Statistical Model for Prediction of Life-Reduction in SAC Lead-free Interconnects during Long-Term High Temperature Storage using Principal Component Regression and Ridge Regression

by

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A thesis submitted to the Graduate Faculty of
Auburn University
in partial fulfillment of the
requirements for the Degree of
Master of Science

Auburn, Alabama
August 6, 2016

Keywords: Regression, Life Prediction, Electronic Packaging

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Abstract

Electronics in military and defense applications may be stored for prolonged period of time prior to deployment in mission critical applications. In addition, electronics in automotive applications may be used underhood, mounted directly on-engine and on-transmission with expectation to survive in excess of 10 years, 100,000 miles of usage in a variety of environments. Previous researchers have studied the microstructure, mechanical response and failure behavior of leadfree solder alloys when subjected to elevated isothermal aging and/or thermal cycling. The effects are most pronounced in the widely used SnAgCu based alloys including SAC105, SAC205, SAC305 and SAC405 solders. Lower silver solders such as the SAC105, often touted for their resistance to transient dynamic shock and vibration, are the most susceptible to thermal aging amongst the SAC solders. The effects have been verified in the solder alloys at both lower strain rates in the neighborhood of 1e-4 sec-1 to 1e-5 per sec typical of thermal cycling, and at 1-to-100 per sec typical of shock and vibration. Degradation in the neighborhood of 50% has been measured at low temperature exposures. In this thesis, accelerated test thermal cycle data collected on SAC leadfree assemblies subjected to high temperature thermal aging for period of up to 1-year has been used for development of model for prediction of life reduction from long term storage at elevated temperatures. The input parameters to the model include geometry parameters including chip size, mold compound thickness, package size, board thickness, solder joint height, pad diameter, die attach thickness, and package pitch. In addition, material parameters considered include coefficient of thermal expansion, elastic modulus, and the

glass transition temperature for all the package elements in the electronic assembly. Principal component regression in conjunction with stepwise regression and Ridge Regression have been used to identify the influential variables, remove the multi co-linearity between the predictor variables, and calculate the sensitivities of the life reduction due to elevated temperature exposure on the predictor variables. The life reduction model has been used to predict the expected life reduction after prolonged storage of 20-40 years.

Acknowledgments

I would like to thank my major professor and advisor, Dr. Pradeep Lall, for his guidance during my master's program. I am grateful to him for providing exceptional facilities to conduct my research and without him this project would not be a success. I also thank him for providing me various opportunities in the field of electronic Packaging.

I would like to express my sincere gratitude to my committee members, Dr. Jeffrey Suhling and Dr. David Beale from department of Mechanical Engineering for being a part of my committee and guiding me in my graduate studies.

I would like to thank all the mechanical engineering students who took part in this research project. Without them, this project would not be a success.

I would like to thank Auburn University and the Department of Mechanical Engineering for providing me financial aid during my graduate studies at Auburn.

Finally, I would like to thank my parents, Duraisamy and Saradhamani, and my sister, Sree Preethi Usha, for their continued support and encouragement which helped me to complete my research.

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Chapter 1

Introduction

1.1 Overview

A faster method of determination of reliability and improvements in electronic packaging are necessary because of the fast paced developments and improvements in the field of electronics. Researches have proven a best method in understanding the behavioral pattern of electronics. By the understanding of the behavioral pattern one could be ready to see what might happen to a package at a certain operating condition. By that knowledge reliability could be improved based on changing or improving the parameter that most contributes for the failure.

1.2 Life Prediction

Parameters includes geometric and material configurations of the package. Researches about the aging effects is been in demand because aging is one of the major concerning factor in electronics. Previously, several efforts have been made to investigate and understand the failure mechanics and thermal reliability of packages under harsh environment. Researches about life prediction [Lall 2004-2008, Farooq 2003, Knecht 1991, Muncy 2003, 2004, Perkins 2003, 2004, Drake 2007, Warner 2004] of BGA's has been made widely.

Shirgaokar.A [Shirgoakar 2008] using Principal Component Regression have demonstrated that thermal-fatigue reliability of area-array packages depends on the design parameters (i.e. I/O pitch, I/O count, package size, substrate thickness etc.), the material sets (properties) used in the construction of a package, and the environmental conditions such as extreme temperatures, dwell times, and ramp-rates encountered by the package during its life but never included Aging effects and Aging temperature effects in his model. Prevalent approaches for reliability prediction were are also made using non-linear finite element methods [Darveaux 1991-1997, 2000, Gustaffson 2000, Goetz 2000, Johnson 1999, Riebling 1996] and first-order closed form models [Clech 1996-1997, Vandevelde 1998, 2002, 2003].

Arunachalam.D [Arunachalam 2011] used Ridge regression based on Development of Acceleration Factors and Closed Form Life Prediction Models for Lead-free Packaging but he never included aging effects.

Hariharan.G [Hariharan 2007] also used Principal component Regression model to present decision-support models for deployment of various ball grid array devices and flip chip electronics under various harsh thermal environments.

Naveen Singh [Singh 2005] used a combination of statistics-based and failure-mechanics based methodology to identify the critical parameters and their sensitivity on the thermal reliability of the BGA packages. There are lot of efforts put by researchers to find aging effects and aging temperature effects.

Jean paul clech [Clech 1996, 1997, 1998] and Vandevelde [Vandevelde 1998, 2002, 2003] tried to add aging effects and aging temperature effects alongside with all parameters that affect the package.

1.3 Regression Analysis

Regression analysis is an effective technique used in the life prediction of electronic package based on historical behavior of the packages. There are various regression techniques used to build the prediction equation. Multiple linear regression is one of those technique which uses two or more explanatory variables and one response variable to develop a relationship by fitting a linear equation to observed data. Each and every value of the independent variable x is associated with a value of the dependent variable y. The regression equation looks like $y = \beta_0 + \beta_1 x_1 + \beta_1 x_2 + \beta_3 x_3 + \ldots.\beta_k x_k$ where the dependent variable y changes corresponding to the changes made in independent variables x.

The observed values of y varies about their mean. Hence the regression equation has term for this variation. RESIDUAL is the error or the variation in observed value from the original value. The DATA is comprised of FIT + RESIDUAL. This deviation is expressed as ε. The FIT represents the regression equation with constants $\beta_0, \beta_1, \beta_2, \beta_3, \ldots.\beta_k$ and variables $y, x_1, x_2, x_3, \ldots\ldots x_k$. In general 'Y' is the response matrix and 'X' is the predictor matrix.

The correlation matrix plays an important role in solving a regression problem using the formula $\beta = (X'X)^{-1} X'Y$. If the determinant of the $X'X$ matrix is nearly one then this method is considered to be effective. But if the columns of the X matrix are related to each other this method won't be effective as the determinant of the $X'X$ matrix tends to move toward zero. In most of the engineering applications, some columns are derived functions of the other. The factors that contribute to the response can be the derived functions of each other. The regular solution to multiple linear regression, $\beta = (X'X)^{-1} X'Y$ would fail in those cases. Over the years, there have been a lot of techniques developed to circumvent the resulting numerical snag. If the co-efficient tends to infinity, it loses the actual meaning and fail to explain the actual significance of the variable. So this method fails if the determinant is zero.

The predominant techniques used are using Principal component Analysis and Ridge Regression. The principal component method transforms the predictors into their principal components and therefore reduces the dimensions of the predictors which nulls the effect of the co-relation. This is an effective method for curve fitting and for low dimension data. When the size of the dataset is large, it loses its accountability. It is more of a curve fitting tool than a prediction tool. Ridge regression on the other side introduces a small positive value called a bias parameter to keep $X'X$ from tending to infinity.

1.4 Scope of Data

The Accelerated test data is accumulated from the open literatures and tests performed at CAVE[3]. The accumulated data is used to run regression analysis to predict life. The accumulated data is based on BGA packages of various material and geometric configurations. Below shown is the Table 1 consisting of accumulated test data

Table 1: Scope of accelerated test data

Solder Alloy	SAC 305
Ball Count	97 to 360
Ball DiaMM	0.20 to 0.40
PCB ThicknessMM	1.57 to 2.36
PitchMM	0.4 to 1
Package SizeMM	5 to 19
Change in Temp(°C)	100 to 165
Dwell TimeMINUTES	10 to 60
AgingDAYS	1 to 360
Aging Temperature(°C)	25 to 125

1.5 Test Vehicle:

The fine-pitch PBGA packages measured 5×5, 10×10, 15×15, and 19 × 19 mm and contained SAC105, SAC305, and Sn–37Pb solder joints for each size fine-pitch PBGA. For better analysis of the aging effect on the reliability PBGA components used NSMD pads. The test vehicle, dubbed TV7, is shown in Fig. FR-406 glass epoxy laminated with a glass transition temperature *Tg* of 170 °C was used in the test vehicle. The board design dimensions were 100.076 × 67.056 mm with a thickness of 1.574 mm (measured laminate to laminate). Four circuit layers along with reasonable copper distribution to provide optimum copper balance and CTE for thermal cycle testing [Hai 2013].

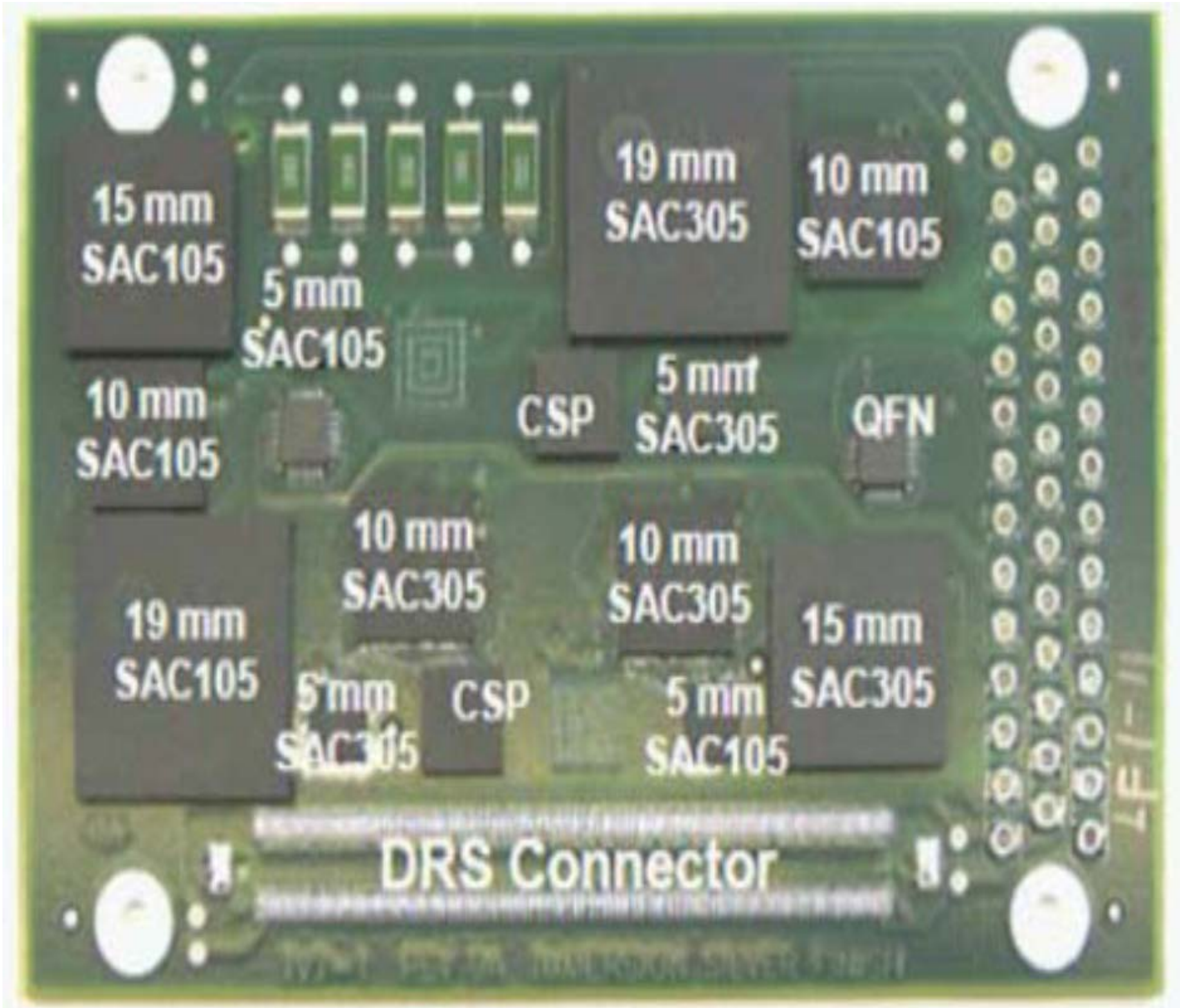


Figure 1: Assembled test Vehicle [Hai 2013]

1.6 Weibull Plots:

The following figures [Hai 2013] shows the weibull plots for the thermal cycling results on isothermaly aged SAC 305 solder alloys of 5mm, 10mm, 15mm, 19mm BGAs. The interesting trends that could be seen from the weibul plots are

1. reliability of SAC alloys are improved by higher Ag content;
2. reliability for finepitch packages decreases with aging;
3. increase in aging temperatures result in increased rates of degradation for the SAC alloy solders tested;
4. small changes in reliability were observed versus board finish.

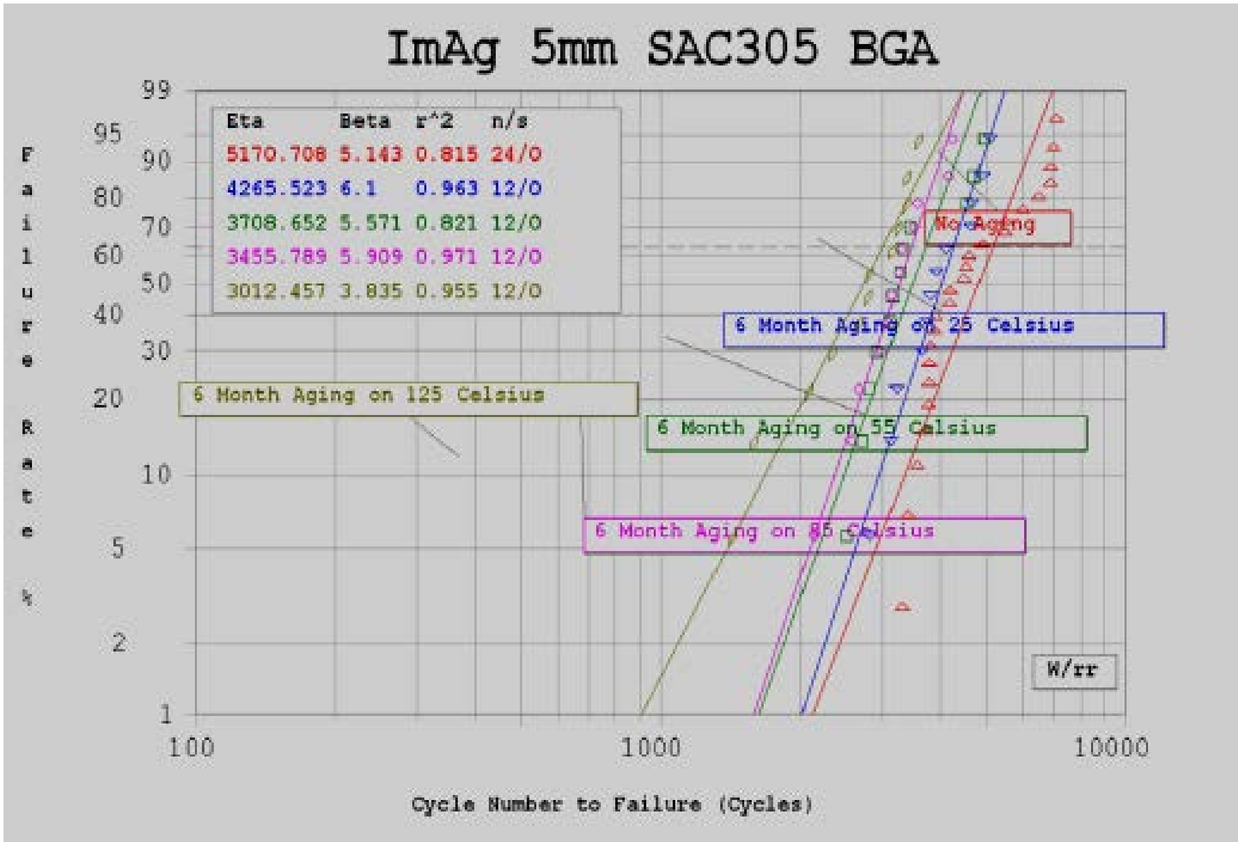


Figure 2 : SAC 305 on ImAg for 5mm BGA [Hai 2013]

Figure 3: SAC 305 on ImSn for 5mm BGA [Hai 2013]

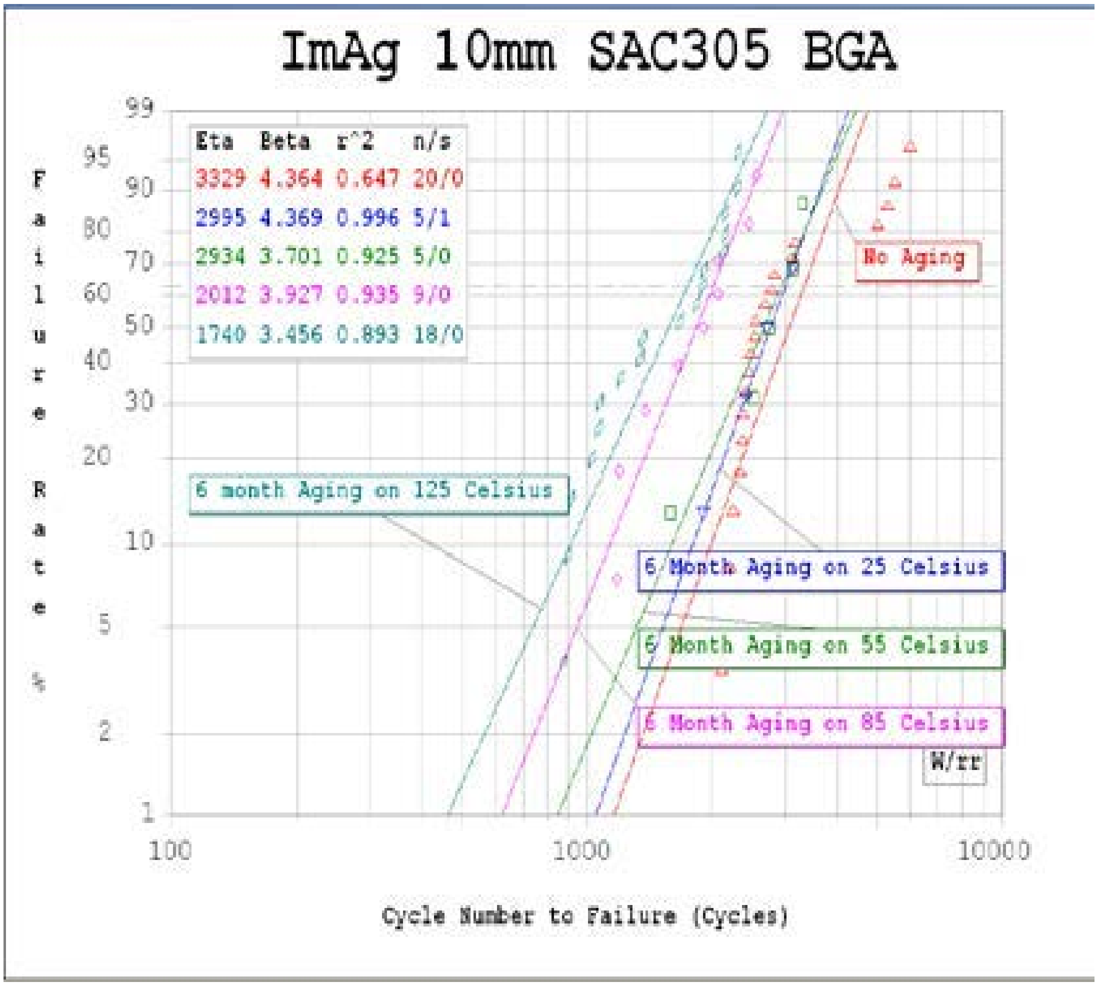


Figure 4: SAC 305 on ImAg for 10mm BGA [Hai 2013]

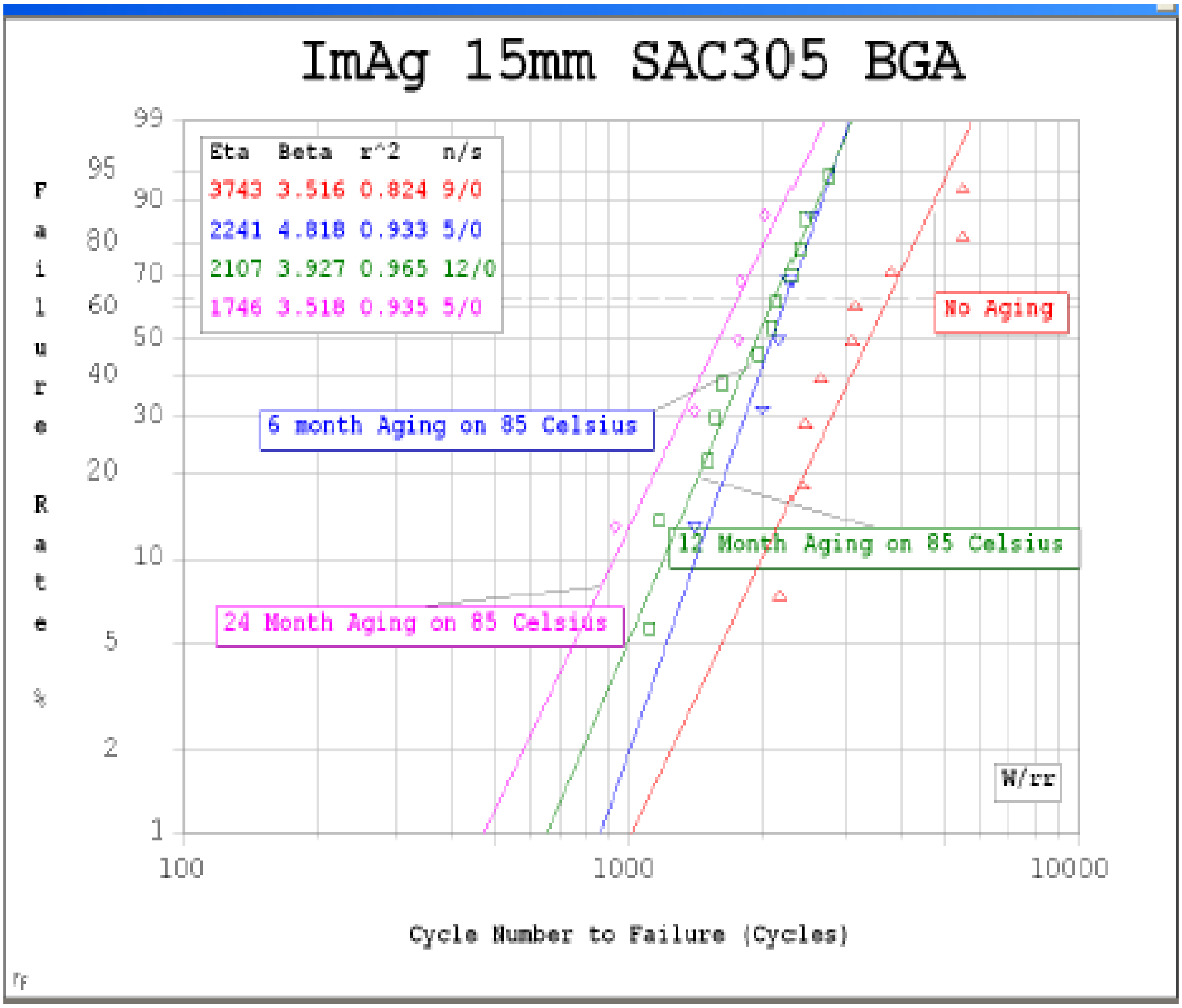


Figure 5: SAC 305 on ImAg for 15mm BGA [Hai 2013]

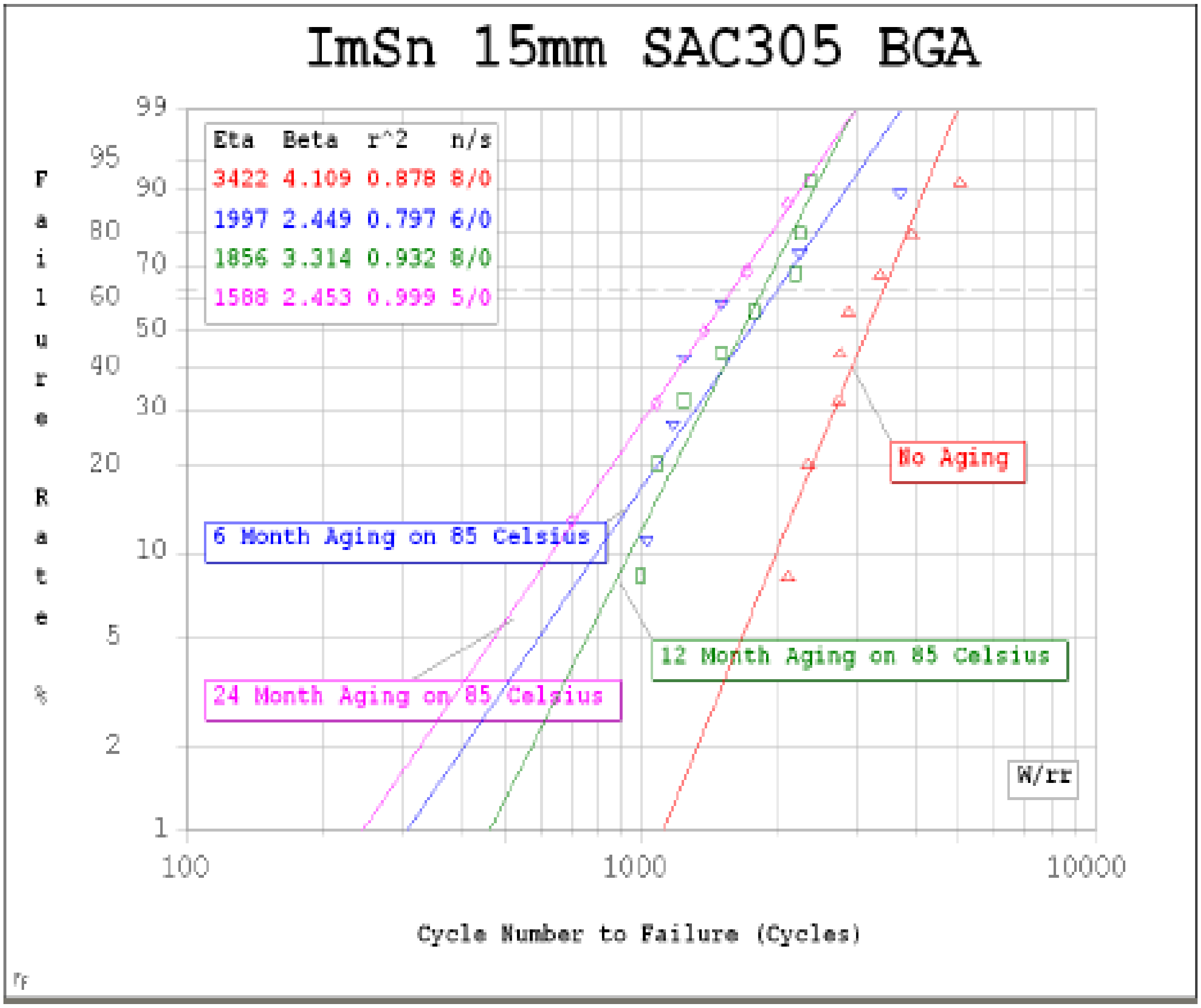


Figure 6: SAC 305 on ImSn for 15mm BGA [Hai 2013]

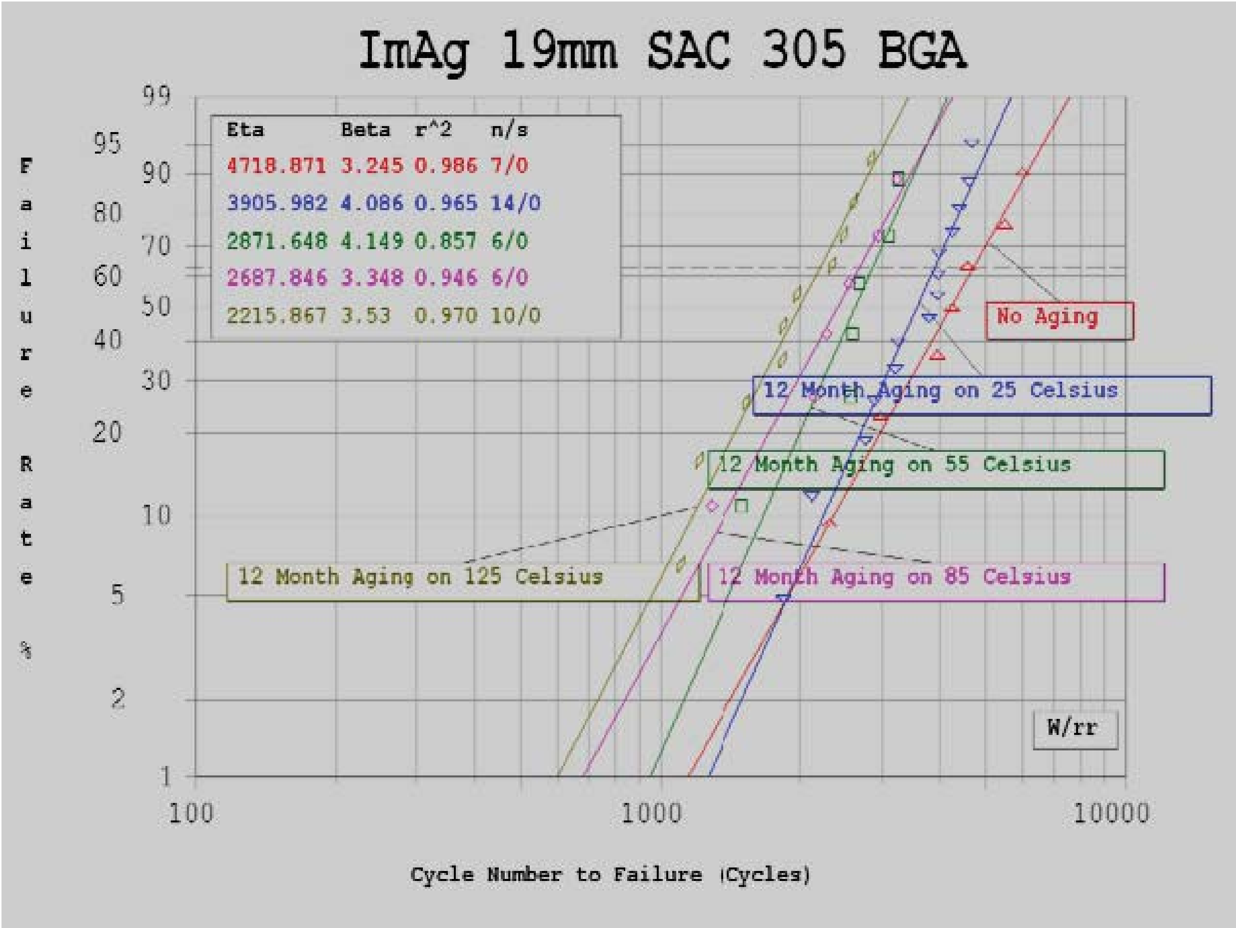


Figure 7: SAC 305 on ImAg for 19mm BGA [Hai 2013]

Figure 8: SAC 305 on ImSn for 19mm BGA [Hai 2013]

Chapter 2

Literature Review

2.1 Solder Alloy:

A metallurgical process in which two or more metallic surfaces are joined together by melting a filler metal at the joint is called Soldering. A solder is a filler metal, which has relatively low melting point, usually below 425 °C. In the ever-developing electronics world, wave soldering and reflow soldering are the two primary soldering techniques used for the mass-production of printed circuit boards (PCBs]. The process in which electronic components are adhered to the PCB temporarily with small dabs of adhesive, then the whole assembly passes over flowing solder in a bulk container is called Wave Soldering. Whereas Reflow soldering is a process where the parts are attached to their designated pads on PCB by using a sticky mixture of powdered solder and flux(solder paste), after that the whole assembly passes through a carefully-controlled oven in which the solder joints between parts and bonding pads are formed. Solder, as a joining material, provides electrical, thermal and mechanical continuity in electronics assemblies. The overall functioning of the assembly is determined by the properties of a solder joint. 63Sn-37Pb (eutectic composition) and 60Sn-40Pb (near eutectic composition) are the most widely used soldering alloys in electronic packaging [Zhang 2010].

The Sn-Pb binary system has a melting eutectic temperature of 183 °C and provides material compatibility with most substrate materials and devices. Pb, as the primary 2 component of Sn-Pb solders has many technical advantages over other alloying elements, like Wetting by reducing the surface tension of Sn-Pb solders is facilitated, Allotropic transformation of Sn is prevented, Pb enhances the diffusion of other joint constituents such as Sn and Cu in the liquid state to help with the formation of intermetallic bonds by serving as a solvent metal, Pb is available readily and cheap [Zhang 2010].

Implementation of eutectic and near eutectic Sn-Pb solders has been well developed in electronic packaging over the past decades because of their knowledge base about mechanical properties, chemical properties and reliability. But the raising concern about using Sn-Pb solders are the dangers caused on human health. Over exposure to Pb are detrimental for human health. So we are in need to use Pb-free solders in electronic packages [Zhang 2010].

2.1.1 Pb-free Solders development

A large number of pb-free alloys have been proposed and sn based pb-free alloys appeared to be the appropriate candidate with sn being the primary constituent [Zhang 2010].

2.1.2 Pb-free solder alloys

Sn-Pb solders have been replaced by the introduction of several Pb-free solder alloys with Sn being the major constituent. The Pb-free solder alloys are developed in such a way that they have lower melting temperature and high reliability by the addition of third or fourth elements [Zhang 2010].

2.1.3 Sn-Ag-Cu (SAC) Alloys

More than a decade researches has been made for replacement of Sn-Pb solders with Pb-free solders. But still there is not a perfect replacement found for Sn-Pb solder alloys. SAC (Sn-Ag-Cu) alloys are the closest and the most promising replacement that is suitable and adopted by many packaging industries. The most predominant SAC alloys used are SAC 105, SAC 305, SAC 405, SAC 387, SAC 396. These are widely accepted in many countries like USA, Japan and European countries [Zhang 2010].

2.1.4 SAC Characteristics and Applications

There are several characteristics that give SAC alloys good compatibility with current electronics packaging infrastructure such as relatively low melting temperatures, superior mechanical and solderability properties, and good tolerance for Pb contamination. In spite of the facts such as SAC alloys wide acceptance in world market and its high market share, they are not the perfect replica of Sn-Pb solders. The several disadvantages of SAC alloys are higher reflow temperatures are needed because of higher melting points, reliability problems caused due to excessive growth of intermetallic compounds and higher material costs of SAC alloys [Zhang 2010].

2.1.5 Pb-free Challenges in Electronic Packaging

The electronics used today are designed to operate at a temperature of 183 °C whereas most of the Pb-free alloys have a melting point over 200 °C. Solder joint reliability risk is higher as a result of this higher melting point temperature of SAC solder alloys which might ultimately cause damage to the electronic assemblies. Although lots of Pb-free solder alloys have been investigated due to the raise in concerns about Sn-Pb solders, the above said are the primary reasons why the liquidus temperature is considered as the first and foremost factor when it comes to electronic manufacturing [Zhang 2010].

2.1.6 Pb-free Reliability Challenges

The most predominant concern in packaging industry is the solder joint reliability. Several testing methods and mathematical models have been proposed to predict the life span of the package by taking various parameter into consideration. This study is concentrated primarily on developing a statistical model which includes aging as one of its major parameter along with Ball count, Ball diameter, PCB thickness, Body size, Pitch, Aging temperature, Dwell time, Coefficient of thermal expansion [Zhang 2010].

2.1.7 Transition from Sn-Pb to Pb-free Solder Alloys

As the result of revolutions against the use of Pb products, the European Union had to initiate a ban on lead. This triggered the need of Pb-free products and various researches and innovations were taken place. The transition from Sn-Pb to Pb-free is still not complete. There are still flaws and critical issues that has to be taken care in the Pb-free transition process. So the Pb-free products used with some Pb-coated components to overcome the flaws and critical issues faced. Until an effective method of finding a completely Pb-free solder is undertaken, this would bridge the gap between Sn-Pb and Pb-free soldering [Zhang 2010].

2.2 Aging:

When it comes to electronics, aging is always a concerning factor. All electronic packages are subjected to high temperatures depending on its purpose. Packages age relative to the temperatures it's been exposed to. Previously researchers have used reliability assessment tools to include parameters that affect solder joint reliability, but aging effects have not been included in those models. If a package is stored at a certain temperature for a certain period of time prior to its usage it is necessary to know the life left within the package. If some packages are to be used for prolonged time period, it is not realistic to test those for a longer time period. A prediction model has to be implemented to predict the lifetime, which includes aging as a parameter along with other parameters that affect solder joint reliability. The Prediction model in this work has 11 parameters, which includes aging as well. Principal Component Regression (PCR) and Ridge Regression models have been implemented to develop the mathematical model.

2.3 Statistical Prediction Models

Statistics-based methodology has been used to identify the critical parameters and their sensitivity on the thermal reliability of the BGA packages that affect solder joint reliability. Sensitivities of reliability to design, material, architecture, and environment parameters have been developed from statistical analysis, and validated against experimental data. The parameters of the BGA's which include Ball count, Ball diameter, Pitch, PCB thickness, Package size, Board finish, Dwell time, Change in Temperature, Coefficient of thermal expansion, aging temperatures (25°C, 55°C, 85°C and 125 °C), aging time (1, 180, and 360 Days) and number of cycles obtained from the experiment are the data which were used to perform Regression. MINITAB, MATLAB and SAS have been used to develop Regression models.

Variables used in building the mathematical model in general are closely related to each other despite their individual significant contribution to the life of the package. Inaccurate results will be given if the dependence between predictor variables are not treated. Parameters might have wrong signs and some parameters with significant contribution to the package life which influence solder joint reliability will be statistically insignificant. Poor regression estimates will be produced when least square method is applied to a collinear data with R-Squared (Goodness of fit) value much lower, Variance Inflation Factor (VIF) values being on the higher because of multi-co linearity. Multi-co linearity should be removed in a model. Several methods have been proposed to eliminate multi co linearity. Correlation between the independent variables can be found by calculating Pearson Correlation matrix. Suitable variables are identified using step wise regression [Mccray 2004, Meiri 2002]. Mathematical equations for parametric sensitivities are developed using multivariate regression, analysis of variance (ANOVA) techniques. Studies have been made about the reliability analysis [Hong 1998, Hou 2001, Jagarkal 2004, Engelmaier 1983, 1984, 1990, Peng 2004] and failure mechanics theory of BGA's to identify the parametric sensitivities [Kang 2004].

2.3.1 Principal Component Regression (PCR) [Montgomery 2012]

The principal components are the linear transformation of set of X predictor variables into new set Z predictor variables. This newly formed set of Z variables are uncorrelated with each other and account for much of variation in X together. A scatter of simple points in the n dimensional space having X as a basis forms the principal axes of the ellipsoid. The principal components corresponds to that axes. A rotation from the original x coordinate system to the system defined by the principal axes of this ellipsoid is therefore called as a principal component transformation. The new orthogonal principal components in the order of their importance are raked using the principal component transformation. Multiple linear regressions (MLR) have been performed with the set of principal components against the original response variable. Using the same linear transformation the principal components estimators are transformed back to their original form. This process is called back transformation. Ordinary least square method has been used on principal components. Hence the new set of predictor coefficients are more reliable as the principal components are pair wise independent. Make an assumption that the dataset spans n-sets from the same package architecture. Let the regression model is of the form

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \ldots\ldots + \beta_k x_{ki} + \varepsilon_i \quad (1)$$

where, x_1 , x_2 ,...,x_k are the k-predictor variables, $\beta_0, \beta_1, \beta_2, \ldots\beta_k$ are the regression coefficients, and ε_i is the model error for the ith dataset. The model can be written in matrix notation as follows:

$$\{y\} = [X]\{\beta\} + \{\varepsilon\} \quad (2)$$

where,

$$\{y\} = \begin{Bmatrix} y_1 \\ y_2 \\ . \\ . \\ . \\ y_n \end{Bmatrix} \quad X = \begin{bmatrix} 1 & x_{11} & x_{21} & . & . & . & x_{k1} \\ 1 & x_{12} & x_{22} & . & . & . & x_{k2} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 1 & x_{1n} & x_{2n} & . & . & . & x_{kn} \end{bmatrix} \quad (3)$$

$$\{\beta\} = \begin{Bmatrix} \beta_1 \\ \beta_2 \\ . \\ . \\ . \\ \beta_n \end{Bmatrix} \{\varepsilon\} = \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ . \\ . \\ . \\ \varepsilon_n \end{Bmatrix} \quad (4)$$

The least squares estimator, {b}, of the regression coefficients, {β}, assuming that [X] is of full column rank

$$\{b\} = \lfloor b_0 \quad b_1 \quad b_2 \quad b_3 \quad . \quad . \quad . \quad . \quad b_n \rfloor^T = \left([X]^T \quad [X]\right)^{-1} [X]^T \{y\} \tag{5}$$

The variance and covariance matrix of the estimated regression coefficients in vector {b} is

$$\mathrm{var}\{b\} = \sigma^2 \left([X]^T \quad [X]\right)^{-1} \tag{6}$$

where each column of [X] is the measurement of a particular predictor variable. Centering or scaling or standardizing the independent variables are done in multiple linear regressions. Such transformation of the geometry, architecture, material properties, and operating conditions predictor variables has an advantage in electronic packaging reliability which allows results from different studies to be comparable. The next step after the independent variables are centered and scaled, then the variable, x_{ji}, is transformed as follows:

$$x_{ji}{}^{*} = \left(\frac{x_{ji} - \overline{x}_j}{s_j} \right) \tag{7}$$

$$s_j = \sqrt{\sum_{i=1}^{n} \left(x_{ji} - \overline{x}_j \right)^2}$$

The process of centering and scaling has been used to develop an alternative formulation as follows:

$$y_i = \beta_0^* + \beta_1^* x_{1i}^* + \beta_2^* x_{2i}^* + \ldots\ldots + \beta_k^* x_{ki}^* \tag{8}$$

$$= \beta_0^* + \beta_1^* \left(\frac{x_{1i} - \overline{x_1}}{s_1} \right) + \ldots\ldots\ldots + \beta_k^* \left(\frac{x_{ki} - \overline{x_k}}{s_k} \right) + \varepsilon_i$$

The equation may be written in matrix format as follows:

$$\{y\} = \beta_0^* \{1\} + [X^*]\{\beta^*\} + \{\varepsilon\} \tag{9}$$

Where {1} is the unit vector, of size n x 1, and β* is the vector of transformed coefficients. Centering and scaling makes $[X^*]^T [X^*]$ the k x k correlation matrix of the independent variables.

$$C = [X^*]^T [X^*] \tag{10}$$

Where C is the correlation matrix. Better prediction results are achieved using principal components regression than ordinary least squares. PCR has been used to combat multi-co linearity. In PCR the original set of k predictor variables have been transformed into a new set of orthogonal or uncorrelated variables called principal components of the correlation matrix. Ranking of the new orthogonal variables in the order of their importance is done using this transformation. Multiple linear regression analysis of original response variables using ordinary least squares have been done against the newly formed principal components. The regression coefficients of the principal components from MLR have then been back transformed into new set of coefficients that correspond to the original correlated variables. The newly obtained coefficients from the back transformation process are called principal component estimators. Eigenvalues of the correlation matrix, [C], have been calculated using the following determinant equation:

$$([C]-\lambda[I])[V] \qquad (11)$$

$$\Rightarrow \left\|[C]-\lambda[I]\right\| = 0 \text{ or } \left[X^*\right]^T[X]^* - \lambda[I]$$

where λ_1, λ_2,..., λ_k are the eigenvalues of the correlation matrix, and [V] is a k x k eigenvector matrix consisting of normalized eigenvectors associated with each eigenvalues. Since the eigen- vector are orthogonal, $[V][V]^T = I$. The regression equation of centered and scaled variables can be written as follows:

$$\{y\} = \beta_0^*\{1\} + \left[X^*\right]\{\beta^*\} + \{\varepsilon\} \qquad (12)$$

$$\Rightarrow \{y\} = \beta_0^*\{1\} + \left[X^*\right][V][V]^T\{\beta^*\} + \{\varepsilon\}$$

$$\{y\} = \beta_0^*\{1\} + [Z]\{\alpha\} + \{\varepsilon\}$$

where [Z] is an n x k matrix of principal components and $\{\alpha\}$ is a vector of new regression coefficients. The new model formulation is then written as follows:

$$y_i\big|_{1<i<n} = \beta_0^* + \alpha_1 Z_1 + \alpha_2 Z_2 + \ldots\ldots + \alpha_k Z_k + \varepsilon_i \qquad (13)$$

where Z_1, Z_2,..., Z_k are the k new variables called principal components of the correlation matrix [C]. The principal components are orthogonal to each other. Each principal component is a linear combination of the transformed predictor variables

$$z_j = \left[x_1^* \quad x_2^* \quad x_3^* \quad . \quad . \quad . \quad . \quad x_k^*\right] \begin{Bmatrix} v_{1j} \\ v_{2j} \\ v_{3j} \\ . \\ . \\ . \\ v_{kj} \end{Bmatrix} \tag{14}$$

The coefficients for the centered and scaled variables are obtained as follows:

$$\begin{aligned} &\{\alpha\}_{k\times 1} = [V]^T \{\beta^*\} \\ &\Rightarrow [V]\{\alpha\}_{k\times 1} = [V][V^T]\{\beta^*\} \\ &\Rightarrow \{\beta\}_{k\times 1} = \{V\}_{k\times k}\{\alpha\}_{k\times 1} \end{aligned} \tag{15}$$

where [V] is the eigenvector matrix, and {α} is a vector of new regression coefficients. Assume that r-variables have been dropped. A principal component analysis has been performed on this original predictor variable matrix X and its eigenvalues and corresponding eigenvectors have been extracted. The back transformation to coefficients of the natural variables is done as follows:

$$b_{j,pc} = \frac{b^*_{j,pc}}{s_j} \tag{16}$$

where, j = 1,2,…..k

The equations used under principal component regression were from the reference [Lall 2008]

2.3.2 Ridge Regression [A.E Hoerl, 1970]

Consider the standard model for multiple linear regression, Y=Xβ+ε, where X is the matrix of predictors and Y is the matrix of the response. β is the regression coefficient matrix which is unknown at this point. The usual procedure of determining the values is called the Gauss-Markov linear functions.

Let B be the estimate of any vector β. The residual sum of squares can be written as,

$$\begin{aligned} F(\beta) &= \varphi = (Y - X\beta)'(Y - X\beta) \\ &= Y'Y + \beta'X'X\beta - 2\beta'X'Y \end{aligned} \tag{17}$$

The difference between the observed and fitted values had been expressed as the estimate. The above equation is differentiated and equated to zero to find the minimum value.

$$\begin{aligned} &\frac{\partial F(\beta)}{\partial \beta} = X'X\beta - X'Y = 0 \\ &X'X\beta = X'Y \\ &\beta = (X'X)^{-1}X'Y \end{aligned} \tag{18}$$

If the determinant of the $X'X$ matrix is nearly one then this method is considered to be effective. But if the columns of the X matrix are related to each other this method won't be effective as the determinant of the $X'X$ matrix tends to move toward zero. In most of the engineering applications, some columns are derived functions of the other. The factors that contribute to the response can be the derived functions of each other. The regular solution to multiple linear regression, $\beta = (X'X)^{-1} X'Y$ would fail in those cases. Over the years, there have been a lot of techniques developed to circumvent the resulting numerical snag. If the co-efficient tends to infinity, it loses the actual meaning and fail to explain the actual significance of the variable. So this method fails if the determinant is zero.

The predominant techniques used are using Principal component Analysis and Ridge Regression. The principal component method transforms the predictors into their principal components and therefore reduces the dimensions of the predictors which nulls the effect of the co-relation. This is an effective method for curve fitting and for low dimension data. When the size of the dataset is large, it loses its accountability. It is more of a curve fitting tool than a prediction tool. Ridge regression on the other side introduces a small positive value called a bias parameter to keep $X'X$ from tending to infinity.

For data where X matrix is not specified to be near co linearity, the dispersion is expressed as,

$$D(\beta) = \sigma^2 (X'X)^{-1} \tag{19}$$

The trace of the dispersion matrix is the total variance, thus,

$$TrD(\beta r) = \sigma^2 \sum_{i=1}^{s} \frac{1}{\lambda_i} \tag{20}$$

Where the λ_i are the non-zero eigen values of $X'X$, So if one or more of the Eigen values are low, the variance inflation is going to be high. Adding the scalar matrix kI to $X'X$ in the least square estimator as suggested by Hoerl and Kennard, is the possible remedy for inflation. Thus the regression equation takes the form

$$\beta = (X'X + kI)^{-1} X'Y \tag{21}$$

Upon calculating the dispersion and variance for the above equation as we did before, we get,

$$\mathrm{TrD}(\beta r) = \sigma^2 \sum_{i=1}^{s} \frac{\lambda_i}{(\lambda_i + k)^2} \tag{22}$$

It is clear that variance inflation will be lesser in the (22) in the event if low Eigen values.

By finding the point on the ellipsoid centered at the LS estimator β, the value of the ridge estimator 'k' is obtained. And by the residual sum of squares the hyper ellipsoid is formed. When there is inflation in the actual values, the residual sum of squares should be reduced. Let B be any estimate of the actual vector β. The residual sum of squares in that case will be.

$$\begin{aligned} F(B) = \varphi &= (Y - XB)'(Y - XB) \\ &= (Y - X\beta)'(Y - X\beta\beta \\ &+ (B - \beta)'X'X(B - \beta) \\ &= \varphi_{min} + \varphi(B) \end{aligned} \tag{23}$$

As we understand, β has inflated to B and hence $\phi(B)$ is the residual that has added because of the inflation. Hence we try to reduce that term.

$$\varphi(B) = F = (B - \beta)'X'X(B - \beta) \tag{24}$$

The ridge trace can be shown to be following a path through sum of the squares so that for a fixed ∅ a single value of B is chosen and that is the one with the minimum length.

Minimize B'B

Subject to $(\mathrm{B}-\beta)'\mathrm{X}'\mathrm{X}(\mathrm{B}-\beta) = \varphi_0$

Solving it using a lagrangian multiplier 'k'.

Minimize,

$$\mathrm{F} = \mathrm{B}'\mathrm{B} + \left(\frac{1}{\mathrm{k}}\right)\left[(\mathrm{B}-\mathrm{B})'\mathrm{X}'\mathrm{X}(\mathrm{B}-\beta)' - \varphi_0\right] \quad (25)$$

Where $\left(\frac{1}{k}\right)$ is the lagrangian multiplier.

$$\frac{\partial \mathrm{F}}{\partial \mathrm{B}} = 2\mathrm{B} + \left(\frac{1}{\mathrm{k}}\right)\left[2(\mathrm{X}'\mathrm{X})\mathrm{B} - 2(\mathrm{X}'\mathrm{X})\beta\right] = 0 \quad (26)$$

Hence it reduces to

$$\mathrm{B} = \left[\mathrm{X}'\mathrm{X} + \mathrm{kI}\right]^{-1}\mathrm{X}'\mathrm{Y} \text{ [A.E Hoerl, 1970]} \quad (27)$$

The value of 'k' is chosen such that k > 0 and then $\emptyset_0$ is computed. In terms of β*, the residual sum of squares becomes,

$$\varphi^*(\mathrm{k}) = (\mathrm{Y}-\mathrm{X}\hat{\beta}^*)'(\mathrm{Y}-\mathrm{X}\hat{\beta}^*) = \varphi_{\min} + \mathrm{k}^2\hat{\beta}^{*\prime}(\mathrm{X}'\mathrm{X})^{-1}\hat{\beta}^* \quad (28)$$

If the squared length of the regression vector B is fixed atR^2, then $\hat{\beta}^*$ is the value of B that gives a minimum sum of squares. Hence $\hat{\beta}^*$is the value of B that minimizes the function,

$$\mathrm{F} = (\mathrm{Y}-\mathrm{XB})'(\mathrm{Y}-\mathrm{YB}) + \left(\frac{1}{\mathrm{k}}\right)(\mathrm{B}'\mathrm{B} - \mathrm{R}^2) \quad (29)$$

The equation $B = [X'X + kI]^{-1} X'Y$ which is used to perform Ridge Regression is just a modification of the original Multiple Linear Regression equation, $\hat{\beta} = [X'X + kI]^{-1} X'Y$, except that we introduce a small positive term, 'k' which is called a bias parameter and is added to the diagonal of the variance-covariance portion of the Regression equation .The original data is retained as there is no change to the second part of the equation, but the variance which is a derived property of the actual data undergoes a small bias addition.

This process of ridge regression is a regular regression process which is started by forming a set of Predictor(X) and Response Variables (Y).An initial range of the Bias Parameters 'k' is chosen . The equation (27) is solved with the range of bias parameter. The stability is looked for in each of the attempts. The model should neither be over-biased nor be under-biased. So care should be taken.

A point where both the Regression co-efficients and the VIF values remain stable or show minimal change upon further biasing is considered to be a perfect bias. A value k is chosen and the results of the equation (27) for the chosen k value will be the results of the Ridge Regression process. The overall adequacy of the model is tested using ANOVA table. The equations used in ridge regression were referenced from [Arunachalam 2011].

The entire flow of the Ridge Regression process can be explained by the flow chart shown in Figure 9.

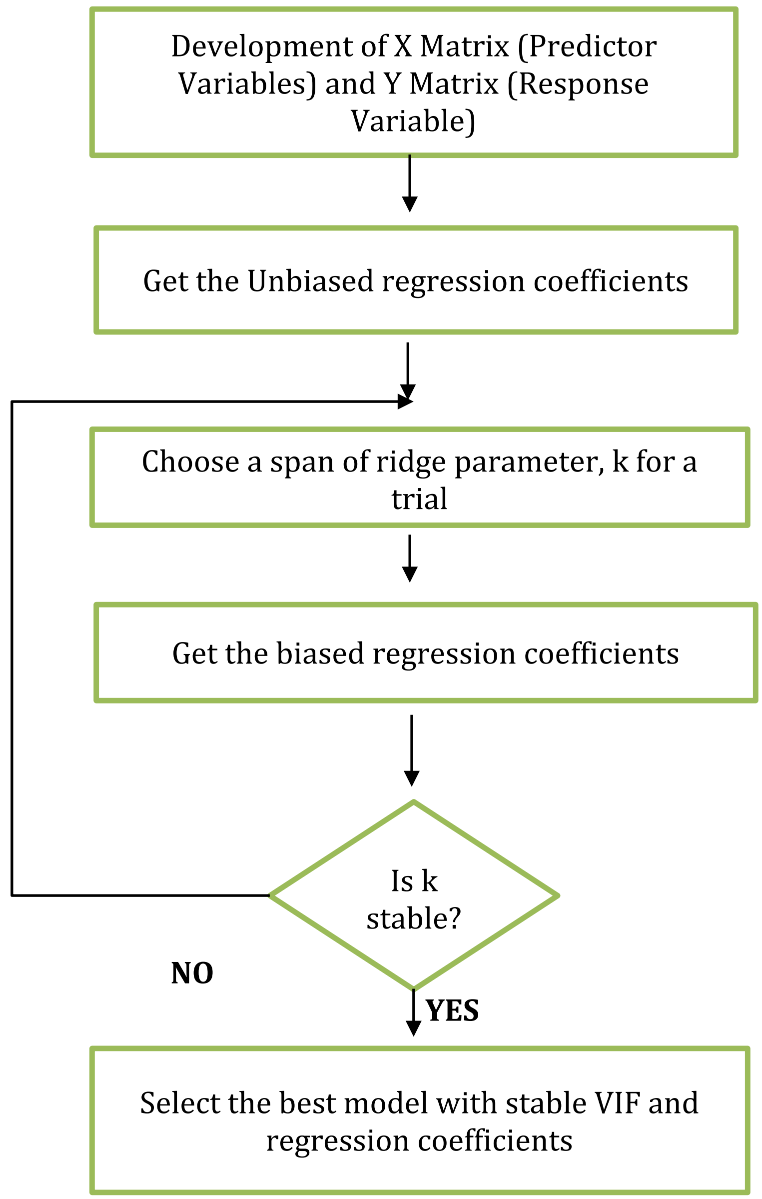


Figure 9: Ridge Process

Overall adequacy of the model is proven from the small P value of the ANOVA table rejecting the null hypothesis. Very small P values from individual T tests on the coefficients of regression of variables indicate the statistical significance of all the predictor variables.

Individual T test are conducted on the coefficients of regression of original variables using the individual T test values of the variables. The test statistic follows a students' T distribution with (n-k-1) degrees of freedom. The P values of individual T tests given by the 'p' values table which are lesser than 0.05 proving the statistical significance of individual regression coefficients of original predictor variables at a 95 % confidence.

Chapter 3

Model Development

Several statistical models have been developed to predict the thermal reliability of Ball Grid Array packages based on studies made about the reliability of BGA packages [Syed 1996, 1997, 2001, 2004, Evans 1997, Banks 1995, Chaeng 2005]. Most parameters which influence solder joint reliability have been included and investigated. These models include linear models and log-log models. The values of cycles to 63.2% failure for different configurations of area array packages has been used to compare the accuracy of the predicted values of each model.

3.1 Initial Model:

So in this study maximum number of parameters that influence solder joint reliability have been investigated using regression analysis. A total of 17 variables were included in the initial model for investigation. The statistical model with all the parameters in the form of mathematical equation is given by,

$$
\begin{aligned}
N_\alpha = \quad & A_0 + A_1\,(\text{BallCount}) + A_2\,(\text{BallDiaMM}) + A_3\,(\text{SJHMM}) + A_4\,(\text{BallHtMM}) \\
& + A_5\,(\text{PCBThickMM}) + A_6\,(\text{PitchMM}) + A_7\,(\text{DietoBody}) \\
& + A_8\,(\text{BodySizeMM}) + A_9\,(\text{DieSizeMM}) + A_{10}\,(\text{DNPMM}) \\
& + A_{11}\,(\text{BoardFinishID}) + A_{12}\,(\Delta T) + A_{13}\,(\text{DwellTimeMIN}) \\
& + A_{14}\,(\text{DeltaAlphaPPMC}) + A_{15}\,(\text{AgingDAYS}) + A_{16}\,(\text{AgingTemp DegC}) \\
& + A_{17}\,(\text{Beta})
\end{aligned}
$$

Where A_0, A_1,……A_{17} are the constants corresponding to the variables. The predictor variables used in building the initial equation are BodyMM (package body size), DietoBodyRatio (ratio of die size to body size), BallCount, BallDiaMM (solder ball diameter), PitchMM (solder ball pitch), SJHMM (Solder Joint Height), BallHtMM (Solder Ball height), PCBthickMM (PCB thickness), BoardFinishID, ΔT (thermal cycling temperature range), DieSize, DNPMM (Distance to Neutral Point), Coefficient of Thermal Expansion, Aging time, Aging temperature and Dwell Time. BoardFinishID do not have numerical values so they were used as dummy variables and each dummy variable was assigned a numerical value.

Table 2: Multiple Linear Regression model with initially used variables

Predictor	**Coeff**	**SE Coeff**	**T**	**P**	**VIF**
Constant	19140	13881	1.38	0.171	
BallCount	-15.29	2.76	-5.53	0.000	27.66
BallDiaMM	71331	79838	0.89	0.374	16552.32
SJHMM	-47880	56105	-0.85	0.396	5231.51
BallHtMM	-23480	25408	-0.92	0.358	1682.10
PCBThickMM	1304	2497	0.52	0.603	187.50
PitchMM	-6765	1577	-4.29	0.000	254.84
DietoPackage	-12798	19438	-0.66	0.512	1957.64
BodySizeMM	-349	1066	-0.33	0.744	12050.32
DieSizeMM	125	1247	0.10	0.920	14110.51
DNPMM	1117	317	3.52	0.001	243.08
BoardFinishID	-304	158	-1.93	0.057	14.75

DeltaT DegC	-18.21	2.83	-6.43	0.000	2.000
DwellTimeMin	-31.60	5.36	-5.89	0.000	1.940
DelAphaPPMC	-865	1724	-0.50	0.617	598.46
AgingDays	-72.50	19.60	-3.70	0.000	3.160
AgingTemp DegC	-14.43	2.50	-5.77	0.000	2.09
Beta	-134.1	58.9	-2.28	0.025	3.47

From the above table it is seen that lots of parameters in spite of influencing solder joint reliability, are not statistically significant. If the p-values are above 0.050 it is considered to be statistically insignificant. Parameters like die to package ratio, Die size and beta have p-values above 0.050 and they are statistically insignificant. The next step is performing Principal Components Analysis Regression and Ridge Regression on these variables and select the variables fit to be used in the model. Statistically insignificant variables are neglected from the model. Those will be discussed on the concerned chapters.

Chapter 4

Prediction Model by Principal Component Regression

Life prediction models have been widely used in electronics to assess the influence of various parameters that affects the life of the electronic package. Principal Component Regression models [King 1999] is been practiced widely in case of multi-co linearity. The PCR model presented in this work has 11 parameters in total.

4.1 Scope of Data:

The dataset includes accelerated test reliability data from open literature for a variety of packaging architectures including, Chip-array ball-grid array (CABGA), Ceramic ball-grid array (CBGA), Plastic ball-grid array (PBGA). Package size ranges from 5mm to 19mm with Ball count ranging from 64 to 360. Data gathered on test assemblies from temperature cycle conditions including, TC1 (0 to 100°C, 10 min dwell), TC2 (-15 to 125°C, 60 min dwell), TC3 (25 to 125°C, 10 dwell), TC4 (-40 to 100°C, 10 min dwell), TC5 (0 to 100°C, 60 min dwell), TC6 (25 to 125°C, 60 min dwell).

4.2 Model Development:

Aging time and aging temperature has been considered as parameters alongside with other parameters that affect solder joint reliability. Ball count, Ball diameter, PCB Thickness, Pitch, Body size, Board finish, cycle conditions such as dwell time and delta T and Coefficient of thermal expansion. Packages used were BGA, CBGA, CABGA, CVBGA and PBGA on SAC 305 alloys were considered for analysis. Initially the data showed a lot of correlation between different sets of variables. The p-values were above 0.05 and the variance inflation factor (VIF) were also so high justifying the seriousness of multi co-linearity. There were a lot of coefficients with values more than 0.5.

The regular multiple regression model has random values with inaccurate results. P values were above 0.05 and VIF's were too high too. This is due to the presence of multi co-linearity between the variables involved. So Principal Component Regression is implemented to eliminate the co-linearity. Regression of transformed variables against life is given in the Table 3 below:

Table 3: Multiple Linear Regression model using Principal Components

Predictor	Coeff	SE Coeff	T	P	VIF
Z1	-0.1997	0.0175	-11.43	0.000	1
Z2	-0.2123	0.0217	-9.80	0.000	1
Z3	-0.0856	0.0273	-3.14	0.002	1
Z4	0.1160	0.0476	-2.44	0.017	1
Z5	- 0.4092	0.0563	-7.27	0.000	1
Z6	0.2660	0.0589	4.52	0.000	1
Z7	0.6171	0.0731	8.44	0.000	1
Z8	-0.4553	0.0906	-5.02	0.000	1
Z9	-0.434	0.114	-3.81	0.000	1
Z10	-0.720	0.209	-3.44	0.001	1
Z11	1.169	0.277	4.22	0.000	1

The above table shows the P values are now less than 0.05 and VIF's are exactly 1, which shows multi co-linearity has been completely removed from the model. The ANOVA Table 4 is tabulated below to check if there is any presence of linear relationship between predictor variables and response variable.

Table 4: Analysis of Variance of Multiple Linear Regression model with Principal Components as variables

Source	**DF**	**SS**	**MS**	**F**	**P**
Regression	11	69.11	6.28	41.69	0.000
Residual Error	102	15.37	0.15		
Total	113	84.49			

Now the coefficients are in a combination of principal components. It needs to be back transformed to its original form using the same process. Table 5 below shows the back transformed regression coefficients.

Table 5: Back transformed Coefficients

Predictor	**Coeff**
Ln(Ball Count)	-0.628
Ln(Ball DiaMM)	1.787
Ln(PCB ThicknessMM)	-0.2421
Ln(PitchMM)	-2.874

Ln(Package SizeMM)	0.77
Ln(Board FinishID)	-0.0394
Ln(Delta T)	-1.28
Ln(Dwell TimeMIN)	-0.188
Ln(DeltaAlpha PPMC)	-0.846
Ln(AgingDAYS)	-0.0349
Ln(Aging Temp C)	-0.2458

The final regression equation is given by,

$$N_\alpha = e^{(20.61)} (BallCount)^{(-0.628)} (BallDiaMM)^{(1.788)} (PCBThicknessMM)^{(-0.242)} (PitchMM)^{(-2.874)} (PackageSizeMM)^{(0.77)} (BoardFinishID)^{(-0.040)} (\Delta T)^{(-1.28)} (DwellTimeMIN)^{(-0.188)} (DeltaAlphaPPMC)^{(-0.850)} (AgingDAYS)^{(-0.03489)} (AgingTempC)^{(-0.24582)}$$

The PCR model shown above predicts the life of BGA package of SAC 305 alloys with high accuracy. The R-Squared value is also high. The Residual plots are used to study about the residuals from the model. The Figure 10 is shown below,

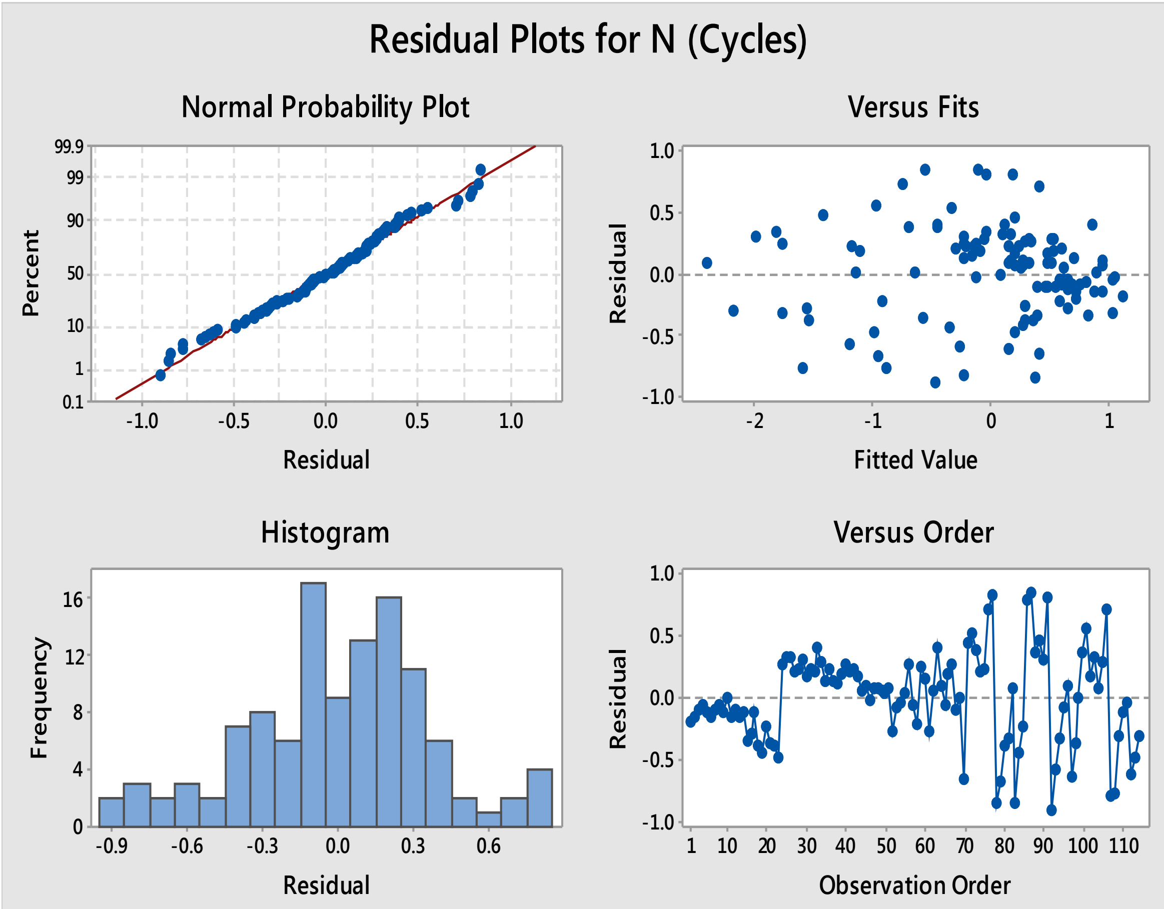


Figure 10: Residual Plot of Principal Components Regression

Normality, constant variance and independence has been checked for the model obtained from PCR. Violations of model assumptions are studied using residual plots. The residual plots studied (Figure 10) include normal probability plot, histogram plot of residuals, plot of residuals against fitted values, plot of residual against regressor. Straight line variation of normal probability plot shows cumulative normal distribution. No violation of constant variance assumption is shown in the horizontal band.

4.3 Model Validation:

The model developed from PCR should be validated to check its accuracy. A set of data has been kept separately before implementing PCR for the purpose of model validation. The main objective of the predictive equation is predicting life, so it's necessary that the validation should be done with data that is not used in model development. The effect of Aging and Aging temperature has been presented. The predictions from the statistical model have been compared with the experimental data.

4.3.1 Aging Effects:

Life of the package decreases with increase in aging. Statistical model has been compared with experimental data, which agrees with the trend. Data compared are for 1, 10, 180, and 360 days of aging and were also predicted for 720 days of aging from the PCR equation, which is shown in following figures. Package size used for model prediction is 5mm at 25, 55, 85 and 125°C temperatures.

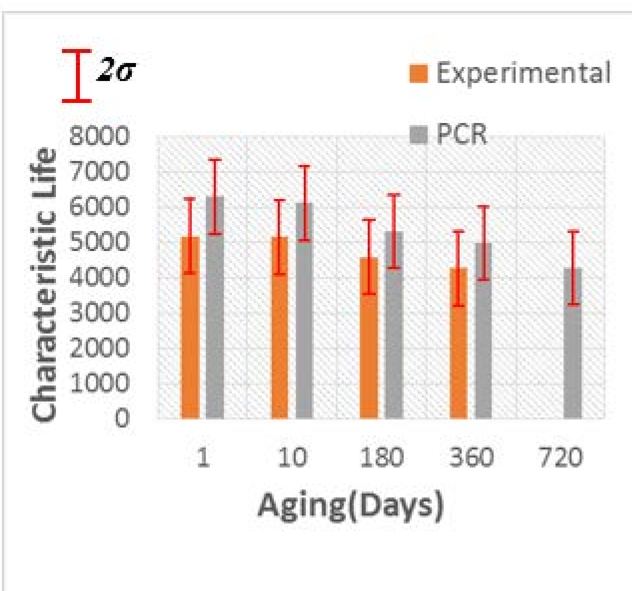


Figure 11: Effect of Aging for 5mm package at 25°C

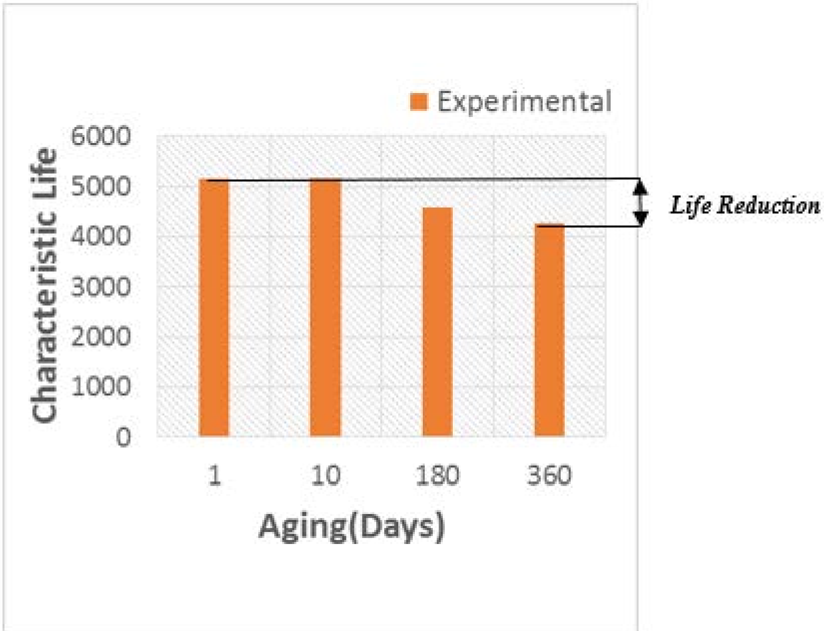
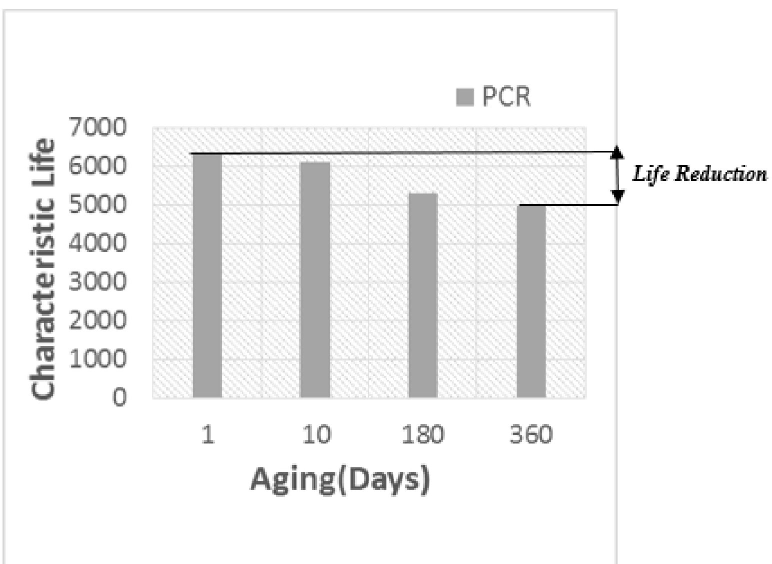


Figure 12: Effect of Aging (Experimental) for 5mm package at 25°C



Life Reduction (1 to 360 days):
Experimental – 17.5%
PCR - 21.05%
Error - 20.28%

Figure 13: Effect of Aging (PCR) for 5mm package at 25°C

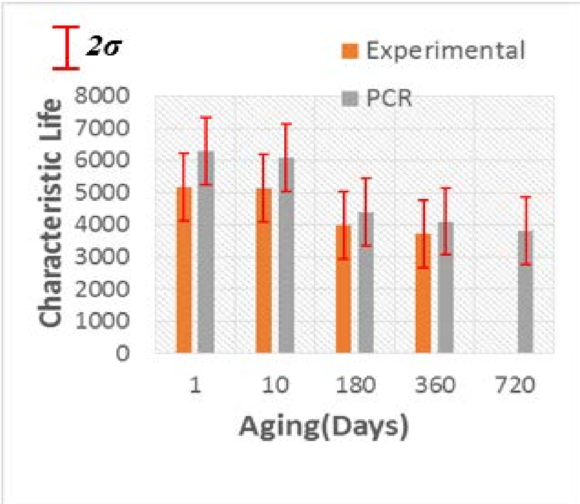


Figure 14: Effect of Aging for 5mm package at 55°C

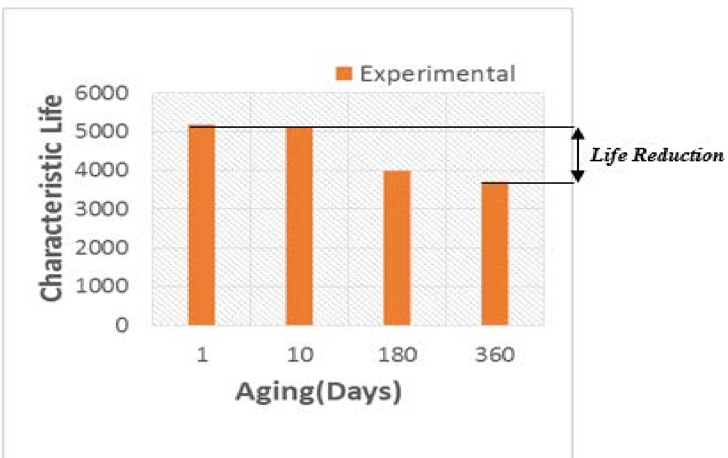
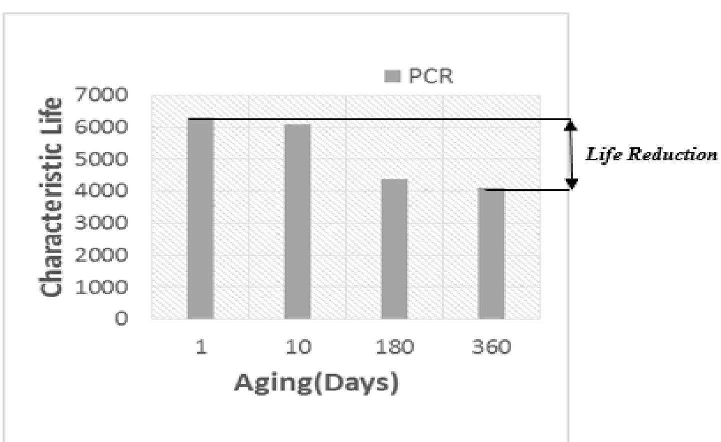


Figure 15: Effect of Aging (Experimental) for 5mm package at 55°C



Life Reduction (1 to 360 Days):
Experimental – 28.2%
PCR - 34.8%
Error - 23.40%

Figure 16: Effect of Aging (PCR) for 5mm package at 55°C

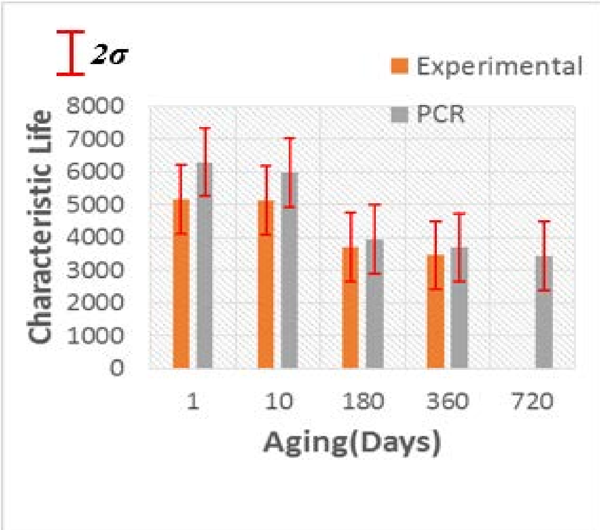


Figure 17: Effect of Aging for 5mm package at 85°C

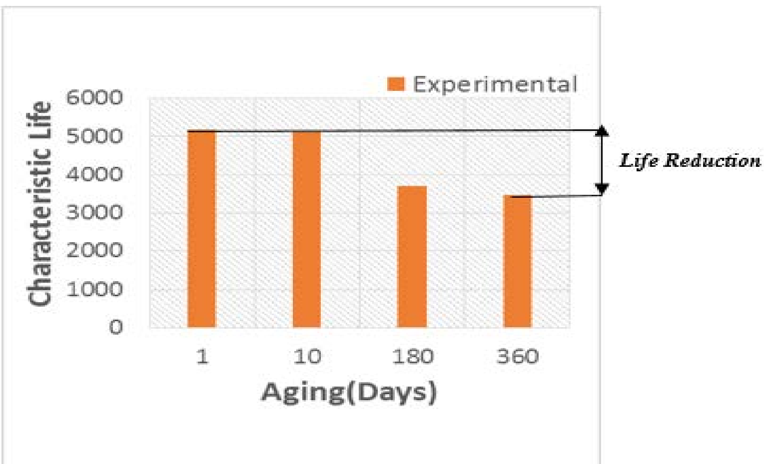
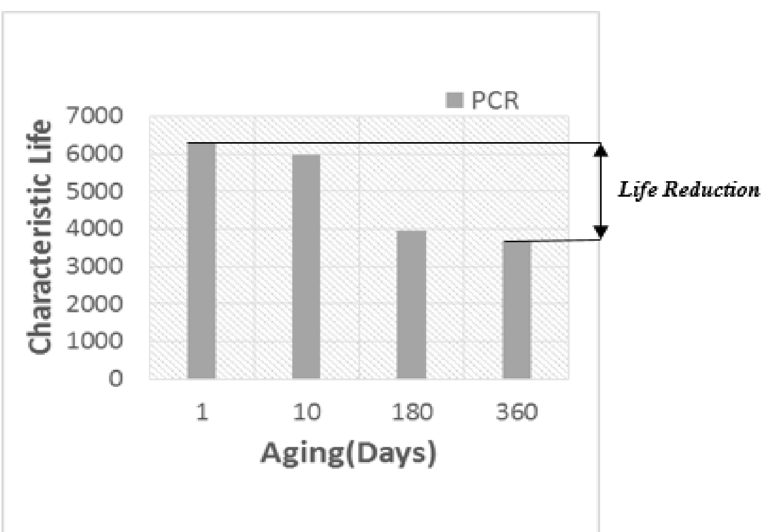


Figure 18: Effect of Aging (Experimental) for 5mm package at 85°C



Life Reduction (1 to 360 Days):
Experimental – 33.12%
PCR - 41.2%
Error - 24.39%

Figure 19: Effect of Aging (PCR) for 5mm package at 85°C

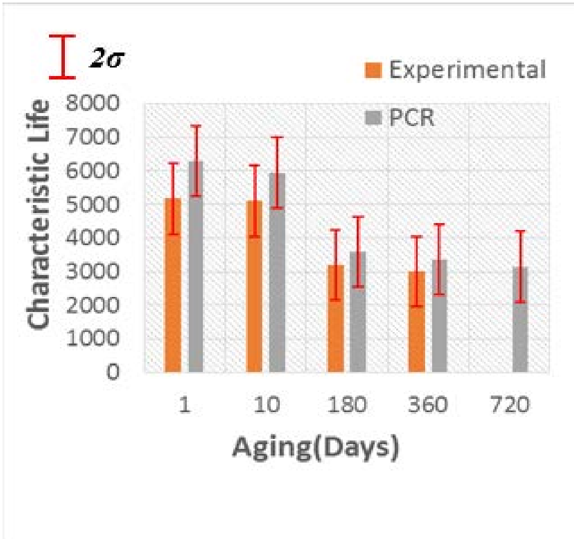


Figure 20: Effect of Aging for 5mm package at 125°C

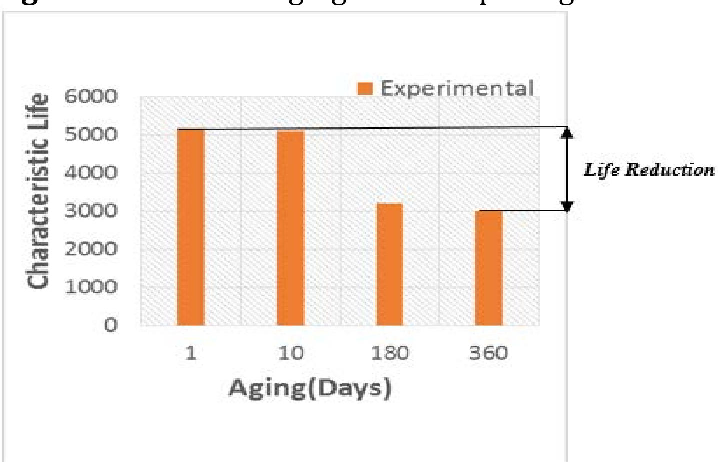
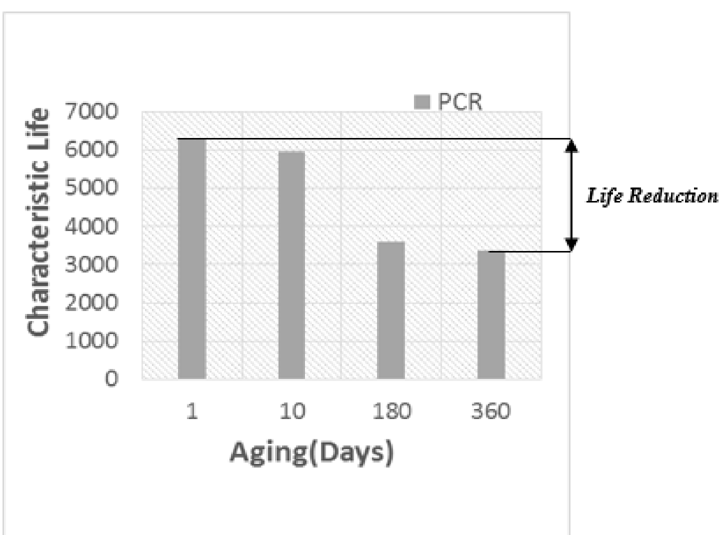


Figure 21: Effect of Aging (Experimental) for 5mm package at 125°C



Life Reduction (1 to 360 Days):
Experimental – 41.35%
PCR - 46.32%
Error - 12.02%

Figure 22: Effect of Aging (PCR) for 5mm package at 125°C

If we need data for a longer term like 20 years it is highly difficult to perform the experiment for that long. Prediction equations can come in handy in these scenarios. So Figure 23, Figure 24, Figure 26 shows the prediction of life for 0, 5, 10 and 20years through PCR equation.

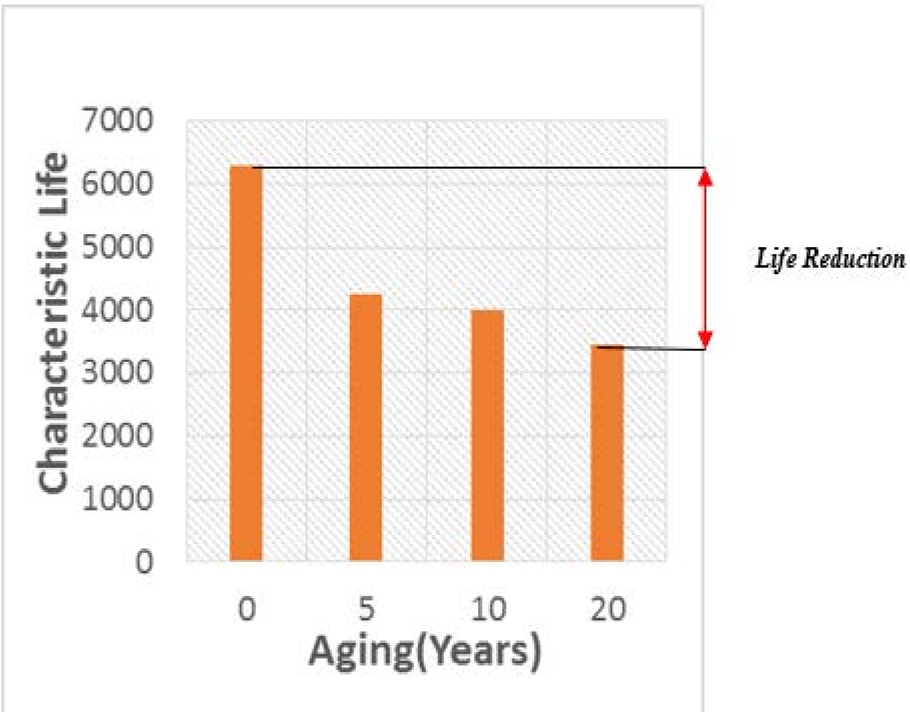


Figure 23: Prediction of Aging (Years) for 5mm package at 25°C

Life Reduction:
PCR - 45.20%

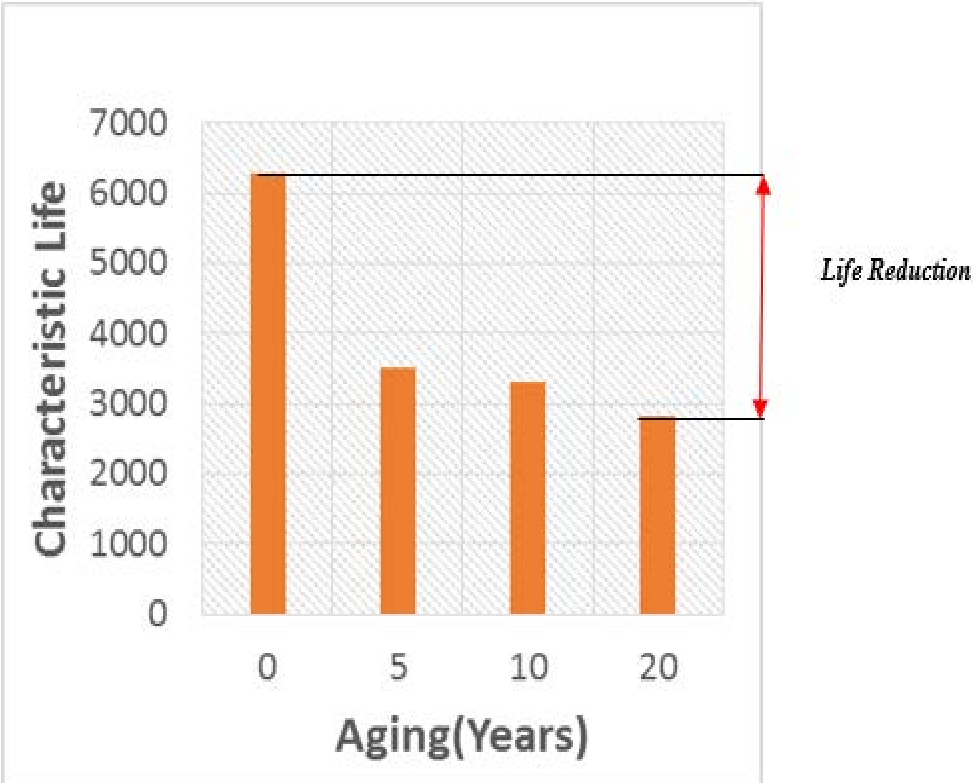


Figure 24: Prediction of Aging (Years) for 5mm package at 55°C

Life Reduction:
PCR - 55.26%

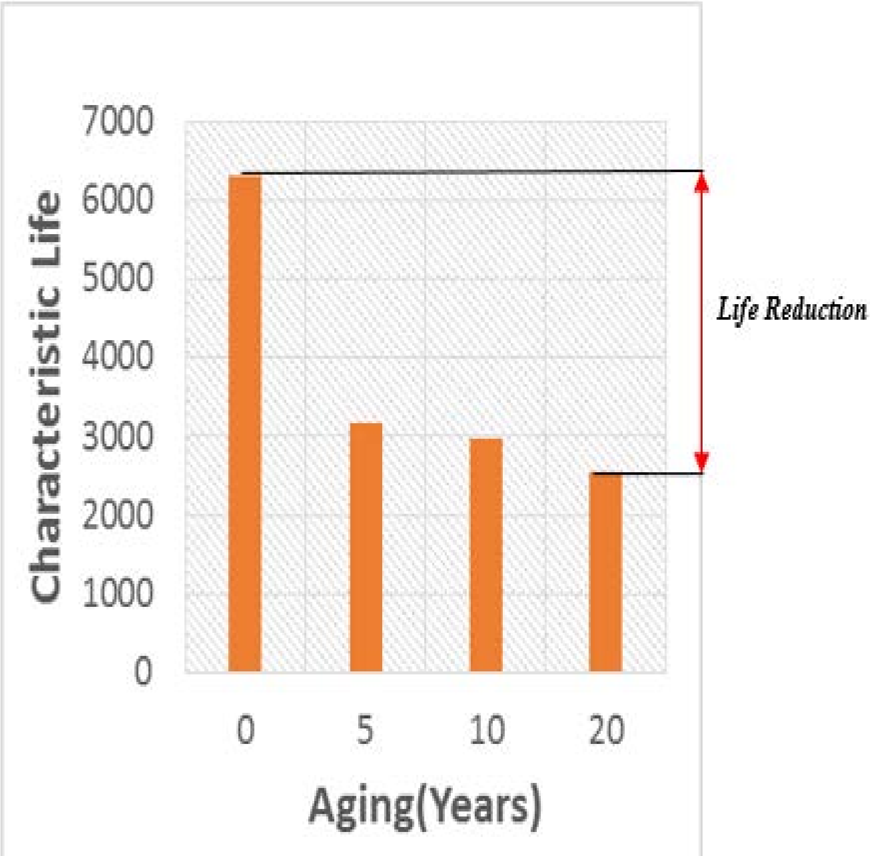


Figure 25: Prediction of Aging (Years) for 5mm package at 85°C

Life Reduction:

PCR - 59.40%

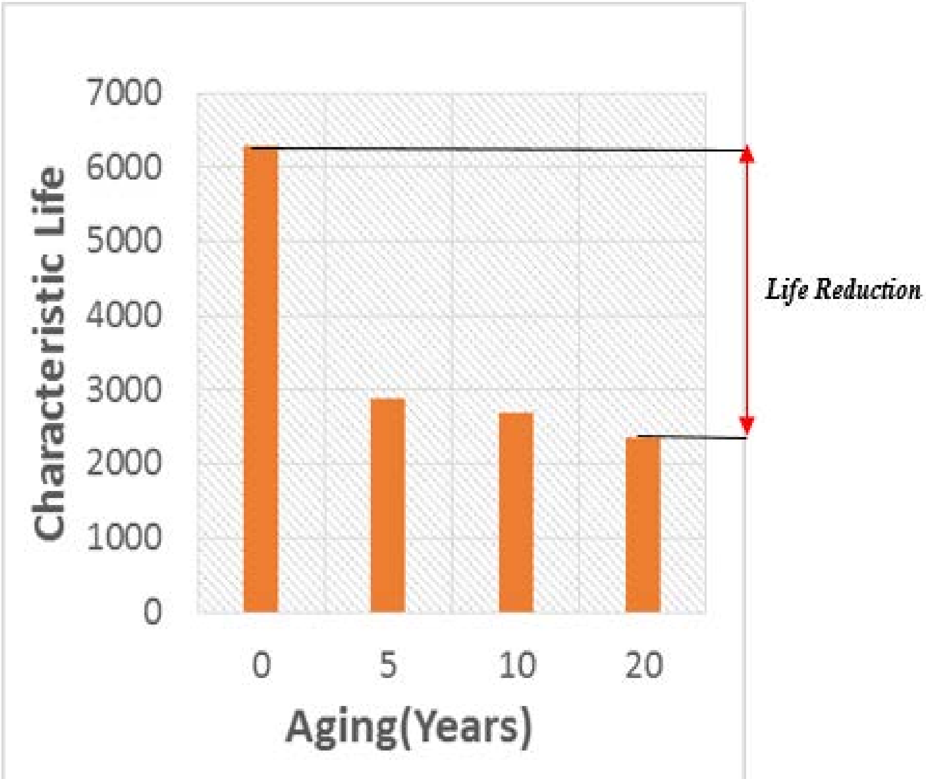


Figure 26: Prediction of Aging (Years) for 5mm package at 125°C

Life Reduction:
PCR - 62.83%

4.3.2 Aging Temperature:

Life of the package decreases with increase in aging temperature. Statistical model has been compared with experimental data, which agrees with the trend, which is shown in Figure 27.

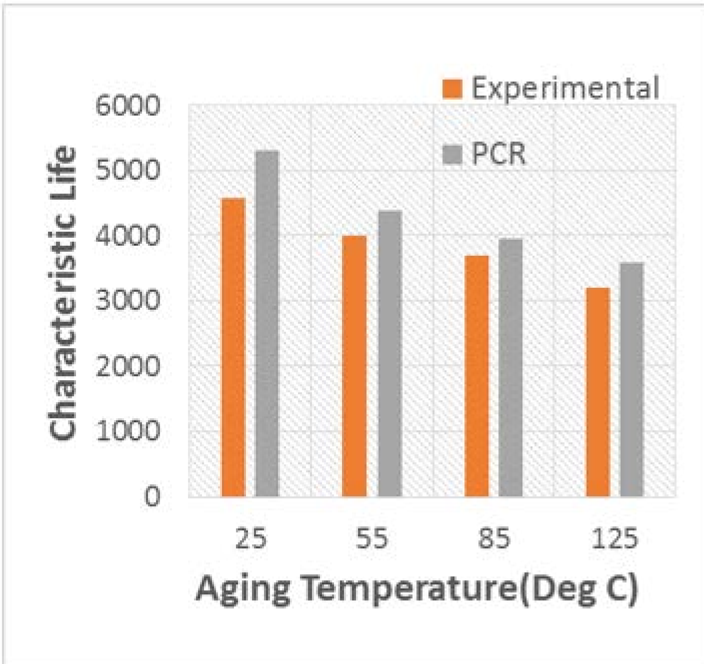


Figure 27: Effect of Aging Temperature for 5mm package at 6 month aging

4.3.3 Ball Count:

Experimental data indicates that thermal reliability of the BGAs decreases with the increase in the ball count. The following Figure 28 shows the general trend of decrease in thermal reliability with the increase in the ball count, which is in agreement with the failure mechanics theory. Increase in the number of solder balls distributes the thermal deformation over a larger number of solder joints reducing the stress level in the individual ball.

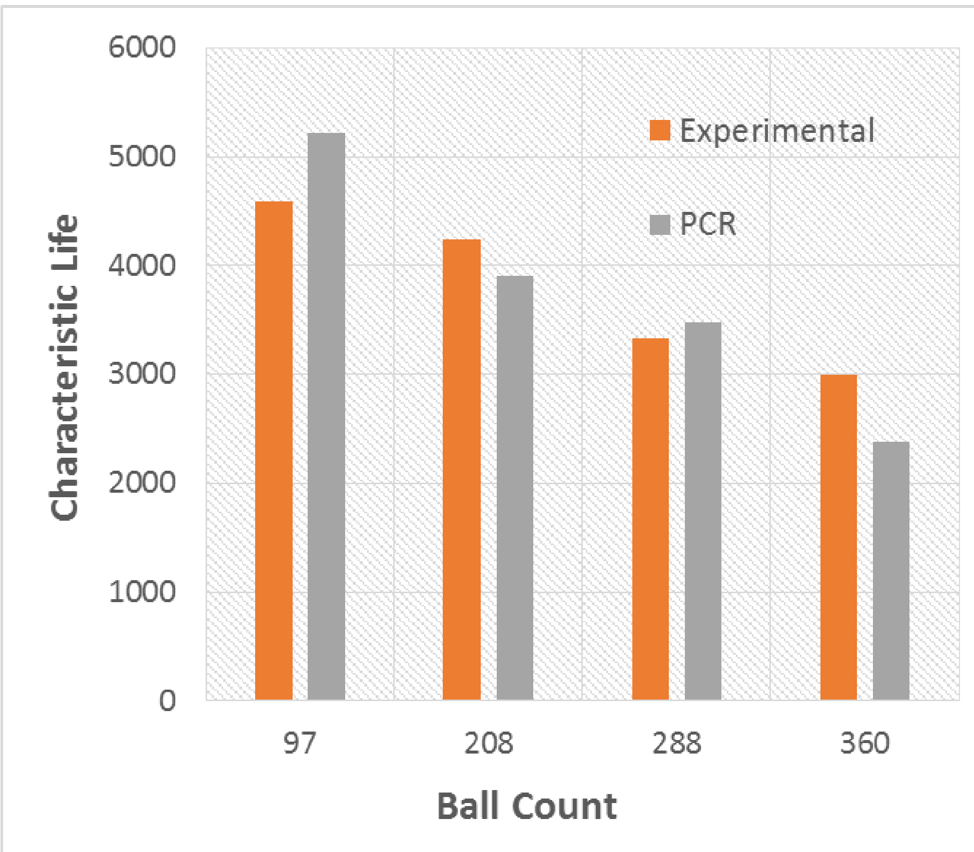


Figure 28: Validation of Ball Count

4.3.4 PCB Thickness:

The decrease in reliability of a ball-grid array package with the increase in the PCB thickness. This effect is consistent from failure mechanics as the increased PCB thickness leads to higher assembly stiffness, which results in higher stresses in the interconnect.

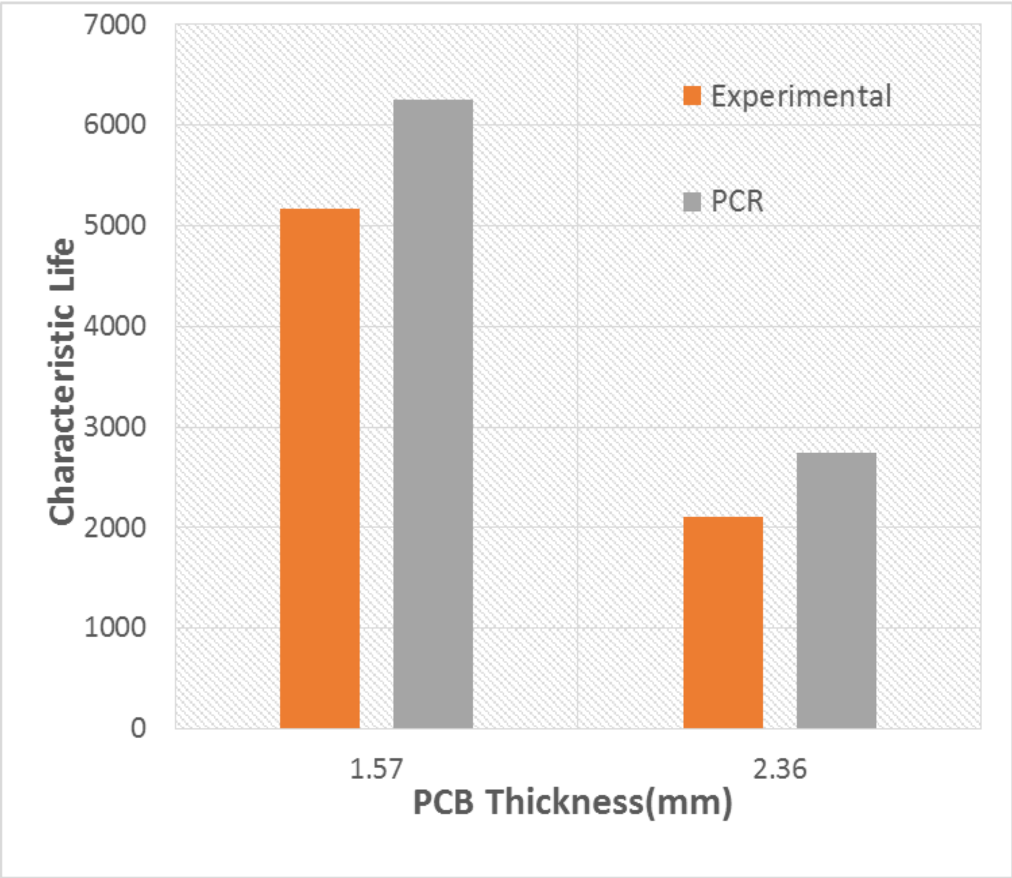


Figure 29: Validation of PCB Thickness

4.3.5 Pitch:

Increase in Pitch decreases life of the package.

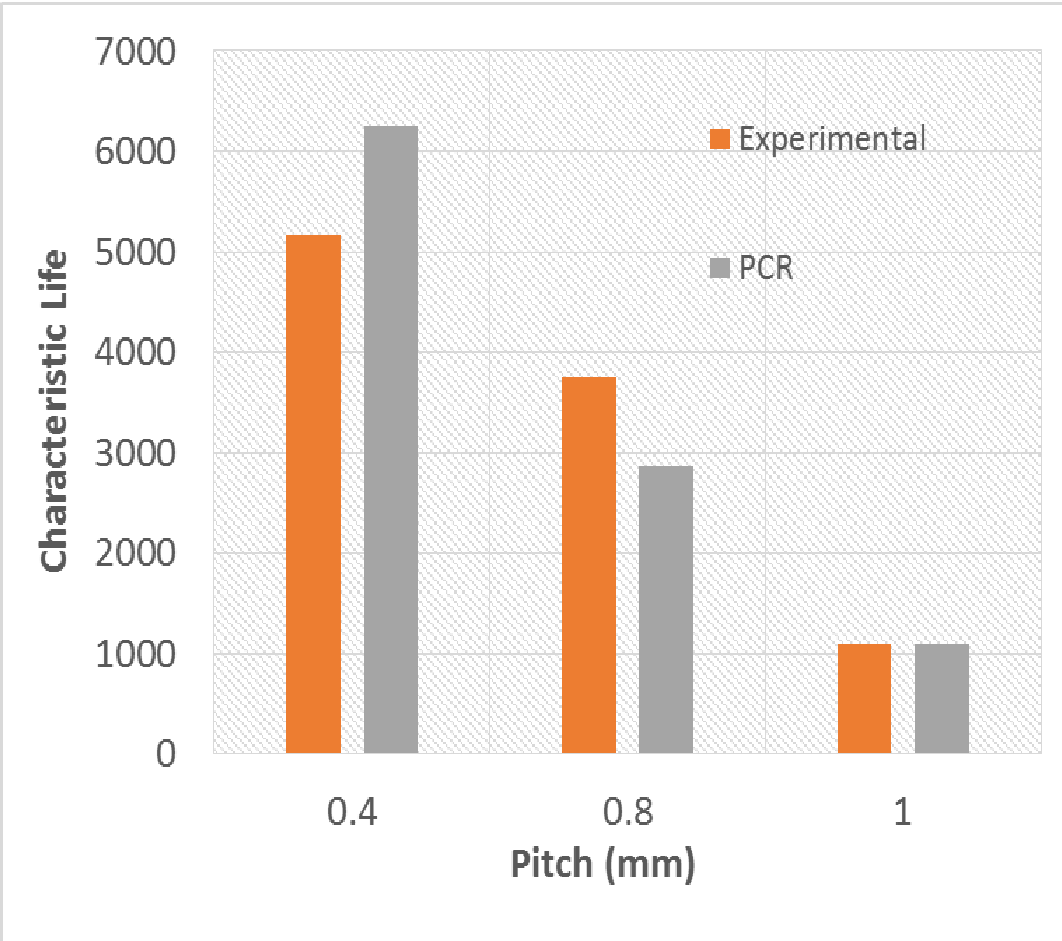


Figure 30: Validation of Pitch

4.3.6 Ball Diameter:

Experimental data indicates that the increase in the ball diameter leads to overall better thermal reliability of the package. This trend is in compliance with the failure mechanics theory as the increase in the solder ball diameter increases the crack area resulting in higher thermal fatigue life.

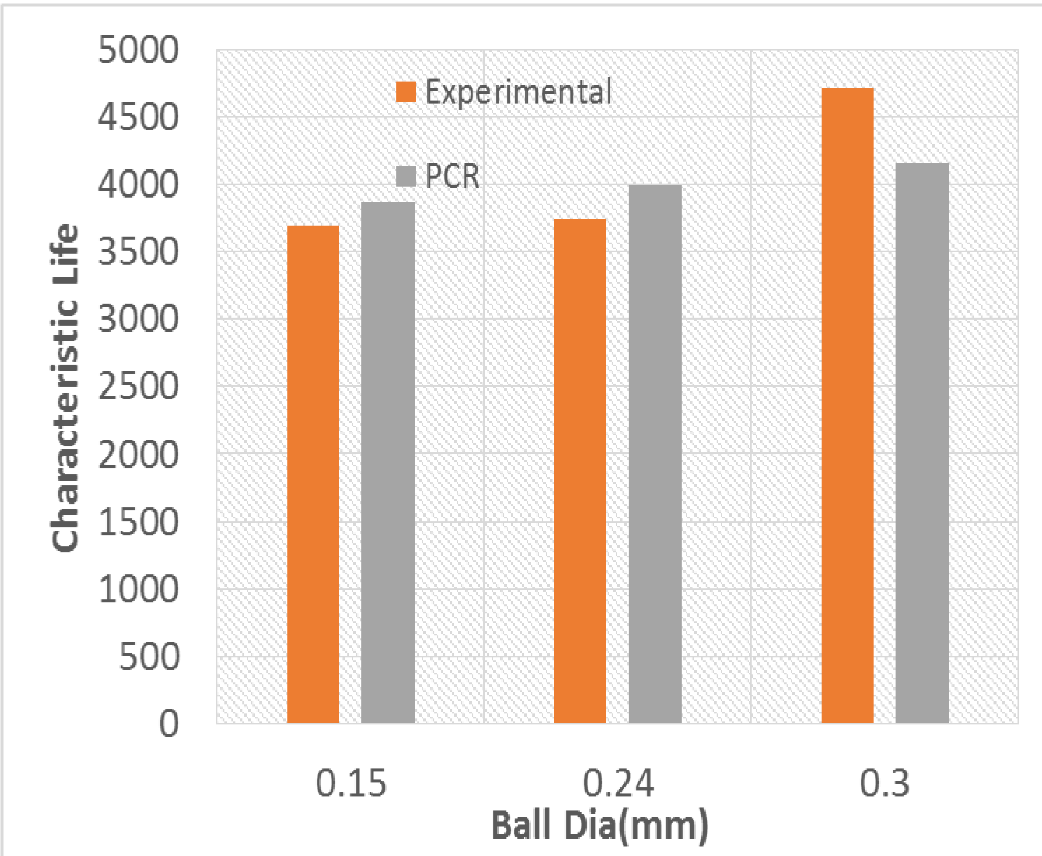


Figure 31: Validation of Ball Diameter

4.3.7 Package Size:

Increase in Package size increases life of the package.

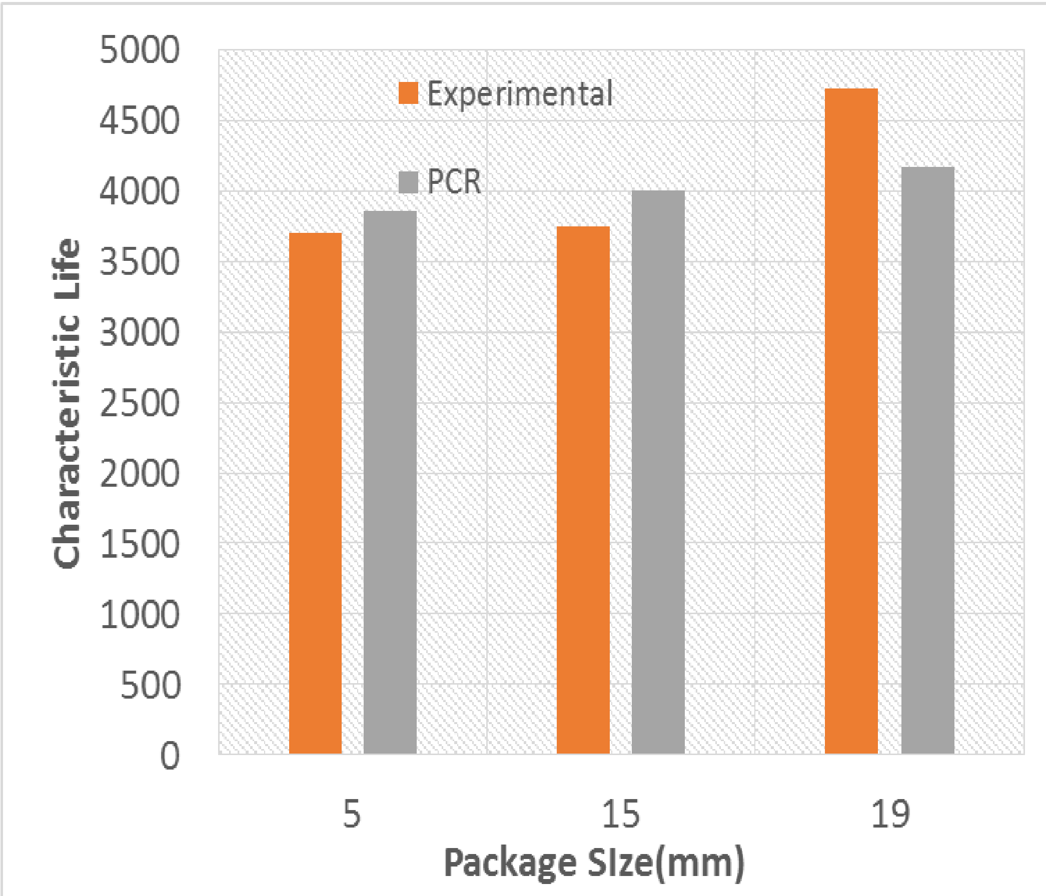


Figure 32: Validation of Package Size

4.3.8 Delta T:

Temperature difference is the most significant parameter in thermo-cycling. The model predicts square negative influence on life.

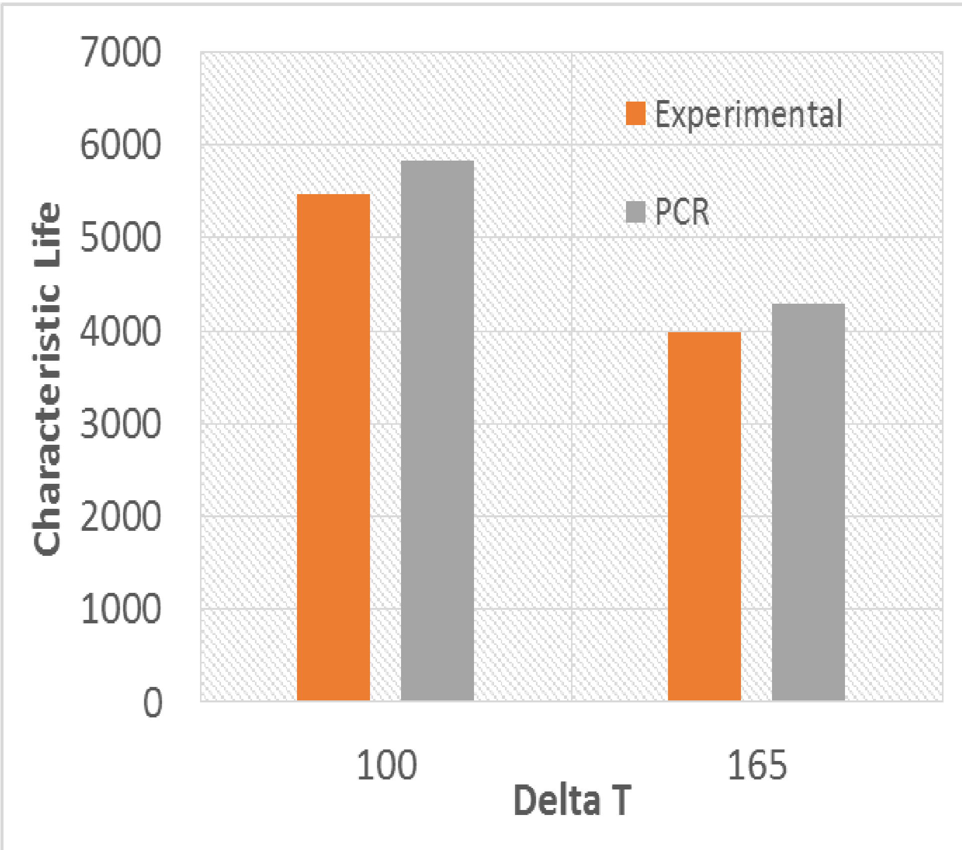


Figure 33: Validation of Delta T

4.3.9 Dwell Time:

Dwell time is a critical contributor to the life of the solder ball and as the model suggests, the increase in dwell time decreases life.

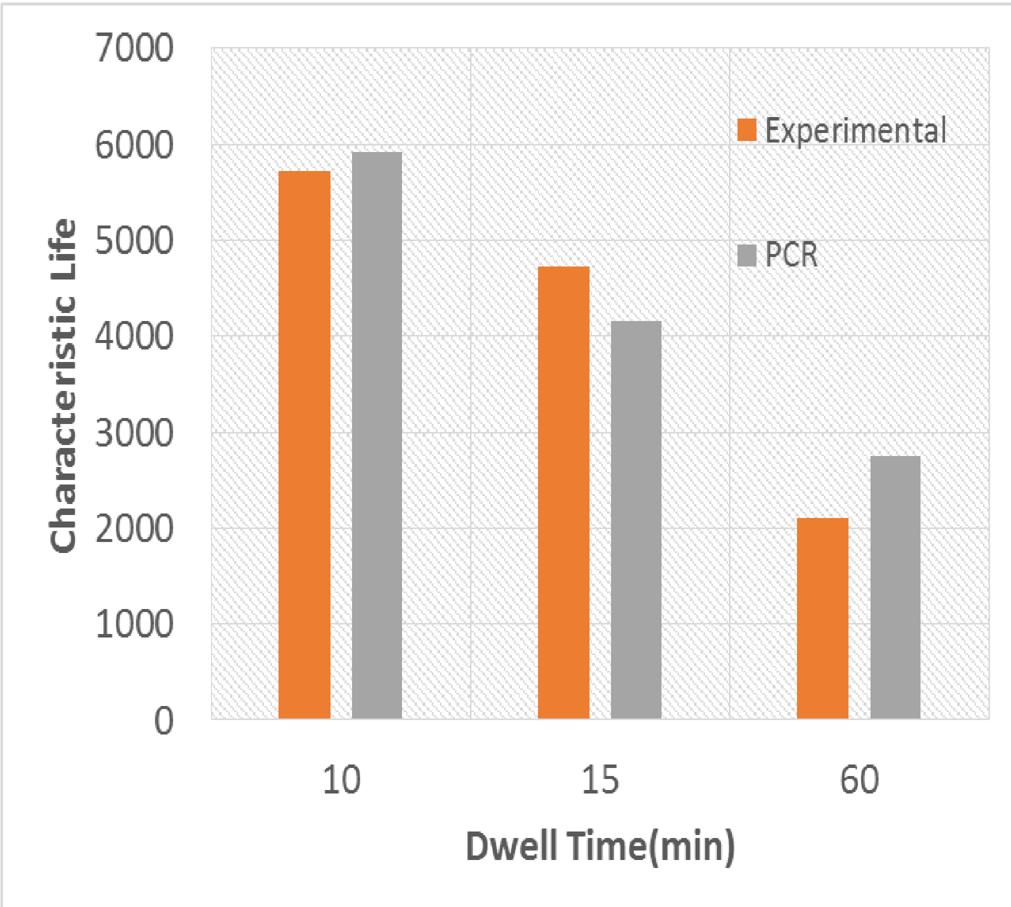


Figure 34: Validation of Dwell Time

4.4 Model Prediction:

The PCR model is predicted and is plotted against the actual N cycles to failure to check if the points lie around the 45-degree line from the center and the dotted lines represent 95% interval,

Experimental data (N): PCR data:

Standard Deviation (Aged) = 1046.433 Standard Deviation (Aged) = 1238.082

2*Standard Deviation (2σ) = 2092.865 2*Standard Deviation (2σ) = 2476.164

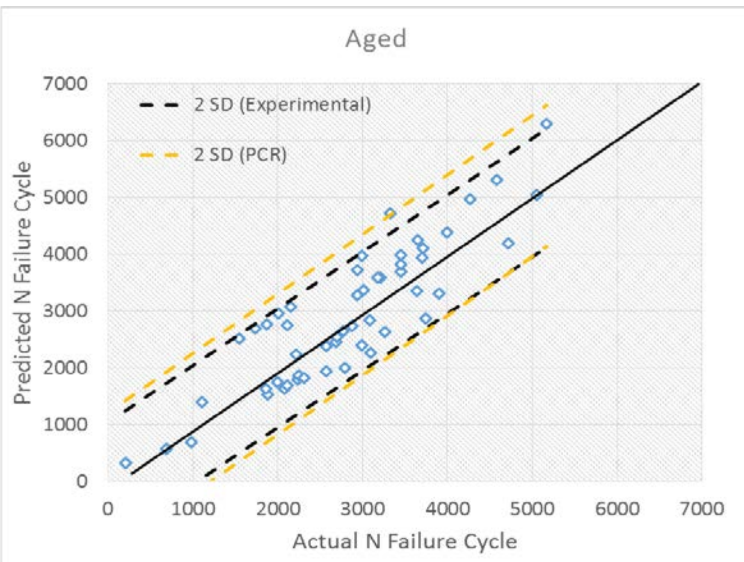


Figure 35: Actual versus predicted failure cycles plot (Aged)

Example calculation for Aged Sample:

Package Size : 5mm
Aging Temperature: 55 Deg C
Aging Time : 180 days

$$N_\alpha = e^{(20.61)} * (97)^{(-0.628)} * (0.15)^{(1.787)} * (1.57)^{(-0.242)} * (0.4)^{(-2.874)} * (5)^{(0.77)} * (2)^{(-0.0394)}$$

$$* (165)^{(-1.28)} * (15)^{(-0.188)} * (6)^{(-0.846)} * (180)^{(-0.0349)} * (55)^{(-0.2458)}$$

Experimental Value : 3650

Model Value of Aged Sample: 4257

Experimental data (N):

Standard Deviation (Aged) = 1712.719

2*Standard Deviation (2σ) = 3425.437

PCR data:

Standard Deviation (Aged) = 1659.53

2*Standard Deviation (2σ) = 3319.06

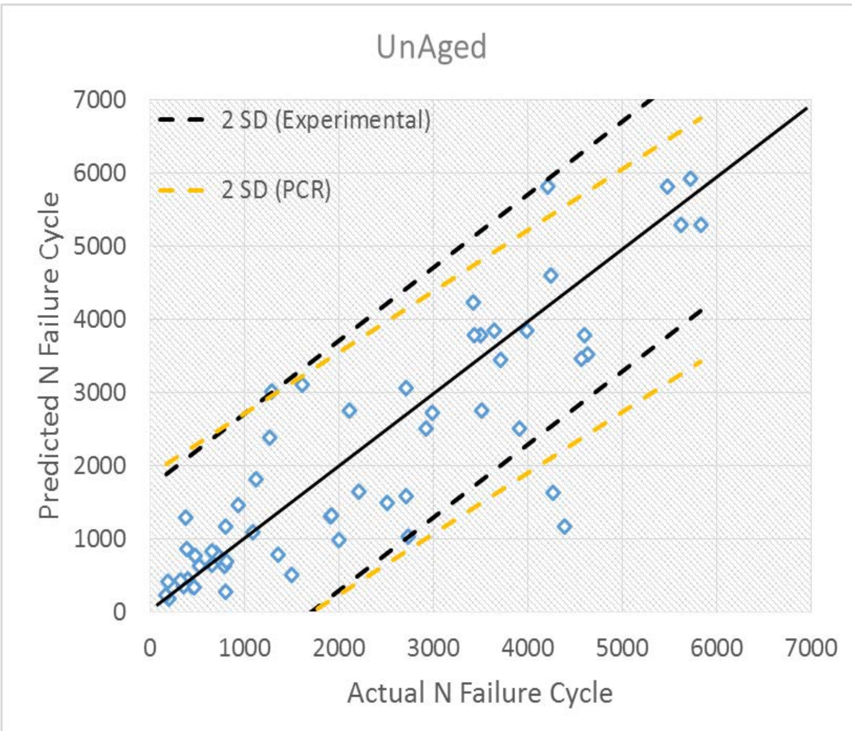


Figure 36: Actual versus predicted failure cycles plot (UnAged)

Example Calculation for UnAged Sample:

Package Size : 5mm
Aging Temperature: 55 Deg C
Aging Time : 1 day

$$N_\alpha = e^{(20.61)} * (97)^{(-0.628)} * (0.15)^{(1.787)} * (1.57)^{(-0.242)} * (0.4)^{(-2.874)} * (5)^{(0.77)} * (2)^{(-0.0394)}$$

$$* (165)^{(-1.28)} * (15)^{(-0.188)} * (6)^{(-0.846)} * (1)^{(-0.0349)} * (55)^{(-0.2458)}$$

Experimental Value : 5055

Model Value of UnAged Sample: 5103

Chapter 5

Prediction Model using Ridge Regression

Prediction model using Ridge Regression is done with the same 11 variables and dataset used in PCR model. Initially, Multiple Linear Regression gave results with irregularities like variables with high variation inflation factor and correlation coefficients with above 0.5. It is evident from the table of Pearson's correlation coefficient, which is given below,

5.1 Selection of Ridge Parameter:

The correlation matrix of the predictor variables shows that lot of predictors are correlated to each other. Below is the Table 6showing Correlation Matrix,

Table 6: Correlation Matrix

	Ball Count	**Ball Dia**	**PCB Thick**	**Pitch**	**Packg Size**	**Board Finish**	**Del T**	**Dwell Time**	**Del Alpha**	**Aging Time**	**Aging Temp**
Ball Count	**1**	**0.46**	**0.06**	**-0.52**	**-0.02**	**-0.75**	**0.53**	**0.32**	**0.08**	**0.62**	**0.53**
Ball Dia	**0.46**	**1**	**0.23**	**0.17**	**0.47**	**-0.08**	**0.11**	**0.17**	**0.25**	**0.06**	**0.07**
PCB Thick	**0.06**	**0.23**	**1**	**-0.22**	**-0.23**	**0.21**	**-0.14**	**0.58**	**0.97**	**-0.34**	**-0.29**
Pitch	**-0.52**	**0.17**	**-0.22**	**1**	**0.84**	**0.70**	**-0.47**	**-0.44**	**-0.21**	**-0.55**	**-0.48**
Packg Size	**-0.02**	**0.47**	**-0.23**	**0.84**	**1**	**0.39**	**-0.26**	**-0.34**	**-0.21**	**-0.30**	**-0.26**
Board Finish	**-0.75**	**-0.08**	**0.21**	**0.70**	**0.39**	**1**	**-0.59**	**-0.22**	**-0.22**	**-0.81**	**-0.66**
Del T	**0.53**	**0.11**	**-0.14**	**-0.47**	**-0.26**	**-0.59**	**1**	**0.13**	**-0.08**	**0.53**	**0.47**

Dwell Time	**0.32**	**0.17**	**0.58**	**-0.44**	**-0.34**	**-0.22**	**0.13**	**1**	**0.57**	**0.10**	**0.08**
Del Alpha	**0.08**	**0.25**	**0.97**	**-0.21**	**-0.21**	**-0.22**	**-0.08**	**0.57**	**1**	**-0.35**	**-0.30**
Aging Time	**0.62**	**0.06**	**-0.34**	**-0.55**	**-0.30**	**-0.81**	**0.53**	**0.10**	**-0.35**	**1**	**0.77**
Aging Temp	**0.53**	**0.07**	**-0.29**	**-0.48**	**-0.26**	**-0.66**	**0.47**	**0.08**	**-0.30**	**0.77**	**1**

Ridge regression is applied to the problem to circumvent the numerical snag. A small positive bias parameter k is introduced to try and reduce the variance and hence the Mean Square Error (MSE). Different bias values from 0 to 0.02 in increments of 0.0025 are tried to see if the regression co-efficients and the variance stabilize. If no stability is achieved, the upper limit of the bias parameter is increased until a point, which records the stability of the parameters, is reached at. Using the k value in the equation (27), the biased models are developed. The process is carried out as mentioned in Figure 9. The regression co-efficients and the VIF are recorded in every compilation. These values are recorded to see if stability is achieved. There are different ways where Stabilization of ridge parameter can be determined. Requirements of the model play a vital role. Since the objective is prediction care is taken that the model chosen to be with stable bias has good prediction accuracy and complied with the physical interpretations of the case. Observing the 'ridge plot' is the most common method. In the ridge plot, the bias parameter is on the X-axis against the β co-efficients themselves. The trade-off can be based on prediction accuracy of the model. Below is a ridge plot of the model based on the parameters mentioned above.

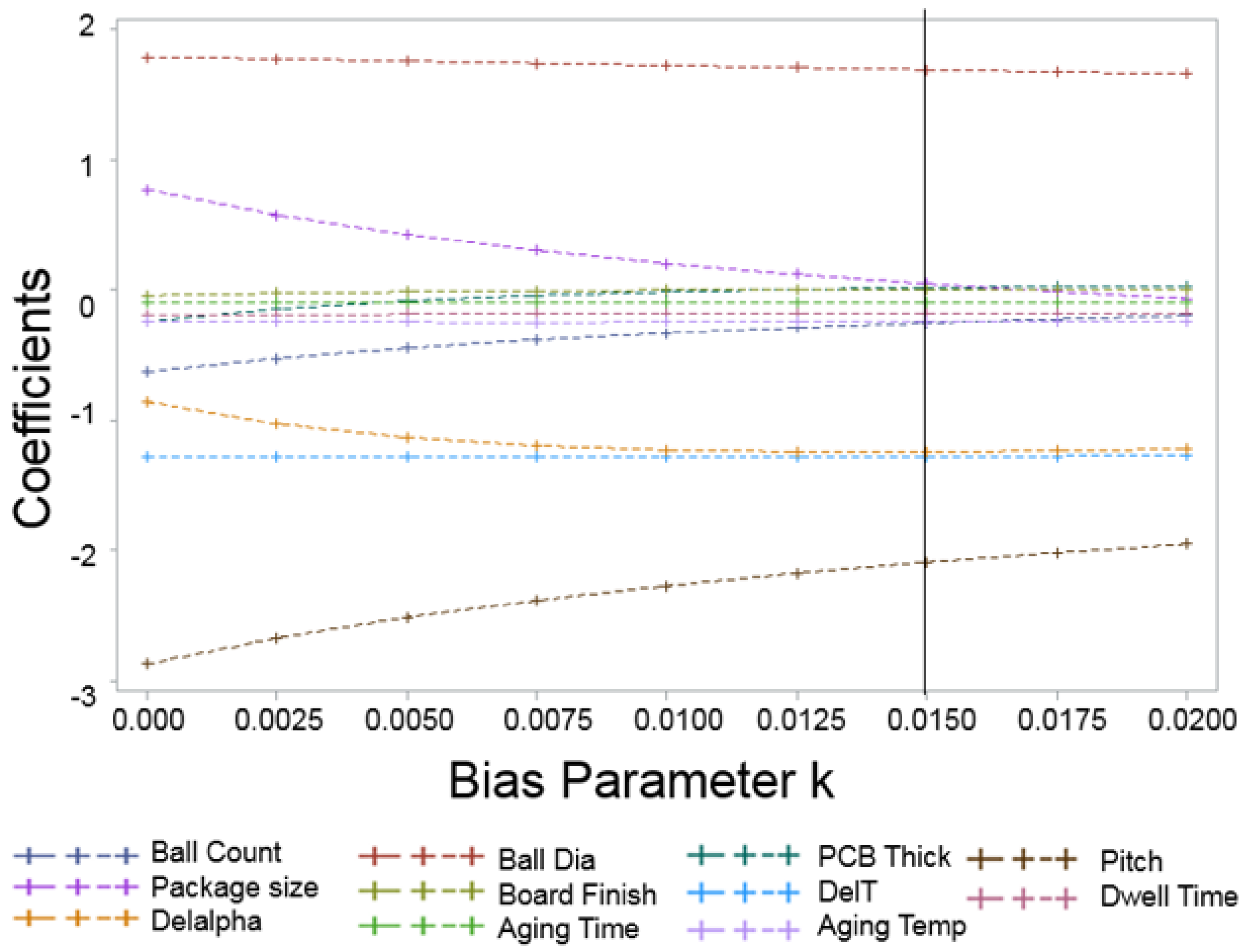


Figure 37: Ridge Plot (Based on coefficients)

A bias at which a stability of the coefficients is seen is chosen to the biasing parameter. In the Figure 37, upon closely observing plot, we see that all the variables stabilize at about 0.015. Hence the bias parameter is chosen to be 0.015. The model corresponding to k=0.015 is given below:

$$
\begin{aligned}
N_{\alpha} = \quad & e^{(21.21)}\,(\mathrm{BallCount})^{(-0.3328)}\,(\mathrm{BallDiaMM})^{(1.7329)}\,(\mathrm{PCBThicknessMM})^{(-0.0146)} \\
& (\mathrm{PitchMM})^{(-2.2718)}\,(\mathrm{PackageSizeMM})^{(0.2019)}\,(\mathrm{BoardFinishID})^{(-0.0028)} \\
& (\Delta T)^{(-1.2843)}\quad (\mathrm{DwellTimeMIN})^{(-0.1829)}\,(\mathrm{DeltaAlphaPPMC})^{(-1.2302)} \\
& (\mathrm{AgingDAYS})^{(-0.0355)}\,(\mathrm{AgingTempC})^{(-0.24607})
\end{aligned}
$$

An Analysis of variance (Table 7) is performed to verify that the results of the Regression process are significant.

Table 7: ANOVA

Source	**DF**	**SS**	**MS**	**F**	**P**
Regression	11	69.11	6.28	41.69	0.000
Residual Error	102	15.37	0.15		
Total	113	84.49			

5.2 Model Validation:

Validation has been performed to check the accuracy of Ridge model. In this validation, ridge model has been compared with experimental data alongside with PCR model results. This is done to check the accuracy of both the model together.

5.2.1 Actual VS Predicted values:

The following graph (Figure 38 and Figure 39) is plotted with actual experimental data against predicted data from the ridge model and the dotted lines represent 95% interval,

Aged:

Experimental data (N):

Standard Deviation (Aged) = 1046.433

2*Standard Deviation (2σ) = 2092.865

Ridge data:

Standard Deviation (Aged) = 1218.798

2*Standard Deviation (2σ) = 2437.596

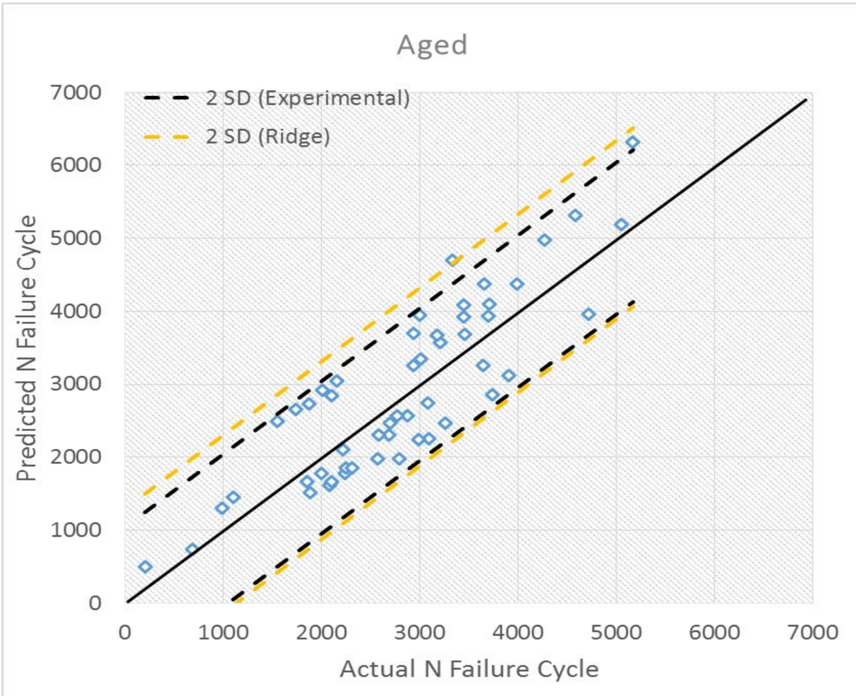


Figure 38: Actual versus predicted failure cycles plot (Aged)

Example calculation for Aged Sample:

Package Size : 5mm
Aging Temperature: 55 Deg C
Aging Time : 180 days

$$N_{\alpha} = e^{(21.21)} * (97)^{(-0.3328)} * (0.15)^{(1.7329)} * (1.57)^{(-0.0146)} * (0.4)^{(-2.2718)} * (5)^{(0.2019)} * (2)^{(-0.0028)} * (165)^{(-1.2843)} * (15)^{(-0.1829)} * (6)^{(-1.2302)} * (180)^{(-0.0355)} * (55)^{(-0.24607)}$$

Experimental Value : 3650

Model Value of Aged Sample: 4318

UnAged:

Experimental data (N):

Standard Deviation (Aged) = 1712.719

2*Standard Deviation (2σ) = 3425.437

Ridge data:

Standard Deviation (Aged) = 1547.48

2*Standard Deviation (2σ) = 3094.96

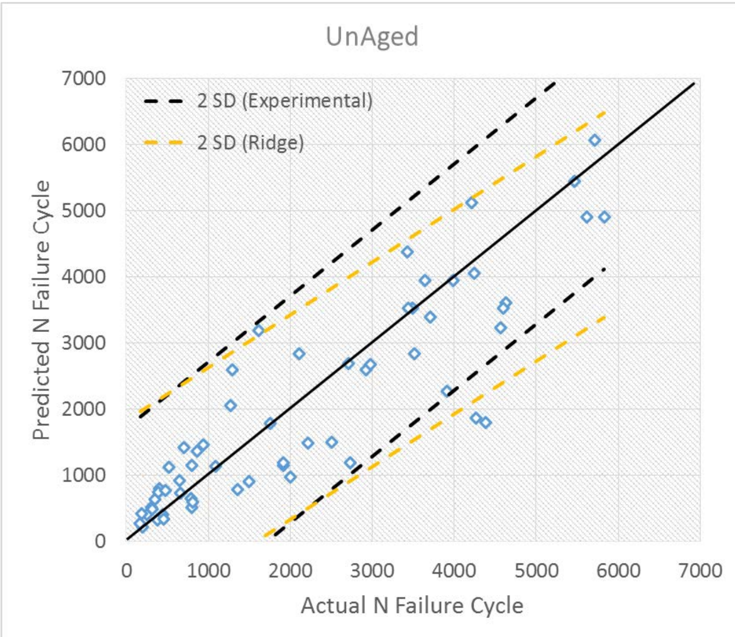


Figure 39: Actual versus predicted failure cycles plot (UnAged)

Example Calculation for UnAged Sample:

Package Size : 5mm
Aging Temperature: 55 Deg C
Aging Time : 1 day

$$N_{\alpha} = e^{(21.21)} * (97)^{(-0.3328)} * (0.15)^{(1.7329)} * (1.57)^{(-0.0146)} * (0.4)^{(-2.2718)} * (5)^{(0.2019)} * (2)^{(-0.0028)}$$

$$* (165)^{(-1.2843)} * (15)^{(-0.1829)} * (6)^{(-1.2302)} * (1)^{(-0.0355)} * (55)^{(-0.24607)}$$

Experimental Value : 5055

Model Value of UnAged Sample: 5192

5.2.2 Aging Effects:

Life of the package decreases with increase in aging. Statistical model has been compared with experimental data, which agrees with the trend. Data compared are for 1, 10, 180, and 360 days of aging and were also predicted for 720 days of aging from the Ridge model and PCR model, which is shown in following figures. Package size used for model prediction is 5mm at 25, 55, 85 and 125°C temperatures.

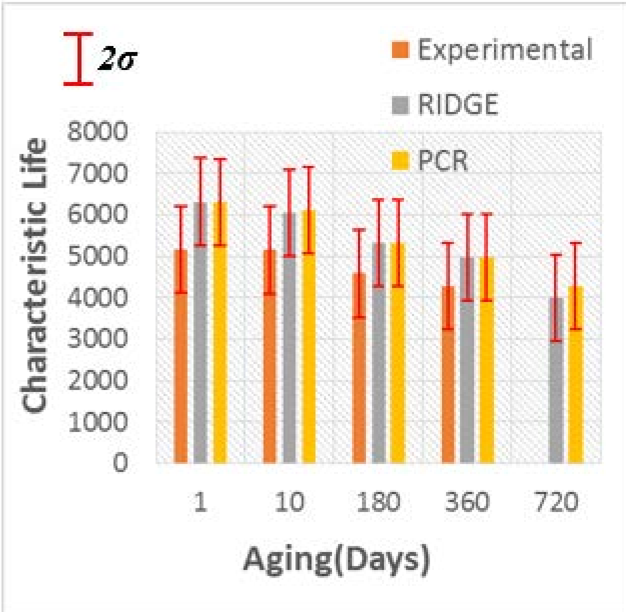
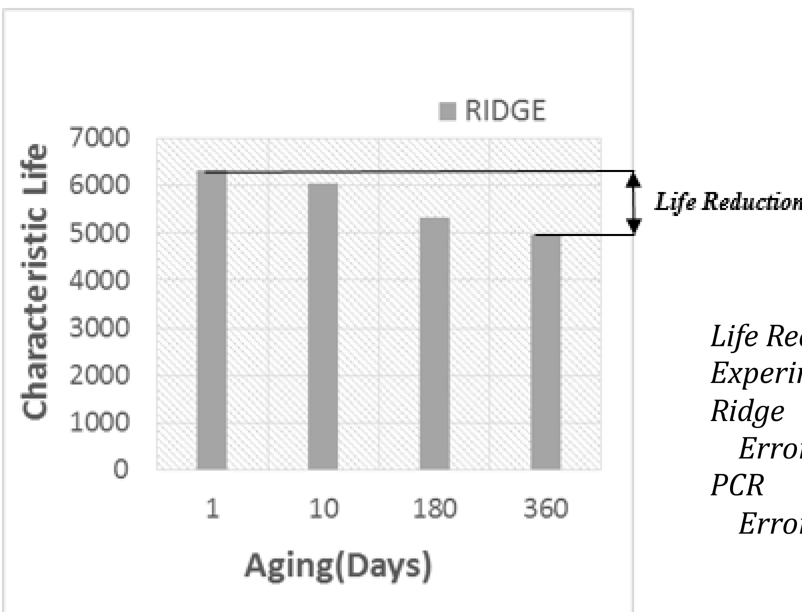


Figure 40: Effect of Aging for 5mm package at 25°C



Life Reduction (1 to 360 Days):
Experimental – 17.5% (Figure 12)
Ridge - 21.27%
Error - 21.54%
PCR - 21.05% (Figure 13)
Error - 20.28%

Figure 41: Effect of Aging (Ridge) for 5mm package at 25°C

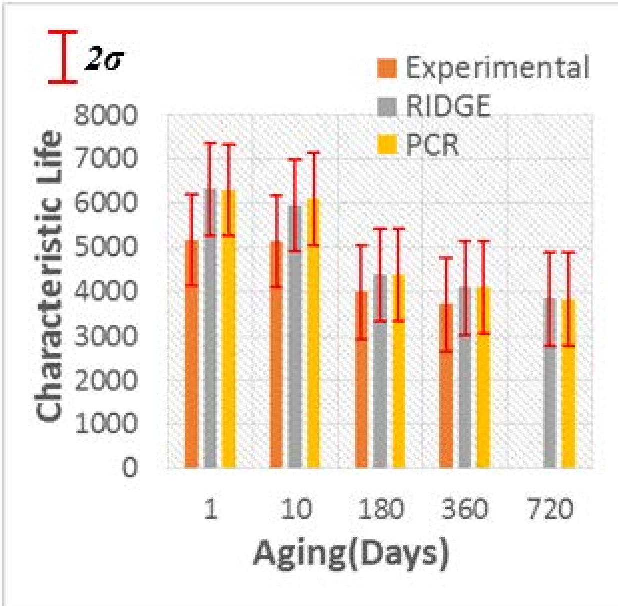
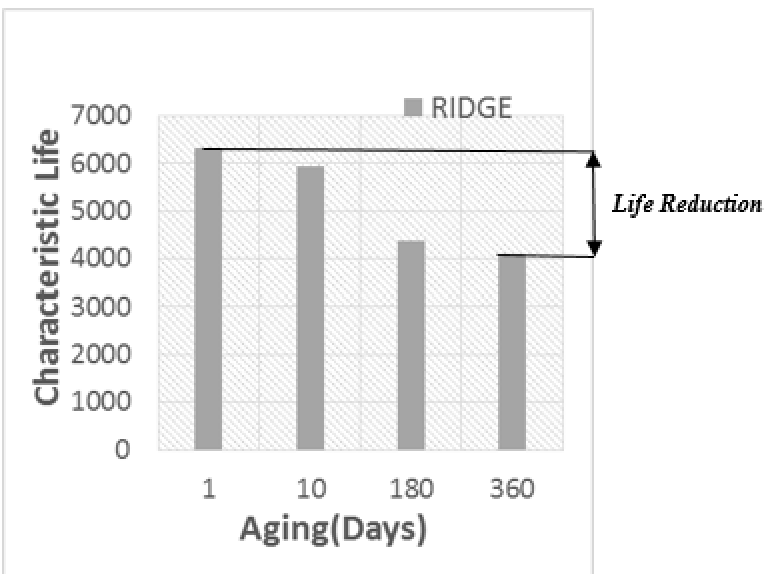


Figure 42: Effect of Aging for 5mm package at 55°C



Life Reduction (1 to 360 Days):
Experimental – 28.2% (Figure 15)
Ridge - 35.19%
Error - 24.78%
PCR - 34.8% (Figure 16)
Error - 23.40%

Figure 43: Effect of Aging (Ridge) for 5mm package at 55°C

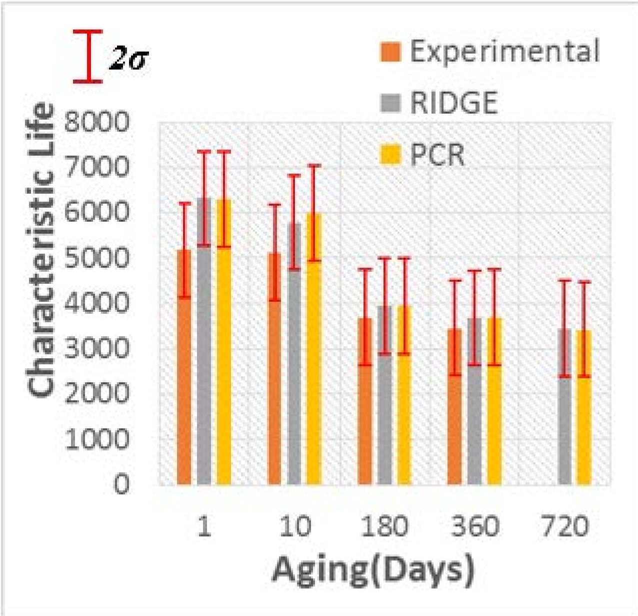
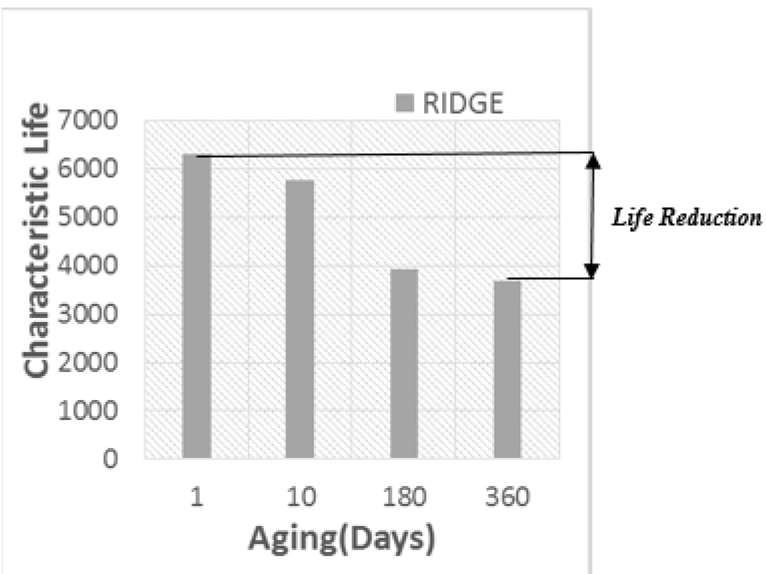


Figure 44: Effect of Aging for 5mm package at 85°C



Life Reduction (1 to 360 Days):
Experimental – 33.12% (Figure 18)
Ridge - 41.73%
Error - 25.99%
PCR - 41.2% (Figure 19)
Error - 24.39%

Figure 45: Effect of Aging (Ridge) for 5mm package at 85°C

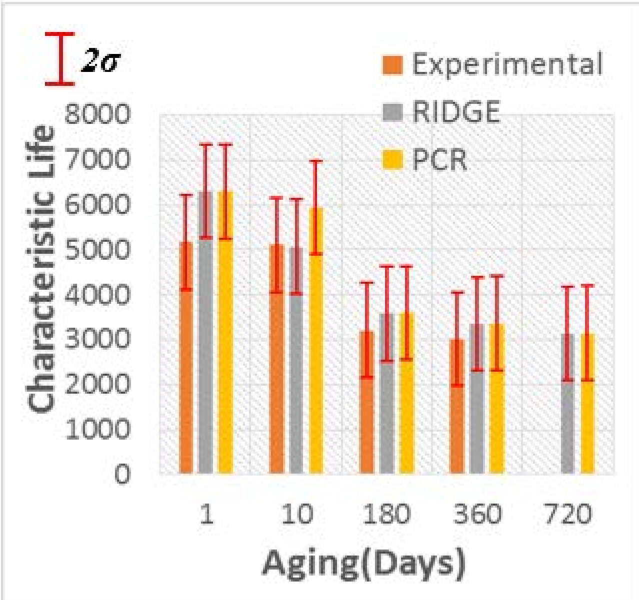
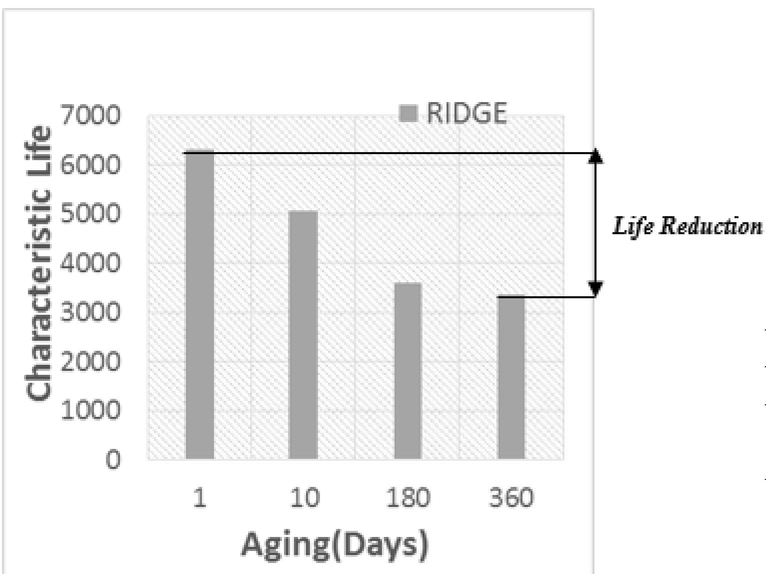


Figure 46: Effect of Aging for 5mm package at 125°C



Life Reduction (1 to 360 Days):
Experimental – 41.35% (Figure 21)
Ridge - 46.94%
Error - 13.51%
PCR - 46.32% (Figure 22)
Error - 12.02%

Figure 47: Effect of Aging (Ridge) for 5mm package at 125°C

If we need data for a longer term like 20 years it is highly difficult to perform the experiment for that long. Prediction equations can come in handy in these scenarios. So Figure 48, Figure 49, Figure 50, Figure 51 shows the prediction of life for 0, 5, 10 and 20years through Ridge model and those were compared with PCR model results.

Figure 48: Prediction of Aging (Years) for 5mm package at 25°C

Life Reduction:
Ridge - 40.86%
PCR - 45.20% (Figure 23)

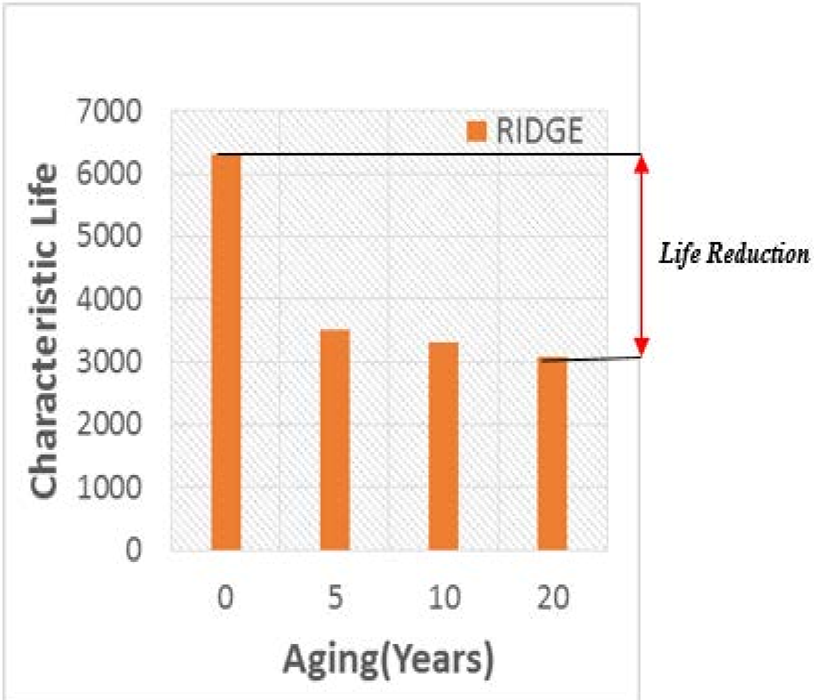


Figure 49: Prediction of Aging (Years) for 5mm package at 55°C

Life Reduction:
Ridge - 51.41%
PCR - 55.26% (Figure 24)

Figure 50: Prediction of Aging (Years) for 5mm package at 85°C

Life Reduction:
Ridge - 56.32%
PCR - 59.40% (Figure 25)

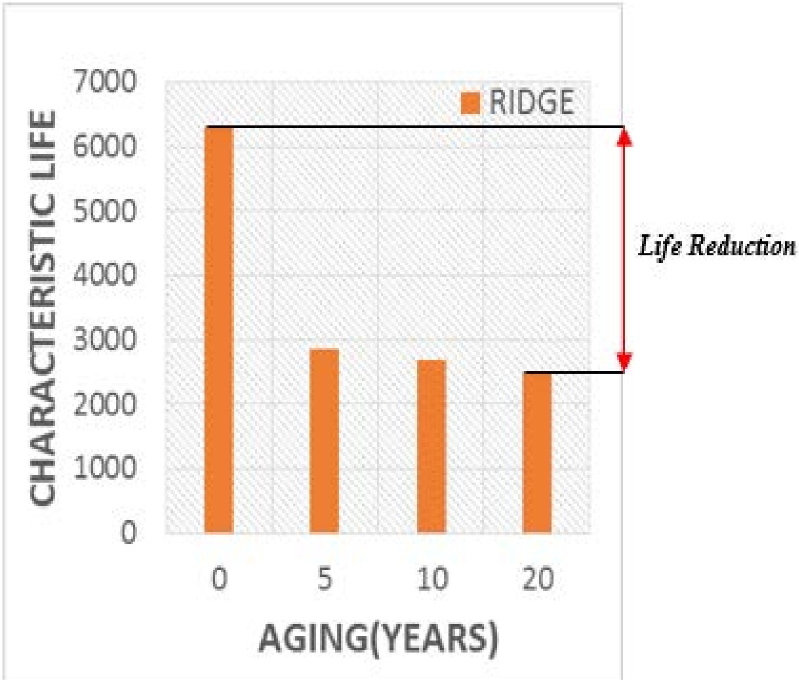


Figure 51: Prediction of Aging (Years) for 5mm package at 125°C

Life Reduction:
Ridge - 60.21%
PCR - 62.83% (Figure 26)

5.2.3 Aging Temperature:

Life of the package decreases with increase in aging temperature. Statistical model has been compared with PCR model and experimental data,

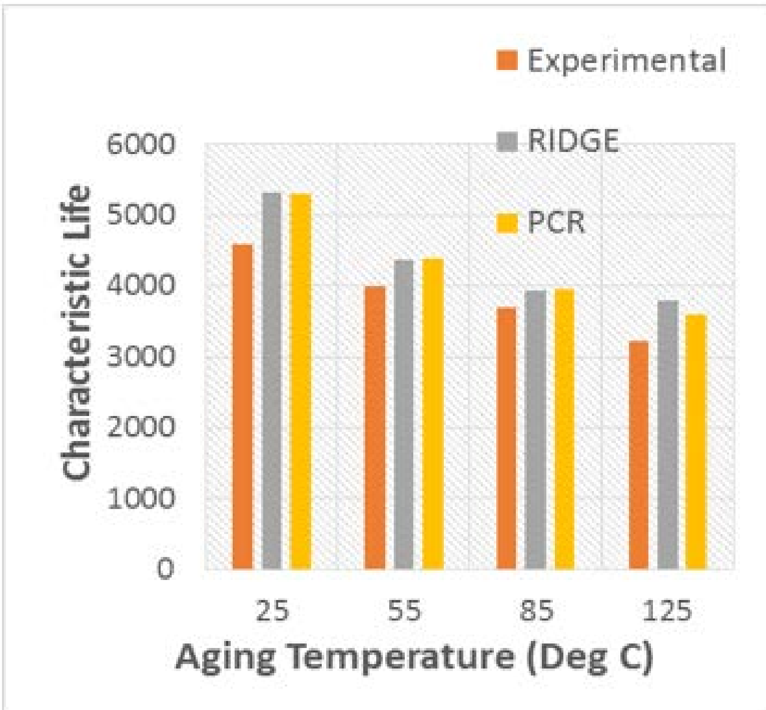


Figure 52: Effect of Aging Temperature for 5mm package at 6 month aging

5.2.4 Ball Count:

Experimental data indicates that thermal reliability of the BGAs decreases with the increase in the ball count. From the Figure 53 it is shown that the general trend of decrease in thermal reliability with the increase in the ball count, which is in agreement with the failure mechanics theory. Increase in the number of solder balls distributes the thermal deformation over a larger number of solder joints reducing the stress level in the individual ball.

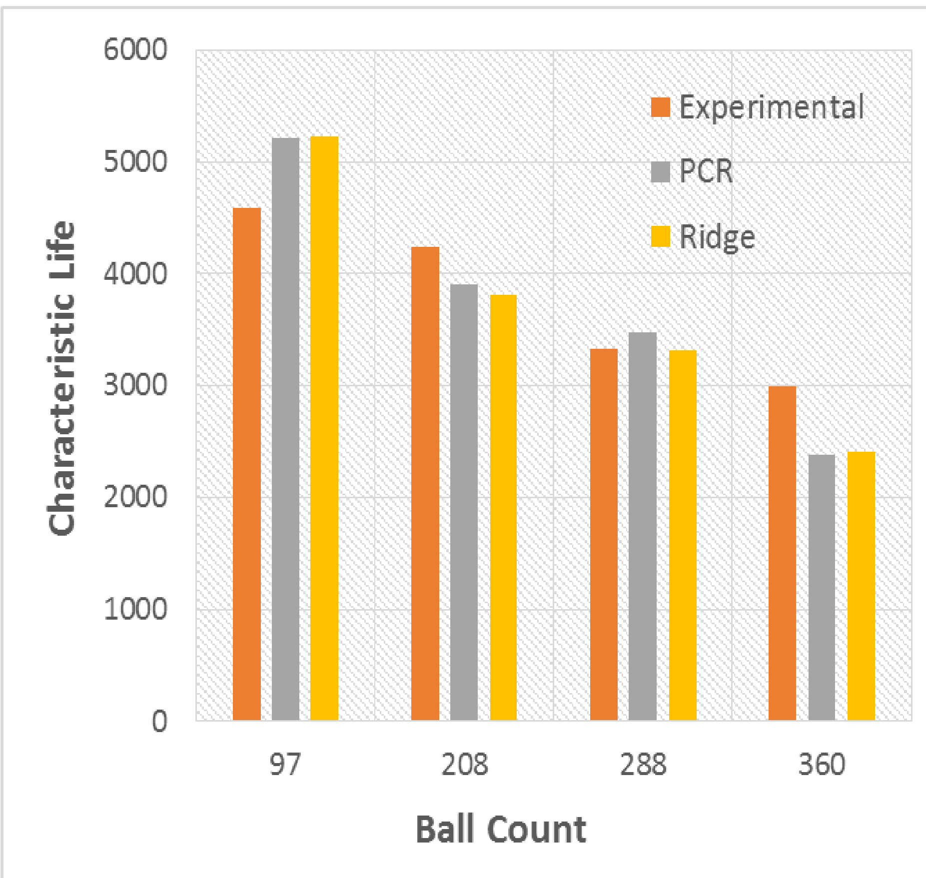


Figure 53: Validation of Ball Count

5.2.5 PCB Thickness:

The decrease in reliability of a ball-grid array package with the increase in the PCB thickness. This effect is consistent from failure mechanics as the increased PCB thickness leads to higher assembly stiffness, which results in higher stresses in the interconnect.

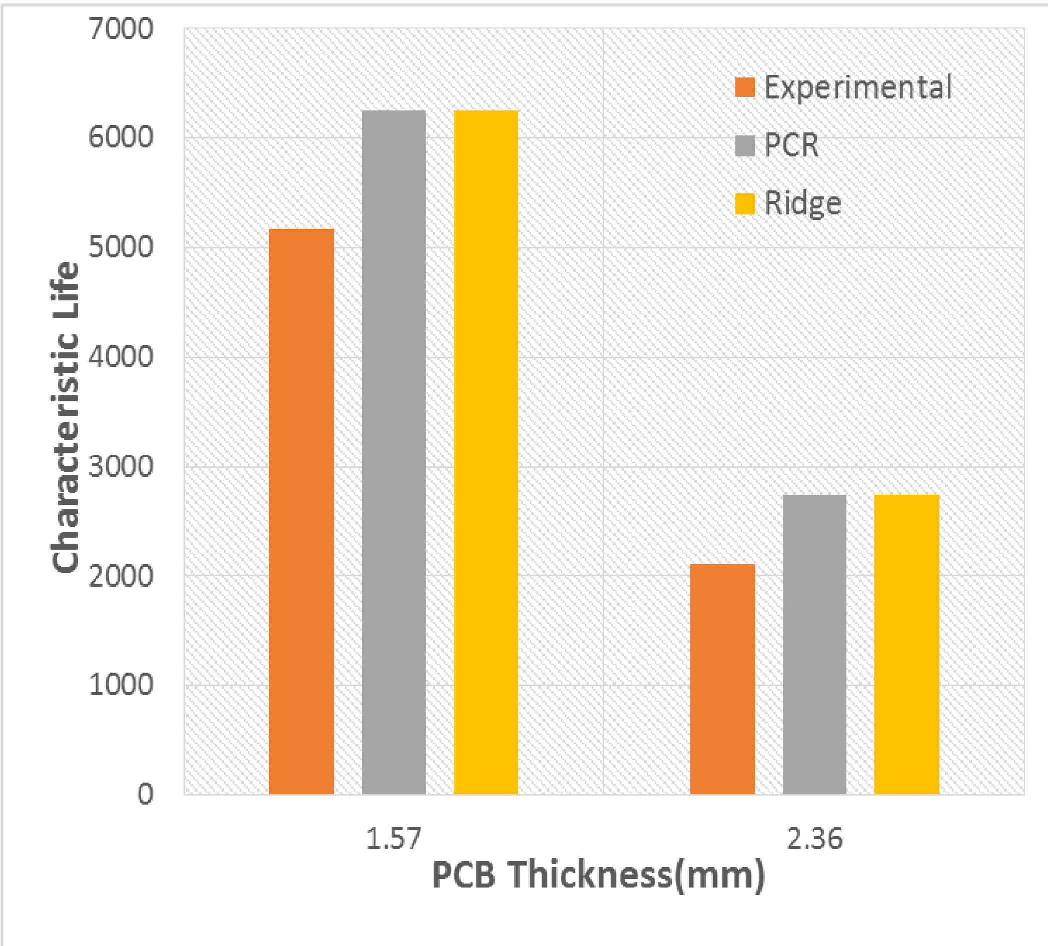


Figure 54: Validation of PCB Thickness

5.2.6 Pitch:

Increase in Pitch decreases life of the package.

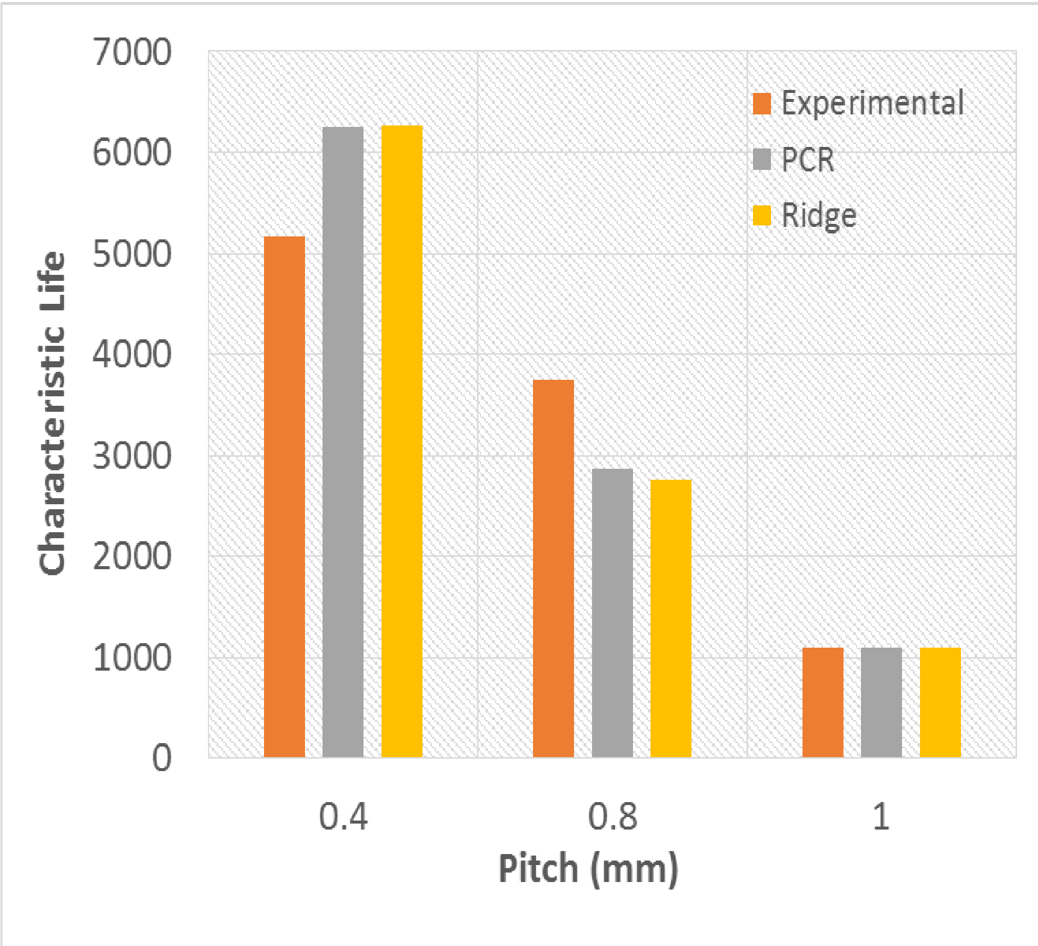


Figure 55: Validation of Pitch

5.2.7 Ball Diameter:

Experimental data indicates that the increase in the ball diameter leads to overall better thermal reliability of the package. This trend is in compliance with the failure mechanics theory as the increase in the solder ball diameter increases the crack area resulting in higher thermal fatigue life.

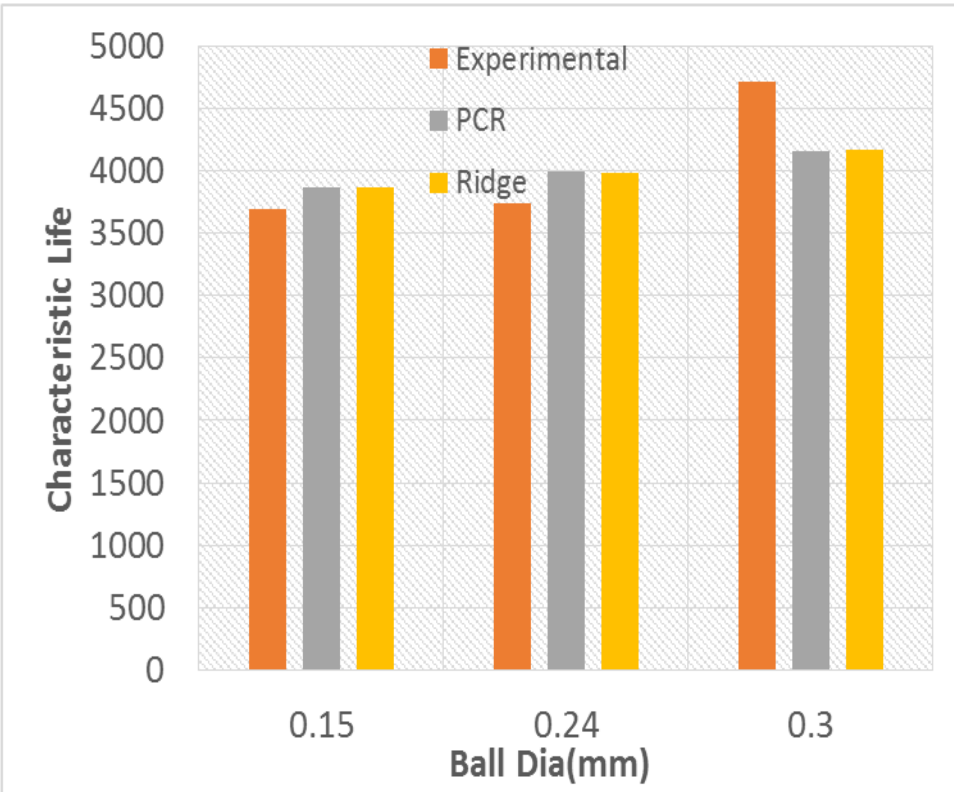


Figure 56: Validation of Ball Diameter

5.2.8 Package Size:

Increase in Package size increases life of the package.

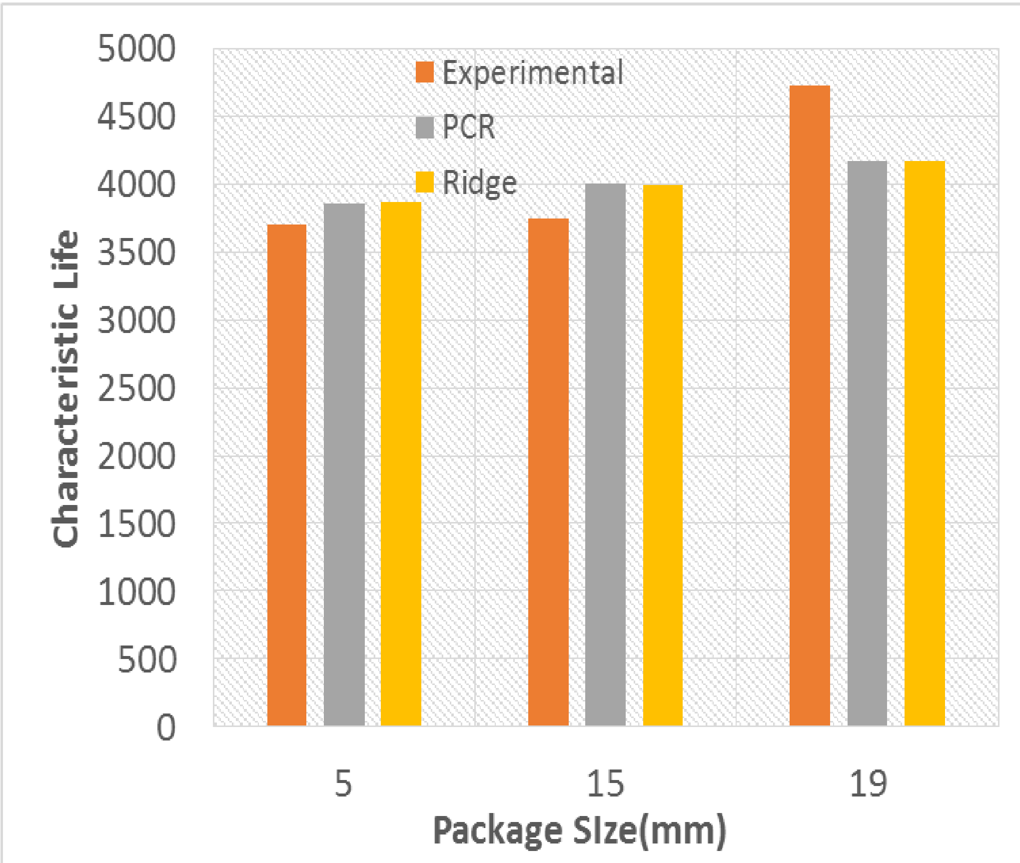


Figure 57: Validation of Package Size

5.2.9 Delta T:

Temperature difference is the most significant parameter in thermo-cycling. The model predicts square negative influence on life.

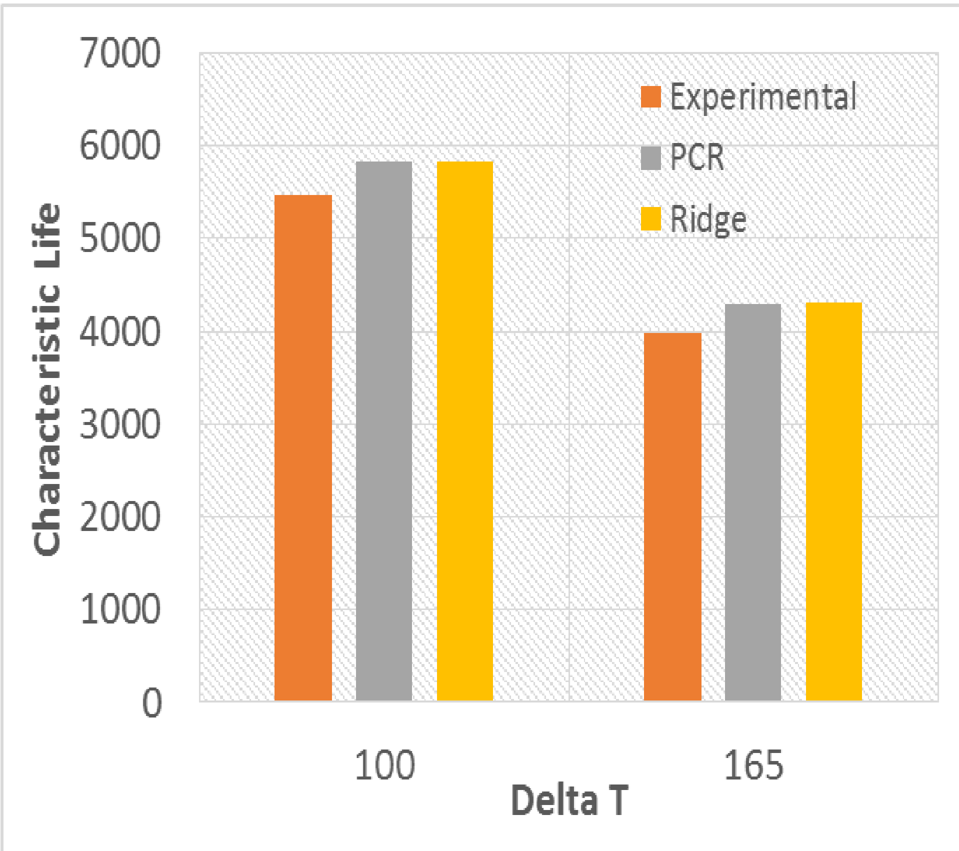


Figure 58: Validation of Delta T

5.2.10 Dwell Time:

Dwell time is a critical contributor to the life of the solder ball and as the model suggests, the increase in dwell time decreases life.

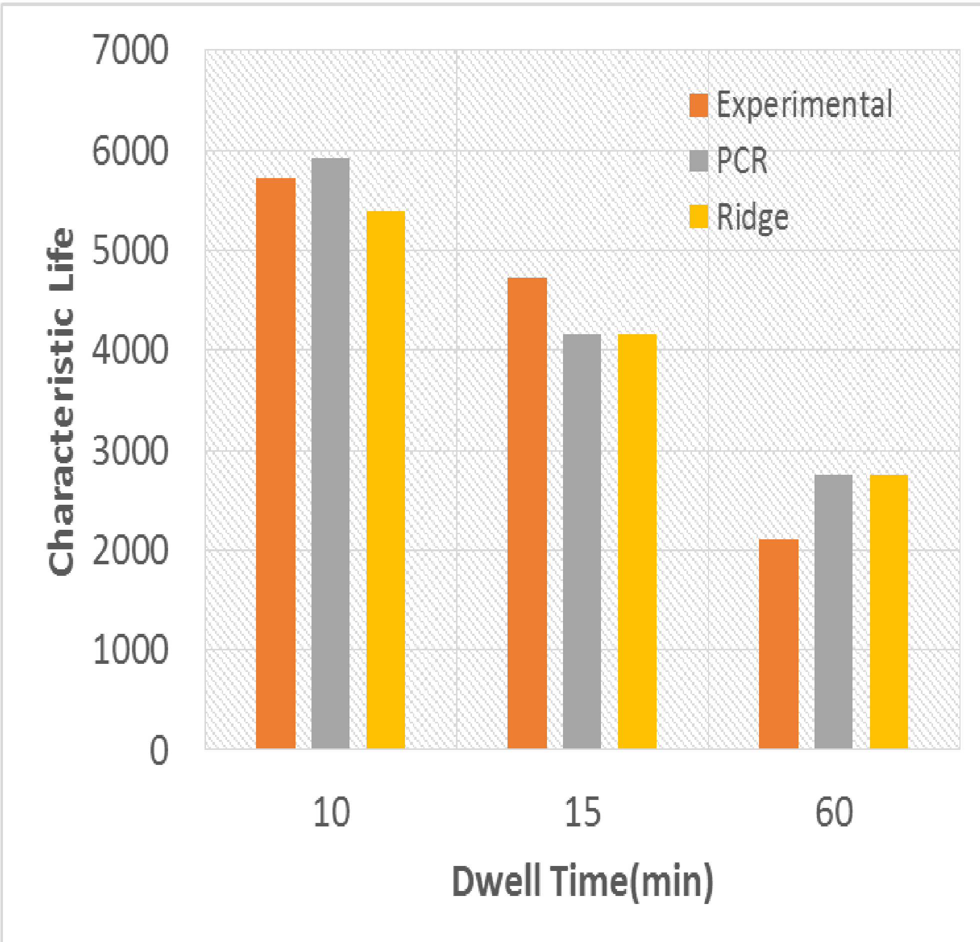


Figure 59: Validation of Dwell Time

5.3 Cross Validation:

Before doing the statistical analysis part of the data were taken separately and put aside for cross validation. To check the accuracy of the model apart from the data used this has been done.

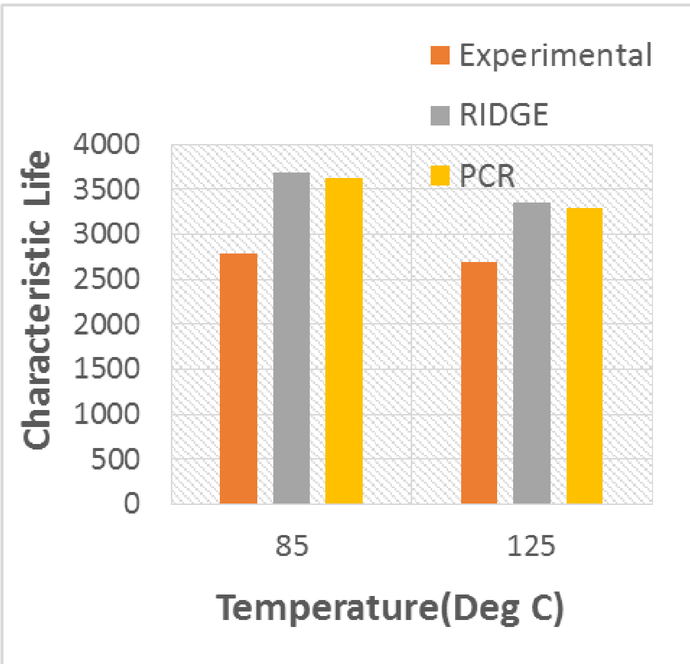


Figure 60: Cross Validation of 5mm Package

Package size – 5mm
Aging time - 720 Days (2 Years)

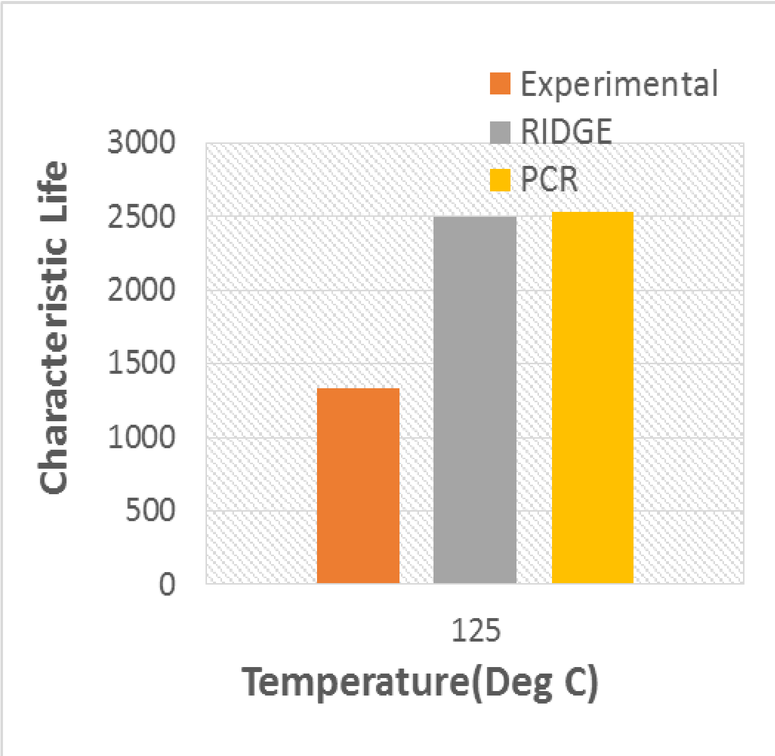


Figure 61: Cross Validation of 10mm Package

Package size – 10mm
Aging time - 720 Days (2 Years)

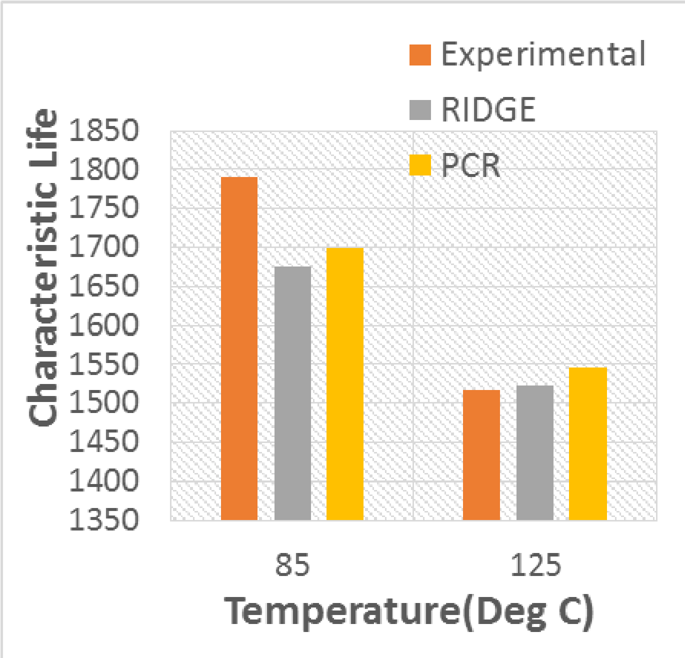


Figure 62: Cross Validation of 15mm Package

Package size – 15mm
Aging time - 720 Days (2 Years)

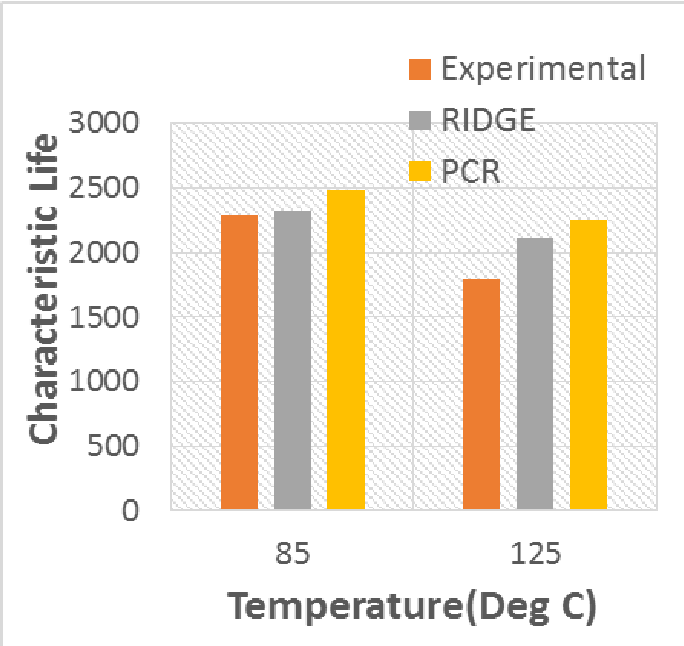


Figure 63: Cross Validation of 19mm Package

Package size – 19mm
Aging time - 720 Days (2 Years)

Chapter 6

Summary and Conclusions

A statistics-based regression modeling methodology has been presented which also included aging effects and aging temperature effects. Using the regression model projection of aging until 20 years for temperatures 25°C, 55°C, 85°C and 125 °C for 5mm package have been presented. It is been found that at 25°C life has been decreased to 45%, at 55°C it's decreased to 54%, at 85°C it's decreased to 59% and at 125 °C it's further decreased to 62%. Ridge model has been compared with PCR model and the graphs been plotted between them. The method provides an extremely cost effective and time effective solution for the thermo-mechanical reliability assessment of PCB assemblies subjected to harsh environments. The developed methodology allows the user to understand the relative impact of various geometric parameters, material properties, aging effects and Aging temperatures of the different configurations of lead free solder joints. The model predictions from statistical models have been validated with the actual experimental data, not used for the model development. A small set of data were set aside before model development for prediction purpose to check the accuracy of the model. For model development a total of 114 observations were used. The convergence between experimental results and the prediction results with higher order of accuracy achieved by any first order closed form models has been demonstrated. Use of these models are recommended for analyzing the relative influence of aging on the thermo-mechanical reliability of the package instead of using them for absolute life calculations which will consume time.

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Appendix-1

List of Symbols

α	Coefficient of Thermal Expansion
β	Coefficient of regression
ΔT	Temperature Cycle Magnitude
E	Model random error
1/TmeanK	Inverse of the mean temperature in Kelvin
[A]	Matrix of Predictor Variables, of full column rank
1/TmeanK	Inverse of the mean temperature in Kelvin
AlphaRelPPMC	Difference in CTE between part and PCB in ppm/C
BGA	Ball Grid Array
BallCount	Number of solder balls in the package
BallDiaMM	Diameter of the solder ball in millimeters
BallHtMM	Height of the solder ball in millimeters
ChipAreaSQMM	Area of the chip in Sq. millimeters
CABGA	Chip array BGA
Coef	Coefficient
DeltaTdegC	Temperature cycle range in degree centigrade
DieLengthMM	Chip Length in millimeters
DietoBodyRatio	Ratio of the length of the chip to the length of the package
ENIG	Electroless Nickel Immersion Gold
fu	frequency of temperature cycle under use conditions
fa	frequency of temperature cycle under accelerated test

	conditions
h	Solder Joint Height
HalfDiagLenMM	Half Diagonal Length of chip in mm.
HASL	Hot Air Solder Leveling
k	number of predictors
m	Empirical Constant in Coffin-Manson Equation
MS_{res}	Mean Square of residuals
n	number of data points
p	number of variables
PitchMM	Solder Ball Pitch in millimeters
Prefix Ln	Natural logarithm
PBGA	Plastic Ball Grid Array
PCB	Printed Circuit Board
PCR	Principal Component Regression
PkgPadDiaMM	Diameter of the package pad in millimeters
PkgPdAreaSQMM	Area of the Package Pad in sq. millimeters
PkgWtGM.	Weight of the package in grams
R^2	Multiple coefficient of determination
R_j^2	Adjusted R Square
s	Standard Deviation
SolderVolCUMM	Volume of the solder in cubic mm
SS_{res}	Sum of Squares of residuals
V	Volume of Solder Joint

[V]	The k x k eigenvector matrix consisting of normalized eigenvectors
VIF	Variance Inflation Factor
X	Predictor Variable
[X]	Scaled and Centered Predictor Variable Matrix
Y	Regressor Variable
[Z]	The n x k matrix of principal components