices was described first in *A Science Framework for K-12 Science Education* (National Research Council, 2011). These eight practices are: (1) asking questions (for science) and defining problems (for engineering), (2) developing and using models, (3) planning and carrying out investigations, (4) analyzing and interpreting data, (5) using mathematics and computational thinking, (6) constructing explanations (for science) and designing solutions (for engineering), (7) engaging in argument from evidence, and (8) obtaining, evaluating, and communicating information (National Research Council, 2011). The NGSS acknowledged and suggested overlap in its practice standards, which was also called for in much mathematics and statistics education literature (NGSS Lead States, 2013).

Though these practices each relate to statistics education, analyzing and interpreting data practices has direct relationship to statistical standards. Analyzing data in K–2 built on prior experiences and progresses to collecting, recording, and sharing observations (NGSS Lead States, 2013). Analyzing and interpreting data in 3–5 built on K–2 experiences and progresses to introducing quantitative approaches to collecting data and conducting multiple trials of *qualitative* observations (NGSS Lead States, 2013). Emphasis in grades 3-5 NGSS practices are on graphical displays such as bar graphs, picto-graphs, pie charts and informal comparisons of recorded data to refine or evaluate a problem (NGSS Lead States, 2013).

Analyzing and interpreting data in NGSS grades 6–8 built on K–5 experiences and progressed to extending quantitative analysis to investigations, distinguishing between correlation and causation, and basic statistical techniques of data and error analysis. Practices in this NGSS area focused on constructing, analyzing, and/or interpreting graphical displays of data and/or large data sets to identify linear, nonlinear, temporal, and spatial relationships while understanding the difference between cause and correlation. NGSS practices in grades 6-8 also emphasized the use of mean, median, mode, and variability as tools to characterize and analyze data then use these tools to make conclusions and improve objects, tools, or processes (NGSS Lead States, 2013).

Analyzing data in NGSS' grades 9–12 built on K–8 experiences and progressed to introducing statistical analysis, comparisons of data sets for consistency, and using models to generate and analyze data (NGSS Lead States, 2013). NGSS (NGSS Lead States, 2013) practices in grades 9-12 include using data to fit slope, intercept, and determine correlation coefficients and apply these values to questions and problems. These NGSS (NGSS Lead States, 2013) practices also required students to consider the limitations of data analysis as well as compare and contrast different types of data sets.

Data analysis and statistics is the connection between mathematics and science. For this reason, it is imperative for both mathematical and science standards to include them in their content standards. The NGSS, CCSS-MNGA, GAISE, and PSSM correlate very strongly with one another in relationship to objectives, pacing, and practice. Though content objectives in the

NGSS differ in their major scope for student learning via statistics, the NGSS' emphasis in practices make them essential within the classroom at all grade levels just as the CCSS-M, GAISE, and PSSM. Though major content objectives differ in these documents, bridges can be built between their standards with the use of collaboration between different content teachers and constructivist teaching principles that use the exploration and collection of real world data (Cobb & Moore, 2000; Garfield & Ben-Zvi, 2008, 2009).

Advanced Placement Statistics. The Advanced Placement (AP) program began in 1957 as a means for high school students to earn college credit for learning college material (Klopfenstein & Thomas, 2009). The intention for the creation of the AP program was to reduce course work repetition in early college years (Klopfenstein & Thomas, 2009). Today the program has broadened in use as an indicator by colleges for motivation and high achievement (Klopfenstein & Thomas, 2009). Though the AP program has been shown to be excellent indicators for higher education, the AP program itself does not cause this effect. When self-selection of individual students and available opportunities for students were introduced into models, Klopfenstein and Thomas (2009) found that the AP program did not contribute to student achievement in higher education.

The AP statistics course was added to the AP program in 1996 as the enrollment of students in statistics courses in higher education had increased by 45% reportedly by the *Chronicle of Higher Education* in 1990. The AP statistics course sponsored by the College Board takes place in many high schools throughout the U.S. Data reported from Rodriguez (2012) and Pierson (2013) showed that the numbers of students taking AP statistics has increased approximately 12% a year but approximately 75% more students are taking AP calculus than AP statistics. The NAEP's 2014 state of education reported that in 1990 only 1% of students were taking statistics and probability in high school and in 2013 the amount of students had increased to 11%.

The topic outline for the AP statistics course encompassed exploring data, sampling and experimenting, anticipating patterns, and making statistical inference (College Board, 2010). The College Board (2010) labels a major topic as exploring data, which pushes students to describe patterns and departures from patterns. The major topic of sampling and experimentation which in the AP statistics objectives (College Board, 2010) required students to plan and conduct experiments. Anticipate patterns in the AP statistics course description (College Board, 2010) required students to understand random phenomena using probability and simulation. The course description (College Board, 2010) for AP statistics courses in the major topic of statistical inference required students to estimate population parameters and test hypothesizes. The AP statistics course outline and descriptions (College Board, 2010) aligned very well with other standards in the CCSS-M and GAISE.

Differences and Similarities in PSSM, GAISE, CCSS-M, and AP Statistics. The PSSM, GAISE report, CCSS-M, and AP statistics program mainly provided a set of content and depth of standards for statistics. Individual course progressions and limitations of how to teach certain concepts were not given in any of these documents although some examples were given in the GAISE. A similar characteristic but not the focus in each of these documents push students to explore and make sense of real world problems. All three sets of these documents push for active learning by the student in which students develop conceptual understanding through engaging experiences. Though the GAISE, PSSM, and AP statistics guidelines were more explicit about the inclusion of active learning principles, the CCSS-M's inclusion of mathematical practice standards for students related to many of the active learning principles found within the GAISE and AP statistics documents. A distinctive feature, however, concerning the difference between the standards for AP statistics and all other three documents was its lack of instructional principles, which was the reason AP statistics, was not included in statistics education. It should be noted, however, that the AP statistics section (College Board, 2010) for instructional practices did cite PSSM (NCTM, 2000) in its discussion of effective teaching practices.

Though the CCSS-M , GAISE report, and PSSM were released at different times, were created for different reasons, and serve currently in different roles for statistics education, there is considerable consistency in respect to the statistical standards of spread, central tendency, and display methods. The introduction to the spread of data, many times described as variance, in all three documents was first found in display techniques like histograms, dot plots, and stem-and-leaf plots in elementary grades. Although the standards did not emphasize the spread of data and implications of these plots, they lead students to the intuition of spread, differences among samples, and randomness. These display techniques were expanded upon in middle grade bands of 6-8 to include box plots and quantitative measures of data variation in the CCSS-M. The CCSS-M moved some standards of PSSM's high school grade bands (9-12) to the middle grade bands (6-8) such as the understanding of variability at a more complex level.

The first quantitative measure for spread within all documents excluding the CCSS-M was range, difference of the largest and smallest value of a data set. The interquartile range, difference between the third and first quartile, was included in the GAISE and AP statistics standards. Interestingly, the CCSS-M did not include range as a content objective in mathematics. Perhaps with reasoning that the nature of the calculation of range is very prone to outliers and does not take into effect other observed data points; however, the interquartile range, the first measure of spread in the CCSS-M, is more robust. AP statistics past open-ended solutions commonly reported range as a measure of spread when comparing distributions.

Though the range and interquartile range are measures commonly reported in statistical analysis, these measures are purely order based and do not take into account each data point of a sample and its effect on the actual spread of data. In fact, statistics based on order is called non-parametric statistics and is usually preferred over parametric methods when outliers may be present and not disregarded. The GAISE and CCSS-M both introduce the mean absolute deviation (MAD), a parametric version of the spread of data, as a quantitative measure for spread at level B and in grade 6 respectively while the AP curriculum does not include it. The GAISE reports that the MAD, the sum of the absolute differences of the sample mean and each data point divided by the number of data points, "serves as a precursor to the standard deviation developed at Level C" (Franklin et. al, 2007, p. 44). AP statistics objectives (College Board, 2010) do not include MAD as a content objective but does discuss range and median.

Actual statistical content objectives for middle grades students from the GAISE, PSSM, and CCSS-M are very similar and in fact the GAISE report was based from the PSSM. High school standards in the CCSS-M, PSSM, and levels B and C within the GAISE strongly relate to AP statistics objectives. The CCSS-M has seventeen standards with nine sub standards (NGA & CCSSO, 2010). The PSSM had four major objectives with ten descriptions of how to meet these major objectives (NCTM, 2000). The GAISE report categorized students' statistical understanding at levels that do not necessarily correlate with grade. The AP curriculum consists of seventy numbered standards that are grouped into four major objectives and fifteen subcategories (College Board, 2010). Although the GAISE report does not imply that levels A, B and C correspond to elementary, middle grades, and high school, the report makes clear that these are concepts that should be obtained by grade 12 for college statistical readiness. There were many similarities of level B in the GAISE report content with middle grades content in the PSSM and CCSS-M.

Probably the most pivotal similarity within these documents and other statistical content frameworks is their categorization of standards. The GAISE required students to formulate questions that can be analyzed by the collection of data, select and use appropriate statistical methods to analyze data, develop and evaluate predictions and inferences based on data, and apply basic properties of probability (Franklin et. al, 2007). The CCSS-M required students to interpret categorical and quantitative data, make inferences and justify conclusions, use rules of probability, and use probability to make decisions (NGA & CCSSO, 2010). The AP statistics standards required students to explore data patterns, plan and implement statistical experiments, explore random phenomena using probability and simulation, and make statistical inference (College Board, 2010). Each of these frameworks subcategorize topics into four very similar sets of standards: asking statistical questions, data collection, analyzing data, and making conclusions.

Another important similarity but also difference between the CCSS-M and GAISE is their setting of specific standards at certain grade levels. The GAISE report does not specify certain grade levels for specific standards; however, the CCSS-M sets standards for specific grade levels from K-8 and does not for grades 9-12. The AP statistics standards required students have taken an advanced high school algebra course before completing the course, thus ensuring most students are near the end of high school before completing the course (College Board, 2010). Though none of these frameworks requires certain curricula, certain curricula foster some of the visions, principles, and practices within these documents. School system and teacher selection of curriculum within the classroom should be chosen wisely to foster specific learning environments (NCTM, 1991).

Programs

Based on the priorities of curriculum, program selection or the intended curriculum plays a key part of teachers' enacted curriculum (Stein, Remillard, and Smith, 2007). The selection of resources and course materials by school leaders lay the foundation for expectations of teachers and students. With this being the case, resources should be chosen that harness the importance of context in statistics and focus on student-centered instruction (Cobb & Moore, 1997). Programs should focus on curricula that foster the development of learning, teaching, and curriculum that have been previously discussed.

Standards based programs and conventional programs are best used with specific goals in mind (Stein, Remillard, and Smith, 2007). Standards based programs focus largely on students building knowledge with the guidance of the teacher while conventional programs lead the transfer, mediation, and creation of knowledge primarily by the teacher (Stein, Remillard, and Smith, 2007). Both standards based and conventional programs share the idea that mathematics should be understood and conceptualized by all. They differ in their emphasis on procedural skills, standard integration, and emphasis on non-routine problems (Stein, Remillard, and Smith, 2007).

Statistical Progressions

The Common Core Standards Writing Team (CCSWT) (2012) has drafted a progressions framework for the standards in the CCSS-M. This progression document outlined subdomains of probability and statistics with connections from the middle school to these domains. In addition to outlining the progression of the CCSS-M, CCSWT (2012) provided contexts to teach certain standards, explanations of standards, and examples for problems in the CCSS-M standards. CCSWT (2012) also provided additional mathematics that students should learn in order to be prepared for more advanced courses. This progression framework related much of the framework provided through the GAISE.

The CCSWT (2012) was geared largely towards practicing teachers and helping them understand standards and their connections within the high school CCSS-M. Clear connections were made to the CCSS-M standards with corresponding descriptions in the margin. The CCSWT could provide to be an excellent resource for teachers to understand the meaning of a particular standard or connect the standard to student's previous knowledge. Using this document may also paint a clearer picture for how one standard may relate to another.

Chance and Rossman (2001) carried on a healthy debate on possible progressions for introductory statistics courses and offer arguments and rebuttals for common progression changes. Their debate focused on four major points: when to teach data analysis in respect to data collection, when to teach descriptive analyses for bivariate data, when to teach inferences for means and proportions, and when to teach significance testing in respect to confidence intervals. Chance and Rossman (2001) did not declare a one size fits all progression, but do elaborate on the difficulties faced in each progression.

If teachers start classes by focusing on the importance of sampling, they mirror the nature of statistics (Chance & Rossman, 2001); however, Cobb and Moore (1997) stated that exploratory data analysis builds confidence for students and introduces them early to one of the key aspects of statistics variation. Chance and Rossman (2001) also argued that performing descriptive analysis of bivariate data allows association and causation relationships to develop at an early stage of the course and be developed throughout. Chance and Rossman (2001) also argued that early bivariate descriptions allow students to build conceptions of variables relating to one another; however, they argue that this can be negative. Students may look at bivariate descriptive analysis as largely on using and manipulating formulas. In addition, by waiting on the introduction of bivariate analysis, students can be introduced to statistical inference of bivariate data as a complete package rather than focusing on descriptive analysis then reverting back later in the course to inferential aspects of bivariate analysis.

Chance and Rossman (2001) similarly argued for the incorporation of inferences for means before or after inferences for proportions. Chance and Rossman (2001) argued that inferences for both mean and proportion could also be introduced to students as side-by-side comparisons because of their strikingly similar conceptual framework. The benefits of proportions rely on one parameter representing both the center and spread of the data and its easy incorporation into statistical investigations. The benefits of performing mean inferences first result in students gaining a better understanding of spread alongside of center because of their separate parameter values (Chance and Rossman, 2001). Though these inference procedures are similar, there are multiple ways to teach and conduct statistical tests. Chance and Rossman (2001) argued that randomized experiments lead naturally into hypothesis testing and empirical rules lead naturally into conceptual frameworks for confidence intervals. Though both hypothesis testing and confidence intervals are different, they tell much of the same story. Chance and Rossman (2001) actually implied that they should be connected for students' conceptual understanding.

Summary

Three major points emerge from this review of statistics curriculum. First, good teaching is not just what you teach, but how you teach. Similarities between PSSM, GAISE, CCSS-M, and the AP statistics frameworks make it clear that teaching should foster active learning principles. What teachers foster inside their classroom through their enacted curriculum will describe what learning takes place (Stein, Remillard, and Smith, 2007). Resources teachers have at hand play an integral part in what was performed inside the classroom, thus district leaders, policy makers, and curriculum leaders who want students to learn how to reason and think statistically need to put curriculums that foster this in the hands of teachers.

Second, the type of program that is chosen in a school significantly impacts what happens in the classroom (Stein, Remillard, and Smith, 2007). The priorities of a curriculum should be well articulated and integrated into the choice of program. Thirdly, progressions of standards should foster students' conceptual understanding. Similarities in PSSM, GAISE, CCSS-M, and NGSS standards progression can be easily extended to AP statistics progressions. Though different progressions are possible, teachers should have reasons for decisions that choose to make in progressions and be aware of misconceptions that may arise in certain progressions (Chance and Rossman, 2001).

In conclusion, progression of standards, goals, and guiding principles should be integrated. Student learning targets, programs, and progressions in statistics should be clearly established through sources such as PSSM, GAISE, and CCSS-M. Programs should be chosen that make seamless expectations between principles of good effective teaching of mathematics for teachers and mathematical standards for students. These programs should integrate with expectations from the priorities of the curriculum with standards, practice, and progressions. School leaders highlight these expectations through the adoption and support of different programs in their districts.

Assessment

Assessments are often used as a means to evaluate students, but assessments should do more (NCTM, 2014; NCTM, 2001; NCTM, 1995). There is a subtle difference between evaluation and assessment. The previous section in teaching: eliciting and using evidence of student thinking developed some key ideas of formative assessment. Evaluations are many times viewed as summative assessments, a final product that demonstrates student conception or understanding of a concept (NCTM, 1995). The following section on assessment reviews the roles different forms of assessment have played in largely the summative or evaluation phase of statistics education literature.

Projects

Though teachers of mathematics use projects many times as a summative assessment tool, projects can also be used as methods for students to develop and construct knowledge. Projects are often used to build and show mastery of the complete statistical process in introductory statistics courses. Carnell (2008) performed a study treating or assigning two classes a group statistical project while leaving other class untreated to determine differences in attitude. To ensure treatment affects were based solely on the given project, Carnell (2008) devoted little in-class time to the project and change in instructional methods. No significant differences were found between the two classes in end of course evaluations and student interest in statistics. Studies such as Carnell's (2008) highlight the role of the interconnection between assessment, instruction, and curriculum in the development of attitudes and conceptual understanding. The inclusion of only a statistical project, with no change in other factors, had no effect on students' conceptions of statistics.

Starkings (1997) highlighted the role the teacher plays in the successful completion of statistical projects in introductory statistics classes. Starkings (1997) suggested limiting questions for projects to ensure adequate completion. Starkings (1997) also suggested the use of collaborative groups and staged assessment to ensure adequate completion of projects. Limiting question types provides for easier grading by the teacher and provides clear goals for students but lowers the taxonomic level needed for students to succeed. Starkings (1997) suggested mixed ability groupings for projects to help students reinforce their own ideas while simultaneously being peer mentors for others. Starkings (1997) recommended staged assessment as a way for the teacher to provide feedback to students as they progress through the project. This in turn provides a means of formative assessment for an assessment tool that was many times used as a culminating summative assessment tool.

Starkings (1997) suggested two models for assessing projects while using a staged assessment strategy. The Analysis, Design, Implementation, and Design divides a project into four stages of assessment that were graded as a group collectively. The 4P model, however, emphasized more writing in the project and was graded individually through a project log, written project report, practical development, and project presentation at different stages set by the instructor (Starkings, 1997). Both of these models could be used differently for projects within statistics classes; however, it should be noted the interconnection between each of these stages. Using projects towards the end of a course could help students make the interconnection of different stages within a statistical study.

Though Starkings (1997) lacked suggestions for specific pedagogical strategies, she did highlight the role of instruction during the project. Starkings (1997) recommended that project work should foster an active environment of problem solving and experimentation. Starking

(1997) also suggested that asking for certain criteria on the project before it was presented in class creates problems with project selection and methods used to complete the project. Sarkings (1997) feedback for students was paramount during and after completion of certain stages and showed how a project can be used as a means of formative assessment to guide and develop student understanding. Starkings (1997) advised using a help factor scale, which allows students to ask for help during the project before the due date with minor deductions. The staged assessment process should be cohesive with instruction. This can help maximize the development of student knowledge when using a project (Starkings, 1997).

Ramirez-Faghih (2012) investigated how students' statistical reasoning and motivation develop when allowed to complete a statistical project. Ramirez-Faghih (2012) projects were completed in a college mathematics one class with 73 participants. Ramirez-Faghih's (2012) project provided students the opportunity to pick their own topics, write their own survey questionnaire, collect their own data, and analyze the results. Results concerning motivation were analyzed qualitatively through Wigfield and Eccles' (as cited by Ramirez-Faghih, 2012) Expectancy-Value theory. Results suggested that students "with high values in terms of their interest, attainment, and utility (positive subjective-task values) can effectively surpass the relative cost of engaging in the statistical investigation" (Ramirez-Faghih, 2012).

Students in Ramirez-Faghih's (2012) study who did not complete the project or had unsatisfactory results had difficulty in developing a research question and writing a survey instrument which further developed difficulties in data analysis and inference. This finding falls in line with GAISE recommendations for guided instruction through three cognitive levels of statistical literacy and misconceptions discussed by Konold and Higgins (2003). This may have also been assisted through Starkings (1997) proposal of staged assessment during the project.

Ramirez-Faghih (2012) found a significant difference between students who completed the project successfully using categorical data and those who used continuous data. This suggests further evidence into the relationship between components of statistical thinking and GAISE process components with informal statistical inference and formal statistical inference.

Ramirez-Faghih's (2012) lack of focus on pedagogical techniques used by the instructor was a weakness. This could have potentially influenced student development to complete the statistical project. The only clear evidence of pedagogy used was homework assignments that were devoted to the four steps of the project as described earlier (Ramirez-Faghih, 2012). The second underlying weakness was Ramirez-Faghih's (2012) pre and post-statistical assessment. Objectives addressed by the assessment tended to be procedural and did not fit the overarching scheme of the report to develop statistical reasoning (Ramirez-Faghih, 2012). This also suggests pedagogical techniques used in the classroom possibly did not support the development of statistical reasoning as proposed by the CCSS-M, AP standards, and GAISE report but instead focused on statistical literacy separate from reasoning and thinking.

Studies reviewed in this section concerning projects have highlighted the use of projects as a means to develop cognition and affection towards statistics. These studies have also stressed the impact of alignment between projects and the goals of instruction (Ramirez-Faghih, 2012). An important characteristic of projects that have been used positively in the statistics classroom have focused on the assessment tool as not just an evaluation instrument but as a tool used to build and assess understanding, blurring the lines of a project being strictly a formative or summative assessment tool.

Portfolios

In statistics and mathematics instruction, portfolios are many times used as summative assessments in which students present some of their best work. Research on the use of projects and portfolios, however, encourages providing ongoing support and feedback for particular phases of the process. Keeler (1997) discussed the use of a portfolio in her graduate level statistics course as a way to develop students' ability to incorporate research models and complete statistical procedures. Keeler's (1997) article was largely divided into four sections: the use of a portfolio in a constructivist-learning environment, the development of a portfolio, a portfolio sketch, and issues and concerns in the use of portfolios as an assessment tool.

The use of portfolios has the ability to tie directly into a constructivist framework of learning. Teachers using a constructivist approach to instruction construct their own understandings instead of inertly absorbing or replicating the understandings of the instructor. Keeler (1997) stated, "learning is the result of an action in which students participate, not the inevitable product of encountering materials" (p. 3). The use of a portfolio maintains this environment in the assessment phase of instruction making teaching in a constructivist manner seamless in the classroom. Students become responsible for their own learning and improvement during assessment.

Keeler (1997) maintained the following three criteria should be communicated to students when using a portfolio as an assessment: the assessment purpose, the criteria or methods for determining what was put into the portfolio, and the evaluation criteria for scoring the pieces or the collection. This directly relates to the openness standard in NCTM's (1995) Assessment Standards. Keeler (1997) stated, "the portfolio can do what traditional assessment in most cases does not do, provide direct evidence for evaluating the student's progress over time, at mastering the essential concepts and techniques of the course" (p. 4). The portfolio becomes meaningless without clearly identifying a goal, incorporating ways for this goal to be accomplished, and determining a way that this can be analyzed in a meaningful way,

Driessen et. al (2006) described a portfolio's goal as a means to measure students' ability to reflect on the professional and personal development. Unique to the portfolio assessment was the students' ability to determine what should be included in the portfolio (Keeler, 1997). Because of this, teachers must be especially careful to provide opportunities for students to engage in meaningful work. Finding meaningful activities and students incorporating these activities in the portfolio can be ways of monitoring student progress in learning.

Keeler's (1997) use of the portfolio in the graduate course also reflected the traditional or longitudinal portfolio where student work was gathered over time in order to demonstrate growth. Keeler (1997) used periodic checks of student work to make the assessment technique more formative and open in grading by providing written feedback to students. Keeler (1997) also exemplified student work that demonstrated goals for the assessment. A portion of Keeler's (1997) portfolio required students to correct their tests and highlight areas of weakness or further study. The incorporation of a reflective journal by Keeler (1997) also allowed the teacher to reflect on the cognitive processes of students. This process also served as a metacognitive process for students in which students learn about how they learn best (Keeler, 1997) and develop the habits of mind of a continuous learner and statistician.

Delcham and Sezer (2010) compared a traditional college statistics course where information was conveyed to students and students provide results with limited in-class support from the instructor to a writing centered classroom where the teacher uses writing assignments to evaluate student understanding throughout the course very similar to a portfolio. Delcham and

Sezer (2010) emphasized the role of writing and reading skills and the need for nurturing of development of these skills in the classroom. Delcham and Sezer (2010) posed nine assignments in this writing intensive class that focused on particular content areas. Delcham and Sezer (2010) writing assignments focused around topics such as explanations of graphical representations and appropriate uses of central tendency to the normal probability distribution and its relationship to the central limit theorem and sampling distributions. Though Delcham and Sezer (2010) did not attempt to compare results of a writing centered class with other approaches, the need for writing within statistics was apparent. Attending to and requiring writing in a statistics had positive consequences for Delcham and Sezer (2010) teaching and their respective students.

This section has highlighted the potential impact of portfolios in a range of classrooms especially a constructivist learning environment. Positive use of portfolios in a classroom should focus around student metacognition of statistics through writing. Portfolios allow opportunities for students to clarify understanding and prove conception of big ideas. These works also provide teachers the opportunity to guide instruction and provide for a means of summative assessment for their course.

Multiple Choice Tests

The Statistical Reasoning Assessment (SRA, Garfield & Chance, 2000) instrument is a multiple choice test in which choices were statements of reasoning for a particular solution. The SRA was developed as part of the ChancePlus Project in an attempt to understand student reasoning by the use of multiple-choice tests (Garfield & Chance, 2000). The SRA (Garfield & Chance, 2000) focused on reasoning about data, statistical measures, samples, uncertainty, and association and misconceptions such as sample size, outcome orientation, averages, law of small numbers, representativeness of a sample, and equiprobability bias.

The Comprehensive Assessment of Outcomes in Statistics (CAOS) was a test designed by DelMas et. al (2007) through an NSF grant entitled Assessment Resource Tools for Improving Statistical Thinking (ARTIST). The test was designed over a three-year process of question construction and test validation with teachers of statistics (delMas et. al, 2007). Test questions required students to think through and reason with statistics rather than compute formulas, recall definitions, or use formulas (delMas et. al, 2007). During the construction, two large class assessments were given to students in high school AP statistics courses and introductory statistics courses at the tertiary level and a smaller assessment was given to 30 teachers of statistics (delMas et. al, 2007). Results from individual portions of the assessment showed little improvement in students understanding of visual and descriptive representations of standard deviation, extrapolation during regression, spread within data given boxplots, probability from a two way table, understanding of sampling variability and confidence intervals (delMas et. al, 2007).

Though Delmas et. al's (2007) article went into detail concerning specific test questions, the focus on overall test results were interesting. The reliability coefficient, Cronbach's alpha, from the latest version was .81 showing acceptable internal consistency in questioning (delMas et. al 2007). The large scale testing showed increases of only approximately 9% from 44.9% to 54.0% in all pre-test and post-test scores (delMas et. al, 2007). Though this increase was not very large, it was found to be significant at the 1% alpha level in the study (delMas et. al, 2007). Disaggregation of testing results showed that questions had gains from 23% to 60% while others had decreases in results from 7% to 15% (delMas et. al, 2007). Although Slauson (2008) found improvement in once class focused on group work and not in one lecture based classroom with the use of the CAOS, results from large-scale implementation beg the question to what were these instructors doing differently than one another to promote achievement on the CAOS assessment.

The AP Statistics exam consists of 40 multiple-choice questions in 90 minutes (College Board, 2010). Questions on the test do not require intensive calculations such as standard deviation, but do require students to use formulas appropriately and technology to calculate these results (College Board, 2010). Questions require students to use statistical output to calculate statistical tests or perform statistical analysis using a graphing calculator (College Board, 2010). Even though these types of questions were intertwined in the assessment and different from SRA and CAOS questions, some questions focus on conceptual ideas of central tendency, graphs, variation, and interpreting results correctly. Different than the SRA and CAOS assessment, the AP multiple-choice section did not provide reasoning for specific answers but does provide distractors for student misconceptions. There were also questions on the AP multiple choice exam (College Board, 2010) that require the calculation of conditional, independent, binomial, and other probabilities that makes it drastically different than the CAOS assessment and more similar to the SRA.

Open Ended Responses

The AP statistics exam (College Board, 2010) is also composed of an open-ended section where students must construct and justify their solutions to problems in 90 minutes. The six questions in this section are divided into different parts that probe students understanding into major components of the AP curriculum (College Board, 2010). One of these five questions is an investigative task involving more extended reasoning (College Board, 2010). The AP College Board (2010) stated that statistics is a discipline where clear and complete communication is essential, thus this section of the exam requires students to use these skills to solve non-routine problems. These problems relate major curriculum topics in the course and requires students to validate their statistical reasoning. Though these problems require the connection of ideas in the AP statistics class, the problems are many times subdivided into specific portions to ensure equitable grading and to provide openness (NCTM, 1991) to students during the assessment process (College Board, 2010). The AP exam weighted each portion of its test as 50% multiple choice, 25% open ended response questions 1-5 and 25% for the investigative task.

Even though open-ended questions elicit the most knowledge of students' statistical reasoning and thinking, they are extremely hard to evaluate on large scales. Garfield and Chance (2000) discuss this extensively. Large groups of teachers grade the AP open-ended statistics test in June and July of each calendar year, and graders have explicit instructions on what to count as partial and essentially correct solutions. In order for students to perform well on this portion of the exam, it is imperative that they understand what graders were looking for when asked specific questions. When asked to compare distributions, students are required to describe differences and similarities in shape, outliers, spread, and center. These questions often did not explicitly state the requirements for full credit. Similarly, students are required when asked to perform statistical tests to check for certain conditions, state null and alternative hypothesizes, perform the calculation correctly, and make appropriate conclusions that are not explicitly stated in directions. Based on these grading procedures, a three-part question many times becomes sub-divided into six graded portions in which the students must understand and explain engrained questions to perform well on the test.

Summary

Though decisions about what assessments or evaluation tools used at district, state, and nation levels may be out of the hands of teachers, teachers can often choose assessment tools in their classroom. Thus, teachers should understand that their use of assessment fosters and encourages what students perceive as worthy within the classroom (Healy & Hoyles, 2000; Chance, 2002). Questions that teachers use can bring out reasoning that helps develop student understanding (Garfield, 2003; Smith & Stein2011). An important conclusion from both Healy and Hoyles (2000) and Brenner, Herman, Ho, and Zimmer (1999) is that students should be encouraged to use multiple representations to show understanding and make connections within mathematics. The use of projects, portfolios, multiple-choice tests, and open-ended responses all have benefits and drawbacks in use and ability to develop student reasoning. Forms of summative assessment in this section have been shown to be valid ways of understanding students understanding, but assessments must do more. Summative assessments should be used to improve mathematics and statistics programs. Using assessments in these ways will in turn improve student learning (NCTM, 1995). It is important for teachers to not only understand the purposes of certain assessments but also use assessments to progress student understanding and program development.

Synthesis

Biggs (2003) and Webb (1997) described the importance of alignment between assessment, teaching, learning, and curriculum in program development. Sowder & Schappelle (2002) categorized mathematics education research into four sections of teaching, learning, assessment, and curriculum, recognizing the overlap within research of these separate but equally important ideas. NCTM's Professional Standards for Teaching Mathematics (1991) similarly described the alignment of curriculum, assessment, teaching, and learning as being pivotal for students, instructors, school systems, and industry. NCTM (2014) discussed the use of learning and teaching with curriculum and assessment as key components to improving mathematics education. Documents such as these have outlined the importance and interconnection of learning, teaching, technology, curriculum, and assessment. These interconnections are just as important in statistics education.

Teachers' goals for instruction play a critical role in what should be taught within the statistics classroom (Chance, 2002). Though literacy is important, it can be argued that reasoning and thinking are just as important (Chance, 2002). If readers and users of statistics are to be truly literate, they must understand all of the components of the statistical process that validate a study or could make a study invalid (Chance, 2002). Reviews from the literature have proposed four focuses of introductory statistics instruction: asking statistical questions, the data collection process, analyzing data, and making conclusions. In order for teachers to develop reasoning and thinking, specific attention should be played to each of these areas of the introductory statistics course.

If teachers want students to be able to think and reason with statistics, teachers need to incorporate reasoning into their assessment practices (Chance, 2002; Garfield, 2003). If students are to reason through statistics, teachers should be give students opportunities to develop this reasoning in class. Teachers need opportunities to elicit and use evidence of student reasoning in the guiding of instruction and discussions. Students should also play a key role in the assessment process. When students are a part of the assessment process, they are more likely to develop reasoning and thinking ability, control for misconceptions, and create an open environment for assessment (Garfield, 2003; Chance, 2002; NCTM, 1995).

Teaching practices should promote student growth through clear learning goals and tasks that promote reasoning, sense making, and connection to prior knowledge (NCTM, 2014). Even though not as much attention to what teachers do in the classroom has been observed in statistics education literature, mathematics education literature has played particular importance to this. Teachers supporting productive struggle, posing purposeful questions, and promoting productive discourse are essential in moving students forward in reasoning and thinking. When teachers attend to teaching practices, they are more likely to promote statistical reasoning and thinking.

Building procedural fluency from conceptual understanding and using representations to foster this process have been studied in statistics education literature (Jacob, 2013; Feldman, Konold, Coulter, & Conroy, 2000; Mokros & Russell, 1995; Konold, 2002; Leman & House, 2012). Students' development of statistical reasoning is prominently tied to visualizations and experiences they have in the classroom. Premature exposure to algorithms has been shown to interfere with students' conceptual understanding of important mathematical and statistical content. For these reasons, it is important for teachers not focus as much on traditional algorithms in introductory statistics courses but on the development of reasoning and thinking skills. Statistical reasoning and thinking skills are vital to the understanding of the complete statistical process.

Teachers should use technology to help develop students' understanding of statistical content that is difficult to visualize and understand in small classroom experiments (Cobb, 2013). Importantly, the use of technology should be used to help develop students understanding through tasks that allow them to explore and make sense of problems themselves (Chance et. al, 2007). For teachers to use technology effectively in the classroom, it has been shown that teachers and teacher educators should attend to the intersection of statistical content knowledge, pedagogical knowledge of statistics, and the knowledge of technology in statistics (Wild and Pfannkuch, 1999). Research has highlighted the need for the integration of statistical content knowledge, pedagogical content knowledge, and statistics technological technology to ensure technology is used effectively (Lee & Hollebrands, 2011; Bayés, Meletiou-Mavrotheris, & Paparistodemou, 2014).

Context and variation have been discussed as important elements separating mathematical and statistical reasoning (delMas, 2004). DelMas (2004) suggested that mathematical problem solving is independent of developing questions, collecting data, and making conclusions in statistics (delMas, 2004). For this reason, it is imperative for instruction in the introductory statistics classroom to allow students to deal with real world data and mathematics simultaneously. The collection of data introduces variability at early stages within a statistics course and allows students to understand this from different points in statistics such as the sampling and analysis stage. This data could be extracted from the classroom or through data sets to help promote meaningful contexts that stimulate certain important concepts such as variation (Shaughnessy et. al, 2009). Using real world data also promotes affective catch and hold strategies that have been shown to increase student affection (Mitchell, 1993). Using meaningful data sets will in essence promote student interest and create meaningful contexts for students to investigate and develop statistical reasoning and thinking.

This review of literature has shown four key ingredients to advancing student achievement in statistics. The first goal for any instructor should be to set clear goals for their students that promote statistical literacy, reasoning, and thinking in the cognitive domain and interest in the affective domain (NCTM, 2014). Once these goals are in place, teaching strategies that promote literacy, reasoning, and thinking should be incorporated (NCTM, 2014). These teaching practices should use curriculum and technology that foster the development of the students' cognitive and affective goals. In the midst of these teaching strategies, assessment should be used to elicit student understanding in obtaining these goals and used to improve individual and program achievement.

Statistical Reasoning Learning Environments

The previous sections have highlighted the need for learning statistics with literacy, reasoning, and thinking. Statistical reasoning and thinking are clear goals for instruction that require much more effort to develop and assess. The use of effective teaching practices that incorporate technology, curriculum, and assessment as learning tools have been said to increase students' statistical reasoning and thinking. The description for the integration of teaching, technology, curriculum, and assessment that Garfield and Ben-Zvi (2008, 2009) have summarized foster statistical reasoning and thinking in a description.

Garfield and Ben-Zvi's (2008, 2009) description of a socio-constructivist approach to instruction was called the Statistical Reasoning Learning Environment (SRLE). Instruction with use of the SRLE alls in close alignment with NCTM teaching principles (2014, 2000) that foster student engagement and classroom discourse between students and the teacher, NCTM professional teaching standards to promote discourse (1991), and other educational research on promoting discourse (Shaughnessy et. al, 2009; Middleton & Jansen, 2011; Smith & Stein, 2011). The SRLE also includes components of assessment that include students in the assessment process and use assessment to advance student reasoning and thinking. The SRLE has six principles of instructional design that are aligned with Cobb and McClain (as cited by Garfield and Ben-Zvi, 2009).

1. Focus on developing central statistical ideas rather than on presenting set of tools and procedures.

2. Promote classroom discourse that includes statistical arguments and sustained exchanges that focus on significant statistical ideas.
3. Use assessment to learn what students know and to monitor the development of their statistical learning as well as to evaluate instructional plans and progress.
4. Integrate the use of appropriate technological tools that allow students to test their conjectures, explore and analyze data, and develop their statistical reasoning.
5. Use real and motivating data sets to engage students in making and testing conjectures.
6. Use classroom activities to support the development of students' reasoning.

Though empirical studies of teaching practices in introductory statistics have been lacking (Baglin, 2013; Loveland, 2014; Chance & Garfield, 2002), there have been many pedagogical practices presented in this literature review that describe ways to develop statistical reasoning and thinking. Garfield and Ben-Zvi (2008, 2009) did not separate their key ideas into teaching, learning, curriculum and assessment categories; however, their inclusion of each of these elements in the SRLE implies that the SRLE framework will be extremely applicable and useful for monitoring statistical classrooms holistically based on this literature review. It is hypothesized that effective use of Garfield and Ben-Zvi's (2008, 2009) Statistical Reasoning Learning Environment (SRLE) can promote statistical reasoning based on this synthesis.

There has been minimal empirical evidence to what extent the SRLE in its entirety does in fact improve students' statistical reasoning and thinking (Baglin, 2013; Loveland, 2014). Garfield and Ben-Zvi (2009) also believed that the SRLE approach to statistical instruction could promote student interest in statistics. This question, however, is not be addressed in this study because of students' current lack of experience with what statistics is before entering an introductory course (Reid & Petocy, 2002; Bulmer & Rolka, 2005; Bond, Perkins, & Ramirez, 2012). This lack of true understanding currently in the population at large makes affective pre-tests not very affective in true measurement of affection on a large scale.

Constructivist Theory

Based on the description of the SRLE's six components and its categorization as a socio-constructivist learning environment, this section briefly describes the constructivist theory. Current views of constructivism come largely from ideas of John Dewey's pragmatism (Brewer, 2007). Dewey's ideas were revolutionary in American Education in the 1920s because most classrooms at the time were teacher centered, controlled by the teacher, lecture and text based, and large amounts of oral written recitation (Brewer, 2007). Dewey believed curriculum should be based on students' interests and involve active experiences (Brewer, 2007). Dewey also believed that active curriculum should be integrated, rather than divided into subject-matter segments (Brewer, 2007). Dewey (Brewer, 2007) believed teachers were responsible for achieving the goals of the school, but the specific topics to be studied to meet those goals cannot be determined in advance because the topics should be of the interest to the children (Brewer, 2007).

Dewey believed that learning was active and schooling was unnecessarily long and restrictive (Neill, 2005). He believed that students should be actively involved in real-life tasks and challenges (Neill, 2005). Dewey's educational philosophy helped forward the progressive education movement (Neil, 2005). This philosophy of education began the development of experiential education programs and experiments (Neill, 2005). These views of pragmatism are very similar to today's view of constructivism.

Though Dewey's ideas of learning are very similar to constructivist principles today, constructivism today is said to center on ideas from Lev Vygtosky (1978) and Jean Piaget

(Wadsworth, 1996; Broody & Davidson, 1998). Piagetian constructivists believe that knowledge is constructed through an individual process (Wadsworth, 1996). Social constructivists, such as Vygtosky (1978), believe that knowledge is not located in individuals but rather in communities. How the learner constructs this knowledge is one of the main differences between constructivist and socio-constructivist theories. Both constructivist methodologies encourage hands-on approaches to teaching; however, the socio constructivist view focuses on this development being constructed in cooperative settings of discourse (Brody & Davidson, 1998). Current movements related to constructivism are very socio-constructivist in nature using learning in cooperative groups and meaningful tasks that promote discourse (Brody & Davidson, 1998). Teachers who promote socio-constructivist views depended on group interaction and learning activities and assignments that foster the development of knowledge. Both constructivist and socio-constructivist approaches encourage trial and error, experimentation, and interaction with learners, guided by instructors, which allow students to develop their own understanding from real life context (Brody & Davidson, 1998).

The introduction of constructivist learning theory into the American education system has not been completely embraced since its inception, thus classroom-teaching philosophies are still very diverse (Klein, 2003). This paradigm has caused researchers to coin the term “reform mathematics” as efforts to shift from the behaviorist approaches to instruction to constructivist approaches. Because of the overlap between teachers’ experiences and teacher education, the ability to monitor “reform mathematics” classrooms has been a challenge for researchers.

Measuring Conformity of Instruction with Constructivist Theory

The Constructivist Learning Environment Survey (CLES) was first presented at the American Education Research Association in 1991. The main purpose or use of the first CLES survey was to monitor reform efforts of teachers towards constructivist instruction (Taylor, Fraser, & White, 1994). Taylor et. al (2004) modified the original CLES toward use in a mathematics classroom. Taylor et. al (1994) discussed how the CLES could also be used as a predictor or criterion variable for other studies. Even though these were the practical uses of the CLES in research, Taylor et. al (1994) largely focused on the viability of the CLES scales to understand the mathematics classroom through triangulation of multiple sources. The CLES scales were personal relevance, shared control, critical voice, student negotiation, and uncertainty. Taylor et. al (1994) believed that a constructivist-learning environment should promote the use of these scales.

Personal relevance questions within the survey attempted to monitor how students' everyday lives were being used as a meaningful context to develop knowledge (Taylor et. al, 2004). The shared control scale monitored how students were developing as autonomous learners by monitoring their own learning and developing learning activities or assessments that build their knowledge (Taylor et. al, 2004). The critical voice scale monitored the extent at which students have a voice to control the pedagogical techniques and instruction within the classroom (Taylor et. al, 2004). The student negotiation scale related very similarly to the other three scales (Taylor et. al, 2004); however, this scale was geared more toward monitoring how well the teacher moves away from individual and cooperative group assignments of working and checking problems, to classrooms where students explain reasoning and justify results from learning activities. The uncertainty scale attempted to monitor the myth that mathematics exist independently from human experience as well as the coexistence of multiple answers and pathways to problems (Taylor et. al, 2004).

Support for the scales was provided quantitatively and qualitatively (Taylor et. al, 2004). Means, standard deviations, and Cronbach's alpha were provided for each scale (Taylor et. al, 2004). A correlation matrix was also provided between the scales (Taylor et. al, 2004). Questions with low correlation or negative correlation within each scale were also discussed (Taylor et. al, 2004). The researchers largely used the statistical analysis to subdivide groups for qualitative interviewing (Taylor et. al, 2004). Researcher's semi-structured interview questions focused on survey questions that had large standard deviation and survey questions that had low or negative correlation with similar scale questions within the pilot class (Taylor et. al, 2004). The interviewer had observed the class many times and attempted to create an atmosphere where students would feel free to share their concerns (Taylor et. al, 2004). The class being investigated was using a 9-week study of how to find the surface area of an egg and one of the researchers involved in the study was that actual teacher with 13 years of teaching experience (Taylor et. al, 2004).

The original CLES consisted of samples of over 500 students; however, the sample for the mathematical revision of the survey consisted of only 34 students in eighth or ninth grade from Australia (Taylor et. al, 2004). The 34 students consisted of only one class in which one of the authors of the study taught (Taylor et. al, 2004). Findings from the study suggested that negatively worded questions on the scale might create conceptual difficulties for students to respond accurately to on a scale that produces negative results (Taylor et. al, 2004). Questions that were geared towards absence of constructivism in the mathematics classroom did not directly relate to the appearance of these criteria possibly because of cultural contexts of mathematics and past experiences in the mathematics classroom (Taylor et. al, 2004). Questioning of student responses after the survey also revealed that classroom pedagogies, constructivism in particular, may not be mutually exclusive from one another and may actually work side by side especially in the perceptions of students involved (Taylor et. al, 2004). Researchers completing this project cautioned others considering use of the survey to monitor the development of the constructivist environment because of students' past experiences and diverse conceptions of their future (Taylor et. al, 2004).

Similar to the CLES, Henry (2003) attempted to measure the learning environment of Florida middle school classrooms through the development of a teacher survey. Henry (2003) created separate scales for both traditional approaches and constructivist. Scales were grouped to encompass classroom management approaches, teaching activities, and assessments that were sub categorized as traditional or constructivist (Henry, 2003). Henry's (2003) sub scales encompassed teacher control, teacher rigidity, and teacher presentation to measure traditional approaches of classroom activities. Student engagement and student control variables were used to quantify the amount of constructivism in classroom activities (Henry, 2003). The amount of constructivism towards assessment was monitored through subscales of teacher or student control and types of assessment used in the classroom (Henry, 2003). Classroom management subscales were teacher control, centrality of instruction, and student interaction (Henry, 2003). Through piloting the survey in a smaller setting, Henry (2003) was able to reduce the questions from 71 to 57 and increase internal reliability by removing questions that were inconsistent with the scale, producing a Cronbach's reliability coefficient well above .6 on all scales. The final version had decreases in reliability coefficients, which would be expected without any bootstrapping or hold out procedures from the pilot study; however, the final reliability for all scales was still over .6 with the exception of teacher control on both the assessment and teaching methods scales that were not included in the original pilot (Henry, 2003). Henry's (2003) scales showed strong positive correlation between constructivist approaches scales and traditional scales. Interestingly, however, Henry (2003) did not report cross correlations between the assessment, management, and activities scales.

Hassad (2011) chose to survey teachers to understand students' learning environment, but chose to focus on specifically introductory statistics classrooms. Hassad (2011) claimed to have developed a first of its kind survey, Teaching of Introductory Statistics Scale (TISS), to understand behaviorist and constructivist approaches of tertiary level statistics teachers. Hassad's (2011) TISS survey was originally constructed of 14 different type items that were reiterated up to 30 times in the survey. These items were shared with different statistical instructors from different fields (Hassad, 2011). Two different forms of qualitative analysis via in person interviews or e-mail correspondence followed this (Hassad, 2011). After the qualitative analysis, questions were revised, added to, and removed based on consensus of interviews and administered to 30 statistical instructors via e-mail (Hassad, 2011). Following this pilot, items were assessed for variability and validity with potential discriminant value for each item (Hassad, 2011). Items believed to need revision were discussed with teachers of introductory statistics courses then finalized to a set of 10 items for large sampling (Hassad, 2011). The final versions sampling consisted of 227 statistical teachers involved with different professional organizations, department heads dissemination in public four-year institutions, and published research in statistics education (Hassad, 2011).

Hassad (2011) claimed orthogonality between the two scales, however it has been shown that behaviorist and constructivist approaches may be mixed in many statistics and mathematics classrooms, especially those of teachers who were experiencing pedagogical transition (Zieffler, Park, Garfield, delMas, & Bjornsdottir, 2012; Fraser, 1998). Interestingly, Hassad (2011) found through international sampling that behaviorist approaches were more common in the United States and with professors having mathematics and engineering degrees. Constructivist approaches were more common with instructors who were affiliated with professional organizations (Hassad, 2011). The final scale factors used in the study were deemed reliable, but difficulty still existed in the non-orthogonal nature of behaviorist and constructivist theories for disaggregation purposes (Hassad, 2011). Note that Hassad's (2011) survey fails to segregate the relationship between instruction, curriculum, and assessment in behaviorist and constructivist pedagogical techniques, which may contribute to lower reliability between the two scales.

Zieffler et. al (2012) similarly chose to survey teachers to understand learning environments of introductory statistics classes in the post-secondary and AP Statistics setting. The Statistics Teaching Inventory (STI) (Zieffler et. al, 2012), was a fifty item survey that underwent different phases before its final revision and large-scale dissemination. Zieffler et. al (2012) briefly discussed similar studies and gave references to similar elementary, secondary, and tertiary schools studies. The aim of this survey was to determine how teachers of statistics within post-secondary education conform to the traditional approach to teaching and reform based instruction (Zieffler et. al, 2012). Zieffler et. al (2012) also associated their survey to the SRLE. Definitions were given for these two methods of teaching with emphasis on the GAISE report for reform-based instruction. Of the 101 participants in the pilot study (response rate 25%), over 75% reported familiarity with the GAISE report and 75% of those reported teaching practices that were either mostly or completely aligned to the GAISE report. Graphs within the report implied a significant portion of those surveyed straddling a statistical instruction fence of behaviorist and constructivist approaches (Zieffler et. al, 2012). Zieffler et al. (2012) suggested one possible reason: administration of the survey might itself cause change within the teacher.

The STI's initial and final survey questions were grouped into mainly four categories not including demographic questions: teaching practice, assessment practice, assessment beliefs, and teaching beliefs (Zieffler et. al, 2012). The original survey pilots contained 102 items that were administered to the Consortium for the Advancement of Journal of Statistics Education Undergraduate Statistics Education (CAUSE) and the Research Advisory Board of CAUSE. NSF advisors to the project also provided written feedback of the pilot version of the test. After the pilot, questions were refined and reduced (Zieffler et. al, 2012). After revision of the initial survey, questions were read aloud to faculty teaching statistics in Florida and analyzed qualitatively to ensure the questions were measuring what was intended (Zieffler et. al, 2012). After interviews and analysis were completed, researchers reduced the survey to 50 questions and completed a small pilot in Minneapolis/St. Paul area to create the version of the STI reported by Zieffler et. al (2012).

Zieffler et. al (2012) mentioned that surveys were not always adequate measures, so an interview process was used to help with this discrepancy. Often what teachers reported was different from what was actually observed in classrooms (Zieffler, 2012). Zieffler et. al (2012) discussed that there was a low volunteer resampling for interviews. Even with a survey followed by an interview process, what happens inside the classroom cannot be appropriately judged without actual observations of classrooms.

Based on misperception of teacher's use of constructivist principles, a student survey has often been used for cross validation of learning environments (Fraser, 1998). A student survey also corresponds with ideas from constructivist theory, that knowledge was not just disseminated from the teacher to the students, but rather created by the students and facilitated by the teacher. Based on this view of the construction of knowledge, the best place to monitor the learning environment should be through the learners who create and live within the environment both the teacher and students. However, a relevant student survey to measure a statistical classroom's learning environment was not found in the literature.

Research has shown that the degree to which a classroom is a constructivist-learning environment can be measured (Taylor et. al, 1994; Hassad, 2011; Zieffler, et. al, 2012); however, difficulties do exist with teacher perception and reality. Taylor et. al's (1994) and Henry's (2003) research on the monitoring of constructivism in mathematics classrooms pointed out the need for clarity in measurement. Hassad (2011) and Zieffler et. al (2012) both focused attention to measurement of constructivism in statistics classrooms. Taylor et. al (1994) and Henry (2003) used much more broad subscales to monitor this development while Hasad (2011) and Zieffler et. al (2012) used much more content specific terminology.

Research Questions

The previous sections have highlighted the role that teaching, learning, curriculum, technology, and assessment have on student achievement. These sections have also highlighted the fact that it is not just what you teach, but how you teach. Current empirical studies have lacked the combination of teaching, learning, curriculum, and assessment in monitoring students' learning outcomes however. Additionally, empirical studies measuring reasoning and thinking are lacking related to teaching, learning, curriculum, and assessment (Baglin, 2013; Loveland, 2014). There have also not been studies monitoring the development of students' statistical reasoning and thinking composed of multiple teachers' use of active learning and lecture based classes. In addition, current statistics instruction teacher surveys (Zieffler et. al, 2012; Hassad, 2011) have not been linked to reasoning and thinking assessments such as the CAOS and SRA.

This study attempts to fill the void in the lack of empirical evidence to monitor the students' increase in statistical reasoning and thinking with instructional practices from the mathematics and statistics education literature. In addition, this research also attempts to fill the gap of generality for best teaching practices by monitoring multiple teachers' use of instructional practices rather than just one. Based on these gaps in the literature, the following research questions were developed:

1) To what extent do students' statistical reasoning and thinking ability improve in classrooms that show high levels of conformity to a Statistical Reasoning Learning Environment?
2) To what extent do students' statistical reasoning and thinking improve in classrooms that do not conform to a Statistical Reasoning Learning Environment?
3) To what extent do students' ability to reason and think about statistics differ between classes that do and do not conform to a Statistical Reasoning Learning Environment?

Chapter 3: Methodology:

The intent of this study was to show the effectiveness of the six components of the Statistical Reasoning Learning Environment (SRLE) (Garfield and Ben-Zvi, 2008, 2009) in developing students' statistical reasoning. This chapter begins with a description of the postpositivist theoretical framework underlying the study. The design section then describes the need and use for quasi-experimentation. The chapter then details instrumentation used to measure conformity to SRLE principles and gains in statistical reasoning. Next, this chapter describes the population and methods used for recruitment. Following this, the chapter details procedures for matching teachers on two levels of teacher demographics and one level of student demographics. Finally, the section presents methods of data analysis to answer the research questions with their required assumptions for use.

Theoretical Framework

Simon (2009) advocated that researchers must be proponents of different theories of knowledge other than their own worldviews or philosophies. Using theories of knowledge as tools and lenses emphasizes the fact that certain theories may be more effective at answering certain questions than others (Simon, 2009; Bernard & Ryan, 2010; Creswell, 2013). Creswell (2013) stated that to build a new theory, one must first advance a theory, collect data to test it, then reflect on whether the theory was confirmed or disconfirmed by the results in the study. The review of literature in this study identified aspects of teaching, learning, technology, curriculum, and assessment that may have influence on students' ability to reason and think about statistics. Based on these pedagogical techniques, the Statistical Reasoning Learning Environment (SRLE) was chosen as an effective framework to represent factors that influence statistical literacy, reasoning, and thinking.

A postpositivist epistemology forms the basis for this study, using quantitative tools to answer the research question (Creswell, 2013; Bernard & Ryan, 2010). A postpositivist epistemology was chosen because this philosophy has elements of logic, cause and effect, and determinism based on a priori theories (Creswell, 2013), in this case examining how well the use of an SRLE, an a priori theory, influences students' statistical reasoning.

Causation is limited by the use of a postpositivist epistemology. Specifically, using a single measure to understand students' development of statistical reasoning, as was the case in this study, may not adequately capture all the effects of the SRLE. This study's use of the Comprehensive Assessment of Outcomes (CAOS) (delMas et. al, 2007) is limited in its ability to measure student reasoning by only containing multiple choice questions. In fact, Garfield (2003) a co-author for both research tools being used to measure statistical reasoning of students and the teachers' conformity levels to the SRLE in this study, the CAOS and STI respectively, discussed the difficulty of developing a tool to measure statistical reasoning.

A postpositivist epistemology also assumes that there is an objective truth that is being measured, students' statistical reasoning ability. Though statistical reasoning has been defined in the previous chapter, statistical reasoning can potentially vary and shift relative to the researcher, reader, or study participants. The continuity of similar authorship and institutions of research tools in this study reduced this limitation. Dr. Joan Garfield coauthored the frameworks for the SRLE, STI, and CAOS, and delMas co-authored the STI and CAOS. At an institutional level, researchers from the University of Minnesota were involved in developing the SRLE, STI, and CAOS. The use of instruments developed by the same authors to monitor statistical reasoning by students and the conformity level of teachers to SRLE principles can potentially reduce the effects of differing definitions of statistical reasoning.

Design

Creswell (2013) as well as Bernard and Ryan (2010) stated that the goals of research should influence the approaches a researcher should take. The goal of this study was to monitor the impact of principles from the SRLE on the development of students' statistical reasoning. For this reason, a quasi-experimental design using comparative groups was used. This study was considered a quasi-experimental design because it had no control over the allocation of treatment to participants and did not use a randomized sampling procedure (Reichardt and Mark, 2004).

In order to adjust for likely dependent relationships that may occur in a quasi-experimental design, matching was used to reduce confounding and bias within the study that potentially occurs through lack of random sampling and assignment (Reichardt and Mark, 2004; Stuart & Rubin, 2008). Teachers were matched on their teaching experience, prior course work in statistics, professional development, and their school's percent of students receiving free and reduced lunch and differences of conformity to SRLE principles. Teachers who were in transition between beliefs and practices or had dissimilarity between beliefs and practices were omitted from the analysis in order to help ensure SRLE and GAISE principles were enacted. Likewise, teachers who did not match on demographics while having contrasting conformity levels compared to other teachers were discarded from the analysis. The matching procedure potentially reduced bias resulting from the lack of randomization by matching teachers and schools who were very similar (Reichardt and Mark, 2004; Stuart & Rubin, 2008).

Instrumentation

This study used two different measurement tools. The first instrument measured teachers' self-reported agreement with beliefs and practices related to the SRLE through an online survey. This survey was also used to identify teacher characteristics for matching purposes. The second instrument assessed students' reasoning and thinking skills through a multiple-choice test that was administered by paper. The second instrument also collected the last completed mathematics course by the student. This information was used to understand the school and student demographics more specifically within Advanced Placement (AP) statistics classrooms.

Statistics Teaching Inventory

In order to assess teachers' level of agreement in belief and practice with the SRLE, the researcher asked teachers to complete a modified version of the Statistics Teaching Inventory (STI) survey (Zieffler, et. al, 2012). This version of the survey was modified by the original authors and was expected to be an even better instrument than the initial version of the STI proposed by Zieffler, et. al (2012). The following section highlights Zieffler, et. al's (2012) development of the published version of the STI. Garfield provided the newest unpublished version of the STI through e-mail for use in this study. The researcher, for purposes of this study, included only questions related to face-to-face instruction and additional questions particular to this study were added. The modified version of the STI used in this study was administered through the online system Qualtrics (2013) and is located in Appendix A. The following four sections describing validity and reliability, the measurement of conformity, demographics, and coding and analyzing the responses relate to this modified STI instrument.

Validity and Reliability of the Statistics Teaching Inventory. The initial version of the STI consisted of 102 items drawn up to align with the GAISE report by a project team consisting of members of the statistics education community (Zieffler, et. al, 2012). This education community consisted of the Consortium for the Advancement of Undergraduate Statistics Education (CAUSE) and the Research Advisory Board of CAUSE (Zieffler, et. al, 2012). The GAISE was correlated with six general principles as defined by the SRLE (Garfield & Ben-Zvi, 2008, 2009). Teachers attending this conference gave written feedback for each item and results were analyzed for consistency, redundancy, and missing topics (Zieffler, et. al, 2012). Using this information, the STI was revised and given as think-a-loud interviews to a group of statistics instructors from post-secondary institutions throughout the United States and two statistics education researchers from the University of Minnesota (Zieffler, et. al, 2012). Based on the interviews, researchers revised the STI to 50 questions consisting of six parts: (1) Teaching Practice; (2) Assessment Practice; (3) Course Characteristics; (4) Teaching Beliefs; (5) Assessment Beliefs; and (6) Additional Information (Zieffler, et. al, 2012).

These 50 items were pilot tested by 101 out of about 400 registrants attending the 2009 United States Conference on Teaching Statistics conference (Zieffler, et. al, 2012). Researchers conducted subsequent qualitative interviews with a group of sixteen teachers for validation purposes (Zieffler, et. al, 2012). Approximately 79% of those participating in the pilot survey had instructional practices that matched their beliefs, and 86% had assessment practices that matched their beliefs (Zieffler, et. al, 2012). Two graduate students rated the sixteen participants on a scale of 1-10 for their conformity to reform methodology as described by the GAISE report (Franklin et. al, 2007) and SRLE principles (Garfield & Ben-Zvi, 2008, 2009; Zieffler, et. al, 2012). After removing 4 participants who showed inconsistency in interviews and STI survey results, raters' scores and STI surveys resulted in a Pearson's correlational (r) value of .77 (Zieffler, et. al, 2012).

Based on the removal of these four outliers in the development of the STI, improvements for the STI were suggested by Zieffler, et. al (2012). Questions 16-25 responses were suggested to be changed from dichotomous values of agree and disagree to interval scales (Zieffler, et. al, 2012). Questions related to technology were revised in order to discriminate between teachers using calculators and presentation software from those using tools and technology to develop statistical reasoning and thinking (Zieffler, et. al, 2012).

Though the STI was assumed a valid and reliable measurement tool for SRLE principles based on past research, methods for analysis are unpublished. The first sections begin with categorization of question types in the STI in practice, beliefs, and demographics in the version of the STI included in this study (Appendix C). The concluding section details the procedure used by the researcher to analyze questions in the STI (Appendix C) to measure conformity to principles of the SRLE. The following section will describe the use of the STI to determine conformity to the SRLE in practice and beliefs. These two sections will be followed by included demographics in the STI and finalize with how these dichotomous values were coded and analyzed for discrimination of conformity to SRLE.

Measuring Conformity to a Statistical Reasoning Learning Environment in Practice and Beliefs. Parts one through four of the STI required teachers to answer questions related to their classroom practice. These questions required teachers to reflect on their practice and answer questions as if they were a student in the class. Questions in part one of the STI emphasized pedagogical practice. Part two of the STI focused on curricular emphasis. The third part of the STI measured the use of technology in the classroom. The fourth part of the STI measured assessment practices.

The fifth portion of the survey required teachers to answer questions related to their belief in effectiveness of SRLE principles and not necessarily their practice. This part of the survey measured a teacher's beliefs about teaching, learning, and assessment in statistics in one unseparated section. The beliefs section was analyzed as a whole because of this grouping of questions.

Demographics Obtained by Statistics Teaching Inventory. Part six and seven of the survey were added by the researcher to gather information about characteristics of the teacher completing the survey for matching purposes. Characteristics of teachers that were included in the STI are years of teaching experience in introductory statistics, graduate course work in statistics, teaching constraints, experience in applied statistics, and number of professional development courses. The matching procedure was used to reduce confounding that occurs with use of quasi-experimentation. Finally, a question about their interest in allowing students to participate in the administration of the student reasoning assessment later in the course was included.

Coding and Analyzing the STI. Table 4 maps coding of the STI into categories of pedagogy, curriculum, technology, assessment, and beliefs. The researcher eliminated some questions in the analysis of the survey because of inability to identify clearly high and low conformity levels to SRLE principles in question wording. Table 4 also summarizes questions that favored conformity to SRLE principles and questions that did not. Questions that did favor SRLE principles were reverse coded to provide consistency in measurement of SRLE principles.

Table 4

Statistics Teaching Inventory Question Direct and Reverse Coding

	Practice				Beliefs
	Part I	Part II	Part III	Part IV	Part V
Questions	Pedagogy	Curriculum	Technology	Assessment	Mixed
Directly transformed	2, 5b, 5c, 5d, 5f	10	3, 4, 5	3, 4, 5, 6	4, 5, 6, 7, 10, 11, 12, 15
Reverse Coded and transformed	1, 3, 5a		1	1, 2	3, 8, 9
Disregarded	4	1,2 3, 4, 5, 6, 7, 8, 9	2		1, 2, 13, 14, 16, 17

Questions in the survey were largely Likert-type scale responses and percentage questions. Based on Zieffler et. al (2012)'s suggestions for improvement, responses on the Liker-type scale within parts two through five of the STI were transformed to decimals between 0 and 1 for comparison between questions that have percentage responses. Questions on the STI incorporate 3, 4, and 5-point scales. This transformation from ordinal data to proportions was completed based on suggestions made for improvement from Zieffler et. al (2012) because of the need for consistent comparison in the instrument. Decimal numbers closer to one imply close conformity to SRLE principles and decimal numbers close to zero represent less conformity to SRLE principles.

Likert scale results in each part of the survey were summed and divided by the total questions in the section. This was completed for parts one through four of the STI survey in

Appendix C to produce conformity levels for each portion of the survey. The averages for parts one through four were then summed and divided by four to produce an overall scale of conformity of practices to the SRLE. Likert scale responses from part five were summed and divided by the total questions to produce a raw score of conformity of beliefs in components of the SRLE. The averages for each participant were then placed as ordered pairs to ensure consistency between practice and beliefs. Teachers who showed inconsistency between practice and beliefs were removed from the study.

Comprehensive Assessment of Student Outcomes in Statistics

The Comprehensive Assessment of Outcomes in Statistics (CAOS) was used to assess students' reasoning and thinking skills. The CAOS is a test designed by delMas et. al (2007) through a National Science Foundation grant entitled Assessment Resource Tools for Improving Statistical Thinking (ARTIST). Questions on the CAOS require students to think through and reason with statistics rather than compute formulas, recall definitions, or use formulas (delMas et. al, 2007). The instrument is included in Appendix B.

Validity and Reliability of the Comprehensive Assessment of Student Outcomes in Statistics. The test was designed over a three-year period of creation and revision (delMas et. al, 2007). The first round consisted of dissemination and discussion between the ARTIST team of researchers and graduate students (delMas et. al, 2007). The second round of revisions were made after giving the test to Advanced Placement (AP) statistics teachers and teachers of introductory statistics at the tertiary level who used the test as a pre and post assessment tool in their classrooms (delMas et. al, 2007). The third round of feedback came from 30 AP statistics exam readers in June 2005 (delMas et. al, 2007).

The final version consists of 40 questions that assess a student's ability to reason and think statistically (delMas et. al, 2007). Students are required to read responses in order to determine which response produces the best reasoning. Students must also read studies using statistical thinking to determine errors or provide improvements for a statistical study. Content objectives in the CAOS focus on data collection and design, descriptive statistics, graphical representations, the normal distribution, bivariate data, probability, sampling variability, confidence intervals, and significance tests (delMas et. al, 2007).

To validate the instrument, 1,470 students completed the final version of the CAOS, which included students enrolled in introductory statistics courses at two-year colleges, four-year colleges, and universities (delMas et. al, 2007). Seven hundred and sixty three of these students completed the instrument as a pretest and posttest. Cronbach's alpha reliability coefficient from this large-scale dissemination was .82 showing acceptable internal consistency for college-level introductory statistics courses (delMas et. al, 2007). The large-scale testing showed increases of approximately 9% from 44.9% to 54% in all pre-test and post-test scores (delMas et. al, 2007). Slauson (2008) reported pre-CAO scores of approximately 43% and post assessment scores of 47% in a combination of students in two classrooms focusing on both active learning and lecture-based principles. Tintle et. al, (2012) reported increases from 48.4% to 57.2% in a traditional lecture-based class and increase from 44.7% to 55.7% for a new curriculum centered on reform. These results highlighted the small effect that instruction has had on increasing statistical reasoning and thinking in many statistical classrooms throughout the United States. These reports also highlighted that baseline scores on the CAOS used as a pretest have ranged from 43% to 48.4% (delMas et. al, 2007).

Using the Comprehensive Assessment of Outcomes. This study used the CAOS as a pre-test and post-test to assess students' statistical reasoning ability. The teachers of the course administered the pre-test after approximately a quarter of the course was completed. Teachers of the course administered the post-test after the course was completed. Disaggregated student data with broad statistical categorization of percent of correct responses was provided to instructors to encourage focus within the course. This same data was provided to teachers at the end of course for program development. Solutions to the CAOS were not provided to teachers in the study before or after the study.

Procedure

The overall procedure for this study required the collection of both teacher and student data from a project supporting the development and success of AP statistics courses. The researcher collected teacher data from the STI at the beginning of the school year and used the data to classify teachers' beliefs and practices relative to SRLE principles. The researcher also used demographic information included on the STI to match pairs of teachers who had high conformity to SRLE principles with teachers who had low conformity to SRLE principles. Teachers were matched if they had the same amount of teaching experience, similar coursework in statistics, similar professional development, and similar school socio-economic status. Teachers who were matched then administered the CAOS as a pre-test and post-test in their course to measure the amount of improvement in statistical reasoning. Students completed pre-assessments after approximately a quarter of the course and post-assessments at the end of the course.

Population

Participants were drawn from the A+ College Ready project being conducted in the southeast region of the United States; see http://www.apluscollegeready.org. This project focused on the development of Advanced Placement (AP) science, English, and mathematics courses. The A+ project aimed to increase the numbers of students taking AP courses and making qualifying exam scores of 3, 4, and 5 on the end of course examinations. For this reason, more teachers from A+ were new to teaching AP courses. To ensure teachers were well prepared for teaching AP courses, the project provided professional development through AP conferences. AP conference presenters presented sessions on AP statistics content as well as activities and tools used to teach the content. Conferences lasted anywhere from four to five days. The A+ project also supported this goal by providing similar professional development for teachers during the school year by local presenters. In addition to providing professional development to teachers involved in the program, monetary benefits existed for teachers and students involved in the program. This may have produced extrinsic motivation for teachers and students. Low-income students also had the benefit of no out-of-pocket expense for the end of course AP examination due to state funding, AP waivers, and cost reimbursement through the program. Similarly, all students received a partial reduction in the normal AP exam cost.

For the purposes of this study, only AP statistics teachers who were a part of the A+ project were included, along with their students. The A+ project promoted principles from the Advanced Placement program (College Board, 2010), which were predicted to increase the likelihood of finding classrooms that conform to the SRLE. These principles are supported through professional development for teachers, and expert teachers who led Saturday sessions for students.

The researcher sent e-mails to AP statistics teachers involved in the A+ project inviting them to participate in the study. Thus, only AP statistics teachers involved in the A+ project in past and present were available to participate. AP statistics teachers could potentially be involved with the A+ project for one to six years at the start of this project. Teachers' involvement in professional development opportunities varied through experience in summer workshops and attendance in various numbers of professional development activities sponsored by the program.

Teacher Participation and Matching

Teachers who responded to the request for participation were asked to complete the STI survey, which was used to identify teachers' beliefs and practices related to the SRLE. The STI's intended use was to measure conformity of beliefs and instructional practice to SRLE principles and the recommendations from the Guidelines for Assessment and Instruction in Statistics Education (GAISE). The STI results were used to divide possible participants into two groups of more conformity and less conformity to the SRLE principles. Teachers who had inconsistent scores on the beliefs and practices subscales on the survey or who were unwilling to participate in the student assessment portion of the study were removed from possible matching and further analysis.

Teachers with more conformity to the SRLE in belief and practice were matched with teachers who showed less conformity to the SRLE based first on teaching experience in statistics. If multiple teachers had the same amount of teaching experience and different conformity levels, they were then compared more closely by matching their amount of completed coursework and professional development in statistics education. If two or more teachers were still available as potential matches, they were then compared by their schools' socio-economic status to make a final selection. Figure 6 summarizes the order for the matching process of differing conformity levels of teachers.

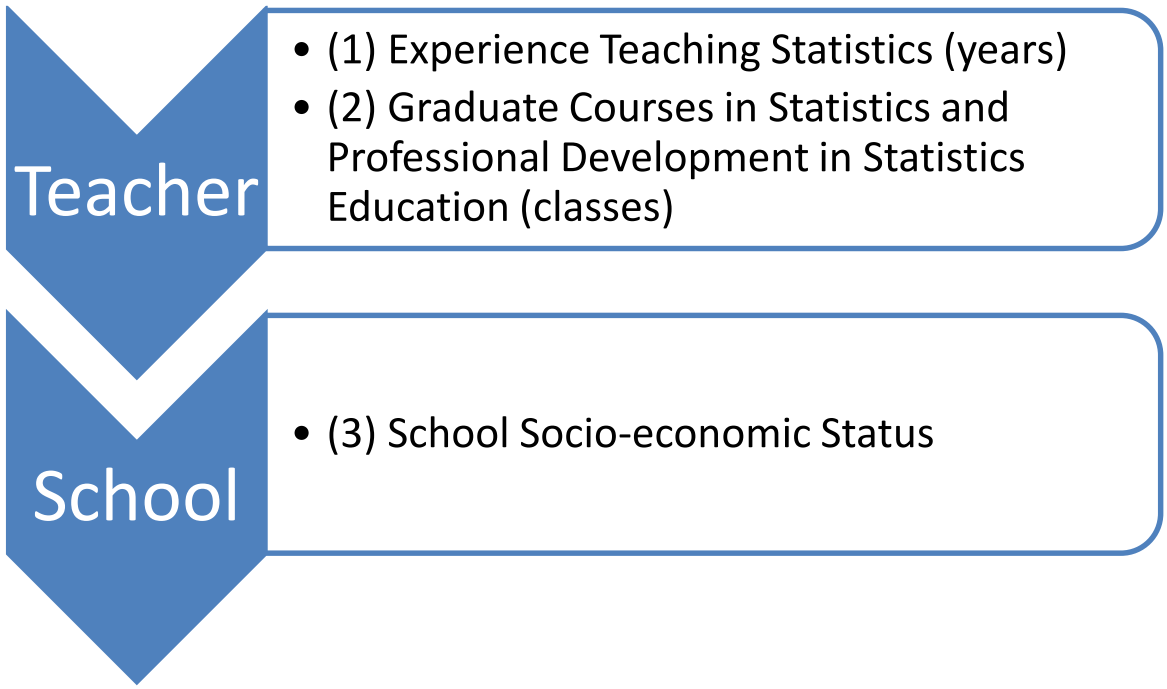

Figure 6. Order of matching procedure for possible participants within study.

First, teachers from the high conforming group and low conforming group were categorized by their experience teaching introductory statistics from 0 years, 1 year, 2 years, and 3 or more years. The teachers in the high and low conformity levels were compared by experience level to establish potential matches. Next, the teachers were compared by the sum of their courses completed in applied and theoretical statistics to refine the set of potential matches further. If more than one potential match existed based on years of experience and completed coursework in statistics, teachers' professional development in statistics education as measured by the sum of hours in workshops, short courses, AP conferences, and other professional development opportunities in the last two years was used for matching.

Finally, the school demographic information of the number of students receiving free and reduced lunch was if necessary to establish final matches. This matching procedure was used to help ensure similar teaching constraints. School demographics was used as the least restrictive in the matching process because student pre-tests were considered as covariates in the Analysis of Covariance (ANCOVA) procedures.

The goal for this matching was to obtain at least five pairs of teachers who had similar teaching experience, courses taken in statistics, professional development, and school demographics but different levels of conformity to the SRLE. Five sets of teachers were used in order to ensure a quasi-random selection of participants and ensure sample size for student participation was large enough to support the statistical analysis.

Student Participation

The researcher contacted the superintendents of the school districts of the five matched pairs of teachers to obtain consent for participation in the study. After superintendents gave permission for the students to participate, teachers sent opt-out letters to students' parents or guardians; see Appendix C. After allowing at least one week for return of opt-out letters, students and parents who chose to participate in the study completed the pre-test CAOS.

Students participating in the study were given the CAOS pre-assessment in late October. The researcher contacted all teachers participating in the study to ensure they administered the pre-assessment after the topics included on the CAOS: graphical representation, boxplots, data collection and design, descriptive statistics, and bivariate data. The beginning of the course was used to create appropriate benchmark scores for individual students that would later be used as covariates in the analysis. The researcher asked teachers to encourage students to perform well on the pre-assessment by providing disaggregated data from the pre-assessment that could be used to make instructional decisions in their course. This was included in the design process to help produce reliable pre-assessment results and provide teachers with information that could potentially increase their effectiveness.

The CAOS post-test was administered approximately one week after the completion of their final unit of instruction before transition to instruction focusing on preparing students for the AP examination, thus maximizing the effects of the SRLE on student reasoning and mitigating effects of test preparation on the CAOS. Teachers were given a summary of the students' scores shortly after returning the student assessments in order to assist teachers in determining certain areas for improvement before the AP test at the beginning of May.

Data Analysis

To answer the research questions, a one-way Analysis of Covariance (ANCOVA) design was used to determine if differences between instructional practice and beliefs groupings of classrooms post-tests differed while controlling for students' pre-test scores as covariates. Cook and Campbell (1979) stated that this design for quasi-experimentation is a more powerful procedure for detecting differences than traditional pre and post-test difference designs. Given the variation between and among groupings in the quasi-experiment, type III sum of squares was used in SPSS to control for differences in sample sizes of students in both groupings by teacher and by high or low conformity.

In order to complete an ANCOVA design, Mertler and Vannatta (2010) list several assumptions that must be verified. The first assumption is of random sampling and reliable covariates. Observations within this study were quasi-experimental, thus lacked true randomization; however, the matching procedure attempted to mitigate this difficulty present in most educational research. In addition, a previously developed and validated assessment instrument was used as a covariate. The second assumption listed by Mertler and Vannatta (2010), stated that in order to perform an ANCOVA, the dependent variable should be approximately normal. Mertler and Vannatta (2010) also stated that the ANCOVA test is not largely affected by this assumption when group sizes are fairly large and equivalent. Though testing procedures for normality are highly sensitive, the procedure was completed as an additional means of analysis. Based on the sensitivity to normality testing, further exploration was given as needed to justify the assumption of normality.

Mertler and Vannatta (2010) also recommended several tests for homogeneity of variances between groups on the post-test. Homogeneity of variances in this study ensures that the variation between each groups' post-test scores were approximately equal. A Levene's test was used in this study to ensure the assumption between groups was not violated. The covariate of pre-test scores given at similar times in the course was expected to be linearly related to the dependent variable of post-test scores making it a reliable linear predictor. This assumption was still verified through scatter plots and calculations of correlation coefficients before each testing procedure.

A critical assumption, as described by Mertler and Vannatta (2010), is the assumption of homogeneous regression slopes during the analysis. Inequality in regression slopes would imply that the relationship of students' pre-test scores and post-test scores were different in different classrooms. Homogeneity of slopes is simply stating that students in one teacher's classrooms has a similar increase in scoring rates from the pre-test to post-test within each other teachers' class. This assumption was tested by conducting an F test for interactions. If no interactions existed, a full factorial model was specified and used for analysis.

The first ANCOVA procedure determined if significant differences were found between student post-test results among teachers that were categorized as high conformity to SRLE principles while controlling for students' pre-test scores. The second ANCOVA procedure determined if significant differences existed between student post-test scores among teachers who had less conformity to SRLE principles while controlling for students' pre-test scores. The third ANCOVA procedure determined if there was a significant difference between post-test scores of teachers with high conformity and low conformity to the SRLE while controlling for pre-test scores.

Summary

This chapter began with a discussion of the theoretical framework used for this study, a postpositivist epistemology and the methodology used to investigate the extent to which students develop statistical reasoning based on more or less conformity of their teachers with SRLE principles. A matching procedure based on teacher and school demographics was used to create a quasi-experimental design. The STI was used to identify teacher beliefs and practices related to instructional practice and the CAOS was used to monitor the development of students' statistical reasoning. Data was analyzed using a series of ANCOVA tests.

Chapter 4: Results

The first section in this chapter details the selection of participants in this study based on the Statistics Teaching Inventory (STI) (Zieffler, et. al, 2012) and the previously described matching procedures using demographic information. The second section of this chapter analyzes post-assessment scores on the Comprehensive Assessment of Outcomes (CAOS) (delMas et. al, 2007) for teachers at low levels and high levels of conformity to Statistical Reasoning Learning Environment principles (SRLE) (Garfield and Ben-Zvi, 2009) after adjusting for pre-assessment scores on the CAOS. The final section in this chapter analyzes post-assessment CAOS scores after adjustment for pre-assessment CAOS scores between groups of teachers at low and high levels and between matched teachers of high and low levels of conformity to SRLE principles.

Responses to the Statistics Teaching Inventory

Twenty teachers of statistics provided responses to the STI from an e-mail list of fifty-two teachers during a two-week collection period (38.4% response rate). One of the respondents did not complete the survey and four were not interested in having their students participate in the second phase of the study, the student assessment. Based on this information, five participants were removed from any further analysis. Responses from the STI survey were coded 0 to 1 with higher values representing higher conformity to SRLE principles as prescribed in Chapter 3. Figure 7 summarizes the average conformity levels for each possible participant in the STI categories of pedagogy, curriculum, technology, and assessment in histograms. These four parts were averaged to create a self-assessment average that anticipated how well teachers believed their classes actually conformed to SRLE principles (see Figure 8). The eleven questions from the beliefs portion of the STI were averaged to create a belief scale in the effectiveness of SRLE principles (see Figure 9).

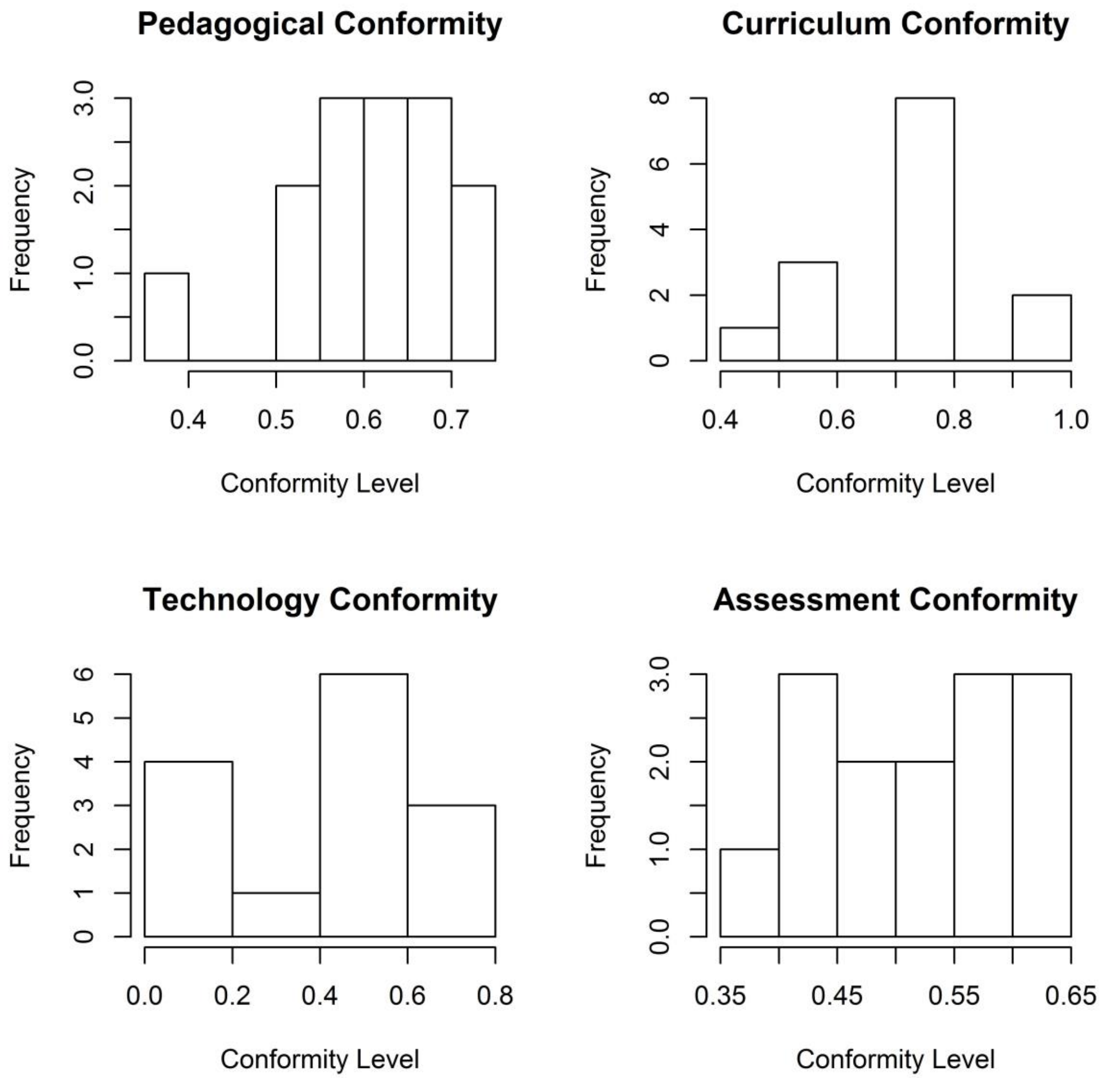

Figure 7. Histograms of self-reported teaching practices for possible participants by subscale.

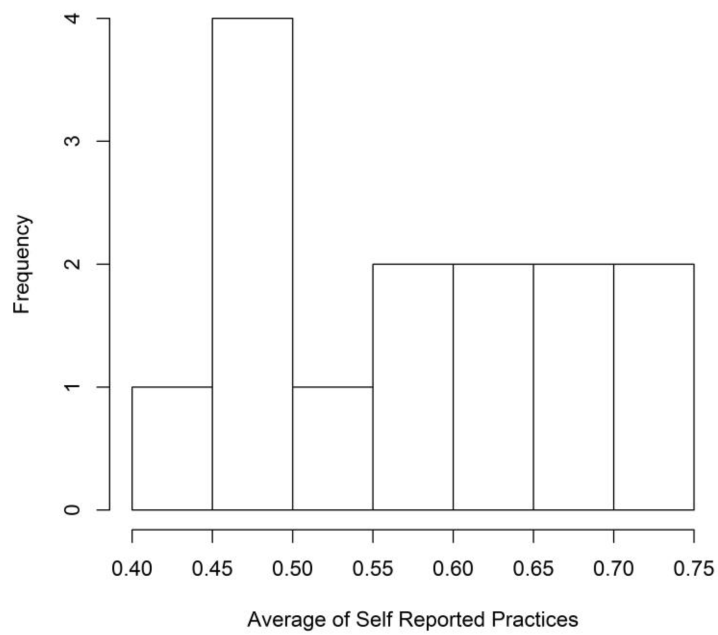

Figure 8. Histogram of possible participants' self-reported practices average.

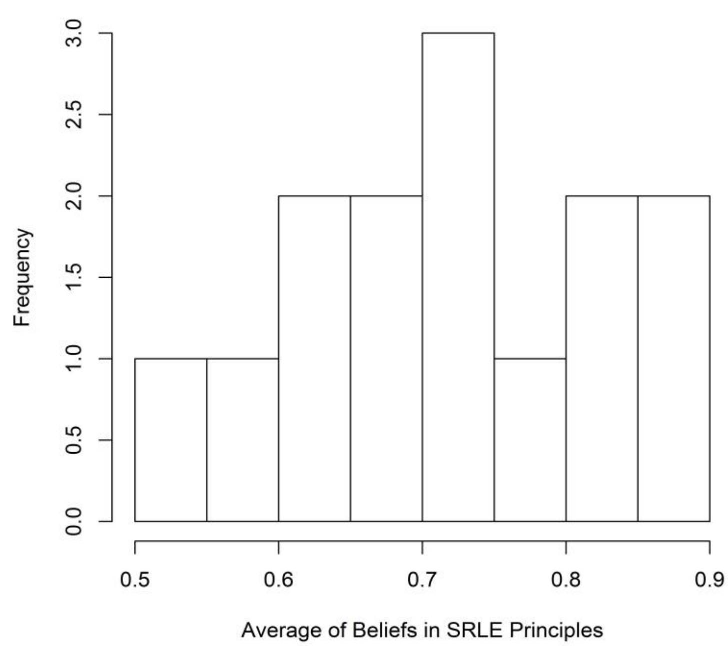

Figure 9. Histogram of possible participants' beliefs in effectiveness of SRLE principles.

A scatter plot (see Figure 10) was constructed for the averages of survey participants' self-assessment results vs. beliefs to help identify teachers with similar beliefs and practice. Because this research project had the goal of evaluating the effect of SRLE principles on student statistical reasoning, participants who were inconsistent in their beliefs and practices were removed from possible matching. Thus, the participant scoring a belief level of .83 and a practice level of .34 was excluded from possible matching and analysis, leaving 14 possible participants for the full study. The Pearson's correlation coefficient for possible participants was .81 between the scores for beliefs and practices.

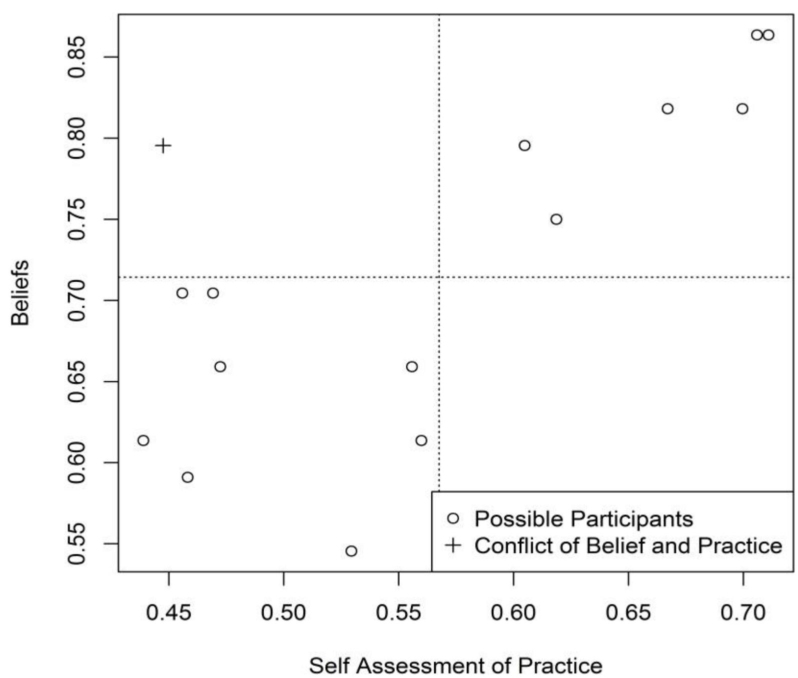

Figure 10. Scatter plot of possible participants' beliefs and practices subscales from the STI.

Teachers were categorized as high conformity when their responses were higher than the typical average response in belief and practice, the upper right quadrant of Figure 10. Teachers were categorized as low conformity to the SRLE principles when their responses were lower than the average in beliefs and practices, the lower left quadrant of Figure 10. Of 14 possible teachers for matching, eight were categorized as low conformity and six were categorized with higher conformity levels. To produce an overall level of conformity with SLRE, participants' beliefs in SRLE principles were averaged with perceived use of SRLE principles in the classroom, thus allowing univariate comparison (see Figure 11).

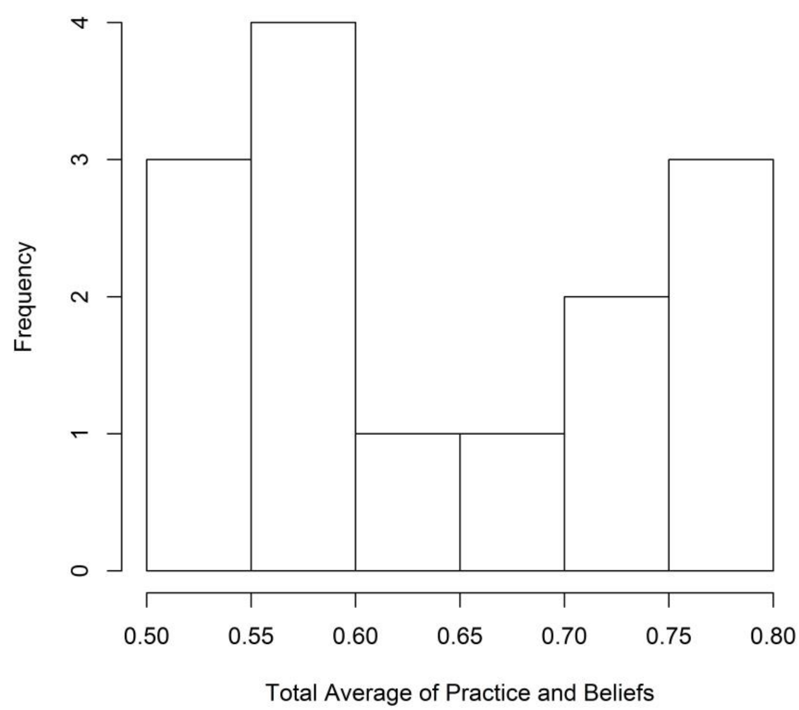

Figure 11. Histogram of total average conformity to SRLE conformity.

Identification of Matched Pairs of Participants

After identifying possible participants for the study and their conformity grouping through the STI, teachers were ordered based on their conformity level and numbered for purposes of anonymity. Teachers were then grouped by zero, one, two, and greater than two years of experience. Based on the groupings it was determined how many potential matches existed between lower conformity and higher conformity classrooms. The best match was determined in each grouping by first observing coursework in statistics, then professional development in statistics education. Given a matching existed, school demographics were compared for each matching.

The first level of separation to create matches identified three teachers with no experience in teaching statistics, three teachers with one year experience, four teachers with two years of experience, and four teachers with more than two years of experience (see

Table 5). Based on this grouping of teachers by years of experience it was possible to select one set of teachers with zero years of experience, one set of teachers with one year of experience, two sets of teachers with two years of experience, and one set of teachers with at least three years of experience.

Table 5

Possible Matches Grouped by Years of Experience

Possible Teacher Matches for Zero Years of Teaching Experience in Statistics

Teacher ID	Total Conformity	Years of Experience	Completed Coursework	Professional Development	Percent Free and Reduced Lunch
1	.537	0	4	13	45%
8	.700	0	4	5	20%
9	.743	0	3	10	75%
Possible Teacher Matches for 1 Year of Teaching Experience in Statistics					
3	.526	1	4	4	60%
6	.605	1	6	5	65%
10	.684	1	4	8	58%
Possible Teacher Matches for 2 Years of Teaching Experience in Statistics					
2	.525	2	5	6	65%
4	.58	2	8	7	0%
14	.759	2	2	6	60%
13	.787	2	12	9	55%
Possible Teacher Matches for greater than 2 Years of Teaching Experience in Statistics					
11	.566	6	2	11	30%
7	.587	4	2	3	50%
12	.587	12	5	7	20%
5	.785	7	5	15	24%

The second round of matching required the pairs of teachers to be matched as closely as possible on previously completed coursework in statistics and professional development in statistics education as shown in

Table 5. Teachers with no experience in teaching statistics consisted of two high conforming teachers and one low conforming. Based on the similarity of completed coursework for all three teachers, professional development in statistics education was used to match Teacher 1 and 9; see

Table 5. This also maximized the distance between conformity levels for teachers with zero years of experience. Although Teacher 1 and 9 were matched at the second level, the number of reported students receiving free and reduced lunch were 30% larger for the higher conformity classroom. However, Teacher 9 was still a better match than Teacher 8 to Teacher 1 on this criterion.

For teachers with one year of experience, the high conformity Teacher 10 could have been matched with either Teacher 6 or 3, both lower conformity, based on years of experience. At the next level of matching Teacher 10 best matched with Teacher 3, based on professional development. This matching also maximized the difference between conformity levels. This was also a better match on reported free and reduced lunch participants. Teacher 10 was thus matched with Teacher 3 based on similarity at all levels of matching.

There were two possible pairings for teachers with 2 years of experience in teaching statistics. Teachers 14 and 13 could have been matched with Teachers 4 or 2. At the second level of matching, Teacher 14 matched most closely with Teacher 2. Teacher 4 matched most closely with Teacher 13. Similarly, Teachers 14 and 2 were more closely matched as well as 13 and 4 based on professional development. Though larger differences existed for completed coursework and professional development for Teacher 4 and 13, it is important to highlight the fact that these two teachers reported the most experience in completed coursework and professional development within the STI survey. Teacher 14 and 2 were very similar on the school students receiving free and reduced lunch while Teacher 4 and 13 were very different. Even though the difference in school socio-economic status existed, Teacher 4 and 13 were matched closely on teaching experience, completed coursework in statistics, and professional development. In conclusion, Teacher 14 matched Teacher 2 because they had similar teacher characteristics and school demographics to qualify for comparison. Teacher 13 matched Teacher 4 largely on teacher demographics but not on school demographics. Though these two Teachers differed on school demographics, school demographics was the less strenuous matching factor in the study.

For teachers who had greater than 2 years of experience, there existed only one high conformity teacher for possible matching. Teacher 5 could have potentially been matched with Teacher 7, 11, or 12 based on years of experience and previously completed coursework. At the second level, professional development, Teacher 11 more closely matched Teacher 5. The matching of Teachers 11 and 5 also maximized the distance of conformity for teachers with three or more years of experience. Although the third level was not used for matching purposes, Teachers 11 and 5 had a difference of approximately 6% in students' receiving free and reduced lunch for teachers with more than two years of experience. The matching of Teachers 11 and 5 were thus matched because of similar years of experience while simultaneously reducing for differences in completed coursework and percent of free and reduced lunch school demographics.

This process resulted in the identification of five sets of teachers with identical teaching experience in statistics, similar coursework in statistics, and similar school demographics. Table 6 summarizes the pairing of teachers in this experiment. Note, however, the mismatch of Teachers 13 and 4 in school demographics.

Table 6

Comparison of Matched Teachers at All Levels

Higher Conformity to SRLE Teacher Number	Lower Conformity to SRLE Teacher Number	Difference in Conformity Levels	Years of Experience Teaching Statistics	Completed Coursework in Statistics		Percent Free and Reduced Lunch	
				Higher	Lower	Higher	Lower
9	1	.206	0	3	4	45%	75%
10	3	.158	1	4	4	58%	60%
14	2	.234	2	2	5	60%	65%
13	4	.207	2	12	8	55%	0%
11	5	.219	>2	5	2	24%	30%

Student Responses

After matching teachers and the collection of opt-out letters from parents, teachers were directed to administer the CAOS as a pre-assessment. These assessments were then mailed to the researcher for analysis. Similarly, teachers administered the CAOS as a post-test at the end of the course. Tests were then mailed to the researcher for analysis. Student absences on the assessment dates reduced the number of students participating in both the pre- and post-test design. Incorrect coding of anonymous identifiers by participants also reduced the total number of participants who had pre- and post-test scores that could be matched. Based on this protocol, exact response rates could not be calculated. An underestimate of response rates for students in a teacher's entire statistics classroom were calculated by using the number of matched pre- and post-tests to the total available from each teachers' classes in Table 7.

Table 7

Student Response Rates

	Low Conformity Classrooms					High Conformity Classrooms				
Match	Teacher Number	Matched Tests	Total Students	Response Rate	Average Class Size	Teacher Number	Matched Tests	Total Students	Response Rate	Average Class Size
1	5	60	69	.87	17.25	11	46	65	.71	20.67
2	3	11	15	.73	15	10	5	7	.71	5
3	4	23	34	.68	12.5	13	12	13	.92	6.5
4	2	14	15	.93	15	14	7	8	.88	7
5	1	70	87	.80	17.5	9	17	23	.74	17
Total		178	220				87	116		

Observations with missing data were excluded from analysis, thus only observations that had matching pre-test and post-test results were used to answer research questions. Table 8 identifies the total number of students taking part in each teachers pre- and post-test and the number of matched samples used in the statistical analysis. Pre-screening of the data did not identify any univariate outliers. Class sizes and the number of students each teacher instructed varied considerably. Class sizes for teachers with less conformity to SRLE principles were generally larger than high conformity classrooms as presented in Table 7. Similarly, the low conformity teachers tended to have more opportunities to teach the content on a given day as seen in Table 8.

Table 8

Teacher Sample Sizes of Pre-tests, Post-tests, and Matched Tests

	Low Conformity					High Conformity				
Match	Teacher Number	Pre-test	Post-test	Matched Tests	Number of Classes	Teacher Number	Pre-test	Post-test	Matched Tests	Number of Classes
1	5	69	61	60	4	11	52	62	46	3
2	3	14	12	11	1	10	7	6	5	1
3	4	28	27	23	2	13	12	13	12	2
4	2	14	15	14	1	14	7	8	7	1
5	1	83	78	70	4	9	20	20	17	1
Total		208	193	178	12		98	109	87	8

Initial Analysis of Pre-Test Results

Based on a later start to the study than expected, pre-tests were administered approximately nine weeks into a thirty six-week school schedule leading up to the administration of the AP exam. To reduce the effects of this late administration, the researcher contacted each teacher to ensure that they had all covered the same material before administration of the pre-tests. Teachers were directed to administer the exam after the topics of graphical representation, boxplots, data collection and design, descriptive statistics, and bivariate data were covered in the course. Due to its late administration in the school year, effects of the SRLE could have been found in pre-test results. These effects could potentially also be misrepresented by prior experience with mathematics coursework because of association of the first few chapters of an introductory statistics course with mathematical standards in the state. For this reason, results on the pre-test results were compared looking both at responses to the entire pre-test and responses to questions on content that had been taught by teachers in the study before the administration of the pre-test. Twenty-four out of forty questions were taught by all teachers in the experiment before administration of the pre-test.

A summary of the average percent of correct responses out of 40 for each class' pre-test is given in Table 9. Table 10 represents the average percent correct of the 24 previously completed questions related graphical representation, boxplots, data collection and design, descriptive statistics, and bivariate data. Differences in percent correct for pre-test results on the entire test were very small between matched teachers. When only looking at questions on the pre-test that had been previously taught, differences between higher conformity teachers and lower conformity teachers were in three of the five matches.

Table 9

Percent Correct on Entire CAOS Pre-Test

	Higher Conformity	Lower Conformity	Difference in Averages
Match 1	46.5%	46.0%	0.5%
Match 2	42.5%	35.5%	7.0%
Match 3	40.0%	52.7%	-12.7%
Match 4	42.5%	42.3%	0.2%
Match 5	44.4%	41.3%	3.1%

Table 10

Percent Correct for 24 CAOS Pre-Test Items Covering Graphical Representation, Boxplots, Data Collection and Design, Descriptive Statistics, and Bivariate Data

	Higher Conformity	Lower Conformity	Difference in Averages
Match 1	50.6%	49.8%	0.8%
Match 2	41.1%	37.8%	3.3%
Match 3	46.2%	62.3%	-16.1%
Match 4	46.4%	44.3%	2.1%
Match 5	48.5%	44.7%	3.8%

Based on pre-test results, four high conformity teachers' classrooms outperformed their matched counterparts. To ensure that this phenomenon was not due to treatment effects of the SRLE, a subsequent analysis included only items on the text addressing previously covered content. Out of the four higher conformity teachers that whose students outperformed their respective matches, three of these teachers' students widened the difference when only analyzing previously covered material. The one match in which a lower conformity teacher's classroom outperformed its respective match also widened its difference when only analyzing previously covered material. Thus, pre-test results were mixed and initial effects of the SRLE were not apparent.

Statistical Reasoning Improvement in High Conformity Classrooms

The first research question in this study asked the extent to which students' statistical reasoning and thinking ability improve in classrooms that show high levels of conformity to SRLE principles. There were 87 students with matching pre-test and post-test results among five different teachers who were identified as having high conformity to SRLE principles. Response rates for student participation were unavailable based on the use of student anonymous identifiers as described by IRB protocol.

Table 11 provides summary information for all pre-test and post-test data received. A histogram of post-test scores for students in high conformity classrooms is presented in Figure 12. An ANCOVA procedure was used to test for differences in post-test scores among teachers categorized as high conformity classrooms while controlling for the students' pre-test scores.

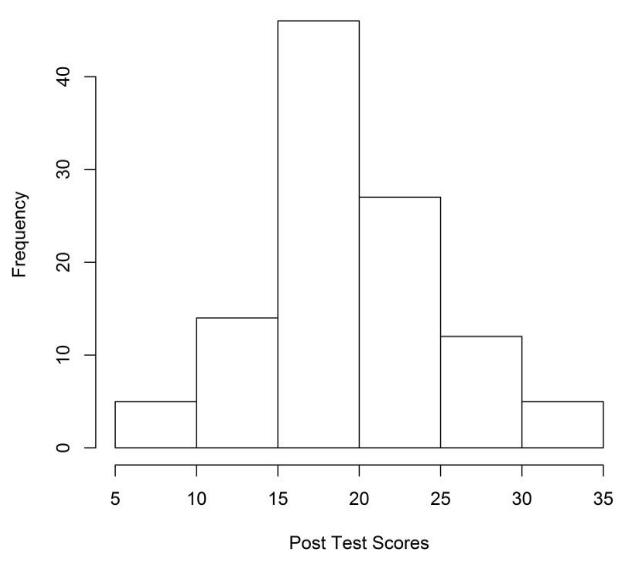

Figure 12. Histogram of post test scores for high conformity.

Table 11

Pre-test and Post-test Summary Statistics for High Conformity Classrooms

	Mean	Standard Deviation	Sample Size
Pre-Test	17.88	4.30	98
Post-Test	19.96	5.305	109

Testing conditions for use of ANCOVA

An ANCOVA was used to determine if differences existed between the number of questions answered correctly on post-test scores after adjusting for pre-test scores among teachers within high conformity classrooms. Before performing the ANCOVA procedure to measure outcomes in the high conformity classrooms, post-test scores were tested for normality. The Shapiro-Wilkson Test for normality failed to reject that the population was normal while the Komogorov-Smirnov test rejected normality at the .05 alpha level. Based on this contradiction and testing sensitivity, this phenomenon was investigated further looking at a histogram of the scores (see Figure 12) and a Q-Q plot (see Figure 13). Based on these plots and observation of kurtosis levels, post-test scores appeared to be approximately normal; consequently, data were used without transformations.

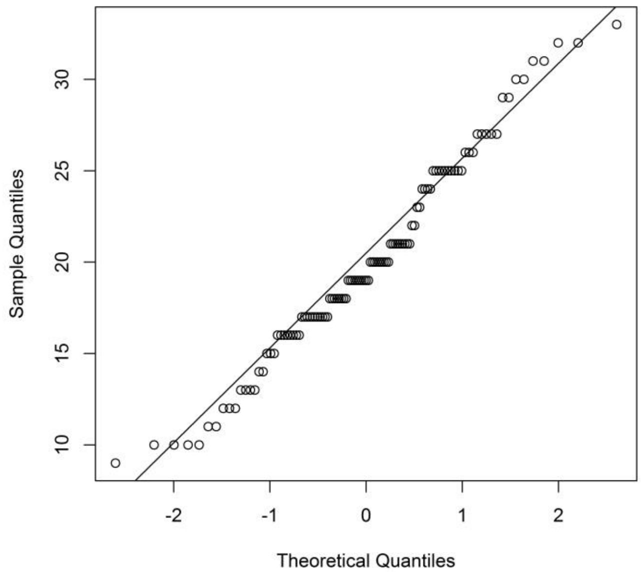

Figure 13. Q-Q plot for high conformity post-test scores.

Mertler and Vannatta (2010) also recommended a test for homogeneity of variances using a Levene’s test. This test was used to ensure that the covariate is linearly related to the dependent variable. The Levene’s test produced F(4, 82) =.959 with a p-value of .435 indicating a failure to reject equality of variances between different teachers at a significance level of .05. A linear relationship was also explored between pre-test and post-test scores. The scatter plot (see Figure 14) and the correlation coefficient of .338 indicated that there was a weakly positive association between students’ pre-tests and post-tests in high conformity classrooms (Salkind, 2004).

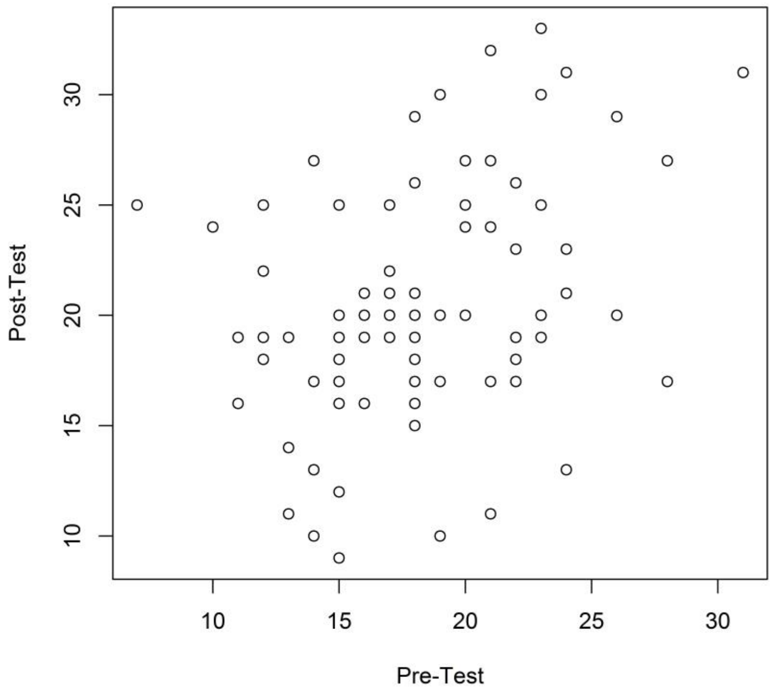

Figure 14. Scatterplot of pre-test vs. post-test for high conformity.

The ANCOVA procedure requires homogeneity of regression slopes for independent factors in the ANCOVA model. Mertler and Vannatta (2010) recommended testing for interaction between these factors by observing the F ratio and p-value for the interaction (see Table 12). The ANCOVA indicated no significant interaction between teachers and the pre-test, $F(4, 77)=1.306$, $p=.275$. Because no significant interaction existed between teachers and their pre-test scores, it was assumed that each teacher in the study improved at similar rates within the study, making it safe to use the ANCOVA procedure (Mertler & Vannatta, 2010).

Table 12

ANCOVA Pre-Screening for Homogenous Slopes

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	915.947	9	101.772	5.208	.000	.378
Intercept	134.941	1	134.941	6.905	.010	.082
Teacher	224.848	4	56.212	2.876	.028	.130
Pretest	256.210	1	256.210	13.110	.001	.145
teacher * pretest[a]	102.104	4	25.526	1.306	.275	.064
Error	1504.765	77	19.542			
Total	38594.000	87				
Corrected Total	2420.713	86				

Note. [a.] The teacher*pretest factor was used for interpretation of interaction

Thus, assumptions for completing the ANCOVA were verified. The assumption of homogenous slopes was not rejected. The covariate of pre-tests was projected to be a significant factor to reduce variability in students' post-test scores. The assumption of homogenous variances between teachers was not rejected. Students' post-test scores were assumed to be from a normal distribution. Based on these assumptions, analysis was performed to test for differences among teachers in high conformity classrooms post-test scores.

Analysis

A full factorial ANCOVA was completed (see Table 13). The ANCOVA procedure found that after adjustment for the significant covariate of pre-tests, post-test scores varied significantly among teachers in high conformity classrooms, $F(4,81)=6.78$, $p<.001$. This statistically significant difference among teachers in high conformity classrooms highlights differences in students reasoning ability among teachers even after adjustment for students' prior knowledge. A partial-Eta Squared of .264 also indicates a moderate effect size (Ferguson, 2009) of teachers within the high conformity grouping. Post-hoc comparisons between teachers in high conformity classrooms were not performed because of confounding factors apparent in the matching process. Factors such as teaching experience may have significant impact on differences found between pairwise comparisons. The adjusted means in Table 14 represent post-test problems correct for each teacher assuming that each teacher were assumed to have students with equal pre-test scores. Comparison of adjusted with non-adjusted teacher means, displayed in Table 14, revealed four teachers' classes increases in adjusted means while one teachers' classes decreased. Increases in four adjusted means with only one decrease elaborates a weighted sample in which one teacher's students potentially had higher pre-test results than others within the group. A lack of student improvement in this teacher's classes may have also contributed to this decrease in adjusted mean. Exploratory analysis was used to understand improvement in high conformity classrooms further.

Table 13

ANCOVA Test for Differences between Teachers' Post-Test Scores while Controlling for Pre-Tests

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	813.844	5	162.769	8.205	.000	.336
Intercept	682.142	1	682.142	34.386	.000	.298
Pretest [a]	467.948	1	467.948	23.589	.000	.226
Teacher [b]	537.999	4	134.500	6.780	.000	.251
Error	1606.869	81	19.838			
Total	38594.000	87				
Corrected Total	2420.713	86				

Notes. [a] The Pretest factor was used to test significance of the covariate. [b.]The Teacher factor was used to test for differences between groups of students' post-test scores by each high conformity teacher.

Table 14

High Conformity Post-test Means and Adjusted Post-Test Means for High Conformity Classrooms

Teacher Number	Adjusted Mean	Non-Adjusted Mean
9	19.103	19.06
13	25.472	24.33
14	19.734	19.14
11	19.048	19.59
10	25.848	24.60
Mean of Teachers' Means	21.841	21.344

Exploratory Analysis

To further understand these differences in adjusted means of the high conformity group, an exploratory analysis was performed looking at raw percent increases. The numbers of students influenced by each teacher was also compared. Teachers with higher adjusted means had student increases from pre-test to post-test of over 20% gains. Table 15 summarizes these results.

Table 15

Percent of Increase from Pre-Test to Post-Test in High Conformity Classrooms

Teacher	Post-Test	Pre-Test	Difference	Number of Students
9	47.7%	44.3%	3.4%	17
13	61%	40%	21%	12
14	47.9%	42.5%	5.4%	7
11	49%	46.5%	2.5%	46
10	61.5%	42.5%	19%	5

The improvements in Teachers 13 and 10 were approximately 21% and 19% respectively on the assessment, whereas the national average for improvement was only 9% for college students. The other three classes' non-adjusted post-test score means in the high conformity grouping were approximately 6% lower than the national average for college students. Teacher 11's forty-six students accounted for over half of the high conformity grouping. Students of this teacher increased by only approximately 2.5% while the Teacher 14 improved by approximately 5.4%.

Of the five high conformity teachers, Teacher 11 had the least improvement, but the teacher also had the least amount of students receiving free and reduced lunch. Of the two highest performing teachers' classes, Teacher 13 completed significantly more coursework than other participants in the study. When finding the mean of each teachers' mean on post-tests, the average for high conformity classrooms was approximately 53%, 2% higher than the national average for college students.

Thus, results from the high conformity grouping showed improvement in statistical reasoning at varied levels. Two teachers had substantial increases in statistical reasoning while three others had relatively small gains. The reduction in Teacher 11's adjusted mean was verified to be related to a low improvement rate from pre-test to post-test. Additionally, a decrease in the adjusted mean of only Teacher 11's post-test scores was related to the large number of students in their class. Other teachers in the high conformity grouping had substantially less students taking AP statistics. In conclusion, high conformity classrooms showed increases in statistical reasoning substantially above the national improvement rates for college students in two cases, and below the national improvement rate for the other 3.

Statistical Reasoning Improvement in Low Conformity Classrooms

Research question two in this study analyzed the extent to which students' statistical reasoning and thinking ability improve in classrooms that show low levels of conformity to SRLE principles. There were 176 students with matched pre-test and post-tests among five different teachers who were identified as having lower conformity to SRLE principles. Response rates for student participation were unavailable based on the use of student anonymous identifiers as described by IRB protocol. Table 16 provides summary information for all pre-test and post-test data received. A histogram of post-test scores for high conformity classrooms is presented in Figure *16*.

Table 16

Pre-test and Post-test Summary Statistics for Low Conformity Classes

	Mean	Standard Deviation	Sample Size
Pre-Test	17.625	4.39	208
Post-Test	20.88	4.91	191

Testing conditions

An ANCOVA was used to determine if differences existed between students' post-test scores after adjusting for pre-test scores among low conformity teachers. Before performing the ANCOVA procedure to measure outcomes in the low conformity classrooms, post-test scores were tested for normality. The Komogorov-Smirnov (p=.027) and Shapiro-Wilkson (p=.030) test rejected normality at the .05 alpha level. Based on the discrete structure of the post-test scores, a histogram (see Figure 16) and Q-Q plot (see Figure 15) was created to further analyze this assumption. The histogram appeared approximately normal and the Q-Q plot was roughly linear. Based on the robustness of the procedure to this assumption and given plots, it was determined to use the dependent variables without transformation and proceed with further assumption testing.

Mertler and Vannatta (2010) also recommended a test for homogeneity of variances using a Levene's test. This test was used to ensure that the covariate was linearly related to the dependent variable. The Levene's test for equality of variances produced F(4,171)=1.601with a p-value of .176 indicating a failure to reject equality of variances between different teachers at a significance level of .05. A linear relationship was also explored between pre-test and post-test scores. The scatter plot (see Figure 17) and the correlation coefficient of .557 indicated that there was an association between students' pre-tests and post-tests. With a moderately high positive correlation, it was assumed that the pre-test scores would serve well as a covariate to reduce or control for the variability in post-test scores.

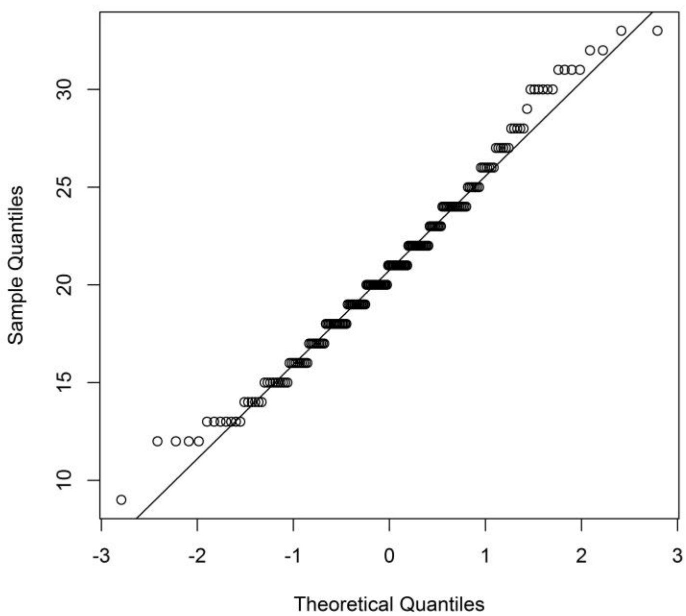

Figure 15. Normal QQ plot for post-pest scores in low conformity classrooms

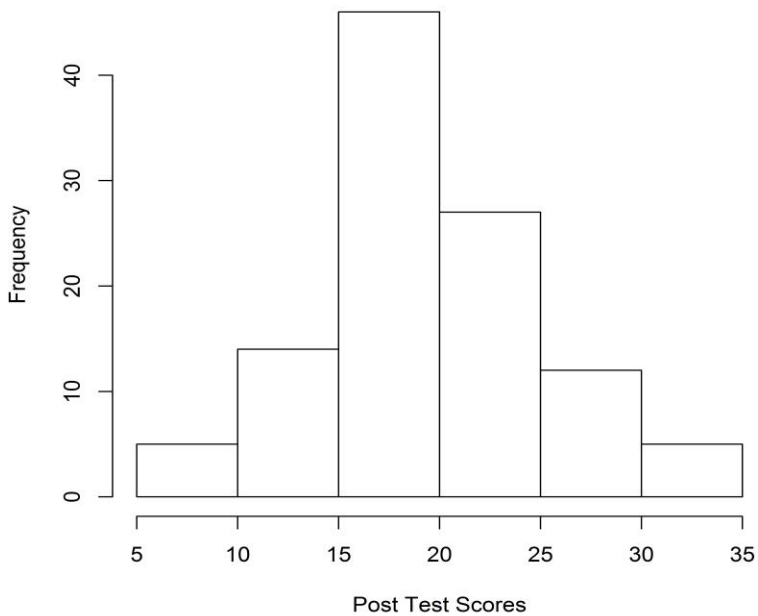

Figure 16. Histogram for post-test scores for low conformity classrooms.

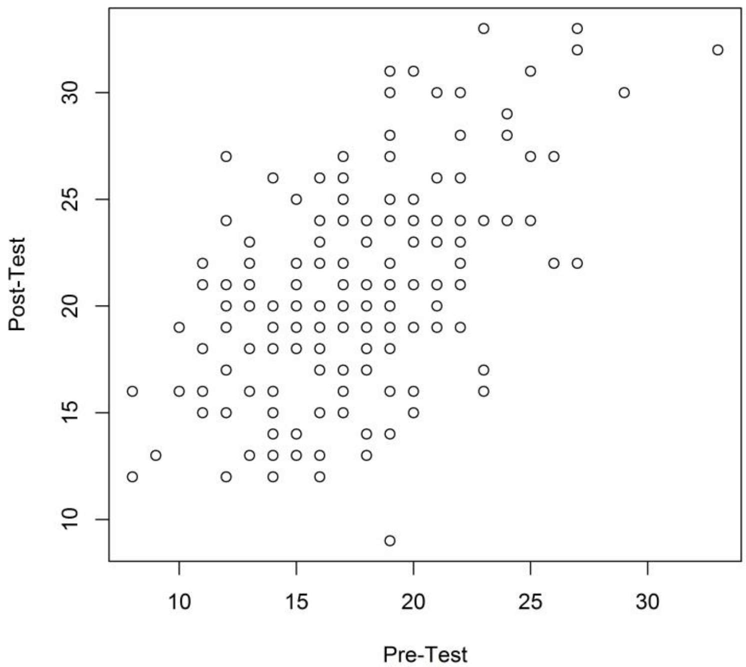

Figure 17. Scatter plot of pre-test vs. post-tests for low conformity classrooms.

The assumption of homogeneity of slopes was tested by observing the interaction between different teachers and their pre-tests by observing the F ratio and p-value for the interaction. The ANCOVA summary indicated no significant interaction between teachers and the pre-test, $F(4, 166)=1.501$, $p=.204$ (see Table 17). With no statistically significant interaction between teachers and their pre-test scores, the assumption for homogeneity of slopes referred to by Mertler and Vannatta (2008) was met for the ANCOVA comparison between teachers procedure.

Table 17

ANCOVA Test for Interaction in Low Conformity Classrooms

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	1626.179[a]	9	180.687	11.488	.000	.384
Intercept	613.854	1	613.854	39.029	.000	.190
Teacher	45.711	4	11.428	.727	.575	.017
Pretest	387.051	1	387.051	24.609	.000	.129
teacher * pretest[a]	94.404	4	23.601	1.501	.204	.035
Error	2610.860	166	15.728			
Total	79643.000	176				
Corrected Total	4237.040	175				

Note. [a]The teacher*pretest factor was used for testing of homogeneity of slopes.

Thus, assumptions for completing the ANCOVA were verified. The assumption of homogenous slopes was not rejected. The covariate of pre-tests was projected to be a significant factor to reduce variability in students' post-test scores. Students' post-test scores were assumed to be from a normal distribution. Assumptions of homogenous variance between teachers was not rejected. Based on these assumptions, analysis was performed to test for differences among low conformity teachers.

Analysis

A full factorial ANCOVA was completed (see Table 18). The ANCOVA procedure found that after adjustment for the significant covariate of pre-tests, post-test scores varied significantly among low conformity teachers, $F(4,166)=6.78$, $p=.010$ (see Table 18). Thus, assuming teachers' students had similar reasoning abilities, post-test reasoning ability was determined to be different. A partial Eta-Squared of .075 revealed a small effect size of teachers in the low conformity grouping (Ferguson, 2009). Match wise comparisons using post-hoc analysis were not performed because of obvious confounding factors present in the matching procedure. Comparison of adjusted teacher means, displayed in Table 18, revealed two low conformity teachers' adjusted student means decreased and three adjusted student means increased. Table 19 presents adjusted means of post-test scores after pre-test score adjustments and raw post test score averages among teachers for comparison purposes. Teacher 4's post-test scores were reduced by nearly three questions indicating a large adjustment based on pre-assessment scores (see Table 19).

Table 18

ANCOVA Test for Differences between Low Conformity Teachers

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	1531.775	5	306.355	19.251	.000	.362
Intercept	969.176	1	969.176	60.903	.000	.264
Pretest	785.560 [a]	1	785.560	49.365	.000	.225
Teacher	218.173 [b]	4	54.543	3.428	.010	.075
Error	2705.264	170	15.913			
Total	79643.000	176				
Corrected Total	4237.040	175				

Note. [a] The Pre-test factor was used to test significance of the covariate. [b.]The Teacher factor was used to test for differences between teachers with low conformity to SRLE principles.

Table 19

Adjusted and Non-Adjusted Post-Test Means for Low Conformity Classrooms

Teacher Number	Adjusted Mean	Non-Adjusted Mean
5	20.672	21.18
3	17.162	16.36
2	19.796	19.57
1	20.274	19.8
4	22.067	25.33
Mean of Teachers’ Means	19.994	20.448

Analysis of low conformity classrooms revealed differences between individual teachers. Though differences existed, the effects of these teachers were relatively small. Teachers in the low conformity grouping thus increased at similar but significantly different rates. Increases in teachers with low conformity classrooms were very similar to national averages for improvement by college students.

Exploratory Analysis

To further understand these differences in adjusted means of the high conformity group, an exploratory analysis was performed with raw percent increases. The number of students influenced by each teacher was also compared. Teachers with higher adjusted means had higher amounts of student increase and teachers with lower adjusted means had lower amounts of student increase in statistical reasoning. Table 20 summarizes these results.

Table 20

Percent of Increase from Pre-Test to Post-Test in Low Conformity Classrooms

Teacher	Post-Test	Pre-Test	Difference	Number of Students
5	53%	46%	7%	70
3	40.9%	36.8%	4.1%	23
2	48.9%	42.3%	5.6%	11
1	49.5%	41.3%	8.2%	60
4	63.3%	52.7%	10.6%	14

The highest performing teacher among low conformity classrooms was Teacher 4. Teacher 4’s school reported 0% free and reduced lunch students and the teacher had significantly more course work in statistics than others in the lower conformity group. Teacher 4 scored approximately 12% higher than the national average. Note, however, that Teacher 4’s pre-test average was approximately 2% higher than the national average on the post-test. This implies that this teacher's students improved by approximately 10%, which is 1%, more improvement than the national average. Though this teacher seemingly had high performances on the post-test, after adjustment for pre-test scores, it was very similar to other teachers within the low conformity grouping.

Teacher 5's teacher had 6 years of experience, and students' increased by approximately 7% and scored 2% above the national average without adjustment for pre-test scores. Post-test averages for Teachers 1 and 2 were approximately 2% below the national average while Teacher 1 improved by approximately 8% and Teacher 2 by 6.5%. The lowest performing class, Teacher 3, improved by 4%, and scored 11% below the national average on the post-test. When finding the mean of students influenced by each teacher, the average for low conformity classrooms was approximately equal to the national average of 51% for college students.

Thus, results from the low conformity grouping showed improvement in statistical reasoning. One teacher had increases in statistical reasoning above the national average while four others had gains less than the national average. Even though teachers had significantly different post-test scores after adjustment for pre-test scores, variation between teachers' students was not large as indicated by the partial Eta-Squared and relatively similar increases to national averages for increase. In conclusion, low conformity classrooms showed increases in statistical reasoning substantially similar to or below national improvement rates for college students.

Comparison of Statistical Reasoning Improvement between High and Low Conformity Classrooms

Research question three in this study analyzed the extent to which students' statistical reasoning and thinking ability differed between teachers that show low and high levels of conformity to SRLE principles. An ANCOVA was used to determine if differences existed between the groupings of high and low conforming classrooms post-test scores after adjusting for pre-test scores.

Testing conditions

Before performing the ANCOVA procedure to measure outcomes, post-test scores were tested for normality. The Komogorov-Smirnov ($p<.001$) and Shapiro-Wilkson ($p=.003$) test rejected normality at the .05 alpha level. A histogram (see Figure 18) and Q-Q plot (see Figure 19) was created to further analyze this assumption. The histogram appeared approximately normal and the Q-Q plot was roughly linear. Based on the robustness of the procedure to this assumption (Mertler and Vannatta, 2010) and given plots, it was determined to use the dependent variables without transformation and proceed with further assumption testing.

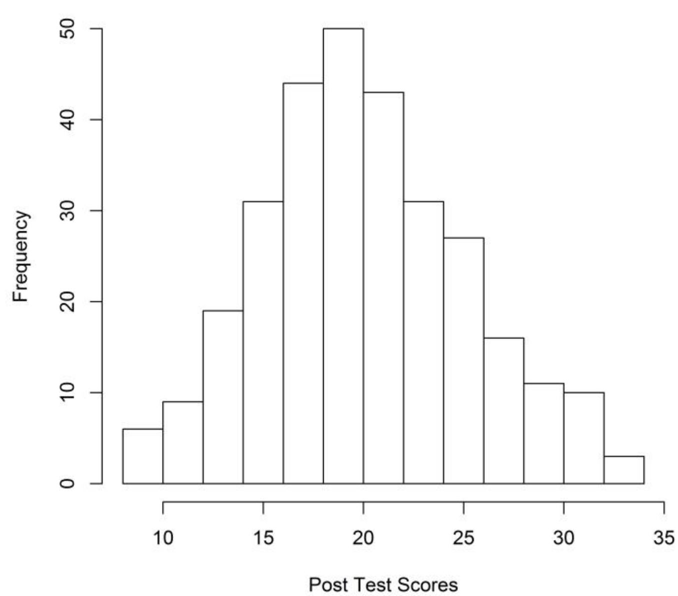

Figure 18. Histogram for all post-test scores.

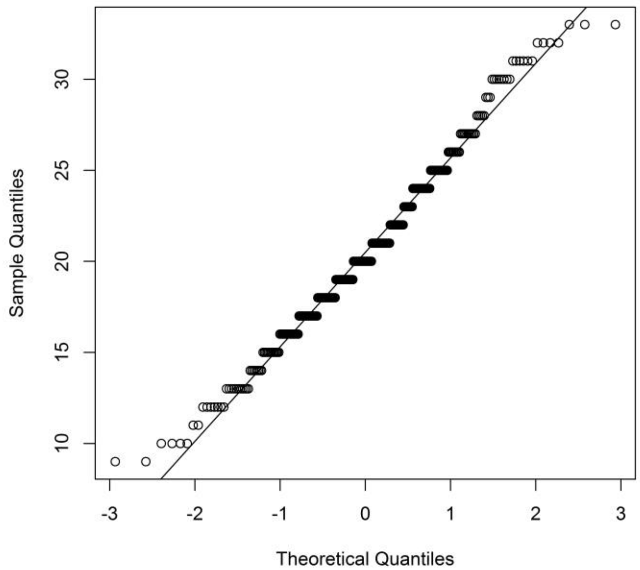

Figure 19. Normal QQ plot for post scores for all participants.

Mertler and Vannatta (2010) also recommended a test for homogeneity of variances using a Levene's test. This test was used to ensure that the covariate is linearly related to the dependent variable. Levene's test produced F(1,261)=3.756 with a p-value of .054 indicating a failure to reject equality of variances between different teachers at a significance level of .05. A linear relationship was also explored between pre-test and post-test scores (see Figure 20). The scatter plot and correlation coefficient of .508 indicated that there was an association between students' pre-tests and post-tests and the inclusion of pre-test scores to be appropriate. Since there was a moderate relationship between students' pre-test and post-test scores, pre-test scores were assumed to serve as a controlling covariate to reduce the variability within students' post-test scores.

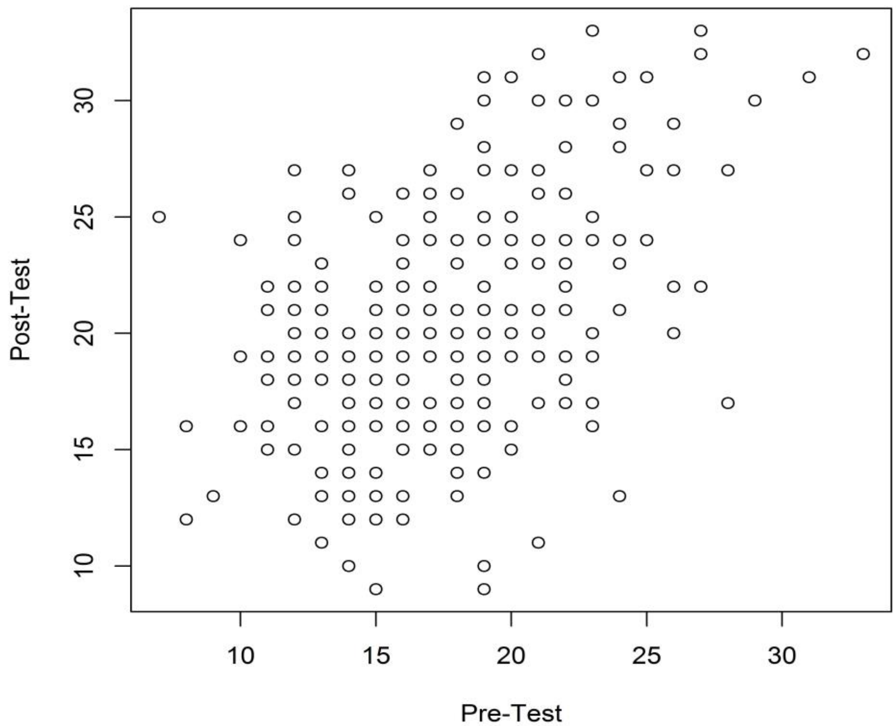

Figure 20. Scatter plot for pre-test vs. post scores for all participants.

The assumption of homogeneity of slopes was tested by observing the interaction between different teachers and their pre-tests by observing the F ratio and p-value for the interaction (see Table 21). The ANCOVA indicated no significant interaction between teachers and the pre-test, $F(1, 259)=62.275$, $p=.076$. No significant interaction between teachers and their pre-tests implied that the assumption of equal slopes for the covariate of pre-test across teachers was approximately the same. With the testing assumption for homogeneity of slopes and others as suggested by Mertler and Vannatta (2008) fulfilled, a full factorial ANCOVA was completed.

Table 21

Pre-Screening Test for Homogeneity of Slopes between Conformity Levels

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	1594.973[a]	3	531.658	27.169	.000	.239
Intercept	1647.314	1	1647.314	84.181	.000	.245
Conformity	44.102	1	44.102	2.254	.135	.009
Pretest	1207.730	1	1207.730	61.717	.000	.192
conformity * pretest[a]	62.275	1	62.275	3.182	.076	.012
Error	5068.305	259	19.569			
Total	118237.000	263				
Corrected Total	6663.278	262				

Note. [a] The conformity*pretest factor tests for homogeneity of slopes.

Thus, assumptions for completing the ANCOVA were verified. The assumption of homogenous slopes was not rejected. The covariate of pre-tests was determined to be used reduce variability in students' post-test scores. Students' post-test scores were assumed to be from a normal distribution based on QQ plots and histograms. Assumptions of homogenous variance between high and low conformity groupings was not rejected. Based on these assumptions, analysis was performed to test for differences between high and low conformity classrooms.

Analysis

The ANCOVA procedure was completed in SPSS using Type III sum of squares to determine if differences existed in students' statistical reasoning between high and low conformity classrooms. The ANCOVA testing procedure found that after adjustment for the significant covariate of pre-tests, post-test scores did not significantly vary between high and low conformity teacher groupings, $F(1,260)=.953$, $p=.330$, $\eta^2=.024$ (see Table 22). Thus, reasoning ability was not statistically different between grouping of teachers into high and low conformity sets adjusting for pre-test scores. A partial Eta Squared of .024 was deemed a low effect size (Ferguson, 2009).

Table 22

ANCOVA Test for Differences between Conformity Levels after Controlling for Pre-Tests

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	1532.698[a]	2	766.349	38.836	.000	.230
Intercept	1609.612	1	1609.612	81.570	.000	.239
Pretest	1527.173	1	1527.173	77.392	.000	.229
conformity	18.797	1	18.797	.953	.330	.004
Error	5130.580	260	19.733			
Total	118237.000	263				
Corrected Total	6663.278	262				

Note. [a] The conformity factor tests for differences between high conformity and low conformity to SRLE principles grouping.

The unadjusted post-test score averages for high conformity classrooms was 20.39 and for low conformity classrooms was 20.7. The adjusted post-test score average for high conformity classrooms was 20.3 and for low conformity classrooms was 20.8. The decrease in adjusted means for high conformity classrooms may potentially be related to higher pre-test scores in the grouping. Though effects of the SRLE were mixed from initial analysis of pre-test results, higher pre-test scores caused a reduction in adjusted student post-test means for high conformity teachers. Similarly, the increase in adjusted means for post-test scores in low conformity classrooms indicated lower than average pre-test scores.

Thus, statistically significant results were not found between students' reasoning ability after adjustment for students' prior reasoning ability as measured by the CAOS. Though statistical differences were not found between the grouping of teachers' students into high and low conformity groupings, the answer to research question three was not clear. The extent to which differences did exist between high and low conformity classrooms needed more detail, because variation between teachers' students in the experiment was quite evident from research questions one and two. Potential weighting by total students' participating in each teachers' classrooms may have skewed differences that existed between matches. To understand further the effects of conformity to SRLE principles on students' statistical reasoning, exploratory analysis was conducted to understand the variation between teachers, specifically matched teachers.

Exploratory Analysis

Given the differences in the number of participating students from each teacher deemed as having high conformity and low conformity to SRLE principles, individual differences between matched teachers within the study were explored. Before assuming that there was a difference between individual teachers however, an ANCOVA was performed in order to determine if significant differences existed between different teachers' post-assessment scores while controlling for pre-test scores. Before performing the ANCOVA for differences between each teachers' classes within the study, testing conditions were checked.

Equality of variances was checked using a Levene's test which resulted in F(9,253)=1.46 and a p-value of .163 giving no evidence to reject equality of variances between teachers. Previously checked assumptions from the ANCOVA on conformity level were sufficient to proceed with the ANCOVA procedure testing for differences between teachers in the experiment after adjusting for students' pre-test scores (see Table 23).

Table 23

ANCOVA Test for Differences between Teachers after Controlling for Pre-Tests

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	2351.098	10	235.110	13.740	.000	.353
Intercept	1678.883	1	1678.883	98.112	.000	.280
Pretest	1253.462	1	1253.462	73.251	.000	.225
Teacher	837.197	9	93.022	5.436	.000	.163
Error	4312.179	252	17.112			
Total	118237.000	263				
Corrected Total	6663.278	262				

Note. [a] The teacher factor tests for differences among all teachers in the study.

After adjusting for the significant covariate of students' pre-test scores, there was a statistically significant difference between teachers' students statistical reasoning on the post-test, $F(9,252)=5.436$, $p<.001$, $\eta^2=.163$. This significant difference meant that differences existed in students' post-test scores between at least one pair of teachers in the study. A η^2 of .163 indicated that teachers had a moderate effect for differences in students' post-test scores. Based on confounding factors between different matches, only students' post-test adjusted means were compared between matched teachers (see Table 24). This post-hoc analysis confirmed three sets of matched classrooms in the experiment that had differences in post-test scores that had a less than 10% probability happening by chance. Ten percent was chosen for further analysis because of the exploratory nature of the analysis and the clear division of p-values between matched participants in the study. Of these, one lower conforming teacher's classes outperformed a higher conforming teacher's classes (Match 1) and two higher conforming teachers' classes outperformed their respective lower conforming teacher's class when accounting for students' pre-test scores (Match 2 and 3). Table 25 was also created to further understand these differences in adjusted means, by comparing increases from pre-test to unadjusted post-test score means between matched teachers.

Table 24

Adjusted Class Mean Comparisons between Matched Teachers

Match	High Conformity Teachers Adjusted Mean	Low Conformity Teachers Adjusted Mean	Difference	P-value for pairwise comparison
1	18.771	20.767	-1.996	.017
2	24.680	17.205	7.475	.003
3	25.149	22.208	2.941	.088
5	18.916	20.335	-1.419	.210
4	20.542	19.846	.696	.730

Table 25

Comparison of Percent of Increase between Matched Teachers

Match	High Conformity				Low Conformity				Difference in Average Increase between Low and High Conformity
	Teacher Number	Pre-test Percent Correct	Post-test Percent Correct	Percent of Increase	Teacher Number	Pre-test Percent Correct	Post-test Percent Correct	Percent of Increase	
1	11	46.5%	49%	2.5%	5	46%	53%	7%	-4.5%
2	10	42.5%	61.5%	19%	3	36.8%	40.9%	4.1%	14.9%
3	13	40%	61%	21%	4	52.7%	63.3%	10.4%	10.6%

Using adjusted means, students of Teachers 10 and 13 had substantial higher statistical reasoning ability on the post-test in comparison to their matched teachers and all others within the study. The average percent correct in these two teachers' classes was over 60%. Similarly, Teachers 10 and 13 also increased on pre-test to post-test scores by 19 and 21% respectively. Both of these teachers were from the higher conformity grouping. This implies that the largest gains on post-test scores were from two teachers within the higher conformity groupings after controlling for pre-test scores. The only other teacher to have improvements higher than the national average of 9.1% was Teacher 4, who had an improvement in its score of 10.6%. In addition, this teacher's class had pre-test results higher than the national average post-test.

The lowest performing high conformity teacher (Teacher 11) represented over half of the total high conformity group. Thus, potential also existed for unequal allocation between matched samples to overweight conformity levels. The matched Teachers 1 and 9 represented approximately 39.3% of the low conformity group and approximately 19.5% of the high conformity group respectively. Because of unequal allocations of students in the high and low conformity groupings by different teachers, potential weighting of means existed in the ANCOVA testing procedure. This can be exhibited by comparing the mean of teachers' means in Table 14 and Table 19 of 21.3 for high conformity and 20.4 for low conformity with the overall low conformity mean of 20.7 and 20.3 and for high conformity classrooms. These results produce mixed interpretations for the increase of statistical reasoning based on weighting of students' post-test scores. We will now take a closer look at the three pairs of matched teachers with statistically significant pairwise differences (see Table 24).

Match 1. Match 1 consisted of Teacher 5 with 70 (33.5% of low conformity group) total student participants and Teacher 11 which had 46 (52.9% of high conformity group) total student participants. Match 1 consisted of teachers with the most experience teaching statistics in the study, six and seven years. These teachers also had similar coursework in statistics and professional development. Both teachers within this match consisted of schools who reported less than 30% free and reduced lunch.

Though Teacher 11 had the most experience teaching statistics, student performance in the class was the lowest within the high conformity grouping considering both adjusted and non-adjusted means. Teacher 5 was the second highest performing low conformity teacher given either the adjusted or the non-adjusted means. Teacher 5 increased by 7% from pre-test to post-test while Teacher 11 increased by 2.5%. The difference between the two teachers' classes was approximately two questions on the post-test given the adjusted and non-adjusted means. The probability of a difference this large or larger to occur by chance was calculated to be approximately 1.7%. Thus, differences found between these two teachers' students were not due to chance alone. Based on the raw percent of increase being low in the high conformity teachers' classes compared to national averages and this study's average, this statistical difference may more likely be due to a low performance of students in the high conformity teachers' classes rather than high performing students in the low conformity teachers' classroom.

Match 2. Match 2 consisted of two teachers with one year of experience. Teacher 3, the the teacher found to have low conformity to SRLE principles, consisted of 11 (6.2% of low conformity group) students and Teacher 10 consisted of five students (5.7% of high conformity group). Teachers of the classrooms both reported similar teaching coursework in statistics while Teacher 10's teacher reported approximately four more hours of professional development. Both schools reported between 58% and 60% free and reduced lunch.

The low conformity Teacher 3 had the lowest student performance within the study and classroom 10 was the highest performing high conformity teacher. Teacher 3's students increased by only 4.1% from pre-test to post-test results while Teacher 10's students increased by 19%. After accounting for pre-test scores, the high conformity teachers' students correctly answered approximately 7.5 more questions, or 19% of the post-test. The probability of a difference this large or larger to occur by chance was calculated to be approximately .3%. Thus, differences found between these two teachers' students were probably not due to chance alone. Though the low conformity teacher was the lowest performing teacher within the low conformity group, Teacher 10 had student improvement rates and post-test scores much higher than the national average. This difference was likely due to high performance in student reasoning in the high conformity teacher's class rather than low performance by students in low conformity teacher's class.

Match 3. Match 3 consisted of Teacher 4 and 13. Teacher 4 had 23 (12.9% of total low conformity group) total students participate and Teacher 13 had 12 (13.8% of high conformity group) total students. Both teachers within this matching reported two years of experience teaching statistics with between 8 and 12 courses in statistics. These teachers also reported very high amounts of professional development in statistics education, 7 and 9 hours. Teacher 13 had higher amounts of coursework and reported professional development in both situations. Teacher 4 reported 0% free and reduced lunch while Teacher 13 reported 50% free and reduced lunch.

Teacher 13, a high conformity teacher, after adjusting for pre-test scores outperformed the lower conformity Teacher 4 by approximately 3 questions or 7.5% higher on the post-test. Teacher 13 increased by 21% from pre-test to post-test while Teacher 4 increased by 10.6%. Given the difference in potential student demographics and opportunity to learn, this matching is eye opening. Students from a lower socio-economic school with low pre-assessment scores increased to nearly the same level of performance on post-test scores as their higher socio-economic status counterparts. The probability of a difference in increase this large or larger to occur by chance was calculated to be approximately 8.8%. Thus, differences found between these two teachers' students were probably not due to chance alone.

Based on individual matched comparisons, significant differences existed in student post-test scores between three matched high and low conformity teachers. The difference observed in favor of lower conformity teachers (Match 1) was likely due to a low performing high conformity teacher rather than the high performance of the high conformity teacher. This is based on a student increase of only 7% in the low conformity teacher's class. Differences in favor of higher conformity teachers were more likely due to high performance of students in the high conformity teachers' classes rather than the low performance of students in the low conformity teachers' classes. This is because Matches 2 and 3 had improvement rates nearly double and triple their matched low conformity counterparts. These differences shed light on implications for researchers, teachers, and teacher educators.

Based on the high performance of students' post-test scores and improvement rates in these teachers' classes, the difference observed between these matchings were likely due to high improvement rates of the high conformity classroom. Matching 3 also illustrated the potential for SRLE principles to cross barriers of school demographics that were measured in this study by free and reduced lunch. Students' pre-test scores were approximately 13% lower in the high conformity classroom, while raw post-test scores were very similar.

In summary, no statistical difference in reasoning ability was found between groupings of low conformity and high conformity classrooms. These results were potentially biased however by weighting of the pool of students in each grouping by the number of students each teacher influenced. Further exploratory analysis revealed statistically different pairwise comparisons between matched teachers. Individual comparisons revealed that these differences were related more to the presence of high variation among teachers with high conformity to SRLE principles. Two high conformity teachers had substantial increases in students' statistical reasoning as compared to their similar low conformity teachers and one high conformity teacher had substantially lower student increases in reasoning than its similar low conformity teacher's students.

Chapter 5: Conclusions and Implications

In this chapter, the limitations and conclusions of the study are presented. This is followed by the implications of the study for teachers and teacher educators. Lastly, directions for future research studies are suggested.

Limitations

This study is quasi-experimental, which has several inherent limitations. Possibly the largest limitation of this type of design is its lack of randomization. The methodology proposed attempted to reduce lack of randomization by matching teachers based on their previous statistical teaching experience, content knowledge of statistics at the teacher level, and on school socio-economic status. Matching of teachers were exact at the first level and varied slightly at the second and third levels. Matching 4 varied largely at the third level around socio-economic status but was included because of the teachers' high levels of statistical content knowledge. Though these matches attempted to produce a quasi-randomization, confounding factors could easily be present. A noteworthy confounding factor present in the study was actual class sizes in Table 7; high conformity teachers' classrooms were smaller on average than low conformity classrooms across matched and non-matched classrooms.

Because of the lack of assignment to particular conformity groups, there were also unequal treatment allocations by each teacher within both high and low conformity groupings. One teacher had five total participants in the study while another in the group with the same conformity had forty-six. This large difference within conformity levels could affect conclusions made in the study. To adjust for this phenomenon, type III sum of squares was used to reduce the effects of weighted means. Analysis was also performed between conformity levels and between teachers with type III sum of squares to reduce this potential for error and maximize potential to observe differences.

Since the study included young students or vulnerable populations, Institutional Review Board approval of the study was delayed. Based on a later start of the study than anticipated, pre-tests were administered approximately after 25% of the course was completed. With this in mind, effects of the Statistical Reasoning Learning Environment (SRLE) (Garfield and Ben-Zvi, 2008, 2009) may have been found in pre-test results. This would have limited the effect size observed from pre-tests to post-tests. Pre-tests were compared between and across matched groups because of this limitation in the study. Though student pre-tests were similar between most matched teachers, the matched teachers with large difference between school socio-economic statuses also had large differences in pre-test results. The large difference in pre-test scores between these two teachers' students could have potentially had maturation bias. Pre-tests being administered at earlier stages in the course could help increase effect size observed in similar future studies.

Though the Statistics Teaching Inventory (STI) (Zieffler, et. al, 2012) was assumed an improvement from its previously completed counterpart, the instrument itself was not fully validated before the study. Thus, teachers identified as being high or low conformity could have been potentially misidentified. The technology component of the STI failed to collect information from teachers based on their identification of calculator use. This may have potentially misrepresented the use of technology within the course. The use of the STI, a purely quantitative instrument based on teachers' perceived use and beliefs in effectiveness of SRLE principles, could have potentially produced biased results.

Participants in this study were largely new teachers of statistics. Remillard (2005) emphasized the impact that teacher experiences, content knowledge, and pedagogical content knowledge play on enacting an intended reform based curriculum. Thus newer teachers and their variation in experiences, content knowledge, and pedagogical content knowledge could have substantially influenced the relationship of the intended and enacted curriculum. Stein, Remillard, and Smith (2007) discussed the differences between the intended and enacted curriculum in education and specifically with two new elementary teachers. The intended curriculum was represented in this study by the beliefs and practice scale in the STI while the enacted curriculum was unmeasured. The discrepancy between the intended and enacted curriculum could have potentially produced higher or lower conformity rates for participants within the study potentially skewing results in unpredictable ways.

Conclusions

Although the data did not yield statistically significantly results between levels of conformity with the SRLE model, there were significant differences within high conformity teachers' classrooms, within low conformity teachers' classrooms, and between matched teachers. Key results for each of the research questions are presented in the sections below.

The Development of Reasoning in High Conforming SRLE Classrooms

Research question one examined the extent to which students' statistical reasoning and thinking ability improve in classrooms that show high levels of conformity to a Statistical Reasoning Learning Environment. For teachers identified as high conformity to SRLE principles, significant differences existed in students' post-test scores after adjustment for their pre-test scores. Results for student improvement and post-test scores among high conformity teachers' varied. Post-hoc analysis revealed two sets teachers which had significantly higher student scores and levels of improvement than three others in the high conformity grouping. These two sets of teachers improved by approximately 20% and had post-test scores 10% higher than others in the group. In addition, these two teachers' students scored 10% higher than national averages for college students in improvement and post-test scores. The three other teachers' students in the high conformity group scored below the national average for college students in both improvement and post-test scores.

Though significant differences did exist between students' development of statistical reasoning among high conformity teachers, these differences must be interpreted with caution. The lack of randomization and quasi-experimental matching between these high conformity teachers substantially limits the ability to draw comparisons. Years of experience, coursework in statistics, professional development in statistics education, and school socio-economic status were not grouped similarly. These findings do suggest however that the use of SRLE principles (Garfield and Ben-Zvi, 2008, 2009) did produce two classrooms with significantly larger test scores and improvements than both the national average for college students and other teachers identified as high conformity in this study. Variation within groupings of high conformity teachers' students suggest either unmeasured discrepancy between belief and practice for participants involved in the study, confounding factors that may have impacted results, or STI factor weighting for identification of SRLE conformity.

The extent to which students' reasoning ability increased among high conformity teachers varied considerably. Two teachers' students had improvement rates roughly three times as much as others in the grouping and twice as much as national averages for improvement.

Three of the teachers in the high conformity grouping had student increases lower than the national average improvement rates.

The Development of Statistical Reasoning in Low Conformity SRLE Classrooms

Research question two examined the extent to which students' statistical reasoning and thinking improve in teachers' classrooms that do not conform to the SRLE. Significant differences between students' post-test scores among teachers in the low conformity grouping existed after adjustment for pre-test scores. A small effect size (Ferguson, 2009), however, indicated minor variation between teachers in the low conformity grouping.

Four of the teachers in the lower conforming group had student improvement rates less than the national average for improvement while one teachers' students improved 1% more than the national average for improvement. Two of the teachers, without adjustment for student pre-test scores, students scored above the national average for college students. Of note, the one low conformity teacher who students improved 1% more than the national average for college students also had a student pre-assessment average as high as national average post-test scores.

Similar to teachers in the high conformity grouping, comparisons between low conformity groupings must be interpreted with caution. Many confounding variables have been identified and uncontrolled for in these comparisons. Unique to the low conformity group however is the effect of maturation for Teacher 4. With a baseline average as high as national averages for college students, students may show growth at different rates than others within the study. The extent to which students reasoning ability increased in low conformity teachers' classrooms was not substantially different than national averages for improvement. One teacher had student improvements larger than the national average while four others had rates smaller.

Results for low conformity teachers' classrooms were also more consistent with national averages for improvement than the high conformity teachers.

Difference between the Development of Statistical Reasoning in Low and High Conformity Classrooms

Research question three examined the extent to which students' ability to reason and think about statistics differ between teachers' classes that do and do not conform to the SRLE principles. No statistical difference in student post-test scores was found between the groupings of high conformity and low conformity teachers' students after adjustment for pre-test scores in the Analysis of Covariance design. Unadjusted and adjusted means were slightly higher for lower conformity teachers' classrooms than high conformity teachers' classrooms. The comparison of high conformity and low conformity groups, however, should be used with caution. Larger numbers of influences on students for certain teachers in the experiment weighted the effects of the teacher in high and low conformity groupings.

Based on the difference in the number of students each teacher influenced, further exploratory analysis was conducted to understand differences between the high and low conformity matchings. Statistical differences among students' post-test scores in the study did exist between three matched teachers in the study while controlling for students' pre-test scores. Post-hoc analysis compared matched teachers in the quasi-experiment.

Two lower conformity teachers' students outperformed their matched high conformity matches and three higher conformity teachers' students outperformed their low conformity teachers' students. Of the two lower conformity teachers whose students outperformed their respective higher conformity classrooms, only one match had statistically significant differences.

Of the five matched teachers, three of the high conformity classrooms outperformed their matched low conformity classrooms. The likelihood for two of these three to occur by chance were less than 10%. Noteworthy from initial pre-test analysis, four of the high conformity classrooms also outperformed their matched lower conformity classrooms.

The extent to which students reasoning ability differed between low and high conformity classrooms was not statistically different. Pairwise comparisons of matched teachers in the experiment revealed three cases in which statistically significant differences did exist. Details of these differences revealed two high conformity classrooms that demonstrated substantially high amounts of improvement in statistical reasoning and one high conformity group that showed low amounts of improvement in statistical reasoning.

In conclusion, this study has found evidence to support the use of SRLE principles to improve students' statistical reasoning. These improvements were both substantially above and below their respective peers. The discrepancy in improvement levels of classrooms categorized as high conformity to SRLE principles provides reason for further investigation. Perhaps certain factors of the SRLE contribute more to the increase in students' statistical reasoning than others or a strong combination of all factors within the STI are equally important. Based on results of this study, the use of the SRLE in its entirety does more to improve students' statistical reasoning than negatively effect it.

Implications

The results from this study can inform both teachers, curriculum writers, and teacher educators who engage in statistics or mathematics education and also suggest possible directions for future research.

Teachers, Curriculum Writers, and Teacher Educators

As teachers, curriculum writers, and teacher educators consider the results of the study, they might consider the potential for increases in reasoning with use of SRLE principles, which are related to the mathematical teaching practices described in Principles to Action (PtA) (NCTM, 2014). In particular, Match 2 and 3 in this experiment revealed potential to narrow achievement gaps present in schools throughout the nation. Furthermore, two classrooms with high amounts of professional development and coursework in statistics had very high post-test scores. These results support the importance of teacher professional development in both pedagogical and content knowledge in statistics.

Equity. Matching 3 in this quasi-experiment differed in socio-economic status of students but both ended at high values of post-test scores in relationship to other classrooms in the study. The high conformity Teacher 10's school also reported free and reduced lunch amounts larger than most participating teachers' schools. Teacher 10 and 13 also had national averages for improvement and post-test scores well above national averages for college students. Students in these two teachers' classrooms were able to perform and improve in statistical reasoning well above the norm and their low conformity counterparts.

These situations exemplify what NCTM (2014) describes as "equity and access." When students have access to engaging mathematics curriculum, effective teaching and learning, and the support and resources needed through organizations like A+ College Ready to maximize their learning potential, they can perform at high levels of statistical reasoning. The two high conformity classrooms with large amounts of improvement and high post-test scores also support the fact that SRLE principles (Garfield and Ben-Zvi, 2008, 2009) and teacher practices in PtA can overcome differences in prior knowledge that may exist between students. Though Teacher 4's class had substantially larger pre-test scores than other sin the study with reportedly no free and reduced lunch, Teacher 10 and 13's classes improved to similar end of course reasoning ability

Based on the levels of improvement and end of course reasoning ability by students in Teachers 10 and 13 class within this study, teachers and teacher educators of statistics should help provide access to statistical courses that promote reasoning and sense making. This can be encouraged through the emphasis of statistical reasoning during development of teacher candidates in the post-secondary setting. All students in statistical classes should be asked to reason and make sense of mathematics regardless of students' prior knowledge. The development of high levels of reasoning and sense making in statistics is achievable in all classes that make this a priority.

Effective Teaching Practices. Substantial student gains and end of course reasoning ability in Teacher 10 and 13's classes also support the relationship between effective teaching practices in mathematics and statistics education. Students' ability to reason about statistics in classrooms with high conformity to SRLE principles exemplify the use of instructional practice and learning environments of students. In addition, these improvements demonstrate the relationship between effective teaching and learning in mathematics and statistics.

The SRLE's first principle requiring the focus of central statistical ideas rather than on presenting set of tools and procedures closely aligns with the PtA's goal of the establishment of goals for learning and the use of mathematical teaching practices. The establishment of clear learning goals for specific outcomes is stressed within PtA. Teachers should set clear learning targets for their students. If teachers believe that reasoning and sense making is important to the development of students, learning should focus around this and not be expected to happen by coincidence. Similarly, teacher educators should stress this during teacher preparation. The SETS document recommended that prospective teachers learn statistics in ways that enable them to develop a deep conceptual understanding of the statistics they will teach through appropriate modeling (Franklin et. al, 2015). Setting student goals that stretch beyond mere statistical competency and recitation of knowledge, that focus on reasoning and sense making in statistics should be a focus of student learning.

The SRLE's (Garfield & Ben-Zvi, 2008, 2009) principles of promoting classroom discourse through statistical arguments and sustained exchanges on significant statistical ideas is key in PtA's teaching practice standards as well. In particular, PtA stressed the use of meaningful mathematical discourse. PtA stated that students should have discourse among students that build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments (NCTM, 2014). Teachers should focus on developing a learning environment that promotes the reasoning and critiquing of others' statistical ideas. Similarly, teacher educators should provide opportunities for teacher candidates to experience this in their own classrooms. This can be illustrated through in-class demonstrations or teaching experiences and practice in field settings. The focus of discourse for teachers and teacher educators based on SRLE principles and PtA teaching practices should however be placed on students.

The use of technology as a tool to build student understanding in mathematics and statistics classes can help build students reasoning (Chance, et. al, 2007; NCTM, 2014). The

SRLE requires students' teachers to integrate the use of appropriate technological tools that allow students to test their conjectures, explore and analyze data, and develop their statistical reasoning. Teachers of statistical classrooms should use technology to build students understanding rather than just use it as a pedagogical tool. Teachers can do this by designing lessons around technology designed for specific learning targets. Activities that begin with smaller experiments can be developed and implemented using technology to gain deeper conceptual understanding then extended to asymptotic structures. Teacher educators can illustrate this in their own programs by providing opportunities to mix statistical content and technology in their coursework and methods courses. Focus should be placed however on the use of the technology to develop specific statistical content rather than programs to perform statistics for students without reasoning.

The use of effective curriculum that builds on student prior knowledge as well as promotes conceptual knowledge can be used to build statistical reasoning (NCTM, 2000). The SRLE requires teachers to use real and motivating data sets to engage students in making and testing conjectures. PtA supports curriculum that develops important mathematics along coherent learning progressions (NCTM, 2014). These progressions should be used to develop connections among areas of mathematical and statistical study and the real world (NCTM, 2014). The SRLE also promotes the use classroom activities to support the development of students' reasoning (Garfield & Ben-Zvi, 2008, 2009). Similarly, PtA encourages the implementation of tasks that promote reasoning and problem solving (NCTM, 2014). Teachers should use tasks that engage students in solving and discussing tasks (NCTM, 2014). These tasks should promote mathematical reasoning and problem solving in mathematics and statistics and allow for multiple entry points and varied solution strategies (NCTM, 2014). Teacher educators can reinforce these ideas by using curriculums in their methods courses that reinforce the creation and critiquing of student knowledge in their lessons.

Using evidence of student thinking through formative and summative assessments to build student understanding and guide instruction promote students' ability to statistically reason (NCTM, 2014). The SRLE required teachers to use assessment to learn what students know and to monitor the development of their statistical learning as well as to evaluate instructional plans and progress (Garfield & Ben-Zvi, 2008, 2009). PtA's last teaching practice promotes the elicitation and use of evidence to promote student thinking and learning (NCTM, 2014). Teachers should use evidence of student thinking to assess progress toward students' development of reasoning and sense making. Teachers can do this and teacher educators can model this by using authentic and performance based assessments as a tool of, for, and as learning.

Teacher Knowledge. Similarly, two of the three teachers (Teachers 4 and 13) who had experienced the most coursework in statistics also had students score substantially higher than the national average. Though more improvement was found in the higher conformity Teacher 13, teacher content knowledge may be a contributing factor to students' statistical reasoning ability (Jacobson & Lehrer, 2000/2002). For this reason, it is imperative for future teachers of statistics to have experiences with statistical content. Given mathematical content standards and trends of these standards, mathematics teacher educators should provide experiences for their teacher candidates to be successful. The Statistics Education of Teachers (SETS) (Franklin et. al, 2015) highlights some of the experiences that may potentially be worthwhile for teacher candidates.

The SETS document recommended that because many currently practicing teachers have not had an opportunity to learn statistics during their pre-service preparation programs, robust professional development opportunities need to be developed to advance in-service teachers' understanding of statistics (Franklin et. al, 2015). Similarly, SETS recommended all courses and professional development experiences for statistics teachers allow teacher candidates to develop the habits of mind of a statistical thinker and reasoner (Franklin et. al, 2015). Institutions need to provide opportunities for the development of currently practicing and future teachers' statistical knowledge at in-depth levels of reasoning and thinking. This development should stretch across disciplines and focus on the improvement of statistics teaching at all levels (Franklin et. al, 2015).

Future Research

Based on previously mentioned limitations, current research, and findings within this study, an extension of this study would be profitable in gaining additional insights into the impact of the SRLE on students' statistical reasoning. Factors that should be considered in an extension should include the effects of teachers taking part in the STI survey itself. It is possible that teachers taking part in the survey before the course could have exhibited reflection that influenced actual practice. This could be addressed by re-administration of the STI after the course. The reasoning assessment itself could be replaced with instruments that specifically monitor statistical reasoning habits such as analyzing a problem, implementing a strategy, monitoring one's progress, seeking and using connections, and reflecting on one's solution (Shaughnessy, Chance, and Kranendonk, 2009). A newer assessment tool using constructed responses and multiple choice items such as the Levels of Conceptual Understanding in Statistics (Jacobbe, 2015) could be used to help monitor the development of students' reasoning and conceptual understanding of statistics, especially at the K-12 levels.

Based on the experience of conducting this research experiment, it would be advisable to especially control for the difference between the intended and enacted curriculum (Stein, Remillard, and Smith, 2007). This could be addressed by a design that included observations of participants' instructional practice and/or a student survey on perceived instructional practice after the course. Using observation or a student survey may potentially reduce conflicts between teachers intended and enacted curriculum. Currently no known student survey exists that measures SRLE principles; thus, a useful first step would be the creation and validation of such a survey. Work by Taylor, Fraser, & White (1994) could be used as an excellent starting point for such a survey. Such a survey could easily be administered before or after the administration of the CAOS.

Two teachers' classes within this study had substantial gains compared to others within the study and to the national improvement rates for college students. Both of these classes' teachers were categorized as high conformity to SRLE principles. Potential exists to examine differences between high conformity classrooms with intentions of understanding the variation observed within high conformity classrooms. Specific questions may address the influence of NCTM's (2014) suggested mathematics teaching practices and principles from the SRLE (Garfield and Ben-Zvi, 2008). The lowest and highest increases in statistical reasoning were found within the high conformity classrooms making the understanding of this variance important for effective use of SRLE principles (Garfield and Ben-Zvi, 2008). Perhaps certain SRLE principles or mathematical teaching practices have differing impact levels on students' ability to reason. This could suggest priorities in professional development and teacher education. In addition, class size may have had more impact in high conformity classrooms than low conformity classrooms based on the need for student interaction and teacher facilitation.

Future research looking at the impact of SRLE principles on statistical reasoning might also explicitly address class size.

Comparison of teachers' classes with different levels of socio-economic status led to interesting results. The one low conformity teacher whose students scored as well as the two high conformity classrooms had extremely high pre-test scores and reported 0% free and reduced lunch. Students within this teacher's classrooms reported taking and completing high levels of math such as AP Calculus AB during their junior year in high school. This fact highlights the role that potential earlier coursework can play on students' ability to reason statistically. Unfortunately, research related to equity in statistics education and the effects of SRLE principles on inequity are very limited. NCTM (2000, 2014) has emphasized the role of equity and access to effective instruction in mathematics classes. Future research looking to understand the effects of SRLE principles for under-represented, at-risk, and/or lower level students could expand and contribute significantly to literature in statistics education.

In conclusion, even though results were not statistically significant when comparing low and high conformity classrooms when grouped as a whole, post hoc comparisons of the classrooms did provide evidence that supported the effectiveness of SRLE principles focusing on research based teaching, learning, assessment, and technology for promoting students' statistical reasoning. Findings from this study suggest the importance of continuing to study the development of students statistical reasoning. In addition, SRLE principles have shown potential to advance students' statistical reasoning.

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Appendix A: Statistics Teaching Inventory

<u>Statistics Teaching Inventory</u>

The following survey will take approximately 15 minute of your time. Please answer each question carefully and candidly and respond to all the questions in reference to an introductory statistics course. Refer to the AP statistics course that you currently teach or have recently taught.

Directions: For questions with a radio button: Select only one answer. For questions with a checkbox: Select all that apply. For some questions, you will be asked for a percentage. Use the slider to mark the percentage for these questions. If you are unsure of an exact percent, please enter an approximation. Use the arrows at the bottom right to move to the next set of questions.

This survey was first created and modified by Andrew Zieffler, Jiyoon Park, Joan Garfield, Robert delMas, and Audbjorg Bjornsdottir through a grant from the National Science Foundation (STEPS Project, NSF DUE-0808862).

Part I.

Consider the total amount of time that you meet face-to-face with your students. Approximately what percentage of this time is spent on each of the following? (Note: The percentages below should add up to 100%)

______ 1. Students meeting together as a whole class (not in small groups) for lecture, discussion, or demonstration:

______ 2. Students working in groups:

______ 3. Students working individually on an activity:

______ 4. Students taking an assessment:

5. Consider a student who was fully engaged in your course. To what extent do you think that student would agree or disagree with the following statements about this course?

	Strongly Disagree	Disagree	Agree	Strongly Agree
a) The content was presented mostly through the instructor's lectures.	❍	❍	❍	❍
b) The instructor asked challenging questions that made me think.	❍	❍	❍	❍
c) The course frequently required students to work together.	❍	❍	❍	❍
d) The content was presented mostly through activities.	❍	❍	❍	❍
e) This course encouraged students to discover ideas on their own.	❍	❍	❍	❍
f) This course often used technology (e.g. web applets, statistical software) to help students understand concepts.	❍	❍	❍	❍

Part II.

The following items will ask you about your curricular emphasis. Consider the entirety of your course as you complete this section. To what extent are the following addressed in your course?

	Seldom or not at all	A few times	Repeatedly
1. The need to base decisions on evidence (data)	❍	❍	❍
2. Difficulties involved in getting good quality data	❍	❍	❍
3. The study of variability is at the core of statistics	❍	❍	❍
4. The need to select an appropriate model for making a statistical inference	❍	❍	❍
5. The process of selecting an appropriate model for making a statistical inference	❍	❍	❍

To what extent do you emphasize each of the following approaches to statistical inference in your course?

	Not at all	To some extent	A major emphasis
6. Parametric methods (e.g. t-test, z-test)	❍	❍	❍
7. Bayesian methods	❍	❍	❍
8. Simulation/resampling (e.g. randomization, bootstrap methods)	❍	❍	❍
9. Other (Please describe):	❍	❍	❍

10. Of all the data sets students see in this course, what portion of them are real data?

❍ None
❍ A few
❍ About half
❍ Most of them
❍ All of them

Part III.

1. Other than hand calculators, do students use technology tools during the course?

❍ Yes
❍ No

If Yes Is Selected, Then Skip To 3. In what settings do students work ...If No Is Selected, Then Skip To 2. What are your reasons for not using...

2. What are your reasons for not using technology other than hand calculators in your course?

(Select all that apply.)

❑ there is no computer technology available
❑ there are departmental constraints on technology use
❑ students are already provided with statistical output
❑ students use hand calculators to compute statistics using formulas
❑ Other ____________________

If 2. What are your reasons fo... Is Greater Than or Equal to 0, Then Skip To End of Block

3. In what settings do students work with each of these technology tools? (Select all that apply.)

	Delivery of course content	Activities and assignments (e.g. homework, projects)	Assessments (e.g. quizzes, exams)
Statistical Analysis Packages (minitab, SPSS, IMP, StatCrunch, etc.)	❑	❑	❑
Graphing calculators with built in statistical functions	❑	❑	❑
Spreadsheet tools (e.g. Excel)	❑	❑	❑
Web Applets	❑	❑	❑
Conceptual Software (e.g. TinkerPlots, Fathom, etc.)	❑	❑	❑
Other:	❑	❑	❑

Questions 4 and 5 ask you to consider how students use technology. In answering these questions, consider the total amount of time students use technology. (These responses do not need to add up to 100%.)

______ 4. What percentage of time that students spend using technology is designed to spent analyzing data?

______ 5. What percentage of time that students spend using technology is designed to be spent understanding statistical concepts?

Part IV.

Consider your total set of assessments that count for a grade in your class. Approximately what percentage of the students' grade is dedicated to evaluating each of the following? (These percentages do not need to add up to 100%.)

______ 1. Students' ability to use formulas to produce numerical summaries of a data set:

______ 2. Students' ability to perform step-by-step calculations to compute answers to problems:

______ 3. Students' ability to critically examine statistics in the media:

______ 4. Students' ability to interpret results of a statistical analysis:

______ 5. Students' ability to reason correctly about important statistical concepts:

______ 6. Students' ability to successfully complete a statistical investigation (e.g., a course project):

______ 7. Other (please describe):

Part V.

Please rate the extent to which you agree or disagree with each of the following statements as they reflect your beliefs (but not necessarily your actual teaching) regarding the teaching, learning, and assessment of introductory statistics:

	Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
1. Rules of probability should be included in an introductory statistics course.	❍	❍	❍	❍	❍
2. The topic of theoretical probability distributions (e.g., the binomial distribution) should be included in an introductory statistics course.	❍	❍	❍	❍	❍
3. Students should learn how to read statistical tables of theoretical distributions (e.g., t-table, F-table).	❍	❍	❍	❍	❍
4. Technology tools should be used to illustrate most abstract statistical concepts.	❍	❍	❍	❍	❍
5. Students should learn the importance of using appropriate methods for collecting data.	❍	❍	❍	❍	❍
6. Students should learn connections between the quality/nature of the data and inferences that are made.	❍	❍	❍	❍	❍
7. Students should learn fewer topics in greater depth instead of learning more topics in less depth.	❍	❍	❍	❍	❍
8. Lectures should be the primary way for students to learn statistical content.	❍	❍	❍	❍	❍
9. Quizzes and exams should be used as the primary way to evaluate student learning.	❍	❍	❍	❍	❍
10. Alternative assessments (e.g., projects, presentations,) should be used to provide important information about student learning.	❍	❍	❍	❍	❍

11. All assessments should be regularly reviewed to see that they are aligned with important student learning goals.	❍	❍	❍	❍	❍
12. Assessments should be used to provide formative feedback to students to improve their learning.	❍	❍	❍	❍	❍
13. Students should be assessed on their ability to complete an open-ended statistical problem.	❍	❍	❍	❍	❍
14. Students should be assessed on their statistical literacy (e.g., ability to read a graph, understand common statistical words, etc.).	❍	❍	❍	❍	❍
15. Students should analyze data primarily using technology.	❍	❍	❍	❍	❍
16. Statistics courses should be updated continually in light of developments such as new technology and common core curriculum requirements.	❍	❍	❍	❍	❍
17. Statistics instructors should be actively engaged in the statistics education community.	❍	❍	❍	❍	❍

Part VI.

1. How many students are enrolled in one typical section of this course?

2. Please indicate the mathematical prerequisite for this course:

- ❍ Calculus
- ❍ Pre-Calculus
- ❍ Algebra II
- ❍ None
- ❍ Other ____________________

3. Identify any constraints that keep you from making changes that you would like to implement to improve your course. (Select all that apply):

- ❑ Personal time constraints
- ❑ Departmental or institutional constraints
- ❑ The teaching assistants you work with
- ❑ Technology constraints (e.g., lack of computer lab, cost of software)
- ❑ Characteristics of students (ability, interest, etc.)
- ❑ Limitations in terms of what can be done within the classroom management system
- ❑ Your own comfort level with the classroom management system
- ❑ Other ____________________

Part VII.

1. How many years have you been teaching an introductory statistics course?

2. In your graduate coursework, how many courses did you take in theoretical statistics (e.g., mathematical statistics, probability)?

- ❍ None
- ❍ 1
- ❍ 2
- ❍ 3
- ❍ 4
- ❍ 5 or more

3. In your graduate coursework, how many courses did you take in applied statistics (i.e., involved the analysis of data)?

- ❍ None
- ❍ 1
- ❍ 2
- ❍ 3
- ❍ 4
- ❍ 5 or more

4. Please rate the amount of experience you have had in analyzing data outside of your coursework in statistics (e.g., in your own research, consulting, etc.).

- ❍ No experience
- ❍ Very little experience
- ❍ Some experience
- ❍ A lot of experience

For items 5-8, please indicate the number of professional development opportunities in which you have participated during the last 2 years to improve your teaching of statistics.

	0	1	2	3	4	5 or more
5. Live or pre-recorded webinars (online seminars):	❍	❍	❍	❍	❍	❍
6. Workshops	❍	❍	❍	❍	❍	❍
7. Short courses/minicourses	❍	❍	❍	❍	❍	❍
8. Other:	❍	❍	❍	❍	❍	❍

9. How many years have you been a part of the A+ College Ready program?

10. Please provide any additional comments in the space below.

11. The purpose of this research is to monitor student reasoning in your statistics class. Teachers whose students participate in this research will be required to administer a pre-assessment and post-assessment that measures students' statistical reasoning ability. In addition, teachers will be required to distribute opt-out letters to students at the beginning of the course to ensure student and parent willingness to participate. The researcher will contact the teachers' district leader responsible for research for approval before the teacher sends home consent letters to parents. Would you be willing to participate further in this research if your district leader approves?

❍ Yes
❍ No

If No Is Selected, Then Skip To End of Survey. If Yes Is Selected, Then Skip To While all responses will remain c...

While all responses will remain confidential, your contact information is needed for further participation in this research.

Name:	
School:	
E-mail:	
Number of Students Enrolled at your school in grades 9-12:	
Number of Students in your AP Statistics course:	
Percent of Students with Free/Reduced Lunch:	

Appendix B: Comprehensive Assessment of Outcomes

Comprehensive Assessment of Outcomes Cover Sheet

Please read and follow these directions to fill out your Scantron.

You have ***up to 1 hour*** to complete this exam. Please use a ***pencil*** on the Scantron form to ensure proper scoring. This test does ***not require a calculator***, but can be used during testing. There are ***40 questions*** on this exam.

In the name section of your Scantron form, use the following information to create your unique anonymous ID.

1. your middle initial - use "X" if you do not have a middle name
2. the first initial of your mother's first name - use "X" if unknown
3. the first initial of your father's first name - use "X" if unknown
4. first digit of your birth month - (01, 02, …, 12)
5. second digit of your birth month - (01, 02, …, 12)
6. number of older siblings - use "9" if greater than nine

Write in the subject the name of your last completed mathematics course.

Write the date you are taking the exam in the date section.

Write in the hour box your teachers name.

When you are finished, please check your Scantron for stray marks and ensure the rectangles are filled completely.

DO NOT TURN THE PAGE UNTIL DIRECTED TO DO SO.

Comprehensive Assessment of Outcomes for a first course in Statistics (CAOS)

CAOS 4

Developed by the Web ARTIST Project https://app.gen.umn.edu/artist/

Funded by a grant from the National Science Foundation NSF
CCLI ASA- 0206571

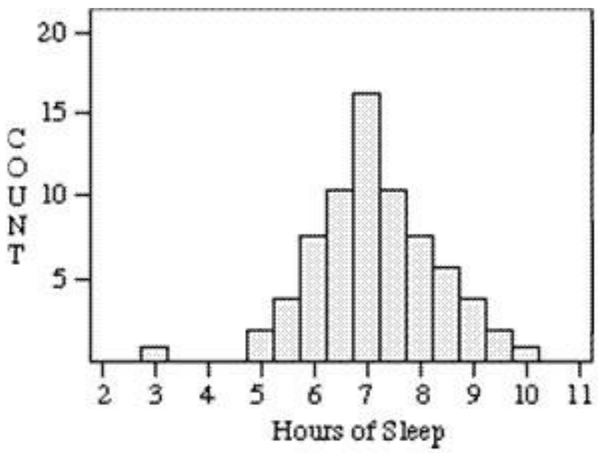
Principal Investigators:
Joan Garfield and Bob delMas, University of Minnesota
Beth Chance, Cal Poly – San Luis Obispo Post-doctoral
Research Assistant:
Ann Ooms, University of Minnesota

Version 31

September 8, 2005

ARTIST CAOS 4 POSTTEST

The following graph shows a distribution of hours slept last night by a group of college students.



1. Select the statement below that gives the most complete description of the graph in a way that demonstrates an understanding of how to statistically describe and interpret the distribution of a variable.

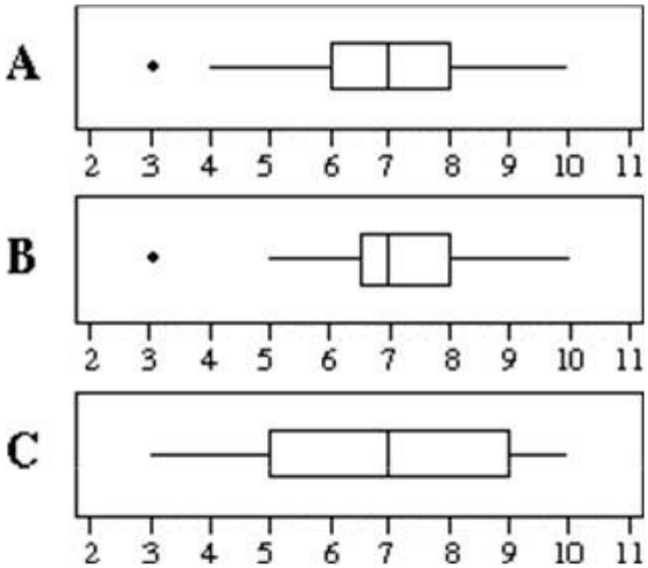
    a. The bars go from 3 to 10, increasing in height to 7, then decreasing to 10. The tallest bar is at 7. There is a gap between three and five.

    b. The distribution is normal, with a mean of about 7 and a standard deviation of about 1.

    c. Most students seem to be getting enough sleep at night, but some students slept more and some slept less. However, one student must have stayed up very late and got very few hours of sleep.

    d. The distribution of hours of sleep is somewhat symmetric and bell-shaped, with an outlier at 3. The typical amount of sleep is about 7 hours and overall range is 7 hours.

2. Which box plot seems to be graphing the same data as the histogram in question 1?

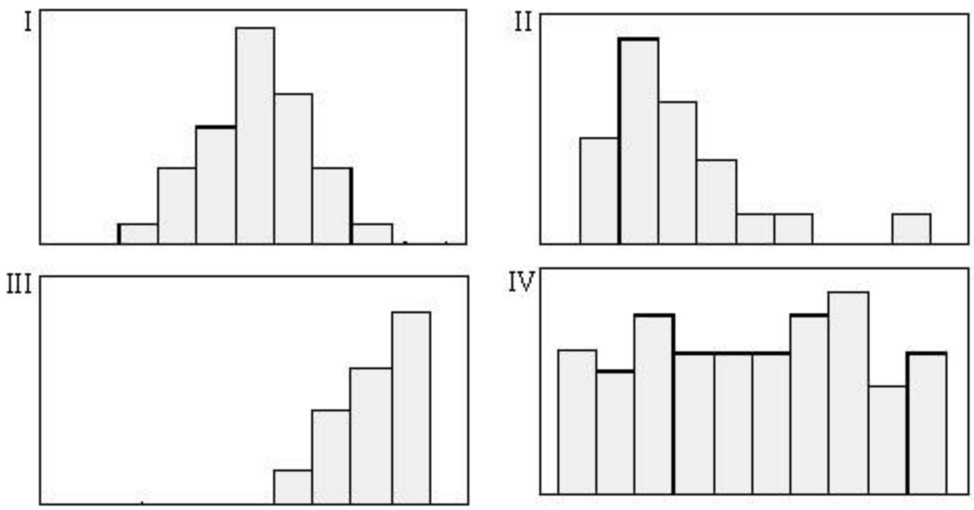


a. Boxplot A.

b. Boxplot B.

c. Boxplot C.

Items 3 to 5 refer to the following situation:
Four histograms are displayed below. For each item, match the description to the appropriate histogram.



3. A distribution for a set of quiz scores where the quiz was very easy is represented by:
   a. Histogram I.
   b. Histogram II.
   c. Histogram III.
   d. Histogram IV.

4. A distribution for a set of wrist circumferences (measured in centimeters) taken from the right wrist of a random sample of newborn female infants is represented by:
   a. Histogram I.
   b. Histogram II.
   c. Histogram III.
   d. Histogram IV.

5. A distribution for the last digit of phone numbers sampled from a phone book (i.e., for the phone number 968-9667, the last digit, 7, would be selected) is represented by: a. Histogram I.

   b. Histogram II.

   c. Histogram III.

   d. Histogram IV.

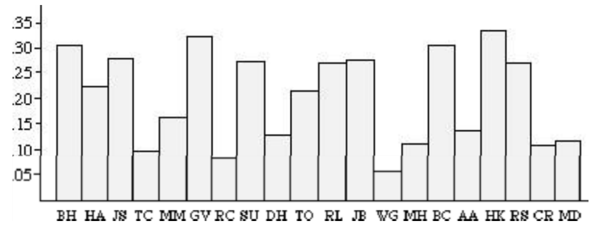
6. A baseball fan likes to keep track of statistics for the local high school baseball team. One of the statistics she recorded is the proportion of hits obtained by each player based on the number of times at bat as shown in the table below. Which of the following graphs gives the best display of the distribution of proportion of hits in that it allows the baseball fan to describe the shape, center and spread of the variable, proportion of hits?

Player	Proportion of hits
BC	0.301
AA	0.143
HK	0.341
RS	0.261
CR	0.115
MD	0.125

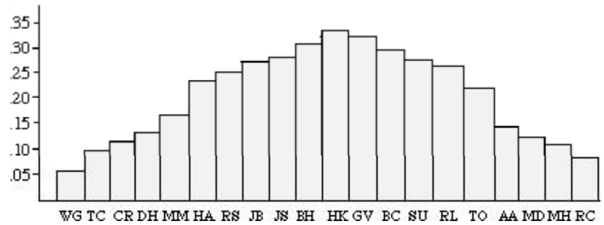
Player	Proportion of hits
BH	0.305
HA	0.229
JS	0.281
TC	0.097
MM	0.167
GV	0.333
RC	0.085

Player	Proportion of hits
SU	0.270
DH	0.136
TO	0.218
RL	0.267
JB	0.270
WG	0.054
MH	0.108

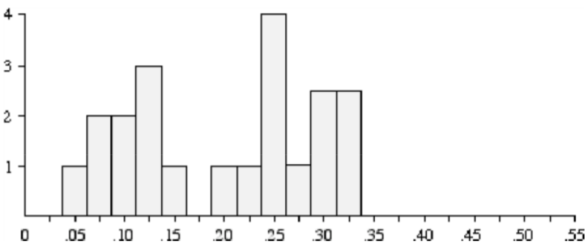
A



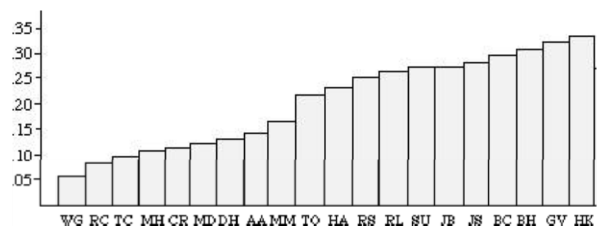
B



C



D



7. A recent research study randomly divided participants into groups who were given different levels of Vitamin E to take daily. One group received only a placebo pill. The research study followed the participants for eight years to see how many developed a particular type of cancer during that time period. Which of the following responses gives the best explanation as to the purpose of randomization in this study?

   a. To increase the accuracy of the research results.

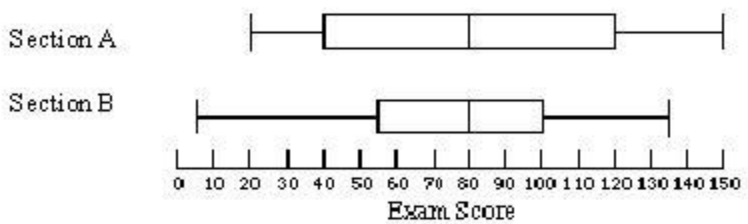
   b. To ensure that all potential cancer patients had an equal chance of being selected for the study.

   c. To reduce the amount of sampling error.

   d. To produce treatment groups with similar characteristics.

   e. To prevent skewness in the results.

Items 8 to 10 refer to the following situation:
The two boxplots below display final exam scores for all students in two different sections of the same course.



8. Which section would you expect to have a greater standard deviation in exam scores?

    a. Section A.

    b. Section B.

    c. Both sections are about equal.

    d. It is impossible to tell.

9. Which data set has a greater percentage of students with scores at or below 30?

    a. Section A.

    b. Section B.

    c. Both sections are about equal.

    d. It is impossible to tell.

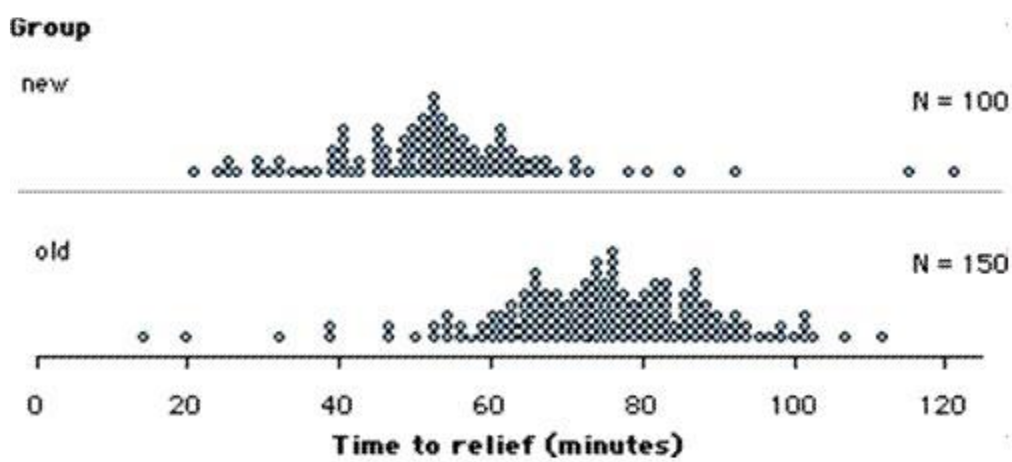
10. Which section has a greater percentage of students with scores at or above 80?

    a. Section A.

    b. Section B.

    c. Both sections are about equal.

Items 11 to 13 refer to the following situation:
A drug company developed a new formula for their headache medication. To test the effectiveness of this new formula, 250 people were randomly selected from a larger population of patients with headaches. 100 of these people were randomly assigned to receive the new formula medication when they had a headache, and the other 150 people received the old formula medication. The time it took, in minutes, for each patient to no longer have a headache was recorded. The results from both of these clinical trials are shown below. Items 11, 12, and 13 present statements made by three different statistics students. For each statement, indicate whether you think the student's conclusion is valid.



11. The old formula works better. Two people who took the old formula felt relief in less than 20 minutes, compared to none who took the new formula. Also, the worst result - near 120 minutes - was with the new formula.

    a. Valid.

    b. Not valid.

12. The average time for the new formula to relieve a headache is lower than the average time for the old formula. I would conclude that people taking the new formula will tend to feel relief about 20 minutes sooner than those taking the old formula.

    a. Valid.

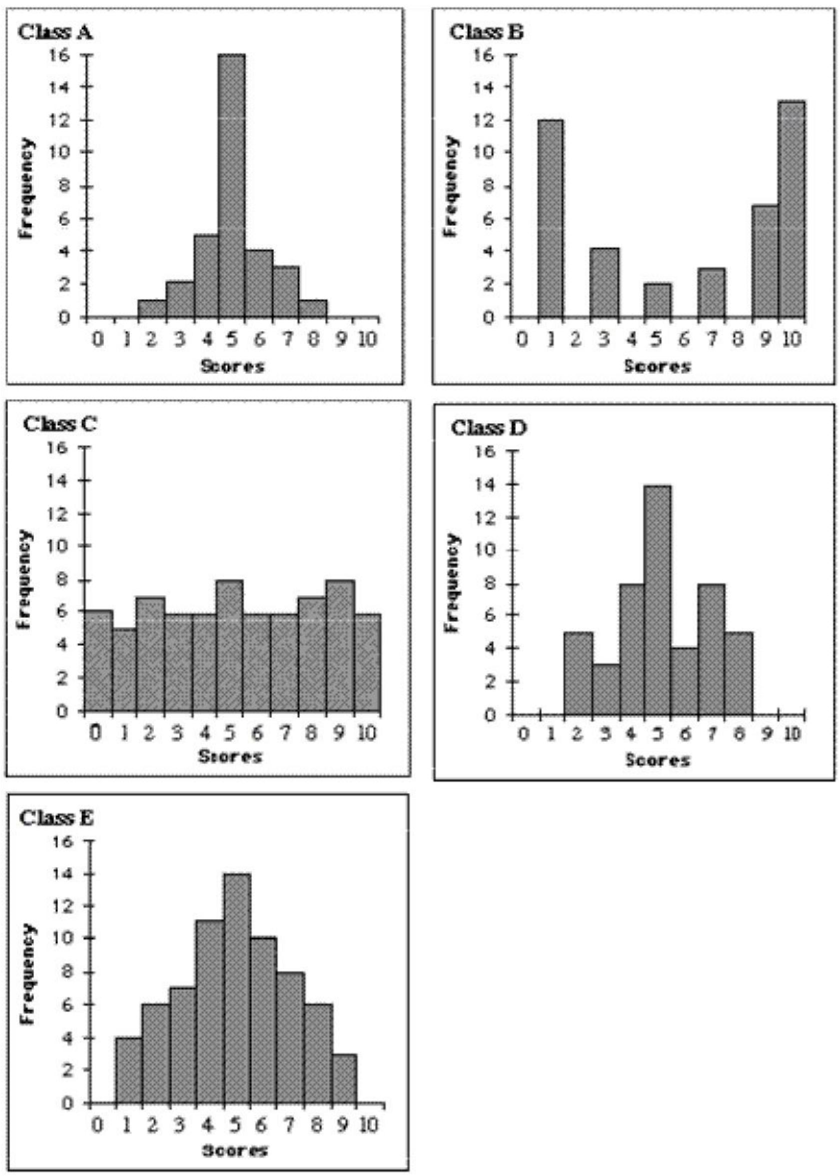
b. Not valid.

13. I would not conclude anything from these data. The number of patients in the two groups is not the same so there is no fair way to compare the two formulas. a. Valid.

b. Not valid.

Items 14 and 15 refer to the following situation:

Five histograms are presented below. Each histogram displays test scores on a scale of 0 to 10 for one of five different statistics classes.



14. Which of the classes would you expect to have the lowest standard deviation, and why?

    a. Class A, because it has the most values close to the mean.

    b. Class B, because it has the smallest number of distinct scores.

c. Class C, because there is no change in scores.

d. Class A and Class D, because they both have the smallest range.

e. Class E, because it looks the most normal.

15. Which of the classes would you expect to have the highest standard deviation, and why?

    a. Class A, because it has the largest difference between the heights of the bars.

    b. Class B, because more of its scores are far from the mean.

    c. Class C, because it has the largest number of different scores.

    d. Class D, because the distribution is very bumpy and irregular.

    e. Class E, because it has a large range and looks normal.

16. A certain manufacturer claims that they produce 50% brown candies. Sam plans to buy a large family size bag of these candies and Kerry plans to buy a small fun size bag. Which bag is more likely to have more than 70% brown candies?

    a. Sam, because there are more candies, so his bag can have more brown candies.

    b. Sam, because there is more variability in the proportion of browns among larger samples.

    c. Kerry, because there is more variability in the proportion of browns among smaller samples.

    d. Kerry, because most small bags will have more than 50% brown candies.

    e. Both have the same chance because they are both random samples.

17. Imagine you have a barrel that contains thousands of candies with several different colors. We know that the manufacturer produces 35% yellow candies. Five students each take a random sample of 20 candies, one at a time, and record the percentage of yellow candies in their sample. Which sequence below is the most plausible for the percent of yellow candies obtained in these five samples?

    a. 30%, 35%, 15%, 40%, 50%.

    b. 35%, 35%, 35%, 35%, 35%.

    c. 5%, 60%, 10%, 50%, 95%.

    d. Any of the above.

18. Jean lives about 10 miles from the college where she plans to attend a 10-week summer class. There are two main routes she can take to the school, one through the city and one through the countryside. The city route is shorter in miles, but has more stoplights. The country route is longer in miles, but has only a few stop signs and stoplights. Jean sets up a randomized experiment where each day she tosses a coin to decide which route to take that day. She records the following data for 5 days of travel on each route.

Country Route - 17, 15, 17, 16, 18

City Route - 18, 13, 20, 10, 16

It is important to Jean to arrive on time for her classes, but she does not want to arrive too early because that would increase her parking fees. Based on the data gathered, which route would you advise her to choose?

a. The Country Route, because the times are consistently between 15 and 18 minutes.

b. The City Route, because she can get there in 10 minutes on a good day and the average time is less than for the Country Route.

c. Because the times on the two routes have so much overlap, neither route is better than the other. She might as well flip a coin.

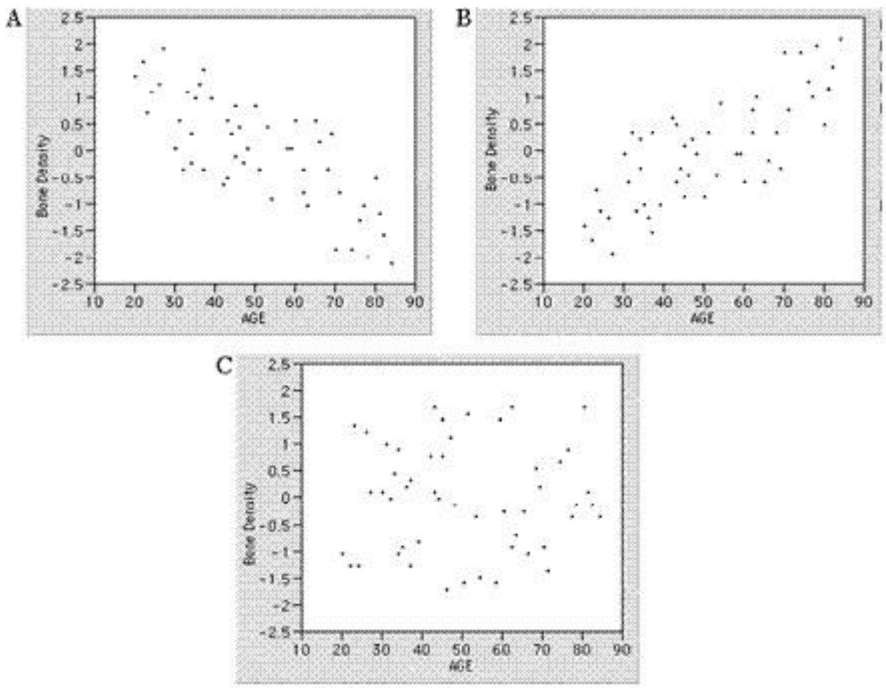
19. A graduate student is designing a research study. She is hoping to show that the results of an experiment are statistically significant. What type of *p*-value would she want to obtain?

a. A large *p*-value.

b. A small *p*-value.

c. The magnitude of a *p*-value has no impact on statistical significance.

20. Bone density is typically measured as a standardized score with a mean of 0 and a standard deviation of 1. Lower scores correspond to lower bone density. Which of the following graphs shows that as women grow older they tend to have lower bone density?

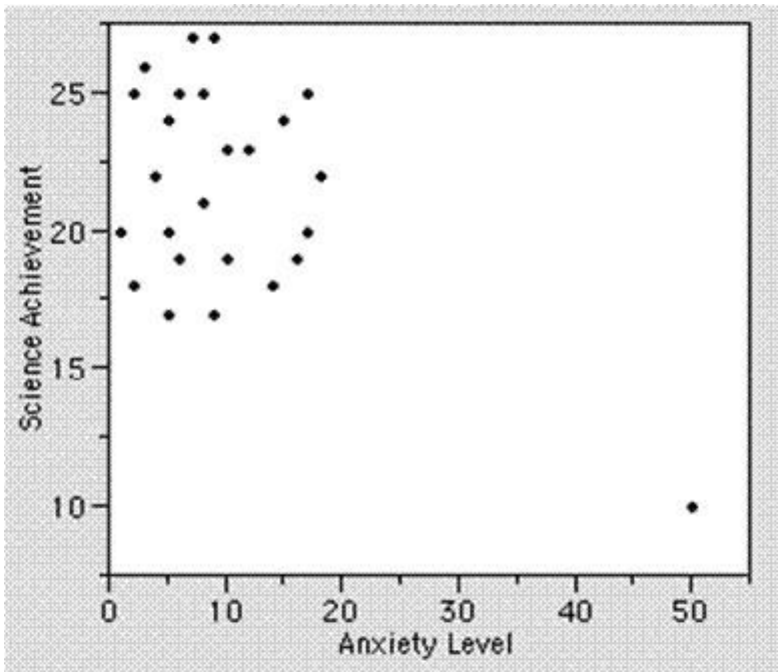


a. Graph A.

b. Graph B.

c. Graph C.

21. The following scatterplot shows the relationship between scores on an anxiety scale and an achievement test for science. Choose the best interpretation of the relationship between anxiety level and science achievement based on the scatterplot.



a. This graph shows a strong negative linear relationship between anxiety and achievement in science.

b. This graph shows a moderate linear relationship between anxiety and achievement in science.

c. This graph shows very little, if any, linear relationship between anxiety and achievement in science.

22. Researchers surveyed 1,000 randomly selected adults in the U.S. A statistically significant, strong positive correlation was found between income level and the number of containers of recycling they typically collect in a week. Please select the best interpretation of this result.

a. We can not conclude whether earning more money causes more recycling among U.S. adults because this type of design does not allow us to infer causation.

b. This sample is too small to draw any conclusions about the relationship between income level and amount of recycling for adults in the U.S.

c. This result indicates that earning more money influences people to recycle more than people who earn less money.

Items 23 and 24 refer to the following situation:

A researcher in environmental science is conducting a study to investigate the impact of a particular herbicide on fish. He has 60 healthy fish and randomly assigns each fish to either a treatment or a control group. The fish in the treatment group showed higher levels of the indicator enzyme.

23. Suppose a test of significance was correctly conducted and showed no statistically significant difference in average enzyme level between the fish that were exposed to the herbicide and those that were not. What conclusion can the graduate student draw from these results?

    a. The researcher must not be interpreting the results correctly; there should be a significant difference.

    b. The sample size may be too small to detect a statistically significant difference.

    c. It must be true that the herbicide does not cause higher levels of the enzyme.

24. Suppose a test of significance was correctly conducted and showed a statistically significant difference in average enzyme level between the fish that were exposed to the herbicide and those that were not. What conclusion can the graduate student draw from these results?

    a. There is evidence of association, but no causal effect of herbicide on enzyme levels.

    b. The sample size is too small to draw a valid conclusion.

    c. He has proven that the herbicide causes higher levels of the enzyme.

    d. There is evidence that the herbicide causes higher levels of the enzyme for these fish.

Items 25 to 27 refer to the following situation:

A research article reports the results of a new drug test. The drug is to be used to decrease vision loss in people with Macular Degeneration. The article gives a *p*-value of .04 in the analysis section. Items 25, 26, and 27 present three different interpretations of this *p*value. Indicate if each interpretation is valid or invalid.

25. The probability of getting results as extreme as or more extreme than the ones in this study if the drug is actually not effective. a. Valid.

b. Invalid.

26. The probability that the drug is not effective.

    a. Valid.
    b. Invalid.

27. The probability that the drug is effective.

    a. Valid.
    b. Invalid.

Items 28 to 31 refer to the following situation:

A high school statistics class wants to estimate the average number of chocolate chips in a generic brand of chocolate chip cookies. They collect a random sample of cookies, count the chips in each cookie, and calculate a 95% confidence interval for the average number of chips per cookie (18.6 to 21.3). Items 28, 29, and 30 present four different interpretations of these results. Indicate if each interpretation is valid or invalid.

28. We are 95% certain that each cookie for this brand has approximately 18.6 to 21.3 chocolate chips.

    a. Valid.
    b. Invalid.

29. We expect 95% of the cookies to have between 18.6 and 21.3 chocolate chips.

    a. Valid.

    b. Invalid.

30. We would expect about 95% of all possible sample means from this population to be between 18.6 and 21.3 chocolate chips.

    a. Valid.

    b. Invalid.

31. We are 95% certain that the confidence interval of 18.6 to 21.3 includes the true average number of chocolate chips per cookie.

    a. Valid.

    b. Invalid.

32. It has been established that under normal environmental conditions, adult largemouth bass in Silver Lake have an average length of 12.3 inches with a standard deviation of 3 inches. People who have been fishing Silver Lake for some time claim that this year they are catching smaller than usual largemouth bass. A research group from the Department of Natural Resources took a random sample of 100 adult largemouth bass from Silver Lake and found the mean of this sample to be 11.2 inches. Which of the following is the most appropriate statistical conclusion?

    a. The researchers cannot conclude that the fish are smaller than what is normal because 11.2 inches is less than one standard deviation from the established mean (12.3 inches) for this species.

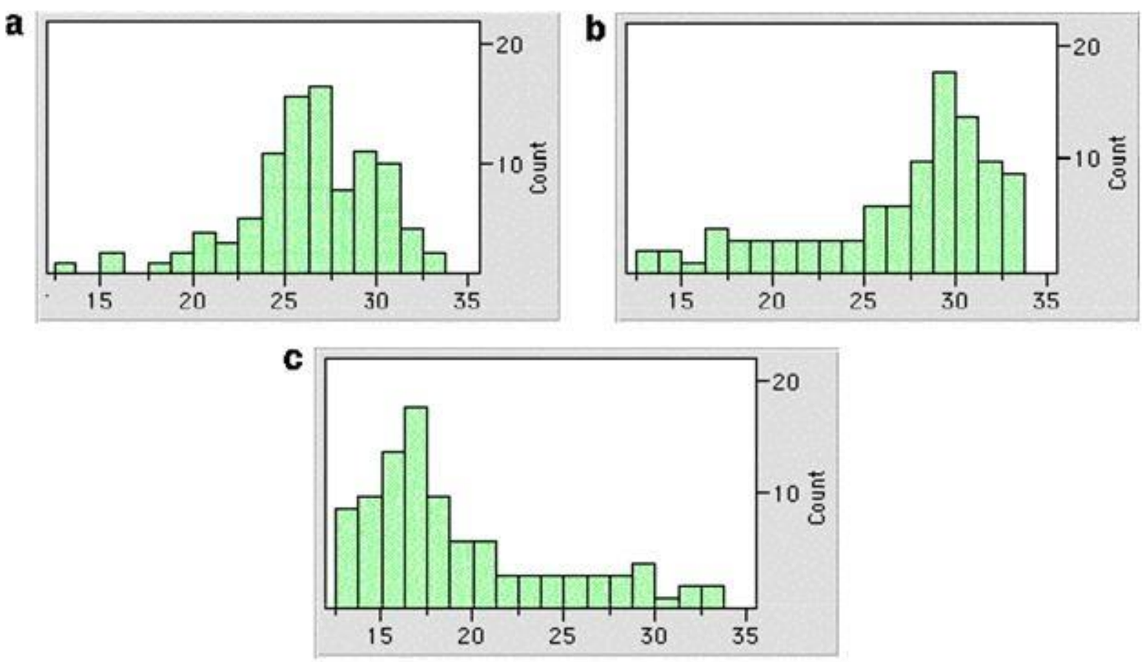
    b. The researchers can conclude that the fish are smaller than what is normal because the sample mean should be almost identical to the population mean with a large sample of 100 fish.

    c. The researchers can conclude that the fish are smaller than what is normal because the difference between 12.3 inches and 11.2 inches is much larger than the expected sampling error.

A study examined the length of a certain species of fish from one lake. The plan was to take a random sample of 100 fish and examine the results. Numerical summaries on lengths of the fish measured in this study are given.

Mean	26.8mm
Median	29.4mm
Standard Deviation	5.0 mm
Minimum	12.mm
Maximum	33.4mm

33. Which of the following histograms is most likely to be the one for these data?



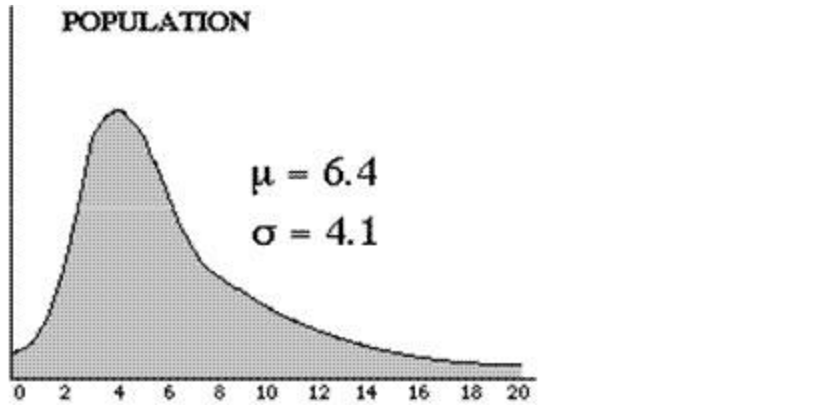
a. Histogram a.

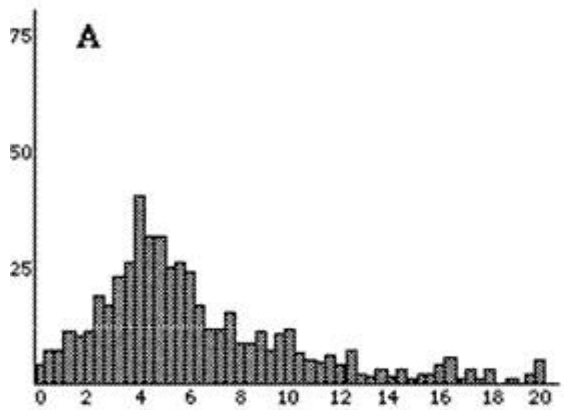
b. Histogram b.

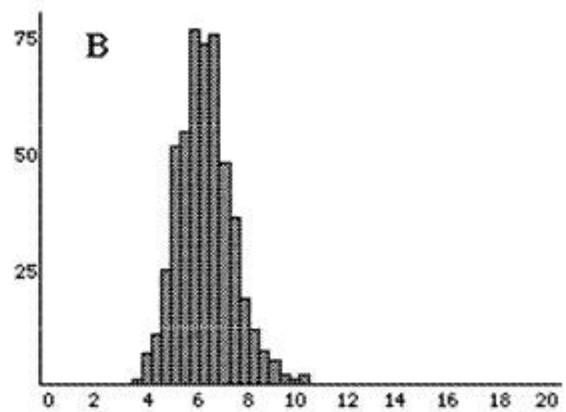
c. Histogram c.

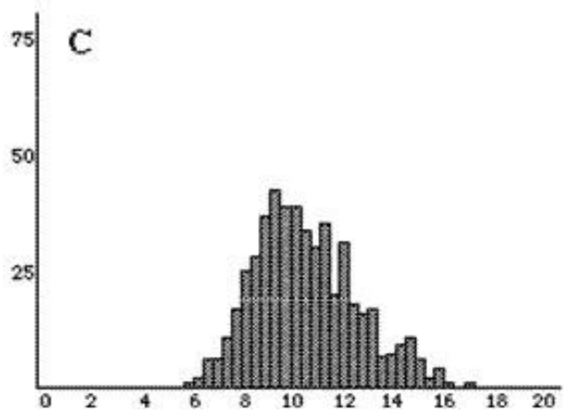
Items 34 and 35 refer to the following situation:

Four graphs are presented below. The graph at the top is a distribution for a population of test scores. The mean score is 6.4 and the standard deviation is 4.1.









34. Which graph (A, B, or C) do you think represents a single random sample of 500 values from this population?

a. Graph A

b. Graph B

c. Graph C

35. Which graph (A, B, or C) do you think represents a distribution of 500 sample means from random samples each of size 9?

a. Graph A

b. Graph B

c. Graph C

36. This table is based on records of accidents compiled by a State Highway Safety and Motor Vehicles Office. The Office wants to decide if people are less likely to have a fatal accident if they are wearing a seatbelt. Which of the following comparisons is most appropriate for supporting this conclusion?

Safety Equipment in Use	Injury		**ROW TOTAL**
	Nonfatal	Fatal	
Seat Belt	412,368	510	412,878
No Seat Belt	162,527	1,601	164,128
COLUMN TOTAL	574,895	2,111	577,006

a. Compare the ratios 510/412,878 and 1,601/164,128

b. Compare the ratios 510/577,006 and 1,601/577,006

c. Compare the numbers 510 and 1,601

37. A student participates in a Coke versus Pepsi taste test. She correctly identifies which soda is which four times out of six tries. She claims that this proves that she can reliably tell the difference between the two soft drinks. You have studied statistics and you want to determine the probability of anyone getting at least four right out of six tries just by chance alone. Which of the following would provide an accurate estimate of that probability?

    a. Have the student repeat this experiment many times and calculate the percentage time she correctly distinguishes between the brands.

    b. Simulate this on the computer with a 50% chance of guessing the correct soft drink on each try, and calculate the percent of times there are four or more correct guesses out of six trials.

    c. Repeat this experiment with a very large sample of people and calculate the percentage of people who make four correct guesses out of six tries.

    d. All of the methods listed above would provide an accurate estimate of the probability.

38. A college official conducted a survey to estimate the proportion of students currently living in dormitories about their preference for single rooms, double rooms, or multiple (more than two people) rooms in the dormitories on campus. Which of the following does NOT affect the college official's ability to generalize the survey results to all dormitory students?

    a. Five thousand students live in dormitories on campus. A random sample of only 500 were sent the survey.

    b. The survey was sent to only first-year students.

    c. Of the 500 students who were sent the survey, only 160 responded.

    d. All of the above present a problem for generalizing the results.

39. The number of people living on American farms has declined steadily during the last century. Data gathered on the U.S. farm population (millions of people) from 1910 to 2000 were used to generate the following regression equation: Predicted Farm Population = 1167 - .59 (YEAR). Which method is best to use to predict the number of people living on farms in 2050?

   a. Substitute the value of 2050 for YEAR in the regression equation, and compute the predicted farm population.

   b. Plot the regression line on a scatterplot, locate 2050 on the horizontal axis, and read off the corresponding value of population on the vertical axis.

   c. Neither method is appropriate for making a prediction for the year 2050 based on these data.

   d. Both methods are appropriate for making a prediction for the year 2050 based on these data.

40. The following situation models the logic of a hypothesis test. An electrician uses an instrument to test whether or not an electrical circuit is defective. The instrument sometimes fails to detect that a circuit is good and working. The null hypothesis is that the circuit is good (not defective). The alternative hypothesis is that the circuit is not good (defective). If the electrician rejects the null hypothesis, which of the following statements is true?

   a. The circuit is definitely not good and needs to be repaired.

   b. The electrician decides that the circuit is defective, but it could be good.

   c. The circuit is definitely good and does not need to be repaired.

   d. The circuit is most likely good, but it could be defective.

Appendix C: Permission Forms

AUBURN
UNIVERSITY

COLLEGE OF EDUCATION

DEPARTMENT OF CURRICULUM AND TEACHING

INFORMATION LETTER FOR PARENTS of a Research Study entitled
"Using Effective Teaching Methods to Develop Students' Statistical Reasoning"

Dear Parent and/or Guardian,

Your child is invited to participate in a research study being conducted by Basil Conway IV, a graduate student in the Department of Curriculum and Teaching at Auburn University. The purpose of the research study is to monitor students' development of statistical reasoning during their AP statistics course during the 2014-2015 school year.

To monitor your child's development of statistical reasoning during the course, your child will be asked to take a pre-test at the beginning and a post-test at the end of the 2014-2015 school year. The reasoning assessment takes no longer than 1 hour to complete and will take place during your child's AP statistics class at the beginning and ending of the school year.

Your child's participation in this research project is completely voluntary. There is no compensation and no costs associated with participation in this research. You may decline to have your child participate altogether, in which case your child will be given an alternative assignment to completing the examinations. There are no known risks to participation beyond those encountered in everyday life. Your child's identity will be anonymous, and data from this research will only be reported in summary form. Your child's teacher will be provided summary information from this assessment to help inform instruction at the beginning of the course and to inform test preparation needs before the AP statistics exam in April 2015.

If for any reason **you do not wish** your son or daughter to participate in the research project, **please sign this form and return it to you child's teacher by __________________________.**

Parent Name: ________________________

Student Name: ________________________

Parent Signature: ________________________

The Auburn University Institutional Review Board has approved this document for use from 9/28/14 to 9/27/15
Protocol # 14-303 EP 1409

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AUBURN

UNIVERSITY

COLLEGE OF EDUCATION

DEPARTMENT OF CURRICULUM AND TEACHING

INFORMATION LETTER FOR STUDENTS of a Research Study entitled "Using Effective Teaching Methods to Develop Students' Statistical Reasoning"

Dear Student,

You are invited to participate in a research study being conducted by Basil Conway IV, a graduate student in Department of Curriculum and Teaching at Auburn University. The purpose of the research study is to monitor students' development of statistical reasoning in their AP statistics course during the 2014-2015 school year.

To monitor your development of statistical reasoning during the course, you will be asked to take a pre-test at the beginning and a post-test at the end of the 2014-2015 school year. The reasoning assessment takes no longer than 1 hour to complete and will take place during your AP statistics class at the beginning and ending of the school year.

Your participation in this research project is completely voluntary. There is no compensation and no costs associated with participation in this research. You may decline to participate altogether, in which case you will be given an alternative assignment during class. There are no known risks to participation beyond those encountered in everyday life. Your identity will be anonymous, and data from this research will only be reported in summary form. Your teacher will be provided summary information from this assessment to help inform instruction at the beginning of the course and to inform test preparation needs before the AP statistics exam in April 2015.

If for any reason **you do not wish** to participate in the research project, **please sign this form and return it to your teacher by ________________________.**

Student Name:

____

Student Signature:

____

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