Trading Strategies and Risk Management for Wind Power in the Electricity Market

by

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Abstract

The use of wind energy, one of the main renewable energy sources, has rapidly expanded around the world in past decades. However, the uncertainty and unpredictability of wind power lead to a number of challenges for both power systems and wind power producers (WPPs). In general, WPPs trade part of their production in the short-term electricity market and are exposed to significant uncertainties due to the volatility of market price and the limited predictability of real-time generation.

This research develops an analytical trading electricity model for WPPs in the shortterm electricity market in the U.S. The model is designed to find the optimal bidding strategy to maximize the expected revenue under the uncertainties. In addition, this research shows how advanced forecasting techniques can be used jointly with the proposed bidding strategy to help WPPs trade energy in the short-term market.

Furthermore, this research also evaluates risk management in the bidding strategy problem for WPPs in the short-term electricity market. The conditional value at risk (CVaR) concept is utilized to develop a Mean-CVaR model to address the risk and uncertainty inherent in wind power trading. Bidding strategies with and without considering risk are compared by the Monte Carlo simulation method using real-world data; the simulations show that the results are almost the same in the long run.

Finally, this research presents a static hedging strategy for WPPs to manage production revenue risks via future contracts. A 2-factor term structure model in a Heath-Jarrow-Morton framework is developed for the electricity future price. The Monte Carlo simulation method is used to develop scenarios for the evolution of future prices and wind power generation to determine the optimal hedging strategy. A series of sensitivity analyses are conducted to show how the optimal hedge ratio changes with respect to various factors.

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List of Abbreviations

- CFDs Contracts For Difference
- CVaR Conditional Value at Risk
- DA Day-Ahead
- FERC Federal Energy Regulatory Commission
- HJM Heath-Jarrow-Morton
- ISO Independent System Operator
- LMP Locational Marginal Price
- LSEs Load-Serving Entities
- NYMEX New York Mercantile Exchange
- PJM Pennsylvania-New Jersey-Maryland Interconnection
- PPA Power Purchasing Agreement
- PTC Production Tax Credit
- RT Real-Time
- RTO Regional Transmission Organization

SSM State Space Model

VaR Value-at-Risk

WPP Wind Power Producer

Chapter 1

Introduction

This chapter introduces relevant background information about electricity markets and wind power generation. The problem statement and objective of this research are provided, followed by an explanation of the organization of this dissertation.

1.1 Background

1.1.1 Wind power

Wind power is the use of air flow through wind turbines to mechanically power generators for electricity. There is no emission during the wind power generation process, which is significantly beneficial to the environment. The rapid development of wind power techniques and government support have made wind power more and more economically attractive in the past few decades, and it has become the fastest growing source of renewable energy in the world.

As of 2015, more than 80 countries around the world were using wind power to supply their electricity grids. As of the end of 2015, global wind power capacity had expanded to 432,000 MW. Yearly wind energy production is also growing rapidly and accounts for around 4% of worldwide electricity usage, 11.4% in the EU. China and the United States are the largest wind generation capacity countries, accounting for 31% and 18% of world wind generation capacity, respectively [1]. Wind power is expected to continue its fast growth in the future, especially as humans are faced with the deterioration of the environment and the gradual depletion of conventional energy sources.

As of the end of 2015, the U.S. nameplate wind power generating capacity was nearly 75,000 MW. For calendar year 2015, the electricity produced from wind power in the U.S. amounted to 190.9 TWh, or 4.67% of all generated electrical energy [2]. The U.S. wind industry has grown by an average of 25.8% a year over the last 10 years (beginning of 2005 - end of 2014) [3]. The development of wind power in the U.S. has been supported mainly through a production tax credit (PTC), which pays producers according to the amount of electricity produced. As of the end of 2015, the five states with the greatest installed wind capacity were: Texas (17,713 MW), Iowa (6,212 MW), California (6,108 MW), Oklahoma (5,184 MW) and Illinois (3,842 MW) [4].

Wind power has benefited from a virtuous cycle of increased deployment bringing about greater economies of scale and manufacturing improvements, increased competition, and falling costs. Nowadays, onshore wind is one of the most competitive sources of electricity. Technology improvements (e.g., higher hub heights and larger swept areas) and declining total installed costs make onshore wind within the same cost range as, or even lower than, that for new fossil fuel capacity. Onshore wind projects around the world are capable of delivering electricity for \$0.04/kWh to \$0.09/kWh, while power generated by fossil fuels costs between \$0.045/kWh and \$0.14/kWh. Power purchase agreement (PPA) announcements made in 2015 and 2016 for future delivery (e.g., 2017 and beyond) imply costs around \$0.04/kWh. Based on global data, the weighted average investment cost for onshore wind fell by slightly more than two-thirds between 1983 and 2015 to \$1,550/kW [1].

1.1.2 Electricity market overview

Markets for delivering power to consumers in the U.S. are split into two systems: traditional regulated markets and market-regulated markets run by independent system operators (ISOs) or regional transmission organizations (RTOs). Traditional wholesale electricity markets exist primarily in the Southeast, Southwest and Northwest, where utilities are responsible for system operations and management, and typically, for providing power to retail consumers. Traditional systems rely on management to make those decisions, usually based on the cost of using the various generation options. In general, utilities in these markets are vertically integrated, as they own the generation, transmission and distribution systems.

Over the last two decades, the electric power industry has experienced a profound liberalization process. In the United States, the Federal Energy Regulatory Commission (FERC) enacted Orders 888 and 889 in 1996 to establish the foundation for developing competitive bulk electricity markets, to which transmission services have nondiscriminatory open access. Along with facilitating open-access to transmission, ISOs operate the transmission system independently of wholesale market participants. In Order 2000, the Commission encouraged utilities to join RTOs, which, like ISOs, operate the transmission systems and develop innovative procedures to manage transmission equitably. ISOs/RTOs are responsible for administering transmission tariffs, coordinating and scheduling maintenance, maintaining system security levels, forecasting electricity demand, and coordinating long-term planning.

ISOs/RTOs market operations encompass multiple services that are needed to provide reliable and economically efficient electric service to customers. Each of these services has its own parameters and pricing. ISOs/RTOs use markets to determine the providers and prices for many of these services. These markets include the day-ahead (DA) electricity market, the real-time (RT) electricity market (sometimes called the balancing market), capacity markets (designed to ensure that enough generation is available to reliably meet peak power demands), ancillary services markets, financial transmission rights (contracts for hedging the cost of limited transmission capability) and virtual trading (financial instruments to create price convergence in the DA and RT markets). This research focuses on the DA electricity market and the RT electricity market.

ISOs/RTOs use bid-based markets to determine the electricity price and economic dispatch. In competitive markets, prices reflect the market fundamentals: the factors driving supply and demand. Rates are determined by costs and market fundamentals, because changes in supply and demand will affect consumers by influencing the cost and reliability of electricity. Supply incorporates generation and transmission, which must be adequate to meet all customers' demands simultaneously, instantaneously and reliably. Consequently, the main supply factors that affect prices include fuel prices, capital costs, transmission capacity, and the operating characteristics of power plants. Prices are also affected by sudden changes in demand and extremely high levels of demand, especially if more-expensive power plants must be turned on to serve load.

ISOs/RTOs use markets to deal with transmission constraints through locational marginal pricing (LMP). ISOs/RTOs calculate a LMP at each location on the power grid. LMP reflects the marginal cost of serving load at the specific location, given the set of generators that are dispatched and the limitations of the transmission system. LMP has three elements: an energy charge, a congestion charge, and a charge for transmission system energy losses.

1.1.3 Time framework for the short-term electricity market

ISOs/RTOs short-term electricity markets include the DA and RT markets. The DA market schedules electricity production and consumption before the operating day, whereas the RT market reconciles any differences between the schedule in the DA market and the RT load while observing reliability criteria, forced or unplanned outages and the electricity flow limits on transmission lines

The DA market produces financially binding schedules for the production and consumption of electricity one day before its production and use in the operating day. The purpose of the DA market is to give generators and load-serving entities (LSEs) a means for scheduling their activities sufficiently prior to their operations, based on a forecast of their needs and consistent with their business strategies. In the DA markets, the schedules for supply and usage of energy are compiled hours ahead of the beginning of the operating day. ISOs/RTOs then run a computerized market model that matches buyers and sellers throughout the geographic market footprint for each hour throughout the day. The model then evaluates the bids and offers of the participants, based on the power flows needed to move the electricity throughout the grid from generators to consumers. The market rules dictate that generators submit supply offers and loads submit demand bids to ISOs/RTOs by a deadline that is typically in the morning of the DA scheduling. Generation and demand bids that are scheduled by the DA market are settled at DA market prices. Typically, 95 percent of all energy transactions are scheduled in the DA market and the rest are scheduled in the RT market.

The RT market is used to balance the differences between the DA scheduled amounts of electricity and the actual RT load. The RT market runs hourly and in five-minute intervals and clears a much smaller volume of energy and ancillary services than the DA market, typically accounting for only 5 percent of scheduled energy. The RT market provides generators with additional opportunities for offering energy into the market. Megawatts over- or under- produced relative to the DA commitments are settled at RT prices. RT market prices are significantly more volatile than DA market prices. This difference stems from demand uncertainty, transmission, and generator forced outages, as well as other unforeseen events. Because the DA market is generally not presented with these events, it produces more stable prices than does the RT market. Also, because the volumes in the RT market are much smaller, supply and demand are more likely to be imbalanced, leading to both positive and negative price movements.

Current Operating Day



Figure 1.1: Time framework for DA and RT market operation in PJM

ISOs/RTOs in the U.S. share similar time frameworks for the short-term electricity market. This research uses Pennsylvania-New Jersey-Maryland interconnection market (PJM) as an example of how the short-term electricity market operates in the U.S. PJM coordinates the flow of wholesale electricity in all or part of 13 states on the eastern seaboard and the District of Columbia, and is the largest wholesale energy market in the U.S. As seen in Fig.1.1, for generation companies to sell electricity in the DA market for the next operating day, they must submit 24 hourly offers before 12:00 noon of the current operating day. After collecting offers and bids from both the supply side and the demand side, ISOs/RTOs develop an hourly schedule for the next day by least-cost security constrained unit commitment and security constrained economic dispatch programs, then post a DA schedule and LMPs at 4 pm. During the next operating day, the RT market is organized to guarantee real time balance between power supplies and demands. If generation companies produce less power than their DA scheduled quantities, they must buy power from the RT market at RT LMPs; if they produce more power than the DA scheduled quantities, they are paid by the market at RT LMPs. To recover uplift costs due to deviations, ISOs/RTOs impose deviation penalty charges on generation companies that cause such problems.

1.2 Problem Statement

In general, WPPs in the U.S. prefer to commit to PPAs to sell the generated wind power at a fixed price, as these arrangements bring in stable cash flows for a relatively long term, typically over 10 years. However, because the national average price of PPAs has been declining in the past years, and it has become more difficult for WPPs to sign PPAs contracts, more and more wind energy is traded in the short-term electricity market. Due to the uncertainties of wind power generation and electricity market prices, it is challenging for WPPs to develop a bidding strategy to sell power in the two short-term electricity markets. Unlike conventional power generators, which can usually control generation outputs, WPPs face unpredictable power generation and frequently must pay penalty fees in some ISOs/RTOs markets. The revenue gained from the two-settlement electricity revenue for WPPs includes three parts: DA market revenue, RT market revenue, and deviation penalty charge.

Because of the uncertainties of wind power and market prices, it is difficult for WPPs to compete with other power producers. The key problem is to determine how much power WPPs should commit to the DA market for each hour of the next operating day. In most cases, WPPs generate at their maximum available capacity in the RT market due to the low marginal production cost. As shown in Fig.1.1, when WPPs submit offers to the DA markets for the next operation day, they need to forecast wind power generation 12 to 36 hours ahead, and in the meantime, they are also unaware of hourly DA LMPs, RT LMPs, and deviation penalty rates. All of these uncertainties make the bidding problem complex, and WPPs face volatile revenues from the short-term electricity markets. It is also a challenge for WPPs to incorporate risk management into their bidding strategy in the real world. Therefore, this research is intended to address the two basic problems: (1) determining the optimal bidding strategy for WPPs to trade power in the short-term market to maximize the expected revenue, and (2) determining the optimal bidding strategy if WPPs consider risk management.

As WPPs are exposed to significant uncertainties due to the volatility of market prices and limited predictability of RT generation, it is essential for them to manage risk. One possible way to manage the price risk and quantity risk is through power derivatives. The main power derivative used by electricity market participants is electricity future contracts, because these contracts allow producers to avoid financial losses in extreme cases by selling their energy productions at fixed prices spanning a pre-specified time period. This research discusses whether and how electricity future contracts can be used to hedge production revenue risks for WPPs.

1.3 Research Objective

The main goal of this research is to help WPPs increase the profitability and manage financial risks at an acceptable level, therefore contributing to the competitiveness of the whole wind power generation industry as well as integration of the wind power to power systems. To accomplish these goals, the specific objectives of this research are as follows:

- 1 Develop a closed-form solution for the bidding strategy problem with the objective of maximizing the expected revenue for WPPs in the short-term electricity market. Uncertainties of wind power generation and electricity prices need to be considered and appropriately modeled. The approach should be easy for WPPs to adopt in the real world, and it should also be tested by real-world data.
- 2 Take advantage of advanced time series models and forecasting techniques to predict the hourly electricity price to help WPPs gain profits in the short-term electricity market. Accurate forecasting plays a crucial role in improving the profitability for WPPs in the short-term electricity market. Therefore, it is critical to develop an appropriate forecasting model to work with the bidding strategy model.

- 3 Incorporate risks in the bidding strategies and evaluate the importance of risk management in wind power trading in the short-term electricity market. Due to the uncertainty of wind power generation and highly volatile electricity prices, the cash flows of wind power production are unstable. WPPs might need to consider risk management while they are making hourly bidding decisions.
- 4 Build a hedging strategy model for WPPs to participate in energy financial markets to hedge price risks and volumetric risks. This study tests whether hedging with electricity future contracts can reduce the risk for WPPs. While it is relatively simple to hedge price risks for a specific amount of generation, it becomes difficult when the generation is uncertain, i.e., when volumetric risks are involved.

1.4 Dissertation Outline

The remainder of the dissertation is organized as follows.

Chapter 2 presents an analytical trading electricity model for WPPs in the short-term electricity market in the U.S. The model is designed to find the optimal bidding strategy to maximize the expected revenue under these uncertainties. This chapter also shows how advanced forecasting techniques can be used with the proposed bidding strategy to help WPPs trade energy in short-term markets. A case study is presented to illustrate the effectiveness of this proposed bidding strategy and advanced forecasting techniques by using a set of real data taken from a wind farm in the PJM electricity market.

Chapter 3 presents a bidding strategy for wind power trading under uncertainty in the short-term electricity market. The conditional value at risk concept is utilized to develop a Mean-CVaR model to address the risk and uncertainty inherent in wind power trading. Bidding strategies with and without considering risk are compared by the Monte Carlo simulation method using real-world data, and the simulations show that the results are almost the same in the long run.

Chapter 4 presents a static hedging strategy for WPPs to manage production revenue risks via future contracts. A 2-factor term structure model in a Heath-Jarrow-Morton framework for the electricity future price is developed. The model also considers the correlation between wind power generation and electricity future prices. A series of sensitivity analyses are conducted to show how the optimal hedge ratio changes with respect to various factors.

Chapter 5 provides the conclusions and contributions of this research and recommends directions for future research.

Chapter 2

Wind Power Bidding Strategy

in the Short-Term Electricity Market

This chapter presents an analytical trading electricity model for WPPs in the shortterm electricity market in the U.S. This model addresses four specific uncertainties: RT wind power generation, DA LMPs, RT LMPs, and deviation penalty rates. The model is designed to find the optimal bidding strategy to maximize the expected revenue under these uncertainties. In addition, this chapter shows show advanced forecasting techniques can be used with the proposed bidding strategy to help WPPs trade energy in the short-term electricity market. A case study is presented to illustrate the effectiveness of this proposed bidding strategy and advanced forecasting techniques using a set of real-world data taken from a wind farm in the PJM electricity market.

2.1 Introduction

Use of wind energy has rapidly expanded around the world. Wind power is expected to continue its fast growth in many countries, including the U.S. and China. However, the uncertainty and variability of wind power give rise to several challenges of power systems and the electricity market.

From WPPs' point of view, it is essential to participate in the electricity market and gain as much revenue as possible. In general, WPPs in the U.S. prefer to commit to PPAs to sell the generated wind power, as these arrangements bring in stable cash flow for a relatively long term, typically over 10 years. Besides PPAs, WPPs can also participate in two other short-term markets, the DA market and the RT market. The DA market operates like a forward market, and the RT market is often referred to as a balancing market. When generation companies commit to sell a certain amount of power in the DA market, imbalances between the DA committed quantity and the actual wind power generation must be rectified in the RT market. Due to the uncertainties of wind power generation and electricity market prices, it is challenging for WPPs to develop a bidding strategy to sell power in those two short-term electricity markets. This chapter intends to determine how much power they should commit to the DA market. WPPs will generate at its maximum available capacity in the RT market due to the low marginal production cost.

Since electricity markets around the world have been reformed and wind generation has penetrated the market only recently, an optimal bidding strategy for WPPs in the short-term electricity market is still a relatively new problem. Two main methods of approaching the problem have been proposed in the literature.

The first approach is to develop a stochastic programming by scenarios generation. Matevosyan and Soder [5] generated scenarios and transferred the problem into a mixed integer linear program model for an imbalance cost scheme in the Nordic electricity market. Their work used an ARMA model and wind power forecast errors. Catalao, et al. [6] proposed a two-stage stochastic programming model which used a hybrid intelligent approach to generate the scenario tree. Morales et al. [7] and Hosseini-Firouz [8] built a multi-stage stochastic programming model which considered risk management by scenarios generation. This model applied ARIMA techniques to predict electricity prices and wind speeds. Their case study used wind speed of the state of Kansas and historical prices of the Iberian Peninsula electricity market for the fitting process in their case study. Botterud, et al. [9] derived optimal DA bids under different assumptions for risk preferences and deviation penalty schemes in the U.S. market. Their model used a generalized reduced gradient algorithm to solve the nonlinear programming problem. They emphasized benefits of using advanced wind power forecasting methods.

The second approach to this bidding strategy problem is the analytical method. Pinson et al. [10] proposed different strategies by the analytical method for point prediction and probabilistic wind forecasting cases. They followed the Dutch electricity pool but used a wind farm in Ireland in their case study. Their paper did not apply forecasting techniques to predict electricity price. Dent et al. [11] presented a bidding strategy for WPPs in the Great Britain market using the analytic method and discussed whether WPPs should be risk- averse. Zhang et al. [12] proposed an analytical method based on assuming normal distribution of wind power generation. They tested bidding strategies on a hypothetical wind farm following Spanish market rules. Bidding strategy for wind power in the electricity market is also addressed in [13–23].

The above literature review shows that most papers address European electricity markets, which introduce short and long prices in the RT market; therefore, short and long imbalance charges are rectified into those two prices. However, U.S. electricity markets differ from European electricity markets in terms of market and price structure, because there is only one RT price in the U.S. electricity market with the same deviation charges for short and long imbalances. Although in some markets, WPPs are exempt from paying imbalance charges, which are penalties for deviation between the RT delivery and DA schedules, other electricity markets, like PJM, request WPPs to pay this imbalance charge.

This research not only presents an analytical model of the optimal DA market bidding strategy for WPPs in U.S. electricity markets; it also takes advantage of forecasting techniques and historical data to help WPPs make bidding decisions. This work suggests the following improvements:

- 1 No need to generate scenarios because it does not require generating a great number of scenario trees or solving complex LP problems.
- 2 Tested the effectiveness of the strategy with real-world data provided by a wind power generation company in the U.S. electricity market.
- 3 Utilized advanced forecasting techniques to improve the accuracy of prediction of revenue streams.
- 4 Incorporated valuable market information, such as forecasted ISO demands, forecasted and generated hourly wind power generations, to help WPPs bid to the short-term market.

2.2 Mathematical Formulation

The main symbols used in this chapter:

- π_h Revenue from energy market, hour $h \ (h = 1 \dots 24)$, [\$].
- $q_{DA,h}$ Quantity bid into the DA market, hour h, [MWh].
- $q_{RT,h}$ Actual delivery, hour h, [MWh].

 q_{max} Wind farm generation capacity, [MW].

 $p_{DA,h}, p_{RT,h}$ DA and RT LMPs, hour h, [\$/MWh].

pen Deviation penalty rate for deviation between DA schedule and RT delivery, [\$/MWh].

 $f(q_{RT,h})$ Probability density function of RT wind power generation.

 $F(q_{RT,h})$ Cumulative distribution function of RT wind power generation.

 q_{pred} Normalized predicted power

The revenue gained from the two-settlement electricity revenue for an hour, h, includes three parts: DA market revenue, RT market revenue, and deviation penalty charge, as shown in Eq.(2.1):

$$\pi_h = p_{DA,h} \cdot q_{DA,h} + p_{RT,h} \cdot (q_{RT,h} - q_{DA,h}) - pen \cdot |q_{RT,h} - q_{DA,h}|$$
(2.1)

In this chapter, it is assumed that WPPs are price takers, i.e., their bidding strategies do not affect market LMPs, as generation outputs of most wind farms are relatively small compared to generation outputs of other resources. Also, wind power is usually not the marginal cost unit in the electricity market. In reality, the above assumption may not hold all the time, especially for those regions with high share of wind power. In this chapter, WPPs are assumed to be risk-neutral, meaning they make hourly decisions based on the expected value of revenue without considering risks.

2.2.1 Analytical model development

This section proposes an analytical method to solve the bidding strategy problem. Eq.(2.1) is the revenue function used to calculate revenue from short-term markets for WPPs. The goal of the proposed model is to maximize the expected value of the revenue:

$$Max \quad E[\pi_h(q_{DA,h})] \tag{2.2}$$

$$s.t. \quad 0 \le q_{DA,h} \le q_{max} \tag{2.3}$$

Due to the assumption of independence between electricity prices and wind power generations, Eq.(2.2) can be converted as follows:

$$E[\pi_{h}(q_{DA,h})]$$

$$= E[p_{DA,h} \cdot q_{DA,h}] + E[p_{RT,h}(q_{RT,h} - q_{DA,h})] - E[pen \cdot |q_{RT,h} - q_{DA,h}|] \qquad (2.4)$$

$$= E[p_{DA,h} \cdot q_{DA,h}] + E[p_{RT,h}] \cdot \left(\int_{0}^{q_{DA,h}} (q_{RT,h} - q_{DA,h})f(q_{RT,h})d(q_{RT,h})\right)$$

$$+ \int_{q_{DA,h}}^{q_{max}} (q_{RT,h} - q_{DA,h})f(q_{RT,h})d(q_{RT,h}))$$

$$- E[pen] \cdot \left(-\int_{0}^{q_{DA,h}} (q_{RT,h} - q_{DA,h})f(q_{RT,h})d(q_{RT,h})\right)$$

$$+ \int_{q_{DA,h}}^{q_{max}} (q_{RT,h} - q_{DA,h})f(q_{RT,h})d(q_{RT,h}) d(q_{RT,h}) d(q_{RT,$$

$$+ (E[p_{RT,h}] + E[pen]) \cdot \int_{0}^{q_{DA,h}} (q_{RT,h} - q_{DA,h}) f(q_{RT,h}) d(q_{RT,h}) + (E[p_{RT,h}] - E[pen]) \cdot \int_{q_{DA,h}}^{q_{max}} (q_{RT,h} - q_{DA,h}) f(q_{RT,h}) d(q_{RT,h})$$
(2.6)

We know that :

$$\frac{d(\int_{0}^{q_{DA,h}}(q_{RT,h} - q_{DA,h})f(q_{RT,h})d(q_{RT,h}))}{d(q_{DA,h})} = (q_{DA,h} - q_{DA,h})f(q_{DA,h}) \cdot 1 - (0 - q_{DA,h})f(0) \cdot 0 \\
+ \int_{0}^{q_{DA,h}}(-1)f(q_{RT,h})d(q_{RT,h}) \\
= -\int_{0}^{q_{DA,h}}f(q_{RT,h})d(q_{RT,h}) \qquad (2.7)$$

$$\frac{d(\int_{q_{DA,h}}^{q_{max}}(q_{RT,h} - q_{DA,h})f(q_{RT,h})d(q_{RT,h}))}{d(q_{DA,h})} = (q_{max} - q_{DA,h}) \cdot f(q_{max}) \cdot 0 - (q_{DA,h} - q_{DA,h}) \cdot f(q_{DA,h}) \cdot 1 \\
+ \int_{q_{DA,h}}^{q_{max}}(-1)f(q_{RT,h})d(q_{RT,h}) \\
= -\int_{q_{DA,h}}^{q_{max}}f(q_{RT,h})d(q_{RT,h}) \tag{2.8}$$

With Eq.(2.7) and Eq.(2.8), the first derivative of Eq.(2.6) with respect to $q_{DA,h}$:

$$\frac{d(E[\pi_h(q_{DA,h})])}{d(q_{DA,h})} = E[p_{DA,h}] - (E[p_{RT,h}] + E[pen]) \cdot \left(\int_0^{q_{DA,h}} f(q_{RT,h})d(q_{RT,h})\right)
+ (E[p_{RT,h}] - E[pen]) \cdot \left(-\int_{q_{DA,h}}^{q_{max}} f(q_{RT,h})d(q_{RT,h})\right)
= E[p_{DA,h}] - (E[p_{RT,h}] + E[pen]) \cdot (F(q_{DA,h}))
+ (E[p_{RT,h}] - E[pen]) \cdot (-(1 - F(q_{DA,h})))
= E[pen] - E[p_{RT,h}] + E[p_{DA,h}] - 2E[pen] \cdot F(q_{DA,h})$$
(2.9)

Note that $F(q_{DA,h})$ is the cumulative distribution function of $q_{RT,h}$ when $q_{RT,h}$ is equal to $q_{DA,h}$.

The second derivative of Eq.(2.6) with respect to $q_{DA,h}$:

$$\frac{d^2(\pi_h(q_{DA,h}))}{d(q_{DA,h})^2} = -2E[pen] \cdot f(q_{DA,h}) \le 0$$
(2.10)

The above result indicates that the objective function is concave in $q_{DA,h}$. According to Karush-Kuhn-Tucker conditions theorem, this problem can be solved as follows [24]:

$$(q_{DA,h})^{*} = \begin{cases} 0, & \text{if } a \leq 0 \\ & \text{or } E[pen] = 0, E[p_{DA,h}] < E[p_{RT,h}] \\ q_{max}, & \text{if } a \geq 1 \\ & \text{or } E[pen] = 0, E[p_{DA,h}] > E[p_{RT,h}] \\ F^{-1}(a), & \text{if } 0 < a < 1 \\ & \text{arbitrary } \text{if } E[pen] = 0, E[p_{DA,h}] = E[p_{RT,h}] \\ & a = \frac{E[pen] + E[p_{DA,h}] - E[p_{RT,h}]}{2E[pen]}$$
(2.12)

Where $F^{-1}(a)$ is the inverse distribution function (also called quantile function) of $q_{RT,h}$. From the above results, we can see that once a is obtained in Eq.(2.12), the optimal $(q_{DA,h})^*$ can be easily obtained according to Eq.(2.11). For instance, if a is 0.35, then the optimal DA bid for that specific hour should be the 35th percentile of wind power generation distribution.

2.2.2 Bidding strategy process

The process of the proposed optimal bidding strategy can be summarized in four steps:

- Step 1 Develop $f(q_{RT,h})$ by the historical data. In this chapter, the pdf of RT wind power generations and the pdf of wind power generations prediction errors are used as $f(q_{RT,h})$.
- Step 2 Estimate the expected values of $q_{DA,h}$, $q_{RT,h}$ and *pen* for each hour of the next operating day, either by prediction techniques or by calculating means of the historical data.
- Step 3 Plug those expected values into Eq.(2.12) to obtain a.
- Step 4 Refer to Eq.(2.11) to find $(q_{DA,h})^*$.

2.2.3 $f(q_{RT,h})$ and beta distribution

From the above analysis, we know that it is critical to develop an appropriate $f(q_{RT,h})$ to find the optimal $(q_{DA,h})^*$. In this chapter, two pdfs are used: the pdf of historical actual wind power generation and the pdf of wind power generation prediction errors. The reason these two pdfs are used is that the historical data of point forecasted hourly wind power generation and actual generated wind power generation is available. Both pdfs are applied in the case study.

Although the Weibull distribution is commonly employed to describe wind power generation, it might be inappropriate to describe wind power generation prediction errors. This chapter uses a beta distribution function to describe wind power generation prediction errors by analyzing the historical hourly data of forecasted wind power generation and generated wind power generation. Beta distribution is used for two reasons. First, the beta distribution can accurately represent the pdf of the wind power prediction errors. Second, this mathematical method can easily find the percentile value $(q_{DA,h})^*$ of other general wind farms if two parameters of the beta distribution are roughly known, rather than exact historical data.

Beta function that models the occurrence of RT wind power generation x if a certain prediction power is given is as follows:

$$f_q(x) = x^{\alpha - 1} (1 - x)^{\beta - 1} * n \tag{2.13}$$

where α and β are two parameters used in beta function, and n is a constant [20]. α and β are calculated based on the available data. Once α and β are fixed, the inverse distribution function of beta distribution can be easily used to find $(q_{DA,h})^*$ in Matlab.

2.2.4 Forecasting techniques

LMPs and deviation penalty rates forecasting are essential in the proposed bidding strategy, as a derived in Eq.(2.12) is determined by their expected values. Four prediction techniques are introduced in this research: the cubic spline, ARMA, ARMAX and state space models (SSM). The latter three models are time series models. Except for ARMA, the other three prediction models include forecasted ISO demands as an explanatory variable.

ARMA model

ARMA relates current prices to past prices, and current errors to previous errors as well. The general ARMA (p,q) model has the following form

$$y_t = \phi_1 y_{t-1} + \dots + \phi_p y_{t-p} - (\theta_1 \varepsilon_{z-1} + \dots + \theta_q \varepsilon_{t-q}) + \varepsilon_t$$
(2.14)

where ε_t is a white noise process, and ϕ_p and θ_q are coefficients of backshift operators [25]. ARMA(3,1) is used in the case study.

ARMAX model

An ARMAX model simply adds one explanatory variable to the right side of an ARMA model; an ARMAX model relates current price to past prices and current forecasted demands. The general ARMAX model has the following form

$$y_t = \beta x_t + \phi_1 y_{t-1} + \dots + \phi_p y_{t-p} - (\theta_1 \varepsilon_{z-1} + \dots + \theta_q \varepsilon_{t-q}) + \varepsilon_t$$
(2.15)

where x_t is the explanatory variable, which represents the forecasted demand in our case, whose coefficient is β [26].

State Space model

The state space model (also called the dynamic linear model) provides a mathematically rigorous treatment of time series modeling based on a Bayesian approach. By allowing for variability in the regression coefficients, this model can let the system properties change in time. The general state space model has the following form

$$\begin{cases} Y_t = F_t \theta_t + \epsilon_t, & \epsilon_t \sim \mathcal{N}(0, V_t) \\ \theta_t = G_t \theta_{t-1} + \omega_t & \omega_t \sim \mathcal{N}(0, W_t) \end{cases}$$
(2.16)

where Y_t is the observation at time t, θ_t is the vector of parameters, F_t is the row vector of covariates at time t, G_t is the evolution matrix, and ϵ_t and ω_t are the observation error and the evolution error at time t, respectively. The upper part is called the observation equation and the lower part is called the evolution equation [27]. In the case study, ARMAX in the state space model is used to forecast LMPs and deviation penalty rates.

Cubic Spline model

A cubic spline model is not a time series model, as it only relates current prices to forecasted demands. A cubic spline is a special function defined piecewise by a cubic polynomial. General cubic spline model has the following form

$$y_h = \beta_0 + \beta_1 x_h + \beta_2 x_h^2 + \beta_3 x_h^3 + \beta_4 (x_h - a_1)_+^3 + \beta_5 (x_h - a_2)_+^3 + \dots + \beta_{k+3} (x_h - a_k)_+^3 \quad (2.17)$$

where k is the number of knots, a_k is the value of kth knot, and β is the coefficient [28].
The above four forecasting models are applied to predict LMPs and deviation penalty rates in the case study in R software, and the relevant results are presented in the following section.

2.3 Case Study

In this section, the proposed bidding strategy model is applied to a large-scale wind farm, which is within PJM in the U.S. and has a total installed generation capacity of 102 MW. This wind farm trades the generated electricity in the short-term electricity market operated by PJM. This section first describes and analyzes the historical data of this wind farm. Then the results of the proposed bidding strategy model test are presented. Finally, the results of bidding strategies with different forecasting techniques are compared.

2.3.1 Description of data

Hourly data from Jan. 1st to Dec. 31th, 2013 is used, including DA LMPs, RT LMPs, deviation penalty rates, forecasted ISO demands, forecasted wind power generation and generated RT wind power generation. To make analyses and predictions more accurately, the hourly data is divided into 24 groups, one for each hour, and the first nine months' data is treated as the training data and the rest is treated as the testing data.



Figure 2.1: Histogram and statics of DA and RT LMPs



Figure 2.2: Histogram and statics of deviation penalty rates



Figure 2.3: Histogram of generated RT wind power generation

The histograms and statics of hourly DA and RT LMPs, deviation penalty rates, and generated wind power generation in 2013 are shown in Figs.2.1-2.3, respectively. From Fig.2.1 we can see the mean of DA LMPs is about \$1 higher than that of RT LMPs; meanwhile, the volatility of DA LMPs is smaller than that of RT LMPs. Number of negative prices of RT LMPs is much larger than DA LMPs. In Fig.2.2, most deviation penalty rates are not more than \$5/MWh. Fig.2.3 shows that about half of the time, this wind farm generated less than 20 MWh in 2013.

Table 2.1 indicates correlation coefficients between each data item. Pred. denotes Predicted, Gen. denotes generated, MW denotes wind power output, and Load means ISO demand. The correlation coefficient between generated RT wind power generation and LMPs is relatively small and negative. The predicted ISO demand is highly correlated with DA LMPs, RT LMPs and deviation penalty rates. Thus, including this factor in price

	DA LMP	RT LMP	pen	Pred. Load	Pred. MW	Gen. MW
DA LMP	1	0.61	0.33	0.64	-0.02	-0.04
RT LMP		1	0.28	0.52	-0.05	-0.08
pen			1	0.49	0.01	-0.02
Pred. Load				1	0.01	-0.04
Pred. MW					1	0.53
Gen. MW						1

Table 2.1: Correlation coefficients

forecasting models is both reasonable and necessary. WPPs in the real world can also access the information about predicted ISO demands before they make bidding decisions in the short-term electricity markets.

2.3.2 The pdf of wind power generation prediction errors

When comparing the hourly forecasted wind power generation and generated wind power generation data, it is found that the point forecast accuracy is not good enough to be used for the bidding purpose. To make the bidding strategy perform better, the pdf of wind power generation prediction errors is developed, which is expressed in the Beta distribution function to represent RT wind power generation distribution when forecasted wind power generation are given.

q_{pred}	0~0.1	0.1~0.2	0.2~0.3	0.3~0.4	0.4~0.5
α	0.35	1.17	1.45	1.71	1.36
β	0.80	6.57	4.52	3.96	2.19
q_{pred}	0.5~0.6	0.6~0.7	0.7~0.8	0.8~0.9	0.9~1.0
α	1.17	1.09	1.39	1.52	1.00
β	1.30	0.97	0.80	0.52	0.31

Table 2.2: Parameters for 10 individual ranges of q_{pred}

The q_{pred} is divided into 10 ranges with an interval of 0.1. α and β values of beta pdfs for these 10 q_{pred} ranges were calculated in Matlab and are presented in Table 2.2. 10 ranges were chosen because if too many ranges were chosen, the available data would be not enough to develop an accurate distribution function for individual ranges, but if too few ranges were chosen, the distribution function for individual ranges would become too rough to represent real distributions.



Figure 2.4: Real pdfs of wind power generation Prediction Error for $q_{pred} = [0, 0.1], [0.1, 0.2], [0.6, 0.7] and [0.9, 1.0]$

The real distributions of wind power generation prediction errors for $q_{pred} = [0, 0.1]$, [0.1, 0.2], [0.6, 0.7] and [0.9, 1.0] are presented as examples in Fig.2.4. Beta distributions of these four ranges are shown in Fig.2.5, accordingly.

2.3.3 Results of bidding strategy model tests

This section tests the effectiveness of the bidding strategy proposed in this chapter. 24 hourly model tests are conducted separately by following the steps mentioned in the bidding



Figure 2.5: Beta pdfs of wind power generation Prediction Error for $q_{pred} = [0, 0.1], [0.1, 0.2], [0.6, 0.7] and [0.9, 1.0]$

strategy process section to see whether the optimal $(q_{DA,h})^*$ by the proposed model can achieve the maximum revenue. Because the purpose of this part of the case study is to test the proposed model, information of forecasted wind power generation and advanced prediction techniques are not used in this part. Therefore electricity LMPs, deviation penalty rates and wind power generations are treated as stochastic variables, whose probability distributions are based on historical data of those variables in 2013. In the test, the study time is one hour and the goal is to find the optimal $(q_{DA,h})^*$ to maximize the expected one-hour revenue. The whole year data is used to do the test, so there are 365 samples for individual hours test.

The result is shown in Table 2.3, where $(q_{DA,h})^*$ and $(q_{DA,h})^{**}$ are the optimal solution obtained by the proposed bidding strategy model and the actual optimal solution obtained by enumeration method, respectively; $\pi(q_{DA,h})^*$ and $\pi(q_{DA,h})^{**}$ denote the revenue by bidding

hours	$\left(a_{DAB}\right)^{*}$	$(a_{D,4,1})^{**}$	Diff	π^*	π^{**}	Diff _
1	(QDA,h) 102	(QDA,h)		850.0	850.0	0.0
1	102	102	0	009.0	009.0	0.0
2	27	28	1	713.7	713.7	0.0
3	67	67	0	651.8	651.8	0.0
4	102	102	0	638.7	638.7	0.0
5	64	63	1	593.7	593.7	0.0
6	65	55	10	636.7	637.4	0.7
7	10	8	2	787.4	787.4	0.0
8	101	101	0	904.4	904.4	0.0
9	47	39	8	825.0	826.0	1.0
10	12	9	3	844.4	844.9	0.5
11	0	0	0	881.8	881.8	0.0
12	17	10	7	881.9	883.5	1.6
13	0	0	0	919.7	919.7	0.0
14	9	4	5	945.3	945.8	0.5
15	102	102	0	1169.3	1169.3	0.0
16	102	102	0	1197.6	1197.6	0.0
17	99	94	5	1193.7	1193.8	0.1
18	102	102	0	1358.5	1358.5	0.0
19	102	102	0	1299.4	1299.4	0.0
20	84	84	0	1143.7	1143.7	0.0
21	48	44	4	1122.3	1122.4	0.1
22	15	17	2	1087.1	1087.2	0.1
23	102	102	0	1056.7	1056.7	0.0
24	91	91	0	907.0	907.0	0.0
	MWh	MWh	MWh	\$	\$	\$

Table 2.3: Comparison for individual hours, real data

 $(q_{DA,h})^*$ and $(q_{DA,h})^{**}$ to the market, respectively; Diff._q and Diff._π denotes the difference between $(q_{DA,h})^*$ and $(q_{DA,h})^{**}$, and the difference between $\pi(q_{DA,h})^*$ and $\pi(q_{DA,h})^{**}$, respectively. To obtain $(q_{DA,h})^*$, means of historical data are used as the expected value of DA LMPs, RT LMPs, and deviation penalty rates to calculate *a* in step 3 of the bidding strategy process; the pdf of RT wind power generation shown in Fig.2.3 is used as $f(q_{RT,h})$ in step 4.

hours	$\left(q_{DA,h}\right)^{*}$	$(q_{DA,h})^{**}$	$\operatorname{Diff.}_q$	π^*	π^{**}	Diff. π
1	102	102	0	860.4	860.4	0.0
2	29	29	0	715.8	715.8	0.0
3	67	65	2	660.0	660.0	0.0
4	102	102	0	653.9	653.9	0.0
5	59	60	1	581.6	581.6	0.0
6	39	40	1	624.8	624.9	0.1
7	5	6	1	768.6	768.6	0.0
8	98	96	2	862.7	862.7	0.0
9	34	35	1	815.2	815.3	0.1
10	7	7	0	824.1	824.1	0.0
11	1	0	1	912.9	913.5	0.6
12	13	13	0	909.0	909.0	0.0
13	1	1	0	989.4	989.4	0.0
14	1	1	0	1019.4	1019.4	0.0
15	102	102	0	1195.7	1195.7	0.0
16	102	102	0	1216.9	1216.9	0.0
17	101	101	0	1278.7	1278.7	0.0
18	102	102	0	1372.5	1372.5	0.0
19	102	102	0	1296.0	1296.0	0.0
20	81	81	0	1136.3	1136.4	0.1
21	33	33	0	1107.1	1107.1	0.0
22	15	15	0	1080.2	1080.2	0.0
23	102	102	0	1035.7	1035.7	0.0
24	99	99	0	924.4	924.4	0.0
unit	MWh	MWh	MWh	\$	\$	\$

Table 2.4: Comparison for individual hours, simulated data

From Table 2.3, we can find that for most hours the difference between $(q_{DA,h})^*$ and $(q_{DA,h})^{**}$ are fairly small. Even for those hours whose optimal $q_{DA,h}$ differences are relatively large, such as 6am and 12pm, their one-hour revenue differences turn out to be small. The

reason these differences exist is partly because the sample size for each hour is not large enough; some extreme values of variables will affect the final result to some extent. When the scenario number increases from 365 to 1,000,000 by Monte Carlo simulation method, both $q_{DA,h}$ differences and the revenue differences are largely reduced, which can be found in Table 2.4. The distributions of those random variables in the simulation case are almost the same as the ones in the previous case. Both the method and procedure to find $(q_{DA,h})^*$ and $(q_{DA,h})^{**}$ in this simulation are the same as the ones used in the previous case.

The test of significance indicates that $(q_{DA,h})^*$ and $(q_{DA,h})^{**}$ are equivalent to each other in a statistic sense. The process of this significance test is presented in Appendix A. The above results indicate the proposed optimal bidding strategy works well in the real world as intended.

2.3.4 Results of wind power generation prediction errors

This part of case study is designed to compare the revenues by applying the same proposed bidding strategy but using different $f(q_{RT,h})$ in step 4 of the bidding strategy process: the pdf of RT wind power generation and the pdf of wind power generation forecasting errors. The main difference between these two pdfs is that the latter one includes information of forecasted wind power generation and relevant historical distribution, whereas the former only represents the distribution of historical wind power generation outputs. Note that in this chapter, forecasting technique is not used to predict wind power generation; all the forecasted wind power generation data is given. It needs to be pointed out that both the pdf of RT wind power generation and the pdf of wind power generation forecasting errors are expressed in the Beta distribution function, and both models use historical means as the expected values of $p_{DA,h}$, $p_{RT,h}$ and *pen* in Eq.(2.12) to obtain the optimal $(q_{DA,h})^*$.



Figure 2.6: Revenues comparison between bidding strategies with the pdf of RT wind power generation and with the pdf of wind power generation prediction errors

Fig.2.6 shows bidding results, where the vertical axis is the whole year revenue for each hour. In Fig.2.6, most groups using the pdf of wind power generation prediction errors as $f(q_{RT,h})$ outperform the ones using the pdf of wind power generation as $f(q_{RT,h})$, and the total revenue difference in 2013 is \$434,578, which accounts for about 6.7% of the total revenue. From the above results, we can see that information of forecasted wind power generation does help WPPs gain the extra revenue in the short-term electricity market if applying the proposed bidding strategy.



Figure 2.7: Revenues comparison between bidding the predicted wind power generation and bidding strategy with the pdf of wind power generation prediction errors

Fig.2.7 shows results of bidding the predicted wind power generation model and the proposed bidding strategy with the pdf of wind power generation prediction errors as $f(q_{RT,h})$. The former just bids the forecasted wind power generation quantity to the DA market without using the proposed strategy model. The latter is the same as in the previous case.

In Fig.2.7, nearly all groups of bidding strategies, with the pdf of wind power generation prediction errors, outperform the ones of bidding predicted wind power generation, and the total revenue difference in 2013 is \$249,797. In other words, the proposed bidding strategy model with the pdf of wind power generation prediction errors as $f(q_{RT,h})$ helps WPPs improve total revenue of bidding predicted wind power quantity to the DA market in 2013 by approximately 3.7%. The above results indicate again that the proposed bidding strategy model works well in the real world.

2.3.5 Results comparison of different bidding strategies and forecasting techniques

In previous cases, it is assumed the expected values of LMPs for the same hour remain constant in 2013, which is oversimplified in the real world, as WPPs can take advantage of valuable information, like forecasted ISO demands, and advanced forecasting techniques to predict next day's LMPs and deviation penalty rates accurately. This part of the case study will show how to incorporate forecasting techniques into the proposed optimal bidding strategy.

The major difference between these bidding strategies with forecasting techniques and previous ones is that the expected values of DA LMPs, RT LMPs, and deviation penalty rates will be predicted rather than being means of historical data. Therefore, for individual hours, each day's $q_{DA,h}$ could be different if the calculated *a* in Eq.(2.12) changes. Four advanced forecasting models are adopted in this research: cubic spline, ARMA, ARMAX and state space models. Besides these four models, five other bidding strategy models are investigated for comparison. The result of 3-month revenues by these nine bidding strategy models are presented in Table 2.5.

Bidding Strategies	Profit /\$	Difference /\$
Perfect forecasting model	1,915,076	18,096
Cubic spline model	$1,\!905,\!708$	8,728
State space model	$1,\!903,\!783$	6,803
Bid historical mean model	1,896,980	0
ARMAX Model	$1,\!877,\!956$	-19,024
Bid predicted wind power generation model	$1,\!841,\!126$	-55,854
ARMA Model	1,833,629	-63,351
Bid full capacity model	$1,\!793,\!951$	-103,029
Bid 0 MWh model	1,775,087	-121,893

Table 2.5: Results of different bidding strategies

In Table 2.5, bid 0 MWh model and bid full capacity model mean that WPPs bid 0 MWh and 102MWh to the DA market, respectively. Bid historical mean model applies the proposed bidding strategy and uses the means of historical data as the expected values of those random variables in Eq.(2.12), which is the same as the bidding strategy with the pdf of wind power generation prediction errors in the previous section. Perfect forecasting model assumes that wind power generation forecasting is 100% accurate, and WPPs bid these forecasted quantities to the DA market. There will be no imbalance cost in this case, and it serves as a benchmark of perfect information, since it is unrealistic in the real world.

Note further that, within these nine bidding strategies, five follow the same proposed optimal bidding strategy process but with different approaches to obtain the expected values of LMPs and deviation penalty rates. These five strategies include four forecasting models and historical mean model. Beta distribution function of wind power generation prediction errors is used in these models. To make accurate predictions, within ARMA, ARMAX and state space models, 24 hourly forecasting models are developed to predict the DA LMPs, RT LMPs and deviation penalty rates of next operation day's individual hours.

Table 2.5 shows that either bidding nothing or bidding full capacity to the DA market are bad strategies, because, as predicted, their deviation costs become relatively high in these cases. Except for the ARMA model, all the models that apply the proposed bidding strategy gain revenues higher than the bid predicted wind power generation model.

Bid historical mean model serves as a benchmark to calculate revenue difference between those bidding models in Table 2.5. Among those models with forecasting techniques, the state space model and the cubic spine model achieve the highest two revenues, gaining about \$6,803 and \$8,728 over the bid historical mean model, respectively. This promising result indicates some advanced forecasting techniques would be beneficial to the proposed optimal bidding strategy. The state space model and the cubic spine model are recommended to predict market prices .

2.4 Conclusion

This chapter presents an optimal bidding strategy model for WPPs to trade power in the short-term electricity market. It is tested in a case study with real-world data and compared with the model by the Monte Carlo simulation method. The results of both cases indicate that the proposed model is promising. The analytic model is much simpler and more computationally efficient than the simulation method which needs to develop a large scenarios set.

The proposed model can also utilize the information of forecasted ISO demands, forecasted and generated wind power generations to help bid. To make the model more accurate and efficient, fitted beta distribution functions are used to describe the error between forecasted value and realized value of wind power generation. This research also shows that advanced forecasting techniques would help the proposed optimal bidding strategy. The state space model and cubic spline model gain higher revenues than other forecasting models in the case study.

Chapter 3

Evaluating Risk Management Strategies

in Wind Power Trading

This chapter presents a bidding strategy for wind power trading under uncertainty in the short-term electricity market. The conditional value at risk (CVaR) concept is utilized to develop a Mean-CVaR model to address the risk and uncertainty inherent in wind power trading. Bidding strategies with and without considering risk are compared by the Monte Carlo simulation method using real-world data, and the simulations show that the results are almost the same in the long run. This chapter suggests that WPPs focus only on maximizing the expected revenue in the short-term market, as this strategy will lead to the highest revenue, as well as optimal risk management in the long run.

3.1 Introduction

As wind power technology matures and the electricity market liberalizes, wind power has increasingly been integrated into electricity systems around the world. However, both power systems and electricity markets have to deal with the challenge of the uncertainties of wind power. WPPs need to find the best trading strategy to sell wind power to electricity markets, considering the fact that its actual generation is constantly different from scheduled production [29]. In general, WPPs commit to sell power in the forward market and buy or sell the deviation in the balancing market. WPPs face the problem every day of deciding how much power they should bid to the DA market. Due to the uncertainty of market prices and actual generation, the volatility of revenue is relatively high. Incorporating risk elements into bidding strategies is necessary but challenging in the real world.

There are a few papers address the bidding strategy problem for WPPs in the shortterm electricity market. Hosseini-Firouz [8] provided a stochastic programming approach to solve the problem by scenario generation, using CVaR for risk management. Moreno, et al. [30] addressed the problem of optimal participation in the wind energy market through a stochastic optimization process. In order to reduce risk, their study proposed a CVaR constraint for the bid that maximizes the expected revenue. They studied a 10-month period case following the Spanish market rules. Morales et al. [7] built a multi-stage stochastic programming model that considers risk management to fit the Iberian Peninsula electricity market. The study period is one hour and CVaR is included in the objective function. Morteza et al. [31] compared two bidding strategies based respectively on a naive use of wind production forecasts and on stochastic programming models. The expected value and CVaR of the revenue were computed on a daily and monthly bases. Botterud, et al. [9] derived optimal DA bids under different assumptions for risk preferences and deviation penalty schemes in the U.S. electricity market. The bidding strategies were tested on one-hour and four-month periods. Catalao, et al. [6] developed a two-stage stochastic programming approach to address the problem and used a hybrid intelligent approach to generate scenario trees. On the other hand, Dent et al. [11] presented a bidding strategy model using the analytic method for WPPs in the Great Britain market. This study also discussed whether wind owners should be risk-averse in the real world although the study used a risk-averse trading strategy. Zhang et al. [12] proposed an analytical method assuming a normal distribution of wind power output. Using a hypothetical wind farm following Spanish market rules, they tested three bidding strategies: the expected profit maximization strategy, the chance-constrained programming based strategy, and the multi-objective bidding strategy. Bidding strategy and risk management for wind power in the electricity market was also addressed in [32–45].

Most of the above literature is intended to address European electricity markets, which are different from U.S. electricity markets in terms of market and price structure. This chapter focuses on bidding problems faced by WPPs in the U.S. market. This chapter discusses risk management in depth, especially how and why risk management affects bidding decisions. This research also presents how the study period influences the bidding strategy. Finally, conclusions are drawn based on simulation results and present related mathematical analyses.

3.2 Mathematical Formulation

The main symbols used in this chapter:

- π_h^m Revenue from energy market, hour h ($h=1\ldots 24$), sample m ($m=1\ldots M$) [\$].
- $q_{DA,h}$ Quantity bid into the DA market, hour h [MWh].
- $q_{RT,h}$ Actual delivery, hour h [MWh].
- q_{max} Wind farm generation capacity [MWh].

 $p_{DA,h}, p_{RT,h}$ DA and RT LMPs, hour h [\$/MWh].

pen Deviation penalty rates for deviation between DA schedule and RT delivery [\$/MWh].

w Trade-off value assigned to CVaR.

 E_h^n Expected revenue for *n* days, hour *h* [\$].

 $prob^m$ Probability of sample m.

CVaR Conditional value at risk [\$].

VaR Value at risk [\$].

 α Confidence level of CVaR and VaR.

 C_h^n Weighted sum of the expected revenue and CVaR for n days, hour h [\$].

F() Cumulative distribution function of revenues.

- μ Revenue mean [\$].
- σ Revenue standard deviation [\$].

This section presents the mathematical bidding decision model to find the optimal DA bid, $(q_{DA,h})^*$, according to the different criterion. Two models are presented: a basic model without considering risk and a model that considers risk. The choice of bidding decision models is dependent on the WPPs' risk preference. As explained in the previous chapter, the revenue of WPPs for a given hour, h, includes three parts as follows:

$$\pi_h = p_{DA,h} \cdot q_{DA,h} + p_{RT,h}(q_{RT,h} - q_{DA,h}) - pen|q_{RT,h} - q_{DA,h}|$$
(3.1)

3.2.1 The Basic model

The basic model is designed for risk-neutral WPPs, that make bidding decisions based only on the expected value. The objective function of the basic model is to maximize the expected revenue for a given hour h in a one-day study period and is expressed as:

$$Max \quad E_h(q_{DA,h}) = \sum_{m=1}^M prob^m \cdot \pi_h^m(q_{DA,h})$$
(3.2)

Subject to:

$$0 \le q_{DA,h} \le q_{max} \tag{3.3}$$

where π_h^m is presented in Eq.(3.1), $q_{DA,h}$ is the only decision variable, and $p_{DA,h}$, $p_{RT,h}$, $q_{RT,h}$, and *pen* in Eq.(3.1) are random variables with their own specific distributions. The above model is for a one-day study period, and the maximized expected revenue of *n* days $E_h^n(q_{DA,h})$ for a specific hour *h* will simply be the summation of every day's $E_h(q_{DA,h})$ during the study period.

3.2.2 The CVaR model

Due to uncertainties of electricity prices and wind power generation in real time, trading revenues are expected to be highly volatile. The most common risk measures used in previous literature are VaR and CVaR; the latter is derived from the former and has been verified as superior to the former as CVaR can be expressed linearly and has good mathematical properties within an optimization problem [7,46].



Figure 3.1: VaR and CVaR concepts illustration

As shown in Fig.3.1, α confidence level (CVaR_{α}) represents the mean of the worst 100(1- α) percentage of possible revenues. α is typically assigned at 0.99 or 0.95. WPPs prefer the CVaR value to be as large as possible when the worst cases occur. This risk model here is developed for risk-averse decision makers.

The CVaR model for a given hour h in a period of n days is expressed as follows:

$$Max \quad C_h^n(q_{DA,h}) = E_h^n(q_{DA,h}) + w \cdot CVaR_\alpha(q_{DA,h}) \tag{3.4}$$

Subject to:

$$0 \le q_{DA,h} \le q_{max} \tag{3.5}$$

where

$$CVaR_{\alpha} = E(\pi_h \mid \pi_h < VaR_{\alpha}) \tag{3.6}$$

$$VaR_{\alpha} = F^{-1}(1-\alpha) \tag{3.7}$$

The objective function (3.4) consists of two parts: the expected total revenue and the risk aspect of CVaR. In other words, this objective function is intended to trade off between the expected revenue and risk by maximizing their weighted sum. Risk is represented by CVaR multiplied by the weighting parameter w, which reflects the WPPs' risk preference. w can be any value greater than or equal to zero, and the higher the value, the greater the risk aversion. When w is zero, the above objective function is exactly the same as the basic model, indicating that WPPs are risk-neutral in this case. This model imposes that the DA bid must be nonnegative and below the normalized capacity of the wind farm in Constraint (3.5). Eq.(3.6) and Eq.(3.7) show mathematical expressions of CVaR and VaR, respectively, as well as the relation between them.

3.2.3 Uncertainties and Monte Carlo Simulation

The uncertainties of the four random variables $(p_{DA,h}, p_{RT,h}, q_{RT,h}, and pen)$ in this chapter are treated as stochastic variables, and their individual distribution for a given hour remains the same during the study period, although WPPs can take advantage of useful information and forecasting techniques to predict the next day's market prices and wind power generation while they are making bidding decisions.

This chapter adopts the Monte Carlo simulation and enumeration method to find the approximate optimal solution to this stochastic problem [47]. A large enough number of equiprobable scenarios are generated to represent the probability distribution of those random variables based on real world data. Note that correlations among market LMPs, wind power generation and deviation penalty rates are not considerred in scenarios generation, as they are beyond the scope of this chapter.

3.3 Case Study

In order to test the proposed model, a wind farm located within the PJM electricity market in the U.S. is chosen. The data used here is the same as the one used in the previous chapter. The total installed wind power generation capacity of the wind farm is 102 MW. This section first introduces basic information about this wind farm and data used for the case study. Then the optimal bidding strategies with and without considering risk for a one-day study period are presented. Finally, the study period is extended to longer than one day and the relevant results are compared.

Table 3.1: Data statistics

	Min	Mean	Max	StDev
DA LMP [\$/MWh]	-0.68	31.74	278.1	13.39
RT LMP [\$/MWh]	-40	30.86	410.4	15.43
Deviation penalty rate [\$/MWh]	0.02	1.4	15.95	1.75
RT wind power generation [MWh]	0	29.87	102	30.41

The statistics of data are displayed in Table 3.1. The mean of DA LMPs is approximately \$1 higher than that of RT LMPs; in the meantime the standard deviation of DA LMPs is smaller than that of RT LMPs. There are more negative LMPs in the RT market than in the DA market. The above analyses indicate that the RT market is more volatile than the DA market.

Note that hourly data is divided into 24 groups based on hour time. Each hourly group has its own individual distribution, which is different from the overall distribution shown in Figs.2.1 - 2.3. Monte Carlo simulations for each hour are based on the individual distribution rather than the overall distribution.

3.3.1 Results for one-day study period

Simulation results of both bidding strategy models with a one-hour study period are presented in Table 3.2 and Fig.3.2. For the CVaR model, α is 0.95, meaning that CVaR takes into account the worst 5% of possible revenue outcomes. w is 0.1 in this case study. Of course, w can take any number to reflect the decision-maker's different risk preferences. For each hour, 100,000 scenarios were generated to find $(q_{DA,h})^*$.

Hours	1	2	3	4	5	6	7	8	
$(q_{DA,h})^*$ of basic model	102	29	68	102	63	58	11	102	
$(q_{DA,h})^*$ of CVaR model	49	14	23	35	19	21	4	10	
Hours	9	10	11	12	13	14	15	16	
$(q_{DA,h})^*$ of basic model	37	7	0	12	1	5	102	102	
$(q_{DA,h})^*$ of CVaR model	4	1	0	1	1	1	5	5	
Hours	17	18	19	20	21	22	23	24	
$(q_{DA,h})^*$ of basic model	93	102	102	89	47	25	102	93	
$(q_{DA,h})^*$ of CVaR model	1	13	29	8	4	1	67	31	
	unit: MWh								

Table 3.2: $(q_{DA,h})^*$ for one-day study period

Table 3.2 shows the optimal $(q_{DA,h})^*$ of the basic model and the CVaR model when the study period is one day. We can see that risk consideration significantly affects the bidding strategy, as the average difference between these two $(q_{DA,h})^*$ s is about 46 MWh, while the capacity of this wind farm is only 102 MW. Optimal DA bids for the CVaR model are lower than the ones for the basic model for most hours. Lower DA bids reduce exposure to shortfalls in the RT wind power generation, which need to be bought back from the highly volatile RT market.

The expected revenues and CVaRs by bidding $(q_{DA,h})^*$ for these two models are presented in Fig.3.2, where *E* denotes the expected revenue. Most CVaRs are negative, meaning



Figure 3.2: Expected revenues and CVaRs of two bidding models with one-day study period that WPPs must face possible revenue losses when the worst 5% of events occur for those hours. It is clear that in order to avoid the worst cases, WPPs must trade off between the expected revenue and risk, as we can see from Fig.3.2 that when WPPs bid $(q_{DA,h})^*$ of the basic model, the CVaR is relatively small. For instance, at 5am, WPPs need to give up \$16 revenue to avoid an occurrence of -\$838 CVaR when bidding 63MWh to the DA market. From the plot we can also see that for most hours, the expected revenue difference between these two models is much smaller than the CVaR difference, which makes sense to those risk-averse WPPs who are willing to make less money to avoid possible large losses.



Figure 3.3: α sensitivity analysis of CVaR models with one-day study period for 5am, 15am, 18pm, and 22pm

Fig.3.3 shows the sensitivity analysis of the relationship between α and $(q_{DA,h})^*$ of the CVaR model. All 24 hour cases share a similar trend, and 4 hour cases are shown in Fig.3.3. The results indicate that WPPs should bid more quantity to the DA market as $(1 - \alpha)$ of CVaR increases. In general, the more $(1 - \alpha)$ is, the less risk-averse WPPs tend to be. Higher DA bids increase exposure to shortfalls in the RT wind power generation, which need to be bought back from the highly volatile RT market. When $(1 - \alpha)$ is 1, the CVaR model will be the same as the basic model.

Fig.3.4 shows part of the results of sensitivity analysis of the relationship between w and $(q_{DA,h})^*$ of the CVaR model. All 24 hour cases also share a similar trend. The results indicate that WPPs should bid less quantity to the DA market as w increases. The underlying reason



Figure 3.4: w sensitivity analysis of CVaR models with one-day study period for 2am, 4am, 5am, and 15pm

is similar to that of the previous case. In general, the higher w is, the more risk-averse people tend to be. When w is 0, the CVaR model will be the same as the basic model.

3.3.2 Results for longer study periods

In this section, the study period is extended from one day to longer time periods. $(q_{DA,h})^*$ s of the basic model with one-day, one-month, and one-year study periods are shown in Fig.3.5. As expected, $(q_{DA,h})^*$ s for different study periods are close to each other for each individual hour, because of the same distribution assumptions of random variables for each day. Theoretically, optimal $(q_{DA,h})^*$ of the basic model should be the same no matter how long the study period is, and the difference in this case study is because of Monte Carlo sampling variations.



🗖 qDA,h of one-day basic model 🗖 qDA,h of one-month basic model 🖾 qDA,h of one-year basic model

Figure 3.5: $(q_{DA,h})^*$ of basic models with different study periods: one-day, one-month, and one-year



Figure 3.6: $(q_{DA,h})^*$ of CVaR models with different study periods, and $(q_{DA,h})^*$ of basic models

The results of the CVaR model with different study periods are presented in Fig.3.6. $(q_{DA,h})^*$ s are no longer close to each other for most hours. For most hours, $(q_{DA,h})^*$ tends

to increase as the study period increases. This result indicates that the study period does affect WPPs' bidding decisions when they consider risk management, even when their risk preference remains the same. For instance, at 6 am, $(q_{DA,h})^*$ of the CVaR model for the one-hour period is 21 MWh, and it increases to 44 and 56 MWh, respectively, when the study period is extended to 30 and 365 days. In other words, when WPPs bid 4 MWh to the DA market for 6 am, they will achieve optimal results from the one-day risk management point of view. However, if they keep bidding 4 MWh to the DA market for more days, they will no longer achieve the optimal result in terms of the same risk preference.



Figure 3.7: CVaR of basic models and one-day CVaR model VS study periods, 1am, 5am, and 18pm

Therefore, it is essential for WPPs to consider the length of the study period they should choose if they want to include risk management in their bidding strategy. Generally speaking, rational WPPs should make bidding decisions on a long-term basis, as they make thousands of hourly bids for one wind farm every year and they usually have multiple wind farms. If they adopt the short-term strategy for a long period of time, compared to a bidding strategy using the basic model, not only will the expected total revenue be lower but the risk might be even higher, as shown by examples in Fig.3.7. CVaRs of one-day CVaR models are exceeded by CVaRs of corresponding basic models as the study period increases to some points.



Figure 3.8: $(q_{DA,h})^*$ s of CVaR models VS study periods, 6am, 9am, 12am, and 24pm

Fig.3.8 shows how $(q_{DA,h})^*$ of the CVaR model changes as the study period changes from 1 day to 500 days for 6am, 9am,12am, and 24pm. From Fig.3.6 and Fig.3.8, it is also found, more importantly, that after rapid growth during the early stage, all the optimal $(q_{DA,h})^*$ s of the CVaR model tend to approach the corresponding optimal $(q_{DA,h})^*$ s of the basic model for each individual hour. In other words, in the long run, the optimal bidding strategy of the basic model which does not consider risk turns out to be fairly close to the optimal bidding strategy of the model which considers risk. Hence, WPPs may focus solely on the expected revenue for short-term trading rather than considering both the expected revenue and risk, as the former will eventually lead to the highest expected revenue as well as the lowest risk. A mathematical analysis of the above conclusion is presented in the following section.

3.3.3 Mathematical analysis

Assuming that for a given hour, the mean and standard deviation of the revenue for one day are μ_1 and σ_1 , respectively, then the mean and standard deviation of n-days revenue will be $n\mu_1$ and $\sqrt{n}\sigma_1$, because long-term revenue follows an approximately normal distribution according to the Central Limit Theorem.

From [48] we know that if $X \sim \mathcal{N}(\mu, \sigma^2)$, then

$$CVaR_{0.95}(X) = \mu - 2.07\sigma$$
 (3.8)

Thus if $X \sim \mathcal{N}$ $(n\mu_1, n\sigma_1^2)$, then

$$CVaR_{0.95}(X) = n\mu_1 - 2.07\sqrt{n}\sigma_1 \tag{3.9}$$

Plugging Eq.(3.9) into Eq.(4.5), we get

$$C_h^n(q_{DA,h}) = (1+w)n\mu_1 - 2.07w\sqrt{n\sigma_1}$$
(3.10)

Let A = $(1 + w)n\mu_1$, B = $2.07w\sqrt{n\sigma_1}$, then

$$\frac{B}{A} = \frac{2.07w\sqrt{n}\sigma_1}{(1+w)n\mu_1} = \frac{2.07w\sigma_1}{(1+w)\mu_1}\frac{1}{\sqrt{n}}$$
(3.11)

$$\lim_{n \to +\infty} \frac{B}{A} = \lim_{n \to +\infty} \frac{2.07 w \sigma_1}{(1+w)\mu_1} \frac{1}{\sqrt{n}} = 0$$
(3.12)

As we can see in Eq.(3.12), when *n* goes infinite, the ratio of B/A will approach 0, indicating that when *n* is large enough, the objective function is dominated by the first part of the model, which is determined by μ_1 . Therefore, when the study period is very long, Eq.(4.5) is almost equivalent to finding out $(q_{DA,h})^*$ to maximize μ_1 . And we already know that the basic model is intended to maximize the expected revenue as well. Therefore, when the study period increases, optimal $(q_{DA,h})^*$ for the CVaR model will be close to the optimal $(q_{DA,h})^*$ for the basic model.

Table 3.3: B/A ratios of different study periods for 8am

n	1	10	100	365	1000	10000
B/A	42.34%	13.39%	4.23%	2.22%	1.34%	0.42%

Take 8 am for an instance to show how the B/A ratio changes as n increases. The results are presented in Table.3.3. Through simulation, we know $\mu_1 \approx 800$ and $\sigma_1 \approx 1800$, and w = 0.1 and $\alpha = 0.95$ as before. From Table.3.3, we can see that the B/A ratio decreases as n increases, and it is only 0.42% when n is 10000.



Figure 3.9: Relationship between the expected revenue, revenue standard deviation, and w*CVaR as the study period change from 1 day to 10 days, 8am



Figure 3.10: Relationship among the expected revenue, revenue standard deviation, and w^*CVaR as the study period changes from 1 day to 1000 days, 8 am

A series of simulations are performed to show how the expected revenue, standard deviation, and w^*CVaR change as the study period increase. The simulation result regarding

the relationship between the expected revenue, standard deviation, and w^*CVaR as the study period change for 8am are presented in Fig.3.9 and Fig.3.10. In this case, WPPs submit $(q_{DA,h})^*$ of the basic model to the DA market. Fig.3.9 shows that the expected revenue, standard deviation and w^*CVaR are comparable to each other for a short study period. However we can see from Fig.3.10 that the expected revenue increases much faster than the standard deviation and w^*CVaR as study period increases. When the study period is 1000 days, the standard deviation and w^*CVaR are relatively small compared to the expected revenue. Both plots of simulation results match well with the mathematical analysis.

Although this research only take 8am as an example, mathematical proof and simulation results are similar for other hours. Fig.(3.12) shows that the B/A ratio will approach 0 when n goes infinite, no matter what values w, σ_1 and μ_1 are. In this chapter, we assume that $\alpha =$ 0.95, but obviously α can be any number and the above conclusions will still hold. CVaR is used as a risk measurement in this chapter, however, the conclusion applies to any bidding model using other risk measurements, such as revenue volatility and value at risk (VaR). The underlying mathematical proofs for these measurements are similar to the one presented in this section.

3.4 Conclusion

This chapter proposes a stochastic programming model for WPPs to trade energy in the short-term electricity market. The Monte Carlo simulation method is employed to generate scenarios to deal with uncertainties of market prices and wind power generation, focusing on evaluating risk management in this bidding problem. Risk management does affect WPPs' bidding strategy on a short-term basis. However, this research recommends that WPPs should not consider risk management and should focus solely on maximizing the expected revenue on an hourly basis, as this strategy will bring about the highest expected revenue and the lowest risk in the long run. This recommendation is based on the fact that WPPs usually have multiple wind farms and make a large number of hourly bids in a year for each wind farm. It will help other researchers simplify mathematical models of this bidding strategy problem, as a model without considering risk is much easier to solve than one that does consider risk either in an analytic way or others.

Chapter 4

Hedging Strategies for Wind Power Producers

Using Electricity Future Contracts

This chapter presents a static hedging strategy for WPPs to manage production revenue risks via future contracts. Due to the uncertainty of wind power and electricity prices, WPPs can take part in the energy financial market to reduce extreme risks. A 2-factor term structure model in a Heath-Jarrow-Morton framework is specified to describe the electricity future price. The Monte Carlo simulation method is used to develop scenarios for the evolution of future prices and wind power generation to determine the optimal hedging strategy. The model also consider the correlation between wind power generation and electricity future prices. A series of sensitivity analyses is conducted to show how the optimal hedge ratio changes with respect to various factors.

4.1 Introduction

Over the last two decades, electricity systems worldwide have experienced a profound liberalization process. As a result, electricity is commonly traded in open electricity markets, in which generation, transmission, and distribution companies participate. Compared with the prices of other commodities such as crude oil or natural gas, electricity prices are highly volatile and may have spikes of several orders of magnitude within a short time because electricity is non-storable, the demand is inelastic, and transmission capacity is limited. The use of wind energy, one of the main renewable energy sources, has rapidly expanded around the world in past decades. However, the uncertainty and intermittence of wind power lead to a number of challenges for both power systems and WPPs. In general, WPPs trade part of their production in the short-term electricity market, and they are exposed to significant uncertainties due to the volatility of market prices and limited predictability of real-time generation.

One possible way to manage the price risk and quantity risk is through the derivatives market. Currently, the market is quite small for wind index futures which specifically hedge production risk in the wind energy industry. In 2007, the U.S. Futures Exchange began offering wind futures; however, the exchange itself closed down one year later and wind futures for U.S. wind power have been unavailable since then. In 2015, Nasdaq Commodities launched German wind index futures contracts that allow power producers, utilities and investors to hedge risks from inconsistent output. However, it is hard to find enough counterparties that are naturally short wind and who can create a demand pull by buying up these contracts from the originators to hedge their own investments. WPPs are also concerned about the basis risk between the actual wind farm location and the index.

There are four main types of electricity derivatives available in the energy market: future contracts, forward contracts, options, and contracts for difference (CFDs). The future and forward contracts are different from traditional future and forward contracts in the financial market, as they have a delivery period of time. The electricity is not delivered at a fixed point in time but over a period. In this sense electricity future/forward contracts correspond to swaps as defined in financial textbooks. European-style call and put options with forward contracts as underlying are also available. However, the low liquidity of these contracts makes
them difficult to use for risk management purposes in practice, and makes transaction costs high because of the bid-ask spreads. CFDs are designed to hedge the difference between the system price and the local area price. However, the liquidity of these contracts is also low.

This research focuses on using electricity future contracts as a hedging tool for WPPs because of their high liquidity in practice. The first electricity futures contract was launched by the New York Mercantile Exchange (NYMEX) in March, 1996 [49]. The Nordic electricity futures market then provided the transnational futures trade [50]. Power producers and consumers are allowed to trade futures contracts in the well-developed electricity markets to hedge the risk [51, 52]. Future contracts allow producers to avoid financial losses in extreme cases by selling their energy productions at fixed prices spanning a pre-specified time period. This chapter develops the optimal hedging strategies for WPPs using electricity future contracts to manage extreme risks.

McKinnon [53] first proposed an optimal hedging strategy for farmers by holding a certain short position in futures to protect them from the output uncertainty and the market price uncertainty. The well-known optimal minimum-variance hedge ratio $H^* = cov(SQ, S)/var(S)$, where S is a spot price, and Q is an output. In the financial field, under the assumption of bivariate normality on spot price and quantity, the optimal minimumvariance hedge ratio is expressed as $H^* = \rho \cdot \sigma_S / \sigma_F$, where ρ is the correlation coefficient between the changes in the spot and futures prices, σ_S is the standard deviation of the changes in the spot price, and σ_F is the standard deviation of the changes in the future price. The underlying ideas of deriving these two optimal hedge ratios are similar. However, this optimal hedging strategy only succeeds when the goal is to minimize the revenue variance. Additional, the traditional assumption that the minimum risk hedge ratio remains the same irrespective of when hedging is undertaken has been proved not to hold for markets characterized by high volatility, irregular correlation between spot and futures returns, and limited and imperfect arbitrage opportunities. In this chapter, the CVaR rather than variance is used as a risk measurement. The former has been used widely for risk management in past decades as it only focuses on the left tail risks.

There is some literature on the subject of hedging strategies for LSEs and generation companies. LSEs are obligated to provide electricity to retail customers at fixed prices [54,55]. However, they need to buy electricity from the electricity market, where prices are highly volatile. In addition to price risks, they also face quantity risks due to the uncertainty of the demands they have to meet in real time.

Nasakkala and Kepoo [56] developed an analytic method of partially hedging electricity cash flows with static future strategies. Their model was intended to minimize the variance of a portfolio's cash flow to find the optimal hedge ratio and timing. They incorporated the correlation between future prices and demands in the model. Oum et al. [57] and Oum and Oren [58] also proposed an analytic model of hedging strategies for LSEs. They addressed the problem of price and quantity risk under an expected utility maximization criterion subject to a variance constraint. They made restrictive assumptions to analytically solve the optimization problem with the variance risk constraint. Xu et al. [59] presented a midterm power portfolio optimization model and the corresponding methodology for LSEs. They introduced risk terms based on semi-variances of spot market transactions. Carrion et al. [60] proposed a risk-constrained stochastic programming model to help LSEs decide which future contracts they should sign and at what price they need to sell electricity in order to maximize the expected profit at a given risk level. Kettunen et al. [61] developed a multistage stochastic optimization approach for electricity retailers to optimize hedging across an intermediate stage in the planning horizon. The model incorporated the correlation between spot prices and loads by the HSS scenario building process.

Power producers are faced with a similar problem, as instead of buying electricity from the electricity market, they sell it to the market at volatile prices. Conventional generators, like thermal plants, need to consider the hedge problem and operation schedule problem jointly. In addition, they are faced with the uncertainty of variable costs. On the other hand, renewable energy producers can ignore variable costs in most cases, but they have to deal with quantity risks because their generation capacities or supply obligations largely depend on weather conditions.

Conejo et al. [62] proposed a two-stage stochastic programming approach for power producers with thermal units to select future contracts to hedge risks, which is modeled using the CVaR methodology. Giacometti et al. [63] proposed a stochastic multistage portfolio mode for the daily hydro-power system scheduling problem. Their approach was developed for both electricity production schedules and trading strategies in the future markets. They claimed that future prices derived from future curve dynamics perform better than ones derived from spot dynamics. Fleten et al. [64] [65] used stochastic programming to find the optimal integrated production schedule as well as dynamic and static hedging strategies for a hydro-power producer. In addition to future contracts, people also suggested to use energy storage devices, financial options to reduce risk [66–68].

However, to our best knowledge, few articles address the hedging strategy problem for WPPs using future contracts. Wind power is much more volatile than other power resources because its real-time generation depends largely on weather conditions. In this chapter, strategies for WPPs to hedge joint price and quantity risks by using electricity future contacts are proposed. A stochastic programming by the Monte Carlo simulation method is used to find the optimal hedging strategy.

This research makes the following major contributions:

- 1 Test whether electricity futures could be used to help WPPs manage price risks and quantity risks.
- 2 Successfully develop a Monte Carlo simulation model to simulate the process of a 2factor term structure future prices and wind power generation, taking into account the correlation between them.
- 3 Conduct a series of sensitivity analyses to show how the optimal hedge ratio varies with respect to the model parameters, which can be used to help WPPs make hedging decisions in real world markets.

4.2 Future Price Model

An electricity future contract is the obligation to buy or sell a specified amount of power - 1MW during base load - at a predetermined price during the settled delivery period. In addition to base-load contracts which cover every hour of the relevant day, peak load and off-peak load contracts are available in markets as well. This chapter only considers base load contracts and assumes that future contracts are settled financially, which means there is no physical delivery involved.

The future contracts are standardized by three things: volume, delivery period, and settlement. Volume is the amount of power underlying the contract; it is equivalent to the total number of hours multiplied by contract base size. For instance, for a 1MW baseload contract with the delivery period of May, the contract volume is 744MWh (1MW \times 31 days \times 24h/days). The value of the future contract is equal to the product of the volume and the price difference between contract open time and contract close time. If the future price is \$10 at open time and \$15 at close time, then the value for this contract would be a \$3,720 profit. Delivery periods may be daily, weekly, monthly, quarterly, or annual within the current calendar year. This research only considers so-called monthly deferred settlement future contracts, which are designed to enhance the efficiency between physical and financial markets with monthly cash settlements where mark-to-market value is accumulated during the trading period and realized in the delivery period. For simplicity, margin calls and transaction fees are not considered in this research. It is assumed that WPPs open the future contact at the beginning of trading time and close it at the end of the delivery period. Based on the above assumptions and the unique settlement structure, the deferred settlement futures provide exactly the same cash flow as do the physical contracts. Therefore, the exchange of the settlement is between the future price fixed at the purchase of the contract and the average spot price during the delivery period.

Academic literature has recognized that the pricing of electricity futures should not be the same as the known traditional future models. In contrast to financial and other commodity markets, where the cost-of-carry method as a non-arbitrage condition can be applied in most cases, electricity markets are different because electricity is not storable, a characteristic which makes the cost-of-carry method inapplicable to price electricity futures. Therefore, finding a mechanism behind price formation in electricity futures markets is highly important and challenging for both academics and practitioners. There are two main ways to price electricity futures. The traditional way concentrates on modeling the stochastic process of the spot price as well as other state variables such as the convenience yield. The problem with this method is that future prices are given endogenously from the spot price dynamics. Empirical evidences show that the electricity spot and future prices are not closely related, as the electricity supply-demand equilibrium depends on the time of the year and changes over time. As a result, theoretical future prices obtained by this method frequently turn out to be inconsistent with observed market future prices.

The second way focuses on modeling the evolution of the whole future curve using a few stochastic factors taking the initial term structure as given. The Heath-Jarrow-Morton(HJM) model was first proposed in the interest rate market in 1992. Clewlow and Strickland [69] extended the HJM model from the fixed income market to the electricity future market. In this chapter, a 2-factor HJM model is adopted to simulate the evolution process of electricity future price. The model is based on the rational expectation hypothesis, which states that the future price is the best prediction of the spot price at delivery. The stochastic differential equation describing the process followed by the unitary future curve F(t,T) is as follows:

$$\frac{dF(t,T)}{F(t,T)} = \sigma_s e^{-a(T-t)} dz_{s,t} + \sigma_l dz_{l,t}$$

$$\tag{4.1}$$

with mean reversion speed a, delivery time T, short-term volatility σ_s , and long-term volatility σ_l and where z_s and z_l are Brownian Motions with correlation ρ . The model indicates that the structure of the market prices is described by two correlated Brownian Motions, which have their own volatility term structure $\sigma_s(t,T)$ and $\sigma_l(t,T)$. These two correlated Brownian Motions can be combined as a new Brownian Motion:

$$\sigma_s e^{-a(T-t)} dz_{s,t} + \sigma_l dz_{l,t} = \Sigma_{inst}(t,T) dZ_t$$
(4.2)

$$\Sigma_{inst}(t,T) = \sqrt{\sigma_s^2 e^{-2a(T-t)} + \sigma_l^2 + 2\rho\sigma_s\sigma_l e^{-a(T-t)}}$$

$$\tag{4.3}$$

The instantaneous volatility function $\Sigma_{inst}(t,T)$ of the unitary future F(t,T) is called the time-to-maturity varying volatility function, which indicates how volatile the price is depending on how far the product is from delivery. Eq.(4.3) indicates that the future price behaves like a Geometric Brownian Motion with instantaneous volatility close to σ_l when time t is far away from T. The instantaneous volatility increases to $\sqrt{\sigma_s^2 + \sigma_l^2 + 2\rho\sigma_s\sigma_l}$ as time t approaches to T. This feature matches well with the fact that future prices tend to be highly volatile when the delivery time is close.

The natural logarithm of the future price is normally distributed as shown:

$$lnF(t,T) \sim \mathcal{N}\left(lnF(0,T) - \frac{1}{2}\int_0^t \Sigma_{inst}(t,T)^2 dt, \int_0^t \Sigma_{inst}(t,T)^2 dt\right)$$

It is noteworthy that the expected value of future prices remains the same as time goes on, although from Eq.(4.4) we know that the expected value of the natural logarithm of future prices decreases as time goes on. The equations above are valid on the assumption of instantaneous product delivery, and they need to be slightly modified for a specified delivery period, such as one month (the period used in this chapter). Given a unitary future curve, the price of a non-infinitesimal future $F(t, T, \theta)$ specified by a delivery period $(T, T + \theta)$ at time t is given by the following equation:

$$F(t,T,\theta) = \frac{1}{\theta} \int_{T}^{T+\theta} F(t,u) du$$
(4.4)

The above equation indicates that the future price $F(t, T, \theta)$ can be viewed as the average price of F(t, T) during the delivery period $(T, T + \theta)$. As the stochastic differential equation and natural logarithm distribution of $F(t, T, \theta)$ are similar to Eq.(4.1) and Eq.(4.4), respectively, they are not presented here. For more details, please refer to [69–71].

Another important issue that needs to be addressed when pricing the future contracts is the risk premium, which is defined as the futures price minus the expected delivery-date spot price. Risk premium can be either positive or negative. It is not easy to actually deduce what the risk premium should be. Some previous research has been done on this topic. For instance, Bessembinder and Lemmon [72] found that electricity future prices in the U.S. have a negative risk premium when expected demand is low and demand variance is moderate, and a positive risk premium when expected demand is high and demand variance is high. Geman and Vasicek [73] found that the risk premium in the PJM electricity market was negative for forward contracts with a short time to delivery and positive for contracts with a long time to delivery. Benth et al. [74] and Furi and Meneu [75] found similar results for the Nord pool and the Spanish power market, respectively. Lucia and Torr [76] found evidence of seasonality in the risk premium at Nord pool. Generally, frequently emerging spot price spikes give the electricity consumer an incentive to pay a premium for hedging the price, while the producer receives a premium because it will lose the benefit from the spikes if the price is hedged. Overall, the empirical findings of these papers suggest that the size and sign of the future risk premium largely depends on the time of delivery. For simplicity, we assume the risk premium is zero in this research.

4.3 Risk Treatment and Monte Carlo Simulation

In this chapter, WPPs are assumed to be risk-averse, implying that they make economic decisions based not only on the expected profit but also on risk factors like reducing the probability of extreme losses. Risks can be measured in various ways, i.e., variance, VaR and CVaR. VaR and CVaR are developed to measure the downside risk. This chapter adopts CVaR of the profit with a confidence level α as the risk measurement. CVaR measures the weighted average loss of the worst cases for a given fractile. Confidence level α can be assigned from 0 to 1 according to WPPs' preference. In this chapter, α is assigned at 0.95, which is a common number used in practice.

In order to properly characterize the uncertainty within the model, the Monte Carlo simulation method is used to generate scenarios. A large enough number of equiprobable scenarios are generated, and within each scenario the future price from time 0 to the end of the delivery period and wind power generation output during the delivery period are simulated. Although Weibull distribution is commonly employed to describe wind power generation, log-normal distribution has also been proved to be a good alternative, especially for long-term outputs [77]. This research only considers monthly future contracts, and assumes the monthly wind power generation follows a log-normal distribution. The distribution of wind power generation during the delivery period will not change no matter how far the delivery time T is from time 0. This assumption is different from our assumption about future prices, as according to Eq.(4.4), their distribution is dependent on t and T. The correlation between prices and wind power generation is also incorporated in this scenario generation method. Please refer to [78] for details about how to generate correlated random variables by the Monte Carlo framework. By this simulation method, the complex optimization problem of the stochastic model is transferred to a deterministic equivalent problem, which can be easily solved by a simple enumeration method in the R software.

4.4 Hedging Strategy Model Formulation

The main symbols used in this chapter:

 $prob^m$ Probability of scenario m occurring

 π^m Profit in scenario m

w Trade-off value assigned to CVaR

 $S^m_{(T,\theta)}$ Spot price during the delivery period $(T, T + \theta)$ in scenario m

 $q^m_{(T,\theta)}$ Wind power generation during the delivery period $(T,T+\theta)$ in scenario m

 $F_{(t,T,\theta)}$ Future price at time t for the delivery period $(T, T + \theta)$

 $X_{(0)}$ The amount of the short position in future contracts at time 0

 $q_{(T,\theta)}^{max}$ Maximum wind power generation during the delivery period $(T, T + \theta)$

The goal of this hedging strategy is to help WPPs reduce the extreme risk of production profit by taking a short position in electricity future contracts. This chapter only considers a static hedging, meaning that the position in futures remains the same over the predetermined trading period. In other words, the positions can not be adjusted even if new market information is available. Therefore, WPPs have to make a decision based on the information available up to time 0. The effect of hedge strategies is mainly determined by the proportion of the production shorted in future contracts. For simplicity, WPPs are assumed to be pricetakers and that they always generate at maximum available capacity without considering schedule or operation problems. The proposed stochastic programming model is formulated below:

$$Max \quad \sum_{m=1}^{M} prob^{m} \cdot \pi^{m} + w \cdot CVaR(\pi^{m}) \tag{4.5}$$

subject to:

$$0 \le X_{(0)} \le q_{(T,\theta)}^{max} \tag{4.6}$$

$$E(S^m_{(T,\theta)}) = F_{(0,T,\theta)}$$
 (4.7)

$$S^m_{(T,\theta)} = F^m_{(T+\theta,T,\theta)} \tag{4.8}$$

where

$$\pi^m = S^m_{(T,\theta)} q^m_{(T,\theta)} + (F_{(0,T,\theta)} - S^m_{(T,\theta)}) X_{(0)}$$
(4.9)

The objective function is intended to find the optimal hedged generation $X_{(0)}$ to maximize the weighted sum of the expected profit and α confidence level CVaR of possible profits. Eq.(4.9) indicates that the total profit consists of two parts: the generation profit from spot markets and the financial profit/loss from future markets. The future price $F_{(0,T,\theta)}$ is known when WPPs make a decision in the future market. Constraint (4.6) shows that WPPs are only allowed to take a short rather than a long position in future contracts, and the position should not exceed the maximum production, which is the product of the generation capacity and the total number of hours during the delivery period. This constraint also implies that the purpose of using future contacts in this chapter is for hedging rather than speculation. Constraint (4.7) indicates that the expected spot price during the delivery period $(T, T+\theta)$ is equal to the future price with the same delivery period at time 0, which reflects the previous assumption of no risk premium. Constraint (4.8) indicates that the spot price during the delivery period $(T, T + \theta)$ is equal to the future price with the same delivery period at close time, based on the assumption that there is no arbitrage opportunity.

Due to the assumption of no risk premium in Constraint (4.7), it can be easily seen that the expected profit in (4.9) remains the same no matter what $X_{(0)}$ is. In other words, the first part of the objective function (4.5) is independent of $X_{(0)}$. As a result, the objective function (4.5) is equivalent to finding $X_{(0)}$ to maximize the profit CVaR, as shown below:

$$Max \quad w \cdot CVaR(\pi^m) \tag{4.10}$$

4.5 Case Study

In this section, one reference case study is presented to test whether the proposed model works and results of several sensitivity analyses.

4.5.1 Reference case

As a reference case for subsequent analyses, an illustrative case study of the optimal hedging strategy discussed above is presented. Consider a case that is characterized as follows:

- 1 The wind farm generation capacity is 100MW, and the expected wind power generation is 30MWh per hour during the delivery period, with lognormal distribution $ln(q_{(T,\theta)}) \sim \mathcal{N}(3.40, 0.15)$.
- 2 Delivery time T is 240 days from now, and the delivery period θ lasts 30 days. The associated future price $F_{(0,T,\theta)}$ is 10/MWh at time 0. The correlation coefficient between future prices and wind power generation is 0.1.
- 3 Parameters in 2-factor HJM model: $\sigma_s = 0.873$, $\sigma_l = 0.195$, $\rho = 0.67$, a = 30. These parameters are similar to [70], which estimated parameters from the data of the U.K. electricity market.
- 4 The step interval in simulating the future price process is 1 day. 100 iterations are run, and 10,000 scenarios are generated in each iteration, with each scenario representing one possibility of future price evolves from day 0 to day 270 as well as wind power generation during the delivery period.

Figs.4.1 - 4.3 show the main characteristics of the 2-factor HJM future prices model in a typical iteration. Fig.4.1 shows how the instantaneous volatility varies as time t goes on. The instantaneous volatility remains around σ_l when time t is not close to T, and it starts increasing when time t approaches the delivery period. This simulation result matches the mathematical Eq.(4.2) for $\Sigma_{inst}(t,T)$. Fig.4.2 and Fig.4.3 show how the mean and standard deviations of future prices change along with time t, respectively. Note that prices are in the natural logarithm. We can see that both the mean and the standard deviation change relatively quickly when the time t approaches the delivery period. These simulation results also match the theoretical values in Eq.(4.4).



Figure 4.1: Volatility term structure in the simulation



Figure 4.2: Mean of future prices in the simulation



Figure 4.3: Standard deviation of future price in the simulation

Fig.4.4 presents how the CVaR changes when the hedge ratio increases from 0 to 1. Hedge ratio is a ratio of hedged generation $X_{(0)}$ divided by $q_{max,\theta}$. The optimal hedge ratio H^* is 0.23 in this case. In other words, to maximize the profit CVaR for the specified month, WPPs have to short 16,560MWh (0.23 * 100MW * 24hours/day * 30days) future contracts. The H^* will change when the predefined assumptions vary, as will be shown in the sensitivity analysis.

A typical profit distributions before and after applying the optimal hedging strategy are presented in Fig.4.5. From the figure, we can see that the variance of profits is smaller when the optimal hedging strategy is applied than when it is not applied. In other words, the proposed hedging strategy leads to the profit with higher certainty, which is preferable to WPPs. More importantly, extreme risks are largely reduced by applying the optimal hedging strategy, as it can be seen that the left-tail area is chopped off greatly after applying



Figure 4.4: Profit CVaR for hedge ratios from 0 to 1



Figure 4.5: Profit distribution before and after applying the optimal hedging strategy

the optimal hedging strategy. Fig.4.6 presents CVaRs before and after applying the optimal hedging strategy in 100 iterations. The hedging strategy raises the CVaR by 8% on the average. Test of statistical significance also indicates that, at =0.05, the CVaR after applying the strategy is higher than the one before applying the strategy. The details about this significance test are presented in Appendix B. It should be pointed out that the means of these two profit distributions are the same, and the right-tail area of high profits is also chopped off. From this case, we find that the proposed hedging strategy does help WPPs reduce extreme risks.



Figure 4.6: CVaR before and after applying the optimal hedging strategy

4.5.2 Sensitivity analysis

To study how the optimal hedge ratio and associated CVaR are affected by various parameters in the model, this section presents a series of sensitivity analyses. The assumptions made in this part are similar to the ones in the reference case.



Figure 4.7: Optimal hedge ratio VS the correlation between generation and future prices

Fig.4.7 plots the relationship between the optimal hedge ratio and the correlation between future prices and wind power generation. The optimal hedge ratio increases almost linearly as the correlation coefficient increases from -1 to 1. This result matches the traditional hedge theory mentioned in the introduction, although it uses the profit variance rather than CVaR as a risk measurement. Fig.4.8 shows how the CVaR changes along with the correlation between prices and wind power generation changes. We can see that the CVaR approaches the minimum when the correlation is weak, and it tends to increase when the correlation is enhanced either positively or negatively. Note that these CVaRs in Fig.4.8 are obtained by applying the optimal hedge ratios of the individual correlation coefficient.

Fig.4.9 shows the sensitivity analysis of the relationship between the optimal hedge ratio and the volatility of wind power generation. Unlike the previous correlation case, this correlation is negative. When the volatility of wind power generation increases, the



Figure 4.8: CVaR VS the correlation between generation and future prices



Figure 4.9: Optimal hedge ratio VS the volatility of wind power generation



standard deviation of wind generation outputs / 1000MWh

Figure 4.10: CVaR VS the volatility of wind power generation

optimal hedge ratio decreases. This finding makes sense because that if WPPs commit a high volume of future contracts when their generation is high-volatile, they will suffer highvolatile profits/loss from the future market, a result which is contrary to the goal of using futures. Fig.4.10 plots how the CVaR changes when the volatility of wind power generation changes. As expected, the relationship is negative. We find that except for the correlation, the CVaR shares the same trend with the optimal hedge ratio in the sensitivity analysis for the rest of model parameters. We will not present the CVaR plots for the rest of the sensitivity analyses.



Figure 4.11: Optimal hedge ratio VS the short-term volatility of future prices



Figure 4.12: Optimal hedge ratio VS the long-term volatility of future prices



Figure 4.13: Optimal hedge ratio VS mean reversion speed a

Fig.4.11 and Fig.4.12 show the sensitivity analysis of the relationship between the optimal hedge ratio and short-term volatility σ_s and long-term volatility σ_l in the future price model, respectively. In the future price model, σ_s and σ_l are main factors that determine the volatility of future prices. From Fig.4.11 and Fig.4.12, it can be seen that both factors have little effect on the optimal hedge ratio, as the optimal hedge ratio remains almost the same as σ_s or σ_l increases from 0 to 1. The above results indicate that the volatility of future prices does not affect the optimal hedge ratio in our case.

Fig.4.13 shows the sensitivity analysis of the relationship between the optimal hedge ratio and the mean reversion speed a. From the figure, we can see that the optimal hedge ratio remains almost the same as a increases from 0 to 200. Fig.4.14 shows the sensitivity analysis of the relationship between optimal hedge ratio and correlation ρ in the future price model. From the figure, we can see that the optimal hedge ratio remains almost the same



Figure 4.14: Optimal hedge ratio VS correlation ρ in the future price model

as ρ increases from -1 to 1. The above results indicate that the mean reversion speed a and correlation ρ in the future price model will not affect the hedge ratio in our case.

Fig.4.15 shows the sensitivity analysis of the relationship between optimal hedge ratio and delivery time T. From the figure, we can see that the optimal hedge ratio remains almost the same as T increases. Note that it is assumed that the volatility of wind power generation does not change no matter how long the delivery time T is, and the volatility of the future price will change along with T. The results match well with the previous sensitivity analyses with respect to the volatility of wind power generation and future price. However, it should be pointed out that the result does not mean that the optimal hedge time would be any day during the delivery time T.



Figure 4.15: Optimal hedge ratio VS the delivery time T



Figure 4.16: Optimal hedge ratio VS α confidence level

Fig.4.16 shows the sensitivity analysis of the relationship between optimal hedge ratio and α confidence level of CVaR. As can be expected that when $(1-\alpha)$ increases, the optimal hedge ratio increases as well. In general, a higher hedge ratio covers more generation and prevents more low-tail risk events.

4.6 Conclusion

In this research, a hedging strategy model is proposed to help WPPs manage risks via electricity futures. The Monte Carlo simulation method is used to determine the optimal hedge ratio by simulating electricity future price and wind power generation. A 2-factor Heath-Jarrow-Morton model is adopted to describe the future price process. This method also considers the correlation between future prices and wind power outputs. Sensitivity analyses of various factors are conducted, and the results could be used to guide WPPs in dealing with the hedge problem under different situations. As unexpected, it is found that the volatility of future prices does not affect the hedge decision when the CVaR is used as the risk measurement.

Chapter 5

Conclusions and Future Work

This chapter summarizes the findings and contributions of this dissertation. The main goal of this research is to help WPPs increase profitability and manage financial risks at an acceptable level, therefore contribute to the competitiveness of the wind power generation industry.

Chapter 2 presents an optimal bidding strategy model for WPPs to trade power in the short-term electricity market. The proposed model is tested in a case study with real-world data and compared with the model by the Monte Carlo simulation method. The results of both cases indicate that the proposed model is promising. The analytic model is much simpler and more computationally efficient than the simulation method which needs to develop a large scenarios set. The proposed model can also help develop bids by using information about forecasted ISO demands and forecasted and generated wind power generations. To make the model more accurate and efficient, fitted beta distribution functions are used to describe the error between forecasted value and realized value of wind power generation. This research also shows that advanced forecasting techniques would help the proposed optimal bidding strategy. The state space model and cubic spline model gain higher revenues than other forecasting models in the case study. Further research will be conducted to improve the accuracy of prediction techniques, and to extend the proposed model to other electricity markets with different market rules. Chapter 3 proposes a stochastic programming model for WPPs to trade energy in the short-term electricity market. The Monte Carlo simulation method is used to generate scenarios to deal with uncertainties of market prices and wind power generation, focusing on evaluating risk management in this bidding problem. Risk management affects WPPs' bidding strategy on a short-term basis; however, this research recommends that WPPs avoid considering risk management and , instead, focus solely on maximizing the expected revenue on an hourly basis, as this strategy will bring about the highest expected revenue and the lowest risk in the long run. This recommendation is based on the fact that WPPs usually have multiple wind farms and make a large number of hourly bids in a year for each wind farm. This finding will help other researchers simplify mathematical models of this bidding strategy problem, as a model without considering risk is much easier to solve than one that considers risk, either in an analytic way or another.

Chapter 4 proposes a hedging strategy model to help WPPs manage risks via electricity futures. The Monte Carlo simulation method is used to find the optimal hedge ratio by simulating electricity future price and wind power generation. A 2-factor Heath-Jarrow-Morton model is adopted to describe the future price process. This method also considers the correlation between future prices and wind power outputs. Sensitivity analyses of various factors are conducted, and the results could be used to guide WPPs in dealing with the hedge problem under different situations. The results show that the volatility of future prices does not affect the hedge decision when CVaR is used as the risk measurement. Although this research only considers monthly futures, the proposed model can be easily extended to other time range futures, such as daily and quarterly. Further research will be conducted on the optimal hedge time and dynamic hedge strategies for WPPs.

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Appendix A

Significance Test for $q_{DA,h}$

Data: $(q_{DA,h})^*$ and $(q_{DA,h})^{**}$ in Table 2.4

Test: Paired t-test

Hypothesis: $H_0: (q_{DA,h})^* = (q_{DA,h})^{**}, \quad H_a: (q_{DA,h})^* \neq (q_{DA,h})^{**}$

Significance level: $\alpha = 0.05$

Degree of freedom: 23

Rejection region: t > 1.714

Result: $t^* = 0.2719$, p = 0.7881. Fail to reject H_0

Conclusion: at $\alpha=0.05$, we don't have evidence to suggest that $\left(q_{DA,h}\right)^{*}$ and $\left(q_{DA,h}\right)^{**}$

are different.

Appendix B

Significance Test for CVaR

Data: The CVaR shown in Fig.4.6

Test: Paired t-test

Hypothesis: H_0 : $CVaR^* = CVaR^{**}$, H_a : $CVaR^* < CVaR^{**}$

 $CVaR^*$ denotes the CVaR before applying the hedging strategy

 $CVaR^{**}$ denotes the CVaR after applying the hedging strategy.

Significance level: $\alpha = 0.05$

Degree of freedom: 99

Rejection region: t > 1.662

Result: $t^* = 97.92, p = 7.281e^{-44}$. Reject H_0

Conclusion: at $\alpha = 0.05$, we conclude that the CVaR after applying the optimal hedging strategy is higher than the CVaR before applying it.