Control of Sawtooth Oscillation Dynamics using Externally Applied Stellarator Transform

by

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Abstract

The control of sawtooth oscillations is an active area of tokamak research. The sawtooth oscillation is driven by ohmic heating of the core plasma until the safety factor drops below one triggering the growth of an m = n = 1 kink-tearing mode. Large sawtooth oscillations need to be avoided in ITER, since they can trigger neoclassical tearing modes and edge localized modes resulting in loss of plasma confinement in some cases. However, small sawtooth oscillations may be beneficial in preventing impurity and helium ash accumulation in the plasma core.

Sawtooth oscillations are observed in the Compact Toroidal Hybrid (CTH), a currentcarrying stellarator/tokamak hybrid device. CTH has the unique ability to change the relative amount of applied vacuum rotational transform from stellarator coils to the rotational transform generated by the plasma current. The vacuum rotational transform is systematically varied from 0.02 to 0.13 to observe changes in the sawtooth oscillation. Three two-color soft x-ray cameras were constructed and installed on CTH. Each two-color camera employs two 20-channel diode arrays to detect the signatures of sawtooth instabilities. The diagnostic primarily measures bremsstrahlung radiation which is dependent on temperature, or thermal kinetic energy, of electrons within the plasma. The sawtooth instability is a periodic rearrangement of the core plasma thermal energy. Therefore, the bremsstrahlung radiation is strongly tied to the dynamics of the sawtooth oscillation.

Sawteeth observed within CTH are tokamak-like despite employing a three-dimensional confining field because: (1) the presence of the m = n = 1 kink-tearing mode, (2) the monotonically decreasing rotational transform profile is dominated by the plasma current not the vacuum rotational transform, and (3) the measured scaling of the normalized inversion surface radius with total rotational edge transform.

The measured sawtooth period decreases by a factor of two over a vacuum rotational transform from 0.02 to 0.13. The sawtooth amplitude is observed to decrease with increasing levels of 3D field, as quantified by the amount of vacuum transform imposed. The measured crash time of the sawtooth oscillation does not appear to depend of the amount of vacuum transform applied, indicating that the final reconnection dynamics of the m = 1 and n = 1 mode are not significantly affected by the 3D stellarator fields.

Previous numerical simulations show that the internal kink mode is significantly destabilized with increasing flux surface elongation of the q = 1 surface. The experimental results indicate that the decrease in sawtooth period and amplitude is correlated to the mean elongation of the non-axisymmetric plasmas within CTH. This dissertation describes the sawtooth theory for an axisymmetric plasma, the development of the two-color diagnostic used to characterize the sawtooth oscillation, and the properties of the sawtooth oscillation observed in the plasmas within CTH.

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Chapter 1

Introduction

Fusion is a process that takes place naturally within the stars of the universe, such as the Sun in our solar system, in which two light nuclei collide into each other producing a heavier nuclei accompanied by the release of energy. The ability to reproduce fusion reactions in a controlled environment and harvesting the energy released during the process would provide a clean energy source for years to come.

Reproducing the fusion process of joining two similarly charged nuclei together and capturing the energy is not without its challenges. To produce a fusion reaction the two light nuclei must have enough kinetic energy to overcome the mutual Coulomb repulsion between the nuclei. The excess energy from the fusion reaction needs to be captured and transferred into electrical energy to be put on the electrical grid efficient enough to have a net gain in energy. Two of the more promising techniques to produce a fusion reaction are internal confinement and magnetic confinement fusion. Internal confinement fusion is the process of rapidly heating a small amount of thermonuclear fuel, usually by compression of the target through x-rays induced by lasers.¹ Magnetic confinement holds a hot plasma with magnetic fields long enough and at high enough density that the random thermal collisions between light nuclei will produce fusion. The largest magnetic confinement device is currently being constructed in France, is known as ITER.²

The plasmas used in magnetic confinement can be created by heating a gas, such as deuterium, increasing the thermal velocity of the atoms. The deuterium gas is pumped into a vacuum within a container (called the vacuum vessel) and typically heated up with electromagnetic waves or by ohmic heating. The hot atoms randomly collide into eachother and some of those collisions result in the stripping of an electron off of the deuterium atom electrically breaking the gas down and forming a plasma. For a fusion reaction, the plasma must contain the correct nuclei, such as deuterium and tritium, at an even higher temperature (100 million degrees centigrade).³ At this temperature the nuclei have enough thermal kinetic energy to overcome the mutual Coulomb repulsion between the nuclei. Since particles within the plasma have an electrical charge, the plasma can be confined using magnetic fields through the Lorentz force. The plasma will cool rapidly if it comes into contact with the vacuum vessel; therefore, the plasma is kept in a magnetic bottle away from the vacuum vessel.

One of the major challenges of magnetic confinement is not only to contain the plasma with magnetic fields but to ensure the stability of the system once it is confined. Destabilizing forces within the plasma arise from current or pressure gradients which may drive an instability and lead to a loss of confinement. One form of a pressure-driven instability which limits the maximum current that can be driven in the plasma core for a given magnetic field is the sawtooth instability. The sawtooth instability primarily effects the core of the plasma and does not lead to a loss of confinement but limits plasma performance and may trigger other deleterious instabilities. The properties of the sawtooth instability are described in detail in this thesis as well as how changing a three-dimensional magnetic confining field alters its characteristics.

1.1 Magnetic confinement of a plasma

The interaction between the magnetic field, \vec{B} , and the charged particles within the plasma is described by the Lorentz force:

$$\vec{\mathbf{F}} = q\vec{v} \times \vec{\mathbf{B}},\tag{1.1}$$

where q is the charge of the particle and \vec{v} is the velocity of the particle. A charged particle with a non-zero component of the velocity perpendicular to the magnetic field will experience an attractive force towards the magnetic field line leading to circular gyro-motion about the field line. In a plasma, the particles follow helical orbits around the field lines due to their combined parallel and perpendicular motion. The particle will proceed to orbit around the field line until an external force interacts with the particle such as collisions with other particles.

To confine a plasma, magnetic field lines are oriented in order for charged particles to not intersect with the vacuum chamber. This can be accomplished using a solenoid forming a cylindrical plasma column known as a θ -pinch. However, the charged particles will escape the cylindrical plasma column at both ends due to their parallel velocity, v_{\parallel} , along the magnetic field lines. The parallel linear velocity results in a short confinement time, approximately equal to the length of the cylinder divided by v_{\parallel} . Therefore, a cylindrical plasma column is not a practical approach to magnetic confinement.

The next logical step to confine a plasma would be to connect the ends of the cylinder together to prevent end-losses, creating a torus. The basic geometry of a torus is shown in figure 1.1. They are described by two different coordinate systems, the first one is from the center of the toroid denoted by the red circle. The major radius, R, is given a subscript if it is in the center of the plasma, R_0 . The vertical distance above the mid-plane is given by Zand the toroidal angle, ϕ , increases counter-clockwise as seen from looking down upon the torus, the negative \hat{z} direction. From the center of the plasma, denoted by the blue square a pseudo-coordinate system is defined by the minor radius, r, measured from the end of the major radius, R_0 , and is at a maximum, a, when it is at the edge of the plasma. At any toroidal location, the poloidal angle, θ , is zero at most outboard side increasing to $\theta = \frac{\pi}{2}$ at the topmost portion of the torus, Z > 0, and to $\theta = \pi$ on the inboard of the torus. The



Figure 1.1: The toroidal magnetic field geometry illustrating the poloidal direction, θ , toroidal direction, ϕ , the distance from the origin, major radius R_0 , plasma minor radius, a, and the vertical distance above the mid-plane, Z.

conversion between the coordinate systems is given by,

$$R = R_0 + r \cos(\theta)$$

$$z = r \sin(\theta)$$
(1.2)

$$\phi = \phi$$

There are many other magnetic field arrangements to confine a plasma but the configurations discussed in this thesis are toroidal in shape, a tokamak and a stellarator.

If a toroidal device only employed toroidal magnetic fields the plasma will experience a net outward drift leading to a loss of confinement. This is due to the effect that Lorentz force (given by equation 1.1) has on positive and negative charged particles. Similarly charged particles will group in the either the top or bottom regions of the plasma. This charge distribution creates and electric field in the \hat{z} direction. The plasma experience and outward force in the radial direction due to $\mathbf{E} \times \mathbf{B}$ drift from the vertical electric field. To counter-act this drift, a poloidal magnetic field is added creating a helical magnetic field configuration, similar to the stripes on a barber pole. A charged particle traveling along the helical magnetic field line still undergoes a drift due to the Lorentz force. However, due to the helical magnetic field the drift is away from the original particle location half of the time and towards the original point the other half. The particles will be confined within the plasma since the net motion as the particle progress along the helical field cancels out. The helicity of the magnetic fields is characterized by the rotational transform:⁴

$$\iota = \frac{\iota}{2\pi} = \frac{\mathrm{d}\varphi_{\mathrm{p}}}{\mathrm{d}\varphi_{\mathrm{t}}}.\tag{1.3}$$

Where φ_p represents the poloidal flux and φ_t is the toroidal flux. The toroidal flux is given by,⁴

$$\varphi_{\rm t} = \int \vec{B} \cdot \mathrm{d}\vec{A}_{\phi} \tag{1.4}$$

and the poloidal flux by,

$$\varphi_{\rm p} = \int \vec{B} \cdot \mathrm{d}\vec{A_{\theta}}.\tag{1.5}$$

The helical field needed for plasma confinement in a tokamak is generated from a combination of toroidal external magnetic field coils and a poloidal magnetic field produced by the toroidal plasma current. The toroidal field is established by external magnetic field coils and the poloidal field is induced by a toroidal current driven through the plasma. This combination of inducing the poloidal fields results in flux surfaces that do not vary with toroidal angle. Figure 1.3 (a) shows the last closed flux surface of an axisymmetric tokamak. One of the largest tokamaks currently under construction is ITER in southern France. ITER is an important stepping stone for experimental tokamaks with the goal of producing more thermal energy from fusion reactions than supplied from auxiliary heating.²

In contrast to a tokamak, a stellarator does not drive an internal plasma current but uses external coils to provide both toroidal and poloidal magnetic fields. Stellarators are nonaxisymmetric but exhibit a stellarator symmetry.^{5,6} Stellarator symmetry has the property that the observer will see the same flux surfaces looking one way down the torus as the opposite direction but will be flipped upside-down relative to the other. Written as functions defining points on the flux surface, R and Z:

$$[R(\Phi,\theta,\phi), Z(\Phi,\theta,\phi)] = [R(\Phi,\theta,-\phi), -Z(\Phi,\theta,-\phi)].$$
(1.6)

R and Z are described in cylindrical coordinates with the poloidal and toroidal angles of θ and ϕ . Φ is the radial flux coordinate.

In addition to stellarator symmetry, stellarators also have a field period symmetry meaning that the magnetic structure is identical at discrete toroidal locations. The field periodic magnetic flux surfaces are related by:

$$[R(\Phi,\theta,\phi), Z(\Phi,\theta,\phi)] = [R(\Phi,\theta,\frac{2\pi n}{N}+\phi), Z(\Phi,\theta,\frac{2\pi n}{N}+\phi)].$$
(1.7)

The number of field periods is N while n is any integer. Figure 1.3 (b) shows the last closed flux surface of a plasma having both a stellarator and a ten-fold field periodicity.

The helical fields of a torus shaped plasma form a set of nested flux surfaces which are also surfaces of constant pressure. The dynamics of the plasma within these magnetic fields may be described by the theory of ideal magnetohydrodynamics (MHD). An excellent review of ideal MHD is given by Friedberg.^{7,8} The relationship between pressure and magnetic flux surfaces can be shown using the ideal MHD force balance equation:

$$\nabla p = \vec{J} \times \vec{B},\tag{1.8}$$

where \vec{J} is the current density and p is the pressure. By definition, ∇p is perpendicular to the constant pressure contours. The dot product of the magnetic field with equation 1.8 gives:

$$\vec{B} \cdot \nabla p = \vec{B} \cdot (\vec{J} \times \vec{B}) = 0.$$
(1.9)



Figure 1.2: (a) The last closed flux surface of an axisymmetric tokamak. (b) The last closed flux surface of a 10 field period stellarator. The colors indicate magnetic field strength. Reprinted from Donald A. Spong. 3D toroidal physics: Testing the boundrys of symmetry breaking. *Physics of Plasmas*, 22(5), 2015, with the permission of AIP Publishing.

Therefore, the magnetic field lines given by \vec{B} are perpendicular to ∇p and must lie on the surfaces of constant pressure. These surfaces of constant pressure lie on the magnetic flux surfaces. In a confined plasma there are many of these surfaces nested within each other. Figure 1.3 shows the nested flux surfaces for the plasma confinement machine used in this thesis, the Compact Toroidal Hybrid (CTH). CTH is a combination of a tokamak and a five-field periodic stellarator and can vary the amount of rotational transform from a near axisymmetric tokamak-like equilibrium to a non-axisymmetric stellarator equilibrium. CTH was designed to investigate the stability limits of current-carrying plasmas while employing strong three-dimensional magnetic field shaping.



Figure 1.3: Nested magnetic flux surfaces on the Compact Toroidal Hybrid. The color gradient represents the magnetic field strength (blue is low, yellow is high) with some magnetic field lines in white.



Figure 1.4: The sawtooth oscillation observed by the SXR diagnostic installed on CTH. The sawtooth consists of a ramp phase, a trigger of a ideal MHD instability within the plasma, and a fast collapse phase expelling thermal energy from the core of the plasma. The oscillation is clearly captured on the central channel signal. The SXR signals outside of the inversion radius reveal inverted sawteeth due to the sudden increase in thermal energy outside the core of the plasma during a sawtooth crash.

1.2 The sawtooth instability

One of the major hurdles with magnetic confinement is the stability of the confined plasma. The stability of the plasma is determined by the growth rate of infinitesimal perturbations that arise within the plasma. One technique to determine the stability of the system is to start with the ideal MHD equations and add infinitesimal perturbations to the velocity, magnetic field, electric field, etc. This technique is described in further detail in chapter 5. The primary conclusion from ideal MHD stability analysis is that the plasma is unstable if any perturbation makes the change in potential energy negative. The type of stability is classified from its free-energy source, typically from pressure and current gradients within the plasma. Instabilities within the plasma need to be avoided or minimized in order to confine the plasma long enough to produce energy from fusion reactions.

This thesis focuses on a particular tokamak phenomenon, the sawtooth oscillation, which is a periodic rearrangement of the core plasma temperature and loss of core plasma thermal confinement due to MHD instability. Sawteeth were first observed though a soft x-ray (SXR) diagnostic in the ST tokamak in 1974.⁹ Three two-color SXR cameras¹⁰ were constructed and installed on CTH to detect the signatures of sawtooth instabilities. A typical sawtoothing oscillation observed by the two-color SXR diagnostic on CTH is shown in figure 1.4. The sawtooth cycle can be divided into three distinct events: (i) the ramp phase; (ii) the initiation of the m = n = 1 instability; (iii) the crash phase. The central channel signal shown in figure 1.4 shows the ramp phase and the crash phase, but the m = n = 1 instability is not clearly evident from these signals. The SXR signals outside of the inversion radius reveal inverted sawteeth due to the sudden increase in thermal energy outside the core of the plasma during a sawtooth crash. The expulsion of the thermal energy from the plasma core during a sawtooth crash has been observed experimentally with two-dimensional images of the electron temperature.¹¹ An in-depth description of the phenomenology of the sawtooth oscillation is found in chapter 5.

Depending on the size of the inversion radius and core temperature drop, sawtooth oscillations are not inherently bad for a confined plasma. Small sawteeth can be beneficial by flushing impurities and helium ash from the core of the plasma.^{12,13} Conversely, sawteeth with a large temperature perturbations and inversion radii have many deleterious effects on tokamak discharges. Plasmas with large sawteeth are more susceptible to Edge Localized Modes (ELMs) and may lead to the degradation of core confinement. ELMs occur in short bursts leading to the relaxation in temperature, pressure, and density gradients at the plasma edge.¹⁴ Long period sawteeth can seed neoclassical tearing modes (NTMs).¹⁵ Tearing modes take the form of magnetic islands which, in the case of NTMs, are long-lived and require a seed perturbation in order to be driven unstable.¹⁶ NTMs are triggered at lower beta (the ratio of plasma pressure to the magnetic field pressure) as the sawtooth period increases limiting the efficiency of which the magnetic field confines the plasma.¹⁷ However, sawteeth with core temperature drops of a fraction of a keV and an inversion radius less than 40% of the plasma radius are thought to be tolerable in ITER.¹⁸ Therefore, the control of sawteeth are an important issue for ITER operation.

There are many good review articles summarizing different aspects of the sawtooth oscillation. An excellent, succinct review of the different theoretical models of the sawtooth oscillation is given by Wesson.³ In contrast, Hastie gives a more complete review of the sawtooth theory and experimental data up to 1997.¹⁹ Migliuolo gives a reivew of the ideal and resistive m = 1 modes in tokamaks.²⁰ The ITER physics review gives an excellent summary of the sawtooth oscillation corresponding to high temperature tokamaks and the effect of alpha particle cooling.¹⁸ A book concerning the magnetohydrodynamic stability in tokamaks was written by Zohm which provides the theoretical derivation behind the ideal MHD instability.²¹ Freidberg wrote about the stability analysis out to the fourth order and contains toroidal corrections to ideal magnetohydrodynamic stability of the sawtooth oscillation.⁸ Finally, Chapman wrote about the recent advances in controlling the sawtooth oscillation.²²

1.3 Thesis overview

The remainder of this dissertation discusses the primary diagnostic used to parameterize the sawtooth oscillations observed in CTH. Chapter 2 is a physical description of the CTH device and a description of the diagnostics used to characterize the plasmas within CTH. An example plasma discharge and typical plasma parameters used in this thesis is presented followed a short description of the equilibrium reconstruction code used to characterize the rotational transform and other plasma parameters.

Chapter 3 contains a complete discussion of the construction and calibration of the two-color SXR diagnostic installed on CTH. The chapter begins by describing the hardware structure of the internal components and the assembled camera. The signal amplifiers as well as a discussion of the design choices for each main component of the amplifier is presented. Relative calibration and measurement of the change in poloidal viewing extent of the diodes, known as the geometric factor, is addressed. Chapter 4 addresses the validity of the two-color measurement. It describes the common x-ray sources emitted from a CTH plasma and models them using an atomic code database, ADAS.²³ The chapter also presents a comparison between the observed line radiation with the simulated ionization states using ADAS. The theory of the electron temperature measurement using the continuum radiation emitted from the plasma is presented. At the end of chapter 4 is an estimation of the electron temperature using the two-color technique. These measurements are then compared to electron temperature inferred from the soft x-ray spectrometer and Spitzer's resistivity.

Chapter 5 presents the observed effect of additional three-dimensional fields have on sawtooth oscillations. It begins by deriving the ideal MHD stability theory implicating a m = n = 1 instability as being the primary factor behind the sawtooth crash. The Kadomtsev and Porcelli models of the sawtooth crash are described with an explanation of the applicability of these models to the sawteeth observed within CTH. The observational results of the addition of three-dimensional fields is presented.

Chapter 6 provides a discussion of the two-color diagnostic as well as the sawteeth observed in CTH as well as suggestions for future work.

Additionally, three appendices are included to explain derivations of some of the parameters used. Appendix A describes the derivation of the geometric factor. The geometric factor is not used in this thesis but is important for tomographic reconstructions that can be preformed with the emissivity or electron temperature data. Appendix B shows the Computer-Aided Drafting (CAD) drawings of the two-color and bolometer components. Appendix C gives a brief description of the computer codes written to analyze the data presented in this thesis.

Chapter 2

The Compact Toroidal Hybrid

The Compact Toroidal Hybrid (CTH) is a combination of a tokamak and a five-field period stellarator. CTH is designed to investigate the stability limits of current-carrying plasmas while employing strong three-dimensional magnetic field shaping. CTH can provide a stable stellarator equilibrium then may induce a plasma current through an central transformer. The CTH external magnetic field coils are discussed in this chapter as well as the ability to vary the amount of poloidal magnetic field generated by external coils relative to that from internal plasma current. What follows is an example discharge showing the creation of a stellarator plasma followed by the induction of a plasma current. The diagnostics used to measure basic plasma parameters are then discussed. The final section describes the computational equilibrium reconstructions used to further diagnose the plasma.

2.1 CTH magnet coil set

Figure 2.1 shows the CTH magnetic field coils. The five-field periodicity of the flux surfaces in CTH is due to a continuously wound Helical Field coil (HF; shown in red). The HF coil wraps around the vacuum vessel five times in the poloidal direction for every two transits in the toroidal direction. CTH uses two vertical field coils in order to produce closed flux surfaces, the Main Vertical Field coil (MVF; shown in red) and the Trim Vertical Field (TVF; shown in green). The MVF is connected in series with the HF and creates flux surfaces that are not radially centered within the vacuum vessel. The addition of the TVF shifts the flux surfaces towards center the vacuum vessel allowing for control of the major radial position of the plasma. The Shaping Vertical Field coil (SVF; shown in magenta) controls the elongation of the plasma but was not used for the data presented in this thesis.



Figure 2.1: Drawing of the magnetic field coils shown in color on CTH. The vacuum vessel is depicted in grey.

The Radial Field coil (RF; shown in dark blue) limits the vertical position of the plasma to keep it centered in the vacuum vessel throughout the discharge.

CTH has a series of ten Toroidal Field coils (TF; shown in yellow). In contrast to a tokamak, the majority of the toroidal field in CTH is generated by the HF coil. The TF coils are used to vary the ratio of the toroidal to poloidal field strength. The HF and TF are independently controlled, allowing a wide range of plasma configurations. CTH is equipped with a central solenoid coil (OH; shown in teal) used to produce a toroidal loop voltage inducing a plasma current, I_p , and generate ohmic heating. An extensive discussion of the magnetic coils on CTH is discussed by Peterson²⁴ and Stevenson.²⁵



Figure 2.2: Prior to plasma breakdown at 1.6 s, nested flux surfaces are created within CTH. Gas is then puffed into the machine and ionized by ECRH creating a stellarator plasma, outlined by the grey box. At 1.62 s, the ohmic transform is discharged producing a toroidal loop voltage, inducing a plasma current and increasing the electron temperature and density.

2.2 An example plasma discharge in CTH

Figure 2.2 is an example plasma discharge within CTH. Prior to plasma breakdown at 1.6 s, nested flux surfaces are created in CTH by energizing the magnetic field coils. Gas is then puffed in, typically hydrogen, and ionized by Electron Cyclotron Resonance Heating (ECRH) creating a stellar ator plasma with an electron temperature $\sim 20 \, \mathrm{eV}$ and a maximum density of $4 \times 10^{18} \text{ m}^{-3}$. The transform of the outermost closed flux surface during this phase of a discharge is referred to as the edge vacuum rotational transform, $t_{vac}(a)$. The edge vacuum rotational transform is used as a proxy for the amount of 3D shaping, higher values correspond to more highly shaped plasmas. CTH can vary the vacuum rotational transform from a near axisymmetric tokamak-like equilibrium, $t_{vac}(a) = 0.02$, to a non-axisymmetric stellarator equilibrium, $t_{vac}(a) = 0.35$. Figure 2.3a shows the effect that different magnetic field strengths from the HF and TF coils have on the vacuum rotational transform. To increase the vacuum rotational transform, the magnitude of the magnetic field from the HF is increased to keep $\langle |B| \rangle \approx 0.53$ or 0.64. These values of $\langle |B| \rangle$ are important to ensure that magnetic fields are resonant with the ECRH in order to ionize the gas puffed into the machine. The direction of the magnetic field produced by the TF coil either adds or subtracts to the field generated by the HF coil to decrease or increase the transform. The x-axis of figure 2.3a corresponds to normalized toroidal flux and can be thought of a radial quantity where a value of one is the Φ enclosed by the last closed flux surface. Figure 2.4a shows the three-dimensional last closed flux surface for a typical ECRH discharge in CTH. The white lines trace several field lines while the red and blue color shading indicates the strength of the magnetic field from stronger to weaker. Flux surfaces as functions of R and Z is shown in figure 2.4b for the half field period, $\phi = \frac{\pi}{10} \pm \frac{n\pi}{5}$, and in figure 2.4c for the full field period, $\phi = \frac{n\pi}{5}.$

CTH can operate purely as a stellarator, but for the discharges in this thesis the OH coil was used to induce a plasma current. The induction of plasma current resistively increases the electron temperature and further increases the electron density. In figure 2.2 the OH coil



Figure 2.3: (a) Adjusting the ratio of the TF and HF coil currents (magnetic field magnitude) allows for variation of the vacuum transform. Increasing the magnetic field strength of the HF coil (orange arrow) increases the vacuum transform. The addition of the magnetic field from the TF coil (purple arrow) will decrease the vacuum transform and if the magnetic fields are in the opposite direction the vacuum transform increases (green arrow). (b) The total rotational transform for a 35 kA discharge shown in black and the green line is the vacuum transform for this discharge.

is energized at 1.62 s. The addition of plasma current will increase the poloidal magnetic field, increasing the poloidal flux.

The poloidal field may be written as a function of the plasma current through Ampère's law. For a large aspect-ratio machine with a circular cross-section, the cylindrical approximation of the rotational transform is defined as:³

$$t_{\rm plasma}(r) = \frac{\mu_0 R_0 I_{\rm p}(r)}{2\pi r^2 B_{\phi}(r)}.$$
(2.1)

The fact that the rotational transform is proportional to the I_p results in a monotonically decreasing tokamak-like profile, shown in black in figure 2.3b. The rotational transform produced at the outermost flux surface produced by the plasma and the external coils is defined as the total rotational transform, $t_{tot}(a)$. It is related to the vacuum rotational transform and the plasma contribution to the rotational transform through:

$$\boldsymbol{t}_{\text{tot}}(a) = \boldsymbol{t}_{\text{vac}}(a) + \boldsymbol{t}_{\text{plasma}}(a).$$
(2.2)

The rotational transform from the addition of plasma current dominates the total rotational transform. In some cases for discharges in this thesis it provides up to ~ 96% of the total rotational transform. The green line in figure 2.3b is the vacuum rotational transform for that particular discharge, illustrating $t_{vac}(a) \gg t_{plasma}(a)$.

An example of magnetic surfaces with plasma current is shown in figure 2.5a. Flux surfaces as a function of R and Z are shown in figure 2.5b for the half field period, in figure 2.5c for the field period. Typical ohmic plasma parameters for discharges used in this thesis can be found in table 2.1.

2.3 CTH diagnostic suite

CTH has a set of diagnostics to measure basic plasma parameters. These diagnostics measure fluctuations in the magnetic field, electron density, x-ray emission, plasma current,

Parameter	Definition	Value
n_e	electron density	$1 - 3 \times 10^{19} \mathrm{m}^{-3}$
T_e	electron temperature	$\sim 150\mathrm{eV}$
I_p	plasma current	$20-50\mathrm{kA}$
a	plasma minor radius	$\sim 0.17\mathrm{m}$
R_0	plasma major radius	$\sim 0.74\mathrm{m}$
$\langle B \rangle$	magnetic field strength	$\leq 0.5\mathrm{T}$
$t_{\rm vac}(a)$	vacuum rotational transform	0.018 - 0.13
$\boldsymbol{t}_{tot}(a)$	total rotational transform	0.21 - 0.58

Table 2.1: Typical parameters for ohmic plasma discharges analyzed in this thesis.



Figure 2.4: Magnetic flux surfaces for an ECRH discharge in CTH. (a) Three-dimensional representation of the last closed flux surface color shaded from red (stronger) to blue (weaker) representing magnetic field strength of the last closed flux surface. The white lines trace several individual magnetic field lines. (b) The flux surfaces at the half field period. (c) The flux surfaces at the field period.



Figure 2.5: Magnetic flux surfaces for a ohmic discharge in CTH. (a) Three-dimensional representation of the last closed flux surface color shaded from red (stronger) to blue (weaker) representing magnetic field strength of the last closed flux surface. The white lines trace several magnetic field lines. (b) The flux surfaces at the half field period. (c) The flux surfaces at the field period.


Figure 2.6: Location of the diagnostics on CTH.

total radiated power, and H-alpha emission. The location of diagnostics on CTH is shown in figure 2.6.

There are 50 magnetic pickup coils mounted inside and outside the vacuum vessel that measure change in the magnetic flux. A complete description of the magnetic diagnostics are discussed by Stevenson²⁵ and Ma,²⁶ but a short description will be given. Some of the magnetic pickup coils, are simple loops wrapped around a cylindrical tube that is then bent to form a circle which are used to measure the currents they enclose, known as Rogowski coils. Since Rogowski coils only measure the current enclosed, the coils within the vacuum vessel measure only the plasma current. The Rogowski coils outside the vacuum vessel measure the current in the vacuum vessel, I_{vv} , as well as the plasma current. The measurements from the outer and inner Rogowski coils are subtracted to give I_{vv} . Knowledge of I_{vv} is important for the eddy current model of the vacuum vessel used in reconstructions (see section 2.4). Eddy currents produce large enough magnetic fields to perturb the magnetic field measurements in the magnetic diagnostics. A discussion of the eddy current model can be found in a paper by Ma²⁶ or Stevenson.²⁵

Segmented Rogowski coils are used in CTH to give a better spatial representation of the magnetic fluctuations due to the plasma and are used to reconstruct the plasma current profile. CTH has two full Rogowski coils at $\phi = 264$ degrees and $\phi = 342$ degrees and two segmented Rogowski coils located at $\phi = 96$ degrees and $\phi = 324$ degrees. Measurement of the radial flux is obtained by trapezoidal saddle coils located within the vacuum vessel at $\phi = 288$ degrees and shown in figure 2.6.

Local measurements of the poloidal and radial magnetic field are performed by six cube coils located at $\phi = 108$ degrees. The cube coils are two solenoids wrapped orthogonal to each other around a cube and are symmetric about the mid-plane. The cube coils are used to determine if the vertical location of the plasma.

A set of seven H-alpha detectors are installed on CTH at the $\phi = 252$ degree port. Halpha detectors measure the radiation due to electrons decaying from the third to the second energy level in hydrogen atom with a wavelength of 656.28 nm.²⁷ The H-alpha detectors indicate the amount of neutral hydrogen contained in the plasma, a cooler plasma will generally have a higher H-alpha signals.

A three-channel millimeter wave interferometer is installed on CTH to measure the line-integrated electron density of the plasma. One of the three chords passes through the horizontal mid-plane, while the other two are symmetric above and below the mid-plane. A complete description of the interferometer system is given by ArchMiller.²⁸

Two visible spectrometers (not shown in figure 2.6) are used to identify plasma impurity content as well as determine which ionization states are present in the plasma. One spectrometer is sensitive to wavelengths between 200 and 600 nm with a ~ 5 Å resolution. The other spectrometer is sensitive to wavelengths between 200 and 300 nm with a much higher ~ 0.7 Å resolution. The spectrometers are used to help quantify the effect of impurities on the soft x-ray measurements developed for this thesis (see section 4.1.3).

CTH has an extensive collection of emissivity diagnostics consisting of three 20-channel two-color soft x-ray cameras¹⁰ and two 20-channel bolometer cameras. The two-color soft xray cameras are all located at $\phi = 252$ degrees at poloidal angles: $\theta = 0, 60$, and 270 degrees. The design, construction, and effects of impurities are discussed in much further detail in chapter 3 and chapter 4. The two bolometer systems are based on the two-color camera design and are located at $\phi = 0$ degrees, $\theta = 270$ degrees and $\phi = 36$ degrees, $\theta = 60$ degrees shown in figure 2.6.

A soft x-ray spectrometer by Amptek²⁹ (model X-123SDD), used for central electron temperature measurements is located at $\phi = 36$ degrees, $\theta = 0$ degrees. The spectrometer primarily measures free-free bremsstrahlung radiation from electron and hydrogen or impurity ion collisions. Photons from 900 eV up to 12 keV are measured during a typical ohmic discharge on the CTH. Combining spectrometer data over many discharges is useful in determining high-Z impurities within CTH. The spectrometer also has the capability to time resolve photons over a eight specific energy ranges.

2.4 Equilibrium reconstructions using V3FIT

Equilibrium reconstruction is the process of determining MHD equilibrium properties given a set of experimental measurements. Computational reconstruction of the plasma equilibrium at specific times during the discharge is useful to estimate plasma parameters, such as the rotational transform, when diagnostics are not available to directly measure a specific parameter. All of the reconstructions contained in this thesis use the V3FIT³⁰ reconstruction code. V3FIT calculates the most probable fit to the plasma current and pressure profiles of a three-dimensional plasma. The code then minimizes the mismatch between the experimental measurements and modeled signals based on the calculated equilibrium. The V3FIT code uses VMEC^{31,32} as the MHD equilibrium solver. VMEC is an ideal MHD solver used to calculate three-dimensional nested flux surfaces in non-axisymmetric plasmas.

Only magnetic diagnostics were used in the reconstructions performed for this thesis. The primary purpose of reconstructions in this thesis is to calculate the vacuum and total rotational transform. Unfortunately, the outer Rogowski coils are no longer operational and a proxy model developed by Scott Massidda and Greg Hartwell is used to calculate the vacuum vessel current:

$$I_{\rm vv} = 1423.7 \, V_{\rm loop} + 0.048446 \, I_{\rm p} + 33.3. \tag{2.3}$$

Where $V_{\rm loop}$ is the average of four loop voltage measurements.

Chapter 3

Design of the two-color SXR diagnostic

The measurement of soft x-ray (SXR) emission from laboratory plasmas is a long standing standard diagnostic for determination of equilibrium emissivity profiles and fluctuations. Various methods are available for estimation of $T_{\rm e}$ such as x-ray pulse height analysis,^{33,34} crystal spectrometer systems,³⁵ and multi-foil filter techniques.³⁶ The SXR camera system to measure the non-axisymmetric plasmas of the Compact Toroidal Hybrid (CTH) experiment uses the two-foil method.^{37,38} Bremsstrahlung radiation is measured by two SXR cameras viewing the same plasma volume through filters having different thicknesses and hence effective photon transmission cut-off energies. The ratio of these signals is related to the line integrated value of $T_{\rm e}$ along the viewing chord of a particular channel of the camera.

The two-foil technique has been used on a number of current-carrying axisymmetric plasmas including reversed-field pinch,^{39,40,41} tokamak,^{42,43} and stellarator plasmas.⁴⁴ Similar systems using multiple (more than two) foils have also been developed recently.⁴⁵ The two-foil system developed on CTH has been designed to measure electron temperatures of order 100 eV Determining the electron temperature from the two-foil system as well as the effect of impurities on the measurement is discussed in chapter 4.

This chapter focuses on the design and calibration of the two-color SXR system on CTH. The first section goes into detail about the modular hardware components of the diagnostic internals. The chordal layout of the cameras is then discussed. Each chord of the two-color diagnostic was designed to have a minimal toroidal and poloidal sampled volumes so as to not have a large asymmetric change of flux surface structure over the viewed portion of the stellarator plasma. What follows is an in depth discussion about the design and selection of the electronic components and the measurement of attenuation of the signal from the filters. A relative diode calibration of each diode was conducted using an integrating sphere to measure the response of each diode observing an identical surface. This calibration important for the two-color diagnostic since it compares signals from different diodes to determine a spatial temperature profile. A measurement of the effect of the apparent change in slit width, the geometric factor, is then discussed.

3.1 Hardware design

The two-color SXR camera system for CTH is composed of three two-color cameras. Each 40-channel camera is mounted on a standard 4.5 inch conflat flange (CF) with two 25-pin electrical feedthroughs. Figure 3.1a shows an assembled camera. The camera system is 1.75 inch in length, highlighted by the underlined camera section. An extension of either 6.9 inch or 4.7 inch is used to ensure the camera views the entire plasma depending on which flange it is mounted on CTH .

Figure 3.1b shows the internal components from the camera section in figure 3.1a Each two-color camera has the same basic modular design, consisting of three stainless steel plates that house the essential components. This modular design allows the slits and filters to be independently changed allowing easy reconfiguration of the viewing solid angles and photon transmission properties without any major modification to the housing structure. Stainless steel tubes, machined to length, define the separation between the plates housing the internal components. A divider is welded to the middle of each tube to ensure optical isolation between the diode arrays. Dowel pins are used to align the component plates and the separation tubes, with threaded rods fixing the camera components securely to the CF flange.

The diode section in figure 3.1b consists of a custom teflon socket that holds two AXUV20ELG diode arrays purchased from the Opto Diode Corporation.⁴⁶ The 20-channel diode arrays are mounted parallel to one another with the diode centers spaced 1.5 cm. The diodes have a relatively flat responsivity of ~ 0.27 A/W for photon energies above 260 eV.



Figure 3.1: (a) A picture of an assembled camera (b) exploded view of the internal components of the camera (right). The camera is composed of three main sections, with two machined tubes defining the distance between each section.



Figure 3.2: (a) A schematic of the channel layout of the diode. The diode has two grounds (GND) but one of them is not connected (NC) to prevent a ground loop. (b) The teflon socket portion of the two-color camera. The custom teflon socket holds 42 gold-plated sockets with Kapton® wire connecting each pin to the corresponding pin on the flange. (c) A diagram of the pin layout for the two-color camera on the air-side of the flange.

Further discussion about the responsivity of the diode array and sensitivity of the two-color camera is found in section 4.7. Above the schematic of the diode section is an assembled picture of the diodes housed in the teflon socket with a stainless steel divider in the middle. The teflon socket is designed to have air gaps around the outside to allow for internal ventilation. This ventilation is necessary to not rupture the filters when CTH is brought between vacuum and atmospheric pressure. The socket houses gold–plated female pins at each diode pin to hold the diode array and provide an electrical connection. Figure 3.2a shows the layout of the pins on the socket with the diode array. The pin layout is relative to the gold dot on the lower right side of the diode array. The diode array has two grounds (GND), one of them is not connected to prevent a ground loop, discussed further in section 3.3. Solid core 26 AWG wire insulted with Kapton® is soldered to the bottom of the female pins in the socket shown in figure 3.2b. The wire is crimped to another gold-plated female pin then connected to the male pin on the flange. The length of the wire was kept to a minimum and is enclosed in a grounded stainless steel tube to reduce the amount of noise pickup before the signal from each diode reaches the amplifiers. An diagram of the pin layout on the air-side of the flange for each two-color camera is shown in figure 3.2a.

The filter/slit section in figure 3.1b holds the filter and the poloidal slit assemblies. The filters are mounted by the Lebow Company⁴⁷ onto custom frames. The transmission properties and filter thicknesses are explored further in section 4.2.1. The filters are screwed tightly to the filter/slit section plate to ensure light tightness. Above the diagram of the filter/slit section is a photograph of the bottom assembled section showing the filter plates with the filters in the center mounted directly to the plate. Each two-color camera uses two 0.2 mm wide poloidal slits. The slits are formed on a single piece of 0.127 mm thick stainless steel using a laser cutting technique. They rest in a precisely machined recessed portion of the plate to make certain that they are centered with respect to the diode. Two stainless steel bars clamp the slits to the mounting plate to ensure that they lie flat. A top (plasma facing) view of the assembled filter/slit section is shown above the cap section in figure 3.1b. The length of the stainless steel tube between the filter/slit section and the diode section as well as the poloidal slit width define the chordal view for each diode array. The chord layout is further discussed in section 3.2.

The right section in figure 3.1b is the cap section of the camera. The cap section is 0.125 inch thick with two 0.125 inch slits cut vertically defining the toroidal extent of the camera view. On top of the cap section are the nuts securing the entire assembly by the threaded rod to the flange.

Each camera is mounted on a rotatable flange to ensure that the camera is viewing a poloidal slice perpendicular to the mid-plane. To prevent light from reflecting off the flange back into the bottom of the camera, a teflon sleeve system was machined to fit snugly between the camera and the inner diameter of the rotatable flange. A picture of the teflon sleeve on the camera is shown in figure 3.1a. The sleeve system consists of two identical teflon pieces with eight thru holes as well as a groove in the bottom of each sleeve. The teflon pieces are offset by 90 degrees with the grooves facing each other to ensure light-tightness while allowing proper ventilation when CTH is brought under vacuum or vented to atmospheric pressure.

3.2 Chord layout of the two-color camera system

Each diode in the diode array has a solid angle geometry defined by the rectangular poloidal and toroidal slits. The poloidal slits were designed to be large enough in the toroidal direction as to not limit the toroidal extent. For simplicity, the viewing chord will be broken up into a poloidal slice and a toroidal extent. The poloidal slice is defined by the width of each diode and poloidal slit as well as the distance between them. Figure 3.3a shows a close up of the diode array with the individual diodes in green and the poloidal slit represented by the thick black line. The chord is defined by two lines, one from each edge of the diode in the diode array to opposite the edge of the slit. The line from the bottom of the diode will intersect the top of the slit. The angle subtended by these two lines is the poloidal extent, Ω , for that diode. Due to the nature of the diode array, as you progress towards the edge of the diode array the distance between the diode and poloidal slit increases. This increase decreases the poloidal span from ~ 3.9 degrees for a central channel, $\Omega_{\rm C}$, to ~ 2.5 degrees for an edge channel, $\Omega_{\rm E}$. At a distance from the camera to the center of the vacuum vessel, the width of the chord decreases from 2.5 to 1.5 cm for the central to the edge channel. The lines of sight for each two-color camera is shown in figure 3.3b. Each line in figure 3.3a is a line bisecting the poloidal viewing extent; therefore, there is only one line per diode. The naming convention for the two-color cameras is: SC(toroidal angle)_(poloidal angle)(thin or thick filter)-(channel). SC252_000_TN-1 refers to channel one on the two-color camera with the



Figure 3.3: (a) The poloidal angle extent for each diode is given by the two lines, one from each edge of the diode to the opposite edge of the slit. Due to the nature of the diode array and slit setup, the poloidal extent from an edge channel, $\Omega_{\rm E}$, is smaller than a central channel, $\Omega_{\rm C}$. (b) Lines of sight for the three two-color cameras with respect to the CTH vacuum vessel.



Figure 3.4: Toroidal overlap, illustrated by the grey region, of the central diode viewing chords at the mid-plane for the SC252_000_TN and SC252_000_TK cameras.

thin filter at a toroidal angle of 252 degrees and poloidal angle of 0 degrees. SC252_000_TK-1 is the thick filter camera designation at the same poloidal and toroidal angle.

The toroidal angular extent for each diode is defined by a similar procedure as the poloidal geometry. The 20-channel diode arrays are mounted parallel to one another, with the slits defined by the cap section of the camera. Since the distance between the toroidal slit and each diode on the diode array is the same, each diode will have an identical ~ 13.5 degrees toroidal view of the plasma. This corresponds to a toroidal chordal width at the center of the vacuum vessel of approximately 8 cm. Figure 3.4 shows the overlap of the toroidal viewing extent (shaded gray region) for each detector array along with the position of the magnetic axis of a typical CTH discharge. The diode arrays view essentially the same section of the plasma, with approximately 83% of their views in common. Due to the helical nature of the CTH plasmas, there is an excursion of the magnetic axis by 1 mm in the radial direction and 1 cm (not shown) in the vertical direction over the viewing volume of a SXR camera.

3.3 Two-color electronics

The signal amplifiers are constructed using four layer circuit boards with ground and power supply planes. Four layer boards are used to reduce noise by minimizing the number of potential pickup loops and to simplify the layout of electronic components. Figure 3.5 is a photograph of the completed electronics attached to the vacuum vessel. Figure 3.6 is the circuit diagram of the signal amplifiers for the two-color system. The amplifiers consist of three separate functional stages: (1) a transimpedance stage, (2) a four pole low-pass Sallen-Key filter,⁴⁸ and (3) a voltage gain stage with channel dependent amplification to allow different gain levels for core and edge channels.

The transimpedance amplifier is used to convert a current from the photodiode to a voltage for the digital acquisition device. The digital acquisition, D-tAcq,⁴⁹ hardware digitizes an analog signal input between ± 10 V. Careful consideration was given to this stage, since any noise will be amplified by the electronics. Ultra low noise voltage regulators, with a noise level of 16 and $45 \,\mu V_{\rm RMS}$, were used to keep the input voltage to the operational amplifiers (op-amp) as constant as possible. To further reduce the noise, the length of the conductive traces on the board before the transimpedance stage as short as possible.

Choosing the op-amp for the circuit was based on two considerations: (1) the slew rate must be large enough to observe 40 kHz observations and (2) the op-amp needs to have a high-gain bandwidth and low input impedance. The slew rate is the rate that the output voltage can change based on the input voltage. The slew rate imposes restrictions on the high-frequency oscillations that can be observed and is given by:

slew rate
$$\geq 2\pi V_{\rm pp} f_{\rm obs}$$
 (3.1)

 $V_{\rm pp}$ is the peak output voltage, and $f_{\rm obs}$ is the maximum input frequency. The op-amps used for the two-color system are ADA 4000-2 by Analog Devices⁵⁰ which have a slew rate of $20 \text{ V}/\mu \text{s}$. The maximum voltage output of the amplifiers is limited to $\pm 10 \text{ V}$ by the D-tAcq.



Figure 3.5: The electronics for the two-color camera system attached to the vacuum vessel. The vacuum vessel is on the right hand side of the picture with the green four-layer circuit boards shown in middle. What is shown is one amplifier board for one diode array within the two-color camera, the amplifier electronics for the other diode array is directly behind this one. The signal leaves the amplifier board through twisted pair cables to the digital acquisition device.



Figure 3.6: Circuit diagram of the transimpedance amplifier. The amplifiers consists of three separate functional stages: (1) a transimpedance stage (highlighted by the orange line), (2) a four pole low-pass Sallen-Key filter (purple line), and (3) a voltage gain stage (green line) with channel dependent amplification to allow different gain levels for core and edge channels.

Therefore, the largest input frequency the op-amp can properly amplify is ~ 318 kHz. Since common experimental signals are less than 40 kHz, the slew rate of the op-amp is more that sufficient.

The second consideration for selecting an op-amp is having a high gain bandwidth product (GBW) and low input impedance. These are primarily due to the bandwidth requirements. The bandwidth describes the range of frequencies where a minimal voltage loss in the signal will occur. This is defined up to the $-3 \, dB$ point where the signal is attenuated to 70.7% of the original value. The bandwidth of the transimpedance amplifier is given by:⁵¹

$$bandwidth = \sqrt{\frac{GBW}{2\pi R_{\rm f}(C_{\rm f} + C_{\rm i})}}$$
(3.2)

Where $R_{\rm f}$ is the feedback resistance and $C_{\rm f}$ is the feedback capacitance. The two-color amplifiers have a GBW of 5 MHz, $R_{\rm f}$ of 5.1 M $\Omega \pm 1\%$, and $C_{\rm f}$ of 0.7 pF \pm 0.1 pF. $C_{\rm i}$ is the sum of the diode, the common and differential mode impedance of the op-amp, and the stray capacitance on the circuit board ($C_{\rm i} = 40 \,\mathrm{pF} + 9.5 \,\mathrm{pF} + 0.3 \,\mathrm{pF} = 49.8 \,\mathrm{pF}$). Therefore, the transimpedance amplifiers have a bandwidth of 55.5 kHz.

Transimpedance amplifiers without a feedback capacitor will oscillate if any noise is introduced in the system. The desired value of the feedback capacitor depends on the desired gain of the stage, given by $R_{\rm f}$, and limits the bandwidth shown in equation 3.2. The equation to calculate the feedback capacitance is given by:⁵¹

$$C_f = \sqrt{\frac{C_i}{\pi R_f GBW}} \tag{3.3}$$

Using the values given above, we require a $C_{\rm f}$ of 0.8 pF. The feedback capacitance value used in the amplifier boards is 0.7 pF, limited by what was available at the time. This stage is particularly sensitive to noise and choosing the right values for $C_{\rm f}$ and $R_{\rm f}$ while retaining the bandwidth requirements is difficult. Therefore, once the design of this stage was done, the components remain the same for every diode in the two-color camera system. Further gains for low signal levels may be achieved with the third stage of the circuit.

The second stage of the circuit, shown in figure 3.6, is a four pole low-pass Sallen-Key filter with a -3 dB cutoff at ~ 50 kHz. The Sallen-Key topology follows a Butterworth filter⁵² having a flat voltage attenuation over the passband.

The signal is filtered before the analog to digital conversion of the signal in order to prevent signal aliasing. Aliasing of the signal occurs for frequencies larger than half of the acquisition frequency, called the Nyquist frequency. To prevent aliasing the signal must be attenuated to the signal to noise level of the D-tAcq at the Nyquist frequency. The D-tAcqs used for the two-color system have a signal to noise level of 86 dB and a Nyquist frequency of 250 kHz. The four pole low-pass Sallen-Key filters used in the two-color amplifiers attenuate the signal by $\sim 56 \, \text{dB}$ at 250 kHz. The transimpedance stage acts as another filter; therefore, the total measured attenuation of the two-color amplifiers at the Nyquist frequency is 80 dB. Signals with frequency greater than 250 kHz are sufficiently filtered to prevent aliasing. The measurement of the attenuation and electronic calibration is discussed further in section 3.4

The third stage of the two-color amplifiers is a non-inverting voltage gain stage. This stage was added to fine tune the output signal of the amplifier to be above 1 mV and less than 10 V before the D-tAcq. Since the edge channels have a smaller étendue and view a colder region of the plasma, it is sometimes desired to have a larger voltage gain on the edge channels compared to the central channels. The voltage gain for this stage is given by:

$$V_{\rm out} = V_{\rm in} \left(1 + \frac{R_{\rm GAIN}}{1\,\rm k\Omega}\right) \tag{3.4}$$

The voltage gain for each channel behind a thin filter is 8.5 while the thick filter channels have a voltage gain of 21. Matching the impedance across the input of the op-amp is important to minimize the noise introduced by the third stage. The impedance matching resistor, $R_{\rm Z}$, is calculated from:

$$\frac{1}{R_{\rm Z}} = \frac{1}{1\,\mathrm{k}\Omega} + \frac{1}{R_{\rm GAIN}}\tag{3.5}$$

The thick and thin filter have a R_Z of $1 \text{ k}\Omega$ or 909Ω . The calculated R_Z values from equation 3.5 are slightly lower, but were not available for purchase or would not fit on the amplifier board. Therefore, an overall gain of $4.335 \times 10^7 \text{ V/A}$ is used for every channel in the thin filters and the thick filters have a gain of $1.071 \times 10^8 \text{ V/A}$.

An important part in designing the electronics for the two-color system is to ensure proper grounding of the system while minimizing pickup loops. One of the most common pickup loops is a ground loop, occurring when two connected parts of the circuit are grounded separately. When a large change in magnetic flux is introduced to this loop, such as the OH transformer in CTH, current will be inductively driven in the loop producing noise. Figure 3.7 shows the grounding setup for the two-color amplifiers. The amplifier electronics are mounted directly to the vacuum flange to minimize lead length and noise pickup prior to the first amplifier stage. Isolation from the vacuum vessel as well as powering the electronics through an isolation transformer reduced the noise in the system from $\sim 340 \text{ mV}$ to $\sim 12 \text{ mV}$. These modifications were confirmed empirically when the magnetic fields were present but there was not a plasma within CTH.

Each diode array within a two-color camera is independently grounded to its amplifier board. The boards are then connected to a common ground connected to the power supply then to Earth ground. Diverging from the amplifier boards are the shielded twisted pair cables leading to the D-tAcq ACQ196CPCI-96-50⁴⁹ data acquisition module with a 100 k Ω input impedance operating at a 500 kHz sampling rate. The twisted pair cables are grounded to the amplifier boards while the other end at the D-tAcq is an open connection to avoid a ground loop.



Figure 3.7: Grounding configuration for the two-color camera system. Through trial and error, it was found that isolating the camera from the vacuum vessel resulted in the least amount of noise. Further reduction of noise was accomplished by avoiding potential pickup loops and shielding the signal path.

3.4 Electronic attenuation

One of the trade-offs of using a Butterworth filter is that the phase of the signal is shifted at higher frequencies. In order to compare high frequency features in the SXR signals with corresponding magnetic signals at lower frequency, the voltage attenuation, and phase shift of the filter must be corrected for. The attenuation due to the amplifier system is shown in figure 3.8. The amplifiers for the thick filters shown in black have a nearly identical phase and voltage attenuation as the amplifiers for the thin filters shown in orange. This is expected, since the transimpedance stage amplifiers have much higher bandwidth than the four-pole Butterworth filters. The theoretical curve was calculated using only the low-pass filter stage and does not include the attenuation due to the transimpedance stage.

Measurement of the attenuation was accomplished by using a function generator creating a sinusoidal wave with a peak-to-peak voltage of 204 mV. The signal was then put through a photodiode equivalent circuit to simulate the output and capacitance load from the photodiodes. The circuit converted the signal to a 204 nA sine wave with a 10 M Ω resistor and has a 40 pF capacitor between the signal and ground. An oscilloscope then compared the input signal and output signal of the amplifier system. The output waveforms were averaged over 100 samples to give an accurate representation of the phase shift and the voltage output. Each channel was measured in δf intervals of 3 kHz to 50 kHz. One channel for each the thick and thin filter amplifier boards is measured out to 250 kHz. The relatively flat voltage attenuation for both amplifier boards shown in figure 3.8a and the phase shift in figure 3.8b. The difference between the measured voltage attenuation and additional phase shift is due to the lack of the filtering due transimpedance stage in the theoretical calculation.

3.5 Bolometer system on CTH

CTH has a system of two bolometer cameras which are based on the two-color diagnostic design. A pair of diode arrays are housed within each of the bolometer system, one having



Figure 3.8: (a) Voltage attenuation for the two-color amplifiers. The orange and black lines are the measured values of the amplifier boards for the thin and thick filter cameras. The theoretical voltage attenuation of the butterworth filter, excluding the transimpedance stage, is shown in teal. (b) Phase shift for the two-color amplifiers. The orange and black lines are the measured values of the amplifier boards for the thin and thick filter cameras. The theoretical phase attenuation of the butterworth filter, excluding the transimpedance stage, is shown in teal.

a metallic filter to select a photon energy range of interest. The other diode does not have a filter, therefore measuring the total radiated power. Since the diode arrays are both mounted within the same housing, a teflon plug as well as a divider between the diodes is used to ensure light-tightness between the diode arrays. An expanded view of the internal components of the bolometer systems is found in the appendix B in figure B.1. To minimize photon reflections off of the stainless steel in the interior of the bolometer cameras, select components are coated in and anti-reflective material (black matte spray paint). Tests on the bench showed that the diode array system is light tight and has minimal reflections. The bolometer systems have a protective cover over the toroidal slits within the vacuum vessel. This cover is controlled by a rotary motion feedthrough mounted to the flange of the bolometer system. The cover is rotated out of the view of the diodes prior to plasma operations within CTH but protects the diode arrays during cleaning of the vacuum vessel.

3.6 Relative brightness calibration

The two-color diagnostic utilizes diodes that are from separate diode arrays. Therefore, it is important to know if each diode in the diode array produce the same current while observing the same source. The relative calibration measurement is important in reducing the error with the two-color measurement as well as if the diode arrays are to be used in tomography. The output of each diode array was measured using the same amplifier board and D-tAcq channel to ensure identical voltage gains. A different amplifier board was used for the bolometer systems since they have different capacitance and shunt resistance than the two-color diodes. The relative calibration was preformed using a visible light source and it is assumed that relative calibrations translate to the x-rays observed during normal operation.

A drawing of the setup for the relative calibration is shown in figure 3.9. The integrating is sphere illuminated by a white light-emitting diode (LED) provided a Lambertian surface



Figure 3.9: Schematic of the relative calibration setup.

for the measurements. A Lambertian surface has the property that the apparent brightness of the surface is the same, independent of observational direction.

A custom machined fixture mounted to integrating sphere securely held the LED. A current limiting 330Ω resistor was in series with the LED. A constant 5.079 V applied across the LED resistor circuit was monitored with a digital mulitmeter before and after each relative calibration measurement.

Each camera was assembled in their respective housing leaving the filter/slit and cap section off. Therefore, only the bare diode array to observe the entire Lambertian surface. To ensure a translation of the diode array parallel to the face of the integrating sphere, a coordinate measuring machine was used to measure its position and plane relative to the diode plane. The camera was translated stopping at several locations to measure the position of the camera relative to the Lambertian surface. The camera system was adjusted until there was less than a $\sim 5 \,\mathrm{mil}$ movement in the y or z-direction over the distance when the measurement was taken, ~ 4 in. To make certain that each diode only observed the Lambertian surface during the calibration, an opaque cloth was draped over the calibration setup. Each diode array was translated across the Lambertian surface and the resulting output is shown in figure 3.10a for each diode. A voltage offset due to the circuit was measured when no light was present and was subtracted from the output. The output for each diode was then fit with a Gaussian distribution to compute the maximum value of the voltage output for each diode. Ideally, one would measure the current output from each diode at the same distance away from the light source. Since each diode array used the housing for their respective SXR camera, that would be difficult. Therefore, each diode array was translated with a known distance between the Lambertian surface and diodes. The distance between the Lambertian surface and diodes was then increased and the measurement was preformed again. Figure 3.10b shows the measured voltage peaks for each channel in the SC252_060_TN diode array at several distances away from the integrating sphere. The resultant peaks from the respective Gaussian fits were then plotted as current vs. the distance



Figure 3.10: (a) Example output of the signal for a diode translating across the Lambertian surface during the relative calibration setup. The output is then fit to a Gaussian function (shown in red) to record the peak voltage. (b) The peak voltage measurements for the SC252_060_TN diode array at several distances away from the Lambertian surface. A linear fit on the data for each channel was performed as a function of distance.

and fitted with a linear fit. Since each diode array observed the same surface over the same solid angle, the slope of the linear fit should be the same for each diode. The differences in the y-offset (voltage offset) for each diode is the comparable value for the relative calibration. Each voltage offset was then scaled to a channel ten in the SC252_000_TN camera. A plot of all of the channels for all of the SXR camera systems is shown in figure 3.11. Every channel except for channels one and twenty are within $\approx 3\%$ of each other. Channels one and twenty are right near the edge of the diode array and the effect of producing a larger signal than every other channel is a product of the manufacturing process. These channels usually have a very low signal during normal plasma operations that they are not used in the data analysis.

3.7 Geometric factor measurement

The poloidal viewing angular extent changes for each diode within each SXR camera as described in section 3.2 and figure 3.3a. This effect is due to the geometry of a flat diode array and the placement of the slit in the center of the diode array. The edge channels are further away from the slit than the central channels resulting in an apparent decrease in slit width. This effect is called the geometric factor and is important to account for when comparing integrated emissivity measurements such as V3FIT and tomography reconstructions. A derivation of the geometric factor is given in appendix A with the primary result given by:

$$f_{g_i} = \frac{A_{\text{diode}} A_{\text{slit}}}{4\pi d^2} \cos^4 \alpha_i.$$
(3.6)

Where A_{diode} is the area of individual diode, A_{slit} is the area of the slit, d is the distance between the diode array and the slit, and α is the angle between a perpendicular line connecting the diode array and slit with a line intersection the individual ith diode and slit.

The geometric factor was measured by completely assembling one camera system with the poloidal and toroidal slits and placing it in the same setup as the relative brightness



Figure 3.11: The relative calibration measurement for the SXR diodes on CTH. Except for the edge channels, each diode has a relatively equal response if they observe the same surface.



Figure 3.12: The measured geometric factor (black) and the theoretical values (orange) for the two-color camera system.

calibration shown in figure 3.9. To reduce internal reflections, the inside of the camera was painted with black matte spray paint. An identical procedure as the relative calibration measurement to ensure a parallel translation across the Lambertian surface was used. The assembled camera was then translated across the Lambertian surface. The output of the signal was different than the relative calibration setup since the solid angle was decreased to view only a portion of the Lambertian surface. Therefore, while translating across the Lambertian surface, the current output from the diodes increased while viewing the surface, remained flat throughout the Lambertian surface, then decreased to zero when the surface was out of view. The flat-top portion of each signal was then averaged and had a standard deviation for the error. Three measurements were then averaged together and corrected for the relative calibration. Figure 3.12 shows the measured geometric factor normalized to a central channel, $f_{g_i}/f_{g_{11}}$, to eliminate A_{diode} , A_{slit} , and d from equation 3.6. The large error bars are due the small signal level output during the measurement, typically on the order of double the noise level in the signal. The measured value is in good agreement with the $\cos^4 \alpha$ dependance. The geometric factor shown in figure 3.12 is from one SXR camera. The geometric factor for the remaining cameras was then calculated based on the measured values of α for each camera.

Chapter 4

Theory, simulations, and electron temperature measurements for the two-color diagnostic

This chapter describes the theory behind the x-ray radiation emitted from a plasma, the temperature calculated from the two-color SXR diagnostic, and the complications that arise due to impurities within the plasma. The two-color diagnostic measures the continuum radiation due to bremsstrahlung radiation while trying to avoid line radiation from impurities in the plasma. A simulation of the radiation for each major ion species in the plasma and spectroscopic measurement of the charge states of impurities in the plasma is discussed. What follows is a description of the theory behind the two-color diagnostic and a calculation of the sensitive range of the camera. Finally, electron temperature estimations from the two-color diagnostic, SXR spectrometer, and a Spitzer resistivity calculation are presented.

4.1 Radiation in plasmas

Electromagnetic radiation emitted from plasmas arise from the dynamics of the charged particles. This includes transitions of electrons between energy levels in an atom, charged particles orbiting magnetic field lines, and from the collisions of particles. The primary radiation discussed in this thesis involves the acceleration of charged particles in the presence of an electric field. Figure 4.1 is an energy level diagram illustrating the common losses in energy undergone by electrons in a plasma that can result in the emission of an x-ray photon. The dashed lines represent the energy of the electrons with more energetic electrons at the top of the figure. A sea of free electrons is at the top of the diagram and an atom with bound electrons is at the bottom. Bremsstrahlung radiation occurs when a charged particle is accelerated through the electric field of another charged particle. The accelerated particle loses kinetic energy in the process which is emitted in the form of a photon. If the final state



Figure 4.1: Drawing of the energy level transitions for common x-ray radiation from electrons in a plasma. The dashed line represents atomic energy levels, increasing towards the top. χ_i is the ionization potential, E_n the energy level of the n^{th} state, and E_0 is the energy of the ground state of the atom. For free-free transitions the electron looses some energy in a interaction with a positive ion. Free-bound transitions occur when an electron becomes captured by an atom into a quantized bound state. Electrons bound to an atom transitioning between energy levels emit line radiation.

of the electron is not bound to an atom, the electron is said to be free and the process is known as free-free (ff) bremsstrahlung radiation. If the electron is captured into a bound state of an atom the process is free-bound (fb) or radiative recombination radiation. The majority of the continuum radiation in a plasma is from the acceleration of electrons simply due to their lighter mass when compared to ions. Recombination and free-free bremsstrahlung radiation is discussed further in section 4.1.1.

Line radiation is due to transitions of electrons within bound states of an ion. Since the bound states are quantized in nature, the transitions between these energy levels are distinct. Therefore, when an electron makes a transition to a lower energy level the photon emitted in the process will have discrete frequency values. In general, the electrons in the bound



Figure 4.2: An illustration of the collision of an electron with a positive ion resulting in an loss of energy due to the acceleration from the Coulomb force of the ion. This process is known as bremsstrahlung radiation and the loss of energy is emitted as a photon.

states are excited to higher energy levels from collisions with other free electrons. Once the electrons are in excited states they can spontaneous make a transition to a lower energy level, emitting photons with discrete energies (line radiation). Line radiation is discussed further in section 4.1.2.

4.1.1 Bremsstrahlung radiation

A free electron approaching a positive ion will undergo an acceleration due to the Coulomb force from the ion, $F_c = Ze^2/4\pi\epsilon_0 r^2$. Where Z is the charge of the ion, e is the electron charge, ϵ_0 is the permittivity constant, and r is the distance between the ion and electron. This process can be described as a classical two-body collision problem, shown in figure 4.2. The electron approaches a positive ion from the left with an initial velocity, v_0 , along a trajectory at a distance b away from the ion, known as the impact parameter. The path of the electron during the collision is described by:

$$r = \frac{4\pi\epsilon_0 m_e v_0^2 b^2}{Ze^2 (1 + \epsilon \cos\theta)}.$$
(4.1)

Where $m_{\rm e}$ is the mass of the electron, θ is described in figure 4.2, and the eccentricity, ϵ , is given by:

$$\epsilon = \sqrt{1 + \left(\frac{4\pi\epsilon_0 m_{\rm e} v_0^2 b}{Ze^2}\right)^2}.$$
(4.2)

The electron will accelerate towards the positive ion losing kinetic energy that is emitted in the form of a photon. Conservation of energy yields the relation:

$$\frac{1}{2}m_{\rm e}{v_0}^2 = \frac{1}{2}m_{\rm e}(\dot{r}^2 + (r\dot{\theta})^2) - \frac{Ze^2}{4\pi\epsilon_0 r}.$$
(4.3)

The radiated energy of an accelerating charge is obtained by writing the electric field as a Fourier integral, applying Parseval's theorem and integrating over the solid angles of the radiation:⁵³

$$\frac{\mathrm{d}W}{\mathrm{d}v} = \frac{e^2}{4\pi\epsilon_0} \frac{4}{3c^3} \bigg| \int_{-\infty}^{\infty} \dot{\boldsymbol{v}} e^{i\omega t} \mathrm{d}t \bigg|^2.$$
(4.4)

Where $\frac{dW}{dv}$ is the radiated energy for one collision between an electron and an ion of charge Z. The acceleration, \dot{v} , is found through the substitution of equation 4.1 into equation 4.3. The electron may undergo many collisions with a random number of ions with density, n_i , at any distance away from an ion, b. To account for this, the radiated energy is multiplied by $n_i v_0$ and integrated over all of the possible impact parameters:

$$\frac{\mathrm{d}P}{\mathrm{d}v} = n_{\mathrm{i}}v_0 \int_0^\infty \frac{\mathrm{d}W}{\mathrm{d}v}(v,b)2\pi b\mathrm{d}b.$$
(4.5)

This is known as the power spectrum and the analytical solutions were found by Kramers.⁵⁴ The total radiation per unit volume per unit frequency is found by multiplying by an electron distribution function, f(v), and integrating over the distribution of velocities:

$$\int \frac{\mathrm{d}P}{\mathrm{d}v} f(v) \mathrm{d}^3 v. \tag{4.6}$$

If the electron distribution is isotropic and in thermal equilibrium the electron distribution function is Maxwellian. The power per unit solid angle per unit frequency per unit volume for free-free bremsstrahlung is given by:⁵³

$$j_{\rm ff}(E) = 1.2161 \times 10^{-39} n_e n_i Z^2 \left(\frac{T}{\rm eV}\right)^{-1/2} \bar{g}_{\rm ff} {\rm e}^{-E/T}$$
(4.7)

The temperature, T, is in eV with the conversion from the temperature in Kelvin, $T_{\rm k}$, to eV given by, $T = k_{\rm b}T_{\rm k}/e$ where $k_{\rm b}$ is the Boltzmann constant. Classical derivations of bremsstrahlung radiation are multiplied by a Gaunt factor,⁵⁵ $\bar{g}_{\rm ff}$. The Gaunt factor accounts for quantum mechanical corrections to the impact parameter when it is small, such as the Heisenberg uncertainty principle. The corrections can be ignored for photon energies much smaller than the initial particle energy, $\hbar\omega \ll \frac{1}{2}m_{\rm e}v_0^2$. Numerical calculations of the Maxwell-averaged Gaunt factor, were done by Karzas and Latter,⁵⁶ and a non-relativistic analysis was done by Sommerfeld.⁵⁷ The values of the Gaunt factor used in this dissertation are calculated using the ADAS²³ codes.

Since a plasma contains electrons in a continuum of energy levels, the photons emitted are continuous in nature. Figure 4.3 shows three bremsstrahlung radiation calculations as a function of photon energy for a pure hydrogen plasma at a temperature of 100 eV and a density of 1.0×10^{19} m⁻³. The teal curve represents the exponential decay of bremsstrahlung radiation, setting the free-free Gaunt factor to one. The red curve is the free-free bremsstrahlung radiation with the Gaunt factor calculated by ADAS. At this temperature the inclusion of the Gaunt factor is important if measuring photons with energies less than 100 eV. The black curve in figure 4.3 is the free-free bremsstrahlung radiation plus the free-bound contribution



Figure 4.3: The spectral power for various bremsstrahlung radiation processes for a 100 eV and $1.0 \times 10^{19} \,\mathrm{m}^{-3}$ pure hydrogen plasma. The teal line assumes the classical bremsstrahlung process while the red line adds the quantum mechanical correction to the classical derivation, the free-free Gaunt factor. The black line adds the free-bound recombination contribution to the free-free bremsstrahlung. A scaled bremsstrahlung emission, $j(E) \times 1\%$, for an oxygen plasma is shown in purple.

for H⁺ calculated by ADAS. The purple curve is the scaled spectral power, $j(E) \times 1\%$, for a pure oxygen plasma at the same density and temperature.

Free-bound radiation is the process in which a free electron undergoes a transition from an initial positive energy state to a bound state of an atom. The final bound state must have discrete energies, known as recombinations edges:

$$E_{\rm n} = \frac{Z^2 R_{\rm y}}{n^2}.$$
 (4.8)

Where R_y is the Rydberg energy and n is the principle quantum number. The emission is continuous but has finite jumps corresponding to the binding energy states of the nucleus. The emission also involves the decay of excited electrons to another bound state, discussed further in section 4.1.2. H⁺ has one large recombination edge for n = 1 at 13.6 eV and much smaller edges at n = 2, 3, etc. More complicated ions such as oxygen shown in the purple line in figure 4.3, have numerous large edges.

The free-bound bremsstrahlung radiation is given by:⁵³

$$j_{\rm fb}(E) = 1.2161 \times 10^{-39} n_e n_i Z^2 \left(\frac{T}{eV}\right)^{-1/2} e^{-E/T} \left[\bar{G}_n \frac{Z^2 R_y}{T} \frac{2}{n^3} e^{Z^2 R_y/n^2 T}\right]$$
(4.9)

Determination of the strength of the recombination radiation compared to free-free radiation is achieved by evaluating the exponent in the multiplication of the exponentials, $e^{-E/T}e^{Z^2R_y/n^2T}$,

$$\frac{1}{T}\left(\frac{Z^2 R_{\rm y}}{n^2} - E\right) = \frac{Z^2 R_{\rm y}}{T} \left(\frac{1}{n^2} - \frac{E}{Z^2 R_{\rm y}}\right). \tag{4.10}$$

Free-bound bremsstrahlung radiation does not take place for $E < Z^2 R_y/n^2$; therefore, equation 4.10 will always be negative and will have an exponential decay for increasing photon energies. For $E \ll Z^2 R_y$ only very high n states will contribute in order for equation 4.10 to be negative. However, due to the n^{-3} dependance in equation 4.9 the recombination radiation becomes negligible. Conversely for $E \ge Z^2 R_y$, any n state is allowed.

To find the total radiation for a given atom within a sea of free electrons we need to add the free-free bremsstrahlung given by equation 4.7 with the free-bound bremsstrahlung given by equation 4.9. Since the Gaunt factors are on the order of one and assuming that n is low, it can be seen that if $Z^2R_y \ge T$ the recombination radiation will dominate. CTH plasmas typically have an electron temperature of ~ 100 eV. For recombination radiation to dominate, Z would have to be greater than ~ 3.3. This is a particularly low value and ionized states as high as O^{4+} (Z = 4) have been observed by spectrometers in CTH.

The total continuum radiation is the sum of the free-free and free-bound contributions for all of the species present within the plasma, $j_{\text{total}} = \sum_{i} j_{i}$. When the recombination term is negligible, the only factor that differs with the sum of equation 4.7 over each species is Z.
Therefore, the total emission is proportional to:

$$\sum_{i} n_{e} n_{i} Z_{i}^{2} = n_{e}^{2} Z_{eff}.$$
(4.11)

Where it was assumed that the electron and ion density are equal. Therefore, the sum of the bremsstrahlung radiation may be written as equation 4.7 multiplied by an effective charge, Z_{eff} . The effective charge is the amount the bremsstrahlung with the additional ions exceeds the hydrogen bremsstrahlung.

4.1.2 Line radiation

Line radiation is produced when an electron changes energy levels within the bound state of an atom. Since the energy levels are quantized, the emitted photon in the process has discrete energies. Therefore, the observed spectra has peaks at specific wavelengths.

Line radiation may occur with the excitation of an electron through a collision with another particle or a photon. One of these processes is free-bound bremsstrahlung where the free electron can be captured by the means of radiative recombination or dielectric recombination. This process leaves the electron in an excited bound state, which can decay to a lower bound state and emit line radiation. Radiative recombination occurs when the free electron is captured to one of the bound quantum states with a simultaneous emission of a photon from the excess energy of the original electron. This process is given by:

$$e^- + X^{++} \to h\nu + X^+.$$
 (4.12)

Where e^- is the electron, X^{++} is a doubly ionized state, X^+ is singly ionized, and the photon of energy is $h\nu$. This photon emitted has a continuous spectra with discrete jumps at the recombination edge where free electrons captured in the next energy level contribute to the spectra. Line radiation from radiative recombination may occur if the captured electron decays to a bound state with lower energy. For dielectric recombination the free electron excites an electron in one bound state to a higher energy state leaving the ion in a doubly excited state, $(X^+)^{**}$. The excited electron decays to a lower state producing a photon in the process:

$$e^- + X^{++} \to (X^+)^{**} \to h\nu + X^+.$$
 (4.13)

The photon emitted in this process has a discrete spectra because the excited electron transitions between bound states within the atom. The total dielectric contribution to the total line radiation is generally a few orders of magnitude less than the bremsstrahlung for temperature ranges in this thesis.

Finding the exact energy of these transitions involves solving Schrödinger's equation to get the structure of the atom which becomes a many-body problem for complex ions. Analytic solutions only exist for hydrogen, but the excitation, ionization, and dielectric recombination transitions are well known for helium. All of the line radiation transitions in this thesis were calculated using the ADAS codes and database.

4.1.3 Impurities present in CTH

CTH is kept under a vacuum with typical base pressures on the order of 10^{-8} Torr to ensure the hydrogen plasma discharges are as pure as possible. Nevertheless, impurities do exist and can be the dominate source of radiation despite consisting of less than 0.1% of the total plasma density. The main impurities expected within a typical discharge in this thesis include water coming off the walls of the vacuum vessel, residual He from glow discharge cleaning, and impurities sputtered from the carbon (C) and molybdenum (Mo) limiters. Measurements from a residual gas analyzer show elements representative of the background base pressure, oxygen (O), nitrogen (N), and argon (Ar).

Spectra between 200 to 600 nm including the ionization states of several impurities from an ohmic discharge on CTH is shown in figure 4.4. The spectra are observed from



Figure 4.4: Spectroscopic measurement of the line radiation of an ohmic hydrogen discharge indicating the presence of impurities in CTH plasmas. The measurements were taken at different toroidal locations on the vacuum vessel, red at $\phi = 180$ degrees and blue at $\phi = 252$ degrees. Spectral lines corresponding to N⁺ to N³⁺, C⁺ to C⁵⁺, and O⁺ to O⁴⁺ are observed. Image courtesy of Curtis Johnson. The blue line is from shot 15052048 and the red line is from shot 15052049.

two different toroidal locations, both viewing limiters during the discharge. The spectrum observed from $\phi = 180$ degrees is shown in red (shot: 15052049) and the spectrum shown in blue line (shot: 15052048) is from the same toroidal location as the two-color diagnostic, 252 degrees. The ionization state for each ion identified in figure 4.4 are denoted by roman numerals. Where, I is neutral, II is singly ionized (e.g. C⁺), III is doubly ionized (e.g. N²⁺), etc. Ionization states ranging from N⁺ to N³⁺, C⁺ to C⁴⁺, and O⁺ to O⁴⁺ are observed. The spectra were acquired by Curtis Johnson and the lines were identified by Curtis Johnson, Dr. Stuart Loch, and Dr. David Ennis.

4.1.4 Simulated ionization states of impurities in CTH

To estimate the ionization states for impurities present within CTH plasmas, the relative concentration of each ionization state is determined using a steady-state ionization balance model where density dependent effects of ionization from excited states were also taken into account.⁵⁸ The steady-state ionization balance uses ionization, S, and recombination rate coefficients, R, to find the fraction of the total population of the ion in each ionization state. The ionization coefficient for a transition from an ionization stage, α , to a stage with an additional electron missing, β , contains the sum of the ionization rates from each energy level within the α ionization stage multiplied by the population fraction of electrons in that energy level with respect to the ground state. The fraction of electrons in each energy level is dependent on the electron temperature and density. For low densities, $< 10^{18} \text{ m}^{-3}$, it is safe to assume that the electrons are overwhelmingly in the ground state which gives rise to the coronal equilibrium model.⁵⁹ As the density approaches $\sim 10^{19} \text{ m}^{-3}$ the excited states have larger populations and can contribute to the effective ionization, in this case the coronal model is no longer valid.⁶⁰

Calculation of the population of each ionization stage can be done if the ionization and recombination rates are known. Consider an element with three ionizations stages (α, β , and γ), the bare nucleus. The population density for ions in each state may be written as,

$$\frac{\mathrm{d}N_{\alpha}}{\mathrm{d}t} = -(S_{\alpha\to\beta} + S_{\alpha\to\gamma})N_{\alpha}N_{\mathrm{e}} + R_{\beta\to\alpha}N_{\beta}N_{\mathrm{e}} + R_{\gamma\to\alpha}N_{\gamma}N_{\mathrm{e}}$$

$$\frac{\mathrm{d}N_{\beta}}{\mathrm{d}t} = S_{\alpha\to\beta}N_{\alpha}N_{\mathrm{e}} - (R_{\beta\to\alpha} + S_{\beta\to\gamma})N_{\beta}N_{\mathrm{e}} + R_{\gamma\to\beta}N_{\gamma}N_{\mathrm{e}}$$

$$\frac{\mathrm{d}N_{\gamma}}{\mathrm{d}t} = S_{\alpha\to\gamma}N_{\alpha}N_{\mathrm{e}} + S_{\beta\to\gamma}N_{\beta}N_{\mathrm{e}} - (R_{\gamma\to\beta} + R_{\gamma\to\alpha})N_{\gamma}N_{\mathrm{e}}$$
(4.14)

Where $N_{\rm e}$ is the density of electrons and the subscripts of the ionization and recombination rate coefficients signify the transition between the respective ionization stages. Recombinations and ionization coefficients for the transitions skipping a state are much smaller than the single ion stage coefficients. Therefore, $S_{\alpha \to \gamma}$ and $R_{\gamma \to \alpha}$ are usually set to zero.

In the equilibrium steady-state approximation the particle density in each state is unchanged, i.e. dN/dt = 0. The fractional abundances are then calculated using pre-generated ionization and recombination rates at specific temperatures and enforcing conservation of the total number of particles, $N_{\text{total}} = N_{\alpha} + N_{\beta} + N_{\gamma}$. Figure 4.5a is a steady-state ionization balance plot for oxygen generated by ADAS which included density dependent effects on the rate coefficients. For a CTH plasma, with electron temperatures of 100 eV to 200 eV, it is evident that fully stripped, H-like (O^{7+}) , and He-like oxygen (O^{6+}) would be present in ionization equilibrium. The equilibrium ionization balance model overestimates the relative charge states of the impurity ions in the plasma in favor of highly ionized states due to two effects. Firstly, temperature profile effects are neglected, using a constant electron temperature over the entire plasma, and secondly, the effects of impurity ion transport and effective residence time in the plasma have been neglected. Time-dependent ionization balance calculations were done to simulate the effect that the effective impurity residence time has on the population of the ionized states. Figure 4.5b shows the time-dependent, $dN/dt \neq 0$, ionization balance calculations for oxygen. The time-dependent ionization balance was calculated using a electron temperature of $100 \,\mathrm{eV}$ and density of $10^{19} \,\mathrm{m}^{-3}$. At approximately $10^{0} \,\mathrm{sec}$ the oxygen plasma reaches the same fractional abundances as the equilibrium ionization balance shown in figure 4.5a. For a typical impurity confinement time in a tokamak plasma,³ 10^{-3} sec, the highest charge states of the O impurity will be the He-like ion states instead of the H-like states predicted by the equilibrium model. Therefore, in the equilibrium model more highly ionized impurities are predicted than are likely present in CTH plasmas.

4.2 Modeled signals of the two-color diagnostic

Two-color SXR cameras attempt to measure the high energy tail of the bremsstrahlung continuum radiation produced by the plasma. This section discusses the modeling of the measured signal based solely on the bremsstrahlung radiation. For temperatures of order 100 eV, line radiation from impurities can also be a significant component of the measured SXR signal. The effects that impurities have on the two-color diagnostic measurements is discussed further in section 4.3.

A SXR signal is produced with a photon with energy E from the plasma passes through a light blocking filter colliding with the photodiode producing an electrical signal. For a given diode, the observed signal, $S(T_{\rm e}, d)$, depends upon the responsivity of the diode, A(E), the transmission function of the filter material of thickness d, $T_{\rm Be}(E, d)$, the bremsstrahlung and line radiation $j(E, T_{\rm e})$ emitted by the plasma, and the geometric factor,³⁷

$$S(T_{\rm e},d) = \int_0^\infty dE \int_V j(E,T_{\rm e})A(E)T(E,d)\frac{d\Omega}{4\pi}dV$$

$$\simeq f_{\rm g} \int_0^\infty dE \int_l j(E,T_{\rm e})A(E)T(E,d)dl.$$
(4.15)

The bremsstrahlung continuum radiation is composed of free-free and free-bound transitions modified by the the Gaunt factor due to the H⁺ main plasma component calculated by ADAS as discussed in section 4.1.1. The filter transmission and diode absorption contributions are discussed in the following subsections.



Figure 4.5: (a) Equilibrium ionization balance plot for oxygen with an electron density of 10^{19} m^{-3} . (b) Time dependent ionization balance plot for oxygen with an electron density of 10^{19} m^{-3} and temperature of 100 eV. For a CTH plasma, with an electron temperature of 100 eV, it is evident that H-like (O⁷⁺) and He-like oxygen (O⁶⁺) would be present in ionization equilibrium. However, using typical impurity confinement times, 10^{-3} sec, the time dependent ionization calculation reveals that the equilibrium balance calculation over estimates the ionization states showing the highest charge state will be He-like oxygen.

4.2.1 Modeling filter transmission for the two-color diagnostic

An accurate model of the filter transmission reduces the error in the temperature estimation. Despite the filter being 99.8% pure beryllium, the transmission of the filter is decreased by the abundance of heavy element impurities.⁶¹ The calculation of the filter transmission as a function of energy begins by expressing the scattering of a photon by an atom. The primary interactions a low energy x-ray has with matter are scattering and photoabsorption. For long wavelengths these interactions may be expressed through an atomic scattering factor,⁶² $f_1 + if_2$. The scattering factor, f_1 , is significant for Z < 10 and high energy photons > 10 keV. Beryllium is the primary filter component (Z = 4) but the high energy photons travel through the detector, see section 4.2.3. The absorption coefficients, f_2 , have been calculated for x-rays from 0.5 to 30 keV for atomic elements from Z = 1 to 92.⁶² The probability of the photon being absorbed into a specific atom is given by the photoabsorption cross section,

$$\mu_{\rm a}(E) = 2 r_0 \lambda f_2(E) / m_{\rm a}. \tag{4.16}$$

Where r_0 is the classical electron radius, λ is the photon wavelength, and m_a is the atomic mass of the element divided by Avrogando's number. The photoabsorption cross section is calculated for the elements present in the Beryllium filters described in table 4.1. The concentrations quoted are the typical impurity manufactured specifications and the maximum allowed tolerance. An effective photoabsorption cross section is found by using the atomic mass of the atom, A_i and a calculated value of the fractional abundance by weight, x_w , through:

$$\mu_{\text{eff}} = \frac{\sum_{i} x_{i} \,\mu_{i} \,A_{i}}{\sum_{i} x_{i} \,A_{i}}.\tag{4.17}$$

The energy dependance on the filter transmission is then calculated using an attenuation function,

$$T = e^{-\rho \,\mu_{\rm eff} \,d}.\tag{4.18}$$

Where ρ is the average density of the material with filter thickness, d. The transmission functions for the two beryllium filters used in the two-color diagnostic, 1.8μ m and 3.0μ m, is shown in figure 4.6. For the thinner Be foils employed on CTH, a low energy (50 – 110 eV) transmission window is seen on the pure beryllium filter (black line). The low energy transmission window significantly alters the ratio of the modeled SXR signals by passing impurity line radiation. Combining the assumed impurities in the Be filter eliminated this low energy transmission window and shifted the transmission of filter significantly. The filter transmissions with a typical impurity (orange-red line) and the maximum allowed (teal line) content is shown in figure 4.6 for the 1.8 μ m filter. The purple line is the transmission of a 3.0 μ m beryllium filter with typical rolled impurity levels.

The two-color diagnostic on CTH initially had beryllium filters until it was discovered that carbon coated the filters in May 2015. It was found that the coating on the filters was due to the movable carbon limiters near the two-color diagnostic. During normal operation, the amount of carbon sputtered on the two-color diagnostic is minimal but a series of experiments was done in May in which the limiters were moved into the plasma significantly increasing the amount of carbon sputtered onto the two-color diagnostic. Carbon coating two-color diagnostics is not a new phenomena and has been observed on MST.⁶³ Since the thickness of the carbon coating was not uniform and not known, the filters were replaced. Simulations were performed to find a suitable replacement for the beryllium filters that would produce a greater edge channel signal while avoiding line radiation. Compound filters with a $0.5 \,\mu$ m carbon layer and either a $1.0 \,\mu$ m or $3.0 \,\mu$ m aluminum layer were selected. Unfortunately, the signal levels from the carbon aluminum filters were an order of magnitude lower than expected. The primary cause of this could be due to additional impurities in the filters, incorrect thickness levels of the materials, or an aluminum oxide growth while the filter was

	Maximum Allowed		Typical	
Element	PPM	$x_{ m w}$	PPM	$x_{\rm w}$
Be	-	99.22	-	99.55
BeO	300	0.083	130	0.036
Fe	300	0.18	130	0.080
\mathbf{C}	300	0.040	200	0.027
Ni	200	0.13	50	0.032
Ca	200	0.088	200	0.089
Al	100	0.030	50	0.015
Si	100	0.031	70	0.022
Zn	100	0.072	100	0.072
Mg	60	0.016	30	0.0081
Cu	50	0.035	40	0.028
Mn	30	0.018	5	0.0030
Cr	25	0.014	10	0.0057
Co	7	0.0045	5.6	0.0036
Pb	7	0.016	5.6	0.012
Mo	7	0.0074	5.6	0.0059
Ag	7	0.0083	5.6	0.0067
Ti	7	0.0037	5.6	0.0030

Table 4.1: Impurity concentrations quoted by the manufacture of the Beryllium filters used in the two-color system on CTH specified as the maximum allowed and typical concentrations. For each specification, the parts per million (ppm) and a calculated value of the fraction abundance, x_w , by weight are reported. The factional abundance by weight for the typical rolled specifications is used for the calculation of the filter transmission curve.



Figure 4.6: The transmission function of a $1.8 \,\mu\text{m}$ beryllium filter with and without heavy atom impurities. The black line is pure beryllium while the teal and orange lines include the typical and maximum allowed impurities from table 4.1. The teal line is the transmission of a $3.0 \,\mu\text{m}$ beryllium filter with typical impurity levels.

not in vacuum (not specified by the manufacture). The unexpected decrease in the signal levels using the carbon and aluminum filters made the temperature measurement for the non-central channel prohibitively challenging.

4.2.2 Diode absorption

Calculating the absorption of the diode is similar to calculating the transmission of a filter explained in section 4.2.1. Since the diodes have an internal 100% quantum efficiency for photons and lack a surface dead region, the calculation of the absorption is one minus the transmission. Equation 4.18 is used to calculate the transmission of silicon with a thickness of $35 \,\mu$ m. Figure 4.7 shows the transmission (black line) and absorption (maroon line) of a diode in the diode array. The responsivity of a diode, is found by⁶⁴

$$A(E) = \frac{0.98A_{\rm Si}(E)}{W}.$$
(4.19)

The 0.98 factor takes into account reabsorbance of fluorescent photons from the silicon detector.⁴⁶ $A_{\rm Si}(E)$ is the absorption of the silicon layer and W is the average energy for electron-hole pair creation in silicon,⁶⁵ 3.66 eV. The calculated absorption, shown in green, is in good agreement with the data provided by the manufacture, shown in light blue, in figure 4.7. The diodes have a flat responsivity ~ 0.27 A/W for photons up to ~ 4 keV. Photons above ~ 4 keV have enough energy to pass through the detector.

4.2.3 X-ray energy range recorded by the two-color SXR cameras

The effective energy range of the detector is found by multiplying the transmission function of the filter by the absorption function of the diode. This function, T(E)A(E), is the fraction of photons that will be detected by the diagnostic. Figure 4.8 shows the effective range of the diode arrays behind beryllium filters of thicknesses 1.8 and 3.0 μ m There is a clear deviation in the fraction of absorbed photons between the two filters for photons less



Figure 4.7: The absorption (maroon line), transmission (black line), and calculated absorption (green line) of a diode used in the two-color camera system. The calculated absorption lines is in good agreement with the data provided by the manufacture (blue line).



Figure 4.8: The filter transmission multiplied by the diode absorption, illustrating the region of the x-ray spectrum the diodes with different Be filter thicknesses detect.

than $\sim 4 \text{ keV}$. It is this difference that is used to estimate the electron temperature given in section 4.2. It is important that the bremsstrahlung over this energy range have minimal impurity and free-bound radiation contributions.

Photons above ~ 4 keV are equally detected through both filter thicknesses, due to the similar filter transmission functions. The decrease in the number of photons detected is due to the photons passing through the detector. From this plot, it is clear that the two-color camera system observes photons from ~ 0.6 keV to ~ 20 keV.

4.2.4 Calculation of the electron temperature with the two-color diagnostic

Calculation of the electron temperature begins by modeling the signals from each diode with equation 4.15. For modeling purposes it is assumed that the electron temperature can



Figure 4.9: Plots of the ratio of the simulated signals from a diode observing the same solid angle of the plasma but through filters of different thickness. The red line represents pure 1.8μ m and 3.0μ m thick beryllium. The teal and black lines have the typical impurity levels expected in the beryllium filters but the observed plasmas have a peaked or broad temperature profile. The purple line is the ratio of the aluminum and carbon filters.

be approximated using a two-power profile with radial dependence,

$$T(r) = T_0 \left(1 - \left(\frac{r}{a}\right)^{\alpha} \right)^{\beta}.$$
(4.20)

The free parameters α and β control the shape of the two-power profile. The α parameter determines the width of the profile while β determines how fast the profile drops off.

The expected signal levels, $S(T_{\rm e}, d)$, are then calculated for temperatures ranging from 10 - 200 eV for each filter thickness. If the detectors share the same line of sight, the line integral in equation 4.15 is identical. The signal ratio, $R(T_{\rm e})$, is taken to cancel out the unknown quantities such as the effective atomic number of the plasma, $Z_{\rm eff}$, and density, $n_{\rm e,i}$, leaving only a functional dependence on the effective cut-off energies of the filters and the electron temperature.

$$R(T_{\rm e}) = \frac{S(T_{\rm e}, d_1)}{S(T_{\rm e}, d_2)}$$
(4.21)

The line integrated electron temperature is determined by comparing the measured ratio to the ratio of the simulated signal. If the filter transmission is approximated as a step function, an exact form of equation 4.21 can be used.^{37,66,67} Figure 4.9 shows the ratios of the simulated signal as a function of electron temperature for four filter combinations. The effect of the low energy transmission window for the pure beryllium filters is clearly evident shown by the red curve as compared to the other curves. The robustness of this measurement derives from the fact that the ratio is weakly dependent on the temperature profile. The signal ratio using the beryllium filters using a typical impurity content within the filter and a peaked temperature profile with $\alpha = 2, \beta = 6$, is shown in black. The teal line uses the same filter transmission functions, but has a broader temperature profile with $\alpha = 4, \beta = 6$. The differences between ratios the of peak and broad profiles, $\delta T_e/T_e$ (not shown), is between 6% and 7% for temperatures ranging from 60 to 160 eV. The signal ratio for the aluminum and carbon filters used for some of the data in this thesis is shown in purple. The thicknesses of the aluminum and carbon filter was chosen to have a dynamic ratio change over the expected temperatures for CTH plasmas.

4.3 Line radiation effect on the two-color diagnostic

Impurity radiation in the energy range where the filter transmissions are different (430 eV to 4 keV for the filters used on CTH) will lead to errors in the simple interpretation of the temperature ratio. To understand and estimate the effects of the impurity line radiation and the sensitivity of these initial two-color estimates of the electron temperature, the total bremsstrahlung and line radiation is calculated using ADAS.



Figure 4.10: Calculated line radiation for each ionization state and the total bremsstrahlung power using ADAS for oxygen. The dashed lines are the total power for each ionization state and bremsstrahlung radiation (black line). The solid lines are the power multiplied by the transmission of the filter and diode absorption functions.

Figure 4.10 shows the total power from the line radiation and bremsstrahlung for one impurity, oxygen. Bremsstrahlung radiation is calculated for each ionization state as a function of electron temperature. The emissivity is then multiplied by the transmission function of the filter and the absorption function of the diode and integrated over the photon energies to give the total power. The power of the bremsstrahlung radiation was then multiplied by the fractional abundance for each ionization state explained in section 4.5. Each value is then added together to give the total bremsstrahlung due to oxygen, represented by the solid black line in figure 4.10. The total unfiltered bremsstrahlung power for oxygen is shown by the dashed black line.

In order to compare the bremsstrahlung radiation with the line radiation, the strongest 50 radiative transitions for each ionization state at a given temperature are calculated using ADAS. ADAS is also used to calculate the photon emissivity coefficient of the corresponding transition within the atom. The photon emissivity coefficient is then multiplied by the value of the transmission of the filter and diode absorption at the wavelength of the transition. All of the filtered transitions are then added together and shown as the solid lines on figure 4.10 for each ionization state. The emissivity for the ionizations states below O^{4+} are filtered sufficiently that they do not show up in this figure. The unfiltered cases are shown by the dashed lines.

For reference, the bremsstrahlung radiation due to hydrogen at 100 eV is on the order of 10^{-35} Acm³. From this, it is clear that the line radiation will dominate the x-ray region for the two-color diagnostic using the filters described in section 4.2.1. However, as discussed in section 4.5, the ionization balance model overestimates the relative charge states of the impurity ions in the plasma in favor of highly ionized states. The highest oxygen ionization state measured by spectroscopic measurements is 4+ which typically radiates at energies less than 50 eV. The filter transmission for photon energies less than 200 eV is less than 1.0×10^{-6} ; therefore, the line radiation should be sufficiently filtered out. Higher values of the effective ion charge can certainly not be ruled out and would lead to a substantial fraction of the

measured SXR signal resulting from line radiation and would require detailed knowledge of the plasma impurity content to correct the measured signal. For higher temperature plasmas, $T_{\rm e} > 1 \, \rm keV$, thicker filters^{68,69,66} can be used that avoid many of the issues.

4.4 Electron temperature measurements in CTH

The estimated electron temperature from the two-color diagnostic is shown in the bottom panel in figure 4.11 (shot: 14110709). The top panel is the plasma current for the discharge reaching a peak of ~ 40 kA and the middle panel is the line integrated estimated electron density. Since the two-color diagnostic is very sensitive to the impurity radiation, it is usually cross calibrated with another electron temperature measurement such as Thomson scattering.⁷⁰ A Thomson scattering diagnostic only provides a few temperature measurements at distinct spatial locations during the discharge. The strength of the two-color diagnostic is that it will provide a fast time resolution, 50 kHz, of the electron temperature per chord, allowing tomographic reconstructions of the electron temperature profile. A Thompson scattering diagnostic is due to be installed on CTH; therefore, the electron temperature measurements presented rely on theoretical calculations. The results are consistent with those predicted by Spitzer resistivity and the SXR spectrometer as shown in the bottom panel of figure 4.11. The following sections describe how the electron temperature estimates were derived from the SXR spectrometer and Spitzer resistivity.

4.4.1 Estimation of plasma temperature from the SXR spectrometer

The SXR spectrometer, described in section 2.3, measures photons from 0.503 to 1.3 keV over eight channels each spanning $100 \,\text{eV}$. An electron temperature estimate can be found by taking the logarithm of the ratio free-free bremsstrahlung,

$$kT_{\rm e} = \frac{E_1 - E_0}{\ln\frac{I_0 g_1}{I_1 g_0}}.$$
(4.22)



Figure 4.11: Electron temperature estimations for a plasma discharge in CTH (shot 14110709). The top panel shows the plasma current and the middle panel is the line integrated electron density. The bottom panel shows the three temperature estimates from Spitzer resistivity (red line), the SXR spectrometer (magenta diamonds), and the two-color diagnostic (black line).

Where, E is the average energy of the specific channel, I is the number of counts observed, and g_0 and g_1 are the Gaunt factors. The spectrometer observes the bremsstrahlung radiation from the core of the plasma during a ohmic discharge in CTH. Every detection observed in each channel is recorded as a square pulse and counted using an algorithm. The number of counts observed are then corrected for the attenuation of the filter by dividing by the transmission of the filter at the average energy. The number of counts are binned into 10 ms intervals before an iterative algorithm is applied to find the electron temperature. The algorithm finds the temperature based on the ratio between every channel, e.g. channel 1 and 2, channel 1 and 3, channel 3 and 4, etc. All of the temperatures calculated from each ratio during the time segment are averaged together to find the final electron temperature. Currently, for the discharge presented in figure 4.11 (shot 14110709), these measurements indicate the plasma core is approximately 100 eV.

4.4.2 Spitzer resistivity electron temperature estimation

An estimation of the electron temperature can be made from the plasma current and loop voltage. The caveats of this temperature estimation are that the loop voltage assumed to be constant across the plasma and the plasma is toroidally symmetric and with a circular cross-section, neither of which are true in CTH. From these assumptions Ohm's law can be written as:

$$J = \frac{I_{\rm p}}{\pi a^2} = \frac{E}{\eta}.\tag{4.23}$$

Where J is the current density and the resitivity, $\eta = Z_{\text{eff}}\eta_{\text{s}}$. The electric field may be written in terms of the loop voltage:

$$E = \frac{V_{\text{loop}}}{2\pi R_0}.\tag{4.24}$$

Spitzer's resistivity is given by:³

$$\eta_{\rm s} = 1.65 \times 10^{-9} ln\Lambda/T_{\rm e}^{3/2}.$$
(4.25)

Where Λ is the Coulomb logarithm, and the electron temperature has units of keV. Combining the three equations and solving for the electron temperature gives:

$$T_{\rm e} = 2.2165 \times 10^{-6} \left(\frac{R_0 \, Z_{\rm eff} \, I_{\rm p} \, {\rm ln}\Lambda}{a^2 V_{\rm loop}} \right)^{\frac{2}{3}} [\rm keV].$$
(4.26)

The electron temperature estimate from Ohm's law and Spitzer's resistivity is strongly dependent on the plasma current as evident by similarity of the red line in figure 4.11 to the plasma current trace. For the discharge presented, the Coulomb logarithm is approximated to be equal to 15 and Z_{eff} is set to 1.5.

Chapter 5

Sawtooth oscillations in the CTH

This chapter discusses the observed behavior of the sawtooth instability in CTH. The sawtooth instability has been the interest of many studies since it was discovered but its properties are still not fully understood. The beginning of this chapter presents an overview of basic sawtooth physics. Then an ideal MHD model is discussed to characterize the main instability responsible for the occurrence of sawteeth and goes into detail about MHD stability providing a more comprehensive picture of the sawtooth instability is presented. The dynamics and two theoretical models describing the onset of the sawtooth crash and the current active area of sawtooth control schemes is then discussed. What follows are the main observational results of the thesis; the effects of varying three-dimensional fields on sawtooth oscillation behavior. These observational results have been reproduced by resistive MHD simulations using NIMROD.⁷¹ Finally, the results are discussed with possible correlations to certain sawtooth theoretical models in order to explain the observed behavior.

5.1 Sawtooth oscillations

Sawteeth are periodic relaxations of the plasma temperature and sometimes plasma density and were first observed in the ST tokamak in 1974.⁹ They are primarily a core phenomenon consisting of three functional stages: (i) the ramp phase, (ii) the precursor phase, and (iii) the crash phase. The top black line in figure 5.1b is a characteristic sawtooth oscillation observed by a SXR emissivity diagnostic looking through the core of a plasma denoted by the light blue color in figure 5.1a. The ramp phase of the sawtooth oscillation begins when the core of the plasma is ohmically heated. Eventually, the increasing core temperature peaks the current profile (due to reduced resistivity) which lowers the central



Figure 5.1: (a) A schematic of a plasma column cross section to illustrate some of the key parameters of a sawtooth oscillation. (b) A basic sawtooth oscillation as observed by several SXR emissivity measurements at different radii. A core chord (black line) has the characteristic sawtooth oscillation while the chord outside the inversion radius (magenta line) displays inverted sawteeth. Further out the in the plasma, the effect of the sawtooth oscillation is smaller in amplitude and delayed in time, indicating heat pulse propagation as seen by the blue line.

safety factor enough to trigger a MHD instability within the plasma core, initiating the precursor phase. The instability grows large enough to cause a rapid crash and re-organization of flux in the core of the plasma. The crash time, t_{crash} , is on the order of 100 μ s for the sawteeth observed in CTH while the rise time, t_{rise} , is typically about four times longer. The sawtooth crash is illustrated by the gold line in figure 5.1b. The thermal energy from the core of the plasma is deposited outside the inversion radius, r_{inv} , during the sawtooth crash. This sudden increase in thermal energy increases the temperature of the surrounding region, increasing the SXR signal, leading to the inverted sawtooth behavior illustrated by the magenta line. The process then repeats itself as essentially nested magnetic flux surfaces are restored in the core of the plasma after the crash phase.

Sawtooth oscillations are primarily a core phenomena and do not, by themselves, lead to a termination of plasma confinement. The deposition of thermal energy from the core during a sawtooth crash increases the temperature out to the mixing radius. SXR emissivity measurements observe a slight increase in signal right outside the inversion radius shown as the light blue line in figure 5.1b as thermal energy propagates outward. The mixing radius is typically 25-50% of the plasma minor radius in a typical tokamak plasma.¹⁸

5.2 Ideal MHD stability

The sawtooth instability plays an important role in determining the plasma profiles and possibly prevents impurity accumulation in the plasma core, but may be detrimental if they are too large. Therefore, there has been much work to understand what drives the instability and methods of controlling the oscillation. The most common technique to investigate stability is to introduce a small perturbation to a plasma that is in equilibrium. A plasma is in equilibrium if there is no net force acting on any volume element. For a plasma in a potential V(x), this corresponds to $d\mathbf{V}/dx = 0$. Introducing a small perturbation produces a set of perturbed forces within the plasma and the direction of these forces determine the stability. If the forces restore the plasma back to its original state, the plasma is unstable.

Linear stability analysis can predict if the perturbation will be unstable and grow exponentially. The analysis starts with the ideal MHD equations:

Mass conservation:
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$
Momentum conservation: $\rho \frac{d \mathbf{v}}{dt} = \mathbf{J} \times \mathbf{B} - \nabla p$ Energy conservation: $\frac{d}{dt} \left(\frac{p}{\rho^{\gamma}}\right) = 0$ Ohm's law: $\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$ Maxwell: $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ $\nabla \cdot \mathbf{B} = 0$

A small perturbation is introduced about a zeroth-order equilibrium in terms of a Taylor expansion around $f(\mathbf{r}_0)$:

$$f(\boldsymbol{r},t) = f_0(\boldsymbol{r}) + \epsilon \tilde{f}_1(\boldsymbol{r},t) + \epsilon^2 \tilde{f}_2(\boldsymbol{r},t) + \cdots .$$
(5.2)

Linear stability analysis keeps the zeroth and first order terms, $f_0(\mathbf{r})$ and $\epsilon \tilde{f}_1(\mathbf{r}, t)$. The nonlinear terms are important to analyze the stability of the sawtooth oscillation but are ignored for now. The expansion is applied to each variable in equation 5.1 with the assumption that the product of two perturbations is negligible. Since the equilibrium quantities, $f_0(\mathbf{r})$, are time independent a Fourier decomposition of the perturbed quantities can be written as:

$$\tilde{f}_1(\boldsymbol{r},t) = f_1(\boldsymbol{r})e^{-i\omega t}.$$
(5.3)

This will replace the time derivatives in the Taylor series expansion with multiplicative factors, $-i\omega$. In general ω can be written as a sum of real, ω_r , and imaginary, ω_i , parts:

$$e^{-i\omega t} = e^{-i\omega_r t + \omega_i t} = e^{-i\omega_r t} e^{\omega_i t}.$$
(5.4)

If ω is imaginary the perturbation given by equation 5.3 will have an exponentially growing or decaying solution. Due to the existence of the exponentially growing solution, the system is unstable. Real ω leads to oscillatory solutions about the equilibrium and the system is stable.

It is convenient to write the first-order perturbation of the system in terms of a displacement vector, $\xi(\mathbf{r}, t)$:

$$\mathbf{v}_1 = \frac{\mathrm{d}\boldsymbol{\xi}}{\mathrm{d}t}.\tag{5.5}$$

A convenient choice of initial conditions for the displacement, magnetic field, pressure, and mass density at t = 0 is to set them to zero. It is assumed that $\tilde{\mathbf{v}}_1(\mathbf{r}, t = 0) \neq 0$; therefore, the plasma is in equilibrium but is moving at a small velocity. Each term in equations 5.1 is then linearized applying the Fourier decomposition discussed. Substitution of terms into the momentum equation yields an eigenvalue equation gives:

$$-\omega^2 \rho_0 \boldsymbol{\xi} = \mathbf{F}(\boldsymbol{\xi}) \tag{5.6}$$

where, the righthand side is the ideal MHD force operator and is given by:

$$\mathbf{F}(\boldsymbol{\xi}) = \frac{1}{\mu_0} (\nabla \times \mathbf{B}_0) \times \mathbf{B}_1 + \frac{1}{\mu_0} (\nabla \times \mathbf{B}_1) \times \mathbf{B}_0 + \nabla (\boldsymbol{\xi}_{\perp} \cdot \nabla p_0 + \gamma p_0 \nabla \cdot \boldsymbol{\xi}).$$
(5.7)

The subscript \perp refers to the direction perpendicular to the equilibrium magnetic field while the 0 and 1 subscripts correspond to the zeroth and first order pertubations. The force operator, $\mathbf{F}(\boldsymbol{\xi})$, has the property that it is self-adjoint.⁸ The eigenvalues, ω^2 , of a self-adjoint operator are real;⁷² hence if $\omega^2 > 0$, ω is real and the system is stable. If $\omega^2 < 0$, then ω is imaginary with both an exponentially growing and decaying solution. The exponentially growing solution yields the instability.

Taking the dot product of equation 5.6 with the complex conjugate of the displacement vector, multiplying it by 1/2, and integrating over the volume gives:

$$\frac{\omega^2}{2} \int \rho_0 |\boldsymbol{\xi}|^2 \mathrm{d} \boldsymbol{V} = -\frac{1}{2} \int \boldsymbol{\xi}^* \mathbf{F}(\boldsymbol{\xi}) \mathrm{d} \boldsymbol{V}.$$
 (5.8)

The factor of 1/2 was multiplied by equation 5.8 in order to make the left hand side proportional to the kinetic energy of the plasma, $K(\boldsymbol{\xi}^*, \boldsymbol{\xi})$ multiplied by ω^2 . The right hand side is the perturbed potential energy of the plasma, $\delta W(\boldsymbol{\xi}^*, \boldsymbol{\xi})$. Solving for ω^2 yields:

$$\omega^2 = \frac{\delta W(\boldsymbol{\xi}^*, \boldsymbol{\xi})}{K(\boldsymbol{\xi}^*, \boldsymbol{\xi})}.$$
(5.9)

Since an arbitrary displacement vector was multiplied by the eigenvalue given by equation 5.6 to derive equation 5.9, ω^2 is no longer an eigenvalue. It can be shown by decomposing $\boldsymbol{\xi}$ into

eigenfunctions that ω^2 is the weighted sum of the eigenvalues of the system.²¹ Therefore, the stability conditions that if $\omega^2 > 0$ the system is stable and if $\omega^2 < 0$ the system is unstable are still valid. Since the kinetic energy is always positive it follows that the system is ideal MHD stable if $\delta W(\boldsymbol{\xi}^*, \boldsymbol{\xi}) > 0$ and unstable if $\delta W(\boldsymbol{\xi}^*, \boldsymbol{\xi}) < 0$.

The MHD equations have been eloquently combined into a potential energy equation which determines the MHD stability of the system. Due to the complexity of $\delta W(\boldsymbol{\xi}^*, \boldsymbol{\xi})$, it is usually analyzed numerically to determine plasma stability. However, an intuitive form has been derived assuming the plasma is surrounded by a vacuum region bounded by a conducting wall. After re-arranging the terms in the potential energy equation accordingly, it may be written as the sum of three physically distinct contributions:

$$\delta W = \delta W_{\rm V} + \delta W_{\rm F} + \delta W_{\rm S}.\tag{5.10}$$

Where V refers to the vacuum, S is the surface, and F is the fluid potential energy. This is known as the "intuitive" form of the energy principle.⁷³ The full expansions of each of the surface and vacuum potential energies can be found in Freidberg's book.⁸ The vacuum potential energy is always positive and is therefore always stabilizing. Since we are concerned with the instabilities in the core of the plasma, such as sawtooth oscillations, we can assume there are no surface currents; therefore, $\delta W_{\rm S} = 0$.

The fluid term is given by:⁸

$$\delta W_{f}(\boldsymbol{\xi}^{*},\boldsymbol{\xi}) = \frac{1}{2\mu_{0}} \int_{P} \left[|\mathbf{Q}_{\perp}|^{2} + \mathbf{B}^{2} |\nabla \cdot \boldsymbol{\xi}_{\perp} + 2\boldsymbol{\xi}_{\perp} \cdot \boldsymbol{\kappa}|^{2} + \mu_{0}\gamma p |\nabla \cdot \boldsymbol{\xi}|^{2} - \mu_{0} [(\boldsymbol{\xi}_{\perp} \cdot \nabla p)(\boldsymbol{\xi}_{\perp}^{*} \cdot \boldsymbol{\kappa}) + (\boldsymbol{\xi}_{\perp}^{*} \cdot \nabla p)(\boldsymbol{\xi}_{\perp} \cdot \boldsymbol{\kappa})] - (\mu_{0}\mathbf{J}_{\parallel}/2)[\boldsymbol{\xi}_{\perp}^{*} \times \mathbf{b} \cdot \mathbf{Q}_{\perp} + \boldsymbol{\xi}_{\perp} \times \mathbf{b} \cdot \mathbf{Q}_{\perp}^{*}] \right] \mathrm{d}\boldsymbol{r}.$$

$$(5.11)$$

Where the integrand is over the plasma volume, $\mathbf{Q}_{\perp} = \nabla \times (\boldsymbol{\xi} \times \mathbf{B})$, $\mathbf{b} = \mathbf{B}/\mathbf{B}$, and the curvature vector is given by $\boldsymbol{\kappa} = \mathbf{b} \cdot \nabla \mathbf{b}$. The terms in the first line are all positive; therefore, they are stabilizing. Physically, they represent the shear and compressional Alfvèn waves

and sound waves within the plasma. The second and third lines are negative and are the two possible sources of a MHD instability. Pressure-driven instabilities occur if the second line is dominate. If the curvature vector and ∇p are parallel to each other, referred to as bad curvature, the term is negative and destabilizing. Interchange instabilities and ballooning modes are two common types of pressure-driven instabilities.

Current-driven modes occur when the dominate destabilizing term is proportional to the parallel current, $\mathbf{J}_{||}$, in the third line in equation 5.11. Current-driven modes are commonly known as kink modes because they lead to kinking of the magnetic surfaces. They are further subdivided by whether the unstable displacement perturbs the plasma surface (external kink modes) or remans internal (internal kink modes). The sawtooth oscillation occurs when an internal kink mode is driven unstable. To understand what drives the sawtooth oscillation we need to apply equation 5.11 to tokamak geometry. The first reduction of the fluid term is accomplished by assuming the plasma is a straight cylinder of length $2\pi R_0$. This configuration is known as the straight tokamak and it is convenient to express the displacement vector as a Fourier decomposition:

$$\boldsymbol{\xi}(r,\theta,\phi) = \boldsymbol{\xi}(r)e^{i(m\theta - n\phi)}.$$
(5.12)

Where m(=1, 2, ...) is the poloidal mode number and n(=1, 2, ...) is the toroidal mode number. The mode numbers are related to the periodicity of the mode. An n = 5, m = 2mode would repeat itself five times in one poloidal circumference and two times in one toroidal circumference. The reduction of δW yields an expansion of the form $\delta W = \delta W_0 + \delta W_2 + \delta W_4 + \cdots$. Each term scales as $\delta W_i \sim \epsilon^i \delta W_0$ where $\delta W_0 \sim B_0^2 R_0 \xi^2 / \mu_0$. Assuming internal modes, $\xi(a) = 0$ the first non-vanishing term is of second order:⁷⁴

$$\frac{\delta W_2}{\epsilon^2 W_0} = \int_0^a \left(\frac{n}{m} - \frac{1}{q}\right)^2 \left[r^2 \xi'^2 + (m^2 - 1)\xi^2\right] r \mathrm{d}r.$$
(5.13)



Figure 5.2: (a) A trial function for the m = 1, n = 1 internal kink mode that causes $\delta W_2 \to 0$ in a straight tokamak. (b) Top, before the q=1 surface is in the plasma. Bottom: the q=1 displacement.

Where q is the inverse of the rotational transform and increases monotonically with r in CTH (and a tokamak). For $m \ge 2$ each term in the integrand is positive and non-zero; therefore, all of the internal modes are ideal MHD stable. For the case where m = 1, if a q = 1 surface exists within the plasma, then a trial function can be constructed which causes $\delta W_2 \rightarrow 0$. Figure 5.2a shows an example trial function of $\xi(r)$ that is rigid or constant within the q = 1 surface and decreases to zero over a width of δ where q = 1. Since the trial function is flat for q < 1, equation 5.13 will be zero for m = n = 1 as $\delta \rightarrow 0$.

Since it is possible for the $\delta W_2 \to 0$ this implies that the plasma would be only marginally stable if a q = 1 surface exists within the plasma. The next lowest order contribution to the stability is of order ϵ^4 and given by:⁷⁵

$$\delta \hat{W}_4 = \frac{\delta W_4}{\epsilon^4 W_0} = \xi_0^2 \int_0^{r_1} \left[r\beta' + \frac{r^2}{R_0} \left(1 - \frac{1}{q} \right) \left(3 + \frac{1}{q} \right) \right] r \mathrm{d}r.$$
(5.14)

The ratio of the plasma pressure to the magnetic pressure is defined by β . Given that pressure profiles typically decrease monotonically to the edge of the plasma, β' is negative.

When q < 1 both terms in the integrand of equation 5.14 are negative and the plasma is unstable. Therefore, where q(r) < 1 an m = n = 1 internal kink mode can exist and is predicted to be unstable in a straight tokamak. The instability takes the form of an internal kink as shown in figure 5.2b. The top plot shows an unperturbed flux surface in the plasma core outlined in blue. For surfaces with q < 1, the instability shifts the core toroidally and poloidally as shown in the bottom plot. Since the m = n = 1 mode stability is determined by fourth order terms in ideal MHD, non-ideal effects such as finite electrical resistivity will play an important role in the sawtooth oscillation.

Derivation of the potential energy terms for a toroidal tokamak system assuming a large aspect ratio, R_0/a , low β , and circular cross section were completed by Bussac *et al.*⁷⁶ Using the same trail function as shown in figure 5.2a, they found that $\delta \hat{W}_2$ can still be zero but the fourth order ideal term is modified by toroidal effects. They derived an expression that can be written in terms of the cylindrical contribution from equation 5.14 plus a toroidal modification:

$$\frac{\delta W_{\rm f}}{W_0} = \epsilon^4 \left[\left(1 - \frac{1}{n^2} \right) \delta \hat{W}_{\rm 4C} + \frac{1}{n^2} \delta \hat{W}_{\rm 4T} \right]. \tag{5.15}$$

Where \hat{W}_{4C} is the cylindrical contribution discussed previously and \hat{W}_{4T} is the toroidal correction. In the limit where n = 1 the cylindrical contribution vanishes and the stability is only dependent on the toroidal contribution:⁸

$$\delta \hat{W}_4 \approx \frac{3n^2 r_1^4}{R_0} (1 - q_0) \left(\frac{13}{144} - \beta_p^2\right). \tag{5.16}$$

For tokamak plasmas, the primary term that drives the plasma unstable is the poloidal beta, $\beta_{\rm p}$. Many tokamak plasmas have shown experimental evidence correlating the m = n = 1kink mode and its nonlinear development with the sawtooth oscillation.⁷⁷



Figure 5.3: An illustration of SXR signals during a sawtoothing oscillation (a). The start of the oscillation begins at t_0 (green line). The current profile peaks at t_1 (red line) triggering an m = n = 1 instability followed by the sawtooth crash at t_2 (black line). The temperature profile (b) peaks as the SXR emission rises which increases the current profile leading to a decrease in the safety factor shown inside the inversion radius (c).

5.3 Phenomenology of the sawtooth oscillation

The sawtooth oscillation is a periodic rearrangement of the core plasma temperature and loss of core plasma thermal confinement due to an m = n = 1 instability. Figure 5.3a depicts a typical sawtoothing SXR signal measuring the bremsstrahlung radiation given by equation 4.7. The bremsstrahlung radiation is a function of electron density and temperature; however, for the sawtooth oscillations observed in CTH the line integrated density does not exhibit a sawtoothing pattern; hence, the SXR emissivity measurements primarily vary due to temperature fluctuations. Figure 5.3b shows the temperature profile based on a normalized radial distance from the plasma core, r/a. Figure 5.3c is the q profile, the inverse of the rotational transform, as a function of the normalized radial distance. The start of the sawtooth oscillation, t_0 , is depicted by the green line in each of the plots in figure 5.3, with a flat electron temperature and q profile for $r < r_{\rm mix}$. Ohmic heating of the plasma core will increase the core temperature profile at a later time, t_1 , increasing the bremsstrahlung radiation measured by the SXR diagnostic. The current density is related to the electron temperature, $J \approx E_0/\eta_{\rm s} \propto T_{\rm e}^{3/2}$; therefore, an increase of the core temperature profile increases the core current density. The safety factor, q(r), is related to the current density by:

$$q(r) = \frac{2B(r)}{\mu_0 R_0 J(r)}.$$
(5.17)

As the core current density increases the value of q decreases as shown by the red line in figure 5.3c. At some point the safety factor drops below one and triggers an m = n = 1 MHD mode as described by the linear analysis in section 5.2.

The amplitude of the internal kink mode grows large enough that during the last stages of its nonlinear evolution, core flux surfaces begin to reconnect leading to the rapid transport of thermal energy out of the core. The rapid loss of thermal energy is observed in the SXR emissivity measurements after t_2 lowering the core safety factor (q) and temperature. The temperature remains constant at the inversion radius which is also the q = 1 surface. The thermal energy from the core of the plasma heats the surrounding plasma outside of the inversion radius, r_{inv} , propagating out as far as the mixing radius, r_{mix} . The sawtooth instability thus limits the plasma current density profile in the core of the plasma.

5.3.1 Sawtooth crash models

The exact mechanism to determine when the m = n = 1 mode will trigger a sawtooth crash and the arrangement of the flux surfaces after a sawtooth crash is still not completely understood. Sawtooth behavior is currently thought to depend on non-ideal effects such as energetic particles and alpha heating. The first accurate prediction of the sawtooth crash time in small axisymmetric plasmas was the Kadmotsev model.⁷⁸ There are two main assumptions in the Kadmotsev model, magnetic surfaces with equal and opposite helical flux reconnect and the toroidal flux is conserved during the process. The helical flux is given by:

$$\frac{\mathrm{d}\varphi^*(r)}{\mathrm{d}r} = B_\theta - (r/R_0)B_\phi = B_\theta(1-q).$$
(5.18)

Since q > 0 in a tokamak, it is evident that the helical flux is at a maximum when q = 1 and will it be positive when q < 1 and negative when q > 1. A qualitative picture outlining the reconnection process is found in figure 5.4 showing the evolving flux surfaces at a poloidal cross section. Initially, the plasma is heated with a set of nested flux surfaces and a q = 1 surface appears within the plasma as discussed in section 5.3 and shown in green in figure 5.4a. The flux surfaces inside and outside have field lines that twist in opposite directions with respect to the q = 1 flux surface. These surfaces are represented by the flux surfaces labeled as 1 and 2. Once the safety factor is below one within the plasma an internal helical kink mode develops that tilts and displaces the flux surfaces within the q = 1 surface. The displacement leads to the formation of an x-point shown in figure 5.4b. The helical magnetic flux surfaces of equal and opposite helical flux reconnect at this location expelling the hot plasma core through the poloidally localized x-point. The mixing of the



Figure 5.4: Images of the flux surfaces at a poloidal cross section illustrating the Kadmotsev model of a sawtooth crash. The formation of a q = 1 surface represented by the green line in (a) triggers the m = n = 1 instability within the plasma. The flux surfaces 1 and 2 have equal and opposite helical flux due to field lines that twist in opposite directions with respect to the q = 1 surface. The m = n = 1 instability displaces the core of the plasma in (b) forming an X-point and starting the reconnection process. The reconnection process continues in (c) with surfaces with equal and opposite magnetic flux until all of the initial surfaces with q < 1 are annihilated and a set a nested flux surfaces form in (d).

hot plasma core near the magnetic axis with the cooler plasma results in a flattening of the temperature profile up to the mixing radius. The reconnection process continues in figure 5.4c until the flux near the original q = 1 surface has moved to the center of the plasma. After the process is done, the initial q = 1 surface becomes the final plasma axis, due to the toroidal flux conservation assumption. All of the flux surfaces initially inside of the q = 1 surface are gone and the flux surfaces return to the normal nested configuration shown in figure 5.4d. The reconnection time scale of the Kadomtsev model is derived from the magnetic pressure driving the outflow of the plasma into the new flux surfaces, given by the Bernoulli equation, and the speed at which the plasma flows into the x-point, given by Ohm's law. The estimation of the reconnection time scale or crash time of the sawtooth oscillation is approximately:

$$\tau_{\rm crash} \approx \sqrt{\tau_{\rm res} \tau_{\rm A}}$$
 (5.19)

Where the Alfvèn time scale is $\tau_{\rm A} = r_1 \sqrt{\mu_0 \rho} / B_0$ and the resistive time is $\tau_{\rm res} = \mu_0 r_1^2 / \eta$. For a CTH plasma with an electron temperature of 150 eV and density of $10^{19} \,{\rm m}^{-3}$: $\rho = 1.67 \times 10^{-8} \,{\rm kg/m^3}$, $r_1 \approx 0.05 \,{\rm m}$, assuming Spitzer's resistivity (equation 4.25) $\eta_{\rm s} = 4.26 \times 10^{-7} \,{\rm \Omega m}$, $B_0 \approx 0.5 \,{\rm T}$. Combining the values gives an approximate crash time predicted by Kadomtsev's model of $10 \,\mu$ s. Typical crash times observed in CTH are on the order of $100 \,\mu$ s. The Kadomtsev model was able to accurately predict the sawtooth crash time in small sized tokamaks but is off by a factor of ten for the three-dimensional fields in CTH. Numerical simulations using resistive MHD equations in a cylindrical equilibrium confirm the basic dynamics of the Kadmotsev model in an axisymmetric plasma.^{79,80,81,82} It was found that for larger tokamaks such as JET, the Kadmostev model predicated the crash time to be on the order of 10 ms but the observed crash time was $100 \,\mu$ s.⁸³ There is also experimental evidence that the flattening of the q profile does not always occur, and in some cases it stays below 1 for the entire sawtooth cycle.^{84,85,86,87,88} Mapping of the q = 1 magnetic surfaces by measuring the SXR emission suggested that a secondary instability interrupted the reconnection process.⁸⁹ There have also been observations of sawteeth that partially
collapse,^{90,91,92} sawteeth that only develop the m = n = 1 mode right before the crash,⁹³ and sawteeth where the m = n = 1 mode is present throughout the sawtooth cycle.^{89,94}

A more comprehensive model of the sawtooth crash phase including non-ideal effects applicable to hotter tokamaks was developed by Porcelli *et al.*⁹⁵ Their model included a criterion for the sawtooth crash trigger and the profile relaxation after. The trigger is based on a modified model of the potential energy which includes MHD effects and plasma shaping, along with kinetic effects like trapped thermal particles and collision-less fast ions. The trapped thermal particles (Kruskal-Oberman correction⁹⁶) and fast ion contributions are stabilizing if they are inside and near the q = 1 surface. The normalized potential energy in the Porcelli model is defined as:

$$\delta \hat{W} \equiv -\frac{4\delta W}{s_1 \xi^2 \epsilon_1^2 R B^2} \tag{5.20}$$

Where the subscript denotes the quantities are evaluated at the q = 1 surface, $\epsilon_1 = \bar{r}_1/R$, the shear is given by $s_1 = \bar{r}_1 q'(\bar{r}_1)$ where $\bar{r}_1 \approx r_1 \sqrt{\kappa_1}$ is the average radius. The elongation κ is defined as the ratio of the horizontal and vertical axes, b/a of the q = 1 surface. The model has three conditions, that if any are met, lead to a sawtooth crash. The three conditions include high-energy trapped particles, the diamagnetic rotation being insufficient to stabilize the mode, and a term depending on the energy drive. Two of the conditions depend on the shear of the q profile, which can be written in terms of a critical shear, $s_{\rm crit}$, for the instability:

$$s_{1} > s_{\text{crit}} = \alpha (S^{1/3} \hat{\rho})^{1/2} (\beta_{\text{i}1} R^{2} / \bar{r}_{1}^{2})^{7/12} (\bar{r}_{1} / r_{\text{n}}) (\bar{r}_{1} / r_{\text{p}})^{1/6} \approx \frac{\alpha (S^{1/3} \rho_{\text{i}})^{1/2} (\beta_{\text{i}1} R^{2})^{7/12}}{\kappa^{1/4} \sqrt{r_{1}} r_{\text{n}} r_{\text{p}}^{1/6}} \qquad (5.21)$$

Where S is the magnetic Lundquist number, $\hat{\rho}_1 = \rho_i/\bar{r}_1$, ρ_i is the thermal ion Larmor radius, β_{i1} is the ion toroidal beta, r_p is the pressure scale length, r_n is the density scale length, $\alpha = 1.5c_*^{-7/6}[\tau/(1+\tau)]^{7/12}$ where c_* is a numerical factor on order of unity, and τ is the ratio of electron to ion temperatures, T_e/T_i . During a sawtooth oscillation the shear of the q profile increases with time as the current density profile peaks. Once the shear reaches a critical value, given by equation 5.21, the m = n = 1 mode is triggered, grows rapidly inducing a sawtooth crash. Pointing out some specific variables, this theory predicts that the critical value of the shear can be lowered if the electron temperature is decreased, by increasing the elongation, or increasing the size of the q = 1 surface. The Porcelli model has been compared to experimental values in the JET tokamak with the simulated sawtooth period found to be within ~ 20% of the measured values.⁹⁷ It should be noted that although the trigger for the sawtooth crash is somewhat understood, the nonlinear dynamics of the crash phase and how the profiles change after the sawtooth crash are still an active area of research. This model was also derived for an axisymmetric tokamak and does not take three-dimensional magnetic surface shaping into account.

5.4 Control of the sawtooth oscillation

Sawteeth can be favorable for the plasma confinement applications by providing some level of central impurity control but are typically detrimental if the energy loss is too large due to large amplitude sawteeth. As tokamaks have increased in size and electron temperatures progresse towards the goal of thermonuclear fusion, sawtooth oscillations within the tokamak have increased in amplitude and are more prone to trigger Edge Localized Modes (ELMs) and Neoclassical Tearing Mode (NTM) instabilities. Therefore, it is beneficial to either avoid sawteeth oscillations all together by keeping q > 1 or to have smaller, more frequent sawteeth to decrease the energy loss in the core of the plasma.

Modifying the dynamics of the sawtooth oscillation, such as the period and amplitude, can be achieved by varying the shear of the q profile,²² changing the toroidal flow,⁹⁸ or heating the ions in the plasma.⁹⁹ Heating the electrons of the plasma through steerable Electron Cyclotron Resonance Heating (ECRH) or Electron Cyclotron Current Drive (ECCD) can be used to control the current profile; hence, controlling the q profile through equation 5.17.^{100,101} However, since the q profile evolves over the corse of a discharge, controlling the sawtooth oscillation in this manner requires real-time calculations of the q profile with active control of the ECRH launch angle during a discharge.

The sawtooth period has been observed to also increase as the toroidal rotation is increased. The toroidal rotation can be decreased when counter-NBI (Neutral Beam Injection) is used for example, which can be interpreted as the torque from counter-NBI inducing a toroidal flow which balances the intrinsic ion diamagnetic rotation of the plasma.⁹⁸

In addition, the sawtooth period has been observed to increase as the electron temperature increases.¹⁰² This can be explained by equation 5.21 of the Porcelli model, increasing the electron temperature increases α , increasing the critical shear.

The shape of the core flux surfaces has been found to significantly modify the sawtooth amplitude and period. Prior to the Porcelli model, numerical studies revealed that the internal kink is significantly destabilized by elongation.¹⁰³ Typically, higher elongation destabilizes the m = n = 1 mode leading to smaller sawteeth¹⁰³ while increasing the triangularity stabilizes the instability.¹⁰⁴ A derivation of the potential energy term including the plasma elongation found the elongation to be the dominating destabilizing term if $\Delta q = 1 - q_0$ is sufficiently small.¹⁰⁵ Sawteeth were eliminated completely above a elongation (κ) of 2.2 to 2.6 depending on the current profile and are replaced with a continuous oscillation in an axisymmetric tokamak.¹⁰⁶ A comparison of bean and oval shaped plasmas on the DIII-D tokamak showed the sawtooth period for bean shaped plasmas is approximately twice the oval case.¹⁰⁷

5.5 Sawtooth observations while changing the amount of 3D field

Currently, the theoretical computations and observations of the sawtooth stability have been completely only for axisymmetric plasmas. This thesis explores, the effects of threedimensional shaping on the sawtooth oscillation. Sawteeth and sawtooth-like behavior have



Figure 5.5: An example discharge showing the plasma current (top panel), line integrated electron density (middle panel), and the SXR signal (bottom panel). The SXR signal from the central channel exhibits clear sawtooth behavior while a signal observing the plasma right outside the inversion radius shows inverted sawteeth.

been observed on non-axisymmetric machines such as the Large Helical Device,¹⁰⁸ Compact Helical System,¹⁰⁹ and Heliotron E.¹¹⁰ The sawteeth observed in these machines are either associated with the q = 2 rational surface, n = 1, m = 2 mode, or with q profiles having a value of 1 at multiple radial locations.

Sawtoothing plasmas are observed during ohmic discharges in CTH. These plasmas in CTH are tokamak-like, with a monotonically decreasing rotational transform as shown previously in figure 2.3. Figure 5.5 shows an example CTH sawtoothing discharge. The top panel is the plasma current peaking at approximately 30 kA in the middle of the discharge. The middle panel shows the electron density to be relatively constant during the discharge at about 2×10^{19} m⁻³. The bottom panel shows two SXR signals from the SC252_000_TN twocolor camera. The central channel is a chord observing the center of the plasma exhibiting clear sawtoothing behavior. The other chord is the signal from right outside the inversion radius demonstrating inverted sawteeth.

Due to the changing values of the density, plasma current, and their radial profiles during the discharge, the analysis of the sawteeth was divided into time segments containing three to five sawteeth. To further filter the sawteeth only time segments near the peak plasma current were selected for the database. Therefore, the scatter plots presented contain data from 144 discharges in CTH each containing three to five sawtooth oscillations. The density for all of the oscillations near peak plasma current ranges from $0.6-3.5 \times 10^{19} \text{ m}^{-3}$. Each line integrated density measurement, $\int n_e dl$, was divided by the reconstructed path length to find the estimated electron density. The plasma current was systematically varied from 15.7 kA to 60.8 kA to scan a range of total rotational transform values. The vacuum rotational transform was varied to study the effects of three-dimensional shaping on the sawtooth oscillation. For the data presented in this thesis, the vacuum rotational transform is used as a proxy for the three-dimensionality of the magnetic configuration. The higher the vacuum rotational transform the more shaped the plasma. Figure 5.6 shows the variety of different rotational and total transforms in the sawtoothing database with each dot representing a different



Figure 5.6: The amount of 3D field (t(a)) as a function of edge rotational transform $(t_{vac}(a))$. 144 discharges were analyzed in this thesis with each dot representing a single discharge.

discharge. The edge rotational transform values are found using V3FIT reconstructions. The edge vacuum rotational transform, $t_{vac}(a)$, is found during the ECRH phase of the discharge (see figure 2.2) while the t(a) is during the center of the sawtoothing window selected for each discharge.

A brief summary of the major observations of this thesis are the following:

- 1. The observed sawtooth period and amplitude decrease with increasing 3D field.
- 2. There is no strong correlation between the sawtooth crash time and an increasing 3D field.
- 3. A decreasing sawtooth period and amplitude are correlated with increasing mean elongation.

- 4. The 1/1 mode observed through analysis of SXR signals does not disappear after sawtooth reconnection events, continues on throughout the cycle, and spikes in amplitude during a sawtooth crash.
- 5. NIMROD resistive MHD simulations capture a similar decreasing trend of the sawtooth cycle period with an increasing 3D field as in experiment.

Further clarification of these results follow in the subsequent sections.

5.5.1 Analysis of sawtooth parameters using CTH SXR emission

The two-color SXR cameras are used to characterize sawtooth oscillations. The amplitude, rise, and crash time for each sawtooth oscillation is calculated from the central chord for each of the three two-color SXR cameras using the thin filter because they have the highest signal levels. The amplitude, rise, and crash times is extracted from the SXR signal by determining the time of the sawtooth crash from the first derivative of the SXR signal. The amplitude, crash, and rise time were averaged together throughout the sawtoothing window for each SXR camera. The final sawtooth amplitude, rise, and crash times were found by averaging the values from the three SXR cameras together. The value of the sawtooth period is defined as the sum of the sawtooth rise and crash times. Each two-color SXR camera calculated the estimated electron temperature during the sawtoothing window, which was then averaged over the observational window. The temperature is primarily used in the estimation of the plasma resistivity through Spitzer's formula discussed in section 4.4.2.

The inversion radius is calculated by performing a singular value decomposition (SVD) with the central SXR camera, SC252_000_TN. The central camera was chosen for this calculation due to the up-down symmetry of the flux surfaces at this toroidal location. SVD separates a spatio-temporal signal, such as a matrix consisting of the signals from a multi-channel SXR diagnostic, into orthogonal temporal modes and orthogonal spatial modes.¹¹¹ Fourier decomposition can be used to identify MHD modes for cylindrical plasma geometries;⁵³ however, SVD analysis is used due to the three-dimensional nature of CTH plasmas.

Each spatial mode corresponds to a unique temporal mode allowing one to distinguish coherent structures and phenomena. Consider a spatio-temporal function, f(x, t), the expansion into two functions dependent on space or time is given by:¹¹²

$$\mathbf{f}(x,t) = \sum_{n} \mathbf{A}_{n} \mathbf{v}_{n}(x) \mathbf{u}_{n}(t)$$
(5.22)

where, the spatial components, $v_n(x)$, and the temporal components, $u_n(t)$, have the property:

$$\mathbf{v}_{\mathbf{i}}(x) \cdot \mathbf{v}_{\mathbf{j}}(x) = \delta_{\mathbf{i}\mathbf{j}},\tag{5.23}$$

$$\mathbf{u}_{\mathbf{i}}(t) \cdot \mathbf{u}_{\mathbf{j}}(t) = \delta_{\mathbf{i}\mathbf{j}}.\tag{5.24}$$

Here δ_{ij} is the Kronecker delta function. The weight, A_n , is a measure of the total contribution of each spatial and temporal function relative to the overall signal. High values of A_n correspond to the modes that are well correlated in time while random patterns in the data will have a small weight.¹¹³ This allows the rejection of uncorrelated noise within the data while keeping physical relevant phenomena. To perform SVD on multiple SXR signals, a matrix of the observational channels is constructed, $\mathbf{F}_{ij} = f(x_i, t_j)$, with $f(x_i, t_j)$ representing the signal from a single channel. The spatial and temporal modes are extracted through, $\mathbf{F} = \mathbf{VAU}^*$, where \mathbf{U}^* is the complex conjugate of the temporal mode. The vectors $\mathbf{V} = [v_1(x), v_2(x), \dots]$ and $\mathbf{U} = [u_1(t), u_2(t), \dots]$ are composed of the spatial and temporal modes given by equation 5.22.

The sawtooth oscillation is driven by ohmic heating of the core plasma peaking the temperature profile driving q < 1 which triggers an m = n = 1 instability leading to a sawtooth crash and a flattening of the temperature profile. The dynamics of the temperature profile and the m = n = 1 internal kink are captured by SVD analysis. Figure 5.7a shows the largest three modes of the SVD analysis for a single discharge (shot: 14081312). The spatial components are on the left and the temporal modes corresponding to the spatial modes are on the right with subscripts referring to the mode number. The three modes were selected based



Figure 5.7: (a) Singular value decomposition of a sawtoothing segment observed in CTH (shot: 14081312, time window: 1.6494 s to 1.6519 s). The spatial modes (left side) and the temporal modes (right side) of the singular value decomposition from the central SXR camera during a sawtoothing interval. The temporal modes have a clear rise and crash phase of the sawtooth oscillation. The third spatial mode shows the m = 1 oscillation is correlated with the sawtooth instability. (b) The weights of the spatial and temporal modes from SVD.

on the weight of modes as shown in figure 5.7b. The largest weight $W_1 = 373$ captures the spatial temporal pair (v_1, u_1) illustrating the time evolution of the sawtooth rise and crash. The addition of the second spatial mode, v_2 , to the first spatial mode captures the effect of the sawtooth oscillation on the overall emissivity profile. Prior to the addition of the spatial modes, the value of the temporal mode at a given time is multiplied by the spatial mode mapping these components back into the space-time domain, through equation 5.22. At the peak of a sawtooth oscillation, indicated by the red line in figure 5.7a the value of the first temporal mode, $u_1(t_{crash})$ is positive while the second temporal mode, $u_2(t_{crash})$, is negative. Therefore, the addition of the first two spatial modes can be written as $v_1 + (-1) \times v_2$ leading the core of the profile to peak. The second spatial profile, v_2 , changes sign outside of channels eight and thirteen relative to the central channels. Therefore, the second spatial mode will subtract from the edge of the profile further peaking the emissivity profile at the peak of the sawtooth oscillation. Since the emissivity profile is related to the temperature, this describes the peaking of the electron temperature profile prior to a sawtooth crash as described in section 5.3

After the crash, denoted by the blue line in figure 5.7a, the value of the first and second temporal modes are both positive. The addition of the first and second modes, $v_1 + v_2$, results in a flattened emissivity profile. From the clearly inverted sawtooth behavior in the temporal modes of the second mode and the effect the spatial mode has on the first spatial mode it is clear that the inversion radius can be interpolated from this mode. The inversion radius is located where the second spatial mode crosses zero. SVD analysis was preformed on each of the sawtoothing intervals for each discharge in this thesis to find the inversion radius. This is further reinforced by the reconstruction using only the first and second modes of singular value decomposition. Figure 5.8 is a contour plot of the reconstructed signal with a linear fit subtracted from each channel to suppress the overall SXR emissivity increase during the course of the discharge. The channel number of the SXR diagnostic is on the y-axis and time is on the x-axis. The figure shows an increase in the core emissivity, until a



Figure 5.8: A contour plot of the SXR signal reconstructed from the first two modes of SVD (shot: 14081312, time window: 1.6494 s to 1.6519 s, camera: SC252_000_TN).

sawtooth crash where the thermal energy is deposited outside the inversion radius increasing the signal measured by the SXR diagnostic outside of the inversion radius while decreasing the the plasma core. The inversion radius calculated from the second spatial mode is denoted by the dashed white lines.

The third spatial mode in figure 5.7a reveals the m = 1 radial fluctuation. The temporal mode component shows the m = 1 mode oscillating in time prior to reaching its maximum amplitude after the sawtooth crash. The m = 1 observed for sawteeth in CTH plasmas is discussed further in section 5.5.3.

5.5.2 Characterization of sawtooth behavior in CTH

Sawtooth oscillations observed in CTH exhibit behavior similar to those in axisymmetric tokamaks. In tokamaks, the normalized inversion radius (inversion radius divided by the minor plasma radius) is proportional to the rotational transform.¹¹⁴ Figure 5.9a shows the normalized inversion radius of the sawtooth discharges presented in this thesis as a function

of the total edge rotational transform. It is evident that the amount of core plasma effected by the sawtooth oscillation scales with the total rotational transform, similar to the behavior observed in tokamaks. However, the observed inversion surface radius does not scale strongly with the amount of 3D shaping, shown in figure 5.9b

5.5.3 Effects of three-dimensional magnetic fields on sawtooth behavior

The sawtooth oscillation can be modified through active means including heating of the electrons near the q = 1 surface, modifying the shear of the q profile, changing the toroidal flow, or heating ions in the plasma, as discussed in section 5.4. Currently, a few studies have been preformed to study the effects of plasma shaping on the sawtooth oscillation. The existing studies focus on increasing the elongation or the triangularity of the q = 1 flux surface in an axisymmetric plasma. Typically, higher elongation destabilizes the m = n = 1 mode leading to smaller sawteeth. This thesis explores the effect of increasing levels of 3D magnetic field from external coils, as quantified by the amount of vacuum rotational transform imposed, on the sawtooth oscillation.

Three discharges were selected to highlight the overall results of the entire sawtooth database presented in section 5.5. The three dischages have similar line integrated electron density, peak plasma current, and central SXR signal levels. Figure 5.10a shows the three discharges with vacuum transforms of 0.023 (black; shot: 16050455), 0.056 (orange; shot: 15102760), and 0.116 (teal; shot: 15100556). The top panel displays the plasma current of each discharge sharply increasing during the initial firing of the OH bank at 0 ms followed by a smooth increase after about 10 ms reaching approximately 23 kA for each discharge then decreasing to zero. The middle panel is the line integrated electron density for each discharge which is calculated using the plasma width determined by V3FIT reconstructions during the middle of each interval of three to five sawteeth. The bottom panel displays a central SXR chord (SC252_000_TN-10) observing the core of the plasma from the camera at the mid-plane. To examine the SXR signals in more detail, figure 5.10b shows a zoomed view of the sawteeth



Figure 5.9: (a) A plot of the normalized inversion radius on the y-axis and the total rotational transform on the x-axis. The normalized inversion radius for sawtoothing plasmas in CTH scales with the total rotational transform. This behavior is also observed in tokamaks. (b) The normalized inversion radius plotted vs. the vacuum rotational transform. The normalized inversion radius appears to be unaffected by changes in the vacuum rotational transform.

for each case. Since the sawteeth dynamics change as the plasma current and density evolves, the signals shown are 3 ms after the start of sawteeth for each discharge illustrated by the green line. A linear fit was subtracted for each case to subtract the equilibrium and were also shifted in time to line up the first sawtooth oscillation for each discharge. Each of the SXR signals in figure 5.10b have a diamond after the crash of the sixth sawtooth oscillation. It is clear that the time required for six sawtooth oscillations is longest for the low vacuum transform case and increasing $t_{vac}(a)$ decreases the sawtooth period. Additionally, with the lowest $t_{vac}(a)$ applied, the amplitude of the sawtooth oscillation is approximately 13 nA in contrast to the 8 nA amplitude for the high vacuum transform discharge.

Since the dynamics of the plasma current, resistivity, density, and flux surface shape evolve over the course of a discharge the sawtoothing parameters also vary. For the sawtoothing discharges observed in CTH the period and amplitude tend to increase as the shot evolves with time. To illustrate how the sawtooth parameters evolve with time, a running average consisting of eight sawtooth periods for the shots displayed in figure 5.10 is shown in figure 5.11a. The running average for the low (black line) and medium (orange line) vacuum transform cases was preformed until the plasma current started to decrease after reaching approximately 23 kA. The running average for the high vacuum transform case (teal line) ceased at the 22 ms mark in figure 5.10 due to a noticeable hesitation in the sawtooth oscillation. It is clear that the sawtooth period for each plasma discharge increases with time, while the highest vacuum transform case increases in sawtooth period only to decrease after 4 ms. The effect of the additional three-dimensional fields on the sawtooth period is also apparent, it is clear that the increase of vacuum transform decreases the sawtoothing period.

Figure 5.11b shows a running average plot of eight sawtooth amplitudes for the three discharges. The average sawtooth amplitude increases with time for each of the three discharges, similar to the trend in the sawtooth period. The sawtooth amplitude in the high vacuum transform case appears to flatten at the same time the period of the oscillation starts to decrease. The exact mechanism behind this is not known but a similar behavior is



Figure 5.10: (a) The plasma current (top plot), line integrated electron density (middle plot), and central SXR channel for three discharges with different vacuum rotational transform. (b) A magnified portion of the SXR signals 3 ms after the onset of sawteeth for each vacuum rotational transform. Notice as the fractional transform increases the frequency of the sawtooth oscillation increases along with a decrease in the sawtooth amplitude.

observed in other high vacuum transform discharges in CTH. From the plots shown in figure 5.11 it is evident that increasing the vacuum transform leads to smaller, more frequent sawteeth.

The decrease of the sawtooth period as a function of vacuum rotational transform is observed throughout the entire sawtooth database as shown in figure 5.12a. It is observed that the sawtooth period decreased by a factor of 2 as the edge vacuum rotational transform increased from 0.02 to 0.14. From the dynamics of the sawtooth over the course of the discharge shown in figure 5.11a it is clear that the sawtooth period is dependent on the time chosen during the discharge. Even though the sawtoothing portion analyzed for each discharge was taken during a sawtoothing interval nearest to the peak plasma current of the discharge, scatter in the data is still expected. The sawtooth period is the sum of the linear rise time due to the ohmic heating of the core of the plasma and the crash time due to the nonlinear reconnection dynamics of the m = n = 1 kink-tearing instability. A decrease in sawtooth period is either due to a decrease in the rise time, the crash time, or some combination of the rise and crash time. Figure 5.12b shows the crash time for the sawtoothing discharges presented in this thesis. The sawtooth crash time appears to be unaffected by the amount of three-dimensional shaping. Therefore, it appears that the linear growth rate of the m = n = 1 mode could be changed by the amount of three-dimensional fields imposed on the plasma.

In axisymmetric tokmaks, increasing the mean elongation was found to destablize the m = n = 1 mode leading to smaller, more frequent sawteeth.¹⁰³ Plasmas within CTH are non-axisymmetric having the elongation, κ , vary depending on the amount of vacuum transform imposed. The elongation typically varied from 1.76 to 1.15 for the full and half field periods at low vacuum transform, $t_{\rm vac} < 0.03$, while at high vacuum transform, $t_{\rm vac} > 0.10$ the elongation would vary between 2.4 at the full field period and 1.32 at the half field period. Therefore, a mean elongation, $\bar{\kappa}$ of the outer most flux surface is used to compare the 3D shaping to the axisymmetric tokamak plasmas. It is assumed that the mean elongation at



Figure 5.11: (a) A running average of eight sawtooth periods for three sawtoothing discharges. The increase of three-dimensional field decreases the overall sawtooth period. For the 0.023 and 0.056 vacuum transform cases, the sawtooth period increases with time but plateaus as the plasma current peaks. (b) A running average of eight sawtooth periods for three sawtoothing discharges. The increase of three-dimensional field decrease the overall amplitude.



Figure 5.12: (a) Sawtooth period systematically decreases with three-dimensional magnetic shaping. Each dot is the averaged sawtooth period for three to five sequential sawtooth oscillations during a discharge. A similar behavior is seen in the high vacuum transform case before the period decreases further. (b) The sawtooth crash time appears to be unaffected by the amount of three-dimensional shaping.

the outermost flux surface directly translates to the elongation at the q = 1 surface. The mean elongation was calculated at the last closed flux surface with V3FIT reconstructions using the volume of the plasma, toroidally averaged cross-sectional area, and the surface area of the plasma.¹¹⁵ Figure 5.13a shows the period as a function of vacuum rotational transform color coated with ranges of mean elongation. Shorter period sawteeth are observed at higher levels of mean elongation. Throughout the database, the large amplitude sawteeth are not observed with high levels of three-dimensional magnetic shaping. The decreasing sawtooth amplitude is also correlated with increasing mean elongation as shown in figure 5.13b.

The entire database of sawtoothing discharges presented in this thesis have an average plasma current from 15.7 to 60.8 kA while also varying the line integrated electron density from 0.6 to $3.5 \times 10^{19} \,\mathrm{m}^{-3}$. The rotational transform is dominated by the contribution due to the plasma current which may have a profound effect on the sawtooth period and amplitude. Figure 5.14a is the sawtooth period and figure 5.14b is the sawtooth amplitude versus edge vacuum rotational transform with ranges of plasma current binned into three ranges (the distributions are not equal in range because of the low number of discharges at high plasma current). Despite having a large distribution of plasma currents throughout the database, the effect on sawtooth period and amplitude observed with edge vacuum rotational transform is independent of the amount of plasma current. If you single out a range of plasma current they all exhibit the a decrease in sawtooth period with increasing edge vacuum rotational transform. The density is known to have a weak effect on the sawtooth period.¹⁰⁴ Therefore, the sawtooth period and amplitude are plotted versus edge vacuum rotational transform in figure 5.15 with the data points color coated based on the measured line integrated electron density. It is clear in figure 5.15a that for edge vacuum rotational transforms less than 0.05 there is a small density dependance on the sawtooth period. The higher density points are at a higher sawtooth period (greater than $0.5 \,\mathrm{ms}$) while the lower electron densities (less than $1.5 \times 10^{19} \,\mathrm{m^{-3}})$ have a range of sawtooth period from 0.3 to 0.55 ms. However, as the vacuum rotational transform increases the scatter of sawtooth period decreases and converges down to approximately 0.35 ms. From the plots shown in figure 5.14 and figure 5.15 it is clear that the change in electron density or plasma current have little or no effect on the sawtooth period and amplitude. Therefore, it is clear that the edge vacuum rotational transform is the dominate contribution to the change sawtooth dynamics observed within CTH.

The dynamics of the m = 1 mode during sawtooth oscillations are different for the 3D shaped plasmas compared to axisymmetric tokamaks. In axisymmetric tokamaks, the m = 1 mode exhibits a growing sinusoidal-like oscillation reaching the maximum amplitude prior to the crash. The mode is then quenched following a sawtooth crash event. However, for the discharge presented in figure 5.7a, the m = 1 mode was present during the entire sawtoothing portion and did not exhibit a clear growing sinusoidal pattern. A lack of a growing sinusoidal-like oscillation of the m = 1 mode is observed the three discharges present in figure 5.10. Figure 5.16 shows the third temporal mode of SVD analysis, u₃, illustrating the m = 1 oscillation with time. Above the third temporal mode for each discharge is the corresponding central SXR signal. The magnitude of the m = 1 mode peaks during each sawtooth crash event and is present throughout the entire cycle for each of the discharges. A Fourier transform of the u_3 mode reveals the dominate frequency of the m = 1 mode is identical to the sawtoothing frequency. The temporal mode is not equal to zero for an extended amount of time; therefore, the m = 1 oscillation is always present during the plasma discharge suggesting that a full Kadomtsev-like reconnection does not take place during the sawtooth crash events.

Section 5.3 explains how the linear growth during the sawtooth rise is dependent on the ohmic heating rate of the plasma core. The decrease in rise time with increasing levels of three-dimensional fields could simply be explained by a systematic increase in the ohmic heating within the q = 1 surface as $t_{\text{vac}}(a)$ also increased. An estimate of the ohmic heating rate inside the q = 1 surface can be made with experimentally measured values. The q = 1 surface in CTH discharges can be indentified by summing the vacuum rotational transform and rotational transform due to the plasma current, $t_{\text{vac}} + t_p = 1$. The t_p can be



Figure 5.13: (a) Increasing the vacuum rotational transform inherently increases the mean elongation of the flux surfaces. Shorter period sawteeth are observed at higher levels of mean elongation. (b) Large amplitude sawteeth are not observed at high levels of mean elongation or vacuum transform.



Figure 5.14: (a) The sawtooth period plotted with varying levels of edge vacuum rotational transform binned into discrete ranges of plasma current. (b) The sawtooth period amplitude with varying levels of edge vacuum rotational transform binned into discrete ranges of plasma current.



Figure 5.15: (a) The sawtooth period plotted with varying levels of edge vacuum rotational transform binned into discrete ranges of line integrated electron density. (b) The sawtooth period amplitude with varying levels of edge vacuum rotational transform binned into discrete ranges of line integrated electron density.



Figure 5.16: The oscillation of the m = 1 instability extracted by the third temporal mode, u_3 of SVD for three discharges at different levels of edge vacuum rotational transform. Above each of the third temporal modes is the central SXR signal to illustrate the peaking in magnitude of the m = 1 oscillation during a sawtooth crash. The m = 1 mode is present throughout the entire discharge for each case.

approximated by a straight cylindrical approximation:

$$\epsilon_{\rm p} = \frac{R_0 B_\theta(r)}{r B_\phi(r)} = 1 - \epsilon_{\rm vac}.$$
 (5.25)

The poloidal magnetic field, B_{θ} , can be found from Ampère's law, $2\pi r B_{\theta} = \mu_0 I_p$. The toroidal magnetic field, B_{ϕ} , is found by averaging the reconstructed toroidal magnetic field on the flux surface closest to the inversion surface determined by singular value decomposition of the SXR signals discussed earlier. Combining equation 5.25 with the poloidal magnetic field written in terms of the plasma current gives:

$$1 - \iota_{\rm vac} = \frac{\mu_0 I_{\rm p} R_0}{2\pi r^2 B_{\phi}}.$$
 (5.26)

Therefore, the average current density inside the t = 1 surface is given by:

$$\frac{2B_{\phi}}{\mu_0 R_0} (1 - t_{\rm vac}) = \frac{I_{\rm p}}{\pi r^2} = J_{\rm z1}.$$
(5.27)

Using this estimate of the central current density, the ohmic heating rate inside the q = 1 surface is estimated by $P_{\text{OH}} = \eta_s J_{z1}^2$, where the Spitzer resistivity is calculated from the estimated electron temperature measured by the two-color measurement. Figure 5.17a shows the estimated ohmic heating power within the q = 1 surface for the entire database. It appears that there is no strong correlation with the estimated ohmic heating inside the q = 1 surface and the vacuum transform. Therefore, the decrease in sawtooth period is most likely an effect due to increasing the three-dimensional fields and not decreasing the ohmic heating rate within the q = 1 surface can also be found by calculating the sawtooth ramp rate multiplied by the density and relating it to the ohmic heating power. The heating rate of a sawtooth oscillation is related to the ramp rate through the one-dimensional electron heat balance equation from

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$$\frac{3}{2}\frac{\partial}{\partial t}n_{\rm e}T_{\rm e} = \frac{1}{r}\frac{\partial}{\partial r}r\chi_{\rm e}\frac{\partial}{\partial r}T_{\rm e} + \eta J_{\rm z}^2 - Q_{\rm ei} - Q_{\rm R}$$
(5.28)

Where χ_e is the electron thermal conductivity perpendicular to the magnetic field, ηJ_z^2 is the ohmic heating rate with J_z as the toroidal current density and η the resistivity, Q_{ei} is the electron-ion energy transfer rate, and Q_R is the impurity radiation loss rate. The time-rate of change of the electron temperature is assumed to be related to the amplitude of the sawtooth oscillation observed by the SXR diagnostic divided by the rise time of the sawtooth oscillation. The assumption is derived from the concept that the SXR diagnostic measures bremsstrahlung radiation which depends on the electron temperature and density. The change in the measured SXR signal is due to variations in the electron temperature because the measured line averaged electron density during a sawtoothing discharge in CTH does not change. Therefore, the left-hand side can be written as $(3n_e/2)(\partial T_e/\partial t)$.

Due to the flattening of the temperature profile after a sawtooth crash, the transport term, $\partial T_e/\partial r$, is small and is ignored. While the electron-ion energy transfer rate, Q_{ei} , is small if the ion and electron temperatures are equal (which is not true for CTH plasmas) and Q_R is difficult to determine without a calibrated bolometer; therefore, they are both neglected for simplicity. The ramp rate in our simplified theory is then directly correlated to the ohmic heating power by $(3n_e/2)(\partial T_e/\partial t) = \eta J_z^2$. Figure 5.17b shows the ramp rate calculated from the observed SXR signal multiplied by the line integrated electron density over the edge vacuum rotational transform. From the estimation of the ohmic heating power using equation 5.27 and by the observed ramp rates, there is no strong correlation with the amount of vacuum transform and the ohmic heating rate within the q = 1 surface. Therefore, the observed sawtooth period and amplitude decrease with increasing vacuum transform is most likely not due to an increased heating rate.

The Porcelli model suggests that the linear rise portion of the sawtooth oscillation increases the shear at the q = 1 surface until it reaches a critical value, given by equation 5.21, followed by a rapid re-organization of the flux (magnetic reconnection). The critical value



Figure 5.17: (a) An estimated ohmic heating power within the q = 1 surface. Note that the heating power does not have a clear correlation with increasing vacuum transform. (b) The calculated ramp rate from the two-color SXR data multiplied by the line averaged electron density.



Figure 5.18: The change in critical shears dependence from Porcelli's model with the average elongation and inversion radius as the vacuum transform increases.

of the shear can be lowered if the elongation or inversion radius decreases. This suggests that if the linear rise or ramp rate of the m = n = 1 instability is kept the same between a low and and high vacuum transform case, the case with the lowest critical shear value would crash more rapidly. As the vacuum transform increases in CTH, the inversion radius decreases and it was shown that the mean elongation increases in figure 5.13. Figure 5.18 shows that there is no strong correlation with the critical shear using the calculated values of $\bar{\kappa}$ at the inversion surface and the inversion radius. This simplified expression of the critical shear neglected unknown variables due to ion effects such at the ion toroidal beta, thermal ion Larmor radius, pressure and density scale lengths. The observed decrease in sawtooth period and amplitude appears to be correlated with increasing $t_{vac}(a)$ (similarly $\bar{\kappa}$) rather than to core equilibrium changes or changes in the central ohmic heating rate.

5.6 Resistive MHD simulations of CTH plasmas

Simulations of sawtoothing CTH plasmas to study the effects of 3D shaping were completed by Nick Roberds.¹¹⁷ The work used a three-dimensional single fluid MHD code NIM- ROD^{71} to model plasmas. The simulations first modeled a pure axisymmetric plasma to ensure the simulations could exhibit sawtoothing behavior with a clear m = n = 1 mode. The values of density and electron temperature in the simulation were close to the experimental values, but other values were adjusted to achieve numerical convergence. Using a set of extended resistive MHD equations the calculations demonstrated a clear m = n = 1 mode in axisymmetric CTH-like plasmas with a period of $\sim 0.52 \,\mathrm{ms}$ and the n = 1 tearing mode acting as the primary factor in the sawtooth crash. The vacuum transform was increased by introducing a current in the helical field coil to study the effects of 3D shaping on the sawteeth. It was found that the as the vacuum transform increased, the sawtooth period decreased, consistent with the experimental results. The addition of vacuum transform, produced a helical deformation of the island and the core of the plasma compared to the axisymmetric case. The simulations also found that a complete magnetic reconnection took place, indicating that q > 1 everywhere after the sawtooth crash. This is not consistent with the observations in CTH because a complete reconnection implies that the m = n = 1mode disappears which disagrees with the singular value decomposition analysis presented in section 5.5.1.

Chapter 6

Discussion

One of the primary results of the sawtooth studies shown in this thesis is that the sawtooth period decreases with increasing vacuum rotational transform. To study the dynamics of the sawtooth oscillation three two-color SXR cameras were constructed. These cameras are designed to determine the electron temperature by measuring the bremsstrahlung radiation within the plasma. This is challenging for low temperature plasmas because the filters are required to be thin in order to provide sufficient output signal but thick enough to avoid line radiation. Chapter 4 is entirely dedicated to the theory and simulations of radiation within the plasma using the ADAS codes. It is shown in figure 4.10 that bremsstrahlung radiation from an oxygen impurity is approximately three orders of magnitude greater than the bremsstrahlung radiation from ionized hydrogen. Even if the impurity content consists of 1% of the entire plasma it will be the majority of the bremsstrahlung radiation detected by the two-color diagnostic. Measuring the impurity bremsstrahlung still results in an accurate temperature measurement due to the exponential dependance on the electron temperature. However, the problem due to using thin filters arises with the line radiation. The signal for the thin filter cameras will allow more radiation through than the thicker filters. This introduces an error within the two-color measurement that is difficult to eliminate without an accurate model of the impurity concentration within the plasma. However, recent simulations show that the ionization state of the impurity ions may not be high enough to effect the measurement. Nevertheless, the temperature measurement presented within this thesis are assume approximate until the diagnostic is cross-calibrated with a Thomson scattering diagnostic. The estimated electron temperatures derived from the two-color diagnostic agree with two other electron temperature estimates discussed in section 4.4 Unfortunately, the Thomson scattering diagnostic is not online at the time of writing this thesis.

The primary use for the two-color diagnostic is to utilize the thin filter in each of the cameras as an emissivity diagnostic measuring sawteeth within CTH. The sawtooth oscillation is a periodic relaxation event driven by ohmic heating of the plasma core. It consists of three functional stages: (i) the ramp phase; (ii) the precursor phase; and (iii) the crash phase. During the ramp phase the core of the plasma is heated through ohmic means resulting in a linear increase in the temperature and a peaking of the radial temperature profile. The precursor phase is shown through ideal MHD stability analysis for an axisymmetric tokamak to begin when q drops below 1 triggering an m = n = 1 internal-kink instability. The crash phase is a rapid re-organization of magnetic flux (reconnection) and crash of the core electron temperature and occasionally the electron density. The properties of sawtooth oscillations are characterized using a two-color SXR emissivity diagnostic measuring the bremsstrahlung radiation. The bremsstrahlung radiation is a function of electron density and temperature; however, for the sawtooth oscillations observed in CTH the line integrated electron density does not exhibit a sawtoothing pattern. Hence, the SXR emissivity measurements primarily vary due to electron temperature fluctuations.

Since the discovery of the sawtooth oscillation in the mid 1970s the physics underlying the growth of the m = n = 1 mode and the final flux surface arrangement following a sawtooth crash is still not completely understood. Two sawtooth crash models are discussed in this thesis, the Kadomtsev model and the Porcelli model. The Kadomtsev model had initial success with low temperature axisymmetric tokamaks but phenomena such as q remaining less than 1 and observations of partial sawteeth demonstrated that the model doesn't provide a complete picture. The Porcelli model included non-ideal effects applicable to hotter tokamaks and has been successful at predicting the sawtooth period.

Modifying the period and amplitude of the sawtooth instability, can be achieved by varying the shear of the q profile. However, since the q profile evolves over the course

of a CTH discharge, controlling the sawtooth oscillation in this manner requires real-time calculations of the q profile with active control of the ECRH launch angle during a discharge. The properties of the sawtooth oscillation also depend on the shape of the flux surfaces at the q = 1 surface. Typically for an axisymmetric plasma, higher elongation destabilizes the m = n = 1 mode leading to smaller more frequent sawteeth.

This thesis explores the effects of three-dimensional shaping on the sawtooth oscillation. The plasmas in CTH are tokamak-like, with a monotonically decreasing rotational transform profile. The sawteeth observed in CTH are also tokamak-like; the amount of core plasma effected by the oscillation scales with the total rotational transform and the m = n = 1internal kink mode as the primary mechanism behind the sawtooth crash.

The measured sawtooth period and amplitude are observed to decrease by a factor of 2 with increasing levels of 3D magnetic field from the external coils. The measured crash time of the sawtooth oscillation is not correlated with the amount of vacuum transform applied, indicating that the nonlinear reconnection dynamics of the MHD kink-tearing instability are not affected. Since the crash time is not substantially changed by the amount of 3D magnetic field applied, the observed decrease in the sawtooth period is concluded to result from a decrease in the rise time of the oscillation. The observed decrease in both sawtooth period and amplitude is correlated with an estimate of the mean elongation of the last closed flux surface, rather than to core equilibrium changes causing a change in the central ohmic heating rate estimated using the two-color diagnostic. Given that the kink-tearing mode is well known to be destabilized by elongation in tokamak plasmas, this observation supports an interpretation of the reduced sawtooth period resulting from a change in the linear stability threshold for the kink-tearing mode responsible for the crash.

6.1 Future work

The decrease in period and amplitude of the sawtooth oscillation with increasing threedimensional magnetic fields is an interesting observation. Ideally, to thoroughly understand the physics, one would need to derive the MHD formulation, which is extremely difficult if not impossible analytically. The results in this thesis point to the increasing mean elongation as one of the primary factors behind the changes in the sawtooth oscillation. However, there could be more complicated phenomenon effecting the sawtooth oscillation that the present diagnostics installed on CTH cannot quantify such as modifying the shear of the rotational transform near the q = 1 surface.

The two-color diagnostic primarily measures the temperature characteristics of the sawtooth oscillation and doesn't provide much information about the rotational transform profile other than the location of the q = 1 surface. The location of the q = 1 surface is an important result, and has already been incorporated into V3FIT reconstructions²⁶ to improve the reconstruction parameters. Using the raw SXR signals and the location of the q = 1surface could give more information about the rotational transform profile, providing insight to the shear at the q = 1 surface. Since V3FIT is an equilibrium solver and the sawtooth oscillation is a non-linear phenomena, one should be careful about using the reconstructed profiles. Simulations using a set of extended resistive MHD equations in NIMROD were discussed in section 5.6. The simulations reproduce sawteeth that are consistent with the experimental results. Extending this work to include plasmas with parameters closer to experimental values could explain what happens to the rotational transform profile, shear, and linear growth rate of the m = n = 1 mode.

Improvements can be made to the exsisting two-color diagnostic to further understand the sawtooth oscillation in CTH. The first priority should be to install thinner filters (probably Be) that avoids line radiation but has a strong signal in the edge channels. By doing so, one could measure the heat pulse propagation during sawtooth crash events. Tomography of the SXR signals and of the line integrated temperature measurement is a clear next step for the SXR diagnostic system on CTH. Using bi-orthogonal decomposition on the spatial emissivity profile derived from tomography would provide actual information about the shape of the q = 1 resonant surface. The ability to quantify the shape of the surface while not relying on the outer flux surface and to detect the dynamics of the sawtooth crash would be beneficial. Tomography could also reveal information about the poloidal rotation of the plasma which has been linked to changing the sawtooth period.

Currently, a new ECRH system is being installed on CTH. There have been previous experiments sweeping the ECRH across the q = 1 surface to suppress the sawtooth oscillation^{100, 101} which would be interesting to repeat on CTH and to determine if the increase in vacuum transform effects this in any way.

CTH is equipped with a Shaping Vertical Field (SVF) which is designed to change the shape of the flux surfaces. Decoupling the mean elongation from the increasing threedimensional fields is a clear next step to determine if the sawtooth dynamics are primarily effected by three-dimensional fields or a two-dimensional flux surface shape.

Finally, for the data presented within this thesis, it was difficult to get sawtoothing oscillations for very high vacuum transforms. Recently, deuterium is being used instead of hydrogen as the primary gas puffed into CTH. Preliminary results are promising, it seems that it is easier to get higher densities and plasma currents at higher vacuum transforms than ever before. Therefore, it would be useful to use deuterium to study the sawtooth oscillation at higher transforms, above 0.13.

Occasionally at high vacuum rotational transforms, the amplitude of the sawteeth would decrease for several oscillations (and increase in frequency). The amplitude of the sawtooth oscillation would proceed increase in amplitude for several more sawtooth oscillations and follow a periodic pattern of small then large sawteeth. This phenomena occurred throughout the course of the discharge for some discharges when the vacuum rotational transform was greater than 0.1 (sample discharges where this occurs: 15100556, 15100561, and 15102747).

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Appendices

Appendix A

Derivation of the geometric factor

The derivation of the geometric factor begins with the equation to calculate the simulated signal of a diode behind a filter. This was first shown as eqn. 4.15, but is repeated here:

$$S(t) = \int_0^\infty \mathrm{d}E \int_V j(E, T_\mathrm{e}) A(E) T_\mathrm{Be}(E, t) \frac{\mathrm{d}\Omega_\mathrm{D}}{4\pi} \mathrm{d}V_\mathrm{plasma}.$$
 (A.1)

The geometric factor relates $\frac{d\Omega_D}{4\pi} dV_{\text{plasma}}$ to the line integral, $f_g dl$ If the distance between the diode and slit is much larger than the width of the slit, the solid angle of the plasma as viewed by the individual diode may be written as:

$$\frac{\mathrm{d}\Omega_{\mathrm{D}}}{4\pi} = \frac{w_{\mathrm{s}}' h_{\mathrm{s}}}{4\pi d'^2}.\tag{A.2}$$

Where d' is distance between the individual diode and slit, $w'_{\rm s}$ is the apparent width of the slit, $h_{\rm s}$ is the height of the slit, and $A_{\rm slit}$ is the area of the slit. The distance between the individual diode and slit, d', can be written in terms of the distance the diode array is from the slit, d. Using the geometry is shown in figure A.1, the distance between the diode and slit can be written as $d' = d/\cos(\alpha)$. Where α is the angle between the normal of the individual and a line intersecting the center of the individual diode and the center of the slit.

Since the diode is at an angle α with respect to the normal of the slit, the apparent width of the slit is related to the slit width by $w'_{\rm s} = w_{\rm s} \cos(\alpha)$ shown in figure A.2. Substituting $w'_{\rm s}$ and d' into equation yields:

$$\frac{\mathrm{d}\Omega_{\mathrm{D}}}{4\pi} = \frac{A_{\mathrm{slit}}\cos^3(\alpha)}{4\pi d^2},\tag{A.3}$$

where the area of the slit is given by, $A_{\text{slit}} = h_{\text{s}} w_{\text{s}}$.



Figure A.1: The geometry for the individual diode and the slit. The distance from the individual diode, d', can be written in terms of d and $\cos(\alpha)$.



Figure A.2: The geometry relating the slit width as seen by the diode at an angle α with respect to the normal of the slit. The apparent slit width, $w_{s'}$ is related to the actual slit width by $w'_{s} = w_{s} \cos(\alpha)$.

The volume of the plasma that views the diode is given by:

$$dV_{\text{plasma}} = l^2 dl \left(\frac{w'_{d} h_{d}}{l^2}\right) = A_{\text{diode}} \cos(\alpha) dl.$$
(A.4)

Where l is distance between the infinitesimally small plasma volume and diode, w'_{d} is the apparent width of the diode, h_{d} is the height of the diode. The apparent width of the diode can be related to the actual width of the diode is given by $w'_{d} = w_{d} \cos(\alpha)$ through the geometry in figure A.2. The width and height of the diode has been combined to the area of the diode, A_{diode} . Combining equations A.3 and A.4 gives:

$$\frac{\mathrm{d}\Omega_{\mathrm{D}}}{4\pi}\mathrm{d}V_{\mathrm{plasma}} = \frac{A_{\mathrm{slit}} A_{\mathrm{diode}} \cos^4(\alpha)}{4\pi d^2} \mathrm{d}l$$

$$= f_g \mathrm{d}l.$$
(A.5)

Finally, the geometric factor is given by:

$$f_g = \frac{A_{\text{diode}} A_{\text{slit}}}{4\pi d^2} \cos^4 \alpha. \tag{A.6}$$

Appendix B

CAD Drawings of the two-color diagnostic

This appendix contains all of the computer assisted design (CAD) drawings for the twocolor system and the additional light-blocking dividers between the didoes for the bolometer. An expanded view of the bolometer system is shown in figure B.1 labeling each of the parts. There are five items not pictured in figure B.1, the flange, the bottom tube, the teffon sleeve, the slit bracket, and the teffon gasket. The bottom tube attaches to the left of the bottom plate and length is dependent on the camera. The cameras located at SC252_060, SC252_300, and SC036_060 have a bottom tube length of 4.7 in. The camera located at SC252_000 has a bottom tube length of 6.9 in. The camera located at SC000_270 has a bottom tube length of 8.41 in. All of the other parts are identical between the cameras.



Figure B.1: An expanded view of the bolometer camera with labels for the individual parts according to the CAD drawing.



Figure B.2: The flanges for the two-color SXR cameras.



Figure B.3: The custom flange for the bolometer system at SC036_060. The other bolometer system uses a 50-pin flange.



Figure B.4: The teflon gaskets used to isolate the SXR cameras from the vacuum vessel.



Figure B.5: The teflon sleeves used to block the light from reflecting off of the flange into the bottom of the camera.



Figure B.6: The bottom tube for the SXR cameras.



Figure B.7: The bottom plate for the SXR cameras.



Figure B.8: The teflon socket for the SXR cameras.



Figure B.9: The diode sleeve that prevents the light from the bolometer from interfering with the filtered signal on the adjacent diode. This piece is specific to the bolometer cameras and the two-color diagnostic does not use the diode sleeve.



Figure B.10: The middle tube for the SXR cameras.



Figure B.11: The filter plates for the SXR cameras.



Figure B.12: The middle plate for the SXR cameras.



Figure B.13: The poloidal slits for the SXR cameras.





Figure B.14: The slit brackets for the SXR cameras. These are placed on top of the slits to ensure they lie flat.



Figure B.15: The top tube for the SXR cameras. The groove in the side of the tube is unique for the bolometer cameras and not present in the two-color cameras.



Figure B.16: The top divider that prevents the light from the bolometer from interfering with the filtered signal on the adjacent diode. This piece is specific to the bolometer cameras and the two-color diagnostic does not use the diode sleeve.



Figure B.17: The top plate for the SXR cameras.

Appendix C

Codes Used

This appendix describes the codes that I wrote to analyze, grab, or filter the data presented in this thesis. The codes are located on CTH group's hard drive known as 'bonnie' under one of the following directories:

- 1. IDLsubroutines/diagnostics/sxr/
- 2. _Users/herfindal/idl_programs
- 3. _Users/herfindal/IDL_Subroutines
- 4. _Users/herfindal/SXR_IDL
- 5. _Users/herfindal/adas_idl_programs

The programs were written using IDL version 8.3.0. Each camera has information such as the physical position on the vacuum vessel is stored on the tree for each CTH shot. This information needs to be updated each time the camera is taken off of the vaccuum vessel or some internal components are changed. To change this information you need to call programs in directory 4. First you need to call removes xrcamera from tree.pro, this program will delete the camera from the tree. Secondly, you need to call adds xrcamerato tree.pro. You need to specify the shot to -1 to save to the model tree and the program will call the function camera Specs__define. The camera Specs__define function will read a text file containing only a list of numbers with the rows defined as:

name name of camera

create_date date file is created
diode_sn	The serial number of the diode array
filtercomp	filter material, e.g. Be or Al
dtacq	dtacq board number
dtacq₋ch	the dtacq channel array
nchords	number of chords in camera
c_ref	the position of the center of the camera, typically the center of the back of the flange in machine coordinates (m)
c_ijk	normal vector i,j,k pointing from cref to the diodes in machine coordinates
d₋diode	distance from c_ref to diode plane
d_p_slit	distance from c_{-} ref to poloidal slit plane
d_t_slit	distance from c_ref to toroidal slit plane
${\rm diode_t_off}$	distance diodes are offset from c_ref. Positive is more in the toroidal direction
diode_off	distance the diodes are offset towards the top or bottom of the camera. Positive is to the top (the direction that poloidal theta increases)
$diode_length$	length of diode element (m). In the toroidal direction
${\rm diode}_{-}{\rm width}$	width of diode element (m). In the poloidal direction
diode_sep	separation between diode elements (m)
${ m slit}_{-}{ m width}$	length of slit in the poloidal direction (m)
${\rm slit_thick}$	thickness of the slit (m)

tor_slit_wid	width of toroidal slit (m)
$tor_slt_t_of$	distance the toroidal slit are offset from c_ref, Positive is more in the toroidal direction
rotation	rotation of diode array from vertical
chordDir	direction of chord creation (1 or -1)
relative_cal	The relative calibration values

The codes used to grab the data from the tree store it in a structure, hereafter scstruc (Soft x-ray Camera structure), designated by the user. After the user uses getsc to grab the data from the tree, the other programs accept scstruc and either modify data within or add elements to the structure. The codes used to grab the data from the tree and general processing of the data are:

getscinfo This program will grab the camera information stored in the tree defined by the list above and stores it into a structure.

getscdata This program grabs the data from the cth tree.

- getsc This program is the user friendly way to grab the SXR camera data from the tree. It calls getscinfo and getscdata and stores it in scstruc. This is the program that the end user should use to grab the data. To plot the data, use the command /plot_data in the call line.
- **getscchords** This program will calculate the chordal information based on the diode position and slit width in the tree and add it to the structure created from the getscprogram. The program is fairly complicated.
- **phase_correct** This program corrects for the voltage attenuation and phase delay due to the signal amplifiers.

filterscdata This program takes a scstruc and will filter the data in every channel using a fourth order Butterworth filter at a cut-off frequency designated by the user.

The program used to draw the camera chords and to quickly plot the data of each channel are:

draw_sxr_chords

This is a function that will accept the scstruc. Usually you call the drawcthvv program prior to using this function.

plotscdata This program will produce a contour plot the data within a scstruc.

The programs used for general analysis of the data are:

scbd This program accepts an scstruc and preforms bi-orthogonal decomposition on the data set.

The primary programs to analyze the sawtoothing discharges used grabbed information from a sawtooth database spreadsheet. The sawtooth database spreadsheets contain the shot number and starting and ending times of the sawtoothing portion while the rest of the spreadsheet is filled in using programs within the run_sawtooth program. The programs primarly used run_sawtooth program are:

run_sawtooth This is the automated program that calls different programs to analyze a sawtooth database spreadsheet.

ave_ipne This program calculates the average line integrated density and plasma current.

grab_mag_info This program will grab the current settings for the HF, TVF, and TF coils. It also grabs the value the voltage the ohmic bank fired at.

- **plasma_center** This program calculates the center in meters of the plasma based on the fractional channel information given by scbd.
- run_bdThis program calls scbd twice and plasma_center once. It is used to accu-
rately find the inversion radius and the center of the plasma.

saw_database_impact

This program takes any channel, e.g. 10.23, and calculates the impact parameter based on the center of the plasma specified by the user.

cthcheckrecon_jeff2

This program will check the reconstruction files and read information from them.

sawtooth_parameters

This program will use the strongest sawtoothing channel to calculate the rise, crash, and amplitude of the sawtooth oscillations. It also calculates the error in the analysis.

saw_info This program is not used by run_sawtooth. This program will read a structre that contains the information about the rise, crash, and amplitude of the sawtooth oscillations from SC252_000_TN, SC252_060_TN, and SC252_300_TN and find the average sawtooth period, rise and crash time, amplitude, ramp rate, current density, and resistivity.

The programs used to calculate the estimated temperature of the two-color diagnostic are:

calc_axuv_responsivity

This program calculates the responsivity of the diode arrays.

calc_filter_transmission_multiple

This program will calculate the filter transmission for filters consisting of boron, aluminum, carbon, and aluminum oxide.

$calc_filter_transmission$

This program calculates the transmission of the beryllium filters with the impurities stated by the manufacture.

chord_emissivity

This program will calculate the expected signal output from the diodes assuming the bremsstrahlung radiation is purely exponential. To derive the temperature ratio you need to run this program using the two different filter transmission functions then just divide the output of this program together.

two_color_temp

This program takes the thin and thick filter data, calculates the ratio, and interpolates the temperature based on the ratio curve. You will need to manually adjust the current_limit variable if you change the filters within the diagnostic. Any values of current less than the current_limit parameter are ignored and the temperature is set to zero.

Programs that utilize the adas database:

- **cth_bremss** This program calls the continu.pro program to calculate the free-free and free-bound bremsstrahlung radiation including the Gaunt factor.
- filter_spectrum This program will calculate the line radiation and bremsstrahlung radiation multiplied by a filter function. It is used to determine how much the line radiation effects the two-color diagnostic. Usually the filter function is a multiplication of the filter and the diode absorption function.