Examining Process-Driven Math: A User Centered Design and Universal Design for Learning Perspective

by

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Abstract

Many students with disabilities who are capable of learning math find limited success because the tools they are using to access mathematical content are not well aligned with their range of abilities. Process-Driven Math (PDM) is a learning support tool that was initially developed for one student who is blind and unable to write or type. PDM uses chunking to simplify the landscape of complex algebraic expressions and reduce the cognitive load on working memory. The method employs a set of communication rules based on appropriate math vocabulary for accurately describing mathematical transformations verbally (Gulley et al., 2017). The PDM method was later adapted for visual learners who also need additional tools to succeed in mathematics.

In this study, PDM was examined using both qualitative and experimental research designs. Qualitative research focused on the experiences of 25 students at three state schools for the blind who interacted with the fully audio PDM method during several class periods. Experimental research evaluated the differences in assessment performance of more than 650 students with and without difficulties related to several categories of disability who were randomly assigned to PDM and non-PDM treatment groups.

This research was approached from both a User Centered Design and a Universal Design for Learning perspective. Findings indicate that cohorts of students at the schools for the blind are diverse groups who collectively require a broad range of tools to access mathematical content. Data indicates a preference for a PDM tool that incorporates visual and tactile elements that can be used or not at will by the student. The experimental data from post-secondary schools shows that students who reported difficulties with concentration scored higher on performance assessments when receiving PDM instruction as opposed to typical classroom instruction.

The development of PDM is an ongoing, iterative process. The qualitative data from this study will direct improvements to PDM so it will better meet the needs and preferences of students who are blind or have visual impairments. The experimental data will be used to develop additional research models to further study PDM with students who report difficulties with concentration.

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List of Abbreviations

AYP	Annual Yearly Progress
CAST	Center for Applied Special Technology
CRA	Concrete-Representational-Abstract
EAHCA	Education for All Handicapped Children
ESSA	Every Student Succeeds Act
ETS	Educational Testing Service
IDEA	Individuals with Disabilities Education Act
IEP	Individualized Education Program
NAEP	National Assessment of Education Progress
NCES	National Center for Education Statistics
NCLB	No Child Left Behind
NSB	National Science Board
NSF	National Science Foundation
PARCC	Partnership for Assessment of Readiness for College and Careers
PDM	Process-Driven Math
SBVIs	Students who are Blind or have Visual Impairments
STEM	Science, Technology, Engineering and Math
SVIs	Students who have Visual Impairments
SWDs	Students with Disabilities
TVIs	Teachers of the Visually Impaired
UCD	User Centered Design
UDL	Universal Design for Learning

Chapter One – Introduction

The United States Congress established the National Science Foundation (NSF) in 1950 in order to pursue “the promotion of basic research and education in the mathematical, physical, medical, biological, engineering, and other sciences” (National Science Foundation [NSF], 1994; National Science Board [NSB], 2020). The National Science Board (NSB), the governing body of the NSF, was tasked with establishing national policies and also serving as an independent body to advise Congress and the President on issues relating to science, technology, engineering, and math (STEM) and STEM education (NSF, 1994). Now, seventy years after the creation of the NSF, the NSB has published its Vision 2030 report (NSB, 2020). In addition to emphasizing the role that STEM has played in our nation’s economic prosperity and security, the first page of the report highlights our nation’s increasing need for a STEM-capable workforce for the decade ahead (NSB, 2020). Also highlighted on the first page is the acknowledgement that mathematics scores of students in the United States are far below those of students in many other nations. The report further indicates that inadequate progress has been made in increasing the representation of women and underrepresented minorities in STEM fields (NSB, 2020). It is important to note that the term “underrepresented minorities” is not exclusive to ethnic and racial minority groups. Individuals with disabilities are considered by many to be the most underrepresented of all the minority groups (Waldman et al., 2014; Schroeder, 2015).

In an earlier 2015 report published by the NSB, individuals with disabilities were specifically mentioned as an underrepresented group facing significant barriers to participation in STEM disciplines (NSB, 2015). Based on the board’s assumption of 25 million jobs requiring significant STEM knowledge, the report indicates that addressing inequities and underrepresentation in the US education system should be a top national priority (NSB, 2015).

All students should have access to a post-secondary education with significant STEM engagement to ensure an adequate workforce in our technology driven global economy. The implications of these priorities are clear; those who are cutoff from STEM pathways in the education system will not have access to STEM avenues of employment where job opportunities are increasing and wages are high. Because of this, it is crucial to identify the specific barriers that students face in STEM education in order to address the problem of decreased access to STEM jobs.

Mathematics is considered the gateway to all STEM fields and those who face barriers to mathematics education will have far fewer job opportunities in STEM fields. Moreover, barriers to mathematical learning can have even greater implications beyond STEM both for individuals and society at large. College level mathematics courses are part of every post-secondary core curriculum. Students who are unable to meet their core math requirement are not only barred from jobs requiring skills in STEM, they are cut off from all jobs requiring a college degree. Within and beyond STEM fields, barriers to mathematics education threaten to decrease opportunities for individuals and reduce the overall strength of the US workforce.

Since the initial point of entry to all STEM pathways is math, it is imperative to address the barriers that math creates. A snapshot of statistics related to mathematics achievement gives a hint at the overall scope of the problem. The US government reports data on math achievement from the National Assessment of Education Progress (NAEP) given every two years for 4th and 8th grade students and every four years to high school seniors. A score designation of "NAEP Proficient" is defined as, "…representing solid academic performance for each grade assessed. Students reaching this level have demonstrated competency over challenging subject matter, including subject-matter knowledge, application of such knowledge to real-world situations, and analytical skills appropriate to the subject matter" (National Assessment of Educational Progress [NAEP], 2019b). In other words, math achievement at or above the level of "NAEP Proficient" would likely indicate that students are capable of successfully navigating STEM pathways. In 2019, 41% of 4th grade students and 34% of 8th grade students received scores that were at or above the level of "NAEP Proficient" (NAEP, 2019a). The last NAEP mathematics data published for 12th grade students comes from 2015 and data indicates that 25% of high school seniors scored at or above the level of "NAEP Proficient" (NAEP, n.d.). Based on these achievement results, many students headed to college are ill-prepared to succeed in college level math. In a 2014 press release titled "Regarding the M in STEM" the NSF calls attention to the poor preparation that US students have in mathematics and the barrier it establishes for college completion and participation in STEM, especially for underrepresented groups. The press release states, "When incoming community college students are tested for their mastery of math, 60-70 percent of them are assigned to developmental mathematics courses" (NSF, 2014). Students taking developmental math do not earn college credit; however, successful completion of developmental courses provides an alternative route for students with low math achievement scores to eventually access college level math. If students are successful in developmental math, they will most likely take college algebra, a course that is generally understood to have a failure rate that hovers around 50 percent (Gordon, 2008). As discouraging as these statistics may be, the situation is even more profound for students with disabilities. Studies evaluating math achievement for students with disabilities indicate that they score well below their peers in the general population on math achievement assessments (Blackorby et al., 2003; Wei et al., 2013). Without improving overall achievement in mathematics, it is impossible to reach the NSB's stated goal of providing all US students, including students with disabilities, access to high quality education with significant STEM engagement.

Identifying and implementing interventions that will significantly reduce barriers to mathematics education for all students, including students with disabilities, requires a comprehensive, long-term effort from a broad spectrum of stakeholders. Current best practices in the creation of academic interventions encourage both individualization as well as curricular development from a Universal Design for Learning (UDL) perspective (CAST, 2018). The goal of UDL is the creation of inclusive interventions and curricula capable of meeting the needs of as many learners as possible. Using a normal distribution for reference, interventions targeting the majority of students found within one standard deviation of the mean score on a mathematics assessment would benefit 68% of students. Based on 2019’s total US enrollment of 76.5 million students (pre-K through college) this approach would exclude over 24 million students from the group targeted for improved math interventions (National Center for Education Statistics [NCES], 2019). Since students with disabilities generally score below their peers in mathematics assessments, it is reasonable to assume that many of them would fall within the group of 24 million students excluded from the intervention. If the intended group was broadened to those falling within two standard deviations of the mean, 95% of students would benefit leaving almost 4 million excluded. Since US education policy, specifically No Child Left Behind (NCLB) and its updated Every Student Succeeds Act (ESSA), allows states to provide alternative below grade level assessments to no more than 1% of the student population, then a goal consistent with that mandate would be the development of interventions that are designed to meet the needs of 99% of the student population (Shaul, 2005).

Statement of the Problem

Designing mathematics interventions to improve education for a simple majority of students is, in and of itself, an enormous challenge. Designing interventions that are inclusive and effective for all students, including those whose learning may be impacted by one or more categories of disability, is truly a formidable task. Currently, and for several decades past, providing appropriate access to education for students with disabilities has included the development of an Individualized Education Program (IEP). IEPs frequently call for existing curricula to be modified in order to meet the accessibility needs of students with disabilities on an individual basis. When technology has been involved, modifications have often required special adaptive equipment added on to standard technology to make it behave differently than its developers intended. Accessibility that is tacked on to an existing piece of technology as an afterthought rarely provides students with disabilities with the same quality educational experience as their peers. In addition, the adaptive equipment that is added on may cause students with disabilities to feel different from their peers and consequently engage less with their technology. Over time it has become clear that accessibility features work best when they are built into technology from the ground up. The UDL approach focusses on establishing a comprehensive understanding of the learning needs and challenges experienced by all students before technology development begins. The goal is that accessibility features will be built in from the start so that students with disabilities can use the same technology as their peers and experience the same engaging educational experiences. For curricula that does not involve technology, the UDL goals are similar – the development of curricula and lesson plans with sufficient flexibility built in to meet the specific learning needs of all students (CAST, 2018). A User Centered Design (UCD) approach to building learning support tools complements the objectives of UDL in its requirement that the target users be involved in all phases of planning and development, thus ensuring the final product is usable and efficient (Norman, 2013).

Paradoxically, the UDL goal of synthesizing the full range of student learning into universal curricula and educational technology cannot take place without a significant investment of time and research aimed at fully understanding the unique needs of individual learners, especially those with disabilities. The work involved in individualization through an IEP and the work involved in creating curricula from a UDL perspective should not be thought of as antithetical. Instead, both should be employed as part of a comprehensive, sequential strategy for the ultimate development of robust and effective UDL curricula, technology, and learning support tools.

Purpose of the Study

The purpose of this study is to capitalize on the development of Process-Driven Math (PDM), a method of math instruction and assessment initially designed to support the learning of one student with disabilities. Many students are capable of succeeding in mathematics, but the tools they need to succeed are not available to them. It is possible that tools developed for one student, or one group of students, could potentially help improve learning outcomes for many other students with and without disabilities. This study examines PDM’s potential to support the learning of a broader cross section of students both with and without disabilities. This research is one part of a comprehensive, iterative process in the development of tools to support mathematical learning within a UDL framework.

Process-Driven Math was developed to meet the needs of a college student who is blind, has fine motor deficits that prevent writing or typing (so he is unable to use braille), and is unable to speak above a whisper. At the time this student matriculated, he had the intellectual capacity to succeed in college level mathematics, but not the tools he needed to demonstrate his abilities. Through a user centered design process, PDM, a fully audio method of math instruction and assessment was developed. PDM uses an individual who functions as both a reader and a scribe to read the math aloud and then accurately record all of the transformations the student directs during the simplification process.

Algebraic expressions can be very complex, some containing well over 100 numbers, variables, and operational symbols. It is impossible for students listening to an audio rendering of this type of expression to remember all of the component parts, much less how they relate to one another (Cowan, 2010; Miller, 1956). Working memory is overwhelmed by the reading of the expression as well as the processes required to make decisions about its simplification. Process-Driven Math uses chunking and the incorporation of discrete and appropriate mathematics vocabulary to communicate complex mathematical expressions without overloading a student’s working memory. PDM provides an accurate rendering of mathematical expressions by chunking it into its largest structural pieces using appropriate math vocabulary. This provides the student with the overall landscape of the problem without overloading working memory. The student is then put in control of the process of requesting more detailed information about specific substructures within the expression. For example, an expression containing more than 100 numbers and symbols might be reduced to the accurate audio rendering: “rational 1 divided by rational 2.” From this chunked audio rendering of the expression, the student understands the broader landscape of the problem and can begin formulating a plan for simplification. The student didn’t actually have to hear any of the numbers, variables, or operation symbols to begin the intellectual work required for simplifying the expression.

A rational is comprised of two substructures, a numerator and a denominator. The student who hears “rational 1 divided by rational 2” might ask to hear what is in the numerator of rational 1. If the numerator contains more numbers and symbols than working memory can support, then the numerator is also chunked into its component substructures. An accurate audio rendering of the numerator of the first rational might be, “factor 1 times factor 2 times factor 3.” The student continues to control the process by choosing to either hear more about each of the three factors in the numerator of rational 1, or asking to hear what is in the other numerator or one of the denominators.

At any point the student may decide to use the same discrete math vocabulary to direct transformations in any one of the substructures of the expression. The transformations are recorded by the scribe and the expression is re-authored, incorporating the student’s transformations into a new and accurate rending of the partially simplified or modified expression. This iterative process is repeated until the student is satisfied that the expression is fully simplified. The scribe’s accurate recording of all transformations allows the student to request a reading of all transformations made to check work and confirm the final answer.

The student for whom the PDM method was developed was able to successfully complete developmental algebra, pre-calculus with trigonometry, and research statistics. Reducing barriers to mathematics through the development of tools that matched his abilities removed significant educational roadblocks. He graduated from college in four years and is currently enrolled in a master’s degree program.

A second student who had a visual impairment and was unable to read braille also used the PDM method to access the content of her college mathematics course. Although the student had low vision tools available to her, she chose to work with the PDM method because the audio chunking gave her a more solid grasp of the expressions than enlarged text. The degree of enlargement the student required would not allow her to take in complex expressions with one look. She found her working memory overloaded as she explored the enlarged text in pieces, trying to synthesize the landscape of complex expressions by memorizing the component parts. The audio chunked PDM rendering provided the complete landscape of each problem and gave the student greater control over the simplification processes without overloading her working memory. A UCD approach was employed while working with this student and the method was tailored to meet her specific needs. She found she was most successful when using the fully audio PDM method to initially solve a problem and then visually checking the work to confirm she had done the math correctly. This student used PDM to successfully complete her College Algebra course.

A third student needing additional tools to succeed in mathematics had dyslexia, dyscalculia, and dysgraphia. The team who developed PDM thought it possible that this third student, though sighted, might share some areas of difficulty with the two students with visual impairments (SVIs). While the SVIs were overwhelmed with a direct audio rendering of math expressions with many numbers, variables, and operation symbols, the third student was overwhelmed by the visual representation of so many pieces of mathematical information. The team adapted the method to meet the needs of this third sighted student. A set of manipulatives were created, and color and shape were used to reinforce the meaning of the chunks that represented mathematical structures. The numbers, variables, and operation symbols were hidden until the student chose to reveal the contents of each of the component substructures. Transformations could be made in one area of the problem without the student having to see all of the information housed in the other substructures. Again, a UCD approach was taken in developing the method to meet this student's particular needs. He used a combination of both the audio and visually adapted PDM method stating that the audio rendering helped assure him that he was interpreting the math correctly without any reversals or substitutions of numbers, letters, and symbols.

This study is undertaken to evaluate the experiences and outcomes of a large cross section of learners, those with and without disabilities, using PDM. This study is one step in a sequence of anticipated studies aimed at the development of mathematics curricula and technology built within a UDL framework.

The specific purposes of this study are twofold:

- To gain understanding of the mathematical experiences of a broader cross section of secondary SVIs, both braille and non-braille users, with PDM. Fundamental research will be conducted using qualitative case study methodology. A UCD lens will be applied to further refine the fully audio PDM method using insights gained from this qualitative research.
- To evaluate the efficacy of the visually adapted PDM method for students with and without disabilities. The visually adapted method was created to support sighted students needing additional tools to support mathematical learning. An experimental study with random assignment will incorporate PDM and non-PDM lessons into College Algebra courses at three post-secondary institutions. Pre- and post-test data will be used to determine the efficacy of PDM to improve learning outcomes for all students including students who may have one or more disabilities.

Research Questions

Qualitative:

RQ1: *How did Students who are Blind or have Visual Impairments (SBVIs) from three different schools for the blind, both braille and non-braille users, describe their feelings about mathematics?*

a. *What are some experiences that shaped their current feelings about mathematics?*
b. *What elements of classroom instruction do they think are most important in the mathematics education of SBVIs?*

RQ2: *How did SBVIs from three different schools for the blind, both braille and non-braille users, describe their perceptions of the fully audio Process-Driven Math method?*

Experimental:

RQ3: *Are there differences in the performance assessments of post-secondary students, both with and without disabilities, who have been taught topics in algebra either with or without the visually adapted Process-Driven Math method?*

Limitations

1. Post-secondary students were taught topics in algebra through a series of teaching videos, either PDM or Non-PDM, delivered through online courses. Students accessed the videos outside of the structured environment of a classroom and self-report was used to determine whether or not students had fully watched all of the teaching videos. While this introduces a variable outside of the control of the researchers into the study, random assignment mitigates its impact under the assumption that students who didn't fully engage with the content were evenly distributed through the different experimental sections.
2. The presence of disabilities was not determined with documentation indicating a diagnosis by a healthcare or education professional. Self-report instruments were used to identify the likelihood that individuals had one or more disabilities in several categories.

Participants were asked to evaluate the degree of difficulty they experienced with specific ordinary tasks to indicate the possible presence of one or more disabilities. This strategy was employed to mitigate the fact that many people who experience difficulties indicative of disability choose not to be tested and possibly diagnosed with a disability.

3. Despite a large sample size of 687 participants at post-secondary institutions, the number of participants who indicated a difficulty that is associated with one or more categories of disability was low. Some categories of disability did not have enough people to allow statistically significant conclusions to be drawn about participants within those categories.

Significance of the Study

Students with disabilities (SWDs) have fought and won many battles that have given them rights within the education system. The right to a free and appropriate public education which is now guaranteed includes the development of and individual education program. IEP's allow for accommodations or modifications of existing curricula to make content accessible to SWDs. In addition, accessibility standards have been established to ensure that technology used in educational contexts meets industry-wide standards. Unfortunately, accessibility often becomes a check box to demonstrate compliance with the law without reference to how usable the accessible technology actually is for the student. Moreover, the implementation of these accommodations, modifications, and accessibility standards can often result in the exclusion of SWDs from mainstream environments with their peers when they are removed from the classroom for specialized instruction.

The next frontier in the battle to improve the educational experiences of SWDs is inclusion through the development of curricula and technologies built within a UDL framework. When curricula and technology are constructed from the outset with the needs of all learners accounted for, greater equity in education is achieved. The development of mathematical tools to improve learning outcomes for all learners, including learners with disabilities, would have significant positive impacts in three key areas. First, the successful development of a UDL curricula that improves mathematics education in one course would open the door for transferability to other courses and potentially other math related disciplines. Second, the development of learning supports to improve outcomes in math for SWDs is likely to increase the participation of SWDs in mathematics and other STEM disciplines. Finally, increasing the representation of SWDs in STEM disciplines would lead to a more diverse STEM workforce which would be a significant step toward achieving some of the goals outlined in the National Science Board's Vision 2030 report for our nation.

Conclusion

For much of the 20th century, students with disabilities faced discrimination. They have been marginalized, treated as scapegoats for underperforming schools, and resented because of additional costs incurred to provide for their educational needs. Progress in the area of civil rights has ushered in significant positive changes, yet SWDs are often still marginalized, standing on the periphery of educational environments that have not been constructed for their inclusion. Many who continue to fight for the rights of students with disabilities want societal change in the perception of disability. Advocates believe that the human condition is lived along a continuum of disability and that all people are likely to experience one or more disabling conditions for at least some portion of their lifespan.

As the stigma of disability has decreased, educators and researchers have come to realize that some of the struggles experienced by SWDs are amplifications of the struggles experienced by many other learners. Understanding how to reduce barriers for SWDs may lead to a deeper understanding of how to reduce barriers faced by many learners, both with and without disabilities. A quote from a 1997 article from the journal *The Mathematics Teacher* speaks to this idea with the following statement: “You may find that the efforts made in helping visually impaired students in learning mathematics can shed new light and insights on difficulties shared by *all* students” (Dick and Kubiak, 1997). This current study was designed to determine whether or not Process-Driven Math, a method developed to support the learning of one student who is blind, could be adapted within a user centered design and universal design for learning framework to improve learning outcomes in mathematics for many diverse learners.

Chapter Two – Review of Literature

Introduction

The provision of a free and appropriate public education (FAPE) is a right guaranteed to all Students with Disabilities (SWDs) in the United States (Zukauskas, 2019). However, FAPE is a moving target, continually debated and reinterpreted as political and educational climates change (Conn, 2017; Johnson, 2003). The history of American public education is punctuated with legal and legislative battles fought to secure improved access to education for SWDs. As people with disabilities joined the civil rights movement, expectations for the experiences of SWDs in school changed, and therefore the view of what FAPE should look like changed also. Some of the most significant victories for SWDs included the guarantee of an IEP and appropriate accommodations to facilitate each student's advancement through the education system. Additional improvements in technology, especially personal computing and mobile technology, also brought unprecedented potential for improved access to educational materials for SWDs. The influence of the Universal Design for Learning (UDL) and User-Centered Design (UCD) movements has also been significant, affecting a shift in the expectations set for the educational experiences of SWDs (Bordac & Rainwater, 2008; Griful-Freixenet et al., 2017). With UDL's rise in prominence, accessibility became a steppingstone toward the broader paradigm of inclusion. Inclusion is a construct that places emphasis on both access to content as well as the benefits of equitably shared educational environments (Meyer & Gordon, 2014). A more in-depth review of the history, advocacy movements, and technological advances for SWDs provides important context for researchers seeking to promote equity in education through the development of educational tools that are both inclusive and evidence-based.

History of Education in the U.S. for Students with Disabilities

In the 2017-2018 school year, 7 million students (14% of all students), were provided with special education services (NCES, 2019). Over one hundred years earlier, during the formative years of our nation's public school systems, students with disabilities were largely excluded from public education (Zukauskas, 2019). Courts in multiple states ruled against SWDs with the argument that their presence in schools had a detrimental effect on the educational experiences of their non-disabled counterparts (Yell et al., 1998).

In 1910, the White House Conference on Children was held, in part, to address the educational needs of SWDs. This conference provided guidance encouraging states to create dedicated classrooms for the education of SWDs in public schools, rather than institutionalizing them (Yell et al., 1998). Due to the influence of the conference, less SWDs were institutionalized; however, the conditions of the self-contained classrooms for these students were often not any more humane than the conditions students had experienced when placed in institutions (Yell et al., 1998).

Substantial leaps forward in the cause of educational rights for SWDs came as a result of the civil rights movement in the United States. In 1954, the Supreme Court ruling on *Brown v. Board of Education* stated that school segregation resulted in an inferior education for African American children as compared to their white counterparts. Segregated schools were found to be in violation of the 14th amendment, which guaranteed equal protection for all U.S. citizens (Ogbonna-Mcgruder et al., 2011). This ruling provided the precedence needed to argue and win landmark court cases for greater inclusion of SWDs within the American public school system (Yell et al., 1998).

In 1965 the Elementary and Secondary Education Act (ESEA) was signed into law to provide federal grant money to help states address the disparities faced by economically underprivileged students in public schools (Casalaspi, 2017; Le Floch et al., 2018). Within a year, the act was amended and additional grant money was given to states to provide for the education of SWDs (Warnes, 2019). The increased priority given to the education of SWDs was evident in the succession of legislation that followed. In 1970, grant money originally allocated through ESEA was reallocated to a new program signed into law titled the Education for the Handicapped Act (Yell et al., 1998). In 1973, Section 504 of the Rehabilitation Act was established which protected people with disabilities from discrimination by any agency receiving federal funds, including both K-12 and postsecondary institutions (Office for Civil Rights [OCR], 2010).

While momentum for the educational rights of SWDs was increasing through policy changes, they continued to face exclusion and discrimination in schools, especially in the area of assessment. In 1969, the United States began the National Assessment of Education Progress (NAEP) to provide a basis for comparing student achievement across states (Shaul, 2005). Despite the new laws directed at improving education for SWDs, it would be almost 30 years before accommodations would be made available for SWDS to participate in the NAEP assessments (Shaul, 2005).

In 1975, the Education for All Handicapped Children Act (EAHCA) was passed to guarantee SWDs an education that was appropriate for the needs of each individual (Leafstedt et al., 2007; Yell et al., 1998). For the first time, testing was specifically addressed within legislation by guaranteeing SWDs the right to non-discriminatory testing (Yell et al., 1998). Of significance was the stipulation that no single test could be used to make decisions of grade placement (Johnson & Kowalski, 1976). Tests were to ensure that appropriate placement and services were rendered, as opposed to using them to stigmatize or discriminate against SWDs

(Johnson & Kowalski, 1976). The EAHCA also mandated that each child with a disability have an IEP, a change that would have a significant impact on the provision of accommodations for decades to come (Johnson & Kowalski, 1976; Ulric, 2014). The rights of SWDs were further validated with the 1986 amendment to EAHCA which gave attorney's fees to parents who had won court battles against schools that had denied their children appropriate educational opportunities guaranteed by the law (Yell et al., 1998).

The increased attention given to issues related to the education of SWDs through successive policy changes began to have a significant impact on research related to access and accommodations for SWDs. In 1986, Educational Testing Services (ETS) produced two separate reports that focused on the SAT and GRE tests given to SWDs. ETS's decision to undertake these studies was directly influenced by educational policy as stated in the preface of each report. "Regulations under Section 504 of the Rehabilitation Act of 1973 impose new requirements on institutional users, and indirectly on admissions test sponsors and developers, in order to protect the rights of handicapped persons" (Braun et al., 1986a, p. i; Braun et al., 1986b, p. i). The reports acknowledged that accommodations for "handicapped examinees" had been implemented without fidelity across contexts and that little research existed to evaluate modifications (Braun et al., 1986a, 1986b).

Disability Rights and Advocacy Movements

These and other educational research studies focusing on issues related to disability were taking place against the backdrop of a larger conversation about disability theories being debated around the world. The two opposing theories were the medical impairment model and the social oppression model (Smith-Chandler & Swart, 2014). Under the medical impairment model, disability was understood as a condition that affected individuals and required medical care

(Shakespeare, 2013). The social oppression model put forth the idea that disability only existed in the context of a society that promoted oppression by erecting barriers that prevented the inclusion of people with disabilities into all aspects of life and culture (Shakespeare, 2013).

In essence, these two theories ran parallel to the different approaches used to find solutions to improve education for SWDs. One approach was to either modify curriculum or find targeted technologies to help SWDs overcome their specific disabilities on an individual basis. These types of strategies are commonly found in IEPs developed to meet the needs of each student on a case-by-case basis. The alternate approach was to produce curricula with technologies built in so that as many students as possible could have access without having to modify the curricula. This latter approach reflects the growth of both the UDL and UCD movements which would usher in a shift in both public policy and best practices across all levels of education.

Universal Design for Learning.

The universal design movement began almost fifty years ago with a man named Ronald Mace. Mace was an architect with disabilities who believed that creating environments to improve safety and functionality for people with disabilities would improve the functionality of those spaces for all people (Mcguire et al., 2006). The framework was broadly adapted and applied to the development of communication tools and other technologies that improve life for all people, including those with disabilities, without the need for specialized equipment. The Center for Applied Special Technology (CAST) adapted Universal design concepts to education and has been on the vanguard of the UDL movement for over 30 years (CAST, 2018). Originally, the research at CAST focused on assessing technology to support the learning of individuals with special needs. Over time, CAST changed its approach, placing research emphasis on developing curricula capable of meeting the needs of all students rather than finding technology fixes to make existing curricula accessible to individual SWDs. This shift in focus laid the foundation for the future development of the three UDL principles (multiple means of engagement, multiple means of representation, and multiple means of action and expression) along with their 32 checkpoints that would have a profound impact on how educators and policy makers sought solutions to the educational barriers faced by SWDs (CAST, 2018).

User-Centered Design.

The UCD movement emerged concurrently with universal design and is built on the idea that the wants and preferences of end-users should drive the design process for new technologies (Norman, 1986; Norman, 2013). UCD finds its theoretical basis in Vysgotsky’s Active Theory, a meta-theory that emphasizes the importance of the social contexts and environmental interactions that influence the usability of a tool or learning aid (Kim, 2010). Donald Norman is credited with establishing the tenets of UCD and authoring works that brought public awareness to the benefits of a design process that included input from end-users throughout development (Norman, 1986; Norman 2013). UCD gained momentum within the disability rights movement because emerging “accessible” technologies intended to support SWDs often did not meet their needs. Too often “accessible” products were developed end to end without any input from people who had the disability for whom the technology was created (Bateman et al., 2018; Huang & Chiu, 2016). Jakob Nielsen (2005), a researcher and business partner of Donald Norman, stated, “The bigger point here, however, concerns a fallacy: the assumption that accessibility exists in a vacuum and can be scored without considering users and their tasks.” The adaptation of UCD to classroom and curricular contexts is sometimes referred to as Human-Centered Design because the learning environments need to account for the needs and preferences of both students and teachers (Shivers-McNair et al., 2018).

Inclusion: Policy and Reality

As disability rights, UDL, and UCD became growing forces of societal change, the Individuals with Disabilities Education Act (IDEA) of 1990, a renaming of the EAHCA, was passed. It ushered in a new era in education affording greater legal power and respect through language for people with disabilities (Russo, 2019; Warnes, 2019). IDEA mandated that all states serve SWDs through local educational agencies. Central to the law was the requirement for SWDs to be identified by the schools and have their IEP created through a multidisciplinary team of educators that included the student's parents (Russo, 2019). The legislation stipulated that students needing special education should receive services in the least restrictive environment possible with full inclusion as the targeted goal (Russo, 2019). IDEA's language incorporated both of the parallel approaches for improving educational outcomes for SWDs. Individualized plans that often included modifications to existing curricula were foundational to the IEP requirements of IDEA. However, IDEA also stipulated that education for SWDs take place in the least restrictive environment possible, language that aligned the legislation with UDL philosophy.

IDEA: Inclusion in Statewide Assessments.

In 1997, an amendment to IDEA required for the first time that the inclusion of SWDs in district and statewide assessments was mandatory (Browder et al., 2005). The amendment stated that decisions involving the use of accommodations and alternate assessments would be made by the student's IEP team (Shaul, 2005). This legislation ensured that students for whom accommodations were not sufficient would be given alternative assessments. Prior to the implementation of this legislation, many states did not have alternate assessments for students with significant cognitive disabilities. The alternate assessments developed in response to this legislation were often functional assessments of basic independent living skills rather than academic assessments linked to the general education curricula (Shaul, 2005). The provisions of this amendment to IDEA were indicative of the momentum that was driving the educational standards for SWDs beyond accessibility toward a higher goal of inclusion.

NCLB: Unintended Consequences for SWDs.

In 2001, the sweeping educational reform legislation titled No Child Left Behind (NCLB) was voted into law. The changes in policy as they related to assessments and SWDs were significant. SWDs were required to participate in the system of accountability through mandatory statewide assessments (Cole, 2006). The test scores of SWDs were reported as a subgroup, and the progress of SWDs was factored into each school's Annual Yearly Progress (AYP) grade (Diorio, 2019). Schools failing to make their AYP for multiple years were threatened with restructuring or state takeover (Diorio, 2019).

The next fifteen years following the implementation of NCLB is a story of both tragedy and triumph for SWDs. With NCLB, 95% of all SWDs were required to participate in statewide assessments (Browder et al., 2005). While mandatory inclusion of SWDs appeared to be a step forward, unintended adverse consequences were seen within the first few years of NCLB's implementation. Due to the high stakes nature of these assessments, SWDs often became scapegoats in schools and districts where test scores lagged and the AYP was not met (Browder et al., 2005). The practice of educational triage was also reported to have an adverse effect on SWDs. Schools trying to ensure their AYP prioritized the education of students who had the potential for greater score increases, and SWDs were given a much lower priority because they were less likely to make significant gains to support the school's AYP (Browder et al., 2005). Some have even suggested that an increase in the drop-out rate for SWDs could have been caused by educators who tacitly encouraged them to drop out in order to maintain the school's AYP (Browder et al., 2005).

NAEP: Unethical Exclusionary Practices for SWDs.

History now makes it clear that pressure for schools to meet their AYP led to unethical practices in the administration of high stakes tests. These unethical practices paradoxically caused greater exclusion for SWDs despite the NCLB's stated intention of increasing inclusion for SWDs. NCLB's mandatory score reporting for SWDs as a standalone subgroup shed light on how states manipulated sores on the NAEP. NCLB made states' participation in the NAEP mandatory and the NAEP then became subject to the regulations for subgroup reporting required in NCLB. Due to the NAEP's requirements for standardization across states, some accommodations normally available to SWDs on other statewide assessments were unavailable to them for the NAEP. States may have used this as a justification to manipulate their exclusion rates of SWDs in an attempt to bolster their national standing on the assessment; NAEP exclusion rates for SWDs between states generally ranged between two and ten percent (Shaul, 2005). A close examination of the 2003-2004 NAEP revealed that Texas required over 60% of its SWDs to take an alternative assessment that was scored with below grade level standards (Shaul, 2005). Texas was in violation of NCLB because its number of alternative test takers far exceeded the 1% limit established by NCLB (Shaul, 2005).

Alignment of Policy and UDL

The continued irregularity in the assessment of SWDs eventually led to legislative changes to address the problems. NCLB 2004 inspired a national conversation that increased research on universal design and testing accommodations for SWDs (Lazarus & Thurlow, 2009). By this time, CAST had established its three principles for UDL which stated that students should be provided with multiple means of engagement, multiple means of representation, and multiple means of action and expression (Orkwis & McLane, 1998). The promotion of these ideas led to the incorporation of UDL principles into the updated legislation. Both NCLB and IDEA were rewritten and an emphasis was placed on the development of large scale assessments created from a universal design framework (Lazar & Briggs, 2015).

After the 2004 NCLB update, the literature reflects increased attention to universally designed assessments to promote the inclusion of SWDs. In a 2006 report titled “A State Guide to the Development of Universally Designed Assessments” the authors cited six key universal design considerations that would promote standard testing conditions. These included, “Intended constructs are measured, respect for the diversity of the assessment population, concise and readable text, clear format for test, clear visuals, and changes allowed to format without changing meaning or difficulty” (Johnstone et.al., 2006, p. 1). Another report from the same year suggested the use of think aloud cognitive labs to evaluate test items for SWDs and English language learners (Johnstone et al., 2006). A 2009 article titled “Modifying Achievement Test Items: A Theory-Guided and Data-Based Approach for Better Measurement of What Students with Disabilities Know” described a UDL approach for the development of alternative assessments and test items (Kettler et al., 2009). While research played a role in improving accessibility and inclusion for SWDs on high stakes assessments, the economic crisis of 2009 led to an unexpected and unprecedented acceleration of the pace at which inclusive high stakes tests with robust accessibility features were developed and deployed.

Common Core and UDL Assessments.

When the great recession hit in 2009, congress acted quickly to extricate the country from the crisis by passing the American Recovery and Reinvestment Act. It contained $4.3 billion dollars of funding for competitive education grants (National Science Teacher's Association [NSTA], 2009). A goal of the competition was to align academic competencies across all states and encourage the adoption of common core standards (National Conference of State Legislatures [NCLS], 2014). An additional $330 million was funded for a separate competitive process to develop high stakes tests that would assess the common core standards (NCLS, 2014). Two multi-state consortia were awarded grants to develop the tests. The Partnership for Assessment of Readiness for College and Careers (PARCC) was awarded $186 million and the Smarter Balanced consortium was awarded $176 million (NCLS, 2014). Four smaller grants were awarded to other consortia for the development of alternative assessments intended for SWDs who were unable to take the high stakes tests developed by PARCC and Smarter Balanced (NCLS, 2014).

PARCC and Smarter Balanced: Rapid Progress in a UDL Framework.

The tests developed by both PARCC and Smarter Balanced leveraged the advances made in recent decades in the areas of assistive technology and universal design. CAST provided specific and comprehensive input to both PARCC and Smarter Balanced during their development phases providing guidance on accessibility features, accommodations, and language (CAST, 2013a; CAST, 2013b). Some of the accessibility tools, like highlighting, were made available to all students taking the tests. Other features, like speech to text, referred to as embedded accommodations, could be enabled on an individual basis through the recommendation of the IEP team (Education Testing Services [ETS], 2018; Bowman et al., 2017). For students with very unique technology needs, additional hardware connected via USB port could also be implemented with a recommendation of the student's IEP team (ETS, 2018; Bowman et al., 2017). Each platform had a robust set of on-board accessibility tools built within a UDL framework that made it possible for many SWDs to take these high stakes tests in the same environment as their peers (ETS, 2018; Bowman et al., 2017). This was a significant step forward in the struggle for inclusion in assessments for SWDs.

In 2013, when the common core tests were first piloted, a backlash against common core resulted and states began withdrawing from their consortia in significant numbers each year (National Center for Learning Disabilities [NCLD], 2018). Within just a few years, the PARCC consortia was fully disbanded, and the Smarter Balanced consortia was reduced to less than half of its original size (Jochim & McGuinn, 2016). Anger over the common core and the failure of NCLB to reach its promised goals of educational proficiency of all students led to the drafting of new legislation.

The Collapse of Common Core: A Step Backward in Inclusion.

In 2015, the Every Student Succeeds Act (ESSA) was passed. While the mandates for statewide testing and reporting established with NCLB remained in place, the prescriptive requirements for annual yearly progress and the corresponding punishments for failure to reach benchmarks were largely eliminated (U.S. Department of Education, n.d.). The rule establishing a maximum limit of 1% of students eligible to take alternative assessments remained in place, and alternative assessments were still required to align with states' content standards (Education and Secondary Education Act [ESEA] Network, 2019). ESSA's overall shift of authority from the federal government to individual states jeopardized some of the federally mandated provisions that were significant for advancing the educational cause of SWDs under NCLB. While NCLB and IDEA were intentionally aligned when rewritten in 2004, the alignment between ESSA and IDEA is less clear and advocates for SWDs were concerned that recent advances would not be maintained (NCLD, 2018). Of great concern was maintaining the forward momentum for universally designed tests with robust accessibility features (Individuals with Disabilities Education Improvement Act, 2004; Peters & Araya, 2011).

Since the dissolution of the PARCC consortia and the decline of the Smarter Balanced consortia, several states switched to the SAT and ACT to meet their state mandated assessment requirements for high school students (Jochim & McGuinn, 2016). A 2016 comparison of the PARCC, Smarter Balanced, ACT, and SAT showed that only the PARCC and Smarter Balanced tests provided onboard tools, referred to as universal features and designated features, in their assessments (Lazarus & Thurlow, 2016). Many consider it important that the federal government continue to lead with policy initiatives requiring third party test developers to adhere to the highest standards of onboard accessibility tools available within a universally designed framework to support the inclusion of SWDs (Koretz & Barton, 2003; Lazarus & Thurlow, 2016).

UDL and Classroom Concerns

While the adaptation of UDL principles in the development of standardized testing has been largely heralded as highly successful, the adaption of the UDL framework by individual teachers in their curricula is less easy to measure. UDL principles have been broadly promoted in education literature as a means to improve access to education, but there is a concern over the lack of empirical evidence demonstrating the efficacy of implementing UDL principles in classroom curricula for SWDs (Kennedy et al., 2014). In addition, despite the widespread support for the UDL framework, there remains confusion about how classroom teachers should implement UDL principles in their lessons (Lowrey et al., 2017; Ralabate et al., 2012). Guidelines for the UDL framework were created to encourage quality instruction for the broadest spectrum of learners in an inclusive setting (CAST, 2018). However, government policy also mandates that special education instruction be individualized, as evidenced by the requirement for SWDs to have an IEP (Kennedy et al., 2014; Yell & Shriner, 1997). From a UDL perspective, if the needs of all learners are taken into account from the inception of lesson preparation, then SWDs need not be excluded from a rich educational experience within the same environment as their peers (CAST, 2018). However, some researchers and educators caution that the move to full inclusion should not take place until evidence is clear that instructional practices will adequately meet the learning needs of all students, especially special education students (Mcguire et al., 2006). Educational researchers seeking to develop interventions to reduce barriers for SWDs must find unique approaches that will meet the needs of many diverse learners and be proven effective across the entire spectrum of learners through empirical testing.

A UDL Approach: Mathematics and Students with Visual Impairments (SVIs)

To develop evidence-based UDL curricula, researchers must strive to understand the barriers facing the entire cross section of students with particular attention given to those who face the greatest barriers. Mathematics is a subject that creates barriers for many learners and Duval (2006) offers an explanation for these difficulties:

> Mathematics is the domain within which we find the largest range of semiotic representation systems, both those common to any kind of thinking such as natural language and those specific to mathematics such as algebraic and formal notations. And that emphasizes the crucial problem of mathematics comprehension for learners (p. 108).

If mathematics creates barriers for students in the general population, it stands to reason that it may be especially difficult for specific populations of students, particularly students with visual impairments (SVIs). The most current data available on mathematics achievement for SVIs comes from the U.S. Department of Education's *National Longitudinal Transition Study-2* (*NLTS2*). According to the study, SVIs were well behind their sighted peers in math achievement. Seventy-five percent of SVIs were more than a full grade level behind sighted peers in math, and twenty percent were five or more grade levels behind their sighted peers (Blackorby et. al., 2003).

SVIs: History and Assistive Technologies

A broad overview of the unique education history and issues facing SVIs, both braille and non-braille users, provides important background for educators and researchers hoping to make progress in developing inclusive, evidence-based mathematics curricula. Over the last few decades, various educational assistive technologies have been developed to address the different types of barriers that SVIs face in the classroom (Rosenblum et al., 2019). The availability of assistive technologies for SVIs has been wide-ranging and rapidly increasing (Kelly & Smith, 2011). Current assistive technologies for SVIs range from low- to high-tech and include supports such as enlarged print materials, magnifying screens to enlarge books and content on a board at the front of the classroom, mobile devices that enlarge font sizes, Perkins braillers, refreshable braille devices, refreshable tactile graphing boards, e-readers, and speech-to-text technologies to name a few (DePountis et al., 2015; Presley & D'Andrea, 2009). These assistive technologies are used by students across the spectrum of visual impairment at both schools for the blind and in mainstream classrooms to help overcome barriers and improve access to educational content (Presley & D'Andrea, 2009). The assistive technologies that support the learning of SVIs who are non-braille users are especially important because they provide these students with alternative means, other than braille, to engage with written content.

Braille literacy, which includes both literary and math braille, has long been considered the gold standard for academic achievement and successful living for people who are blind or have visual impairments (Tobin & Hill, 2015). The number of braille users has decreased significantly over the last several decades (Schroeder, 1989; Johnson, 1996; Silverman & Bell, 2018). A 2019 report from American Printing House for the Blind indicates that less than 8% of SVIs are braille readers (American Printing House [APH], 2019). Many attribute the decrease in braille literacy to recent trends in education that favor educating SVIs in mainstream classrooms rather than at schools for the blind. Currently, over 80% of SVIs are educated in mainstream classrooms (APH, 2019). SVIs are taught braille in mainstream classrooms by Teachers of the Visually Impaired (TVIs), some of whom are itinerant and serve students in multiple schools. SVIs in mainstream schools lack the daily immersion in braille (both literary and math) that is provided at schools for the blind (Johnson, 1996). The challenges are even greater when addressing adequate access to mathematics braille code for SVIs because TVI training in math braille is inconsistent and inadequate (Amato, 2002; Rosenblum & Amato, 2004). There are concerns from many in the field that the high caseloads of under resourced TVIs assigned to serve multiple schools results in a poor education for SVIs (Forster & Holbrook, 2005; Steele et al., 2006). Advocates for SVIs also raise concerns that the model of the itinerant TVIs to support SVIs in mainstream settings has been implemented over the last several decades without adequate evidence based evaluation of its efficacy (Forster & Holbrook, 2005; Steele et al., 2006). Although issues related to braille literacy are most prominent in mainstream schools, they also exist at the schools for the blind. SVIs may transfer between mainstream schools and schools for the blind across the trajectory of their K-12 education, affecting their proficiency in braille. Students who begin at mainstream schools as enlarged print readers may later transfer to a school for the blind having had no exposure braille; therefore, even at schools for the blind there may be many non-braille users.

Decades have passed since educators first identified the crisis in braille literacy and the ramifications of the crisis are severe (Johnson, 1996; Silverman & Bell, 2018). A 2009 report from the National Federation for the Blind attributes the 50% high school dropout rate, as well as the 70% unemployment rate for people who are blind or have visual impairments, to the braille literacy crisis (National Federation of the Blind [NFB], 2009). As an alternative strategy for literacy, many SVIs have become auditory readers, an option that is increasingly accessible as text-to-speech technologies have improved over the last few decades (Papadopoulos & Koutsoklenis, 2009). SVIs can now access written content through recorded books, screen readers on computers, and mobile devices. While reversing the downward trend for braille use is an important goal, alternate technologies must continue to be made available to give access to written content for the students who have not been trained to use braille code. There are now more SVIs who are auditory readers than braille readers (APH, 2019). However, the auditory delivery of mathematical content with screen readers has been a much greater challenge. Due to the non-linear nature of mathematical expressions, standard screen readers often do not read mathematical content accurately, especially as the complexity of the content increases, and students are unable to navigate or interact with the expressions (Frankel et al., 2016). Over the last several years, audio technologies that read mathematical expressions accurately have been developed. These include MathSpeak™, a software application that reproduces the Nemeth Code (math braille) in audio format, and ClearSpeak, an automated synthetic speech style that replicates the way math is spoken in the classroom (Frankel et al., 2017).

Statistics on mathematics achievement for SVIs do not differentiate between braille versus non-braille users, and assessing the scope of the problem relating to mathematics achievement for all SVIs is difficult due to a lack of empirical evidence (Giesen et al., 2012). However, the link between braille literacy and successful living would suggest that SVIs who are non-braille users would face greater barriers in mathematics than their peers who use braille. Experts in the field state, "Without the ability to read and write the symbols that represent mathematical concepts, the field of mathematics is closed to persons who are visually impaired (that is, are blind or have low vision)" (Kapperman & Sticken, 2003). Research also indicates a crisis in mathematics education for SVIs in Brazil and around the world, noting that gaps in mathematical knowledge between sixth and ninth grade are so significant, that they close the door to future studies in mathematics for many students (Pinho et al., 2016). The need for additional tools to improve mathematical learning for SVIs, especially those who are non-braille users, is clear. Best practices for the development of any new learning support for SVIs, or any SWDs, indicate that the goal is interventions that will be both evidence-based and inclusive. A UCD approach is also considered a best practice for ensuring the practical usability of newly designed interventions.

Process-Driven Math (PDM)

With the goal of improving mathematical learning for SVIs who do not use braille, a research group developed PDM, a fully audio method of math instruction and assessment designed to support SVIs who do not use braille (Gulley et al., 2017). From its inception, the method was developed within a UCD framework; one of the team members who helped develop the method is both blind and unable to use braille. The method relies heavily on “chunking,” a mechanism that reduces the cognitive load on working memory (Cowan, 2010; Gobet et al., 2001). The chunking used in PDM is built around idea that numbers and symbols in each unit of a problem can be represented by appropriate mathematical vocabulary. This simplifies the aural delivery of complex expressions so that a student's working memory is not overloaded. PDM then gives students the ability to interact with elements in the chunked problems one at a time, simplifying them in discrete, user-controlled steps. All of the student-directed transformations are recorded by a scribe and substituted back into the problem. The problem is re-chunked after each transformation and read back to the student so that the new structure of the problem reflects the simplifications the student made up to that point (Gulley et al., 2017). Two students, one who was blind and one who had a visual impairment, used the PDM method to access their college mathematics curricula. Each was successful in the college mathematics courses in which they used PDM (Gulley et al., 2017).

Adapting PDM with a UDL Perspective.

Based on the researchers' understanding that additional groups of students might benefit from the chunking mechanism built into PDM, the method was adapted for visual learners (Phillips et al., 2018). Researchers identified multiple groups of learners who might benefit from the visually adapted PDM method including students with math disabilities, students with math anxiety, and students who have difficulty concentrating (Phillips et al., 2018). Central to the visually adapted PDM method is the use of physical manipulatives to parse mathematical expressions into chunks. The method uses both color and shape to reinforce the mathematical meaning of the individual chunks of math problems (Phillips et al., 2018).

The use of manipulatives to support mathematical learning for individuals with disabilities as a means of explicit instruction is supported by a strong base of research (Avant & Heller, 2011). The Concrete-Representational-Abstract (CRA) method is a sequential instructional approach specifically aimed at reducing barriers faced by students with learning disabilities in mathematics (Bouck & Sprick, 2019; Gibbs et al., 2018; Satsangi, Hammer, & Evmenova, 2018). In the concrete phase, students use physical manipulatives, in the representational phase students make drawings of the physical manipulatives, and in the abstract phase students use standard mathematical notation (Satsangi, Hammer, & Hogan, 2018).

In recent years, a modification of the CRA intervention, CRA-I, has been studied to evaluate the components of the method when they are not used in a strictly sequential order. In this approach, abstract mathematical notation is incorporated into the concrete and representational stages (Satsangi, Hammer, & Hogan, 2018). Studies have shown that the CRA-I sequence can be a valuable tool for teaching mathematical content because the incorporation abstract notation throughout the lesson may assist students in the transition from concrete manipulatives to abstract notation. In addition, the incorporation of abstract notation throughout the teaching sequence may speed the process (Satsangi, Hammer, & Hogan, 2018). Similarities exist between the CRA-I and PDM methods. PDM uses concrete manipulatives with wipe off surfaces and the abstract mathematical notations are written directly onto the concrete manipulatives throughout the simplification process (Phillips et al., 2018).

More recently, another modification to CRA, called VRA, has been explored. The VRA approach employs virtual manipulatives in the place of concrete manipulatives (Bouck & Sprick, 2019). The virtual manipulatives can be controlled by students using a mouse, keyboard, touch pad, or touch screen (Satsangi, Hammer, & Evmenova, 2018). Depending on the individual structures being represented, students may be able to move, rotate, zoom in, or enlarge the virtual manipulatives. Moreover, manipulatives available on digital platforms, especially mobile platforms, may increase accessibility for SWDs. Virtual manipulatives may also be advantageous because they can be used on a device that does not draw additional attention to the user. Some suggest that SWDs may avoid using concrete manipulatives because those physical manipulatives draw additional attention to the student's need for the intervention (Satsangi, Hammer, & Hogan, 2018). The use of virtual manipulatives may help to mitigate student resistance to the use of physical manipulatives because the virtual manipulatives can be used discretely (Satsangi, Hammer, & Hogan, 2018). In two recent studies, the use of a virtual manipulative as a tool for high school SWDs solving multiple step algebraic equations was evaluated (Satsangi, Hammer, & Evmenova, 2018; Satsangi, Hammer, & Hogan, 2018). In both studies, the virtual manipulative was a highly effective tool for improving the ability of students to solve multi-step algebraic equations. The researchers who created the PDM method suggest that future work may include the development of virtual manipulatives built into an inclusive software application to support the mathematical learning of students with and without disabilities (Gulley et al., 2017).

Summary

Understanding how students learn has been an area of academic interest for centuries (Grinder, 1989). Student learning outcomes are influenced heavily by teachers, curricula, assessments, and technology; however, the influence the social environments in which students are situated during instruction and assessment also has a profound influence on student learning as well (Mahn, 1999; Mahn & John-Steiner, 2012). For the vast majority of human history, SWDs have been marginalized and excluded from shared educational environments, and the more profound the disability, the greater the exclusion (Yell et al., 1998). Over the last century, advocacy movements for SWDs have had an enormous impact on the access that SWDs have to our education system. The progression of hard-fought battles that won increased rights for SWDs over the past several decades provides essential context to inform the development of new interventions designed to support the learning needs of SWDs. Today’s best practices are a paradoxical mixture of both legal mandates for individualization and social reforms that call for inclusion. The development of academic tools that are both individualized and inclusive requires a robust research design. Interventions should be developed from a UCD perspective ensuring that supports are usable by the students for whom they are created. Interventions should also be evidence-based, demonstrating that they are effective for the students for whom they are developed. Finally, interventions should be developed within a UDL framework so as many students as possible can participate in shared educational spaces with their peers.

Almost a century ago, Lev Vygotsky, a Russian psychologist, developed his cultural-historical theory of development. Within this framework he identified the importance of social, collaborative environments on the construction of knowledge. Vygostky theorized that the developmental deficits seen in students with disabilities was a result of their removal from the social and cultural environments in which other children were educated (Gredler, 2005; Zimmerman et al., 2003). Forty years later, Albert Bandura developed his social learning theory of self-efficacy. Self-efficacy, a person’s belief about their own ability to achieve, is significantly influenced by students’ evaluation of their own past performance (Bandura, 1977; Crain, 2005). Combined, Vysgotsky’s and Bandura’s social constructivist theories point to two elements in education that are essential for student learning. The first is students’ participation in the social, educational environments of their peers. The second is the provision of proper tools so students have what they need to succeed and develop a positive sense of self-efficacy. These elements are foundational tenets of Universal Design for Learning principles, indicative of the movement’s grounding in two of the most important educational theories developed over the last century.

Chapter Three - Methods

Introduction

An examination of this study's purpose lays the foundation for understanding its significance. The purpose informed the development of three research questions that set the boundaries for the examination of student experiences with mathematical learning. To answer the research questions, two distinct methodological approaches, qualitative and experimental, were employed. A detailed description of the procedural elements and data analysis used in each portion of the study provide the background necessary for an examination of the evidence and a discussion of its implications found in subsequent chapters.

Context

This study is an initial step in a sequence of anticipated studies to help direct the development of mathematics curricula and technology built within a UDL framework. Students with disabilities have long been reported to fall well behind their peers in mathematical learning (Blackorby et al., 2003). Students who are blind or visually impaired (SBVIs) face an array of barriers that make mathematical learning difficult. For some students the main barrier may be a lack of instruction and immersion in math braille (Nemeth Code or UEB Math) (Schroeder, 2015). For others, the degree of enlargement required to see a math problem may cause that problem to exceed the dimensions of the page or digital screen displaying the content. This makes it difficult for the student to interact meaningfully with large and complex math problems. The development of additional tools to support the mathematical learning of SBVIs is important for their educational attainment. Many other students, with and without disabilities, also face barriers to mathematical learning. Identifying elements in methods of instructions and assessment that support the learning of a diverse population of learners, those with and without disabilities, will help promote the development of learning support tools within a Universal Design for Learning framework (CAST, 2018).

The development of PDM, a mathematical learning support tool, began with the creation of a method of instruction and assessment for one student who is blind, unable to write or type, and unable to speak above a whisper. This student had the intellectual capability to learn the content in a college algebra course, but tools did not exist to make the course content accessible for him. Process-Driven Math was developed to provide a means for this student to apprehend large and complex algebraic expressions and execute successive transformations, arriving at final simplifications. Another student with limited vision used the PDM method to improve access to College Algebra course content, and a third student with dyslexia, dyscalculia, and dysgraphia used a visual adaptation of the PDM method for access to College Algebra course content. The experiences of these three students with very diverse learning needs brought to light the possibility of creating PDM tools for mathematical learning within a Universal Design for Learning framework to promote inclusive learning environments. A team of researchers funded by the National Science Foundation began working to investigate PDM and explore its use with diverse student populations. This current study was undertaken as an initial step in a long-term project of creating curricula to support the mathematical education of many diverse learners.

The specific purposes of this study are twofold:

- To gain understanding of the mathematical experiences of a broader cross section of secondary SBVIs, both braille and non-braille users, with PDM. Fundamental research is conducted using qualitative case study methodology. A user centered design (UCD) lens is applied so that findings can be used to further refine the PDM method using insights gained from this qualitative research.

- To evaluate the efficacy of the visually adapted PDM method for students with and without disabilities. The visually adapted method was created to support sighted students needing additional tools to support mathematical learning. An experimental study with random assignment incorporates PDM and non-PDM lessons into college algebra courses at three post-secondary institutions. Pre- and post-test data is used to determine the efficacy of PDM to improve learning outcomes for all students including students who may have disabilities.

Qualitative Research at State Schools for the Blind

To further evaluate PDM with a larger group of SBVIs, I initiated the current study with fellow researchers on this NSF funded project. Due to the low incidence of blindness and visual impairment in students on college campuses, I chose to work with high school algebra and pre-calculus students being educated at secondary schools for the blind. I selected a qualitative design so that the experiences of students with visual disabilities could be used to answer the first and second research questions.

Research Questions

RQ1: *How did SBVIs from three different schools for the blind, both braille and non-braille users, describe their feelings about mathematics?*

  c. *What are some experiences that shaped their current feelings about mathematics?*
  d. *What elements of classroom instruction do they think most important in the mathematics education of SBVIs?*

RQ2: *How did SBVIs from three different schools for the blind, both braille and non-braille users, describe their perceptions of the fully audio Process-Driven Math method?*

Sample and Participant Selection

Case selection occurred at three levels: the schools, the mathematics classes, and the individual participants in those classes. Institutional Review Board (IRB) approval for all research activities associated with this qualitative research was sought and granted from Auburn University at Montgomery. The research settings were bounded at three secondary schools for the blind in the United States. I made the decision to work at state schools for the blind because they provide the greatest opportunity to interact with multiple students with visual impairments in an academic setting. Therefore, the research setting was primarily convenience based. Institutional Review Board (IRB) approval for all research activities conducted at these schools was sought and received. Administrators at each participant institution selected 8th – 12th grade mathematics classes they considered suitable for the research. These decisions were based on the experience level of the instructors as well as the willingness of the instructors to allow the research to be conducted in their classes. Instructors were provided thorough information about the study and signed consent forms in all classes selected for the study. Finally, both students and their parents from these classes were provided thorough information about the proposed study and were given the opportunity to choose to participate or not participate based on their preferences. All documents were provided in accessible formats including braille and enlarged print. Only students who signed assent forms and whose parents/guardians also signed consent forms were included in the research.

The result of the case selection process was the participation of 25 high school students taking pre-algebra, algebra, and pre-calculus courses at three state schools for students who are blind or have visual impairments. We (two other members of the NSF funded research team and myself) conducted the research across three consecutive semesters (Spring 2018, Fall 2018, and Spring 2019). Seven students participated twice during both the Fall 2018 and Spring 2019 semesters. Each student participant agreed to take part in either an in-person focus group or a virtual interview to answer questions about their learning experiences with (PDM). Students who attended the focus groups or interviews were given $50 gift cards as incentives for participation.

Research Design

Throughout the semester, I taught between three and five mathematics lessons to SBVIs using the fully audio PDM method to teach lessons and work through practice problems. Our research team developed open ended research questions designed to help us learn about the experiences of SBVIs, both braille and non-braille users, with mathematics in general and PDM. Focus groups and interviews were conducted by two other members of the research team after I taught the last PDM lesson. Responses to questions about student experiences with mathematics education in general were categorized into positive and negative perceptions. Responses to questions about PDM were coded using a User-Centered Design (UCD) lens which places the needs, wants, and preferences of the end-user at the center of the design process (Dorrington, 2016; Norman, 2013).

We screened the focus group and interview questions with an advisory committee made up of mathematics educators, a special education instructor, and an external project evaluator with expertise in STEM education. We incorporated six questions into the interview. The first question – *What is your favorite subject in school?* – was designed as an ice breaker to get students comfortable in the focus group or interview. I analyzed student responses to the subsequent five questions/statements:

1. *Complete this sentence. Mathematics makes me feel….*

   If I asked you this same question sometime before taking this class would your answer be any different?

2. *Think back to any time before now.*

   (a) What were some good times you have had with mathematics?

   (b) What were some bad times you have had with mathematics?

3. *Think about the math class you're attending right now.*

   (a) What are some things you like about this class?

   (b) What are some things you dislike about this class?

4. *Someone visited your class to teach a lesson using Process-Driven Math.*

   (a) What did you think of Process-Driven Math compared to the way you normally learn math?

   (b) What are some things you like about Process-Driven Math?

   (c) What are some things you dislike about Process-Driven Math?

   (d) What do you think would make Process-Driven Math better, or more useful, to you and your classmates?

5. *Let's suppose you were a group that helped mathematics teachers in your school teach mathematics better.*

   What are some things you would tell them to help teach mathematics better?

Procedure

Prior to the focus groups and interviews, I taught students algebra lessons using Process-Driven Math (PDM) for four or five class periods. Students who chose not to participate in the focus groups or interviews were still required to be present for the PDM math lessons because the lessons were considered a part of their regular class instruction. I taught two or three of the PDM class sessions virtually via Skype or speakerphone, and the last two lessons I taught in person. I worked with the classroom teachers to select topics that were a part of the unit being covered by the class at the time that I taught. The classroom teachers provided me with example problems two to five days prior to the scheduled date of instruction.

Topics students encountered varied due to the differences in course content and lesson plans at each school on the day I taught the lesson. For example, I taught students at one school a lesson using PDM to solve problems with the quadratic formula and taught students at another school a lesson using PDM to solve problems using the point-slope equation. Although lesson topics were different, the PDM method was employed according to established standards and consistency was maintained in the administration of the method (Gulley et al., 2017). Students progressed through problems that I chunked into component substructures using appropriate math vocabulary. I revealed the chunks of the problems gradually as students asked to hear the contents of the substructures in each chunk. Students performed mathematical operations verbally using the same PDM language I modeled for them to precisely communicate the transformations they wanted to make. I wrote down all of the transformations each student described and substituted those transformations back into problem. Then, I spoke the contents of the partially simplified problem again, chunking it into its component substructures with appropriate math vocabulary. That process was repeated as many times as necessary until each problem was fully simplified. Lessons lasted between 35 and 50 minutes.

Two other members of the research team observed the last lesson at each school. These same researchers conducted the interviews and focus groups with the student participants. Student interviews and focus groups were held after the last day of instructions and lasted approximately 75 minutes. The results from all groups are presented together in the analysis. We did not ask specific questions about students’ visual impairments, although some provided that information during the interviews and focus groups as they contextualized their experiences with mathematics.

Data Analysis

Interview data was transcribed using NVivo software and transcripts were de-identified. Names were replaced with randomly generated participant numbers. A member of the research team listened to portions of the original recordings and made corrections in areas of the software transcriptions that were in question. I did an inductive analysis of the focus group and interview data employing a User Centered Design lens. I took this approach so that the data could influence the ongoing development of PDM. Students answered interview questions about their experiences with mathematics in general, their experiences with PDM, their thoughts about improving PDM, and their thoughts about how math could be taught better for students with visual impairments. Students were aware that their feedback was part of an ongoing process to develop tools for mathematical learning for students who are blind or have visual impairments.

Before coding, I read the focus group transcripts at least three times to get an understanding of each group's dynamic and how students reacted to one another's statements. I read the interview transcripts through at least twice before coding. On the subsequent passes through the focus group and interview transcripts I added margin notes summarizing students' positive and negative feelings and perceptions about their experiences with mathematics and PDM. I also noted their thoughts about improving PDM and improving math education in general. Next, I compiled separate documents for each of the students who had participated in focus groups. These individual documents were a mixture of direct quotes and paraphrased summaries of all that the student said in response to both the interview questions and the statements of other students. I compiled these documents for each individual so I could hear the continuity of each individual's story with mathematics and PDM. These individual documents also allowed me to hear with clarity the voices of students who spoke less than others; in the individual documents their contributions were not overshadowed by the responses of students who spoke more. At the end of each individual document I wrote a researcher's note to summarize the student's feelings about math, both positive and negative, and the experiences that were significant in shaping those feelings. I summarized the student's overall perception of PDM and the specific things the student liked and disliked about the method. Student recommendations to improve PDM and math teaching in general were also summarized.

From a User-Centered Design perspective, statements that students made regarding what they liked and disliked about PDM were grouped into categories and related categories were combined into themes. Some of the themes are comprised of categories that can be thought of as opposite sides of the same coin. For example, several students felt that PDM was too slow, so "too slow" became a category. Other students said they liked the slower pace of PDM so "slower pace is good" became another category. These categories were then combined into one theme called "Pacing." A similar process of creating categories and collapsing categories into themes was used for the question about what students would recommend to math teachers to help them improve their teaching.

Limitations

1. Student Variability: We were given access to different mathematics courses at each school ranging from 8th grade pre-algebra to 12th grade pre-calculus. There were many different types of visual disabilities represented in each class along with significant differences in the degree of vision available to each student. Students used a broad assortment of assistive technologies including braillers, enlarged print text, enlarged

digital text, and white boards. A basic premise that undergirds UDL research is that one size does not fit all when it comes to the development of education resources. While the broad range of student abilities, vision, and experiences within classes reinforced that premise, it also presented challenges in giving all students the exposure they needed to PDM to be able to effectively evaluate its potential as a learning support tool.

2. Class Variability: Variabilities between the classes across the three schools was significant. Class sizes ranged between four and ten students. The length of class periods ranged from 35 minutes to 50 minutes between the different schools. Both of these factors significantly impacted the amount of time each student spent interacting aurally with the PDM instructor during the group lessons.
3. Virtual Instruction Variability: Student participants were introduced to the fully audio PDM method in virtual environments. If a computer was available in the classroom, the lesson was taught over Skype. If a computer was not available, a speaker phone was used to teach the lesson. I occasionally experienced difficulties hearing students if they were not near, or in the right direction of, the source’s microphone. Since the method relies fully on verbal communication, the difficulties in hearing created by the technology may have slowed the pace of some of the lessons or made it more difficult for students to remain fully engaged throughout the lesson.

Delimitations

1. Students at the schools for the blind were all exposed to the fully audio PDM method. While the visually adapted method could have been implemented for some of the students with varying degrees of vision, it would not have been possible for one instructor to provide both the fully audio and the visually adapted method to different

students in the same class period. That type of flexibility is envisioned for future UDL support tools that will be built from knowledge generated within this and other studies.

2. Students were exposed to a maximum of three virtual lessons and two in-person lessons. Although additional lessons would have been beneficial in increasing the degree of exposure and comfort students had with PDM, it was important to minimize the disruption to the normal teaching sequences in each class.

Summary

Students with visual impairments are presented with significant challenges in their education, especially when it comes to engaging with content that is highly visual in nature such as mathematics. As technology advances, so does the possibility for the development of new tools to support mathematical learning for SBVIs. However, in order to develop solutions that will be truly effective, researchers should understand how students across the spectrum of vision loss experience new approaches to mathematical learning during the early stages of the development process because it gives SBVIs an opportunity to make significant contributions in shaping a tool being developed. There is great diversity within the community of students who are blind or have visual impairments. SBVIs may be braille users who use Nemeth Code or UEB Mathematics Braille to access their mathematics content, or they may be non-braille users with a completely different set of tools that they use to access math. The development of tools to support their learning cannot be one size fits all. The qualitative research described in this study gives students with a wide range of vision using a broad array of supports a voice in creating solutions to their needs.

In comparison, user feedback collected after development is complete will primarily indicate whether or not an intervention improves outcomes. Problems found at the end of development may be remedied with a post-development add on to improve accessibility. This approach is generally much less effective than prioritizing the incorporation of accessibility options from the ground up. A qualitative study employed toward the beginning of design and development provides insight into why specific approaches may or may not be beneficial to particular groups of students. The incorporation of these perspectives early on in the development process increases the likelihood that the end product will meet the needs of the students for whom it is intended.

Experimental Research at Post-Secondary Schools

Research Question

RQ3: *Are there differences in the performance assessments of post-secondary students, both with and without disabilities, who have been taught topics in algebra either with or without the visually adapted Process-Driven Math method?*

Sample and Participation Selection

Student participants were recruited from developmental algebra, college algebra, and pre-calculus algebra at three post-secondary institutions in Alabama: Institutional Review Board (IRB) approval for all research activities associated with this experimental research was sought and granted by Auburn University at Montgomery. The three schools are located in the southeastern United States and include a large land-grant university with approximately 25,000 students, a regional university campus with approximately 5,000 students, and a community college with approximately 5,000 students. At all three institutions research was conducted with the support of the mathematics department chairs and course instructors. At the land-grant institution, research was conducted in all sections of a developmental algebra course and all sections of a pre-calculus math course. At the regional university campus research was conducted in all sections of a developmental algebra course and all sections of a college algebra course. At the community college research was conducted in two sections of a pre-calculus course. All students in these courses were co-enrolled and randomly assigned to one of three online Functions Supplement courses created by the research team and made available to students through their institution's Learning Management System (LMS). Informed Consent documents were provided to all students through the Functions Supplement courses and a total of 687 students provided digital signatures for consent.

Research Design

Functions Supplement Courses.

Students were randomly assigned to one of the three Functions Supplement courses (A, B, and C). The pre and post assessments for Functions Supplement B were distinct and dissimilar from those of the other two treatment groups. Only Functions Supplements A and C had identical pre and post assessments and those two treatment groups are compared in the statistical analysis of this study. The courses were built using each institution's LMS. Canvas was used at two of the post-secondary institutions and Blackboard was used at the third. Having random assignment within each course section allowed researchers to control for variables such as differences in teaching styles between instructors and differences in student performance based on days of the week and times of the day that classes would meet. The Functions Supplement courses were online companion courses to the students' regular math courses in which they were enrolled at their institutions. Enrollment in the Functions Supplement companion courses was mandatory and counted for 5% of students' grades. This was designed to provide students with an incentive to engage meaningfully with the course materials. Students who did not sign informed consent were still required to complete the assignments, but their data was not collected for research purposes. Assignments were graded by the research team and grades forwarded to the course instructors. Students were given multiple attempts to improve their grades on Functions Supplement assessments to address any ethical dilemma arising from the different teaching methods. Scores for the purpose of the research data were only taken from a student's first attempt. A student's best score from multiple attempts was sent to instructors to be factored into the student's grade for the course.

Functions Supplement A was the control section in which course content was both taught and assessed in a manner that approximated a typical classroom format of math instruction and assessment. Functions Supplement C was an experimental section in which course content was taught using Process-Driven Math and pre and post assessments were identical those given to Group A. Functions Supplement B was an experimental section in which course content was both taught and assessed using Process-Driven Math. This group was not included in the statistical analysis for this study because the group was given alternative pre and post assessments that were not identical to the pre and post assessments given to Groups A and C.

Each of the three Functions Supplement courses contained a survey, a pre-assessment, five teaching modules, and a post-assessment. Course content was comprised of five closely clustered topics related to algebraic functions: evaluation of functions, combination of functions, composition of functions, inverse functions, and even and odd functions. Each teaching module contained a teaching video, an animated PowerPoint example problem, and a homework assessment.

Survey.

All students in each Functions Supplement course took the same survey. The first item in the survey contained Informed Consent documentation and a place for a digital signature indicating consent. If a student did not give consent then the survey ended and no data was collected from any of the assignments the student completed in their Functions Supplement course. If a student gave consent, then the remainder of the items in the survey were made available to the student. In addition to collecting demographic information, the survey contained seven items designed to help identify the presence of difficulties related to different categories of disability. For these seven items, participants were asked to respond to a statement and indicate the usual degree of difficulty experienced with specific actions. The first six items were taken from the NSF Survey of Earned Doctorates (NSF, 2017 p. 13). The seven survey items were:

1. **Seeing** words or letters in ordinary newsprint (with glasses/contact lenses, if you usually wear them)
2. **Hearing** what is normally said in conversation with another person (with hearing aid, if you usually wear one)
3. **Walking** without human or mechanical assistance or using stairs
4. **Lifting** or **carrying** something as heavy as 10 pounds, such as a bag of groceries
5. **Concentrating, remembering, or making decisions** because of a physical, cognitive, or emotional condition
6. **Writing** with a pen or pencil or **typing** on a keyboard or screen without human or mechanical assistance
7. **Reading** words and/or mathematics expressions because of seeing letters, words, or symbols that appear reversed, substituted, omitted, or added

Multiple choice options for each of these items were: None, Slight, Moderate, Severe, Unable to Do. Surveys were de-identified with numbers that were linked to students' performance and homework assessments.

Teaching Videos.

All teaching videos, both experimental and control, were created to have a high degree of fidelity between the two types. As many elements as possible were kept similar between the control and experimental videos. Variables that were controlled for include:

- Format of Instruction: Both types of teaching videos contained an introductory segment that provided an explanation of the topic and a practical application segment to work through an example problem step by step. The same expressions and equations were used as examples in each type of video.
- Scripting: Teaching videos were fully scripted and narrations differed between the two types of videos only in instances where Process-Driven Math language was employed in the experimental videos.
- Pacing: In both types of videos instruction was not rushed. Example problems were executed in a step by step manner without skipping or combining steps in the simplification process. The numbers, variables, and operation symbols were written or placed on the white board at the same time that the narration spoke each of those elements in an expression or equation.
- Narration: The same instructor narrated both the experimental and control videos moderating voice and tone to be as similar as possible in both the experimental and control videos.
- Visual Presentation: The same instructor was used in both video types. All of the action was focused on a whiteboard that was lying flat on a horizontal surface. Only the instructor’s hands and forearms were on camera in both the experimental and control videos.

- Length of Time: Videos were approximately the same length ranging from 6 to 12 minutes depending on the topic. The differences in time between the control videos and their corresponding experimental videos were within 10% of each other for all topics.

The experimental factor that differentiated the two videos is the method of teaching, either Non-PDM Instruction (typical classroom instruction) or Process-Driven Math instruction.

Non-PDM Teaching Videos.

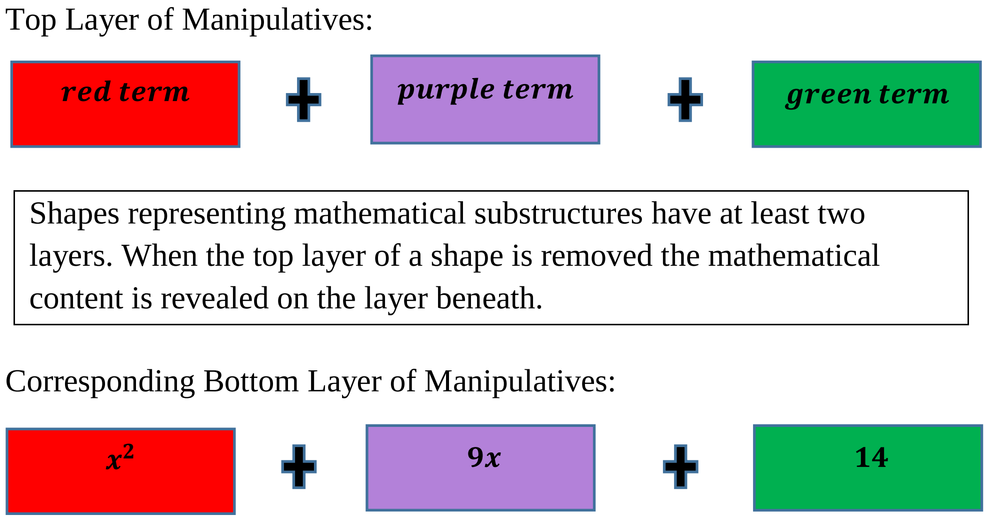
The control videos employed the Non-PDM instruction typical of classroom instruction. These videos were seen by students in Treatment Group A who were enrolled in the Functions Supplement A course. In these videos, all mathematical content was written out neatly and clearly on the whiteboard while the narration was ongoing. After each step, an explanation was given for the next step and then the math expression was re-written incorporating the required transformation. Next steps were written slowly and neatly below the previous line and numbers, variables, and symbols were written at the same time that they were spoken during the narration.

PDM Teaching Videos.

PDM instructional videos used the visually adapted PDM method. These videos were seen by students in Treatment Group C who were enrolled in the Functions Supplement C course. In these videos, all mathematical content was written out neatly and clearly on manipulatives that were applied to the whiteboard while the narration was ongoing. The manipulatives were used to chunk the problem into its component substructures. The chunking was presented through the combined use of shape, color, and appropriate math vocabulary. Manipulatives were created from magnetic wipe off materials that adhered to magnetic whiteboards. The manipulatives were placed on a whiteboard in double layers. The top layer indicated the identity of the chunk as either a term, factor, rational or other mathematical element. The bottom layer contained the numbers, variables, and operation symbols that comprised each individual chunk. The shape of the manipulatives reinforced the identity of the chunk. For example, rectangular shapes represent terms which are elements that are either being added or subtracted. Shapes with curved edges (circles or ovals) represent factors which are elements that are either being multiplied or divided. If an expression was made up of three elements being added together, the PDM narration would be read, "a term plus a term plus a term." Visually this would be represented with three colored rectangles, each with the word "term" written on it with plus signs separating the rectangles. Different colors would be used to emphasize that these were each distinct elements in the expression as seen in Figure 1.

Figure 1

The Use of Color, Shape and Mathematics Vocabulary in PDM Chunking



Shapes representing mathematical substructures have at least two layers. When the top layer of a shape is removed the mathematical content is revealed on the layer beneath.

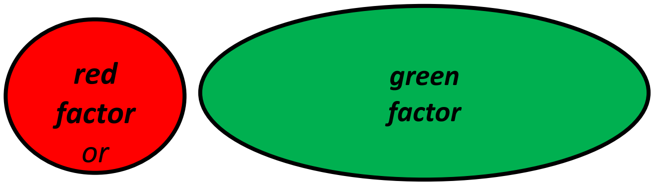
In more complex problems, the top layers of the manipulatives remained in place unless a manipulative was in a chunk of the problem where a simplification was being executed. The content inside a chunk was revealed when the top layer of a manipulative was removed to display the mathematical elements written on the bottom layer. The other chunks of the expression remained covered with their top layers to prevent students from becoming distracted by the numbers, variables, and operation symbols in other parts of the problem. In the PDM teaching videos, next steps were not written below the previous steps. Instead, substitutions and transformations were carried out directly on the bottom layer of the wipe-off manipulative shapes. The mathematical elements written on a manipulative could be erased and replaced to demonstrate the next step taken in solving a problem.

More complex problems may have substructures that are more than one layer deep requiring the manipulatives have additional layers. For example, a factor (oval shaped) could be comprised of two terms being added together. Removing the factor's the top layer would reveal two terms (rectangular shapes) separated by a plus sign. The rectangles would sit on an oval base making it clear that they are components of the oval factor. The rectangular terms might also have top layers that would be removed to reveal their mathematical content. Sometimes transformations change the basic substructures within a chunk of a problem, and this requires a change in the shape used to chunk the math. An example of the sequential revelation of mathematical content in multiple layers followed by a transformation that results in a change of shape can be seen in Figure 2.

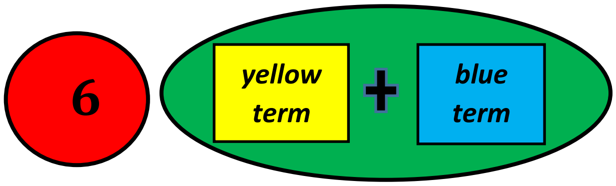
Figure 2

Multiple Layers and Changes of Shape with PDM Chunking

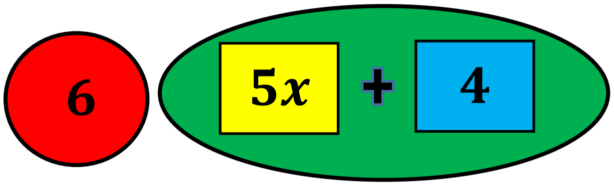
Factors are substructures that are multiplied together and their shapes are circular or oval. Factors placed next to each other are always multiplied; therefore an operation symbol for multiplication is not necessary. In the expression below the red factor is being multiplied by the green factor.



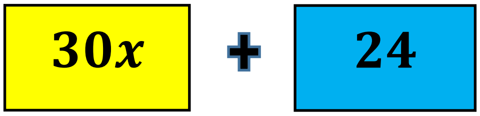
When the top layer of each factor is removed, greater detail of the mathematical content in each factor is revealed. The red factor is the number 6 and has no additional substructures. The green factor is comprised of two terms being added together



When the top layer of the yellow and blue terms is removed the mathematical content in each term is revealed. The yellow term is the number 5 multiplied by the variable *x*. The blue term is the number 4.



The mathematical transformation required to simplify this expression is a distribution. The factor 6 is multiplied by each of the terms that comprise the green factor. The resulting products are added together. Mathematical elements that are added together are terms. In PDM terms are designated with square or rectangular shapes. Therefore this simplification results in two rectangles that contain the products of the distribution.



Animated Example Problems.

All animated PowerPoint videos, both experimental and control, were created to have a high degree of fidelity between the two types. As many elements as possible were kept similar between the control and experimental animated PowerPoint videos. Variables that were controlled for included:

- Format of Instruction: Both types of teaching videos contained an introductory statement indicating the type of example problem that would be solved. The same expressions and equations were used in both types of animated PowerPoint videos.
- Scripting: The animated PowerPoint videos were fully scripted and narrations differed between the two types only in instances where Process-Driven Math language was employed in the experimental section.
- Pacing: In both types of animated PowerPoint videos instruction was not rushed. Example problems were executed in a step by step manner without skipping or combining steps in the simplification process. The words, numbers, variables, and operation symbols appeared on the screen at the same time that the narration spoke each of those elements in an expression or equation.
- Narration: The same instructor narrated both the experimental and control videos moderating voice and tone to be as similar as possible in both the experimental and control animated PowerPoint videos.
- Visual Presentation: Elements in each type of animated PowerPoint video appeared on the screen one at a time and coincided with the narration.
- Length of Time: Videos were approximately the same length ranging from one to three minutes depending on the topic being taught. The differences in time between the control

animated PowerPoint videos and their corresponding experimental videos were within 10% of each other for all topics.

The experimental factor that differentiated the two animated PowerPoint videos was the method of teaching, either Non-PDM instruction (typical classroom instruction) or Process-Driven Math instruction.

Non-PDM Animated Example Problems.

These animated PowerPoint videos were seen by students in Treatment Group A who were enrolled in the Functions Supplement A course. These control animated PowerPoints presented numbers, letters, and symbols appearing as though written by hand (one digit, letter, or symbol at a time). Next steps appeared below the previous steps and were written out in the same manner without combining or skipping steps. Audio narration provided a description of the mathematical transformations, and numbers, letters, and symbols appeared as they were spoken.

PDM Animated Example Problems.

These animated PowerPoint videos were seen by students in Treatment Group C who were enrolled in the Functions Supplement C course. PDM animated PowerPoint videos incorporated digital representations of the manipulatives for chunking that were used in the PDM teaching videos. Expressions and equations were presented in a chunked format using shape, color, and appropriate math vocabulary to represent the mathematical substructures of a problem. Audio narration coincided with the appearance of the chunked problems and the incremental revealing of their contents. Audio narration provided a description of the mathematical transformations of numbers, letters, and symbols as they were superimposed on the digital representations of the manipulatives. While work was being done in one chunk of a problem, the other chunks were represented as blank shapes without text to eliminate distraction. Next steps did not appear below the previous step. Instead, substitutions and transformations were carried out directly in the chunk of the original problem in which the change was being made. Digital representations of the manipulatives moved out or disappeared and replacements moved in or appeared when transformations changed substructures within the problem.

Pre and Post Performance Assessments.

The pre and post-performance assessments were identical for students in Treatment Group A and Treatment Group C. Assessments provided type written math problems with multiple choice answers, visually consistent with what would be found in a typical multiple choice assessment. For each of the three topics on the assessment, students were asked to solve one problem that was subdivided into three steps. The questions for the second and third steps of each problem provided students with the correct answer to their previous step. Students were not able to go back and see or change previous answers during the assessment. Prior to seeing the three problems that they would be asked to solve, students were given three sets of self-efficacy questions, each set consisting of two survey items. The format selected for self-efficacy items is consistent with standards established by Bandura (2006) that were later modified Bonne and Lawes (2016). The first item in each set displayed the exact problem that students would later be asked to solve. Students were asked to respond to the statement, “I could solve this problem” with the following multiple choice options:

- I agree because I have successfully solved this type of problem before.
- I disagree because I have not been successful in solving this type of problem before.
- I disagree because I have never seen this type of problem before.

The second self-efficacy question in each set asked, “How strongly do you feel about your response to the last question?” The following multiple choice options were given:

- Strongly
- Not strongly at all

The full assessment is provided in the Appendix.

Procedures

Students were given directions by their instructors to access the material in their Functions Supplement Course during the first week of class. The Functions Supplement courses were designed so that each item (informed consent, kickoff survey, pre assessment, the 5 teaching modules, and the post assessment) remained locked and unavailable to students until the previous item had been completed. The first item, the informed consent document assignment, was open to all students when they entered their course. The second item, the survey, was only available after the informed consent assignment was completed. The third item, the pre-assessment, was only available after the survey had been completed. Following the pre-assessment, students began working sequentially through each teaching module. Within each module, the first assignment contained a teaching video followed by a yes/no question requiring students to indicate that they either had or had not watched the entire video. The second assignment would not open until the teaching video assignment was completed. The second assignment contained an animated PowerPoint video followed by a yes/no question requiring students to indicate that they either had or had not watched the entire animated PowerPoint video. The final assignment in each module, which would not open until the animated PowerPoint assignment was completed, was the homework assessment for the module. After module 5 was completed, then the post-assessment for the entire course would be available to the students. Students were given four weeks to complete all of the assignments in the Functions Supplement course.

Data Analysis

Student data from the survey was linked to their assessment data and downloaded into SPSS to run Mixed Analysis of Variances (Mixed ANOVAs). The three independent variables were: Treatment (Group A or Group C), Time (pre-test to post-test), and Perceived Difficulty (“none” or “slight or more”). Three categories of Perceived Difficulty were examined: difficulty seeing, difficulty concentrating, and difficulty reading words and symbols. The four dependent variables were the changes in assessment scores (pre to post) for: All Topics Combined, Topic 1 (Evaluation of Functions), Topic 2 (Composition of Functions), and Topic 3 (Inverse Functions). Mixed ANOVAs were run to look for differences in Treatment by Time by Perceived Difficulty for each of the four dependent variables using self-efficacy as a covariate. A set of Mixed ANOVAs was run for each of the three categories of Perceived Difficulty. Within each set, four Mixed ANOVAs were run for each of the four dependent variables. Mixed ANOVAs were run to evaluate differences in performance assessments by time, treatment, and difficulty.

Limitations

1. Post-secondary students were taught topics in algebra through a series of teaching videos, either PDM or Non-PDM, delivered through online courses. Students accessed the videos outside of the structured environment of a classroom and self-report was used to determine whether or not students had fully watched all of the teaching videos. While this introduces a variable outside of the control of the researchers into the study, random assignment mitigates its impact under the assumption that students who didn’t fully engage with the content were evenly distributed through the different sections.
2. The presence of disabilities were not determined with documentation indicating a diagnosis by a healthcare or education professional. Self-report instruments were used to

identify the likelihood that individuals had one or more disabilities in several categories. Participants were asked to evaluate the degree of difficulty they experienced with specific ordinary tasks to indicate the possible presence of one or more disabilities. This strategy was employed to mitigate the fact that many people who experience difficulties indicative of disability choose not to be tested and diagnosed with a disability.

3. Despite a large sample size of 687 participants at post-secondary institutions, the number or participants who indicated difficulty associated some categories of disability was low. Some categories of disability did not have enough people to allow statistically significant conclusions to be drawn about participants within those categories. Those categories were eliminated from the statistical analysis.

Delimitations

1. Researchers chose a multiple choice format for performance assessments to reduce bias and expedite the grading process.
2. Researchers included only three of the five topics taught in the Functions Supplement courses in the pre and post assessments. This decision was made to reduce the amount of time required for students to complete the pre and post assessments so that students would not become fatigued during the assessments. The goal was to try to keep course completion rates as high as possible since only data from students who completed every assignment in the course was included in the data analysis.

Summary

Improving math education for students with disabilities requires the development of evidence-based interventions. Statistically significant evidence-based interventions are difficult to research because of the underdiagnoses, underreporting, and low incidence of students with disabilities in postsecondary institutions. This study’s large number of postsecondary student participants increased the likelihood of achieving statistically significant numbers of participants with disabilities. In addition, the method of identifying students with potential disabilities in the survey increased the likelihood of including cases that were either undiagnosed or unreported.

Chapter Four - Results

Introduction

Data from the qualitative and experimental arms of this project are presented in this chapter. The scope of the entire project is broad; over 700 students at three post-secondary institutions and three schools for the blind were enrolled over three semesters. The breadth of work required to create curricular content, build courses, work in partnership with schools, departments and instructors, enroll students, manage data, and conduct analyses required teamwork. I initiated the project and managed the research activities at the lead institution. I was joined by a team of collaborative researchers from my own institution as well as other institutions that were a part of the NSF award. I contributed extensively to all aspects of the qualitative and experimental research as did other collaborative researchers. In describing the activities and results of the qualitative research, I use the first person pronouns “I” and “we” to indicate the work I was solely responsible for and the work I contributed to with others.

The qualitative data from the schools for the blind are organized into nine data tables, each table encapsulating a significant aspect of student experience with math education in general and Process-Driven Math. Two of the questions were coded and the themes that emerged from the coding are reported. I held discussions with two other researchers who were very familiar with the data and we came to a consensus of the themes that emerged from my analysis. The quantitative data from the work with post-secondary students are reported as a series of Mixed ANOVAs that were run to determine the effects of treatment by time by perception of difficulty with self-efficacy as a covariate. These Mixed ANOVAs were run for all four dependent variables: evaluation of functions, composition of functions, inverse functions, and all topics combined. Significant results are further described with accompanying graphs.

Qualitative Research Findings

This qualitative report stems from an inductive analysis of interview and focus group data from 25 student participants at three state schools for the blind. I worked with students in math classes that covered topics in algebra for three consecutive semesters. In-person focus groups conducted by two other researchers on this NSF funded project lasted 75 minutes and included 2 to 6 classmates at a time. Individual interviews were held to accommodate scheduling conflicts; one Skype and one in-person interview were held. Seven of the 25 students participated twice during two consecutive semesters of the same academic year. Each student was given a randomly generated number to de-identify their data and these numbers are used in the data presented in this section. Of the 25 students we interviewed, twelve were braille users. Our data collection did not include questions about students' degree or proficiency with math braille (either Nemeth Code or UEB Math).

The qualitative data tells a story about the relationships these students have had with their mathematics education. Because we used a User Centered Design lens in our examination of the data, our emphasis is on hearing the voices of the students and taking their experiences at face value. The initial portion of the protocol asked students to make a statement to summarize their feelings about math. Subsequent questions focused on their individual lived experiences that shed light on the feelings they expressed about mathematics. Student quotes are reported in data tables organized around salient issues that had significant impact on their math education. Students made meaning of their own math experiences, so no attempt was made to interpret additional meaning from questions focused on their lived experiences. Individually, they are experts of their own journeys through one or more math education systems.

The data from students' experiences with math education provide background to contextualize their thoughts about the fully audio Process-Driven Math method. Each student interacted with PDM from their own situated perspective influenced by years of positive and negative experiences with mathematics education. The questions from the last section of the interview protocol asked students to give their opinions about using and improving PDM as well as improving math education in general. I report these data as themes supported by quotes; two other researchers who are very familiar with the data agreed with the themes I identified.

At the beginning of the interview protocol researchers asked students to complete the sentence, "Mathematics makes me feel…." Six students had positive responses, four had neutral or mixed responses, and 14 had negative responses. Each student's response, as well as the frequency of responses in each category, are included in Table 1.

Table 1

Student Feelings about Mathematics

Perception & Student	Braille	Complete the Sentence: Mathematics makes me feel…	Frequency *n*, (%)
Positive			7, (28)
9984	No	Prepared and ready for life and what hits me	
7196	Yes	Like learning more and more	
2527	Yes	Like learning	
5906	No	Confident	
1372	No	Sophisticated	
0442 [a]	Yes	Like I'm learning something	
5306	Yes	Like I've accomplished something	
Neutral/Mixed			4, (16)
2401	Yes	Makes me feel like different depending on, like, how…what kind of math.	
8397	Yes	Very neutral towards it. It just makes me feel a lot of things.	
8220	Yes	Intrigued / Confused but also relieved [a]	
5755	No	Like a bunch of numbers / Decent [a]	

Perception & Student	Braille	Complete the Sentence: Mathematics makes me feel…	Frequency *n*, (%)
Negative			14, (56)
3984	No	Brain dead	
0170	No	Tired / Stressed [a]	
4031	No	Anxious / Stressed [a]	
6140	Yes	Tired / Stressed [a]	
5856	No	Incompetent / Frustrated [a]	
3926	No	Like I'm worthless and slow	
5252	Yes	Like I only want to look at the clock so I can know when the class is going to end	
7676	Yes	Me too (agreeing with a negative response)	
2724	No	Very annoyed	
3791	unknown	A little bit nervous	
5112	Yes	Bored	
7667	No	Frustrated	
6699	Yes	Aggravated	
8416	No	Anxious	

[a] Students who participated during two consecutive semesters

Some students had negative experiences in various math classes over the years that significantly impacted their math education. Students reported disinterest from some of their teachers, a lack of reasonable accommodations in the classroom, problems with assistive technology, and a breakdown in the referral system for students with disabilities. Student quotes about these experiences are included in Table 2.

Table 2

Negative Experiences in Mathematics Classes

Student	Braille	Supporting Quote
5856	No	I remember feeling inadequate because I could never finish the whole paper in time. … In algebra I never had a teacher who actually cared to sit down and explain things so it's always just kind of been a struggle to figure out what's going on.

Student	Braille	Supporting Quote
0442	Yes	My freshman year at my public school, sometimes the math teacher didn't read like the problems or anything out loud. He just wrote it on the board, so everyone else knew what the problem was. A lot of times when I asked what the problem was, he said that he would tell me later and he was just going to move on to the next one and later on he said it wasn't that big of a deal. But it ended up affecting my grade sometimes because they would do worksheets where…he would write the problems on the board and they would write it down on a piece of paper and turn it in and it would count as a participation grade type thing, but he never ended up actually telling me what the problems were, so it impacted my grade, which kind of made me dislike math a little bit.
3984	No	It was…7th grade. We had this horrible math teacher. Like she would literally just pass out worksheets like, 'All right there you go.' …You go ask her for help and she'd be like, 'Oh you know go ask the teacher next door.' …But yeah. That's probably where I got discouraged from math.
8397	Yes	Basically the whole 5th grade was pretty rough. I didn't like it because that's when my school really started to lack accessibility because I was the only blind child at that school
4031	No	None of my teachers have understood my visual impairment and that like made it really hard to learn stuff because they would just be like writing things on the board and I would be like, I can't. Like what do you expect from me? Those darn green markers! … I remember my teachers would always discard my visual impairment…It was just constantly fighting. It was a constant tug of war of being like, 'Hey, can you please write that bigger?' Or, 'Hey, can you enlarge that?'…It got to a point to where I just gave up because nobody would help. … I have been in public school until about a month ago…I have been completely public school and I didn't get an IEP until about 8th grade and have been legally blind since I was four.
7667	No	I think I was fine...up until the graphing and variables and complexity of it all…sometimes I just can't fit the whole problem under VisioBook or whatever.

Several students reported positive experiences with their mathematics classes. Generally positive experiences focused on individualized attention and the ability for students to go at their own speed. Quotes about positive experiences in mathematics classes are included in Table 3.

Table 3

Positive Experiences in Mathematics Classes

Student	Braille	Supporting Quote
4031	No	Honestly the earliest memory I have of enjoying mathematics was about a month ago when my current teacher sat down with me and didn’t force me to go any particular speed and that was my junior year.
5856	No	I appreciate…the smaller class size because there’s a lot less judgment when you’re around four or five people versus when you’re in a room of 30 and you’re the only one behind. And you can take things at your own pace here. …It’s really about you as the individual.
6140	Yes	I would say my earliest memory of, uh, enjoying mathematics is, um, when I learned like how to actually do algebra because algebra was really hard for me at first. The person that helped me was my current teacher. …She took it at our speed…She waited for us to understand what we were working with. That was like 7th or 8th grade.
0170	No	I like how small the class is. Even though I’ve always been in small classes. But the minute I started going into bigger classes I fall behind very quickly. …No matter how frustrated you get they’re always there to help you through it.
3984	No	I like how, you know, you can just talk about the math problem and you can explain where you’re struggling and she will sit down with you and just explain it all.
5755	No	Our high school math teacher…she would…go with the flow, like go at your own pace. …And she’s going smooth and not being rushed.
0442	Yes	Some of the good experiences that I’ve had with math are at my school when we are learning different things and people have different learning styles and we accommodate to help each person with their individual learning style so that everyone kind of figures it out. Even if it’s a self-paced kind of thing we all figure it out eventually, which made it easier for me to associate math with a positive thing.

Student	Braille	Supporting Quote
7667	No	I like the intimate setting where it is only 5 people in a classroom. And you know most of the time I do like the way the teacher teaches.
5112	Yes	I like the small classes for the one-on-one with teachers to get more help when needed.

During our focus group discussions, one specific content area, geometry, was mentioned by several students. The majority of these students expressed negative feelings about geometry while one student talked about a positive experience with tactile shapes. Student quotes about their experiences with geometry are included in Table 4.

Table 4

Student Experiences with Geometry

Student	Braille	Supporting Quote
4031	No	My struggle was with geometry. I hated geometry. I never want to do it again. It's pointless. You never need it. Ever. …I had a hard time trying to like figure out angles. Like I can tell you what a shape is, but it's hard for me because of my blindness to determine how something looks. Like a sighted or visually impaired person can look at something and…determine how something is supposed to look and I have to use my hands and it's kind of difficult for me sometimes.
0170	No	I had the hardest time in geometry only because I don't see the point on knowing the triangles…learning the angles for shapes and that I don't get why we should know that.
5856	No	It was just a difficult subject. …even if I'm not blind, it's really hard to get differences between angles and so much subtlety.
6140	Yes	I think my worst time with math was geometry because like geometry was really hard for me to understand because I can't see it. I can't physically see it so I like I would just get confused because there were so many numbers. And there were so many angles and it was just – it was really hard for me to understand what was going on.

Student	Braille	Supporting Quote
0442	Yes	In geometry…when we had the circles and had to identify all the different angles in the circles and the chords and all of those things, it helped having actual pictures – tactile pictures – of a circle and all the lines and stuff to figure out. Like what made each angle and where exactly… how to identify it.

We asked students to evaluate their experiences with Process-Driven Math. Their responses during the focus groups and interviews were based on the experiences they had when I taught them PDM lessons. Each semester I taught students between three and five lessons using the fully audio Process-Driven Math method. I functioned as the reader/scribe for the students and revealed what was in each chunk of their math problems using appropriate math vocabulary and precise language that is not ambiguous. I encouraged all students to participate in the iterative dialogue that is a central component of the method. For the students, their portion of the dialogue was comprised of specific questions about the contents of individual chunks in a problem as well as directions for making transformations in those chunks. The students had to use the same appropriate math vocabulary and precise language that I used. After students described a transformation they wanted to make, I restated the contents of that particular chunk incorporating the student directed transformations.

Of the 25 students who participated, 22 provided answers to the questions about Process-Driven Math. Most students identified elements of PDM that they liked and elements that they didn’t like. We read through the full range of comments each student made and classified their perspectives about PDM as being either primarily positive, primarily neutral or mixed, or primarily negative. Fourteen students whose feelings about PDM were primarily positive seemed to appreciate the logic of the process, the structure for going through a problem without skipping steps, and the time to think through strategies before implementing them. Quotes from students who had an overall positive perspective about PDM are included in Table 5.

Table 5

Positive Student Perceptions about Process-Driven Math

Student	Braille	Supporting Quote
6140	Yes	You're not forgetting any steps or leaving any steps out…I like that it helps me focus and retain information … I'm just like, 'Oh now I remember because um I'm focused on that problem.'
8397	Yes	I loved it because it gave me time to figure out what the heck was going on with the problem so it's not all smashed in my face at once like a giant brick.
3984	No	I like that it was easier to understand. And it helped. It really helped me understand how the math works better…I feel like having the alternative there really helped and it really engaged with me if you will.
6906	No	It's good for people, not just with low vision. People, even people who are struggling with math might need Process-Driven Math because it helps them to go through the steps of what they need to do in order to get their answer.
5112	Yes	Um I just like how it's more sanctioned off, so you can um like go part by part. And you're not trying to speed through it as much. Gotta actually slow down.
0442	Yes	I like that there's a sort of map I guess that you have to follow to get through the problem… I like that it's a relatively simple process that can help people that aren't necessarily visual learners.
8416	No	I feel like it helps me to retain it better. Trying to get through it you know I'm skipping all the steps but I can't really do that with Process-Driven Math so I feel like it definitely helps me retain the information better.
7667	No	The thing I like about Process-Driven Math is that you can be more independent…It also allows you to focus more on what you are saying.

Eight of the students indicated an overall neutral or mixed perception of PDM. Some students liked the unique, systematic approach, but also felt it was too complex and time consuming. Others who liked elements of PDM said they would prefer using it one-on-one rather than in a group or that they wanted to have a written element to accompany the audio rendering. Quotes from students who had a mixed or neutral feelings about PDM are included in Table 6.

Table 6

Mixed / Neutral Student Perceptions about Process-Driven Math

Student	Braille	Supporting Quote
9984	No	So Process-Driven Math for me is just really slow…Things I like is that it's very intuitive
3926	No	The things that this whole thing is about helped me understand it [the quadratic equation] a tiny bit better, but I still really don't understand it.
7196	Yes	Something I like is that is how she solved that equation and I learned it…but I would like to do it one-on-one.
6699	Yes	I like it…But for me, personally, it just seems a bit complex.
1372	No	I think it is a different way to do the math. It's more complex in my opinion. It does make you think more. And so, like instead of writing it out…it allows you to think more instead of letting your hand do the talking
2527	Yes	See I would like to do Process-Driven Math, but I would like a copy of the problem and maybe a paper to take notes on.
2724	No	Let's say…we were all at the same level and doing it – it would be so much easier doing that way because we would all know it and do it.
8220	Yes	So what I liked about PDM was that it was a little more formulaic…It made sense. Since I have um read braille all of my life…it seems that the way that I have done it is faster for me.

Six of the students indicated an overall negative perception of PDM. Generally these students disliked the fact that they only heard the math and didn't get to see it or touch it. Some found that hearing and speaking the math created confusion and they found the vocabulary requirements cumbersome. Quotes from students who had overall negative perceptions of PDM are included in Table 7.

Table 7

Negative Student Perceptions about Process-Driven Math

Student	Braille	Supporting Quote
0170	No	I'm a very visual person so it's harder for me to understand.
4031	No	I am definitely more of a visual person and when I hear things like audibly, or whatever, it just doesn't…I feel like I have audio dyslexia because it just gets mixed up in my head.
5856	No	It introduces you to this form of co-dependence where you have to rely on somebody else to feed you information while everything is slow.
7676	Yes	I don't like math whether it's process-driven – (interrupted)
2401	Yes	...it's kind of slow and there are a lot of terms I'm supposed to say, like wait, 'what am I supposed to say?'
5306	Yes	I consider myself a very visual learner, which is ironic, I know, since I am blind. I have to see it, have it physically in front of me and be able to work through it in order for it to make sense

Themes emerged after I conducted an iterative, inductive analysis of what students liked and disliked about PDM and what they would do to improve it. Two other researcher who were very familiar with the data agreed with the themes of Pacing, Control, Visual or Tactile Support, and Good for Others. Several students disliked the slow pace of the PDM simplification process. Conversely, other students indicated that they enjoyed the slowness because it gave them the time they needed to think. I identified the theme of Pacing from these comments. Some students felt like the method gave them more autonomy because the reader/scribe followed their lead as they made decisions about how to approach the solution process. Others felt that having a person serve as an intermediary between the student and the math reduced their autonomy leading to feelings of frustration. I identified the theme of Control from these statements. Many students expressed a desire to have the problems presented in both an auditory as well as a visual or tactile format. Some students felt that having the math written or in braille would increase their understanding of the problem. For others, they wanted a tactile element to help increase focus and prevent boredom. I identified the theme of Visual or Tactile Support for these comments. Finally, some students who found value in the way PDM is structured didn't think they would want to use it themselves. They identified other groups that they thought would benefit from PDM. I identified the theme of Good for Others from these comments. Quotes supporting each category of themes about PDM are included in Table 8.

Table 8

Themes that Emerged about Using and Improving Process-Driven Math

Theme & Student	Braille	Supporting Quotes
Pacing		
2401	Yes	I know it's supposed to be this way, but for me it's just very, very slow.
2527	Yes	I think what would've helped was one-one-one, so we can go at our own pace.
3984	No	Because of all the moving around of stuff and you know getting everything to where you can do the problem. That alone takes a good five minutes.
8416	No	It takes too long. I get a little antsy. I get a little impatient.

Theme & Student	Braille	Supporting Quotes
Control		
8397	Yes	I like the idea of walking people through things ‘cause normally I’m the one being walked through it and I like having some freedom where there’d be like, ‘This is what we’re supposed to do, we’re supposed to do this or that.’
4031	No	Some sense of control is taken away when you can’t look at it and when you are only given pieces at a time.
5306	Yes	I also struggle with not having the whole scope of the equation. Getting handed things in chunks doesn’t work for my brain. I get a little paranoid about what’s the rest of the equation, what the problem holds, and it’s hard for me to focus on the little thing.
Visual/Tactile		
0170	No	I feel like for those who are visual, if they can see it, it would be better…and be able to separate it out and be able to see it in the process.
5856	No	It’s easier to just have it written down in front of you and be able to organize it almost like a puzzle. It’s hard to do that like mentally and visually in your head.
5112	Yes	I think a way to make it better would be to have the students do something physically so they’re not going to space out or be bored sitting there listening.
0442	Yes	I think that if you have some way for the people that are maybe a little bit more visual learners to follow along with it and to have the problem in front of them, like if you had some sort of refreshable braille display for the braille readers or some kind of a, like, screen for the sighted people.
7667	No	Sometimes it may be easier to space out since you’re not physically per se doing anything.
Good for Others		
5856	No	I think when it comes to when you’re first learning a topic, PDM is really good, but for somebody who knows what they’re learning, it further complicates it and it causes a lot of frustration.

Theme & Student	Braille	Supporting Quotes
3791	unknown	So like people who don't understand it that well, I think they should be in Process-Driven Math.
5906	No	But kids who need help writing down problems or need help figuring out how to solve them…like they would need Process-Driven Math.
6699	Yes	I think it is very useful and can be used for many people, but for me personally I just like to do things on my own and read the problems myself.

In the last question of the protocol, students were asked to pretend that they were a part of a committee established to give advice to help math teachers teach better. Two other researchers who were very familiar with the data agreed with the themes that emerged from this data: Individualize Instruction, Try Different Approaches, and Incorporate Student Interests. Comments pointing to the importance of individualized instruction were the most prevalent of all statements made during this section of the interview. This is not unexpected given that so many of the students' statements about their positive classroom experiences were focused on the benefits of the one-on-one one attention they had received from teachers. Quotes supporting the identification of these three themes are included in Table 9.

Table 9

Themes that Emerged about Improving Math Teaching

Theme & Student	Braille	Supporting Quotes
Individualize Instruction		
2401	Yes	I guess as a general rule…maybe try to individualize it
4031	No	Teachers need to sit down one-on-one and figure out what they're struggling with and break it down in a way they will understand.

Theme & Student	Braille	Supporting Quotes
6140	Yes	Or just try to sit down one-one-one with them and if they're struggling with something try to figure out what they are trying to do and how they are trying to work out the problem and try to break it down for them in a way that they could understand.
7676	Yes	Oh, um we just need – well I need one-one-one because everyone else does it better.
5856	No	So instead of assuming what a child knows, actually go through the problem with them, almost like process-driven math, and figure out where in the step-by-step process they are messing up or what they don't understand in the step-by-step process.
7667	No	I think the way to help them teach better is to get them more engaged and really walk around to the students and see if they need help by looking over their should and seeing what they're doing.
0442	Yes	Just to make sure that you know how to go at the pace of the learners if necessary. Like slow down if it needs to be, or go faster if it needs to be depending on how each person processes.
Try Different Approaches		
6699	Yes	So if they're like just giving us worksheets every single day, maybe they could be more – maybe they could throw in an activity or throw in maybe a video of the subject – just alternate ways of learning would probably help.
8397	Yes	I know for me with the assessments, I like to do things orally…So maybe during assessments, if like a student has a different way they would take the test just pull them aside into an off room or something and have them do it that way.
1372	No	Different students may learn in different ways, so try to maybe adapt to the way those kids learn. Not necessarily adapt, but try and teach the way they would understand.

Theme & Student	Braille	Supporting Quotes
Incorporate Student Interests		
3791	unknown	Like I enjoy guinea pigs. And you know we were talking one day, me and the math teacher were doing a math problem, and we put guinea pigs in it. I forget what it was, but it made it fun and it made me want to do it.
5906	No	Get to know what they like, who they are. And then incorporate the stuff that they like or what they've done or fun moments into the math and make it more exciting and more relatable.

The lived experiences of SBVIs in math classes, their perspectives on Process-Driven Math, and their views on improving math education are valuable data. Barriers that SBVIs experience in their math education cannot be reduced without an in-depth understanding of their needs, wants and perspectives. Listening to what these students have to say is a critical step in the development of effective strategies to support their mathematical learning. The incorporation of this data into ongoing efforts to improve math education for SBVIs will be further discussed in Chapter 5.

Experimental Results

The quantitative portion of this study is an expansion of the investigation of Process-Driven Math within a Universal Design for Learning framework. Researchers used an experimental design to evaluate the efficacy of the visually adapted PDM method as an instructional approach for multiple groups of students. Groups were identified by students' self-reports of difficulties experienced with everyday tasks – difficulties that could be indicative of a disability. The experiment was run to determine the extent to which student performance was dependent on type of instruction and perceived difficulty. A total of 687 students participated and were randomly assigned to two treatment groups. Student performance was evaluated by analyzing scores on pre and post assessments in two randomly assigned treatment groups. Treatment Group A received instruction that was consistent with typical classroom instruction and Treatment Group C received Process-Driven Math instruction. The characteristics of these instructional methods and detailed descriptions of the teaching materials and assessments for each treatment group are found in Chapter 3. Prior to any assessment or instruction, all students answered survey questions indicating the likelihood of a disability in seven categories: seeing, hearing, walking, lifting, concentrating, reading, and writing. Four of the seven categories (hearing, walking, lifting, and writing) did not produce sufficient numbers to yield statistical significance and were not included in this analysis. The categories of concentrating, seeing, and reading had sufficient frequency to support statistical analysis. In addition to the questions about disability, responses to self-efficacy questions were included in the analysis in order to account for its influence as a covariate on student performance.

Students in Treatment Group A were taught and assessed with materials that closely approximated typical classroom instruction and assessment. Students in Treatment Group C were taught using the visually adapted Process-Driven Math methodology. Both groups were given identical pre and post assessments. Students in a third group, Treatment Group B, were given alternative pre and post assessments formatted using PDM; only Treatment Groups A and C, which received identical pre and post assessments, were included in this statistical analysis. The pre and post assessments contained three multistep problems from three topics related to algebraic functions: evaluation of functions, composition of functions, and inverse functions. Between the pre and post assessments, students were exposed to five teaching modules, each consisting of a teaching video (approximately 10 min), an animated video with an example problem (approximately 2 minutes), and a homework assignment requiring students to solve one multistep problem in three discrete stages. Three of the five teaching modules were aligned to the same three topics represented in the problems on the pre and post assessments. Two additional modules covered related topics that were not represented on the pre and post assessments.

Statistical analyses were carried out using the SPSS (Statistical Package for the Social Sciences) package. Both descriptive statistics and mixed ANOVAs (Analysis of Variance) were run. The mixed ANOVA afforded an opportunity to determine the extent to which performance differences changed from pre to post and were due to instructional approach (A vs. C) or perceived difficulty. Statistical significance for all effects was set *a priori* to $p < .05$. Estimated marginal means were used to determine the influence of self-efficacy on main effects. This analysis was conducted to answer the following research question:

> RQ3: *Are there differences in the performance assessments of post-secondary students, both with and without disabilities, who have been taught topics in algebra either with or without the visually adapted Process-Driven Math method?*

Frequencies

A total of 360 students were in Treatment Group A and 327 students were in Treatment Group C. Students in both treatment groups were asked to indicate the degree of difficulty they experienced with the following actions:

1. **Concentrating, remembering, or making decisions** because of a physical, cognitive, or emotional condition
2. **Seeing** words or letters in ordinary newsprint (with glasses/contact lenses, if you usually wear them)

3. **Reading** words and/or mathematics expressions because of seeing letters, words, or symbols that appear reversed, substituted, omitted, or added

Students were given the following options for their responses: none, slight, moderate, severe, and unable to do. Responses of slight, moderate, severe, and unable to do were combined into one category and renamed "slight or more" in order to reach numbers that would yield statistical significance.

The frequencies of students in each treatment group whose self-report survey answers were "none" or "slight or more" were counted and percentages determined. For the difficulty concentrating category, 241 (66.9%) in Treatment Group A and 213 (65.1%) in Treatment Group C indicated their degree of difficulty was none; 119 (33.1%) in Treatment Group A and 114 (34.9%) in Treatment Group C indicated their degree of difficulty was slight or more. For the difficulty seeing category, 267 (74.2%) in Treatment Group A and 230 (70.3%) in Treatment Group C indicated their degree of difficulty was none; 93 (25.8%) in Treatment Group A and 97 (29.7%) in Treatment Group C indicated their degree of difficulty was slight or more. For the difficulty reading category, 308 (85.6%) in Treatment Group A and 281 (85.9%) in Treatment Group B indicated their degree of difficulty was none; 52 (14.4%) in Treatment Group A and 46 (14.1%) in Treatment Group C indicated their degree of difficulty was slight or more. For a summary of frequencies in each treatment group by degree of difficulty in each perceived difficulty category see Table 10.

Table 10

Frequency of Each Perceived Difficulty by Treatment Group

Degree of Perceived Difficulty	Treatment A (N=360)		Treatment C (N=327)	
	Frequency	Percent	Frequency	Percent
	Concentrating			
None	241	66.9	213	65.1
Slight or more	119	33.1	114	34.9
	Seeing			
None	267	74.2	230	70.3
Slight or more	93	25.8	97	29.7
	Reading Words and Symbols			
None	308	85.6	281	85.9
Slight or more	52	14.4	46	14.1

Performance Assessment Scoring

Student performance was measured before and after instruction modules with identical pre and post assessments. The assessments contained three problems from three separate topics: evaluation of functions, composition of functions, and inverse functions. Each problem was divided into three individual steps with each step counting for 1 point. Therefore, each topic on the assessment was worth a total of 3 points and the overall assessment for all topics combined was worth 9 points.

Descriptive Statistics

For each perceived difficulty the mean and standard deviation for overall performance as well as performance on each of the three topics were determined. In each treatment group these values were reported in two categories: students who indicated their perceived difficulty was “none” and students who indicated their perceived difficulty was “slight or more.” No statistical differences were found between the scores of the two treatment groups in the categories of perceived difficulty seeing or perceived difficulty reading. However, a statistical difference between Treatment Group A and Treatment Group C was observed when analyzing those scores in the category of perceived difficulty concentrating.

Descriptive Statistics for Difficulty Concentrating by Treatment and Time

For the All Topics Combined scores, students in Treatment Group A who reported no difficulty concentrating showed improvement from pre assessment (*M* = 4.76, *SD* =2.28) to post assessment (*M* = 6.81, *SD* = 2.19); those who reported slight or more difficulty concentrating showed less improvement in their mean scores from the pre assessment (*M* = 4.98, *SD* =2.26) to post assessment (*M* = 6.50, *SD* = 2.28). For the All Topic Combined scores, students in Treatment Group C who reported no difficulty concentrating showed improvement from pre assessment (*M* = 4.95, *SD* =2.26) to post assessment (*M* = 6.88, *SD* = 2.06); those who reported slight or more difficulty concentrating showed more improvement in their mean scores from the pre assessment (*M* = 4.42, *SD* = 2.35) to post assessment (*M* = 6.95, *SD* = 2.19). These differences were shown to be statistically significant in subsequent tests.

For the Topic 1 Evaluation of Functions scores, students in Treatment Group A who reported no difficulty concentrating showed improvement from pre assessment (M = 1.92, *SD* = 1.07) to post assessment (M = 2.43, *SD* = 0.81); those who reported slight of more difficulty concentrating improved less in their mean scores from the pre assessment (*M* = 2.05, *SD* = 0.92) to post assessment (*M* = 2.30, *SD* = 0.90). For the Topic 1 Evaluation of Functions scores, students in Treatment Group C who reported no difficulty concentrating showed improvement from pre assessment (*M* = 2.10, *SD* = 0.91) to post assessment (*M* = 2.47, *SD* = 0.83); those who reported slight of more difficulty concentrating improved more in their mean scores from the pre assessment (*M* =1.99, *SD* = 0.98) to post assessment (*M* = 2.47, *SD* = 0.78). These differences were shown to be statistically significant in subsequent tests.

For the Topic 2 Composition of Functions scores, students in Treatment Group A who reported no difficulty concentrating showed improvement from pre assessment (*M* = 1.47, *SD* = 1.02) to post assessment (*M* = 2.19, *SD* = 0.95); those who reported slight of more difficulty concentrating improved less in their mean scores from the pre assessment (*M* = 1.62, *SD* = 1.02) to post assessment (*M* = 2.21, *SD* = 0.86). For the Topic 2 Composition of Functions scores, students in Treatment Group C who reported no difficulty concentrating improved from pre assessment (*M* = 1.72, *SD* = 1.03) to post assessment (*M* = 2.32, *SD* = 0.87); those who reported slight of more difficulty concentrating improved more in their mean scores from the pre assessment (*M* =1.47, *SD* = 1.04) to post assessment (*M* = 2.19, *SD* = 0.96). These differences were not shown to be statistically significant for Topic 2 Composition of Functions; however, these differences did make a contribution to the statistical significance observed in the All Topics Combined analysis.

For the Topic 3 Inverse Functions scores, students in Treatment Group A who reported no difficulty concentrating showed improvement from pre assessment (*M* = 0.87, *SD* = 0.97) to post assessment (*M* = 1.96, *SD* = 0.94); those who reported slight of more difficulty concentrating improved less in their mean scores from the pre assessment (*M* = 0.87, *SD* = 1.03) to post assessment (*M* = 1.91, *SD* = 0.96). For the Topic 3 Inverse Functions scores, students in Treatment Group C who reported no difficulty concentrating showed improvement from pre assessment (*M* = 0.88, *SD* = 0.95) to post assessment (*M* = 1.93, *SD* = 0.98); those who reported slight of more difficulty concentrating improved more in their mean scores from pre assessment

(*M* = 0.61, *SD* = 0.83) to post assessment (*M* = 1.91, *SD* = 0.96). These differences were not shown to be statistically significant for Topic 3 Inverse Functions; however, these differences did make a contribution to the statistical significance observed in the All Topics Combined analysis. For a summary of mean scores and standard deviations for the perceived difficulty concentrating variable see Table 11.

Table 11

Descriptive Statistics of Pre & Post Scores for Difficulty Concentrating

Variable	N	Pre-Assessment		Post-Assessment	
		Mean	SD	Mean	SD
All Topics Combined					
Treatment A [a]					
No difficulty	185	4.76	2.28	6.81	2.19
Slight or more difficulty	90	4.98	2.26	6.50	2.28
Treatment C [a]					
No difficulty	176	4.95	2.26	6.88	2.06
Slight or more difficulty	77	4.42	2.35	6.95	2.19
Topic 1 Evaluation of Functions					
Treatment A [b]					
No difficulty	241	1.92	1.07	2.43	0.81
Slight or more difficulty	119	2.05	0.92	2.30	0.90
Treatment C [b]					
No difficulty	213	2.10	0.91	2.47	0.83
Slight or more difficulty	114	1.99	0.98	2.47	0.78
Topic 2 Composition of Functions					
Treatment A [b]					
No difficulty	241	1.47	1.02	2.19	0.95
Slight or more difficulty	119	1.62	1.02	2.21	0.86
Treatment C [b]					
No difficulty	213	1.72	1.03	2.32	0.87
Slight or more difficulty	114	1.47	1.04	2.19	0.96
Topic 3 Inverse Functions					
Treatment A [b]					
No difficulty	241	0.87	0.97	1.96	0.94
Slight or more difficulty	119	0.87	1.03	1.91	0.96
Treatment C [b]					
No difficulty	213	0.88	0.95	1.93	0.98
Slight or more difficulty	114	0.61	0.83	1.91	0.96

[a] *Scores range from 0 to 9*

[b] *Scores range from 0 to 3*

Mixed ANOVAs

Across both treatments and all perceived difficulties, time (from pre to post assessment) had the greatest effect on student performance. Whether students were in Treatment Group A or Treatment Group C, group means for all topics increased after the instructional period between the pre and post assessments. The second largest and consistent contributor to student performance was the covariate self-efficacy. Students with high self-efficacy scored higher on assessments across treatment, time, and perceived difficulties.

Mixed ANOVA for Difficulty Concentrating, Treatment, and Time with Self-efficacy Covariate

In the mixed ANOVA for difficulty concentrating analysis, the variable of time from pre to post assessment had the most significant impact on student performance with medium to large effect sizes for the four dependent variables: All Topics Combined ($F = 103.95$, $p < .001$, $ES = .116$), Topic 1 Evaluation of Functions ($F = 66.12$, $p < .001$, $ES = .088$), Topic 2 Composition of Functions ($F = 167.71$, $p <$, $ES = .197$), Topic 3 Inverse Functions ($F = 297.09$, $p < .001$, $ES = .303$). Self-efficacy had the second most significant impact on student performance with small effect sizes for the four dependent variables: All Topics Combined ($F = 13.77$, $p < .001$, $ES = .026$), Topic 1 Evaluation of Functions ($F = 16.59$, $p < .001$, $ES = .024$), Topic 2 Composition of Functions ($F = 24.25$, $p < .001$, $ES = .034$), Topic 3 Inverse Functions ($F = 10.54$, $p = .001$, $ES = .015$). One significant finding for treatment by time had a small effect size and appeared only for the All Topics Combined dependent variable ($F = 5.43$, $p = .020$, $ES = .010$). No significant findings were associated with perceived difficulty concentrating by time in any of the dependent variables. Two significant findings for perceived difficulty concentrating by treatment by time also had small effect sizes: All Topics Combined ($F = 7.641$, $p = .006$, $ES = .014$) and Topic 1 Evaluation of Functions ($F = 4.67$, $p = .031$, $ES = .010$). All Mixed ANOVA results for perceived difficulty concentrating are summarized in Table 12.

Table 12

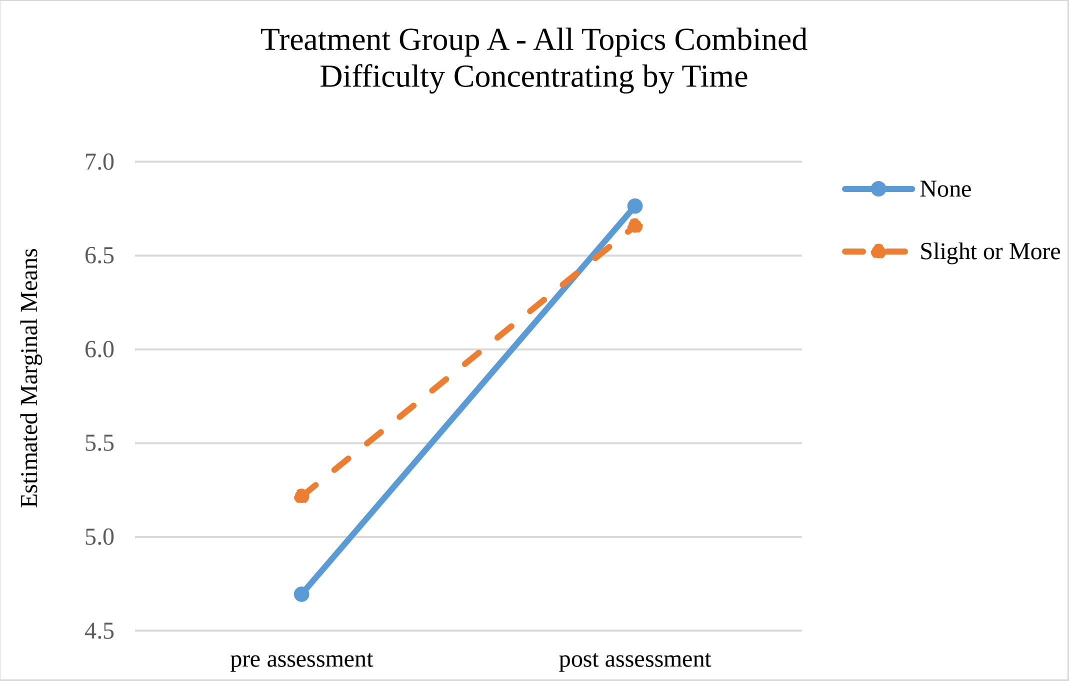
Summary of Mixed ANOVA for Difficulty Concentrating by Treatment

Variable	Total Topics		Topic 1 Evaluation of Functions		Topic 2 Composition of Functions		Topic 3 Inverse Functions	
	F (p)	*ES*	*F (p)*	*ES*	*F (p)*	*ES*	*F (p)*	*ES*
				Within Subjects				
Time (pre to post)	103.95 (<.001)	.166	66.12 (<.001)	.088	167.71 (<.001)	.197	297.09 (<.001)	.303
Self-efficacy by time	13.77 (<.001)	.026	16.59 (<.001)	.024	24.25 (<.001)	.034	10.54 (.001)	.015
Treatment by time	5.43 (.020)	.010	.401 (.527)	.001	.039 (.843)	<.001	1.26 (.263)	.002
Difficulty by time	.179 (.672)	<.001	2.27 (.132)	.003	.480 (.489)	.001	.697 (.404)	.001
Treatment by difficulty by time	7.64 (.006)	.014	4.67 (.031)	.007	.965 (.326)	.001	2.27 (.132)	.003
				Between Subjects				
Self-efficacy	116.51 (<.001)	.233	152.98 (<.001)	.183	157.874 (<.001)	.188	25.82 (<.001)	.434
Treatment AC	0.00 (.993)	<.001	2.25 (.134)	.003	.509 (.476)	.001	2.24 (.135)	.003
Difficulty concentrating	1.01 (.315)	.002	1.58 (.210)	.002	.483 (.487)	.001	.265 (.607)	<.001
Treatment AC by difficulty interaction	0.05 (.818)	<.001	.079 (.778)	<.001	1.02 (.314)	.001	1.628 (.202)	.002

Graphical representations of the statistically significant differences found in perceived difficulty concentrating by treatment and time for All Topics Combined are shown in Figures 3 through 6. Figure 3 demonstrates that students in Treatment Group A (Traditional Non-PDM) who reported no difficulty concentrating improved more from pre to post assessment than students who reported slight or more difficulty concentrating.

Figure 3

Group A (Traditional Non-PDM) by Difficulty Concentrating for All Topics



In contrast to Figure 3, Figure 4 shows that students in Treatment Group C (PDM) who reported slight or more difficulty concentrating improved more from pre to post assessment than students who reported no difficulty concentrating.

Figure 4

Group C (PDM) by Difficulty Concentrating for All Topics

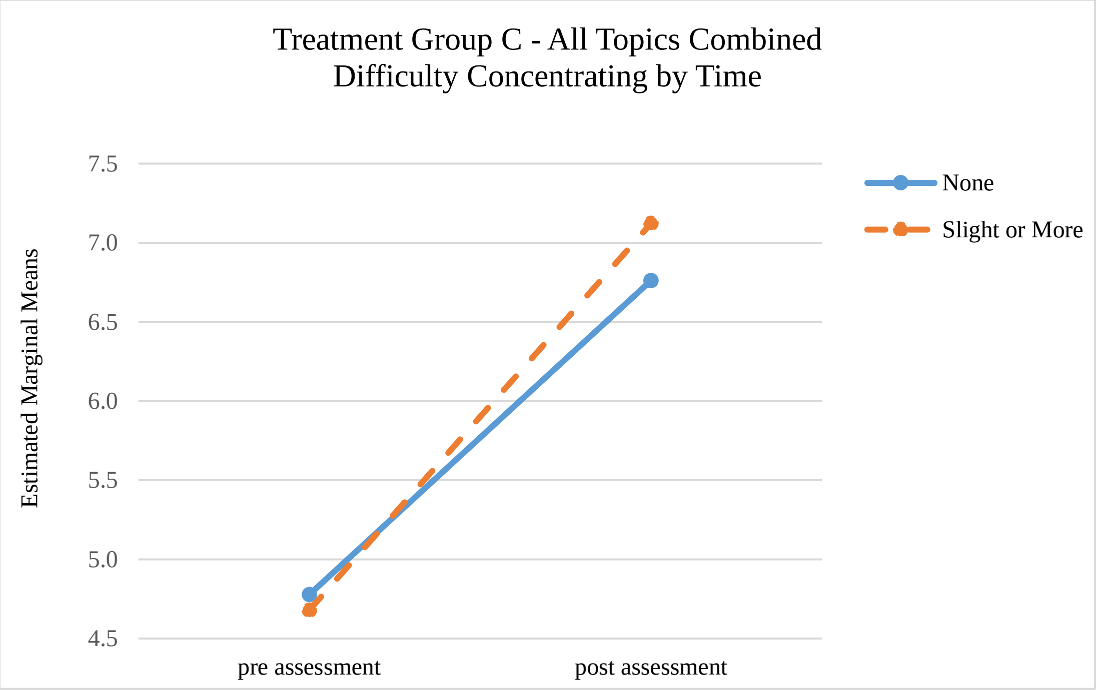


Figure 5 shows that for students who reported no difficulty concentrating there is almost no difference in improvement from pre to post assessment between Treatment Group A (Traditional Non-PDM) and Treatment Group C (PDM).

Figure 5

No Difficulty Concentrating by Treatment for All Topics

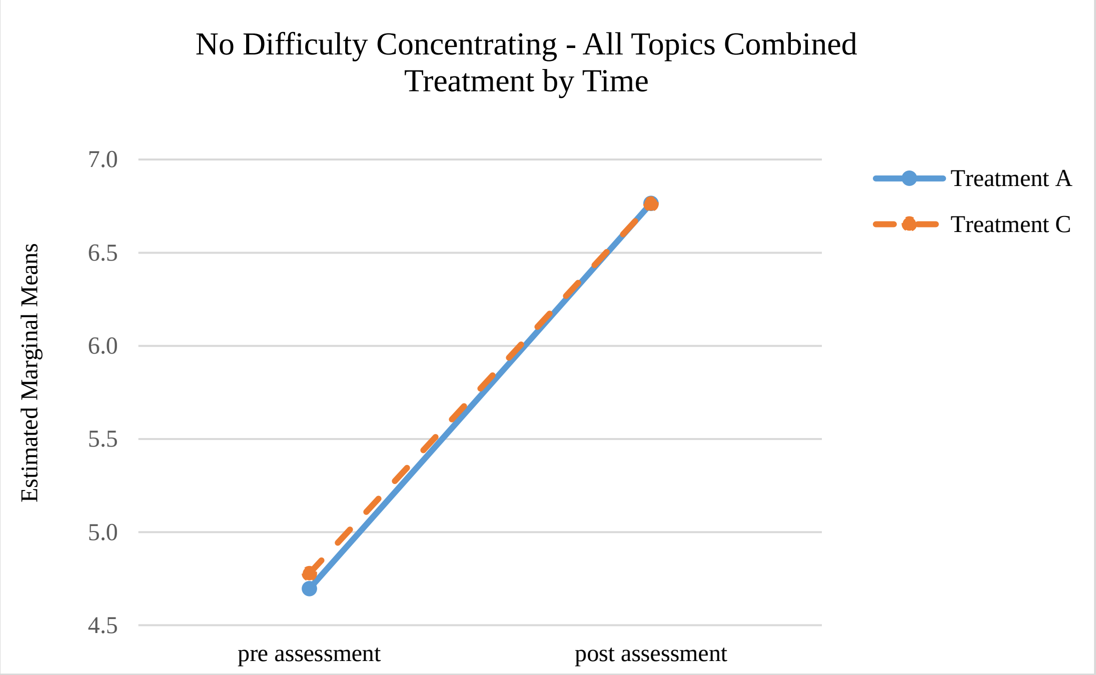
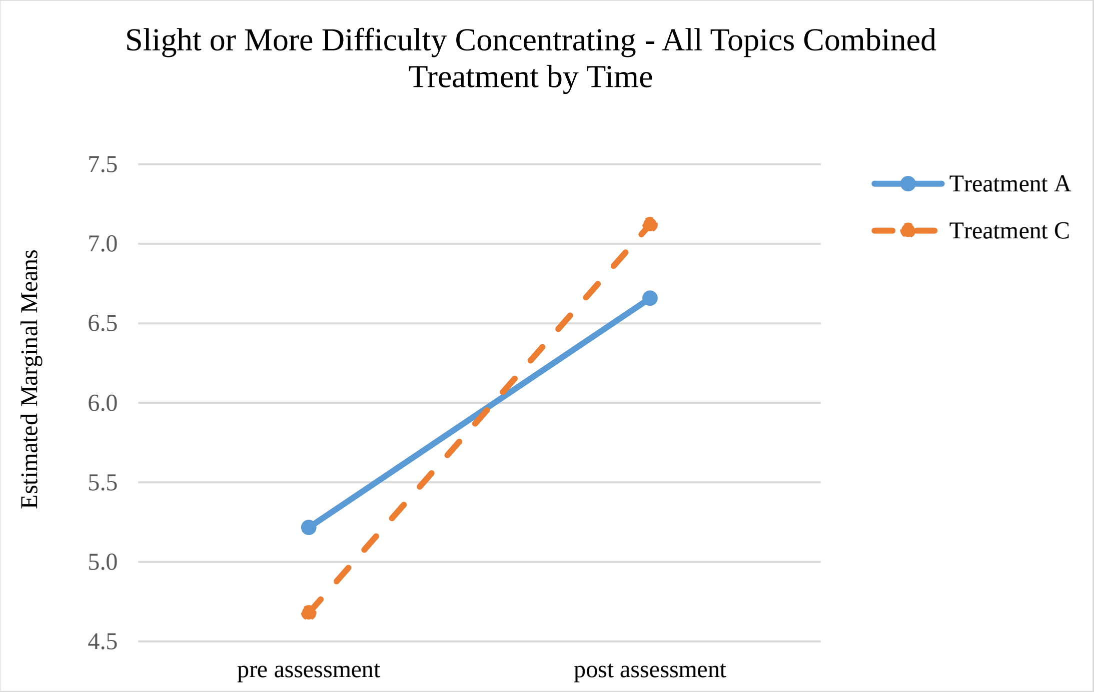


Figure 6 shows that for students who reported slight of more difficulty concentrating, those in group C (PDM) showed greater improvement from pre to post assessment than those in Treatment Group A (Traditional Non-PDM).

Figure 6

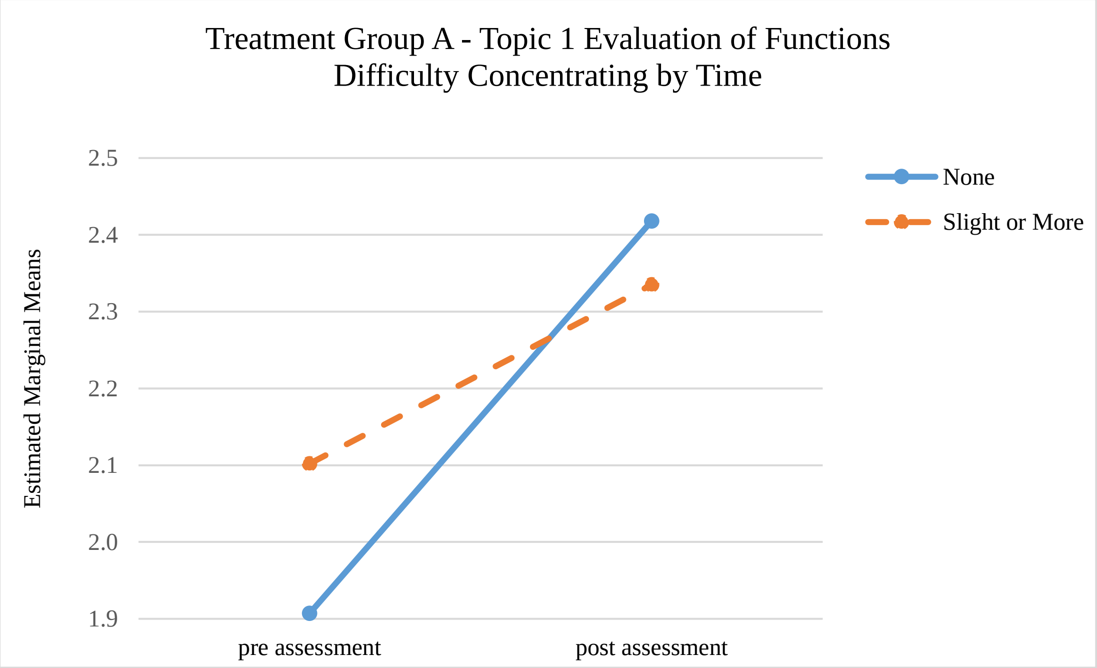
Some Difficulty Concentrating by Treatment for All Topics



Graphical representations of the statistically significant differences found in perceived difficulty concentrating by treatment and time for Topic 1 Evaluation of Functions are shown in Figures 7 through 10. Figure 7 demonstrates that students in Treatment Group A (Traditional Non-PDM) who reported no difficulty concentrating improved more from pre to post assessment than students who reported slight or more difficulty concentrating.

Figure 7

Group A (Traditional Non-PDM) by Difficulty Concentrating for Topic 1



In contrast, Figure 8 shows that students in Treatment Group C (PDM) who reported slight or more difficulty concentrating improved more from pre to post assessment than students who reported no difficulty concentrating.

Figure 8

Group C (PDM) by Difficulty Concentrating for Topic 1

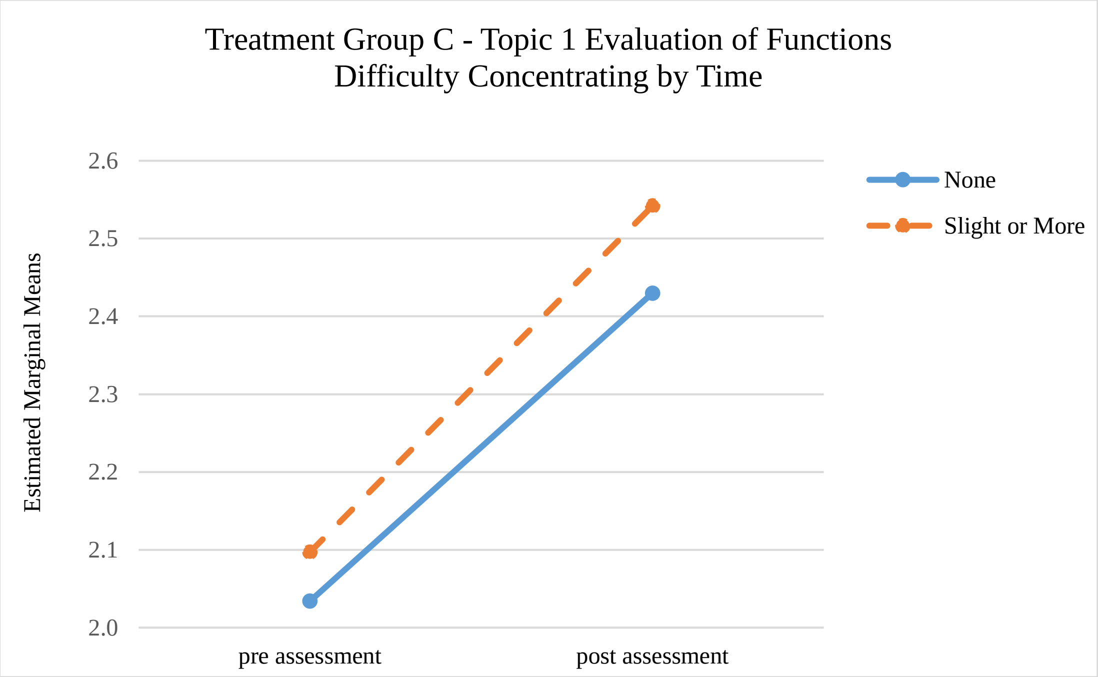


Figure 9 shows that for students who reported no difficulty concentrating there was greater improvement from pre to post assessment in Treatment Group A (Traditional Non-PDM) than those in Treatment Group C (PDM).

Figure 9

No Difficulty Concentrating by Treatment for Topic 1

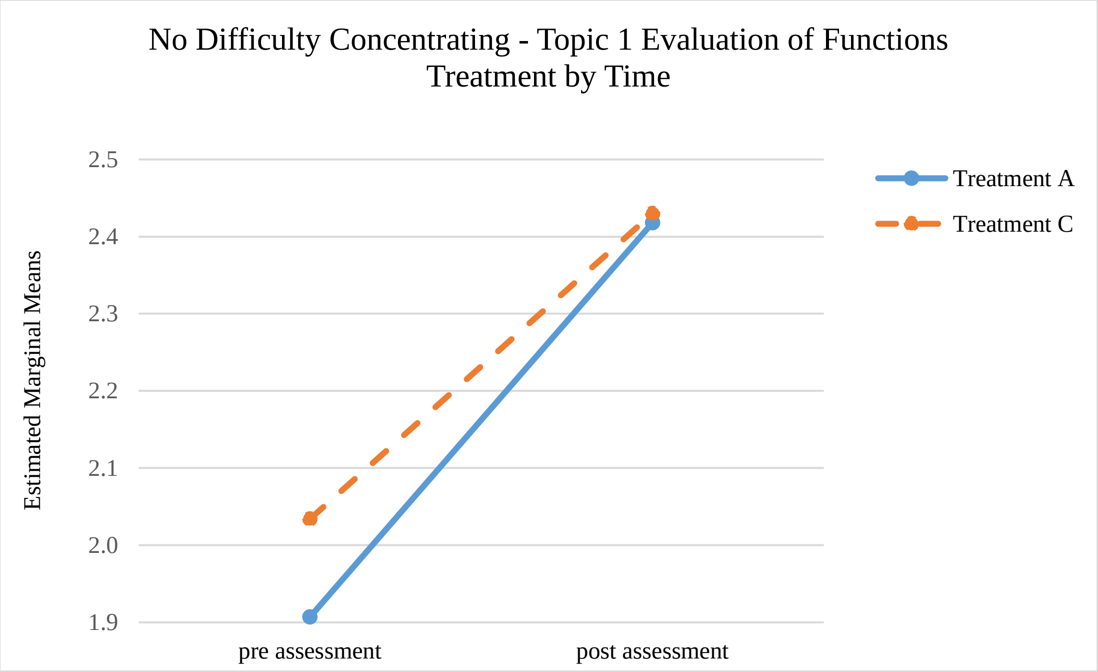
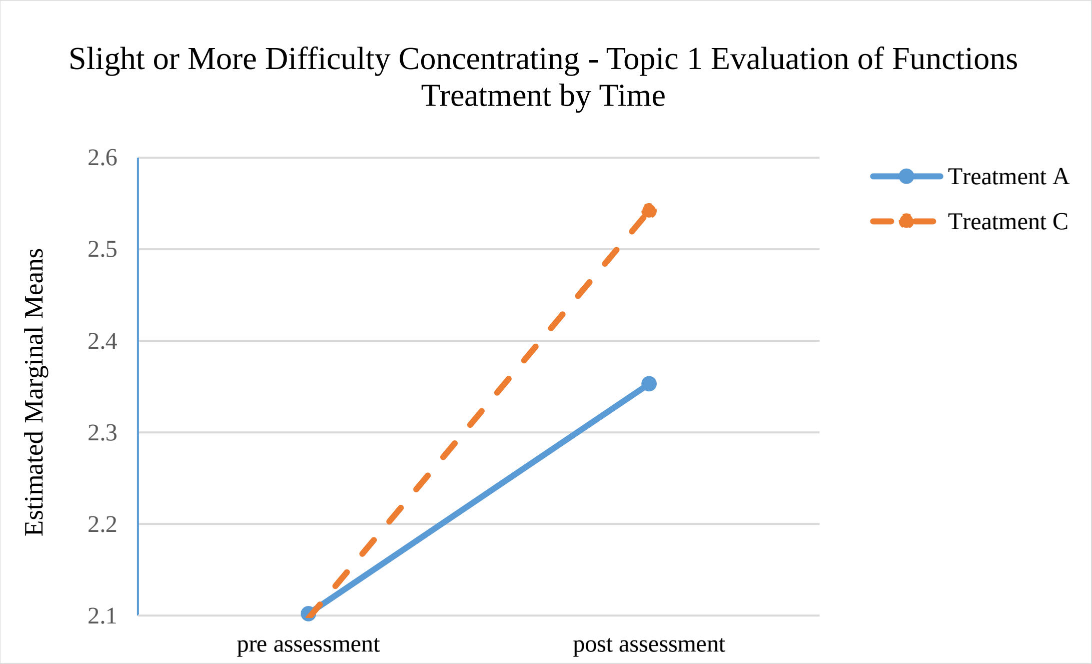


Figure 10 shows that for students who reported slight of more difficulty concentrating, those in Treatment Group C (PDM) showed greater improvement from pre to post assessment than those in Treatment Group A (Traditional Non-PDM).

Figure 10

Some Difficulty Concentrating by Treatment for Topic 1



Results for Perceived Difficulty Seeing and Perceived Difficulty Reading

No statistically significant findings related to the effect of perceived difficulty seeing or perceived difficulty reading were found in this study. Within these categories of perceived difficulty, time from pre to post assessment and self-efficacy made consistent and significant contributions to student performance in both treatment groups for all four dependent variables. These findings are summarized in Tables 13 through 16.

Table 13

Descriptive Statistics of Pre & Post Scores – Difficulty Seeing

Variable	N	Pre-Assessment		Post-Assessment	
		Mean	SD	Mean	SD
		All Topics Combined			
Treatment A [a]					
No difficulty	205	4.87	2.27	6.73	2.25
Slight or more difficulty	70	4.70	2.31	6.64	2.16
Treatment C [a]					
No difficulty	179	4.88	2.18	6.87	2.09
Slight or more difficulty	74	4.57	2.55	6.96	2.12
		Topic 1 Evaluation of Functions			
Treatment A [b]					
No difficulty	267	1.99	0.99	2.39	0.83
Slight or more difficulty	93	1.47	1.01	2.20	0.93
Treatment C [b]					
No difficulty	230	1.68	1.02	2.28	0.91
Slight or more difficulty	97	1.53	1.09	2.27	0.88
		Topic 2 Composition of Functions			
Treatment A [b]					
No difficulty	267	1.54	1.03	2.19	0.92
Slight or more difficulty	93	1.47	1.01	2.20	0.93
Treatment C [b]					
No difficulty	230	1.68	1.02	2.28	0.91
Slight or more difficulty	97	1.53	1.09	2.27	0.88
		Topic 3 Inverse Functions			
Treatment A [b]					
No difficulty	267	.91	1.00	1.99	0.96
Slight or more difficulty	93	0.76	0.96	1.85	0.90
Treatment C [b]					
No difficulty	230	0.80	0.90	1.93	1.00
Slight or more difficulty	97	0.75	0.96	1.91	0.90

[a] *Scores range from 0 to 9*

[b] *Scores range from 0 to 3*

Table 14

Summary of Mixed ANOVA for Difficulty Seeing by Treatment

Variable	Total Topics		Topic 1 Evaluation of Functions		Topic 2 Composition of Functions		Topic 3 Inverse Functions	
	F (p)	*ES*	*F (p)*	*ES*	*F (p)*	*ES*	*F (p)*	*ES*
				Within Subjects				
Time (pre to post)	103.59 (<.001)	.165	68.82 (<.001)	.092	170.92 (<.001)	.200	276.74 (<.001)	.289
Self-efficacy by time	13.19 (<.001)	.025	15.51 (<.001)	.022	24.16 (<.001)	.034	11.16 (.001)	.016
Treatment by time	2.44 (.119)	.005	<.001 (.982)	<.001	.004 (.947)	<.001	.552 (.448)	.001
Difficulty by time	.701 (.403)	.001	.666 (.415)	.001	.813 (.368)	.001	.001 (.971)	<.001
Treatment by difficulty by time	.554 (.457)	.001	.098 (.754)	000	.045 (.831)	<.001	.083 (.074)	<.001
				Between Subjects				
Self-efficacy	115.75 (<.001)	.181	151.16 (<.001)	.393	160.86 (<.001)	.191	25.853 (<.001)	.037
Treatment AC	.008 (.930)	<.001	.827 (<.363)	.001	0.907 (.341)	.001	.523 (.470)	.001
Difficulty seeing	.097 (.756)	<.001	<.001 (.992)	<.001	<.001 (.991)	<.001	1.577 (.210)	.002
Treatment AC by difficulty interaction	.004 (.952)	<.001	.997 (.318)	.001	.007 (.933)	<.001	.348 (.555)	.001

Table 15

Descriptive Statistics of Pre & Post Scores for Difficulty Reading

Variable	N	Pre-Assessment		Post-Assessment	
		Mean	SD	Mean	SD
All Topics Combined					
Treatment A [a]					
No difficulty	234	4.90	2.28	6.78	2.17
Slight or more difficulty	41	4.44	2.24	6.27	2.45
Treatment C [a]					
No difficulty	224	4.77	2.28	6.90	2.03
Slight or more difficulty	29	4.93	2.45	6.90	2.58
Topic 1 Evaluation of Functions					
Treatment A [b]					
No difficulty	308	1.98	1.03	2.42	0.81
Slight or more difficulty	52	1.88	1.00	2.19	1.03
Treatment C [b]					
No difficulty	281	2.07	0.93	2.48	0.80
Slight or more difficulty	46	2.02	1.00	2.46	0.89
Topic 2 Composition of Functions					
Treatment A [b]					
No difficulty	308	1.51	1.03	2.21	0.93
Slight or more difficulty	52	1.54	1.00	2.12	0.90
Treatment C [b]					
No difficulty	281	1.63	1.03	2.29	0.87
Slight or more difficulty	46	1.63	1.08	2.20	1.09
Topic 3 Inverse Functions					
Treatment A [b]					
No difficulty	308	0.90	0.99	1.96	0.95
Slight or more difficulty	52	0.69	0.96	1.90	0.89
Treatment C [b]					
No difficulty	281	0.80	0.91	1.94	0.97
Slight or more difficulty	46	0.67	0.92	1.83	1.00

[a] *Scores range from 0 to 9*

[b] *Scores range from 0 to 3*

Table 16

Summary of Mixed ANOVA for Difficulty Reading Words and Symbols by Treatment

Variable	All Topics Combined		Topic 1 Evaluation of Functions		Topic 2 Composition of Functions		Topic 3 Inverse Functions	
	F (p)	*ES*	*F (p)*	*ES*	*F (p)*	*ES*	*F (p)*	*ES*
				Within Subjects				
Time (pre to post)	94.72 (<.001)	.153	57.67 (<.001)	.078	131.97 (<.001)	.162	240.07 (<.001)	.260
Self-efficacy by time	14.51 (<.001)	.027	16.63 (<.001)	.024	26.57 (<.001)	.038	10.72 (.001)	.015
Treatment by time	.686 (.408)	.001	.201 (.654)	<.001	.032 (.857)	<.001	.022 (.882)	<.001
Difficulty by time	.820 (.365)	.002	.789 (.375)	.001	2.46 (.117)	.004	.114 (.736)	<.001
Treatment by difficulty by time	.015 (.903)	<.001	.523 (.470)	.001	.014 (.907)	<.001	.218 (.640)	<.001
				Between Subjects				
Self-efficacy	114.90 (<.001)	.180	150.40 (<.001)	.373	163.06 (<.001)	.193	25.23 (<.001)	.036
Treatment AC	.679 (.410)	.001	3.219 (.073)	.005	1.19 (.276)	.002	.667 (.414)	.001
Difficulty Reading	.316 (.574)	.001	.015 (.903)	<.001	1.11 (.292)	.002	1.13 (.287)	.002
Treatment AC by difficulty interaction	1.04 (.308)	.002	1.059 (.304)	.002	.196 (.658)	<.001	<.001 (.993)	<.001

Chapter Five - Discussion

Introduction

Equity and inclusion in our education system are topics of increasing prominence in our national discourse. Over the last 70 years, both the civil rights and disability rights movements have played a key role in shining a light on educational inequality. The stakes could not be higher. Quality of life is directly related to quality of education and a poor education puts people, especially those with disabilities, at risk of poverty and dependence.

Issues of equity and inclusion are particularly important with regard to STEM education. An increasing number of jobs in our technologically driven economy are dependent on a STEM prepared workforce and people with disabilities continue to be underrepresented in those jobs. Mathematics is the gateway to all STEM fields and must, therefore, be a focal point of efforts to improve equity and inclusion. The barriers that people with disabilities face in math education severely limit their access to STEM fields and STEM related jobs.

Leveling the educational playing field is contingent upon the development of effective curricula and tools created within a Universal Design for Learning framework. Key to the success of these efforts is the incorporation of a User Centered Design perspective during development. The needs, wants, and perspectives of end users must be central to the development process if the strategies, tools, and technologies developed are to benefit those for whom they are designed (Dorrington et al., 2016; Norman, 2013).

Summary of Study

The purpose of this study was twofold. First, we sought to learn about the experiences that SBVIs had with mathematics education and Process-Driven Math using a qualitative User Centered Design approach. Second, evidence was sought to determine whether or not the visually adapted Process-Driven Math could improve learning outcomes for students with and without indications of disability.

The Process-Driven Math method was originally designed to support the learning of one student who is blind and unable to write, type, or speak above a whisper. This fully audio method of instruction and assessment provided the tools necessary for this student to fully control all of the intellectual processes involved in simplifying complex algebraic expressions. The process involved an iterative question and answer process dependent on another person functioning as a reader/scribe (Gulley et. al., 2017). The reader/scribe would chunk the math into its component substructures, or chunks, and then describe those chunks using appropriate math vocabulary. The student could then choose which chunk of the problem to work in, ask for the numerical values and variables within the chunk, and give instructions for mathematical transformations that the reader/scribe would then incorporate into the expression.

In the qualitative arm of this study, a User Centered Design approach was implemented. Students who are blind or visually impaired were given multiple opportunities to interact with a reader/scribe using Process-Driven Math. During interviews and focus groups, students were encouraged to share their feelings about mathematics and their experiences that shaped those feelings. This information provided context for how students perceived their experiences with PDM. The body of data from this phase of the study sheds light on the needs, wants, and preferences of SBVIs with regard to mathematics education and interventions. Their stories provide understanding about the environments and tools that these SBVIs think are most important for their learning.

The adaptation of the Process-Driven Math method to a visual platform provided the opportunity for learners who are sighted to interact with the method. In the experimental arm of the study, students were randomly assigned to two treatment groups – one was provided traditional instructional materials and the other was provided PDM instructional materials. Participants indicated their degree of difficulty with several different daily activities which could be indicative of various disabilities. An experiment was run to determine the extent to which performance was dependent on the type of treatment and perceived difficulty. The results indicate which students might benefit from further development of the visually adapted Process-Driven Math tool.

Findings from Qualitative Research

The findings from the interview and focus groups indicated that the student participants valued their education and wanted to learn mathematics in an environment that was conducive to their specific needs. The students who participated in the qualitative study had varied educational backgrounds. Some students had gone to public schools for several years and then transferred to their school for the blind. Others attended both their local public school and their school for the blind concurrently, taking different classes at each location. A few students had been at their school for the blind since they started formal schooling.

Some students who had attended public schools indicated that their schools did not provide adequate accommodations for their visual disability. These students shared frustrating and painful experiences of not having access to the curriculum because they couldn’t read the board or because content that was written for the sighted students was not provided verbally or in braille for the student who was blind. The students who shared these stories indicated that they had been the only student at their school who was blind or visually impaired. It is likely that these difficult experiences were instrumental in producing the strong feelings that they expressed about the tools and teaching styles that would benefit them the most.

Student feedback about their experiences with PDM was varied and most elements of the method were not universally agreed upon. Some students were very positive about the way the problems were laid out and the logical, stepwise approach to simplifying expressions while other students were frustrated by the complexity of the communication. Some students liked the slower pace because it gave them adequate time to think through their options and choose a strategy beforehand while others felt the slower pace was a drawback that caused exasperation. This data elucidates the challenge of building curricula within a UDL framework because supports that benefit one student may end up creating additional challenges for another (Griful-Freixenet et al., 2017).

Whether they liked or disliked PDM, almost all of the students expressed a desire to have a visual or tactile component available to accompany the audio delivery. Originally, PDM was developed to meet the needs of one student who is blind and unable to write or type. His particular set of abilities necessitated a completely audio-based mathematics tool in which all mathematical content was read to him. However, if a linear reading of complex math expressions had been employed, it would have overloaded this student’s working memory. Instead, chunking was incorporated into the verbal rendering of the mathematics to reduce the cognitive load on his working memory (Cowan, 2010; Gobet et al., 2001). Conversely, for students who could write or type, the fully audio rendering made it more difficult for them to apprehend the meaning of the math problems, even with chunking employed.

The feedback these students provided through the User Centered Design framework reinforces the purpose of selecting this methodological approach. UCD based development is agile and allows student feedback to override preconceived ideas and ultimately change the direction of development (Bordac & Rainwater, 2008). Some students indicated an appreciation for the chunking employed in PDM as well as a frustration over the absence of written or tactile access. Their input indicates there may be benefit in intertwining the audio delivery of PDM along with the visual and tactile PDM supports for a multisensory delivery of math content. Thus, adding the visual and tactile components while maintaining the chunking inherent to the PDM method would be a high priority in subsequent iterations of development. In so doing, the method would have greater flexibility and move closer toward UDL principles by providing multiple means of engagement, representation, and action and expression (CAST, 2018).

When giving their opinions about the best strategies for teaching SBVIs, most said that individualized instruction was essential. When talking about their current math classes, students said they liked the one-on-one attention that was available to them when they needed it. Students stressed the importance of knowing that they could get individualized help when they were stuck.

The qualitative research was conducted to gain a better understanding of the needs and preferences of SBVIs with regard to math education. They are the experts of their own blindness or visual impairment and are most qualified to indicate what helps them learn. Understanding the struggles and barriers they have faced provides important information about which barriers should be prioritized when developing new tools. Their opinions about PDM – what they liked, what they disliked, and what they would change – will determine the ongoing development of PDM to support the mathematical learning of SBVIs.

Limitations of Qualitative Research

Students at the schools for the blind had intermittent exposure to five PDM lessons across each semester and three of the lessons were conducted over Skype or speakerphone. The lessons taught over Skype or speakerphone were not as effective as in-person teaching because the engaging dynamic between teacher and students is not the same. In addition, there were occasional difficulties with the audio transmission during the remote lessons. All five lessons taught in-person and on consecutive days would have been a more effective way to give students an immersive experience with PDM.

Across all three schools there was a wide age range of the participants. A more narrow case selection at the class level would have allowed for a more similar teaching experience for the participants at all three schools.

Findings from Experimental Research

The experiment run at post-secondary institutions was designed to determine whether or not PDM instruction would lead to better outcomes on performance assessments for students with and without perceived difficulties. Questions on a student survey asked student participants to indicate their degree of difficulty with everyday tasks in seven categories that relate to disabilities. Despite having almost 700 students enrolled, there were insufficient numbers of students reporting difficulty in four of the categories and those categories were eliminated from the data analysis. Four of the categories of perceived difficulty that were completely excluded due to an insufficient sample size were hearing, walking, lifting or carrying, and writing or typing. Since students with disabilities have lower high school graduation rates than students without disabilities, it is not surprising that the incidence of some categories of disability are especially low at post-secondary institutions (NFB, 2009; Shifrer et al., 2013).

The three categories that remained a part of the analysis were concentrating, seeing, and reading words and symbols. Students who reported difficulty concentrating performed better with Process-Driven Math instruction than with traditional classroom instruction methods, with a small effect size. No differences were observed in the performance outcomes by treatment of students who reported difficulty seeing and students who reported difficulty reading words and symbols. The number of students who reported difficulty in the categories of seeing and reading words and symbols was smaller than the number of students who reported difficulty concentrating. It is possible that larger sample sizes of students reporting difficulties with seeing and reading words could have led to different experimental outcomes than what was seen in this study.

Limitations from Experimental Research

The decision to provide all content through online companion Functions Supplement courses had substantial benefits in terms of increasing the sample size, random assignment, and mitigating the effects of class differences. The alternate design was to have all students experience the teaching materials during in-person classroom instruction. While the second model would have provided greater assurance that students had experienced the treatment, it also would have introduced significant complications in terms of communication with instructors and ensuring appropriate technology in all classrooms. Using the online companion course model meant that the research content was accessed independently and outside of the classroom. Self-report was relied upon to indicate that participants had actually accessed the materials.

The questions in the survey about perceived difficulties with various everyday tasks were designed to indicate the potential that a respondent could have a disability. This approach was taken because people may not know they have a disability or they may not have had that disability formally diagnosed. Students who indicated that they experienced difficulty in a category may or may not have actually had a corresponding disability.

Finally, the degree of exposure that students had with both the control and PDM methods for the five modules was not immersive. Each teaching video was approximately 10 minutes and each animated PowerPoint video was approximately 2 minutes, which is far less time than one class instructional period. If greater exposure to the control and experimental treatments had been provided, it is possible that more statistically significant differences between students in the two treatment groups would have been observed.

Future Research

Future research at schools for the blind should continue using both UCD and UDL frameworks and focus on the development of PDM lessons with multiple options for companion materials to accompany the audio PDM delivery. These would include: braille rendered companion materials (both UEB Math and Nemeth Code, depending on a student's training), altered text size companion materials, and tactile manipulative companion materials. Exposure to multiple days of the audio PDM lessons with companion materials matched to each student's specific needs would be followed by a performance assessment using an alternating treatment single case study design. After the conclusion of on-site research activities, individual interviews could be scheduled to add context to the assessment data. Results from this type of mixed method study would inform further development of PDM, including the possibility of a PDM software application with multiple accessibility companion options built in from the ground up.

A longitudinal study exploring the impact of repeated PDM exposure with the same group of students at a school for the blind across several consecutive grades would also be of value. A study that delivered PDM lessons intermittently from the 6th through the 9th grades would provide insight into how exposure to the method in earlier grades could impact learning once students begin to encounter more complex algebraic content in the 9th grade. Data from 9th graders with no prior exposure to the PDM method could be compared to data from 9th graders with repeated exposure over several years.

In addition, since SBVIs placed a high priority on their interactions with teachers, it follows that teachers should be incorporated into future research efforts. Tools that are developed to support SBVIs need to be valued by teachers because they will be responsible for incorporating them into the classroom experience. A Human Centered Design research approach (as opposed to a User Centered Design approach) focusses on learning about the perspectives of both students and teachers because both are invested in the learning experiences that take place within classrooms (Shivers-McNair et al., 2018).

Future research should also include additional semesters of the current experimental study in order to obtain data about students who reported difficulty in the four categories that didn’t render a sufficient number of participants for data analysis. Another approach for collecting data from students in low incidence disability groups at post-secondary institutions is an alternating treatment single case study research design. In addition, secondary institutions, where the incidence of students with some of these perceived difficulties may be higher could also be considered as sites for future research.

Building on the foundation of this study, additional research could focus on providing greater saturation with the PDM method for students who report difficulty concentrating to see if increased exposure increases the effect size of the PDM treatment. Qualitative data collection should also be included to provide context for differences observed between treatment groups. In addition, research that incorporates physical manipulatives used by students who report difficulty concentrating should be undertaken. A few studies indicate that the use of tactile elements during math instruction may provide some level of improvement in the math performance in children with concentration deficits (Matthews et al., 2020; Suneeta et al., 2007). Future studies could look at the effect of PDM instruction with and without manipulatives to determine whether or not the tactile component improves outcomes for these students.

Implications

The implications of improving math education for students, both those with and without disabilities, are significant. A PDM learning support tool with multiple accessibility options built in from the ground up holds great promise for learning and inclusion. Several researchers and educators familiar with the project have expressed optimism that introducing the PDM method to students earlier in their education may improve long term learning outcomes. In PDM chunking, the mathematical content in the component substructures of a problem is initially hidden and students are only given the broad landscape of the problem. Without the distraction of hearing or seeing specific content within the chunks, students can focus on the strategy of their first step and then engage with the actual numbers, variables, and symbols. Although the chunking was primarily developed to protect a student’s working memory from being overwhelmed, it affords other benefits to learners as well. Chunking with appropriate math vocabulary may help students develop a deeper understanding of the structural relationships that exist between elements in mathematical expressions as well as the rules that define the interactions between those elements.

Difficulty concentrating is a prevalent condition that makes learning more difficult for many students. The most recent data available from the Centers for Disease Control and the National Institute of Mental Health indicate that 9.4% of children and 4.4% of adults have been diagnosed with Attention Deficit Hyperactivity Disorder (ADHD) (Centers for Disease Control and Prevention [CDC], 2020; National Institute of Mental Health [NIMH], 2017). Finding new methods of math instruction that improve outcomes for these students could also improve their overall educational attainment. In the experimental portion of the study, students who experienced difficulties with concentration learned specific algebra content better with the visually adapted PDM method than with traditional teaching. Although the effect of the gains for students with perceived difficulty concentrating in the PDM treatment group was modest, the prevalence of children and young adults who have difficulty concentrating suggests that improving math education for these students could have a broad and significant impact.

A PDM experience that is customizable for each individual would promote shared educational spaces for all learners. Prioritizing those elements that emerged as themes within this research would help ensure the inclusion of SBVIs in classrooms in both mainstream schools and schools for the blind. During individual practice, control of the technology that either speaks or displays the math is important because each individual’s preferred pace plays a role in their learning. Users should have the option to make multiple smaller transformations in one step and PDM manipulatives should be available for learners who benefit from a tactile approach.

Ultimately, if PDM were built into a flexible software application with multiple accessibility options, then it could further enhance the mathematical learning experiences of students who have difficulty concentrating, SBVIs, and other learners as well. Not only could diverse learners all use the same base technology in the same shared spaces, but more SBVIs would be able to access complex mathematical content independently from home. Daily practice outside of math class is an important part of building mathematical competencies; however, many SBVIs who are not proficient with Nemeth Code or Math UEB are unable to reinforce the day’s learning with independent practice at home. A PDM software application could render complex mathematical expressions without overwhelming a student’s working memory. Students could opt for the fully audio delivery or a combined delivery that incorporated the visually adapted PDM, refreshable braille, or enlarged text. Color, font size, and font type could all be customizable to fit the specific preferences of each learner. A software application would have to deliver the math in an accurate and accessible format, allowing for user-controlled simplifications in individual chunks while recording all steps taken.

This type of robust application could potentially move the needle toward achieving some of the goals outlined in the National Science Board's Vision 2030 report (NSB, 2020). More students would have the tools for success in mathematics which could mean more students meeting their core math requirement in college, more students engaging in STEM content, and ultimately more diversity in jobs requiring STEM knowledge.

Discussion

From its inception, Process-Driven Math was developed with the intent of serving many different types of students through its development within frameworks of both Universal Design for Learning and User Centered Design practice. The fully audio method was initially developed to support the learning of one student who is blind, unable to type or write, and unable to speak above a whisper. This student used the fully audio Process-Driven Math method to access all course content for mathematics courses and was successful in both Intermediate Algebra and Pre-Calculus with Trigonometry. Subsequently a second student with a visual impairment used the fully audio Process-Driven Math method to access the majority of her course content in College Algebra and she was also successful in her math course. I wanted to have a better understanding of the needs and preferences of a larger cross section of SBVIs so they could be incorporated into the ongoing development of PDM. The qualitative research design was developed as a means to increase our knowledge about the tools that would most benefit SBVIs in their math education.

Working with SBVIs at the schools for the blind revealed a very diverse group of students with a broad range of accessibility needs and preferences. Some students have visual impairments that require low light inside a room while others need bright light to maximize their vision. Some need their print extra small because they have lost peripheral vision while others need their print enlarged. Students who need enlargement require different degrees of magnification. Some have vision that is decreasing so the tools that they use are changing over time. Those who are blind use a completely different set of tools than those who have visual impairments.

The data from the qualitative findings indicates that there is potential for PDM to be modified to be more useful to a broader cross section of SBVIs. The students we worked with had very limited exposure to PDM, a method that is significantly different from anything they had previously experienced. Several found merit in the approach; however, the majority of the students said they wanted the written or braille counterpart to accompany the audio PDM that they had experienced. Creating lessons that incorporate the written and braille companion elements is likely to be more useful and appealing to SBVIs than the fully audio PDM alone.

The PDM method was visually adapted to support the learning of sighted students. A student with dyslexia, dysgraphia, and dyscalculia used the visually adapted PDM method. The student used the same PDM manipulatives that are seen in the PDM teaching videos created for the Functions Supplement C course used in this study. The manipulatives are made of magnetic, colored dry erase material cut into shapes that represent specific mathematical elements. The manipulatives adhere to magnetic white boards and the math content is written directly on the wipe-off surfaces of the manipulative shapes. This student used these PDM tools to access almost all course content in Intermediate Algebra and College Algebra and was successful in both courses. He appreciated having both the audio rendering of the math accompanied by the visual and tactile components. The student indicated that the PDM audio rendering of the math provided important confirmation that he had accurately processed the written content. He also indicated that using the manipulatives to chunk problems let him focus on one area of the problem at a time without distraction from other areas.

After working with the students at the schools for the blind and hearing their perspectives through the interview data, it seems plausible that many of them could benefit from interacting with the manipulatives and the visually adapted PDM method. The manipulatives can be customized and the size of the writing applied to them can be adjusted depending on students' needs. Some students in the focus groups expressed a desire to have something to do with their hands during the lesson. The manipulatives would serve a two-fold purpose. First, they would provide a surface for written math content. Second, students who indicated a preference for a kinesthetic component in their mathematics lessons would benefit from the movement involved in using the manipulatives.

The intended goal for Process-Driven Math is the development of a flexible tool with an array of accessibility options that can meet the mathematical learning needs of a diverse cross section of students. Students with disabilities have struggled with exclusion and marginalization in the American education system for over a century (Zukauskas, 2019). Rooted in ideas of social constructivism promoted by Vygotsky, modern movements of inclusion seek to promote the understanding that shared educational spaces benefit all learners – those with an without disabilities (CAST, 2018). A student participant we interviewed at one of the schools for the blind provides a glimpse of what that kind of environment is like:

> Some of the good experiences I've had with math are at my school when we are learning different things and people have different learning styles. And we accommodate to help each person with their individual learning style so that everyone kind of figures it out

> even if it’s a self-paced kind of thing. We all figure it out eventually, which made it easier for me to associate math with a positive thing.

Often in discourse about shared educational spaces, the primary focus is the perceived benefit for students with disabilities. I think many people, those with and without disabilities, would consider the supportive and enriching environment described by this student to be an ideal setting for learning mathematics. In many ways it describes an environment in which the objectives of the UDL framework are met. The picture we see from this quote shows us a diverse population of students sharing the same educational space. Their learning is self-paced and they are using a broad range of tools and assistive technologies which give the group multiple means of representation, engagement, and expression (CAST, 2018). The students are modelling good learning behaviors for one another and even providing scaffolding for each other, key tenets of Vygotsky’s cultural historical theory (Crain, 2005; Gredler, 2005). They are experiencing success and encouraging each other, conditions that motivate students and lead to increased self-efficacy (Bandura, 1977). The environment described by this student is in a school for the blind. The goal of developing a math education tool within a Universal Design for Learning framework and from a User Centered Design perspective is to promote the creation of similar shared educational spaces in many different types of schools so that in every educational setting, diverse learners can share educational spaces and learn together in inclusive, enriching environments.

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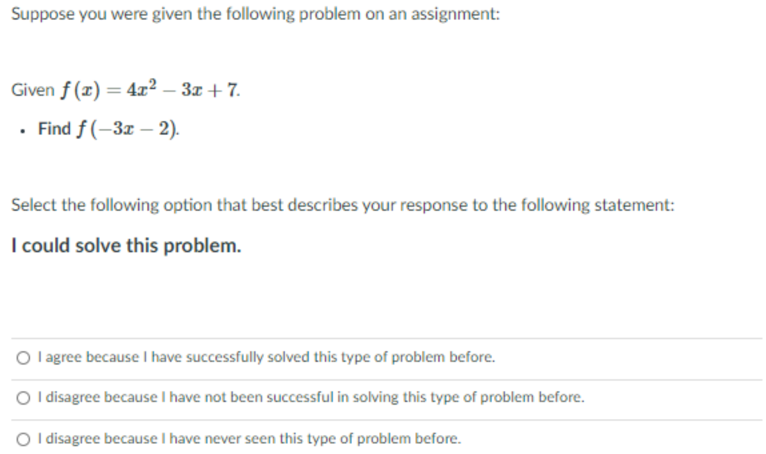
Appendix

Pre and Post Assessment for Experimental Research

Students were unable to go back to previous problems while taking the assessment.

Self-Efficacy Question 1



Suppose you were given the following problem on an assignment:

Given $f(x) = 4x^2 - 3x + 7$.

- **Find $f(-3x-2)$.**

Select the following option that best describes your response to the following statement:

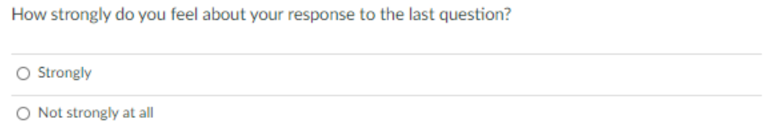
I could solve this problem.

○ I agree because I have successfully solved this type of problem before.

○ I disagree because I have not been successful in solving this type of problem before.

○ I disagree because I have never seen this type of problem before.

Self-Efficacy Question 2

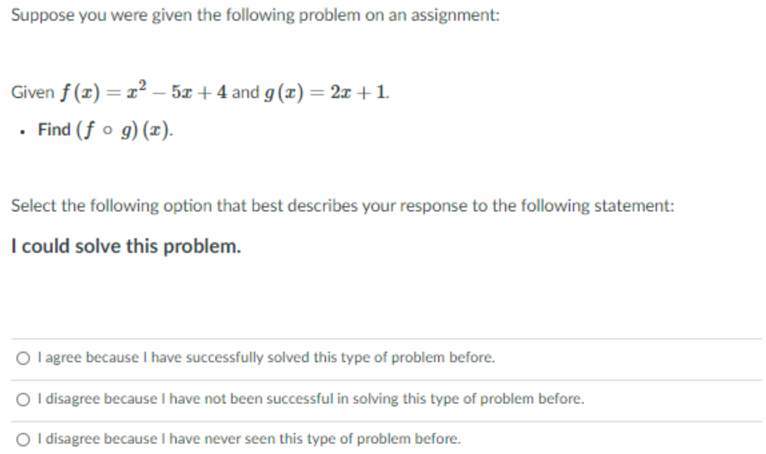


How strongly do you feel about your response to the last question?

○ Strongly

○ Not strongly at all

Self-Efficacy Question 3



Suppose you were given the following problem on an assignment:

Given $f(x) = x^2 - 5x + 4$ and $g(x) = 2x + 1$.

- **Find $(f \circ g)(x)$.**

Select the following option that best describes your response to the following statement:

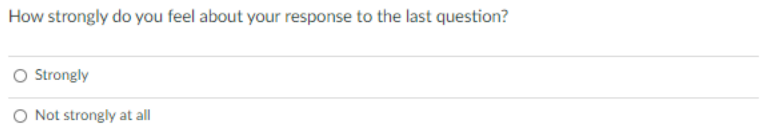
I could solve this problem.

○ I agree because I have successfully solved this type of problem before.

○ I disagree because I have not been successful in solving this type of problem before.

○ I disagree because I have never seen this type of problem before.

Self-Efficacy Question 4

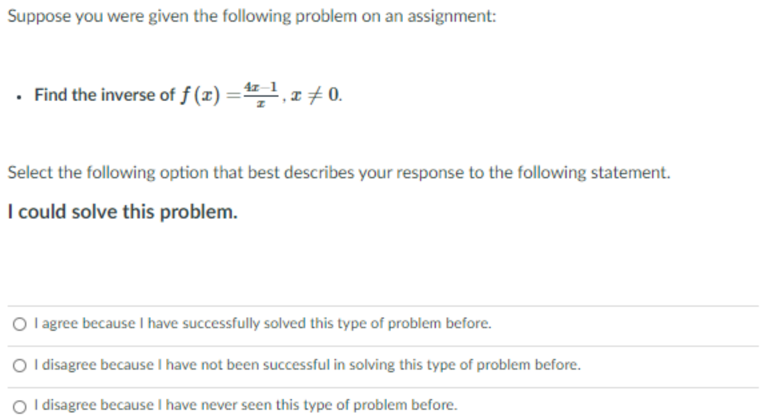


How strongly do you feel about your response to the last question?

○ Strongly

○ Not strongly at all

Self-Efficacy Question 5



Suppose you were given the following problem on an assignment:

- **Find the inverse of $f(x) = \frac{4x-1}{x}$, $x \neq 0$.**

Select the following option that best describes your response to the following statement.

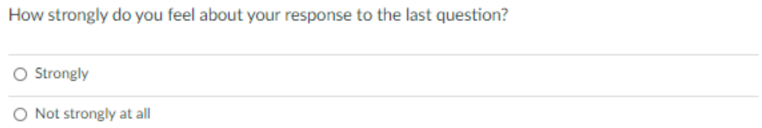
I could solve this problem.

○ I agree because I have successfully solved this type of problem before.

○ I disagree because I have not been successful in solving this type of problem before.

○ I disagree because I have never seen this type of problem before.

Self-Efficacy Question 6

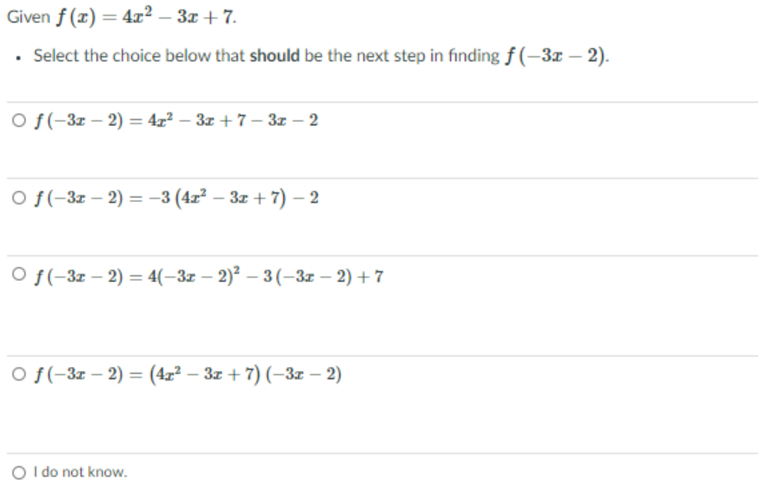


How strongly do you feel about your response to the last question?

○ Strongly

○ Not strongly at all

Assessment Topic 1 Question 1



Given $f(x) = 4x^2 - 3x + 7$.

- Select the choice below that **should** be the next step in finding $f(-3x-2)$.

○ $f(-3x-2) = 4x^2 - 3x + 7 - 3x - 2$

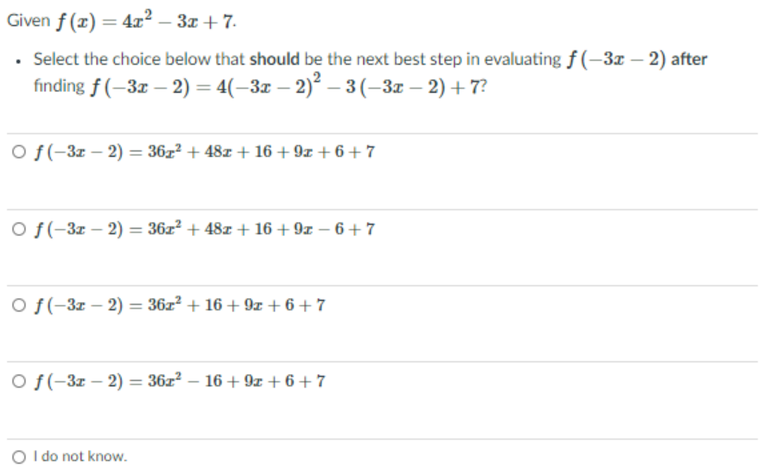
○ $f(-3x-2) = -3(4x^2 - 3x + 7) - 2$

○ $f(-3x-2) = 4(-3x-2)^2 - 3(-3x-2) + 7$

○ $f(-3x-2) = (4x^2 - 3x + 7)(-3x-2)$

○ I do not know.

Assessment Topic 1 Question 2



Given $f(x) = 4x^2 - 3x + 7$.

- Select the choice below that **should** be the next best step in evaluating $f(-3x - 2)$ **after** finding $f(-3x-2) = 4(-3x-2)^2 - 3(-3x-2) + 7$?

○ $f(-3x-2) = 36x^2 + 48x + 16 + 9x + 6 + 7$

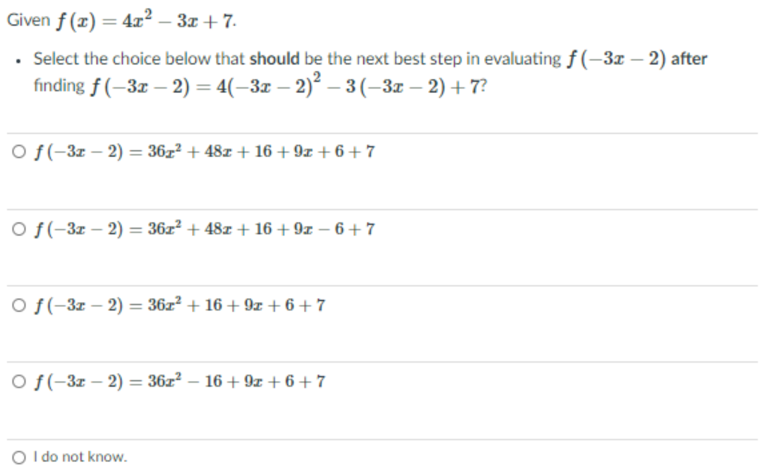
○ $f(-3x-2) = 36x^2 + 48x + 16 + 9x - 6 + 7$

○ $f(-3x-2) = 36x^2 + 16 + 9x + 6 + 7$

○ $f(-3x-2) = 36x^2 - 16 + 9x + 6 + 7$

○ I do not know.

Assessment Topic 1 Question 3



Given $f(x) = 4x^2 - 3x + 7$.

- Select the choice below that **should** be the next best step in evaluating $f(-3x - 2)$ **after** finding $f(-3x-2) = 4(-3x-2)^2 - 3(-3x-2) + 7$?

○ $f(-3x-2) = 36x^2 + 48x + 16 + 9x + 6 + 7$

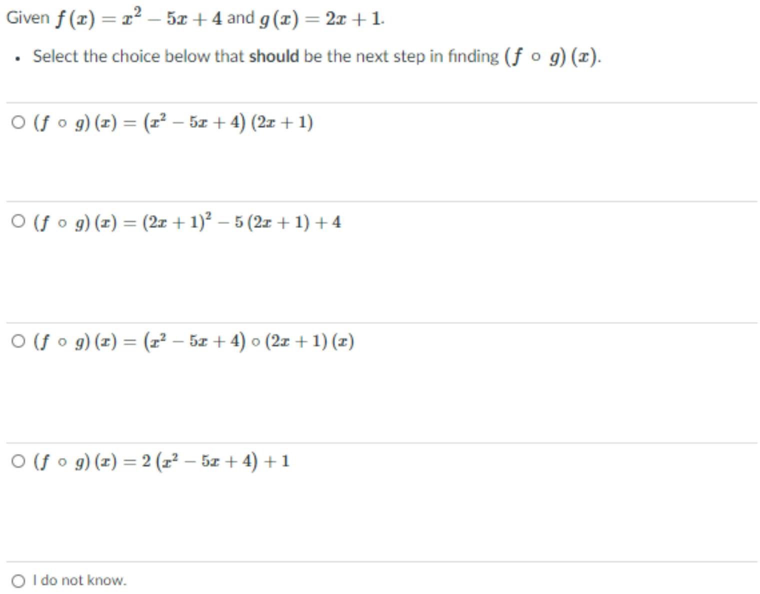
○ $f(-3x-2) = 36x^2 + 48x + 16 + 9x - 6 + 7$

○ $f(-3x-2) = 36x^2 + 16 + 9x + 6 + 7$

○ $f(-3x-2) = 36x^2 - 16 + 9x + 6 + 7$

○ I do not know.

Assessment Topic 2 Question 1



Given $f(x) = x^2 - 5x + 4$ and $g(x) = 2x + 1$.

- Select the choice below that **should** be the next step in finding $(f \circ g)(x)$.

○ $(f \circ g)(x) = (x^2 - 5x + 4)(2x + 1)$

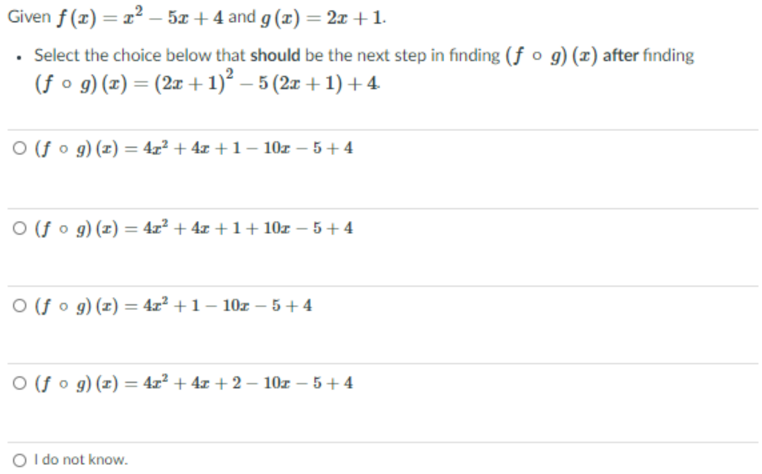
○ $(f \circ g)(x) = (2x + 1)^2 - 5(2x + 1) + 4$

○ $(f \circ g)(x) = (x^2 - 5x + 4) \circ (2x + 1)(x)$

○ $(f \circ g)(x) = 2(x^2 - 5x + 4) + 1$

○ I do not know.

Assessment Topic 2 Question 2



Given $f(x) = x^2 - 5x + 4$ and $g(x) = 2x + 1$.

- Select the choice below that **should** be the next step in finding $(f \circ g)(x)$ **after** finding $(f \circ g)(x) = (2x + 1)^2 - 5(2x + 1) + 4$.

○ $(f \circ g)(x) = 4x^2 + 4x + 1 - 10x - 5 + 4$

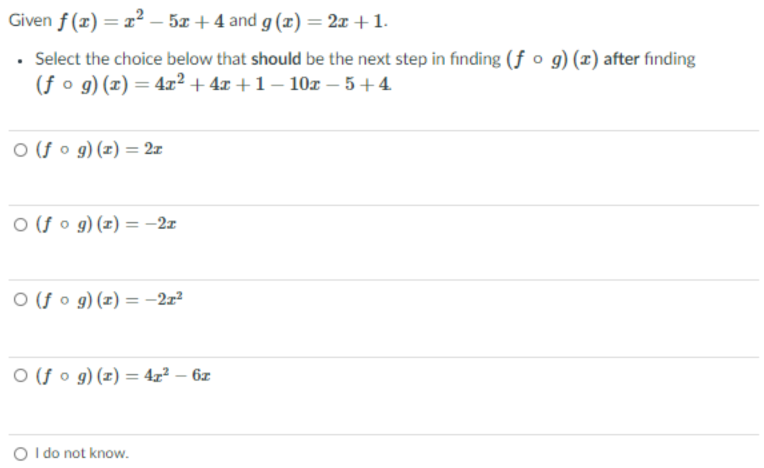
○ $(f \circ g)(x) = 4x^2 + 4x + 1 + 10x - 5 + 4$

○ $(f \circ g)(x) = 4x^2 + 1 - 10x - 5 + 4$

○ $(f \circ g)(x) = 4x^2 + 4x + 2 - 10x - 5 + 4$

○ I do not know.

Assessment Topic 2 Question 3



Given $f(x) = x^2 - 5x + 4$ and $g(x) = 2x + 1$.

- Select the choice below that **should** be the next step in finding $(f \circ g)(x)$ **after** finding $(f \circ g)(x) = 4x^2 + 4x + 1 - 10x - 5 + 4$.

○ $(f \circ g)(x) = 2x$

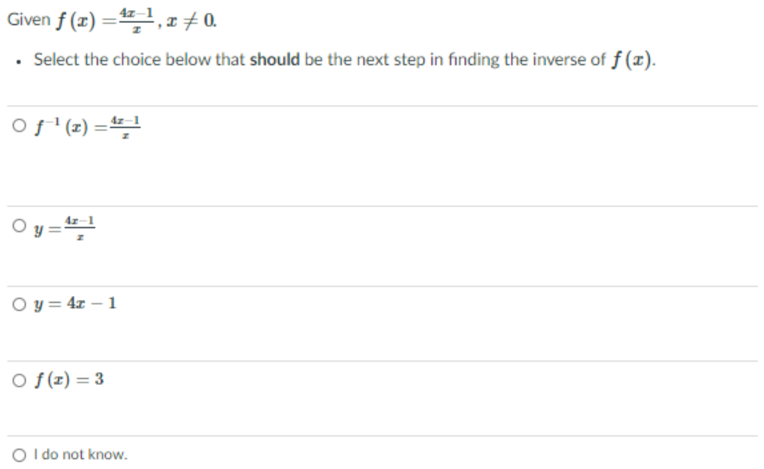
○ $(f \circ g)(x) = -2x$

○ $(f \circ g)(x) = -2x^2$

○ $(f \circ g)(x) = 4x^2 - 6x$

○ I do not know.

Assessment Topic 3 Question 1



Given $f(x) = \frac{4x-1}{x}, x \neq 0$.

- Select the choice below that **should** be the next step in finding the inverse of $f(x)$.

○ $f^{-1}(x) = \frac{4x-1}{x}$

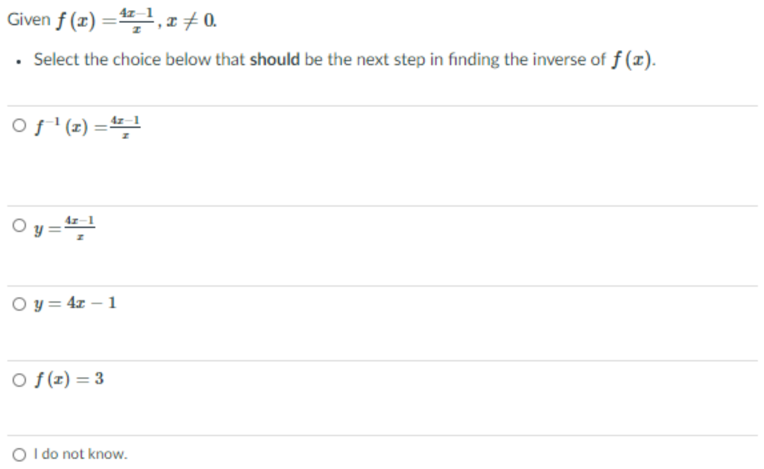
○ $y = \frac{4x-1}{x}$

○ $y = 4x - 1$

○ $f(x) = 3$

○ I do not know.

Assessment Topic 3 Question 2



Given $f(x) = \frac{4x-1}{x}, x \neq 0$.

- Select the choice below that **should** be the next step in finding the inverse of $f(x)$.

○ $f^{-1}(x) = \frac{4x-1}{x}$

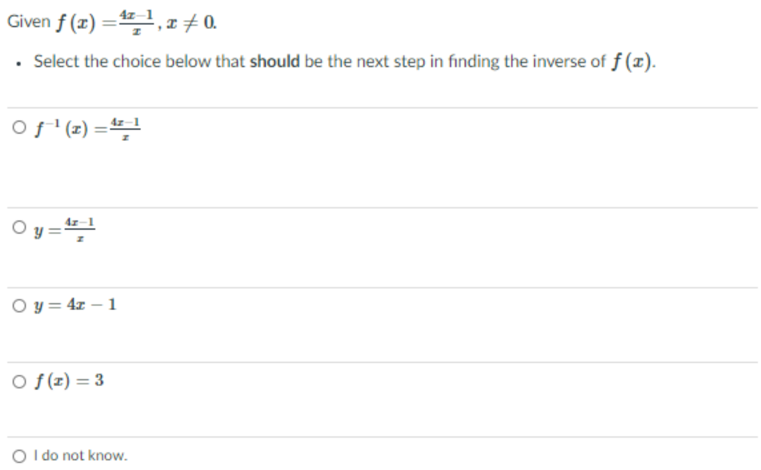
○ $y = \frac{4x-1}{x}$

○ $y = 4x - 1$

○ $f(x) = 3$

○ I do not know.

Assessment Topic 3 Question 3



Given $f(x) = \frac{4x-1}{x}, x \neq 0$.

- Select the choice below that **should** be the next step in finding the inverse of $f(x)$.

○ $f^{-1}(x) = \frac{4x-1}{x}$

○ $y = \frac{4x-1}{x}$

○ $y = 4x - 1$

○ $f(x) = 3$

○ I do not know.