Profile Monitoring of Multivariate Processes for Efficient Detection of Parameter Changes

by

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Abstract

Profile monitoring has been extensively studied when the profile of a quality characteristic is normally distributed. There are a limited number of studies for the case when a profile follows other distributions such as Weibull, lognormal or Gamma. A profile, having these last three distributions, has many practical applications. It is also of interest to determine how well profile-monitoring of estimators can detect changes in parameters of an underlying distribution. Control chart methods are utilized to monitor such parameter estimates. The performance of a few monitoring statistics is investigated in this dissertation. A form of the Hotelling's T^2 statistic and another that utilizes the concept of an exponentially weighted moving average are investigated. As a performance measure, the mean and standard deviation of the time to first detection of shift in process parameters, are explored. We also investigated a method that utilizes estimated percentiles of the distribution for profile monitoring.

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EWMA	Exponentially Weighted Moving Average
CUSUM	Cumulative Sum
Q-Q Plot	Quantile-Quantile Plot
CL	Center Line
LCL	Lower Control Limit
UCL	Upper Control Limit
QCH	Quality Characteristics
MFC	Mass Flow Controller
VDP	Vertical Density Profile
ARL	Average Run Length
SDRL	Standard Deviation of Run Length
MEWMA	Multivariate Exponentially Weighted Moving Average

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Chapter 1

Introduction

Measurement data and information availability has been increasing in the manufacturing environment today because of increased usage of sensors for automatic collection of data/measurements in industries. Transactional and customer relevant information is also growing at a rapid rate in service industries. It is important to monitor and control processes in such an environment. Often, quality of a process can be characterized by some relationship between a response variable and explanatory variable(s). Such a relationship is also referred to as a profile (Kang & Albin, 2000). Profile monitoring is a technique to monitor such profiles instead of monitoring individual quality characteristics.

As an example, a machine vision system is used in the electronics manufacturing industry to check the quality of LCD displays. A typical LCD consists of thousands of pixels which are individually lit and are independent of each other. As explained by Wang and Tsung (2005), amount of luminance of each pixel follows a normal distribution. Hence testing of a typical LCD display would involve measuring luminance of thousands of pixels. Presence of any defects in LCD display would result in violation of normality on measured luminance. Luminance measurement here is an example of a profile and quality testing of LCDs to make sure luminance measurement for each LCD display follows a normal distribution, with known parameters, is an example of profile monitoring.

As discussed by Walker and Wright (2002), in manufacturing of particleboard and fiberboard, the density of a board is very important for its machinability. A profilometer with a laser device is used to measure the density of board at fixed depths across thickness. A sample of 2 square inch board would consist of 314 vertical density profile (VDP) at every 0.002 inches distance. Monitoring VDP, in this case, is another example of profile monitoring.

Profile monitoring generally consists of two phases. In Phase I, stability of a process is evaluated, any outliers with assignable causes are removed and in-control process parameters are estimated. Hence, the characteristics of an in-control profile are estimated in phase I. Phase II is used to either verify that the process or profile of the quality characteristic is still in control or to detect shifts in the quality characteristics from an in-control state.

In Phase II, often distribution parameters are used to monitor profiles given that parameter estimates are independent. Traditional control charting techniques like Shewhart chart, exponentially weighted moving average (EWMA) chart or cumulative sum (CUSUM) charts are used with control limits set at a specified distance from the mean to detect shifts in the mean. Mean of the parameter estimates that fall within control limits normally indicate that process is in control and indicates presence of common cause variation only. If the mean of parameter estimates falls outside the control limits or if there are non-random patterns it indicates the likely presence of special causes. Such a process is considered to be out of control.

Chicken et al. (2009) and Wang and Tsung (2005) have indicated that there is a possibility that profile parameters or estimates may be the same even though the profile has changed. Sometimes it may be difficult to estimate parameters because of the difficulty in identification of the correct model that fits the quality characteristic. Wang and Tsung (2005) have shown an example of a machine vision system for inspecting phone display where 5000 points are measured at the rate of 10 seconds per display. Because of a huge amount data in their case, dark corners and bad pixels could be averaged out failing to be detected by conventional charting method. Hence they suggested that instead of monitoring summary statistics, a profile could be characterized by a Quantile-Quantile (Q-Q) plot and a Q-Q plot could be monitored through profile monitoring techniques. Wang and Tsung (2005) proposed monitoring the slope and intercept of a Q-Q plot by using EWMA charts for the quality characteristic which is normally distributed. Since the normal distribution is a two-parameter distribution, a Q-Q plot using the slope and intercept is adequate in representing the distribution. Estimates of the mean and standard deviation of the normal distribution uniquely impacts the slope and intercept in a Q-Q plot.

In profile monitoring in a data-rich environment, a quality characteristic or a dependent variable could come from any of the defined distributions. For example, mobile phone display inspection by a machine vision system as used in Wang and Tsung (2005) follows a normal distribution. Physiological measurements like blood pressure of adult humans seem to follow a normal distribution. Over-voltage occurring in an electrical system, manufacturing and delivery times in the case of industrial engineering, and wind speeds are often seen to match a Weibull distribution (Jangamshetti & Rau, 1999; Osmokrovic, Krivokapic, MatijaSevic, & Kartalovic, 1996). Exchange rates, price indices, and stock market indices have been seen to follow a lognormal distribution. In reliability analysis, time to repair a maintainable system is seen to follow a lognormal distribution. A Gamma distribution has been used to model aggregate insurance claims and amount of rainfall accumulated in a reservoir.

Though there are several studies that have monitored linear and non-linear profiles, there are very few studies that have been performed to monitor a profile which could be modeled by a known distribution such as a Weibull, lognormal, Gamma or a general profile in a data-rich environment.

There are many cases where quality characteristics could be correlated. As an example, extrusion is widely used in plastic manufacturing. Flow rate per unit length and mass per unit area are two important quality characteristics affected by mold temperatures. These quality characteristics are correlated (Rauwendaal, 2013). Customer evaluation of a product may be characterized by numerous quality characteristics that are correlated. Functional performance of an automotive component may be characterized by multiple correlated quality characteristics. In such cases monitoring individual quality characteristics may be misleading. Multivariate control charts like Hotelling T^2 chart and multivariate EWMA charts are commonly used to monitor correlated quality characteristics.

In the manufacturing context, as production rate increases and cost per part is high, it becomes more and more important for any shift in process parameters, due to special causes, to be detected as soon as possible. The cost of non-detection or slower detection may result in increased scrap or additional cost related to customer complaints.

This study aims to address the following research question:

Demonstration of profile monitoring when the profile is known to follow known distribution such as normal, Weibull, lognormal or gamma. Through the use of simulation, the study will compare various techniques of profile monitoring and detection of shift in process parameters. The objective will be to propose a method that is faster/efficient in detection. Some new monitoring statistics will be proposed, and their performance will be investigated via a simulation procedure. Also, for unknown distributions that are estimated through a chosen number of percentiles, similar monitoring techniques may be used.

Chapter 2

Literature Review

2.1 Control Charts

The use of statistical control charts to monitor processes has been quite common since Shewhart (1931) introduced the technique. Since then, product and process quality characteristics have been monitored by using various forms of univariate and multivariate control charting techniques. Using profiles to monitor a quality characteristic (QCH) is relatively new. This last method is becoming more and more common with the volume and ease of capturing measurement data where corresponding variables may be correlated and/or follow a certain statistical distribution.

A Shewhart control chart is a technique that displays a quality characteristics vs sample or subgroup number i (i = 1, 2,……., m), where random samples, either individual observations or m subgroups are chosen from a process at intervals (such as hourly, daily, or per shift, etc.). A Shewhart chart consists of a Center Line (CL), Lower Control Limit (LCL) and Upper Control Limit (UCL) calculated as

$$LCL = \mu_w - L\,\sigma_w \tag{2.1}$$

$$CL = \mu_w \tag{2.2}$$

$$UCL = \mu_w + L\,\sigma_w \tag{2.3}$$

where w is a QCH, μ_w is the mean of w, σ_w is the standard deviation of w and L is the distance of control limits from the center line in units of standard deviations of the monitoring statistic w. There are two types of control charts: 1: for targeted parameters, and 2: developed from an initial observations of m subgroups each of size n. When subgroups of observations are selected, a measure of variability of observations is the sample range (R) given by $(X_{max} - X_{min})$, the difference between the maximum and the minimum values of observations in a subgroup.

Traditionally, two of the most common control charts to monitor a process are the R and $\bar{X}$ charts. The multiplier "3" in the equations for the lower and upper control limits is based on the assumption of normality of a charting statistic. When control limits are placed at the three (so-called) standard deviations (sigma) away from the mean, the probability of a "false alarm" is roughly 0.0027. This implies the probability of reaching a false conclusion that the process is out of control is approximately 0.27%. Such an error is labeled as a Type I in the context of decision making from control charts. Tightening of control limits, say to two sigma around the mean, will increase the chances of a "false alarm" or Type I error. A Type II error in the context of decision making from control charts, is to conclude that the process is in-control when it actually is out-of-control. Tightening of the control limits will lead to reducing the probability of a type II error. Hence, an inverse relationship exists between the probabilities of a Type I and II errors assuming other parameters of a control chart, such as n are held constant.

Two systems of causes are assumed in a control charting scheme – common or chance causes, and special or assignable causes. Common causes are inherently part of a system. They occur because of confounding effects that may impact process parameters. They are assumed to be random in nature. Special (or assignable) causes are due to underlying changes in the process that, when identified, lead to remedial actions that must be taken. Their effect is not generally random in nature.

2.2 Phases of Monitoring

In Phase I of process monitoring, observations from an assumed "in-control" process are obtained and monitored via the corresponding control charts. Suppose, for example, R and $\bar{X}$ charts are being used to monitor process variability and mean. If some values of R or ($\bar{X}$) fall outside control limits and assignable causes are removed, then the corresponding observations are deleted leading to a recalculation of center line and the control limits. Further, based on recalculated control limits, if other values of R or ($\bar{X}$) are outside the revised limits, they are consequently deleted under the assumption of presence of assignable causes. When no further values of the monitoring statistic, R and ($\bar{X}$), fall outside revised control limits, the revision process is terminated. At this juncture, we assume that observations originate from an "in-control" process. This revision-procedure of limits until no further values of the monitoring statistics are outside control limits is termed as Phase I. Hence, at the end of Phase I, estimates of process parameters, such as the process standard deviation (σ) and the process mean (μ) are obtained. These estimates are used to calculate control limits at the end of Phase I. Further, the control limits found at the end of Phase I, are used to monitor a process in the next phase.

2.3 Common Rules of Detection of Process Changes

In Phase II, as new observations are taken from a process, they are monitored using the control limits obtained at the end of Phase I. Hence, a determination is now made to identify if there has been a change in process parameters. If a monitoring statistic plots outside control limits or an identifiable pattern is observed, the inference is made that the associated process parameter has changed from its "in-control" value, only if assignable causes are found.

A Shewhart control chart has a major disadvantage that ignores information from a sequence of m (rational) subgroups but uses the very last sample or subgroup to decide whether the process is in-control. This makes the Shewhart chart insensitive to shifts smaller than 1.5 standard deviations in magnitude Montgomery (2013). In order to overcome this disadvantage, several supplemental sensitizing rules have been proposed since the mid-1950's, e.g., Page (1955), Western Electric (1956), Roberts (1958), and Bissel (1978). According to Montgomery (2013) and Minitab documentation, the most widely used runs rules, in addition to the first four suggested by Western Electric, are as follows:

1) One or more points greater than 3-sigma from the center line (where sigma represents the standard deviation of the monitoring statistic)

2) Two out of three points greater than 2-sigma from the center line on the same side within 3-sigma limit
3) Four out of five points greater than 1-sigma from the center line on the same side but within 3-sigma limit
4) Eight points in a row on one side of the center line (i.e., a run of length 8)
5) Six points in a row all increasing or all decreasing (runs of length 6 up or down)
6) Fifteen points in a row within 1-sigma of the center line on either side
7) Fourteen consecutive points occurring up and down from the center line
8) Eight points in a row greater than 1-sigma on either side
9) Any non-random pattern in the data
10) One or more points near a warning limit (placed at ± 2-sigma from the center line).

Use of all of the above sensitizing techniques can reduce the average run length of detecting change in a process parameter and improve the ease of decision making using Shewhart control charts (Montgomery, 2013). However, the chance of a “false alarm” also increases with an increase in the number of rules that are used.

Two other control charts to magnify the degree of change in a monitoring statistic have been subsequently developed. They are the cumulative sum (CUSUM) control chart and the exponentially weighted moving average (EWMA) control chart. CUSUM charts were developed by Page (1954), while EWMA charts were developed by Roberts (1959). In an EWMA chart, the weight given to the past observations decreases as the time from the current period increases. Hence, observations that occur 20 periods prior to the current will receive a weight that is much smaller than the weight given to an observation that occurs 10 periods prior to the current. The past weights decrease exponentially with time, in a geometric fashion, and hence the name EWMA given to the associated control chart. CUSUM and EWMA charts are used to detect smaller shifts in process parameters of a quality characteristics being monitored.

2.4 Cumulative Sum (CUSUM) Control Chart

A CUSUM uses two statistics: One sided lower C^- and one-sided upper C^+, which are calculated by accumulating deviations from the mean that are below and above a specified target, respectively. They are defined as follows:

$$C_i^- = max[0, (\mu_0 - K) - X_i + C_{i-1}^+] \quad (2.4)$$

$$C_i^+ = max[0, X_i - (\mu_0 + K) + C_{i-1}^+] \quad (2.5)$$

where X_i (i= 1, 2, ….) is the i^{th} observation from a process. At an in-control state, it is assumed that X_i follows a normal distribution with targeted mean μ_0 and standard deviation σ_0. The starting values of C_i^- *and* C_i^+ are 0, K is the reference value and is one half of the magnitude of the shift that we wish to detect. The statistics C_i^- *and* C_i^+

accumulate deviations from the target value that are greater than *K* and are reset to zero when they become negative. When C_i^- and C_i^+ exceed the desired interval, the process is considered to be out of control. The desired interval, *H*, is generally considered to be 5 times σ_0. Since the deviations from a chosen target are being accumulated successively, as the monitoring statistics C_i^- and C_i^+ are calculated, the CUSUM statistic may inflate the impact of a shift in a process parameter. Usually, for small shifts, the average run length (ARL) for detection using the CUSUM chart is smaller than that for a corresponding Shewhart chart.

According to Montgomery (2013), subgroups should be selected in such a manner that maximize variability between subgroups and simultaneously minimize variation within a subgroup. Grouping of observations in the above manner is known as rational subgrouping. In case of monitoring rational subgroups where more than one observation is chosen in a subgroup, X_i in the above equations for CUSUM statistics is replaced by $\bar{X}_i$, which is the average of the observations in subgroup *i* and sigma is replaced by $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$.

2.5 Exponentially Weighted Moving Average (EWMA) Chart

EWMA charts, like CUSUM charts, are also used to detect smaller shifts in a process mean. EWMA charts are easier to set up and their performance is comparable to CUSUM charts. The EWMA monitoring statistic in period *i* is defined by the following equation (Montgomery, 2013)

$$M_i = \lambda X_i + (1-\lambda)M_{i-1} \tag{2.6}$$

where $0 < \lambda < 1$ is a charting constant and $M_0 = \mu_0$ is the target-value . The constant, λ, is often termed as the smoothing constant. When $\lambda = 1$, the EWMA chart reduces to the appropriate Shewhart chart.

If the observations X_i are independent random variables with variance σ^2, then the variance of M_i is given by

$$\sigma_{M_i}^2 = \sigma^2 \left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right] \tag{2.7}$$

The control limits for the targeted EWMA control chart are given by

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} \tag{2.8}$$

$$CL = \mu_0 \tag{2.9}$$

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} \tag{2.10}$$

where *L* is the selected number of standard deviations away from the mean based on a chosen level of Type I error. As the time period *i* increases, the term $\left[1-(1-\lambda)^{2i}\right]$ approaches 1, hence the EWMA chart parameters reduce to

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}} \tag{2.11}$$

$$CL = \mu_0 \tag{2.12}$$

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}} \tag{2.13}$$

2.6 Control Charts for Multivariate Observations

There are numerous manufacturing processes where simultaneous monitoring of more than one quality characteristic is necessary. As pointed out by Montgomery (2013), monitoring two or more QCH that are correlated, with individual control charts can be misleading. Hence when two or more QCH are measured from the same sampling unit, multivariate quality control techniques are used to monitor a process. Hotelling (1947) introduced a method also known as Hotelling T^2 control chart using bombsight data.

For individual observations, Hotelling's multi-variate T^2 is calculated as

$$T^2 = (X - \bar{X})' S^{-1} (X - \bar{X}) \tag{2.14}$$

where X is the sample observations vector given by $\left[X_1,\ X_2\ , \dots \dots\ ,\ X_p\right]^T$ and S is the variance - covariance matrix respectively of the observations, and $\bar{X} = \left[\bar{X}_1, \bar{X}_2, \dots \dots, \bar{X}_p\right]^T$ is the mean vector of the p quality characteristics. Hotelling T^2 requires the assumption that underlying distribution is multivariate normal.

For Phase I, the control limits are given by the following equation (Tracy, Young, & Mason, 1992)

$$LCL = 0 \tag{2.15}$$

$$UCL = \frac{(m-1)^2}{m} \beta_{\alpha, p/2, (m-p-1)/2} \tag{2.16}$$

where m is the number of subgroups, α is the chosen level of significance, and β represents the upper α^{th} percentile of the Beta distribution with appropriate parameters. For Phase II, control limits are given by

$$LCL = 0 \tag{2.17}$$

$$UCL = \frac{p(m+1)(m-1)}{m^2 - mp} F_{\alpha,p,m-p} \tag{2.18}$$

where $F_{\alpha,p,m-p}$ represents the upper percentile of the F-distribution with an upper tail probability of α, numerator df = p, and denominator df = m - p.

In the event that the number of preliminary subgroups m is large, i.e., say greater than 30, the control limits can be approximated as

$$LCL = 0 \tag{2.19}$$

$$UCL = \frac{p(m-1)}{m-p} F_{\alpha,p,m-p} \tag{2.20}$$

or

$$UCL = \chi^2_{\alpha,p} \tag{2.21}$$

For data collected using subgroups, the Hotelling T^2 is calculated as

$$T^2 = n(\bar{X} - \bar{\bar{X}})' S^{-1} \left(\bar{X} - \bar{\bar{X}}\right) \ , \tag{2.22}$$

where $\bar{X}$ is the mean vector of p quality characteristics, and $\bar{\bar{X}}$ is the mean vector of $\bar{X}_j$ and S is the sample covariance matrix of the vector of means.

Alt (1985) proposed control limits based on the phase in which the chart is used. In Phase I, stability of a process is evaluated using historical data, where outliers with assignable causes are removed and process parameters are re-estimated. Process parameters estimated from Phase I are subsequently used to obtain revised control limits. Phase II is used to either verify that the process is still in control or to detect shifts in process parameters from an in-control state.

In Phase I, the control limits are given by

$$LCL = 0 \tag{2.23}$$

$$UCL = \frac{p(m-1)(n-1)}{mn-m-p+1} F_{\alpha,p,mn-m-p+1} \tag{2.24}$$

The control limits in Phase II are given by

$$LCL = 0 \tag{2.25}$$

$$UCL = \frac{p(m+1)(n-1)}{mn-m-p+1} F_{\alpha,p,mn-m-p+1} \tag{2.26}$$

2.7 Monitoring of Profiles

As discussed in Chapter 1, a profile is defined as a relationship between a response variable and explanatory variable(s) that characterizes the quality of a process (Kang & Albin, 2000). LCD display, for example, consists of thousands of pixels each having luminance that are independently and normally distributed. A defect in an LCD display may show a shift in distribution parameters on luminance measurements.

Many studies in the past have used Hotelling T^2 in some form to monitor a profile. Multiple researchers have proposed using T^2 control charts differently. For example, Stover and Brill (1998) proposed a T^2 calculation for a linear profile using a regression line represented by $y = ax + b$ as

$$T_i^2 = [s_b\ (a_i - a_0)^2 + s_a\ (b_i - b_0)^2 - \frac{2s_{ab}(a_i - a_0)(b_i - b_0)}{[s_a s_b - s_{ab}^2]} \tag{2.27}$$

where, a_i and b_i are the slope and intercept based on the i^{th} observation, a_0 and b_0 are the average slope and intercept over all the observations, s_a^2 is the slope variance, s_b^2 is the intercept variance, and s_{ab} is the covariance between slope and intercept. The upper control limit for the T^2 control chart is calculated as

$$T^2 \sim [p(n+1)(n-1)F(p, n-p, \alpha)]/[n(n-p)] \tag{2.28}$$

where, $F(p, n\text{-}p)$ has p numerator df and $(n - p)$ denominator df. Dimension number in each multivariate observation is represented by p, the number of estimated parameters, and n is the number of vector observations in each subgroup.

Kang and Albin (2000) may be one of the earliest researchers to propose profile monitoring. They propose two control charting methods where quality characteristics of interest could be characterized by a profile. Their study focused on a profile that was linear in nature. They discuss the application of linear profiles in semiconductor manufacturing where a mass flow controller (MFC) regulates the flow of gases. The pressure in the chamber is a linear function of flow rate, X. Their linear profile is expressed as

$$\text{P} = P_0 + \left(\frac{Q_{max}RTt}{V}\right)X \tag{2.29}$$

where the intercept P_0 is the base pressure and the slope consists of Q_{max} which is the maximum flow rate, R is the type of gas, T is temperature, t represents time, and V is the volume of chamber.

First they proposed the use of Hotelling T^2 control chart to monitor the slope and intercept of linear profiles. Their proposed T^2 statistic is

$$T_j^2 = \left(M_j - U\right)^T \Sigma^{-1}(M_j - U) \tag{2.30}$$

where the vector $M_j = (a_{0j}, a_{1j})^T$, a_{0j}, a_{1j} are least squares estimators for slope and intercept, vector $U = (A_0, A_1)^T$, A_0 is the mean intercept, A_1 is the mean slope, the variance-covariance matrix $\Sigma = \begin{pmatrix} \sigma_0^2 & \sigma_{01}^2 \\ \sigma_{01}^2 & \sigma_1^2 \end{pmatrix}$, σ_0^2 is the variance of a_0, σ_1^2 is the variance of a_1, and σ_{01}^2 is the covariance of a_{0j} and a_{1j}.

Kang and Albin (2000) used an upper control limit as a chi-square variable with 2 degrees of freedom for a bivariate response for a specified significance level α.

Their second method was to use an EWMA chart and an *R*-chart based on regression residuals between sample and reference line to detect shifts in the intercept, slope, and residual variance.

Jin and Shi (2001) proposed monitoring process-faults using wavelet analysis for diagnostic systems without prior information. They indicated that tapping torque signals, welding force signals, and stamping tonnage signals are in waveform that are cycle based and each cycle has different segments representing process stages.

Walker and Wright (2002) discuss numerous examples including manufacturing of particleboard and fiberboard where the density of a board is very important for its machinability. A profilometer with a laser device is used to measure the density of board at fixed depths across thickness. A sample of 2 square inch board would consist of 314 vertical density profile (VDP) at every 0.002 inches distance. The profile, in this case, has a complicated form having hundreds of points of interest which cannot be modeled by a polynomial profile. They proposed the use of generalized additive models (GAMs) to monitor a profile. Complicated curves are fit using GAMs and difference between the curves were evaluated using parametric models.

Kim, Mahmoud and Woodall (2003) propose an alternate approach to Kang and Albin (2000). Instead of considering residuals between sample and reference line to detect a shift in intercept, slope and variance as proposed by Kang and Albin (2000), they propose to code the independent variables in their simple linear regression model. Coding makes estimators of y-intercept and slope independent and thereby allowing separate Shewhart type chart for regression parameters.

Mahmoud and Woodall (2004) proposed a method based on a multiple regression model using indicator variables. Using simulation, they compared the performance of their proposed F test method based on indicator variables with the performance of a T^2 control chart proposed by Stover and Brill (1998), T^2 control chart proposed by Kang and Albin (2000), and Shewhart type control charts proposed by Kim et al. (2003). Through simulation they showed the effectiveness of their method, as determined by the overall probability of an out-of-control signal, is better than that of the T^2 control chart proposed by Stover and Brill (1998). The method proposed by Kang and Albin (2000) was also ineffective in detecting shifts in process standard deviation.

Wang and Tsung (2005) discussed the application in a machine vision system used in electronics manufacturing industry to check the quality of an LCD display. They indicated homogeneity of the quality characteristic within a sample, such as luminance of the pixels in this context, may be violated when the sample size is large. Traditional control charting techniques to monitor process variability and mean are less effective as measured by the average run length to detection of out-of-control condition. They proposed the usage of a Quantile-Quantile (Q-Q) plot to monitor a profile.

Gupta et al. (2006) compared the performance of a control charting method proposed by Croarkin and Varner (1982) with Kim et al. (2003) using a simulation method. Their simulation study showed that Croarkin and Varner's method performed poorly compared to the control charting scheme proposed by Kim et al.

Mahmoud et al. (2007) proposed a change point approach based on a segmented regression technique to detect changes in Phase I linear profiles. Using simulation, they compared their proposed method with an F-test proposed by Mahmoud and Woodall (2004) and a method proposed by Kim et al. (2003). The authors concluded that the change point method performs better under sustained step shift in a regression parameter. But, the change point method proposed by Mahmoud et al. (2007) was insensitive to randomly scattered un-sustained shifts in process parameters in which case the method by Kim et al. (2003) and the F-test performed better than the change point method.

Majority of research on profile monitoring till 2007 was focused mainly on a simple linear profile. Zou, Tsung and Wang (2007) proposed a multivariate exponentially weighted moving average method based on Lowry et al. (1992) to monitor general linear profiles that can be represented by multiple linear regression models. Lowry et al. (1992) defined a multivariate EWMA model as

$$M_i = RX_i + (I - R)M_{i-1} \tag{2.31}$$

where M_i is the EWMA statistic in period i, X_i = observation in period i, i = 1,2 , ……, $M_0 = 0$ and R = diag (r_1, r_2, ……r_p), $0 < r_j \leq 1$, j = 1,2,…..p, where p represents the number of independent variables. An out-of-control signal is determined by the monitoring statistic, T_i^2, if it exceeds a chosen upper bound, h_4. This is given by the relation :

$$T_i^2 = M_i' \Sigma_{M_i}^{-1} M_i > h_4 \tag{2.32}$$

where h_4 is chosen based on a desired in-control ARL (Average Run Length), Σ_{M_i} is the variance covariance matrix of M_i. The weights r_1, $r_2 \ldots r_p$ are generally considered equal unless there is a reason to consider different weights based on a sequence of individual observations. Considering equal weights where $r_1 = r_2 = r$, a multivariate EWMA statistic in Eq. (2.31) reduces to

$$M_i = rX_i + (1-r)M_{i-1} \tag{2.33}$$

The covariance matrix of M_i is given by

$$\Sigma_{M_i} = \{r[1-(1-r)^{2i}]/(2-r)\}\Sigma \tag{2.34}$$

For the larger values of i, the above covariance matrix simplifies to

$$\Sigma_{M_i} = \left\{\frac{r}{(2-r)}\right\}\Sigma \tag{2.35}$$

Zou, Tsung and Wang (2008) proposed a multivariate exponentially weighted moving average procedure along with a generalized likelihood ratio test to monitor non linear profiles. Saghaei, Mehrjoo and Amiri (2009) proposed a method based on cumulative sum statistics to monitor linear profiles in Phase II.

Zhang and Albin (2009) proposed using a chi-square control chart to monitor complex profiles to detect outliers. Their proposed method does not require fitting a regression model. They compared their method with a non-linear regression method and showed that their method performed better and had a lower level of mis-identification.

Many studies on profile monitoring have been based on the assumption that the corresponding response variable is normally distributed. However, some researchers have studied profile monitoring when a response variable follows a distribution other than normal. Amiri et al. (2011) proposed a method where the response variable is binary. The number of responses for a given occurrence rate is described by a Poisson distribution. They use the log link function to model a failure rate as a function linked to the number of occurrences. The authors monitor a process through a T^2- type statistic that uses the estimated parameters of the log link function. This was proposed for Phase I analysis. Later in 2012, Amiri et al. (2012) used a T^2 based method for monitoring a Gamma response profile in Phase I.

Zhang and Albin (2009) used a χ^2control chart to identify outliers for the case where profiles could not be characterized by a specific function. They used pairwise differences among medians to estimate the variance of profiles. Their method showed better performance with fewer false alarms compared to other existing non-linear regressions method.

Noorossana and Ayoubi (2011) propose a non-parametric bootstrap control chart for simple linear profiles in Phase II based on the T^2 statistic. Their simulation showed that the bootstrap control chart performed better with increasing data size. They indicated that their method was applicable to multiple, polynomial, and nonlinear profiles in addition to simple linear profiles. Nikoo and Noorossana (2012) used nonparametric regression with wavelets for monitoring nonlinear profiles in Phase II.

Adibi et al. (2014) use a *p*-value approach to monitor linear profiles in Phase II. Out-of-control state is determined when the *p*-value is less than some pre-determined significance level. They compared their procedure with a Shewhart-based method, and their simulation showed that their method performed satisfactorily.

Mitra and Clark (2014) propose an aggregate method instead of individual control charts for each element of variance-covariance matrix for monitoring variability in multivariate processes. They also compared the performance of their method with that of the traditional method using mean time to first detection of shift in process variability.

While comparing the performance of different methods in profile monitoring, majority of research have focused on ARL or mean time to first detection of shift in process mean. Limited research has focused on the standard deviation of run length (SDRL) as a performance metric. Aly et al. (2015) used SDRL as a performance metric in Phase II to compare methods proposed by Kang and Albin (2000), Kim et al. (2003)

and Mahmoud et al. (2010) and their findings indicated that the method proposed by Kim et al. (2003) performed better in terms of SDRL.

Kazemzadeh et al. (2016) proposed using an adaptive variable sample size scheme to monitor simple linear profiles to improve the performance of conventional control charts. Their study showed that using an adaptive feature improves the performance in detecting parameter shifts.

Ghashghael and Amiri (2017a) propose a Max-MEWMA (Multivariate Exponentially Weighted Moving Average) and a Max-MCUSUM control charts for multivariate linear regression profiles in Phase II. Ghashghael and Amiri (2017b) also proposed a sum of squares control charts for Phase II monitoring of multivariate linear regression profiles. Both of their proposed control charts detects whether an out-of-control signal is caused by a location shift or scale shift.

The objective of this dissertation is to develop techniques to monitor profiles from known or unknown distributions. Four forms of the T^2 statistic will be used. Details of the four monitoring methods are described in Chapter 3.

Chapter 3

Profile Monitoring Through Control Charts

3.1 Methodology

The T^2 chart was introduced by Hotelling (1947) to jointly monitor multiple quality characteristics at least two of which are correlated. Control charts based on T^2 have been used in several studies including those by Alt and Smith (1988) and Wierda (1994).

Without loss of generality, consider distributions that are from the two-parameter family. The normal distribution, the two-parameter Weibull, the lognormal, and the two-parameter Gamma are examples of such distributions. A profile random variable (prv) following a known underlying distribution may be monitored by using control charts that utilize the distribution parameters estimated from chosen sample or subgroups. A T^2 chart may be used for this purpose. Another approach is to monitor a chosen number of quantiles of the selected distribution through a joint T^2 control chart. These approaches are subsequently described.

3.2 Monitoring Methods

We first present two methods of jointly monitoring estimated parameters from a selected distribution.

For a j^{th} subgroup, let the parameter estimates of the distribution be represented by the vector $M_j = (a_{0j}, a_{1j})^T$; further the estimated variance-covariance matrix of the vector M_j is denoted by S_1. The monitoring sample statistic, ${T_j}^2$, for the j^{th} subgroup is given by

$$ {T_j}^2 = (M_j - \bar{M})^T S_1^{-1} (M_j - \bar{M}) \tag{3.1} $$

where $\bar{M}$ represents the estimated average value of the vector M over all subgroups. The corresponding variance-covariance matrix S_1 is given by

$$ S_1 = \frac{1}{(m-1)} \sum_{j=1}^{m} (M_j - \bar{M})(M_j - \bar{M})^T \tag{3.2} $$

Wierda (1994) concluded that a T^2 control chart has the advantage over univariate charts since it takes the correlation structure into account.

A chart statistic, for monitoring a target parameter vector M_0, for a univariate EWMA chart is given by

$$ M_{ewma_j} = r\big(X_j\big) + (1-r) M_{ewma_{j-1}} \tag{3.3} $$

where r represents a smoothing constant, $0 \leq r \leq 1$. Usually M_{ewma0} is chosen as X_1, the first observed value of the estimated statistic at subgroup j =1.

A multivariate EWMA chart, which we will refer here-after as T^2-EWMA (MEWMA), is an extension of a univariate EWMA. The MEWMA –statistic is calculated as suggested by Lowry et. al. (1992):

$$M_{ewma_j} = RX_j + (\mathrm{I} - R)M_{ewma_{j-1}}, \tag{3.4}$$

where X_j is the estimated parameter for subgroup j, and R is a diagonal matrix of the smoothing constants. In the special case when the smoothing constants have a common value r, the charting statistic for subgroup j is given by

$$T_{ewma_j}{}^2 = (M_{ewma_j} - \overline{M_{ewma}})^T S_2^{-1}(M_{ewma_j} - \overline{M_{ewma}}), \tag{3.5}$$

where,

$$S_2 = \frac{1}{(m-1)}\sum_{i=1}^{m}(M_{ewma_j} - \overline{M_{ewma}})(M_{ewma_j} - \overline{M_{ewma}})^T \tag{3.6}$$

$\overline{M_{ewma}}$ represents the average value of the vector M_{ewma_j} and S_2 represents the estimated variance-covariance matrix of the vector M_{ewma}.

When a profile function is distribution free, a general approach can be used to estimate the unknown distribution characteristics. In this context, the corresponding quantiles serve to identify the profile. For the case of general profiles, instead of using parameter estimates, quantile estimates can be used to calculate the corresponding T^2 and T^2-EWMA statistics. Such statistics may also be calculated using quantile estimates from a known underlying distribution to determine the performance of the monitoring method.

For the vector $M_{pj} = (a_{1j}, \ldots\ldots, a_{pj})^{\mathrm{T}}$, let $a_{1j}, \ldots\ldots, a_{pj}$ represent the estimates of the selected p quantiles from an empirical distribution. The sample statistic, $T_{pj}{}^2$, for subgroup j, is given by

$$T_{pj}{}^2 = (M_{pj} - \overline{M_p})^T S_3^{-1} (M_{pj} - \overline{M_p}), \tag{3.7}$$

where $\overline{M_p}$ represents the average value of the vector M_p. The variance-covariance matrix S_3 is given by

$$S_3 = \frac{1}{(m-1)} \sum_{j=1}^{m} (M_{pj} - \overline{M_p})(M_{pj} - \overline{M_p})^T, \tag{3.8}$$

The statistic in Eq. (3.7) represents the third monitoring method considered in this dissertation for detecting shifts in process parameters.

Using a similar concept as before, an EWMA of estimated quantiles will be developed and statistically examined. An EWMA extension of the above equations is given by

$$M_{pewma_j} = RX_{pj} + (\mathrm{I} - R) M_{pewma_{j-1}}\ , \tag{3.9}$$

where X_{pj} is the estimated percentile for subgroup *j*. In the special case, when the smoothing constants have a common value *r*, we have, the fourth monitoring statistic for subgroup j, as

$$T_{pewma_j}{}^2 = (M_{pewma_j} - \overline{M_{pewma}})^T S_4^{-1} (M_{pewma_j} - \overline{M_{pewma}}), \tag{3.10}$$

where,

$$S_4 = \frac{1}{(m-1)} \sum_{j=1}^{m} (M_{pewma_j} - \overline{M_{pewma}})(M_{pewma_j} - \overline{M_{pewma}})^T, \tag{3.11}$$

$\overline{M_{pewma}}$ represents the average value of the vector M_{pewma_j} and S_4 represents the estimated variance-covariance matrix of the vector M_{pewma_j}.

3.3 Simulation Procedure

A simulation will be used to calculate the control chart limits for monitoring each of the above four statistics. Figure 1 describes the flow chart for calculation of limits. The limits are calculated using the following steps:

1) Let number of rows represent the number of subgroups, *m*.

   Generate *n* observations from a specified statistical distribution with known parameters.

2) The model error ε is assumed N(0, 0.01).

3) Distribution parameter and percentiles are estimated using the results from step 2, for each subgroup *j*.

4) The four monitoring statistics, ${T_j}^2$, ${T_{ewma_j}}^2$, ${T_{pj}}^2$ and ${T_{pewma_j}}^2$, respectively, are calculated as described in the previous equations.

5) Steps 2 – 4 are replicated 10,000 times.

6) For a chosen level of significance α, say, of 5%, the 95th percentiles of ${T_j}^2$, ${T_{ewma_j}}^2$, ${T_{pj}}^2$ and ${T_{pewma_j}}^2$ respectively, are recorded for a given number of subgroups, say *m* = 100.

7) Steps 1 – 6 are repeated with the number of subgroups varying between 100 – 500 with an increment of 1such as 101, 102…..500. Clearly, this procedure

provides the empirical limits of a monitoring statistic, for a chosen level of significance as well as a chosen number of subgroups.

In order to detect a shift in process parameters, it is important to first have an in-control process. Outliers are removed (by utilizing the established empirical control limits) and in control process parameters are estimated. The flow chart for removing outliers and obtaining parameter estimates, in Phase I, is shown in Figure 2 described in the following steps:

1) Let number of rows represent the number of subgroups, m.

   Generate n observations from a given distribution with known parameters for each subgroup.

2) The model error ε is assumed N(0, 0.01).

3) Distribution parameters are estimated using the results from step 2, for each subgroup j.

4) The four monitoring statistics, ${T_j}^2$, ${T_{ewma_j}}^2$, ${T_{pj}}^2$ and ${T_{pewma_j}}^2$, respectively, are calculated as described in previous equations.

5) The previously described monitoring statistic calculated in step 4 is compared with empirical limits, for the appropriate subgroup size for the corresponding distribution (normal or Weibull).

6) The subgroups that are detected out-of-control are removed.

7) The remaining subgroups are considered to represent an in-control state of the process. Estimates of in-control process parameters are calculated using remaining observations.

The flow chart for detecting shifts in parameter estimates is shown in Figure 3. Phase II is a continuation of Phase I and is simulated using the following steps:

8) A sample of *n* observations are generated from a specified statistical distribution with predetermined shift in process parameters.
9) The four monitoring statistics, ${T_j}^2$, ${T_{ewma_j}}^2$, ${T_{pj}}^2$ and ${T_{pewma_j}}^2$, respectively are calculated using the addition of the subgroup data from a distribution with shifted parameters and each is compared with their respective control limits calculated from the Matlab program as described in Figure 1.
10) The RL to first detection is recorded for ${T_j}^2$, ${T_{ewma_j}}^2$, ${T_{pj}}^2$ and ${T_{pewma_j}}^2$, respectively, when each monitoring statistic is greater than its respective limit. Note that the monitoring statistics may first detect the shift at different (times) subgroups. So, to monitor the performance of the statistic, records are maintained for the first time that particular statistic detects the shift.
11) Steps 9 – 10 are repeated until each method has a first detection.

12) Steps 9 – 11 are repeated 10,000, 20,000 or 50,000 times to determine the mean time to first detection and the standard deviation of the time to first detection (SDRL) for each method. The mean time to first detection for an out-of-control state is often represented as the ARL for detection and used as a measure of performance.
13) All the steps in Phase I, and Steps 9 – 12 in Phase II are repeated for the values of exponentially weighted moving average constant (*r*) ranging from 0.1 to 0.9, at an increment of 0.1.
14) The ARL and SDRL are used to evaluate the performance of each of the four monitoring methods.

3.4 Applications of Profile Monitoring to Different Distributions

The normal distribution is one of the most widely used distribution in manufacturing settings. The probability density function (pdf) of a random variable X for a normal distribution is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \tag{3.12}$$

where, μ represents the mean and σ^2 represents the variance of the random variable. The notation N(μ, σ^2) is used to represent a normal distribution with mean μ and variance σ^2.

A quality characteristic or a dependent variable can originate from any distribution. For example, mobile phone display inspection by a machine vision system of the variable luminance, as used in Wang and Tsung (2005), can be modeled by a normal distribution. Physiological measurements such as blood pressure of adult humans seem to closely follow a normal distribution.

A Weibull distribution, on the other hand, is widely used in reliability engineering to model time to failure. The pdf of a Weibull random variable *T* with minimum life zero is given by

$$f(t) = \frac{k}{\lambda}\left(\frac{t}{\lambda}\right)^{k-1} exp\left[-\left(\frac{t}{\lambda}\right)^{k}\right], \tag{3.13}$$

where k is a shape parameter and λ is a scale parameter. Both parameters are greater than zero. The mean and variance of a Weibull distribution are given by

$$\mu = \lambda\Gamma\left(1 + \frac{1}{k}\right), \tag{3.14}$$

$$\sigma^2 = \lambda^2\left[\Gamma\left(1 + \frac{2}{k}\right) - \left\{\Gamma\left(1 + \frac{1}{k}\right)\right\}^2\right], \tag{3.15}$$

Over-voltage occurrence in an electrical system, manufacturing and delivery times in the case of industrial engineering, and wind speeds are often found to fit a Weibull distribution (Jangamshetti & Rau, 1999; Osmokrovic, Krivokapic, MatijaSevic, & Kartalovic, 1996).

Exchange rates, price indices, and stock market indices have been found to follow a lognormal distribution. In reliability analysis, time to repair of maintainable systems has been established to follow a lognormal distribution. A Gamma distribution has been used to model aggregate insurance claims and amount of rainfall accumulated in a reservoir. In this dissertation, monitoring profiles of characteristics from some selected distributions, such as the normal and Weibull, are considered.

3.5 Performance Measures

The ARL for first detection of an out-of-control condition along with SDRL will be used as measures of performance of the monitoring statistics.

3.5.1 Confidence Intervals for ARL

Confidence intervals for ARL may be computed, using principles from the Central Limit Theorem, as :

$$ARL \pm t_{\alpha/2} * SDRL/\sqrt{N_{rep}}, \tag{3.16}$$

where N_{rep} represents the number of replications to compute ARL, for a given parameter combination of the distribution, and $t_{\alpha/2}$ is the quantile of the *t*-distribution, with a confidence level of (1- α) and degrees of freedom of $(N_{rep} - 1)$. Since the number of simulation replications for a given parameter combination, is quite large (at least 10,000), the *t*-statistic can be replaced by the corresponding standard normal $(z_{\alpha/2})$ statistic.

For a given parameter combination, comparison of the relative performances of four monitoring statistics maybe accomplished through a comparison of the confidence intervals for ARL for statistical significance.

Results from the simulations conducted in the study, as will be discussed in the next chapter, will indicate ARL and SDRL of the time to first detection of an out-of-control condition for each monitoring statistic. Obviously, smaller values of ARL are desirable as they will indicate a faster detection of out-of-control conditions in the process. A smaller value of SDRL is also desirable as this will indicate the consistency or precision with which detection of an out-of-control condition is accomplished.

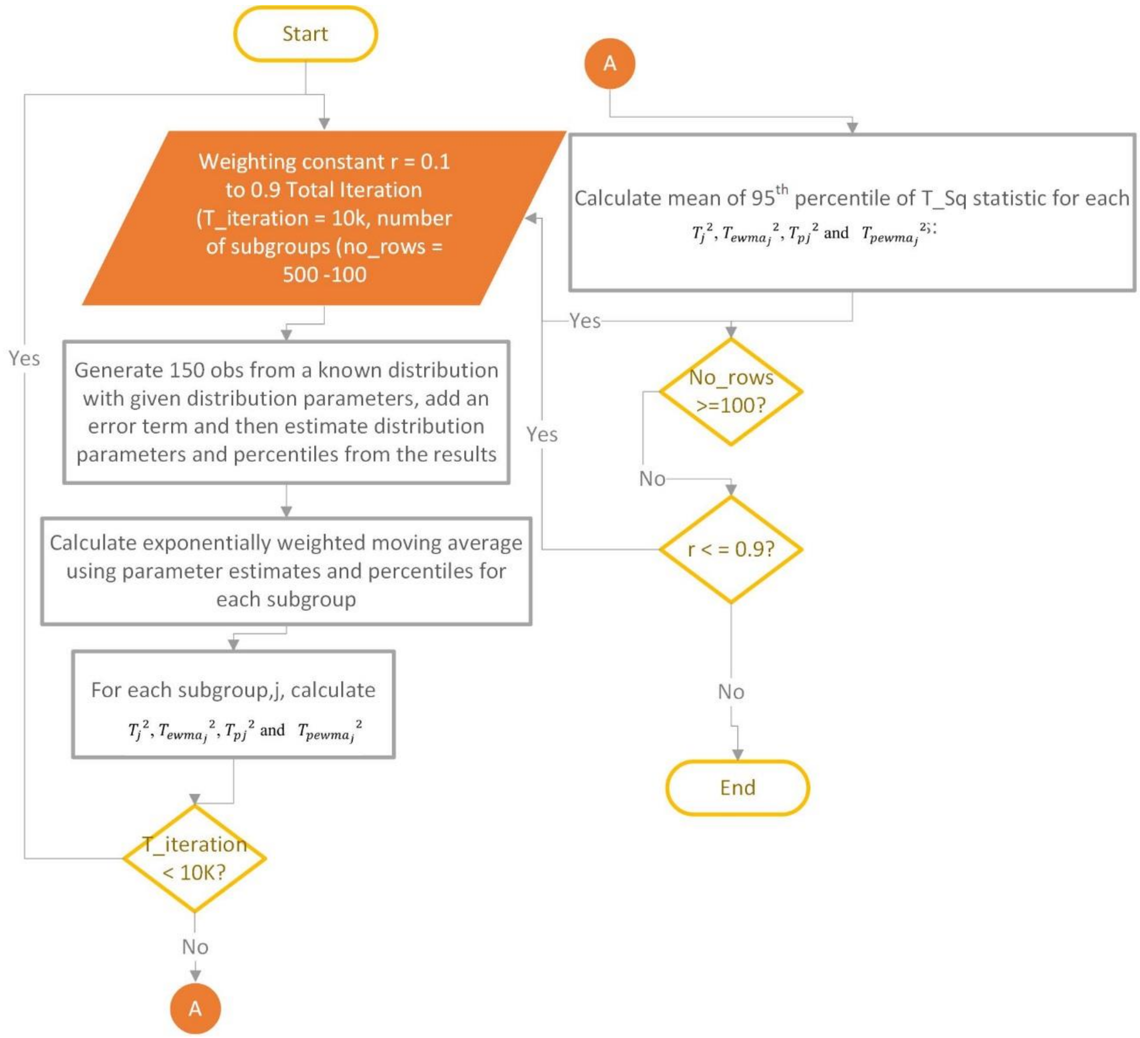


Figure 1: Flow chart for calculation of control limit, based on a chosen level of significance

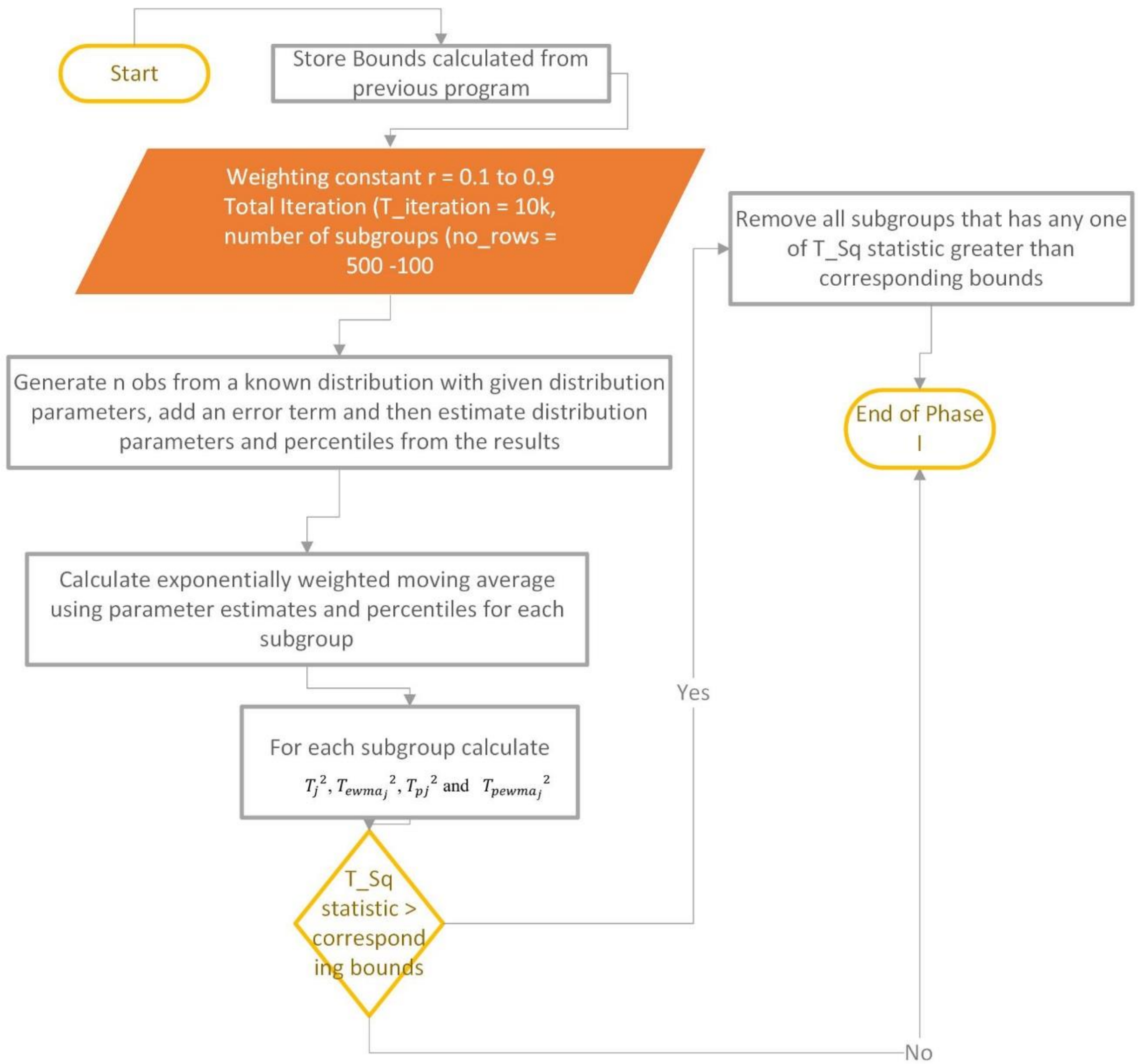


Figure 2: Flow chart of Phase I removing outliers and defining in-control process

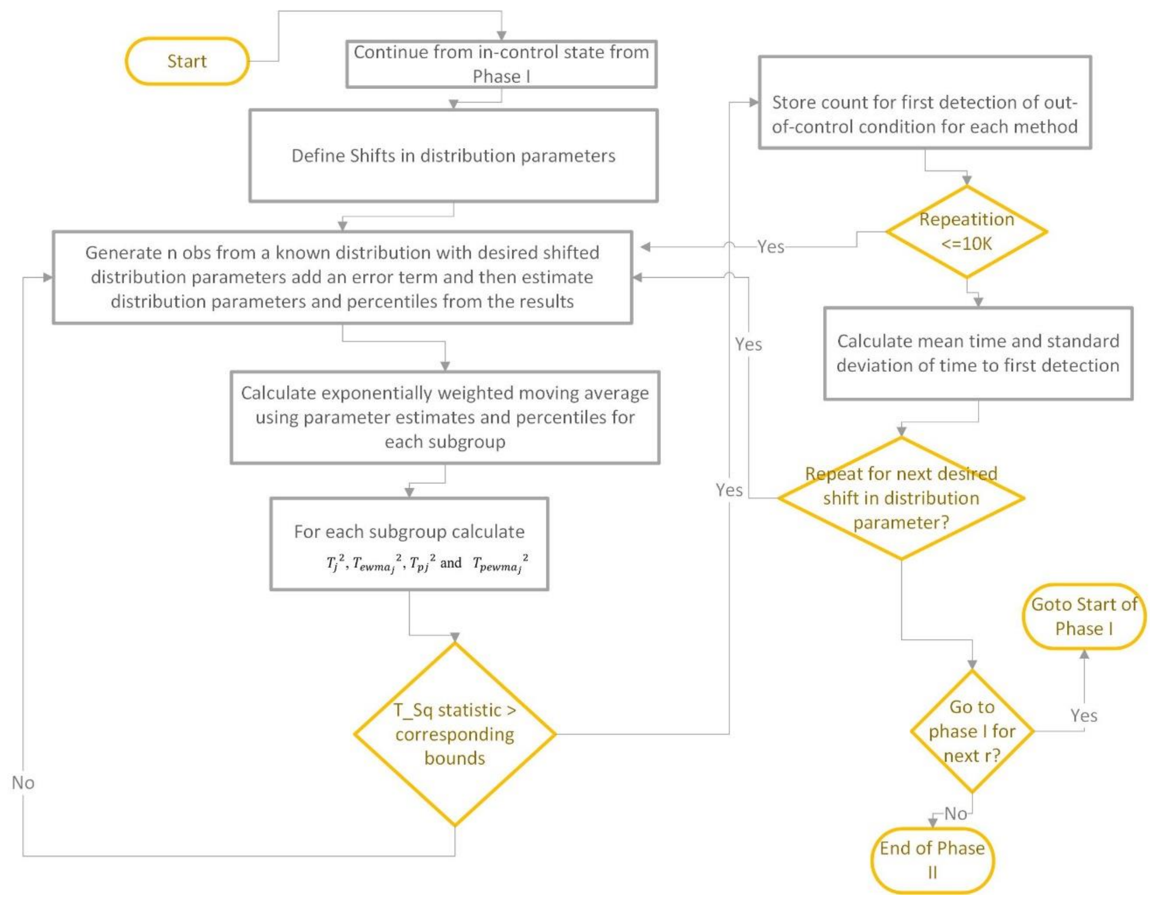


Figure 3: Flow chart of Phase II for detection of shifts in process parameters

Chapter 4

Results and Conclusion

4.1 Control Limits

For each parameter combination, the upper control limits for each monitoring statistic, are first determined. These limits are then utilized in Phase II of the monitoring process to determine the time to detection of an out-of-control condition.

As described in Chapter 3, Figure 1: Flow chart for calculation of limits based on a chosen level of significance, a Matlab program was run to calculate control limits and are reported in Appendix A. The number of individual observations in each subgroup was considered to be 150 in order to ensure a sufficient subgroup size for estimation of the distribution parameters. The Simulation was replicated 10,000 times to calculate the 95th percentile of the T^2 statistic for each method of the four monitoring statistics, ${T_j}^2$, ${T_{ewma_j}}^2$, ${T_{pj}}^2$ and ${T_{pewma_j}}^2$. The control limits calculation was repeated for number of subgroups within 100 – 500 in steps of 1, and for EWMA constant values *r* ranging from 0.1 to 0.9 at an increment of 0.1.

4.2 Discussion of Simulation Runs

Simulation runs of 10,000, 20,000 and 50,000 were compared to determine if the results from 10,000 runs were sufficiently close to those of the other two runs. A conservative approach of simulation runs as high as 50,000 was chosen based on control chart type simulations performed by other researchers such as Mahmound and

Maravelakis (2010), Amin and Miller (1993) and Tatum (1997). After comparing results of the three run sizes, we determined that 10,000 replications were sufficiently large to compare performance ranking between the four methods. See Figure 4 below for an illustrative example. The figure shows the distribution of ARL as a function of the size of shift in the process mean. The empirical distribution of the ARL exhibits a similar behavior for each of three simulation runs. Due to the fact that 50,000 replications have already been run for the evaluation of all desired shifts, results from 50,000 will be discussed in this chapter.

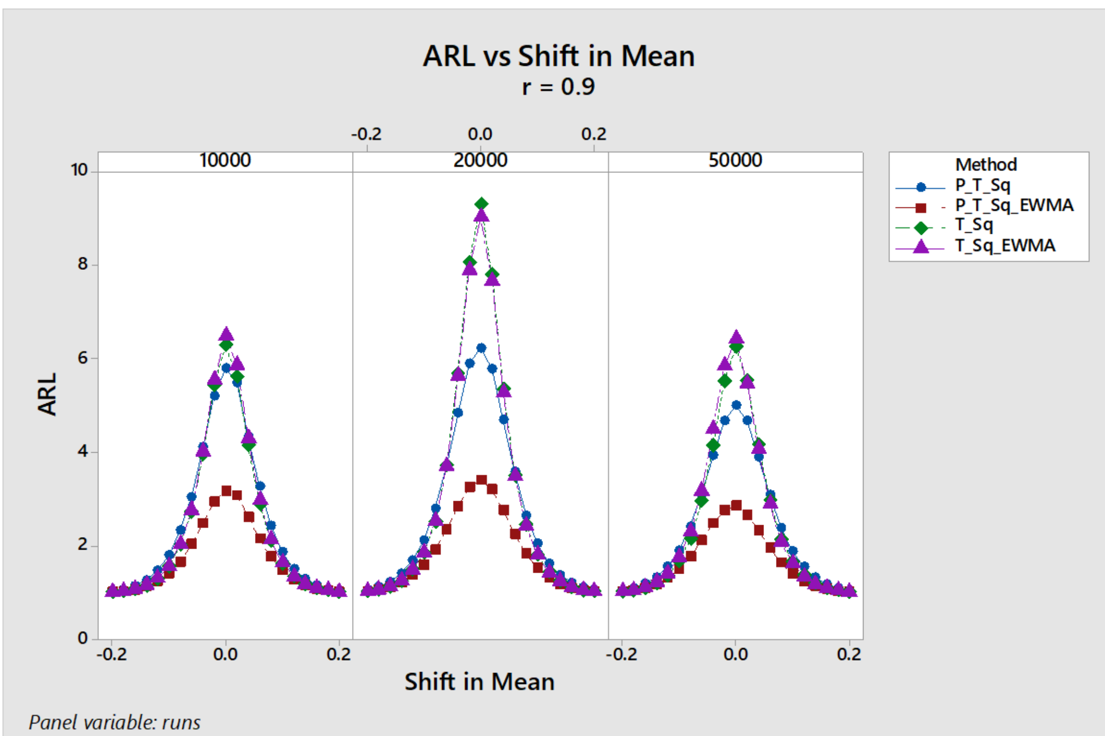


Figure 4: Comparison of ARL distributions between 10,000, 20,000 and 50,000 runs for the shift in mean with exponentially weighting constant of *r* = 0.9

4.3 Repeatability of Simulation

In order to ensure that performance ranking observed is repeatable, simulation of each of 10,000, 20,000 and 50,000 runs was repeated for the mean shift of a Normal distribution from zero. The relative performance ranking amongst the four methods was found to be repeatable; see Figure 5, where the ARL as a function of the shift in the mean, from a normal distribution, is shown. Note a similar pattern in the distribution for the two replicates.

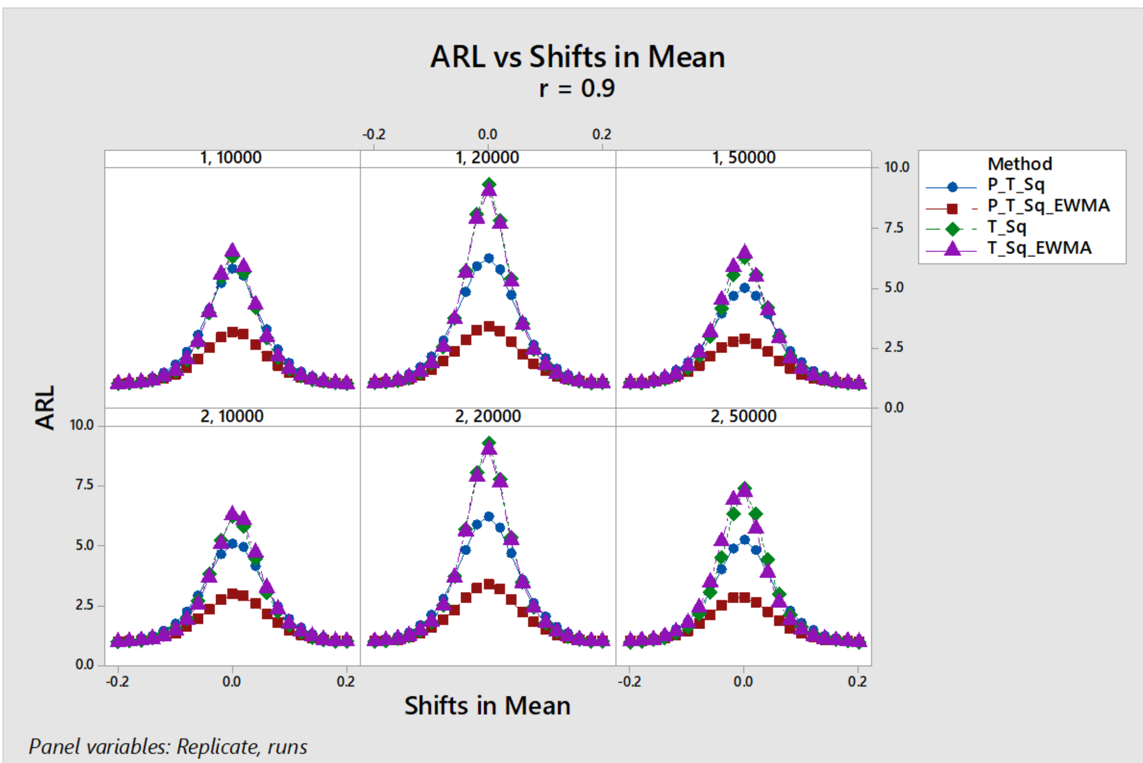


Figure 5: Comparison of two replicates for the mean shift with exponentially weighted moving average constant *r* = 0.9

It is observed from figure 5 that using the ARL as a performance measure determined that the monitoring statistic $T^2_{pewma_j}$ performs best for smaller shifts within 0.15. For shifts in the process mean of magnitude greater than 0.15, the performance of all four monitoring statistics are similar in nature. The next two sections will evaluate shifts in process parameters for the two underlying distributions.

4.4 Shifts in the Normal Distribution Parameters

4.4.1 Shift in the Mean N(μ, 1)

In order to detect a shift in the mean, simulations were run 50,000 times for different shifts. In-control state of the process was the N(0, 1). Shifts in the mean from –0.2 to +0.2, at an increment of 0.02, were systematically varied. The ARL and SDRL of were found for each of the four monitoring statistics. Only two of the monitoring statistics, i.e., $T^2_{ewma_j}$ and $T^2_{pewma_j}$, are impacted by the chosen value of the weighting constant, *r*. The value of *r* was varied between 0.1 to 0.9, in an increment of 0.1.

Simulation results from 50,000 runs showed that $T^2_{pewma_j}$ performed best compared to all other methods for all higher values of $r \geq 0.40$ (See Figure 6a). As *r* decreases below 0.4, both EWMA methods showed inconclusive results (See Figure 6b). The monitoring statistic $T^2_{p_j}$, which does not incorporate the weighting constant, seemed to be the next best statistic for faster detection. Simulation results showed that the performance of all four methods were practically the same when the shift in mean was at least 0.2.

In order to determine if there is a significant difference in the ARLs of the 4 monitoring statistics, for a given shift in the process mean. Confidence intervals may be obtained using Equation (3.16), for a chosen level of confidence. If the confidence intervals do not overlap, it may be concluded that a difference exists in the corresponding ARLs of the monitoring statistics.

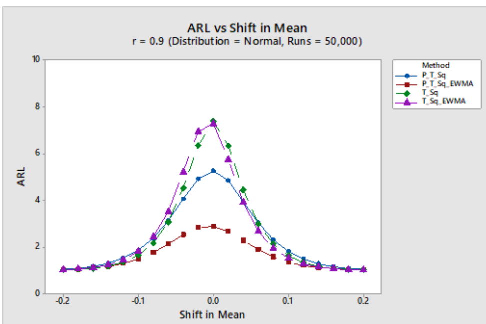


Figure 6a

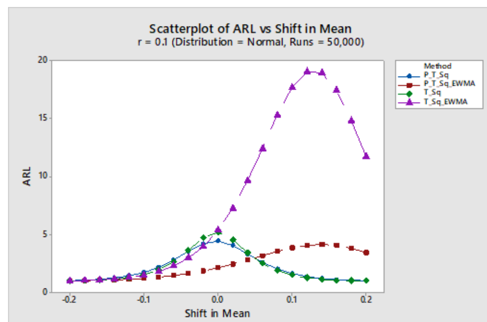


Figure 6b

Figure 6: ARL for the shift in mean for different *r*

Figures 7a and 7b show the confidence intervals for the ARLs for the case when the smoothing constant r is 0.9 and 0.1, respectively. For a given choice of the mean shift, say 0.02, it is found that the confidence intervals do not overlap, indicating a significant difference in the performance of the monitoring statistics when measured by ARL. As an example, 95% confidence limits for the ARLs of all four methods, when the mean shift is 0.02, using Equation (3.16) are shown in Table 1.

Note that it is sufficient for the confidence intervals not to overlap but not necessary to indicate a significant difference (Maghsoodloo & Huang, 2010). A conservative approach is taken by checking confidence interval not overlapping.

Method	ARL	SDRL	LCL	UCL
$T_j{}^2$ (T_Sq)	6.330	5.793	6.279	6.380
$T^2_{ewma_j}$ (T_Sq_EWMA)	5.748	5.229	5.702	5.794
$T^2_{p_j}$, (P_T_Sq)	4.847	4.301	4.809	4.884
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	2.665	2.117	2.647	2.684

Table 1: Example of confidence interval calculation (*r*=0.9, shift in mean 0.02, number of simulations = 50,000)

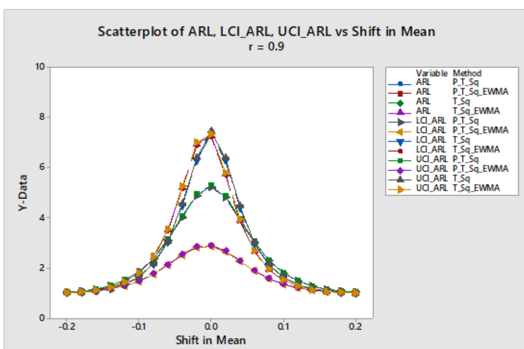


Figure 7a

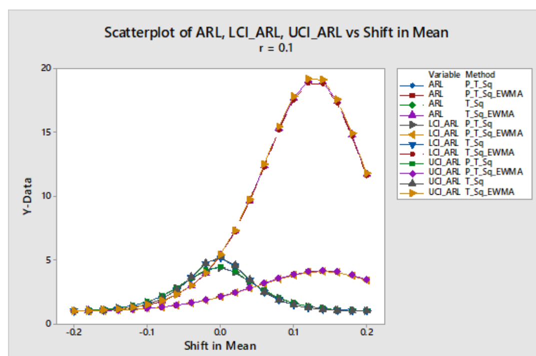


Figure 7b

Figure 7: ARL for the shift in mean for different *r* with confidence interval

Figures 8a, 8b, 8c and 8d show the effect of *r* on ARL performance of different methods. Figure 8a and 8c, and similarly, Figures 8b and 8d, indicate the same values of ARL but different scales for ARL. Note that Figures 8c and 8d do not show the ARL at *r* = 0.2 for visual comparison of ARLs at different *r*. These four figures clearly show that, for smaller shifts, the method $T^2_{pewma_j}$ performs better, compared to all others, when *r* is greater than 0.5. For larger shifts, e.g. 0.2 or beyond shown in the following Figures 8b and 8d, all methods perform equally well for *r* greater than 0.3.

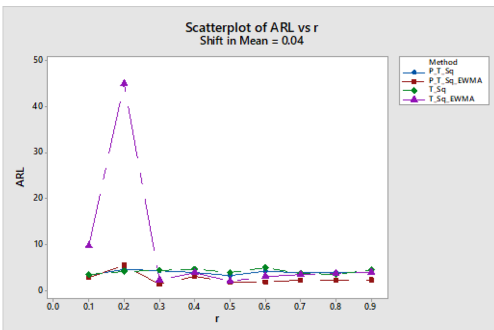


Figure 8a

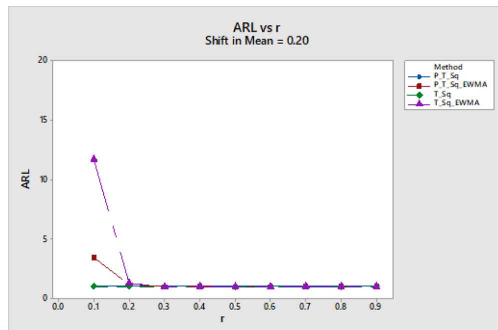


Figure 8b

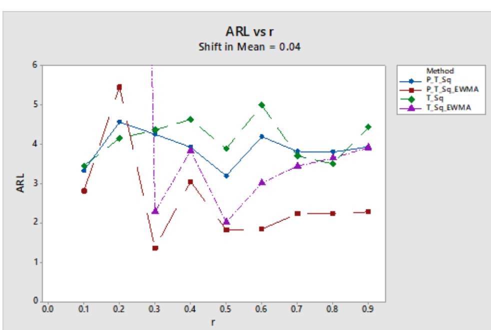


Figure 8c

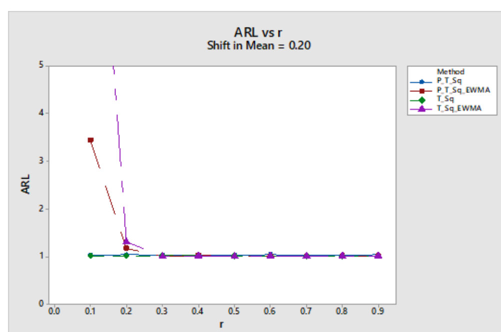


Figure 8d

Figure 8: ARL vs *r* for different shifts in mean

4.4.2 Shifts in Standard Deviation σ

The impact of a shift in the standard deviation, with the mean being kept at the in-control value, is also investigated. The standard deviation was varied from the in-control value of 1, in increments of 0.02. For the two EWMA-type monitoring statistics, the impact of the weighting constant *r* was studied. Figures 9a, 9b, 9c, and 9d show the performance of the statistics, for shifts in standard deviation, for large values of *r* (0.9 and 0.8) and small values of *r* (0.2 and 0.1), respectively.

For higher values of *r* and shifts beyond σ =1 in standard deviation, $T^2_{pewma_j}$ seemed to have smaller ARL for smaller shifts. As the shift in standard deviation gets larger, all methods seemed to perform equally as well. For smaller values of *r* (0.2 and 0.1), both T^2 statistics with exponentially weighted moving average showed inconsistent results. The ARLs were exceedingly high being greater than 40.

For higher values of *r* and shifts below σ =1 (i.e., decrease in process standard deviation), ${T_{ewma_j}}^2$ seemed to perform better than ${T_{pewma_j}}^2$ in terms of ARL (See Figures 9a and 9b)

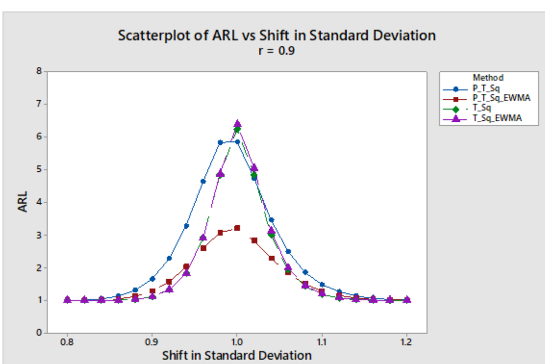


Figure 9a

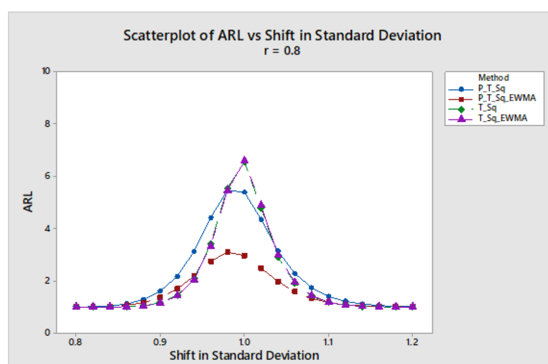
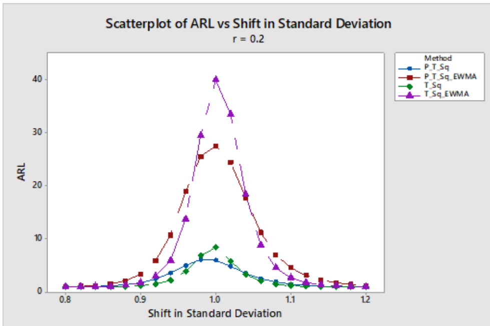


Figure 9b



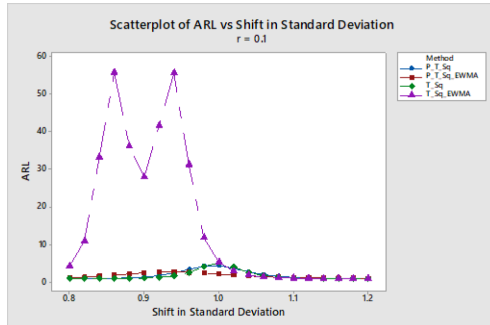
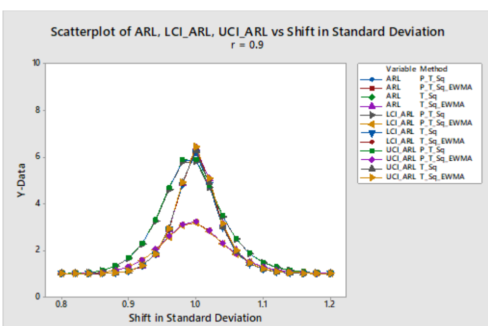


Figure 9c

Figure 9d

Figure 9: ARL for shift in standard deviation for different *r*

As before, differences in the ARLs of the monitoring statistics are investigated using confidence intervals given by Equation (3.16). Figures 10a, 10b, 10c and 10d, show the ARLs and the confidence intervals for specified shifts in σ. It was found that the confidence intervals do not overlap for standard deviation values in the range of 0.9 to 1.1.



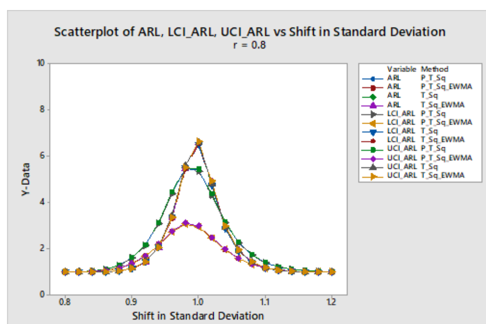
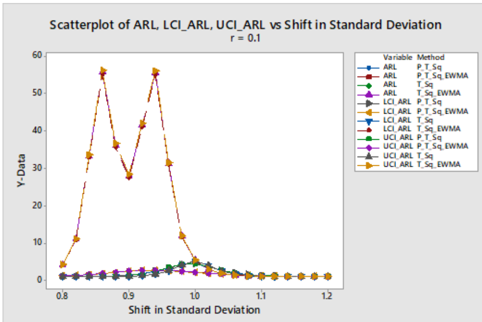


Figure 10a

Figure 10b



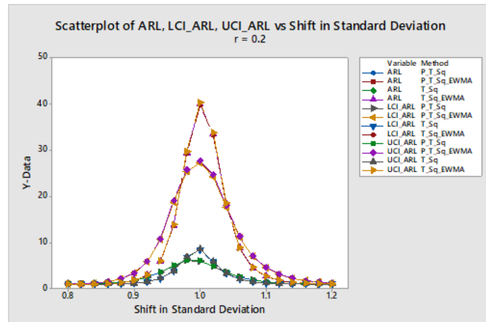


Figure 10c Figure 10d

Figure 10: ARL for the shift in standard deviation for different *r* with CI

4.5 Weibull Distribution

4.5.1 Shifts in the Scale Parameter λ

The two-parameter Weibull distribution is impacted by the scale parameter λ and the shape parameter *k*; see Equation (3.13). We first determine the impact of a change in the scale, where λ is varied from 0.4 to 0.6, in increments of 0.02. The in-control value of λ is 0.5. As before, the impact of the weighting constant, *r*, on the EWMA-type monitoring statistics is explored.

Simulation results with 50,000 runs showed that ${T_{pewma_j}}^2$ performed better than all the other methods, for higher values of *r* and smaller shift in scale (see Figures 11a and 11b). For larger shifts in λ beyond 0.2, all four methods performed equally well. From our simulation results, it is not recommended to use either of the EWMA methods for smaller values of *r* and smaller shifts in λ (shifts smaller than 0.05 in magnitude). All methods performed equally well for larger shifts in λ even when chosen value of r is very small – say 0.1 or 0.2 (See Figures 11a, 11b, 11c and 11d).

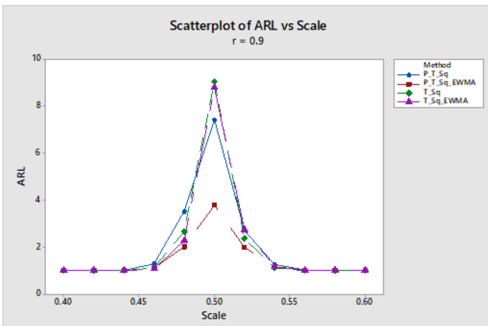


Figure 11a

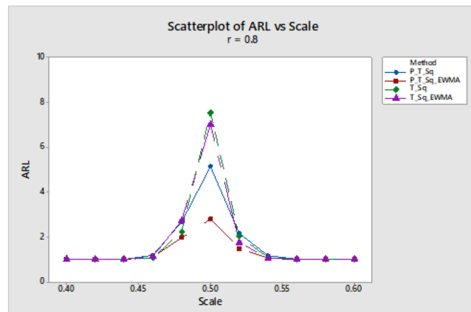


Figure 11b

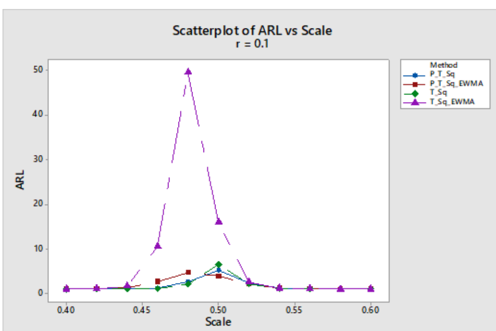


Figure 11c

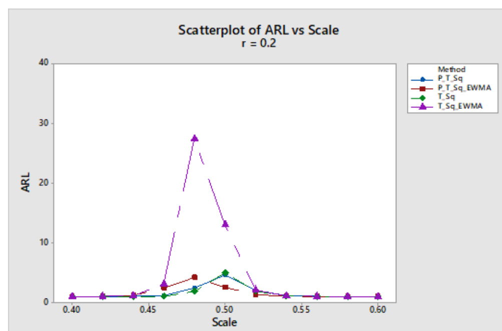


Figure 11d

Figure 11: ARL for the shift in scale factor for different *r*

ARL comparisons between the four monitoring methods are conducted using confidence intervals as given by Equation (3.16). Figures 12a, 12b, 12c and 12d show the ARLs as well as their corresponding confidence intervals. For small shifts in λ (between

± 0.05 from the target value), the confidence intervals of the monitoring methods do not overlap, indicating a significant difference in the ARLs.

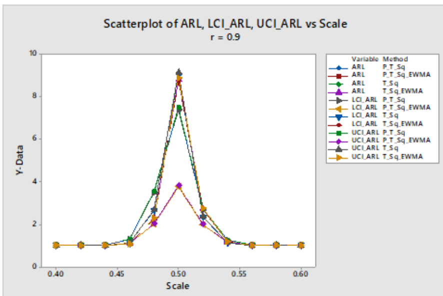


Figure 12a

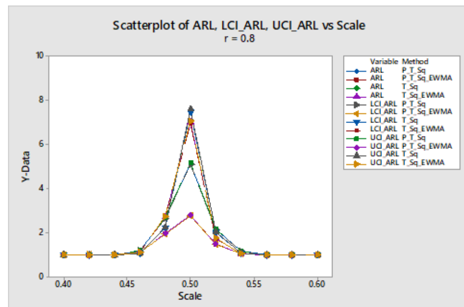


Figure 12b

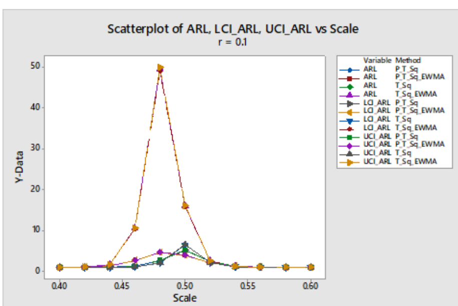


Figure 12c

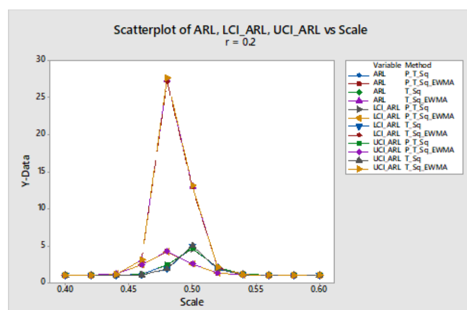


Figure 12d

Figure 12: ARL for the shift in scale factor for different *r* with CI

4.5.2 Shifts in the Shape Parameter

The shape parameter *k* was varied from 1.6 to 2.4, in increments of 0.08. The in-control value of *k* is 2, although our program can handle other values of *k*. As before, the impact of the weighting constant, *r*, on the EWMA-type monitoring statistics is explored. Simulation results with 50,000 runs showed that ${T_{pewma_j}}^2$ performed better than all the other methods for higher values of r ≥ 0.6 and smaller shift (= 0.08) in the shape parameter (see Figures 13a and 13b). For larger shifts in shape parameter all four methods performed equally well.

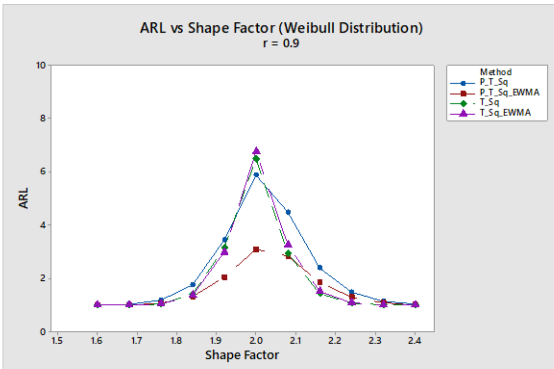


Figure 13a

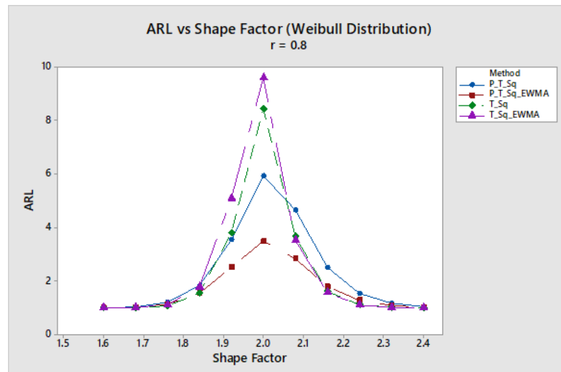


Figure 13b

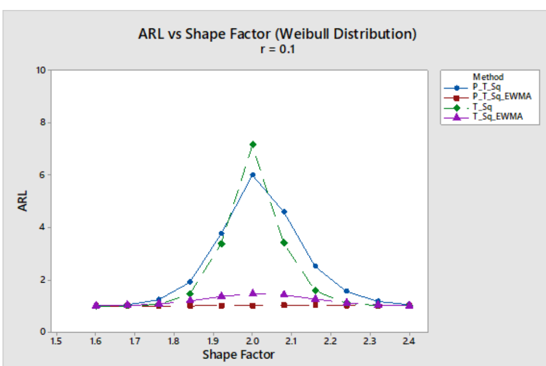


Figure 13c

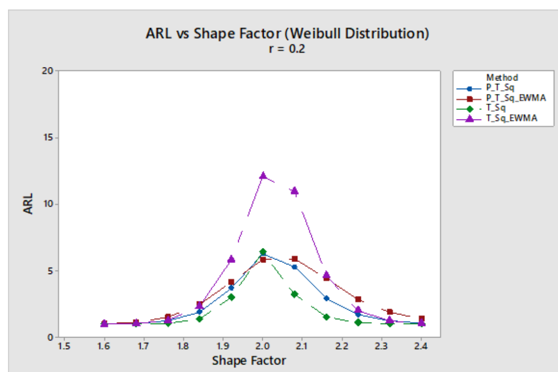


Figure 13d

Figure 13: ARL for the shift in shape factor for different *r*

From our simulation results, it is not recommended to use either of the exponentially weighted moving average type methods for smaller values of *r* and smaller shifts in scale. All methods performed equally well for larger shifts in shape even when chosen value of r is very small – say 0.1 or 0.2 (See Figures 13c and 13d).

ARL comparisons between the four monitoring methods are conducted using confidence intervals as given by Equation (3.16). Figures 14a, 14b, 14c and 14d show the ARLs as well as their corresponding confidence intervals. For small shifts in shape parameter (between ± 0.3 from the target value), the confidence intervals of the monitoring methods do not overlap, indicating a significant difference in the ARL of the monitoring statistics.

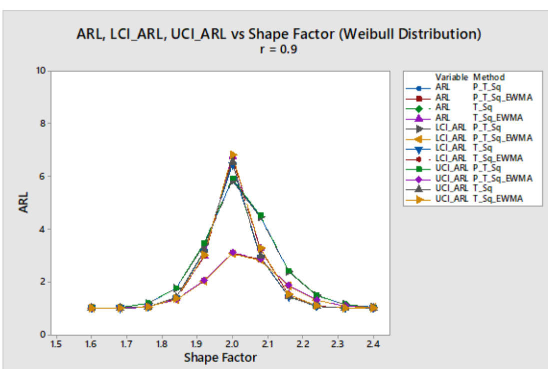


Figure 14a

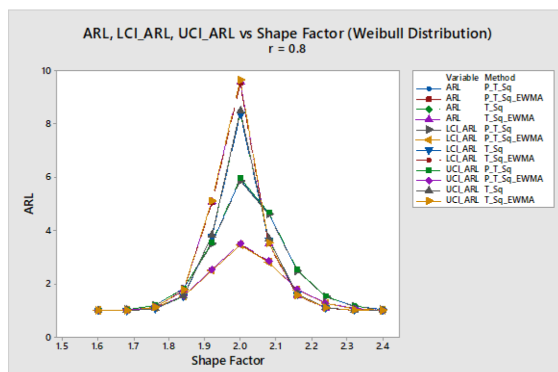
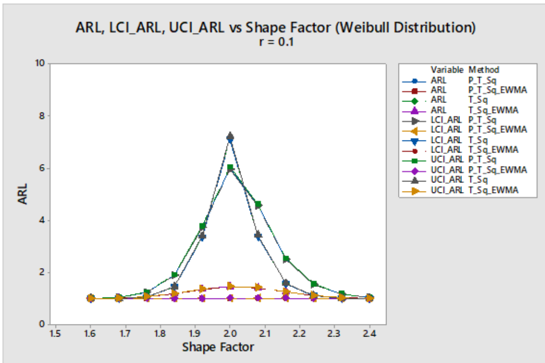


Figure 14b



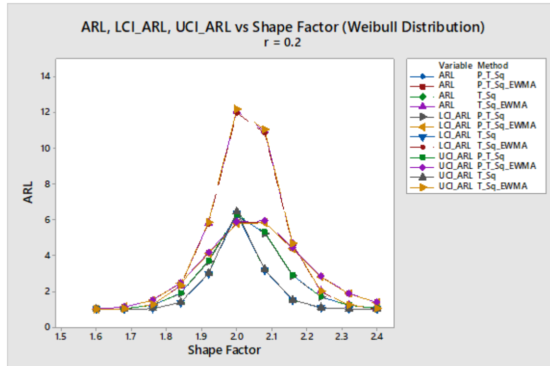


Figure 14c

Figure 14d

Figure 14: ARL for the shift in shape factor for different *r* with CI

Chapter 5

Limitations and Further Study

5.1 Number of Percentile Points

In this dissertation, two percentile-based monitoring methods i.e., ${T_{pj}}^2$ and ${T_{pewma_j}}^2$ used five estimated percentile points (10^{th} , 30^{th} , 50^{th} , 70^{th} and 90^{th} percentiles) to represent a profile. The result showed ${T_{pewma_j}}^2$ method performed better in terms of ARL to detect out-of-control points when the choice of weighting constant *r* was greater than 0.5. Effect of the change in the number of estimated percentile points on the performance of ${T_{pj}}^2$ and ${T_{pewma_j}}^2$ was not explored. The 5 percentiles used ignored the tails of the underlying distribution; however, increasing the number of percentile-points will increase the computation time and false-alarm rate. As the number of estimated percentile points in the monitoring statistic increases, a better representation of the profile should be obtained. Hence, an area of further research will be to determine an optimal number of percentile points in a monitoring statistic.

5.2 Application of Monitoring Methods to Unknown Distributions

Two percentile-based monitoring methods i.e., ${T_{pj}}^2$ and ${T_{pewma_j}}^2$ were explored for two underlying distributions; however, the methodology could have potential application for cases where the underlying distribution is unknown.

5.3 Other Distributions

Performance comparison of the four monitoring methods for other underlying distributions than the normal and Weibull was not explored. Profile monitoring when the underlying distribution is Lognormal may have applications in finance when monitoring exchange rates, price indices, and stock market indices. The lognormal underlying distribution has also applications in time to repair maintainable systems. Similarly, in an insurance industry, it may be important to monitor aggregate insurance claims. This characteristic is known to follow a Gamma distribution.

5.4 CUSUM Control Chart

Some authors have used CUSUM-based methods in profile monitoring. Performance comparison of our methods with those of CUSUM-based have not been explored. This will be another area of future research.

5.5 Effect of Skewness

The Weibull distribution has a wide-range of skewness depending on its shape parameter *k*, which can be used to study the impact of skewness on performance of the four profile-monitoring methods. Other skewed distributions such as Lognormal and Gamma also provide wide-range of skewness. Effects of skewness on ARL performance has not been explored and may be an area for future study.

5.6 Error Distribution

In this dissertation, the error distribution was assumed to be normal with a mean of 0 and a standard deviation of 0.01. The effect of varying the standard deviation from

0.01 on performance of the four methods was not explored. This may constitute a future area of research.

5.7 Simultaneous Shifts in Parameters

This dissertation results are based on shift in only one parameter at a time. In real-life situations, it may be possible for two or more parameters to shift simultaneously. The Effect of such shifts on performance of the methods has not been explored.

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Appendix A: Control Limits for Monitoring Statistics

Appendix A.1: Limits for Normal Distribution

		Limits for Normal Distribution			
Weighting Constant *r* (A)	Number of Subgroups (B)	Limits T^2 (C)	Limits T^2_{EWMA} (D)	Limits $T_p^{\ 2}$ (E)	Limits T^2_{pewma} (F)
0.1	500	5.9773	5.8601	11.0956	10.8563
0.1	490	5.9647	5.8434	11.0720	10.6415
0.1	480	5.9627	5.8361	11.0695	10.4316
0.1	470	5.9609	5.8296	11.0688	10.2261
0.1	460	5.9604	5.8228	11.0658	10.0257
0.1	450	5.9595	5.8155	11.0628	9.8302
0.1	440	5.9584	5.8086	11.0607	9.6404
0.1	430	5.9572	5.8029	11.0576	9.4559
0.1	420	5.9562	5.7983	11.0560	9.2763
0.1	410	5.9546	5.7912	11.0524	9.1031
0.1	400	5.9538	5.7844	11.0510	8.9358
0.1	390	5.9523	5.7783	11.0482	8.7747
0.1	380	5.9513	5.7741	11.0473	8.6195
0.1	370	5.9502	5.7693	11.0436	8.4713
0.1	360	5.9490	5.7643	11.0407	8.3300
0.1	350	5.9479	5.7586	11.0388	8.1941
0.1	340	5.9470	5.7539	11.0350	8.0656
0.1	330	5.9451	5.7489	11.0321	7.9451
0.1	320	5.9443	5.7441	11.0299	7.8314
0.1	310	5.9440	5.7391	11.0276	7.7247
0.1	300	5.9423	5.7338	11.0248	7.6256
0.1	290	5.9410	5.7285	11.0215	7.5329
0.1	280	5.9401	5.7228	11.0196	7.4477
0.1	270	5.9388	5.7173	11.0153	7.3695
0.1	260	5.9374	5.7115	11.0115	7.2985
0.1	250	5.9355	5.7053	11.0068	7.2335
0.1	240	5.9330	5.6996	11.0033	7.1747
0.1	230	5.9309	5.6937	10.9993	7.1215
0.1	220	5.9295	5.6863	10.9937	7.0740
0.1	210	5.9270	5.6819	10.9898	7.0305
0.1	200	5.9263	5.6758	10.9843	6.9904
0.1	190	5.9248	5.6691	10.9799	6.9540
0.1	180	5.9222	5.6656	10.9756	6.9202
0.1	170	5.9202	5.6600	10.9713	6.8880
0.1	160	5.9179	5.6542	10.9648	6.8571
0.1	150	5.9158	5.6486	10.9577	6.8269
0.1	140	5.9123	5.6429	10.9503	6.7959
0.1	130	5.9100	5.6379	10.9429	6.7646

		Limits for Normal Distribution			
Weighting Constant *r* (A)	Number of Subgroups (B)	Limits T^2 (C)	Limits T^2_{EWMA} (D)	Limits $T_p^{\,2}$ (E)	Limits T^2_{pewma} (F)
0.1	120	5.9066	5.6332	10.9357	6.7333
0.1	110	5.9028	5.6252	10.9289	6.7010
0.1	100	5.8990	5.6158	10.9211	6.6680
0.2	500	5.9716	5.9191	11.0899	10.9262
0.2	490	5.9588	5.9045	11.0662	10.7097
0.2	480	5.9579	5.8995	11.0643	10.4977
0.2	470	5.9569	5.8957	11.0634	10.2903
0.2	460	5.9560	5.8909	11.0598	10.0875
0.2	450	5.9553	5.8883	11.0579	9.8896
0.2	440	5.9548	5.8854	11.0557	9.6980
0.2	430	5.9536	5.8808	11.0524	9.5117
0.2	420	5.9519	5.8778	11.0511	9.3306
0.2	410	5.9509	5.8732	11.0480	9.1555
0.2	400	5.9493	5.8677	11.0455	8.9867
0.2	390	5.9491	5.8650	11.0423	8.8240
0.2	380	5.9491	5.8624	11.0408	8.6677
0.2	370	5.9475	5.8585	11.0389	8.5183
0.2	360	5.9469	5.8542	11.0371	8.3745
0.2	350	5.9451	5.8504	11.0350	8.2365
0.2	340	5.9450	5.8476	11.0336	8.1066
0.2	330	5.9435	5.8434	11.0310	7.9843
0.2	320	5.9418	5.8387	11.0275	7.8698
0.2	310	5.9398	5.8343	11.0239	7.7612
0.2	300	5.9393	5.8302	11.0203	7.6620
0.2	290	5.9380	5.8265	11.0157	7.5697
0.2	280	5.9367	5.8214	11.0121	7.4845
0.2	270	5.9357	5.8172	11.0070	7.4072
0.2	260	5.9343	5.8111	11.0037	7.3381
0.2	250	5.9324	5.8062	11.0018	7.2762
0.2	240	5.9300	5.8004	10.9982	7.2213
0.2	230	5.9271	5.7963	10.9938	7.1740
0.2	220	5.9241	5.7905	10.9892	7.1331
0.2	210	5.9227	5.7851	10.9850	7.0980
0.2	200	5.9208	5.7806	10.9798	7.0674
0.2	190	5.9183	5.7765	10.9755	7.0419
0.2	180	5.9160	5.7696	10.9713	7.0194
0.2	170	5.9140	5.7629	10.9656	7.0002
0.2	160	5.9122	5.7582	10.9615	6.9831

		Limits for Normal Distribution			
Weighting Constant *r* (A)	Number of Subgroups (B)	Limits T^2 (C)	Limits T^2_{EWMA} (D)	Limits T_p^2 (E)	Limits T^2_{pewma} (F)
0.2	150	5.9101	5.7539	10.9566	6.9675
0.2	140	5.9065	5.7486	10.9495	6.9529
0.2	130	5.9032	5.7421	10.9422	6.9380
0.2	120	5.9001	5.7370	10.9344	6.9242
0.2	110	5.8971	5.7310	10.9281	6.9107
0.2	100	5.8936	5.7260	10.9194	6.8961
0.3	500	5.9755	5.9565	11.0896	10.9898
0.3	490	5.9629	5.9422	11.0647	10.7719
0.3	480	5.9620	5.9394	11.0630	10.5586
0.3	470	5.9604	5.9359	11.0596	10.3498
0.3	460	5.9587	5.9324	11.0581	10.1461
0.3	450	5.9584	5.9314	11.0560	9.9476
0.3	440	5.9579	5.9288	11.0548	9.7547
0.3	430	5.9570	5.9262	11.0514	9.5666
0.3	420	5.9554	5.9239	11.0493	9.3842
0.3	410	5.9537	5.9211	11.0464	9.2079
0.3	400	5.9529	5.9196	11.0437	9.0379
0.3	390	5.9524	5.9169	11.0420	8.8739
0.3	380	5.9506	5.9147	11.0392	8.7166
0.3	370	5.9496	5.9117	11.0354	8.5657
0.3	360	5.9485	5.9101	11.0319	8.4204
0.3	350	5.9482	5.9078	11.0304	8.2828
0.3	340	5.9472	5.9045	11.0280	8.1521
0.3	330	5.9469	5.9018	11.0252	8.0276
0.3	320	5.9451	5.8981	11.0208	7.9111
0.3	310	5.9436	5.8944	11.0175	7.8017
0.3	300	5.9426	5.8915	11.0134	7.6988
0.3	290	5.9416	5.8893	11.0110	7.6049
0.3	280	5.9401	5.8871	11.0071	7.5189
0.3	270	5.9391	5.8862	11.0026	7.4409
0.3	260	5.9378	5.8826	11.0008	7.3712
0.3	250	5.9358	5.8801	10.9989	7.3088
0.3	240	5.9360	5.8765	10.9961	7.2551
0.3	230	5.9340	5.8728	10.9919	7.2095
0.3	220	5.9320	5.8698	10.9877	7.1705
0.3	210	5.9309	5.8659	10.9828	7.1372
0.3	200	5.9281	5.8604	10.9772	7.1100
0.3	190	5.9257	5.8555	10.9753	7.0879

		Limits for Normal Distribution			
Weighting Constant *r* (A)	Number of Subgroups (B)	Limits T^2 (C)	Limits T^2_{EWMA} (D)	Limits $T_p^{\ 2}$ (E)	Limits T^2_{pewma} (F)
0.3	180	5.9236	5.8520	10.9712	7.0694
0.3	170	5.9213	5.8488	10.9665	7.0534
0.3	160	5.9183	5.8424	10.9603	7.0412
0.3	150	5.9165	5.8367	10.9532	7.0309
0.3	140	5.9146	5.8335	10.9464	7.0220
0.3	130	5.9114	5.8274	10.9400	7.0133
0.3	120	5.9077	5.8228	10.9342	7.0061
0.3	110	5.9040	5.8168	10.9265	6.9985
0.3	100	5.9009	5.8129	10.9173	6.9922
0.4	500	5.9707	5.9642	11.0842	11.0278
0.4	490	5.9581	5.9494	11.0611	10.8093
0.4	480	5.9575	5.9479	11.0587	10.5954
0.4	470	5.9575	5.9468	11.0564	10.3858
0.4	460	5.9559	5.9445	11.0538	10.1812
0.4	450	5.9552	5.9430	11.0517	9.9817
0.4	440	5.9542	5.9409	11.0494	9.7886
0.4	430	5.9540	5.9397	11.0462	9.6006
0.4	420	5.9523	5.9375	11.0438	9.4182
0.4	410	5.9518	5.9351	11.0408	9.2411
0.4	400	5.9510	5.9334	11.0396	9.0691
0.4	390	5.9502	5.9311	11.0369	8.9028
0.4	380	5.9491	5.9288	11.0350	8.7443
0.4	370	5.9480	5.9263	11.0327	8.5926
0.4	360	5.9473	5.9243	11.0293	8.4471
0.4	350	5.9473	5.9223	11.0271	8.3086
0.4	340	5.9466	5.9204	11.0246	8.1767
0.4	330	5.9463	5.9182	11.0224	8.0523
0.4	320	5.9450	5.9163	11.0201	7.9350
0.4	310	5.9438	5.9136	11.0160	7.8254
0.4	300	5.9428	5.9107	11.0129	7.7218
0.4	290	5.9419	5.9076	11.0106	7.6270
0.4	280	5.9405	5.9056	11.0059	7.5402
0.4	270	5.9391	5.9029	11.0037	7.4615
0.4	260	5.9365	5.9003	11.0004	7.3905
0.4	250	5.9345	5.8982	10.9974	7.3279
0.4	240	5.9328	5.8955	10.9926	7.2736
0.4	230	5.9317	5.8936	10.9885	7.2268
0.4	220	5.9294	5.8896	10.9847	7.1867

		Limits for Normal Distribution			
Weighting Constant *r* (A)	Number of Subgroups (B)	Limits T^2 (C)	Limits T^2_{EWMA} (D)	Limits T_p^2 (E)	Limits T^2_{pewma} (F)
0.4	210	5.9285	5.8872	10.9822	7.1532
0.4	200	5.9268	5.8846	10.9787	7.1270
0.4	190	5.9251	5.8803	10.9728	7.1061
0.4	180	5.9231	5.8762	10.9692	7.0901
0.4	170	5.9210	5.8729	10.9624	7.0773
0.4	160	5.9197	5.8702	10.9563	7.0670
0.4	150	5.9167	5.8656	10.9506	7.0589
0.4	140	5.9148	5.8613	10.9440	7.0526
0.4	130	5.9117	5.8578	10.9383	7.0473
0.4	120	5.9079	5.8512	10.9304	7.0429
0.4	110	5.9040	5.8459	10.9230	7.0392
0.4	100	5.9014	5.8411	10.9134	7.0360
0.5	500	5.9747	5.9732	11.0885	11.0546
0.5	490	5.9624	5.9610	11.0650	10.8355
0.5	480	5.9618	5.9594	11.0628	10.6210
0.5	470	5.9608	5.9581	11.0611	10.4106
0.5	460	5.9604	5.9557	11.0603	10.2060
0.5	450	5.9588	5.9533	11.0577	10.0064
0.5	440	5.9571	5.9509	11.0569	9.8121
0.5	430	5.9556	5.9498	11.0546	9.6234
0.5	420	5.9552	5.9486	11.0507	9.4399
0.5	410	5.9542	5.9467	11.0478	9.2617
0.5	400	5.9519	5.9443	11.0444	9.0896
0.5	390	5.9517	5.9429	11.0423	8.9230
0.5	380	5.9506	5.9407	11.0401	8.7631
0.5	370	5.9492	5.9395	11.0378	8.6101
0.5	360	5.9483	5.9370	11.0363	8.4639
0.5	350	5.9473	5.9354	11.0318	8.3254
0.5	340	5.9461	5.9342	11.0286	8.1937
0.5	330	5.9452	5.9330	11.0258	8.0691
0.5	320	5.9439	5.9313	11.0225	7.9521
0.5	310	5.9429	5.9299	11.0211	7.8427
0.5	300	5.9421	5.9271	11.0174	7.7402
0.5	290	5.9421	5.9255	11.0147	7.6454
0.5	280	5.9402	5.9235	11.0120	7.5591
0.5	270	5.9383	5.9215	11.0085	7.4807
0.5	260	5.9360	5.9190	11.0044	7.4104
0.5	250	5.9350	5.9169	11.0009	7.3481

		Limits for Normal Distribution			
Weighting Constant *r* (A)	Number of Subgroups (B)	Limits T^2 (C)	Limits T^2_{EWMA} (D)	Limits $T_p^{\ 2}$ (E)	Limits T^2_{pewma} (F)
0.5	240	5.9342	5.9134	10.9963	7.2937
0.5	230	5.9327	5.9105	10.9926	7.2466
0.5	220	5.9301	5.9081	10.9867	7.2063
0.5	210	5.9283	5.9053	10.9814	7.1732
0.5	200	5.9272	5.9037	10.9759	7.1470
0.5	190	5.9261	5.9011	10.9714	7.1267
0.5	180	5.9240	5.8976	10.9669	7.1109
0.5	170	5.9207	5.8929	10.9623	7.0988
0.5	160	5.9186	5.8906	10.9578	7.0898
0.5	150	5.9168	5.8865	10.9503	7.0829
0.5	140	5.9136	5.8819	10.9431	7.0779
0.5	130	5.9111	5.8761	10.9363	7.0744
0.5	120	5.9090	5.8722	10.9297	7.0717
0.5	110	5.9062	5.8670	10.9211	7.0695
0.5	100	5.9030	5.8626	10.9138	7.0681
0.6	500	5.9746	5.9716	11.0936	11.0740
0.6	490	5.9617	5.9586	11.0696	10.8546
0.6	480	5.9613	5.9586	11.0695	10.6396
0.6	470	5.9593	5.9568	11.0662	10.4295
0.6	460	5.9584	5.9546	11.0639	10.2246
0.6	450	5.9575	5.9523	11.0602	10.0250
0.6	440	5.9555	5.9506	11.0590	9.8304
0.6	430	5.9533	5.9484	11.0561	9.6405
0.6	420	5.9513	5.9470	11.0533	9.4565
0.6	410	5.9514	5.9462	11.0504	9.2787
0.6	400	5.9515	5.9455	11.0484	9.1071
0.6	390	5.9503	5.9438	11.0459	8.9412
0.6	380	5.9495	5.9418	11.0434	8.7816
0.6	370	5.9483	5.9409	11.0396	8.6289
0.6	360	5.9473	5.9395	11.0375	8.4833
0.6	350	5.9467	5.9385	11.0358	8.3437
0.6	340	5.9454	5.9363	11.0331	8.2117
0.6	330	5.9448	5.9350	11.0304	8.0857
0.6	320	5.9441	5.9337	11.0281	7.9677
0.6	310	5.9425	5.9317	11.0249	7.8573
0.6	300	5.9421	5.9301	11.0221	7.7548
0.6	290	5.9400	5.9274	11.0195	7.6601
0.6	280	5.9388	5.9257	11.0157	7.5737

		Limits for Normal Distribution			
Weighting Constant *r* (A)	Number of Subgroups (B)	Limits T^2 (C)	Limits T^2_{EWMA} (D)	Limits $T_p^{\ 2}$ (E)	Limits T^2_{pewma} (F)
0.6	270	5.9370	5.9239	11.0131	7.4944
0.6	260	5.9358	5.9216	11.0102	7.4231
0.6	250	5.9338	5.9180	11.0052	7.3595
0.6	240	5.9332	5.9168	11.0014	7.3043
0.6	230	5.9306	5.9146	10.9959	7.2567
0.6	220	5.9282	5.9111	10.9915	7.2176
0.6	210	5.9270	5.9093	10.9868	7.1845
0.6	200	5.9254	5.9076	10.9828	7.1583
0.6	190	5.9227	5.9045	10.9792	7.1377
0.6	180	5.9215	5.9028	10.9741	7.1218
0.6	170	5.9201	5.9011	10.9682	7.1106
0.6	160	5.9178	5.8975	10.9628	7.1024
0.6	150	5.9147	5.8949	10.9560	7.0969
0.6	140	5.9128	5.8905	10.9495	7.0931
0.6	130	5.9110	5.8880	10.9430	7.0906
0.6	120	5.9074	5.8837	10.9361	7.0891
0.6	110	5.9039	5.8788	10.9271	7.0881
0.6	100	5.9017	5.8739	10.9184	7.0874
0.7	500	5.9740	5.9699	11.0882	11.0727
0.7	490	5.9609	5.9567	11.0639	10.8533
0.7	480	5.9596	5.9552	11.0608	10.6388
0.7	470	5.9589	5.9553	11.0585	10.4288
0.7	460	5.9574	5.9535	11.0563	10.2234
0.7	450	5.9565	5.9525	11.0552	10.0232
0.7	440	5.9555	5.9519	11.0520	9.8294
0.7	430	5.9560	5.9523	11.0514	9.6404
0.7	420	5.9559	5.9527	11.0505	9.4564
0.7	410	5.9548	5.9519	11.0483	9.2780
0.7	400	5.9544	5.9502	11.0459	9.1064
0.7	390	5.9529	5.9484	11.0433	8.9411
0.7	380	5.9519	5.9467	11.0420	8.7827
0.7	370	5.9500	5.9452	11.0376	8.6303
0.7	360	5.9491	5.9441	11.0350	8.4837
0.7	350	5.9475	5.9423	11.0328	8.3441
0.7	340	5.9458	5.9404	11.0282	8.2116
0.7	330	5.9444	5.9390	11.0243	8.0857
0.7	320	5.9429	5.9362	11.0222	7.9684
0.7	310	5.9417	5.9339	11.0186	7.8583

		Limits for Normal Distribution			
Weighting Constant *r* (A)	Number of Subgroups (B)	Limits T^2 (C)	Limits T^2_{EWMA} (D)	Limits $T_p^{\ 2}$ (E)	Limits T^2_{pewma} (F)
0.7	300	5.9408	5.9328	11.0155	7.7549
0.7	290	5.9376	5.9305	11.0118	7.6603
0.7	280	5.9354	5.9278	11.0085	7.5729
0.7	270	5.9340	5.9254	11.0047	7.4938
0.7	260	5.9328	5.9242	11.0035	7.4218
0.7	250	5.9319	5.9234	10.9993	7.3582
0.7	240	5.9296	5.9211	10.9966	7.3029
0.7	230	5.9276	5.9183	10.9935	7.2560
0.7	220	5.9266	5.9165	10.9891	7.2170
0.7	210	5.9249	5.9137	10.9836	7.1855
0.7	200	5.9233	5.9115	10.9794	7.1598
0.7	190	5.9214	5.9094	10.9762	7.1398
0.7	180	5.9199	5.9067	10.9720	7.1250
0.7	170	5.9179	5.9044	10.9666	7.1143
0.7	160	5.9158	5.9013	10.9608	7.1068
0.7	150	5.9131	5.8978	10.9553	7.1017
0.7	140	5.9103	5.8943	10.9482	7.0987
0.7	130	5.9081	5.8917	10.9439	7.0967
0.7	120	5.9045	5.8878	10.9357	7.0953
0.7	110	5.9012	5.8834	10.9276	7.0946
0.7	100	5.8971	5.8805	10.9199	7.0944
0.8	500	5.9692	5.9685	11.0863	11.0797
0.8	490	5.9558	5.9554	11.0619	10.8601
0.8	480	5.9544	5.9536	11.0601	10.6455
0.8	470	5.9536	5.9523	11.0590	10.4354
0.8	460	5.9529	5.9518	11.0568	10.2303
0.8	450	5.9520	5.9513	11.0535	10.0303
0.8	440	5.9514	5.9497	11.0513	9.8358
0.8	430	5.9502	5.9483	11.0503	9.6460
0.8	420	5.9495	5.9478	11.0483	9.4620
0.8	410	5.9492	5.9476	11.0452	9.2837
0.8	400	5.9476	5.9463	11.0429	9.1113
0.8	390	5.9465	5.9450	11.0406	8.9464
0.8	380	5.9458	5.9441	11.0371	8.7872
0.8	370	5.9452	5.9431	11.0350	8.6337
0.8	360	5.9433	5.9410	11.0318	8.4875
0.8	350	5.9423	5.9398	11.0288	8.3480
0.8	340	5.9413	5.9390	11.0252	8.2157

		Limits for Normal Distribution			
Weighting Constant *r* (A)	Number of Subgroups (B)	Limits T^2 (C)	Limits T^2_{EWMA} (D)	Limits $T_p^{\ 2}$ (E)	Limits T^2_{pewma} (F)
0.8	330	5.9407	5.9396	11.0226	8.0909
0.8	320	5.9396	5.9379	11.0196	7.9739
0.8	310	5.9385	5.9363	11.0170	7.8638
0.8	300	5.9383	5.9352	11.0130	7.7616
0.8	290	5.9365	5.9335	11.0100	7.6673
0.8	280	5.9352	5.9319	11.0083	7.5809
0.8	270	5.9337	5.9303	11.0055	7.5014
0.8	260	5.9316	5.9288	11.0029	7.4307
0.8	250	5.9296	5.9265	10.9998	7.3683
0.8	240	5.9281	5.9251	10.9957	7.3141
0.8	230	5.9267	5.9234	10.9911	7.2676
0.8	220	5.9245	5.9221	10.9868	7.2281
0.8	210	5.9221	5.9195	10.9831	7.1970
0.8	200	5.9209	5.9163	10.9781	7.1707
0.8	190	5.9195	5.9145	10.9724	7.1505
0.8	180	5.9183	5.9132	10.9689	7.1352
0.8	170	5.9166	5.9111	10.9629	7.1246
0.8	160	5.9153	5.9094	10.9582	7.1172
0.8	150	5.9125	5.9067	10.9522	7.1119
0.8	140	5.9099	5.9036	10.9461	7.1087
0.8	130	5.9066	5.9006	10.9392	7.1069
0.8	120	5.9041	5.8979	10.9320	7.1060
0.8	110	5.9018	5.8949	10.9235	7.1055
0.8	100	5.8973	5.8908	10.9163	7.1053
0.9	500	5.9762	5.9738	11.0910	11.0913
0.9	490	5.9642	5.9614	11.0667	10.8713
0.9	480	5.9636	5.9612	11.0642	10.6558
0.9	470	5.9636	5.9605	11.0628	10.4451
0.9	460	5.9622	5.9591	11.0605	10.2397
0.9	450	5.9610	5.9584	11.0592	10.0395
0.9	440	5.9602	5.9577	11.0568	9.8451
0.9	430	5.9588	5.9564	11.0544	9.6556
0.9	420	5.9568	5.9543	11.0513	9.4720
0.9	410	5.9560	5.9532	11.0494	9.2938
0.9	400	5.9549	5.9519	11.0478	9.1216
0.9	390	5.9544	5.9512	11.0458	8.9558
0.9	380	5.9528	5.9501	11.0440	8.7962
0.9	370	5.9509	5.9487	11.0412	8.6442

		Limits for Normal Distribution			
Weighting Constant *r* (A)	Number of Subgroups (B)	Limits T^2 (C)	Limits T^2_{EWMA} (D)	Limits $T_p^{\ 2}$ (E)	Limits T^2_{pewma} (F)
0.9	360	5.9507	5.9476	11.0390	8.4982
0.9	350	5.9498	5.9471	11.0366	8.3595
0.9	340	5.9481	5.9464	11.0320	8.2283
0.9	330	5.9463	5.9444	11.0292	8.1027
0.9	320	5.9460	5.9439	11.0269	7.9829
0.9	310	5.9447	5.9417	11.0227	7.8717
0.9	300	5.9437	5.9401	11.0196	7.7689
0.9	290	5.9425	5.9391	11.0168	7.6736
0.9	280	5.9418	5.9375	11.0132	7.5862
0.9	270	5.9395	5.9356	11.0091	7.5073
0.9	260	5.9388	5.9353	11.0057	7.4365
0.9	250	5.9373	5.9334	11.0005	7.3742
0.9	240	5.9358	5.9326	10.9965	7.3204
0.9	230	5.9332	5.9307	10.9933	7.2741
0.9	220	5.9321	5.9293	10.9901	7.2350
0.9	210	5.9313	5.9275	10.9865	7.2029
0.9	200	5.9289	5.9248	10.9822	7.1773
0.9	190	5.9269	5.9238	10.9795	7.1575
0.9	180	5.9259	5.9220	10.9751	7.1429
0.9	170	5.9233	5.9187	10.9699	7.1324
0.9	160	5.9211	5.9167	10.9637	7.1250
0.9	150	5.9182	5.9145	10.9584	7.1201
0.9	140	5.9151	5.9112	10.9512	7.1170
0.9	130	5.9126	5.9093	10.9452	7.1153
0.9	120	5.9096	5.9058	10.9386	7.1144
0.9	110	5.9066	5.9020	10.9289	7.1140
0.9	100	5.9042	5.8979	10.9207	7.1138

Appendix A.2: Limits for Weibull Distribution

		Limits for Weibull Distribution			
Weighting Constant *r* (A)	Number of Subgroups (B)	Limits T^2 (C)	Limits T^2_{EWMA} (D)	Limits T_p^2 (E)	Limits T^2_{pewma} (F)
0.1	500	5.9965	5.8742	11.0661	10.8503
0.1	490	5.9839	5.8576	11.0424	10.6354
0.1	480	5.9826	5.8524	11.0399	10.4251
0.1	470	5.9820	5.8476	11.0385	10.2197
0.1	460	5.9809	5.8423	11.0380	10.0195
0.1	450	5.9797	5.8374	11.0347	9.8238
0.1	440	5.9791	5.8308	11.0331	9.6333
0.1	430	5.9780	5.8269	11.0305	9.4485
0.1	420	5.9774	5.8235	11.0287	9.2690
0.1	410	5.9765	5.8199	11.0261	9.0950
0.1	400	5.9757	5.8138	11.0248	8.9270
0.1	390	5.9748	5.8073	11.0217	8.7657
0.1	380	5.9734	5.8012	11.0190	8.6107
0.1	370	5.9730	5.7971	11.0170	8.4625
0.1	360	5.9709	5.7921	11.0141	8.3203
0.1	350	5.9697	5.7868	11.0117	8.1859
0.1	340	5.9694	5.7806	11.0094	8.0581
0.1	330	5.9681	5.7751	11.0062	7.9368
0.1	320	5.9666	5.7705	11.0034	7.8229
0.1	310	5.9657	5.7664	11.0011	7.7170
0.1	300	5.9646	5.7629	10.9991	7.6183
0.1	290	5.9636	5.7576	10.9941	7.5270
0.1	280	5.9624	5.7514	10.9912	7.4436
0.1	270	5.9610	5.7453	10.9883	7.3676
0.1	260	5.9593	5.7404	10.9864	7.2987
0.1	250	5.9581	5.7372	10.9837	7.2357
0.1	240	5.9575	5.7336	10.9800	7.1776
0.1	230	5.9567	5.7280	10.9764	7.1256
0.1	220	5.9544	5.7219	10.9721	7.0797
0.1	210	5.9527	5.7144	10.9671	7.0381
0.1	200	5.9500	5.7077	10.9614	7.0001
0.1	190	5.9493	5.7012	10.9576	6.9648
0.1	180	5.9478	5.6948	10.9534	6.9310
0.1	170	5.9457	5.6881	10.9486	6.8993
0.1	160	5.9432	5.6811	10.9431	6.8680
0.1	150	5.9404	5.6755	10.9372	6.8375
0.1	140	5.9381	5.6688	10.9312	6.8070

		Limits for Weibull Distribution			
Weighting Constant *r* (A)	Number of Subgroups (B)	Limits T^2 (C)	Limits T^2_{EWMA} (D)	Limits T_p^2 (E)	Limits T^2_{pewma} (F)
0.1	130	5.9354	5.6625	10.9244	6.7758
0.1	120	5.9330	5.6564	10.9177	6.7446
0.1	110	5.9302	5.6487	10.9070	6.7125
0.1	100	5.9266	5.6398	10.8991	6.6795
0.2	500	5.9927	5.9287	11.0625	10.9204
0.2	490	5.9797	5.9142	11.0382	10.7039
0.2	480	5.9802	5.9121	11.0362	10.4920
0.2	470	5.9790	5.9083	11.0341	10.2849
0.2	460	5.9786	5.9024	11.0333	10.0829
0.2	450	5.9780	5.8988	11.0313	9.8860
0.2	440	5.9771	5.8970	11.0303	9.6939
0.2	430	5.9756	5.8930	11.0267	9.5078
0.2	420	5.9747	5.8889	11.0250	9.3271
0.2	410	5.9738	5.8852	11.0237	9.1522
0.2	400	5.9739	5.8824	11.0214	8.9830
0.2	390	5.9725	5.8785	11.0185	8.8192
0.2	380	5.9719	5.8757	11.0153	8.6620
0.2	370	5.9711	5.8724	11.0129	8.5111
0.2	360	5.9700	5.8694	11.0113	8.3662
0.2	350	5.9693	5.8661	11.0096	8.2293
0.2	340	5.9687	5.8635	11.0068	8.0990
0.2	330	5.9683	5.8595	11.0047	7.9759
0.2	320	5.9676	5.8555	11.0007	7.8612
0.2	310	5.9667	5.8514	10.9981	7.7535
0.2	300	5.9654	5.8470	10.9949	7.6527
0.2	290	5.9639	5.8434	10.9922	7.5593
0.2	280	5.9628	5.8402	10.9876	7.4742
0.2	270	5.9616	5.8374	10.9853	7.3977
0.2	260	5.9612	5.8336	10.9811	7.3285
0.2	250	5.9596	5.8299	10.9768	7.2667
0.2	240	5.9584	5.8248	10.9732	7.2120
0.2	230	5.9573	5.8215	10.9698	7.1636
0.2	220	5.9557	5.8171	10.9659	7.1225
0.2	210	5.9544	5.8119	10.9615	7.0877
0.2	200	5.9527	5.8062	10.9562	7.0581
0.2	190	5.9501	5.8013	10.9526	7.0321
0.2	180	5.9489	5.7973	10.9479	7.0102

		Limits for Weibull Distribution			
Weighting Constant *r* (A)	Number of Subgroups (B)	Limits T^2 (C)	Limits T^2_{EWMA} (D)	Limits T_p^2 (E)	Limits T^2_{pewma} (F)
0.2	170	5.9463	5.7927	10.9427	6.9915
0.2	160	5.9436	5.7858	10.9369	6.9744
0.2	150	5.9416	5.7802	10.9311	6.9587
0.2	140	5.9392	5.7756	10.9233	6.9449
0.2	130	5.9355	5.7705	10.9170	6.9310
0.2	120	5.9337	5.7653	10.9102	6.9171
0.2	110	5.9297	5.7593	10.9017	6.9035
0.2	100	5.9272	5.7528	10.8937	6.8899
0.3	500	5.9972	5.9626	11.0704	10.9855
0.3	490	5.9852	5.9499	11.0449	10.7679
0.3	480	5.9848	5.9475	11.0441	10.5550
0.3	470	5.9841	5.9459	11.0413	10.3463
0.3	460	5.9833	5.9436	11.0388	10.1426
0.3	450	5.9826	5.9417	11.0368	9.9443
0.3	440	5.9811	5.9394	11.0339	9.7509
0.3	430	5.9807	5.9383	11.0317	9.5628
0.3	420	5.9793	5.9354	11.0282	9.3807
0.3	410	5.9779	5.9331	11.0264	9.2051
0.3	400	5.9768	5.9310	11.0237	9.0348
0.3	390	5.9761	5.9290	11.0227	8.8700
0.3	380	5.9754	5.9261	11.0209	8.7110
0.3	370	5.9749	5.9242	11.0189	8.5593
0.3	360	5.9745	5.9211	11.0171	8.4144
0.3	350	5.9732	5.9180	11.0143	8.2765
0.3	340	5.9719	5.9141	11.0115	8.1457
0.3	330	5.9707	5.9129	11.0088	8.0216
0.3	320	5.9695	5.9106	11.0052	7.9056
0.3	310	5.9686	5.9078	11.0025	7.7959
0.3	300	5.9675	5.9045	10.9985	7.6942
0.3	290	5.9659	5.9007	10.9954	7.6008
0.3	280	5.9645	5.8970	10.9919	7.5159
0.3	270	5.9625	5.8933	10.9868	7.4387
0.3	260	5.9626	5.8897	10.9826	7.3694
0.3	250	5.9607	5.8873	10.9788	7.3080
0.3	240	5.9590	5.8844	10.9754	7.2546
0.3	230	5.9575	5.8812	10.9723	7.2086
0.3	220	5.9555	5.8753	10.9683	7.1700

		Limits for Weibull Distribution			
Weighting Constant *r* (A)	Number of Subgroups (B)	Limits T^2 (C)	Limits T^2_{EWMA} (D)	Limits T_p^2 (E)	Limits T^2_{pewma} (F)
0.3	210	5.9548	5.8727	10.9640	7.1369
0.3	200	5.9524	5.8673	10.9593	7.1102
0.3	190	5.9500	5.8629	10.9524	7.0878
0.3	180	5.9480	5.8602	10.9490	7.0698
0.3	170	5.9461	5.8549	10.9431	7.0548
0.3	160	5.9450	5.8513	10.9374	7.0430
0.3	150	5.9425	5.8471	10.9315	7.0331
0.3	140	5.9402	5.8449	10.9249	7.0243
0.3	130	5.9373	5.8386	10.9174	7.0162
0.3	120	5.9344	5.8331	10.9103	7.0089
0.3	110	5.9314	5.8279	10.9015	7.0017
0.3	100	5.9281	5.8216	10.8936	6.9954
0.4	500	5.9926	5.9773	11.0676	11.0188
0.4	490	5.9800	5.9635	11.0433	10.8005
0.4	480	5.9777	5.9620	11.0409	10.5870
0.4	470	5.9767	5.9604	11.0405	10.3780
0.4	460	5.9767	5.9580	11.0383	10.1731
0.4	450	5.9758	5.9564	11.0352	9.9740
0.4	440	5.9756	5.9563	11.0333	9.7810
0.4	430	5.9748	5.9541	11.0310	9.5933
0.4	420	5.9735	5.9523	11.0284	9.4111
0.4	410	5.9727	5.9510	11.0258	9.2346
0.4	400	5.9721	5.9493	11.0236	9.0641
0.4	390	5.9721	5.9479	11.0218	8.8994
0.4	380	5.9707	5.9455	11.0190	8.7413
0.4	370	5.9696	5.9430	11.0161	8.5891
0.4	360	5.9695	5.9414	11.0136	8.4418
0.4	350	5.9676	5.9374	11.0113	8.3020
0.4	340	5.9662	5.9359	11.0089	8.1704
0.4	330	5.9654	5.9343	11.0069	8.0444
0.4	320	5.9639	5.9319	11.0031	7.9282
0.4	310	5.9621	5.9278	10.9995	7.8196
0.4	300	5.9608	5.9261	10.9980	7.7175
0.4	290	5.9597	5.9233	10.9950	7.6228
0.4	280	5.9588	5.9210	10.9916	7.5371
0.4	270	5.9568	5.9164	10.9883	7.4596
0.4	260	5.9552	5.9139	10.9842	7.3900

		Limits for Weibull Distribution			
Weighting Constant *r* (A)	Number of Subgroups (B)	Limits T^2 (C)	Limits T^2_{EWMA} (D)	Limits T_p^2 (E)	Limits T^2_{pewma} (F)
0.4	250	5.9545	5.9122	10.9799	7.3288
0.4	240	5.9534	5.9098	10.9769	7.2757
0.4	230	5.9520	5.9071	10.9719	7.2300
0.4	220	5.9498	5.9050	10.9682	7.1913
0.4	210	5.9490	5.9035	10.9651	7.1591
0.4	200	5.9472	5.9005	10.9608	7.1332
0.4	190	5.9451	5.8968	10.9560	7.1120
0.4	180	5.9432	5.8937	10.9532	7.0961
0.4	170	5.9409	5.8891	10.9483	7.0833
0.4	160	5.9390	5.8848	10.9429	7.0731
0.4	150	5.9361	5.8812	10.9352	7.0656
0.4	140	5.9343	5.8774	10.9299	7.0593
0.4	130	5.9315	5.8730	10.9219	7.0543
0.4	120	5.9281	5.8682	10.9147	7.0498
0.4	110	5.9248	5.8628	10.9055	7.0458
0.4	100	5.9211	5.8577	10.8971	7.0426
0.5	500	5.9959	5.9892	11.0665	11.0406
0.5	490	5.9827	5.9764	11.0422	10.8218
0.5	480	5.9823	5.9758	11.0396	10.6078
0.5	470	5.9810	5.9733	11.0371	10.3984
0.5	460	5.9805	5.9723	11.0356	10.1941
0.5	450	5.9803	5.9714	11.0345	9.9944
0.5	440	5.9794	5.9697	11.0308	9.8001
0.5	430	5.9786	5.9683	11.0291	9.6103
0.5	420	5.9779	5.9664	11.0268	9.4268
0.5	410	5.9764	5.9636	11.0248	9.2488
0.5	400	5.9753	5.9618	11.0238	9.0772
0.5	390	5.9744	5.9602	11.0205	8.9118
0.5	380	5.9734	5.9590	11.0196	8.7528
0.5	370	5.9713	5.9564	11.0179	8.6005
0.5	360	5.9710	5.9546	11.0150	8.4550
0.5	350	5.9714	5.9547	11.0120	8.3141
0.5	340	5.9712	5.9542	11.0104	8.1815
0.5	330	5.9701	5.9535	11.0082	8.0570
0.5	320	5.9686	5.9517	11.0044	7.9397
0.5	310	5.9675	5.9495	11.0015	7.8302
0.5	300	5.9669	5.9474	10.9969	7.7281

		Limits for Weibull Distribution			
Weighting Constant *r* (A)	Number of Subgroups (B)	Limits T^2 (C)	Limits T^2_{EWMA} (D)	Limits T_p^2 (E)	Limits T^2_{pewma} (F)
0.5	290	5.9666	5.9458	10.9934	7.6336
0.5	280	5.9659	5.9433	10.9898	7.5465
0.5	270	5.9642	5.9408	10.9863	7.4685
0.5	260	5.9630	5.9383	10.9822	7.3980
0.5	250	5.9607	5.9363	10.9791	7.3358
0.5	240	5.9597	5.9345	10.9759	7.2816
0.5	230	5.9587	5.9328	10.9725	7.2349
0.5	220	5.9570	5.9299	10.9689	7.1949
0.5	210	5.9557	5.9271	10.9652	7.1638
0.5	200	5.9544	5.9256	10.9604	7.1385
0.5	190	5.9518	5.9226	10.9557	7.1185
0.5	180	5.9497	5.9208	10.9514	7.1029
0.5	170	5.9487	5.9174	10.9450	7.0911
0.5	160	5.9461	5.9133	10.9399	7.0816
0.5	150	5.9440	5.9086	10.9338	7.0752
0.5	140	5.9414	5.9044	10.9261	7.0708
0.5	130	5.9376	5.9002	10.9191	7.0672
0.5	120	5.9343	5.8960	10.9133	7.0643
0.5	110	5.9316	5.8914	10.9068	7.0622
0.5	100	5.9278	5.8851	10.8980	7.0605
0.6	500	5.9920	5.9877	11.0614	11.0525
0.6	490	5.9802	5.9751	11.0384	10.8334
0.6	480	5.9779	5.9735	11.0358	10.6188
0.6	470	5.9768	5.9729	11.0335	10.4091
0.6	460	5.9759	5.9721	11.0319	10.2049
0.6	450	5.9752	5.9700	11.0294	10.0055
0.6	440	5.9743	5.9692	11.0270	9.8116
0.6	430	5.9739	5.9683	11.0250	9.6227
0.6	420	5.9729	5.9669	11.0226	9.4396
0.6	410	5.9723	5.9655	11.0212	9.2623
0.6	400	5.9720	5.9640	11.0177	9.0902
0.6	390	5.9708	5.9618	11.0158	8.9241
0.6	380	5.9694	5.9609	11.0138	8.7652
0.6	370	5.9684	5.9595	11.0117	8.6125
0.6	360	5.9666	5.9581	11.0084	8.4662
0.6	350	5.9648	5.9563	11.0058	8.3261
0.6	340	5.9642	5.9551	11.0040	8.1928

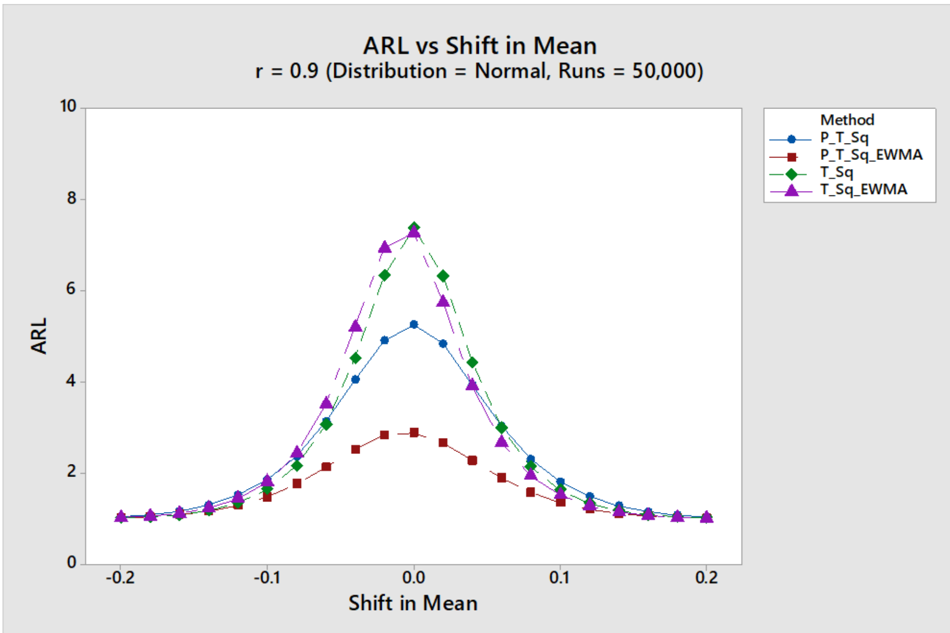
		Limits for Weibull Distribution			
Weighting Constant *r* (A)	Number of Subgroups (B)	Limits T^2 (C)	Limits T^2_{EWMA} (D)	Limits T_p^2 (E)	Limits T^2_{pewma} (F)
0.6	330	5.9636	5.9539	11.0021	8.0668
0.6	320	5.9622	5.9529	11.0005	7.9495
0.6	310	5.9603	5.9514	10.9983	7.8397
0.6	300	5.9598	5.9506	10.9956	7.7388
0.6	290	5.9580	5.9485	10.9921	7.6456
0.6	280	5.9579	5.9465	10.9875	7.5591
0.6	270	5.9564	5.9436	10.9843	7.4806
0.6	260	5.9552	5.9407	10.9800	7.4105
0.6	250	5.9542	5.9385	10.9760	7.3477
0.6	240	5.9513	5.9358	10.9721	7.2934
0.6	230	5.9503	5.9338	10.9673	7.2469
0.6	220	5.9500	5.9324	10.9646	7.2086
0.6	210	5.9473	5.9290	10.9598	7.1756
0.6	200	5.9460	5.9265	10.9554	7.1493
0.6	190	5.9449	5.9240	10.9503	7.1288
0.6	180	5.9426	5.9208	10.9460	7.1133
0.6	170	5.9401	5.9176	10.9393	7.1020
0.6	160	5.9377	5.9141	10.9356	7.0938
0.6	150	5.9354	5.9112	10.9302	7.0879
0.6	140	5.9335	5.9088	10.9221	7.0840
0.6	130	5.9317	5.9045	10.9156	7.0813
0.6	120	5.9298	5.9003	10.9090	7.0798
0.6	110	5.9259	5.8946	10.8994	7.0788
0.6	100	5.9225	5.8897	10.8910	7.0781
0.7	500	5.9987	5.9932	11.0695	11.0564
0.7	490	5.9862	5.9803	11.0446	10.8374
0.7	480	5.9852	5.9788	11.0420	10.6228
0.7	470	5.9851	5.9783	11.0401	10.4130
0.7	460	5.9844	5.9783	11.0381	10.2087
0.7	450	5.9837	5.9772	11.0361	10.0094
0.7	440	5.9834	5.9764	11.0343	9.8158
0.7	430	5.9821	5.9746	11.0317	9.6281
0.7	420	5.9815	5.9740	11.0288	9.4454
0.7	410	5.9802	5.9728	11.0266	9.2689
0.7	400	5.9790	5.9718	11.0229	9.0974
0.7	390	5.9781	5.9704	11.0213	8.9308
0.7	380	5.9772	5.9683	11.0181	8.7708

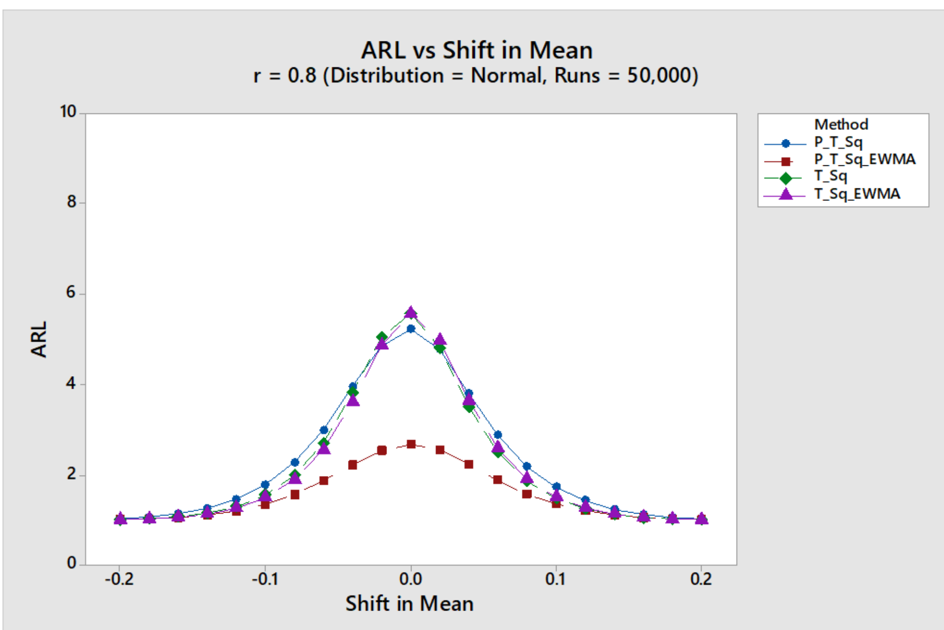
		Limits for Weibull Distribution			
Weighting Constant *r* (A)	Number of Subgroups (B)	Limits T^2 (C)	Limits T^2_{EWMA} (D)	Limits $T_p^{\ 2}$ (E)	Limits T^2_{pewma} (F)
0.7	370	5.9759	5.9666	11.0164	8.6178
0.7	360	5.9747	5.9656	11.0139	8.4716
0.7	350	5.9733	5.9643	11.0119	8.3326
0.7	340	5.9727	5.9626	11.0095	8.2006
0.7	330	5.9715	5.9621	11.0070	8.0746
0.7	320	5.9706	5.9606	11.0055	7.9551
0.7	310	5.9698	5.9590	11.0046	7.8442
0.7	300	5.9683	5.9572	10.9998	7.7423
0.7	290	5.9666	5.9552	10.9957	7.6473
0.7	280	5.9657	5.9537	10.9920	7.5603
0.7	270	5.9647	5.9516	10.9893	7.4814
0.7	260	5.9636	5.9509	10.9863	7.4104
0.7	250	5.9624	5.9485	10.9820	7.3470
0.7	240	5.9613	5.9465	10.9784	7.2921
0.7	230	5.9590	5.9452	10.9737	7.2457
0.7	220	5.9575	5.9442	10.9697	7.2067
0.7	210	5.9567	5.9422	10.9654	7.1750
0.7	200	5.9553	5.9395	10.9622	7.1498
0.7	190	5.9530	5.9368	10.9572	7.1294
0.7	180	5.9509	5.9339	10.9510	7.1148
0.7	170	5.9482	5.9308	10.9454	7.1046
0.7	160	5.9467	5.9286	10.9395	7.0975
0.7	150	5.9441	5.9244	10.9319	7.0929
0.7	140	5.9423	5.9222	10.9263	7.0901
0.7	130	5.9400	5.9193	10.9209	7.0882
0.7	120	5.9371	5.9162	10.9137	7.0871
0.7	110	5.9340	5.9133	10.9063	7.0865
0.7	100	5.9304	5.9067	10.8977	7.0862
0.8	500	5.9949	5.9943	11.0611	11.0604
0.8	490	5.9825	5.9817	11.0369	10.8413
0.8	480	5.9817	5.9809	11.0353	10.6273
0.8	470	5.9811	5.9802	11.0337	10.4175
0.8	460	5.9796	5.9786	11.0313	10.2123
0.8	450	5.9793	5.9792	11.0286	10.0126
0.8	440	5.9793	5.9788	11.0277	9.8186
0.8	430	5.9780	5.9769	11.0241	9.6302
0.8	420	5.9775	5.9758	11.0218	9.4471

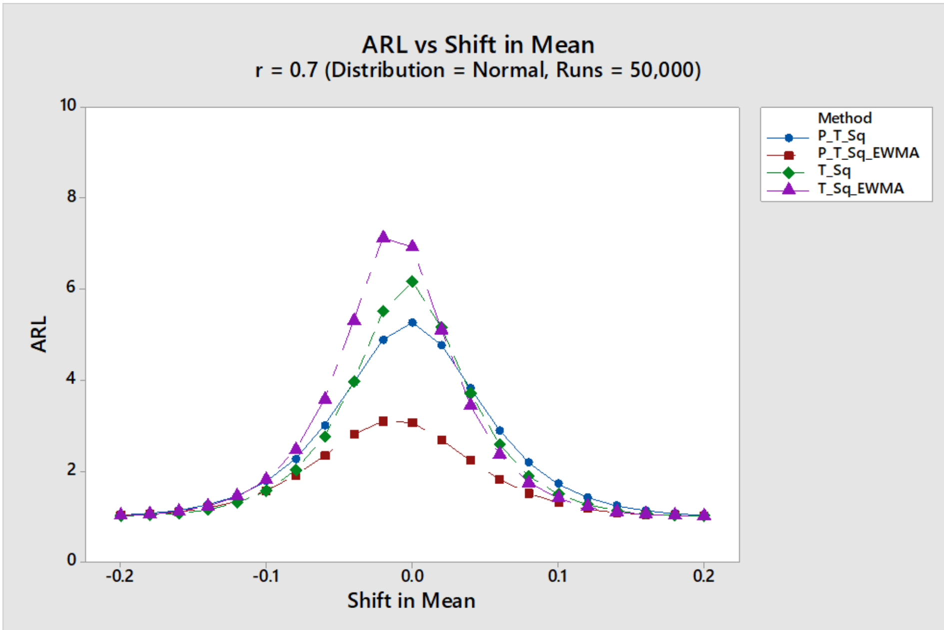
		Limits for Weibull Distribution			
Weighting Constant *r* (A)	Number of Subgroups (B)	Limits T^2 (C)	Limits T^2_{EWMA} (D)	Limits T_p^2 (E)	Limits T^2_{pewma} (F)
0.8	410	5.9765	5.9740	11.0199	9.2702
0.8	400	5.9756	5.9729	11.0170	9.0990
0.8	390	5.9749	5.9717	11.0143	8.9337
0.8	380	5.9746	5.9719	11.0113	8.7750
0.8	370	5.9747	5.9698	11.0084	8.6211
0.8	360	5.9735	5.9687	11.0065	8.4743
0.8	350	5.9727	5.9677	11.0034	8.3347
0.8	340	5.9711	5.9662	11.0005	8.2025
0.8	330	5.9697	5.9650	10.9979	8.0769
0.8	320	5.9692	5.9642	10.9952	7.9587
0.8	310	5.9686	5.9631	10.9914	7.8487
0.8	300	5.9680	5.9631	10.9888	7.7470
0.8	290	5.9666	5.9608	10.9846	7.6519
0.8	280	5.9652	5.9587	10.9820	7.5653
0.8	270	5.9630	5.9575	10.9796	7.4862
0.8	260	5.9616	5.9554	10.9760	7.4168
0.8	250	5.9600	5.9539	10.9713	7.3547
0.8	240	5.9590	5.9529	10.9673	7.3013
0.8	230	5.9578	5.9515	10.9631	7.2560
0.8	220	5.9571	5.9491	10.9583	7.2172
0.8	210	5.9554	5.9473	10.9551	7.1862
0.8	200	5.9537	5.9450	10.9519	7.1616
0.8	190	5.9521	5.9438	10.9464	7.1424
0.8	180	5.9505	5.9418	10.9419	7.1279
0.8	170	5.9483	5.9389	10.9353	7.1176
0.8	160	5.9472	5.9365	10.9307	7.1099
0.8	150	5.9434	5.9339	10.9255	7.1050
0.8	140	5.9413	5.9316	10.9196	7.1019
0.8	130	5.9391	5.9279	10.9108	7.1001
0.8	120	5.9352	5.9245	10.9042	7.0990
0.8	110	5.9322	5.9205	10.8968	7.0984
0.8	100	5.9298	5.9179	10.8890	7.0983
0.9	500	5.9970	5.9972	11.0633	11.0627
0.9	490	5.9837	5.9844	11.0392	10.8433
0.9	480	5.9839	5.9842	11.0385	10.6287
0.9	470	5.9831	5.9832	11.0350	10.4183
0.9	460	5.9824	5.9829	11.0330	10.2132

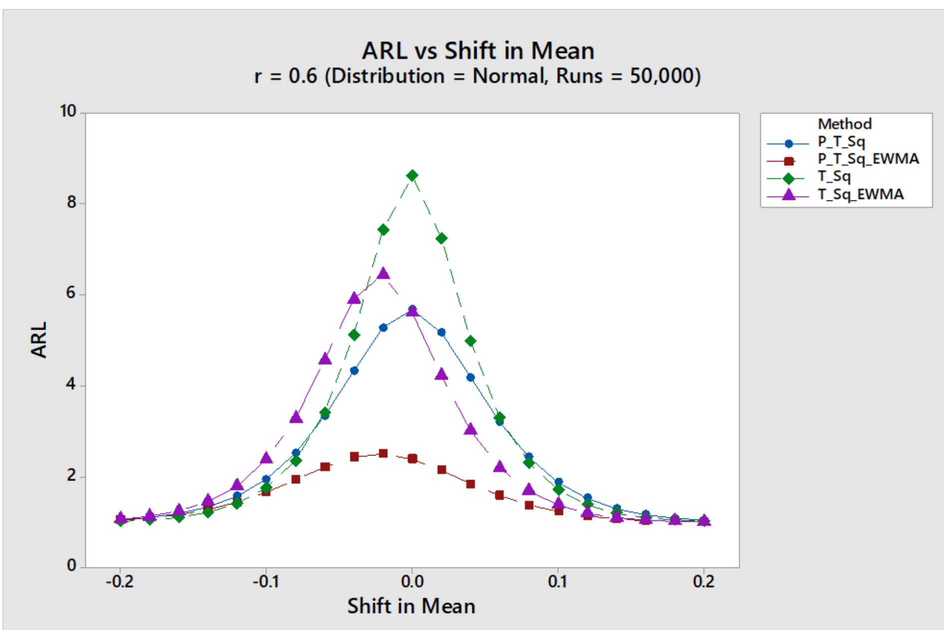
		Limits for Weibull Distribution			
Weighting Constant *r* (A)	Number of Subgroups (B)	Limits T^2 (C)	Limits T^2_{EWMA} (D)	Limits $T_p^{\ 2}$ (E)	Limits T^2_{pewma} (F)
0.9	450	5.9811	5.9820	11.0314	10.0134
0.9	440	5.9804	5.9815	11.0293	9.8191
0.9	430	5.9799	5.9810	11.0272	9.6302
0.9	420	5.9794	5.9803	11.0267	9.4476
0.9	410	5.9772	5.9793	11.0236	9.2715
0.9	400	5.9768	5.9784	11.0206	9.1000
0.9	390	5.9761	5.9762	11.0182	8.9343
0.9	380	5.9743	5.9752	11.0165	8.7756
0.9	370	5.9731	5.9741	11.0140	8.6229
0.9	360	5.9710	5.9721	11.0092	8.4761
0.9	350	5.9691	5.9711	11.0071	8.3360
0.9	340	5.9682	5.9693	11.0042	8.2028
0.9	330	5.9673	5.9689	11.0012	8.0777
0.9	320	5.9662	5.9674	11.0001	7.9599
0.9	310	5.9656	5.9661	10.9985	7.8498
0.9	300	5.9655	5.9664	10.9951	7.7482
0.9	290	5.9655	5.9653	10.9911	7.6531
0.9	280	5.9635	5.9637	10.9884	7.5665
0.9	270	5.9617	5.9625	10.9836	7.4890
0.9	260	5.9600	5.9601	10.9818	7.4194
0.9	250	5.9595	5.9580	10.9769	7.3570
0.9	240	5.9579	5.9575	10.9725	7.3033
0.9	230	5.9566	5.9555	10.9675	7.2578
0.9	220	5.9544	5.9538	10.9642	7.2192
0.9	210	5.9529	5.9528	10.9590	7.1879
0.9	200	5.9513	5.9507	10.9545	7.1627
0.9	190	5.9499	5.9493	10.9490	7.1431
0.9	180	5.9480	5.9471	10.9420	7.1284
0.9	170	5.9448	5.9440	10.9369	7.1178
0.9	160	5.9424	5.9413	10.9321	7.1106
0.9	150	5.9411	5.9387	10.9257	7.1061
0.9	140	5.9379	5.9363	10.9199	7.1030
0.9	130	5.9359	5.9328	10.9123	7.1014
0.9	120	5.9325	5.9297	10.9050	7.1006
0.9	110	5.9287	5.9251	10.8969	7.1000
0.9	100	5.9247	5.9220	10.8892	7.0999

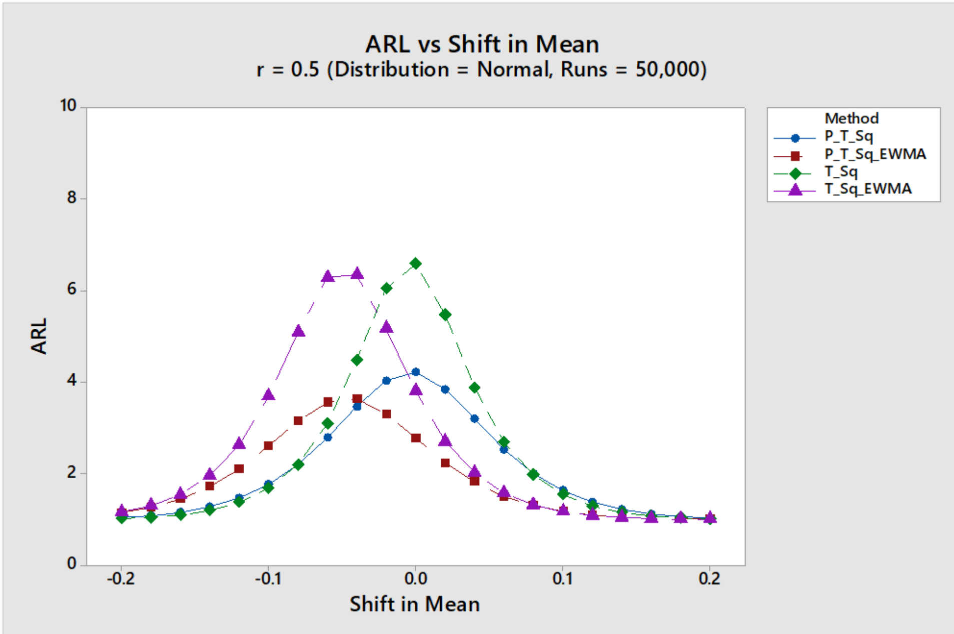
Appendix B: Comparison of ARL for Detecting Shift in Mean, Normal Distribution, 50,000 runs for Different Values of *r*

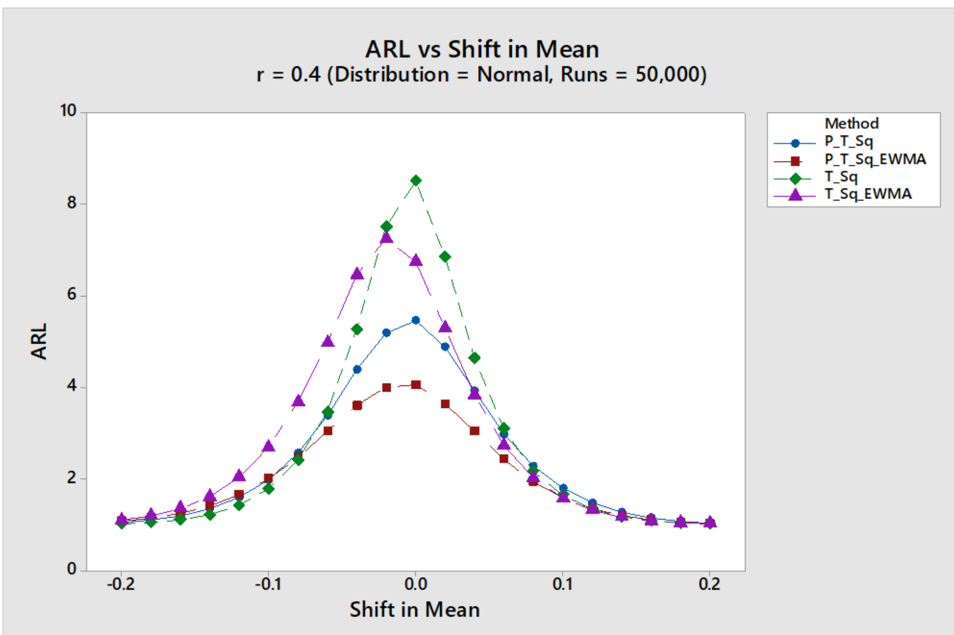


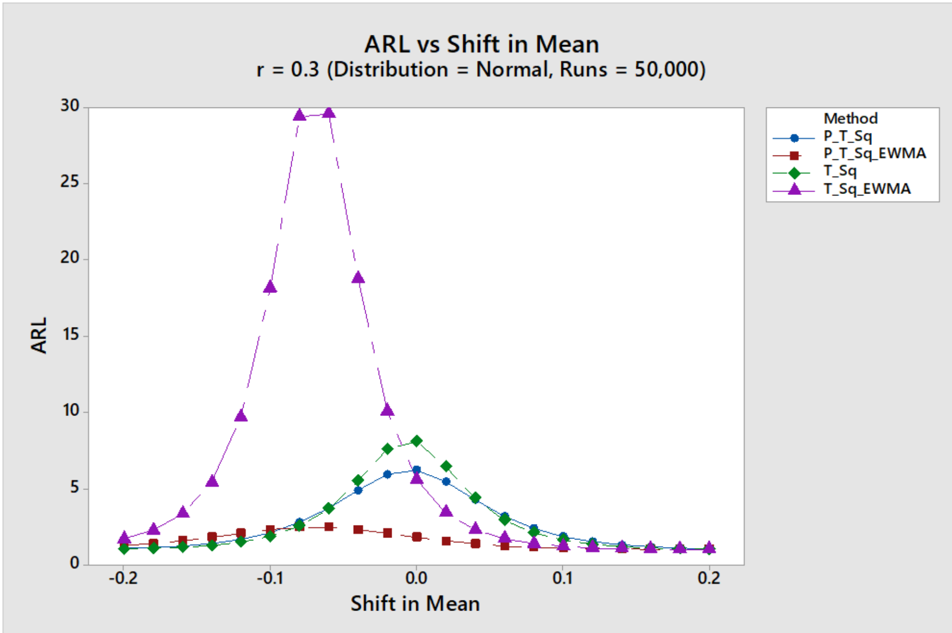


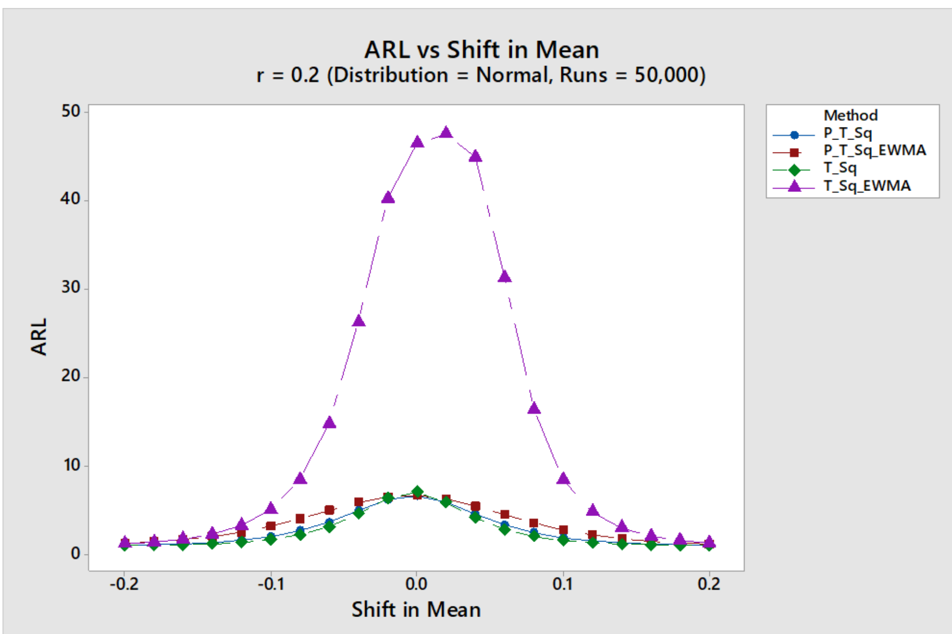

ARL vs Shift in Mean
r = 0.7 (Distribution = Normal, Runs = 50,000)
ARL
Shift in Mean
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA
0
2
4
6
8
10
-0.2
-0.1
0.0
0.1
0.2

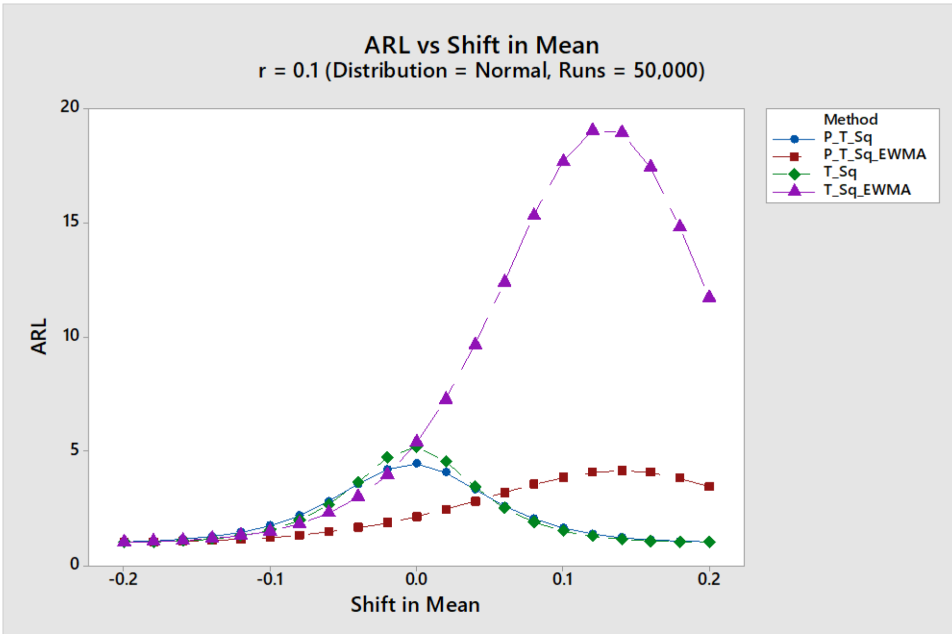

ARL vs Shift in Mean
r = 0.6 (Distribution = Normal, Runs = 50,000)
ARL
Shift in Mean
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA
0
2
4
6
8
10
-0.2
-0.1
0.0
0.1
0.2


ARL vs Shift in Mean
r = 0.5 (Distribution = Normal, Runs = 50,000)
ARL
Shift in Mean
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA
0
2
4
6
8
10
-0.2
-0.1
0.0
0.1
0.2

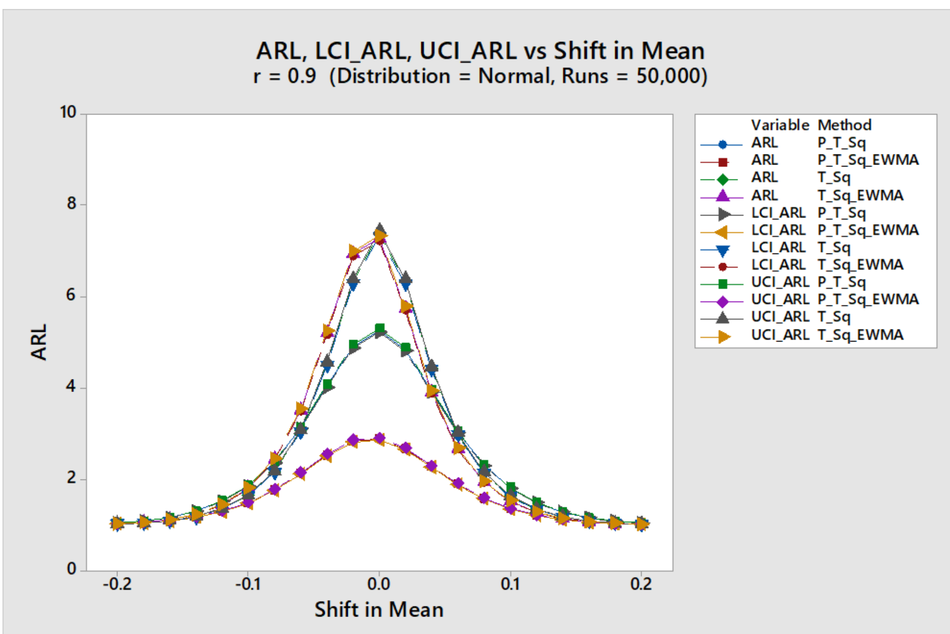

ARL vs Shift in Mean
r = 0.4 (Distribution = Normal, Runs = 50,000)
ARL
Shift in Mean
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA
0
2
4
6
8
10
-0.2
-0.1
0.0
0.1
0.2


ARL vs Shift in Mean
r = 0.3 (Distribution = Normal, Runs = 50,000)
ARL
Shift in Mean
30
25
20
15
10
5
0
-0.2
-0.1
0.0
0.1
0.2
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA

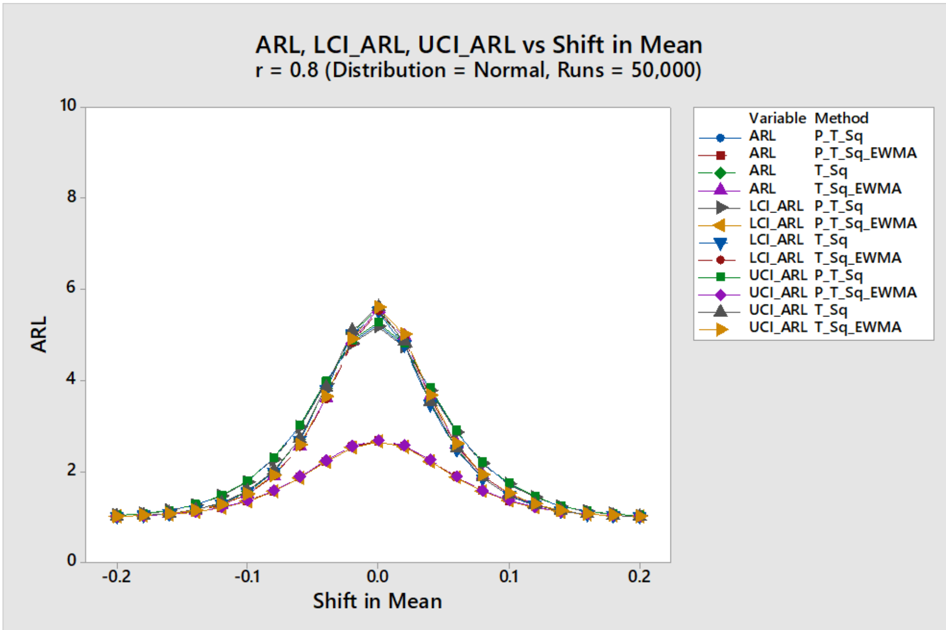

ARL vs Shift in Mean
r = 0.2 (Distribution = Normal, Runs = 50,000)
ARL
Shift in Mean
50
40
30
20
10
0
-0.2
-0.1
0.0
0.1
0.2
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA

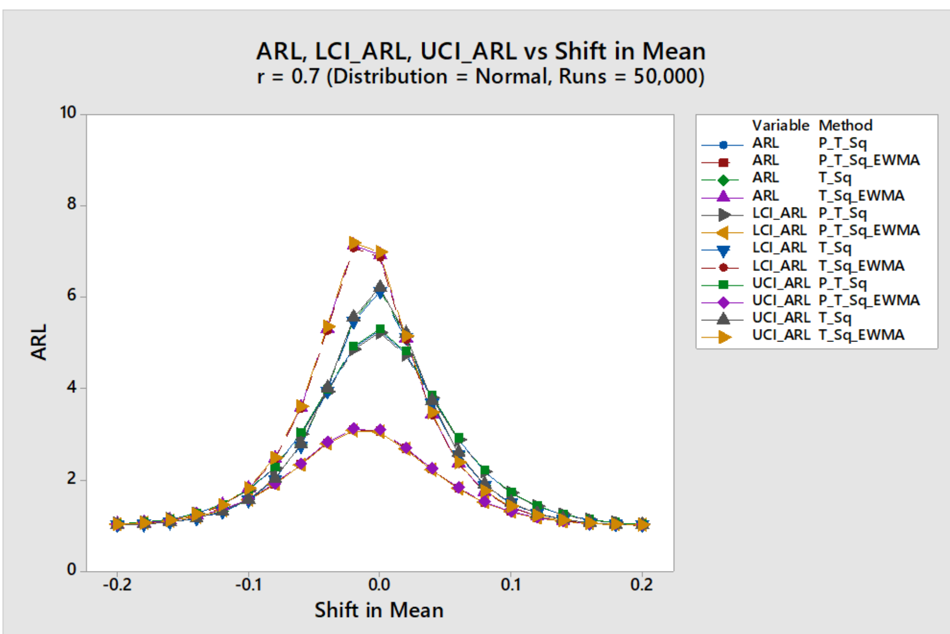


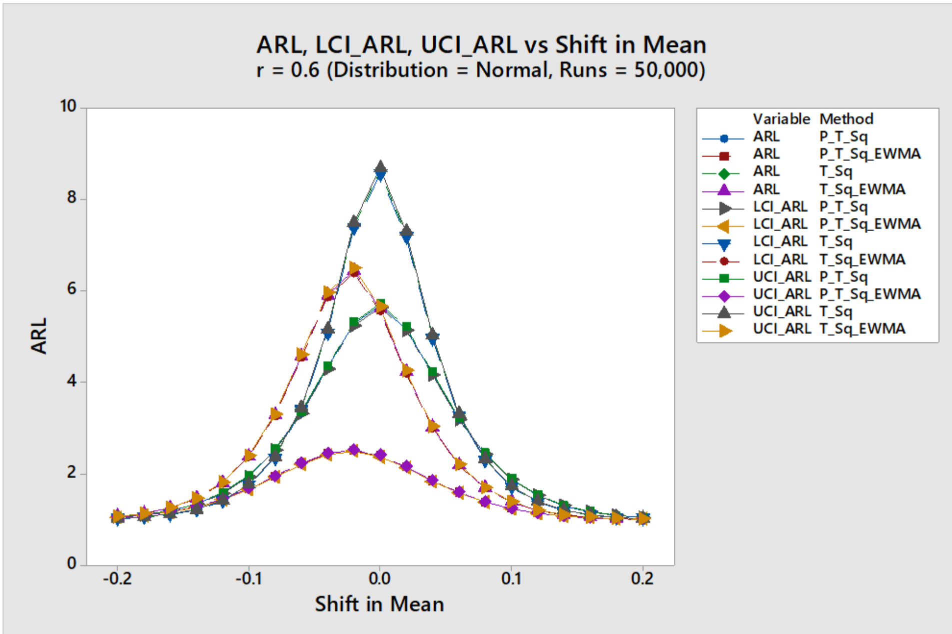
ARL vs Shift in Mean
r = 0.1 (Distribution = Normal, Runs = 50,000)
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA
ARL
Shift in Mean

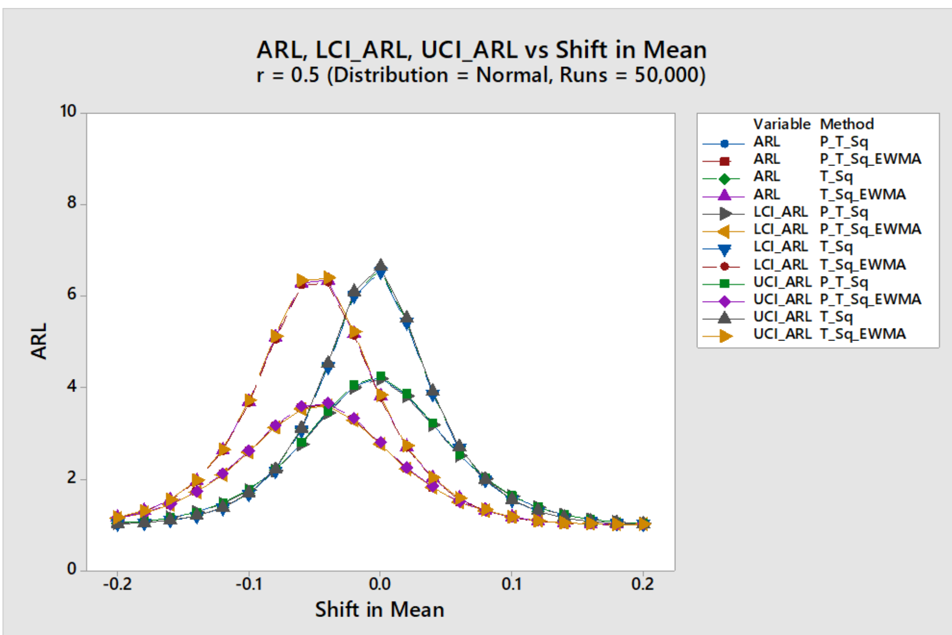


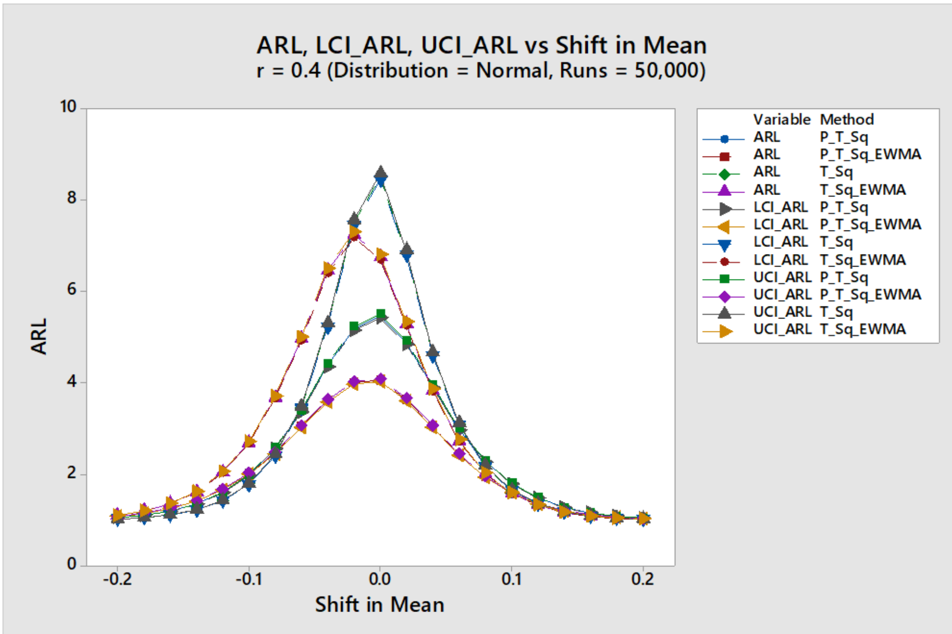
ARL, LCI_ARL, UCI_ARL vs Shift in Mean
r = 0.9 (Distribution = Normal, Runs = 50,000)
Variable Method
ARL P_T_Sq
ARL P_T_Sq_EWMA
ARL T_Sq
ARL T_Sq_EWMA
LCI_ARL P_T_Sq
LCI_ARL P_T_Sq_EWMA
LCI_ARL T_Sq
LCI_ARL T_Sq_EWMA
UCI_ARL P_T_Sq
UCI_ARL P_T_Sq_EWMA
UCI_ARL T_Sq
UCI_ARL T_Sq_EWMA
ARL
Shift in Mean

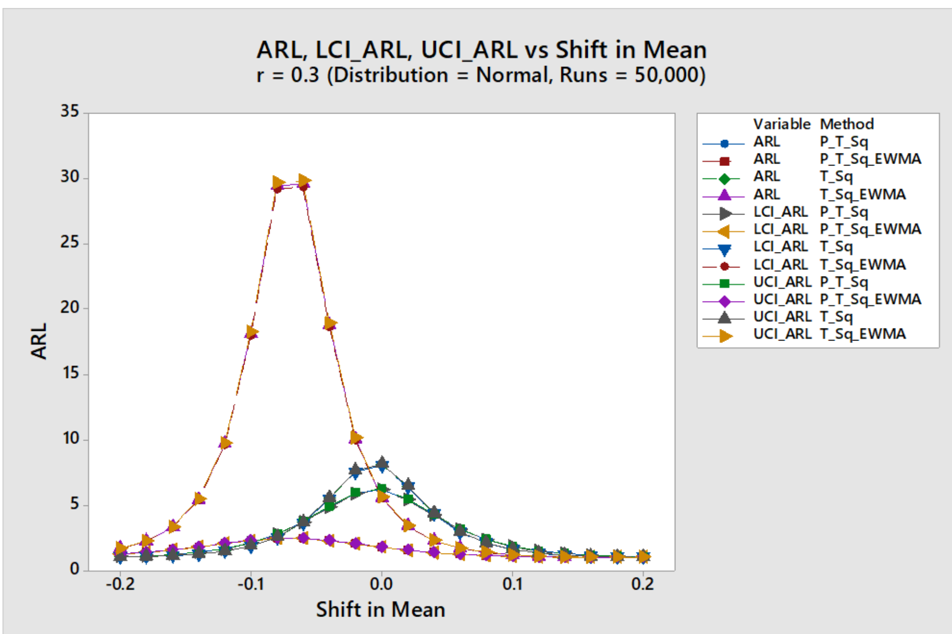

ARL, LCI_ARL, UCI_ARL vs Shift in Mean
r = 0.8 (Distribution = Normal, Runs = 50,000)
ARL
Shift in Mean
10
8
6
4
2
0
-0.2
-0.1
0.0
0.1
0.2
Variable Method
ARL P_T_Sq
ARL P_T_Sq_EWMA
ARL T_Sq
ARL T_Sq_EWMA
LCI_ARL P_T_Sq
LCI_ARL P_T_Sq_EWMA
LCI_ARL T_Sq
LCI_ARL T_Sq_EWMA
UCI_ARL P_T_Sq
UCI_ARL P_T_Sq_EWMA
UCI_ARL T_Sq
UCI_ARL T_Sq_EWMA

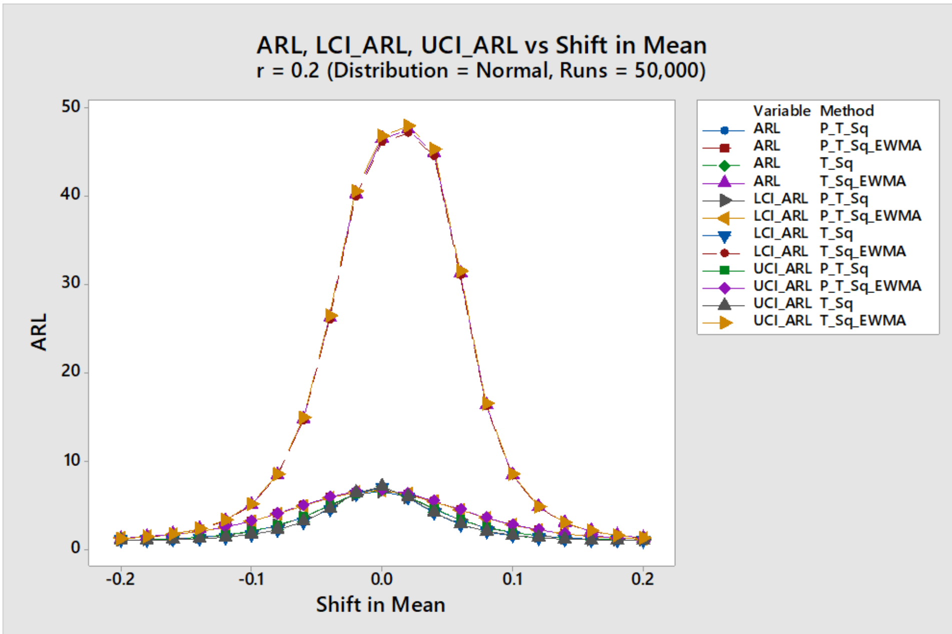

ARL, LCI_ARL, UCI_ARL vs Shift in Mean
r = 0.7 (Distribution = Normal, Runs = 50,000)
ARL
Shift in Mean
10
8
6
4
2
0
-0.2
-0.1
0.0
0.1
0.2
Variable Method
ARL P_T_Sq
ARL P_T_Sq_EWMA
ARL T_Sq
ARL T_Sq_EWMA
LCI_ARL P_T_Sq
LCI_ARL P_T_Sq_EWMA
LCI_ARL T_Sq
LCI_ARL T_Sq_EWMA
UCI_ARL P_T_Sq
UCI_ARL P_T_Sq_EWMA
UCI_ARL T_Sq
UCI_ARL T_Sq_EWMA

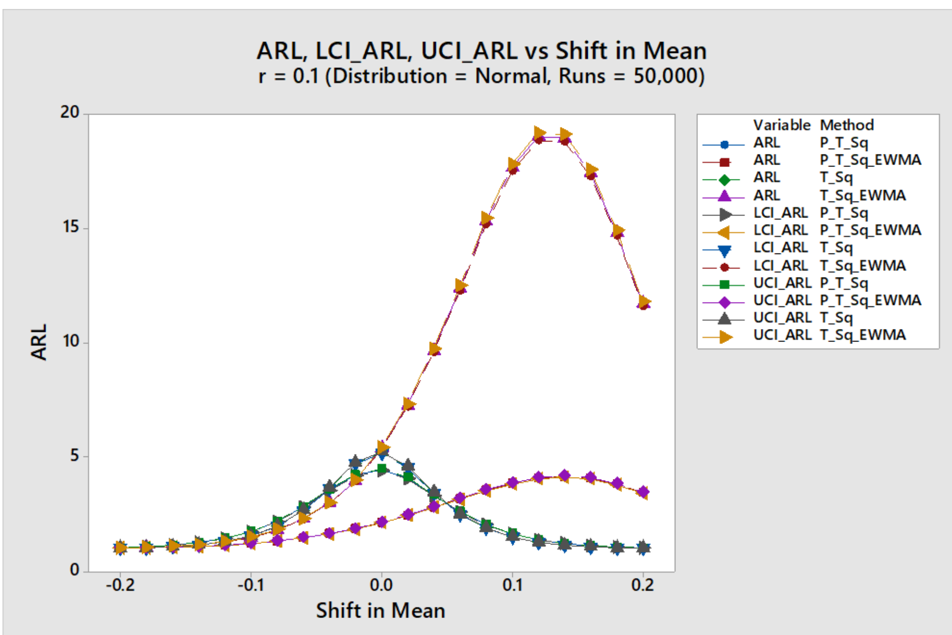

ARL, LCI_ARL, UCI_ARL vs Shift in Mean
r = 0.6 (Distribution = Normal, Runs = 50,000)
ARL
Shift in Mean
Variable Method
ARL P_T_Sq
ARL P_T_Sq_EWMA
ARL T_Sq
ARL T_Sq_EWMA
LCI_ARL P_T_Sq
LCI_ARL P_T_Sq_EWMA
LCI_ARL T_Sq
LCI_ARL T_Sq_EWMA
UCI_ARL P_T_Sq
UCI_ARL P_T_Sq_EWMA
UCI_ARL T_Sq
UCI_ARL T_Sq_EWMA


ARL, LCI_ARL, UCI_ARL vs Shift in Mean
r = 0.5 (Distribution = Normal, Runs = 50,000)
ARL
Shift in Mean
Variable Method
ARL P_T_Sq
ARL P_T_Sq_EWMA
ARL T_Sq
ARL T_Sq_EWMA
LCI_ARL P_T_Sq
LCI_ARL P_T_Sq_EWMA
LCI_ARL T_Sq
LCI_ARL T_Sq_EWMA
UCI_ARL P_T_Sq
UCI_ARL P_T_Sq_EWMA
UCI_ARL T_Sq
UCI_ARL T_Sq_EWMA

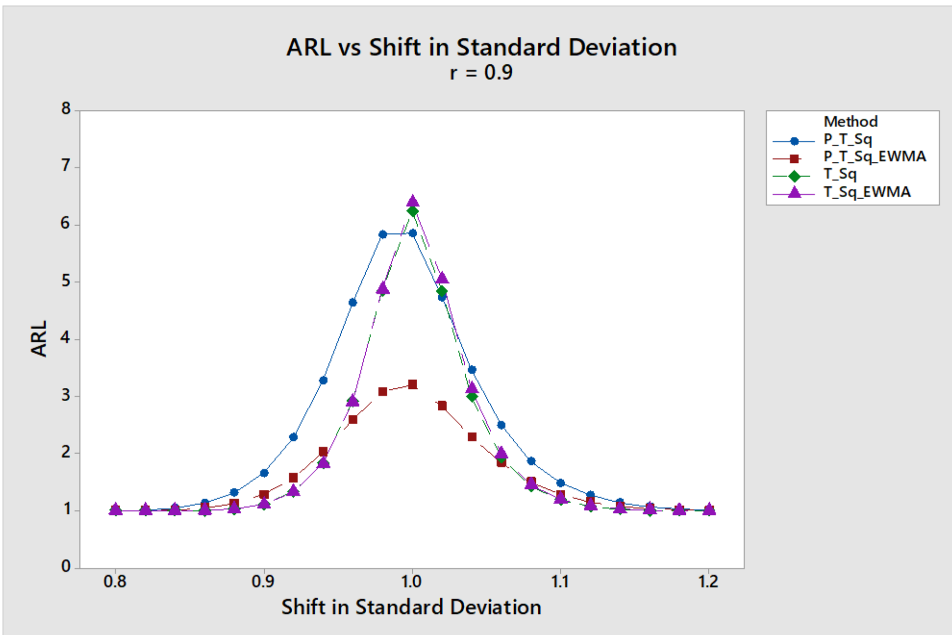

ARL, LCI_ARL, UCI_ARL vs Shift in Mean
r = 0.4 (Distribution = Normal, Runs = 50,000)
ARL
Shift in Mean
Variable Method
ARL P_T_Sq
ARL P_T_Sq_EWMA
ARL T_Sq
ARL T_Sq_EWMA
LCI_ARL P_T_Sq
LCI_ARL P_T_Sq_EWMA
LCI_ARL T_Sq
LCI_ARL T_Sq_EWMA
UCI_ARL P_T_Sq
UCI_ARL P_T_Sq_EWMA
UCI_ARL T_Sq
UCI_ARL T_Sq_EWMA

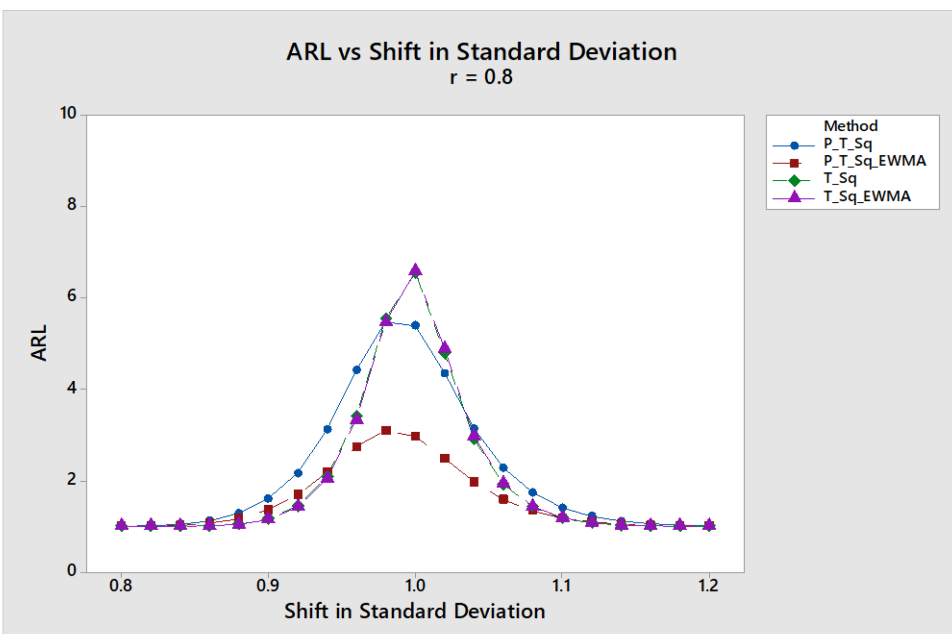

ARL, LCI_ARL, UCI_ARL vs Shift in Mean
r = 0.3 (Distribution = Normal, Runs = 50,000)
ARL
Shift in Mean
Variable Method
ARL P_T_Sq
ARL P_T_Sq_EWMA
ARL T_Sq
ARL T_Sq_EWMA
LCI_ARL P_T_Sq
LCI_ARL P_T_Sq_EWMA
LCI_ARL T_Sq
LCI_ARL T_Sq_EWMA
UCI_ARL P_T_Sq
UCI_ARL P_T_Sq_EWMA
UCI_ARL T_Sq
UCI_ARL T_Sq_EWMA

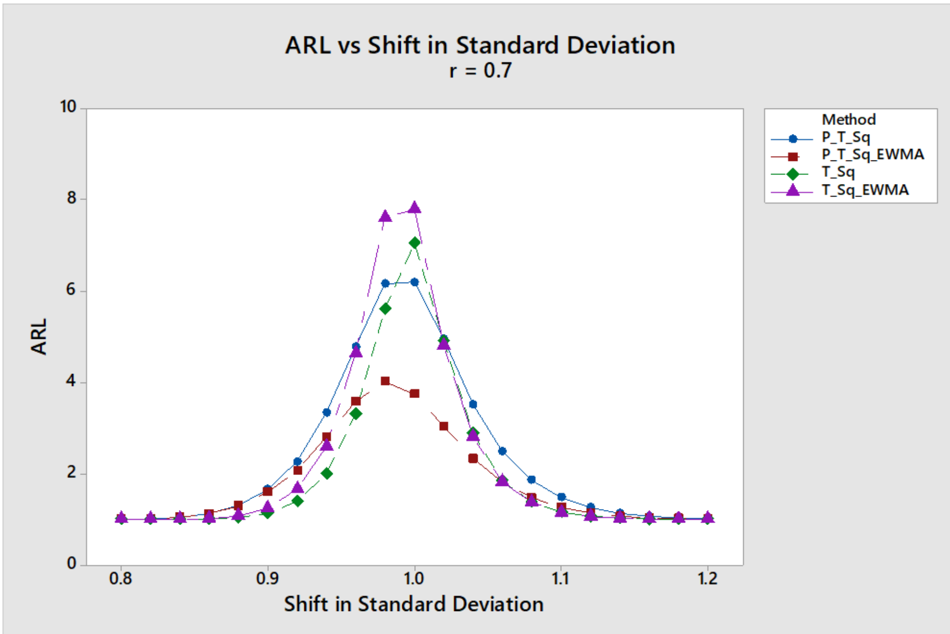

ARL, LCI_ARL, UCI_ARL vs Shift in Mean
r = 0.2 (Distribution = Normal, Runs = 50,000)
ARL
50
40
30
20
10
0
-0.2
-0.1
0.0
0.1
0.2
Shift in Mean
Variable Method
ARL P_T_Sq
ARL P_T_Sq_EWMA
ARL T_Sq
ARL T_Sq_EWMA
LCI_ARL P_T_Sq
LCI_ARL P_T_Sq_EWMA
LCI_ARL T_Sq
LCI_ARL T_Sq_EWMA
UCI_ARL P_T_Sq
UCI_ARL P_T_Sq_EWMA
UCI_ARL T_Sq
UCI_ARL T_Sq_EWMA

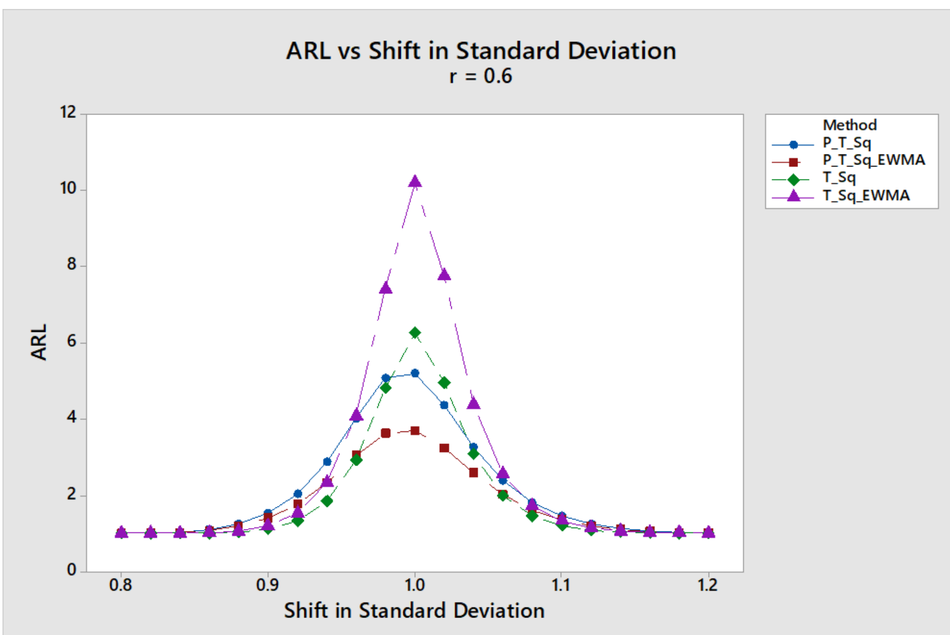

ARL, LCI_ARL, UCI_ARL vs Shift in Mean
r = 0.1 (Distribution = Normal, Runs = 50,000)
ARL
20
15
10
5
0
-0.2
-0.1
0.0
0.1
0.2
Shift in Mean
Variable Method
ARL P_T_Sq
ARL P_T_Sq_EWMA
ARL T_Sq
ARL T_Sq_EWMA
LCI_ARL P_T_Sq
LCI_ARL P_T_Sq_EWMA
LCI_ARL T_Sq
LCI_ARL T_Sq_EWMA
UCI_ARL P_T_Sq
UCI_ARL P_T_Sq_EWMA
UCI_ARL T_Sq
UCI_ARL T_Sq_EWMA

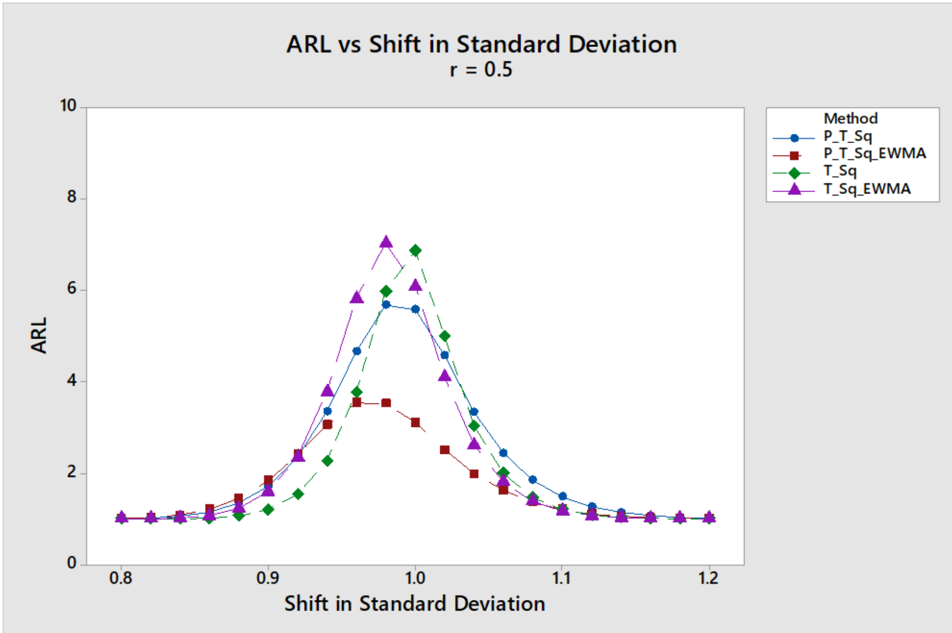
Appendix C: Comparison of ARL for Detecting Shift in Standard Deviation, Normal Distribution, 50,000 runs for Different Values of *r*

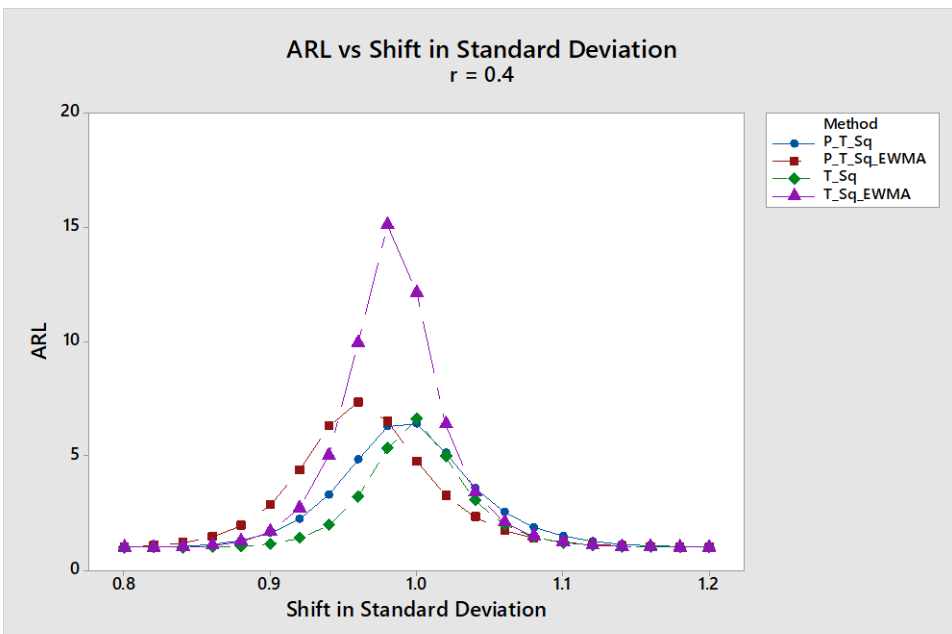


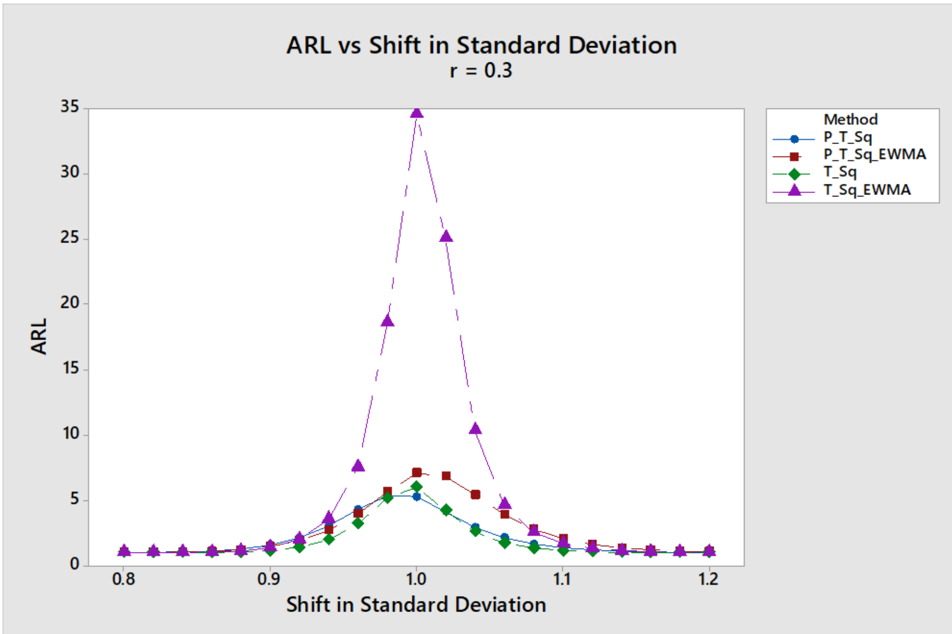


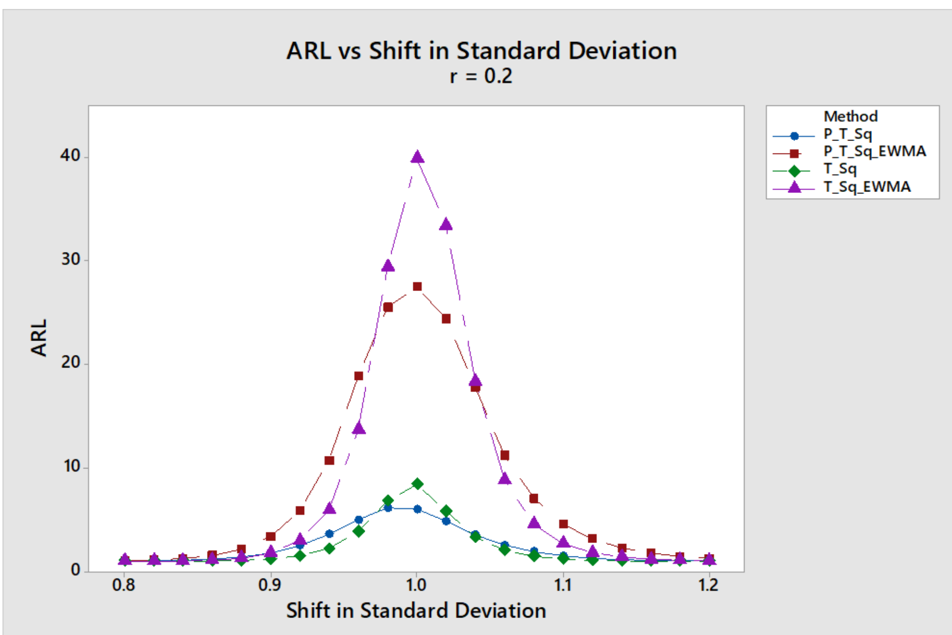

ARL vs Shift in Standard Deviation
r = 0.7
ARL
Shift in Standard Deviation
0
2
4
6
8
10
0.8
0.9
1.0
1.1
1.2
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA

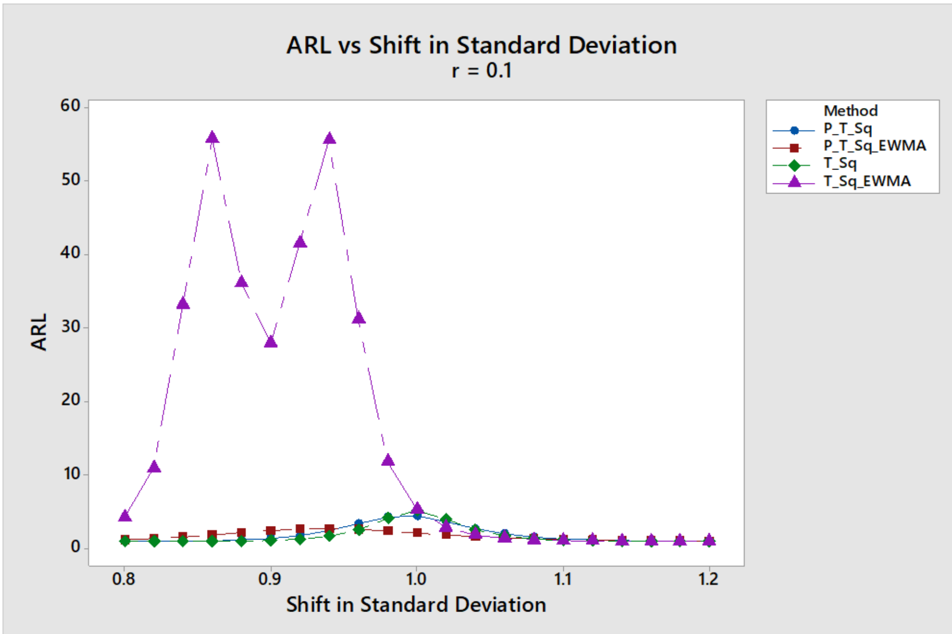

ARL vs Shift in Standard Deviation
r = 0.6
ARL
Shift in Standard Deviation
0
2
4
6
8
10
12
0.8
0.9
1.0
1.1
1.2
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA


ARL vs Shift in Standard Deviation
r = 0.5
ARL
Shift in Standard Deviation
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA

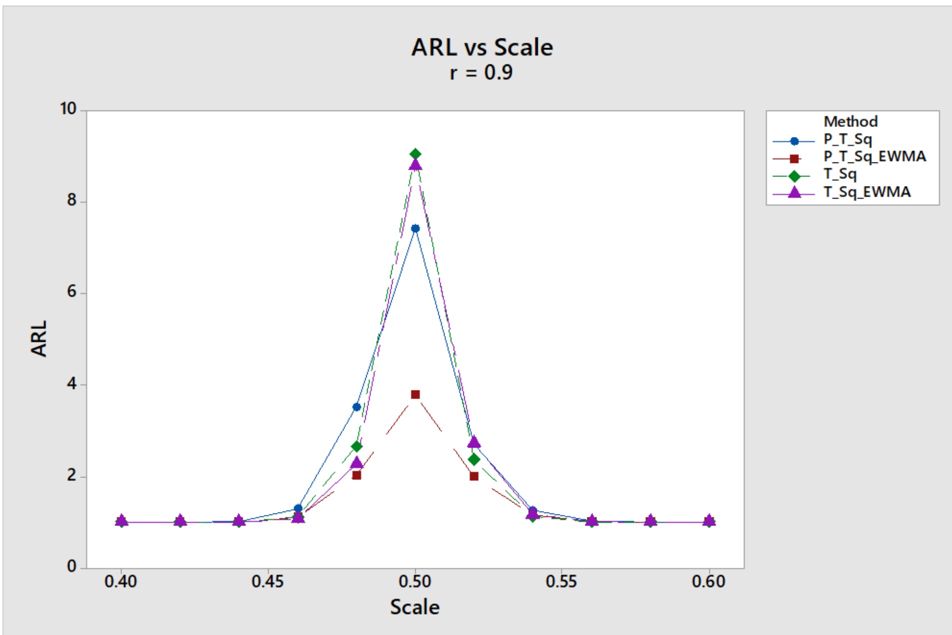

ARL vs Shift in Standard Deviation
r = 0.4
ARL
Shift in Standard Deviation
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA

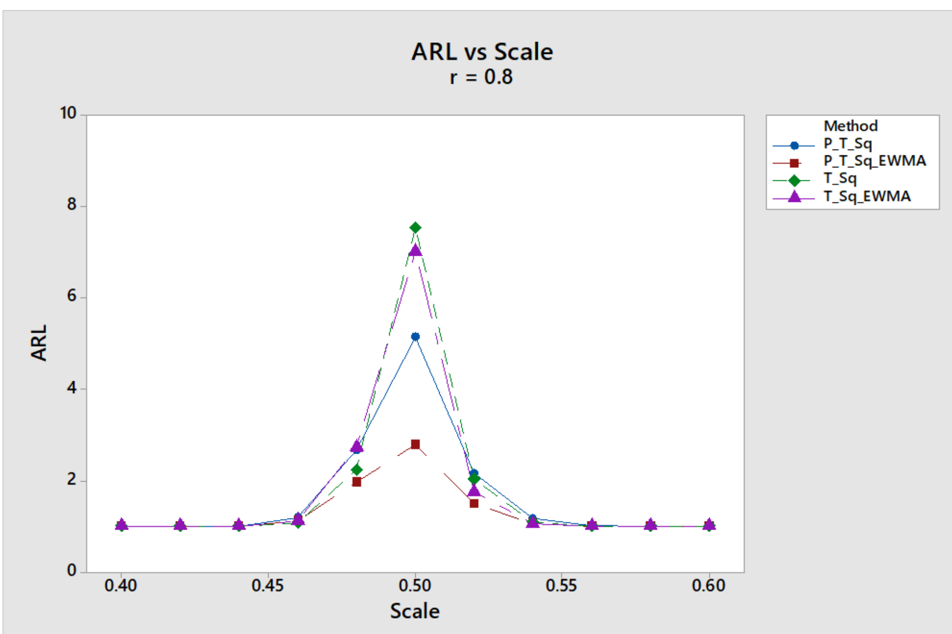

ARL vs Shift in Standard Deviation
r = 0.3
ARL
Shift in Standard Deviation
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA
0
5
10
15
20
25
30
35
0.8
0.9
1.0
1.1
1.2

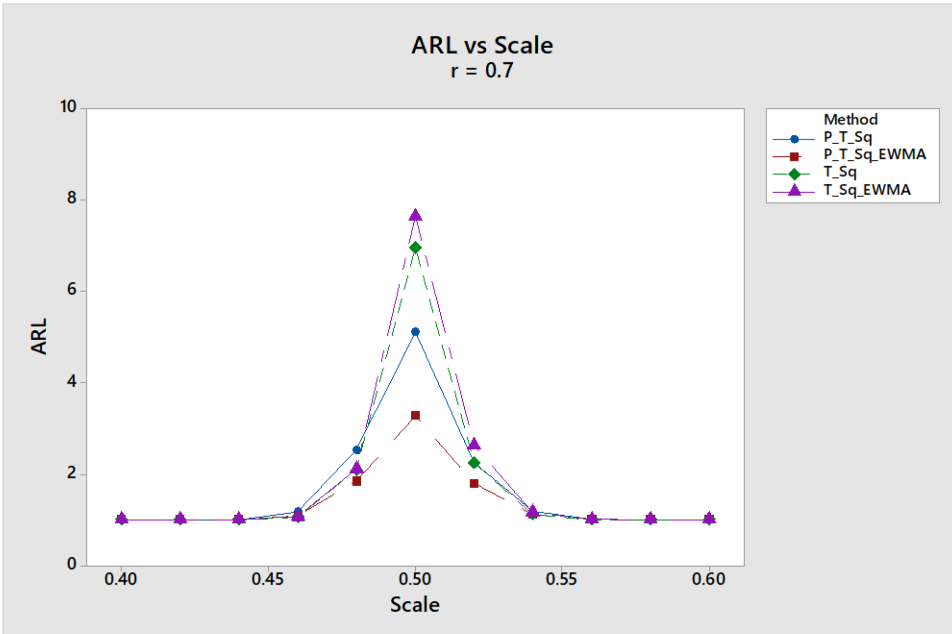

ARL vs Shift in Standard Deviation
r = 0.2
ARL
Shift in Standard Deviation
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA
0
10
20
30
40
0.8
0.9
1.0
1.1
1.2

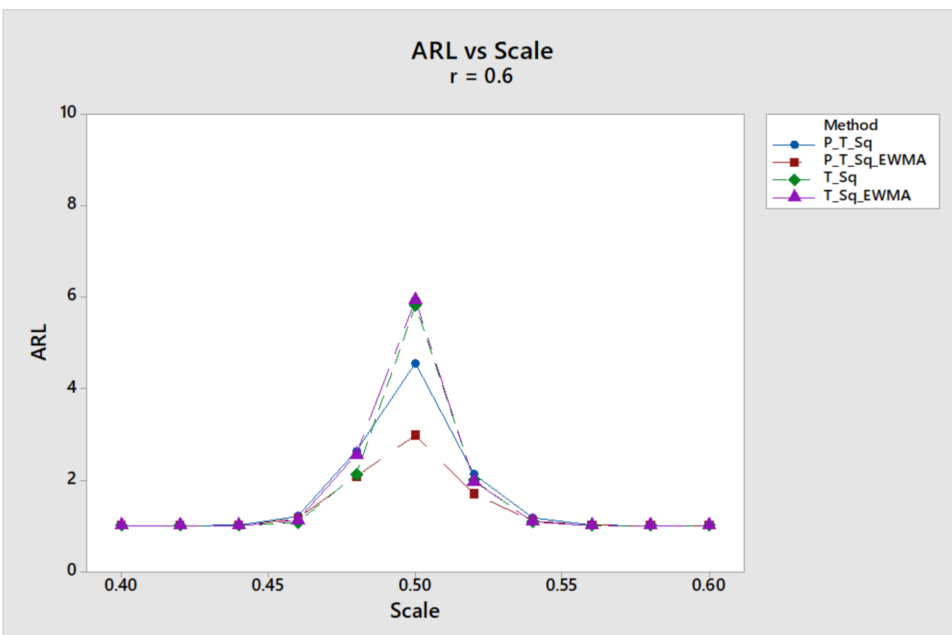

ARL vs Shift in Standard Deviation
r = 0.1
ARL
Shift in Standard Deviation
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA
60
50
40
30
20
10
0
0.8
0.9
1.0
1.1
1.2

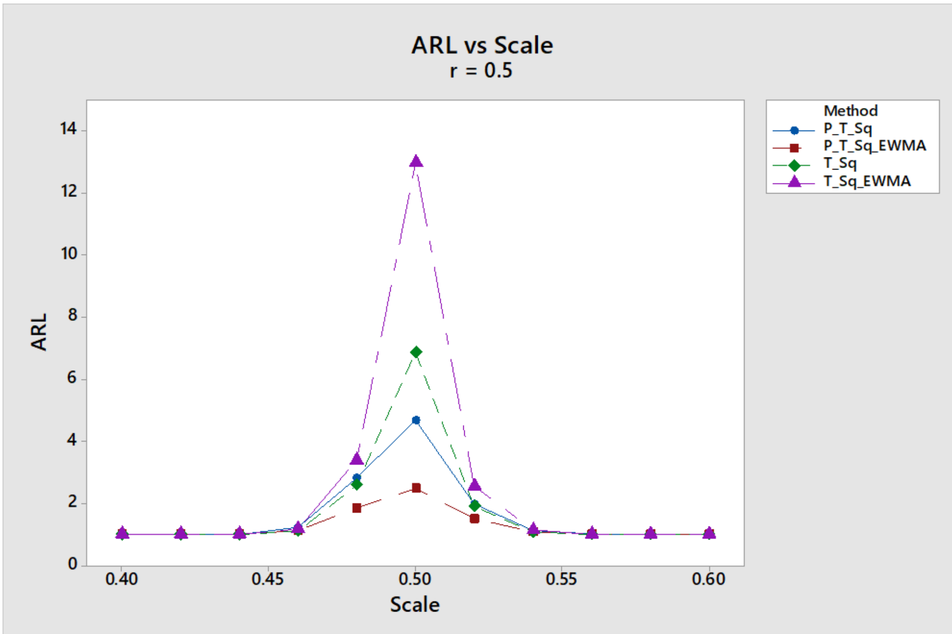
Appendix D: Comparison of ARL for Detecting Shift in Scale Parameter, Weibull Distribution, 50,000 runs for Different values of *r*

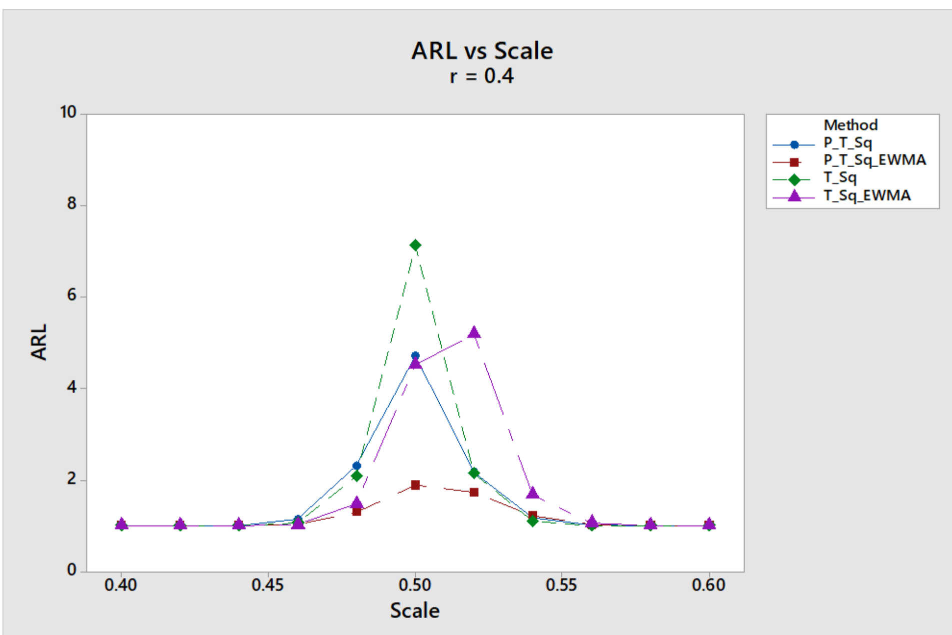


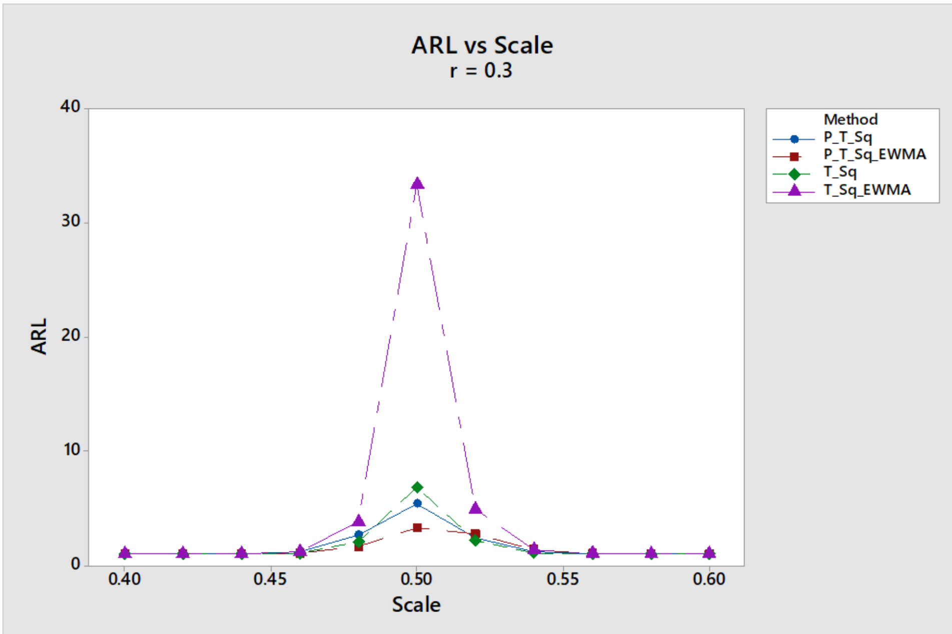


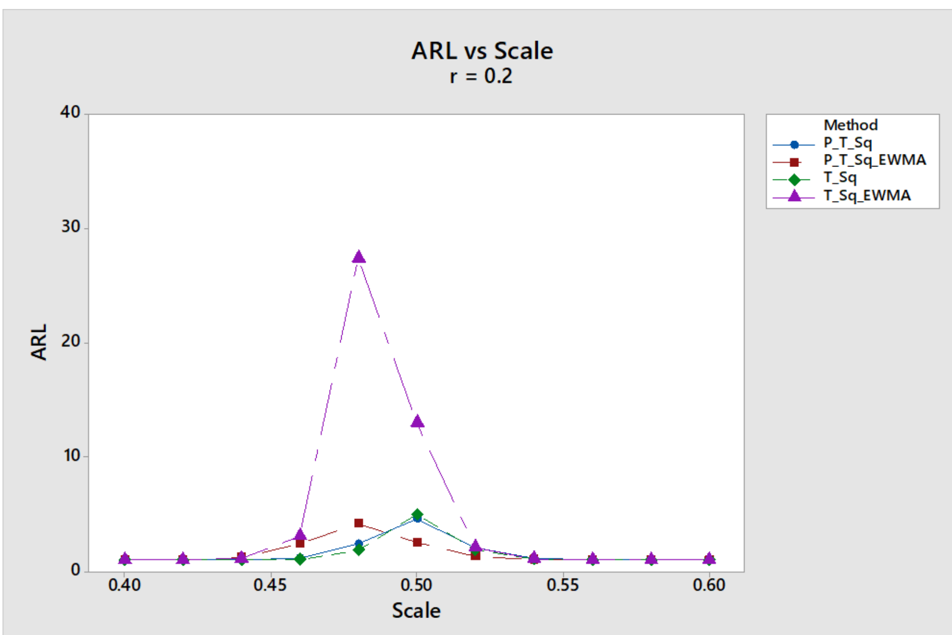

ARL vs Scale
r = 0.7
ARL
Scale
0
2
4
6
8
10
0.40
0.45
0.50
0.55
0.60
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA

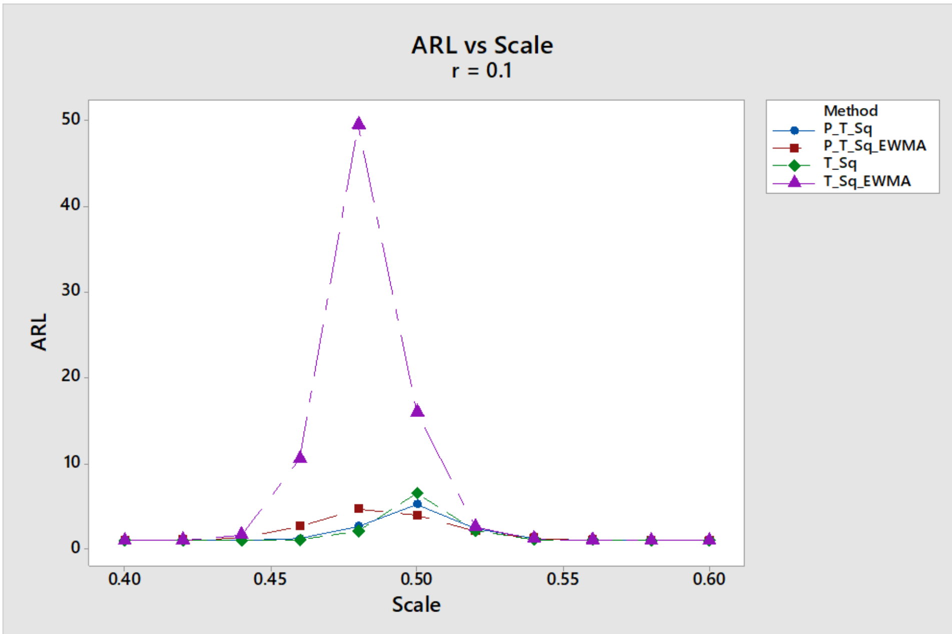

ARL vs Scale
r = 0.6
ARL
Scale
0
2
4
6
8
10
0.40
0.45
0.50
0.55
0.60
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA


ARL vs Scale
r = 0.5
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA
ARL
Scale
0
2
4
6
8
10
12
14
0.40
0.45
0.50
0.55
0.60

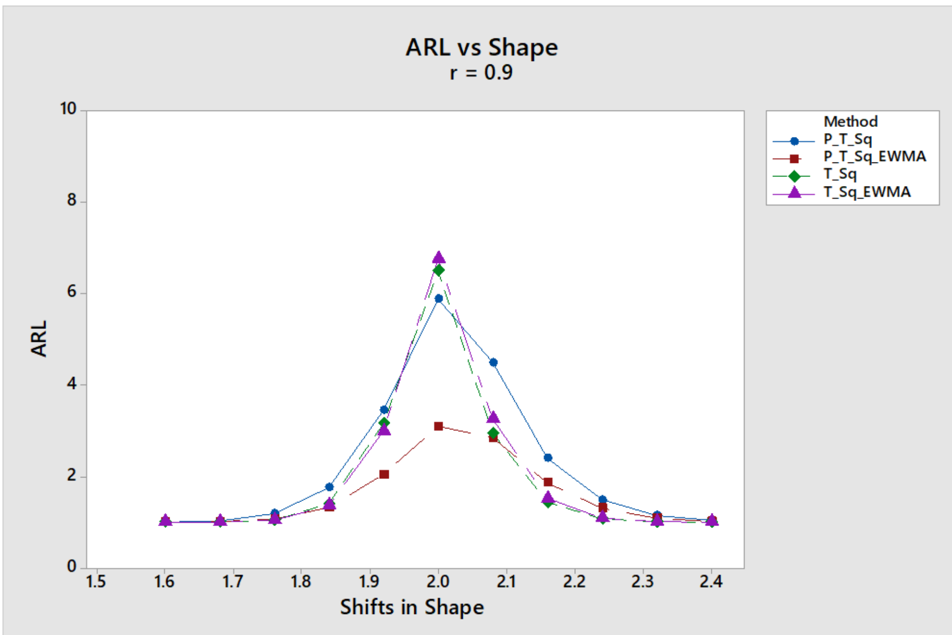

ARL vs Scale
r = 0.4
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA
ARL
Scale
0
2
4
6
8
10
0.40
0.45
0.50
0.55
0.60

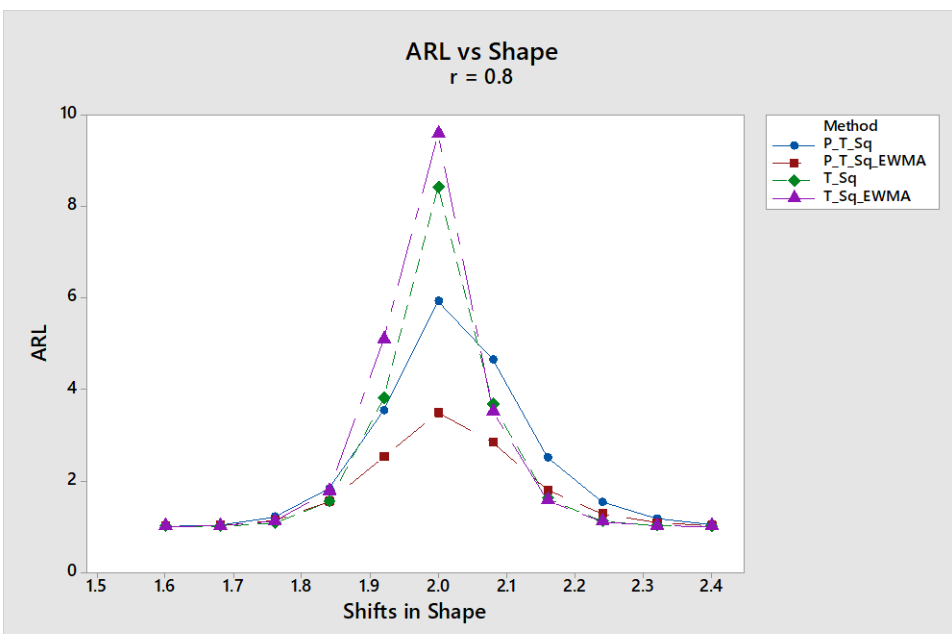

ARL vs Scale
r = 0.3
40
30
20
10
0
ARL
0.40
0.45
0.50
0.55
0.60
Scale
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA

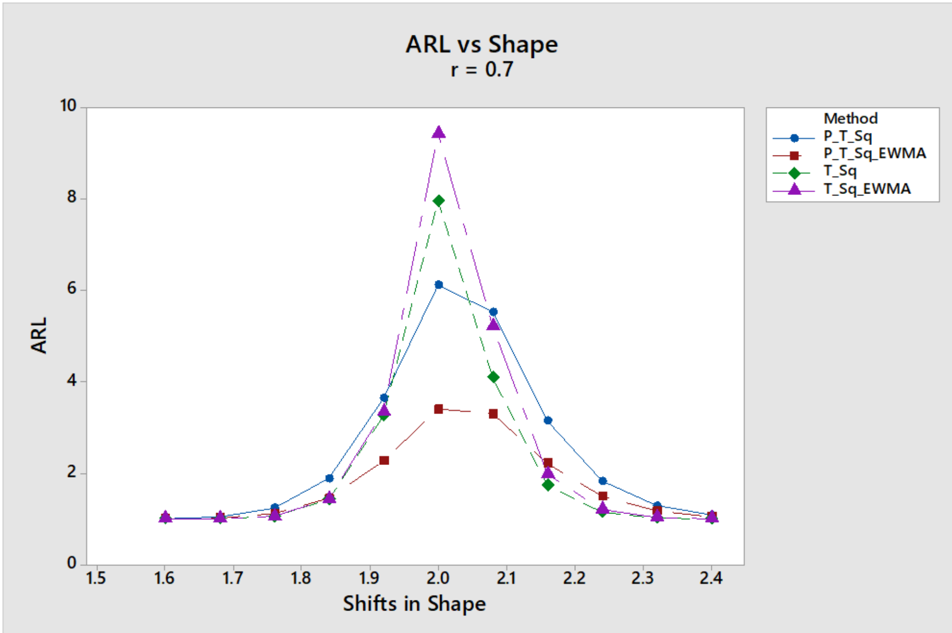

ARL vs Scale
r = 0.2
40
30
20
10
0
ARL
0.40
0.45
0.50
0.55
0.60
Scale
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA

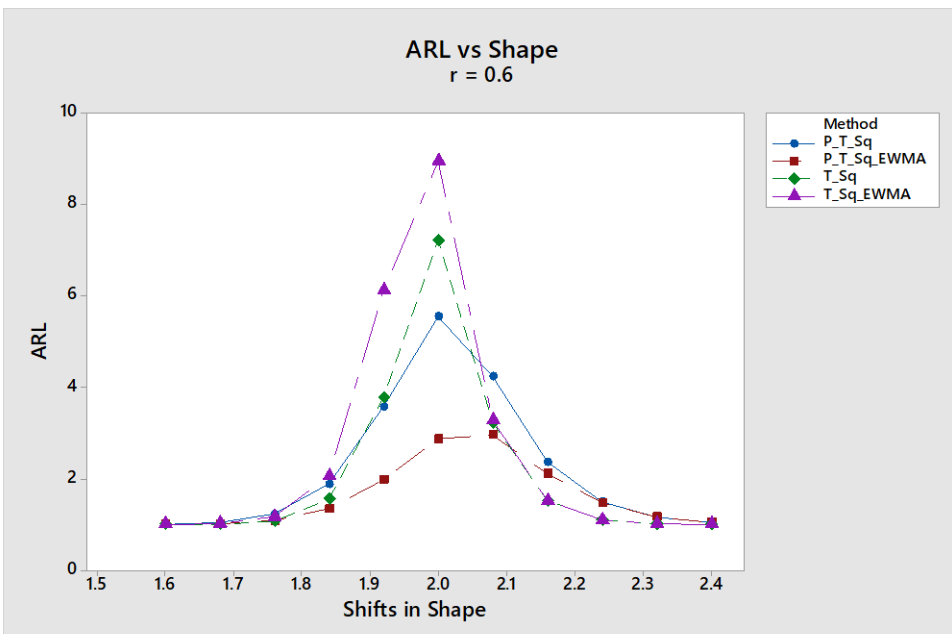

ARL vs Scale
r = 0.1
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA
ARL
50
40
30
20
10
0
0.40
0.45
0.50
0.55
0.60
Scale

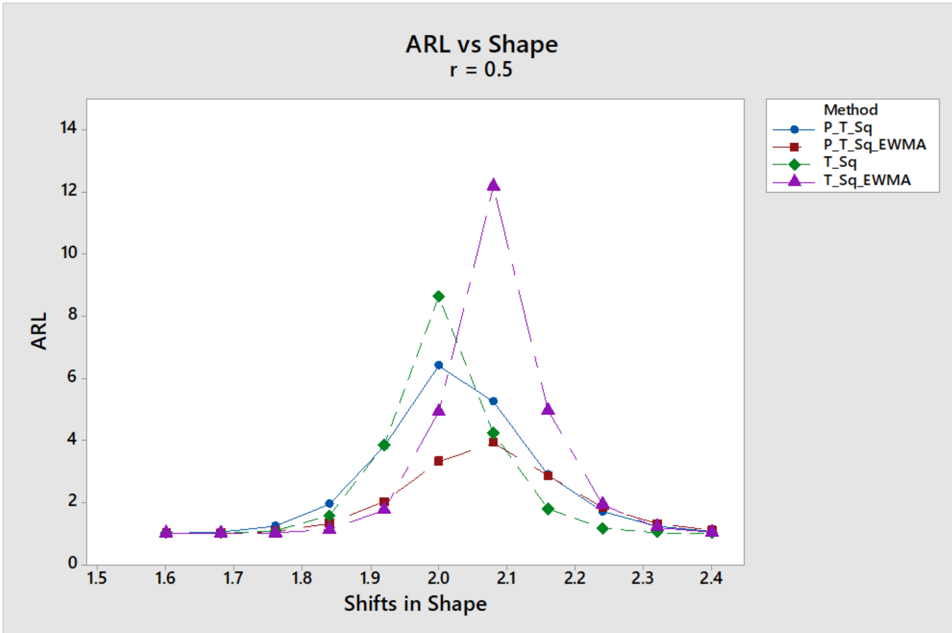
Appendix E: Comparison of ARL for Detecting Shift in Shape Parameter, Weibull Distribution, 50,000 runs for Different Values of *r*

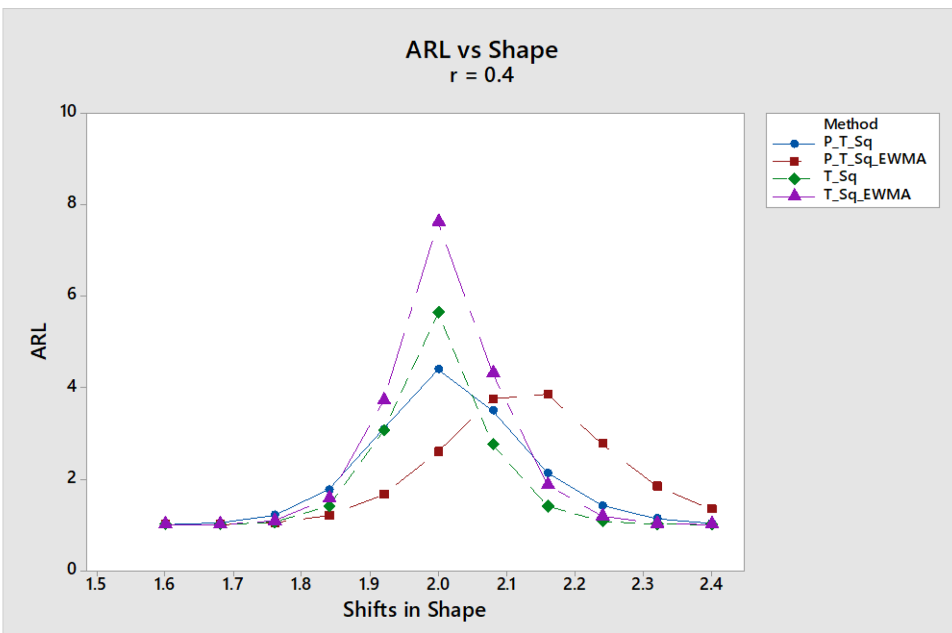


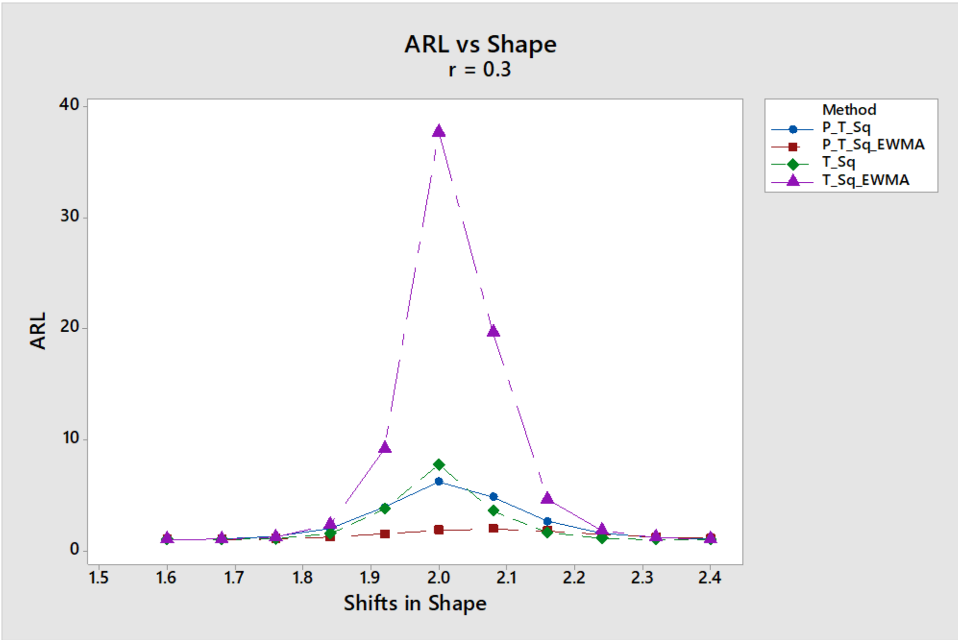


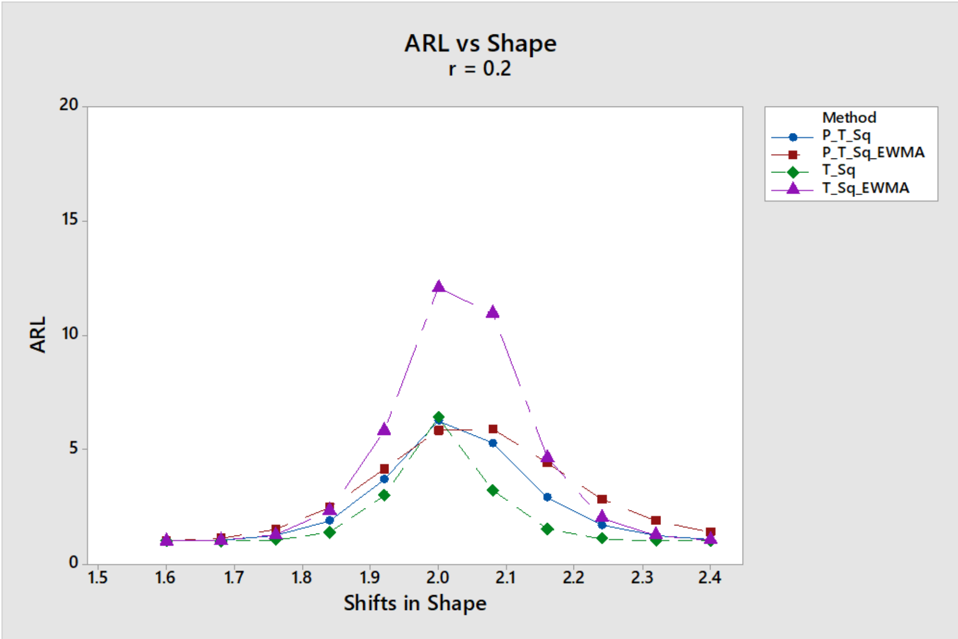

ARL vs Shape
r = 0.7
ARL
Shifts in Shape
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA

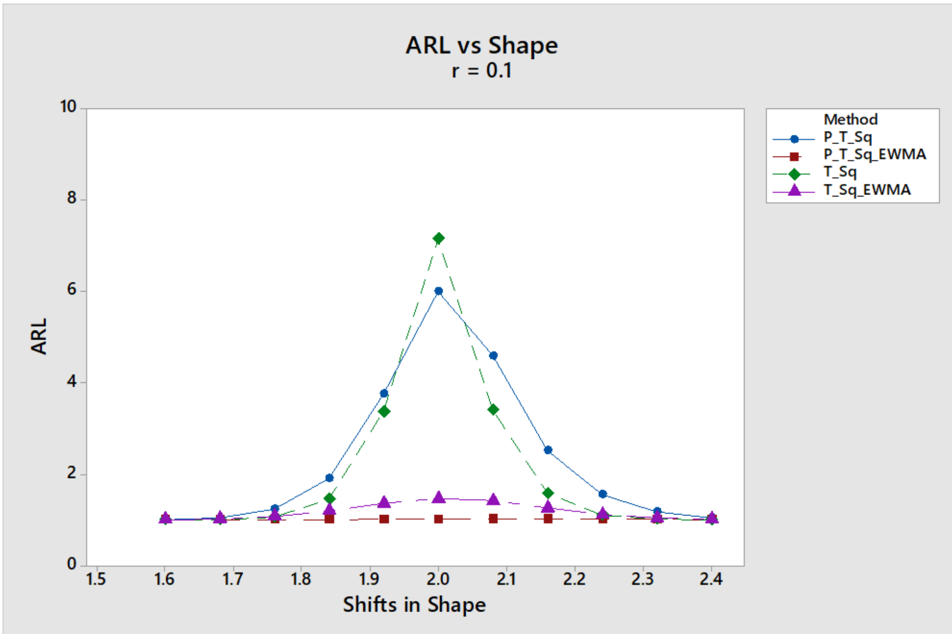

ARL vs Shape
r = 0.6
ARL
Shifts in Shape
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA


ARL vs Shape
r = 0.5
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA
ARL
0
2
4
6
8
10
12
14
1.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3
2.4
Shifts in Shape


ARL vs Shape
r = 0.4
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA
ARL
0
2
4
6
8
10
1.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3
2.4
Shifts in Shape


ARL vs Shape
r = 0.3
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA
ARL
0
10
20
30
40
1.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3
2.4
Shifts in Shape


ARL vs Shape
r = 0.2
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA
ARL
0
5
10
15
20
1.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3
2.4
Shifts in Shape


ARL vs Shape
r = 0.1
ARL
Shifts in Shape
Method
P_T_Sq
P_T_Sq_EWMA
T_Sq
T_Sq_EWMA
0
2
4
6
8
10
1.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3
2.4

Appendix F: Results

Appendix F.1 ARL for Shift in Mean, Normal Distribution, 50,000 runs

r = 0.1, Shift in Mean = - 0.2

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.0314	0.1792	1.0298	1.0330
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0146	0.1219	1.0136	1.0157
$T_j{}^2$ *(T_Sq)*	1.0116	0.1091	1.0106	1.0125
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0205	0.1463	1.0193	1.0218

r = 0.1, Shift in Mean = - 0.18

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.0678	0.2686	1.0655	1.0702
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0286	0.1716	1.0271	1.0301
$T_j{}^2$ *(T_Sq)*	1.0315	0.1807	1.0299	1.0331
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0454	0.2159	1.0435	1.0473

r = 0.1, Shift in Mean = - 0.16

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.1412	0.4022	1.1377	1.1447
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0516	0.2314	1.0496	1.0536
$T_j{}^2$ *(T_Sq)*	1.0761	0.2861	1.0736	1.0786
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0920	0.3157	1.0892	1.0947

r = 0.1, Shift in Mean = - 0.14

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.2574	0.5696	1.2524	1.2624
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0894	0.3112	1.0867	1.0921
${T_j}^2$ *(T_Sq)*	1.1532	0.4215	1.1495	1.1569
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.1726	0.4467	1.1686	1.1765

r = 0.1, Shift in Mean = - 0.12

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.4458	0.8035	1.4388	1.4528
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.1405	0.3980	1.1370	1.1440
${T_j}^2$ *(T_Sq)*	1.2987	0.6237	1.2933	1.3042
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.3019	0.6271	1.2964	1.3074

r = 0.1, Shift in Mean = - 0.10

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.7367	1.1363	1.7268	1.7467
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.2176	0.5140	1.2131	1.2221
${T_j}^2$ *(T_Sq)*	1.5413	0.9155	1.5333	1.5493
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.4942	0.8511	1.4867	1.5016

r = 0.1, Shift in Mean = - 0.08

Method	**ARL**	**SDRL**	**LCL**	**UCL**
$T_{p_j}^2$, (P_T_Sq)	2.1817	1.6023	2.1677	2.1958
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.3224	0.6524	1.3167	1.3281
$T_j{}^2$ *(T_Sq)*	1.9696	1.3872	1.9574	1.9817
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.8131	1.2139	1.8024	1.8237

r = 0.1, Shift in Mean = - 0.06

Method	**ARL**	**SDRL**	**LCL**	**UCL**
$T_{p_j}^2$, (P_T_Sq)	2.8100	2.2450	2.7903	2.8297
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.4675	0.8334	1.4602	1.4748
$T_j{}^2$ *(T_Sq)*	2.6565	2.0889	2.6382	2.6748
$T_{ewma_j}^2$ (T_Sq_EWMA)	2.2925	1.7354	2.2773	2.3078

r = 0.1, Shift in Mean = - 0.04

Method	**ARL**	**SDRL**	**LCL**	**UCL**
$T_{p_j}^2$, (P_T_Sq)	3.5605	3.0231	3.5340	3.5870
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.6509	1.0409	1.6418	1.6601
$T_j{}^2$ *(T_Sq)*	3.6340	3.0947	3.6069	3.6611
$T_{ewma_j}^2$ (T_Sq_EWMA)	2.9953	2.4465	2.9738	3.0167

r = 0.1, Shift in Mean = - 0.02

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	4.1988	3.6792	4.1666	4.2311
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.8570	1.2730	1.8458	1.8681
$T_j{}^2$ *(T_Sq)*	4.7259	4.2033	4.6890	4.7627
$T_{ewma_j}^2$ (T_Sq_EWMA)	3.9594	3.4301	3.9293	3.9894

r = 0.1, Shift in Mean = 0

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	4.4468	3.9260	4.4124	4.4812
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	2.1360	1.5614	2.1223	2.1497
$T_j{}^2$ *(T_Sq)*	5.2039	4.7014	5.1627	5.2451
$T_{ewma_j}^2$ (T_Sq_EWMA)	5.3828	4.8550	5.3402	5.4253

r = 0.1, Shift in Mean = 0.02

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	4.0808	3.5414	4.0497	4.1118
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	2.4536	1.8890	2.4371	2.4702
$T_j{}^2$ *(T_Sq)*	4.5617	4.0412	4.5263	4.5971
$T_{ewma_j}^2$ (T_Sq_EWMA)	7.2647	6.7897	7.2052	7.3242

r = 0.1, Shift in Mean = 0.04

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	3.3245	2.7637	3.3003	3.3488
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	2.8044	2.2821	2.7844	2.8244
${T_j}^2$ *(T_Sq)*	3.4470	2.8944	3.4216	3.4724
$T^2_{ewma_j}$ (T_Sq_EWMA)	9.6622	9.1019	9.5824	9.7420

r = 0.1, Shift in Mean = 0.06

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	2.6071	2.0510	2.5892	2.6251
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	3.1800	2.6608	3.1567	3.2033
${T_j}^2$ *(T_Sq)*	2.4993	1.9286	2.4824	2.5162
$T^2_{ewma_j}$ (T_Sq_EWMA)	12.3890	11.9503	12.2843	12.4938

r = 0.1, Shift in Mean = 0.08

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	2.0532	1.4688	2.0403	2.0661
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	3.5492	3.0731	3.5223	3.5761
${T_j}^2$ *(T_Sq)*	1.8978	1.3019	1.8864	1.9092
$T^2_{ewma_j}$ (T_Sq_EWMA)	15.3162	15.0683	15.1842	15.4483

r = 0.1, Shift in Mean = 0.10

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.6439	1.0323	1.6349	1.6530
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	3.8412	3.3660	3.8117	3.8707
${T_j}^2$ *(T_Sq)*	1.5030	0.8689	1.4954	1.5107
$T^2_{ewma_j}$ (T_Sq_EWMA)	17.6692	17.3295	17.5173	17.8211

r = 0.1, Shift in Mean = 0.12

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.3801	0.7287	1.3738	1.3865
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	4.0755	3.5962	4.0440	4.1070
${T_j}^2$ *(T_Sq)*	1.2700	0.5931	1.2648	1.2752
$T^2_{ewma_j}$ (T_Sq_EWMA)	19.0253	18.6144	18.8621	19.1884

r = 0.1, Shift in Mean = 0.14

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.2181	0.5154	1.2136	1.2226
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	4.1535	3.6741	4.1213	4.1857
${T_j}^2$ *(T_Sq)*	1.1407	0.3978	1.1372	1.1442
$T^2_{ewma_j}$ (T_Sq_EWMA)	18.9613	18.4670	18.7995	19.1232

r = 0.1, Shift in Mean = 0.16

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.1125	0.3533	1.1094	1.1156
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	4.0791	3.6037	4.0476	4.1107
${T_j}^2$ *(T_Sq)*	1.0637	0.2605	1.0615	1.0660
$T^2_{ewma_j}$ (T_Sq_EWMA)	17.4281	17.1052	17.2782	17.5781

r = 0.1, Shift in Mean = 0.18

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.0583	0.2498	1.0561	1.0604
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	3.8139	3.3408	3.7846	3.8432
${T_j}^2$ *(T_Sq)*	1.0283	0.1708	1.0268	1.0298
$T^2_{ewma_j}$ (T_Sq_EWMA)	14.8082	14.4030	14.6820	14.9345

r = 0.1, Shift in Mean = 0.20

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.0255	0.1608	1.0241	1.0269
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	3.4503	2.9686	3.4243	3.4763
${T_j}^2$ *(T_Sq)*	1.0102	0.1023	1.0093	1.0111
$T^2_{ewma_j}$ (T_Sq_EWMA)	11.7084	11.2922	11.6094	11.8073

r = 0.9, Shift in Mean = - 0.2

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.0379	0.1986	1.0362	1.0397
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0187	0.1381	1.0175	1.0200
${T_j}^2$ *(T_Sq)*	1.0147	0.1214	1.0137	1.0158
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0204	0.1437	1.0192	1.0217

r = 0.9, Shift in Mean = - 0.18

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.0815	0.2954	1.0789	1.0841
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0417	0.2085	1.0399	1.0435
${T_j}^2$ *(T_Sq)*	1.0372	0.1946	1.0355	1.0389
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0505	0.2289	1.0485	1.0525

r = 0.9, Shift in Mean = - 0.16

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.1623	0.4332	1.1585	1.1661
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0870	0.3101	1.0842	1.0897
${T_j}^2$ *(T_Sq)*	1.0841	0.3034	1.0815	1.0868
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.1088	0.3485	1.1058	1.1119

r = 0.9, Shift in Mean = - 0.14

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.2984	0.6211	1.2930	1.3039
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.1661	0.4371	1.1622	1.1699
$T_j{}^2$ *(T_Sq)*	1.1774	0.4549	1.1734	1.1813
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.2267	0.5249	1.2221	1.2313

r = 0.9, Shift in Mean = - 0.12

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.5179	0.8862	1.5101	1.5257
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.2931	0.6168	1.2877	1.2985
$T_j{}^2$ *(T_Sq)*	1.3529	0.6908	1.3468	1.3589
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.4408	0.7954	1.4338	1.4478

r = 0.9, Shift in Mean = - 0.10

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.8541	1.2593	1.8430	1.8651
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.4791	0.8457	1.4717	1.4866
$T_j{}^2$ *(T_Sq)*	1.6451	1.0240	1.6361	1.6541
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.8024	1.2036	1.7918	1.8129

r = 0.9, Shift in Mean = - 0.08

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	2.3660	1.7996	2.3502	2.3818
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.7630	1.1597	1.7529	1.7732
$T_j^{\,2}$ *(T_Sq)*	2.1592	1.5827	2.1453	2.1731
$T^2_{ewma_j}$ (T_Sq_EWMA)	2.4343	1.8805	2.4178	2.4508

r = 0.9, Shift in Mean = - 0.06

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	3.1298	2.5891	3.1071	3.1525
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	2.1250	1.5628	2.1113	2.1387
$T_j^{\,2}$ *(T_Sq)*	3.0549	2.5186	3.0328	3.0769
$T^2_{ewma_j}$ (T_Sq_EWMA)	3.5090	2.9735	3.4829	3.5351

r = 0.9, Shift in Mean = - 0.04

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	4.0461	3.5026	4.0154	4.0768
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	2.5289	1.9907	2.5115	2.5463
$T_j^{\,2}$ *(T_Sq)*	4.5237	3.9946	4.4886	4.5587
$T^2_{ewma_j}$ (T_Sq_EWMA)	5.2083	4.6910	5.1672	5.2494

r = 0.9, Shift in Mean = - 0.02

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	4.9122	4.3972	4.8736	4.9507
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	2.8330	2.2811	2.8130	2.8530
$T_j{}^2$ *(T_Sq)*	6.3393	5.8558	6.2879	6.3906
$T^2_{ewma_j}$ (T_Sq_EWMA)	6.9391	6.4695	6.8824	6.9958

r = 0.9, Shift in Mean = 0

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	5.2611	4.7683	5.2193	5.3029
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	2.8820	2.3409	2.8615	2.9026
$T_j{}^2$ *(T_Sq)*	7.3848	6.8754	7.3245	7.4450
$T^2_{ewma_j}$ (T_Sq_EWMA)	7.2761	6.7476	7.2169	7.3352

r = 0.9, Shift in Mean = 0.02

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	4.8467	4.3009	4.8090	4.8844
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	2.6651	2.1171	2.6465	2.6837
$T_j{}^2$ *(T_Sq)*	6.3297	5.7927	6.2789	6.3805
$T^2_{ewma_j}$ (T_Sq_EWMA)	5.7479	5.2286	5.7020	5.7937

r = 0.9, Shift in Mean = 0.04

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	3.9301	3.3985	3.9003	3.9599
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	2.2761	1.7098	2.2612	2.2911
$T_j{}^2$ *(T_Sq)*	4.4324	3.9103	4.3981	4.4666
$T_{ewma_j}^2$ (T_Sq_EWMA)	3.9064	3.3731	3.8768	3.9360

r = 0.9, Shift in Mean = 0.06

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	3.0330	2.4689	3.0113	3.0546
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.8913	1.3024	1.8799	1.9028
$T_j{}^2$ *(T_Sq)*	2.9948	2.4456	2.9733	3.0162
$T_{ewma_j}^2$ (T_Sq_EWMA)	2.6682	2.1090	2.6497	2.6867

r = 0.9, Shift in Mean = 0.08

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	2.3072	1.7405	2.2920	2.3225
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.5796	0.9608	1.5711	1.5880
$T_j{}^2$ *(T_Sq)*	2.1367	1.5631	2.1230	2.1504
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.9386	1.3507	1.9267	1.9504

r = 0.9, Shift in Mean = 0.10

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.8078	1.2079	1.7972	1.8184
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.3533	0.6852	1.3473	1.3593
$T_j{}^2$ *(T_Sq)*	1.6256	1.0067	1.6168	1.6344
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.5183	0.8867	1.5105	1.5261

r = 0.9, Shift in Mean = 0.12

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.4926	0.8598	1.4851	1.5002
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.2073	0.4985	1.2030	1.2117
$T_j{}^2$ *(T_Sq)*	1.3382	0.6728	1.3323	1.3441
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.2758	0.5924	1.2706	1.2810

r = 0.9, Shift in Mean = 0.14

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.2812	0.6033	1.2760	1.2865
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.1131	0.3550	1.1100	1.1162
$T_j{}^2$ *(T_Sq)*	1.1754	0.4568	1.1714	1.1794
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.1403	0.4021	1.1367	1.1438

r = 0.9, Shift in Mean = 0.16

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.1544	0.4216	1.1507	1.1581
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.0577	0.2451	1.0555	1.0598
${T_j}^2$ *(T_Sq)*	1.0846	0.3014	1.0819	1.0872
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.0668	0.2665	1.0644	1.0691

r = 0.9, Shift in Mean = 0.18

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.0778	0.2900	1.0753	1.0803
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.0257	0.1616	1.0243	1.0272
${T_j}^2$ *(T_Sq)*	1.0361	0.1930	1.0344	1.0378
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.0274	0.1683	1.0259	1.0289

r = 0.9, Shift in Mean = 0.20

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.0367	0.1945	1.0350	1.0384
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.0107	0.1034	1.0098	1.0116
${T_j}^2$ *(T_Sq)*	1.0137	0.1180	1.0127	1.0148
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.0099	0.1002	1.0090	1.0108

Appendix F.2 ARL for Shift in Standard Deviation, Normal Distribution, 50,000 runs

r = 0.1, Shift in Standard Deviation = - 0.2

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.0005	0.0219	1.0003	1.0007
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.1849	0.4677	1.1808	1.1890
${T_j}^2$ *(T_Sq)*	1.0000	0.0000	1.0000	1.0000
$T_{ewma_j}^2$ (T_Sq_EWMA)	4.2281	3.7085	4.1956	4.2606

r = 0.1, Shift in Standard Deviation = - 0.18

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.0043	0.0654	1.0037	1.0048
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.3464	0.6869	1.3404	1.3525
${T_j}^2$ *(T_Sq)*	1.0001	0.0077	1.0000	1.0001
$T_{ewma_j}^2$ (T_Sq_EWMA)	10.9768	10.5579	10.8843	11.0694

r = 0.1, Shift in Standard Deviation = - 0.16

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.0185	0.1375	1.0173	1.0197
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.5671	0.9443	1.5588	1.5754
${T_j}^2$ *(T_Sq)*	1.0003	0.0173	1.0001	1.0005
$T_{ewma_j}^2$ (T_Sq_EWMA)	33.2947	31.9318	33.0148	33.5745

r = 0.1, Shift in Standard Deviation = - 0.14

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.0633	0.2591	1.0610	1.0656
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.8629	1.2678	1.8518	1.8740
${T_j}^2$ *(T_Sq)*	1.0037	0.0604	1.0031	1.0042
$T^2_{ewma_j}$ (T_Sq_EWMA)	55.8010	53.5902	55.3313	56.2707

r = 0.1, Shift in Standard Deviation = - 0.12

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.1694	0.4440	1.1655	1.1733
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	2.1664	1.6012	2.1524	2.1805
${T_j}^2$ *(T_Sq)*	1.0186	0.1385	1.0174	1.0199
$T^2_{ewma_j}$ (T_Sq_EWMA)	36.1758	54.7271	35.6961	36.6555

r = 0.1, Shift in Standard Deviation = - 0.10

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.3884	0.7332	1.3819	1.3948
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	2.4593	1.9338	2.4424	2.4763
${T_j}^2$ *(T_Sq)*	1.0810	0.2953	1.0784	1.0836
$T^2_{ewma_j}$ (T_Sq_EWMA)	27.9866	51.2993	27.5370	28.4363

r = 0.1, Shift in Standard Deviation = - 0.08

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.7807	1.1803	1.7704	1.7911
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	2.6599	2.1277	2.6413	2.6786
${T_j}^2$ *(T_Sq)*	1.2544	0.5630	1.2494	1.2593
$T^2_{ewma_j}$ (T_Sq_EWMA)	41.5965	56.4779	41.1015	42.0916

r = 0.1, Shift in Standard Deviation = - 0.06

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	2.4689	1.8943	2.4523	2.4855
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	2.7086	2.1771	2.6895	2.7277
${T_j}^2$ *(T_Sq)*	1.6630	1.0429	1.6539	1.6722
$T^2_{ewma_j}$ (T_Sq_EWMA)	55.6301	51.3466	55.1800	56.0802

r = 0.1, Shift in Standard Deviation = - 0.04

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	3.4209	2.8740	3.3957	3.4461
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	2.6152	2.0814	2.5970	2.6334
${T_j}^2$ *(T_Sq)*	2.5495	1.9770	2.5322	2.5668
$T^2_{ewma_j}$ (T_Sq_EWMA)	31.2938	30.2591	31.0286	31.5590

r = 0.1, Shift in Standard Deviation = - 0.02

Method	**ARL**	**SDRL**	**LCL**	**UCL**
$T_{p_j}^2$, (P_T_Sq)	4.2941	3.7420	4.2613	4.3269
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	2.4015	1.8665	2.3852	2.4179
$T_j{}^2$ *(T_Sq)*	4.1097	3.5720	4.0784	4.1410
$T_{ewma_j}^2$ (T_Sq_EWMA)	11.8314	11.4935	11.7306	11.9321

r = 0.1, Shift in Standard Deviation = 0

Method	**ARL**	**SDRL**	**LCL**	**UCL**
$T_{p_j}^2$, (P_T_Sq)	4.4363	3.8922	4.4022	4.4704
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	2.1360	1.5660	2.1222	2.1497
$T_j{}^2$ *(T_Sq)*	5.1687	4.6280	5.1281	5.2093
$T_{ewma_j}^2$ (T_Sq_EWMA)	5.3638	4.8525	5.3212	5.4063

r = 0.1, Shift in Standard Deviation = 0.02

Method	**ARL**	**SDRL**	**LCL**	**UCL**
$T_{p_j}^2$, (P_T_Sq)	3.6509	3.0966	3.6237	3.6780
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.8717	1.2760	1.8605	1.8828
$T_j{}^2$ *(T_Sq)*	3.9823	3.4422	3.9521	4.0125
$T_{ewma_j}^2$ (T_Sq_EWMA)	2.9050	2.3480	2.8844	2.9255

r = 0.1, Shift in Standard Deviation = 0.04

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	2.7250	2.1656	2.7060	2.7440
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.6469	1.0302	1.6379	1.6559
$T_j{}^2$ *(T_Sq)*	2.5206	1.9566	2.5035	2.5378
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.8725	1.2851	1.8612	1.8837

r = 0.1, Shift in Standard Deviation = 0.06

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	2.0244	1.4411	2.0117	2.0370
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.4543	0.8122	1.4472	1.4614
$T_j{}^2$ *(T_Sq)*	1.7161	1.1114	1.7064	1.7258
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.4001	0.7535	1.3935	1.4067

r = 0.1, Shift in Standard Deviation = 0.08

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.5735	0.9471	1.5652	1.5818
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.3107	0.6394	1.3051	1.3163
$T_j{}^2$ *(T_Sq)*	1.3167	0.6457	1.3111	1.3224
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.1727	0.4497	1.1688	1.1766

r = 0.1, Shift in Standard Deviation = 0.10

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.3079	0.6343	1.3024	1.3135
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.1964	0.4854	1.1922	1.2007
${T_j}^2$ *(T_Sq)*	1.1297	0.3828	1.1263	1.1331
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0687	0.2717	1.0663	1.0711

r = 0.1, Shift in Standard Deviation = 0.12

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.1655	0.4400	1.1617	1.1694
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.1234	0.3737	1.1201	1.1267
${T_j}^2$ *(T_Sq)*	1.0496	0.2282	1.0476	1.0516
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0245	0.1599	1.0231	1.0259

r = 0.1, Shift in Standard Deviation = 0.14

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.0821	0.2981	1.0794	1.0847
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0716	0.2753	1.0691	1.0740
${T_j}^2$ *(T_Sq)*	1.0180	0.1359	1.0168	1.0192
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0086	0.0927	1.0078	1.0094

r = 0.1, Shift in Standard Deviation = 0.16

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.0362	0.1936	1.0345	1.0379
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0396	0.2042	1.0378	1.0414
${T_j}^2$ *(T_Sq)*	1.0053	0.0727	1.0047	1.0060
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0022	0.0477	1.0018	1.0027

r = 0.1, Shift in Standard Deviation = 0.18

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.0167	0.1305	1.0155	1.0178
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0219	0.1503	1.0205	1.0232
${T_j}^2$ *(T_Sq)*	1.0012	0.0346	1.0009	1.0015
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0004	0.0210	1.0003	1.0006

r = 0.1, Shift in Standard Deviation = 0.20

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.0063	0.0798	1.0056	1.0070
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0103	0.1015	1.0094	1.0112
${T_j}^2$ *(T_Sq)*	1.0002	0.0155	1.0001	1.0004
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0002	0.0126	1.0000	1.0003

r = 0.9, Shift in Standard Deviation = - 0.2

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.0021	0.0460	1.0017	1.0025
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.0004	0.0190	1.0002	1.0005
${T_j}^2$ *(T_Sq)*	1.0000	0.0000	1.0000	1.0000
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.0000	0.0000	1.0000	1.0000

r = 0.9, Shift in Standard Deviation = - 0.18

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.0124	0.1125	1.0114	1.0134
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.0030	0.0547	1.0025	1.0034
${T_j}^2$ *(T_Sq)*	1.0001	0.0077	1.0000	1.0001
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.0001	0.0077	1.0000	1.0001

r = 0.9, Shift in Standard Deviation = - 0.16

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.0471	0.2203	1.0451	1.0490
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.0141	0.1188	1.0130	1.0151
${T_j}^2$ *(T_Sq)*	1.0006	0.0245	1.0004	1.0008
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.0006	0.0237	1.0004	1.0008

r = 0.9, Shift in Standard Deviation = - 0.14

Method	**ARL**	**SDRL**	**LCL**	**UCL**
$T^2_{p_j}$, (P_T_Sq)	1.1325	0.3901	1.1290	1.1359
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0493	0.2295	1.0473	1.0514
$T_j{}^2$ *(T_Sq)*	1.0058	0.0759	1.0051	1.0065
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0057	0.0750	1.0050	1.0063

r = 0.9, Shift in Standard Deviation = - 0.12

Method	**ARL**	**SDRL**	**LCL**	**UCL**
$T^2_{p_j}$, (P_T_Sq)	1.3143	0.6444	1.3087	1.3199
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.1285	0.3790	1.1252	1.1319
$T_j{}^2$ *(T_Sq)*	1.0288	0.1723	1.0273	1.0304
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0284	0.1711	1.0269	1.0299

r = 0.9, Shift in Standard Deviation = - 0.10

Method	**ARL**	**SDRL**	**LCL**	**UCL**
$T^2_{p_j}$, (P_T_Sq)	1.6599	1.0511	1.6507	1.6691
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.2882	0.6129	1.2829	1.2936
$T_j{}^2$ *(T_Sq)*	1.1114	0.3499	1.1084	1.1145
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.1094	0.3462	1.1063	1.1124

r = 0.9, Shift in Standard Deviation = - 0.08

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	2.2806	1.7154	2.2656	2.2957
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.5725	0.9472	1.5642	1.5808
$T_j{}^2$ *(T_Sq)*	1.3337	0.6713	1.3278	1.3395
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.3289	0.6656	1.3230	1.3347

r = 0.9, Shift in Standard Deviation = - 0.06

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	3.2797	2.7192	3.2559	3.3036
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	2.0238	1.4443	2.0112	2.0365
$T_j{}^2$ *(T_Sq)*	1.8300	1.2365	1.8192	1.8409
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.8237	1.2342	1.8129	1.8345

r = 0.9, Shift in Standard Deviation = - 0.04

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	4.6447	4.1042	4.6087	4.6807
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	2.5895	2.0354	2.5716	2.6073
$T_j{}^2$ *(T_Sq)*	2.9165	2.3547	2.8959	2.9372
$T^2_{ewma_j}$ (T_Sq_EWMA)	2.9061	2.3464	2.8856	2.9267

r = 0.9, Shift in Standard Deviation = - 0.02

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	5.8313	5.3439	5.7845	5.8782
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	3.0841	2.5859	3.0615	3.1068
${T_j}^2$ *(T_Sq)*	4.8555	4.2985	4.8178	4.8931
$T_{ewma_j}^2$ (T_Sq_EWMA)	4.8750	4.3125	4.8372	4.9128

r = 0.9, Shift in Standard Deviation = 0

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	5.8600	5.3196	5.8134	5.9066
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	3.2049	2.6919	3.1813	3.2285
${T_j}^2$ *(T_Sq)*	6.2391	5.7273	6.1889	6.2893
$T_{ewma_j}^2$ (T_Sq_EWMA)	6.3946	5.8593	6.3432	6.4459

r = 0.9, Shift in Standard Deviation = 0.02

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	4.8467	4.3009	4.8090	4.8844
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	2.6651	2.1171	2.6465	2.6837
${T_j}^2$ *(T_Sq)*	6.3297	5.7927	6.2789	6.3805
$T_{ewma_j}^2$ (T_Sq_EWMA)	5.7479	5.2286	5.7020	5.7937

r = 0.9, Shift in Standard Deviation = 0.04

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	3.4672	2.9398	3.4414	3.4929
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	2.2906	1.7332	2.2754	2.3058
${T_j}^2$ *(T_Sq)*	2.9961	2.4340	2.9748	3.0175
$T^2_{ewma_j}$ (T_Sq_EWMA)	3.1227	2.5584	3.1003	3.1452

r = 0.9, Shift in Standard Deviation = 0.06

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	2.4971	1.9207	2.4803	2.5140
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.8384	1.2463	1.8275	1.8493
${T_j}^2$ *(T_Sq)*	1.9340	1.3465	1.9222	1.9458
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.9912	1.4084	1.9789	2.0036

r = 0.9, Shift in Standard Deviation = 0.08

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.8636	1.2683	1.8524	1.8747
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.4996	0.8731	1.4919	1.5072
${T_j}^2$ *(T_Sq)*	1.4279	0.7832	1.4211	1.4348
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.4554	0.8154	1.4483	1.4626

r = 0.9, Shift in Standard Deviation = 0.10

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.4847	0.8472	1.4773	1.4922
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.2834	0.6045	1.2781	1.2887
${T_j}^2$ *(T_Sq)*	1.1859	0.4684	1.1818	1.1900
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.1995	0.4881	1.1952	1.2037

r = 0.9, Shift in Standard Deviation = 0.12

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.2722	0.5889	1.2670	1.2773
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.1538	0.4205	1.1501	1.1575
${T_j}^2$ *(T_Sq)*	1.0732	0.2802	1.0707	1.0756
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0788	0.2917	1.0763	1.0814

r = 0.9, Shift in Standard Deviation = 0.14

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.1386	0.3948	1.1351	1.1420
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0773	0.2875	1.0748	1.0798
${T_j}^2$ *(T_Sq)*	1.0259	0.1628	1.0245	1.0273
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0285	0.1714	1.0270	1.0300

r = 0.9, Shift in Standard Deviation = 0.16

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.0697	0.2756	1.0673	1.0721
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0370	0.1978	1.0352	1.0387
${T_j}^2$ *(T_Sq)*	1.0078	0.0888	1.0070	1.0086
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0089	0.0948	1.0080	1.0097

r = 0.9, Shift in Standard Deviation = 0.18

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.0316	0.1813	1.0301	1.0332
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0154	0.1248	1.0143	1.0165
${T_j}^2$ *(T_Sq)*	1.0019	0.0442	1.0015	1.0023
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0022	0.0477	1.0018	1.0027

r = 0.9, Shift in Standard Deviation = 0.20

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.0130	0.1148	1.0120	1.0140
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0063	0.0799	1.0056	1.0070
${T_j}^2$ *(T_Sq)*	1.0005	0.0224	1.0003	1.0007
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0005	0.0237	1.0003	1.0007

Appendix F.3 ARL for Shift in Scale Parameter, Weibull Distribution, 50,000 runs

r = 0.1, Shift in Scale Parameter = - 0.10

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.0000	0.0000	1.0000	1.0000
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.0002	0.0134	1.0001	1.0003
$T_j^{\ 2}$ *(T_Sq)*	1.0000	0.0000	1.0000	1.0000
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.0000	0.0045	1.0000	1.0001

r = 0.1, Shift in Scale Parameter = - 0.08

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.0000	0.0045	1.0000	1.0001
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.0202	0.1430	1.0189	1.0214
$T_j^{\ 2}$ *(T_Sq)*	1.0000	0.0000	1.0000	1.0000
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.0192	0.1403	1.0179	1.0204

r = 0.1, Shift in Scale Parameter = - 0.06

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.0064	0.0797	1.0057	1.0071
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.3267	0.6551	1.3210	1.3324
$T_j^{\ 2}$ *(T_Sq)*	1.0005	0.0214	1.0003	1.0006
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.6472	1.0208	1.6383	1.6561

r = 0.1, Shift in Scale Parameter = - 0.04

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.2008	0.4874	1.1965	1.2051
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	2.6704	2.1561	2.6515	2.6893
$T_j^{\ 2}$ *(T_Sq)*	1.0637	0.2584	1.0614	1.0659
$T_{ewma_j}^2$ (T_Sq_EWMA)	10.4919	10.0325	10.4039	10.5798

r = 0.1, Shift in Scale Parameter = - 0.02

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	2.6645	2.1009	2.6461	2.6829
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	4.6723	4.2312	4.6352	4.7094
$T_j^{\ 2}$ *(T_Sq)*	2.0684	1.4913	2.0554	2.0815
$T_{ewma_j}^2$ (T_Sq_EWMA)	49.5250	45.0152	49.1305	49.9196

r = 0.1, Shift in Scale Parameter = 0

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	5.2067	4.6770	5.1657	5.2477
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	3.9374	3.4505	3.9072	3.9677
$T_j^{\ 2}$ *(T_Sq)*	6.5114	5.9933	6.4588	6.5639
$T_{ewma_j}^2$ (T_Sq_EWMA)	15.9962	15.5204	15.8602	16.1323

r = 0.1, Shift in Scale Parameter = 0.02

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	2.3788	1.8102	2.3630	2.3947
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	2.1140	1.5326	2.1005	2.1274
$T_j{}^2$ *(T_Sq)*	2.1661	1.5912	2.1521	2.1800
$T^2_{ewma_j}$ (T_Sq_EWMA)	2.5786	2.0152	2.5609	2.5962

r = 0.1, Shift in Scale Parameter = 0.04

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.2133	0.5116	1.2088	1.2178
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.2563	0.5695	1.2513	1.2613
$T_j{}^2$ *(T_Sq)*	1.1064	0.3455	1.1034	1.1094
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.1733	0.4536	1.1693	1.1772

r = 0.1, Shift in Scale Parameter = 0.06

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.0175	0.1334	1.0164	1.0187
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0339	0.1863	1.0323	1.0355
$T_j{}^2$ *(T_Sq)*	1.0039	0.0626	1.0034	1.0044
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0094	0.0969	1.0085	1.0102

r = 0.1, Shift in Scale Parameter = 0.08

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.0006	0.0241	1.0004	1.0008
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0018	0.0424	1.0014	1.0022
${T_j}^2$ *(T_Sq)*	1.0001	0.0077	1.0000	1.0001
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0001	0.0118	1.0000	1.0002

r = 0.1, Shift in Scale Parameter = 0.10

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.0000	0.0000	1.0000	1.0000
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0001	0.0110	1.0000	1.0002
${T_j}^2$ *(T_Sq)*	1.0000	0.0000	1.0000	1.0000
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0000	0.0000	1.0000	1.0000

r = 0.9, Shift in Scale Parameter = - 0.10

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.0000	0.0000	1.0000	1.0000
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0000	0.0000	1.0000	1.0000
${T_j}^2$ *(T_Sq)*	1.0000	0.0000	1.0000	1.0000
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0000	0.0000	1.0000	1.0000

r = 0.9, Shift in Scale Parameter = - 0.08

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.0000	0.0063	1.0000	1.0001
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.0000	0.0000	1.0000	1.0000
$T_j^{\ 2}$ *(T_Sq)*	1.0000	0.0000	1.0000	1.0000
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.0000	0.0000	1.0000	1.0000

r = 0.9, Shift in Scale Parameter = - 0.06

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.0116	0.1081	1.0107	1.0126
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.0022	0.0466	1.0018	1.0026
$T_j^{\ 2}$ *(T_Sq)*	1.0010	0.0316	1.0007	1.0013
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.0006	0.0249	1.0004	1.0008

r = 0.9, Shift in Scale Parameter = - 0.04

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.2958	0.6184	1.2904	1.3012
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.1069	0.3418	1.1039	1.1099
$T_j^{\ 2}$ *(T_Sq)*	1.1140	0.3573	1.1108	1.1171
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.0782	0.2905	1.0757	1.0808

r = 0.9, Shift in Scale Parameter = - 0.02

Method	**ARL**	**SDRL**	**LCL**	**UCL**
$T_{p_j}^2$, (P_T_Sq)	3.5156	2.9832	3.4895	3.5418
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	2.0117	1.4411	1.9991	2.0244
${T_j}^2$ *(T_Sq)*	2.6556	2.0991	2.6372	2.6740
$T_{ewma_j}^2$ (T_Sq_EWMA)	2.2681	1.6947	2.2532	2.2830

r = 0.9, Shift in Scale Parameter = 0

Method	**ARL**	**SDRL**	**LCL**	**UCL**
$T_{p_j}^2$, (P_T_Sq)	7.4235	6.8464	7.3635	7.4835
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	3.7863	3.3236	3.7571	3.8154
${T_j}^2$ *(T_Sq)*	9.0547	8.6075	8.9792	9.1301
$T_{ewma_j}^2$ (T_Sq_EWMA)	8.7934	8.3955	8.7198	8.8670

r = 0.9, Shift in Scale Parameter = 0.02

Method	**ARL**	**SDRL**	**LCL**	**UCL**
$T_{p_j}^2$, (P_T_Sq)	2.7111	2.1595	2.6922	2.7300
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.9942	1.4160	1.9818	2.0066
${T_j}^2$ *(T_Sq)*	2.3564	1.8089	2.3406	2.3723
$T_{ewma_j}^2$ (T_Sq_EWMA)	2.7214	2.1856	2.7023	2.7406

r = 0.9, Shift in Scale Parameter = 0.04

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.2537	0.5648	1.2488	1.2587
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.1538	0.4231	1.1501	1.1575
${T_j}^2$ *(T_Sq)*	1.1275	0.3789	1.1241	1.1308
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.1709	0.4486	1.1670	1.1749

r = 0.9, Shift in Scale Parameter = 0.06

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.0213	0.1470	1.0200	1.0226
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.0108	0.1046	1.0099	1.0117
${T_j}^2$ *(T_Sq)*	1.0050	0.0705	1.0043	1.0056
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.0079	0.0886	1.0071	1.0087

r = 0.9, Shift in Scale Parameter = 0.08

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.0006	0.0237	1.0004	1.0008
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.0003	0.0167	1.0001	1.0004
${T_j}^2$ *(T_Sq)*	1.0000	0.0045	1.0000	1.0001
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.0000	0.0045	1.0000	1.0001

r = 0.9, Shift in Scale Parameter = 0.10

Method	**ARL**	**SDRL**	**LCL**	**UCL**
$T^2_{p_j}$, (P_T_Sq)	1.0000	0.0045	1.0000	1.0001
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0000	0.0045	1.0000	1.0001
$T_j{}^2$ *(T_Sq)*	1.0000	0.0000	1.0000	1.0000
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0000	0.0045	1.0000	1.0001

Appendix F.4 ARL for Shift in Shape Parameter, Weibull Distribution, 50,000 runs

r = 0.1, Shift in Shape Parameter = - 0.4

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.0031	0.0554	1.0026	1.0036
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.0000	0.0000	1.0000	1.0000
${T_j}^2$ *(T_Sq)*	1.0000	0.0000	1.0000	1.0000
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.0028	0.0525	1.0023	1.0032

r = 0.1, Shift in Shape Parameter = - 0.32

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.0408	0.2055	1.0390	1.0426
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.0001	0.0077	1.0000	1.0001
${T_j}^2$ *(T_Sq)*	1.0031	0.0567	1.0026	1.0036
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.0187	0.1378	1.0175	1.0199

r = 0.1, Shift in Shape Parameter = - 0.24

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.2375	0.5393	1.2328	1.2422
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.0007	0.0268	1.0005	1.0010
${T_j}^2$ *(T_Sq)*	1.0595	0.2519	1.0573	1.0617
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.0761	0.2849	1.0736	1.0786

r = 0.1, Shift in Shape Parameter = - 0.16

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.9031	1.3093	1.8916	1.9146
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.0029	0.0541	1.0025	1.0034
${T_j}^2$ *(T_Sq)*	1.4556	0.8172	1.4484	1.4628
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.1933	0.4840	1.1890	1.1975

r = 0.1, Shift in Shape Parameter = - 0.08

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	3.7552	3.2123	3.7271	3.7834
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.0072	0.0853	1.0065	1.0080
${T_j}^2$ *(T_Sq)*	3.3726	2.8357	3.3478	3.3975
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.3601	0.6995	1.3539	1.3662

r = 0.1, Shift in Shape Parameter = 0

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	6.0030	5.4276	5.9554	6.0506
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.0126	0.1134	1.0116	1.0136
${T_j}^2$ *(T_Sq)*	7.1506	6.6261	7.0925	7.2086
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.4606	0.8212	1.4534	1.4678

r = 0.1, Shift in Shape Parameter = 0.08

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	4.5985	4.1042	4.5625	4.6345
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.0160	0.1269	1.0148	1.0171
T_j^2 *(T_Sq)*	3.4080	2.8519	3.3830	3.4330
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.4171	0.7682	1.4103	1.4238

r = 0.1, Shift in Shape Parameter = 0.16

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	2.5211	1.9570	2.5040	2.5383
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.0149	0.1236	1.0139	1.0160
T_j^2 *(T_Sq)*	1.5769	0.9482	1.5686	1.5852
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.2600	0.5779	1.2550	1.2651

r = 0.1, Shift in Shape Parameter = 0.24

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.5526	0.9291	1.5445	1.5608
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.0108	0.1051	1.0099	1.0118
T_j^2 *(T_Sq)*	1.1061	0.3442	1.1031	1.1091
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.1135	0.3524	1.1105	1.1166

r = 0.1, Shift in Shape Parameter = 0.32

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.1767	0.4563	1.1727	1.1807
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0069	0.0831	1.0062	1.0076
$T_j^{\ 2}$ *(T_Sq)*	1.0137	0.1169	1.0127	1.0147
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0354	0.1910	1.0337	1.0371

r = 0.1, Shift in Shape Parameter = 0.4

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.0454	0.2187	1.0435	1.0474
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0031	0.0563	1.0026	1.0036
$T_j^{\ 2}$ *(T_Sq)*	1.0006	0.0253	1.0004	1.0009
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0066	0.0813	1.0059	1.0073

r = 0.9, Shift in Shape Parameter = - 0.4

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.0023	0.0477	1.0019	1.0027
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0005	0.0219	1.0003	1.0007
$T_j^{\ 2}$ *(T_Sq)*	1.0000	0.0063	1.0000	1.0001
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0000	0.0045	1.0000	1.0001

r = 0.9, Shift in Shape Parameter = - 0.32

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.0282	0.1695	1.0267	1.0297
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0078	0.0883	1.0070	1.0086
${T_j}^2$ *(T_Sq)*	1.0021	0.0466	1.0017	1.0025
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0018	0.0424	1.0014	1.0022

r = 0.9, Shift in Shape Parameter = - 0.24

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.1883	0.4762	1.1842	1.1925
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0711	0.2766	1.0687	1.0735
${T_j}^2$ *(T_Sq)*	1.0492	0.2262	1.0472	1.0512
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0436	0.2126	1.0417	1.0455

r = 0.9, Shift in Shape Parameter = - 0.16

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.7600	1.1675	1.7498	1.7702
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.3219	0.6517	1.3162	1.3276
${T_j}^2$ *(T_Sq)*	1.4041	0.7554	1.3975	1.4107
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.3659	0.7092	1.3596	1.3721

r = 0.9, Shift in Shape Parameter = - 0.08

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	3.4503	2.9316	3.4246	3.4760
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	2.0436	1.4770	2.0307	2.0566
$T_j{}^2$ *(T_Sq)*	3.1624	2.6136	3.1395	3.1853
$T^2_{ewma_j}$ (T_Sq_EWMA)	2.9894	2.4218	2.9682	3.0107

r = 0.9, Shift in Shape Parameter = 0

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	5.8837	5.3559	5.8368	5.9306
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	3.0965	2.5790	3.0739	3.1191
$T_j{}^2$ *(T_Sq)*	6.4975	6.0070	6.4448	6.5501
$T^2_{ewma_j}$ (T_Sq_EWMA)	6.7693	6.2687	6.7144	6.8243

r = 0.9, Shift in Shape Parameter = 0.08

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	4.4924	3.9384	4.4579	4.5269
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	2.8447	2.3144	2.8244	2.8649
$T_j{}^2$ *(T_Sq)*	2.9489	2.3852	2.9280	2.9698
$T^2_{ewma_j}$ (T_Sq_EWMA)	3.2632	2.7139	3.2394	3.2870

r = 0.9, Shift in Shape Parameter = 0.16

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	2.4030	1.8316	2.3869	2.4190
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.8603	1.2672	1.8492	1.8714
${T_j}^2$ *(T_Sq)*	1.4404	0.7992	1.4334	1.4474
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.5189	0.8901	1.5111	1.5267

r = 0.9, Shift in Shape Parameter = 0.24

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.4903	0.8546	1.4828	1.4978
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.3091	0.6346	1.3035	1.3146
${T_j}^2$ *(T_Sq)*	1.0748	0.2826	1.0724	1.0773
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.0923	0.3170	1.0895	1.0951

r = 0.9, Shift in Shape Parameter = 0.32

Method	ARL	SDRL	LCL	UCL
$T_{p_j}^2$, (P_T_Sq)	1.1480	0.4124	1.1444	1.1516
$T_{pewma_j}^2$ (P_T_Sq_EWMA)	1.0878	0.3090	1.0851	1.0905
${T_j}^2$ *(T_Sq)*	1.0074	0.0865	1.0066	1.0081
$T_{ewma_j}^2$ (T_Sq_EWMA)	1.0102	0.1021	1.0093	1.0111

r = 0.9, Shift in Shape Parameter = 0.4

Method	ARL	SDRL	LCL	UCL
$T^2_{p_j}$, (P_T_Sq)	1.0350	0.1891	1.0333	1.0367
$T^2_{pewma_j}$ (P_T_Sq_EWMA)	1.0200	0.1414	1.0187	1.0212
${T_j}^2$ *(T_Sq)*	1.0004	0.0190	1.0002	1.0005
$T^2_{ewma_j}$ (T_Sq_EWMA)	1.0006	0.0241	1.0004	1.0008