Using Real Options Analytics to Improve the Capital Budgeting Process under Uncertainty

by

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Abstract

Capital allocation processes are complex and time consuming in large organizations because of the diverse choices from projects proposed by various departments within the organization. Invariably these projects reveal various types of uncertainties in the amounts of investment needed, timing over which such investments are made, and technical and regulatory risks as well as ultimate benefits accruing to the firm. In practice, companies utilize basic financial evaluation tools such as the expected Net Present Value (NPV), what-if scenarios, and risk simulation along with qualitative judgments to judiciously allocate the limited resources and capital available to them. However, these basic tools have shortcomings of ignoring investment flexibility embedded in the investment projects. Many attempts have been made to remedy these shortcomings, but these attempts have created more methodology confusions. On the other hand, these also have perpetuated the practice of using the expected value criterion under the assumption that things will work out as expected over the long-run. However, it is important to recognize that real investments are not single decisions without future flexibility, but rather a basket of interacting options driven by many different uncertainties. At a single project level, real options thinking has been proposed in aiding this important evaluation process. That is, investments into products, systems or technologies, have a changing economic value showing downside risk and upside potential over the project life. However, this option framework has not been explicitly considered in allocating the limited capital among competing risky projects. This research is to address these needs and attempt to improve capital budgeting processes by developing a decision criterion which

explicitly considers both the changing option values associated with each project and other financial analytics.

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List of Abbreviations

CAPM	Capital Asset Pricing Model
CDF	Cumulative Distribution Function
CFaR	Cash Flow at Risk
CNPVaR	Conditional Net Present Value at Risk
CVaR	Conditional Value at Risk
ES	Expected Shortfall
EVPI	Expected Value of Perfect Information
FNPV	Flexible Net Present Value
GP	Goal Programming
ILP	Integer Linear Programming
IRR	Internal Rate of Return
K-P	Kim-Park
MEL	Mean Excess Loss
MOPM	Multi-Objective Mathematical Programming
NFNPV	Net Flexible Net Present Value
NPV	Net Present Value
NPVaR	Net Present Value at Risk
PAR	Project at Risk
PV	Project Value / Present Value

QQ	Quantile-Quantile
ROP	Real Option Price
ROV	Real Option Value
VaR	Value at Risk
WACC	Weighted Average Cost of Capital

Chapter 1. Background and Problem Statements

Capital budgeting decisions are one of the most challenging decisions in business as they have to make decisions under uncertain market dynamics. In particular, many industries must address regulatory uncertainty, increasing competition, and limited capability to access capital markets. As the degree of investment decisions become more complex due to environmental issues, multinational exposures, and fluctuating currency risks, managers increasingly resort to more sophisticated analytical techniques. One of them is the real options decision framework where a decision maker does not commit a decision immediately but delays the decision until the decision maker has a better idea of what is going to happen to the investment outcome. Clearly capital budgeting process takes many different stages of deliberations, but this study is primarily to focus on the analytical techniques based on real options in creating the best project portfolio for funding. Capital-investment performance can have an enormous impact on an organization's value, and it can drive growth and increase overall returns on invested capital. The best companies use a clear capital-allocation strategy to build winning portfolios. Also, the practice of allocating the limited funds in this fashion is known to bring the most capital preservation to the business over the planning horizon. This research reinforces the importance of the real options decision framework in capital budgeting.

1.1 Statement of the Problem

1.1.1 Decision Process

Traditionally, typical capital expenditure decisions are based on three fundamental questions when evaluating individual projects: (1) Is this project contributing something toward the goal of the organization, and does the project generate sufficient cash flows to meet the required minimum returns? (2) Do we have the sufficient funds to invest in this project now? (3) Are there competing alternatives worth considering? Then, the next consideration is to select the most attractive projects given the capital position and budget. This is a typical once-and-for-all type single-period investment decision.

What we are interested in concerning this research is, however, to consider a decision process where a firm makes capital investment decisions on a periodic basis over a long period of time. This is a closer representation of a capital budgeting process for on-going firms. In each decision period, a set of investment proposals is submitted for funding consideration. Using some decision criteria, the firm screens out the projects worthy of funding and then selects the best mix of projects within the funds budgeted for that period. For the next decision period, cash flows generated from all the projects funded in the prior periods will be available for future funding and may be added on to the funds budgeted for that period. Then a new set of investment proposals would be considered for funding out of this newly created budget. The process will repeat until the predetermined horizon time. This whole repeating process is known as a multi-stage capital budgeting decision problem.

In each stage, the firm must decide what action to take next in order to optimize overall capital performance – either continue or abandon ongoing projects in addition to selecting a new set of projects submitted for that period. Ultimately, managers are interested in creating maximum value to their firms by achieving higher lifecycle returns through the capital projects funded over the horizon. Research by McKinsey, across a range of industries, has shown that more active resource reallocation such as described above correlates with higher shareholder returns [1].

1.1.2 Uncertainty about Future Investment Opportunities

With the multi-stage capital budgeting framework, one of the important considerations is the amount of information the decision maker can obtain about the future. This study assumes that the decision maker has some expectation as to future investment opportunities without requiring specific knowledge about particular investment opportunities. This view describes some middle ground concerning the availability of information regarding the outcomes of future investments [2]. One of the more critical issues is to react quickly when opportunities come up during the course of a year. Two factors are critical – availability of funds and investment review or capital budgeting cycle. First, budget issues may be met by creating a reserve fund by reinvesting some or all the cash flows generated from the ongoing projects in addition to the budget allocated to that funding period. Or if allowed, the firm could borrow money to fund the critical projects. Second, by shortening the investment review cycles (let say from a year to semiannual or even quarter), the firm would be in a better position to scale up production in a unit that suddenly takes off, to launch a marketing campaign to meet an unexpected wave of customer demand, or even to acquire a facility that comes abruptly onto the market [3]. This is known as agile budgeting. This research is to consider these types of avenues in crating the project portfolios.

1.1.3 Uncertainty about Future Cash Flows

In this research, we will assume that all the investment projects proposed for funding consideration will be described by some sort of probabilistic cash flows with estimated means and variances in each period. One way to reflect the degree of risk involved in each project is either to vary the degree of periodic variance of cash flow or to assume the different type of probability distributions. When the future is truly uncertain, a complete stochastic description of the uncertainty may not be feasible. However, rarely do managers know absolutely nothing of the future, even in the most uncertain environments. In that situation, a uniform distribution might be suitable, as we may only have information with less detailed structure, such as boundaries on the magnitude of the uncertain quantities. This modeling flexibility will allow us to take advantage of the opportunities that higher levels of uncertainty provide.

1.1.4 Investment Flexibility over Time

One of the primary interests in considering the real options decision framework is to incorporate uncertainty resolution by various option strategies. For example, by deferring an investment decision, we may get to know more about the project as the market dynamics change over time. That allows us to make a better decision as we get to revise cash flows projected at the time the investment is first proposed. The missing ingredient in many capital budgeting decisions is a careful consideration of this value of flexibility. Companies can respond more quickly to changing conditions and outperform competitors - the greater the level of uncertainty, the greater the value of flexibility. Real option theory maintains that flexibility becomes more important when volatility is more intense. To capture this value and gain the best position for responding to future economic changes, all companies should integrate flexibility into their capital budgeting decisions. Then clearly, we are interested in knowing how uncertainty about the project's future changes over time. In a sequential decision-making process where future budgets are influenced by current decisions, this information about the proposal's level of uncertainty becomes extremely valuable to the decision maker. Therefore, the idea is, "how can this time-phased information regarding uncertainty about the cash flows be captured by real options and utilized in the investment decision-making process?"

1.2 Scope of the Research

The paramount problem that confronts the decision maker is determining how the available capital should be allocated to the proposed risky projects. Therefore, the main purpose of this research is to develop a methodology for incorporating real option value into the improvement of the capital allocation decision problem. Another purpose of this research is to investigate the way of having an agile budget in each period through funds generated from cash flows of the projects undertaken during prior budget periods. By having this budget flexibility, the firm would be able to undertake a new investment, expand or scale up the projects as it deems necessary. Of particular concern is the development of a decision criterion to be used in making investment decisions where current decisions are influenced by future investment opportunities, and future budgets will be influenced by current decisions. These goals are accomplished in three ways.

- We develop and incorporate a principle concerning the measure of investment flexibility based on projected cash flows into a decision criterion, which can be used in making investment decisions on a periodic basis. We name this decision criterion as the Kim-Park method, or simply the K-P criterion.
- We develop a simulation model to compare the long-term effectiveness of the K-P criterion along with other traditional decision criteria by applying them to the identical groups of projects in each decision period. We need to utilize a computer simulation to investigate a large class of investment settings, as they were not normally manageable by available analytical techniques.
- We generate and collect the data describing important features of this type of periodic decision process through the simulation model. This allows us to provide a better understanding of particular characteristics of the model's performance. This will also provide ample economic interpretations of the simulation results.

1.3 Objectives of this Research

The goal of this research is to develop a multi-stage capital budgeting decision model to allocate limited resources to competing projects under uncertainty in order to maximize the total value of the firm. The research framework will be based on optimization models and real options. Four objectives are outlined toward this research goal.

1.3.1 Objective 1: Develop a practical real option valuation method with a loss function

Despite the fact that the NPV method is known to underestimate the project value by ignoring the flexibility, it is still widely used in capital budgeting. Real option is a risk-hedging tool by evaluating a project's embedded uncertainty associated with changes in future investment opportunities. Real options analysis has been applied to a broad range of investment decision problems with various types of real options strategies from manufacturing industries to service sectors. However, many companies are reluctant to adopt real options analysis in their decision process. There are two main reasons why the NPV method is still favorable for non-financial industries. The first reason is that the NPV method has been used for a long time in their business and is difficult to change; the second reason is that the real options approach is too complicated to understand mathematically and, there are also some practical difficulties of applying the financial option methodology to real projects. One top of conceptual and practical difficulties, estimating project volatility is always a challenging issue to most decision makers.

Real option valuation methodology is based on financial option valuation, which assumes that value of an underlying asset is lognormally distributed because the asset, such as the stock price, never becomes negative. However, a real project value estimated by aggregating discounted future cash flows can be negative. Then our question is, "why do we use the Black-Scholes (B-S) model to price real option value?" Moreover, in highly uncertain circumstances, the investment flexibility tends to change over time. Therefore, it is important to re-evaluate the changing project value when circumstances change. We need a more practical real option valuation approach for non-financial business, so that we can react to market changes in a timely manner when new information arrives.

1.3.2 Objective 2: Determine a relationship between CNPVaR and real options pricing

An important part of capital budgeting is to calibrate the amount of risk and uncertainty embedded in a project. Generally, we begin analyzing project risk by determining the inherent uncertainty in a project's cash flow. Once we obtain the net present value (NPV) distribution by aggregating future cash flows over the investment life, we may be able to estimate the range of possible losses, which may exceed an investor's acceptable loss. To determine whether or not this loss is within the acceptable range, we may bring the concept of the conditional value at risk (CVaR). It is basically the expected loss in the range of possible losses on an investment; the range of possible losses are also bound to change over a given time period at a specified degree of confidence. If a typical business is willing to accept the investment with the estimated possible losses, we may view this amount (CVaR) as the firm's risk tolerance associated with the project. To measure the risk of a project, we propose the modified term the Conditional Net Present Value at Risk (CNPVaR), which is basically the CVaR concept on the distribution of project value at the decision point.

It is important to recognize that the uncertainty not only brings possible losses but also potential surprise benefits. Traditionally, due to investors' risk aversion, much literature has been focused on avoiding the potential downside risk at the expense of ignoring the upside potential gains. Real investments are not a single decision without future flexibility but rather a set of opportunities to change initial decisions even for on-going projects when circumstances change. Due to various uncertainties, the economic value of the investment changes over the project life. Real options analysis is the tool that considers both consequences of uncertainty: the downside risk and the upside potential benefits. Thus, investors may be interested in hedging this expected loss (CNPVaR) by either delaying the project or using other features of various real options. Then, the question becomes, “what is the right price to pay for hedging this risk?” “is there a way to price this option value based on the CNPVaR?” Our objective is to address these before developing a formal decision criterion for risky project evaluation.

1.3.3 Objective 3: Develop a capital budgeting model based on the real option approach

At a single project level, we may consider three critical elements to determine the merit of project worthiness. They are profitability, variability and flexibility. If we measure the profitability by the expected value of the project’s NPV distribution, the variability by the variance of the NPV distribution, and the flexibility by the option value, we may be able to come up with an index that integrates these three elements. Then our objective is to utilize this index in allocating the limited capital among competing risky projects in the multi-stage capital budgeting process. This research aims to address these needs and attempts to maximize the long-term value of the firm by explicitly considering the real option value associated with each project and other financial constraints.

1.3.4 Objective 4: Develop a simulation model to test the effectiveness of the K-P criterion

Once we have developed a decision criterion to allocate a limited capital, we need to test how effective the proposed decision criterion is. There is no way of predicting future investment opportunities in any precise fashion. One alternative is to simulate the future investment environment by creating various plausible investment scenarios. The simulation process can be divided into three phases:

- Phase 1: Generate a set of investment proposals in each period by considering a different mix of risks from low to high.
- Phase 2: Apply the three decision criteria to the set of investment proposals submitted in each period. For those selected for funding by each criterion, the realizations of cash flows will be of source of additional funds available for future budget periods. In addition, given these realizations of the proposal's cash flows, whether or not any options can be exercised in each period, and that information will be incorporated in the next decision period.
- Phase 3: Once Phases 1 and 2 are complete, the realizations of cash flows of all the projects generated during the study period are preserved. Given these realizations of cash flows over the entire study period, we will be able to determine the effectiveness of the K-P criterion as a value maximization tool in capital budgeting.

1.4 Plan of Study

Chapter 2 provides a review of the literature related to the issues raised in the proposed research problems. The literature review indicates that there is a critical need to systematically incorporate real option framework into capital budgeting process.

Chapter 3 investigates a measure of option value based on project cash flows, which departs from the traditional return-based measure, namely the Black-Scholes formula. This measure uses the loss function concept commonly adopted in applied statistical decision theory. By computing the option value based on the project cash flow, it eliminates the need of determining the project volatility which is one of the most challenging tasks in real options analysis.

Chapter 4 examines what the correct price to pay is to retain an option. The option values determined through loss functions only serve the maximum price to pay. By borrowing the concept of conditional value at risk (CVaR), we first develop the net present value at risk. By knowing this value, we will be able to determine an appropriate price to pay for option. Also, we develop a decision criterion which integrates three critical elements associated with a risky investment: profitability, variability, and flexibility. Here for each project, we determine profitability by the expected value of the NPV distribution, variability by the variance of the NPV distribution, and flexibility by the real option value which is determined in Chapter 3. We call this new measure the K-P index. Then, the K-P criterion is simply the decision criterion to rank the risky investment projects by using the K-P index. To test the effectiveness of the K-P criterion as a decision criterion, we will examine two traditional criteria, namely the expected NPV criterion and Mean-CVaR criterion.

Chapter 5 describes the various features and assumptions of the simulation model which is used to test the effectiveness of three decision criteria – expected NPV criterion, mean-CVaR criterion, and the K-P criterion – in selecting (or creating) best investment portfolio in each decision period for multi-stage capital budgeting problems. To do this, we develop simulation models for each decision criterion and use a MATLAB to obtain the solution to the multi-stage decision problems.

Chapter 6 presents the process of simulating the investment decision process, by describing the input data generation methods and creating various plausible investment scenarios (or investment settings) where we apply each decision criterion to select the proposals in each period.

To determine the effectiveness of each decision criterion, we use the terminal wealth, the cash accumulated at the beginning of horizon (or study period), here the terminal wealth will be also a random variable as each simulation run generates one terminal wealth. By repeating the iterations, we obtain a terminal wealth distribution associated with each decision criterion. Then we are left to compare these terminal wealth distributions by the stochastic dominance rules. To examine the effects of critical input parameters on the performance of each decision criterion, we conduct a series of statistical tests and sensitivity analyses. Detailed economic interpretations of simulation results are also provided.

Chapter 7 contains summaries, conclusions, and recommendations for further research. A complete list of references is also presented in Appendix.

Chapter 2. Literature Review

The capital budgeting decision, which is the process of evaluating and allocating limited resources to competing projects, is one of the most important decisions in business. In order to increase the value of the firm, the firm needs to generate cash flows through various investment activities. Managing capital investment wisely means better cash flow, faster growth, and competitive advantage in market place. Capital investment performance can have an enormous impact on an organization's value, and it can drive growth and increase the overall return on invested capital [4][5]. As a result, the numerous efforts toward finding and applying sophisticated capital budgeting approaches have been made [6][7][8][9][10].

2.1 Capital Budgeting Decision by Mathematical Programming

In early mid-50's, Gunther may be one of early adopters of a simple linear programming model to solve a capital budgeting problem [8]. Then it followed by Lorie and Savage who showed the difference between maximizing the present worth and ranking projects by rate of return in selecting the best projects [11]. Markowitz and Manne [12] formulated a more sophisticated linear programming model to allocate limited sources and provided various economic interpretations on the outputs. Several researchers adopted a single-stage linear programming model to solve various forms of capital budgeting problems [13][14][15], and then many authors extended the linear programming model to solve multi-period capital budgeting problems [16][17][18]. However, Weingartner [19][20][21] is one of the first researchers who examined the shortcomings of the Lorie and Savage's pure capital budgeting model – maximization of net present value as an objective function with budget constraints, pointing out the conceptual difficulty in determining an appropriate discount rate to use in NPV calculation to go with the objective function under capital rationing environment. The issue of determining the correct discount rate is governed by what kinds of projects are included in the budget. But to determine the projects to be included in the budget we need to know the discount rate. This is so called a chicken and egg problem. One approach he advocated is the horizon model which is basically a multi-period mixed integer programming model that allows reinvestment of cash receipts from the projects selected by lending unused budget to outside, and borrowing the needed capital so that all good projects will be funded in full. The model's ultimate objective is to maximize the horizon value at a future time. Another contribution of the horizon model formulation to the body of capital budgeting literature is that the way of his modeling avoids the need of discount rate. However, his horizon model is still based on the assumptions that all required parameters are known with certainty. Therefore, many attempts have been made to consider capital allocation problems under risky or uncertain business environments, and this research also addresses the issue of capital budgeting decisions under uncertainty.

The purpose of this research is to develop a multi-stage capital budgeting decision model under uncertainty by incorporating the investment flexibility through real options framework. To develop such an allocation model, our review of literature will be composed of three parts: First, overview of previous works on capital budgeting decision problems which consider explicitly risk and uncertainty. Second, we examine the various risk measures that can be incorporated into a practical capital budgeting decision models. Third, we review the capital budgeting models which incorporate the investment flexibility through the real options approach.

2.2 Overview of the Capital Budgeting Problems under Uncertainty

With the development of mathematical programming techniques, capital budgeting models become more sophisticated to deal with many complex decision variables as well as resource constraints. In particular, these models attempt to incorporate numerous stochastic approaches to address risk and uncertainty common in business investment environment. There are several well-known tools for capital budgeting such as the capital asset pricing model (CAPM) [22] and the risk-adjusted discount rate [23]. However, these approaches mainly focus on asset valuations under risk through finding an appropriate discount rate instead of focusing on the capital allocation decision under budget limits.

In terms of capital allocation decision where there are multiple competing goals to achieve, multi-objective mathematical programming techniques have been proposed ranging from deterministic models to probabilistic models. First, the goal programming (GP) is one of them and has been applied to capital allocation decisions with multiple goals to satisfy such as minimizing the cost of production, increasing productivity, and allocating available resources. Hawkins and Adams [24] demonstrated the application of the GP to a capital budgeting problem based on the linear programming and integer programming. Keown and Martin [15], Keown and Taylor [25], De et al. [26] formulated a GP to select a set of investments with budget constraints. Ahern and Anadrajah [27] developed a weighted integer GP model for new railway projects requiring the different capital investment levels. Tang and Chang [28] explored to formulate a multiple criteria decision-making model utilizing the goal programming and fuzzy analytics. Bi-Level linear programming has been also attempted to deal with multiple goals by pre-prioritizing the goals in two levels [29][30], but all these models did not consider investment flexibility in any explicit manner.

The fuzzy set theory also introduced in mathematics to deal with incomplete or imprecise information. The fuzzy set theory has been proposed as one of the capital budgeting decision methods since late eighties [31][32]. The fuzzy discounted cash flow and fuzzy present value techniques were studied by several researchers [33][34], along with ranking investment proposals based on a possibility distribution which is defined by fuzzy numbers [35]. Then numerous papers addressing the fuzzy capital budgeting techniques have been published [36][37][38][39][40][41]. However, all these models are more or less some sort of extensions of fuzzy mathematics to capital budgeting problems, but they do not much deal with multi-stage capital budgeting problems under uncertainty.

A more quantitative approach to deal with uncertainty, multi-stage stochastic programming models were also attempted. Beraldi et al. introduced a customized branch-and-bound method to solve their stochastic programming model [42], and Rafiee et al. studied multi-period project selection along with scheduling in an uncertain environment [43]. However, these papers proposed a scenario tree model to represent the uncertain progression, so it is very difficult to obtain the analytical solutions to these models in the presence of a larger number of stochastic decision variables.

One of the alternative techniques for stochastic programming is the robust optimization, which uses the ranges of possibilities instead of the probability distributions of the parameters [44]. Goldfarb and Iyengar [45] formulated the robust portfolio selection problems based on the mean and variance model. Kachani and Langella [46] proposed the robust formulation for a capital budgeting problem based on a range of possible values associated with uncertain cash flows. Albadi and Koosha [47] studied on a marketing budgeting allocation model considering several parameters such as size of the market segments, marginal contribution, discount rate, etc. The main difference between the robust optimization and the stochastic programming is that the robust optimization does not require any probability distributions of the parameters.

Myers [48] is the first author who introduced the concept of real options by recognizing the additional value from having investment flexibility, similar to the financial option valuation. Ever since, it has been widely advocated as one of the sophisticated real asset valuation techniques to quantify uncertain investment opportunities in dollars. One of the main reasons why the real options decision framework is getting attention in business communities is to accept the investment risk as is (rather than avoiding it), then focus more on how to manage or hedge the downside risk while retaining the upside potential by paying some form of option premium. The option premium is determined by valuing the managerial flexibility associated with the uncertainty. Even though many researchers have extended the concept of real options in making capital budgeting decisions under uncertain business environments [49][50][51][52][53][54][55][56][57], but most of these publications focus on asset valuations for single-period capital allocation problems, rather than multi-stage periodic decisions.

2.3 Risk Measurements

2.3.1 Utility Theory and Mean – Variance Model

Risk analysis has been extensively studied as one of the most important considerations in any financial investments. In order to consider uncertainty in capital budgeting decision, all risk measurements are based on the utility theory. Von Neumann and Morgenstern [58] presented that when there are choices under uncertainty, an individual makes a decision to maximize the expected utility. Friedman and Savage [59] discussed the different degrees of risk in the alternatives and tested the hypothesis proposed by Von Neumann and Morgenstern utility theory. Utility theory is both a *prescriptive* and a *descriptive* approach to decision making. The theory tells us how individuals and corporations *should* make decisions, as well as predicting how they *do* make decisions [60]. It is just like the Khun-Tucker conditions in optimization theory – whether or not we reach an optimal solution, any mathematical programming algorithm must satisfy the Khun-Tucker conditions. Likewise, the utility theory will tell us whether or not we reach a rational decision. The utility theory has an important role in the mean-variance model for analyzing risky assets [61] and many other financial decision makings.

A portfolio selection model proposed by Markowitz [62] was originally developed for asset allocation in financial investments such as securities and bonds. To select an optimal portfolio, it should reflect an investor's risk preference based on the utility theory. The mean – variance model seeks a trade-off between the return and the risk, and this model has been extensively implicated for capital budgeting decision problems. The development of portfolio theory had led to the capital market theory or the capital asset pricing model to allocate shares of their investment amounts to risky financial assets [22][63]. In earlier studies which adapted the portfolio selection model into capital budgeting decision, the expected present value and variance to the firm were considered to make an optimal combination of investment proposals [64][65]. In parallel, many authors extended the portfolio model to consider the multi-period investment decisions [66][67]. However, all these portfolio-theory-based models are limited to project selection solely based on the trade-off between risk and return, without considering any investment flexibility.

2.3.2 Downside Risk Measurements

In more recently, the concept of the mean – risk model in capital budgeting problem has been extended along with development of downside risk management. In regard to more prudent risk management in capital budgeting problem, decision makers must consider the direction of an investment's movement. For most investors, unexpected gains above the mean value are no cause of concern – instead risk is about the odds of losing money. One of the most popular risk measures to address these investors' concern is the Value at Risk (VaR). This was initially developed by JP Morgan [68] to measure the potential loss in value of a risky asset or portfolio over a target horizon time (a holding period) within a given confidence level. The VaR describes the maximum expected loss over a given period of time and with a confidence level which is defined by the certain quantile of the return probability distribution of an investment portfolio. When all portfolios have the same expected return, minimizing the VaR of the portfolio is conceptually equivalent to the Markowitz's mean – variance model [69]. While the VaR measure has been accepted as a standard measure of risk in financial industry, it has been also criticized and proven that the VaR measure does not satisfy the requirements to be a coherent risk measure [70][71][72][73]. Consequently, Rockafella and Uryasev [74] proposed an alternative risk measure known as the conditional value at risk (CVaR): The CVaR simply describes the expected loss beyond the VaR with a given confidence level, and it is also known as the tail conditional expectation, the mean excess loss or the expected shortfall [75][76].

The CVaR has been widely accepted as a return based risk measure in the financial industry and, to some extent in the non-financial industry as well. However, non-financial companies needed to adjust the return based concept of CVaR to a cash flow based measure, referred to Cash Flow at Risk (CFaR). The CFaR measures the degree of risk associated with a cash flow shortfall in any operational period [77][78][79][80]. Several McKinsey reports also found many real world applications of the CFaR [81][82][83]. While the CFaR focuses on each operational period, the Net Present Value at Risk (NPVaR) measures the overall risks associated with undertaking an investment project. Ye and Tiong [84] proposed the NPVaR approach by combining the weighted average cost of capital and the return – risk model in the investment evaluation of financed infrastructure projects. On the other hand, Lu et al. [85] suggested a Project at Risk (RAR) model as a decision-support criterion, which is the concept similar to the CFaR and the NPVaR, to quantify the possible losses. Furthermore, Pergler and Rasmussen [86] illustrated the NPVaR analysis to analyze a company's current performance risk in the capital-intensive industries. Since many businesses or investors are interested in how to manage the downside risk, our research will address this hedging strategy as a way to manage the downside risk.

2.4 Capital Budgeting Problems to consider Real Options

2.4.1 Real Option Approach into Investment Decision Making

The concept of real options is originally from valuing financial options in the seventies. Black and Scholes [87], and Merton [88] presented a mathematical option-pricing formula to determine a fair price or theoretical value for the financial options. Then the financial option theory provided substantial insight into capital investment decision making: Additional value arising from flexibility could be captured for valuing other assets as well. In the early stage of the real option analysis, many researchers have tried to value real assets by borrowing the concept of financial options. As mentioned earlier, Myers [48] first used the term "real options" to value an investment opportunity, then later the more complete form of real option framework was developed by McDonald and Seigel [89]. Dixit and Pindyck [50] first came up with an analogy between "defer an opportunity to invest" and the financial call option. Trigeorgis [90][91][51] discussed managerial flexibility embedded in investment opportunities and labeled the five types of real options – 1) defer, 2) abandonment, 3) contract, 4) expand, and 5) switch.

Luehrman [92][93] viewed the real option strategy as a complement to the traditional discounted cash flow and illustrated how to treat a series of business strategies in the real option framework. Lander and Pinches [94] offered reasons why corporate managers and practitioners are reluctant to use a real options model in their project valuation or capital budgeting decision, and they suggested ways to improve some of their misconceptions. Amram and Kulatilaka [95][96] addressed how to frame the real options thinking according to different types of managers to facilitate their understanding of decision making process. Copeland and Antikarov [52] provided many real case examples to demonstrate how real options analyses enrich asset valuations in business for the practitioners. Nembhard et al. [97] developed a real options model in manufacturing system to improve operational decision making. Park and Herath [98] explained more clear idea on how to apply the real options approach to various engineering economic decision problems under uncertainty and provided various modeling applications with numerical examples.

Many creative ways of valuing real assets through real options framework have expanded to various industries. One of the early modeling applications is found in research and development (R&D) areas to decide whether or not it is worth taking R&D effort now for possible commercialization of the product in the future [99][100][101][102][103][104][105][106][107][108]. The reason why the real option framework has a great deal of attentions in the R&D is that it has a sequence of investment strategies which can depend on future investment opportunities.

This leads to our research question – real options decision framework as a way of valuing future investment opportunities and using them in resource allocations under uncertainty.

More recently, Triantis and Borison [109] reported that certain industries related with oil and gas, mining, and life sciences have more interests in incorporating real options in their investment decisions. Stout et al. [110] illustrated how to incorporate real options and the traditional discount cash flow into the capital budgeting process with a case study of a rental car company. Fernandes et al. [111] discussed the real option approach to deal with a high level of competitiveness and consequent market uncertainty in energy sector investment. Gazheil and Bergh [112] summarized more than 50 articles of the real option applications in the renewable energy sector from 2003 to 2016. However, no article addressed or developed a reasonable technique to extend the real option framework beyond a single asset valuation.

One of the serious attempts to use real options in capital budgeting under uncertainty is by Meier et al. [113]. The authors developed an integer programming model to create an optimal portfolio using real option values. This scenario-based model becomes mathematically intractable because the possibilities of feasible portfolios grows extremely vast.

2.4.2 Real Option Valuation

As mentioned earlier, the origin of real option valuation is from the financial option pricing: the Black-Scholes formula which assumes the underlying asset's price movement can be described by the geometric Brownian motion [50]. However, there are many limitations in valuing real assets through the Black-Scholes formula: It requires a high level of mathematical knowledge, and needs to accept the many underlying assumptions of the financial option pricing [96][114]. On top of these concerns, unlike financial options, estimating a project volatility is another challenging issue in real options.

Another approach to value real option is by the conventional decision tree analysis [52], [115]. Decision tree analysis represents the investment alternatives, and it uses decision nodes to reflect managerial flexibility [116]. However, as we add more decision options or investment opportunities in the tree, the analysis becomes quickly getting out of hand – just impractical to use. More practical approach to value real options may be using the binomial lattice framework, and it has been proven that with a large number of nodes, the option value converges to the same value obtained by the Black-Scholes formula [117][118][119][120]. The binomial lattice, eventually, has the same outcome as the Black-Scholes formula because many underlying parameters defining the binomial lattice is based on the geometric Brownian motion.

Most real options analyses assume a constant volatility, but Herath and Park [121] developed a modeling scheme to consider some form of changing volatility over time. Furthermore, Herath and Park [122] considered a Bayesian approach to real option valuations and presented the relationship between the opportunity loss concept and the expected value of perfect information (EVPI) in the real option valuation.

2.5 Summary

Real Option analysis has received a great deal of attention from both academics and industries because its decision framework resembles many real-world decision environments. However, due to mathematical sophistication required in understanding the modeling process, many companies have difficulties in adopting the valuation method in their investment decisions. According to recent surveys for the U.S. in 2007 and Canadian companies in 2010, only 14.3% and 16.8% of survey respondents answered that they use real options in making capital budgeting decisions, respectively [123][124]. Moreover, these surveys showed that the lack of top managers' support and mathematical complexity required in understanding the real option is the dominant reason why it is slow to adopt the method in capital budgeting decisions.

Although there have been significant developments of capital investment decision models in many different directions and numerous attempts have been made to integrate the real option analysis into the traditional capital budgeting decision, the literature review indicates that there is a critical need to develop a capital budgeting model with a practical real option valuation in dynamic investment environment. This leads to our research goal – real options decision framework as a way of valuing future investment opportunities and using them in resource allocations under uncertainty.

Chapter 3. Determination of Real Options Value based on Loss Function Approach

Before developing a new decision criterion, we need to examine the current methods of calculating real option values and present the practical issues associated with the current valuation. Real option value calculation based on either the binomial lattice approach or the Black and Scholes method requires a lognormality of project value distribution and volatility of the percent change in project values. This approach poses some conceptual problems as the project value, which is the aggregate sum of discounted cash flows, can take a negative value. Since the periodic project cash flow contains many elements of random variables such as unit price, demand, variable cost, and others, the resulting cash flow distribution could be normally distributed. Furthermore, the sum of the discounted periodic cash flows will also be normally distributed. Therefore, we may need an alternative real option valuation where the project value is normally distributed, which is different from a lognormal distribution assumption used in financial options. In fact, the proposed methodology based on the loss function approach in this chapter is not limited to a normal distribution type, it can be of any distribution.

3.1 Real Options Thinking

Traditional measures of investment worth such as net present value (NPV) and internal rate of return (IRR) are based on the most likely values of project cash flows with the assumption of an immediate project commitment and no changes in investment direction. Under these circumstances, the decision maker, typically, estimates the most likely values based on the best information available at the time of investment decision. Clearly, it is much straightforward in terms of analysis, but these measures do not consider any potential value associated with investment flexibility offered by changing course of actions that could occur over the project life. For example, managers and executives are in a position to reassess the viability of project in progress after receiving new pieces of market information – to continue or phase out, expand or scale back the operation. In other words, we may keep the project when things are moving in the right direction but can get out of the project when things are going in other direction. If any project under consideration comes with these types of investment flexibility, managers can limit the financial risk to the level they can manage. Clearly, the traditional approaches without considering these types of investment flexibility systematically undervalue projects having such embedded opportunities. How valuable is it to have this investment flexibility? And how much are you willing to pay for this flexibility when you consider the investment? The approach proposed in this chapter is to address these questions and others.

3.2 The Real Option Valuation based on the Black-Scholes Model

In this section, we will examine the current option valuation model based on the geometric Brownian motion, namely, the Black and Scholes model, and we will discuss the conceptual issues using the Black and Scholes model in valuing real options.

3.2.1 Financial Options and Black-Scholes Formula

Real options concept which was first proposed by Myers (1977) is largely based on financial options [48]; you have the right but not the obligation to take an action on real assets. Even though the Black and Scholes formula was originally developed to price financial options

[87], it has been also widely utilized in pricing real options as well. The formulation of call options proposed by Black and Scholes can be written as

$$c = S_0 N(d_1) - K e^{-rt} N(d_2) \tag{3-1}$$

where

$$d_1 = \frac{\ln\left({S_0}/{K}\right) + \left(r + {\sigma^2}/{2}\right)t}{\sigma\sqrt{t}},\ d_2 = d_1 - \sigma\sqrt{t} \tag{3-2}$$

$N(x)$ denotes the standard normal cumulative distribution function.

Now in order to use the Black and Scholes formula to value real options, we need to identify the corresponding one to one parameter matching. The underlying asset in real options, which is the project itself, is equivalent to the financial assets such as stock in financial option. The required investment cost is viewed as the strike or exercise price of stock in financial option. The option life in real asset or the time duration offered to make an investment decision is viewed as the maturity of financial option. The volatility of project value is viewed as the volatility of return of stock. Table 3-1 summarizes five key parameters required in pricing option for both financial option and real option.

Table 3-1 Summary of parameters for options

Parameters	Financial option	Real option
Initial value of underlying asset	S_0	$PV_0 = E[PV_t] \cdot e^{-rt}$
Strike price / Investment cost	K	I
Discount rate	r	r
Horizon period	t	t
Volatility of underlying asset	σ	σ

Using the Black and Scholes formula with the real option parameters defined in Table 3-1, the real option value (ROV) – the call-option-type real option such as a defer option or a growth option – can be written as

$$ROV\_call = PV_0 N(d_1) - I \cdot e^{-rt} N(d_2) \quad (3\text{-}3)$$

where

$$\boldsymbol{d_1} = \frac{\ln\left(\frac{PV_0}{I}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}, \; \boldsymbol{d_2} = \boldsymbol{d_1} - \boldsymbol{\sigma\sqrt{t}} \quad (3\text{-}4)$$

For a put-option-type real option, we can use the similar analogies to find the option value as follows:

$$ROV\_put = -I \cdot e^{-rt} N(-d_2) - PV_0 N(-d_1) \quad (3\text{-}5)$$

where d_1 and d_2 are as defined in Equation (3-3).

3.2.2 Estimating Project Volatility

Volatility is one of the important variables in valuation of options. In the financial options analysis, volatility is commonly estimated by tracking the historical data of stock price movement. For real assets, unlike a stock, it is not likely to have any historical data available to estimate the annualized changes in project value. There have been several attempts to estimate the project volatility based on project cash flows [52]. Han and Park [125] proposed a method by using an analytical relationship between the project volatility (σ) and the parameters of project value distribution at the option life. In any project analysis, we need to estimate the anticipated cash flows associated with undertaking an investment. To determine the project value at the option life, we may attempt to estimate what the future cash flow series look like beyond the option life. Since these project cash flows are random variables described by probabilistic distributions, we may find the project value distribution by aggregating these random cash flow distributions. If we know the exact probability distributions, we may obtain the project value distribution by either an analytical method or using the Monte Carlo simulation method.

A method proposed by Han and Park [125] uses the latter approach to develop a project value distribution first, and then utilize the mean and variance of the project value distribution to estimate the volatility. The mathematical relationship between the project volatility and the project value distribution parameters can be developed based on properties of the lognormal distribution in the geometric Brownian motion. In that sense, Han and Park method still assumes implicitly that the project value be lognormally distributed.

To illustrate how the volatility related to the parameters of the project value distribution, we may start with a stochastic process known as the geometric Brownian motion, which is the special case of the simple Brownian motion with a drift. Suppose, the process of exponential Brownian motions, X_t, which is suggested by Black, Scholes and Merton [87][126] is a lognormal process with drift μ and volatility σ, and it can be denoted in the stochastic differential equation [50][127]:

$$dX_t = \mu X_t dt + \sigma X_t dB_t \tag{3-6}$$

where μ and σ are constants. Since the changes in rate of return from the asset is normally distributed, the absolute changes in X_t are also lognormally distributed. The Equation (3-6) can be written with a simple Brownian motion with drift as follow:

$$dF = \left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma dB_t \tag{3-7}$$

where $F(x) = \log X_t$. It tells us that the change in the logarithm of X_t is normally distributed with the mean rate of return, $\left(\mu - \frac{1}{2}\sigma^2\right)$, and the volatility, σ, which is a measure of the riskiness of the asset. Therefore, we assume that X_t is a value of risky project at time *t* which is given by geometric Brownian motion of the form:

$$X_t = X_0 \cdot e^{\mu t + \sigma B_t}, t \geq 0 \tag{3-8}$$

This process X_t has the expected value $E[X(t)]$, and variance $Var[X(t)]$, at time t.

$$E[X_t] = X_0 \cdot e^{\mu t} \tag{3-9}$$

$$Var[X_t] = X_0^2 \cdot e^{2\mu t}\left(e^{\sigma^2 t} - 1\right) = E[X_t]^2 \cdot \left(e^{\sigma^2 t} - 1\right) \tag{3-10}$$

Then the volatility (σ) is derived from the relationship between the parameters of lognormal and normal distribution.

$$\sigma = \sqrt{\frac{\ln\left(\frac{Var[X_t]}{E[X_t]^2} + 1\right)}{t}} \tag{3-11}$$

It shows that if we know the expected value of project $E[X_t]$ and variance $Var[X_t]$ at some future point in time t, we could calculate the project volatility σ using Equation (3-10). In real options analysis, the expected value $E[X_t]$ and variance $Var[X_t]$, can be obtained typically from the project value distribution aggregated by the discounted future cash flows of a project at the end of option life. But one caveat is that the project value distribution must be a lognormal distribution to use Equation (3-10). Consequently, it would be necessary to develop a real option valuation method when the project value distribution takes a non-lognormal type.

3.3 Develop a Practical Real Option Evaluation Method with Loss Function

As seen in the previous section, using the Black-Scholes formula to calculate the real option value implicitly assumes the project value to be lognormally distributed. This assumption is empirically validated for financial assets such as a stock price movement, which is never below zero. For real assets, however, the project (or present) value (PV) distribution could be normally distributed or take any shape other than a lognormal distribution. In this section, we will introduce the loss function approach to determine the option value that does not require an assumption of a lognormal PV distribution. It is shown that the loss function approach is an analytically and theoretically correct approach that can produce identical results consistent with the Black-Scholes formula, if we assumed a lognormal present value distribution.

3.3.1 The Concept of Loss Function

The concept of loss functions found in Bayesian statistics is that they measure how *bad* our current estimate is: The larger the loss, the worse the estimate is according to the loss function. A simple, and very common, example of a loss function is the linear loss, a type of loss function that increases linearly with the difference. Or a quadratic loss function is also frequently used in estimators like linear regression and calculation of unbiased statistics. In real options analysis, we adopt the linear loss function concept.

To illustrate the loss function concept, let's consider a popular economic reorder quantity model found in the supply chain. A stock-out is one of the things to avoid in inventory management as it can cause various negative effects resulting in economic loss as well as opportunity cost. In practice, however, the future demand is a random variable which is not easy to estimate in any precise fashion. Therefore, what we can do is to estimate the possible shortage and come up with a strategy to limit a stock-out at an acceptable level. A popular inventory strategy used in practice for this purpose is known as the lot-size, reorder point inventory model (Q, R) [128]. In this model, the expected number of shortage is simply the number of excess demand x over reorder level R. It can be expressed as

$$E[\max(x-R,0)] = \int_R^{\infty}(x-R)f(x)dx \qquad (3\text{-}12)$$

where $f(x)$ is the demand distribution function. If the demand is normally distributed, the expected number of shortage is computed by using the standardized loss function. (Conceptually, demand cannot be negative just like a stock price, but if we consider the returns from the customer who purchased the product earlier period, we may view this return as a negative demand. Then a normal distribution assumption may hold.) The standardized loss function $L(z)$ is defined as

$$L(z) = \int_{z}^{\infty} (x - z)\phi(x)dx \tag{3-13}$$

where $\phi(x)$ is the standard normal density function. If the demand follows a normal distribution with mean μ and standard deviation σ, then the average number of shortage can be expressed as

$$n(R) = \sigma L\left(\frac{R - \mu}{\sigma}\right) = \sigma L(z) \tag{3-14}$$

The standardized variate z is equal to $(R - \mu)/\sigma$.

Once the amount of average loss is determined, we can estimate the magnitude of economic loss in monetary terms. If the financial loss is proportional to the magnitude of the average loss, we can use a linear loss function. If the loss increases quadratically, a quadratic loss function would be more suitable to determine the economic loss. In real options analysis, a linear loss function is more appropriate as any miss from the target is all equally undesirable.

3.3.2 Valuing Real Option with the Standardized Loss Function Approach

In this section, we propose a practical framework to estimate the real option value based on the project value distribution. We explicitly assume a non-lognormal distribution for the project value (PV) at the option life, and the required investment cost (I) to generate that project value distribution at the option life. These two parameters can be viewed as the demand (x) and the reorder level (R)respectively in the expected number of shortage model.

Real Call Option Model

Let's revisit the basic financial call option as described in Figure 3.1. A call option is the option to buy the underlying stock at a strike price (K) by a maturity date (T). The option buyer has the right to buy shares at the strike price, and this will happen when the stock price at the maturity date (S_T) is greater than K. In other words, the call option has value in the blue-shaded area.

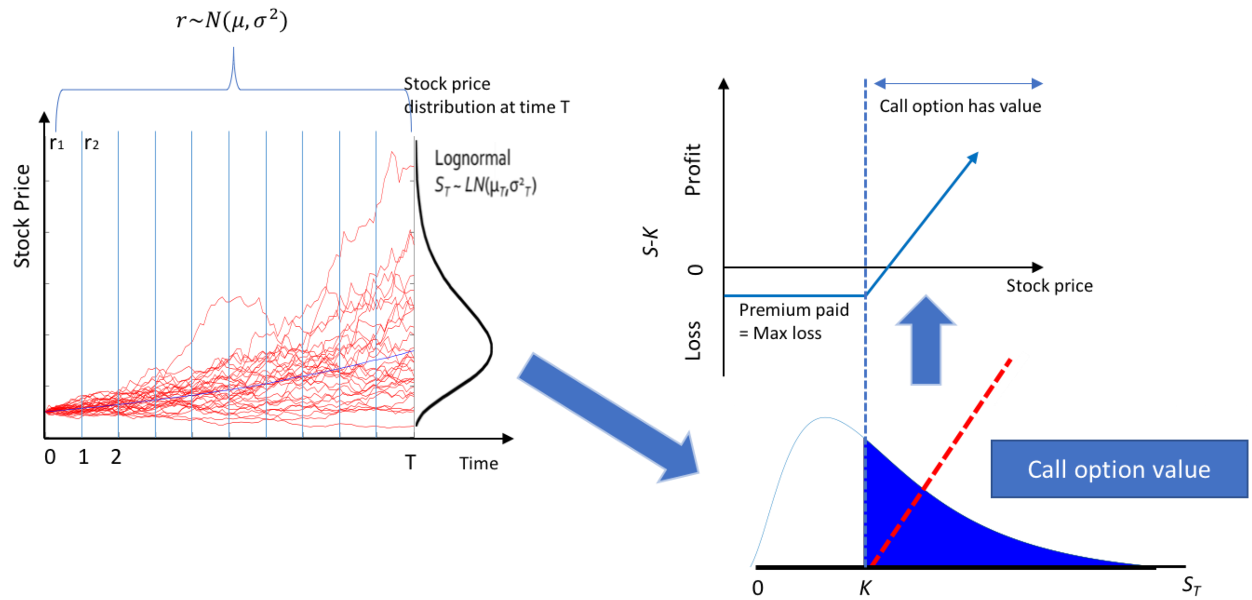


Figure 3-1 Financial call option where it has its option value

From a real options point of view, the option to delay an investment is equivalent to a call option. In the abstract, assume that a project requires an initial investment up-front investment of I_0 and that the present value of expected cash inflows (or we call project value) computed now is PV_0. Then, the net present value of this project is determined by:

$$\text{NPV}_0 = PV_0 - I_0$$

Instead of making an investment decision immediately based on this NPV_0 value, let's assume that the firm has exclusive rights to this project for the next T years, meaning that the firm can undertake this project whenever it deems worthwhile within T years. Certainly, the project value can change over this period as market dynamics tends to fluctuate. Thus, the project may have a negative present value right now, but it could turn into a positive value if things are moving in the right direction while the firm waits. If we define PV_T as the project value at the end of wait (T), which is the present value of the expected future cash inflows, we summarize the firm's investment decision at the end of wait (T) as follows:

If $PV_T > I$, invest in the project,

$PV_T < I$, do not invest in the project.

If the firm does not invest in the project, it will simply lose what it originally invested in the project, or any money paid to hold the project till T (more precisely, the option premium paid for this project). The relationship can be presented in a payoff diagram of project value assuming that the firm holds out until the end of option period. (See Figure 3-2.)

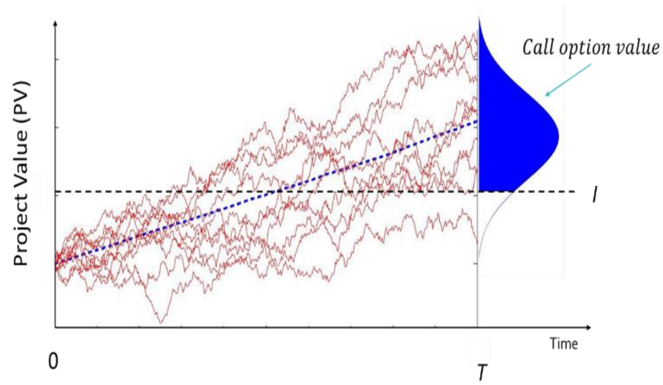


Figure 3-2 Real call option value

Call Option Value Calculation Based on the Loss Function

The expected number of inventory shortage model is equivalent to a call-option type in real option analysis as shown in Figure 3-3. The opportunity cost (or regret) is the blue - shaded area when the firm did not take action because they thought the project value at time T is less than the investment cost. Therefore, the expected present value over the investment cost could be calculated by

$$E[\max(PV - I, 0)] = \int_I^\infty (PV - I)f(PV)dPV \tag{3-15}$$

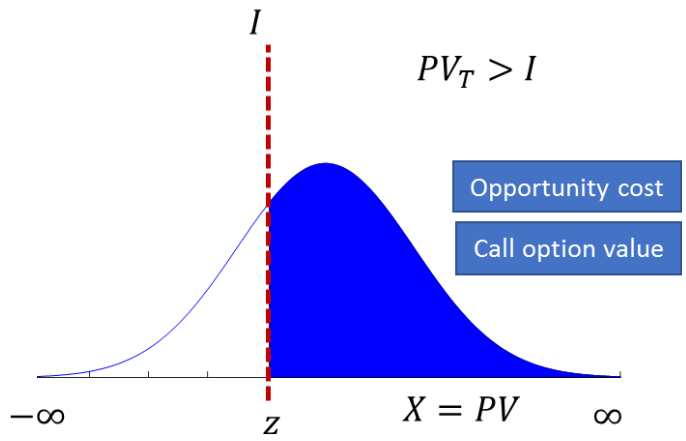


Figure 3-3 Real call option value and opportunity cost

When the project value is normally distributed with mean μ and standard deviation σ, then the real option value (ROV) with the investment cost I can be shown as

$$ROV(I) = \sigma[PV] \cdot L\left(\frac{I - E[PV]}{\sigma[PV]}\right) = \sigma[PV] \cdot L(z) \quad (3\text{-}16)$$

where

$$L(z)_{call} = \int_{z}^{\infty} (x - z)\phi(x)dx = \phi(z) - z\big(1 - \Phi(z)\big) \quad (3\text{-}17)$$

where $\Phi(x)$ is the cumulative standard normal distribution function. This is the same theory of call-option type real option analysis; the option value can be obtained by the discounted partial expectation of the PV distribution.

3.3.3 Real Put Option Model

The put option is just the opposite case of a call option – you have the right to sell the underlying asset at a predetermined price (K) at option life (T) but you have no obligation to do it. This situation is described in Figure 3-4. The option buyer has the right to sell shares at the strike price, and this will happen when the stock price at the maturity date (S_T) is less than *K*. In other words, the put option has value in the shaded area in Figure 3-4.

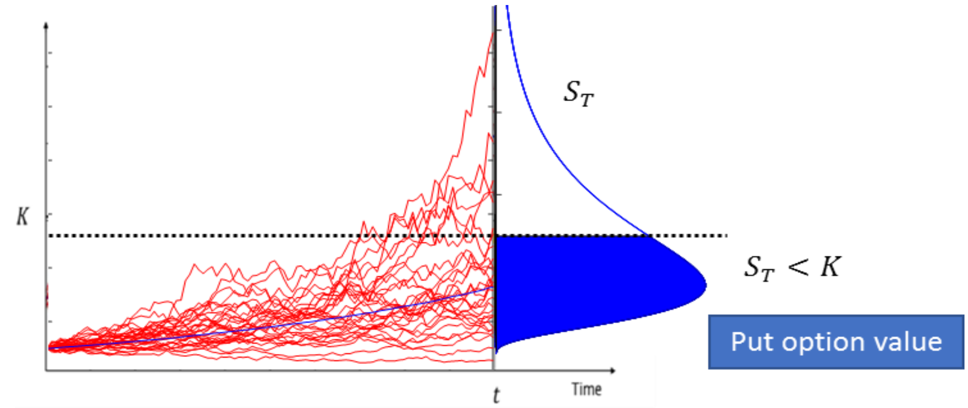


Figure 3-4 Put option value in financial option

Real Put Option

In terms of real put option application, we may consider abandoning a project when its cash flows do not measure up to expectation. The project value (PV) is then the remaining value of the project at the time of abandonment consideration. Then we explore the liquidation or abandonment value (*S*) (or the market price offered by other business to acquire the project) for the same project at the same point in time. If the project has a life of *n* years, we can compare the value of continuing the project with the liquidation value. In terms of decision, we could consider liquidating the project if $PV < S$, the payoff from owning an abandonment option is

$$\text{Put option value} = 0, \text{ if } PV > S$$

$$= S - PV, \text{ if } PV < S$$

Then, the relationship of profit is illustrated in Figure 3-5.

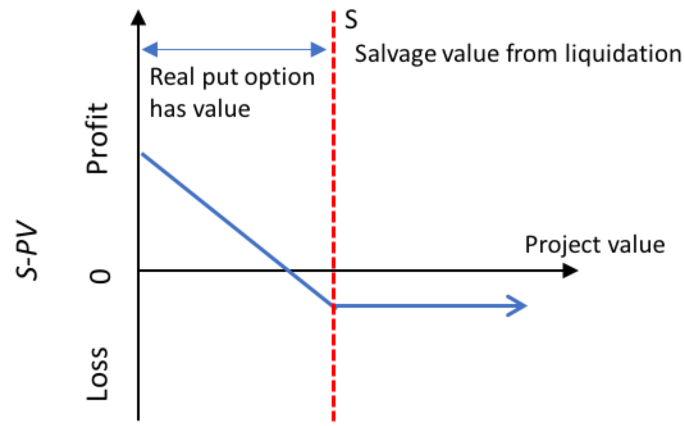


Figure 3-5 Real put option value from abandoning a project

Real Put Option Value Calculation based on Loss Function

The standardized loss function can be modified for a put-option type real option with partial expectation on the left-tail as shown in Figure 3-6.

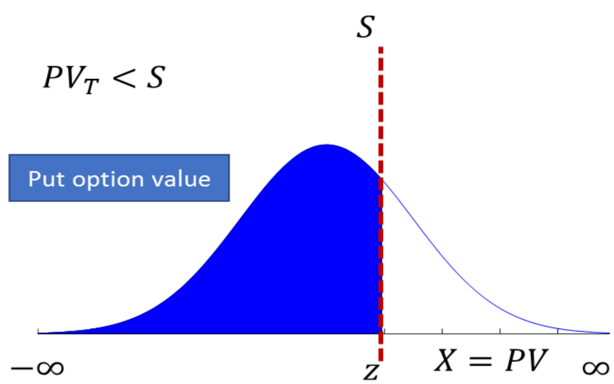


Figure 3-6 Real put option value calculation based on Loss Function

The standardized loss function for a put option is

$$L(z)_{put} = \int_{-\infty}^{z} (z - x)\phi(x)dx = \int_{-\infty}^{z} z\phi(x)dx - \int_{-\infty}^{z} x\phi(x)dx = z\Phi(z) + \phi(z) \quad (3\text{-}18)$$

These results indicate that we can value of real options without requiring a lognormal assumption for a project value distribution.

3.4 An Illustrating Example

To demonstrate the computational process of obtaining call and put option values through the standardized loss function, we will present a numerical example taken from Copeland and Antikarov [52].

3.4.1 Investment Environment

Suppose a manufacturing company already came up with future cash flows associated with a project but it has three years to decide whether or not to invest. With various future uncertainties, the company considers a deferral option based on the estimated future cash flow. Table 3-2 shows estimating future cash flows over the project life from operating activities.

Table 3-2 Estimating future cash flows over the project life

t=3	t+1	t+2	t+3	t+4	t+5	t+6	t+7
Revenue	$1300	1300	1300	1300	1300	1300	1300
Sales price/unit	13	13	13	13	13	13	13
Unit sales	100	100	100	100	100	100	100
Expenses	-920	-920	-920	-920	-920	-920	-920
Variable cost/unit	7	7	7	7	7	7	7
-Variable costs	-700	-700	-700	-700	-700	-700	-700
-Fixed costs	-20	-20	-20	-20	-20	-20	-20
-Depreciation	-200	-200	-200	-200	-200	-200	-200
EBIT	380	380	380	380	380	380	380
-Cash Taxes	-114	-114	-114	-114	-114	-114	-114
+Depreciation	200	200	200	200	200	200	200
-Working capital	-300	0	0	0	0	0	300
Expected cash flow	166	466	466	466	466	466	766
Standard Deviation	52.49	147.36	147.36	147.36	147.36	147.36	242.23

With the expected NPV criterion, a project could be accepted when the expected NPV obtained by discounting the future cash flows is greater than 0, and each cash flow (a_n) is treated as most-likely values of each period.

$$\sum_{n=t+1}^{N} \frac{a_n}{(1+r)^n} - I > 0 \quad (3\text{-}19)$$

However, considering uncertainty, the PV could be a random variable which has a mean and variance instead of a static value as illustrated in Figure 3-7.

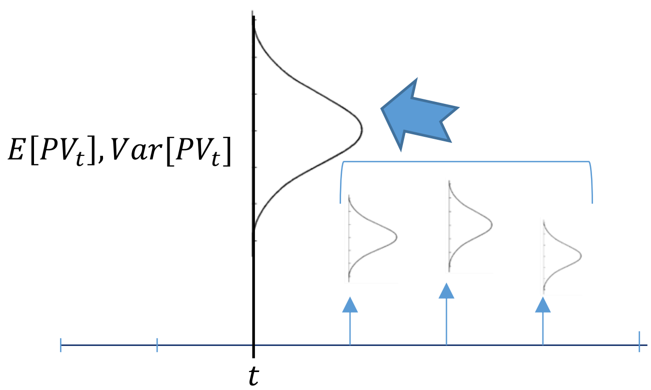


Figure 3-7 PV distribution by aggregating discounted cash flow

Suppose, the company assumes a 32% of standard deviation – of each cash flow and 12% of weighted average cost of capital (WACC), then the expected present value and variance of the project value will be calculated by

$$E[PV_t] = \sum_{n=t+1}^{N} \frac{E[a_n]}{(1+WACC)^n} = \$1{,}994.56$$

$$Var[PV_t] = \sum_{n=t+1}^{N} \frac{(st.d[a_n])^2}{(1+WACC)^{2n}} = (312.18)^2$$

When the investment cost for the project is $1,600, the project is acceptable in the conventional view point because the expected NPV is greater than 0. Nonetheless, there are possible losses as the shaded area in Figure 3-8, that should be considered. The real option strategy could hedge these potential losses.

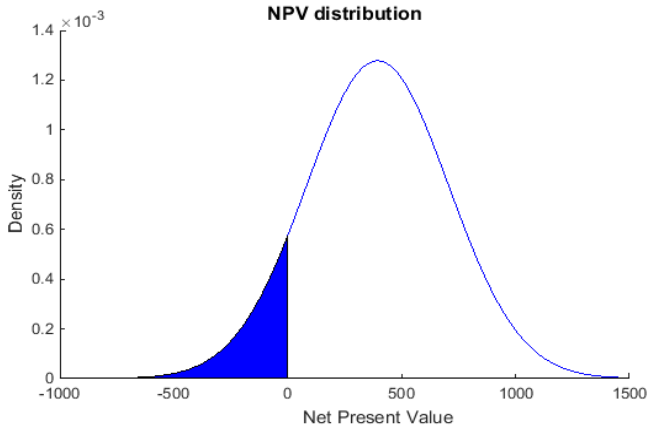


Figure 3-8 Net Present Value distribution

In addition, we conducted 1,000 iterations of the Monte Carlo simulation to obtain the project values with the random sale price and variable cost. Figure 3-9 displays a normal quantile-quantile (QQ) plot, which shows the quantiles of the project values versus the theoretical quantiles values from a normal distribution. As we can see that the points seem to fall along a straight line in the QQ plot, there is a strong evidence that the project value is normally distributed.

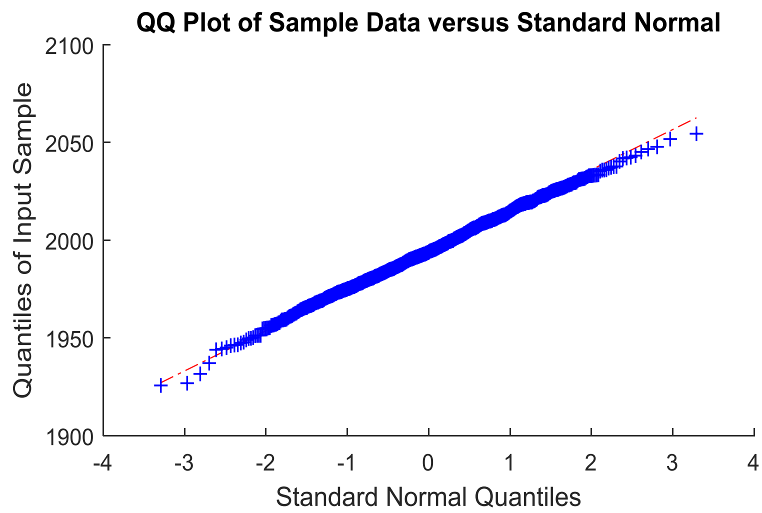


Figure 3-9 QQ plot of project value with 1000 times iteration

3.4.2 Procedure of real option evaluation with the standardized loss function

To implement the loss function approach, we may use the following computational steps:

- Step 1: The generation of future cash flow at each period
- Step 2: The estimation of project value from the discounted cash flows at time T, option life
- Step 3: The determination of the expected value and variance of the project value distribution
- Step 4: The calculation of real option value at time T with the standard loss function (partial expectation)
- Step 5: The calculation of real option value at time 0 by discounting the value obtained in Step 4.

3.4.3 Call-option type real option: defer options

With a defer option for the project, the company can wait and see. If the project value is greater than the investment cost, then the company will invest in the project. If the PV distribution could be assumed as a normal distribution, the option value could be calculated with the partial expectation which is the same as the expected number of shortage model. It can be written as

$$E[\max(PV_t - I, 0)] = \int_I^{\infty} (PV_t - I) f(PV_t) dPV_t \tag{3-20}$$

Here, we could substitute the expected value and standard deviation obtained from the estimated cash flows into the defer option valuation model.

$$ROV_t(I) = \sigma[PV_t] \cdot L\left(\frac{I - E[PV_t]}{\sigma[PV_t]}\right) = \sigma[PV_t] \cdot L(z)_{call} \tag{3-21}$$

$$z = \frac{I - E[PV_t]}{\sigma[PV_t]} = \frac{\$1{,}600 - \$1{,}994.56}{\$312.18} = -1.2639$$

$$ROV_t(I) = \sigma[PV_t] \cdot \left[\phi(z) - z\big(1 - \Phi(z)\big)\right] = \$409.0$$

where $\phi(z)$ is the standard normal distribution density function and $\Phi(z)$ is the standard normal cumulative distribution function. This real option value represents the value at time t, thus we need to bring this value at time 0 by discounting at a risk-free interest rate.

$$ROV_{call}(I) = ROV_t(I) \cdot e^{-rt} = \$342$$

Implied Volatility in Black-Scholes

The call option value calculated by the Black-Scholes formula gives almost the same value with the standard loss function. To calculate the real option value with the Black-Scholes formula, the important parameters to be defined are the initial value and the volatility of the project. These values could be obtained from the project value distribution. Therefore, once the project value distribution with the expected value and variance is generated, the implied project volatility could be calculated with Equation (3-11).

$$\sigma = \sqrt{\frac{\ln\left(\frac{Var[PV_t]}{E[PV_t]^2}+1\right)}{t}} = \sqrt{\frac{\ln\left(\frac{(312.18)^2}{(1{,}994.56)^2}+1\right)}{3}} = 0.09$$

Then the call-option value with the Black-Scholes formula will be

$$\begin{aligned} ROV_{call} &= PV_0 N(d_1) - I\cdot e^{-rt}N(d_2) \\ &= E[PV_t]\cdot e^{-rt}N(d_1) - I\cdot e^{-rt}N(d_2) = \$337.75 \end{aligned}$$

It shows that how robust this loss function approach is as an alternative to the Black and Scholes method.

3.4.4 Put-option type real option: abandonment options

If the project value is less than a resale / salvage value at time *t*, the company with an abandonment option can terminate an on-going project before its project life. As the same assumption as a delay option, when the PV distribution follows a normal distribution, the abandonment option could be the expected value of the difference between the salvage value and the project value at time *t*. When a resale or salvage value is denoted by K, it can be written as

$$E[\max(K - PV_t, 0)\,] = \int_{\infty}^{I} (K - PV_t) f(PV_t) dPV_t \tag{3-22}$$

Equation (3-18) shows the partial expectation which can be obtained by the standard loss function for put-option value.

$$ROV_t(K) = \sigma[PV_t]\cdot L\left(\frac{K - E[PV_t]}{\sigma[PV_t]}\right) = \sigma[PV_t]\cdot L(z)_{put} \tag{3-23}$$

$$ROV_t(K) = \sigma[PV_t]\cdot L(z)_{put} = \sigma[PV_t]\cdot\left[\phi(z) - z\big(1-\Phi(z)\big)\right] \tag{3-24}$$

The put-type real option should be exercised when a remaining project value is less than a salvage value or resale value. For example, if the expected revenue decreases during the option life, the expected project value at time t should be decreased. In this case, the abandonment option could provide a protection against a downside risk.

Suppose that the expected revenue in the previous example (see Table 3-2) decreases from \$1,300 to \$1,100, the project value distribution would have a new expected value and a variance: $E[PV_t] = \$1{,}355.63$, and $Var[PV_t] = (226.25)^2$. With these new expected project value and the standard deviation, we could calculate the abandonment option value with the standard loss function as the same procedures as a defer option. Suppose the salvage value, K, is \$1,600, then the normalizing value, z, can be obtained

$$z = \frac{K - E[PV_t]}{\sigma[PV_t]} = \frac{\$1{,}600 - \$1{,}355.63}{\$226.25} = 1.08$$

The abandonment option value of the project can be obtained by the standard loss function approach.

$$ROV_t(K) = \sigma[PV_t] \cdot L(z)_{put} = \sigma[PV_t] \cdot \left[\phi(z) - z\big(1 - \Phi(z)\big)\right] = \$260.51$$

$$ROV_{put}(K) = ROV_t(K) \cdot e^{-rt} = \$217.60$$

Once again, it could be compared with the put option value with the Black-Scholes formula:

$$ROV_{put} = Ke^{-rT}N(-d_2) - PV_0N(-d_1) = \$221.07$$

In consequence, this standardized loss function approach could replace the Black-Scholes method for not only the call-type option, but also the put-type option valuation.

3.4.5 Sensitivity Analysis

Recall the example cash flows of the manufacturing company. If the risk-free interest rate is 6%, the real option value calculated from the Black-Scholes formula is

$$ROV = PV_0N(d_1) - I \cdot e^{-rt}N(d_2) = \$337.75$$

The discounted option value from the expected shortage model with the standardized loss function shows the similar value as the option price from the Black-Scholes formula.

$$ROV_0(I) = ROV_t(I) \cdot e^{-rt} = \$342.38$$

For practical purpose, these two methods produce almost identical values. The difference is coming from the different assumption of project value distribution; thus, we conduct the sensitivity analysis by varying the expected project value, investment cost, option life, and discount rate. In Figures 3-10 and 3-11, the dotted lines show the option values based on the standardized loss function approach; the solid lines show option values from the Black-Scholes formula.

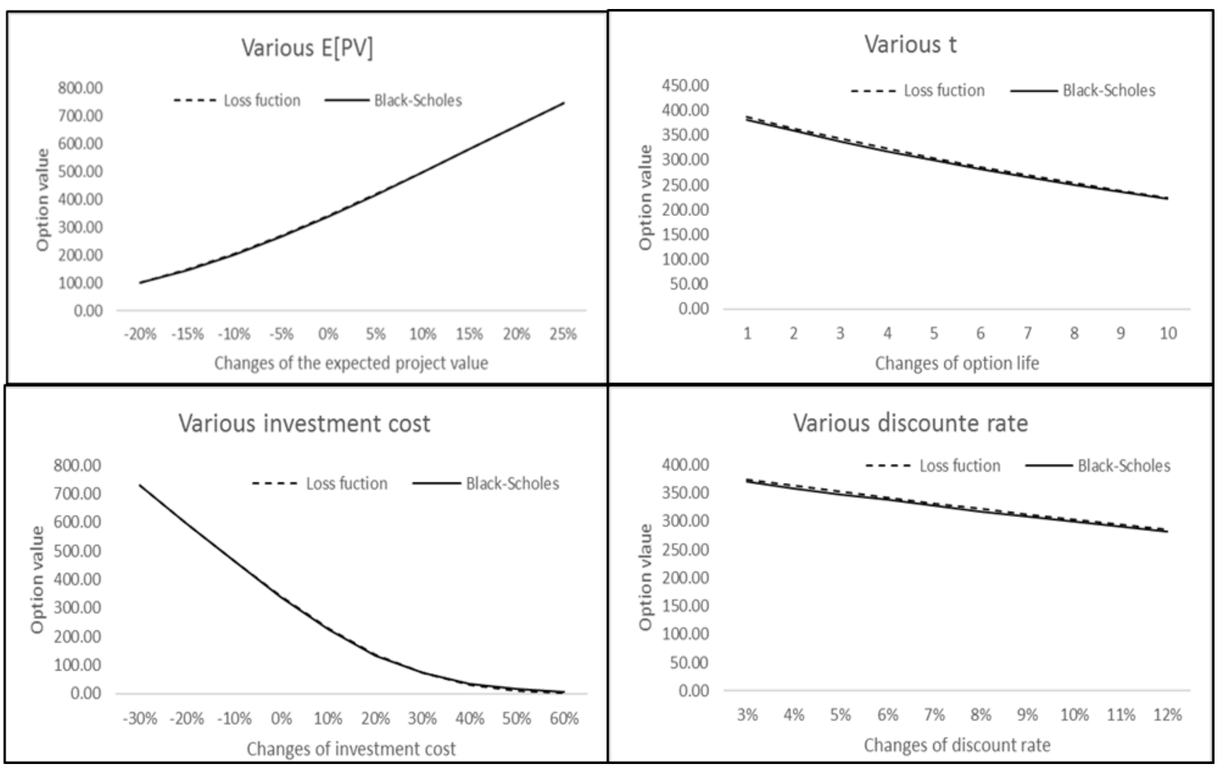


Figure 3-10 Comparison of call-option values based on B-S formula and loss function

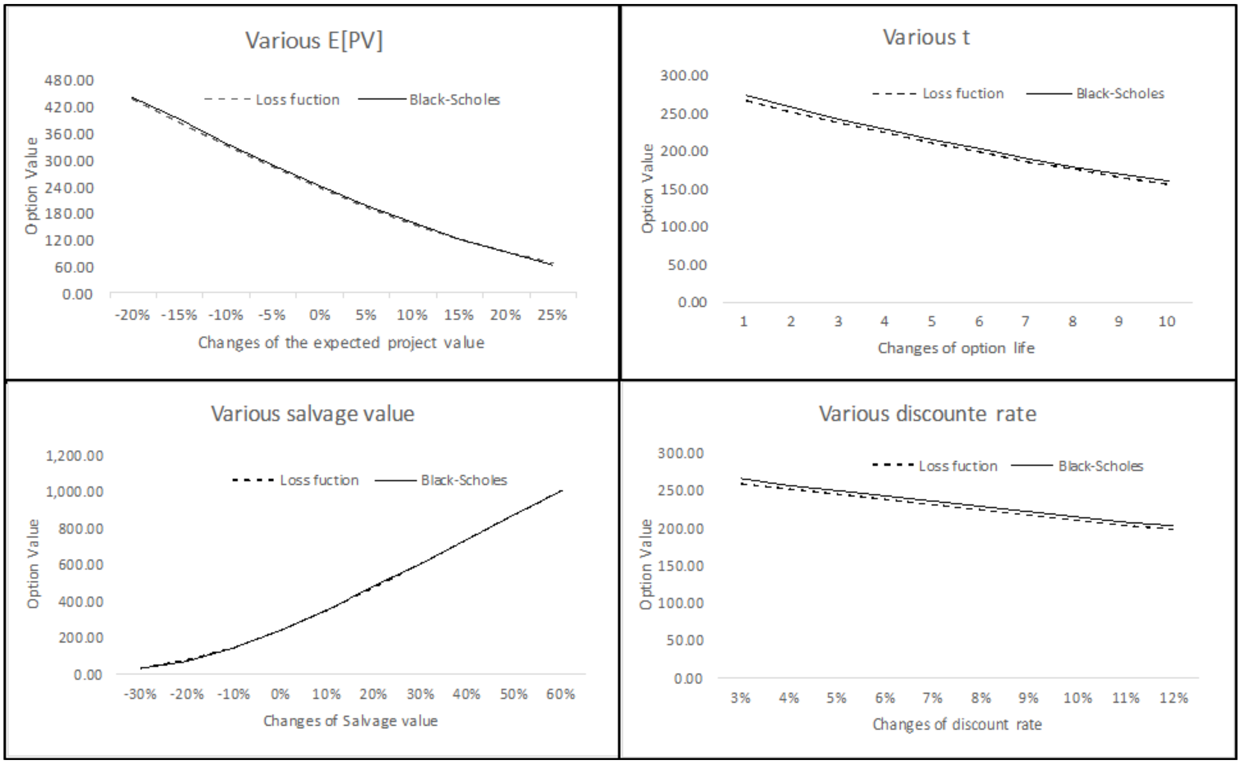


Figure 3-11 Comparison of put-option values based on B-S formula and loss function

The results in Figures 3-10 and 3-11 indicate that the loss function approach almost duplicates the results from the Black-Scholes method. In our future task is to develop a series of option value formulas driven by various non-lognormal distributions, so that one can make a quick option value calculation without resorting to the Black-Scholes method.

3.5 Summary

In summary, in the traditional project evaluation, the first step is to estimate the future cash flows by integrating various cash flow components. From there, we calculate the project value distribution, which may not be a lognormal distribution. To explain a stock price movement, the financial option analysis assumes the distribution of underlying asset to be a lognormal distribution, whereas the present value distribution of the cash flows to be a normal type can be a more realistic assumption for real projects. Moreover, we can easily verify that the real option value using the standardized loss function approach could produce the identical value with the value obtained from the well-known Black-Scholes formula, if we assume the same lognormal PV distribution. Therefore, based on a partial expectation of the normal distribution, the new real option evaluation method can be a more practical method. In addition, this alternative approach is easy to understand the process of option valuation and update the real option value when a company receives new pieces of information for the project in concern.

Chapter 4. Determine a Relationship between CNPVaR and Real Options Value

In Chapter 3, we have developed an alternative procedure to determine the option value based on the loss function approach. The option value sets simply the maximum amount to pay for retaining investment flexibility. The next step is to determine an appropriate price to pay to obtain the option. Then we are ready to develop a decision criterion to consider three critical elements – profitability, variability, and flexibility. Once again, profitability will be measured by the expected net present value, the variability by the variance of the NPV, and the flexibility by the real option value. To integrate these three elements in a decision criterion, we propose the K-P criterion, which will be used as a decision criterion to select the best portfolio in each decision period over the planning horizon.

4.1 Determine a Relationship between CNPVaR and Real Options Value

Evaluating an investment project solely based on the expected NPV ignores the possibility of losing big money for a risky investment due to uncertainty associated with project cash flows. To gauge the extent of loss, financial institutions have adopted a risk measure known as the conditional value at risk (CVaR). The CVaR intuitively shows how much risk exists in an investment project. If an investor or business is willing to take that risk, then we can view this value as a kind of risk tolerance to the investor. With this information available, we may be interested in coming up with a strategy to hedge the further risk through any option mechanisms such as real options. Naturally, one of the important questions is to determine an appropriate amount of real option premium to pay for a given level of risk tolerance. Therefore, before developing any capital budgeting decision model, it is essential to explore the relationship between the risk tolerance implied and the realistic real option premium to pay. In this research, we will explore this relationship through the conditional value at risk concept.

4.1.1 Value at Risk and Conditional Value at Risk

The Value at Risk (VaR) is one of the popular risk measures developed by JP Morgan [68] to assess for market risks of financial assets or portfolios based on statistical return data of the financial assets. It is expressed as the quantile of the future losses within a given confidence level over a certain time duration. The purpose of VaR is to establish a common reference point for market risk measurements, such that it gives to investors a better understanding of the risk exposure. In financial institutions such as banks and investment firms, it plays the role of a standard risk indicator. Even though the VaR has become a standard measure of risk exposure, it has been criticized for its shortcomings [72]. There are two main arguments: First, the VaR failed one of the requirements of the coherent risk measure, which is the sub-additivity property. Second, the assumptions of VaR cannot be maintained for the valid value under an extreme loss beyond the specified confidence level. Due to the shortcomings of the VaR, the Conditional Value at Risk (CVaR) was suggested by Rockafellar and S. Uryasev [74]. It measures the expected loss beyond a specified confidence level and is known as the Expected Shortfall (ES) or Mean Excess Loss (MEL). We will adopt the CVaR concept as a risk measure in our research.

The VaR measures the maximum expected loss at a given investor's confidence level (p) over the given time horizon (t), thus, the VaR corresponds to the lower-tail level $(1-p)$. If a random variable X represents the value of an asset and the F_X is the cumulative distribution function of X, VaR and CVaR are denoted by

$$VaR_p(X) = inf\{x|F_X\ (x) \geq 1 - p\} = F_X^{-1}(1 - p) \tag{4-1}$$

$$\begin{aligned} CVaR_p(X) &= inf\left\{VaR_p(X) + \left(\frac{1}{1-p}\right) \cdot E[X - VaR_p(X)]^{-}\right\} \\ &= E\left[X\middle|X \leq VaR_p(X)\right] \end{aligned} \tag{4-2}$$

Since the Value at Risk (VaR) was originally developed for financial institutions, similar terms as shown in Figure 4-1 have been introduced for non-financial institutions to meet their different circumstances. The Cash Flow at Risk (CFaR) measures the degree of risk associated with a cash flow shortfall in any operational period [129][78][79]. The Net Present Value at Risk (NPVaR), on the other hand, measures the overall risk associated with undertaking an investment project [84][130][85].

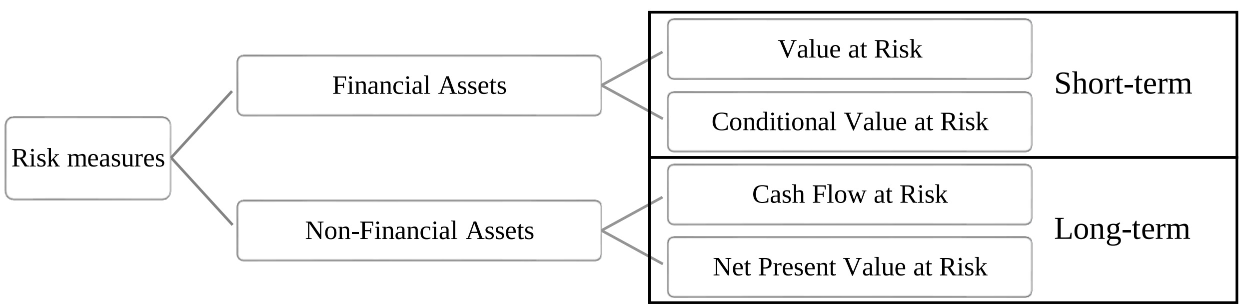


Figure 4-1 Summary of risk measures based on Value at Risk

Since we are dealing with investments in real assets, our focus would be more on cash flow realizations of the investments. We propose the term, Conditional Net Present Value at Risk (CNPVaR), which has applied the CVaR concept on a project value distribution. The CNPVaR represents the average of losses beyond the critical point with a given tolerance level. Especially, the value does not need to be interpreted as only loss; it could be any value lower than the critical point or simply a degree of undesirability.

4.1.2 Conditional Net Present Value at Risk (CNPVaR)

For any investment project, it is always possible to realize a negative NPV even though the expected NPV might be positive. So, we would be interested in knowing how much NPV is at risk for the project under consideration. In real options analysis, we typically separate the investment cost and project value, which is the sum of the discounted cash flows after investment. As mentioned in Chapter 3, the project value distribution may not follow a lognormal distribution. Instead, the project value (PV) is often normally distributed with aggregation of a sufficient number of random cash flow components.

To develop a dollar-based measure, the Net Present Value at Risk (NPVaR) and Conditional Net Present Value at Risk (CNPVaR) with a confidence level (p) could be modified from the forms of the VaR and the CVaR for a normal distribution as follows:

$$NPVaR_p(X) = E[NPV] + \Phi^{-1}(1-p)\sigma[NPV] = E[NPV] + z\sigma[NPV] \qquad (4\text{-}3)$$

$$CNPVaR_p(X) = E[NPV] - \left(\frac{\phi(z)}{\Phi(z)}\right) \cdot \sigma[NPV] = E[NPV] - \left(\frac{\phi(z)}{1-p}\right) \cdot \sigma[NPV] \qquad (4\text{-}4)$$

where $\Phi^{-1}(1-p) = z$. Now we can easily calculate $CNPVaR$, once we know the mean and variance of project value distribution.

To illustrate the computational process, recall Section 3.4.1, the project value distribution with mean and variance were

$$E[NPV] = \sum_{n=1}^{N} \frac{E[a_n]}{(1+WACC)^n} - I = \$1{,}994.56 - \$1{,}600 = \$394.56$$

$$Var[NPV] = \sum_{n=1}^{N} \frac{(st.d[a_n])^2}{(1+WACC)^{2n}} = (312.18)^2$$

where WACC is 12%. With the mean and variance from the NPV distribution, the NPVaR and CNPVaR with a 95% confidence level over the project life can be calculated by

$$NPVaR_p(X) = E[NPV] + z\sigma[NPV] = -\$118.93$$

$$CNPVaR_p(X) = E[NPV] - \left(\frac{\phi(z)}{1-p}\right) \cdot \sigma[NPV] = -\$249.38$$

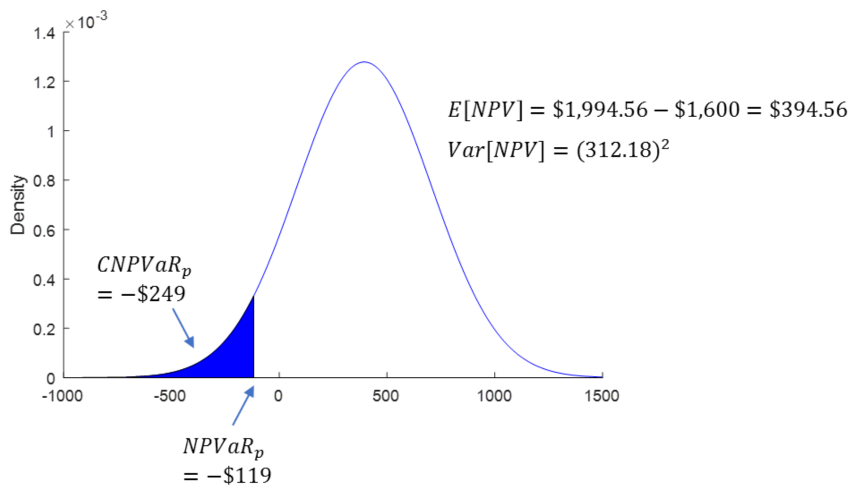


Figure 4-2 NPVaR and CNPVaR with a 95% confidence level

Figure 4-2 shows that the maximum losses are \$119 with a 95% confidence level and the average extreme losses beyond \$119 is around \$249 at the same confidence level.

4.1.3 Real option pricing with CVaR approach

Once we have determined the option value associated with an investment project, our question is what the right price to pay for the option is. Conceptually, we know that the maximum amount to pay should not exceed the option value. If the payment is greater than the real option value itself, the option no longer has any benefit.

The first step toward the option pricing for an investor (business) who reveals a certain degree of risk tolerance, we may revisit the CNPVaR, which captures the downside risk exposure in the investment project. For example, in the previous section, we obtained the NPVaR and CNPVaR at 95% confidence level from the NPV distribution. Those values could be interpreted:

- The most an investor expect to lose in present dollars over the project life is \$118.93 with 95% confidence.
- The average loss amount exceeding \$118.93 is \$249.38 with 95% confidence.

If the investor is willing to accept the project with the possible losses up to \$249.38, it could be the maximum payment for acquiring real option. Recall the example in Chapter 3 where the real option value of the proposed approach is

$$ROV_0(I) = ROV_t(I) \cdot e^{-rt} = \$342.38$$

In this example, the maximum option price could be CNPVaR; Option price < \$249 < \$342. Figure 4-3 illustrates the relationship between the CNPVaR and the option premium (price).

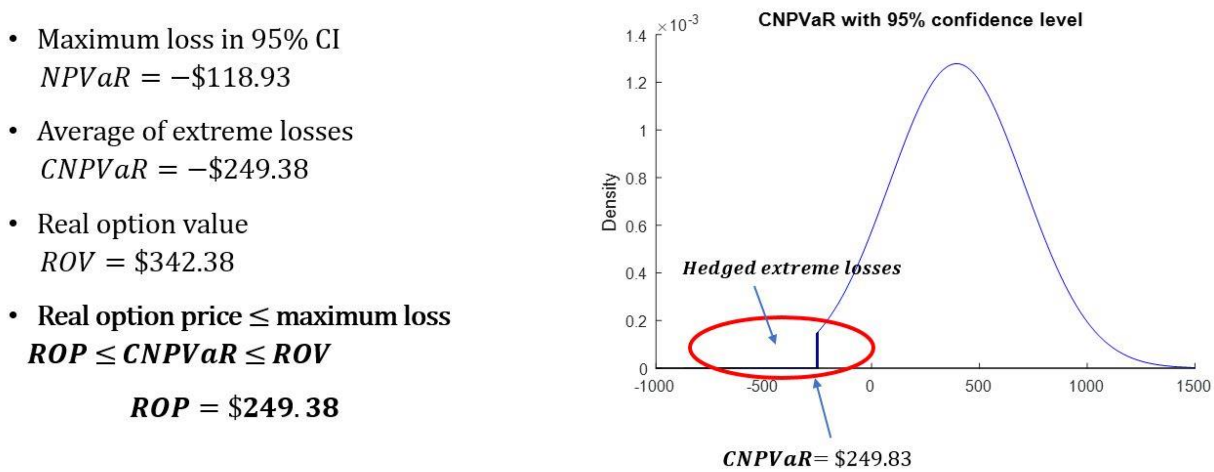


Figure 4-3 Real option pricing with CNPVaR with a 95% confidence level

Now in the previous example, if the investor requires a much tighter confidence level, say 99%, instead of 95%, what would be the proper option price?

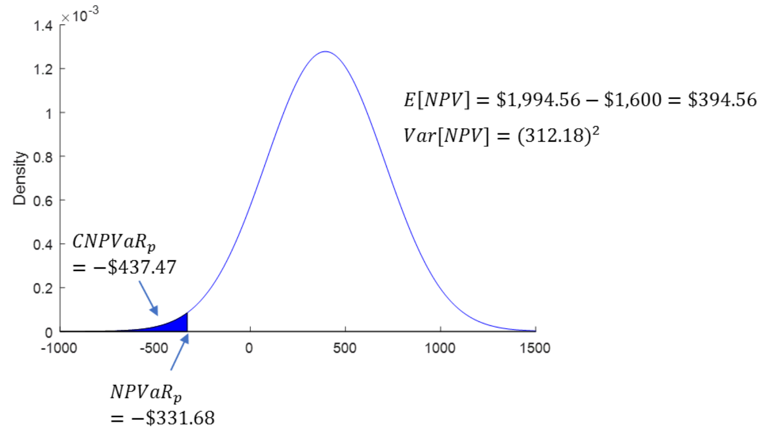


Figure 4-4 NPVaR and CNPVaR with 99% confidence level

As shown in Figure 4-4, the revised *NPVaR* and *CNPVaR* are now -$331.68 and -$437.47 respectively. With an increased confidence level, the maximum loss and the average loss beyond NPVaR are much greater than the case of 95% level. Figure 4-5 illustrates the revised option pricing for this investor with 99% confidence level is now $342.38. In other words, the investor could pay up to the maximum amount to have an assurance to limit the loss at the option premium of $342.38.

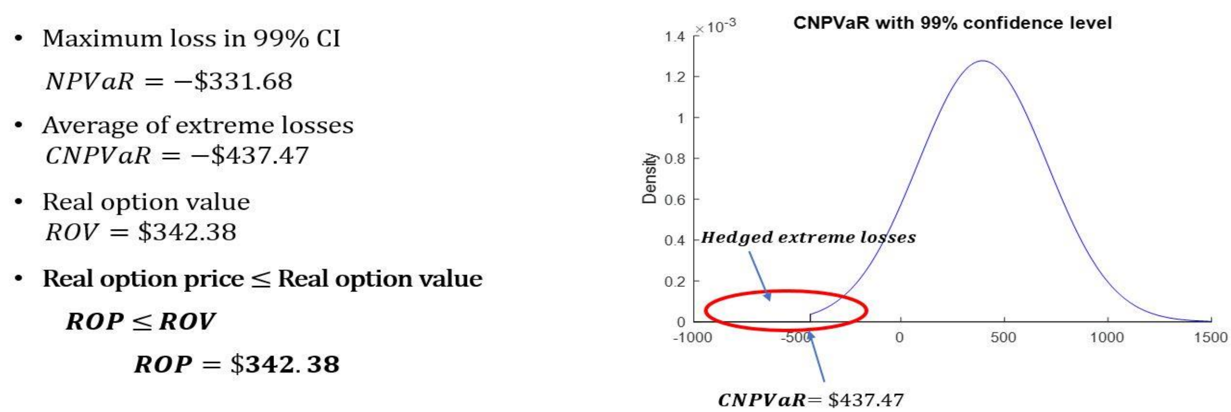


Figure 4-5 Real option pricing with CVaR with a 99% confidence level

Therefore, the maximum amount of real option premium will be the minimum value between option value and CNPVaR, it can be written by

$$Real\ Option\ Price\ (ROP) \leq \min(ROV, CNPVaR) \qquad (4\text{-}5)$$

In summary, the Value at Risk (VaR) is one of the well-known risk measures used by financial institutions to capture the potential loss in value of their traded portfolios from market movements over a specified period. Once the potential loss amount is determined, then this can be compared to their available capital and cash reserves to see if the losses can be covered without putting the firms at risk. When the concept is applied to non-financial firms, the Net Present Value at Risk (NPVaR) represents the worst outcome at a given confidence level on the NPV distribution. We propose to adopt the loss information obtained from the CNPVaR as a basis to determine what the appropriate price to pay is to acquire an option associated with the investment project under consideration for funding.

4.2 Development of the K-P Criterion

In the previous section, we developed a framework for calculating the investment flexibility (or option premium) based on the CVaR. In this section, we formally propose the decision criterion, called the Kim-Park method (or namely K-P criterion), and describe its application to a sequential decision process. To facilitate the understanding of the potential advantage of utilizing the K-P criterion when decisions are made on a periodic basis, we present the logical derivation of the K-P criterion in several steps. In Section 4.2.1, we describe the specific capital budgeting decision environment in which the K-P criterion is to be applied. In Section 4.2.2, we develop a single measure as an operational decision rule to seek a practical trade-off among the three major investment factors described earlier.

4.2.1 Capital Budgeting Decision Environment

Revisit the investment situation that is described in Chapter 1. The firm has to make capital investment decisions on a periodic basis. The frequency of investment decisions can vary depending on the type of business sector, but it could be done quarterly, semiannually, or annually. The firm's objective is to maximize the value creation through successful investments at the end of some predefined time frame. At each time of deliberation, all units of the organization submit the capital investment proposals to the capital expenditure committee detailing the anticipated cash flows from the proposals and any other relevant information worth considering. All cash flow estimates are based on the best information available at the time of submission, knowing that these estimates are subject to change due to considerable uncertainty. To factor the degree of uncertainty embedded in the projects, all future cash flows are described by either ranges of low and high or various types of probability distributions. Furthermore, the firm has no precise knowledge of the investment proposals that will be submitted for consideration in future periods. More specifically, we will assume the following investment situations:

- The firm will call for capital expenditure requests from all divisions within the organization on regular basis. Then among the proposals submitted for funding, the firm will consider which proposals should be funded within the budget. The firm will only know the proposals submitted for that decision period, but no specific knowledge regarding the future investment opportunities in advance. The firm has to make decision based on the most likely estimates of cash flows or probabilistic description of cash flows contained in the proposals at that time.
- The firm does not have information about the types of investment proposals (investment opportunities) that would be submitted in future budget period in advance.

- The firm knows exactly how much budget be allocated at the starting period ($t = 0$) and has a reliable future budget in each future decision period. Normally the future budget for capital expenditure would be a function of expected sales or operating profit to be generated.
- The actual future budget can fluctuate depending upon the acceptance or rejection of projects available in the previous decision periods. Any realization of cash flows from the project funded in previous periods would be reinvested in the business – meaning that the future budget will comprise of two sources: regularly budgeted amount plus the cash flows from the previous investments. Certainly, this assumption can be relaxed if any firm has a different investment policy regarding the cash infusion from the investment activities.
- The firm will make every effort to maintain a relatively constant capital structure such that the weighted average cost of capital (discount rate) will remain steady during the study period.
- The firm's ultimate objective is to create the largest value to the shareholders and the planning horizon is a purely function of management policy. For example, the firm could set a short-term goal with a planning horizon of 3 years or a relatively long-term goal with a planning horizon of 5 years. Most public utility firms might have much longer planning horizons as their capital expenditure programs to be more capital extensive requiring a longer time to recover the capital.

For these investment situations, the firm is faced with selecting a set of projects at each decision period which would result in the maximization of return on invested capital at the horizon time.

4.2.2 Description of Three Key Investment Factors to Be Considered

A typical capital budgeting decision still could be made solely on the basis of expected cash flows. However, in a context of sequential capital budgeting problems, our goal is to consider the investment flexibility embedded in each project to prepare for the unforeseen investment risk or opportunity that could arise in the future.

As pointed out earlier, when comparing sets of uncertain cash flow sequences over a multi-period investment horizon, each decision ought to consider three factors. They are:

- The expected terminal profitability (profitability)
- The magnitude of expected loss (variability)
- The flexibility in future investment activity (flexibility).

The new proposed decision criterion (K-P criterion) seeks some way to reflect these three elements in a single measure. The approach taken in this study is as follows: the terminal profitability is measured by the expected net present value of the project; the variability in the net present value is expressed in terms of standard deviation from the expected net present value and its loss magnitude determined by CVaR; and the investment flexibility preference is measured by the size of option value. Thus, conceptually, the overlapped space in Figure 4-6 represents the project desirability based on these three factors. In fact, the overlapped area created by the profitability and flexibility represents the flexible NPV in real options, and the overlapped area created by the profitability and variability represents the mean-CVaR measure. The overlapped area between the flexibility and variability is considered to determine an appropriate real option premium.

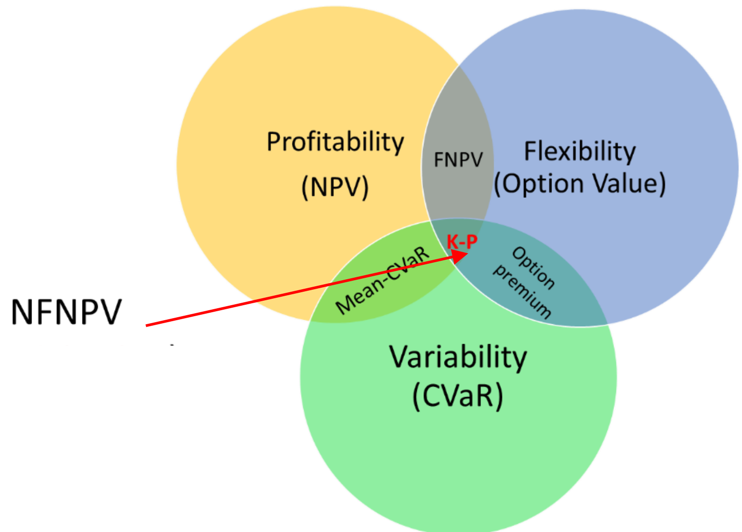


Figure 4-6 Three elements for the K-P criterion and their relationships

To develop an operational and practical decision rule which indicates whether or not a project should be included in the budget, we need a single dollar measure which brings these major parameters together. Therefore, subsequent sections examine what type of integration of these three factors should result in a single dollar measure.

4.2.3 Devising a Single Value - Net Flexible Net Present Value (NFNPV)

In real options, the flexible NPV (FNPV) is defined as the NPV of investment project with options, which captures strategic concerns.

$$\text{FNPV} = \text{NPV (without option)} + \text{Value of options} \quad (4\text{-}6)$$

Conceptually, we see that increases in an investment's expected value increase its desirability, while increases in the option value also tend to increase an investment's desirability as well. Clearly, the higher FNPV is, the better investment is. In ranking investment projects, we will also prefer an investment with a larger value of FNPV to the one with a lower FNPV. In order to create such an option value, it requires an option premium to pay, which should be the smaller than the option value itself. Otherwise, it does not make any economic sense to retain the option.

The issue here is how much option premium should be paid to retain the investment flexibility when an average expected loss is established by the investor at a certain level of confidence. In Section 4.1, we have developed what the appropriate amount a firm should pay as an option premium based on the value of CVaR. Therefore, if we determine the option premium based on the CVaR, in fact, we are incorporating three elements of investment factors by the following relationship:

$$\text{NFNPV} = \text{NPV} + \text{Value of options} - \text{Option premium} \tag{4-7}$$

where NFNPV refers to the net FNPV. In this research, we propose the NFNPV as a measure of economic desirability of an investment. In other words, an uncertain prospect is considered to be characterized by a single index on which accept-reject decisions can be made.

4.3 Mathematical Programming Model base on the K-P Criterion

4.3.1 Background of Mathematical Programming Model

In capital budgeting problems, the linear programming (LP) technique is widely utilized within a mathematical framework of optimization with a linear objective function and subject to a set of linear constraints, especially restricted by limited capital [131][11][12][16][18][132][133]. The objective function represents the total discounted cash flows of the multi-investment projects selected in multi-period. We develop an integer program (IP) capital allocation model [134][135][14][15][113][136], in which decisions of project selection are explicitly made by binary variables. In other word, a decision variable of "0" for a certain project means a firm does not accept the investment project, a decision variable of "1" means that the project will be accepted or funded in capital allocation problem. In other words, no partial funding is allowed. Therefore, we use the integer linear programming (ILP) to maximize the NFNPV.

4.3.2 The Kim-Park model

We call the process of measuring the worthiness of project and allocating resources based on the NFNPV to be the Kim-Park method or the K-P criterion. The problem of selecting the combination of investments that maximizes the NFNPV for a particular set X of investment proposals is equivalent to solving the following ILP problem at each decision period:

The conceptual objective function is comprised of maximizing FNPV (NPV + option value) and minimizing Option price, therefore, the mathematical capital allocation model for a group of projects in a single period will be shown as:

$$W_0 = max \sum_{j=1}^{J} (NPV_{0j}x_{0j} + ROV_{0j}y_{0j} - \lambda ROP_{0j}y_{0j}) \tag{4-8}$$

$$s.t. \quad \sum_{j=1}^{J} (I_{0j}x_{0j} + ROP_{0j}y_{0j}) \le B_0 \tag{4-9}$$

x_{tj}: project selection variable for project j at time t / decision variable

y_{tj}: option selection variable for project j at time t / decision variable

ROV_j: real option value for project j

ROP_j: real option price for project j

λ: coefficient of real option price

I_{tj}: the initial investment cost for project j at time t

B_t: budget allowed at time t

Recall that valuation of the real option with the loss function approach described in Chapter 3 is shown as,

$$ROV_j = \sigma[PV_j] \cdot \left[\phi(z_j) - z_j\left(1 - \Phi(z_j)\right)\right] \tag{3-24}$$

where,

$$z_j = \frac{I_j - E[PV_j]}{\sigma[PV_j]}$$

and the option premium based on the relationship with the CNPVaR was shown ealier as,

$$ROP_j \leq \min(ROV_j, CNPVaR_j) \tag{4-5}$$

Furthermore, since we assume a periodic investment decision environment, multiple proposals will be proposed over horizon time. Thus, the proposed model after the first budget period will be shown as

$$max \sum_{t=1}^{T} \frac{W_t}{(1+r)^t}, t \geq 1 \tag{4-10}$$

From the second period, delayed projects from the previous period should be considered to compete with new proposed projects. Moreover, when they are considered, the NPV and investment cost of the delayed projects should be updated with new information.

$$W_t = max \sum_{j=1}^{J} (NPV_{tj}x_{tj} + ROV_{tj}y_{tj} - ROP_{tj}y_{tj}) + \sum_{d=1}^{D} UNPV_{td}x_{td} \tag{4-11}$$

$$s.t.\ Total\ investment\ (\ projects + options + delayed\ projects(if\ applicable))$$
$$\leq (assinged\ budget)\ + (budget\ left\ over\ from\ previous\ period)$$
$$+ (cash\ generated\ from\ selected\ projects)$$

$$\sum_{j=1}^{J}(I_{tj}x_{tj} + ROP_{tj}y_{tj}) + \sum_{d=1}^{D} UI_{td}x_{td} \leq B_t + \left(B_{t-1} - \sum_{j=1}^{J} I_{(t-1)j}x_{(t-1)j}\right)e^{(1+r)} + \sum_{g=1}^{G}\sum_{j=1}^{J} a_{tj}x_{tj} \tag{4-12}$$

$$x_{tj} + y_{tj} \leq 1, for\ delay\ option$$

$$x_{tj} - y_{tj} \geq 0, for\ abandonment\ option$$

$$x_{tj}, y_{tj} = \{0,1\}, \forall t, \forall j$$

J: the number of projects

D: the number of delayed projects from the previous period

G: the number of considered groups; new projects are being proposed at each period

T: the horizon time

a_{tj}: the expected cash-flow for project j at time t

I_{tj}: the initial investment cost for project j at time t

r: the WACC (Weighted Average Cost of Capital)

$UNPV_{td}$: updated NPV value for delayed project d from previous period

UI_{td}: updated investment cost value for project d from previous period

Since the real option mitigates the risk, the risk of worst case is smaller than CNPVaR or option premium, because the firm would just lose the option premium.

4.3.3 Consideration of Early Termination.

The abandonment option considers the value from the possibility of early termination when the resale value is greater than the remaining project value, which is ignored by the traditional

NPV analysis. Therefore, the early termination time will be decided by comparing the remaining project value with the resale project value, and we may accomplish this by adding the following constraint.

$$\sum_{n=m}^{N} \frac{a_n}{(1+r)^n} - \frac{I_j\left(1+q_j\right)^{-m}}{(1+r)^m} \leq 0, m \leq N \tag{4-13}$$

n: the current study periods

N: the total life of project j

m: early termination time of project j

a_n: cash flow of project j at time n

q: discount rate for resale value of project j

Then the NPV formulation associated with the abandonment option should be modified:

$$NPV_j = -I_j + \sum_{n=0}^{m} \frac{a_n}{(1+r)^n} + \frac{I_j(1+q)^{-m}}{(1+r)^m},\ m \leq N \tag{4-14}$$

4.3.4 Consideration of Interrelated Proposals.

The K-P criterion can easily handle dependent relationships among proposals such as mutually exclusiveness and contingency. We may add the following restrictions for each set of mutually exclusive proposals:

$$\sum_{i \in J} X_i \leq 1 \tag{4-15}$$

where the summation is over a set J of mutually exclusive projects in each decision period. This restriction ensures that at most only one proposal of the set J will be accepted if we do not allow any partial project funding. To handle contingency relationships, we may add another constraint in the following form:

$$X_j \leq X_m \tag{4-16}$$

where acceptance of proposal j assumed to be conditional upon acceptance of proposal m [19].

4.3.5 Consideration of Covariance among Proposals.

Even though the K-P criterion assumes independence among proposals for modeling simplicity, but we can consider any interdependences within project cash flows and among the projects as well. This implies that covariance between the cash flows and among the proposals in the combination must be recognized to determine the overall risk. This will change the dynamics of calculating the option value as well.

4.4 Summary

In summary, in this chapter we have developed an option pricing method based on the CVaR concept, which will be used as a basis to measure investment flexibility. Then we have devised the K-P index which integrates three critical elements as a single measure in evaluating a risky investment project. The three elements are profitability, variability, and flexibility. By this K-P measure, we prefer a large profitability, a smaller variability, and a larger flexibility. However, the variability and flexibility are positively correlated – in other words, a smaller variability translates into a smaller flexibility, and vice versa. However, when we are comparing different investment projects, the K-P measure clearly ranks a larger K-P value higher than the smaller one, so the overall project portfolio selection based on this measure would lead to better value creation for the firm. To demonstrate how effective the K-P criterion is, we develop a simulation model in Chapter 5.

Chapter 5. Description of the Simulation Models

In Chapter 4, we presented the three key elements to define the K-P criterion: the maximization of the expected present worth (profitability), the minimization of the expected loss (variability), and the maximization of the flexibility in future investment activity (flexibility). Even though our effort to integrate these three elements into a single measure is worth exploring, it is important to demonstrate the effectiveness of the criterion in much more an elaborate simulated capital budgeting environment. For this purpose, this chapter describes the computer simulation models to create the investment opportunities in each decision period, select the projects by applying the K-P criterion and track the wealth accumulation from the invested projects. Due to the stochastic nature of the investment environment as described in Chapter 1, project cash flows will be generated as a function of many random elements – unit price, demand quantity, variable cost, and others – all having some type of predefined probability distributions. These randomness in cash flows will create a wide range of possibilities in terms of project selections. Since investment decisions are made on a periodic basis over a long period of time, an investment decision made at the current period will affect the decisions on future periods. As a result, it is almost implausible to obtain a practical solution to this type of multi-stage capital allocation decision problems by any analytical means. So, we have to resort on computer simulation.

The chapter is organized as follows: First, assumptions for the simulation model will be defined. Second, the background of simulation process will be presented in two parts: 1) descriptions of the basic mathematical formulations, 2) the procedure of creating investment opportunities. Third, the description of simulation process will be presented using the flow charts.

5.1 Assumptions of the Simulation Model

In order to build an efficient simulation model based on the mathematical model presented in Chapter 4, we define the following assumptions:

- The firm's goal is to create the largest wealth through various investment activities.
- The action of the capital investment decision is recognized by a sequence of discrete events, which occurs periodically and affects future decisions. For each period, a new set of projects are being proposed for funding consideration. However, the firm has no ability to predict the forthcoming investment opportunities in advance.
- When new projects are proposed, the firm gets to know the initial investment cost, the project life, and the means and variances of periodic cash flows. If allowed, a project with a delay option must be exercised within one period. However, a project with an abandonment option, once selected, can be exercised at any time over the life of the project.
- Neither partial funding of a project nor any partial payment of option premium is allowed.
- Since the project cash flows are a function of many random variables –unit price, demand quantity, variable cost, and others – the resulting project value distribution, that is obtained by aggregating the discounted random cash flows, is normally distributed based on the central limit theorem.
- In obtaining the project value distribution, a weighted average cost of capital (WACC) is adopted under the assumption of a normal business risk reflected in the discount rate.

- The confidence level that represents a risk tolerance, also known as a risk preference or a risk appetite, is the parameter predetermined by a company's guideline or policy. The guideline or policy matters could be a collective decision of board members or risk management committee in the company.
- The term "project value (PV)" refers to the net present value of the project less its investment costs.
- To reflect the increasing uncertainty about the cash flows occurring further out in the future, the variance of periodic cash flow is gradually increasing proportionally over time based on the Brownian motion.
- In generating the periodic cash flow realizations, two situations are considered:
  - Mutually independent cash flows: cash flows are randomly generated based on a normal distribution with given mean and variance of each cash flow.
  - Perfectly positively correlated cash flows: cash flow at period *n* is perfectly positively correlated with cash flow at period *n*-1. This is simply for modeling convenience as it is rather difficult to estimate cross correlation coefficients among project cash flows for projects being proposed at each decision period.
  - By examining the two extreme cases – mutually independent and perfectly correlated cash flows, we can know of the partially correlated situation as its results should be bounded between mutually independent cash flows and perfect correlated cash flows.
- Capital budgets consist of two parts – budgets allocated in each decision period, and cash infusion from the projects undertaken prior periods. Any unused funds will be temporarily

invested in assets to earn a risk-free interest and returned to the next budget period to be added on that period of budget.

5.2 Background of the Simulation Model

5.2.1 The Basics of Mathematical Formulations

In order to explain the details of simulation model, we need to briefly summarize the mathematical formulations of three key elements that define the K-P criterion: the maximization of the expected net present worth (profitability), the maximization of the flexibility in future investment activity (flexibility), and the minimization of the expected loss (variability).

The Maximization of the Expected Net Present Worth

In section 4.1.2, the net present value (NPV) of each project is calculated by aggregating the discounted cash flows in each period. When a cash flow in each period is a random variable, the expected net present value is denoted by

$$E\left[NPV_j\right] = \sum_{n=1}^{N} \frac{E[a_n]x_j}{(1+r)^n} - I_j$$

j: the number of projects

x: the decision variable for project j

N: the project life for project j

r: the discount rate

I_j: the initial investment cost for project j

The Maximization of the Flexibility in Future Investment Activity

In section 3.3.2, the real option valuation based on the standardized loss function is introduced. Real Option Value (ROV) with investment cost (I) is presented by

$$ROV(I) = \sigma[PV] \cdot L\left(\frac{I - E[PV]}{\sigma[PV]}\right) = \sigma[PV] \cdot L(z) \tag{3-16}$$

where, $z = (I - E[PV])/\sigma[PV] = -E[NPV]/\sigma[PV]$

I : the initial investment cost

$L(z)$: the standardized loss function

$E[PV]$: the expected project value, $E[PV] = E[NPV] - I$

$\sigma[PV]$: the standard deviation of the project value, $\sigma[PV] = \sigma[NPV]$

Two different loss functions are presented to calculate the option value for delay option and abandonment option respectively. The standardized loss function $L(z)_{call}$ for the delay option is the partial expectation of the PV distribution when the realization is greater than a threshold x. The standardized loss function $L(z)_{put}$ for the abandonment option value is the partial expectation of the PV distribution when the realization is less than a threshold.

$$L(z)_{call} = \int_z^\infty (x - z)\phi(x)dx = \phi(z) - z\big(1 - \Phi(z)\big) \tag{3-17}$$

$$L(z)_{put} = \int_{-\infty}^z (z - x)\phi(x)dx = \int_{-\infty}^z z\phi(x)dx - \int_{-\infty}^z x\phi(x)dx = z\Phi(z) + \phi(z) \tag{3-18}$$

The Minimization of the Expected Loss

In section 4.1.2, the Net Present Value at Risk (NPVaR) and the Conditional Net Present Value at Risk (CNPVaR) were introduced along with Value at Risk (VaR) and the Conditional Value at Risk (CVaR) as a risk measure. They are presented as follow:

$$NPVaR_p(X) = E[NPV] + \Phi^{-1}(1-p)\sigma[NPV] = E[NPV] + z\sigma[NPV] \tag{4-3}$$

$$CNPVaR_p(X) = E[NPV] - \left(\frac{\phi(z)}{\Phi(z)}\right) \cdot \sigma[NPV] = E[NPV] - \left(\frac{\phi(z)}{1-p}\right) \cdot \sigma[NPV] \tag{4-4}$$

where

p: the confidence level

$\Phi^{-1}(x)$: the inverse cumulative distribution function of a normal distribution.

z: the correspondence of the inverse cumulative distribution function of a normal distribution.

$\phi(x)$: the probability density function of a normal distribution.

Therefore, once we obtain the expected value and standard deviation of NPV distribution, we can determine the NPVaR and the CNPVaR.

We also have shown what the reasonable real option price to pay is by comparing the conditional net present value at risk (CNPVaR) and real option value (ROV) in Section 4.1. The CNPVaR captures the average of extreme losses in the investment project based on a firm's risk tolerance level, and the ROV represents the maximum real option payment. Therefore, if the CNPVaR is greater than the ROV, the maximum real option price will be the same as the ROV, but if the CNPVaR is smaller than the ROV, the maximum real option price will be the CNPVaR not the ROV because CNPVaR is the maximum loss the firm could take.

$$Real\ Option\ Price\ (ROP) \leq \min(ROV, CNPVaR) \tag{4-5}$$

5.2.2 Description of Key Logics of the Simulation model

Generation of the Basic Cash Flows

In this research, we assume that a group of ten projects is being proposed in each period. Certainly, we could increase the number of proposals to consider in each period, but our main focus is to understand the dynamics of the K-P criterion in the multi-stage capital allocation process. In Section 3.4, we presented the example of a simple project cash flow statement as it included many elements such as sales price, unit sale, variable cost, fixed cost, depreciation, and tax. To seek a variety of cash flow compositions, our basic simulation model generates project cash flows during the first period as shown in Table 5-1. Some of the characteristics for the project group are:

Table 5-1 Example of Cash flows and additional information

n	P001	P002	P003	P004	P005	P006	P007	P008	P009	P010
0	$ 0	0	0	0	0	0	0	0	0	0
1	600	800	800	1000	700	800	600	1200	1000	700
2	620	600	600	500	600	600	750	400	600	700
3	610	500	800	800	700	700	600	500	300	700
4	600	800	600	500	500	800	750	500	800	700
5	630	400	700	800	700	700	600	800	600	700
IRR(%)	16%	18	23	25	18	24	19	23	21	22
E[PV]	$2203	2284	2541	2633	2317	2596	2378	2518	2434	2523
σ[PV]	800	938	721	445	379	948	388	1008	730	409

- The internal rate of return (IRR) of each project ranges between 15% and 25%.
- All projects have the same initial investment of $2,000 and project life of 5 periods. The time unit can be of any duration, such as a month, a quarter, a semi-annual or an annual.
- According to the Brownian motion [137][127], the variance is basically increasing proportional to time. We assume that the variance of each cash flow is increasing at the rate of 10% over the previous period.

- The risk-free rate is 6%, and the weighted average cost of capital (WACC) is 12%.
- Three periods of cash flows are used in this simulation and their descriptions are shown in Appendix 1.

Mapping projects

In order to allocate the limited capital more effectively, the K-P criterion classifies the proposed projects into two groups: projects that should be considered for immediate funding and projects that could be delayed or abandoned in the future periods. For example, for a project with a high expected net present value but having a low option value, it could be more advantageous to fund the project without any further delay. On the other hand, for a project with a small expected net present value but having a high option value, it would be a better strategy to defer our decision until we have more information about the project. The question is how we classify the projects into two groups. One way to accomplish this task is to introduce an E[NPV]-Option Value chart as shown in Figure 5-1 where all projects are positioned according to their expected NPV and option values. To facilitate this grouping, we need to establish a dividing line that separates the projects into two areas. For example, a 45 degree diagonal line in Figure 5-1 means a slope of 1 or E[NPV] = Option Value. With this slope, any projects located above the line put into the "invest project without option" group and all other projects are placed in the group for future funding or abandonment consideration.

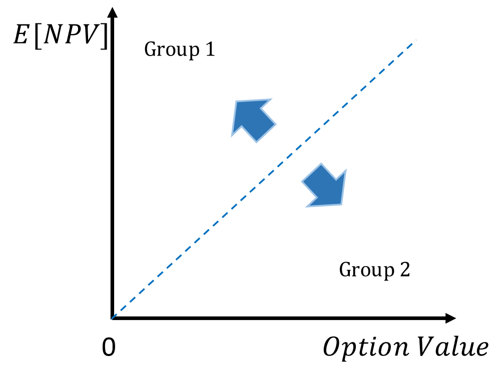


Figure 5-1 Classifying the projects into two groups in the simulation model

In terms of grouping projects, we could consider the Luehrman's method [92][93] which divides the option space based on the two metrics, the value-to-cost on the horizontal axis and the volatility[1] on the vertical axis. Each project is grouped into one of six different areas (or investment strategies) in the chart. However, it is rather difficult to determine the variance of the project return, which is one of the parameters to calculate the option value using the Black-Scholes formula. On the other hand, our research adopts the standardized loss function approach to determine the option value, which does not require estimating the variance of project value return. It only requires the mean and variance of project value distribution. Therefore, our approach to divide the projects according to the slope parameter based on E[NPV] and option value serves our mapping task better in practical sense.

An example of mapping projects given in Table 5-1 is shown in Figure 5-2. In Figure 5-2(a) uses the expected NPV and standard deviation of the NPV distribution, where Figure 5-2(b) replaces the standard deviation with the option value. Even though our intention is to adopt the format of Figure 5-2(b) in our research, we simply present Figure 5-2(a) for comparison purpose.

[1] He called it "Cumulative Volatility" as a measure of uncertainty by standard deviation times square of the number of periods, or $\sigma\sqrt{t}$. He used the variance of project return instead of project value.

For example, project 004 (P004) has a high expected net present value with a low standard deviation, but it also has relatively a higher real option value. It basically means that P004 is a good project, worth investing now in the traditional capital budgeting framework but also could be even better deferring to add more value to the firm as it commands a high option value. This type of information is not revealed in the traditional mean-variance chart.

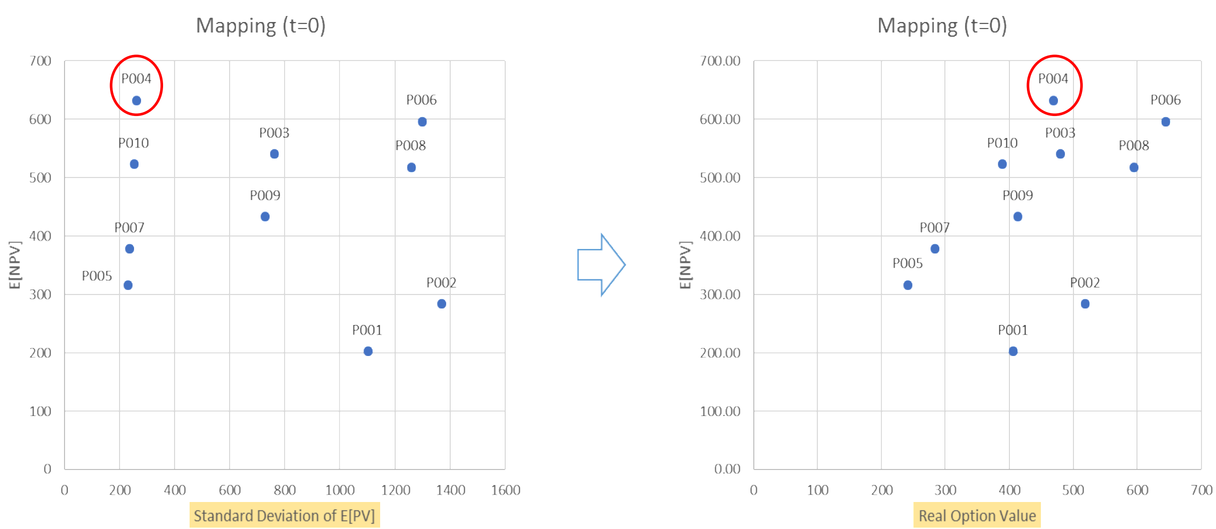


Figure 5-2 Mapping projects with two different coordinates

Deciding the threshold s* to determine investment strategies

In Figure 5-1, we have shown the conceptual idea on how to divide the projects into two groups through establishing a slope determined by the decision maker (or the firm). To use the K-P criterion, we need to assign the value of the slope parameter. We formally define such a slope parameter as

$$slope = s = \frac{Expected\ Net\ Present\ Value}{Real\ Option\ Value} = \frac{E[NPV]}{ROV} \quad (5\text{-}1)$$

For each project, we compute the slope according to Equation (5-1). Depending on the value of the slope, we may interpret the project as follows:

- Project with a higher slope value: It has relatively a smaller option value, so the embedded option strategy may be ignored or make investment decisions based on the expected NPV.
- Project with a smaller slope value: It has relatively a higher option value, so it is worth exploring the option strategy.

To create the largest possible terminal value to the firm through investment activities, we need to consider both strategies: some projects should be undertaken without real options and some projects should be considered with real options. The question then arises: how to decide a proper strategy for each investment project - whether considering real options or not? To answer this question, we may establish a threshold slope, s^*, to separate the projects into two groups. We call Group 1 for the projects without considering option strategies, and Group 2 for projects with option strategies.

- With establishing a larger s^* implies that more projects with option strategies would be considered.
- With choosing a smaller s^* means that more projects without option strategies would be considered.

As an example, consider Figure 5-3 where we set a threshold slope $s^*$to one. Then, P003, P004, P005, P007, P009, and P010 belong to Group 1, and P001, P002, P006, and P008 are placed into Group 2. This grouping will serve a very important role in our simulation model. Any funding commitments to projects with real options (either delay or abandonment) will be from Group 2.

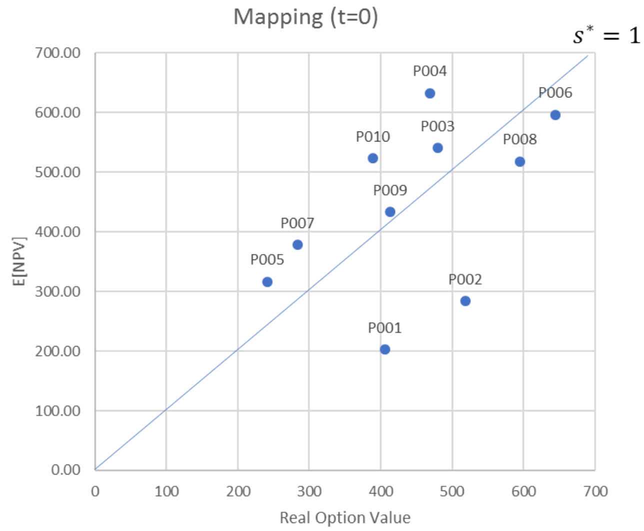


Figure 5-3 An example of grouping projects based on a threshold slope, s*=1

A threshold slope could be also determined by the budget availability at each decision point as well as the risk tolerance determined by the firm. In this research, we will conduct a sensitivity analysis to see how sensitive the choice of s* on the value creation of the firm is.

Updating the cash flows for the delayed projects

In our multi-stage decision process, a set of investment projects is being proposed in each period, each project will be grouped into either Group 1 or 2. If the budget allows, we will select some projects from Group 2 by paying option premiums. Now the question is what the consequences of these delayed projects in the following budget period are. First, we will have some updated information about these delayed projects, so we need to update the cash flows of these delayed projects based on the best information available at that time. Then these delayed projects will be considered along with the new set of projects proposed in that period. In terms of simulation logic, the updating process would be as follows:

- Update the cash flows using Monte Carlo simulation starting with the means and variances of the initial cash flows of delayed projects. For example, if project 006 (P006) in Table 5-1 is delayed for one period, the updated cash flow in each period is generated by Monte Carlo samplings from the respective normal distribution with the periodic mean (which is the same as initial estimate) and periodic standard deviation. Table 5-2 presents the updated cash flows along with the data which is used in Monte Carlo sampling.
- Even though the means remain the same at their original estimates, the updated cash flows will take different cash flow sequences as the variance figures are changing at the rate of 10% over the previous period. Consequently, we will also have changing values of E[PV], and the IRR, due to random samplings.

Table 5-2 An example of updated cash flows for project 006

n	Initial cash flows ($)	Standard deviation ($)	Updated cash flows ($)
0	0	0.00	0
1	800	593.30	1166.3
2	600	466.69	898.49
3	700	571.05	1082.98
4	800	684.48	124.22
5	700	628.15	1006.93
IRR(%)	24%	-	37%
E[PV]	2,596	-	3,052
σ[PV]	948.94	-	983.46

Computing the resale value and the remaining value to determine an early termination

For all on-going projects, the firm needs to decide whether or not to continue or abandon the project. The abandonment option gives the firm an operating flexibility to terminate projects prematurely if the expected returns from terminating projects are greater than the estimated remaining (salvage or market) values at the decision point. The remaining value can be obtained by aggregating the discounted cash flows that are expected f we kept the project until its service life, and the expected value from terminating projects are the offers received to buy out these projects.

When $\widetilde{a_n}$ represents the updated cash flow at period t, the remaining value can be obtained by

$$Remaining\ value\ (RM) = \sum_{n=t+1}^{N} \frac{E[\widetilde{a_n}]}{(1+r)^n} \qquad (5\text{-}2)$$

$\widetilde{a_n}$: The updated cash flow at time n

N: The project life

t: The time of decision to be made

r: WACC (Weighted Average Cost of Capital)

Additionally, in order to estimate a resale value (market value) in this simulation, we assume that a resale value in each period decreases by a constant rate from the initial investment cost. Therefore, when q represents a constant decreasing rate for the resale value, it can be obtained by

$$Resale\ value\ (RS) = \frac{I}{(1+q)^n} \qquad (5\text{-}3)$$

I: The initial investment cost of project

q: The decreasing rate of project

Therefore, if the estimated current resale value (or market offer) is better than the remaining value, a project which has an abandonment option will be terminated (or exercised) before its project life.

5.3 Description of Simulation Process

Now we have defined the general assumptions and simulation logics to go with the K-P criterion, we describe two specific simulation models in detail. First model is designed to accommodate the delay options and the second model for abandonment options. We could develop a more general simulation model to consider two different types of options simultaneously, but we want to isolate each option in the model to see how effective the K-P criterion is in each type of investment environment. By knowing the performance of the K-P criterion in each case, we can easily draw a general conclusion for the mixed case.

5.3.1 The Simulation Process associated with Delay Options

When a delay option is available to a project, there is a critical question to be asked before making the final investment decision: Is it worth delaying? By delaying we have an opportunity to acquire a new piece of information about the project, so that we could avoid or limit the downside risk of the project. On the other hand, by delaying the project, we could incur some sort of opportunity cost if the project in fact was a good one and could capture more market share without delaying. It is always the trade-off between limiting the downside risk and the cost of delaying. Conceptually, it is worth delaying if the gain from hedging the potential downside risk is higher than the cost of delay. Since we do not know the future investment opportunities, the simulation model will provide some clue whether or not considering real options would be effective strategies.

Procedure

The process of the simulation model for projects with delay options is as follows:

Step 1: Estimate the cash flows of the projects.

Step 2: Calculate the decision parameters: net present value, conditional value at risk, real option value, and option price.

Step 3: Determine a threshold slope, s^*.

Step 4: Group the projects into Group 1 and Group 2 by applying s^*.

Step 5: Select projects to invest now from Group 1 based on the maximization of the expected NPV.

Step 6: Select projects to delay from Group 2 based on the maximization of the K-P criterion

Step 7: Reset the decision period (or advance the clock) from period t to period t+1.

Step 8: Estimate cash flows for newly proposed projects at period t+1.

Step 9: Identify the delayed projects from period t.

Step 10: Update the cash flows of the delayed projects from period t.

Step 11: Combine the newly proposed projects from period t+1 with the delayed projects from period t.

Step 12: If $t < T$ and go to Step 2. Repeat until it reaches $t = T$.

The simulation process for delay option is illustrated in Figure 5-4.

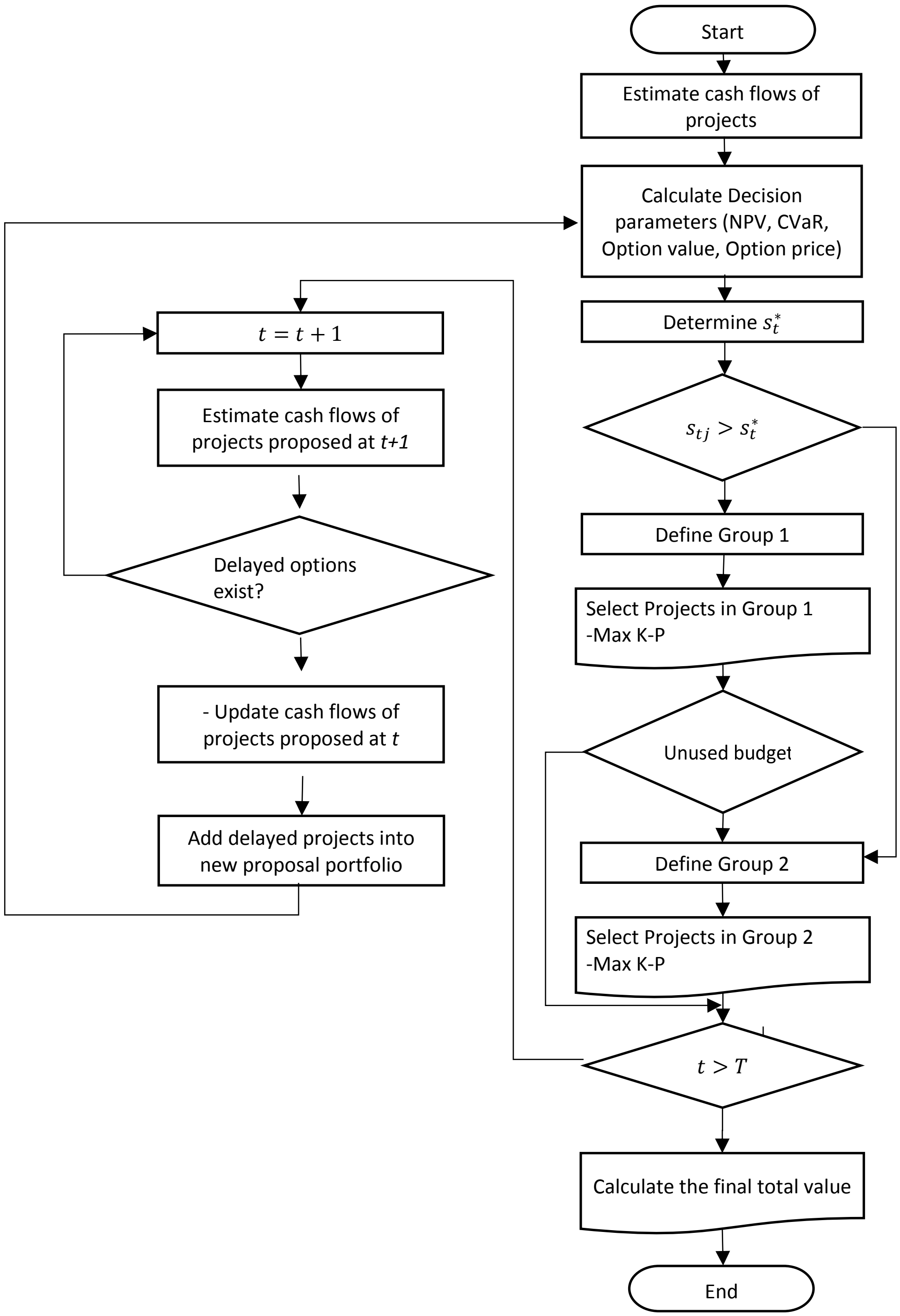


Figure 5-4 Simulation flow chart for projects associated with delay options

5.3.2 The Simulation Process associated with Abandonment Options

Under uncertainty, the project value at some future time can be less than the value originally estimated. An abandonment option allows us to terminate an investment project prematurely. In this simulation model, the resale value (market value) of the project will be compared with the remaining value from the project if continued.

Procedure

The process of the simulation model for projects with abandonment options is as follows:

Step 1: Generate the cash flows of the projects.

Step 2: Calculate the decision parameters: net present value, conditional value at risk, option value and option price.

Step 3: Determine a threshold slope, s* for abandonment options.

Step 4: Group the projects into Group 1 or Group 2 by applying the s^*.

Step 5: Select projects to invest now from Group 1 based on the maximization of expected NPV.

Step 6: Select projects to invest with abandonment options from Group 2 based on the maximization of the K-P criterion.

Step 7 Reset the decision period (or advance the clock) from period t to period t+1.

Step 8: Estimate cash flows for newly proposed projects at period t+1.

Step 9: Identify the projects with abandonment options.

Step 10: Estimate the resale value and the remaining value of each project with abandonment option.

Step 11: Compare between the remaining project value and resale value (market value) for projects in Group 2 that have an abandonment option.

Step 12: If the resale value > the remaining value, then the specific project is terminated.

Step 13: If $t < T$ and go to Step 2. Repeat until it reaches $t = T$

The simulation process for abandonment option is illustrated in Figure 5-5.

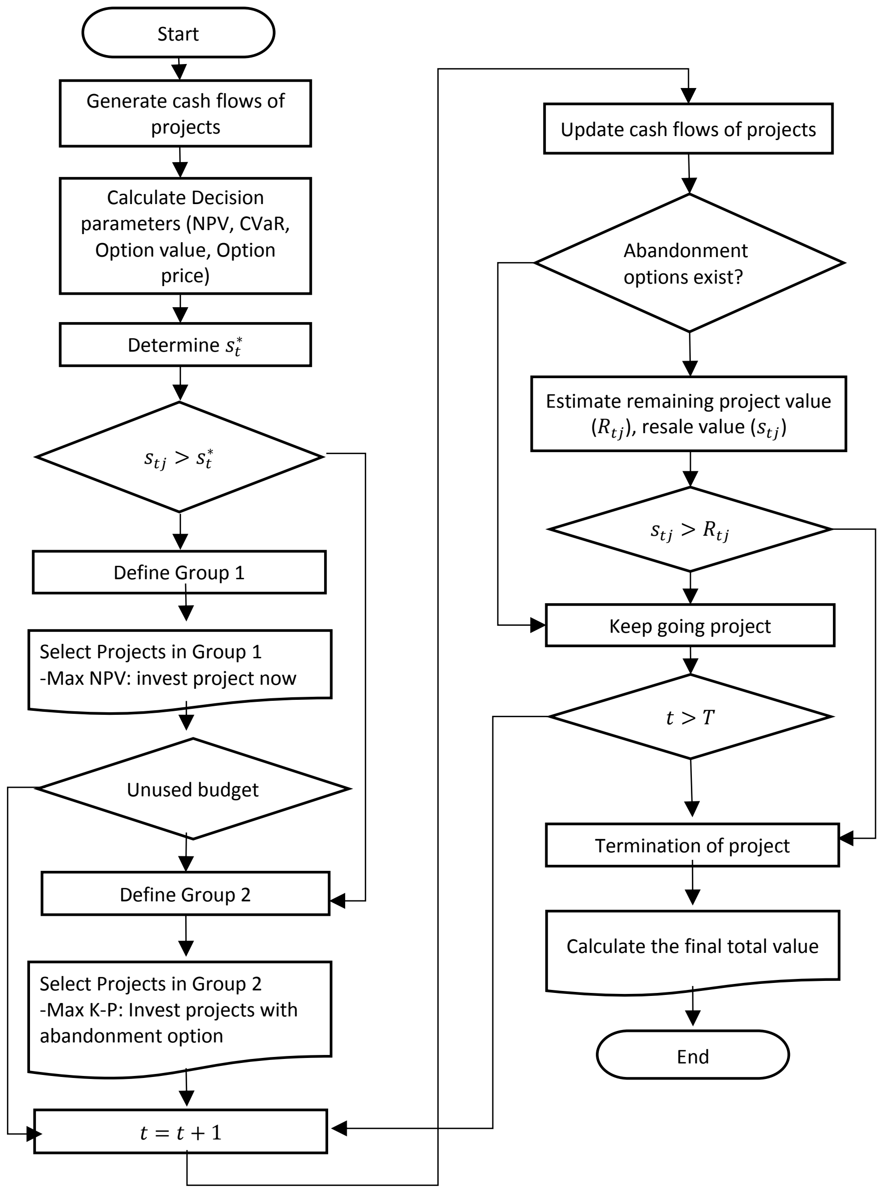


Figure 5-5 Simulation flow chart for projects associated with abandonment options

5.4 Summary

In summary, in this chapter we have described the general assumptions and simulation logics to use in testing the effectiveness of the K-P criterion. In doing so, we have presented the cash flow generation process and a process of grouping projects into two groups for funding consideration. Then we have developed two specific simulations models – one designed to consider the delay options strategies, and the other for abandonment option strategies.

Chapter 6. Simulation Results and Comparison of the K-P Criterion with Other Criteria

Having described the simulation models in the previous chapter, now we are ready to run the simulation models to see how effective the K-P criterion is when compared with other traditional models. The traditional models that will be compared with are the expected net present value criterion and the mean-risk model. In particular, we will present the details of simulation results and provide an in-depth analysis of the performance of the K-P criterion under varying investment environments.

This chapter is organized as follows: Section 6.1 describes the conceptual performance among the productivity, flexibility, and variability in the capital budgeting decision problem. Section 6.2 examines the characteristics of the different decision criteria in terms of the project selections. Section 6.3 presents the simulation results of each decision criterion as a capital budgeting tool for long-term wealth creation. Section 6.4 presents various sensitivity analyses for the key input parameters. All computational results are obtained by Matlab 9.2_R2017a.

6.1 Preference of Performance

Recall that there are three key investment factors to be considered in the capital budgeting in Chapter 4.

- We prefer a larger profitability, if all other things being equal.
- We prefer a larger flexibility, if all other things being equal.

- We prefer a smaller variability, if all other things being equal.

In Figure 6-1, we illustrate the conceptual preference among the three elements in a 3-D format. If the profitability and the flexibility are the same, then we may prefer a smaller variability. For example, we would prefer Y over X, in other words, Y dominates X. In general, a larger flexibility is associated with a larger variability, so we need a further trade-off between the variability and the flexibility. Thus, the K-P index developed in earlier chapter is to seek a trade-off among these three factors.

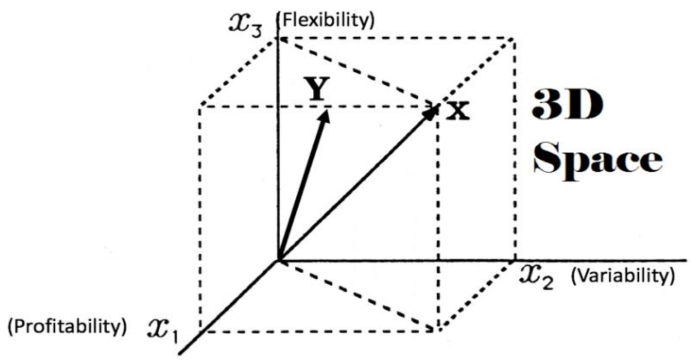


Figure 6-1 A 3D preference coordinate system

6.2 Short-Term Performance of the K-P Criterion

Many investment performance measures have been studied in finance and business. Well-known performance measures are alpha, beta, standard deviation, and the Sharpe ratio. All of these measures are designed to provide investors with the risk-return characteristics of an investment. In particular, the Sharpe ratio is the average return earned in excess of the risk-free rate per unit of volatility or total risk [139]. The Sharpe ratio has become the most widely used method for calculating risk-adjusted return. Even though the Sharpe ratio is designed for a risk-return measure of financial assets, we could borrow the similar concept to see what kind of risk-return characteristics each project portfolio created by the three different decision criteria has. The use of

Sharpe ratio will only be limited to gauge the periodic performance of the decision criteria on a short-term basis. We are simply interested in knowing whether or not the K-P criterion alters the return-risk characteristics in any drastic manner in each decision period, when compared with the traditional criteria.

6.2.1 The Sharpe ratio on Project Portfolio

The Sharpe ratio measures the portfolio performance based on both the rate of return represented a profitability and the portfolio risk as measured by the standard deviation of the portfolio return. It can be denoted as:

$$Sharpe\ ratio = \frac{(Average\ Protfolio\ Return - Risk\ free\ rate)}{Standard\ deviation\ of\ portfolio\ return} \quad (6\text{-}1)$$

A higher Sharpe ratio indicates a better adjusted return per given level of risk. Therefore, to increase the Sharpe ratio of an investment portfolio, we need to select projects which have a higher return or lower risk. However, it is difficult to apply for the real project because the standard deviation of portfolio return is rather complicate to determine. The reason is that the standard deviation of return in financial investment can be obtained by analyzing historical data, but in real projects there is no historical data to go by. Therefore, we could adopt the implied volatility for a real project, which is obtained by Equation (3-10) as a standard deviation of portfolio return.

$$\sigma = \sqrt{\frac{\ln\left(\frac{Var[X_t]}{E[X_t]^2} + 1\right)}{t}} \quad (3\text{-}10)$$

For each selected project in the portfolio, we compute the volatility of expected return using Equation (3-10). To compute the overall standard deviation of portfolio return, we need to aggregate the individual standard deviation by the proportion of the project in the portfolio. One thing that makes this process complicate is that each project in the portfolio may have a different life, and scale of investment could be different. Nevertheless, our objective is to approximate or gauge the degree of magnitude of return-risk of the portfolio created by each criterion in a controlled business environment.

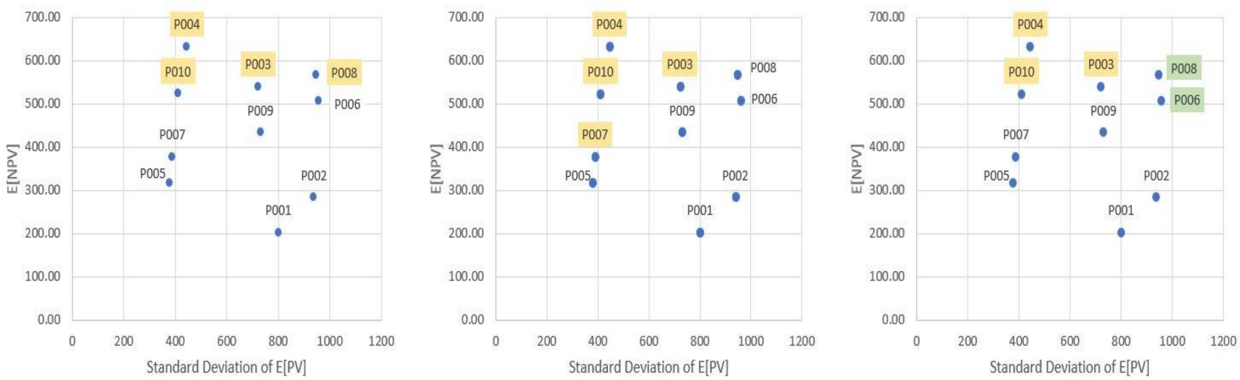
6.2.2 Comparisons of the Project Selections

As described in Chapter 5, in each decision period, we have a set of ten investment proposals submitted for funding consideration, and so we have a total of 30 investment proposals over three budget periods. Now just focusing on the first budget period, by applying the three decision criteria to the same set of investment proposals, the project selections by each decision criterion are shown in Table 6.1.

Table 6-1 Project selections for the first budget period

Criterion	P001	P002	P003	P004	P005	P006	P007	P008	P009	P010
max NPV	0	0	1	1	0	0	0	1	0	1
max NPV-CVaR	0	0	1	1	0	0	1	0	0	1
max K-P	0	0	1	1	0	0	0	0	0	1
Delay option	0	0	0	0	0	1	0	1	0	0

Figure 6-2 illustrates the selection of projects on the coordinate grid with standard deviation for the horizontal-axis and the expected net present value for the vertical-axis. The points refer to the position of each project reflecting the profitability and variability. Points in yellow color represent the projects selected for funding by each decision criterion. Points in green in Figure 6-2(c) denote the projects selected with delay option.



(a) max NPV (b) max NPV-CVaR (c) max K-P

Figure 6-2 Project selections by the three decision criteria during the first budget period

Case 1: Maximize the expected net present value

In Figure 6-2(a), the projects selected by the maximization of NPV are shown in yellow color. As we can see, selected projects – P003, P004, P008, and P010 – have higher NPVs among the ten proposals. It is not surprising as the expected NPV criterion just ignores any variability associated with the project.

Case 2: Maximize the net present value -Conditional Value at Risk

Figure 6-2(b) presents all project selections done by the typical mean-risk rule. Clearly projects P003, P004, P007, and P010 are non-dominant projects – higher means but smaller variances.

Case 3: Maximize the K-P criterion

Figure 6-2(c) depicts the situation where the K-P criterion would select the best projects during the first budget period. Projects P003, P004 and P010 are recommended for funding immediately but P006 and P008 are also included in the portfolio for future consideration – that is, they are delayed for one budget period. In the second budget period, these two delayed projects would be added on to the new set of projects proposed, resulting in a total of 12 projects competing.

The reason why these two projects are included for consideration in the second budget period is because of the potential upside profit. To delay these two projects, the firm had to pay option premium out of the budget during the first period. If any of delayed projects were not considered in the second period, the firm would simply lose the option premium associated with that project. This process continues on the next decision period.

6.2.3 The Sharpe ratio for the Three Portfolios

After selection of projects (or creating a funding portfolio) in each budget period, we may be interested in knowing how the firm's risk position has changed. To determine the Sharpe ratio for portfolios of real investment, we may need to come up with the parameters that define the Sharpe ratio. For this purpose, we do the following:

- The average portfolio return is obtained by the arithmetic mean of the IRR. Conceptually, once we identify all the expected cash flows in the portfolio against the total investment required to create that portfolio, we can determine the expected return on that portfolio.
- The risk-free rate is assumed at 6% in the simulation model throughout. This is higher than the current risk-free rate in the market place, but it can be changed to reflect the actual investment environment.
- The individual standard deviation of each project is calculated based on the logic described in earlier Section 6.2.1, but a volatility of project portfolio is obtained by aggregating these individual standard deviations.

$$Var\left(\sum_{i=1}^{N} a_i X_i\right) = \sum_{i=1}^{N} a_i^2 Var(X_i) + \sum_{i \neq j} a_i a_j Cov(X_i, X_j) \qquad (6\text{-}2)$$

If we assume that all projects in the portfolio are mutually independent, the covariance term drops out resulting in:

$$Var\left(\sum_{i=1}^{N} a_i X_i\right) = \sum_{i=1}^{N} a_i^2 Var(X_i) \tag{6-3}$$

N: The number of selected projects

a_i: The weight of project i, which is $1/N$ because the investment cost is assumed the same

$Var(X_i)$: The variance of project i, which is obtained by Equation (3-10)

Table 6-2 provides the modified Sharpe ratio of each portfolio from the three criteria along with the average return and the volatility. (Additional data of IRR, $E[NPV]$, and $\sigma[NPV]$ for each project can be found in Appendix 1.) As we can see, the maximization of NPV has the highest average return, but it has the highest volatility as well since it does not consider any risk in project selection. On the contrary, the maximization of NPV – CVaR has the lowest average of IRR and also lowest volatility because it considers only the down-side of risk, whereas the K-P lies in between the two.

Table 6-2 The Sharpe ratio and basis data of three criteria

	Max NPV	Max NPV-CVaR	Max K-P
Average return	23.35%	22.26%	23.24%
Volatility	0.2548	0.1989	0.2096
Sharpe ratio	0.6811	0.8178	0.8223

Table 6-2 further indicates that the K-P criterion has the highest modified Sharpe ratio, even though the difference between the K-P and NPV – CVaR is relatively small. Since the greater Sharpe ratio is desirable, the use of K-P criterion results in the best excess return per risk as it can hedge the potential risk through the option mechanism.

Another interesting observation is that both criteria (K-P and NPV-CVaR) have different mixes of projects in its portfolio, indicating that the use of the K-P criterion as a capital budgeting tool may improve the profitability without increasing the risk profile. Of course, it is premature to make this claim with this limited observation. Our intention is simply to point out that if such a short-term performance comparison is desired we could adopt the Sharpe ratio. Since our objective is not to measure the short-term performance of the K-P criterion, we do not intend to further extend the Sharpe ratio analysis.

6.3 Long-Term Performance of the K-P Criterion

Our research objective is to determine if the K-P criterion can be of an effective long-term capital budgeting tool in the face of uncertainty. Our first task is to see which decision criterion can bring the maximum wealth creation at the end of study period, by simulating the plausible investment scenarios described in Chapter 5. In specific, we adopted the simulation logic described in Chapter 5 as follows:

- Using the random number generation function in Matlab, we tracked cash flows generated for each investment proposal and recalculated the quantitative financial metrics, such as the expected net present value, real option value, and conditional value at risk. These values provide the basis to determine the K-P index and other measures.
- Each criterion may select a different set of investment projects based on its investment measure. Then the simulation model determines the total terminal wealth of each criterion in terms of cash accumulation in each iteration. In each iteration, a new set of random numbers is used, so we will have a new set of terminal wealth figures for the three criteria. This process repeats for 1,000 times.

- Since we have two basic real option strategies, we have two types of simulation model. One model is designed to consider only the delay options and the other model for the abandonment options.
- We present the simulation results according to the option strategy and give economic interpretations of the simulation outcomes for each decision criterion.

6.3.1 The Simulation results associated with delay options

We first consider the delay option strategy. In other words, all projects submitted for funding come with some sort of delay options. Either the expected NPV or NPV – CVaR does not take advantage of any investment flexibility in creating a project portfolio. Only the K-P criterion explicitly factors this investment flexibility into the funding decision. Figure 6-3 illustrates the final terminal values obtained by the three criteria through 1,000 simulation runs. Each simulation run creates one terminal value for each criterion based on the same set of investment proposals. In Figure 6-3, the terminal values in blue are produced by the expected NPV, the terminal values in red by the NPV-CVaR criterion, and the terminal values in yellow by the K-P criterion. Clearly, we see that the K-P criterion produces higher terminal values in a large percentage of the simulation runs, but we need a further statistical verification.

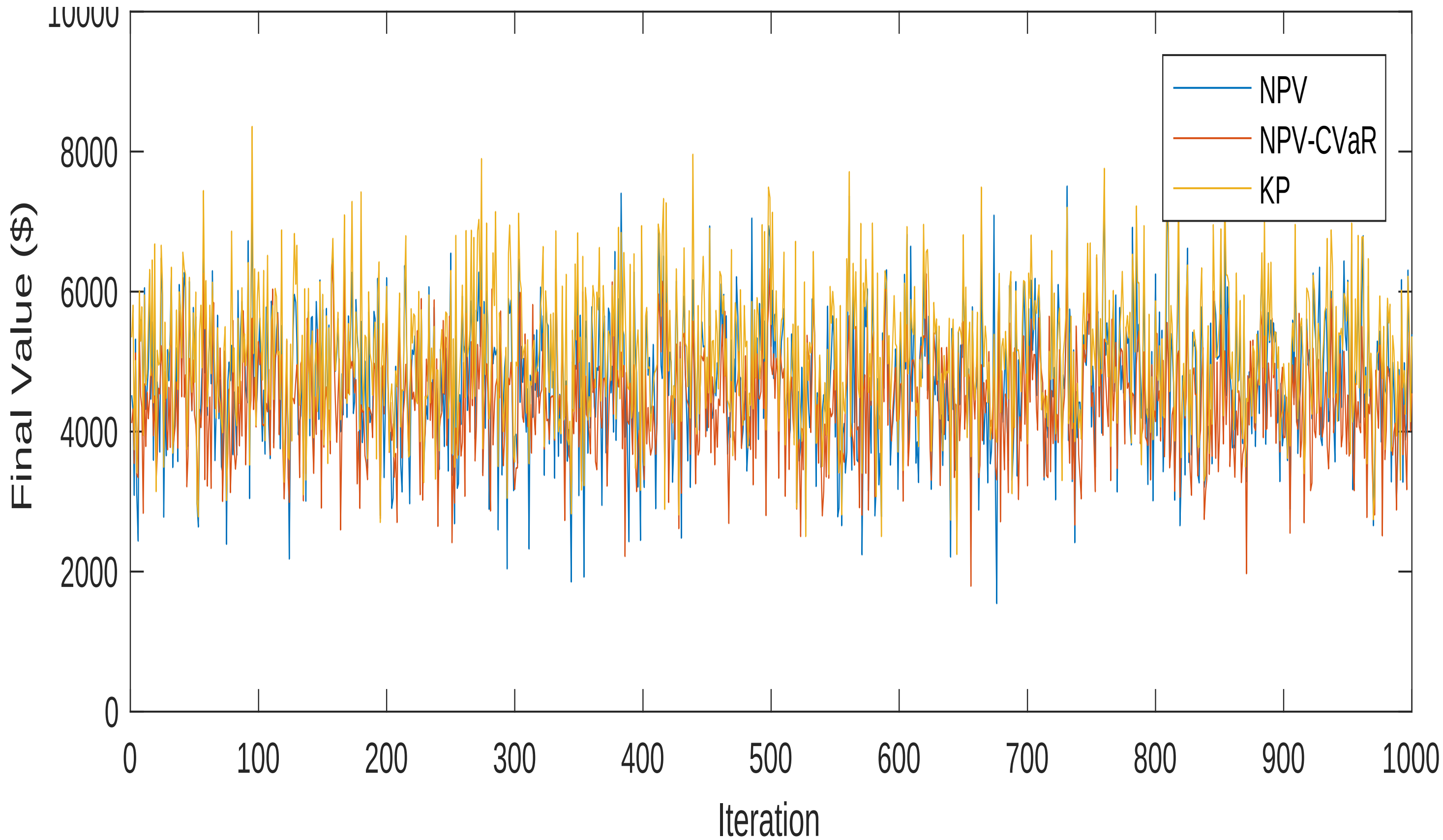


Figure 6-3 The total terminal wealth created by the three decision criteria with delay options

One way to visualize the performance of the K-P criterion is to compare the terminal values produced by the K-P criterion with those by the other two criteria. In Figure 6-4 (a), we first sort the 1,000 final values produced by the expected NPV criterion in ascending order. Then for each final value associated with the expected NPV criterion, we identify the final values attained by the K-P and NPV – CVaR for the same set of investment setting. In Figure 6-4(a), we clearly see that a large percentage of the final values produced by the K-P criterion lie above those by the NPV and NPV – CVaR, indicating a possible dominance. On the other hand, Figure 6-4(b) is basically the same as Figure 6-4(a) but the final values are ordered by the K-P criterion. Both figures show how the option plays an important role in creating value to the firm.

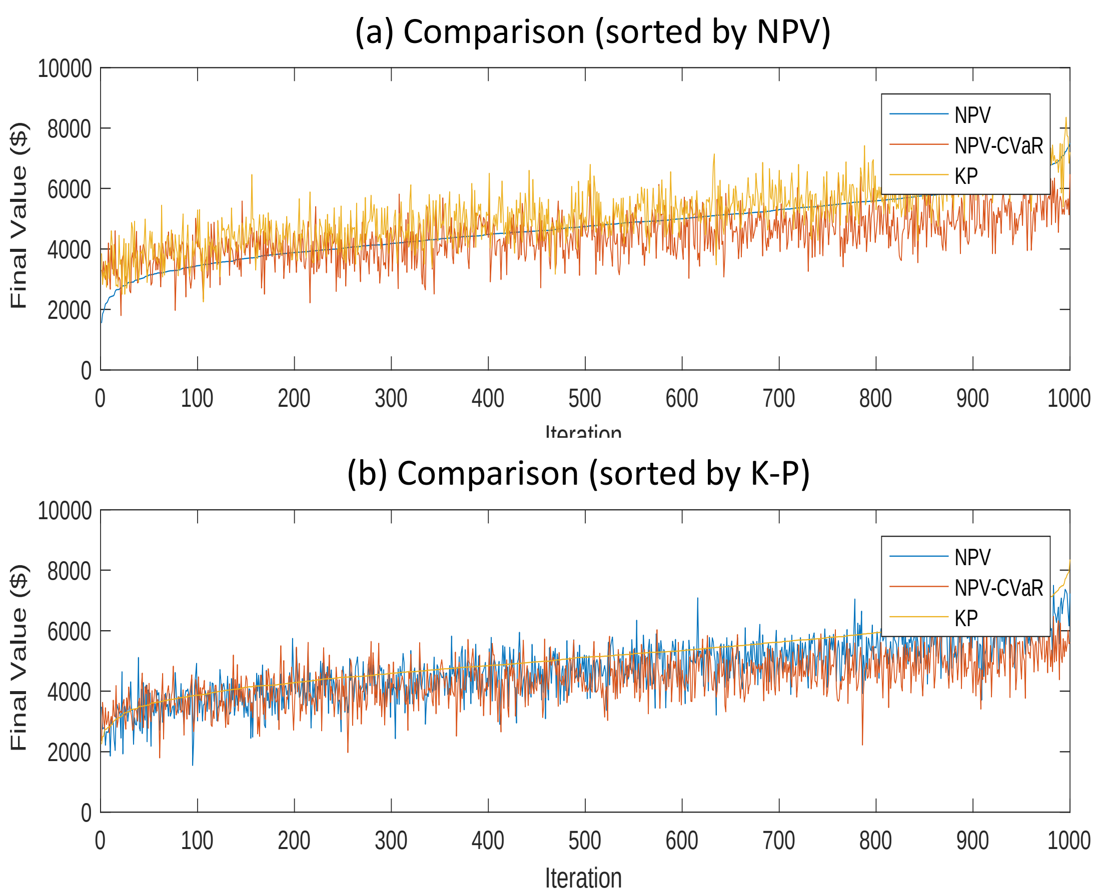


Figure 6-4 comparison of the total terminal wealth sorted by NPV and K-P criteria

Range of Final Value Distributions

Now instead of plotting the final values in terms of iteration sequence, we may compute the mean value of the final value distribution along with the range of the final values. Figure 6-5 depicts these statistics in box plots with the median represented by a red line and the mean value in a blue diamond. In this case, it happens that the mean and median values are very close each other.

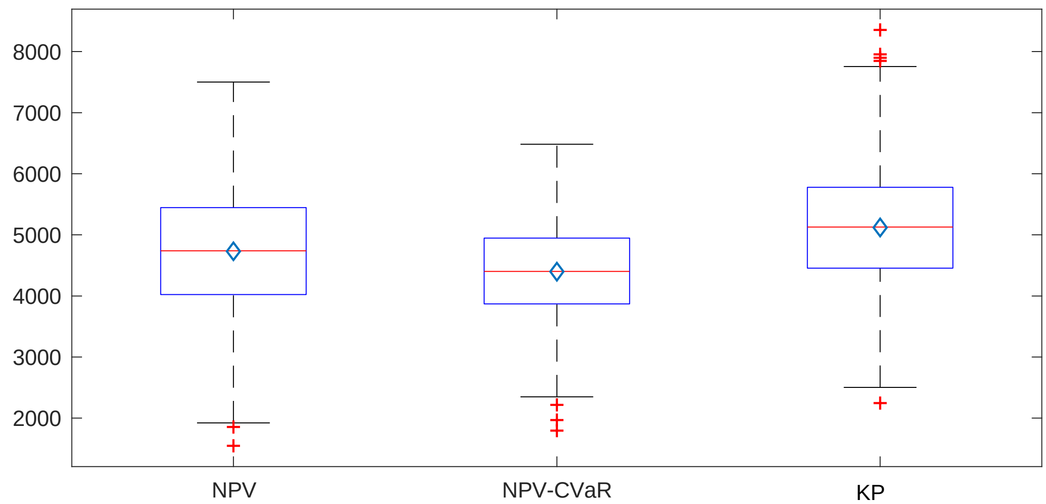


Figure 6-5 The box plot and average of the total terminal value from 1000 iteration

We see in Figure 6-5 that

- The median and average value of the K-P criterion are greater than other criteria,
- The worst final value produced by the K-P criterion is still higher than those of other criteria, and
- The NPV-CVaR has the narrowest range of final value distribution among the three criteria, indicating that the higher return is compromised at the expense of limiting the risk at the appropriate level.

Another statistic of interests is to see how many times the K-P criterion produced the higher terminal values than those two criteria out of 1,000 simulation runs. Table 6-3 shows that the K-P criterion produced the best terminal values 601 times out of 1,000, about 60% of the cases. As expected, the expected NPV criterion outperforms over the NPV – CVaR, implying that the expected NPV criterion works fine as a decision tool in a long-term capital budgeting environment in this example.

Table 6-3 The number of the best cases out of 1000 iteration when delay option involved

	Iteration	Max NPV	Max NPV-CVaR	Max K-P
# of best case	1000	257	142	601

Stochastic Dominance

Now we formally explore a more theoretical examination of the performance of the three criteria through the concept of the stochastic dominance. The histograms based on 1000 iterations for the terminal value are shown in Figure 6-6, and the differences among the three histograms are shown by overlapping the distributions on the same chart. The means and variances of the terminal value distributions from the expected NPV, the NPV – CVaR, and the K-P are as follows:

Table 6-4 The summary of distribution information associated with delay option

	Final Value Distribution		
Delay Option	NPV	NPV-CVaR	K-P
Mean	$4,691	4,295	5,167
Standard Deviation	926	727	998

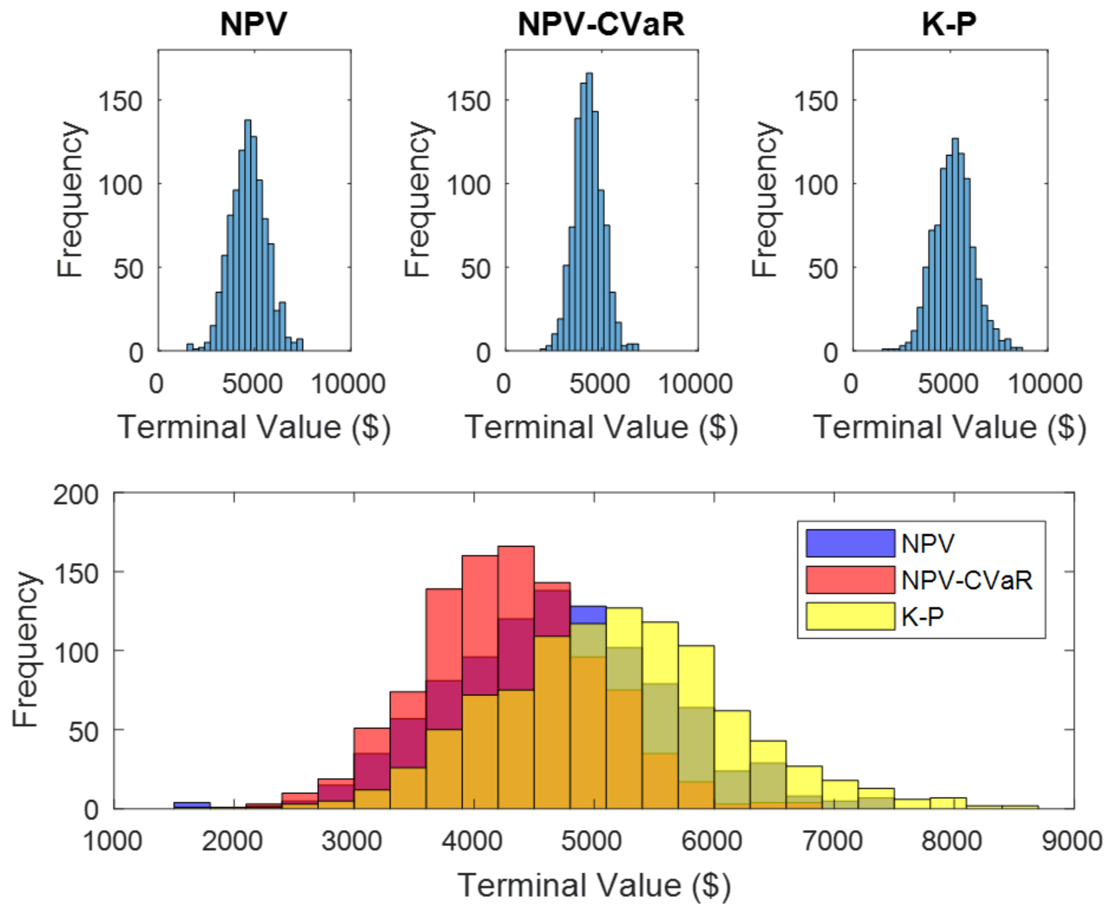


Figure 6-6 Terminal value histograms where the delay options are allowed

Then Figure 6-7 illustrates the normal-fitted probability distribution of the terminal values based on simulation results of 1000 iterations.

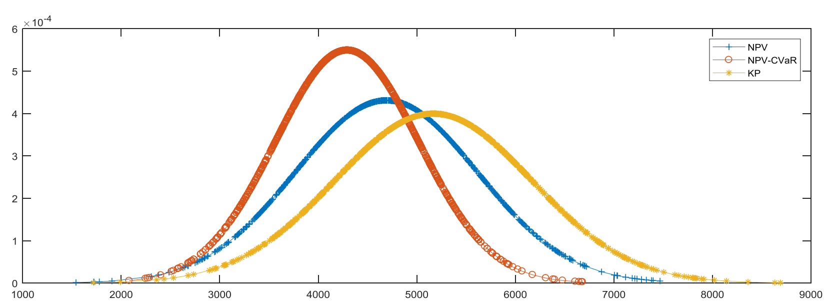


Figure 6-7 Normal-fitted probability distributions with delay options

Mean-Variance Rules:

We begin with the mean-variance rules to detect any obvious dominance among the three criteria. The dominance rules for the mean-variance criterion are:

- If project A has mean value the same as or higher than that of project B, and has a lower variance than B, we prefer project A.
- If project A has variance the same as or lower than that of project B and has higher mean than B, we prefer project A.

When we apply these mean-variance rules to the three final value distributions, we do not see a clear-cut dominance of a decision criterion over others.

- K-P versus NPV:
  - Mean value \$5,167 > \$4,691 and variance \$998 > \$926, no dominance
- K-P versus NPV – CVaR:
  - Mean value \$5,167 > \$4,295 and variance \$998 > \$727, no dominance
- NPV versus NPV – CVaR:
  - Mean value \$4,691 > \$4,295 and variance \$926 > \$727, no dominance.

The ultimate choice between the K-P and the NPV or between the K-P and the NPV – CVaR depends usually on the trade-off between the mean and variance for the decision maker. However, if the mean-variance rule fails, we can further examine the possibility of any stochastic dominance (probabilistic dominance) among the criteria [60].

Definition: First-Degree Stochastic Dominance. Given two random variables X and Y with cumulative probability distribution functions $F(x)$ and $G(y)$, we say that X D Y (X dominates Y) or F D G [$F(x)$ dominates $G(y)$], if the following conditions hold:

$$F(x) \leq G(y) \quad for\ every\ x$$
$$and\ F(x) < G(y) \quad for\ some\ x$$
$$E[X] > E[Y]$$

The theorem states that the cumulative distribution function (CDF) of X must lie below that of Y for at least one value and must lie nowhere above it, and the mean value of X must be greater than the mean of Y. The theorem is equally valid for continuous and discrete probability distributions. The only requirement for this first-degree stochastic dominance to hold is that the decision maker (or business) has a non-decreasing utility function of wealth, or simply the more profit the project generates the more valuable the project to the firm is.

Figure 6-8 clearly shows that the K-P criterion dominates both the NPV and NPV-CVaR probabilistically. In other words, the K-P criterion is the most effective in terms of wealth creation when it is applied to multi-stage capital budgeting decision problems. The higher variance of the terminal value distribution of the K-P criterion is worth taking considering the potential upside value.

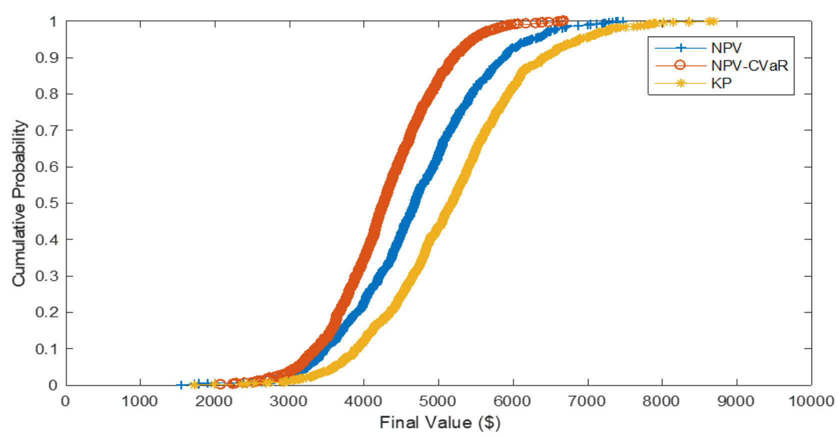


Figure 6-8 Cumulative terminal value distributions for the three decision criteria

6.3.2 The results associated with abandonment options

Now we consider the abandonment option strategy. In other words, projects submitted for funding come with some sort of abandonment options. As before, neither the expected NPV nor the NPV-CVaR takes advantage of any investment flexibility associated with project abandonment in creating a funding project portfolio. Only the K-P criterion explicitly factors this investment flexibility into the funding decision.

Figure 6-9 illustrates the terminal values produced by the three criteria through 1,000 simulation runs. Each simulation run creates one terminal value for each criterion based on the same set of investment proposals. In Figure 6-9, the terminal values in blue are produced by the expected NPV, the terminal values in red by the NPV-CVaR criterion, and the terminal values in yellow by the K-P criterion. Clearly, as with the case of delay option, we see that the K-P criterion produces higher terminal values in large percentages of the simulation runs, which begs for further statistical verification.

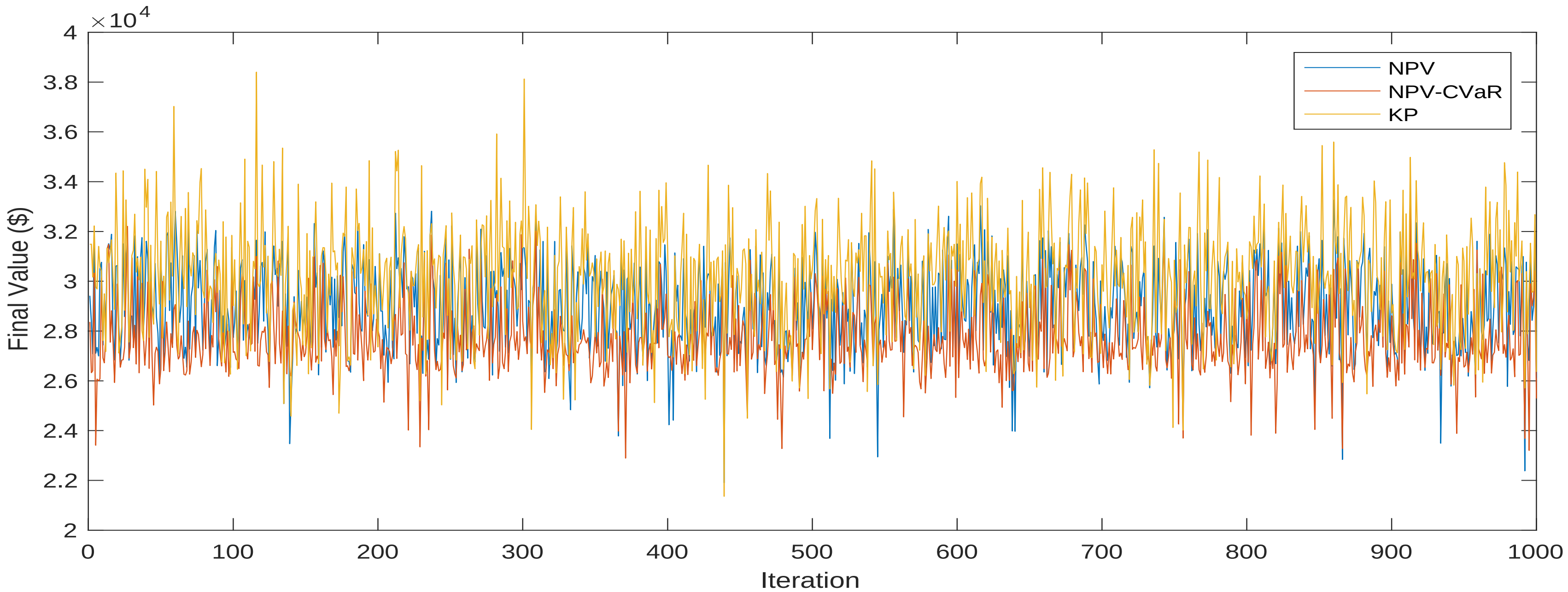


Figure 6-9 The terminal wealth produced by the three criteria with abandonment options

Figure 6-10 shows the results of the simulation model sorted by the terminal value obtained from the NPV and the K-P, and it presents clearly that the K-P criterion outperforms over other criteria.

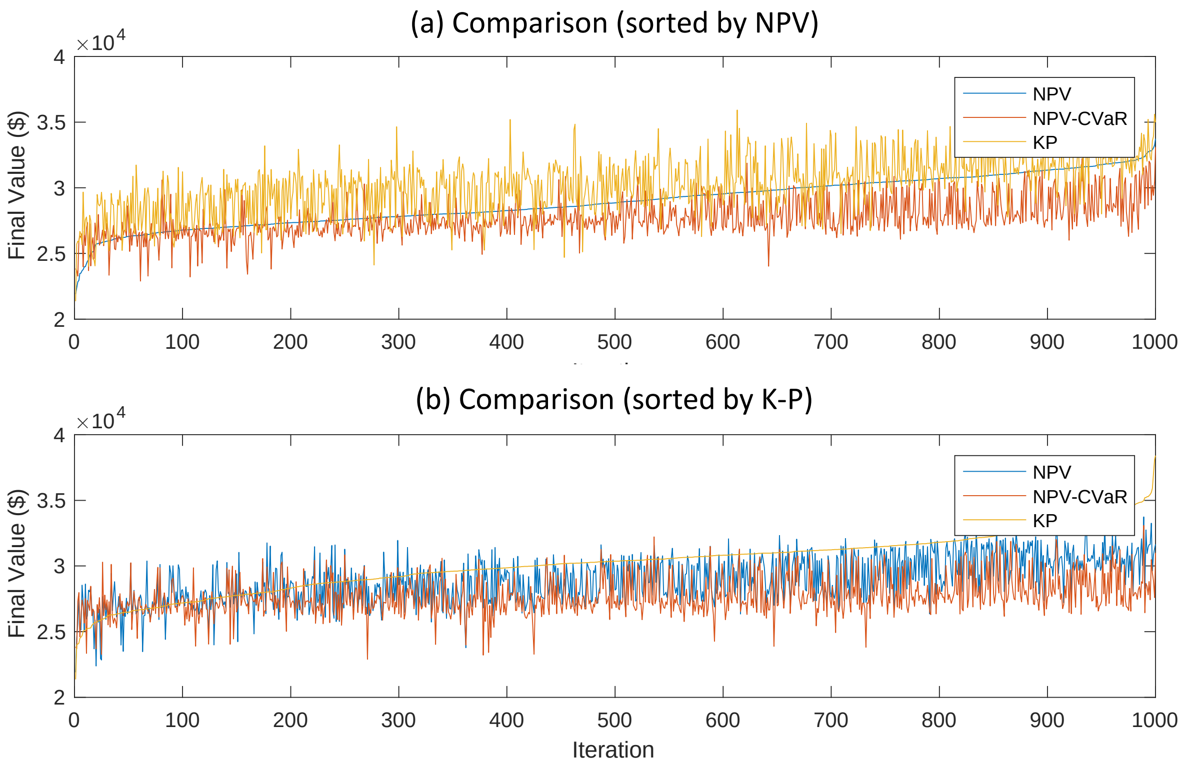


Figure 6-10 Comparison of the total terminal values sorted by NPV and K-P criteria

If we use a box plot to compare the performance of each decision criterion, we obtain Figure 6-11, which is very similar to Figure 6-5 for the delay option case. Once again, with abandonment options in the projects, the K-P criterion produced the highest average wealth creation compared with other criteria.

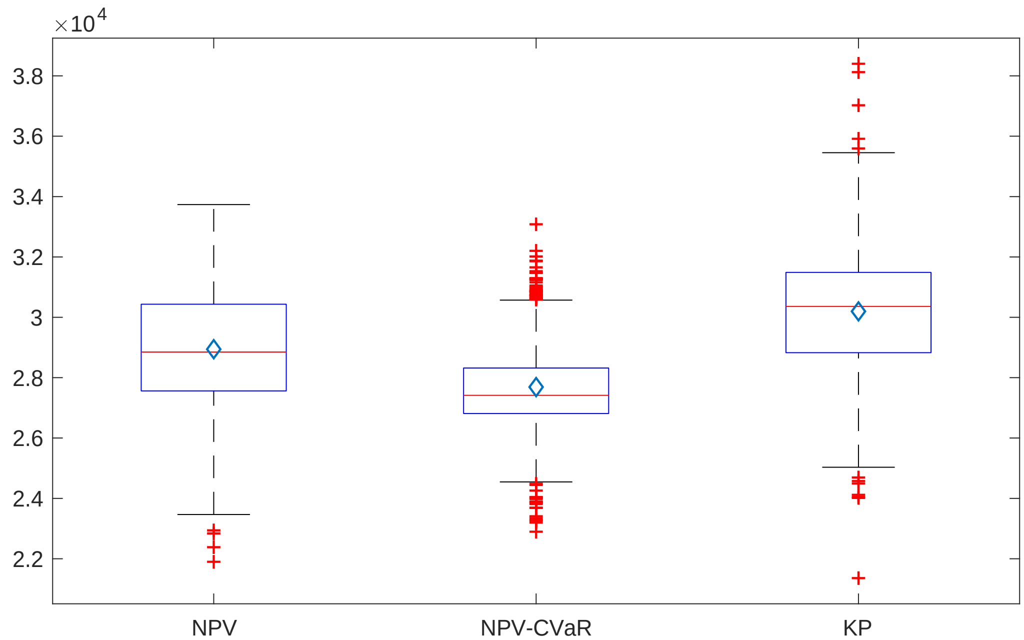


Figure 6-11 The box plot and average of the total terminal value from 1000 iteration

Once again, as we can see in Table 6-5, the maximization of the K-P criterion provides the best performance: 596 times out of 1000 iterations. Almost 60% of the times, which is the outcome similar to the delay option case in Table 6-3.

Table 6-5 The number of the best case when abandonment option involved

	Iteration	Max NPV	Max NPV-CVaR	Max K-P
# of best case	1000	235	169	596

Stochastic Dominance

Figure 6-12 shows each histogram for the terminal value and we can place the three histograms on the same chart to see how much areas are overlapped. The means and variances of the terminal value distributions from the expected NPV, the NPV-CVaR, and the K-P are as follows:

Table 6-6 The summary of the terminal value distributions with abandonment option

	Terminal Value Distribution		
Abandonment option	NPV	NPV-CVaR	K-P
Mean	$5,313	4,959	6,034
Standard Deviation	921	759	1,145

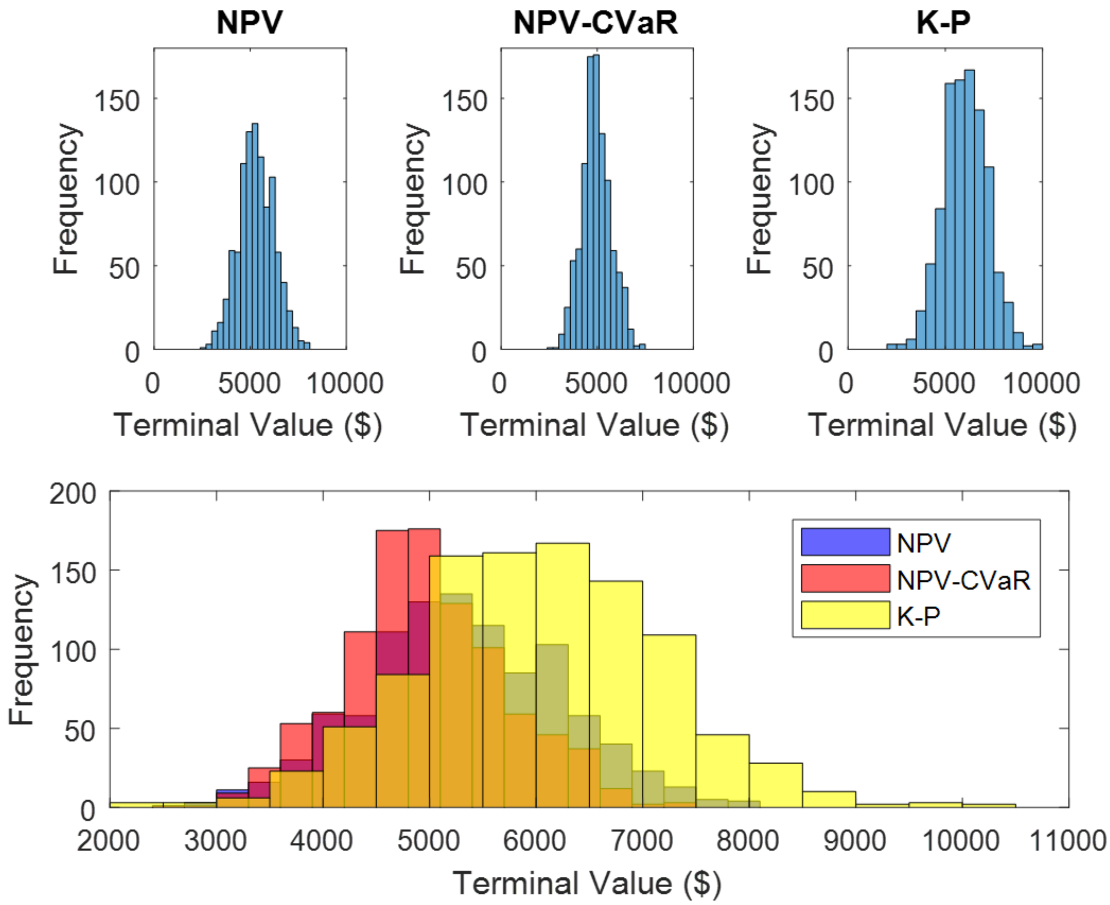


Figure 6-12 Terminal value histograms with abandonment options

Then Figure 6-13 illustrates the normal-fitted probability distributions of the terminal values based on 1000 iterations.

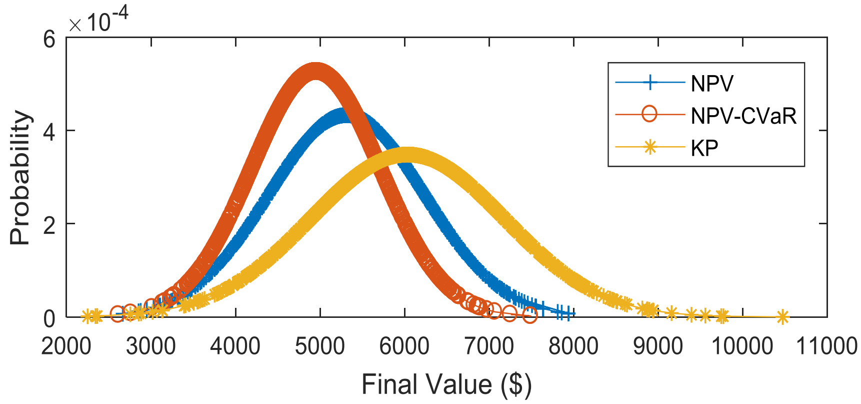


Figure 6-13 Normal-fitted probability distributions with abandonment options

With the cumulative terminal distributions plotted in Figure 6-14, we clearly see that the cumulative final value distribution from the K-P criterion lies underneath of the other two criteria, indicating that the K-P criterion dominates both the NPV and NPV-CVaR probabilistically. In other words, just like the case of the delay option, the K-P criterion is the most effective in terms of wealth creation when it is applied to multi-stage capital budgeting decision problems. The higher variance of the terminal value distribution of the K-P criterion is not something to avoid.

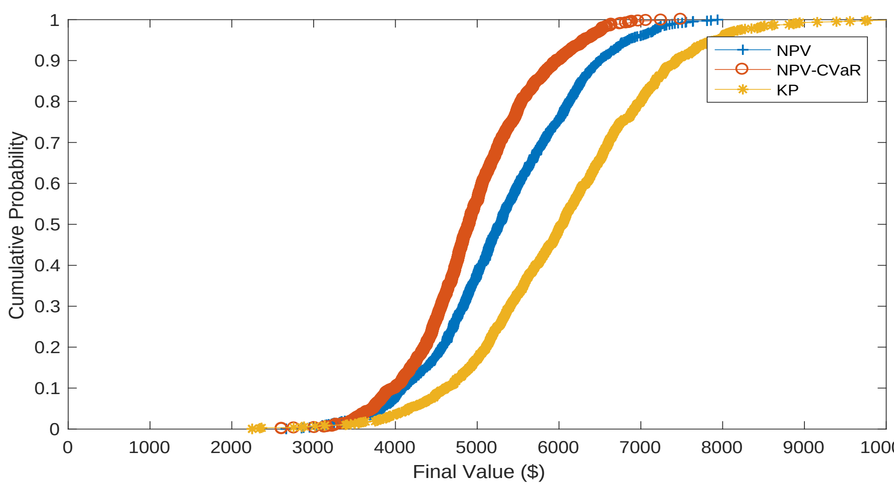


Figure 6-14 Cumulative final value distributions for the three decision criteria

6.4 Sensitivity Analysis

In the previous section, we have shown that the K-P criterion can be of a very effective decision rule in selecting projects by considering the investment flexibility. Since we have assumed certain sets of investment conditions or decision parameters in the simulation model, it is desirable to relax certain assumptions to see how those would affect the simulation outcomes. For this purpose, we will consider the following issues:

- We assumed that cash flows between the periods within the project and among the projects were mutually independent. How does the correlations among cash flows within the project affect the outcomes?
- We assumed random cash flow patterns for individual projects. Does any particular pattern of cash flow series affect the simulation outcomes?
- We assumed a 95% confidence level when we calculate the conditional value at risk. How does the level of risk tolerance affect the simulation outcomes?
- For projects associated with real options, we needed to set the threshold slope parameter to group the projects into two categories: projects without consideration of real options and with real options for future consideration. We will see how sensitive it is to select the slope parameter in terms of simulation outcomes.

6.4.1 Correlation of Cash flows

In the original simulation models, we assumed a statistical independence among cash flows within the project. If we assume some sort of correlations among the cash flows, we need to modify the simulation model to reflect these dependencies in generating random cash flows. For example, if the firm can predict the direction of future cash flows based on the realization of the prior period's cash flows, the firm would be in much better position on what to do for the delayed projects or abandonment decision for the on-going projects. To illustrate, let's assume that the cash flows at n is just depending on what happens to the cash flows at $n-1$. This one-period dependence assumption is more realistic in practice. Recall that the cash flows for the investment projects submitted at the first budget period shown in Table 6-7.

Table 6-7 Example of cash flows at the first budget period

n	P001	P002	P003	P004	P005	P006	P007	P008	P009	P010
0	$ 0	0	0	0	0	0	0	0	0	0
1	600	800	800	1000	700	800	600	1200	1000	700
2	620	600	600	500	600	600	750	400	600	700
3	610	500	800	800	700	700	600	500	300	700
4	600	800	600	500	500	800	750	500	800	700
5	630	400	700	800	700	700	600	800	600	700
σ[PV]	50%	60	30	10	10	50	10	50	30	10

In our earlier simulation, each cash flow is randomly generated by a normal random deviate with the predetermined mean and variance. For example, a cash flow of project 001 (P001) at the first period is randomly generated by a normal distribution with $N(\$600, 300^2)$, where the standard deviation is determined by 50% of the mean, then $N(\$620, 310^2)$ at the second period, and so forth, regardless what happened to the prior period. Suppose that there is a perfectly positively correlation between successive cash flows. If a realization of cash flow at the first period is greater than the initially estimated value, the cash flow to be generated at the second period will be adjusted higher by the percentage in difference, that is, by Equation (6-4).

$$diff = \frac{x_{n-1} - E[x_{n-1}]}{E[x_{n-1}]} \tag{6-4}$$

x_{n-1}: The realized cash flow at time $n-1$

$E[x_{n-1}]$: The expected cash flow at time $n-1$ which is predetermined

Then a successive cash flow will be generated from a normal distribution with the modified mean value and variance. The modified mean value is

$$E[x_n] \cdot (1 + diff) \quad (6\text{-}5)$$

Table 6-8 summarizes the results with both independent and dependent cash flow assumptions. The results show that with these kind of perfectly positively correlations, the K-P performance is even more pronounced. The results are not surprising. If we can have this much of information about the future, the K-P criterion will make timely decisions on whether or not to exercise the options in each period, which will lead to a better project selection in the coming periods. However, cash flows having this kind of perfectly positively correlation or mutual independence is not realistic. A more plausible case may take the middle ground that cash flows are somehow partially correlated. In that case, we can postulate that the performance of the K-P criterion would be somewhere between the two outcomes. That means the K-P criterion still outperforms over other criteria.

Table 6-8 Summary of the simulation results with perfect correlation of cash flows

Option Strategy	Cash flows	Iteration	Max NPV	Max NPV-CVaR	Max K-P
Delay Options	Independent	1000	257	142	601
	Dependent	1000	14	44	942
Abandonment Options	Independent	1000	235	169	596
	Dependent	1000	86	47	867

6.4.2 Generation of different cash flows patterns

In the simulation model, we assumed that all cash flows are random in nature – no recognizable patterns in the cash flows. Suppose that we can conjecture some sort of cash flow patterns, in other words, all projects are taking some sort of recognizable patterns even though the actual realization would deviate somehow from these known patterns: 1) Random, 2) Uniform

Series, 3) Gradient Series (Increasing), and 4) Gradient Series (Decreasing). With the random pattern being the base, Table 6-9 summarizes the simulation results.

Table 6-9 Summary of the results based on the different cash flow patterns

Cash Flows	Option Strategy	Iteration	Max NPV	Max NPV-CVaR	Max K-P
Random	Delay Options	1000	257	142	601
	Abandonment Options	1000	235	169	596
Uniform Series	Delay Options	1000	101	516	383
	Abandonment Options	1000	354	265	281
Increasing (10%)	Delay Options	1000	137	158	705
	Abandonment Options	1000	421	340	239
Decreasing (10%)	Delay Options	1000	424	381	195
	Abandonment Options	1000	97	55	848

Once again, the results are not surprising. Under the assumption of uniform series, the K-P criterion does not perform as good as the other criteria. This is expected as the K-P criterion does not have much room to take advantage of investment flexibility with almost known futures. On the other hand, with the increasing cash flow pattern, the K-P criterion with the delay option strategy is providing the best performance but the maximization of NPV provides the better performance than the K-P criterion with abandonment option, it is intuitively expected because the abandonment option is not much valuable when project value increases over time. In contrary, with the decreasing cash flow pattern, the K-P model with abandonment option strategy brings the best outcome and the delay option strategy is not providing a good result. In that case, there is no benefit of delaying when decreasing cash flows are expected. In that regards, the K-P criterion performs exactly the way we have expected.

6.4.3 Risk Tolerance Level

There are two types of risk preference parameters in this simulation model: First, confidence level in conditional value at risk. Second, the coefficient of risk in the mean-risk model. With the option premium determined based on the CVaR, it is our interest to see if the risk tolerance has any effect on the simulation outcome. In our simulation model, we assumed that the firm sets the confidence level at 95% in determining the CVaR. Recall in our optimization model for the NPV – CVaR, we used a risk-return tradeoff parameter (λ) and our interest is to examine the sensitivity of this parameter as well.

First, as we vary the degree of confidence level from 95% (base) to either 90% or 99%, the simulation results are summarized in Table 6-10. As we can observe, the confidence parameter does not affect the simulation results in any significant way. As we are more risk averse (say 99%), the performance of the K-P criterion is in fact performed a little better.

Table 6-10 Summary of the results based on the confidence level

Confidence level	Iteration	Max NPV	Max NPV-CVaR	Max K-P
90%	1000	231	151	618
95%	1000	257	142	601
99%	1000	209	168	623

Second, the initial coefficient (λ) of risk aversion was 30% in both the mean-risk model and the K-P criterion: maximization of $NPV - \lambda CVaR$ and maximization of $FNPV - \lambda ROP$. Table 6-11 presents the results of the sensitivity analysis of coefficient (λ) by varying the parameter from 0.1 to 0.5. A smaller coefficient provides the results which are favor to the maximization of K-P criterion, but a greater coefficient affects less favor to the K-P criterion. It means that the max K-P criterion still outperforms over other criteria, but requiring higher option premiums reduces the overall profitability of investments.

Table 6-11 Summary of the results based on the coefficient of risk variable

Coefficient (λ)	Iteration	Max NPV	Max NPV-CVaR	Max K-P
0.1	1000	99	30	871
0.3	1000	257	142	601
0.5	1000	325	135	540

6.4.4 The threshold slopes

Recall that the threshold slope in Figure 5-3. The threshold determines real option strategies according to projects' coordinate positions. Based on the example cash flows in Appendix 1, the slope ranges from 0.59 to 1.34. Therefore, we conduct a sensitivity analysis of the threshold slope from 0.5 to 1.5, so that the results cover both extreme cases. A greater slope means that we are giving a more weight to considering the option strategies, and a less weight means favoring the expected net present value method.

- **K-P Criterion with Delay Options**: In Table 6-12, we observe the following: 1) When the threshold is too small, the result of the K-P criterion is the same as the NPV method because the investment decision does not consider the real option strategies. 2) When the threshold is too large, the K-P criterion delays all the projects, incurring too much opportunity costs due to not funding the good projects on time.

Table 6-12 The results based on the threshold slope when the delay option involved

Threshold slope	Iteration	Max NPV	Max NPV-CVaR	Max K-P
0.5	1000	706	294	706
1	1000	257	142	601
1.5	1000	684	316	0

- **The K-P Criterion with Abandonment Options**: In Table 6-13, for the projects associated with abandonment options, a greater slope causes better results for the K-P criterion as more projects to be considered for funding. Therefore, the K-P criterion provides better results 835 times out of 1000.

Table 6-13 The results based on the threshold slope when the abandonment option involved

Threshold slope	Iteration	Max NPV	Max NPV-CVaR	Max K-P
0.5	1000	638	366	638
1	1000	235	169	596
1.5	1000	131	34	835

6.5 Summary

Based on the simulation model developed in Chapter 5, we have presented the simulation results and given various economic interpretations of the simulation results. We have shown that

- The K-P criterion can serve as a very effective capital allocation tool for short-term as well as long-term investment environment. In particular, the K-P criterion is more suitable for the multi-stage capital budgeting tool to create the most wealth creation to the firm.
- It has shown that the K-P criterion dominates the conventional capital budgeting tools probabilistically – namely, the expected NPV and NPV – CVaR for the sets of investment situations simulated.
- The sensitivity analyses on various input parameters reinforce the findings on how effective the K-P criterion would be as a capital budgeting decision criterion under uncertainty.

Chapter 7. Conclusions

The primary purpose of this research is to examine how critical it is to consider option value in multi-stage capital budgeting decision problems under uncertainty. Of particular interest is how to integrate the three key elements (profitability, variability, and flexibility) in a single measure so that we can select the projects based on this measure. A complete summary is given in Section 7.1, followed by conclusions in Section 7.2, and future research recommendations in Section 7.3.

7.1 Summary of this research

This study begins with the reviews on the decision criteria considering risk and uncertainty in the capital budgeting literature. Our particular interest is to know of any specific efforts to incorporate real option strategies into multi-stage capital budgeting decisions under uncertainty. As is evidenced by the review of the literature, limited attention is given to the development of a methodology to consider the option values explicitly in capital allocation process. Therefore, this research has accomplished the following tasks to answer the ultimate question – should we consider option values associated with projects in capital allocation decisions?

Development a practical real option valuation method with the loss function

Despite of many improvements in the real option analysis, there are many practical difficulties of formulating real option strategies based on the financial option framework. The reason is that the financial option assumes the project value distribution in any time to be a lognormal. According to the central limit theorem, a normal distribution assumption can be more reasonable for project value as the project cash flows are determined by a linear sum of many random variables. Therefore, we proposed an alternative method for valuing real options with the standardized loss function. Moreover, the valuation of real option with the loss function approach is more intuitively understandable and serves practical needs better in business community, so that managers can easily apply the computational framework in routine project evaluation and react in a timely manner in highly uncertain investment environment.

Development a real option pricing model based on the CVaR concept

We have developed a real option pricing method based on the CVaR concept. The CVaR measures the average of extreme losses beyond the maximum estimated loss at a specified confidence level of an investor or a firm. If a typical investor is willing to accept an investment, we may view this amount (CVaR) as an investor's risk tolerance associated with the project. In order to determine an appropriate amount of real option premium to pay for a given level of risk tolerance, we investigated the relationship between the CVaR and the real option premium. Since the CVaR represents the maximum amount of losses a firm could accept to take, the maximum price to keep the real option should be less than the CVaR. Obviously, the real option premium should not be greater than the real option value, and the CVaR would give a reference point on how much could be paid for the real option in terms of firm's risk tolerance.

Development a capital budgeting decision model with the real option approach

We identified three critical elements in evaluating a risky investment project: profitability, variability, and flexibility. In order to integrate these three elements in a single measure, we developed the K-P criterion to allocate the limited capital in the multi-stage capital budgeting process. The K-P criterion reflects these three elements as follows: 1) the terminal profitability is obtained by the expected net present value of the project, 2) the variability is measured its loss magnitude determined by CVaR, and 3) the investment flexibility is obtained by real option value with the loss function approach. Moreover, an elaborate mathematical programming model was developed to select investment projects based on the K-P criterion.

Development of the Simulation Models

We have developed a simulation model to test the long-term effectiveness of the K-P criterion along with other traditional decision criteria. In order to examine a large class of investment settings with various future uncertainties, we needed to employ the computer simulation approach because they were not normally manageable by available analytical techniques. We have described the general assumptions and simulation logics to use in testing the effectiveness of the K-P criterion. In doing so, we have presented the cash flow generation process and a process of grouping projects into two groups for funding consideration. Then we have developed two specific simulations models – one designed to consider the delay options strategies, and the other for abandonment strategies.

7.2 Conclusion

The main purpose of the simulation experiments is to answer the following specific question: *Should we consider option values in capital budgeting decisions under uncertainty?* According to the simulation results presented in Chapter 6, the K-P criterion outperformed over other traditional measures for the investment settings we have tested. The effectiveness of the K-P criterion is examined in three specific areas:

As a short-term measure

The K-P criterion has shown to be a very effective decision rule in selecting projects under uncertainty. The three criteria including the K-P criterion selected different mixes of projects in each decision point, and they were illustrated on the coordinate grid with the expected net present value and the standard deviation corresponding to profitability and variability. The modified Sharpe ratio measured each portfolio performance, and it indicated that the K-P criterion provided the better Sharpe ratio.

As a long-term measure

The K-P criterion produced the higher terminal values among the three criteria in the same set of investment proposals by creating additional values from the investment flexibility. Besides the higher terminal values in most of the simulation runs, we showed that the K-P criterion dominates both the NPV and NPV – CVaR probabilistically based on the first-degree stochastic dominance.

Specific Findings

We conducted the sensitivity analyses on various input parameters to investigate how effective the K-P criterion would be in the different conditions. 1) The K-P criterion performance is even more pronounced when there are positive correlations among cash flows. It can be interpreted that if there is more information available, the real option strategies worked by capturing the investment flexibility. 2) In the increasing cash flow series pattern, the K-P criterion with delay option strategy provided the best performance but the abandonment option strategy did not offer a more favorable result. The results were the other way around in the decreasing cash flow patterns. These results are not surprising as we expect a better cash flow in the future, the abandonment option will improve the terminal values, and with an expectation of decreasing cash flow series, the delay option does not provide any advantage. 3) In terms of the risk tolerance level, we examined two types of risk preference parameters. First, there was not significant difference in the results from the 90% to 99% confidence level. Second, there was a less favorable trend for the K-P criterion when the coefficient (λ) of risk aversion increases. 4) The threshold slope separates proposals into two groups: One group will only include projects without considering real options, and the other group with real options strategies. The results showed that both the NPV and K-P criteria have the same outcome when the threshold slope gets smaller than all proposals because of no consideration of option strategies. That means the expected NPV method can be a special case of the K-P criterion.

7.3 Recommendations for Future Research

The proposed decision model can be extended to consider other types of real option strategies such as compound options or switching options in the project portfolios. For more capital intensive investment projects, a sequential or staged investment decision would be more common with ever-changing market dynamics. A compound options would be more appropriate to manage such dynamic situations in hedging investment risks. It would be even more interesting to see how the K-P criterion would fair with the traditional measures. Another area of research to make our simulation model more comprehensive is to incorporate the Bayesian framework to update the option values as we receive new pieces of information. Clearly the future research direction remains the same – how important it is to consider option values in capital budgeting decisions under uncertainty.

References

[1] M. Engel, C. Musso, and J. Wallach, “The nimble prosper: Dynamic resource allocation in chemicals | McKinsey & Company,” 2015.

[2] C. S. Park and G. J. Thuesen, “The Effectiveness of Using Uncertainty Resolution to Measure Investment Flexibility in Sequential Capital Rationing,” *E Trans.*, vol. 11, no. 3, pp. 207–220, Sep. 1979.

[3] T. Koller, D. Lovallo, and Z. Williams, “How to catch those fleeting investment opportunities | McKinsey & Company,” 2014.

[4] D. Sull, “Why Good Companies Go Bad,” *Harvard Business Review*, 01-Jul-1999.

[5] T. Dorsey, “Exxon Mobil Corporation Is The Profitable Investment In Natural Gas,” *Seeking Alpha*, 02-May-2013.

[6] T. Klammer, “Empirical evidence of the adoption of sophisticated capital budgeting techniques,” *J. Bus.*, vol. 45, no. 3, pp. 387–397, 1972.

[7] L. D. Schall, G. L. Sundem, and W. R. Geijsbeek, “Survey and analysis of capital budgeting methods,” *J. Finance*, vol. 33, no. 1, pp. 281–287, 1978.

[8] R. Aggarwal, “Corporate use of sophisticated capital budgeting techniques: a strategic perspective and a critique of survey results,” *Interfaces*, vol. 10, no. 2, pp. 31–34, 1980.

[9] R. Pike, “Do sophisticated capital budgeting approaches improve investment decision-making effectiveness?,” *Eng. Econ.*, vol. 34, no. 2, pp. 149–161, 1989.

[10] R. Burns and J. Walker, “Capital budgeting surveys: the future is now,” 2015.

[11] J. H. Lorie and L. J. Savage, "Three problems in rationing capital," *J. Bus.*, vol. 28, no. 4, pp. 229–239, 1955.

[12] H. M. Markowitz and A. S. Manne, "On the Solution of Discrete Programming Problems," *Econometrica*, vol. 25, no. 1, p. 84, Jan. 1957.

[13] W. J. Baumol and R. E. Quandt, "Investment and Discount Rates Under Capital Rationing--A Programming Approach," *Econ. J.*, vol. 75, no. 298, p. 317, Jun. 1965.

[14] S. C. Myers, "A Note on Linear Programming and Capital Budgeting," *J. Finance*, vol. 27, no. 1, p. 89, Mar. 1972.

[15] A. J. Keown and J. D. Martin, "An integer goal programming model for capital budgeting in hospitals," *Financ. Manag.*, pp. 28–35, 1976.

[16] S. P. Bradley and S. C. Frey, "Equivalent Mathematical Programming Models of Pure Capital Rationing," *J. Financ. Quant. Anal.*, vol. 13, no. 2, pp. 345–361, 1978.

[17] P. Lusztig and B. Schwab, "A Note on the Application of Linear Programming to Capital Budgeting," *J. Financ. Quant. Anal.*, vol. 3, no. 4, pp. 427–431, 1968.

[18] D. J. Ashton and D. R. Atkins, "Rules of Thumb and the Impact of Debt in Capital Budgeting Models," *J. Oper. Res. Soc.*, vol. 30, no. 1, pp. 55–61, 1979.

[19] H. M. Weingartner, *Mathematical programming and the analysis of capital budgeting problems*. Prentice-Hall, Inc., 1962.

[20] H. M. Weingartner, "Criteria for programming investment project selection," *J. Ind. Econ.*, pp. 65–76, 1966.

[21] H. M. Weingartner, "Capital budgeting of interrelated projects: survey and synthesis," *Manag. Sci.*, vol. 12, no. 7, pp. 485–516, 1966.

[22] W. F. Sharpe, “Capital asset prices: A theory of market equilibrium under conditions of risk,” *J. Finance*, vol. 19, no. 3, pp. 425–442, 1964.

[23] E. F. Fama, “Risk-adjusted discount rates and capital budgeting under uncertainty,” *J. Financ. Econ.*, vol. 5, no. 1, pp. 3–24, Aug. 1977.

[24] C. A. Hawkins and R. A. Adams, “A Goal Programming Model for Capital Budgeting,” *Financ. Manag.*, vol. 3, no. 1, pp. 52–57, 1974.

[25] A. J. Keown and B. W. Taylor, “A chance-constrained integer goal programming model for capital budgeting in the production area,” *J. Oper. Res. Soc.*, vol. 31, no. 7, pp. 579–589, 1980.

[26] P. K. De, D. Acharya, and K. C. Sahu, “A Chance-Constrained Goal Programming Model for Capital Budgeting,” *J. Oper. Res. Soc.*, vol. 33, no. 7, pp. 635–638, Jul. 1982.

[27] A. Ahern and G. Anandarajah, “Railway projects prioritisation for investment: Application of goal programming,” *Transp. Policy*, vol. 14, no. 1, pp. 70–80, 2007.

[28] Y.-C. Tang and C.-T. Chang, “Multicriteria decision-making based on goal programming and fuzzy analytic hierarchy process: An application to capital budgeting problem,” *Knowl.-Based Syst.*, vol. 26, pp. 288–293, Feb. 2012.

[29] O. Ben-Ayed and C. E. Blair, “Computational difficulties of bilevel linear programming,” *Oper. Res.*, vol. 38, no. 3, pp. 556–560, 1990.

[30] S. Dempe, *Foundations of bilevel programming*. Springer Science & Business Media, 2002.

[31] J. J. Buckley, “The fuzzy mathematics of finance,” *Fuzzy Sets Syst.*, vol. 21, no. 3, pp. 257–273, 1987.

[32] A. Kaufmann and M. M. Gupta, *Fuzzy mathematical models in engineering and management science*. Elsevier Science Inc., 1988.

[33] T. L. Ward, “Discounted fuzzy cash flow analysis,” in *1985 Fall Industrial Engineering Conference Proceedings*, 1985, pp. 476–481.

[34] E. E. Karsak, “Measures of Liquidity Risk Supplementing Fuzzy Discounted Cash Flow Analysis,” *Eng. Econ.*, vol. 43, no. 4, p. 331, Summer 1998.

[35] J. J. Buckley, “Portfolio analysis using possibility distributions,” in *Approximate reasoning in intelligent systems, decision and control*, Elsevier, 1987, pp. 69–76.

[36] C.-Y. Chiu and C. S. Park, “Capital Budgeting Decisions With Fuzzy Projects,” *Eng. Econ.*, vol. 43, no. 2, pp. 125–150, Jan. 1998.

[37] C. Carlsson and R. Fullér, “Capital budgeting problems with fuzzy cash flows,” *Mathw. Soft Comput.*, vol. 6, no. 1, pp. 81–89, 1999.

[38] C. Kahraman, D. Ruan, and E. Tolga, “Capital budgeting techniques using discounted fuzzy versus probabilistic cash flows,” *Inf. Sci.*, vol. 142, no. 1, pp. 57–76, 2002.

[39] A. L. Sanches, O. E. Pamplona, and J. A. B. Montevechi, “Capital budgeting using triangular fuzzy numbers,” *V Encuentro Int. Finanz.*, vol. 19, 2005.

[40] X. Huang, “Chance-constrained programming models for capital budgeting with NPV as fuzzy parameters,” *J. Comput. Appl. Math.*, vol. 198, no. 1, pp. 149–159, Jan. 2007.

[41] I. U. Sari and C. Kahraman, “Interval type-2 fuzzy capital budgeting,” *Int. J. Fuzzy Syst.*, vol. 17, no. 4, pp. 635–646, 2015.

[42] P. Beraldi, A. Violi, F. De Simone, M. Costabile, I. Massabò, and E. Russo, “A multistage stochastic programming approach for capital budgeting problems under uncertainty,” *IMA J. Manag. Math.*, vol. 24, no. 1, pp. 89–110, 2013.

[43] M. Rafiee, F. Kianfar, and M. Farhadkhani, "A multistage stochastic programming approach in project selection and scheduling," *Int. J. Adv. Manuf. Technol.*, vol. 70, no. 9–12, pp. 2125–2137, Feb. 2014.

[44] J. M. Mulvey, R. J. Vanderbei, and S. A. Zenios, "Robust optimization of large-scale systems," *Oper. Res.*, vol. 43, no. 2, pp. 264–281, 1995.

[45] D. Goldfarb and G. Iyengar, "Robust Portfolio Selection Problems," *Math. Oper. Res.*, vol. 28, no. 1, pp. 1–38, Feb. 2003.

[46] S. Kachani and J. Langella, "A robust optimization approach to capital rationing and capital budgeting," *Eng. Econ.*, vol. 50, no. 3, pp. 195–229, 2005.

[47] A. Albadvi and H. Koosha, "A robust optimization approach to allocation of marketing budgets," *Manag. Decis.*, vol. 49, no. 4, pp. 601–621, May 2011.

[48] S. C. Myers, "Determinants of corporate borrowing," *J. Financ. Econ.*, vol. 5, no. 2, pp. 147–175, 1977.

[49] L. Trigeorgis, *Real options in capital investment: Models, strategies, and applications*. Greenwood Publishing Group, 1995.

[50] A. K. Dixit and R. S. Pindyck, *Investment under uncertainty*. Princeton university press, 1994.

[51] L. Trigeorgis, "Real Options," *MIT Press*, 1996.

[52] T. E. Copeland and V. Antikarov, *Real options: a practitioner's guide*. Texere New York, 2001.

[53] J. Mun, *Real options analysis: Tools and techniques for valuing strategic investments and decisions*, vol. 137. John Wiley & Sons, 2002.

[54] M. A. Brach, *Real options in practice*, vol. 198. John Wiley & Sons, 2003.

[55] Smit and Trigeorgis, *Strategic Investment: Real Options and Games.* Princeton university press, 2004.

[56] R. L. Shockley, *An applied course in real options valuation*. Thomson Higher Education, 2007.

[57] G. Guthrie, *Real options in theory and practice*. Oxford University Press, 2009.

[58] J. Von Neumann and O. Morgenstern, "Theory of games and economic behavior," *Bull Amer Math Soc*, vol. 51, no. 7, pp. 498–504, 1945.

[59] M. Friedman and L. J. Savage, "The utility analysis of choices involving risk," *J. Polit. Econ.*, vol. 56, no. 4, pp. 279–304, 1948.

[60] C. S. Park and G. P. Sharp-Bette, *Advanced engineering economics*. Wiley, 1990.

[61] J. C. Mao, "Models of capital budgeting, EV vs ES," *J. Financ. Quant. Anal.*, vol. 4, no. 5, pp. 657–675, 1970.

[62] H. Markowitz, "Portfolio Selection," *J. Finance,* vol. 7, no. 1, pp. 77–91, 1952.

[63] W. F. Sharpe, "The sharpe ratio," *J. Portf. Manag.*, vol. 21, no. 1, pp. 49–58, 1994.

[64] J. Lintner, "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *Rev. Econ. Stat.*, vol. 47, no. 1, p. 13, Feb. 1965.

[65] J. Van Horne, "Capital-budgeting decisions involving combinations of risky investments," *Manag. Sci.*, vol. 13, no. 2, p. B–84, 1966.

[66] J. Mossin, "Optimal multiperiod portfolio policies," *J. Bus.*, vol. 41, no. 2, pp. 215–229, 1968.

[67] E. F. Fama, "Multiperiod consumption-investment decisions: A correction," *Am. Econ. Rev.*, vol. 66, no. 4, pp. 723–724, 1976.

[68] JP Morgan, "RiskMetrics-Technical Document, value at risk." 1995.

[69] P. Embrechts, E. Mcneil, and D. Straumann, “Correlation: pitfalls and alternatives,” in *Risk Magazine*, 1999.

[70] P. Jorion, “Risk2: Measuring the risk in value at risk,” *Financ. Anal. J.*, vol. 52, no. 6, pp. 47–56, 1996.

[71] A. Lucas and P. Klaassen, “Extreme returns, downside risk, and optimal asset allocation,” *J. Portf. Manag.*, vol. 25, no. 1, p. 71–+, 1998.

[72] P. Artzner, F. Delbaen, J.-M. Eber, and D. Heath, “Coherent Measures of Risk,” *Math. Finance*, vol. 9, no. 3, pp. 203–228, Jul. 1999.

[73] F. Delbaen, “Coherent risk measures,” *Bl. DGVFM*, vol. 24, no. 4, pp. 733–739, 2000.

[74] R. T. Rockafellar and S. Uryasev, “Optimization of conditional value-at-risk,” *J. Risk*, vol. 2, pp. 21–42, 2000.

[75] C. Acerbi and D. Tasche, “On the coherence of expected shortfall,” *J. Bank. Finance*, vol. 26, no. 7, pp. 1487–1503, 2002.

[76] R. Campbell, R. Huisman, and K. Koedijk, “Optimal portfolio selection in a Value-at-Risk framework,” *J. Bank. Finance*, vol. 25, no. 9, pp. 1789–1804, Sep. 2001.

[77] D. A. LaGattuta, J. C. Stein, M. L. Tennican, S. E. Usher, and J. Youngen, “Cashflow-at-Risk and Financial Policy for Electricity Companies in the New World Order,” *Electr. J.*, vol. 13, no. 10, pp. 15–20, Dec. 2000.

[78] J. C. Stein, S. E. Usher, D. LaGattuta, and J. Youngen, “A Comparables Approach to measuring CashFlow-At-Risk for Non-Financial firms,” *J. Appl. Corp. Finance*, vol. 13, no. 4, pp. 100–109, 2001.

[79] N. Andrén, H. akan Jankensg\a ard, and L. Oxelheim, “Exposure-Based Cash-Flow-at-Risk: An Alternative to VaR for Industrial Companies,” *J. Appl. Corp. Finance*, vol. 17, no. 3, pp. 76–86, 2005.

[80] J. Kim, A. M. Malz, and J. Mina, *Longrun: Technical Document*. RiskMetrics Group New York, 1999.

[81] T. E. Copeland and P. T. Keenan, “How much is flexibility worth?,” *McKinsey Q.*, no. 2, pp. 38–50, 1998.

[82] K. S. Buehler and G. Pritsch, “Running with risk,” *McKinsey Q.*, no. 4, pp. 40–49, 2003.

[83] M. Pergler and A. Rasmussen, “Strategic commodity and cash-flow-at-risk modeling for corporates | McKinsey & Company,” 2013. .

[84] S. Ye and R. L. Tiong, “NPV-at-risk method in infrastructure project investment evaluation,” *J. Constr. Eng. Manag.*, vol. 126, no. 3, pp. 227–233, 2000.

[85] Y.-C. Lu, S. Wu, D.-H. Chen, and Y.-Y. Lin, “BOT projects in Taiwan: financial modeling risk, term structure of net cash flows, and project at risk analysis,” *J. Struct. Finance*, vol. 5, no. 4, pp. 53–63, 2000.

[86] M. Pergler and A. Rasmussen, “Making better decisions about the risks of capital projects,” *McKinsey Insights Publ. May*, 2014.

[87] F. Black and M. Scholes, “The pricing of options and corporate liabilities,” *J. Polit. Econ.*, pp. 637–654, 1973.

[88] R. C. Merton, “Theory of rational option pricing,” *Bell J. Econ. Manag. Sci.*, pp. 141–183, 1973.

[89] R. L. McDonald and D. Siegel, *The value of waiting to invest*. National Bureau of Economic Research Cambridge, Mass., USA, 1986.

[90] L. Trigeorgis, “Real options and interactions with financial flexibility,” *Financ. Manag.*, pp. 202–224, 1993.

[91] L. Trigeorgis, “The Nature of Option Interactions and the Valuation of Investments with Multiple Real Options,” *J. Financ. Quant. Anal.*, vol. 28, no. 1, p. 1, Mar. 1993.

[92] T. A. Luehrman, “Investment opportunities as real options: Getting started on the numbers,” *Harv. Bus. Rev.*, vol. 76, pp. 51–66, 1998.

[93] T. A. Luehrman, “Strategy as a portfolio of real options,” *Harv. Bus. Rev.*, vol. 76, pp. 89–101, 1998.

[94] D. M. Lander and G. E. Pinches, “Challenges to the practical implementation of modeling and valuing real options,” *Q. Rev. Econ. Finance*, vol. 38, no. 3, Part 2, pp. 537–567, Jan. 1998.

[95] M. Amram and N. Kulatilaka, “Real options: Managing strategic investment in an uncertain world,” *OUP Cat.*, 1998.

[96] M. Amram and N. Kulatilaka, *Real options*. Harvard Business School Press Boston, MA, 1999.

[97] H. B. Nembhard, L. Shi, and C. S. Park, “Real option models for managing manufacturing system changes in the new economy,” *Eng. Econ.*, vol. 45, no. 3, pp. 232–258, 2000.

[98] C. S. Park and H. S. Herath, “Exploiting uncertainty—investment opportunities as real options: a new way of thinking in engineering economics,” *Eng. Econ.*, vol. 45, no. 1, pp. 1–36, 2000.

[99] G. R. Mitchell and W. F. Hamilton, “Managing R&D as a strategic option,” *Res.-Technol. Manag.*, vol. 31, no. 3, pp. 15–22, 1988.

[100] T. W. Faulkner, "Applying 'options thinking'to R&D valuation," *Res.-Technol. Manag.*, vol. 39, no. 3, pp. 50–56, 1996.

[101] E. Pennings and O. Lint, "The option value of advanced R & D," *Eur. J. Oper. Res.*, vol. 103, no. 1, pp. 83–94, 1997.

[102] H. S. Herath and C. S. Park, "Economic analysis of R&D projects: an options approach," *Eng. Econ.*, vol. 44, no. 1, pp. 1–35, 1999.

[103] P. D. Childs and A. J. Triantis, "Dynamic R&D investment policies," *Manag. Sci.*, vol. 45, no. 10, pp. 1359–1377, 1999.

[104] E. H. Bowman and G. T. Moskowitz, "Real options analysis and strategic decision making," *Organ. Sci.*, vol. 12, no. 6, pp. 772–777, 2001.

[105] A. Huchzermeier and C. H. Loch, "Project management under risk: Using the real options approach to evaluate flexibility in R… D," *Manag. Sci.*, vol. 47, no. 1, pp. 85–101, 2001.

[106] H. Weeds, "Strategic delay in a real options model of R&D competition," *Rev. Econ. Stud.*, vol. 69, no. 3, pp. 729–747, 2002.

[107] R. G. McGrath, W. J. Ferrier, and A. L. Mendelow, "Real options as engines of choice and heterogeneity," *Acad. Manage. Rev.*, vol. 29, no. 1, pp. 86–101, 2004.

[108] M. Schneider, M. Tejeda, G. Dondi, F. Herzog, S. Keel, and H. Geering, "Making real options work for practitioners: a generic model for valuing R&D projects," *Rd Manag.*, vol. 38, no. 1, pp. 85–106, 2008.

[109] A. Triantis and A. Borison, "Real options: state of the practice," *J. Appl. Corp. Finance*, vol. 14, no. 2, pp. 8–24, 2001.

[110] D. E. Stout, Yan Alice Xie, and Howard Qi, "Improving Capital Budgeting Decisions: With Real Options," *Manag. Account. Q.*, vol. 9, no. 4, pp. 1–10, Summer 2008.

[111] B. Fernandes, J. Cunha, and P. Ferreira, "The use of real options approach in energy sector investments," *Renew. Sustain. Energy Rev.*, vol. 15, no. 9, pp. 4491–4497, 2011.

[112] A. Gazheli and J. van den Bergh, "Real options analysis of investment in solar vs. wind energy: Diversification strategies under uncertain prices and costs," *Renew. Sustain. Energy Rev.*, vol. 82, pp. 2693–2704, Feb. 2018.

[113] H. Meier, N. Christofides, and G. Salkin, "Capital budgeting under uncertainty-an integrated approach using contingent claims analysis and integer programming," *Oper. Res.*, vol. 49, no. 2, pp. 196–206, 2001.

[114] M. J. Brennan and E. S. Schwartz, "Evaluating Natural Resource Investments," *J. Bus.*, vol. 58, no. 2, pp. 135–157, 1985.

[115] T. Copeland, T. Koller, J. Murrin, and M. Valuation, "Managing the Value of Companies," *McKinsey Co. Lnc John Willey Sons Lnc USA Ganada*, vol. 3, 2000.

[116] A. Borison, "Real options analysis: Where are the emperor's clothes?," *J. Appl. Corp. Finance*, vol. 17, no. 2, pp. 17–31, 2005.

[117] L. E. Brandão, J. S. Dyer, and W. J. Hahn, "Using Binomial Decision Trees to Solve Real-Option Valuation Problems," *Decis. Anal.*, vol. 2, no. 2, pp. 69–88, Jun. 2005.

[118] J. C. Hull, *Fundamentals of Futures and Options Markets*, 8 edition. Boston: Pearson, 2013.

[119] J. C. Hull, *Options, Futures, and Other Derivatives*, 10th ed. Pearson, 2017.

[120] D. G. Luenberger, "Investment science," *OUP Cat.*, 1997.

[121] H. S. Herath and C. S. Park, “Multi-stage capital investment opportunities as compound real options,” *Eng. Econ.*, vol. 47, no. 1, pp. 1–27, 2002.

[122] H. S. Herath and C. S. Park, “Real options valuation and its relationship to Bayesian decision-making methods,” *Eng. Econ.*, vol. 46, no. 1, pp. 1–32, 2001.

[123] S. Block, “Are ‘Real Options’ Actually Used in the Real World?,” *Eng. Econ.*, vol. 52, no. 3, pp. 255–267, Aug. 2007.

[124] H. K. Baker, S. Dutta, and S. Saadi, “Management Views on Real Options in Capital Budgeting,” Social Science Research Network, Rochester, NY, SSRN Scholarly Paper ID 1617927, May 2010.

[125] H. J. Han, *Estimating project volatility and developing decision support system in real options analysis*. Auburn University, 2007.

[126] R. C. Merton, “On the pricing of contingent claims and the Modigliani-Miller theorem,” *J. Financ. Econ.*, vol. 5, no. 2, pp. 241–249, 1977.

[127] T. Mikosch, *Elementary stochastic calculus, with finance in view*, vol. 6. World Scientific Publishing Co Inc, 1998.

[128] S. Nahmias and T. L. Olsen, *Production and operations analysis*. Waveland Press, 2015.

[129] D. A. LaGattuta, J. C. Stein, M. L. Tennican, S. E. Usher, and J. Youngen, “Cashflow-at-risk and financial policy for electricity companies in the new world order,” *Electr. J.*, vol. 13, no. 10, pp. 15–20, 2000.

[130] S. K. Bagui and A. Ghosh, “Road Project Investment Evaluation Using Net Present Value (NPV) at Risk Method,” *Jordan J. Civ. Eng.*, vol. 6, no. 2, 2012.

[131] P. Gunther, “Use of linear programming in capital budgeting,” *J. Oper. Res. Soc. Am.*, vol. 3, no. 2, pp. 219–224, 1955.

[132] J. T. Blake and M. W. Carter, “A goal programming approach to strategic resource allocation in acute care hospitals,” *Eur. J. Oper. Res.*, vol. 140, no. 3, pp. 541–561, Aug. 2002.

[133] J. L. Corner, R. F. Deckro, and R. W. Spahr, “Multiple-objective linear programming in capital budgeting,” *Adv. Math. Program. Financ. Plan.*, vol. 3, pp. 241–264, 1993.

[134] H. M. Weingartner, “Capital Budgeting of Interrelated Projects: Survey and Synthesis,” *Manag. Sci.*, vol. 12, no. 7, pp. 485–516, Mar. 1966.

[135] H. M. Weingartner, “Capital Rationing: n Authors in Search of a Plot,” *J. Finance*, vol. 32, no. 5, p. 1403, Dec. 1977.

[136] K. Iwamura and B. Liu, “Chance constrained integer programming models for capital budgeting in fuzzy environments,” *J. Oper. Res. Soc.*, vol. 49, no. 8, pp. 854–860, 1998.

[137] M. Liao, *Applied Stochastic Processes*, 1 edition. Boca Raton: Chapman and Hall/CRC, 2013.

[138] “Elementary Stochastic Calculus, with Finance in View,” *World Scientific Publishing Company*. [Online]. Available: http://www.worldscientific.com/worldscibooks/10.1142/3856. [Accessed: 23-Jan-2018].

[139] W. F. Sharpe, “Mutual fund performance,” *J. Bus.*, vol. 39, no. 1, pp. 119–138, 1966.

Appendix 1: Example of three groups of proposals

1. Proposals at time 0

n	P001	P002	P003	P004	P005	P006	P007	P008	P009	P010
0	$ 0	0	0	0	0	0	0	0	0	0
1	600	800	800	1000	700	800	600	1200	1000	700
2	620	600	600	500	600	600	750	400	600	700
3	610	500	800	800	700	700	600	500	300	700
4	600	800	600	500	500	800	750	500	800	700
5	630	400	700	800	700	700	600	800	600	700
IRR(%)	16%	18	23	25	18	24	19	23	21	22
E[PV]	$2203	2284	2541	2633	2317	2596	2378	2518	2434	2523
σ[PV]	800.27	938.52	721.78	445.01	379.32	948.94	388.62	1008.4	730.66	409.96

2. Proposals at time 1

n	P101	P102	P103	P104	P105	P106	P107	P108	P109	P110
0	$ 0	0	0	0	0	0	0	0	0	0
1	500	700	700	700	700	700	1000	700	600	650
2	700	650	500	750	600	500	600	800	700	700
3	800	700	400	700	700	700	600	550	500	700
4	700	650	600	750	750	500	400	700	800	600
5	800	600	900	700	700	800	900	600	600	700
IRR(%)	21%	20	16	23	21	18	24	21	18	20
E[PV]	$2473	2395	2200	2595	2475	2294	2563	2440	2299	2415
σ[PV]	247.27	718.49	1100	259.50	1361	458.71	897.10	1219	1149	241.51

3. Proposals at time 2

n	P201	P202	P203	P204	P205	P206	P207	P208	P209	P210
0	$ 0	0	0	0	0	0	0	0	0	0
1	750	650	1000	800	700	600	750	700	1000	700
2	600	650	400	700	600	620	600	600	500	600
3	700	600	500	600	800	610	500	700	800	700
4	600	750	500	700	800	600	800	600	500	600
5	700	600	800	600	700	630	400	700	800	700
IRR(%)	20%	19	19	22	23	16	17	19	25	19
E[PV]	2425	2343	2339	2485	2578	2203	2239	2380	2633	2380
σ[PV]	484.94	585.67	1403.6	993.88	128.92	220.30	671.77	952.03	789.77	119.00