Essays on Option Hedging and Application of the Black-Scholes Model

by

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Abstract

This dissertation is composed of three essays related to hedging and application of the Black-Scholes model. The first essay is motivated by the short-lived arbitrage model, which has been shown to significantly improve in-sample option pricing fit relative to the Black-Scholes model. We imply both volatility and virtual interest rates to adjust minimum variance hedge ratios. Using several error metrics, we find that the hedging model significantly outperforms the traditional delta hedge and a current bench-mark hedge based on the practitioner Black-Scholes model. Our applications include hedges of index options, individual stock options and commodity futures options. Hedges on gold and silver are especially sensitive to virtual interest rates.

The second essay analyzes the optimal capital structure of a nation from a corporate finance perspective. In particular, we draw an analogy between a nation's fiat money and corporate equity following Bolton and Huang (2018). Based on dynamic capital structure theory, we develop a stochastic model to determine the optimal combination of fiat money and foreign-currency debt used by a nation to fund its investments. The optimal capital structure of a nation depends on the trade-off between the inflation risk of fiat money and the default risk of foreign-currency debt. Introducing outstanding debt to our model sheds light on how a nation dynamically adjusts its capital structure over time. Based on an analysis of 22 emerging economies, the empirical results support our theoretical model of the capital structure of a nation.

The third essay studies the impact of changes in the Federal funds rate target on option prices. I find that, on average, an unanticipated 25-basis-point cut in the Federal funds rate target is associated with an 18.5% increase in S&P 500 index call options prices and an 18.6% decrease in the put options prices. However, the result is reversed in the 2008 financial crisis, during

which an unexpected cut in the Federal funds rate target raises market concerns. I conduct a quantitative analysis of the transmission channels in terms of underlying security price, volatility and interest rates. Evidence shows that FOMC monetary policy has more influence on the options market during the 2008 financial crisis. Bank equity options are more sensitive to the changes in the Federal funds rate target than S&P 500 index options.

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1.1 Introduction

Because market participants are generally risk averse, individuals and firms employ hedging strategies that reduce price or return volatility. Investment managers and firm CEO's cannot control various inputs and outputs such as commodity prices, currency prices, equity prices, and bond prices. In such cases, options (and/or futures) are used to take positions that reduce volatility. Options are highly leveraged, have limited liability and are an efficient way to offset, or hedge, this volatility. Thus, there is significant demand for call and put options. In some cases, option demand can be satisfied by standardized exchange traded options. In many cases, however, customized options are required and this demand is satisfied by over-the-counter option writers.

The over-the-counter (OTC) market for derivatives is documented by the Bank for International Settlements (BIS). According to BIS, the notional amount of outstanding equity linked OTC contracts was 2,930 billion US dollars in 2018[1]. The OTC equity options account for more than 60 percent of this amount. The outstanding amount of OTC commodity contracts was 1,898 billion US dollars with 7 percent of this amount in gold option contracts ($376 billion).

Over-the-counter writers are exposed to risk by virtue of their short positions. To reduce exposure, the writer is obliged to take positions to offset the risk of the written options. By far the most basic and widely used method for reducing this risk is the delta hedge as calculated from the celebrated Black-Scholes (1973) model. The delta hedge can be easily implemented since

[1] www.bis.org/statisticsd8.pdf

managing the risk in a portfolio of options involves only trades in the underlying asset and a risk-free account.

Despite its landmark contribution to the financial industry, some Black-Scholes assumptions cannot be empirically justified. Over the past 45 years, researchers have extended or otherwise altered the Black-Scholes model in attempts to improve hedging and pricing. Some models extend the Black-Scholes model to better describe the dynamics of the underlying asset. For example, the jump-diffusion model of stock prices was introduced by Merton (1976) and further extended by a number of authors including Ball and Torous (1985), Naik and Lee (1990), and Bates (1991). Hull and White (1987, 1988), and Heston (1993) were early proponents of models that include both the underlying and stochastic volatility as state variables. Bates (1996), and Duffie, Pan, and Singleton (2000) develop models that includes both jumps and stochastic volatility. In a landmark study, Bakshi, Cao, and Chen (1997) investigated several models and hedging techniques.

Theoretical models that effectively hedge jump-diffusions require the addition of options to the short term portfolio. See, for example, Andersen and Andreasen (2000) or He et al. (2006). In addition, models with multiple hedging instruments also theoretically improve hedging performance for smooth diffusions. In practice, however, multiple instruments pose significant issues. For example, a hedge portfolio with the underlying and an option would be efficacious only if there were synchronous or near synchronous trades of the target (hedged) option, the underlying, and the hedging option(s). Since options are typically less liquid than the underlying, synchronicity is a significant problem. Because of these difficulties and other considerations such as transaction costs, the single instrument hedge using the underlying remains the preferred approach among practitioners.

Notwithstanding improvements in modeling prices, the workhorse for pricing and single instrument hedging is the Practitioner Black-Scholes model. A key element of this model is a volatility surface that is derived by fitting implied volatilities to moneyness and maturity (Christoffersen and Jacobs, 2004). Other approaches have been developed to improve hedging performance by adjusting the Black-Scholes delta. One alter-native that has found traction in the literature is the minimum variance (MV) hedge ratio. The MV hedge remedies the failure of the delta hedge to account for multiple state variables that are correlated with the underlying. Shortcomings of the delta hedge have been addressed by a number of authors including Hagan, et al. (2002), Bartlett (2006), Alexander and Nogueira (2007), and Hull and White (2017). Other considerations also favor the MV hedge over the delta hedge. Christoffersen and Jacobs (2004) provide a compelling argument for choosing a hedging loss function that is consistent with the parameter estimation loss function. In its simplest incarnation, this means that a least squares loss function should be used to evaluate hedging performance if parameters are chosen to minimize in-sample squared errors.

Minimum-variance deltas have been developed using stochastic volatility as an additional state variable. Hull and White (HW, 2017) document that a number of researchers have shown improved hedging performance by combining the Practitioner Black-Scholes delta and Practitioner Black-Scholes vega. In these models, "Practitioner" implies that a formal stochastic process is not assumed but implied from the data. Hull and White develop a model of this nature by empirically determining the partial derivative of expected volatility with respect to price (vega). The model is tested against the SABR stochastic volatility model (Hagan et al., 2002) and the local volatility model using data from widely traded stocks, commodities, and ETFs. Their primary focus is hedging options on the S&P500 index. According to their performance metric ("gain"), the HW hedging model shows significant improvements over the SABR model and the local volatility model. The HW hedge performance was better for calls than puts and better for indices than individual stocks.

We develop a different MV hedge based on a practitioner version of the short-lived arbitrage (SLA) model of Otto (2000). The formula for the SLA model is essentially the same as that of the Merton Stochastic Interest Rate model. However, it differs in one crucial aspect. The short-lived arbitrage is latent but expressed as a state variable in the Merton bond price term. Since it is unobservable, Otto refers the Merton bond term as a "virtual bond." Hilliard and Hilliard (HH, 2017), extend the Otto model for correlations with the underlying and find that jointly implied volatility and virtual bond price significantly improves fit vis-á-vis the Black-Scholes pricing model with implied volatility. They also find that put options have higher prices than call options relative to their Black-Scholes counterparts, consistent with the empirical findings of Cremers and Weinbaum (2010) and HH. Motivated by significantly better in-sample fit, we use the SLA model to develop a MV hedge ratio using jointly implied volatility and virtual bond price that takes delta, vega, gamma and rho exposure into consideration. Consistent with the SLA model, Greeks are computed using implied yield-to-maturity rather than observed rates.

Using an extensive dataset from selected indices, individual stocks, and commodities, we demonstrate that SLA hedge ratios produce MV hedges that dominate delta hedges and our benchmark, the Hull-White (2017) hedge. In particular, compared to alternative hedges, the SLA hedge significantly reduces mean absolute errors, standard deviations, and the RMSE of daily hedging errors. In terms of point estimates, error metrics for stock indices from the SLA hedge were less than those of alternative hedging methods for calls in 36 out of 36 metrics. For puts, error metrics for the SLA hedge were better in 34 of 36 cases. Results were essentially the same for stocks and commodities. We use the Diebold-Marino statistic to test for significance between the SLA hedge and alternatives hedges. Counts of significance for the SLA hedge versus alternatives at the 0.01 or 0.05 level for calls were as follows: 29 of 36 (indices), 20 of 36 (stocks), and 22 of 36 (commodities). For puts, the corresponding counts were: 18 of 36 (indices), 7 of 36 (stocks) and 26 of 36 (commodities). There were no cases where an alternative hedge was significantly better than the SLA hedge.

We decomposed the MV hedge ratio into component parts corresponding to the Greeks. The composition of the ratios was similar for both puts and calls. The BS delta terms contributed about 85 percent to the MV hedge ratio for equity indices and stocks and about 60 percent to the hedge ratio for commodities. The vega (ν) term contributed about 20 percent to the crude oil hedge ratio. For calls, rho (ρ) contributed about 15 percent to gold and silver hedge ratios, consistent with evidence that precious metal returns are correlated with interest rates. This supports Bailey (1987) who used the Ramaswamy and Sundaresen (1985) stochastic interest model to study Gold Comex Options. He found average errors of \$43 per contract with a stochastic interest rate model while a constant interest rate model had average errors of \$96 dollars per contract.

1.2 Minimum Variance Hedging

Under the classical Black-Scholes model, the underlying asset follows a geometric Brownian motion (gBm) $dS = S\mu dt + S\sigma dZ_s$ with constant drift and volatility. By construction, the delta of a call ($\delta_{BS} = \frac{\partial C}{\partial S} \coloneqq C_s$) nulls out random effects in a portfolio consisting of a call and the underlying asset.

Equivalently, delta minimizes the variance of an investment portfolio consisting of an option, stock and bond written as

$$H = -C + \delta S + B\,, \tag{1}$$

where C is a call, S is the underlying spot price, B is a risk-free bond, and δ is the hedging ratio. The local change is

$$dH = -dC + \delta dS + dB$$

with variance

$$Var[dH] = Var[dC] + \delta^2 Var[dS] - 2\delta Cov[dC, dS].$$

The first order condition for minimum variance ($\frac{\partial Var}{\partial \delta}$=0) leads to the result that

$$\delta_{MV} = \frac{Cov[dC, dS]}{Var[dS]} = \frac{Cov[C_s dS + (\cdot)dt, dS]}{Var[dS]} = C_{s.}$$

So that the minimum variance delta, δ_{MV}, is equal to the standard Black-Scholes delta, δ_{BS}, when the only state variable is the underlying.

1.2.1 Minimum-Variance Hedge Ratios Under Multiple State Variables

Consider an option pricing model based on diffusions in the underlying, volatility and interest rates. Denote the equilibrium (call) option price as $C = C(S, \sigma, r, t)$ with local change

$$dC = C_S dS + C_\sigma d\sigma + C_r dr + (\cdot)dt.$$

Using the same setup as in equation (1), the minimum variance hedge ratio $\delta_{MV} = \frac{Cov[dC,dS]}{Var[dS]}$ can be expressed as

$$\delta_{MV} = \frac{Cov[C_S dS + C_\sigma d\sigma + C_r dr, dS]}{Var[dS]}\ , \tag{2}$$

$$\delta_{MV} = C_S + C_\sigma \frac{Cov[d\sigma, dS]}{Var[dS]} + C_r \frac{Cov[dr, dS]}{Var[dS]}.$$

The easy result is that $\delta_{MV} = C_S$ when covariance are zero but otherwise δ_{MV} depends on covariances, vega (C_σ) and rho (C_r). But the result is not without complications. Specifically, how do we compute the partials? Do we require the multistate equilibrium model or will Black-Scholes partials suffice? The answer to the latter question is a qualified yes. Bates (2005) and Alexander and Nogueira (AN, 2007) note that a sufficient condition for model free hedge ratios for options is that the underlying process be scale invariant. This condition is satisfied by most option pricing models, with notable exceptions being models based on arithmetic Brownian motion or the CEV model. When the underlying is scale independent, pure vanilla options are homogeneous of degree one (Merton, 1973) and a well known result of Euler gives

$$\frac{\partial C}{\partial S} = \frac{C}{S} - \frac{K}{S}\frac{\partial C}{\partial K}. \tag{3}$$

The notion that delta ($\frac{\partial C}{\partial S} = \delta$) is model independent follows from the fact that, if traded, C is observable and $\frac{\partial C}{\partial K}$ can be estimated from a set of prices with different strikes. Ergo, the delta is model independent. Similar model independence can be inferred for gamma ($\gamma = \frac{\partial^2 C}{\partial S^2}$)See Bates (2005).

Alexander and Nogueira (2007) state that if competing models are scale invariant "the only reason their price ratios differ is because they have a different …t to the market prices of options." In fact, BS hedge ratios can be quite different from those of smile consistent models. The good news is that for the data studied, AN report that BS delta and gamma hedges perform better than model independent hedges for equity index options. We assume that MV approximations in equation (2) are useful even when Greeks from multi-state equilibrium models are replaced by BS Greeks. And, in any case, we test their efficacy against alternatives on a database of heavily traded indices, stocks and commodities.

There is also a growing literature on how MV hedge ratios can be obtained by adjusting BS deltas. As noted by AN and outlined in equation (2), the adjustment depends on local covariances between the underlying and the additional state variables. Adjustments have been investigated by HW (2017), Englemann, Fengler, and Schwendner (2006) and Alexander and Kaeck (2012). Alexander and Kaeck use the chain rule to compute the smile adjustment $\delta_{SA} = \frac{\partial H}{\partial S} - \frac{\partial H}{\partial \sigma}\frac{\partial \sigma}{\partial S}$. The covariance adjustment and smile adjustment are approximately the same since $\sigma = \sigma(S)$ implies $d\sigma = dS\frac{\partial \sigma}{\partial S} + O(dS^2)$ and thus $\frac{Cov[d\sigma, dS]}{Var[dS]} \approx \frac{\partial \sigma}{\partial S}$. Alexander and Kaeck result can be obtained for three state variables by a straightforward extension of the chain rule. In the HW model, $\frac{\partial \sigma}{\partial S}$ is approximated by $\frac{\partial E[\sigma_I]}{\partial S}$ where σ_I is implied volatility.

1.2.2 The Model of Short-Lived Arbitrage

An important component of our hedging approach are the comparative statics suggested by a short-lived arbitrage model that depends on the value of a virtual bond. The Black-Scholes model is based on number of perfect capital market assumptions and fundamentally on the assumption of the absence of arbitrage. However, empirical studies have documented short-lived arbitrage opportunities in otherwise well functioning markets. See Sofianos (1993), HH and others. In a world of rational economic agents, the existence of arbitrageurs is prima facia evidence of arbitrage opportunities. Otto's proxy for (latent) short-lived arbitrage is an Ornstein-Uhlenbeck process of the form

$$dx = \kappa(\theta - x)dt + \sigma_x dZ_x, \tag{4}$$

that is pinned to zero at option expiration (T) and with long term mean $\theta = 0$. The arbitrage variable is thus a bridge between current value x_0 and $x_T = 0$. Using conditional probabilities, HH (2017) convert the OU diffusion to a bridge diffusion and extend the Otto model to account for correlations between the arbitrage and underlying diffusion.

Assuming gBm for the underlying and using the Garman (1977) setup for pricing with multiple state variables, the instantaneous change in the candidate portfolio sans delta risk is set equal to $y(t) = r + x(t)$, where r is the risk-free rate. Although cast as a short-lived-arbitrage model, the variable $x(t)$ proxies for net frictions that act to perturb the risk-free rate. The net result is a pricing model that is isomorphic to Merton's stochastic interest rate model ($y(t)$ is the instantaneous virtual rate) where the bond is a non-traded "virtual bond." The virtual bond is of the form $V = B^r B^x$ where B^r is the risk-free bond and B^x is a function of the parameters of the bridge diffusion. For the functional form of $V = V(x_t, \kappa, \sigma_x, T - t)$ see Otto or HH.

The SLA model for a call option (C) is written

$$C = SN(d_1) - VKN(d_2), \tag{5}$$

where S is the underlying spot price, K is the strike price, $V \coloneqq e^{-RT}$ is the bond and R is the virtual yield. The arguments d_1 and d_2 are standard BS calculations when volatilities and yields are implied. The pertubation from the constant rate may be because of short lived arbitrage, market frictions, or stochastic interest rates. It is not the same as a pure stochastic interest rate model, however, since the stochastic interest rate is the same for all firms in the economy while the virtual yield is unique to the frictions on the equity of a firm. For margined options, greek formulas do not depend on the risk-free rate[2]. For these options, the perturbed rate is $y(t) = x(t)$, the virtual bond is, $B^x = e^{-R^*(T-t)}$ and R^* is the pertubation due to frictions net of interest rates.

Jointly implying volatility and the virtual bond, HH find that the SLA model significantly improves in-sample fits relative to the Black-Scholes model for each of a select set of stocks. Better SLA pricing performance suggests that "practitioner" SLA based hedges may also perform better. And this means specifically that implying latent virtual yields might lead to improved hedges. For more detail on the issue of hedging and model fit see Alexander and Kaeck (2012).

[2] The Brent Crude oil options that we use are margined options.

1.2.3 Practitioner Short-Lived Arbitrage Dynamics

We consider non-local time and the role of gamma in developing the SLA hedge ratio. Local terms (dS) are replaced by finite terms (ΔS) and changes in $C = C(S, \sigma, R, t)$ are written

$$\Delta C = C_S \Delta S + C_\sigma \Delta\sigma + C_R \Delta R + \frac{1}{2} C_{SS} \Delta S^2 + O(\Delta^2), \tag{6}$$

and so

$$\delta_{MV} = \frac{Cov\left[C_S \Delta S + C_\sigma \Delta\sigma + C_R \Delta R + \frac{1}{2} C_{SS} \Delta S^2, \Delta S\right]}{Var[dS]}, \tag{7}$$

Assuming ΔS is normal with mean $\mu S \Delta$ and variance $\sigma^2 S^2 \Delta^2$, $\frac{Cov[\Delta S^2, \Delta S]}{Var[\Delta S]} = 2\mu S \Delta$ and the minimum variance hedge ratio is

$$\delta_{MV} = C_S + \beta_1 S C_{SS} + \beta_2 C_\sigma + \beta_3 C_R + O(\Delta^2), \tag{8}$$

where $\beta_1 = \mu\Delta, \beta_2 = \frac{Cov[\Delta\sigma, \Delta S]}{Var[dS]}$, and $\beta_3 = \frac{Cov[\Delta R, \Delta S]}{Var[dS]}$. After σ and R are jointly implied using the SLA model, we compute Greeks and use equation (8) in a regression setup to estimate coefficients β_1, β_2, and β_3.

1.2.4 Data

We test the proposed hedging method for options on selected indices, individual stocks, and commodity futures. The indices are the S&P 500 (SPX), the Nasdaq 100 (NDX), and the Russell 2000 (RUT); the individual stocks are Amazon (AMZN), Google (GOOGL), and Berkshire Hathaway (BRK.A)[3]; the three commodity futures are Brent Crude Oil Futures, 100 oz Gold Futures, and 5,000 oz Silver Futures. These assets and contracts are highly liquid and have been widely studied in the financial economics literature.

[3] Google essentially renamed itself as "Alphabet" in August 2015. It continues to trade under the same GOOG symbol.

We obtain daily interest rates, equity prices, and option data from OptionMetrics. Dividend yields on the underlying indices are from FactSet. The data for the commodity futures are from the Intercontinental Exchange (ICE). The options on indices are European style while options on individual stocks and commodities are American style. Options on Brent Crude futures are margined style and have no early-exercise premiums under weak assumptions (Hilliard and Hilliard, 2019). The options on commodities from the ICE do not contain information on Greeks and so the test design is slightly different from the test design for options on indices and individual stocks. Except for Gold and Silver, the data is from the period January 2, 2013 through June 27, 2019. Data for options on 100 oz Gold Futures and 5,000 oz Silver Futures was first available on January 2, 2014.

For options on indices and individual stocks, we retain only those with available bid price, offer price, implied volatility, delta, gamma, vega, and theta. We also delete options with no volume, except for options on Gold and Silver since available data does not report volume. To evaluate daily hedging performance, we select options that have at least two successive trading days. Options are retained if the maturity is at least 14 calendar days and the average of best bid price and best offer price is at least 25 cents. To avoid deep in- and out-of-the-money options, we delete call options with delta less than 0.05 or greater than 0.95, and put options with delta less than -0.95 or greater than -0.05.

We summarize the data for indices, individual stocks, and commodities in tables I.1 through table I.4. In table I.1 we present summary statistics for options on indices. More puts are traded than calls and short-term options dominate long-term options. There are 443,168 observations on S&P 500 call options and 800,210 observations on S&P 500 put options. The average price of call options is higher than the average price for put options for all indices.

For individual stocks shown in table I.2, there are more call contracts traded than put contracts. And there are more contracts on Amazon options than on Google or Berkshire Hathaway. Individual stocks also tend to have longer maturities. The average maturities for calls on the S&P 500, the NASDAQ 100 and the Russell 2000 are, respectively 51, 47, and 48 days while average maturities on Amazon, Google and Berkshire Hathaway are, respectively, 121, 111, and 198 days. Longer maturities and the reversal in contract volume is consistent with the notion that there is more hedging activity in indices and speculative activity in individual stocks.

Descriptive statistics for options on commodity futures are given in Table I.3. Statistics for crude oil are based on the number of actual trades. Unfortunately, the ICE does not report volume on Gold and Silver so statistics here are derived from all open contracts. Summary statistics for the underlying commodity futures are given in Table I.4.

1.3 Hedge Ratio Estimation and Error Metrics

Estimating the MV delta for the Short Lived Arbitrage (SLA) model and testing for errors is done in several steps:

1.3.1 Estimate Implied Parameters

Using daily data we jointly estimate implied volatility and virtual yield for each day (t). All observations surviving screens are used to calculate the $\hat{\sigma}$ and $\hat{R}$ that minimizes the sum of squared errors, L_t. For day-t with n_t observations let

$$L_t = \sum_{i=1}^{n_t} \left(C_{obs_i} - C_i(\sigma_t, R_t) \right)^2 , \tag{9}$$

so that $\hat{\sigma}$ and $\hat{R}$ are given by $ArgMin\ L_t(\hat{\sigma}, \hat{R})$ where C_{obs_i} is the observed call option price and $C_i(\sigma_t, R_t)$ is the call option price under the SLA model with implied volatility σ_t and virtual yield R_t. The subscript i is unique to options with different strikes and maturities. The optimal $\hat{\sigma}_t$ and

$\hat{R}_t$ are thus the jointly implied volatility and virtual yield to maturity, respectively. To obtain a volatility parameter unique to a given contract we then fix $\hat{R}_t$ and recompute $\hat{\sigma}_i$, such that $C_i(\hat{\sigma}_i, \hat{R}_t) = C_{obs_i}$. The end result of this procedure is a set of implied parameters $\hat{R}_t$ and $\hat{\sigma}_i$ for each observation.

1.3.2 Compute Greeks

Use the parameters implied in step 1. and the SLA model to compute Greeks delta (δ), vega (ν), rho (ρ), and gamma (γ). For convenience, functional forms for equities, futures options and margined options are given in the Appendix.

1.3.3 OLS Regressions

Using 26 weeks of data, OLS regressions are used to estimate β coefficients required for the MV delta. We estimate β's for the MV hedge ratios using non-local changes as follows: For each 26 week segment, use all observations to compute

$$\Delta C_i - Delta_i \Delta S_{t(i)} = \beta_\sigma Vega_i \Delta S_{t(i)} + \beta_R Rho_i \Delta S_{t(i)} + \beta_\gamma S_{t(i)} Gamma_i \Delta S_{t(i)} + \epsilon_i, \quad (10)$$

where $t = 1, 2, \dots, 130$ days, and $t(i)$ is the day-t that corresponds to option-i. Specifically $t(i) = t$ such that option i is contained in the set $I_t = \{options\ on\ day - t\}$. There are 260 segments of weekly rolling regressions.

1.3.4 Compute Hedge Ratios

Using β's and implied parameters from step 3 compute the SLA MV hedge ratios for each observation. To compute the hedge ratio for option-i, we take the β coefficients from equation (10) and compute the SLA MV hedge ratio as

$$\delta_{SLA_i} \coloneqq \delta^0 = Delta_i + \beta_\sigma Vega_i + \beta_R Rho_i + \beta_\gamma S_{t(i)} Gamma_i. \quad (11)$$

We also compute four other hedge ratios. They are: 1) δ_{BS}: Delta computed as a BS delta with observed short-term interest rates, 2) δ_{BS}^I: Delta computed as BS delta with implied yields $\hat{R}_t$,

3) δ_{HW}: the Hull-White delta and estimation procedure computed with observed interest rates, and

4) δ_{HW}^{I}: The Hull-White MV delta and estimation procedure computed with implied yields $\hat{R}_t$.

1.3.5 Compute Errors

For each option (i) and for each of the five ratios δ^j we compute daily out-of-sample errors for the next five days. Specifically, the daily error for observation-i with hedge set on day-t is computed as

$$\epsilon_i^j \coloneqq \Delta C_i - \delta_i^j \Delta S_{t(i)}. \tag{12}$$

Although the β's computed are fixed, the hedge ratios for contracts will typically be different because Greeks depend on strikes and maturities.

1.3.6 Error Metrics

Using errors computed in step 5 we use all observations and compute mean absolute errors, the standard deviations of errors, and the root-mean-square of errors (RMSE). We focus more on RMSE errors because 1) Christoffersen and Jacobs (2004) persuasively argue that the loss function should be the same as the loss function used to estimate parameters and 2) it is a measure of fit commonly used by other researchers. Results from the standard deviation metric are included but they are almost the same as RMSE results (they are mathematically the same if the sample mean is zero). We include the absolute error metric.

1.3.7 Significance Tests

The Diebold-Marino (1995) setup for a sequence of prediction errors is a follows: Let p = observed out-of-sample price and $\hat{p}$ be estimated price. The pricing error for observation-i and estimation methodology $j, j = 0, 1, \dots, 4$ is

$$e_i^j = p_i - \hat{p}_i^j.$$

The error metric is defined as $g(e_i^j)$. The average error metric for week-k is $\frac{\sum_{i=1}^{n_k} g(e_i^j)}{n_k} = \bar{g}_k^j$ where n_k is the number of observations in week-k. In the sample there are $T = 338$ weeks[4]. Let $j = 0$ correspond to the SLA hedge. Other hedges, $j = 1, 2, \ldots, 4$, are defined in step 4. We use a Heteroscedasticity Autocorrelation Consistent (HAC) estimator to compute standard errors of α_j in the regression $d_k^j = \alpha_j + \epsilon_k$ where $d_k^j = \bar{g}_k^j - \bar{g}_k^0, k = 1, 2, \ldots, T.$[5] The t-stat on α_j is the DM test statistic.

1.4 Results

We use several error metrics to compare hedging results using the BS delta, the HW delta, and the SLA delta. The BS delta is the Greek delta while the HW delta and SLA delta are minimum variance approximations. Hedge ratios are designated by: δ_{BS}(BS hedge), δ_{HW} (HW hedge), and δ_{SLA} (SLA hedge). Greeks for the BS delta and HW deltas are computed using both observed and implied interest rates. Greeks for the SLA hedge are computed using implied yields.

1.4.1 Point Estimates

As precursor to detailed significance tests, we first present a general overview of hedging error metrics for options on the S&P 500 index for different delta buckets in figures one through four[6]. There are 10 buckets containing observations for call deltas between 0.05 and 0.95. The 0.1 bucket consists of deltas between 0.05 and 0.15. Other buckets are similarly centered. Error metrics are normalized by the average price of options in the bucket. Put buckets are similarly defined with negative deltas.

[4] The dataset contains 338 observation weeks for indices, stocks and oil. There are 271 observation weeks for gold and silver.
[5] We use the Dmarino module in Stata and Bartlett HAC standard errors to compute t-statistics (Baum, 2003).
[6] The delta bucket is defined as the usual Black-Scholes delta.adjusted for dividends.

Mean absolute value errors for calls are shown in figure I.1. The metric for the δ_{SLA} hedge is smaller than the corresponding metric for the δ_{HW} hedge for all delta buckets with near equality for the 0.2 bucket. The error metric for δ_{BS} hedge is larger than that of the δ_{SLA} hedge for all buckets and also larger than that of the δ_{HW} hedge except for the largest four buckets (0.6, 0.7, 0.8 and 0.9).

RMSE errors for calls are shown in figure I.2. Results are similar to those of the absolute value metric. The error metric for the δ_{SLA} hedge is smaller than those of other hedges for all buckets. Similar to figure I.1, δ_{SLA} and δ_{HW} hedges strictly dominate the δ_{BS} hedge with the exception of the metric for buckets 0.8 and 0.9 where the δ_{BS} hedge metric is better than the δ_{HW} metric.

Mean absolute errors for puts are shown in figure I.3. The error metric for the δ_{SLA} hedge is less than that of the δ_{BS} for all buckets except the -0.1 bucket and less than that of the δ_{HW} hedge for all buckets except for the -0.6 and -0.5 bucket where the difference is negligible.

RMSE errors for puts are shown in figure I.4. The δ_{SLA} hedge has lower metrics than the δ_{BS} and the δ_{HW} for all buckets except equality with δ_{HW} at the -0.6 bucket.

1.4.2 Diebold-Marino Test Statistics

In tables I.5, I.6 and I.7 we present error metrics for hedging calls on selected indices, stocks, and commodities. In these tables we consider five different hedging ratios: The Black-Scholes delta with observed interest rates (δ_{BS}); the Black-Scholes delta with implied interest rates (δ_{BS}^{I}); the Hull-White minimum variance delta with observed interest rates (δ_{HW}), the Hull-White minimum variance delta with implied interest rates (δ_{HW}^{I}) and the Short-Lived-Arbitrage minimum variance delta (δ_{SLA}). All ratios with implied interest rates are jointly implied with volatility.

We use the Diebold-Marino statistic (see Section 3) to test for significant differences between error metrics. The difference in error metrics is computed by subtracting the sample mean of the δ_{SLA} error metric from that of the candidate error metrics. A positive DM statistic corresponds to a smaller δ_{SLA} hedging error.

1.4.2.1 Results for Hedging Calls on Indices

Results for hedging calls on the S&P 500, Russell 2000 and NASDAQ indices are given in table I.5. We focus our discussion on RMSE errors but results are similar for standard deviations and for absolute errors. Panel A gives results for the S&P 500. The δ_{SLA} hedge gives the smallest RMSE and is significantly less than other hedges by the DM test on 12 of 12 comparisons. Panel B in table I.5 shows similar but somewhat weaker results for the Russell 2000. RMSE errors for the δ_{SLA} hedge are all smaller and differences are significant at the 0.01 or 0.05 for 9 of 12 comparisons. NASDAQ results are given in panel C. Tests statistics are positive for all metrics and significant for the RMSE metric versus all alternatives.

1.4.2.2 Results for Hedging Calls on Stocks

Results for hedging calls on Amazon, Google and Berkshire stocks are given in table I.6. With one exception, the DM test statistics for the absolute value, standard deviation, and RMSE metrics are all positive, indicating lower average errors for the δ_{SLA} hedge. The results for Berkshire, panel C, are strongest with the δ_{SLA} error metric being significant at the 0.01 (0.05) level for 10 (2) of the twelve DM test statistics.

1.4.2.3 Results for Hedging Calls on Commodities

Results for oil, gold, and silver call options are given in table I.7. The DM test statistic is positive for all metrics and all commodities. For gold and silver, the DM statistic is significant at 0.01or 0.05 in 9 of 12 comparisons. The results are weakest for oil, where the DM test statistic is positive and significant at the 0.05 level in 4 of 12 cases. Compared to gold and silver, oil is a bit different since it has stochastic convenience yield while silver and especially gold are generally considered investment assets. Thus a Black-Scholes type model is more appropriate for gold and silver than for oil.

1.4.2.4 Hedging Gains for Calls

As in Hull-White (2017), we construct a measure of the improvement of the hedge using the δ_{SLA} hedge as the benchmark. The measure is computed as

$$Gain := \left(\frac{Comparison\ hedge\ error\ metric}{\delta_{SLA}\ hedge\ error\ metric}\right)^2. \quad (13)$$

The error metrics are average absolute error, standard deviation, and RMSE. Comparison hedges use the two δ_{BS} ratios and the two δ_{HW} ratios. A gain greater than one corresponds to a smaller error metric using the δ_{SLA} hedge ratio. Results are given in table I.8. Gains are greater than one for all indices, stocks, and commodities. The greatest RMSE gain is versus Berkshire Hathaway and δ_{HW} hedge (Gain=1.6448). The smallest RMSE gain is versus Amazon and the δ_{HW}^{I} hedge (Gain=1.0017). The δ_{SLA} hedge versus crude oil and gold was uniformly large versus all alternative hedging methods. The smallest RMSE gains were against indices, stocks and the δ_{HW}^{I} hedge ratio.

1.4.2.5 Results for Put Hedges

Tables I.9, I.10, and I.11 give results for put hedges. Overall, the results for hedging puts on indices, table I.10, are similar to the results for calls with the δ_{SLA} hedge having a positive DM statistics in every comparison except one. Counts and significance levels for the S&P 500 and NASDAQ were relatively weaker however. Significance counts at 0.01 or 0.05 were: S&P 500 (6 of 12), Russell 2000 (9 of 12) and NASDAQ (3 of 12). While DM t-statistics were positive, the δ_{SLA} hedge was not significantly better than the δ_{HW} hedge for the S&P 500 and NASDAQ.

The results for puts on stocks, table I.10, were also weaker than the results for calls. While positive, DM statistics were not significant at 0.01 or 0.05 for Amazon and in only two cases for Google. Anomalous to all other results, the DM statistic for the RMSE metric versus the δ_{HW}^{I} hedge was negative albeit not significant for Amazon.

Results for hedging puts on commodities were strong (table I.11). All DM-statistics were positive and significant at the 0.01 or the 0.05 level in 26 of 36 cases. But, similar to calls, t-stats for put hedges were weaker for oil than for gold and silver. Hedging Gains for Puts.

Gain results for puts are given in table I.12. Gains for commodities were all greater than one and generally larger than gains for stocks. Gains are also greater than one for all indices with the exception of the absolute error metric and δ_{BS} and δ_{BS}^{I} hedges for NASDAQ. All gains are greater than one for Amazon and Berkshire but less than one for the absolute error metric for Google and the δ_{BS} and δ_{HW} hedges. In summary, gains are positive in 32 of 36 cases.

1.4.3 Parameters and Hedging Coefficients

Hedging improvements are a result of using implied interest rates and adding greeks beyond the Black-Scholes delta. Percentage contributions to the average SLA hedge ratios are depicted in figures I.5 and I.6. For calls, figure I.5, the Black-Scholes delta term contributes about 80 percent to the δ_{SLA} ratio for indices and 85% for stocks. Vega and gamma terms contribute slightly more than the rho term. But commodities are different. For hedge ratios on oil, the average contribution of the delta term falls to about 55 percent while vega contribution increases to about 25 percent. Rho and gamma each contribute about 10 percent. For gold and silver, the delta contribution increases to about 65 percent while rho increases to about 15 percent. The combined contribution of vega and gamma is about 20 percent.

The pattern of greek contributions to hedge ratios for puts shown in figure I.6 is similar to that of calls for indices and stocks. However, rho is less heavily weighted for silver and gold, contributing just over 5 percent to the hedge ratio.

The importance of rho, for hedging gold and silver calls and puts contrasts with its negligible contribution to the hedge ratio for indices and stocks. This is consistent with evidence that precious metal returns are correlated with interest rates. Precious metals, especially gold, are used as effective hedges against inflation and economic uncertainty. In addition, gold is denominated in US dollars in international markets and is influenced by U.S. interest rate policies (Wang, 2013). Thus, stochastic interest rates are likely an important state variable in equilibrium option pricing models for gold and silver. In a study of Gold Comex Futures Options, Bailey (1987) found that the Ramaswamy and Sundaresen (1985) stochastic interest rate model had an average error of $43 per contract while a constant interest rate model had an error of $96 dollars per contract.

1.5 Conclusion

The effect of the length of the hedging horizon and level of volatility warrant further investigation. We extended the hedging horizon from 5 to 20 days and examined equity (oil) hedges in periods of extreme volatility January 1, 2007 to December 31 (September 1, 2008 to December 31,2010). Because of data limitations and for economy of presentation, we look at subsets of all scenarios examined in earlier sections.

1.5.1 Extended Hedging Horizon

Using the same setup as before, we examined hedges with horizons of 5, 10, and 20 days for puts and calls. We only compute RMSE errors for HW and SLA hedges. The results are shown in table I.13. For indices (Panel A), there is virtually no change in RMSE as the horizon is extended. For Amazon, the daily RMSEs over 5, 10, and 15 day horizons are 2.7453, 2.7577 and 2.7470.

The HW hedge has a smaller RMSE than the SLA hedge at only one of 18 data points (the 20 day horizon for the Russell 2000).

Results for stocks are shown in Panel B. The RMSEs for all SLA hedges are smaller than the HW RMSEs for puts and calls at all horizons. The horizon effect is negligible. At first blush it seems curious that Berkshire Hathaway RMSEs are about are 1/4 and 1/10 those of Google and Amazon, respectively. But from table I.2, average Berkshire Hathaway option prices follow approximately the same ratio.

Commodity results are shown in Panel C. Increases in RMSEs are usually found only in the third significant place. In fact, for silver and gold there is no difference over horizons until the fifth or sixth place (not shown). Point estimates of SLA RMSEs are smaller than HW RMSEs in every case.

1.5.2 Volatility Effects

Our initial focus was on data from January 2, 2013 to June 27, 2019 (benchmark period). Here we investigate hedging performance during the high volatility period from January 1, 2006 through December 31, 2010. This period includes the subprime crisis and recession dated from December 2007 to June 2009. We do not have options data for silver and gold during this period and to keep the number of tables manageable we only investigate the S&P 500, Amazon and Brent Crude.

Results for calls are shown in table I.14. The differences in the magnitude of errors and DM statistics in the volatile versus the benchmark period are small and follow no systematic pattern. RMSEs for the SLA hedge and the benchmark (volatile) period are: S&P 500 2.0269 (1.6792), Google 1.0941 (1.3570) and Brent Crude 0.1169 (0.2096). The SLA hedge still dominates alternatives in the volatile period. All DM statistics are positive and are significant at 0.01 or 0.05 as follows: S&P 500 (12 of 12), Google (2 of 12) and Brent Crude Oil (6 of 12).

Results for puts are shown in table I.15. RMSEs are: Benchmark (volatile) period: Brent Crude 0.1152, (0.1623), S&P 500 2.7453, (1.7338) and Google 1.2814, (1.2412). All DM statistics are positive and DM statistics are significant at 0.01 or 0.05 as follows: S&P 500 (10 of 12), Google (0 of 12), and Brent Crude Oil (2 of 12). Results are a bit weaker for puts compared to those in the benchmark period.

In summary, the SLA hedge performed well for calls in periods of high volatility. It does less well for puts though DM statistics remain positive for all cases. Only Brent Crude RMSE errors were larger for both put and calls during the volatile period.

1.6 Conclusion

Motivated by its derivation, the original benchmark for hedging custom option positions is the Black-Scholes delta. Based on least square loss functions, numerous authors have proposed choosing hedge ratios based on least squares approximations. These ratios depend on the pricing model used and the type of approximation employed. We develop a ratio based on a so-called "short lived arbitrage" model. In this model, the interest rate is latent and embodies deviations from model assumptions such as the possibility of arbitrage. We refer to this rate as the virtual interest rate. As such, we imply interest rates in the same manner that volatilities are implied. Using a least squares formulation we develop a ratio that is a function of the greeks delta, vega, rho, and gamma. The greeks are computed using the implied interest rate.

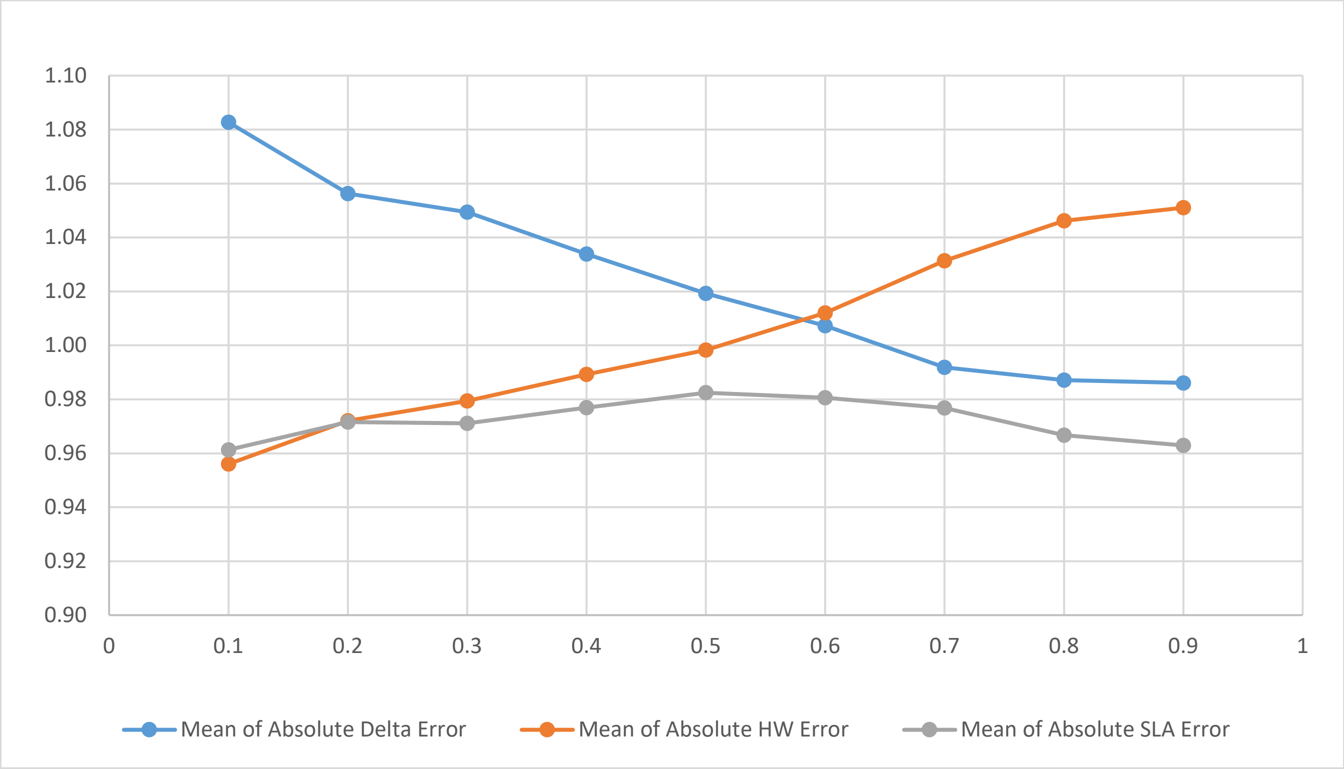
Using a large data set that includes seven years of data and up to 800,000 observations, we evaluate the performance of the SLA hedge on a set of indices, stocks and commodities. We evaluate performance using the absolute value, standard deviation, and RMSE error metrics. In terms of point estimates, error metrics from the SLA hedge bests alternatives for calls on indices in 36 of 36 metrics. For puts, error metrics from the SLA hedge was better in 34 of 36 cases.

We use the Diebold-Marino statistic to test for significance between the SLA hedge and alternative hedges. Counts of significance at the 0.01 or 0.05 level for calls were as follows: 29 of 36 (indices), 20 of 36 (stocks), and 22 of 36 (commodities). For puts the corresponding counts are: 18 of 36 (indices), 7 of 36 (stocks) and 26 of 36 (commodities). Results were generally robust to periods of market stress and extended hedging horizons.

The SLA hedge ratios were composed of four greeks. For calls, delta makes the strongest contribution, averaging about 85 percent of the hedge ratio for indices and stocks. The contributions to the hedge ratio for commodity hedges is a bit different. Delta accounted for about 55 percent of the SLA hedge and rho up to 15 percent. Contrasted to its role in hedging indices and stocks, rho was especially important in gold and silver hedges, perhaps due to the strong relationship between precious metals and interest rates.

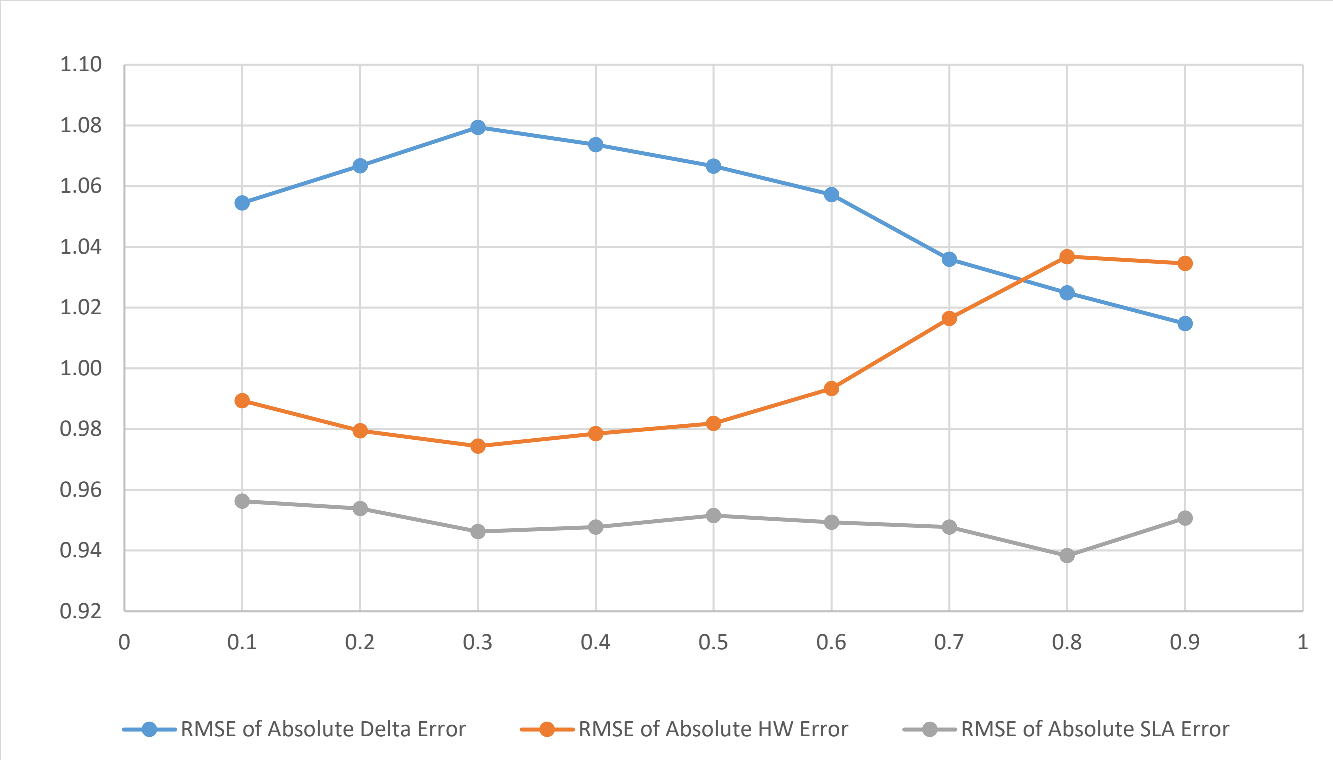
The main contribution of the paper is the novel use of the virtual interest rate. Due to violations of standard pricing assumptions, the virtual interest rates is in effect a pertubation of the observed interest rate. We find that the perturbed rate can be used to improve hedge ratios in a practitioner Black-Scholes model.

Figure I.1SPX Absolute Value Hedging Error for Calls by Delta Category



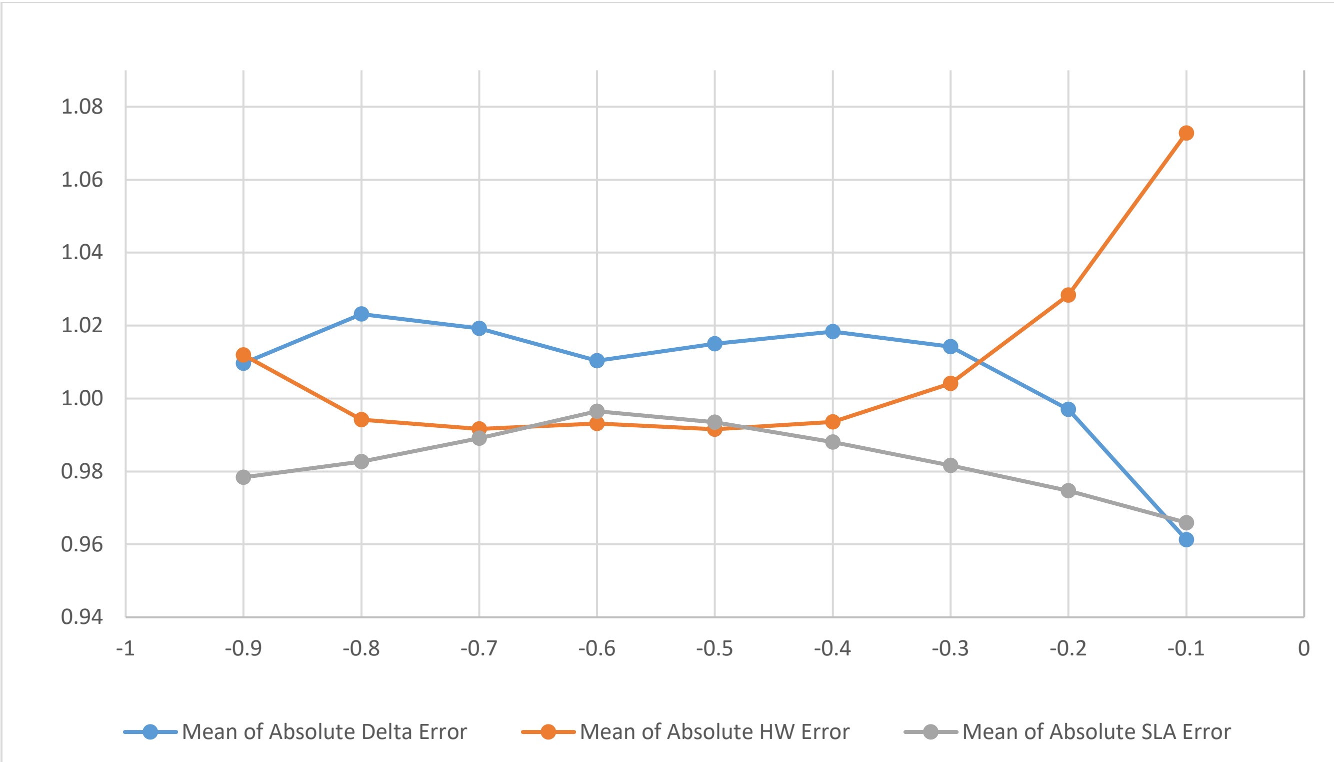
Mean Absolute Errors normalized by average call price. Hedge ratios are set by the Black-Scholes delta (δBS), Hull-White Minimum Variance delta (δHW) and Short Lived Arbitrage Model delta (δSLA). For data description see notes, Table I.5.

Figure I.2. SPX RMSE Hedging Errors for Calls by Delta Category



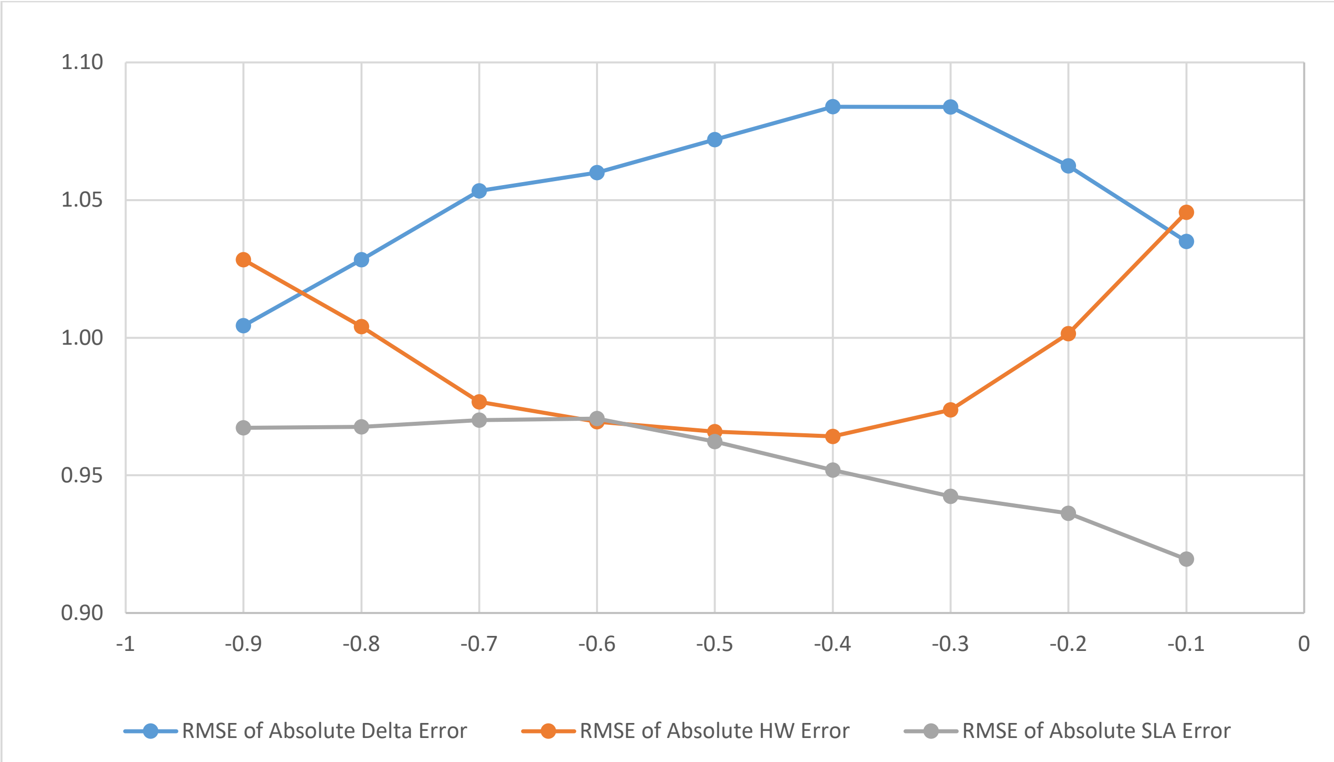
RMSE Errors normalized by average call price. See notes in Figure I.1.

Figure I.3. SPX Absolute Value Hedging Error for Calls by Delta Category



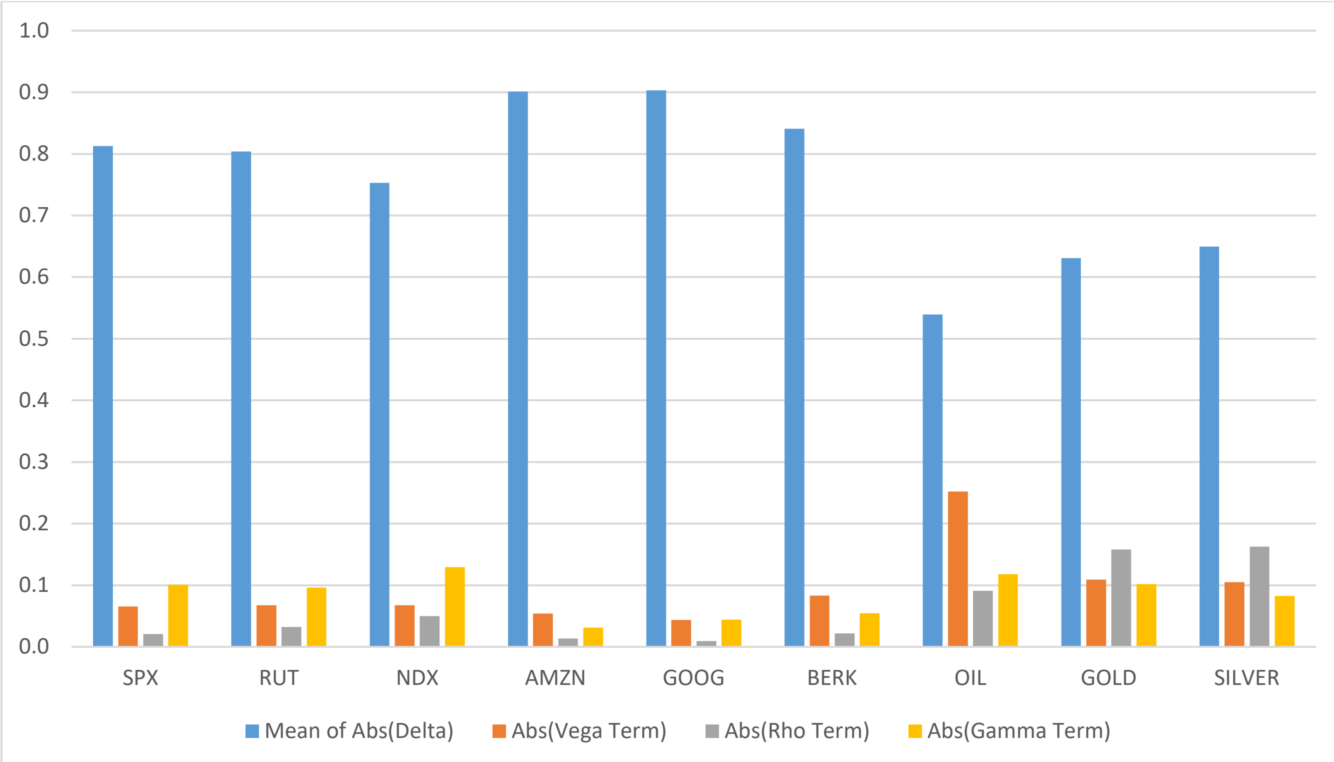
Mean Absolute Errors normalized by average put price. See notes Figure I.1.

Figure I.4. SPX RMSE Hedging Errors for Puts by Delta Category



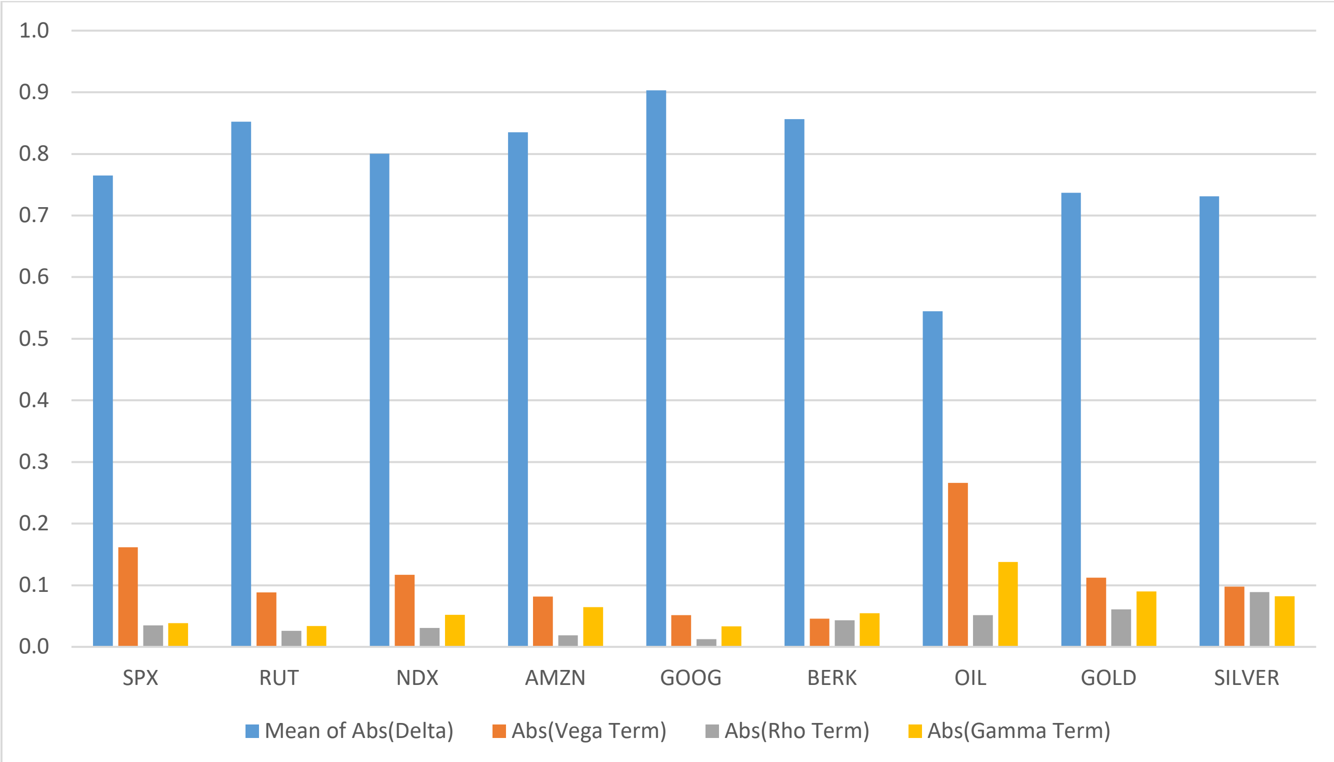
RMSE Errors normalized by average put price. See notes in Figure I.1.

Figure I.5. Average Contribution of Greek Terms to SLA Hedge Ratio for Calls



See notes, Table I.5.

Figure I.6. Average Contribution of Greek Terms to SLA Hedge Ratio for Puts



See notes, Table I.5.

Table I.1.Summary Statistics for Index Options

	S&P 500			RUSSELL 2000			NASDAQ 100		
Maturity	Number	Price Mean	Maturity Mean	Number	Price Mean	Maturity Mean	Number	Price Mean	Maturity Mean
Panel A: Call Options									
All	443,168	35.820	50.781	143,477	22.308	48.332	99,092	80.903	46.699
F14	169,001	26.092	22.417	59,357	16.952	22.170	43,742	56.567	21.699
F31	178,926	32.809	43.607	52,233	23.948	42.641	36,495	75.281	42.534
F61	53,659	48.428	73.280	19,965	26.727	73.426	10,591	125.481	73.387
F91	20,227	57.336	101.615	5,655	28.461	102.932	3,699	177.720	102.449
F123	8,963	70.395	148.387	3,643	32.792	147.071	2,410	159.454	148.118
F183	10,463	91.181	251.244	2,241	37.993	244.323	1,842	191.984	241.441
F366	1,929	130.098	501.476	383	82.066	513.292	313	240.549	551.824
Panel B: Put Options									
All	800,210	27.429	54.505	239,274	18.034	47.632	131,858	61.360	39.113
F14	268,728	18.629	22.510	83,425	12.758	22.832	60,747	48.014	21.665
F31	327,976	24.277	44.054	101,739	18.101	43.071	52,731	63.530	42.919
F61	109,416	34.269	73.161	40,664	22.882	72.724	13,938	87.654	72.318
F91	45,317	39.580	102.721	6,955	26.947	102.582	3,144	121.657	101.117
F123	24,693	50.704	147.968	3,667	34.725	146.909	843	120.836	144.005
F183	20,274	80.961	249.195	2,101	49.361	251.839	402	252.320	241.070
F366	3,806	142.754	460.180	723	83.162	514.221	53	312.511	640.132

The data is from OptionMetrics and includes daily observations from January 2, 2013 January 28, 2019. F14 are options with maturities between 14 and 30 days. F31 are options with maturities between 31 and 60 days. F31 are options with maturities between 31 and 60 days. Similarly, F61 have maturities from 61 to 90days, F91 from 91 to 122 days, F123 from 123 to 182 days, F183 from 183 to 365 days, and F366 over 365 days.

Table I.2. Sample Descriptions

	AMAZON			GOOGLE			BERKSHIRE HATHAWAY		
Maturity	Number	Price Mean	Maturity Mean	Number	Price Mean	Maturity Mean	Number	Price Mean	Maturity Mean
Panel A: Call Options									
All	381,791	57.048	120.965	235,021	34.831	110.870	49,088	7.671	198.390
F14	103,519	30.205	22.028	65,215	17.508	21.788	7,765	3.160	22.908
F31	96,063	39.650	42.406	60,547	24.366	43.432	9,870	3.775	42.409
F61	42,429	49.536	74.644	31,776	31.603	74.454	3,129	5.357	75.452
F91	28,792	65.898	105.033	16,330	39.980	104.948	3,611	5.870	105.888
F123	35,680	69.586	151.261	20,480	45.135	150.114	5,902	6.174	151.653
F183	43,968	90.463	252.278	24,913	59.874	254.488	9,364	9.016	255.658
F366	31,340	139.921	547.190	15,760	94.916	540.102	9,447	16.505	554.104
Panel B: Put Options									
All	340,026	48.345	129.006	173,426	23.718	86.145	45,605	4.213	190.049
F14	73,176	31.276	22.757	52,183	14.905	21.527	7,695	2.140	22.301
F31	92,337	35.970	42.864	50,193	19.525	44.306	8,985	2.486	43.425
F61	42,556	41.624	74.624	26,805	25.237	74.144	3,241	3.250	75.329
F91	27,111	51.570	105.046	10,844	29.588	104.332	3,541	3.369	105.550
F123	33,051	54.869	151.638	12,584	32.171	149.341	5,329	3.844	150.492
F183	41,270	69.013	255.968	15,509	40.753	264.573	8,856	5.075	260.629
F366	30,525	98.195	545.227	5,308	60.534	469.337	7,958	8.224	550.066

The data is from OptionMetrics and includes daily observations from January 2, 2013 January 28, 2019. F14 are options with maturities between 14 and 30 days. F31 are options with maturities between 31 and 60 days. F31 are options with maturities between 31 and 60 days. Similarly, F61 have maturities from 61 to 90days, F91 from 91 to 122 days, F123 from 123 to 182 days, F183 from 183 to 365 days, and F366 over 365 days.

Table I.3. Summary Statistics on Options on Commodity Futures

	Brent Crude Oil			100 oz Gold			5,000 oz Silver		
Maturity	Number	Price Mean	Maturity Mean	Number	Price Mean	Maturity Mean	Number	Price Mean	Maturity Mean
Panel A: Call Options									
All	70,676	2.101	85.895	145,717	117.669	289.501	57,702	2.031	349.979
F14	10,317	1.691	23.407	13,184	123.955	21.517	2,878	1.488	21.422
F31	22,089	1.612	45.151	14,022	117.177	44.957	3,419	1.513	45.861
F61	15,508	1.948	74.327	7,892	117.065	74.132	1,848	1.631	75.356
F91	8,968	2.360	105.243	6,175	113.682	105.473	1,982	1.846	106.344
F123	8,036	2.894	148.762	11,379	106.182	153.045	3,654	1.966	153.224
F183	5,126	3.544	244.651	38,692	110.848	274.827	15,082	2.083	277.939
F366	632	4.154	452.294	54,373	124.070	508.704	28,839	2.167	515.769
Panel B: Put Options									
All	75,345	2.129	80.768	149,683	183.439	285.261	91,092	5.393	293.673
F14	14,135	1.656	22.446	12,706	163.077	21.637	7,731	5.937	21.479
F31	22,796	1.690	44.940	15,230	177.740	44.949	9,134	5.737	45.063
F61	15,987	2.066	74.529	8,524	186.432	74.213	4,854	5.889	74.774
F91	9,520	2.329	105.106	6,817	196.819	105.426	4,361	5.949	105.930
F123	7,375	2.984	148.246	12,508	197.691	152.781	7,153	5.831	151.979
F183	5,026	3.743	243.107	39,572	184.766	273.985	22,599	5.410	274.254
F366	506	4.901	467.381	54,326	183.403	508.686	35,260	4.947	512.299

The data is from ICE and includes daily observations from January 2, 2013 January 28, 2019 (Brent Crude Oil) and from January 2, 2014 to June 28, 2019 (Gold and Silver). F14 are options with maturities between 14 and 30 days. F31 are options with maturities between 31 and 60 days. F31 are options with maturities between 31 and 60 days. Similarly, F61 have maturities from 61 to 90days, F91 from 91 to 122 days, F123 from 123 to 182 days, F183 from 183 to 365 days, and F366 over 365 days.

Table I.4. Summary Statistics on Commodity Futures

	Brent Crude Oil			100 oz Gold			5,000 oz Silver		
Maturity	Number	Price Mean	Maturity Mean	Number	Price Mean	Maturity Mean	Number	Price Mean	Maturity Mean
All	9,494	62.496	140.376	5,752	1283.557	268.409	5,775	16.624	269.302
F14	865	62.727	21.972	712	1283.613	21.463	697	16.410	21.311
F31	1,474	63.060	45.695	714	1271.234	44.723	711	16.398	44.827
F61	1,449	63.169	75.311	342	1278.208	73.792	331	16.396	74.601
F91	1,357	62.253	105.898	246	1281.362	105.301	276	16.549	105.641
F123	1,966	62.347	150.395	429	1267.800	152.632	428	16.362	152.056
F183	1,931	61.743	248.541	1,332	1294.935	274.209	1,332	16.682	273.796
F366	452	62.653	482.135	1,977	1284.941	513.307	2,000	16.845	512.434

The data is from ICE and includes daily observations from January 2, 2013 January 28, 2019 (Brent Crude Oil) and from January 2, 2014 to June 28, 2019 (Gold and Silver). F14 are options with maturities between 14 and 30 days. F31 are options with maturities between 31 and 60 days. F31 are options with maturities between 31 and 60 days. Similarly, F61 have maturities from 61 to 90days, F91 from 91 to 122 days, F123 from 123 to 182 days, F183 from 183 to 365 days, and F366 over 365 days.

Table I.5. Error Metrics and Significance Tests for Calls on Indices

Panel A. S&P 500			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	443,168	1.3370	2.2666	2.2668	2.769***	2.922***	2.828***
Delta Error	Implied	443,168	1.3662	2.2772	2.2781	3.002***	3.091***	3.021***
HW Error	Observed	443,168	1.2967	2.1135	2.1137	2.248**	2.142**	2.202**
HW Error	Implied	443,168	1.3066	2.1054	2.1055	2.081**	2.05**	2.102**
SLA Error	Implied	443,168	1.2673	2.0268	2.0269			

Panel B. Russel 2000			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	143,477	0.8806	1.4254	1.4258	2.626***	2.997***	2.594***
Delta Error	Implied	143,477	0.9434	1.5156	1.5160	4.000***	4.828***	4.246***
HW Error	Observed	143,477	0.8485	1.3446	1.3447	1.885*	2.023**	2.353**
HW Error	Implied	143,477	0.8400	1.3192	1.3193	2.033**	1.410	1.955*
SLA Error	Implied	143,477	0.8322	1.3057	1.3059			

Panel C. NASDAQ			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	99,092	3.6516	6.2788	6.2818	3.109***	2.535**	3.045***
Delta Error	Implied	99,092	4.0204	6.8713	6.8746	5.384***	4.955***	6.369***
HW Error	Observed	99,092	3.4848	6.0338	6.0389	0.660	1.305	2.775***
HW Error	Implied	99,092	3.4870	6.0371	6.0420	0.419	1.051	3.339***
SLA Error	Implied	99,092	3.4500	5.8252	5.8303			

Hedge ratios are BSM Delta, Hull-White (HW, 2017) minimum variance ratios and Short-Lived-Arbitrage (δSLA) minimum variance ratios. Greeks are computed using observed interest rates and implied virtual rates as denoted. The test statistics are Diebold-Marino using weekly average of error metrics versus the Short Lived Arbitrage (δSLA) metric over 260 weeks. Metrics are the mean error, the average of absolute errors (Avg|Error|), the standard deviation of errors (Std Dev) and the root mean square of errors (RMSE). Obs is the total number of observations over all days. The data is from the period January 2, 2013 through June 28, 2019.

Table I.6. Error Metrics and Significance Tests for Calls on Stocks

Panel A. Amazon			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	381,791	1.2933	2.0501	2.0571	1.664*	1.506	1.632
Delta Error	Implied	381,791	1.2848	1.9925	2.0030	2.139**	2.1**	2.181**
HW Error	Observed	381,791	1.2784	2.0676	2.0696	1.255	1.381	1.347
HW Error	Implied	381,791	1.2457	1.9894	1.9935	0.129	0.114	0.149
SLA Error	Implied	381,791	1.2446	1.9682	1.9755			

Panel B. Google			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	235,021	0.8345	1.1146	1.1164	2.33**	1.493	1.556
Delta Error	Implied	235,021	0.8484	1.1239	1.1260	6.097***	5.118***	5.785***
HW Error	Observed	235,021	0.8344	1.1145	1.1163	2.324**	1.487	1.548
HW Error	Implied	235,021	0.8483	1.1238	1.1259	6.09***	5.107***	5.775***
SLA Error	Implied	235,021	0.8171	1.0927	1.0941			

Panel C. Berkshire Hathaway			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	49,088	0.2023	0.2970	0.2973	2.209**	1.938*	1.668*
Delta Error	Implied	49,088	0.2125	0.3105	0.3109	3.594***	3.769***	3.375***
HW Error	Observed	49,088	0.2164	0.3679	0.3679	2.325**	2.454**	2.445**
HW Error	Implied	49,088	0.1973	0.2936	0.2936	2.741***	1.945*	2.552**
SLA Error	Implied	49,088	0.1929	0.2868	0.2869			

Hedge ratios are BSM Delta, Hull-White (HW, 2017) minimum variance ratios and Short-Lived-Arbitrage (δSLA) minimum variance ratios. Greeks are computed using observed interest rates and implied virtual rates as denoted. The test statistics are Diebold-Marino using weekly average of error metrics versus the Short Lived Arbitrage (δSLA) metric over 260 weeks. Metrics are the mean error, the average of absolute errors (Avg|Error|), the standard deviation of errors (Std Dev) and the root mean square of errors (RMSE). Obs is the total number of observations over all days. The data is from the period January 2, 2013 through June 28, 2019.

Table I.7. Error Metrics and Significance Tests for Calls on Commodities

Panel A. Brent Crude Oil			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	70,676	0.0885	0.1242	0.1364	1.291	1.693*	1.819*
Delta Error	Implied	70,676	0.0912	0.1376	0.1493	2.357**	2.118**	2.075**
HW Error	Observed	70,676	0.0876	0.1241	0.137	1.219	1.841*	1.981**
HW Error	Implied	70,676	0.0875	0.1244	0.1373	1.166	1.786*	1.911*
SLA Error	Implied	70,676	0.0849	0.1027	0.1169			

Panel B. Gold			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	145,717	0.9559	7.3609	7.401	1.979**	1.159	1.127
Delta Error	Implied	145,717	0.8841	7.0666	7.0856	2.527**	3.495***	3.276**
HW Error	Observed	145,717	0.8581	7.3609	7.3912	1.334	2.729**	2.762***
HW Error	Implied	145,717	0.8841	7.0666	7.0855	2.489**	3.493***	3.284***
SLA Error	Implied	145,717	0.8256	6.9804	6.9995			

Panel C. Silver			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	57,702	0.0343	0.1766	0.1779	1.889*	2.388**	2.266**
Delta Error	Implied	57,702	0.0388	0.1815	0.1828	2.671***	4.712***	4.348***
HW Error	Observed	57,702	0.0345	0.1777	0.179	2.213**	1.834*	1.734*
HW Error	Implied	57,702	0.0398	0.182	0.1834	3.673***	4.700***	4.249***
SLA Error	Implied	57,702	0.0324	0.1702	0.1717			

Hedge ratios are BSM Delta, Hull-White (HW, 2017) minimum variance ratios and Short-Lived-Arbitrage (δSLA) minimum variance ratios. Greeks are computed using observed interest rates and implied virtual rates as denoted. The test statistics are Diebold-Marino using weekly average of error metrics versus the Short Lived Arbitrage (δSLA) metric over 260 weeks. Metrics are the mean error, the average of absolute errors (Avg|Error|), the standard deviation of errors (Std Dev) and the root mean square of errors (RMSE). Obs is the total number of observations over all days. The data is from the period January 2, 2013 through June 28, 2019 for Brent Crude oil and January 2, 2014 through June 28, 2019 for gold and silver.

Table I.8. Gain in Squared Error Metric Ratios for Calls Using SLA

Panel A. Indices		Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Hedge	Rates	S&P 500			Russel 2000			NASDAQ		
Delta Error	Observed	1.1130	1.2506	1.2507	1.1198	1.1918	1.1920	1.1203	1.1618	1.1609
Delta Error	Implied	1.1622	1.2623	1.2633	1.2852	1.3474	1.3476	1.3580	1.3914	1.3903
HW Error	Observed	1.0471	1.0874	1.0875	1.0398	1.0605	1.0603	1.0202	1.0729	1.0728
HW Error	Implied	1.0630	1.0790	1.0791	1.0190	1.0208	1.0206	1.0216	1.0741	1.0739
SLA Error	Implied	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Panel B. Stocks		Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Hedge	Rates	Amazon			Google			Berkshire Hathaway		
Delta Error	Observed	1.0797	1.0849	1.0843	1.0432	1.0405	1.0411	1.0999	1.0720	1.0733
Delta Error	Implied	1.0657	1.0248	1.0279	1.0783	1.0580	1.0591	1.2132	1.1717	1.1738
HW Error	Observed	1.0550	1.1035	1.0975	1.0430	1.0403	1.0408	1.2584	1.6448	1.6441
HW Error	Implied	1.0017	1.0216	1.0182	1.0780	1.0578	1.0589	1.0456	1.0477	1.0472
SLA Error	Implied	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Panel C. Commodities		Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Hedge	Rates	Brent Crude Oil			Gold			Silver		
Delta Error	Observed	1.0880	1.4643	1.3615	1.3406	1.1120	1.1180	1.1219	1.0765	1.0744
Delta Error	Implied	1.1549	1.7959	1.6291	1.1467	1.0249	1.0247	1.4309	1.1370	1.1347
HW Error	Observed	1.0647	1.4615	1.3726	1.0804	1.1120	1.1151	1.1325	1.0890	1.0870
HW Error	Implied	1.0634	1.4688	1.3786	1.1468	1.0248	1.0247	1.5102	1.1430	1.1410
SLA Error	Implied	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table I.9. Error Metrics and Significance Tests for Puts on Indices

Panel A. S&P 500			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	800,210	0.9562	2.8465	2.8466	2.425**	2.011**	2.300**
Delta Error	Implied	800,210	0.9665	2.8006	2.8006	2.518**	2.135**	2.323**
HW Error	Observed	800,210	0.9491	2.788	2.7881	1.874*	0.386	0.574
HW Error	Implied	800,210	0.9503	2.7881	2.7882	1.524	0.201	0.099
SLA Error	Implied	800,210	0.9398	2.7451	2.7453			

Panel B. Russel 2000			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	239,274	0.9767	1.2117	1.2157	1.454	2.140**	1.954*
Delta Error	Implied	239,274	0.9829	1.2199	1.2247	1.935*	2.345**	2.238**
HW Error	Observed	239,274	0.9557	1.2062	1.2092	3.014***	2.578***	2.780***
HW Error	Implied	239,274	0.9534	1.2089	1.2126	4.357***	4.103***	3.881***
SLA Error	Implied	239,274	0.9508	1.2033	1.2071			

Panel C. NASDAQ			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	131,858	3.3467	6.2605	6.2638	1.005	1.466	1.384
Delta Error	Implied	131,858	3.3605	6.2302	6.2335	0.52	0.911	1.993**
HW Error	Observed	131,858	3.4061	6.0839	6.0906	2.142**	2.009**	-0.374
HW Error	Implied	131,858	3.4245	6.0628	6.07	1.474	1.443	0.59
SLA Error	Implied	131,858	3.3972	6.0458	6.0536			

Hedge ratios are BSM Delta, Hull-White (HW, 2017) minimum variance ratios and Short-Lived-Arbitrage (δSLA) minimum variance ratios. Greeks are computed using observed interest rates and implied virtual rates as denoted. The test statistics are Diebold-Marino using weekly average of error metrics versus the Short Lived Arbitrage (δSLA) metric over 260 weeks. Metrics are the mean error, the average of absolute errors (Avg|Error|), the standard deviation of errors (Std Dev) and the root mean square of errors (RMSE). Obs is the total number of observations over all days. The data is from the period January 2, 2013 through June 28, 2019.

Table I.10. Error Metrics and Significance Tests for Puts on Stocks

Panel A. Amazon			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	340,026	1.5485	2.917	2.9189	0.875	1.198	1.117
Delta Error	Implied	340,026	1.623	2.9202	2.9211	1.105	0.966	0.915
HW Error	Observed	340,026	1.5711	2.9964	2.9971	1.353	1.691*	1.608
HW Error	Implied	340,026	1.4951	2.6164	2.6172	0.007	0.619	-0.289
SLA Error	Implied	340,026	1.4735	2.6008	2.6021			

Panel B. Google			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	239,274	173,426	0.8454	1.2955	1.407	2.082**	2.308**
Delta Error	Implied	239,274	173,426	0.8443	1.2839	0.831	1.234	1.069
HW Error	Observed	239,274	173,426	0.8439	1.2981	1.399	1.472	1.916*
HW Error	Implied	239,274	173,426	0.8451	1.2888	1.375	1.241	1.393
SLA Error	Implied	239,274	173,426	0.845	1.2809			

Panel C. Berkshire Hathaway			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	45,605	0.1864	0.3134	0.3135	0.834	2.185**	2.179**
Delta Error	Implied	45,605	0.1883	0.3045	0.3047	1.49	1.831*	1.425
HW Error	Observed	45,605	0.1881	0.3123	0.3124	1.696*	2.371**	2.23**
HW Error	Implied	45,605	0.188	0.313	0.3132	1.633	2.232**	1.945*
SLA Error	Implied	45,605	0.1859	0.3035	0.3036			

Hedge ratios are BSM Delta, Hull-White (HW, 2017) minimum variance ratios and Short-Lived-Arbitrage (δSLA) minimum variance ratios. Greeks are computed using observed interest rates and implied virtual rates as denoted. The test statistics are Diebold-Marino using weekly average of error metrics versus the Short Lived Arbitrage (δSLA) metric over 260 weeks. Metrics are the mean error, the average of absolute errors (Avg|Error|), the standard deviation of errors (Std Dev) and the root mean square of errors (RMSE). Obs is the total number of observations over all days. The data is from the period January 2, 2013 through June 28, 2019.

Table I.11. Error Metrics and Significance Tests for Puts on Commodities

Panel A. Brent Crude Oil			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	75,345	0.082	0.117	0.1298	1.082	1.448	1.839*
Delta Error	Implied	75,345	0.0826	0.117	0.1169	2.081**	2.522**	2.355**
HW Error	Observed	75,345	0.0831	0.119	0.1312	1.424	1.832*	2.052**
HW Error	Implied	75,345	0.0865	0.120	0.1295	2.383**	2.708***	2.573**
SLA Error	Implied	75,345	0.0819	0.103	0.1152			

Panel B. Gold			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	149,683	1.0481	11.953	11.96111	1.769*	2.123**	1.892*
Delta Error	Implied	149,683	1.0426	11.962	11.97106	1.800*	2.683***	2.457**
HW Error	Observed	149,683	1.0201	12.014	12.02233	1.838*	2.889***	2.887***
HW Error	Implied	149,683	1.0273	12.007	12.0161	1.889*	3.797***	3.835***
SLA Error	Implied	149,683	0.8252	11.466	11.4757			

Panel C. Silver			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	91,092	0.0276	0.367	0.3678	3.006***	2.632***	2.274**
Delta Error	Implied	91,092	0.0271	0.345	0.3453	4.571***	3.473***	3.323***
HW Error	Observed	91,092	0.0272	0.368	0.3682	2.261**	2.134**	1.994**
HW Error	Implied	91,092	0.028	0.345	0.3457	3.386***	3.339***	3.144***
SLA Error	Implied	91,092	0.0259	0.306	0.3061			

Hedge ratios are BSM Delta, Hull-White (HW, 2017) minimum variance ratios and Short-Lived-Arbitrage (δSLA) minimum variance ratios. Greeks are computed using observed interest rates and implied virtual rates as denoted. The test statistics are Diebold-Marino using weekly average of error metrics versus the Short Lived Arbitrage (δSLA) metric over 260 weeks. Metrics are the mean error, the average of absolute errors (Avg|Error|), the standard deviation of errors (Std Dev) and the root mean square of errors (RMSE). Obs is the total number of observations over all days. The data is from the period January 2, 2013 through June 28, 2019 for Brent Crude Oil and January 2, 2014 through June 28, 2019 for Gold and Silver.

Table I.12. Gain in Squared Error Metric Ratios for Puts Using SLA

Panel A. Indices		Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Hedge	Rates	S&P 500			Russel 2000			NASDAQ		
Delta Error	Observed	1.0350	1.0750	1.0750	1.0550	1.0140	1.0140	0.9710	1.0720	1.0710
Delta Error	Implied	1.0580	1.0410	1.0410	1.0690	1.0280	1.0290	0.9790	1.0620	1.0600
HW Error	Observed	1.0200	1.0310	1.0310	1.0100	1.0050	1.0030	1.0050	1.0130	1.0120
HW Error	Implied	1.0230	1.0320	1.0320	1.0060	1.0090	1.0090	1.0160	1.0060	1.0050
SLA Error	Implied	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Panel B. Stocks		Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Hedge	Rates	Amazon			Google			Berkshire Hathaway		
Delta Error	Observed	1.1040	1.2580	1.2580	1.0010	1.0230	1.0230	1.0050	1.0670	1.0660
Delta Error	Implied	1.2130	1.2610	1.2600	0.9980	1.0050	1.0050	1.0260	1.0070	1.0070
HW Error	Observed	1.1370	1.3270	1.3270	0.9970	1.0270	1.0270	1.0240	1.0590	1.0590
HW Error	Implied	1.0300	1.0120	1.0120	1.0000	1.0120	1.0130	1.0230	1.0640	1.0640
SLA Error	Implied	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Panel C. Commodities		Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Hedge	Rates	Brent Crude Oil			Gold			Silver		
Delta Error	Observed	1.0020	1.2870	1.2710	1.4950	1.0870	1.0860	1.1280	1.4430	1.4440
Delta Error	Implied	1.0170	1.2970	1.0290	1.4990	1.0880	1.0880	1.0930	1.2730	1.2730
HW Error	Observed	1.0290	1.3240	1.2990	1.2560	1.0980	1.0980	1.0960	1.4470	1.4470
HW Error	Implied	1.1150	1.3480	1.2640	1.2600	1.0970	1.0960	1.1650	1.2760	1.2760
SLA Error	Implied	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Results for calls. Gain is (comparison metric/SLA metric)^2. See notes Table I.5.

Table I.13. Extended Horizon RMSE Results: Comparison of Hull-White MV Ratios and Short-Lived-Arbitrage MV Ratios

	Hedging horizon	RMSE Call Errors			RMSE Put Errors		
		Obs	HW	SLA	Obs	HW	SLA
Panel A. Index Options							
S&P500	5 days	443,168	2.1137	2.0269	800,210	2.7881	2.7453
S&P500	10 days	443,168	2.1503	2.0697	800,210	2.9012	2.7577
S&P500	20 days	443,168	2.1442	2.069	800,210	2.8836	2.747
Russell	5 days	143,477	1.3447	1.3059	239,274	1.2092	1.2071
Russell	10 days	143,477	1.3481	1.3007	239,274	1.225	1.225
Russell	20 days	143,477	1.347	1.2993	239,274	1.2226	1.2275
NASDAQ	5 days	99,092	6.0389	5.8303	131,858	6.0906	6.0536
NASDAQ	10 days	99,092	6.0824	5.9533	131,858	6.1148	6.0613
NASDAQ	20 days	99,092	6.0491	5.9202	131,858	6.0875	6.0536
Panel B. Stock Options							
Amazon	5 days	381,791	2.0696	1.9755	340,026	2.9971	2.6021
Amazon	10 days	381,791	2.0875	1.9847	340,026	3.0461	2.624
Amazon	20 days	381,791	2.0822	1.9906	340,026	3.071	2.6482
Google	5 days	235,021	1.1163	1.0941	173,426	1.2988	1.2814
Google	10 days	235,021	1.1163	1.0949	173,426	1.2971	1.2768
Google	20 days	235,021	1.1163	1.0937	173,426	1.3001	1.2773
Berkshire	5 days	49,088	0.3679	0.2869	45,605	0.3124	0.3036
Berkshire	10 days	49,088	0.3615	0.2885	45,605	0.3127	0.3037
Berkshire	20 days	49,088	0.3579	0.2869	45,605	0.3138	0.3045
Panel C. Commodities							
Brent Crude	5 days	70,676	0.137	0.1169	75,345	0.1312	0.1152
Brent Crude	10 days	70,676	0.1367	0.1178	75,345	0.1309	0.1156
Brent Crude	20 days	70,676	0.1371	0.1183	75,345	0.1313	0.1167
Gold	5 days	145,717	7.3912	6.9995	149,683	12.02233	11.4757
Gold	10 days	145,717	7.3912	6.9996	149,683	12.0215	11.4756
Gold	20 days	145,717	7.3912	7.0024	149,683	12.0204	11.4764
Silver	5 days	57,702	0.179	0.1717	91,092	0.3678	0.3061
Silver	10 days	57,702	0.179	0.1717	91,092	0.3682	0.3061
Silver	20 days	57,702	0.179	0.1717	91,092	0.3682	0.3129

See notes, Table I.5.

Table I.14. Error Metrics and Significance Tests for Calls During High Volatility

Panel A. S&P 500			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	85,116	1.2947	2.1322	2.1327	2.710**	2.481**	2.408**
Delta Error	Implied	85,116	1.3684	2.1323	2.1332	4.726***	4.660***	4.440***
HW Error	Observed	85,116	1.0820	1.7110	1.7136	2.442**	2.482**	2.296**
HW Error	Implied	85,116	1.0842	1.7144	1.7171	2.567**	2.643***	2.908***
SLA Error	Implied	85,116	1.0677	1.6769	1.6792			

Panel B. Google			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	109,224	0.8378	1.3890	1.3890	1.4280	1.0350	0.8900
Delta Error	Implied	109,224	0.8405	1.3692	1.3692	2.609***	1.3220	1.0080
HW Error	Observed	109,224	0.8377	1.3889	1.3889	1.4250	1.0320	0.8880
HW Error	Implied	109,224	0.8404	1.3691	1.3691	2.605***	1.3200	1.0060
SLA Error	Implied	109,224	0.8169	1.3570	1.3570			

Panel C. Brent Crude Oil			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	1,155,549	0.1298	0.2634	0.2634	1.1750	1.886*	1.940*
Delta Error	Implied	1,155,549	0.1517	0.2478	0.2478	2.105**	1.96**	2.192**
HW Error	Observed	1,155,549	0.1261	0.2615	0.2615	0.6680	1.745*	1.769*
HW Error	Implied	1,155,549	0.1494	0.2457	0.2457	2.689***	2.322**	2.796***
SLA Error	Implied	1,155,549	0.1286	0.2096	0.2096			

Hedge ratios are BSM Delta, Hull-White (HW, 2017) minimum variance ratios and Short-Lived-Arbitrage (δSLA) minimum variance ratios. Greeks are computed using observed interest rates and implied virtual rates as denoted. The test statistics are Diebold-Marino using weekly average of error metrics versus the Short Lived Arbitrage (δSLA) metric over 260 weeks. Metrics are the mean error, the average of absolute errors (Avg|Error|), the standard deviation of errors (Std Dev) and the root mean square of errors (RMSE). Obs is the total number of observations over all days. The data is from the period January 1, 2006 through December 31, 2010.

Table I.15. Error Metrics and Significance Tests for Puts During High Volatility

Panel A. S&P 500			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	95,623	1.1519	1.9180	1.9180	2.3900**	2.156**	2.212**
Delta Error	Implied	95,623	1.1226	1.8497	1.8497	2.086**	1.716*	1.840*
HW Error	Observed	95,623	1.0884	1.7429	1.7429	3.126***	2.871***	2.645**
HW Error	Implied	95,623	1.0888	1.7438	1.7439	3.162***	2.912***	2.575**
SLA Error	Implied	95,623	1.0752	1.7338	1.7338			

Panel B. Google			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	86,680	0.7579	1.2594	1.2603	0.205	0.937	0.767
Delta Error	Implied	86,680	0.7545	1.2458	1.2471	0.355	0.443	0.337
HW Error	Observed	86,680	0.7578	1.2594	1.2603	0.199	0.934	0.765
HW Error	Implied	86,680	0.7578	1.2594	1.2603	0.199	0.935	0.765
SLA Error	Implied	86,680	0.7535	1.2397	1.2412			

Panel C. Brent Crude Oil			Error			DM Test		
Hedge	Rates	Obs	Avg|Error|	Std Dev	RMSE	Avg|Error|	Std Dev	RMSE
Delta Error	Observed	1,563,921	0.1018	0.1695	0.1695	2.496**	1.812*	1.820*
Delta Error	Implied	1,563,921	0.1059	0.1651	0.1651	1.913*	1.802*	1.677*
HW Error	Observed	1,563,921	0.1004	0.1677	0.1677	2.506**	1.912*	1.857*
HW Error	Implied	1,563,921	0.1056	0.1656	0.1656	1.786*	1.674*	1.660*
SLA Error	Implied	1,563,921	0.0837	0.1622	0.1623			

Hedge ratios are BSM Delta, Hull-White (HW, 2017) minimum variance ratios and Short-Lived-Arbitrage (δSLA) minimum variance ratios. Greeks are computed using observed interest rates and implied virtual rates as denoted. The test statistics are Diebold-Marino using weekly average of error metrics versus the Short Lived Arbitrage (δSLA) metric over 260 weeks. Metrics are the mean error, the average of absolute errors (Avg|Error|), the standard deviation of errors (Std Dev) and the root mean square of errors (RMSE). Obs is the total number of observations over all days. The data is from the period January 1, 2006 through December 31, 2010.

Chapter II On the Dynamic Capital Structure of Nations: Theory and Empirics

2.1 Introduction

It is always interesting to come across new and novel ways to view different parts of the world in which one lives. Consider, for example, an article published in the October 2017 issue of The Economist: “Why national accounts might be like a corporate balance-sheet?" This article clearly caught our attention. It seemed at first glance to be a comparison stretched tremendously. Yet, after reading the paper by Bolton and Huang (2018), which was the basis for the article, it did not require a suspension of belief as regards the reasonableness of comparing the funding sources of a government to those of a company. Indeed, as they argue, fiat money is a claim on the output of a nation and is a reasonable equivalent to the equity of a firm, which is a claim on profits. Foreign-currency debt of a nation, moreover, is an analogy to a firm’s debt, which provides external funding. This similarity between the debt and equity of firms and the debt and fiat money of nations seems compelling enough to us to explore such a comparison further, following Bolton and Huang (2018).

In the corporate finance world, the optimal capital structure is typically determined by maximizing the value of a firm. In particular, the solution to such a maximization problem enables one to determine the optimal combination of debt and equity to fund a firm’s investments. In a perfect capital market, in which the Modigliano-Miller (1958) theorem holds, the value of a firm is independent of its capital structure, or any particular combination of debt and equity. Importantly, Bolton and Huang (2016) establish an analog to this theorem for a nation. More specifically, they point out that a nation’s investments can be financed by issuing fiat money, domestic-currency debt (which is a money-like claim), and foreign-currency debt, or a combination of the three. In a frictionless world, they argue it should not matter which combination of these funding options a nation chooses.

The classical trade-off theory in choosing the optimal capital structure for corporations involves weighing the tax advantage of debt against the financial distress it creates in the event of a default. This particular approach does not directly apply to nations because there is no tax advantage associated with foreign-currency debt. Instead, the trade-off for a nation is between the inflation risk of fiat money and the default risk of foreign-currency debt. When a nation issues more fiat money than the production of goods and services, the value of fiat money depreciates due to the resulting inflation. As is the case when existing equity holders find their holdings diluted with the issuance of new equity, some of the wealth of those holding fiat money is transferred to the holders of newly issued fiat money. Consequently, a nation provides a premium when issuing money-like claims (e.g., domestic-currency debt) to those acquiring such claims to compensate for inflation risk. In the case of foreign-currency debt, however, its value is not diluted when fiat money is printed. Yet, foreign-currency debt is subject to default risk, as Argentina's and Venezuela's recent defaults on U.S. dollar-denominated sovereign debts amply demonstrates.

The purpose of this paper is to pursue further the intriguing analogy identified by Bolton and Huang (2018). We do so by developing a dynamic model in order to determine the optimal combination of fiat money and foreign-currency debt used by a nation to fund its investments. The model that is used here was originally introduced by Brennan and Schwartz (1978) to quantitatively solve for the optimal capital structure of corporate firms. The use of a dynamic capital structure model of corporations or nations overcomes the defects of the static capital structure model in Bolton and Huang (2018). The defect is that financial decisions are inherently dynamic. For example, a firm typically adjusts its debt-to-equity ratio over time as the financial and economic environment in which it operates changes. Bolton and Huang's (2018) static model ignores a nation's option to seek additional funding and fails to consider the impact of existing debt on new financing decisions. It is not surprise that increases in the amount of existing debt contribute to the default risk of newly issued debt, thus affect the financing decisions. Also, traditional corporate finance models typically only provide qualitative implications. For example, Miller (1977) compares the cost of a bankruptcy to the tax benefit of debt as the "horse and rabbit stew", but no proof is provided as to the optimal size of either the horse or the rabbit.

Importantly, Brennan and Schwartz (1978) are the first to develop a model to determine the optimal capital structure quantitatively. Based upon a diffusion processes to model firm value over time and a numerical solution approach, they are able to determine the optimal mix of debt and equity. Fischer, et al. (1989) extend the paper by Brennan and Schwartz (1978) by developing a dynamic capital structure model that allows for changes over time in firm value, cash flows, transaction costs, interest rates and the macroeconomic environment, more generally. Goldstein, et al. (2001) also develop a dynamic capital structure model related to that of Fischer, et al. (1989), and find that the optimal debt and equity mix based on the model is close to the observed actual mix.

The dynamic model used in this paper is a stochastic control problem and follows from the work of Goldstein, et al. (2001). In particular, it is assumed that a nation maximizes the value of its investments by choosing the optimal combination of fiat money (and money-like domestic-currency debt) and foreign-currency debt. Based on Miller's (1977) seminal paper, one is able to obtain a closed-form solution to our stochastic control problem. In general, we find increased inflation risk leads to investments funded more by foreign-currency debt, while greater default risk leads to investments funded by more fiat money or money-like domestic-currency debt.

Our paper contributes to the existing literature along two dimensions. First, this is the first paper that studies the dynamic capital structure of a nation from a corporate perspective. This brings new framework in studying how a nation finances its economy. Second, Bolton and Huang (2018) is an innovative theoretical work but lacks a thorough empirical support. We collect data of fiat money and debts of 22 emerging economies, which lend empirical supports to our hypotheses implied from the dynamic model.

The remainder of the paper proceeds as follows. Section 2 develops the basic dynamic capital structure model for a nation, provides the optimal mix of fiat money and debt, and discusses the implications for our empirical work. Section 3 explains the hypotheses that flow from the model, describes the sample data used to test the hypotheses, and provides the results of the tests performed.

2.2 Optimal Capital Structure for Nations

2.2.1 Model Setup

Consider a nation with a small open economy that plans to invest V_0in a project at time zero to enhance productive capability. A representative agent runs the project and its value V_t evolves over time according to a geometric Brownian motion process as follows:

$$\frac{dV_t}{V_t} = \mu dt + \sigma dB_t, \tag{1}$$

where B_t is a Brownian motion with Normal distribution $N(0, \sigma^2 t)$ and the initial value of investment is V_0.

A nation can issue fiat money, domestic-currency debt, foreign-currency debt, or a combination of the three to fund the investment. For simplicity, assume the two types of debt are perpetual. In this regard, according to Bloomberg data and BMI Research Report, perpetual bonds globally totaled \$215 billion for 784 deals in 2017. Nations that issue perpetual bonds, some through state-owned banks and corporations, include Argentina, Brazil, China, and India. Assume foreign-currency debt contributes a portion, p, of the investment, domestic-currency debt a portion, q, and the remaining portion, l ($l = 1 - p - q$), is provided by fiat money.

When the government does not default on its debt, any claim on the investment project continuously pays a nonnegative coupon, C, per instant of time, with an interest rate, r. Denoting the value of such a claim by $F(V_t, t)$, based on Black and Cox (1976), it satisfies the following partial differential equation (PDE):

$$\mu V_t F_v(V_t,t) + \frac{1}{2}\sigma^2 V_t^2 F_{vv}(V_t,t) - rF(V_t,t) + F_t(V_t,t) + C = 0. \quad (2)$$

For a debt that pays a constant time-independent cash flow, $F_t(V_t, t) = 0$, so the PDE for debt reduces to the following:

$$\mu V_t F_v(V_t,t) + \frac{1}{2}\sigma^2 V_t^2 F_{vv}(V_t,t) - rF(V_t,t) + C = 0, \quad (3)$$

which has a solution of the following form,

$$F(V_t,t) = A_0 + A_1 V_t^{-x} + A_2 V_t^{-y}, \quad (4)$$

where $x = \frac{1}{\sigma^2}\left[\left(\mu - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right]$ and $y = \frac{1}{\sigma^2}\left[\left(\mu - \frac{\sigma^2}{2}\right) - \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right]$.

Note that $A_0, A_1,$ and A_2 are determined by boundary values and $x > 0$ and $y < 0$.

2.2.2 Foreign- and Domestic-Currency Debt and Fiat Money

Clearly, foreign-currency debt has default risk. If an investment project does not generate sufficient output a nation can decide to default on the debt, and, if so, debtholders suffer losses. Debtholders are not in a position to seize a nation's assets. For this reason, if a threshold, V_B, exists, a nation declares default if an investment project has a value, V_t, that is below the threshold. Denoting foreign-currency debt by $D_f(V_t, t)$, then

$$lim_{V_t \to V_B} D_f(V_t,t) = 0 \text{ and } lim_{V_t \to \infty} D_f(V_t,t) = \frac{C_f}{r},$$

where c_f is the coupon of foreign-currency debt and *r* is the interest rate. Thus, the value of foreign-currency debt is as follows:

$$D_f(V_t, t) = \frac{c_f}{r}\left[1 - \left(\frac{V_B}{V_t}\right)^x\right]. \qquad (5)$$

Assume a nation only issues foreign-currency debt and $\frac{c_f}{r} = V_0$. Debtholders require a premium to compensate for the default risk associated with the debt. Such a premium depends on the investment level V_0 and the portion of foreign-currency debt, *p* (*p=1* in this case). Denote the total cost (DC_f) of such debt, including principal and interest, as follows:

$$DC_f(V_0, V_B, V_t, p) = \alpha_1(V_0, p)\left[1 - \left(\frac{V_B}{V_t}\right)^x\right]. \qquad (6)$$

Included in this equation is the risk premium, which is $\frac{\alpha_1(V_0,p)}{c_f/r} - 1$. It is intuitive to assume that $\alpha_1(V_0, p) \geq \frac{c_f}{r}$ because the premium should be nonnegative. In addition, $\frac{\partial\alpha_1(V_0,p)}{\partial V_0} > 0$, and $\frac{\partial\alpha_1(V_0,p)}{\partial p} > 0$, since greater investment funded by debt contributes to higher default risk.

In contrast, domestic-currency debt contributes to inflation risk, since a nation can print fiat money not only to fund an investment project but also satisfy its domestic-currency debt obligations. To compensate for any potential losses in purchasing power, domestic debtholders will require a premium (higher yield) to compensate for the de facto default risk due to potential inflation. By a similar argument used for foreign-currency debt, the total cost of domestic-currency debt incorporating an inflation factor β is expressed as follows:

$$DC_d(V_0, V_B, V_t, q, \beta) = \alpha_2(\beta, V_0, q)\left[1 - \left(\frac{\psi V_B}{V_t}\right)^x\right]. \qquad (7)$$

Again, $\frac{\partial\alpha_2(\beta,V_0,p)}{\partial V_0} > 0$, $\frac{\partial\alpha_2(\beta,V_0,p)}{\partial q} > 0$, and $\frac{\partial\alpha_2(\beta,V_0,p)}{\partial \beta} > 0$. Let ψ indicate that the default thresholds of foreign- and domestic-currency debt are correlated so that the threshold for the latter debt is ψV_B. In general, when an investment project underperforms, thereby lowing the value of the debt used to finance the project, a nation is more likely to repay its foreign-currency debt before its domestic-currency debt. The reason is that a nation can print fiat money to repay its domestic-currency debt but this option is not available in the case of foreign-currency debt.

For fiat money to be acceptable as a store of value, a nation provides a premium to compensate for inflation risk. The total cost of issuing fiat money as a funding source is expressed as: $FC(V_0, l, \beta) = \alpha_3(\beta, V_0, l)$, where $\alpha_3(\beta, V_0, l)$ has same properties as $\alpha_2(\beta, V_0, q)$.

The residual claims or equity value, which belongs to the nation, of the investment project at time t after subtracting both the foreign- and domestic-currency debt is expressed as follows:

$$\mathrm{E}(V_0, V_B, V_t, p, q, l, \beta) = \mathrm{E}\left[V_t - DC_f(V_0, V_B, V_t, p) - DC_d(V_0, V_B, V_t, q, \beta) - FC(\beta, V_0, l)\right]. \quad (8)$$

2.2.3 Optimization Problem and Solution

What is the optimal combination of fiat money, domestic- and foreign-currency debt? The answer involves solving the following optimization problem:

$$\max_{\substack{p,q,l\in(0,1) \\ p+q+l=1}} \mathrm{E}(V_0, V_B, V_t, p, q, l, \beta) \quad (9)$$

at time 0 when a nation decides on the mixture of funding sources. To proceed, assume $\alpha_1(V_0, p) = (V_0 p)^\theta$, $\alpha_2(\beta, V_0, q) = (\beta V_0 q)^\theta$ and $\alpha_3(\beta, V_0, l) = (\beta V_0 l)^\theta$ for some $\theta \in (1,2)$ for simplicity. We choose the range $(1,2)$ for θ for two purposes. First, $\theta \in (1,2)$ assures that, taking domestic-currency debt for example, the premium $(\beta V_0 q)^{\theta-1} - 1$ is positive. Second, risk-averse investors require a larger premium for holding additional debt. Apparently, the premium $(\beta V_0 q)^{\theta-1} - 1$ is a concave function with respect to the face value. The first order derivative $(\theta - 1)(\beta V_0 q)^{\theta-2}$ is positive and the second order derivative $(\theta - 1)(\theta - 2)(\beta V_0 q)^{\theta-3}$ is negative when $\theta \in (1,2)$.

Based on the following first-order conditions (FOC),

$$0=\frac{\partial \mathrm{E}(V_0,V_B,V_t,p,q,l,\beta)}{\partial p}=-\left[1-\left(\frac{V_B}{V_0}\right)^x\right]\theta p^{\theta-1}V_0^\theta+V_0^\theta\beta^\theta\theta(1-p-q)^{\theta-1},$$

$$0=\frac{\partial \mathrm{E}(V_0,V_B,V_t,p,q,l,\beta)}{\partial q}=-\left[1-\left(\frac{\psi V_B}{V_0}\right)^x\right]\theta(1-p)^{\theta-1}(\beta V_0)^\theta+V_0^\theta\beta^\theta\theta(1-p-q)^{\theta-1},$$

the optimal foreign-currency debt portion is

$$p^*(V_0,V_B,\beta)=\frac{\left[1-\left(\frac{\psi V_B}{V_0}\right)^x\right]^{\frac{1}{\theta-1}}\beta^{\frac{\theta}{\theta-1}}}{\left[1-\left(\frac{\psi V_B}{V_0}\right)^x\right]^{\frac{1}{\theta-1}}\beta^{\frac{\theta}{\theta-1}}+\left[1-\left(\frac{V_B}{V_0}\right)^x\right]^{\frac{1}{\theta-1}}}\,, \tag{10}$$

the optimal domestic-currency debt portion is

$$q^*(V_0,V_B,\beta)=\frac{\left[1-\left(\frac{V_B}{V_0}\right)^x\right]^{\frac{1}{\theta-1}}-\left[1-\left(\frac{\psi V_B}{V_0}\right)^x\right]^{\frac{1}{\theta-1}}\left[1-\left(\frac{V_B}{V_0}\right)^x\right]^{\frac{1}{\theta-1}}}{\left[1-\left(\frac{\psi V_B}{V_0}\right)^x\right]^{\frac{1}{\theta-1}}\beta^{\frac{\theta}{\theta-1}}+\left[1-\left(\frac{V_B}{V_0}\right)^x\right]^{\frac{1}{\theta-1}}}, \tag{11}$$

and the optimal fiat money portion is

$$l^*(V_0,V_B,\beta)=\frac{\left[1-\left(\frac{\psi V_B}{V_0}\right)^x\right]^{\frac{1}{\theta-1}}\left[1-\left(\frac{V_B}{V_0}\right)^x\right]^{\frac{1}{\theta-1}}}{\left[1-\left(\frac{\psi V_B}{V_0}\right)^x\right]^{\frac{1}{\theta-1}}\beta^{\frac{\theta}{\theta-1}}+\left[1-\left(\frac{V_B}{V_0}\right)^x\right]^{\frac{1}{\theta-1}}}. \tag{12}$$

The second-order condition (SOC) indicates that the maximum is obtained at p^*, q^* and l^*.

2.2.4 Dynamic Capital Structure of Nations

The above analysis assumes that a nation has no other investment or debt at time 0. However, it may have multiple investment projects and related outstanding debt when issuing new debt for a new investment project. To derive the dynamic optimal debt structure, assume that a nation has an existing investment project $\bar{V}_t$, that follows the same geometric Brownian motion as V_t. In this case, coupons $\bar{C}_f$ and $\bar{C}_d$ are paid on outstanding foreign- and domestic-currency debt, respectively. If a nation decides to invest V_0 for a new project, the total investment value at time *t* becomes $V_t + \bar{V}_t$.

Since the additional debt contributes to a higher default risk and greater inflation potential, investors require a higher yield on the new debt due to the greater outstanding debt. Denote the greater outstanding debt influence on the new debt premium by the function *f*. The new foreign-currency debt cost is

$$\widehat{DC}_f(V_0, V_B, V_t, p) = f(\bar{C}_f, r_f)\alpha_1(V_0, p)\left[1 - \left(\frac{V_B}{V_t + \bar{V}_t}\right)^x\right], \tag{13}$$

where $f(\bar{C}_f, r_f)$ is the impact of the outstanding foreign-currency debt on the additional foreign-currency debt premium. It is reasonable to assume the following about the premium:

$$f(\bar{C}_f, r_f) > 1 \text{ and } \frac{\partial f(\bar{C}_f, r_f)}{\partial C_f} > 0.$$

To elaborate, $f(\bar{C}_f, r_f)$ essentially serves as a borrowing constraint in that it prevents countries from entering into a situation of "debt immiseration"[7]. As the debt premium increases with additional debt issuance, it becomes a penalty and thereby incentivizes countries from ever increasing their debt as well as defaulting on existing debt.

Similarly, the new domestic-currency debt cost is

$$\widehat{DC}_d(V_0, V_B, V_t, q) = f(\bar{C}_d, r_d)\alpha_2(\beta, V_0, q)\left[1 - \left(\frac{\psi V_B}{V_t + \bar{V}_t}\right)^x\right]. \tag{14}$$

The outstanding foreign- and domestic-debt costs follow equations 6 and 7, but assuming debt is perpetual,

$$\overline{DC}_f(\bar{C}_f, r_f, V_B, V_t) = \frac{\bar{C}_f}{r_f}\left[1 - \left(\frac{V_B}{V_t + \bar{V}_t}\right)^x\right], \tag{15}$$

and

[7] See Campbell and Hercowitz (2019) for a discussion of this issue.

$$\overline{DC}_d(\bar{C}_d, r_d, V_B, V_t) = \frac{\bar{C}_d}{r_d}\left[1 - \left(\frac{\psi V_B}{V_t + \bar{V}_t}\right)^x\right]. \quad (16)$$

What is the dynamic optimal capital structure of a nation when outstanding debt exists? The answer is found by maximizing the following:

$$\max_{\substack{p,q,l \in (0,1) \\ p+q+l=1}} \mathrm{E}\left(V_0, \bar{V}_0, V_B, \bar{V}_B, V_t, \bar{C}_f, \bar{C}_d, p, q, l, \beta\right), \quad (17)$$

where

$$\mathrm{E}\left(V_0, \bar{V}_0, V_B, \bar{V}_B, V_t, \bar{C}_f, \bar{C}_d, p, q, l, \beta\right) = V_t + \hat{V}_t - \widehat{DC}_f(V_0, V_B, V_t, p) - \widehat{DC}_d(V_0, V_B, V_t, q)$$
$$-\overline{DC}_f\left(\bar{C}_f, r_f, V_B, V_t\right) - \overline{DC}_d(\bar{C}_d, r_d, V_B, V_t). \quad (18)$$

By implementing the FOCs, the optimal solution is

$$p^*\left(V_0, V_B, \bar{C}_f, \bar{C}_d, \beta\right) = \frac{\left[f(\bar{C}_d, r_d)\left[1 - \left(\frac{\psi V_B}{V_t + \bar{V}_t}\right)^x\right]\beta^\theta\right]^{\frac{1}{\theta-1}}}{\left[f(\bar{C}_d, r_d)\left[1 - \left(\frac{\psi V_B}{V_t + \bar{V}_t}\right)^x\right]\beta^\theta\right]^{\frac{1}{\theta-1}} + \left[f\left(\bar{C}_f, r_f\right)\left[1 - \left(\frac{V_B}{V_t + \bar{V}_t}\right)^x\right]\right]^{\frac{1}{\theta-1}}}, \quad (19)$$

$$q^*\left(V_0, V_B, \bar{C}_f, \bar{C}_d, \beta\right) = \frac{\left[f\left(\bar{C}_f, r_f\right)\left[1 - \left(\frac{V_B}{V_t + \bar{V}_t}\right)^x\right]\right]^{\frac{1}{\theta-1}}}{\left[f(\bar{C}_d, r_d)\left[1 - \left(\frac{\psi V_B}{V_t + \bar{V}_t}\right)^x\right]\beta^\theta\right]^{\frac{1}{\theta-1}} + \left[f\left(\bar{C}_f, r_f\right)\left[1 - \left(\frac{V_B}{V_t + \bar{V}_t}\right)^x\right]\right]^{\frac{1}{\theta-1}}}$$
$$- \frac{\left[f(\bar{C}_d, r_d)\left[1 - \left(\frac{\psi V_B}{V_t + \bar{V}_t}\right)^x\right]\right]^{\frac{1}{\theta-1}}\left[f\left(\bar{C}_f, r_f\right)\left[1 - \left(\frac{V_B}{V_t + \bar{V}_t}\right)^x\right]\right]^{\frac{1}{\theta-1}}}{\left[f(\bar{C}_d, r_d)\left[1 - \left(\frac{\psi V_B}{V_t + \bar{V}_t}\right)^x\right]\beta^\theta\right]^{\frac{1}{\theta-1}} + \left[f\left(\bar{C}_f, r_f\right)\left[1 - \left(\frac{V_B}{V_t + \bar{V}_t}\right)^x\right]\right]^{\frac{1}{\theta-1}}}, \quad (20)$$

and

$$l^*\left(V_0, V_B, \bar{C}_f, \bar{C}_d, \beta\right) = \frac{\left[f(\bar{C}_d, r_d)\left[1 - \left(\frac{\psi V_B}{V_t + \bar{V}_t}\right)^x\right]\right]^{\frac{1}{\theta-1}}\left[f\left(\bar{C}_f, r_f\right)\left[1 - \left(\frac{V_B}{V_t + \bar{V}_t}\right)^x\right]\right]^{\frac{1}{\theta-1}}}{\left[f(\bar{C}_d, r_d)\left[1 - \left(\frac{\psi V_B}{V_t + \bar{V}_t}\right)^x\right]\beta^\theta\right]^{\frac{1}{\theta-1}} + \left[f\left(\bar{C}_f, r_f\right)\left[1 - \left(\frac{V_B}{V_t + \bar{V}_t}\right)^x\right]\right]^{\frac{1}{\theta-1}}}. \quad (21)$$

2.3 Hypotheses, Data and Empirical Results

2.3.1 Capital Structure Hypotheses

Inflation is an essential determinant of the composition of sovereign debt. A government with domestic-currency debt has an incentive to inflate away so as to mitigate the debt burden. A higher level of domestic-currency debt increases such incentive. Inflation reduces the purchasing power of fiat money as well as the value of domestic-currency debt, thereby shifting investor preferences for foreign-currency debt. Claessens et al. (2007) use panel data from 35 countries to study the determinants of development of domestic-currency government debt market. They find that lower inflation rates associate with lower volatilities of inflation and, consequently, a lower tendency for government to dilute its debt through inflation. The mitigated inflation risk contributes to the development and expansion of domestic-currency government debt markets. Their result is consistent with Ize and Yeyati's (2003) finding that, in minimum variance portfolio equilibria, less volatility in inflation contributes to the deepening of financial dollarization. Burger and Warnock (2006) also share a similar finding that countries with stable inflation rates and strong creditor rights have more developed local bond markets and rely less on foreign-currency-denominated bonds.

An alternative explanation of the negative relationship between the level of inflation and domestic-currency debt issuance is that governments with high inflation do not need to issue large amounts of domestic-currency debt, as the inflation tax is a major source of government revenue. At the same time, governments with more fixed regimes may want to signal the credibility of their regime by issuing relatively more foreign currency debt. Torre et al. (2003) claim that issuing domestic-currency debt is more costly than foreign-currency debt, the additional cost incurred could be interpreted as a hedge against a future devaluation.

Above analysis is consistent with our model since, following equation 19-21,

$$\frac{\partial p^*}{\partial \beta} > 0, \frac{\partial q^*}{\partial \beta} < 0, \text{ and } \frac{\partial l^*}{\partial \beta} < 0.$$

Our first hypothesis is a direct description of our model's prediction regarding inflation's effects on a nation's financing decision:

H1: Higher inflation risk increases the issuance share of foreign-currency debt but reduces the issuance shares of both domestic-currency debt and fiat money.

The ability and the willingness to pay is another important factor that determines the composition of sovereign debt. Given a nation's willingness to pay, its foreign reserves partially determine its ability to pay off its foreign-currency debt and, consequently, the default risk. Paolo and Roubini (2009) find that a nation with relatively higher default risk is described by a handful of economic characteristics: high ratio of total external debt to the capacity to pay, high short-term debt over foreign reserves, etc.

The increased default risk contributes to the additional cost of the debt. Using panel data from 16 emerging countries between year 1998 and 2002, Rowland and Torres (2004) reveal that a nation with less foreign reserves has to provide a higher yield for its debt. Min (1998) applies cross sectional analysis to sovereign bond data from 11 countries between year 1991 and 1995. He concludes that higher yield spreads are associated with a lower ratio of foreign reserves to GDP and other macroeconomic fundamentals. Similar findings also can be found in Budina and Mantchev (2000) for Bulgarian Brady bonds and Rojas and Jaque (2003) for Chilean sovereign bonds.

Given a nation's willingness to pay, a lower level of default chance, which is measured by the threshold V_B, implied by sufficient foreign reserves mitigates the default risk of foreign-currency debt and contributes to the development of foreign-currency bond market. This analysis is reflected in equation 19-21:

$$\frac{\partial p^*}{\partial V_B} < 0, \frac{\partial q^*}{\partial V_B} > 0, \text{ and } \frac{\partial l^*}{\partial V_B} > 0;$$

and implies our second hypothesis:

H2: Higher default risk reduces the issuance share of foreign-currency debt but increases the proportion of new money-like claims.

A higher level of existing debt brings debt service difficulties and, consequently, possible liquidity problems. It is expected that a higher debt service level compromises the degree of creditworthiness, resulting in a higher yield spread (Min, 1998). An increase in foreign-currency debt leads to higher default risk, while greater accumulated domestic-currency debt increases inflation risk. The increased risk of both have a negative impact on future debt issuance. This is consistent with the model since

$$\frac{\partial p^*}{\partial \bar{C}_f} < 0,\ \frac{\partial q^*}{\partial \bar{C}_d} < 0, \text{ and } \frac{\partial l^*}{\partial \bar{C}_f} > 0.$$

Our third hypothesis is a direct description of the above partial derivatives:

H3: Outstanding debt reduces the issuance shares of both foreign-currency and domestic-currency debt.

Eichengreen and Mody (1998) study almost 1,000 developing country bonds issued over 1991 to 1996 and find that the debt launches depend on not only the existing debt level measured by debt-to-GDP level but also the issue size. If a nation issues too much debt, it may be unable to handle the burden the debt imposes due to the associated principle and interest payments. If this happens, the nation is less likely to pay its domestic-currency debt but instead focus more on repaying its foreign-currency debt. In the same situation, investors prefer to hold foreign-currency debt, since:

$$\frac{\partial p^*}{\partial V_0} > 0,\ \frac{\partial q^*}{\partial V_0} < 0 \text{ and } \frac{\partial l^*}{\partial V_0} < 0,$$

which implies our fourth hypothesis:

H4: The larger the initial investment the lower the issuance of domestic-currency debt, while the higher the issuance of foreign-currency debt.

2.3.2 Sample and Data

The sample used for the empirical analysis consists of quarterly data for domestic- and foreign-currency debt of the central government of 22 emerging countries, including Argentina, Brazil, Bulgaria, Chile, China, Columbia, Hungary, India, Indonesia, Latvia, Lithuania, Malaysia, Mexico, Peru, Philippines, Poland, Romania, South Africa, Thailand, Turkey, Ukraine and Uruguay, and covers the period 2004-2015. Data for sovereign bonds issued by the same countries come from Bloomberg. Information for each bond includes the institutional name of the debtor, the issue date, maturity, face value, coupon, price, yield to maturity, credit default swap (CDS) spread, and currency denomination. Inflation and GDP data come from World Bank Indicators, while the source of quarterly sovereign debt data is a new dataset constructed by Arslanalp and Tsuda (2014). Summary statistics for the variables are in Table II.1.

Table II.2 contains basic information on sovereign debt and its composition in terms of foreign and domestic type by country. The first column shows the average annual level of debt issuance as a percentage of GDP for the various countries in the sample. Total issuance is the sum of the face value of debt issued in a given year for a given country, and the average level of issuance is 7.63 percent of GDP. The second column shows the average share of debt issuance denominated in domestic-currency for the sample period, which is 68.65 percent.

2.3.3 Empirical Results

The first hypothesis test involves the relation between a change in the share of domestic-currency debt and inflation. Higher inflation should lower the value of domestic-currency debt, so that investors are less likely to purchase additional such debt. The following three regressions are therefore estimated:

$$Pct_f = \alpha_1 + \alpha_2 Inflation + controls + e,$$

$$Pct_d = \alpha_1 + \alpha_2 Inflation + controls + e,$$

$$Pct_m = \alpha_1 + \alpha_2 Inflation + controls + e,$$

where Pct_f, Pct_d and Pct_m denote the shares of foreign debt, domestic debt and fiat money, respectively. *Inflation* is the lagged quarterly inflation rate, using the consumer price index. Table II.3 shows there is a significantly positive relationship between inflation and the change in the proportion of foreign- currency debt, as hypothesized. In terms of economic significance, a one percentage point increase in inflation is associated with a 3.56 percent increase in the share of foreign-currency debt, which is about 10 percent of the average value of such debt for the countries in our sample. Inflation and both an increase in the issuance of domestic- currency debt and fiat money are negatively related, but the relationships are not statistically significant.

The second hypothesis test involves the relationship of default risk to the composition of funding sources. The value-weighted Credit Default Swap (CDS) spread is used as a proxy for market default risk. Thus, the following two regressions are estimated:

$$Pct_f = \alpha_1 + \alpha_2 CDS\ Spread + \alpha_3 CDS\ Spread^2 + controls + e,$$

$$Pct_d + Pct_m = \alpha_1 + \alpha_2 CDS\ Spread + \alpha_3 CDS\ Spread^2 + controls + e,$$

where *CDS_Spread* is the average, value-weighted CDS spread for foreign-currency debt issued during a quarter. Table II.4. Panel A confirms the hypothesis of a negative relationship between default risk and the additional issuance of foreign-currency debt, with a nation tilting towards money-like claims for financing an investment project. The results indicate, moreover, that a ten-basis point increase in the CDS spread associated with foreign-currency debt is related to 3-percentage point decline in the share of foreign-currency debt. In addition, the ratio of foreign exchange reserves-to-total debt is used as an alternative proxy for market default risk. The results in Table II.4, Panel B show a significantly positive relationship between the ratio of foreign exchange reserves-to-total debt and the proportion of newly issued foreign-currency debt, as hypothesized.

The third hypothesis states that the larger the outstanding domestic-currency debt the higher the inflation risk, which means a nation has a greater incentive to dilute domestic-currency debt by issuing fiat money. At the same time, an increase in foreign-currency debt increases default risk. The following regressions are therefore estimated:

$$Pct_f = \alpha_1 + \alpha_2 Outstanding\ Debt + controls + e,$$

$$Pct_d = \alpha_1 + \alpha_2 Outstanding\ Debt + controls + e,$$

$$Pct_m = \alpha_1 + \alpha_2 Outstanding\ Debt + controls + e.$$

Table II.5 presents the empirical results for these three equations. They show that more debt outstanding is associated with both less newly issued domestic- and foreign-currency debt. A one-percentage point increase in the ratio of outstanding foreign-currency debt to GDP is associated with 0.472 percent decrease in the share of foreign-currency debt. The results when including inflation, default risk and outstanding debt together in the same equation for explaining the different new funding portions are reported in Tables II.6 (Panel A and Panel B). The results are consistent with the previous and more disaggregated results.

The fourth and last hypothesis involves a concern among investors that too much investment may be undertaken. To test this hypothesis, the following regressions are estimated (using both OLS and Logit) based on individual issuances of both foreign and domestic debt:

$$Foreign_{dummy} = \alpha_1 + \alpha_2 \log(Invest\ Amount) + Maturity + Price + Coupon + controls + e,$$

$$Domestic_{dummy} = \alpha_1 + \alpha_2 \log(Invest\ Amount) + Maturity + Price + Coupon + controls + e,$$

where the *Invest Amount* is the amount for each issue of new debt, whether foreign or domestic. The dependent variables are dummy variables. For example, *Foreign_dummy* equals one if the debt is foreign debt, otherwise zero. As is shown in Table II.7, he results indicate a greater issuance of foreign-currency debt when the amount of the initial investment requiring funding is larger.

2.4 Conclusions

A theoretical model of the optimal dynamic capital structure of a nation is developed. The model compares a nation to a company, following the novel paper by Bolton and Huang (2018). To make the model dynamic rather than static, we draw upon the earlier and important papers by Goldstein, et al. (2001) and Miller (1977). Based on the optimal mix of fiat money (and money-like debt) and foreign-currency debt derived from the model, we derive several hypotheses. First, higher inflation increases the issuance share of foreign-currency debt but reduces the issuance shares of both domestic-currency debt and fiat money. Second, higher default risk reduces the issuance share of foreign-currency debt but increases the proportion of new money-like claims. Third, outstanding debt reduces the issuance share of both foreign-currency and domestic-currency debt. Lastly, the larger the initial investment the lower the issuance of domestic-currency debt, while at the same time the larger the issuance of foreign-currency debt.

Each of these four hypotheses is tested based upon fiat money and debt data for 22 countries over the period 2004 – 2016. The results are consistent with the predictions of the model. In particular, there is a significantly positive inflation relationship to the change in the proportion of foreign-currency debt, as predicted. Also, there is a significantly negative relationship between default risk and the additional issuance of foreign-currency debt, with a nation tilting towards money-like claims for financing an investment project. In addition, there is a significantly positive relationship between the ratio of foreign reserves-to-total debt and the proportion of newly issued foreign-currency debt. Furthermore, more debt outstanding is significantly associated with both less newly issued domestic- and foreign-currency debt. Lastly, the results indicate a greater issuance of foreign-currency debt when the amount of the initial investment requiring funding is larger.

In summary, the comparison of a nation to a company seems reasonable, albeit novel, as the dynamic capital structure model and empirical results presented here seem to indicate.

Table II.1. Statistical Summary for Variables

Variable	Obs.	Mean	Std. Dev.	Min	Max
Foreign-Currency Debt/(Total Debt + Fiat Money)	319	22.81	26.59	0.06	99.96
Domestic-Currency Debt/(Total Debt + Fiat Money)	319	65.74	32.07	0.04	99.94
Fiat Money/(Total Debt + Fiat Money)	319	11.44	18.71	0.00	90.80
Log(GDP in $ billions)	340	25.59	1.25	22.57	28.43
Total Debt/GDP	340	44.43	19.09	8.19	117.93
Inflation	332	1.29	0.75	-0.35	3.39
CDS Spread	302	123.10	85.51	39.00	449.97
Yield-to-Maturity	340	2.38	4.47	-8.58	20.50

Table II.2. Sovereign Debt Information for Sample Countries

Country Name	Annual Debt Issuance Average (% of GDP)	Share of Issuance in Domestic-Currency Debt Average (% of Issuance)	Share of Issuance in Foreign-Currency Debt Average (% of Issuance)
Argentina	7.66	32.00	68.00
Brazil	10.53	68.84	31.16
Bulgaria	2.58	36.42	63.58
Chile	1.20	67.85	32.15
China	20.36	87.53	12.47
Colombia	2.91	79.53	20.47
Hungary	7.52	70.10	29.90
India	11.32	100.00	0
Indonesia	1.88	80.51	19.49
Latvia	5.78	36.69	63.31
Lithuania	6.18	30.57	69.43
Malaysia	23.74	94.89	5.11
Mexico	4.43	94.74	5.26
Peru	5.79	37.71	62.29
Philippines	5.68	92.25	7.75
Poland	10.09	77.82	22.18
Romania	8.25	59.49	40.51
South Africa	11.05	77.48	22.52
Thailand	1.13	97.67	2.33
Turkey	12.66	78.90	21.10
Ukraine	2.95	52.38	47.62
Uruguay	4.23	57.05	42.95
Average	7.63	68.65	31.35

Table II.3. Inflation Risk and the Composition of Newly Issued Sovereign Debt and Fiat Money

Variables	(1) Foreign-Currency Debt	(2) Domestic-Currency Debt	(3) Fiat Money	(4) Fiat Money Plus Domestic-Currency Debt	(5) Domestic-Currency Debt /Total Debt
Inflation	3.560*	-3.047	-0.513	-3.560*	-4.264*
	(1.939)	(2.483)	(1.627)	(1.939)	(2.336)
Fiat Money	-4.720*	3.301	1.419	4.720*	4.595
	(2.542)	(3.254)	(2.132)	(2.542)	(3.061)
Log(GDP)	-4.819*	6.488*	-1.668	4.819*	6.148*
	(2.712)	(3.472)	(2.275)	(2.712)	(3.266)
Yield to Maturity	-1.714***	1.818***	-0.104	1.714***	1.944***
	(0.363)	(0.465)	(0.305)	(0.363)	(0.437)
Constant	251.891***	-174.012***	22.121	-151.891***	-185.453***
	(26.689)	(34.169)	(22.387)	(26.689)	(32.143)
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes
Observations	319	319	319	319	319
R-squared	0.317	0.231	0.030	0.317	0.290

Note: Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Table II.4. Default Risk and the Composition of Newly Issued Sovereign Debt and Fiat Money

Panel A. using Credit Default Swap Spread

Variables	(1) Foreign-Currency Debt	(2) Domestic-Currency Debt	(3) Fiat Money	(4) Fiat Money Plus Domestic-Currency Debt	(5) Domestic-Currency Debt /Total Debt
CDS Spread	-30.134**	42.720**	-12.586	30.134**	44.734***
	(11.608)	(19.713)	(10.219)	(11.608)	(15.469)
CDS Spread Squared	7.961***	-11.469**	3.507	-7.961***	-11.993***
	(2.438)	(4.137)	(2.110)	(2.438)	(3.208)
Fiat Money	-14.886**	19.143	-4.257	14.886**	20.536**
	(6.857)	(11.408)	(5.354)	(6.857)	(9.262)
Log(GDP)	5.006	-9.705	4.699	-5.006	-10.001
	(7.866)	(12.742)	(5.662)	(7.866)	(10.418)
Yield to Maturity	-1.357***	1.361**	-0.003	1.357***	1.428**
	(0.449)	(0.553)	(0.444)	(0.449)	(0.522)
Constant	263.032***	-162.55**	-0.479	-163.032***	-180.083***
	(51.811)	(68.261)	(28.918)	(51.811)	(60.317)
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes
Observations	283	283	283	283	283
R-squared	0.438	0.369	0.071	0.438	0.445

Panel B. Using Foreign Exchange Reserves/Debt

Variables	(1) Foreign-Currency Debt	(2) Domestic-Currency Debt	(3) Fiat Money	(4) Fiat Money Plus Domestic-Currency Debt	(5) Domestic-Currency Debt /Total Debt
Foreign Exchange	22.465***	-24.438***	1.974	-22.465***	-22.838***
Reserves/Debt	(6.007)	(7.734)	(5.170)	(6.007)	(7.248)
Foreign Exchange	-5.078***	5.515**	-0.437	5.078***	4.652**
Reserves/Debt Squared	(1.724)	(2.220)	(1.484)	(1.724)	(2.081)
Fiat Money	-11.190***	10.124***	1.066	11.190***	12.510***
	(2.756)	(3.548)	(2.372)	(2.756)	(3.326)
Log(GDP)	0.915	0.474	-1.389	-0.915	-0.763
	(2.812)	(3.621)	(2.421)	(2.812)	(3.393)
Yield to Maturity	-1.050***	1.151***	-0.101	1.050***	1.174***
	(0.344)	(0.443)	(0.296)	(0.344)	(0.415)
Constant	249.503***	-171.134***	21.631	-149.503***	-186.955***
	(26.445)	(34.050)	(22.763)	(26.445)	(31.913)
Year Fixed Effect	Yes	Yes	Yes	Yes	Yes
Observations	319	319	319	319	319
R-squared	0.352	0.262	0.031	0.352	0.323

Note: Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1.

Table II.5. Outstanding Sovereign Debt and the Composition of Newly Issued Sovereign Debt

	(1)	(2)	(3)	(4)	(5)
Variables	Foreign-Currency Debt	Domestic-Currency Debt	Fiat Money	Fiat Money Plus Domestic-Currency Debt	Domestic-Currency Debt /Total Debt
Outstanding Foreign-	-0.472***	-2.016***	0.953**	-1.063**	-1.707***
Currency Debt/GDP	(0.070)	(0.661)	(0.428)	(0.523)	(0.622)
Lagged Fiat Money	-4.410*	5.256	0.354	5.610**	6.556**
	(2.500)	(3.369)	(2.184)	(2.669)	(3.171)
Log(GDP)	-5.066*	5.122	-0.919	4.203	4.642
	(2.705)	(3.724)	(2.414)	(2.950)	(3.505)
Yield to Maturity	-1.281***	1.561***	0.011	1.572***	1.677***
	(0.334)	(0.446)	(0.289)	(0.353)	(0.420)
Constant	281.022***	-175.080***	19.898	-155.182***	-186.294***
	(26.806)	(36.778)	(23.840)	(29.130)	(34.609)
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes
Observations	319	301	301	301	301
R-squared	0.386	0.222	0.050	0.299	0.278

Note: Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Table II.6. Inflation Risk, Default Risk, and Outstanding Debt and the Composition of Newly Issued Sovereign Debt

Panel A. Using Credit Default Swap Spread

Variables	(1) Foreign-Currency Debt	(2) Domestic-Currency Debt	(3) Fiat Money	(4) Fiat Money Plus Domestic-Currency Debt	(5) Domestic-Currency Debt /Total Debt
Inflation	5.277**	2.954	-5.041**	-2.087	0.381
	(2.128)	(2.914)	(1.984)	(2.327)	(2.696)
CDS Spread	-27.091***	32.818***	-1.739	31.078***	39.002***
	(6.813)	(9.190)	(6.257)	(7.339)	(8.505)
CDS Spread Squared	7.259***	-10.212***	1.443	-8.768***	-11.418***
	(1.593)	(2.084)	(1.419)	(1.665)	(1.929)
Outstanding Foreign-	-0.421***	-2.740***	1.493***	-1.247**	-2.146***
Currency Debt/GDP	(0.073)	(0.688)	(0.468)	(0.549)	(0.637)
Fiat Money	-9.815***	17.857***	-3.610	14.246***	19.336***
	(3.310)	(4.233)	(2.882)	(3.380)	(3.917)
Log(GDP)	-0.128	-7.113	3.105	-4.008	-7.845*
	(3.540)	(4.615)	(3.142)	(3.685)	(4.271)
Yield to Maturity	-1.498***	1.530***	0.152	1.681***	1.690***
	(0.368)	(0.484)	(0.329)	(0.386)	(0.448)
Constant	292.517***	-179.617***	13.315	-166.302***	-191.836***
	(26.753)	(35.184)	(23.955)	(28.098)	(32.561)
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes
Observations	267	267	267	267	267
R-squared	0.494	0.375	0.098	0.438	0.441

Panel B. Using Reserve/Debt

Variables	(1) Foreign-Currency Debt	(2) Domestic-Currency Debt	(3) Fiat Money	(4) Fiat Money Plus Domestic-Currency Debt	(5) Domestic-Currency Debt /Total Debt
Inflation	5.488***	-2.924	-2.770	-5.694***	-4.892**
	(1.913)	(2.606)	(1.717)	(2.009)	(2.425)
Foreign Exchange	17.717***	-27.893***	-2.599	-30.493***	-28.764***
Reserves/Debt	(6.819)	(8.762)	(5.775)	(6.757)	(8.156)
Foreign Exchange	-5.728***	6.916***	0.451	7.367***	6.608***
Reserves/Debt Squared	(1.901)	(2.626)	(1.731)	(2.025)	(2.445)
Outstanding Debt/GDP	-0.480***	-1.770**	0.969**	-0.800	-1.552**
	(0.085)	(0.693)	(0.456)	(0.534)	(0.645)
Fiat Money	-2.482	9.194**	0.298	9.492***	11.129***
	(3.034)	(3.767)	(2.483)	(2.905)	(3.506)
Log(GDP)	-6.818**	1.966	-0.610	1.356	1.343
	(3.034)	(3.906)	(2.574)	(3.012)	(3.635)
Yield to Maturity	-1.501***	1.273***	0.143	1.415***	1.452***
	(0.355)	(0.485)	(0.320)	(0.374)	(0.451)
Constant	267.104***	-170.840***	17.799	-153.041***	-189.560***
	(26.736)	(38.474)	(25.358)	(29.669)	(35.813)
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes
Observations	301	301	301	301	301
R-squared	0.423	0.255	0.059	0.364	0.323

Note: Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Table II.7. Initial Amount of Investment Funded by Debt and Type of Debt Issued: Foreign or Domestic-Currency Debt

Variables	OLS Foreign -Currency Debt	OLS Domestic -Currency Debt	Logit Foreign -Currency Debt	Logit Domestic -Currency Debt
Log(Amount Issued)	0.01405***	-0.01405***	0.359***	-0.359***
	(0.002)	(0.002)	(0.058)	(0.058)
Maturity	0.00448***	-0.00448***	0.074***	-0.074***
	(0.000)	(0.000)	(0.007)	(0.007)
Price	-0.00024	0.00024	-48.368	48.368
	(0.002)	(0.002)	(50.403)	(50.403)
Coupon	0.00373***	-0.00373***	0.076***	-0.076***
	(0.001)	(0.001)	(0.017)	(0.017)
Log(Lagged Quarterly	-0.01174***	0.01174***	-0.455***	0.455***
Issued Debt)	(0.002)	(0.002)	(0.068)	(0.068)
Lagged Fiat Money	-0.61994**	0.61994**	-19.316**	19.316**
	(0.265)	(0.265)	(8.571)	(8.571)
Log(Lagged Total Debt)	-0.04867***	0.04867***	-0.382***	0.382***
	(0.006)	(0.006)	(0.137)	(0.137)
Log(Quarterly GDP)	0.02035***	-0.02035***	-0.025	0.025
	(0.005)	(0.005)	(0.125)	(0.125)
Constant	-0.27967***	1.27967***	2.433	-2.433
	(0.105)	(0.105)	(2.997)	(2.997)
Year Fixed Effects	Yes	Yes	Yes	Yes
Observations	8,295	8,295	8,295	8,295
R-squared (Pseudo)	0.083	0.083	0.214	0.214

Note: Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1. In this tab, independent variables are dummy variables. For example, Foreign equals one if the issued debt is foreign debt, otherwise it is zero.

Chapter III The Reaction of Option Prices to the Changes in the Federal Funds Rate Target

"I think that there is a problem with cutting rates because it shows a sense of alarm."[8]

— *Robert Shiller*

3.1 Introduction

The statutory objectives for monetary policy in the Federal Reserve Act are maximum employment, stable prices, and moderate long-term interest rates. However, the immediate effects of the Federal Open Market Committee's (FOMC) announcements are found in financial markets. Bernanke and Kuttner (2005) find that, on average, an unanticipated 25-basis-point cut in the Federal funds rate target is associated with about a 1% increase in broad stock indexes. But Kontonikas et al. (2013) claim that, throughout the 2008 financial crisis, stocks did not react positively to unexpected Federal funds rate cuts, which were interpreted as signals of the desperation of central bankers and the subdued future economic growth. Vahamaa and Aijo (2011) and Gospodinov and Jamali (2012) show that the surprise changes in the Federal funds rate target significantly increase the volatility of the S&P 500 index. Brenner et al. (2009) study the short-term response of U.S. stock, Treasury, and corporate bond markets to the release of U.S. macroeconomic information, including FOMC's target rate changes. Investors, researchers and policymakers have been interested in the impacts of monetary policy on the security markets and understanding the link between security pricing and monetary policy.

In recent decades, the options markets have been developing with a dramatic increase in trading volume. As highly leveraged financial instruments, options have been widely exploited by market participants to manage risks, discover equity prices, forecast market movements, etc. Chakravarty et al. (2004) document that the contribution of options markets to price discovery is

[8] CNBC interview: https://www.cnbc.com/2019/08/20/robert-shiller-says-the-feds-rate-cut-had-the-opposite-intended-effect-sparked-recession-alarm.html

about 17% on average. Faccini et al. (2015) extract information from S&P 500 index option prices to predict U.S. real economic activities. Despite its huge trading volume and close connections with other financial markets and the macroeconomy, there is limited research discussing how options markets respond to monetary policy decisions. In this study, I aim to contribute to the literature by examining the reactions of options prices to the changes in the Federal funds rate target.

The efficient market hypothesis suggests that anticipated monetary policy decisions should have already been incorporated in security prices. Market participants only react to the surprise changes in the Federal funds rate target. Following the method from Kuttner (2001), I use Federal funds futures contracts to measure the unexpected Federal funds rate changes and am therefore able to estimate the impact of monetary policy shocks on options prices. This paper applies an "event-study" analysis style based on Bernanke and Kuttner (2005), the sample of events is the union of all days when the FOMC announces the Federal funds rate target decisions from 2003 to 2017.

Baseline results show that during expansionary periods a 25-basis-point unanticipated cut in the Federal funds rate is associated with an 18.5% increase the S&P 500 index call options prices and an 18.6% decrease in the put options prices. But the result is reversed during the 2008 financial crisis, showing a clear structural change in the options market reaction to monetary policy changes. The reversed reactions are consistent with Kontonikas et al. (2013)'s finding that unexpected Federal funds rate target cuts signal worsening future economic activity that triggers market downturns. I also find that the effects of unanticipated cuts account for larger part of the response of S&P 500 index options prices in the 2008 financial crisis than in expansionary periods. When there is unusual uncertainty in the market, both option buyers and writers wait for the information from the Federal Reserve to assess the state of current economic environment and to provide guidance through turbulent waters.

Based on the Black-Scholes pricing model, the theoretical price for a call or a put option is a non-linear function of various inputs, including security price, strike price, security volatility, interest rates, and maturity. I apply the short-lived arbitrage model (Hilliard et al., 2019) to estimate how the changes in the Federal funds rate target drive option prices through the underlying security price, its volatility and interest rates. The results suggest that the majority of changes in S&P 500 index options prices can be attributed to the movements of the underlying index. The direct influence of interest rates accounts for 10% to 18% of option prices changes depending on option types and business cycles. I also find that the options market does not fully incorporate available interest rate information before FOMC meetings and is less efficient during the 2008 financial crisis.

Though the monetary policy decisions from FOMC have broad market impact, banks are particularly sensitive to the changes in the Federal funds rate change because of the nature of their business. I find that options on three major U.S. bank holding companies, including JP Morgan, Citigroup, and Wells Fargo, experience more volatility on FOMC meeting days. The regression results show that bank equity options respond more aggressively to the Federal funds rate changes in two senses – larger coefficient magnitudes and higher R-squared values.

Endogeneity may impose potential econometric problems because monetary policy may react to the movements of financial markets, including the options market. However, as pointed out by Kontonikas et al. (2013), the endogeneity problem should be less of a concern when daily data are used within an event study framework. The FOMC's decisions on Federal funds rate target are unlikely to be affected by the dynamics of financial markets on the same day, thus the chance that my results are compromised by reverse causality running from options prices to changes in monetary policy is minimal.

This study contributes to the existing literature on the relationship between monetary policy decisions and financial markets. I examine how options prices react to FOMC monetary policy decisions and reveal a structural change in the options market reaction during the 2008 financial crisis. Using the short-lived arbitrage model, I quantitatively measure the sources of options prices changes, providing a more detailed understanding of the movements in the options market. To my best knowledge, this is the first comprehensive study, both qualitatively and quantitatively, that directly connects the options prices and the Federal funds rate target. This paper also contributes to the debate of monetary policy effects over business cycles (Christiano et al., 2002; Gregoriou et al, 2009). I show that the expansionary Federal funds rate shocks during 2008 financial crisis were associated with market downturns, indicating the inefficiency of monetary policy when the market was filled with fears. In the end, this study adds to the literature that examines the impact of monetary policy decisions on banking sector (Yin et al., 2010; Ornella, 2015; Borio et al., 2017). I find a strong relationship between banks and interest rates from the reactions of stocks and options on three major bank holding companies to monetary policy shocks.

The remainder of the paper is organized as follows. Section 2 discusses the construction of variables and the data used in the following empirical analysis. Section 3 revisits previous studies and reveal new findings under different sample periods. Section 4 reports and discusses the results from the empirical analysis. Section 5 concludes.

3.2 Model and Data

3.2.1 The Expected and Unexpected Components of the Fed Funds Rate Target Changes

Asset markets are forward looking and tend to incorporate any information about anticipated policy changes. However, it is difficult to separate the unexpected policy change that may generate a market response. For a monetary policy decision made on day d of month m, Kuttner (2001) creates a measure for the unexpected change in the Federal funds rate target as below

$$\Delta i^u = \frac{D}{D-d}\left(f_{m,d}^0 - f_{m,d-1}^0\right), \tag{1}$$

where $f_{m,d}^0$ is the current-month 30 Day Federal funds futures rate and D is the number of days in the month. The scale $\frac{D}{D-d}$ is introduced because the contract settlement prices are based on monthly averages. The expected component of the rate changes is defined as the actual change minus the unexpected change, or

$$\Delta i^e = \Delta i - \Delta i^u. \tag{2}$$

3.2.2 Short-Lived Arbitrage Option Pricing Model

To study the channels of monetary transmission mechanism, I need an option pricing model to estimate how much the changes in security price, volatility, and interest rates contributes to the changes in option prices, respectively.

The Black-Scholes model assumes constant volatility and interest rates, resulting in a simple and elegant option pricing formula. The Black-Scholes formula, however, does not fit the settings of this study. To estimate how the changes in the Federal funds rate target affect option prices through the underlying security price, volatility and interest rates, I use a model relaxing the constant constraints on volatility and interest rate. Hilliard and Hilliard (2017) show the advantage of the Short-Lived Arbitrage model in pricing options. Following the spirit of short-lived arbitrage model, I use the call pricing formula:

$$C(S_t,\ K, \sigma_t^*,\ r_t^*, t, T) = N(d_1)S_t - N(d_2)Ke^{-r_t^*(T-t)}, \tag{3}$$

where $d_1 = \frac{ln\left(\frac{S_t}{K}\right)+\left(r_t^*+\frac{\sigma_t^{*2}}{2}\right)(T-t)}{\sigma_t^*\sqrt{T-t}}$, $d_2 = d_1 - \sigma_t^*\sqrt{T-t}$.

Using the method from Hilliard et al. (2019), I calculate the Greeks using the jointly implied volatility σ^* and interest rate r^*:

$$Delta = \delta = \frac{\partial C}{\partial S} = N(d_1^*), \qquad Vega = \nu = \frac{\partial C}{\partial \sigma} = SN'(d_1^*)\sqrt{T-t},$$
$$Rho = \rho = \frac{\partial C}{\partial r} = K(T-t)e^{-r_t^*(T-t)}N(d_2^*). \tag{4}$$

3.2.3 Data

The data used in the empirical analysis consist of the S&P 500 index and its options, the VIX index, the stock prices and options of three major U.S. bank holding companies – JP Morgan, Citigroup, and Wells Fargo, Federal funds future prices, and the Federal funds rate target. OptionMetrics provide all data related to the prices and options of indices and stocks. Federal funds future prices are from FactSet and the Federal funds target rates are from the Federal Reserve Bank of St. Louis website.

For options on the S&P 500 index and three major U.S. bank holding companies, I retain only those with at least two successive trading days to evaluate daily option price changes. I drop options with no trading volume, options whose price is less than 25 cents, and short-term options whose maturity is less than 14 days. I also delete deep in- and out-of-money options whose delta has absolute value less than 0.05 or greater than 0.95.

Since my research focuses on the response of asset prices to specific events, I follow the "event-study" style analysis from Bernanke and Kuttner (2005). For the purpose of this research, the sample of events is defined as the union of all days when decisions on the funds rate target are announced, with days corresponding to the scheduled FOMC meetings, from January 29, 2003 to December 13, 2017. I drop the FOMC meeting on December 16, 2008 because the FOMC shifted from a target number to a target range for the Federal funds rate. There are 8 scheduled FOMC meeting each year, therefore, there are 119 events in the sample.

3.3 Previous Empirical Studies

3.3.1 Data

Since option price is a function of its underlying security's price, the stock market is a channel through which a monetary policy shock exerts its influence on the options market. To understand the policy transmission mechanism, I apply the event study approach in Bernanke and Kuttner (2005) to estimate the impact of monetary policy on stock price. The idea is to regress the changes in stock price on the changes in the Federal funds rate target on FOMC meeting days:

$$\Delta S_t / S_t = a + b \Delta i_t + \epsilon_t \tag{5}$$

and

$$\Delta S_t / S_t = a + b^e \Delta i_t^e + b^u \Delta i_t^u + \epsilon_t. \tag{6}$$

where $\Delta S_t = S_t - S_{t-1}$, S_t is the closing price on the FOMC meeting day and $S_{t-1}$is the closing price one-day before the FOMC meeting day.

Regression (5) estimates the sensitivity of stock price to the changes in the Federal funds rate target, Bernanke and Kuttner (2005) apply the regression on the CRSP value-weighted return over the FOMC meeting days between June 1989 and December 2002 (see column (a) and (b) in Table III.1). Because the available 30-Day Federal Funds Futures data provided by FactSet is limited, my replication only applies to the period from February 1994 to December 2002 (see column (c) and (d)). My replication result suggests that the stock market only responds to the surprise components significantly and that the estimated coefficient of the surprise change is -2.56 that is significant at the 0.01 level, which is very similar to the finding in Bernanke and Kuttner (2005).

Applying the same analysis to our sample period from January 2003 to December 2017, I find the opposite result for economic periods that are classified as expansionary versus recessionary. The recessionary periods (2008 financial crisis) last from the third quarter of 2007 to the second quarter of 2009 according to the definition by the Federal Reserve Bank.[9] The rest of the periods from 2003 to 2017 are classified as expansionary periods. During expansionary periods the S&P 500 index negatively responds to the unexpected changes, consistent with Bernanke and Kuttner's (2005) finding, as shown in Table III.2 column (b). However, over recessionary periods the S&P 500 index positively responds to the unexpected changes, which means that an unanticipated cut in the Federal funds rate target associate with a market downturn thus fails to stimulate the market (see Table III.2 column (d)). The positive correlation is consistent with Vahamaa and Aijo's (2011) finding that an unexpected cut in the Federal funds rate has different effects on market volatility between expansive and restrictive policy cycles.

3.3.2 FOMC Effect on the VIX index

Volatility is an important factor that directly influences the price of an option. The VIX index, derived from S&P 500 index call and put options prices, is a proxy of the implied volatility of the S&P 500 index. Vahamaa and Aijo (2011) examine how the Federal Reserve's monetary policy decisions affect the returns on the VIX index. They find a positive relationship between policy surprises and the VIX index and the impact of monetary policy varies between expansive

[9] Dates of U.S. recessions: https://fred.stlouisfed.org/series/JHDUSRGDPBR

and restrictive policy periods. I loosely replicated their research approach and find similar results. Then I repeat the regression for VIX index returns:

$$\Delta VIX_t/VIX_t = a + b\Delta i_t + \epsilon_t \tag{7}$$

and

$$\Delta VIX_t/VIX_t = a + b^e\Delta i_t^e + b^u\Delta i_t^u + \epsilon_t. \tag{8}$$

As shown in Table III.3, the VIX index responds positively to the unexpected changes in the Federal funds rate in expansionary periods, but the correlation is reversed in the 2008 financial crisis. This is not surprising because the VIX index moved in the opposite direction of the S&P 500 index about 80% according to a CBOE report[10]. The negative relationship between the VIX index and the unexpected fund rate cuts shows that the expansionary monetary policy in an economic recession raises concerns about further economic slowdown.

3.4 Empirical Studies

3.4.1 FOMC Effect on S&P 500 Index Options

The most straightforward way to study the effect of changes in the Federal funds rate on S&P 500 index options is to repeat the regressions as in the above section. But options should be treated differently because there are multiple options with different maturities and moneyness traded at different prices on the underlying security each day. I use open interest weighted option return to measure the overall options reaction to the change in the Federal funds rate target on each FOMC meeting day. Taking call options on the S&P 500 index for example, the option return on FOMC meeting date t is

[10] CBOE report, http://www.cboe.com/products/vix-index-volatility/vix-options-and-futures/vix-index/the-relationship-of-the-spx-and-the-vix-index

$$\Delta Call_t = \frac{\sum_i \left(Call_{i,t+1} - Call_{i,t}\right) \times W_{i,t}}{\sum_i Call_{i,t} \times W_{i,t}}, \tag{9}$$

where $W_{i,t}$ is the open interest of option i on e FOMC meeting vent date t. Because of liquidity considerations, I only retain options with maturity between 14 and 90 days.

Next, I run the following two regressions:

$$\Delta Call_t = a + b\Delta i_t + \epsilon_t \tag{10}$$

and

$$\Delta Call_t = a + b^e \Delta i_t^e + b^u \Delta i_t^u + \epsilon_t. \tag{11}$$

The regression results reported in Table III.4 Panel A show that the S&P500 index call options only respond to unexpected changes in the Federal funds rate target. In expansionary periods, a 25-basis-point unanticipated cut is associated with about an 18.5% increase in the call options prices. However, the result is reversed in recessionary periods where a 25-basis-point unanticipated cut is associated with about a 48.1% decrease in the call options prices. The unexpected cut in the Federal funds rate target in an economic recession is interpreted as a negative signal because investors take such cuts as a prediction of a future market declines. This interpretation is consistent with past studies that interest rate cuts during the crisis were perceived by market participants as a signal of deteriorating economic prospects and reinforced "flight to safety" trading (Florackis et al., 2014; Kontonikas et al., 2013). The options market is very sensitive to a cut in recessionary periods with a 48.1% drop in the call options prices versus an 18.5% increase in the put options prices during expansionary periods. The put options on S&P 500 index behave similarly with the opposite direction. A 25-basis-point unanticipated cut is associated with a 18.6% drop in the put options prices in expansionary periods versus a 42.1% surge during an economic recession see Table III.4 Panel B. The larger reaction magnitudes in the economic recession relate to the finding that the possibility of black swans is initially ignored, but ultimately turns into an overstated fear (Gennaioli et al., 2015). This result is also consistent with the finding in Bondt and Thaler (1985) that most people tend to overreact to unexpected and dramatic news events.

The R-squared value is much higher for regressions in economic recessions than for those in economic expansions. This implies that the FOMC's monetary policy decisions have more influence on the financial markets during recessionary periods. Perhaps the market participants, already in panic, wait for the macroeconomic forecast from the Federal Reserve and trade accordingly.

The linear regression results describe the overall response of options to the changes in the Federal funds rates target. I take a different approach here because option prices are not a linear function of security price, security volatility or interest rates. To gain a more detailed understanding, I analyze how options prices react to the changes in the Federal funds rate through three channels: underlying security price, volatility, and interest rates. To do so, I apply the following first order approximation

$$dCall_{i,t} = \frac{\partial Call_{i,t}}{\partial S_t} \times dS_t + \frac{\partial Call_{i,t}}{\partial \sigma_{i,t}} \times d\sigma_{i,t} + \frac{\partial Call_{i,t}}{\partial r_t} \times dr_t. \quad (12)$$

As shown in the equation (12), the changes in the option prices can be attributed to three Greek terms: delta $= \frac{\partial Call_{i,t}}{\partial S_t} \times dS_t$, vega $= \frac{\partial Call_{i,t}}{\partial \sigma_{i,t}} \times d\sigma_{i,t}$, and rho $= \frac{\partial Call_{i,t}}{\partial r_t} \times dr_t$. The absolute value of each Greek term is applied to avoid negative terms.

Based on implied volatility and interest rates and using the three Greek terms from the Short-Lived Arbitrage model, I find that the delta term plays the key role while the vega and rho terms account for a significant part in determining the changes in the S&P 500 index call options. The rho term contributes about 18% in expansionary periods and 15% in recessionary periods (see Figure III.1). For S&P 500 index put options, the delta term still dominates and the rho term accounts for 10% to 12% depending on the business cycle.

The delta term plays a more dominant role in recessionary periods than in expansionary periods. The explanation rests in the coefficients of the unexpected change found in Table III.2: The S&P 500 index responds more aggressively to the Federal funds rate change in an economic recession. To confirm this explanation, I regress each Greek term on the changes in Federal funds rate target:

$$Greek\ term = a + b\Delta i_t + \epsilon_t \tag{10}$$

and

$$Greek\ term = a + b^e\Delta i_t^e + b^u\Delta i_t^u + \epsilon_t. \tag{11}$$

Table III.5 shows that both delta and vega terms only respond to the unexpected part of target rate changes in expansionary periods. But the rho term, which is the sensitivity of an option to the interest rate, reacts not only to the unexpected changes in the Federal funds rate target but also to the expected changes. Table III.4 and 5 together show that though the S&P500 index options do not respond to raw funds rate changes, rho terms which relate to the interest rate respond to such shocks. This implies that, unlike the equity market, the options market does not fully reflect available information related to interest rates before FOMC announcements.

I repeat the analysis for Greek terms in recessionary periods. Table III.5 and 6 show that in general Greek terms respond to the unexpected changes in the Federal funds rate target more aggressively in recessionary periods. For example, in expansionary periods, a 25-basis-point unexpected cut associates with a 1.78% increase in the rho term of call options on the S&P 500 index. The same cut associates with a 4.37% increase in the rho term in recessionary periods. As highly leverage financial assets, options are more sensitive to many market innovations than equities. Being exposed to options in an economic recession is even more risky. Table III.5 and 6 together also confirm the opposite effect of monetary policy on the S&P 500 index and the VIX index during expansionary versus recessionary periods. The delta term of a S&P 500 call option on average responds to an unexpected 25-basis-point funds rate cut with an increase of 22.53% in expansionary periods while a 25.55% drop in recessionary periods. The vega term of a S&P 500 put option on average responds to an unexpected 25-basis-point funds rate cut with a 4.83% increase in expansionary periods while a 5.04% drop in recessionary periods.

Information loss occurs when conducting an aggregate analysis. Regressing option prices changes to the changes in the Federal funds rate target fails to discover the dynamics of individual components related to security price, volatility, and interest rates. Above analysis of Greek terms reveals how each component contributes to the option prices around the FOMC announcements. This method allows market participants to both qualitatively and quantitatively understand the driving forces of option prices.

3.4.2 Different FOMC Effects on S&P 500 Index Options Across Moneyness

Trading asymmetries are found among different moneyness levels. Out-of-the-money (OTM) options on market indices are more heavily traded than other options are. Market participants purchase more OTM options on the S&P 500 index to hedge against a potential market crash. I examine whether options across different moneyness levels respond to FOMC decisions differently.

Define the moneyness of a call option by the ratio of the underlying security price and the strike price, that is, $moneyness = S_t/K$. For a put option, the moneyness is defined inversely: $moneyness = K/S_t$. An option with moneyness between 0.98 and 1.02 is defined as an at-the-money (ATM) option. The moneyness of an in-the-money (ITM) option is greater than 1.02 and the moneyness of an out-the-money (OTM) option is less than 0.98.

Tables III. 7 and III. 8 return consistent results that OTM options, both calls and puts, are most sensitive to the unexpected changes in the Federal funds rate target, while ITM options are least sensitive. All regressions report insignificant coefficients of raw funds rate change and expected changes. It is true that OTM options have the least sensitivity of option price to the interest rate, rho. However, the returns of OTM options are more sensitive to the interest rate changes because they are cheaper than ATM and ITM options. Because OTM options have higher level of liquidity, stronger responses to interest rate shocks, and lower prices, they serve as cost-efficient financial instruments to hedge or speculate the risks involved in FOMC monetary policy decisions. This is in line with the finding in Chen et al. (2005) that informed traders should transact OTM options to extract as much value as possible from their private information.

In recessionary periods, options respond more to the unexpected Federal funds rate changes than they do during expansionary periods. The high R-squared value also indicates that FOMC monetary policies have more influence over other information sources when there is fear in the market. Unfortunately, the amplified influence of Federal funds rate cuts during the crisis is against the initial intend of the Federal Reserve, showing the inefficiency of conventional monetary policy.

3.4.3 FOMC Effect on Options of Banks

My analysis by far has estimated the impacts of the Federal funds rate target changes on the option prices of a broad stock market index – the S&P 500 index. Now I turn to the impacts on banking industry. Because the majority of its assets and liabilities depends on the interest rate, banks are more sensitive to interest rate changes (Du et al., 2018). I examine this hypothesis based on stock prices of three major U.S. bank holding companies – JP Morgan, Citigroup, and Wells Fargo.

Table III.9 show that bank stock prices behave differently from the S&P 500 index in two aspects. First, in general, bank stock prices are sensitive to both the expected changes and the unexpected changes in the changes in the Federal funds rate target. But the S&P500 index only react to the unexpected changes as shown in Table III.2. This difference suggests the higher sensitivity of bank equities to FOMC monetary policy changes (Fraser et al., 2002). Second, bank stock prices experience more volatility than the S&P 500 index on FOMC announcement days. On average, a 25-basis-point unexpected cut is followed by a 0.92% increase in the S&P 500 index in expansionary periods, the same cut is associated with increases of 1.43% for JP Morgan, 1.52% for Citigroup, and 1.20% for Wells Fargo stock prices. The difference is larger during the 2008 financial crisis: a 25-basis-point unexpected cut in the crisis is associated with a 2.91% drop in the S&P 500 index versus a 7.84% decrease in JP Morgan stock price, a 9.24% decrease for Citigroup, and a 12.14% drop for Wells Fargo.

Following the same logics, options on banks are expected to have a larger response magnitude on FOMC meeting days than those on the S&P 500 index. Furthermore, the rho term is expected to play a more important role in determining option price movements. Table III.10 shows that on FOMC meeting days, the average absolute return of S&P 500 call options is 25.51%, while the average absolute return of call options on Wells Fargo is as high as 33.48%. Call options on JP Morgan and Citigroup are also more volatile than their S&P 500 counterparts. Such differences are smaller for put options, but in general options on three major banks' stocks have more volatility than S&P 500 index options across different moneyness.

As shown in Table III.11, options on bank equities respond to not only the unexpected changes but also the expected changes in the Federal funds rate target during both expansionary periods and the 2008 financial crisis. The significant coefficients of the expected change indicate that the options on bank equities do not fully incorporate available information before FOMC announcements. Options on bank equities are more sensitive to the changes and regressions return higher R-squared during financial crises. The combined evidence suggests that changes in the Federal funds rate target have more influence on banking sector during a financial crisis. One explanation is that market investors overreact in the chaotic financial environment where the market efficiency is severely compromised (Lim et al., 2008). In particular, during periods of high interest rate volatility, interest rate volatility has more impact on stocks of financial intermediaries than non-financial firms (Yourougou, 1990). This finding is also consistent with the result in Choi et al. (1992) that the interest rate sensitivity of bank stock returns differs in the pre- and post-1979 periods.

Similar to the analysis for options on S&P 500 index, I plot the Greek terms for options on bank equities (see Figure III.2, III.3, and III.4). Compared to their counterparts on S&P 500 index, the changes in option prices on bank equities are less attributed to the delta term but more to the rho term as expected. Put options in recessionary periods are the exception. I conclude that the interest rate plays a more important role in bank equity options pricing than it does in S&P 500 index options pricing.

3.5 Conclusions

This paper studies the impact of monetary policy decisions on the options market. I find that an unexpected cut in the Federal funds rate target moves the options market in opposite directions between periods of expansion and recession. In expansionary periods, a 25-basis-point unanticipated cut is associated with about an 18.5% increase in S&P 500 index call options prices and an 18.6% drop in S&P 500 index put options prices. However, during the 2008 financial crisis, a 25-basis-point unanticipated cut is associated with about a 48.1% decrease in the S&P 500 index call options prices and a 42.1% increase in S&P 500 index put options prices. The higher R-squared and larger coefficients in regressions show that monetary policy decisions have more influence on the financial markets during the 2008 financial crisis.

Using the Short-Lived Arbitrage model, I quantitatively measure three transmission channels for rate changes – equity price, market volatility and interest rates. Evidence show that the changes in the Federal funds rate target does not only indirectly moves the options market through equity price and its volatility, but also through interest rates directly. Bank equity options are more sensitive to monetary policy shocks, exhibiting more volatility than the options on the S&P 500 index in FOMC meeting days. Compared with options on the S&P 500 index, bank equity options react stronger to the changes in the Federal funds rate.

The overall evidence suggests that cutting the Federal funds rate target, a conventional monetary policy tool, fails to boost the market in the 2008 financial crisis. Indeed, it is interpreted as a negative sign of future economy and contributes to market turmoil. As an empirical study on the effects of monetary policy on options markets, this paper helps investors, policymakers, and researchers better understand how options markets respond and incorporate the information from monetary policy decisions.

Figure III.1.Average Contribution of Greek Terms to Changes in S&P 500 Index Option Price

Figure III.2. Average Contribution of Greek Terms to Changes in JP Morgan Option Price

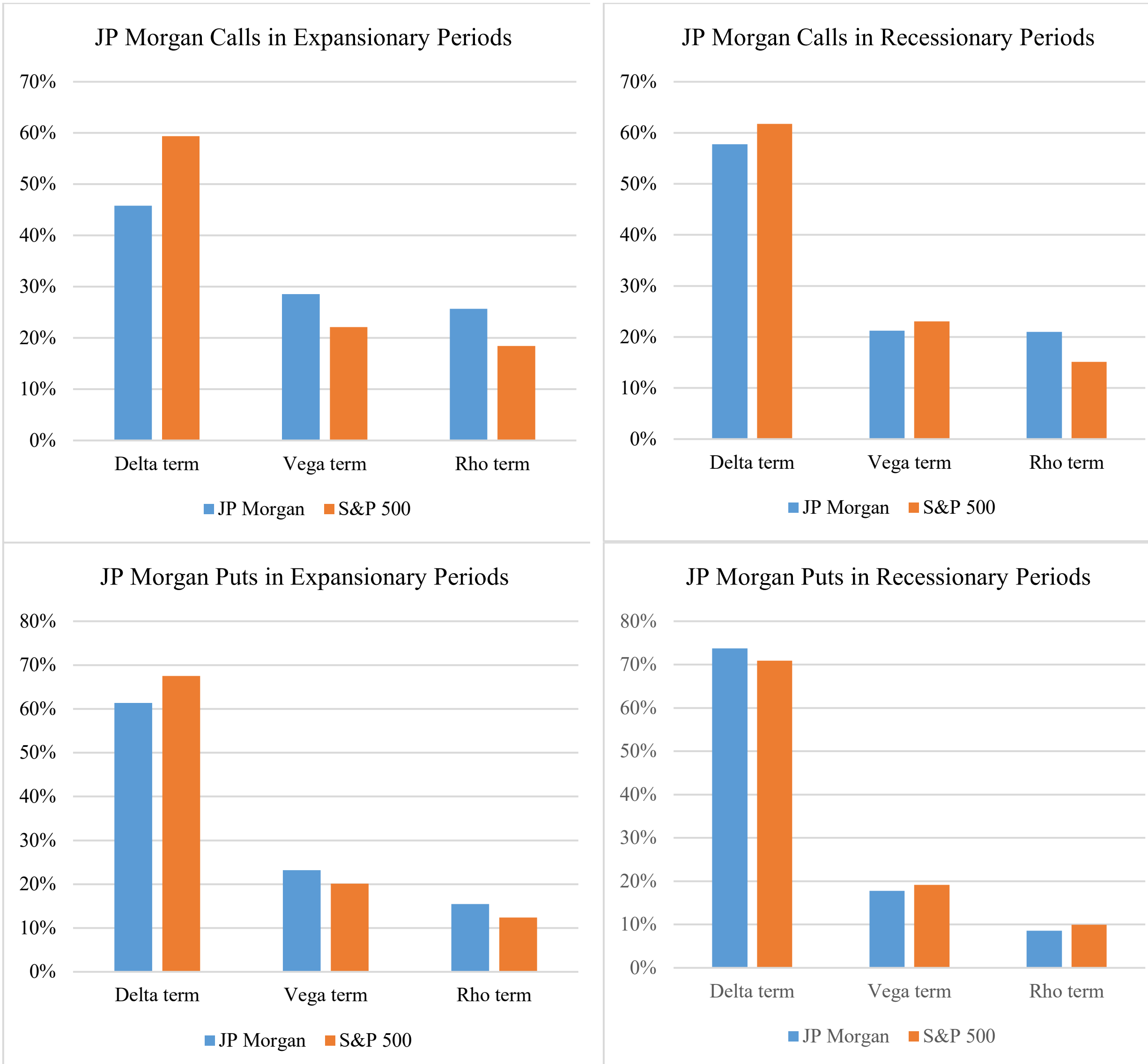


Figure III.3. Average Contribution of Greek Terms to Changes in Citigroup Option Price

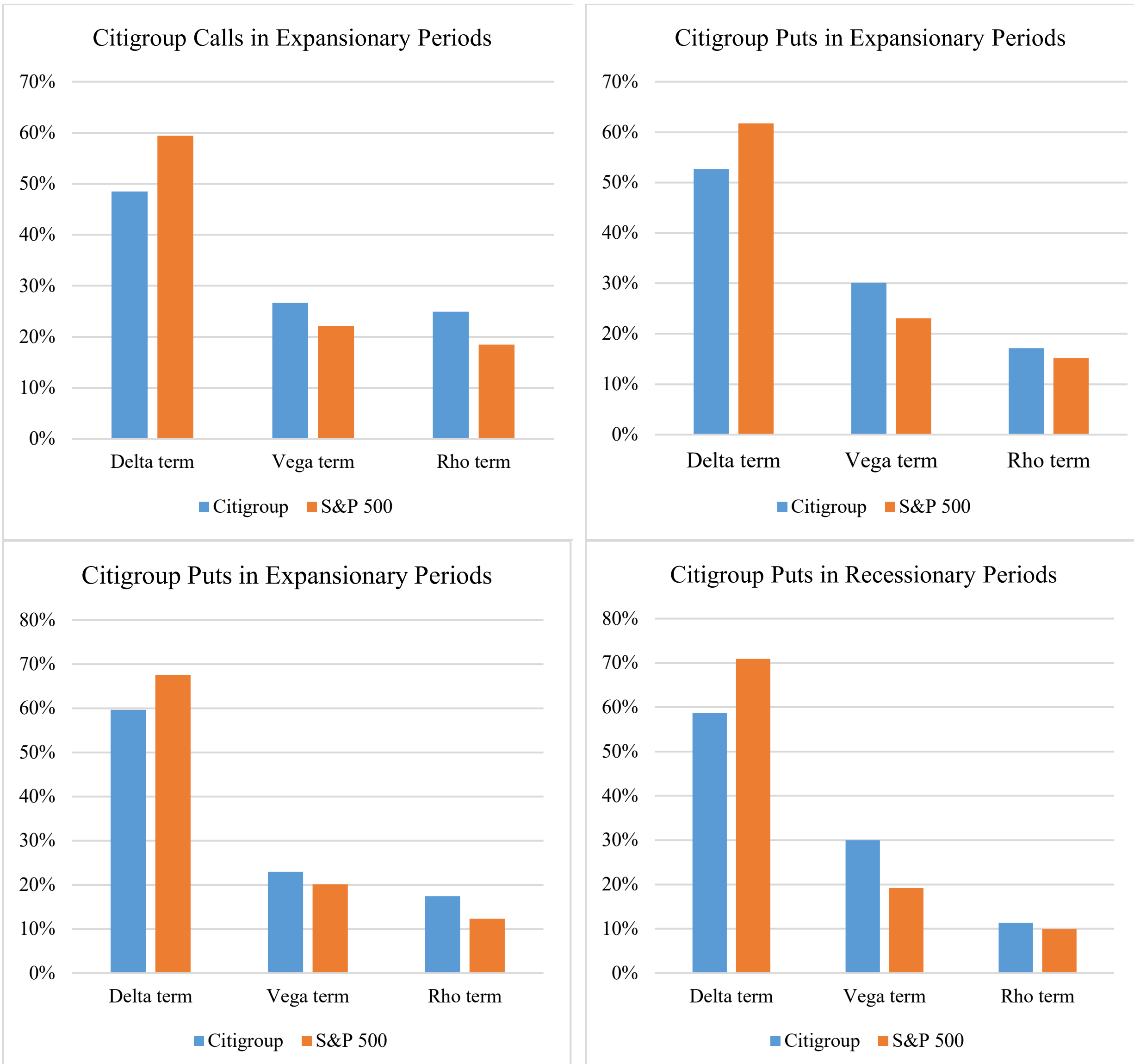


Figure III.4. Average Contribution of Greek Terms to Changes in Wells Fargo Option Price

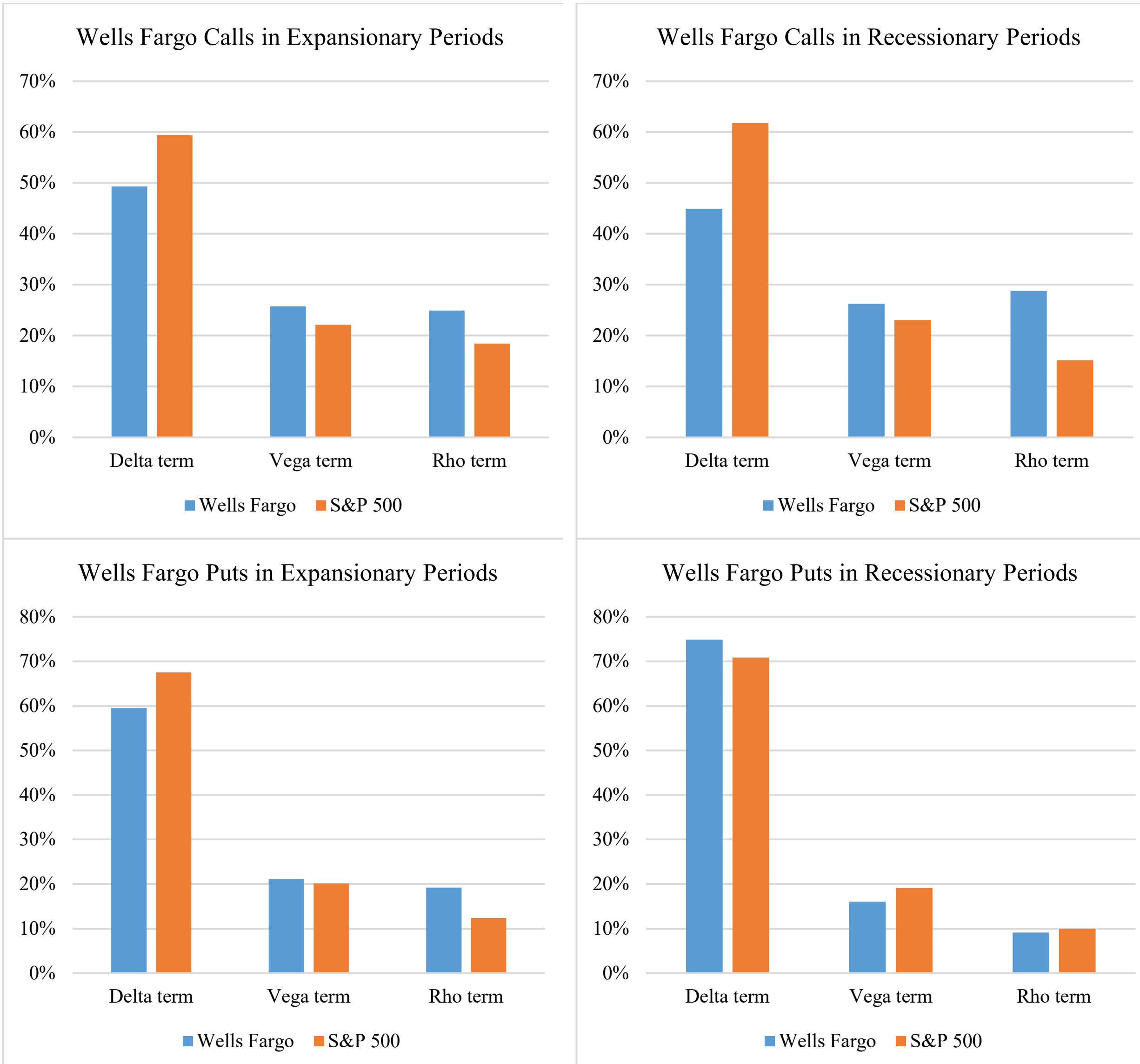


Table III.1.Market Index and Federal Funds Rate Changes. A Replication of Bernanke and Kuttner (2005)

	1989 - 2002 (B&K 2005, Original)		1994 - 2002 (B&K 2005, Replicate)	
Regressor	(a)	(b)	(c)	(d)
Raw funds rate change	-0.11		-0.803	
	(0.355)		(0.584)	
Expected change		0.67		0.0268
		(0.406)		(0.621)
Unexpected change		-2.55***		-2.560***
		(0.914)		(0.809)
Constant	0.17**	0.11	0.328**	0.276*
	(0.079)	(0.080)	(0.145)	(0.139)
R-squared	-0.007	0.049	0.025	0.131

The table reports the results of regressing S&P 500 index returns on changes in the Federal funds rate, column (c), and on the expected and unexpected components of the funds rate change, column (d). Columns (a) and (b) depict results for 1-day CRSP value-weighted equity from Bernanke and Kuttner's (2005) for comparison.

Table III.2. S&P 500 Index and Federal Funds Rate Changes

	2003 - 2017 (exclude 2008 financial crisis)		2008 financial crisis	
Regressor	(a)	(b)	(c)	(d)
Raw funds rate change	-1.036		1.055	
	(0.817)		(2.254)	
Expected change		-0.875		3.029
		(0.801)		(1.881)
Unexpected change		-3.662***		11.64**
		(1.344)		(4.082)
Constant	0.288***	0.297***	1.311*	1.407**
	(0.103)	(0.101)	(0.699)	(0.544)
R-squared	0.015	0.069	0.020	0.463

The table depicts the results of regressing S&P 500 index returns on changes in the Federal funds rate, column (a) and (c), and on the expected and unexpected components of the funds rate change, column (b) and (d).

Table III.3. VIX Index and Federal Funds Rate Changes

	2003 - 2017 (exclude 2008 financial crisis)		2008 financial crisis	
Regressor	(a)	(b)	(c)	(d)
Raw funds rate change	7.452		3.971	
	(6.067)		(10.24)	
Expected change		6.352		-4.819
		(5.975)		(8.673)
Unexpected change		25.42**		-43.17**
		(10.02)		(18.83)
Constant	-3.092***	-3.153***	-3.345	-3.775
	(0.765)	(0.752)	(3.174)	(2.508)
R-squared	0.014	0.060	0.013	0.442

The table reports the results of regressing VIX index returns on changes in the Federal funds rate, column (a) and (c), and on the expected and unexpected components of the funds rate change, column (b) and (d).

Table III.4. S&P 500 Option Prices and Federal Funds Rate Changes

Panel A. Call Options

	2003 - 2017 (exclude 2008 financial crisis)		2008 financial crisis	
Regressor	(a)	(b)	(c)	(d)
Raw funds rate change	-17.08		6.159	
	(23.65)		(41.45)	
Expected change		-13.61		40.89
		(23.48)		(35.76)
Unexpected change		-73.89*		192.4**
		(39.38)		(77.63)
Constant	6.466**	6.660**	17.58	19.28*
	(2.983)	(2.954)	(12.85)	(10.34)
Observations	106	106	13	13
R-squared	0.005	0.035	0.002	0.415

Panel B. Put Options

	2003 - 2017 (exclude 2008 financial crisis)		2008 financial crisis	
Regressor	(a)	(b)	(c)	(d)
Raw funds rate change	19.76		-20.11	
	(19.08)		(42.97)	
Expected change		16.42		-47.77
		(18.82)		(42.09)
Unexpected change		74.42**		-168.5*
		(31.56)		(91.36)
Constant	-9.123***	-9.310***	-13.53	-14.88
	(2.407)	(2.368)	(13.32)	(12.17)
Observations	106	106	13	13
R-squared	0.010	0.053	0.020	0.259

Panel A reports the results of regressing S&P 500 index call options on the Federal funds rate changes, column (a) and (c), and on the expected and unexpected components of the Federal funds rate changes, column (b) and (d). Panel B reports results for S&P 500 index put options.

Table III.5. Greek Terms of the S&P 500 Option Prices and Federal Funds Rate Changes Excluding the 2008 Financial Crisis

Panel A. Call Options

Regressor	Delta term	Delta term	Vega term	Vega term	Rho term	Rho term
Raw funds rate change	-22.36		-0.387		7.735**	
	(16.05)		(5.183)		(3.419)	
Expected change		-14.74		-1.687		7.127**
		(16.21)		(5.271)		(3.482)
Unexpected change		-90.12***		11.17		13.15*
		(31.49)		(10.24)		(6.765)
Moneyness	-3.546**	-3.414*	-1.269**	-1.291**	-0.619	-0.630*
	(1.766)	(1.752)	(0.570)	(0.570)	(0.376)	(0.376)
Constant	9.486***	9.337***	-0.801	-0.775	-1.121**	-1.109**
	(2.191)	(2.174)	(0.707)	(0.707)	(0.467)	(0.467)
Observations	318	318	318	318	318	318
R-squared	0.018	0.037	0.015	0.021	0.025	0.028

Panel B. Put Options

Regressor	Delta term	Delta term	Vega term	Vega term	Rho term	Rho term
Raw funds rate change	10.15		1.534		-4.865***	
	(13.09)		(3.083)		(1.809)	
Expected change		4.268		-0.320		-4.479**
		(13.15)		(3.067)		(1.832)
Unexpected change		66.59***		19.32***		-8.572**
		(24.64)		(5.748)		(3.433)
Moneyness	-2.128	-1.995	-0.495	-0.453	0.129	0.120
	(1.345)	(1.332)	(0.317)	(0.311)	(0.186)	(0.186)
Constant	-0.310	-0.503	-1.273**	-1.334**	0.517	0.529*
	(2.283)	(2.262)	(0.538)	(0.528)	(0.315)	(0.315)
Observations	315	315	315	315	315	315
R-squared	0.010	0.033	0.009	0.049	0.025	0.030

The table depicts the results of regressing Greek terms of options on the Federal funds rate.

Table III.6. Greek Terms of the S&P 500 Option Prices and Federal Funds Rate Changes During the 2008 Financial Crisis

Panel A. Call Options

Regressor	Delta term	Delta term	Vega term	Vega term	Rho term	Rho term
Raw funds rate change	0.834		6.532		16.73***	
	(23.15)		(8.943)		(4.649)	
Expected change		2.305		-2.606		17.48***
		(17.23)		(8.781)		(4.482)
Unexpected change		102.2***		-42.48**		34.91***
		(27.14)		(19.06)		(10.13)
Moneyness	3.415	-1.956	-2.075	-2.075	-2.711**	-2.528*
	(6.898)	(7.149)	(2.664)	(2.435)	(1.303)	(1.255)
Constant	16.69	0.577	0.473	0.0265	2.940	2.893
	(9.957)	(7.753)	(3.846)	(3.518)	(2.157)	(2.073)
Observations	39	39	39	39	39	39
R-squared	0.007	0.356	0.031	0.213	0.336	0.404

Panel B. Put Options

Regressor	Delta term	Delta term	Vega term	Vega term	Rho term	Rho term
Raw funds rate change	-11.41		-1.946		-5.035***	
	(17.02)		(3.918)		(1.724)	
Expected change		-34.48**		-5.510		-5.269***
		(15.17)		(3.938)		(1.667)
Unexpected change		-135.1***		-21.06**		-11.38***
		(32.94)		(8.549)		(3.680)
Moneyness	-1.350	-1.350	-0.151	-0.151	-0.0349	-0.0264
	(5.071)	(4.208)	(1.167)	(1.092)	(0.503)	(0.485)
Constant	-12.36*	-13.49**	-0.306	-0.480	0.850	0.821
	(7.319)	(6.079)	(1.685)	(1.578)	(0.899)	(0.867)
Observations	39	39	39	39	39	39
R-squared	0.014	0.340	0.007	0.155	0.193	0.271

The table depicts the results of regressing Greek terms of options on the Federal funds rate.

Table III.7. S&P 500 Option Prices and Federal Funds Rate Changes by Moneyness Excluding the 2008 Financial Crisis

Panel A. Call Options

	ITM		ATM		OTM	
Regressor	(a)	(b)	(c)	(d)	(e)	(f)
Raw funds rate change	-10.36		-21.97		-29.51	
	(12.43)		(24.40)		(32.26)	
Expected change		-8.060		-18.34		-25.35
		(12.23)		(24.22)		(32.14)
Unexpected change		-47.85**		-81.26**		-97.42*
		(20.52)		(40.62)		(53.91)
Constant	4.825***	4.953***	9.411***	9.614***	8.551**	8.783**
	(1.569)	(1.539)	(3.078)	(3.047)	(4.070)	(4.044)
Observations	106	106	106	106	106	106
R-squared	0.007	0.054	0.008	0.038	0.008	0.031

Panel B. Put Options

	ITM		ATM		OTM	
Regressor	(a)	(b)	(c)	(d)	(e)	(f)
Raw funds rate change	18.24		18.06		17.19	
	(13.89)		(18.02)		(19.83)	
Expected change		15.56		15.07		13.82
		(13.63)		(17.81)		(19.58)
Unexpected change		62.09***		66.91**		72.28**
		(22.86)		(29.88)		(32.85)
Constant	-3.549**	-3.701**	-6.793***	-6.960***	-9.847***	-10.04***
	(1.761)	(1.724)	(2.273)	(2.242)	(2.502)	(2.464)
Observations	105	105	106	106	106	106
R-squared	0.016	0.068	0.010	0.048	0.007	0.047

The table depicts the results of regressing call and put option returns on the Federal funds rate changes.

Table III.8. S&P 500 Options and Federal Funds Rate Changes by Moneyness Excluding the 2008 Financial Crisis

Panel A. Call Options

	ITM		ATM		OTM	
Regressor	(a)	(b)	(c)	(d)	(e)	(f)
Raw funds rate change	1.979		2.169		12.69	
	(28.88)		(38.62)		(49.94)	
Expected change		27.57		36.66		52.34
		(23.85)		(31.69)		(44.62)
Unexpected change		139.3**		187.1**		225.3**
		(51.78)		(68.79)		(96.86)
Constant	13.38	14.64*	18.09	19.78*	21.71	23.65*
	(8.956)	(6.898)	(11.98)	(9.163)	(15.49)	(12.90)
Observations	13	13	13	13	13	13
R-squared	0.000	0.463	0.000	0.470	0.006	0.375

Panel B. Put Options

	ITM		ATM		OTM	
Regressor	(a)	(b)	(c)	(d)	(e)	(f)
Raw funds rate change	-14.92		-19.39		-24.41	
	(30.70)		(37.50)		(47.43)	
Expected change		-35.49		-45.10		-54.57
		(29.68)		(35.94)		(46.65)
Unexpected change		-125.2*		-157.3*		-186.2*
		(64.42)		(78.01)		(101.3)
Constant	-9.062	-10.07	-12.23	-13.48	-14.91	-16.39
	(9.521)	(8.582)	(11.63)	(10.39)	(14.71)	(13.49)
Observations	13	13	13	13	13	13
R-squared	0.021	0.280	0.024	0.294	0.024	0.256

Panels A and B report the results regressing S&P 500 index call and put option returns on the Federal funds rate changes.

Table III.9. Bank Stock Prices and Federal Funds Rate Changes

Panel A. JP Morgan

	2003 - 2017 (exclude 2008 financial crisis)		2008 financial crisis	
Regressor	(a)	(b)	(c)	(d)
Raw funds rate change	-2.089		7.353	
	(1.863)		(6.858)	
Expected change		-1.866		11.83**
		(1.383)		(4.193)
Unexpected change		-5.723**		31.36***
		(2.656)		(8.312)
Constant	0.535**	0.547***	4.319**	4.537**
	(0.216)	(0.200)	(1.614)	(1.440)
R-squared	0.018	0.049	0.148	0.505

Panel B. Citigroup

	2003 - 2017 (exclude 2008 financial crisis)		2008 financial crisis	
Regressor	(a)	(b)	(c)	(d)
Raw funds rate change	-2.870*		9.085	
	(1.608)		(11.52)	
Expected change		-2.674**		14.28
		(1.221)		(8.248)
Unexpected change		-6.062***		36.96**
		(2.044)		(15.06)
Constant	0.609**	0.620**	6.165*	6.419*
	(0.258)	(0.250)	(3.264)	(3.487)
R-squared	0.022	0.037	0.071	0.221

Panel C. Wells Fargo

	2003 - 2017 (exclude 2008 financial crisis)		2008 financial crisis	
Regressor	(a)	(b)	(c)	(d)
Raw funds rate change	-2.916**		13.53	
	(1.287)		(12.63)	
Expected change		-2.801***		20.06*
		(1.027)		(9.534)
Unexpected change		-4.796**		48.55***
		(1.931)		(15.17)
Constant	0.342*	0.348*	7.764*	8.083*
	(0.187)	(0.182)	(3.974)	(4.036)
R-squared	0.043	0.052	0.111	0.278

The table reports the results of regressing bank stock returns on the Federal funds rate changes, column (a) and (c), and on the expected and unexpected components of the funds rate change, column (b) and (d).

Table III.10. Absolute Percentage Change of Options and Federal Funds Rate Changes

	Call Options				Put Options			
Options	All	ITM	ATM	OTM	All	ITM	ATM	OTM
S&P 500	25.51	16.01	28.73	30.02	23.16	17.40	23.92	25.45
JP Morgan	28.76	21.43	34.38	29.70	24.78	18.19	26.34	27.47
Citigroup	28.38	20.97	35.35	28.14	24.99	17.05	26.73	28.76
Wells Fargo	33.48	23.88	43.08	31.69	25.95	18.16	28.04	27.95

The table reports the average absolute percentage return of options on S&P500 index, JP Morgan, Citigroup, and Wells Fargo stocks on FOMC event days.

Table III.11.Bank Equity Options and Federal Funds Rate Changes

Panel A. Call Options on JP Morgan

	2003 – 2017 (exclude 2008 financial crisis)		2008 financial crisis	
Regressor	(a)	(b)	(c)	(d)
Raw funds rate change	-65.76**		80.51**	
	(30.37)		(30.56)	
Expected change		-56.27**		109.3***
		(23.20)		(26.09)
Unexpected change		-141.9***		221.2***
		(53.17)		(40.87)
Moneyness	0.781	0.788	-2.879	-2.879
	(3.315)	(3.305)	(8.455)	(7.999)
Constant	14.46***	14.34***	47.86***	49.02***
	(5.511)	(5.398)	(16.53)	(16.27)
R-squared	0.018	0.030	0.148	0.244

Panel B. Put Options on JP Morgan

	2003 - 2017 (exclude 2008 financial crisis)		2008 financial crisis	
Regressor	(a)	(b)	(c)	(d)
Raw funds rate change	25.02		-18.54	
	(16.43)		(25.11)	
Expected change		20.64		-50.74***
		(13.67)		(10.19)
Unexpected change		59.04**		-176.1***
		(26.11)		(20.07)
Moneyness	4.218**	4.202**	5.386	5.386
	(1.722)	(1.713)	(5.628)	(4.559)
Constant	-11.95***	-11.89***	-26.29***	-27.60***
	(2.714)	(2.640)	(7.669)	(6.588)
R-squared	0.024	0.034	0.052	0.455

Panel C. Call Options on Citigroup

Regressor	2003 - 2017 (exclude 2008 financial crisis) (a)	(b)	2008 financial crisis (c)	(d)
Raw funds rate change	-65.48***		25.21	
	(20.17)		(45.03)	
Expected change		-62.56***		45.62
		(17.24)		(33.37)
Unexpected change		-110.0***		157.7**
		(33.17)		(64.33)
Moneyness	1.482	1.470	11.89	11.95
	(2.234)	(2.235)	(10.13)	(9.777)
Constant	10.94***	11.12***	7.247	8.802
	(4.049)	(4.020)	(13.59)	(12.61)
R-squared	0.027	0.035	0.067	0.176

Panel D. Put Options on Citigroup

Regressor	2003 - 2017 (exclude 2008 financial crisis) (a)	(b)	2008 financial crisis (c)	(d)
Raw funds rate change	42.17***		-14.02	
	(14.19)		(27.54)	
Expected change		39.88***		-34.77**
		(12.11)		(13.86)
Unexpected change		76.02***		-149.8***
		(17.66)		(31.54)
Moneyness	4.192**	4.187**	5.569	5.569
	(1.625)	(1.617)	(5.554)	(4.469)
Constant	-11.91***	-12.03***	-13.24	-14.81**
	(2.601)	(2.545)	(7.810)	(6.087)
R-squared	0.046	0.059	0.050	0.384

Panel E. Call Options on Wells Fargo

Regressor	2003 - 2017 (exclude 2008 financial crisis) (a)	(b)	2008 financial crisis (c)	(d)
Raw funds rate change	-82.11***		114.6	
	(20.51)		(83.94)	
Expected change		-77.60***		150.4**
		(16.90)		(58.13)
Unexpected change		-114.5***		419.3***
		(34.33)		(73.81)
Moneyness	0.946	0.802	-4.479	-4.479
	(2.844)	(2.859)	(20.57)	(18.94)
Constant	10.37**	10.48**	82.87**	83.35**
	(4.784)	(4.778)	(38.38)	(37.18)
R-squared	0.036	0.040	0.059	0.163

Panel F. Call Options on Wells Fargo

Regressor	2003 - 2017 (exclude 2008 financial crisis) (a)	(b)	2008 financial crisis (c)	(d)
Raw funds rate change	52.09***		-37.97	
	(15.59)		(29.29)	
Expected change		50.97***		-55.71***
		(15.59)		(17.89)
Unexpected change		60.41**		-159.3***
		(25.06)		(40.85)
Moneyness	3.305*	3.347*	4.625	3.741
	(1.846)	(1.841)	(7.263)	(6.615)
Constant	-8.632***	-8.659***	-24.14***	-25.39***
	(2.537)	(2.527)	(8.789)	(8.446)
R-squared	0.049	0.049	0.090	0.251

The table depicts the results of regressing bank equity option returns on the Federal funds rate changes.

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Appendix: Greeks for the Short Lived Arbitrage Model

A1: Call options on spot indices or stocks with continuous dividend q

$$C = e^{-q(T-t)}SN(d_1) - e^{-R(T-t)}KN(d_2), \tag{14}$$

$$P = e^{-R(T-t)}KN(-d_2) - e^{-q(T-t)}SN(-d_1), \tag{15}$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(R - q + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \tag{16}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}. \tag{17}$$

$$Delta\ (\delta):\ \frac{\partial C}{\partial S} = e^{-q(T-t)}N(d_1), \qquad \frac{\partial P}{\partial S} = e^{-q(T-t)}(N(d_1) - 1),$$

$$Vega(\nu):\ \frac{\partial C}{\partial \sigma} = \frac{\partial P}{\partial \sigma} = e^{-q(T-t)}S\sqrt{T-t}n(d_1),$$

$$Rho(\rho):\ \frac{\partial C}{\partial R} = K(T-t)e^{-R(T-t)}N(d_2),\ \frac{\partial P}{\partial R} = -K(T-t)e^{-R(T-t)}N(-d_2),$$

$$Gamma(\gamma):\ \frac{\partial^2 S}{\partial S^2} = \frac{\partial^2 P}{\partial S^2} = \frac{e^{-q(T-t)}n(d_1)}{S\sigma\sqrt{T-t}},$$

where $n(d_1)$ is the standard normal density, N is the cumulative standard normal distribution, and R and σ are implied.

A2: Call options on gold and silver futures contracts

$$C = e^{-R(T-t)}[FN(d_1) - KN(d_2)], \tag{18}$$

$$P = e^{-R(T-t)}[KN(-d_2) - SN(-d_1)], \tag{19}$$

$$d_1 = \frac{\ln\left(\frac{F}{K}\right) + \frac{\sigma^2}{2}(T-t)}{\sigma\sqrt{T-t}}, \tag{20}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}. \tag{21}$$

$$Delta\ (\delta):\ \frac{\partial C}{\partial S} = e^{-R(T-t)}N(d_1), \qquad \frac{\partial P}{\partial S} = e^{-R(T-t)}(N(d_1)-1),$$

$$Vega(\nu):\ \frac{\partial C}{\partial \sigma} = \frac{\partial P}{\partial \sigma} = e^{-R(T-t)}K\sqrt{T-t}n(d_2),$$

$$Rho(\rho):\ \frac{\partial C}{\partial R} = -(T-t)C,\ \frac{\partial P}{\partial R} = -(T-t)P,$$

$$Gamma(\gamma):\ \frac{\partial^2 S}{\partial F^2} = \frac{\partial^2 P}{\partial F^2} = \frac{e^{-R(T-t)}n(d_1)}{F\sigma\sqrt{T-t}}.$$

A3: Margined Options on Brent Crude Oil

Interest rates do not appear in the gBm option pricing formula for margined options. The Greeks as the same as Greeks for equity style futures options except implied $R = R^*$ is no longer virtual yield but a market friction perturbed from zero.