Elementary Teacher Beliefs of Mathematics Discourse During Whole Class Discussions

by

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A dissertation submitted to the Graduate Faculty of
Auburn University
in partial fulfillment of the
requirements for the Degree of
Doctor of Philosophy

Auburn, Alabama
December 11, 2021

Keywords: Mathematical Discourse, Teacher Beliefs

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Abstract

The art of teaching and learning mathematics is rooted in the discourse that takes place between the teacher and students and among students. This discourse creates understanding, recognition of confusion, and corrections that lead to forward progress in learning. However, the type of discourse that occurs in mathematics today differs greatly from classroom to classroom. While some teachers facilitate student led discourse, others rely on the traditional instructional strategy of lecturing.

The purpose of this study was to explore the level of elementary mathematics teacher beliefs about dialogic discourse and teachers' beliefs about the role of mathematical discourse during whole-class discussions. The impact of years of teaching experience, level of degree, teaching status, and grade of instruction on the participants' level of elementary mathematics teacher beliefs about dialogic discourse were investigated. Previous literature was explored to understand the history of mathematics discourse, the development of today's definition of this term, and the importance of comprehending teacher beliefs.

Teachers' beliefs about dialogic discourse were measured using the Beliefs in Mathematics Discourse (BMD) Survey for the 88 participants ($M = 2.54$, $SD = 0.53$), and a semi-structured follow-up interview was conducted with four participants. Years of teaching experience was statistically significant when a simple regression model was used to examine its' relationship to teachers' beliefs about mathematics discourse ($F = 5.365$, $p = .02$). There was a significant difference in BMD survey scores among teachers who had earned different degrees ($F_{4,82} = 3.415$, $p = .012$) and the groups with a different teaching status ($F_{2,82} = 5.746$, $p = .005$). Those with a master's degree scored significantly higher than those with a high school/associate degree ($p = .006$), and in-service teachers scored significantly higher than pre-service teachers (p

= .004). During the interviews, two predominant themes emerged including the role of the teacher and the role of the student during these interactions. Secondary themes emerged within these ideas including teacher questioning, teaching planning, time, student questioning, student engagement, emotions, formative assessments, and inclusivity.

This research study is significant because the participants were able to use their voice to express their opinions and contribute to the research on mathematics discourse. The results confirm that the range of teacher beliefs about mathematics discourse fall along a continuum. The results demonstrate a need for more training on mathematics discourse, especially among pre-service and novice teachers. Future research is needed to explore the impact of professional development and teacher training on teacher beliefs on mathematics discourse to determine the most effective instructional methods.

Acknowledgements

I would like to first thank all of the participants in this study who gave their time during the chaos of the school year to take part in this study, especially during these unprecedented times with all of the added responsibilities and stress of caring for yourself and your students' health in addition to their education. I learned so much from each of you, and without your input, this research study could never have been completed. I would especially like to thank the four interview participants who gave me even more of their time. Their willingness to share added so much to this study, and I appreciate them greatly.

Thank you also to Dr. Jang for allowing me to use her incredible survey instrument, the Beliefs in Mathematics Discourse (BMD) Survey. I am thrilled with the results of this study, and I am grateful you developed this tool and allowed me to use it for this research study.

I would also like to thank my dissertation committee. Thank you for taking the time to provide me with valuable feedback to make this study a worthwhile learning experience. Thank you to Dr. Lakin and Dr. Wang for sharing your methodology expertise. Thank you to Dr. Cardullo and Dr. Tripp for your time and feedback. I truly appreciate all of the wisdom and knowledge that you each shared with me.

I would like to express my sincere gratitude for my dear friend and former coworker Allie Aldrich who, as always, encouraged me and helped me every step of the way. You worked with me side-by-side on this project like so many others. Thank you for sharing your knowledge, giving up your time for this endeavor, and thank you always for your friendship.

I would like to give a special recognition to my advisor, Dr. Megan Burton. Words cannot describe how thankful I am to have you as my advisor. You recommended me for scholarships and provided me with valuable advice for my dissertation and my future career.

You are dedicated, knowledgeable, and you were always in my corner cheering me on each step of the way. I never ceased to be amazed by how quickly you would respond to my emails to ensure I had everything I needed to keep moving forward to complete my dissertation. You went above and beyond your responsibilities, and words cannot express my gratitude for all that you have done for me.

I would also like to thank my family. My dad, an Auburn graduate, taught me to believe in Auburn and love it. I am beyond happy to finally earn a degree from Auburn University. My mom spent numerous hours caring for my children so I could write, and I will always remember and appreciate what she did to help me finish this research study. Thank you to my brother Blake who encouraged me along the way and shares a love for Auburn.

To my children, Audrey and KC, I hope you know that you can accomplish anything, and I will be by your side through it all. I love you both always. To my husband, Kent, I am eternally grateful for your support and encouragement. You always believed in me. I pursued and completed my PhD because of you, and you willingly sacrificed whatever was needed to help me make this dream come true. Your voice of encouragement has been my strength through this entire process. I love you always and forever.

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List of Abbreviations

BMD	Beliefs in Mathematics Discourse Survey
CCSSM	Common Core State Standards for Mathematics
NCTM	National Council of Teachers of Mathematics
NRC	National Research Council
OECD	Organization for Economic Cooperation and Development
PADM	Preservice Teachers' Attitudes about Discourse in the Mathematics Classroom
PISA	Programme for International Student Assessment
SFL	Systemic Functional Linguistics
SMP	Standards for Mathematical Practice
TIMSS	Trends in International Mathematics and Science Study
TTPS	Teaching Through Problem Solving

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CHAPTER I. INTRODUCTION

The art of teaching and learning mathematics is rooted in the discourse that takes place between the teacher and students and among students. Mathematical discourse should be considered more than a aid to thinking; it is nearly tantamount to thinking itself (Sfard, 2001). This discourse creates understanding, recognition of confusion, and corrections that lead to forward progress in learning. However, the type of discourse that occurs in mathematics today differs greatly from classroom to classroom. While some teachers facilitate student led discourse, others rely on the traditional instructional strategy of lecturing.

According to the Trends in International Mathematics and Science Study (TIMSS), mathematics instruction in the United States has been characterized as procedural, teacher-centered, and lacking in the area of engaging students in thinking and justifying their reasoning (Mullis et al., 2000). In addition, international assessments such as the TIMSS and the Programme for International Student Assessment (PISA) rank the United States far below other advanced nations in K-12 mathematics (Gonzales et al., 2004, p. 101). In 2015, the average mathematics score for U.S. students on the PISA ranked 36 of 69, meaning they scored lower than more than half of the other education systems that tested and lower than the Organization for Economic Cooperation and Development (OECD) average (Kastberg et al., 2016). The inadequate mathematics achievement of students in the U.S. is a complicated problem that will require complex solutions (Jang, 2010).

According to Ambrose et al. (2004) teacher beliefs about mathematics and teaching mathematics are shared factors across multiple research studies that are identified as contributing to the poor performance. Teachers' beliefs directly shape their instructional practice, and as a result, student learning is also influenced (Carnegie, 2014; Hofer, 2001; Muis et al., 2006;

Pajares, 1992; Spiegel, 2012). Jang (2010) points out that while educators may be introduced to a new curriculum or instructional resources, putting any education reform into action hinges on teacher beliefs on that subject matter, student learning, and teaching in general.

The Common Core State Standards for Mathematics (CCSSM) published in 2010 outlined what all "students should understand and be able to do" (Common Core State Standards Initiative (CCSSI), 2010, p. 4) by describing grade-level standards that included both conceptual understanding and procedural knowledge. The Standards for Mathematical Practice (SMP) were also included in this document that defined skills that all students should develop and utilize when learning and doing mathematics. The third standard, states that students should "construct viable arguments and critique the reasoning of others." (CCSSI, 2010, p. 6). However, these were not new processes, prior to the publication of this document, these processes were acknowledged as traits of mathematically proficient students and skills that prepare students to solve problems and become effective users of mathematics in society (National Council of Teachers of Mathematics (NCTM), 2000a; National Research Council (NRC), 2001).

In 2014, the National Council of Teachers of Mathematics (NCTM) published *Principles to Actions* which contended "Mathematical discourse among students is central to meaningful learning of mathematics" (p. 35). In order to engage students in the mathematical processes outlined in the SMP, effective teachers employ instructional practices that support students in constructing knowledge and reasoning with mathematics topics (Khan, 2012; NCTM, 2014). As a result, NCTM (2014) recognized eight effective teaching practices that teachers should utilize to enhance their mathematics instruction, including the practice of facilitating meaningful mathematics discourse. Webb et al. (2014) concluded that student achievement was higher for students who participated in purposeful mathematics discourse, meaning they questioned the strategy or solution presented by another student or proposed a way for that student to improve his or her work, than students with limited or no engagement in the discussion.

Many researchers have drawn the conclusion, based on their studies, that teachers should not only encourage student discussions but students should lead the discussions themselves (Gonzalez & DeJarnette, 2013; Hufferd-Ackles et al., 2004; Otten, Cirillo, et al., 2015; Otten, Engledowl, et al., 2015). However, O'Brian (1999) found that teachers in the United States persist in stating information instead of supporting students in constructing a mathematical understanding through discourse. This study will focus on gaining an understanding of teacher beliefs about mathematics discourse during whole-group discussions.

Statement of the Problem

In order for the United States to stay a global economy leader, students should be ready for a competitive job market, which means they are prepared with the necessary skills to excel in problem solving, communicating with others, analyzing and rational thinking (Baroody, 2011; Elizabeth et al., 2012). The purpose of education is to create thinkers, not workers. The National Council of Teachers of Mathematics [NCTM] (2000a), points out that these skills are necessary for functioning in everyday life not just the workplace. Smith and Stein (2011) conclude that social interaction "provides us with the opportunity to use others as resources, to share our ideas with others, and to participate in the join construction of knowledge" (p. 1). However, Dyer (2016) determined that supporting effective mathematics discourse among students was challenging for teachers. Thus, mathematics classrooms concentrate on the content standards with limited attention given to the SMPS and the process standards, which, according to NCTM (2000a), includes representation, connections, communication, proof, reasoning, and problem solving (Boaler, 2015; McGatha & Bay-Williams, 2013).

In 2014, NCTM again reiterated the importance of discourse on students' development of conceptual understanding of mathematics topics by stating, that "Learners should have experiences that enable them to construct knowledge socially, through discourse, activity, and interaction" (p. 9). NCTM also asserts that students benefit from participating in meaningful discourse rather than simply retelling how they solved a problem (McGatha & Bay-Williams, 2013; NCTM, 2000a; NCTM 2000b; NCTM, 2014). A shift from a focus on correct answers toward productive discussions that concentrate on why, how, and when a certain strategy is appropriate and useful is necessary (McGatha & Bay-Williams, 2013). Productive discourse occurs when students "seriously consider their peers' mathematical perspectives as a way to construct mathematical understandings" in addition to expressing their individual thoughts. (NCTM, 2010, p.1) When teachers prioritize productive discourse, students develop experience with a powerful tool that will help them succeed in mathematics and other subject areas as well (NCTM, 2000b; NCTM, 2014; Rumsey & Langrall, 2016).

When investigating discourse, researchers have studied various aspects and impacts of the communication between students and teachers in mathematics classrooms (Aziza, 2018; Cavanna et al., 2015; Eddy & Kuehnert, 2018; Forman et al., 1998; Gresalfi et al., 2008; Hamm & Perry, 2002; Leinhardt & Steele, 2005; Nathan & Knuth, 2003; Mullins, 2018). Qualitative studies have focused on the actions of teachers determined to change their teaching practices to better facilitate productive discourse (Forman et al., 1998; Nathan & Knuth, 2003). Additionally, the impact of various actions taken by the teacher, such as teacher questioning, including the classification of various types of questions have been studied to learn more about classroom discourse. (Aziza, 2018; Eddy & Kuehnert, 2018). Researchers have also observed and analyzed classroom discourse to study other concepts including competency, authority, and the complexity of classroom discourse (Hamm & Perry, 2002; Leinhardt & Steele, 2005; Gresalfi et al., 2008). The culture of the learning community within a classroom can have a large effect on the learning that takes place. This culture is impacted by teacher beliefs. For this reason, some classroom discourse studies have investigated how the expectations within a classroom relate to the mathematical discourse that takes place (Cavanna et al., 2015; Mullins, 2018). Other studies on this topic focus on essential practices for educators to implement when facilitating productive discourse (Stein et al., 2008; Cirillo, 2013; Alwarsh, 2018).

While this topic has been extremely popular and well researched, there is a gap in the current literature. There is limited data examining teacher beliefs with a large sample of participants. Examining a large sample of responses on a survey instrument with open-ended response items in conjunction with interviews that provide participants an additional opportunity to voice their beliefs, can contribute to what we know about teacher beliefs and practices in this space. This study adds to current research by contributing a quantitative view of teachers' beliefs of mathematics discourse in combination with a qualitative view gleaned from follow-up interviews with participants and examination of the individual open ended responses.

Purpose of the Study

The purpose of this study is to explore the level of elementary mathematics teacher beliefs about dialogic discourse elementary mathematics teacher beliefs about discourse during whole-class discussions among the current sample. In addition, this study will investigate elementary mathematics teachers' beliefs about the role of mathematical discourse. This study will also consider if years of teaching experience, educational status, and grade of instruction impact teachers' beliefs of the role of mathematical discourse.

Research Questions

The research study will answer the following questions to explore teacher beliefs about mathematics discourse:

1. What is the level of elementary mathematics teacher beliefs about dialogic discourse during whole-class discussions among the current sample?
2. Do years of teaching experience impact teachers' beliefs of the role of mathematics discourse during whole-class discussions?
3. Do teachers with various levels of degrees have different levels of beliefs about dialogic discourse during whole-class discussions?
4. Do pre-service and in-service teachers have different levels of beliefs about dialogic discourse during whole-class discussions?
5. Do teachers in different grade levels have different levels of beliefs about dialogic discourse during whole-class discussions?
6. What are elementary mathematics teachers' beliefs about the role of mathematical discourse?

Significance of the Study

This research will provide a unique insight into teacher beliefs about mathematics discourse by using a survey instrument with open-ended items as well as multiple-choice items rather than a traditional Likert-scale items. According to Fisher (1993), social desirability bias is an error due to the eagerness of participants to positively portray themselves and avoid embarrassment, a common tendency in social science research. To mitigate this bias and better capture the complexity of teacher beliefs, the BMD survey instrument utilized in this survey presents hypothetical situations that include students sharing both correct and incorrect reasoning and participants are prompted to describe how the teacher should react in order to lead a mathematical discussion (Jang, 2010). Furthermore, by utilizing this survey instrument, the beliefs of a large number of participants can be investigated, in comparison to the small number of participants used in a case study approach. This study will further add to current research on teacher beliefs of mathematical discourse by gathering additional insights through a follow-up interview that will allow pre-service and in-service teachers additional opportunities to express their views, allowing their voice to be included in the research on this topic. Overall, by utilizing a unique way to explore a range of teacher beliefs about mathematics discourse and examining differences in those beliefs based on specified characteristics, this study will contribute to progressing the discussions about teacher beliefs about mathematical discourse and its relationship with instructional practices and student learning.

Limitations of the Study

This study will have several limitations. The first limitation relates to drawing conclusions about teachers' beliefs about mathematical discourse based on self-reported data. For this study, participants will be asked how a teacher should respond to a hypothetical situation, and the researcher will deduce and categorize their beliefs from the responses provided. While the BMD instrument allows participants to respond in their own words, different scenarios may produce difference replies.

Next, the instrument is created for a limited audience. While the instrument was designed for pre-service elementary teachers, in-service teachers were also part of pilot work for designing the BMD instrument. Jang (2010) notes that the survey is not designed for use with secondary teachers. Additionally, the survey, which contains items that cover material from grades K through 5, will be sent to elementary school teachers who may only have an early childhood certification. As a result, teachers who are certified in grades K though 2 will be asked questions for grades 3 through 5. The same is true of teachers with elementary education certification, which includes grades 3 through 6, who will be asked about situations in grades K through 2.

Sampling may also be a limitation of the study. A representation of all relevant populations is necessary to establish the reliability and validity of an instrument (Jang, 2010), but a majority of the participants for this study will most likely be from one state in the southeastern United States.

The final limitation of the study is the small number of participants who are anticipated for the follow-up interview. Participants will be selected for this part of the study based on their voluntary participation. As a result, the degree to which their responses represent the entire sample may be taken into consideration.

Definition of Terms

Beliefs

According to Jang (2010), a person's beliefs are difficult to measure because belief is an underlying construct which cannot be openly observed. For the purposes of this study, belief will be defined as "The conscious and unconscious ideas and thoughts teachers have about how teachers and students should participate in classroom discussion to build mathematical knowledge" (Jang, 2010, p. 1).

Dialogical Discourse

Dialogical discourse is the highest level of discourse defined in this study. Teachers, at this level, encourage students to communicate with one another as a way to construct an understanding of mathematical ideas. Students are encouraged to actively participate in whole class discussions and subsequent instruction is influenced by these conversations (Scott & Mortimer, 2005; Steffe & D'Ambrosio, 1995).

Discourse

According to NCTM's (2014) *Principles to Actions: Ensuring Mathematical Success for All*, classroom discourse is defined as, "the purposeful exchange of ideas through classroom discussion as well as through other forms of verbal, visual, and written communication" (p. 29). In addition, discourse is defined by Otten, Cirillo, et al. (2015) as using communication through written or spoken language to convey a message. In this study, discourse is defined as the manner in which teachers communicate with students during whole class discussions to develop mathematical ideas.

Educational Status

For this study, education status is defined as the highest level of degree a participant has attained including an undergraduate, masters, Educational Specialist, or Ph.D. Additionally, participants can indicate if they are currently working on a degree and specify what type of degree they are working toward.

Emerging Dialogical Discourse

Emerging dialogical discourse, the next highest level, describes classroom discussions that involve numerous students sharing their thinking with the teacher. Communication, at this level, is restricted to interactions between the students and the teacher and is focused on providing assistance or sharing the students' thinking (Cobb et al., 1997). While teachers, at this level, believe it is important to explore student thinking through questioning, students are not encouraged to communicate with one another during whole-class discussions.

Grade of Instruction

Grade of instruction, in this study, refers to the grade level(s) that the participant is currently teaching including kindergarten, first, second, third, fourth, fifth grade or sixth.

In-service Elementary Teacher

An in-service elementary teacher, in this study, is someone who is currently teaching mathematics in a general education classroom for students in any of the grades Kindergarten through sixth-grade.

Partial Univocal Discourse

Partial univocal discourse is the level above univocal discourse. Information, at this level, is still sent in one direction from the teacher to the students, but the teacher also converses with students as a method of checking students' understanding. While teachers at this level may give students an opportunity to speak to one another in pairs or small groups, students are not encouraged to converse with one another during whole-class discussions. The teacher dominates the classroom discussions with explanations that focus on telling students the correct method for solving problems, and teacher questioning focuses on correcting students rather than exploring their understanding.

Pre-service Elementary Teacher

In this study, a pre-service elementary teacher is defined as someone who is currently enrolled in an elementary teacher education program at an accredited university in the southeastern region of the United States at the time of the study. Specifically, these students are enrolled in the math methods courses at the time of the study.

Professional Development

For this study, professional development is defined as ongoing learning opportunities and training specifically designed to provide mathematics educators with information about an instructional practice or mathematics content.

Teaching Experience

Teaching experience, for the purposes of this study, describes the number of years a participant has worked as a certified teacher in an elementary classroom.

Univocal Discourse

Univocal discourse is the lowest level of discourse on the construct map (Figure 1) that is utilized in this study. According to Jang (2010), when discourse is univocal, knowledge is transmitted in one direction. Teachers at this level believe their role is to communicate the correct definition of a mathematical concept, strategy for solving a problem, or solution for a problem to students. Students are not encouraged to verbalize their approach to solving a problem or converse with their classmates. Instead, the teacher is the one who provides the explanations including pointing out when students' answers are incorrect.

Summary

Professional organizations, such as NCTM, advocate for productive mathematical discourse in the classroom that encourages students to discuss mathematical ideas and strategies with one another (NCTM, 2000a; NCTM 2000b; NCTM, 2014). In addition, research shows the effectiveness of this type of instruction (Webb et al., 2014). However, in my classrooms, teachers still explain mathematical concepts or demonstrate specific strategies that should be used to solve problems (O'Brian, 1999; Truxaw & DeFranco, 2007). Teacher beliefs are at the heart of this issue. Teacher beliefs influence the instructional practices they choose to utilize in the classroom, which, in turn, affects student learning. This means that in order to change teaching practices, teacher beliefs must change, and in order to change teachers' beliefs, you must first fully understand what those are. This study will utilize a unique survey instrument to more fully understand the beliefs of a numerous pre-service and in-service teachers regarding mathematics discourse and use a follow-up interview to further explore those beliefs, allowing participants an additional opportunity to voice their beliefs.

Organization of the Study

The study was organized in the following way: Chapter I introduces the study by stating the problem, the purpose, and the research questions guiding the study. Chapter I also includes the significance and limitations of the study as well as definitions of key terms. Chapter II presents a review of the literature relevant to this study including a history of mathematical discourse, the development of today's definition of mathematical discourse, discourse and student achievement, the student's role in discourse, the teacher's role in discourse, the role of teacher beliefs in teaching mathematics, measuring teacher beliefs of the role of mathematical discourse, and the theoretical framework for this study. Chapter III describes the methodology used in this study including the role of the researcher, participants and recruitment, the data collection, the instruments used, and the data analysis. Chapter IV will present the findings of this study, and Chapter V will present a summary of the findings, a discussion of the results, conclusions, implications, and recommendations for future research.

Chapter II. REVIEW OF THE LITERATURE

Introduction

Previous research on mathematical discourse, teacher beliefs, and the relationship between mathematical discourse and teacher beliefs provide insights on the progression of mathematical discourse in the classroom over the past several decades. The history of mathematical discourse as well as the development of today's definition are explored in this chapter to position this study within the context of this growing body of research. Studies that examined tools for improving discourse, the student's role in discourse, the teacher's role in discourse, and discourse and student achievement are analyzed to indicate the significance of the topic of discourse. An overview of the previous literature that defines teacher beliefs, the role of teacher beliefs in teaching mathematics, and measuring teacher beliefs of the role of mathematical discourse is included to articulate the significance of the impact of teacher beliefs on teaching practices and student learning. Finally, the theoretical framework that frames this study is explained.

Mathematical Discourse

In 1935, Brownell published an appeal for "full recognition of the value of children's experiences" and for using "arithmetic less a challenge to pupil's memory and more a challenge to his intelligence" (p. 31). More than eight decades ago, this innovative and almost defiant proposal indicated a landmark shift in the history of educational research turning from the behaviorist era and beginning a new direction in the study of human understanding (Sfard, 2002). In 1987, Vygotsky advocated for discourse in the mathematics classroom to promote interaction between students and educators. He hypothesized that children social experience is crucial in cognitive development because the interactions within the learning community present circumstances that allow children to plan, practice, and reason. He noted that, "Students grow into the intellectual life around them," (p. 88) so the quality of the discourse between students and teacher is an integral part of what and how they learn (Vygotsky 1987; Pierson, 2008).

In, 1989, The National Council of Teachers of Mathematics (NCTM), announced and published *Curriculum and Evaluation Standards for School Mathematics*, an innovative document that aimed attention at a shift in mathematics education toward critical thinking. Communication along with problem solving, connections, and reasoning, were emphasized throughout the document, and as a result, a focus on communication, along with the other focal points, started to grow (Tuck, 2018). According to Boaler (2002), this signaled a shift from the traditional sit-and-get model previously used toward a reform in mathematics.

NCTM later extended this document in 1991 to *Professional Standards for Teaching Mathematics*, and in 1995 published *Assessment Standards for School Mathematics*. In 2000, *Principles and Standards for School Mathematics* was published. In these books, the authors examine the teacher's role in discourse, the student's role in discourse, and tools for improving discourse (NCTM, 1991; NCTM, 1995; NCTM, 2000a). In addition, the authors of *Principles and Standards for School Mathematics* added to these three ideas of mathematical discourse the standards of worthwhile mathematical tasks, the learning environment, and the analysis of teaching and learning to the effective math teaching principle (NCTM, 2000b). In 2014, NCTM published *Principles to Actions*, a historical document, which called for students to be actively engaged in discourse to learn mathematics. The third of the eight Standards for Mathematical Practice, which described what students should be doing as they learn mathematics, stated that students should "(c) Construct viable arguments and critique the reasoning of others" (NCTM, 2014; p. 8). Furthermore, the text outlined eight Mathematics Teaching Practices that should be consistent elements of every mathematics lesson, and several of these practices are rooted in discourse. Teacher should “Facilitate meaningful mathematical discourse”, “Pose purposeful questions”, “Support productive struggle in learning mathematics”, “Elicit and use evidence of student thinking” (NCTM, 2014, p. 3). These publications (NCTM, 1989, 1991, 1995, 2000a, 2000b, 2014) demonstrate the organization’s focus on the importance of discourse, thinking, reasoning as central themes in teaching in learning. Discourse became a more integral focus of the research of reform mathematics.

Over time, the research on mathematical discourse has expanded since the pivotal work of NCTM (1989) that emphasized communication to include several aspects of mathematical discourse. Several studies have explored the teacher’s role in discourse (Cirillo, 2013; Kimani et al., 2016; SanGiovanni, 2016; Smith & Stein, 2011) as well as the student’s role in discourse (Carter, 2008; Gonzalez & DeJarnette, 2013; Hoffman et al., 2009; Leinhardt & Steele, 2005; Kilic et al., 2010; Knuth & Peressini, 2001; Wagganer, 2015). Other studies have focused on the tools for improving discourse including specific practices for teachers to be more effective in facilitating productive mathematics discourse (Alwarsh, 2018; Boaler, 2015, 2016; Cirillo, 2013; Stein et al., 2008) as well as the practices of teachers who determined to make a change in their practice toward more productive discourse (Forman et al., 1998; Nathan & Knuth, 2003). Additionally, the relationship between discourse and student achievement has also been investigated (Boaler, 2002, 2008; Boaler, Williams, & Confer, n.d.). Today, the definition of mathematical discourse has been influenced by these studies and others including the work of Stein, Engle, Smith, and Hughes (2008) who developed a framework for mathematical discourse that is highly recognized by educators and researchers today.

Development of Today's Definition of Mathematical Discourse

According to NCTM's (2014) *Principles to Actions: Ensuring Mathematical Success for All*, classroom discourse is defined as, "the purposeful exchange of ideas through classroom discussion as well as through other forms of verbal, visual, and written communication" (p. 29). Mathematical discourse is much more than getting students to talk (Alwarsh 2018). Sammons (2012) asserted that understanding mathematics requires social interactions. In the classroom, discourse provides opportunities for those interactions to take place and supports students in agreeing or disagreeing with one another to construct meaning (Bauersfeld, 1995).

Through classroom discourse, students' thinking is verbalized and can be further developed as a result of the questions and thorough examination other others as well as listening to the ideas of their classmates (Cobb et al., 1992; Cobb, 1997; Krummheuer, 1995; Wood et al., 1991). The phrase "exchange of ideas" is important to note in the NCTM (2014) definition because mathematical discourse, from a traditional viewpoint, may have been viewed as a one-way method of conveying a message from the teacher to the students (Saglam et al., 2015). A discussion of ideas, reasoning, and thinking goes beyond simply communicating one person's viewpoint, thinking, or beliefs. Instead, active participation, engagement, and the exchange of ideas, reasoning, and question about a meaningful topic of study are necessary for discourse (Manouchehri & St. John, 2006). Rather than a teacher simply providing explanations for students, discourse includes questioning, inserting various opinions into the conversation, and monitoring reasoning (Kilic et al., 2010). When creating the *Teacher Belief about Mathematical Discourse* (BMD) measure, Jang (2010) defined discourse as "the way that teachers interact with students in the process of developing mathematical ideas during open discussion," (p. 5).

The research and strategies for today's definition of mathematical discourse continued to expand. In 2008, Stein et al. introduce five key practices that teachers can use, including anticipating, monitoring, selecting, sequencing, and making connections, as part of a pedagogical framework that teachers can utilize to use student responses to mathematical tasks more effectively in discussions. The five practices were later unpacked in greater detail in the book *5 Practices for Orchestrating Productive Mathematics Discussions* (Smith & Stein, 2011). The authors propose that this research-based model is necessary due to the challenges faced by teachers who select student-centered, inquiry-based instructional tasks that go beyond identifying and implementing these tasks in the classroom. When using rich mathematical tasks, multiple solutions and solution paths are possible, and as a result, students often solve the problems in unexpected and unique ways. The teacher is responsible for making sense of the students' thinking and start to align it with recognized understandings of the nature of mathematics. For this reason, discourse is critical in understanding the solutions and solution paths students utilize. Because this is challenging for most teachers, the authors define each practice and demonstrate how a class discussion about a cognitively demanding task could be enhanced through the use of each practice. The authors support each practice with current theory, and they describe how implementing these practices can make a discussion-based pedagogy manageable for educators and productive for teaching and learning (Stein et al., 2008).

The Teacher's Role in Discourse

The role of the teacher is crucial in relation to the effect of mathematical discourse on student learning (NCTM, 2000a; NCTM, 2014; Springer & Dick, 2006; Truxaw & DeFranco, 2007). As a result, teachers with productive beliefs about mathematical discourse, recognize that their role is to engross students in meaningful tasks that encourage problem solving and reasoning through discourse which results in a shared understanding of mathematics (NCTM, 2014). Through discourse, teachers help students to be mindful of and develop their informal, implicit understanding (NCTM, 2000a), and prompting students to explain their mathematical thinking is a powerful method of formative assessment (Anderson et al., 2011). In order to facilitate productive discourse, teachers encourage open discussions of student thinking without using his or her position of authority to impose a different idea or strategy (Bahr & Bahr, 2017).

In one study, Hamm & Perry (2002) investigated the classroom processes for discourse and participatory structures that allow sources of mathematical authority in six first-grade classrooms. Each teacher's mathematics lessons were videotaped for five consecutive lessons, and the authors analyzing the lessons coding for behaviors such as teacher questioning and how student responses were confirmed. The authors posed that understanding how mathematical ideas are created and validated in the classroom are integral to understanding mathematics, and various sources, including the teacher, the community of learners, the discipline of mathematics, or the text, are implicitly or explicitly given authority in the mathematics classroom by the way the teacher communicates through discourse patterns and participatory structures. The authors found that in all but one classroom, the teachers positioned themselves as the ultimate source of authority in the classroom, but the one exception inspires hope for shifting mathematics instruction from formal instruction to a growth-and-change culture that allows students to be part of a mathematics learning community (Hamm & Perry, 2002).

One important aspect of the teacher's role in discourse is selecting a meaningful, high-level mathematical task for students (Cirillo, 2013; Kimani et al., 2016; NCTM, 2000a; SanGiovanni, 2016; Smith & Stein, 2011). Mates (2016) refers to this as the "site of interaction between students and mathematics" (p. 98). In addition, teachers should maintain the complexity of the task while routinely and proactively encouraging student thinking (Henningsen & Stein, 1997). In order for the teacher to fulfill this role, he or she should be able to understand the rationale behind student errors, and this requires the teacher to construct a profound understanding of mathematical fundamentals and concepts (Maguire & Neill, 2006).

While the teacher sets and maintains high expectations for students to productively struggle through challenging and meaningful tasks, it is imperative for students to have a safe place for learning (Jabari, 2016, Lampert, 1988). As a result, several studies about classroom discourse have investigated how the expectations within a classroom relate to the mathematical discourse that takes place (Cavanna et al., 2015; Mullins, 2018). Students and teachers work together to form a learning community, and the culture of that group, which can be influenced by teacher beliefs, can have a large effect on the learning that takes place.

Students and teachers work together to form a learning community, and the culture of that group, which can be influenced by teacher beliefs, can have a large effect on the learning that takes place. Studies about classroom discourse have investigated how the expectations within a classroom relate to the mathematical discourse that takes place (Cavanna et al., 2015; Mullins, 2018). Cavanna et al. (2015) focused on the teacher's role in mathematics classroom discourse by examining one teacher's opinion of unexpected moments that occurred during instruction as she purposefully attempted to change her mathematics classroom discourse practices. Through an exploratory case study, the authors noted that the teacher identified three areas as unexpected including features of lesson enactment, characteristics of student learning, and her own intentionality or purposefulness. The authors used a systemic functional linguistics (SFL) appraisal framework to investigate how the teacher gauged the moments, and through this exploratory case study, they observed how her opinions changed over a two-year collaboration during which the teacher and her colleagues participated in a professional development. Throughout the study, Brenda, the teacher, changed her beliefs regarding her students and mathematics to a more positive view of what kind of mathematics her students were able to do which, in turn, affected the way she planned mathematics classroom instruction including the type of tasks and how they were introduced (Cavanna et al., 2015).

Mullins (2018) pointed out a need for those in positions of authority, such as researchers, educations, state and national organizations, and policy makers, to focus on mending the disconnect between simply initiating mathematical discussions to developing mathematical discourse in the classroom. She advocated for the implementation of social norms in the mathematics classroom in order to encourage students to actively participate, which is necessary for productive discussions. Through a synthesis of education research on social norms in the mathematics classroom, she identified essential factors that educators can concentrate on to establish social norms that create and maintain an inquiry-based classroom. As a result, Mullins (2018) found that the norms should be developed through a collaboration of the teacher and students, the classroom environment must support mathematical discussion, and lastly, norms should be negotiated or renegotiated to help influence students' thinking. Both findings from both studies by Cavanna et al. (2015) and Mullins (2018) provide insights into what a mathematics classroom could look like if teacher beliefs are changed to have higher expectations for students and if appropriate social norms are implemented.

Teachers "assess and advance student understanding, provide opportunities for productive struggle, and facilitate discourse to foster conceptual understanding and procedural fluency" when they use effective questioning (NCTM, 2014, p. 114). Teacher questioning, including the classification of various types of questions, has been the focus of several studies

(Aziza, 2018; Eddy & Kuehnert, 2018; Gonzalez & DeJarnette, 2013, Kimani et al., 2016). According to Gonzalez & DeJarnette (2013) and Kimani et al. (2016), the questions teachers select to ask are essential to promoting productive discourse. Productive questioning prompts students to deepen their understanding (Chval et al., 2013), These questions should involve higher-level thinking from students because they are process questions instead of product questions that go beyond remembering a fact (Reinhart, 2000).

The Student's Role in Discourse

Several research studies have examined the student's role in discourse (Carter, 2008; Gonzalez & DeJarnette, 2013; Hoffman et al., 2009; Leinhardt & Steele, 2005; Kilic et al., 2010; Knuth & Peressini, 2001; Wagganer, 2015). In the mathematics classroom, it is important for students to comprehend the value of engaging in mathematical discussions to investigate each other's mathematical reasoning (Gonzalez & DeJarnette, 2013). According to NCTM (2000a), students listen and reply to each other as well as their teacher in order for productive discourse to take place. In addition, everyone involved should be an active listener who responds honestly, respectfully, and willingly (Wagganer, 2015). When students ask one another questions or restate someone else's mathematical thinking, they provide feedback to each other as well as their teacher (Carter, 2008; Kilic et al., 2010; Wagganer, 2015).

Mathematical discourse that supports students in engaging with their classmates and teachers to develop deep mathematical knowledge has been central to the reforms in mathematics education (Hiebert, 1992: Silver & Smith, 1996). Participating in classroom discourse promotes student thinking and communication skills which effects other content areas as well (McGraw-Hill Education, 2018). Through discourse, students are actively working to prove their thinking to convince themselves and their classmates of their solution and strategy

(Kilic et al., 2010; NCTM, 2000a). As a result, students will increase their understanding and establish ownership when they have the opportunity to discuss different viewpoints (Gonzalez & DeJarnette, 2013). Students search for patterns and inconsistencies, initiate questions and problems, and build on one another's ideas (Tuck, 2018). In an ideal classroom with productive mathematical discourse, "students question one another, explain and justify their ideas, and work together to compare strategies and solutions" (Hoffman et al., 2009, p. 234).

Analyzing the role of students in classroom discourse is a complex subject matter. Leinhardt and Steele (2005) examine the complexity of using instructional dialogues in mathematics teaching by tracking a ten-lesson unit in a fifth-grade classroom that studied functions and their graphs. The authors found that the teacher's communication served two purposes including developing co-constructed meaning of important mathematical ideas and allowing the class to navigate a meaningful path through the appropriate mathematics. As a class, the group flagged central questions, coordinated and differentiated between ideas, anticipated and discussed openly potential errors. Through their dialogue, students were able to point out and resolve misconceptions. Agendas, the conditions that allowed adjustments to the agendas, and problematizing of the mathematics supported the path through the mathematics. The authors found that the instructional dialogue was supported by routines, metatalk, the creation of and maintenance of the intellectual climate (Leinhardt & Steele, 2005).

Tools for Improving Discourse

Facilitating meaningful mathematics discourse is a complex process that requires purposeful and thoughtful planning. As a result, studies have focused on the tools for improving discourse including specific practices for teachers to be more effective in facilitating productive mathematics discourse (Alwarsh, 2018; Boaler, 2015, 2016; Cirillo, 2013; Stein et al., 2008) as well as the practices of teachers who determined to make a change in their practice toward more productive discourse (Forman et al., 1998; Nathan & Knuth, 2003). Steele (2019) concludes that classroom communication is more productive when appropriate tools are used to assist in this planning, and he recommends two research-based tools, *5 Practices for Orchestrating Productive Mathematics Discussions* (Smith & Stein, 2018), which has already been discussed, and the Teacher Discourse Moves that are discussed in *Mathematics Discussions in Secondary Classrooms* (Herbel-Eisenmann et al., 2017). These moves include waiting, inviting, revoicing, asking, probing, and creating. Waiting involves giving students think time after a question has been posed or a student has responded allowing students time to process what has been said, develop new ideas, and holds students responsible for their mathematical thinking. Inviting involves welcoming the viewpoints of all students and emphasizes the role of students as mathematicians who are working together to construct a mathematical understanding. Next, revoicing provides an opportunity to strengthen, define, or expand a mathematical idea that has been shared while asking involves prompting students to revoice serves a similar purpose. Probing a student's mathematical thinking can help them to verbalize their reasoning, emphasize appropriate mathematical language, and provide other students access to these mathematical ideas. Finally, creating refers to the opportunities designed to engage in discussions about their classmates' strategies, which encourages students to listen actively and reason about the similarities and distinctness of various strategies (Steele, 2019).

As mentioned previously, some teachers in mathematics have made the decision to change their practice to include a more student-centered, productive discourse. Therefore, some studies on this topic focus on essential practices for educators to implement when facilitating productive discourse (Cirillo, 2013; Alwarsh, 2018). Alwarsh (2018) relates mathematical discourse to teaching through problem solving (TTPS). The aim of TTPS is to facilitate an environment in which students can construct reasoning and connect their solution strategies with new mathematical ideas. Alwarsh (2018) points out that educators can enhance mathematical classroom discussions by applying socio-mathematical norms, using specific talk moves, and planning for discussions in advance. Mathematical discourse is much more than getting students to talk. Alwarsh (2018) makes connections to the work of Stein, et al. (2008) by providing specific examples and making connections to a fourth-grade classroom scenario that is presented in the article. Because productive discussions that are facilitated rather than dictated by the teacher have a significant impact on the quality of mathematics instruction, this article provides important support for mathematics educators (Alwarsh, 2018).

In her research brief, Cirillo (2013) provides an overview of much of the research on mathematics discourse by discussing and expanding upon nine strategies for facilitating proactive discussions in the mathematics classroom. First, educators should attend to the classroom culture by including students in establishing respectful discussion norms. Next, educators must choose high-level mathematics tasks in order for students to have something worthwhile to discuss. Cirillo (2013) then expands on the work of Smith & Stein (2011) to discuss how educators should also anticipate strategies that students might use to solve the tasks and monitor their work. Because knowledge should be co-constructed in group discussions, teachers should allow student thinking to shape discussions. Teachers should carefully examine and plan well-formulated questions that will open rather than close the class discussion. Cirillo (2013) provides alternatives to directly telling students information because educators should be strategic about "telling" new information. By exploring incorrect solutions, teachers can help remove the stigma of being wrong to help students learn from mistakes. Cirillo (2013) goes back to the work of Smith and Stein (2011) to discuss how teachers should select and sequence the ideas to be shared in the discussion to move the mathematics forward and help students draw connections and summarize the discussion.

Discourse and Student Achievement

Several research studies have concluded that mathematics achievement is related to classroom communication and discourse (Hufferd-Ackles et al., 2004; Whitenack & Yackel, 2002). Stinson (2012) suggested that scholarly debate is necessary and constructive for academic progress and success. Hattie et al. (2016) found that classroom discourse has an effect size of 0.82 in comparison to direct instruction which had an effect size of 0.59 and stated that teachers "would be wise to focus their energy on building classroom discourse rather than attempting to teach test-taking" (p. 41). High levels of achievement in mathematics have been directly related to students who actively engage in mathematics (Boaler, 2002, 2008; Boaler et al., n.d.; Hattie et al., 2016). Hiebert and Wearne (1993) also noted that when classroom discourse happened in combination with problem-solving tasks, student achievement was positively affected. Hung (2015) observed a relationship between students' self-image, engagement in discourse, and success in mathematics. Not surprisingly, the students who perceived themselves to be competent in mathematics, achieved performed higher on classwork and assessments. These studies support the need for discourse in the mathematics classroom to increase engagement, support confidence, and push students to critically examine the mathematics they are examining (Boaler, 2002, 2008; Boaler et al., n.d.; Hattie et al., 2016; Hiebert & Wearne, 1993; Hung, 2015)

Teacher Beliefs in Teaching Mathematics

Teacher beliefs is a difficult construct to define, and as a result, a common definition has yet to be agreed upon (Cross, 2009). Often the term belief is used interchangeably with words such as “disposition,” “opinion,” “value,” “attitude,” and “perception” (Jang, 2010). As far back as 1933, Dewey stated that beliefs “cover all the matters of which we have no sure knowledge and yet which we are sufficiently confident of to act upon and also the matters that we now accept as certainly true, as knowledge, but which nevertheless may be questioned in the future…” (p. 6) and in1968, belief was defined by Rokeach as “any simple proposition, conscious or unconscious inferred from what a person says or does” (p. 113). Belief has also be defined as “embodied conscious and unconscious ideas and thoughts about oneself, the world, and one’s position in it, which are considered by the individual to be true” (Pajares, 1992; Thompson, 1992). In this study, beliefs will be defined as the ideas and opinions, both conscious and unconscious, regarding the way teachers and students should interact during classroom discourse to construct mathematical understanding.

Previous literature on the significance of teachers’ beliefs in relation to their teaching practice and knowledge of subject matter express the value of studying this topic (Carnegie, 2014; Cavanna et al., 2015; Hofer, 2001; Muis et al., 2006; Pajares, 1992; Sarason, 1971). Brendefur and Frykholm (2000), note that due to the significance of communication in mathematics education research, it is critical for teacher educators to not only understand pre-service teachers’ beliefs about discourse as a tool for developing student understanding of mathematics but also discern how to support teachers in developing practices that promote mathematical discourse. Thus, in order to change teacher practice in the classroom, it is important to understand their beliefs of the topic. Research shows that what teachers think influences their teaching practices (Cavanna et al, 2015; Carnegie, 2014; Hofer, 2001; Muis et al., 2006; Pajares, 1992). Consistently, research has illuminated the manner in which teachers "translate their knowledge of mathematics and pedagogy into practice through the filter of their beliefs" (Manouchehri, 1997, p. 198; Thompson, 1992). According to Ball (1998a, 1988b, 1990) and Bush (1986), often teacher beliefs about mathematics, students' learning, and teaching, are developed and solidified long before potential teacher candidates enter teacher education programs.

For this reason, it is important to investigate the beliefs of both current teachers and pre-service teachers regarding mathematical discourse. Cooney (1994) noted that it is important for teacher educators to acknowledge that their students' beliefs may not align with those of the teacher education program. Understanding what pre-service teachers believe as well as how their beliefs are structured and defended is crucial and changing prospective teachers' beliefs is imperative to their development (Cooney et al., 1998). In order to change teaching practices, teacher beliefs must change because as Sarason (1971) stated, changes in education rely on what teachers think and how they operate in their teaching practice. Mathematical discourse is crucial in reform-based classrooms, and as a result, Brendefur & Frykholm (2000) suggest it is important to understand pre-service teachers' beliefs about mathematical discourse and take time during teacher education programs to influence teacher beliefs about this crucial topic so that they may establish better opportunities for student learning.

For example, Cavanna et al. (2015) focused on the teacher's role in mathematics classroom discourse by examining one teacher's opinion of unexpected moments that occurred during instruction as she purposefully attempted to change her mathematics classroom discourse practices. Through this exploratory case study, the authors noted that the teacher identified three areas as unexpected including features of lesson enactment, characteristics of student learning, and her own intentionality or purposefulness. The authors used a systemic functional linguistics (SFL) appraisal framework to investigate how the teacher gauged the moments, and through this exploratory case study, they observed how her opinions changed over a two-year collaboration during which the teacher and her colleagues participated in a professional development through classroom observations, debriefings, and interviews. Throughout the study, Brenda, the teacher, changed her beliefs regarding her students and mathematics to a more positive view of what kind of mathematics her students were able to do which, in turn, affected the way she planned mathematics classroom instruction including the type of tasks and how they were introduced (Cavanna et al., 2015). Jang (2010) asserts that achieving any reform hinges on what teachers believe about that topic as well as teaching and learning overall regardless of what current research, resources, and mathematics curriculum materials may suggest about innovations and improvements to mathematics education.

Previous research has investigated to what extent teacher beliefs change over time with experience and professional development. Studies by Cooney et al. (1998) as well as Cross (2009) have suggested that teachers' beliefs about mathematics have been unreceptive to change, and the work of Pajares (1992) indicates that this is true regardless of additional training or experience. Nespor (1987) argued that beliefs do not require internal consistency or a general agreement on its validity, and although it seems as if reasoning and evidence would alter one's beliefs, only a "conversion or gestalt shift" that prompts a change in beliefs (p. 321). This study will investigate if years of teaching experience, among other factors, affect elementary teacher beliefs of the role of mathematics discourse.

Measuring Teacher Beliefs of the Role of Mathematical Discourse

While the importance of teacher beliefs as well as their influence on teacher practice has been investigated (Ernest, 1989; Fang, 1996; Stipek et al., 2001; Thompson, 1992), more research is needed on teacher beliefs of the role of mathematical discourse. Previous studies usually utilize a case study approach that include small sample sizes, which according to Erickson (1986) are not focused on generalization. As a result, surveys with Likert-scale items have been used in an effort to focus on the limitations of previous studies. These surveys are easy to work with, and they are often chosen as a measurement tool for research studies because participants are able to quickly submit their responses. Jang (2010), however, points out that these tools also have their limitations including the researcher's ability to fully understand participants' responses. In addition, participants are selecting a numerical response that best fits the opinion they chose to report rather than having an opportunity to fully express their point of view.

Theoretical Framework

Constructivism and Social Constructivism

At the heart of recognizing the importance of discourse in the mathematics classroom, an understanding of how an individual learns is of vital importance. This study is framed around social constructivism that evolved from the learning theory known as constructivism. Constructivism originated within Piaget's Theory of Cognitive Development and is described by Kamii (1985) as "the theory according to which each child builds his own knowledge from inside, through his mental activity, in interaction with the environment" (p. 6). Knowledge is actively constructed by the individual rather than passively received (von Glaserfield, 1990; Walker & Shore, 2015). According to Clements and Battista (2009), a constructivist teacher, by contrast, guides student learning by selecting purposeful tasks and presenting situations that welcome discourse.

Piaget and Vygotsky both supported constructivism (Smith, 2018), but while Piaget focused on the individual's construction of knowledge, Vygotsky (1978, 1986) concluded that learning is a social process. Vygotsky's theory of social constructivism contains the idea that social interactions, including dialogue among children and between adults and children, strongly influence an individual's construction of his or her own knowledge (Bereiter, 1994; Bruning et al., 1995; Vygotsky, 1978, 1986; Walker & Shore, 2015). In a constructivist classroom, the teacher scaffolds learning by serving as a facilitator or coach instead of the traditional role of transmitting information (Walker & Shore, 2015). Through discourse, students in this type of classroom are treated as a community of learners because they form ideas, test their ideas, and attempt to make sense of their ideas and those of their peers (Brown & Compione, 1994; Smith, et al., 2000). If students construct their own individual knowledge through interactions with their teachers and peers, it is important to understand what type of discourse is optimal for student learning. As Nathan and Knuth (2003) point out, the presence of discussions in the classroom does not guarantee that learning will be constructed. Instead, the quality and type of discourse influence the possibility for encouraging purposeful mathematical understanding and strategies (Truxaw, 2020). Constructivism, along with Vygotsky's theory of social constructivism, disputed the traditional idea that the teacher is the giver of knowledge (Bickhard, 1998; Mikusa & Lewellen, 1999; Walker & Shore, 2015). This study is based on exploring teachers' beliefs around discourse utilizing the theory of social constructivist lens.

Dialogic Discourse

The idea of dialogic discourse, including the dichotomous alternative of authoritative discourse, can be traced back Bakhtin's (1935, 1981) theory of dialogism, in which it was referred to as internally persuasive discourse (Bosser & Lindahl, 2021; Saglam et al., 2015; Scott et al., 2006; Truxaw, 2020). Later, Mortimer and Scott (2003) redefined what Bakhtin (1935, 1981) referred to as internally persuasive discourse as dialogic discourse (Bosser & Lindahl, 2021; Scott et al., 2006). Dialogic discourse is characterized by the presence of at least two difference perspectives at once (Bakhtin, 1981; Wegerif, 2011). Truxaw (2020) refers to dialogic discourse as a "give and take communication where students actively construct meaning" (p. 122). While authoritative discourse requires acceptance because meaning is not debatable, dialogic discourse is correlated to meaning making because discourse is viewed as negotiable (Wells, 2007; Wertsch, 1991) and open to different viewpoints and perceptions (Bakhtin, 1981; Bosser & Lindahl, 2021; Lotman, 2000; Scott et al., 2006).

The ideas of power, control, and authority also arise when exploring the type of discourse used in a classroom. Authoritative discourse typically prompts a scenario where a teacher initiation-student response-teacher evaluation (IRE) pattern takes place (Mortimer & Scott, 2003), and as a result, the teacher is usually viewed as the primary source of knowledge in the classroom while maintaining command of the classroom discourse (Bosser & Lindahl, 2021). In contrast, dialogic discourse is typified by teachers asking questions to elicit student thinking and build upon their responses (Scott et al., 2006; Nystrand, 1997). Thus, students are viewed as critical contributors to classroom discourse in these situations (Bosser & Lindahl, 2021). Dialogic discourse, therefore, is essential for establishing an optimal learning environment for students to construct their own understanding of mathematical ideas.

Summary

A review of the literature provided an understanding of previous research on mathematical discourse, teacher perceptions, and the relationship between mathematical discourse and teacher perception. Additionally, the theoretically framework that frames this study was described. The research studies cited in this chapter lay the groundwork of understanding as the study develops.

Chapter III. METHODS

Introduction

The purpose of this mixed methods study was to examine the level of elementary mathematics teacher beliefs about dialogic discourse during whole-class discussions among the current sample. This study also explored if years of teaching experience impacted teachers' beliefs of the role of mathematical discourse, and if teachers in different grade levels and teachers with various levels of degrees have different levels of beliefs about dialogic discourse. In addition, this study explored elementary mathematics teachers' beliefs about the role of mathematical discourse. The methodology that was used to explore teacher beliefs about mathematics discourse is discussed including the role of the researcher, a detailed description of the participants, the data that was collected, and how the data was analyzed is outlined and described.

The research study will answer the following questions.

1. What is the level of elementary mathematics teacher beliefs about dialogic discourse during whole-class discussions among the current sample?
2. Do years of teaching experience impact teachers' beliefs of the role of mathematics discourse during whole-class discussions?
3. Do pre-service and in-service teachers have different levels of beliefs about dialogic discourse during whole-class discussions?
4. Do teachers in different grade levels have different levels of beliefs about dialogic discourse during whole-class discussions?
5. Do teachers with various levels of degrees have different levels of beliefs about dialogic discourse during whole-class discussions?

6. What are elementary mathematics teachers' beliefs about the role of mathematical discourse?

The Role of the Researcher

As the researcher in this study, I served as both interviewer and transcriber. Since the interview was semi-structured, I made decisions about how to ask the questions, make inferences about teacher beliefs from the data collected, and draw conclusions as a partner with participants to create knowledge. As a result, I became a key instrument in the data collection process (Creswell, 2018). Thus, my background in mathematics education along with a description of my beliefs on mathematical discourse is included. For six years, I taught fourth-grade mathematics in a school district in the southeastern United States. Prior to teaching fourth grade, I served as an aide in a third-grade inclusion classroom to support the special education teacher and filled long-term substitute positions in second-grade classrooms within the same school district the previous year. As a fourth-grade teacher, I was a part of a Curriculum Design Team that was awarded a grant to write and pilot a new district math curriculum for second through fifth grade. I led professional development workshop for teachers and administrators, and I was a presenter at the 2015 National Council of Teachers of Mathematics Regional Conference in Nashville, Tennessee, which included an additional session at the request of the conference to accommodate the teachers who were unable to attend the first session that was filled to capacity. I was also certified as a National Board for Professional Teaching Standard, and I was a finalist for the Presidential Award for Excellence in Mathematics and Science Teaching. Following my time as a fourth-grade teacher, I served as the District

Mathematics Instructional Coach within the same school district. Later, I also served as a Curriculum Consultant to assist in completing the writing of the district math curriculum.

My beliefs about how students learn mathematics and mathematic discourse were heavily influenced by Dr. Constance Kamii and Jean Piaget as well as the work of Lev Vygotsky. Throughout my work toward my Master's and Educational Specialist's degrees, I took every class that Dr. Kamii offered. As a result, I read her publications including *Young Children Reinvent Arithmetic (2nd ed.)* (2000) which encouraged teachers to require students to produce and share arguments with their peers during whole class discussions, to construct mathematical knowledge rather than the teacher acting as the authority on mathematical knowledge in the classroom. During class, we read Piaget's original work translated from French to find the errors in translation in order to develop a strong understanding of his theory. Piaget's Theory of Cognitive Development describes how knowledge is constructed from within which contrasted the traditional theories such as behaviorism (Kamii, 1984; Kamii, 2000). Piaget and Vygotsky both supported constructivism (Smith, 2018), but while Piaget focused on the individual's construction of knowledge, Vygotsky (1978) concluded that learning is a social process. Vygotsky's social constructivist theory incorporates the idea that individuals learn from one another meaning the learner should be engaged in the learning process. As a result, I believe the role of the teacher is to make students think as much as possible, which includes having students agree or disagree with one another during a mathematics discussion rather than the teacher fulfilling the role of the ultimate source of knowledge in the classroom. When I was a classroom teacher, I sat alongside my students as a participant during whole class discussions. I acted as facilitator, as students who were purposefully selected led the discussion by sharing their thinking and strategies for solving the problem that was being discussed. This practice led school leaders to send other teachers from within our school to observe this learning environment.

Student led discussions were an integral part of how students constructed an understanding of mathematics in my classroom, and as a result, I believe it is one of the most significant instructional practices that teachers can develop. I will do my best to interview and analyze findings objectively while acknowledging this is through the lens of a teacher and my past experiences in my classroom. While I acknowledge my personal beliefs about mathematics discourse, I realize that all teachers have different experiences and perspectives that are valid, meaningful, and worth working to understand.

Participants and Recruitment

In this study, the targeted survey population included participants who were teaching mathematics in elementary school classrooms, enrolled in an elementary teacher education program or recent elementary education graduates. Participants varied in years of teaching experience, teaching status, highest degree attained, and grade level they were teaching. All participants had the opportunity to enter a random drawing for a $5 gift card as motivation to participant in the survey.

Participants for the study were recruited using convenience and snowball sampling. An information letter (Appendix A) and email (Appendix B) that contained the online survey link including information regarding the survey was distributed to school principals and university instructors who taught pre-service teachers. Teachers were encouraged to share it with other educators they knew who met the criteria for the study, and the university instructors were encouraged to share this opportunity with peers teaching elementary mathematics methods courses at other universities. The survey information was also distributed and shared via social media. There were a total of 227 responses to the survey, but only 88 completed surveys were recorded and used for data analysis, with a 38.77% completion rate. Incomplete surveys were removed from analysis.

Participating teacher demographics are reported in Table 1. Most of the participants were in-service teachers, and the largest subgroup of in-service teachers had one to five years of teaching experience. Most participants have a master's degree. For the question, "What is the highest level of degree that you have attained?", 15 participants selected that they were "Currently working on a degree." Of those responses, 12 indicated that they were working on a bachelor's degree and three noted they were working on a master's degree. Since this should have been a separate question, the groups of "High school diploma" and "Associate degree" were combined, and the 12 responses that noted they were working on a bachelor's degree were added to this category. Additionally, the three responses who indicated that they were working on master's degree were added to the category of bachelor's degree showing this was their highest level of degree attained. There were two pre-service teachers who indicated their highest level of degree attained was an undergraduate degree, but these responses were not changed even though they were currently working on an undergraduate degree. Most in-service teachers were currently teaching second or fourth grade.

Table 1

Descriptive Statistics for Categorical Variables

		Frequency	Percent
Teaching Status	In-service teacher	56	63.6
	Pre-service teacher	28	31.8
	Recent graduate	1	1.1
	n/a	3	3.4
	Total	88	100
Teaching Experience	None (Pre-service teachers)	28	31.8
	1-5 years	18	20.5
	6-10 years	9	10.2
	11-15 years	9	10.2
	16-20 years	9	10.2
	21-25 years	5	5.7
	26+ years	10	11.4
	Total	88	100

Degree	High school diploma/Associate Degree	25	28.4
	Bachelor's Degree	26	29.5
	Master's Degree	28	31.8
	Educational Specialist	7	8.0
	PhD	1	1.1
	n/a	1	1.1
	Total	88	100
Grade of Instruction	None (Pre-service teacher)	29	33.0
	Kindergarten	8	9.1
	First grade	8	9.1
	Second grade	11	12.5
	Third grade	8	9.1
	Fourth grade	11	12.5
	Fifth grade	6	6.8
	Sixth grade	4	4.5
	n/a	3	3.4
	Total	88	100

In order to recruit pre-service and in-service teachers to participate in a follow-up interview to learn more about their beliefs of mathematical discourse in the classroom, participants were given information regarding this portion of the study while taking the online survey. If the pre-service and in-service teachers agreed to participate in the interview, they provided their name and email address in order to be contacted by the researcher. An email (Appendix C) with information and a link to setup an interview date and time was sent to the participants who volunteered. All participants had the opportunity to enter a random drawing for a $15 gift card as motivation to participant in the survey. The targeted interview population included both pre-service and in-service teachers, teachers who had various amounts of teaching experience, attained various levels of degrees, and taught various grade levels from kindergarten through sixth grade. Out of the 88 survey participants, four in-service teachers agreed to participate in the interview, which indicates a 5% response rate.

Instrumentation

In this study, the Beliefs of Mathematics Discourse (BMD) survey was utilized to measure teacher beliefs and a semi-structured post-interview was used to explore the level of elementary teacher beliefs about dialogic discourse in the classroom during whole class discussions. This section describes the instruments used for this study.

The Beliefs about Mathematics Discourse (BMD) Survey

The BMD (Appendix D) created by Jang (2010) measures the level of elementary teacher beliefs about dialogic discourse using mostly open-ended questions that were scored using a scale that ranged from one to four, with four being the highest level of dialogic discourse. Dr. Jang gave permission for this survey instrument to be used in this study (Appendix E). This survey was selected because of the nature of its design. Teachers were presented with a hypothetical teaching situation that describes students' correct and incorrect thinking, and the teacher was then asked to explain how he or she would respond in order to lead a mathematical discussion (Jang, 2010). This type of survey contrasts others that commonly use Likert-scale items that include a statement and a standard set of responses from which participants choose, such as strongly agree, agree, neutral, disagree, or strongly disagree (Jang, 2010). Jang (2010) believed the use of indirect questioning instead of the participant simply selecting a statement with which he or she agrees would reduce social desirability, which was evident in the results of her study and is discussed in greater detail regarding evidence of external validity, but research on this topic is incomplete (Fisher, 1993).

For this study, the third and final version of the BMD survey created by Jang (2010), which includes ten items, was used. These items depict a variety of complicated elementary mathematics topics from kindergarten to fifth grade including number and operations. The first seven items presented teachers with a scenario and asked participants to explain how a teacher should respond to the situation. For items 8-10, teachers were asked to select which of the choices provided best describes what they believe is most important in response to the situation described. For these multiple-choice questions, it is important to note that participants were given a final option of “other” and if participants selected this choice, they were asked to specify what they mean by this response.

Validity and Reliability

Jang (2010) presented evidence of the validity and reliability of the BMD measure including content validity, response process validity, internal structure validity, external validity, and reliability. First, a panel of experts that included mathematics teachers, master educators, and teacher educators analyzed the evidence for content validity. The evidence consisted of the construct map, item design, outcome space and Wright map, a measurement model, and the panel of experts determined that content coverage of the BMD instrument was acceptable (Jang, 2010).

Validity evidence for the response process included an analysis of the post-hoc think-aloud interviews. During the creation of the BMD instrument, participants were invited to take part in an interview after completing the survey. At this time, participants were asked to verbalize their thinking aloud as they reflected on each task within the survey. Based on participant responses during this interview process, certain items within the BMD measure were revised throughout the design process. For example, participants noted that the definition of the word discussion was unclear and certain items lacked impartiality regarding the subject of discourse. As a result, Jang (2010) clarified the definition of what constituted a discussion, and the items that lacked neutrality were deleted or revised.

Analysis of the Wright Map provided evidence of internal structure validity because the empirical results supported the theoretical framework of the construct map. This tool by Wright and Masters (1982) is utilized to create a visual depiction of data (Wilson, 2005). The four levels of belief, which are univocal, partial univocal, emerging dialogical, and dialogical, were shown in the Wright map, and distinctions within these four levels were evident. Jang (2010) noted that the differentiation between emerging dialogical (level 3) and dialogical (level 4) was not as obvious as the differentiation between univocal (level 1) and partial univocal (level 2). She noted that additional research is needed to clarify the construct map and the survey instrument to measure a spectrum of teacher beliefs about mathematics discourse.

Evidence of external validity consisted of a comparison of participant responses to the BMD measure and the Preservice Teachers' Attitudes about Discourse in the Mathematics Classroom (PADM) measure (Casa et al., 2007). During the study, Jang (2010) included both instruments in the survey. The PADM instrument is a Likert-type survey designed to measure preservice teachers' attitudes toward discourse in the mathematics classroom. Because the two instruments measure similar constructs, a high correlation is expected. Analysis of the post-hoc interviews showed that the PADM provided an insight into teachers' overall beliefs about discourse in the mathematics classroom, but the BMD measure was able to characterize more complex beliefs about mathematics discourse. Jang (2010) believed this was because participants were able to express their beliefs about how teachers should respond to specific classroom scenarios when responding to the BMD measure rather than merely selecting a statement in the PADM measure. In addition, Jang (2010) found that the PADM measure may be susceptible to social desirability bias because 96% of participants (157 out of 163) scored higher on the PADM measure than the BMD measure. These results would imply that teachers were more supportive of mathematics discourse when taking the PADM measure than when taking the BMD measure.

To address the reliability evidence, Jang (2010) used Cronbach's alpha, Pearson Separation, and included data for inter-rater reliability. Cronbach's alpha, which was measured at 0.72, and Pearson Separation reliability, which was measured at 0.75) suggested an acceptable reliability level. For each item, the inter-rater reliability ranged from 0.50 to 0.97, which Jang (2010) noted indicated a need for more rater training.

Post- Semi-Structured Interviews

Following the completion of the online survey, four participants volunteered to participate in an interview to explore elementary mathematics teachers' beliefs about mathematical discourse. Data from this part of the study provided an opportunity to better understand teacher beliefs regarding the role of discourse in the classroom. All teacher-participants were invited to participate, but participation was voluntary. Participants who choose to participate in the interview were contacted by the researcher via email to setup a convenient time for that portion of the study. These participants completed one audio-recorded, semi-structured interview via Zoom with the researcher, and the interviews were transcribed verbatim. The interviews last approximately 30-45 minutes, and no demographic information were collected at this time. The semi-structured interview protocol appears in Appendix F, and a codebook that includes a definition and example of the themes and subthemes that emerged from the interview transcripts can be found in Appendix G.

Validity and Reliability

Validity was established for the semi-structured post-interviews through triangulation of data. During data collection, the information was substantiated through the sources of data collected and between research participants. Any contradictory data was reported for the purposes of transparency and to ensure validity. Reliability was enhanced through the use of a digital recorder to store a quality recording of the interviews to be transcribed. Participants' interview responses were also compared to their BMD survey scores and responses. Also, reflexivity was also used to acknowledge how my background and beliefs about mathematics discourse influence the study.

Data Collection and Analysis

Construct Map

At the lowest level, Univocal discourse is focused on conveying an exact message, and teachers at this level, aim to transmit precise meanings and procedures through explanations and demonstrations of correct methods to solve a problem (Frykholm, 1999; Knuth & Peressini, 2001; Scott & Mortimer, 2005, Truxaw & DeFranco, 2008). Additionally, communication is univocal when the teacher does not seek to understand the student's reasoning (Knuth & Peressini, 2001). Jang (2010) stated teachers who engage in univocal discourse with students concentrate on the accuracy of student answers instead of their reasoning. Rather than asking students to elaborate on their thinking or engage in communicating with other students, teachers, at this level, believe their role is to explain a correct procedure to students, point out incorrect student answers followed by further explanation of the correct response (Jang, 2010). They believe the role of explaining belongs to the teacher because students may confuse one another due to their incomplete and difficult to understand explanations (Chapin et al., 2003).

Partial Univocal, the level above Univocal discourse, is characterized by teachers who still seek to tell students exact methods for thinking and problem solving, but in addition, they aim to check students' comprehension (Jang, 2010). The teacher still controls the discourse in the classroom through explanations and leading the questioning that takes place (Bearne, 1999). These questions typically seek to correct rather than understand student reasoning. While students may have an opportunity, to communicate in pairs or small groups, teachers, at this level, do not encourage students to communicate with one another during whole-class discussions. Jang (2010) described this group of teachers as committed to the belief that providing students step-by-step instructions for solving a problem are essential for student understanding.

Emerging Dialogue, the next highest level, includes a group of teachers who believe it is necessary to inquire about student reasoning, but they lack an understanding of the significance of student-to-student communication during whole class discussions (Jang, 2010). Teachers, at this level, draw out the reasoning of individual students (Scott & Mortimer, 2005). These conversations are restricted to sharing or support among students and between student and teacher (Cobb et al., 1997). Jang (2010) points out the efforts of these teachers to assist students in expressing their thinking, but explains they are often discussed with the teacher only since these teachers seldom promote student-to-student communication during a whole-class discussion.

Dialogical, the highest level, includes a group of teachers who believe the role of the teacher is to facilitate mathematical discourse “as a way to learn mathematics” by supporting students to collaborate with their classmates, clearly share their thinking in a coherent manner, and explore mathematical concepts (Jang, 2010, p. 17). These teachers facilitate active student participation and elicit student thinking for the purpose of constructing mathematical knowledge and guiding mathematical instruction that follows (Scott & Mortimer, 2005; Steffe & D’Ambrosio, 1995). Furthermore, teachers at this level, encourage and support student-to-student discussions for the purpose of sharing mathematical strategies and productively struggling about various mathematical ideas during whole-class interactions (Jang, 2010).

Prior to the creation of the BMD measure by Jang (2010), few survey instruments existed to measure teacher beliefs about mathematical discourse. Previous surveys, such as the one developed by DeFranco et al. (2008) and the Preservice Teachers' Attitude about Discourse in the Mathematics Classroom, or PADM (Casa et al., 2007) were developed based on the assumption that two dichotomous approaches to mathematical discourse existed – namely univocal or dialogical approach. These 5-point Likert-type surveys are easy to work with to assess teachers' beliefs of mathematical discourse but fall short in measuring teacher beliefs along a continuum from univocal to dialogical. Jang recognized the complexity in this scale and the need for more categories on the continuum. As a result, Jang (2010) created the BMD measure to accomplish this task.

The work of Brendefur and Frykholm (2000) developed a framework of four constructs including unidirectional, contributive, reflective, and instructive communication utilized to analyze assorted types of classroom discourse. This organization from simple to more complex types of communication was influential in the work of Jang (2010) in the creation of the BMD measure, but since the descriptions of each level lacked a complete explanation, improvements were made. The work of Scott and Mortimer (2005) established two valuable dimensions of classroom discourse, which are the dialogic-authoritative dimension and the interactive-noninteractive dimension. While these dimensions were informative, they were still organized as a dichotomy rather than a continuum (Jang, 2010). These frameworks were integrated by Jang (2010) to inform the creation of the BMD measure utilized in this study.

Figure 1

Construct Map from Jang (2010)

Belief in Whole-Class Discussion

Respondent	Responses
	More Belief ↑
Dialogical	It is important for the teacher to engage students in conversation with one another to justify their mathematical thinking and argue about mathematical ideas.
Emerging dialogical	It is important for the teacher to encourage students to explore multiple ways of thinking about mathematics and to share these thoughts with one another.
Partial univocal	It is important for the teacher to ask questions to check students' mathematical reasoning and to guide them to the correct answer.
Univocal	It is important for the teacher to explain mathematical ideas and to show to how to solve mathematical problems using a step-by-step process.
	↓ Less Belief

The Beliefs about Mathematics Discourse (BMD) Survey (Appendix D)

The survey data for this study was collected using an online survey administered using Qualtrics. Participants received the link to the online survey, and the informed consent was displayed in an introduction when participants clicked the link to begin the survey. In addition to participant responses to the survey, demographic information were collected including teaching experience, highest degree attained, teaching status, and grade level of instruction. The survey took participants approximately 15-20 minutes to complete.

A power analysis was completed to determine the minimum number of participants needed to validate the online-survey portion of the study. An a-priori sample size calculator for simple regression was used with an anticipated effect size (f^2) of .15, a desired statistical power level of .8, one predictor, and a probability level (*p*) of .05. This power analysis calculation indicated that a minimum of 47 participants for the online survey were needed to validate the study. For the ANOVA model used with the third research question with a significance level (α) of .05, a desired statistical power level of .8, six predictors, and a large effect size (f^2) of .4, the sample size calculator indicated that a minimum of 86 participants for the online survey were needed to validate the study. For the fourth research question, the same calculation was used with four predictors, and the results stated that a minimum of 73 participants were needed to validate the study. For the fifth research question with nine predictors, the sample size calculation indicated that a minimum of 102 participants were needed to validate the study.

Instead of using the item-specific scoring guide for the third and final version of the BMD provided by Jang (2010), the researcher used the results of a pilot study to create the Beliefs in Mathematics Discourse (BMD) Survey Scoring Guide to score each survey item (Appendix H). One rubric was used to score every question rather than using a different rubric for each question. For this study, two raters scored each survey, and inter-rater reliability, the degree to which the raters agree in the scoring of the survey items, was checked after all of the surveys were scored. The first rater was the researcher. The second rater was an elementary teacher with mathematics teaching experience. Prior to scoring the surveys individually, the two raters met to train using the BMD survey scoring guide.

First, the raters reviewed and discussed each section of the item specific scoring guide. For the final three questions, which are multiple choice, the raters agreed on a score for each answer choice. If the participant provided an explanation, the researchers agreed to read the explanation to determine if the participants written response confirmed or contradicted the answer choice selected. If the participant's written response differed significantly from the answer choice selected, the participants agreed to change the score based on the written response.

The raters met six times during the months of March-June to score the survey responses. During the first session, the raters scored questions one through eight for the first ten survey responses together. During the second session, the raters scored questions one through six and questions nine and ten together for the next ten survey responses. This strategy gave the raters an opportunity to score different questions independently and discuss their scores in order to score more consistently. During the third session, the raters scored the first three questions for the next 37 survey responses together. At the fourth session, the raters discussed all of the survey items they had scored independently clearing up issues that arose. For example, the raters determined how to respond if the participant's response included the word "discuss" as in "discuss place value." One rater believed this should be scored a one because she assumed the teacher would be doing the talking and explaining of the mathematical concept. The raters decided, however, to score these type responses as a two because the word "discuss" implies that both parties would be communicating, but that score could change if the remainder of the participants response warranted a different score. For example, if the rest of the response clearly indicated that the teacher would be explaining the mathematical idea to the student, the raters agreed to score the response a one, but if the rest of the response indicated the participant thought the teacher should include strategies from multiple students in the discussion, the raters agreed to score that response a three. After this discussion, the raters went back to all previous responses and made corrections to ensure consistency with this decision. The raters also realized they had each incorrectly scored one of the multiple-choice items, meaning the raters agreed on their response, but recorded this response incorrectly. Those errors were corrected immediately.

After all of these errors were addressed, the participants scored the remaining 25 survey responses independently. They met a fifth time to discuss their scoring for these survey responses, and if any of the scores were different, the raters reread the question and the participant's response. After a brief discussion, the raters decided on a final score which was recorded. The inter-rater reliability for this portion of the survey responses was 83% indicating a need to adjust the rubric for future research studies in order for raters to have a better understanding of how to score each response.

RStudio was used as the interface to run R code, a free open-source statistical software used to complete the statistical analyses. Prior to analysis, the dependent variables were tested for meeting the assumptions of normality and homogeneity of variance and were screened for outliers. To determine the level of elementary mathematics beliefs about dialogic discourse during whole class discussions among the current sample, the median, standard deviation, and range of the BMD survey scores were determined. Next, a simple regression model was used to examine the relationship between elementary years of teaching experience and teachers' beliefs about mathematics discourse. A one-way ANOVA was used to determine levels of beliefs about dialogic discourse during whole-class discussion differed among the teachers who had achieved various levels of degrees. A one-way ANOVA was also used to determine levels of beliefs about dialogic discourse during whole-class discussion differed among participants with various teaching statuses, the majority of whom were pre-service and in-service teachers and among the teachers who taught different grade levels. Descriptive statistics were analyzed for each research question including the mean and standard deviation.

Post- Semi-Structured Interviews

The interview data for this study was collected using recordings of the Zoom. Four interviews were conducted, transcribed verbatim, and read in their entirety. Throughout the content analysis, memos were developed to encompass ideas that emerged during the initial reading. Next, the interviews were analyzed line-by-line and themes were developed after multiple phases of coding. Pieces of the teacher's interview were be used to provide a more detailed description of their beliefs about the role of math discourse. The responses from teachers of similar demographic groups were also compared to one another to look for trends, and the trends were compared between the various demographic groups including the type of degree and years of teaching experience of participants.

Interpretation of Data Analysis

Following the analysis of the data collected from the BMD survey instruments results and the semi-structured post-interviews, the two data sets were analyzed together to draw conclusions. The qualitative data collected from the interview process were used to support the generalizability of the information quantitative data analysis. Exerts from teacher written responses were used to provide a more detailed description of their perceptions of the role of math discourse.

Summary

This study used multiple types of data collection and data analysis to explore teacher beliefs about mathematics discourse. The choice of using an open-ended item response survey instrument allowed teacher beliefs about how a teacher should respond to a hypothetical situation to lead a mathematics discussion to be more fully explained. This type of data collection addressed some of the limitations of the commonly used Likert-scale items. In addition, a large number of participants can be included in contrast to a case study type approach. Since the researcher still inferred teacher beliefs based on the participants' responses, a follow-up interview provided an additional opportunity for teachers to express their beliefs.

CHAPTER IV. RESULTS

Introduction

This mixed methods study examined elementary mathematics teacher beliefs about discourse during whole-class discussions. Each participant's level of beliefs about dialogic discourse were scored using the BMD survey. Relationships between this score and participants' highest level of degree attained, teaching status, and grade level taught were explored. In addition, voluntary follow-up interviews were conducted with four participants, and data was analyzed to provide further evidence of teacher beliefs of discourse. This chapter describes the data analysis and a detailed response for each research question.

Data Screening

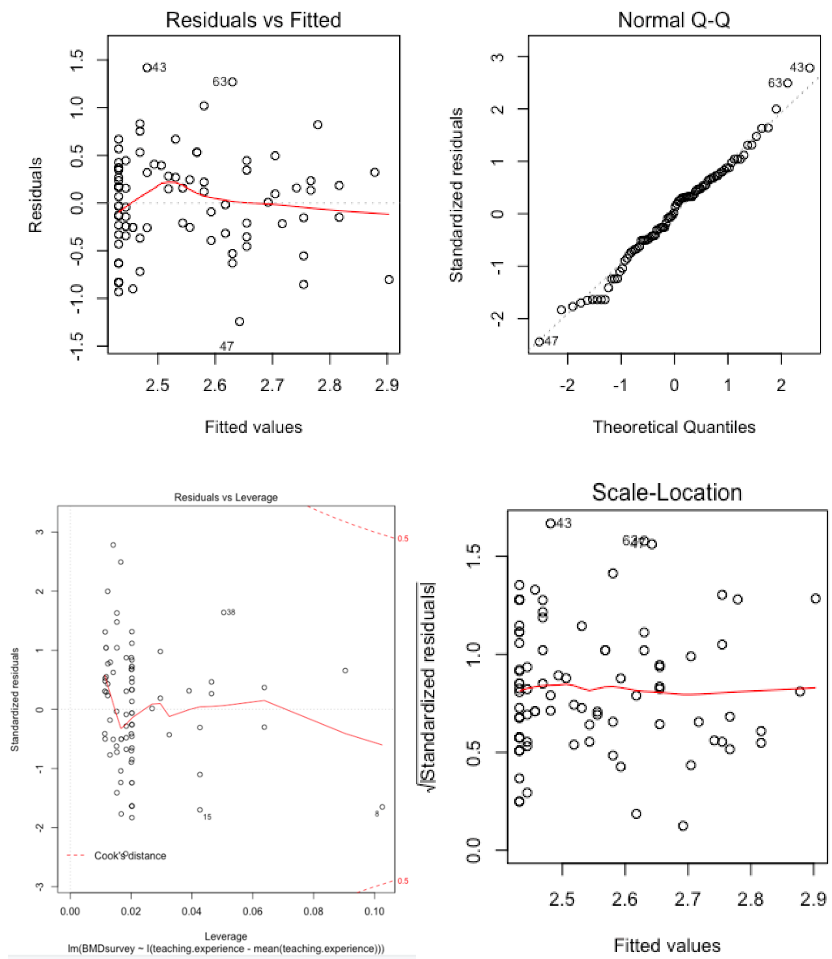
Prior to analysis of the data, the dependent variables were tested for meeting the assumptions of normality and homogeneity of variance and were screened for outliers. For the simple regression model, visualizing the residuals was used to diagnose potential problems in the model. For the one-way ANOVA analyses, boxplots and histograms were examined to evaluate outliers, the skewness and kurtosis of the data was used assessed normality, and Levene's test evaluated homogeneity of variances.

Several diagnostic plots were analyzed for the simple regression that examined the relationship between elementary years of teaching experience and teachers' beliefs about mathematics discourse (Figure 2). A linear relationship between can be assumed between the predictors and outcome variables due to the Residuals vs Fitted plot (top left) because the data mostly fits along a horizontal line. The normality assumption was visually assessed using the Q-Q plot of residuals (top right). High leverage points were evaluated using the Residuals vs Leverage plot (bottom left). The three most extreme points (#8, #15, and #38) were still within

Cook's distance lines. Three outliers were identified, but I decided to include all teachers' beliefs in the analysis. Homogeneity of variance can also be assumed because the residuals are primarily spread along the range of predictors with a predominantly red line fit across the data on the Scale-Location plot (bottom right).

Figure 2

Diagnostic Plots for Years of Teaching Experience



Only one outlier was identified in the boxplots (Figure 3), but I decided to include all scores from the BMD survey in the analysis. The outlier was within the grade level analysis. Scores on the BMD survey, which indicate a participant's level of beliefs about dialogic discourse during whole-class mathematics discussions, were normally distributed with a skewness of 0.06 and kurtosis of -0.07 (*SD* = 0.5). The assumption of homogeneity of variance was met for all groupings of educational level, teaching status, and grade level taught as determined by Levene's test. The outcome of Levene's test were not significant which indicated the assumption of homogeneity of variance had been met (Table 2).

Figure 3

Box Plots for Level of Degree, Teaching Status, and Grade Level

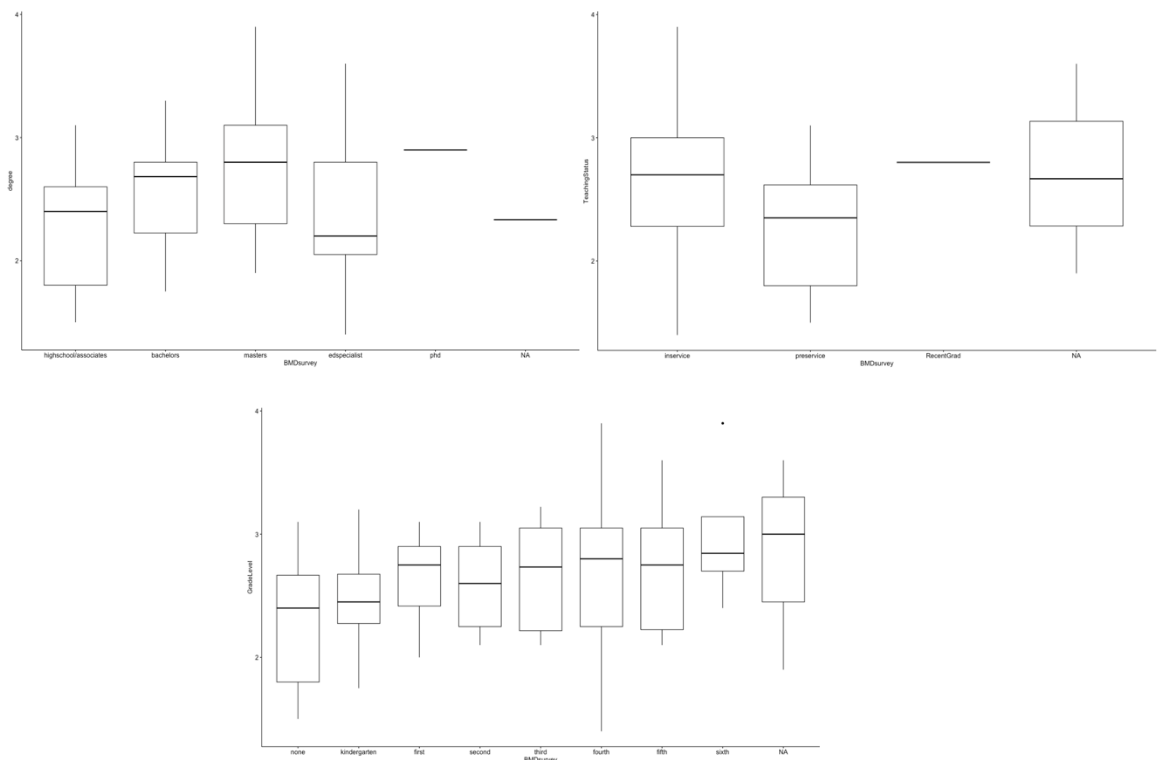


Table 2

Levene's Test for Homogeneity of Variance

	Levene Statistic	p-value
Degree	0.96	.43
Teaching Status	0.96	.39
Grade Level	0.65	.71

Results of Statistical Analyses

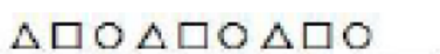
Research Question 1: What is the level of elementary mathematics teacher beliefs about dialogic discourse during whole-class discussions among the current sample?

Elementary mathematics teachers' beliefs about dialogic discourse were measured using the BMD Survey and questions were scored using a scale that ranged from one to four ($M = 2.5$, $SD = 0.5$). For each participant, an average score for the BMD Survey was calculated. Scores ranged from 1.4 to 3.9 with four being the highest level of dialogic discourse. Scores of zero indicated a missing entry and scores of 99 indicated a nonsense response. Both were removed for data analysis. Participants' level of belief of dialogic discourse was evident in the range of responses for the first question. The first question (see Figure 4) asked about responding to a student regarding a pattern problem.

Figure 4

BMD Survey Question One

1. A kindergarten teacher asks students to copy a pattern series and fill in the blank to continue the pattern.



The teacher reconvenes the class and asks Sarah to come up to the board to fill in the blank. Sarah fills in the blank with a circle.

What should the teacher SAY and DO to guide students toward an understanding of the correct pattern?

One participant responded that the teacher should, "Explain what a pattern is to the class and ensure Sarah also understands. Then, give her (an) opportunity to retry her answer. If she answers incorrectly again explain to her and the class the correct answer and why this is the correct answer. Next give another example for her/the class to answer." This response was scored as a one, the lowest level of belief of dialogic discourse because the knowledge and understanding of how to solve this problem is being transmitted in one direction. According to the scoring rubric, this would be univocal discourse because the teacher believes her role is to explain the correct definition of a mathematical concept, strategy for solving a problem, or solution without student encouraging the student to vocalize their thoughts. In this example, the student is a listener rather than a contributor to the discussion.

Another participant responded that the teacher should, “The teachers should ask Sarah what the pattern is in order to understand Sarah’s thinking. After that, Sarah may be able to understand what the pattern is to correct herself, but if she does not, the teacher can explain what a pattern is and guide Sarah to an understanding of the pattern and how to find patterns.” This response was scored as a two, the next to lowest level of belief of dialogic discourse because the teacher is still the primary voice for sharing knowledge with little student input. In keeping with the scoring rubric, this response would be partial univocal discourse because when a teacher does prompt student contribution, the purpose is for the teacher to check student understanding with the hope that students will self-correct their incorrect responses. The focus of this example is in reaching the correct answer rather than encouraging the student to construct an understanding. During the interviews, one participant noted that in her observations, students seemed to immediately react negatively when they were questioned, meaning they automatically assumed their answer was incorrect if it was questioned. The participated noted that perhaps this was due to the fact that often incorrect responses were questioned while correct responses are commended and the instruction proceeds without further discussion as is evident in partial univocal discourse.

A different participant said, “The teacher should ask students if they have any other answers and have them explain their thought process. Then circle around and ask if Sarah now understands." This response was scored a three, the next to highest level of belief of dialogic discourse because while multiple students are encouraged to share their thinking, the focus is still communicating between the teacher and student rather than including student to student conversations as a part of the whole class discussion. In keeping with the scoring rubric, this response would be considered emerging dialogic discourse because the teacher believes students can learn from listening to other students explain their thinking. The teacher values multiple strategies and multiple solutions, but the teacher is still leading the discussion rather than allowing students to converse with one another during whole class discussions.

Another participant responded, "The teacher should ask if the class agrees or disagrees with Sarah's answer. Then allow a student that disagrees to share why and have a discussion with Sarah about how she chose a circle. Through explaining their choices the class can gain a deeper understanding of why the pattern would need a triangle next." This response would be scored as a four, the highest level of belief of dialogic discourse because the students are encouraged to communicate with one another to construct an understanding of mathematics. According to the scoring rubric this reply aligns with dialogic discourse because the student is communicating with Sarah to explain why he/she disagrees with her in order to convince her to change her thinking. Students are believed to be capable of both listening and contributing to the mathematical discussion including being able to converse with their peers while the teacher steps back into a facilitator role rather than the central control of the discourse.

While these are only four example responses to the first question, it is evident that there is a range of teacher beliefs of mathematical discourse. This finding was evident in the analysis of the interview transcriptions as well. Even though all four interview participants expressed their belief that discourse was important to student learning, a difference in their level of belief of dialogic discourse was evident.

During the interview, participants were asked, “In your opinion who should be doing most of the talking during a whole class mathematics discussion and why?” Participant 4 responded that she believed the conversation should be 50/50 between the teacher and students while the other three participants believed students should be doing most of the talking. This participant added that “during the beginning of the week, it’s going to be more teacher conversation or guided,” working toward that “back and forth” as students became more comfortable with the material. She explained that they teach one concept per week noting that some concepts stretch to two weeks of instruction. These beliefs would be described as partial univocal because the teacher’s role is to explain while the student’s role is to ask questions to deepen their understanding of the teacher’s instruction. She adamantly explained that discourse was important stating that a quiet classroom in which the students listening to one person, the teacher, talk “doesn’t bode well.” While this teacher believes discourse is important, there is a difference in her level of belief from the other participants. For example, the other three participants responded to the same question, which is, “In your opinion who should be doing most of the talking during a whole class mathematics discussion and why?”, that they believed that whole class discussions should be more student voice than teacher voice. In other words, students should be doing more of the explaining and questioning, but even amongst these three participants, differences were evident among their beliefs. While they all agreed that the teacher should serve more as a facilitator of the classroom discourse and learning, Participants 2 and 3 said the whole class discussions should be led by students, meaning the student presenting should have the responsibility of asking his or her peers if they have any questions and calling on them. I believe these two participants beliefs could be classified as dialogic discourse because students are expected to discuss mathematical ideas, strategies, and problems for the purpose of learning from one another and constructing and understanding of what is being deliberated. These differences amongst the participants beliefs confirm the need for a range of categories to categorize teacher beliefs of mathematics discourse rather than the traditional dichotomous view that teachers either believed in univocal or dialogic discourse.

Research Question 2: Do years of teaching experience impact teachers' beliefs of the role of mathematics discourse during whole-class discussions?

A simple regression model was used to examine the relationship between elementary years of teaching experience and teachers' beliefs about mathematics discourse. Measurements of teachers' beliefs of the role of mathematics discourse during whole-class discussions were an average of their score from the BMD Survey previously discussed (M = 2.5, SD = 0.5, scores range from one to four). In addition, the predictor of years of teaching experience was self-reported by participants (M = 9.1, SD = 1.3). Descriptive statistics for years of teaching experience and BMD survey scores are reported in Table 3. The main effect of years of teaching experience was included in the full model, as shown in (1) while the standardized score form was included in (2).

$$\mathrm{Y_i} = \beta_0 + \beta_1 \mathrm{Experience_i} + \varepsilon_\mathrm{i} \quad (1)$$

$$Y_i = \beta_0 + \beta_1 Experience_i \quad (2)$$

A correlation table is shown in Table 4, and the results of the regression model are given in Table 5. A significant regression was found ($F_{1,86} = 5.37$, $p = .02$) with years of teaching experience explaining about 6% of the variance in BMD Survey scores ($R^2 = .06$). The intercept ($\beta_0 = 2.54$, $p < .001$) is the expected BMD Survey score for a participant with an average number of years of teaching experience. The main effect of years of teaching experience ($\beta_1 = 0.01$, $p = .023$) indicates that participants' scores increased, on average, 0.01 points for additional each year of teaching experience. The estimated equation that includes this regression coefficient is included in (3).

$$\hat{Y} = 2.54 + 0.01x_i \quad (3)$$

Years of teaching experience explained about 6% of the variance ($R^2 = .06$). Based on the residual plots shown in Figure 2, a non-linear model may be a better fit for this data since the curved line suggests a quadratic term may be needed. An effect plot for years of teaching experience is shown in Figure 5, and a residual plot for years of teaching experience is included in Figure 6.

Table 3

Descriptive Statistics for Numerical Variables

	vars	*n*	mean	*sd*	min	max	range	se
teaching.experience	1	88	9.1	10.3	0.0	38.0	38.0	1.10
BMDsurvey	10	88	2.5	0.5	1.4	3.9	2.5	0.06

Table 4

Correlation Table

	Teaching experience
BMD survey	0.24*

*** p < .001, ** p < .01, * p < .05

Table 5

Simple Regression Model Predicting Teachers' Beliefs of Discourse

	Full Model
(Intercept)	2.54***
	(0.05)
Teaching experience (centered at the mean)	0.01*
	(0.01)
R^2	.06
Adj. R^2	.05
Num. obs.	88
RMSE	0.51

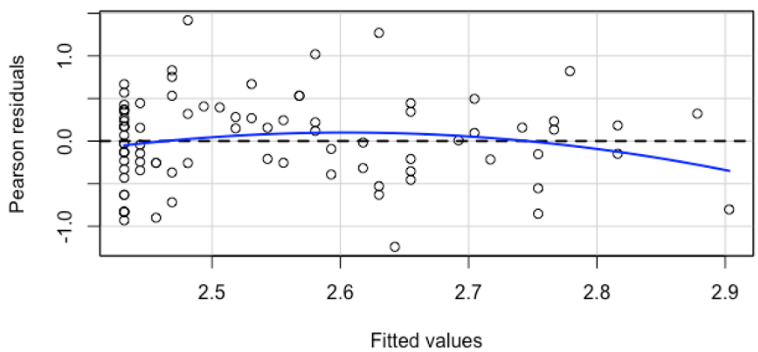
*** p < .001, ** p < .01, * p < .05

Figure 5

Years of Teaching Experience Effect Plot

Figure 6

Residual Plots for Years of Teaching Experience



Research Question 3: Do teachers with various levels of degrees have different levels of beliefs about dialogic discourse during whole-class discussions?

A one-way ANOVA was used to determine levels of beliefs about dialogic discourse during whole-class discussion differed among the teachers who had achieved various levels of degrees. There was a significant difference in BMD survey scores among the five groups of teachers who had earned different degrees ($F_{4,82} = 3.42$, $p = .012$). About 10% of the variance in scores was explained by the different degrees ($\omega^2 = .10$). To follow-up on the significant omnibus test, a Tukey multiple pairwise comparison post-hoc test was used. Based on the Tukey post-hoc test, those with a master's degree, scored significantly higher than those with a high school/associate degree ($p = .006$). It is important to note that participants with a high school/associate degree are preservice teachers. There was no significant difference between those with a bachelor's degree versus those with a high school/associate degree ($p = .233$), those with an educational specialist degree versus those with a high school/associate degree ($p = .973$), or those with a doctoral degree versus those with a high school/associate degree ($p = .748$). There was also no significant difference between those with a master's degree versus those with a bachelor's degree (p = .610), those with an educational specialist degree versus those with a master's degree (p = .941), or those with a doctoral degree versus those with a bachelor's degree (p = .970). There was also no significance between those with an educational specialist degree versus those with a master's degree (p = .445), those with a doctoral degree versus those with a master's degree (p = .999) or those with a doctoral degree versus those with an educational specialist degree (p = .894). However, because the sample size for PhD degree was small, the results may be biased for that group. See Table 6 for descriptive statistics by group. In the present sample, participants with a master's degree were associated with a higher level of beliefs about dialogic discourse during whole-class mathematics discussions.

Table 6

Descriptive Statistics by Highest Degree Attained

	N	*M*	*SD*
High school/Associate degree	25	2.3	0.5
Bachelor's degree	26	2.6	0.4
Master's degree	28	2.8	0.5
Educational Specialist degree	7	2.4	0.7
PhD	1	2.9	N/A
N/A	1	2.3	N/A
Total	88	2.5	0.5

Research Question 4: Do pre-service and in-service teachers have different levels of beliefs about dialogic discourse during whole-class discussions?

A one-way ANOVA was used to determine levels of beliefs about dialogic discourse during whole-class discussion differed among participants with various teaching statuses, the majority of whom were pre-service and in-service teachers. There was a significant difference in BMD survey scores among the groups with a different teaching status ($F_{2,82}$ = 5.746, p = .005). About 10% of the variance in scores were explained by teaching status (ω^2 = 0.10). To follow-up the significant omnibus test, the Tukey multiple pairwise comparison post-hoc test was used. In-service teachers scored significantly higher than pre-service teachers (p = .004). There was no significant difference between in-service teachers and the recent graduate who had not yet started teaching (p = .958), or between pre-service teachers and recent graduates (p = .555). See Table 7 for descriptive statistics by group. In the present sample, in-services teachers were associated with a higher level of belief of dialogic discourse than pre-service teachers during whole-class mathematics discussions.

Table 7

Descriptive Statistics by Teaching Status

	N	*M*	*SD*
Recent graduate/not yet teaching	1	2.8	N/A
Pre-service teacher	56	2.7	0.5
In-service teacher	28	2.3	0.5
N/A	3	2.7	0.9
Total	88	2.5	0.5

Research Question 5: Do teachers in different grade levels have different levels of beliefs about dialogic discourse during whole-class discussions?

A one-way ANOVA was used to determine if levels of beliefs about dialogic discourse during whole-class discussion differed among the teachers who taught different grade levels. There was no significant difference in BMD survey scores among participants who taught various grade levels ($F_{7,77}$ = 1.89, p = .082). According to the power analysis, 102 participants were needed to validate the study, and because only 88 participants completed the BMD survey, the limited number of participants may have contributed to this result. See Table 8 for descriptive statistics by group.

Table 8

Descriptive Statistics by Grade Level

	N	*M*	*SD*
None (Pre-service teachers)	29	2.3	0.5
Kindergarten	8	2.5	0.4
First grade	8	2.7	0.4
Second grade	11	2.6	0.4
Third grade	8	2.7	0.5
Fourth grade	11	2.7	0.7
Fifth grade	6	2.7	0.6
Sixth grade	4	3.0	0.6
N/A	3	2.8	0.9
Total	88	2.5	0.5

Research Question 6: What are elementary mathematics teachers' beliefs about mathematical discourse?

Voluntary semi-structured post-interviews were conducted after participants finished the BMD survey to understand teacher beliefs about mathematical discourse further. I conducted four interviews, and all the volunteers were white, female, in-service teachers with ten or more years of teaching experience. See Table 9 for descriptive statistics for each participant.

Table 9

Descriptive Statistics for Interview Participants

	BMD Survey Score	*Years of Teaching Experience*	*Level of Degree*
Participant 1	2.6	26	Educational Specialist
Participant 2	2.8	10	Bachelor's
Participant 3	3.6	12	Master's
Participant 4	2.3	15	Master's
Average	2.8	16	N/A

Participant 1 was a second-grade teacher with an Educational Specialist degree from Ottowa, Canada. At the time of the interview, she was teaching virtually due to the COVID pandemic. Participant 2 was a fourth-grade teacher with an undergraduate degree who had also been teaching virtually for over a year at the time of the interview. Participant 3 was a fifth grade teacher with a master's degree. Participant 4 was a fourth grade Special Education teacher with a master's degree. At the time of the interview, she was teaching in a blended learning format meaning that some of her students were virtual learners and the rest met in person.

The interviews were transcribed verbatim and read in their entirety. Throughout the content analysis, memos were developed to encompass ideas that emerged during the initial reading. Next, the interviews were analyzed line-by-line and themes were developed after multiple phases of coding. The primary focus of the analysis was to determine each participant's beliefs about mathematical discourse and what it should look like during whole-class discussions. The predominant themes that emerged were the role of the teacher and the role of the student during these interactions. Several other secondary themes emerged within these ideas including teacher questioning, teaching planning, time, student questioning, student engagement, emotions, formative assessments, and inclusivity. A codebook that includes a definition and example from the interview transcripts can be found in Appendix G. These will all be described within the frames of the role of the teacher and student during whole-class mathematics discussions.

The Role of the Teacher

The four participants varied in their beliefs of the role of the teacher during a whole class mathematics discussion. Participants 1, 2, and 3 characterized the teacher's role as someone who would allow students to share their strategies for solving a problem the teacher purposefully selected and posed to the class. They described the teacher as a facilitator who guides the conversation and keeps the discussion "on track." Participant 1 expressed that the discourse should be more student talking than teacher talking while the other two went as far as saying the discourse should be student-led. Participants 2 and 3 went as far as describing the need for teachers to hold back before they interject to prevent intervening too soon and one of these participants described how she mandated a teacher wait time for herself. Even though it appears all three of these participants believe in dialogic discourse, this indicates a difference in their level of belief of dialogic discourse.

Teacher Questioning

The theme of teacher questioning was evident across all participants, but they disagreed on the purpose and types of questions the teacher asked. Participants 1, 2, and 3 believed the primary purpose of teacher questioning was to further prompt student thinking and reasoning as indicated by the type of questions that they suggested that teachers should ask students. These questions were open-ended and began with words like, "Why?" The first three participants suggested that the teacher's role is to pose questions to the extent of even redirecting misunderstandings with questions. Participant 3 described this by saying, "I feel like the job of the teacher when a kid says something or asks a question, that the teacher's job is to question back to get the kids thinking about it rather than just giving the answer because I feel like when the teacher tells the answer, you stop the discussion, and it ends there."

Participant 4, in contrast to the participants mentioned in the previous paragraph, described the role of the teacher as someone who would question students on the extent to which they followed their "success criteria," the steps recorded for solving the problem independently. She described the teacher as a moderator who has talking points, and she suggested the ratio of teacher to student talking should be 50/50, meaning the teacher talks half of the time and the remaining half is split between all learners. This participant emphasized that discourse should not be all teacher talk, and teachers should not talk at kids. To explain what this meant, she asserted that within this conversation between teacher and student, both sides should be talking adding that the teacher should avoid, '*Charlie Brown teachering'* at me (the student) all the time because the student would not get it. She laughed as she referenced this fictitious character from the Charles Schulz *Peanuts* comic strip, later turned into a cartoon, who with an unintelligible "wah wah" voice that was easy to tune out. The participant also stressed the importance teacher questioning, but the type of questions that she posed differed from the other participants. In contrast to the other participants, who discussed teacher to student, student to teacher, and student to student questions about the student constructed strategies used to solve a problem, this participant questioned students recall of their prior knowledge and their understanding of the tools, practices, and steps they have been shown. She accentuated that the student should understand why they do something a certain way so students will have a deeper understanding of the mathematics. While this participant strongly believes in students learning through mathematical discourse, these beliefs indicate a difference in her level of belief of dialogic discourse from the other participants.

Teacher Planning

All four participants discussed the importance of teacher planning for discourse, but they differed in what that planning looks like for teachers. All believed that teachers should purposefully select problems that have an entry point for all students and can be solved with multiple strategies. For example, when students are constructing an understanding of multiplication, they may be asked to determine the number of square foot tiles needed for the flooring of a small storefront that is 18 feet long and 9 feet wide. While some students may count every single time, others may use repeated addition to add the tiles in each row or column. Additionally, a student could separate the space into two smaller sections. One section would be

10 feet by 9 feet and the second space would be 8 feet by 9 feet. After calculating the number of tiles needed for each space, the student would add them together, establishing a foundation for area models, the distributive property, and distributing binomials in algebraic equations. Participant 1 suggested occasionally using problems that have multiple solutions. For example, if a farmer has 24 feet of fence, what size pen could he make for pigs? There are multiple dimensions possible for the pig pen with the given materials, and the problem could prompt a discussion on which solution was the most practical. Participants 2 and 3 noted that teachers should also purposefully select students to explain their strategies for solving the problem being discussed, and Participant 2 went as far as to say the teacher should order these from “rudimentary to complex.” Participant 3 expressed the difficulty in planning for discourse due to its unpredictable nature may deter some teachers from engaging in it with students. She suggested that teachers plan questions and anticipate how students might respond as a part of their lesson planning. Participant 4 described teacher planning as preparing talking points that need to be said. These beliefs again signify a difference in teachers’ level of beliefs of dialogic discourse in the classroom.

Time

Participants 2 and 3 brought up the idea of time as an obstacle for teachers engaging in student led discourse. These two participants acknowledged the lengthy list of standards that teachers are required to teach, and the time required to allow mathematical discourse to develop organically from the students. The human element to this method of student learning is takes time to develop and time is a very valuable commodity in education.

Formative Assessments

Formative assessments also emerged as a theme from all participants when discussing their beliefs about mathematical discourse during whole class discussions. They discussed the use of hand signals as a way to formatively assess students during this part of instruction. As previously mentioned, Participant 2 encouraged students to use a hand signal to communicate when they needed the presenter to restate what they had just said. Participant 1 described a list of hand signals including a thumb to the chest to indicate when a student has a response, students hold up a finger for each strategy they have used to solve the problem, and students wiggle their fist to indicate they are still working. Participants 1, 2, and 3 explained how they use thumbs up or thumbs down for students to vote for their choice between two answer choices. Additionally, Participant 1 said she also encourages her students to put their thumb in the middle if they are uncertain. Participant 2 mentioned purposefully sequencing student strategies that are presented during whole class discussions from basic to complex solutions, and in order to do so, the teacher must formatively assess student work prior to the whole class discussion to purposefully select the student work that will promote productive discourse on the mathematical ideas the teacher has anticipated will be brought out in the student work. Participants 3 and 4 also discussed using turn-and-talks during whole class discussions and the importance of students being able to restate what their classmate had said. Describing the importance of this type of discourse between students indicates their belief in the importance of students learning from their communications with one another.

Inclusivity

Finally, all participants described their belief that mathematical discourse provided an opportunity for all students to learn, and it is the teacher’s responsibility to setup the classroom discourse in a way to promotes everyone's participation. They described how seeing multiple strategies from other students for solving a problem opens up learning opportunities for more students. Participant 1 described the importance of students being comfortable expressing confusion, and stating, "I don't know YET, but I'm going to learn it." This communicates the expectation that everyone can learn and is expected to learn. She also communicated the importance of the teacher purposefully selecting problems that all students can jump into solving. Participant 4, who came from a special education background, was passionate about encouraging teachers to have high expectations for all students. She stated, "Just make sure everyone is talking, and don't assume because a kid has a label or whatever, don't have low expectations. Have the same expectations and figure out how everyone can meet those expectations." These statements indicate their beliefs that mathematical discourse is possible for all students and essential to each student's learning.

The Role of the Student

Participants diverged on some of their beliefs on the role of students during whole class discussions, but they agreed on several others. All participants expressed the importance of discourse for student engagement, students expressing confusion and students being able to explain mathematical ideas more aptly to their peers than the teacher in some cases. Participants differed in the extent to which students led the conversation and the type of discourse they engaged in with their peers.

Student Questioning

Each participant voiced the significance of students communicating when they are confused. Participant 2 explained that students asking questions, rather than just the teacher, was "one million percent" beneficial because, "It's that you've thought enough to articulate a question." She went on to say that questioning means engagement. This teacher even had a hand signal that students could use to silently voice their confusion. By holding up their hand in the shape of the letter "c" and wiggling it back and forth, students can respectfully ask the presenter to say what they just said again. Three of the participants recounted the value of training students in asking appropriate questions at the beginning of the year. Participant 3 explained that she would write 2 + 2 = 7 on her board at the beginning of the year. She used this to prompt discussion with her students about asking appropriate questions and respectfully disagreeing because students had to convince her to change her answer. This indicates the value participants place in student questioning because they are willing to invest time in training students to do this well. For participants 1, 2, and 3, this could even mean asking a classmate for clarification or questioning their classmates about the particular strategy or solution he or she shared which goes beyond the idea of the teacher as the ultimate source of knowledge in the classroom to whom all questions are directed. For participant 4, she believed that it was essential for students to express their confusion to the teacher or classmates, and she explained that student questioning was expected not just encouraged.

Student Engagement

All participants believed that discourse was essential for student engagement and understanding. They believed these classroom conversations served to deepen students' mathematical understanding. Participant 4 stated that, "Just because they're staring at you and that they're quiet, that doesn't mean that they're engaged or they're even learning anything. That's a big concern." Participant 3 stated, "I think that it's the only way that our kids are going to actually retain and learn." In order to engage students in the discourse, the participants described various student responsibilities including asking for clarification when confused and expressing disagreement. In addition to asking questions, students were expected to be able to restate someone else's thought or strategy. Participants 1, 2, and 3 also explained the importance of students debating to respectfully resolve disagreements. This indicates their level of belief of dialogic discourse because students are communicating and learning from one another.

Emotions

Throughout the interviews, participants described various emotions that students expressed during whole class mathematical discussions. All participants described positive student emotions, such as excitement, associated with the student being successful in some way during the mathematical discussion. Participants 2 and 4 mentioned the thrill that comes with learning something new and the desire to want to share it with others while the other two referred to student confidence as a result of taking ownership of one's strategy for solving a problem and being able to defend it to classmates. Participants 2 and 3 spoke of breaking down the hesitation with mathematics. Participant 3 explained this by saying that when students begin talking about their misconceptions or see their peers had similar misconceptions, they realize they are not alone. In other words, making mistakes and determining how to correct those mistakes is normalized as a part of learning rather than a source of embarrassment or shame. As a result, students are more eager to engage in the mathematics without fear of failure because mistakes are part of learning. Participant 3 described this as positive peer pressure when students see the impossible is possible, and students benefit from seeing other students share their strategies for solving a problem they thought was unsolvable. Additionally, students benefit from seeing a peer struggle with the same misconception because it is comforting to know someone else struggled with the idea too. All participants described the positive impact of students hearing a mathematical concept or strategy explained by their peers. Participant 4 explained that peers use "kid friendly language" and Participant 1 stated, "I think it sinks in more than if I do it. I think it lets them feel like, 'Oh, ok!' rather than 'Oh I got it wrong.'" Participant 4 purposed a possible reason for this interaction that takes place. In the teacher and student relationship, the teacher is viewed as an authority figure. She explained that for some students the adults in their life are the ones who are verbally abusive, so an adult correcting you may cause anxiety in this situation. The excitement that students feel when they have learned something new could spark interest from their peers. Regardless, all participants believed that students listen better to their peers' explanations. Participant 3 also noted the emotions that arose in her classroom when students were questioned. She explained that students sometimes shut down as soon as they were questioned about their work, and she posed that perhaps this was due to previous school experienced. Due to time constraints, teachers often move on quickly when students have the correct answer, so when students are questioned, they automatically believe they did something wrong. She explained that it takes time to help students understand that questions are asked to help someone understand rather than always point out errors.

Summary

There was a significant difference in BMD survey scores among the five groups of teachers who had achieved different degrees as well as among the groups with a different teaching status. In particular, those who had a master's degree, scored significantly higher than those with a high school/associate degree, and in-service teachers scored significantly higher than pre-service teachers. The semi-structured post interviews provided more descriptions of the various levels of teacher beliefs of dialogic discourse during whole class mathematics discussions. These findings will be discussed further in Chapter 5 along with limitations of the study and recommendations for future research.

CHAPTER V. DISCUSSION, IMPLICATIONS, LIMITATIONS, AND CONCLUSIONS

Introduction

The purpose of this study was to explore the level of elementary mathematics teacher beliefs about dialogic discourse and teachers' beliefs about the role of mathematical discourse during whole-class discussions. The impact of years of teaching experience, level of degree, teaching status, and grade of instruction on the participants' level of elementary mathematics teacher beliefs about dialogic discourse were investigated. Teacher beliefs were the focus of the study. Participants' scores from the Beliefs in Mathematics Discourse survey and follow-up interviews with four participants were the data examined for this study.

This study explored teacher beliefs about mathematical discourse using a survey with open-ended items as well as multiple-choice items rather than a traditional Likert-scale items. Data was collected with a large number of participants in contrast to the case studies that are common when studying teacher beliefs. Teachers' beliefs about mathematics may shape the discourse in the classroom, and subsequently affect the mathematics instruction (Truxaw & DeFranco, 2017). According to Smith & Stein (2011), the teacher's role in discourse is pivotal, a claim with which all four interview participants for this study agreed. The survey data in combination with the follow-up interviews expand the current research-based knowledge on teacher beliefs of mathematics discourse. After data was collected, the two data sets were analyzed together, and several similarities and differences were noted between the qualitative and quantitative data collected for this study. This chapter discusses the findings, implications, limitations and conclusions of the study.

Findings

Research Question 1: What is the level of elementary mathematics teacher beliefs about dialogic discourse during whole-class discussions among the current sample?

Both the survey and interview data indicated a range in teachers' level of belief of dialogic discourse. The BMD Survey questions were scored using a scale that ranged from one to four (M = 2.5, SD = 0.5), and participant scores ranged from 1.4 to 3.9 with four being the highest level of dialogic discourse. Survey responses included a range of responses including replies that aligned with univocal discourse, partial univocal, emerging dialogic, and dialogic, the levels of discourse on Jang's (2010) construct map (Figure 1), meaning the average BMD survey score (M = 2.5) fell between partial univocal and emerging dialogic discourse.

In this research study, the mean score of 2.5 for participants on the BMD survey, with four being the highest level of dialogic discourse, indicated there is room for teachers to progress and grow in their level of beliefs about dialogic discourse during whole class mathematical discussions. McGatha & Bay-Williams (2013) noted discourse is essential to teaching for mathematical proficiency, and Drageset (2015) noted that one of the aims of mathematical discourse is to promote mathematical reasoning rather than the transmission of facts and strategies. The data from this research study indicated that teachers have mixed beliefs on these statements and the level of teacher beliefs of dialogic discourse still vary considerably. While some agreed whole heartedly believing students can lead and contribute to the classroom discourse to construct an understanding of the mathematical ideas being discussed, others disagreed believing the teacher is still the one who shares the mathematical knowledge and strategies with students. This finding that participants in the study had a large range in their level of beliefs of dialogic discourse suggests that within a cohort of teachers, their level of beliefs of dialogic discourse can vary significantly. The research of Brendefur and Frykholm (2000) supported the finding that participants had a large range in their level of beliefs of dialogic discourse in their study of two preservice teachers' conceptions and practices. In these case studies, the two participants contrasted one another in their beliefs about mathematical discourse and their capacity and enthusiasm for implementing discourse in the classroom.

The BMD survey could be used as a self-assessment tool at the beginning of a mathematics methods course, when working with an instructional coach within a school district, or at the beginning of a professional development course, and the education course, coaching cycle, or professional development in order to identify and meet individual teacher needs. Changing teachers' instructional practices involving productive whole class mathematical discussions can be a lengthy and difficult endeavor (Hoffman et al., 2009; McKeown & Beck, 1999), but it would be especially difficult for a teacher to set goals for improvement without first knowing his or her starting point. For example, one of the interview participants expressed strong beliefs about the importance of students participating in whole class discussions, and she talked about student led discussions as an aim for what discourse could look like ten years from now, while two of the other participants described how they utilized student led discussions in their classrooms regularly. She was unaware that student led discussions were already taking place in classrooms. The fourth participant also expressed her strong beliefs in student participant during classroom discourse, but the instructional practices she described would score a two on the BMD survey rubric, with four being the highest level of belief of dialogic discourse. She was confident in her believes that students should be able to ask questions in the classroom, but she was unaware that students could do much more such as leading whole class discussions. In order for these teachers to make changes in their instructional strategies to promote a more productive mathematics discourse, a self-assessment tool, such as the BMD survey, is needed to inform them of where their current beliefs fell along the continuum of dialogic discourse and the areas in which they had room for improvement. An analysis of the teacher's responses could help to determine how their level of belief of dialogic discourse changes based on the mathematics content, meaning is subtraction a more difficult topic for the teacher to promote discourse? Once a teacher realizes he or she has room to grow and progress in their beliefs of productive whole class discussions, a plan for professional growth in this area could be established and put into action.

Research Question 2: Do years of teaching experience impact teachers' beliefs of the role of mathematics discourse during whole-class discussions?

A simple regression model found that participants' years of teaching experience were statistically significant in relation to their score on the BMD survey. The main effect of years of teaching experience ($\beta_1 = .01$, p = .023) indicated that participants' scores increased, on average, 0.01 points for each additional year of teaching experience with years of teaching experience explaining about 6% of the variance in BMD Survey scores ($R^2 = 0.06$).

The interview data differed from the survey data which could be due to the smaller sample interviewed. The two interviewed teachers with the least amount of experience were more vocal about describing student led whole classroom discussions, an idea that characterizes dialogic discourse because students are actively driving the conversation and learning through conversing with one another. These two participants had 10 years and 12 year of teaching experience and scored 2.8 and 3.6 respectively, with four being the highest possible score, on the BMD survey. Among these two participants, only one had a master's degree. While they had the least experience, they were experienced teachers. According to Caspari-Sadeghi & Konig, (2018) and Palmer et al. (2005) expertise in the teaching field requires more than five years of teaching experience and a role or position that indicated their expertise such as serving as a leader for the grade level in which they taught or mentoring new teachers. Both of these participants met these qualifications, and thus, could be considered an expert teacher. The other two participants had 15 years and 26 years of teaching experience and scored 2.3 and 2.6 respectively, with four being the highest possible score on the BMD survey. These two participants had a master's degree and an Educational Specialist degree respectively. Among these four interview participants, more years of teaching experience did not necessarily correlate with a higher score on the BMD survey. It would be interesting to explore how the amount of professional development on mathematical discourse impacts teacher beliefs on this topic in future research.

This finding that participants years of teaching experience were statistically significant in relation to their BMD survey score could be explained by a research study by McAninch (2015) who noted differences between experienced teachers and novice teachers finding that teachers with more experience asked questions and provided feedback more often than novice teachers. The research of Hufferd-Ackles et al. (2004) disagreed with this finding, however, and concluded that both novice and experienced teachers can establish classroom communities that advocate discourse, so perhaps another factor contributes to this result rather than years of experience alone. Jansen (2009) purposed those differences between experienced and novice teachers could be because teachers with more years of experience may have taught the same grade level for numerous years, and as a result, they have had more time to study the mathematics content for that grade level. According to Caspari-Sadeghi & Konig, (2018) and Palmer et al. (2005), a teacher is no longer considered a novice teacher after he or she has taught for five years. Content knowledge contributes to a teacher's confidence level and thus his or her ability to engage students in the classroom discourse.

Research Question 3: Do teachers with various levels of degrees have different levels of beliefs about dialogic discourse during whole-class discussions?

There was a significant difference in BMD survey scores among the five groups of teachers who had earned different degrees ($F_{4,82} = 3.42$, $p = .012$), and 10% of the variance in scores was explained by the different degrees ($\omega^2 = .10$). It is interesting to note that both years of teaching experience and teachers' level of degree were statistically significant in relation to participants' BMD survey scores. When two factors, years of teaching experience and highest level of degree attained, are analyzed together, these results may indicate that professional development and teacher education on the topic of discourse has an impact on teacher beliefs about mathematical discourse rather than experience alone.

Hanushek (1986) and Goldhaber and Brewer (1997) found that teachers' years of experience and level of degree were unimportant predictors of student achievement. These findings disagree with the results of the current research study. Goldhaber and Brewer (1996), studied student achievement at the secondary level and found subject-specific training had a positive impact on student test scores in the area of mathematics and science. While the elementary teachers who participated in this study do not have subject-specific degrees, these findings indicate the importance of specialized professional development and teacher training. DuFour & Mattos (2013) affirmed professional learning and teacher education regarding effective instructional practices is one of the most effective ways to boost student achievement, and Leatham et al. (2015) agree that teacher support in shifting to dialogic discourse and their belief in the value of this instructional practice is critical. These findings agree with the results of this study that imply that teacher training and professional learning is necessary to change teachers' beliefs and instructional practices, which impact student achievement. This conclusion has implications for future research that will be discussed later in this chapter.

Research Question 4: Do pre-service and in-service teachers have different levels of beliefs about dialogic discourse during whole-class discussions?

A one-way ANOVA was used to determine levels of beliefs about dialogic discourse during whole-class discussion differed among participants with various teaching statuses, the majority of whom were pre-service and in-service teachers. There was a significant difference in BMD survey scores among the groups with a different teaching status ($F_{2,82} = 5.75$, $p = .005$), and 10% of the variance in scores was explained by teaching status ($\omega^2 = 0.10$). In-service teachers scored significantly higher than pre-service teachers ($p = .004$). This indicates that in-service teachers have a higher level of belief of dialogic discourse during whole-class discussions as compared to pre-service teachers.

This finding that in-service teachers scored significantly higher on the BMD survey agrees with the research of Blanton et al. (2001) that found that novice teachers engaged in more univocal than dialogic discourse as compared to experienced teachers because teachers put into practice what they believe. This finding indicates the importance of the design of teacher education programs. A research study by Franke et al. (2001) discussed the importance of integrating both mathematical content and pedagogy. While his study focused on teachers participating in a professional development, I believe this idea applies to teacher education programs as well. Pre-service teachers should have the opportunity to learn both mathematical content and effective teaching practices for helping students to construct an understanding of this content. It is important to note that most of the pre-service teachers who participated in this study completed the survey at the beginning of their mathematics methods course before they had much experience teaching mathematics. This may have contributed to their scores being statistically lower than in-service teachers.

Research Question 5: Do teachers in different grade levels have different levels of beliefs about dialogic discourse during whole-class discussions?

A one-way ANOVA was used to determine if levels of beliefs about dialogic discourse during whole-class discussion differed among the teachers who taught different grade levels and found there was no significant difference in BMD survey scores among participants who taught various grade levels ($F_{7,77} = 1.89$, $p = .082$). This indicated that the grade level taught did not influence the participants' level of beliefs of dialogic discourse.

This finding of no significant difference in BMD survey scores among participants who taught various grade levels agrees with the research of Tuck (2018) who also found no significant difference in teacher perceptions of univocal and general mathematics discourse based on grade of instruction. Although, research by Tuck (2018) did find a significant difference among participants perceptions of dialogic discourse among participants who taught different grade levels. In her study, Tuck decided to band together multiple grade levels into two bands, elementary and middle grades. Two of the interview participants expressed an interest in the results of this portion of the research study. They wanted to know how teachers' beliefs of mathematics discourse differed among participants who taught different grade levels. As elementary teachers, they believed based on their experience that as students progressed to higher grades, the importance of discourse in the mathematics classroom diminished. While this research study did not find a significant difference in BMD survey scores among participants who taught various grade levels, this is likely the result of the small sample size of participants.

The data from this study could be used to band together multiple grade levels to explore differences in early childhood (kindergarten through second grade) and elementary (third through sixth grade). Banding together multiple grade levels would create a larger sample size.

Research Question 6: What are elementary mathematics teachers' beliefs about mathematical discourse?

Four voluntary semi-structured post-interviews were conducted after participants finished the BMD survey to understand teacher beliefs about mathematical discourse further, and the findings from the interview portion of this research study were explored. The first significant finding was the way in which the teacher sequences student work that is presented during whole class discussions is crucial to the productivity of the discussion and student learning. After students are given ample time to independently, with a partner, or as a small group, the teacher purposefully selects the work to be presented and the order in which it is discussed. The next major finding was that student discourse during whole-class discussions is imperative to learning that lasts. Teachers often express frustration over the concepts that students do not retain long after it has been taught such as place value or basic multiplication facts. Educators lament that this knowledge seems evaporate. In contrast, the interview participants discussed the important role that student discourse plays in their construction of knowledge that remains and serves as a foundation and building block for new learning.

Sequencing Student Presenters

One significant finding from the interview portion of this research study is the idea of strategically sequencing the students who presented their solutions for the mathematical problems being deliberated during whole class discussions based on the complexity of their work. Stein et al. (2008) agree with the importance of this finding, discussing various strategies for purposefully sequencing student working including starting with the most common strategy used by students that is based on a misconception because first clearing up that error would allow students to progress in their understanding of how to solve the problem. One participant noted that during her student led whole class discussions, she would ask multiple students who had solved a particular problem in different ways to explain their strategies and solutions, and she sequenced these from “rudimentary to the more complex.” She added that her reasoning for starting with the simple solution was in order to ensure there was an entry point for even the student who performs the lowest in mathematics. This participant was a fourth-grade teacher who briefly included an example of starting the whole class discussion by asking a student who had solved the problem using a drawing to present first which could lead to subsequent presenters who used a more efficient strategies. For example, if students were asked to help a local bakery determine the number of boxes needed to fill muffin orders that are sold in packages of 10, a student may draw the 132 blueberry muffins then outline groups of 10 to determine the number of boxes needed. Alternatively, another student may repeatedly write the number 10 on their paper, keeping track of their progress, working their way up until they reach the closest possible number then determine how many cupcakes would be in the last unfilled box and finally count the total number of boxes needed. Stein et al. (2008), recognize the importance of this conclusion, asserting that teachers who purposefully select and sequence students to explain their strategies make the classroom discourse more meaningful and make it easier to anticipate how to discussion will progress.

Learning that Lasts

In this research study, participants voiced their belief that mathematical discourse is vital for lasting learning. One interview participant noted, “I think it’s the only way that our kids are going to actually retain and learn." Duckworth (1996) agrees with this finding, concluding that students develop confidence in their own thinking when their teacher or peers value what they have to say. Another stated that she advocates for students to be able to understand the reasoning behind the mathematical ideas they discuss because "it's way more important than learning just an algorithm and the steps that way." She added that she hoped that students having the opportunity to ask questions and understand the "why" would never change. A third participant said, "They have to be involved in their learning or they don't learn" adding "they don't learn as well, and they don't retain it as well." She added, "The more active they're involved, the more they retain it." All four interview participants noted students were more active participants when they were involved in the discourse. One participant pointed out the value of having to "organize your thought enough to explain to another person," and she later said the same thing about the mental exercise of formulating a question. She also explained that she believed "you learn more when you're describing it and have to communicate it." When students are active participants in the classroom who elicit and reflect on the presenter's thinking, they have an opportunity to reflect prompting and increased metacognitive awareness, and Webb (1991) notes, "This cognitive restructuring may help the explainer to understand the material better, as well as help him or her recognize gaps in understanding" (p. 368). This participant went on to describe the excitement of student's taking ownership of their learning through participating in the whole class discussions by saying, "It's everything when you've achieved it. Then, it's yours. You get to keep that knowledge now."

Limitations of the Study

This study has three main limitations including self-reporting, and self-selection bias, and lack of generalizability.

Self-Reported Survey Data

Because the responses to the BMD survey were self-reported by participants, the research study is limited by a dependence on what participants are willing to disclose. Participants were asked to respond to teaching scenario from which the researcher could infer their beliefs about mathematical discourse, and because this instrument relies on inference (Jang, 2010), participants responses could look different if other scenarios were presented. For example, some of the participants included the word "discuss" in their response without further detail, and the scorers had to determine a consistent way to score these responses. When a participant said something such as, "Discuss place value," the scorers decided to rate this as a two using the BMD survey rubric. The term "discuss" implied a conversation would take place, so the scorers determined a one would not be the best fit for this response. The BMD survey was designed to limit social desirability bias (Jang, 2010), but this limitation was still possible. Participants could still describe what they believe the teacher "should do" rather than how they would actually respond to the situation presented. The results of the study could have been impacted for these reasons.

Self-Selection Bias

The research study was shared with a large number of pre-service and in-service teachers via convenience and snowball sampling, but they chose whether or not to participate in the study. The teachers who were interested in the topic of mathematical discourse were more likely to respond and invest time in a survey such as this one. Questions on the BMD survey required participants to respond in their own words, and as a result this unique instrument stands out in comparison to the typical Likert scale survey questions. Additionally, those who are confident in their beliefs about this topic were more likely to disclose their opinions.

Generalizability

The results of this research study are a representation of the current sample. They are not generalizable to all elementary pre-service and in-service teachers because the study only reached those contacted by convenience and snowball sampling. In order to represent the entire population of pre-service and in-service mathematics teachers, all of these teachers would need to be surveyed.

Recommendations for Future Research

It may be beneficial to examine whether certain mathematical concepts or topics better lend themselves to setting up discourse. Participants seemed to struggle with describing how to promote discourse with certain topics such as subtraction. Traditionally, students have been taught to subtract with regrouping, and these types of lessons would tend to have a lower level of dialogic discourse as the teacher is explaining the steps of the subtraction process to students. It would be beneficial to analyze participants' scores to determine if certain questions were more challenging to describe how to promote discourse than others.

The relationship between teacher beliefs and their current classroom practices could also be explored. The BMD survey asks participants to explain what the teacher should do to promote discourse in the situation presented, and it would be beneficial to compare this to how the teacher promotes discourse in his or her classroom. It would also be beneficial to compare these results to participants who completed a Likert-scale type survey to determine if the BMD survey reduces social-desirability bias.

For this study, I sought to explore the relationship between the amount of professional development or coursework on discourse a participant had received and their level of beliefs of dialogic discourse during whole-class discussions, but I failed to specify a common unit of measurement for participants to report this information. As a result, some participants recorded that they had received 100 hours of training on discourse while another participant reported completing one college course. It may be beneficial to investigate this relationship to determine if professional development or college courses on mathematical discourse impact participants' beliefs on this topic including the number of hours, type and other variations in training.

Some of the items on the BMD survey presented a situation where the student(s) correctly answered the math problem posed while others described a situation where the student(s) had answered incorrectly. It may be beneficial to analyze the relationship between the student accuracy and the teacher's level of belief of dialogic discourse demonstrated in the response. As one of the interviewed participants noted, teachers tend to respond to correct answers with affirmation then move on to other points of discussion without questioning that student. While scoring the BMD surveys, the second scorer and I noticed that some participants scored higher on these questions because their response was to tell the student they did a great job then ask other students to share a different strategy for solving the problem. This response would seem to score a three on the rubric, but more research is needed to determine if this is because the participant's belief of discourse should be scored at that level or if it is due to the nature of the question.

As previously stated, future research could further explore the difference in levels of beliefs about dialogic discourse during whole-class discussion among the teachers who taught different grade levels by creating grade level bands that include a larger number of participants. During the interview portion of the research study, one participant expressed an interest in exploring this question because she perceived a change in instructional practices once students went to sixth grade. Another participant expressed an interest in learning the differences in beliefs about dialogic discourse among early childhood teachers (kindergarten through second grade) and elementary teachers (third through fifth grade) because she recognized differences in teacher beliefs among those groups at the school where she taught. Additional interview questions could also explore teacher beliefs about their expectations for students at difference ages. In other words, does that teacher believe a kindergarten student is capable of engaging in classroom discourse or does the student need to be a certain age before this instructional strategy is appropriate? It would be beneficial to further investigate teacher beliefs about how a student's age affects his or her ability to participate in whole class discourse.

Almost all of the participants in this study were white females, which is not an accurate representation of the current population of elementary mathematics teachers. Future research could address the lack of diversity among the participants of this study. By expanding upon this study to include a more diverse selection of participants, future studies could learn even more about teacher beliefs about mathematical discourse during whole class discussions.

The creator of the BMD survey (Jang, 2010) encouraged using this unique tool to track changes in teacher beliefs over time, so a longitudinal study would be beneficial to investigate how teachers' beliefs change over time. Future research could present the BMD survey at the beginning and end of a mathematics methods course to assess changes in their beliefs after completing the curriculum. Jang (2010) proposed the BMD can be a beneficial tool for documenting changes in teacher beliefs even after the complete the teaching program when participants become a teacher and throughout their career. This current research study could include pre-service teachers in a mathematics methods course or participants in a professional development focused on discourse over the course of a year. Researchers would also be able to investigate strategies for developing productive discourse within these classrooms.

Conclusions

According to NCTM (2014), one of the most important teaching practices for educators is to "facilitate meaningful mathematical discourse" (p. 3). When discussing the Standards for Mathematical Practice, NCTM (2014) asserted that students should "construct viable arguments and critique the reasoning of others" in order to promote student learning (p. 8). As a result, it can be concluded that discourse is a critical topic of discussion and paramount to student learning. This research study utilized a unique survey instrument and follow-up interviews to explore teacher beliefs about whole class mathematics discourse. The range of teacher beliefs about mathematics discourse evident in this research study indicated that these opinions fall along a continuum rather than a binary option. Therefore, within a cohort of teachers, their level of beliefs of dialogic discourse can vary significantly. A tool, such as the BMD survey, can be used to help teachers who aim to improve mathematics discourse in their classroom establish a baseline for their level of beliefs of dialogic discourse from which to measure growth. In this study, participants with more years of teaching experience scored higher on the BMD survey indicating a higher level of belief of dialogic discourse ($F = 5.365$, $p = .02$). Additionally, participants with a master's degree scored significantly higher than those with a high school/associate degree ($p = .006$), and in-service teachers scored significantly higher than pre-service teachers ($p = .004$). The results demonstrate a need for more training on mathematics discourse, especially among pre-service and novice teachers. Future research is needed to explore the impact of professional development and teacher training on teacher beliefs on mathematics discourse to determine the most effective instructional methods. Additionally, the themes that emerged during the interviews include two predominant themes, including 1) the role of the teacher and 2) the role of the student, as well as several secondary themes within these ideas including teacher questioning, teaching planning, time, student questioning, student engagement, emotions, formative assessments, and inclusivity. These themes are important ideas that should be included in teacher training and professional development on mathematics discourse.

This research study was significant because the participants were able to use their voice to express their opinions and contribute to the research on mathematics discourse. In order to advance teachers' beliefs, and subsequently their instructional practices, related to mathematics discourse, it is imperative to first understand current beliefs. The results of this study will further the understanding of this topic as well as influence future studies using this unique survey instrument.

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Appendix A

Information Letter

AUBURN
UNIVERSITY

COLLEGE OF EDUCATION
CURRICULUM & TEACHING

(NOTE: DO NOT AGREE TO PARTICIPATE UNLESS IRB APPROVAL INFORMATION WITH CURRENT DATES HAS BEEN ADDED TO THIS DOCUMENT.)

INFORMATION LETTER
for a Research Study entitled
"Elementary Teacher Beliefs of Mathematics Discourse During Whole Class Discussions"

You are invited to participate in a research study to explore teacher beliefs about mathematics discourse in the classroom. This study is being conducted by Jana Walls, doctoral candidate, under the direction of Dr. Megan Burton in the Auburn University Department of Curriculum and Teaching. You were selected as a possible participate because you are a current elementary mathematics educator or a student who intends to become an elementary mathematics educator.

What will be involved if you participate? If you decide to participate in this research study, you will be asked to complete an online survey regarding your beliefs of mathematical classroom discourse. Your total time commitment will be approximately 15-20 minutes. Additionally, at the end of the survey, you may volunteer to complete a follow-up interview regarding your believes of mathematical classroom discourse. The researcher will contact you to setup an interview time, and your time commitment for the interview will be approximately 30-45 minutes. Your total time commitment will be approximately 45-65 minutes.

Are there any risks or discomforts? The risks associated with participating in the survey from this study are the same as you experience through an everyday use of the internet. To minimize these risks, we will allow you to submit your survey anonymously. The risks associated with participating in the interview portion of this study are a breach of confidentiality. To minimize these risks, the contact information you provide will only be used to setup the interview and interview responses will be recorded and stored separately from any identifiable information.

Are there any benefits to yourself or others? Participants do not directly benefit for participation. If you participate in this study, you can expect to better understand your personal beliefs about productive mathematical classroom discourse. We/I cannot promise you that you will receive any or all of the benefits described. However, by participating, you will be contributing to the knowledge base about teacher beliefs.

Will you receive compensation for participating? To thank you for your time completing the survey portion of this study, you may choose to enter a drawing for one of five $5 gift cards, and to thank you for your time completing the interview portion of this study, you may choose to enter a drawing for one of five $15 gift cards. To enter the drawing, you will provide your name and email address at the end of the survey. The recipients of the eGift Cards will be randomly selected, and the gift cards will be sent via email. This same process will be used after the interviews have been conducted.

5040 HALEY CENTER
AUBURN, AL 36849-5212
TELEPHONE:
334-844-4434
FAX:
334-844-6789
www.auburn.edu

The Auburn University Institutional Review Board has approved this Document for use from 10/30/2020 to --------- Protocol # 20-496 EP 2010

Are there any costs? If you decide to participate, you will not incur any costs to you.

If you change your mind about participating, you can withdraw at any time during the study by closing your web browser during the survey or informing the researcher via email that you no longer wish to participate in the Zoom interview portion of the study. During the interview, you can withdraw at any time by telling the researcher that you no longer wish to participate and closing your web browser. Your participation is completely voluntary. If you choose to withdraw, your data can be withdrawn as long as it is identifiable. Once you've submitted anonymous data, it cannot be withdrawn since it will be unidentifiable. Your decision about whether or not to participate or to stop participating will not jeopardize your future relations with Auburn University or the Department of Education. Your privacy will be protected.

Any information obtained in connection with this study will remain anonymous. Information obtained through your participation may be used to complete an educational requirement, published in a professional journal, or presented at a professional conference.

If you have questions about this study, please contact Jana Walls at jww0043@auburn.edu or Dr. Megan Burton at meb0042@auburn.edu.

If you have questions about your rights as a research participant, you may contact the Auburn University Office of Research Compliance or the Institutional Review Board by phone (334)-844-5966 or e-mail at IRBadmin@auburn.edu or IRBChair@auburn.edu.

HAVING READ THE INFORMATION ABOVE, YOU MUST DECIDE IF YOU WANT TO PARTICIPATE IN THIS RESEARCH PROJECT. IF YOU DECIDE TO PARTICIPATE, PLEASE CLICK ON THE LINK BELOW.
YOU MAY PRINT A COPY OF THIS LETTER TO KEEP.

Jana Walls
Student Principal Investigator

Dr. Megan Burton
Faculty Principal Investigator/Advisor

The Auburn University Institutional Review Board has approved this document for use from __________ to _________. Protocol #________

https://auburn.qualtrics.com/jfe/form/SV_b9q8Wkw7Es7YPHL

Version Date: 1/22/2021

The Auburn University Institutional Review Board has approved this Document for use from 10/30/2020 to --------- Protocol # 20-496 EP 2010

Appendix B

Survey Recruitment Email

To Whom It May Concern:

I am a doctoral candidate in the Department of Curriculum and Teaching at Auburn University. I would like to invite teachers or students who are working to become teachers to participate in my research study entitled "Elementary Teacher Beliefs of Mathematics Discourse During Whole Class Discussions." Teachers may participate if they are certified elementary mathematics teachers or currently enrolled in an elementary teacher education program.

Participants will be asked to complete an online survey which should take approximately 15-20 minutes to complete. Survey responses will be kept anonymous. Participants may choose to be entered in a drawing for one of five $5 gift cards for participating in the study.

If you would like to know more information about this study, an informational letter is attached to this email. If you decide to participate after reading the letter, you can access the survey from a link in the letter.

If you have any questions, please contact me at jww0043@auburn.edu or my advisor, Dr. Burton, at meb0042@auburn.edu.

Thank you for your consideration,
Jana Walls
Auburn University
jww0043@auburn.edu
205-910-9744

Appendix C

Interview Recruitment Email

Dear Participants,

Thank you for expressing interest in the follow-up interview for my research study "Elementary Teacher Beliefs of Mathematical Discourse During Whole Class Discussions". You may participate if you are a certified elementary mathematics teacher or enrolled an elementary teacher education program.

Participants will be asked to complete a follow-up interview which should take approximately 30-45 minutes to complete. Interview responses will be kept anonymous. Participants may choose to be entered in a drawing for one of five $15 gift cards for participating in the study.

Please click on the link below to setup a date and time for your interview.

If you have any questions, please contact me at jww0043@auburn.edu or my advisor, Dr. Burton, at meb0042@auburn.edu.

https://doodle.com/poll/ax65r4an5x6m6zgr

Thank you for your consideration,
Jana Walls
Auburn University
jww0043@auburn.edu
205-910-9744

Appendix D

BMD Measure (Version 3)

(Jang, 2010)

You are invited to participate in a research study to explore teacher beliefs about mathematics discourse in the classroom. This study is being conducted by Jana Walls, doctoral student, under the direction of Dr. Megan Burton in the Auburn University Department of Curriculum and Teaching. You were selected as a possible participate because you are a current or future elementary mathematics educator.

What will be involved if you participate? If you decide to participate in this research study, you will be asked to complete an online survey regarding your beliefs of mathematical classroom discourse. Your total time commitment will be approximately 15-20 minutes.

Are there any risks or discomforts? The risks associated with participating in this study are the same as you experience through an everyday use of the internet. To minimize these risks, we will allow you to submit your survey anonymously.

Are there any benefits to yourself or others? If you participate in this study, you can expect to better understand your personal beliefs about productive mathematical classroom discourse. We/I cannot promise you that you will receive any or all of the benefits described. However, by participating, you will be contributing to the knowledge base about teacher beliefs.

Will you receive compensation for participating? To thank you for your time, you may choose to enter a drawing for one of five $15 gift cards.

Are there any costs? If you decide to participate, you will not incur any costs to you.

If you change your mind about participating, you can withdraw at any time during the study by closing your browser window. If you choose to withdraw, your data can be withdrawn as long as it is identifiable. Once you've submitted anonymous data, it cannot be withdrawn since it will be unidentifiable. Your decision about whether or not to participate or to stop participating will not jeopardize your future relations with Auburn University or the Department of Education. Your privacy will be protected.

Any information obtained in connection with this study will remain anonymous. Information obtained through your participation may be used to complete an educational requirement, published in a professional journal, or presented at a professional conference.

If you have questions about this study, please contact Jana Walls at jww0043@auburn.edu or Dr. Megan Burton at meb0042@auburn.edu.

If you have questions about your rights as a research participant, you may contact the Auburn University Office of Research Compliance or the Institutional Review Board by phone (334)-844-5966 or e-mail at IRBadmin@auburn.edu or IRBChair@auburn.edu.

HAVING READ THE INFORMATION ABOVE, YOU MUST DECIDE IF YOU WANT TO PARTICIPATE IN THIS RESEARCH PROJECT. IF YOU DECIDE TO PARTICIPATE, PLEASE CLICK ON THE LINK BELOW.

About You

Information on your background will help me understand your opinions. All responses are confidential.

Gender

- [ ] Male
- [ ] Female

Age _______

Ethnicity

- [ ] African American
- [ ] Asian
- [ ] Caucasian
- [ ] Hispanic
- [ ] Pacific Islander
- [ ] Native American
- [ ] Other

How many years have you taught? _______

Approximately how many professional development sessions or methods class periods have you attended on mathematics discourse? ________

If you have attended one or more professional development sessions on mathematical discourse, please provide a brief description (at my school, NCTM conference, etc.).

What is the highest level of degree you have attained?

- [ ] High school
- [ ] Associates
- [ ] Undergraduate
- [ ] Masters
- [ ] Educational Specialist
- [ ] Ph.D.
- [ ] I am currently working on a degree.

If you are currently working on a degree, please specify what kind:

What is your certification level?

- [ ] Early Childhood (K-2)
- [ ] Elementary (K-6)
- [ ] Other

If you have another certification, please specify what kind:

What grade level(s) do you currently teach?

- [ ] None. I am a pre-service teacher.

	Kindergarten
	First grade
	Second grade
	Third grade
	Fourth grade
	Fifth grade

What is your current status?

	Recent graduate of teacher education program, but have not entered teaching yet
	In-service teacher
	Pre-service teacher

What kind of community do you currently teach?

	None. I am a pre-service teacher.
	Inner city/Urban
	Suburb
	Rural

Directions: Questions 1-7 start with a teaching scenario and ask for your opinion about teaching strategies. There are no right or wrong answers to any of these questions. Please provide "clear and detailed" answers to help me to understand your opinions.

1. A kindergarten teacher asks students to copy a pattern series and fill in the blank to continue the pattern.

△□○△□○△□○ ___

The teacher reconvenes the class and asks Sarah to come up to the board to fill in the blank. Sarah fills in the blank with a circle.

What should the teacher SAY and DO to guide students toward an understanding of the correct pattern?

2. A fourth grade teacher asks students to work individually to write a number sentence and solve the problem below. These students already learned about subtraction.

[Problem] You had $19.50 and bought a sweatshirt for $17.70. How much money did you have left?

When the teacher reconvenes the class, the first student the teacher calls on, Leandra, writes a correct number sentence and gives a coherent explanation.

In this situation, what should the teacher do next?

3. Second grade students are working on a subtraction problem. The tacher calls on Ben to present his computation to the class.

Below Ben shows his work.

12
- 8
16

What should the teacher SAY and DO to guide students toward an understanding of subtraction?

4. A third grade teacher asks students to work individually on the problem 7 x 15. The teacher reconvenes the class and calls on three students to present their computations on the board.

Niral	Carlton	Jasmine
15	15	15
X 7	X 7	X 7
35	35	105
+ 7	+70	
42	105	

How should the teacher guide a discussion about the three computations?

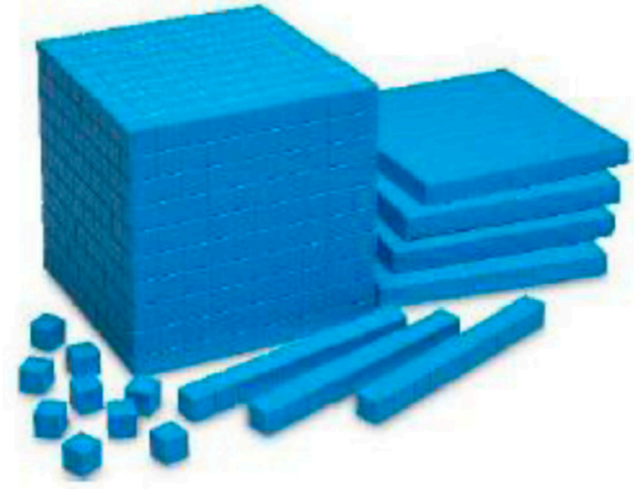
5. A second grade teacher gives each student pair 2 quarters, 5 dimes, 15 nickels and asks each pair to make one dollar using the least amount of coins.

Below students show their work.
Pair 1: 2 quarters, 5 dimes
Pair 2: 1 quarter, 15 nickels
Pair 3: 5 dimes, 10 nickels

When the teacher reconvenes the class, how should he/she lead a discussion of students' answers?

6. A class of second grade students have been introduced to base ten blocks for the past few weeks. A second grade teacher asks students to work in pairs to show the number 406 with base ten blocks.

Base ten blocks



When the teacher reconvenes the class, how should he/she lead a discussion of students' answers?

7. A fourth grade teacher calls on students to present how they set up the subtraction problem 25 – 17.7. Two students volunteer.

Juan	Sara
25	25
-17.7	- 17.7

When the teacher reconvenes the class, how should he/she lead a discussion of students' answers?

For questions 8-10, please choose ONE answer that best describes what you believe is MOST important. For each question, please explain why your choice is most important as clearly as possible. Remember there are no right or wrong answers.

8. The mathematical term "multiples" was introduced to students in third grade. This year, their fourth grade teacher asks students to work individually to find multiples among numbers on the board.

8, 12, 16, 20, 24, 28, 32, 36

When the teacher reconvenes the class, what is MOST important for the teacher to do with the class?

	ask for answers, and highlight correct answers while reviewing the definition of "multiple."
	ask students to agree or disagree with one another's ideas and explanations
	guide students to identify multiples of 4, and then ask whether all numbers are also multiples of 8.
	Elicit several students' ideas about multiples, asking questions to check for understanding
	Other

Other (please specify):

Explain why your choice is most important.

---

9. A third grade teacher launches a geometry unit with the following sorting task.

[Problem] Sort your polygons into two groups: triangles and other shapes.

Students work in pairs. Their polygons include triangles (scalene, equilateral, isosceles, and right triangles) and other polygons (squares, rectangles, hexagons, and parallelogams). The teacher notices that some students sort into three groups: equilateral triangles, all other triangles, and all other polygons.

In this situation, the MOST important thing the teacher should do with the class is…

- ☐ ask students to share their groupings and ask questions to check for understanding
- ☐ Explain the properties of triangles and have students fix their errors
- ☐ Ask students to explain their sorts, and invite other students to ask the presenters questions and comment
- ☐ Ask questions about the properties of triangles to check for student understanding
- ☐ Other

Other (please specify)

Explain why your choice is most important.

---

10. A second grade teacher asks students to find solutions for a pair of equations. The teacher explains that the square and triangle in both equations stand for the same numbers.

$$\square - \triangle = 15$$
$$\square + \triangle + \triangle = 21$$

During partner work, the teacher notices that some students cannot find a solution that works for both equations.

In this situation, the MOST important thing the teacher should do with the class is…

- ☐ lead students to share their answers and make comments on one another's answers.
- ☐ re-explain the problem
- ☐ ask several students to share their answers (right or wrong) and explain their reasoning
- ☐ model another similar (easier) problem and then return to the problem
- ☐ Other

Other (please specify)

Explain why your choice is most important.

Thank you for completing the survey for my research study to explore teacher beliefs about mathematics discourse during whole class discussions. If you would like to entered in a drawing for one of five $5 gift cards, please provide your contact information.

☐	No, I would not like to be entered in the drawing for one of five $5 gift cards.
☐	Yes, please enter me in the drawing.

First name:
Last name:
Email address:

Would you be willing to complete a follow-up interview for my research study to further explore teacher beliefs about mathematics discourse during whole class discussions? The interview should take approximately 30-45 minutes to complete via Zoom, and participants may choose to be entered in a drawing for one of five $15 gift cards for participating.

If you would like to know more information about this interview, an email will be sent to you.

☐	No, I would not like to be entered in the drawing for one of five $15 gift cards.
☐	Yes, please enter me in the drawing.

Appendix E

Permission to use the BMD survey instrument from Jang (2010)

From: **Jang, Heeju** jangh@smccd.edu
Subject: Re: [EXTERNAL] Measuring Teacher Beliefs about Mathematics Discourse Instrument
Date: September 22, 2019 at 10:26 PM
To: Jana Walls jww0043@auburn.edu

HJ

Hi Jana,

Thank you for your email. Sorry for a late reply. I was out of country. I feel very happy to hear that my humble work could be helpful to you. Yes. Please do so.

Wish you all the best,
Heeju

From: Jana Walls <jww0043@auburn.edu>
Sent: Thursday, September 19, 2019 2:27:32 PM
To: Jang, Heeju <jangh@smccd.edu>
Subject: [EXTERNAL] Measuring Teacher Beliefs about Mathematics Discourse Instrument

Dr. Jang,

Good afternoon! I am a doctoral student at Auburn University writing my dissertation proposal about elementary teachers' perspectives on mathematics discourse. I taught fourth grade mathematics for six years, and I also served as the district mathematics instructional coach at Trussville City Schools in Trussville, Alabama. I believe facilitating student discourse was one of my strengths as a mathematics educator, and I am passionate about helping other educators believe in the benefits of discourse, with the hopes that it would also change their practice and improve student learning.

I would like your permission to use you instrument titled "BMD (Teacher Beliefs about Mathematics Discourse) Version 3" from your dissertation titled, "Measuring Teacher Beliefs about Mathematics Discourse: An Item Theory Approach." I will acknowledge your work in any publications or presentations that I may produce. I will also gladly send you a copy of my completely research study upon its completion and approval from the university.

I look forward to hearing from you and perhaps speaking with you regarding any changes that have been made to this instrument since it was published in May 2010.

Respectfully,

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Appendix F

Semi-structured Interview Protocol

Semi-structured Interview Protocol

1. How would you define productive mathematics discourse during whole-class discussions in the classroom?

2. What does a productive whole class discussion look like in the classroom?

3. In your opinion, who should be doing most of the talking during a whole class mathematics discussion and why?

4. What is the role of the teacher during a whole class discussion?

5. What is the role of the students during a whole class discussion?

6. Are there benefits from students explaining their thinking to each other and asking questions of one another to convince their classmates of a certain strategy or solution? If so, what are they?

7. Are there any drawbacks to students explaining their thinking to each other and asking questions of one another to convince their classmates of a certain strategy or solution? If so, what are they?

8. In ten years, how will whole class mathematics discussions in the classroom look?

9. In 2014, NCTM published *Principles to Actions* which included the eight Effective Mathematics Teaching Practices. The fourth practice standard is that students should "Facilitate meaningful mathematical discourse." Do you believe this will still be important in ten years? Why or why not?

10. Is there anything else you would like to add about your beliefs about mathematics discourse?

Appendix G

Beliefs in Mathematics Discourse Codebook

Beliefs in Mathematics Discourse Codebook

Theme	Secondary Theme	Definition	Example from Interview Transcripts
The Role of the Teacher	Teacher Questioning	Any evidence referring to the teacher asking students for information relevant to their learning of mathematics.	"… when a kid says something or asks a question, the teacher's job is to question back to get the kids thinking about it rather than just giving the answer."
	Teacher planning	Any evidence recognizing preparations made by the teacher that will promote discourse during whole class discussions.	"… first off, the teacher has to have a target in mind of what is the purpose. They can't just go into (the lesson) without knowing what is the purpose."
	Time	Any evidence referring to the temporal length of opportunities that teachers have to engage in discourse with students.	"It takes time. That's the deal. It's human, and that's why it takes time. That's why it's hard to (have discourse) sometimes in the classroom because we're pressured to get things done."
	Formative assessments	Any evidence referring to teachers' eliciting and using evidence of student learning to improve student understanding.	"… when the teacher is guiding it, she has to pull as much student input in as she can and also just doing some formative assessments through that so like the whole sides of the room idea … where the kids maybe have confusion or disagreement on (a) concept that the teacher use something such as, 'If you feel this way, stand over here. If you feel that way stand over here (motioning to a different side with her hands)… then give each side the chance to try to convince the kids to come over to their side…"
	Inclusivity	Any evidence recognizing the teacher's efforts to include all students, especially who may otherwise be excluded.	"I also think that the benefits (of discourse) would be they (students) realize that there's not just one way to tackle it (solve the problem). They all have an entry point to whatever mathematics situation is in front of them because there's not just a single correct approach to start into a problem and I feel it gives kids

			confidence in their thoughts because they start talking about it …”
The Role of the Student	Student questioning	Any evidence referring to a student inquiry.	“The student’s role is to be curious and ask questions to understand more deeply like if they’re confused about something to be willing to say that, to bring it up to the class or if they think differently then somebody…”
	Student engagement	Any evidence referring to the extent of student attention, interest, and curiosity during discourse.	“They have to be involved in their learning or they don’t learn… The more active they’re involved, the more they retain it.”
	Emotions	Any evidence referring to strong student reactions or feelings as a result of discourse.	“… when they hear this child had the same misconception as them it makes them realize, ok, well I’m not alone here… and they don’t feel so… they don’t have that hesitation with math I believe. I think it breaks that down.”

Appendix H

Beliefs in Mathematics Discourse (BMD) Survey Scoring Guide

Created August 2020

Score	Description
4	The teacher encourages students to agree or disagree with one another and explain why to convince each other with explanations. The teacher encourages students to make comments to each other, restate another student's answer, and ask each other questions.
3	The teacher asks students to explain their thinking. The teacher asks if anyone has a different way to solve the problem or answer and asks them to share their explanations with the class.
2	The teacher asks questions of the students to guide them to the correct answer or to help students catch their own mistake.
1	The teacher explains the correct definition of a mathematical concept, strategy for solving a problem, or solution for a problem to students including pointing out when students' answers are incorrect.
99	Irrelevant response. Nonsense.
0	Missing response