A Computational System to Solve the Warehouse Aisle Design Problem

by

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Abstract

Order picking is the most costly operation in a warehouse. However, current warehouse design practices have been using the same design principles for more than sixty years: straight rows with parallel pick aisles and perpendicular cross aisles that reduce the travel distance between pick locations. Gue and Meller (2009) altered this design mentality by proposing a fishbone layout that offered reductions in travel distance of up to 20% in unit-load warehouses. However, research in finding alternative layouts for order picking warehouses is lacking. The main result of this dissertation is to show that there are non-traditional designs that reduce the cost of order picking operation by changing the aisle orientation, aspect ratio, and placement of depot simultaneously. We present designs that achieve reductions in travel distance of up to 5.3%.

Order picking warehouse layout optimization is computationally complex. Three problems need to be solved: layout design, product allocation, and pick routing. In this dissertation, we focus on layout optimization but we propose new methods for product allocation, routing, and certain speed-up techniques for routing algorithms.

We need to solve multiple traveling salesman problems (TSP) to find the expected distance traveled for order picking. We develop parallel and distributed computing techniques to solve large batches of TSPs simultaneously. We compare two well known TSP solvers and various machine settings (serial, parallel, and distributed). Distributed computing techniques only show their real benefits when the TSP instances have more than 50 locations so that the network and file read/write overhead is relatively low. Our results also show that for both real data and generated data, a scheduling algorithm like longest processing time performs better than a naïve method like the equal distribution rule even though the method used for estimating processing times of TSPs is crude (TSP size, in this case).

In all of the layout literature, simulation and analytical models assume a simple travel rule: order pickers follow the aisle centers. Following aisle centers leads to longer travel distances when an order picker picks items within the pick aisles that have angles other than 90 degrees between cross aisles. By using a visibility graph, we show that paths are more reasonable in most layout settings, and comparisons between traditional and non-traditional layouts for order picking operations are affected. Our results show that the visibility graph method impacts the assessment of well-known non-traditional layouts compared to traditional counterparts. Moreover, it also changes the rank order of the three most common traditional layouts.

Most importantly, we develop a warehouse layout optimization system that models layouts, allocates products to storage locations, calculates routing distances, and performs heuristic optimization over a comprehensive set of layout design parameters. The system searches over 19 different design classes simultaneously by using a layout encoding. We propose improved non-traditional aisle designs for different pick list sizes and demand skewness. The proposed designs can shorten the average walking distance up to 5.3% compared to traditional layouts.

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Table of Contents

List of Figures

List of Tables

List of Abbreviations

3PL	Third party logistics
AS/RS	Automated storage and retrieval system
COI	Cube-Per-Order index
ES	Evolution strategies
ESPP	Euclidean shortest path problem
FRP	Forward-reserve problem
I/O	Input/output
P&D	Pickup and deposit
PSO	Particle swarm optimization
SKU	Stock keeping unit
SPP	Shortest path problem
TSP	Traveling salesman problem
WMS	Warehouse management system

Chapter 1

Introduction

During the last few decades, globalization has become the main driving factor in business. This phenomenon has created a market dynamic that increases competition and demands a higher level of efficiency. In this new global market, complex supply chain partnerships have simply become inevitable to compete with other companies in the same business due to efficiency and cost effectiveness. Despite the attempts industry has made to eliminate different kinds of inventories to reduce cost, it is necessary to build warehouses that contribute to a multitude of the missions in the supply chain. These tasks include inventory stockpiling, consolidation, distribution, production logistics, and stock mixing. For example, a customer in the United States orders an item, which is manufactured in China, and wants to get the item in three days. If the item comes directly from China, then the manufactured item might travel by truck, then by plane, and by another truck again. The shipping cost will become so high that the customer might be paying shipping expenses which cost more than the actual item. So how do companies like Amazon offer next-day shipping with such a small cost? The answer is warehousing. The item is warehoused in the United States, probably in multiple locations, so that it can be delivered quicker and cheaper with less variation in lead time. This example relates to distribution. Warehouses are also used in the stock mixing and the consolidation process as evident by their use by many retail stores. Walmart, the leading retailer in the world (Gensler, 2016), has thousands of stores around the world that are supplied by thousands of vendors. If each vendor sends items directly to each store then those items would normally move at highly individual rates. Instead, vendors send items in a full truckload to a warehouse or a distribution center and items are sorted and prepared for storage in the warehouse. The justification for the warehouse is the cost reduction in transportation with fewer shipments from vendors to retail stores.

To compete in global business, companies are looking for new ways to deliver a quality product quickly at a low cost. One way to trim costs is to eliminate the need for warehousing. Frazelle (2002) has stated that warehousing might be eliminated if the other four areas of logistics (customer service and order processing, inventory planning and management, supply, transportation) are well planned. Yet this is not the case in many companies because of long supplier lead times, and manufacturers are forced to serve customers from inventory instead of making to order. Therefore, they are left with two options, either choosing a third-party warehousing company to fulfill their warehousing needs or perform continuous improvement in their warehousing operations to meet all of the requirements of the supply chain process. As companies have seen the benefits of outsourcing their logistics functions and concentrating on their core business, the number of third-party logistics companies has increased and has started to offer an increasing number and variety of value-added services including labeling, kitting and special packaging. ReportsnReports.com (2017) reports that global 3PL (third party logistics) market size hit \$759.6 billion in 2016, a 4.5% increase from previous year with a market size of \$721 billion. ReportsnReports.com (2017) predicts that 3PL occupy 10% of the logistics market size in 2020 with a market size of \$900 billion. Armstrong reports that among the top 20 third-party logistics warehouses in 2015 there was an 3.3% growth on their combined space, moving from 591.2 to 610.7 millions of square feet (Bond, 2015). This substantial increase in total square footage leads to greater operations and maintenance cost for 3PL companies.

Another way to deal with this competition is to decrease the shipment size and deliver products to customers quicker. According to the Bureau of Transportation, smaller sized shipments increased by 56% from 1993 to 2002, which supports efficient, just-in-time inventory systems. These can reduce inventory carrying costs and overall logistics costs. Manufacturers are trying to progress along the lean manufacturing principles by reducing their inventory levels. Retail stores and wholesalers are realizing they can still be profitable with less inventory. This phenomenon in both retail stores and manufacturing operations leads to an increase in labor cost for warehouses.

Although warehousing represents only 1.76% of the cost of sales for major manufacturing companies (Ross and Pregner, 2011), that statistic becomes more significant when it comes to retail stores and third-party logistics and warehousing companies. Since their core operations include warehousing, any improvement in warehousing operations will significantly affect their business compared to manufacturing companies that are using in-house warehousing. Mainly for this reason, our target audience is the warehouses, retail stores, e-commerce companies, and all companies that perform order picking operations.

In a typical warehouse, items are received from suppliers, they are stored in storage locations, order pickers fulfill customer orders and assemble them for shipment, and completed orders are shipped to customers. Order picking is the retrieval of items from the storage area to fulfill customer orders. It involves the process of grouping and scheduling the customer orders, releasing them to the order pickers, the picking of the items from storage locations, and the disposal of the picked items (De Koster et al., 2007).

This dissertation offers an approach that minimizes the costs of the most costly operation in a warehouse, order picking. Although the order picking operation is the most costly operation in a warehouse, current warehouse design practices have been using the same design principles for more than sixty years: straight rows with parallel pick aisles and perpendicular cross aisles that reduce the travel distance between pick locations (Vaughan and Petersen, 1999; Petersen, 1999). Gue and Meller (2009) recently changed these assumptions and achieved reductions in travel distance up to 20% in unit-load warehouses. This dissertation is approaching the same question for order picking warehouses: “Can we achieve an improvement in cost of the order picking operation with non-traditional designs?” To answer this question we need to pose other questions that are presented in the following section.

1.1 Problem Statement

Our research considers different aisle orientations to facilitate material flow between depot location and pick locations and in between pick locations to minimize expected travel distance for a given set of orders. There is a variety of studies on methods, policies, or techniques developed to improve the overall order picking procedure. The decisions usually concern policies for the picking of the products, the routing of the order pickers, and the allocation of products to storage locations. In our research, we assume a turnover-based storage policy which reduces travel by dedicating the most convenient storage locations to items with the highest turnover frequency and compared its performance with a random storage policy. We select this storage policy because the turnover-based storage policy outperforms other storage policies in manual order-picking systems with regards to travel distance (De Koster et al., 2007). Furthermore, we assume that the routes are optimal or near-optimal in order picking operations and pick lists have been determined. Further information related to the turnover-based storage policy and routing algorithms are in the next chapter.

Problem 1 *What arrangement of pick aisles, cross aisles and depot location minimizes labor costs in an order picking operation?*

This is the major question in our research. We investigate different warehouse design classes. We define a warehouse design class by the number of exterior and interior nodes and number of cross aisle segments. For example in Figure 1.1, the 2-0-1 class means that there are 2 exterior nodes, no interior nodes and 1 cross aisle segment. Each design has a number of pick aisle regions that are bounded by exterior boundaries of the warehouse and the cross aisle(s). These regions have parallel pick aisles specified by the region angle. Each pick aisle has pick locations where an order picker can pick an item. We develop a graph-based network model which represents these pick locations, pick aisles, regions, cross aisles, warehouse boundaries, and a depot location. Figure 1.2 shows an example of this graph-based network model. Although, representation of a warehouse is an important part of the answer to this question, we need to answer a related and a more difficult question.

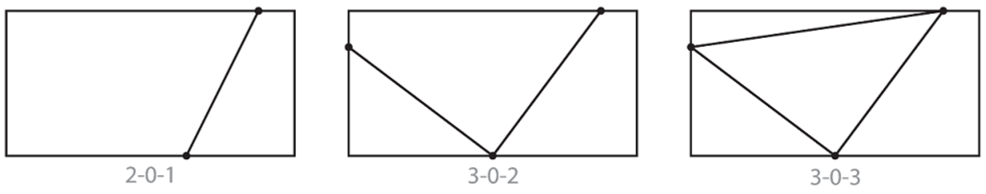


Figure 1.1: Some warehouse design classes

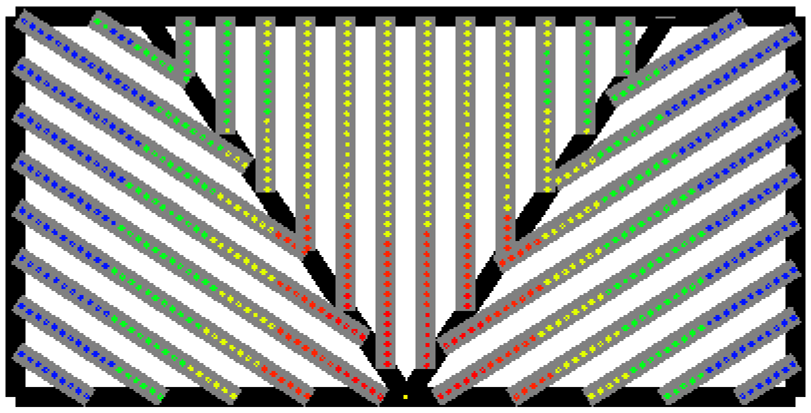


Figure 1.2: A 3-0-2 class with a graph-based network model

Problem 2 *How to measure the potential effectiveness of a design for a particular order picking operation?*

In an order picking operation, the number of picks and their locations may change for every order picking tour. Compared to unit-load operations, where analytical models exist for both traditional (Francis, 1967; Bassan et al., 1980) and non-traditional warehouses (Gue and Meller, 2009; Öztürkoğlu et al., 2014), there are no analytical models for more than two cross aisles for order picking operations and the existing research has only focused on traditional warehouse designs (Rosenblatt and Roll, 1984; De Koster et al., 2007). The designs we develop in this research will not support an analytical model because of their complex and stochastic nature. It is complex because solving each picking tour optimally is a case of the traveling salesman problem (TSP), which has been solved optimally for one and two block warehouses (Ratliff and Rosenthal, 1983; Roodbergen and De Koster, 2001b). It is stochastic because evaluating only a single design is equivalent to solving hundreds of TSP problems and evaluating a single tour would provide little information about the quality of a design for an “expected order”. For these reasons, it is computationally hard to find an optimal solution that minimizes the expected travel distance with a given order set and design class.

Establishing efficient ways to evaluate designs, create more reasonable pick paths, and search over solution space are three challenging computational problems. The first challenge is mostly related to parallel and distributed computing techniques. Writing an efficient and scalable parallel program is a complex task. However, C# parallel libraries provide the power of parallel computing with simple changes in the implementation of finding a set of optimal/near-optimal routes if a certain condition is met: the steps inside the operation must be independent. Solving large batches of traveling salesman problems is an example of such independent operations. In Chapter 3, we give details about the large batch of TSPs and its solution techniques.

Even though warehouse design optimization has been studied since the early 1960s, all design and routing techniques use a very common travel rule: workers follow the centers of pick and cross aisles. But we propose the visibility graph method to address the second challenge. The visibility graph method constructs a graph whose nodes are either vertices of obstacles or attractions and whose edges are pairs of mutually visible nodes. In other words, for any pair of nodes if the line segment that connects them does not pass through an obstacle, an edge is created between them. Then we can run Dijkstra’s algorithm to find the shortest path between any two nodes in this graph. In Chapter 4, we describe the visibility graph method and compare it with the aisle centers method.

For the third challenge we propose Evolution Strategies (ES) as a heuristic optimizer. ES is a meta-heuristic that works well with continuous problems. We present an encoding that uses a string of continuous variables to define locations of the cross aisle endpoints, the angles of picking aisles, and the location of the depot. Details of this algorithm and encoding are given in Chapter 5.

Finally, we describe our design of experiments and its results in Chapter 6. We also give a brief summary of the dissertation in this final chapter.

1.2 Major Contributions

Until Gue and Meller (2009) introduced the Flying-V and Fishbone layouts, the warehousing literature almost always assumed traditional warehouses (presented in Figure 4.4d) and their slight variations. Öztürkoğlu et al. (2014) extended this work by further relaxing the established rules of warehouse design by using angled pick aisles. These designs resulted in up to 22.52% savings over traditional unit-load warehouses. We are extending this existing work by considering new aisle designs for order-picking warehouses.

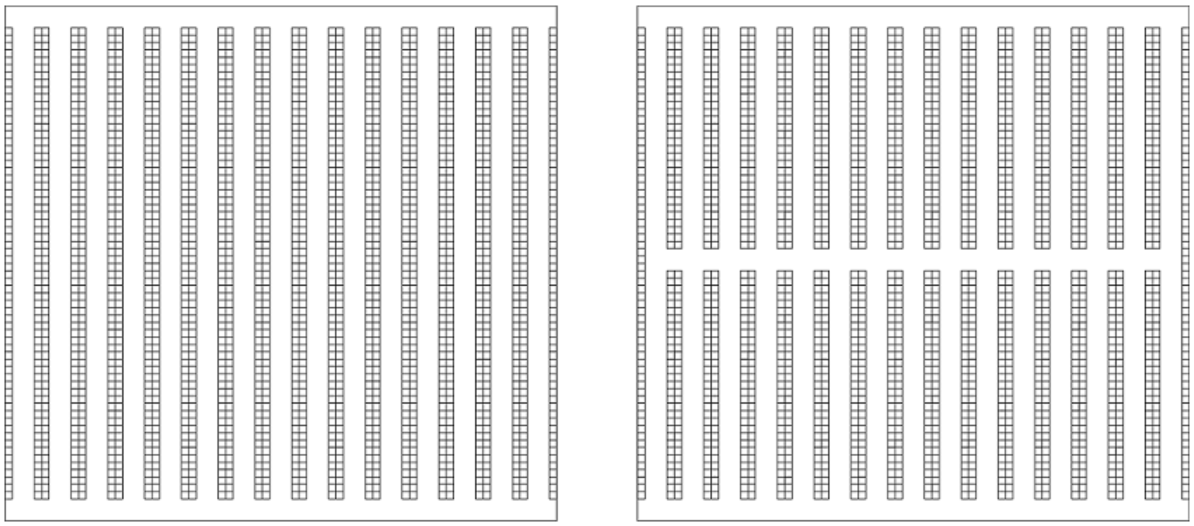


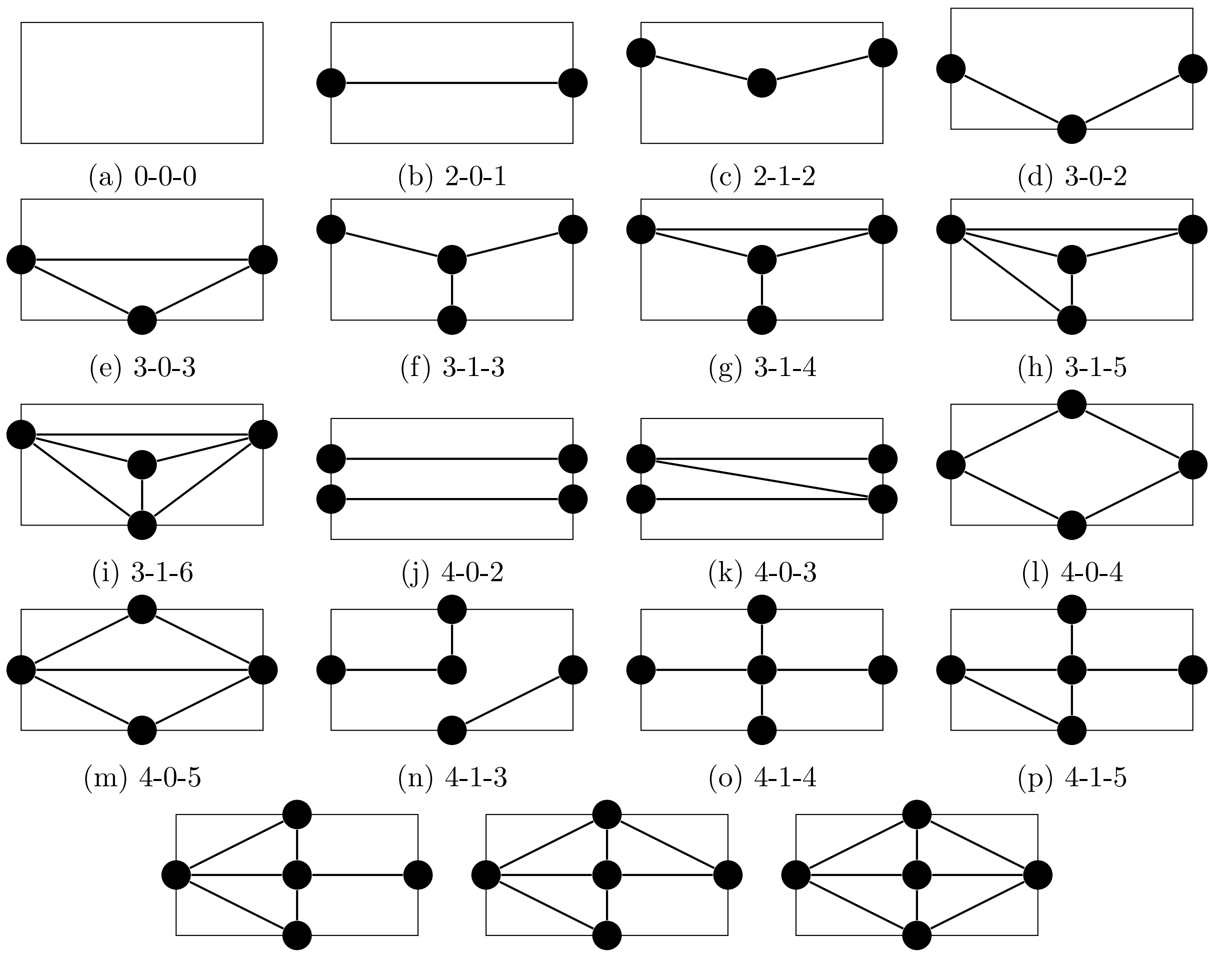
Figure 1.3: Two traditional warehouse designs for order picking

Contribution 1 *We develop near-optimal designs for different order-picking warehouse categories by searching through nineteen different warehouse design classes.*

When we say order-picking warehouse category, we mean the order-picking warehouses that show similar characteristics based on area, average number of stock keeping units (SKUs) , average number of units (such as pieces or cases that are less-than-unit load) per pick line, etc. Frazelle (2002) describes different profiling techniques to characterize warehouses. We relax traditional warehouse design assumptions and allow regions that have parallel pick aisles inside their boundaries to take any angle and allow cross aisles to be oriented freely. We also permit the depot location to float along a side of the warehouse and look for its optimal location. However, we do not consider multiple depots in our research. With given storage, routing and picking policy and set of orders, we propose to find optimal designs by searching through nineteen different warehouse design classes: 0-0-0, 2-0-1, 2-1-2, 3-0-2, 3-0-3, 3-1-3, 3-1-4, 3-1-5, 3-1-6, 4-0-2, 4-0-3, 4-0-4, 4-0-5, 4-1-3, 4-1-4, 4-1-5, 4-1-6, 4-1-7, and 4-1-8 (see Figure 1.4).

We develop a graph-based network model that is used in the detailed evaluator function of the two meta-heuristics, Particle Swarm Optimization (PSO) and ES. PSO and ES are two meta-heuristics that work well with continuous problems. Öztürkoğlu et al. (2014) used PSO to find optimal angles for unit-load warehouses and this is the main reason for our PSO selection. However, based on our experiments PSO did not perform well for the 2-0-1 design class, therefore, we implemented ES. The comparison showed that ES performs better than PSO for the 2-0-1 design class for unit-load operations with the randomized storage policy. Figure 1.5 is an example of this comparison with 300 iterations. ES performs much better and results in a Chevron design (see Figure 1.6) which is proven to be optimal for unit-load warehouses with a single cross aisle (Öztürkoğlu et al., 2014). ES performed better because we think the problem has a good neighborhood structure and ES can better exploit that. Therefore we will only consider ES as our meta-heuristic optimizer.

Contribution 2 *We develop more reasonable paths using the visibility graph method.*


(a) 0-0-0 (b) 2-0-1 (c) 2-1-2 (d) 3-0-2

(e) 3-0-3 (f) 3-1-3 (g) 3-1-4 (h) 3-1-5

(i) 3-1-6 (j) 4-0-2 (k) 4-0-3 (l) 4-0-4

(m) 4-0-5 (n) 4-1-3 (o) 4-1-4 (p) 4-1-5

(q) 4-1-6 (r) 4-1-7 (s) 4-1-8

Figure 1.4: Design classes that are being searched

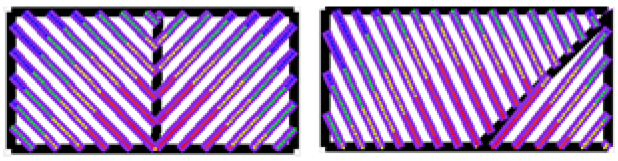


Figure 1.5: ES (left) vs PSO (right) with 300 iterations

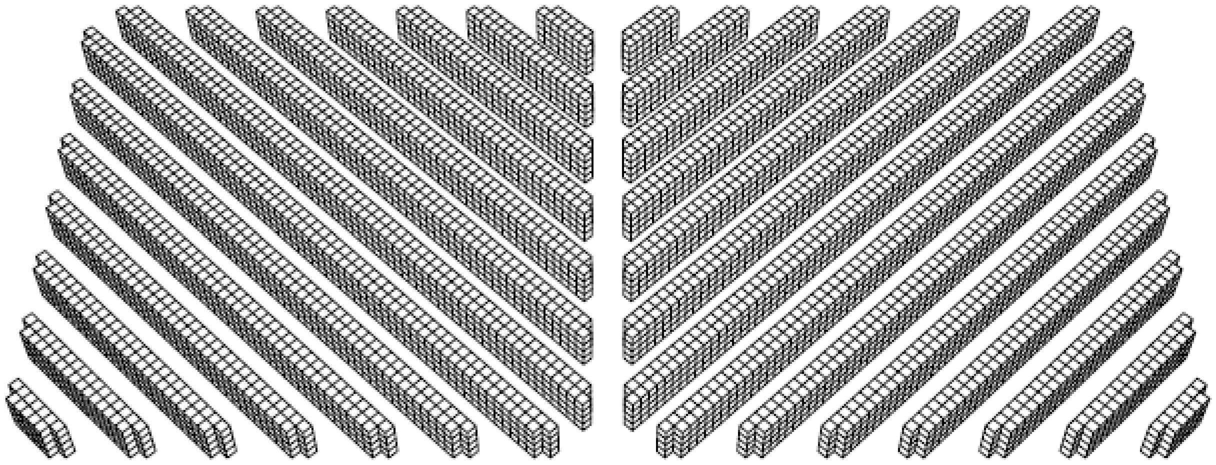


Figure 1.6: The Chevron design

These new paths impact performance comparisons between traditional and non-traditional layouts.

Contribution 3 *We develop a warehouse design system that can optimize the layout for a given set of orders with highly efficient parallel/distributed algorithms.*

One of our main contributions to industry and academia is the development of a warehouse design tool. This tool is capable of:

- Creating a graph-based network model of a warehouse design using both center aisles and visibility graph,
- Searching the solution space (designs in a specific design class) by using Particle Swarm Optimizer or Evolution Strategies,

- Showing a graphical representation of the warehouse with cross and pick aisles, and pick locations,
- Showing the most and the least convenient storage locations in different colors,
- Interfacing with TSP Solver Concorde and Lin-Kernighan-Helsgaun to find the optimal/near-optimal routing per pick tour,
- Importing order pick list data in order to allocate products and to perform design optimization,
- Importing a list of different warehouse designs in Excel format to calculate their objective functions and export the list of designs with their objective function values. This capability is helpful to someone who wants to create a design of experiments to perform regression or use it as a training data set for ANNs or another meta-modeling technique,
- Searching nineteen different warehouse design classes,
- Calculating the wasted space used by cross aisles,
- Solving large batches of TSPs in parallel and distributed computing environments.

Chapter 2

Literature Review

2.1 Warehouse Operations

A warehouse is defined as a place for storing or buffering large quantities of raw materials, work-in-process items or finished goods. Warehouses are used by manufacturers, wholesalers, importers, exporters, etc. Figure 2.1 shows the functional areas and operations within warehouses. In a typical warehouse, items are received from suppliers, they are stored in storage locations, order pickers fulfill customer orders and assemble them for shipment, and completed orders are shipped to customers. A typical warehouse shares many functions with a cross-docking warehouse. However, storage is an important function in a typical warehouse, while in a cross-docking warehouse, there is little or no storage (Bartholdi and Hackman, 2011).

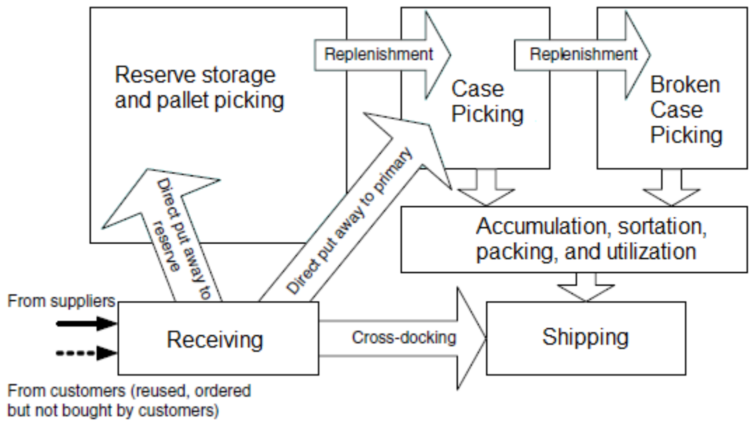

Reserve storage and pallet picking
Replenishment
Case Picking
Replenishment
Broken Case Picking
Direct put away to reserve
Direct put away to primary
Accumulation, sortation, packing, and utilization
From suppliers
Receiving
Cross-docking
Shipping
From customers (reused, ordered but not bought by customers)

Figure 2.1: Typical warehouse flows and operations (Tompkins, 2010)

2.1.1 Receiving and Shipping

Receiving is the first activity in a warehouse. When an inbound trucker phones the warehouse to get a delivery appointment, the receiving operation begins. The trucker arrives at the assigned receiving dock, goods are unloaded and inspected at a place before being stored in an assigned location. This place is called the "pickup and deposit (P&D) point". For cross-docking warehouses, received items are sent directly from the receiving docks to the shipping docks. For typical warehouses, received items are put into storage. Then orders are picked from storage, prepared for shipment, and shipped to customers through shipping docks. The receiving and shipping operations are integrated with the storage and order picking operations and this makes the receiving and shipping operations more complicated in typical warehouses than in cross-docking warehouses (Gu et al., 2007). The literature on receiving and shipping is very limited and has been focused on the truck-to-dock assignment problem for cross-docking warehouses (see Gu et al. (2007), for details).

2.1.2 Storage

Storage is the physical containment of an item while it is awaiting a demand (Tompkins, 2010). According to Bartholdi and Hackman (2011), a storage mode means a region of storage or a piece of equipment for which the costs to pick/restock from any location are all approximately equal. The storage mode of items depends on the size and quantity of the items in inventory and the handling characteristics of the product or its container. Storage modes can be grouped into two main categories: storage modes for unit-load operations and storage modes for less-than-unit-load operations. A unit-load combines individual items or items in shipping containers into single units that can be moved easily with a pallet jack or forklift truck. The typical unit-load is a pallet. Pallets are either stored in block stacking or pallet racks (and their slight variations). Because pallets are (mostly) standardized and are generally large, they are mostly handled one at a time in unit-load operations. Third-party transshipment warehouses are typically unit-load warehouses that receive, store, and forward pallets. Many warehouses also perform unit-load operations in some portion of their activities. Storage modes for less-than-unit-load storage include items that are generally stored in cases or cartons (and sometimes pieces). The most popular less-than-unit-load storage mode systems are bin shelving, modular storage drawers/cabinets, and gravity flow racks (Frazelle, 2002). Selection of storage mode generally includes these objectives:

- Efficient space use,
- Efficient material handling,
- The most economical storage in relation to costs of equipment use of space, damage of material, handling labor, and operational safety,
- Maximum flexibility in order to meet changing storage and handling requirements, and
- A good model of the organization of SKUs throughout the warehouse.

Detailed literature about storage mode selection is in Frazelle (2002) and Bartholdi and Hackman (2011).

When the storage mode is selected for each item, a decision must be made for storage assignment for each item. The product allocation problem concerns the assignment of items to storage locations. There are five frequently used product allocation policies: random storage, closest-open location storage, class-based storage, full-turnover storage, and dedicated storage (De Koster et al., 2007). Random storage, closest-open location storage, and class-based storage are also called *shared storage.*

The random storage policy allows items to be stored anywhere in the storage area with equal probability. This policy has an advantage of higher space utilization (or low space requirement) at the expense of increased travel distance (Choe and Sharp, 1991), but it is harder to manage, because locations of items change over time. The closest-open location storage policy allows order pickers to choose the location for storage themselves. This policy leads to a storage area where storage locations are full around the depot location and gradually empty towards the back. The class-based storage policy distributes the items based on some measure of demand frequency, among a number of classes, where each class represents a region within the storage area. The idea is based on Pareto's method: *85% of the turnover will be a result of 15% of the items stored.* To increase the order picking efficiency, the most popular 15% of items should be stored such that travel distance is minimized. Storage within a region is random.

The full-turnover storage policy distributes items over the storage area according to their turnover. Items with the highest turnover rates are stored at the"closer to depot" locations or "easily accessible" locations. Slow moving items are stored closer to the back of the storage area. The dedicated storage policy assigns each product to a specific location. Material handling workers know in advance where to store items and order pickers become familiar with the locations of items which reduces time wasted for searching. On the other hand, average storage capacity is only about 50% utilized (Bartholdi and Hackman, 2011). Petersen and Aase (2004) show that with regard to travel distance in a manual order-picking system, full-turnover storage and class-based storage outperform random storage. However, these policies may also increase picker congestion within aisles containing the most popular SKUs. Also, they may require periodic movement of SKUs because of demand seasonality of SKUs. Cube-per-order index (COI) is one of the most well known implementations of the full turnover-based storage policy (Heskett, 1963, 1964). The COI of a product is the ratio of the product's total required space to the number of trips required to satisfy its demand per period. The ones with the lowest COI are located closest to the depot. Goetschalckx and Ratliff (1990) consider shared storage and show that a duration-of-stay policy is optimal under an assumption of perfectly balanced inputs and outputs. The duration-of-stay policy requires arrival/departure information on individual items of a particular product, whereas the turnover-based and class-based storage policies require only turnover rate information at the product level. In this research we assume we know only product information.

Each storage policy has advantages and disadvantages. Some disadvantages can be easily eliminated by using a warehouse management system (WMS) that gives information to material handling workers and reduces search and travel time. But this may not be the case in practice. According to Bartholdi and Hackman (2011), shared storage requires greater software support and more disciplined warehouse processes. For example, a worker might pick the item from a more convenient location rather than a farther location that is given by the WMS.

2.1.3 Order Picking

Order picking is the retrieval of items from a storage area to fulfill customer orders. It involves the process of grouping and scheduling customer orders, releasing them to the order pickers, the picking of the items from storage locations, and the disposal of the picked items (De Koster et al., 2007). The main objective of order picking is to maximize the service level subject to resource constraints such as labor, machines, and capital (Goetschalckx and Ashayeri, 1989). The faster an order can be retrieved, the sooner it is available for shipping to the customer. If the warehouse cannot process orders quickly, effectively, and accurately then service level optimization efforts will suffer. Any inefficiency in order picking can lead some orders to miss their shipping due time. These orders either have to wait until the next shipping period or have to be shipped with expedited shipping. Either way, the warehouse will suffer from customer dissatisfaction or additional cost. Therefore, minimizing order retrieval time is a key to a successful warehouse. Figure 5.1 shows the order picking time components in a typical warehouse. 50% of the order picker's time is travel time. Travel time is a direct expense but it does not add value, therefore it is a waste (Bartholdi and Hackman, 2011). For manual-pick order picking systems, the travel time is an increasing function of the travel distance (De Koster et al., 2007). For these reasons, we select minimizing travel distance as an objective for improvement.

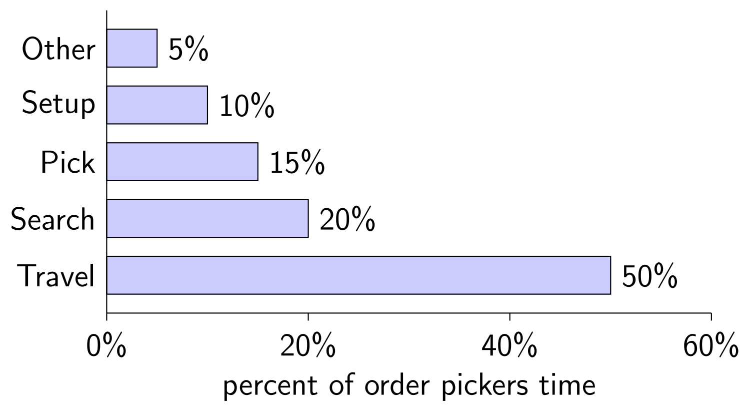


Figure 2.2: Typical distribution of an order picker's time (Tompkins, 2010)

2.1.3.1 Order Picking Systems

Warehouses can have multiple order picking systems. These systems are either manual or automated (see figure 2.3). Tompkins (2010) states that there must be a balance between the level of automated systems (which are inflexible) and labor in order to respond to future business requirements without sacrificing logical labor savings today. Because automation is capital intensive and inflexible, the majority of warehouses employ humans for order picking (Le-Duc, 2005).

The *picker-to-parts system*, where the order picker walks or drives along aisles to pick items, is most common among manual order picking systems. De Koster et al. (2007) distinguish two types of picker-to-parts systems according to picking height level: *low-level* picking and *high-level* picking. In low-level order picking systems, an order picker picks items from bin shelving storage, modular storage drawers/cabinets, or gravity flow racks. Cart picking and tote picking are the most common retrieval methods. High-level order picking systems (sometimes called man-aboard order picking systems) improve storage density. An order picker retrieves items by using a lift truck, an order picker truck or a man-aboard automated storage and retrieval system. High-level order picking systems are less productive compared to low-level order picking systems because of the vertical travel time. An order picker can perform 70-120 picks per hour with cart picking compared to 50-100 picks per hour by using an order picker truck (Frazelle, 2002).

Parts-to-picker systems include carousels and automated storage and retrieval systems (AS/RS). Carousels are mechanical devices that hold and rotate items for order picking. Horizontal and vertical carousels are most common. In an AS/RS, a storage and retrieval (S/R) machine travels horizontally and vertically simultaneously in a picking aisle, transporting one or more unit loads (pallets or bins) to and from the input/output (I/O) point located at the end of the system. The order picker takes the required number of items when the S/R machine arrives at the I/O point. The S/R machine can work under different operating modes: single, dual and multiple command cycles. The single command cycle performs either a storage or a retrieval between successive visits to the I/O point. In a dual command cycle, a load is deposited at an empty location from the I/O point, then another load is retrieved from the rack. In multiple command cycles, the S/R machines have more than one shuttle and can pick up and deposit multiple loads in one cycle.

Put systems combine the principles of parts-to-picker and picker-to parts order picking systems. In these systems, item retrieval can be performed either in a parts-to-picker or a picker-to-parts manner. After the items are retrieved, an order picker takes the carrier (usually a bin) associated with these items, then distributes the items over customer orders. According to De Koster et al. (2007), put systems can be very efficient in well-managed warehouses such that an order picker can pick up to 1000 small items in an hour. In this research we limit ourselves to low-level picker-to-parts systems with multiple picks per route. These systems are very common in practice, especially in Western Europe where 80% of order picking systems are of this type (De Koster et al., 2007). Detailed design and selection methods for order picking systems are given in Frazelle (2002) and Dallari et al. (2009).

2.1.3.2 Order Picking Methods

In picker-to-parts systems, there are several methods of order picking (see Table 2.1).

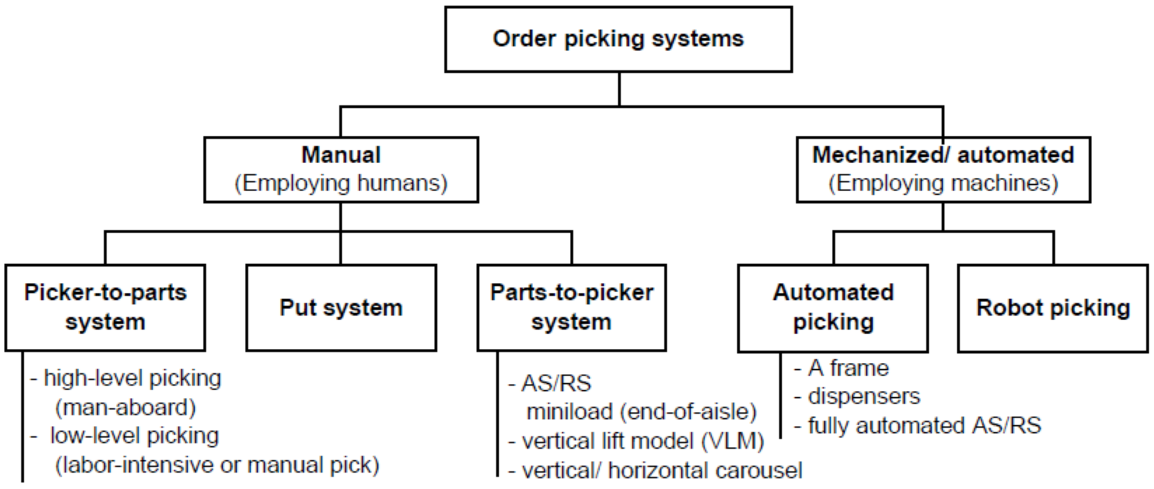


Figure 2.3: Classification of order picking systems (De Koster et al., 2007)

Discrete picking is the most common order picking methodology because of its simplicity (Tompkins, 2010). An order picker completes a tour through the warehouse to pick all items for a single order. Because the risk of omitting merchandise from an order is reduced and it provides the fastest customer response in a service window environment, this methodology is often preferred. Especially in warehouses with large orders (those with greater than ten-line items), discrete picking may yield an efficient picking tour (Frazelle, 2002). However, it is the least productive method due to excessive travel time per pick compared with other methods.

Batch picking is another order picking methodology that is commonly used for case and broken-case picking. Instead of traveling throughout the warehouse to finish a single order, an order picker can complete several orders with a single trip. By increasing the number of orders per tour, the number of items picked per tour by an order picker increases. Hence, it reduces the travel time per pick and reduces the total travel time to complete all orders. On the other hand, orders need to be sorted either by the order picker (*sort-while-pick*), or the sorting takes place in a designated area after the pick process has finished (*pick-and-sort*). Therefore, the benefits of reduced travel time must be compared against the cost of sortation. Batching single-line orders is a common strategy because it uses the advantage of shorter travel time per pick and there is no need to sort items.

Zone picking divides the order picking area into distinct sections and assigns order pickers to picking zones. Picking zones should not be mistaken for storage zones, which are defined to facilitate efficient and safe storage and are specified in slotting. In zone picking, an order picker works on one order at a time and picks all lines that are located within that zone for that order. Then, these items are brought to an area for consolidation before shipment. Traversal of a smaller area, reduced traffic congestion, good housekeeping in the order picker's zone (items are better organized in picking zone), and the order pickers' familiarity with the item locations are the main advantages of zoning. The main disadvantages of zoning are the cost of sorting and the potential for order-filling errors. Zoning might be necessary because of the different skills or equipment associated with a warehouse. Also product properties such as size, weight, and safety requirements might force a warehouse manager to define zones in the warehouse.

There are two variations for establishing order integrity in zone picking. *Sequential zone picking* (or progressive zoning) is picking one zone at a time, then passing the order to next zone until the order is completed. The contents of the order generally move in a tote and the order picker hands the tote and pick list to the next picker. Some intelligent systems may skip zones where no items need to be picked (*zone skipping*). In *simultaneous zone picking* (or synchronized zoning), a number of order pickers within their zones start on the same order in parallel, then these partial orders are consolidated at a designated location. *Wave picking* is same as discrete picking except that a selected group of orders is scheduled to be picked during a specific time horizon. The other three methodologies are combinations of the methodologies described above, therefore they are more complex and require more control.

2.1.3.3 Routing Methods

Routing methods determine the picking sequence of items on the pick list to ensure a good route through the warehouse. The problem of sequencing and routing order pickers is a special case of the TSP, classified as the *Steiner* TSP. In a classical TSP, given distances

Table 2.1: Methods of order picking (Tompkins, 2010)

Procedure	Pickers per Orders	Line Items per Pick	Periods per Shift
Discrete	Single	Single	Single
Zone	Multiple	Single	Single
Batch	Single	Multiple	Single
Wave	Single	Single	Multiple
Zone-Batch	Multiple	Multiple	Single
Zone-Wave	Multiple	Single	Multiple
Zone-Batch-Wave	Multiple	Multiple	Multiple

between each cities, a salesperson needs to find the shortest possible route that visits each city exactly once and returns to the origin. Similarly, in order picking, the order picker starts at the depot location (origin), visits all pick locations in his/her pick list and then returns to the depot location. However, some differences exist between the classical TSP and the Steiner TSP. In the Steiner TSP, some cities do not have to be visited at all but some other cities can be visited more than once. The Steiner TSP is, in general, not solvable in polynomial time (De Koster et al., 2007). However, Ratliff and Rosenthal (1983) found the problem of order picking in a rectangular warehouse as a solvable case of the TSP, and proposed a polynomial-time dynamic programming algorithm to solve it optimally. Their algorithm uses an approach that starts from the left-most pick aisle, enumerates all possible equivalence classes (i.e., the possible degree parities of the corner nodes of the pick aisles and the number of connected subtours in the current partial solution) for each picking aisle. Then it finds the best equivalence class solutions for the right-most pick aisle, and finds the optimal route by backward recursion from these solutions. For the case of single block warehouses, there are seven possible equivalence classes that need to be considered. De Koster and Poort (1998) showed that this exact algorithm can be extended in such a way that the shortest order picking routes can be found in both warehouses with a central depot and warehouses with decentralized depositing. Roodbergen and De Koster (2001b) extended the algorithm by Ratliff and Rosenthal (1983) to warehouse settings with two blocks (i.e., three cross aisles or one middle aisle). Çelik and Süral (2014) showed that the multi-item order picking problem can be solved in polynomial time for both fishbone and flying-V layouts. The main idea behind their algorithm is to transform the fishbone layout into an equivalent warehouse setting with two blocks. For warehouses with three or more blocks, the number of possible equivalence classes increases exponentially (Çelik and Süral, 2014). Therefore, extending the algorithm is of little of use.

The existing literature has largely been devoted to finding efficient heuristics because efficient optimal algorithms are not available for every layout, and optimal routes may not consider real-world problems in order picking such as aisle congestion. For example, an S-shape can avoid aisle congestion because it has a single direction if the pick density is sufficiently high (i.e., there is at least one pick in every aisle). Many routing policies described in the literature have been analyzed for four types of warehouse systems (i.e., conventional multi-parallel-aisle systems, man-on-board AS/RS, unit-load AS/RS, and carousel systems) (Gu et al., 2007). When using a S-shape heuristic, order pickers must completely traverse the entire aisle containing at least one pick. Aisles without picks are not visited. From the last entered aisle, the order picker returns to the depot. In the return method, the order picker enters and leaves each aisle from the same end and only visits aisles with picks. The midpoint policy divides the warehouse into two areas (see Figure 2.4). The heuristic collects all the items in the upper section, after which the lower section is dealt with. In the largest gap method, the order picker traverses the first and last aisle with picks entirely. All the other aisles are entered from the front and back in such a way that the non-traversed distance between two adjacent locations of items to be picked in the aisle is maximized. In the composite heuristic, aisles with picks are visited, but dynamic programming is used to decide either entirely traverse or enter and left at the same end (see Roodbergen and De Koster (2001a)). Petersen (1997) analyzed six routing heuristics (S-shape, return, midpoint, largest gap, composite, and optimal) for single-block warehouses and concluded that the best heuristic solution is on average 5% more than the optimal solution (see Figure 2.4).

These routing heuristics are not suitable for our research for two reasons. First, these heuristics are designed for single-block warehouses and some (aisle-by-aisle, S-shape, largest gap, combined) can be modified for multiple-block warehouses, but they are not designed for non-traditional designs. Therefore, the routing might not work in some non-traditional designs. Second and most important, these heuristics are fairly simple construction heuristics which construct a feasible solution, without attempting any improvement by means of local search or meta-heuristic search.

Makris and Giakoumakis (2003) present Lin and Kernighan (1973)s TSP-based k-interchange methodology for single-block warehouses and show that their procedure outperformed the S-shape heuristic in seven of eleven examined cases. Theys et al. (2010) extended their research for multi-block warehouses and achieved average savings in route distance of up to 47% when using the Lin-Kerhighan-Helsgaun (LKH) heuristic (Helsgaun, 2000) compared to S-shape, largest gap, combined, and aisle-by-aisle heuristics. The quality of the solutions by the LKH heuristic is, on average, clearly superior to the other routing heuristics. Although the LKH heuristic's average computation time (0.25 seconds) is more than these heuristics (the calculation time is negligible for these heuristics) it is 36 times faster than the exact TSP algorithm "Concorde" (9.23 seconds). Moreover, LKH's solutions deviate on average only 0.1% from optimum. A detailed description of these routing policies and their variations can be found in De Koster et al. (2007), Gu et al. (2007), and Helsgaun (2000).

2.1.3.4 Complexity of Order Picking Systems

Figure 2.5 helps us to identify the complexity of order picking systems. If the system is further from the origin, it is harder to design and manage. In our research we are focused on manual, static, many aisles, continuous, no-zoning, pick-by-order order picking systems. We are interested only in turnover-based storage policy, because it outperforms random and class-based storage policies with regards to travel distance (Petersen and Aase, 2004). We

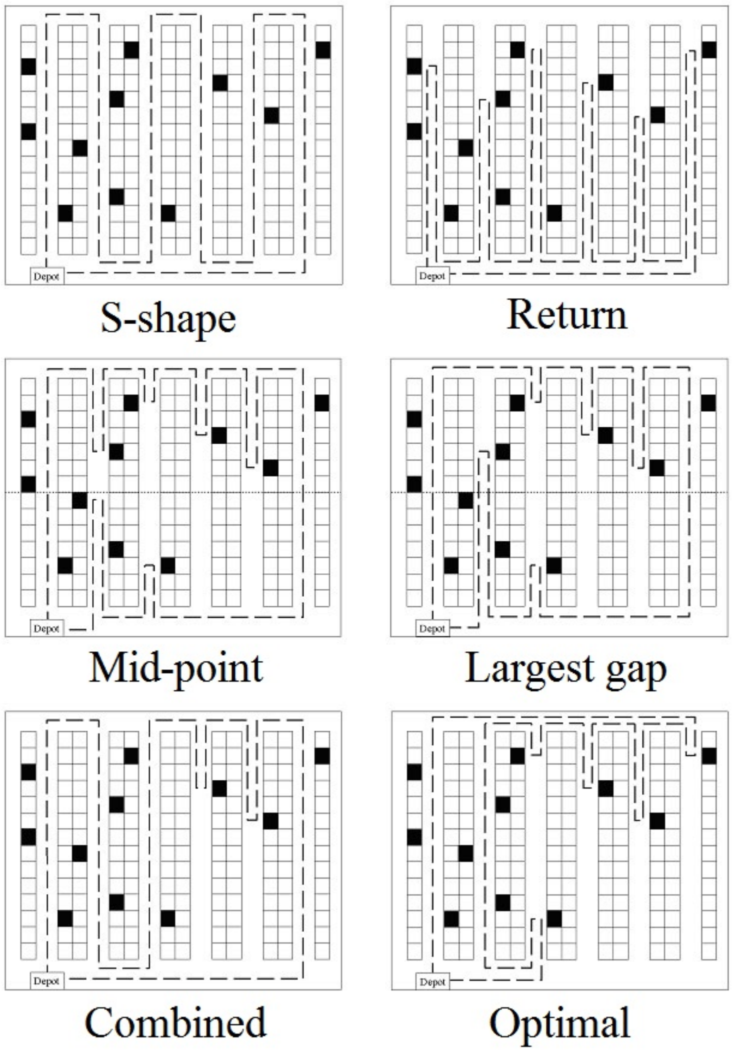


Figure 2.4: Routing methods for single-block warehouses (De Koster et al., 2007)

are focused on optimal routing, because computational methods for solving routing problems are very efficient and because most heuristics in the literature are dependent on traditional aisle structures. We select manual picking, many aisles, because that is the most common warehouse in practice as we mentioned earlier in Chapter 1 (De Koster et al., 2007). We select pick-by-order strategy because order batching strategies need more computation and our focus is on the layout design rather than batching strategies. As a consequence of the pick-by-order strategy, there is no-zoning either.

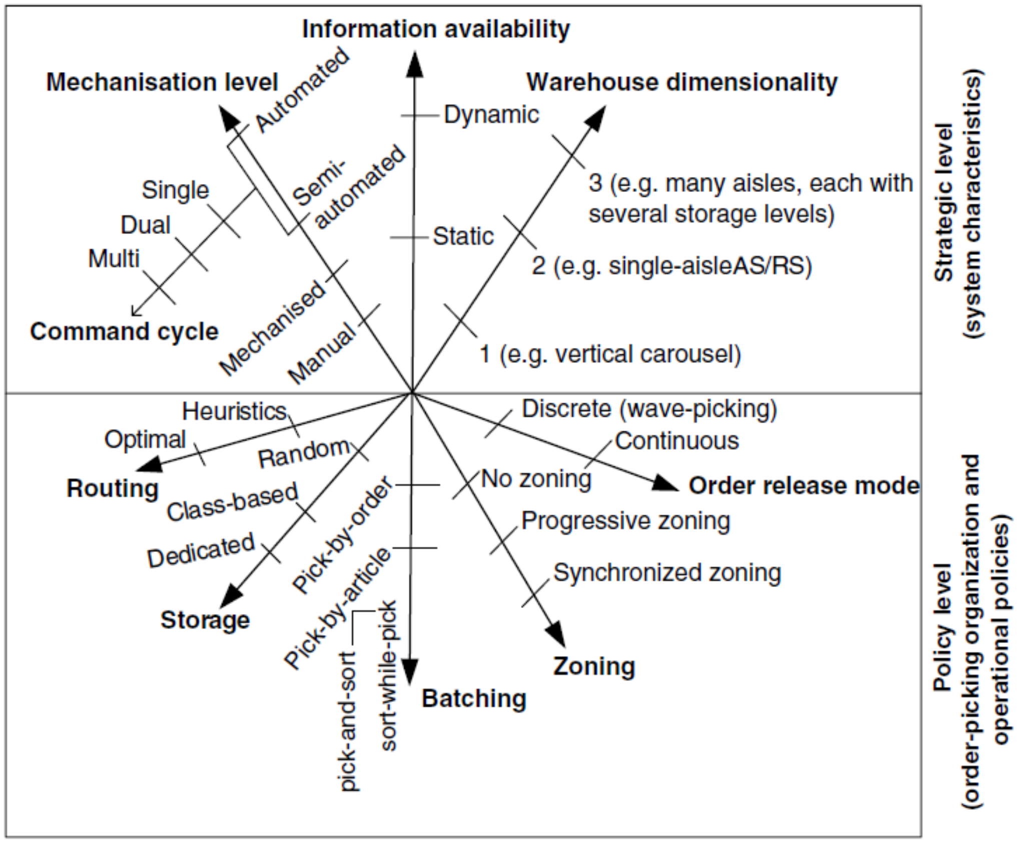


Figure 2.5: Complexity of order picking systems (De Koster et al., 2007)

2.2 Warehouse Design

2.2.1 Warehouse Layout

De Koster et al. (2007) divide layout design problems in the context of order picking into two sub-problems: the layout of the facility containing the order picking system and the layout within the order picking system. The first problem is related to the *facility layout problem* and the main objective is to determine the location of various departments such as receiving, picking, storage, sorting, and shipping that minimizes the handling cost related to the activities between departments. The second problem can also be called the *aisle configuration problem*. The warehouse layout within the order picking system serves as a master plan for storage. It indicates the locations and widths of the aisles, traffic flow, dock locations, and storage bay depths. It may specify areas for storage of certain types of products. The layout may also include specialized areas for order picking, high security storage, or temperature controlled space. Space is often lost because of the need for access to each item on the pick list (honeycombing), and this loss can be minimized by an effective layout (Ackerman et al., 1972). The most common objective function is travel distance (De Koster et al., 2007). Our research focuses on this second sub-problem.

The literature related to the aisle configuration problem is mostly related to traditional warehouses. Several researchers used different combinations of P&D locations. Bassan et al. (1980) showed that the depot should be centrally located under the condition of random storage to minimize average travel distance. Kunder and Gudehus (1975) and Hall (1993) considered a centrally located P&D location and derive analytical travel models for manual order-picking. Bartholdi and Hackman (2011) divide the layout problem within the order picking system into three types: *layout of a unit-load area*, *layout of a carton-pick-from pallet area*, and *layout of a piece-pick-from-carton area.*

A unit-load warehouse means that only a single unit (typically a pallet) of material is handled at a time. It is the simplest type of warehouse. Third party warehouses and import warehouses are examples of unit-load warehouses that receive, store and ship pallets. Many warehouses also devote some portion of their activity to unit-load operations. In single-command operations, one item is picked or stored during one trip by an operator. Hence, operators travel empty either when they return to a P&D location or go to a storage location for picking (dead-heading). Single-command operations are common in unit-load warehouses and unit-load replenishment in the reserve area. Francis (1967) and Bassan et al. (1980) modeled single-command travel distance in traditional warehouses and present some well-known results on optimal warehouse shape and P&D location. In order to reduce dead-heading, dual-command operations can be used. In dual-command operations, an operator deposits an item and then goes to another location to pick another item, then returns back to a P&D location. Malmborg and Bhaskaran (1987) considered dual-command travel in traditional warehouses. Pohl et al. (2009a) develop expected travel distance equations in traditional designs with a middle cross aisle for dual-command operations. The literature on the layout design problem for unit-load warehouses has mainly focused on AS/RS (see Sarker and Babu (1995) and Roodbergen and Vis (2009) for extended literature reviews on AS/RS).

The second type of layout problem is carton-pick-from-pallet-area where the storage or restocking operation is done by pallets but the picking is done with cartons or cases, which makes it different than unit-load warehouses. The handling unit (a carton or case) weighs between 5 and 50 pounds and the picking operation can be handled by one person. The warehouse is divided into two areas: *forward area* and *reserve area*. Popular SKUs are usually placed in the forward area for case-picking and replenished from the reserve area. The problem of deciding which products should be stored in the forward area and in what quantities is called the *forward-reserve problem* (FRP). If a product is not stored in the forward area, then it needs to be picked from the reserve area. The most common pick area is the ground floor of a pallet rack. Generally, dedicated storage is used in the forward area. Hackman et al. (1990) presented a heuristic method for the FRP that attempts to minimize the total costs for picking and replenishing. Frazelle et al. (1994) extended their work to determine the size of the forward area together with the allocated products. Van den Berg et al. (1998) considered busy and idle periods for order picking operations to find an allocation of product quantities in the forward area which minimizes the expected labor time during the picking period. Reducing the number of replenishments in busy periods by performing replenishments in idle periods not only increases throughput during the busy periods but also reduces possible congestion. Gu (2005) proposes a bi-level hierarchical heuristic approach that is very efficient in generating near optimal solutions for sizing and dimensioning of a forward-reserve warehouse.

A third type of layout problem is piece-pick-from-carton-area where storing or restocking operations are done by cartons but the picking operation is done with pieces. It is the most labor intensive activity in the warehouse because the product is handled at the smallest unit-of-measure. Moreover, the importance of piece-picking operation has increased in the past 20 years because of pressure to reduce inventory and use the remaining space to expand product variety. A general rule of thumb in this type of warehouse is to separate the storage and the picking activities. A *fast-pick area* is a sub-region of the warehouse in which picks and orders are concentrated within a small physical space. This leads to reduced pick costs and quick response to customer demand.

2.2.2 Non-Traditional Designs

There has been much work done in the literature related to order picking warehouses and how to modify their designs. However, most previous research assumed two design rules:

- Picking aisles must be straight and parallel to each other.
- If present, cross aisles must be straight, parallel to each other and orthogonal to the picking aisles.

In the late 2000s, the non-traditional warehouse layout problem became a main focus. Gue and Meller (2009) proposed two non-traditional unit-load aisle configurations: the flying-V and fishbone layouts (see Figure 2.6). The former has two nonlinear cross aisles inserted into a traditional layout and offers about 10% improvement over the traditional design. The latter has two angled cross aisles at 45° and 135°, with picking aisles at 0°, 90°, and 180° in the regions divided by the angled cross aisles and offers about 20% improvement over the traditional design. Pohl et al. (2009b) modeled a dual-command expected travel distance expression for the fishbone layout and observed that the fishbone design still reduces dual-command travel by approximately 10%-15%. They also offered a fishbone-triangle for dual-command operations by inserting an additional cross aisle segment between two diagonal cross aisles for dual-command operations. Öztürkoğlu et al. (2014) extended the work by Gue and Meller (2009) to the case of angled picking aisles. They showed that up to

22.52% reduction in single-command travel distance is possible, as compared to a traditional warehouse with parallel aisles.

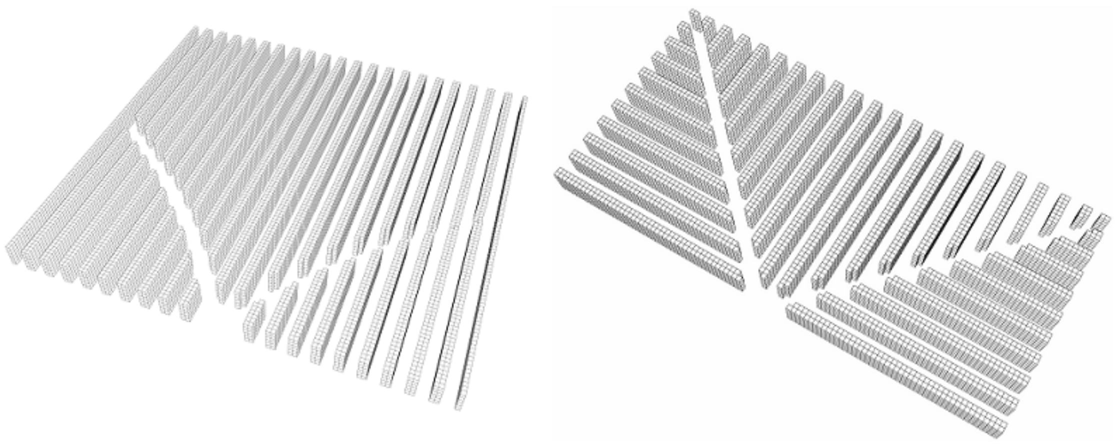

(a) Flying-V (b) Fishbone

Figure 2.6: Flying-V and Fishbone layouts proposed by Gue and Meller (2009)

To the best of our knowledge, there exists only a few studies that discuss the performance of the fishbone layout under multiple-item picks. Dukic and Opetuk (2008) analyzed the fishbone layout and concluded that it results in larger routes than the traditional layout under multiple-item picks. However, their analysis is based on the results of an S-shape heuristic modified for fishbone layout and they only considered a random storage policy. Çelik and Süral (2014) filled the gap by presenting a polynomial-time algorithm that optimally solves the picker routing problem and presented alternative heuristic methods. They also presented their analyses for different warehouse sizes and shapes, and under random and turnover-based storage policies. They showed that as the size of the pick list grows, the fishbone is outperformed by the traditional layout under random storage, with a maximum gap of around 36%. They observed that the relative performance of the fishbone layout is better under random demand for single-command operations, whereas it performs better under more skewed demand when the pick list size grows larger.

2.3 Heuristic Optimization

2.3.1 Particle Swarm Optimization

Particle swarm optimization (PSO) is one of the most popular evolutionary algorithms. It was introduced by Kennedy and Eberhart (1995) as an alternative to other evolutionary algorithms, such as genetic algorithms, evolution strategies, and differential evolution to solve continuous non-linear functions. The paradigm is inspired by the flocking of birds and schooling of fish.

In the original PSO algorithm, each particle represents a solution that moves toward its previous best position and the global best position found in the population so far. After initializing parameters and generating the initial population randomly, each particle is evaluated by the fitness function. After evaluation, each particle updates its position, velocity, and fitness value. If there is an improvement in its fitness value, it updates its personal best. If the best particle in the population improves the global best (i.e., the best particle found so far in the population), then the global best is also updated. Next, the velocity of the particle is updated by using its previous velocity, personal best and the global best to move the particle to another (hopefully, better) place in the search space. The algorithm performs these steps repeatedly until it is terminated by a stopping criterion such as maximum number of iterations. Shi and Eberhart (1998a,b) introduced an inertia weight to the algorithm. It is denoted as w and is used to balance global and local search. It can be a positive constant or a function of iteration. A larger w means that higher importance is given to global search, while smaller values of w favors local search. Some application areas of PSO are chemistry and chemical engineering (Cedeno and Agrafiotis, 2003; Shen et al., 2004), function optimization (Kennedy and Eberhart, 1995; Parsopulos and Vrahatis, 2002), operations research (Baltar and Fontane, 2006), and machine learning (Meissner et al., 2006).

Several studies have been done related to warehouse layout optimization using PSO (Onut et al., 2008; Sooksaksun et al., 2012; Öztürkoğlu et al., 2014). However, research focusing on warehouse design optimization using PSO has been either focused on order picking operations for traditional warehouse layouts or unit-load operations for non-traditional layout designs. To the best of our knowledge, there is a lack of research combining these two aspects.

2.3.2 Evolution Strategies

Evolution strategies (ES) were introduced by Rechenberg (1965) and Schwefel (1965) and imitate the principles of natural evolution as a method to solve parameter optimization problems. ES is a population-based meta-heuristic optimization algorithm that uses biology-inspired mechanisms such as mutation, crossover, and survival of the fittest in order to refine a set of solution candidates iteratively. The advantage of ES compared to other optimization methods is its “black box” character that makes only few assumptions about the underlying objective functions. Moreover, objective function definitions generally require less insight into the structure of the problem space than the manual construction of an admissible heuristic. Therefore, the performance of ESs is generally good for many problem classes. Some application areas of ES are data mining and data analysis (Cordon et al., 1998), scheduling (Huang et al., 1999), chemistry and chemical engineering (Roosen and Meyer, 1992; Cutello et al., 2005; Emmerich et al., 2000), resource minimization and environment surveillance/protection (O’Brien et al., 2003), combinatorial optimization (Nissen and Krause, 1994; Beyer, 1992), geometry and physics (Keller et al., 1999; Weinert and Mehnen, 2001), and optics and image processing (Greiner, 1996; Back and Schutz, 1995; Wiesmann et al., 1998; Li et al., 2006). There are several variants of ES based on population strategy. The *(1+1)-ES* consists of a single individual which is reproduced. Both the elder and the offspring compete and the better individual survives to form the next population. The *(1,1)-ES* is equivalent to random walk because in every generation the parent is replaced with its offspring and there is no survival of the fittest. The *(μ + 1)-ES* contains μ individuals from which one is drawn randomly. This individual is reproduced from the current population, and the worst fit individual is removed from the the joint set of its offspring and the current population. This strategy is also called "Elimination of the worst". The *(μ + λ)-ES* uses reproduction operations and from μ parent individuals $\lambda \geq \mu$ offspring are created. From the joint set of parents and offspring, only the μ fittest ones survive (Schwefel, 1975, 1977). In *(μ, λ)-ES*, introduced by Schwefel (1981), $\lambda \geq \mu$ offspring are created from μ parents. The parents are subsequently replaced with the μ fittest from the λ offspring individuals. ES uses primarily mutation and selection as search operators. The operators are applied in a loop and an iteration of the loop is called a generation. The sequence of generations is continued until a termination condition is met. As a termination condition, distance measures in the fitness or maximum number generations can be used. For continuous search spaces, mutation is normally performed by adding a normally distributed random value to each element of the vector. The mutated strategy parameter σ (i.e., the standard deviation of the normal distribution) controls the mutation strength. Larger σ values increase the scatter of the mutation and smaller σ values narrow down the mutation. Rechenberg (1973) stated that the quotient of the number of successful mutations to the total number of mutations should be approximately $\frac{1}{5}$. This is also called the $\frac{1}{5}$ success rule (Schwefel, 1993). If the quotient is larger, σ should be increased (increasing the scatter of the mutation), else σ should be decreased to narrow down the mutation. Schwefel (1993) suggested dividing the σ by the factor 0.85 when the quotient is larger, and multiplying it by 0.85 if the quotient is smaller.

2.4 Conclusions

As we mentioned before, non-traditional warehouse design is a new area. The majority of work has been concentrated on unit-load warehouses, with only two studies done in order picking operations. Dukic and Opetuk (2008) also state that more research is needed regarding other routing policies, storage methods, and the shape and size of warehouse to completely validate fishbone layout.

In conclusion, the following potential research areas emerge from the analysis of the literature in warehouse design and optimization:

- Order picking systems are the heart of the warehouse. However, investigation of combining storage and routing policies is mostly limited to traditional warehouse designs in order picking operations. A way to combine storage and routing policies with non-traditional warehouse design is needed.

- Development of new methodologies that bring together the known effects of warehouse design, that can evaluate different layouts according to the needs of the enterprise, and that can create optimal or improved warehouse designs is needed. Using meta-modeling and heuristic optimization techniques together in warehouse design creates an emerging approach that has not been used in the field.

- Petersen and Aase (2004) showed that a good storage policy can decrease the travel time up to 30%. Pohl et al. (2011); Çelik and Süral (2014) used the optimal strategy for single-command operations to analyze multi-command operations because an optimal strategy for multi-command operations is not known. Analysis of other storage policies and finding an effective storage policy for non-traditional designs in order picking operations is an open area.

Chapter 3

Large Batches of Traveling Salesman Problems

3.1 Introduction

With the arrival of multi-core processors in 2005, computers gained more power by providing more clock cycles per CPU. However, most software implementations are still inefficient single processor programs (Ismail et al., 2011). Writing an efficient and scalable parallel program is a complex task. However, C# parallel libraries provide the power of parallel computing with simple changes in the implementation if a certain condition is met: the steps inside the operation must be independent (i.e., they must not write to memory locations or files that are accessed by other steps). Solving large batches of Traveling Salesman Problems is an example of such independent operations. Each TSP instance can be solved by calling a TSP Solver in parallel. Applications of large batches of TSPs include design of order picking warehouses (Ozden et al., 2017), large scale distribution network simulation (Kubota et al., 1999; Sakurai et al., 2006), case-based reasoning for repetitive TSPs (Kraay and Harker, 1997), and delivery route optimization (Sakurai et al., 2011). In these applications the TSP solving consumes most of the computational effort.

We use both the LinKernighan Heuristic (LKH) and the Concorde exact TSP Solver (Concorde). The methods we describe are applicable to optimization problems that must be solved repetitively in an overall algorithm. In this paper, we present two example problems that solve large batches of TSPs and give implementation details in the context of warehouse design for order picking operations. The main result of this paper is to show that doing the parallelism at the TSP level instead of the TSP Solvers' implementation level (Ismail et al., 2011) provides better CPU utilization. A parallel implementation generally achieves better CPU execution times than serial implementations, but an improved CPU utilization is not easily achievable. To the best of our knowledge, this is the first work that presents CPU utilizations for solving large batches of TSPs in serial, parallel, and distributed computing environments.

This work is organized as follows. In Section 3.2, we give a technical description of the Traveling Salesman Problem (TSP) with solution techniques and its variant of large batches of Traveling Salesman Problems. In Section 3.3, we describe our implementation of serial, parallel, and distributed large batches of TSPs solvers. In Section 3.4, we present the computational results, and in Section 3.5 we offer conclusions.

3.2 Traveling Salesman Problem and Solution Techniques

The Traveling Salesman Problem (TSP) is an NP-hard (Garey and Johnson, 1979) combinatorial optimization problem where a salesman has to visit n cities only once and then return to the starting city with minimum travel cost (or travel distance). It is one the most famous and widely studied combinatorial problems (Rocki and Suda, 2013). Solving the problem with a brute force approach requires a factorial execution time $O(n!)$ by permuting all the possible tours through n cities and therefore checking $(n-1)!$ possible tours. Given a starting city, there can be $n-1$ choices for the second city, $n-2$ choices for the third city, and so on. In the symmetric TSP, the number of possible solutions is halved because every sequence has the same distance when traveled in reverse order. If n is only 20, there are approximately 10^{18} possible tours. In the asymmetric TSP, costs on an arc might depend on the direction of travel (streets might be one way or traffic might be considered).

Using an integer linear programming formulation (Ismail et al., 2011), the TSP can be defined as:

$$\min \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij} \tag{3.1}$$

$$\sum_{j \in V} x_{ij} = 1, i \in V \tag{3.2}$$

$$\sum_{i \in V} x_{ij} = 1, j \in V \tag{3.3}$$

$$\sum_{i \in S} \sum_{j \in S} x_{ij} \leq |S| - 1, \forall S \subset V, S \neq \emptyset \tag{3.4}$$

$$x_{ij} \in \{0, 1\}, \forall i, j \in V \tag{3.5}$$

where $x_{ij} = 1$ if the path goes from city i to city j and 0 otherwise. V is a set of cities, S is a subset of V, and c_{ij} is the cost of moving from city i to city j. The first set of equalities enforces that each city be arrived at from exactly one city, and the second set enforces that from each city there is a departure to exactly one other city. The third set of constraints ensures that a single tour is created which spans all cities.

TSP is a widely studied problem where solution methods can be classified as Exact Algorithms, TSP Heuristics, or Meta-Heuristics. Exact algorithms are guaranteed to find an optimal solution in a bounded number of steps. Enumeration is only good for solving small instances up to 10 cities. The dynamic programming algorithm of Held and Karp (1962) and the branch-and-bound algorithm are some well known algorithms in this class. They are good for solving instances up to 60 cities. Concorde is a code for solving symmetric TSPs and related network optimization problems exactly using branch-and-bound and problem specific branch-and-cut techniques (Applegate et al., 2007; Cook, 2014). This algorithm is the current exact method of choice for solving large instances. Concorde was able to solve a 85,900-city problem in TSPLIB (2013).

Heuristics are used when the problem is large and a solution is needed in a limited amount of time. We can categorize these heuristics into two groups: "constructive heuristics" and "improvement heuristics." Constructive heuristics start with an empty tour and repeatedly extend the tour until it is complete. The most popular constructive heuristic is the nearest neighbor algorithm, where the salesman chooses the nearest unvisited city as the next move and finally returns to the first city. Improvement heuristics start with a complete tour and then try to improve it with local moves. The most popular and easily implementable heuristic is the pairwise exchange, or 2-*opt*, which iteratively removes two edges and replaces them with two different edges to obtain a shorter tour. The algorithm continues until no more improvement is possible. *k-opt* is a generalization which forms the basis of one of the most effective heuristics for solving the symmetric TSP, the Lin-Kernighan Heuristic (Lin and Kernighan, 1973). *k-opt* is based on the concept of *k-optimality*: "A tour is said to be *k-optimal* if it is impossible to obtain a shorter tour by replacing any *k* of its links by any other set of *k* links" (Helsgaun, 2000). For a more detailed review of these algorithms refer to (Helsgaun, 2000).

Meta-heuristic algorithms are designed for solving a problem more quickly than exact algorithms but are not specifically designed for any particular problem class. Most of these meta-heuristics implement some form of stochastic optimization. The solution is dependent on the set of random numbers generated. Meta-heuristics' ability to find their way out of local optima contributes to their current popularity. Specific meta-heuristics used for solving the TSP include simulated annealing (Kirkpatrick, 1984), genetic algorithms (Grefenstette et al., 1985), tabu search (Knox, 1994), ant colony optimization (Dorigo and Gambardella, 1997), iterated local search (Lourenço et al., 2003), particle swarm optimization (Shi et al., 2007), nested partitions (Shi et al., 1999), and neural networks (Angeniol et al., 1988). There are many variants and hybrids of these meta-heuristics designed to solve the TSP (Lazarova and Borovska, 2008).

3.2.1 Parallel/Distributed Implementations

The algorithms mentioned in this section solve a single TSP using parallel/distributed techniques. A parallel and concurrent version of the Lin-Kernighan-Helsgaun heuristic using SPC^3 programming is implemented by Ismail et al. (2011). SPC^3 is a newly developed parallel programming model (Serial, Parallel, and Concurrent Core to Core Programming Model) developed for multi-core processors. Developers can easily write new parallel code or convert existing code written for a single processor. All of their speed-ups were less than 2 times compared to single thread runs, even when using a 24-core machine. The computational time of each individual task parallelized was insignificantly small, therefore the overhead of the parallelization prevented achievement close to the theoretical boundaries of the speed-up (MSDN, 2016c). Aziz et al. (2009) developed a sequential algorithm for solving TSP and converted into a parallel algorithm by integrating it with the Message Passing Interface (MPI) libraries. The authors use a dynamic two dimensional array and store the costs of all possible paths. They decompose the task of filling this 2D array into subroutines to parallelize the algorithm using MPI. The Message Passing Interface provides the subroutines needed to decompose the tasks involved in the TSP solving process into subproblems that can be distributed among the available nodes for processing. Experimental results conducted on a Beowulf cluster show that their speed-ups were less than 3.5 times on a 32 processor cluster. Another technique to implement parallel heuristics for the geometric TSP (symmetric and Euclidean distances between cities), called the divide and conquer strategy, is proposed in (Cesari, 1996). Cesari (1996) subdivides the set of cities into smaller sets and computes an optimal subtour for each subset. Each subtour is then combined to obtain the tour for the entire problem. The author was able to achieve between 3.0 and 7.2 times speed-up on a 16 core machine.

3.2.2 Large Batches of Traveling Salesman Problems

Solving a single TSP gives the best path for a certain instance. However, this assumes that the location of the cities (visited points) are fixed. In situations where the problem consists of finding the optimal locations of these cities (visited points), numerous TSPs must be solved to assess a certain design, (e.g, a warehouse layout or a distribution network). Large batches of TSPs are different from the multiple traveling salesman problem (mTSP) which consists of determining a set of routes for m salesmen who all start from and return back to a depot. In large batches of TSPs, to find the expected distance traveled (or another relevant statistic of the distribution of tour lengths), we need to evaluate a large set of tours. Solving large batches of TSPs provides a more accurate estimate and a wider information set than solving only a single TSP. This type of problem can be found in design of order picking warehouses, large scale distribution network simulation (Kubota et al., 1999; Sakurai et al., 2006), case-based reasoning for repetitive TSPs (Kraay and Harker, 1997), and delivery route optimization (Sakurai et al., 2011). Since our focus is solving large batches of TSPs, parallelizing each task at the TSP level will lead to better speed-ups than solving a single TSP using parallel techniques. To best of our knowledge, ours is the first comprehensive comparison of serial, parallel, and distributed large batches of TSPs solvers.

3.3 Methodology

The methodology for solving large batches of TSPs includes three main steps. In Step 1, problem specific conditions are set (e.g., a warehouse layout structure or a distribution network structure). This typically involves creating locations and calculating distances between locations. We need to assess the total or average cost of TSPs for the given configuration of Step 1. Therefore, multiple TSPs are created and evaluated with a TSP solver in Step 2. In Step 3, the total or average cost of the TSPs is calculated. In the next subsection, we provide an example from the literature in the context of a large-scale distribution network

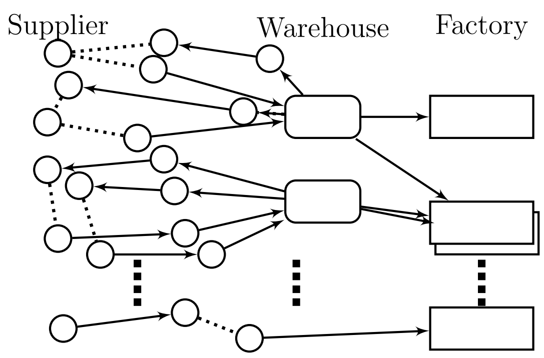


Figure 3.1: Large-scale distribution network Kubota et al. (1999).

simulation (Kubota et al., 1999). In the later subsections, we discuss this problem in the context of design of order picking warehouses and give implementation details.

3.3.1 Large-scale Distribution Network Simulation

A large scale distribution network may look like in Figure 3.1. This network includes multiple manufacturing enterprises. Parts are delivered from suppliers directly to factories or through warehouses. Direct delivery from the supplier to a warehouse or a factory would be inefficient. Therefore, a truck visits several suppliers and collects parts.

According to Kubota et al. (1999), the total cost of distribution must be calculated. Therefore, several hundreds of distributing routes are created for differing conditions to find the best large-scale distribution network. Kubota et al. (1999) note that efficiency (i.e., solving overall problem quickly) and precision (i.e., solving close to optimal) are important. The methodology herein is applicable to this class of distribution network problem.

3.3.2 Design of Order Picking Warehouses

The warehouse design software that motivated this study creates a warehouse layout for given parameters, calculates shortest path distances between storage locations and between storage locations to a pick and deposit location (i.e., a location where a TSP tour starts and ends), creates a TSP file based on the product orders (i.e., each order is a pick tour or a

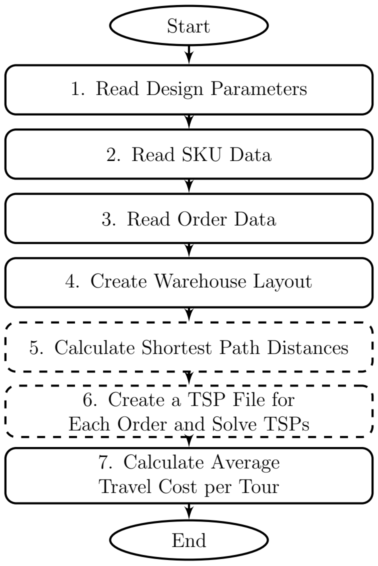

Start
1. Read Design Parameters
2. Read SKU Data
3. Read Order Data
4. Create Warehouse Layout
5. Calculate Shortest Path Distances
6. Create a TSP File for Each Order and Solve TSPs
7. Calculate Average Travel Cost per Tour
End

Figure 3.2: Stages in warehouse design creation and fitness assessment.

TSP), sends these to the Concorde or LKH solver and reads the resulting TSP distance from these solvers (Ozden et al., 2017). To find the best designs, we need to consider numerous product orders that represent the order picking activity of the warehouse. We identify the design that gives the minimum average travel cost.

Figure 3.2 shows the seven stages in the warehouse layout creation and fitness assessment. Boxes with dashed lines represent the most time consuming parts of the overall process. In this paper, we focus on Stage 6: "Create a TSP File for Each Order and Solve TSPs." We will describe how we implemented parallel and distributed computing techniques for this stage. Our approach is not specific to warehouse design and can be used for any application that requires solving large batches of TSPs.

We use the C# programming language and the .NET environment. We use C# parallel class methods (MSDN, 2016b) for parallel computing. For distributed computing, we implement a modified version of the asynchronous client-server example from the Microsoft Developer Network (MSDN) (MSDN, 2016a).

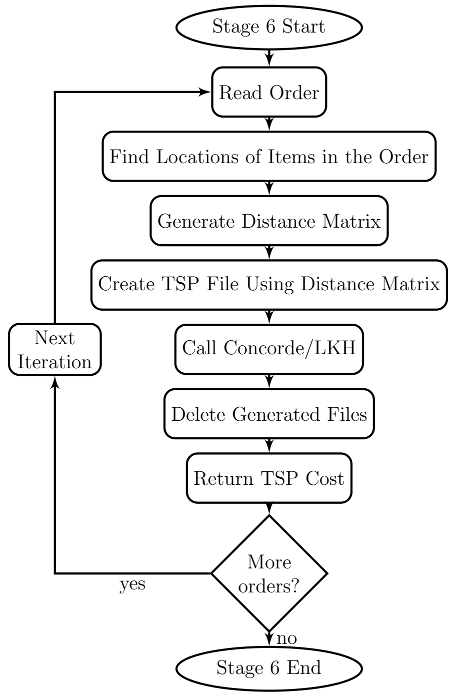


Figure 3.3: Serial execution of Concorde/LKH solvers.

3.3.3 Serial Execution of Concorde/LKH

In this case, we send a set of orders (pick lists) to a wrapper function one by one in serial (see Figure 3.3). For each order, we find the products and their locations in the warehouse and generate a distance matrix that contains only the items in this particular order. Because the shortest path distances are already calculated in the previous stage, we only generate a sub-matrix of the main distance matrix which contains the all-pairs shortest path distances between every storage location and the pick and deposit location in the warehouse. Based on this distance matrix, we generate a file in the TSP file format (TSPLIB, 2013). Concorde or LKH is called to solve the corresponding TSP file, and to read and keep the distance after the execution. Finally, we delete the generated TSP file and any generated files from Concorde/LKH and continue to the next order. Stage 6 ends when all orders are evaluated.

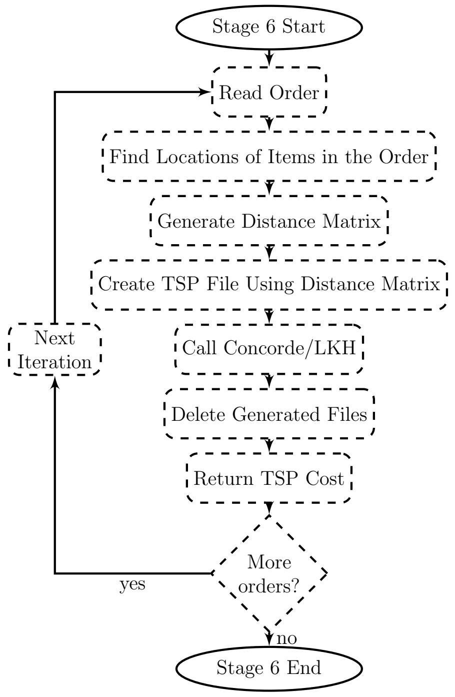


Figure 3.4: Parallel execution of Concorde/LKH solvers.

3.3.4 Parallel Execution of Multiple Concorde/LKH solvers

Since the operation inside the loop of Figure 3.3 is independent of any other, we can use “Parallel For Loop”MSDN (2016c) and send a set of orders to the wrapper function in parallel. The rest of the operations are the same as a serial execution, but they are performed in parallel until all orders are completed. In Figure 3.4, blocks with dashed lines represent the operations that are performed concurrently.

3.3.5 Parallel and Distributed Execution of Multiple Concorde/LKH Solvers

In this case, we have a master-slave architecture to perform distributed computing. We use a static load balancing methodology to distribute TSPs evenly among machines because dynamic load balancing methodologies increase the network overhead by sending and receiving the status of each processor of each slave machine. Also in our first set of experiments, we analyze TSPs of the same size in one batch, which makes dynamic load balancing less effective. We first create TSP files of each order in the master machine in parallel and distribute the TSP files to each slave machine using the TCP/IP protocol with given workload percentage. If the master machine requires provably optimal solutions, then it sends the TSP files using port 8888 otherwise it uses port 8889. Two processes are running on the slave machine. The first listens to port 8888 and solves the TSP files that are sent by the master machine with Concorde in parallel. The second listens to port 8889 and solves the TSP files that are sent by the master machine with the LKH in parallel. A slave machine receives the TSP files over the TCP/IP protocol and keeps the files until Stage 6 ends. The slave machine waits for a "DONE" signal to start TSP runs in parallel, then returns the TSP distance over TCP/IP with the "DONE" signal at the end, closes the communication between the master and slave machines. After the master machine receives all TSP distances, Stage 6 ends. Figure 3.5 shows the parallel and distributed execution of multiple Concorde/LKH solvers. Blocks with dashed lines represent the operations that are performed concurrently.

3.4 Computational Results

3.4.1 Fixed Size TSP Instances

We have selected fixed sized TSP instances generated using our warehouse optimization software. For the execution of the algorithms, a Lenovo workstation with a six core hyper-threaded Intel Xeon E5-1650 processor is used. Each workstation has 64GB of RAM and 256GB of Solid State Drive. The operating system is 64 bit Windows 7, Enterprise Edition. The total number of parallel threads that can be executed is $2 \times 6 = 12$. Table 4.3 gives a summary of the parameter settings used in the set of experiments. To address the variability in execution times as a result of the background processes of the operating system, we perform five runs for each experiment and calculate average execution time. We use default settings for Concorde. For LKH, we set "RUNS" (the total number of runs) to 1, "TIME_LIMIT" to 1 s and "MOVE_TYPE" to 4 which means that a sequential 4-opt move is to be used.

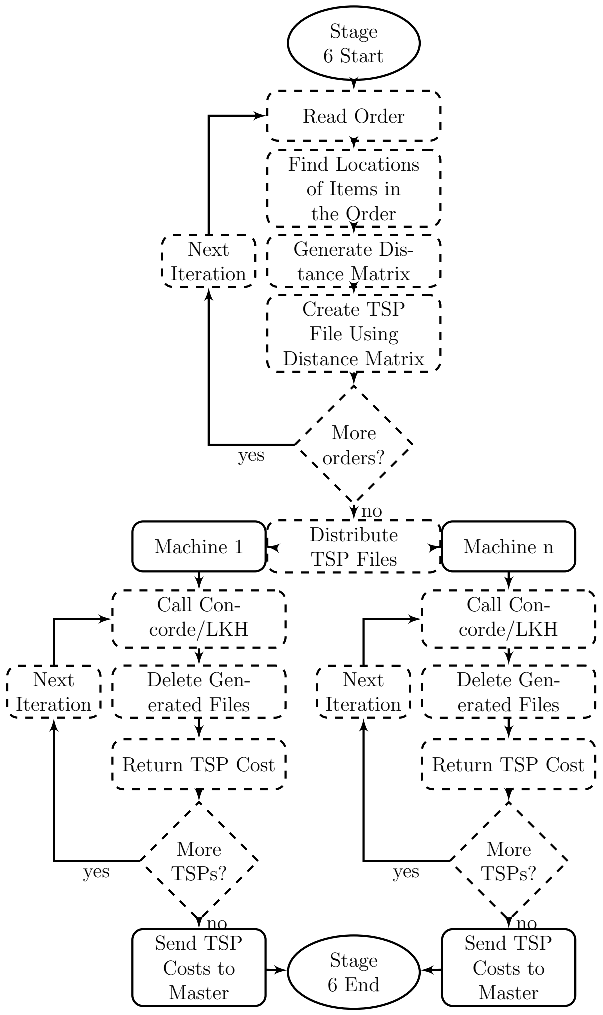


Figure 3.5: Parallel and distributed execution of multiple Concorde/LKH solvers.

Table 3.1: Problem parameters used for the computational experiments.

Number of cities	5, 25, 50, 100, 150, 200
Number of TSPs Solved	10, 100, 1000, 10,000
Execution mode	Serial, parallel, distributed with two or three machines

Table 3.2 shows the average execution time speed up over serial execution. Speed up is the ratio of the execution time of the serial algorithm to the execution time of the parallel/distributed algorithm. Both LKH and Concorde better utilize computing resources when the TSP size increases (see Figures 3.6 and 3.7). LKH uses parallel executions better than Concorde, because it is set to run with a maximum execution time for each TSP instance. As a result, all concurrent operations have similar execution times, which enables a better workload balance among CPU cores. When solving with Concorde, some TSP instances are harder, therefore the time to find an optimal solution varies, leading to a poor workload balance among cores (and CPUs for distributed computing).

However, Concorde uses distributed-2 and distributed-3 executions more efficiently because the overhead of sending TSP files to slaves has less effect in execution of Concorde than LKH (see Figures 3.8 and 3.9). In Figure 3.6, the speed-up decreases when solving for instances with fewer than 50 cities and more than 100 TSPs. This is because the network overhead of sending many TSP files becomes inefficient for few cities with Concorde. The same is true for LKH, but in this case the speed-up decreases even for a 50 city TSP. There is a point between the number of TSPs (many) and the city size (low) for both TSP Solvers where distributed computing becomes inefficient.

Table 3.3 shows the average speed-up/physical core ratio (SPR) of the additional CPU cores for parallel, distributed-2, and distributed-3 executions. It is important to note that serial execution uses one of the six physical cores instead of one of the twelve logical cores. A hyperthreaded processor can achieve 30 percent increased performance compared to a non-hyperthreaded processor (Casey, 2016). Therefore we should be able to see SPR values as high as 130%. Values less then 100% mean that executions with parallel or distributed techniques do not effectively use the physical cores. Values higher than 100% mean that executions with parallel or distributed techniques use computing resources more effectively than serial execution because of hyper-threading. This means that parallelization done at

Table 3.2: Average Speed-ups (Multiplier)

		Parallel		Distributed 2		Distributed 3	
Cities	**Batch**	**Concorde**	**LKH**	**Concorde**	**LKH**	**Concorde**	**LKH**
5	**10**	3.68	3.13	5.87	4.30	7.17	5.80
	100	4.63	3.77	7.82	5.16	10.31	6.96
	1000	3.42	4.25	6.28	6.30	8.01	7.89
	10000	3.34	4.03	5.31	4.68	6.39	5.48
25	**10**	4.69	3.95	6.28	4.40	6.31	4.87
	100	4.54	4.44	9.87	6.09	12.69	7.40
	1000	3.73	5.63	10.35	7.56	13.58	9.29
	10000	3.73	5.16	9.22	5.87	12.76	7.70
50	**10**	3.79	4.01	4.78	4.64	4.12	5.35
	100	3.73	6.14	7.09	8.10	6.39	9.73
	1000	4.65	6.75	10.03	9.19	13.70	12.25
	10000	4.58	6.80	10.52	8.85	15.02	11.57
100	**10**	4.62	4.13	4.70	4.47	5.25	4.66
	100	5.44	6.71	8.06	10.48	10.15	12.95
	1000	6.62	6.98	12.03	11.34	16.87	15.90
	10000	6.80	7.22	13.21	12.08	19.35	16.74
150	**10**	1.46	4.16	1.25	3.98	1.36	4.34
	100	5.51	6.58	5.87	9.46	5.60	11.81
	1000	6.69	7.07	11.06	10.80	14.51	14.33
	10000	7.20	7.37	12.53	11.50	17.76	14.79
200	**10**	2.47	3.64	1.45	3.47	2.83	3.59
	100	5.25	6.80	5.20	7.72	5.95	9.37
	1000	6.49	7.13	10.60	9.28	12.51	11.86
	10000	6.07	7.49	12.30	10.13	15.01	12.92

a higher level (solving each entire TSP in parallel) improves CPU utilization over parallelization done at Stage 6 (finding an optimal tour by performing a number of trials where each trial attempts to improve the initial tour) for LKH using SPC^3 (Ismail et al., 2011). Moreover, a parallel implementation can solve more than six times faster than a serial implementation on the same 6-core machine. We used the following formula to calculate the SPR values in Table 3.3:

$$spr = \frac{su}{npc} \tag{3.6}$$

where *spr* is the speed-up/physical core ratio, *su* is the average execution time speed-up over serial execution, and *npc* is the number of physical cores available per execution. *npc* values for parallel, distributed-2, and distributed-3 executions are 6, 12, and 18, respectively. Table 3.4 shows a 4-way ANOVA using Minitab 17.0 statistical software. The differences between the group means of the main effects (TSP size, number of TSPs (batch), algorithm type (parallel, distributed-2, and distributed-3), solver type (LKH or Concorde)) and their interactions (two, three, and four level interactions) are all statistically significant. The model can explain 94.72% of the variability of the response data around its mean.

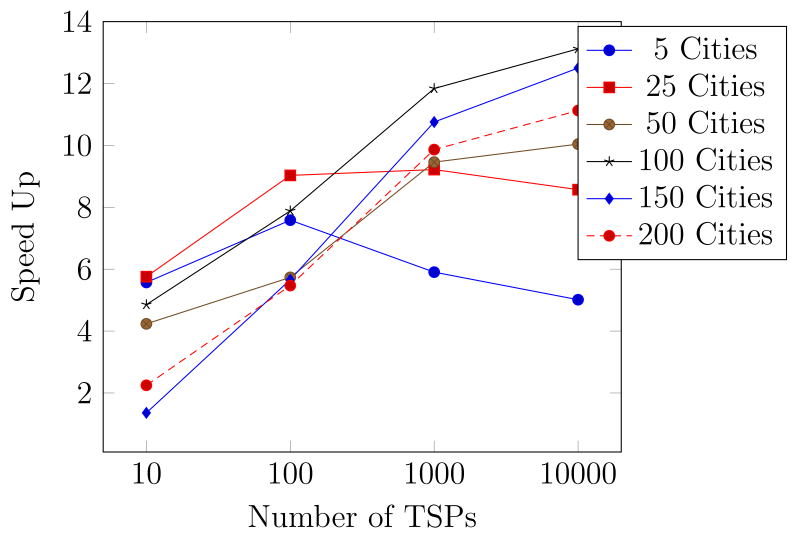


Figure 3.6: Number of TSPs vs. Number of Cities for Speed Up (Concorde)

Tables 3.5 and 3.6 show the average execution times and average tour costs of Concorde and LKH. In all experiments, we use a single cross aisle traditional warehouse layout. The warehouse can accommodate 1000 items and these items are randomly distributed throughout the storage locations. We generated 10, 100, 1000, and 10000 orders with 4, 24, 49, 99, 149, and 199 items, and we calculated the distance of each order using TSP solvers. Therefore the average cost for 5 cities and 10 # of TSPs is the average distance of completing 10

Table 3.3: Average Speed-up/Physical Core Ratio (Percent) and 95% Confidence Intervals on SPR

		Parallel		**Distributed 2**		**Distributed 3**	
City	**Batch**	**Concorde**	**LKH**	**Concorde**	**LKH**	**Concorde**	**LKH**
5	**10**	61(52,70)	52(41,63)	49(41,56)	36(25,47)	40(27,53)	32(22,43)
	10^2	77(75,79)	63(59,66)	65(62,68)	43(42,44)	57(55,60)	39(37,41)
	10^3	57(57,57)	71(70,72)	52(51,54)	52(52,53)	45(43,46)	44(43,44)
	10^4	56(55,56)	67(67,68)	44(44,45)	39(38,40)	36(35,36)	30(30,31)
25	**10**	78(73,83)	66(58,74)	52(46,58)	37(36,38)	35(30,40)	27(21,33)
	10^2	76(72,79)	74(71,77)	82(80,85)	51(48,54)	70(67,74)	41(40,42)
	10^3	62(60,64)	94(89,99)	86(85,87)	63(61,65)	75(72,79)	52(50,53)
	10^4	62(62,63)	86(85,87)	77(76,77)	49(48,50)	71(70,72)	43(41,44)
50	**10**	63(52,74)	67(64,70)	40(36,44)	39(36,41)	23(21,25)	30(28,32)
	10^2	62(54,71)	102(98,106)	59(55,64)	67(66,69)	35(30,41)	54(51,57)
	10^3	77(72,83)	113(111,114)	84(79,88)	77(76,77)	76(67,86)	68(67,69)
	10^4	76(76,77)	113(112,115)	88(85,90)	74(73,74)	83(82,85)	64(63,66)
100	**10**	77(46,108)	69(64,73)	39(33,45)	37(36,39)	29(23,35)	26(26,26)
	10^2	91(78,103)	112(107,117)	67(55,80)	87(82,92)	56(48,65)	72(66,78)
	10^3	110(108,113)	116(114,118)	100(89,111)	95(93,96)	94(90,98)	88(86,90)
	10^4	113(113,114)	120(119,122)	110(109,112)	101(98,103)	107(104,110)	93(92,94)
150	**10**	24(22,27)	69(61,78)	10(5,15)	33(30,37)	8(6,9)	24(24,25)
	10^2	92(79,104)	110(102,117)	49(39,59)	79(75,83)	31(21,41)	66(61,70)
	10^3	112(110,113)	118(116,120)	92(88,97)	90(88,92)	81(74,87)	80(75,84)
	10^4	120(118,122)	123(122,124)	104(99,110)	96(95,97)	99(92,105)	82(81,84)
200	**10**	41(25,57)	61(58,63)	12(6,18)	29(28,30)	16(8,23)	20(19,21)
	10^2	88(76,100)	113(111,116)	43(22,65)	64(59,69)	33(22,44)	52(46,58)
	10^3	108(103,114)	119(116,121)	88(82,94)	77(75,80)	69(46,93)	66(63,68)
	10^4	101(90,113)	125(124,125)	103(96,109)	84(84,85)	83(67,100)	72(71,73)

orders where each order has 4 items. Since each order has to start and end at the depot, picking 4 items in a warehouse is a 5-city TSP.

3.4.2 Variable Size TSP Instances

We should emphasize that our methodology is not bounded to fixed size TSP instances. In this set of experiments, we demonstrate our methodology's ability to solve variable size

Table 3.4: ANOVA for SPR versus Size, Batch, Algorithm, Solver

Source	**DF**	**Adj SS**	**Adj MS**	**F-Value**	**P-Value**
Size	5	7.6132	1.52264	275.88	0
Batch	3	21.4787	7.15956	1297.22	0
Algorithm	2	12.0637	6.03184	1092.89	0
Solver	1	0.078	0.078	14.13	0
Size*Batch	15	7.7685	0.5179	93.84	0
Size*Algorithm	10	0.7865	0.07865	14.25	0
Size*Solver	5	1.3363	0.26725	48.42	0
Batch*Algorithm	6	0.4232	0.07054	12.78	0
Batch*Solver	3	0.3713	0.12377	22.42	0
Algorithm*Solver	2	1.2914	0.64569	116.99	0
Size*Batch*Algorithm	30	0.7685	0.02562	4.64	0
Size*Batch*Solver	15	1.524	0.1016	18.41	0
Size*Algorithm*Solver	10	0.4784	0.04784	8.67	0
Batch*Algorithm*Solver	6	0.5061	0.08435	15.28	0
Size*Batch*Algorithm*Solver	30	0.4957	0.01652	2.99	0
Error	576	3.179	0.00552		
Total	719	60.1625			
	S	**R-sq**	**R-sq(adj)**	**R-sq(pred)**	
	0.0742909	94.72%	93.40%	91.74%	

Table 3.5: Concorde Average Execution Times and Average Cost per Tour

		Average Execution Time (seconds)				
City	**Batch**	**Single**	**Parallel**	**Distributed 2**	**Distributed 3**	**Avg. Cost**
5	**10**	0.27	0.07	0.05	0.04	740.20
	100	2.73	0.59	0.35	0.27	782.50
	1000	27.83	8.13	4.44	3.48	788.10
	10000	281.20	84.19	52.98	44.00	786.20
25	**10**	0.66	0.14	0.11	0.11	1466.80
	100	6.71	1.48	0.68	0.53	1490.90
	1000	70.33	18.89	6.80	5.19	1499.60
	10000	684.64	183.48	74.28	53.68	1499.80
50	**10**	1.53	0.41	0.32	0.37	2215.50
	100	16.33	4.47	2.33	2.63	2203.90
	1000	180.18	38.91	17.99	13.35	2208.80
	10000	1763.84	384.82	167.74	117.51	2208.00
100	**10**	6.21	1.44	1.32	1.20	2747.80
	100	150.80	28.02	19.20	14.99	2826.10
	1000	1484.11	224.42	124.68	88.17	2807.20
	10000	14081.30	2070.64	1065.72	728.33	2807.40
150	**10**	21.25	14.84	20.66	15.85	3055.32
	100	358.97	66.28	63.61	70.01	3055.05
	1000	3882.53	580.35	351.87	269.67	3053.73
	10000	39560.49	5492.05	3164.54	2238.90	3050.34
200	**10**	66.58	30.09	131.03	29.17	3148.05
	100	743.70	143.93	309.28	139.23	3161.00
	1000	7594.09	1173.47	719.45	799.00	3160.18
	10000	74592.12	12502.08	6082.22	5193.40	3157.99

Table 3.6: LKH Average Execution Times and Average Cost per Tour

		Average Execution Time (seconds)				
City	**Batch**	**Single**	**Parallel**	**Distributed 2**	**Distributed 3**	**Avg. Cost**
5	**10**	0.11	0.04	0.03	0.02	740.20
	100	1.13	0.30	0.22	0.16	782.50
	1000	11.29	2.66	1.79	1.43	788.10
	10000	114.15	28.29	24.40	20.84	786.20
25	**10**	0.21	0.05	0.05	0.04	1466.80
	100	1.80	0.41	0.30	0.24	1490.90
	1000	19.00	3.39	2.52	2.05	1499.60
	10000	185.82	36.03	31.68	24.18	1499.80
50	**10**	0.65	0.16	0.14	0.12	2215.50
	100	5.51	0.90	0.68	0.57	2204.10
	1000	55.85	8.27	6.07	4.56	2209.00
	10000	551.92	81.21	62.35	47.73	2208.20
100	**10**	2.05	0.50	0.46	0.44	2752.90
	100	25.72	3.84	2.46	2.00	2829.20
	1000	249.86	35.78	22.03	15.73	2809.80
	10000	2514.98	348.26	208.26	150.25	2809.90
150	**10**	2.90	0.71	0.74	0.67	3064.88
	100	31.64	4.83	2.46	2.69	3065.36
	1000	331.69	46.91	30.73	23.22	3063.17
	10000	3310.92	449.05	287.87	223.98	3060.00
200	**10**	3.67	1.01	1.06	1.02	3158.04
	100	30.64	4.51	4.00	3.31	3169.81
	1000	291.98	40.98	31.48	24.66	3170.03
	10000	3060.64	408.72	302.00	236.98	3168.13

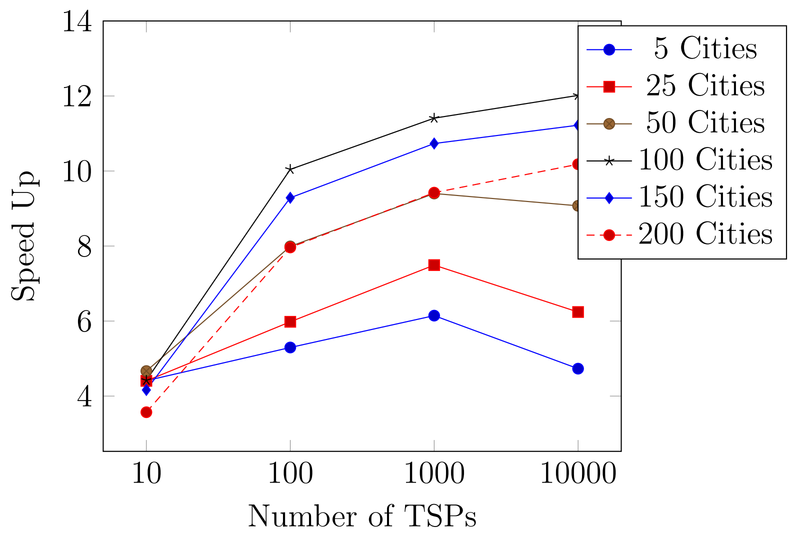


Figure 3.7: Number of TSPs vs. Number of Cities for Speed Up (LKH)

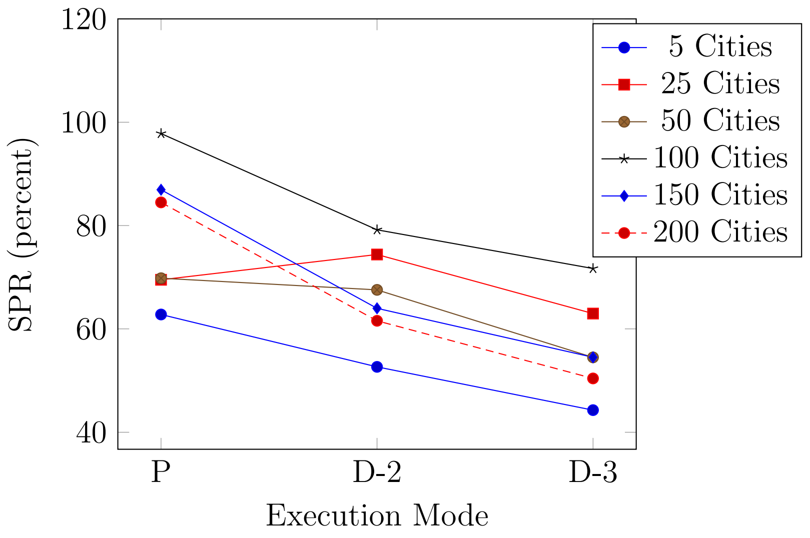


Figure 3.8: Execution Mode vs. Number of Cities for SPR (Concorde)

TSP instances. These instances are from real order picking data where TSP sizes are different. This real order data set has 10,967 TSPs, the average number of cities visited per TSP is 10.12, and the largest TSP has 164 cities. The frequency of TSP sizes is shown in Figure 3.10. Because of variable TSP sizes, equal distribution of TSPs among identical machines may create overloaded machines and increase the total makespan. Therefore, we compare an equal distribution rule (EDR) against a well known task assignment rule, the longest processing time (LPT) rule (Graham, 1969). In LPT, jobs (TSPs) are sorted by

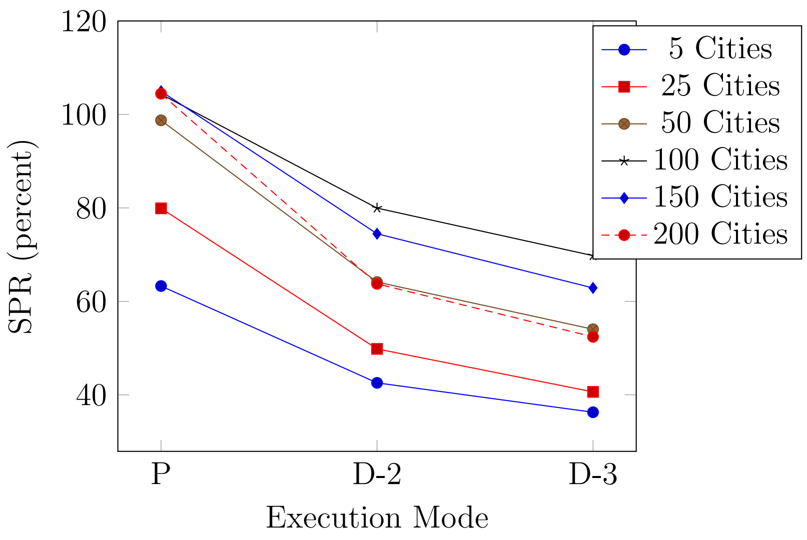


Figure 3.9: Execution Mode vs. Number of Cities for SPR (LKH)

problem size (as a proxy for processing time, which we do not know yet) and assigned to the machine with the earliest end time so far. In worst case scenario, the algorithm achieves a makespan of $4/3 - 1/3(m)OPT$ where m is the number of machines (i.e., LPT produces a makespan that is at most 33% worse than optimal assignment). We perform five replications for each experiment. Two important things affect the results of these runs: the order of the TSPs in EDR and the estimation of processing time by size of TSP. In some cases, a 40-city TSP can be much harder than a 41-city TSP. Also, a two machine EDR may perform close to optimal, whereas a three machine EDR can be quite suboptimal. The 3-way ANOVA in Table 3.8 shows that all main factors and the Machines*Solver interaction are significant. Other two-way interactions and the three way interaction are insignificant. The model explains 99.65% of the variability of the response data around its mean. Figure 3.11 shows the statistically significant effects — all three main effects and the two way interaction for Machines*Solver. The difference between the means of the TSP solvers is more pronounced than the other two main effects; LKH is nearly three times faster than Concorde. Obviously, increasing from two to three machines reduces the time needed. The 1.8 second difference between the means of EDR and LPT is statistically significant, showing the benefit of using

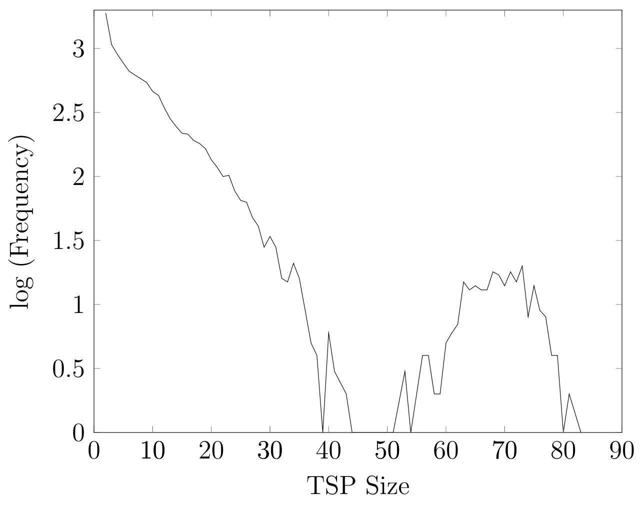


Figure 3.10: TSP Size Frequency for 10,967 TSPs (Three outliers, 162, 163, and 164 are omitted from the graph)

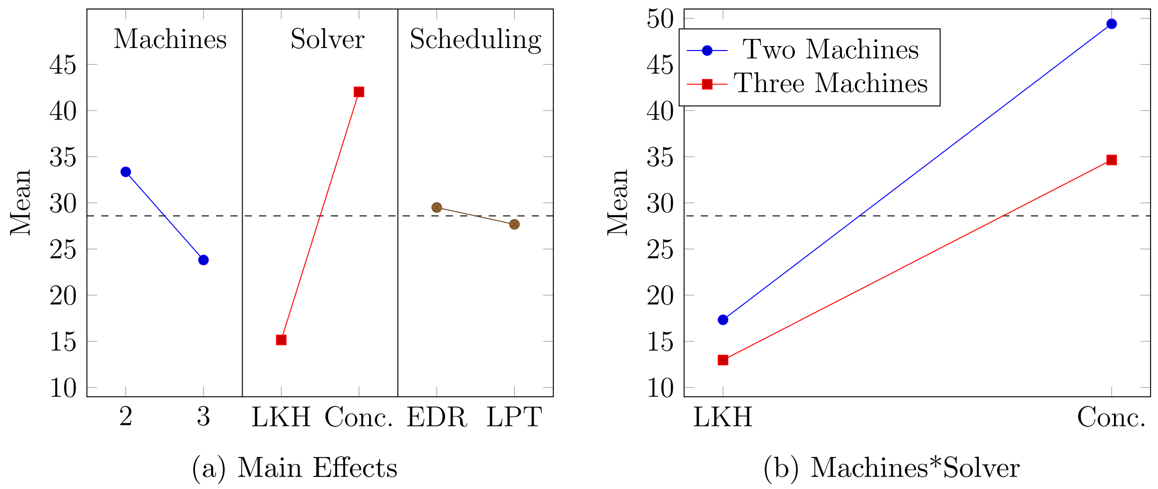


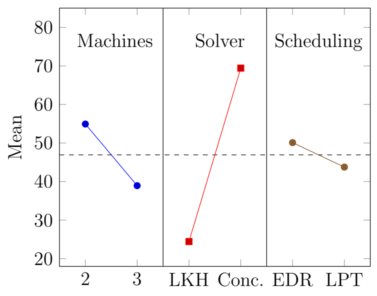
Figure 3.11: Main Effects and Machines*Solver Interaction Plots for Time (Real Data)

LPT for unequal sizes of TSPs. The two way effect lines are not parallel, showing that Concorde benefits relatively more from using three machines than does LKH.

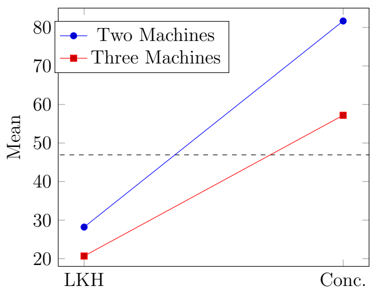
In order to show LPTs effectiveness, we create a more controlled experiment with generated TSPs. In this case, we create TSPs in the following order repeatedly for 3,840 times: size 51, size 11, size 6. In this way, we know that the EDR method will assign all 51-city TSPs to the first machine, all 11-city TSPs to the second machine, and all 6-city TSPs to the third machine. In this experiment, there are 11,520 TSPs and average TSP size is

Table 3.7: ANOVA: Time versus Machines, Algorithm, Solver (Real Data)

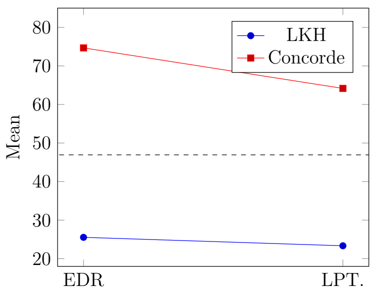
Source	**DF**	**Adj SS**	**Adj MS**	**F-Value**	**P-Value**
Machines	1	912.11	912.11	996.58	0.000
Solver	1	7219.01	7219.01	7887.54	0.000
Scheduling	1	33.31	33.31	36.40	0.000
Machines*Solver	1	270.03	270.03	295.03	0.000
Machines*Scheduling	1	0.82	0.82	0.90	0.350
Solver*Scheduling	1	2.35	2.35	2.57	0.119
Machines*Solver*Scheduling	1	0.07	0.07	0.08	0.779
Error	32	29.29	0.92		
Total	39	8467.01			
	S	**R-sq**	**R-sq(adj)**	**R-sq(pred)**	
	0.956683	99.65%	99.58%	99.46%	



(a) Main Effects



(b) Machines*Solver



(c) Solver*Scheduling

Figure 3.12: Main Effects and Machines*Solver Interaction Plots for Time (Generated Data)

22.67. All main effects, and two and three way interactions are statistically significant except for the Machines*Scheduling interaction. Figure 3.12 shows the three main effects and the significant two way interactions, Machines*Algorithm and Algorithm*Scheduling. The results are congruent with those from the real data. LKH is 2.84 times faster than Concorde on average. LPT finishes on the average 6.35 seconds earlier than EDR, again showing its benefit. The choice of the scheduling approach is more important when the TSP solver is Concorde because of the longer and non-uniform processing times for this exact method.

Table 3.8: ANOVA: Time versus Machines, Algorithm, Solver (Generated Data)

Source	**DF**	**Adj SS**	**Adj MS**	**F-Value**	**P-Value**
Machines	1	2555.9	2555.9	404.55	0.000
Algorithm	1	20244.7	20244.7	3204.36	0.000
Scheduling	1	403.5	403.5	63.87	0.000
Machines*Solver	1	718.9	718.9	113.79	0.000
Machines*Scheduling	1	8.2	8.2	1.29	0.264
Solver*Scheduling	1	173.1	173.1	27.40	0.000
Machines*Solver*Scheduling	1	36.2	36.2	5.73	0.023
Error	32	29.29	0.92		
Total	39	8467.01			
	S	**R-sq**	**R-sq(adj)**	**R-sq(pred)**	
	0.956683	99.65%	99.58%	99.46%	

3.5 Conclusions

We presented how parallel computing techniques can significantly decrease the overall computational time and increase the CPU utilization for solving large batches of TSPs using simple and effective parallel class methods in C#. Moreover, we showed our results for distributed and parallel computing methods with two and three slave machines. Using distributed computing techniques requires some background in C# socket programming but simple examples can be found on the web (MSDN, 2016a).

Solving large batches of TSPs is the most computationally intensive step in many applications involving routing. Our results show that using C# parallel class methods is a simple, effective and scalable way to parallelize solving large batches of TSPs. The programmer can write wrapper functions for Concorde and LKH, and implement "parallel for loops" to leverage the multi-core processors. However, distributed computing techniques only show their real benefits when the TSP instances have more than 50 cities so that the network and file read/write overhead is relatively negligible. Our results also show that for both real data and generated data, a scheduling algorithm like LPT performs better than a naïve method like EDR even though the method used for estimating processing times of TSPs is not very accurate (TSP size, in this case).

The Lin-Kernighan Heuristic (LKH) can be selected over the Concorde TSP Solver when optimality is desired but not required. In our results, LKH is 24.37, 30.59, 20.14, and 21.91 times faster than Concorde on average in single, parallel, 2-computer distributed, and 3-computer distributed runs for solving 10,000 200-city TSPs, respectively. The average optimality gap is less than 0.34% per run.

Chapter 4

Calculating the Length of an Order Picking Path

4.1 Introduction

An important part of warehouse modeling is the simple task of determining the distance between two points. In almost every warehousing paper we know of, researchers use rectilinear or Manhattan distance to estimate the distance between two points in a warehouse. In many settings, workers “cut corners” and therefore do not make turns at right angles (see Figure 4.1). The effects of this imprecision in the literature have been negligible because the opportunity to cut corners in a traditional warehouse are very limited. Furthermore, the layouts in most papers are fixed, so whatever errors are introduced by assuming right-angle travel are consistent across experiments, making the results “relatively” correct.

Gue and Meller (2009) assumed a network in which nodes represent picking points (in front of pallet locations, for example), the intersections of cross aisles and picking aisles, and the pick up and deposit point (P&D). Without explicitly saying so, they defined a graph network using an “aisle centers method” in which the path between points in different picking aisles or between a pick location and the P&D point passed through the single point of intersection between the picking aisle(s) and the cross aisle (see Figure 4.2). This assumption has limited effect on their problem, but only because they are interested in paths between

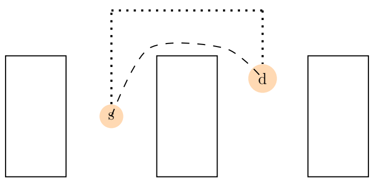


Figure 4.1: Example of a shortest path using a rectilinear distance vs. a path that cuts the corners

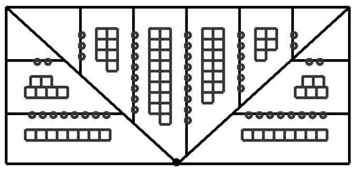
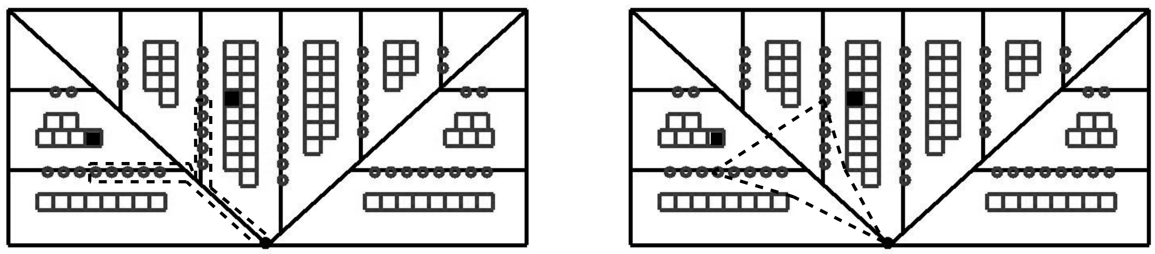


Figure 4.2: A graph network using aisle centers method defined by Gue and Meller (2009) for fishbone layout

locations and the P&D point, and the layouts generated by their models have "favorable" angles similar to those one could imagine being taken by a worker.

When modeling an order picking warehouse, however, we care very much about the distances between storage locations because they constitute the length of a picking tour. Therefore, every "favorable" angle has a corresponding "unfavorable" angle that poorly represents what might be expected of ordinary worker travel patterns. Using the naïve network model of Gue and Meller (2009) especially overestimates the distance of tours that require turns of more than 90 degrees, as we would expect in any non-traditional design (see Figure 4.3a).

We propose a new method of modeling distances between points in a warehouse that uses the concept of a visibility graph to represent reasonable levels of "corner cutting" that one would expect of productive order pickers. The visibility graph is a special graph whose nodes are either the vertices of the obstacles or attractions and whose edges are pairs of mutually visible nodes. Figure 4.3b shows an example of a pick path using a visibility graph where obstacles are the storage locations (racks or pallets). These proposed paths also serve as a lower bound on travel distances since no shorter path exists without passing through the obstacles. However, this path example shown in Figure 4.3b has its own problem. Because obstacles are defined as storage locations, paths have no safe distance to storage locations. An order picker will run into these pallets because she or he has a volume in space. We create a buffer area to tackle this problem and present the effects of the length of the buffer distance on visibility graph density in Section 4.3.



(a) A pick path by following aisle centers (b) A pick path by using visibility graph

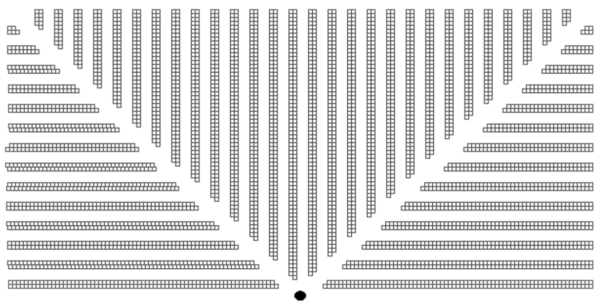
Figure 4.3: Pick path examples for picking two items from storage locations shown in bold and returning to depot

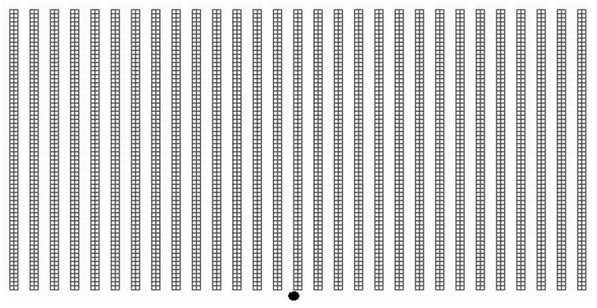
Çelik and Süral (2014) used the aisle centers method and compared the fishbone layout to a traditional layout for order picking operations. They generated test instances and calculated the average tour costs for fishbone and traditional layouts. They found that fishbone can perform as high as 30% worse than an equivalent traditional layout as the pick list increases. Roodbergen et al. (2008) also used the aisle centers method to optimize the traditional layout by finding the optimal number of pick and cross aisles, and the length of a pick aisle excluding the width of the cross aisles. Our main motivation is to find the effect of modeling travel on a visibility graph in performance comparisons between the fishbone layout (see Figure 4.4a) and a traditional layout (see Figure 4.4c) as well as performance comparisons between traditional layouts with zero (see Figure 4.4b), one (see Figure 4.4c), and two (see Figure 4.4d) cross aisles.

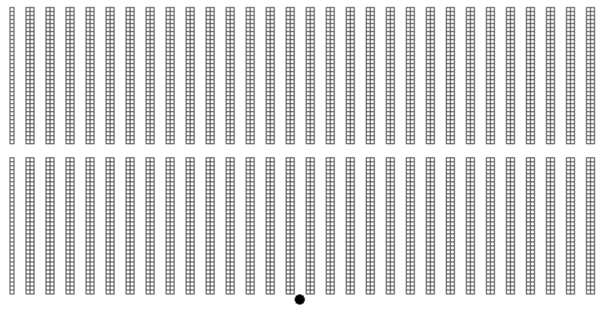
This chapter is organized as follows. Detailed information about visibility graph and other shortest path methods is given in Section 4.2. We describe the methodology in Section 4.3. We present the computational results in Section 4.4 and offer conclusions in Section 4.5.

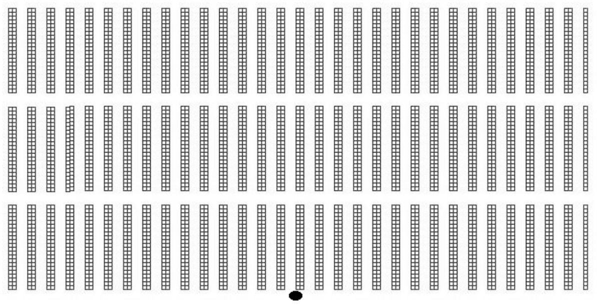
4.2 The Shortest Path Problem

The shortest path problem (SPP) is the problem of finding a path from a specific origin to a specific destination in a network such that the total cost associated with the path is


(a) Fishbone Layout


(b) Traditional Layout A


(c) Traditional Layout B


(d) Traditional Layout C

minimized. The SPP has widespread applications including vehicle routing in transportation systems (Zhan and Noon, 1998), telecommunications (Moy, 1994), VLSI design (Peyer et al., 2009), and path planning in robotics (Soueres and Laumond, 1996). There are well-known algorithms for solving SPP including Dijkstra's algorithm for solving single source SPP with non-negative weights (Dijkstra, 1959), Bellman-Ford-Moore algorithm for solving single source SPP with possible negative weights (Bellman, 1958; Ford, 1956; Moore, 1959), A* search algorithm for solving single source SPP by using heuristics for fast evaluation (Hart et al., 1968), Floyd-Warshall algorithm for solving all-pairs SPP (Floyd, 1962), and Johnson's algorithm for solving all-pairs SPP on sparse graphs (Johnson, 1977). The details of these algorithms can be found in Gallo and Pallottino (1986). These traditional algorithms have a common requirement: they all need a predefined graph network with a list of edges and their weights. Depending on how this graph network is generated, the shortest path distances between the same two points may change significantly. Until now, researchers in the warehouse design area assumed that workers follow the aisle centers, therefore, the network was a simple network with strict restrictions on paths that order pickers can follow. We change this assumption and allow workers to walk freely inside the aisles. Our problem is to find shortest path distances between two points in a warehouse such that workers can walk freely inside the aisles but need to go around the storage locations or any obstacles. This problem is a well-known problem in computational geometry called the Euclidean shortest path problem.

The Euclidean shortest path problem (ESPP) seeks the shortest path between two points in Euclidean space that does not intersect with any of a given set of obstacles. There exist efficient exact algorithms with $O(n \log n)$ time complexity for two dimensional space where n is the number of nodes in the graph (Hershberger and Suri, 1999). In three or more dimensions the problem is NP-hard (Canny and Reif, 1987). In this chapter, we are only interested in exact algorithms that work in two dimensional space with multiple obstacles. Hershberger and Suri (1999) state that there have been two fundamentally different approaches to this problem: the visibility graph method and the shortest path map method.

The visibility graph method produces a graph (called a visibility graph) whose nodes are either the vertices of the obstacles or attractions and whose edges are pairs of mutually visible nodes. In other words, for any pair of nodes if the line segment that connects them does not pass through an obstacle, an edge is created between them. Once the graph is defined we can run Dijkstra's algorithm produces the shortest path between any two nodes. The run time complexity of this approach is $O(n \log n + E)$ for sparse graphs where E is the number of edges in the graph (Ghosh and Mount, 1991). However, the visibility graph can have n^2 edges in the worst case; therefore algorithms that depend on the visibility graph method will have a similar worst-case run time.

The shortest path map method decomposes the plane into regions such that all points d in a region have the same sequence of obstacle vertices in their shortest path to s. The last obstacle vertex along the shortest $s - t$ path is the root r of the cell containing d. The root r can see all the points within its region. Figure 4.5 shows an example of the shortest path

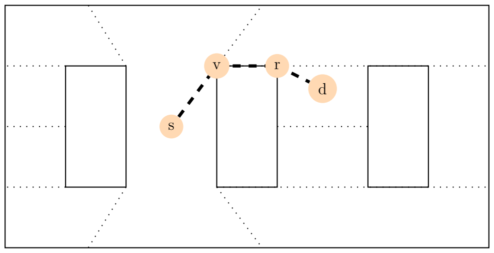


Figure 4.5: A shortest path map with respect to source point s within a polygonal domain with 3 obstacles. The heavy dashed path indicates the shortest $s-d$ path, which reaches d via the root r of its cell. Extension segments are shown thin and dotted.

map. Point s reaches d via obstacle vertices v and r. In this example, vertex r is the root of the region containing d. Hershberger's algorithm computes shortest paths in the presence of polygonal obstacles in $O(n \log n)$ time and space where n is the number of nodes in graph. For a detailed review of algorithms that use the visibility graph method or the shortest path map method, see Toth et al. (2004).

Although Hershberger's algorithm has a better worst-case running time, we implemented the visibility graph method because it is easier to understand and implement than the shortest path map method. Our main aim is to show how paths using the visibility graph method can affect the performance comparisons between traditional and fishbone layouts as well as performance comparisons between traditional layouts with zero, one, and two cross aisles. Computational efficiency is not the main target of this chapter.

4.3 Methodology

In our first set of experiments, we would like to see if the visibility graph method affects the comparisons between fishbone and traditional layout. We use the average tour cost for a set of test instances as the performance metric. We developed a warehouse modeling system to calculate the average tour cost of test instances for a given warehouse environment. This system can compare any warehouse layout with the visibility graph method or the aisle centers method. The following nine steps are performed in this system to calculate the average tour cost.

In Step 1, we create exterior aisles for a given width and depth of the warehouse. These exterior aisles serve as the boundaries of the rectangle-shaped warehouse.

In Step 2, we create cross aisles using exterior and interior points. An exterior point lies on the boundaries of the warehouse (i.e., exterior aisles) and an interior point lies inside the boundaries of the warehouse. Cross aisles divide the warehouse into regions.

In Step 3, we create pick aisles, pick locations, and storage locations for each region. We assume that items on both sides of a pick aisle may be accessed with negligible lateral movement. Figure 4.6 illustrates a warehouse.

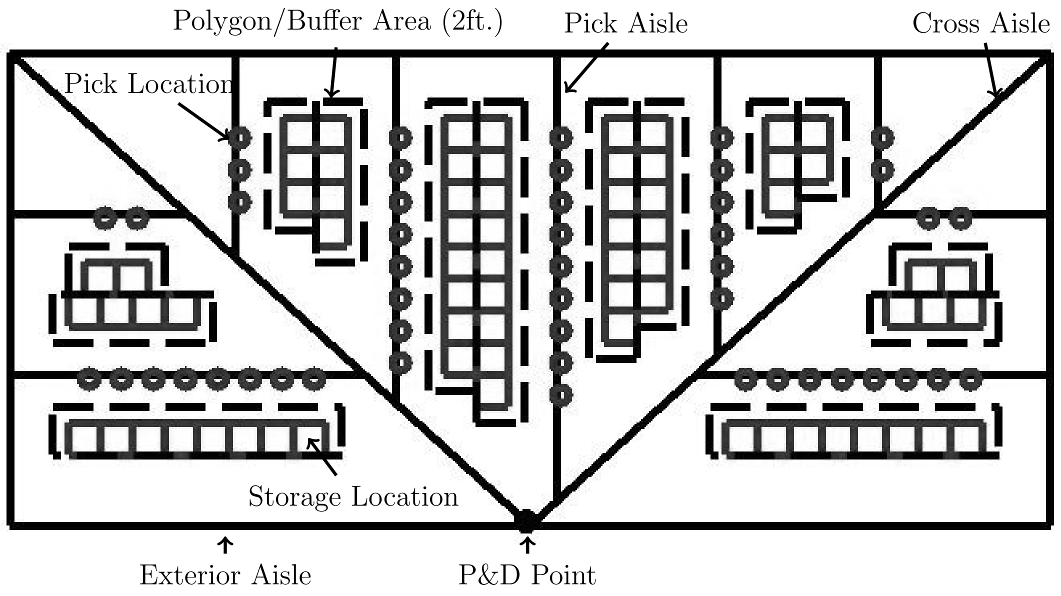


Figure 4.6: Representation of a warehouse. This particular fishbone layout has a single P&D point, 67 storage locations, 51 pick locations, 9 pick aisles, 2 cross aisles, and 4 exterior aisles. The lines that represent the aisle centers are both used for building the warehouse structure (i.e., storage locations and pick locations) and finding the shortest path distances with the aisle centers method.

In Step 4, we calculate polygons that will be used in the visibility graph calculation. In this step, we create the polygons considering a buffer distance. Corners of the polygons are also included as a node in the visibility graph. When buffer distance increases the polygons (buffer areas) become larger which decreases visibility. Figure 4.7 shows an example of a visibility graph created for a fishbone layout with a 2 ft. buffer distance. In Figures 4.8a and 4.8b, we present two examples of visibility between pick locations. If buffer distance is 1 ft., then pick location 1 is visible to both pick location 2 and pick location 3. However, visibility between pick location 1 and pick location 2 is lost when buffer distance is 2 ft. We can increase the buffer distance when modeling a warehouse with order pickers using forklifts. We can decrease it as necessary if they are picking the items by walking.

It is important to note that the density of the visibility graph decreases with greater buffer distance. Table 4.1 shows the number of arcs created for various buffer distances. When the buffer area becomes sufficiently large, the visibility graph becomes similar to the graph generated with the aisle centers method.

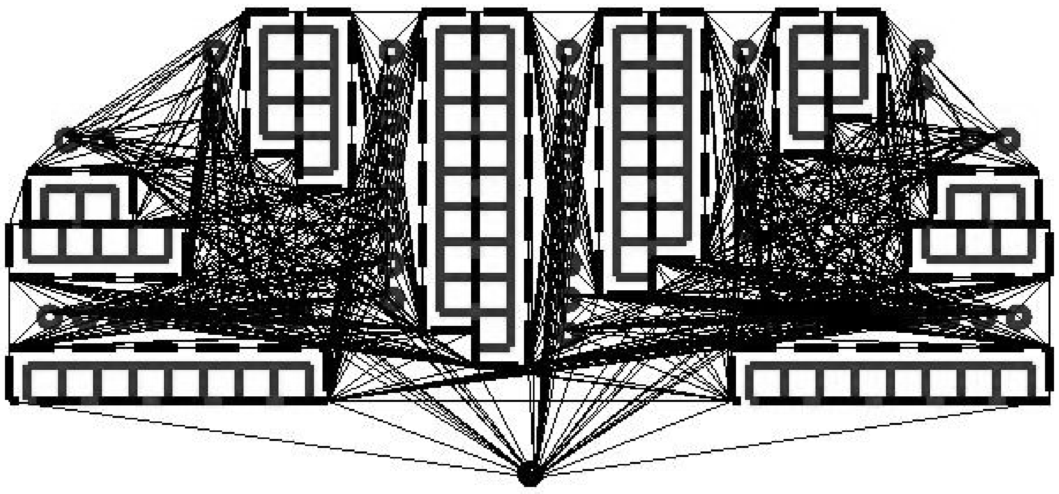
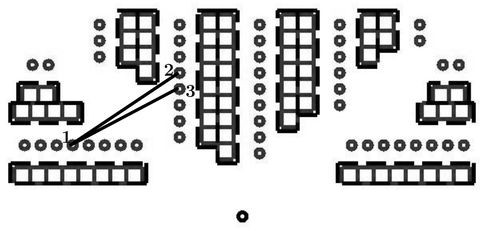


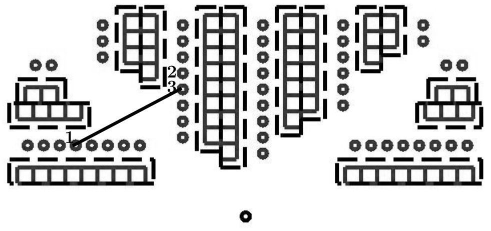
Figure 4.7: Visibility Graph of a fishbone layout (buffer distance is 2 ft.)

Table 4.1: Number of edges created for each buffer distance

Buffer Distance (ft.)	Number of Arcs
0.5	1046
1	1014
2	930



(a) Buffer Distance (1 ft.), more visibility



(b) Buffer Distance (2 ft.), less visibility

Figure 4.8: Increasing the buffer distance decreases the visibility between pick locations

In Step 5, we use Algorithm 1 to define the visibility graph. The "CreateVisibility-Graph" function checks each pair of graph nodes for a line of sight. The "Visible" function checks each edge in a set of polygons (P) if the edges of those polygons intersect with the line between $n1$ and $n2$. If there is an intersection, the function immediately returns false without further investigation. If it cannot find any intersecting edges after checking all edges in all polygons then there is a line of sight between $n1$ and $n2$. Then it defines an arc between those two nodes by using the connect function. The worst case complexity of this algorithm is $O(n^3)$ where n is the number of nodes.

In Step 6, we calculate the all-pairs shortest path distances using Algorithm 2. In this case, the shortest path vertices include all vertices in the visibility graph.

In Step 7, once the all-pairs shortest path distances are calculated and stored in the *dist* matrix, we allocate products to storage locations using *distance based slotting* (Pohl et al., 2011). This type of allocation stores the most frequently demanded products to the storage locations that are nearest to the P&D point.

Algorithm 1 Visibility Graph Calculation

```
function CreateVisibilityGraph(void)
    for ($i := 0$;$i < Count(G)$;$i \leftarrow i + 1$) do
        for ($j := 0$;$j < i$;$j \leftarrow j + 1$) do
            if $Visible(G[i], G[j])$ then
                $Connect(G[i], G[j])$
            end if
        end for
    end for
end function

function Visible($n1, n2$)
    for ($i := 0$;$i < Count(P)$;$i \leftarrow i + 1$) do
        for all ($edge$ in $P[i]$) do
            if $Intersect(n1, n2, edge)$ then
                return $false$
            end if
        end for
    end for
    return $true$
end function

function Connect($n1, n2$)
    $n1.AddtoConnectionList(n2)$
    $n2.AddtoConnectionList(n1)$
end function
```

Algorithm 2 All-Pairs Shortest Path

```
function AllPairsShortestPath(void)
    for ($i := 0$;$i < Count(G)$;$i \leftarrow i + 1$) do
        $DijkstrasShortestPath(G, i)$
    end for
end function

function DijkstrasShortestPath($G, k$) $Q \leftarrow \emptyset$
    for ($i := 0$;$i < Count(G)$;$i \leftarrow i + 1$) do
        $dist[k, i] \leftarrow \infty$
        Add $G[i]$ to $Q$
    end for
    $dist[k, k] \leftarrow 0$
    while Q is not empty do
        $u \leftarrow min(Q)$
        remove $u$ from $Q$
        for all $v$ in neighbor of $u$ do
            $shortdist \leftarrow dist[k, u] + length(u, v)$
            if $shortdist < dist[k, v]$ then
                $dist[k, v] \leftarrow shortdist$
            end if
        end for
    end while
end function
```

In Step 8, we generate a distance matrix for each order in the list of orders by using a subset of the *dist* matrix. This distance matrix contains the shortest path distances between every storage location that needs to be visited in the warehouse and the P&D point. Finding the shortest tour distance in a warehouse for an order is an example of a Traveling Salesman Problem (TSP). We obtain optimal travel distances using the Concorde TSP solver (Applegate et al., 2007).

In Step 9, the average of all TSP costs is used as a fitness function for a given warehouse layout. This is the final step for calculating the performance metric of a given test instance for a given warehouse layout.

4.4 Results

In this section, we describe our computational experiments aimed at achieving three objectives. First, we would like to see how the results of fishbone and traditional layout comparison would change for various pick lists with a turnover based storage policy and random storage policy if the visibility graph methods is used, rather than the aisle centers method. Second, we would like to see how the size of the warehouse affects the comparison of the visibility graph method and the aisle centers method under a random storage policy. Lastly, we would like to compare zero, one, and two cross aisle traditional layouts with both methods (the visibility graph method and the aisle centers method) and see if the best layout that achieves the minimum average tour cost changes for various warehouse sizes and demand skewness.

To evaluate the effect of the visibility graph method in average travel cost, the percent improvement of a fishbone layout ($PI_{Fishbone}$) over a traditional one cross aisle layout (see Figure 4.4c) is calculated as follows:

$$PI_{Fishbone} = \frac{Z_{Traditional} - Z_{Fishbone}}{Z_{Traditional}} \tag{4.1}$$

Table 4.2: Warehouse parameters used for the computational experiments

Layout	Number of Locations	Area	Aspect Ratio	Aisle Angles
Traditional B	4012	191260	0.51	90, 90
Fishbone	4000	196420	0.5	0,90,0

where $Z_{Fishbone}$ and $Z_{Traditional}$ refer to the average travel cost for the fishbone layout and the traditional layout, respectively.

Table 4.2 shows the warehouse parameters used for each layout. For each instance, the width of the pick and cross aisles is set to 12 ft, and the width and the depth of storage locations are set to 4 ft. Buffer distance is set to 1 ft.

We use the same order generation method used by Çelik and Süral (2014). However, instead of evaluating 100 orders for each pick list size as Çelik and Süral (2014) did, we change the number of orders for each pick list size. For small pick lists, tours have a larger variance in travel distance. Therefore, we increase the sample size for small pick lists to decrease the variability. Table 4.3 shows the number of orders sampled for each pick list size. For all situations considered in this chapter, the number of orders sampled for each pick list size is sufficient to guarantee a relative error of at most 1% with a probability 95% to estimate the mean travel distance.

Table 4.3: Number of orders evaluated for each pick list size

Pick List Size	Number of Orders
1	10000
2	5000
3	3333
5	2000
10	1000
30	333

For small pick lists, especially in single-command operations, the fishbone layout's performance improvement over the traditional layout B decreases from 18.28% to 14.16%. This is because the traditional layout B benefits from using the diagonal paths within the picking and cross aisles. However, the fishbone layout is already designed to benefit for having a direct access to P&D point. Therefore, the visibility graph method does not decrease the travel distance to P&D point as much as it decreased it for the traditional layout B. There is a significant difference between the visibility graph method and the aisle centers method for large pick lists for very skewed demand patterns (i.e., 20/80 and pick list size of 30). Because the most frequently picked items are located closer to end points of pick aisles where most of the "corner cutting" happens with the visibility graph method. In uniform demand with large pick lists, items in the middle of the pick aisles are picked as frequently as the items at the end points of pick aisles. Therefore, "corner cutting" is not happening as frequently as it does with more skewed demand patterns.

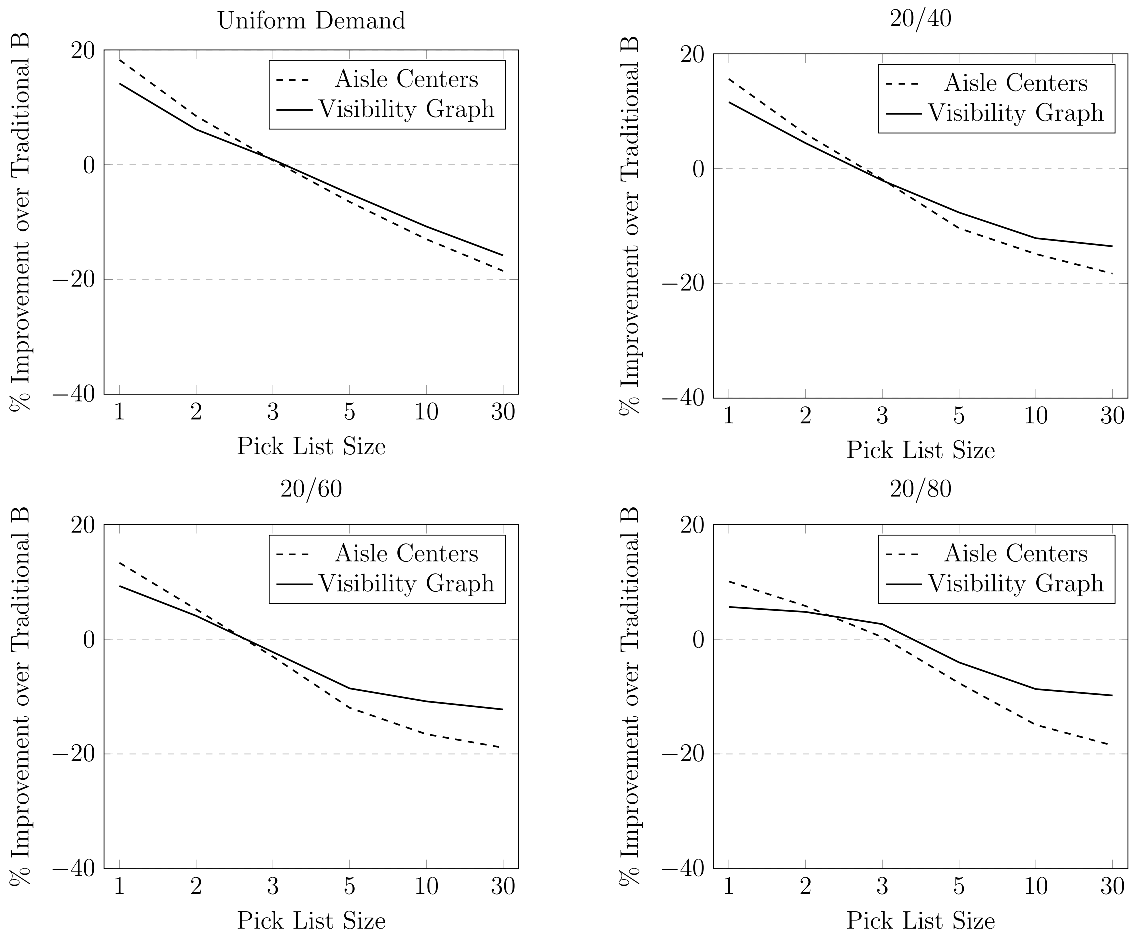


Figure 4.9: Fishbone Percent Improvement over Traditional Layout B

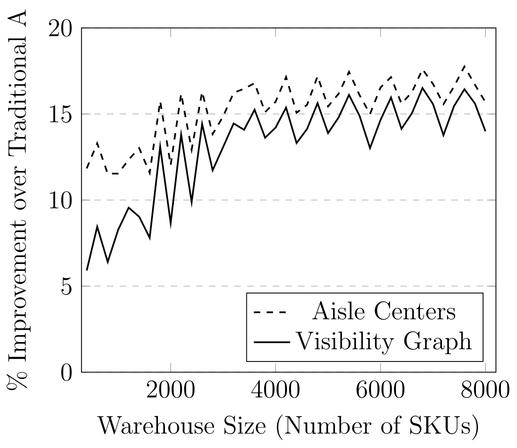


Figure 4.10: Fishbone Percent Improvement over Traditional for Unit-load Operations Under Varying Warehouse Sizes

In a second set of experiments, we analyzed the percent improvement of the fishbone layout over traditional layout A (see Figure 4.4b) for unit-load operations under varying warehouse sizes with both methods (the aisle centers method and the visibility graph method). Figure 4.10 shows the results of this analysis. For small size warehouses the difference between the two distance estimation methods for percent improvement over traditional layout A is as high as 6%. As one would expect, the storage density in small size warehouses is lower than the large size warehouses because aisles consume a larger portion of the warehouse area. The shortest paths from pick locations to the P&D point in traditional layout A are slightly towards the P&D point which does not exist with the aisle centers method. Small sized traditional layout A benefits more from these diagonal paths than the fishbone layout because the fishbone layout is designed to have direct access to the P&D point. For medium to large warehouses the difference is around 2%.

In our last set of experiments, we analyze the effect of the visibility graph method for selecting the best traditional layout. Traditional layouts A, B, and C (see Figures 4.4b, 4.4c, 4.4d) are very common layouts in warehousing. We calculated the average tour distances for various size warehouses from 200 SKUs to 1000 SKUs for various numbers of lines in a pick list. We assumed uniform demand among items in the warehouse. Table 4.4 shows the rank order of traditional layouts A, B, and C with both methods. As we have seen in the previous experiment, smaller warehouses have a larger percentage gap in average tour distance between the aisle centers method and the visibility graph method. We see a similar increase when pick lists are larger. Twelve out of 30 rank orders change. More importantly, in 13% of the cases the best layout with respect to average tour cost changes. This means that some layouts gain more advantage than other layouts when using the visibility graph method. Otherwise we would reach the same conclusion when finding the best warehouse layout design, which is not the case. In other words, the estimation of shortest distances in a warehouse is very important.

Table 4.4: Average tour distance for traditional A, B, and C with the aisle centers method (ACM) and the visibility graph method (VGM)

		Average tour distance (ft.)							
		Traditional A		Traditional B		Traditional C		Rank Order[1]	
Size	SKUs	ACM	VGM	ACM	VGM	ACM	VGM	ACM	VGM
1	200	137.7	121.0	148.1	124.8	168.0	135.4	A, B, C	A, B, C
1	400	191.0	172.4	202.5	173.6	224.3	183.6	A, B, C	A, B, C
1	600	230.7	211.5	254.0	217.7	254.6	214.8	A, B, C	A, C, B*
1	800	269.8	247.2	289.1	250.3	294.5	250.1	A, B, C	A, C, B*
1	1000	292.8	274.5	311.8	275.9	333.5	282.4	A, B, C	A, B, C
2	200	232.4	197.6	232.8	192.9	258.4	207.5	A, B, C	B, A, C*
2	400	323.0	284.2	315.3	269.7	342.5	282.8	B, A, C	B, C, A*
2	600	388.0	350.1	393.7	335.8	392.9	331.2	A, C, B	C, B, A*
2	800	451.8	407.6	445.8	386.0	449.6	384.7	B, C, A	C, B, A*
2	1000	493.5	453.8	484.6	427.0	508.7	434.7	B, A, C	B, C, A*
3	200	299.3	249.6	286.8	237.0	317.2	255.3	B, A, C	B, A, C
3	400	415.7	359.5	389.9	331.4	419.3	344.4	B, A, C	B, C, A*
3	600	499.3	444.3	484.7	412.5	478.4	405.9	C, B, A	C, B, A
3	800	581.5	516.8	551.9	474.6	550.0	471.5	C, B, A	C, B, A
3	1000	639.4	580.2	595.2	530.1	620.4	531.9	B, C, A	B, C, A
5	200	393.8	317.9	364.3	293.3	396.7	316.0	B, A, C	B, C, A*

Continued on next page

Table 4.4 – continued from previous page

		Average tour distance (ft.)							
		Traditional A		Traditional B		Traditional C		Rank Order[1]	
Size	SKUs	ACM	VGM	ACM	VGM	ACM	VGM	ACM	VGM
5	400	547.2	464.3	495.9	412.8	524.8	427.9	B, C, A	B, C, A
5	600	665.2	575.1	615.1	517.3	602.2	501.6	C, B, A	C, B, A
5	800	775.5	672.9	702.4	597.2	686.7	585.3	C, B, A	C, B, A
5	1000	848.4	754.7	760.2	662.9	777.3	661.0	B, C, A	C, B, A*
10	200	542.1	426.9	494.8	380.8	523.8	404.6	B, C, A	B, C, A
10	400	777.2	634.1	681.3	548.4	698.9	553.6	B, C, A	B, C, A
10	600	951.4	797.9	845.5	683.5	799.3	653.3	C, B, A	C, B, A
10	800	1121.7	943.1	972.2	795.0	919.0	760.0	C, B, A	C, B, A
10	1000	1235.0	1070.9	1052.4	885.6	1033.2	858.7	C, B, A	C, B, A
30	200	764.4	610.2	740.8	549.7	807.3	593.9	B, A, C	B, C, A*
30	400	1189.0	969.2	1105.2	838.1	1119.2	837.2	B, C, A	C, B, A*
30	600	1528.9	1271.9	1400.5	1074.6	1300.7	1006.7	C, B, A	C, B, A
30	800	1873.6	1538.8	1655.3	1271.8	1508.4	1179.1	C, B, A	C, B, A
30	1000	2096.6	1801.0	1808.9	1432.9	1722.4	1343.8	C, B, A	C, B, A

4.5 Conclusion

In conclusion, the visibility graph method affects the previous results of Gue and Meller (2009) and Çelik and Süral (2014). First, the performance improvements of the fishbone layout varies from the aisle centers method as much as 9%. Second, the visibility graph method has a larger impact for small size warehouses when comparing the fishbone layout to the traditional layout. This indicates the importance of a good distance estimation for order picking operations. Third, it also affects the best layout selection in a traditional layout setting.

[1](*) indicates that the rank order has changed

Chapter 5

A Computational System to Solve the Warehouse Aisle Design Problem

5.1 Introduction

Order picking is the most costly operation in a warehouse. However, current warehouse design practices have been using the same design principles for more than sixty years: straight rows with parallel pick aisles and perpendicular cross aisles that reduce the travel distance between pick locations (Vaughan and Petersen, 1999; Petersen, 1999). Gue and Meller (2009) recently challenged these assumptions by proposing the fishbone layout, which achieved reductions in travel distance up to 20% in unit-load warehouses. The fishbone layout has been effectively applied to newly built warehouses (Meller and Gue, 2009). Warehouses that are already constructed may also get a high return on investment with a warehouse re-design if there is a highly increased efficiency in order picking operations. Berglund and Batta (2012) state that cross aisle configurations may be changed without incurring prohibitive costs. Some order picking operations are performed by picking from a pallet storage which can be easily reconfigured to re-orient the cross aisle and pick aisle positions.

Dukic and Opetuk (2008) and Çelik and Süral (2014) analyze the fishbone layout for order picking operations with different routing policies and show that the fishbone layout can perform as much as 30% worse than a traditional layout. However, the research in finding optimal layouts for order picking warehouses is lacking. The main result of this paper is to show that there are non-traditional designs that reduce the cost of order picking operation by changing the aisle orientation, aspect ratio, and placement of depot simultaneously.

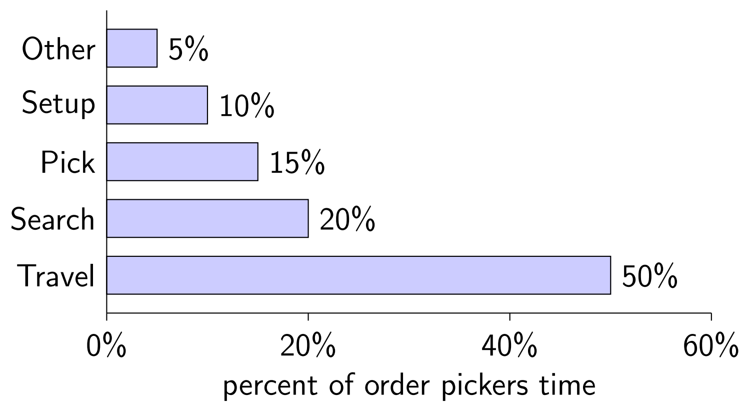


Figure 5.1: Typical distribution of an order picker's time (Tompkins, 2010)

5.2 Literature Review

Order picking is the retrieval of items from a storage area to fulfill customer orders. It involves the process of grouping and scheduling customer orders, releasing them to the order pickers, picking of the items from storage locations, and the disposal of the picked items (De Koster et al., 2007). The faster an order can be retrieved, the sooner it is available for shipping to the customer. Therefore, minimizing order retrieval time is a key to a successful warehouse. Figure 5.1 shows the order picking time components in a typical warehouse. 50% of the order picker's time is travel time. For manual-pick order picking systems, travel time is an increasing function of the travel distance (De Koster et al., 2007). For these reasons, we select minimizing travel distance as an objective for improvement.

Warehouses can have multiple order picking systems. These systems are either manual or automated. Tompkins (2010) states that there must be a balance between the level of automated systems (which are inflexible) and labor in order to respond to future business requirements without sacrificing logical labor savings today. Because automation is capital intensive and inflexible, the majority of warehouses employ humans for order picking (Le-Duc, 2005). In this paper we limit ourselves to low-level picker-to-parts systems with multiple picks per route. In these systems, an order picker picks items from bin shelving storage, pallet storage on the floor, modular storage drawers/cabinets, or gravity flow racks. Cart picking and tote picking are the most common retrieval methods. These systems are very common in practice, especially in Western Europe where 80% of order picking systems are of this type (De Koster et al., 2007).

In picker-to-parts systems, discrete picking is the most common order picking methodology because of its simplicity (Tompkins, 2010). An order picker completes a tour through the warehouse to pick all items for a single order. Because the risk of omitting the merchandise from an order is reduced and it provides the fastest customer response in a service window environment, this method is often preferred. Especially in warehouses with large orders (those with more than ten line items), discrete picking may yield an efficient picking tour (Frazelle, 2002). For other methods of order picking we refer to Tompkins (2010).

Routing methods determine the picking sequence of items on the pick list to ensure a good route through the warehouse. The problem of sequencing and routing order pickers is a special case of the TSP, classified as the Steiner TSP. In a classical TSP, given distances between each cities, a salesperson needs to find the shortest possible route that visits each city exactly once and returns to the origin. Similarly, in order picking, the order picker starts at the depot location (origin), visits all pick locations in his/her pick list and then returns to the depot location. However, some differences exist between the classical TSP and the Steiner TSP. In the Steiner TSP, some cities do not have to be visited at all and some other cities can be visited more than once. The Steiner TSP is not solvable in polynomial time in general (De Koster et al., 2007). However, Ratliff and Rosenthal (1983) show that the problem of order picking in a rectangular warehouse is a solvable case of the TSP. They proposed a polynomial-time dynamic programming algorithm to optimally solve this problem. Their algorithm uses an approach that starts from the left-most pick aisle, enumerates all possible equivalence classes (i.e., the possible degree parities of the corner nodes of the pick aisles and the number of connected subtours in the current partial solution) for each picking aisle. Then it finds the best equivalence class solutions for the right-most pick aisle, and find the optimal route by backward recursion from these solutions. For the case of single block warehouses, there are seven possible equivalence classes that need to be considered. De Koster and Poort (1998) show that this exact algorithm can be extended in such a way that the shortest order picking routes can be found in both warehouses with a central depot and warehouses with decentralized depositing. Roodbergen and De Koster (2001b) extend the algorithm by Ratliff and Rosenthal (1983) to warehouse settings with two blocks (i.e., three cross aisles or one middle aisle). Çelik and Süral (2014) show that the multi-item order picking problem can be solved in polynomial time for both fishbone and flying-V layouts. The main idea behind their algorithm is to transform the fishbone layout into an equivalent warehouse setting with two blocks. For warehouses with three or more blocks, the number of possible equivalence classes increases quickly (Çelik and Süral, 2014). Therefore, extending the algorithm is of little of use.

The existing literature has largely been devoted to finding efficient heuristics because efficient optimal algorithms are not available for every layout, and optimal routes may not consider real-world problems in order picking such as aisle congestion. For example, an S-shape can avoid aisle congestion because it has a single direction if the pick density is sufficiently high (i.e., there is at least one pick in every aisle). Many routing policies described in the literature have been analyzed for four types of warehouse systems (i.e., conventional multi-parallel-aisle systems, man-on-board AS/RS, unit-load AS/RS, and carousel systems) (Gu et al., 2007). When using a S-shape heuristic, order pickers must completely traverse the entire aisle containing at least one pick. Aisles without picks are not visited. From the last entered aisle, the order picker returns to the depot. In the return method, the order picker enters and leaves each aisle from the same end and only visits aisles with picks. The midpoint policy divides the warehouse into two areas. The heuristic collects all the items in the upper section, after which the lower section is dealt with. In the largest gap method, the order picker traverses the first and last aisle with picks entirely. All the other aisles are entered from the front and back in such a way that the non-traversed distance between two adjacent locations of items to be picked in the aisle is maximized. In the composite heuristic, aisles with picks are visited, but dynamic programming is used to decide either entirely traverse or enter and left at the same end (see Roodbergen and De Koster (2001a)). Petersen (1997) analyzes six routing heuristics (S-shape, return, midpoint, largest gap, composite, and optimal) for single-block warehouses and concludes that the best heuristic solution is on average 5% more than the optimal solution.

These routing heuristics are not suitable for our research for two reasons. First, these heuristics are designed for single-block warehouses and some (aisle-by-aisle, S-shape, largest gap, combined) can be modified for multiple-block warehouses, but they are not designed for non-traditional designs. Therefore, the routing might not work in some non-traditional designs. Second and most important, these heuristics are fairly simple construction heuristics which construct a feasible solution, without attempting any improvement by means of local search or meta-heuristic search.

Makris and Giakoumakis (2003) present Lin and Kernighan (1973)s TSP-based k-interchange methodology for single-block warehouses and show that their procedure outperformed the S-shape heuristic in seven of eleven examined cases. Theys et al. (2010) extend their research for multi-block warehouses and achieve average savings in route distance of up to 47% when using the Lin-Kerhighan-Helsgaun (LKH) heuristic (Helsgaun, 2000) compared to S-shape, largest gap, combined, and aisle-by-aisle heuristics. The quality of the solutions by the LKH heuristic is, on average, clearly superior to the other routing heuristics. Although the LKH heuristic's average computation time (0.25 seconds) is more than these heuristics (the calculation time is negligible for these heuristics) it is 36 times faster than the exact TSP algorithm "Concorde" (9.23 seconds). Moreover, LKH's solutions deviate on average only 0.1% from optimum. Therefore, we select LKH as our routing algorithm. A detailed description of these routing policies and their variations can be found in De Koster et al. (2007), Gu et al. (2007), and Helsgaun (2000).

5.3 Methodology

5.3.1 General Framework

Our solution approach has the following steps as given in Figure 5.2. First we import order profile data of the warehouse (this could be historical data from the enterprise or simulated data generated from an order profile). Then we define search boundaries such as size and aspect ratio. Then the ES algorithm searches over 19 design classes using our encoding scheme. With the results for all promising designs and their neighboring designs, we choose a design that has the lowest expected travel distance among competing designs.

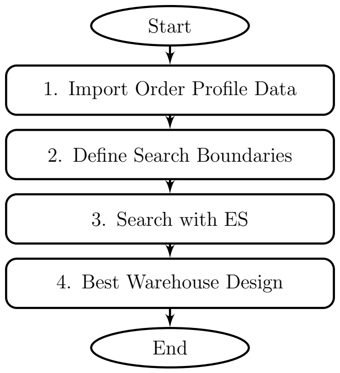


Figure 5.2: The solution approach

5.3.2 Assumptions

The order picking process as analyzed in this research is subject to a number of assumptions. First, we select the turnover-based storage policy as our storage policy. For warehouses that keep their product popularity information updated on a timely basis, turnover-based storage is better if there is little congestion. Our modeling approach assumes that the turnover frequency of each product is known and constant through time. We also assume that the capacity of the allocated space for each product is sufficient. In practice, demand rates are varying. Therefore, warehouses using a turnover-based storage policy need to reassign products to storage locations over time. However, we use constant order data which leads to a fixed demand curve over all the layouts considered to provide a consistent basis for comparison.

Second, we consider only the straight line distance within an aisle, and not the lateral movements within a picking aisle (Goetschalckx and Ratliff, 1988).

Third, the picking route is assumed to start and end at a single depot, located anywhere along the periphery of the warehouse. We do not consider multiple depot locations.

Fourth, the routing computations do not account for similar products stored at different locations. In this situation, a choice has to be made from which location the products have to be retrieved. A model for the problem of simultaneous assignment of products to locations and routing of order pickers is given in Daniels et al. (1998).

Finally, the capacity of the order picker is assumed to be sufficient for all necessary items to be picked during a single tour.

5.3.3 Importing Order Data

In the first phase, we either get simulated orders generated from an order database or use real order data over a given period. Generating orders is beyond the scope of this research, however using real order data might be very time consuming because we need to solve thousands of TSP-like routing problems to evaluate one particular design instance. Therefore, generating simulated orders that represent real order data can save computational time. Because we assume that we only know turnover rate information at the product level, our import phase only needs these two parameters: order ID and SKU number. In Table 5.1, we give an example of order data.

After order data is imported, we extract each unique SKU number and order ID. We also calculate the average number of SKUs per order which is later used by the product allocation algorithm.

Table 5.1: Example of order data

Order ID	SKU Number
554468267	5161503
554468267	5161484
554468267	5161233
554469595	5140361
554469595	5058449
554469621	5058449

5.3.4 Warehouse Design Classes

We classify a warehouse design according to three components: exterior nodes, interior nodes, and cross aisle segments. A node is defined as a point of intersection of a cross aisle segment and the exterior boundary of the design space, or the intersection of three or more cross aisle segments in the interior. We do not allow interior nodes with degree less than two. We do not restrict aspect ratio, but we do require cross aisle and picking aisle segments to be straight lines. We combine design classes in a single search to find the best layout among multiple design classes for a given order profile.

The 0-0-0 design is the most basic warehouse design class, with no cross aisles and only one region. A traditional one-block warehouse is a subset of this design class (see Figure 5.3). This design is assumed to be the best design in practice for unit-load warehouses until more promising layouts were proposed by Gue and Meller (2009). The 2-0-1 design is a warehouse design class with two exterior nodes, no interior node and one cross aisle. A traditional two-block warehouse is a subset of this design class (see Figure 5.4) and it is the most common design in practice for order picking. The 3-0-2 design has three exterior nodes, no interior node, and two cross aisles. The fishbone layout (see Figure 5.5) is a member of this class. It has been investigated by Dukic and Opetuk (2008) and Çelik and Süral (2014) for order picking operations. The 4-0-2 design is a warehouse design class with four exterior nodes, no interior node, and two cross aisles. A traditional three-block warehouse is a subset of this design class (see Figure 5.6). Several authors have considered these design as a promising layout for order picking operations (Hsieh and Tsai, 2006; Roodbergen et al., 2008; Chen et al., 2013). We also investigate classes 2-1-2, 3-0-3, 3-1-3, 3-1-4, 3-1-5, 3-1-6, 4-0-3, 4-0-4, 4-0-5, 4-1-3, 4-1-4, 4-1-5, 4-1-6, 4-1-7, and 4-1-8 (see Figure 5.7).

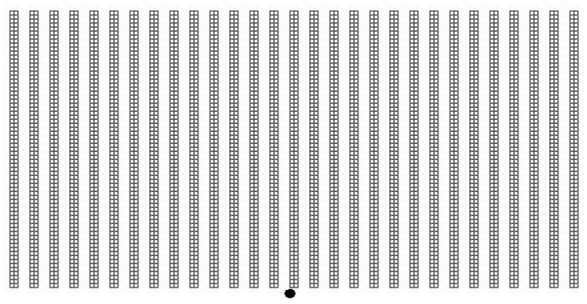


Figure 5.3: Traditional one-block warehouse

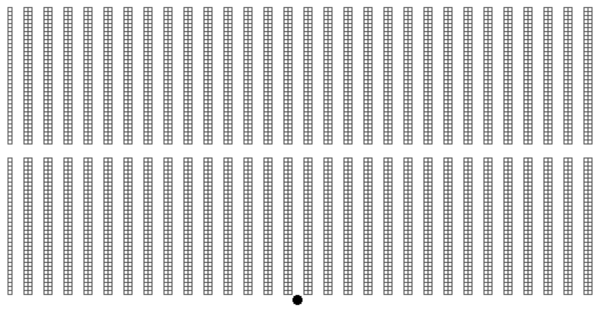


Figure 5.4: Traditional two-block warehouse

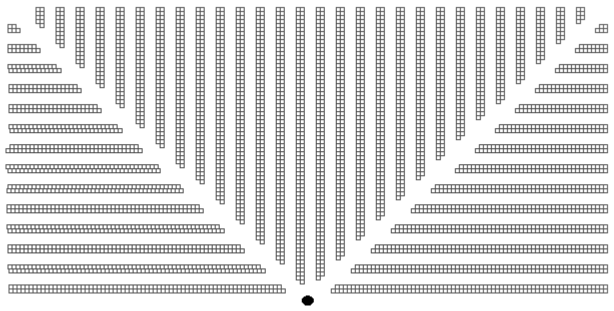


Figure 5.5: Fishbone layout

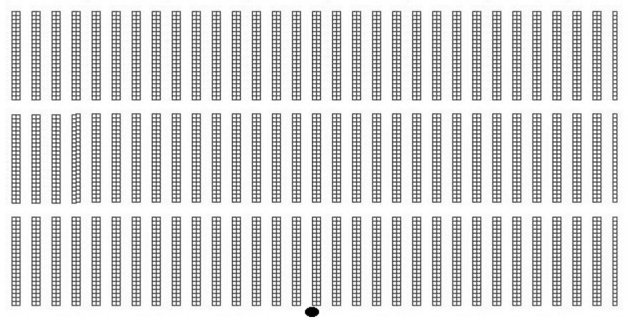


Figure 5.6: Traditional three-block warehouse

5.3.5 Searching the Design Space

Because the search space of possible designs is large, the use of brute force search is impractical. A non-linear objective, the large search space, and continuous variables require a meta-heuristic that can effectively search a complex objective function surface. The ES is an effective algorithm for such contexts.

Öztürkoğlu et al. (2014) introduced the use of a warehouse encoding of continuous variables. Our encoding uses for each class a string of continuous variables that defines locations of the cross aisle endpoints, the angles of picking aisles in each region, and the location of the depot. The upper-left corner is defined as the origin 0. The upper-right, lower-right, and lower-left corners are defined as 0.25, 0.5, 0.75, respectively. An interior node is defined in two dimensional space and the upper-left corner is defined as the origin (0, 0) and the lower-right corner is defined as (1, 1). When evaluating the design, we convert the encoding of the cross aisles and depot to an (x, y) point in the coordinate system. An example of the encoding and the relevant layout are given in Table 5.2 and Figure 5.8, respectively. Independent variables are type 1, parameters are type 2, and dependent variables are type 3. E1, E2, E3, and E4 reflect the position of exterior nodes along a clockwise path of length 1 beginning and ending at the upper left corner. IX and IY are the normalized coordinate location of the interior node. D is the location of the depot using the same encoding system as used for exterior nodes. A1 through A8 are the angles of the picking aisles for pick aisle regions. HA1 to HA8 and VA1 to VA8 are the

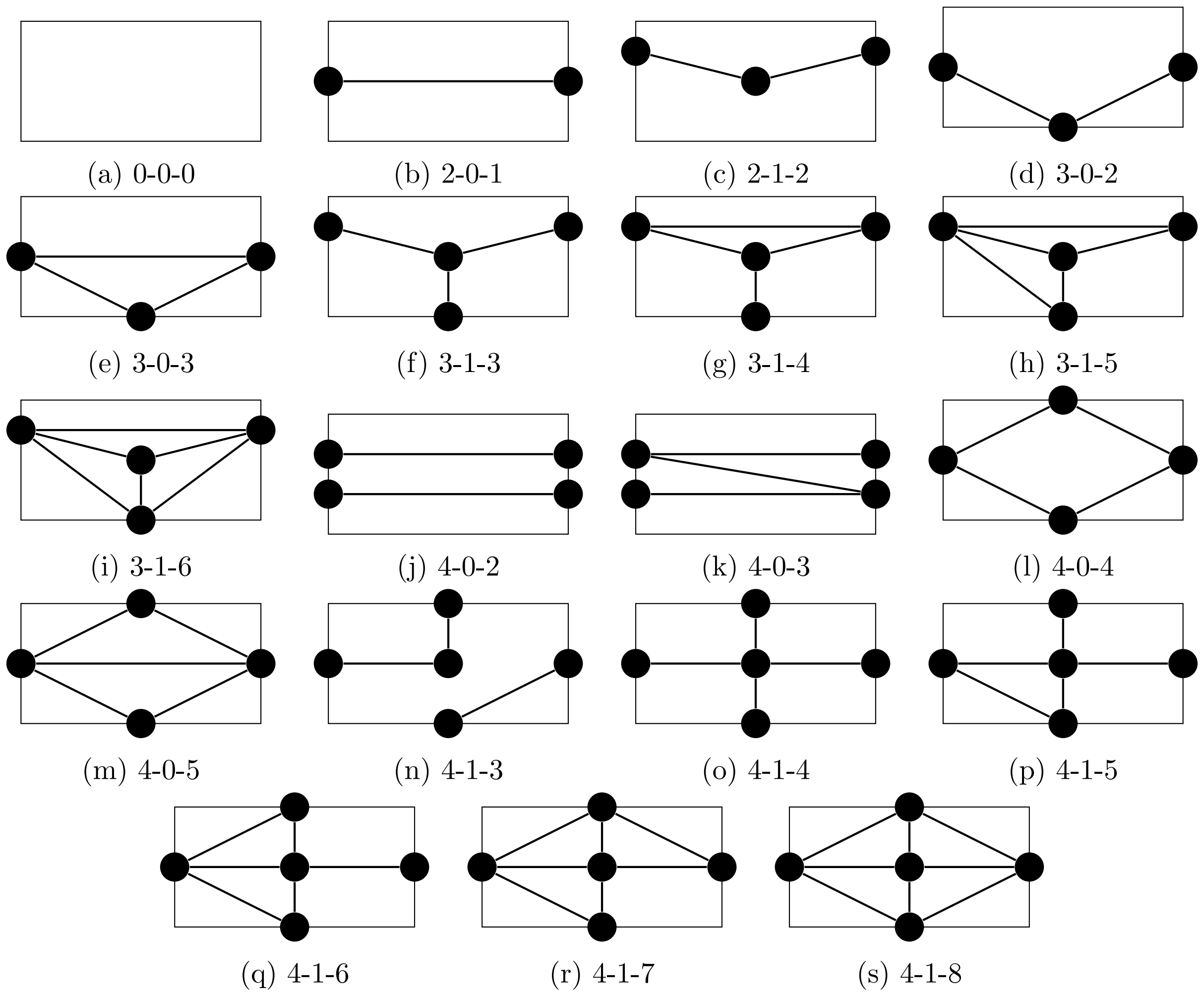


Figure 5.7: Design classes that are being searched

horizontal and vertical adjuster variables, respectively. Each adjuster variable shifts the parallel pick aisle inside the region in horizontal or vertical directions without changing their angle. These adjuster variables help to determine the best positions for the pick aisles. A pick aisle can be shifted horizontally at most by the distance between two parallel pick aisles in the same region. It can be shifted vertically at most by the width of the storage location opening. We standardized this distance and set the range between 0 and 1. Therefore each adjuster variable can take a value in this range. We control the creation of each cross aisle segment with PC parameters. For example, the likelihood of having a cross aisle segment between E1 and E2 nodes are determined by PC12 parameter. A value of 1 means that it will always create that cross aisle segment. Optimization algorithm decreases or increases the probability of these variables to determine the optimal design class. In order to keep the actual design, we need more information than the likelihood of cross aisle segment creation. Hence, we keep the realization of the corresponding connection.

Table 5.2: Encoding Example

Name	Type	Range	Value	Description
SLW	2	$(0,\infty)$	4	Storage Location Width
SLD	2	$(0,\infty)$	4	Storage Location Depth
CAW	2	$(0,\infty)$	12	Cross Aisle Width
PAW	2	$(0,\infty)$	12	Pick Aisle Width
WW	3	$(0,\infty)$	400	Warehouse Width
WD	3	$(0,\infty)$	200	Warehouse Depth
WA	2&3	$(0,\infty)$	80000	Warehouse Area
AR	1	$(0,\infty]$	0.5	Aspect Ratio
E1	1	$[0,1)$	0.0416	Exterior Node 1
E2	1	$[0,1)$	0.2083	Exterior Node 2
E3	1	$[0,1)$	0.6250	Exterior Node 3

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Table 5.2 – continued from previous page

Name	Type	Range	Value	Description
E4	1	[0,1)	0.6250	Exterior Node 4
IX	1	(0,1)	0.5000	Interior Node X Axis
IY	1	(0,1)	0.5000	Interior Node Y Axis
D	1	(0,1)	0.5	Depot
A1	1	[0,1)	0.1777	Region 1 Angle (32°)
A2	1	[0,1)	0.8166	Region 2 Angle (147°)
A3	1	[0,1)	0.5000	Region 3 Angle (90°)
A4	1	[0,1)	0.5000	Region 4 Angle (90°)
A5	1	[0,1)	0.5000	Region 5 Angle (90°)
A6	1	[0,1)	0.5000	Region 6 Angle (90°)
A7	1	[0,1)	0.5000	Region 7 Angle (90°)
A8	1	[0,1)	0.5000	Region 8 Angle (90°)
HA1	1	[0,1)	0.5000	Horizontal Adjuster 1
HA2	1	[0,1)	0.5000	Horizontal Adjuster 2
HA3	1	[0,1)	0.5000	Horizontal Adjuster 3
HA4	1	[0,1)	0.5000	Horizontal Adjuster 4
HA5	1	[0,1)	0.5000	Horizontal Adjuster 5
HA6	1	[0,1)	0.5000	Horizontal Adjuster 6
HA7	1	[0,1)	0.5000	Horizontal Adjuster 7
HA8	1	[0,1)	0.5000	Horizontal Adjuster 8
VA1	1	[0,1)	0.5000	Vertical Adjuster 1
VA2	1	[0,1)	0.5000	Vertical Adjuster 2
VA3	1	[0,1)	0.5000	Vertical Adjuster 3
VA4	1	[0,1)	0.5000	Vertical Adjuster 4

Continued on next page

Table 5.2 – continued from previous page

Name	Type	Range	Value	Description
VA5	1	[0,1)	0.5000	Vertical Adjuster 5
VA6	1	[0,1)	0.5000	Vertical Adjuster 6
VA7	1	[0,1)	0.5000	Vertical Adjuster 7
VA8	1	[0,1)	0.5000	Vertical Adjuster 8
PC12	1	[0,1]	0	Probability of E1-E2 connection
PC13	1	[0,1]	1	Probability of E1-E3 connection
PC14	1	[0,1]	0.0001	Probability of E1-E4 connection
PC15	1	[0,1]	0	Probability of E1-I connection
PC23	1	[0,1]	1	Probability of E2-E3 connection
PC24	1	[0,1]	0	Probability of E2-E4 connection
PC25	1	[0,1]	0	Probability of E2-I connection
PC34	1	[0,1]	0	Probability of E3-E4 connection
PC35	1	[0,1]	0	Probability of E3-I connection
PC45	1	[0,1]	0	Probability of E4-I connection
C12	3	0 or 1	0	Realization of E1-E2 connection
C13	3	0 or 1	1	Realization of E1-E3 connection
C14	3	0 or 1	0	Realization of E1-E4 connection
C15	3	0 or 1	0	Realization of E1-I connection
C23	3	0 or 1	1	Realization of E2-E3 connection
C24	3	0 or 1	0	Realization of E2-E4 connection
C25	3	0 or 1	0	Realization of E2-I connection
C34	3	0 or 1	0	Realization of E3-E4 connection
C35	3	0 or 1	0	Realization of E3-I connection
C45	3	0 or 1	0	Realization of E4-I connection

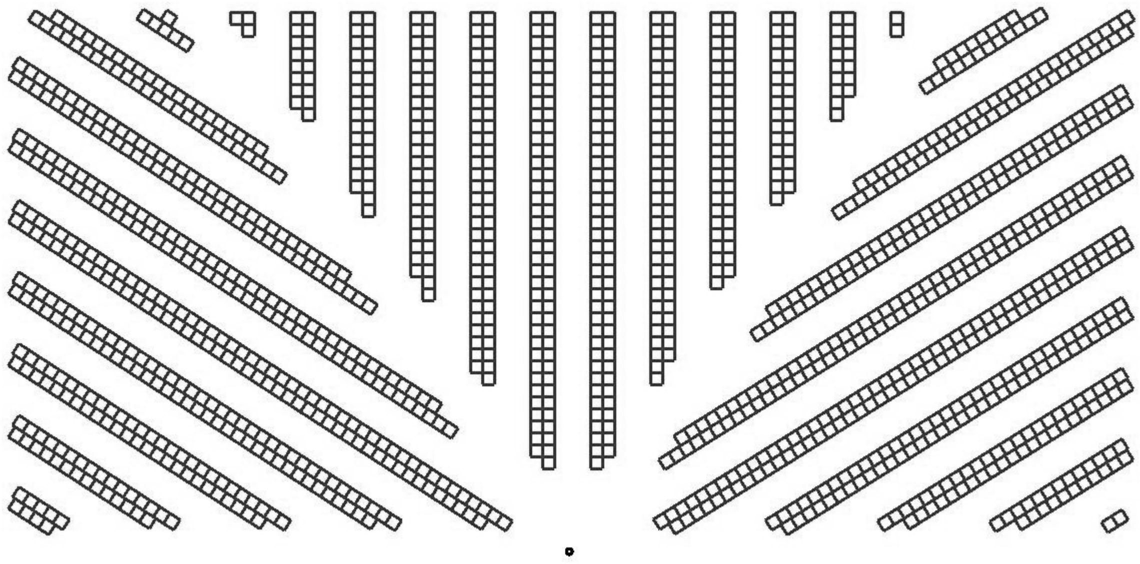


Figure 5.8: Corresponding representation of the encoding

5.3.5.1 ES Algorithm

In our implementation of the basic ES algorithm (only selection and mutation, no recombination), we select (μ + λ)-ES as the population strategy. The *(μ + λ)-ES* uses reproduction operations and from μ parent individuals $\lambda \geq \mu$ offspring are created. From the joint set of parents and offspring, only the μ fittest ones survive (Schwefel, 1975, 1977). We use a single σ value for the population. The σ value changes according to a modified $\frac{1}{5}$ rule. We decrease $\frac{1}{5}$ ratio into $\frac{1}{20}$ because success rate at each iteration is much lower than 0.2. Therefore keeping the original $\frac{1}{5}$ rule leads to an early convergence during optimization process. We use an early termination rule if the optimization does not improve the best solution more than 0.5% for the last 100 iterations. Figure 5.9 depicts the main steps of the ES algorithm. Algorithm 3 shows the pseudo-code of the heuristic optimizer.

Algorithm 3 Pseudo-code for the ES algorithm

```
for all parents $i$ in population of size $\mu$ do
    Initialize $x$-vector randomly between its bounds
    Calculate $x$-fitness value
end for
while maximum iterations not reached do
    for all offspring $j$ in children population do
        Select a parent $x$ randomly
        Draw $z$-vector from the normal distribution N(0, $\sigma^2$)
        $y$-vector = $x + z$
        if $f(y) < f(x)$ then
            increase success rate
        else
            decrease success rate
        end if
    end for
    Join parent and children population and select $\mu$ fittest for the next generation
    $successratecounter = successratecounter + 1$
    if $successratecounter = 10$ then
        $successratecounter = 0$ //Reset counter
        if $successrate > 0.05$ then
            $\sigma = \sigma/0.85$ //Increase sigma
        else
            $\sigma = \sigma * 0.85$ //Decrease sigma
        end if
    end if
    if No significant (less than 0.5 percent) improvement over last 100 iterations then
        break;
    end if
end while
```

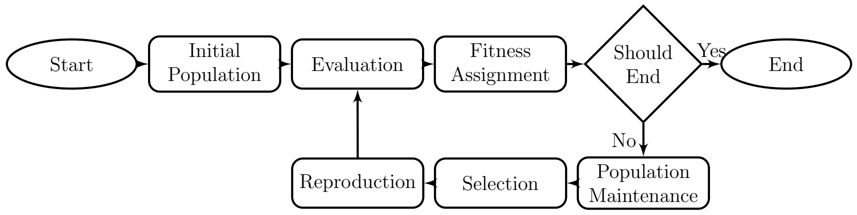


Figure 5.9: ES Algorithm

5.3.5.2 Evaluation

We developed a module to create a graph that represents the warehouse as a network (see Figure 5.10). We have two different network representations: aisle centers and visibility graph. In aisle centers method, order pickers follow center of the aisles to perform the picking. In visibility graph method order pickers follow paths in a visibility graph. These methods are described in detail in Chapter 4. The nodes of the network include the depot location (n1), exterior nodes (n2), interior nodes (n3), the beginning or ending points of the picking aisles (n4), pick locations (n5), and corner nodes that connect the exterior boundary edges (n6). The edges of the network include exterior boundary edges (ed1), region edges (ed2), picking aisles (ed3), picking aisle connection edges (ed4), and depot connection edges (ed5). A pick location is placed on the center line of the appropriate picking aisle and it provides access to the center of the corresponding storage locations. The distance between a pick location and its connected storage locations are assumed to be zero, because these storage locations are actually served from the same coordinate.

A connection between two nodes creates an edge, but types of the nodes being connected define the types of the edges being created. Table 5.3 lists the connections between types of nodes and types of edges being created by these connections. Categorizing nodes and edges in the graph based network representation is useful for many reasons. First, region edges can be used to define regions in the graph based network. A region is an area bounded by region edges that can be used to place angled picking aisles that are parallel to each other. Second, an efficient Dijkstra's shortest path algorithm can be implemented by using only the

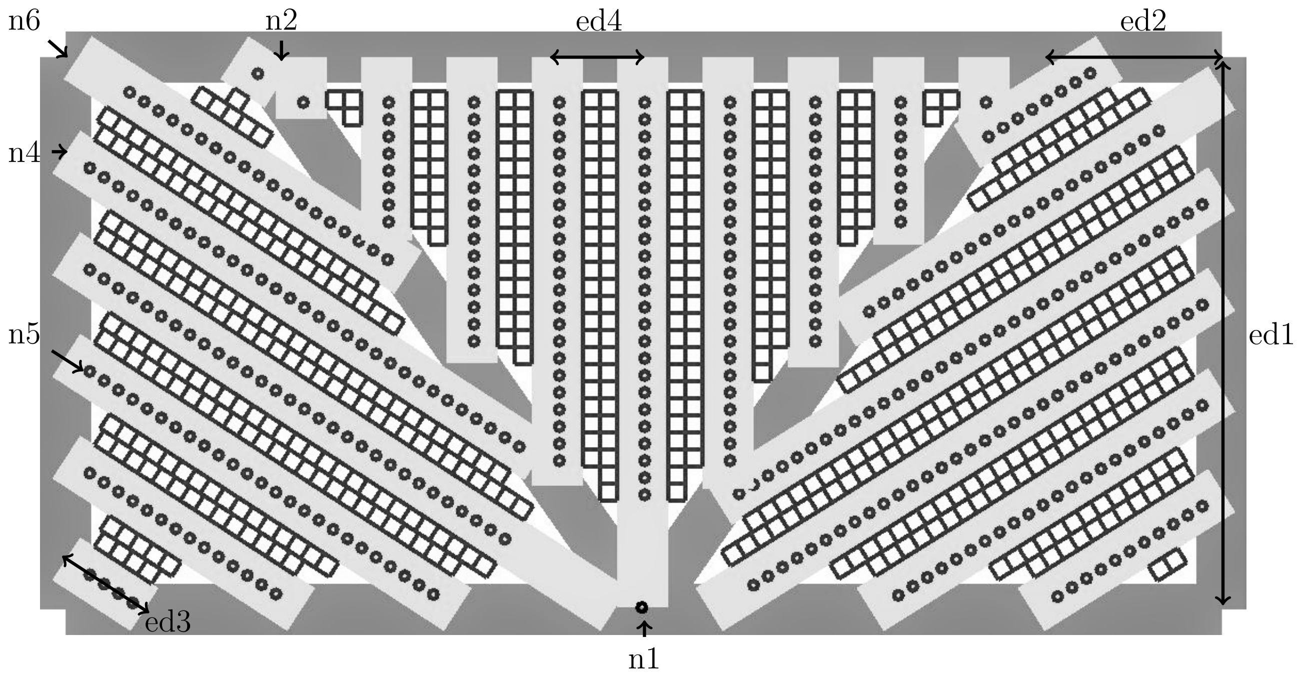


Figure 5.10: Warehouse graph based network representation

subset of the graph (for the aisle centers method). Third, different methods like visibility graph do not use all the nodes or edges defined in Table 5.3, so this would make it easier to develop different path finding methods in a warehouse. We use Dijkstra's shortest path algorithm to find shortest distances between any two pick locations or any pick location and depot location.

5.3.5.3 One-to-Many Problem

The one-to-many problem, also called single-cycle command in warehouse terminology, is to visit only a single location to pick an item and return to the depot location. The one-to-many problem is an important problem in order picking operations for warehouses that are frequently performing single picks per order. Dijkstra's algorithm allows us to compute point-point shortest path queries for any design instance (Dijkstra, 1959). The worst-case running time for the Dijkstra algorithm on a graph with n nodes and m edges is $O(n^2)$ (Schrijver, 2005).

Table 5.3: List of node connections and name of the connections as types of edges

Node 1	Node 2	Edge	Description
n6	n6	ed1	Connection of two corner nodes is an exterior boundary edge.
n6	n6	ed2	If there is no exterior node lying on the exterior boundary edge, then an exterior boundary edge becomes also a region edge.
n6	n2	ed2	Connection of an exterior node and a corner node is a region edge.
n2	n2	ed2	Connection of two exterior nodes is a region edge.
n2	n3	ed2	Connection of an exterior node and an interior node is a region edge.
n4	n4	ed3	Connection of two beginning and ending points of pick aisles is a pick aisle if beginning or ending points of pick aisles are located on the different region edges.
n4	n4	ed4	Connection of two beginning and ending points of pick aisles is a pick aisle if beginning or ending points of pick aisles are located on the same region edges.
n1	n2	ed5	Connection of a depot location and an exterior node is a depot connection edge.
n1	n6	ed5	Connection of a depot location and a corner node is a depot connection edge.
n1	n4	ed5	Connection of a depot location and a beginning or ending point of a pick aisle is a depot connection edge.

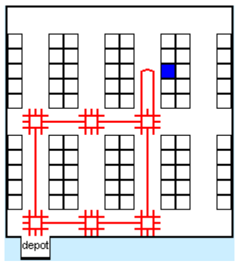


Figure 5.11: Single cycle command example

5.3.5.4 Many-to-Many Problem

The many-to-many problem, also called the many-to-many shortest path, is to calculate the pick location to pick location shortest path distances to find locations that are closest to any other location on average. In other words, by calculating many-to-many distances, we find the pick locations that are close to the centroid of the design space. We still use Dijkstra's algorithm to calculate many-to-many distances. However, calculation of many-to-many distances takes much more time compared to the calculation of one-to-many distances. If we assume n is the total number of pick locations, the worst-case running time would be O(n^3). After calculating the total travel distances to every location for each location, we sort the locations from minimum to maximum to find the most convenient locations for multiple order picking operations. Algorithm 4 shows the details of all pairs shortest path (i.e., many-to-many distances).

5.3.5.5 Storage (Product Allocation)

We select turnover-based as our storage policy. The most convenient locations for multiple-command order picking for the turnover-based storage policy are not known in the literature (Pohl et al., 2011). Therefore, we develop our own algorithm to allocate the products to storage locations.

In the importing order data phase, we calculate the average order size. Because we assume that the pick lists have been determined, each order picker travels through the

Algorithm 4 All-Pairs Shortest Path

```
function ALLPAIRSSHORTESTPATH(void)
    for ($i := 0$;$i < Count(G)$;$i \leftarrow i + 1$) do
        $DijkstrasShortestPath(G, i)$
    end for
end function

function DIJKSTRASSHORTESTPATH($G, k$) $Q \leftarrow \emptyset$
    for ($i := 0$;$i < Count(G)$;$i \leftarrow i + 1$) do
        $dist[k, i] \leftarrow \infty$
        Add $G[i]$ to $Q$
    end for
    $dist[k, k] \leftarrow 0$
    while Q is not empty do
        $u \leftarrow min(Q)$
        remove $u$ from $Q$
        for all $v$ in neighbor of $u$ do
            $shortdist \leftarrow dist[k, u] + length(u, v)$
            if $shortdist < dist[k, v]$ then
                $dist[k, v] \leftarrow shortdist$
            end if
        end for
    end while
end function
```

warehouse to pick the items on the pick list. For any order that consists of $n > 1$ picks, an order picker needs to travel between $n - 1$ pick locations. In order to minimize the travel distances, popular products should be located close to each other. In other words, they should be close to the centroid of the warehouse layout. Because the many-to-many algorithm finds the convenient locations for the multiple order picking case, we need to consider these locations in our product allocation algorithm when the average order size is greater than 1.

Let do_k and dm_k denote one-to-many and many-to-many travel distances for each pick location k. We can find normalized one-to-many sdo_k and many-to-many travel distances sdm_k for each pick location k by using Equation 5.1 and Equation 5.2 respectively.

$$sdo_k = \frac{do_k - do_{min}}{do_{max} - do_{min}} \tag{5.1}$$

where do_{min} and do_{max} are the minimum and maximum values for one-to-many travel distances respectively.

$$sdm_k = \frac{dm_k - dm_{min}}{dm_{max} - dm_{min}} \tag{5.2}$$

where dm_{min} and dm_{max} are the minimum and maximum values for many-to-many travel distances respectively.

These normalized values are between 0 and 1, 0 means the most convenient location and 1 means the least convenient location for each problem. Then we use a linear combination of the normalized values of one-to-many and many-to-many travel distance values. If average order size is 1, it means that 100% of the orders are single-command. If the average order size is 2, it means that on the average, orders are dual-command cycle. A dual-command cycle consists of two edges that belong to a single-command cycle (one-to-many problem) and one edge that belongs to travel between locations (many-to-many problem). Therefore, we assume that on the average $\frac{2}{3}$ of the travel distances in order picking operations belong to one-to-many problem and $\frac{1}{3}$ of the travel distances in order picking operations belong to many-to-many problem. Let θ denote the fraction of the travel between distances (many-to-many problem) on expected travel distances and aos denote average order size. We can calculate θ by using:

$$\theta = \frac{aos - 1}{aos + 1}. \tag{5.3}$$

We can use this θ to calculate the convenience c_k of each pick location k as a linear combination of the one-to-many and the many-to-many problem by using Equation 5.4:

$$c_k = (1 - \theta) * sdo_k + \theta * sdm_k \tag{5.4}$$

According to this algorithm, the most convenient locations have values close to 0 and the least convenient locations have values close to 1. Then we allocate the most popular products to the most convenient pick locations. We can assign either a single SKU or multiple SKUs to one pick location.

Routing is the most computationally time consuming part of our design algorithm. For a given number of orders n, we need to solve n TSP problems to calculate the objective function. Since LKH is promising according to Theys et al. (2010), we select LKH TSP heuristic as our routing method. Our interface with LKH can run in a parallel computing environment.

5.3.6 Validation

Because of the complexity of the objective function, we validated the ES by solving a single-command unit-load warehouse problem, which has been optimally solved by Öztürkoğlu et al. (2012) for one, two, and three cross aisle designs. The objective is to minimize the expected travel distance for single-command operations under random storage (i.e. the shortest path from each location to P&D point). We used the aisle centers method described in Chapter 4 for calculating pick paths during the optimization.

We used the warehouse design parameters shown in Table 5.4 for first validation experiment. The settings for the ES are given in Table 5.5.

Table 5.4: Warehouse design parameters for the first experiment

Parameter	Value	Description
Warehouse Width	300	Width of the warehouse.
Warehouse Depth	150	Depth of the warehouse.
Location Width	4	Distance between two pick locations within a picking aisle.
Location Depth	4	Picking aisle width, this parameter determines the distance between two pick locations that are located in two neighboring picking aisles in the same region.
Cross Aisle Width	10	Width of a cross aisle.
Picking Aisle Width	10	Width of a picking aisle.

Table 5.5: ES settings for the first experiment

Parameter	Value	Description
μ	3	Number of parents.
λ	30	Number of offspring created from parents.
Max Iterations	300	Termination condition.
σ	1	Standard deviation of normal distribution.
Sigma alteration	5	Alter sigma every this many iterations.

As can be seen in Figure 5.12, the final solution of the ES is similar to Chevron. However, the final result is not exactly same as the Chevron design. There are three reasons for this. First, the original problem is not divided into sub-problems as it was done by Öztürkoğlu et al. (2014). Dividing it into sub-problems makes the surface of the search space much smoother, because exterior nodes do not cross the corner points of the warehouse (which might create swaps in the region angles). Second, we allow the depot to move around freely, also we add the adjuster parameters, which increase the solution space and therefore need to increase the number of iterations. Third, our designs are discrete design models, which means that a slight change in the angles or cross aisles might make some pick locations disappear or appear. Therefore, the solution of the ES shown in Table 5.6 might be slightly better than Chevron in this discrete case. The third situation is described in (Öztürkoğlu et al., 2014).

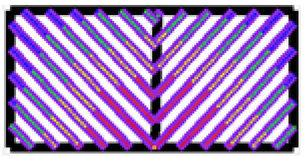


Figure 5.12: ES design with 300 iterations

Table 5.6: Solutions for the first experiment and the objective function values

Parameter	Value
Angle 1	47.153
Angle 2	133.955
Adjuster 1	0.159
Adjuster 2	0.907
Exterior Node 1	0.622
Exterior Node 2	0.129
Depot	0.622
Objective	115.02

In a second experiment, we altered warehouse design parameters to analyze how these the ES performs with a larger warehouse (see Table 5.7). The settings for the ES are given in Table 5.8. We increased the number of iterations for ES but the rest of the parameters stayed the same. The ES found a Chevron-like design again (see Figure 5.13). Table 5.9 shows the result of the ES algorithm.

Other than validating the optimization algorithm, we also validated the following features of the system: shortest path calculation between two points and optimal route distance calculation. We created twenty random samples for two different layouts and checked the distances with debugging and also measuring on paper.

Table 5.7: Warehouse design parameters for the second experiment

Parameter	Value	Description
Warehouse Width	900	Width of the warehouse.
Warehouse Depth	450	Depth of the warehouse.
Location Width	4	Distance between two pick locations within picking aisle.
Location Depth	4	Picking aisle width, this parameter determines the distance between two pick locations that are located in two neighboring picking aisles in the same region.
Cross Aisle Width	10	Width of a cross aisle.
Picking Aisle Width	10	Width of a picking aisle.

Table 5.8: ES settings for the second experiment

Parameter	Value	Description
μ	3	Number of parents.
λ	30	Number of offspring created from parents.
Max Iterations	500	Termination condition.
σ	1	Standard deviation of normal distribution.
Sigma alteration	5	Alter sigma every this many iterations.

5.4 A Computational Experiment with Real Order Data Set

After validating our system, we optimized a layout with real order data set. This real order data set has 11,438 TSPs, the average number of items per order is 9.62, and the largest order has 413 items. The frequency of order sizes is shown in Figure 5.14. The settings for the ES are given in Table 5.10. The optimization terminated after 220 iterations because for the last 100 iterations, the improvement was less than 0.5%. Table 5.12 shows the result of the ES algorithm. The optimized layout achieved 5.3% shorter travel distance on average

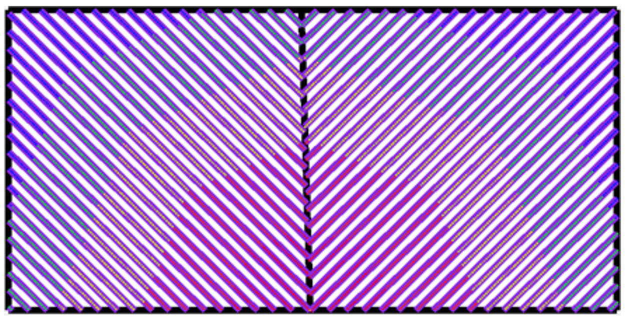


Figure 5.13: Final ES solution for the second experiment

Table 5.9: Solutions for the second experiment and the objective function values

Parameter	ES
Angle 1	44.601
Angle 2	135.384
Adjuster 1	0.927
Adjuster 2	0.206
Exterior Node 1	0.626
Exterior Node 2	0.120
Depot	0.626
Objective	366.864

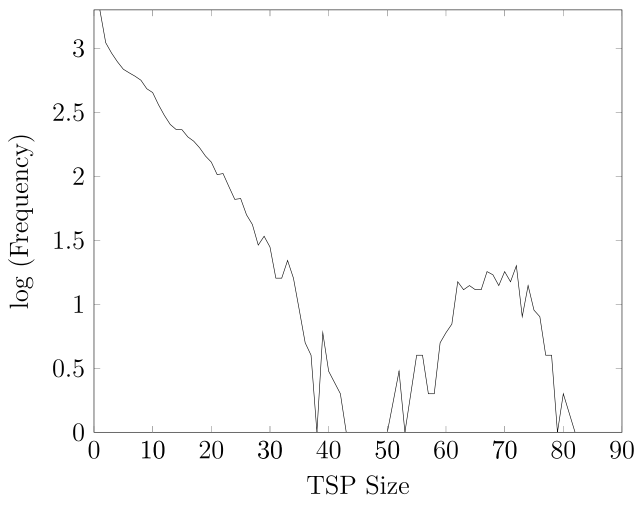


Figure 5.14: Order Size Frequency for 11,438 orders

compared to an optimized traditional three block layout (see Table 5.11). The optimization run finished in 12 days.

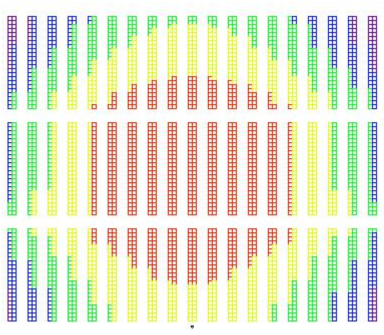
5.5 Conclusions

In this chapter, we described the details of our warehouse design and optimization system. Order picking is the most costly operation in the warehouse, yet we still don't know the effects of non-traditional layouts on order picking. Layout optimization problem for order picking warehouses considers the layout design, product allocation, and routing simultaneously. Even solving the routing optimally is NP-hard. Therefore, we described a system that solves this problem using simulation and heuristic optimization.

Table 5.10: ES settings for the real order data experiment

Parameter	Value	Description
μ	20	Number of parents.
λ	120	Number of offspring created from parents.
Max Iterations	500	Termination condition.
σ	0.5	Standard deviation of normal distribution.
Sigma alteration	10	Alter sigma every this many iterations.

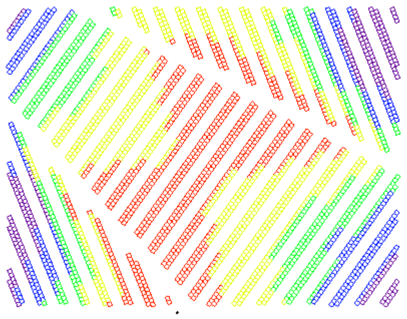
Table 5.11: Best Traditional Layout with Real Order Data

Parameter	Value	Design
Avg. PL Size	9.62	
E1, E2, E3, E4	0.041, 0.630, 0.921, 0.651	
IX, IY	N/A	
Depot	0.625	
Angles	90, 90, 90	
# Locations	2622	
Aspect Ratio	0.8	
Width	412.90	
Depth	330.31	
Area	136386.22	
H.Adjusters	0.5, 0.5, 0.5	
V. Adjusters	0.5, 0.5, 0.5	
Avg. Travel Cost	1026.50	

The system architecture is built on many components such as graph networks, a novel product allocation algorithm, an extended version of encoding proposed by Öztürkoğlu et al. (2014), and parallel & distributed computing. With our improved encoding, ES can learn the promising design classes and eliminate the inferior ones earlier without wasting computing resources. Moreover, before doing experiments, we validated the ES algorithm and distance calculations.

The system is scalable which means that certain calculations such as routing can be distributed to run on multiple computers to decrease the overall computational time. It can solve optimization experiments or single design assessments in batch. In this way, researchers can create design of experiments in Excel and import it to the system to get the results.

Table 5.12: Optimized Layout with Real Order Data

Parameter	Value	Design
Avg. PL Size	9.62	
E1, E2, E3, E4	0.041, 0.630, 0.921, 0.651	
IX, IY	N/A	
Depot	0.641	
Angles	111, 54, 111	
# Locations	2602	
Aspect Ratio	0.78	
Width	417.24	
Depth	324.68	
Area	135470.22	
H.Adjusters	0.57, 0.88, 0.63	
V. Adjusters	0.34, 0.97, 0.59	
Avg. Travel Cost	972.06	

The resulting data set can be used as a training data set for artificial neural network or for regression analysis.

Chapter 6
Non-traditional Warehouse Design Optimization and Their Effects on Order Picking Operations

6.1 Introduction

In this chapter, we describe our computational experiments aimed at achieving two objectives. First, we would like to see how optimized non-traditional layouts compare to common traditional layouts under near-optimal routing of pickers when multi-item pick lists are present with various demand skewness from generated orders of fixed pick list size. Second we would like to see if there are certain patterns when analyzing these results (i.e., how layouts are changing when pick list sizes are increasing/decreasing or demand skewness is increasing/decreasing).

6.2 Order Generation Method

We use the model by Bender (1981) to calculate the probability of demand for each SKU. For each SKU s, the model uses the following analytical function:

$$F(s) = \frac{(1+A)s}{A+s} \tag{6.1}$$

where A is a shape factor that depends on the skewness of the demand. For demand skewness pattern of 20/40 (i.e., twenty percent of most frequently picked SKUs constitute forty percent of the picks), the value of A is 0.60. A is 0.20 and 0.07 for demand skewness patterns of 20/60 and 20/80, respectively. The probability p_s of demand for SKU s is determined from

Table 6.1: Parameters used for the experiments under skewed demand and turnover-based storage

Demand Skewness	Number of Orders	Pick List Size
Random	4000	2
Random	2666	3
Random	1600	5
Random	800	10
Random	266	30
20/40	4000	2
20/40	2666	3
20/40	1600	5
20/40	800	10
20/40	266	30
20/60	4000	2
20/60	2666	3
20/60	1600	5
20/60	800	10
20/60	266	30
20/80	4000	2
20/80	2666	3
20/80	1600	5
20/80	800	10
20/80	266	30

$$p_s = F\left(\frac{s}{N}\right) - F\left(\frac{s-1}{N}\right) \tag{6.2}$$

where N is the number of storage locations in a warehouse. This method is also used by Çelik and Süral (2014). However, instead of evaluating 100 orders for each pick list size as Çelik and Süral (2014) did, we change the number of orders for each pick list size. For small pick list sizes, each tour has a larger variance in travel distance. Therefore, we increase the sample size for small pick list sizes to decrease the width of the confidence interval. For all situations considered in this chapter, the number of orders sampled for each pick list size is sufficient to guarantee a relative error of at most 1% with a probability 95% to estimate the mean travel distance.

Table 6.2: Roodbergen's Optimizer Parameters

Parameter	Value
Total aisle length	1004
Number of picks per route	2
Aisle width	6.096
Cross aisle width	3.6576

6.3 Experiment Settings

Because of the time complexity of every optimization run, we perform two replications for each set of parameters given in Table 6.1. We set the size of warehouse as 2000 SKUs, because smaller size warehouses generally fail to gain improvements from cross aisles and larger warehouses are extremely hard to solve. So we seek a size that can show the benefit of adding cross aisles while having a reasonable computational time. In some cases, we increase the number of replications to four because of high variation in best and worst results.

In order to assess the performance of the optimized layouts, we compare them with two different traditional layouts, traditional two block and traditional three block layouts, given in Figures 5.4 and 5.6. For traditional layouts, the depot location that minimizes the average travel distance is the exact middle of the front cross aisle (Roodbergen and Vis, 2006). Therefore, we only change the aspect ratio starting from 0.2 with 0.1 increments until 1 (i.e., a square warehouse). Moreover, we test certain cases with Roodbergen's "Warehouse layout optimizer" and add those designs to our comparison. However, Roodbergen's optimizer assumes uniform demand and the depot is located in the lower left corner. Roodbergen's optimizer may create layouts that have more than three blocks which is a limit in our search space (i.e., a 4-0-2 design). Roodbergen's optimizer creates a warehouse with more than three blocks when the pick list size is greater than 2. Therefore the only result we are able to compare is when the pick list size is 2. For this particular case, we use the parameters given in Table 6.2 for Roodbergen's layout optimization tool. The tool produces the design shown in Table 6.3.

Table 6.3: Optimal Traditional Layout with Roodbergen's Layout Optimizer

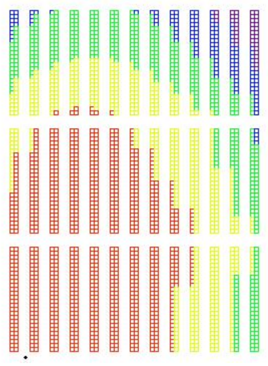
Parameter	Value	Design
Skewness	Random	
PL Size	2	
E1, E2, E3, E4	0.916, 0.333, 0.833, 0.416	
IX, IY	N/A	
Depot	0.72	
Angles	90, 90, 90	
# Locations	2028	
Aspect Ratio	1.25	
Width	293.09	
Depth	366.37	
Area	107379.93	
H.Adjusters	0.6, 0.6, 0.6	
V. Adjusters	0.5, 0.5, 0.5	
Avg. Travel Cost	712.20	

Table 6.4 shows the results for generated orders and best traditional layouts selected from a set of aspect ratio values and two different design classes (2-0-1, and 4-0-2). These experiments are performed on four Lenovo workstations. Each workstation has a six core hyperthreaded Intel Xeon E5-1650 processor. Workstations have 64GB of RAM and 256GB of Solid State Drive. The operating system is 64-bit Windows 7, Enterprise Edition. The total number of parallel threads that can be executed is $2 \times 6 = 12$.

6.3.1 Uniform Demand

In uniform demand, we achieve up to 3.2 percent improvement over traditional layouts. The design we found for pick list size 3 is an interesting design with two cross aisles that meets at the lower right corner. First cross aisle is on the middle of the back cross aisle and second one is the 2/3rd of the left cross aisle. Both traditional three block layout and optimized layout have similar aspect ratio and depot location (0.125 and 0.625 are symmetric). We emphasize that this design is significantly different than previous non-traditional designs in the literature and needs more elaboration. For other pick list sizes, the designs are not significantly cost effective compared to their traditional counterparts. In some cases the

Table 6.4: Optimization Results for Generated Order Data

		ES Seed				Best Traditional		
Skewness	PL Size	0	1	2	3	Asp. Rat.	Design	Cost
Random	2	582.10	585.39			0.7	4-0-2	589.23
Random	3	689.31	707.67			0.7	4-0-2	712.09
Random	5	907.29	870.12			0.8	4-0-2	874.09
Random	10	1197.10	1156.44			0.7	4-0-2	1158.91
Random	30	1894.16	1882.48			0.5	4-0-2	1909.23
20-40	2	458.51	482.72	457.75		0.7	4-0-2	476.88
20-40	3	595.56	590.40			0.7	4-0-2	591.17
20-40	5	747.57	773.66			0.8	4-0-2	745.77
20-40	10	1013.25	1018.07			0.8	4-0-2	1031.44
20-40	30	1762.28	1721.86			0.5	4-0-2	1762.81
20-60	2	373.55	390.21			0.5	4-0-2	383.83
20-60	3	478.03	478.93			0.5	4-0-2	484.26
20-60	5	622.99	646.31			0.5	4-0-2	635.04
20-60	10	876.71	915.63			0.7	4-0-2	894.47
20-60	30	1512.71	1509.17			0.6	4-0-2	1559.15
20-80	2	285.24	288.03	283.90	277.92	0.5	4-0-2	282.37
20-80	3			357.21	357.21	0.4	4-0-2	365.52
20-80	5			476.68	490.68	0.5	4-0-2	494.65
20-80	10			686.08	695.15	0.5	4-0-2	713.14
20-80	30			1206.49	1210.16	0.6	4-0-2	1225.32

optimizer tries to create a third middle cross aisle by using the interior node and therefore getting more benefits by the additional cross aisle.

Table 6.5: Optimization Results for Uniform Demand

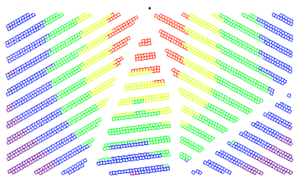
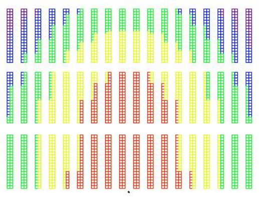
Parameter	Near-optimal	Traditional	Design
Skewness	Random	Random	
PL Size	2	2	
E1, E2, E3, E4	0.68, 0.35, 0.12, 0.60	0.916, 0.333, 0.833, 0.416	
IX, IY	0.59, 0.54	N/A	
Depot	0.126	0.625	
Angles	29, 152, 152, 7	90, 90, 90	
# Locations	2001	2016	
Aspect Ratio	0.58	0.7	
Width	434.57	399.78	
Depth	254.18	279.85	
Area	110459.59	111876.30	
H.Adjusters	0.93, 0.21, 0.64, 0.86	0.7, 0.7, 0.7	
V. Adjusters	0.04, 0.51, 0.25, 0.27	0.3, 0.3, 0.5	
Cost	582.10 (-1.2%)	589.23	

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Table 6.5 – continued from previous page

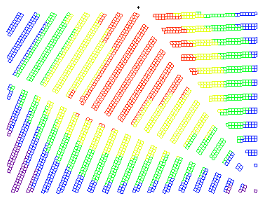
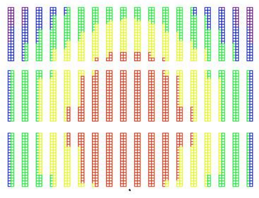
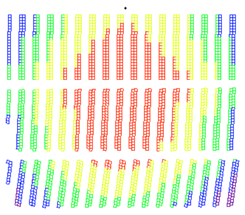
Parameter	Near-optimal	Traditional	Design
Skewness	Random	Random	
PL Size	3	3	
E1, E2, E3, E4	0.49, 0.91, N/A, 0.13	0.916, 0.333, 0.833, 0.416	
IX, IY	N/A	N/A	
Depot	0.131	0.625	
Angles	67, 3, 58	90, 90, 90	
# Locations	2000	2016	
Aspect Ratio	0.74	0.7	
Width	382.23	399.78	
Depth	283.54	279.85	
Area	108379.64	111876.30	
H.Adjusters	0.42, 0.21, 0.86	0.7, 0.7, 0.7	
V. Adjusters	0.3, 0.3, 0.5	0.3, 0.3, 0.5	
Cost	689.31 (-3.2%)	712.09	
Skewness	Random	Random	
PL Size	5	5	
E1, E2, E3, E4	0.82, 0.34, 0.43, 0.91	0.916, 0.333, 0.833, 0.416	
IX, IY	N/A	N/A	
Depot	0.125	0.625	
Angles	90, 82, 87	90, 90, 90	
# Locations	2000	2006	

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Table 6.5 – continued from previous page

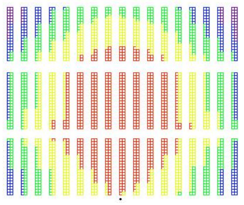
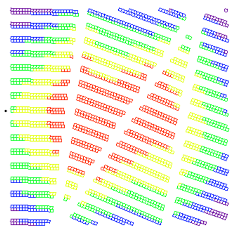
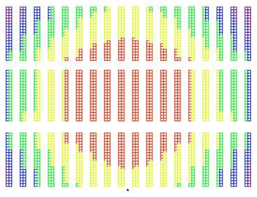
Parameter	Near-optimal	Traditional	Design
Aspect Ratio	0.83	0.8	
Width	362.63	367.84	
Depth	301.88	294.27	
Area	109469.42	108242.85	
H.Adjusters	0.94, 0.08, 0.94	0.5, 0.5, 0.5	
V. Adjusters	0.65, 0.87, 0.20	0.7, 0.5, 0.5	
Cost	870.12 (-0.5%)	874.09	
Skewness	Random	Random	
PL Size	10	10	
E1, E2, E3, E4	0.09, 0.22, 0.57, 0.70	0.916, 0.333, 0.833, 0.416	
IX, IY	N/A	N/A	
Depot	0.125	0.625	
Angles	161, 178, 160, 157	90, 90, 90	
# Locations	2000	2016	
Aspect Ratio	1.00	0.7	
Width	335.02	399.78	
Depth	334.36	279.85	
Area	112017.69	111876.30	
H.Adjusters	0.76, 0.10, 0.89, 0.93	0.7, 0.7, 0.7	
V. Adjusters	0.20, 0.70, 0.57, 0.60	0.3, 0.3, 0.5	
Cost	1156.44 (-0.2%)	1158.91	

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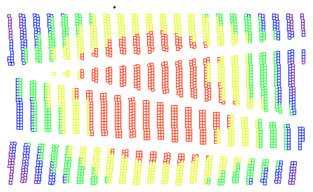
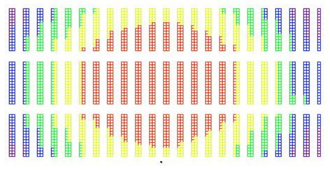
Table 6.5 – continued from previous page

Parameter	Near-optimal	Traditional	Design
Skewness	Random	Random	
PL Size	30	30	
E1, E2, E3, E4	0.91, 0.30, 0.82, 0.41	0.916, 0.333, 0.833, 0.416	
IX, IY	0.99, 0.82	N/A	
Depot	0.091	0.625	
Angles	94, 84, 92, 91	90, 90, 90	
# Locations	2000	2025	
Aspect Ratio	0.60	0.5	
Width	447.49	478.26	
Depth	270.37	239.13	
Area	120988.48	114366.11	
H.Adjusters	0.07, 0.24, 0.98, 0.68	0.5, 0.5, 0.5	
V. Adjusters	0.81, 0.78, 0.74, 0.74	0.5, 0.5, 0.5	
Cost	1882.48 (-1.4%)	1909.23	

6.3.2 20/40 Demand Skewness Pattern

In 20/40 demand skewness case, up to 4 percent improvement is achieved over traditional layouts. The final design for pick list size 2 has similarities with leaf layout but it has a second supporting cross aisle that ends at the top right corner. Also the intersection of the two cross aisles are at 2/3 of the bottom cross aisle. For pick list size 3, the final design is moving towards a traditional layout. However, this design is not able to achieve significant improvement over traditional counterpart. For pick list size 5, the final design is slightly inferior to the three block traditional layout. For pick list sizes 10 and 30, optimization is trying to make a traditional layout with more cross aisles by using an interior node to achieve shorter average walking distance per tour.

Table 6.6: Optimization Results for 20/40 Demand Skewness

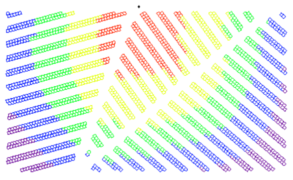
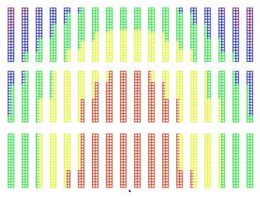
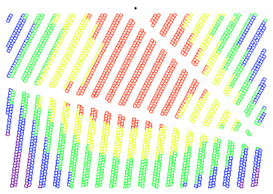
Parameter	Near-optimal	Traditional	Design
Skewness	20/40	20/40	
PL Size	2	2	
E1, E2, E3, E4	0.24, N/A, 0.11, 0.69	0.916, 0.333, 0.833, 0.416	
IX, IY	N/A	N/A	
Depot	0.119	0.625	
Angles	140, 15, 126	90, 90, 90	
# Locations	2000	2016	
Aspect Ratio	0.59	0.7	
Width	425.75	399.78	
Depth	252.53	279.85	
Area	107515.63	111876.30	
H.Adjusters	0.94, 0.16, 0.32	0.7, 0.7, 0.7	
V. Adjusters	0.84, 0.30, 0.31	0.3, 0.3, 0.5	
Avg. Travel Cost	457.75 (-4.0%)	476.88	
Skewness	20/40	20/40	
PL Size	3	3	
E1, E2, E3, E4	0.43, N/A, 0.90, 0.12	0.916, 0.333, 0.833, 0.416	
IX, IY	N/A	N/A	
Depot	0.124	0.625	
Angles	83, 61, 63	90, 90, 90	
# Locations	2001	2016	

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Table 6.6 – continued from previous page

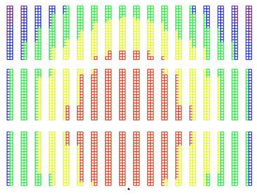
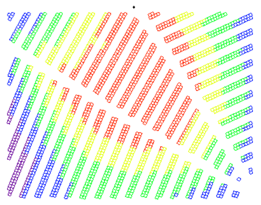
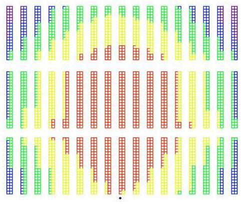
Parameter	Near-optimal	Traditional	Design
Aspect Ratio	0.69	0.7	
Width	396.63	399.78	
Depth	273.53	279.85	
Area	108488.13	111876.30	
H.Adjusters	0.31, 0.59, 0.21	0.7, 0.7, 0.7	
V. Adjusters	0.36, 0.82, 0.82	0.3, 0.3, 0.5	
Avg. Travel Cost	590.40 (-0.1%)	591.17	
Skewness	20/40	20/40	
PL Size	5	5	
E1, E2, E3, E4	0.13, N/A 0.49, 0.95	0.916, 0.333, 0.833, 0.416	
IX, IY	0.83, 0.59	N/A	
Depot	0.129	0.625	
Angles	68, 61, 25	90, 90, 90	
# Locations	2000	2006	
Aspect Ratio	0.77	0.8	
Width	375.98	367.84	
Depth	290.87	294.27	
Area	109359.95	108242.85	
H.Adjusters	0.25, 0.47, 0.44	0.5, 0.5, 0.5	
V. Adjusters	0.78, 0.76, 0.90	0.7, 0.5, 0.5	
Avg. Travel Cost	747.57 (0.2%)	745.77	

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Table 6.6 – continued from previous page

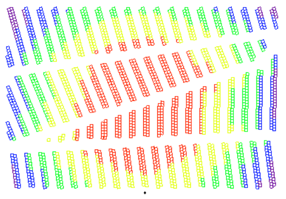
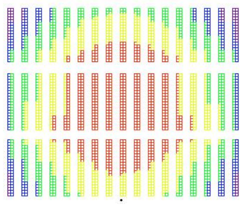
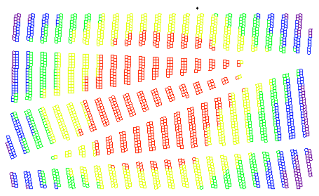
Parameter	Near-optimal	Traditional	Design
Skewness	20/40	20/40	
PL Size	10	10	
E1, E2, E3, E4	0.29, 0.91, 0.42, 0.81	0.916, 0.333, 0.833, 0.416	
IX, IY	0.94, 0.31	N/A	
Depot	0.622	0.625	
Angles	105, 98, 89, 112	90, 90, 90	
# Locations	2000	2006	
Aspect Ratio	0.69	0.8	
Width	412.26	367.84	
Depth	283.94	294.27	
Area	117059.09	108242.85	
H.Adjusters	0.33, 0.71, 0.76, 0.51	0.5, 0.5, 0.5	
V. Adjusters	0.74, 0.48, 0.06, 0.78	0.7, 0.5, 0.5	
Avg. Travel Cost	1013.25 (-1.8%)	1031.44	
Skewness	20/40	20/40	
PL Size	30	30	
E1, E2, E3, E4	0.95, 0.43, 0.86, 0.78	0.916, 0.333, 0.833, 0.416	
IX, IY	0.98, 0.27	N/A	
Depot	0.155	0.625	
Angles	85, 99, 89, 96, 109	90, 90, 90	
# Locations	2000	2025	

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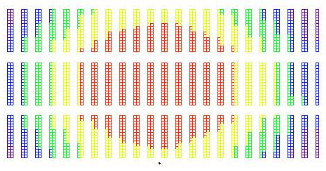
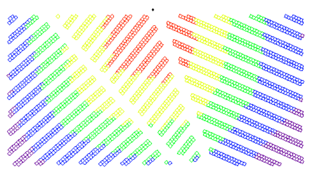
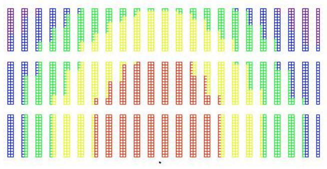
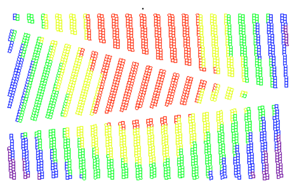
Table 6.6 – continued from previous page

Parameter	Near-optimal	Traditional	Design
Aspect Ratio	0.60	0.5	
Width	460.84	478.26	
Depth	274.90	239.13	
Area	126686.85	114366.11	
H.Adjusters	0.67, 0.53, 0.51, 0.33, 0.58	0.5, 0.5, 0.5	
V. Adjusters	0.53, 0.56, 0.24, 0.21, 0.76	0.5, 0.5, 0.5	
Avg. Travel Cost	1721.86 (-2.3%)	1762.81	

6.3.3 20/60 Demand Skewness Pattern

In the 20/60 demand skewness case, the best improvement over traditional is 3 percent which is achieved with pick list size 30. However, this design is resembling another traditional layout with the difference of a diagonal cross aisle between left and right end points of the other two cross aisles making a Z-shape. In other cases, we think the designs are significantly different than traditional designs. This design achieves 2 percent improvement over traditional counterpart which seems insignificant. For pick list size 2 and 3 cases, the optimizer found a traditional three block layout in the middle of the search process, but moved to a better 3-0-2 design in both cases.

Table 6.7: Optimization Results for 20/60 Demand Skewness

Parameter	Near-optimal	Traditional	Design
Skewness	20/60	20/60	
PL Size	2	2	
E1, E2, E3, E4	0.03, 0.13, 0.58, 0.59	0.916, 0.333, 0.833, 0.416	
IX, IY	N/A	N/A	
Depot	0.123	0.625	
Angles	156, 41, 51	90, 90, 90	
# Locations	2000	2025	
Aspect Ratio	0.54	0.5	
Width	447.39	478.26	
Depth	239.84	239.13	
Area	107300.71	114366.11	
H.Adjusters	0.78, 0.41, 0.56	0.5, 0.5, 0.5	
V. Adjusters	0.22, 0.26, 0.48	0.5, 0.5, 0.5	
Avg. Travel Cost	373.55 (-2.7%)	383.83	
Skewness	20/60		
PL Size	3	3	
E1, E2, E3, E4	0.83, N/A, N/A, 0.98	0.916, 0.333, 0.833, 0.416	
IX, IY	0.98, 0.51	N/A	
Depot	0.121	0.625	
Angles	94, 75	90, 90, 90	
# Locations	2000	2025	

Continued on next page

Table 6.7 – continued from previous page

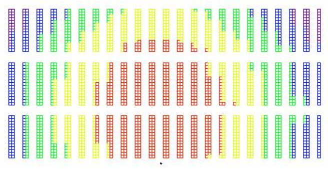
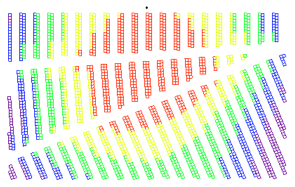
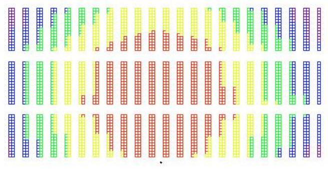
Parameter	Near-optimal	Traditional	Design
Aspect Ratio	0.62	0.5	
Width	423.70	478.26	
Depth	260.96	239.13	
Area	110570.16	114366.11	
H.Adjusters	0.18, 0.34	0.5, 0.5, 0.5	
V. Adjusters	0.45, 0.07	0.5, 0.5, 0.5	
Avg. Travel Cost	478.03 (-1.2%)	484.26	
Skewness	20/60	20/60	
PL Size	5	5	
E1, E2, E3, E4	0.78, N/A, 0.30, 0.92	0.916, 0.333, 0.833, 0.416	
IX, IY	N/A	N/A	
Depot	0.125	0.625	
Angles	91, 113, 95	90, 90, 90	
# Locations	2000	2025	
Aspect Ratio	0.62	0.5	
Width	421.44	478.26	
Depth	262.36	239.13	
Area	110570.16	114366.11	
H.Adjusters	0.87, 0.35, 0.17	0.5, 0.5, 0.5	
V. Adjusters	0.40, 0.17, 0.20	0.5, 0.5, 0.5	
Avg. Travel Cost	622.99 (-1.8%)	635.04	

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Table 6.7 – continued from previous page

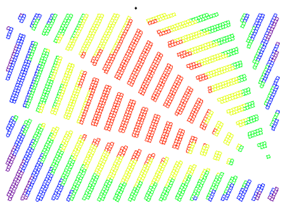
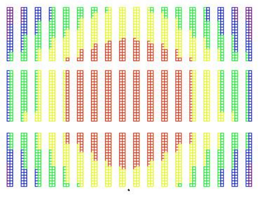
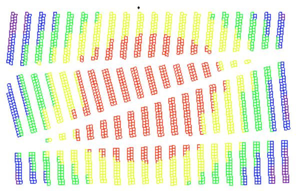
Parameter	Near-optimal	Traditional	Design
Skewness	20/60	20/60	
PL Size	10	10	
E1, E2, E3, E4	0.88, 0.99, 0.11, 0.20	0.916, 0.333, 0.833, 0.416	
IX, IY	0.996 0.89	N/A	
Depot	0.119	0.625	
Angles	66, 70, 64, 19	90, 90, 90	
# Locations	2000	2016	
Aspect Ratio	0.71	0.7	
Width	413.62	399.78	
Depth	293.69	279.85	
Area	121473.64	111876.30	
H.Adjusters	0.66, 0.37, 0.07, 0.87	0.7, 0.7, 0.7	
V. Adjusters	0.76, 0.89, 0.85, 0.23	0.3, 0.3, 0.5	
Avg. Travel Cost	876.71 (-2.0%)	894.47	
Skewness	20/60	20/60	
PL Size	30	30	
E1, E2, E3, E4	0.42, 0.91, 0.31, 0.81	0.916, 0.333, 0.833, 0.416	
IX, IY	N/A	N/A	
Depot	0.116	0.625	
Angles	85, 92, 95, 104	90, 90, 90	
# Locations	2000	2009	

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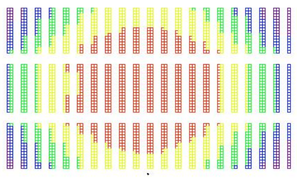
Table 6.7 – continued from previous page

Parameter	Near-optimal	Traditional	Design
Aspect Ratio	0.63	0.6	
Width	432.11	430.30	
Depth	270.90	258.18	
Area	117059.08	111095.51	
H.Adjusters	0.37, 0.70, 0.83, 0.96	0.1, 0.1, 0.1	
V. Adjusters	0.90, 0.32, 0.75, 0.77	0.9, 0.9, 0.5	
Avg. Travel Cost	1512.72 (-3.0%)	1559.15	

6.3.4 20/80 Demand Skewness Pattern

In the 20/80 demand skewness case, maximum improvement over traditional is 3.8 percent in pick list size 10. We can definitely see a pattern in the designs which create a diagonal aisle between 1/2 or 2/3 of the left or right cross aisles. The region closest to the depot location becomes 90 degrees with the region next to it is about 15-20 degrees rotated to one of the sides (75, 101, or 111 degrees). Farthest regions to the depot location are more similar to traditional one block layout.

Table 6.8: Optimization Results for 20/80 Demand Skewness

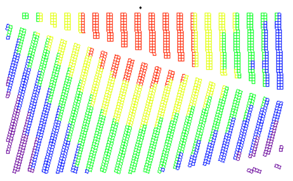
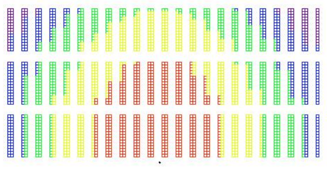
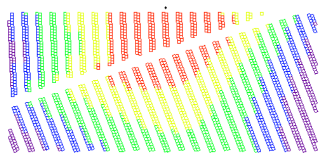
Parameter	Near-optimal	Traditional	Design
Skewness	20/80	20/80	
PL Size	2	2	
E1, E2, E3, E4	0.47, 0.38, 0.99, 0.61	0.916, 0.333, 0.833, 0.416	
IX, IY	N/A	N/A	
Depot	0.122	0.625	
Angles	92, 76, 159	90, 90, 90	
# Locations	2000	2025	
Aspect Ratio	0.60	0.5	
Width	419.45	478.26	
Depth	253.27	239.13	
Area	106232.52	114366.11	
H.Adjusters	0.19, 0.21, 0.51	0.5, 0.5, 0.5	
V. Adjusters	0.39, 0.12, 0.18	0.5, 0.5, 0.5	
Avg. Travel Cost	277.92 (-1.6%)	282.37	
Skewness	20/80	20/80	
PL Size	3	3	
E1, E2, E3, E4	0.83, 0.84, 0.24, 0.91	0.916, 0.333, 0.833, 0.416	
IX, IY	N/A	N/A	
Depot	0.128	0.625	
Angles	111, 93, 60	90, 90, 90	
# Locations	2000	2028	

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Table 6.8 – continued from previous page

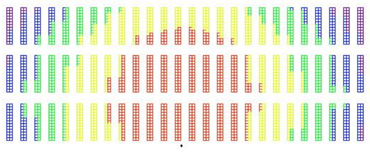
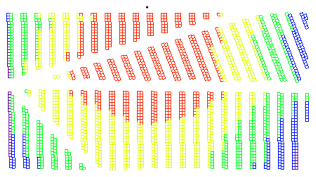
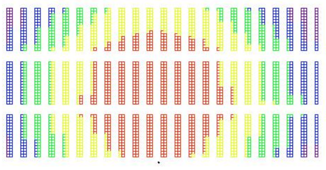
Parameter	Near-optimal	Traditional	Design
Aspect Ratio	0.48	0.4	
Width	465.81	542.52	
Depth	223.32	217.01	
Area	104023.80	117733.00	
H.Adjusters	0.52, 0.28, 0.80	0.5, 0.5, 0.5	
V. Adjusters	0.21, 0.17, 0.23	0.5, 0.5, 0.5	
Avg. Travel Cost	357.21 (-2.2%)	365.52	
Skewness	20/80	20/80	
PL Size	5	5	
E1, E2, E3, E4	0.37, N/A, 0.67, 0.21	0.916, 0.333, 0.833, 0.416	
IX, IY	0.04, 0.46	N/A	
Depot	0.120	0.625	
Angles	91, 111, 91	90, 90, 90	
# Locations	2000	2025	
Aspect Ratio	0.54	0.5	
Width	464.20	478.26	
Depth	249.17	239.13	
Area	115662.07	114366.11	
H.Adjusters	0.69, 0.82, 0.78	0.5, 0.5, 0.5	
V. Adjusters	0.33, 0.62, 0.09	0.5, 0.5, 0.5	
Avg. Travel Cost	476.68 (-3.6%)	494.65	

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Table 6.8 – continued from previous page

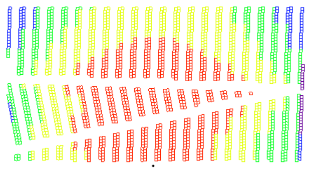
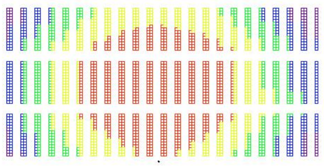
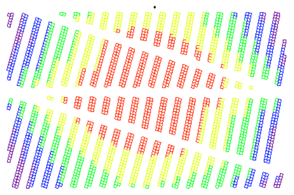
Parameter	Near-optimal	Traditional	Design
Skewness	20/80	20/80	
PL Size	10	10	
E1, E2, E3, E4	N/A, 0.88, N/A, 0.77	0.916, 0.333, 0.833, 0.416	
IX, IY	0.97, 0.53	N/A	
Depot	0.627	0.625	
Angles	88, 101	90, 90, 90	
# Locations	2000	2025	
Aspect Ratio	0.55	0.5	
Width	447.98	478.26	
Depth	247.56	239.13	
Area	110902.54	114366.11	
H.Adjusters	0.95, 0.76	0.5, 0.5, 0.5	
V. Adjusters	0.86, 0.75	0.5, 0.5, 0.5	
Avg. Travel Cost	686.08 (-3.8%)	713.14	
Skewness	20/80	20/80	
PL Size	30	30	
E1, E2, E3, E4	0.01, N/A, 0.37, 0.89	0.916, 0.333, 0.833, 0.416	
IX, IY	0.96, 0.89	N/A	
Depot	0.132	0.625	
Angles	75, 75, 79, 79	90, 90, 90	
# Locations	2000	2009	

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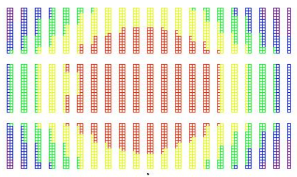
Table 6.8 – continued from previous page

Parameter	Near-optimal	Traditional	Design
Aspect Ratio	0.65	0.6	
Width	422.47	430.30	
Depth	275.70	258.18	
Area	116474.96	111095.51	
H.Adjusters	0.14, 0.18, 0.60, 0.93	0.1, 0.1, 0.1	
V. Adjusters	0.06, 0.16, 0.86, 0.82	0.9, 0.9, 0.5	
Avg. Travel Cost	1206.49 (-1.5%)	1225.32	

6.4 Summary

This research has shown that some non-traditional layouts are robust for several pick list sizes and demand skewnesses. To the author's knowledge, this is the first reported application of layout optimization for order picking operations, comparisons of parallel and distributed large batches of TSP solvers, and realistic path finding in warehouses.

In Chapter 3, parallel and distributed computing of well known heuristic and exact TSP solvers (LKH and Concorde) are proposed. They provide higher CPU utilization compared to other published parallel TSP solvers. Various applications of large batches of TSPs and a detailed graphic representation of methodology are given. We compared serial, parallel, and distributed solver implementations. These implementations are found to be easy to program and highly efficient compared to serial implementations. Our results indicate that parallel computing using hyper-threading for solving 150- and 200-city TSPs can increase the overall utilization of computer resources up to 25 percent compared to single thread computing. The resulting speed-up/physical core ratios are as much as ten times better than the parallel and concurrent version of the LKH heuristic using SPC^3 in the literature. For variable TSP sizes, a longest processing time first heuristic performs better than an equal distribution rule. We illustrated our approach with an application in the design of order picking warehouses.

In Chapter 4, we introduce the visibility graph as a new way of estimating the length of a route traveled by order pickers in a warehouse. Following aisle centers leads to longer travel distances when an order picker picks items within picking aisles which have angles other than 90 degrees between cross aisles. Details of the visibility graph implementation are given in the context of warehouse design. We present and compare results for traditional and fishbone layouts. Our results show that the visibility graph method changes the assessment of the fishbone layout as high as 9% compared to the traditional counterpart. Moreover, we analyze the affect of the visibility graph method for selecting the best layout that minimizes the average travel cost from three most common traditional layouts. By using the visibility graph method, we find a different layout in 13% of the cases than the previous distance estimation method used in the literature. We anticipate that visibility graph method can be readily integrated into warehouse layout optimization and comparison. Furthermore, distance estimation is an important aspect of retail layout design and other applications in facility layout design, and the visibility graph method will be also relevant for such applications.

In Chapter 5, we present a computational system to solve the warehouse aisle design problem. This system allows cross aisles and pick aisles to take on any angle. It produces near-optimal designs for nineteen design classes including design classes with an interior point.

In Chapter 6, we show that these new non-traditional designs can decrease average travel distance up to 4% compared to traditional counterparts. For small pick list sizes, because of the greater importance of the depot location to travel cost, layouts are designed with vertical cross aisles. As the pick list size increases, these cross aisles become horizontal allowing better access between storage locations. We anticipate our approach to be a starting point for more detailed research for warehouse layout optimization.

During our experiments we evaluated over 4.8 billion TSPs. These calculations took over two months period on four workstations. We believe this can be improved by eliminating LKH and implementing our own TSP solver inside our system. Also, graphical processing units (GPUs) can be used in complex calculations and the design search space can be enlarged by adding more external and internal nodes to potentially find better designs for order picking operations. We can investigate other routing and storage policies and embed these into the encoding scheme as variables so that we can choose a layout, a routing and a storage policy for a given environment.

For future research, we will implement a more efficient shortest path mapping method and compare computational times between two methods for different size warehouses. Moreover, we will develop paths and travel time models that consider not only linear paths but also non-linear paths around the corners of pick aisles.

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Appendices

Appendix A

Region Finding Algorithm

The region finding algorithm finds the regions inside a warehouse. Region edges are either exterior aisles or cross aisles. The number of regions changes based on how cross aisles are formed.

Algorithm 5 Region Finding Algorithm

```
$i \leftarrow 0$
$j \leftarrow 0$
$regions \leftarrow \emptyset$
while ($Count(edges) > 0$) do
    $current \leftarrow edges[i]$
    $start \leftarrow current$
    $regionedges \leftarrow \emptyset$
    $k \leftarrow 0$
    repeat
        $regionedges[k] := undirect(current)$
        $remove(current, edges)$
        $current := rightmost(current)$
        $k \leftarrow k + 1$
    until $current = start$
    $regions[j] := regionedges$
end while
$maxarea := 0$
$largestregion \leftarrow \emptyset$
for all $region$ in $regions$ do
    if $area(region) > maxarea$ then
        $maxarea := area(region)$
        $largestregion := region$
    end if
    $delete(largestregion)$
end for
```