THE RELATIONSHIP OF TEACHERS' CONTENT KNOWLEDGE AND

PEDAGOGICAL CONTENT KNOWLEDGE IN ALGEBRA, AND

CHANGES IN BOTH TYPES OF KNOWLEDGE AS A

RESULT OF PROFESSIONAL DEVELOPMENT

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RESULT OF PROFESSIONAL DEVELOPMENT

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In seeking to improve the mathematics education of all students, it is important to understand the connection between the content knowledge and pedagogical content knowledge of mathematics teachers, and how professional can provide growth in both of these types of knowledge. We do not have an answer about the interplay of content knowledge and pedagogical content knowledge in successful instructional practices in the mathematics classroom. For a period of a year and a half surveys were administered to a number of secondary mathematics teachers and surveys, observations, and interviews focused on four secondary mathematics teachers. All of the teachers taught mathematics and particularly the four teachers all taught Algebra I at their respective schools.

Assessment of the teachers' content knowledge, pedagogical content knowledge, and changes of instructional practices also included interviews with professional development presenters.

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I. STATEMENT OF THE PROBLEM

Successful competition by the United States in the global economy depends on having adults who are well prepared in mathematics and science (National Center for Education Statistics, 2001). Adults of the twenty-first century need to be mathematically proficient in order to be productive members of our society (Ball, 2003a), and the need for mathematics in everyday life has never been greater and will indeed continue to increase (National Council of Teachers of Mathematics [NCTM], 2000). Mathematical competence can open doors to a productive future, while the doors to a productive future can remain closed for those students lacking in mathematical competence (NCTM, 2000).

As a nation, however, the U. S. is not providing its students with the mathematical preparation needed to be successful. According to the results of the Third International Mathematics and Science Study (TIMSS), our students in the United States only achieve at average levels when compared to students in other countries (NCES, 2003a). In addition, according the National Assessment of Educational Progress (NAEP) results, less than 20% of twelfth grade students and about one-third of eighth grade students had achieved mathematical proficiency (Pehle et al., 2004).

A critical juncture in the mathematical preparation of students is the high school algebra course. Algebra serves as a gatekeeper, offering differential opportunities for entry into advanced mathematics courses (Ball, 2003a), for preparation for college

(Pascopella, 2000, Lawton, 1997, Chevigny, 1996, Silver, 1997, Olson, 1994), and for preparation for the world of work (Silver, 1997). Olson (1994) pointed to a study by the College Board, which suggested that students who take Algebra in high school are two and one-half times more likely to attend college than those students who do not. When minority students complete Algebra in high school, the gap between the percentages of minority and non-minority students who attend college virtually disappears. Students who are not proficient in Algebra do not have access to a full range of educational and career opportunities (Ball, 2003a).

Algebra should be embedded throughout the K-12 curriculum in order to provide the opportunities for students to develop a solid foundation for the content (NCTM, 2000). Students need to develop a deep understanding of the algebra content, and mathematics in general. Students need to develop fluency with procedures as well as conceptual understanding for why those procedures work (RAND, 2003). In addition a students' content knowledge must include the processes of problem solving, reasoning and proof, communication, making connections, and using representations (NCTM, 2000). Content knowledge provides the ability to know, understand, and have the ability to use mathematics (NCTM, 2000).

Teachers play a key role in ensuring that all students have the experiences needed to learn the mathematics necessary for success in future educational opportunities and careers (Mewborn, 2003). In particular, we must consider the types of knowledge mathematics teachers need to provide all students with equitable opportunities to learn algebra. While all might agree teachers need content knowledge of the subject they will be teaching, there is not a common definition of content knowledge on which everyone agrees. Content knowledge has often been defined by the number of university level courses taken (Even, 1993, No Child Left Behind Act [NCLB], 2001), the grade point average at the collegiate level (Even, 1993), or scores on a state mandated test (NCLB, 2001). This, however, is limited.

Teachers need a deep understanding of the mathematics they will teach (CBMS, 2001). Like students, teacher's content knowledge should include both procedural knowledge and conceptual knowledge, an understanding how this knowledge is structured and generated throughout the domain of mathematics (Shulman, 1986). If students are expected to develop mathematical proficiency and to apply mathematics in real world situations, no less can be expected of their teachers (CBMS, 2001). However, studies have found teachers' content knowledge is often thin and inadequate to provide instruction for students in today's classrooms (Ball, 1988a, 2003b; Ball & Bass, 2000; Fuller, 1996; Ma, 1999; Mewborn, 2001; Stacey, et al, 2001).

However, good content knowledge alone is not sufficient to successfully teach students mathematics (Even, 1993). Teachers also need knowledge of mathematics used specifically to facilitate the learning of mathematics by students (Sherin, 2002). At the turn of the twentieth century Dewey (1974) proposed the importance of a proper relationship between knowledge of content and pedagogy, and this tension still remains at the turn of the twenty-first century. Shulman (1987) first referred to this mixture of content knowledge and knowledge of pedagogy that belongs exclusively to teachers as "pedagogical content knowledge." Pedagogical content knowledge, also referred to as the "mathematical knowledge for teaching" (Ball et al. 2003), bundles mathematics knowledge with the knowledge teachers have about learners, learning, and pedagogy

(Ball et al. 2001). Pedagogical content knowledge can help teachers anticipate where students will have difficulties and be ready with alternative methods, explanations and representations related to a mathematical topic (Ball et al. 2001). In addition, pedagogical content knowledge includes representations that are most useful for teaching mathematics content (Ball et al. 2003).

We need to understand both teacher's content knowledge as well as their pedagogical content knowledge. Some research has been conducted in the areas of teacher content knowledge and pedagogical content knowledge of elementary teachers. The Learning Mathematics for Teaching project found that elementary teachers who scored higher on instruments measuring content knowledge and pedagogical content knowledge produced larger gains in student achievement in mathematics (Ball et al. 2005). However, far fewer studies have focused on these same types of knowledge for secondary teachers including teachers of algebra. Stump (1997), Even (1993), and Llinares (2000) conducted studies related to measuring teachers' content knowledge of specific algebraic topics and its relationship to their pedagogical content knowledge. These studies suggest that teachers' content knowledge and pedagogical content knowledge do make a difference in students learning mathematics. However, much work remains to be done in understanding the content knowledge and pedagogical content knowledge of teachers and how they effect student learning.

The RAND report (Ball, 2003a) calls for additional research to further explore content-specific knowledge and pedagogical content knowledge for teaching mathematics, and how and where such knowledge makes a difference in successful teaching. Efforts to improve mathematics education for all students will be limited without a better understanding of both content knowledge and pedagogical content knowledge, as well as the connection between these types of knowledge.

We also need to have a better understanding of how content knowledge and pedagogical content knowledge are used during mathematics instruction. If content knowledge and pedagogical content knowledge do make a difference in student achievement, understanding how these knowledge types are used in instruction remains an important issue to be solved (Ball, 2003a). What ultimately matters is how teachers are able to use their knowledge in the course of teaching (Ball et al., 2001). More research is needed to understand the relationship of content knowledge and pedagogical content knowledge and how it is used in instructional contexts (Ball, 2003a).

An additional question lies in the extent to which professional development can change both content knowledge and pedagogical content knowledge of teachers. Professional development should provide opportunities that support learning both content knowledge and pedagogical content knowledge as well as how to effectively use them (Ball, 2003a), and efforts have been made over the past decade to organize professional development to develop better mathematical knowledge (Ball, 2003a). Methods for developing content knowledge and pedagogical content knowledge for teaching remains an unsolved problem for the improvement of mathematics teaching and learning (Ball et al., 2001).

Furthermore, if professional development can facilitate changes in both content knowledge and pedagogical, we do not fully understand how these types of knowledge might influence changes in teachers’ instructional practices. Certainly possessing content knowledge and pedagogical content knowledge will not automatically translate into successful mathematics instruction (Ball et al. 2001).

In order to better understand the impact of teacher knowledge on mathematics instruction, which in turn affects the future success of students, the following questions will be addressed in this study: What content knowledge and pedagogical content knowledge of algebra do high school algebra teachers possess? How are high school algebra teachers' content knowledge and pedagogical content knowledge reflected in their instructional practices? What growth in content knowledge and pedagogical content knowledge can be seen from participating in professional development that includes attention to increasing both content knowledge and pedagogical content knowledge? How are changes in content knowledge and pedagogical content knowledge reflected in instructional practice?

II. REVIEW OF RELATED LITERATURE

Mathematics is important in ensuring the future success of all students (RAND, 2003) and algebra serves as a gatekeeper course to differing opportunities for advanced mathematics courses (RAND, 2003), to college preparation (Chevigny, 1996; Lawton, 1997; Olson, 1994; Pacopella, 2000; Silver, 1997) and for the preparation for the world of work (Silver, 1997). Without algebra, students have limited chances of success (RAND, 2003). We will begin this chapter by examining student knowledge, first more broadly, then more specifically related to secondary students. Given that the success of students largely depends on the quality of the classroom teacher (Mewborn, 2003) we will next turn our attention to teacher knowledge necessary for teaching, focusing on content knowledge, including both procedural and conceptual knowledge and the processes , and the pedagogical content knowledge needed to teach algebra. Finally, we will consider professional development and how it can be used to promote growth in both of these types of knowledge.

Student Knowledge

This section will begin by addressing the importance of algebra for all students, followed by a broad overview of what is meant by algebra and the specific content that high school students need to learn. We next address the need for conceptual understanding, along with procedural understanding, mathematical processes, and how they should learn it. Finally, we focus on what is meant by the Algebra I course.

Principles and Standards for School Mathematics, released by the National Council of Teachers of Mathematics (NCTM) in 2000, declares, “Algebra competence is important in adult life, both on the job and as preparation for postsecondary education” (p. 37). Without algebraic proficiency, students do not have access to a full range of educational and career opportunities (RAND, 2003). Many schools in the United States require students to demonstrate proficiency in algebra (RAND, 2003), in part due to many states adopting higher standards resulting from public pressure for more accountability and higher standards (RAND, 2003). The No Child Left Behind Act has also reinforced these higher standards (RAND, 2003).

The National Assessment of Educational Progress (National Center for Educational Statistics [NCES], 2000) shows that students who have taken more advanced classes such as algebra and geometry scored higher than students who had not taken these courses. Data on the 1996 NAEP assessment also affirmed that achievement was strongly related to the coursetaking of students (Strutchens et al., 2000). Clearly algebra is an important part of students’ mathematical preparation, and it should be available to all students. Moses and Cobb (2001) argued that the access of algebra to all students should be “the new civil right.” They stated:

> …once solely in place as the gatekeeper for higher math and the priesthood who gained access to it, [algebra] now is the gatekeeper for citizenship, and people who don’t have it are like the people who couldn’t read and write in the industrial age….[Lack of access to algebra] has become not a barrier to college entrance, but a barrier to citizenship. That’s the importance of algebra that has emerged with the new higher technology. (Moses & Cobb in Ball et al., 2003, p. 47)

High school Algebra serves as a gatekeeper course, offering different opportunities for entry into advanced mathematics courses (RAND, 2003), for preparation for college (Pascopella, 2000, Lawton, 1997, Chevigny, 1996, Silver, 1997, Olson, 1994), and for preparation for the world of work (Silver, 1997). Students who take Algebra are more likely to attend college than those student who do not (Olson, 1994). When minority students complete Algebra in high school, the gap between the percentages of minority and non-minority students who attend college virtually disappears. Students who are not proficient in Algebra do not have access to a full range of educational and career opportunities (RAND, 2003).

The Algebra Strand

Principles and Standards for School Mathematics (PSSM), the national standards for school mathematics released by the National Council of Teachers of Mathematics (NCTM) in 2000 recommended that all students should learn algebra and it should be embedded in mathematical topics from kindergarten through twelfth grade. Through the algebra strand, "teachers can help students build a solid foundation of understanding and experience as a preparation for more sophisticated work in algebra in the middle grades and high school" (NCTM, 2000, p. 37).

Let us first look at the broad overview of the mathematics students should learn, contained within four major areas of its algebra strand (NCTM, 2000). These areas are:

> Understand patterns, relations, and functions; represent and analyze mathematical situations and structures using algebraic symbols; use mathematical models to represent and understand quantitative relationships, and analyze change in various contexts. (p. 296)

Experiencing algebra across the grade levels will not necessarily resemble a formal course in algebra. For example, consider the approach of analyzing changes in various contexts at different grade levels. "In prekindergarten through grades 2, students can, at first, describe qualitative change ('I grew taller over the summer') and then quantitative changes ('I grew two inches taller in the last year')" (NCTM, 2000, p. 40). Students in grades 3-5 should be able to "look at sequences and be able to distinguish between arithmetic growth (2, 5, 8, 11, 14, …) and geometric growth (2, 4, 8, 16, …)" (NCTM, 2000, p. 40). "With a strong middle-grades focus on linearity, students should learn about the idea that slope represents the constant rate of change in linear functions" (NCTM, 2000, p. 40) and be ready in high school to learn "about classes of functions with nonconstant rates of change" (NCTM, 2000, p. 40).

Algebra Content in High School

In addition to the general picture of algebra across grades K-12 presented in the previous section, specific algebra content has been specified for grades 9-12. I will draw on the recommendations from two sources: the RAND report (2003) and *Principles and Standards for School Mathematics* (NCTM, 2000).

The RAND report (2003) is a strategic plan for research and development of mathematics education in the United States for developing the mathematical proficiency of all students**.** Four areas are specified in this report (RAND, 2003): working flexibly and meaningfully with formulas or algebraic relations, operating with and representations of numbers, understanding of function, and identifying and using variables in quantitative contexts as well as patterns. *The Principles and Standards for School Mathematics*

(NCTM, 2000) also contains specific recommendations for grades 9-12 falling within the four major areas within its algebra standard.

First, students should understand patterns, relations, and functions (NCTM, 2000); the RAND report (2003) refers to this having a “robust understanding” of functions. In grades 9-12, students should be able to use various representations for functions and consider the advantages and disadvantages of each representation relative to the context of the problem (NCTM, 2000). Students should explore different classes of functions and how the parameters affect the graphs for each of these functions (NCTM, 2000).

Second, students at the 9-12 grade band should be able to represent and analyze mathematical situations and structures using algebraic symbols (NCTM, 2000). This includes having the ability to work flexibly and meaningfully with algebraic symbolism in representing and solving problems (NCTM, 2000, RAND, 2003). Students should be able to operate with algebraic symbols as well as be able to understand where to use them (NCTM, 2000). Students should be able to work flexibly and meaningfully with formulas or algebraic relations, representing situations, manipulating them, and solving equations (RAND, 2003). In addition, students should be able to use symbols, formulas, and functions in representing quantitative context (RAND, 2003).

Third, students at the 9-12 grade band should be able to use mathematical models to represent and understand quantitative relationships (NCTM, 2000). Students should be able to identify the class or classes of functions that might model a situation, use symbolic representations, and draw conclusions about a situation being modeled.

Last, students should be able to analyze change in various contexts (NCTM, 2000). Students should be able to approximate change and interpret rates of changes using graphs and numerical data (NCTM, 2000). Students should be able to use functions in analyzing the change of one quantity in relation to another (RAND, 2003).

Thus, the algebra that students in grades 9-12 should learn is much broader than the traditional emphasis on symbolic manipulation. Their knowledge should include: a robust understanding of functions, should develop fluency in using algebraic structures in relationship to expressions, equations, and inequalities, using algebraic reasoning in relationship to other mathematical fields, and analyze change in various contexts.

Conceptual and Procedural Knowledge

In order to achieve these content goals, students need to have a deeper understanding of mathematics. They must know more than procedural knowledge, which Hiebert and Lefevre (1986) defined as knowing the rules and algorithms of mathematics. Procedural knowledge includes a series of actions (Hiebert & Carpenter, 2002) and within the context of school mathematics often consists of a series of symbolic manipulations in performing step-by-step procedures. They must also have conceptual knowledge, which includes the understanding of how and why mathematical algorithms work (RAND, 2003). The RAND report (2003) explicitly includes attention to both procedural and conceptual fluency, and this can also be inferred from *Principles and Standards* (2000).

Skemp (1976) referred to procedural knowledge as instrumental understanding and to relational understanding as the union of both procedural and conceptual knowledge. Skemp (1976) pointed out the relative advantages of both relational and instrumental understanding. With instrumental understanding, he asserted that students find mathematics easier to understand, the rewards are more immediate and apparent, and correct answers are more quickly obtained. With relational mathematics, however, students can more easily adapt their skills to new tasks, their skills are easier to remember, relational mathematics can be a goal within itself, and growth and explorations into new areas are more likely to occur. Thus to be truly successful, students need both conceptual knowledge and procedural knowledge of mathematics. Learning mathematics with understanding enables students to actively build new knowledge from their experiences and prior knowledge (NCTM, 2000). Thus, students need to develop both procedural knowledge as well as the conceptual knowledge for why procedures work.

Mathematical Processes

In looking at the mathematical knowledge that students need, we must also consider the means through which students should acquire and be able to use mathematics. The RAND report (2003) says for students to be competent in mathematics they must have conceptual understanding and procedural fluency, which have already been addressed in the previous section. In addition, student competency should include strategic competence, adaptive reasoning, and productive disposition. Productive disposition is the inclination for students to see the usefulness of mathematics and to believe in one's own efficacy.

Principles and Standards (NCTM, 2000) explicates this kind knowledge in its "process standards." The document proposes five process standards: problem solving, reasoning and proof, communication, connections, and representations. These processes highlight the ways of acquiring and using mathematical content, including algebra (NCTM, 2000). I briefly describe each of this in the following.

First, problem solving should provide students with the means to apply and use a variety of methods for solving problems (NCTM, 2000). Problem solving refers to solving a task for which a solution method is not immediately known (NCTM, 2000). Problem solving is an integral part of mathematics learning (NCTM, 2000). Through strategic competence, students should have the ability to formulate and solve mathematical problems (RAND, 2003).

Second, students should be able to use the algebra content in reasoning and proof (NCTM, 2000). Students should be able to develop, investigate, and evaluate mathematical conjectures, which are fundamental aspects of mathematics (NCTM, 2000). Being able to reason is essential to mathematics (NCTM, 2000) and must be developed through consistent use (NCTM, 2000). Students should have the ability to use logical thought, be able to reflect, explain, and justify (RAND, 2003).

Third, students should be able to communicate mathematics clearly so that others can understand their thinking (NCTM, 2000). In addition, communication should also allow students to evaluate the mathematical thinking of others (NCTM, 2000). Communication helps build meaning and through communication students will build better mathematical understanding (NCTM, 2000).

Fourth, students should be able to recognize connections between mathematical topics (NCTM, 2000) and to other areas of mathematics. They should also be able to recognize and apply mathematics in contexts outside mathematics. By making connections between mathematical ideas, students will develop a deeper understanding of the mathematics they are studying (NCTM, 2000).

Last, students need to be about to create, understand, and use a variety of methods to represent mathematical situations (NCTM, 2000). Students should be able to use representations in modeling and interpreting phenomena (NCTM, 2000). How students represent mathematics is fundamental in how they understand and use these ideas (NCTM, 2000).

Process skills must be interwoven with the mathematics content. Students cannot solve problems without understanding and using mathematical content. Likewise, processes can be learned in the content. For example, "establishing geometric knowledge calls for reasoning. The concepts of algebra can be examined and communicated through representations" (NCTM, 2000, p. 31). Thus, students need to be proficient in algebraic concepts using all of the mathematical processes. These processes enable students to understand and use the mathematics content in solving problems in various contexts (NCTM, 2000).

Algebra I Course

In the previous sections a broad view of Algebra was provided, in which it was presented as a strand that students should learn in all K-12 grade levels, as well as specific expectations for the 9-12 grade band. At the high school level algebra topics are typically taught in a specific course, although some states teach high school-level algebra as part of an integrated curriculum (NCES, 2007). All students in Alabama, the state in which this study was conducted, must satisfy educational requirements set by the Alabama State Department of Education (ASDE), which include completing an algebra one course. Algebra I is required because of its importance in the development of mathematical empowerment (ASDE, 2003). The course is defined in the *Alabama Course of Study: Mathematics* (ASDE, 2003) as follows:

> Algebra I is a formal, in-depth study of algebraic concepts and the real number system. In this course students develop a greater understanding of and appreciation for algebraic properties and operations. Algebra I reinforces concepts presented in earlier courses and permits students to explore new, more challenging content which prepares them for further study in mathematics. The course focuses on the useful application of course content and on the development of student understanding of central concepts. Appropriate use of technology allows students opportunities to work to improve concept development. As a result, students are empowered to perform mathematically, both with and without the use of technological tools. (p. 49)

This document also outlines two alternatives for meeting this requirement: "To better meet the needs of students of varying abilities, school systems may offer Algebra I (140 hours/one credit) or Algebra IA and IB (280 hours/two credits)" (ASDE, 2003, p. 49). The two year sequence for Algebra I suggests that it was designed students who need more time in which to develop algebraic proficiency (Paul, 2005).

Conclusion

Students will be limited in their opportunities for future success both in the world of academics and workplace without algebraic proficiency (RAND, 2003, Chevigny, 1996, Lawton, 1997, Olson, 1994, Pacopella, 2000, Silver, 1997). Learning algebra is important to all students, and emphasis should be placed on the development of algebra skills throughout the K-12 curriculum (NCTM, 2000, RAND, 2003) Algebra includes not only the knowledge of the content, but also the processes by which students acquire and are able to do mathematics (NCTM, 2000). Finally, for students to be successful in learning algebra, the content should be addressed within contexts where students can develop procedural knowledge as well as the conceptual underpinnings for these procedures.

Teacher Knowledge

The classroom teacher is the key factor in influencing the mathematical success of students (Mewborn, 2003). Given the complexities of teaching, it is difficult to simply create a list of attributes and types of knowledge that an effective mathematics teacher should possess, at least one on which everyone would agree. For example, school age students were asked to provide their views of the qualities effective teachers should possess (Ball, 1988a). Students replied they felt teachers need to be enthusiastic, helpful, and strict, but they also need to be knowledgeable of their subject matter (Ball, 1988a). Teachers need to know the mathematics content they will be teaching; Lee (et al, 2003) noted that if a teacher does not know mathematics, he/she will not teach it. We will focus on the aspects of teacher knowledge in the following sections. We will first look at content knowledge, how it is defined, and algebra content knowledge in particular. Next we will focus on pedagogical content knowledge and its definition.

Content Knowledge

Teachers should be knowledgeable in the content areas for which they are responsible to teach. This must include a deep understanding of the mathematics they are teaching (NCTM, 2000), including both mathematical concepts and procedures (Conference Board of the Mathematical Sciences [CBMS], 2001). It will difficult for any teacher to teach others about a subject if the teacher does not know the content himself/herself. The same is true for mathematics teachers. Mathematics teachers should know the mathematics they are teaching. But what exactly is it that they should know and how should they know it? The following section will address the definition of content knowledge, algebra content knowledge and a synthesis of the two types of knowledge.

Definition of Content Knowledge

There is considerable variation in what people mean when they refer a teacher's subject matter knowledge, what some also refer to as content knowledge. Earlier definitions measured subject matter knowledge of pre-service and in-service teachers in terms of the number of mathematics courses taken at the university level and/or the teacher's grade point average (Even, 1993). Some may argue that the number of mathematics courses measures the content knowledge of teachers, and this mathematical knowledge can be increased by taking more college-level mathematics courses (Noddings, 1998). The belief that a teacher's content knowledge is determined by the number of college level mathematics courses taken may guide policy decisions (Mewborn, 2001). For example, with the passage of the No Child Left Behind legislation the number of college mathematics courses taken or score on a state mandated test, is used as a determining factor in defining what a "highly qualified" teacher is (NCLB, 2001). This, however, can be misleading. For example, Noddings (1998) suggested that having adequate subject matter knowledge is not always insured by the presence of a mathematics major.

The CBMS report (2001) contains specific recommendations for the preparation of mathematics teachers at each grade level. Emphasis from these recommendations recognized the need for teachers to develop a deep understanding of mathematics, develop a connection of mathematical topics, and develop basic mathematical ideas (CBMS, 2001).

Teachers should possess at least the depth of mathematical knowledge that students need to learn (RAND, 2003). Teachers must have a deep understanding of the mathematical ideas in the 9-12, including both procedural knowledge as well as the conceptual knowledge underlying why the procedures work (CBMS, 2001).

Teachers' content knowledge should also contain the mathematical proficiency in using mathematical processes and applying mathematics in real world situations (NCTM, 2000, RAND, 2003). Teachers should be able to recognize and apply mathematics in contexts outside of mathematics (NCTM, 2000).

Algebra Content Knowledge

The CBMS report (2001) specifically addresses the algebra content knowledge teachers need to teach mathematics to all students. The focus of the five areas of algebra and number theory included in the section for recommendations for high school teacher preparation follow:

> To be well-prepared to teach such high school curricula, mathematics teachers need: 1) Understanding of the properties of the natural, integer, rational, real, and complex number systems. 2) Understanding of the ways that basic ideas of number theory and algebraic structures underlie rules for operations on expressions, equations, and inequalities. 3) Understanding and skill in using

> algebra to model and reason about real-world situations. 4) Ability to use algebraic reasoning effectively for problem solving and proof in number theory, geometry, discrete mathematics, and statistics. 5) Understanding of ways to use graphing calculators, computer algebra-systems, and spreadsheets to explore algebraic ideas and algebraic representations of information, and in solving problems. (p. 40)

Synthesis

For the purpose of this study, content knowledge will be defined as both the procedural and conceptual knowledge as well as the mathematical processes for using mathematics. This includes the teachers' ability to solve problems using a variety of methods, adapting to different contexts (NCTM, 2000). In addition, content knowledge includes the ability to use reasoning and proof to make and investigate conjectures and evaluate mathematical arguments (NCTM, 2000) and be able to use algebraic reasoning in relationship to other mathematical topics (CBMS, 2001). Teachers should have the ability to communicate mathematics so that others can learn and be able to listen to how others think about mathematics (NCTM, 2000). Teachers should have the ability to make connections between mathematical topics, between the areas of mathematics, and to real-world problems (NCTM, 2000). Teachers should be able to access different representations in organizing mathematics problems and should be able to translate between the mathematical representations (NCTM, 2000) and be able to model algebra in real world context (CBMS, 2001). In addition, algebra teachers need an understanding of the use of technology in solving problems, and technology for exploring algebraic ideas and representations (NCTM, 2000).

Pedagogical Content Knowledge

Content knowledge, however, is not sufficient and there is a difference between knowing mathematics and being able to teach it (Mewborn, 2001). Teachers also need knowledge of mathematics specifically used to facilitate the learning of mathematics by students (Sherin, 2002). Shulman (1987) first referred to this mixture of content knowledge and knowledge of pedagogy that belongs exclusively to teachers as "pedagogical content knowledge." Pedagogical content knowledge (Ball et al. 2003), bundles mathematics knowledge with the knowledge teachers have about learners, learning, and pedagogy (Ball et al. 2001). Pedagogical content knowledge can help teachers anticipate where students will have difficulties and be ready with alternative methods, explanations and representations related to a mathematical topic (Ball et al. 2001). In addition, pedagogical content knowledge includes representations that are most useful to teaching mathematics content (Ball et al. 2003).

Definitions of pedagogical content knowledge

The term "pedagogical content knowledge" was introduced by Shulman (1987) as the mixture of content knowledge and pedagogy that belongs exclusively to teachers and has since been used by many authors (Ball, 1988a; Ball, 2000; Borko, et al., 2000; Even, 1993; Fuller, 1996; McGowen, et al., 2002; McNamara, 1991; Sherin, 2002). Shulman (1987) referred to pedagogical content knowledge as the professional understanding of how content and pedagogy fit together. Sherin (2002) described pedagogical content knowledge as the knowledge used specifically for teaching the content, including not only knowing how to present the domain to facilitate the learning by students, but also knowing the typical understandings and misunderstandings of the students being taught.

Ball (1988a) defined pedagogical content knowledge as the intertwining of content knowledge and pedagogy to form a subject-specific body of knowledge. Pedagogical content knowledge also includes the mathematical knowledge needed to manage both routine and non-routine problems (Ball, 2000). Llinares (2000) felt the results of his study on functions reaffirmed earlier definitions of pedagogical content knowledge which included the integration of content knowledge and the knowledge the teacher had of the students as learners in the classroom environment. Pedagogical content knowledge bundles the knowledge of learners, learning, and pedagogy with mathematical content knowledge.

Ma (1999) termed pedagogical content knowledge as the knowledge teachers use to "unwrap" mathematical topics in order to present the content in ways to enable students to be successful in learning mathematics. Ma (1999) studied the difference in Chinese and U. S. teachers' knowledge of mathematics for teaching and how the Chinese Based on her conversations with the Chinese teachers, Ma (1999) referred to "knowledge packages" as the organization and development of mathematical ideas within a mathematics domain and involved "the longitudinal process of opening up and cultivating such a field in students' minds" (Ma, 1999, p. 114). For example, a knowledge package for subtraction with regrouping may contain other links such as "the composition of numbers within 100, subtraction without regrouping, composing and decomposing a higher value unit" (Ma, 1999, p. 19).

In addition, teachers have to be aware of the sequence in which mathematical ideas are developed. For example, in developing the meaning of division of fractions, an understanding should first be developed from a "meaning of addition, to the meaning of multiplication of whole numbers, to the meaning of multiplication with fractions, to a meaning of division with fractions" (Ma, 1999, p. 114). Finally, the Chinese teachers within the study referred to key elements within a knowledge package as "concept knots" that link related mathematical concepts. For example, within the knowledge package of division of fractions, Chinese teachers referred to the meaning of multiplication, modeling division of whole numbers, the concepts of a fraction and a whole, and the meaning of multiplication of fractions as concept knots (Ma, 1999). Teachers must be able to understand the "packing" of specifics within a knowledge package as well as the "unpacking" of these same specifics and how they relate to each other (Ma, 1999). Procedural knowledge as well as conceptual knowledge are interwoven within the knowledge packages (Ma, 1999).

In later writings, Ball (CBMS, 2001) referred to pedagogical content knowledge as the "mathematical knowledge for teaching." Ball and Bass (2003) addressed the major areas that need to be considered in the mathematical knowledge of teaching. First, the teaching of mathematics involves substantial mathematical work (Ball & Bass, 2003) such as the following:

- Design mathematically accurate explanations that are comprehensible and useful for students,
- Use mathematically appropriate and comprehensible definitions;
- Represent ideas carefully mapping between a physical or graphical model, the symbolic notation, and the operation or process;
- Interpret and make mathematical and pedagogical judgments about students' questions, solutions, problems, and insights (both predictable and unusual);

- Be able to respond productively to students' mathematical questions and curiosities;
- Make judgments about the mathematical quality of instructional materials and modify as necessary;
- Be able to pose good mathematical questions and problems that are productive for students' learning;
- Assess students' mathematics learning and take next steps. (Ball & Bass, 2003, p. 11)

Next, teachers need to be able to unpack mathematical ideas and understand how mathematical ideas connect across a mathematical domain. For example, when teaching the multiplication of 35 and 25, this problem can be represented as 35 groups of 25 objects or as a rectangular array with dimensions of 35 and 25 and an area of 875 square units (Ball & Bass, 2003). Teachers need to be able to access different representations for mathematical problems that may arise within the instruction. Teachers have to understand the different representations of and the meaning behind the algorithm for multiplying whole numbers (Ball et al. 2005), as seen in the previous example.

Teachers need to understand how mathematical ideas change and grow. For example, teaching elementary students who are subtracting whole numbers that you cannot subtract a larger number from a smaller number soon becomes false when students began to consider the set of integers (Ball & Bass, 2003). Teachers must be able to evaluate different methods for representing and solving mathematical problems whose generalizability and validity may not be immediately clear (Ball et al. 2005). See figure 1 for an example of the type of item developed for the Learning Mathematics for Teaching project [LMT] to evaluate the ability of teachers to appraise and validate unconventional methods used by students (Ball et al., 2005).

> Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
35	35	35
x 25	x 25	x 25
125	175	25
+ 75	+ 700	150
875	875	100
		+ 600
		875

> Which of these students is using a method that could be used to multiply any two whole numbers?

	Method would work for all whole numbers	Method would NOT work for all whole numbers	I'm not sure
a) Method A	1	2	3
b) Method B	1	2	3
c) Method C	1	2	3

Figure 1. Item for measuring specialized content knowledge (Ball et al. 2005, p. 43)

In addition, teachers need to be able to size up a typical wrong answer and be able to analyze the source of the student's error. An example of such an error follows in figure 2.

```
  35
x25
175
 70
245
```

Figure 2: Incorrect mathematical procedure for multiplying two whole numbers. (Ball et al. 2005, p. 17)

All of these considerations fall within the domain of the mathematical knowledge of teaching (Ball & Bass, 2003). The CBMS report (2001) posits that mathematical knowledge for teaching will allow teachers to better assess student's work while being able to understand where and why students make errors, and also to further the students' creative thinking. With pedagogical content knowledge, teachers are better able to anticipate where students will have difficulties and be prepared with alternate explanations or models. Teachers also need to be able to consider the mathematical ideas of their students and consider the value of different representations.

Synthesis

For the purpose of this study the following definition of pedagogical content knowledge will be used. Pedagogical content knowledge is viewed as the knowledge of a teacher to use his/her knowledge of mathematics to "unwrap" the mathematical topics and present the content in ways for students to successfully learn the mathematics (Ma, 1999). This knowledge includes the teacher's ability to use content knowledge to access different representations, as well as different methods for solving mathematics problems that may arise within the mathematics instruction. Pedagogical content knowledge also includes the ability of teachers to direct students to make connections between mathematical topics as well as helping them to see the connectedness of different representations for those same topics. Pedagogical content knowledge includes the ability of teachers to understand where and why students make errors and be prepared with alternative explanations and models. Pedagogical content knowledge also includes the ability of teachers to respond productively to students' questions and pose problems and questions that are productive to student learning.

Instruction to Promote Student Success in Algebra

Teaching mathematics to all students needs to address not only "what you teach but how you teach it" (Silva et al, 1990, p. 379). Teachers will need to approach teaching in different ways. In the following sections we will focus on instrumental understanding and relational understanding and how classroom instruction should promote them. In addition, we will look at two projects that were successful at teaching algebra to all students.

Instrumental Understanding and Relational Understanding

Skemp (1976) discussed how the balance of attention to instrumental understanding and relational understanding can effect classroom instruction. He describes instrumental understanding as being synonymous with procedural knowledge, while relational understanding captures both procedural and conceptual understanding and their interrelationship. Skemp (1976) provided the following analogy. Suppose students were taught music in two different situations. The first group used pencil and paper with no musical instruments involved. The students studied the notes, how to write them and what combinations make up a melody. At the end of the study period the students were asked to write a short melody. The second group began with instruments and related sound to the notes on paper. This group was also asked to write a short melody. Skemp (1976) suggested that very few of the first group would be able to successfully write a melody and most would hate the music class, whereas the majority of the second group would be successful in the melody writing and enjoy it at the same time. In the same sense, students who are passive learners of mathematics instruction will not be as successful as students who are actively involved in their classroom instruction.

Skemp (1976) points out when where relational and instrumental understanding can cause problems in the classroom. In the first case, teachers want to develop relational understanding and the students want only instrumental understanding. For example, a student may say that he/she does not wish to understand how a particular rule works, just tell me the rule for how to solve a problem. Even though this may be frustrating for the teacher, Skemp (1976) felt that less damage is done in this situation than when the order of learning is reversed. If a student wishes to learn relationally and the teacher is teaching instrumentally, the student may have difficulties in applying rules and formulas simply because he/she does not understand why he/she is using the particular rules or formulas.

In contrast, Skemp (1976) also points out reasons why teachers may prefer to teach mathematics instrumentally. They may feel relational understanding takes too long and is difficult to achieve, that skills must first be developed in order to later develop relational understanding can later be developed, and that there is little encouragement for newer teachers to teach relationally when everyone else is teaching instrumental mathematics.

Pesek and Kirshner (2002) designed a study to address the issue that teaching to develop relational understanding will take too much time (Skemp, 1976). Pesek and Kirshner (2002) noted that teachers sometimes adopt a two-track strategy for teaching mathematics. Part of instructional time is devoted to teaching students mathematics for meaning (relational) while the remainder of the time is spent in teaching mathematics for recall and procedures (instrumental). This two-track strategy might be the result of teachers feeling pressured into covering larger amounts of material, along with having an attitude that teaching students relationally takes more time than simply presenting material instrumentally (Pesek & Kirshner, 2002).

Pesek and Kirshner (2002) devised a study to investigate whether students who only learned mathematics relationally could perform as well as a group of students who were taught instrumentally first and then relationally. The instrumental instruction consisted of five days of instruction on finding formulas for perimeters and areas of various geometric shapes and how to use them to solve for area and perimeter. During this time frame the second group of students reviewed material that was not related to the unit on perimeter and area. After the first five days of instrumental instruction only, all students in both groups received three days of relational instruction on the same topics, in which "connections were developed through concrete materials, questioning, student communication, and problem solving" (Pesek & Kirshner, 2002, p. 103). Teachers providing the relational instruction never used formulas or gave specific strategies for students to find area and perimeter. Student strategies evolved from their own understanding of area and perimeter (Pesek & Kirshner, 2002).

Following the instruction, students who received only three days of relational instruction only and were not exposed to the area and perimeter formulas were more able to explain why the formulas worked than the group that had direct instruction with the formulas. These students also scored higher on the posttest and retention test than those students receiving both types of instruction (Pesek & Kirshner, 2002).

Conclusion

It is important for students to develop both the procedures for doing mathematics as well as the conceptual understanding for why the algorithms work. Developing relational understanding, which includes both procedural as well as conceptual understanding, takes less time than instrumental and students who have only relational understanding do better in mathematics that those students who had received both relational understanding as well as instrumental.

Projects to Improve Algebra Instruction

The Algebra Project and the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) Project both promoted teaching mathematics in ways that aligned with the ideas of NCTM (2000). The mathematics achievement of the students in both projects were higher than the mathematics achievement of students in traditional mathematics classrooms.

The Algebra Project

Moses (2000) asserted that algebraic thinking skills should be encouraged at every grade level. Furthermore, by teaching algebraic skills to all students, he felt students will be provided a way of understanding and interpreting occasions that may arise in daily living. The Algebra Project was begun out of a desire by Robert Moses to ensure his children would achieve mathematical literacy (Moses et al., 1989). Components of the Algebra Project included a curriculum that would address the conceptual leap from arithmetic to algebra, a curricular process which was experimental and linked familiar physical experiences to abstract mathematics, the expectation for success which was reinforced by students, parents, teachers, and administrators (Silva et al., 1990).

There were three broad goals of the Algebra Project (Silva et al. 1990). First, the Algebra Project had the goal of developing the mathematical ability of middle school students so that they would be prepared for the study of algebra at the high school level and could master the skills of college preparation mathematics courses (Silva et al. 1990). A second goal of the Algebra Project was to change the way mathematics teachers constructed their learning environments by producing teachers able to facilitate the learning of algebra related to real life experiences of the students (Silva et al. 1990). A third goal of the Algebra Project was to build a larger community of individuals including parents, community volunteers, and administrators and involve them in enhancing mathematics instructional programs (Silva et al. 1990). Achieving the goals of the Algebra Project involved a four-fold approach: development of a curriculum at the middle school level, development of a method of training teachers in both “what to teach and how to teach it” (Silva et al, 1990, p. 379), sharing the project with other communities, and development of a strategy of nurturing support among parents, organizations within the community and the broader community.

As a result of the Algebra Project, the teacher's role was now that of a facilitator and the sixth grade curriculum was developed to teach algebraic skills in real world contexts. Students who were involved in the first year of the project were all placed in a higher level mathematics course above algebra one when entering high school. About thirty-nine percent were placed in Honors Geometry or Honors Algebra (Moses et al., 1989).

The QUASAR Project

A second project has had success in teaching algebra to all students. The QUASAR project asserted that an emphasis should be placed on students learning mathematics by engaging in activities that are embedded in the student's social and cultural context (Silver et al. 1993). Mathematical instruction must address the connection between what is taught in school with the social lives of the children asked to learn it (Silver et al. 1993). The goals of the QUASAR project aimed at producing students who:

> not only can accurately execute algorithms and recall factual knowledge but also have the capacity to impose meaning and structure on new situations, to generate hypotheses and critically examine evidence, and to select the most appropriate from among a repertoire of strategic alternatives. (Silver et al. 1993, p. 14)

The vision of the QUASAR project was for the mathematics classroom to become a community of collaborative, reflective practice where students are actively engaged in the mathematics they are learning, as well as challenged to think deeply about mathematical ideas (Silver et al. 1993). At one of the middle schools involved in the QUASAR project, which had a reputation for low-performance in mathematics, change was evident in the first grading period when compared to the other twenty middle schools within the district (Silver, 1995). The QUASAR students in this comparison had the second highest scores and at the end of the course passed at much higher rates than students who were at district schools demographically similar to their own (Silver, 1995). Credit for the exceptional performance was attributed to mathematical instruction that emphasized thinking, reasoning, communication, and problem solving (Silver, 1995), the same processes deemed important by NCTM (2000).

Conclusion. All students can learn algebra when the content is developed across the K-12 curriculum and learned in real world context. Students should be allowed to develop procedural knowledge as well as conceptual fluency using a variety of methods, representations, and processes within the learning process. Furthermore, students can be successful when they are actively engaged in the learning process and challenged to think deeply about mathematical ideas (Silver et al. 1993).

Research Related to Content Knowledge and Pedagogical Content Knowledge

A better understanding of teachers' content knowledge and pedagogical content knowledge is necessary to further our ability to provide all students with experiences that promote success in algebra. In this section, we will look at some research studies addressing content knowledge and pedagogical content knowledge of teachers, as well as studies that have focused on changing content knowledge and pedagogical content knowledge. The first part of the section focuses on elementary teachers, followed by studies on secondary teachers. Our attention will then turn attempts at measuring teacher knowledge, and this section will end with studies addressing changes in teacher knowledge.

Content Knowledge Studies with Elementary Teachers

Several studies have focused on the knowledge of elementary pre-service and in-service teachers in the content areas of multiplication of multi-digit numbers and place value (Ball, 1988a, Ma, 1999, Ball & Bass, 2000), subtraction and division of fractions (Ball, 1988a, Ma, 1999, Ball & Bass, 2000), decimal numeration (Stacy et al., 2001), and geometry (Ma, 1999, Swafford, Jones, & Thornton, 1997, Borko et al., 2000). The general consensus of these studies is that while teachers could use algorithms to teach specific topics, they were unable to provide adequate explanations for why those algorithms work. They were unable to provide concrete examples or other conceptual approaches that might promote student learning. In addition, teachers had difficulty recognizing and understanding errors made by students.

Several studies explored teachers' knowledge of multi-digit multiplication, and how place value related to the multiplication process (Ball, 1988a, Ma, 1999, and Ball & Bass 2000). Teachers in each of the studies had procedural knowledge of the area, but the majority of both pre-service and in-service teachers were unable to provide an adequate explanation for why the multi-digit multiplication algorithm worked. Most explanations revolved around the placement of zero place holders. Some teachers actually suggested using pictures of apples or oranges as place holders so the students would not get confused and think the zero place holder was actually a part of the answer (Ball & Bass, 2000). The majority of the teachers did not provide explanations that included knowledge about the numbers as multiples of ten and one hundred (Ball & Bass, 2000; Ma, 1999).

Ball (1988a) further pointed out when teachers strictly adhere to teaching the shortcut algorithm, the conceptual understanding of multi-digit multiplication is hidden.

These studies also probed the teachers' knowledge of subtraction and division of fractions (Ball, 1988a; Ma, 1999; Ball & Bass, 2000). The results were typically the same as the studies conducted with multi-digit multiplication. Teachers had procedural knowledge but were generally unable to provide conceptual examples. Furthermore, teachers were unable to provide an adequate explanation for why the algorithms for subtraction and division of fractions work. In every area, elementary teachers from the United States were less likely to give conceptual explanations when explaining the mathematics in question and were more likely to give further clarification on the rules and procedures. On the other hand, a majority of the Chinese teachers provided conceptual explanations of the mathematical topics covered. When interviewed, the Chinese teachers stressed the importance of knowing not only how to work problems but also why the procedures that were used worked (Ma, 1999).

Stacy et al. (2001) studied preservice teachers' knowledge of student difficulties with decimal numeration. Participants were asked to compare decimals and determine which items would be difficult for students (Stacy et al., 2001). A significant number of these preservice teachers had an inadequate knowledge of decimals, particularly in comparing decimal numbers with different numbers of digits and their relationship to zero. For example, 0.123 is greater than 0.6 because 0.123 has more digits than 0.6. In addition, children may consider the number with the most digits to be larger, so 0.116 would be larger than 0.2. Stacy et al. (2001) found the explanations of the participants fell into three distinct categories. First, the participants thought students would have problems with particular problems but gave no explanations for why they felt this way (Stacy et al., 2001). Second, participants identified a surface reason for a difficulty, such as one number has more decimal places but gave no further explanations. Third, participants explained why the features of the problem would cause difficulty (Stacy et al., 2001). Thus, many participants could identify problems that would cause students difficulties but could not adequately explain why.

Ma (1999) and Borko et al. (2000) conducted studies involving teacher knowledge of geometry. Ma (1999) presented the teachers in her study with the scenario of a student claiming that as the perimeter of a figure got larger so did the area of the figure. Only one of the 23 U. S. teachers investigated the claim and provided a counterexample for the student's claim. On the other hand, the majority of the Chinese teachers investigated the student's claim.

Borko et al. (2000) conducted a case study of a preservice teacher in order to see how she used her mathematical content knowledge in selecting worthwhile tasks for her students and in conducting student discourse. The tasks she selected involved circles, their areas, circumference, and π and involved both hands-on investigations by the students as well as investigations using Geometer Sketchpad. The teacher in this situation was confident of her mathematical abilities. Borko et al. (2000) found that teachers who possess greater content knowledge are more likely to emphasize problem solving, student inquiry, and conceptual knowledge. Teachers with greater content knowledge promoted student discourse and asked questions that required higher order thinking skills of their students (Borko et al., 2000).

These studies found that elementary teachers had a procedural knowledge of the mathematics topics but lacked conceptual understanding. Teachers with greater content knowledge are more likely to emphasize problem solving, student inquiry, conceptual knowledge, and higher order thinking skills.

Content Knowledge Studies with Secondary Teachers

Relatively few studies have focused on the content knowledge of secondary mathematics teachers, perhaps because of the belief that content knowledge may not be a problem at the secondary level because of secondary teacher's specialized knowledge of mathematics (Ball et al., 2001). However, the available research, while limited, has served to reveal the fallacy of this assumption (Ball et al., 2001). At the secondary level, studies have considered teachers' content knowledge and pedagogical content knowledge in the areas of slope (Stump, 1997; Sherin, 2002) and functions (Even, 1993; Llinares, 2000; Sherin, 2002). Nathan & Koedinger (2000) studied teachers' perceptions about algebraic reasoning. These studies included both pre-service and in-service teachers.

Stump (1997) used slope as the basis for studying the teacher's knowledge of the subject since slope is a critical topic in algebra. It appears throughout the secondary mathematics curriculum, and mathematics teachers need to have a deep understanding of the topic and be able to communicate about slope correctly to the students. The study explored whether teachers understood the different representations of slope and the different approaches taken in teaching this topic. Stump (1997) also asked teachers to include any illustrations, examples, analogies and explanations they would employ while teaching this concept. The study focused on how the teachers were able to relate different types of representations of slope, such as algebraic and geometric representations or functional and trigonometric representations. While in-service teachers could reflect on the difficulties they had observed across the years they had actually taught slope in the algebra classroom, pre-service teachers could only speculate on what difficulties students might have with conceptually understanding slope.

Stump (1997) found the majority of teachers experienced difficulties when slope was related to problems involving functional and trigonometric representations. She found no difference in the mathematical understanding of slope in pre-service and in-service teachers. Physically representing slope held more importance than a geometric representation for most teachers, but none of the teachers mentioned that a physical representation was a prerequisite of slope. While teaching experience may provide teachers with some growth in content knowledge, this growth is not insured by experience alone (Stump, 1997). Stump (1997) found teachers to have a range of levels of content knowledge, with most weaknesses occurring in the connection between the different types of representations of slope, such as algebraic and geometric representations or geometric and trigonometric representations.

Even (1993) studied pre-service secondary teachers' content knowledge and pedagogical content knowledge of functions. Questionnaires were collected from 152 participants. These questionnaires contained nine nonstandard problems addressing different aspects of content knowledge about functions, as well as six items related to students' misunderstandings or misconceptions about functions (Even, 1993). For example, participants were asked to provide a definition of function and to supply an alternative definition if students did not understand the first one (Even, 1993). Follow-up interviews were designed to develop a more accurate and detailed picture of the participants' content knowledge and pedagogical content knowledge (Even, 1993). Reactions to students' misconceptions were also addressed. For example, "Is it important to teach the vertical line test for graphs of functions to students? Why? (What is the (vertical) line test? What would you teach to your students? How would you teach it? Can you give me an example?)" (Even, 1993, p. 100). Even (1993) found that most participants viewed functions as having a "nice" graph or one that can be described by an expression. In addition, participants could select between functions and non-functions, but had difficulty in explaining why it was important to know the difference between functions and non-functions (Even, 1993). This limited understanding of functions can put teachers in unfamiliar situations and affect questions they ask, mathematical tasks they select, and the direction of a student's mathematical thinking (Even, 1993).

Llinares (2000) explored teachers' knowledge of functions video recordings of classes, and interviews that called for the analysis of hypothetical situations and analysis of textbook problems with his case study subjects. The findings of the study were summed up in three themes: teacher knowledge and the pupils' images of the concept of function, the flexibility of teacher knowledge, and conceptions about how students learn mathematics. The teacher expressed the desire for her students to learn conceptually about functions but felt that students had to learn through the memorization of certain procedures, practice on examples, and that the conceptual understanding would come later (Llinares, 2000).

Nathan and Koedinger (2000) found teacher's perceptions about the algebraic reasoning of their students could have an effect on the tasks teachers chose for students. Nathan and Koedinger (2000) asked teachers to make predictions of the level of difficulty that students would have when given different types of problems to solve. Participants in the study spanned the United States and taught in various school settings. Teachers ranked twelve problems, six of which were based on arithmetic and the other six were based on algebra. The six problems in each group were basically the same problem , but written in different contexts. For example, an arithmetic (result unknown) problem follows: “Starting with 81.9, if I subtract 66 and then divide by 6, I get a number. What is it?” (Nathan & Koedinger, 2000, p. 170). An example of an algebra (start-unknown) problem follows: “Solve for x: 6x + 66 = 81.90” (Nathan & Koedinger, 2000, p. 170). Ninth grade Algebra I students were asked to solve the problems and their difficulties were compared to the teacher predictions.

Surprisingly, teachers felt that students could more easily solve an algebraic equation than a word problem, but in reality students were more able to get a correct answer for a word problem than solve the algebraic equation. Nathan and Koedinger (2000) noted that for most teachers, textbooks were the primary resource and in many cases, they were the only resource used by the teachers. Evidence in the teacher knowledge about their students and their proposed difficulty level of problems may have stemmed from the sequence of how students were introduced to these types of problems in their own classrooms (Nathan & Koedinger, 2000). For example, if story problems were introduced first, students they would not be as difficult for the students. Furthermore, Nathan and Koedinger (2000) suggested the order that the problems are presented in the textbook might have been an influencing factor in the teachers’ perceptions about the difficulties that students would have, rather than the reality their study portrayed.

Thus, much like the studies conducted with elementary teachers, weaknesses in content knowledge were found in the studies with secondary teachers. Teachers experienced difficulty in understanding slope, function, and difficulty in considering where students might have difficulties in mathematical tasks.

Measuring Mathematical Knowledge for Teaching

To better understand the content knowledge and pedagogical content knowledge of teachers, we need to be able to measure these types of knowledge. Researchers involved in the Study of Instructional Improvement (Hill, Schilling, & Ball, 2004) began developing mathematics items related to the K-6 curriculum that would be used in surveying teachers to help in answering the question “What mathematical knowledge is needed to help students learn mathematics?” Three test forms were developed and piloted with participants from the California’s Mathematics Professional Development Institutes, a project which was aimed at boosting the teachers’ subject matter knowledge of mathematics.

The researchers set out to explore what subject matter knowledge is needed for teaching and how it is used. Mathematics items were formulated in two different contexts. Some of the questions required participants to rely on their content knowledge of mathematics, such as finding decimals that were halfway between two given decimal numbers. Other questions, referred to as the mathematics-as-used-in-teaching items, required participants to rely on specialized knowledge of mathematics. This specialized knowledge included evaluating unconventional methods used by students for solving mathematics problems, using or examining alternative representations, and providing explanations why the methods or representations work. One such question, related to this last category of questions, involved participants evaluating three different methods for multiplying two two-digit numbers. Teachers needed to inspect each step in each of the processes to understand what was done at each step and decide whether the method would be mathematically correct for each situation. This type of mathematical situation arises regularly during classroom instruction, although it may not necessarily occur for other adults who regularly use mathematics.

Initial findings of this study suggest "that teachers' knowledge of mathematics for teaching is at least partly domain specific rather than simply related to a general factor such as overall intelligence, mathematical ability, or teaching ability" (Hill, Schilling, & Ball, 2004, p. 26). Evidence from this study supports the conjecture that teachers need more mathematical content knowledge than the well-educated adult. Teachers need more depth in their mathematical knowledge beyond the knowledge related to the content in a textbook. Having strong content knowledge does matter, but teachers also need to know why mathematical statements are true, be able to represent mathematical ideas using multiple representations to model phenomena and select what method is best, what constitutes an appropriate definition for a concept or term, and ways for teachers to evaluate other mathematical procedures, representations, and solutions (Hill, Schilling, & Ball, 2004).

In a later study, Ball, Hill, and Bass (2005) relating student achievement to the Study of Instructional Improvement which was engaged in study school reform. Student scores were collected on the mathematics portion of the Terra Nova as well as teachers' performance scores on the mathematical knowledge for teaching instruments (Ball et al.

2005). Initial findings found the teachers' scores predicted the size of the gain in scores of students on their respective test (Ball et al. 2005).

Changes in Teacher Knowledge

Studies have also been conducted to explore changes in teacher knowledge. Sanders and Morris (2000) studied the mathematical skills of pre-service teachers whereas Sherin (2002) and Arbaugh and Brown (2002) used teacher study groups as a basis for improving teacher knowledge.

Even when pre-service teachers are confronted with results that indicate weak conceptual knowledge in their own mathematical abilities, they sometimes fail to do anything to correct the problem. Sanders and Morris (2000) studied how the exposure of the depth of content knowledge empowered or debilitated pre-service teachers. Students involved in the study were administered a scale to determine their confidence in tackling certain types of questions. Those students who had poor results were advised to attend class. Students were expected to take responsibility for making improvements in their own performances. The researchers included informal conversations with the students as well as more-formal interviews to clarify their observations. Some of the pre-service teachers in this study failed to admit that they had mathematics inadequacies, while others stayed away from tasks they knew would be problematic, and still others admitted to their inadequacies but did little to correct the problem. Just a few pre-service teachers admitted they were having mathematical difficulties and worked to make improvements in their mathematical understanding. Sanders and Morris (2000) felt a critical component of their study was the need to encourage pre-service teachers to admit that they had gaps in their mathematical skills and knowledge.

Sherin (2002) found teachers developed deeper content knowledge through reflection on the development of student understanding in a lesson, looking at student learning, and knowledge of mathematics. Along with videotaping the group reflection sessions, participants were interviewed to develop an understanding of their knowledge about linear functions. The content knowledge of the participants was examined within three classes of interaction involving the teachers' content knowledge and the implementation of the curriculum (Sherin, 2002). First, some teachers changed the design of the mathematics lesson but the content knowledge of the teacher did not change. Second, some teachers used the lesson as it was designed and in doing so they learned new content knowledge. Third, some teachers developed new content knowledge as well as made changes in the lesson as it developed during classroom instruction. This type of interaction involves instructional strategies that are new to both the teacher and lesson.

Arbaugh and Brown (2002) conducted a study in which the researcher was included in the study group and was considered the "expert" other. The responsibilities of the expert other included providing articles that were of interest to the study group, asking probing questions, challenging the teachers to reflect on their knowledge and teaching. Both pre- and post-interviews were conducted with the participants. Task sorting was included as part of the interview process with follow up questions to give the teachers an opportunity to explain their process in the activity. Task sorting was placed into four categories: lower-level demands (memorization), lower-level demands (procedures without connecting to meaning), higher-level demands (procedures with connections to meaning), and higher-level demands (doing mathematics) (Arbaugh & Brown, 2002). Through classroom observations, data was collected on the types of tasks teachers in the study were using in their classroom instruction. A portion of the study group time was devoted to a discussion of the tapes from classroom observations. Arbaugh and Brown (2002) found that working with the teachers helped them learn about the levels of cognitive demands. Progress was made in how the teachers were thinking about the tasks they were choosing for their students, giving consideration to the thinking and reasoning skills of their students. Through these tasks, teachers began to select and incorporate more high-level tasks into their teaching practices.

Thus, when pre-service teachers realize they lack content knowledge, they don't always do anything to correct the problem. Teachers who reflected on their own teaching tasks developed deeper content knowledge and changes were made in instructional practices.

However, much more research is needed. Very little of this research has linked teacher knowledge with actual classroom practice. Results from these studies seem to indicate the content knowledge and pedagogical content knowledge are limited and inadequate in providing the types of instruction necessary for teaching mathematics and particularly algebra to all students.

Professional Development

Results from the studies done of the content knowledge of teachers suggest a lack and depth in the understanding of the topics they are expected to teach, as well as weaknesses in their pedagogical content knowledge. Knowing what to teach and how to teach it are crucial for student learning. A need for change is evident from the success of projects such as the Algebra project and the QUASAR project. However, teachers cannot be expected to change their instructional practices just because they are told to do so (Mewborn, 2003).

Professional development must be provided that affords opportunities for teachers to experience growth in these types of knowledge that can be transferred to classroom practice. Guskey (2002) noted that "professional development programs are systemic efforts to bring about change in the classroom practices of teachers, in their attitudes, and beliefs, and in the learning outcomes of students." Teacher thinking needs to be at the center of professional development (Mewborn, 2003). Lee (2004/2005) proposes that requirements for successful professional development need to include participants as decision makers and consumers, should recruit teachers from the same context, and build a partnership between university, public, schools, and local education agencies.

Borasi and Fonzi noted these components necessary for successful professional development. First professional development should be sustained and intensive (Borasi & Fonzi). Professional development should be long-term and should provide a safe environment for teachers to come together and discuss their successes and frustrations (Mewborn, 2003)

Second, professional development should be informed by how people learn best (Borasi & Fonzi). Three cycles of learning should be considered (Borasi & Fonzi)

> 1) engaging actively in situations that provoke cognitive dissonance, thus initiating new constructions of meaning;
>
> 2) sharing and discussing these constructions with a group to arrive at consensus and generalizations;
>
> 3) applying these generalizations to new situations. (p. 30)

These learning cycles should focus on knowledge of and about mathematics, theories of mathematics learning, how students develop particular mathematical ideas, ability to plan instruction, and the ability to effectively interact with students (Borasi & Fonzi).

Third, professional development should focus on activities that are critical to learning and teaching and should focus on offering a rich set of diverse experiences (Borasi & Fonzi). In addition, teachers need to learn mathematics the way they are expected to teach it (Schifter, 1998). Teachers need help to develop their knowledge of mathematics beyond what nonprofessional functioning adults need (Hill, Schilling, & Ball, 2004). Teachers need to revisit mathematical concepts to help them gain conceptual understanding of the mathematical ideas and help them make connections to other mathematical topics (Mewborn, 2003). Teachers need to be involved in experiencing a broader version of mathematics, so they can break away from the way they have traditionally thought about mathematics (Acquarelli & Mumme, 1996). Teachers have to learn mathematics differently in order to teach it differently (Lee, 2004/2005). Professional development needs to help prepare teachers for the tasks that they will encounter on the job (Hill, Schilling, & Ball, 2004). Professional development should offer a rich set of diverse experiences (Borasi & Fonzi).

Fourth, professional development should focus on developing collaboration within a community of learners (Borasi & Fonzi). The professional environment needs to be designed to develop a supportive community among the teachers (Heaton, 2000; Romagnano, 1994). Professional development should involve multiple grade levels so that teachers can feel a part of the K-12 mathematics education system rather than feeling like they are at an isolated grade level (Ruopp, Cuoco, Rasala, & Kelemanik, 1999).

Teachers should attend professional development as a collective group from the same school, rather than as individual teachers, in order to build a stronger teacher community (Lee, 2004/2005). Furthermore, by working as a community it is easier for teachers to address issues that might promote or inhibit mathematics teacher reform (Acquarelli & Mumme, 1996). Teacher leaders need to grapple with the same types of issues that the teachers that they work with will experience (Acquarelli & Mumme, 1996).

Sowder, Philipp, Armstrong, and Schappelle (1998) found that teachers' instructional practice changes as their mathematical knowledge changes. Teachers became more independent and depended less on the prescribed curricula. These teachers saw their students as being capable mathematics students and expected more conceptual understanding of mathematics by their students. This resulted in students showing a deeper mathematical understanding of the mathematics they were studying.

The Renaissance program was a large scale professional development program to empower mathematics teachers to transform students into mathematical thinkers (Acquarelli & Mumme, 1996). After the third year of the Renaissance professional development program teachers were seen using cooperative learning, asking students to explain their mathematical thinking through writing, encouraging students to use concrete materials in mathematical tasks, and providing calculators for students to use (Acquarelli & Mumme, 1996). Since the majority of teachers has learned mathematics in traditional ways, they need ample time to make changes in their instructional practices. Mathematics teachers need opportunities to construct new understandings of mathematics and teaching so they can in turn provide similar opportunities for their students (Acquarelli & Mumme, 1996).

Conclusion

Professional development can be successful in increasing content knowledge and pedagogical content knowledge of teachers. Teachers need to be active participants in learning, both in the role of a student experiencing mathematics and in the role of a facilitator learning to understanding how to use this knowledge in the successful teaching practices. Teachers need to revisit the "big" ideas of the mathematics they are teaching and be able to develop both a depth in understanding as well as how topics relate to other areas of mathematics. Professional development needs to be long term and provide opportunities for teachers to build a rapport with other teachers in discussing their failures and successes. However, even if professional development does increase content knowledge and pedagogical content knowledge of teachers, there is no assurance that changes in this knowledge will be evident in their instructional practices.

Research Questions

In this section, a synthesis of the literature review is provided, from which the research questions for the study emerged. Students need to be successful in mathematics, particularly in algebra, which is the gatekeeper course to future opportunities for success (RAND, 2003). To increase student learning, teachers need to possess both a deep knowledge and understanding of the mathematics content as well as pedagogical content knowledge. However, numerous studies suggest that teachers lack both the content knowledge and pedagogical content knowledge to be effective in successfully providing the types of opportunities students need to learn mathematics. Most of this research has been done at the elementary level, but one might argue that these same conclusions are true about secondary teachers. Further study is needed to verify whether secondary teachers have the necessary content knowledge and pedagogical content knowledge to be effective.

Opportunities to develop greater content knowledge and pedagogical content knowledge and how to effectively use them should be provided through professional development (RAND, 2003). How to effectively develop content knowledge and pedagogical content knowledge for teaching remains an unsolved problem for the improvement of mathematics teaching and learning (Ball et al., 2001). While research suggests that professional development can increase content knowledge and pedagogical content knowledge of teachers, we do not know if and how these changes will be reflected in instructional practices within the mathematics classroom.

To address these issues, the following research questions were proposed to guide the study.

1. What content knowledge and pedagogical content knowledge of algebra do high school algebra teachers possess?
2. How are high school algebra teachers' content knowledge and pedagogical content knowledge reflected in their teaching practices?
3. What growth in content knowledge and pedagogical content knowledge can be seen from participating in professional development that includes attention to increasing both content knowledge and pedagogical content knowledge?
4. How are changes in content knowledge and pedagogical content knowledge reflected in teachers' instructional practices?

III. THEORETICAL UNDERPINNINGS AND METHODOLOGIES

The purpose of this study is to better understand the content knowledge and pedagogical content knowledge of mathematics teachers, how these types of knowledge are reflected in instructional practices, how intensive professional development promotes changes in these types of knowledge, and how changes in both types of knowledge are reflected in instructional practice. In order to answer these questions, four areas of research design have to be addressed. First, theoretical underpinnings for the study will be considered. Next, the general design of the study will be described. This study consisted of two parts, survey research and multi-individual case studies. Then the context of the study will be addressed. Finally, I will describe the methodologies and methods used in the two parts of the study.

Theoretical Underpinnings

This study is qualitative in nature, therefore the analysis is both the creation and interpretation of the researcher (Denzin & Lincoln, 2000). Interpretation of all research is guided by the set of beliefs and feelings the research holds about how the world should be studied and understood (Denzin & Lincoln, 1994). However, qualitative researchers attempt to study phenomenon in “natural settings while attempting to make sense of phenomena in terms of meanings people bring to them” (Denzin & Lincoln, 1994, p. 2). Intrepretations from qualitative research are constructed to provide a conduit through which the voices of those being researched can be heard (Denzin & Lincoln, 2000).

Crotty (1998) denotes four elements that drive qualitative research, as follows:

- Epistemology: the theory of knowledge embedded in the theoretical perspective and thereby in the methodology.
- Theoretical perspective: the philosophical stance informing the methodology and thus providing a context for the process and grounding its logic and criteria.
- Methodology: the strategy, plan of action, process or design lying behind the choice and use of particular methods and linking the choice and use of methods to the desired outcomes.
- Methods: the techniques or procedures used to gather and analyze data related to some research question or hypothesis. (p. 3)

We begin first with the epistemology underlying this research study, which gives us a way to understand what we gain from a research project and helps us to explain what we know (Crotty, 1998). Epistemology determines how we go about our research: “Epistemology is concerned with providing a philosophical grounding for deciding what kinds of knowledge are possible and how we can ensure that they are both adequate and legitimate” (Crotty, 1998, p. 8). The epistemological stance taken in this research study is constructivism. Crotty (1998) defined constructivism as “the meaning-making activity of the individual mind” or the individual engaging with research and making sense of it.

Constructivism takes into consideration our own individual experiences and suggests that how we make sense of the world is as worthy and valid as how any other person makes sense of it (Crotty, 1998). Meaning is constructed through our engagement with the realities of the world (Crotty, 1998), and the meaning of a phenomenon must be interpreted to be fully understood (Denzin & Lincoln, 1994).

Constructivism suggests that knowledge is not simply imprinted onto the mind, but rather concepts and abstractions are formed by the active mind working with the impressions it receives of phenomena (Denzin & Lincoln, 1994). Different people may construct different meaning from the same event (Crotty, 1998). Thus, the constructivist may tentatively come to interpret the language and actions of others through his/her own constructs, but he/she has to acknowledge that others have realities that are different from their own interpretations (Ernest, 1998). Ernest (1998) pointed out two principles of constructivism. First, knowledge cannot be directly transferred from the world or persons to the mind of the learner or knower. New knowledge is actively constructed from pre-images within the mind of the learner based on stimuli from the environment. Therefore, individual learners construct unique interpretations even when exposed to the same stimuli (Ernest, 1998). Second, knowledge is constructed and cannot reveal anything definitive about the world or any other domain (Ernest, 1998). For example, observers of the same phenomenon may offer different interpretations related to the person's background and experiences.

The theoretical perspective for this study can be identified as phenomenology. The actual meaning of phenomenology can be confusing and diluted since its use is so popular and widespread. As Patton (2002) suggests, "Phenomenology can refer to a philosophy, an inquiry paradigm, an interpretive theory, a social science analytical perspective or orientation, a major qualitative tradition, or a research methods framework" (p. 104). Phenomenology is reflected in most qualitative research and aides the researcher in attempting "to understand the meaning of events and interactions to ordinary people in particular situations" (Bogdan & Biklen, 1998, p. 23). Phenomenology "requires us to engage with phenomena in our world and make sense of them directly and immediately" (Crotty, 1998). In addition, phenomenology suggests that we attempt to lay aside the previous understandings we have about the phenomena, look at our immediate experiences with it, and possibly develop new meanings from it through a fresh set of eyes (Crotty, 1998). As Gall et al. (2003) state,

> Phenomenology is the study of the world as it appears to individuals when they place themselves in a state of consciousness that reflects an effort to be free of everyday biases and beliefs. As such, phenomenology shares the goal of other qualitative research traditions to understand how individuals construct, and are constructed by, social reality. (p. 481)

Patton (2002) suggests, "All our understanding comes from sensory experience of phenomena, but that experience must be described, explicated, and interpreted" (p. 106). However, interpretations and descriptions of experiences can become so intertwined they often become one (Patton, 2002). To understand an experience, interpretation is essential and experience is included in the interpretation (Patton, 2002). Phenomenology focuses on how we put the phenomena together in a way to make sense of the world (Patton, 2002).

The last of the two elements driving qualitative research are methodology and methods (Crotty, 1998). These elements will be addressed in the following sections.

Study Design

Understanding how the content knowledge and pedagogical content knowledge of mathematics teachers relate to each other is not simple. Since this relationship is complex, one cannot develop a full understanding by looking at these types of knowledge through one method of data collection. While some understanding can be gained from data collection from any given source, a deeper and better understanding of the content knowledge and pedagogical content knowledge of teachers and how they relate to each can be developed by studying these types of knowledge from multiple data sources (Crotty, 1998).

This study consisted of two complementary investigations. First, survey research was conducted with a large pool of teachers in order to understand both their content knowledge and pedagogical content knowledge. In addition, this phase explored the attitudes teachers have about how students should learn mathematics. Second, a multi-case study was used to provide an in-depth examination of these same types of knowledge, as well as to probe how teachers use their knowledge in mathematical instruction. The cases also offered opportunities to understand how content knowledge and pedagogical content knowledge changed as a result professional development and to see what how these changes are reflected in mathematics instructional practices. More detail on these two investigations is provided in later sections.

Context for the Study

Conducting research in order to answer proposed research questions requires an available and willing source of teachers. The Multi-District Mathematics Systematic Improvement Program (MDMSIP) provided a context within which the research questions could be addressed. The project activities include measuring content knowledge and pedagogical content knowledge of teachers in grades K-12, as well as providing professional development which included attention to increasing both of these types of knowledge.

Multi-District Mathematics Systematic Improvement Program (MDMSIP)

Multi-District Mathematics Systematic Improvement Program (MDMSIP) is a partnership between two universities and twelve school districts in East Alabama. Members of the faculty of both the College of Education and the Mathematics Department at East University and the Mathematics Department of South University are included in the partnership. The goal of MDMSIP is to improve mathematics education within the partnership districts, with a focus on increasing overall student achievement while addressing gaps between demographic groups. These improvements will be met by aligning the K-12 curriculum, enhancing professional knowledge of practicing mathematics teachers, ensuing consistency in teaching, developing a cadre of knowledgeable teachers, and improving the preparation at the university level for future mathematics teachers (MDMSIP, 2003b). MDMSIP began work with an initial $100,000 grant from East University while submitting a proposal for funding from the National Science Foundation (NSF). MDMSIP subsequently received a five-year $9,000,000 grant from NSF beginning in October 2003 and continuing through 2008. The following mission statement was adopted for the MDMSIP project.

> To enable all students to understand, utilize, communicate, and appreciate mathematics as a tool in everyday situations in order to become life-long learners and productive citizens by (MDMSIP). The mission will be met by:
>
> - Aligning the curriculum K-12
> - Ensuring consistency in teaching
> - Providing professional development
> - Redesigning preparation of new teachers. (MDMSIP, 2003b, p. 1)

Schools from the twelve school districts of MDMSIP were given the opportunity to apply for two weeks of professional development during the summer of 2004. The Cohort Selection Committee of MDMSIP was comprised of university mathematics education professors, university mathematics professors, and persons affiliated with MDMSIP who had background in K-12 education. The selection of the schools to participate in the first summer of professional development was based on the following three criteria: eighty percent of the mathematics teachers were going to attend for the two weeks, administrative support was evident, and a school-based strategic plan had been developed for implementing instructional change.

From this selection process, a total of 24 schools, including nine schools that included grades 9-12, were selected to join the first cohort (Cohort I) of professional development, meaning they would be included in the baseline data collection for the project. All other schools in the MDMSIP partnership were asked to participate in baseline data collection for the first year of this project, and one additional secondary school volunteered to be included.

Once the selection process had been completed and schools were notified of their acceptance into Cohort I, principals were asked to give preferred dates for data collection at their schools. Initial data collection at the secondary schools included a grade-appropriate measurement of teacher knowledge -- the Algebra Content Knowledge Instrument (ALCKIN) for grades 9-12 -- and the Teacher Attitude Survey (TAS), which will be discussed later in this chapter. These instruments were administered at all the cohort schools and the volunteer non-cohort schools. They were also subsequently administered the following years to measure progress. Classroom observations of selected teachers were conducted throughout the year and following years using the Research Teaching Observation Protocol, an instrument developed by SOURCE (YEAR). Finally, an instrument designed to measure the attitudes of students towards mathematics and mathematics teaching and learning was administered at selected grades.

Professional development provided by MDMSIP focused on engaging each of the teachers in the project in 160 contact hours over the span of the five-year project. Activities provided through the professional development included a two-week summer institute, with a one-week follow-up summer institute the following summer. Professional development provided by the MDSMIP during the two week of summer training involved several key components. The content at each level focused on the “big picture” of the content at that level, instructional strategies to help all students be successful, assessment that aligned with instructional practices, methods for developing support within the community and with parents, and other issues that arise from the effort to implement institutional changes related to reform mathematics education (MDMSIP, 2003b).

Teachers from Cohort schools were also expected to attend quarterly follow-up meetings which met one Saturday morning before the beginning of each academic quarter. These meetings were designed to help teachers implement the ideas from the summer institute while focusing on the content areas to be covered in the next nine weeks of student instruction.

Teachers could also select from a variety of optional activities. Planned activities included the following:

- A course designed to increase the content knowledge of teachers
- An online support course designed to help teachers who are beginning implementation
- Specially designed course for new teachers
- Mini courses to address special interest or ideas that arise
- A mini course to address parental and community involvement
- A mini course on problem solving and critical thinking for K-grade 8 (MDMSIP, 2003b)

Throughout all activities, focus was placed on a curriculum guide adopted by the MDMSIP partnership, and attention was consistently given to the research base for the project (MDMSIP, 2003b). Broader curricular and pedagogical issues were also focuses of the project (MDMSIP, 2003b). Emphasis was placed on increasing the participants' content knowledge beyond what they are required to teach (MDMSIP, 2003b).

School-based Teacher Leaders (STLs) were selected at each school within the partnership. STLs were given the responsibilities of meeting with teachers, planning and conducting school-based planning and inquiry groups, and working with individual teachers to improve their skills. In addition, School-based Teacher Leaders across the partnership were encouraged to attend the Cohort I training.

Thus, the MDMSIP project provided an available source of high school teachers who would be involved in professional development that provided attention to increasing content knowledge and pedagogical content knowledge. For these reasons, MDMSIP was selected as the setting for this study of teachers' content knowledge and pedagogical content knowledge before and after professional development. The project also provided an opportunity to study the teachers' content knowledge and pedagogical content knowledge through classroom observations in order to better understand how these same types of knowledge are reflected in classroom instruction.

The following sections address the methodologies and methods used in the two investigations included in this study.

Survey Research

Survey research provides the opportunity to study a large population of teachers in order to develop a broad picture of the phenomenon being studied. In survey research, a sample population is studied and inferences can then be made to a larger population (Ary, Jacobs, & Razavieh, 2002). Survey research can be used to measure intangibles such as attitudes, opinions, or other values and can more easily be used with a larger sample (Ary et al. 2002). Surveys and questionnaires can be reduced to quantifiable data if they are used alone within the research process. Survey research can entail two different types of questions which require differing types of analysis (Ary, Jacobs, & Razavieh, 2002). Close-ended questions are more difficult to construct but can be more easily coded and scanned for analysis; moreover, they lead to easily-quantifiable results (Ary, Jacobs, & Razavieh, 2002). In contrast, open-ended questions are more easily constructed but are more difficult to analyze (Ary, Jacobs & Razavieh, 2002). In open-ended types of questions, participants have the freedom of responding in any manner they choose, which requires more time to analyze (Ary, Jacobs & Razavieh, 2002) since the responses cannot be easily quantified. A mixture of open- and closed-ended questions was used used in this investigation to provide both quantifiable results and deeper analysis of the subjects.

Subject Selection

Teachers from the nine secondary schools who were accepted to be included in the summer professional development training as part of Cohort I of MDMSIP, along with the additional school that volunteered to be a part of the baseline data collection of MDMSIP, were used as the subjects for the survey research portion of this study. Participants in the survey research at these participating schools included all teachers who taught mathematics. Each teacher at the participating schools gave consent to be included in the data collection of MDMSIP, and this study was embedded within that larger data collection effort.

Instrumentation

Two instruments were used in developing an overall picture of the content knowledge and pedagogical content knowledge of subjects, the Algebra Content

Knowledge Instrument (ALCKIN) and the Teacher Attitude Survey (TAS). Further description of both of these surveys will be addressed in the following sections.

Algebra Content Knowledge Instrument (ALCKIN)

At the K-8 level, instruments to measure the mathematical knowledge of participants in MSMSIP were available from other projects. However, the MDMSIP was unable to identify an instrument for measuring the content knowledge and pedagogical content knowledge of secondary school teachers. Thus, I volunteered to develop an instrument that would be used by the project and for this study, and the Algebra Content Knowledge Instrument (ALCKIN) resulted. The following sections describe the content knowledge and pedagogical content knowledge that was included on the ALCKIN, development of the test items, field testing of the ALCKIN.

Content knowledge and pedagogical content knowledge. In order to develop the test items for this instrument, consideration was given to the types of knowledge that teachers should possess in order to teach algebra to high school students along with the major algebraic topics teachers are expected to teach their students. I used five key sources to make decisions about what algebra content knowledge should be included. The CBMS report (2000) gives recommended topic areas for the preparation of teachers. The RAND report (2003) and the PSSM (2000) both contain big ideas in relationship to the types of algebraic knowledge in which students should be proficient. The last two documents, the Alabama Course of Study (2003) and the Curriculum Guide from the MDMSIP project (2003a), provide more specific objectives to be covered in the algebra courses at the secondary level.

The recommendations from the CBMS report (2000), the RAND report (2003), and the PSSM (2000) have already been discussed in detail in the literature review. Based on the big ideas from these documents, the following content areas of algebra were given consideration: families of functions, using algebraic structures in relationship to expressions, equations, and inequalities, analyzing change in various contexts, using algebraic reasoning in relationship to other mathematical fields, and properties of number systems. Attention to conceptual and procedural knowledge (SOURCE) was included, as were the processes of problem solving, reasoning and proof, making connections, communicating mathematically, and representing mathematics in a number of ways (NCTM, 2000).

Teachers' pedagogical content knowledge includes the ability of the teacher to use his/her content knowledge in different representations as well as different methods for solving mathematics that may arise in classroom instruction. Therefore, the instrument included items that asked the participants to assess students' work, including the use of technology, communication, reasoning and proof, real-world applications, and relating algebra to other areas of mathematics.

Test format. The process for developing the format for the instrument began by considering items from other assessments. The items in content instruments developed by the Learning Mathematics for Teaching (LMT) project (Hill, Schilling, & Ball, 2003) had formats that would provide the kinds of information desired. These instruments were multiple-choice in format, and were written so that researchers could get an overall picture of how teachers could solve mathematics problems that arise in the classroom (Hill, Schilling, & Ball, 2003). However, I felt that additional understanding of how their mathematical knowledge would be gained by asking them to provide explanations. Thus, the quantitative summaries possible with close-ended items were merged with the deeper insights provided by open-ended responses.

Item development. A pool of thirty-five algebra items covering the identified content areas was developed for consideration to be used on the ALCKIN. Some of the mathematics problems were drawn from National Assessment of Educational Practices [NAEP] (NCES, 2003b), RAND (2003), PSSM (NCTM, 2000), Stump (1997), Llinares (2000), and the LMT project (Hill, Schilling, & Ball, 2003). Additional items were developed to address areas of algebra content that were not covered by these items.

Field testing. The initial pool of items was field tested in ensure the items were not confusing and to assure the instrument could be completed in a thirty-minute timeframe. The ALCKIN was field tested with two undergraduate secondary mathematics education students, one person with an undergraduate degree in secondary mathematics education, and four mathematics education doctoral students. All of the mathematics education doctoral students had previously taught mathematics at the secondary level. Each participant was asked to complete all items and to record the amount of time required to complete each test item. They were also asked to note anything they found confusing in the way the test items were written or in the accompanying diagrams. Successive revisions were made until the instrument was of an appropriate length, with well-designed tasks addressing the identified areas.

The final ALCKIN, designed for teachers to complete in a thirty-minute session, contained 10 questions. Table 1 describes the format and content of the items, and Appendix A contains the actual instrument.

Table 1

Item Format of the Algebra Content Knowledge Instrument (ALCKIN)

Question	Algebraic Content	Item Format	Processes
1	Functions	Agree, disagree or I'm not sure with teacher explanation	Representations including multiple methods Communication Reasoning and Proof Connections
2	Simplifying algebraic expressions	Multiple-choice with teacher explanation	Representations including multiple methods Communication Reasoning and Proof Problem Solving
3	General algebraic knowledge	Always true, not always true, or I'm not sure	Reasoning and Proof Connections
4	Understanding of number systems	Agree, disagree, or I'm not sure	Connections Reasoning and Proof
5	Solving an equation	Short Answer with teacher explanation	Representations including multiple methods and the use of technology Communication Reasoning and Proof
6	Simplifying algebraic expressions	Teacher explanation	Problem Solving Representations Communications
7	Using algebraic reasoning in relationship to geometry	Multiple choice with teacher explanation	Real-world applications Representations including multiple methods Reasoning and Proof Connections Communications
8	Rate of change	Multiple choice with teacher explanation	Real-world applications Reasoning and Proof Communication Connections Representations

Question	Algebraic Content	Item Format	Processes
9	Families of Functions	Multiple choice with teacher explanation	Reasoning and Proof Communication Representations Connections
10	Algebraic representations within an equation	Teacher explanation	Representations Communication Connections Reasoning and Proof

Teacher Attitude Survey

An attitude survey was developed by the Evaluation Planning Team of the MDMSIP partnership to assess teachers' attitudes and beliefs in several areas, as follow. The attitude survey was used to access their attitudes toward mathematics and to measure the teachers' beliefs about teaching and learning. It was also designed to find out about the teachers' instructional and assessment practices used in their classrooms and their involvement in professional development. In addition, the instrument was designed to address their impressions of parental involvement and overall school effectiveness. Furthermore, its design measured the expectations of change and to measure the amount of involvement with the MDMSIP project and to gather demographic information. Sources used by the evaluation committee included the RAND report (Ball, 2003a), the National Assessment of Educational Practices (NAEP, 2003), and several other federal projects.

A preliminary pool of items was selected. The committee did further revisions of the items selected and narrowed the list until teachers could complete the teacher attitude survey in twenty to thirty minutes. A final selection was made and the attitude survey was finalized. Data from the TAS provided information about how teachers viewed their classroom practices and how they felt students best learn mathematics in their classrooms. See Appendix B for the complete Teacher Attitude Survey (TAS). In particular, in this investigation I will focus on items pertaining to classroom practice and how each teacher felt students best learn mathematics in their classroom.

Procedure for Survey Study

As a member of the Evaluation Planning Team of MDMSIP, I chose to take the leadership role in collecting the baseline data for grades 9–12 for several reasons. First, since part of the data that was to be collected would be included in this study, it ensured that all of the data was collected in a systematic and consistent manner. Second, since I would be collecting all the data from the teachers, I would have first-hand knowledge of what happened and could document any similarities or differences in the methods used as the data collection process proceeded. The collection of data from mathematics teachers of students in grades 9–12 served as part of the overall baseline data for the MDMSIP project as well as the basis for this study.

Administration of Surveys

Ten secondary schools were included in the first year's baseline data collection for the MDMSIP partnership and furthermore served the basis for this study. Within these schools, all secondary teachers who taught a mathematics class were to complete the attitude survey and content instrument part of the data collection process. Eight of the schools included grades 9-12, and the other two schools contained grades kindergarten through the twelfth grade. The MDMSIP Evaluation Planning Team determined that all the secondary teachers in K-12 schools who taught mathematics classes from the seventh to twelfth grade were to complete the surveys. In addition, student surveys were administered at the same schools. MDMSIP wanted the data collection process to be as unobtrusive as possible in the participating schools. For this reason, principals at each of the participating schools were allowed to make the decision on how best to go about administering the student and teacher surveys.

The TAS and ALCKIN were administered at each of the ten schools. Initial contact was made at each school via a telephone conversation with the principal. During the conversation, a meeting was set with the principal along with a request for the number of students to be surveyed and how the permission letters should be bundled for ease in distribution. At each of the initial meetings permission letters were delivered and return dates were set for the administration of the student surveys and teacher TAS and ALCKIN. A second visit was made to each school at least two days prior to the student survey administration to deliver the surveys. This was done so that the person or persons administering the surveys would have an opportunity to see them before administering them and to relate any questions they had to the MDMSIP office. Each principal made his/her own decision as to the best way to do the data collection at their respective schools. Different methods were employed at each school with both the student and teacher surveys, but my discussion will focus primarily on the teacher surveys.

Surveys from additional participants were collected during the two weeks of professional development during the summer of 2004. This collection of additional data included 12 cohort I teachers who were absent during the original data collection or who had recently been hired for the 2004-05 school year. Two additional teachers were School Teacher Leaders (STLs) at non-Cohort I schools; all STLs were invited to attend the Summer Institute.

Of the seventy teachers involved in the data collection, sixty-seven teachers completed the TAS, and sixty-five completed the ALCKIN. Teachers were asked to provide demographic information on the TAS. The participants of the 67 TAS contained 20 males and 47 females. The 67 participants were also categorized into two racial groups: 20 were 'Black' and 47 were 'White'. Table 2 contains information regarding the number of years of teaching experience of the participants; note that a mix of levels of experience is evident.

Table 2

Years of Teaching Experience

No. of Years	No. of Participants
0 – 5	22
6 – 10	13
11 – 15	18
16 – 20	2
Over 20	12

Data Analysis

Multiple choice responses were tabulated from the ALCKIN and from the TAS. Written explanations from the ALCKIN surveys were entered in Atlas.ti (Muhr, 1991) for coding. Analysis and conclusions drawn from these instruments are addressed in full detail in chapter four. Results from the analysis of these documents provide a general understanding of the content knowledge and pedagogical content knowledge that high school algebra teachers possess.

Multi-Individual Case Studies

Case studies offer an opportunity for detailed, in-depth data collection over a period of time, drawing on multiple sources of information (Cresswell, 1998). Having multiple cases provides the opportunity to compare and contrast the cases in order to show generalizability or diversity between the cases (Bogdan & Biklen, 1998).

Case studies were incorporated to develop a deeper understanding of the content knowledge and pedagogical content knowledge of the teachers involved in the cases. Beyond the information gained from written instruments, additional sources of data collection allowed for a deeper probe into these same types of knowledge, as well as observing how these same types of knowledge related to the teachers' instructional practices. Furthermore, the cases incorporated attention to how these types of knowledge changed as a result of professional development that included focus on increasing teacher knowledge. In addition, the cases provided insight into how changes in both types of knowledge are reflected within instructional practices. Data was analyzed and used to develop a descriptive model that encompassed all factors related to the cases (Bogdan & Biklen, 1998).

Subject Selection

Teachers selected for the multi individual case studies were chosen from high school mathematics teachers involved in MDMSIP in east Alabama who were planning to participate in summer professional development provided by MDMSIP. Given their involvement in the MDMSIP, these teachers were both a convenient data source and were willing to participate in the data collection for this research.

When selecting subjects for individual case studies, Cresswell (1998) advocates selecting a range of subjects. Not having any prior knowledge of the teachers involved in MDMSIP, the high schools in the project joining Cohort I were used as a means of selecting a varied set of subjects. Although the school was not the unit of analysis for this study, it was used to begin the process of selecting teachers. The available teachers in the selection process taught diverse student populations in relationship to student achievement on the Alabama High School Graduation Examination [AHSGE] (ASDE, 2003). Racial and socioeconomic factors of each school were taken into consideration. Table 3 contains information about the demographics, the number of students tested, the passing percentages of those students tested, the percent of 'Black' students tested, and the percent of students tested qualifying for the lunch program as obtained from the Alabama State Department of Education website for the AHSGE in 2003.

Table 3

School Information[1] Regarding Demographics and Results of the Alabama High School Graduation Examination

School[1]	No. Students Tested	% Students Passing Mathematics Portion of the AHSGE	% Black Students	% Student on Lunch Program
*Abbott	183	95.08%	32.24%	24.04%
Youngs	37	75.68%	10.81%	24.32%
*Dover	317	72.24%	17.98%	19.24%
Capstone	181	71.27%	25.41%	24.86%
*Sandsfield	151	64.24%	35.10%	41.72%
Newsville	64	62.50%	65.63%	51.56%
Porter	113	61.06%	24.78%	33.63%
Frazier	31	45.16%	87.10%	61.29%
*Clarion	201	41.29%	100%	61.69%

[1]Pseudonyms are used

*Source of subjects for case studies.

From the nine high schools available, teachers were selected from a school with the highest achieving students, Abbott High School, and from a school with the lowest achieving students, Clarion High School. In addition, two schools falling between the schools with the highest and lowest achieving students were selected, Dover High School and Sandsfield High School.

The majority of the school systems within the MDMSIP serve rural schools and have small mathematics departments, thus limiting the possible pool of teachers from which to select. With the importance of Algebra I for all students, the pool of possible teachers from each of the mathematics departments was narrowed to include only teachers who taught Algebra I, Algebra IA, or Algebra IB.

A further narrowing occurred by the type of algebra course taught. In Alabama, Algebra I is taught in 140 hours as one credit; alternatively, students can complete the Algebra I course in two courses, Algebra IA and Algebra IB, which counts 280 hours and two credits. With more time allowed to teach the Algebra I content in the two courses, mathematics instruction could possibly be different in the single-course scenario. Thus, preference was given to teachers who teach Algebra I as well as Algebra IA and/or Algebra IB. Two of the chosen schools had only two teachers that taught one or more classes of Algebra I, Algebra IA, or Algebra IB. At the remaining two schools, the principals were asked to help select two Algebra I, Algebra IA, or Algebra IB teachers who employed different instructional methods in classroom instruction. This process yielded eight teachers. Initial classroom observations were made of all the eight teachers.

However, the pool subsequently further contracted. At two of the schools, one of the two teachers observed was not going to be rehired for the upcoming academic year, meaning that continuing data collection would not be possible. At the third school, one of the two teachers did not plan to participate in the summer professional development, meaning that it was less likely that there would be any changes in their knowledge.

Finally, at the highest achieving school, one of the teachers was only teaching Algebra I, not Algebra IA or IB, which meant that variations in mathematics instruction would less likely to be observed. Thus, there were four teachers in the final pool. These subjects gave their assent to participate in the study as an extension of the MDMSIP. The demographics for the four teachers are given in Table 4.

Table 4

Demographics of Case Study Participants

Name[1]	School	Years of Experience	Race	Gender	Degrees	Courses Taught
Mrs. Cotney	Abbott High School	28	W	F	BS – Mathematics Education MS – Language Arts Education	Algebra I Algebra IA
Mrs. Willoughby	Sandsfield High School	5	W	F	BS – Mathematics Education	Algebra I Calculus Informal geometry
Mrs. Pitchford	Clarion High School	4	B	F	BS & MS – Mathematics Education	Algebra I
Mrs. Colley	Dover High School	6	W	F	BS & MS – Mathematics Education	Algebra IB

[1]Pseudonyms are used

Data Sources and Procedure

Triangulation of data sources was used to reduce the chance of misinterpretation of the data. All my data sources provided a useful but complementary perspective in understanding the teachers' content knowledge and pedagogical content knowledge. In addition, these data sources helped me understand how content knowledge and pedagogical content knowledge are reflected in instructional practice and how these types of knowledge change as a result of professional development. Finally, these data sources provided the opportunity to see how changes in content knowledge and pedagogical content knowledge are reflected in instructional practices.

Data was collected during three timeframes. The pre-professional development timeframe refers to the spring of 2004 before the participants had participated in professional development. The professional development timeframe includes the two week Cohort I summer professional development as well as the follow up quarterly meetings held in August and September. The post-professional development timeframe refers to the spring of 2005, after the participants had participated in initial professional development. Table 5 summarizes the data collection sources for each of these three timeframes. The following sections provide further details.

Table 5

Case Study Time Frames and Data Collection Points

Time Frame	Data Collection Points
Pre-Professional Development	Algebra Content Knowledge Instrument (ALCKIN)
	Teacher Attitude Survey (TAS)
	Classroom Observations
	-Reformed Teaching Observation Protocol (RTOP)
	-Field Notes
	Teacher Interviews
	Journal Entries
Professional Development	Document Analysis
	Professional Development Presenter Interviews
	Quarterly Meeting Observations
	-Field Notes
	Journal Entries
Post Professional Development	Classroom Observations
	-Reformed Teaching Observation Protocol (RTOP)
	-Field Notes
	Teacher Interviews
	Journal Entries

Pre-Professional Development

This timeframe of data collection refers to the spring of 2004 prior to involvement of the subjects in the two-week summer professional development. Data sources included the ALCKIN and TAS, which they had completed as part of the first investigation, as well as classroom observations and interviews. Each of these data sources will be addressed in the following section, along with the procedures and/or protocols related to each of these sources.

Algebra Content Knowledge Instrument and Teacher Attitude Survey. The teachers were previously involved in the survey research conducted as the first part of this study. Thus, their responses to the Algebra Content Knowledge Instrument (ALCKIN) and the Teacher Attitude Survey (TAS) were included. Descriptions of these instruments are included in the previous section.

Classroom observations. Classroom observations of each participant were made during the spring of 2004. The classroom observations were conducted to further develop an understanding of the case study teachers' content knowledge and pedagogical content knowledge. Observations focused on evidence of content knowledge and pedagogical content knowledge and how it was used in their mathematics instruction. These observations were audiotaped and field notes were kept; both of these were transcribed in order to be included in the data analysis. Finally, an observation instrument was used, which is described in the next section. These sources of data are described in the following paragraphs.

Field notes were used to record those items which would not be recorded on audio tape. For example, when teachers and/or students worked mathematics problems on the whiteboard or used the overhead projector, the work was written in the field notes. Sketches of classroom settings, notations of dates and times, and other non-verbal communications were also recorded. For example, if a student nodded his head in agreement to an answer or if a teacher worked the problem without any verbal explanation, these were recorded in the field notes taken from the observation.

The Reformed Teaching Observation Protocol (RTOP) was used as an observation instrument to collect data about the case study teachers' classroom practices relative to reform practice. The Evaluation Planning Team of MDSMIP made the decision to use the Reformed Teaching Observation Protocol (RTOP) as part of the project's teacher observation process. This instrument was developed by the Arizona Collaborative for Excellence in the Preparation of Teachers (ACEPT) at Arizona State University (Sawade et al., 2000). The purpose of the instrument is to assess the degree to which the instructional practice of observed mathematics teachers is "reformed". The RTOP instrument contains twenty-five items which are scored on a scale rating of zero to four for a possible total of one hundred points. Five areas are included on the RTOP: lesson design and implementation, propositional knowledge, procedural knowledge, communicative interactions, and student/teacher relationships (Sawade et al., 2000). The instrument also contained space for a drawing of the classroom setting as well as blank pages for comments to be recorded related to aspects of the classroom instruction. The comments were used to rate the twenty-five items. The higher the score indicated the higher the degree of reform. Training was provided by the MDSMIP project on use of the RTOP before classroom observations began. See Appendix C for the complete RTOP instrument.

Initial classroom observations were included for each of the eight teachers in the original pool. An addition observation was made of the final pool of four subjects. Classroom observations at Abbott High School, Sandsfield High School and Dover High School lasted ninety minutes, while the classroom observations at Clarion High School lasted fifty minutes. Artifacts related to classroom observations such as worksheets and/or review sheets were collected at the time of the observations. RTOP forms related to each classroom observations were processed and average scores from these forms were assigned to each participant for each particular phase of data collection.

Teacher interviews. Interviews of the case study participants were used to further clarify the depth of content knowledge of algebra they possess as well as the self perception they had of their own knowledge. These interviews were also used to probe how teachers used their pedagogical content knowledge to assess how students worked mathematical problems using non-procedural methods including technology, as well as their attitudes about whether their students could learn algebra well. In deciding what questions and/or problems to include in the interviews, a preliminary analysis was conducted of both the Teacher Attitude Survey (TAS) and the Algebra Content Knowledge Instrument (ALCKIN). Five mathematical tasks and three questions related to these instruments were used in the teacher interview.

Two tasks on the ALCKIN seemed to present the greatest variation of responses from the teachers who had completed this instrument in the field testing. Question five (See Appendix A) asked teachers to use a spreadsheet in solving a quadratic equation. Since the equation did not have integer answers, teachers were asked to use the spreadsheet to give approximate solutions to the problem and to explain why they selected their particular answers. Question six (See Appendix A) required the teachers to provide multiple methods for showing and/or explaining how two expressions are equal without using the collection of like terms or the distributive property. These two questions were included in the interview to see if teachers could use different methods and representations in solving problems.

A third question was taken from the original pool of items on the ALCKIN. The question required teachers to work a word problem using three different methods. While it elicited interesting responses, it was not included in the final instrument due to the time required to complete the task. Teachers were asked to solve the task, as shown in Figure 3. After solving the problem, they were asked to provide a second method for solving the same problem, and, if one was provided, a third method of solving was requested.

> Jameel looked out in the parking lot and decided to count cars and motorcycles by the number of their wheels, excluding spares. He saw that there were 17 vehicles with a total of 56 wheels. How many motorcycles did he see?

Figure 3: First Mathematics Task in the Teacher Interview

Teachers were also asked how they would approach teaching this type of problem to their algebra students and how they felt their students would solve the problem. Finally, two non-procedural student solutions to the problem were presented to the teachers, and they were asked to comment on how the student had worked the problem and if they would accept this solution from their own students. They were also asked to explain why they would or why they would not accept the solution. This question was used because 65% of the teachers in the survey investigation had either strongly agreed or agreed with statement six on the Teacher Attitude Survey (TAS) which stated, “In the mathematics class, each student’s solution process should be accepted and valued.” The entire task is given in Appendix D.

A fourth mathematical task was posed pertained to solving a quadratic equation. Given the difficulty teachers had in solving the quadratic equation using the spreadsheet in the ALCKIN, this question was designed to further probe other ways of solving quadratic equations. Teachers were asked to respond to two different possible student solutions. The first student graphed the two sides of the equation and found the solution from the resulting intersecting graphs. The second student contained incomplete work but did contain a correct answer. Teachers were asked if they would accept the work and solutions presented in these two cases and to explain why they would or why they would not accept them. (See Appendix D.)

Finally, two additional questions from the TAS pertaining to classroom practice and how each teacher felt students best learn mathematics in their classroom were included. The two statements follow:

2. It is important for students to figure out how to solve mathematics problems for themselves.

16. Teachers should model and demonstrate mathematical procedures and then, ideally, time should be allowed for the students to have the opportunity to practice those procedures.

Teachers in the survey study generally either strongly agreed or agreed with both of these statements. Since these two statements are in direct contrast to each other, additional probing was designed to see how teachers felt both of these types of instructional practices could coexist within mathematics instruction and formed the basis for two of the interview questions. (See Appendix D.)

Finally, given the importance of algebra for all students, teachers were asked what percentage of the students in all of their Algebra I classes they felt could learn algebra well. Teachers were asked to explain why they felt this way. For a complete interview protocol, problems and student solutions see Appendix D.

Each of the case study teachers was interviewed during the spring of 2004. Each interview lasted about thirty minutes. All interviews were conducted during the planning period of the teacher either before or following a classroom observation that was scheduled for the same day. Interviews were audio taped and transcribed to be used in the data analysis. Teacher work pertaining to the mathematical tasks was also collected.

Professional Development

The focus of this time frame was to understand the professional development in which the case study teachers were involved and to seek understanding of how presenters viewed the mathematical knowledge of the case study teachers and the changes in the content knowledge they may have observed during professional development. Data sources included document analysis of the presenters guides for Algebra I, Geometry, and Algebra II in order to better understand the training they received, interviews with their professional development presenters, and observations of their participation in quarterly meetings. Each of these data sources are described in the following sections.

Document analysis. Presenters guides from the Cohort I summer professional development were analyzed. General sessions which related to all three groups were noted. For example, sessions on assessment and cooperative learning were attended by all of the secondary teachers. Each presenter guide was then reviewed for the types of algebra content that was addressed.

Professional development presenter interviews. Presenters of the MDMSIP summer professional development training were interviewed to better understand the case study participants. Presenters were asked questions related to how well the participants were involved in the activities during the professional development, as well as the attitudes they exhibited regarding the professional development training they received. Presenters were also asked about the type of algebra content knowledge they felt the case study participant possessed, and if they observed any changes in the teacher's content knowledge. Presenters were also asked their opinions as to whether they felt the participants would return to the classroom and implement the types of changes promoted by MDMSIP. For a complete presenter interview protocol and questions see Appendix F.

Interviews with the presenters of the summer professional development were conducted during the fall of 2005. Presenters who were teaching in the K-12 system were interviewed either before school, after school, or during their planning period. Two of the summer professional development presenters were working at East University and a third was employed at South University. These presenters were interviewed on the campus of East University when a convenient time and place could be arranged between the presenter and researcher. All interviews with the presenters were audiotaped and transcribed. Each of the interviews was about twenty minutes in length.

Quarterly meetings. I attended three quarterly meetings held in September and October of 2004 and March of 2005. The first meeting was held at a local junior high school and the last two meetings were held on the campus of East University. Field notes were taken at all three meetings, focusing on content knowledge, pedagogical content knowledge, and hints that these particular teachers were making changes in their instructional practices. Since audiotaping was not used at quarterly meetings, field notes served as the primary source of recorded data.

Post-Professional Development

The last time frame considered changes in the content knowledge and pedagogical content knowledge of the case study teachers as they returned to the classroom following the professional development. Data sources included classroom observations and interviews. Details of each of these follow.

Classroom observations. One classroom observation of the four case study teachers was completed during the spring of 2005. These classroom observations focused on changes in the content knowledge and pedagogical content knowledge of the case study teachers as well as how those changes were reflected in instructional practices. The lengths of the classroom observations and how they were conducted were the same as those done during pre-professional development. Audio taping, field notes, RTOP protocols were again used.

Interviews. A follow-up interview with the teachers took place during the spring of 2005. The interview addressed the following areas. First, teachers were asked the questions from the pre-professional development interview related to how well they felt their students for this academic school year could learn algebra during this school year. Second, participants were asked questions related to the professional development they received from MDMSIP, or if they did not participate in the summer professional development of MDMSIP, questions were asked related to coursework taken toward an advanced degree. They were asked about what type of algebra content knowledge they felt they possessed and how they felt this knowledge had changed as a result of professional development.

Third, participants were asked to solve a quadratic equation from the pre-professional development interview (see Appendix D). The participants were asked to solve this problem using a second method and if one was provided, a third method was requested. This request for different methods could be used to further develop an understanding of the depth of the teachers' content knowledge and how it might have changed. For a complete interview protocol, see Appendix E.

Each interview lasted approximately thirty minutes. Interviews were conducted during the planning period of the teacher and were audiotaped. The audio tapes were transcribed. Teacher work pertaining to mathematical tasks was also collected.

Journaling

A journal was kept related to all aspects of the study. My own personal thoughts about the research, the data collection process, the teachers I was observing, as well as my insights, my anxieties, and my doubts. This written record of feelings would allow me to see how my subjective impressions were affecting the conclusions I was forming about the research. It was a way to record my feelings at the time of data collection, those things that you may feel differently about as time as passed from the actual events. This notes were transcribed and included in the data analysis.

Data Analysis

The data analysis was organized around the three phases of data collection involved in the multi-individual case studies. Analysis of the pre-professional development focused on assessing content knowledge and pedagogical content knowledge before professional development. The professional development phase focused on understanding the professional development received by participants and to further understand any growth in content knowledge and pedagogical content knowledge that had occurred. The post-professional development phase was used to consider the changes in content knowledge and pedagogical content knowledge of the case study teachers.

Pre-Professional Development

Coding from the ALCKIN documents was used as the initial basis for coding of the classroom observations and teacher interviews conducted during the pre-professional development timeframe. Additional codes were added as needed, especially to describe classroom situations that might not have arisen in written instruments.

Professional Development

In considering the professional development timeframe, presenter interviews and field notes related to the quarterly meetings were entered into Atlas.ti (Muhr, 1991) and coded. The code list developed in the pre-professional phase was used, and additional codes were added as warranted from classroom observations and teacher interviews.

Post-Professional Development

During the post-professional development timeframe, classroom observations and teacher interviews were again entered into Atlas.ti (Muhr, 1991) for analysis. The code lists used to analyze these documents began with the same as the codes used in the spring of 2004 for classroom observations. Additional codes related to changes in content knowledge as well as pedagogical content knowledge were added as needed. This analysis focused on changes in the teacher's content knowledge and pedagogical content knowledge.

Data from all sources was used to develop a picture of each individual teacher including their content knowledge and pedagogical content knowledge and how the two types of knowledge relate to each other during classroom instruction. Furthermore, the pictures of the teachers included the changes they made in both of these types of knowledge as a result of participation in professional development as well as changes that occurred in mathematics instruction within their classrooms. Triangulation of data sources was used to reduce the chance of misinterpretation of the data. This triangulation of data was done as a process of clarifying meanings through identifying the different ways in which the phenomenon can be viewed along with increasing the repeatability of an observation or interpretation (Denzin & Lincoln, 2000). Cases will be considered individually and will be considered collective addressing similarities and differences between them.

Conclusion

Interpretation of analysis related to the survey research and synthesis across the cases provided the basis for answering the research questions related to this study. The content knowledge and pedagogical content knowledge of teachers will be analyzed by looking at the combination of information gained from both of the studies. Cases will be viewed looking for depth of both of these types of knowledge. Cases will also provide an insight into the how content knowledge and pedagogical content knowledge are reflected in instructional practice. Changes in both of these types of knowledge will be found by looking at the cases. Finally, case will provide the answer to how any changes in both of these types of knowledge are reflected in instructional practice.

IV. RESULTS OF THE SURVEY RESEARCH

This chapter reports results from the first part of the study, related to the analysis of the Algebra Content Knowledge Instrument [ALCKIN] and the Teacher Attitude Survey (TAS) for 54 of subjects. Each question of the ALCKIN was analyzed separately to address the correctness of the participants' responses, as well as the types of explanations they used in justifying their answer choices. Results from the individual qustions were then compiled to draw overall conclusions relating to the teachers' content knowledge and pedagogical content knowledge. Analysis of the TAS will involve only the five questions related to this study. The following sections begin with an overview of the process of coding, then present the results from item-level analyses, and conclude with general results drawn across these analyses.

Coding

The analysis of data began with coding of the written explanations provided on the ALCKIN. Written explanations were entered into Atlas.ti (Muhr, 1991), a program used to analyze qualitative data and codes iteratively developed to capture new responses. Many codes were item-specific. For example, the use of the term "slope" pertained primarily to question eight, and "vertical line test" was only used when referring to question one. Other codes applied to more than one of the questions. For example, teachers may have stated that drawings could be used, or they may have actually used a drawing in clarifying their explanation, resulting in the code of Drawing.

The recoding of documents was an ongoing process as new codes were entered into the code list from subsequent documents. After all the documents had been coded, the documents were all revisited to ensure that any additional codes were used when appropriate. Table 6 contains a list of the codes as well as the frequencies for each code; detailed definitions are provided in Appendix G.

Table 6

Codes for the Algebra Content Knowledge Instrument

Code	Frequency
Counterexample	39
Definition	76
Denotes Asymptote	3
Denotes Student Error	5
Discontinuous Function	1
Drawing/Modeling	12
Explanation of Expression	15
Understanding of Expression	35
Formula Manipulation	6

(table continues)

Table 6 (continued)

Code	Frequency
Function	45
Graph	11
I Don't Know	23
Incorrect Reasoning/Answer	74
Uses Manipulatives	13
Multiple Reasons	85
No Explanation Given	176
Procedural Knowledge	28
Provides Example	4
Slope Reference	33
Spreadsheet Analysis	30
Subject Matter Taught	8
Synthetic Division	4
Table	5
Uses Division	18
Uses Factoring	33
Uses Quadratic Formula	11
Value Substitution	22
Vertical Line Test	48

The following sections present the analysis of responses for each task. For each, a summary of the participants' responses will be provided, followed by an analysis of correct, incorrect, and ambiguous responses.

Question One - Functions

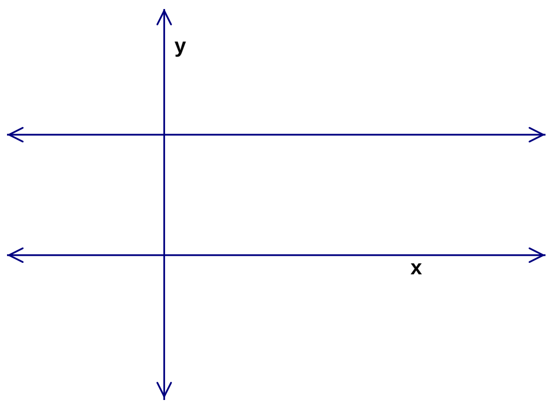
In developing the Algebra Content Knowledge Instrument (ALCKIN), functions were one of the content areas to be included and were the basis for the first question. This task presented three scenarios in which a fictitious student stated that a graph or equation was not a function. The student then provided a true statement about the problem which had nothing to do with whether the graph or equation was a function. The participants were then teacher was asked to analyze the student's thinking. Regardless of the answer selected, participants were requested to provide an explanation about why they chose their particular answer. Figure 4 contains question one from the ALCKIN.

Analysis of the Results from Question One

Each part of question one represents a function since each element in the domain is assigned to exactly one element in the range. Therefore, participants should have selected "I disagree" for each part of the question. In explaining their responses, it was hoped that participants would address statements made by the students, identifying misunderstandings students may have about what defines a function.

1. Mr. Farrow asked his algebra students to determine which of the following expressions/graphs are functions. For each of the following decide if you agree, disagree, or you're not sure about the student's assessment.

A.

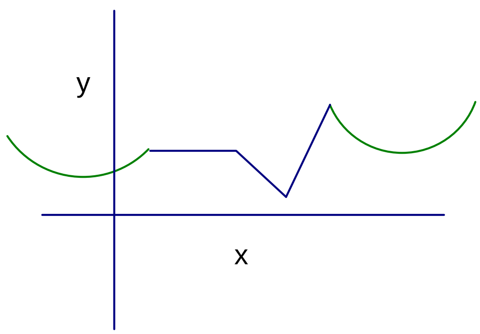


Noah said A was not a function because every x value corresponds to the same y value.

I agree	I disagree	I'm not sure
1	2	3

Explain your answer.

B.



Bart said B was not a function because it had a "strange shape" and you could not find an equation for it.

I agree	I disagree	I'm not sure
1	2	3

Explain your answer.

C.

$$y = \frac{1}{x}$$

Sanchez said C was not a function because it was undefined when x = 0.

I agree	I disagree	I'm not sure
1	2	3

Explain your answer.

Figure 4. Question One from the Algebra Content Knowledge Instrument.

Table 7 shows how the sixty-five participants selected their answers and the number of participants from each particular group who did not include an explanation for each of the three parts.

Table 7

Responses to Question One on Functions

Answer	Part A		Part B		Part C	
	Number of responses	Number with no explanation	Number of responses	Number with no explanation	Number of responses	Number with no explanation
Agree	8	7	6	4	21	13
Disagree	*52	5	*56	11	*34	7
Not sure	5	2	3	3	7	3
No answer	0	0	0	0	3	2

*Correct response.

Both parts A and part B contained graphs, and the explanations given by the participants were quite similar. Thus, the analyses of these two parts were combined. The majority of their explanations fell within three distinct categories: definition of a function, vertical line test, and restatement of the fact that it is a function. See a summary of responses in table 8. Note that some of the participants used two or more explanations to support their responses. Thus, one participant's explanation could have been counted more than once in the table. For example, a participant who explained by stating the graph was a function and the graph passed the vertical line test would be counted in the category stated "It is a Function" as well as the category Vertical Line Test. Each explanation will be elaborated in the coming paragraphs.

Table 8

Explanations for Question 1A and Question 1B

Reason	Number Part A	Number Part B
Definition of a Function	21	16
Vertical Line Test	13	20
Stated "It is a function."	13	10

We will first consider acceptable responses to these parts. The teachers who used the definition of a function for their explanation either wrote "definition of a function" or made a statement referring to the definition of a function. The following are examples of these types of statements. One teacher wrote, "A function assigns each x value with exactly 1 y (i.e. not 2). It's ok for 2 x's to have the same y." A second teacher went even further with her definition, "Every x value can correspond to the same y but every y value cannot correspond to the same x value."

The vertical line test was given as an explanation by several of the teachers. However, one participant provided an explanation for the vertical line test in writing, "B is a function because there is no place where a vertical line may be drawn to intersect at more than one point", although he did not clarify with what it was intersecting.

The following show how some of the participants used a combination of more than one of the aforementioned reasons in their explanations. One teacher wrote, "It is a function. It passes the vertical line test." Another teacher explained, "All horizontal lines are functions because they pass the vertical line test. Each x-value is only being used once." One participant related to domain and range as well as the x and y values and wrote, "It is a function because each y is paired with a unique x value; it passes the vertical line test. For each element of the domain there is only one corresponding range value."

Several other explanations were provided by one or more teachers. One teacher stated for part A, "Line should be vertical to not be a function" as a counterexample to the horizontal line graphs that would be function. From part B, still another stated "Every function is not a straight line, some functions are curves." One participant noted, "This is a piece-wise function." There were explanations which could not be classified. On both parts A and B participants used what they called 'the pencil test', 'straight line test', and 'parallel test' for their explanations. While they may have been referring to the vertical line test, no other explanation was provided.

In other cases, errors appeared in the thinking of the participants, suggesting that some of the participants did not have a deep understanding of functions. Examples of these errors from part A include the statement by a teacher who wrote, "Fails 'pencil test', (Unique 1-1 correspondence) test." Although one teacher wasn't sure whether she agreed with the student or not, she wrote, "I'm not sure I understand the graph, is this a plane or what?" She went on to state, "If this is a line, yes it is a function." One teacher agreed that the graph in part B was not a function but was unsure of the explanation and wrote, "Function is a relationship, I think." Even though two teachers both disagreed with the student, their explanations contained errors in their thinking about what a function is. One teacher explained, "A function is a one to one corresponds (sic). The range and domain have that one to one correspondt (sic)." Another teacher stated, "This is a function. You can find a shape."

A little over half of the sixty-five participants selected the correct answer for part C. The explanations for part C could not be grouped into specific categories as in parts A and B, and more errors were found in the thinking of the participants than in the previous two parts. The majority of the participants had idiosyncratic ways of expressing why they choose a particular answer. Along with using the definition of a function or stating "It is a function," some participants also included sketches of the graph and/or a table of values. A decision could not be made if the sketches were included to support their explanation or to help the participants in selecting their answers. The following are examples of explanations that did not contain errors in the thinking of the participant. One participant wrote, "It's not a continuous function but it is a function," while another participant stated, "Since each x-value would correspond to exactly one y-value this is a function. It is just not a continuous function, $x = 0$ is the exception."

As in parts A and B, the explanations written by some participants contained errors or information that made it difficult to determine whether the participant truly understood functions. One of the misconceptions involved thinking that a function had to be continuous and if an equation could represent a function if it has a restriction on x.

Other misconceptions included if y can undefined at some x value and still be a function and if functions have to have a one to one correspondence. Still others felt that is y was undefined this represented a value for y. One teacher's explanation contained the following information "A function assigns exactly 2 y to each x; therefore, each x gives only 1 answer. Since the 'answer' for x = 0 is that y is undefined, & this is the only answer, then this is a function." Other teachers who disagreed provided the following. One teacher wrote "C is a function when x ≠ 0." Another teacher stated "The function still has a one to one correspondce (sic)" as well as the sketch in Figure 5.

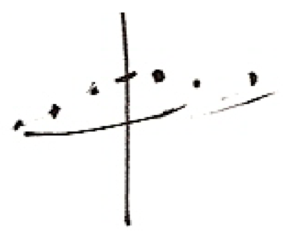


Figure 5. Teacher's incorrect sketch for the graph y = $\frac{1}{x}$.

Some teachers agreed that the equation did not represent a function, but provided explanations based on incorrect reasoning. For example, one teacher stated, "A function cannot be graphed with an 'undefined'." Two participants felt the student was correct in his thinking, because "It is also not a function because the variable has a negative exponent." Another participant stated, "Yes, if the denominator is zero the function is undefined." Still another participant wrote "Sanchez understands that C would not be a continuous function because he realized that the graph would be undefined when x = 0."

Finally, one teacher provided this explanation, “Functions cannot have an unknown without restrictions. You cannot divide by zero.”

Others participants were not sure whether to agree or disagree with the student but included the following types of explanations. One teacher questioned her own thinking as she wrote, “It is undefined when x = 0, that I agree with. I believe that the graph would pass the vertical line test. I’m not sure,” while a second participant suggested, “I think this is a function but maybe it needs a limit or has an asymptote.” One participant stated, “Function is unclear to me.”

Still other participants elected to support their explanations in different ways. For example, one teacher chose to point out that this topic is not introduced in Algebra I but did give suggestions for an approach in helping students understand what happens with this particular equation. He wrote “In Alg I they have not been introduced to vert. isomtopes so they may see it as non-function but when get them build table or graph should make more clear.” Although one participant did not select an answer he wrote in the explanation space, “C is a function with domain: $\{x \mid x \neq 0\}$.”

Only a few of the teachers responded to the parts of question one by referring to what the students stated. Those that did responded with correct mathematical thining. For example, one teacher stated, “Yes, every x does correspond to the same y value, but that doesn’t make it not a function. Non-functions pair diff y-values with the same x & it passes the vertical line test,” and provided an accompanying sketch shown in figure 6.

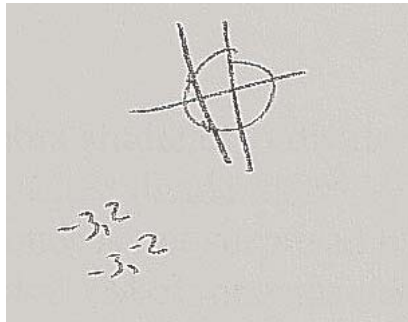


Figure 6. Example of a graph not passing the vertical line test.

Still another participant explained what part of the student statement in part B he agreed with as well as what part he was in disagreement with by stating, “He is right about not finding an equation, but wrong about it not being a function.” A teacher used the fact that the graph would have an asymptote within his explanation: “True, but you have an asymptote at x = 0. Still a function.” Another teacher explained her answer with the following statement, “It is a function, simply a rational function. Yes it is undefined at x = 0 but this only yields an asymptote. While it is not a continuous function, it is still a function.”

Conclusion

The results from question one of the ALCKIN suggest that the participants do not have a deep understanding of functions. When presented with a graphical representation, the participants were more likely to select the correct answer. Explanations were generally given as statements such as repeating the definition of a function, stating that a given situation was a function, or using procedural knowledge such as the vertical line test. Some of the participants did use other forms of representations such as drawing pictures.

The reasoning presented by some participants further suggests that they did not have a thorough understanding of functions. Errors were observed in whether functions includes one-to-one correspondence, continuity, can equations with undefined solutions be a function, and graphing. Some of the participants had difficulty relating that the statement $y = \frac{1}{x}$ was undefined when $x = 0$ to what this fact might have to do with whether or not it is a function relationship.

Question Two - Simplifying Algebraic Expressions

Question two addressed simplifying algebraic expressions, in particular the simplification of the rational expression $(x^2 - 4)/(x + 2)$. See Figure 7 for question two of the Algebra Content Knowledge Instrument.

2. Mrs. Jones asked her algebra students to divide $x^2 - 4$ by $x + 2$. Seth said, "I have an easy method, Mrs. Jones. I just divide the x^2 by x and the 4 by the 2. I get $x - 2$, which is correct." Mrs. Jones is not surprised by this as she had seen students do this before. What did she know? (Mark one answer.)
   a. She knew that Seth's <u>method</u> was wrong, even though he happened to get the <u>right</u> answer for this problem.
   b. She knew that Seth's <u>answer</u> was actually wrong.
   c. She knew that Seth's <u>method</u> was right, but that for many algebraic fraction division problems this would produce a messy <u>answer</u>.
   d. She knew that Seth's <u>method</u> only works for some algebraic fractions.
   e. I'm not sure.

Explain your answer.

Figure 7: Question two from the Algebra Content Knowledge Instrument.

Analysis of Results of Question Two

Table 9 shows a summary of the answers given. There are two responses that could be considered correct to this particular question. If the method was analyzed for correctness in all situations, only answer choice "A" would be considered correct. However, the wording of answer choice "D" indicates that there are certain situations in which Seth's method would work. This particular method works only in a very few special cases. Examples can be provided for which the method would work for Seth, but there are many counterexamples for which the method would not work.

Table 9

Question Two Responses to Simplifying Algebraic Expressions

Answer	Number of Responses
A. Method was wrong, although answer was correct	*45
B. Answer was wrong	0
C. Method was correct, but other problems would produce messy answers	2
D. Method only works for some algebraic expressions	**14
E. I'm not sure	3
No answer selected	1
Selected two answers	1

*Correct Response
** Works in limited cases

For those participants who selected answer choice A, the majority of the explanations fell into three major procedural categories. These explanations related to factorization of the numerator and cancellation of common factors (n = 19), long polynomial division (n = 11), and synthetic division (n = 4). The teachers emphasized these procedural methods for showing how the student should approach simplifying the expression instead of addressing his invented method. This was true for the majority of participants whether they recognized the method was correct or incorrect. Most participants emphasized that the student should remember the rules or procedures for simplifying these types of algebraic expressions. This was further demonstrated by participants simplifying the expression using one or more of the procedural methods and/or writing an explanation for why they should use these procedures. None of the teachers used any type of conceptual explanation for why the method proposed in the task is not valid when simplifying any type of algebraic or numerical fraction. The following are examples of the types of explanations given. One teacher stated:

> That is the right answer (x – 2), but division by common terms is incorrect. It needs to be factored and then divided. An x^2 can only be divided by an x if it is by itself or being multiplied (not added or subtracted).

Another teacher, closest to addressing the issue involved in the problem, stated, “Seth needs to understand that x + 2 represents one factor and x and 2 should not be considered separately.” A third teacher responded, “The correct method is to use long division, synthetic division, or factor the numerator [$(x^2 – 4)/(x + 2)$] and cancel out common factors.” Sample work from a participant can be found in Figure 8.

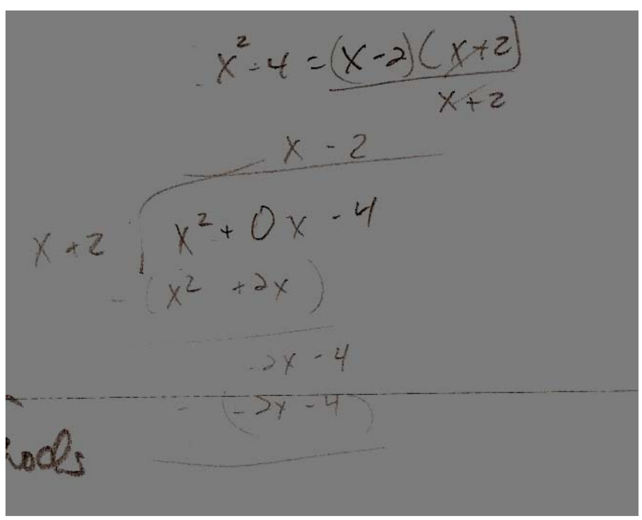


Figure 8. Participant’s simplification of the algebraic expression by factorization and division.

She followed her work with “Not all binomials are the difference of two squares. Neither are all quadratics. Factoring or synthetic division would be correct methods to solve the problem.” Yet another participant wrote:

> In order to divide a polynomial by another polynomial, you must factor the dividend first. (If it is not factorable, you can use long division or possibly synthetic division.) Only if a factor of the dividend is identical to the divisor can you cancel those factors out.

The participants (n = 14) who selected answer choice D thought Seth’s method would work but in only certain situations. Those that selected this answer generally provided examples that would work as well as counterexamples. One of these teachers noted that students can make errors but arrive at correct solutions: “Sometimes students get the right answer, but the methods are not true for all questions.” Still other participants gave examples showing this method would only work for the difference of two squares. One teacher wrote the problem in division form, then explained, “If there had been a middle term, his method would not have worked.” Another teacher provided examples where Seth’s method would work along with counterexamples where the method would not work. She wrote “Works on these types: $[(x^2 – 4)/(x + 2)]$; $[(x^2 – 9)/(x + 3)]$; $[(x^2 – 16)/(x + 4)]$; Counterexample: $[(x^3 – 9)/(x + 3)] \neq x^2 – 3$.” Yet another teacher reiterated this same thinking in her response: “It may have worked for $[(x^2 – 4)/(x + 2)]$ but does not work for $[(x^2 – 8)/(x + 2)]$. This method only works when the numerator is a difference of perfect squares & the denominator is a factor of the numerator.” Note that providing examples work and/or counterexamples does not address the issue for why the method will or will not work.

Conclusion

The majority of participants could simplify the algebraic fraction using at least one procedural method. Furthermore, teachers emphasized the students knowing a correct procedure for the simplification of the given algebraic expression. Participants who selected the option that Seth's method would work in some situations often provided examples that worked and counterexamples where his method would not work. However, none of these participants suggested that they would encourage Seth to look for counterexamples so that he could see that his method was not valid in all situations. None of the participants offered any type of explanation that would help students conceptually understand why the method he employed would or would not work.

Question Three - General Algebraic Knowledge

Question three involved three of the four areas that were used to select or develop items for the ALCKIN: using algebraic structures in relationship to expressions, equations, and inequalities; families of functions; and properties of number systems. The purpose of this particular question was to see if teachers could use their mathematical knowledge in evaluating statements indicated as "rules of thumb." In the course of classroom instruction teachers often offer students "rules of thumb" to help them remember particular mathematical ideas or procedures. Sometimes, however, these handy memory devices are not actually true, or they are not true in all situations. The participants were not asked to explain any of their answer choices. Figure 9 contains question three from the Algebra Content Knowledge Instrument

3. Teachers often offer students “rules of thumb” to help them remember particular mathematical ideas or procedures. Sometimes, however, these handy memory devices are not actually true, or they are not true in all situations. For each of the following, decide whether it is true all of the time or not. (Mark TRUE FOR ALL SITUATIONS, NOT ALWAYS TRUE, or I’M NOT SURE.)

		True for ALL Situations	NOT Always True	I’m Not Sure
A.	A binomial made up of two perfect squares cannot be factored into two binomials unless the two terms have a subtraction sign between them.	1	2	3
B.	An asymptote is a line that a graph approaches but never crosses.	1	2	3
C.	Any number to the zero power is equal to one.	1	2	3
D.	When graphing linear inequalities, if the inequality sign is “<” you shade below the line and if the inequality sign is “>” you shade above the line.	1	2	3

Figure 9. Question three from the Algebra Content Knowledge Instrument.

Table 10 shows how the teachers responded to each answer choice. In each scenario, participants should have selected “NOT Always True”. Counterexamples can be provided for each of the four statements. A counterexample for statement A would be $-16 + x^2$, since this is a difference of squares but not written with a subtraction sign. For statement B, consider a function with a horizontal asymptote that can be crossed such as $y = \frac{x}{x^2 - 1}$. For statement C consider that $0^0 \neq 1$, and for statement D consider the graph for 3 > x + y, where the shading of the graph would be below the line even though a ">" inequality is used.

Table 10

Question Three Responses to General Algebraic Knowledge

Answer	Number of Participants A	B	C	D
True for All Situations	41	41	47	16
NOT always true	*17	*11	*16	*44
I'm not sure	6	11	1	3
No answer selected	2	2	1	2

*Correct Answer

Conclusion

Participants had a difficult time in recognizing that the "rules of thumb" are not valid in all situations. Approximately one fourth of the participants were able to decide that the first three situations were not true all of the time while about sixty-seven percent answered that the last situation was not true for all situations. These numbers seem to suggest that the participants did not realize the exceptions to these "rules of thumbs." Although the participants were not asked to respond to any of the above statements or explain their reasoning, some of the participants did provide the types of counterexamples that were mentioned above.

Question Four - Understanding of Number Systems

One of the four areas used in selecting and developing test items for the ALCKIN was properties of number systems. To develop a better understanding on how well participants understand the number systems, question four made specific statements involving numbers belonging to certain sets. Figure 10 contains question four from the Algebra Content Knowledge Instrument.

4. Consider the following statements related to different sets of numbers. After reading each statement decide if you agree, disagree, or if you are not sure.

	Statement	I agree	I disagree	I'm not sure
A.	-3 is a rational number	1	2	3
B.	2/3 is a real number	1	2	3
C.	.010010001… is a rational number.	1	2	3
D.	√5 is a complex number.	1	2	3

Figure 10. Question four from the Algebra Content Knowledge Instrument.

Participants should have agreed with statements A, B, and D, disagreed with statement C. Negative three is a rational number since it can be written as a fraction $\frac{-3}{1}$.

For part B, two-thirds is a real number because the set of real numbers is the union of all rational numbers and irrational numbers. Since two-thirds is a rational number, it is also a real number. In part C, 0.010010001… is not a rational number because a fraction cannot be found to represent this decimal number. Finally, for part D, since all real numbers are complex numbers which can be written in the form a + b*i* where b is equal to zero, √5 is a complex number. Table 11 shows the results of how participants responded to each statement.

Table 11

Question Four Responses to Understanding of Number Systems

Answer Choice	Number of Participants			
	A	B	C	D
I agree	*56	*54	19	*20
I disagree	8	10	*42	41
I'm not sure	1	1	4	4

* Correct Answer

Conclusion

The majority of the participants were able to correctly identify the rational number and the real number, but only about two-thirds of them knew that the decimal in part C was not a rational number and little more than thirty percent knew that the square root of five was a complex number. The results suggest the participants do not necessarily have a complete understanding of the number systems and how each system is built from the other.

Question Five - Solving an Equation

One area of consideration in the ALCKIN was using algebraic structures in relationship to expressions, equations, and inequalities. The emphasis for the fifth problem on the ALCKIN was not on testing the participants' ability to solve algebraic equations but rather to give the participants an opportunity to find the solutions for an equation in a format which may be different from the usual method to which teachers and students may be accustomed. The participants were told that a teacher had asked his students to solve the quadratic equation $3x^2 = 4 - 2x$ using a spreadsheet. This particular problem was selected with the intent purpose that the resulting polynomial not be factorable and that the solution would involve a radical. A spreadsheet table was presented in which a range of values for x had been substituted in the expressions $3x^2$ and $4-2x$. This table shows the approximate solution for the quadratic equation should be between -1.5 and -1.6. Note that there is a second solution between 0.8 and 0.9, which is not shown in the table. Figure 11 contains question five from the Algebra Content Knowledge Instrument.

5. Mr. Casteel is using spreadsheets in his Algebra class to find solutions for quadratic equations. What approximate solution(s) for the equation $3x^2 = 4\text{-}2x$ should Mr. Casteel's students give using the following spreadsheet?

X	$3x^2$	4-2x
-1.8	9.72	7.6
-1.7	8.67	7.4
-1.6	7.68	7.2
-1.5	6.75	7
-1.4	5.88	6.8
-1.3	5.07	6.6
-1.2	4.32	6.4
-1.1	3.63	6.2
-1	3	6
-0.9	2.43	5.8
-0.8	1.92	5.6
-0.7	1.47	5.4
-0.6	1.08	5.2
-0.5	0.75	5
-0.4	0.48	4.8

Solution(s):

Explain your answer.

Figure 11. Question five from the Algebra Content Knowledge Instrument.

Analysis of the Results From Question Five

The solution for the quadratic equation includes the values for x which make $3x^2$ equal to 4-2x. The participants should have looked to see where the second and third columns are closest to being equal. Twenty-seven of the sixty-five participants (41%) gave answers that fell within the correct interval of x values, "between -1.5 and -1.6", or reported in different ways that conveyed this thought.

The following are examples of participants who used the spreadsheet to find the approximate solutions without relying on another procedural method in their justification. One teacher stated "When x = -1.6, $3x^2$ is greater than 4 – 2x. When x = -1.5, $3x^2$ is less than 4 – 2x. Somewhere between these two x-values $3x^2$ should equal 4 – 2x." A second teacher explained:

> Look for a row or rows in which the entries in the 2nd and 3rd columns are equal. If there are not any, look for a pair of rows where the larger value "switches" from column 2 to column 3. The root will be between the x-values for those rows.

Even though these participants relied on the spreadsheet to find and support their solutions, the order in which work was recorded on their instrument suggested six of them may have began the process by trying to use an algebra procedure similar to one of those discussed in the next section.

While we cannot completely infer what the participants did from the written work, the order in which they presented their work suggests some of the participants began by setting the equation equal to zero and factoring. For example, one participant factored $3x^2 + 2x – 4 = 0$ into (3x – 2) and (x + 2) and wrote x = 2/3 and x = -2 as her solutions and used her work as the explanation for her answer. Another teacher also made errors in his factorization when he wrote (3x + 4) and (x – 1) as his factors and he did not provide any further answer or explanation for what he was doing. One participant exhibited more than one error in his thinking as he indicated an attempt to factor the quadratic expression, wrote x = -2/5, defended his answer by stating "both sides of the equation match", and drew an arrow pointing to the row that contains the values x = -0.4, $3x^2$ = .48 and 4 – 2x = 4.8. One teacher began by showing 3x-2 and x+2 as factors for the quadratic expression. He proceeded to use the quadratic formula but made a mathematical error and ended up with a two in the denominator instead of three and his solutions of "1.25, -2.25" were not in the chart thus prompting him to provide "???" as his explanation. Another teacher also tried factoring but marked through his work and wrote “x = -1.5, x = -1.6” and did not provide any explanation for why he picked those particular solutions.

The order of the work of one teacher suggests that two procedural processes were tried before using the spreadsheet provided to solve the problem. She attempted to factor and had written the following parentheses, “(3x 4) and (x 1)”. She also began to substitute into the quadratic formula with “$-2 \pm \sqrt{\ }$” and ended up simply writing “$-1.5 < x < -1.6$”. She indicated that when $x = -1.5$, $3x^2 < 4 - 2x$ and when $x = -1.6$, $3x^2 > 4 - 2x$. Even though this participant began by using other methods to find the solution for the equation, it appeared that she was successful in using the spreadsheet in finding the approximate solution for the equation. It should be further noted that she made an error in writing the inequality she used for her solution.

Nine participants tried to use the quadratic formula to find the solutions for the equation. These solutions were left in varying forms of simplification. Participants using the quadratic formula instead of the spreadsheet also exhibited errors either in substituting in the formula or in the simplification of the expression. After her unsuccessful attempt with the quadratic formula, one teacher wrote as her explanation, “I guess I do not understand exactly what this is asking for. *You would plug each value into & check to see which one is correct.” Another teacher actually found the solutions to be 0.87 and -1.54 but did not provide any explanation other than the work used when substituting into the quadratic formula and did not refer back to the table.

Some of the respondents either did not attempt to answer the problem or wrote such responses as “I don’t know”, “not sure”, “?”, “can’t do this” or simply rewrote the problem. Other participants wrote incorrect solutions for the equation. One teacher, who gave "-1, 3, and 6" as her solution wrote, "Because they are all whole numbers, and decimals throw students for a loop the majority of the time. They have the misconception that an answer is usually not a decimal."

Conclusion

The participants had a difficult time finding an approximate solution to the quadratic equation using the spreadsheet provided. While about one-half of the participants were able to give a correct approximate solution, only one-third found the approximate solution without displaying some algebraic procedure such as factorization or the quadratic formula. Many procedural errors were made by the participants including incorrect factorizations, incorrect substitution in the quadratic formula, incorrect writing of an inequality solution, and incorrect simplification of an algebraic expression involving a radical. Even though some of the participants actually found two solutions for the equation when using other methods other than looking at the spreadsheet, none of the sixty-five participants mentioned that there were two solutions for the equation and that one of these solutions was not contained in the spreadsheet.

Question Six - Simplifying Algebraic Expressions

Question six addresses the content area of using algebraic structures in relationship to expressions, equations, and inequalities. Figure 12 contains question six from the Algebra Content Knowledge Instrument.

6. Without using algebraic manipulation such as collecting like terms or using the distributive property, what other methods could you as a teacher use to justify to your students that the expressions 3x + 5 + 5x -3 and 4 (2x + 1/2) are equivalent to each other?

Figure 12. Question six from the Algebra Content Knowledge Instrument.

Analysis of the Results from Question Six

Responses from the participants fell within three general categories; see table 12. Ten of the participants used a combination of two of the three categories. Examples of each of these categories follow. Note that only half were able to provide a satisfactory response.

Table 12

Question Six Responses

Response	Number of Participants
*Substitute in value(s) for x	18
*Use Manipulatives	13
*Drawing/Modeling	12
Didn’t indicate another way	24
No Response	11

*Correct Answer

The first category includes participants who suggested that a value or values be substituted in the place of x to show that equal values resulted. For example, one teacher who responded with, “Substitute the same value for x,” and a second teacher wrote, “Try plugging in values for x. If when simplified the answers are the same, then the expressions can be equivalent. When x = 2, then both exp = 18.” Another teacher realized showing the expressions equal did not actually prove them equal, and explaining, “Substitute values in place of x and simplify using the order of operations, although you would have to point out that you cannot <u>prove</u> they are equivalent that way because you cannot check <u>all</u> real numbers.” Along with value substitution, one participant offered an additional method for showing the two expressions equivalent: “I would use a graphing calculator to graph both equations to see if we get the same line, or we could simply pick a number for x, plug it in and see if we get the same solution.”

Participants also suggested using some type of manipulative to show students that the two expressions are equivalent. For example, one teacher replied, “Use an object to represent x and let them discover the relationship”. Other participants were more specific in the type of manipulatives they would use to show equivalence such as one participant who suggested, “Use physical manipulatives such as algebra tiles”.

Participants also either suggested using drawing or modeling. All but one of these participants drew a representation to show how the two expressions are equivalent. An example is given in Figure 13.

Use manipulatives.

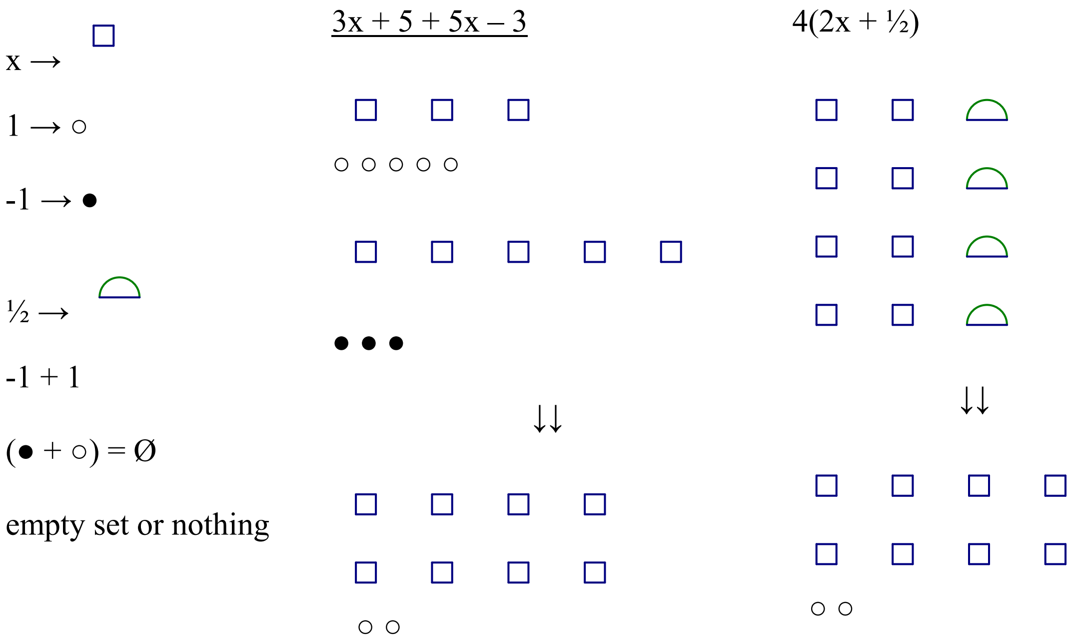


Figure 13. Participant's pictorial representation to show two algebraic expressions are equivalent.

However, more than half of the participants (n = 35) were unable to answer this particular item as requested. Twenty-four of these participants either did not respond, wrote "I don't know", "I'm not sure", or "Can't think of anything", while ten other participants simplified the two expressions by using the distributive property and the collection of like terms.

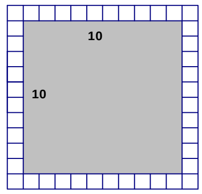
Conclusion

Over half of the participants had difficulty offering any way for teachers to help students understand that two algebraic expressions were equivalent or simply relied on procedures that in the instructions they were asked not to use. The majority of the participants who attempted to offer a way to show that the expressions were equivalent used a procedure where students would substitute a value or values into the place of x. Even fewer of the participants offered concrete examples such as various manipulatives or making a drawing or sketch of the two expressions.

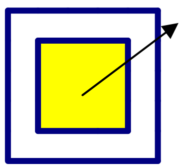
Question Seven - Using Algebraic Reasoning in Relationship to Geometry

Question 7 asked the participants to use algebraic reasoning in other mathematical fields such as geometry. The focus of question seven related to forming an algebraic expression to represent a geometric model. Participants were asked to respond to solutions of two students who had looked at the problem in different ways and had approached working the fictitious problem differently. Note that the first three answer choices contained equivalent algebraic expressions. Figure 14 contains question seven from the Algebra Content Knowledge Instrument.

7. Students who are in Mrs. Simpson's algebra class were working on the following problem: You are going to build a square garden and surround its border with square tiles. Each tile is 1 foot by 1 foot. For example, if the dimensions of the garden are 10 feet by 10 feet, then you will need 44 tiles for the border.

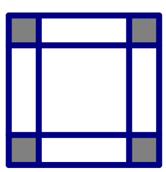


How many tiles would you need for a garden that is n feet by n feet? Two students gave the following diagrams, representations, and reasons for their answers.



James stated you could find the number of tiles needed for the border by finding the area of the entire garden and border and subtract out the area of the garden.
Which representation best reflects his method?

A. $4n + 4$
B. $4(n + 1)$
C. $(n + 2)^2 – n^2$
D. All are equally correct



Ann said you could take the entire length of each side of the border which is n and multiply by four, then add in each of the corners.
Which representation best reflects her method?

A. $4n + 4$
B. $4(n + 1)$
C. $(n + 2)^2 – n^2$
D. All are equally correct

Which student has the best understanding of the tile problem?

A. James
B. Ann
C. Both show equal understanding.

Explain your answer:

Figure 14. Question seven from the Algebra Content Knowledge Instrument.

Analysis of the Results from Question Seven

Responses to the first two parts of question seven are summarized in Table 13. The correct algebraic expression for James' interpretation should have been answer choice "C". Since the border adds two to the length and width of the garden, the area of the garden and border would be $(n + 2)^2$ and the area of the garden would be n^2. Since the first area also includes the area of the garden, James removed this and is left with the area of the border. The correct algebraic expression for Ann's interpretation should have been answer choice "A". Since the border of the garden is n tiles on each side, four n would represent the total tiles on the sides of the garden. Ann then includes the tiles from the corners. Note that while all three expressions are equivalent, only one correctly represents the interpretation given by each student.

Table 13

Question Seven Responses on Using Algebraic Reasoning in Relationship to Geometry

	Number of Participants	
Answer Choice	James	Ann
A. $4n + 4$	1	*50
B. $4(n + 1)$	2	4
C. $(n + 2)^2 – n^2$	*51	2
D. All are equally correct	7	5
E. No response	4	4

*Correct Answer

Almost eighty percent of the participants were able to select the correct expression for James' interpretation, and about seventy-eight percent were able to select Ann's algebraic expression that fit her interpretation. Twelve of the participants noted that all three of the algebraic expressions are equivalent.

The participants were then asked to choose which of the two students had a better understanding of the problem and to explain why they chose that particular student. Participants should have stated that both students had an equal understanding of the problem, since both correctly represented a solution. Table 14 summarizes the responses and those with no explanation of their choice.

Table 14

Question Seven, Part C Responses

Answer Choices	Number of Participants	Contained No Explanation
James	12	8
Ann	11	5
Both show equal understanding	*37	11
No response	5	5

*Correct Answer

Participants who felt both students understood the problem equally well explained their answers using some way of stating that there are various ways of approaching and solving problems in mathematics and were more likely to point out that both students arrived at the same solution. For example, one teacher stated: “Answer A best represents Ann’s view while choice C is representative of James’ view. Since the 2 are equal one might assume their understanding is equal.” A second teacher explained, “Either method produces the same correct result, and each student shows critical thinking skills.” Another teacher actually showed that the two expressions $(n + 2)^2 – n^2$ and $4n + 4$ were equivalent to each other, while another stated: “Although they approached it from different viewpoints, they both are correct in their view of the problem. Furthermore, they get the same result when you simplify.”

After selecting James as having the best understanding of the problem, one teacher wrote, “This is the easiest way to explain it to a class.” A second teacher explained:

> I feel that James seems to better understand, because he seems to know it is more logical to get a more correct answer by using what you really know. With Ann’s method, how do you really know the “measure” of the corners.

The following are examples from participants who selected Ann as having the better understanding. One participant wrote “Ann is finding perimeter. James is finding area of tiles.” A second participant explained with “James’ answer involves area; Ann’s involves perimeter, which is what a ‘border’ is.”

Still other participants focused on the geometric aspects, perimeter and area, as factors in their explanations. For example, one participant wrote, “Both ways are acceptable. Ann is more focused on perimeter whereas James is focused on area, but both are ok.” Another participant explained:

James' approach involves area, while Ann's involves perimeter. → As long as only 1x1 tiles are used in <u>one</u> row around garden, each method works, although Ann's seems to be simpler. However, if the tiles are in <u>two</u> or more rows, problem becomes more difficult for Ann.

Other participants focused on the practicality of the two methods. For example, one teacher felt "Ann's method is more practical than James" and another teacher explained "James' answer is more complicated, but also works."

Errors can be noted in some of the explanations of the participants who selected Ann as having the best understanding. For example, one teacher failed to realize that since the area of the tiles was one that the area of the border would also be equal to the number of tiles in the border. She stated, "Even though there eq are the same, Ann's method will give you the # of 1x1 tiles, James' way will give you the area of the border (not the # of tiles)." A second teacher felt James was missing something as she stated, "James is close to understanding, but is missing the 4 corner pieces." Another teacher seemed to think the numbers used in the problem promoted which method was better when he wrote, "As long as the dimensions of the garden are both even numbers, then James' method will do nicely. Otherwise, Ann has the better approach."

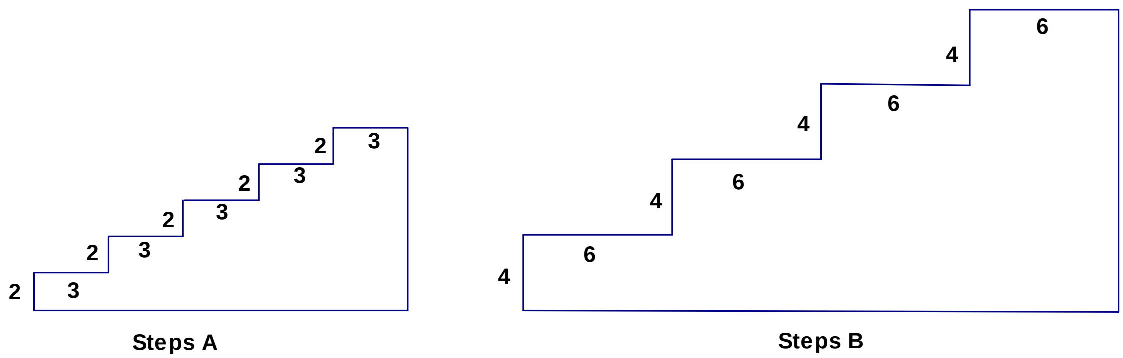
Conclusion

More than three-fourths of the participants were able to correctly identify the algebraic expressions representing the geometric interpretations of the two students. However, only about half of the participants felt that both students had an equal understanding of the problem. The data suggest that these teachers felt there is more than one method for solving mathematical problems, some even pointing out procedurally that the two algebraic expressions representing the two students' choices are equivalent. Errors in the thinking of the participants could be seen by the indication of some of the participants who felt that if you talk about the border of a patio you have to relate it to perimeter and not to area.

Question Eight – Rate of Change

Question eight addressed the content area of rate of change was written within a context of steepness of steps. The participants were asked to determine which group or groups had a correct Figure 15 contains question eight from the Algebra Content Knowledge Instrument.

8. Mrs. Ledbetter gave her algebra students the following two diagrams of steps and asked them to determine which set of steps were the steepest. The students could not agree on an answer and gave the following answers and reasons for their answers.



A. **Group 1** said that both sets of steps have the same steepness. “First connect the points at the edges of the steps with a line and measure the angle formed between this line and the ground. The measure of this angle is the same in both set of steps. Therefore they have the same steepness.”

B. **Group 2** said Steps B were steeper because the steps in Steps B are taller than in Steps A.

C. **Group 3** thought both sets of steps had the same steepness since doubling both the length and height of the steps does not affect the steepness.

D. **Group 4** said both sets of steps had the same steepness. “If you sketch in a line that contains the edges of the steps and determine the slope of the line in each set of steps, you will find they have equal slope. Steps A have a slope of 2/3 and Steps B have a slope of 4/6 which is equivalent to 2/3. Therefore, they have the same steepness.”

Which group(s) have a correct understanding of slope? (Circle all that apply).

A. Group 1 B. Group 2 C. Group 3 D. Group 4

Of the four groups which group has the best understanding of slope?

____

Explain why you chose this group.

Figure 15. Question eight from the Algebra Content Knowledge Instrument.

Analysis of the Result from Question Eight

Table 15 summarizes whether the participants felt the students did or did not understand slope from the explanation given. Six of the participants chose not to respond to the question.

Table 15

Question Eight Responses on Rate of Change

Answer Choices	Number of Participants		
	Understand	Do Not Understand	No Response
A. Group 1	*40	19	6
B. Group 2	5	*54	6
C. Group 3	*38	21	6
D. Group 4	*54	5	6

*Correct Answer

Group 1 in part A described a trigonometric representation for slope. This group understood that the steps had the same slope because the tangent of the angle formed with the ground would be the same in both sets of steps. Group B did not understand steepness relates to both vertical change as well as horizontal change. Steepness is not based on “tallness.” Group C understood that doubling the ratio of the change in the height of the rows as related to the change in the length of the steps, results in equivalent ratios. Group D understood slope and steepness and defended their answer in a traditional view of finding slope of a line.

For Group A, the trigonometric representation 61% of the participants correctly selected that the participants understood slope. For Group B, 82% were correct in selecting that the group did not understand slope. For Group C, 58% of the participants selected that the group understood slope. For Group D, 82% of the participants said the group understood slope.

After the participants chose the group or groups they felt had a good understanding of slope, they were asked to select which of the four groups had the best understanding of slope and to explain why they felt this group had the best understanding. Table 16 shows how the participants selected which group or in some cases groups had the best understanding of slope.

Table 16

Group with the Best Understanding of Slope

Group	Number of Participants
A	1
B	1
C	5
D	45
A and D	1
A, C, and D	*1
No Response	11

*Correct Answer

The following are examples from the participants explaining why they selected each group or groups. One teacher selected Group A and explained his answer by stating, "Group A brought in the idea of angle w/ the ground which is a key thought for slope although group 4 could do slope group 1 had a better working knowledge." A second teacher felt that Group B had a better understanding and wrote, "The steps are higher." Another teacher selected group C and stated, "It's the simplest answer," and yet another explained, "Because they understand how the slope will not change if you double both numbers."

Of the participants who chose Group D as having the best understanding and provided an explanation, most fell into one of two distinct categories. Their statements of explanation either pointed out that the students used terminology related to slope or the students actually calculated the slope. The following are examples from some of the participants who explained their choice with terminology. One teacher wrote, "Slope is discussed as rise/run, which they thoroughly explained," while a second teacher explained, "I feel all students understand well, but group D uses the terminology in context and more fully explains problems." One participant stated "Group C understood the concept of change in slope but Group D understood exactly what a slope and how to determine how to get the slope to compare the steepness." A second participant wrote "Group 1 & 3 showed an understanding, but group 4 actually related the problem back to slope and used slope in their explanation."

The other category of participants used the idea that the students actually calculated the value for the slope and thus had a better understanding of slope than the other groups. The following are examples of these types of explanations. One teacher explained, "They calculated slope instead of measuring angles." A second teacher goes even further than just stating the students know how to compute slope when she wrote, "Group D has a better understanding because they know how to find slope, and they understand what slope represents. They also can recognize equivalent fractions. Group 1 understands that grade and slope go together, but they did not actually find it." Another teacher offered, "They have shown they understand the concept of the slope graphically (drawing the lines) and algebraically (calculating the slopes and showing they are equivalent)."

There were other explanations that did not fall into these two categories. For example, one participant wrote "Their reasoning best meats (sic) my expectations of understanding slope." A second participant stated, "The easiest way to measure steepness is slope," while another teacher wrote, "Understands slope as rise/run. Same slope means same steepness."

Two of the participants opted to select more than one group as having the best understanding of slope. One teacher selected both groups A and D and stated, "The 2 groups I circled both show understanding of what it means to have steep stairs" as his explanation. A second teacher selected groups A, C, and D and explained, "All three have an equal understanding. Each deals w/ a different concept of slope & its app."

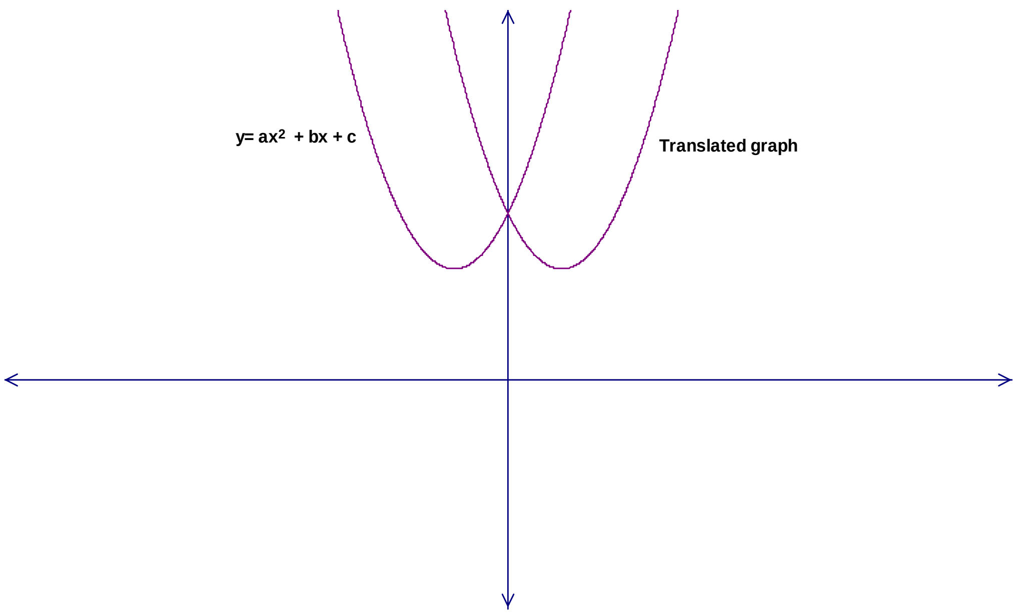
Conclusion

Participants were just as likely to select the incorrect group (Group B) as they were to select the group that found the numerical slope (Group D). A little more than half of them selected the group using the trigonometric representation as understanding. Even fewer of the participants realized that doubling the rise and run (Group C) was a valid understanding of slope More participants (n = 54) selected group D as understanding slope than either group A (n = 40) and group C (n = 38). Furthermore, their overwhelming selection of group D as having the best understanding suggests that participants felt students had to find the numerical value of slope and/or use terminology such as slope or rise/run in order to have an understanding of slope. Only one participant felt that all three groups had an equal understanding of slope.

Question Nine - Family of Functions

Families of functions were the focus of the selected content areas of the ALCKIN for question nine. To further understand what the participants know about a family of functions, this question was represented by a parabola and its translation in the coordinate plane. Figure 16 contains question nine from the Algebra Content Knowledge Instrument.

9. Mr. Seng's algebra class is studying the graph of $y = ax^2 + bx + c$ and how changing the parameters a, b, and c will cause different translations of the original graph.



How do you think Mr. Seng will explain the translation of the original graph $y = ax^2 + bx + c$ to the translated graph?

A. Only the **a** value changed

B. Only the **c** value changed

C. Only the **b** value changed

D. At least two of the parameters changed.

E. You cannot generate the translated graph by changing any of the parameters.

Explain your answer choice:

Figure 16. Question nine from the Algebra Content Knowledge Instrument.

Analysis of Results of Question Nine

After selecting an answer, participants were asked to explain why they selected their particular answer. Answer choice "D" which states that at least two parameters changed is the more general answer for generating any translation of a graph. However, changing both the b and c parameters would result in graphs which do not share a common y-intercept. The problem did not directly specify that the two graphs shared the same y-intercept, so participants would have to infer this from looking at the graph to select "C" as the correct response. Thus, answer choice "D" would be accepted as the general response, which might reflect some knowledge of how changing both the parameters translates graphs on the coordinate plane. On the other hand, if participants inferred from the graphs that they shared the same y-intercept, then answer choice "C" might also be considered correct. Table 17 showed how the distribution of responses to the question.

Table 17

Question Nine Responses to Families of Functions

Answer Choice	Number of Participants
Only the **a** value changed	6
Only the **b** value changed	**15
Only the **c** value changed	8
At least two of the parameters changed	*17
You cannot generate the translated graph by changing any of the parameters	6
No response	13

* Correct response – General Case.
**Correct response – Special Case.

This question appeared to cause the participants a lot of difficulty in both their answer selection, as well as in the explanations provided for those answers. Note that 13 participants did not even attempt to answer the problem, and many more did not explain their answers. No matter which answer choice was selected by the participants, there was not a lot of commonality in the explanations provided. Some explanations contained statements about what changes the parameters would cause in the original equation, in attempting to explain what parameters must have changed. One teacher noted "c ↑ ↓ and a makes it skinnier & fatter" while another teacher acknowledged "a = changes the width and direction of opening, c = shifts the parabola up & down …" A somewhat more complete explanation follows: "The steepness of the two graphs did not change. The vertex is all that changed & the b-value is what switched the vertex from one side of the y-axis to the other." Another participant suggested, "The axis of symmetry x = -b/2a is what changed. b was originally positive and changed to a negative. a & c stayed the same."

Still others used the analysis of the general form for the equation for a parabola. One teacher had a question mark and "maybe" written by her answer choice with the following statements as her reasons:

> $$y = a(x^2 + \frac{b}{a}x) + c$$
> $$y = a(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}) + c - \frac{b^2}{4a^2}$$
> $$y = a(x + \underbrace{\frac{b}{2a}}_{a})^2 + \frac{4a^2c - b^2}{4a^2}$$
>
> negative shifts it to the right. You would have to change b/2a to shift the graph left or right, but I can't decide if you can do that without shifting the graph up or down simultaneously! I would need more time to play with some numbers.

There were errors in the thinking of some of the participants. For example, two participants, who stated that the b value changed, stated "…the b value moves the graph left and right…" Two other teachers, who indicated that only the c value changed, used the following as their explanation: "Changing the c value shifts the graph left or right." Another teacher chose to simply reiterate the answer he selected and stated, "You must change two parameters in order to translate the particular graph." One participant wrote "Changing the coefficient of the x^2 term changes the width of the curve. These are not from the same family of graphs." Additional errors came from participants who said the graph could not be translated by changing any of the parameters. For example, a second participant felt "Δ horz by adding or sub from quadratic as a whole not changing quadratic by coefficient value" and another participant wrote, "Translation is a shift of axis here, not a change in graph's parameters."

It was difficult to determine what the participants were thinking in some of the explanations. For example, one teacher declared "Axis of symmetry is –b/2a and a is positive → neg/pos = neg and a is negative → neg/neg = positive." Another teacher stated, "The vertex changed, involving 2 points (h, k)" and one teacher stated "a" affects the orientation and steepness of curve along with "Another format for quadratic: $(x – h)^2 + k$, vertex at (h, k); $x^2 – 2hx + h^2 + k$; -2k = b; $(h^2 + k) = c$; in this format, h alone has changed, from + to -."

Conclusion

The participants had a difficult time with this particular problem, suggesting that they do not have a full understanding of how families of functions work and the effects that are caused by changes in parameters. Participants were more likely to list any and all information they thought they knew about the problem, and very few of the participants were able to provide an adequate explanation for which parameter or parameters caused the change in the graph.

Question 10 - Algebraic Representations within an Equation

This particular question on the ALCKIN comes from the content area of using algebraic structures in relationship to expressions, equations, and inequalities. The item first gave background information in the format of a word problem related the amount of time it took to bake chocolate chip cookies and plain cookies, then asked them to interpret one part of an equation representing that relationship. Figure 17 contains question ten from the Algebra Content Knowledge Instrument.

10. Mrs. Westbrook's algebra students were working on writing algebraic equations for problems similar to the following:

It takes 0.2 hour to bake a dozen chocolate chip cookies and 0.15 hour to bake a dozen plain cookies, how many dozen cookies can be baked in fifteen hours?

Mrs. Westbrook's students came up with the following equation:

$0.2x + 0.15y = 15$

where x = number of dozens of chocolate chip cookies and y = number of dozens of plain cookies.

What does the 0.2x represent in the equation?

Figure 17. Question ten from the Algebra Content Knowledge Instrument.

Analysis of Results of Question Ten

Participants were asked to give an explanation as to what the 0.2x represented in this particular equation. A correct response should have indicated that 0.2x represented the time to bake the chocolate chip cookies and/or the part of the fifteen hours need to bake the chocolate chip cookies. The majority of the responses to this question fell into the categories listed in Table 18.

Table 18

Question Ten Responses to Algebraic Representations within an Equation

Response Category	Number of Participants
Translated the expression into Words	11
Analyzed the Expression Correctly	*31
Total Number of Chocolate Chip Cookies	11
Did Not Answer	3
Other	9

*Answered Correctly

Less than half of the participants were able to analyze what the expression represented in the equation. The following are examples of analyses given by some of these participants. The majority were very much like one teacher who wrote, "The total amount of time it would take to bake x dozen chocolate chip cookies." A second teacher explained with "The amount of time of the 15 hours used to bake the chocolate chip cookies depending on the # of dozens of chocolate chip cookies." Another teacher simply stated, "Time used in baking chocolate chip cookies!"

Errors were also found in the explanations of this expression. For example, one participant wrote "# of hours needed to bake a dozen choc. chip cookies," and a second participant stated "The time it takes to make x number of chocolate chip cookies." Another participant used "Rate = time in hours per one dozen choc. chip cookies." One teacher explained “The number of hours that it takes to bake cookies” not specifying chocolate chip cookies.

Examples of how the participants tried translating the expression into words are as follows. One wrote “.2 hours times the number of chocolate chip cookies.” Another explained with “The number of minutes x the number of dozens of choc chip cookies.” Other examples were written similar to one teacher who stated “0.2 hours per 1 dozen chocolate chip cookies” and still other explanations were similar to a second teacher who wrote “0.2x represents the fact that it takes 2/10 hr. to bake one dozen chocolate ch.”

Eleven of the participants elected to explain the representation by saying it either represented the number of chocolate chip cookies or number of dozens of chocolate chip cookies that were baked. An example of these types of explanations came from one teacher who wrote “The total number of dozens of chocolate chip cookies that can be baked in 15 hours.”

Still other participants used explanations that were difficult to categorize. For example, one teacher wrote “2/10 of the total number of cookies” and a second teacher stated “.2 hour and x is how many dozen chocolate chip cookies.” Another teacher used “The percentage of chocolate chip cookies in 15 hours.”

Conclusion

Less than half of the participants gave a plausible explanation for the algebraic expression. This suggests that participants had a difficult time in determining a correct explanation for an algebraic expression that is part of an algebraic equation. It appears that teachers do not have a full understanding of how the expressions contribute to the overall equation used to solve problems.

Teacher Attitude Survey

There were five questions on the TAS that directly pertained to classroom practice and how each teacher felt students best learn mathematics in their classroom. The following five statements were considered:

2. It is important for students to figure out how to solve mathematics problems for themselves.
6. In a mathematics class, each student's solution process should be accepted and valued.
7. Students learn mathematics best from their teacher's demonstrations and explanations.
16. Teachers should model and demonstrate mathematical procedures and then, ideally, time should be allowed for the students to have the opportunity to practice those procedures.
17. Rather than demonstrating how to solve a problem, a teacher should allow students to figure out and explain their own ways of solving mathematics problems, including word problems.

Teachers were asked to respond to the above five statements which appeared on the Teacher Attitude Survey (TAS) with "strongly agree", "agree", "neutral", "disagree", and "strongly disagree". Table 19 summarizes how the 54 participants responded to each part.

Table 19

Teacher Attitude Survey Summary

Statement	Number of Participants					
	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree	No Answer
Statement 2	20	26	6	0	1	1
Statement 6	14	22	12	6	0	0
Statement 7	3	16	18	17	0	0
Statement 16	14	33	5	2	0	0
Statement 17	6	21	16	9	1	1

The participants were generally in agreement with the statement that students should figure out how to solve mathematics problems for themselves. About two-thirds agreed that each student's solution process should be accepted and valued, while only about 35% thought that students learn best by teacher's demonstrations and explanations. Note that this is in direct contrast to the results for statement 16, where almost 90% thought teachers should model and demonstrate mathematical procedures and allow time for students to practice these procedures. Half of the participants agreed that students should figure out and explain their own ways of solving mathematics, which is in contrast to their agreement with statement 2.

Conclusion

Had the intention of this instrument only been to see if the participants had the necessary content knowledge to answer questions, there is little doubt that the participants would have done relatively well on a majority of the questions. However, when using a format that required them to look at mathematics in the context of a variety of lenses such as student work and solving problems using technology, a different picture emerged. The purpose of the ALCKIN was to gain understanding of the content knowledge and pedagogical content knowledge of the participants.

Content Knowledge

The analysis of their content knowledge includes both the procedural knowledge and conceptual knowledge along with the processes of: reasoning and proof, communication, connections, problem solving, use of technology (NCTM, 2000), and real world applications (CBMS, 2000).

In only seven of the twenty-five tasks where teachers were asked select an answer were more than 80% of the participants able to select the correct answer. The content knowledge exhibited tended to be procedural in nature, since the majority of the reasoning given, when any at all was given, was algebraic manipulations. For example, participants used or suggested the procedures of “vertical line test”, “factoring binomials”, “synthetic division”, and “quadratic formula.” On every item of the ALCKIN instances can be noted where teachers made errors in mathematical computations or in the reasoning they provided for their answer choices or where they exhibited a significant lack of understanding of the mathematical topic in question. Even though participants could generally write algebraic expressions, many had difficulty in expressing what algebraic expressions meant in equations.

The majority of the participants exhibited a limited ability to view mathematics in multiple ways or in ways that demonstrated the conceptual basis for the problem. Multiple methods of solving problems suggested involved different procedures related to the same topic such as factorization or long division. A few conceptual examples were used, such as drawing pictures. While some participants did suggest the use of manipulatives, a conclusion cannot be drawn about whether this would be specifically to develop mathematics conceptually or used as another procedural method.

Pedagogical Content Knowledge

There was very little evidence of pedagogical content knowledge in on the responses to the items of the ALCKIN. The value teachers placed on doing mathematics procedurally was very evident when the majority of participants selected the group that found the numerical value for slope, and only one of the participants recognized that three of the four groups had equal understanding of their representations of slope. The majority of the participants seemed to think that a student had to find the value of slope to really understand it.

Participants did not respond to the errors in student reasoning given on the ALCKIN. Since they did not respond to these errors, alternative explanations or models with not offered by the participants.

In conclusion, participants in this part of the study had a procedural knowledge of mathematics but displayed little conceptual knowledge of the same mathematical topics. Participants made mathematical errors, gave incorrect reasoning, and exhibited a lack of understanding in some of the content areas. The pedagogical content knowledge of the participants was very limited in these participants. There were few alternative representations or methods used, except those that were procedural in nature. Connections were not made between mathematical topics. Student errors were generally ignored and not responded to. They emphasized the use of procedural knowledge and suggested how students should have worked problems using set procedures.

In the next chapter, results from four case study teachers will be presented in an effort to gain a better understanding of content knowledge and pedagogical content knowledge. Examination of the instructional practices of the case study teachers also provides further insight into how both of these types of knowledge are used in mathematics instruction.

V. CASES

This chapter contains case studies of four secondary mathematics teachers who taught Algebra I at the high school level. Multiple data sources were used to develop a deep understanding of their content knowledge and pedagogical content knowledge. In addition, the cases help us understand how content knowledge and pedagogical content knowledge of the cases are reflected in instructional practices. Furthermore, the cases will further provide information about the changes in their content knowledge and pedagogical content knowledge resulting from professional development. In addition, the cases will further provide information on how changes in bother of these types of knowledge are reflected in instructional practices. Data collection sources were surveys, classroom observations, interviews, and quarterly meeting observations.

The structure of each case consists of three main sections of analysis: pre-professional development, professional development, and post-professional development. First, each subject's content knowledge and pedagogical content knowledge was analyzed prior to professional development, referred to as pre-professional development. The supporting data for this analysis was collected in the spring of 2004 and included surveys, written instruments, classroom observations, and interviews.

The second analysis, referred to as professional development, involved the teachers' participation in professional development, which included attention to increasing content knowledge and pedagogical content knowledge. Data for this analysis was collected during the summer of 2004, fall of 2004, and the spring of 2005 and included both teacher and presenter interviews, analysis of professional development presenter documents, and quarterly meeting observations.

The third section of analysis, referred to as post-professional development, focused on changes in both content knowledge and pedagogical content knowledge of the case teachers after professional development, as well as how changes in these types of knowledge are seen in instructional practice. Data for this section of analysis was collected during the spring of 2005 and included both classroom observations and interviews.

The following sections begin by describing the professional development and the coding of the documents related to the case study teachers. Each case will be described organized around the timeframes of pre-professional development, professional development, and post-professional development. Finally, conclusions will be drawn on each case.

Description of Professional Development

Professional development at the MDMSIP summer institute was provided for Algebra I, Geometry, and Algebra II, addressing content related to specific areas. In addition, there were general sessions that all secondary teachers attended during the summer professional development that did not directly relate to content knowledge of these content areas. These sessions dealt with pedagogical issues, which included assessment, working with diverse students, cooperative learning, incorporating technology into classroom instruction, questioning, teaching via problem solving, and developing mathematical understanding. Presenters for the Multi-District Mathematics Systemic Improvement Project [MDMISP] summer professional developmental training modeled the types of instructional strategies they were encouraging the teachers to implement into their own classrooms.

Participants were expected to serve in the roles of both students and teachers while actively participating in the activities, as well as reflecting on how each of these pedagogical issues might appear within their own classroom. Issues related to how they might facilitate changes to improve the learning of mathematics for their own students were also addressed. Presenters for the MDMSIP summer institute also worked with participating teachers on transforming procedural lessons into investigative lessons, as well as facilitating mathematical lessons from Interactive Mathematics Program (IMP) units (Fendel et al., 2000).

During quarterly meetings, case study teachers met to topics that would be covered in the next grading period. Opportunities were also provided further develop content knowledge and pedagogical content knowledge.

Coding

Documents related to the case studies were entered into Atlas.ti (Muhr, 1991). The data from the case study participants was already included from the first part of this study. Additional documents included field notes from classroom observations, and transcriptions of interviews of teachers and professional development presenters. The codes developed in the survey study were used as a beginning point for coding the case study documents. New codes were added to this list of codes as the text from the documents was analyzed. The frequencies for some of the codes became so large it became obvious that additional codes needed to be added to further clarify the existing code. For example, Teacher Question was one of the initial codes. Different types of teacher questions began to evolve as additional documents were analyzed, and the code Teacher Question was broken down into the types of questions that a teacher asked such as "Teacher questions involves asking students to name the next step or procedure" or Teacher Questions 'why' or 'how'".

Additional codes were also added to capture events that may normally be observed in classroom observations but may not be a part of written instruments. For example, student conversation was a part of classroom observations and codes such as

“Student Gives Correct Answer” or “Student Indicates They Don’t Understand” had to be included.

Once all of the documents had been coded, each document was revisited to ensure consistency of coding throughout all of the documents. Table 20 contains a list of the codes that evolved as well as the frequency of each code. Appendix H provides more detailed explanations of how each code was used.

Table 20

Codes for Case Studies

Code	Frequency
Checks Work	18
Conceptual Knowledge	24
Definition	52
Denotes Using Inverse Operations	2
Formula Manipulation	7
Gives Praise to Students	30

Multiple Reasons	35
Problem Instructions	2
Provides Example	19
Step by Step Procedures	225
Student Agrees with Teacher	137
Student Answers “Why” or “How”	41
Student Gives Correct Answer	177
Student Gives Correct Procedure	86
Student Gives Incorrect Answer	31
Student Gives Incorrect Procedure or Next Step	15
Student Indicates They Don’t Understand	17
Student Practice, Practice Problems or Quiz	12

(table continues)

Table 20 (continued)

Code	Frequency
Student Question	96
Student Recognition of an Error	16
Student Recognizes Teacher Error	3
Student Short Answers to Arithmetic or Algebraic Question	236
Teacher Answers Own Question	76
Teacher Answers Student Question	25
Teacher Can’t Recognize Student Error	3

Teacher Asks Clarifying Question	174
Teacher Comment or Question Indicates a Task is Easy	23
Teacher Gives Hint	117
Teacher Gives Non Mathematical Instructions	121
Teacher Ignores Incorrect Student Procedure	18
Teacher Ignores Student's Question	6
Teacher Gives Incorrect Information	6
Teacher Question Involves Asking for an Answer to an Arithmetic Problem or Simplifying an Algebraic Expression	134
Teacher Question Involves Asking if Student has Other Questions or Understands	168
Teacher Questions "How" or "Why"	133

(table continues)

Table 20 (continued)

Code	Frequency
Teacher Questions Involves Asking Students to Name the Next Step or Procedure	220
Teacher Recognizes Correct Student Procedure	35
Teacher Recognizes Incorrect Student Answer	30
Teacher Recognizes Incorrect Student Procedure	37
Teacher Reiterates Students' Reply	210
Teacher Reminds Students of "Steps" or "Rules"	90

Uses Manipulatives	20
Uses Mnemonics	17

Each case will be considered in the following sections. Analysis of each case study teacher will be considered in the three phases of analysis: pre-professional development, professional development, and post-professional development. Each phase of analysis will consider the content knowledge and pedagogical content knowledge of the cases as well as looking for changes in these types of knowledge resulting from their involvement in professional development. The cases will be summarized in an effort to understand how their content knowledge and pedagogical content knowledge are used in instructional practices both before and after professional development.

Mrs. Cotney

Mrs. Cotney is a White female and when this research study began she was completing her twenty-eighth year as a classroom teacher. Mrs. Cotney was short in stature and soft spoken. Her educational background included a Bachelor of Science degree in mathematics education and a Master's degree in language arts education. She taught at Abbott High School, a ninth through twelfth grade school, where she was primarily responsible for teaching Algebra I and Algebra IA. Abbott High School had 95% of students passing the Alabama High School Graduation Exam (ASHGE), 32% of its students were Black, and 24% of the students were on the free or reduced lunch program.

Pre-Professional Development

In this section, we will consider Mrs. Cotney's classroom environment, how she viewed her students' ability to learn algebra and responses to statements contained on the Teacher Attitude Survey. Next, Mrs. Cotney's content knowledge and pedagogical content knowledge will be addressed from the ALCKIN and interview. Finally, a further look at her content knowledge and pedagogical content knowledge will be drawn from classroom observations, which includes how these types of knowledge were reflected in her instructional practices.

Classroom Environment

Mrs. Cotney had a smile on her face for the students that she was teaching and gave encouragement to all of her students. For example, comments such as "Good for you" or "You're trying so hard" and "You had it (the answer), you were saying it, take some credit" were often heard as she observed pairs or small groups of students working on their practice problems. Mrs. Cotney frequently moved among her students to observe how her students were working problems, offered encouragement, and answered their questions.

Her classroom consisted of student desks in straight rows facing the front of the room where the overhead projector, pull-down screen, and whiteboard were. This particular classroom setting suggested that Mrs. Cotney used a traditional approach to teaching mathematics. However, students were observed forming pairs or small groups when time was allowed for them to practice the mathematical instruction Mrs. Cotney had provided. The daily homework assignments for each of her classes were written on a portion of the whiteboard. Looking from the back of the classroom, the right wall was covered with bookcases containing extra textbooks. Mrs. Cotney's teacher desk sat at the back of the classroom along with an additional table with chairs.

Mrs. Cotney had four function calculators available for students to use in the classroom, which could be found in a basket at the back of the room. The left-hand wall had an additional whiteboard. Spaces above the whiteboard were covered with mathematical posters and what Mrs. Cotney called "helping devices" for her students. One such poster contained a table of counting numbers along with their squares. Still another poster consisted of the mnemonic, "Please Excuse My Dear Aunt Sally," meant to help students remember the order of operations of parentheses, exponents, multiplication, division, addition and subtraction.

Student Ability

Mrs. Cotney had different thoughts about the abilities her students possess to learn mathematics based on whether they were in Algebra I or Algebra IA. For those students who are in the Algebra I classes, she felt that a large majority of them could learn algebra well because they have:

> Enough math background that I can build on. Those who don't when they are sitting in a class with kids who seem to understand push themselves a little harder to get it, and the other ones will come in before school for help because they don't want to look bad with the other ones.

In contrast, Mrs. Cotney felt that only half of her students in her Algebra IA and Algebra IB classes could learn algebra well. For the half from this group that could learn algebra, she commented,

> They are the ones that have gotten through, you know they've learned enough. I don't know that any of them this past semester could have passed regular Algebra I. They couldn't have done it in eighteen weeks what we did in eighteen weeks. Ah, but going slowly, giving the repetition, we could build on what they did have.

On the other hand, she explained why she felt the others could not learn algebra well as follows:

> The fifty percent that can't when I looked back at their school history most of them have not passed a math class since maybe second grade. So they are really, really, really lacking in skills. They are so weak and they have no concept of number. They don't understand. I mean you can take half of twelve and they are perplexed. They don't understand number period. They are just the sad ones.

Thus, Mrs. Cotney's comments suggest she felt if student have been successful with mathematics in their past courses, they can learn algebra. In addition, students need to have this background in order for her to build on it.

Teacher Attitude Survey

Mrs. Cotney was questioned about the two conflicting statements from the TAS related to related to instructional practices. When asked, "It is important for students to figure out how to solve mathematics problems for themselves?" she replied:

> Oh, I agree that is one of my, I agree absolutely. (Laughs). It's hard. When we are learning something new, systems of equations. And I have given them some tools you know I have shown them ways to solve different kinds, maybe not all the different methods. I'll try to give them some that don't look like we've been doing and ask them in groups sometimes because some of them are insecure, you

> know. Ah, ask them to figure out how to solve this. You know, use what you know about how systems work together, use what you know about equations, numbers in general, and come up with a strategy.

Mrs. Cotney was also asked to reply to the second statement, “Teachers should model and demonstrate mathematical procedures and then ideally time should be allowed for the students to have the opportunity to practice those procedures.” In addition, she was asked how she thought they could coexist in mathematics instruction. She replied:

> Oh they can. The second, the second part you read, I think is important when you are introducing new skills. You know they need to see it modeled. And then let them go with it after they have an understanding of the process.

Thus, Mrs. Cotney’s agreement with the statements suggest she felt teachers should model and demonstrate for students and that by allowing time for student to practice those procedures she was allowing them to develop an understanding of the mathematics.

Content Knowledge

Mrs. Cotney’s content knowledge will be viewed following the definition of content knowledge in the literature review, considering in turn procedural knowledge, conceptual knowledge, and mathematical processes.

Procedural knowledge. Mrs. Cotney exhibited some procedural knowledge on the ALCKIN. She correctly answered 14 of the 25 mathematical tasks. On question five on the ALCKIN, she was unable to provide any approximate solution to the quadratic problem involving the spreadsheet. However, she exhibited strong procedural content knowledge during her interview. She was able to correctly solve the wheels and vehicles problem by using the set procedure of solving a system of linear equations by elimination. The problem follows:

> Jameel looked out in the parking lot and decided to count cars and motorcycles by the number of their wheels, excluding spares. He saw that there were 17 vehicles with a total of 56 wheels. How many motorcycles did he see?

She provided the written work in Figure18 and commented, "Because that is the kind of thing I would give them when we are working with a system of equations. I think we do one with chickens and cows."

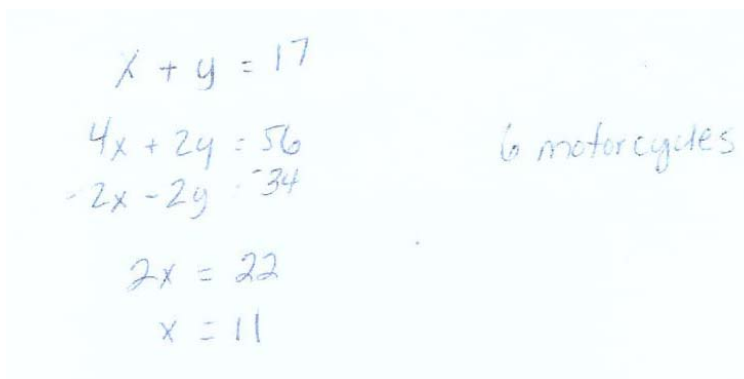


Figure18. Mrs. Cotney's work for the wheels and vehicles problem.

Conceptual knowledge. Conceptual knowledge is the ability to know why and how algorithms work. Mrs. Cotney exhibited weak conceptual content knowledge during this same timeframe. She was able to give examples for showing the two algebraic expressions were equivalent from question six on the Algebra Content Knowledge

Instrument [ALCKIN] (see Appendix A), which may suggest she may have had some conceptual understanding of this topic. Mrs. Cotney wrote, “Manipulatives (algebra tiles, money, for instance), draw pictures.” Other than this one particular incident, there were no other examples where she demonstrated conceptual understanding of the mathematics being addressed.

Mathematical processes. When asked to solve the wheels and vehicles problem (see Appendix D) using a different method than the procedural method she had already used, Mrs. Cotney was unable to provide one, and stated, “I’m sure there are like I tell my kids fifty different ways to do it but right this minute, no I can’t.” Mrs. Cotney stated that she believed there are many different ways to solve mathematics problems and students should be allowed to solve problems using methods of their own choosing as long as they were mathematically sound. She stated that students did not have to work problems in her classroom using her particular method, “I’m just not big on you have to follow my twelve steps.”

Thus, Mrs. Cotney’s content knowledge consisted of strong procedural knowledge and limited conceptual knowledge. Observations of her use of mathematical processes were limited to problem solving in which a set procedure was used. There was no evidence of her mathematical reasoning, use of connections, or ability to use multiple representations or methods.

Pedagogical Content Knowledge

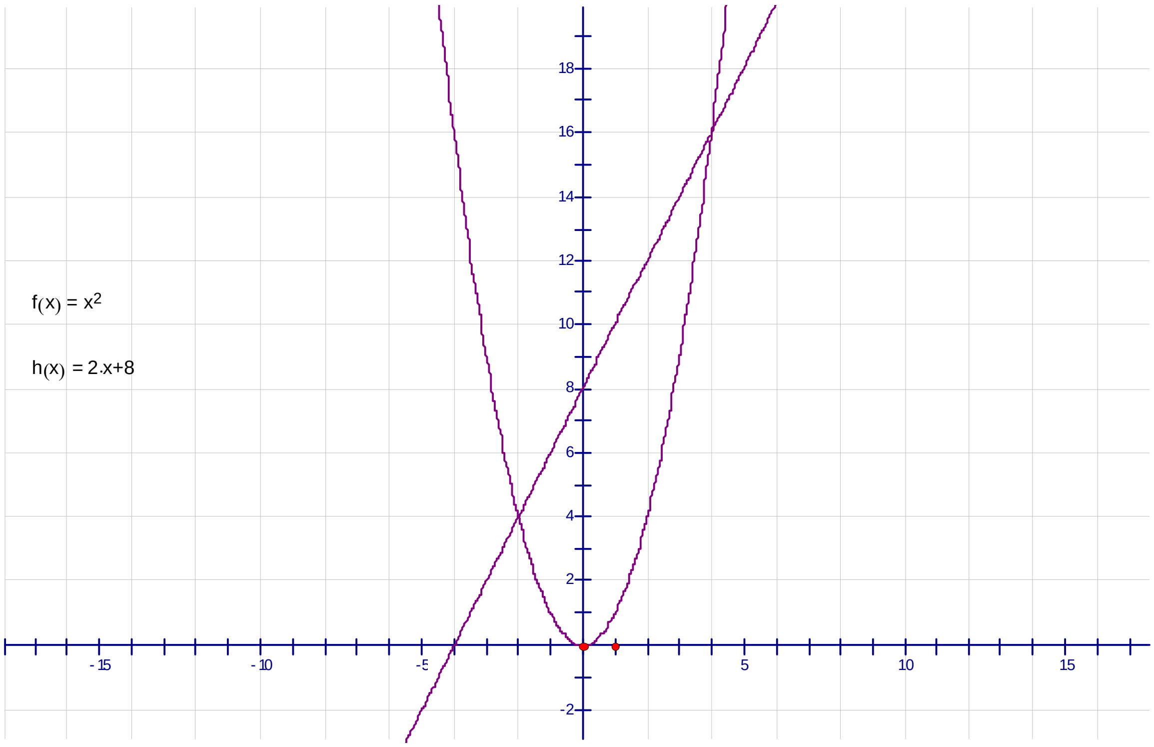
Pedagogical content knowledge will be examined through the lens of the definition provided in Chapter 2. Pedagogical content knowledge is the ability to “unwrap” and present mathematical topics so that students can be successful in learning mathematics. Pedagogical content knowledge includes the ability to access different representations as well as methods for solving mathematics problems. Pedagogical content knowledge is the ability to recognize student errors and be able to respond to them with alternative models and explanations. Pedagogical content knowledge includes the ability to responds to questions, and to pose questions and problems that are productive to students learning mathematics.

Mrs. Cotney had difficulty in recognizing errors made by students on the ALCKIN. For example, on question one from the ALCKIN (see Appendix A), she disagreed with the student in part B and agreed with the student in part C without providing any explanation for what she thought about what the students had stated. In addition from question seven on the ALCKIN (see Appendix A), Mrs. Cotney was unable to recognize that both students had presented viable methods for finding the number of tiles in the border of the patio. She stated, “James is close to understanding but missing the 4 corner pieces.” In general, she had a hard time generating alternative solution methods.

On question eight from the ALCKIN, Mrs. Cotney selected only two of the three correct groups, Group 1 and Group 4, as having a good understanding of slope. Furthermore, she felt Group 4 had the best understanding of slope because they were the only group that found the slope by using rise over run and stated, “They understand the relationship of rise to run.”

However, Mrs. Cotney did not have any difficulty in assessing the two elementary non-procedural approaches used by students to solve the wheels and vehicle problem in the interview (see Appendix D). She felt both were viable and would be methods she would accept in her own classroom. Mrs. Cotney showed hesitancy in accepting the correctness of a non-procedural method (see figure 19) used to solve the quadratic problem $x^2 = 2x + 8$.

To find the solution I graphed each side of the equation.



The two graphs intersected at two points (-2,4) and (4, 16). From these I can find the two solutions for the equation $x^2 = 2x + 8$. x would be equal to -2 and 4.

Figure 20. Student's non-procedural method for solving $x^2 = 2x + 8$.

Mrs. Cotney admitted that she would have to further investigate the graphing method to convince herself that the process would always work. Her thoughts on the graphing approach were:

> Graph each side of the equation. (Pauses) Oh, Okay. (Pauses) It's working out. We've never done one like this before. I can't figure out what (Pause) exactly she is doing, what she's doing. (Pauses) I'm going to try and work it in my head and

make sure it's working out some way. (Laughs) Yea, (Pauses) Looks like it

works. (Pauses) After you leave I'm going to pull some out and play with them.

She ended up by checking the answer to the problem by solving the problem by factorization and stated factorization would be an easier way to solve the problem. This indicated that Mrs. Cotney relied on procedures and place a value on procedures.

Thus, Mrs. Cotney was able to correctly evaluate different representations, as well as different methods of working problems presented by fictitious students. However, she did have difficulty in accessing errors made by students on the ALCKIN.

Further insights Mrs. Cotney's pedagogical content knowledge will be provided in the following section describing her classroom instruction, as along with insights into her ability to apply this knowledge in her classroom.

Classroom instruction

Mrs. Cotney's used limited pedagogical content knowledge in presenting mathematics, which was evident in her typical method of providing step-by-step instructions to her students on how to solve a problem. Mrs. Cotney was competent with the procedural knowledge of the topics she taught during classroom observations. She was not observed making any mathematical errors. The following is an example of a classroom dialogue when Mrs. Cotney's Algebra I class was simplifying radicals:

Mrs. Cotney: Look at this one. These are simple. Take the square root of five over eight. One of the ways you are going to see this you can write this, okay. We can't keep a radical in our denominator can we?

Student: No.

Mrs. Cotney: Isn't that one of our steps for simplifying radicals.

Student: What do we do?

Mrs. Cotney: Let me show you what to do. This is easy. This is very easy. What you do to get rid of the fraction, ah the radical in the denominator is multiply by 1. That doesn't change the problem does it?

Student: No.

Mrs. Cotney: But instead of writing one over one but we're going to write it (writes square root of eight over the square root of eight) Isn't this one, isn't this one by itself? So I'm not changing the problem.

Student: Why don't you use the five?

Mrs. Cotney: Because we don't care if we have a fraction, a radical in our numerator. Now think back to the very beginning before Jimmy started falling asleep. We can multiply a radical times a radical. What is the square root of five times the square root of eight?

Student: Square root of forty.

Mrs. Cotney: Very good. We're getting silly. Square root of eight times square root of eight is? Don't you tell me the square root of sixty-four. Think.

Student: Eight.

Mrs. Cotney: Eight. Remember when you square a square root you get the number. We're not through.

Student: If you have a perfect square in the radical do you still have the radical?

Mrs. Cotney: It's cool on the bottom, right? Remember this is what we want. What perfect square will divide into forty?

Student: Five.

Mrs. Cotney: Five's not a perfect square is it?

Student: Four.

Mrs. Cotney: Okay. Four. What is the square root of four?

Student: Two.

Mrs. Cotney: Think of radicals just like you do variables. If we had two x over eight we would simplify it, right? To one x over four, ignore the radical. Two into itself goes?

Students: One.

Mrs. Cotney: And two goes into eight?

Student: Four.

Mrs. Cotney: You got it? So that's the square root of ten over four. Simplified. Okay. Make sure you've got it. The main thing, the main thing you've got to remember is it's the denominator we're trying to get rid of, alright. Not the denominator but the radical in the denominator we're trying to get rid of so we multiply by that radical over itself so that gives us the one which won't change the problem but will let us work with it.

Student: What if the denominator is already a perfect square?

Mrs. Cotney: If it's already a perfect square, well at least you're done.

After providing the step-by-step instruction, Mrs. Cotney allowed time for students to work on a problem very similar to the one that she had just demonstrated. After working through some examples with the entire class, she assigned additional problems for students to work. Students were able to work with a partner or within a small group on problems which were provided from their textbook or as part of a worksheet to be completed during class.

Mrs. Cotney was not observed giving her students opportunities to develop an understanding of the mathematics they were taking. Even when opportunities arose in her classroom, she did not give her students the chance to fully explore the purpose of why they were doing particular procedures. For example, when solving two simultaneous equations with two unknowns she pointed out,

> Mrs. Cotney: What is it you are looking for when you are solving a system of equations?
>
> Student: The ordered pair.
>
> Mrs. Cotney: The ordered pair when the lines cross.

Mrs. Cotney had an average score of 32 on the RTOP instrument from classroom observations during the spring of 2004. This score suggests that she did not use reformed teaching practices as a strong point in her mathematics instruction. In general, classroom instruction reflected a very limited use of pedagogical content knowledge.

Student questioning. Mrs. Cotney used questioning that appeared to reinforce the procedural knowledge of her students. The majority of her questions involved asking students to give answers to mathematical problems and/or provide what the next step would be in a series of steps to simplify expressions or solve algebraic equations. The following was a teacher and student dialogue which involved learning how to simplify radicals.

> Mrs. Cotney: There you go. I would like for you to try one all on your own. Let's see what you could do. The square root of eighty. Think what will divide this

number? Mentally, you know, you don't need to list it. That's kind of a crutch but it might make you feel a little more comfortable. What is the biggest number on the left side that will divide into eighty? (As the student tells the Mrs. Cotney what to do, she works the problem on the overhead projector.)

Student: Sixteen. Five stays in. Sixteen squared is four. Four goes on the outside.

Mrs. Cotney: So you're saying sixteen times five and that gives you

Student: Eighty.

Mrs. Cotney: What is the square root of sixteen, don't you say eighty.

Student: Four.

Thus, Mrs. Cotney posed questions during her classroom instruction that required students to provide answers to mathematical problems, simplifying algebraic expressions, or solving algebraic problems. She was able to respond to her students' questions but the questions were similar to the same types of questions she asked in class. Mrs. Cotney's questions were not the types of questions or problems that would produce success in student learning.

Analyzing student errors. While Mrs. Cotney had difficulty in recognizing student errors on the ALCKIN, she could recognize and react to errors made by her students in executing mathematical procedures. In the following excerpt, students were solving for the lengths of unknown sides in a right triangle using the Pythagorean Theorem.

Mrs. Cotney: No calculators! Oh, my? The a is six, lets say centimeters and the b is eight centimeters and we want to find out what c is, it's coming back to you now.

Student: Um humph.

Mrs. Cotney: Pretty much?

Student: Um humph.

Mrs. Cotney: Yes.

Student: thirty-six plus sixty-four equals a hundred and then you

Another Student: c equals a hundred.

Mrs. Cotney: No, use common sense. If this thing is six centimeters and this is eight centimeters, do you really think this is gonna be a hundred? (Referring to the hypotenuse of a right triangle.)

Student: No.

Another Student: Do the radical thing.

Mrs. Cotney: That's c squared. We don't want c squared, we want c.

Thus, this shows that Mrs. Cotney was able to recognize and respond to the errors her students made in carrying out procedures, although she did try to get her students to think about the reasonableness of their answer. However, her response to student errors did not provide them with alternative explanations or other models that might help them better understand their errors.

Multiple methods of solving problems. In the interview, Mrs. Cotney stated that students did not have to follow her step-by-step instructions and she valued other methods used by her students in solving problems. In her classroom, she was able to assess and accept different ways of solving problems. However, she was often quick to point out which method was the easiest for students to use. For example, when solving systems of linear equations involving two variables using substitution she stated, "I promise if you hang in there with me we're going to find an easier way to do this, but we have to go through this." However, the majority of different methods observed from the work exhibited by students consisted of putting extra steps into procedures as opposed to omitting steps.

On one occasion when students were finding the area of irregular shaped polygons Mrs. Cotney told students a particular way to solve a problem but agreed that the way some of the students had worked the problem was also correct, as in the following:

> Mrs. Cotney: If we get it into two rectangles, we'll be in good shape. (Sketched in a dotted line to separate the irregular shaped polygon into two rectangles.) Number 8. First I would draw it on paper, I sure would. I'm at an advantage because I can mark on these (referring to the overhead sheet she was working on) and you can't mark in the book. Actually we've got one big area then we have to subtract this. That's how David is doing his right now and it's worked perfectly.
>
> Student: That's what I was doing.
>
> Mrs. Cotney: Does that mean that I'm wrong and you're right? Maybe? It just means there are different ways you can work the very same problem which is kind of good and kind of bad too. Sometimes it easier if you've got one. The way I was going to do it was just to find this part and this part. (Indicates the two small rectangles formed by her dotted line.) Find the area of this skinny rectangle and the area of this box and then add them together. Do you know what I am saying? The way you two guys did it was to get the area of the great big rectangle and then subtract this one. What did ya'll get?

Also while solving for unknown sides of a right triangle using the Pythagorean Theorem, she stated

> Okay where are you? You know what c is. You can set it up in an equation $a^2 + b^2 = c^2$ and solve it but it's easier to do $c^2 - a^2 = b^2$. You subtract the a squared from this c squared. Good.

Even though Mrs. Cotney indicated she valued methods students used in solving problems, she often pointed out the easiest method to use. The methods she termed as "different" tended to very similar ways to the procedures demonstrated for solving problems, where students simply used slightly different steps. For example, although students may have divided a polygon in slightly different ways to find its area, methods to arrive at the answer were the same.

Conclusion

Mrs. Cotney felt that the students who had a good mathematics background could learn algebra well, while the students that did not have a mathematics background could not learn algebra well. Mrs. Cotney exhibited strong procedural knowledge but limited conceptual knowledge of the algebra content. There was little evidence of use of mathematical processes. Her classroom instruction primarily consisted of teaching students step-by-step procedures for simplifying algebraic expressions or for solving algebraic equations. Explanations on the ALCKIN were procedural in nature and the only exception related to the problem where she suggested students use manipulatives in showing the two algebraic expressions were equivalent. She did not provide opportunities for her students to develop a conceptual understanding of the mathematics they were taught.

Mrs. Cotney was able to recognize errors that her students were making. However, it appeared she had difficulty in recognizing and assessing errors made in student statements on the ALCKIN, since she generally did not make any response to them, possibly these statements extended beyond merely following set procedures. Her questions to students generally called on their procedural knowledge. She recognized students in her class who worked problems in what she termed different ways, although these differences generally related to minor variations of the procedural methods other students were using. Her content knowledge was reflected in her instructional practices but her pedagogical content knowledge was limited. Although she used questioning, recognized and responded to student errors, and noted "different" methods used by students, they were rather superficial representations of pedagogical content knowledge.

Professional Development

Mrs. Cotney participated in the Algebra I section of the summer of 2004 professional development training of Cohort I. Mathematics content addressed within the Algebra I group involved working in the Interactive Mathematics Program (IMP) Baker's Choice (Fendel et al., 2000) and Solve It units (Fendel et al., 2000). Teachers were expected to be familiar with the concepts of how to graph an equation, and how to use graphing and at least one other method for solving a system of linear equations involving two variables. Participants were involved in investigating inequalities and developing the subsequent rules that apply to inequalities. From the Solve It unit (Fendel et al., 2000), teachers used different approaches to solving equations.

Mrs. Cotney appeared to be enthusiastic about the training she received through MDSMIP. She indicated that she had taught algebra for several years and was looking for different ways to teach: “I thought it was wonderful. I really did. We all came back just charged up. We were so excited. I don’t know what to say, it was just great.” One of the presenters for the summer institute, stated:

> Out of all the participants that I had, she was definitely one that stood out as having a very positive attitude about the training. A lot of people were there just because they had to be there and she did not have that attitude at all. She seemed to really enjoy the sessions and really wanted to know more about it. She asked a lot of questions and she was kind of a quiet person but she, you could just tell the way that she was acting that she was enthusiastic about the training. Definitely.

The presenter also expected that Mrs. Cotney would try the things she had learned from the institute in her classroom. On the other hand, another of Mrs. Cotney’s presenters was hesitant in thinking that Mrs. Cotney would make any changes in her instructional practices:

> I don’t know if I would go that far in how comfortable she was. Ah, I think she would try them. I think that she ah made a connection. I think she could see how it would work. I don’t know if she went as far as to, I don’t know if she would implement it in her classroom or not.

The presenters of the Algebra I sessions of the MDSMIP Cohort I summer professional development training agreed that Mrs. Cotney was knowledgeable about the Algebra I content. One presenter stated, “I don’t think she was lacking in content knowledge at all…I think she was strong.”

In addition to the MDSMIP summer development, Mrs. Cotney took Internet classes offered through Harvard University. She had taken a class on differentiating instruction and was presently taking a class on fostering algebraic thinking in high school classes. When Mrs. Cotney was questioned about the impact these courses may have had on her classroom instruction, she replied:

> I was telling Brittany Johnson (another teacher at her school). Monday I was sitting here doing my homework, cause it was due Tuesday and I was showing her what was on there and I said Dr. Allen (project director for MDSMIP) and MDSMIP could have just written this course because it is MDSMIP. I mean it is everything, everything we read, talked about, philosophy, it's just reiterated. I mean they are talking about Core Plus, CMP (Connected Mathematics Program, IMP and modeling, internal models and I'm going I didn't need to take this but it's reaffirming. (Note that pseudonyms were used.)

While participating in a small group discussion at a quarterly meeting, Mrs. Cotney indicated that she may have been experiencing some difficulties with implementing the different methods of representation. Mrs. Cotney was overhead telling her group:

> I've never used them (algebra tiles) until this year so I'm not real comfortable with them. I have two Algebra IB classes, one of them I don't enjoy. They cannot read it (IMP unit). Alice (IMP unit) does get into rules but…. I want them to know five and the square root of five are not the same.

Thus, the professional development presenters thought Mrs. Cotney was knowledgeable of the Algebra I content. She was an active participant in the professional development, but the presenters were split on whether she would implement any instructional changes. Growth in pedagogical content knowledge was noted in her conversation about algebra tiles since she had not been familiar with them prior to professional development.

Post-Professional Development

During post-professional development, the topics of changes in classroom environment, changes in student ability, changes in content knowledge, changes in pedagogical content knowledge, and how the changes of content knowledge and pedagogical content knowledge are reflected in instructional practices will be addressed. Each of these will be addressed in the following sections:

Classroom Environment

The layout of the classroom during post-professional development remained unchanged. Mrs. Cotney continued to encourage all of her students as she had in the pre-professional development timeframe.

Student Ability

In the follow-up interview, Mrs. Cotney indicated she felt a large percentage of her students could learn algebra well. However, she made the following comments about why some of them may not learn algebra well.

> Mrs. Cotney: I think ninety-five percent of them can learn algebra well. They're not. But they can.
>
> Interviewer: So why do you think they can even though they're not?
>
> Mrs. Cotney: OK. A few things. Major immaturity for ninth graders. I mean it's just this group, not just mine, the whole ninth grade, we're seeing it across the board. They have been spoon fed and I'm not pointing fingers, I spoon feed too. Not as much now but you know and if I can't show them every possible variation

> on a process they don't understand because they don't know how to think (laughs). I mean they don't know how to think it through and, not they don't know how, they haven't been required too much and so they don't want to and if you don't give them all the steps they go home and you're not teaching them. So they can learn, they really can. I don't think I have a slow one in the algebra one class but they are not being successful.

Thus, while Mrs. Cotney previously felt her students could not learn algebra well if they were not mathematically prepared, during the post-professional development phase, her views had changed somewhat. She now suggested that her students are immature and can't learn algebra well unless she "spoon feeds" them, meaning she needs to give them step-by-step instructions.

Changes in Content Knowledge

Mrs. Cotney was confident that she knew the content knowledge of Algebra I well and did not feel she learned any additional algebra content as a result of MDSMIP summer professional development training. However, she did feel that she had developed a better understanding of how the content fits together. No changes in her procedural knowledge or conceptual knowledge were observed during the interview and classroom observation.

During an interview in the spring of 2005, Mrs. Cotney was asked to solve the quadratic problem $x^2 = 2x + 8$, which she was able to do by setting the equation equal to zero and using factorization. Before continuing she checked her answer for correctness by using substitution. Previously during an interview in the spring of 2004 when asked to provide another method for solving an algebraic problem, she was unable to do so.

However, on this occasion she provided other procedural methods for solving the quadratic problem, such as graphing or setting the equation equal to zero and using the quadratic formula. Mrs. Cotney also suggested a non-procedural scheme of using the trial and error method where students could substitute in values for x until they found which number or numbers made the statement true.

Thus, Mrs. Cotney did exhibit some changes in the mathematical processes related to her content knowledge. Previously she had been unable to offer different ways of solving problems, but now she offered more than one procedural way, as well as a non-procedural way for solving the same problem. This may also reflect a deeper conceptual understanding of the mathematics.

Changes in Pedagogical Content Knowledge

No changes in her pedagogical content knowledge were evident in observations of her classroom. She used the same types of strategies when recognizing student errors or the types of questions asked of students after attending professional development. There was no additional evidence, such as displayed student work or projects that indicated students were involved in IMP units (Fendel et al., 2000).

She still pointed out student errors instead of taking advantage of these opportunities to challenge her students to look at the work of other students to decide if the answers, methods, and/or procedures used were correct. For example, during a review of homework problems, a student wrote the following work on the whiteboard corresponding to homework problem twenty-six:

$(5x - 6)(5x - 6)$.

26. $(5m - 6)(5m - 6)$

$$
\begin{array}{l}
5m(5m - 6) - 6(5m - 6) \\
25m^2 - 30m - 30m + 36 \\
\quad\quad +30m\ \ +30m \\
\hline
25m^2 \quad\quad\quad\quad\quad + 36
\end{array}
$$

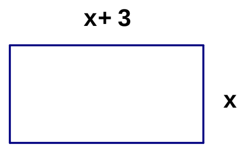
Instead of asking the students if they agreed with what the student had done, Mrs. Cotney erased the two 30m terms which had been added, and the following classroom discussion took place:

> Mrs. Cotney: Why don't we do this?
>
> Student: There's no equal sign.
>
> Mrs. Cotney: There's no equal sign. So you didn't swap them from one side of the equal sign to the other did you and you know not to change signs. So what should it be?
>
> (Student went back to the whiteboard and wrote $25m^2 - 60m + 36$.)

Mrs. Cotney's questions still focused on having students provide answers to mathematical problems or to provide the next step in a procedure. She did not always allow adequate wait time for students to answer questions. In some cases questions were rephrased to give students hints for the answer, and in other cases Mrs. Cotney merely answered her own question. For example, Mrs. Cotney and her students were working with the following diagram to represent the area of the rectangle. She said:

> So if I wanted to I could go in and calculate the area right now. And how would I do that? (No pause.) I would just say that times that right? (Indicates the x and x +

3.) We could calculate area right now. (Writes l = x + 3 and w = x on the overhead sheet.)



So we agree that our length right now is x plus three and the width is x?

Although Mrs. Cotney continued to state that her students could work problems in any manner they chose, I continued to frequently observe her telling students a particular way to proceed with problems, rather than allowing them to access different representations and different methods for solving problems. For instance in responding to a student answer, Mrs. Cotney stated, “You didn’t sketch it,” indicating the student should have made a geometric sketch of the problem in order to be able to solve it. In one particular discussion, Mrs. Cotney questioned the students about a different way of writing (2x-3)(2x-3). She asked:

Mrs. Cotney: How else could I write this? It’s a really tricky question; you’re going to have to think. What’s another way I could have written this problem other than doing it that way?

Student: (3-2x)(3-2x).

Mrs. Cotney: Without changing signs? Okay so let’s fill out the variable part. What’s another easy way I can write this problem?

Students: (Talking at once)

Mrs. Cotney: So if I multiply something by itself, I’m squaring it correct? (Wrote $(2x - 3)^2$ next to the problem.)

Thus, Mrs. Cotney's pedagogical content seemed largely unchanged from the period before professional development, as reflected both in the interview and classroom observations.

Reflection of Changes in Knowledge in Instructional Practices

Mrs. Cotney indicated that she was using different instructional practices and when she expressed an excitement about what was going on with her Algebra IB class. She said, "I'm having a good time. You know last year I cried over my Algebra IB class. These are my favorite classes. The students are getting so excited about learning." She was further asked if she felt her students were retaining the material better, and she answered, "Oh, yes! I'm having to reteach some of the Algebra IA material but they are remembering the new material better." However, changes in instructional practices, much less changes in content knowledge and pedagogical content knowledge, were not observed during the spring of 2005.

Mrs. Cotney was questioned concerning the types of changes she had made in her classroom practices as a result of professional development:

> I am doing more, oh gosh (laughs) I'm regressing this semester but until recently a lot more I call them open ended activities in class. I'll always do team work and team projects and things but I'm not giving step by step here's how you do it. I mean I'm having them work toward coming up with processes.

When asked to provide an example, Mrs. Cotney said:

> I'm going to pull in one of the IMP things. With my Algebra B students rather than talk about here's how you make your in out table, here's what you do with it, you know because some of that should be review for them, I presented you know

> each days unit or each days activities, they got with their teams and together they would come up with how's the best way to get this information. How can we and what they were doing was learning functions. I mean that's what they were learning but it wasn't until way, way down the road that we used the words functions, domains, and ranges but they understood what it meant when we got to them. You know so it more was indirect teaching as opposed to what I'm doing right now standing in front of the room, datta, datta, da.

Mrs. Cotney further commented on her difficulty in making changes in her instructional practices, contradicting the successes she reported she was having with her class, when she stated:

> You know it was really, really hard first semester to try to implement things and to get kids at this age to buy into a new way of doing things. I mean we were in tears half the time, not the kids but the teachers. (Laughs) Going (Sighs) It's not working, they're not doing anything and so you start letting it go and dropping back into your old ways. You don't need this, I do.

The RTOP score of 35 was recorded from the classroom observation of Mrs. Cotney during the spring of 2005. This reflects a slight increase from the RTOP score of 32 received during the pre-professional development timeframe.

Mrs. Cotney indicated she had used "in-out tables" during classroom instruction suggesting growth in pedagogical content knowledge had provided other ways to present mathematics instruction to her students. Thus, some changes in instructional practices resulted from growth of pedagogical content knowledge.

Conclusion

While Mrs. Cotney thought the summer professional development training was good, the presenters were split in their thoughts on whether Mrs. Cotney would implement any instructional changes into her classroom instruction. Mrs. Cotney did exhibit some changes in her content knowledge and pedagogical content knowledge, when she discussed using “in-out tables”, algebra tiles, and when she offered multiple procedural as well as non-procedural methods students could use in solving problems. However, the changes in her content knowledge and pedagogical content knowledge were not observed in her instructional practices. Mrs. Cotney reported that she had tried implementing the IMP units (Fendel et al., 2000) during the fall semester of 2004 but fell back, in the spring semester of 2005, into the same type of instructional practices she had used in the 2003-2004 academic school year. She indicated that the students and their parents felt she was not teaching her students mathematics if she did not show them all of the steps. RTOP scores from the spring of 2004 and the spring of 2005 were very similar.

Conclusion of Case

Mrs. Cotney felt her Algebra IA students could learn algebra if they had the mathematical background on which to build, but considered all of her Algebra I student capable of learning. Mrs. Cotney stated she demonstrated and explained mathematics to her students and allowed time for them to practice, in turn helping them to understand mathematics. Mrs. Cotney had strong content knowledge but somewhat limited pedagogical content knowledge. Growth in both her content knowledge and pedagogical content knowledge was observed as a result of professional development. Content knowledge and limited pedagogical content knowledge were observed during her instructional practices during the post-professional development timeframe, but they were generally unchanged from the pre-professional development timeframe. However, she did indicate that observations growth in these types of knowledge could have been observed in classroom instructional practices during the previous semester.

Mrs. Willoughby

Mrs. Willoughby is an older White female who was teaching high school mathematics as a second career. Mrs. Willoughby stands at an average height and talks with a normal level of voice. At the beginning of this study, Mrs. Willoughby was in her fifth year of teaching at Sandsfield High School. She holds a Bachelor of Science degree in mathematics education and was working on her Master's degree in mathematics education. At Sandsfield High School, a ninth through twelfth grade school, her primary teaching responsibilities were Algebra I, Calculus and informal geometry. Passing rate on the Alabama High School Graduation Exam at Sandsfield High School was 64%, the student population was 35% Black, and 41% of the students were on the free or reduced lunch program.

Pre-Professional Development

In this section, we will consider Willoughby's classroom environment, how she viewed her students' ability to learn algebra, and her responses to statements contained on the Teacher Attitude Survey. Next, Mrs. Willoughby's content knowledge and pedagogical content knowledge will be addressed from the ALCKIN and interview. Finally, a further look at her content knowledge and pedagogical content knowledge will be considered from classroom observations, which included how these types of knowledge were reflected in her instructional practices.

Classroom Environment

Her classroom appeared to be a student-friendly environment and her students seemed at ease, although she dominated the classroom conversation. Student participation was minimal and generally consisted of Mrs. Willoughby asking her students questions that could be answered quickly and with short answers.

The student desks in Mrs. Willoughby's classroom formed straight lines which faced the whiteboard at the front of her classroom. This type of classroom setting might suggest that Mrs. Willoughby used a traditional approach to teaching mathematics and nothing from classroom observations contradicted this assessment. The teacher's desk was at the front of the classroom. From the back of the classroom, the left wall was covered with windows while the back wall contained storage cabinets. The right hand wall held both a bulletin board and a whiteboard. During classroom visits work done by students such as projects from her geometry classes were displayed. Although calculators were not easily accessible to students in Mrs. Willoughby's algebra classes, they were able to easily borrow one from her when the need arose.

Student Ability

Mrs. Willoughby felt almost all of her algebra I students could learn algebra well. She said, "I teach advanced algebra. And so in the advanced algebra I would say ninety-seven, eight percent, learn it well. They're dedicated. They'll study. They'll retake test. They ask questions. Those are the kids that succeed." Thus, Mrs. Willoughby felt the majority of her students could learn algebra well because they would put in the necessary effort; they are the students that will generally succeed.

Teacher Attitude Survey

Mrs. Willoughby was asked to respond to the two conflicting statements from the TAS with which a majority of teachers had agreed. Her response to, "It is important for students to figure out how to solve mathematics problems for themselves" was, "First I demonstrate a concept and then I give them I call guided practice and I give them work to do and let them work at those problems." When asked to respond to "Teachers should model and demonstrate mathematical procedures and then, ideally, time should be allowed for students to have the opportunity to practice those procedures" and how this statement and the first statement could coexist in mathematics instruction, she replied, "Ah, now I don't know how well they will coexist because if one is letting them figure out how to let them work it themselves and another is modeling it, I model but I don't know how it would coexist unless some concepts you might have to teach. I'm not sure."

Thus, Mrs. Willoughby responded that she actually taught a concept and let her students practice it which is what the second statement implied. Since the second response contradicted what she had previously stated, it suggests that she does not think you can allow students solve mathematics problems for themselves and model for them in instructional practices.

Content Knowledge

Mrs. Willoughby's content knowledge will be viewed in terms of procedural knowledge, conceptual knowledge, and mathematical processes. These will be addressed in the following sections.

Procedural knowledge. Mrs. Willoughby correctly answered 15 of the 25 mathematical tasks on the ALCKIN. However, Mrs. Willoughby exhibited strong procedural knowledge of the algebra one content during interviews. When asked to work the wheels and vehicles task during an interview in the spring of 2004 (see Appendix D), she was able to take the information from the problem, put it in a table representing the number of wheels per vehicle, number of vehicles with each type of wheel, and the total number of wheels from each type of vehicle (see Figure 20).

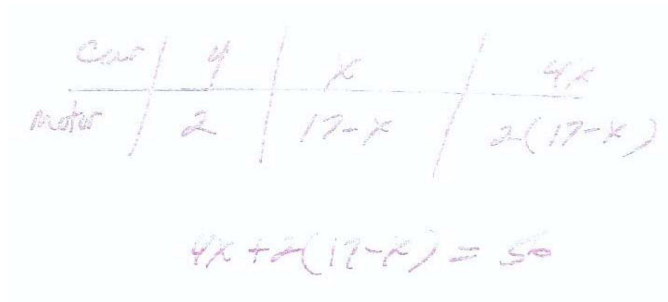


Figure 20. Mrs. Willoughby's setup to solve the wheels and vehicles problem.

She used only one variable to represent the number of cars, then represented the other quantities in the problem in terms of that variable. She set up an equation by adding the resulting expressions to total fifty-six, the number of wheels for the two types of vehicles. From classroom observations, Mrs. Willoughby possessed procedural knowledge of the mathematical topics she taught and no observations were made of Mrs. Willoughby making mathematical errors.

One significant error can be pointed out in Mrs. Willoughby's content knowledge. On question three of the ALCKIN (see appendix A), Mrs. Willoughby noted that the statement, "Any number to the zero power was equal to one" was true for all situations.

Conceptual knowledge. Mrs. Willoughby exhibited some gaps in conceptual knowledge in the data collected during the spring of 2004. On question six of the ALCKIN (see Appendix A), she could not offer any other method to show the two algebraic expressions were equivalent except the procedural methods indicated in the instructions. On question ten from the ALCKIN (see Appendix A), she explained the 0.2x in the equation as "the time (0.2 hour) times the number of dozens of chocolate chip cookies", indicating a verbal description of the expression, instead of what the expression represents in the equation. Thus, Mrs. Willoughby appeared to have some limits in her conceptual understanding of the mathemaics. Showing two algebraic expressions equivalent was limited to a set procedure, and she did not exhibit understanding of the algebraic expression in an equation.

Mathematical processes. Use of mathematical processes was not evident in the ALCKIN and interview. From question five on the ALCKIN (see Appendix A), Mrs. Willoughby was able to find a correct solution to the quadratic equation by using the spreadsheet and justified her answer with "at that point answers are closest." In question eight (see Appendix A), she was able to correctly identify only two of the three groups that correctly described slope. The group that she did not select as understanding slope was group one, which related slope to the tangent of the angle. Furthermore, she selected the group that found the value for slope as having the best understanding, explaining that "(group) D is actually calculating the actual slope." Thus, Mrs. Willoughby displayed limited use of the mathematical processes. Some different representations she could understand such as the spreadsheet, while she did not recognize all representations of slope.

Pedagogical Content Knowledge

Her pedagogical content knowledge will be examined by in the definition from the literature review. Pedagogical content knowledge is the ability to "unwrap" and present mathematical topics so that students can be successful in learning mathematics. Pedagogical content knowledge includes the ability to access different representations as well as methods for solving mathematics problems. Pedagogical content knowledge is the ability to recognize student errors and be able to respond to them with alternative models and explanations. Pedagogical content knowledge includes the ability to responds to questions, and to pose questions and problems that are productive to students learning mathematics.

Mrs. Willoughby's pedagogical content knowledge as exhibited on the ALCKIN, seemed to be based on knowledge that was primarily procedural in nature. She tended to ignore errors in reasoning made by students. During an interview Mrs. Willoughby was asked to respond to student work on a quadratic equation problem where the student had graphed both sides of the equation $x^2 = 2x + 8$ (see Appendix D). Her analysis follows:

> (Long pause) I would have never thought of that. I mean because I teach them what the graph actually looks like instead of what they are doing. Ah, does it work every time? I would have to check. (Laughs.) Pretty good. I would have never thought of that. But I see. Ah, Yea because that would be like a system of equations and where they, yea because where they cross would be, I think so.

Mrs. Willoughby believed students should be allowed to work problems using methods or procedural steps different from her own. She also suggested that students could use the guess and check method in finding the solution to the wheels and vehicles problem (see Appendix D). Mrs. Willoughby noted that Seth's method in question two on the ALCKIN (see Appendix A) would work for some algebraic fractions. She explained with "works only on difference of squares." Thus, Mrs. Willoughby did exhibit limited pedagogical content knowledge. She was able to access different forms and representations of problems by fictitious students both in the interview and on the ALCKIN. However, she did not respond to the errors made by students on the ALCKIN.

Further clarification of Mrs. Willoughby's pedagogical content knowledge will be focused in the following section describing her classroom instruction, as well as her ability to apply this knowledge in her classroom.

Classroom instruction

Mrs. Willoughby's content knowledge was largely reflected in mathematics instruction where students were given step-by-step instructions for working problems. For example when instructing her students on how to find the distance between ordered pairs such as (9, -2) and (3, -4), she told her students:

> Label these x sub one, y sub one, x sub two, y sub two. (Above nine she placed x sub one, above negative two she put y sub one and above three she placed x sub two and followed y sub two above the negative four.) So I've got three minus nine squared plus negative four minus negative two squared. When I put that minus negative, what do you when you're subtracting? Add the inverse. … Every time she sees subtraction, she automatically thinks add the inverse. So I've got the square root of negative six squared plus negative two squared. That's the square root of thirty-six plus four which is the square root of forty. You've got to factor

now, right? Forty factors into two times two times two times five. I've got a perfect square so I can factor out two square roots of ten.

After Mrs. Willoughby gave students the "steps" for simplifying expressions or for solving equations, she proceeded to work more examples using the same general pattern described above. Students worked the examples as she worked them at the whiteboard. After working a few examples she questioned the students as she worked the problems.

Instead of accessing different methods or representations, she stressed that students should know the steps in each procedure and was often heard reminding students of what rules they should follow or steps they should take next. For example when simplifying problems such as $(5\sqrt{2}+3\sqrt{5})(2\sqrt{10}-3)$ using the First, Outer, Inner and Last (FOIL) method she remarked, "You can't multiply a number under the radical because it is an irrational number not a…. Be careful never, never multiply inside and outside. Let's do it in order."

Step-by-step procedures were also stressed in the following conversation concerning the Pythagorean Theorem:

> Okay. Ah first thing, the first thing we did in this chapter was where we used the Pythagorean Theorem…Okay a is square root of five, b is six, and we're going to find c. Now remember on the Pythagorean Theorem what is c? (Students were trying to answer but Mrs. Willoughby never slowed up in her talking.) c is what? It's the hypotenuse. It's the one that is always by itself. Okay? So if I'm not wrong, features you need to keep in mind, unless I specify different, c is the hypotenuse. So the Pythagorean Theorem is a squared plus b squared is equal to c squared.

Mrs. Willoughby's classroom instruction did not offer students the opportunities to develop a conceptual understanding of the mathematical procedures they were learning. Mrs. Willoughby had an average RTOP score of 26.5 from the classroom observations done during this timeframe of the research study, suggesting her instructional practices were not aligned with reform mathematical instructional practices.

Thus, Mrs. Willoughby's content knowledge was reflected in instructional practices, but limited pedagogical content knowledge was evident. She emphasized students doing step-by-step procedures instead of accessing different representations and procedures that students could use.

Student questioning. Mrs. Willoughby did not ask questions or pose problems productive in helping students learn mathematics. Questions generally prompted students to remember procedures. For example, the following classroom dialogue took place when students were solving equations involving radicals.

> Mrs. Willoughby: What about the last one? It's the only one that is a little bit difficult. We've got the radical isolated so we've got square root of x plus two is equal to x minus four. What do we need to do?
>
> Student: Square both sides.
>
> Mrs. Willoughby: So I've got x plus two. Is that equal to x squared plus eight? x squared plus sixteen?
>
> Student: x squared minus
>
> Mrs. Willoughby: x squared minus what?
>
> Student: You have to foil it out. Eight x.
>
> Mrs. Willoughby: Eight x plus….Okay. Now what do we do?

Student: Subtract

Mrs. Willoughby: Subtract

Another S: Set it equal to zero.

Mrs. Willoughby: Okay. (Pause) So I can do that at the same time so zero is equal to x squared minus nine x plus fourteen. Okay now we're factoring. What are two factors of fourteen, what can we tell about the factor, positive, negative, one of each?

Mrs. Willoughby also often asked her students to provide answers to problems. For example when solving for a in the equation $\sqrt{a} = 5\sqrt{2}$, the following classroom dialogue took place.

Mrs. Willoughby: Okay. What do we do about a radical, what should we do? (Pause some mumbling of answers) How do we solve number one? What do we do? (No pause for students to respond.) We're going to square both sides because if we have two radicals but they are both by themselves, right? They are both isolated. One on one side, one on the other. So if I square this side I'm going to get?

Student: a

Mrs. Willoughby: a. Okay if I square this side what do I get? (Students can be heard trying to answer but Mrs. Willoughby continues.) What's five squared?

Student: Twenty-five

Mrs. Willoughby: Twenty-five times the square root of two squared.

Student: Two

Mrs. Willoughby: Okay…

As in previous examples of classroom dialogue, note that Mrs. Willoughby repeated statements made by her students.

Thus, these example show Mrs. Willoughby questioned her students largely to remember steps in procedures or to provide answers to problems. These types of questions are not likely to be productive in increasing students' conceptual understanding.

Analyzing student errors. Mrs. Willoughby was able to respond to her students who made mathematical errors or used incorrect procedures. However, she did not respond with alternative explanations or models. For example, when solving the equation $10-\sqrt{3y}=1$ the class had manipulated the equation to the point of having $-\sqrt{3y}=-9$. The following classroom discussion subsequently took place:

Mrs. Willoughby: Square both sides will give you three y is equal to eighty-one. Now this is what I do because I look at things a little bit different. The first thing I do is get rid of my negative and I multiply by negative one. But what happens? I get the same answer. Okay so you could square both sides. I can see that negative there and ooh I'm going to get rid of it because I don't like it. One way is no better than the other. Then what do we do?

Student: Shouldn't it be negative three y?

Mrs. Willoughby: No and that's fine, you get rid of the negative sometimes is because when you negative square root of three y times negative square root of three y. What's a negative times negative? [Shows $\left(-\sqrt{3y}\right)\left(-\sqrt{3y}\right)$=]

Student: Positive

Mrs. Willoughby: It's a positive…

As another example, she recognized incorrect methods suggested by students when asking for the next step in solving the equation $\sqrt{a^2 - 6x + 73} = 10$.

Mrs. Willoughby: …And now what do I do?

Student: Square the radical

Mrs. Willoughby: No, I square both sides…

Thus, Mrs. Willougbhy was able to recognize and respond to her students' errors. However, her responses generally consisted of correcting students, rather than offering them an alternative explanation or model that might help them to see the source of their error.

Multiple methods of solving problems. Mrs. Willoughby was able to recognize when students were using other procedural methods for solving problems which were different from her own. Two examples follow. First, the following classroom discussion took place involving solving the equation $\sqrt{\frac{4x}{5}} = 12$.

Student: I have no idea what I did but somehow I had twenty and twenty-five in there and still got one hundred and eighty. (One hundred and eighty was the correct answer to the problem.)

Mrs. Willoughby: Somehow you got twenty and twenty-five and then you still got one eighty. Let's see if we can name a new math concept after you. (Mrs. Willoughby moved to the student, looked at her work and walked to whiteboard and proceeded to show others what the student had done.) Okay let me show you.

I think what you did, I don't, you forgot what you did but I see exactly what you did. Okay let's look at it another way. We've got, we're back to this point. Okay? Now this is what Augustine, I had Augustine and I'm thinking of somebody Amber, I've got Amber that sits over there. This is what she did. Okay. She did, same as Jarrett, okay and then she said okay, times square root of five over the square root of five. She going to rationalize this denominator is equal to twelve. So she's got the square root of twenty x over five is equal to twelve. Okay? Then she squared both sides and got twenty x over twenty-five is equal to one forty-four. Okay? Is that valid?

Student: Um humph.

Mrs. Willoughby: Of course it is. If we go on and multiply by twenty-five and divide by twenty we will still get one eighty.

$$\sqrt{\frac{4x}{5}} = 12$$

$$\frac{\sqrt{4x}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = 12$$

$$\left(\frac{\sqrt{20x}}{5}\right)^2 = (12)^2$$

$$\frac{20x}{25} = 144$$

Mrs. Willoughby: Which is the best?

Student: Shortest one

(Student laughs)

Mrs. Willoughby: The best is what works for you. Okay. Just because I do it simple and Jarrett went this way and Amber did this way. What happens? We all get the same answer. Okay? So there is no one way where you stop and say okay that's it, I can't do anything else…

Mrs. Willoughby noted when procedures suggested by students were probably not the easiest ones, such as the one above for simplifying expressions. She stated when they should not need to use a suggested procedure, even though she did not offer any explanation for why it should not be used, as in the following:

Mrs. Willoughby: What about this one? What are we going to do? What do I need to do on that one?

$$\sqrt{\frac{4x}{5}} - 9 = 3$$

Student: Separate (Indicates separating $\sqrt{\frac{4x}{5}}$ into $\frac{\sqrt{4x}}{\sqrt{5}}$) the four x and five.

Mrs. Willoughby: Separate. We're not going to do that now. How am I going to get this nine out of my way?

Another student says: Square both sides.

Mrs. Willoughby: Okay we're going to square this but do we want to square it first? (Student mumbling) I'm going to add nine. I knew folks were thinking about it and I wanted you to say it for me. Okay. So we've got the square root of four x over five is equal to twelve. Now we've got a radical is equal to something. We square both sides. If I square this side what do I have?

Although the following classroom discussion shows three different procedural approaches to simplifying the expression given to the class, it demonstrates that Mrs.

Willoughby recognized and took instructional time to show students each of the methods. The work shown on the whiteboard during the discussion is interspersed:

> Mrs. Willoughby: There are quite a few ways to approach this problem. (Refers to the following problem.) Okay. Let's start with the first time, let's separate the radical. Okay? We're going to do it that way. I'm going to do it a couple ways and let you see what you like. So I've got the square root of five x to the fourth divided by the square root of four n to the fifth. Now can I take the square root of four?
>
> Student: Two
>
> Mrs. Willoughby: Yea, it's two but I can do this I can take the square root of five x to the fourth divided by two n square roots of n, right? Is that okay? Does everyone agree with that?
>
> Student: n squared
>
> Mrs. Willoughby: n squared instead of n. Ya'll okay with that? Then what would we have to do?
>
> Student: Multiply by the square root of n
>
> Mrs. Willoughby: Multiply by the square root of n. We do not want a radical on the bottom. So we've got the square root of five x to the fourth n divided by two n cubed. n squared times n.

$$\sqrt{\frac{5x^4}{4n^5}}$$

$$\frac{\sqrt{5x^4}}{\sqrt{4n^5}}$$

$$\frac{\sqrt{5x^4}}{2n^2\sqrt{n}} \cdot \frac{\sqrt{n}}{\sqrt{n}}$$

$$\frac{\sqrt{5x^4 n}}{2n^3}$$

Mrs. Willoughby: Is it simplified?

Student: No

Mrs. Willoughby: What do I need to do? What's wrong with this?

Student: Too many x's

Mrs. Willoughby: There's too many x's up there. I can take the square root of that and what do you…x squared times the square root of five n over two n cubed. That's one way to do it. Okay.

$$\frac{\sqrt{5x^4 n}}{2n^3} = \frac{x^2\sqrt{5n}}{2n^3}$$

(Goes back to the same problem for other suggested ways of simplifying the problem.)

Mrs. Willoughby: Alright Jarrett you don't like my work.

Jarrett: That's not the way I worked it.

Mrs. Willoughby: Okay

Jarrett: I got the wrong answer.

Mrs. Willoughby: What's another way to do it?

Jarrett: Separate the radical

Mrs. Willoughby: You want to separate what?

Jarrett: Like the square root five x, the square root of five x to the fourth over four n to the fifth.

Mrs. Willoughby: Now what do you want to do Jarrett?

Jarrett: Then multiply square root of four n by

Mrs. Willoughby: Okay so we've got the square root of twenty x to the fourth n to the fifth over four n to the fifth, right? We're in good shape. That's right Jarrett. But where did you go from there because we've got to get an answer. Okay let's do a prime factorization of twenty, how would you factor. Two goes twenty ten times, and two goes in five times. So I've got two times two times five times x four times and y, I mean n five times. Okay so I'm going to circle my pairs. So I've got two x square n squared and I've got five n under that radical over four n to the fifth. Did you get that? (Could not understand what the student said.) Okay, now look at it, is it simplified? The simplest it can be is simplified. What else can I do to this? (Pause) Can I simplify that? Okay what do I do?

Jarrett: the n

Mrs. Willoughby: the n, I've got two n's up here and five down here, right? So the rule says I would have x squared n squared minus five over two so I've x squared n to the negative three square roots of five n over two, right? So x squared square root of five n over two n cubed. So you were on the right track you just didn't take, you didn't.

Jarrett: simplify.

Mrs. Willoughby: You didn't go on and get it in simplest form. (Student laughs) Okay Jarrett remember if you got a n in two places, something is wrong unless one's under the radical, if they're both outside the radical then you've got to do something. You don't want an n in two places, top and bottom unless one is under the radical. Okay?

$$\sqrt{\frac{5x^4}{4n^5}}$$

$$\frac{\sqrt{5x^4}}{\sqrt{4n^5}} \cdot \frac{\sqrt{4n^5}}{\sqrt{4n^5}}$$

$$\frac{\sqrt{20x^4n^5}}{4n^5}$$

$$\frac{2x^2n^2\sqrt{5n}}{4n^5}$$

$$\frac{x^2n^{2-5}\sqrt{5n}}{2}$$

$$\frac{x^2\sqrt{5n}}{2n^3}$$

Mrs. Willoughby: So yea you were doing it right. You just started out and stopped way to early. Okay…

Thus, Mrs. Willoughby did acknowledge different methods used by her students even though they were variations of the same general procedure. She also took instructional time to demonstrate these methods to students. However, this is a limited way of accessing different representations or different models.

Conclusion

Mrs. Willoughby felt that all of her students could learn algebra. She exhibited strong procedural knowledge but limited conceptual knowledge of the algebra content. She used procedural explanations on a majority of the items on the ALCKIN and felt students who computed slope had a better understanding. She showed two algebraic expressions were equivalent using procedures (which she was asked not to use in the instructions) and agreed that Seth's method worked only on certain algebraic expressions. Mrs. Willoughby was able to work the quadratic problem on the ALCKIN using the spreadsheet.

Her classroom instruction exhibited her content knowledge as she provided step-by-step instruction of procedures. However, the student methods recognized by Mrs. Willoughby as "different" from her own but were variations of the same procedure and were not accessing truly different methods. Mrs. Willoughby recognized and responded to errors of her own students but did not respond with alternative explanations and models. Students were asked questions related to answers for arithmetic procedures, simplification of algebraic expressions, or telling her the steps in algebraic procedures. These types of questions were not the type that would be productive to successful learning of mathematics by her students. Therefore, Mrs. Willoughby exhibited limited pedagogical content knowledge in her instruction.

Professional Development

Mrs. Willoughby attended the geometry sessions during the two weeks of summer of 2004 professional development provided by MDMSIP. The general topics covered in the geometry sessions were the same as in the Algebra I sessions. Mathematics related to algebra covered within the IMP units (Fendel et al., 2000) used during the geometry sessions included proportional reasoning and constructing "in-out" tables, as well as discussions on how to introduce students to the terminology related to the IMP units (Fendel et all, 2000).

When Mrs. Willoughby was questioned about the professional development she received during the summer professional development training, she stated,

> Mrs. Willoughby: Overall I was very impressed with it because to start I did not think it would work and I had some …., now I've not used some of the IMP units like they did but I have used the concepts of those and used the discovery and have definitely found out that it is for the lower students as well.
>
> Interviewer: What was your attitude toward attending the summer professional development training?
>
> Mrs. Willoughby: It was very lousy. I just thought, I just thought I can teach and I didn't think that that changing was, I didn't see anything wrong with what I did. And so I thought that it was just a waste of my time and that I was just going for two weeks and in my journal I started out letting them know what I thought and then you know after the first week I thought you know this is pretty good and Dr. Thompson (MDMSIP presenter) even said he could tell the difference in what I wrote in that two weeks, how I changed from thinking you know they're wasting my time to, you know, I can see where I can use this.

One of Mrs. Willoughby's presenters of professional development commented on her attitude as well as her engagement in the institute during the two week period.

> Ah, to begin with Veronica was very resistant about any of the possible methods that we were discussing. She was openly resistant. She said I'm here but I don't know that I'm going to get anything out of it. As the days progressed and as the

> activities progressed she began to see how this could work in her classroom and by the end of institute she was very excited about going back to the classroom and trying some of these things. She was very engaged. She got involved in all of the activities. She asked questions about well, how would this work in a regular classroom and what kind of modifications could be made, and she made suggestions about how she might be able to use things herself, rather than asking "What can I do?" It was a drastic change. She came in very negative, not wanting to learn anything, not wanting to get anything out of the workshop, but she left with a lot of new techniques, a lot of new things to try and she was very positive at the end of the workshop.

This same presenter also felt that Mrs. Willoughby would implement the instructional strategies learned during the two week professional development in her own classroom practices.

During the follow-up quarterly meeting Mrs. Willoughby participated in the sessions and shared strategies that she found successful during classroom instruction. For example, she shared the following after one presenter began a lesson plan with algebra tiles.

> The tiles are absolutely wonderful. The algebra tiles are absolutely wonderful! I even have students now who will sketch out on an algebra test the problem (Mrs. Willoughby does an algebra tile drawing on the board) who don't have the rules down but they get the right answer. On their quiz, I tell them I don't want them to just factor but I want them to draw it. You go from the tiles to factoring.

Mrs. Willoughby expressed that before the professional development she did not know how to use algebra tiles.

At another quarterly meeting, Mrs. Willoughby used algebra tiles to represent the following expressions along with different colors to represent the positive and negative tiles. She drew the tiles and stated:

> What is that? (Referring to her drawing.) The students will come up with
>
> $2x^2 – 2x + 2$
> $-x^2 + 3x + 1$
>
> (They can zero out a pair of similar tiles that have different colors.) Then they can zero it and come up with $x^2 + x + 3$. Different shapes and different colors. I discovered algebra tiles this year. I did not know how to use them before this year. I solve equations with them. I started them off with positive and negative numbers. They came up with the rules, but if they forget the rules they can go back and draw the circles with + and – in them.

Mrs. Willoughby expressed that she felt the methods learned at the summer institute were more helpful for some of her students than others. She stated:

> You know I think that the things I learned during the summer at institute helped me more with my ah (Algebra) A/B classes and my remediation class. I mean my remediation class I used hands on with them so much to introduce topics and, you know, I had kids tell me after, you know, after we did algebra tiles, I had a little boy come and tell me that that was the first time he understood what it meant to multiply binomials.

One of the presenters gave this assessment of Mrs. Willoughby's algebraic knowledge:

> I think she has a very good content knowledge of algebra one. I know she's taught algebra one also and she could make the connections between the geometry and the algebra one. I'm having trouble remembering exactly what we did that actually would have centered on algebra since this was a geometry workshop … But we did do some, we looked at patterns, we looked at in out tables and how to introduce students to the terminology of the IMP where they used in out tables, which is something the kids probably had not called that before, and so we did do that in the workshop but we basically added the content.

Thus, professional development presenters felt Mrs. Willoughby possessed good content knowledge of algebra and could make connections between the algebra content and geometry content. Presenters also felt she would implement changes into her instructional practices. Changes in content knowledge and pedagogical knowledge were demonstrated through her ability to use algebra tiles to model algebraic manipulations and willingness to use them in her instruction.

Post-Professional Development

During the post-professional development timeframe, changes in several areas will be addressed, including classroom environment, her views of student ability, content knowledge, and pedagogical content knowledge. How the changes in content knowledge and pedagogical content knowledge are reflected in instructional practices will be addressed. Each of these areas will be addressed in the following sections.

Classroom Environment

The classroom layout remained unchanged, and no changes not noted in Mrs. Willoughby's demeanor. However, student work from the IMP units (Fendel et al., 2000) was displayed on the classroom walls, evidence that some changes in instructional practices may have occurred.

Student Ability

When Mrs. Willoughby was questioned about how well her students could learn algebra, she commented:

> I went to the counselor first, after the first test and said some of the students did not need to be in the advanced (Algebra I) class because that's a fact they needed to be in the more hands on Algebra A/B and they struggled…

She also felt that the students who are in the Algebra IA and Algebra IB classes could also learn algebra but maybe not at the same pace as those students in the Algebra I classes. She stated, "You know I think it all, I think it more depends on the individual student and the background they have. Whether they can learn at the pace that we set in the advanced classes (Algebra I)."

Thus, Mrs. Willoughby feels all of her students can learn algebra, no matter which section of algebra they are taking. She did note that her students in Algebra IA and Algebra IB would need to learn algebra at a different pace. Moreover, she seemed to think that use of "hands-on" methods was more important for the lower level courses than for Algebra I.

Changes in Content Knowledge

Mrs. Willoughby felt that she had good algebra content knowledge, stating, “I’m real comfortable with algebra. I enjoy algebra and kids don’t seem to find it as exciting as I do. I enjoy it. I enjoy algebra.” Mrs. Willoughby did not feel she learned any new algebra content as a result of the professional development, pointing out:

> Well, they did the Pythagorean Theorem this summer in and so I did not see, I was not able to teach it this last nine weeks, so I didn’t get to use but I have introduced the Pythagorean theorem by one of the games that we used in the geometry.

When asked to solve a quadratic equation during her final interview, Mrs. Willoughby was not only able to solve the equation by setting the equation equal to zero, factoring the quadratic expression, and setting each factor equal to zero but she was able to offer other methods that could be used on the same problem. For example, she stated, “Let’s see you could do it where, now I could do it with x squared minus two x minus eight is equal to zero and x squared and (Pauses while she is working) and these are, I’ve got two negative x’s, a negative….I could do it with algebra tiles” and provided a drawing for her method. Other techniques were also suggested:

> Mrs. Willoughby: Well you could use the quadratic formula, you could graph it.
>
> Interviewer: Okay so what do you mean by graphing it?
>
> Mrs. Willoughby: To me this is more algebra two. It’s just choose points and for x and solve and if you know it’s a parabola and you know what a parabola looks like and I for one note that they would have to randomly choose points and they could get it.

Thus, Mrs. Willoughby demonstrated different ways of representing and solving mathematical tasks using a variety of techniques, both procedural and conceptual. This indicated a growth in her content knowledge from the pre-professional development phase.

Changes in Pedagogical Content Knowledge

Mrs. Willoughby praised students when they used methods that she herself had not thought of using. For example, during a problem in which students were given the ordered pairs for two points and asked to find the equation of a line that contained these two points, Mrs. Willoughby commented:

> You know but if I see something new like Marquette, I saw how he got that problem today. I never thought of about. All he had was two points and I, but one of the two points was the y intercept and I never thought about that's a way to get the slope and he's real bright and you know I enjoy, but I thought you know something different, that's something that because I like to, you know, and I don't if what I do is right or wrong because I like to show different techniques for the same, to do the same thing because different people think different ways and so I like to know different ideas and that's what I try to. That's what I want to get from MDSMIP is some different ways to teach the same things we've taught.

Thus, Mrs. Willoughby was able to understand a different representation her student had for finding the equation of the line.

Mrs. Willoughby indicated she used algebra tiles in developing an understanding for understanding why algebraic rules she used. She said:

> I picked it up (algebra tiles) somewhere else because I saw that the that the hands on worked … it's because I tried to show them what I was doing before I put, actually put it on the board and I would give them a test, a quiz on multiply this, drawing out the tiles because I would tell them you can't use these on the exit exam. So we did a lot and they would come in, are we going to be able to do algebra tiles.

Thus, Mrs. Willoughby demonstrated access to different representations as well as different methods for developing an understanding for algebraic rules. This suggests growth in her pedagogical content knowledge

One error that was pointed out in the mathematical knowledge of Mrs. Willoughby on the ALCKIN also was manifested in the knowledge constructed by her students. A group of students had concluded that any number to the zero power was equal to one. This was written by students on their displayed work from the IMP Alice unit (Fendel et al., 2000). Thus, her lack of understanding may be reflected in the mathematics her students learn. Thus, Mrs. Willoughby did exhibit growth in pedagogical content knowledge as evidenced through the demonstration of how to use algebra tiles and build the bridge between the physical models, to drawings, and to developing an understanding for the rules.

Reflection of Changes in Knowledge in Instructional Practices

Mrs. Willoughby had reported using hands on techniques with her classes; however, this was not observed. When asked if she was had used these same types of strategies with the particular Algebra I class that was observed, she responded:

> With this class in eighteen weeks to cover the course of study, I did very little. I did probably maybe four or five activities. I mean you know but not near as much as with the geometry class where with the geometry B that I've got where you have eighteen weeks with half the book. In eighteen weeks to cover everything in the algebra one content you don't have time to do a lot of activities. I did Alice with them, that was one I used with them, and I used probably about four or five different activities in that.

Her follow-up RTOP score during the spring of 2005 was 41, which was somewhat higher than her RTOP score from the previous spring and suggests that some instructional change may have occurred. The same comments made about how her content knowledge and pedagogical content knowledge were reflected in her instructional practices prior to professional development could be said about these types of knowledge post-professional development.

Thus, no changes in her content and pedagogical content knowledge were actually observed in her instructional practices. Although she indicated she had used algebra tiles with some of her classes, there was no evidence in the follow up observations that Mrs. Willoughby had made changes within the algebra one class.

Conclusion

Mrs. Willoughby possessed content knowledge which includes strong procedural knowledge and limited conceptual knowledge. Mrs. Willoughby had limited pedagogical content knowledge Growth in mathematical processes was observed through problem solving in different ways and using different representations such as algebra tiles. It was expected that Mrs. Willoughby would make instructional changes as a result of professional development. An error made by Mrs. Willoughby was manifested in the work constructed by her students. Growth in both her content knowledge and pedagogical content knowledge resulted from professional development. Although growth in content knowledge and pedagogical content knowledge was demonstrated in professional development activities, it was not reflected in her instructional practices.

Conclusion of Case

Mrs. Willoughby felt that all of her students could learn algebra but maybe at different paces. She felt she had good algebra content knowledge, and while she exhibited strong content knowledge, evidence of pedagogical content knowledge was limited. Evidence of growth in both of these types of knowledge was seen in the post-professional development timeframe. Mrs. Willoughby demonstrated the use of algebra tiles during the quarterly meetings and discussed how to use them in instruction. Thetudent work displayed also suggested she was implementing instructional changes. However, how these changes in her content knowledge and pedagogical content knowledge were actually reflected in her instructional practices was not clear, because it was not evident in the classroom observations I did.

Mrs. Pitchford

Mrs. Pitchford is a Black female, and she was in her fourth year of teaching mathematics when this study began. She was shorter than average and had a normal speaking voice that was easily heard in her classroom. Mrs. Pitchford held a Bachelor of Science degree, as well as a Master’s degree in mathematics education. During this research study, Mrs. Pitchford was working on her Education Specialist’s degree in mathematics education. At Clarion High School, a ninth through twelfth grade school, her teaching responsibilities included Algebra I. Clarion High School had a passing rate of 41% on the Alabama High School Graduation Exam, the student population was 100% Black, and 61% of its students were on the free or reduced lunch program.

Pre-Professional Development

In this section, we will consider Mrs. Pitchford's classroom environment, how she viewed her students' ability to learn algebra and responses to statements contained on the Teacher Attitude Survey. Next, Mrs. Pitchford's content knowledge and pedagogical content knowledge will be addressed from the ALCKIN and interview. Finally, a further look at her content knowledge and pedagogical content knowledge will be considered from classroom observations, which includes how these types of knowledge were reflected in her instructional practices.

Classroom Environment

She generally sat at her desk while her students worked in pairs or small groups. She instructed her students that one person from each pair or group could come and ask her for help when they needed it. Only once during any classroom observations was she observed moving between the pairs and small groups to observe what the students were doing and to answer their questions.

The student desks in Mrs. Pitchford's classroom formed two sets of straight rows, each set facing toward the center of the classroom. This type of setting might suggest that Mrs. Pitchford used a traditional approach to teaching mathematics. However, since there was very little mathematics instruction, classroom observations did not contradict nor confirm this belief. The back wall contained a whiteboard with the teacher desk at the far corner of this particular wall. From this reference point, the right hand wall had a whiteboard while the left hand wall contained windows. The front of the room contained a television, computer, overhead projector and pull down screen. Posters depicting famous Black Americans and pertinent information about each adorned the remaining wall space in the classroom.

Mrs. Pitchford encouraged her students to borrow calculators for classroom use. During one classroom observation she made the following statements.

> …It's not three times two, it's three, it's negative three squared. That's negative three times negative three. That's why I told you're going to need a calculator tomorrow. You're going to need it because when you do the square root, you've got to take the square root. It's not always a perfect square root for the number. That means it doesn't work out. That's what I'm going to be looking for tomorrow. So how many of you have found you a calculator, a scientific calculator? What are ya'll going to do tomorrow? Huh…

Student Ability

Mrs. Pitchford felt only a little more than half of her students could learn algebra well because ,"I was looking at them as (having) more general math skills." In regards to the other students who she did not feel could learn algebra well, she indicated that they were lacking in the mathematical skills they should have already learned in previous mathematics classes. Thus, she largely saw their success as depending on their prior knowledge.

Teacher Attitude Survey

Mrs. Pitchford was also asked to reflect on two conflicting statements from the TAS. She was first asked to respond to, "It is important for students to figure out how to solve mathematics problems for themselves." She indicated that this was a part of her instructional practice and stated, "By letting them work individually especially with practice problems and really trying to get them to use math, mental math rather than depending on their calculators." In response to "Teachers should model and demonstrate mathematical procedures then ideally time should be allowed for students to have the opportunity to practice these procedures" and how the two statements can coexist in the mathematics classroom, she replied, "That's how, that's how I really, how I teach my class. By modeling you know the examples and showing them how to work them and go through the steps and then after I've done this, I let them, you know, practice on working the problems themselves."

Thus, Mrs. Pitchford felt she was allowing students to figure out how to solve mathematics for themselves by modeling and demonstrating examples. She then allowed students to practice these same types of examples.

Content Knowledge

Mrs. Pitchford's content knowledge will be viewed through the lenses of procedural knowledge, conceptual knowledge, and mathematical processes. Each of these will be addressed in the following sections.

Procedural knowledge. Mrs. Pitchford correctly answered 9 of the 25 mathematical tasks on the ALCKIN, suggesting weak overall knowledge of algebra. She exhibited significant weaknesses in her procedural knowledge. On parts B and C of question one (see Appendix A), she agreed with the students that neither of the situations represented a function and offered no explanations to defend her incorrect answers suggesting she may not understand functions.

When presented with the wheels and vehicles problem (see Appendix D), she was unable to find any method to correctly solve the problem. She responded with the following:

> Mrs. Pitchford: (Pauses) I think I would, seventeen vehicles, fifty-six wheels, and you know that cars have four wheels and you would divide four into the fifty-six and what is left should be the two wheels on the motorcycle. That's how I think I would teach.
>
> Interviewer: Okay so will you work it for me. Just work it on there (indicated the sheet of paper that the problem was written on) will be fine.
>
> Mrs. Pitchford: (Pauses) Humph. Still not going to balance out. It going to come out to be fourteen. (Pauses) Humph. That's the only logical reason I could see how to, how to figure it out. You know a car has four wheels.

Thus, Mrs. Pitchford exhibited weak procedural knowledge in selecting answers on the ALCKIN. She was also unable to solve the wheels and vehicles problem using any procedure.

Conceptual knowledge. Mrs. Pitchford also exhibited limited conceptual knowledge. On question six on the ALCKIN she was unable to provide a method for showing that the two algebraic expressions were equivalent and left the question blank (see Appendix A). From question eight on the same instrument, she selected the group using the tangent of the angle as the only group which had an understanding of slope.

Since she elected not to support her answer with an explanation, it is not clear why she selected this particular group. From question ten on the ALCKIN, she explained the 0.2x with “the # of chocolate chip cookies that can be baked in 15 hrs” which is not what the expression represents within in the equation. These examples suggest that Mrs. Pitchford had very limited conceptual understanding.

Mathematical processes. In question five on the ALCKIN (see Appendix A) Mrs. Pitchford was unable to correctly provide a solution to the quadratic problem using part of a spreadsheet. She substituted -1.8 into the expression 4-2x and got 6.7. She explained her answer as follows: “Student’s should use the value of x in each place in x in column’s 2 & 3.” This example suggests that Mrs. Pitchford had limited ability to use mathematical processes.

Pedagogical Content Knowledge

Very little could be determined about Mrs. Pitchford’s pedagogical content knowledge. Since few of her answers on the ALCKIN were supported with written explanations, it was difficult to assess her pedagogical content knowledge.

In the interview, Mrs. Pitchford was asked to respond to the student work involving graphing both sides of the quadratic problem $x^2 = 2x + 8$ to obtain its solution (see Appendix D). It wasn’t clear whether she understood what the student had done and she responded with the following:

> Interviewer: …Here’s the first one and tell me what you think about what this student did.
>
> Mrs. Pitchford: (Pauses – Long Pause) Humph.
>
> Interviewer: What do you think of that?

Mrs. Pitchford: If I presented it to my class, it would be difficult for them.

Interviewer: So do you think it is an okay way to work it?

Mrs. Pitchford: It's an okay to work it but I think more or less they, I believe they let the computer do it. (Laughs) That's what I am thinking, I mean you know.

She was unable to provide a method of solving the wheels and vehicles problem during the interview. However, she recognized that both of the elementary methods she was asked to analyze were suitable ways of solving the same problem (see Appendix D).

Thus, Mrs. Pitchford exhibited limited pedagogical content knowledge in the ALCKIN and interview. Further assessment of her pedagogical content knowledge was done through her classroom instruction which will be detailed in the following section.

Classroom Instruction

There was little evidence of pedagogical content knowledge in her classroom instruction. During the majority of the time students put homework problems on the whiteboard, worked in pairs or small groups with little instruction provided by Mrs. Pitchford. When procedural instructions were given, it was apparent that the students had already previously worked on these same types of problems. For example:

Mrs. Pitchford: Does anyone remember what we are doing?

Students: (Many different answers are given.)

Mrs. Pitchford: Add exponents. (She wrote $a^2(a^3)(a^6)$ on the whiteboard.) Multiply ones, add exponents. (She wrote a^{2+3+6} and a^{11}.)

(Without any additional comments, she moved to another problem and wrote $(l^2k^2)(l^3k)$ on the whiteboard, Mrs. Pitchford: Separate them. (She wrote $l^{2+3}k^{2+1} = l^5k^3$.).

Mrs. Pitchform: Next problem $(10^2)^3$ (Even though she asked the students for the next step she wrote 10^6 and continued.) You're just going to multiply your exponents.

Mrs. Pitchford typically gave her students verbal instructions on what steps to use, then simply demonstrated solutions to sample problems on the whiteboard. For example:

Mrs. Pitchford: These are multiple step equations so you use opposite operations. (She then proceeded to work the problem on the whiteboard for the students.)

$$\frac{3x+6}{4} = 3$$
$$4 \cdot \frac{3x+6}{4} = 3 \cdot 4$$
$$3x + 6 - 6 = 12 - 6$$
$$\frac{3x}{3} = \frac{12}{3}$$
$$x = 2$$

Thus, Mrs. Pitchford was able to use her content knowledge to work these particular problems. However, she was not posing problems productive to learning or presenting them in ways so students could successfully learn.

When a student requested that a particular problem be worked, Mrs. Pitchford frequently exhibited weak content knowledge. Furthermore, she offered no explanations for the procedures she used. For example, when asking students if they had any questions about the problems on a worksheet, this exchange followed:

Mrs. Pitchford: What about the quadratics?

Students: Quadratics?

Mrs. Pitchford: Starts on the second page.

Student: Seventeen, no number eighteen.

Mrs. Pitchford: (Writes $4x^2 – 11x – 3 = 0$ on the whiteboard.) You're going to get it in the foil method. You're going to factor it out. (Writes on the whiteboard: (4x 3)(x 3) = 0. She then erased the 3 in the first parentheses and wrote (4x 1)(x 3) = 0, Followed by $4x^2$ 12x 1x 3.)

Mrs. Pitchford: We want a -11x so 12x has to be negative. (She wrote, $4x^2 – 12x + 1x – 3$.) You come back to the original $4x^2 – 11x – 3$. What signs go in the parentheses?

Students: (Guess both positive and negative.)

Mrs. Pitchford: (Puts a subtraction sign in front of the three and an addition sign in front of the one. Fills in original parentheses and writes (4x + 1)(x – 3) = 0.)

Thus, even though Mrs. Pitchford's exhibited correct content knowledge in her instructional practices, she exhibited weaknesses in this knowledge.

In addition, Mrs. Pitchford indicated to the students when a procedure should be used on all the assigned problems. For example, in a lesson where students were simplifying expressions involving the power rule for exponents, the following dialogue took place:

Student: Hey don't you multiply the whole numbers? (Called Mrs. Pitchford over to her desk.) Do you multiply or add?

Mrs. Pitchford: Multiply. You multiply on all of these. (To another student.) The only time you multiply is when you have a number on the outside.

Simply telling students what to do is not presenting mathematics so that students can successfully learn it.

Mrs. Pitchford had an average RTOP score of 9 1/3 based on classroom observations during the spring of 2004 suggesting that her instructional practices were not at all conducive to helping students develop a conceptual understanding of the mathematics that was taught during classroom instruction.

Student questioning. Very little dialogue took place in Mrs. Pitchford's classroom. Thus, it was difficult to determine evidence of her pedagogical content knowledge in her questioning. The typical types of questions she generally asked did not require much mathematical thought and could be answered with short answers. For example:

> Mrs. Pitchford: Factor it out, you know two times three and one times six is six. We want to get seventeen in the middle. We know what numbers multiply to seven?
>
> Student: one and seven

In another classroom observation Mrs. Pitchford asked, "What is the square root of thirty-six?" and the student answered "six." That was the extent of the interaction.

Thus, there was very little opportunity to observe how Mrs. Pitchford responded to questions of her students. In addition, Mrs. Pitchford was not observed posing problems and questions that would be productive to student learning.

Analyzing student errors. There was little evidence that Mrs. Pitchford could use her mathematical knowledge to recognize the errors of her students. She had difficulty in responding to students who had made errors. For example, a student went to the whiteboard and showed the following work:

Student: $-4(cs^2t)^2$

$-4(c^2s^4t^2)$

$-4c^2-4s^4-4t^2$

Mrs. Pitchford: Who did this? The answer should be negative four c squared s to the fourth t squared. I don't know where you got all those extra negative fours.

Thus, Mrs. Pitchford had difficulty in recognizing where and why her students made errors. When she did recognize an incorrect answer, she did not understand why the student made the error nor did she respond with alternative explanations and models.

Multiple methods of solving problems. Mrs. Pitchford was not observed working mathematical problems using different methods nor did she encourage her students to use alternative ways of solving problems during classroom instruction.

Conclusion

Mrs. Pitchford expressed that not all of her students could learn algebra well because they lacked the necessary general mathematics skills needed for learning algebra. Mrs. Pitchford showed limited procedural knowledge and limited conceptual knowledge. Mrs. Pitchford was observed on more than one occasion making mathematical errors or showing a lack of understanding for the algebraic topics she was teaching. Errors were also exhibited in the few explanations that she provided on the ALCKIN and neither did she assess any of the errors in the statements made by students on the same instrument. She exhibited difficulty in recognizing why students made the types of mathematical errors they were making. Her classroom instruction consisted of showing students procedures to work problems while giving very little verbal instruction, displaying very little evidence of pedagogical content knowledge. Mrs. Pitchford asked her students questions that required primarily required them to use arithmetic. It appeared that Mrs. Pitchford lacked the ability to solve problems using a variety of methods and there was no evidence that working problems in different ways was valued in classroom instruction.

Professional Development

Mrs. Pitchford did not attend the two-week MDSMIP professional development training with the other teachers from her school. Instead, she participated in classes at a local university that were required for her education specialist degree in mathematics education. In the interview, she was asked to describe the classes she had completed and how these classes had promoted any change in her classroom instructional practices. She gave no indication that these classes helped her make any changes. Mrs. Pitchford said, "From the classes, well the classes I'm taking right now are dealing with curriculum. I took an advanced curriculum class which I had to design my own curriculum and then the other class I had was a statistics class." She further indicated that what she had learned in her statistics class could not be used in the classroom:

> Those higher level classes I'm taking (laughs) higher level math classes have nothing to do with algebra to be honest with you. I mean I'm taking like I said six hundred level courses but they're more or less like this statistics class it's dealing with variance and this here kind of stuff that I don't get to present to them.

Thus, Mrs. Pitchford indicated that the courses she was taking for her advanced degree were not beneficial to her instructional practices.

Post-Professional Development

We will now consider changes that occurred during the post-professional development timeframe, including changes in classroom environment, view of student ability, content knowledge, and pedagogical content knowledge. We will also consider how the changes of content knowledge and pedagogical content knowledge were reflected in changes in instructional practices. Each of areas will be addressed in the following sections:

Classroom Environment

Mrs. Pitchford exhibited the patterns of behavior during this timeframe that she did during pre-professional development timeframe. The classroom layout remained unchanged. There was no other evidence of change in the classroom environment.

Student Ability

Mrs. Pitchford felt a majority of her students could learn not algebra well. Regarding the ability of her students in her algebra classroom who she did not feel could learn algebra well, she stated:

> To be honest with you they are terrible. You build upon and you have to come in with something to, you know, to get something. I'm seeing children come in here without any type of computation skills, division, just basic math skills. They don't have, they're not having it. They're not having the mental skills that they need to have to work problems quickly without having their hands on a calculator, but if you don't know how to put the information into the calculator, it's still not going to come out correctly.

Thus, Mrs. Pitchford felt her students do not have the necessary computational skills to learn algebra well. She stated that these skills were necessary, so that she could build upon them to help her students learn algebra.

Changes in Content Knowledge

When questioned during an interview in the spring of 2005, Mrs. Pitchford stated that she was confident that she knows the algebra content well. She commented with "Oh, very well." However, continued weaknesses in her content knowledge were still observed.

In the interview, Mrs. Pitchford used a set procedure to correctly solve the quadratic problem $x^2 = 2x + 8$ she was given (see Figure 21) and provided a second procedural way for students to solve the problem involving using the quadratic formula.

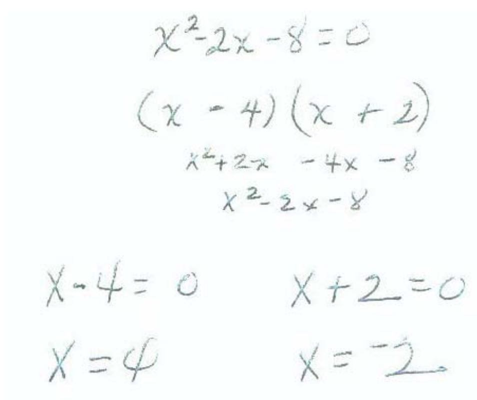


Figure 21. Mrs. Pitchford's work on the quadratic problem.

Her third method for solving the problem reflected incorrect mathematical knowledge:

Interviewer: Can you think of a third way?

Mrs. Pitchford: Do ah, take the square of both sides, yea take the square of both sides and you might be able to do it that way. I think. I'm not sure.

Thus, while Mrs. Pitchford was able to correctly solve the mathematical task this time in two different ways, she suggested a third method that did not reflect correct mathematics.

Changes in Pedagogical Content Knowledge

In one classroom observation in the spring of 2005, students had apparently been working on using the quadratic formula to solve equations. Mrs. Pitchford began the class as follows:

> Mrs. Pitchford: Now what I'm going to do is I'm going to put the quadratic formula on the board and also the formula you use to find the determinants. Now the determinant is easy. That's why I could not figure out why ya'll could not do those problems where you were finding the determinant. Now that homework I gave you, ya'll remember that is some of your quiz grade. So did ya'll do it?
>
> Students: Yes
>
> Mrs. Pitchford: Alright everybody turn it in?
>
> Student: No
>
> (On whiteboard Mrs. Pitchford had written.)
>
> -b – 4ac
>
> $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
>
> Mrs. Pitchford: When you find the determinant, when you're trying to find out how many roots you have this is all you use. (Indicates the first expression written.) Right here, I believe or is it b squared?
>
> Students: b^2

Mrs. Pitchford: Are you sure?

Student: No that's right, that's right.

Student: b squared

Mrs. Pitchford: (Picks up a textbook, looks it up and changes the b on the whiteboard to $b^2 – 4ac$.) b squared minus four a.

Mrs. Pitchford did not appear to know the algebraic expression for the determinant and the relationship between the determinant and the quadratic formula since she had correctly written the quadratic formula on the whiteboard.

Thus, no particular changes were evident in Mrs. Pitchford's pedagogical content knowledge. However, she did exhibit weakness in making connections between mathematical topics.

Reflection of Changes in Knowledge in Instructional Practices

Mrs. Pitchford had a RTOP score of eleven on the classroom observation done during the spring of 2005 which was about the same as the previous spring. This might not be surprising since no changes were observed in her content knowledge or pedagogical content knowledge. Mrs. Pitchford generally sat at her desk and gave verbal instructions when her students had a question. When students gave incorrect solutions for the problems they were assigned to work on, the following exchange resulted:

Mrs. Pitchford: Are ya'll finished with number three? What is the answer?

Student: Negative two point sixty-one.

Teacher: No

Students: (Other answers are called out.)

Mrs. Pitchford: Didn't you have a negative number under the radical? You've got to add the…. What did you have up under the radical?

Students: (Mumbling)

Mrs. Pitchford: Sixteen and what?

Student: Sixteen and thirty-six? (More student mumbling)

Mrs. Pitchford: Sixteen minus twenty-four is equal to what? Negative what?

Student: No

Mrs. Pitchford: What is sixteen minus twenty-four?

Student: Negative eight.

Mrs. Pitchford: Negative eight! (Student talking) Any time you end up with a negative number up under the radical it's no solution. Because you are going to end up with an imaginary number. So you draw the little circle and put a slash through it and means no solution to that problem. So any time you come up with a negative number under the radical it's no solution, you stop. Any questions about that? (No questions.) Okay did you guys move on to number four?

Conclusion

Mrs. Pitchford did not participate in the MDSMIP professional development. Rather, she was involved in taking classes for her education specialist degree in mathematics education. She did not feel these classes helped her in making changes in her instructional practices, and no real changes were observed. The comparison of the RTOP scores from the spring of 2004 and the spring of 2005 were almost the same. In classroom instruction, Mrs. Pitchford used the same general types of instructions as she has used the previous spring. She continued to struggle with the algebraic content, making significant algebraic errors, and no changes in content knowledge or pedagogical content knowledge were evident.

Conclusion of Case

Mrs. Pitchford felt that students who had general mathematics skills could learn algebra well. Mrs. Pitchford exhibited some weaknesses in her content knowledge and limited pedagogical content knowledge. She was seldom observed giving any type of mathematics instruction to her classes. She made mathematical errors and errors in her mathematical reasoning and was unable to solve problems using a variety of methods. Mrs. Pitchford did not attend the summer professional development training, and no changes in her knowledge were seen as a result of the coursework she took her education specialist degree. No changes were seen in her content knowledge and pedagogical content knowledge, and she continued to make algebraic errors.

Mrs. Colley

Mrs. Colley is a White female who was in in her sixth year of teaching when this study began. Mrs. Colley was tall in statue and had a booming voice to complement her height. She holds both a Bachelor of Science degree as well as a Master’s degree in mathematics education. Mrs. Colley taught at Dover High School, a ninth through twelfth grade school, where her primary teaching responsibility was Algebra IB. Dover High School had a 72% passing rate on the Alabama High School Graduation Exam, the student population was 18% Black, and 19% of the students were on the free or reduced lunch program.

Pre-Professional Development

In this section, we will consider Mrs. Colley's classroom environment, how she viewed her students' ability to learn algebra and responses to statements contained on the Teacher Attitude Survey. Next, Mrs. Colley's content knowledge and pedagogical content knowledge will be addressed from the ALCKIN and interview. Finally, a further look at her content knowledge and pedagogical content knowledge will be considered from classroom observations, focusing on how these types of knowledge were reflected in her instructional practices.

Classroom Environment

Mrs. Colley exhibited a pleasing personality and set a comfortable classroom setting where her students appeared to be at ease. She exhibited patience in working with her students. Mrs. Colley's classroom consisted of student desks arranged in straight rows facing the front of the classroom. This setting suggested that Mrs. Colley used a traditional format for mathematical instruction, and during the spring of 2004, Mrs. Colley was the dominant character in facilitating mathematics instruction. The front of the classroom contained a whiteboard, overhead projector and a pull-down screen, the teacher desk sitting on the left end of this wall. The right-hand wall also had a whiteboard along with a bulletin board. The left-hand wall contained windows. During one classroom observation, student dodecahedrons hung from the ceiling. Students had researched a mathematician and had included facts pertaining to their mathematician on the regular polygonal faces.

Student Ability

Mrs. Colley's felt that only about one fourth of her students could learn algebra well. She stated during an interview in the spring of 2004,

> I guess they're learning it well the way I'm teaching it. The other ones are struggling. You know what I'm saying? Ah and that's what the MDSMIP thing is, it's learning it how to teach it so the other half can get it, the other seventy-five percent but those that it comes, the twenty-five percent is just that it comes easy to, those that just pick up on it. Those are the ones to me that are going to learn it and retain it, where the other seventy-five percent may struggle. They'll learn it long enough to take the test and then be done.

Thus, Mrs. Colley did not feel many of her students could learn algebra well. She indicated that they did not learn well from the instructional practices she was employing. Her students focused on learning material for the test.

Teacher Attitude Survey

Mrs. Colley was asked to respond to "It is important for students to figure out how to solve mathematics problems for themselves." She replied:

> Humph. Let's see. Humph. Well they have to work their own stuff. Ah, I usually after we've done notes we ah, after we've done notes I usually give them classwork. And during their classwork we're, I don't let them work in so called groups but I don't mind them doing peer tutoring across the row or anything like that and then I'm always up here and around helping out where you need help but it's mostly them having to work the problems. So, yes, then the next day we usually come in and I give a daily quiz and that's independent and if you know if

> you've done the classwork the day before you should be successful in the daily quiz.

Mrs. Colley was asked respond to "Teachers should model and demonstrate mathematical procedures and then ideally time should be allowed for the students to have the opportunity to practice those procedures" and how both of these statements could coexist in the mathematics classroom. Her response follows:

> See I don't know. That's that to me is what scares me cause I know a lot of the MDMSIP stuff and a lot of the stuff that I've seen, cause I came on board a little late but a lot of the stuff I've seen has been letting them explore and come up their own way of doing it. I know there was a tower thing that we did over at West State University where and it's on video tape and I've seen it on public television and all of that sort of stuff where the kids had to come up with how many different varieties of two color towers could there be as long as they are five tall and it's just seeing the kids work through it. But to me, you know, the objectives in the course of study say the kids have to be able to do slope, you know or the kids in the course of study have to be able do this. And then the graduation exam is going to present it written form, test form, that sort of thing. … [T]hat's one thing I'm you know, I'm looking, I'm hoping that's clarified for me, because how can they go from well this and this goes together, but how am I supposed to take what I've figured out here to work this problem? I don't, I don't, I don't get how they coexist.

Thus, Mrs. Colley allows students to work together in figuring how to do mathematics. But she contradicts this statement in implying that she does not know how students can make sense of the mathematics themselves if teaching are modeling and demonstrating for them.

Content Knowledge

Mrs. Colley's content knowledge will be examined from the perspectives of procedural knowledge, conceptual knowledge, and mathematical processes in the following sections.

Procedural knowledge. Mrs. Colley exhibited strong procedural knowledge. The majority of her explanations used procedures on the ALCKIN. Mrs. Colley successfully solved the wheels and vehicles problem procedurally (see Appendix D) by setting up two equations, solving for one variable in terms of the other, and solving the problem by substitution. Figure 22 contains Mrs. Colley's work on this problem.

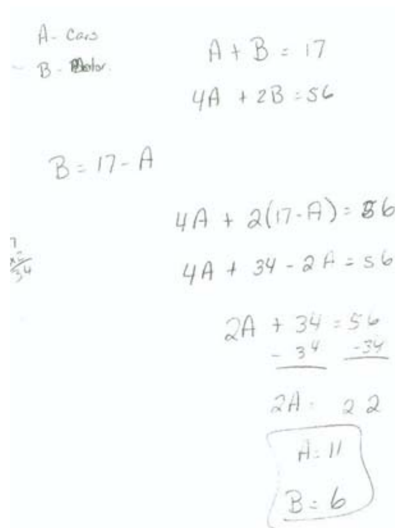


Figure 22. Mrs. Colley's work on the wheels and vehicles problem.

It should be noted, however, that Mrs. Colley made errors on question four on the ALCKIN that related to sets of numbers (see Appendix A). She disagreed that 2/3 was a real number, agreed that .010010001… was a rational number, and disagreed that $\sqrt{5}$ was a complex number. This reflects limited knowledge involving number systems and how they are related.

Conceptual knowledge. Mrs. Colley exhibited limited conceptual knowledge. Mrs. Colley offered no additional way for showing the two algebraic expressions were equivalent on question six on the ALCKIN (see Appendix A). Mrs. Colley was able to correctly identify the groups of students that understood slope from question eight on the same instrument. She stated that group four had the best understanding of slope and wrote the following as her explanation "Group 1 & 4 both understand slope. It's just that Group 4 was able to right the solution down better and convince the reader of the understanding. They also were able to show the two slopes were equivalent by reducing."

Mrs. Colley provided a conceptual approach for solving the wheels and vehicles problem as her second method. She stated:

> Mrs. Colley: Hum, I can think of it but it doesn't come to me as quickly as this one did. The other way would be to use (pause) see I'm having trouble thinking it through. It would just trial and error, just basically drawing cars or having a cube to represent a car and counting up that way.
>
> Interviewer: Ah just trial and error and using manipulatives?
>
> Mrs. Colley: Right.
>
> Interviewer: Are those two different ways?

Mrs. Colley: No, I'm using it together because I'm a visual. Yes, either I'd have to draw a picture or I'd have to use something. Do you know what I'm saying? To do trial and error.

So, Mrs. Colley exhibited limited conceptual knowledge. However, she was able to provide a conceptual example for solving the wheels and vehicles problem.

Pedagogical Content Knowledge

Her pedagogical content knowledge will be examined by how it is defined in the literature review, looking at the following abilities: to "unwrap" and present mathematical topics so that students can be successful in learning mathematics, to access different representations as well as methods for solving mathematics problems, to recognize student errors and be able to respond to them with alternative models and explanations, to respond to questions, and to pose questions and problems that are productive to students learning mathematics.

Mrs. Colley's pedagogical content knowledge appeared limited based on her responses to the ALCKIN. Mrs. Colley's was able to recognize that Seth was using a procedure that was not mathematically correct on question two of the ALCKIN (see Appendix A). She selected the correct answer and followed her selection with the following explanation, "Many students do not realize that you cannot cancel #'s when they are being added/subtracted to other #'s You can only cancel when #'s are be multiplied to one another."

Mrs. Colley was unable to find the solution for the quadratic problem using the spreadsheet which was problem five on the ALCKIN (see Appendix A). She recognized that the methods used by both students on question seven of the same instrument were correct and explained her answer choice by stating, "They both found correct solutions to the problem by looking at patterns & how the original problem was worked."

However, during an interview Mrs. Colley recognized that using graphing to find the solution for the problem $x^2 = 2x + 8$ (see Appendix D) was feasible, declaring:

> Humph, wait a minute. Now I see what they did. OK. How'd they get two equations out of this one equation? OK I got it. (Laughs) OK, I'll get it – OH. It's different. (Pause) I would have to say I've never worked it that way before and it looks to me like it's valid but I would have never thought of it myself.

In the wheels and vehicles problem in the interview (see Appendix D), Mrs. Colley recognized that both the elementary methods and solutions were viable and indicated that she would accept them as ways to work the problem in her classroom. However, she pointed out that she felt that students needed to know the usual procedures for working the problem, as follows:

> Yea, because to me this is the trial and error. Because like I said I'd have to draw pictures or use manipulatives. I don't know that I would thought of it this way I mean this makes sense but like I said before I was having trouble, I'm so used to doing it this (points to her work) doing this is a stretch even for me. But it would still concern me that they need to be able to do it this way (Indicates the way she worked the problem.) because this is the way the graduation exam, the SAT's, the ACT's are all going to be set up so it concerns me that yes you can rationally think it, but what if the problem was given to you in this form and you didn't have the written thing. Could they still get to the solution?

Thus, Mrs. Colley exhibited limited pedagogical content knowledge on the ALCKIN. However, she was able to determine the elementary methods for solving the wheels and vehicles were acceptable ways of solving problems.

Further clarification of Mrs.Colley's pedagogical content knowledge will be evident in the following section describing her classroom instruction, as well as her ability to apply this knowledge in her classroom.

Classroom instruction

Mrs. Colley's content knowledge was primarily reflected in her providing students with step-by-step instructions on how to solve and/or simplify algebraic expressions. No errors in her content knowledge were observed. Mrs. Colley showed an understanding of each of a variety of algebraic topics as well as the ability to procedurally work related problems. For example, the following classroom dialogue took place when Mrs. Colley was reviewing how to solve quadratic equations by factoring in preparation for a semester exam:

> Mrs. Colley: Okay let's start off by looking at the factoring. Okay? (She wrote ex1) $x^2 – x – 12 = 0$ on the whiteboard.) The first thing we need to check for when we're given the problem is it is in standard form? Who can tell me what is standard form of a quadratic is?
>
> Student: $ax^2 + bx + c = 0$
>
> Mrs. Colley: Close. a x squared plus b x plus c. Okay. And all that needs to be on one side of the equal sign, right? The whole thing needs to be set equal to zero in order for us to solve it? Okay. (Writes formula and reminds students it needs to be set equal to zero.) Now the first problems done with factoring were problems

where there’s one in front of the x squared. A one for our a value. Right? Those were the first type we looked at. Okay and for those we took the c value, right, and in this case the c value is negative twelve and we looked at it’s factors, one and twelve, two and six, three and four, right? (She writes these off to the side on the whiteboard.)

Student: Um humph.

Student: Yes.

Mrs. Colley: Alright, then what did we look at when we saw all these factors we had listed?

Student: The middle number.

Mrs. Colley: Which one of these could equal the b value, the middle number, right?

Student: negative four and three

Mrs. Colley: Negative four and three. Okay. Multiply together they do give us the negative twelve, added together they do give us the negative one, right? So our factors for this problem were what?

Student: x plus three and x minus four.

Mrs. Colley: x plus three and x minus four. So when you’re solving by factoring you’re not done yet are you?

Student: No

Mrs. Colley: So what do we do? (Students mumbling.) Set them both equal to zero. Good. And then we solve each of them independently, right? How do you get x by itself here?

Student: Minus three

Mrs. Colley: You subtract three on both sides. So I get x is equal to negative three. How do I get x by itself here?

Students: Add four

Mrs. Colley: Add four. So x is equal to four. Those are your two solutions, alright?

After giving students the “steps”, Mrs. Colley gave the students a similar problem to work and walked around the room observing and answering any questions that they might have.

Classroom instruction did not seem designed to encourage students to develop conceptual understanding for the algebraic topics they were. The average of the RTOP scores from the classroom observations during the spring of 2004 was 27.5.

Thus, classroom instruction revealed generally strong content knowledge but limited pedagogical content knowledge. Mrs. Colley was not observed making errors of her own.

Student questioning. Mrs. Colley’s questions primarily focused on answers to arithmetic problems, providing the next step in the procedure, or the simplification to an algebraic expression. The following conversation is typical:

Mrs. Colley: (Wrote 34) $\frac{1}{2}+\frac{2}{x}=\frac{1}{x}$ on the overhead projector.) Alright what’s the first thing we need to do?

Student: Find a common denominator.

Mrs. Colley: Find what the common denominator is right? See what they have in common. What is the common denominator?

Student: two x

Mrs. Colley: Good. Two x. Two goes into two x how many times.

Student: x

Mrs. Colley: x times. x times one is?

Student: x

Mrs. Colley: x. x goes into two x

Student: two times

Mrs. Colley: two times. Two times two is?

Student: Four

Mrs. Colley: x goes into two x

Student: two

Mrs. Colley: Two times. Two times one is?

Student: two

Mrs. Colley: Does anybody need to see that redone? (Goes back through changing the fractions to common denominators again.) Okay. (Writes on the overhead) $\frac{x+4=2}{2x}$ Now we can just look at the numerator. x plus four is equal to two. So how do I solve for x?

Student: Subtract four.

Mrs. Colley: subtract four. So x is equal to negative two. Alright?

It should be pointed out, as in the previous teacher/student dialogue, Mrs. Colley frequently repeated her students' statements.

Thus, Mrs. Colley questioned her students often but her questions did not require much mathematical thought. They were not the types of questions or problems posed to be productive in helping students learn mathematics.

Analyzing student errors. Within her own classroom, Mrs. Colley handled the errors made by her students in two distinct ways. First, Mrs. Colley would give further comments on questions asked by her students. For example, when using the quadratic formula to solve quadratic equations students had taken the square root of sixteen and got four. One student wanted to take the square root again and Mrs. Colley responded with "No, once you've taken the square root you don't take the square root again." Secondly, Mrs. Colley responded with yes/ no responses without any further explanation. For example, Mrs. Colley had asked her students for three methods to use when solving quadratic equations, and the following classroom dialogue took place:

> Mrs. Colley: …..Alright. Let's go back and look at where we started this nine weeks. We started this nine weeks in chapter ten. Okay. In chapter ten that was that horrible word factoring. Alright? Ya'll just thought that was horrible.Okay, alright so we started off looking at the three ways to solve quadratics. Alright the first way we learned how to use and that was factoring, right?
>
> Student: Yea.
>
> Mrs. Colley: Does anybody remember the other two ways?
>
> Student: Elimination
>
> Mrs. Colley: No

Student: Substitution

Mrs. Colley: Those two things would be the same thing as factoring.

Student: The quadratic equation.

Mrs. Colley: The quadratic equation, the quadratic formula and the other one was completing the square. Okay do ya'll remember the quadratic formula?

Students: Yes

Thus, Mrs. Colley was able to recognize and respond to the errors her students were making. However, she did not respond by suggesting alternative explanations and models.

Multiple methods of solving problems. Mrs. Colley's classroom instruction did not include attention to accessing different representations or different methods for solving a problem. Mrs. Colley controlled the instruction within her own classroom with little input from the students so it was difficult to determine whether the students were allowed to work problems using methods other than the ones she stressed in class.

Conclusion

Mrs. Colley felt that all students given time and the proper type of instruction could learn algebra. Mrs. Colley exhibited strong procedural knowledge but limited conceptual knowledge. Explanations on the ALCKIN were all procedural in nature, and Mrs. Colley could not offer another way to show the two algebraic expressions were equivalent on the same instrument.

Mrs. Colley appeared to know the algebra content she was teaching and did not appear to make mathematical errors. Her classroom instruction consisted of giving step-by-step procedures. The types of questions Mrs. Colley asked her students were ones that could be answered with a mathematical solution or the simplification of an algebraic expression or simply stating what should be the next step in a mathematical process.

Mrs. Colley was limited in her ability to work algebra problems using a variety of methods. Moreover, on the ALCKIN she was unable to solve the quadratic problem using the spreadsheet. She appeared to be able to recognize when students were giving incorrect answers or when students suggested the use of incorrect methods for solving problems. Responses on the ALCKIN indicate she did not always choose to respond to the errors in the statements provided by the students.

Professional Development

During the summer of 2004, Mrs. Colley attended the algebra two sessions of the two week MDSMIP summer professional development training. Mrs. Colley's content sessions included covering the IMP Fireworks unit (Fendel et al., 2000) along with parts of the Pennant Fever unit (Fendel et al., 2000). Content covered within the Fireworks unit (Fendel et al., 2000) dealt with methods for solving quadratic equations which involved factoring, root finding, completing the square, and using algebra tiles. The content covered within the Pennant Fever unit (Fendel et al., 2000) dealt with different types of probability.

Mrs. Colley's felt her attitude changed as the professional development progressed during the summer sessions, as follows:

> Mrs. Colley: Ah, I don't know, I don't want to give up two weeks of my summer.
>
> Interviewer: So how did your attitude change over the two weeks you were there?

> Mrs. Colley: I realized that once I got in there I was getting a lot of good stuff. And that was worth it.

The presenters saw little or no change in Mrs. Colley's attitude as she continued to attend the professional development training. One presenter felt that Mrs. Colley was "a little apprehensive" at the beginning of the professional development. A second presenter stated:

> I don't know that I saw her attitude change. I think she came in with a good attitude and she sounded like she wanted to learn something. She did not, she did not appear resistant to being there and there were occasions that she discussed how her students were and how they might react to what we were doing, but she didn't seem like she was arguing about it. She seemed like she was willing to give it a go and see if she could do this with her students.

The presenters expressed that they felt Mrs. Colley showed a willingness to make changes in her classroom instruction as a result of the professional development. One of the presenters stated:

> She seemed to be open to it. There were others in the class that argued about that their students couldn't possibly do it, what we were doing but she didn't come across with that kind of attitude in the class, like she was willing to do something different.

Another presenter pointed out "Yes, yes but she was, did keep questioning the problems that all of us have. For instance how does she make her students want to do this?" A third presenter further reiterated what the other two presenters had to say about Mrs. Colley.

> Well I think she was very receptive to the MDMSIP model of learning and ah I see her as a very receptive person overall and would be someone who would be, you know, someone who would be willing to try new things and try a new style of teaching and things like that.

Mrs. Colley attended all of the quarterly meetings during the fall of 2004 and the spring of 2005. She felt like she got more from the quarterly meetings than she did from the two weeks of summer professional development training; see the following excerpt from the interview:

> Mrs. Colley: I actually got more out of the Saturday meetings than I did from the others.
>
> Interviewer: Okay, tell me how it was more helpful.
>
> Mrs. Colley: How, well the others teachers were coming back and saying, you know, I used this, this and this and it worked.
>
> Interviewer: Okay
>
> Mrs. Colley: So I came back and I was immediately able to put into play and saw that it really did work instead of having to wait until the time came to do it because we're all just about in this same area as far as where we are in teaching the curriculum. So when you go to one of the Saturdays and say hey you know last week we did such and such and we did it this way in my class and it really worked and so you're able to go back and say hey, you know, look I've got this new way to do this. Look at it this way and I really got more, I get more from the

> Saturdays than I do from the other because I'm able to immediately come back and use it.

Mrs. Colley shared her experiences with other teachers in order to help them make better decisions regarding their own classrooms based on what had happened with her. For example,

> Algebra B, trying to put IMP units in is overwhelming! We goofed. We did the way we usually do. But use About Alice (referring to All About Alice, an IMP unit), use sections eight one and eight two, days one through seven, and section eight three, days sixteen and seventeen. The rest is longs and fractional exponents. We (indicating the teachers at her school) are discussing what we are going to do because they are not getting it!

One presenter noted that Mrs. Colley "knows her algebra." When asked about Mrs. Colley's content knowledge, another presenter stated, "I believe all of the basic knowledge." A third presenter assessed her knowledge in terms of the types of mathematics that were covered during the two weeks of summer professional development training provided by MDSMIP. She stated:

> For the quadratic stuff she seemed to be perfectly at ease with all of it. I mean there wasn't, there wasn't any difficulty with that. And she seemed to be at ease with the, with the other part also. I mean she was I think particularly in that group she was very good.

Thus, Mrs. Colley was very involved in the professional development training . Presenters felt her knowledge of the algebra content was good. They also felt she would try to implement changes into her instructional practice. She was willing to share success and failures with the other teachers at the quarterly meetings.

Post-Professional Development

During the post-professional development timeframe, we will consider changes in her classroom environment, in her views of student ability, in her content knowledge, and in her pedagogical content knowledge. Moreover, we will consider how changes in content knowledge and pedagogical content knowledge are reflected in instructional practices. Each of these areas is addressed in the following sections:

Classroom Environment

The layout of Mrs. Colley remained unchanged from the previous school year. She continued to be personable and exhibited a patience attitude with her students. Group work from the Fireworks IMP unit (Fendel et al., 2000) was displayed on the hallway wall outside of Mrs. Colley's classroom, suggesting that ….

Student Ability

In the spring of 2005, she stated that she felt all of her students could learn algebra well and supported this statement as follows:

> The percentage of them that can learn algebra well given the desire to do so. It would be one hundred percent. They all could do learn to do it. But a lot of them don't have the desire to do it. …Ah, I think a lot of it has to do with how it is presented and see I'm looking at the kids in my class, I'm not looking overall. And I really and truly believe that I can get to all of them. But I think at some point it does fall on the students shoulders. They have to be willing to accept me helping them to get it.

Thus, Mrs. Colley felt that all of her students can learn algebra but learning may depend on how it is presented to students. However, students need to be willing to seek help.

Changes in Content Knowledge

Mrs. Colley was asked to assess her own content knowledge of algebra. She responded as follows:

> Not very good. Algebra one I've got. Algebra one I could battle with just about anybody else on the planet. Algebra two I'm still struggling…

Mrs. Colley felt that the professional development training did have an affect on her content knowledge of algebra, and she reaffirmed that she had learned new content when she stated:

> Ah, let's see. Yea I completely got where the completed square form came from. And I don't think I had really gotten that before. I had seen it done but I didn't really get it. I could do it, I could manipulate it the way somebody showed it to me to manipulate it but I didn't get it. And I got it.

She further stated that the training "…helped in making connections. But it also gave me a chance to realize that I'm not the only blooming idiot out there. There are other people who are struggling with this stuff just like I am."

It should be noted that during an interview in the spring of 2005 that Mrs. Colley was able to solve the quadratic equation problem $x^2 = 2x + 8$ (see Appendix E) using factoring, using the "magic square" method, and using algebra tiles (see Figure 23).

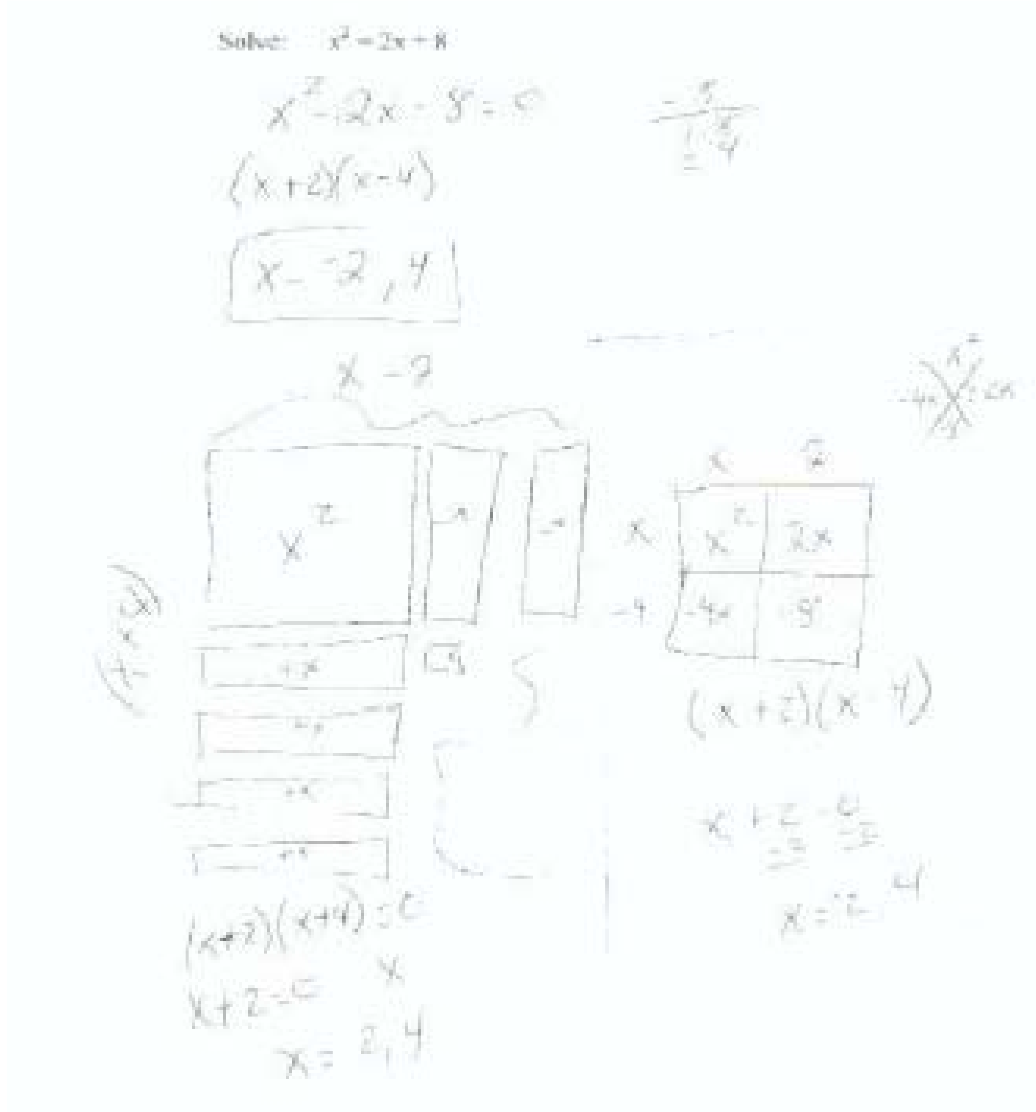


Figure 23. Mrs. Colley's work for the quadratic problem.

The three methods all involved factoring but in each method she used a different type of factoring.

She noted the effect these different methods had on the success of her students within her own classroom. Her conversation related to this transition went as follows.

Mrs. Colley: I taught it all three ways. I started off with the algebra tiles. And then once they got the fact that ah they began to see the pattern of the fact that hey the sides are multiples of the c value. They started making that transition and so then they started working it that way and so then I showed them the magic square way. And some of them have jumped to the first way that I done it just because they have developed the complete understanding and can skip sections. But I've learn to let them do that on their own. But some of them are still having difficulty making the jump from the tiles to that.

Interviewer: Last year did you do the algebra tiles?

Mrs. Colley: No

Interviewer: Last year did you do all three ways?

Mrs. Colley: No I did the first one.

Interviewer: Okay, so have you seen a difference in what your students are doing?

Mrs. Colley: Oh yea and I can see where I'm getting to different ones of them without them having to bang their heads. And them getting frustrated. Octavia in fact would never get the first way. They got the algebra tiles.

Interviewer: Okay, so this way (points to her drawing of algebra tiles)

Mrs. Colley: That way, yes, and they are still there. They're one of the ones having a hard time making it to here (magic square).

Interviewer: But they could solve this? (Points to the quadratic equation problem.)

Mrs. Colley: They could solve that using it. If they had enough algebra tiles to work the large numbers, that's the way they would choose to do it.

Thus, Mrs. Colley demonstrated changes in her content knowledge. While she may have previously suggested the use of manipulatives, she demonstrated solving a quadratics problem by using them. In addition, she was bridging from the physical manipulatives, to drawing pictures, to her “magic square”, and eventually to applying rules for factoring. Furthermore, she gave an example of content that she better understood as a result of professional development.

Changes in Pedagogical Content Knowledge

Growth in her content knowledge directly connected to changes in Mrs. Colley’s pedagogical content knowledge. This growth in pedagogical content knowledge was evident from the previous example in her use of algebra tiles and in helping students make connections between physical objects and the rules for factoring polynomials. From her conservation, it suggests that her increased pedagogical content knowledge aided in the success of her students learning mathematics.

Reflection of Changes in Knowledge in Instructional Practices

With growth in both her content knowledge and pedagogical content knowledge, one might expect that these changes were reflected in her instructional practices. In one instance, Mrs. Colley had problems for her students to factor written on the whiteboard as students came into her classroom. One of the students factored $16a^2 – 24a + 5$ into factors $(4a + 1)$ and $(4a + 5)$. The following classroom dialogue took place:

> Mrs. Colley: Added together gives you the negative twenty-four. Correct? Sixteen a squared minus four a, or negative four have a four a in common. Right? Twenty a and five, have a five. Sixteen a squared, negative twenty a have a four a.

Negative four a and five have a one. So this is your answer (4a + 1)(4a + 5), whoa what's wrong here?

Student: The 4a should be negative.

Another Student: The four is not negative.

Mrs. Colley: Which four?

Student: In your graph, in the box on the side with your first negative. One of the four a's has to be negative because in your second block on the top, the four a is negative. Then make your four a negative, then make your other four a negative so when you multiply you get negative twenty a

Mrs. Colley: Humph?

(Students think one or both of 4a's should be negative)

Student: Do you want me to show you?

Mrs. Colley: Show us.

Student: This right here has to be negative so when you multiply these right here you get negative. Okay make this negative but then you come right here and this will make this negative so you add this one negative and that works out down here and gives you negative twenty a so in that way it comes in a negative twenty a. (Student goes to whiteboard and puts in negative on both 4a's and comes up with this.)

	-4a	1
-4a	16a	-4a
5	-20a	5

Mrs. Colley: Alright, do you see what she did?

Student: No.

Mrs. Colley tried to explain what the student had done but had to check the answer by multiplying the two factors together to make sure that they equaled the original algebraic expression. The factors did in fact give the correct answer and she proceeded to go over what the student had done. Mrs. Colley then asked if there was another way to factor the same expression, and a student came up with factoring out negative one and negative five and writing the factors as (4a – 5) and (4a – 1). She agreed that both sets of factors were correct in the following discussion:

Mrs. Colley: Okay you could have done your four a minus one and your four a minus five. And still works, right? That's positive sixteen a squared, that's negative four a, that's negative twenty a, and that's negative five. And that's what threw me off because I'm used to doing it that way instead of the other way.

Student: That's five

Mrs. Colley: Both, they're both right. Okay. So in that case your answer would be four a minus one and four a minus five, right? Okay, Good!

Thus, from the use of different representations and different methods for factoring polynomials, Mrs. Colley was able to respond to her student's error. What resulted was a different representation of the factors of the trinomial that she did not immediately recognize as correct. She was able to determine that the student had factored the trinomial correctly, even though she checked it with procedures.

Further evidence of use of a different kind of mathematical knowledge was evident in a lesson in which Mrs. Colley used a hands on activity related to the difference of two squares in her Algebra IB class during the spring of 2005. The paper square manipulatives had another square marked within the upper right hand corner. The first square had numerical measurements, while the second square had variable measurements. The class worked as a whole group and Mrs. Colley took time during instruction to allow students to make and record observations which were discussed within the class. Mrs. Colley instructed the students to physically remove the smaller square and find the lengths of the sides after the smaller square was removed, area of the original large square, area of the smaller square, and the area of the remaining polygon. The students took the remaining polygon and divided it along a diagonal so that the two pieces formed a rectangle. Dimensions of the rectangle were found and the area computed. The students surmised from the exploration that the area formed by the region from the differences of a squared and b squared had the same area as the rectangle whose dimensions were a plus b and a subtract b. The RTOP score during this classroom observation was fifty-two which was significantly higher than her previous RTOP scores.

Conclusion

Mrs. Colley exhibited a positive attitude during professional development and the presenters felt she would be willing to make instructional changes based on this professional development. Growth was seen in both her content knowledge and pedagogical content knowledge. This growth was seen in her ability to work problems incorporating non-standard methods, as well as recognizing student answers she was not expecting. There was evidence from classroom observations, quarterly meetings, and work exhibited by her students that she was implementing changes in her instructional practices. Although she was allowing students to do activities, she directed instruction and still had students do mathematics in a procedural way. Colley RTOP scores showed an increase from the classroom observations made during the spring of 2004 to the spring of 2005.

Conclusion of Case

Mrs. Colley felt that all of her students could algebra given the proper type of instruction, this attitude did not change. She felt she knew the algebra I content well. Prior to the professional development, she exhibited strong procedural knowledge and limited conceptual knowledge. Her pedagogical content knowledge was also limited. She was not observed making mathematical errors. However, changes in both her content knowledge and pedagogical content were observed following the professional development. Using algebra tiles, making drawing, moving to a “magic square”, and bridging these methods to formal rules were all evident. Evidence that this type of instruction had been used was seen in the types of work displayed by students, which was not observed during pre-professional development.

Conclusion

The content knowledge and pedagogical content knowledge of these cases prior to professional development will first be addressed. All of the participants in the case studies felt they had good algebraic content knowledge. Three of the case study teachers, Mrs. Cotney, Mrs. Willoughby, and Mrs. Colley, exhibited strong procedural knowledge while Mrs. Pitchford exhibited weaknesses in her procedural knowledge. However, all of the subjects exhibited limited conceptual knowledge. None of the case study participants could work problems using multiple methods. However, all of the cases could recognize that the elementary approaches provided in the wheel and vehicle problem were viable ways of working the problem. Little evidence of pedagogical content knowledge was was seen in either the ALCKIN or interviews.

We next consider how the participants used their content knowledge and pedagogical content knowledge during classroom instruction prior to the professional development. Their procedural content knowledge was reflected in the instructional practices, which largely consisted of presenting step-by-step procedures to their students. Mrs. Cotney, Mrs. Willoughby, and Mrs. Colley did not make exhibit errors in their content knowledge during instruction, while Mrs. Pitchford did. All of the case study participants asked questions involving answers to arithmetic problems, simplifying algebraic expressions, or naming the next step in a procedure. However, these types of questions were not of the type most productive in promoting successful learning of mathematics by students. None of the participants were observed providing classroom instruction that was conducive to students developing a conceptual understanding of mathematics.

Three of the participants, Mrs. Cotney, Mrs. Willoughby, and Mrs. Colley, provided step-by-step mathematical instruction to their students. They were able to recognize the errors in the thinking of their students and respond to them. However, their responses did not include alternative explanations or models for representing problems. They were able to recognize when students worked problems using a variation of the procedure they presented, but no solutions significantly different from the presented solution were given or encouraged. The average RTOP scores during the spring of 2004 for these three case study teachers were similar.

Mrs. Pitchford was different from the previous three participants, since she provided very little instruction in the classes that were observed. Thus, it was difficult to determine how her content knowledge and pedagogical content knowledge were used in instruction. In the limited classroom instruction that was observed, she made mathematical errors. She also had difficulty in recognizing and responding to why her students were making errors. The RTOP score on this case study teacher was lower than the other three cases.

Classroom observations revealed similarities in the pedagogical content knowledge all four cases used. Their instructional practices involved procedures and emphasizing procedural knowledge through the questioning they provided.

Three of the participants, Mrs. Cotney, Mrs. Willoughby, and Mrs. Colley, attended the summer professional development training. All of these case study teachers agreed that the professional development training was beneficial. None of the participants felt their content knowledge had changed as a result of professional development, a view with which the presenters agreed. Mrs. Pitchford did not attend the summer professional development training but was finishing up coursework for her education specialist degree.

The three case study participants who attended the professional development, Mrs. Cotney, Mrs. Colley, and Mrs. Willoughby, all exhibited changes in content knowledge and/or pedagogical content knowledge following the professional development. Mrs. Cotney referred to “in/out” tables to before graphing linear equations, and mention that she better understood how the algebra content fit together. Mrs. Colley and Mrs. Willoughby both emphasized the use of algebra tiles when working with algebraic expressions. All three could offer more than one method to work the quadratic equation during the second interview. Mrs. Colley and Mrs. Willoughby also worked the quadratic problem using conceptual methods such as algebra tiles or drawing pictures.

The other two case study teachers did exhibit changes in mathematics instruction. Mrs. Willoughby demonstrated the use of algebra tiles at a quarterly meeting. She shared how she used them in her instruction with the other teachers. Mrs. Colley used algebra tiles, drawings and her “magic square” when factoring trinomials with her students. Both of these participants felt that all students could learn algebra but at different paces. These two teachers had increased RTOP scores from the spring of 2004 until the spring of 2005. Presenters of both of these teachers felt they would implement changes in their classroom instruction, and observations suggested that they did.

However, even though three of the cases exhibited changes in their content knowledge and pedagogical content knowledge and two of them exhibited some changes in their instructional practices, the pedagogical content knowledge of all the case study teachers was still quite limited. No changes in content knowledge and pedagogical content knowledge of Mrs. Willoughby were observed in her instructional practices. Mrs. Colley’s students used “magic squares” when displaying work on the board. However, the changes in content knowledge and pedagogical content knowledge were not observed in classroom observations. It can be said that they had used manipulatives or “in-out tables”, which was accessing different representations, and in some instances students were making the connections from these to the rules. However, it cannot be said that the manipulatives or other forms of representations were used in a different way from teaching procedures since their introduction was not observed. In addition, other areas considered in pedagogical content knowledge were not obvious. How they “unwrapped” the mathematical topics and presented them so students could successfully learn mathematics was not observed. How these teachers made connections between mathematical topics was not seen. The teachers did not respond to student errors with alternative explanations and models, although it might be argued that using manipulatives was using alternative models. Neither was using them in response to student errors obvious. How the teachers responded to questions still remained the same as before changes in knowledge. It did not appear that these teachers posed problems and questions that facilitated the successful learning of mathematics.

VI. SUMMARY AND RECOMMENDATIONS

To become productive members of society, adults of the twenty-first century need to be proficient in mathematics (Ball, 2003a), and the need for mathematics in our everyday lives continues to grow (NCTM, 2000). However, in the United States we are not preparing our students for the demands to be mathematically proficient. According to results of the Third International Mathematics and Science Study (TIMSS) (NCES, 2003a) and the National Assessment of Educational Practices (NCTM, 2004), our students do not achieve at the levels necessary for mathematics success. Mathematics preparation of students reaches a critical juncture with the Algebra I course. Algebra serves as a gatekeeper course, offering differential opportunities for entry into advanced mathematics courses (Ball, 2003a), for preparation for college (Pascopella, 2000, Lawton, 1997, Chevigny, 1996, Silver, 1997, Olson, 1994), and for preparation to enter the world of work (Silver, 1997).

Teachers play a key role in ensuring that all students have the opportunities and experiences needed to learn mathematics (Mewborn, 2003). Therefore, we must consider the types of knowledge needed to provide all students with equitable opportunities to learn algebra. Studies have shown that teacher's content knowledge is often thin and inadequate to provide the instructional opportunities needed for students to successfully learn mathematics (Ball, 1998a, 2003b, Ball & Bass, 2000; Fuller, 1996, Ma, 1999, Mewborn, 2001, Stacy, et al., 2001). Moreover, teachers need to know how to use their mathematics knowledge in facilitating the learning of mathematics by their students (Sherin, 2002), which is often referred to as pedagogical content knowledge (Shulman, 1987).

This study seeks to better understand teacher's content knowledge and pedagogical content knowledge. While some research has been conducted in this area at the elementary level (cf. Ball et al. 2005), far fewer studies have been done with secondary teachers. We need a better understanding of content-specific knowledge and pedagogical content knowledge (RAND, 2000), as well as how they are used during mathematics instruction (Ball, 2003a).

An additional question lies in the extent to which professional development can change both content knowledge and pedagogical content knowledge of teachers. Effectively developing content knowledge and pedagogical content knowledge for teaching remains an unsolved problem for the improvement of mathematics teaching and learning (Ball et al., 2001). Furthermore, even if professional development can facilitate changes in both content knowledge and pedagogical, we do not understand how these types of knowledge might influence changes in teachers' instructional practices. The possession of content knowledge and pedagogical content knowledge do not automatically mean successful mathematics instruction (Ball et al. 2001).

This study was design to answer the following specific research questions related to the broader issues raised above:

1. What content knowledge and pedagogical content knowledge of algebra do high school algebra teachers possess?

2. How are high school algebra teachers' content knowledge and pedagogical content knowledge reflected in their teaching practices?

3. What growth in content knowledge and pedagogical content knowledge can be seen from participating in professional development that includes attention to increasing both content knowledge and pedagogical content knowledge?

4. How are changes in content knowledge and pedagogical content knowledge reflected in teachers' instructional practices?

In the following sections, answers to these questions will be presented, based on the findings of this study. We will then consider limitations of the research and implications for the field of practice. Finally, we will explore areas for further research.

Summary

This study was comprised of two major investigations, one involving survey research and the other multi-case studies. Survey research provided the opportunity to study the content knowledge, pedagogical content knowledge and attitudes of a large group of teachers. The multi-case studies provided the opportunity for an in-depth study of four teachers and included the opportunity to understand how the case study teachers used their content knowledge and pedagogical content knowledge in their instructional practices. The case studies also provided the opportunity to probe for depth in content knowledge and pedagogical content knowledge. The results of each of these investigations have been summarized in previous chapters, and we will now review what we have learned about of the research questions as a culmination of both of these methodologies.

1. What content knowledge of algebra and pedagogical content knowledge do algebra teachers possess?

We will focus our attention to the areas that make up content knowledge. They are procedural knowledge, conceptual knowledge, and the mathematical processes. Did participants demonstrate the ability to use the rules and algorithms of mathematics, or procedural knowledge? On the parts of the ALCKIN that required a selection of answers, only seven of the twenty-five tasks were answered correctly by more than eighty percent of the participants. This ability to do mathematics procedurally was predominant in explanations and in the procedures suggested in clarifying answer choices. The majority of the case study teachers exhibited strong procedural knowledge and performed additional mathematical tasks using procedures.

However, there were errors made in the procedures used by some of the participants. This was true for all the items on the ALCKIN. This included computational errors and errors in mathematical notations. The majority of the case study participants did not make procedural errors.

Was conceptual knowledge evident in the participants? The conceptual knowledge of all the participants in both studies was limited. A limited view of conceptual knowledge was exhibited by using drawings or algebra tiles or through the suggestion of using manipulatives. Gaps were exhibited in the understanding of topics on the ALCKIN, such as understanding of algebraic expressions, understanding of functions, and slope.

Did participants exhibit use of the mathematical processes? Participants in both studies had a difficult time in providing different representations for mathematical situations. This was true with the case study teachers as well. Some of the participants did not recognize that algebraic expressions, although equivalent, cannot be used to describe different interpretations by students. Less than half of the participants could explain the meaning an algebraic expression within the context of an algebraic equation. In addition, the majority of participants did not provide or use multiple methods for solving mathematical situations, and they tended to be quite procedural in nature. When the case study teachers were pushed to provide other ways of solving mathematical tasks, even alternative procedural methods could not be provided.

It might also be noted that a mathematics education degree does not always ensure the content knowledge and pedagogical content knowledge to teach mathematics (Ball et al., 2001). This was observed in the case of the teacher who was completing her education specialist degree in mathematics education. She exhibited weak content knowledge, both in procedural and conceptual knowledge. She had difficulty in recognizing why her students made errors and difficulty herself in working mathematical tasks. In addition, she exhibited a lack of understanding for the connections between mathematical topics and difficulty in using and recognizing different representations as well as methods for solving mathematical problems. One would hope that this teacher is an isolated case!

In considering the pedagogical content knowledge of teachers, our examination will be guided by how the definition from the literature review, which briefly follows. Pedagogical content knowledge is the ability to "unwrap" and present mathematical topics so that students can be successful in learning mathematics. Pedagogical content knowledge includes the ability to access different representations as well as methods for solving mathematics problems. Pedagogical content knowledge is the ability to recognize student errors and be able to respond to them with alternative models and explanations. Pedagogical content knowledge includes the ability to responds to questions, and to pose questions and problems that are productive to students learning mathematics.

First, was the ability to "unwrap" mathematics topics and present them so students can be successful in learning mathematics evident? Actual mathematical instruction was not used in answering this question, however if participants lack an understanding of mathematics topics it could be argued that they do not have the ability for unwrapping them and presenting them in ways so students can be successful.

Next, do teachers have the ability to access different representations as well as methods for solving mathematics problems that arise within mathematics instruction? Participants had a difficult time in solving the spreadsheet problem on the ALCKIN. The different representations of slope also caused a majority of the participants' problems in recognizing they were equally valid. The case study teachers were able to use their pedagogical content knowledge in analyzing non-procedural methods used by hypothetical students within problem solving context during interviews.

Did the participants have the ability to recognize student errors and be able to respond to them with alternative models and explanations? Participants had difficulty in assessing student errors on the ALCKIN. Instead of responding to errors or statements made by students, participants generally ignored them or responded how the student should have worked the problem using a set algebraic manipulation, rather than providing alternative explanations or models.

Did the participants have the ability to respond to questions, and to pose questions and problems that are productive to students learning mathematics? While the design of the ALCKIN did directly address this issue, none of the case study teachers exhibited this particular part of pedagogical content knowledge in interviews or in classroom observations.

Overall, it can be stated teachers in this research study had content knowledge. However, their knowledge was primarily procedural and they had limited conceptual knowledge. Also lacking was their ability to use various representations as well as use different methods to solve problems. We can claim that the participants did not have a deep understanding of the algebra content. Participants also exhibited limited pedagogical content knowledge. This was evident in all areas addressed related to the definition of pedagogical content knowledge.

2. How are high school algebra teachers' content knowledge and pedagogical content knowledge reflected in their teaching practices?

Again we return to what constitutes content knowledge as defined in the literature refiew. First, procedural knowledge was predominant within instructional practice. Case study teachers demonstrated use of procedures within a variety of mathematics topics in almost all of the classroom observations. Three of the cases were not observing making procedural mistakes, while one case study teacher did. However, when we consider their conceptual knowledge, it was seen to be very limited. There was not an occasion that could be identified where the case study teachers exhibited they knew why rules and algorithms worked. Were the mathematical processes evident in instructional practice? Case study teachers were not observed using different methods of working problems. Conjectures were not investigated, nor was the case study teachers' ability to develop and evaluate arguments observed either. Teachers did communicate ideas and clarify what they meant but it was in superficial ways and relating to providing procedures and explaining what to do next. Case study teachers were not observed making connections between mathematical topics. Representations of mathematics did not go beyond the procedures for doing mathematics.

Let us return to the definition of pedagogical content knowledge and see how that definition is reflected in instructional practice. First, consider the ability to "unwrap" the mathematical topics and present the content in ways for students to successfully learn the mathematics. My conclusion is that there was no unwrapping of mathematical topics or ideas, only presentation of particular procedures.

Second, did the teachers access different representations as well as different methods for solving mathematics problems that may arise within the mathematics instruction? Once again this was not obvious. While teachers may have felt that their students were using alternative methods, in actuality they were generally the same procedures used by the teacher, but using a different order of steps or omitting some steps altogether. Since the teacher was dominant in mathematics instruction, little opportunity was given for other types of mathematical problems to arise that would necessitate different representations.

Next, did the teacher use her ability to direct students in making connections between mathematical topics as well as the connectedness of different representations for those same topics? Connections between mathematical topics were not noted and since the case study teachers did not use, or have students use, different representations for mathematical topics, this facet of pedagogical content knowledge was not present either.

What was the ability of teachers to understand where and why students make errors and be prepared with alternative explanations and models? Analysis of student errors within the context of mathematical instruction generally related to incorrect mathematical computation errors, the teacher recognizing incorrect answers to algebraic expressions or equations or, or using incorrect procedures in completely algebraic tasks. All student errors related to the mathematical topics covered during a particular day's instructions. Neither did results show teachers offering alternative explanations or models.

Did the pedagogical content knowledge of the case study teachers include their ability to respond productively to students' questions and pose problems and questions that are productive to student learning? Case study teachers could respond to questions their students asked during instruction, but the questions were generally about the correctness of an answer, if a student was doing the work correctly, or what the student should do next in solving the mathematical task. Moreover, teachers did ask their students questions productive in successful learning of mathematics, but they generally related to asking students to give short answers to arithmetic problems, the simplification of algebraic expressions, or to name the next step in a procedural process. Therefore, once again the type of questions required of a teachers' pedagogical content knowledge was limited since these questions were not the type that would be productive to students learning mathematics in the ways they need to.

From the Teacher Attitude Survey, participants felt they had to model and demonstrate mathematics and then allow time for students to practice these procedures. This is how mathematical instruction was reflected in a majority of the case study teachers. Half of the cases felt that by allowing students time to practice procedures, this was the same as helping them make sense of the mathematics. This view of mathematics instruction is limited when compared to what pedagogical content knowledge includes. This prevalent attitude along with the lack of depth in content knowledge may further suggest why the pedagogical content knowledge was limited in instructional practices. Content knowledge, but not a deep understanding of it, was reflected in the instructional practices of the case study teachers. However, limited pedagogical content knowledge was observed during these classroom settings.

3. What growth in content knowledge and pedagogical content knowledge can be seen from participating in professional development that includes attention to increasing both content knowledge and pedagogical content knowledge?

Changes in the procedural knowledge of the case study teachers were not observed. However, changes in conceptual knowledge were evident, which was expressed either by demonstration and/or in conversations. Mrs. Willoughby demonstrated the use of algebra tiles in showing the addition of algebraic expressions. Mrs. Colley drew pictures of algebra tiles and a “magic square” in solving a quadratic problem. Both of these examples show that the case study teachers could solve problems in different ways and could communicate their understanding clearly. They were also able to makes connections to the procedures used and it was obvious they used different representations in demonstrating they understood how to solve the quadratic problem using different methods. All of these case study teachers were able to suggest other non-procedural ways of solving the quadratic equation. Two of them offered conceptual ways of solving the problem. In pre-professional development only one of the case was able to supply more than one way of solving problems.

Growth in pedagogical content knowledge was also evident. Mrs. Willoughby presented the use of algebra tiles in ways she said she used them in instructional practices. She provided the connections to using algebra tiles which included: using the tiles, drawing pictures and the bridging to the algorithms related to their use. Mrs. Colley also demonstrated that she was using algebra tiles for the multiplication of binomials and factorization. Although instruction was not observed using algebra tiles, it was evident that students had learned with them from the work they did on the board.

Conversations with one of the case study teachers included such terminology as "in-out" tables and how they could be used before introducing terms such as function, domain, and range. She also indicated that she had used them during instruction.

4. How are changes in content knowledge and pedagogical content knowledge reflected in teachers' instructional practices?

Strictly limiting the answer to this question to how changes in content knowledge and pedagogical content knowledge were observed in classroom observations, the answer would be simple. There was very little difference in how the content knowledge and pedagogical content knowledge was reflected through during classroom observations between pre- and post-professional development. It can be noted that one of the cases was using an activity to help students develop the conceptual understanding for factoring the differences of two squares, but even in this one instance, she controlled the classroom conversation and limiting the opportunities for her student to really develop the understanding they needed. However, when we consider the types of observations such as the demonstration of algebra tiles with the two case study teachers and the conversation about "in-out" tables, we might be able to suggest that it was possibly reflected in their instructional practice just not on the days of observation. Also the display of student work from the IMP units (Fendel et al., 2000) would suggest that changes in both of these types of knowledge are reflected in the case study teachers' instructional practices. These are limited views of pedagogical content knowledge, because they do not get everything the definitions of these types of knowledge entail. Furthermore, without these observations within the classroom context, we cannot say that it is used as a tool in helping students make sense of the mathematics or as just another procedure.

Limitations

Although survey research and multi individual case studies were considered the best methodologies for this study there were several limitations for each. First, survey research allows for a sample population to be studied and inferences made about the general population (Ary, Jacobs, & Razavieh, 2002). However, this pool of participants may not be representative of the population of the state in which this study was conducted, much less to all algebra teachers in the nation. Moreover, despite the fact that some of the data could be reduced to simple numerical analysis such as percentages, the explanations of the participants were coded using software designed for qualitative analysis and was subjective to my own interpretations (Bogdan & Biklen, 1998).

Second, while the use of multiple case studies allows for a detailed, in-depth data collection over a long period of time drawing from multiple sources of information (Cresswell, 1998) with the ability to compare two or more cases in order to show generalizability or diversity between the cases (Bogdan & Biklen, 1998), use of the case study limits the number of participants in the study. While conclusions will be drawn about the four individual cases, these generalizations cannot be presumed true about the population from which these cases were selected, much less to the general population of all algebra teachers in the nation (Bogdan & Biklen, 1998).

Third, the longevity of the study was a limitation of this study (Bogdan & Biklen, 1998). Data collection occurred over a span of one and a half years. One phase of the study involved assessing changes in content knowledge and pedagogical content knowledge of the cases. It cannot be assumed that sufficient time had elapsed for changes to occur in either type of knowledge as well as in both of them. Since the reflection of changes in both of these types of knowledge was considered within instructional practice, the length of this study may not have allowed enough time for the changes in both types of knowledge to be reflected in instructional practice. In addition, even though classroom observations were announced, there is still the possibility that changes in both of these types of knowledge were more observable when the research was not present.

Finally, while I tried to minimize my initial feelings about the phenomena I was studying, as a researcher, I brought my own bias to the interpretations of the study (Bogdan & Biklen, 1998). First, I have my own set of experiences as a mathematics classroom teacher and beliefs about the mathematical tasks performed on both the ALCKIN and during teacher interviews. Although I tried to lay aside my preconceived notions about what I expected to see, it is difficult to separate myself from the phenomena I are studying (Bogdan & Biklen, 1998). I was involved in all parts of the data collection process. First the instruments were all administered in my presence. There was personal interaction with all participants, including the case study teachers and the presenters of professional development. Classroom observations may have also been influenced by my presence, not only from the teacher's standpoint but also in the reactions of the students as well (Crotty, 1998). While interviews had certain protocols to follow and data from the all sources of collection including classroom observations, quarterly meeting observations, or interviews were entered into software designed for qualitative analysis, the coding was created by me and was subject to my own interpretations (Bogdan & Biklen, 1998). Final conclusions and inferences were also drawn from my own perceptions of this analysis.

Implications of the Study

In the following section I will address the implications of this research study. They include implications for teachers, teacher education programs, professional development, and for policy makers.

Implications for Teachers

Teachers need to recognize that while they may know procedures, they may not have the deep understanding of content knowledge and pedagogical content knowledge for teaching mathematics. Without strong content knowledge including a deep understanding, pedagogical content knowledge will suffer and affect successful mathematics instruction. Realizing and working toward increasing content knowledge can and should influence the way in which mathematics is presented within the classroom domain. Recognizing this lack of knowledge should influence teachers to seek professional development that offers them the opportunity to develop a deep understanding of the mathematics they are teaching. They should seek professional development that offers them the opportunity to learn mathematics in a different ways if they are going to teach it differently. Professional development should include the revisiting of the big ideas that teachers are expected to learn. Professional development cannot be considered a quick fix to the problem and should be considered a long-term investment.

With increases in both content knowledge and pedagogical content knowledge, teachers really need to use it. Why would you want to learn something new if you did not intend to put it to use? Both content knowledge and pedagogical content knowledge should be reflected in the instructional practices of teachers. With strong content knowledge and pedagogical content knowledge, teachers can move from being providers of mathematical procedures to facilitators of instruction that provide students with tasks to build mathematical knowledge along with the types of understanding necessary for them to be successful in mathematics.

Change for any of us is never easy. Committing to change will be necessary for effective instructional change to take place. Saying that it will be easy for teachers to implement different instructional strategies that research has proven to be effective is not realistic. The majority of us may remember our first year of teaching and how everything did not always go well, but we persevered. The same will be true for those working toward creating the seamless connection between knowledge and practice that will make a difference in the success students have at learning mathematics which in turn have a direct affect on their future opportunities for success (Ball et al., 2001).

Implications for Teacher Education

Preparing pre-service teachers who have strong content knowledge, including a deep understanding of the content, and strong pedagogical content knowledge should be the goal of teacher education programs. Coursework should be designed so that they may revisit the big ideas of the mathematics they will be teaching, using the types pedagogical practices they will be expected to use. We cannot expect changes to take place in the K-12 setting if professors at the university level are not willing to provide those teachers with opportunities to develop both content knowledge and pedagogical content knowledge through courses in both mathematics and mathematics education.

Pre-service teachers should be paired with in-service teachers who demonstrate strong content knowledge and strong pedagogical content knowledge that is reflected in their instructional practices. This pairing will ensure continuity between the coursework taken at the university and reinforcement in how both of these types of knowledge should be effectively used. These situations also offered the opportunity for interns tp use and further develop their content knowledge and pedagogical content knowledge under the guidance of an experienced in-service teacher. In this way, pre-service teachers' knowledge will be more likely to reflect in their instructional practices.

Faculty from both the mathematics department and the mathematics education department must collaborate to ensure that the courses taught within the two departments align with common goals in preparing pre-service teachers. Pre-service teachers need to know that content knowledge and pedagogical content knowledge are both valued within both departments. Teamwork will make a stronger program in preparing future teachers to teach mathematics.

Implications for Professional Development

Teachers need to be provided professional development where they can experience growth in both content knowledge and pedagogical content knowledge in ways that are transferable to their instructional practices. Professional development should be intensive and long-term providing teachers with a safe environment to discuss their success and failures (Mewborn, 2003). Teachers need to be provided with opportunities to revisit mathematical concepts to help them build their conceptual knowledge of those topics. Opportunities should also be provided to help teachers make connections to other mathematical topics (Mewborn, 2003). Professional development needs to prepare teachers for the tasks they will encounter on the job (Hill, Schilling, & Ball, 2004). Moreover, we cannot expect "quick fixes" to these problems. While some evidence of change was seen in this study, a much longer timespan might be necessary to see the profound changes I had hoped to see.

Implications for Policy Makers

Policy can effect what happens in the classroom. Taking a set number of courses in mathematics does not equate to a pre-service teacher having the strong content knowledge, with deep understanding, that will be sufficient to be successful in teaching students mathematics. Nor is students only gaining exposure to the types of pedagogy sufficient in helping them to use their content knowledge in assuring successful learning of mathematics for all students. Nor does a score on a test (NCLB, 2001) assure that mathematics teachers are adequately prepared for the job they will be called to do. Any of these in isolation can undermine the importance of the kinds of knowledge teachers need to have to be successful in teaching students mathematics. Policy makers need to consider what knowledge teachers need to be successful and require coursework that will develop that knowledge. These courses should include opportunities for the intertwining of the domains of content knowledge and pedagogical content knowledge. Coursework should be required of students to revisit the topics they will be expected to teach and in ways that will be useful in the classroom, adding depth to their knowledge of the mathematical topics.

Implications for Future Research

Conclusions drawn from this study suggest we do not know nearly enough about the content knowledge and pedagogical content knowledge of high school algebra teachers. Neither do we now have a full understanding about how changes in these types of knowledge are reflected in instructional practices. This study suggests results similar to studies done with elementary teachers (Ball, 1988a, Ma, 1999, Ball & Bass, 2000, Stacy et al., 2001. Swafford, Jones, & Thornton, 1997, Borko et al., 2000), algebra teachers have inadequate content knowledge and pedagogical content knowledge for successful mathematics instruction. Additional research needs to be conducted to further develop an understanding of the content knowledge and pedagogical content knowledge that high school teachers possess. Results from this study suggest that both of these types of knowledge are limited in their use in instructional practice. Therefore, more research needs to be done in understanding how to create a seamless connection between mathematical knowledge and instructional practice (Ball et al., 2001). In addition, we need to develop a better understanding of what content knowledge and pedagogical content knowledge of algebra pre-service teachers have. If the mathematics and mathematics education faculty members collaborate to ensure that pre-service teachers develop content knowledge and pedagogical content knowledge, what kind of content knowledge and pedagogical content knowledge will result? Furthermore, how does the the mathematical knowledge of pre-service teachers compare to that of pre-service teachers attending universities where the mathematics department and mathematics education departments do not collaborate to ensure the acquisition of both of these types of knowledge?

Conclusion

Do you hear the doors closing on the productive futures of many of students in the U. S.? Without adequate preparation in mathematics in general and algebra in particular, those doors to future educational and career opportunities will remain closed. A central key in opening this door lies in learning algebra and learning it with deep understanding. But the key to these opportunities lies in the hands of teachers who have the strong content knowledge and strong pedagogical content knowledge needed to be successful in opening that door for the adults of the twenty-first century. However, along with many past studies, this study demonstrates that many teachers do not possess these types of knowledge. This study also demonstrates that providing teachers with strong content knowledge and strong pedagogical content knowledge is possible, and it can influence instructional practices. This process of change, however, is neither simple nor direct. Nonetheless, we need to maintain our commitment to finding ways of accomplishing this goal, which in turn affects the learning of students and provides them all the opportunities they all deserve.

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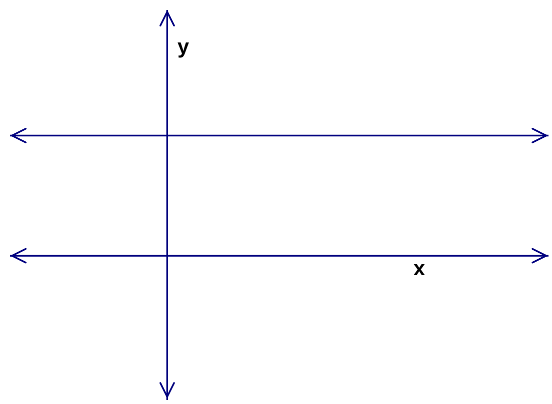
APPENDICES

APPENDIX A

ALGEBRA CONTENT KNOWLEDGE INSTRUMENT (ALCKIN)

11. Mr. Farrow asked his algebra students to determine which of the following expressions/graphs are functions. For each of the following decide if you agree, disagree, or you're not sure about the student's assessment.

A.

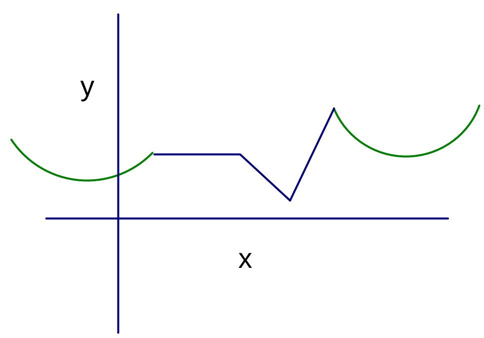


Noah said A was not a function because every x value corresponds to the same y value.

I agree	I disagree	I'm not sure
1	2	3

Explain your answer.

B.



Bart said B was not a function because it had a "strange shape" and you could not find an equation for it.

I agree	I disagree	I'm not sure
1	2	3

Explain your answer.

C. $y = \frac{1}{x}$

Sanchez said C was not a function because it was undefined when $x = 0$.

I agree	I disagree	I'm not sure
1	2	3

Explain your answer.

12. Mrs. Jones asked her algebra students to divide $x^2 - 4$ by $x + 2$. Seth said, "I have an easy method, Mrs. Jones. I just divide the x^2 by x and the 4 by the 2. I get $x - 2$, which is correct." Mrs. Jones is not surprised by this as she had seen students do this before. What did she know? (Mark one answer.)

    f. She knew that Seth's <u>method</u> was wrong, even though he happened to get the <u>right</u> answer for this problem.

    g. She knew that Seth's <u>answer</u> was actually wrong.

    h. She knew that Seth's <u>method</u> was right, but that for many algebraic fraction division problems this would produce a messy <u>answer</u>.

    i. She knew that Seth's <u>method</u> only works for some algebraic fractions.

    j. I'm not sure.

Explain your answer.

13. Teachers often offer students "rules of thumb" to help them remember particular mathematical ideas or procedures. Sometimes, however, these handy memory devices are not actually true, or they are not true in all situations. For each of the following, decide whether it is true all of the time or not. (Mark TRUE FOR ALL SITUATIONS, NOT ALWAYS TRUE, or I'M NOT SURE.)

		True for ALL Situations	NOT Always True	I'm Not Sure
A.	A binomial made up of two perfect squares cannot be factored into two binomials unless the two terms have a subtraction sign between them.	1	2	3
B.	An asymptote is a line that a graph approaches but never crosses.	1	2	3
C.	Any number to the zero power is equal to one.	1	2	3
D.	When graphing linear inequalities, if the inequality sign is "<" you shade below the line and if the inequality sign is ">" you shade above the line.	1	2	3

14. Consider the following statements related to different sets of numbers. After reading each statement decide if you agree, disagree, or if you are not sure.

	Statement	I agree	I disagree	I'm not sure
A.	-3 is a rational number	1	2	3
B.	2/3 is a real number	1	2	3
C.	.010010001… is a rational number.	1	2	3
D.	√5 is a complex number.	1	2	3

15. Mr. Casteel is using spreadsheets in his Algebra class to find solutions for quadratic equations. What approximate solution(s) for the equation $3x^2 = 4-2x$ should Mr. Casteel's students give using the following spreadsheet?

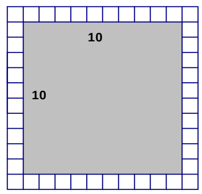
X	$3x^2$	4-2x
-1.8	9.72	7.6
-1.7	8.67	7.4
-1.6	7.68	7.2
-1.5	6.75	7
-1.4	5.88	6.8
-1.3	5.07	6.6
-1.2	4.32	6.4
-1.1	3.63	6.2
-1	3	6
-0.9	2.43	5.8
-0.8	1.92	5.6
-0.7	1.47	5.4
-0.6	1.08	5.2
-0.5	0.75	5
-0.4	0.48	4.8

Solution(s):

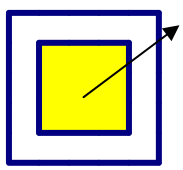
Explain your answer.

16. Without using algebraic manipulation such as collecting like terms or using the distributive property, what other methods could you as a teacher use to justify to your students that the expressions 3x + 5 + 5x -3 and 4 (2x + 1/2) are equivalent to each other?

17. Students who are in Mrs. Simpson's algebra class were working on the following problem: You are going to build a square garden and surround its border with square tiles. Each tile is 1 foot by 1 foot. For example, if the dimensions of the garden are 10 feet by 10 feet, then you will need 44 tiles for the border.

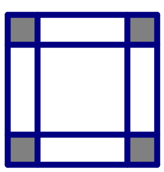


How many tiles would you need for a garden that is n feet by n feet? Two students gave the following diagrams, representations, and reasons for their answers.



James stated you could find the number of tiles needed for the border by finding the area of the entire garden and border and subtract out the area of the garden.
Which representation best reflects his method?

A. $4n + 4$
B. $4(n + 1)$
C. $(n + 2)^2 - n^2$
D. All are equally correct



Ann said you could take the entire length of each side of the border which is n and multiply by four, then add in each of the corners.
Which representation best reflects her method?

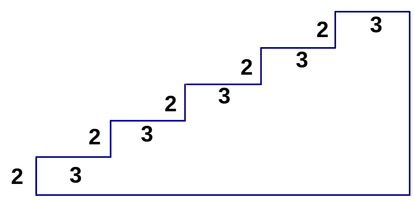
A. $4n + 4$
B. $4(n + 1)$
C. $(n + 2)^2 - n^2$
D. All are equally correct

Which student has the best understanding of the tile problem?

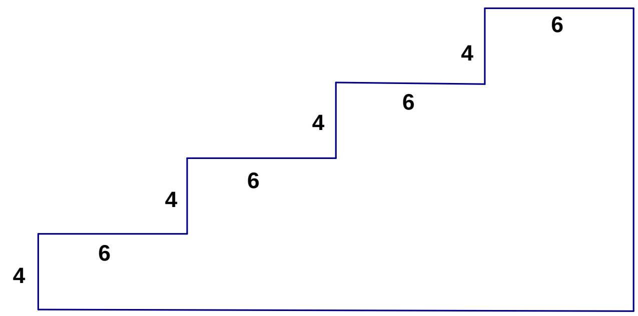
A. James
B. Ann
C. Both show equal understanding.

Explain your answer:

18. Mrs. Ledbetter gave her algebra students the following two diagrams of steps and asked them to determine which set of steps were the steepest. The students could not agree on an answer and gave the following answers and reasons for their answers.



Steps A



Steps B

E. **Group 1** said that both sets of steps have the same steepness. "First connect the points at the edges of the steps with a line and measure the angle formed between this line and the ground. The measure of this angle is the same in both set of steps. Therefore they have the same steepness."

F. **Group 2** said Steps B were steeper because the steps in Steps B are taller than in Steps A.

G. **Group 3** thought both sets of steps had the same steepness since doubling both the length and height of the steps does not affect the steepness.

H. **Group 4** said both sets of steps had the same steepness. "If you sketch in a line that contains the edges of the steps and determine the slope of the line in each set of steps, you will find they have equal slope. Steps A have a slope of 2/3 and Steps B have a slope of 4/6 which is equivalent to 2/3. Therefore, they have the same steepness."

Which group(s) have a correct understanding of slope? (Circle all that apply).

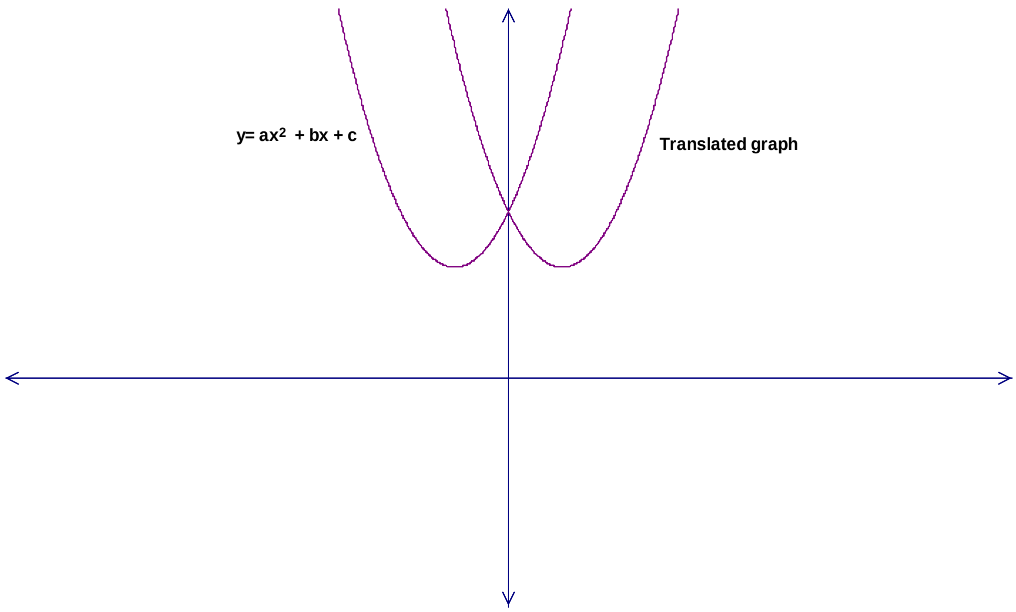
A. Group 1   B. Group 2   C. Group 3   D. Group 4

Of the four groups which group has the best understanding of slope?

____

Explain why you chose this group.

19. Mr. Seng's algebra class is studying the graph of $y = ax^2 + bx + c$ and how changing the parameters a, b, and c will cause different translations of the original graph.



How do you think Mr. Seng will explain the translation of the original graph $y = ax^2 + bx + c$ to the translated graph?

F. Only the **a** value changed

G. Only the **c** value changed

H. Only the **b** value changed

I. At least two of the parameters changed.

J. You cannot generate the translated graph by changing any of the parameters.

Explain your answer choice:

20. Mrs. Westbrook's algebra students were working on writing algebraic equations for problems similar to the following:

It takes 0.2 hour to bake a dozen chocolate chip cookies and 0.15 hour to bake a dozen plain cookies, how many dozen cookies can be baked in fifteen hours?

Mrs. Westbrook's students came up with the following equation:

$0.2x + 0.15y = 15$

where x = number of dozens of chocolate chip cookies and y = number of dozens of plain cookies.

What does the $0.2x$ represent in the equation?

APPENDIX B

TEACHER ATTITUDE SURVEY

SECTION A – Your Beliefs About Mathematics and Learning Mathematics

The following questions are to be answered according to the degree to which you agree or disagree with each of the sentences.

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
1. Teachers should ensure as much as possible that students experience success in mathematics by clearly explaining and modeling how to complete each day's assignment, by closely monitoring students' work, and by continually providing feedback including, if necessary, supplementary detailed explanation of how to solve a problem.	○	○	○	○	○
2. It is important for students to figure out how to solve mathematics problems for themselves.	○	○	○	○	○
3. Time should be spent practicing mathematical procedures before students spend much time solving mathematics problems.	○	○	○	○	○
4. I feel relaxed and confident when teaching mathematics.	○	○	○	○	○
5. If the class is going to use a model of a mathematical situation, I usually prefer first to show my students how to use the model.	○	○	○	○	○
6. In a mathematics class, each student's solution process should be accepted and valued.	○	○	○	○	○
7. Students learn mathematics best from their teacher's demonstrations and explanations.	○	○	○	○	○
8. Students should have many informal experiences (such as solving word problems) with a mathematical concept before they are expected to master that concept.	○	○	○	○	○
9. I feel more comfortable teaching algebra than teaching geometry, probability, or data analysis.	○	○	○	○	○
10. I feel comfortable teaching data analysis and statistics.	○	○	○	○	○
11. Students must confront a mathematical idea many times before they will understand it, so teachers must provide a variety of mathematics problems addressing that idea and challenge the students to figure out how to solve those problems.	○	○	○	○	○
12. No student should associate mathematics with frustration, so a teacher should limit the questions he or she asks of the student to those that the teacher is reasonably confident that the student can answer correctly.	○	○	○	○	○

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
13. When planning a mathematics lesson, I know that I am able to provide mathematics activities that are relevant to my students' lives.	○	○	○	○	○
14. If a student is going to be a good problem solver, then it is important for that student to know how to follow directions.	○	○	○	○	○
15. Students should understand the meaning of a mathematical concept before they memorize the definitions and procedures associated with that concept.	○	○	○	○	○
16. Teachers should model and demonstrate mathematical procedures and then, ideally, time should be allowed for the students to have the opportunity to practice those procedures.	○	○	○	○	○
17. I don't feel that I need to recall all of the answers to all of the questions that my students may have about mathematics because I know that I will be able to figure out a solution as my students and I work on a question.	○	○	○	○	○
18. No matter whether I am teaching mathematics to the whole class or to one group at a time, I am most comfortable when I first model the activity, then provide some practice and immediate feedback, and, finally, clarify what the assignment is and how I expect it to be completed.	○	○	○	○	○
19. Students achieve mathematical understanding through the direct personal experience of figuring out their own solutions to problems and then verifying their thinking for themselves.	○	○	○	○	○
20. When students are grouped for instruction on the basis of their past mathematical performance, each student may then receive the level of mathematics instruction that is most appropriate for that student.	○	○	○	○	○
21. Rather than demonstrating how to solve a problem, a teacher should allow students to figure out and explain their own ways of solving mathematics problems, including word problems.	○	○	○	○	○
22. Students will not understand a mathematical concept until they have memorized the definitions and procedures associated with that concept.	○	○	○	○	○

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
23. When I work with a small group of students during a mathematics lesson, I know that I will be able to assess their understanding as I observe them working on mathematical problems and interacting with each other to complete a mathematics task.	O	O	O	O	O
24. I feel that most mathematics teachers who teach the same grade level as me have a better understanding of mathematics than I have.	O	O	O	O	O
25. Teachers should incorporate students' diverse ideas and personal experiences into mathematics instruction that encourages greater student-student and student-teacher interaction.	O	O	O	O	O

SECTION B - Information About The Mathematics Classes You Teach

For this section of the survey, we would like you to report on a SPECIFIC MATHEMATICS class that you teach. We will call this class your TARGET MATH CLASS. To identify your TARGET MATH CLASS, please read the following instructions carefully. If you teach more than one math class, your FIRST CLASS OF THE WEEK in which MATHEMATICS is taught is your **TARGET MATH CLASS** (e.g., 1st period Monday, 3rd period Monday, etc.)

Target Class Information

Name of Class: ______________________ **Days and Times it Meets:** ___________________________

1 How many different subject or course preparations are you responsible for?

1	2	3	4	5 or more
O	O	O	O	O

For the current school year, list the courses you are teaching or have taught, including both mathematics and non-mathematics classes:

____

Please answer the following questions regarding your TARGET MATH CLASS.

2. What is the grade level(s) of the students in your TARGET MATH CLASS?

Pre-K	1	2	3	4	5	6	7	8	9	10	11	12	Non-graded
O	O	O	O	O	O	O	O	O	O	O	O	O	O

3. On a typical day, how long is your TARGET MATH CLASS?

< 40 minutes	41-50 minutes	51-60 minutes	61-70 minutes	71-80 minutes	81-90 minutes	Over 90 minutes
O	O	O	O	O	O	O

4. How many students are in your TARGET MATH CLASS?

For example, if you had 18 students: | Number of students in your class:

1	8
0O	0O
1◉	1O
2O	2O
3O	3O
4O	4O
	5O
	6O
	7O
	8◉
	9O

0O	0O
1O	1O
2O	2O
3O	3O
4O	4O
	5O
	6O
	7O
	8O
	9O

5. Please describe the range of student ability in your TARGET MATH CLASS. (mark only one)

O Mostly below grade level
O Mostly below or at grade level
O A balance of students at, below, and above grade level
O Mostly at or above grade level
O Mostly above grade level

6. Approximately what proportion of students in this TARGET MATH CLASS are:

	None	**1-25%**	**26-50%**	**51-75%**	**76-100%**	**Not sure**
Title 1 Math	O	O	O	O	O	O
Students with disabilities	O	O	O	O	O	O
ESL students	O	O	O	O	O	O
Gifted and talented	O	O	O	O	O	O
African-Americans	O	O	O	O	O	O
White	O	O	O	O	O	O
Hispanic	O	O	O	O	O	O

7. Approximately what proportion of students in this TARGET MATH CLASS do you expect to:

	None	**1-25%**	**26-50%**	**51-75%**	**76-100%**
Graduate from high school	O	O	O	O	O
Attend a trade school or technical school	O	O	O	O	O
Attend a junior college	O	O	O	O	O
Attend a four-year college or university	O	O	O	O	O

8. How much time do you spend preparing students in your TARGET MATH CLASS for standardized tests, such as the SAT-10? (mark only one)

O 1 day or less
O 2 days
O 3 days
O 4 days
O 1 week
O 2-3 weeks
O 1 month or more

Please answer the following questions regarding your **INSTRUCTION** in your **TARGET MATH CLASS.**
Use the following scale:

Never
Rarely (a few times a year)
Sometimes (once or twice a month)
Often (once of twice a week)
All or almost all math lessons

9. About how often do you do each of the following in your mathematics instruction in the TARGET CLASS?

	Never	Rarely	Some-times	Often	All or Almost All
a. Introduce content through formal presentations.	O	O	O	O	O
b. Arrange seating to facilitate student discussion.	O	O	O	O	O
c. Use open-ended questions.	O	O	O	O	O
d. Require students to explain their reasoning when giving an answer.	O	O	O	O	O
e. Encourage students to communicate mathematically.	O	O	O	O	O
f. Encourage students to explore alternative methods for solutions.	O	O	O	O	O
g. Encourage students to use multiple representations (e.g., numeric, graphic, or geometric representations.)	O	O	O	O	O
h. Allow students to work at their own pace.	O	O	O	O	O
i. Help students see connections between mathematics and other disciplines.	O	O	O	O	O
j. Use assessment to find out what students know before or during a unit.	O	O	O	O	O
k. Embed assessment in regular class activities.	O	O	O	O	O
l. Assign mathematics homework.	O	O	O	O	O
m. Read and comment on what students have written in their notebooks or journals.	O	O	O	O	O

10. About how often do students in the TARGET CLASS take part in each of the following types of activities as part of their mathematics instruction?

	Never	Rarely	Some-times	Often	All or Almost All
a. Use a calculator?	O	O	O	O	O
b. Work math problems with a partner or in a small group?	O	O	O	O	O
c. Write a few sentences about how they solved a math problem?	O	O	O	O	O
d. Explain how they solved a problem to the class?	O	O	O	O	O
e. Use wooden or plastic blocks, rods, shapes or other objects to solve a math problem?	O	O	O	O	O
f. Work on one math problem or question for more than 10 minutes?	O	O	O	O	O
g. Do math problems that are challenging?	O	O	O	O	O
h. Think about why something in math class is true?	O	O	O	O	O
i. Do math projects or investigations that take several days to complete?	O	O	O	O	O
j. Apply math situations to life outside of school?	O	O	O	O	O
k. Take tests where they have to explain their answers?	O	O	O	O	O
l. Copy notes or problems off the board?	O	O	O	O	O
m. Do 10 or more practice problems by themselves?	O	O	O	O	O
n. Memorize math facts for a test or quiz?	O	O	O	O	O
o. Listen to me lecture about math?	O	O	O	O	O
p. Complete many math problems quickly?	O	O	O	O	O
q. Practice to take a standardized-test, like the SAT-10?	O	O	O	O	O
r. Use a computer to practice their math?	O	O	O	O	O
s. Take multiple-choice tests?	O	O	O	O	O

SECTION C - Background Information

1. How many years have you: (round up to the nearest year)

Example:		Years taught at this school?		Total years as a teacher?	
2	5				
0O	0O	0O	0O	0O	0O
1O	1O	1O	1O	1O	1O
2⦿	2O	2O	2O	2O	2O
3O	3O	3O	3O	3O	3O
4O	4O	4O	4O	4O	4O
	5⦿		5O		5O
	6O		6O		6O
	7O		7O		7O
	8O		8O		8O
	9O		9O		9O

2. Have you ever taught outside this school system?

O Yes O No

If Yes, where did you teach? (mark all that apply)

O An urban public school district
O A suburban public school district
O A rural public school district
O A private school

3. Are you a graduate of this school system?

O Yes O No

4. What is the highest level of education you have completed?

O Bachelors degree
O Bachelors with additional graduate credits
O Masters degree
O Masters + 15 credits
O Masters + 30 credits
O Masters + 45 credits
O Specialist
O Doctorate

5. Please indicate whether your degree(s) were in Mathematics Education, Mathematics, or another discipline:

Bachelors:	O Math Ed	O Math	O Other, please specify: ________
Masters:	O Math Ed	O Math	O Other, please specify: ________
Specialist:	O Math Ed	O Math	O Other, please specify: ________
Doctorate:	O Math Ed	O Math	O Other, please specify: ________

6. Approximately how much time overall have you spent in professional development during the past year?

Less than 20 hours	20-40 hours	41-80 hours	81-120 hours	121-160 hours	Over 160 hours
O	O	O	O	O	O

7. Approximately how much time have you spent in professional development associated with TEAM-Math during the past year?

None	Less than 5 hours	6-10 hours	11-15 hours	16-20 hours	Over 20 hours
O	O	O	O	O	O

8. Approximately much time have you spent in mathematics-specific professional development (other than that offered by TEAM-Math) during the past year?

None	Less than 5 hours	6-10 hours	11-15 hours	16-20 hours	Over 20 hours
O	O	O	O	O	O

9. Are you:

O Female O Male

10. Are you:

O African-American (Black)
O Asian-American
O Biracial/Multiethnic
O Hispanic
O Native American
O White, Non-Hispanic
O Other (please specify): ________

SECTION D - Involvement in TEAM-Math

Please answer the following questions to the degree to which you agree or disagree with each of the sentences.

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
1. Members of our school meet regularly to discuss the progress of our efforts.	O	O	O	O	O
2. I am more enthusiastic about our school now that I am involved in this project.	O	O	O	O	O
3. I personally support the TEAM-Math mission and vision.	O	O	O	O	O
4. Our school has enough staff, time, and other resources to really make this project pay off for the school.	O	O	O	O	O
5. The assistance of the TEAM-Math project team was valuable in preparing for this project.	O	O	O	O	O
6. The TEAM-Math mission and vision are understood by members of our school.	O	O	O	O	O
7. I am satisfied with our relationship with members of the TEAM-Math project team.	O	O	O	O	O

8. To what extent have you personally been involved in this project? (Mark all that apply.)
   - O Not involved at all
   - O Not directly involved, but I have heard about TEAM-Math from colleagues who are involved
   - O Involved in extensive discussions about TEAM-Math with colleagues
   - O Involved in implementing aspects of TEAM-Math in my school
   - O School Teacher Leader
   - O District Teacher Leader
   - O Other (please specify): ______________________

9. In what ways would you like to become further involved with the TEAM-Math Project?

APPENDIX C

RTOP CLASSROOM OBSERVATION SURVEY

Reformed Teaching Observation Protocol (RTOP)

Daiyo Sawada
External Evaluator

Michael Piburn
Internal Evaluator

and

Kathleen Falconer, Jeff Turley, Russell Benford and Irene Bloom
Evaluation Facilitation Group (EFG)

Technical Report No. IN00-1
Arizona Collaborative for Excellence in the Preparation of Teachers
Arizona State University

I. BACKGROUND INFORMATION

Name of teacher ____________________ Announced Observation? ____________________ (yes, no, or explain)

Location of class ____ (district, school, room)

Years of Teaching ____________________ Teaching Certification ____________________ (K-8 or 7-12)

Subject observed ____________________ Grade level ____________________

Observer ____________________ Date of observation ____________________

Start time ____________________ End time ____________________

II. CONTEXTUAL BACKGROUND AND ACTIVITIES

In the space provided below please give a brief description of the lesson observed, the classroom setting in which the lesson took place (space, seating arrangements, etc.), and any relevant details about the students (number, gender, ethnicity) and teacher that you think are important. Use diagrams if they seem appropriate.

2000 Revision

Record here events which may help in documenting the ratings.

Time	Description of Events

2000 Revision

III. LESSON DESIGN AND IMPLEMENTATION

		Never Occurred				Very Descriptive
1)	The instructional strategies and activities respected students' prior knowledge and the preconceptions inherent therein.	0	1	2	3	4
2)	The lesson was designed to engage students as members of a learning community.	0	1	2	3	4
3)	In this lesson, student exploration preceded formal presentation.	0	1	2	3	4
4)	This lesson encouraged students to seek and value alternative modes of investigation or of problem solving.	0	1	2	3	4
5)	The focus and direction of the lesson was often determined by ideas originating with students.	0	1	2	3	4

IV. CONTENT

Propositional knowledge

6)	The lesson involved fundamental concepts of the subject.	0	1	2	3	4
7)	The lesson promoted strongly coherent conceptual understanding.	0	1	2	3	4
8)	The teacher had a solid grasp of the subject matter content inherent in the lesson.	0	1	2	3	4
9)	Elements of abstraction (i.e., symbolic representations, theory building) were encouraged when it was important to do so.	0	1	2	3	4
10)	Connections with other content disciplines and/or real world phenomena were explored and valued.	0	1	2	3	4

Procedural Knowledge

11)	Students used a variety of means (models, drawings, graphs, concrete materials, manipulatives, etc.) to represent phenomena.	0	1	2	3	4
12)	Students made predictions, estimations and/or hypotheses and devised means for testing them.	0	1	2	3	4
13)	Students were actively engaged in thought-provoking activity that often involved the critical assessment of procedures.	0	1	2	3	4
14)	Students were reflective about their learning.	0	1	2	3	4
15)	Intellectual rigor, constructive criticism, and the challenging of ideas were valued.	0	1	2	3	4

Continue recording salient events here.

Time	Description of Events

2000 Revision

V. CLASSROOM CULTURE

	Communicative Interactions	Never Occurred				Very Descriptive
16)	Students were involved in the communication of their ideas to others using a variety of means and media.	0	1	2	3	4
17)	The teacher's questions triggered divergent modes of thinking.	0	1	2	3	4
18)	There was a high proportion of student talk and a significant amount of it occurred between and among students.	0	1	2	3	4
19)	Student questions and comments often determined the focus and direction of classroom discourse.	0	1	2	3	4
20)	There was a climate of respect for what others had to say.	0	1	2	3	4
	Student/Teacher Relationships					
21)	Active participation of students was encouraged and valued.	0	1	2	3	4
22)	Students were encouraged to generate conjectures, alternative solution strategies, and ways of interpreting evidence.	0	1	2	3	4
23)	In general the teacher was patient with students.	0	1	2	3	4
24)	The teacher acted as a resource person, working to support and enhance student investigations.	0	1	2	3	4
25)	The metaphor "teacher as listener" was very characteristic of this classroom.	0	1	2	3	4

Additional comments you may wish to make about this lesson.

APPENDIX D

FIRST TEACHER INTERVIEW PROTOCOL

- My name is Joy Black, and I am from Auburn University. I work with the MDSMIP project which is trying to understand and improvement mathematics in your school.
- We are interested in finding out what teachers think about math and how they feel about teaching math.
- During this interview I will ask questions pertaining to your opinions and beliefs about how you and your students would work mathematics problems and questions about how you would approach teaching certain algebraic topics. This is not a test, so there are no right and wrong answers; it's only your opinions about methods you use in your classroom that matter.
- Everything you tell me is completely confidential. That means, we never share your answers with anyone.
- First, I need to get your permission to participate in this interview.

1) Give two copies of the permission form to the teacher and ask them to sign one and return to you. The second permission form is for the teacher to keep.

2) Say: "I want you to think about the students you have in your algebra classes. What percent of them do you think can learn algebra well?
    a. If the teacher answers all or almost all ask: "Why do you think almost all of the students can learn algebra well?"
    b. For other answers ask: "Why do you think this percentage of your students can learn algebra well?" After they answer ask this: "Why do you think the other students cannot learn algebra well?"

3) Say: "88% of the teachers surveyed agreed or strongly agreed with the following statement: It is important for students to figure out how to solve mathematics problems for themselves. How are you carrying this out in your own classroom?"

4) Say: "On the other hand 88% of these same teachers also agreed or strongly agreed with the following statement: Teachers should model and demonstrate mathematical procedures and then, ideally, time should be allowed for the students to have the opportunity to practice those procedures. In what ways can these coexist in the mathematics classroom?"

5) Give the teacher the following problem (on a separate sheet of paper) and ask him/her to work it:

   Jameel looked out in the parking lot and decided to count cars and motorcycles by the number of their wheels, excluding spares. He saw that there were 17 vehicles with a total of 56 wheels. How many motorcycles did he see?

6) When the teacher finishes working the problem ask him/her to work the problem using a different method.

7) When the teacher finishes working the problem a second way, ask the teacher to work the problem using a third method. If the first two methods appear to basically be the same such as setting up two equations – solving the first by elimination and the second by substitution, say “Can you work the problem in any other way that is really different from the first two ways you have worked the problem?”

8) Give time for the teacher to work the problem a third way. If the teacher can not quickly come up with a third way move on to the next question.

9) Ask the teacher how he/she would teach this problem to his/her algebra classes.

10) When she is finished say: “If you gave this problem to your algebra students to solve, which of the methods that you used do you think your students would use?”

11) After the teacher chooses a method say: “Why do you think your students would use this method?” If the teacher says that the students would use none of the three methods ask: “What method do you think your students would use to solve the problem and why do you think they would use this method?”

12) If the teacher did not give a picture solution to the problem, give her the picture solution you have prepared (See Interview Protocol page 4). Say: “Look at this student solution and tell me what you think about it.” After the teacher replies ask: “Would you accept this as a solution to this problem?” Depending on their answer ask “Why or why not?” (If the teacher gave a picture solution, go directly to the question “Would you accept this as a solution to the problem?” and follow it by “Why or why not?”)

13) Give them the second solution which is a reasoned solution (See Interview Protocol page 5). Say: “Look at this student solution and tell me what you think about it.” After the teacher replies ask: “Would you accept this as a solution to the problem?” Depending on their answer ask “Why or why not?” (If the teacher gave a reasoning type solution, go directly to the question “Would you accept this as a solution to the problem?” and follow it by “Why or why not?”)

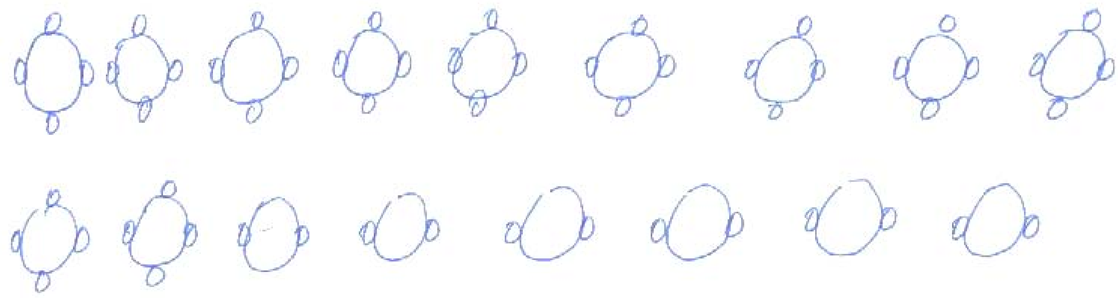
14) Give the teacher the following problem on a separate sheet of paper:

$$x^2 = 2x + 8$$

Give the teacher the first student solution where the student graphed both sides of the equation (See Interview Protocol page 6). Say “Explain what you think about what this student did to solve the problem.” After the teacher explains, ask: “Do you think this is an acceptable way for a student to work the problem? Why or why not?”

15) Give the teacher the second student solution where the student shows some work then comes up with the correct solution (See Interview Protocol page 7). Say "Explain what you think about what this student did to solve the problem." After the teacher explains, ask: "Do you think this is an acceptable way for a student to work the problem? Why or why not?"

16) Thank the teacher for his/her participation.



First I drew seventeen circles to represent the motorcycles and cars. I went back and put two wheels on each circle – the cars and motorcycles all have two wheels. Then I put two more wheels on as many of the circles until I had used all 56 wheels. So there are 11 cars and 6 motor cycles.

There are 17 cars and motorcycles.
Each has at least two wheels.

$17 \times 2 = 34$ wheels

$56 - 34 = 22$ wheels left over.

Each car has two more wheels.

$$2\overline{)22}\quad 11$$

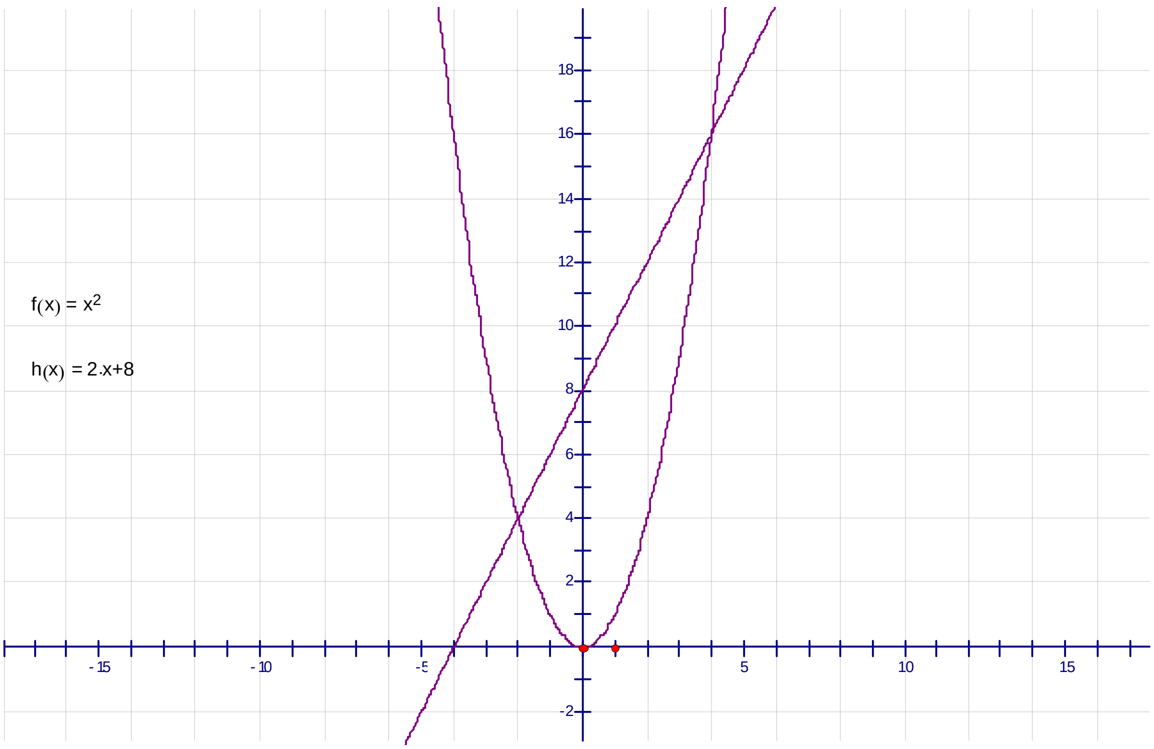
So there are 11 cars

$17 - 11 = 6$ motorcycles.

1. Solve for x:

$$x^2 = 2x + 8$$

To find the solution I graphed each side of the equation.



The two graphs intersected at two points (-2,4) and (4, 16). From these I can find the two solutions for the equation $x^2 = 2x + 8$. x would be equal to -2 and 4.

$$x^2 = 2x + 8$$

$$x^2 - 2x = 8$$

$$x(x-2) = 8$$

$$x(x-2) - 8 = 0$$

$$-2 \cdot 4 = -8$$

So $x = 4$ and $x = -2$

APPENDIX E

PROTOCOL FOR SECOND TEACHER INTERVIEW

- My name is Joy Black, and I am from Auburn University. I work with the MDSMIP project which is trying to understand and improvement mathematics in your school.
- We are interested in finding out what teachers think about math and how they feel about teaching math.
- During this interview I will ask questions pertaining to your opinions and beliefs about how you and your students would work mathematics problems and questions about how you would approach teaching certain algebraic topics. This is not a test, so there are no right and wrong answers; it's only your opinions about methods you use in your classroom that matter.
- Everything you tell me is completely confidential. That means, we never share your answers with anyone.
- First, I need to get your permission to participate in this interview.
- Give two copies of the permission form to the teacher and ask them to sign one and return to you. The second permission form is for the teacher to keep.

1. Give me your overall impression on the cohort I professional development training you received this summer through MDSMIP.

As the teacher gives her impressions, I will be looking for answers to the following questions. If the teacher does not address the questions, these will be used as follow up questions to the first overall question.

2. Prior to the summer cohort I training, what was your attitude toward attending the summer professional development training? In what ways do you think it changed as the two weeks progressed?

3. What changes have you made in your classroom practices as a result of the professional development training you have received? Give me an example. Do you think it was successful? Why or why not? Will you use this approach again? How do you think you will change it? Why?

4. What kind of content knowledge of algebra do you think you possess? Did you learn anything new about algebra this summer? (If yes) Give me an example. In what ways do you think the professional development training affected your content knowledge of algebra?

5. Last spring I asked you about the percentage of students you had in your algebra classes that could really learn algebra well. Think about the students you have in your algebra classes this year. What percent of them do you think can learn algebra well? If the teacher answers all or almost all ask: "Why do you think almost all of the students can learn algebra well?" For other answers ask: "Why do you think this percentage of your students can learn algebra well?" After they answer ask this: "Why do you think the other students cannot learn algebra well?"

6. Was there any part of the summer professional development that may have changed your perception about the percentage of your students who can learn algebra well? If so, give me an example.

7. Did you participate in any other professional development training besides the two week cohort I professional development training? If yes, what training did you participate in? Do you think this professional development training had an impact on your classroom practices for this academic school year? What impact has it had? Can you give me an example?

8. You've participated in follow-up cohort I meetings, give me your impression of these meetings. Were they helpful? If yes, in what ways and why do you think it was helpful. If the answer was no, why do you think it was not helpful?

9. Give the teacher the following problem (on a separate sheet of paper) and ask him/her to work it:

$x^2 = 2x + 8$

10. When the teacher finishes working the problem ask him/her to work the problem using a different method.

11. When the teacher finishes working the problem a second way, ask the teacher to work the problem using a third method. If the first two methods appear to basically be the same such as setting up two equations – solving the first by elimination and the second by substitution, say "Can you work the problem in any other way that is really different from the first two ways you have worked the problem?"

12. Give time for the teacher to work the problem a third way. If the teacher can not quickly come up with a third way move on to the next question.

13. Ask the teacher how he/she would teach this problem to his/her algebra classes.

14. When she is finished say: "If you gave this problem to your algebra students to solve, which of the methods that you used do you think your students would use?"

15. After the teacher chooses a method say: "Why do you think your students would use this method?" If the teacher says that the students would use none of the three methods ask:"What method do you think your students would use to solve the problem and why do you think they would use this method?"

APPENDIX F

PROFESSIONAL DEVELOPMENT PRESENTERS INTERVIEW PROTOCOL

1. Tell me about your overall impression on how (the teacher) participated in the cohort I summer professional development training.

As the presenter gives the impression on the teacher, I will be looking for answers to the following questions. If the presenter does not address the questions, these will be used as follow up questions to the first overall question.

2. How engaged was this teacher during the professional development? Can you give me an example?

3. How did you see her attitude change during the summer professional development training? Can you give me an example?

4. Did you think this teacher was open to the ideas presented during the summer training? Why or why not? Did she seem to have an openness to implement the types of classroom instruction that were being encouraged through the professional development? Why or why not?

5. What kind of algebraic content knowledge was addressed during the summer professional development training? What do you think this is the kind of algebraic content knowledge this teacher possesses?

APPENDIX G

CODE LIST FOR THE ALGEBRA CONTENT KNOWLEDGE INSTRUMENT

CODES FOR ALGEBRA CONTENT KNOWLEDGE INSTRUMENT

Code	Frequency	Explanation of Code
Counterexample	39	Provides an example that would make a statement false such as $0^0 \neq 1$.
Definition	76	Use the definition of a term as an explanation.
Denotes Asymptote	3	Denotes that the function has an asymptote.
Denotes Student Error	5	"Maybe they were thinking" type statements.
Discontinuous Function	1	States the function is not continuous
Drawing/Modeling	12	States or draws a model for the explanation.
Explanation of Expression	15	Put into words such as two tenths times x.
Understanding of Expression	35	Indicates an understanding that 0.2 x means the number of hours it would take to bake x dozen chocolate chip cookies.
Formula Manipulation	6	Analyzes general quadratic equation to see which parameter reflects the graph.
Function	45	Indicates that the graph or problem is a function.
Graph	11	Teacher sketches a graph in the Cartesian plane to use in the answer decision or to clarify the selected answer.
I Don't Know	23	Teacher writes "I don't know. I'm not sure! Or Not sure! Took a Guess.
Incorrect Reasoning/Answer	74	The reasoning is incorrect; teacher gave an incorrect answer; teacher gave a partial answer, teacher doesn't really explain the answer; partial explanation; answer has nothing to do with the question.

Code	Frequency	Explanation of Code
Use Manipulatives	13	Teacher suggests some type of manipulative such as algebra tiles or counters in the explanation.
Multiple Reasons	85	Teacher uses more than one method in supporting his/her explanation.
No Explanation Given	176	No explanation was provided or participant did not choose an answer but provided an explanation.
Procedural Knowledge	28	Uses procedures for explanation.
Provides Example	4	Gives a number example in the explanation.
Slope Reference	33	Noted that a particular group used slope.
Spreadsheet Analysis	30	Uses the spreadsheet correctly to find the solution for x in the quadratic equation.
Subject Matter Taught	8	Relates explanations to the fact that their students do not learn this or the teacher does not teach this particular concept.
Synthetic Division	4	States or demonstrates synthetic division in the explanation.
Table	5	Creates a chart or suggests that a chart be used.
Uses Division	18	States or demonstrates polynomial division in the explanation.
Uses Factoring	33	Breaks the expressions down into factors and cancels out the common factors.
Uses Quadratic Formula	11	Uses the quadratic formula in an attempt to find the solution instead of using the spreadsheet.

Code	Frequency	Explanation of Code
Value Substitution	22	Suggest that students can use the substitution of a value or values for the variable to show two algebraic expressions are equivalent.
Vertical Line Test	48	Denotes or demonstrates that the vertical line test in testing for functions.

APPENDIX H

CODE LIST FOR THE CASE STUDIES

CODES FOR CASE STUDIES

Code	Frequency	Code Explanation
Checks Work	18	Teacher demonstrates or encourages students to check their solutions.
Conceptual Knowledge	24	Teacher makes a statement that refers to the conceptual understanding of the mathematical topic being discussed in class such as referring to algebra tiles.
Definition	52	Defines terms, gives formulas, or teacher asks the students for the definition of terms or formula.
Denotes Using Inverse Operations	2	Teacher indicates that the procedural step in solving an algebraic equation would involve doing an inverse operation such as taking a square root of both sides of an equation.
Formula Manipulation	7	Includes correct substitution into formula and solving for unknowns
Gives Praise to Students	30	Gives praise or encouragement to students. You can do this! Think! Teacher expresses that students can do the work because they are advanced students.
Multiple Reasons	35	Multiple ways of working problems procedurally.
Problem Instructions	2	Gives the students instructions on their assignment.
Provides Example	19	Begins procedural instruction by working an example.
Step by Step Procedures	225	Teacher goes through step by step procedures for working a problem.

Code	Frequency	Code Explanation
Student Agrees with Teacher	137	Yes madam, um humph, etc.
Student Answers "Why" or "How"	41	Teacher asks a question for why he did something. Student explains how to do something procedurally or why he/she chose a particular procedure.
Student Gives Correct Answer	177	Student gives a correct answer to an arithmetic problem, correct solution to an equation, or correct simplification to an algebraic expression. For example, "What is five squared?"
Student Gives Correct Procedure	86	"Find a common denominator" "Subtract forty-eight." "Just divide everything by two."
Student Gives Incorrect Answer	31	Student answers a procedural question incorrectly.
Student Gives Incorrect Procedure or Next Step	15	"You added the four sides together"
Student Indicates They Don't Understand	17	Student tell teacher they don't understand a procedure.
Student Practice, Practice Problems or Quiz	12	Students are given class time to work several practice problems or a quiz over previously covered material. Students work problems after procedural instruction from the teacher. Students put homework problems on the whiteboard.
Student Question	96	Student questions such as "Are these bonus questions?"
Student Recognition of an Error	16	Student recognizes own error or the error of a classmate.

Code	Frequency	Code Explanation
Student Recognizes Teacher Error	3	Student recognizes when the teacher writes something incorrectly on the whiteboard.
Student Short Answers to Arithmetic or Algebraic Question	236	Student answers the teacher's request for an answer to an arithmetic procedure or for the simplification of an algebraic expression.
Teacher Answers Own Question	76	Teacher asked a question but does not allow time for students to answer. Teacher answers her own question.
Teacher Answers Student Question	25	Answers student question or responds to an answer given by the student.
Teacher Can't Recognize Student Error	3	Can't or does not recognize student error.
Teacher Asks Clarifying Question	174	Teacher may ask "Do you mean?" What did you get?"
Teacher Comment or Question Indicates a Task is Easy Or Can be done quicker	23	That was easy. Or Okay a quick way to do this is…
Teacher Given Hint	117	It's a multi-step equation.
Teacher Gives Non Mathematical Instructions	121	Think before you speak. Your assignment is…
Teacher Ignores Incorrect Student Procedure	18	Teacher ignores incorrect student suggestion for working problem.
Teacher Ignores Student Question	6	Teacher ignores student question of comment.
Teacher Gives Incorrect Information	6	$b – 4ac$ (instead of $b^2 – 4ac$)

Code	Frequency	Code Explanation
Teacher Question Involves Asking for an Answer to an Arithmetic Problem or Simplifying an Algebraic Expression	134	Procedure, step, names a formula, ask for agreement from the students.
Teacher Question Involves Asking if Students have other Questions or Understands	168	For example: Alright? Do you have any questions? Do you understand? Which problems do you need worked?
Teacher Questions "How" or "Why"	133	Why you cannot do something, how you did something.
Teacher Question Involves Asking To Name the Next Step or Procedure	220	As the teacher is working, she asked students to tell the next step or procedure.
Teacher Recognizes Correct Student Procedure	35	Correct. Um humph.
Teacher Recognizes Incorrect Student Answer	30	Denotes that student answer is incorrect.
Teacher Recognizes Incorrect Student Procedure	37	Teacher denotes that student procedure is incorrect.
Teacher Reiterates Students' Reply	210	Teacher repeats what the student just said.
Teacher Reminds Students of "Steps" or "Rules"	90	Reminds students of steps that should be taken to solve problem or reminds students of rules associated with working a problem.
Uses Manipulatives	20	Uses manipulatives, drawings, graphs, etc.
Uses neumonics	17	Uses a procedure considered to be a neumonic. For example: Please excuse my dear aunt Sally or Remember these are the problems we put in a chart.