The Effects of the Concrete-Representational-Abstract-Integration Sequence to Teach Students with Disabilities Binomial Multiplication and Quadratic Transformations into Factored Form

by

Alexcia J. Moore

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Approved by

Margaret M. Flores, Chair, Professor, Special Education, Rehabilitation and Counselling
Caroline Dunn, Professor, Special Education, Rehabilitation and Counselling
Margaret Shippen, Professor, Special Education, Rehabilitation and Counselling
Craig Darch, Professor, Special Education, Rehabilitation and Counselling

Abstract

Algebra is the foundational mathematics course that students take as they begin their high school career and is often difficult for the average learner. In order for students with disabilities and those with mathematics difficulties to meet the high expectations of future mathematical courses, providing quality instruction by implementing evidenced based practices within the inclusive setting is essential to improving the academic achievement of students in courses such as algebra. The concrete-representational-abstract (CRA) sequence is an evidenced based practice that is proven effective in increasing students' mathematical skills. The development of a modified version of the CRA sequence, the concrete-representation-abstract-integration sequence (CRA-I), was shown to be effective in teaching advanced mathematics skills to three students with disabilities (Strickland, 2012). However, the research in teaching secondary students with disabilities advanced mathematical skills is lacking. Therefore, this study investigated if receiving supplemental algebra instruction using the CRA-I sequence, a graphic organizer, the BOX Method, and mnemonic instruction would affect students' performance in multiplying binomial expressions to form quadratic expressions and transforming quadratic expressions into factored form. Similar to previous CRA research, this intervention taught students two mnemonics, FACTOR and HUMP BACK FACTOR, to develop procedural fluency in the transformation of quadratic expressions into factored form. There was no current research that implemented the use of CRA-I and mnemonic strategy instruction for advanced algebra skills, although CRA and mnemonic instruction is shown effective for improving basic algebra skills.

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Chapter 1. Introduction

A shift in the focus of how to provide high quality education for all began in the United States in the mid-1980s when schools failed to prepare students to compete against their counterparts across the world in reading and mathematics (Brownell, Ross, Colon, & McCallum, 2005). In the decade prior, students with disabilities acquired rights to receive a free and appropriate education with the passage of the Education for all Handicapped Children Act in 1975. The passage of this law opened the doors for all students to receive a free and appropriate education. This was the beginning of inclusive education. As schools began to provide inclusive educational settings to all students, parents and educators across the nation saw that all students, not merely those with disabilities, were not learning and educators failed to provide adequate instruction. Subsequent educational laws such as No Child Left Behind (NCLB, 2001) and the Every Student Succeeds Act (ESSA, 2014) focused on increasing the standards for students and ensuring that students received effective instruction to increase student academic growth. The shift and focus on high quality instruction to meet more rigorous standards led to the development of the Common Core State Standards Initiative (CCSSI, 2010). The CCSSI published standards in 2010 and by 2013 forty-five states, the District of Columbia, and several U.S. territories adopted the standards for full implementation (CCSSI, 2010). There are forty-one states, the District of Columbia and several U.S. territories that are implementing CCSSI (CCSSI, 2019). Therefore, the majority of students across the nation must meet more rigorous standards. For students with disabilities, the Individuals with Education Act (IDEA) of 1990 and 2004 emphasized the requirement that students with disabilities have equal access to the general education curriculum and are served in their least restrictive environment (Strickland & Maccini, 2013). Therefore, students with disabilities are also held to the same standards.

Most states in the U.S. adopted the Common Core State Standards (CCSS) with a purpose to identify standards that would prepare students to be college and career ready, develop higher order thinking skills, prepare students to succeed globally, and implement practices that are research based and evidenced based (CCSSI, 2001). Specifically, in the area of mathematics, the CCSSI standards requires that a greater emphasis is put on the development of students' conceptual understanding in basic mathematical skills (Watt, Watkins, & Abbitt, 2016). Beginning in a third-grade, the CCSS focuses on the development of reasoning skills beyond computation only. Students develop numerical and multiplicative reasoning skills in addition to the early development of reasoning in algebra. Effective instructional practices at the elementary level must develop students' computation skills, specifically in numerical computation, through the use of manipulatives to build conceptual understanding of mathematical concepts and relate them to the real world (Pratt, 2018, Scheuremann, Deshler, & Schumaker, 2009). The CCSS focuses on developing these mathematical skills through a progression that builds conceptual understanding so students can find more success in problem-solving, computation, and procedural knowledge (CCSSI, 2001).

As a response to the need for educational reform, several organizations provided guidance to address the growing needs of students. Specifically, the National Council for Teachers of Mathematics (NCTM) responded to the call to improve mathematics instruction by publishing the *Principles and Standards for Mathematics in 1989* (NCTM, 2014). Since then, NCTM published an updated version in 2000 and the *Principles to*

Actions in 2014, imploring educators to develop students' understanding and reasoning in mathematics though the use of a variety of tools and hands-on activities not only in elementary school but throughout grades K-12 (Pratt, 2018). Furthermore, the NCTM (2014) provided guidance to educators to meet the demands of students who come from culturally, linguistically, and economically diverse backgrounds, all which impact student success.

Researchers, such as Witzel, Smith, and Brownell (2001) and Miller and Hudson (2006), responded to the call for more researched based interventions to improve student learning. Witzel and colleagues demonstrated that students with disabilities who struggle with mathematics benefitted from hands-on experiences and strategy instruction to build conceptual knowledge and increase skills in more advanced mathematical concepts. These hands-on experiences, according to Witzel et al. (2001), allowed students to develop conceptual understanding of the operations, numerical symbols, and abstract equations. Miller and Hudson (2006) identified that the use of multiple representations was an important step in helping students with disabilities develop conceptual understanding in the mathematics classroom. In addition, Miller and Hudson (2006) identified the concrete-representational-abstract sequence (CRA) as an instructional approach that provided students with opportunities to engage in instruction that used multiple representations and provided hands-on experiences for students. The CRA sequence is a three-step process and students learned to progress through skills going from concrete application to abstract application. During the first step students learned to use objects during the concrete phase to learn mathematical concepts. During the second step, representational, students learned to use picture representations of mathematical concepts. Lastly, during the third step, abstract, students learned to use numbers and symbols to represent mathematical concepts.

Statement of the Research Problem

Research regarding effective interventions to develop advanced algebra skills for secondary students with disabilities is lacking. There were two studies conducted with secondary students with disabilities using strategies including graphic organizers and the explicit inquiry routine (EIR) to improve mathematical performance of secondary students with disabilities (Ives, 2007; and Scheuermann et al., 2009). There were only three studies that investigated using the concrete- representational-abstract (CRA) sequence with secondary students to teach basic algebra concepts. Maccini and Ruhl (2001), Maccini and Hughes (2000), and Witzel, Mercer, and Miller (2003) illustrated that supplemental CRA instruction was an effective way of increasing algebra skills for students with disabilities and who struggle in mathematics.

Strickland and Maccini (2012) further extended the line of CRA research and investigated teaching mathematics skills to solve more complex algebraic problems. Strickland and Maccini (2012) used a modified version of the CRA sequence, the concrete-representation-abstract-integration sequence (CRA-I), to teach mathematics skills to solve complex algebraic problems involving quadratic expressions. Strickland and Maccini implemented the use of a graphic organizer and CRA-I to teach students to multiply linear expressions embedded within word problems. This is the only research conducted investigating improving students with disabilities mathematical skills beyond linear equations. Therefore, as a response to these findings in literature, the development of this study focuses on the implementation of mnemonic instruction and graphic organizers taught using CRA-I sequence to multiply linear expressions to form quadratic expressions and to transform quadratic expressions into factored form. In addition, this study focuses on improving students' accuracy and investigating whether students will generalize their skills to word problems.

Justification for the Study

ESSA (2014), NCTM (2000), and the National Mathematics Advisory Panel (NMAP, 2008) emphasized the need to provide effective interventions for students in mathematics during the school day in order to impact instruction. NMAP (2008) stated that algebra teachers should be prepared to meet the needs of students who lack basic skills in algebra such as the commutative property, distributive property, solving algebraic equations, and transforming algebraic equations. The NCTM (2014) *Principles to Actions* identified eight mathematics teaching practices that all effective mathematics teachers should use daily when providing instruction. These teaching practices include: (a) establishing clear goals, (b) promoting problem-solving and reasoning, (c) making connections through the use of mathematical representations, (d) engaging in meaningful mathematical discourse, (e) purposeful questioning, (f) developing conceptual understanding to build procedural fluency, (g) consistently support productive mathematical struggle, and (h) elicit and use student ideas. Furthermore, NCTM (2014) stated that the use of manipulatives across grades k-12 is a necessity for improving student success in solving complex algebraic problems.

Watt et al. (2016), conducted a systematic review of literature and identified five effective algebra interventions for teaching students within the domains of the common core state standards. Watt and colleagues identified CRA, tutoring, mnemonic instruction, enhanced anchored instruction, and graphic organizers as effective interventions for teaching students with mathematical difficulties. Most studies found in the review used a combination of interventions to improve learning. Ives (2007) implemented the use of graphic organizers taught within strategy instruction in mathematics classrooms for students in the seventh through twelfth grades to learn how to solve systems of linear equations. This study found a higher level of conceptual understanding for solving systems of linear equations with students who used graphic organizers compared to those who did not use graphic organizers. Scheuermann et al. (2009), investigated students' performance in solving one-variable equations embedded within word problems using the explicit inquiry routine (EIR) which combines mathematical teaching approaches such as CRA and explicit instruction. Students in this study improved in solving one-variable equations presented in a variety of word problems.

Maccini and Ruhl (2001) and Maccini and Hughes (2000) used the CRA sequence in combination with mnemonic strategy instruction to improve basic algebra skills in students with disabilities. In both studies, students learned the STAR strategy to create and solve linear equations from word problems within the CRA sequence. Maccini and Ruhl (2001) investigated the effects of CRA strategy and the mnemonic strategy on the problem-solving performance of secondary students with disabilities to improve the algebraic subtraction of integers embedded within word problems. Maccini and Hughes (2000) extended this line of research and implemented the CRA sequence and the mnemonic strategy to improve students' performance in solving algebraic word problems involving the addition, subtraction, multiplication, and division of integers. Witzel et al.

(2003) implemented the CRA strategy and taught a group of students how to solve linear equations for one variable.

In the Strickland and Maccini (2012) study, students learned advanced algebraic skills using the CRA-I sequence in combination with a graphic organizer. The students in the Strickland and Maccini (2012) study learned binomial multiplication using the CRA-I sequence and a graphic organizer, The BOX Method, following the acquisition of conceptual knowledge of binomial multiplication. The graphic organizer helped students build conceptual understanding and develop procedural fluency in binomial multiplication. Therefore, to develop advanced algebraic skills, effective instructional interventions must be used in combination to provide students with the greatest probability of acquiring and maintaining skills.

The use of mnemonic strategy instruction in combination with the CRA sequence was effective for students with disabilities when teaching basic algebraic skills (Maccini & Hughes, 2000; Maccini & Ruhl, 2001; Witzel et al., 2003) As evidenced in studies conducted by Maccini and Hughes (2000) and Maccini and Ruhl (2001), when CRA instruction included mnemonic strategy instruction, students learned to develop a plan to perform operations on algebraic concepts. In addition, mnemonic instruction helped to improve students with disabilities' memory issues by drawing on their strengths rather than weaknesses by making connections to familiar and meaningful concepts (Scruggs & Mastropieri, 1990). Therefore, these methods may improve student performance when attempting more complex algebraic problems.

Purpose of the Study

The purpose of this study was to determine if receiving supplemental algebra instruction using the CRA-I sequence, graphic organizers, and mnemonic instruction would affect students' performance in multiplying binomial expressions to form quadratic expressions and transforming quadratic expressions into factored form. Probes assessed accuracy in binomial multiplication, quadratic transformations into factored form when the coefficient in the first term is equal to one, and quadratic transformations into factored form when the coefficient in the first term is more than one. The generalization probe determined if any skills learned in the intervention would transfer to word problems.

Research Questions

1. What are the effects of instruction using the concrete-representational-abstract-integration strategy (CRA-I) and a graphic organizer, the BOX Method, on students' accuracy in multiplying binomial expressions to form quadratic expressions?
2. What are the effects of instruction using the mnemonic strategy, FACTOR, taught within the CRA-I strategy on students' accuracy in completing problems involving the transformation of quadratic expressions into factored form when the coefficient in the first term is equal to one?
3. What are the effects of instruction using the mnemonic strategy, HUMP BACK FACTOR, taught within the CRA-I strategy on students' accuracy in completing problems involving the transformation of quadratic expressions into factored form when the coefficient in the first term is more than one?

4. What are the effects of the CRA-I strategy and the mnemonic instruction on students' generalization to problems including word problems involving the multiplication of binomial expressions and transforming quadratic expressions into factored form?

Definition of Terms

Algebraic Expression: a mathematical sentence which contains at least one variable and symbols. EX: 9n+5 (4x+3) -2x.

Binomial: an algebraic expression representing the sum or difference of two terms.

Binomial multiplication: The multiplication of two binomials.

BOX Method: A graphic organizer with a four-quadrant representation to teach students to organize and self-monitor computations when performing binomial multiplication and transformation of quadratics into factored form.

Coefficient: a number placed in front of a variable that indicates that the number (constant) is multiplied to the unknown quantity.

Combine Like Terms: also known as simplifying algebraic expressions, is the process of adding the coefficient of like terms.

Constant: A number that is not in front of a variable. The known number in an algebraic expression.

Denominator: the bottom number in a fraction; the divisor.

Expression: a mathematical sentence without an equal sign.

Equation: a mathematical sentence with an equal sign.

Factor: a number or algebraic expression that is multiplied by another number or algebraic expression to acquire a product.

Factored Form of Quadratic Expressions: A polynomial expression written as the product of two binomials. **Transform into factored form** refers to the transformations of functions.

Factoring: The process used to write a polynomial expression as a product of its factors.

Integer: a positive or negative whole number that is not a fraction.

Inverse Operations: Operations that undo each other. For example, the inverse operation of addition is subtraction.

Leading Coefficient: The coefficient of the first term.

Like Terms: Terms that have the same variable raised to the same power (exponent).

Linear Expression: A polynomial expression written in standard form with the highest power being 1. For example, $2x+3$ is a linear expression because the power on the term $2x$ is an understood 1.

Monomial: an algebraic expression with one term that is either a number, a variable or a product of a number(s) and variable(s).

Numerator: the top number in a fraction.

Numerical Expression: a mathematical sentence which contains numbers and symbols representing operations. EX: $9+5$ or $(4+3)-2$

Polynomial- an algebraic expression consisting of more than one monomial term or the sum or difference of monomial terms. For example, $3x+2y-4$ or $3x^2+2x+2$.

Quadratic Expression: A polynomial expression written in standard form with the highest power being 2. For example, $2x^2+3x+6$ is a linear expression because the power on the term $2x^2$ is 2.

Standard Form of a polynomial: a polynomial expression containing variables with exponents (powers) that are written from highest exponent (power) to lowest exponent (power).

Terms: a number, variable, or the product of a number and variable that are separated by addition or subtraction. For example, in $2x^2+3x+6$ there are three terms. $2x^2$ is one term.

Trinomial: an algebraic expression representing the sum or difference of three terms.

Variable: a symbol that represents one or more numbers, most often a letter such as x

Limitations of the Study

The current study is limited in the number of participants. There were two participants, so the results of this study cannot be generalized. In addition, the students received the intervention individually in a separate setting. Therefore, it is unclear how effective the intervention is when delivered to a group of students in the general education setting. Furthermore, the study does not compare the intervention to another form of instruction. Therefore, the effectiveness of the intervention compared to others cannot be determined. The researcher implemented all of the lessons in the intervention. Therefore, it cannot be determined that the same results would be achieved if a teacher implemented the intervention.

Summary

This study sought to extend previous research conducted by Strickland and Maccini (2012). Similar to the previous study, this study taught algebra skills through the use of the CRA-I strategy to move students through the sequence of skills and work with quadratic expressions within in the appropriate pace of the algebra curriculum. Unlike the previous studies, this study introduced the use of the graphic organizer, the BOX Method, during the first lesson of instruction to make connections earlier between conceptual understanding and procedural fluency. Furthermore, the intervention taught students two mnemonics, FACTOR and HUMP BACK FACTOR, to develop procedural fluency in the transformation of quadratic expressions into factored form. There was no current research that implemented the use of CRA-I and mnemonic strategy instruction for advanced algebra skills, although CRA and mnemonic instruction is shown effective for improving basic algebra skills. In addition, previous studies focused on developing conceptual understanding through the context of word problems, however, this current study focused on using array representation to build conceptual understanding of the multiplication and the distributive property to increase student accuracy in binomial multiplication and transformation of quadratics into factored form.

Chapter 2. Review of Literature

Introduction

Mathematics is often difficult for the average learner and the level of difficulty increases as students matriculate through high school mathematics courses such as Algebra, Geometry, and Algebra 2 (Strickland, 2017). Currently, students with disabilities are expected to enroll in courses beyond Algebra 1 and to meet rigorous standards for mathematical coursework (Strickland, 2017; Strickland & Maccini, 2010). In addition to meeting the minimum course requirements for high school mathematics, students with disabilities and those with mathematical difficulties who have a goal of enrolling in a 2- or 4-year post-secondary education program will need to enroll in courses in high school that will make them college ready. Courses such as Algebra 2, Pre-calculus, and Calculus are all mathematics courses that will benefit students in high school and make them college ready, especially for students wanting to pursue careers in the Science, Technology, Engineering, and Mathematics (STEM) field (Wade, Sonnert, Sadler, & Hazari, 2017). However, many students, especially those with disabilities and mathematics difficulties, are not prepared to enroll in advanced high school mathematics courses nor college mathematics courses, and many are unable to be successful once enrolled in these courses.

According to the National Council of Teachers of Mathematics (NCTM, 2000), if students want to be successful in college mathematics courses, educators must be trained to provide students with the skills to develop conceptual understanding and procedural fluency in mathematics while in high school (NCTM, 2000). When students are not provided with instruction that develops conceptual understanding and procedural fluency, they will often have to enroll in remediation courses while in college. According to the *Remedial Coursetaking at U.S. Public 2- and 4-Year Institutions: Scope, Experiences, and Outcomes Statistical Analysis Report* (Chen*,* 2016) from those individuals enrolled in 2-and 4-year post-secondary institutions between 2003-2009, 59.3% of students enrolled at 2-year institutions and 32.6% of students enrolled at 4-year institutions were enrolled at some point in remedial mathematics courses during their post-secondary enrollment. In addition, researchers have found that college professors believe that often students are unable to gain success in college-level mathematics courses because they have had a lack in instruction that develops students' ability to understand concepts and make connections to the appropriate procedures to use (Wade et al., 2017). According to research conducted by Wade and colleagues, the result of lack of instruction in developing conceptual understanding is that many students must then alter their career plans and pursue other fields that do not require the more advanced college mathematics courses.

Therefore, to meet the high expectations of future mathematical courses, providing quality instruction to a group of diverse learners by implementing evidenced based practices within the inclusive setting is essential to improving the academic achievement of students in mathematics courses such as algebra (Strickland, 2014). Algebra is the foundational mathematics course that students take as they begin their high school career and is the course that continues to build conceptual knowledge and procedural fluency for students to be able to pursue more advanced mathematics courses. This chapter presents research regarding algebra interventions that improved algebraic skills of secondary students with disabilities and secondary students who struggle in mathematics. This literature review is presented in the following major sections: algebra skill development, mnemonic strategy instruction, and CRA for algebra. The CRA section is divided into two segments: CRA for basic algebra concepts and CRA-I for complex algebra concepts.

Research Regarding Effective Strategies to Develop Algebraic Thinking

The NCTM (2000) standards emphasized the need for all students to receive high quality mathematics instruction that provides access to the general education curriculum. Students in the secondary setting often struggle with making the connection between arithmetic and algebraic thinking. According to Cai, Lew, Morris, Moyer, Ng, and Schmittau (2005) students in the United States (U.S.) are often not introduced to the formal use of algebraic thinking skills until the eighth and ninth grade. This lack of formal instruction in the use of algebraic thinking skills until later grades results in students who struggle in secondary mathematics when asked to apply algebraic thinking skills fluently. Cai et al. (2008) identified that curriculum should promote algebraic thinking skills in the elementary grades. However, when students are unable to learn essential skills of algebraic thinking in the elementary grades, quality instructional practices at the secondary level become vital to the future success of secondary students with mathematics difficulties.

Maccini, Strickland, Gagnon, and Malmgren (2008) identified six mathematical principles and practices that would improve the performance of secondary students with mathematical difficulties. These practices included explicit/direct instruction, strategy instruction, real-world activities, technology, graduated instructional sequence, grouping for instruction, and other approaches to adapting instruction including cue cards, graphic organizers, and monitoring of academic tasks (Maccini et al., 2008). Results from Maccini and colleagues' study were consistent with results found in Gagnon and Maccini (2007), a study on highly effective special education teachers in the inclusive setting and are consistently identified in research as effective in improving secondary students' mathematical skills.

One instructional method, graphic organizers, was effective in improving mathematical skills for secondary students in Ives' (2007) study. According to Ives (2007), the use of graphic organizers in the secondary setting for students with mathematical difficulties was a useful technique used primarily in literature to improve reading comprehension. The researcher suggested that graphic organizers should be used in higher level mathematics courses so that students gain a broader understanding of the mathematical symbols, expressions, and equations and relations that exist between these mathematical concepts. According to Ives, graphic organizers provided students with a way to represent complex mathematical concepts graphically. Essentially, Ives conducted a study that compared groups of students' abilities to solve systems of two linear equations with two variables and systems of three linear equations with three variables.

Ives (2007) sought to answer three questions. The first question asked if students with learning disabilities or attention deficit disorder would perform better on the skills and concepts after learning to use a graphic organizer to solve two linear equations in two variables. The next question asked if there were a difference between the performance of those in the graphic organizer group (treatment) and the control group two to three weeks after instruction had ended. Finally, the author investigated whether the study could be replicated using the graphic organizers to solve three linear equations in three variables.

In order to address the research questions, the researcher conducted two studies and used a two-group comparison experimental design to analyze the data. Using this type of design allowed the researcher to determine if there were differences observed in the groups as well as the effectiveness of using the graphic organizer to teach students with mathematical difficulties how to solve the two types of systems of linear equations. The first study answered the first two research questions. The second study answered the first and last research questions. The setting for the study was a private school for students with learning disabilities in grades six through 12. For the first study, there were 26 participants with 14 participants in the graphic organizer group and 16 participants in the control group. For the second study, there were a total of 20 participants with 10 participants in each group. In both studies, participants included students with attention deficit hyperactivity disorder and learning disabilities in reading, language, mathematics, and written ability.

The researcher developed a graphic organizer used in both studies that students used to learn to solve systems of linear equations. The graphic organizer consisted of a chart organized with two rows by three columns, with each column labeled III, II, and I. The first study focused on students learning how to solve two linear equations in two variables; therefore, they used only the first two columns (II and I). In the second study, students used all three columns and rows because students completed complex systems of linear equations. Using the graphic organizer, students worked in a clock-wise motion from cell to cell using the column headings as a guide. They worked from left to right to eliminate variables and solve the system of equations (Ives, 2007). The top row of the graphic organizer guided students as they eliminated the variables according to the column heading. For example, the students began in the first cell that has a III as the heading. The graphic organizer prompted the student to reduce the system of equations with three variables in the first cell to a system of equations with two variables in the cell under heading II. The last cell on the top row had a one heading and prompted the student to solve the system of equations with two variables in the previous cell for one variable and place the solution in the cell with heading I. The bottom row guided students to solve the equations for the variables using the column headings to determine the number of solutions for each cell. For example, once the student completed the top row, the student placed the solution from the cell on the top row in column I in the cell on the bottom row under column I. Next, the graphic organizer prompted the student to move to the next cell on the bottom row in column II and substitute the solution to the variable from the previous cell into the equation on the top row in column II. Once the student completed the problem, the student had two solutions on the bottom row in column II. The students continued this until they solved all variables.

The researchers used three measures with content related to prerequisite skills, justification skills, and classroom instructional objectives to collect data in the study. The prerequisite skills test measured the extent to which students were adequately prepared for the lessons. The content skills test assessed students’ justification of procedures used to solve systems of equations as well as their ability to solve for systems of equations. Finally, the last measure used in the study was an end-of-unit test that the teachers developed and gave prior to and after the study concluded. This measure assessed the content covered in class during the time of the study.

The researcher developed the lesson plans that included strategy instruction. According to Ives (2007), strategy instruction included verbal modeling, dialogue, and rich explanations (Ives, 2007). The teachers used direct instruction that included feedback, asking questions, and giving probes. Lesson one focused on providing students with an assessment and review of prerequisite skills. Lessons two and three provided students with the introduction to simple linear equations leading to working with a variety of systems of linear equations. Although the researcher began with structured lesson plans, they revised the lesson plans throughout the study based on the needs of the students. Revisions in the lesson plans were consistent across all the courses. The researcher included advance preparation procedures in which he attended all of the classes where the study was going to be implemented in order for the students to become familiar with him.

The study began with administration of the *Test of Prerequisite Skills*. Then instruction began with a review of the pre-requisite skills. After this review, the researcher implemented the lessons according to the lesson plan procedures. On the last date of instruction, the researcher administered students the *Test of Content Skills*. Two to three weeks after instruction, the students completed the content assessment again. The researcher administered the teacher-generated unit test one week after instruction.

The researcher analyzed the data with an analysis of variance (ANOVA) to compare group means on the teacher-generated and content skills tests (Ives, 2007). Results from the teacher-generated test in study one indicated that there was a statistical difference observed in the group means of the graphic organizer group and the control group. For the graphic organizer group, the group means were significantly higher than the group means for the control group and there was a medium effect size observed. On the content skills test, the researcher garnered two scores from the assessment. First, the researcher analyzed the scores from the concepts section followed by an analysis of scores from the solving systems section of the assessment. Results for the first section score, the concepts section, indicated that the group means for the graphic organizer group were significantly higher on the posttest and the follow-up test compared to the group means for the control group. There was a large effect size on both the posttest and the follow-up test. However, on the solving systems section of the posttest and follow-up test, there was not a statistically significant difference in the group mean scores for the graphic organizer group and the control group. Therefore, results from study one indicated that students in the graphic organizer group's performance improved more than students' performance in the control group. In addition, students in the graphic organizer group had a better conceptual understanding of the content and maintained these skills two to three weeks after instruction ended.

Results from study two addressed research questions one and three. Question one investigated whether students who received instruction using graphic organizers outperformed those who did not receive instruction using a graphic organizer. Question three investigated if results from the second study were replicated from the first study. Study two had a smaller sample size and therefore it was expected that statistical analysis would not yield any statistically significant results due to the reduction in power. To deal with this, the researchers used a "what if analysis". Results from the data analysis using the what if procedure yielded statistically significant results with a large effect size. The researchers used visual analysis of bar graphs to answer the third question and identify if results from the first study were replicated in study two. Visual analysis of the graphs confirmed that results from study one in study two. Visual analysis also confirmed that for every assessment measure the graphic organizer group outperformed students in the control group. However, the researcher also noted that there was not any statistically significant difference in the group means in study one on the teacher generated test.

Results from Ives' (2007) study showed the promising benefits of using a graphic organizer to teach students with mathematical difficulties in the secondary setting. Results from the study provided evidence that students who used a graphic organizer had better conceptual understanding of mathematical concepts and skills compared to those who did not use a graphic organizer. Students who used graphic organizers consistently solved systems of equations with a higher accuracy rate than those who did not use graphic organizers. The use of the graphic organizers also suggested that students who had other difficulties, such as in language and writing, benefitted from having a way to organize the vocabulary and language often used in instruction in the secondary mathematics classroom.

Graphic organizers were shown as effective for students with mathematics difficulties who also experienced language and writing deficits because they helped students better understand the meaning of the mathematics they learned. However, utilizing one representation to teach new mathematical concepts and skills cannot promote generalization of skills (Miller & Hudson, 2006). Miller and Hudson's (2006) first guideline for helping students better understand the meaning of mathematics was use a variety of representations to convey the meaningfulness of the mathematics learned. The use of a variety of representations in the mathematics classroom was important because it promoted the development of conceptual understanding. The National Council of Teachers of Mathematics (NCTM, 2000) identified conceptual understanding as a main goal of mathematics instruction.

Array representation and area models used in the elementary level when teaching multiplication promotes multiplicative understanding and algebraic reasoning (Day & Hurrell, 2015). According to Day and Hurrell, array representations and area models of multiplication promote the understanding of multiples, factors, and makes comparisons from the concrete level to the abstract level easier. In addition, the use of array representation fosters the understanding of the commutative property, which states that with addition or multiplication the order of the terms does not matter because the results will be the same. Barmby, Harries, Higgins, and Suggate (2009) stated that array representations develop a greater depth of conceptual understanding and procedural knowledge in students. Furthermore, the CCSS identified in the third-grade students must receive instruction on using array representation and area models of multiplication to develop students understanding of the distributive property (Kinzer & Stanford, 2014).

The use of array representation must be implemented in mathematics instruction for secondary students especially those who failed to acquire fluency in multiplication at the elementary level. According to Kinzer and Stanford (2014), the distributive property is the center of multiplication and area models of multiplication are effective instructional methods for developing algebraic reasoning. However, mathematics teachers lack thorough understanding of why multiplication is performed and how multiplication is performed (Pratt, 2018). There is a lack of conceptual understanding of multiplication, relations, and connections to other mathematics concepts. Therefore, Pratt (2018) investigated whether developing preservice mathematics teachers' understanding of multiplication as area models would foster conceptual understanding for the multiplication of integers and binomials and impact future instruction. Results from the study illustrated that after implementing the use of area models of multiplication and manipulatives, most participants' conceptual understanding of multiplication changed.

Specifically, Pratt (2018) implemented a collective case study that spanned three academic years. Using a design approach, each year the researcher planned a series of tasks where participants engaged in multiplication of integers and binomials. These scaffolded tasks throughout the year progressed participants from multiplying integers to multiplying binomials. At the end of the academic year the researcher reviewed the activities and changed the focus of the tasks in year two. During the second year the focus shifted to scaffolding the development of the participants' conceptual understanding. First, participants engaged in using area models of multiplication to develop understanding of whole number multiplication. These tasks used whole numbers and binomials then participants transitioned to multiplication of integer sets. Based on the findings from the second year, participants in the third year followed the same implementation plan; however, algebra blocks and base ten blocks supported instruction. The researcher modeled whole numbers with base ten blocks and algebraic representations with algebra blocks prior to engaging in tasks involving multiplication of integers and multiplication of binomials. Comparisons between the whole number and binomial representations and then participants engaged in the remainder of the tasks of the study. Participants in the third year showed the most growth in conceptual understanding, illustrating the significant impact that a variety of representations, including area models of multiplication and manipulatives, have in developing conceptual understanding of multiplication. Although research says that multiple representations are effective in developing algebraic reasoning skills in secondary students, mnemonic strategy instruction is also a practice found to be effective in providing high quality instruction for students with disabilities in the inclusive setting (Mastropieri & Scruggs, 1998; Scruggs et al., 2010). Specifically, mnemonic instruction helps to improve students with disabilities' memory issues by drawing on their strengths rather than weaknesses by making connections to familiar and meaningful concepts (Scruggs et al., 1998). Therefore, secondary mathematics instruction must implement additional strategies, such as mnemonic instruction, to teach students more complex algebraic skills.

Research Regarding Mnemonic Instruction

Mnemonics are strategies or procedures that improve memory (Scruggs et al., 2010). Mnemonic strategies are highly effective in improving the amount and length of time students store and recall information (Mastropieri & Scruggs, 1998). According to Mastropieri and Scruggs, mnemonic strategies were most successful when new information is related to information already stored in one's long-term memory. Mnemonic strategy methods include the keyword, peg word, reconstructive elaborations, and letter strategies. Letter strategies are the most familiar to students and include mathematics strategies such as FOIL and PEMDAS.

Mnemonic strategies were shown effective for students with and without disabilities as well as in a variety of content areas (Mastropieri & Scruggs, 1998). Maccini et al. (2007) conducted a systematic review of literature on mathematics interventions for secondary students with disabilities. Mnemonic instruction was an efficient and effective method for teaching mathematics concepts to secondary students. One study included in the review targeted secondary students with disabilities and taught students the LAP mnemonic to improve skills in addition and subtraction of fractions (Test & Ellis, 2005). The LAP mnemonic means: "*Look at the sign and denominator. Ask yourself the question, Will the smallest denominator divide into the largest denominator an even number of times? Pick your fraction type*" (Test & Ellis, 2005). Instruction with the LAP mnemonic strategy improved students' accuracy in adding and subtracting fractions.

Another study with secondary students in mathematics focused on problem solving and used a mnemonic strategy called SOLVE (Freeman-Green et al., 2015). The SOLVE strategy means: "*Study the problem, Organize the facts, Line up a plan, and Examine your results*" (Freeman-Green et al., 2015). The nine lessons in the intervention was implemented using explicit instruction. There were eight phases of the intervention which included the following: (1) pretest, (2) describe, (3) model, (4) verbal practice, (5) controlled practice and feedback, (6) advanced practice and feedback, (7) posttest procedures, and (8) maintenance and generalization procedures (Freeman-Green et.al., 2015). Once students completed all phases of the intervention they took a standardized assessment. This study illustrated that students with disabilities can develop problem solving skills using mnemonic strategies taught with explicit instruction. The use of the SOLVE strategy in this study further illustrated that when students learned strategies that taught how to develop, create, and execute a plan, while also self-monitoring one's own behaviors, they were more successful in learning how to solve word problems. Although mnemonic instruction is effective in developing procedural fluency in students there is a need for evidence of effective practices that improved secondary students with disabilities and math difficulties in developing more complex algebraic skills. According to Maccini and colleagues (2008) The concrete-representational-abstract sequence (CRA) was a three-step process and students learned to progress through skills going from concrete application to abstract application. The CRA sequence includes many characteristics that are aligned to the NCTM standard of using hands on activities. In addition, CRA research in algebra implemented mnemonic instruction within the CRA sequence to develop procedural knowledge after acquiring conceptual understanding. Therefore, the remaining major sections will review research in the area of the CRA sequence of instruction for secondary students with disabilities.

Research Regarding CRA for Mathematics

Maccini et al. (2007), Watt et al. (2016), and Marita and Hord (2017) reviewed literature regarding effective mathematics instruction for secondary students with disabilities. Findings from these reviews of literature demonstrated the effectiveness of the concrete-representational-abstract sequence. Studies included in the reviews used concrete objects and pictorial representations to develop conceptual understanding in various mathematical skills prior to engaging in abstract instruction. Peterson, Mercer, and O’Shea (1988) conducted the initial research on the CRA sequence. The study investigated the effect of teaching students with disabilities place value using the CRA sequence compared to teaching students with disabilities place value at the abstract level only. The researchers also investigated the effects of the CRA sequence on the maintenance and generalization of the skills.

There was a total of twenty-four (24) participants in the study. All participants were elementary or middle school students with a learning disability. There were twenty (20) male and four (4) female participants with ages that ranged from eight (8) to thirteen (13). All of the participants received instruction for mathematics in the special education classroom. The classroom teachers implemented the intervention in their classrooms.

The study consisted of three phases that included teacher training, instruction, and post treatment. During the teacher training phase of the intervention five teachers participated in training workshops. During the training workshops, the teachers reviewed research on effective teaching, direct instruction model, and concrete-to- abstract teaching (Peterson et al., 1988). In addition, during the trainings the researcher modeled examples and non-examples of concrete-to-abstract teaching and one of the scripted lessons. The teachers then modeled one of the scripted lessons. Prior to implementing the lesson in their own classroom, the teachers demonstrated mastery of the instructional procedures.

There was a total of nine instructional lessons implemented during the instruction phase of the intervention. Each lesson was ten (10) to fifteen (15) minutes in length and was delivered to groups of three students. The nine lessons followed the direct instruction model which included, (a) an advanced organizer, (b) demonstration and model of skill, (c) guided practice, and (d) independent practice. During the instruction phase of the intervention, the only difference in instruction between the treatment group and control group was the inclusion of manipulatives and pictorial representations during the concrete and representational lessons of the intervention. The control group received all nine lessons during instruction at the abstract level while the treatment group received three lessons at each level of instruction (concrete, representation, and abstract).

The first three lessons during the instruction phase of the intervention taught the skill at the concrete level using manipulatives that included blocks, place value strips, and place value sticks. During these lessons, participants used the concrete objects to develop conceptual understanding of place value. The next three lessons were implemented during the representational level. During the representational level of instruction participants created pictures to represent and demonstrate place value. The representational level served as the bridge between the concrete level and the abstract level. The final three lessons of the intervention taught the skill at the abstract level.

The final phase of the intervention was the posttreatment phase. During this phase each participant completed a post-test that assessed skill acquisition and generalization. One week after the administration of the post-test, participants completed another assessment of maintenance, which again assessed skill acquisition and generalization.

A multivariate analysis of variance (MANOVA) procedure tested the effects of the intervention (CRA) on place value acquisition and place value generalization. Results from the MANOVA generated a significant main effect for treatment (F (2, 21) = 4.49, $p < .05$). Follow-up tests were conducted utilizing the univariate analysis of variance procedure. Results from the follow-up tests generated a significant main effect for place value acquisition (F (1, 22) = 8.79, $p < .01$). Therefore, students who received the intervention demonstrated understanding of place value of ones and tens better than students who did not receive the intervention. There were no significant effects for place value generalization on the follow-up tests (F (1, 22) = 1.66, $p > .05$). Therefore, participants in both groups performed similar when identifying place value in multi-digit numbers.

Results from Peterson et al. (1988) illustrated that mathematics instruction using the concrete-representational-abstract sequence was more effective than abstract only instruction. This study was the first to demonstrate the use of concrete objects and pictorial representations to facilitate skill acquisition and maintenance. Furthermore, this was the first study to suggest that additional research in the future should focus on teaching new skills using the CRA sequence to validate the strategy's effectiveness.

Since the initial study on CRA research, several studies have extended the strategy to other mathematic skills including basic facts (Miller & Mercer, 1993), addition and subtraction with regrouping (Flores, 2009; Flores 2010; Kaffar & Miller, 2011) multiplication and division (Harris, Miller, & Mercer, 1995; Milton, Flores, Moore, Taylor, & Burton, 2019), and multiplication with regrouping using the partial products algorithm (Flores, Moore, & Meyer, 2020).

Research Regarding CRA for Basic Algebraic Problem Solving

Prior to the year 2000, there was a lack of research regarding effective problem-solving instruction for students with disabilities in algebra (Maccini & Ruhl, 2001). Therefore, Maccini and Ruhl developed a new way for students to make meaning of beginning algebra topics within word problems using CRA. In this pilot study, Maccini and Ruhl investigated the effects of implementing explicit, strategic instruction with a problem-solving strategy to solve algebraic problems that involved the subtraction of integers. The researchers also intended to improve the participants' abilities to generalize skills to more complex problems involving the subtraction of integers.

The participants in Maccini and Ruhl's (2001) multiple-probe design study were three eighth-grade students with specific learning disabilities (SLD) who had a history of difficulties in mathematics. The researchers implemented the intervention for 20-30 minutes during study sessions outside of the general education class (Maccini & Ruhl, 2001). Students began the study in baseline and completed probes that comprised of five problems assessing their ability to subtract integers and solve word problems involving the subtraction of integers. Each of the students completed four probes sporadically during baseline to determine the need for intervention and to assess students' performance stability. When the students' performance was stable, the first student began the first phase of intervention. When the first student improved and the researcher observed stability of data points during this phase of instruction, the second student began the first phase of instruction. The third student entered the first phase of instruction once the second student improved and the researchers observed stability in his performance during the first phase of instruction. There were three phases of instruction for each student and each moved from one phase to the next based on individual mastery defined as 80% mastery on two consecutive test probes. The students' progression through each phase was not dependent upon other students (Jitendra et. al., 1999). A description of instruction and the three phases follows.

The students learned a mnemonic strategy to create mathematical equations from word problems. The strategy prompted students to: Search the word problem, Translate the words in picture and mathematical expression form, Answer the problem, and Review the solution (STAR). The STAR strategy included the use of the CRA sequence of instruction, self-monitoring strategies, and general problem-solving skills. The researchers explicitly taught the STAR strategy through three phases of instruction within the CRA sequence. Each of the instructional lessons consisted of the components of explicit instruction. Explicit instruction was identified as an essential element in promoting understanding of challenging concepts for students with disabilities (Miller & Hudson, 2006). Miller and Hudson stated that explicit instruction provided students with prerequisite skills and motivation to be successful in learning new mathematical concepts. Explicit instruction also gradually increased the student responsibility for performance away from teacher demonstrations (Miller & Hudson, 2006). The instructional lessons in this pilot study included the following components: advance organizer, model, guided practice, independent practice, posttest, and feedback/rewards (Maccini & Ruhl, 2001). The researchers adapted the six components from the instructional procedures outlined in the *Strategic Math Series* (Mercer & Miller, 1991).

The advance organizer identified the new skill, provided students with a rationale for learning the new skill, and made connections to skills learned in previous lessons to the current lesson. During modeling, the researcher used think-aloud techniques to demonstrate how to complete problems using the prompts in the STAR strategy. The researcher first modeled how to ask and answer questions about the problem using the prompts from the strategy with one to two problems. Next, the researcher modeled how to use the STAR strategy with three problems while also involving the students in the process. Guided practice consisted of students receiving three problems to practice the strategy while the researcher assisted as needed. The level of assistance that students received during guided practice was eventually faded.

Following guided practice, students demonstrated their understanding of concepts learned during independent practice. During independent practice, students completed problems without any cues or prompts. Next, the students completed a post-test that consisted of five problems. If students did not master 80% accuracy on two consecutive probes then students were provided with modeling or guided practice of additional problems as needed. In the final component of the lesson, feedback/rewards, the researchers provided students with positive and corrective feedback. The five-step process to giving corrective feedback included both the researcher and student documenting student performance, targeting error patterns and incorrect responses, modeling procedures for error correction, practicing procedures for error correction, and closing session with positive feedback. Occasionally, students earned edible rewards for displaying on-task behaviors.

The researchers used explicit instruction to teach each level of CRA. There were three instructional phases. The three instructional phases were concrete application, semi-concrete application, and abstract application. Phase one, concrete application, provided students with instruction on using the first two steps of the STAR strategy, search the word problem and translate the words in picture form. During the concrete application phase of instruction, the students used algebra tiles as concrete manipulatives. In this phase, students learned self-monitoring and problem-solving techniques that cued them to translate the equations into a visual representation using the algebra tiles. Students learned during the first step, search the word problem, how they must carefully read the problem first. Once they carefully read the problem, students learned to self-regulate their thinking by using self-questioning techniques. Finally, students learned to write down the facts from the problem as a problem-solving technique. The teacher modeled all of the strategies and techniques using think aloud strategies during instruction. During the next step, translate the words in picture form, students learned to represent the equations identified in the word problems using the algebra tiles. Students then solved the equations using the algebra tiles.

In the next phase, semi-concrete or representational, students reviewed the first two steps of STAR. There were the same instructions for the first step, search the word problem. However, for the second step, translate the words in picture form, the students translated the words into drawings of the algebra tiles on paper instead of using the manipulatives. During this step emphasis was on students' performance in creating drawings of the algebra tiles on paper rather than the manipulation of the algebra tiles. Students created drawings of the algebra tiles to represent equations, then solve the problems with the pictorial representations. In the final phase of instruction, abstract, students used the two remaining steps of STAR, answer the problem, and review the solution. Within these steps, students learned how to apply integer rules for addition and subtraction to solving the problem and checking the solution. The third step, answer the problem, provided students with a flowchart of integer rules that students used to apply the rules when subtracting integers. The students practiced using the flowchart to apply the integer rules to a variety of problems. The final step, review the solution, taught students how to review the correctness and reasonableness of the solution to the problem.

Maccini and Ruhl (2001) trained two graduate assistants to complete reliability checks to establish intervention fidelity. These reliability checks observed how well the instructor followed the intervention procedures, as well as how students were scored on the measurement tools. The researchers conducted reliability checks on a total of 20% of lessons within each phase of the intervention. The intervention procedures reliability scores were 96%, 97%, and 80% respectively for the testing sessions, think- aloud protocol, and instructional procedures. The researchers also conducted interrater reliability on scoring procedures. Interrater reliability for scoring tests and strategy use were 95% and 82%. Furthermore, measures of social validity were conducted to determine participants opinion of the efficiency, effectiveness, and acceptability of the intervention. After students completed all instructional phases, they completed a Likert-scale questionnaire to gather data on their opinions of the intervention. The results showed that the students believed that the strategy was effective in improving their skills in solving word problems involving the subtraction of integers.

To determine student growth, the researchers developed measurement tools to assess: (a) percent strategy use, (b) percent correct on problem representation, and (c) percent correct on problem solution. Results of Maccini and Ruhl's (2001) study indicated that all students improved in representing and solving algebraic word problems involving the subtraction of integers. All students increased their performance on percent strategy-use from baseline through each instructional phase. Students' percent of strategy use increased from a range of 13%-46%. Data for percent accuracy on problem representation illustrated that all students increased their mean scores from baseline through all instructional phases as well. The students' mean difference in percent accuracy on problem representation scores ranged from 38.25 percentage points to 72.5 percentage points. All students increased their percent accuracy on problem solution with increases of 43.5%, 50.5%, and 69%. Therefore, the researchers demonstrated a functional relation between the intervention and students' percent strategy use, percent correct on problem representation, and percent correct on problem solution with demonstrations of effect across all three students at three different points in time.

The students completed generalization and maintenance probes following mastery on problem representation in the last phase of the intervention. The researchers assessed generalization tasks with items that were similar and more complex. For example, on the near transfer generalization tasks, items were similar in the structure/type of problem as seen in instruction, however there were different storylines used in the word problems. On the far transfer generalization tasks, items were structurally different in that students solved for an additional unknown quantity. On the near transfer generalization probes students' accuracy on problem representation ranged from 64%-80% with a mean accuracy of 73%. Students' accuracy on problem solution on the near transfer generalization probes ranged from 50%-81% with a mean accuracy of 67%. On items that were more complex and different from those presented in instruction, the students did not show as much success as they illustrated on near transfer generalization probes. Students' accuracy on problem representation ranged from 0%-44% with a mean accuracy of 29%. Students' accuracy on problem solution on the far transfer generalization probes ranged from 7%-40% with a mean accuracy of 28.7%. The students completed maintenance tests two and three weeks after instruction and they continued to show an increase in their performance from baseline. Each student maintained 100% accuracy on problem representation and problem solution on the maintenance probes.

Results of Maccini and Ruhl's (2001) study indicated that students with SLD can be successful in learning how to represent word problems that include the subtraction of integers through the use of the CRA sequence combined with explicit instruction and problem-solving strategies. This pilot study was the first in the literature that illustrated that secondary students with SLD could learn algebraic skills through the use of the CRA sequence and self-regulation training. This study demonstrated the benefits of concrete and pictorial representations to teach complex problem solving including algebraic skills. The study provided a foundation for further CRA and strategy instruction research to improve the mathematical skills of all students.

To further extend the research conducted by Maccini and Ruhl (2001), Maccini and Hughes (2000) conducted a study that also used the CRA sequence and the mnemonic strategy, STAR, to teach problem solving skills to students in a high school setting. They extended Maccini and Ruhl's study by including multiplication and division of integers along with addition and subtraction.The purpose of this study was to investigate the effects of instruction using CRA and STAR on students' performance in solving algebraic word problems. In this study, six high school students with SLD in mathematics participated in the intervention for 20-30-minute sessions that were outside of their general education class. The criteria for participation was: (a) inclusion of mathematical goals on their IEP's, (b) scores of less than 80% on a pretest, and (c) needing specific help in acquiring mathematical skills in order to progress to advanced mathematics courses.

The researchers used a multiple-probe design. Students began in baseline in which they completed a minimum of four probes given intermittently to determine students' accuracy in problem representation and problem solution. Once one group of students had a stable baseline of less than 80% accuracy on the dependent measures, they moved to the instructional phases while the remaining students stayed in baseline. Once the first group demonstrated 80% accuracy on two consecutive probes in the first instructional phase, the second group moved to the instruction phases. The researchers used the same procedures for the second group. Students moved through the instructional phases as a group once students showed mastery of 80% or higher for two consecutive probes.

The researchers gathered data on students' use of think-aloud strategies while solving the problems using audio recordings of sessions. The researchers developed think-aloud protocols to measure students' ability to use the think-aloud strategies without prompting as well as to determine the teacher's reliability and fidelity in modeling the think-aloud strategies. Students in this study followed similar instructional phases as in Maccini and Ruhl (2001) with important differences. First, all instructional lessons followed the explicit lesson framework with six critical components: (1) advance organizer, (2) describe and model, (3) guided practice, (4) independent practice, (5) post-test, and (6) provide feedback (Maccini & Hughes, 2000). The researchers in this study emphasized student feedback. Student feedback was not only positive, but also corrective. Feedback followed five steps that included the students graphing their performance, targeting specific areas of weaknesses, model and re-teaching, practice, and positive feedback. This study added materials and supports to instruction that were different from Maccini and Ruhl (2001). Maccini and Hughes (2000) study provided students with a sheet that showed the steps and sub steps of the STAR strategy. The students also had a work mat that provided a visual representation of positive and negative numbers.

Within the instructional lessons, the researchers taught students the steps of the STAR strategy through concrete, semi-concrete, and abstract phases. Specifically, when students learned the second step in the STAR strategy, translate into picture form, they learned to use the algebra tiles with the work mats to show a physical representation of the equation during the concrete phase (Maccini & Hughes, 2000). During the semi-concrete phase, the students learned how to use the structured worksheet to use drawings of the algebra tiles to represent the equation. Finally, the students learned the final two steps during the abstract phase and translated representations into an algebraic equation.

The researchers implemented several measures to establish intervention fidelity. The researcher trained a graduate student to complete reliability checks and conducted reliability checks on 25% of the instructional phases of the intervention (Maccini & Hughes, 2000). Results for probe sessions and think-aloud procedures were 99.4% and 97% respectively. Intervention fidelity for instructional procedures received 100%. The researchers conducted interrater agreement on all probes during all phases of the intervention with an agreement of 93%. The researchers also measured social validity by giving students and teachers a questionnaire to determine their perspectives on the intervention. In addition to the Likert-scale questions, the students and teachers also responded to open-ended questions.

The researchers gathered data on percentage of strategy use, accuracy of problem representation and solution, accuracy on generalization and maintenance tests, and social validity (Maccini & Hughes, 2000). Results showed an increase in students' percentage of strategy use when using all integer operations to solve algebraic word problems. Upon entering the first instructional phase, students showed increases in their performance on strategy usage and responded to the intervention immediately. For example, during baseline for multiplication of integers, students' average use of the strategy was 29% and increased to 70% during concrete instruction, 67% at semi-concrete instruction, and 79% at abstract instruction. This trend continued for all other integer operations for strategy usage.

The students' percentage accuracy on problem representation also improved. Visual analysis showed immediate effects when students entered concrete instruction; their accuracy on problem representation improved. The mean percentage correct during baseline for all phases ranged from 10.04%-33.38% and increased to a range of 93%-97% during concrete instruction. Semi-concrete and abstract instruction showed the same results with students' mean accuracy on problem representation having a range of 90%-100%. Percentage accuracy on problem solution improved a great deal from baseline through the instructional phases. Student averages ranged from 90.1%-98.9% by the end of the abstract instructional phase. Generalization assessments on test problems similar to those in the instructional phases and more complex than the problems seen previously showed favorable results as well. For the generalization tests on similar problems, students' percentage of accuracy scores were 64.3% on average and 72.5% on average for problem representation. Student performance on more complex problems was an average of 52% for mean percentage accuracy and 61.3% average for problem representation. Maintenance probes occurred up until10 weeks after the intervention. On average students' percentage correct was 75% and 91% for problem solution. Visual analysis of each of the students' data path indicated a functional relation between CRA, STAR and student performance. The researchers observed an immediacy of effect, increase in trend, and increase in level for each student after implementation of the intervention. This was maintained throughout the implementation of the remaining phases of the intervention.

Overall, students benefitted from receiving instruction using CRA and the STAR strategy. Maccini and Hughes (2000) extended literature by including multiplication and division into word problem instruction. This study replicated previous research indicating that secondary students with SLD can develop algebraic skills through the use of the CRA sequence and a mnemonic to solve problems using basic algebra. Results from both studies indicate the promising benefits for educators to implement strategies such as these to improve the mathematical skills of students with SLD.

Research Regarding CRA for Solving Linear Equations

In response to Maccini and Ruhl (2001) and Maccini and Hughes (2000), Witzel, Mercer, and Miller (2003) designed an intervention to extend the line of research by addressing more advanced algebra skills. These researchers believed that Maccini and Ruhl's and Maccini and Hughes's use of the CRA sequence did not adequately address the needs of students with SLD struggling with algebra concepts. Therefore, Witzel et al. developed a new way for students to make meaning of both beginning and advanced algebra topics using the CRA sequence. The CRA strategy used in this study taught students a five-step progression of skills to solve linear equations. Students used CRA to reduce expressions, use inverse operations, solve for variables that were negative or in the denominator, complete transformations on one side of the equal sign, and complete transformation across the equal sign.

Witzel et al.'s (2003) study used a pre-post-follow up design to determine the extent of students' growth achieved using the CRA strategy compared to traditional abstract instruction. In contrast to the previously mentioned studies that employed single case designs, Witzel et al. investigated the growth of 34 matched pairs of sixth and seventh grade students who had a learning disability or were labeled as at risk for algebra failure. In the treatment group and the comparison group, the matched student pairs displayed similar characteristics such as achievement in previous math courses, achievement on state assessments, age and grade level, the at-risk or disability label, and accuracy within one item on the pre-test. The students in both the treatment and comparison groups received instruction in inclusive settings with students with and without disabilities.

Teachers provided instruction to both groups of students during 50-minutes classes and used explicit instruction. The researchers defined explicit instruction as the following steps: introduce the lesson, model the skill, conduct guided practice, and provide independent practice (Witzel et al., 2003). The instructional lessons followed this progression of skills: reducing expressions, inverse operations, negative and divisor variables, variables on one side, and variables on both sides. The teachers of the treatment group provided instruction using manipulatives (concrete) and pictures (representational), while the teachers of the comparison group used only traditional abstract instruction using just numbers and symbols. All students in both the treatment and comparison groups used the same materials and questions throughout instruction.

The researchers developed an assessment tool and described the process through which they ensured the validity of the test items. The researchers began with 70 questions that went through an expert review. After the expert review, the researchers asked 32 students who had successfully completed pre-algebra with a "C" or better to complete 63 questions deemed valid by the expert reviewers. After students completed the items, the researchers determined that 27 questions had medium difficulty. The researchers used these items to as their assessment tool. To ensure that the teachers implemented the intervention with fidelity, the researchers observed each teacher four times throughout the intervention utilizing a researcher-developed fidelity checklist. The researchers observed each teacher during each phase of the intervention on how well the instructional components were delivered, with all teachers showing that the intervention was delivered to fidelity.

Witzel et.al. (2003) used repeated measures analysis of variance (ANOVA) to observe the interaction between two levels of instruction, CRA and abstract only, and three levels of occasions, pretest, posttest, and follow-up. Results from the study indicated that there was a significant difference in student achievement from the pre-test to post-test to follow-up. Results from the follow-up analyses indicated that students who received the CRA strategy outperformed similar students who received traditional abstract-only instruction on both the post-test and follow-up test after three weeks. The authors also conducted an error analysis from the assessments and identified that both groups had difficulties with negative numbers. Error analysis of the examinations also illustrated that the abstract group struggled more with solving equations with variables on both sides of the equal sign compared to the experimental group.

Results from this study further extended the line of research in Maccini and Ruhl's (2000) study that showed the CRA and strategy instruction was effective in teaching students how to solve complex algebraic problems that go beyond solving simple equations using inverse operations. Students learned to use CRA to solve algebraic problems where multiple variables were present on both sides of the equal sign and students had to use multiplication and division to solve for variables where the coefficient is greater than one. Results from this study provided teachers with further evidence that incorporating instructional lessons that presented mathematical concepts in a CRA sequence is more beneficial than abstract only instruction.

Research Regarding Modifying CRA Algebra Instruction

Research conducted by Maccini and Hughes (2000), Maccini and Ruhl (2001), and Witzel, Mercer, and Miller (2003) showed the positive outcomes for using CRA in algebra instruction for students with disabilities and mathematical difficulties. These studies illustrated that students can learn various algebraic skills that include using CRA for basic algebra and linear equations. However, students with disabilities will require that future interventions address the need to improve students' ability to acquire more advanced algebraic skills, such as working with quadratic equations. According to Strickland and Maccini (2013), students with disabilities are required to be exposed to the general education curriculum and therefore will have to learn more advanced skills to be college and career ready. Strickland and Maccini (2013) also stated that the *Organizing Instruction and Study to Improve Student Learning IES Practice Guide* (Pashler, et al., 2007) suggested that students should learn abstract and concrete representations together to help make the transition to abstract notation only smooth and to generalize concepts to other novel situations. Therefore, a modification of the CRA method, the concrete-representation-abstract-integration (CRA-I) sequence, addressed the need for students to acquire more advanced algebra skills and to make a smooth transition to abstract notation only (Strickland & Maccini, 2012). Through this modification, students learned concepts by simultaneously being taught to use concrete representation, pictorial representation, and abstract representation when learning a new skill.

Research Regarding CRA-I and Quadratic Equations

Prior to the year 2012, only three studies used CRA to teach students algebra concepts. These studies focused on improving basic algebra skills and working with linear equations. Therefore, Strickland and Maccini (2012) conducted a study using the modified version of CRA, CRA-I, with a purpose to examine the effects of CRA-I on secondary students with SLD's ability to multiply linear algebraic expressions to form quadratic expression that were embedded within contextualized area word problems. Three male students in eighth and ninth-grade who attended a non-public day school for students with SLD participated in the study. The researchers used a multiple-probe across students design to determine students' ability to perform multiplication on linear expressions to form quadratic expressions, maintain the skills learned three to six weeks after the study, and transfer these skills to more complex and novel problems such as factoring quadratic expressions. The researchers also sought to investigate the benefits and usefulness of the strategy for the participants.

Participants received all instruction from the primary researcher individually in an office outside of their mathematics classroom (Strickland & Maccini, 2012). The study began with the administration of probes to establish baseline data. The researcher administered the tests individually and gave students a calculator, pencil, paper, and algebra tiles during the testing sessions. The researcher provided no prompts and only read word problems when requested. Students moved from baseline to the intervention phase once the researcher observed a stable baseline and trend. The first student only established two data points during baseline and the researchers moved him to intervention phase early due to his low performance and time constraints imposed in the study. Once the first student completed all instructional lessons and showed a response to the intervention during the additional domain probes, the next student moved from baseline to instruction. After completing all instructional lessons, this student completed probes to determine his response to the intervention and the final student moved from baseline to intervention. After all students completed instruction and domain probes, they entered into the generalization and maintenance phases of the intervention.

The researchers developed the instructional lessons that consisted of an introductory lesson and three target lessons. The introductory lesson was a 30-minute lesson that focused on the participants' understanding the use of the Algebra Lab Gear (Picciotto, 1990) algebra tiles to form linear expressions as well as ensuring students understood how to solve area word problems using whole numbers. The target lessons were 40-minute lessons broken into the following objectives: multiplication of linear expressions with positive terms only with the use of algebra tiles, multiplication of linear expressions with positive and negative terms using algebra tiles, and multiplication of linear expressions with positive and negative terms using the box method graphic organizer (Strickland & Maccini, 2012). The researcher implemented all lessons using explicit instruction. First, the researcher provided an activity that activated the participants' prior knowledge and provided them with instruction on prerequisite skills needed for the lesson. Next, the researcher modeled completing a problem using think aloud and questioning strategies. The third step was guided practice in which students completed a similar problem. During guided practice, the researcher prompted the student as needed to ensure the student completed the problem successfully. Lastly, the researcher reminded the student of the lesson objective and asked the student to complete independent practice of the target skill.

Lessons one and two focused on how to use the algebra tiles and pictorial representations to multiply two linear expressions. According to Strickland & Maccini (2012) these lessons focused on procedural knowledge of using the algebra tiles to determine the correct answer. Materials for the lessons included algebra tiles, corner piece, and lesson worksheets. Lesson worksheets consisted of an area word problem, a blank table, a section to sketch the blocks, and a section to write the quadratic equation. The students learned to first identify and write the two linear expressions as the dimensions, length and width, of a room. Next, the students used the algebra tiles and the corner piece to multiply the two linear expressions. The "t" shaped corner piece organized the algebra tiles and prompted students to multiply. The students learned to place the linear equations on the outside of the corner piece to represent the dimensions. As the students multiplied the linear equations they placed the algebra tiles in the area inside of the corner piece. This area represented to answer to the problem. In addition, students learned to sketch the algebra tiles on the worksheet while also physically manipulating the algebra tiles. Finally, students used abstract notation to write the area equation as a quadratic equation.

Lesson three focused on students' transition from using the algebra tiles and drawings to using the Box Method. The Box Method was described as a graphic organizer that is divided into boxes to organize terms when multiplying linear expressions. During lesson three, the researchers introduced the box method to the students through a discussion that explored the natural relationship between the box and the algebra tiles (Strickland & Maccini, 2012). The researchers followed four instructional procedures to guide students through the lesson and discussion. First, the researchers required the students to complete a problem using the algebra tiles. Next, the students placed the algebra tiles on the box template, then wrote the abstract notation for the algebra tile in the corresponding box. Finally, students constructed their own box and multiplied the two linear expressions. In addition, the researcher provided participants with incentives for completing probes that included one dollar given towards a gift card for each completed probe.

The researchers conducted inter-rater reliability on 33% of all domain probes to ensure that the intervention measures were scored consistently. Inter-rater reliability was 100% after differences were discussed. The researcher also conducted social validity measures to determine the usefulness of the intervention, how the students enjoyed using the intervention, and areas of improvement for the intervention. Participants completed a questionnaire for social validity with Likert-scale and open-ended questions and results indicated that all participants enjoyed the intervention and found it useful. Suggestions for future improvement included spending more time on the topics. The researchers conducted treatment fidelity on one-third of the instructional lessons in the intervention. The researcher trained two observers to complete fidelity checklist to ensure the intervention was implemented to fidelity. Results indicated that the intervention was implemented to fidelity 100% of the time with 100% interobserver agreement.

The researchers used visual analysis to analyze students' data. They determined that there was a functional relation shown between CRA-I and multiplying two linear expressions to form quadratic equations. Visual analysis identified an immediacy of effect upon entering each instructional phase, with a strong effect, and a percentage of non-overlapping data points at 100% (Strickland & Maccini, 2012). Therefore, the results of the intervention indicated that the use of the CRA-I is a very effective intervention for these participants. This study extended the line of research in using CRA for more complex algebraic concepts and introduced a more effective way of implementing the CRA sequence.

Research Regarding CRA-I and Factoring Quadratic Equations

To further extend Strickland and Maccini's (2012) research, Strickland and Maccini (2013) conducted a study using CRA-I and the Box Method to improve students' conceptual understanding of quadratic expressions. Like Strickland and Maccini (2012), the researchers used a multiple-probe design to investigate the effects of CRA-I; however in this study, the researchers went across two groups of students. In addition, this study targeted students with mathematical difficulties, unlike in the Strickland and Maccini (2012) study that focused on students with learning disabilities. The purpose of this study was to investigate students with mathematical difficulties' development of skills to transform quadratic expressions into factored form, maintenance of skills learned four to six weeks after the intervention ended, transference of skills to more complex problems, and the extent that CRA-I and the Box Method were found useful.

There was a total of five participants in the study. Two participants were in group one and three participants were in group two. In addition, three of the participants had a learning disability. The study took place in a private high school during the students' scheduled Algebra II mathematics class. The students received all phases of the intervention outside of their mathematics classroom in an alternate setting. The study consisted of two phases: baseline and intervention. The researchers developed domain probes and used parallel versions for baseline and post-test measures. During the baseline phase, two teachers at the school administered the baseline probes and gave students a calculator, pencil, paper, and algebra tiles during the sessions. The test administrators provided no prompts and only when requested read word problems. Each group moved from baseline phase to intervention phase after the researcher observed a stable level and trend during baseline. The researcher defined stability as all students in the group performing at 60% or below on at least two baseline probes with limited variability and no significant increase in scores (Strickland & Maccini, 2013). The intervention phase consisted of nine instructional lessons. At the end of each lesson, the students completed lesson probes to assess student progress towards lesson objectives. Each group progressed from one lesson to the next once all students in the group attained 80% mastery on the lesson probe. Once the first group completed all instructional lessons and attained mastery on the lesson probes, the next group moved from baseline to intervention. After both groups completed instruction and post-test domain probes, they entered into the generalization and maintenance phases of the intervention.

The researchers developed the nine instructional lessons in the intervention and organized each lesson into four parts. The four parts of the lesson were: introduce and activate prior knowledge, investigate the problem, practice the problem, and close and summarize main ideas. The researchers conducted each of the nine instructional lessons in 45-minute sessions. In addition to the nine instructional lessons, the researchers developed an introductory lesson that was implemented prior to the nine target lessons. Like the introductory lesson in the previous study, Strickland and Maccini developed a 45-minute lesson to review prerequisite skills and introduce students to the Algebra Lab Gear (Picciotto, 1990) manipulatives. Lessons one through four of the instructional unit focused on multiplying linear expressions to form quadratic expressions while lessons five through nine focused on factoring quadratic expressions (Strickland & Maccini, 2013). Lessons one through three were like the lessons in the Strickland and Maccini (2012) study. Lessons one and two focused on students engaging in multiplying the linear expressions using the algebra tiles and pictorial representations and lesson three focused on students' transition to abstract notation only using the Box Method.

The researchers developed lesson four as a bridge between lessons one through three and lessons five through nine. Lesson four focused on finding solutions to contextualized area problems. In lessons five and six the students learned to factor quadratic expressions using the algebra tiles and pictorial representations. Students learned how to represent the quadratic expression using the algebra tiles and manipulated the tiles inside of the corner piece until all the tiles formed a square. After completing the square, the students observed the layout of the algebra tiles on the inside of the corner piece and determined the two linear expressions that created the quadratic expression. While manipulating the algebra tiles the students also created pictorial representations. In addition, during lesson six, students wrote the problem solutions in abstract notation.

In lesson seven, students discovered the rules to factoring by investigating the impact of changing values in the linear (middle) or constant (last) term of the quadratic expression. Lesson eight focused on students' transition to using the Box Method to factor the quadratic expressions. During lesson nine students displayed their factoring skills using abstract notation only.

The researchers conducted treatment fidelity on 33% of the instructional lessons to ensure that the intervention was implemented as intended. The researcher created a fidelity checklist and trained two observers to complete treatment fidelity observations. Results indicated that the intervention was implemented to fidelity 100% of the time with 100% interobserver agreement after differences were discussed. The researchers conducted inter-rater reliability on 33% of all domain probes, lesson probes, and transfer probes to ensure that the intervention measures consistently. After differences were discussed, inter-rater reliability was 100%. Lastly, the researcher conducted social validity using a Likert-scale questionnaire to determine the extent that students found the intervention useful. Results indicated that all students found the intervention to be useful.

Strickland and Maccini (2013) used visual analysis to analyze students' data and they determined that there was a functional relation shown between CRA-I, the Box Method and transforming quadratic expressions to factored form. Visual analysis identified an immediacy of effect upon entering the instructional phase. The researchers observed the percentage of non-overlapping data points at 100%. Results also indicated that students increased accuracy on tasks and maintained skills four to six weeks after the intervention ended. In addition, results indicated some students transferred skills to other algebraic tasks. Results of the intervention indicated that the use of CRA-I and the Box

Method were effective for these participants. The students in this study demonstrated the impact that CRA-I and the Box Method had on development of conceptual understanding of quadratics and students' skills in transforming quadratic expressions into their factored form. In addition, this study extended the line of research in using CRA-I and the Box Method for complex algebraic concepts.

As evidenced in these studies CRA instruction can be effective in improving student success with more complex algebraic concepts. Students who received instruction using the CRA sequence were more successful when attempting more complex algebraic problems.

Chapter 3. Methodology

Method

Participants

Two twelfth grade students, Susie and Carly participated in the study. The criteria for participation were: (a) a history of failure or low achievement in mathematics; (b) recommended by his or her teacher as a student who would benefit from the intervention; (c) signed parent consent and student assent; (d) identified as a student with a disability according to the eligibility determination of the state education system; and (e) scored 50% or less on researcher made pre-test. The researcher sent parent consent letters and an information sheet home with students enrolled in an algebra 2 co-taught mathematics course. The researcher did not contact parents to return consent forms.

Both students received instruction in the general education classroom for mathematics instruction and were scheduled into a reinforced instruction class one block per day to provide additional support for deficits in mathematics. Both students qualified for special education services under the category of Other Health Impairment. The participants attended a regular public high school. The school did not receive Title 1 funds and consisted of a population of 1,992 students. Student characteristics and school demographics are described in table 1 and table 2.

An on-site teacher participated in the study and received training on the intervention and how to assist with giving assessment probes throughout the intervention. The researcher delivered all instructional lessons to the students individually in a resource classroom and administered the majority of the probes.

Table 1

Participant Characteristics

Name	Age	Grade	Eligibility	Race	Ethnicity	Cognitive Ability[a]	Math Achievement[b]
Carly	18	12	OHI	African American	Non-Hispanic	68	69
Susie	19	12	OHI	White	Non-Hispanic	70	64

a. Standard score on the Stanford- Binet Intelligence Scales, Fifth Edition (Roid & Pomplun, 2012).

b. Total Broad Math standard score on the Woodcock Johnson III Test of Achievement (Woodcock, McGrew, & Mather, 2001)

Table 2

School Demographics by Race and Ethnicity [a]

Group	Total	% Hispanic	% Asian	% Black/ African American	% American Indian/ Alaska Native	% Native Hawaiian/ Pacific Islander	%White	% Two or more
All Students	1,992	11.35	< 1	22.19	< 1	< 1	66.67	< 1
Students w/ Disabilities	131	13.74	0	40.46	5.34	0	45.80	0

a. 2018-2019 School year

Setting

The study took place in a high school in a rural town in the Southeastern United States. The researcher implemented the intervention during the participants' scheduled remediation block in a special education resource room. This allowed for a minimal loss of instructional time. The remediation block was a total of 86 minutes.The researcher provided the intervention to the participants individually. The researcher was a certified special education teacher with seven years of teaching experience.

Materials

Materials for the intervention consisted of the Algebra Lab Gear (ALG) manipulatives (Picciotto, 1990), assessment sheets, learning sheets, lesson plans, FACTOR and HUMP BACK FACTOR mnemonic cue cards, TI- 30XS calculator, and pencils. The ALG manipulatives were effective for use with improving students' algebraic skills when working with quadratic expressions (Strickland & Maccini, 2012; Strickland, 2017). The manipulatives represented the terms x^2, x, and a constant number. A large blue square tile represented the x^2, a blue rectangle represented the x, and a small yellow square represented the constant. The length of the x tile was the same length as the x^2 tile and the width of the was the same as the constant tile. The ALG manipulatives also consisted of a concrete piece in the shape of a lower-case *t.* The concrete piece organized the tiles during multiplication.

The researcher created the lesson plans, assessments, learning sheets, and cue cards. Lesson plans provided an overview of lesson objectives and a script with suggestions for implementation. Instructional lessons consisted of the components of explicit instruction (Maccini & Hughes, 2006) and included the following components:

(a) advanced organizer, (b) model/demonstration, (c) guided practice, (d) independent practice, and (e) feedback. FACTOR and HUMP BACK FACTOR mnemonic cue cards consisted of the steps used to implement the strategy when used to solve factoring problems.

Learning sheets were guides to the instructional lessons and consisted of (a) problems that assessed pre-requisite skills, (b) guided practice problems, and (c) independent practice problems. Lessons one through three involved the use of the ALG tiles and concrete piece. Learning sheets for lesson one included three pre-requisite skills/review problems, four guided practice problems, and five independent practice problems. Each section consisted of written directions. Directions for pre-requisite skills/review were *Simplify the following polynomial expressions by combining like terms*. Blanks spaces were underneath each problem to write the final answer. Directions for guided practice and independent practice problems were *Use your Algebra tiles, drawings, and/or graphic organizer to multiply binomials*. There were two rows with two problems each in the guided practice section. There were three rows with two problems in the first two rows and one problem in the last row in the independent practice section. Blank spaces and a drawing of the concrete piece were underneath each problem. These were places to write the answer and organize drawings of the pictorial representations. Each problem consisted of positive terms only.

The learning sheets for lesson two included two pre-requisite skills/review problems, four guided practice problems, and five independent practice problems. Directions for each section were *Use your Algebra tiles, drawings, and/or graphic organizer to multiply binomials*. Each problem consisted of positive and negative terms.

Lesson three learning sheets consisted of two pre-requisite skills/review problems, two guided practice problems, and five independent practice problems. Directions for pre-requisite skills/review were *Use your Algebra tiles, drawings, and/or graphic organizer to multiply binomials.* Directions for guided practice and independent practice problems were *Use your graphic organizer to multiply binomials*. Each problem consisted of positive and negative terms. The spaces underneath the problem included the BOX Method graphic organizer. The graphic organizer was the shape of a square split into four quadrants.

Learning sheets for lessons four through six consisted of the same items described above. Lessons four and five included three pre-requisite skills/review problems, four guided practice problems, and five independent practice problems. Blanks spaces were underneath each problem to write the final answer and did not include the concrete piece pictorial representation or the BOX Method graphic organizer. Directions for pre-requisite skills/review were *Identify the factors for the following numbers. Remember factors are numbers multiplied together to produce the number given. Factors can be positive or negative.* Directions for guided and independent practice were *Use your Algebra tiles, drawings, and/or graphic organizer to factor quadratics.* Lesson six learning sheet included two pre-requisite skills/review problems, two guided practice problems, and five independent practice problems. Directions for pre-requisite skills/review were *Use your Algebra tiles, drawings, and/or graphic organizer to factor quadratics.* Directions for guided and independent practice were *Use your Algebra tiles, drawings, the graphic organizer and/or the FACTOR mnemonic cue card to factor quadratic expressions*. The FACTOR cue card stated: **F**orm parenthesis; **A**dd variables to the parenthesis; **C**heck signs of the constant c then of bx, then add signs to the parentheses (If “c” is “+c” then add the signs in front of bx to both parentheses. If “c” is “-c” then add a + to one parenthesis and a – to the other parenthesis); **T**hink of factor pairs of c; **O**bserve factor pair of c that adds to bx; **R**ecord your answer by putting factors from the factor pair in the parenthesis matching the correct signs. Steps to the FACTOR strategy are in Table 3.

Lessons seven and eight consisted of the same items described above. Lessons seven and eight included four pre-requisite skills/review problems, two guided practice problems, and five independent practice problems. Each problem in lesson seven consisted of positive terms. Each problem in lesson eight consisted of positive and negative terms. Directions for pre-requisite skills/review were *Simplify the fractions.* Directions for guided and independent practice were *Use your Algebra tiles, drawings, the graphic organizer and/or the HUMP BACK FACTOR mnemonic cue card to factor quadratic expressions*. The HUMP BACK FACTOR cue card stated: **H**and ‘a’ to ‘c’ by multiplication; **U**ndo ‘a’ from the beginning; **M**ove ‘a’ * ‘c’ to the end by rewriting the problem; **P**roceed to **F.A.C.T.O.R**; **B**ring ‘a’ back to each binomial’s constant and create a fraction; **A**ssess the fraction for simplification; **C**arry the denominator to the binomial’s variable; and **K**ick back and record your answer. Steps to the HUMP BACK FACTOR strategy are in Table 4.

Table 3

Steps of FACTOR Strategy

Steps	Description of FACTOR
Step 1	**F**orm parenthesis
Step 2	**A**dd variables to the parenthesis
Step 3	**C**heck signs of the constant c then of bx, then add signs to the parentheses (If “c” is “+c” then add the signs in front of bx to both parentheses. If “c” is “-c” then add a + to one parenthesis and a – to the other parenthesis)
Step 4	**T**hink of factor pairs of c
Step 5	**O**bserve factor pair of c that adds to bx
Step 6	**R**ecord your answer by putting factors from the factor pair in the parenthesis matching the correct signs

Table 4

Steps of HUMP BACK FACTOR Strategy

Steps	Description of HUMP BACK FACTOR
Step 1	**H**and ‘a’ to ‘c’ by multiplication
Step 2	**U**ndo ‘a’ from the beginning
Step 3	**M**ove ‘a’ * ‘c’ to the end by rewriting the problem
Step 4	**P**roceed to **F.A.C.T.O. R**
Step 5	**B**ring ‘a’ back to each binomial’s constant and create a fraction
Step 6	**A**ssess the fraction for simplification
Step 7	**C**arry the denominator to the binomial’s variable
Step 8	**K**ick back and record your answer

Instructional Procedures

Instructional sessions were thirty to forty-five minutes, scheduled four days per week. In addition to mathematics instruction in the general education classroom, the students received remediation during a regularly scheduled reinforced instruction class. The study lasted five weeks.

Multiplication of Binomial Expressions

Prior to lesson one, the teacher implemented an introductory lesson. The introductory lesson began with an advanced organizer that introduced the topic and stated behavior expectations. Specifically, the advanced organizer included the teacher and student reading and signing a learning contract that stated their commitment to learning the new skills. Next, the teacher modeled how to combine like terms to demonstrate the use of the algebra tiles. Specifically, the teacher modeled how to simplify the expressions $4x + 7x$, $6 + 2x +6x + 7$, and $4x^2 + x + 2x^2 + 3$. When the teacher modeled $4x^2 + x + 2x^2 + 3+6x$, she represented each term with the ALG tiles. The teacher demonstrated by counting out four x^2 tiles, one x tile, two x^2 tiles, three yellow tiles, and six x tiles. Next, the teacher discussed that to simplify the expression the students must combine like terms. The teacher discussed and demonstrated grouping all x^2 tiles, x tiles, and constant tiles. The teacher then demonstrated ordering the tiles from largest to smallest. The teacher wrote the term underneath each grouping. For example, she wrote $4x^2$ underneath the x^2 tiles, $7x$ underneath the x tiles, and 3 underneath the constant tiles. Finally, the teacher discussed writing operation symbols to separate the terms. The teacher pointed to the $7x$ tiles and discussed that the $7x$ and 3 were positive terms. The teacher explained that because the terms were positive that she placed a "+" symbol in front of the term. Then, the teacher wrote a "+" symbol in between the first two terms, $4x^2$ and $7x$, and the last two terms, $7x$ and 3. Finally, the teacher demonstrated reading the expression.

During guided practice, the student used her own ALG tiles. During these problems, the teacher guided the student through the steps and the student repeated the teacher's actions. The teacher provided prompts when needed and asked questions that engaged the student. The student identified like terms and demonstrated combining like terms using the ALG tiles. During independent practice, the student combined like terms and simplified expressions independently. After independent practice, the teacher scored the five problems and provided feedback to the student. If the student incorrectly answered a problem the teacher reviewed the problem with the student.

Lessons one through three followed the same format as the introductory lesson. Lesson one introduced the student to the multiplication of binomial expressions. The advanced organizer consisted of an overview of the lesson, behavior expectations, and review problems of the pre-requisite skill for the lesson, review of combining like terms. After the advanced organizer, the instructor modeled the steps to complete the problem. First, the teacher modeled how to use the ALG tiles to multiply integers. For example, the teacher modeled 2 × 4 with the ALG tiles and concrete piece. To begin, the teacher led a discussion on the shape of the concrete piece and pointed to the top corner where the two sides of the concrete piece created an "x" which prompted multiplication. The teacher discussed that the multiplicand and multiplier were placed on the outside of the concrete piece and that because of the commutative property changing the position did not change the problem. The teacher then modeled 2 × 4 and placed two yellow constant tiles on the top of the concrete piece and four yellow constant tiles on the side of the concrete piece. The teacher demonstrated the commutative property and placed the four yellow constant tiles on the top of the concrete piece and the two yellow constant tiles on the side of the concrete piece. At the end of the discussion the teacher returned the tiles to their original position. Next, the teacher performed multiplication using the concrete pieces. The teacher discussed as she placed tiles inside of the concrete piece that the objective was to create a square on the inside. The teacher discussed that each constant was broken into 1's and that to multiply 2 × 4 using the structure of the shapes of the concrete piece on the inside to multiply. The teacher demonstrated that 8 yellow constant tiles completed the square on the inside of the concrete piece. Lastly, she placed 8 yellow constant tiles on the inside and led a discussion on the area on the inside of the concrete piece.

Next, the teacher modeled the multiplication of binomial expressions to form quadratic expressions with positive terms. Examples of the representational and abstract phase of binomial multiplication are in Table 5. As the instructor modeled using the ALG manipulatives, she also modeled how to draw the representations of the ALG on the worksheet and how to translate the ALG manipulatives and drawings to abstract notation using the BOX Method graphic organizer. For example, the teacher modeled $(x + 1)(x + 3)$ by placing 1 green x tile and 1 yellow constant tile on the top of the concrete piece and 1 green x tile and 3 yellow constant tiles on the side of the concrete piece. The teacher also created pictorial representations and wrote $x + 1$ on the top of the graphic organizer and $x + 3$ on the side of the graphic organizer. Next, the teacher placed 1 x^2 tile on the inside of the concrete piece. The teacher then led a discussion on the area on the inside of the concrete piece. The teacher drew 1 x^2 tile and wrote x^2 in the first quadrant on the graphic organizer. Next, the teacher placed 1 x tile next to the x^2 tile, drew 1 x tile, and wrote 1x in the second quadrant on the graphic organizer. The teacher then placed 3 x tiles underneath the x^2 tile, drew 3 x tiles, and wrote 3x in the third quadrant on the graphic organizer. Next, the teacher placed 3 constant tiles underneath the 1 x tile and next to the 3 x tiles to complete the square. The teacher drew 3 constant tiles and wrote 3 in the last quadrant on the graphic organizer. Finally, the teacher discussed how to write the final answer. The teacher wrote under the ALG tiles while she discussed and illustrated combining like terms with the ALG tiles. The teacher drew a circle around the drawings of the x tiles to group them and illustrate combining like terms. She then wrote the answer underneath the drawing. Lastly, the teacher circled the 1x and 3x in the graphic organizer and wrote a + in the middle and indicated the operation of addition and combining like terms. The teacher then wrote the final answer underneath the graphic organizer.

Following modeling, the student and teacher engaged in guided practice. During guided practice, the student used her own ALG tiles, drawings, and/or the graphic organizer. During these problems, the teacher guided the student through the steps and the student repeated the teacher's actions. The student demonstrated multiplying terms and combining like terms using the ALG tiles. The teacher asked questions that engaged the student and prompted the student to explain her thinking. When working on the third and fourth examples, the teacher asked the student to verbalize her thinking as she performed the tasks. When the student did not explain the step although she correctly performed the step, the teacher orally explained the step. When needed, the teacher provided additional prompts. During independent practice, the student multiplied binomial expressions with positive terms independently. After independent practice, the teacher scored the five problems and provided feedback to the student. If the student incorrectly answered a problem the teacher reviewed the problem with the student.

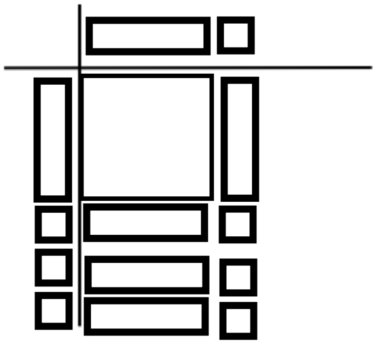
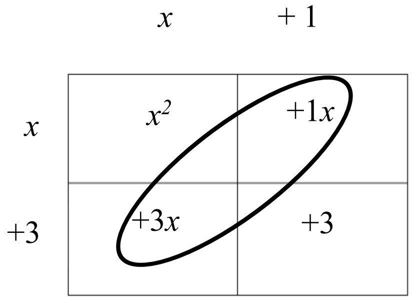
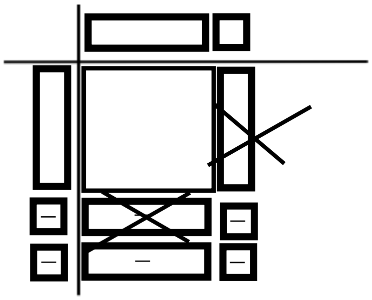
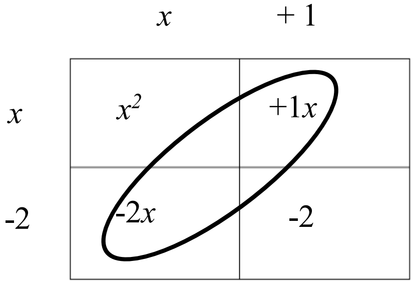
Lesson two introduced the student to the multiplication of binomial expressions with positive and negative terms. The advanced organizer consisted of review problems of the pre-requisite skill for the lesson, multiplication of positive and negative integers. After the advanced organizer, the instructor modeled the steps to complete the problem. First, the teacher introduced a new concept, zero pairs. She then modeled how to create zero pairs with positive and negative integers. To begin, the teacher explained to the student that 1 + (-1), 1-1, and -1 + 1 equal zero. She demonstrated creating zero pairs with opposite integers, such as 1 and -1, 4 and -4, and 5 and -5 using the ALG tiles. Next, she demonstrated addition equation such as 4 + -3 =__. For example, she placed 4 yellow constant tiles on the table and 3 yellow constant tiles with the negative sign on top. The teacher demonstrated creating zero pairs and explained that when you have a tile left over that is the answer. She then completed the addition equation 4 + -3 = 1.

Next, the teacher modeled how to multiply binomial expressions to form quadratic expressions with positive and negative terms. The procedures for multiplication of binomial expressions with positive and negative terms were the same as lesson one, with one difference in the procedure for combining like terms. When the teacher modeled combining like terms she explained that the x term would create zero pairs. For example, when the teacher modeled $(x + 1)(x - 2)$ with the ALG tiles, she removed one zero pair (1x and -1x) from the concrete piece and placed them to the side and pointed to the -1x tile left. The teacher then wrote $x^2 - x - 2$ underneath the ALG tiles. On the pictorial representation the teacher demonstrated crossing out the zero pairs and on the graphic organizer the teacher modeled adding the two numbers. After modeling, the guided practice and independent practice followed the same procedures in lesson one.

Lesson three transitioned students from the use of ALG manipulatives and pictorial representations to the use of the BOX Method graphic organizer only. The advanced organizer consisted of review problems of the pre-requisite skill for the lesson, multiplication of binomial expressions with positive and negative terms. After the advanced organizer, the teacher modeled the multiplication of binomial expressions with positive and negative terms using the BOX Method only, referring to the ALG manipulatives and/or drawings as needed. After modeling, the guided practice and independent practice followed the same procedures in lessons one and two.

Table 5

Examples of Representational and Abstract Phase of Binomial Multiplication

Problem	Representational Phase	Abstract Phase
$(x + 1)(x + 3)$		 $= x^2 + 4x + 3$
$(x + 1)(x - 2)$		 $= x^2 - x - 2$

Transformation of Quadratic Expressions when a = 1

Lessons four through six transitioned to a new skill, the transformation of quadratic expressions into factored form when a=1. Lessons four through six followed the same format as lessons one through three. Lesson four consisted of the transformation of quadratic expressions when a=1 and c was positive. The advanced organizer included a review identifying factors of numbers, which was the pre-requisite skill for the lesson. After the advanced organizer, the teacher modeled the steps to the problem using manipulatives, drawings, the BOX Method graphic organizer. The teacher began with a discussion the standard form of a quadratic expression, ax^2+bx+c. Next, the teacher modeled the steps. For example, when the teacher modeled the steps to factor $x^2 + 5x + 6$, the teacher represented each term with the ALG tiles first. Next, she explained that the objective was to determine the two binomial expressions multiplied to form the quadratic expression. The teacher led a discussion on the characteristics of the ALG tiles and concrete piece while she modeled how to use the ALG tiles to determine the factored form. The teacher placed the x^2 tile inside of the concrete piece, drew the x^2 tile, and wrote x^2 in the first quadrant in the graphic organizer. Next, she placed the constant tiles in various row and column order and explained the importance of the constant. Finally, she demonstrated completing the square with the x tiles based on the constant tiles row and column order. Once the teacher completed the square she placed the ALG tiles on the top and bottom of the concrete piece, drew the pictorial representations, and wrote $(x + 2)(x + 3)$ on the graphic organizer. The teacher then provided guided practice and independent practiced using the same procedures previously described.

Lesson five focused on the transformation of quadratic expressions when a=1 and c was negative. Lesson five followed the same procedures as in lesson four, however when the teacher modeled the steps to the problem she reintroduced the concept of zero pairs. For example, when the teacher modeled $x^2 + x - 6$ she demonstrated that there was no way to place the ALG tiles inside of the concrete piece to form a square. The teacher explained that zero pairs of x's that were taken out during multiplication were brought back to complete the square during factoring. The teacher placed zero pairs inside of the concrete piece one at a time until she completed the square. Furthermore, the teacher led a discussion on and related the organization of positive and negative x terms to the previous multiplication lessons.

Lesson six transitioned the students from using the ALG manipulatives, pictorial representations, and graphic organizer to using a mnemonic strategy. The strategy was: **F**orm parentheses; **A**dd variables to the parentheses; **C**heck signs of the constant c then of bx, then add signs to the parentheses (If "c" is "+c" then add the signs in front of bx to both parentheses. If "c" is "-c" then add a + to one parenthesis and a – to the other parenthesis); **T**hink of factor pairs of c; **O**bserve factor pair of c that adds to bx; **R**ecord your answer by putting factors from the factor pair in the parenthesis matching the correct signs (FACTOR). The teacher demonstrated factoring with the strategy. For example, to transform $x^2 + 11x + 28$ into factored form the teacher orally discussed each step. The teacher said, "The first step is to form parentheses. That means that we create two pairs of parentheses." The next step is to add variables to the parentheses. The teacher stated, "The variable is x". The next step is to check the signs of c. The teacher said, "C is positive therefore we must put the sign in front of the bx term, +, in both parentheses". The next step is to think of factor pairs of c. The teacher stated, "Our c is 28. The factor pairs of 28 are 1 and 28, 2 and 14, and 4 and 7". The next step is to observe factor pairs of c that add to bx. The teacher stated, "The factor pairs must add to the bx. The bx is 11x so our factor pairs must add to 11. The only factor pair that add to 11 are 4 and 7 because 1+28=29 and 2 +14 = 16". The last step is to record your answer by putting factors from the factor pair in the parenthesis matching the correct signs. The teacher demonstrated writing the answer $(x + 4)(x + 7)$. The teacher modeled additional problems with positive and negative terms. During guided practice and independent practice, the student practiced the strategy with positive and negative terms. Examples of the abstract phase of transformations of quadratic expressions into factored form when a = 1 using the FACTOR strategy are in Table 6.

Table 6

Examples of Abstract Phase of Transformation of Quadratic Expressions when a = 1

Problem	Abstract Phase		
		$x^2 + 11x + 28$	
$x^2 + 11x + 28$		$\downarrow$	
	Form parenthesis	( ) ( )	
		$\downarrow$	
	Add variables to the parentheses	$(x\ \ )(x\ \ )$	
		$\downarrow$	*If "c" is "+c" then add the signs in front of bx to both parentheses. *If "c" is "-c" then add a + to one parenthesis and a – to the other parenthesis
	Check signs of the constant c then of bx, then add signs to the parentheses	$(x +\ \ )(x+\ )$	
		$\downarrow$	
	Think of factor pairs of c	<u>28</u> 1 * 28 2 * 14	
	Observe factor pair of c that adds to bx	$\boxed{4 * 7}$	
		$\downarrow$	
	Record your answer by putting factors from the factor pair in the parenthesis matching the correct signs	$(x + 4)(x + 7)$	
		$x^2 - 2x - 15$	
$x^2 - 2x - 15$		$\downarrow$	
	Form parenthesis	( ) ( )	
		$\downarrow$	
	Add variables to the parentheses	$(x\ \ )(x\ \ )$	
		$\downarrow$	
	Check signs of the constant c then of bx, then add signs to the parentheses	$(x +\ \ )(x -\ )$	*If "c" is "+c" then add the signs in front of bx to both parentheses. *If "c" is "-c" then add a + to one parenthesis and a – to the other parenthesis
		$\downarrow$	
	Think of factor pairs of c	<u>-15</u> 1 * -15	
	Observe factor pair of c that adds to bx	$\boxed{3 * -5}$	
		$\downarrow$	
	Record your answer by putting factors from the factor pair in the parenthesis matching the correct signs	$(x + 3)(x - 2)$	

Transformation of Quadratic Expressions when a > 1

Lesson seven introduced a mnemonic strategy to transform quadratic expressions into factored form when a > 1 and c is positive. The strategy was: **H**and 'a' to 'c' by multiplication; **U**ndo 'a' from the beginning; **M**ove 'a' * 'c' to the end by rewriting the problem; **P**roceed to **F.A.C.T.O.R**; **B**ring 'a' back to each binomial's constant and create a fraction; **A**ssess the fraction for simplification; **C**arry the denominator to the binomial's variable; and **K**ick back and record your answer (HUMP BACK FACTOR). The teacher modeled the strategy. For example, to transform $2x^2 + 11x + 15$ into factored form the teacher orally discussed each step. The teacher said, "The first step is to hand 'a' to 'c' by multiplication. This means that we must multiply our 'a' and 'c'" . The teacher demonstrated circling the coefficient in the first term, 2, then creating a hump and crossing out the 15. The teacher wrote 30 next to the 15. The next step two steps are to undo 'a' from the beginning…; move 'a' * 'c' to the end by rewriting the problem. The teacher stated, "The next two steps are performed together". The teacher modeled rewriting the problem as $x^2 + 11x + 30.$ The next step is to proceed to FACTOR. The teacher said, "We have created a new expression where we can use the FACTOR strategy that we learned". The teacher demonstrated using the FACTOR strategy and wrote $(x + 5)(x + 6)$. The next step is to bring 'a' back to each binomial's constant and create a fraction. The teacher stated, "The a = 2. The constant in our first binomial is 5 and the constant in the second binomial is 6. We create a fraction with the 2 as the denominator. The new constant terms are $\frac{5}{2}$ and $\frac{6}{2}$". The next step is to assess the fraction for simplification. The teacher demonstrated using the calculator to determine if the fraction was simplified using the fraction function on the calculator. The teacher demonstrated that $\frac{5}{2}$ was already simplified and that $\frac{6}{2}$ was simplified to 3. The next step is to carry the denominator to the binomial's variable. The teacher said, "The denominator in the first binomial is 2 so we write that in front of the x. Our second constant simplified to a whole number so the denominator is 1." The last step is to kick back and record your answer. The teacher demonstrated writing the answer $(2x + 5)(x + 3)$. The teacher modeled additional problems with positive and negative bx terms. During guided practice and independent practice, the student practiced the strategy with positive and negative bx terms.

Lesson eight consisted of practice with the HUMP BACK FACTOR mnemonic strategy to transform quadratic expressions into factored form when a > 1 and c is negative. The procedures for this lesson were the same as lesson seven. The teacher modeled problems with positive and negative bx terms and negative c terms. During guided practice and independent practice, the student practiced the strategy with positive and negative bx terms and negative c terms.

Table 7

Example of Abstract Phase of Transformation of Quadratic Expressions when a > 1

Problem	Abstract Phase	
$2x^2 + 11x + 15$		$2x^2 + 11x + 15$ $\downarrow$
	Hand 'a' to 'c' by multiplication	$\textcircled{2}x^2 + 11x + \cancel{15}$ 30 $\downarrow$
	Undo 'a' from the beginning and ... **M**ove 'a' * 'c' to the end by rewriting the problem	$x^2 + 11x + 30$ $\downarrow$
	Proceed to **F.A.C.T.O.R**	
	F	$(\quad)(\quad)$ $\downarrow$
	A	$(x \quad)(x \quad)$ $\downarrow$
	C	$(\ x+\ )(\ x+\ )$ $\downarrow$
	T	$\underline{30}$ 1 * 20 2 * 15 3 * 10
	O	$\textcircled{5 * 6}$ $\downarrow$
	R	$(x+5)(x+6)$ $\downarrow$
	Bring 'a' back to each binomial's constant and create a fraction;	$\left(x+\frac{5}{2}\right)\left(x+\frac{6}{2}\right)$ $\downarrow$
	Assess the fraction for simplification	$\left(x+\frac{5}{2}\right)\left(x+\frac{3}{1}\right)$ $\downarrow$
	Carry the denominator to the binomial's variable	$\left(x+\frac{5}{\textcircled{2}}\right)\left(x+\frac{3}{\textcircled{1}}\right)$ $\downarrow$
	Kick back and record your answer	$(2x+5)(x+3)$

Assessment Tools

The researcher developed assessment tools from the high school mathematics curriculum. The researcher developed a pool of 45 questions. Each probe consisted of one sheet of paper with fifteen problems. There were five problems in each section of the assessment requiring students to demonstrate their ability to multiply binomial expressions, factor quadratic expressions when the coefficient “a” is one, and factor quadratic expressions when the coefficient “a” is more than one. Assessments consisted of positive and negative terms. Assessments only differed in the terms used. Each item on the assessment received 1 point for each correct answer and 0 points for each incorrect answer.

For content validity, the researcher distributed the pool of questions to three high school algebra teachers for expert review. All of the teachers earned a master’s degree from an accredited university. Teaching experience ranged from seven years to twenty-seven years. The teachers scored each problem for content relevance and difficulty level. The teachers rated content relevance as: 1= not relevant; 2= somewhat relevant; 3= quite relevant; and 4= highly relevant (Davis, 1992). To compute the item-level content validity index (I-CVI), the researcher computed the number of experts giving a 3 or 4 (dichotomizing the ordinal scale into relevant and not relevant) and divided that number by the total experts (Polit and Beck, 2006). All expert reviewers rated the questions as relevant to binomial multiplication and quadratic transformations into factored form. The I-CVI for the pool of questions was 1.00.

The teachers rated the difficulty level as: 1= easy; 2 =medium; and 3= difficult. The researcher required a consistency of two of the three raters on the difficulty level for each item. The experts consistently rated the difficulty level of the items. All three raters rated twenty-eight (28) of the items consistently as: 9- easy; 4-medium; and 15- difficult. Two of the three raters rated seventeen (17) of the items consistently as 14-medium and 3- difficult. Based on teacher ratings, the researcher did not remove any problems from the pool of questions.

Teacher Training

The researcher provided one-to-one instruction to an on-site teacher prior to implementation of the intervention. The researcher provided the teacher with three one-hour training sessions on the procedures of the intervention and how to administer assessment probes. During the training sessions, the researcher modeled assessment procedures. The teacher then modeled the procedures and received feedback for improvement. The researcher used the treatment fidelity checklist and rated the teacher on the following: a) all materials ready prior to lesson; b) provided necessary instruction to start the probe and explained what he/she would do and why; c) monitored work, provided verbal prompts only, and did not offer answers; and d) closed with a positive statement about performance in the feedback process and mentioned future lesson and expectations. When the teacher received a rating of 100% on the treatment fidelity checklist, the study began.

Research Design

The research design for this study was a multiple-probe across behaviors design (Horner & Baer, 1978). According to Horner and Baer, the multiple-probe design was an adequate choice because of the design's ability to be implemented over an extended period of time when conducting multiple baselines. Furthermore, using a well-designed multiple-probe design meets the quality indicators according to the *CEC Standards for Classifying the Evidence Base of Practices in Special Education* (Cook et al., 2015). The researcher observed the effects of (a) CRA-I and a graphic organizer, the BOX Method, across multiplying binomial expressions to form quadratic expressions, (b) CRA-I and the FACTOR mnemonic strategy on the transformation of quadratic expressions into factored form when the coefficient in the first term is equal to one, and (c) CRA-I and the HUMP BACK FACTOR mnemonic strategy on the transformation of quadratic expressions into factored form when the coefficient in the first term is more than one. All of the participants began at baseline until they reached stability. The researcher defined stability as no more than 20% variability from the mean of baseline. Once each participant showed stability in baseline, the participants received the first phase of the intervention, multiplying binomial expressions to form quadratic expressions. The researcher continued to collect baseline data on the other two behaviors. Once the participant achieved three (3) successful probes at 80% accuracy or above, the participants received the second phase of instruction, transforming quadratic expressions into factored form when the coefficient in the first term is equal to one using CRA-I and the FACTOR strategy. The researcher continued to collect baseline data regarding the third behavior. Once participants achieved three (3) successful probes at 80% accuracy or above during the second phase, participants received the third intervention phase of instruction, transforming quadratic expressions into factored form when the coefficient in the first term is more than one using the HUMP BACK FACTOR strategy. Once participants achieved three (3) successful probes at 80% accuracy or above and the generalization phase began. During the generalization phase, participants received a probe with word problems involving the multiplication of quadratic expressions and transformation of quadratic expressions into factored form.

Treatment Integrity and Inter-Observer Agreement

The researcher conducted treatment fidelity through direct observation of the implementation of the intervention using video recording. According to Lane et al. (2004), this is the most direct method to ensure that the intervention was implemented to fidelity. Video recording only consisted of recording the participants' work and responses during the implementation of the lesson components, limiting the amount of identifiable information from being present on the video. Smith, Saunic, and Taylor (2007) identified five measures to ensure treatment fidelity for evidenced-based practices. According to Smith et al., it is important that beginning procedures are explicitly outlined prior to implementing the intervention. During implementation of the intervention the teacher recorded all lessons.

The researcher and a graduate student familiar with the study conducted treatment integrity on 30% of all lessons across all phases using an integrity checklist developed by the researcher (Horner et. al, 2005; What Works Clearing House, 2010). According to What Works Clearing House (2010) high quality single case design research must conduct treatment integrity on at least 20% of all sessions. The treatment integrity checklist consisted of nine questions rated by checking yes or no. Behaviors assessed included preparation of materials, implementation of components of explicit instruction, implementation of the phases of instruction, and student engagement. The researcher compared the results from the observers' checklist and assessed for agreement.

The researcher conducted inter-rater agreement on 80% of the assessment probes given across each phase. According to Horner et al. (2005) it is important to assess inter-rater agreement frequently to ensure that the dependent variables are being assessed consistently. Two raters, the researcher and a graduate student familiar with the study, conducted the inter-rater agreement. The researcher calculated the agreement using the point-by-point ratio. To calculate, the researcher calculated the total number of agreements and divided the number of agreements by the total number of agreements and the total number of disagreements. Then the researcher multiplied the number by 100.

Social Validity

The researcher administered a social validity survey to the participants and determined the usefulness of the intervention. The participants completed a survey after instruction on the last lesson of the intervention. The student survey consisted of 21 questions with yes and no questions and open-ended response type questions.

Chapter 4. Effects of CRA-I, FACTOR, and HUMP-BACK FACTOR

Results

The researcher used a multiple-probe across behaviors design to evaluate the effects of (a) CRA-I and a graphic organizer, the BOX Method, across multiplying binomial expressions to form quadratic expressions, (b) CRA-I and the FACTOR mnemonic strategy on the transformation of quadratic expressions into factored form when the coefficient in the first term is equal to one, and (c) CRA-I and the HUMP BACK FACTOR mnemonic strategy on the transformation of quadratic expressions into factored form when the coefficient in the first term is more than one. The researcher used visual analysis to interpret data. The researcher noted the following upon visual inspection: percent of overlap between baseline and treatment, level and trend of data paths, immediacy of effect, and the number of data points from the beginning of the intervention to criterion. Figures 1 and 2 display results for Susie and Carly.

Baseline Data

Prior to the intervention, students completed baseline probes together. Susie’s and Carly’s baseline were stable across all behaviors. The researcher defined stability as no more than 20% variability from the mean of baseline. On all three baseline probes, both students scored zero problems correct. After baseline, the researcher began the intervention using the CRA-I sequence, a graphic organizer, the BOX Method, and FACTOR and HUMP BACK FACTOR mnemonic strategies.

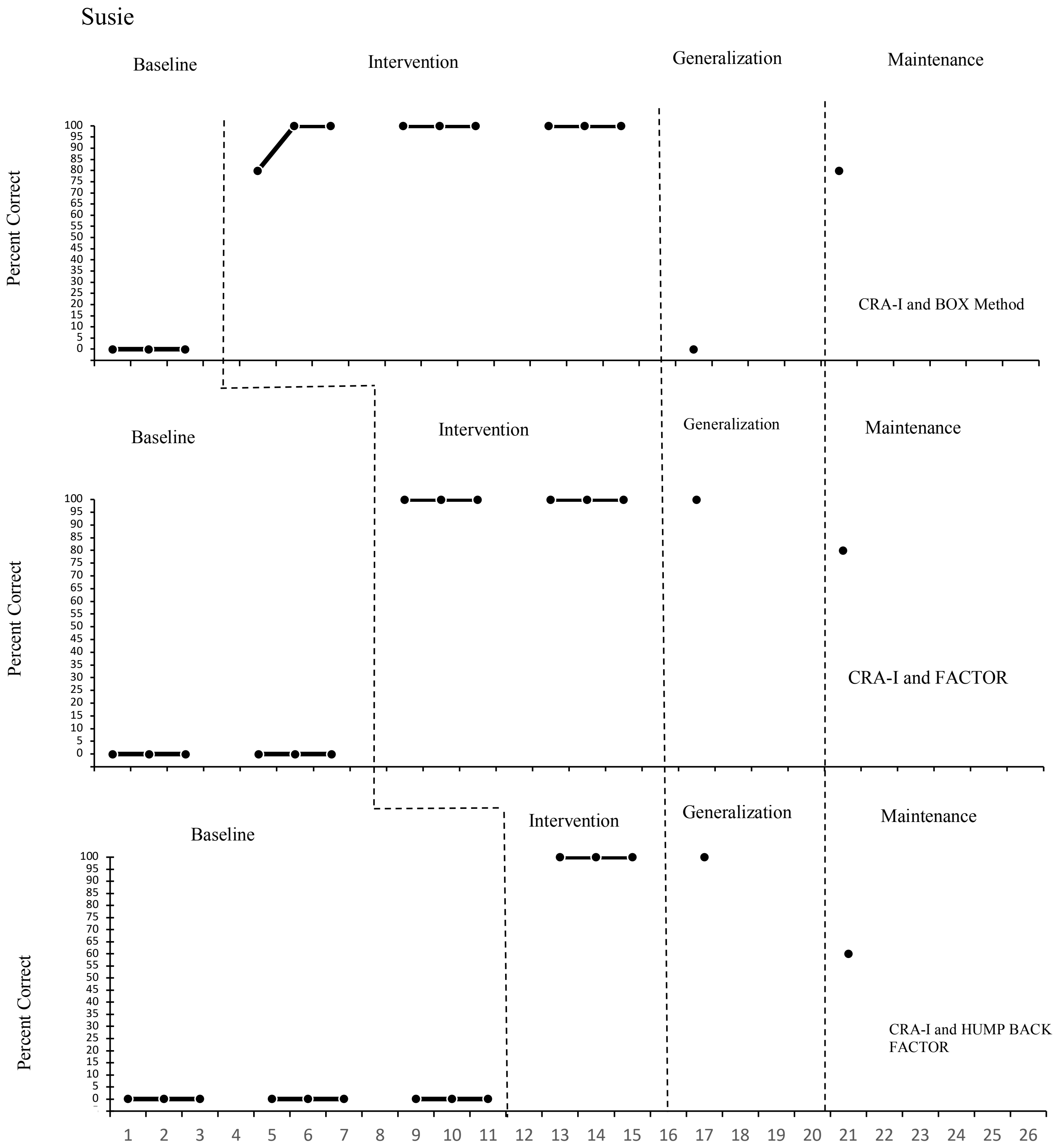


Figure 1. Susie's Results

Performance after Instruction

Susie: Susie reached criterion for binomial multiplication after three probes (80% accuracy or above on three probes). There was an immediate effect observed after baseline with an increase from 0% to 80%. The level for the first phase was 93.3% with a range from 80% to 100%. The data showed an increasing data path. There were no overlapping data points. Percent of non-overlapping data (PND) (Tawney & Gast, 1984) from baseline to binomial multiplication was 100%. For the second behavior, transformation of quadratic expressions into factored form using CRA-I and the FACTOR mnemonic strategy, Susie reached criterion after three probes. There was an immediacy of effect observed after baseline with an increase from 0% to 100%. The level for the first phase was 100% with a range from 100% to 100%. The data showed an increasing data path. There were no overlapping data points. PND from baseline to transformation of quadratic expressions into factored form using CRA-I and the FACTOR mnemonic strategy was 100%. For the third behavior, transformation of quadratic expressions into factored form using CRA-I and the HUMP BACK FACTOR mnemonic strategy, Susie reached criterion after three probes. There was an immediacy of effect observed after baseline with an increase from 0% to 100%. The level for the third phase was 93.3% with a range from 80% to 100%. The data showed an increasing data path. There were no overlapping data points. PND from baseline to transformation of quadratic expressions into factored form using CRA-I and the HUMP BACK FACTOR mnemonic strategy was 100%.

Carly

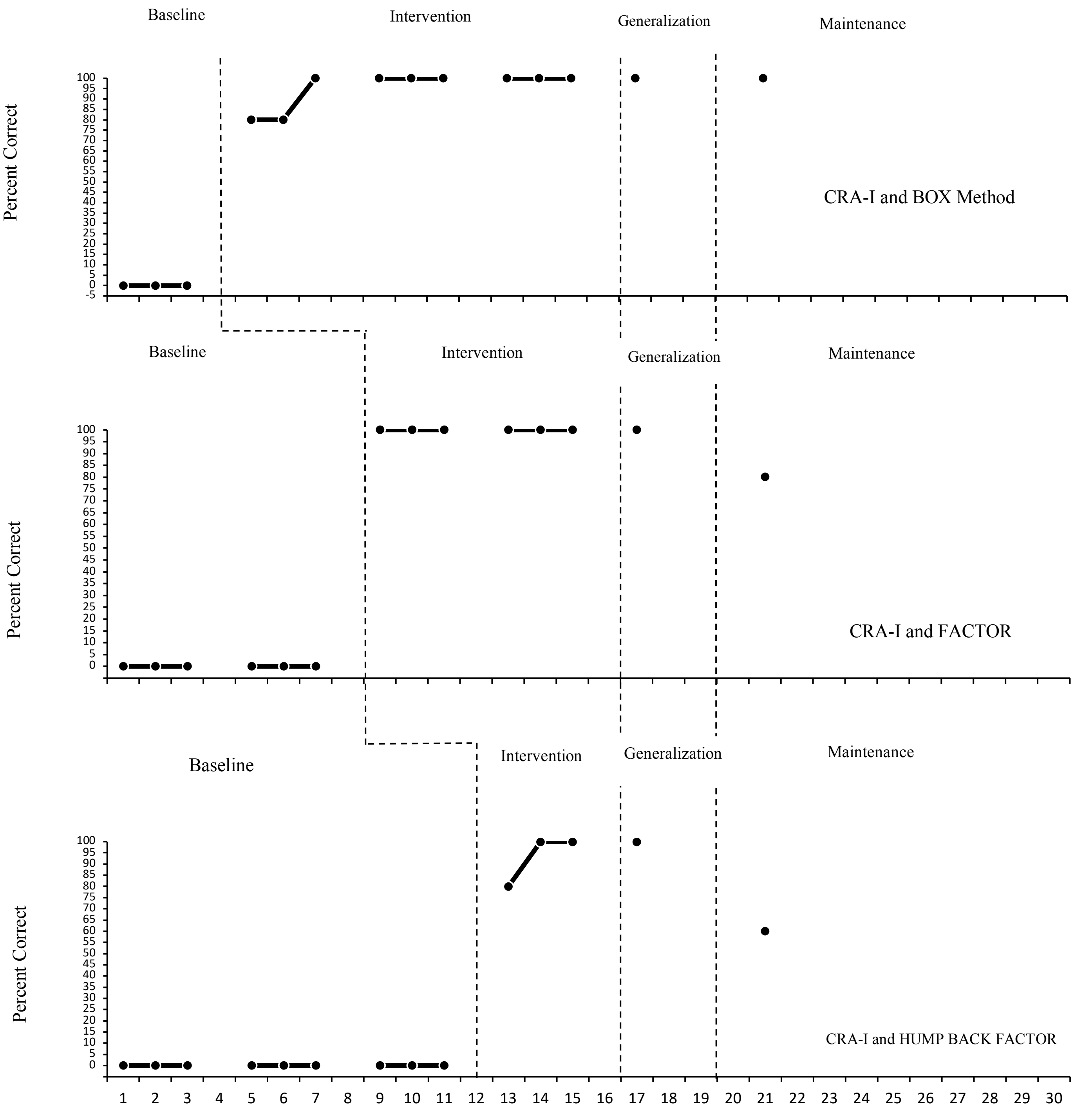


Figure 2. Carly’s Results

Carly: Carly reached criterion for binomial multiplication after three probes (80% accuracy or above on three probes). There was an immediacy of effect observed after baseline with an increase from 0% to 80%. The level for the first phase was 86.7%. with a range from 80% to 100%. The data showed an increasing data path. There were no overlapping data points. PND from baseline to binomial multiplication was 100%. For the second behavior, transformation of quadratic expressions into factored form using CRA-I and the FACTOR mnemonic strategy, Carly reached criterion after three probes. There was an immediacy of effect observed after baseline with an increase from 0% to 100%. The level for the first phase was 100% with a range from 100% to 100%. The data showed an increasing data path. There were no overlapping data points. PND from baseline to transformation of quadratic expressions into factored form using CRA-I and the FACTOR mnemonic strategy was 100%. For the third behavior, transformation of quadratic expressions into factored form using CRA-I and the HUMP BACK FACTOR mnemonic strategy, Carly reached criterion after three probes. There was an immediacy of effect observed after baseline with an increase from 0% to 80%. The level for the first phase was 93.3% with a range from 80% to 100%. The data showed an increasing data path. There were no overlapping data points. PND from baseline to transformation of quadratic expressions into factored form using CRA-I and the HUMP BACK FACTOR mnemonic strategy was 100%.

Generalization Performance

The researcher collected generalization data the day after the intervention ended. Susie completed a probe with three (3) word problems involving the multiplication of binomial expressions and transformation of quadratic expressions into factored form. Susie scored 0% correct for multiplication of binomial expressions, 100% for transformation of quadratic expressions into factored form when a =1, and 100% for transformation of quadratic expressions into factored form when a > 1. Carly completed a probe with word problems involving the multiplication of binomial expressions and transformation of quadratic expressions into factored form. Carly scored 100% correct for multiplication of binomial expressions, 100% for transformation of quadratic expressions into factored form when a =1, and 100% for transformation of quadratic expressions into factored form when a > 1.

Maintenance Performance

The researcher collected maintenance data one to nine weeks after instruction ended for each behavior. The researcher collected Susie's maintenance data for binomial multiplication nine weeks after instruction ended, transformation of quadratic expressions into factored form when a = 1 four weeks after instruction ended, and transformation of quadratic expressions into factored form when a > 1 two weeks after instruction ended. The researcher collected Carly's maintenance data for binomial multiplication seven weeks after instruction ended, transformation of quadratic expressions into factored form when a = 1 three weeks after instruction ended, and transformation of quadratic expressions into factored form when a > 1 one and a half weeks (12 days) after instruction ended. Susie completed a probe for all three behaviors. Susie scored 80%

correct for multiplication of binomial expressions, 80% correct for transformation of quadratic expressions into factored form when a =1, and 60% correct for transformation of quadratic expressions into factored form when a > 1. Carly completed a probe for all three behaviors. Carly scored 100% correct for multiplication of binomial expressions, 80% correct for transformation of quadratic expressions into factored form when a =1, and 60% correct for transformation of quadratic expressions into factored form when a > 1.

Treatment Integrity and Inter-Observer Agreement

The researcher conducted treatment integrity on 30% of all lessons across all phases using an integrity checklist developed by the researcher. The researcher and a graduate student familiar with the study conducted treatment integrity through completion of the checklist and direct observation of the implementation of the intervention using video recording. To calculate inter-observer agreement for treatment integrity, the researcher calculated the total number of agreements and divided the number of agreements by the total number of agreements and the total number of disagreements. Then the researcher multiplied the number by 100. Treatment integrity for multiplication of binomial expressions was 100%, for transformation of quadratic expressions into factored form when a =1 was 100%, and for transformation of quadratic expressions into factored form when a > 1 was 100%. The agreement between observers was 100%.

The researcher conducted inter-rater reliability on 80% of the assessment probes given across each phase. Two raters, the researcher and a graduate student familiar with the study, conducted the inter-rater agreement. The researcher calculated agreement using the point-by-point ratio. To calculate inter-rater reliability, the researcher calculated the total number of agreements and divided the number of agreements by the total number of agreements and the total number of disagreements. Then the researcher multiplied the number by 100. Inter-rater reliability was 100% for each behavior for both students.

Social Validity

The researcher administered a social validity survey to the participants and determined the usefulness of the intervention. The participants completed a survey after instruction on the last day of the intervention. The student survey consisted of questions with yes and no type questions and open-ended response type questions. Susie and Carly indicated that the strategies were useful and improved their skills in binomial multiplication and quadratic transformations. Both students indicated that they liked using the ALG tiles during instruction. Susie stated that the Box Method strategy as her favorite part about learning how to multiply and factor, while Carly stated that learning about all of the strategies as her favorite part. When asked if they thought other students should learn the FACTOR and HUMP BACK FACTOR strategies, both students agreed that the strategy should be taught to other students and that the strategies made factoring problems easier.

Chapter 5. Conclusions and Recommendations

Discussion

The purpose of this study was to investigate the effects of the CRA-I sequence and the BOX Method graphic organizer, CRA-I and the FACTOR mnemonic strategy, and CRA-I and the HUMP BACK FACTOR mnemonic strategy on students' performance in binomial multiplication and the transformation of quadratic expressions into factored form. The research design for this study was a multiple probe across three behaviors. The researcher showed a functional relation between the CRA-I sequence and algebra performance with effects shown across three different behaviors at three different points in time. The students increased their accuracy in the areas of binomial multiplication, transformation of quadratic expressions when $a = 1$, and transformation of quadratic expressions when $a > 1$. The students generalized their performance to the transformation of quadratic expressions embedded within word problems. Carly generalized her performance to multiplication of binomial expressions embedded within word problems. The students maintained their performance for binomial multiplication, transformation of quadratic expressions when $a = 1$, and transformation of quadratic expressions when $a > 1$ one to nine weeks after instruction ended.

Findings Related to CRA-I with the BOX Method, FACTOR, and HUMP BACK FACTOR and Algebra

The findings in this study are important because two secondary students with disabilities and a history of mathematics difficulties learned complex algebraic skills and maintained the skills over time. According to the NCTM, secondary mathematics instruction must develop conceptual understanding and procedural fluency in mathematics trhough the use of a variety of tools and hands on activities (NCTM, 2000).

NCTM also recommends that mathematics strategies build upon student's prior knowledge. This study used a variety of tools and strategies including, the CRA-I sequence, graphic organizers, and mnemonic strategy instruction, and developed conceptual understanding and procedural fluency in binomial multiplication and quadratic transformations. In addition, this study consisted of daily pre-requisite skills problems to relate skills previously learned in mathematics courses and instructional lessons in the intervention.

During baseline both students demonstrated some knowledge of multiplication and the relation between multiplication, the distributive property, and binomial multiplication. However, students did not demonstrate knowledge of the steps to complete the problem. When Susie and Carly completed factoring problems, they recognized that the variable x appeared in the expression twice and performed the steps to solving multi-step equations. However, it was apparent that both students lacked conceptual understanding because they could not differentiate between unlike terms and did not recognize that each term represented different quantities. Furthermore, neither student differentiated between the factoring problem types and completed all factoring problems by solving for the variable x.

During the binomial multiplication phase, students did not experience difficulty with the transition to abstract notation and preferred the BOX method when constant terms were too large, such as in $(x + 8)(x + 9)$ and $(x + 7)(x + 6)$.This was similar to the findings in Strickland's (2014) case study. Strickland (2014) conducted a case study analysis and investigated one student's use of CRA-I and a graphic organizer, the BOX, to perform binomial multiplication and to transform quadratic expressions into factored form. Strickland discovered that although the student resisted giving up the use of ALG manipulatives, when presented with problems with large terms, the student eventually used the graphic organizer to complete the problems instead of the manipulatives. Susie and Carly also resisted giving up the use of the manipulatives. During the lessons, Susie performed calculations using the manipulatives, then would check her answer with the graphic organizer and calculator. When presented with problems that included larger terms, Susie did not attempt to represent the multiplication of the constant terms using the manipulatives. Instead, she used the calculator to compute the constant terms. Eventually, Susie resorted to using the graphic organizer and calculator only to complete the problems. Carly transitioned to using the graphic organizer without difficulty although she consistently referred to the manipulatives when completing the problems. Specifically, when combining like terms Carly consistently referred to creating zero pairs to perform addition of positive and negative integers. She would then use the calculator to check her computations.

Furthermore, in this study students' access to a calculator limited the effect of computational errors in the addition and multiplication of positive and negative integers on students' procedural fluency. Both students preferred to use the ALG tiles when all terms were positive and resorted to the BOX method and used the calculator to check computations when computing problems with positive and negative terms. Susie received four lessons during this phase. Susie preferred to complete problems using the ALG tiles and the BOX method. The researcher drew this conclusion about her preference because she rarely drew representations. During all lessons she used the ALG tiles and used her manipulatives to check her computations prior to using the calculator.

Carly received four lessons during this phase. Carly never drew representations and preferred to use the ALG tiles until the third lesson. Carly used both the calculator and ALG tiles to check her computations; however, when she received instruction in lesson three, she used her calculator to check her computations. Both students improved significantly from baseline to the first phase of the intervention. Susie scored 80%, 100%, and 100% on her probes following instruction and this trend continued throughout the intervention. Carly scored 80%, 80%, and 100% on her probes following instruction and this trend continued throughout the intervention.

During the second and third phases of instruction, transformation of quadratic expressions into factored form when a =1 and transformation of quadratic expressions into factored form when a > 1, the FACTOR and HUMP BACK FACTOR strategies provided students with procedural knowledge to complete complex algebraic problems. The FACTOR and HUMP BACK FACTOR strategies cued students to think about the steps to complete the problems and assisted in the successful completion. This study extends research by Strickland and Maccini (2012) in the use of CRA-I to improve performance of complex algebra skills for students. Strickland and Maccini taught students how to perform binomial multiplication and quadratic transformations into factored form embedded within word problems. The researchers used the CRA-I sequences and the BOX method graphic organizer and developed students' conceptual understanding through area representation. This present study did not embed binomial multiplication and quadratic transformations into factored form problems within word problems. Conversely, the researcher focused on developing students' conceptual understanding through array representation and used the graphic organizer and mnemonic instruction to develop procedural knowledge.

Furthermore, this study extends research by Maccini and Hughes (2000), Maccini and Ruhl (2001) that illustrated the use of CRA with mnemonic strategy instruction were effective for basic algebra skills instruction. Both students improved significantly from baseline to intervention in both phases. Both Susie and Carly scored 100% on all three probes following transformation of quadratic expressions into factored form when $a =1$ instruction and this trend continued throughout the intervention. Susie scored 80%, 100%, and 100% on her probes following transformation of quadratic expressions into factored form when $a > 1$ instruction and this trend continued throughout the intervention. Carly scored 80%, 100%, and 100% on her probes following transformation of quadratic expressions into factored form when $a > 1$ instruction and this trend continued throughout the intervention.

During the generalization probe Carly demonstrated success with binomial multiplication embedded within word problems. Both Carly and Susie demonstrated success with quadratic transformations into factored form embedded within word problems, although they did not receive explicit instruction on these problem types. In the Strickland and Maccini (2013) study, the researchers explicitly taught the steps to create binomial expressions embedded in word problems and focused on area representation of real word problems. However, in this study the researcher explicitly taught students binomial multiplication and quadratic transformations through array representation to develop conceptual understanding and used a graphic organizer and mnemonic strategies to develop procedural fluency. When completing the generalization probe, Carly demonstrated conceptual understanding of quadratic expressions in several ways although she did not receive formal instruction. First, when performing binomial multiplication Carly drew a square to represent the dimensions of the room described in the word problem. Once Carly labeled the dimensions she correctly wrote two binomial expressions. After Carly created the binomial expressions, she drew the BOX method graphic organizer and completed the problem successfully. Although Susie was not successful on the generalization probe with binomial multiplication, she attempted to create a quadratic expression using the units in the problem. Neither student drew representations when completing the quadratic transformation problems to demonstrate conceptual understanding, however both students illustrated the use of the mnemonic strategies to complete the problem.

This study is consistent with the Strickland and Maccini (2012, 2013) studies on the effectiveness of CRA-I and the BOX method graphic organizer of the intervention over a period of time. The study extends the study to the effectiveness of CRA-I and the FACTOR and HUMP BACK FACTOR mnemonic strategy on complex algebraic skills over a period of time. Susie and Carly showed a high level of retention of the material on the maintenance probes. Nine weeks after instruction ended, Susie scored 80% correct on binomial multiplication. Seven weeks after instruction ended, Carly scored 100% correct for multiplication of binomial expressions. After further analysis through visual inspection of student work, Susie’s error occurred in multiplying positive and negative terms, although she performed the steps to binomial multiplication correctly. Four weeks after instruction ended, Susie scored 80% correct for transformation of quadratic expressions into factored form when a =1. Three weeks after instruction ended Carly also scored 80% correct for transformation of quadratic expressions into factored form when a =1. Both students made errors in the determination of the correct positive and negative factors. For example, Susie correctly performed the steps to the FACTOR strategy, however instead of writing $(x + 5)(x - 1)$, she wrote $(x + 1)(x - 5)$. Four weeks after instruction ended on transformation of quadratic expressions into factored form when a > 1, Susie scored 60% correct. Twelve days after instruction ended on transformation of quadratic expressions into factored form when a > 1, Carly scored 60% correct. Again, both students performed the steps to the HUMP BACK FACTOR strategy correctly and only made computational errors.

Limitations and Suggestions for Future Research

The research design of the study presented limitations because no comparisons are made between CRA-I, the BOX method, FACTOR instruction, and HUMP BACK FACTOR instruction and another form of mathematics instruction. Therefore, there may be other instructional programs or strategies that are more effective. In addition, the researcher implemented all of the lessons in the intervention. Therefore, replication with other teachers, researchers, and instructors in a variety of settings is necessary to validate these results. Furthermore, there were only two participants in the study. The participants showed improvement with the instruction however, these results cannot be generalized. Therefore, additional research is required to generalize the results for students' binomial multiplication and transformation of quadratic expressions. In addition, the students received the intervention individually in a separate setting. Therefore, it is unclear how effective the intervention is when delivered to a group of students in the general education setting.

Additional research replicating this intervention with a larger group of students is needed. This study needs to be replicated with a variety of researchers, teachers, and in a variety of settings to determine the effectiveness of this intervention. Furthermore, this intervention should be replicated and compared to other mathematical strategies and interventions.

Conclusions and Recommendations

The No Child Left Behind Act (NCLB; 2002) and the Every Student Succeeds Act (ESSA; 2014) required that students with disabilities meet the same standards as their peers. The Individuals with Education Act (IDEA) of 2004 emphasized the increasing need for high quality instruction that provided equal access to the general education curriculum for students with disabilities. Research on effective secondary mathematics instruction illustrated the need to use a variety of tools and strategies so that students can develop skills in complex concepts (Watt et al., 2016) and that these interventions must be implemented during the school day to impact instruction (NMAP, 2008). Although research illustrated the need for effective intervention for students across grades k-12, students with disabilities continue to struggle with gaining advanced mathematical skills.

Teaching advanced mathematical concepts to students with disabilities is a difficult task. Teachers must be adequately trained and exhibit a comfortability level with teaching the content. The National Council of Teachers of Mathematics (NCTM, 2000) stated that educators must be trained to provide students with the skills to develop conceptual understanding and procedural fluency in mathematics while in high school and when educators are not adequately trained students often enrolled in more remediation courses in college (NCTM, 2000). The NCTM consistently emphasized the need for developing conceptual understanding and procedural fluency throughout the *Principles and Standards for Mathematics* (2000) as well as the *Principles to Actions* (2014). This current study examined the CRA-I, graphic organizer, FACTOR mnemonic, and HUMP BACK FACTOR mnemonic instruction for students with disabilities who struggle in mathematics. The following research questions were examined:

1. What are the effects of instruction using the concrete-representational-abstract-integration strategy (CRA-I) and a graphic organizer, the BOX Method, on students' accuracy in multiplying binomial expressions to form quadratic expressions?
2. What are the effects of instruction using the mnemonic strategy, FACTOR, taught within the CRA-I strategy on students' accuracy in completing problems involving the transformation of quadratic expressions into factored form when the coefficient in the first term is equal to one?
3. What are the effects of instruction using the mnemonic strategy, HUMP BACK FACTOR, taught within the CRA-I strategy on students' accuracy in completing problems involving the transformation of quadratic expressions into factored form when the coefficient in the first term is more than one?
4. What are the effects of the CRA-I strategy and the mnemonic instruction on students' generalization to problems including word problems involving the multiplication of binomial expressions and transforming quadratic expressions into factored form?

The independent variables in this study were CRA-I, the graphic organizer, FACTOR, and HUMP BACK FACTOR. The dependent variables were the percent of problems correct on binomial multiplication, transformation of quadratic expressions into factored form when $a =1$, and transformation of quadratic expressions into factored form when $a >1$ probe. The researcher used a multiple-probe across behaviors design to determine the effectiveness of CRA-I, the graphic organizer, FACTOR, and HUMP BACK FACTOR instruction with secondary students with disabilities who struggle in mathematics. The researcher showed a functional relation between the CRA-I sequence and algebra performance with effects shown across three different behaviors at three different points in time with two participants. Additionally, the students generalized the skills learned to quadratic transformations embedded within word problems. Lastly, both students maintained the skills learned over time.

It is recommended that prior to instruction that the teacher assess students' fluency in addition, subtraction, multiplication, division, and simplifying fractions. A pretest of students' integer skills is also warranted. Although students used calculators in this current study, the time to complete problems were lengthen because students could not easily recall multiplication facts. Students made computational errors when simplifying fractions during the HUMP BACK FACTOR strategy. Therefore, if students do not have these skills, teachers should use other CRA materials to reteach these skills prior to implementing complex algebraic instruction.

Furthermore, it is recommended that future research teaches differentiation between problems types as well as problem-solving skills. Student must develop the knowledge and ability to apply skills to more in-depth problem types (Freeman-Green et al., 2015). Hudson and Miller (2006) identified schema-based instruction as an effective strategy to improve the problem-solving abilities of students with disabilities. Schema-based strategies used representational instruction, such as diagram mapping, to enable students to see patterns and relations in problems, which lead to the ability to translate words into mathematical symbols and find a solution (Jitendra, Hoff, & Beck., 1999). Therefore, it is recommended that future research continue to develop strategies so secondary students can be successful in acquiring advanced mathematical problem-solving skills and teachers are adequately equipped with the tools for effective instruction.

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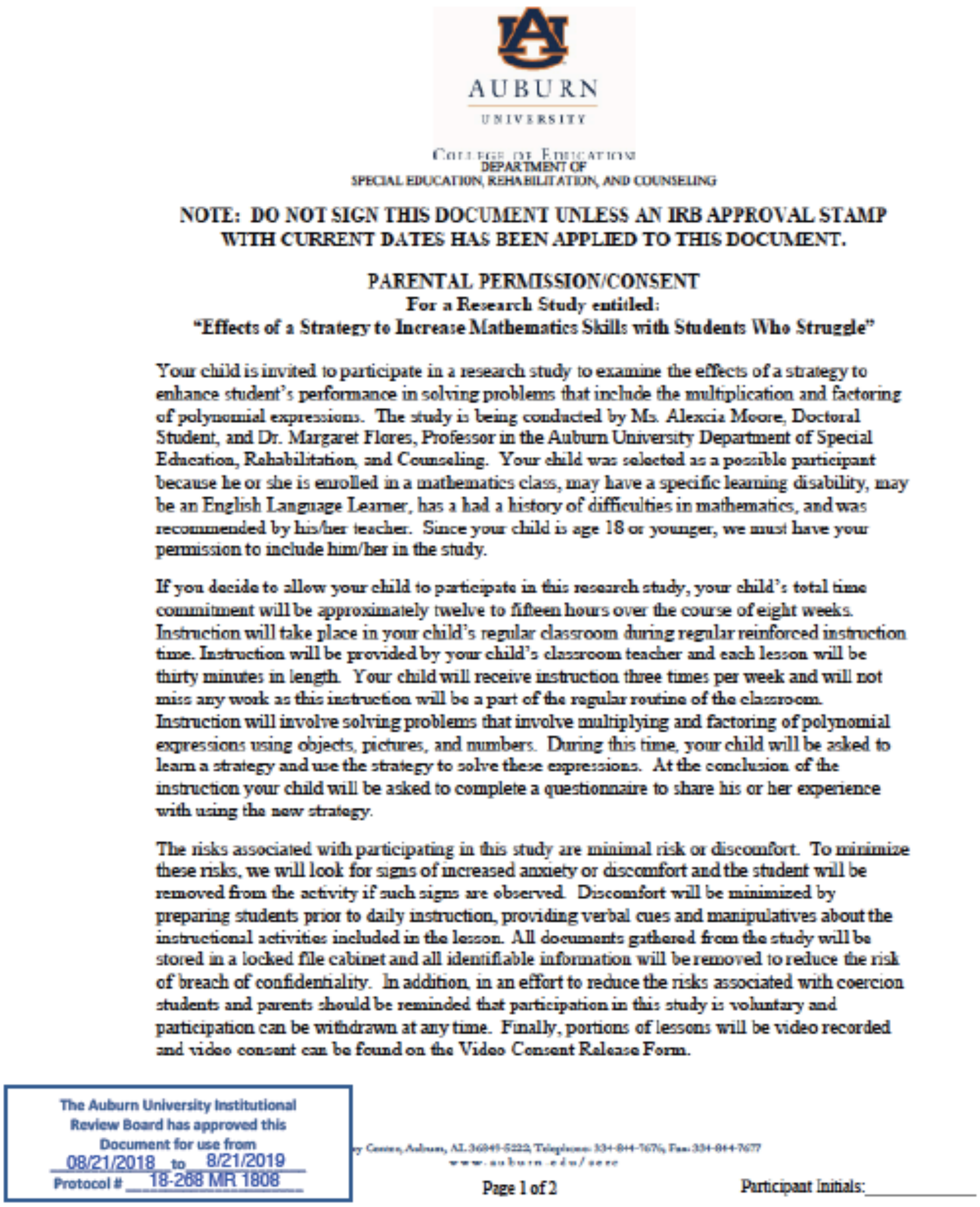
Appendix 1

Parental Permission/Consent Form



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PARENTAL PERMISSION/CONSENT

For a Research Study entitled:
"Effects of a Strategy to Increase Mathematics Skills with Students Who Struggle"

Your child is invited to participate in a research study to examine the effects of a strategy to enhance student's performance in solving problems that include the multiplication and factoring of polynomial expressions. The study is being conducted by Ms. Alexcia Moore, Doctoral Student, and Dr. Margaret Flores, Professor in the Auburn University Department of Special Education, Rehabilitation, and Counseling. Your child was selected as a possible participant because he or she is enrolled in a mathematics class, may have a specific learning disability, may be an English Language Learner, has a had a history of difficulties in mathematics, and was recommended by his/her teacher. Since your child is age 18 or younger, we must have your permission to include him/her in the study.

If you decide to allow your child to participate in this research study, your child's total time commitment will be approximately twelve to fifteen hours over the course of eight weeks. Instruction will take place in your child's regular classroom during regular reinforced instruction time. Instruction will be provided by your child's classroom teacher and each lesson will be thirty minutes in length. Your child will receive instruction three times per week and will not miss any work as this instruction will be a part of the regular routine of the classroom. Instruction will involve solving problems that involve multiplying and factoring of polynomial expressions using objects, pictures, and numbers. During this time, your child will be asked to learn a strategy and use the strategy to solve these expressions. At the conclusion of the instruction your child will be asked to complete a questionnaire to share his or her experience with using the new strategy.

The risks associated with participating in this study are minimal risk or discomfort. To minimize these risks, we will look for signs of increased anxiety or discomfort and the student will be removed from the activity if such signs are observed. Discomfort will be minimized by preparing students prior to daily instruction, providing verbal cues and manipulatives about the instructional activities included in the lesson. All documents gathered from the study will be stored in a locked file cabinet and all identifiable information will be removed to reduce the risk of breach of confidentiality. In addition, in an effort to reduce the risks associated with coercion students and parents should be reminded that participation in this study is voluntary and participation can be withdrawn at any time. Finally, portions of lessons will be video recorded and video consent can be found on the Video Consent Release Form.

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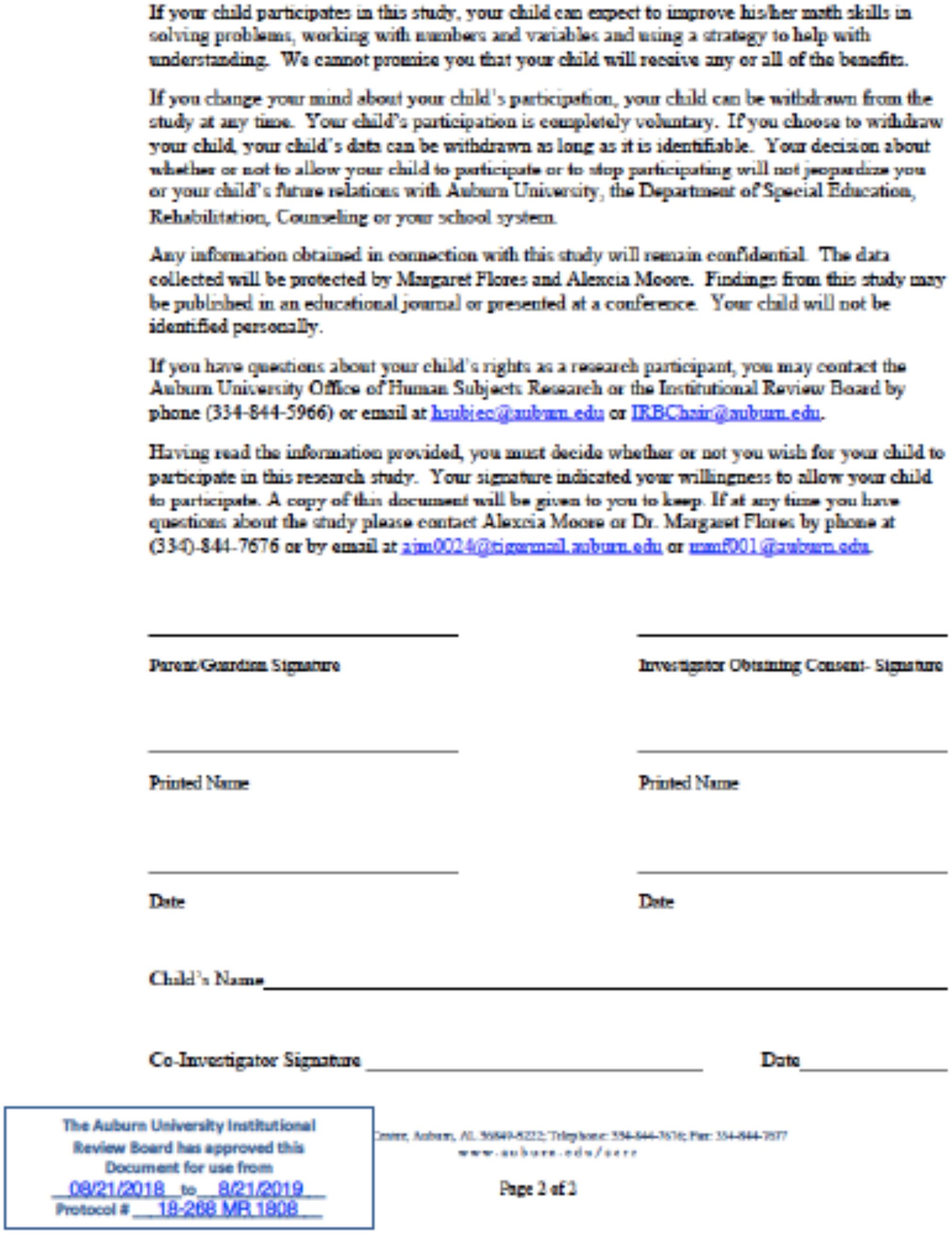
Participant Initials:____________



If your child participates in this study, your child can expect to improve his/her math skills in solving problems, working with numbers and variables and using a strategy to help with understanding. We cannot promise you that your child will receive any or all of the benefits.

If you change your mind about your child's participation, your child can be withdrawn from the study at any time. Your child's participation is completely voluntary. If you choose to withdraw your child, your child's data can be withdrawn as long as it is identifiable. Your decision about whether or not to allow your child to participate or to stop participating will not jeopardize you or your child's future relations with Auburn University, the Department of Special Education, Rehabilitation, Counseling or your school system.

Any information obtained in connection with this study will remain confidential. The data collected will be protected by Margaret Flores and Alexcia Moore. Findings from this study may be published in an educational journal or presented at a conference. Your child will not be identified personally.

If you have questions about your child's rights as a research participant, you may contact the Auburn University Office of Human Subjects Research or the Institutional Review Board by phone (334-844-5966) or email at hsubjec@auburn.edu or IRBChair@auburn.edu.

Having read the information provided, you must decide whether or not you wish for your child to participate in this research study. Your signature indicated your willingness to allow your child to participate. A copy of this document will be given to you to keep. If at any time you have questions about the study please contact Alexcia Moore or Dr. Margaret Flores by phone at (334)-844-7676 or by email at ajm0024@tigermail.auburn.edu or mmf001@auburn.edu.

Parent/Guardian Signature	Investigator Obtaining Consent- Signature
Printed Name	Printed Name
Date	Date

Child's Name____

Co-Investigator Signature __________________________ Date____________

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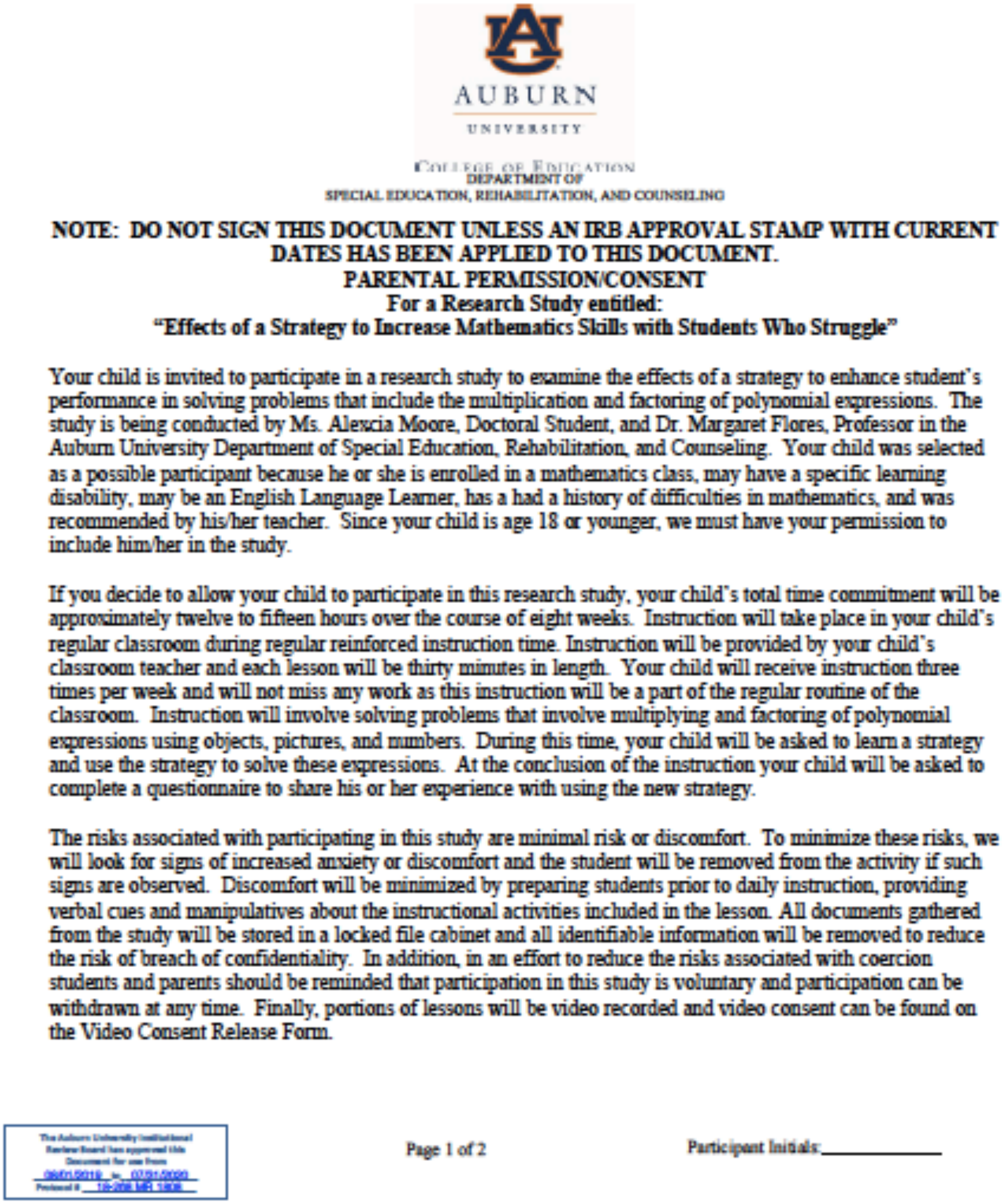

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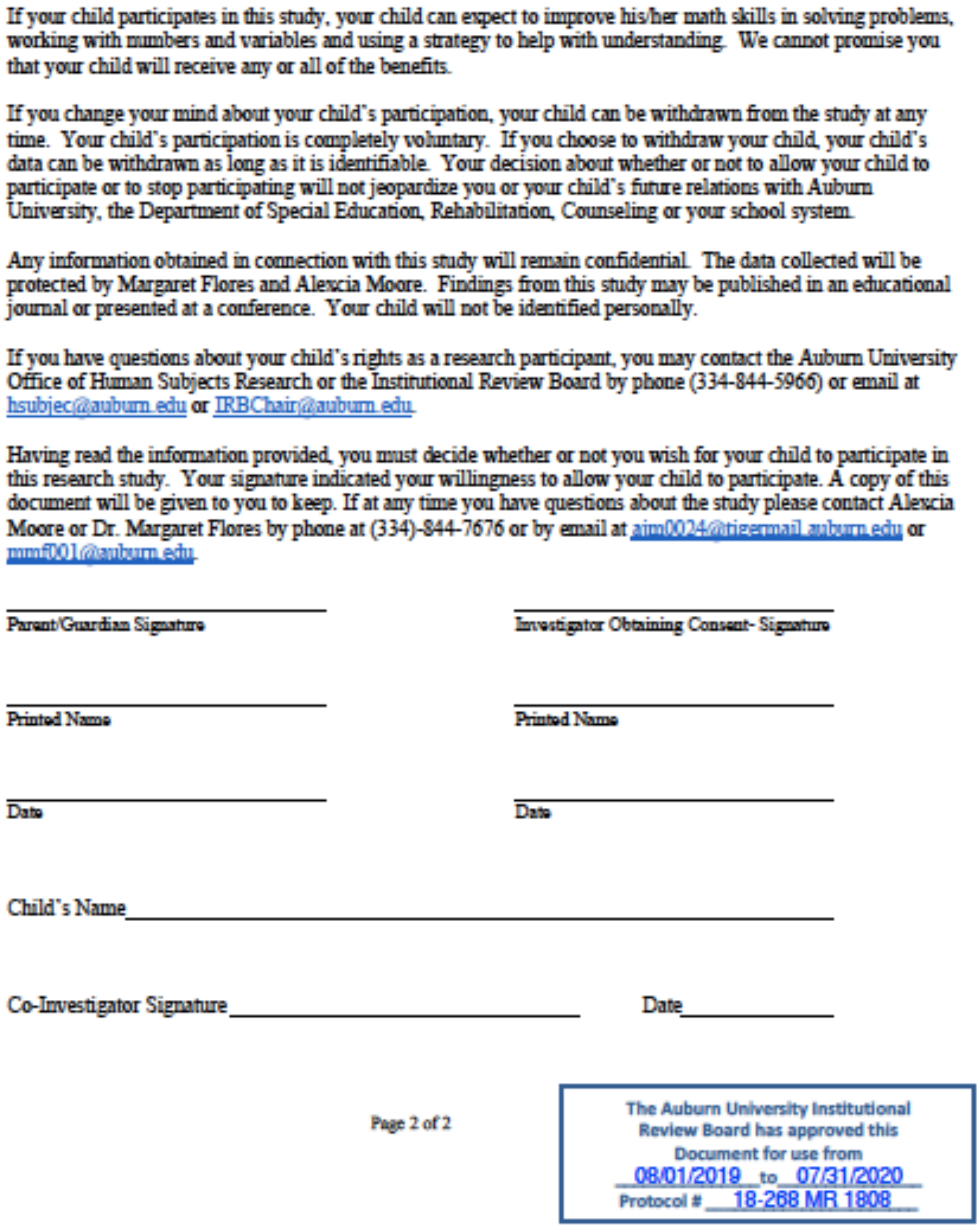
Participant Initials:____________



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Parent/Guardian Signature	Investigator Obtaining Consent- Signature
Printed Name	Printed Name
Date	Date

Child's Name____

Co-Investigator Signature_________________________ Date__________

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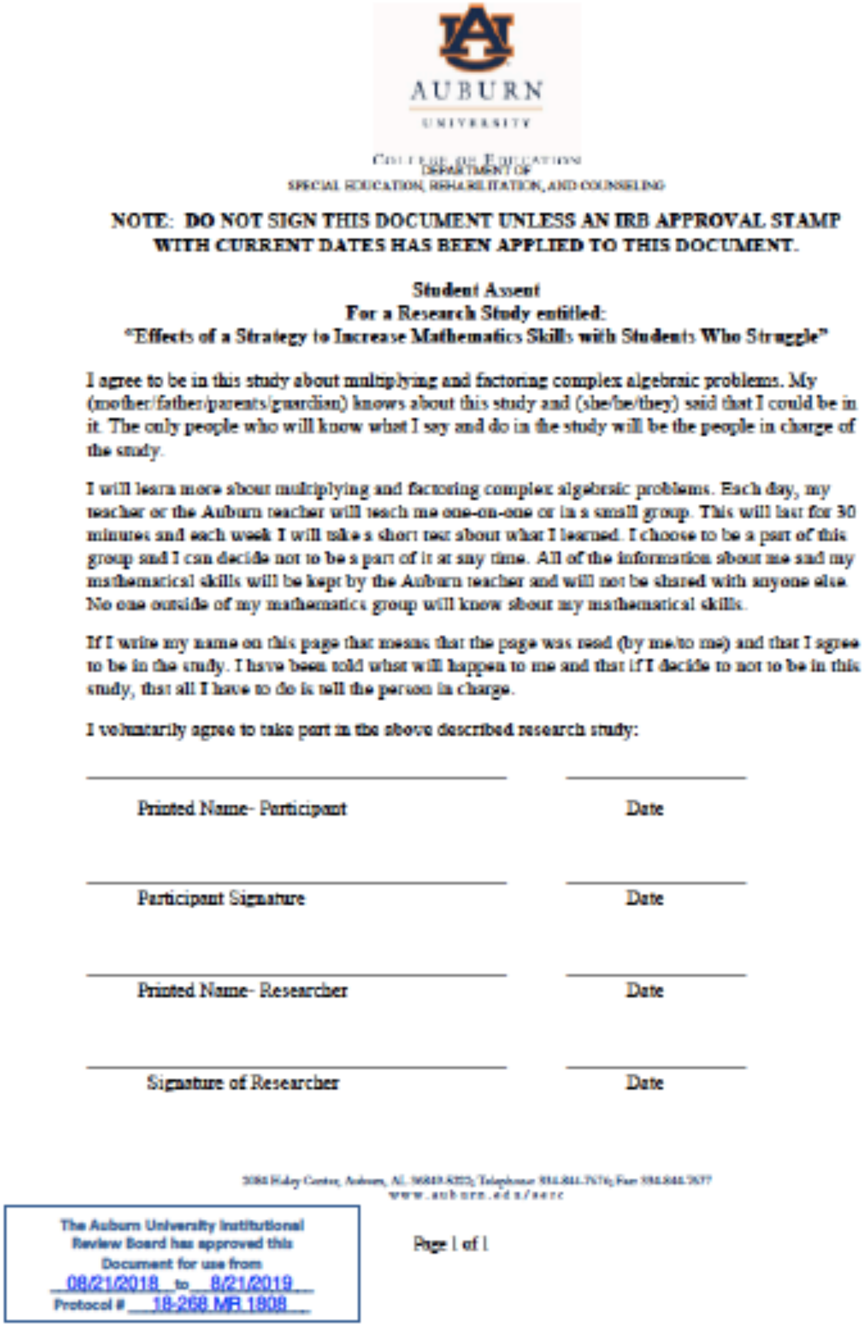
Appendix 2

Student Assent Forms



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Student Assent
For a Research Study entitled:
"Effects of a Strategy to Increase Mathematics Skills with Students Who Struggle"

I agree to be in this study about multiplying and factoring complex algebraic problems. My (mother/father/parents/guardian) knows about this study and (she/he/they) said that I could be in it. The only people who will know what I say and do in the study will be the people in charge of the study.

I will learn more about multiplying and factoring complex algebraic problems. Each day, my teacher or the Auburn teacher will teach me one-on-one or in a small group. This will last for 30 minutes and each week I will take a short test about what I learned. I choose to be a part of this group and I can decide not to be a part of it at any time. All of the information about me and my mathematical skills will be kept by the Auburn teacher and will not be shared with anyone else. No one outside of my mathematics group will know about my mathematical skills.

If I write my name on this page that means that the page was read (by me/to me) and that I agree to be in the study. I have been told what will happen to me and that if I decide to not to be in this study, that all I have to do is tell the person in charge.

I voluntarily agree to take part in the above described research study:

Printed Name- Participant Date

Participant Signature Date

Printed Name- Researcher Date

Signature of Researcher Date

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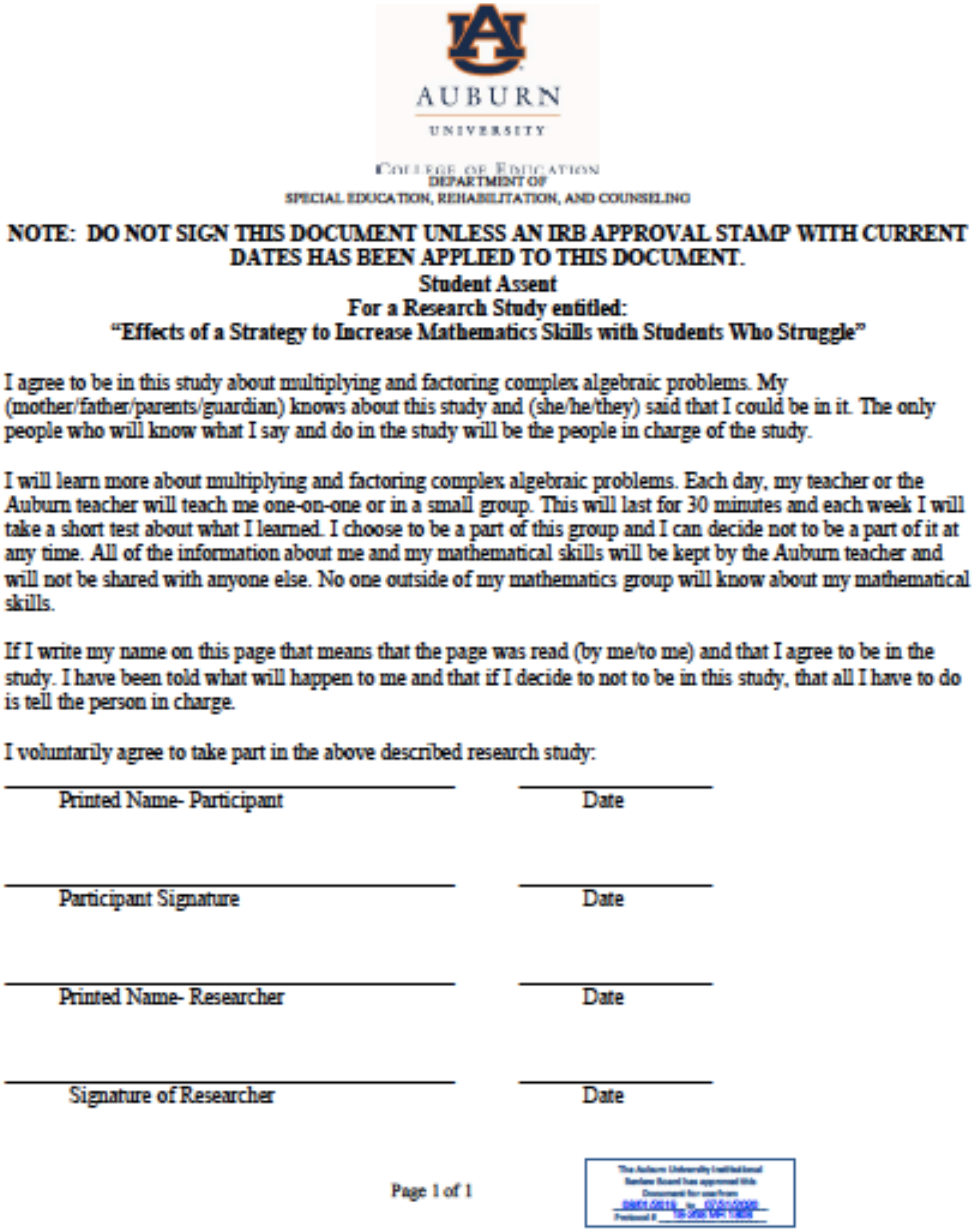

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Printed Name- Researcher Date

______________________________ ____________
Signature of Researcher Date

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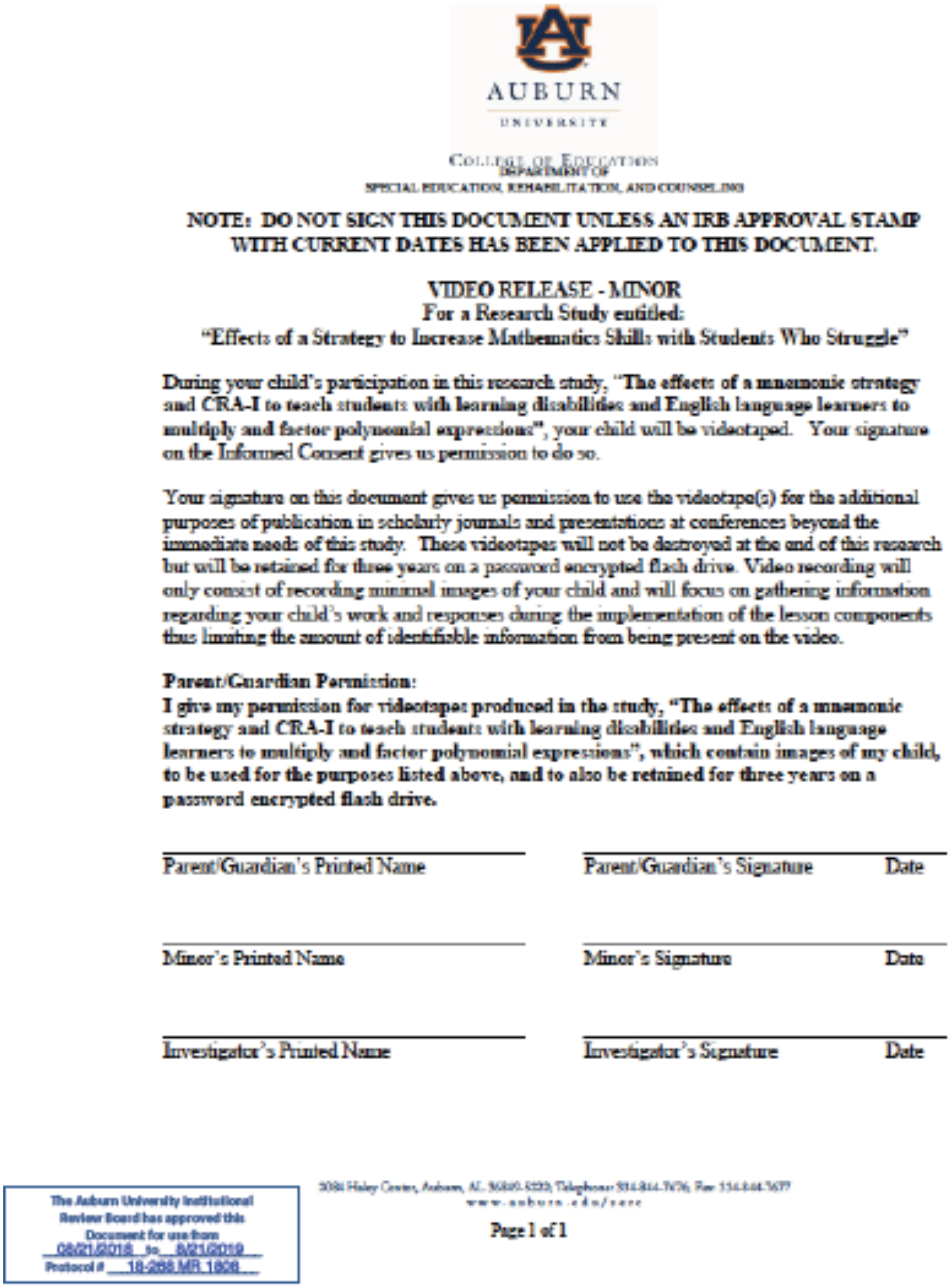
Appendix 3

Video Release Permission Forms



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VIDEO RELEASE - MINOR
For a Research Study entitled:
"Effects of a Strategy to Increase Mathematics Skills with Students Who Struggle"

During your child's participation in this research study, **"The effects of a mnemonic strategy and CRA-I to teach students with learning disabilities and English language learners to multiply and factor polynomial expressions"**, your child will be videotaped. Your signature on the Informed Consent gives us permission to do so.

Your signature on this document gives us permission to use the videotape(s) for the additional purposes of publication in scholarly journals and presentations at conferences beyond the immediate needs of this study. These videotapes will not be destroyed at the end of this research but will be retained for three years on a password encrypted flash drive. Video recording will only consist of recording minimal images of your child and will focus on gathering information regarding your child's work and responses during the implementation of the lesson components thus limiting the amount of identifiable information from being present on the video.

Parent/Guardian Permission:
I give my permission for videotapes produced in the study, "The effects of a mnemonic strategy and CRA-I to teach students with learning disabilities and English language learners to multiply and factor polynomial expressions", which contain images of my child, to be used for the purposes listed above, and to also be retained for three years on a password encrypted flash drive.

Parent/Guardian's Printed Name ____________ Parent/Guardian's Signature ____________ Date

Minor's Printed Name ____________ Minor's Signature ____________ Date

Investigator's Printed Name ____________ Investigator's Signature ____________ Date

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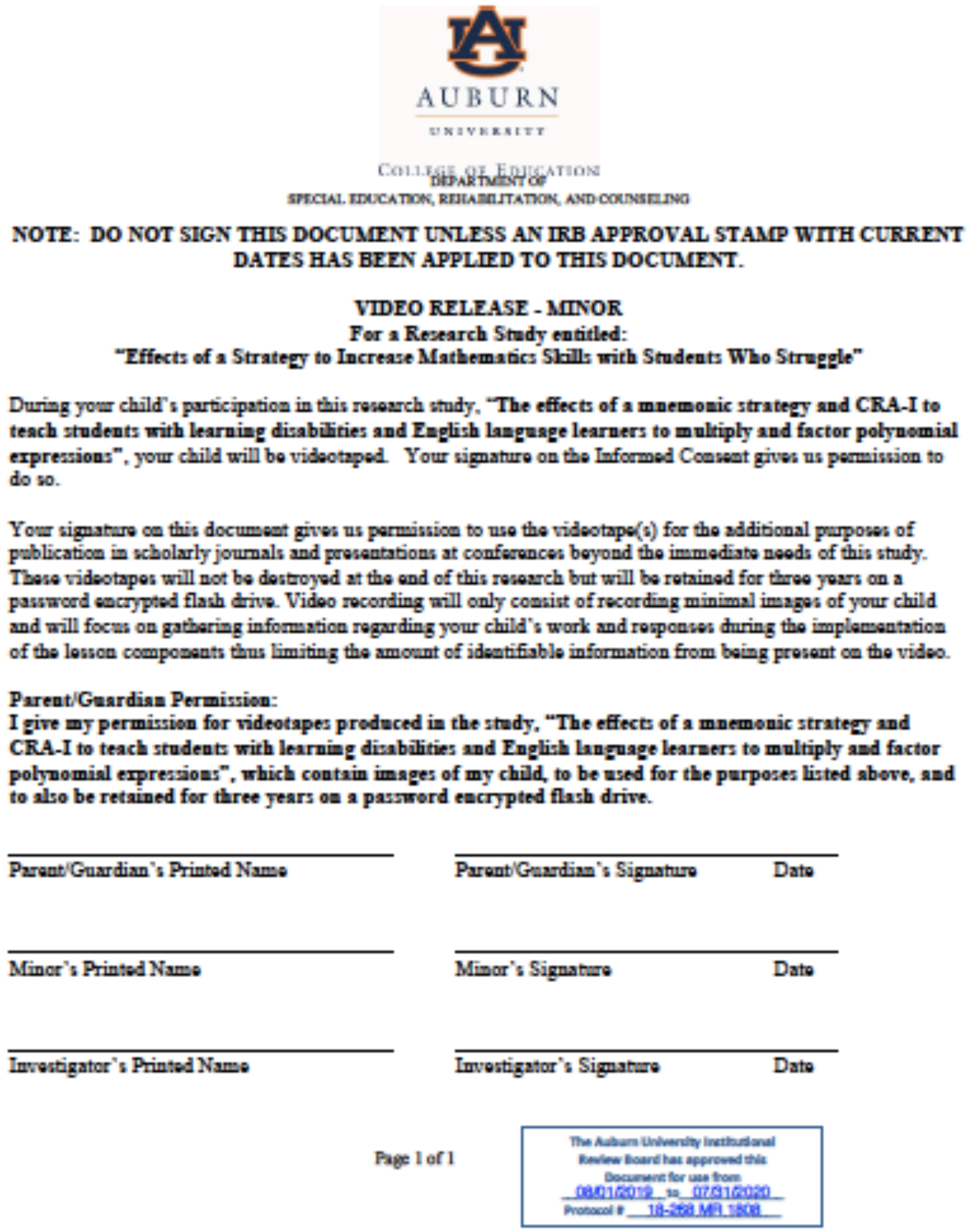

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Parent/Guardian's Printed Name — Parent/Guardian's Signature — Date

Minor's Printed Name — Minor's Signature — Date

Investigator's Printed Name — Investigator's Signature — Date

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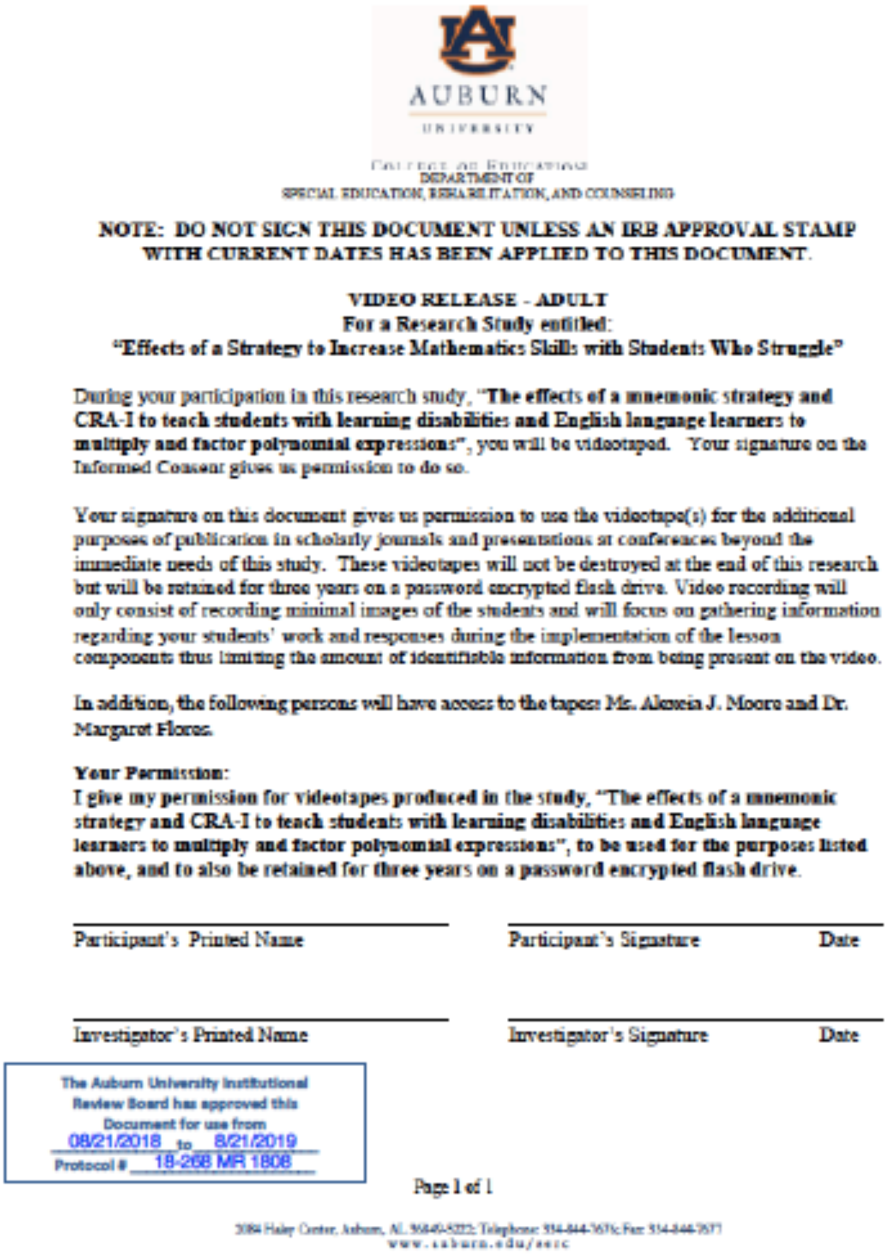

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VIDEO RELEASE - ADULT
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"Effects of a Strategy to Increase Mathematics Skills with Students Who Struggle"

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In addition, the following persons will have access to the tapes: Ms. Alexeia J. Moore and Dr. Margaret Flores.

Your Permission:
I give my permission for videotapes produced in the study, "The effects of a mnemonic strategy and CRA-I to teach students with learning disabilities and English language learners to multiply and factor polynomial expressions", to be used for the purposes listed above, and to also be retained for three years on a password encrypted flash drive.

Participant's Printed Name

Participant's Signature Date

Investigator's Printed Name

Investigator's Signature Date

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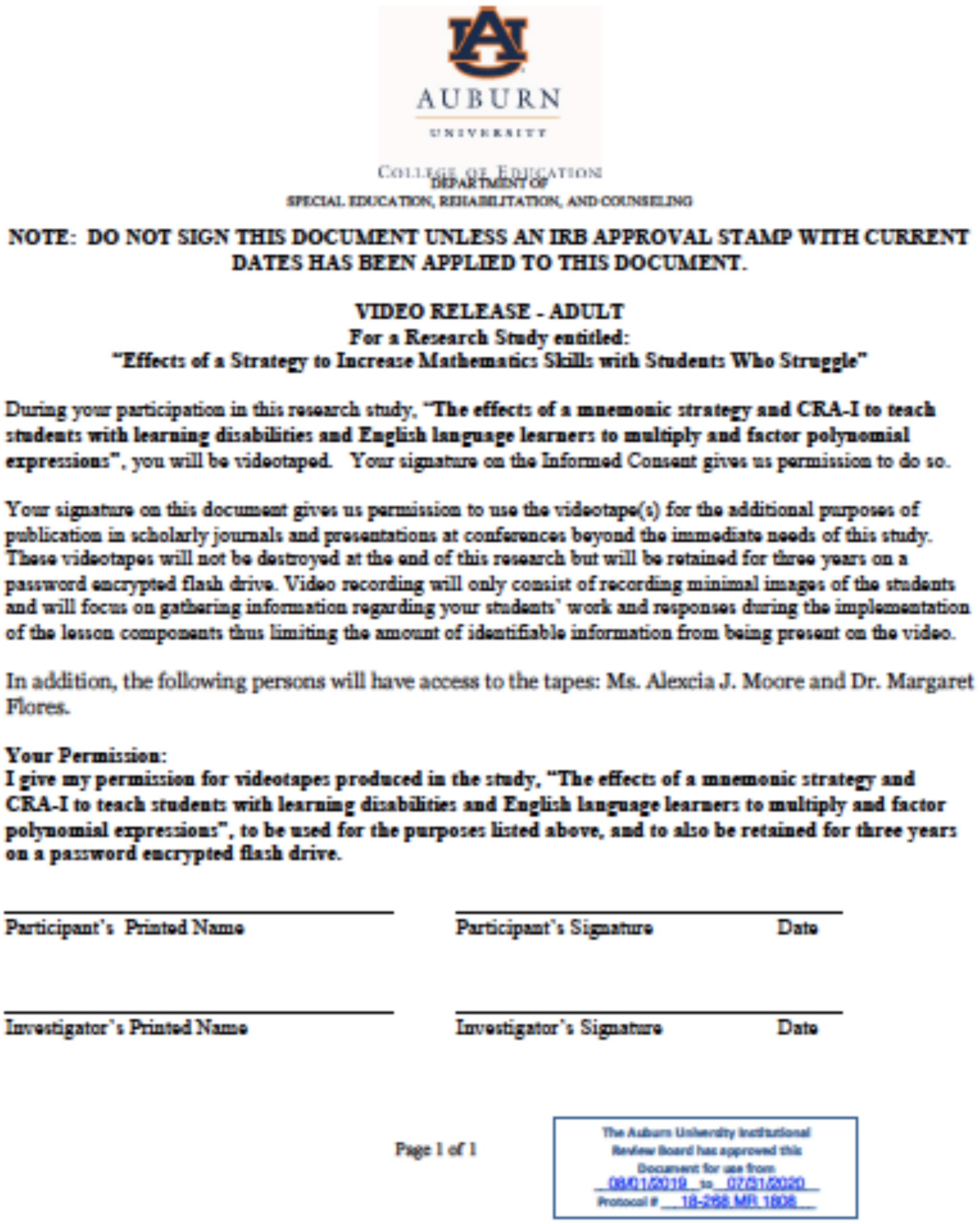

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Participant's Printed Name	Participant's Signature Date
Investigator's Printed Name	Investigator's Signature Date

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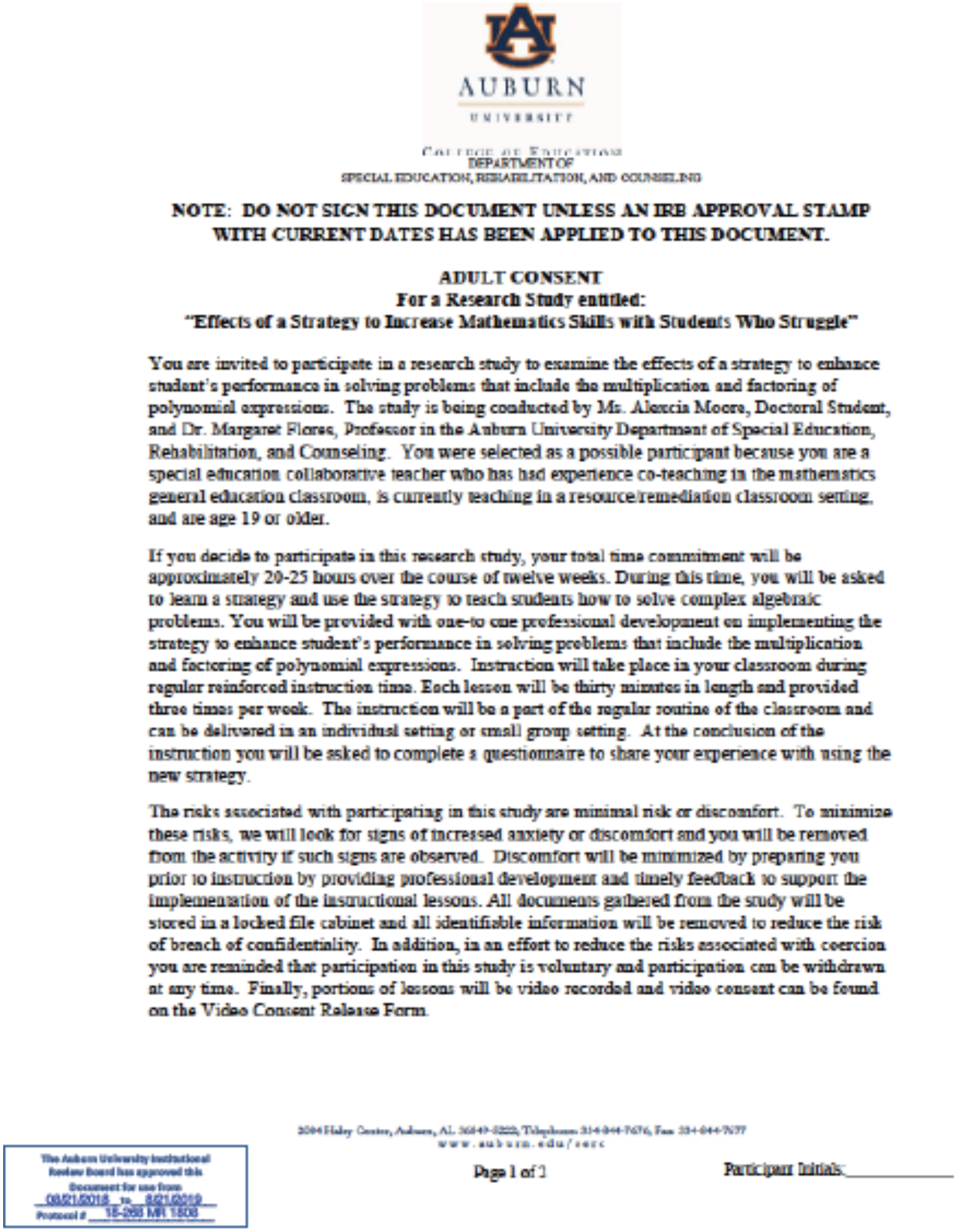
Appendix 4

Adult Consent Forms



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ADULT CONSENT
For a Research Study entitled:
"Effects of a Strategy to Increase Mathematics Skills with Students Who Struggle"

You are invited to participate in a research study to examine the effects of a strategy to enhance student's performance in solving problems that include the multiplication and factoring of polynomial expressions. The study is being conducted by Ms. Alexcia Moore, Doctoral Student, and Dr. Margaret Flores, Professor in the Auburn University Department of Special Education, Rehabilitation, and Counseling. You were selected as a possible participant because you are a special education collaborative teacher who has had experience co-teaching in the mathematics general education classroom, is currently teaching in a resource/remediation classroom setting, and are age 19 or older.

If you decide to participate in this research study, your total time commitment will be approximately 20-25 hours over the course of twelve weeks. During this time, you will be asked to learn a strategy and use the strategy to teach students how to solve complex algebraic problems. You will be provided with one-to one professional development on implementing the strategy to enhance student's performance in solving problems that include the multiplication and factoring of polynomial expressions. Instruction will take place in your classroom during regular reinforced instruction time. Each lesson will be thirty minutes in length and provided three times per week. The instruction will be a part of the regular routine of the classroom and can be delivered in an individual setting or small group setting. At the conclusion of the instruction you will be asked to complete a questionnaire to share your experience with using the new strategy.

The risks associated with participating in this study are minimal risk or discomfort. To minimize these risks, we will look for signs of increased anxiety or discomfort and you will be removed from the activity if such signs are observed. Discomfort will be minimized by preparing you prior to instruction by providing professional development and timely feedback to support the implementation of the instructional lessons. All documents gathered from the study will be stored in a locked file cabinet and all identifiable information will be removed to reduce the risk of breach of confidentiality. In addition, in an effort to reduce the risks associated with coercion you are reminded that participation in this study is voluntary and participation can be withdrawn at any time. Finally, portions of lessons will be video recorded and video consent can be found on the Video Consent Release Form.

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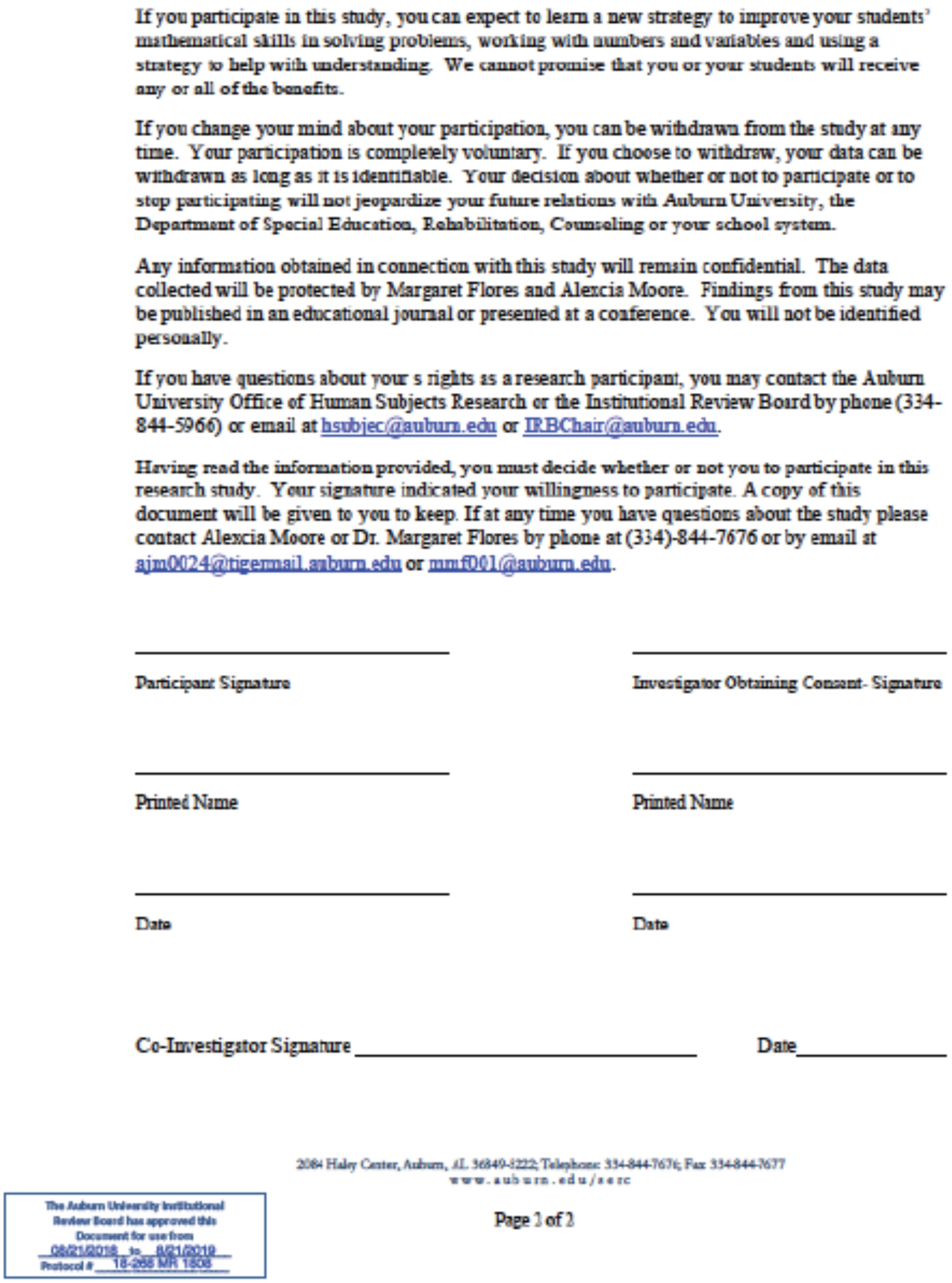
Participant Initials:____________



If you participate in this study, you can expect to learn a new strategy to improve your students' mathematical skills in solving problems, working with numbers and variables and using a strategy to help with understanding. We cannot promise that you or your students will receive any or all of the benefits.

If you change your mind about your participation, you can be withdrawn from the study at any time. Your participation is completely voluntary. If you choose to withdraw, your data can be withdrawn as long as it is identifiable. Your decision about whether or not to participate or to stop participating will not jeopardize your future relations with Auburn University, the Department of Special Education, Rehabilitation, Counseling or your school system.

Any information obtained in connection with this study will remain confidential. The data collected will be protected by Margaret Flores and Alexcia Moore. Findings from this study may be published in an educational journal or presented at a conference. You will not be identified personally.

If you have questions about your s rights as a research participant, you may contact the Auburn University Office of Human Subjects Research or the Institutional Review Board by phone (334-844-5966) or email at hsubjec@auburn.edu or IRBChair@auburn.edu.

Having read the information provided, you must decide whether or not you to participate in this research study. Your signature indicated your willingness to participate. A copy of this document will be given to you to keep. If at any time you have questions about the study please contact Alexcia Moore or Dr. Margaret Flores by phone at (334)-844-7676 or by email at ajm0024@tigermail.auburn.edu or mmf001@auburn.edu.

______________________________	______________________________
Participant Signature	Investigator Obtaining Consent- Signature
______________________________	______________________________
Printed Name	Printed Name
______________________________	______________________________
Date	Date

Co-Investigator Signature ________________________________ Date______________

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Protocol # [illegible]

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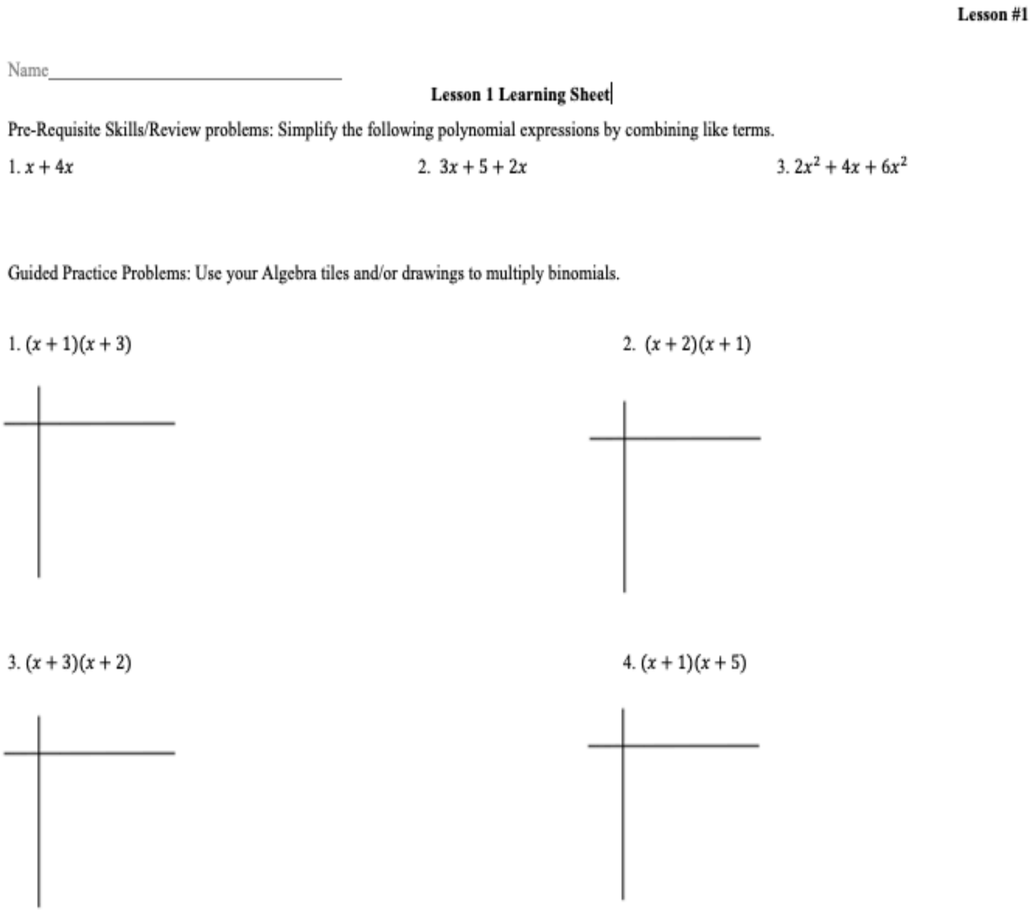
______________________________ Participant Signature	______________________________ Investigator Obtaining Consent- Signature
______________________________ Printed Name	______________________________ Printed Name
______________________________ Date	______________________________ Date

Co-Investigator Signature ________________________________ Date______________

Appendix 5

Binomial Multiplication Learning Sheets



Lesson #1

Name______________________________

Lesson 1 Learning Sheet

Pre-Requisite Skills/Review problems: Simplify the following polynomial expressions by combining like terms.

1. $x + 4x$
2. $3x + 5 + 2x$
3. $2x^2 + 4x + 6x^2$

Guided Practice Problems: Use your Algebra tiles and/or drawings to multiply binomials.

1. $(x + 1)(x + 3)$
2. $(x + 2)(x + 1)$
3. $(x + 3)(x + 2)$
4. $(x + 1)(x + 5)$

Name________________________________

Lesson 1 – Learning Sheet 1

Directions: Use your Algebra tiles and/or drawings to multiply monomials and binomials.

1. $(x+1)(x+3)$

2. $(x+2)(x+2)$

3. $(x+1)(x+4)$

4. $(x+5)(x+2)$

Name________________________________

5. $(x+4)(x+3)$

Name________________________________

Lesson 2 Learning Sheet

Pre-Requisite Skills/Review problems: Multiply.

1. $(x+2)(x+4)$

2.$(x+3)(x+1)$

Guided Practice Problems: Use your Algebra tiles, drawings, and/or the graphic organizer to multiply monomials and binomials.

1. $(x-1)(x-5)$

2. $(x+3)(x-2)$

3. $(x-3)(x-2)$

4. $(x+1)(x-6)$

Name________________________________

Lesson 2 – Learning Sheet 1

Directions: Use your Algebra tiles, drawings, and/or the graphic organizer to multiply binomials.

1. $(x+3)(x-1)$

2. $(x-5)(x+2)$

3. $(x-1)(x-4)$

4. $(x-2)(x-2)$

Name________________________________

5. $(x-4)(x+3)$

Name________________________________

Lesson 3 – Learning Sheet

Pre-Requisite Skills/Review problems: Simplify the following polynomial expressions by multiplying.

1.$(x+1)(x+7)$

2. $(x+3)(x-2)$

Guided Practice Problems: Use your graphic organizer to multiply polynomials.

3. $(x+4)(x-2)$

4. $(x+6)(x-1)$

Name________________________________

Lesson 3 - Learning Sheet 1

Directions: Use your graphic organizer to multiply binomials.

1. $(x+6)(x-1)$

2. $(x-4)(x+2)$

3. $(x-3)(x-2)$

4. $(x-1)(x-1)$

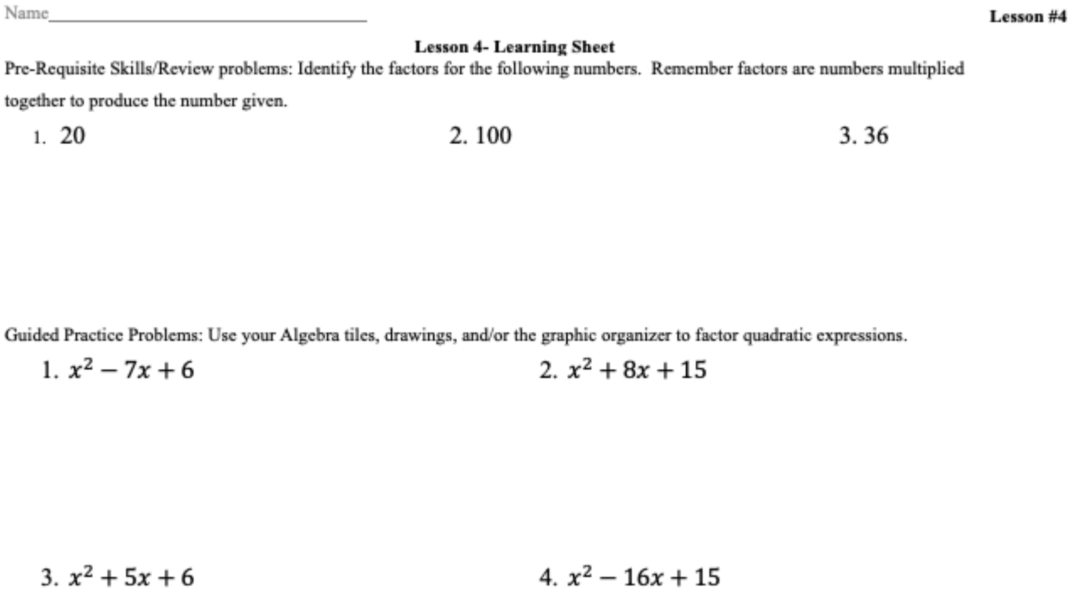
Lesson #3

Name________________________________

5. $(x - 7)(x + 4)$

Appendix 6

Quadratic Transformations into Factored Form when a = 1
Learning Sheets



Name________________________________ **Lesson #4**

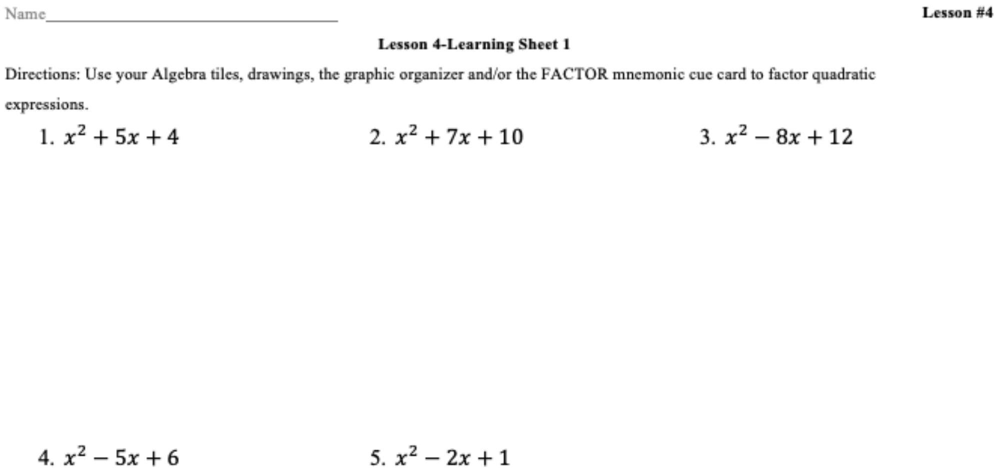
Lesson 4- Learning Sheet

Pre-Requisite Skills/Review problems: Identify the factors for the following numbers. Remember factors are numbers multiplied together to produce the number given.

1. 20
2. 100
3. 36

Guided Practice Problems: Use your Algebra tiles, drawings, and/or the graphic organizer to factor quadratic expressions.

1. $x^2 - 7x + 6$
2. $x^2 + 8x + 15$
3. $x^2 + 5x + 6$
4. $x^2 - 16x + 15$



Name________________________________ **Lesson #4**

Lesson 4-Learning Sheet 1

Directions: Use your Algebra tiles, drawings, the graphic organizer and/or the FACTOR mnemonic cue card to factor quadratic expressions.

1. $x^2 + 5x + 4$
2. $x^2 + 7x + 10$
3. $x^2 - 8x + 12$
4. $x^2 - 5x + 6$
5. $x^2 - 2x + 1$

Name________________________________

Lesson 5 Learning Sheet

Pre-Requisite Skills/Review problems: Identify the factors for the following numbers. Remember factors are numbers multiplied together to produce the number given. Factors can be positive or negative.

1. -10
2. -45
3. -81

Guided Practice Problems: Use your Algebra tiles, drawings, and/or the graphic organizer to factor quadratic expressions.

1. $x^2 + 2x - 15$
2. $x^2 - 2x - 15$
3. $x^2 - 5x - 6$
4. $x^2 - 2x - 15$

Name________________________________

Lesson #5

Lesson 5-Learning Sheet 1

Directions: Use your Algebra tiles, drawings, and/or the graphic organizer to factor quadratic expressions.

1. $x^2 + 3x - 4$
2. $x^2 + 3x - 18$
3. $x^2 + 4x - 45$
4. $x^2 - 2x - 8$
5. $x^2 - 11x - 12$

Name________________________________ **Lesson #6**

Lesson 6-Learning Sheet

Pre-Requisite Skills/Review problems: Use your Algebra tiles, drawings, the graphic organizer and/or the FACTOR mnemonic cue card to factor quadratic expressions.

1. $x^2 + 11x + 28$

2. $x^2 + 14x - 32$

Guided Practice Problems: Use your Algebra tiles, drawings, the graphic organizer and/or the FACTOR mnemonic cue card to factor quadratic expressions.

3. $x^2 - 6x - 27$

4. $x^2 - 10x + 25$

Name________________________________ **Lesson #6**

Lesson 6- Learning Sheet 1

Independent Practice: Use your Algebra tiles, drawings, the graphic organizer and/or the FACTOR mnemonic cue card to factor quadratic expressions.

1. $x^2 + 3x - 4$

2. $x^2 - 12x - 13$

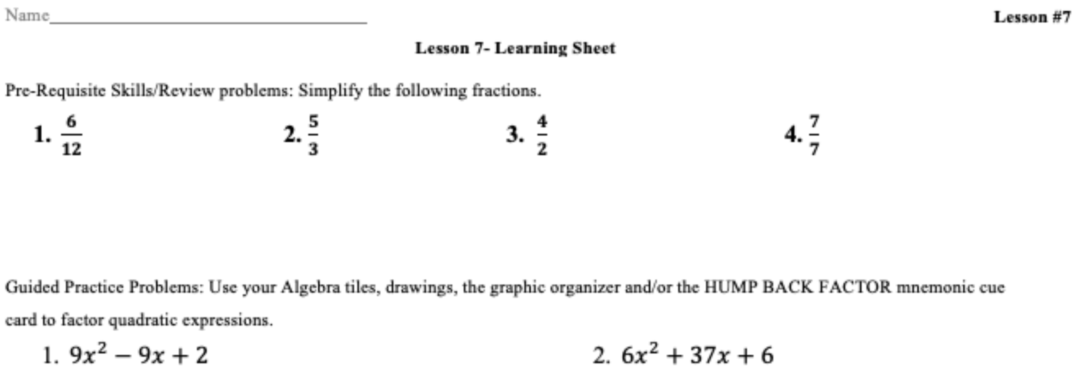
3. $x^2 + 7x + 10$

4. $x^2 - 11x + 18$

5. $x^2 + 10x + 16$

Appendix 7

Quadratic Transformations into Factored Form when a > 1
Learning Sheets



Name________________________________ **Lesson #7**

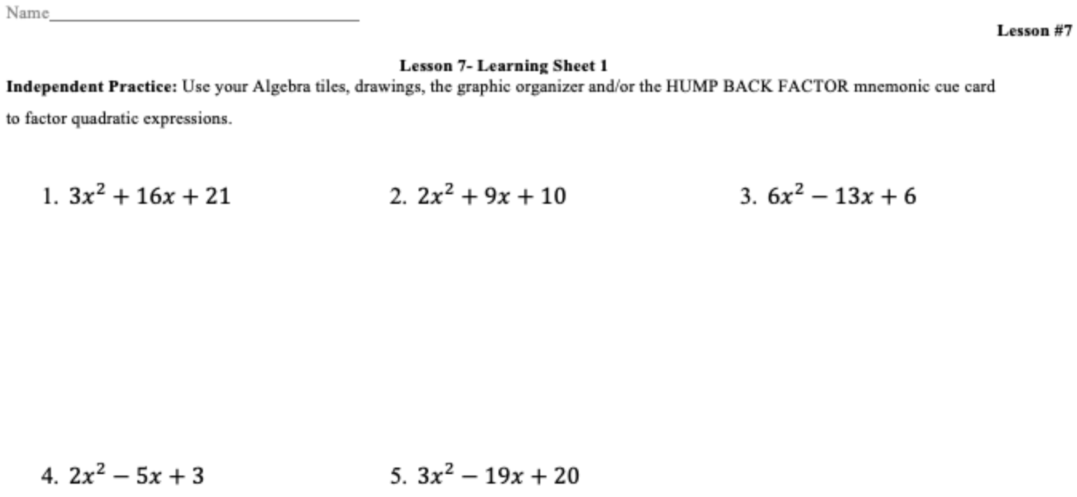
Lesson 7- Learning Sheet

Pre-Requisite Skills/Review problems: Simplify the following fractions.

1. $\frac{6}{12}$ **2.** $\frac{5}{3}$ **3.** $\frac{4}{2}$ **4.** $\frac{7}{7}$

Guided Practice Problems: Use your Algebra tiles, drawings, the graphic organizer and/or the HUMP BACK FACTOR mnemonic cue card to factor quadratic expressions.

1. $9x^2 - 9x + 2$
2. $6x^2 + 37x + 6$



Name________________________________

Lesson #7

Lesson 7- Learning Sheet 1

Independent Practice: Use your Algebra tiles, drawings, the graphic organizer and/or the HUMP BACK FACTOR mnemonic cue card to factor quadratic expressions.

1. $3x^2 + 16x + 21$
2. $2x^2 + 9x + 10$
3. $6x^2 - 13x + 6$
4. $2x^2 - 5x + 3$
5. $3x^2 - 19x + 20$

Name________________________________

Lesson #8

Lesson 8- Learning Sheet

Pre-Requisite Skills/Review problems: Use your Algebra tiles, drawings, the graphic organizer and/or the HUMP BACK FACTOR mnemonic cue card to factor quadratic expressions.

1. $5x^2 + 9x + 4$

2. $3x^2 - 11x + 6$

Guided Practice Problems: Use your Algebra tiles, drawings, the graphic organizer and/or the HUMP BACK FACTOR mnemonic cue card to factor quadratic expressions.

1. $3x^2 - 2x - 21$

2. $9x^2 + 16x - 4$

Name________________________________

Lesson #8

Lesson 8- Learning Sheet 1

Independent Practice: Use your Algebra tiles, drawings, the graphic organizer and/or the HUMP BACK FACTOR mnemonic cue card to factor quadratic expressions.

1. $4x^2 + 7x - 2$

2. $6x^2 + x - 2$

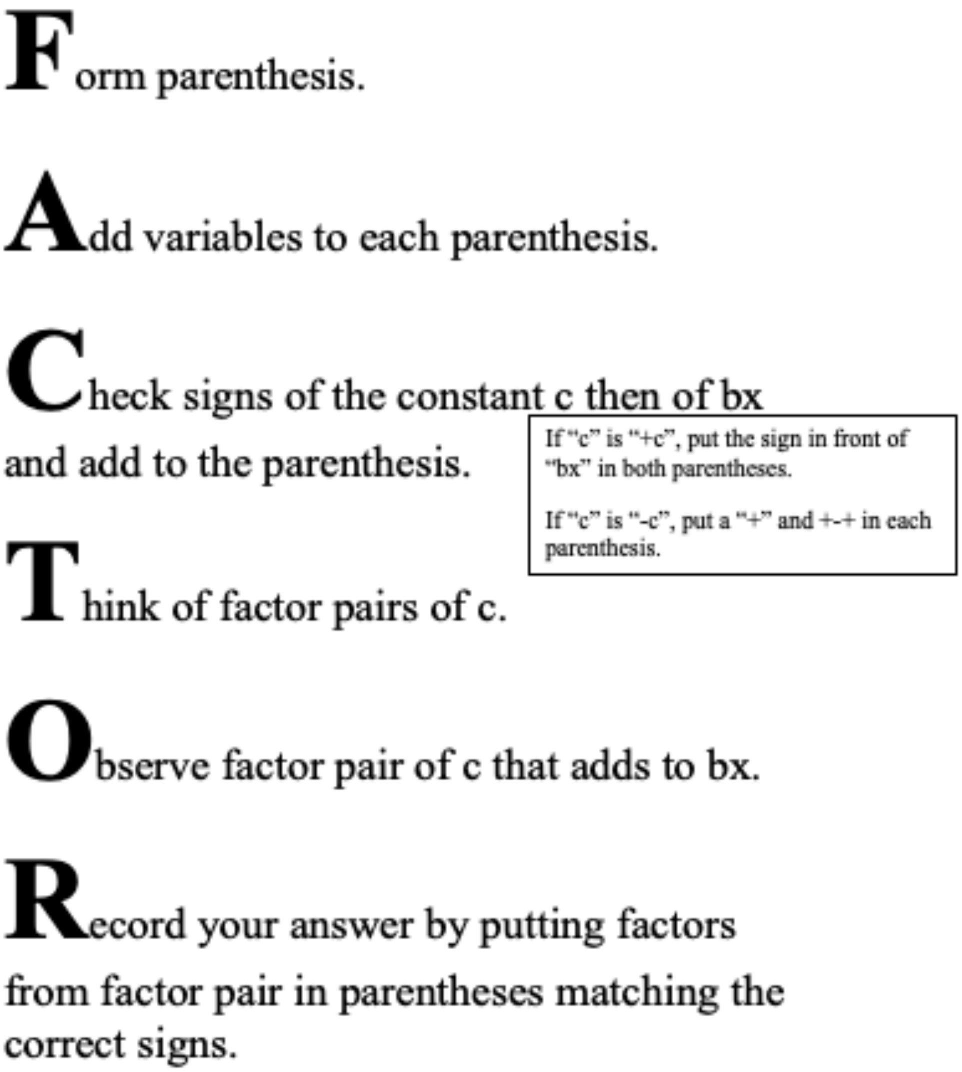
3. $12x^2 + 5x - 3$

4. $5x^2 - 7x - 6$

5. $3x^2 - 4x - 15$

Appendix 8

Mnemonic Strategy- FACTOR



Form parenthesis.

Add variables to each parenthesis.

Check signs of the constant c then of bx and add to the parenthesis.

> If "c" is "+c", put the sign in front of "bx" in both parentheses.
>
> If "c" is "-c", put a "+" and +-+ in each parenthesis.

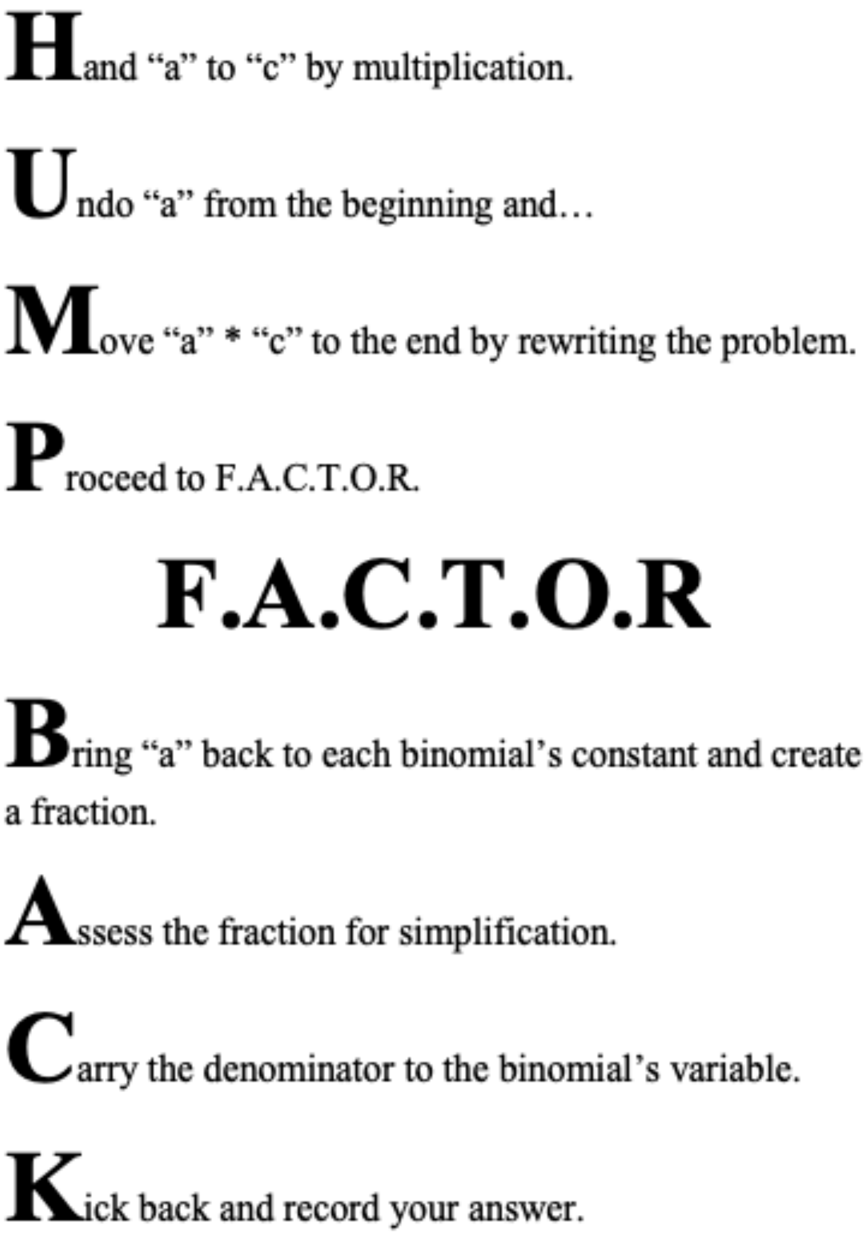
Think of factor pairs of c.

Observe factor pair of c that adds to bx.

Record your answer by putting factors from factor pair in parentheses matching the correct signs.

Appendix 9

Mnemonic Strategy- HUMP BACK FACTOR



Hand “a” to “c” by multiplication.

Undo “a” from the beginning and…

Move “a” * “c” to the end by rewriting the problem.

Proceed to F.A.C.T.O.R.

F.A.C.T.O.R

Bring “a” back to each binomial’s constant and create a fraction.

Assess the fraction for simplification.

Carry the denominator to the binomial’s variable.

Kick back and record your answer.

Appendix 10

Pretest

Name__________________________________ **Pretest**

Pre-Test

Multiply Polynomials:

1. (2x+1) (4x+2)
2. (2x +2) (x-4)
3. (x+4) (x+5)
4. (x-1) (x-2)
5. (3x+1) (x- 8)

Factor Polynomials:

6. $x^2 + 5x + 6$
7. $x^2 - 6x + 8$
8. $x^2 + 4x - 5$

Name________________________________ **Pretest**

9. $x^2 - 9x - 10$

10. $x^2 + 11x + 28$

11. $3x^2 + 2x - 16$

12. $4x^2 + 11x + 7$

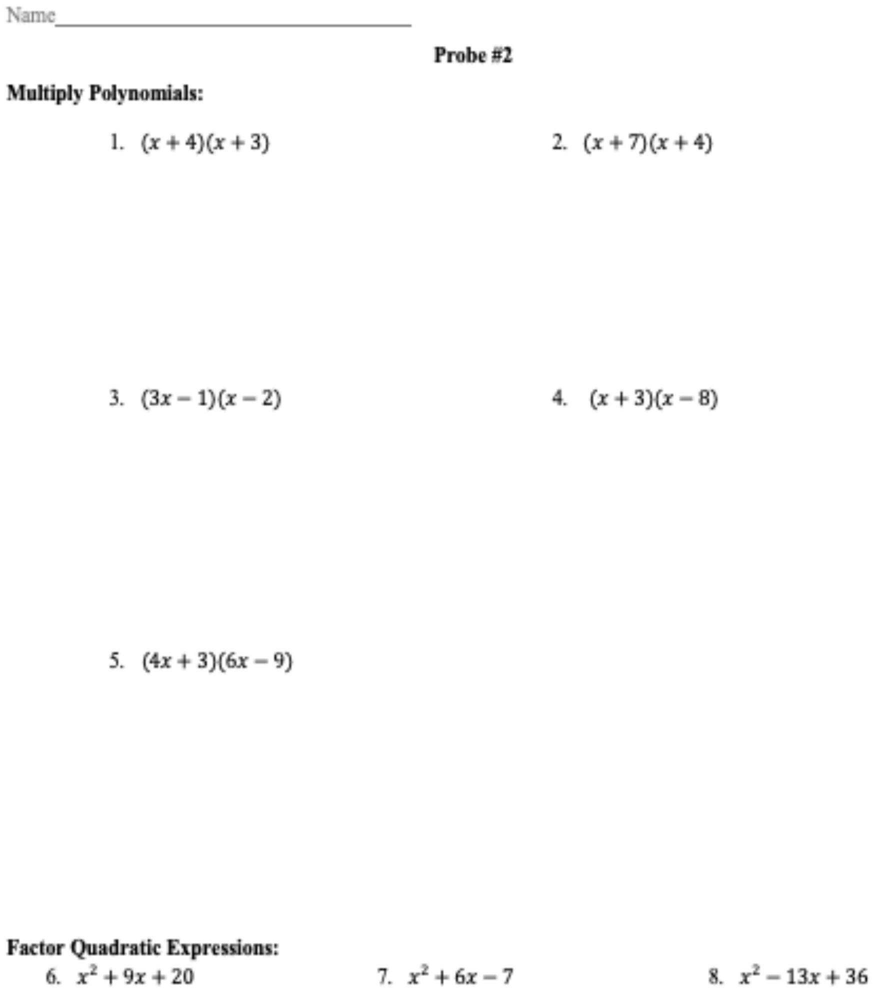
13. $8x^2 - 73x + 9$

14. $2x^2 + 13x + 15$

15. $2x^2 - 4x - 6$

Appendix 11

Intervention Probes



Name________________________________

Probe #2

Multiply Polynomials:

1. $(x+4)(x+3)$
2. $(x+7)(x+4)$
3. $(3x-1)(x-2)$
4. $(x+3)(x-8)$
5. $(4x+3)(6x-9)$

Factor Quadratic Expressions:

6. $x^2+9x+20$
7. x^2+6x-7
8. $x^2-13x+36$

Name__________________________________

Probe #2

9. $x^2 - x - 12$

10. $x^2 + 11x - 60$

11. $2x^2 - 9x + 10$

12. $5x^2 + x - 6$

13. $8x^2 - 13x - 6$

14. $3x^2 + 20x + 12$

15. $9x^2 - 49x - 30$

Name_______________________________

Probe #

Multiply Polynomials:

1. $(3x + 2)(x + 2)$

2. $(x + 6)(x + 8)$

3. $(4x + 3)(x - 1)$

4. . $(x - 3)(x - 5)$

5. $(3x - 6)(x - 4)$

Factor Polynomials:

6. $x^2 + 3x - 10$

7. $x^2 + 6x + 9$

8. $x^2 + 11x + 10$

Name________________________________ **Probe**

9. $x^2 - 2x - 15$

10. $x^2 - 8x + 15$

11. $2x^2 + 7x + 5$

12. $4x^2 - 12x + 9$

13. $2x^2 + 9x + 10$

14. $10x^2 + 11x + 3$

15. $6x^2 + 19x + 14$

Name________________________________

Probe #

Multiply Polynomials:

1. $(x+4)(x+3)$

2. $(x+7)(x+4)$

3. $(x-1)(x-2)$

4. $(x+3)(x-4)$

5. $(x+3)(x-2)$

Factor Quadratic Expressions:

6. $x^2+9x+20$

7. x^2+6x-7

8. $x^2-13x+36$

Name__________________________________ **Probe #**

9. $x^2 - x - 12$

10. $x^2 + 11x - 60$

11. $2x^2 - 9x + 10$

12. $5x^2 + x - 6$

13. $8x^2 - 13x - 6$

14. $3x^2 + 20x + 12$

15. $9x^2 - 49x - 30$

Name____________________________

Probe #

Multiply Polynomials:

1. $(x+2)(x+2)$
2. $(x+6)(x+8)$
3. $(x+3)(x-1)$
4. . $(x-3)(x-5)$
5. $(x-6)(x-4)$

Factor Polynomials:

6. $x^2+3x-10$
7. x^2+6x+9
8. $x^2+11x+10$

Name___________________________________ **Probe #**

9. $x^2 - 2x - 15$

10. $x^2 - 8x + 15$

11. $2x^2 + 7x + 5$

12. $4x^2 - 12x + 9$

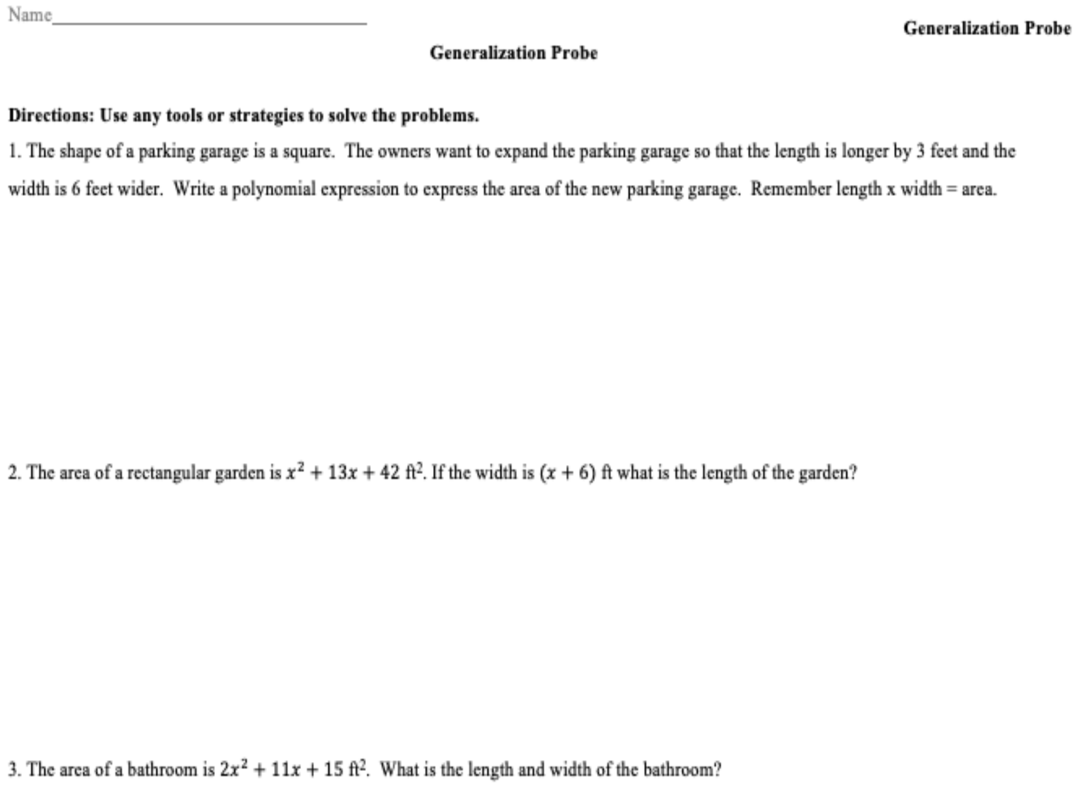
13. $2x^2 + 9x + 10$

14. $10x^2 + 11x + 3$

15. $6x^2 + 19x + 14$

Appendix 12

Generalization Probe



Name________________________________

Generalization Probe

Generalization Probe

Directions: Use any tools or strategies to solve the problems.

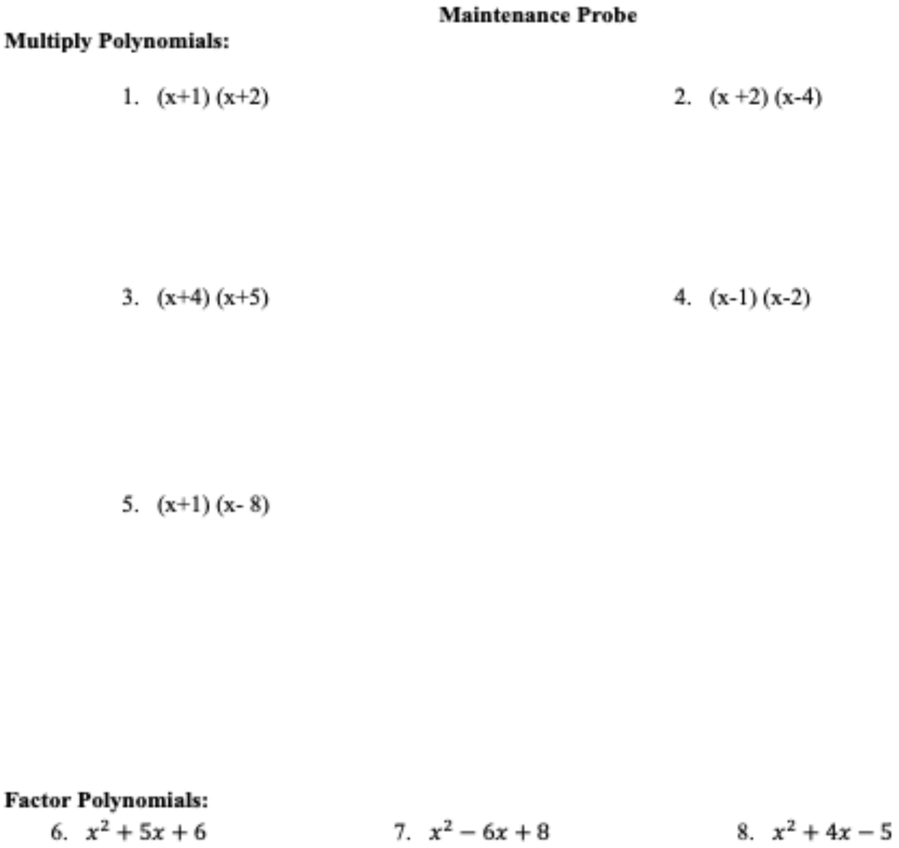
1. The shape of a parking garage is a square. The owners want to expand the parking garage so that the length is longer by 3 feet and the width is 6 feet wider. Write a polynomial expression to express the area of the new parking garage. Remember length x width = area.

2. The area of a rectangular garden is $x^2 + 13x + 42$ ft^2. If the width is $(x + 6)$ ft what is the length of the garden?

3. The area of a bathroom is $2x^2 + 11x + 15$ ft^2. What is the length and width of the bathroom?

Appendix 13

Maintenance Probe



Maintenance Probe

Multiply Polynomials:

1. $(x+1) (x+2)$
2. $(x +2) (x-4)$
3. $(x+4) (x+5)$
4. $(x-1) (x-2)$
5. $(x+1) (x- 8)$

Factor Polynomials:

6. $x^2 + 5x + 6$
7. $x^2 - 6x + 8$
8. $x^2 + 4x - 5$

Name___________________________________

9. $x^2 - 9x - 10$

10. $x^2 + 11x + 28$

11. $3x^2 + 2x - 16$

12. $4x^2 + 11x + 7$

13. $8x^2 - 73x + 9$

14. $2x^2 + 13x + 15$

15. $2x^2 - 4x - 6$

Appendix 14

Treatment Fidelity Checklist

CRA Treatment Integrity Checklist: Multiplying Polynomial Expressions
Date: ________________________________

Instructor Behavior		Yes	No
1	All materials ready prior to lesson		
2	Provides necessary instruction to students to start the lesson or gives an advance organizer, tells the student what he/she will be doing and why.		
3	Teacher demonstrations are accurate according to intervention procedures. • Concrete: uses ALG manipulatives to demonstrate operation according to description in manual. • Representational: uses drawings to represent the operation. • Abstract: Uses BOX METHOD to accurately represent the operation and thinks aloud when solving problems.		
4	Engages students in instruction during demonstration and guided practice by prompting their participation by asking questions, etc.		
5	During independent practice, instructs students to solve problems without guidance.		
6	Monitors the students' work during independent practice while they solve problems without guidance. Provides verbal prompts only if the student has difficulty. Does not offer answers.		
7	The instructor will close the lesson with a positive statement about the student's performance in the feedback process, reviews lesson, and mentions future lesson and expectations.		
8	Follows lesson plans and paraphrases suggested script with materials.		

CRA Treatment Integrity Checklist: Transformations of Quadratic Expressions into Factored Form

Date: ________________________________

Instructor Behavior		**Yes**	**No**
1	All materials ready prior to lesson		
2	Provides necessary instruction to students to start the lesson or gives an advance organizer, tells the student what he/she will be doing and why.		
3	Teacher demonstrations are accurate according to intervention procedures. • Concrete: uses ALG manipulatives to demonstrate operation according to procedures in manual. • Representational: uses drawings to represent the operation. • Abstract: Uses FACTOR and HUMP BACK FACTOR accurately and thinks aloud when solving problems.		
4	Engages students in instruction during demonstration and guided practice by prompting their participation by asking questions, etc.		
5	During independent practice, instructs students to solve problems without guidance.		
6	Monitors the students' work during independent practice while they solve problems without guidance. Provides verbal prompts only if the student has difficulty. Does not offer answers.		
7	The instructor will close the lesson with a positive statement about the student's performance in the feedback process, reviews lesson, and mentions future lesson and expectations.		
8	Follows lesson plans and paraphrases suggested script with materials.		

Appendix 15

Social Validity Survey

Student Name____________________________

	Yes, I agree	No, I do NOT agree
1. Before I learned about multiplying binomial expressions using algebra tiles and drawings, I thought multiplying polynomial expressions were hard.		
2. During multiplication lessons, I did not like using algebra tiles.		
3. Using algebra tiles made solving multiplication problems easier.		
4. Before I learned about multiplying binomial expressions using algebra tiles and drawings, I thought multiplying binomial expressions were easy.		
5. I liked using algebra tiles and drawings to solve multiplication problems.		
6. Multiplication lessons were too long.		
7. Before I learned about factoring quadratic expressions using algebra tiles and drawings, I thought factoring quadratic expressions were hard.		
8. Using the FACTOR and HUMP BACK FACTOR mnemonics made factoring problems easier.		
9. Before I learned about factoring quadratic expressions using algebra tiles and drawings, I thought factoring quadratic expressions were easy.		
10. During factoring lessons, I did not like using algebra tiles.		
11. I think other students should learn to solve problems involving factoring of quadratic expressions with the FACTOR and HUMP BACK FACTOR mnemonics.		
12. Using algebra tiles made solving factoring problems easier.		
13. Factoring lessons were too long.		
14. I think other students should learn to solve problems involving multiplying and factoring quadratic expressions with algebra tiles and drawings.		

15. I liked using algebra tiles and drawings to solve factoring problems.		
16. After learning about multiplying and factoring quadratics expressions with the instructor, I am better at solving problems with the multiplication and factoring of quadratic expressions.		
17. I have used what I learned about multiplying and factoring quadratic expressions in other classes.		
18. After learning about multiplying and factoring quadratic expressions with the instructor, I still have a hard time solving problems with multiplying and factoring quadratic expressions.		
19. Multiplication and factoring lessons using algebra tiles and drawings were boring.		

What was your favorite part about learning how to multiply and factor quadratic expressions?

What part of learning how to multiply and factor quadratic expressions did you **not** like?