Supporting the Development of Teachers' Attributes Needed for the Selection and Implementation of Cognitively Demanding Tasks through Engagement with the MathTwitterBlogosphere

by

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Abstract

The purpose of this study was to determine if and how engagement with an online community supports the development of three attributes identified by Wilhelm (2014) that teachers need to have in order to consistently select and implement cognitively demanding tasks, including mathematical knowledge for teaching, visions of high quality mathematics instruction, and views for supporting struggling students (Wilhelm, 2014). A qualitative case study was undertaken of the MathTwitterBlogosphere (MTBoS), an online community of mathematics educators primarily hosted within Twitter and the blogosphere. A qualitative content analysis was used to better understand the community's content across Twitter and the blogosphere, while a qualitative interview analysis was used to understand the perceptions of five representative community members regarding the support they received from engaging with the community.

The teacher attribute of visions of high quality mathematics instruction was most frequently addressed within community interactions, followed by mathematical knowledge for teaching; views for supporting struggling students was only addressed in about 3% of the community content. Likewise, a majority of the community members interviewed perceived the MTBoS to have strengthened their mathematical knowledge for teaching and visions of high quality mathematics instruction but less so their ability to support struggling students' learning of mathematics. Thus, the MathTwitterBlogosphere community is a promising avenue for providing support to teachers in selecting and implementing cognitively demanding tasks, although it may provide less support for engaging struggling students.

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List of Abbreviations

CCSSO	Council of Chief State School Officers State
CKTM	Content Knowledge for Teaching Mathematics
DOK	Depth-of-Knowledge
MKT	Mathematical Knowledge for Teaching
NCTM	National Council of Teachers of Mathematics
NRC	National Research Council
NGA	National Governors Association
NAGB	National Assessment Governing Board
PCK	Pedagogical Content Knowledge
PLCs	Professional Learning Communities
SMK	Subject Matter Knowledge
VHQMI	Visions of High Quality Mathematics Instruction
VSSS	Views for Supporting Struggling Students

Chapter 1: Introduction

Current mathematics classrooms are dominated by teacher-led, direct instruction (Hiebert & Stigler, 2000; Silver, Mesa, Star, & Benken, 2009; Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013). In such classrooms, students leave unprepared to engage with mathematics beyond the recall of exact procedures and algorithms and often view the mathematics they encounter in the real world as disconnected from the mathematics they learn in the classroom (Boaler, 2002; Boaler & Staples, 2008). As a result of such mathematics instruction, only 44% of the United States high school graduates are prepared for college level mathematics (ACT, 2013; College Board, 2013).

To combat such ineffective teaching and learning outcomes, an array of reform-oriented documents has been published. Beginning in 1980, *An Agenda for Action* (National Council of Teachers of Mathematics [NCTM], 1980) prompted the first of many calls for an increase in problem solving within the mathematics classroom (National Commission on Excellence in Education, 1983; National Research Council [NRC], 1989; NCTM, 1989; NCTM, 1991; NCTM, 1995; NCTM, 2000; NCTM, 2009; National Governors Association Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010; NCTM, 2014), which would increase the depth of students' knowledge and their ability to use the mathematics they are learning (Boaler, 2002; Boaler & Staples, 2008). A range of mathematics education researchers have agreed that the greatest factor affecting students' understanding of mathematics is the cognitive demand of a task (Stein & Lane, 1996; Stein, Grover, & Henningsen, 1996; Boston & Smith, 2009). However, only a few teachers are consistently selecting and implementing cognitively demanding tasks into their mathematics classrooms (Hiebert & Stigler, 2000; Silver et al., 2009; Jackson et al., 2013).

Statement of the Problem

The quality and quantity of cognitively demanding tasks within the secondary mathematics classroom needs to increase (Hiebert & Stigler, 2000; Silver et al., 2009; Jackson et al., 2013). Best practices related to the selection and implementation of cognitively demanding tasks have been established (Stein et al., 1996; Smith & Henningsen, 1997; Jackson et al., 2013); when educators have engaged with these best practices, studies have documented an increase in students' mathematics achievement (Stein & Lane, 1996; Boaler, 2002; Boaler & Staples, 2008). Likewise, Wilhelm (2014) determined attributes of a mathematics teacher needed to consistently select and implement cognitively demanding tasks; the teacher attributes include: content knowledge for teaching mathematics, visions of high quality mathematics instruction, and views for supporting struggling students.

Studies (Arbaugh & Brown, 2005; Boston & Smith, 2009; Boston & Smith, 2011) have shown that with well-designed professional development, teachers can make significant improvements in their selection and implementation of cognitively demanding tasks. In these studies, the professional development was sustained over a period of time and remained focused on the enactment of cognitively demanding tasks (Arbaugh & Brown, 2005; Boston & Smith, 2009). Although the reported professional development was effective in supporting teachers' improvement in their selection and implementation of cognitively demanding tasks, both financial and stakeholder support was needed (Arbaugh & Brown, 2005; Boston & Smith, 2009). Alternatively, professional learning communities (PLCs) have been identified as a context for providing teachers with opportunities for continuous professional learning (Hord, 2008) that may mitigate some of these limitations associated with more formalized professional development. Furthermore, some PLCs are being established within online platforms (McCulloch, McIntosh, & Barrett, 2011; Luehmann & Borasi, 2011; Sakamoto, 2012). For example, one online community, the MathTwitterBlogosphere (MTBoS), is primarily hosted within the social media platform Twitter and the connected blogs of those who engage with the community (Johnson, 2015). The mission of the MTBoS is to help participating teachers improve their teaching practices each year ("Profiles of Math Teachers Who Blog and Twitter", n.d.; Milou, n.d.). Although numerous mathematics teachers, mathematics coaches, and mathematics teacher educators are engaged within the MTBoS and other online communities, research documenting how teachers are being supported by such communities is limited, particularly how it supports their selection and implementation of cognitively demanding tasks.

Purpose of the Study

This study will determine if and how engagement with online communities supports the development of teacher attributes needed to consistently select and implement cognitively demanding tasks, in particular the MathTwitterBlogosphere.

This study will be guided by one overarching question: Does engagement with the MathTwitterBlogosphere support the development of teacher attributes supporting effective use of cognitively demanding tasks? Two subquestions will be addressed as follows:

1. In what ways, if any, does the content of the MTBoS address teacher attributes needed to select and implement cognitively demanding tasks, such as, but not limited to, mathematical knowledge for teaching, visions of high quality mathematics instruction, and views for supporting struggling students?

2. How do members of the MathTwitterBlogosphere community perceive the effects of the MathTwitterBlogosphere on their development of teacher attributes needed to select and implement cognitively demanding tasks, such as, but not limited to, mathematical knowledge for teaching, visions of high quality mathematics instruction, and views for supporting struggling students?

Chapter 2: Review of Related Literature

In this chapter, I will discuss literature related to the need for problem solving in the mathematics classroom, the teacher attributes needed to enact cognitively demanding tasks, and how teachers build their knowledge of mathematics teaching.

In the first section of the literature review, I will discuss the calls for problem solving in mathematics education, the use of cognitively demanding tasks to enact problem solving, the frameworks associated with the selection and implementation of cognitively demanding tasks, and the methods used to prepare teachers to enact cognitively demanding tasks. Next, I discuss the findings of Wilhelm's (2014) research related to the enactment of cognitively demanding tasks and teachers' attributes that support their selection and implementation of cognitively demanding tasks. Lastly, I consider how teachers may develop the attributes needed to support the selection and implementation of cognitively demanding tasks within online learning communities is discussed.

A Need for Problem Solving in Mathematics Education

Problem solving has been an important theme in mathematics education since the year 1980 (NCTM, 1980, NCTM, 1989; NCTM, 1991; NCTM, 1995; NCTM, 2000; NGA & CCSSO, 2010; NCTM, 2014). To better understand this theme, I will begin this section with a historical review of the calls for reform in mathematics teaching, with an emphasis on the gradual increase in calls for cognitively demanding tasks as a way of ensuring problem solving occurs in the mathematics classroom. Second, I will identify a common definition of cognitively demanding tasks and introduce related frameworks. Third, I will examine the past and current state of America’s mathematics classroom. Fourth, I will present the learning affordances available to students when cognitively demanding tasks are consistently selected and implemented (Boaler, 2002; Boaler & Staples, 2008). Fifth, I will introduce the Mathematical Task Framework (Stein et al., 1996; Stein, Smith, Henningsen, & Silver, 2000). To conclude this section, I will present research demonstrating how the use of two frameworks supported mathematics teachers in their selection and implementation of cognitively demanding tasks (Arbaugh & Brown, 2005; Boston & Smith, 2009; Boston & Smith, 2011).

Progression through reform-oriented documents. Beginning in year 1980, reform-oriented documents prompted the first of many calls for problem solving within the mathematics classroom (NCTM, 1980, National Commission on Excellence in Education, 1983; NRC, 1989; NCTM,1989; NCTM, 1991; NCTM, 1995; NCTM, 2000; NCTM, 2009; NGA & CCSSO, 2010; NCTM, 2014). In many of the documents, problem solving is not framed as an additional activity to be included within the classroom, but a means by which mathematics learning should take place (NCTM, 1989; NCTM, 1991; NCTM, 2000; NCTM, 2014). Within the progression of the documents, the specificity of how problem solving may be accomplished has become increasingly clear: from the definition of worthwhile tasks in year 1991, to the call for an increase in the implementation of tasks that promote reasoning and problem solving in year 2014 (NCTM, 1980; NCTM, 1989, NCTM, 1991; NCTM, 2000; NCTM, 2009, NCTM, 2014). A brief overview of major reform document is included below.

An Agenda for Action. The United States is approaching thirty-six years since the initial publication of a nationally accepted mathematics education reform document, *An Agenda for Action* (NCTM, 1980). Although the text did not include a set of mathematical content standards, the document provided empirically supported recommendations for mathematics education over the next decade. Within *An Agenda for Action* (1980), NCTM's first recommendation called for an increase in opportunities for problem solving; the document stated, "The development of problem-solving ability should direct the efforts of mathematics educators through the next decade" (NCTM, 1980, p. 2). To support the efforts of embedding problem solving within the mathematics curriculum, several actions were recommended (NCTM, 1980). One such strategy encouraged educators to create a classroom environment where problem solving could flourish (NCTM, 1980). A combination of these recommended actions focused on changing teachers' classrooms and curriculums to support task enactment that promoted problem solving at all grade levels (NCTM, 1980).

Curriculum and Evaluation Standards for School Mathematics. Following the publication of *An Agenda for Action* (1980), increased efforts were directed at the dissemination and publicity needed to affect the mathematics education community with its messages. As a result, NCTM (1989) released *Curriculum and Evaluation Standards for School Mathematics*. Within this document, a vision for America's students became increasingly clear; students should be guided in, "(1) learning to value mathematics; (2) becoming confident in one's ability; (3) *becoming a mathematical problem solver* [emphasis added]; (4) learning to communicate mathematically, and; (5) learning to reason mathematically" (NCTM, 1989, p. 6). NCTM (1989) identified problem solving as the means by which learning should take place, as stated in the following:

> The introduction of new topics and most subsumed objectives should, whenever possible, be embedded in problem situations posed in an environment that encourages students to explore, formulate and test conjectures, prove generalizations, and discuss and apply the results of their investigations. Such an instructional setting enables students to approach

> the learning of mathematics both creatively and independently and thereby strengthen their confidence and skill in doing mathematics. (p. 128)

The authors also stated that the problem situations (e.g., tasks) that promote problem solving are the fabric that constructs and reinforces learning mathematics (NCTM, 1989).

Professional Standards for Teaching Mathematics. The *Professional Standards for Teaching Mathematics* (NCTM, 1991) was published to describe the instructional practices needed to support the demands set forth by the *Curriculum and Evaluation Standards* (NCTM, 1989). The first standard within the document addressed the importance of worthwhile mathematical tasks, including the content of the task, the attributes of the task that would promote student actions, and the way in which students learn mathematics (NCTM, 1991). Worthwhile mathematical tasks were defined as follows:

> The teacher of mathematics should pose tasks that are based on—
>
> - sound and significant mathematics;
> - knowledge of students' understanding, interests, and experiences;
> - knowledge of the range of ways that diverse students learn mathematics;
>
> and that
>
> - engage students' mathematical understanding and skills;
> - develop students' mathematical understanding and skills;
> - stimulate students to make connections and develop a coherent framework for mathematical ideas;
> - call for problem formulation, problem solving, and mathematical reasoning;
> - promote communication about mathematics;
> - represent mathematics as an ongoing human activity;

- display sensitivity to, and draw on, students' diverse background experiences and dispositions;
- promote the development of all students' dispositions to do mathematics. (NCTM, 1991, p. 25)

To ensure the importance of tasks was communicated, the document connected the task selected to the opportunities students would have to learn specific mathematics (NCTM, 1991). Likewise, it was communicated that worthwhile tasks should be the catalyst used for students to consider specific mathematics concepts, the relationships associated with the concept, and any real world application of the concept (NCTM, 1991).

Assessment Standards for School Mathematics. The *Assessment Standards for School Mathematics* (NCTM, 1995) was also designed and published to describe the assessment practices and policies needed to support the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). One of the central uses for assessment included using tasks within assessments to demonstrate growth in one's ability to explore, conjecture, and solve nonroutine problems (NCTM, 1995). Furthermore, the authors also emphasized the importance of selecting tasks the provided an entry point for all learners (NCTM, 1995).

Principles and Standards for School Mathematics. Following a decade of focused and coherent efforts to improve mathematics education, NCTM continued its push for improvement through the release of *Principles and Standards for School Mathematics* (2000), recognizing that if its standards were to remain pertinent to students' needs, the visions and goals the standards embody must periodically be examined, evaluated, and revised by the community of mathematics educators (NCTM, 2000). Within the first few pages of the newly released document, NCTM (2000) included one of the clearest visions of effective mathematics instruction to date. Within the vision, school mathematics was described as including the following:

- engages all in mathematics instruction;
- ambitious expectations for all;
- knowledgeable teachers with adequate resources;
- a mathematically rich curriculum;
- opportunities to learn procedures alongside conceptual understanding;
- *engaging and complex mathematics tasks* [emphasis added];
- opportunities to make, refine, and explore conjectures (NCTM, 2000).

From this list of characteristics, engaging and complex mathematical tasks lead students into the disposition of problem solving (NCTM, 2000). When instructional tasks are connected to the real-experiences of students, or embedded within the contexts of pure mathematics, they should pique students interest through intellectual challenge, thus, inviting enough speculation to ensure students work hard to solve the mathematical problems (NCTM, 2000). To ensure this intellectual challenge is not only available to a select few, NCTM (2000) again recommended that worthwhile tasks include multiple entry points. In continuing the recommendation of past documents, use of tasks that promote problem solving remained the primary tool for learning mathematics content and making connections across the content (NCTM, 2000).

Adding it Up. Not long after the publication of *Principles and Standards for School Mathematics* (NCTM, 2000), the National Research Council published *Adding It Up: Helping Children Learn Mathematics* (2001). Within this document, the National Research Council (2001) clearly defined what it means to become proficient in mathematics. The text stated, “We have chosen mathematical proficiency to capture what we believe is necessary for anyone to learn mathematics successfully” (p. 116). The learning of mathematics was defined using five interdependent and interrelated strands, the strands included: (1) conceptual understanding, (2) procedural fluency, (3) strategic competence, (4) adaptive reasoning, and (5) productive disposition (NRC, 2001). Although tasks were not discussed within the document, its conceptualization of mathematical proficiency provided additional rationale for the pivotal role of tasks in the mathematics classroom (NCTM, 2014).

Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics. In 2006, NCTM published *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*. The text was published to provide one possible answer to an essential question, “What mathematics should be the focus of instruction and learning at particular grade levels of the pre-K-12 educational system?” (NCTM, 2006, p. 3). As one may infer from the document’s title, the document provided focus for instruction and learning beginning in prekindergarten and concluding in grade 8 mathematics (NCTM, 2006). Curriculum focal points were designed to communicate the recommended focus of instruction and learning at each grade-level; curriculum focal points serve to lay as the foundation for further mathematics learning (NCTM, 2006). The document advocated a focus on problem solving: “They (curriculum focal points) are indispensable elements in developing problem solving, reasoning, and critical thinking skills, which are important to all mathematics learning” (p. 5). Likewise, curriculum focal points were designed and identified to provide the mathematics needed for students to solve problems (NCTM, 2006). A strong focus on problem solving continues throughout the document, at the start of each grade band, educators are reminded of the utility of problem solving; the document repeatedly stated, “It is essential that these focal points be addressed in context that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations” (NCTM, 2006, p. 11).

Focus in High School Mathematics: Reasoning and Sense Making. NCTM’s efforts to ensure students learn through tasks that promote problem solving did not deteriorate with time, in year 2009, NCTM released *Focus in High School Mathematics: Reasoning and Sense Making.* Within the document, NCTM (2009) defined reasoning and sense making as follows, “Reasoning can be thought of as the process of drawing conclusions on the basis of evidence or statement assumptions,” and “we define sense making as developing understanding of a situation, context, or concept by connecting it with existing knowledge” (p. 4). Combined, reasoning and sense making support the notion of problem solving within the classroom; one cannot solve a task without reasoning, and it is through this reasoning that tasks promote the needed opportunities for students to make sense of mathematical ideas (NCTM, 2009).

Common Core State Standards for Mathematics. Following the publication of the *Focus in High School Mathematics: Reasoning and Sense Making* (NCTM, 2009), a state-led effort to develop a nationally accepted set of mathematics standards concluded with the release of the Common Core State Standards (NGA & CCSSO, 2010). The writers of the Common Core quickly identified a major goal in developing the mathematics standards, to provide focus and coherence to mathematics education (NGA & CCSSO, 2010). The writers of the Common Core also recognized the substantial work of mathematics educators; the published standards are a culmination of information drawn from states, international models of practices, and research input from all stakeholders (NGA & CCSSO, 2010). The design of the standards ensured focus and coherence was returned to mathematics education (NGA & CCSSO, 2010). Also important to the development of the content standards were their deep roots in research based learning progressions; learning progressions provide information "detailing what is known today about how students' mathematical knowledge, skill, and understanding develop over time" (NGA & CCSSO, 2010, p. 4).

In addition to content standards, the Common Core Standards for Mathematics described standards for mathematical practice (NGA & CCSSO, 2010). The standards for mathematical practice are a combination of NCTM's (2000) process standards and the previously mentioned strands of *Adding it Up* (2001). Following are the practices that all mathematics educators should strive to develop within their students:

1. *Make sense of problems and persevere in solving them* [emphasis added].
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and making use of structure.
8. Look for and express regularity in repeated reasoning. (NGA & CCSSO, 2010)

Note that if the first standard, make sense of problems and persevere in solving them, becomes the focus of mathematics education, the remaining seven practices will follow suit (Koestler, Felton, Bieda, & Otten, 2013). As mentioned previously, cognitively demanding tasks are the means for which problem solving takes place. Following is an excerpt that connects each standard of mathematical practice back to the first, a focus on problem solving.

> …as students work to solve an authentic problem, they may use quantitative reasoning skills (practice 2) or various tools (practice 5) to make progress in their solution, and

within their work they may look for structure (practice 7) and regularity in reasoning (practice 8). Furthermore, students may be expected to explain and justify their solution rather than simply arrive at an answer, and explaining one's thinking is likely to involve the construction of an argument (practice 3) and an attention to precision (practice 6) in the act of communication. It may also be the case that the problem at hand necessitates that students generate and use a mathematical model (practice 4). In other words, students who engage in the first standard for mathematical practice are likely to be employing the other practices as well. (Koestler et al., 2013, p. 121)

Throughout this progression of documents, tasks that promote problems solving must become the means through which learning the content of mathematics takes place (NCTM, 1980; NCTM, 1989; NCTM, 2000; NGA & CCSSO, 2010; NCTM, 2014).

Principles to Actions: Ensuring Mathematical Success for All. To help support the implementation of the Common Core Standards for Mathematics and other rigorous standards, NCTM's *Principles to Actions: Ensuring Mathematical Success for All* (2014) described an effective mathematics program as one that "engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically" (p. 7). More specifically, learners should be provided opportunities that enable them to—

- *engage with challenging tasks that involve active meaning making and support meaningful learning* [emphasis added]*;*
- connect new learning with prior knowledge and informal reasoning and, in the process, address preconceptions and misconceptions;

- acquire conceptual knowledge as well as procedural knowledge, so that they can meaningfully organize their knowledge, acquire new knowledge, and transfer and apply knowledge to new situations;
- construct knowledge socially, through discourse, activity, and interaction related to meaningful problems;
- receive descriptive and timely feedback so that they can reflect on and revise their work, thinking, and understandings; and
- develop metacognitive awareness of themselves as learners, thinkers, and problem solvers, and learn to monitor their learning and performance. (NCTM, 2014, p. 9)

These targets suggest that the interactions within the classroom directly affect the learning of mathematics (Ball and Forzani, 2011). To help mathematics educators ensure students' learning is characteristic of mathematical proficiency, as defined by the National Research Council (2001), NCTM (2014) developed eight mathematics teaching practices. These practices "represent a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics" (NCTM, 2014, p. 9). The mathematics teaching practices follow:

- Establish mathematics goals to focus learning;
- *Implement tasks that promote reasoning and problem solving* [emphasis added];
- Use and connect mathematical representations;
- Facilitate meaningful mathematical discourse;
- Pose purposeful questions;
- Build procedural fluency from conceptual understanding;
- Support productive struggle in learning mathematics; and
- Elicit and use evidence of student thinking. (NCTM, 2014, p. 10)

Given one of the eight teaching practices is centered on implementing tasks that promote reasoning and problems solving, if this practice is applied effectively, then following a similar argument made by Koestler and colleagues (2013) above, the remaining seven practices become visible and viable within the mathematics classroom.

Conclusion. Calls for reform over the past decades have consistently maintained a common mission; in essence, “An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experience that promote their ability to make sense of mathematical ideas and reason mathematically” (NCTM, 2014, p. 7). The means to accomplish this goal has also been consistent, to engage students in tasks that promote problem solving; NCTM (2014) stated, “Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies” (p. 17). The effect such tasks will have on the opportunities for student learning has also been discussed, the task draws students’ attention to the mathematics, often piquing their interest and promoting hard work on understanding important mathematical ideas (NCTM, 1991; NCTM, 2000; NCTM, 2014).

Cognitively demanding tasks defined. Cognitively demanding tasks are the medium through which problem solving can occur within the mathematics classroom (NCTM, 1980; NCTM, 1989; NCTM, 1991; NCTM, 1995; NCTM, 2000; NCTM, 2006; NCTM, 2009; NCTM, 2014). I begin by establishing common language around cognitively demanding tasks and the frameworks used to assess the cognitive demand of a task.

Definition. Two terms are essential to define a cognitively demanding task: mathematical task and cognitive demand. Walter Doyle (1988) first defined an academic task in terms of the following components:

> (a) a goal state or end product to be achieved, (b) a problem space or set of conditions and resources available to accomplish the task, (c) the operations involved in assembling and using resources to reach the goal state or generate the product, and (d) the importance of the task in the overall work system of the class. (p. 169)

Others have sense provided more specific definitions of a mathematical task. Stein, Smith, Henningsen, and Silver (2009) defined a mathematical task as a mathematical problem or set of problems that address a related mathematical idea or content. Similarly, within *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014), mathematical tasks were defined as “tasks [that] can range from a set of routine exercises to a complex and challenging problem that focuses students’ attention on a particular mathematical idea” (p. 17). For the purposes of this research, the most recently published definition of a mathematical task by NCTM (2014) will be used throughout the remainder the dissertation.

The second portion of the definition, cognitive demand, includes numerous variations, all with similar characteristics; for the purposes of this research, the cognitive demand of an instructional task will be defined as the cognitive processes or reasoning required to solve or participate in a given activity (Doyle, 1988; Jackson et al., 2013; Hsu & Silver, 2014; Stein et al., 1996; Stein et al., 2009). Frameworks used to assign cognitive demand to a mathematics task are discussed next.

Cognitive demand frameworks. Multiple frameworks have been designed and developed to assess the cognitive demand of a mathematical task. In the following section, I discuss several key frameworks.

Bloom's taxonomy. Benjamin Bloom, alongside fellow educators, developed and published the document containing Bloom's taxonomy in 1956, *Taxonomy of Educational Objectives: The Classification of Educational Goals. Handbook I: Cognitive Domain.* During the 1990s, a former student of Benjamin Bloom began the revision process in hopes of adapting the taxonomy to the 21st century; the new taxonomy was published in 2001 (Forehand, 2005). The revised taxonomy (see Figure 1) maintained the six original categories, but three categories were renamed, two categories were interchanged, and those categories with unchanged names were changed to verb form (Krathwohl, 2002).

	Cognitive Processes					
The Knowledge Dimensions	1. Remember	2. Understand	3. Apply	4. Analyze	5. Evaluate	6. Create
Factual						
Conceptual						
Procedural						
Metacognitive						

Figure 1. The Taxonomy Table (Wilson, 2013).

The revised categories and descriptions follow:

1.0 Remember – Retrieving relevant knowledge from long-term memory.

2.0 Understand – Determining the meaning of instructional messages, including oral, written, and graphic communication.

3. 0 Apply – Carrying out or using a procedure in a given situation.

4.0 Analyze – Breaking material into its constituent parts and detecting how the parts related to one another and to an overall structure or purpose.

5.0 Evaluate – Making judgments based on criteria and standards.

6.0 Create – Putting elements together to form a novel, coherent whole or make an original product. (Krathwohl, 2002)

The dividing lines of each domain within the revised hierarchy are more flexible than the dividing lines found within the original taxonomy, Krathwohl (2002) stated, “If, however, one were to locate the ‘center point’ of each of the six major categories on a scale of judged complexity, they would likely form a scale from simple to complex” (p. 215). Beyond changes within the six original levels, the greatest change was the addition of a knowledge dimension (Walsh & Sattes, 2005). Within the knowledge dimension, the levels included: factual knowledge, conceptual knowledge, procedural knowledge, and metacognitive knowledge (Wilson, 2013; Walsh & Sattes, 2005). Given a two-dimensional framework, Krathwohl (2002) stated, “Any objective [learning objective] could be classified in the Taxonomy Table in one or more cells that correspond with the intersection of the column(s) appropriate for categorizing the verb(s) and the row(s) appropriate for categorizing the noun(s) or noun phrase(s)” (p. 215).

Depth of knowledge. Coordinated by the Council of Chief State School Officers (CCSSO) and with the help of the National Institute for Science Education (NISE), Webb (1999) was charged with leading a team to determine the consistency between mathematics and science standards and the assessments that were administered within these disciplines. He developed the depth-of-knowledge (DOK) continuum as a part of that analysis, which can be used to categorize tasks by level of cognitive demand. The first level, recall, “includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula" (Webb, 2002, p. 5). The second level, skill/concept requires students to move beyond a habitual or rote response; students must decide how to approach the problem (Webb, 2002). The third level, strategic thinking requires reasoning, planning, and the use of evidence; tasks requiring students to justify their thinking are typically classified within this third level (Webb, 2002). The last level, extended thinking requires even more complex reasoning, thinking, and planning, but over an extended period of time (Webb, 2002). Webb (2002) further added, "Students should be required to make several connections—relate ideas within the content area, or among content areas—and have to select one approach among many alternatives on how the situation should be solved" (p. 6).

Figure 2 below outlines how the DOK levels may be applied within the mathematics classroom (Kentucky Department of Education, 2007).

Recall & Reproduction (DOK 1)	**Skills & Concepts/ Basic Reasoning (DOK 2)**	**Strategic Thinking/ Complex Reasoning (DOK 3)**	**Extended Thinking/ Reasoning (DOK 4)**
• Identify who, when, what where, and why • Recall facts, terms, concepts, trends, generalizations and theories • Use a variety of tools • Recognize or identify specific information contained in graphics. • Identify specific information in artworks • Define • Describe (recall, recite or reproduce information)	• Describe or explain how or why • Give an example • Describe and explain issues and problems, purposes, patterns, sources, reasons, points of view or processes • Compare artworks and concepts used in artworks • Classify, sort items into meaningful categories • Convert information from one form to another	• Use concepts to solve problems • Use evidence to justify • Propose and evaluate solutions to problems • Recognize and explain misconceptions • Cite evidence and develop a logical argument for concepts • Reason and draw conclusions • Disseminate among plausible answers • Analyze similarities and differences in issues and problems • Apply concepts to new situations • Make connections across time and place to explain a concept or big idea • Recognize and explain	• Connect and relate ideas and concepts within the content area or among content areas • Examine and explain alternative perspectives across a variety of sources • Describe and illustrate how common themes and concepts are found across time and place • Make predictions with evidence as support • Develop a logical argument • Plan and develop solutions to problems • Analyze and synthesize information from multiple sources • Complex reasoning with planning, investigating or developing a product that will most likely require an extended period of time-must require applying significant conceptual understanding and higher-order thinking

Figure 2. Webb's Depth of Knowledge Levels (Kentucky Department of Education, 2007, p. 5).

Task analysis guide. Stein and colleagues (2000) also devised a framework that allows problems to be differentiated by cognitive demand. Within this framework (see

Figure 3), a task may be assigned to one of four levels (Stein et al., 2009). First, *memorization* tasks lack

Low-Level Cognitive Demand	**High-Level Cognitive Demand**
Memorization Tasks • Involve either producing previously learned facts, rules, formulae, *or* definitions or committing facts, rules, formulae, or definitions to memory. • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • Are not ambiguous—such tasks involves exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlay the facts, rules, formulae, or definitions being learned or reproduced. *Procedures Without Connections Tasks* • Are algorithmic. Use of the procedure is specifically called for or its use is evidence based on prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • Have no connection to the concepts or meaning that underlie the procedure being used. • Are focused on producing correct answers rather than developing mathematical understanding. • Require no explanation or explanations that focus solely on describing the procedure that was used.	*Procedures With Connections Tasks* • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problems situations). Making connections among multiple representations helps to develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding. *Doing Mathematics Tasks* • Require complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • Require students to explore and to understand the nature of mathematical concepts, processes, or relationships. • Demand self-monitoring or self-regulation of one's own cognitive processes. • Require students to access relevant knowledge in working through the task. • Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • Require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Figure 3. The Task Analysis Guide (Stein et al., 2000).

connection to concepts; these tasks only require the recall of facts, formulas, or definitions (Stein et al., 2000). Second, *procedures without connection* tasks also lack connections to concepts, but are instead algorithmic; the tasks typically state a given procedure to perform (Stein et al., 2000). Third, *procedures with connections* tasks use the procedure to develop a deeper level of conceptual understanding; procedures with connections tasks often contain multiple representations and will suggest a broad direction or path (Stein et al., 2000). Lastly, *doing mathematics* tasks are non-algorithmic and require students to examine and explore the problem at hand. Memorization and procedures without connections are described as requiring low-levels of cognitive demand; procedures with connections and doing mathematics are described as requiring high-levels of demand (Stein et al., 2000).

Porter's spectrum of cognitive demand. Similar to Webb (1999), Porter (2002) was interested in the cognitive demand alignment between content standards and assessments. Porter (2002) believed that to fully understand content, both descriptors of topics and categories of cognitive demand must be examined. Porter developed five hierarchical cognitive demand categories (see Figure 4). The first category, memorize, includes the memorization of facts, definitions, or formulas (Porter, 2002). The second category, procedures, requires fluency within solving/applying procedures for routine problems (Porter, 2002). The third category, communicate understanding, requires a student to explain one's reasoning (Porter, 2002).

A	B	C	D	E
Memorize facts, definitions, formulas	Perform procedures/solve routine problems	Communicate understanding of concepts	Solve nonroutine problems/make connections	Conjecture, generalize, prove
• Recognize • Identify • Recall • Recite • Name • Tell	• Do computations • Make observations • Take measurements • Compare • Develop fluency	• Communicate mathematical ideas • Use representations to model mathematical ideas • Explain findings and results from statistical analyses • Explain reasoning • Describe • Select	• Apply and adapt a variety of appropriate strategies to solve nonroutine problems • Apply mathematics in context outside of mathematics • Analyze data, recognize patterns • Explore • Judge	• Complete proofs • Make and investigate mathematical conjectures • Infer from data and predict • Determine the truth of a mathematical pattern or proposition

Figure 4. Language Frequently Associated with Porter's Performance Goals (Porter, 2002).

The fourth category, solve nonroutine problems, requires the application of mathematics to unfamiliar or real-world problems (Porter, 2002). The last category, conjecture/generalize/prove, requires students to generalize from a set of mathematical patterns or engage with proofs (Porter, 2002).

Trends in International Mathematics and Science Study (TIMSS) framework. The Trends in Mathematics and Science Study (TIMSS) provides international assessments every four years (Mullis & Martin, 2013). Specific to mathematics, TIMSS developed three assessments, TIMSS Mathematics – Fourth Grade, TIMSS Mathematics – Eighth Grade, and TIMSS Numeracy (Mullis & Martin, 2013). To describe the level of thinking students should be engaged within each content domain, TIMSS developed three cognitive domains. The first level, knowing, covers the recall of facts, concepts, and procedures (Mullis & Martin, 2013). Without the first cognitive domain, reasoning and problem solving becomes impossible (Mullis & Martin, 2013). The second level, applying, requires students to apply knowledge or conceptual understanding to answer questions or solve problems (Mullis & Martin, 2013). Problem solving is heavily emphasized within the apply domain and will often require students to solve a real-world problem, or one that is purely mathematical (Mullis & Martin, 2013). The last domain, reasoning, extends problem solving beyond routine problems; students are required to apply knowledge of understanding to unfamiliar situations (Mullis & Martin, 2013). As with the apply domain, problems may be set in real-world situations or purely mathematical (Mullis & Martin, 2013).

National Assessment of Educational Progress (NAEP) framework. The National Assessment of Educational Progress (NAEP) annually provides a standardized metric of mathematics performance for all states (National Assessment Governing Board [NAGB], 2013). Within the NAEP mathematics assessments, two components exist, the first component strives to identify long-term trends in students' ages 9, 13, and 17 (NAGB, 2013). The second component, or main portion of the NAEP mathematics assessment, is administered to fourth, eighth, and twelfth grade students at the national, state, and district levels. The NAEP mathematics assessments provide valuable information regarding student performance to our nation; as stated within the framework, "The NAEP Assessments provide a rich, broad, and deep picture of student mathematics achievement in the United States" (NAGB, 2013, p. 1). NAEP examined the demand on student thinking alongside the content domain. To assign meaning to differing levels of cognitive demand, NAEP developed three levels of complexity; low complexity, moderate complexity, and high complexity. Low complexity requires students to recall a concept or procedure; students are prompted as to which procedure to carry out (NAGB, 2013). Moderate complexity requires students to determine what to do and how to do it; a choice among solution paths must be made (NAGB, 2013). The last domain, high complexity, requires students "to use reasoning, planning, analysis, judgment, and creative thought" (NAGB, 2013, p. 42).

Analysis of frameworks. The six frameworks describe a remarkably similar trajectory of cognitive demand, see Table 1. Each begins with a level of cognitive demand related to recall of basic facts. Next, the level of cognitive demand associated with applying an algorithm varies in placement across the frameworks. Porter (2002) and the Task Analysis Guide (Stein et al., 2000) both placed procedures in the second cognitive demand category, whereas Bloom's taxonomy placed procedures in the third cognitive demand category (Krathwhol, 2002); the remaining frameworks placed procedures in the first cognitive demand category (Webb, 2002; NAGB, 2013; Mullis & Martin, 2015). Next, the level of cognitive demand associated with conceptual understanding also varies across the frameworks. A task that develops or requires the application of conceptual understanding was placed on the latter half of continuum for all frameworks except

Table 1

Level of cognitive demand associated with task requirements for cognitive demand frameworks.

	Cognitive Demand Framework					
Action needed to solve a task	Bloom's Taxonomy	Depth of Knowledge	Task Analysis Guide	Porter's Spectrum of Cognitive Demand	TIMSS	NAEP
Recall of Basic Fact	Level 1	Level 1	Level 1	Level 1	Level 1	Level 1
Apply Procedure	Level 3	Level 1	Level 2	Level 2	Level 1	Level 1
Develop or Apply Conceptual Understanding	Level 4	Level 2	Level 3	Level 3	Level 2	Level 2
Solving non-routine problems		Level 3	Level 4	Level 4	Level 3	Level 3
Mathematical Proof	Level 6	Level 4		Level 5		

[a]Level 2 and Level 5 for Bloom's taxonomy were not included within the Table. Level 2 requires understanding the instructions or problem and Level 5 requires assigning value based on judgement.

for DOK; within the DOK framework, conceptual understanding was classified under the second cognitive demand domain, skills/concept (Webb, 2002).

Solving non-routine or non-algorithmic problems was the next required level of cognitive demand across the frameworks. For the following three frameworks, the TIMSS framework (Mullis & Martin, 2015), the Task Analysis Guide (Stein et al., 2000), and the, NAEP framework (NAGB, 2013), solving non-routine problems was where the framework stopped. For both Webb's (2002) DOK levels and Porter's spectrum of cognitive demand, solving non-routine problems occupied the second from last category. In contrast, as Bloom's Taxonomy was non-domain specific, the idea of nonroutine tasks were nonexistent. A last level of cognitive demand, only classified by a few frameworks, was that of proof, the frameworks included: Webb's (2002) DOK, Bloom's taxonomy (Krathwhol, 2002), and Porter's spectrum of cognitive demand (2002). Although Bloom's taxonomy does not specifically discuss proof, the idea of pulling together multiple elements to create a cohesive whole (Krathwohl, 2002) aligns well with the notion of proof.

Given their similarities, choosing one framework over another would not significantly alter how cognitive demand is assessed within this study. However, in considering which cognitive demand framework might be most productive for the purposes of this study, several distinctions can be drawn. First, note that Bloom's taxonomy (Krathwohl, 2002) was non-specific to mathematics content, and few mathematics education researchers have used this framework in research on mathematics tasks. Webb's (1999) DOK levels, the TIMSS framework (Mullis & Martin, 2015), and the NAEP framework (NAGB, 2013) each require high levels of cognitive demand early within the progression of the framework; as much of the mathematics classrooms within the United States is dominated by the performance of procedures (Hiebert & Stigler, 2000; Silver et al., 2009; Jackson et al., 2013), having only one category, recall, does not provide the differentiation needed to classify tasks with lower levels of cognitive demand. Of the remaining frameworks, Porter's (2002) Spectrum of Cognitive Demand and Stein and colleagues' (2000) Task Analysis Guide are closely related. However, Stein et al. (2000) was widely used in related studies (e.g., Boston & Smith, 2009; Charlambous, 2010; NCTM, 2014). Thus, the Task Analysis Guide appears most appropriate for this study.

Current and past state of America's mathematics education. In contrast to our nation's mathematics reform documents and the high-levels of cognitive demand advocated within the literature, multiple research findings paint a different picture of the typical United States' secondary mathematics classroom (Schoen & Charles, 2003; Boaler, 2002; Hiebert & Stigler, 2000). To illuminate the issues within the classroom, I will describe both the characteristics of the past and current mathematics classroom; as will be demonstrated, the frequency of cognitively demanding tasks being implemented within mathematics classrooms has remained unchanged over time. Finally, I will discuss the culture for teaching in the United States.

Characteristics of the classroom. In the preface of *Teaching Mathematics through Problem Solving: Grades 6 – 12*, Schoen and Charles (2003) gave a vivid description of many mathematics classrooms. Common characteristics included: examples and homework from the textbook, short and out of context tasks, an emphasis on mastering and maintaining procedural skills, direct instruction, students positioned as passive receivers of knowledge, and if word problems are included, they are briefly stated and presented directly after the procedures students are expected to use.

Boaler (2002) further described the state of many mathematics classrooms. Within the study, two secondary mathematics departments, Phoenix Park and Amber Hill, were examined closely for three years; a longitudinal cohort analysis was conducted as each group of students' progressed through grades 8, 9, and 10 (Boaler, 2002). Amber Hill is the focus of this discussion; details related to both schools will be discussed later in the chapter.

Mathematics instruction at Amber Hill was similar to that of what Schoen and Charles (2003) described above; instruction was presented from the front of the classroom, students spent a majority of their time practicing procedures; in addition, students attending Amber Hill were heavily tracked by ability (Boaler, 2002). As the actions in the classroom have a direct effect on student learning (Ball & Foranzi, 2011), Amber Hill's model of direct instruction produced a mix of student outcomes (Boaler, 2002). Although Amber Hill's students experienced success on assessments requiring the application of learned procedures, when these same students were presented with new learning situations, they could not do anything with the problem, even though the students knew they had learned an appropriate method (Boaler, 2002). Students viewed mathematics as an abstract set of rules and procedures to memorize, and students perceived the mathematics they encountered in the real world as mutually exclusive from the mathematics they encounter in the classroom (Boaler, 2002).

Silver et al. (2009) examined teachers' selection and implementation of cognitively demanding tasks, removed from the context of the classroom, which produced similar results (Silver et al., 2009). They examined the portfolios of teachers applying for the National Board Certification in Early Adolescence/Mathematics in 1998 - 1999. Teacher submitted portfolios which provided researchers opportunities to examine samples of teaching practices, including videotapes, classroom artifacts, and samples of student work (Silver et al., 2009). Of the thirtytwo individuals within the research sample, only half included an instructional task with a high-level of cognitive demand. Silver et al., (2009) concluded, “The fact that about half of the teachers in our sample failed to include in their portfolio entries even a single task that was judged to be cognitively demanding can also be viewed as disappointing because these teachers were showcasing their best practices” (p. 520).

A more recent study continued the trend of past findings; Jackson et al., (2013) examined 165 middle school mathematics classrooms from Fall 2009 through Spring 2011. Each participating teacher was videoed teaching two consecutive lessons within the spring of each year. Teachers were encouraged to include a problem solving activity and a related whole-class discussion within the recorded lessons (Jackson et al., 2013). Of the 460 observed lessons, the tasks used in 274 lessons were labeled as having the potential to elicit high levels of cognitive demand (Jackson et al., 2013). Although task selection held promise, less than half – 40.1% – of the tasks maintained high levels of cognitive demand within set up of the problem-solving activity (Jackson et al., 2013). Research findings continue to conclude that teachers need additional support in enacting tasks in a high quality manner (Boston & Smith, 2009; Jackson et al., 2013).

Culture of teaching. The culture of teaching mathematics in the United States may contribute to the quality of teachers’ mathematics instruction. Hiebert and Stigler (2000) remarked, “The way in which teaching is conducted within a culture is so widely shared that anyone who has grown up in the culture probably could enter a classroom tomorrow and act like a teacher” (p. 8). As prospective teachers enter into schools of education, these students have already received at least 12,000 hours of training in what it means to teach, manage, and learn (Hoy, Davis, & Pape, 2006). Even as new knowledge is developed and learned during one’s college education, as teachers enter their first classroom, they feel bound by the typical U.S. teaching script (Hiebert & Stigler, 2000). This notion of being bound and pressured is not only an internal feeling, parents and colleagues believe students should be taught as they were taught, by way of direct instruction (NCTM, 2014).

Conclusion. As seen, the typical United States mathematics classroom is dominated by direct, teacher-led instruction (Boaler, 2002; Hiebert & Stigler, 2000; Silver et al., 2009; Jackson et al., 2013). In both Silver et al. (2009) and Jackson et al. (2013), only about half of the teachers were able to select tasks that required high-levels of cognitive demand; in Jackson et al., (2013), the cognitive demand of less than half of these tasks were maintained. Furthermore, this overemphasis on learning procedures without understanding is only preparing 44% of U.S. high school graduates for college level mathematics (ACT, 2013; College Board, 2013). Hiebert & Wearne (2003) made a powerful conclusion:

> It is not even enough to memorize procedures for all these things and execute them with blinding speed. It is not enough, because knowing how to execute procedures does not ensure that students understand what they are doing. To understand, students must get inside these topics; become curious about how everything works; figure out how this topic is the same as, and different from, a topic they already studied; and become confident that they could handle problems about the topic, even new problems they have not seen. (p. 3)

In contrast to instruction focused on developing proficiency with procedures, many mathematics education researchers have concluded that implementing instructional tasks that maintain high cognitive demand can help develop students' mathematics understanding, sustain students' interest, and encourage discussion of mathematics (NCTM, 1991; NCTM, 2000: Stein et al., 1996; Boston & Smith, 2009).

Cognitively demanding tasks and student learning. The type of instructional tasks found within a mathematics classroom has direct implications for student learning (Doyle, 1983). To begin, mathematical tasks are what students complete within a classroom; students only learn what they are provided the opportunity to engage in or complete (Doyle, 1983). Likewise, students have much less opportunity to learn what is not required of them to complete (Doyle, 1983). Mathematical tasks also draw students' attention to specific aspects of the mathematics discipline (Doyle, 1983). The content and requirements within the task determine the ways in which a student conceptualizes mathematics (Stein et al., 1996). As both of these factors affecting instructional tasks combine, students' perceptions of mathematics are formed; Stein and colleagues (2009) stated, "Day-in and day-out, the cumulative effect of students' experiences with instructional tasks is students' implicit development of ideas about the nature of mathematics" (p. 1). Instructional tasks affect both student opportunities to learn mathematics and students' perceptions about mathematics (Doyle, 1983; Stein et al., 2009).

As seen in the examination of cognitive demand frameworks, not all tasks are created equal; different types of instructional tasks provide different opportunities for student thinking, reasoning, and thus learning (Stein & Lane, 1996; Stein et al., 1996). Mathematics education research has unanimously agreed that the greatest factor affecting student understanding is the cognitive demand of a task; in order for a task to be enacted with high cognitive demand, tasks must start with a high level of cognitive demand (Stein & Lane, 1996; Stein et al., 1996; Silver et al., 2009; Hsu & Silver, 2014; Boston & Smith, 2009; NCTM, 1991). As such, mathematics teachers must understand the differing domains of cognitive demand and many of the features associated within each domain (Arbaugh & Brown, 2005; Stein et al., 1996; Stein & Lane, 1996). Stein and colleagues (2000) stated, “Acquiring the ability to think with precision about mathematical tasks and their use in class can equip teachers with more developed skills in the ways they select, modify, and enact mathematical tasks with their students” (p. xii). Unfortunately, teachers do not typically adhere to examining tasks for cognitive demand; tasks are selected as a result of superficial characteristics, skills and concepts that need to be covered, or similarities within the mathematics content (Arbaugh & Brown, 2002).

Research on the effects of the use of cognitively demanding tasks. To demonstrate the effects of cognitively demanding tasks on students’ learning and students’ perceptions of mathematics, Boaler (2002) and Boaler and Staples (2008) investigated a large number of students, teachers, and classrooms across multiple secondary schools; the case of Amber Hill was described above and the contrasting case of Phoenix Park will be described in this section. Both were examined closely for three years; within each school, a longitudinal cohort analysis was conducted as each group of students’ progressed through grades 8, 9, and 10 (Boaler, 2002). At the end of grade 10, each student completed the General Certificate of Secondary Education (GCSE) exam; this standardized assessment provided an additional source of data when examining the mathematics learned in the different school approaches (Boaler, 2002). Other data included student interviews and student questionnaires (Boaler, 2002). All students within each longitudinal cohort received a questionnaire while only portions of the students were interviewed at varying times throughout the study (Boaler, 2002).

In comparison to the direct instruction received at Amber Hill, Phoenix Park students were introduced to mathematics through large-scale instructional tasks, which students explored in mixed-ability groups. Each project lasted between two and three weeks (Boaler, 2002).

Cognitive demand was maintained throughout the project as students were not given specific solutions paths; instead, students were expected to turn the presented problems into extended pieces of mathematics work (Boaler, 2002). As a result of the learning environment, when students from Phoenix Park were asked about their perceptions of instruction, their statements included comments regarding mathematics as being useful in new and different situations (Boaler, 2002). These same students had an impressive ability to transfer mathematics outside of the classroom (Boaler, 2002). In examining students standardized mathematics scores, Phoenix Park had a higher percentage of students pass the assessment than did Amber Hill (Boaler, 2002). Boaler (2002) emphasized that students from Phoenix Park did not necessarily know more mathematics than those from Amber Hill, but as a result of the learning opportunities available through the large-scale instructional tasks, these students developed different forms of knowledge and understanding.

Boaler and Staples (2008) report on a study that is part of a larger project, the Standard Mathematics Teaching and Learning Study; the study spanned five years and included three high schools, which they label Greendale, Hilltop, and Railside (Boaler & Staples, 2008). The selection of the three high schools was purposeful, allowing researchers to examine different approaches to mathematics instruction. At Greendale and Hilltop schools, parents and students had a choice of which math class to attend, either geometry or a more remediation-oriented course (Boaler & Staples, 2008). In contrast, students entering Railside were required to take algebra (Boaler & Staples, 2008). The educators of Railside were committed to mixed-ability grouping and effective mathematics teaching practices (Boaler & Staples, 2008). A mixed methods data analysis approach was used (Boaler & Staples, 2008).

Students at Hilltop and Greendale schools were taught using the methods commonly found within the United States classrooms; teachers lectured and students completed short, out of context problems (Boaler & Staples, 2008). In contrast, teachers at Railside selected and implemented instructional tasks with high levels of cognitive demand; student learning was further supported by the combination of student presentations and teacher questioning (Boaler & Staples, 2008). Students were in mixed-ability groups and teachers rarely lectured (Boaler & Staples, 2008). At the start of the study, the students attending Railside achieved considerably lower in comparison to those attending Hilltop or Greendale (Boaler & Staples, 2008). Following the first year of consistent experience with instructional tasks, students at Railside were performing similarly to Hilltop and Greendale students (Boaler & Staples, 2008). Following the second year of the study, students at Railside were outperforming the students receiving traditional instruction (Boaler & Staples, 2008). Differences in perceptions also became evident among students in the different instructional approaches (Boaler & Staples, 2008). Questionnaires were given to all students, and Railside students were consistently more positive about their experiences with mathematics compared to those from other schools (Boaler & Staples, 2008). Considering postsecondary plans, 39% of students at Railside planned a future that involved mathematics, while only 5% of students receiving traditional instruction had such plans (Boaler & Staples, 2008). Boaler and Staples (2008) concluded, “The students at Railside school enjoyed mathematics more than students taught more traditionally, they achieved at high levels on curriculum-aligned tests, and the achievement gap between students of different ethnic and cultural groups was lower than those at other schools” (p. 625).

Conclusion. The consistent enactment of cognitively demanding tasks has great implications for student learning and understanding (Boaler, 2002; Boaler & Staples, 2008). The day-in and day-out cumulative effect of cognitively demanding tasks improves students' perceptions of mathematics, allows for a wide application of mathematics content, and improves standardized testing performance (Boaler, 2002; Boaler & Staples, 2008).

Enacting cognitively-demanding mathematical tasks. Numerous variables and factors influence how a task progresses from mathematical task, as represented in curriculum or instructional materials, to student learning (Stein et al., 1996). Unfortunately, the cognitive demand of many tasks decline as they are enacted within the classroom (Stein et al., 1996; Stein & Lane, 1996; Jackson et al., 2013). Stein and colleagues (1996) developed a Mathematics Task Framework (see Figure 5) to help model the stages of a task throughout instruction; areas of which cognitive demand is likely to falter can be found in the circles below the progression (Boston & Smith, 2009).

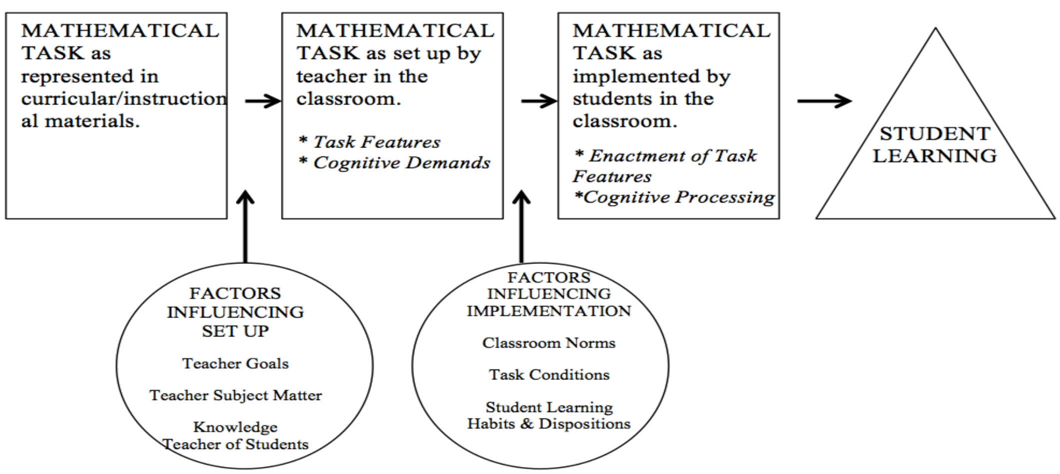


Figure 5. Mathematical Task Framework (Stein et al., 1996).

The first phase of the Mathematical Task Framework suggests that mathematical tasks originate from a variety of teacher resources, including: standards-based curricula, conventional curricula, supplemental materials, assessments, or teacher-created materials (Boston & Smith, 2009; Hsu & Silver, 2014). Regardless of the source from which the teacher selects the task, the higher the initial cognitive demand, the greater the opportunity for student learning (Stein & Lane, 1996). During task selection, characteristics that lead to student engagement, thinking, and reasoning include “the existence of multiple-solution strategies, the extent to which the task lends itself to multiple representations, and the extent to which the task demands explanations and/or justifications from the students” (Stein et al., 1996, p. 461).

The second phase of the Mathematical Task Framework relates to how a mathematical task is introduced by the teacher. Task introduction may range from simply inviting students to get started, to elaborate explanations of context, prerequisite skills, and available resources (Stein et al., 2009; Jackson et al., 2013). Although the students have not begun working on the problem, educators cannot overlook this portion of the framework (Stein et al., 2009; Jackson et al., 2013). Jackson and colleagues (2013) have determined two findings related to task set up (Jackson et al., 2013). First, task set up directly affects which students can participate in solving the problem and in what ways (Jackson et al., 2013). Jackson and colleagues (2013) determined the following four aspects greatly affect task set up:

> 1. Key contextual features of the task scenario are explicitly discussed. 2. Key mathematical ideas and relationships, as represented in the task statement, are explicitly discussed. 3. Common language is developed to describe contextual features, mathematical ideas and relationships. 4. The cognitive demand of the task is maintained over the course of the setup. (p. 652)

Of these, the two most indicative of student learning were mathematical relationships and level of cognitive demand (Jackson, et al., 2013). Results suggested that the greater attention to establishing mathematical relationships in the task set-up, the greater the extent and number of students who could participate (Jackson et al., 2013). The mathematical relationships must be discussed in a manner that produces a shared-knowledge (Jackson et al., 2013). Maintaining cognitive demand during task set up was of equal importance (Jackson et al., 2013). A key in maintaining the desired level of cognitive demand was to determine in which ways the teacher can engage students in the mathematics without compromising the students' opportunities to reason and learn (Jackson et al., 2013). Along with Stein and Lane (1996), Jackson et al. (2013) found that the higher the task's initial level of cognitive demand, the more likely a high level of cognitive demand will be maintained throughout the task.

Second, task setup determines the kind of work the teacher engages in during implementation. If the task set up is done well, students will be able to start working immediately. In contrast, if the task setup is insufficient or lacking, the teacher needs to reintroduce the problem to multiple students, detracting from the time needed to guide other students (Jackson et al., 2013). Within Jackson and colleagues (2013) research, only 6.7% of the 165 observed lessons met these conditions; although rare, teachers must know it is possible to select and maintain cognitively demanding tasks..

In the third phase of the Mathematical Task Framework, students engage with the implemented mathematical task (Stein et al., 1996). In this phase of the process, the problem becomes intertwined with the classroom setting, learning goals, and classroom activity (Stein et al., 2009). Several factors influence either the decline or maintenance of cognitively demanding tasks. The factors affecting the decline of cognitive demand in tasks include: inappropriateness of task for students, shift in focus to obtaining correct answer, providing too much or too little time to work on the task, lack of accountability by students, and teachers' classroom management. Similarly, the factors affecting the maintenance of cognitive demand in tasks include an appropriate amount of time to complete the task, appropriate teacher scaffolding, pressure for explanations and justifications, modeling of high-level thinking by the teacher or peers, supportive classroom environment, and an assessment focused on mathematics understanding. Once students have reached appropriate solutions and understanding, classroom discussion should nurture intellectual risk-taking through respect and valuing of student thinking (NCTM, 1991). With the proper enactment of classroom factors and previous framework phases, students were reported to utilize multiple-solution strategies, multiple representations, and produce explanations and justifications (Stein et al., 1996).

In summary, although a task with high levels of cognitive demand is selected, multiple phases throughout the implementation of a task provide opportunities for cognitive demand to decline. To ensure the greatest opportunities for student learning are provided, the task selected should require high levels of cognitive demand, and the task should be introduced to the class and mathematical relationships should be established (Stein & Lane, 1996; Stein et al., 1996; Smith, Bill, & Hughes, 2008; Jackson et al., 2013). Once the students begin to engage with the task, appropriately scaffolding must be provided while ensuring students are pressed to provide explanations and justifications (Stein et al., 1996; NCTM, 1991).

Teacher change in enacting cognitively demanding task. Given the difficulty of successfully selecting and enacting a cognitively demanding task, determining ways to prepare teachers to select and implement high-cognitive demand tasks is necessary. Arbaugh and Brown (2005) examined how learning to critically examine the cognitive demand of tasks would affect how teachers select tasks for their own classroom. In the Spring of 1999, the mathematics department at Ericson Valley High School (EVHS) received a Toyota Time Grant to support site-specific professional development (Arbaugh & Brown, 2005). In the Fall of 1999, eight of the nine mathematics teachers at EVHS, and one additional teacher from a school within the district met to form the *Toyota Time Study Group* (Arbaugh & Brown, 2005). Many of the mathematics teachers at EVHS had not participated in professional development that required deep reflection on teaching practice; in addition, as classroom observations are often intended for evaluation, many of the mathematics teachers were not used to opening up their classroom for non-evaluative purposes (Arbaugh & Brown, 2005). With both of these factors in mind, Arbaugh and Brown (2005) decided that a productive way to guide the teachers in an initial examination of their instructional practices was to have the teachers critically analyze instructional tasks based on levels of cognitive demand; both the Task Analysis Guide (Stein et al., 2000) and Mathematical Task Framework (Stein et al., 1996) were used to frame the intervention.

Of the nine mathematics teachers attending the *Toyota Time Study Group*, seven participated in the research portions of the project (Arbaugh & Brown, 2005). Data collection included teacher interviews, artifacts from teacher interviews, instructional tasks used in teachers' classrooms the first and last week of the study, instructional tasks teachers used to supplement their textbook throughout the study, and audio-recorded accounts of study group meetings (Arbaugh & Brown, 2005). Prior to the initial and final interviews, teachers completed two task card sorts; task sorts include prompting teachers to sort a provided set of tasks by any criteria they wish. During the interview, teachers were prompted to discuss their decisions for categorizing the cards in the manner they chose (Arbaugh & Brown, 2005). A mixed methods approach was used: qualitative data analysis was used when coding initial and final teacher interviews; quantitative data analysis was used to examine changes in task characteristics (Arbaugh & Brown, 2005).

In comparing the initial and final task sorts, teacher-created categories changed drastically; during the initial task sort, most teachers sorted cards based on observable actions, such as computations (Arbaugh & Brown, 2005). In contrast, during the final task sort, 20 of the 36 teacher-created categories were directly related to levels of cognitive demand (Arbaugh & Brown, 2005). Regarding teachers' thoughts about instructional tasks, Arbaugh and Brown (2005) stated,

> Their use of the LCD [Levels of Cognitive Demand] to complete the final interview task-sorts together with comments made in final interviews provide some evidence that 5 of the 7 teachers had begun to use the LCD to think about differences in the way a task is written, particularly for this activity (the task-sorts). (p. 519)

When analyzing the cognitive demand of tasks used in teachers' classroom the during the first and last week of the study, no significant increase was found (Arbaugh & Brown, 2005). However, upon closer examination, it became clear that some of the teachers actually did make changes in the tasks they chose to use (Arbaugh & Brown, 2005). Three of the seven teachers showed considerable gains in the cognitive demand of the tasks they selected, and one of the teachers was already implementing high-level tasks; this teacher was conducting review and a test within the initial data collection, thus, skewing the cognitive demand of the collected tasks (Arbaugh & Brown, 2005). Arbaugh and Brown (2005) concluded, "It appears that engaging in this professional development experience and learning about the LCD [Levels of Cognitive Demand] criteria supported a majority of these teachers in thinking more deeply about the relationship between mathematical tasks and the work of students in their classes" (p. 525).

Boston and Smith (2009) also conducted a study examining teacher change associated with the selection and implementation of cognitively demanding tasks. Boston and Smith (2009) studied 18 middle school mathematics teachers who were a part of a larger research study, Enhancing Secondary Mathematics Teacher Preparation (ESP). The goal of ESP was to prepare practicing teachers to mentor pre-service teachers during internship; to do so, the practicing teachers were supported in improving their mathematics teaching beliefs and practices (Boston & Smith, 2009). The ESP workshop took place from September 2004 to March 2006 (Boston & Smith, 2009). In the first year of the project, participating teachers attended six full days of ESP workshops focused on improvements in instructional practice and one week designated to building teachers' capacity to mentor pre-service teachers; in the second year of the project, participating teachers attended five half-day ESP workshops focused on building a shared vision of effective mathematics teaching (Boston & Smith, 2011). In its entirety, the ESP workshop was carefully designed as described in the following: "At the heart of the ESP professional development sessions were ongoing opportunities for teachers to solve mathematical tasks, to assess the cognitive demands of mathematical tasks, and to analyze the implementation of mathematical tasks during instructional episodes" (Boston & Smith, 2009, p. 129).

Both the Task Analysis Guide (Stein et al., 2000) and Mathematical Task Framework (Stein et al., 1996) were used to frame the professional development (Boston & Smith, 2009). Data collection included instructional tasks for five consecutive days, student-completed work for three of the tasks within the five consecutive days, and lesson observations (Boston & Smith, 2009). Measures included the "the Instructional Quality Assessment (IQA) Academic Rigor (AR) in Mathematics Rubrics for Potential of the Task and Overall Implementation" (Boston & Smith, 2009, p. 133). An additional IQA checklist was also used to provide data within classroom observations (Boston & Smith, 2009). Initially, data was collected at two time points, during Fall 2004 (T1) and during Spring 2005 (T2). Quantitative methods were used to identify increases in teachers' selection and implementation of cognitively demanding tasks (Boston &

Smith, 2009). To determine the effects of the ESP professional development, a control group of ten teachers—five from each of two schools, was used (Boston & Smith, 2009).

For teachers participating in the ESP workshops, the selection of cognitively demanding tasks increased significantly; using the Task Analysis Guide, from T1 to T2, the task mean improved from 2.45 to 3.01, a difference of 0.47 (Boston & Smith, 2009). Looking specifically at the number of cognitively demanding tasks selected within the school year, during the T1 data collection period, only 44% of submitted tasks required high cognitive demand, but by T2, 73% of submitted tasks required high cognitive demand (Boston & Smith, 2009). Examining the implementation of cognitively demanding tasks through student work samples, the number of high-level tasks increased significantly from T1 to T2, 25% to 67% respectively (Boston & Smith, 2009). Classroom observations were conducted for 11 of the 18 ESP teachers and for the 10 control group participants (Boston & Smith, 2009). From T1 to T2, a significant increase in task implementation did not occur; the lack of significance is contributed to both, a small sample size and the fact that four of the eleven teachers were already maintaining tasks at a high level (Boston & Smith, 2009). In comparison to the contrast group, the ESP participants' observation scores were significantly greater than the contrast group's observation scores (Boston & Smith, 2009).

A follow up study (Boston & Smith, 2011) explored the *residue* of thinking and learning that took place as a result of the ESP project, particularly examining how teachers' selection and implementation of cognitively demanding tasks had progressed two years after the end of the professional development. During Spring 2007 (T3) seven teachers submitted an additional five days of instructional tasks, three sets of student work, and agreed to one observation during the five-day period (Boston & Smith, 2011). A mixed-methods analysis was used, quantitative methods were used to determine changes in the task selection and implementation, and qualitative methods were used to create case studies illustrating different possible projections of teacher change (Boston & Smith, 2011).

They found that ESP teachers' task selection and task implementation increased significantly (Boston & Smith, 2011). At T1, 12 out of 51, or 23.5% of students' work sets were labeled as requiring high-levels of cognitive demand, while at T3, 15 out of 21, or 71.4% of student work sets were labeled as requiring high levels of cognitive demand (Boston & Smith, 2011). Increases in ESP teachers' ability to select cognitively demanding tasks (44.4% to 85.7%) and maintain high levels of cognitive demand during lesson observations (36.4% to 64.2%) were also observed (Boston & Smith, 2011).

They observed four patterns of instructional change (Boston & Smith, 2011), as follows:

> (1) Teachers who selected high-level instructional tasks prior to the ESP project and demonstrated continual improvement in their task implementation throughout the study; (2) teachers who improved in both the selection and implementation of high-level tasks across T1 and T2 and sustained the improvements to T3; (3) teachers who improved their selection and implementation of high-level tasks between T1 and T2, but with the improvements regressed by T3; and (4) teachers who showed no significant improvements at T2, but showed significant improvements at T3. (Boston & Smith, 2011, p. 969)

Of the seven teachers, two teachers aligned with the first pattern, two teachers aligned with the second pattern, two teachers aligned with the third pattern, and one teacher aligned with the last pattern (Boston & Smith, 2011). Results of the study provide promise for the mathematics education community; it is now evident that professional development can enact significant changes in teachers' selection and implementation of cognitively demanding tasks over an extended period of time, thus increasing opportunities for student learning (Boston & Smith, 2009; Boston & Smith, 2011).

In conclusion, mathematics teachers can improve their selection and implementation of cognitively demanding tasks (Arbaugh & Brown, 2005; Boston & Smith, 2009; Boston & Smith, 2011). Pivotal to both studies were the frameworks provided through the Task Analysis Guide (Stein et al., 2000) and the Mathematical Task Framework (Stein et al., 1996).

Conclusion. Problem solving has been on the forefront of mathematics education since the year 1980 (NCTM, 1980; NCTM, 1989; NCTM, 1991; NCTM, 1995; NCTM, 2000; NCTM, 2006; NCTM, 2009; NCTM, 2014). However, teachers consistently struggle to move beyond direct instruction and provide students opportunities to problem solve (Hiebert & Stigler, 2000; Silver et al., 2009; Jackson et al., 2013). A lack of consistent task selection and implementation becomes an issue as those students who have regularly engaged with cognitively demanding tasks have shown great gains in both, mathematics achievement and attitude towards mathematics (Boaler, 2002; Boaler & Staples, 2008). To better help mathematics educators understand the progression of implementing a task within the classroom, Stein and colleagues (1996) developed the Mathematical Task Framework. Research suggests that professional development framed by both the Task Analysis Guide and the Mathematical Task Framework has a possibility of significant teacher change in selecting and implementing tasks (Arbaugh & Brown, 2005; Boston & Smith, 2009; Boston & Smith, 2011).

Teacher Attributes Needed for Task Enactment

Developing instruction centered on the selection and implementation of cognitively demanding tasks requires changes in teachers' knowledge, beliefs, and teaching practices

(Arbaugh & Brown, 2005; Boston & Smith, 2009; Boston & Smith, 2011). Wilhelm (2014), built on this work to describe why different teachers interact with cognitively demanding tasks the way they do. To consistently select and implement cognitively demanding tasks, Wilhelm (2014) concluded that three teacher attributes are needed: *content knowledge for teaching mathematics, visions of high quality mathematics instruction*, and *views about how to support struggling students*. Details related to her research methodology follow.

Wilhelm (2014) strived to determine how teachers' mathematical knowledge for teaching and conceptions of teaching and learning mathematics related to teachers' selection and implementation of cognitively demanding tasks; to do so, 213 middle school mathematics teachers' enactment of cognitively demanding tasks were examined. The primary data source included video recordings of the teachers' classroom instruction, assessment of content knowledge, and interviews (Wilhelm, 2014). Content knowledge was measured each March through paper and pencil assessments adopted from the Learning Mathematics for Teaching project at the University of Michigan (Hill, 2007; Wilhelm, 2014). The assessment measured common content knowledge and specialized content knowledge; both subtests scores were averaged to form a single score for each participant (Hill, 2007; Wilhelm, 2014). The quality of teachers' instructional practices was measured using instruments developed through Boston and Wolf's (2006) Instructional Quality Assessment, specifically, the Task Potential and Task Implementation rubrics (Wilhelm, 2014). Teachers' perceptions of high quality instruction were documented using the Visions of High Quality Mathematics Instruction (VHQMI) instrument (Munter, 2014; Wilhelm, 2014). Questions embedded within the interviews were used to document teachers' views towards supporting struggling students (Wilhelm, 2014). I revisit each of the three teacher attributes below.

Mathematical knowledge for teaching. Content knowledge for teaching mathematics has a direct effect on a mathematics teacher's ability to select and maintain cognitively demanding tasks (Wilhelm, 2014). In this section I examine two teacher knowledge frameworks and I discuss research related to the impact of mathematical knowledge for teaching on one's ability to select and implement cognitively demanding tasks.

Defining mathematical knowledge for teaching. Shulman (1986) and Ball, Hill, and Bass (2005) set out to answer similar questions, where does teachers' knowledge come from? And, is this body of knowledge specialized to teachers? In answering these questions, each author/s developed a knowledge framework; each framework is discussed below.

Shulman's Types of Teacher Knowledge Framework. Much of what is valued within teacher knowledge is visible through teacher examinations. During an exploration of multiple states' teacher examinations, Shulman (1986) was bothered by the overemphasis on pedagogy and the absence of subject-specific content. Striving to reconceptualize teacher knowledge, Shulman (1986) set out to answer multiple questions, "Where do teachers' explanations come from? How do teachers decide what to teach, how to represent it, how to question students about it, and how to deal with problems of misunderstanding?" (p. 8). In essence, "What does a teacher know and when did he or she come to know it?" (Shulman, 1986, p. 8). Specific to what a teacher knows, Shulman (1986) specified three categories of content knowledge: subject matter content knowledge, pedagogical content knowledge, and curricular knowledge. Shulman's (1986) conceptualization of subject matter content knowledge extends well beyond procedural recall; teachers must be able to explain the conceptual underpinnings of a concept, its relation to other areas within mathematics, and why the concept is worth studying. Shulman (1986) also stated that the teacher's level of subject matter content knowledge should be equal to that of their peers who only majors in the subject matter.

Shulman (1986) defined pedagogical content knowledge, the second category of teacher knowledge, as follows:

> The most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, and the most powerful analogies, illustrations, examples, explanations, and demonstrations—in a word, the ways of representing and formulating the subject that make it comprehensible to others. (p. 9)

A teacher with high-levels of pedagogical content knowledge would also know what makes specific concepts within the content easy or difficult to understand for students and any preconceptions or misconceptions students may possess (Shulman, 1986). The last category of knowledge, curricular knowledge, requires educators to understand the full range of programs available to a specific subject (Shulman, 1986). As one would not want a physician to be limited in his or her scope of treatments, Shulman (1986) argued that an educator should likewise be familiar with multiple material resources. A second portion of the curricular knowledge domain requires educators to also know and understand the content taught adjacent to one's own curriculum within the high school; a broad spectrum of understanding allows the educator to connect his or her own material to that of his or her colleagues (Shulman, 1986).

Mathematical Knowledge for Teaching Framework. Ball, Hill, and Bass (2005) set out to answer two pivotal questions, "Is there a body of mathematical knowledge for teaching that is specialized for the work that teachers do? And does it have a demonstrable effect on student achievement?" (p. 22). Hill, Rowan, and Ball (2005) defined mathematical knowledge for teaching as "the mathematical knowledge used to carry out the work of teaching mathematics"

(p. 373). Hill, Ball, and Schilling (2008) further conceptualized Shulman's (1986) initial categories; Figure 6 below shows the authors widely accepted model of mathematical knowledge for teaching. Each of the six strands combine to form mathematical knowledge for teaching (Hill et al., 2008).

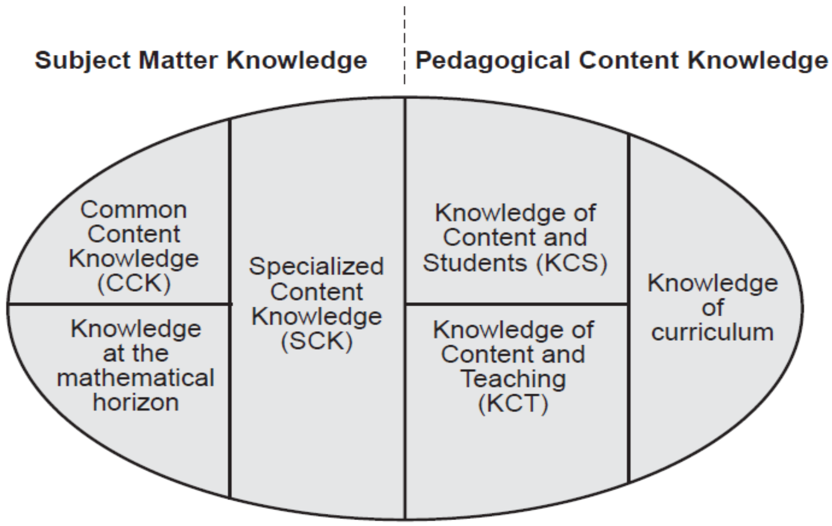


Figure 6. Domain map for mathematical knowledge for teaching (Hill et al., 2008).

The left side of the oval, labeled *subject matter knowledge*, contains three strands of knowledge, common content knowledge (CCK), specialized content knowledge (SCK), and knowledge at the mathematical horizon. The knowledge within these three domains is purely mathematical and does not require additional knowledge of students and teaching (Ball, Thames, & Phelps, 2008). The first strand, common content knowledge, is described as the mathematical knowledge any well-educated adult may possess (Ball et al., 2005). Common content knowledge allows mathematics educators to complete the work being assigned to students (Ball et al., 2008). In contrast to common content knowledge, specialized content knowledge is a newer conceptualization and includes mathematics beyond that of any well-educated adult (Ball et al., 2008; Hill et al., 2008). Specific teaching tasks requiring specialized content knowledge may include: “connecting a topic being taught to topics from prior or future, years, modifying tasks to be either easier or harder, explaining mathematical goals and purposes to parents, giving or evaluating mathematical explanations, and selecting representations for particular purposes” (Ball et al., 2008, p. 10). The last strand within the subject matter knowledge, knowledge at the mathematical horizon, was defined by Ball and Bass (2009) using four elements: “(1) A sense of the mathematical environment surrounding the current ‘location’ in instruction (2) Major disciplinary ideas and structures (3) Key mathematical practices, and (4) Core mathematics values and sensibilities” (p. 5).

The right side of the oval, labeled *pedagogical content knowledge* also contains three strands of knowledge, knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of curriculum (Ball et al., 2008). Pedagogical content knowledge remains connected with the knowledge of mathematics, but more specifically, the content knowledge most connected to instruction (Ball et al., 2008; Shulman, 1986). The first strand within pedagogical content knowledge is knowledge of content and students; this strand of knowledge is described as intertwining the knowledge of mathematics with the knowledge of how students think about, know, or learn about specific areas of mathematics (Hill et al., 2008). Educators having the ability to identify students’ misconceptions align with the knowledge of content and students’ domains; this ability derives from having experiences with students’ thinking and specific areas of the mathematics content (Ball et al., 2008).

The second strand classified within pedagogical content knowledge is knowledge of content and teaching; this strand of knowledge is described as the combination of mathematics knowledge and the design of mathematics instruction (Ball et al., 2008). Specifically,

> Teachers need to sequence particular content for instruction, deciding which example to start with and which examples to use to take students deeper into the content. They need to evaluate the instructional advantages and disadvantages or representations used to teach a specific idea. (Ball et al., 2008, p. 9)

The last strand within pedagogical content knowledge is curricular knowledge (Hill et al., 2008). Similar to Shulman (1986), curricular knowledge is described as the knowledge of specific mathematics programs, knowledge of the variety of instructional materials available within the curriculum, and knowledge of the characteristics of the curriculum that lends itself to particular circumstances within the classroom (Ball et al., 2008).

Although each of the six types of knowledge above is described as mutually exclusive, the lines between each type of knowledge may be subtle. To connect each of the six domains, Ball, Thames, and Phelps (2008) concluded with the following statement:

> Recognizing a wrong answer is common content knowledge, while sizing up the nature of the error may be either specialized content knowledge or knowledge of content and students, depending on whether a teacher draws predominantly from her knowledge of mathematics and her ability to carry out a kind of mathematical analysis or instead draws from experience with students and familiarity with common student errors. Deciding how best to remediate the error may require knowledge of content and teaching. (p. 9)

In conclusion, much of the work completed by Ball and colleagues (2005) is an extension of Shulman's (1986) original framework. Hill, Ball, and Schilling (2008) provided a more detailed conceptualization of both, subject matter knowledge and pedagogical content knowledge. The Mathematical Knowledge for Teaching (MKT) framework, initiated by

Shulman (1986) and finalized by Hill, Ball, and Schilling (2008), will be used throughout the remainder of the dissertation.

Affordances of high MKT within the mathematics classroom. Mathematical knowledge for teaching provides numerous affordances within the mathematics classroom (Ball et al., 2005). Ball, Hill, and Bass (2005) stated that the quality of mathematics teaching in relation to an educator's level of mathematical knowledge for teaching should be of no surprise, "How well teachers know mathematics is central to their capacity to use instructional materials wisely, to assess students' progress, and to make sound judgments about presentation, emphasis, and sequencing" (p. 14). Furthermore, teachers' own mathematical work may be indicative of how the teacher explains or presents these same concepts to students (Matsuura, Sword, Plecham, Stevens & Cuoco, 2013). In general, when educators select a task, knowledge of students must predict what will be of interest and motivation; when implementing a task, the teacher must anticipate how students will respond and the perceived level of difficulty (Ball et al., 2008). Likewise, teachers must be attuned to students' discourse to interpret any emerging and incomplete thinking (Ball et al., 2008). Ball, Thames, and Phelps (2008) concluded, "Each of these tasks requires an interaction between specific mathematical understanding and familiarity with students and their mathematical thinking" (p. 9). Moreover, multiple studies point to the importance of mathematical knowledge for teaching on the selection and implementation of cognitively demanding task.

MKT and cognitively demanding tasks. Charalambous (2010) investigated the association between teachers' mathematics knowledge for teaching (MKT) and the enactment of cognitively demanding tasks in two elementary mathematics classrooms. The study utilized a multiple-case approach, focusing on two teachers, each with differing MKT levels; both participants were sampled from a larger study (see Hill, Blunk, Charalambous, Lewis, Phelps, Sleep et al., 2008). Data sources included eighteen-videotaped lessons—nine from each participant, MKT scores from the paper-and-pencil Learning Mathematics for Teaching assessment, and interviews (Charalambous, 2010). At the start of the study, a clinical interview was conducted following the MKT assessment in which the participants were asked to reflect aloud on their responses from the paper-and-pencil assessment (Charalambous, 2010). When needed, the interviewer asked further questions to help illuminate each participant's reasoning (Charalambous, 2010). Years of experience and disposition towards mathematics were controlled for within the research (Charalambous, 2010). The two participants were Karen, who scored in the 93rd percentile on the MKT assessment and has been teaching for 37 years, and Lisa, who scored in the 35th percentile and had been teaching for 23 years (Charalambous, 2010). Karen adapted the curriculum mandated by her school, mainly using it for ideas, while Lisa built most of her lessons around a self-created curriculum (Charalambous, 2010).

Forty percent of the tasks presented in Karen's class were labeled as intellectually challenging (Charalambous, 2010). In Karen's classroom, 53% of the intellectual time was spent on cognitively demanding tasks (Charalambous, 2010). In the clinical interview, Karen solved problems by reasoning and attending to mathematical concepts rather than simply applying procedures (Charalambous, 2010). In contrast, 17% of the tasks presented in Lisa's class were labeled as intellectually challenging and 81% of the time was spent on less demanding tasks (Charalambous, 2010). In the clinical interview, Lisa focused on rules, algorithms, and key words (Charalambous, 2010).

The analysis of the clinical interviews suggested the differences in task enactment were related to the teacher's MKT; as Charalambous (2010) stated, "The manner in which the two teachers worked on and reasoned through these items is reminiscent of how they worked with their students in the instructional episodes, and consequently, the cognitive level at which the tasks in these episodes were enacted" (p. 269). Charalambous (2010) provided strong evidence for the importance of high-levels of MKT in the mathematics classroom.

Wilhelm (2014) also examined the role of content knowledge for teaching mathematics (CKTM), which she defined to include both, "common content knowledge and specialized content knowledge" (p. 641). She concluded that CKTM had an effect on the implementation of cognitively demanding tasks, even when teacher experience and conceptions are controlled (Wilhelm, 2014). Although teachers' CKTM had no effect on task selection, teachers with scores in the top quartile for CKTM were better able to maintain the cognitive demand of tasks. Wilhelm (2014) stated, "This finding is consistent with the strong conceptual evidence that teachers' CKTM is integral to their decision making during task implementation" (p. 663), which may include orchestrating whole group discourse or guiding students in the evaluation of strengths and weaknesses related to a solution method.

Visions of high quality mathematics instruction. The second component Wilhelm (2014) identified as contributing to a teacher's ability to select and maintain cognitively demanding tasks was visions of high quality mathematics instruction (VHQMI). VHQMI is one of many constructs or lenses within mathematics education that exists to define or examine a teacher's beliefs (Munter, 2014). The construct of vision was selected over a belief construct for the following reasons.

Munter (2014) stated the construct of beliefs may not be productive to use within research given the lack of consensus about the definition of the construct, a view also shared by Philipp (2007). In addition, although researchers agree that teacher beliefs have some influence on how teachers teach (Nathan & Koedinger, 2000), there is often a lack a consistency between beliefs and practice (Raymond, Philipp, 2007; Skott, 2001). Within teachers' beliefs structures, certain beliefs may be held to a higher degree than others, thus, having a greater effect on instruction than others (Philipp, 2007). In two related studies, institutional factors, such as standardized testing and classroom management, were shown to outweigh beliefs on practice (Raymond, 1997; Skott, 2001). As inconsistencies in teachers' beliefs and practices exist, beliefs may not hold productive to use within research.

In contrast, a teacher's vision does not imply perfect alignment with their classroom practices (Munter, 2014). Hammerness (2001) defined a teacher's vision as his or her images of ideal instruction, and Sherin (2001) defined a teacher's professional vision as a teacher's "ability to notice and interpret significant interactions in a classroom" (p. 28). For the purposes of this research, vision will be defined following Hammerness (2001): "Vision consists of images of what teachers' hope could be or might be in their classrooms, their schools, their community, and in some cases, even society...vision can provide a sense of 'reach' that inspires and motivates" (p. 145). With the adoption of visions, a concern of a teacher's proclaimed beliefs aligning or not aligning with instruction becomes irrelevant; in contrast, a teacher's vision determines what he or she identifies as high quality instruction (Munter, 2014).

Components of a vision of high quality mathematics instruction. Expanding on Hammerness' (2001) definition of vision, visions of high quality mathematics instruction includes three closely-related components: the role of the teacher, classroom discourse, and the mathematical task (Munter, 2014); see Table 2.

Teacher beliefs associated with the enactment of cognitively demanding tasks. Wilhelm (2014) presented significant findings related to the effects of VHQMI and teachers' enactment of

Table 2

A description of the three components forming visions of high quality mathematics instruction (Munter, 2014)

VHQMI Component	Description
Teacher Role	• Describes the role of the teacher as proactively supporting students' learning through coparticipation. Stresses the importance of designing learning environments that support problematizing mathematical ideas, giving students mathematical authority, holding students accountable to others and to shared disciplinarily norms, and providing students with relevant resources (Engle & Conant, 2002; as cited in Munter, 2014).
Classroom Discourse	• Patterns/Structure of Classroom Talk – Promotes whole-class conversations, including student-to-student talk that is student initiated, not depending on the teacher (Huffered-Ackles, Fuson, & Sherin, 2004); promotes developing-and supporting a "mathematical discourse community" (Lampert, 1990; as cited in Munter, 2014). • Nature of Classroom Talk – Suggests that classroom talk should be conceptually oriented—including articulating/refining conjectures and arguments for explaining mathematical phenomena—for the purpose of supporting students in "doing mathematics" and/or spawning new investigations (Munter, 2014, p. 5). • Student Questions – Promotes student questions that drive instruction, leading to new mathematical investigations, questions, characteristic of "doing mathematics" (e.g., generalization) (Munter, 2014, p. 5). • Teacher Questions – Describes the role of the teacher questions that are conceptually oriented ("why" questions) in driving investigations, helping students explain their problem-solving strategies, and/or helping the teacher understand students' thinking (Borko, 2004; as cited in Munter, 2014). • Student Explanation – Student explanations include both explanation and justification (Kazemi & Stipek, 2001; as cited in Munter, 2014) with little prompting from the teacher (Hufferd-Ackles, Fuson, & Sherin, 2004).
Mathematical Tasks	• Emphasizes tasks that have the potential to engage students in 'doing mathematics' (Stein et al., 1996; Smith & Stein, 1998), allowing for "insights into the structure of mathematics and "strategies or methods for solving problems" (Hiebert et al., 1997; as cited in Munter, 2014).

cognitively demanding tasks. Teachers scoring in the top quartile for VHQMI were more than three times as likely to select a level four task over a level three task than a teacher who scored in the bottom quartile for VHQMI (Wilhelm, 2014). Of note, within the Task Analysis Guide (Stein et al., 2000), a level four task is *doing mathematics* and a level three task is *procedures with connections*, both of which require high levels of cognitive demand. VHQMI had a much lower effect on maintenance of instructional tasks; no considerable differences were seen starting in the second quartile for VHQMI (Wilhelm, 2014). Furthermore, there was an interaction between VHQMI and CKTM on teachers' selection and implementation of cognitively demanding tasks; when a teacher scored in the third quartile on the VHQMI scale, as the teacher's CKTM level increased, the likelihood of choosing a level 3 task over a lower-level task also increased (Wilhelm, 2014).

Beyond Wilhelm's (2014) research, studies connecting cognitively demanding tasks and teachers' visions are limited. However, two studies examined how beliefs affect both the achievement of students completing mathematical tasks and the types of tasks teachers select and implement.

Šapkova (2014) studies espoused beliefs, which are quite similar to visions; espoused beliefs are defined as "teacher's subjective vision of effective, comfortable teaching adjusted to teachers and learners that may be implemented in practice or has been put to practice" (Šapkova, 2014, p. 129). Šapkova (2014) investigated the effects of the espoused beliefs on 190 seventh through ninth grade mathematics teachers in Latvia on their students' achievement, based on their ability to complete 10 mathematical tasks on the Singapore National Education Institute Project. Teachers were asked to complete the NorBa instruction, which examined mathematics teacher beliefs; all teacher and student data were collected in Fall 2010 (Šapkova, 2014). Šapkova (2014) found that a correlation exists between a teacher's espoused beliefs about mathematics teaching and student achievement: "Cluster analysis showed that teachers whose students demonstrated the highest achievement in solving 10 mathematics tasks express the least agreement with traditional ideas on teaching and learning as compared to teachers whose students belong to other clusters" (p. 139). Those students with low achievement scores were connected to teachers with traditional beliefs about mathematics teaching and were often required to memorize procedures and formulas (Šapkova, 2014). Although the research examined teachers self-reported espoused beliefs, the connection of constructivist beliefs about mathematics teaching—allowing students to develop their own understanding through problematic mathematical tasks (Simon & Schifter, 1991)— and student achievement strengthened the importance of examining teachers' visions or espoused beliefs in relation to cognitively demanding tasks (Šapkova, 2014).

Son and Kim (2015) investigated issues or beliefs that affected three teachers' selection and implementation of cognitively demanding tasks. The first two participants, Brad and Karen, were both fourth-grade teachers who used the same reform-oriented curriculum, *Math Trailblazers*; they were selected for the study as they used the same curriculum differently. The third participant, John, was a fifth grade teacher who used *Scott Foresman-Addison Wesley Mathematics*, a more traditional curriculum. Brad, Karen, and John were selected as they all perceived their students' mathematics competence as medium on a low-medium-high scale (Son & Kim, 2015). Data included surveys, teaching observations, lesson plans and artifacts from each observation, and an interview following each observation. Each teacher was observed teaching fractions four times in two consecutive class periods in the fall and two consecutive class periods in the spring. Teacher beliefs as related to philosophies of education were analyzed using surveys and interviews.

Son and Kim (2015) found that teacher beliefs affected the cognitive demand of the problems selected and implemented within each of the classrooms. Brad had a constructivist view on teaching and learning mathematics, including the roles of the instructor, student, and curriculum, and consistently selected and implemented problems at a high-level of cognitive demand. Karen had a constructivist view on teaching and learning mathematics, but a more traditional view on the roles of the instructor, student, and curriculum (Son & Kim, 2015). While she selected problems at a high level of cognitive demand, she consistently lowered the cognitive demand of the problems within the instruction; when students did not provide Karen with the response she was expecting, she told the students the answer or procedures to use (Son & Kim, 2015). She placed heavy emphasis on procedures and described the role of the teacher as an "explainer" (Son & Kim, 2015). Finally, John had a traditional view on teaching and learning mathematics, including the roles of the instructor, student, and curriculum (Son & Kim, 2015). John consistently selected and implemented problems at a low level of cognitive demand.

Son and Kim (2015) concluded, "Despite the efforts in teacher education with NCTM's Standards documents and innovative curriculum materials, Karen and John still had limited view of mathematics and mathematics teaching and learning, and their teaching practice seemed far from reform in mathematics education" (p. 513). This limited view, or belief, of mathematics teaching and learning had clear implications for Karen and John's selection and implementation of cognitively demanding tasks (Son & Kim, 2015).

Conclusion. Implications from Wilhelm's (2014) research show that visions have a strong effect on teachers' use of cognitively demanding tasks, more so on the task selection than the task implementation. From the last two studies, it becomes clear that holding a constructivist oriented set of beliefs towards mathematics teaching and learning—mathematics learning should be actively constructed through problematic tasks (Simon & Schifter, 1991)—affects both, student achievement and the ability to select and maintain cognitively demanding tasks (Šapkova, 2014; Son & Kim, 2015). Thus, VHQMI affects teachers' enactment of cognitively demanding tasks and their students' achievement.

Views for supporting struggling students. Teachers' views about how to support struggling students (VSSS) is the final component of Wilhelm's framework, related to their conceptions of who can and cannot engage with cognitively demanding tasks (Wilhelm, 2014). Research related to mathematics teachers' views of how to support struggling students in relation to mathematics instruction is limited; as a result, a few studies discussed below were conducted across multiple disciplines.

Shepard (1991) found that many educators believe students learn best by breaking complex materials into smaller parts, then guiding students sequentially through the parts. Learners are only ready to move to more advanced thinking after all prior parts have been learned and mastered (Shepard, 1991). As Shepard (1991) concluded, "Perhaps the most serious consequence of the programmed learning or master learning model of instruction is that higher order skills, which occur later in the hierarchies, are not introduced until after prerequisite skills have been mastered" (p. 6). This model of sequential learning may limit learning opportunities for students as many of the higher-order learning objectives are found in more advanced courses; unless a student advances to upper level mathematics, he or she may lack the opportunities to engage with higher-order instructional objectives (Peterson, 1988). Raudenbush, Rowan, and Cheong (1993) found that teachers holding beliefs about learning as a sequential progression are more likely to hold unproductive views of how to support struggling students. In contrast, Shepard (1991) stated, “We can think of learning as a process whereby students take in information, interpret it, connect it to what they already know, and if necessary, reorganize their mental structures to accommodate new understandings” (p. 8). Peterson (1988) suggested an increased focus on teaching higher-order thinking skills to all students should be in place, specifically, for those students who may be labeled as low achieving.

Teacher VSSS and opportunities for higher order thinking. Wilhelm (2014) found that a productive view of supporting struggling students includes the belief that all students can engage with higher order mathematics (Jackson et al., in press). Wilhelm (2014) reported that a productive VSSS was significantly related to the maintenance of cognitively demanding task (Wilhelm 2014). In regards to task selection, Wilhelm (2014) found that a teacher’s VSSS disposition had no effect on task selection.

Zohar and colleagues (2001) explored whether the knowledge educators possess about teaching and learning related to low-achieving students’ instructional opportunities for higher order thinking; if a teacher believes that the goals of higher-order instruction are beyond what lower-achieving students can do, many students become deprived of opportunities to gain the affordances of learning mathematics through cognitively demanding tasks (Zohar et al., 2001). The subjects of the research study included 40 teachers from two schools located in Israel, twenty from each school. One of the schools served high school students—grades 10 through 12—while the second school served junior high and high school students—grades 7 through 12 (Zohar et al., 2001). Both schools were non-selective schools, with students from many diverse backgrounds (Zohar et al., 2001). Data included semi-structured interviews focusing on practices associated with teaching and learning, specifically, how instruction may shift or change when teaching low-achieving versus high-achieving students (Zohar et al., 2001). A mixed methods approach was used to analyze data; interviews were analyzed both longitudinally, looking at a teacher's interview in its entirety, and horizontally, looking at one question across all interviews (Zohar et al., 2001).

> Within the longitudinal analysis, interviews were divided into three categories as follows: *Distinguishing consistently (DC)*, The teacher was consistent in drawing a distinction between low-achieving and high-achieving students; *Not-distinguishing consistently (NDC)*, The teacher was consistent in not drawing a distinction between low-achieving and high-achieving students; *Inconsistency (INC)*, The teacher drew a distinction between low-achieving and high-achieving students with respect to instruction of higher order thinking in some parts of the interview, but did not draw that distinction in other parts of the interview. (Zohar et al., 2001, p. 474 – 475)

Eight of the 40 teachers were categorized as NDC, 18 were categorized as DC, and the remaining 14 were categorized as INC (Zohar et al., 2001).

In the horizontal analysis, the researchers analyzed the responses to a question in which teachers were asked to weigh the pros and cons of sequencing instruction of a new concept, either knowledge first (transmission of knowledge), or thinking first (teaching of higher order thinking) (Zohar et al., 2001). Eighteen teachers stated that a disadvantage of teaching higher order thinking is alienating lower-achieving students within classroom, thus, confirming their beliefs that tasks requiring higher order thinking are inappropriate for lower achieving students (Zohar et al., 2001). In contrast, twelve of the forty teachers expressed beliefs that lower achieving students could be successful with tasks that require higher order thinking when supported appropriately (Zohar et al., 2001). The authors defined these supports as *special pedagogical means* (Zohar et al., 2001), which included:

> Breaking up a complex task into several simpler components; leading students through a sequence of steps necessary to solve a problem; giving clues; adding more examples, and; letting students work in groups of mixed ability so that peers can learn from each other. (Zohar et al., 2001, p. 479)

Teachers enacting special pedagogical means viewed themselves as supporting lower achieving students and many hoped that with time, lower achieving students would be able to participate in higher-order thinking activities independently (Zohar et al., 2001).

Jackson, Gibbons, and Dunlap (in press) investigated how middle school mathematics teachers diagnostically and prognostically framed students struggling in the mathematics classroom. Rather than focusing on how teachers identified and supported students' problems with general mathematics instruction, they were instead interested in how teachers identified or supported struggling students in regards to reform-oriented mathematics. Jackson and colleagues (in press) stated, "We were interested in whether they viewed their students as capable of engaging in what we referred to as rigorous mathematical activity" (p. 11). The authors defined rigorous mathematical activity as developing procedural fluency as a product of prior conceptual understanding of key mathematics, mathematical reasoning and sense making, and the ability to communicate mathematical ideas (Jackson et al., in press).

The research was completed within two large urban school districts. The schools were selected as they represented many challenges urban school districts face, including high teacher turnover and a majority of students identified as low achieving; however, both schools were unusual in their efforts to enact mathematics reform aligned with NCTM's (2000) *Principles and*

*Standards for School Mathematics (*Jackson et al., in press). In total, 122 teachers, about 60 from each district, participated in the study (Jackson et al., in press). Semi-structured interviews were conducted, focusing specifically on teachers' views of their students' mathematical capabilities (Jackson et al., in press). In analyzing the responses, teachers were labeled as productive versus unproductive based on their diagnosis and prognosis of struggling students. Teachers attributing students' struggles with mathematics to the nature of instruction or previous learning opportunities were labeled as holding productive views (Jackson et al., in press). In contrast, teachers attributing students' struggles with mathematics to characteristics of those students, such as laziness or lacking motivation, were labeled as holding unproductive views; Jackson and colleagues (in press) stated, "We use these terms to signal that the former [productive] suggests the teacher is positioned to examine and perhaps alter her instruction, whereas the latter [unproductive] suggests the teacher is unlikely to do so" (p. 16). Teachers espousing a mix of both productive and unproductive views were coded as holding mixed views (Jackson et al., in press).

A majority of the teachers within the study attributed students' difficulties with mathematics to sources outside of the classroom (Jackson et al., in press). Of the 100 interviews for which a code was assigned, 28 teachers held productive views about why students struggled within the mathematics classroom, such as instruction or schooling opportunities. In contrast, 18 teachers held unproductive views about why students struggled within the mathematical classroom, namely, student or family characteristics. The remaining 54 teachers wavered in their diagnostic features, attributing struggles to both productive and unproductive reasons (Jackson et al., in press).

When teachers were asked to describe supports used to help struggling students, responses were only included in that code if the function of the support was clear (Jackson et al., in press). For example, simply stating *using manipulatives* was not sufficient; a teacher would have needed to state, *using manipulatives to build connections between multiple representations* (Jackson et al., in press). Among the 74 interviews included in the analysis, 14 participants were labeled as describing productive supports for students, thus “the teacher views that student as capable of participating in rigorous mathematical work, albeit with targeted support” (Jackson et al., in press, p. 18). In contrast, 52 of the 74 teachers were labeled as describing unproductive supports for students (Jackson et al., in press). An unproductive description would mean a teacher lowers the cognitive demand of a task, likely providing instruction on which procedures to perform with no mention of the conceptual understanding associated with those procedures (Jackson et al., in press). Lastly, 8 of 74 teachers described mixed supports; “most teachers whose supports were characterized as mixed prioritized drilling basic skills as a necessary prerequisite to supporting students to engage in rigorous mathematical activity” (Jackson et al., in press, p. 24). Thus, supporting students while maintaining the cognitive demand of a task is not easy; those teachers who held productive views about selecting appropriate supports for struggling students acknowledged this challenge (Jackson et al., in press).

Higher-order thinking and low-achieving students. Zohar and Dori (2003) examined if low-achieving students actually benefit when given tasks that require higher levels of cognitive demand, and if they do benefit, to what extent. They examined four different research projects conducted in secondary science classrooms in Israel with a common goal: “To develop students’ higher order thinking skills as an essential component of science learning” (Zohar & Dori, 2003, p. 154). Although the study was not conducted within a mathematics classroom, Zohar and Dori

(2003) provide the only research connecting teachers' views for supporting struggling students with student achievement. Furthermore, Zohar and Dori (2003) were each director of two of the projects, thus, for every project included, one of the two authors was closely involved.

The first of the four projects studied student gains from participating in *The Quality of Air Around Us* module (Zohar & Dori, 2003). The research was conducted in seven different 10th grade classrooms from five different schools. The module used five case studies where each cooperative group was responsible for one of the cases (Zohar & Dori, 2003). After reading their assigned case, "students were requested to analyze data, solve complex problems, pose questions, conduct critical group discussions, play different roles, and write creative titles and passages with regard to controversial issues" (Zohar & Dori, 2003, p. 158). The focus of the research was to examine how students shifted in the questions asked before and after the completion of the module (Zohar & Dori, 2003). When examining pre/posttest assessment, all students improved significantly on all three components; the three components included: number of questions asked, question orientation, and question complexity (Zohar & Dori, 2003).

The second study included the examination of the unit, *The Genetic Revolution – Discussion of Moral Dilemmas* (Zohar & Dori, 2003). The unit required 12 hours and included the examination of ten moral issues related to modern technologies in genetics; within the unit, both biological knowledge and argumentation were both addressed (Zohar & Dori, 2003). The study set out to determine how biological knowledge and argumentation skills were affected by the implementation of the unit (Zohar & Dori, 2003). The research was conducted with ninth grade students from two schools; the design included an experimental and control group (Zohar & Dori, 2003). Data was collected through audio recording and group tests. With regards to analysis, for an argument to be accepted, the argument must have included a conclusion with at least one relevant justification (Zohar & Dori, 2003). To examine the data, previous Biology scores were used to group students into low-, medium- and high-achieving groups (Zohar & Dori, 2003). The difference in pre- and post-assessments were significant for all groups and confirmed that low-achieving students were able to advance their higher-order thinking skills (Zohar & Dori, 2003).

The third study investigated the implementation of the Biotechnology, Environment, and Related Issues module (Zohar & Dori, 2003). The module utilized a combined case study and moral dilemma design to provide both learning and assessment (Zohar & Dori, 2003). The dilemmas inspired by biotechnology research and their applications related to the environment prompted much debated within the eight tenth through twelfth grade science classes located at six different schools (Zohar & Dori, 2003). The goal of the research was to assess improvement in students' knowledge and understanding of scientific issues and higher-order thinking abilities – e.g. argumentation and posing questions (Zohar & Dori, 2003). When examining the post assessments related to students' knowledge and understanding of scientific issues, low-achieving students outperformed high performing students (Zohar & Dori, 2003). When looking at the post assessment related to higher-order thinking abilities, although students labeled as low-achieving did not outperform students labeled as high-achieving, low-achieving students' net gain was greater than the high achieving students' net gain (Zohar & Dori, 2003).

The focus of the final module was on developing critical and scientific thinking (Zohar & Dori, 2003). The module contained three portions, lab experiments, the critical analysis of articles found in the media, and narratives that required students to role play the position of a scientist while solving different segments of a larger problem (Zohar & Dori, 2003). As in the last study, the design called for a treatment and control group as researchers strived to determine if methods used within the unit contributed to the development of critical and scientific thinking skills (Zohar & Dori, 2003). Researchers used pre-post test instruments to collect data on students' knowledge of biology and reasoning skills (Zohar & Dori, 2003). A third element of data resulted from teacher feedback; teachers stated that the module had engaged all students, including those students who had never participated in class before (Zohar & Dori, 2003). All students made significant gains in regards to achievement and reasoning; thus, it can be concluded that the engagement of tasks requiring higher order thinking increased the development of all students thinking (Zohar & Dori, 2003).

In conclusion, it is clear that far too few teachers hold productive views about why students do not learn as expected and how to support those students who do not learn as expected (Zohar et al., 2001; Jackson et al., in press). Unproductive views become detrimental in the mathematics classroom as it becomes the difference in teachers altering instruction to ensure students learn, to placing the blame on students and maintaining ineffective instruction (Jackson et al., in press). The learning affordances gained from consistently engaging in cognitively demanding tasks are too great to ignore (Boaler, 2002; Boaler & Staples, 2008). When cognitively demanding tasks are consistently selected and implemented, at worse, lower achieving students improve significantly, at best, lower achieving students outperform higher achieving students (Zohar & Dori, 2003). To ensure all students have opportunities to engage in cognitively demanding tasks, teachers' perceptions of teaching lower achieving students need to be addressed within teacher education programs and continued professional development opportunities (Zohar et al., 2001).

Conclusion. In this section, I introduced Wilhelm's (2014) framework of teacher attributes needed to select and implement cognitively demanding tasks, which include mathematical knowledge for teaching, visions of high quality mathematics instruction, and visions for supporting struggling students. These recommended teacher attributes are supported well beyond the findings of one study. Mathematical knowledge for teaching affects both a teacher's instructional practices and a teacher's ability to maintain the cognitive demand of a task (Charalambous, 2010; Wilhelm, 2014). Visions of high quality mathematics instruction affects a teacher's selection of cognitively demanding tasks (Wilhelm, 2014; Son & Kim, 2015). Lastly, views for supporting struggling students affects a teacher's willingness to engage lower-achieving students with tasks requiring higher order thinking (Zohar et al., 2001; Jackson et al., in press; Zohar & Dori, 2003; Wilhelm, 2014). In this study, I will use the three attributes proposed by Wilhelm (2014) to frame the identification of content that may support a teacher's selection and implementation of cognitively demanding tasks.

Teacher Learning within Professional Learning Communities

I will now consider how teachers' build their knowledge of mathematics teaching, including the attributes needed to support the selection and implementation of cognitively demanding tasks. To begin, I will examine research on current professional development practices in the United States and the use of learning communities or networks as a context for teacher learning. As access to knowledge and information is available anywhere and at any time, some of these professional learning communities are becoming established within online social platforms, and I will present research examining the affordances available to educators through three online platforms, discussion forums, Twitter, and the blogosphere.

Current professional development practices. Teacher quality is often improved through professional learning (Hirsh, 2009). During the 2003 – 2004 academic year, 83% of educators participated in learning related to academic content (Darling-Hammond et al., 2009).

However, most learning opportunities are not intensive in nature; more than half of the teachers received less than two days of training within a 12-month period (Darling-Hammond et al., 2009). Thus, many workshops adopt a *one-shot* approach that focus on educators learning a prescribed set of skills or procedures (Clarke & Hollingsworth, 2001). As Lieberman and Mace (2010) reported, “Teachers have long perceived professional development, though well intentioned, to be fragmented, disconnected, and irrelevant to the real problems of their classroom practice” (p. 77). Conversely, research supports the effectiveness of professional development that focuses on the concrete, everyday challenges associated with teaching and learning one’s academic content (Darling-Hammond et al., 2009). Improvements in the professional development practices are needed as researchers have concluded that both teacher practice and student achievement are affected by professional development (Darling-Hammond et al., 2009).

Communities and networks. One way in which teachers are working to improve practice is through two closely related entities, communities of practice and personal learning networks (Lave & Wenger, 1991; Hord, 2008; Horn & Little, 2010; Wenger et al., 2011). These communities and networks are more generally referenced as professional learning communities; Hord (2008) stated, “The most promising context for continuous professional learning is the professional learning community. The three words explain the concept: professionals coming together in a group—a community—to learn” (p. 10).

Communities of practice. Wenger (1998) identified three dimensions that characterize a community: (1) mutual engagement references a community engaging in common actions together, (2) a joint enterprise is the mutual accountability of a common goal, and (3) a shared repertoire are the artifacts or “resources for negotiating meaning” within the group (Wenger, 1998, p. 82). Likewise, Wenger, Trayner and de Laat (2011) defined communities of practice in terms of the social structures in which learning takes place, thus “the community aspect refers to the development of a shared identity around a topic or set of challenges. It represents a collective intention—however tacit and distributed—to steward a domain of knowledge and to sustain learning about it” (Wengner et al., 2011, para. 7).

A shared commitment to a common goal among a group of people is what makes the community such a potentially powerful domain for learning as community member’s identities become anchored in one another (Wegner et al., 2011). Wegner (1998) concluded, “It is not easy to become a radically new person in the same community of practice. Conversely, it is not easy to transform oneself without the support of a community” (p. 89).

Personal learning networks. The construct of professional or personal learning networks (PLNs) have received much less attention than that of communities of practice and many of the definitions are anecdotal in nature (Couros, 2010). When PLNs have been formally cited, Couros’s (2010) definition is widely used and accepted: “Personal learning networks are the sum of all social capital and connections that result in the development and facilitation of a personal learning environment” (p. 125). A personal learning environment is defined as “the tools, artefacts [*sic*], processes, and physical conditions that allow learners to control and manage their learning” (Couros, 2010, p. 125).

Wenger and colleagues (2011) defined networks in relation to social structures in which learning takes place as follows:

> The network aspect refers to the set of relationships, personal interactions, and connections among participants who have personal reasons to connect. It is viewed as a

> set of nodes and links with affordances for learning, such as information flows, helpful linkages, joint problem solving, and knowledge creation. (Wenger et al., 2011, para. 6)

A large set of nodes and connections allow network members to quickly solve problems (Wenger et al., 2011). It should be noted that in the sense of a *pure* network, members may not even be aware of others existence even though they are connected within the same network (Wenger et al., 2011). This ambiguity within the network allows for either direct connections, or a series of connections to reach requested solutions or assistance (Wenger et al., 2011).

Combining communities and networks. Although distinct definitions of communities and networks are available, very few groups exist where either aspect, community or network, clearly dominates; thus, one would be hard pressed to identify a *pure community of practice* or a *pure social network* (Wenger et al., 2011). For most groups, Wenger et al. (2011) stated, "The two aspects are combined in various ways. A community usually involves a network of relationships. And many networks exist because participants are all committed to some kind of joint enterprise or domain, even if not expressed in collective terms" (Wegner et al., 2011, para. 10). Just as communities of practice and personal learning networks are closely related in the literature, a lack of delineation between the two structures is also evident; some researchers studying similar online groups use network, while others use community (Hur & Brush, 2009; Duncan-Howell, 2010; LaLonde, 2011; Deyamport, 2013).

Legitimate peripheral participation. Lave and Wenger (1991) were both concerned with issues of what learning is available through community of network interactions. To address these issues, they used ideas related to craft apprenticeship found in West Africa, reexamining them to ensure their ideas related to productive learning practices could be understood by all. They also sought to encompass a learning theory that extended beyond the simplicity of a *learn* *by doing* framework, leading them to explore learning as situated learning, with an "emphasis on comprehensive understanding involving the whole person rather than 'receiving' a body of factual knowledge about the world" (Lave & Wegner, 1991, p. 33). They built on this to formulate a theoretical framework that removed the lack of clarity associated with *situatedness* [*sic*], concluding that "learning is an integral and inseparable aspect of social practice" (Lave & Wegner, 1991, p. 31). As such, Lave and Wenger (1991) defined legitimate peripheral participation in order "to draw attention to the point that learners inevitably participate in communities of practitioners and that the mastery of knowledge and skill requires newcomers to move toward full participation in the sociocultural practices of a community" (p. 29). In essence, the process of learning occurs through the experiences and interactions between newcomer and seasoned community member (Lave & Wenger, 1991).

A key to learning through legitimate peripheral participation is to ensure access to newcomers; as newcomers move towards full participation, they will need access to "a range of ongoing activity, old-timers, and other members of the community; and to information, resources, and opportunities for participation" (Lave & Wenger, 1991, p. 101). The idea of old-timers is central to this formulation; within such communities, very little observable teaching takes place. Instead, the phenomenon of the interactions between the community members is the learning that takes place (Lave & Wenger, 1991) As newcomers work alongside old-timers, newcomers become acquainted with the community's shared repertoire, intensify their efforts, increase their time within the community, take on additional responsibilities and tasks, participate with the culture of practice, and absorb and become absorbed within the community of practice, the identity of master practitioner will begin to form (Lave & Wenger, 1991).

Research on learning through social networks. In this section, three studies provide empirical evidence for the idea that teachers can learn from more accomplished colleagues, as suggested by legitimate peripheral participation.

The first two studies discussed are the result of a larger investigation, the Middle-school Mathematics and Institution Setting of Teaching (MIST) project. Sun and colleagues (2014) looked specifically at how a teacher's network affects his or her development of mathematical knowledge for teaching (MKT) and instructional practices. MKT was measured through an assessment adopted from the Learning Mathematics for Teaching (Hill, 2007; Wilhelm, 2014); instructional practices were measured through observations using the Instructional Quality Assessment (IQA) instrument. To identify teachers' networks, participants were asked to record who they turned to for advice about teaching mathematics. Hierarchical linear models were used to analyze the data. The conclusions of the study were that a teacher's MKT was not affected by access to colleagues with expertise in MKT although a teacher's instructional practices were positively affected by access to colleagues with further developed instructional practices (Sun et al., 2014). The authors suggested that these results may not be surprising given that most of the teachers' conversations were related to teaching practices rather than how to solve mathematics problems, which might build MKT (Sun et al., 2014).

The second study from the MIST project looked specifically at what are the characteristics of the teachers seeking advice and from whom are they seeking advice (Wilhelm et al., 2016). Data included interviews, surveys, classroom observations, and student achievement data (Wilhelm et al., 2016). Teachers' networks, MKT, and instructional practices were measured using the same procedures as in the Sun et al. (2014) investigation described above. In addition, a teacher's tendency towards inquiry-oriented teaching methods was measured through the VHQMI interview protocol developed by Munter (2014). Again, hierarchical general linear model was used to analyze the data. The authors of the study concluded that novice teachers, teachers with underdeveloped MKT, and teachers with a greater capacity to teach with inquiry-oriented methods were more likely to seek advice from colleagues, specifically, from colleagues with higher student achievement gains (Wilhelm et al., 2016). They suggest that it is not surprising that new teachers may seek advice considering they often perceive themselves as in need of assistance (Wilhelm et al., 2016). Second, teachers with underdeveloped MKT were in school districts that valued inquiry-oriented teaching, which may have led them to seek help in determining how students think about solving a particular task or what common misunderstanding students may have. Third, teachers with a greater capacity to teach with inquiry-oriented methods often sought advice as they are "most interested in improving their instruction while teachers whose current methods are more traditional are more satisfied with current instruction" (Wilhelm, 2016, p. 22).

In a third study, Horn and Little (2010) examined how conversation within departmentalized learning communities may lead to teacher learning. The study took place within the mathematics department of East High School, which reported high levels of autonomy, including the freedom to select the frequency and use of departmental meeting times (Horn & Little, 2010). Within the mathematics department, the Algebra Group was examined; "The Algebra Group comprised nine math teachers who were working to detrack the ninth grade algebra classes" (Horn & Little, 2010, p. 183). The Algebra Group met after school for 90 minutes weekly (Horn & Little, 2010). Primary data sources included the audio and video recordings from each weekly meeting; although teachers communicated frequently throughout the remainder of the week, these meetings provided the most time focused on problems of the classroom (Horn & Little, 2010).

Within the Algebra Group, Horn and Little (2010) chose to include one episode within their manuscript; a brief synopsis of the episode is provided below to ensure the context needed to make sense of the research findings is included. An episode referenced as *Alice's Mayhem* opened up the Algebra Group's meeting; Alice was a new teacher who at the beginning of the meeting was prompted to share the developments in her classroom. Alice's account focused on the disparities between her intentions and the realities of the classroom (Horn & Little, 2010). Alice described her frustrations with group work, specifically, students not staying focused within their group. She also wrestled with whether or not to help students whole group, fearing not all students would pay attention, or conversely, trying to help at each individual group, recognizing that all students may not be reached. Finally, she was concerned that a lack of content was being covered and that students level of focus was declining daily (Horn & Little, 2010).

To start, the Algebra Group quickly normalized Alice's issue, providing comments such as "'That would be my fourth block'" (Horn & Little, 2010, p. 194). However, Alice's situation did not conclude with a "don't worry about it, it happens to the best of us" approach; instead, the group used the normalization of the issue as the starting place for a deeper discussion (Horn & Little, 2010). Alice was asked to further specify the problem, "'Alice, can you identify the source of the squirreliness?'" (Horn & Little, 2010, p. 195), which provoked Alice to reconsider why her class acted the way it did (Horn & Little, 2010). As she pondered, she moved into what has been identified as *rough draft* talk. Horn and Little (2010) described *rough draft* talk as, "There were pauses, several unfinished sentences, expression of uncertainty, and explicit revisions, all of which indicated that is in an emerging version of what happened in the classroom that had not yet been closely considered” (p. 195). A last element of the conversation routine, and perhaps the element providing the greatest opportunity for teacher learning, was the generalization of the problem; at this point, the conversation moved between the specifics of Alice’s teaching episode and the general principles of teaching (Horn & Little, 2010).

***Conclusion*.** Communities of practice and personal learning networks can be fundamental to learning (Lave & Wenger, 1991). Through legitimate peripheral participation, teachers can be positioned to learn from their relatively more accomplished colleagues. Such learning was demonstrated within the reported improvements in instructional practices (Sun et al., 2014) and within the Algebra Group (Horn & Little, 2010). It was also determined that novice teachers, teachers with underdeveloped MKT, and teachers with a greater capacity to teach with inquiry-oriented methods were more likely to seek advice from their colleagues. In closing, Wilhelm and colleagues (2016) stated, “Teachers’ interactions with other teachers in their social network have the potential to support their learning” (p. 25).

Moving teacher communities online. While communities of practice or personal learning networks are not new to educators (Sakamoto, 2012), how some educators are interacting with their communities or networks is quickly transforming. Sakamoto (2012) stated, “How I meet other teachers, where we discuss ideas, and how we share information has changed. Significantly. I meet them online. I learn from them online. I share with them online” (para. 16). As communities move to online platforms, the physical limitations of meeting face-to-face are removed and flexibility is added; members may interact from different schools, states, or countries, at whatever time is most convenient for the participating teachers (Duncan-Howell, 2009; Blitz, 2013). In addition, all learning goals and objectives of face-to-face PLCSs can be obtained through online communities; Blitz (2013) further stated, “The literature finds that teachers who collaborate online are engaged with the group, develop a sense of community, improve their knowledge of subject and pedagogical content, and intend to modify their instructional practices accordingly” (p. i). This section provides a detailed overview of three different online social platforms and how communities or networks engage within each online platform.

Online forums. Online forums, also known as discussion boards, discussion groups, discussion forums, or message boards, provide a virtual space for forum members to post messages, which are open for replies; chains of responses can take on the form of an ongoing conversation (Rouse, 2011). Users may also lurk, only reading the posts and accompanying replies without replying (Rouse, 2011). Studies of the interactions in online forums are discussed in detail below.

Online communities within forums. Duncan-Howell (2010) reported findings related to online learning communities and many of the affordances these communities may have for teachers’ professional learning. The communities examined within the research included one local Australian community, one national Australian community, and one international community (Duncan-Howell, 2010). All members of each community were invited to participate in an online survey; following three weeks of survey availability, 98 community members completed the survey (Duncan-Howell, 2010). The survey included 25 open- and close-ended questions organized into four sections, “(1) background, (2) PD (professional development), (3) online communities, and (4) ICT (information and communication technology) use” (Duncan-Howell, 2010, p. 327). A mixed-methods approach was used to analyze the data: themes were identified within the responses to open-ended survey questions and response frequencies were provided for close-ended survey questions (Duncan-Howell, 2010). Responses to the survey suggested that a widespread lack of funding for teachers to participate in in-person professional development makes the free, online communities an attractive source of professional development (Duncan-Howell, 2010).

When asked to rank eight statements related to the aims of professional development, positive change to teaching practice and an improvement in student learning ranked the highest among the teachers (Duncan-Howell, 2010). When asked to estimate the amount of time per week spent engaged with the online community, the majority of participants selected between 1 and 3 hours (Duncan-Howell, 2010). Note that if a community member averaged 1.5 hours per week, the user would have engaged in 60 hours of professional development following a school year (Duncan-Howell, 2010). When asked why they maintained their membership in the on-line community, many teachers listed professional development or classroom/student needs (Duncan-Howell, 2010). For those teachers selecting classroom/student needs, many cited the support received, including, “access to subject-specific resources, handy hints for the classroom, new relevant content, access to expertise to solve classroom problems, sharing lesson ideas and support for classroom problems” (Duncan-Howell, 2010, p. 335) as well as more emotional reasons related to availability of help, support from fellow members, and a sense of camaraderie (Duncan-Howell, 2010). Others cited the flexibility of time to access the community and the relevancy of many of the topics to actual classroom activities, as well as dialogue with other users (Duncan-Howell, 2010). In closing, online communities appeared to provide teachers a rich source of professional learning (Duncan-Howell, 2010).

Hur and Brush (2009) investigated the motivating factors of teachers who engaged in self-regulated online communities of teachers using a case study methodology (Hur & Brush, 2009). For a community to be selected as a case, eight criteria must have been met, including: most community members should be K-12 educators, the community must contain a minimum of 1000 members, the community should meet the community requirements set out by Wenger (1998), the community should be active longer than one year, participants must be voluntary, the community should be organized by its own members, the community must be web-based, and the community must be open to the researchers (Hur & Brush, 2009). Three communities were studied: Teacher Focus, WeTheTeachers, and Teaching community in LiveJournal (T-LJ). The researchers spent approximately one month becoming familiar with characteristics and members of each community (Hur & Brush, 2009). To ensure a range of teachers' views were included, a balanced number of members were sampled from each community; 9 teachers from TeacherFocus, 8 teachers from WeTheTeachers, and 6 teachers from T-LJ agreed to provide further data for the study (Hur & Brush, 2009). Across all participants, 13 were identified as active members (more than 30 postings), 8 were identified as infrequent members (5 – 30 postings), and 2 were identified as lurkers (less than 5 postings) (Hur & Brush, 2009).

The researchers identified five reasons that teachers engage with self-directed, online communities (Hur & Brush, 2009). First was the opportunity to share both, positive and negative emotions related to teaching (Hur & Brush, 2009). Posts related to emotions were well received and support was often offered in a variety of manners (Hur & Brush, 2009). Second was the avenue online spaces provided for teachers to safely discuss issues that may not be accepted at the local school site (Hur & Brush, 2009). If the issues were discussed on the job, many of teachers feared they would be viewed as incapable (Hur & Brush, 2009). A large audience and wide-range of experiences contributed to this support offered to community members (Hur & Brush, 2009). A third reason was to overcome isolation and to interact with teachers who understand their teaching related issues (Hur & Brush, 2009). Fourth was the opportunities to search for new ideas; regardless of teaching level, many teachers not only searched for new ideas, but ideas specific to their exact needs (Hur & Brush, 2009). The last reason was the sense of camaraderie that was developed (Hur & Brush, 2009). Although teachers may have initially approached the community in search of teaching resources, it was the friendships developed within the community that motivated extended engagement within the online community (Hur & Brush, 2009).

Twitter. Twitter is a platform commonly used to host online communities or networks of interest is Twitter; Twitter is a social networking website that allows users to share short, 140 character messages, called tweets (McMahon, 2015). A Twitter user may decide to follow a specific account, although the followed user is not obligated to follow the user back (Mathematics educators on Twitter, 2015). Many view Twitter as microblogging since tweets take the form of a short blog post (McMahon, 2015); others view Twitter as an opportunity to chat in small groups (Mathematics educators on Twitter, 2015). To promote online dialogue, a key feature of Twitter is the use of hashtags: using a hashtag or "#" before a word or phrase and including no spaces marks that tweet in a way that allows other Twitter users to find that and other related tweets (McMahon, 2015). Other features may include direct messaging or retweeting; retweeting may be defined as reposting a tweet to one's followers (McMahon, 2015).

*Educators using Twitter**.*** Internationally and nationally, educators have begun using Twitter to take charge of their own professional growth; McCulloch, McIntosh, and Barrett (2011) stated, "Teachers are beginning to take control of their own professional development, finding new ways to learn from each other, to reflect on their own practice, and to develop learning and support networks of like-minded professionals all over the world" (p. 4). While social media at times receives negative connotations (McCulloch et al., 2011), technology opens the door for people to make both wise and poor decisions in their use of an online platform (Cope, Kalantzis, & Lankshear, 2005). Although many teachers are using Twitter to plan, reflect, and collaborate with one another, empirical research examining educators' use of Twitter is limited (McCulloch et al., 2011). Both, Deyamport (2013) and LaLonde (2011) examined Twitter-hosted personal learning networks, as described in the following sections.

Personal learning networks within Twitter. LaLonde (2011) investigated the role in which Twitter acts in supporting the formation and maintenance of personal learning networks (PLNs) among educators using an Interpretative Phenomenological Analysis (IPA) methodology; as an active user of Twitter himself, an IPA allowed LaLonde to use his own preconceptions and beliefs to help interpret experiences and data within the research. Participants of the study were limited to those who were educators, maintained a conceptual understanding of a PLN, actively used Twitter, and contained contact information within their Twitter profile (LaLonde, 2011). To help identify eligible participants, LaLonde (2011) used a weekly Twitter chat coordinated by the members of The Educators PLN, EdChat. Following a sample of four randomly chosen weeks between September 2009 to September 2010, 2,818 unique eligible educators were identified. LaLonde (2011) randomly chose 20 of the 2,818 users per sampling round, inquired about their participation within his research, and continued the process until seven research participants were secured. Of the seven participants, three were K-12 educators while the other four were post-secondary educators; of the four post-secondary educators, three had prior K-12 teaching experience (LaLonde, 2011). Data within the research included in-depth interviews with each participant regarding their use of Twitter within a PLN.

LaLonde (2011) identified four distinct ways in which Twitter supported the formation and maintenance of educators' PLNs. First, "Twitter allows participants to engage in sustained and consistent dialogue with their PLN, which deepens the relationship between the participant and their PLN" (LaLonde, 2011, p. 57). A key to this sustained and consistent dialogue is the public nature of conversations on Twitter, to engage in a new or existing conversation requires no invitation (LaLonde, 2011). Second, "Twitter provides a way for participants to access the collective knowledge of their PLN" (p. 73). The primary avenue through which the collective knowledge of the PLN was accessed was through the sharing, exchanging, and requesting of resources (LaLonde, 2011). What they felt made resources within a PLN more beneficial than those found using a search engine was that the resources that surface within a PLN have been peer reviewed by the network (LaLonde, 2011).

Third, "Twitter provides participants the ability to amplify and promote deeper thoughts and ideas to a large audience" (LaLonde, 2011, p. 83). Within the research, all seven participants were active bloggers and reported tweeting the links to newly written blog posts (LaLonde, 2011). It was also stated that many of the interactions and ideas that surfaced on Twitter would provide ideas and thoughts prompting a new blog post (LaLonde, 2011). Finally, "specific features of Twitter help to expand PLN" (LaLonde, 2011, p. 86). These features include hashtags, retweets and lists (LaLonde, 2011). These findings suggest that Twitter has the potential to play a substantial role in the development and maintenance of an educator's PLN (LaLonde, 2011).

Deyamport (2013) conducted an action research study to examine the question, "In what ways, if any, can the use of a Twitter-supported personal learning network enhance teachers' personal professional development?" (p. i). Deyamport (2013) enacted a six-week action research study with eight teachers from the same elementary school not currently using Twitter to support their PLN. Deyamport (2013) initially conducted a training to demonstrate both, the functionality of Twitter and how to find related active educators using Twitter. As part of the study, Deyamport (2013) held weekly, individual meetings, and bi-weekly whole-group focus group interviews; these meetings were not only intended for data collection but to also help teachers gain the most from their Twitter-supported PLN. Data sources included participants Twitter feeds, focus group interviews, in-person meetings, the researcher's journal, and the end of study survey; data analysis included a mixed methods approach (Deyamport, 2013).

The findings were mixed; three of the teachers fully engaged with their Twitter-supported PLN, three of the teachers did not fully engage with their Twitter-supported PLN, but instead only used Twitter to find resources related to their personal professional development goals, and the last two teachers did not engage in the Twitter-supported PLN. The three teachers who were fully engaged with Twitter posted 96% of the group's tweets, and the remaining five teachers only tweeted during in-person meetings held with Deyamport (2013). Reasons given for not regularly tweeting ranged from a lack of time to getting easily distracted on the Internet (Deyamport, 2013). Of those teachers who adopted Twitter, common characteristics emerged, including: "a) openness to new experiences, b) social personalities, c) willingness to share things about themselves, and d) teaching subjects that have a robust network on Twitter" (Deyamport, 2013, p. 83). Additional affordances mentioned by those teachers who fully engaged with Twitter included access to a large group of discipline-specific educators and the number of resources relevant to their teaching duties (Deyamport, 2013).

Although not all users fully engaged with the Twitter-supported PLN, the end-of-study survey revealed promising findings for the continued use of Twitter to support teacher learning

(Deyamport, 2013). All of the participants agreed that Twitter was an effective platform for developing a PLN and 88% of these participants stated that they would continue to use Twitter as a PLN (Deyamport, 2013). Additionally, 63% of teachers found that their PLN had a positive impact on their classroom practice and that they had obtained resources or teaching practices that they had used within their classroom.

Several factors adversely affected the outcomes of Deyamport's (2013) study and should be considered when designing future research related to teachers and the use of Twitter-supported PLNs. First, a considerable amount of time was needed for teachers to become acquainted with both the functionality of Twitter and how educators can utilize a Twitter-supported PLN (Deyamport, 2013). Second, the time of the year in which the study was conducted produced unfavorable dynamics, such as preparation for standardized testing and the pressure of covering content standards (Deyamport, 2013). Finally, the researcher reported difficulties in disconnecting his own passions and uses of Twitter from the expectations he held for the research participants (Deyamport, 2013). These research findings suggest that it is very possible that not all educators will view Twitter as beneficial (Deyamport, 2013).

Blogs/blogosphere. The last social platform used to host the interactions of communities and networks of interest are teachers' blogs, sometimes referred to as the blogosphere. Web logs, from here after referred to as blogs, are frequently-updated, personal pages that update in reverse chronological order (Nardi, Schiano, & Gumbrecht, 2004). The author of a blog is a *blogger* (Luehmann & Borasi, 2011), and the posts within a blog typically organize around a general theme (Nardi et al., 2004). Posts are generally made up of text but may also include photographs or other multimedia content, hyperlinks to other websites, hyperlinks to related blogs, or places for viewers to comment (Nardi et al., 2004; Merchant, 2009). As blogs are available for everyone on the internet to see, Nardi and colleagues (2004) remarked, “Blogs are more like radio shows than they are diaries” (p. 222). A commonly used word within the literature is blogosphere; the blogosphere is defined as the collection of all of the blogs on the internet (“Blogosphere”, n.d.).

Blogs within education. While many educators are using blogs, research examining why and how educators are using blogs is limited (Luehmann, 2008; Williams & Jacobs, 2004). NCTM (2014) recognized the use of teachers’ blogs as contributing to the organization and sharing of resources through social media sites, such as Twitter. Luehmann and Borasi (2011) have written about mathematics and science teacher blogging, but information related specifically to mathematics teacher blogging is not included and while the mathematics classroom is examined within their analysis, students within the class engaged with the blogs, not teachers.

Luehmann (2008) examined the autonomous blogging of Ms. Frizzle, a sixth grade urban science teacher (Luehmann, 2008). Luehmann (2011a) used work by Darling-Hammond and Hammerness (2005) to frame teacher learning using actions described below:

> (1) Awareness and consideration of personal educational autobiography; (2) Engagement in critical inquiry-based reflection; (3) Engagement in community-based interactions; (4) Studying practice in a way that is connected to, yet removed form content-specific daily practice; (5) Consideration and integration of an expert voice; and, (6) Engagement in thoughtful intentional professional practices over a long term and in sustained ways. (as cited in Luehmann, 2011a, p. 169 – 170)

Within the study, Luehmann (2008) set out to investigate three research questions, (1) how Ms. Frizzle used her blog for one school year, (2) how Ms. Frizzle used blogging to help her face the issues of working as an urban reform-oriented science teacher, and (3) how Ms. Frizzle perceived the benefits of blogging on her practice and development. Data sources included Ms. Frizzle's blog posts for the 2004 – 2005 school year, comments on those posts from December 2005 through March 2006, emails correspondence between Ms. Frizzle and Luehmann, and transcribed phone conversations between Luehmann and two of Ms. Frizzle's colleagues (Luehmann, 2011b). A mixed methods approach was used to analyze the data based on grounded theory; other data analysis methods included documenting the frequency and content of blog posts (Luehmann, 2008).

Results of the study included that Ms. Frizzle was above average in both the length and richness of her blog posts (Luehmann, 2008). Ms. Frizzle wrote between 22 and 40 blog posts a month, for a total of 316 blog posts for the school year (Luehmann, 2008). Ms. Frizzle not only wrote a lot of posts, but on average, her posts were 51% longer than the standard blog post (Herring, Scheidt, Bonus, and Wright, 2004; as cited in Luehmann, 2008). A combination of both the frequency of posts and length of posts demonstrates the commitment Ms. Frizzle had to her blog and her audience (Luehmann, 2008). Ms. Frizzle used her blogging to help face the issues associated with working as an urban, reform-oriented science teacher through the establishment and development of a community. Based on the work by Darling-Hammond and Hammerness (2005), Ms. Frizzle was reported to most frequently engage in the third teacher learning practice, "Interacting with a like-minded community who can and will push one's thinking" (Luehmann, 2008, p. 308).

A main avenue through which Ms. Frizzle developed a community of practice around her blog was sharing of resources (Luehmann, 2008). The impact sharing resources had on the community far exceeded the materials themselves; distributing resources positioned Ms. Frizzle to interact with her community in additional ways: "(a) It placed her in the position of knowledge broker, (b) it helped her further articulate and advocate for her vision of urban and/or science education, and (c) it helped her nurture her community as she provided resources others found valuable" (Luehmann, 2008, p. 308). Knowledge brokering extended beyond providing one's audience with resources to sharing her opinion on the relevance and value that a resource may provide one's classroom (Luehmann, 2008). Mentoring was a second means that Ms. Frizzle used to nurture her community, providing detailed, step-by-step instructions to many of the science teachers within her community (Luehmann, 2008), thus strengthening her identify as a reform-oriented science teacher (Luehmann, 2008).

A last result of interest was how Ms. Frizzle perceived the effects of blogging on her teaching practice and development (Luehmann, 2008). In analyzing the content of all blog posts, Ms. Frizzle was reported to have blogged about blogging in 14% of her posts; Luehmann (2008) concluded, "The very fact that Ms. Frizzle explicitly commented on the practice of blogging in 44 posts is by itself an indication of the importance of this practice for her" (p. 328). Email correspondences between Luehmann (2008) and Ms. Frizzle revealed more about her perspectives. For example, Ms. Frizzle wrote,

> 'Being a part of a network of bloggers, reading others' blogs, exposes me to the articles and websites that people link to, which I might not otherwise find…I can't PROVE that blogging has improved my teaching. It's been an important part of my maturation as a teacher, that is for certain' (November 6, 2006). (p. 329)

Luehmann (2008) concluded, "Ms. Frizzle is aware of the importance of blogging in her development as a teacher, even if she has difficult pinpointing exactly how and to what extent her growth can be attributed to blogging alone" (p. 329).

In conclusion, communities participating within discussion forums, Twitter, and the blogosphere offer teachers multiple opportunities for learning (Duncan-Howell, 2010; Hur & Brush, 2009; LaLonde, 2011; Deyamport, 2013; Luehmann & Tinelli, 2011). A consistent finding across all studies is the importance of engagement within the community, including connecting with like-minded educators, dialogue following a blog post, and emotional support (Duncan-Howell, 2010; Hur & Brush, 2009; LaLonde, 2011; Deyamport, 2013; Luehmann, 2008). Many teachers reported that the online communities provided a safe avenue for them to be transparent with many issues that they would not be comfortable discussing with colleagues at school (Hur & Brush, 2009; Duncan-Howell, 2010; LaLonde, 2011). As communities and networks formed, a sense of camaraderie was evident across all studies (Duncan-Howell, 2010; Hur & Brush, 2009; LaLonde, 2011; Deyamport, 2013; Luehmann, 2008). A last common affordance was the availability of resources (Duncan-Howell, 2010; Hur & Brush, 2009; LaLonde, 2011; Deyamport, 2013; Luehmann, 2008). Beyond simply providing fellow educators materials, Luehmann (2008) reported on the idea of knowledge brokering, extending the sharing of resources to include views on context and value. Hur and Brush (2009) reported that even if a teacher's initial motivation for becoming involved within a connected network was to find resources, the friendships created kept the teacher engaged within the community.

Conclusion. Professional development that adopts a *one-shot* approach has proven ineffective (Darling-Hammond et al., 2009). In contrast, educators should be provided with learning opportunities that focus on the concrete, everyday challenges associated with teaching and learning one's academic content (Darling-Hammond et al., 2009). One way in which educators are provided the opportunities to discuss and reflect upon these everyday practice is through communities of practice (DuFour, 2004; Hord, 2008; Horn & Little, 2010; Lieberman &

Mace, 2010). Learning within communities of practice most commonly involved improved instructional practices (Sun et al., 2014; Wilhelm et al., 2016; Horn & Little, 2010). Some educators are moving community conversations and interactions to online platforms (Sakamoto, 2012; Blitz, 2013). Within these online platforms, time and distance become moot, as teachers have great flexibility in when and how they engage with learning communities (Sakamoto, 2012; Blitz, 2013). Research has examined these online communities and networks that take place within online forums, Twitter, and the blogosphere. These platforms support the development of both communities and networks, and opportunities for teacher learning are substantial (Duncan-Howell, 2010; Hur & Brush, 2009; LaLonde, 2011; Deyamport, 2013; Luehmann, 2008). Common affordances reported across all platforms include: community approved resources, emotional support, dialogue, connections to other like-minded educators, and a sense of camaraderie (Duncan-Howell, 2010; Hur & Brush, 2009; LaLonde, 2011; Deyamport, 2013; Luehmann, 2008).

Synthesis of Literature Review

Beginning in1980, NCTM's *An Agenda for Action* (NCTM, 1980) was the first of many calls for an increase in problem solving in the mathematics classroom. As the call for teachers' to embed problem solving within mathematics instruction continued throughout the years, cognitively demanding tasks were identified as a means to ensure problem solving occurs in the mathematics classroom (NCTM, 1989; NCTM, 1991; NCTM, 2000; NCTM, 2014)

I define *mathematical tasks* following NCTM (2014): "Mathematical tasks can range from a set of routine exercises to a complex and challenging problem that focuses students' attention on a particular mathematical idea" (p. 17). Second, I define *cognitive demand* as the cognitive processes or reasoning required solving or participating in a given activity (Doyle, 1988; Jackson et al., 2013; Hsu & Silver, 2014; Stein et al., 1996; Stein et al., 2009). Much research has concluded that few teachers are consistently selecting and enacting cognitively demanding tasks (Hiebert & Stigler, 2000; Silver et al., 2009; Jackson et al., 2013). A lack of task enactment becomes an issue as a range of mathematics education researchers have agreed that the greatest factor-affecting student understanding is the cognitive demand of a task (Stein & Lane, 1996; Stein et al., 1996; Hsu & Silver, 2014; Boston & Smith, 2009).

When cognitively demanding tasks are consistently enacted, students' achievement and attitudes in mathematics increase significantly (Boaler, 2002; Boaler & Staples, 2008). As the difficulty of successfully selecting and enacting cognitively demanding tasks has been demonstrated, determining methods of preparing teachers to enact tasks that provide increase in student achievement and attitudes is of great interest. Two studies (Arbaugh & Brown, 2005; Boston & Smith, 2009) documented interventions that were effective in improving teachers' enactment of cognitively demanding tasks. Wilhelm (2014) identified three factors affecting how teachers use cognitively demanding tasks, content knowledge for teaching mathematics, visions of high quality mathematics instruction, and views about how to support struggling students. Although Wilhelm (2014) was the first to examine each of these attributes in unison, other studies examined each of the attributes independently.

Teachers' learning within communities of practice and personal learning networks takes place as participants engage with the practice of a community (Lave & Wenger, 1991; Wegner, 1998; Wegner et al., 2011). The learning and engagement within communities and networks are shifting to online platforms; both Twitter and the blogosphere are gaining interest amongst educators (McCulloch et al., 2011; Luehmann & Borasi, 2011). Affordances available to educators through online communities are great in number and consistent across multiple studies;

affordances included: sense of community, emotional support, availability of resources, large audience, and opportunities for dialogue (Veletsianos, 2011; McCulloch et al., 2011; Luehmann, 2008; Luehmann & Borasi, 2011; Luehmann & Tinelli, 2011).

Research documenting how educators are improving the enactment of tasks without formalized professional development is limited. And although numerous mathematics educators are engaged with an online learning community of practice, research documenting how mathematics teachers learn through online communities is also limited. Additional research examining how mathematics teachers are being supported by online communities in the development of the attributes needed to select and implement cognitively demanding tasks is needed.

This study will be guided by one overarching question: Does engagement with the MathTwitterBlogosphere (MTBoS) support the development of teacher attributes supporting effective use of cognitively demanding tasks? The MathTwitterBlogosphere is a community of mathematics educators who use social platforms to share, connect, and communicate with other mathematics educators; a more detailed description of the community is included in Chapter 3. Two subquestions will be addressed as follows:

1. In what ways, if any, does the content of the MTBoS address teacher attributes needed to selected and implement cognitively demanding tasks, such as, but not limited to, mathematical knowledge for teaching, visions of high quality mathematics instruction, and views for supporting struggling students?
2. How do members of the MTBoS community perceive the effects of the MathTwitterBlogosphere on their development of teacher attributes needed to selected and implement cognitively demanding tasks, such as, but not limited to, mathematical

knowledge for teaching, visions of high quality mathematics instruction, and views for supporting struggling students?

Chapter 3: Methodology

In this chapter, I describe the methods used to examine the support provided by the MathTwitterBlogosphere community. I begin with a description of the theoretical framework and epistemology underlying this study. In the research design section, I describe the qualitative case study methodology used to study the case considered, the MathTwitterBlogosphere. I then detail the procedures of the study, including the development of an a priori coding framework and the data collection. Next, I describe the methods used to analyze community interactions and interviews with community members. Finally, I discuss the reliability and validity of the research methodology.

Theoretical Framework

The goal of the research was to better understand how the development of teacher attributes supporting effective use of cognitively demanding tasks was supported through the participation in an online community of teachers. Teacher attributes promoting selection and implementation of cognitively demanding tasks described by Wilhelm (2014) were used as an a priori framework to identify content that may support a teacher's selection and implementation of cognitively demanding tasks. (Wilhelm, 2014). Similarly, legitimate peripheral participation was used as a framework to understand teacher learning related to these specified teacher attributes (Lave & Wegner, 1991). Legitimate peripheral participation posits that "learning is an integral and inseparable aspect of social practice" (p. 31); thus, as educators interact with the content and members of a community, teachers' learning occurs.

Given the use of an a priori framework to address the research questions, a postpositivism epistemology formed the basis of this study (Creswell, 2013). A postpositivist epistemology was selected because first, logical and rigorous steps were used to collect and analyze data, and second, a range of perspectives from participants within the study were obtained (Creswell, 2013).

Research Design

To answer the research questions, a qualitative case study with an embedded design was conducted (Yin, 2014). A qualitative case study is a preferred research methodology when examining contemporary events, especially events that cannot be manipulated (Yin, 2014); in this study, those events include the everyday interactions within the MathTwitterBlogosphere community. Yin (2014) identified two elements that define a case study:

1. A case study is an empirical inquiry that
    a. investigates a contemporary phenomenon (the "case") in depth and within its real-world context, especially when
    b. the boundaries between phenomenon and context may not be clearly evident.
2. A case study inquiry
    a. copes with the technically distinctive situation in which there will be many more variables of interest than data points, and as one result
    b. relies on multiple sources of evidence, with data needing to converge in a triangulating fashion, and as another result
    c. benefits from the prior development of theoretical propositions to guide data collection and analysis. (loc. 973)

Berg and Lune (2012) discussed the case study of a community, noting that a researcher needs to collect enough data about the community to make inferences regarding the who, why, how and what of the group's interactions.

An embedded design is needed as there are multiple units of analysis (Berg & Lune, 2012). The two units of analysis for this study are (1) the content produced by members of the community, and (2) members of the community. A qualitative content analysis was conducted to determine how specified teacher attributes were addressed within the content of the online community being studied. Interviews with members of the community were analyzed to determine how they perceived engaging with the MTBoS supported their development of specified teacher attributes.

Case selection. A fundamental component in conducting a case study is defining the case (Yin, 2014). Yin (2014) recommended completing two different steps, "defining the case and bounding the case" (loc. 1291).

Defining the case. In this study, the single case under examination is the MathTwitterBlogosphere community. The MathTwitterBlogosphere (MTBoS) is a community of mathematics educators who use social platforms to share, connect, and communicate with other mathematics educators. The MTBoS community is primarily hosted within the social media platform Twitter, and the connected blogs of those who engage with the community (Johnson, 2015). The mission of the MTBoS community is to help each associated teacher improve his or her teaching craft ("Profiles of Math Teachers Who Blog and Twitter", n.d.; Milou, n.d.). Membership within the MTBoS community is open to any and all mathematics teachers ("MTBoS Directory", n.d.).

To further describe the MTBoS community, I disseminated an informal survey on Twitter during Fall of 2015 to which 42 responses were collected. When community members were asked to described their current role in mathematics education, over 70% identified as a high school mathematics teacher (grades 9 – 12), and 19% identified as a middle school mathematics teacher (grades 6 – 8). On average, the sample of community members had been teaching for 11 years, ranging from year 1 of teaching to year 30 of teaching.

The community has made additional efforts to extend the MTBoS beyond the confines of online platforms. First, Twitter Math Camp is an opportunity for those within the online community to come together for face-to-face interaction ("Twitter Math Camp", 2015). Twitter Math Camp is organized by the teachers of the MTBoS community and has grown from 37 participants in 2012, to 185 participants in 2015 (L. Henry, personal communication, September 25, 2015). Additionally, a MTBoS booth was organized by individuals within the community at the NCTM Annual Meetings in the Spring of both, 2015 and 2016. Specific to 2015, the booth was organized by the MTBoS campaign, *Exploring the MathTwitterBlogosphere* ("Exploring the MathTwitterBlogosphere", 2015). The campaign was designed to help introduce mathematics teachers to the MTBoS through an introduction to the community's members and shared resources; likewise, the goal of the booth was described on the campaign's website as follows, "Share the wonders of our community with the wider world of math teachers. The hope is that the booth will be an entry point for teachers who might not find our online community otherwise" ("Exploring the MathTwitterBlogosphere", 2015, para. 3). With both of these events, the MTBoS community tried to extend beyond online collaboration.

Bounding the case. In bounding the case, Yin (2014) stated, "If the unit of analysis is a small group, for instance, the persons to be included within the group must be distinguished from those who are outside of it" (loc. 1356). Two methods were used to identify community members. The first was using the MTBoS directory, which is a community-created directory of mathematics educators who identify as members of the MTBoS ("MTBoS Directory", n.d.). The directory is not safeguarded by any formal membership process or requirements; any mathematics educator who wishes to register their Twitter profile and/or blog may do so ("MTBoS Directory", n.d.). Thus, community members included those who self-identified as a member of the MTBoS through the creation of an entry within the community directory. A second method used to identify community members was use of the specific hashtag, #MTBoS, in tweets. The significance of this particular hashtag was described within possible interactions as a part of the above-mentioned campaign, *Exploring the MathTwitterBlogosphere*, including:

> (1) Announce and introduce yourself in a tweet and include the hashtag #MTBoS. (2) Announce a blog post you've written, new or old. Include #MTBoS. (3) Share a blog post that you've read recently that blew you away. Include #MTBoS. (4) Share a question that's been on your mind about your classroom practice. Include #MTBoS. (5) Share an online math resources you really love. Include #MTBoS. (6) Tweet a favorite quotation or fact about mathematics. #MTBoS is up. (7) Share something awesome about your day of teaching. #MTBoS. (8) Share something hard about your days of teaching. #MTBoS. ("Exploring the MathTwitterBlogosphere", 2015)

As seen, the hashtag #MTBoS is used for various reasons, all related to the work and interactions of the mathematics educators within the MTBoS community. Although other hashtags related to mathematics education are used within Twitter— e.g., #mathchat, #alg2chat, #elemmathchat— #MTBoS was not only promoted as the hashtag of choice within the campaign, but MTBoS is also the abbreviated name of the community.

It is important to note that the community is not a collection of tweets, but instead, a group of mathematics educators who may compose tweets with some variation of the hashtag #MTBoS. Further note that this definition of the MTBoS may not be all encompassing; it is very possible that some members of the community may not have included #MTBoS in a tweet during the data collection period, even though they were active during other periods.

Researcher involvement with the MTBoS. Since I have been involved with the MTBoS community since the Summer of 2014, it is important to understand how that may impact my biases in conducting this study. During my initial interactions with the MTBoS community, I was both a graduate student and a full-time secondary mathematics teacher. Within my graduate program, I was encountering new ways to think about the teaching and learning of mathematics but lacked the classroom support and resources needed to enact these teaching practices in my high school mathematics classroom. I initially found the MTBoS community as I searched for classroom tasks that I would have described a cognitively demanding. I soon realized that not only did the members of this community share a repertoire of such tasks, but they also discussed many other practices I was encountering within my graduate studies. Following a year of online participation, I attended Twitter Math Camp during the Summer of 2015. Through my participation in the camp, I not only received professional development, but I also developed many relationships with the members of the MTBoS community. Since the Spring of 2015, I have presented about the MTBoS community to both preservice and inservice teachers.

Thus, I was personally involved in the community in which I was studying, and I conducted the research from the point of participant observer. A potential danger that comes with a researcher's prolonged engagement with the community under study is an inability to maintain professional judgments during data collection and data analysis (Shenton, 2004). Safeguards that were used to maintain professional judgment included debriefing sessions between researcher and advisors, peer scrutiny of the project, and disclosure of my own beliefs and assumptions (Guba, 1981; Shenton, 2004).

Procedure

This section outlines the procedures used to conduct the study. First, an a priori coding framework was developed and piloted. Next, Phase 1 included the collection of MTBoS community interactions in Fall of 2015. Lastly, Phase 2 included the completion of semi-structured interviews with members of the MTBoS community in Spring 2016.

The coding framework. To systematically describe the meaning of the MTBoS community interactions, the literature from Wilhelm (2014), Munter (2014), and Schreier (2012) was used to develop an a priori coding framework. The coding frame included two components, categories and subcategories. Main categories are the aspects on which the research is focused; subcategories provide a means to further analyze what is being said about each of the main categories (Schreier, 2012).

The criterion of exhaustiveness was also considered during the development of the categories and subcategories; “A coding frame is said to be exhaustive if you are able to assign each unit of coding in your material to at least one subcategory in your coding frame” (Schreier, 2012, p. 76). To accomplish exhaustion, two strategies were implemented. First, a main category labeled irrelevant was added to help reduce the data to only that which was related to the research questions (Schreier, 2012; Riffe et al., 2014). For example, the following tweet was identified as irrelevant, “Can you beat my score of 2636 points at #hextris? http://hextris.github.io/hextris Looks like a cool site, too #mtbos #math.” Without further information or clicking on the link within this tweet, it would not be possible to determine if the tweet addresses the teacher attributes or the MTBoS community. I acted with extreme caution when assigning data to the irrelevant category, as once I coded content as irrelevant, all further opportunities to analyze that data were lost. In this case, the linked content was deemed as having no relevance to the study. Second, residual categories, often labeled miscellaneous, were included; these residual categories "function as containers for all unanticipated information that is relevant to your research question, but does not fit into any of your substantive categories" (Schreier, 2012, p. 93). Residual categories appeared at both levels of my coding frame—main categories and subcategories (Schreier, 2012).

With the above criteria in mind, the main categories within the coding framework included mathematical knowledge for teaching, visions for high quality mathematics instruction, and views for supporting struggling students (Wilhelm, 2014). In addition, a miscellaneous category and an irrelevant category were both included to ensure exhaustion in the framework.

Subcategories within two of the categories were also identified based on the literature review. With the category of mathematical knowledge for teaching, subcategories included subject matter knowledge (Ball et al., 2008; Hill et al., 2008) and pedagogical content knowledge (Ball et al., 2008; Hill et al., 2008); a miscellaneous subcategory was also included. With the second category of visions for high quality mathematics instruction, the subcategories included teacher role, classroom discourse, and mathematical tasks (Munter, 2014); again, a miscellaneous subcategory was included. The remaining categories did not have subcategories based on the literature. Figure 7 provides an overview of the categories and subcategories within the initial a priori coding framework.

Coding Framework	
Categories	**Subcategories**
Mathematical Knowledge for Teaching (MKT)	Subject Matter Knowledge
	Pedagogical Content Knowledge
	MKT miscellaneous
Visions for High Quality Mathematics Instruction (VHQMI)	Teacher Role
	Classroom Discourse
	Mathematical Task
	VHQMI miscellaneous
Views for Supporting Struggling Students (VSSS)	
Miscellaneous	
Irrelevant	

Figure 7. Initial a priori coding framework.

Pilot phase. Even the most cautious construction of a coding framework is likely to have areas of needed improvement; Schreier (2012) stated, “It is impossible to think of all the pitfalls that may occur in actual research practice – only practice itself will show” (p. 147). Thus, a pilot phase was enacted. In December 2015, I first used the initial coding framework to code 200 tweets including hashtag #MTBoS from September 2015; from this, I was able to add representative examples and coding rules to my coding framework.

I then completed three additional iterations of coding to further refine the framework. First, I piloted the coding framework along with a colleague also in graduate school in order to further clarify definitions within the coding frame. My colleague was also studying secondary mathematics education, had experience analyzing qualitative data, and was familiar with the goals of my dissertation research. To trial code the materials, I provided my colleague with an explanation of the coding frame, 105 tweets including hashtag #MTBoS, and one linked blog post from December 2015, representing about 10% of the size of the data set I expected to use in my study. We independently applied the coding framework to the 105 tweets in two iterations; we first coded 53 tweets, discussed our assigned codes, coded 52 tweets, and again discussed our assigned codes. We next discussed how to segment the blog post into units by topic change, independently coded the segments forming the blog post, and discussed our assigned codes. Feedback provided by my colleague throughout the discussions were used to revise the coding framework.

Second, I independently coded another 105 tweets including the hashtag #MTBoS and all corresponding blog posts from December 2015. As recommended by Schreier (2012), I then waited 15 days and recoded the same 105 tweets and corresponding blog posts. Codes were compared across the two points in time and revisions were subsequently made to the coding framework; revisions further clarified definitions and coding rules within the framework. I completed a final iteration of the pilot coding in January 2016 with a new set of data consisting of 105 tweets including hashtag #MTBoS and corresponding blog posts from October 2015. Following a 10-day break, I then recoded the same 105 tweets and blog posts. This led to final revisions to my a priori coding framework that further clarified definitions and coding rules within the framework. See Figure 8 for an abbreviated description of the coding framework, and see Appendix A for the expanded coding framework.

Phase 1. In order to be able to analyze the MTBoS community interactions, I collected two weeks of tweets that included hashtag #MTBoS and any blog posts to which those tweets included a link. Blogs were identified as posts related to the subject of mathematics within a personal blog; neither links to webpages nor hyperlinked pages within blog posts were included.

Coding Framework		
Categories	**Subcategories**	**Definitions**
Mathematical Knowledge for Teaching (MKT)	Subject Matter Knowledge	The knowledge is purely mathematical in nature and does not require additional knowledge of students and teaching (Ball et al., 2008).
	Pedagogical Content Knowledge	Pedagogical knowledge remains connected with the knowledge of mathematics, but more specifically, the content knowledge that includes aspects of the content most connected to instruction (Shulman, 1986).
	MKT miscellaneous	Intended for content related to mathematical knowledge for teaching, but not specifically related to subject matter knowledge or pedagogical content knowledge.
Visions for High Quality Mathematics Instruction (VHQMI)	Teacher Role	Describes the role of the teacher as proactively supporting students' learning through coparticipation. Stresses the importance of designing learning environments that support problematizing mathematical ideas and providing students with relevant resources (Munter, 2014).
	Classroom Discourse	Includes: patterns/structure of classroom talk, nature of classroom talk, student questions, teacher questions, and details related to student explanation.
	Mathematical Task	"Mathematical tasks can range from a set of routine exercises to a complex and challenging problem that focuses on students' attention on a particular mathematical idea" (NCTM, 2014, p. 17).
	VHQMI miscellaneous	Related to instruction in the classroom, but no specifically related to teacher role, classroom discourse, or mathematical tasks.
Views for Supporting Struggling Students (VSSS)		VSSS is related to a teacher's conception of who can and cannot engage with cognitively demanding tasks/higher order thinking (Wilhelm, 2014).
Miscellaneous		All data which is related to components of the MTBoS, but does not fit into any of the above categories.
Irrelevant		All data which is not related to the research question.

Figure 8. A priori coding framework with definitions of each category and subcategory.

A random number generator was used to select the two weeks between September 21, 2015 and November 15, 2015; the weeks of October 5 (Week A) and November 9 (Week B) were identified. All tweets with the hashtag #MTBoS from those two weeks were archived using Tweet Archiver (Agarwal, 2015) Week A included 932 tweets and Week B included 1043 tweets; on average, each day included 141 tweets. To further bound the data set, 75 tweets were randomly selected from each day of data collection; all tweets for a given day were randomly assigned a number between 0 and 1 in Microsoft Excel and ordered from least to greatest. The first 75 tweets were transferred to tables in Microsoft Word for coding. Day 7 within Week A only had 73 tweets, thus random sampling was not needed for this day. Blog posts included within the 75 tweets for each day were transferred into a qualitative software package, Atlas.ti (Friese, 2013). In total, 1048 tweets and 95 blog posts were included in the data analysis.

Phase 2. In this section, I discuss the procedure for the interview phase of the study, including subject selection, instrument development, and the conduct of the interviews.

Subject selection. The MTBoS directory served as a sample space for which interview participants were randomly selected. In selecting potential interview participants, two criteria were considered. First, potential participants needed to be contactable; within the MTBoS directory, the email address was included for most community members. Second, the participation level of potential interviewees within the community was considered; as the goal of the research was to understand the perceptions of the community as a whole, I found it important to hear from a range of MTBoS participants. To measure a member’s level of community participation, the number of Twitter followers for each was considered, as continued community contributions on Twitter will often lead to an increased following of a Twitter profile. To help quantify levels of participation by Twitter followers, I randomly selected 50 community members from the MTBoS directory, recorded the number of their Twitter followers, and then stratified the sample by Twitter followers into thirds. For the purposes of this research, community members with 200 or fewer followers were classified as semi-active members, community members with 201 to 900 followers were classified as active members, and community members with more than 900 followers were classified as highly-active members.

The goal of sampling was to secure a minimum of five participants, with at least one participant from each level of community participation. I selected the goal of five participants to ensure a range of views were heard. The random sampling was designed in rounds, although only one round was needed. In the first round of sampling, twelve potential participants were emailed the recruitment letter, four from each level of participation; see Appendix B. The email address of a community member identified as highly-active was rejected as incorrect, thus, eleven participants received the recruitment letter. Six of the eleven community members, two from each level of participation, responded and returned the consent letter signed; See Appendix C. If the minimum number of community members had not responded, plans included sending a follow up message; see Appendix D. The follow up message was not needed, nor were additional rounds of sampling needed.

Interview protocol. The interviews were guided by an interview script, which includes "an outline of topics to be covered with suggested questions" (Kvale & Brinkmann, 2009, p. 130). The major topics within the interview included participant demographics, engagement with the MTBoS community, perceived development of the teacher attributes identified by Wilhelm (2014), and perceptions related to the selection and implementation of tasks from the community. In alignment with Berg and Lune (2012), under each major topic, other questions were included to probe beyond the initial question. To close the interview, each participant was given an opportunity to make any final closing remarks (Kvale & Brinkmann, 2009; Berg & Lune, 2012).

In the Fall of 2015, the draft interview questions were presented to a community of STEM faculty and graduate students that met biweekly to discuss current issues in STEM education for feedback. The community aligns well with Shenton's (2004) idea of peers acting as "a sounding board for the investigator to test his or her developing ideas and interpretations, and probing from others may help the researcher recognize his or her own biases and preferences" (p. 67). Two iterations of the interview protocol were subsequently pilot tested in February and March of 2016 with mathematics educators participating in the MTBoS community to varying degrees. At the end of each pilot interview, the participants were asked for suggestions on improving the protocol. Feedback from the pilot test was considered in the final revisions of the interview protocol; see Appendix E.

Conduct of interviews. In March of 2016, semi-structured interviews were conducted with all six of the participants. As members of the community span across the United States and various countries, interviews were conducted using Skype, a software application that allowed for audio and video communication ("Skype", n.d.). Five of the six Skype interviews were recorded for transcription purposes; an interview with one active members did not record and was omitted from the study. Callnote ("Callnote Premium", n.d.) was used to record the video and audio portions of the Skype interviews. The interviews ranged in length from 17 minutes to 48 minutes. Within five days of each interview, I transcribed the interview and emailed the transcription to the interview participant. I requested that each participant ensure the interview transcription communicated what he/she intended, also called member checking (Guba & Lincoln, 1981). Three of the five participants responded to this request; requested changes were minor in nature.

Participants. Information related to each participant's current role in mathematics education and each participant's involvement in the MathTwitterBlogosphere is discussed below; pseudonyms are used in all cases. An overview of the interview participants is provided in Table 3.

Table 3

Overview of each interview participant

Participants	Level of Community Participation	Years in Education	Current Role in Education	Current Grade Band
Kathy	Semi-Active	17	Classroom Teacher	Secondary
Nancy	Semi-Active	12	Classroom Teacher	Secondary
Scott	Active	37	Math Coach	Elementary
Eric	Highly-Active	10	Math Coach	Secondary
Sydney	Highly-Active	6	Classroom Teacher	Secondary

The participants for this study are representative of the community members described within the initial survey of the community. Eighty-nine percent of those who responded to that survey identified as a secondary mathematics teacher, four of the five interview participants are involved secondary mathematics. Furthermore, although Scott is currently coaching in elementary grades mathematics, a majority of his teaching career was spent in the secondary mathematics classroom.

A detailed description of each participant follows.

Kathy had less than 200 Twitter followers and was identified as a semi-active community member. Kathy has been a grades 9 – 12 classroom teacher for approximately 17 years. She is currently the department head of a large mathematics department, and in addition to teaching three classes, provides support to 20 other mathematics teachers. Kathy described the students she works with as talented and motivated, “In the honors classes, they are almost able, if I wanted them to, to sort of function by themselves.” Kathy described her involvement in the MTBoS community as passive, as in the following:

> Often it [engagement with the community] ends up being on the weekend, I’ll just kind of read through posts and see what catches my eye, read blogs, occasionally I’ll ask a question here or there or post something about what I am doing, but mostly I’m a consumer I guess.

Nancy had less than 200 Twitter followers and was identified as a semi-active community member. Nancy is currently in her 12th year of teaching mathematics and 5th year at her current school, which is an independent school. She has taught courses ranging from pre-algebra through pre-calculus. Nancy described her current school as fairly diverse; student demographics were reported as 50% Caucasian, 25% Latino, and 25% African American. Although students must pass an entrance exam for admittance into the school, Nancy described the students she works with as ranging in their mathematics abilities, “So we have students who struggle quite a bit in mathematics, in particular and sometimes across the board. We have some very very strong mathematics students who are eager to take all the math we have to offer.” Nancy described her involvement in the MTBoS as specific to the blogosphere; she rarely uses Twitter. Below Nancy described how she streamlines the content she reads,

> I ended up subscribing to a blog feed that kind of summarizes what has been going on in the MathTwitterBlogosphere throughout the week, and it posts every Friday. So, that seemed like an easier way for me to keep up because, it’s a lot of stuff.

Scott was identified as an active community member with between 201 and 900 Twitter followers. Scott taught for 37 years in middle and secondary grades classroom and is currently working as an education consultant and mathematics coach in the elementary grades setting. Scott works with a range of students, many living in middle-class neighborhoods. Additionally, Scott's district is heavily multicultural with about 60% of the students being Asian. Scott described the students he works with as mixed ability, many are motivated while others need extra support. Scott's involvement with the MTBoS community includes disseminating interesting content, cohosting Twitter chats, and connecting with community members. In addition, Scott also looks for opportunities to interact with others in the community: "I'm taking to the opportunity to respond to blogs and to respond to ideas and to answer questions and to provide ideas." Scott stated that he has both the time and desire to help others within the community.

Eric was identified as a highly-active community member, with more than 901 Twitter followers. Eric is in his 10th year of mathematics education and 2nd year working as a mathematics coach. In his role as a coach, Eric meets or teaches with each of the 10 secondary mathematics teachers he supports on a weekly basis. Eric reported that he currently works in a fairly diverse district and supports teachers with classes ranging from honors courses to remediation courses. Eric remarked that even though classroom issues arise, the students he works with "want to be taught, want to learn, want to be engaged and so that is a really good thing." Eric currently interacts with the MTBoS community by seeking or providing feedback on lessons or activities. Eric mentioned that as his schedule has allowed for more flexibility, providing timely feedback to community members is something he enjoys, as stated in the following:

> And so I have a little more flexibility in my schedule these days so when a teacher sends me a notification or tweet, I'm fortunate that I can get back to them and hopefully give them some real-time feedback so that they can apply it to their next class period.

Eric also reported that he has been speaking more frequently at conferences. At conferences, he often interacts with the community through timed tweets that may disseminate important statements or information at specific moments within his presentation.

Lastly, *Sydney* was identified as a highly-active community member, with more than 901 Twitter followers. Sydney taught secondary math for 6 years in both public and private schools and is currently teaching physics. Sydney mentioned that there is no reason she could not go back to the mathematics classroom if she desired. Sydney described the students she works with as ranging in socioeconomic status, as some come from wealthy backgrounds while others attend school on financial aid; she also stated that "they're [students] more white and definitely more affluent than average." Sydney described her students as highly motivated and stated that they traditionally perform well in school. Sydney reported that she has been involved with the MTBoS community since around 2011 and currently enjoys supporting the community through moderating a weekly online meeting. She also enjoys engaging with many colleagues within the community which whom she has developed relationships with over the years.

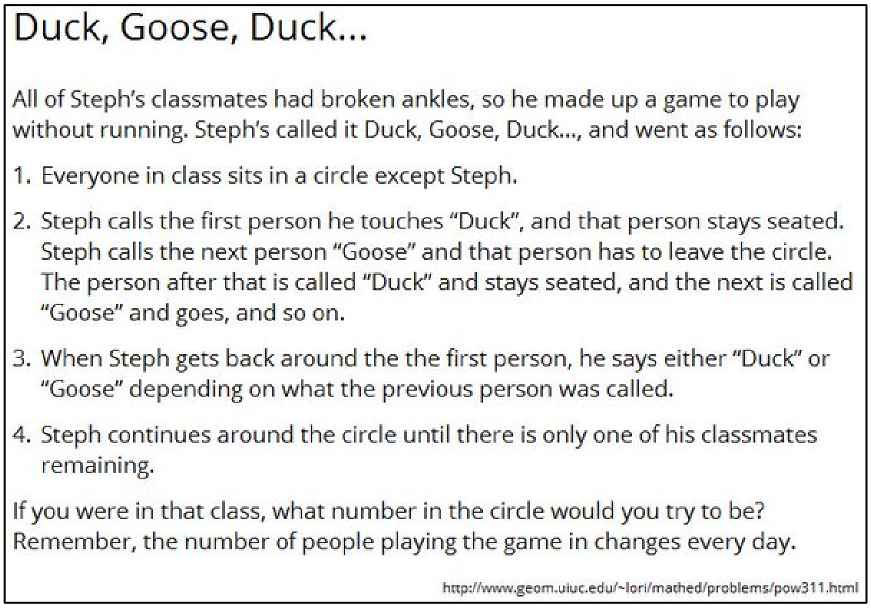
Data Analysis

The data analysis corresponded with the two phases of data collection. First, I conducted a qualitative content analysis of community interactions from Twitter and the blogosphere (Schreier, 2012). Second, I analyzed interviews with provisional coding methods (Saldaña, 2013).

Qualitative content analysis. To provide a description of the collected tweets and blog posts within the MTBoS community, I conducted a qualitative content analysis; as Schreier, (2012) stated, “Qualitative content analysis is a method for systematically describing the meaning of qualitative material. It is done by classifying material as instances of the categories of a coding frame” (p. 1). A major difference between a qualitative content analysis and a quantitative content analysis is the level of interpretation needed is to classify materials. With highly standardized data, minimal interpretation is needed and a quantitative content analysis would be best (Schreier, 2012). In contrast, qualitative content analysis is most suitable for content with less obvious meaning, the material requiring some degree of interpretation (Schreier, 2012). As human communication is often far from standardized (Schreier, 2012), the everyday discussions of the MathTwitterBlogosphere community will best be described through qualitative content analysis.

I will begin the description of the analysis with a discussion of how the data was segmented for coding. Next, I detail how the a priori coding framework was applied to collected data. Lastly, I discuss details describing the inductive coding of community interactions within each category of the coding framework are discussed.

Units of coding. Units of coding differed for content within Twitter and the blogosphere. First, each tweet, retweet, or quoted tweet in Twitter was considered a unit of coding. Furthermore, many tweets contained pictures and videos. As Twitter limits tweets to 140 characters, a user may post an image of text to supplement their tweet; images with text were considered as part of the tweet and were viewed as one unit of coding. To demonstrate, consider the following tweet and image (see Figure 9), “Here’s the #probchat problem for this week’s slow chat! #MTBoS #slowmathchat #mathchat #mathschat.”



Duck, Goose, Duck...

All of Steph's classmates had broken ankles, so he made up a game to play without running. Steph's called it Duck, Goose, Duck..., and went as follows:

1. Everyone in class sits in a circle except Steph.
2. Steph calls the first person he touches "Duck", and that person stays seated. Steph calls the next person "Goose" and that person has to leave the circle. The person after that is called "Duck" and stays seated, and the next is called "Goose" and goes, and so on.
3. When Steph gets back around the the first person, he says either "Duck" or "Goose" depending on what the previous person was called.
4. Steph continues around the circle until there is only one of his classmates remaining.

If you were in that class, what number in the circle would you try to be? Remember, the number of people playing the game in changes every day.

http://www.geom.uiuc.edu/~lori/mathed/problems/pow311.html

Figure 9. Tweet with image containing text.

Tweets that contained non-text based images were also coded alongside the original tweet. When videos were included within tweets, if possible, the central theme of the video was coded. Other possibilities within Twitter included retweets and quoted retweets. Although quoted tweets include the tweet of the original author, the content in addition to the original tweet was identified as the unit of coding; the original tweet was only referenced for additional context.

Within blog posts, units of coding were decided by topic change; as Schreier (2012) stated, "Topic changes signal the end of one unit [of coding] and the beginning of another" (p. 136). Similar to tweets, if videos were included within blogs, the central theme of videos were coded.

Application of a priori coding framework. The goal of the qualitative content analysis was to determine how each teacher attribute was addressed within the MTBoS community; this goal was accomplished by connecting each unit of coding to a category or subcategory within the a priori coding framework (see Figure 8). For example, consider the following tweet with associated image shown in Figure 10, “Day 22: Solving Equations w/ [with] SolveMe Mobiles and @explainevrythng Sour Patch Kids Challenges in #pearce1D #MTBoS.”

Figure 10. Image accompanying a tweet representative of the subcategory mathematical task. This tweet was coded as *mathematical tasks* because a mathematics task related to solving a system of equations was described within the tweet and is visible within the image (see Figure 10). For each day of data collection, the a priori framework was applied first to all 75 tweets, and second, to all linked blog posts. Following the analysis of all 14 days of collected content, coding frequencies for each category and subcategory were determined.

Additionally, two subunits of analysis were considered within the data analysis, the week in which the data was collected and the platform on which it was shared, either Twitter or the blogosphere. Subunits were identified to better understand if patterns of codes varied across the context for which tweets and blogposts were collected (Riffe, Lacy, & Fico2014). Coding frequencies were also determined for each subunit of analysis.

Inductive analysis within the a priori coding framework. To provide a more-detailed description of how each teacher attribute was addressed within the MTBoS, the content within each category and subcategory was inductively coded. Inductive, or open coding, is the process in which concepts within the data are discovered and labeled with codes (Strauss & Corbin, 1998). The codes that emerged within each subcategory and category were organized into themes. For codes to emerge as themes, two criteria were applied. First, the inductive codes, regardless of frequency, must have been similar in topic. Second, the inductive codes must have been related to the definition or description of the category or subcategory in which they were located. For example, the following three codes: *recommended curriculum resources, requests for curriculum resources,* and *discussions of curriculum resources,* were combined into one theme, curriculum resources. The codes were all similar in topic, relating to curriculum, and the codes related to the description of the subcategory for which they were located, a knowledge of the curriculum within the subcategory of pedagogical content knowledge. Thus, the full framework includes categories, subcategories, inductively developed themes, and inductive codes.

Analysis of interviews. The interviews conducted with members of the MTBoS community were analyzed to understand how participants perceived engagement with the MTBoS affected their development of teacher attributes and how the participants described or characterized the content within the MTBoS. Berg and Lune (2012) defined qualitative interviews as "a conversation with a purpose" (p. 105). Through these conversations, a qualitative research interview provides an understanding of a subject's daily world through his or her own perspective (Kvale & Brinkmann, 2009).

The interviews were analyzed using provisional coding in which the inductive codes developed from the qualitative content analysis were used as a *start list* of codes for the interviews (Saldaña, 2013). The list of inductive codes was expanded and modified as needed to capture additional ideas from the interviews. Units of coding were decided by topic change.

After the coding was complete, the resulting codes were compared to the previously-developed themes from the qualitative content analysis. If codes used to build themes in the qualitative content analysis were visible within the analysis of the interviews, then the corresponding code from the interviews was included within that theme. If codes from the interviews were similar in topic to one another, but not directly related to an existing theme, a new theme was developed. The content of new themes were then compared to the a priori framework to determine their position within a subcategory or category of the overall framework (see Figure 8). For example, the theme "development of mathematics knowledge" was specific to the interview analysis; since this theme related to the development of knowledge that was purely mathematical in nature (Ball et al., 2008), the theme was included within the subcategory of subject matter knowledge; see Appendix F for an overview of the themes and codes within the coding framework.

In Table 4 below, I summarize how data sources and data analysis combine to answer each research question.

Reliability and Validity

Trustworthiness can be established within qualitative research; numerous criteria for ensuring a rigorous research methods design have been recognized for many years (Guba, 1981; Lincoln & Guba, 1981; Shenton, 2004). Even so, the trustworthiness of qualitative research has often been criticized by those with rationalistic or positivist paradigms, in which truth is viewed as being confirmable through experimentation (Shenton, 2004). Following is a discussion of the criteria needed to establish trustworthiness within qualitative research and the relevant strategies embedded within each of these criteria. All of the strategies within the discussed criteria are directly related to my discussed research methods.

Table 4

Research questions by units of analysis and methods of analysis

Research Questions	Units of Analysis	Method of Data Analysis
1. In what ways, if any, does the content of the MTBoS address teacher attributes needed to selected and implement cognitively demanding tasks, such as, but not limited to, mathematical knowledge for teaching, visions of high quality mathematics instruction, and views for supporting struggling students?	• Community Interactions o Archived tweets containing #MTBoS o The linked blogs of those tweets containing #MTBoS. • Interviews with Members of the MTBoS community	• Qualitative Content Analysis (Schreier, 2012) • Provisional Coding (Saldaña, 2013)
2. How do members of the MTBoS community perceive the effects of the MathTwitterBlogosphere on their development of teacher attributes needed to selected and implement cognitively demanding tasks, such as, but not limited to, mathematical knowledge for teaching, visions of high quality mathematics instruction, and views for supporting struggling students?	• Interviews with members of the MTBoS community.	• Provisional Coding (Saldaña, 2013)

Guba and Lincoln (1981) identified four criteria related to trustworthiness in research, including credibility, transferability, dependability, and confirmability. Prior to examining each criterion in detail, it is important to understand that the methods associated with a qualitative study are not absolutely reliable or absolutely valid, but are viewed as a matter of degree (Shenton, 2004; Schreier, 2012). When strategies associated with reliability or validity are included within a study, the degree of trustworthiness increases (Shenton, 2004; Schreier, 2012).

Credibility (internal validity). Credibility, or internal validity, is concerned with ensuring the intended phenomena is what is actually being measured (Shenton, 2004). Guba (1981) related the idea of credibility to the "truth" of the research findings. The specific strategies used to increase the degree of credibility within my research included random sampling of individuals, triangulation of data sources, frequent debriefing sessions between researcher and mentors, opportunities for scrutiny of the research project, and member checking (Guba, 1981; Shenton, 2004).

The first methods I used to ensure credibility of the data was random sampling (Shenton, 2004). Shenton (2004) remarked, “A random approach may negate charges of researcher bias in the selection of participants” (p. 65). As seen in the procedures above, random sampling was utilized during the data collection for both Phase 1 and Phase 2. A next practice that was used to ensure credibility of the proposed research was triangulation of data sources (Shenton, 2004). Triangulation was accomplished as various sources of data were utilized within the research; data sources included archived Twitter interactions, blog posts, and interviews with MTBoS community members.

Secondly, I held frequent debriefing sessions between myself and my research advisors; fortunately, such meetings are a natural part of the dissertation process (Guba, 1981). In these meetings, the development of themes from inductive codes were often discussed. Furthermore, Shenton (2004) also recommended using peers to scrutinize the research design and research findings. Both the proposed research design and preliminary research findings were presented to a community of STEM faculty and graduate students.

Finally, I provided interview participants the opportunity to read the transcript of the interview dialogue in which he or she participated, also called member checking. Guba and Lincoln (1981) suggested that member checking is the single most important provision a researcher can make in improving the trustworthiness of the study. Following the transcription of the interviews, participants were provided their interview transcript and asked to ensure what they intended to say was properly documented. Three of the five participants returned verified interview transcripts. One participant remarked that the transcript was sufficient. A second participant made minor grammatical edits. A third participant made slight changes to ensure the transcript was grammatically correct and added a few statements in parenthesis throughout to provide further context to statements. Since no significant misinterpretations were identified in three of the five transcriptions, I proceeded with the analysis of all five transcripts.

Transferability (generalizability). From a rationalistic paradigm, transferability is concerned with how the current findings may be applicable to a wider population (Shenton, 2004). From a naturalistic perspective, some feel that generalizations of this sort are impossible; others strive to report findings in a way that allows the readers to determine the “fit” between the presented context and their own context (Guba, 1981; Shenton, 2004). Regarding the latter of the two views, if the reader is to make such contextual judgment, the researcher must report sufficient contextual information (Guba, 1981; Shenton, 2004). Once sufficient context has been provided, Shenton (2004) commented, “Readers must determine how far they can be confident in transferring to other situations the results and conclusions presented” (p. 70). One manner in which to provide sufficient content is through the “provision of background data to establish context of study and detailed description of phenomenon in question to allow comparison to be made” (Shenton, 2004, p. 73). If the needed contextual features are to be provided, thick description is needed (Shenton, 2004). Within the reported research, a “thick” description of the MTBoS community will be evident in the described community interactions and members’ perceptions.

Dependability (reliability). From a rationalistic paradigm, reliability is viewed as the degree to which, if the methods were repeated, the findings would be reproduced (Shenton, 2004). To address the issue of dependability, an in-depth methodological description has been included within the above portions of this chapter (Shenton, 2014). A last practice related to the consistency, or internal reliability, of the coding frame. As I was the only coder within the main analysis, internal reliability was assessed across points in time (Schreier, 2012). Within the main analysis, after a minimum of 10 to 14 days following initial coding, I used the coding frame to recode 10% of the data from Week A and then 10% of the data from Week B. Once the data had been recoded, a percentage of agreement coefficient was calculated; percentage of agreement = $\frac{Number\ of\ units\ of\ coding\ on\ which\ the\ codes\ agress}{Total\ number\ of\ unis\ of\ coding}$ x 100 (Schreier, 2012). For Week A, the percentage of agreement was 89% for Twitter, and 91% for the blogs. Similarly, for Week B, the percentage of agreement was 87% for Twitter, and 96% for the blogs. In all cases of disagreement, the unit of coding and coding framework were reconsidered and a most appropriate code was identified.

Confirmability (objectivity). Confirmability is concerned with ensuring that the results of the study are a result of the subjects and the conditions of the inquiry opposed to the investigator's biases, professional judgments, perspectives, or motivations (Guba, 1981). Specific to confirmability, the following provisions were made: triangulation of data sources to reduce effect of investigator bias, admission of researcher's beliefs and assumptions, and in-depth methodological description (Shenton, 2004); both triangulation and in-depth methodological description were previously discussed.

The degree of confirmability is the admission of researcher's beliefs and assumptions related to the research (Guba, 1981; Shenton, 2004). To overcome such biases, I have explicated the biases that may relate to the research at hand; my personal background related to the MTBoS community has been included above.

Anticipated ethical issues. Anticipated ethical issues for the proposed research study are few in number. One anticipated ethical issue is the breach of participant confidentiality (Kvale & Brinkmann, 2009). To help protect the participants' confidentiality, pseudonyms were assigned to all interview participants, and only non-specific details were used to describe research participants (Kvale & Brinkmann, 2009). Similarly, when presenting findings from the qualitative content analysis, I quoted tweets that were more general in nature and did not include details that might be used to identify the original author of the tweet. The same precautions were used when describing blogs.

Expected outcomes. I expected all three identified teacher characteristics associated with the selection and implementation of cognitively demanding tasks, mathematical knowledge for teaching, visions of high quality mathematics instruction, and views for supporting struggling students, to be found within the community's interactions. I expected material related to the views for supporting struggling students to appear the least, and I expected material related to visions of high quality mathematics instruction to appear the most, specifically, as related to mathematical tasks. Within the interviews, I expected interview participants to believe the MathTwitterBlogosphere community has supported their selection and implementation of cognitively demanding tasks.

Chapter 4: Findings

This study is designed to address the overarching question, does engagement with the MathTwitterBlogosphere (MTBoS) support the development of teacher attributes supporting effective use of cognitively demanding tasks? To answer this question, I explored two subquestions. First, in what ways, if any, does the content of the MTBoS address teacher attributes needed to select and implement cognitively demanding tasks, such as, but not limited to, mathematical knowledge for teaching, visions of high quality mathematics instruction, and views for supporting struggling students? Second, how do members of the MTBoS community perceive the effects of the MTBoS on their development of teacher attributes needed to select and implement cognitively demanding tasks, such as, but not limited to, mathematical knowledge for teaching, visions of high quality mathematics instruction, and views for supporting struggling students?

In this chapter, I describe the findings relative to these questions in two steps. First, I report the frequency counts and percentages for each category and subcategory in the a priori framework. Second, I describe the community content and interview responses by categories, which are organized into themes that were developed in open coding.

Frequency Counts Across the Framework

To better understand how often each teacher attribute was addressed within the content of the MTBoS community, I counted frequencies for each subcategory and category (see Table 5). Of the relevant content, just over 88% related to teacher attributes, while the remaining 12% related to the MTBoS community, which was captured within the miscellaneous category. Of the teacher attributes, VHQMI (46.70%) was most frequently addressed, followed by MKT (38.53%) and VSSS (3.07%).

Table 5

Coding frequencies and percentages for each category of the coding framework

Mathematical Knowledge for Teaching	Visions for High Quality Mathematics Instruction	VSSS	Miscellaneous	Totals
1155 (38.53%)	1400 (46.70%)	92 (3.07%)	351 (11.71%)	2998

Furthermore, to determine if patterns of codes varied across contexts, Riffe and colleagues (2014) recommended identifying subunits of analysis within the data. In looking at the data from Twitter and the blogosphere, I considered two subunits of analysis – the week in which the data was collected and platform on which it was shared, either Twitter or the blogosphere. Table 6 provides an overview of how the data was distributed across each subunit of analysis.

Table 6

Units of coding per subunit of analysis

	Weeks		
Platform	Week A	Week B	Total
Twitter	477	470	947
Blogosphere	913	1138	2051
Total	1390	1608	2998

Far more units of coding were included within the blogosphere than on twitter. Also, similar units of coding were included across Week A and Week B for Twitter whereas more units of coding were included in Week B than Week A for the blogosphere. This variation is likely a result of the sampling and segmentation across each platform. For data collected from

Twitter, 75 tweets were randomly selected for each of the 14 days of data collection, and each tweet, retweet, or quoted tweet was viewed as one unit of coding. The total units of coding for Twitter were limited by these parameters, thus a similar number of tweets were collected for Week A and Week B. In contrast, although only those blogs linked within a collected tweet were coded, units of coding for blog posts were decided by topic change. As such, there were very few limitations on the number of units per coding per blog post.

Patterns of responses by category and subcategory were also compared by subunits, both by week and by platform. Table 7 includes the coding frequencies organized per week and Table 8 includes the coding frequencies organized by platform.

Table 7

Coding frequencies and percentages for each category of the coding framework by week

	Mathematical Knowledge for Teaching	Visions for High Quality Mathematics Instruction	VSSS	Miscellaneous	Totals
Week A Totals	436 (31.37%)	769 (55.32%)	34 (2.45%)	151 (10.86%)	1390 (46.36%)
Week B Totals	719 (44.71%)	631 (39.24%)	58 (3.61%)	200 (12.44%)	1608 (69.97%)
Grand Totals	1155 (38.53%)	1400 (46.70%)	92 (3.07%)	351 (11.71%)	2998

Table 8

Coding frequencies and percentages for each category of the coding framework by platform

	Mathematical Knowledge for Teaching	Visions for High Quality Mathematics Instruction	VSSS	Miscellaneous	Totals
Twitter Totals	394 (41.61%)	318 (33.58%)	29 (3.06%)	206 (21.75%)	947 (31.59%)
Blog Totals	761 (37.10%)	1082 (52.75%)	63 (3.07%)	145 (7.07%)	2051 (68.41%)
Grand Totals	1155 (38.53)	1400 (46.70%)	92 (3.07%)	351 (11.71%)	2998

When looking at the subunits organized by week of data collection, about half of the categories increased significantly from Week A to Week B. The remaining categories decreased significantly, or remained largely unchanged. Specifically, the MKT category increased significantly from Week A to Week B, as did VSSS. In contrast, the overall category of VHQMI decreased, and the miscellaneous category was essentially unchanged. Thus, variation did exist between the categories from Week A to Week B. However, this may not be surprising as events and interests of the community shift, the content of the community interactions will also shift. For example, in Week B, the National Council of Teachers of Mathematics held a regional conference in Minneapolis, MN; the content within Week B was reflective of this conference.

When looking at the subunits organized by platform, similar percentages of content were distributed across both platforms for the categories of MKT and VSSS. In contrast, a higher percentage of content from the blogosphere addressed VHQMI. A first hypothesis for why the content varied across the platforms is the nature of each platform. Twitter is limited to 140 character statements, minimizing the depth and details included within the content. In contrast, blog posts do not have restrictions and can include more depth and details. As such, the blogosphere may have better suited the content within VHQMI as details are often needed to effectively discuss classroom instruction.

Teacher Attributes Addressed within the MTBoS Community

In the following section, I combine results from both analyses to provide an understanding of how engagement with the MathTwitterBlogosphere addresses and supports the development of teacher attributes supporting effective use of cognitively demanding tasks. To report the results, I describe the community content and interview responses by categories, which are organized into themes that were developed in open coding. See Appendix F for an overview of the themes and codes within the framework.

Mathematical knowledge for teaching. Mathematical knowledge for teaching (MKT) is the first category within the teacher attribute framework. Hill, Rowan, and Ball (2005) defined mathematical knowledge for teaching as the "mathematical knowledge used to carry out the work of teaching mathematics" (p. 373). Three subcategories were included in MKT. First, subject matter knowledge is related to knowledge of pure mathematics (Ball et al., 2008) and second, pedagogical content knowledge is related to knowledge of how to teach mathematics (Shulman, 1986). Third, a residual subcategory of MKT miscellaneous was included for all content related to the mathematical knowledge needed for teaching, but not included within a previous subcategory. MKT was addressed in 38.53% (N = 1155) of the collected data from Twitter and the blogosphere (see Table 5).

In the sections below, themes developed from the open coding in each of the three subcategories will be discussed.

Subject matter knowledge. The first subcategory within the category of MKT is subject matter knowledge, which is knowledge that is purely mathematical in nature (Ball et al., 2008). This includes both knowledge of the mathematics progressions (Ball & Bass, 2009), and the mathematics knowledge of and beyond that of any well-educated adult (Ball et al., 2005; Shulman, 1986). Subject matter knowledge accounted for 25.54% (N = 295) of the collected content within the category of MKT. Three themes, along with constituent codes, will be discussed; Table 9 shows their distribution within the two phases of the study.

Knowledge of mathematics progressions. Knowledge of mathematics progressions, the first theme within subject matter knowledge, is related to understanding the mathematics surrounding a particular mathematics concept, an important component of subject matter knowledge (Ball & Bass, 2009). Two codes comprised this theme. The first code, *content standards and goals*, connected the mathematics used to solve tasks with content standards in the mathematics progressions. All content within this code came from the blogosphere. Most

Table 9

Themes Related to Subject Matter Knowledge

Themes and Codes	Frequency of Units with Code within Community Interactions	Number of Interviewees included in Code
Knowledge of Mathematics Progressions		
Content Standards and Goals	57	
Discussion of Content Standards	4	
Total	61	
Engagement with Mathematics Content		
Sharing Mathematics Knowledge	201	
Doing Mathematics	29	
Thinking about Mathematics	4	
Total	234	
Development of Mathematics Knowledge		5

commonly, mathematics standards were referenced by mathematical content domain, grade level, or standard number. For example, a blog post entitled "Big Nickel" included a mathematics task and the following statement indicating relevant content standards, "Common Core State Standard Alignment: Geometry [G-C.8, G-GMD.3]" The task prompted students to first determine how many actual Canadian nickels would fill the Big Nickel (see Figure 11) and to determine how the value of those nickels would compare to the actual cost of building the Big Nickel. Giving the relevant content standards connected the mathematics needed to solve the Big Nickel task with a mathematical progression.

Figure 11. The Big Nickel monument; a prompt within the Big Nickel task.

A second code, *discussions of content standards*, included discussions of the inclusion or placement of specific standards or goals within the mathematics progressions. For example, consider the reason a teacher gave for introducing completing the square algorithmically, "Because, honestly, 'completing the square' ONLY has a use in high school at THIS particular instant in time. (We don't do conics in Ontario aside from polynomial parabolas.)" In this example, the blogger demonstrated a knowledge of where to place a particular concept within a progression of mathematics.

Engagement with mathematics content. This second theme within the subcategory of subject matter knowledge includes interactions of community members in which they demonstrated a mathematics knowledge of, or beyond that of, any well-educated adult (Ball et al., 2005; Shulman, 1986). Three codes were included in the theme. The first code was *sharing mathematics knowledge*. For example, a blogger described the various manners of calculating statistical measures,

> For the reader who remembers or has had exposure to statistics, our Pythagorean distance measure is very closely tied to "standard deviation" and "variance". Recall that the population variance is just
>
> $$var = \sigma^2 = \frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}$$

where μ is the population mean. Notice the similarity? The division by N is just "averaging" the total square distance (the numerator). Standard deviation is just $\sqrt{var} = \sigma$.

Mathematics knowledge shared within the blogosphere often included a detailed explanation of mathematics content. In contrast, mathematics knowledge shared on Twitter frequently included less detail; for example, consider the following tweet, "Interesting to think of polynomial functions as sums of power functions #MTBoS." Other interactions within this code included requests for other participants to share desired mathematics knowledge, as illustrated in the following tweet: "I have a gr. 11 [grade 11] student who's convinced that quadratics aren't useful in real life. Her passion is music. Any suggestions? #mathchat #MTBoS." In other cases, knowledge was shared for both elementary and secondary mathematics content.

The second code within the theme, *doing mathematics*, included how community members interacted with the mathematics content themselves. For example, consider the following tweet and corresponding image shown in Figure 12: "Exploring ratios on Photo/iMAC #mathchat #MTBoS #edchat #engagemath." This shows how one community member used the aspect ratio within a computer program to explore how ratios affect photo size.

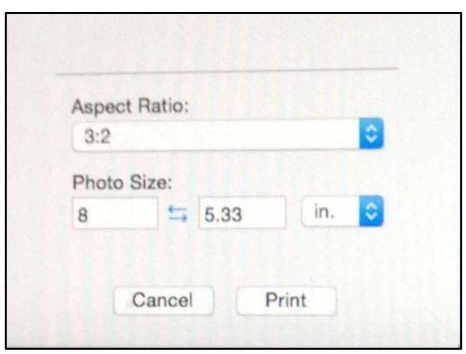


Figure 12. Image demonstrating how a community member may explore mathematics concepts.

In other cases, community members shared their solution for mathematics tasks, often using images or videos. Consider the video show in *Figure 13*, which was embedded in a blog post.

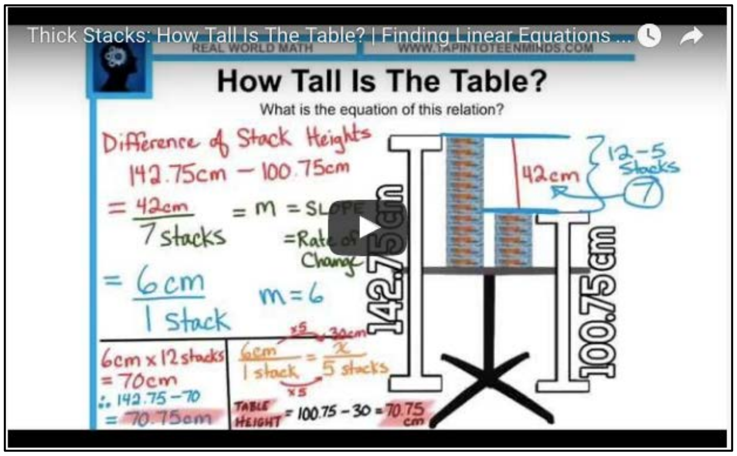


Figure 13. Image of a video embedded within a blog post.

Note that before watching the video, the mathematics tasks to be solved is already visible. Upon hitting play, *Figure 14* provides a progression of three screen shots to illustrate how the author shared his/her solutions of the problem.

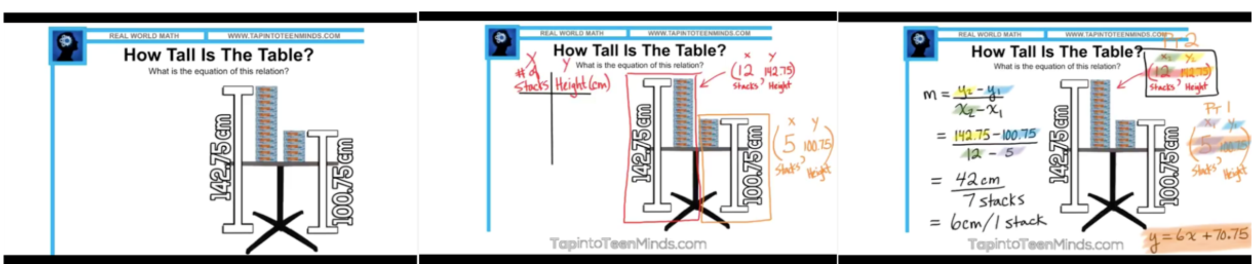


Figure 14. Screen progression of video embedded within a blog post.

The final code in this theme, *thinking about mathematics*, included how community members recorded their thinking about mathematics. For example, a blog post titled “Fraction/Percent Equivalents” described the blogger’s thinking when students were asked how they could shade 1/4 of a 10x10 grid:

> As I was walking around, I head a pair talking about shading a 5x5 in that grid. I saw this [shading fractions] as a beautiful connection to the volume unit they just completed in which they were adjusting dimensions and seeing the effect on the volume.

This example illustrates how the author of the blog showed how he/she made connections between the current and past units of mathematics instruction.

Development of mathematics knowledge. The third theme within the subcategory of subject matter knowledge is drawn from the interviewees' perceptions of how the MTBoS supported their development of subject matter knowledge. When they were asked if they felt their knowledge of mathematics content had improved, Scott, Eric and Sydney all stated that they believed that their content knowledge had improved from engagement with the community, while Kathy and Nancy did not report that their content knowledge had improved.

Those who felt that the community had a positive effect most commonly referred to their exploration of mathematics-specific blog posts. For example, Scott recalled coming across unfamiliar mathematics vocabulary while reading a blog post; specifically, he recalled asking himself, "What do these math words mean?" His curiosity led him to pursue the topic; he stated, "I've found they were opening up things like, Euler's sphere and different types of points, but I had never heard of a Euler sphere before so I was pursuing what that meant." His initial exposure to unfamiliar mathematics vocabulary prompted him to learn new mathematics content. Eric reported that many of the mathematics teachers that he supports teach courses beyond his skill set; as a result, he feels challenged to go back and revisit many of the more advanced topics by exploring various blog posts. He stated, "I'll read their [high school teachers] blog posts and be like, 'Ah man', and I've found myself numerous times either saying it to myself or saying it to them, ‘Dude, I wish I was in your class.’” He reported that exploration of blog posts increased his content knowledge.

Eric also reported developing mathematics knowledge through explorations of Desmos, an in-browser graphing calculator (“Desmos”, n.d.). For example, he stated that when he sits down to design a new Desmos activity, he may not understand or know the exact mathematics needed to create a particular lesson. Exploring other community members’ Desmos activities has helped further his knowledge, as stated in the following:

> And so a lot of learning has happened for me in regards to, ‘Whoa, somebody made this activity and I really like how that functions on this one screen’ so I can duplicate the activity and I can go in like I’m editing it and I can see all of their equations or their constraints, or whatever they did and I learn from that and my mathematical understanding also is strengthened.

Finally, Sydney reported that mathematics problems collaboratively worked out by community members also helped increase her content knowledge. Furthermore, she reported that in the past she struggled to understand higher level mathematics and as a result, she did not retain a significant portion of that knowledge. Engaging with mathematics problems alongside community members helped compensate for that lack of knowledge. In contrast, Kathy and Nancy stated that their mathematics understanding had not been improved by engaging with the community. For example, Nancy stated,

> It might be the way I choose the things I read though. Because I don’t recall enjoying reading about mathematics as much as I enjoy reading about teaching. So I would probably ignore those opportunities, I like talking about mathematics, I just don’t like reading about it.

Thus, the decision to ignore opportunities to read further about mathematics limited her development of her mathematics understanding.

Conclusion about subject matter knowledge. Subject matter knowledge was addressed within the content of the MTBoS community through knowledge that they shared about mathematics progressions and their engagement with mathematics content. First, knowledge of the mathematics progressions was demonstrated through connecting mathematical tasks with content standards as well as discussions about the placement or inclusion of a concept within a mathematics progression. Second, community members also demonstrated a knowledge of, and beyond that of, any well-educated adult by sharing mathematics knowledge, doing mathematics, and thinking about mathematics. Of the methods used to demonstrate knowledge, sharing mathematics knowledge was most commonly observed within the community's interactions.

In the interviews, three of the five community members reported improving their subject matter knowledge by engaging with the MTBoS community, with reading mathematics blog posts as a common approach between the interviewees. One of the two community members who did not perceive improvements of subject matter knowledge stated that she simply did not like to read about mathematics but instead focused on pedagogical issues. While not consistent, engagement in the community can improve subject matter knowledge.

Pedagogical content knowledge. The second subcategory within the category of MKT is pedagogical content knowledge (PCK), which is the mathematics knowledge most connected to instruction (Shulman, 1986). More specifically, PCK includes knowledge of instructional materials and the characteristics of those instructional materials available within the curriculum (Ball et al., 2008; Shulman, 1986), knowledge of how to appropriately sequence mathematics content (Ball et al., 2008), knowledge of mathematics and the design of instruction related to that mathematics (Ball et al., 2008), and knowledge of how students may think or learn about specific mathematics content (Hill et al., 2008). Each of these four components align respectively with the first four themes within the subcategory. Two additional themes are more comprehensive in nature. PCK accounted for 59.48% (N = 687) of the content within the category of MKT. The six themes, along with constituent codes, will be discussed; Table 10 shows their distribution across the two phases of the study.

Curriculum resources. Curriculum resources, the first theme within pedagogical content knowledge, is related to the knowledge of instructional materials and the characteristics of those instructional materials available within the curriculum (Ball et al., 2008). Three codes were used to create the theme. The first code, *recommended curriculum resources*, related to the sharing of recommended resources within the community. Consider the following tweet and corresponding image shown in Figure 15, "Math Tweeps! Looking for @desmos explorations to use this week? Look no further https://www.desmos.com/explore #mtbos #tlap." The included hyperlink was used to point out curriculum materials to community members. All the tweets and blog posts shared resources in a similar manner, with the use of a hyperlink.

Table 10

Themes Related to Pedagogical Content Knowledge

Themes and Codes	Frequency of Units with Code within Community Interactions	Number of Interviewees included in Code
Curriculum Resources		
Recommended Curriculum Resources	209	1
Requests for Curriculum Resources	25	
Discussion of Curriculum Resources	64	
Total	298	1
Curriculum Planning		
Planning Instruction	77	
Sequencing of Content	24	

Total	101	
Methods of Teaching Specific Mathematics	181	
Knowledge of Student Thinking	90	
Underlying Philosophy of Teaching[a]	17	
Development of Pedagogical Content Knowledge		2

[a]Also included as a theme for the VHQMI category

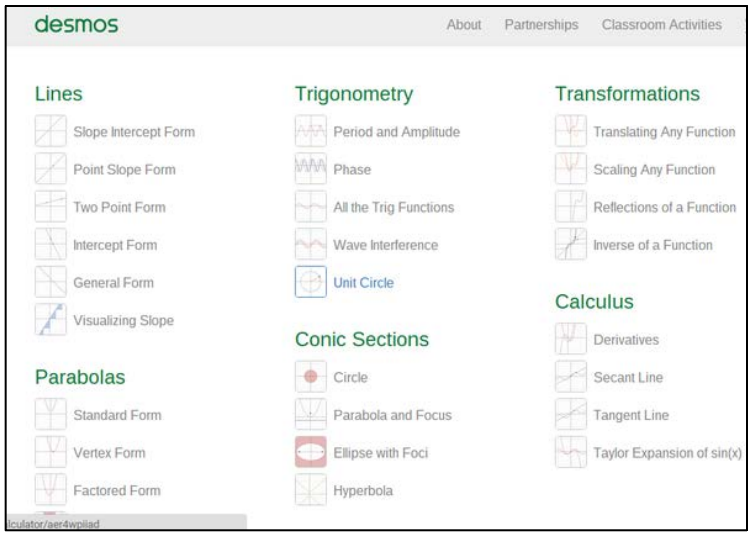


Figure 15. Image of available Desmos explorations by mathematics topic.

In the interviews, Eric discussed how he supported the mathematics teachers within his school with curriculum resources from the MTBoS community. He described the high frequency of shared resources in the community as a “fire hose of information,” making the ability to archive material very important. He went on to state,

> On my computer, I am extremely organized and so the fact—in the digital age that I can grab a link, grab a blog post, and I can archive it somewhere, I can store it somewhere that I know it will always be there, and I can reference.

A second code, *requests for curriculum resources*, related to requests for recommendations of resources specific to a concept, problem type, course, or student type. In some cases, a request was made for materials related to a particular mathematics concept, as in the following, “#MTBoS Help! Anyone have a fun way to review rational number operations? Cannot seem to find anything!” In other cases, materials specific to a problem type were requested, as in the following: “Does anyone have a list of Always Sometimes or Never conjectures for Middle school Ss [students]? #MTBoS #msmathchat #ElemMathchat #math #slowmathchat.” Note that “Always Sometimes or Never conjectures” is an activity that requires students to decide if a mathematics conjecture is always, sometimes, or never true.

A final code within the theme was *discussion of curriculum resources*. These discussions most commonly addressed the limitations of a curriculum. For example, the following comment and image (see Figure 16) refers to a published mathematics task, suggesting that it is over scaffolded:

> This is where most, if not all, of the publisher’s curriculum gets it wrong. Most of the tasks they give do not ask students to make sense of problems and persevere in solving them nor attend to precision. There is far too much suggestion about how a student should solve a problem, and there is no room for students to work things out.

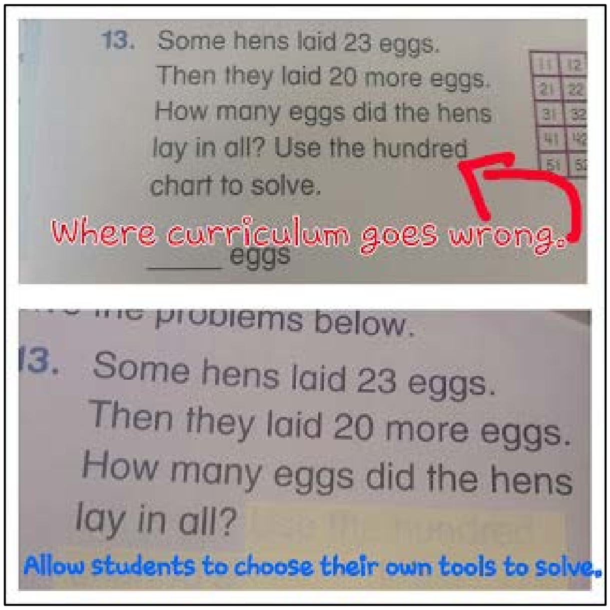


Figure 16. Image demonstrating both a critique of and suggestion for improving the curriculum. In other cases, the code included more general descriptions of curriculum resources; the discussions most commonly addressed the coherence between mathematics tasks.

Curriculum planning. The second theme within the subcategory of pedagogical content knowledge was curriculum planning, including how to appropriately sequence the mathematics curriculum, an important component of PCK (Ball et al., 2008). The theme was built from two codes. The first code was *sequencing of content,* specifically how concepts fits within the mathematics curriculum. Consider the following statement from a blog post: “Today, awesome hook for instant interest in logs; tomorrow, inverse functions to show why logs work”. This post shows the author’s thoughts on sequencing content by discussing what happened that day, and what will happen tomorrow, thus reporting a two-day sequence of instruction. Requests for help in sequencing content was also included in the code, such as the following tweet, “Any ideas for introducing evaluating expressions to Math 6 students using Clothesline? What examples would you start with? #MTBoS." This tweets showed how community members requested help in starting a sequence of instruction.

The second code, *planning*, included both individual and collaborative planning efforts. For example, in the following reflection on students' performance, the blogger gives insights into how he might plan for future instruction: "From what I'm seeing with this group, I should be able to jump into interior and exterior angles of n-sided polygons from an inquiry standpoint without much trouble." Collaborative planning was evident in the "Big Nickel" task previously discussed in the subject matter knowledge subcategory (see Figure 11); the author blogged and included an image (see Figure 17) to demonstrate how Twitter was used to engage others in planning: "This time, rather than trying to develop a question on my own, I tossed it out to the Twitterverse:"

Figure 17. Image demonstrating how collaborative planning was prompted within the community.

The blog post then embeds an image (see Figure 18) of the following tweets, “As expected, I had some ideas coming to me within minutes including these:”

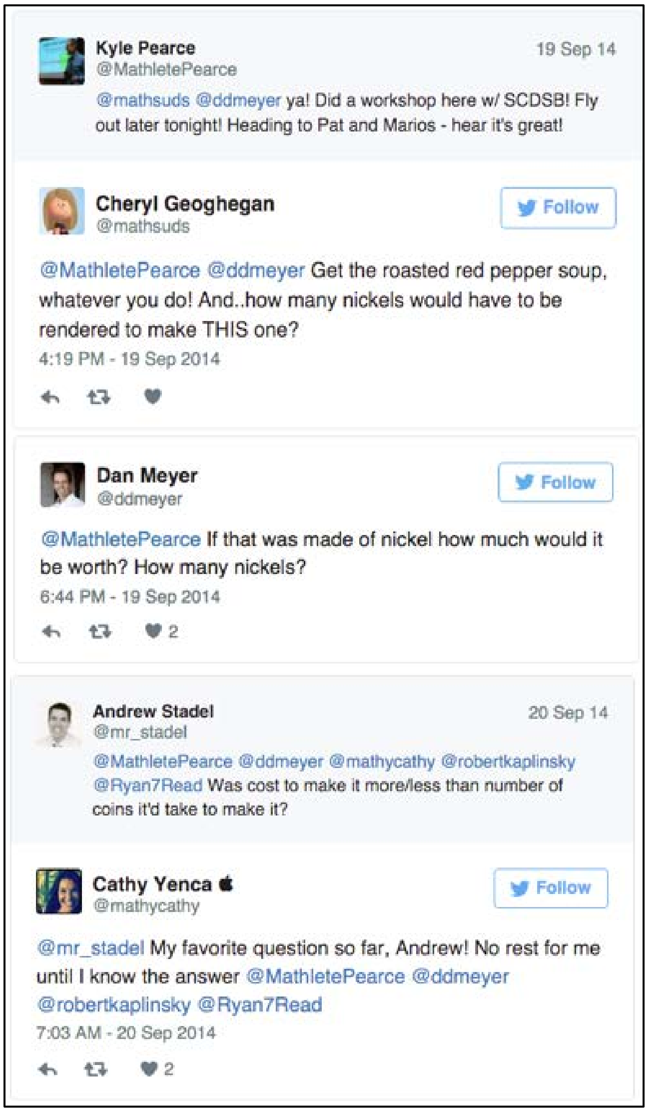


Figure 18. Image demonstrating how community members responded to a prompt for planning a mathematics task.

Methods of teaching specific mathematics. Methods of teaching specific mathematics, the third theme within the subcategory of pedagogical content knowledge, connects the knowledge of mathematics with the design of instruction (Ball et al., 2008). The theme was built from a single code and all discussions were specific to a method of teaching a particular mathematics concept. For example, consider the following tweet and image shown in Figure 19, “Mr. Saladino using #Desmos to discuss square roots!! #redlandsud #mtbos @MonarchsMoore.”

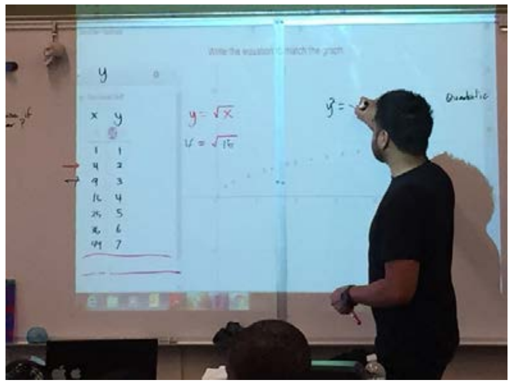


Figure 19. Image demonstrating how a particular method of teaching mathematics was shared with the community.

In the tweet, the method of teaching, using graphical representations within Desmos, was connected to a specific mathematics concept, square roots.

Knowledge of student thinking. Knowledge of student thinking, the fourth theme within the subcategory of pedagogical content knowledge, was related to the knowledge of how students may think or learn about specific areas of mathematics (Hill et al., 2008). The theme included a single code primarily focused on anticipating student thinking. For example, in the following statement and image (see Figure 20), a blogger discussing geometry suggested, “Ask a student to define parallel lines and they may say something like ‘lines that never touch,’ and in their head are visualizing this:”

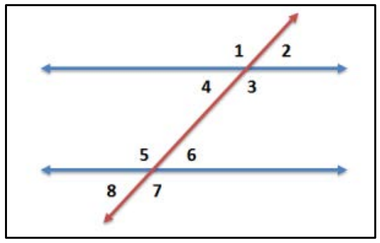


Figure 20. Image demonstrating how a teacher may anticipate students' thinking of a particular mathematics concept.

The author anticipated students' thinking of parallel lines. The anticipation of how students' may think about solving a particular problem was also observed. For example, in describing the implementation of a mathematics task about eating hot dogs, a blogger stated,

> Something important to note is that there are many ways to get the answers. For example, with question 7, one student might notice that Kobayashi easts one less hot dog each round and so after n rounds, he has to eat n hot dogs to catch up.

In other cases, student thinking was anticipated by referencing learning progressions; as in the following,

> As I do with many lessons, in thinking about their strategies beforehand, I referred to the Learning Progressions to see how students' progress through algebra reasoning. If they didn't know the addition expression from memory, like 3 + 3 or 5 + 5, this clip from the progressions [see Figure 21] best describes how I was seeing students arrive at the first expression written for each given sum.

> The advance from Level 1 methods to Level 2 methods can be clearly seen in the context of situations with unknown addends.[1] These are the situations that can be represented by an addition equation with one unknown addend, e.g., $9 + \square = 13$. Students can solve some unknown addend problems by trial and error or by knowing the relevant decomposition of the total. But a Level 2 counting on solution involves seeing the 9 as part of 13, and understanding that counting the 9 things can be "taken as done" if we begin the count from 9: thus the student may say,
>
> | | | | | |
> |---|---|---|---|---|
> | "Niiiiine, | ten, | eleven, | twelve, | thirteen." |
> | | 1 | 2 | 3 | 4 |
>
> Students keep track of how many they counted on (here, 4) with fingers, mental images, or physical actions such as head bobs. Elongating the first counting word ("Niiiiine...") is natural and indicates that the student differentiates between the first addend and the counts for the second addend. Counting on enables students to add and subtract easily within 20 because they do not have to use fingers to show totals of more than 10 which is difficult. Students might also use the commutative property to shorten tasks, by counting on from the larger addend even if it is second (e.g., for $4 + 9$, counting on from 9 instead of from 4).

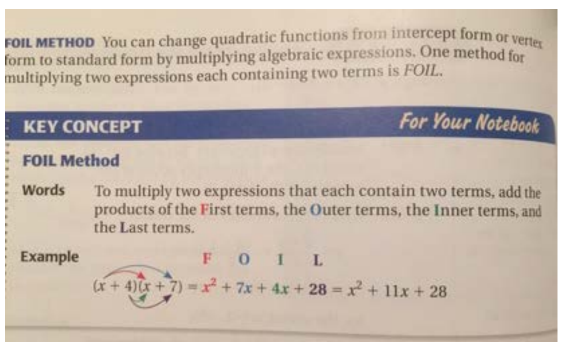
Figure 21. Image demonstrating how the learning progressions may be referenced to understand how a student may think about a particular mathematics topic.

The code also included interpretations of student thinking. For example, one blogger stated, "He had gotten the 'right answer' but his method of solving the problem showed a lack of place value understanding which would cause him to make a mistake if the tens place value had been over 100." Through interpreting the student's method of solving the problem, the blogger recognized that the student lacked an understanding of place value.

Finally, some discussions within the code addressed student mistakes and misconceptions. Consider how a blogger anticipated student mistakes in the following post, "Just this morning my colleagues and I were talking about the struggle that students have when faced with an expression. They are programmed to solve, so they insert an equal sign where ever [*sic*] they can. Even Algebra II students." Related discussions also included strategies for combating and responding to student mistakes.

Underlying philosophy of teaching. Underlying philosophy of teaching, a fifth theme within the subcategory of pedagogical content knowledge, included discussions of beliefs related to the teaching and learning of mathematics. This theme also appears later in the MKT miscellaneous subcategory, as well as in the discussion of VHQMI. As specifically related to pedagogical content knowledge, a single code was included in which the philosophy of teaching is connected with particular mathematics concepts.

Some content within the code related to the importance of students having a conceptual understanding of a mathematical concept. For example, consider a tweet and its related image in Figure 22, “After banning ‘FOIL’ from my classroom, I find this in the text. #ugh #MTBoS #nixthetrix.”



FOIL METHOD You can change quadratic functions from intercept form or vertex form to standard form by multiplying algebraic expressions. One method for multiplying two expressions each containing two terms is *FOIL*.

KEY CONCEPT *For Your Notebook*

FOIL Method

Words To multiply two expressions that each contain two terms, add the products of the First terms, the Outer terms, the Inner terms, and the Last terms.

Example

F O I L

$(x + 4)(x + 7) = x^2 + 7x + 4x + 28 = x^2 + 11x + 28$

Figure 22. Image demonstrating a distaste for teaching tricks in mathematics.

Note, the hashtag “#nixthetrix” references a book developed by members of the MTBoS community, *Nix the Tricks: A Guide to Avoiding Shortcuts that Cut Out Math Concept Development* (Cardone, 2015). Discussions also focused on providing students opportunities to develop their own mathematics knowledge through problem solving. For example, one blogger reported,

> We started exploring scientific notation and I tried to go about giving them a more conceptual understanding of what’s happening when you multiply by 10 to some power,

> instead of just saying ‘now count the place values and that’s your exponent!’ Some of the kids made that discovery on there [*sic*] own which I love, but I didn’t want to just teach the shortcut without exploring the concept a little more deeply.

Although the task was not included, this example demonstrated students being provided an opportunity to develop their own understanding of scientific notation.

Development of pedagogical content knowledge. Development of pedagogical content knowledge, a final theme within the subcategory, relates to how MTBoS community members perceived their pedagogical content knowledge had improved as a result of engaging with the community. When questioned about pedagogical content knowledge, three of the five interviewees reported improvements in pedagogical practices, although their responses did not connect the improved practice with specific mathematics concepts. Two participants discussed improvements in their pedagogical content knowledge at other points in the interviews, particularly their gained knowledge of concept specific teaching practices. For example, when Scott was asked if he benefited from engaging with the MTBoS, he described learning how to use the Singapore Bar method to teach proportional reasoning concepts, as in the following:

> They teach proportional lengths on bars all the way from kindergarten right through secondary, so it is one idea of proportional reasoning that they take all the way through, and I found that was a powerful way. And so I started to create some relational bars myself and would change a value in one of the boxes, and the students would have to try and figure what the lengths of all of the other boxes are. Sort of like Cuisenaire rods; you know if this one is 2, what are all the rest in the puzzle.

Similarly, Nancy also described a content-specific teaching practice when asked if she felt she benefited from engaging with the MTBoS. She reported using error analysis while teaching geometry. In this method, students are given geometry proofs that include intentional mistakes; the students would have to identify and correct the mistakes within each proof. She downloaded the proofs from a blog post, made minor adjustments for her students, and then implemented this teaching method in her classroom.

Conclusion about pedagogical content knowledge. Pedagogical content knowledge was addressed in the content of the MTBoS in a variety of ways. First, knowledge of curriculum was demonstrated through sharing of recommended curriculum resources, frequently through a hyperlink, as well as through discussions of curriculum, most of which included critical critiques. Second, planning of the curriculum was also visible through both individual and collaborative efforts and included sharing knowledge about sequencing the curriculum. Third, a knowledge of how to teach mathematics was seen in discussions of methods of teaching mathematics, all of which were specific to a particular mathematics concept. Fourth, a knowledge of student thinking was demonstrated through anticipating student thinking, interpreting student thinking, and discussing student mistakes; these discussions most commonly anticipated student thinking. Lastly, discussions that communicated an underlying philosophy of teaching focused on the importance of understanding mathematics and constructing knowledge through problem solving.

In the interviews, when asked about their development of PCK, the interviewees did not connect improved teaching practices with mathematics concepts. At a later point in the interviews, two participants discussed learning concept-specific teaching practices from the community. Engagement in the community may have some potential to improve pedagogical content knowledge.

MKT miscellaneous. The last subcategory within the category of MKT is MKT miscellaneous, which includes content related to mathematical knowledge for teaching, but not included within the previous subcategories. Multiple areas of mathematics knowledge were addressed in this subcategory. MKT miscellaneous accounted for 14.98% (N = 173) of the content with the category of MKT. Three themes, along with constituent codes, will be discussed; Table 11 shows their distribution across the two phases of the study.

Table 11

Themes for Miscellaneous Responses in the MKT Category

Themes and Codes	Frequency of Units with Code within Community Interactions	Number of Interviewees included in Code
General Knowledge of Teaching and Learning Mathematics		
General Knowledge of Teaching	63	1
General Knowledge of Student Learning	31	1
General Knowledge of Technology	24	
Total	118	2
Learning about Teaching		
Personal Learning	29	
Reference to Education Literature on Research	14	
Total	43	
Underlying Philosophy of Teaching[a]	12	

[a]Also included as a theme for the VHQMI category

General knowledge of teaching and learning mathematics. General knowledge of teaching and learning mathematics, the first theme within the category of MKT miscellaneous, included knowledge that was widely applicable within mathematics, not specific to a particular mathematics concept. Three codes were used to build this theme. The first code, *general knowledge of teaching*, includes a knowledge of mathematics teaching practices; the teaching practices shared within this code are generalizable to multiple concepts within mathematics. Consider the following tweet and corresponding image shown in Figure 23, “Been using this date pattern for days on end, yet every day a S [student] asks if I messed up… #ZeroAsAnExponent #MTBoS."

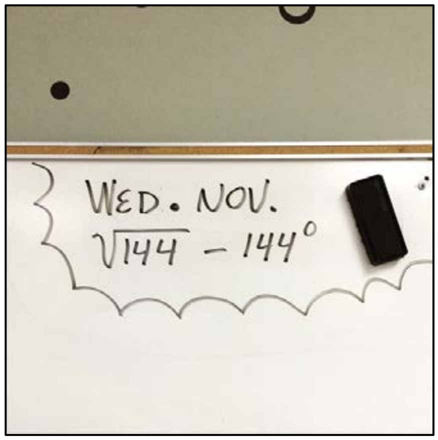


Figure 23. Image demonstrating a generalized teaching practice.

While the method for teaching, using the date to practice procedures, was particularly related to square roots and law of exponents, this approach may be applied to multiple areas within mathematics.

An interviewee also made comments related to *general knowledge of teaching*, particularly how knowledge of teaching is typically discussed within the community. In talking about the blogosphere, Eric stated, "I try to take on that lens of when I blogged, here is something that worked, for the most part. Like here are some problems and here is where I would go for the next time I do it with refinement." Eric did not share actual knowledge of teaching, but instead described how knowledge of teaching is framed and discussed within the community.

The second code, *general knowledge of student learning,* includes content demonstrating a general understanding of student learning. For example, one blogger stated,

> Since learning math is a process and every student is at a different place on their learning journey, offering multiple opportunities to demonstrate learning will help the student understand that they will not suffer if they struggle with a concept initially.

This example demonstrates attention to student learning across the discipline of mathematics. Other discussions extended a knowledge of student learning beyond the subject of mathematics.

An interviewee also demonstrated a *general knowledge of student learning*. For example, when Nancy was asked how she decided to use curriculum resources from the MTBoS community, she stated, “I have to consider the background of my students, is the language used and the type of problems presented, would those make sense to my students in their experience in the math classroom.” This reflects a general knowledge of students and how they learn.

A final code, *general knowledge of technology*, addresses how to use technology within the classroom. For example, a blogger described how Knowledgehook, a whole-group formative assessment tool (“Knowledgehook”, n.d.), may be used to support instruction. In a second example, a tweet and image (see Figure 24) shared the possibility of embedding GeoGebra, dynamic geometry software (“Geogebra”, n.d.), within PearDeck, a presentation platform (“Peardeck”, n.d.) “Embedding @geogebra inside my @PearDeck!! A geometry teacher’s dream!!! #mtbos #geomchat #ntchat.”

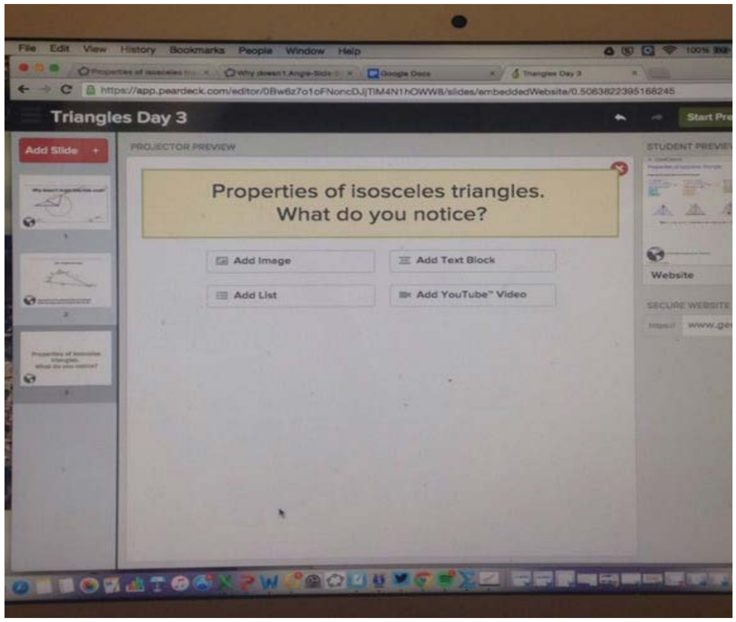


Figure 24. Image demonstrating a knowledge of technology to support instruction.

Other discussions requested help or further information about using technology to support instruction.

Learning about teaching. Learning about teaching, the second theme within MKT miscellaneous, related to learning about the knowledge needed to teach mathematics. The theme included two codes. In the first code, *personal learning*, teachers expressed a desire to discuss learning as it related to mathematics or mathematics education. Consider the following tweet and corresponding image shown in Figure 25, "#Learn Desmos: Use the connect dot feature in tables to draw line segments https://www.desmos.com/calculators/x1mdmornhp…#mathchat #mtbos."

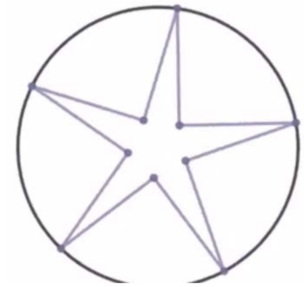
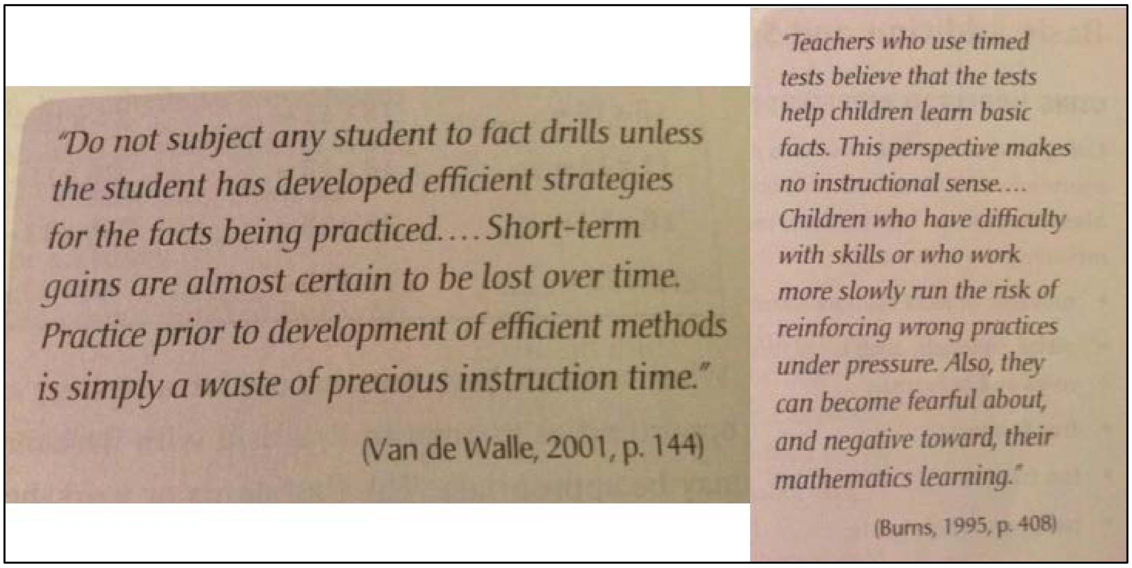


Figure 25. Image demonstrating an opportunity to learn about Desmos within the community. This example illustrates how learning opportunities were shared within the community. Other technologies addressed included hand-held calculators or Kahoot, a game-based learning platform (kahoot.it). Additional discussions reported previous learning or expressed a desire to learn, as seen in the following:

> I have a growing interest in the transition of students from middle school to high school because many of the tasks I use or create get at middle years content. I'm wondering what knowledge students come to my room with and what atmosphere it was learned in. Both have huge impacts on how students operate in my room.

This blogger expressed a desire to learn about his/her students' previous mathematics experiences.

A second code within the theme was *reference to education literature or research*. To demonstrate, consider a tweet and image shown in Figure 26, "Memorizing without understanding? Short-term gains…To be lost over time. #mtbos #elememathchat #mathchat #math"



"Do not subject any student to fact drills unless the student has developed efficient strategies for the facts being practiced.... Short-term gains are almost certain to be lost over time. Practice prior to development of efficient methods is simply a waste of precious instruction time."

(Van de Walle, 2001, p. 144)

"Teachers who use timed tests believe that the tests help children learn basic facts. This perspective makes no instructional sense.... Children who have difficulty with skills or who work more slowly run the risk of reinforcing wrong practices under pressure. Also, they can become fearful about, and negative toward, their mathematics learning."

(Burns, 1995, p. 408)

Figure 26. Image demonstrating how education literature was shared with the community. In this example, published statements by two mathematics education researchers were shared with the community.

Underlying philosophy of teaching. Underlying philosophy of teaching, a final theme within the subcategory of MKT miscellaneous, included discussions of beliefs related to the teaching and learning of mathematics; this theme was also discussed within the context of the PCK subcategory and will appear again in the context of VHQMI. In this subcategory, a single code was included in which the underlying belief that students should understand mathematics relationally was connected with a broad range of mathematics concepts. For example, consider the following tweet and image shown in Figure 27: “Tips Moving From Procedures to Understanding in Math Class https://tapintoteenminds.com/tips-moving-from-math-procedures-to-understanding/ … #MTBoS #ADEchat #edchat #maths.”

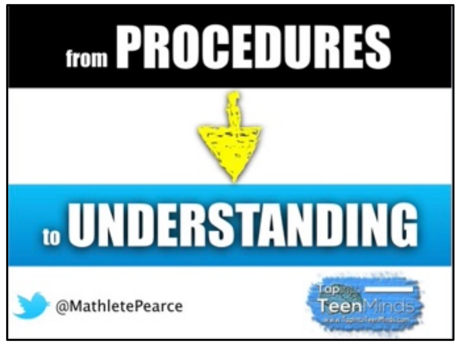


Figure 27. Image demonstrating the importance of moving from procedures to a conceptual understanding of mathematics.

Here, a shift in instruction from a focus on procedures to a focus on understanding was addressed.

Conclusion about MKT miscellaneous. The category MKT miscellaneous included several themes not directly connected to any of the MKT subcategories. First, a general knowledge needed to teach and learn mathematics was demonstrated through discussion of teaching practices, students, and technology; a general knowledge of teaching practices was most commonly discussed. Second, opportunities to learn further about the knowledge needed to teach mathematics were often provided; these opportunities also included references to education literature or research. Lastly, discussions that communicated an underlying philosophy of teaching prioritized students' having a relational understanding mathematics.

Conclusion about MKT. The teacher attribute of mathematics knowledge for teaching was addressed within 38.53% of the MTBoS community content. Across the category, the knowledge needed to teach mathematics was consistently shared with members of the community. This shared knowledge ranged in applicability to one or more mathematics concepts; for example, shared teaching practices included those specific to a particular mathematics concept, to those practices applicable to any area of mathematics. Additionally, many requests for the knowledge needed to teach mathematics were also observed. For example, requests may have ranged from needing help applying mathematics knowledge, to requesting curriculum materials for a specific concept. Of the three subcategories within MKT, PCK was most commonly observed within the community interactions.

In the interviews, three of the five community members reported improving their subject matter knowledge from engaging with the MTBoS community. Also, two of the five interviewees reported improving their pedagogical content knowledge from engaging with the MTBoS community. As a whole, some evidence supports the development of mathematics knowledge for teaching from engaging with the MTBoS community.

Visions of high quality mathematics instruction. Visions of high quality mathematics instruction (VHQMI) is the second category within the teacher attribute framework. Three subcategories are included: teacher role, classroom discourse, and mathematical task (Munter, 2014). In addition, a residual subcategory of VHQMI miscellaneous was included for all content related to instruction, but not included within a previous subcategory. VHQMI accounted for 46.70% (N = 1400) of the collected data from Twitter and the blogosphere (see Table 5).

In the sections below, themes developed from the open coding in each of the four subcategories will be discussed.

Teacher Role. The first subcategory within the category of VHQMI is teacher role, which includes three components (Munter, 2014): creating a classroom culture that supports the problematizing of mathematics, facilitating student learning through coparticipation, and supporting students with appropriate resources. Two themes emerged in alignment with these components; the first theme corresponds to a combination of the first two components, and the second theme corresponds to the third component. Teacher role only accounted for 3.29% (N =

102) of the content within the category of VHQMI. Two themes, along with constituent codes, will be discussed; Table 12 shows their distribution across the two phases of the study.

Table 12

Themes Related to Teacher Role

Themes and Codes	Frequency of Units with Code within Community Interactions	Number of Interviewees included in Code
Teacher as Facilitator		
Problematizing the Mathematics Content	41	
Coparticipation alongside Students	26	2
Total	67	2
Supporting All Student	35	

Teacher as facilitator. Teacher as facilitator, the first theme within VHQMI, includes teacher actions and decisions as they relate to facilitating classroom instruction. Two codes were used to build this theme. The first code, *problematizing the mathematics content*, related to a teacher selecting or staging content in a manner that prompts students to problem solve. For example, in reflecting on the implementation of a task, a blogger stated, “Last year I wanted to really get students asking the questions so I created the following models [see Figure 28] and let students decide what we were trying to find out…”

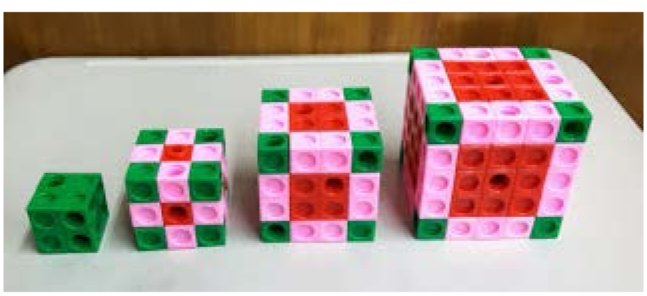


Figure 28. Image demonstrating how teachers may provide students with scenarios to determine which mathematics problem to solve.

The author of the blog did not provide students with the question to answer but provided a context in which the students could decide what problems to solve. Additional posts included showing students a video, object, or image, and then providing an opportunity for the students to determine what question from the provided scenario they wish to solve.

A second code, *coparticipation alongside students*, related to educators acting as facilitators within the mathematics classroom. Consider how one blogger positioned him/herself as a facilitator in the following statement,

> I find using Gameshow makes it even easier for me to be the facilitator of the learning rather than the gatekeeper of knowledge. Typically, the solution students share is enough to consolidate most tasks and I can minimize the amount of direct instruction I must deliver.

The blogger describes how acting in this role of facilitator minimized direct instruction and removed him/herself as the source of mathematical authority. Additionally, when teachers act as facilitators, students' attention may need to be focused on specific mathematics concepts. Consider how one blogger described focusing students' attention during presentations, "As students explain their strategies to the class I translate their words into small equations…All with the goal in mind of sneaking in equation solving."

The interviews also included content coded as *coparticipation alongside students*. When asked if how he implemented tasks had improved from engaging within the MTBoS, Eric responded, "I gravitate towards blogposts that are more conversational, classrooms where the teacher and the student are both learning together. And so, that helps me implement it because, it feels real."

Supporting all students. Supporting all students, the second theme within the subcategory of VHQMI, related to supporting students with appropriate resources or information needed to learn mathematics and consists of a single code. This theme also appears later in the VSSS category. The support provided ranged from additional information needed to solve a problem, to more tangible items, such as a ruler. Support with additional information was particularly visible in discussions of three-act tasks. Within these tasks, act one includes showing students a situation or scenario and then providing an opportunity for them to determine the question they wish to solve using mathematics. In act two, additional information needed to solve the established mathematics question is provided. In the final act, the solution is revealed (Meyer, 2011). This need for additional information within the second act was described in a post about three-act tasks;

> This is a really important moment in the lesson. Student have attempted to solve a problem without the necessary information which has now created a need for that information. In my room, I say 'What's wrong? Why can't you figure this out?' Kids will respond telling me they need more information, and they're usually pretty great at telling me what they need.

A blog post titled "Thick Stacks" provides a more concrete example; Figure 29 provides the context for the post.

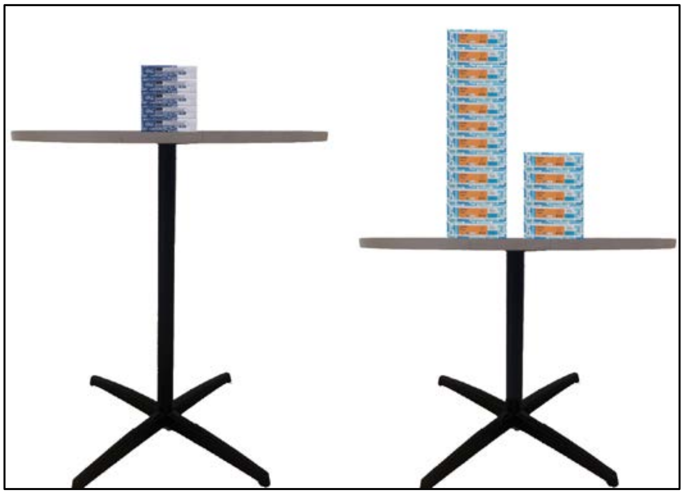


Figure 29. An initial scenario within the three-act mathematics task, Thick Stacks.

In describing the task, the blogger encourages teachers to provide students with an opportunity to discuss potential questions related to this scenario shown in Figure 29, students were encouraged to determine the height of the right, or shorter of the two tables in *Figure 29*. As students tried to solve this problem, the teacher is prompted to support the students with the additional information as shown in Figure 30.

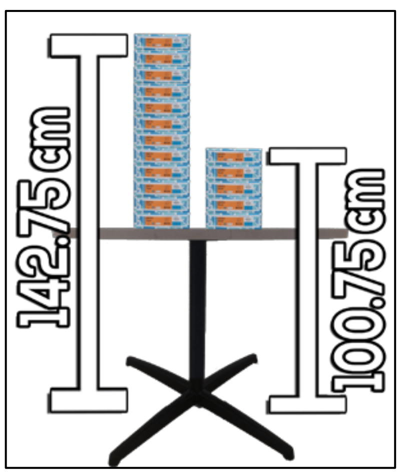


Figure 30. The information students need to solve a mathematics problem.

With this additional information, students are now able to determine the height of the table on the right.

Conclusion about teacher role. Teacher role was addressed through discussions of the teacher as facilitator in supporting all students. First, the role of teacher as facilitator was seen in two specific teacher actions, problematizing the mathematics content for students and supporting student learning through coparticipation. Second, supporting all students to learn mathematics was demonstrated through providing students with resources, most commonly, with the additional information needed to solve a problem.

Classroom discourse. The second subcategory within the category of VHQMI is classroom discourse, which includes whole-class conversation, teacher questions, student-to-student dialogue, and student explanations, all of which should remain conceptually oriented (Munter, 2014). Classroom discourse accounted for 27.29% (N = 382) of the content within the category of VHQMI. One theme, along with constituent codes, will be discussed; Table 13 shows their distribution across the two phases of the study.

Table 13

Codes within the theme of promoting discourse

Codes	Frequency of Units with Code within Community Interactions	Number of Interviewees included in Code
Whole-Class Discourse	65	
Teacher Role in Discourse	124	3
Student Discourse	193	
Total	382	3

The first code, *whole-class discourse*, included interactions that centered on both spoken and written dialogue within a whole-class context. Consider how a blogger described the classroom discourse that followed a functions activity, “After working the [function] machine for about 20 minutes, we had a great discussion about finding the function rules.” In other cases, the classroom discourse was documented through writing. For example, in exploring a task related to gender bias within Lego sets, a blogger used the following image to represent discourse within the classroom (see Figure 31).

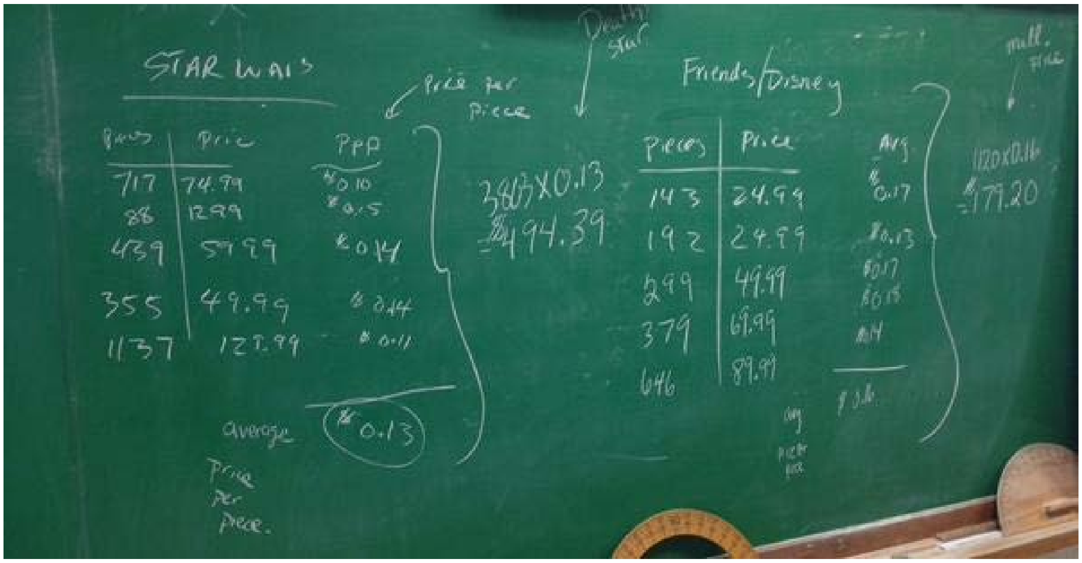


Figure 31. Image demonstrating whole-class, written discourse.

A second code, *teacher role in discourse*, related to teacher questions or strategies used during classroom discourse. In the following example, a blogger described the questions he/she included within his/her planning for a mathematics task:

> During the work at their seats, I will be walking around, and asking questions when necessary to generate conversation and looking at strategies. Questions: How did you arrive at your answer? Does everyone at the table agree? Where do you see (the ears, people, eyes, fingers) in your work? Is there an equation to match your work?

Reflections on teacher questions were also included within this code. Consider a post describing the enactment of a three-act task: “The math coach asked a great question, I think, in the second classroom visit trying Humpty Dumpty: ‘Do you have any questions that we could solve by counting?’ These seemed to help orient them toward countable features.”

Teachers' enactment of strategies to support discourse were also discussed, as in the following post:

> After the groups have arrived at an answer, I will have a couple students swap seats and explain to the new table how they arrived at their answer. They will then discuss what was the same and different about the problems and ways they solved their problems. After they share among tables, I will bring them to the carpet for a group discussion about these similarities and differences.

This illustrates how discourse strategies were shared within the community.

Content coded as *teacher role in discourse* was also discussed within the interviews, primarily related to perceived improvements in discourse practices. Although a question specific to improvements in discourse practices was not asked, three interviewees discussed how engaging with the MTBoS improved their discourse practices. Sydney, Scott, and Eric all reported improved teacher questioning. For example, when asked if learning had occurred for her while engaged with the MTBoS, Sydney commented.

> One guy in particular…, he publishes a lot of videos of his kids thinking through problems, like he is at a white board and he is asking a kid, his own children, problems. And from there, I see a lot of good questioning strategies that have been helpful.

Other content from the interviews related to strategies used to promote classroom discourse. When Eric was asked if there were any classroom practices that he had begun or strengthened from engagement with the MTBoS community, he discussed teacher listening and wait time, as in the following:

> Questioning. Listening. My listening skills have definitely strengthened, which I don't know if most people consider a teacher practice. But I do now and have for a few years

> and that is a big result of, if I don't listen to students then my instruction suffers. My teaching practices suffers, so sometimes you just come in as a teacher—it is very easy to come in with a learning objective and say here is how you do it and then you really don't care or you don't listen to – you might care but you just don't listen to. Like a student will say something and it's not what you had scripted in your head so you think it's wrong when they're not wrong, it's just weird. So listening is insane, so the more you can listen to students – that has improved my teaching practices because then it forces me to respond in real time to what they're interested in.

Eric also discussed improvements he has made with using wait time, as in the following:

> Also, what's the right word, spacing, pausing. That's been huge, asking a question and not speaking after it, even if there's like this awkward silence of kids are like –not saying anything, someone will speak up. So if I break the silence first, then I've ruined the whole thing. So that's a huge teaching practice and where we think time is valuable but at the same time, when we take the time to pause after asking a question and let kids think and process and then respond, that's really influenced my teaching practice and so a lot of those ideas have come from being involved with the MathTwitterBlogosphere, directly or indirectly.

A final code, *student discourse*, related to students' engagement in classroom discourse. Within the code, student-to-student discussions and students' written thinking were reported or shared, as in the following: "Awesome morning!! Kids trying to convince each other that their solution is right!! Not giving up!! #lovewhatyoudo #MTBoS." This tweet illustrates how student-to-student discourse was reported within the community. Images of students' written work were a prevalent means of sharing students' engagement in classroom discourse. Consider the following tweet and image shown in Figure 32, “Beyond excited to have S [student] journal writings to inspire blogging tmrw [tomorrow]! #elemmathchat #MTBoS #5thchat #2ndchat #3rdchat.”

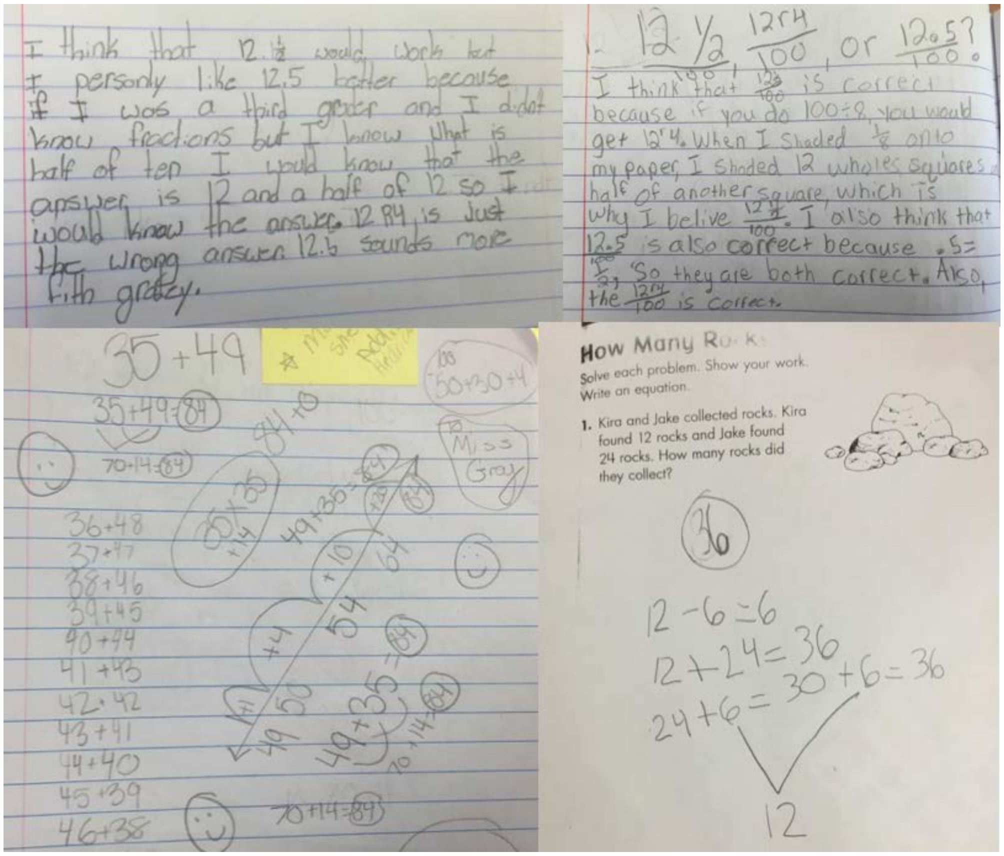


Figure 32. Image demonstrating how students’ thinking was shared within the community. This tweet includes the thinking of four students, expressed through written responses.

Conclusion about classroom discourse. Classroom discourse was addressed within the content of the MTBoS community through promoting discourse of three different groups within the classroom: whole-class, teachers, and students. First, whole-class discourse was demonstrated through both verbal and written dialogue. Second, the teacher’s role in classroom discourse included both questioning and discourse strategies. Lastly, student discourse was demonstrated through reports of student-to-student conversations, student questions, and written student thinking; of these, images of students' written thinking were most common.

In the interviews, three of the five community members reported improving their discourse practices by engaging with the MTBoS community; Sydney, Scott, and Eric all reported improved teacher questioning. Note that no interview question specifically addressed classroom discourse, so that all responses were initiated by the community members themselves. This provides some evidence that engagement in the community can support efforts to improve classroom discourse practices.

Mathematical tasks. The third subcategory within the category of VHQMI related to mathematical tasks. As defined within the review of literature, "Mathematical tasks can range from a set of routine exercises to a complex and challenging problem that focuses students' attention on a particular mathematical idea" (NCTM, 2014, p. 17). Mathematical task accounted for 26.29% (N = 368) of the content within the VHQMI category. The theme in this category, along with constituent codes, will be discussed; Table 14 shows their distribution across the two phases of the study.

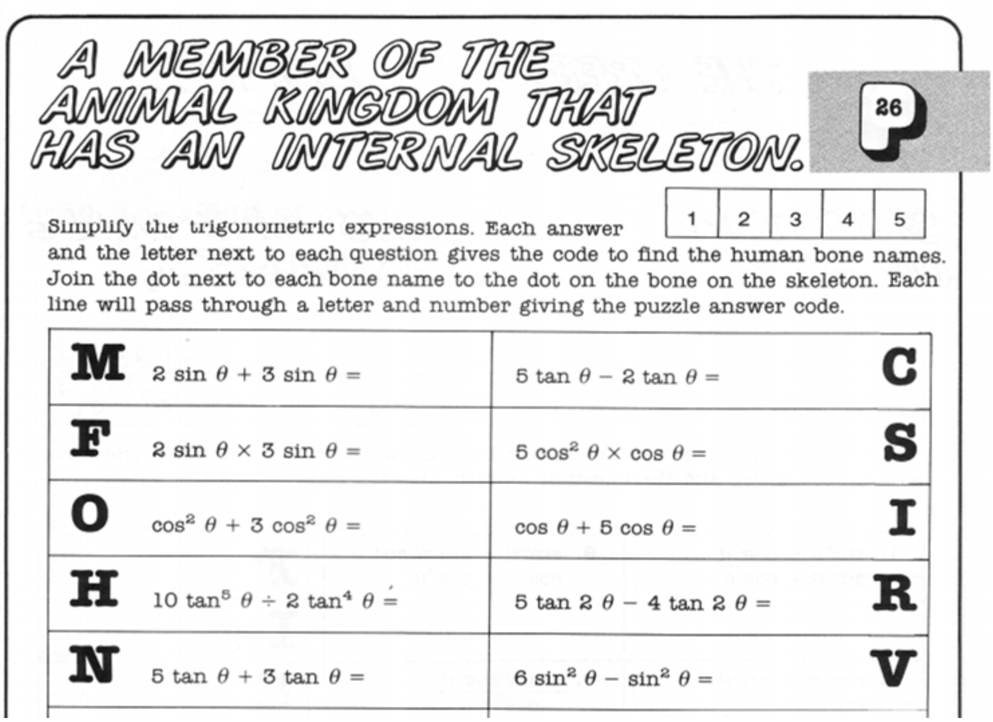
Table 14

Codes within the theme of mathematics tasks

Codes	Frequency of Units within Community Interactions	Number of Interviewees included in Code
Cognitive Demand of Mathematics Task	186	
Nature of Mathematics Tasks		5
Implementation of Tasks	93	5
Sharing a Task	89	
Total	368	10

Four codes were included in the single theme related to mathematics tasks. A first code, *cognitive demand of mathematics tasks*, categorized the level of cognitive demand of the 186 mathematics tasks identified in the content using the Task Analysis Guide (Stein et al., 2000); a more-detailed description of the Task Analysis Guide is provided in Chapter 2. Of the 186 tasks, 28 tasks (15.05%) were identified as low cognitive demand and the remaining 158 tasks (84.95%) were identified as high cognitive demand.

The task in Figure 33, drawn from a blog post titled "A Level Resources, "illustrates a typical low-demand task.


A MEMBER OF THE ANIMAL KINGDOM THAT HAS AN INTERNAL SKELETON.

26 P

1	2	3	4	5

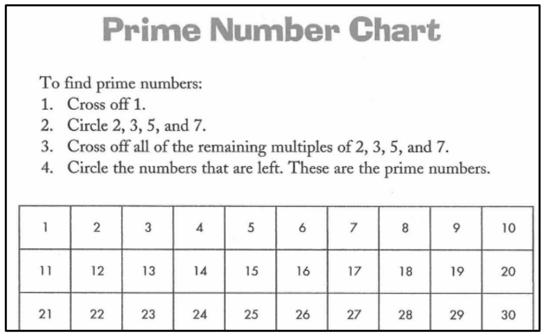
Simplify the trigonometric expressions. Each answer and the letter next to each question gives the code to find the human bone names. Join the dot next to each bone name to the dot on the bone on the skeleton. Each line will pass through a letter and number giving the puzzle answer code.

M	$2 \sin \theta + 3 \sin \theta =$	$5 \tan \theta - 2 \tan \theta =$	**C**
F	$2 \sin \theta \times 3 \sin \theta =$	$5 \cos^2 \theta \times \cos \theta =$	**S**
O	$\cos^2 \theta + 3 \cos^2 \theta =$	$\cos \theta + 5 \cos \theta =$	**I**
H	$10 \tan^5 \theta \div 2 \tan^4 \theta =$	$5 \tan 2\theta - 4 \tan 2\theta =$	**R**
N	$5 \tan \theta + 3 \tan \theta =$	$6 \sin^2 \theta - \sin^2 \theta =$	**V**

Figure 33. Image demonstrating a task requiring a low-level of cognitive demand.

In this task, even though students are required to solve a riddle using the solutions to mathematics problems, students are only required to practice procedures. Similarly, many of the low-demand tasks focused on application of a procedure. In one tweet, a low-demand task was included as a counter-example. Consider the task within the following tweet and image shown in

Figure 34, “That’s ALL the prime numbers? A prime example of an activity with little opportunity for sense-making. #mtbos.”



Prime Number Chart

To find prime numbers:

1. Cross off 1.
2. Circle 2, 3, 5, and 7.
3. Cross off all of the remaining multiples of 2, 3, 5, and 7.
4. Circle the numbers that are left. These are the prime numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

Figure 34. Image demonstrating a counter-example of a cognitively demanding task.

In contrast, consider the following high-demand task taken from a blog post titled, “A Favorite Task: Painted Cube for ‘Exploring the MTBoS’”:

> I imagine most of you have heard of the ‘Painted Cube’ task…if you create a cube out of ‘x’ 1 inch blocks, paint it, and take it apart, how many of the block [sic] will be painted on 3 faces, 2 faces, 1 face, or no face.

This task provides a high-level of cognitive demand as no clear solution path was specified, a key characteristic of tasks identified as high-cognitive demand (Stein et al., 2000). Other tasks were drawn from the content of the elementary grades; for example, the following task was from a blog post titled, “The Beginning of Arrays in 3rd Grade”:

> You have 12 chairs to arrange in straight rows for an audience to watch a class play. You want to arrange the chairs so that there will be the same number in every row with no chairs left over. How many arrangements can you make?

Again, a clear solution path was not specified within the task.

A second code, *nature of mathematics tasks*, includes the interviewees’ descriptions of the mathematics tasks discussed within the community, most in response to the following question: “How would you describe the types of mathematics tasks you find on the MathTwitterBlogosphere?” Kathy expressed a lack of confidence in her response that a majority of the tasks were for elementary aged students. She also communicated a disconnect between the tasks she would use as a secondary teacher and the tasks included within the MTBoS, saying, “I don’t feel like there is a big presence from college professors at all. So there isn’t that piece [college level content] and to some degree, some of the classes that we offer at South High School are more like a college course, so you know the Calculus or Multivariable Calculus kind of thing.” Kathy closed in saying that she could be doing a better job herself in contributing more advanced mathematics tasks to the community.

Nancy felt the tasks within the community possessed a classroom-ready quality, as stated in the following,

> Well the ones [tasks] that I’m drawn to and the ones that you know—I read carefully and remember, tend to be sort of classroom ready, ‘I did this thing in my class today, and this was my results, this is what I liked, this is what I didn’t like, this is what I would do differently if I did it again.’ I like that sort of in-the-trenches reporting.

She also commented that details related to the implementation of the tasks were more important than an actual file or worksheet with the task.

Sydney reported that the tasks within the community were thoughtful and contained good pedagogy. She also discussed two additional characteristics of the tasks, peer-recommended and crowd-sourced. In the following, she explains what she means by peer-recommended:

> If I find something for instance, on somebody’s blog that they’re talking about on Twitter. What I’ve got there is now, somebody I respect wrote it, somebody I know endorsed it and so I know that that material, if I were to just pull it down and print it off

> and hand it out in class tomorrow because if happened to fit, I could trust that it would probably go pretty well.

She further commented that because the community was self-selecting as related to common pedagogies, “somebody is going to go at it [design a task] the same way that I might have.” In discussing the benefits of a crowd-sourced mathematics task, she explained that because a task is crowd-sourced, many people contributed to its creation and thus, it would likely include fewer mistakes and fewer opportunities for it to not go well in the classroom. In closing, Sydney stated, “So the quality [of crowd-sourced tasks] is the highest I’ve found anywhere.”

Eric described the tasks discussed within the community in two ways, teacher-created and teacher-adapted. Teacher-created tasks were described as follows,

> So teacher-created would be a full on –let’s say a three-act task where somebody thought of something and they went out and made it happened with a picture or with a video, like they just bought their own supplies and made the idea happen.

In contrast, teacher-adapted tasks included making changes to tasks from a variety of sources, as in the following example he provided: “They subtracted some scaffolds, they made it more accessible to students, they made an activity that was more student focused as opposed to teacher led.”

The participants also spoke about how the tasks located within the MTBOS compared to those tasks found in other sources. A first point of discussion related to the design of and components included within the MTBoS tasks. For example, Scott stated,

> So I would say that the blogosphere material tends to be more visual than a lot of it, because a lot of the stuff you see in other avenues are text based, two-dimensional, flat, not dynamic whereas you’ve got a lot of the Desmos and Geometer’s Sketchpad things

showing up [in MTBoS tasks], allowing for a bit more flexibility and showing the dynamic nature of the mathematics.

Sydney also commented about the design and aesthetic aspect of the tasks found within the MTBoS community in the following:

> For instance, I care a lot about design of a page, so like typography and a good use of white space and what not. So graphic design stuff, it's [task from MTBoS] going to look as good as something from a professional publisher, where as I go find something from a random teacher on Google. I am more likely to find you know, Comic Sans font that are way too big or you know, just abuse the space on the page. They look cheap and unprofessional.

Nancy felt that tasks similar to those discussed within the MTBoS could be found in other sources, such as mathematics education magazines or at conferences. Even so, she concluded that similar tasks outside the MTBoS were more difficult to locate.

A third code, *implementation of tasks*, includes discussions of how a task was implemented or the details needed to support the implementation of a task. For example, the implementation of a task was evident in a post related to students using mathematics to bungee jump a Barbie doll, the blogger included the following comment and image shown in Figure 35:

> After a couple of years, I started requiring some certain work in order to meet my objectives. Each student must gather data, make a scatter plot, create a line of best fit, find the equation for the line of best fit, and show work for finding the number of rubber bands they want to use.

Figure 35. Image demonstrating how the implementation of a mathematics task was visible within the community.

Other cases reported plans for implementing a task; such as the following tweet, “Tomorrow, it will be BIG NICKEL with @knowledgehook Gameshow! Grab it: http://kylep.ca/1PAoUQP or Try it: http://kylep.ca/1PAoSIN #MTBoS”

Other discussions included in this code provided details needed to support the implementation of a task. Consider the details and image (see Figure 36) a blogger provided for a particular mathematics task:

> If you stop the video at 0:12, you can sort of count the hot dogs. I can clearly see four columns with at least eight hot dogs per column. For the sake of this problem, I am going to pretend that they each have exactly nine hot dogs in each of the four columns. The screen shots below help make this clearer:

Figure 36. Image demonstrating how details needed to support the implementation of a task are shared within the community.

Conversations within the interviews were also coded as *implementation of tasks*, particularly related to perceived improvements in implementing tasks in response to the following question: "Has the MTBoS community affected the way you implement tasks?" Sydney, Eric, and Nancy reported improvement in their implementation of tasks. For example, Eric reported that his ability to implement tasks has improved as a result of others sharing their reflections and experiences with a particular task. Nancy commented that she was confident her implementation of tasks had been affected by engagement with the community but had trouble "putting her finger on" exactly how it had affected her implementation.

In contrast, two interviewees did not report any change. Scott attributed his lack of change to the similarities between his own philosophy of education and the community's philosophy of education; he reported feeling "vindicated" in the teaching practices he used to implement mathematics tasks in past years. Kathy reported that because educators put their own personal touch on the implementation of tasks, she felt she could instead borrow the idea of the task and then adapt it to her own style of teaching and to what worked with her own students.

A final code, *sharing a task*, included intentional sharing of a mathematics task. In this code, the actual task may not have been visible within the collected content. For example, "Check out my @Desmos activity Translation! https:teacher.desmos.com/activitybuilder/custom/560ea7273be09dfe051813fe… Made for my Math 8 teachers. Feedback welcome. #mtbos @mjfenton." This example illustrates how a task may be shared, but not visible within the tweet or blog post.

Conclusion about mathematical tasks. Mathematical tasks were addressed within the content of the community in various manners. First, of the 186 mathematics tasks discussed in this subcategory, almost 85% were identified as cognitively demanding. Second, the implementation of tasks was reported, planned, and supported through various descriptions. Third, plans to discuss teaching with cognitively demanding tasks were visible within Twitter. Lastly, many mathematics tasks were shared within the community, even if the task was not visible in the collected data.

When the interviewees were asked how they would describe the tasks within the community, they discussed qualities such as classroom ready, peer-recommended, crowd-sourced, teacher-created, and teacher-adapted. Note that none referred to the cognitive demand of a task. Also, three of the five community members felt their ability to implement mathematics tasks had improved, with one more stating that he was already implementing tasks in a manner congruent with the philosophy of teaching visible in the MTBoS.

VHQMI miscellaneous. The final subcategory within the category of VHQMI is VHQMI miscellaneous, which includes content related to multiple areas of instruction, but not included within the previous subcategories. VHQMI miscellaneous accounted for 39.14% (N = 548) of the content within VHQMI. Two themes, along with constituent codes, will be discussed; Table 15 shows their distribution across the two phases of the study.

Table 15

Themes Related to Miscellaneous Responses in the VHQMI Category

Themes and Codes	Frequency of Units with Code within Community Interactions	Number of Interviewees included in Code
Instruction		
Sharing Instruction	101	
Assessment in Instruction	102	2
Reflection on Instruction	204	2
Improving Instruction	72	
Total	479	4
Underlying Philosophy of Teaching[a]	57	

[a]Also included as a theme for the MKT category

Instruction. Instruction, the first theme within the category of VHQMI miscellaneous, and includes various aspects of classroom instruction captured in four codes. The first code, *sharing instruction*, includes making descriptions and images of classroom instruction available for the community, as seen in the following:

> I stand in the middle of the room. I use my projector and my bamboo tablet to show how to graph a basic linear equation. This takes less than 2 minutes. Then I give them another equation. I tell them, 'partner 1 in the group, please graph this equation. You may consult your partner.' I write the new equation on my computer that is projected. They get loud and things go on their board.

Such an exact retelling of instruction was often shared with the community. Classroom instruction was also shared using Twitter; for example, consider the following tweet, "Wk – 6 [Week 6] reflection – CheezIts, no grades on tests, and economy chaos. http://wp.me/p3LdGYeo @jreulbach @TeachingChannel #msmathchat #MTBoS." In this tweet, an overview of one week of instruction was shared. Lastly, classroom instruction was also shared through images of students engaged during instruction. For example, see Figure 37.

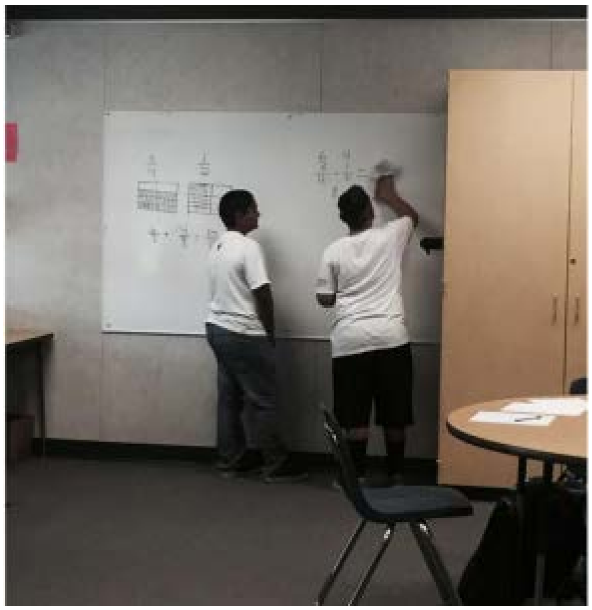


Figure 37. Image demonstrating how student engagement during instruction is often shared with the community.

A second code, *assessment in instruction*, included descriptions and shared knowledge of assessment practices within instruction. For example, the reporting of assessment practices was included in the following example, "I am undertaking a new assessment method with a colleague of 'feedback first and then grades.'" In other cases, knowledge or practices related to assessment was shared within the community. For example, one blogger described how he/she is now scheduling individual feedback sessions with students in the following:

> Schedule 20 minute bi-weekly meeting with each student (he [the author of a book] recommended meeting with 2 students at the same time, which I would do if I had a more typical class load). I meet with students before school, during lunch, during free periods (if they overlap with mine), during tutorial, and after school.

This post illustrates how assessment practices were shared with the community.

Content coded as *assessment in instruction* was also discussed within the interviews, particularly related to improved assessment practices. When asked if she had begun or strengthened any teacher practices through engagement with the MTBoS community, Sydney stated that the use of quiz corrections was her favorite teaching practice that she had gotten from the community; when her kids finish taking a quiz, they have an opportunity to look at an answer key, trade their pencil for a pen, and then make corrections for reduced credit. She described this as an opportunity to provide immediate feedback, “So like, self-correction, not for grading purposes, but for immediate feedback purposes, without putting me in the middle of that ‘wait for me to grade something’ cycle, has been hugely helpful.”

A third code, *reflection on instruction*, included reflections on students and teaching within the context of classroom instruction. Consider the following example in which the blogger reflected on students’ thinking following a logarithms exploration:

> Some of my students had heard of logarithms, and already had a negative opinion of them. But today they were all happy to meet logs because it was such an easier method. And now they were very curious to know what the heck this log thing was and WHY it worked. Boom.

In other cases, the bloggers’ reflections were specific to their own teaching. For example, the following reflection was in response to students requesting a formula versus thinking through the mathematics, the blogger stated, “However, thinking back on this exchange I realize that I was not effectively making a point here, I was simply showing my frustration.” This post shows how the blogger reflected on his/her own actions during instruction.

A final code was *improving instruction*. Consider the following example of a discussion related to improving teaching included within this code:

> 'We teach the way we teach because it is the way we have always taught.' Matt Larson, NCTM President-Elect. We all need to rethink our teaching and our teaching practices to help students develop deep understanding of mathematics.

This blogger communicated a need to rethink and improve instruction for the sake of student understanding. In other cases, requests were made for feedback or suggestions on instruction. For example, one blogger stated, "Have you found a way to help students understand why you don't want them to simply memorize? Help us out by leaving a comment below!"

Several responses within the interviews were coded as *improving instruction*, particularly related to seeking feedback on instruction. Eric stated that one of the benefits of the MTBoS community were the opportunities to received feedback, as stated in the following:

> The feedback, when I was in the classroom is insane. And it's again, I've already mentioned this but the opportunity to toss something out there, if you were willing, you want to make yourself vulnerable. And get some feedback—you start developing relationships or connections with people that you can count on.

Sydney also reported opportunities to receive feedback was a motivating factor in sharing resources, "So if I share a resource, part of my motivation in doing it is that feedback. People are going to use it; people are going to let me know where they struggled or their kids struggled."

Underlying philosophy of teaching. Underlying philosophy of teaching, the second theme within VHQMI miscellaneous, includes discussion of beliefs related to the teaching and learning of mathematics; this theme also appeared in the discussion of MKT. This theme consists of a single code. Discussions within the code related to the importance of students having a conceptual understanding of mathematics concepts. To demonstrate, consider the following tweet, "'Math answers aren't math understanding any more than the destination of your car trip indicates the route you took.' - @ddmeyer blog #MTBoS." This tweet illustrates how an emphasis was placed on understanding the mathematics versus arriving at an answer. Additional discussions provided commentary on teaching through problem solving. For example, one blogger stated, "One thing I know: once educators choose to take a risk, transfer power in their classrooms to their students, create an inquiry-based environment for learning, they do not go back to traditional teaching."

Conclusion about VHQMI miscellaneous. VHQMI miscellaneous included two themes not included in the VHQMI subcategories. First, multiple aspects of instruction were shared, discussed, and reflected upon within the community. The content ranged from reporting what happened during instruction, to intentional efforts to improve instruction. Second, discussions that communicated an underlying philosophy of teaching demonstrated a desire for students to have an understanding of mathematics and for students to develop their own knowledge through problem solving.

Conclusion about VHQMI. VHQMI was addressed within 46.70% of the MTBoS community content. The content most commonly included reports or reflections on a range of topics related to classroom instruction. For example, student discourse was primarily reported through images of students' written work. In contrast, the content within the subcategory of mathematical tasks often provided tasks or problems for the community. Of the subcategories within VHQMI, VHQMI miscellaneous was most common while mathematical tasks and classroom discourse were addressed in about the same frequency.

Three of the interviewees described improved discourse practices, including teacher questioning, teacher listening, and wait time. Furthermore, three of community members reported an improved ability to implement tasks from engaging with the MTBoS community, with another one stating that he was already implementing tasks in a manner congruent with the philosophy of teaching visible in the MTBoS. Lastly, one community member described improved assessment practices from engaging with the community. Thus, some evidence supports the development of VHQMI from engaging with the MTBoS community.

Views for supporting struggling students. Views for supporting struggling students (VSSS) is the third category within the teacher attribute framework. Wilhelm (2014) defined VSSS as a teacher's conception of who can and cannot engage with tasks requiring higher order thinking. VSSS only accounted for 3.07% (N = 92) of all collected data from Twitter and the blogosphere. Two themes developed from the open coding will be discussed; Table 16 shows their distribution across the two phases of the study.

Table 16

Themes Related to Views for Supporting Struggling Students

Themes and Codes	Frequency of Units with Code within Community Interactions	Number of Interviewees included in Code
Equitable View of All Learners		
Commitment to All Students	18	
Gender Equity	4	
Mindset	61	
Total	83	
Supporting All Students	9	5

[a]Also included as a theme for the VHQMI category

Equitable view of all learners. Three codes were included in the theme equitable view of all learners. A first code, *commitment to all students*, included views that all students can learn mathematics. Consider the following tweet and image shown in Figure 38, “Seen on the chalkboard of a classroom I visited this week. #mtbos.”

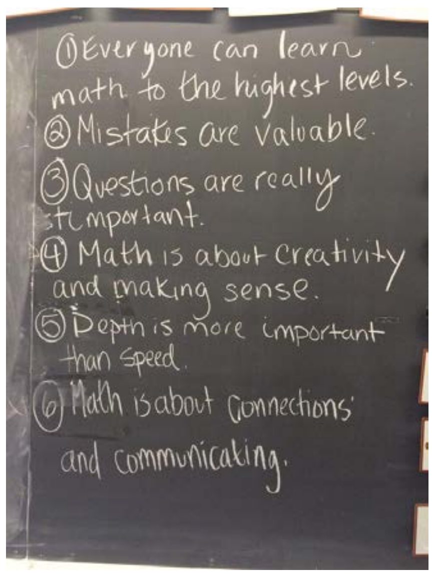


Figure 38. Image demonstrating the idea that all students can learn mathematics.

Although multiple elements relating to education are visible within this image (see Figure 38), the first statement communicates that all students can learn challenging mathematics.

A second code, *gender equity*, related to the role and perception of gender in learning mathematics. For example, one blogger shared thoughts on the effect of gender on learning mathematics following an NCTM conference,

> Calculus I faculty across the country are somehow (unintentionally) hurting students’ confidence in their mathematical ability, which is affecting students taking Calculus II. This happens more frequently for female students.

Although it is unclear what caused the discrepancy in student confidence, students’ opportunities to study advanced mathematics may be affected by gender.

A final code, *mindset,* related to a view of learning mathematics as either fixed or having the ability to improve with effort. For example, consider how a blogger reflected on a set of "habits of mathematicians" posters hanging in his/her classroom,

> I also like think that by seeing them [habits of a mathematician posters] on a daily basis, we all (myself included) are reminded that our capacity for mathematical thinking is not fixed, but rather can increase as we seek out ways to develop it.

This post illustrates the view that both students and educators can grow in capacity for mathematical thinking.

Supporting all students. Supporting all students, the second theme within the category of VSSS, related to content in which strategies intended to help all students engage with mathematics are discussed. Note that this theme also appeared within the teacher role subcategory and consists of a single code. Use of appropriate strategies was one approach discussed to support struggling students. Consider how one blogger described supporting students while implementing a specific task: "When students struggled (or just needed the extra visual or kinesthetic support) I brought out sets of cubes and had students build models." Designing lessons in order to support students was also discussed. For example, one blogger stated, "I have found that by creating more challenges for a student to think through has benefited more of the Not Good at Procedures because they try different things."

Content coded as *supporting all students* was also visible within the interviews. When they were asked if they felt their ability to support the needs of all their learners had improved based on their engagement with the community, only Kathy and Scott replied that they believed it had. Kathy commented that she often accommodated the needs of different learning styles by adapting materials created or shared from the elementary mathematics teachers in the community. She reported borrowing the materials to first, try to get her students up and moving, and second, to have them think about the mathematics differently. Scott listed teaching activities or strategies from the community that he felt reached all students, such as Which One Doesn't Belong? (WODB). WODB has been described as "thought-provoking puzzles for math teachers and students alike. There are not answers provided as there are many different, correct ways of choosing which one doesn't belong" (Bourassa, 2013).

In the interviews, Nancy, Eric, and Sydney all stated that they had received limited support from the community in accommodating the needs of all students. Nancy mentioned that since she knew and understood the needs of her students, she was more likely to only borrow materials or lessons from the community and then alter those to meet her students' needs. Sydney stated that most tasks within the community are targeted towards her more advanced students. She stated,

> There are some people out there doing great work with kids with different types of learning differences but they are not so much at the forefront. We've had a couple of Global Math Meetings about them, but I don't know that I've felt a lot in the way of differentiated resources.

Lastly, Eric commented that he was not aware of community members with experiences accommodating the needs of all learners. Eric recalled that when he first moved to his current district where approximately 95% of the students are on free and reduced lunch, he developed an awareness of the need for materials targeting all students, as stated in the following,

> Like how do I help my students with—how do I help my English Language Learners? You know, what can I do to make the learning more accessible? I felt like that's where I got—I had less responses, no fault to the MathTwitterBlogosphere, but that's where I

think it would be great—I would be extremely grateful if we had more people that could share those experiences.

Conclusion about VSSS. The teacher attribute of VSSS was addressed within 3.07% of the MTBoS community through sharing an equitable view of all learners and supporting all students. An equitable view of all learners was demonstrated through discussions that all students can engage with mathematics. Also, a knowledge of how to support students was visible through appropriate teaching strategies and lesson design.

In the interviews, two of the five community members reported improving their ability to support all learners. Of the remaining three interviewees, two responses indicated that people within community may be doing great work with a range of learners, but these community members are not at the forefront of community conversations.

Miscellaneous. The last category within the framework is miscellaneous; the category provides a greater description of the case under study, the MTBoS community, not related to the components identified a priori. Miscellaneous accounted for 11.71% (N = 351) of the data from Twitter and the blogosphere. Two themes developed from the open coding will be discussed; Table 17 shows their distribution across the two phases of the study.

Online learning community. The first theme within the category of miscellaneous addressed relationships and interactions within the MTBoS community. Three codes were used to build the theme. A first code within the theme was *MTBoS community*, which included discussions of and about the MTBoS community as in the following blog post: "I'm inspired by a lot of the great things I've found in the Math Twitter Blogosphere #MTBoS and a lot of the great folks I've met as part of that community." In other cases, community efforts extended beyond Twitter and the blogosphere. For example, in reference to an MTBoS booth at a National

Table 17

Themes Related to Miscellaneous Category

Themes and Codes	Frequency of Units with Code within Community Interactions	Number of Interviewees included in Code
Online Learning Community		
MTBoS Community	94	2
Personal Engagement in a Learning Community	107	2
Professional Growth	150	3
Total	351	7
Affordances of the MTBoS Community		4

Council of Teachers of Mathematics conference in November 2015, one community member stated, “Thanks to all volunteers who made the #MTBoS booth possible. Was great connecting with you all.”

The *MTBoS community* was also discussed in the interviews. Sydney and Scott both mentioned the like-mindedness visible throughout the MTBoS community. For example, when asked if she felt she benefited from engaging with the MTBoS, Sydney responded,

> I think mostly as a group of like-minded colleagues. Right, so these are people who skew towards progressive education. So it tends to be, you get it, you understand, you understand for instance why I think there are problems with Kahn Academy as a schooling method.

Scott also made comments about the similar philosophies of education that he perceived between himself and other members of the community.

A second code, *personal engagement in a learning community*, related to the personal aspects and interactions within the MTBoS community, but more rarely related to mathematics. For example, consider the following tweet, “A beautiful ceremony at school today. So flattered by the kind words of a student. Brief post at http://bit.ly/1OiCqJS #MTBoS." This tweet illustrates how personal celebrations were shared within the community. Additional discussions demonstrated personal engagement between members of the community. Consider the following tweet, "Burdens are lighter when they're shared, right? Lots of good vibes will be coming your way from the awesome #mtbos community." Lastly, personal engagement with the community also extended beyond Twitter and the blogosphere. For example, "Tweet Up at Brits Pub 5:30 #NCTMregionals all are welcome! #MTBoS 110 Nicollet." Of note, a "Tweet Up" is where people who interact on Twitter gather in person.

Content coded as *personal engagement in a learning community* was also included within the interviews. For example, when asked how he became involved with the MTBoS community, Scott responded,

> So, just an opportunity to, with this retirement, semi-retirement, whatever you want to call it, an opportunity to help out younger teachers. You know, if they post a blog, a lot of people won't have time to read it, I tend to read more of the blogs than if I was a full time educator.

This idea of helping others permeated throughout the interview with Scott. Eric also felt it was important to help others, as stated in the following,

> Especially reply back to math teachers that have questions for me. In other words, like 'Hey Eric, I have used your lesson here, I've used this. Do you have any advice?' And I felt like when I first started using Twitter, one of the most beneficial things that I had, or one of the most beneficial things about using it was people would actually reply during the school day. And so I have a little more flexibility in my schedule these days so when a teacher sends me a notification or Tweet, I'm fortunate that I can get back to them and

> hopefully give them some real-time feedback so that they can apply it to their next class period let's say if they are a single subject teacher. So I really want to give back to the people that are interested and some advice or some feedback that can help benefit them that day or the next day, whatever it might be.

A final code, *professional growth*, related to professional development opportunities within the MTBoS. Several variations of this code were observed. First, conversations from conferences were often extended to the MTBoS community. For example, consider the following tweet, "What actions will you take from this conference back to your Classroom/Team/School/District? #NCTMregionals #MTBoS." This tweet illustrates how a prompt for discussion or action following a conference was observed within the community. Other content referred to professional development presentations or workshops being shared with the MTBoS community. For example, one blogger included the following statement at the start of a blog post, "At #mathsconf15 I delivered a workshop entitled 'Resources for Teaching Maths at AS Level'. I thought it might be helpful to share highlights from that presentation here." Following this statement, the author shared his/her top 10 resources included within that presentation. Other cases included reflections on received professional development or attended conferences. Opportunities for virtual professional development were also discussed, such as Twitter chats, podcasts, and Global Math Department meetings. Of note, Global Math Department meetings are weekly webinars that provide mathematics educators an opportunity to share and discuss teaching practices ("Global Math Department", n.d.). Specific to Twitter chats, most of the interactions included a solicitation for a specific chat. For example, consider the following tweet and corresponding image shown in Figure 39, "This morning[']s #satchat questions about being a #connected educator #CEOct #mtbos Join in '#Satchat Questions are posted. Topic: Connected Educators Month [,] Guest Moderators: @patrickmlarkin @andycinek."

Time:	*Question:*
Q1 7:36	How did you come to be a Connected Educator and who first connected you? #satchat #CEM15
Q2 7:44	How has being a connected educator helped shape or inform your practice? #satchat #CEM15
Q3 7:52	In what ways does your school or district embrace and use connected learning? #satchat #CEM15
Q4 8:00	How do you encourage others to particpate in connected learning? #satchat #CEM15
Q5 8:08	When something WORKS – whether a practice or a product – what's the most effective way to share it w/ educators? #satchat #CEM15
Q6 8:18	Can you share one resource or story which every educator should know about? #satchat #CEM15

Figure 39. Image demonstrating how Twitter chats are advertised within the community.

Responses within the interviews related to opportunities for professional growth within the MTBoS. Two of the five interviewees expressed an appreciation for the conversations that take place within Twitter chats. Scott was more specific in stating that he enjoys slow-chats and book studies hosted on Twitter. A slow-chat discusses one posted question a day, discussions are typically organized around a specified hashtag (Fenton, n.d.); similarly, a book study includes reading and discussing a common book, these discussions are also typically organized around a specified hashtag. Lastly, Sydney mentioned that the Global Math Department has played a large role in how she currently engages with the community.

Affordances of the MTBoS community. Affordances of the MTBoS community, a second theme within the subcategory of miscellaneous, is related to the perceived benefits or affordances of engaging with the MTBoS community beyond those discussed within the previous categories and included only one code. The convenience of the MTBoS was discussed as a benefit. Kathy mentioned that she appreciated the short length of posts on Twitter and that it fit into her life on a regular basis. Similarly, Nancy commented that the community is free and does not require travel. An additional aspect of convenience includes timely responses on questions or requested feedback. Both Kathy and Eric discussed the responsive nature of the community. For example, Eric commented that he has a good following on Twitter and when he posts a question or puts something out there, he will at least have a couple of people respond.

A second benefit of engaging with the community included "stirring excitement" for teaching, as discussed by Nancy in the following:

> It does get me excited about teaching, which is maybe the best thing it could do right, when I'm reading other people's ideas and thoughts, it makes me glad to be teacher and excited to go do it again tomorrow.

A final benefit of the MTBoS related to staying connected with current events in mathematics education. Nancy stated that she enjoyed thinking and reading about mathematics education and because of this enjoyment, the community provides her with a sense of personal satisfaction. Similarly, Kathy mentioned that since she is not reading mathematics education books or magazines, the MTBoS supplements her lack of engagement in current literature. Lastly, Sydney stated that engaging in the community, specifically Twitter, was her filter to the internet. She commented that since she does not have time to read every blog post published, she trusts that if something relevant is being discussed elsewhere in the community, it would also be shared on Twitter.

Conclusion about miscellaneous. The miscellaneous category, based on content not included in other categories, included discussions of and about the interactions within the

MTBoS community. Discussions within the theme included personal stories, connections between members of the community, and opportunities for professional growth.

Within the interviews, all participants reported that they benefited from engaging with the MTBoS community. The most commonly discussed benefit was the convenience of the MTBOS community. Additional affordances included the MTBoS acting as an inspiration for teaching and as a source of literature on mathematics education.

Conclusion

All four categories were addressed within the content of the MTBoS community. First, within MKT, the knowledge needed to teach mathematics was both shared and requests for such knowledge were modeled, most commonly as related to a knowledge of the curriculum. Second, within VHQMI, classroom discourse and mathematical tasks were addressed within similar proportions of the content; classroom discourse, primarily included reports and reflections on discourse and mathematical tasks included 186 mathematics tasks and the details needed to implement those tasks. Third, within VSSS, an equitable view of all learners was discussed through a commitment to all students learning challenging mathematics, regardless of perceived identity or gender. Lastly, miscellaneous provided a greater understanding of the community under study, the MTBoS. Of the four categories, VHQMI was addressed most frequently, while VSSS was addressed least frequently.

In the interviews, a minimum of two interviewees perceived improvements for each of the three teacher attributes. More specifically, as related to mathematical knowledge for teaching, while three of the interviewees reported improving their subject matter knowledge, those who reported improved pedagogical content knowledge did not connect their improved teaching practices with specific mathematics concepts. At a later point in the interviews, two of the participants discussed learning concept-specific teaching practices from the community, related to pedagogical content knowledge. With respect to VHQMI, three interviewees reported improved classroom discourse practices, three interviewees reported improved task implantation, and one interviewee reported improved assessment practices. For VSSS, only two interviewees felt the community has supported their ability to address the needs of all learners. Lastly, all five interviewees felt they benefited from engaging with the MTBoS community.

Chapter 5: Conclusions and Implications

In this study, I set out to determine if engaging with the MTBoS community supports the development of teacher attributes needed to effectively select and implement cognitively demanding tasks. In this chapter, I report the limitations and conclusions of the study, as well as implications of the study for various audiences, including mathematics teachers, mathematics teacher educators, and MTBoS community members. Lastly, I provide suggestions for future studies.

Limitations

There were three main limitations to this study. First, during a given time period, all tweets including the hashtag #MTBoS were archived for analysis. In the interviews, the community members perceived the hashtag #MTBoS as being primarily used to ask a question, share a curriculum resource, or solicit feedback on an activity. As community interactions may extend beyond sharing resources and requesting help, these additional interactions were not captured in the data. For example, instead of using the hashtag #MTBoS, members may tweet to specific community members using Twitter handles (synonymous with a username). Or they may use a different hashtag that is also accessed by members of the community.

A second limitation was that interactions within the community were not purposefully captured, only individual communications. Individual tweets including hashtag #MTBoS were collected without regard to whether it was a part of a larger chain of related tweets. Similarly, the comments of the blogs were not collected. Viewing all related tweets or comments within a community conversation would have been useful in better understanding how the community responded to or discussed topics within mathematics education.

A final limitation was the sample size. Only two weeks of community interactions were collected, one of which occurred during a large mathematics education conference. Similarly, only five community members were interviewed; of the five participants, only one was included in the middle stratum of participation, between 201 and 900 Twitter followers. Increasing both, the number of collected weeks of community interactions and the number of interviewed participants could have provided additional insights into the consistency of community interactions and community member perceptions.

Conclusions

This study was designed to answer the general question of does engagement with the MathTwitterBlogosphere support the development of teacher attributes supporting effective use of cognitively demanding tasks? Two subquestions considered (1) how specified teacher attributes were addressed within the content of the community and (2) how members of the community perceived engaging with the MTBoS supported their development of specified teacher attributes. I begin by presenting my conclusions for the two subquestions, followed by overall conclusions to the general research question.

Subquestion 1: Community content. The first research subquestion asked, In what ways, if any, does the content of the MathTwitterBlogosphere address teacher attributes needed to select and implement cognitively demanding tasks, such as, but not limited to, mathematical knowledge for teaching, visions of high quality mathematics instruction, and views for supporting struggling students? In the following sections, I discuss findings specific to each teacher attribute.

Mathematical knowledge for teaching. Mathematical knowledge for teaching (MKT) was addressed within the content of the MTBoS community. First, just over a quarter of the data within MKT addressed knowledge that was purely mathematical in nature, or subject matter knowledge. Within this content, community members demonstrated knowledge of, and beyond that of, any well-educated adult by sharing mathematics knowledge, doing mathematics, and thinking about mathematics. For example, a community member shared an explanation of how standard deviation and Pythagorean distances are related. Although mathematics tasks were rarely visible within this content, increasing subject matter knowledge alone improves one's ability to maintain the cognitive demand of a task (Wilhelm, 2014); "Teachers' CKTM [content knowledge for teaching mathematics] is integral to their decision making during task implementation" (p. 663).

Second, a majority of the community interactions within MKT addressed the mathematics knowledge most closely connected to instruction, or pedagogical content knowledge. Knowledge of curriculum was primarily demonstrated through sharing of recommended curriculum resources and planning the curriculum, both of which often related to cognitively demanding tasks. For example, a tweet used to demonstrate how resources were shared within the community included a hyperlink to Desmos specific explorations. The central importance of sharing resources was commonly reported in the research about other online communities (Duncan-Howell, 2010; Hur & Brush, 2009; LaLonde, 2011; Deyamport, 2013; Luehmann, 2008).

Within pedagogical content knowledge, knowledge of student thinking was often demonstrated, including how students may think about a particular mathematics topic or solving a particular mathematics task. Such knowledge of student thinking is key in selecting and implementing cognitively demanding tasks; doing so “requires an interaction between specific mathematical understanding and familiarity with students and their mathematical thinking” (Ball et al., 2008). In selecting a task, knowledge of students is used to predict what will be of interest, motivation, and an appropriate level of difficulty for those students (Ball et al., 2008). In implementing a task, knowledge of students allows a teacher within classroom discourse to interpret any emerging or incomplete thinking (Ball et al., 2008).

Visions of high quality mathematics instruction. Visions of high quality mathematics instruction (VHQMI) was clearly addressed within the content of the MTBoS community. Teacher role, classroom discourse, and mathematical task combine to form visions of high quality mathematics instruction (Munter, 2014). The teacher’s role in high quality mathematics instruction was discussed, primarily as facilitator of instruction. Within this content, the teachers’ role was primarily observed as facilitator during instruction. For example, the teacher was often positioned to support students with the additional information needed to solve a three-act mathematics tasks.

Content related to classroom discourse primarily focused on teacher questions and student thinking. For example, one blog post included the planning of teacher questions for a given task. Careful planning of teacher questions is key as these questions either help maintain or decline the cognitive demand of an implemented mathematics task (Son & Kim, 2015). Students’ written responses to prompts or tasks were also common within classroom discourse, with many of them including justifications to support their mathematical thinking. Tasks requiring students to justify their own thinking is key in ensuring students have opportunities to reason and make sense of the mathematics (Stein et al., 1996).

The content related to mathematical tasks primarily included the mathematics tasks visible in the community; of these tasks, almost 85% were identified as requiring high-levels of cognitive demand. This finding is significant because in order for a task to be enacted with high cognitive demand, the task must start with a high level of cognitive demand (Stein & Lane, 1996; Stein et al., Boston & Smith, 2009; NCTM, 1991). Similarly, even if tasks begin as cognitively demanding, but decline in cognitive demand during implementation, research has still reported moderate gains in student achievement (Stein & Lane, 1996). In conclusion, all three aspects of VHQMI were addressed, reported by Wilhelm (2014) as having a significant impact on a teacher's selection and implementation of cognitively demanding tasks.

Views for supporting struggling students. Views for supporting struggling students (VSSS) were addressed sparingly within the content of the MTBoS community. The content related to VSSS primarily addressed having an equitable view of all learners. For example, one included image communicated that all students can learn mathematics to the highest level. Although additional instances of this view were sparse, a productive view of all students enhances a teacher's ability to select and maintain the cognitive demand of tasks (Wilhelm, 2014).

Conclusions about subquestion 1. In answering the first research subquestion, the content of the MTBoS addressed mathematical knowledge for teaching and visions of high quality mathematics instruction. In contrast, the content of the MTBoS addressed views for supporting struggling students at a lower level. In conclusion, the content of the MTBoS suggests that it may have promise in addressing the attributes teachers need to have in order to effectively implement high-cognitive demand tasks.

Subquestion 2: Community member interviews. The second research subquestion asked, How do members of the MathTwitterBlogosphere community perceive the effects of the MathTwitterBlogosphere on their development of teacher attributes needed to select and implement cognitively demanding tasks, such as, but not limited to, mathematical knowledge for teaching, visions of high quality mathematics instruction, and views for supporting struggling students? In the following sections, I again discuss findings specific to each teacher attribute.

Mathematical knowledge for teaching. A majority of community members perceived the effects of the MTBoS to have strengthened their development of mathematical knowledge for teaching. Reading mathematics related blog posts was the only common method of improving content knowledge reported among the interviewees. In contrast, one interviewee stated that she had not developed content knowledge because she preferred to read content about teaching mathematics versus pure mathematics; this echoes the observation of Sun et al. (2014) that when colleagues sought advice from their peers, they were less likely to discuss how to solve mathematics problems as compared to discussing instructional practices. Interviewees were also asked about how the community supports development of pedagogical content knowledge. Responses mainly focused on discussions of improved teaching practices without clear connections to particular mathematics concepts, although two responses addressed the development of concept-specific teaching practices. Overall, there is evidence that engagement in the community can improve mathematics knowledge for teaching.

Visions of high quality mathematics instruction. A majority of community members perceived that the MTBoS strengthened their development of visions of high quality mathematics instruction. They described their development related to mathematical tasks, classroom discourse, and more general instructional practices. However, when interviewees were asked how they would describe the tasks within the community, none referenced the cognitive demand of a task. Instead, tasks were described as classroom ready, peer-recommended, crowd-sourced, teacher-created, and teacher-adapted. This finding is not completely surprising as Arbaugh and Brown (2002) also found that teachers do not typically examine tasks for cognitive demand, but instead focus on more superficial characteristics. A majority also reported improvements in task implementation; one mentioned that he found community members' reflection on the implementation of tasks within blog posts particularly helpful. An additional interviewee reported no change but had a "sense of vindication" as his philosophy of education already aligned with that of the community as a whole. Interviewees also discussed the impact of the community on their development of classroom discourse practices, including teacher questioning, wait time, and teacher listening. Finally, interviewees discussed how engagement with the community has helped improve other aspects of their classroom instruction. For example, one interviewee discussed improvements in how she provides feedback to students on quizzes.

Views for supporting struggling students. A majority of the community members perceived that participation in the MTBoS has only minimally impacted their ability to support struggling students. Three of the interviewees recalled few, if any, community members who share materials targeted for a range of learners; in contrast, two of the interviewees felt the community had helped develop teaching practices that would support all learners.

Conclusion about subquestion 2. In answering the second research subquestion, a majority of the community members perceived engaging with the MTBoS to support their development of mathematical knowledge for teaching and visions of high quality mathematics instruction. In contrast, community members perceived the MTBoS to have provided less support for their development of views for supporting struggling students. In conclusion, responses from the MTBoS community members suggest that it may be helpful in supporting the development of attributes teachers need in order to effectively implement high-cognitive demand tasks.

Other findings. A number of observations based on both the community content and the interviews provide additional insights into the MTBoS community itself. First, there were a number of comments related directly to the MTBoS community and the personal nature of the MTBoS. In other cases, opportunities for professional growth were observed within the community interactions. These opportunities may have included discussions following a conference, or organized discussions within Twitter, such as Twitter chats. Within the conducted interviews, community members also described the affordances of engaging with the MTBoS, including the convenience of the community, a source of inspiration for teaching, and viewing the content of the community as a reliable source of mathematics education literature. All interviewees spoke positively about their experiences of engaging with the MTBoS community.

Second, the like-mindedness of the MTBoS community was visible in both the community content and interview responses. A common teaching philosophy was largely visible across the community content, namely teaching mathematics for understanding by allowing students to develop their own knowledge. This was especially evident within the theme of underlying teaching philosophy, found within the categories of MKT and VHQMI. Similarly, three interviewees mentioned the like-mindedness visible throughout the community. Research completed by Wilhelm and colleagues (2016) and Gamoran and colleagues (2000) showed that teachers who teach for understanding and in inquiry-oriented manners had a greater desire to improve instruction, and thus were more likely to seek advice from their colleagues when compared to those who teach in more traditional methods. From this, it might be hypothesized that because many members of the MTBoS community teach for understanding through inquiry-oriented methods, they sought out and participated in the MTBoS community as a source of advice and resources to support their efforts to improve instruction.

Conclusions about the overall research question. The overarching research question asked, does engagement with the MathTwitterBlogosphere support the development of teacher attributes supporting effective use of cognitively demanding tasks? Data from both phases of the study lead to a common conclusion: the MTBoS community provides support in developing both mathematical knowledge for teaching and visions of high quality mathematics instruction, but less support for developing views for supporting struggling students. Thus, the MTBoS community is one possible avenue that teachers may explore to receive support in selecting and implementing cognitively demanding tasks within the mathematics classroom.

Implications

The results from this study can inform mathematics teachers, mathematics teacher educators, and MTBoS community members. The results of this study also provide direction for future research.

Mathematics teachers. As mathematics teachers consider the results of this study, they might consider engaging with the MathTwitterBlogosphere to support their development of subject matter knowledge, ability to select and implement cognitively demanding tasks, and development of professionalism practices.

Subject matter knowledge. The MTBoS community might be a good source to develop mathematics teachers' subject matter knowledge. However, to develop subject matter knowledge, mathematics teachers need to intentionally engage with the mathematics content of the MTBoS community, unlike one interviewee who steered away from that content. More specifically, teachers might read mathematics related blog posts, attend mathematics focused Global Math Department meetings, and explore Desmos activities. Developing mathematics content knowledge is important because the manner in which a mathematics teacher reasons through mathematics content themselves will be reminiscent of how they work with their own students on related concepts (Charlambous, 2010). Furthermore, a high level of subject matter knowledge is key to improving mathematics teachers' ability to maintain the cognitive demand of a task (Wilhelm, 2014).

Cognitively demanding tasks. The MTBoS community might be a good source for both cognitively demanding tasks and discussions and reflections on how to implement cognitively demanding tasks, given the prevalence of such tasks in the community. Implementing and engaging with the tasks from the MTBoS community can support student understanding of mathematics, since the greatest factor in building their understanding is the cognitive demand of an enacted task (Stein & Lane, 1995; Stein et al., 1996; Boston & Smith, 2009).

Professionalism. Engaging with the MTBoS community might provide opportunities to develop professional relationships with other mathematics educators. All interviewees alluded to the community aspect of the MTBoS. This sense of community included, building relationships within the MTBoS, having a reliable group of community members to provide feedback on created activities or lessons, and the like-mindedness in philosophy of teaching mathematics. *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014) clearly states that teachers should not work in isolation. As such, engaging with the MTBoS community is one-way mathematics educators may interact with other teachers. Furthermore, when mathematics educators teach mathematics for understanding, they are more likely to seek and need a supportive environment with like-minded colleagues (Wilhelm et al., 2016; Gamoran, Secada, & Marrett, 2000). A community similar to the MTBoS can be particularly convenient and helpful.

Mathematics teacher educators. As mathematics teacher educators, including professional development providers, consider the results of this study, they might consider engaging with the MTBoS community as a source for cognitively demanding tasks and to help inform the focus of their work; they may also encourage their students to participate in the MTBoS community. Lastly, mathematics teacher educators should ensure their students or participants develop equity-oriented dispositions related to teaching and learning mathematics, since these issues seem to emerge less naturally within community conversations.

Cognitively demanding tasks. The MTBoS community might be a good source for cognitively demanding tasks. As a majority of the mathematics tasks within the MTBoS were identified as cognitively demanding, mathematics teacher educators need to consider using the mathematics tasks from the MTBoS to help develop their students' curricular knowledge. Similarly, mathematics teacher educators might consider encouraging their students to use the tasks from within the MTBoS community in their own teaching. Lastly, mathematics teacher educators should use the tasks within the MTBoS to prompt discussions of best practices related to the selection and implementation of cognitively demanding tasks. It is clear such discussions are needed as the interviewees did not discuss the cognitive demand of a task when asked to classify the tasks from within the community.

Current focus of practicing classroom teachers. The MTBoS community might be a good source to learn of inservice teachers' current concerns and interests. The collected blog posts and tweets included mathematics educators' day-to-day reflections, concerns, questions, conversations, and activities. Similarly, multiple interviewees described a reliance on the community to keep them informed of current developments and issues within mathematics education. Mathematics teacher educators might engage with the community to determine the current focus of inservice mathematics teachers. Understanding the current concerns and focus of inservice teachers will ensure teacher preparation and professional development remains connected with the current needs of the mathematics classroom.

Encourage students to engage with the MTBoS community. The MTBoS might serve as a helpful resource and community for mathematics teacher educators to share with their students. One reason is because the philosophy of teaching visible within the content of the MTBoS community aligns well with many of the goals discussed in research supported documents, such as *Principles to Actions: Ensuring Mathematics Success for All* (NCTM, 2014). Mathematics teacher educators may also encourage students to participate with the community as it has been described to positively affect the teaching attributes needed to effectively enact cognitively demanding tasks. Lastly, mathematics teacher educators may also encourage students to participate with the community as it has been reported to serve as a source of inspiration and encouragement for those teaching mathematics.

Equity-oriented dispositions. Engaging with the MTBoS community may not provide opportunities for community members to develop equity-oriented dispositions related to mathematics education. My findings allowed me to imply that opportunities to support all students learning of mathematics were minimal within the MTBoS community. From this, mathematics teacher educators must ensure their students and participants are supported in developing the needed dispositions to support each and every student to learn mathematics within teacher education.

MTBoS community members. As MTBoS community members consider the results of this study, they should feel proud of their efforts and contributions positively affecting students learning of mathematics. In Jackson et al. (2013), only 59% of the tasks teachers selected were identified as having the potential to elicit high levels of cognitive demand; in contrast, almost 85% of the tasks discussed and shared within the MTBoS had this same potential. However, members of the MTBoS community might also consider more intentionally engaging with or producing content related to supporting the learning of mathematics by all students, which was not found to be well addressed within the community. When sharing or describing a mathematics task within a blog post, members of the community should find ways to explicitly discuss how all students may be supported in engaging with that task. Similarly, when reflecting on the implementation of a task within a blog post, future supports for students who may have struggled in their learning should be discussed. Lastly, when providing others with feedback on a activity or task, community members should challenge one another to ensure the activity or task allows each and every student to learn the intended mathematics.

Future research. While this study provided potentially useful information about how an online community may support the development of certain teacher attributes, a number of extensions might be helpful. First, future studies might consider whether engaging with the community actually supported the development of teacher attributes supporting effective enactment of cognitively demanding tasks beyond self-reported data. The instruments discussed by Wilhelm (2014) and Munter (2014) might be used to determine members' achievement of MKT, VHQMI, and VSSS, which would help strengthen the understanding of the development of each teacher attribute. Completing classroom observations of community members would also help determine if members of the community are not only selecting, but maintaining the cognitive demand of tasks from within the community.

An additional series of research studies could further examine content within Twitter and the blogosphere. First, what content of the MTBoS community receives the greatest community response? For instance, what is the content of the 10 most favorited tweets across a week long period? Similarly, what is the content of the blog posts with the greatest number of comments? Second, what is the content of other hashtags commonly accessed and used by members of the MTBoS community? For example, are other hashtags used when discussing specific topics within mathematics education? Lastly, tracking and analyzing interactions within the community might further inform our understanding of how the MTBoS community responds to or discusses topics within mathematics education. For instance, around what topics do interactions and discussions tend to occur? Are some members of the community particularly influential? To help explore these issues, a procedure used by Sun and colleagues (2014) might be helpful; members of the MTBoS would be asked to list the 10 community members they turn to for advice about teaching mathematics.

In conclusion, I have found that engaging with the MTBoS supports teachers in their development of mathematical knowledge for teaching and visions of high quality mathematics instruction. However, the MTBoS provides less support for teachers in their development of views supporting struggling students. Thus, the MTBoS is one possible community teachers may explore to receive support in selecting and implementing cognitively demanding tasks. The MTBoS has much potential as a resource for both locating cognitively demanding tasks and discussing how to implement cognitively demanding tasks. Given the cognitive demand of a task is the greatest factor affecting students' understanding of mathematics (Stein & Lane, 1996;

Stein et al., 1996; Boston & Smith, 2009), engaging with the MathTwitterBlogosphere is a great way to ensure mathematics educators are supported in teaching each and every student mathematics. Additionally, participating in the MTBoS provides an effective alternative to more formalized professional development, engaging with the MTBoS community is both convenient and cost effective. Following this study, further investigation is needed to determine how a larger population of mathematics teachers may benefit from participating in similar communities, and thus, positively affect their students learning. Further investigation is also needed in how to prompt or engage mathematics teachers in conversations related to supporting all student learn mathematics.

References

ACT. (2013). *ACT profile report: Graduating class 2013 – National.* Iowa City, IA: Author.

Agarwal, A. (2015, October 21). How to save tweets for any Twitter hashtag in a Google sheet [Web log post]. Retrieved from http://www.labnol.org/internet/save-twitter-hashtag-tweets/6505/

Arbaugh, F., & Brown, C. (2005). Analyzing mathematical tasks: A catalyst for change? *Journal of Mathematics Teacher Education, 8*, 499-536.

Arbaugh, F., & Brown, C. A. (2002). *Influences of the mathematical tasks framework on high school mathematics teachers' knowledge, thinking, and teaching*. Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, LA.

Ball, D. L., & Bass, H. (2009). *With an eye on the mathematical horizon: Knowing mathematics for teaching to learners' mathematical futures*. Paper prepared based on the keynote address at the 43rd Jahrestagung der Gesellschaft fur Didaktik der Mathematik, Oldenburg, Germany.

Ball, D. L., & Forzani, F. (2011). Building a common core for learning to teach and connecting professional learning to practice. *American Educator, 35*(2), 17-21.

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who know mathematics well enough to teach third grade, and how can we decide?. *American Educator, 29*(1), 14-16, 20-22, 43-46.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 398-407.

Berg, B. & Lune, H. (2012). *Qualitative research methods for the social sciences* (8th ed.). Pearson: Boston, MA.

Blitz, C. L. (2013). *Can online learning communities achieve the goals of traditional professional learning communities? What the literature says*. Washington, DC: Regional Educational Laboratory Mid-Atlantic.

Blogosphere. (n.d.). Retrieved from http://www.merriam-webster.com/dictionary/blogosphere

Boaler, J. (2002). *Experiencing school mathematics: Traditional and reform approaches to teaching and their impact on student learning*: Lawrence Erlbaum Associates, Inc.

Boaler, J., & Staples, M. (2008). Creating mathematical futures through an equitable teaching approach: The case of Railside school. *Teachers College Record, 110*(3), 608-645.

Boston, M. D., & Smith, M. S. (2009). Transforming secondary mathematics teaching: Increasing the cognitive demands of instructional tasks used in teachers' classrooms. *Journal for Research in Mathematics Education, 40*(2), 119-156.

Boston, M. D., & Smith, M. S. (2011). A 'task-centric approach' to professional development: Enhancing and sustaining mathematics teachers' ability to implement cognitively challenging mathematical tasks. *The International Journal on Mathematics Education, 43*(6-7), 965-977.

Boston, M. D., & Wolf, M. K. (2006). *Assessing academic rigor in mathematics instruction: The development of the instructional quality assessment toolkit* (CSE Technical Report 672). Retrieved from http://cse.ucla.edu/products/reports/r669.pdf

Bourassa, M. (2013). Which one doesn’t belong?. Retrieved from wodb.ca

Callnote premium [Computer Software]. (n.d.). Retrieved from http://www.kandasoft.com/home/kanda-apps/skype-call-recorder.html

Cardone, T. (2015). *Nix the tricks: A guide to avoiding shortcuts that cut out math concept development* (*2nd ed.).* Retrieved from nixthetricks.com

Charalambous, C. Y. (2010). Mathematical knowledge for teaching and task unfolding: An exploratory study. *The Elementary School Journal*, *110*(3), 247-278.

Clarke, D., & Hollingsworth, H. (2002). Elaborating a model of teacher professional growth. *Teaching and Teacher Education*, *18*(8), 947-967.

College Board. (2013). *2013 SAT report on college and career readiness*. New York, NY: Author.

Cope, B., Kalantzis, M., & Lankshear, C. (2005). Bill Cope, Mary Kalantzis, and Colin Lankshear. A contemporary project: An interview. *E-Learning*, *2*(2), 192-207.

Couros, A. (2010). Developing personal learning networks for open and social learning. In G. Veletsianos (Ed.), *Emerging technologies in distance education* (pp. 109-128). Edmonton, AB: Athabasca University Press.

Creswell, J.W. (2013). *Qualitative inquiry and research design: Choosing among five approaches.* Thousand Oaks, CA: Sage Publications, Inc.

Darling-Hammond, L., Wei, R. C., Andree, A., Richardson, N., & Orphanos, S. (2009). *Professional learning in the learning profession*. Washington, DC: National Staff Development Council.

Desmos. (n.d.). Retrieved from www. desmos.com

Deyamport, W. H. (2013). *An implementation of a Twitter-supported personal learning network to individualize teacher professional development* (Doctoral dissertation). Retrieved from ProQest Dissertations and Theses database. (Order No. 3559871)

Doyle, W. (1983). Academic work. *Review of Educational Research*, *53*(2), 159-199.

Doyle, W. (1988). The context of students' thinking during instruction. *Educational Psychologist*, *23*(2), 167-180.

DuFour. (2004). What is a professional learning community? *Schools as Learning Communities, 61*(8), 6-11.

Duncan-Howell, J. A. (2009). Online professional communities: Understanding the effects of membership on teacher practice. *The International Journal of Learning, 16*(5), 601-613.

Duncan-Howell, J. A. (2010). Teachers making connections: Online communities as a source of professional learning. *British Journal of Educational Technology, 41*(2), 324-340.

Exploring the MathTwitterBlogosphere. (2015). Retrieved from https://exploremtbos.wordpress.com/

Fenton, M. (n.d.). Join the conversation: #slowmathchat. Retrieved from http://reasonandwonder.com/slowmathchat/

Flanigan, R. L. (2011, October 24). Professional learning networks taking off [Web log post]. Retrieved from http://www.edweek.org/ew/articles/2011/10/26/09edtech-network.h31.html

Forehand, M. (2005*). Bloom's taxonomy: Original and revised*. In M. Orey (Ed.), Emerging perspectives on learning, teaching, and technology. Retrieved from http://epltt.coe.uga.edu/index.php?title=Bloom%27s_Taxonomy

Friese, S. (2013). *ATLAS.ti 7 user guide and reference*. Berlin, Germany: ATLAS.ti Scientific Software Development GmbH.

Gamoran, A., Secada, W. G., & Marrett, C. B. (2000). The organizational context of teaching and learning: Changing theoretical perspectives. In M. T. Hallinan (Ed.), *Handbook of the sociology of education* (pp. 37-59). New York, NY: Kluwer Academic/ Plenum Publishers.

Geogebra. (n.d.). Retrieved from www.geogebra.org

Global math department. (n.d.). Retrieved from https://www.bigmarker.com/communities/GlobalMathDept/about

Guba, E. G. (1981). Criteria for assessing the trustworthiness of naturalistic inquiries. *Educational Communication and Technology*, *29*(2), 75-91.

Guba, E. G., & Lincoln, Y. S. (1981). *Effective evaluation: Improving the usefulness of evaluation results through responsive and naturalistic approaches*. Jossey-Bass: San Francisco, CA.

Hammerness, K. (2001). Teachers' visions: The role of personal ideals in school reform. *Journal of Educational Change, 2*, 143-163.

Hiebert, J., & Stigler, J. W. (2000). A proposal for improving classroom teaching: Lessons from TIMSS video study. *The Elementary School Journal, 101*(1), 3-20.

Hiebert, J., & Wearne, D. (2003). Developing understanding through problem solving. In H. L. Schoen & R. I. Charles (Eds.), *Teaching mathematics through problem solving: Grades 6-12* (pp. 3-14). Reston, VA: National Council of Teachers of Mathematics.

Hill, H. C. (2007). Mathematical knowledge of middle school teachers: Implications for the No Child Left Behind policy. *Educational Evaluation and Policy Analysis, 229*(2), 95-114.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372-400.

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching the mathematical quality of instruction: An exploratory study. *Cognition and Instruction, 26*(4), 430-511.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406.

Hirsh, S. (2009). A new definition. *Journal of Staff Development*, *30*(4), 10-16.

Hord, S. M. (2008). Evolution of the professional learning community. *National Staff Development Council*, *29*(3), 10-13.

Horn, I. S., Little, J. W. (2010). Attending to problems of practice: Routines and resources for professional learning in teachers' workplace interactions. *American Educational Research Journal*, *47*(1), 181-217.

Hoy, A. W., Davis, H., & Pape, S. J. (2006). Teacher Knowledge and Beliefs. In P. A. Alexander & P. H. Winne (Eds.), *Handbook of educational psychology* (Second ed.) (pp.715-737). Mahway, NJ: Lawrence Erlbaum Associates, Inc., Publishers.

Hsu, H., & Silver, E., A. (2014). Cognitive complexity of mathematics instructional tasks in a Taiwanese classroom: An examination of task sources. *Journal for Research in Mathematics Education*, *45*(4), 460-496.

Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of math-talk learning community. *Journal for Research in Mathematics Education*, *35*(2), 81-116.

Hur, J. W., & Brush, T. A. (2009). Teacher participation in online communities: Why do teachers want to participate in self-generated online communities of K-12 teachers?. *Journal of Research on Technology in Education, 41*(3), 279-303.

Jackson, K., Garrison, A., Wilson, J., Gibbons, L., & Shahan, E. (2013). Exploring relationships between setting up complex tasks and opportunities to learn in concluding whole-class discussions in middle-grades mathematics instruction. *Journal for Research in Mathematics Education, 44*(4), 646-682.

Jackson, K., Gibbons, L., & Dunlap, C. (in press). Teachers' views of students' mathematical capabilities: A challenge for accomplishing ambitious reform. *Teachers College Record.*

Johnson, R. (2015, December 3). Mathematics educators on Twitter [Web log post]. Retrieved from mathed.net/wiki/Mathematics_educators_on_Twitter

Kentucky Department of Education. (2007). *Support materials for core content for assessment: Version 4.1 mathematics*. Frankfort, KY: Authors

Knowledgehook. (n.d.). Retrieved from https://www.knowledgehook.com/

Koestler, C., Felton, M., Bieda, K. N., & Otten, S. (2013). *Connecting the NCTM process standards and the CCSSM practices*. Reston, VA: National Council of Teachers of Mathematics.

Krathwhol, D. R. (2002). A revision of bloom's taxonomy: An overview. *Theory into Practice, 41*(4), 212-218.

Kvale, S., & Brinkmann, S. (2009). *Interviews: Learning the craft of qualitative research interviewing*. Thousand Oaks, CA: SAGE Publications.

LaLonde, C. (2011). *The Twitter experience: The role of Twitter in the formation and maintenance of personal learning networks* (Master's thesis, Royal Roads University). Retrieved from dspace.royalroads.ca/docs/handle/10170/451

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. New York, NY: Cambridge University Press.

Lieberman, A., & Mace, D. P. (2010). Making practice public: Teacher learning in the 21st century. *Journal of Teacher Education*. *61*(1-2), 77-88.

Luehmann, A. L. (2008). Using blogging in support of teacher professional identity development: A case study. *The Journal of Learning Sciences*, *17*, 287-337.

Luehmann, A. L. (2011a). Reform-minded teaching: Goals and challenges. In A. L. Luehmann & R. Borasi (Eds.), *Blogging as change: Transforming science & math education through new media literacies* (pp. 163-171). New York, NY: Peter Lang Publishing Inc.

Luehmann, A. L. (2011b). The ways we studied blogging. In A. L. Luehmann & R. Borasi (Eds.), *Blogging as change: Transforming science & math education through new media literacies* (pp. 33-59). New York, NY: Peter Lang Publishing Inc.

Luehmann, A. L., & Borasi, R. (Eds.). (2011). *Blogging as change: Transforming science & math education through new media literacies*. New York, NY: Peter Lang Publishing, Inc.

Luehmann, A. L., & Tinelli, L. (2011). Supporting practicing science teachers' growth: The case of blogging in the context of an action research seminar. In A. L. Luehmann & R. Borasi (Eds.), *Blogging as change: Transforming science & math education through new media literacies* (pp. 206-235). New York, NY: Peter Lang Publishing Inc.

Matsuura, R., Sword, S., Piecham, M. B., Stevens, G., & Cuoco, A. (2013). Mathematical habits of mind for teaching: Using language in algebra classrooms. *The Mathematics Enthusiast, 10*(3), 735-776.

McCulloch, J., McIntosh, E., & Barrett, T. (2011). *Tweeting for teachers: How can social media support teacher professional development?* Retrieved from https://research.pearson.com/content/plc/prkc/uk/open-ideas/en/articles/teachers-on-twitter/_jcr_content/par/articledownloadcompo/file.res/Tweeting%20for%20teachers.pdf

McMahon, D. (2015, December). Twitter. Retrieved from http://whatis.techtarget.com/definition/Twitter

Merchant, G. (2009). Web 2.0, new literacies, and the idea of learning through participation. *English Teaching: Practice and Critique*, *8*(3), 107-122.

Meyer, D. (2011, May 11). The three acts of a mathematical story [Web log post]. Retrieved from http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/

Milou, E. (n.d.). What is MTBoS?. Retrieved from www.ericmilou.net

MTBoS directory (n.d.). Retrieved from https://sites.google.com/site/mtbosdirectory/

Mullis, I. V., & Martin, M. O (Eds.). (2013). *TIMSS 2015 assessment framework*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center.

Munter, C. (2014). Developing visions of high-quality mathematics instruction. *Journal for Research in Mathematics Education*, *5*(45), 584-635.

Nathan, M. J., & Koedinger, K. R. (2000). Teachers' and researchers' beliefs about the development of algebraic reasoning. *Journal for Research in Mathematics Education*, *31*(2), 168-190.

Nardi, B., Schiano, D. J., & Gumbrecht, M. (2004). *Blogging as social activity, or, would you let 900 million people read your diary?* Paper presented at the Computer Supported Collaborative Learning, Chicago, IL.

National Assessment Governing Board. (2013). *Mathematics framework for the 2013 national assessment of educational progress.* Washington, DC: Author.

National Commission on Excellence in Education. (1983). A nation at risk: The imperative for educational reform. *The Elementary School Journal*, *84*(2), 113-130.

National Council of Teachers of Mathematics. (1980) *An agenda for action*. Reston, VA: Author.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (1991) *Professional standards for teaching mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (1995). *Assessment standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2006) *Curriculum focal points for prekindergarten through grade 8 mathematics: A quest for coherence*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2009). *Focus in High School Mathematics: Reasoning and Sense Making*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Washington, DC: Authors.

National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education*. Washington, DC: The National Academy Press.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: The National Academy Press.

Peardeck. (n.d.). Retrieved from www.peardeck.com

Peterson, P. L. (1988). Teaching for higher order thinking in mathematics: The challenge for the next decade. In D. A. Grouws & T. J. Cooney (Eds.), *Perspectives on research on effective mathematics teaching* (pp. 2-26). Reston, VA: Erlbaum.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257-315). Reston, VA: National Council Teachers of Mathematics.

Porter, A. C. (2002). Measuring the content of instruction: Uses in research and practice. *Educational Researcher, 31*(7), 3-14.

Profiles of math teachers who blog and twitter. (n.d.). Retrieved from mathtwitterblogosphere.weebly.com

Raudenbush, S. W., Rowan, B., & Cheong, Y. F. (1993). Higher order instructional goals in secondary schools: Class, teacher, and school influences. *American Educational Research Journal, 30*(3), 523-553.

Raymond, A. M. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching practices. *Journal for Research in Mathematics Education, 28*(5), 550-576.

Riffe, D., Lacy, S., & Fico, F. (2014). *Analyzing media messages: Using quantitative content analysis in research* (3rd ed.). New York, NY: Taylor & Fancis.

Rouse, M. (2011, May). Discussion board (discussion group, message board, online forum). Retrieved from http://whatis.techtarget.com/definition/discussion-board-discussion-group-message-board-online-forum

Sakamoto, B. H. (2012, January 3). What is a PLN, anyway? [Web log post]. Retrieved from http://www.teachingvillage.org/2012/01/03/what-is-a-pln-anyway/

Saldaña, J. (2013). *The coding manual for qualitative researchers.* Thousand Oaks, CA: SAGE Publications.

Šapkova, A. (2014). System of mathematics teachers' beliefs about effective teaching/learning and practice as a complex adaptive system. *Journal of Studies in Education, 4*(3), 15-31.

Schoen, H. L., & Charles, R. I. (2003). Preface. In H. L. Schoen & R. I. Charles (Eds.), *Teaching mathematics through problem solving: Grades 6 – 12* (pp. ix-xvii). Reston, VA: Author.

Schreier, M. (2012). *Qualitative content analysis in practice*. Thousand Oaks, CA: SAGE Publications.

Shenton, A. K. (2004). Strategies for ensuring trustworthiness in qualitative research projects. *Education for Information*, *22*(2), 63-75.

Shepard, L. A. (1991). Psychometricians' beliefs about learning. *Educational Researcher*, *20*(7), 2-16.

Sherin, M. G. (2001). Developing a professional vision of classroom events. In T. Wood, S. B. Nelson, & J. Warfield (Eds.), *Beyond classical pedagogy: Teaching elementary school mathematics* (pp. 75-93). Mahwah, NJ: Erlbaum.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4-14.

Silver, E., A., Mesa, V. M., Morris, K. A., Star, J. R., & Benken, B. M. (2009). Teaching mathematics for understanding: An analysis of lessons submitted by teachers seeking NBPTS certification. *American Educational Research Journal*, *46*(2), 501-531.

Simon, M. A., & Schifter, D. (1991). Towards a constructivist perspective: An intervention study of mathematics teacher development. *Educational Studies in Mathematics, 22*, 309-331.

Skott, J. (2001). The emerging practices of a novice teacher: The roles of his school mathematics images. *Journal of Mathematics Teacher Education, 4*, 3-28.

Skype (n.d.). Retrieved from http://www.oxforddictionaries.com/us/definition/american_english/Skype

Smith, M. S., Bill, V., & Hughes, E., K. (2008). Thinking through a lesson: Successfully implementing high-level tasks. *Mathematics Teaching in the Middle School, 14*(3), 132-138.

Smith, M. S., & Henningsen, M. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education, 28*(5), 524-549.

Son, J. & Kim, O. (2015). Teachers' selection and enactment of mathematical problems from textbooks. *Mathematics Education Research Journal*, *27*(4), 491-518.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455-488.

Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematical project. *Educational Research and Evaluation, 2*(1), 50-80.

Stein, M. K., Smith, M. S., Henningsen, M., & Silver, E. A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development*. New York, NY: Teachers College Press.

Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E., A. (2009). *Implementing standards-based mathematics instruction: A casebook for professional development* (Vol. 2). New York, NY: Teachers College Press.

Strauss, A. & Corbin, J. (1998) *Basics of qualitative research: Techniques and procedures for developing grounded theory.* Thousand Oaks, CA: SAGE Publications.

Sun, M., Wilhelm, A. G., Larson, C. J., & Frank, K. A. (2014). Exploring colleagues' professional influence on mathematics teachers' learning. *Teachers College Record*, *116*(6), 1-30.

Traxler, J. (2009). Current state of mobile learning. In M. Ally (Ed.), *Mobile learning: Transforming the delivery of education and training* (pp. 9-24). Edmonton, AB: AU Press.

Twitter math camp. (2015). Retrieved from http://www.twittermathcamp.com/

Veletsianos, G. (2011). Higher education scholars' participation and practices on Twitter. *Journal of Computer Assisted Learning*, *28*, 336-349.

Walsh, J. A., & Sattes, B. D. (2005). *Quality questioning: Research-based practices to engage every learner*. Thousand Oaks, CA: Corwin Press.

Webb, N. L. (1999). *Alignment of science and mathematics standards and assessments in four states.* (Research Monograph No. 18). Washington, DC: Council of Chief State School Officers.

Webb, N. L. (2002). *An analysis of the alignment between mathematics standards and assessments for three states*. Presentation to the Annual Meeting of the American Educational Research Association, New Orleans, LA.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. New York, NY: Cambridge University Press.

Wenger, E., Trayner, B., & de Laat, M. (2011). *Promoting and assessing value creating in communities and networks: A conceptual framework*. Netherlands: Ruud de Moor Centrum.

Wilhelm, A. G. (2014). Mathematics teachers' enactment of cognitively demanding tasks: Investigating links to teachers' knowledge and conceptions. *Journal for Research in Mathematics Education, 45*(5), 636-674.

Wilhelm, A. G., Chen, I, Smith, T. M., & Frank, K. A. (2016). Selecting expertise in context: Middle school mathematics teachers' selection of new sources of instructional advice. *American Educational Research Journal*, 53(3), 1-36.

Williams, J. B., & Jacobs, J. (2004). Exploring the use of blogs as learning spaces in the higher education sector. *Australasian Journal of Educational Technology, 20*(2), 232-247.

Wilson, L. O. (2013). A succulent discussion of the revisions to Bloom's classic cognitive taxonomy by Anderson and Krathwohl and how to use them effectively. Retrieved from http://thesecondprinciple.com/teaching-essentials/beyond-bloom-cognitive-taxonomy-revised/

Yin, R. K. (2014). *Case study research: Design and method*s. Thousand Oaks, CA: SAGE Publications.

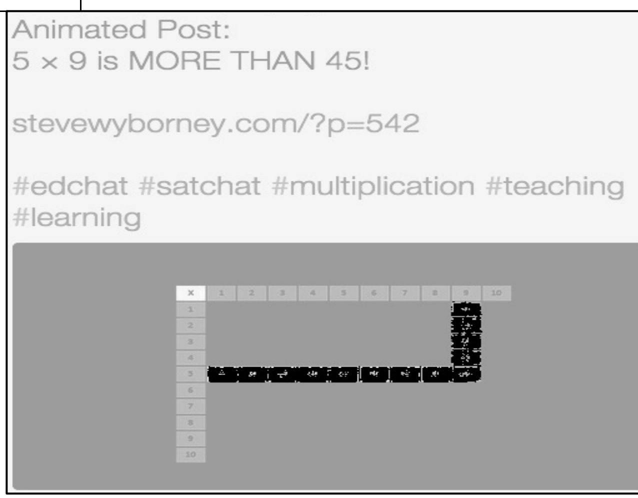
Zohar, A., & Dori, Y. J. (2003). Higher order thinking skills and low-achieving students: Are they mutually exclusive?. *The Journal of the Learning Sciences, 12*(2), 145-181.

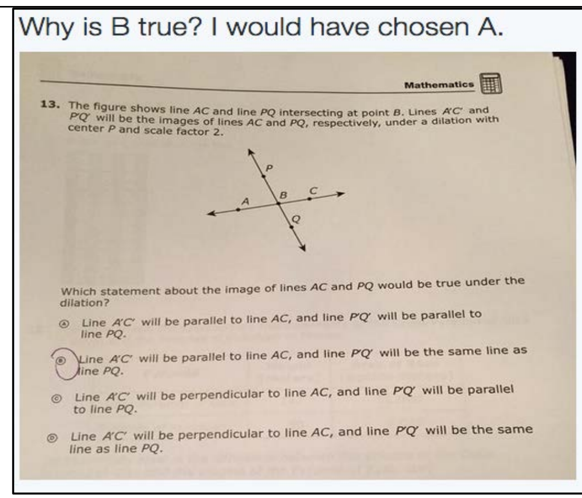
Zohar, A., Degani, A., & Vaaknin, E. (2001). Teachers' beliefs about low-achieving students and higher order thinking. *Teaching and Teacher Education, 17*, 469 - 485.

Appendix A

Coding Framework

Main Cat	Name	Description
Mathematical Knowledge for Teaching (MKT)	Subject Matter Knowledge	• The knowledge is purely mathematical and does not require additional knowledge of students and teaching (Ball, Thames, & Phelps, 2008). o The content of and beyond any well-educated adult o Teaching tasks that may require specialized content knowledge include: connecting a topic being taught to topics from prior or future years, modifying tasks to be easier or harder, explaining mathematical goals and purposes to parents, giving or evaluating mathematical explanations, and selecting representations for particular purposes (Ball et al., 200, p. 10). o Mathematical Horizon – "(1) a sense of the mathematical environments surrounding the current 'location' in instruction (2) Major disciplinary ideas and structures (3) key mathematical practices, and (4) core mathematics values and sensibilities" (Ball & Bass, 2009, p. 5). Coding Rules • If a task is being posed, and the teacher is asking mathematics questions in need of help, then SMK. • A teacher doing math themselves.
Examples:		



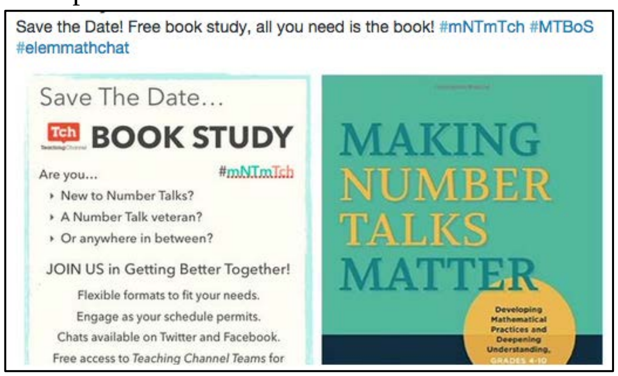


Mathematical Knowledge for Teaching (MKT)	Pedagogical Content Knowledge	• Pedagogical knowledge **remains connected with the knowledge of mathematics**, but more specifically, the content knowledge that includes aspects of the content most connected to instruction (Shulman, 1986). o Intertwining the knowledge of mathematics with the knowledge of how students think about, know, or learn about **specific areas of mathematics** (KCS) (Hill et al., 2008). o Focus on students' misconceptions (KCS) (Ball et al., 2008). o Combination of mathematics knowledge and the design of mathematics instruction, "Teachers need to sequence particular content for instruction, deciding which example to start with and which examples to use to take students deeper into the content. They need to evaluate the instructional advantages and disadvantages or representation used to teach a specific idea" (Ball et al., 2008, p. 9). o Knowledge of specific mathematics programs, knowledge of the variety of instructional materials available with the curriculum, and knowledge of the characteristics of the curriculum that lends itself to particular circumstances within the classroom (Ball et al., 2008). o An educator should be familiar with multiple material resources (Shulman, 1986). ▪ **This includes planning & time frames associated with planning.**
		Examples • Instead we began with the "work" portion of the workshop model then moved to the "share" portion followed by our "mini-lesson" about making sense of the relationship between the operator symbol and the sign of the second operand, which led to great discussion of what it meant to take away negative or add more negative. • Complex Numbers Foldable and Kahoot #alg2chat #mtbos http://ispeakmath.org/2015/12/16/complex-numbers-foldable …
	Miscellaneous	• Intended for content related to mathematical knowledge for teaching ["The mathematical knowledge used to carry out the work of teaching mathematics" (Hill, Rowan, & Ball, 2005, p. 373)] but not specifically related to subject matter knowledge or pedagogical content knowledge. • Not content or situation specific, for example, DOK levels. • May be related to general pedagogy.

Visions for High Quality Mathematics Instruction (VHQMI)	Teacher Role	• Describes **the role of the teacher** as proactively supporting students' learning through coparticipation. Stresses the importance of designing learning environments that support problematizing mathematical ideas and providing students with relevant resources (Munter, 2014). o The act of problematizing mathematical ideas is very important. • Teacher decisions or actions that affect the environment, design, and/or culture of the math classroom in relation to learning.
Examples • Always striving to be the facilitator and setting up norms from the beginning to achieve just that #mtbos • Start the students with a hook, get them chewing on some good math, then open it up for the reveal. Three simple acts, one amazing result. (Directed towards the teacher) • Groups of 3 students sitting together. New partners & new desks every day. I used playing cards given out at random as students entered class to assign students to tables – with hanging numbers indicating which tables made which group. More details about VRGs here.		

Visions for High Quality Mathematics Instruction (VHQMI)	Classroom Discourse	• Patterns/Structure of Classroom Talk – Promotes whole-class conversations, including student-to-student talk that is student initiated, not depending on the teacher (Huffered-Ackles, Fuson, & Sherin, 2004); promotes developing-and supporting a “mathematical discourse community” (Lampert, 1990; as cited in Munter, 2014). • Nature of Classroom Talk – Suggests that classroom talk should be conceptually oriented—including articulating/refining conjectures and arguments for explaining mathematical phenomena—for the purpose of supporting students in *doing mathematics* and/or spawning new investigations (Munter, 2014, p. 5). • Student Questions – Promotes student questions that drive instruction, leading to new mathematical investigations, questions, characteristic of “doing mathematics” (e.g., generalization) (Munter, 2014, p. 5). • Teacher Questions – Describes the role of the teacher questions that are conceptually oriented (why questions) in driving investigations, helping students explain their problem-solving strategies, and/or helping the teacher understand students’ thinking (Borko, 2004; as cited in Munter, 2014). • <u>Details related to Student Explanation</u> – Student explanations include both explanation and justification (Kazemi & Stipek, 2001; as cited in Munter, 2014) with little prompting from the teacher (Hufferd-Ackles, Fuson, & Sherin, 2004). • Mathematical discourse includes the purposeful exchange of ideas through classroom discussion, as well as through other forms of verbal, visual, and written communication (NCTM, 2014). – this includes photos of student work.

Examples:



- Yesterday’s blog: Using Hypernom to get kids talking about math wp.me/p46FUF-1Fs #math #mathchat #tmwyk
- #MTBoS Any HS Ts reading Number Talks & thinking about using them this year? Concerned about daily time/Regents curriculum but want to use.
- We also used it after looking at the graph of y=2*3^x. Students reasoned f(x) is a continuous graph while a sequence is a function but a discrete graph of whole numbers, as Nuri put it. (Importance of student reasoning)

<table>
<tr>
<td>Visions for High Quality Mathematics Instruction (VHQMI)</td>
<td>Mathematical Tasks</td>
<td>

- “Mathematical tasks can range from a set of routine exercises to a complex and challenging problem that focuses students’ attention on a particular mathematical idea” (NCTM, 2014, p. 17).
- Emphasizes tasks that have the potential to engage students in “doing mathematics”, allowing for “insights into the structure of mathematics and “strategies for methods for solving problems” (Munter, 2014).
- “These tasks [that promote reasoning and problem solving] encourage reasoning and access to the mathematics through multiple entry points, including the use of different representations and tools, and they foster the solving of problems through varied solution strategies” (NCTM, 2014, p. 17).

Coding Rules

- **Discussion of procedures specific to the task.**
- A picture or a GIF of a math task, even if not further details given.

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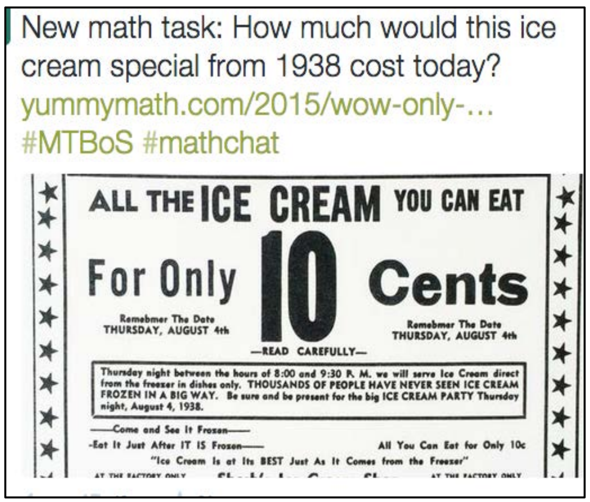
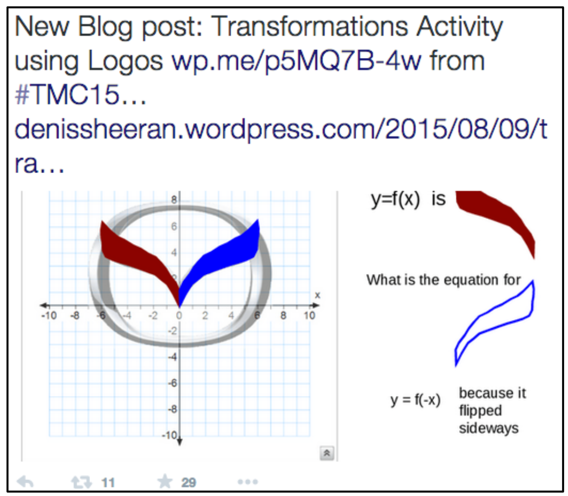
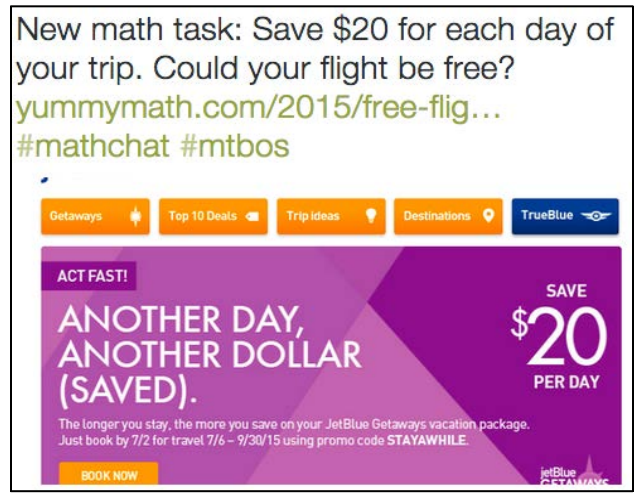
Examples

(Discussion of procedures specific to the task) - There are four sets of cards (each group would get a set so they aren't all working on the same questions). Each set of cards will have answers that have the same characteristics (Set 1 - answer is a trinomial, Set 2 - answer is a binomial with no squared term, Set 3 answer is a binomial with no x term, Set 4 - answer is a binomial with no constant term). A set should contain 12 cards. Four long expressions, four algebraic answers and four algebra tile answers.







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<tr>
<td>VHQMI</td>
<td>Miscellaneous</td>
<td>
Related to instruction in the classroom, but not specifically related to teacher role, classroom discourse, or mathematical tasks.
Coding Rules

May be related to assessment practices.
May be related to <u>REFLETION on instruction/student learning/planning</u>.
May be related to decisions related to learning (ex. My desks are easily movable, based on the activity we are doing).
Includes discussion of procedures related to how to proceed with the task, but not specific to the content of the task.

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<tr>
<td>Views for Supporting Struggling Students (VSSS)</td>
<td>
VSSS is related to a teacher’s conception of who can and cannot engage with cognitively demanding tasks/higher order thinking (Wilhelm, 2014).
Coding Rules

Conversations encouraging students to try or overcome fear.
Any support for a learner/or learners
Mention of supports used to help struggling students.
Discussion of student identities in the math classroom.

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Examples:

In regards to the math, how do students work backwards to generate questions for a given image? Would rephrasing the directions help them think about it differently? If we asked them to create a quiz for the teachers based on the graph, would that have helped? How is wondering about an image different than generating questions for it?
Most students think their role in math classrooms is to get questions right. Theatln.tc/1Opd67 #mathed #mathchat #MTBoS

MrsAGilliam: "Low achievers are often thought of as slow learners, when in fact they are not learning the same thing." @joboaler #msmathchat #MTBoS
</td>
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</table>

Miscellaneous	• All data which is related to the teacher attributes, cognitively demanding tasks, or other components of the MTBoS but does not fit into any of the above categories.
Irrelevant	• All data which is not easily comprehended without further context.

Appendix B

Initial Recruitment Letter

My name is Christopher Parrish, I am an active member of the MTBoS community and a PhD student at Auburn University in Auburn, AL, United States.

I am contacting you to participate in research I am conducting as part of my dissertation. The purpose of my research is to better understand if and how members of the MathTwitterBlogosphere (MTBoS) perceive they benefit from the MTBoS community on their development as a mathematics educator. You were selected as part of a random sample of participants from the MTBoS Directory.

If you decide to participate, I will conduct an in depth, interview (approximately 45 minutes to 1 hour) with you; the interview will be conducted via the web and will be recorded to better analyze your thoughts. The purpose of the interview will be to discuss your perceptions of using the MTBoS community to support your development as a mathematics educator.

Participation within this research is completely voluntary. If you decide to participate, all personally identifying information will be removed from the interview transcripts and reported as with pseudonyms. Great care will be taken to protect your confidentiality within the reporting of findings from the interview data.

I understand that mathematics educators are extremely busy and that an hour of your time would require a large commitment. However, there is minimal research on how teachers are using online communities to support professional growth; your willingness to participate would make a substantial contribution to research on the forefront of teacher learning.

If you would like to participate or have any questions about the study, please reply to this message or email me at CWP0003@tigermail.auburn.edu.

Thank you for your time.

Sincerely,
Christopher Parrish

Appendix C

INFORMED CONSENT
for a Research Study entitled
"Community Members' Perceptions of the MathTwitterBlogosphere"

You are invited to participate in a research study to understand if and how engagement with the MathTwitterBlogosphere community supports the development of the teacher attributes needed to consistently select and implement cognitively demanding tasks. The study is being conducted by Christopher Parrish; under the direction of Dr. W. Gary Martin in the Auburn University Department of Curriculum and Teaching. You were selected as a possible participant because you are a registered member on the MTBoS directory and are age 19 or older.

What will be involved if you participate? If you decide to participate in this research study, you will be asked to participate in a recorded interview, via the web. Your total time commitment will be approximately one hour. The interview will be conducted at a date and time convenient for the interview participant. The interview will be conducted through Skype and recorded with Callnote.

Are there any risks or discomforts? The risks associated with participating in this study includes the breach of participant confidentiality within the reporting of findings from the interview data. To minimize these risks, I will make every effort to ensure the interview transcriptions remain confidential. To do so, transcriptions will include pseudonyms, rather than names, and all other identifying information will be removed. No information will be included in publications, presentations, or reports that could be used to personally identify you. Video recordings of the interview will be deleted upon the completion and verification of the transcript; this will occur on or before July 31, 2017.

Are there any benefits to yourself or others? If you participate in this study, you can expect to increase the understanding of how mathematics teachers support their professional development through the MathTwitterBlogosphere. I cannot promise you that you will receive any or all of the benefits described.

Will you receive compensation for participating? There will be no compensation for participating.

Are there any costs? There is no cost for your participation in the study.

If you change your mind about participating, you can withdraw at any time during the study. Your participation is completely voluntary. If you choose to withdraw, your data can be withdrawn as long as it is identifiable. Your decision about whether or not to participate or to stop participating will not jeopardize your future relations with Auburn University, the Department of Curriculum and Teaching.

Participant's initials ______ Page 1 of 2

Your privacy will be protected. Any information obtained in connection with this study will remain confidential. To ensure confidentiality, I will make every effort to ensure the interview transcriptions remain confidential. To do so, transcriptions will include pseudonyms, rather than names, and all other identifying information will be removed. No information will be included in publications, presentations, or reports that could be used to personally identify you. Video recordings of the interview will be deleted upon the completion and verification of the transcript; this will occur on or before July 31, 2017. Information obtained through your participation may be published in a professional journal, and/or presented at a professional meeting.

If you have questions about this study, please contact Christopher Parrish at CWP0003@tigermail.auburn.edu or Dr. W. Gary Martin at martiwg@auburn.edu. A signed copy of this document will be returned to you through email.

If you have questions about your rights as a research participant, you may contact the Auburn University Office of Research Compliance or the Institutional Review Board by phone (334)-844-5966 or e-mail at IRBadmin@auburn.edu or IRBChair@auburn.edu.

HAVING READ THE INFORMATION PROVIDED, YOU MUST DECIDE WHETHER OR NOT YOU WISH TO PARTICIPATE IN THIS RESEARCH STUDY. YOUR SIGNATURE INDICATES YOUR WILLINGNESS TO PARTICIPATE.

__________________________	__________________________
Participant's signature Date	Investigator obtaining consent Date
__________________________	__________________________
Printed Name	Printed Name
	__________________________
	Co-Investigator Date
	__________________________
	Printed Name

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Appendix D

Follow Up Recruitment Letter

Dear (participant),

A few days ago I sent an email requesting your participation within a research project related to the the MathTwitterBlogosphere Community. I would like to follow up to determine if you have you had an opportunity to review the initial invitation and would be interested in participating within the study.

I am including the content of the original letter below:

My name is Christopher Parrish, I am an active member of the MTBoS community and a PhD student at Auburn University in Auburn, AL, United States.

I am contacting you to participate in research I am conducting as part of my dissertation. The purpose of my research is to better understand if and how members of the MathTwitterBlogosphere (MTBoS) perceive they benefit from the MTBoS community on their development as a mathematics educator. You were selected as part of a random sample of participants from the MTBoS Directory.

If you decide to participate, I will need to conduct an in depth, interview (approximately 45 minutes to 1 hour) with you; the interview would be conducted via the web and would need to be recorded. The purpose of the interview will be to discuss your perceptions of using the MTBoS community to support your development as a mathematics educator.

Participation within this research is completely voluntary. If you decide to participate, all personally identifying information will be removed from the interview transcripts and reported as with pseudonyms. Great care will be taken to protect your confidentiality within the reporting of findings from the interview data.

I understand that mathematics educators are extremely busy and that an hour of your time would require a large commitment. However, there is minimal research on how teachers are using online communities to support professional growth; your willingness to participate would make a substantial contribution to research on the forefront of teacher learning.

If you would like to participate or have any questions about the study, please reply to this message or email me at CWP0003@tigermail.auburn.edu.

Thanks for your time.

Sincerely,
Christopher Parrish

Appendix E

Qualitative Interview Protocol

Thank you for agreeing to participate in this study. I am conducting this study to better understand if and how members of the MathTwitterBlogosphere perceive they benefit from the MathTwitterBlogosphere community on their development as a mathematics educator.

As a reminder, the interview will be recorded. Once the transcription of the interview has been completed and verified, the video recording will be deleted on or before July 31, 2017. Also, no personal information about you will be disclosed as the notes and transcripts will not be tied to any specific participant. If you decide to withdraw your participation from the study, you are free to do so at any time.

Do you have any questions or concerns?

Do you mind if I turn on the recorder?

Demographics	1. Please tell me a little bit about yourself and your current role in mathematics education *Probing Questions:* • *In what state do you currently teach?* • *What grades or specific subjects do you teach?* • *How long have you been a mathematics educator?* 2. How would you describe the students with whom you interact? *Probing Questions:* • *What are the demographics of the students who attend your courses?*
Use of MathTwitterBlogosphere community	3. How did you become involved with the MTBoS community? *Probing Question:* • *Why did you decide to become connected with the community?* • *How do you typically interact with the community?* 4. What other kinds of professional development are you involved with? *Probing Question:* • *How does this professional development relate to the MTBoS community?* 5. Do you feel you benefit from engaging with the MTBoS community? *Probing Questions:*

	• *What are the benefits?* • *How valuable is the #MTBoS to you?* • *What about this community is so beneficial to you?* 6. Do you use the hashtag, "#MTBoS"? *Probing Question:* • *When do you use the hashtag, "#MTBoS"?*
Selection of Tasks Implementation of Tasks	7. How would you describe the types of mathematics tasks shared within the MTBoS community? *Probing Questions:* • *How are they different from (or the same as) activities you may get from other sources?* • *How frequently do you implement a task found within the MTBoS community?* • *How do you decide which tasks to use?* • *How do the tasks that you select fit with the learning goals that you have for your students?* • *Is there a specific blog or portion of the MTBoS that primarily affects the activities you select?* 8. Has the MTBoS community affected the way you implement tasks? *Probing Questions:* • *In what ways? (what ideas did you implement?)* o *How did your students respond?* • *What aspects of the community do you think affects your implementation of the task?* • *Is there a specific blog or portion of the MTBoS that primarily affects how you implement the task?*
Teacher Attributes Subject Matter Knowledge	9. Can you think of a particular situation in which the MTBoS has helped you think differently about some area of your practice? *Probing Questions:* • *Please describe the situation and how the MTBoS community influenced your thinking?* 10. Has the MTBoS affected your understanding of mathematics? *Probing Questions:* • *In what ways?* • *Can you give an example?*

Pedagogical Content Knowledge Visions for High Quality Mathematics Instruction Views for Supporting Struggling Students	11. How has the MTBoS affected the way in which you teach the mathematics content? *Probing Questions:* • *In what ways?* • *Can you give an example?* 12. Can you describe any specific teaching practices you have begun or strengthened from engagement with the MTBoS? *Probing Questions:* • *What aspects of the community affected these practices?* 13. Have the materials within the community helped you better accommodate for the needs of a range of students in your class? *Probing Questions:* • *In what ways?* • *What aspects of the community have had the greatest impact in how you accommodate for all students?* • *How has the MTBoS affected how you address your (minority/ELL) students' needs?*
Summative	14. Are there other examples of how the MTBoS community has impacted your practice? 15. How would you summarize the impact of the MTBoS community on your practice? *Probing Questions:* • *More generally?*

Appendix F

Overview of Themes and Codes within the Coding Framework

Category	Subcategory	Theme	Code	Definition	Phase 1	Phase 2
Mathematical Knowledge for Teaching	Subject Matter Knowledge	Knowledge of Mathematics Progression	Content Standards and Goals	Connects the mathematics used to solve tasks with content standards in larger mathematics progressions	X	
			Discussion of Content Standards	Discussion related to the inclusion or placement of specific standards or goals within larger mathematics progressions	X	
		Engagement with Mathematics Content	Sharing Mathematics Knowledge	Demonstrated how community members shared knowledge that was purely mathematical in nature	X	
			Doing Mathematics	Demonstrated how community members interacted with mathematics content themselves	X	
			Thinking about Mathematics	Demonstrated how community members recorded their thinking about mathematics	X	
		Development of Mathematics Knowledge	Development of Mathematics Knowledge	Community members' perceptions of how MTBoS supported their development of subject matter knowledge		X
	Pedagogical Content Knowledge	Curriculum Resources	Recommended Curricular Resources	The sharing of recommended resources within the community	X	X
			Requests for Curriculum Resources	Requests or recommendations for resources specific to a concept, problem type, course, or student type	X	
			Discussion of Curriculum Resources	Description or critique of curriculum resources	X	
		Curriculum Planning	Planning Instruction	Individual and collaborative planning efforts	X	
			Sequencing of Content	The knowledge of how concepts fit within the mathematics curriculum	X	
		Methods of Teaching Specific Mathematics	Methods of Teaching Mathematics	Connects the knowledge of mathematics with the knowledge of designing instruction (Ball et al., 2008)	X	

		Knowledge of Student Thinking	Knowledge of Student Thinking	Knowledge of how students may think or learn about specific areas of mathematics (Hill et al., 2008)	X	
		Underlying Philosophy of Teaching	Underlying Philosophy of Teaching	Discussion of beliefs related to the teaching and of mathematics	X	
		Development of Pedagogical Content Knowledge	Development of Pedagogical Content Knowledge	Community members' perceptions of how MTBoS supported their development of pedagogical content knowledge		X
	MKT Miscellaneous	General Knowledge of Teaching and Learning Mathematics	General Knowledge of Teaching	General knowledge of mathematics teaching practices	X	X
			General Knowledge of Student Learning	A general understanding of student learning	X	X
			General Knowledge of Technology	General knowledge of how to use technology within the classroom	X	
		Learning about Teaching	Personal Learning	Learning opportunities or discussions related to mathematics or mathematics education	X	
			Reference to Education Literature on Research	Referencing to education literature or research	X	
		Underlying Philosophy of Teaching	Underlying Philosophy of Teaching	Discussion of beliefs related to the teaching and of mathematics	X	
Visions of High Quality Mathematics Instruction	Teacher Role	Teacher as Facilitator	Problematizing the Mathematics Content	A teacher selecting or staging content in a manner that prompts students to problem solve	X	
			Coparticipation Alongside Students	Educators acting as a facilitator within the mathematics classroom	X	X
		Supporting all Students	Supporting All Students in Mathematics	Supporting students with the appropriate resources or information needed to learn mathematics	X	
	Classroom Discourse	Promoting Discourse	Whole-Class Discourse	Whole-class discourse, including both spoken or written dialogue	X	
			Teacher Role in Discourse	Teacher questions or strategies used during classroom discourse	X	X
			Student Discourse	Students' engagement in classroom discourse	X	
	Mathematical Tasks	Mathematics Tasks	Cognitive Demand of Mathematics Tasks	Included the mathematics tasks from within the content	X	
			Nature of Mathematics Tasks	Community members' descriptions of mathematics tasks located within the community		X

			Implementation of Tasks	Discussions of how a task was implemented, or the details needed to support the implementation of a task	X	X
			Sharing a Task	The intentional sharing of a mathematics task	X	
	VHQMI Miscellaneous	Instruction	Reported Instruction	Making descriptions and images of classroom instruction available for the community	X	
			Assessment in Instruction	Descriptions and shared knowledge on assessment practices	X	X
			Reflection on Instruction	Reflections on students and teaching, each with respect to classroom instruction	X	
			Improving Instruction	Conversations or requests for feedback related to improving instruction	X	X
		Underlying Philosophy of Teaching	Underlying Philosophy of Teaching	Discussion of beliefs related to the teaching and of mathematics	X	
Views for Supporting Struggling Students		Equitable View of All Learners	Commitment to All Students	An equitable view of all students as learners of mathematics	X	
			Gender Equity	The role and perceptions of gender in learning mathematics	X	
			Mindset	A view of learning as either fixed or having the ability to improve with effort	X	
		Supporting All Students	Supporting All Students	Supporting students with the appropriate resources or information needed to learn mathematics	X	X
Miscellaneous		Online Learning Community	MTBoS Community	Content related to the MTBoS community as a whole	X	X
			Personal Engagement in Learning Community	Personal aspects and interactions within the MTBoS community	X	X
			Professional Growth	Professional development opportunities within the MTBoS	X	X
		Affordances of the MTBoS Community	Affordances of the MTBoS Community	The perceived benefits and affordances of engaging with the MTBoS community.		X